Grade 11 Applied Mathematics (30S)

A Course for Independent Study

Manitoba

GRADE 11
APPLIED MATHEMATICS (30S)

A Course for
Independent Study

2001

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Course Writer

Harland Garinger	Hapnot Collegiate	Flin Flon S.D. No. 46

Course Editor

Kathy Innes	Winnipeg, Manitoba

Members of the Development Team

Cam Bennet	Dauphin Regional Comprehensive Secondary	Dauphin Ochre Area No. 1 S.D. No. 33
Harland Garinger	Hapnot Collegiate	Flin Flon S.D. No. 46
Robert Hayes	Red River Community College	
Jim Hoger	Treherne Collegiate	Tiger Hills S.D. No. 29
Kathy Innes	Kelvin High School	Winnipeg S.D. No. 1
Steve Khan	St. Norbert Collegiate	Seine River S.D. No. 14
Eric MacPherson	Faculty of Education	University of Manitoba
Fred Pauls	Mennonite Brethren Collegiate Institute	Independent School
Irvin Peters	Garden Valley Collegiate	Garden Valley S.D. No. 26
Roy Spivak	Grant Park High School	Winnipeg S.D. No. 1
Mila Stout	Swan Valley Regional Secondary	Swan Valley S.D. No. 35
Katharine Tetlock	Distance Learning and Information Technologies Unit	Manitoba Education, Training and Youth
Wayne Watt	Curriculum Unit	Manitoba Education, Training and Youth

Manitoba Education, Training and Youth Staff
Schools Division

Lee-Ila Bothe	Coordinator	Production Support Unit Program Development Branch
Paul Cuthbert	Project Manager	Distance Learning and Information Technologies Unit Program Development Branch
Mark Gavard	Independent Study Program Coordinator	Distance Learning and Information Technologies Unit Program Development Branch
Reisha Golden	Publications Editor (until August 1999)	Production Support Unit Program Development Branch
Janet Long	Desktop Publisher	Production Support Unit Program Development Branch
Grant Moore	Publications Editor (after August 1999)	Production Support Unit Program Development Branch
Katharine Tetlock	Project Leader	Distance Learning and Information Technologies Unit Program Development Branch
Wayne Watt	Consultant	Curriculum Unit Program Development Branch
Murielle White	Desktop Publisher	Production Support Unit Program Development Branch

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Grade 11 Applied Mathematics
Introduction

Welcome to *Grade 11 Applied Mathematics (30S): A Course for Independent Study,* offered through Manitoba Education, Training and Youth.

As a student in a course for distance education delivery, you have taken on a dual role—that of student and of teacher. As a student, you are responsible for mastering the lessons and completing the exercises assigned at the end of each lesson. As a teacher, you are responsible for checking your work carefully and noting the nature of your errors. Finally, you must work diligently to overcome your difficulties.

You should seek out a study partner for this course. Most students find that a study partner helps them complete the course with greater success. This study partner can help you correct your assignments and module Self-Tests, as well as help you prepare for the examinations. It will also be necessary from time to time to test some of your work on a partner. This does not always have to be the same person; at times you may find it beneficial to have more than one person helping you. In some modules of the course, you will also be performing mathematical experiments and gathering data. For this work particularly, a partner could be vital to your success.

The applied mathematics curriculum has been developed in response to changing mathematical requirements. These requirements have changed because of the increased use of technology in everyday life, post-secondary education, and the workplace. Business and industry require responsible independent learners who are

- able to communicate mathematical ideas
- flexible
- capable of teamwork
- computer literate
- skilled in problem-solving techniques
- self-reliant

In *Grade 11 Applied Mathematics*, students will master essential skills in topics that have become important in post-secondary institutions, technology-based industries, and daily living.

By taking *Grade 11 Applied Mathematics,* you will gain invaluable skills for living in the 21st Century.

It is not mandatory that you have taken *Grade 10 Applied Mathematics*, but it is highly recommended. If this is impossible, it may be very advantageous for you to at least go through the "Spreadsheets" and "Exploring Math Using Technology" modules of the *Grade 10 Applied Mathematics* course, as the knowledge gained from those modules is a prerequisite for this course. Even if you are familiar with spreadsheets, you will likely find it helpful to go over that particular module in the *Grade 10 Applied Mathematics* course.

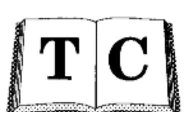
Technical Communication is the clear, understandable transfer of information about a technical situation, process, concept, or service to audiences with varying levels of technical knowledge. This information can be expressed through written, oral, or visual media.

It is becoming increasingly important to be able to explain complex ideas in simple terms, as well as to understand technical information. For example, you may have tried to order a part for a car, snow machine, or household appliance, but could not describe to the technician what you needed. You also may have wanted to perform a function on a VCR or other piece of electronic equipment but could not figure out how to complete the task. For these situations, it is extremely helpful to be able to understand the technical information written in a user's manual.

As technology evolves, so must your technical communication skills. The goal of this course is to strengthen your ability to communicate technical information, and enable you to continually improve these skills throughout your entire life.

Because learning technical communication is an ongoing process, the technical communication projects assigned for this module have been incorporated into several modules throughout the entire course, just as they were in the *Grade 10 Applied Mathematics* course. You have the opportunity to practice and improve your skills in various contexts.

When the following guide graphic appears, it signifies that you are required to complete a technical communications project.



There are six technical communications projects throughout *Grade 11 Applied Mathematics*, located in Modules 1, 2, 3, 5, 6, and 7. Four of these projects are hand-in assignments that will each be worth 5% of your final mark for this course.

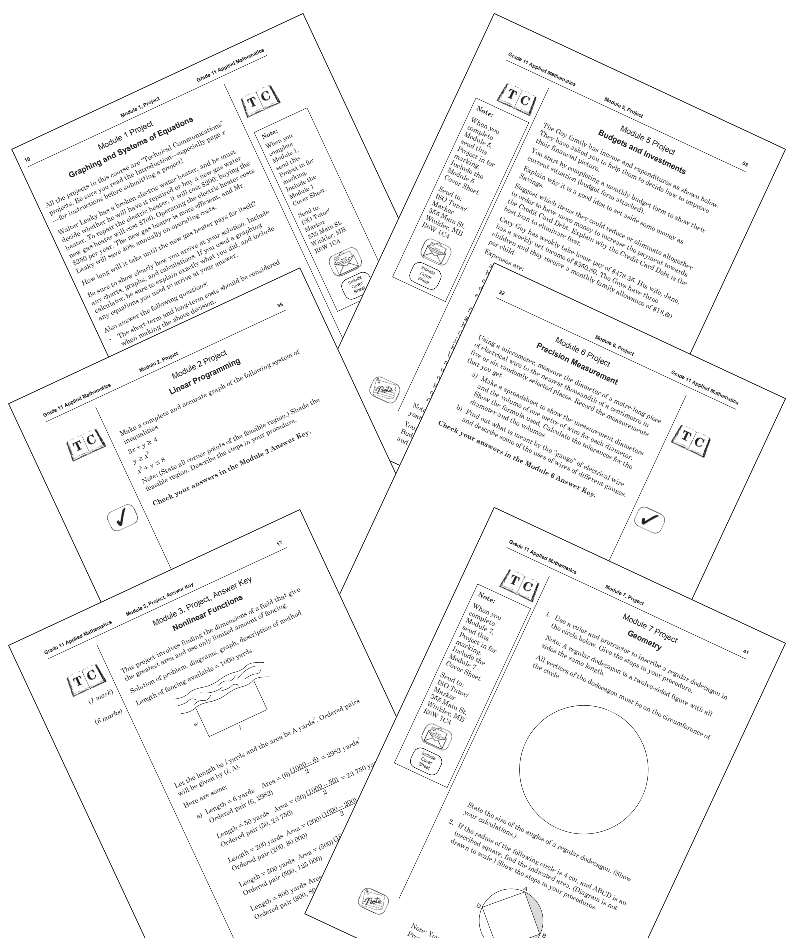
This introduction provides a brief outline of what you are expected to complete for the Project assignments. Occasionally, you may need someone with whom to work. You may choose different people to assist you.

Each project should contain the following sections:

1. Introductory statements.
2. Mathematical solution to the problem presented. Clear explanations are required.
3. Responses to any questions or directions given.
4. Concluding statement.

Each project will be marked out of 10.

Credit will be given for each section indicated above, as well as for organization of the project.



Course Description

The course is divided into eight modules. Modules 1 through 8 contain lessons and assignments. It is recommended that you complete all of the assignments. Answer keys are provided at the end of each module for these exercises.

The eight modules of *Grade 11 Applied Mathematics* are as follows:

Module 1 Graphing and Systems of Equations

Module 2 Linear Programming

Module 3 Nonlinear Functions

Module 4 Personal Finance

Module 5 Budgets and Investments

Module 6 Precision Measurement

Module 7 Geometry

Module 8 Data Management and Analysis

The Table of Contents outlines the major topics and sub-topics found in this course. Every student enrolled in this program is required to complete all eight of these modules. Each module ends with a Self-Test. **The module Self-Tests are all cumulative.** In this way, you will have the chance to continually review material throughout the program. This is done to help you prepare for the examinations.

Self-tests should be written without the aid of any books. Your performance on these tests will give you an indication of how well you understand the material. Your study partner could help you in marking these tests.

Note:

The Self-Tests for Modules 2 and 6 and the Projects for Modules 1, 3, 5, and 7 must be sent to the Tutor/ Marker at the following address as soon as each one is completed.

ISO Tutor/ Marker,
555 Main Street,
Winkler MB
R6W 1C4.

Students must include the appropriate Cover Sheet with each hand-in assignment. Cover Sheets are included on the pages following this introduction.

Your Tutor/ Marker will review the results with you.

The Midterm and Final Examinations must be written under direct supervision and will be sent to your Tutor/ Marker for marking.

Evaluation

Your final mark will be based on the following:

• Module 1 Project	5%
• Module 2 Self-Test	10%
• Module 3 Project	5%
• Midterm Examination (based on Modules 1 to 4)	20%
• Module 5 Project	5%
• Module 6 Self-Test	10%
• Module 7 Project	5%
• Final Examination (based on Modules 1 to 8)	40%
	100%

Your final grade in this course will be based on the results of the Self-Test at the end of Module 2 (Linear Programming), and the Self-Test at the end of Module 6 (Precision Measurement), each worth 10% of the final grade; four projects, each worth 5% of the final grade; and two examinations, a Midterm and a Final Examination. The Midterm Examination will be done after completing Module 4 (Personal Finance) and will be worth 20% of your final grade. The Final Examination is cumulative and, therefore, based on Modules 1 to 8. The Final Examination is worth 40% of your final grade.

You are required to send a cover sheet with each module's completed hand-in assignments. Cover sheets can be found on the pages following this introduction.

Good Luck.

Additional Resources

The *Grade 11 Applied Mathematics* distance education course requires additional materials. All materials can be purchased through the Manitoba Text Book Bureau (MTBB), phone number: 1-866-771-6822.

Item	MTBB Catalogue Number
Graphing Calculator* TI-83	6300

* The TI-83 (Texas Instruments) is the graphing calculator used in the *Grade 11 Applied Mathematics* distance education course.

Access to a personal computer (either Macintosh or Windows) and a spreadsheet program is essential for the course. The student materials have been written using *Clarisworks*, although most spreadsheet programs are compatible with the lessons.

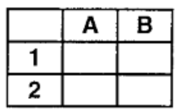
Cautionary Note

Some of the learning experiences in this course involve chance and probability. In some communities, parents may not approve of using playing cards or dice to demonstrate examples of probability. As an alternative to playing cards and dice, numbered index cards and number cubes can be used.

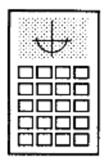
Guide Graphics

Guide graphics have been placed inside the margins of the course to identify a specific task. Each graphic has a specific purpose to help guide you. The significance of each graphic is described below.

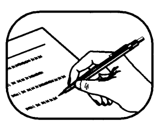
Technical communication project: This guide graphic signifies that the exercise is a technical communications project.



Spreadsheet application: Some activities are easier to complete using a spreadsheet application.



Graphing calculator applications: When an exercise requires the use of a graphing calculator, this graphic will appear. The illustrations and examples shown here use the T1-83 graphing calculator. Other calculators may be used. The screens and symbols vary, and you will need to consult the manual for specific instructions.



Assignment: You are required to do the assignment questions that accompany this graphic.

EXAM TIME!

Exam Time: When this graphic appears, it is time to prepare for an examination.

TEST TIME!

Test Time: When this graphic appears, it is time to write a test.

Check the answer key: After completing an assignment or test, it is important to check your answers. Your partner may help you with this.

Include Cover Sheet

Send in/Include cover sheet: These graphics are to remind you that you must send in the assignment or module test for correcting, and to please include the required cover sheet.

Note: This graphic will appear when there is a direction or explanation that you should note carefully.

Notes

Grade 11 Applied Mathematics

Module 1: Project

Cover Sheet

Please place on top of your work to assist in proper recording of your term work.

Send to:

ISO Tutor/Marker
555 Main St.
Winkler, MB R6W 1C4

Name: ____

Address: ____

Town: _____________________________ **Postal Code:**________________

Attending School: ❑ **No** ❑ **Yes (name)** ___________________________ **School**

For Office Use Only

Module 1 Hand-in Assignment

Date Received: ____________________ Date Returned: ___________________

	Marks
❑ Module 1 Project	/5%

Remarks:

Grade 11 Applied Mathematics

Module 2: Self-Test

Cover Sheet

Please place on top of your work to assist in proper recording of your term work.

Send to:

ISO Tutor/Marker
555 Main St.
Winkler, MB R6W 1C4

Name: ______________________________

Address: ______________________________

Town: ____________________ **Postal Code:** ____________

Attending School: ❑ **No** ❑ **Yes (name)** ____________________ **School**

For Office Use Only

Module 2 Hand-in Assignment

Date Received: ____________ Date Returned: ____________

Marks

❑ Module 2 Self-Test /10%

Remarks:

Grade 11 Applied Mathematics

Module 3: Project

Cover Sheet

Please place on top of your work to assist in proper recording of your term work.

Send to:

ISO Tutor/Marker
555 Main St.
Winkler, MB R6W 1C4

Name: ______________________________

Address: ______________________________

Town: ____________________ **Postal Code:** ____________

Attending School: ❑ **No** ❑ **Yes (name)** ____________________ **School**

For Office Use Only

Module 3 Hand-in Assignment

Date Received: ____________ Date Returned: ____________

	Marks
❑ Module 3 Project	/5%

Remarks:

Grade 11 Applied Mathematics

Module 5: Project

Cover Sheet

Please place on top of your work to assist in proper recording of your term work.

Send to:

ISO Tutor/Marker
555 Main St.
Winkler, MB R6W 1C4

Name: ______________________________

Address: ______________________________

Town: ____________________ **Postal Code:** ______________

Attending School: ❑ **No** ❑ **Yes (name)** ____________________ **School**

For Office Use Only

Module 5 Hand-in Assignment

Date Received: ______________ Date Returned: ______________

	Marks
❑ Module 5 Project	/5%

Remarks:

Grade 11 Applied Mathematics

Module 6: Self-Test

Cover Sheet

Please place on top of your work to assist in proper recording of your term work.

Send to:

ISO Tutor/Marker
555 Main St.
Winkler, MB R6W 1C4

Name: ____

Address: ____

Town: ______________________________ **Postal Code:** ________________

Attending School: ❑ **No** ❑ **Yes (name)** ___________________________ **School**

For Office Use Only

Module 6 Hand-in Assignment

Date Received: ____________________ Date Returned: ____________________

	Marks
❑ Module 6 Self-Test	/10%

Remarks:

Grade 11 Applied Mathematics

Module 7: Project

Cover Sheet

Please place on top of your work to assist in proper recording of your term work.

Send to:

ISO Tutor/Marker
555 Main St.
Winkler, MB R6W 1C4

Name: ____

Address: ____

Town: ______________________________ **Postal Code:** ________________

Attending School: ❑ **No** ❑ **Yes (name)** __________________________ **School**

For Office Use Only

Module 7 Hand-in Assignment

Date Received: ____________________ Date Returned: ____________________

	Marks
❑ Module 7 Project	/5%

Remarks:

Printed in Canada
Imprimé au Canada

GRADE 11
APPLIED MATHEMATICS

Module 1
Graphing and Systems of Equations

Module 1

Graphing and Systems of Equations

Introduction

In *Grade 10 Applied Mathematics*, you explored everyday linear relationships. In this module, you will examine situations where two linear relationships are related through a common factor. You will learn to solve the system of linear equations, which will help you to collect information for making decisions about daily phenomena. You will also learn to use a graphing calculator to solve quadratic and other types of equations. The instructions and graphs shown are related to the TI-83 graphing calculator. Other calculators may be used but you will need to consult the user's manual for details of procedures.

This module will require a strong knowledge of topics learned previously, such as solving equations and graphing functions. This module will also review and extend your comprehension of these topics.

Outline

Lesson 1	Review of Simple Equation-Solving Techniques
Project	Graphing and Systems of Equations
Lesson 2	Using a Graphing Calculator to Solve Equations
Lesson 3	Solving Systems of Linear Equations Graphically
Lesson 4	Solving Systems of Linear Equations Algebraically
Lesson 5	Applications of Systems of Linear Equations
Lesson 6	Solving Systems Involving Nonlinear Equations and Their Applications

Occasionally, you may need someone with whom to work. You may choose different people to assist you.

Notes

Lesson 1

Review of Simple Equation-Solving Techniques

Objectives

When you complete this lesson, you will be able to

- use simple equation-solving techniques to solve linear equations (that is, the highest power of x is 1)
- identify types of nonlinear equations

Overview

This lesson reviews simple to complex linear equations and introduces how to solve nonlinear equations. The examples given review basic equation-solving techniques.

Example 1

Solve:

$3x + 4 = 7$

Solution

$3x+4=7$	
$3x+4-4=7-4$	Subtract 4 from both sides
$3x=3$	
$\frac{3x}{3}=\frac{3}{3}$	Divide both sides by 3
$x=1$	Solution

Example 2

Solve:

$2(5x - 4) = 7(x - 2)$

Solution

$2(5x-4)=7(x-2)$	
$10x-8=7x-14$	Distribute property
$10x-7x-8=7x-7x-14$	Subtract $7x$ from both sides
$3x-8=-14$	
$3x-8+8=-14+8$	Add 8 to both sides
$3x=-6$	
$\frac{3x}{3}=\frac{-6}{3}$	Divide both sides by 3
$x=-2$	Solution

Example 3

Solve:

$$\frac{3x}{2}-2=\frac{2}{5}$$

Solution

$\frac{3x}{2}-2=\frac{2}{5}$	
$\frac{3x}{2}-2+2=\frac{2}{5}+2$	Add 2 to both sides
$\frac{3x}{2}=\frac{2}{5}+\frac{10}{5}$	Find common denominator for right side (i.e., 5)
$\frac{3x}{2}=\frac{12}{5}$	Simplify right side
$\left(\frac{2}{3}\right)\left(\frac{3x}{2}\right)=\frac{2}{3}\left(\frac{12}{5}\right)$	Multiply both sides by $\frac{2}{3}$ to obtain x
$x=\frac{8}{5}$	Solution

Example 4

Solve:

$$\frac{4x+1}{5}=\frac{6x-7}{2}$$

Solution

$\frac{4x+1}{5}=\frac{6x-7}{2}$	
$8x+2=30x-35$	Cross multiply
$8x-30x+2=30x-30x-35$	Subtract 30*x* from both sides
$-22x+2=-35$	
$-22x+2-2=-35-2$	Subtract 2 from both sides
$-22x=-37$	
$\frac{-22x}{-22}=\frac{-37}{-22}$	Divide both sides by –22
$x=\frac{37}{22}$	Solution

Procedures for solving quadratic and cubic equations are as follows:

Example 5

Solve:

$x^2 + 5x + 6 = 0$

Solution

$x^2 + 5x + 6 = 0$	
$(x + 3)(x + 2) = 0$	Factor the trinomial
$x + 3 = 0$ or $x + 2 = 0$	
$x = -3$, or $x = -2$	Solution

Example 6

Solve:

$2x^3 - 3x^2 - 3x + 2 = 0$

Solution

$2x^3 - 3x^2 - 3x + 2 = 0$	
$(2x-1)(x-2)(x+1) = 0$	Factor the cubic expression
$2x - 1 = 0$ or $x - 2 = 0$ or $x + 1 = 0$	
$x = \frac{1}{2}$ or $x = 2$ or $x = -1$	Solutions

In Example 5, the equation is quadratic (that is, x^2, highest exponent of x is 2); in Example 6, the equation is cubic (that is, x^3, highest exponent of x is 3). A quadratic equation may have as many as two answers, whereas a cubic equation may have as many as three answers.

Notice the following pattern:

Equation	Maximum Number of Solutions
Linear	1
Quadratic	2
Cubic	3

You will learn to use your graphing calculator to solve such cubic and quadratic equations. This method will be explained in Lesson 2.

Exponential, trigonometric, and logarithmic equations are examples of other complex equations that can be solved algebraically or by using the graphing calculator.

Exponential	$3^{2x} = 5$	x is an exponent
Trigonometric	$5\cos x - 4 = 0$	x is an angle in a cosine equation
Logarithmic	$5\log x = 7$	x is a number of which the logarithm is being taken

In Lesson 2, you will solve these three types of equations using a graphing calculator.

Assignment

1. Use equation-solving techniques outlined in this lesson to solve the following linear equations.

   a) $3x - 4 = 12$

   b) $\frac{2}{3}x + \frac{5}{8} = -6$

   c) $4(3x - 5) = 7(x + 1)$

   d) $\frac{7x - 3}{4} = \frac{5x + 4}{9}$

   e) $\frac{5x - 7}{6x + 5} = \frac{-3}{4}$

   f) $\frac{-7}{5}x + 9 = 6$

   g) $\frac{2(3x - 4)}{7} = \frac{5(2x + 3)}{6}$

2. Identify the type of equation in each of the following (you may want to refer to Lesson 1).

   a) $x^2 = -7x - 12$

   b) $2^{2x} = 16$

   c) $2x^3 + 7x^2 + 2x - 3 = 0$

   d) $2\sin x + 3 = 4$

   e) $7\log 2x - 5 = 0$

   f) $\cos^2 x = \frac{3}{5}$

3. What is the greatest possible number of solutions for 2a? For 2c? How do you know?

Check your answers in the Module 1 Answer Key.

Module 1 Project

Graphing and Systems of Equations

All the projects in this course are “Technical Communications” projects. Be sure you read the Introduction—especially page *x* —for instructions before submitting a project!

Walter Leaky has a broken electric water heater, and he must decide whether he will have it repaired or buy a new gas water heater. To repair the electric heater, it will cost $200; buying the new gas heater will cost $700. Operating the electric heater costs $250 per year. The new gas heater is more efficient, and Mr. Leaky will save 40% annually on operating costs.

How long will it take until the new gas heater pays for itself?

Be sure to show clearly how you arrive at your solution. Include any charts, graphs, and calculations. If you used a graphing calculator, be sure to explain exactly what you did, and include any equations you used to arrive at your answer.

Also answer the following questions:

- The short-term and long-term costs should be considered when making the above decision.
- What other factors should Mr. Leaky consider before making his decision?
- Which heater would you advise Mr. Leaky to buy? Explain fully.

Note:

When you complete Module 1, send this Project in for marking. Include the Module 1 Cover Sheet.

Send to:
ISO Tutor/
Marker
555 Main St.
Winkler, MB
R6W 1C4

Include Cover Sheet

Lesson 2

Using a Graphing Calculator to Solve Equations

Objectives

When you complete this lesson, you will be able to use a graphing calculator to

- solve linear equations
- solve any solvable equation

Overview

In *Grade 10 Applied Mathematics*, you learned how to graph an equation (function) of the form $y = mx + b$ on a TI-83 graphing calculator. As a review, the following examples and keying sequences are provided as well as Attachment 2.1.

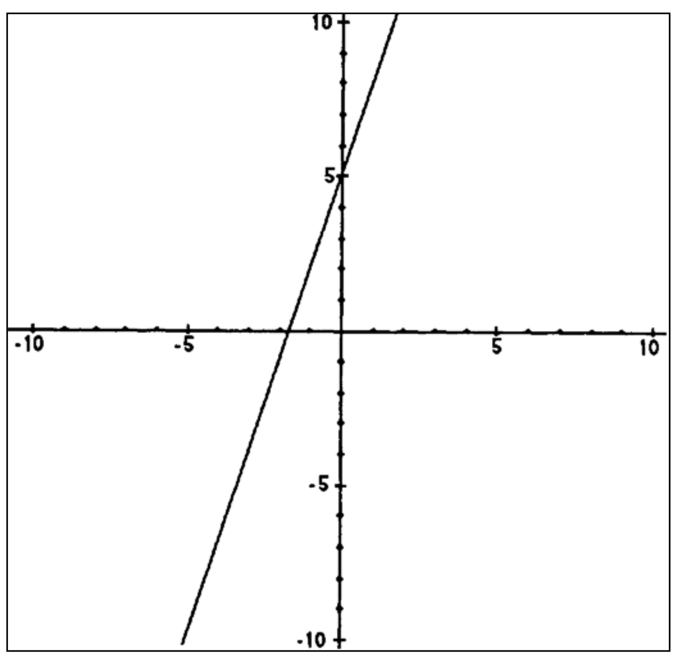
For these examples, set your WINDOW to *default* by pressing [ZOOM] [6].

Example 1

Graph: $y = 3x + 5$

Solution

[Y=] [3] [X,T,θ,n] [+] [5] [GRAPH] yields the following screen (based on default zoom):



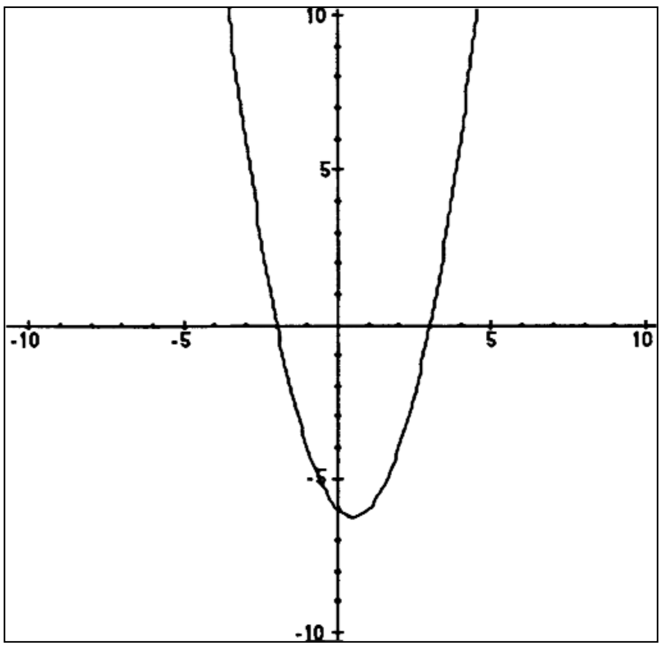
Example 2

Graph: y = $x^2 - x - 6$

Solution

[Y=][X,T,θ,n] [^] [2] [–] [X,T,θ,n] [–] [6] [GRAPH]

This graph would yield the following screen:



The following example provides a simple method for solving all equations graphically, regardless of their difficulty. It is important that you learn the technique outlined.

Example 3

Solve:

$$\frac{3x}{2} - 2 = \frac{2}{5}$$

Solution

Steps:

1. Rearrange to obtain $\frac{3x}{2} - 2 - \frac{2}{5} = 0$ (Equation 1), having 0 on the right-hand side.

2. Form the function $y = \frac{3x}{2} - 2 - \frac{2}{5}$.

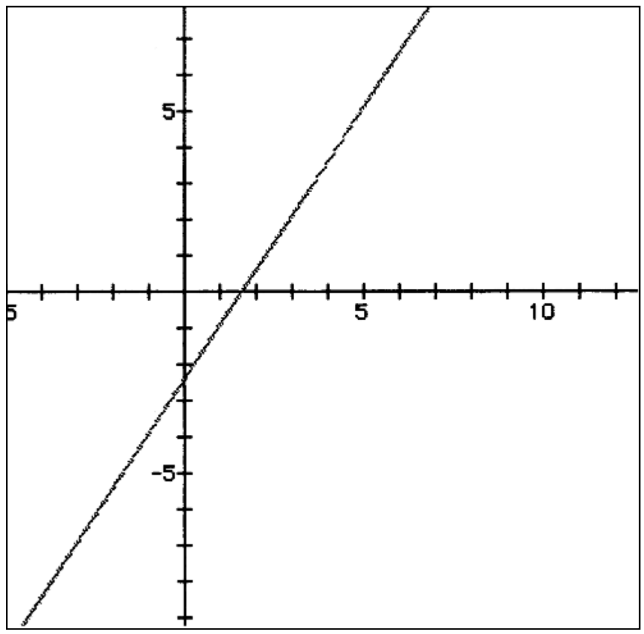
   As in Examples 1 and 2, you are replacing the zero (0) in Equation 1 with y.

3. Graph the function.

   Keying sequence:

   [Y=] [3] [X,T,θ,*n*] [÷] [2] [–] [2] [–] [2] [÷] [5] [GRAPH].

   This yields the following graph:



4. Find the answers (roots) of the equation $\frac{3x}{2} - 2 - \frac{2}{5} = 0$ by finding the zeros of the function $y = \frac{3x}{2} - 2 - \frac{2}{5}$. The Trace Method was used in *Grade 10 Applied Mathematics* but it is not as efficient. The following method is preferred.

   Keying sequence:

   - Press [2nd] [Calc] [2].
   - Set the left and right bounds for x:
     — The left bound would be an x-value less than the x-value of the point where the graph crosses the x-axis (here, $x = 1$).
     — The right bound would be an x-value greater than the actual root (e.g., $x = 2$).

To set the left bound, press [◀] until the cursor reaches the desired value, and then press [ENTER], or enter the value directly by entering 1.

To set the right bound, press [▶] until the cursor reaches the desired value, and then press [ENTER], or enter the value directly by entering 2.

When asked to guess, either enter a value or press [ENTER] and the root will appear, that is, $x = 1.6$.

Example 4

Solve:

$2(5x - 4) = 7(x - 2)$

Solution

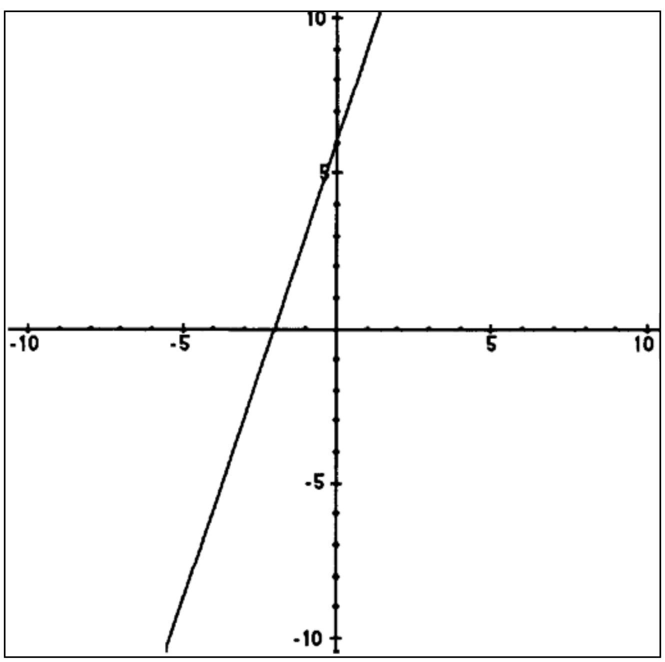
$$2(5x - 4) = 7(x - 2)$$

$$2(5x - 4) - 7(x - 2) = 0$$

Follow steps in Example 3 to obtain the required function.

$$y = 2(5x - 4) - 7(x - 2)$$

Graph the function.
Keying sequence: [Y=] [2] [( ] [5] [X,T,θ,n] [–] [4] [ )] [–] [7] [( ] [X,T,θ,n] [–] [2] [)] [GRAPH]



Find the zero of the function.

Keying sequence: [2nd] [CALC] [2]
Set left bound to $x = –3$. Press [ENTER].
Set right bound to $x = –1$. Press [ENTER].
When asked to guess, press [ENTER].
Zero is $x = –2$.

Example 5

Solve:

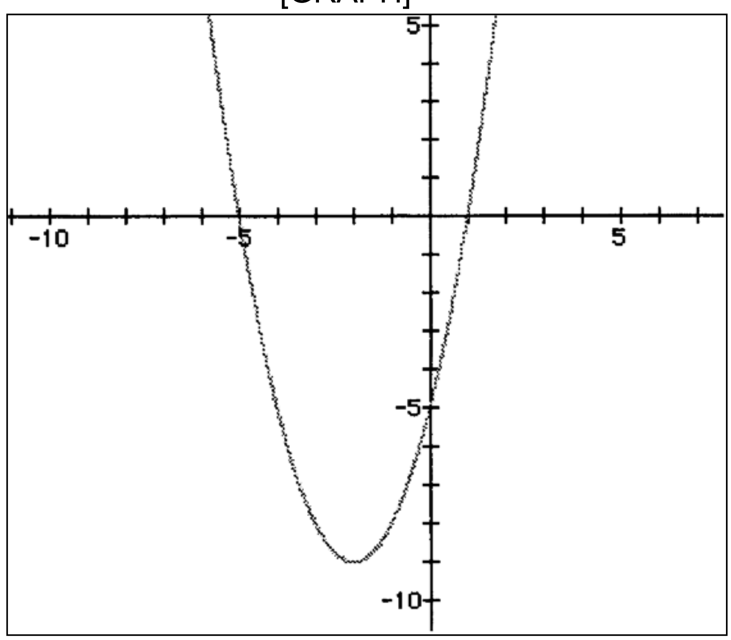
$x^2 + 4x – 5 = 0$.

Solution

$x^2 + 4x – 5 = 0$

$y = x^2 + 4x – 5$

Graph the function.
Keying sequence: [Y=] [X,T,θ,*n*] [x^2] [+] [4] [X,T,θ,*n*] [–] [5] [GRAPH]



Find the points where the graph crosses the x-axis by using the following keying sequence: [2nd] [CALC] [2].

Set left bound to –6. Press [ENTER].
Set right bound to –4. Press [ENTER].
When asked to guess, press [ENTER].

The root $x = -5$ will appear.
Repeat the procedure: [2nd] [CALC] [2].
Set left bound to 0. Press [ENTER].
Set right bound to 2. Press [ENTER].
When asked to guess, press [ENTER].
The root $x = 1$ will appear.

Solution

$x = -5$, $x = 1$

Example 6

Solve:

$3^{2x} = 5$

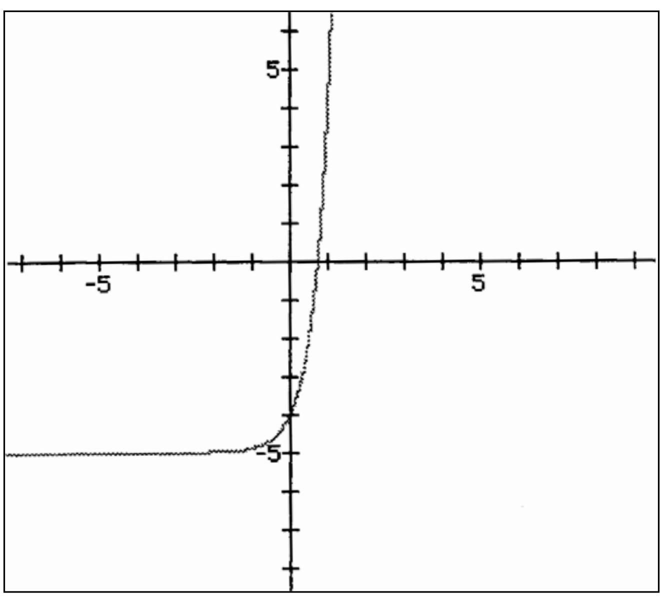
Solution

$3^{2x} - 5 = 0$

$y = 3^{2x} - 5$

Graph the equation.

Keying sequence:
[Y=] [3] [^] [( ] [2] [X,T,θ,n] [ )] [–] [5] [GRAPH].



Notice the graph has only one zero.
Find the root: [2nd] [CALC] [2].
Set the lower bound to $x = 0$. Press [ENTER].
Set the upper bound to $x = 1$. Press [ENTER].

Press [ENTER] again to show the answer: $x = 0.7325$.

Example 7

Solve:

$5\cos x - 4 = 0$

Solution

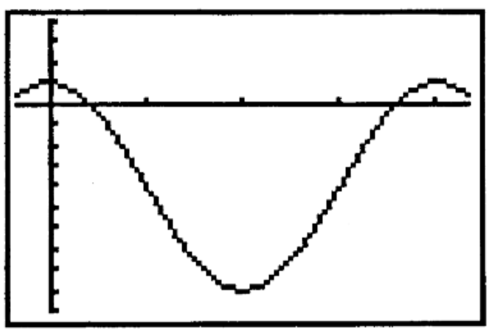
Graph: $y = 5 \cos x - 4$

For this function, set the MODE to degrees. Press [MODE]. Cursor down to the Radian Degree line and highlight *Degree*. Press [ENTER].

Graph the equation.

Keying sequence:
[Y=] [5] [cos] [X,T,θ,n] [ )] [–] [4] [GRAPH].

Press [WINDOW] and change x min = –30, x max = 390, x Scl = 90, y min = –10, y max = 4, y Scl = 1. Press [GRAPH].



This graph is a wave that crosses the x-axis many times. Consider only the answers that lie between 0º and 360º.

Keying sequence: [2nd] [CALC] [2]
Set left bound to $x = 0º$. Press [ENTER].
Set right bound to $x = 180º$. Press [ENTER].
Press [ENTER] again to show the answer: $x = 36.9º$.

Repeat keying sequence: [2nd] (CALC) [2].
Set left bound to $x = 180º$. Press [ENTER].
Set right bound to $x = 350º$. Press [ENTER].
Press [ENTER] again to show the answer: $x = 323.1º$.

Assignment

Using your graphing calculator, find the solutions to the following equations. Give solutions to two decimal places.

1. $\frac{7}{3}x - \frac{5}{8} = 0$
2. $3(x + 3) = 5(-2x + 1)$
3. $3^{2x-3} = 5$
4. $2x^2 - x - 6 = 0$
5. $x^2 = -5x + 1$ (to two decimal places)
6. $4\sin x + 3 = 5$ (between 0° and 180°)
7. $2\cos x = \frac{3}{5}$ (between 0° and 180° to one decimal place)
8. $3\log x - 1 = 2$
9. $2^{3x} = 7^{x+2}$ (Try WINDOW settings of: Xmin = 0
   Xmax = 35
   Xscl = 5
   Ymin = 0
   Ymax = 1 000 000 000 000
   Yscl = 1 000 000

   By reading the x-axis scale, you should be able to set the left bound and right bound values (zero is between 25 and 30).
10. $7\cos x - 5 = 0$ (between 0° and 360° to one decimal place)

Check your answers in the Module 1 Answer Key.

Lesson 3

Solving Systems of Linear Equations Graphically

Outcomes

When you complete this lesson, you will be able to

- solve a system of linear equations graphically
- classify systems as consistent, inconsistent, or dependent

Overview

In *Grade 10 Applied Mathematics*, you examined everyday linear relationships. You gathered data on those relationships, plotted corresponding points, found lines of best fit, and developed linear models for the relationships.

Consider the following examples:

Example 1

You are offered a job as a courier that will pay you in one of two ways:

Option 1 – Base wage of \$140 per week plus \$2 per delivery

Option 2 – Base wage of \$100 per week plus \$3 per delivery

You can expect to make 25 to 60 deliveries each week.

Which option is better? Does a situation exist in which it would not matter which option you chose?

Solution

Each option may be described by a linear function, which may be determined using the *line of best fit* capability of your graphing calculator. (See Page 45 of Module 1 Attachment for Linear Regression directions.) For example, in Option 1, let the number of deliveries be x and the pay be y.

When $x = 30$, $y = (2)(30) + 140$
$= 60 + 140$
$= 200$
So, (30, 200) is one point.

When $x = 50$, $y = 2(50) + 140$
$= 240$
So, (50, 240) is another point.

Calculate some more points in the same way. Plot the points and use the line of best fit to find the equation. This one comes to $y = 2x + 140$.

Similarly, for Option 2, the line comes to $y = 3x + 100$.

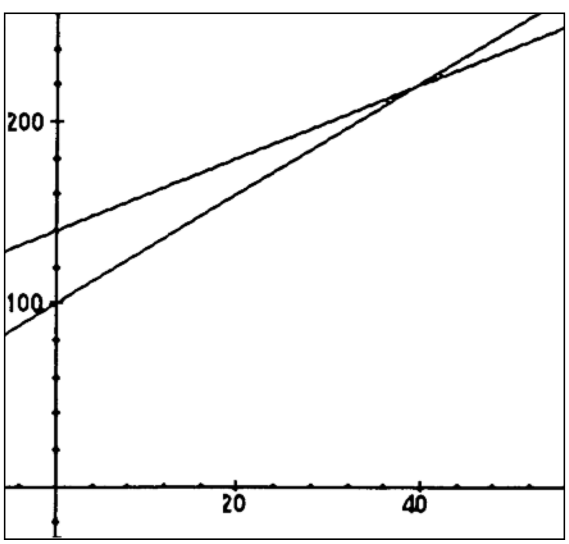
It is also possible to determine the equations from the information. Let x be the number of deliveries.

Option 1: Wage is \$140 plus \$42 for each delivery, which gives the total \$$(140 + 2x)$. The required equation is $y_1 = 140 + 2x$. Similarly, Option 2 gives $y_2 = 100 + 3x$.

Graph these two lines on the same grid using your graphing calculator.

1. Set WINDOW: x from –10 to 60,
   y from –10 to 300
2. In Y= function, check that no other items are entered.
3. Enter the equations:
   a) Press [Y =] and enter $Y_1 = 2x + 140$
      Keying sequence: [Y=] [2] [X,T,θ,n] [+] [140] [ENTER]
      $Y_2 = 3x + 100$

      Keying sequence: [3] [X,T,θ,n] [+] [100]
      Make sure both = signs are toggled on.
   b) Check that you know which line represents which option.
4. Graph the equations: Press [GRAPH].



This graph shows that the pay is the same at the point where the lines cross (the "break-even" point). This graph clearly indicates that Option 1 pays more when the number of deliveries is less than the break-even point and that Option 2 pays more when the number of deliveries is greater than the break-even point.

To decide which option is better, find the break-even point (that is, the intersection point of the two graphs) by pressing [2nd] [CALC] [5]. When prompted by the calculator to identify the first equation, press [ENTER]. When prompted by the calculator to identify the second equation, press [ENTER]. When prompted for a guess, press [ENTER]. At this stage the calculator's guess will be adequate to provide the answer.

Note: The keying sequence [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER] will give the required answers when you have only two equations entered.

Notice the following:

When you press [2nd] [CALC] [5], the calculator displays the equation $y_1 = 2x + 140$ with the cursor on this line. The calculator asks for the first curve. When you press [ENTER], the cursor moves to the other curve and displays the equation $y_2 = 3x + 100$. The calculator asks for the second curve. Press [ENTER]. The calculator understands that you want the intersection of the two lines and asks you to guess. When you press [ENTER] again, the coordinates of the point of intersection are shown. In this case $x = 40$, $y = 220$.

The intersection point $x = 40$, $y = 220$ can be understood as follows:

If you were to make 40 deliveries per week, you would make $220 regardless of the pay scheme. However, for fewer than 40 deliveries per week, Option 1 is better; for greater than 40 deliveries per week, Option 2 is better.

Example 2

Find the intersection point (break even) for the lines

$y = 5x - 4$ and $2x + 3y = 22$

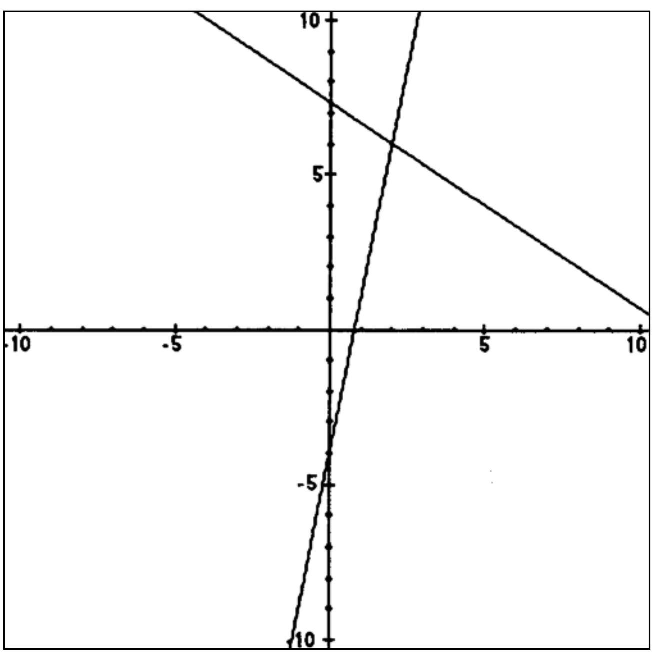
Solution

Reset the window. [ZOOM] [6] does this quickly.
Change the form of the second equation before entering it into your graphing calculator.

$$\begin{aligned} 2x + 3y &= 22 \\ 3y &= 22 - 2x && \text{Subtract } 2x \text{ from both sides} \\ y &= \frac{22}{3} - \frac{2}{3}x && \text{Divide all terms by 3} \end{aligned}$$

Graph the equations on the same grid.

Keying sequence:
[Y=] [5] [X,T,θ,n] [–] [4] [ENTER] [22] [÷] [3] [–] [2] [÷] [3] [x] [X,T,θ,n]
Press [GRAPH].



Find the point of intersection by keying in
[2nd] [CALC] [5] [ENTER] [ENTER] [ENTER].
This gives $x = 2$, $y = 6$. The point (2, 6) is the only point that lies on both lines.

Example 3

Graph and find the point of intersection of the lines

$2x + 3y = 6$
$2x + 3y = -4$

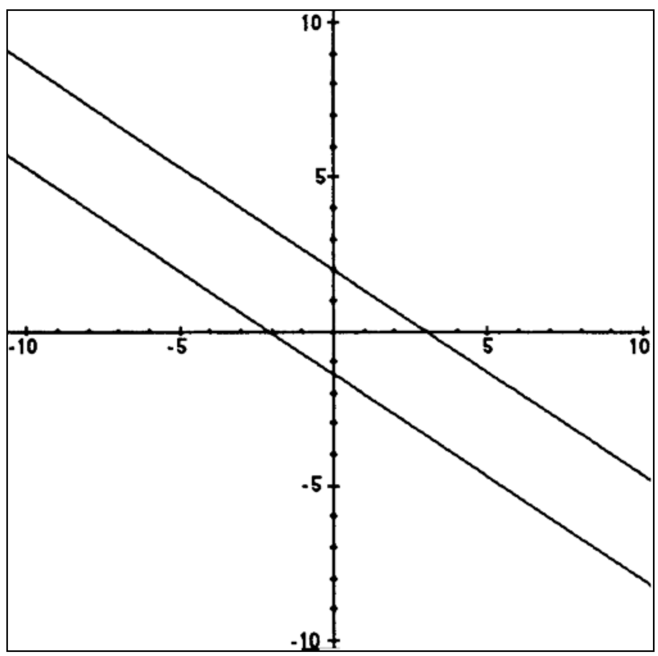
Solution

Change the form of each equation before entering it into the graphing calculator.

$$2x + 3y = 6 \qquad\qquad 2x + 3y = -4$$
$$3y = 6 - 2x \qquad\qquad 3y = -4 - 2x$$
$$y = 2 - \frac{2x}{3} \qquad\qquad y = \frac{-4}{3} - \frac{2}{3}x$$

Graph the equations and find the point of intersection.

Keying sequence: [Y=] [2] [–] [2] [X,T,θ,n] [÷] [3] [ENTER]
[(–)] [4] [÷] [3] [–] [2] [x] [X,T,θ,n] [÷] [3]



The lines are parallel. No intersection point exists.

If you try the [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER] procedure, the calculator indicates an ERROR because they run parallel. This example represents a system of equations that never meet. It is called an **inconsistent** system. Although this can occur when comparing everyday linear models, this type of system is not as useful for decision making as those in Examples 1 and 2.

A similar situation occurs in the following example.

Example 4

Graph and find the point of intersection.

$$y = 2x - 1$$
$$4x - 2y = 2$$

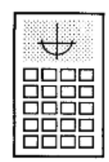
Solution

Change the form of the second equation.

$$-2y = -4x + 2$$
$$y = 2x - 1$$

Even without graphing, you can see that the two lines are identical, which means that the system will meet at every point. This type of system is called **dependent**. A system of two lines that cross at one point is **consistent** and **independent**. These systems are most useful in helping to make decisions.

Assignment



Using your calculator, find the points of intersection for the following systems of linear equations. If the lines run parallel, state **inconsistent**. If the equations represent the same line, state **dependent**. If necessary, round answers to two decimal places.

1. $y = 2x + 3$
   $x + y = 6$

2. $x = 2 - y$
   $3x + 2 = y + 5$

3. $4x - y = 5$
   $2y = 8x - 1$

4. $5x - 6y = 31$
   $6x - 3y = 33$

5. $y = 7x - 3$
   $14x - 2y = 6$

6. $8x + 9y = 7$
   $10x + 21y = 12$

Check your answers in the Module 1 Answer Key.

Lesson 4

Solving Systems of Linear Equations Algebraically

Outcomes

When you complete this lesson, you will be able to solve a linear system of equations using

- the addition/subtraction method
- the substitution method

Overview

It is a useful skill to be able to solve a system of equations without the benefit of a calculator. You should also be able to solve the system algebraically, using either of these methods:

- addition/subtraction
- substitution

Addition/Subtraction Method

1. Multiply one equation or the other or both by the appropriate number(s) to make the coefficients of the x- or y-values the same.
2. If signs of like terms are the same, subtract the two equations. If signs are different, add the two equations.
3. Solve for the remaining variable.
4. Substitute the value found in step 3 into either original equation to find the value of the other variable.

Example 1

Solve:

$$\begin{aligned} 2x + 3y &= 7 \\ x + 2y &= 4 \end{aligned}$$

Solution

$2x + 3y = 7$	
$\underline{2x + 4y = 8}$	Multiply second equation by 2.
$-y = -1$	Subtract the equations.
$y = 1$	Solve for y.

$$
\begin{aligned}
2x + 3(1) &= 7 && \text{Substitute } y = 1 \text{ into first equation.}\\
2x + 3 &= 7 &&\\
2x + 3 - 3 &= 7 - 3 && \text{Subtract 3 from each side.}\\
2x &= 4 &&\\
\frac{2x}{2} &= \frac{4}{2} &&\\
\boldsymbol{x} &\mathbf{= 2} && \text{Solve for } x.
\end{aligned}
$$

Solution

(2, 1)

Example 2

Solve:

$2x + 3y = 1$
$5x - 4y = 14$

Solution

$$
\begin{aligned}
8x + 12y &= 4 && \text{Multiply first equation by 4.}\\
\underline{15x - 12y} &\underline{= 42} && \text{Multiply second equation by 3.}\\
23x \qquad &= 46 && \text{Add the equations.}\\
\frac{23x}{23} &= \frac{46}{23} && \text{Divide both sides by 23.}\\
\boldsymbol{x} &\mathbf{= 2} &&
\end{aligned}
$$

$$
\begin{aligned}
2(2) + 3y &= 1 && \text{Substitute } x = 2 \text{ into first equation.}\\
4 + 3y &= 1 &&\\
4 - 4 + 3y &= 1 - 4 && \text{Subtract 4 from both sides.}\\
3y &= -3 &&\\
\frac{3y}{3} &= \frac{-3}{3} && \text{Divide both sides by 3.}\\
\boldsymbol{y} &\mathbf{= -1} &&
\end{aligned}
$$

Solution

(2, –1)

The second algebraic technique is called the **substitution method.**

Substitution Method

1. Solve either equation for x or y (your choice).
2. Substitute that value into the other equation and solve for the resulting variable.
3. Substitute the resulting value into either original equation to find the second variable.

Example 3

Solve:

$3x + 4y = -2$
$2x - y = 17$

Solution

$2x - 2x - y = 17 - 2x$	Subtract $2x$ from both sides of second equation.
$-y = 17 - 2x$	
$y = 2x - 17$	Divide by –1.
$3x + 4(2x - 17) = -2$	Substitute $2x - 17$ for y in other equation.
$3x + 8x - 68 = -2$	
$11x - 68 + 68 = -2 + 68$	Add 68 to both sides.
$11x = 66$	
$\frac{11x}{11} = \frac{66}{11}$	Divide by 11.
$\boldsymbol{x = 6}$	
$3(6) + 4y = -2$	Substitute $x = 6$ into first equation.
$18 - 18 + 4y = -2 - 18$	Subtract 18 from both sides.
$4y = -20$	
$\frac{4y}{4} = \frac{-20}{4}$	Divide both sides by 4.
$\boldsymbol{y = -5}$	

Solution

(6, –5)

Example 4

Solve:

$4x + 3y = 24$
$3x + 4y = 25$

Solution

$$4x+3y-3y=24-3y$$ Subtract $3y$ from both sides.

$$4x=24-3y$$

$$\frac{4x}{4}=\frac{24}{4}-\frac{3y}{4}$$ Divide both sides by 4.

$$x=6-\frac{3y}{4}$$

$$3\left(6-\frac{3y}{4}\right)+4y=25$$ Substitute $6-\frac{3y}{4}$ for x in second equation

$$18-\frac{9y}{4}+4y=25$$

$$18+\frac{7y}{4}=25$$ Combine y terms.

$$18-18+\frac{7y}{4}=25-18$$ Subtract 18 from both sides.

$$\frac{7y}{4}=7$$

$$\frac{\frac{7}{4}y}{\frac{7}{4}}=\frac{7}{\frac{7}{4}}$$ Divide by $\frac{7}{4}$.

$$\boldsymbol{y = 4}$$

$$4x + 3(4) = 24$$ Substitute $y = 4$ in first equation.

$$4x + 12 - 12 = 24 - 12$$ Subtract 12 from both sides.

$$4x = 12$$

$$\frac{4x}{4} = \frac{12}{4}$$ Divide by 4.

$$\boldsymbol{x = 3}$$

Solution

(3, 4)

Assignment

1. Solve the following systems using the addition/subtraction method.

   a) $5x + 2y = 11$
      $3x + 4y = 1$

   b) $2x + y = 3$
      $3x - y = 2$

   c) $3x + 2y = 11$
      $4x - 3y = 9$

   d) $7x + 3y = 27$
      $2x + 5y = 16$

2. Solve the following systems using the substitution method.

   a) $5x - y = 8$
      $3x + 2y = 10$

   b) $2x - 3y = -1$
      $5x - 2y = 14$

   c) $2x + y = 8$
      $x + y = 5$

   d) $3x + 5y = 37$
      $2x - 3y = 12$

Check your answers in the Module 1 Answer Key.

Notes

Lesson 5

Applications of Systems of Linear Equations

Outcomes

When you complete this lesson, you will be able to apply systems of linear equations to solve problems involving

- numbers
- age
- mixtures
- investments
- rates of travel

Overview

You will now apply the methods of solving a system of linear equations to solve everyday problems. In this procedure, you must decide what the two unknowns represent and let x and y symbolize them. Then set up two equations relating x and y from the given information. Find the solution of the system to answer the problem. Use the graphing calculator method.

Consider the following examples.

Example 1

Find two numbers such that two times the larger number added to three times the smaller is 88, and one-half the larger number minus one-third the smaller is 9.

Solution

Let x = larger number
Let y = smaller number

Set up two equations:
Solve each equation for y

$2x+3y=88$	$\frac{1}{2}x-\frac{1}{3}y=9$
$3y=88-2x$	$\frac{-1}{3}y=9-\frac{1}{2}x$
$y=\frac{88}{3}-\frac{2}{3}x$	$y=-27+\frac{3}{2}x$

Use your graphing calculator to solve the system (see Lesson 2). Zoom out to see graphs. The answer is (26, 12) or the numbers are 26 and 12.

Example 2

In four years, Ashton will be as old as Kehlin is now. Seven years ago, the sum of their ages was 22. What are their present ages?

Solution

Let x = Ashton's age now
Let y = Kehlin's age now

Set up two equations:

$x + 4 = y$	$x + 4$ is Ashton's age in four years.
$x - 7 + y - 7 = 22$	$x - 7$ and $y - 7$ are the boys' ages seven years ago.

Solve each equation for y:

$y = x + 4$

$y = 36 - x$

Using the graphing calculator, we find that $x = 16$ and $y = 20$. Ashton is 16 and Kehlin is 20. (Zoom out to see point of intersection.)

Example 3

Twelve litres of antifreeze solution contains 30% alcohol and 70% water. How many litres of pure alcohol must be added to raise the concentration of alcohol to 40%?

Solution

Let x litres of alcohol be added to the antifreeze.
Let y litres be the total volume.

Set up two equations:

$y = 12 + x$	Total volume of solution
$x + 3.6 = 0.4y$	Total volume of alcohol 30% of 12 = 0.30 x 12 = 3.6

Solve:

Use the graphing calculator to find x and y

$y = 12 + x$

$$y = \frac{x}{0.4} + \frac{3.6}{0.4}$$

Solution

$x = 2$ and $y = 14$

Answer: Two litres of alcohol must be added to 12 litres of solution for a total of 14 litres to achieve the 40% solution.

Example 4

The student council deposited part of the $1200 it earned from yearbook sales into a savings account receiving 9% interest, and the remainder into a chequing account receiving 4% interest. If the total interest for the year is $88, how much was deposited into each account?

Solution

Let $\$x$ = amount invested in savings account
Let $\$y$ = amount invested in chequing account

Set up two equations:

$x + y = 1200$ Total investment

$0.09x + 0.04y = 88$ Total interest calculated using $I = prt$

Note:

where I = interest
p = amount invested
r = rate of interest
t = number of years of interest

Savings $I = prt$
$I = x(0.09)(1)$
$I = 0.09x$

Chequing $I = prt$
$I = y(0.04)(1)$
$I = 0.04y$

Solve each equation for y:

$$y = 1200 - x$$

$$y = \frac{88}{0.04} - \frac{0.09x}{0.04}$$

Solve for x

$$
\begin{aligned}
1200 - x &= \frac{88}{0.04} - \frac{0.09x}{0.04} \\
48 - 0.04x &= 88 - 0.09x \qquad \text{multiply by } 0.04 \\
0.5x &= 40 \\
x &= 800
\end{aligned}
$$

Using a graphing calculator, you will find the two lines intersect at (800, 400). Therefore, $800 is invested in the savings account and $400 in the chequing account.

Example 5

A riverboat travels eight miles in one hour and returns the same distance in one and one-half hours (1.5 hours). Find the rate of the boat in still water and the rate of the river current.

Let x miles per hour = rate of boat in still water
Let y miles per hour = rate of river current

Downstream
Actual rate of travel = $x + y$ miles per hour.
We also know that the boat travelled eight miles in one hour so that the rate of travel is eight miles per hour.

Upstream
Actual rate of travel = $x - y$ miles per hour.
We also know that the boat travelled eight miles in 1.5 hours at r miles per hour so that

$$
\begin{aligned}
8 &= 1.5r \\
\frac{8}{1.5} &= r \\
r &= \frac{16}{3} \\
\text{Therefore, } x - y &= \frac{16}{3}
\end{aligned}
$$

Solve:

$$
\begin{aligned}
x + y &= 8 \\
x - y &= \frac{16}{3}
\end{aligned}
$$

You may use either the algebraic method or the graphing calculator method. Here the addition/subtraction method is shown.

$2x = 8 + \frac{16}{3}$ Add the two equations.

$2x = \frac{40}{3}$ Simplify right-hand side.

$x = \frac{40}{3} \div 2$ Divide both sides by 2.

$x = \frac{20}{3}$

Substitute x in first equation. $\frac{20}{3} + y = 8$

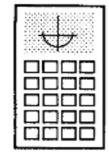
$$y = 8 - \frac{20}{3}$$
$$= \frac{24 - 20}{3} = \frac{4}{3}$$

Rate of boat in still water $= \frac{20}{3}$ mph

and rate of river current $= \frac{4}{3}$ mph

Note: If you use the graphing calculator you will get 6.67 mph for rate of boat in still water and 1.33 mph for rate of river current.

These examples represent a few of the applications of systems of equations. Other problems can be solved using the same procedures.



Assignment

1. The sum of two numbers exceeds twice the smaller number by seven. One-half the larger number is three less than the smaller number. Find the two numbers.
2. Add the smaller of two numbers to one-half the greater and the sum is 11. Subtract the smaller from one-quarter of the greater and the difference is –1. Find the numbers.
3. Jane’s father is four times as old as Jane. Five years ago her father was nine times as old as Jane was then. What are their present ages?
4. Pete has $3.75 in nickels and quarters. If he has a total of 31 coins, how many nickels does he have?
5. A man has $12 000 to invest. He invests part in 3 1/4% bonds and part in 4 1/2% bonds. If his total annual income from the interest is $490, how much does he invest in each bond issue?
6. Susan invested part of her $1800 savings at 9% and the remainder at 6%. In one year, the 9% investment earned $102 more than the 6% investment. How much did she invest at each rate?
7. Simco Gas sells regular gas at 23 cents/L and premium gas at 25 cents/L. It also sells a mixture of the regular gas and the premium gas at a medium price of 24.2 cents/L. If a 10 000 L tank is to be filled with the medium-price gas, how much regular and premium gas should be used?
8. A chemical firm produces an 80% iron alloy by combining 90% iron ore and 60% iron ore by mass. If the company wishes to fill an order for 150 kg of this alloy, how much of each type of ore must be used?
9. A commercial airliner travelled 2200 miles with the wind in 7 1/3 hours and took 10 hours to return the same distance against the wind. Find its airspeed in still air and the wind velocity.
10. The perimeter of an isosceles triangle is 9 inches. Each of the congruent sides is 13 times longer than the base. Find the measure of each side of the triangle.

Check your answers in the Module 1 Answer Key. ✓

Lesson 6

Solving Systems Involving Nonlinear Equations and Their Applications

Outcomes

When you complete this lesson, you will be able to

- solve systems involving nonlinear equations
- apply systems of nonlinear equations to solve problems

Overview

Solving nonlinear systems of equations is similar to solving linear systems of equations except that you may arrive at more than one point of intersection. Use the graphing calculator to find solutions to the systems.

Consider the following examples.

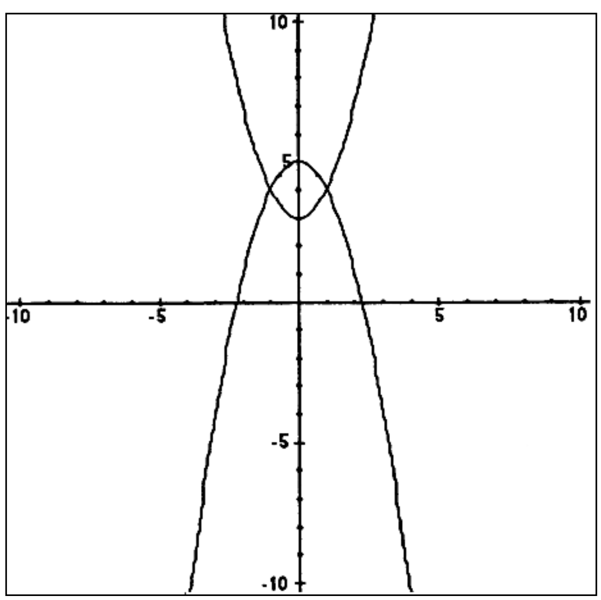
Example 1

Solution

Reset window by keying [ZOOM] [6]
Enter both equations and graph.

$y = x^2 + 3$
$y = -x^2 + 5$

Keying sequence:
[Y=] [X,T,θ,n] [^] [2] [+] [3] [ENTER] [(–)] [X,T,θ,n] [^] [2] [+] [5]
Press [GRAPH]



Find the first point of intersection.

Keying sequence: [2nd] [CALC] [5]

Use the cursor buttons [◀] and [▶] to place cursor close to one of the intersection points.

Press [ENTER] [ENTER] [ENTER] to get the first point (–1, 4).
For the second point, repeat the keying sequence [2nd] [CALC] [5].
Use the cursor buttons to place the cursor close to the other intersection point.

Press [ENTER] [ENTER] [ENTER] to get the second point (1, 4).
Solutions are (–1, 4) and (1, 4).

Example 2

The sum of two numbers is 27 while their product is 126. What are the numbers?

Let x = one number
Let y = other number

$x + y = 27$
$xy = 126$

Solve both equations for y:

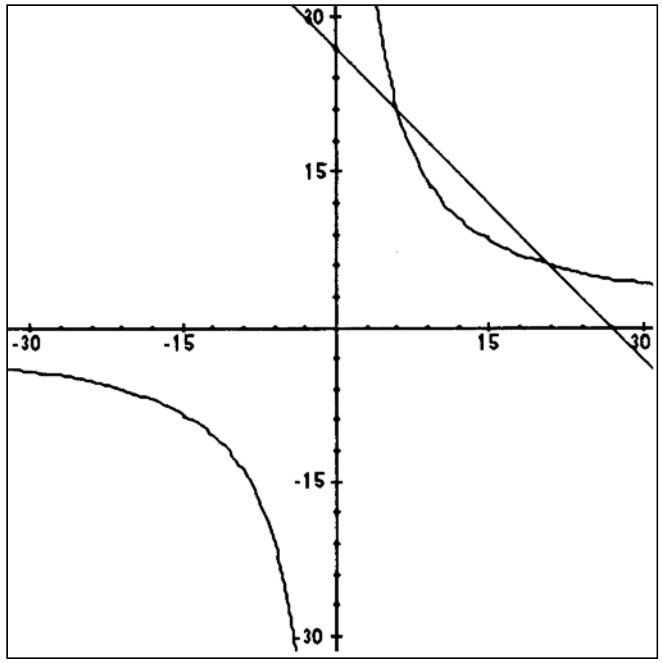
$$y = 27 - x$$

$$y = \frac{126}{x}$$

Use your graphing calculator to graph these equations.
Enter the two equations.

Keying sequence: [Y=] [27] [–] [X,T,θ,n] [ENTER] [126] [÷] [X,T,θ,n]
Press [GRAPH]

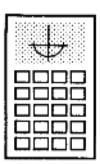
Zoom out to see graphs.



There are two points of intersection.

Use [2nd] [CALC] [5] to obtain the coordinates of the first point, (6, 21), and repeat to give second point (21, 6). Therefore, the two numbers are 6 and 21.

Assignment



Solve the following systems of equations using the graphing calculator.

1. $y = x^2$
   $x + y = -4$

2. $y = x^2$
   $y - 2x = 3$

3. $x^2 + 8xy = 4$
   $x = y$

4. $y = 3x^2 + 1$
   $y = x^2 + 5$

5. $y + 7x = 0$
   $7x^2 = y$

6. The square of one number is equal to a second number. The sum of the two numbers is 2. What are the numbers?

7. The sum of two numbers is $1\frac{7}{15}$. The product of their reciprocals is $1\frac{7}{8}$. Find the two numbers.

8. The product of two positive numbers is 8. The sum of their reciprocals is $\frac{3}{4}$. What are the numbers?

Check your answers in the Module 1 Answer Key.

Attachment A-1

TI-83 Graphing Calculator Procedures

TI-83 Utilities

Note that the 2nd key activates the functions above the keys, which are identified with parentheses ( ).

ON — Press ON, located on the lower left-hand corner.

OFF — Press 2nd (Off).

CLEAR — Press CLEAR to clear the current screen and return to the previous screen.

QUIT — Press 2nd (Quit) to return to the home screen.

INS/DEL — Press 2nd (INS) to insert a character *before* the cursor.

Press DEL to delete the character that the cursor is on.

FUNCTION REGISTER — Press Y= to display the function register. This allows you to store up to 10 functions. They can be graphed one or more at a time.

Press ▼ to move the cursor to the first function. You may want to clear, edit, or define it.

To clear or remove the function, press CLEAR.

To define a new function, e.g., $x^2 - x - 2$, press X,T,θ,n x^2 – X,T,θ,n – 2.

Use the cursor to move to the next function, which is Y_2. Enter $x + 5$.

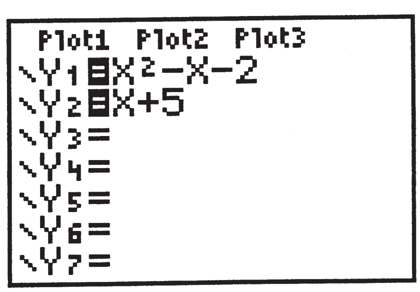


TABLE — This gives a table of values for the function. To see a table, press 2nd (TABLE).

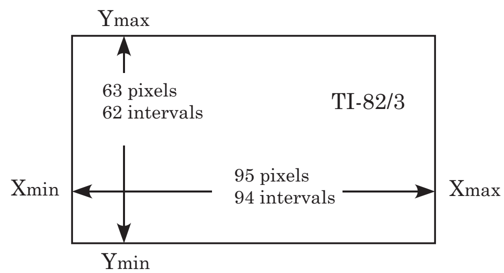
TRACE	Press TRACE. Notice that the cursor is placed directly on the graph. Use the right ▶ or left ◀ arrow keys to move the cursor away from one plotted point to the next along a graphed function. Notice how you can use the cursor to find corresponding *x*- and *y*-values, which are displayed at the bottom of the screen.
ZOOM	Press ZOOM . Some features in this window are as follows: • 2:Zoom In — to enlarge an area of a graph. Press TRACE to place the cursor on a particular point of the graph. Now select Zoom In — that part of the graph will enlarge. This is like looking at a particular speck on the ground from a standing position and then kneeling down to take a better look. • 3:Zoom Out — to attain a greater view of the graph. Press TRACE to place the cursor on a particular point of the graph, and then select Zoom Out. This is similar to moving up and away from the ground in an airplane and surveying a wider and wider view of the ground beneath you. • 6:Zoom Standard — to (re)set the graph to standard size
RESET MEMORY	Allows you to clear all memory or to reset to factory default settings. Press 2nd , (MEM) and then 5 .
CONTRAST	You can adjust the contrast to suit the lighting conditions. The settings are from 0 (lightest) to 9 (darkest). To change the contrast, press and release 2nd. Then, 1. press and hold ▲ which lightens the screen or 2. press and hold ▼ which darkens the screen
ALPHA (A-LOCK)	The alpha function of each key is printed in green above each key. When you press the green ALPHA key, it activates the alpha function for the next keystroke. For example, if you press ALPHA and then TAN, the letter G is entered. The (A-LOCK) key locks in the alpha function.
GRAPH	Press Y=, enter the equation, and press GRAPH . Then you can press ZOOM or TRACE .
WINDOW	Sets the range of values for the viewing windows. X_{SCL} (X scale) and Y_{SCL} (Y scale) define the distance between tick marks on each axis. X_{res} sets pixel resolution (1 through 8) for function graphs. Default is 1. To change a value: 1 use ▶ or ▼ to move the cursor to the variable you want to change 2. edit the value 3. press ENTER

FRIENDLY WINDOW

The TI-83 viewing window has 94 intervals from left to right, so 94 is the magic number. Select Xmin and Xmax so that

$$x = \frac{\text{Xmax} - \text{Xmin}}{94}$$

is an integer or "nice" decimal, such as 0.1, 0.2, 0.25, and so on. This happens when Xmax – Xmin is either a factor or a multiple of 94 (decimal factors are okay). For example, if Xmax – Xmin equals 94, 188, 47, 23.5, 9.4, 18.8, 4.7, or 0.47, the window will be "friendly."



SCATTERPLOT

Scatterplots allow you to plot statistical data from lists.

To create a scatterplot, do the following:

1. **Clear previous data in lists**

   Press STAT 1 to edit the lists. Data already entered in the lists should be cleared. To clear a list, place the cursor at the top of the list on the symbol L_1. Press CLEAR and then ▼. This clears L_1. Repeat this procedure for L_2 to clear it (see Figure 1).

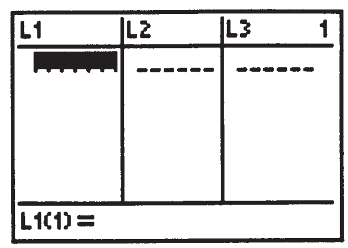


Figure 1

2. **Enter data**

   Use the cursor to move to the first cell of L_1. Enter a value, e.g., 2.5, and press ENTER to move down to the next cell. Continue entering the rest of the data for L_1 (see Figure 2). After making the last entry, use the right arrow ▶ to move the cursor to the first cell of L_2. Enter the data for L_2.

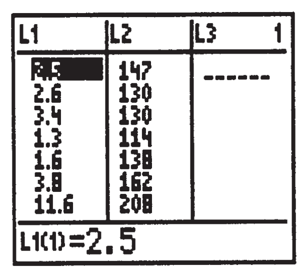


Figure 2

3. **Show the scatterplot**

   Press 2nd Y= ENTER to access the menu for scatterplots. Use your arrow keys if necessary to make your screen look like Figure 3. This figure shows that you want a scatterplot with the Xlist on L_1 and Ylist on L_2. The marker for each point will be a square. To ensure that the data all fit in the window, press ZOOM 9 to set the right window for these data. Press GRAPH to see the scatterplot (see Figure 4).

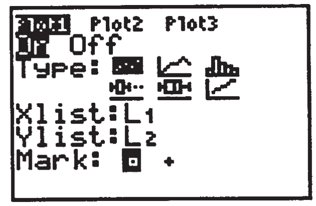


Figure 3

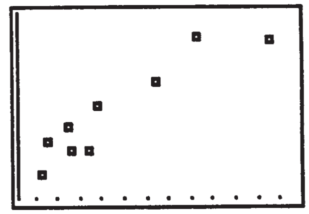
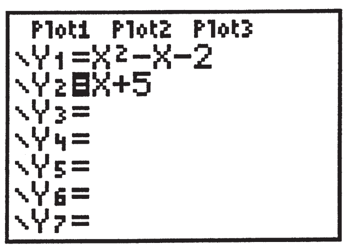


Figure 4

SELECT OR DESELECT A FUNCTION

This feature allows you to “turn on” or “turn off” a function in the register. A function is selected when the = sign is highlighted.

To turn on/off a function, move the cursor to the function, then use ◀ to move the cursor to the function’s = sign. Press ENTER to highlight or remove the highlighting of the function. As shown below, Y_1 is deselected and Y_2 is selected.

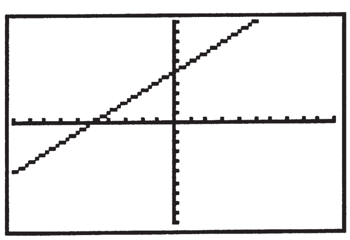


GRAPH A FUNCTION

When a function, e.g., Y_2 is *turned on*, that is, the = sign is highlighted in the function register, it can be graphed.

Press ZOOM 6 to set the screen to the standard setting.

Press GRAPH. All highlighted functions will be graphed.



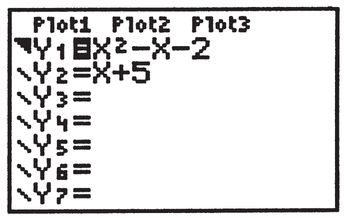
GRAPH STYLES IN THE REGISTER

There are seven graph styles available. They can be used to distinguish between different functions or to create shading above or below the graph.

Press ▲ or ▼ to move the cursor to the function.

Press ▶ or ◀ to move the cursor left, beyond the = sign, to the graph style icon. The default is \.

Press ENTER repeatedly to rotate through the seven different graph styles. Set up your screen to look like this and press GRAPH.



REGRESSION LINE EQUATION

Your calculator can compute the equation of the linear regression line.

Press STAT ▶ to get the menu with the list of regression techniques.

Select a desired regression, e.g., 4:LinReg(ax+b), and press ENTER (see Figure 1).

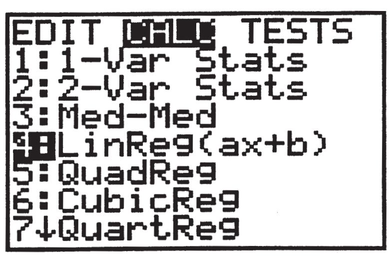


Figure 1

Another screen with LinReg(ax+b) appears. If you want to do a regression on lists L_1 and L_2, key in (L_1,L_2) at the position of the cursor (be sure to include the comma).

Press ENTER to perform the calculation.

The equation will appear.

The slope is “a” and the *y*-intercept is “b.”

To see this equation on your scatterplot, press Y= to get the function register.

The equation should be pasted to the first available location, e.g., (Y_2).

Move the cursor down to the space after the = sign on the line Y_2.

Press $\boxed{\text{VARS}}$ $\boxed{5}$ $\boxed{\blacktriangleright}$ $\boxed{\blacktriangleright}$ $\boxed{\text{ENTER}}$ to paste the least squares equation into the function register (see Figure 2).

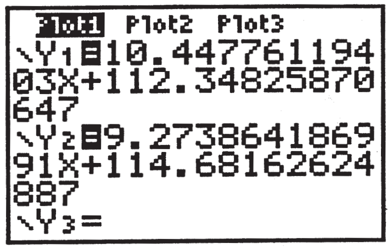


Figure 2

Press $\boxed{\text{GRAPH}}$ to see the least squares line and the scatterplot simultaneously (see Figure 3).

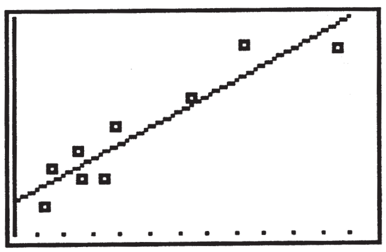


Figure 3

LINK

LINK allows you to send information from one TI-83 to another TI-83. One is the *sending unit* and the other is the *receiving unit*. Connect the two TI-83s with the linking cable.

On the *receiving unit*, press $\boxed{\text{2nd}}$ (LINK) $\boxed{\blacktriangleright}$ to get the RECEIVE menu and select 1:Receive.

Wait.

On the *sending unit*, press $\boxed{\text{2nd}}$ (LINK) to get the SEND menu.

Select 2:All to get the SELECT screen (see Figure 1).

All allows to select from all entries.

Use $\boxed{\blacktriangledown}$ to go to L_1 and press $\boxed{\text{ENTER}}$ to select it.

Use $\boxed{\blacktriangle}$ to go to L_2 and press $\boxed{\text{ENTER}}$ to select it.

A **square dot** next to each indicates that each has been selected to send (see Figure 2).

Press ▶ to get to the TRANSMIT menu

Press 1:Transmit

If Duplicate Name appears, choose one of the following

1. Rename
2. Overwrite

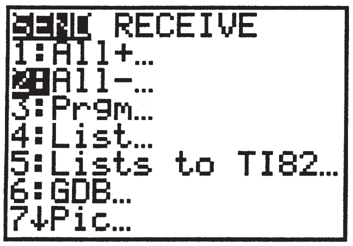


Figure 1

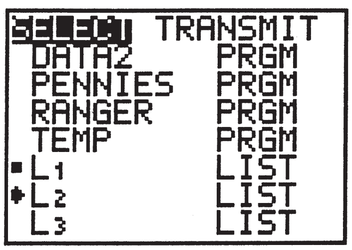


Figure 2

Self-Test

Module 1 – Graphing and Systems of Equations

/64

(10 marks)

TEST TIME!

1. Using your graphing calculator, solve the following equations for "x". (Round answers to the nearest tenth where necessary.)

   a) $\frac{4}{3}(2x - 5) = 6(x + 1)$

   b) $3x^2 = 8x - 5$

   c) $7\sin x + 2 = 5 \quad (0^\circ - 180^\circ)$

   d) $8^{2x+1} = 9$

   e) $7\log x - 5 = 0$

(8 marks)

2. Solve the following systems of linear equations by addition or subtraction.

   a) $2x + 3y = 6$
   $3x - 2y = 7$

   b) $3x = 2y + 7$
   $5y = 2x - 12$

3. Solve the following systems of linear equations by substitution. *(8 marks)*

   a) $3x - 2y = 6$
   $x + 2y = -2$

   b) $7x + 3y = 27$
   $2x + 5y = 16$

4. Solve the following systems of equations using your calculator. *(8 marks)*

   a) $-2x + 3y = 6$
   $6x + 5y = 30$

   b) $\frac{x}{5} + \frac{y}{6} = 3$
   $\frac{x}{10} - \frac{y}{3} = -1$

   c) $x^2 - 5x - y + 4 = 0$
   $x - 4y = 1$

   d) $3xy + 6y = 4$
   $2y - 3x - 4 = 0$

5. Solve the following word problems by setting up a system of equations and using your calculator where necessary to solve the resulting system.

(5 marks)

a) Janet wishes to invest a total of $660 in two parts so that the interest from a 10% investment is equal to the interest from a 12% bond. How much should be invested at each rate?

(5 marks)

b) A grocer bought oranges, some at 54 cents a dozen and some at 60 cents a dozen. He paid $20.10 for them altogether. He sold them all at 75 cents a dozen and made a profit of $6.15. How many oranges did he buy at each price?

c) A nut mixture containing 10% cashews is mixed with another nut mixture containing 30% cashews. The result is 10 kg of a nut mixture containing 24% cashews. How much of each nut mixture was used? *(5 marks)*

d) Find two numbers such that one-half their sum is 11 and one-third their difference is 2. *(5 marks)*

(5 marks)

e) A glider takes two hours to travel 480 km with the wind, but takes three hours to make the same trip against the wind. Find the speed of the plane and the speed of the wind.

(5 marks)

f) The difference between two numbers is 15, and their product is 100. Find the numbers.

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Notes

GRADE 11
APPLIED MATHEMATICS

Module 2
Linear Programming

Module 2

Linear Programming

Introduction

Decisions are often made in an attempt to maximize or minimize particular quantities. For example, businesses attempt to maximize profits while minimizing costs. Similarly, car owners attempt to maximize gas mileage while minimizing costs.

In every maximizing or minimizing situation a number of factors must be considered. These factors are called **constraints**. When a relationship can be expressed as a linear equation and the constraints can be represented by linear inequalities, the maximizing or minimizing can be accomplished using linear programming.

In this module, you will explore linear programming and its applications. You will use a graphing calculator to draw the lines related to the constraints and to find the intersection points required. You will then use algebraic reasoning to find solutions to the problems.

Outline

Lesson 1	Graphing Linear Inequalities
Lesson 2	Graphing Systems of Linear Inequalities
Lesson 3	Determining the Corner Points of a Feasible Region
Lesson 4	Finding Maximum and Minimum Values from Corner Points
Project	Linear Programming
Lesson 5	Application of Linear Programming

Notes

Lesson 1

Graphing Linear Inequalities

Outcomes

When you complete this lesson, you will be able to

- determine the solution set of a linear inequality

Overview

To use linear programming to solve maximizing or minimizing problems, you have to graph linear inequalities, such as

$2x + 3 < 6$ $3x - y \geq 4$ $4x + 5y > 8$ $x \leq 5$

To graph one of these inequalities, consider what happens when you graph a line (such as $Ax + By = C$) on a grid. When you draw any line on a grid, the line divides the grid into three regions:

- Region 1 includes all points on the line, represented by $Ax + By = C$
- Region 2 includes all points to one side of the line, represented by $Ax + By > C$
- Region 3 includes all points on the other side of the line, represented by $Ax + By < C$

Consider the graph below.

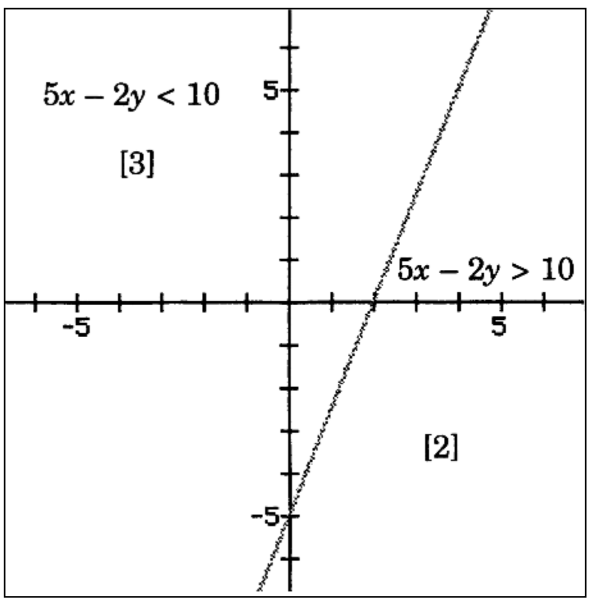
The line $5x - 2y = 10$ separates the grid into three regions:

- Region 1 includes all points on the line $5x - 2y = 10$.
- Region 2 includes all points on one side of the line.

  The inequality $5x - 2y > 10$ represents all points in this region.
- Region 3 includes all points on the other side of the line. The inequality $5x - 2y < 10$ represents all points in this region.

$5x - 2y = 10$

x	0	2
y	–5	0



To verify this, consider the point (0, 0) in Region [3].
For this point, $5x - 2y = 5(0) - 2(0) = 0$, which is < 10.
Region [3] is represented by $5x - 2y < 10$.

To graph an inequality Ax + By > C, follow the procedure here.

Step 1. Draw the related line, Ax + By = C. Use the 2-intercept method. Let $x = 0$ and find the value of y to give the point on the y-axis. Let $y = 0$ and find the value of x to give the point on the x-axis

x	0	?
y	?	0

or rearrange the equation to give $y = -\frac{A}{B}x + \frac{C}{B}$ and use the graphing calculator. Copy the diagram onto a grid.

Step 2. Select a point that is not on the line. Evaluate Ax + By for this point. If the value calculated is > C, then the point selected is in the required region. Shade that region. If the value calculated is < C, then the point is not in the required region. Shade the region on the other side of the line.

For Ax + By > C (and for Ax + By < C), the points of the line are **not** included and the line is dotted.

For Ax + By ≥ C (and for Ax + By ≤ C), the points of the line **are** included and the line is solid.

The examples show how this works.

Example 1

$2x + 3y < 6$

The related equation is $2x + 3y = 6$.

Solution

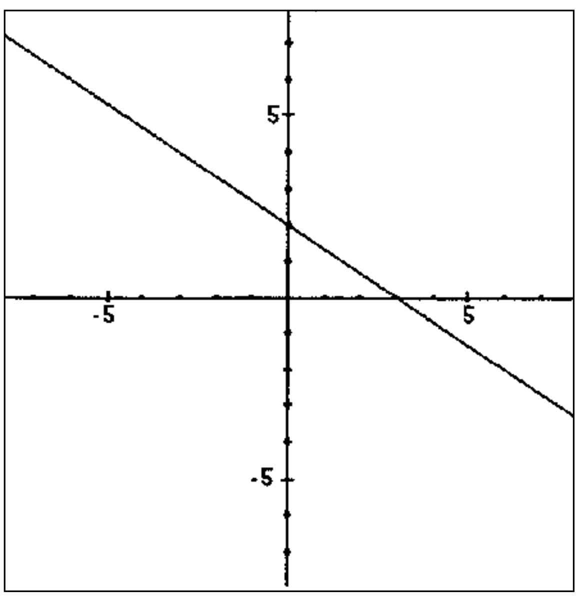
Graph using the x-intercept ($y = 0$) and y-intercept ($x = 0$)

x	0	3
y	2	0

or use the graphing calculator.

$$\begin{aligned} 2x+3y &= 6 \\ 2x-2x+3y &= -2x+6 \\ 3y &= -2x+6 \\ y &= -\frac{2}{3}x+2 \end{aligned}$$

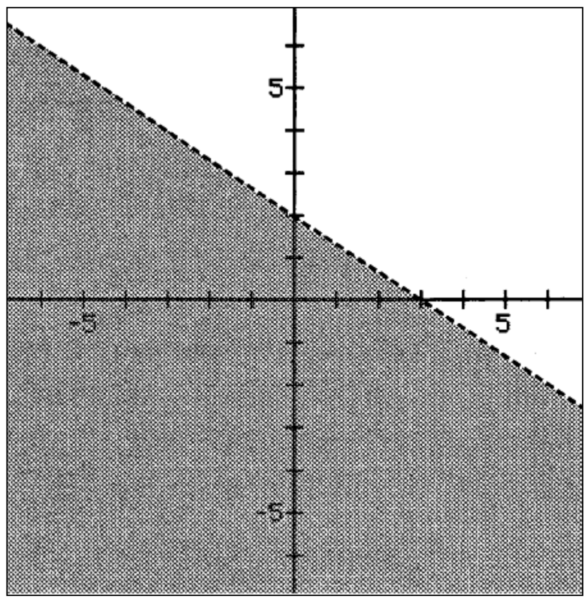
Enter the equation and graph. Copy the line onto your own grid.



To determine which side of the line represents the inequality, select a point. In this example, we will use (0, 0), a point that is clearly not on the line.

$$
\begin{aligned}
& 2x + 3y \\
= \; & 2(0) + 3(0) \\
= \; & 0
\end{aligned}
$$

This is less than 6 and so (0, 0) lies in the required region. Shade this region.



Since (0, 0) satisfies the original inequality, the area shaded is the required region.

Note: the line is dotted because the points on the line do not satisfy the inequality $2x + 3y < 6$.

Example 2

$3x - y \geq 4$

The related equation is $3x - y = 4$.

Solution

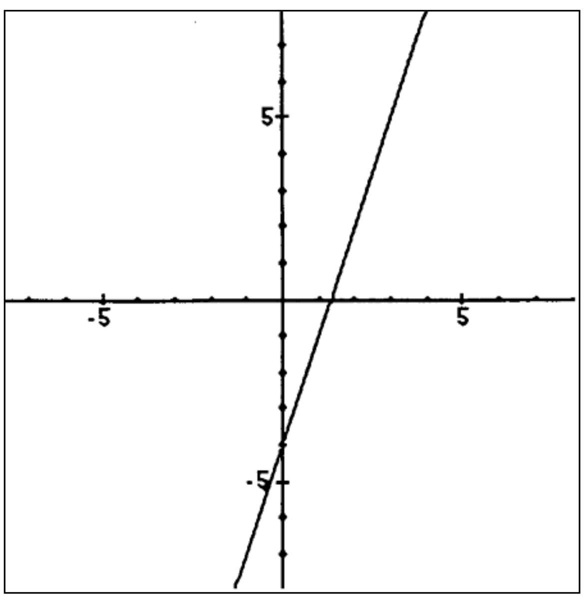
Graph using the x-intercept ($y = 0$) and y-intercept ($x = 0$)

x	0	$\frac{4}{3}$
y	–4	0

or rearrange to use the graphing calculator.

$$\begin{aligned} 3x - y &= 4 \\ 3x - 3x - y &= -3x + 4 \\ -y &= -3x + 4 \\ y &= 3x - 4 \end{aligned}$$

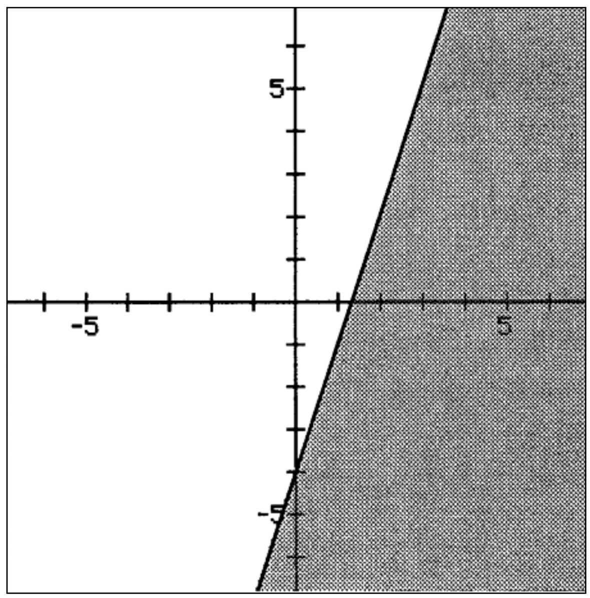
Enter the equation and graph. Copy the graph onto your own grid.



To determine which side of the line represents the inequality, select a point. In this example, we will once again use (0, 0).

$3x - y$
$= 3(0) - 0$
$= 0$

$0 < 4$ and so (0, 0) does **not** lie in the required region. Therefore, the points are on the opposite side of the line from the required region.



Since the inequality is $3x - y \geq 4$, the points on the line $3x - y = 4$ are also included and the line is solid.

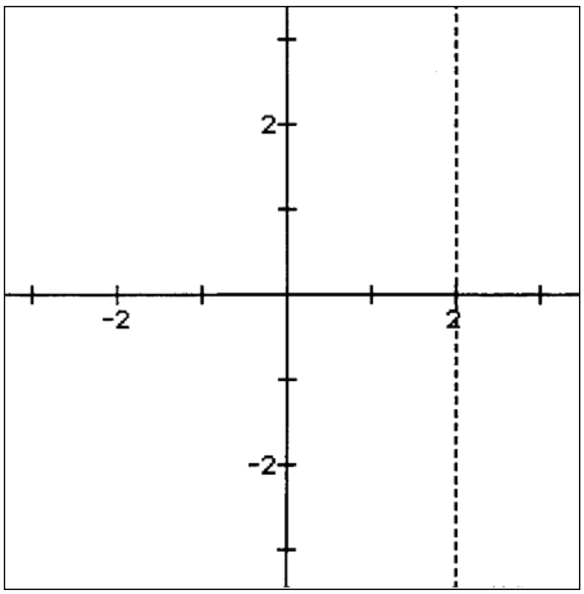
Example 3

$x > 2$

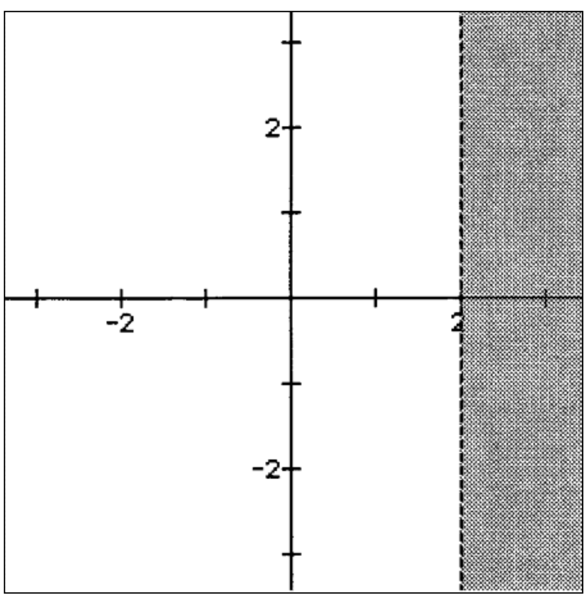
Solution

The related line is $x = 2$.

For $x > 2$, shade all the points to the right of the line.



The region shaded represents all the points for which $x > 2$.



Example 4

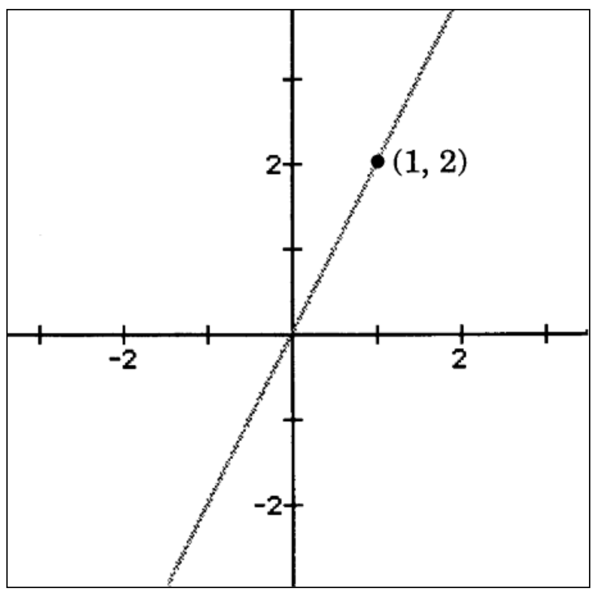
$2x - y > 0$

Solution

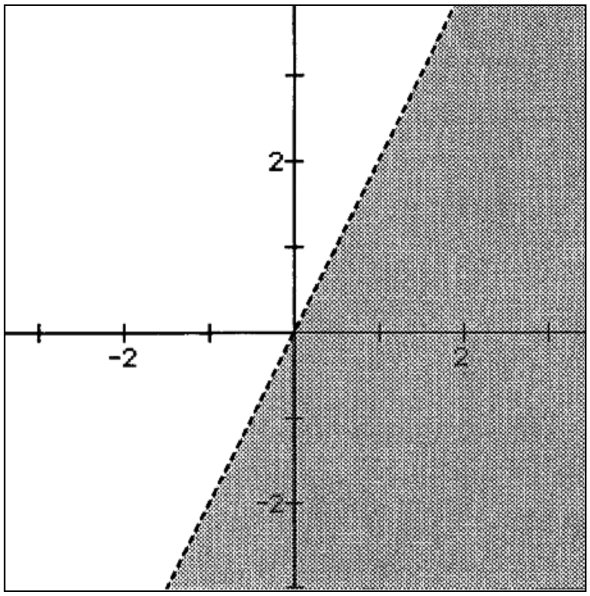
The related line is $2x - y = 0$ or $y = 2x$.

x	0	0
y	0	0

Choose another x-value.
$x = 1$ gives $y = 2$ and (1, 2). In this case (0, 0) lies on the line.
So choose another point. For example, (5, 0)
$2x - y = 2(5) - 0 = 10$
For this point $2x - y > 0$ and so (5, 0) lies in the required region.



The shaded region represents $2x - y > 0$. The line is dotted because the points of the line are **not** included.



Assignment

1. $5x - 2y \geq 4$
2. $2x + 7y < 6$
3. $5y \leq 3x + 1$
4. $x \geq 6y + 5$
5. $2x + 3y > 0$
6. $7x \leq y + 4$
7. $x - y > 6$
8. $4 \leq 2x + 3y$
9. $x < 4$
10. $y \geq -2$

Check your answers in the Module 2 Answer Key.

Lesson 2

Graphing Systems of Linear Inequalities

Outcomes

When you complete this lesson, you will be able to

- graph a system of two linear inequalities

Overview

In the last lesson, you graphed one linear inequality. In this lesson, you will graph two linear inequalities (which together form a system of inequalities) on the same grid. The area where the two shadings overlap gives the required region.

Example 1

$y - x - 3 \leq 0$

$2x + 3y > -6$

Solution

Related equations $y - x - 3 = 0$ $2x + 3y = -6$

Intercept method

x	0	–3
y	3	0

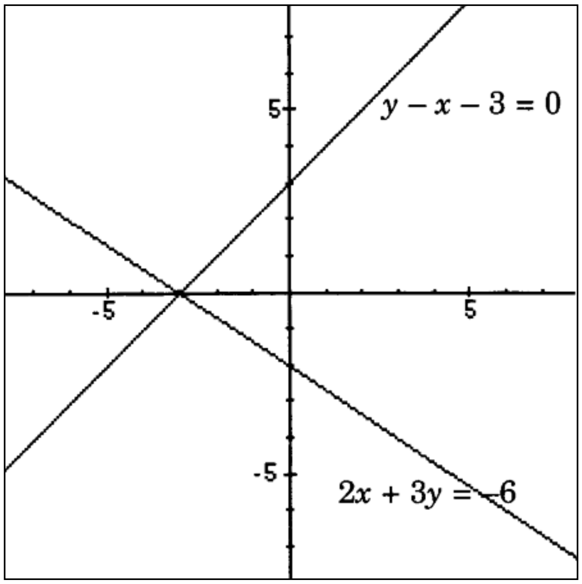
x	0	–3
y	–2	0

or for the graphing calculator.

$y = x + 3$ and $3y = -2x - 6$

$$y = -\frac{2}{3}x - 2$$

If you use the graphing calculator to find the lines, copy the graphs onto your grid. Be sure that you know which is which.



Determine which regions represent the inequalities as before.

Use (0, 0).

$$\begin{aligned} & y-x-3 \\ &= 0-0-3 \\ &= -3 \\ -3 &< 0 \end{aligned}$$

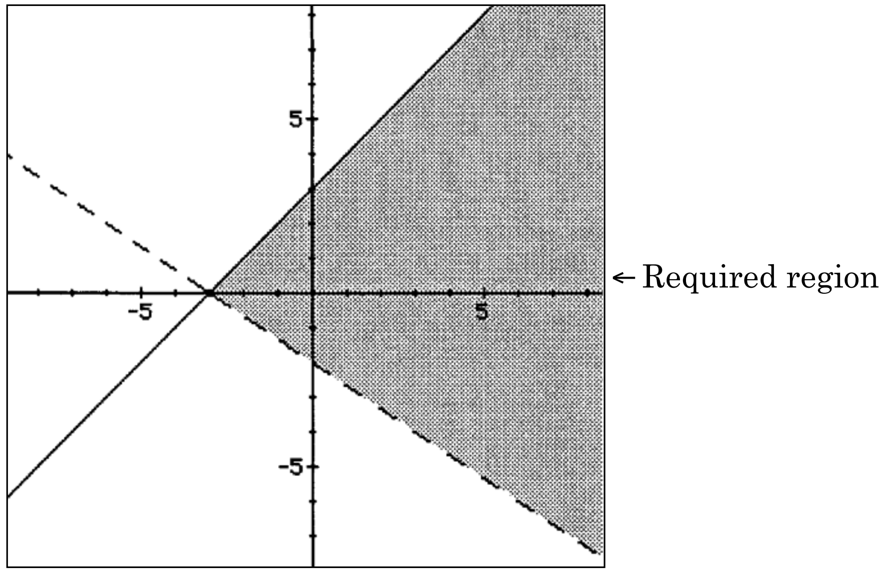
Therefore, (0, 0) lies in the required region. Shade (lightly) the region on the same side of the line.

Use (0, 0) again.

$$\begin{aligned} & 2x+3y \\ &= 2(0)+3(0) \\ &= 0 \\ 0 &> -6 \end{aligned}$$

Therefore, (0, 0) lies in the required region.

Shade the region on the same side of the second line.



The area where the shadings overlap represents the system of inequalities. The line $y - x - 3 = 0$ is solid because the points satisfy $y - x - 3 \leq 0$. The line $2x + 3y = -6$ is dotted because the points are not in $2x + 3y > -6$.

Example 2

$3x - 4y < -12$

$3x + 4y > 0$

Solution

Related lines $3x - 4y = -12$ $3x + 4y = 0$

Intercept Method

x	0	–4
y	3	0

x	0	0
y	0	0

Note that x- and y-intercepts will not work for the second line because you get only one point to plot. Solve the equation for y and find another point.

$4y = -3x$ Subtract $3x$ from both sides.

$y = \dfrac{-3x}{4}$ Divide by 4.

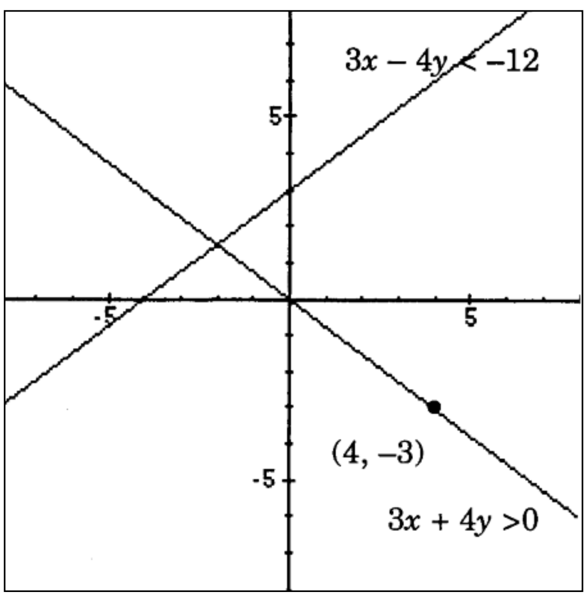
Let $x = 4$, then $y = -\dfrac{3(4)}{4} = -3$.

(4, –3) is a suitable point.

This will create a graph or use the graphing calculator. The first line becomes

$$\begin{aligned} 3x - 4y &= -12 \\ -4y &= -3x = 12 \\ y &= \frac{-3}{-4}x - \frac{12}{-4} \\ y &= \frac{3}{4}x + 3 \end{aligned}$$

Enter both equations in the graphing calculator and graph. Copy the graph onto your grid.



Determine which sides of the lines to shade. Choose points not on the lines.

Use (0, 0) to test $3x - 4y < -12$.

$3x - 4y$
$= 3(0) - 4(0)$
$= 0$
$0 < -12$

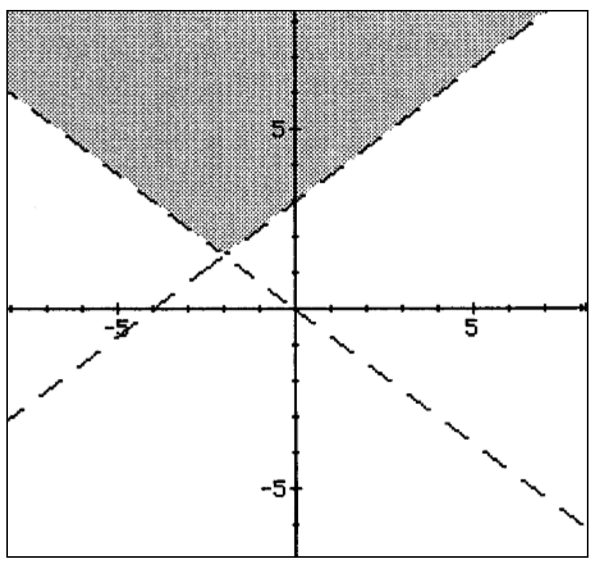
Not true.
Therefore, shade area on other side of line.

Use (1, 1) to test $3x + 4y > 0$

$3x + 4y$
$= 3(1) + 4(1)$
$= 3 + 4$
$= 7$
$7 > 0$

Yes.
Shade area on that side of the line.

Note: (0, 0) can't be used so try something else. Try (1, 1).

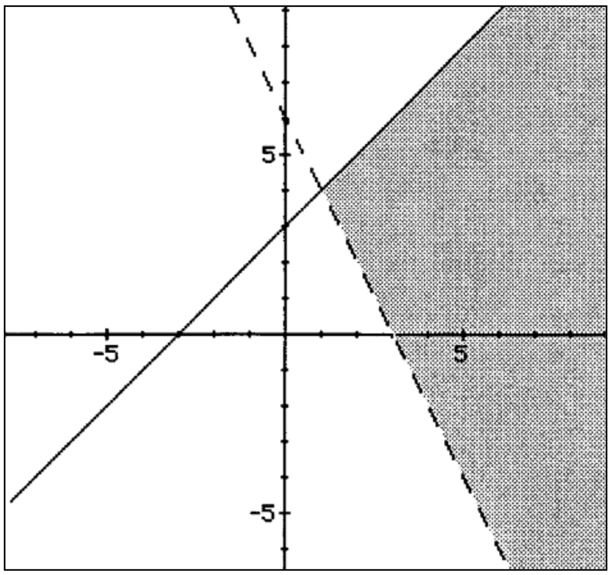


Area where shading overlaps (the darkest region) is the required area.

Note: Both lines are dotted.

Example 3

Find the inequalities that lead to the following overlapping regions.



Line 1: Points (–3, 0) and (0, 3) are on this line.

Slope = $\dfrac{\text{rise}}{\text{run}} = \dfrac{0-3}{-3-0} = 1$, y-intercept = 3

Equation is $y = 1x + 3$

Write equation in the following form: $Ax + By = C$.

$-x + y = 3$

Point (6, 1) lies in region.

For (6, 1), $-x + y = -6 + 1 = -5$

$-5 < 3$

Therefore, inequality is $-x + y < 3$. Line is dotted. Therefore, line is not included. Therefore, required inequality is $-x + y < 3$.

Line 2: Points (0, 6) and (3, 0)

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{6-0}{0-3} = -2,\ y\text{-intercept} = 6$$

Equation is $y = -2x + 6$

Rearrange to $2x + y = 6$

Point (6, 1) lies in region.

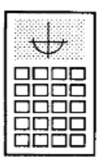
For (6, 1), $2x + y = 2(6) + 1 = 13$

$13 > 6$

Therefore, inequality is $2x + y > 6$. However, the line is solid for this one. Therefore, the required inequality is $2x + y \geq 6$.

Therefore, the system required is

$$\begin{aligned} 2x + y &\geq 6 \\ -x + y &< 3 \end{aligned}$$



Assignment

Graph the following systems of inequalities and shade so that the required area is shown where shading overlaps.

1. $2x - y > 2$
   $3x + y > 1$

2. $\dfrac{2x + y}{3} \leq 1$
   $2x + y \geq 0$

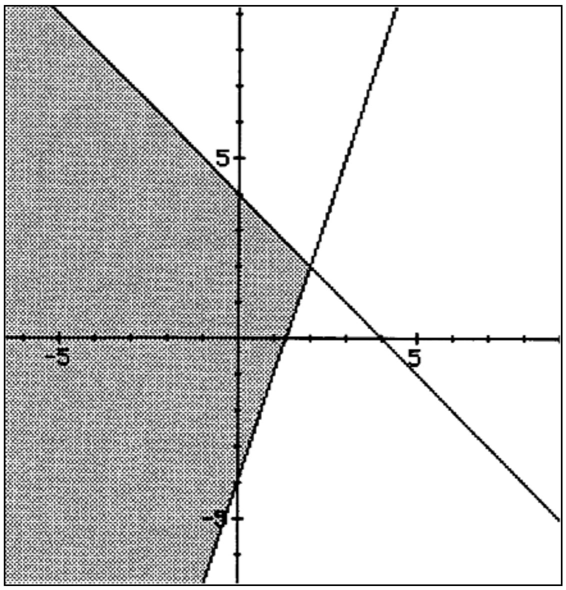
3. $4x - 3y \leq 10$
   $2x + 5y > 6$

4. $7x + 3y < 27$
   $4x - 5y > 8$

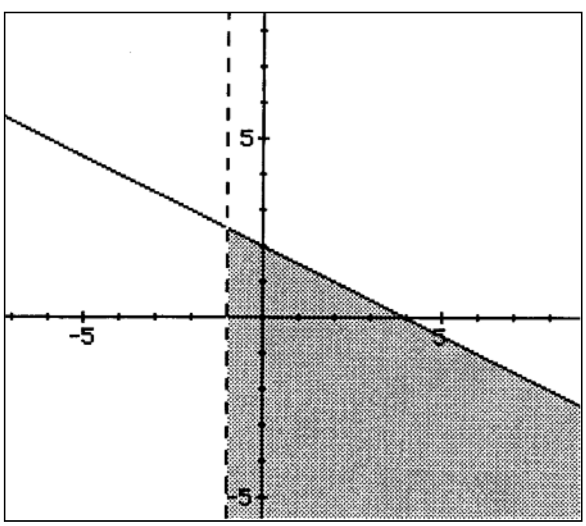
5. $6x - 5y < 30$
   $2x + 3y \geq 1$

6. $x - 2y \leq 7$
   $8x + 3y \geq 6$

7. Find a system of inequalities representing the overlapping regions shown in graph.



8. Find a system of inequalities representing the overlapping regions shown in graph.



Check your answers in the Module 2 Answer Key.

Lesson 3

Determining the Corner Points of a Feasible Region

Outcomes

When you complete this lesson, you will be able to

- determine a feasible region and its corner points for a system of linear inequalities

Overview

The solution set of a system of inequalities is called the **feasible region**. Only the feasible region contains the points that satisfy all the inequalities in the system. This region is created by a geometric figure made of lines. The points where the lines meet are called the **corner points** or vertices of the feasible region. The inequalities represent the constraints on the situation.

If a maximum or minimum value of a linear expression, $\mathrm{A}x + \mathrm{B}y - \mathrm{C}$, exists within the constraints of the system of inequalities, **it will occur at one of the corner points**.

Consider the following examples.

Example 1

Find the feasible region and identify corner points for the system of constraints.

$x \geq 2y + 4$

$x + y \geq -2$

$4x + y \leq 4$

Solution

Steps:

1. Draw the line related to each inequality. Use either the intercept method or a graphing calculator.

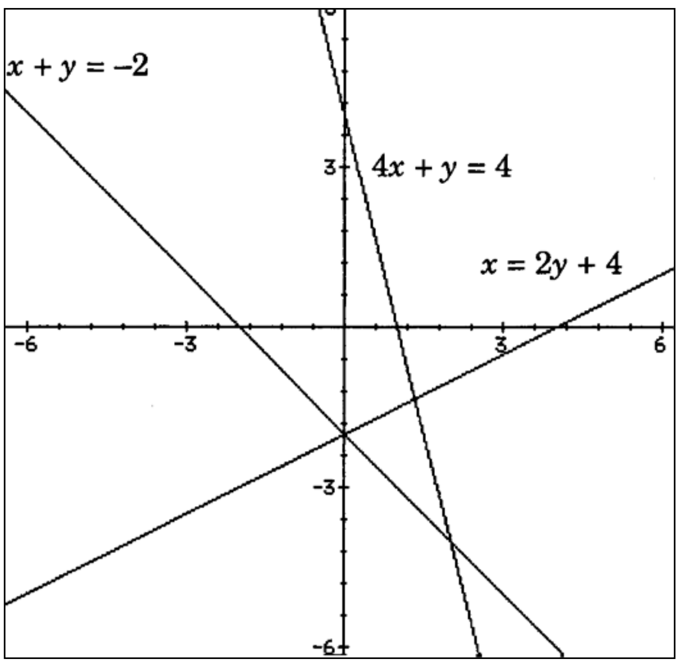
Inequalities	$x \geq 2y + 4$	$x + y \geq -2$	$4x + y \leq 4$
Related lines	$x = 2y + 4$	$x + y = -2$	$4x + y = 4$

Intercept method

x	0	4
y	-2	0

x	0	-2
y	-2	0

x	0	1
y	4	0



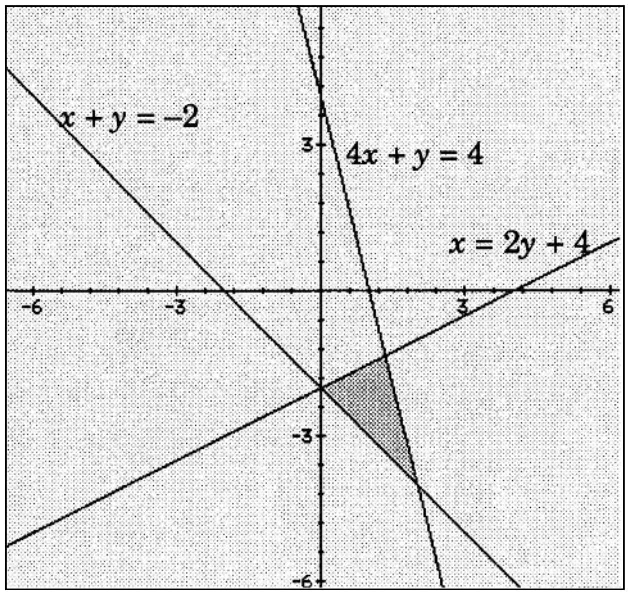
2. Choose a point, e.g., (0, 0), to check to find the region that represents each inequality.

For $x \geq 2y + 4$
$x = 0$
$2y + 4$
$= 2(0) + 4$
$= 4$
$0 < 4$
(0, 0) is not in the required region.

For $x + y \geq -2$
$x + y = 0 + 0 = 0$
$0 \geq -2$
Therefore, (0, 0) is in the required region. Shade that region.

For $4x + y \leq 4$
$4x + y = 4(0) + 0 = 0$
$0 \leq 4$
Therefore, (0, 0) is in the required region. Shade that region.

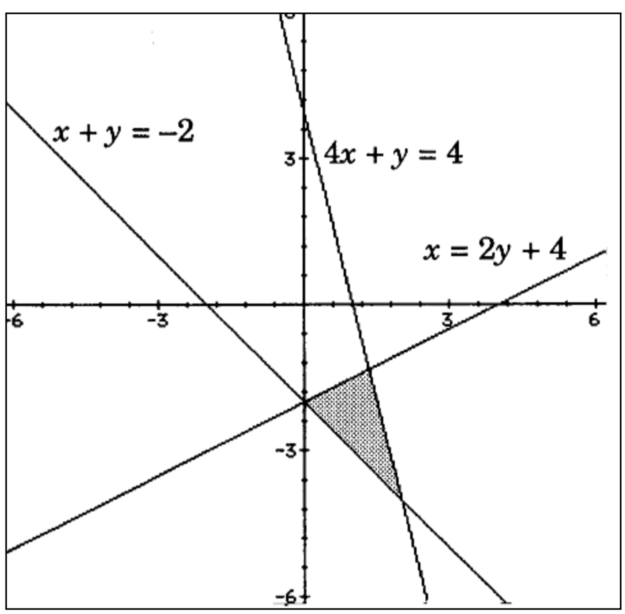
3. Note: Shade lightly because you wish to identify the area where all three inequalities overlap.



The area required is the small triangle where they all overlap. This is the **feasible** region. Note that all lines are solid.

You must find the points of the region that are the corner points.

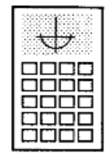
Now show the graph with **only** the feasible region shaded.



Sometimes a corner point is easy to see. For example, the lines $x + y = -2$ and $x = 2y + 4$ cross at (0, –2). You can see this from the intercepts or from your graphing calculator. Other points are not so obvious.

To find the other two corner points, solve the systems of equations representing the lines.

$4x + y = 4$
$x = 2y + 4$



Recall methods from Module 2. You can use the graphing calculator method.

$4x + y = 4$ is arranged to give

$$y = -4x + 4$$

$x = 2y + 4$ is rearranged to give

$$x - 4 = 2y$$

$$\frac{1}{2}x - 2 = y, \text{ or}$$

$$y = \frac{1}{2}x - 2$$

Using the T1-83 graphing calculator

1. Turn on the calculator. Set window to “default”.

   Press [ZOOM] [6].
2. Press [Y=]. Clear any other functions.
3. Enter $Y_1 = -4x + 4$ and
   $Y_2 = 0.5x - 2$

   Make sure that the “=” signs are highlighted, otherwise the lines will not appear on the graph.
4. Graph lines.
5. Find the point of intersection. (Recall from Module 2 or from your calculator manual.) Press [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER].

   This gives $x = 1.333$, $y = -1.333$.

   Note: If you use a non-calculator method, you will get $\left(\frac{4}{3}, \frac{-4}{3}\right)$.

You now have (0, –2) $\left(\frac{4}{3}, \frac{-4}{3}\right)$ as two of the corner points.

To find the third “corner point”, repeat the method for finding the point of intersection of $x + y = -2$ and $4x + y = 4$.

This time enter $Y_1 = -4x + 4$ and
$Y_2 = -x - 2$.

Graph the lines.

Use [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER] to find the third corner point.

This is (2, –4). The three corner points are (0, –2), (1.333, –1.333), and (2, –4).

Note: You may prefer to use the algebraic method to find the corner points.

Assignment.

1. $x \geq 0$
   $y \geq 0$
   $3x + 4y \leq 12$

2. $x \geq -2$
   $y \leq 6$
   $x \leq 3$
   $x - 2y \leq -2$

3. $x \geq 0$
   $y \geq 0$
   $2x + y \leq 6$
   $x + 2y \leq 6$

4. $y \leq x + 2$
   $y \geq x - 2$
   $-2 \leq y$
   $y \leq 3$

5. $x \geq 0$
   $y \geq 0$
   $x + y \geq 6$
   $2x + y \geq 8$
   $2x + 3y \geq 14$

Check your answers in the Module 2 Answer Key.

Notes

Lesson 4

Finding Maximum and Minimum Values from Corner Points

Outcomes

When you complete this lesson, you will be able to

- find maximum and minimum values for a linear expression from a feasible region that is defined by a system of inequalities

Overview

As stated in Lesson 3, it can be proven that a maximum or minimum value for a linear expression can be found from the corner points of a feasible region (defined by a system of inequalities). The method involves finding the corner points and substituting each point into the linear expression to find the maximum value or both values. The linear expression is of the form: $M = Ax + By$.

Consider the following example.

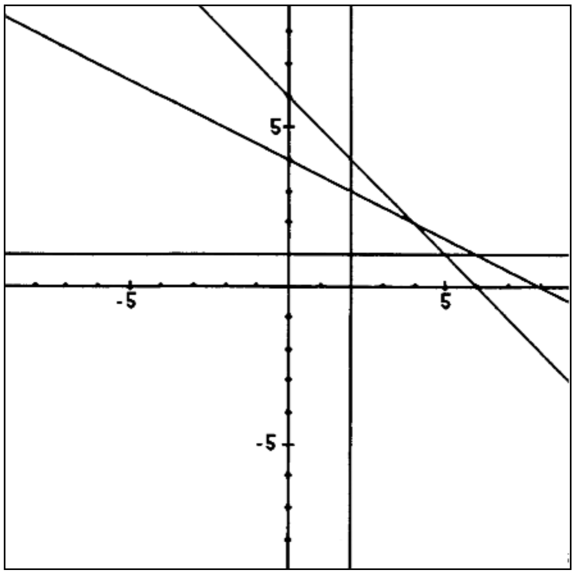
Example 1

Find x and y to maximize the linear expression $M = x + 3y$ subject to the constraints $x \geq 2$, $y \geq 1$, $x + 2y \leq 8$, and $x + y \leq 6$.

Solution

Steps:

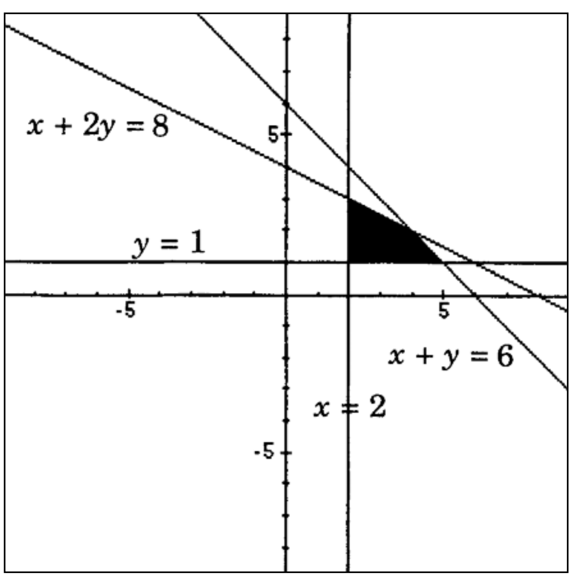
1. Graph the four lines, $x = 2$, $y = 1$, $x + 2y = 8$, and $x + y = 6$.



2. Find the regions representing $x \geq 2$, $y \geq 1$, $x + 2y \leq 8$, and $x + y \leq 6$.

   You can use the point (0, 0) to check each inequality in turn and shade **only** the area where the four inequalities are **all** shaded.

3. Indicate clearly the feasible region.



4. Find the corner points
   a) $y = 1$ and $x = 2$ clearly cross at (2, 1).
   b) $y = 1$ and $x + y = 6$ cross where $y = 1$.
      So, $x + 1 = 6$, therefore, $x = 5$.
      The point is (5, 1).
   c) $x = 2$ and $x + 2y = 8$ cross where $x = 2$.
      So $2 + 2y = 8$, therefore, $2y = 6$, $y = 3$.
      The point is (2, 3).
   d) Use the graphing calculator to find where $x + y = 6$ and $x + 2y = 8$.

      Clear functions. Check window. Rearrange equations:

      $y = -x + 6$ and $y = -\frac{1}{2}x + 4$

      Keying sequence to find intersection point:
      [Y=] [(–)] [X,T,θ,n] [+] [6] [ENTER] [(–)] [.] [5] [X,T,θ,n] [+] [4] [GRAPH] [2nd] [CALC] [ENTER] [ENTER] [ENTER]

      The point is (4, 2).

      Note: You can also find this point by using an algebraic method as in Module 1.

5. Find which of the four points generates the greatest value of $M = x + 3y$.

   Calculate M for the point (2, 1): $M = 2 + 3(1) = 5$
   Calculate M for the point (5, 1): $M = 5 + 3(1) = 8$
   Calculate M for the point (2, 3): $M = 2 + 3(3) = 11$
   Calculate M for the point (4, 2): $M = 4 + 3(2) = 10$

   The greatest value of M occurs when $x = 2$ and $y = 3$. We say that $M = x + 3y$ is **maximized** at (2, 3).

Assignment

1. a) Determine the corner points of the feasible region defined by the following constraints.

   $x \geq 0$
   $y \geq 0$
   $3x + 2y \geq 6$
   $2x + 3y \geq 6$

   b) Determine the coordinates of the corner points of the feasible region that minimizes the expression $4x + y$.

2. a) Determine the corner points of the feasible region defined by the following constraints.

   $x + y \leq 4$
   $x + 4y \geq 7$
   $-x + 2y \leq 5$

   b) Determine the corner points of the feasible region that minimize and maximize the expression $3x + 2y$.

3. a) Determine the corner points of the feasible region defined by the following constraints.

   $x + y \leq 0$
   $-x + 3y + 6 \geq 0$
   $x + 2y - 3 \leq 0$
   $2x + y + 9 \geq 0$

   b) Determine the corner points of the feasible region that minimize and maximize the expression $3x + 2y - 1$.

4. The constraints for manufacturing two types of hockey skates are given by the following region.

   $x \geq 0$
   $y \geq 0$
   $x + 4y \leq 41$
   $y \geq 2x - 10$
   $7x + 64 \geq 11y$

   Find the maximum value of Q over the region if $Q = 3x + 5y$.

5. The constraints for manufacturing basketballs and soccer balls are given by the following region.

   $x \geq 0$
   $y \geq 0$
   $2x + y \leq 260$
   $x + 5y \leq 400$

   Find the maximum value for expenses E over the region if $E = 5x + 4y$.

Check your answers in the Module 2 Answer Key.

Notes

Module 2 Project

Linear Programming

Make a complete and accurate graph of the following system of inequalities.

$3x + y \geq 4$

$y \geq x^2$

$x^2 + y \leq 8$

Note: (State all corner points of the feasible region.) Shade the feasible region. Describe the steps in your procedure.

Check your answers in the Module 2 Answer Key.

Notes

Lesson 5

Application of Linear Programming

Outcomes

When you complete this lesson, you will be able to

- apply linear programming to constraint situations

Overview

In this module you have learned how to find minimum or maximum values for a given linear expression subject to the constraints defined by given linear inequalities. In this lesson you will have to determine the linear expression and the constraints. You will be given a business or other real-life problem to set up appropriate linear inequalities from which to find a feasible region. You will use the vertices of the region to maximize or minimize a given expression.

The procedure is as follows:

1. Read the problem carefully.
2. Define the linear expression to be maximized or minimized (determine what x and y represent).
3. Define the constraints represented by linear inequalities.
4. Draw the graph to determine the feasible region and its corner points.
5. Find the maximum or minimum value.
   a) Substitute each corner point into the linear expression to be maximized or minimized.
   b) Find the corner point that gives you the maximum or minimum value.
   c) Explain your solution.

Example 1

A manufacturer makes air and oil filters for automobiles. Each filter requires the use of three machines: A, B, and C.

- Each air filter requires four minutes on Machine A, four minutes on Machine B, and five minutes on Machine C.
- Each oil filter requires five minutes on Machine A, one minute on Machine B, and six minutes on Machine C.

The manufacturer makes a **profit** of $8.00 on each air filter and $5.00 on each oil filter. However, the number of oil filters produced must not be less than half the number of air filters produced.

Furthermore, the manufacturer has available only 120 minutes on Machine A, 72 minutes on Machine B, and 180 minutes on Machine C for the production of air and oil filters.

How many of each kind of filter should the manufacturer produce to maximize profits?

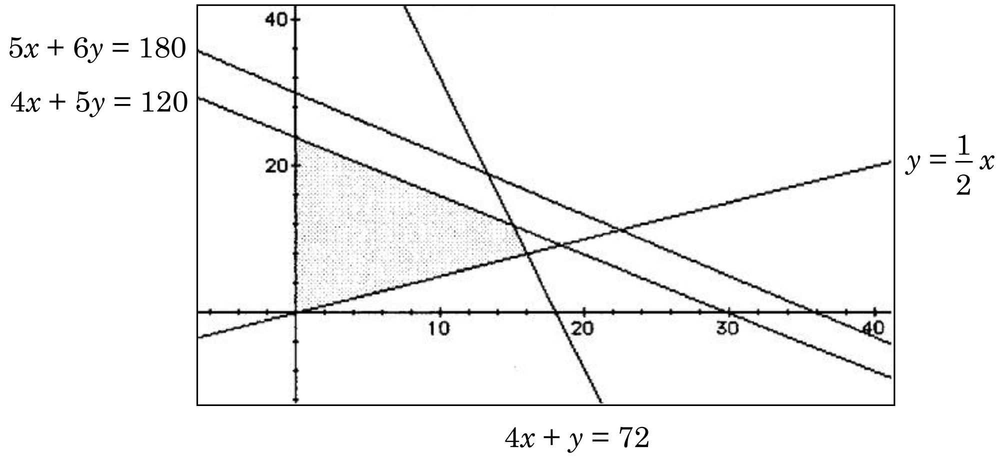
Solution

Steps:

1. Read the problem carefully.
2. Find the linear expression you want to maximize that defines the total profit to be made.
   Let x = number of air filters
   Let y = number of oil filters
   \$P = total profit on air and oil filters

   Therefore, total profit P = $8x + 5y$. This represents the linear expression you want to maximize.
3. Define the constraints:

$4x + 5y \leq 120$	Total time on Machine A must not exceed 120 mins.
$4x + y \leq 72$	Total time on Machine B must not exceed 72 mins.
$5x + 6y \leq 180$	Total time on Machine C must not exceed 180 mins.
$y \geq \frac{1x}{2}$	Number of oil filters must not be less than one-half the number of air filters.
$x \geq 0$ $y \geq 0$	A negative number of filters (oil or air) cannot be produced.



Note: Feasible region is shaded.

4. Set the window: x from –10 to + 40
   y from –10 to + 40

   Use the linear programming techniques in Lesson 3 of this module to determine the feasible region.

   Note: Only the **feasible** region is shaded.

   The corner points are joined as follows:

   a) $x = 0$, $y = 0$ and $y = \frac{1}{2}x$ all cross at (0, 0).

   b) $4x + 5y = 120$ crosses $4x + y = 72$

   $$y = -\frac{4}{5}x + 24 \qquad y = -4x + 72$$

   Keying sequence:
   [Y=] [(–)] [4] [÷] [5] [X,T,θ,n] [+] [24] [ENTER] [(–)] [4] [X,T,θ,n] [+] [72] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER].

   The point is (15, 12).

   c) $y = \frac{1}{2}x$ crosses $4x + y = 72$. As before, $y = -4x + 72$.

   Keying sequence:
   [Y=] [(–)] [4] [X,T,θ,n] [+] [72] [ENTER] [X,T,θ,n] [÷] [2] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER].

   The point is (16, 8).

d. $4x + 5y = 120$ crosses $x = 0$ where $5y = 120$
The point is (0, 24). $y = 24$

Note: The line $5x + 6y = 180$ seems to make no difference to the situation.

5. Find the maximum or minimum value.
   a) Substitute the coordinates of each corner point into the linear expression to be maximized or minimized.
   b) Find the corner point that gives you the maximum or minimum value.
   c) Explain your solution.

Corner points: (0, 24), (0, 0), (15, 12), (16, 8)

Substitute each point into $P = 8x + 5y$ to find a maximum value.

(0, 24) gives

$P = 8(0) + 5(24)$
$P = 120$

(0, 0) gives

$P = 8(0) + 5(0)$
$P = 0$

(15, 12) gives

$P = 8(15) + 5(12)$
$P = 120 + 60$
$P = 180$

(16, 8) gives

$P = 8(16) + 5(8)$
$P = 128 + 40$
$P = 168$

Therefore, (15, 12) gives maximum value of P. Therefore, maximum profit is $180.00 by making 15 air filters and 12 oil filters daily.

Example 2

To produce a top quality yield of carrots, a vegetable farmer must spray his crop with at least 5.3 L of Nutrient A and 3.2 L of Nutrient B per field. Two suppliers provide the nutrients in a special blend, but the amounts of Nutrients A and B in each litre they provide vary by the following percentages:

Supplier	Amount of A	Amount of B
Top Grade	30%	70%
Quality	80%	20%

If the price of Top Grade Brand is \$3.20/L and for Quality Brand is \$3.80/L, then

1. determine how many litres of each brand must be bought from each supplier to provide the required amounts of each nutrient per field at a minimum cost
2. determine what the minimum cost will be for the nutrients bought.

Solution

1. Let x = number of litres of Top Grade fertilizer
   Let y = number of litres of Quality fertilizer
   Let \$C = total cost of fertilizer
2. The total cost of fertilizer will be $C = 3.20x + 3.80y$ and this will be the linear expression that you will want to minimize.
3. Define the constraints:

$x \geq 0$ $y \geq 0$	A negative amount of fertilizer cannot be used.
$0.30x + 0.80y \geq 5.3$	Total amount of Nutrient A must be at least 5.3 L.
$0.70x + 0.20y \geq 3.2$	Total amount of Nutrient B must be at least 3.2 L.

4. Use the graphing calculator and the linear programming method as in the previous example to determine the feasible region, the corner points, and the value of C at each corner point.

   The related lines are $x = 0$, $y = 0$.

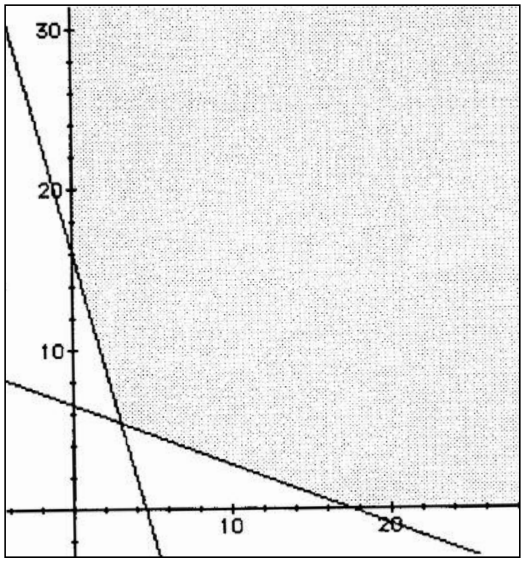
   $0.30x + 0.80y = 5.3$ rearranged as $y = -\dfrac{0.30}{0.80}x + \dfrac{5.3}{0.80}$

   $0.70x + 0.20y = 3.2$ rearranged as $y = -\dfrac{0.70}{0.20}x + \dfrac{3.2}{0.20}$

   Test to find which regions represent the inequalities. Find the feasible region.

In the diagram, only the feasible region is shaded.

$0.30x + 0.80y = 5.3$



$x = 0$ $0.70x + 0.20y = 3.2$ $y = 0$

5. Find the corner points:

   a) $y = 0$ crosses $0.30x + 0.80y = 5.3$
   where $0.30x + 0.80(0) = 5.3$

   $$x = \frac{5.3}{0.30} = \frac{53}{3}$$

   The point is $\left(\frac{53}{3}, 0\right)$.

   b) $x = 0$ crosses $0.70x + 0.20y = 3.2$
   where $0.70(0) + 0.20y = 3.2$

   $$y = \frac{3.2}{0.20} = 16$$

   The point is (0, 16).

   c) Use graphing calculator.

   Keying sequence:
   [Y=] [(–)] [0.30] [÷] [0.80] [X,T,θ,n] [+] [5.3] [÷] [0.80] [ENTER]
   [(–)] [0.70] [÷] [0.20] [X,T,θ,n] [+] [3.2] [÷] [0.20] [GRAPH]
   [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER].

   The point is (3, 5.5).

Substitute the coordinates of the points into the expression which needs to be minimized, C = 3.20x + 3.80y.

(0, 16) gives

$$C = 3.2(0) + 3.8(16)$$
$$C = \$60.80$$

(3, 5.5) gives

$$C = 3.2(3) + 3.8(5.5)$$
$$C = \$30.50$$

$\left(\frac{53}{3}, 0\right)$ gives

$$C = 3.2\left(\frac{53}{3}\right) + 3.8(0)$$
$$C = \$56.53$$

Minimize cost by using three litres of Top Grade and 5.5 litres of Quality Fertilizer for a total cost of $30.50.

Assignment

1. Burlington Runners repairs sports shoes, particularly tennis and jogging shoes. Two operations are required on each shoe and the times required on each operation are shown.

	Strip	**Re-sew**
Tennis	16 min	12 min
Jogging	8 min	16 min

If two people repair these shoes, each working eight hours per day, and if the profits on tennis and jogging shoes are $6.00 and $10.00 respectively per pair, how many pairs of each type of shoe should be repaired daily to maximize the profits?

2. It is recommended that cattle have the nutrients Iron and Riboflavin in their diet. Each animal should get at least 1.9 g of iron and 1.2 g of Riboflavin. Two kinds of feed are available. The quantities of each nutrient are shown in the chart below.

Feed	**Iron**	**Riboflavin**
Husky	5%	2%
Vibrant	2%	3%

Husky Feed sells for $25/kg and Vibrant Feed sells for $32/kg.

a) How many kilograms of each feed are required to feed 100 cattle per day as economically as possible?

b) What is the cost of these quantities in part a?

3. A firm manufactures 2-bulb bedroom lamps and 4-bulb living room lamps. Each day they get a consignment of 480 bulbs and 180 shades. Determine how many of each lamp they should manufacture to yield a maximum profit if their profit on a 2-bulb lamp is $20 and on a 4-bulb lamp is $35. What is the maximum profit?

4. The Sundial Watch Company manufactures two types of watches: a self-winding and an automatic model. In any day there are three hours of machine time and seven hours of jeweller time available. The self-winding model requires 1.5 hours of machine time and one hour of jeweller time, while the automatic model requires 30 minutes of machine time and two hours of jeweller time. If the profit on the self-winding model is $25 and the profit on the automatic model is $18, how many of each model should be manufactured daily for a maximum profit? What is the maximum profit?

5. John and his son service electrical appliances such as toasters and kettles. John can service a toaster in 15 minutes and a kettle in 12 minutes, while his son can service the same appliances in 10 minutes and 20 minutes respectively. If John is paid $24 per hour and his son earns $8 per hour, how long should each work on an order for 50 toasters and 40 kettles so as to minimize the cost of labour?

6. Two workers, George and Janet, make rake and shovel handles. George can make six rake and four shovel handles per hour and Janet can make 10 rake and four shovel handles per hour. If George earns $6.50 per hour and Janet $8.00 per hour, how many hours should each person work to fill an order for 60 rake and 32 shovel handles at the minimum labour cost?

7. Steve and Sandra work for a sports firm that manufactures racquets. In one hour Steve can put grips on four tennis racquets and two squash racquets, while Sandra can put grips on two tennis racquets and three squash racquets. Each day they must put grips on a minimum of 24 tennis racquets and 20 squash racquets. Steve earns $6.00 per hour and Sandra earns $5.00 per hour.

   a) How long should each person spend putting grips on each type of racquet in order to minimize costs?

   b) How many of each type of racquet will have grips put on them at this minimal labour cost?

8. a) A farmer plants two crops, corn and soybeans. The expenses are $6 for each acre of corn planted and $12 for each acre of soybeans planted. Each acre of corn requires 12 bushels of storage, and each acre of soybeans requires 16 bushels of storage. The farmer has at most 3600 bushels of storage available and $2400 to spend on the expenses. Choose variables for the number of acres of corn and soybeans planted.

   b) Write four inequalities that express the conditions of the problem.

   c) Graph the solution set of the system of inequalities described in part a. State the coordinates of the corner points for this feasible region.

   d) Suppose that the farmer earns a profit of $24 for each acre of corn and $48 for each acre of soybeans. Find two ways the farmer can satisfy the conditions while maximizing the profits. (Notice that a linear programming problem can have more than one solution.)

Check your answers in the Module 2 Answer Key.

Notes

TEST TIME!

Self-Test

Module 2 – Linear Programming

/52

1. Using your graphing calculator, solve the following equations for "x". (Round answers to the nearest tenth where necessary.)

(2 marks) a) $3(5x - 7) = 2(4x + 3)$

(2 marks) b) $5x^2 - 7x = -2$

(4 marks) 2. Solve the following systems of linear equations by addition or subtraction.

$72x + 9y = -216$
$15x - 17y = -196$

Note:

When you complete this Self-Test, send it in for marking. Include the Module 2 Cover Sheet.

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Winkler, MB
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Include Cover Sheet

3. Solve the following systems of equations using your calculator.

   a) $-9x + 5y = 76$ *(2 marks)*
      $36x + 9y = -333$

   b) $3x + 4y = 25$ *(2 marks)*
      $y = \sqrt{25 - x^2}$

4. Solve the following word problems by setting up a system of equations and using your calculator where necessary to solve the resulting system.

   a) In the playoffs, the Leafs played 13 games before being eliminated. If they won three more games than they lost, how many games were won and lost? *(5 marks)*

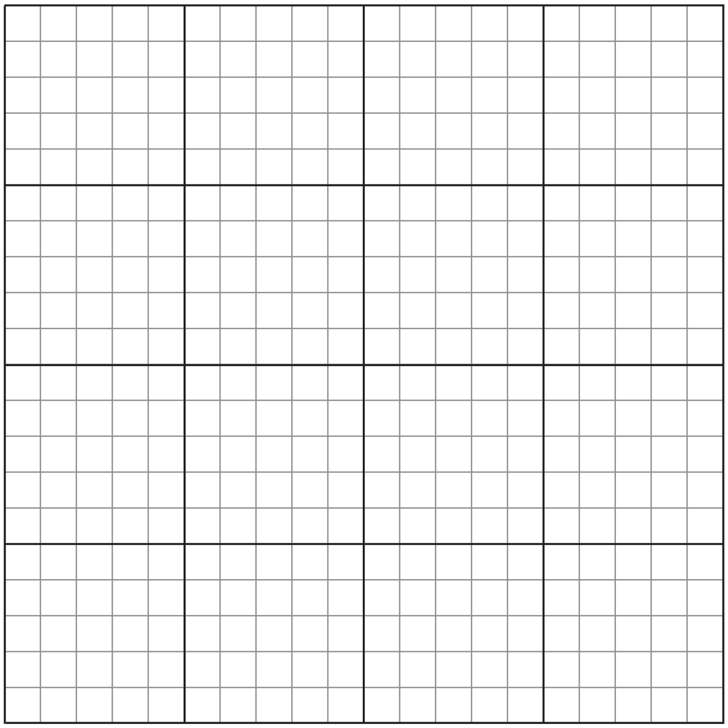
(5 marks)

b) The average of two numbers is 36. If the larger number is twice the smaller, find the two numbers.

5. Graph the following systems of inequalities, double shading your answer area.

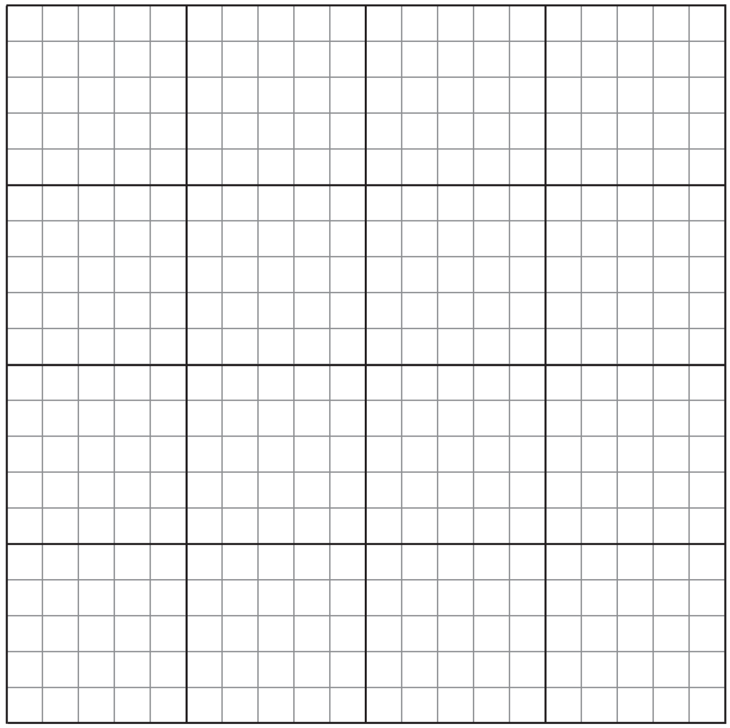
   (5 marks)

   a) $3x - y \leq 5$
      $4x + 3y > 6$



(5 marks)

b) $x + 5y > 7$
$2x - 3y \leq 4$

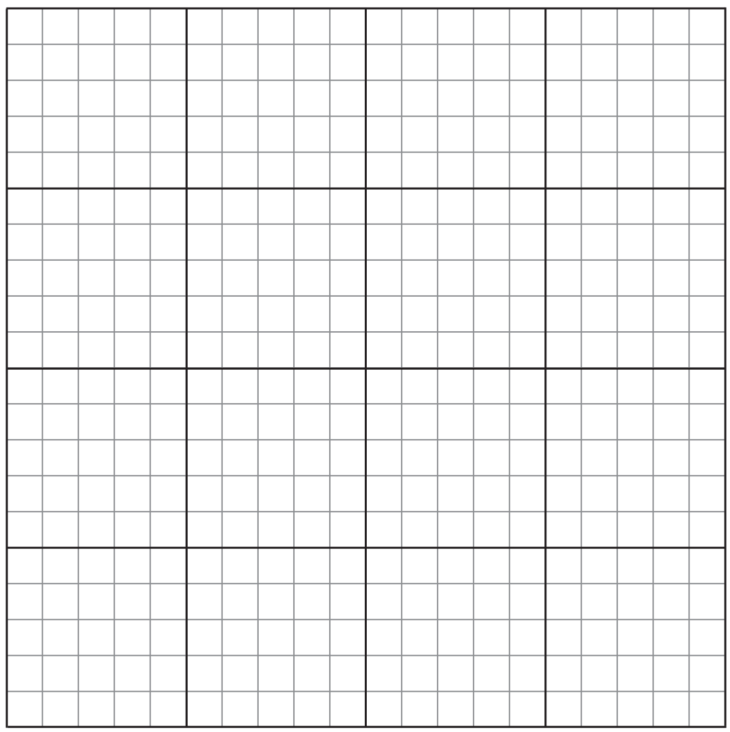


6. Solve the following maximum/minimum application(s).

   a) A manufacturer markets two products. Each unit of Product A requires three hours in the moulding department, four hours in the paint shop, and one hour in finishing. Each unit of Product B requires three hours in moulding, two hours in painting, and two hours in finishing. Each week there are 210 hours available in moulding, 200 hours in painting, and 120 hours in finishing. Shipping can handle no more than 40 units of Product A per week. Each unit of Product A contributes \$20 to profit, while each unit of Product B contributes \$30. Determine how many units of each product should be manufactured per week to maximize profit. *(10 marks)*

(10 marks)

b) Birgitta is advised by her doctor to take extra vitamin A and vitamin B. The minimum requirement of vitamin A is 11 mg and of vitamin B is 24 mg. Two brands of pills that contain vitamins A and B are recommended: Supavit at 5¢ per pill and Vitatoo at 8¢ per pill. Each Supavit pill contains 2 mg of vitamin A and 5 mg of vitamin B. Each Vitatoo pill contains 3 mg of vitamin A and 4 mg of vitamin B. How many pills of each brand should be taken each day to provide the minimum daily need at the least expensive price?



Note:

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Notes

Grade 11 Applied Mathematics (30S)

Module 3
Reasoning to Solve Problems

Module 3: Reasoning to Solve Problems

Introduction

Patterns are a common occurrence in the world around you, and you have seen and used numerical and geometric patterns in many contexts. For example, you used a numerical pattern when you learned to recognize whether a number is a multiple of three. You now know that a number is always a multiple of three if the sum of the digits of that number is a multiple of three. This numerical pattern is always true. You will discover and use similar patterns in this module.

You have already observed some geometric patterns in polygons. For example, you know that the sum of the interior angles in a triangle is always 180˚. Similarly, the sum of the interior angles in a quadrilateral is always 360˚. You know that this pattern continues indefinitely, even for polygons with larger numbers of sides.

Some patterns are only valid in certain circumstances, while other patterns are valid in general. Throughout this module, you will be using two types of reasoning in relation to patterns: inductive reasoning and deductive reasoning. You will use inductive reasoning to examine the changes that occur in each step of a pattern to identify a general rule that the pattern follows. For example, if you see the numbers 1, 2, 4, . . ., you might think that the pattern rule is to double each number and use inductive reasoning to conclude that the pattern will continue as 1, 2, 4, 8, 16 . . . . However, with inductive reasoning, the rule many not always be true. Alternatively, you will use deductive reasoning to complete a pattern when you have been given the general rule. Since the rule on which the pattern is based is known, patterns based on deductive reasoning will always be true.

Assignments in Module 3

When you have completed the assignments for Module 3, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Logic Puzzles
2	Assignment 3.1	Inductive and Deductive Reasoning
4	Assignment 3.2	Invalid Proofs and Spatial Reasoning

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, and can be either handwritten or typewritten. Both sides of the sheet may be filled. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as guides.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 3. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by recording the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1 to 4 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 4.

Resource Sheet for Module 3

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Module 3 Cover Assignment: Logic Puzzles

Logic Grid Puzzles

You have probably seen a logic grid puzzle before, such as the one below. If you are adept at logical reasoning, you probably had success with solving these types of puzzles. If not, here are some tips for solving logic grid puzzles.

- Go through the clues one by one.
- Once you have used a clue, cross it off.
- If you can't use the clue yet, come back to it later.
- Fill in the chart with a check mark ("✓") in the corresponding box that matches two pieces of information.
- Fill in the chart with an "X" in the corresponding box where two pieces of information are not related, or where information can be eliminated.
- If you have a check mark in one row, all of the other symbols in that row must be an "X."

Consider the following puzzle:

Sam, Sally, and Sandra have different favourite colours: purple, yellow, or orange. They also each have different favourite fruits: strawberries, oranges, or bananas. Match Sam, Sally, and Sandra with his or her favourite colour and his or her favourite fruit using the following clues.

	Purple	Yellow	Red	Strawberries	Oranges	Bananas
Sam						
Sally						
Sandra						
Strawberries						
Oranges						
Bananas						

1. Sam likes a fruit that is the same colour as his favourite colour.
2. The person who likes the colour purple enjoys eating strawberries.
3. Sally does not like the colour purple.

Step 1: Sam can like either strawberries because they are red and red is an available colour, or bananas because they are yellow and yellow is an available colour. Therefore, Sam cannot like the colour purple or oranges. Put an "X" in these boxes.

	Purple	Yellow	Red	Strawberries	Oranges	Bananas
Sam	X				X	
Sally						
Sandra						
Strawberries						
Oranges						
Bananas						

Step 2: The person who likes the colour purple enjoys eating strawberries. Therefore, Sam cannot like eating strawberries because he does not like the colour purple. Thus, Sam likes bananas. Also, since bananas are yellow, Sam likes the colour yellow.

	Purple	Yellow	Red	Strawberries	Oranges	Bananas
Sam	X	✓	X	X	X	✓
Sally						
Sandra						
Strawberries						
Oranges						
Bananas						

Since each person has a different favourite colour and fruit, no one else can select the colour yellow or the bananas, so these squares can be marked "X."

	Purple	Yellow	Red	Strawberries	Oranges	Bananas
Sam	X	✓	X	X	X	✓
Sally		X				X
Sandra		X				X
Strawberries						
Oranges						
Bananas						

Step 3: Sally does not like the colour purple. Therefore, she must like the colour red. This means no one else can like the colour red so Sandra must like the colour purple.

	Purple	Yellow	Red	Strawberries	Oranges	Bananas
Sam	X	✓	X	X	X	✓
Sally	X	X	✓			X
Sandra	✓	X	X			X
Strawberries						
Oranges						
Bananas						

Step 4: Go back to Clue 2. You know the person who likes the colour purple also likes strawberries. Therefore, Sandra must like strawberries and Sally must like oranges.

	Purple	Yellow	Red	Strawberries	Oranges	Bananas
Sam	X	✓	X	X	X	✓
Sally	X	X	✓	X	✓	X
Sandra	✓	X	X	✓	X	X
Strawberries	✓	X	X			
Oranges	X	X	✓			
Bananas	X	✓	X			

Therefore, Sam likes the colour yellow and bananas. Sally likes the colour red and oranges. Sandra likes the colour purple and strawberries.

Sudoku Puzzles

Sudoku puzzles are grid puzzles that you have probably seen before, either in a newspaper or online. Some grids are 6 × 6, some are 9 × 9, and sometimes very challenging Sudoku puzzles have even larger dimensions. To solve this 9 × 9 puzzle, the numbers 1 through 9 can only be used once in each vertical and horizontal line, and in each 3 × 3 "cage" outlined with the heavy lines.

1		4	9		3		2	5
	6	2		4		3		
3				1	2		6	
2		9			7	1	4	8
	5	6	1		4			
	1			3	9			7
		1		7		5		2
6	2				5	4		
		5		2		9		6

To begin to solve this Sudoku, look for lines or boxes that are almost complete. For example, the sixth column is almost complete but is missing the numbers 1, 6, and 8. You know the number 6 cannot go in the second row because there is already a number 6 in the second row. Similarly, the number 6 cannot go in the last row. Therefore, the number 6 must go in the seventh row.

Now, the number 1 can be placed. The number 1 cannot go in the dark cage in the top section because there is already a 1 in that cage. Therefore, the number 1 must go in the last row. This leaves only one place for the 8 in the second row. The sixth column is now complete.

1		4	9		3		2	5
	6	2		4	8	3		
3				1	2		6	
2		9			7	1	4	8
	5	6	1		4			
	1			3	9			7
		1		7	6	5		2
6	2				5	4		
		5		2	1	9		6

As well, if most of the numbers in one cage are provided, you can use similar reasoning to find the missing number to complete that cage. This process can be continued to complete the Sudoku as shown below.

1	8	4	9	6	3	7	2	5
5	6	2	7	4	8	3	1	9
3	9	7	5	1	2	8	6	4
2	3	9	6	5	7	1	4	8
7	5	6	1	8	4	2	9	3
4	1	8	2	3	9	6	5	7
9	4	1	3	7	6	5	8	2
6	2	3	8	9	5	4	7	1
8	7	5	4	2	1	9	3	6

Module 3 Cover Assignment

Logic Puzzles

Total: 10 marks

This is a hand-in assignment. Clearly show the steps in your solution on the question sheets below and submit these pages when you send in your assignments for marking.

1. Using the following clues, match these players (Jake, Kailen, Mark, and Brittny) with their favourite sport, the colour of their jersey, and the number of their jersey. *(4 marks)*

	Hockey	Soccer	Football	Baseball	Red	Blue	Green	Orange	23	56	7	81
Jake												
Kailen												
Mark												
Brittny												
23												
56												
7												
81												
Red												
Blue												
Green												
Orange												

- Jake, who wore a blue jersey, was not number 23 or 56.
- Kailen hates the sport of hockey but loves the number 7.
- The person who wore a green football jersey had the number 23 on the back of his or her jersey.
- Brittny's favourite sport is soccer.
- The person who played baseball wore a red jersey.

Module 3 Cover Assignment: Logic Puzzles (continued)

2. Explain one strategy you used to solve the Logic Grid puzzle in Question 1. *(1 mark)*

3. Complete the following Sudoku puzzle. *(4 marks)*

7		2	3		9			1
	8			2		5		9
4		9		8	1		7	6
8					2		5	
	9	4		7	5			2
5			6		8	9		4
	1		4				2	
9		3			7	1		8
		5	8	1			9	

4. Explain one strategy you used to solve the Sudoku puzzle in Question 3. *(1 mark)*

Lesson 1: Conjectures and Counter-examples

Lesson Focus

In this lesson, you will

- ❑ make conjectures by observing patterns and using inductive reasoning
- ❑ analyze conjectures using inductive reasoning
- ❑ explain why inductive reasoning may lead to a false conjecture
- ❑ find and explain a counter-example to disprove a conjecture

Lesson Introduction

You have probably heard someone state an all-encompassing fact, such as "everyone in Winnipeg is a Winnipeg Jets fan." This type of statement is called a *conjecture* as it is a statement that has not been proven to be true. Once you hear a statement like this, you may try to think of someone in Winnipeg who is not a Winnipeg Jets fan. If you can find such a person in Winnipeg who is not a Winnipeg Jets fan, this person would be a *counter-example* to the original statement.

In order to come up with a conjecture, you use a type of thinking based on observations called inductive reasoning. For example, if you look around and see that everyone is wearing Winnipeg Jets apparel, you could reason inductively to make the conjecture that everyone in Winnipeg is a Winnipeg Jets fan. You will be learning about inductive reasoning, conjectures, and counter-examples in this lesson.

Conjectures

A **conjecture** is a statement that has not yet been proven to be true. A conjecture is based upon evidence or patterns that have been gathered or observed and appear to be true. It is also sometimes called an educated guess. You may wish to add this definition to your resource sheet.

Example 1

Jacob's math teacher states that the product of two odd integers is always odd.

a) State the conjecture made in the above statement.

b) List three examples that demonstrate the conjecture may be true.

c) Is this example reasonable? Explain.

Solution

a) The conjecture is, "The product of two odd integers is always odd."

b) To find three examples, first choose three sets of odd numbers.

Let's choose the following sets of numbers:

- 7 and −11
- 23 and 39
- −15 and −45

The product of each set of numbers is:

- $(7)(-11) = -77$
- $(23)(39) = 897$
- $(-15)(-45) = 675$

In all three cases, −77, 897, and 675 are all odd.

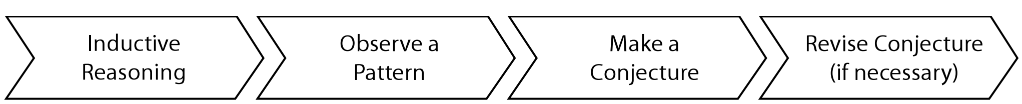
c) Yes, the example is reasonable because three sets of random odd numbers all produced odd products. The conjecture may be true since a counter-example has not yet been found.

Inductive Reasoning

To come up with a conjecture, **inductive reasoning** is often used. Inductive reasoning is defined as drawing a conclusion or making a conjecture after observing a pattern and identifying important properties about the pattern.

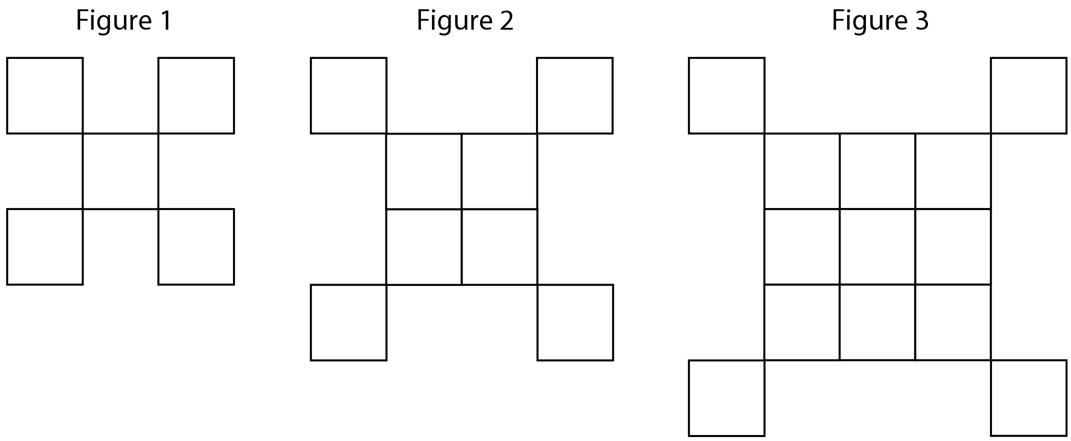
You may wish to add this definition to your resource sheet.

Another way to think about inductive reasoning is by considering the following flow chart.



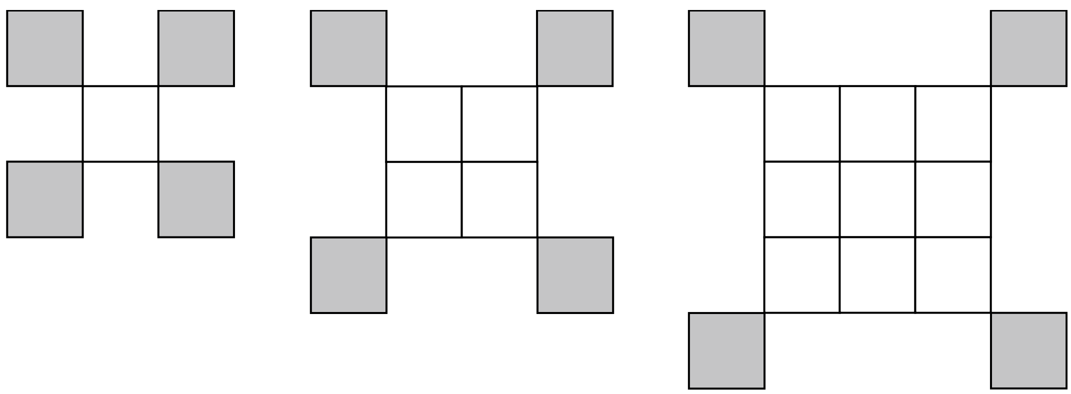
Example 2

Consider the following pattern. Use inductive reasoning to make a conjecture about how many squares will be in the tenth figure.



Solution

Each figure has four squares on the outside edges.



Inside each figure, there is a square. The squares have side lengths equal to the figure number.

For example, the number of squares inside each figure can be shown as:

Figure 1	Figure 2	Figure 3
$1^2 = 1$	$2^2 = 4$	$3^2 = 9$

Therefore, Figure 10 will have a 10×10 square on the inside, plus four squares at each corner. This gives us a total of 104 squares.

A conjecture you can make about Figure 10 will be that it will have 104 squares.

Example 3

Emily works at a used car dealership during evenings and weekends. Over the past month, she has helped six high school students purchase their first car. She has also helped three university students purchase vehicles.

Using inductive reasoning, state two conjectures you could possibly make with regard to the type of people who buy used cars.

Solution

Since Emily only sold vehicles to high school and university students, she could make a conjecture stating that only young people purchase used vehicles.

Additionally, Emily could make a conjecture stating that only people in school purchase used vehicles.

Emily could also reason that inexperienced drivers purchase used vehicles just in case they get into an accident and don't want to damage a brand new car.

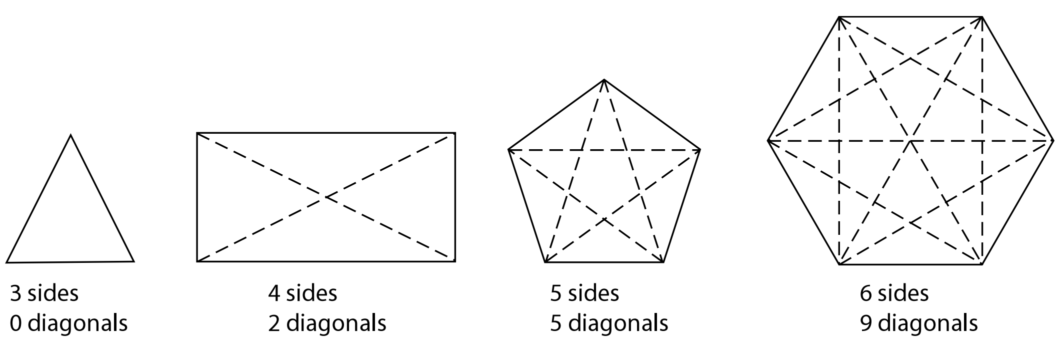
Thus, there are three possible conjectures:

- Only young people purchase used vehicles.
- Only people in school purchase used vehicles.
- Inexperienced drivers purchase used vehicles.

Conjectures can also be made about polygons and other geometric shapes. Consider the following example.

Example 4

Jade drew four polygons—a triangle, a rectangle, a pentagon, and a hexagon. Inside each polygon, she drew diagonals from each vertex.



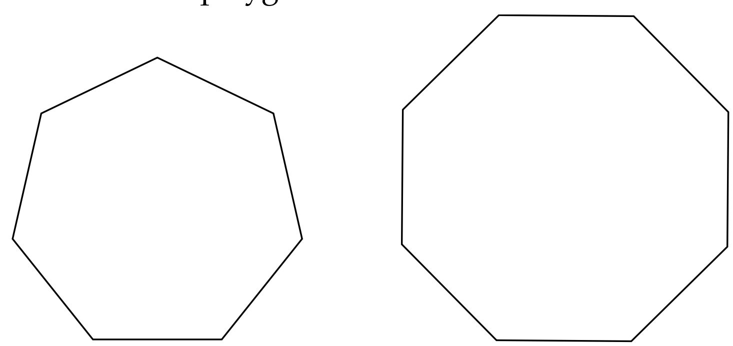
Jade made the conjecture that the number of diagonals that can be drawn in a polygon with n sides is $\frac{n(n-3)}{2}$.

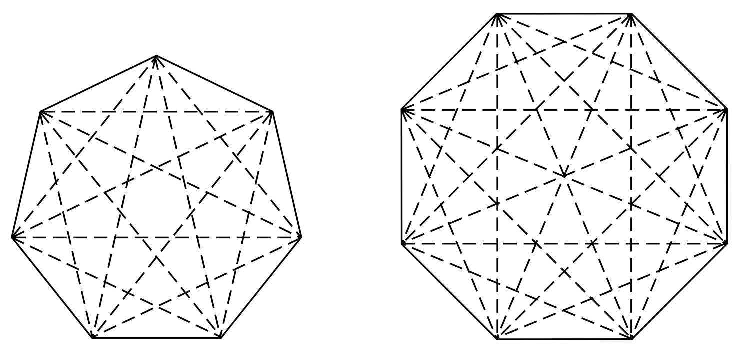
a) Continue with Jade's reasoning by drawing a heptagon and an octagon. Draw the diagonals for each shape.

b) Is this conjecture reasonable? Explain.

Solution

a) Try filling in the diagonals in the shapes below, counting each diagonal as you add it to the polygon.





7 sides
14 diagonals

8 sides
20 diagonals

b) To determine if the conjecture is reasonable, use Jade's formula to calculate the number of diagonals that should appear in shapes with 3, 4, 5, 6, 7, and 8 sides.

3 sides, $n = 3$

$$\text{Number of diagonals} = \frac{3(3-3)}{2} = \frac{3(0)}{2} = \frac{0}{2} = 0$$

4 sides, $n = 4$

$$\text{Number of diagonals} = \frac{4(4-3)}{2} = \frac{4(1)}{2} = \frac{4}{2} = 2$$

5 sides, $n = 5$

$$\text{Number of diagonals} = \frac{5(5-3)}{2} = \frac{5(2)}{2} = \frac{10}{2} = 5$$

6 sides, $n = 6$

$$\text{Number of diagonals} = \frac{6(6-3)}{2} = \frac{6(3)}{2} = \frac{18}{2} = 9$$

7 sides, $n = 7$

$$\text{Number of diagonals} = \frac{7(7-3)}{2} = \frac{7(4)}{2} = \frac{28}{2} = 14$$

8 sides, $n = 8$

$$\text{Number of diagonals} = \frac{8(8-3)}{2} = \frac{8(5)}{2} = \frac{40}{2} = 20$$

Jade's conjecture is reasonable, since each formula gives the number of diagonals drawn in each shape.

Example 5

Try this sequence of steps two times, each time starting with a different number. Make a conjecture about the result.

- Choose a number.
- Double the number.
- Subtract 3.
- Add the original number.
- Divide by 3.
- Add 1.

Solution

Choose two numbers, say 3 and -4.

When you choose 3:

- Double the number to get 6.
- Subtract 3 to get 3.
- Add the original number to get $3 + 3 = 6$.
- Divide by 3 to get 2.
- Add 1 to get 3.

When you choose −4:

- Double the number to get −8.
- Subtract 3 to get −11.
- Add the original number to get −11 + (−4) = −15.
- Divide by 3 to get −5.
- Add 1 to get −4.

In both cases, you end up with your original number.

A conjecture could be that the result of completing the above five mathematical steps, on any chosen number, is the number with which you started.

Counter-examples

Sometimes conjectures can be proven to be true using deductive reasoning. You will learn about deductive reasoning in Lesson 2. However, not all conjectures are true. Sometimes when you use inductive reasoning, you can come to a false conclusion. In this case, it is possible to find a counter-example. A counter-example is a situation or an example that proves the conjecture to be false. As soon as you find one counter-example to a conjecture, the entire conjecture is proven to be false.

Example 6

Eva made the following conjecture: Everyone over the age of 16 in Winnipeg has a driver's license. Is Eva's conjecture reasonable? Explain.

Solution

Eva's conjecture is not reasonable because you do not automatically get your driver's license at the age of 16 in Winnipeg. Some people choose not to drive or not to get their license for other reasons. Therefore, any person who is over the age of 16 and does not have a driver's license would be a counter-example to this conjecture.

Can you or your learning partner think of someone you know who is over the age of 16 and does not have a driver's license?

Example 7

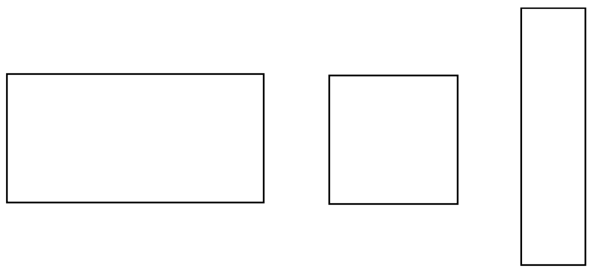
Is the following conjecture true? Explain.

Every rectangle is a square.

Solution

A rectangle has four sides with two sets of sides being parallel to each other. Also, all interior angles are 90°.

The following shapes meet those characteristics.



However, only the middle shape is a square. Therefore, not all rectangles are squares. The conjecture is false. The first and third rectangles are counter-examples to the conjecture that every rectangle is a square.

Example 8

Find a counter-example, if possible, for the following conjecture.

The square root of a number is always less than the number.

Solution

Try taking the square root of some values to see what happens.

$$\sqrt{100} = 10$$

$$\sqrt{25} = 5$$

$$\sqrt{32} = 5.65685\ldots$$

All of these values are less than the original number. However, what happens if you take the square root of a decimal between 0 and 1?

$$\sqrt{0.5} = 0.707106\ldots$$

$$\sqrt{0.02} = 0.141421\ldots$$

Both of these values are greater than the original number. Therefore, the conjecture is false.

You could also consider the following two examples:

$$\sqrt{1} = 1$$

$$\sqrt{0} = 0$$

In both of these cases, the square root value is equal to the original value. These two examples also prove the conjecture to be false.

Therefore, the conjecture is false. Some counter-examples to the conjecture that the square root of a number is always less than the number are the square roots of 0.5, 0.2, 0, and 1.

To practise determining conjectures and counter-examples, try the following learning activity.

Learning Activity 3.1

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

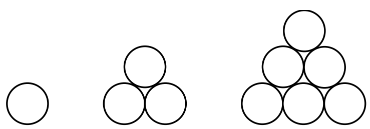
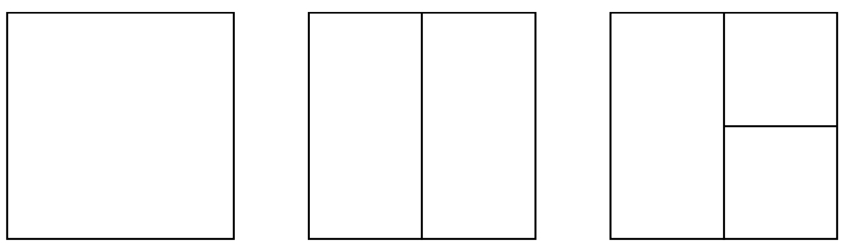
1. Determine the next two terms in this sequence: 2, 4, 8, . . .
2. Determine the next two terms in this sequence: 27, 9, 3, . . .
3. Solve: $2x - 7 = 13$
4. Solve: $1 - \frac{1}{3}x = 2$
5. Solve: $x^2 + 1 = 37$
6. Factor: $x^2 + 7x + 6$
7. How many metres are in 1.23 kilometres?
8. Find the roots of the equation $\frac{x}{2} + 3 = 0$.

continued

Learning Activity 3.1 (continued)

Part B: Conjectures and Counter-examples

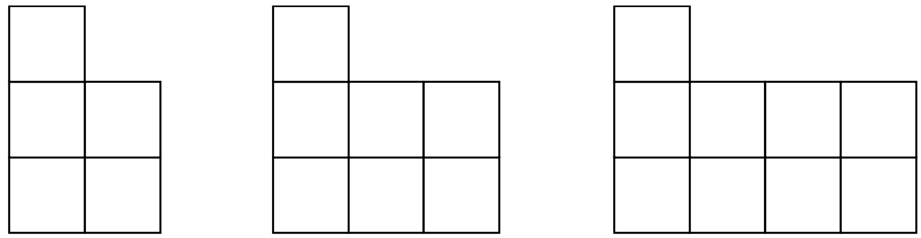
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Using inductive reasoning, determine the next three numbers or images in each pattern. State the rule you used to determine the next numbers or images.
   a) $\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \ldots$
   b) 13, 10, 12, 9, 11, 8, 10 . . .
   c) 144, 121, 100, 81, . . .
   d)
   
   e)
   
2. Shaundra states that the sum of a multiple of 2 and a multiple of 4 is always a multiple of 2.
   a) State the conjecture made in the above statement.
   b) List three examples that demonstrate the conjecture may be true.
   c) Is this conjecture reasonable? Explain.
3. Jessica walks into a clothing store and notices that there are many dresses and skirts on sale. State two conjectures Jessica could make about the sales of this clothing store.

continued

Learning Activity 3.1 (continued)

4. Consider the following pattern.



   a) Make a conjecture as to how many squares will be in the next two figures.
   b) Make a conjecture that describes how many squares are in each figure in relation to the figure number.
   c) Make a conjecture as to how many squares will be in the 30th figure.

5. It was been stated that a natural number (counting numbers greater than or equal to one) can be written as the sum of two or more consecutive numbers. For example:

   $12 = 3 + 4 + 5$

   $17 = 8 + 9$

   $11 = 5 + 6$

   a) Find two more examples that demonstrate this conjecture may be true.
   b) Is there a counter-example that disproves this conjecture? Explain.

6. Consider the following example: "The sum of two numbers is always greater than the larger of the two numbers." Determine whether this conjecture is reasonable. If possible, determine a counter-example to the conjecture.

7. Try this sequence of steps two times, starting with different numbers. Make a conjecture about the result.
   - Choose a number.
   - Triple the number.
   - Add 6.
   - Divide by 3.
   - Subtract 2.

8. In math class, Paula made the following conjecture: "As you travel south, the weather gets warmer." Is this conjecture true? Explain.

continued

9. Mike believes that the sum of two integers is always positive. Find a counter-example to prove him wrong.
10. Georgie is convinced that the sum of a multiple of 2 and a multiple of 4 is a multiple of 4. If possible, find a counter-example that would disprove her conjecture.
11. Before a hockey game, each of the five starting players from both teams shakes hands with the five players on the opposing team. How many handshakes occur?
12. Find two counter-examples to the conjecture: "All positive fractions are between 0 and 1."

Lesson Summary

In this lesson, you learned how to make a conjecture using inductive reasoning by observing patterns in numbers and shapes. You also learned how to find a counter-example to a conjecture to show that it is incorrect. In the next lesson, you will be learning about deductive reasoning. Deductive reasoning is a type of reasoning that is used to prove a conjecture based on accepted truths such as formulas and definitions.

Lesson 2: Deductive Reasoning

Lesson Focus

In this lesson, you will

- ❑ recognize the difference between inductive and deductive reasoning
- ❑ analyze and prove a conjecture, using deductive reasoning
- ❑ prove algebraic and number relationships using deductive reasoning

Lesson Introduction

Your football coach made a rule during the first practice that stated: “If lightning is seen from the field, the practice will be delayed or cancelled.” At the following practice, the coach observed flashes of lightning. What can you conclude about the practice?

If you answered that the practice will be delayed or cancelled, you used deductive reasoning. Deductive reasoning involves reasoning from a set of rules or known facts. You will learn how to use deductive reasoning throughout this lesson.

Deductive Reasoning

Deductive reasoning is different from inductive reasoning, which you learned about in Lesson 1. When you used inductive reasoning, you reasoned from your observations to develop a conjecture, which may or may not always be true. **Deductive reasoning** involves reasoning from accepted truths, facts, and definitions using logic and accepted mathematical strategies. When you use deductive reasoning, the conclusions that you come to are certain or always true (as long as you agree with the accepted facts and definitions). Because of this, you can use deductive reasoning to prove that conjectures are true and that you will never find a counter-example.

You may wish to add a description of deductive reasoning to your resource sheet.

Example 1

Use the definitions of deductive and inductive reasoning to determine whether each scenario is a representation of deductive or inductive reasoning.

a) John uses algebraic strategies to show that the solution to $2x + 3 = 7$ is $x = 2$.

b) Nikole observes that Marissa is late coming to class and that her hairdo is not up to her usual standards. Nikole knows that Marissa is very particular about her hair and concludes that Marissa must have woken up late.

c) Kyra, a student in Mr. Kooning's math class, has observed that Mr. Kooning gave a quiz the last three Fridays in a row. She concludes that there will be a quiz this Friday as well.

d) Shaundra knows that all elephants are mammals. She also knows that all mammals are warm-blooded. She concludes that all elephants are warm-blooded.

Solution

a) As John used accepted mathematical rules to prove that the solution to $2x + 3 = 7$ is $x = 2$, he used deductive reasoning.

b) Nikole observed and made inferences based on prior experience. She did not reason from any known facts. Therefore, Nikole used inductive reasoning.

c) Kyra reasoned from past examples to come to her conclusion. Therefore, she used inductive reasoning.

d) Shaundra reasoned from known facts about mammals to determine that all elephants are warm-blooded. Therefore, she used deductive reasoning.

Example 2

Use deductive reasoning to make a conclusion for each scenario below.

a) All of the women in this room are hairstylists. All hairstylists wear comfortable shoes while working because they are on their feet all day. What can you deduce about Andrea, a woman in this room?

b) Bears hibernate during the winter. All bears have fur coats. What can you deduce about animals who have fur coats?

c) All multiples of 4 are multiples of 2 and 132 is a multiple of 4. What can you deduce about the number 132?

d) All snakes shed their skin. Slithers is a snake. What can you deduce about Slithers?

Solution

a) Andrea must be wearing comfortable shoes. Since all women in the room are hairstylists, Andrea must be a hairstylist. You know that all hairstylists wear comfortable shoes. Therefore, Andrea must be wearing comfortable shoes.

(**Note:** This deductive reasoning relies on the acceptance of the statement, "hairstylists all wear comfortable shoes while working." In deductive reasoning, you base the truth of future conjectures on previously defined statements or accepted truths.)

b) You cannot deduce anything about animals with fur coats. This set of statements does not include any fact stating that "All animals with fur coats . . . ." Therefore, there is nothing to conclude from this set of statements.

(**Note:** Be careful in this type of a situation as many people like to reason backwards and conclude that all animals with fur coats hibernate. This type of reasoning is logically incorrect.)

c) 132 is a multiple of 2. Since 132 is a multiple of 4 and it is stated that all multiples of 4 are multiples of 2, then 132 must be a multiple of 2.

d) Slithers sheds its skin. Since Slithers is a snake and all snakes shed their skin, you can conclude that Slithers will shed its skin.

Using Deductive Reasoning to Prove Conjectures

Throughout this course, you will be using deductive reasoning to prove conjectures to be true. Many students find the concept of proofs difficult. For this reason, it is important that you concentrate on this section and make sure you understand the proofs before continuing. If you encounter any difficulties, remember you can always ask your learning partner and/or your tutor/marker for assistance.

A **proof** is a mathematical argument that shows that a statement is always true or always valid. In other words, it is impossible to find a counter-example.

In order to prove a mathematical statement, it will be helpful to know the following relationships:

- **Even Integers** are numbers that can be written in the form $2m$, where m is an integer. (Even numbers have a factor of two or even numbers are always double another integer.)
- **Odd Integers** are numbers that can be written in the form $2n + 1$, where n is an integer. (One more than an even number will always produce an odd number.)

- **Consecutive Integers** are numbers that can be written in the form n, $n + 1$, $n + 2$, etc, where n is an integer. (By adding one more to the previous number, you will get the next consecutive integer.)

You may wish to add these definitions to your resource sheet.

Example 3

Using deductive reasoning, prove that the product of an odd integer and an even integer is always even.

Solution

Always start to prove these types of statements by introducing your variables and writing an expression for each type of number the statement discusses. Remember, do not use the same variable for different number statements.

It may also be helpful to use a **two-column proof** when proving these types of statements. In a two-column proof, the left-hand side contains all of your mathematical calculations. On the right-hand side, write an explanation for each step.

Let $2n + 1$ be an odd integer and let $2m$ be an even integer, where m and n are integers.	Introduce your variables.
The product of the odd and even integers is $(2n + 1)(2m)$.	Write their product.
$(2n)(2m) + 1(2m)$	Use the distributive property to multiply.
$4mn + 2m$	Simplify.
$2(2mn + m)$	Factor the expression by removing a common factor of 2 from each term.
By the definition above, 2 multiplied by any integer (in this case, $2mn + m$) is an even number. Therefore, the product is even.	This proves our original statement that the product of an even number and an odd number is even.

Example 4

Prove that the sum of two multiples of 4 is a multiple of 4.

Solution

To begin, write two statements to represent the different multiples of 4. Remember, do not use the same variable for each statement!

Let the first multiple of 4 be $4n$. Let the second multiple of 4 be $4m$, where m and n are integers.	Introduce your variables.
$4n + 4m$	Write their sum.
$4(n + m)$	Factor the expression by removing a common factor of 4 from each term.
Therefore, since 4 is a factor of the sum, the sum is a multiple of 4.	This proves our original statement that the sum of two multiples of 4 is a multiple of 4.

It is possible to prove the divisibility rules that you learned during your early years of schooling. Recall:

- A number is divisible by 2 if the last digit is even.
- A number is divisible by 3 if the sum of the digits is divisible by 3.
- A number is divisible by 4 if the last two digits are divisible by 4.
- A number is divisible by 5 if the last digit is a 0 or a 5.
- A number is divisible by 6 if it is divisible both by 2 and by 3.
- A number is divisible by 8 if the last three digits are divisible by 8.
- A number is divisible by 9 if the sum of the digits is divisible by 9.
- A number is divisible by 10 if the last digit is a zero.

Note: There is no simple divisibility rule for 7.

In order to prove divisibility rules, it is useful to know how to expand the digits of a number into a sum based on powers of 10. Any two-digit number, ab, can be written as:

$$ab = a(10) + b(1)$$

For example, 56 can be written as:

$$56 = 5(10) + 6(1)$$

You can use the same pattern with a three-digit number, *abc*.

$abc = a(100) + b(10) + c(1)$

For example, 523 can be written as:

$523 = 5(100) + 2(10) + 3(1)$

Consider the following example.

Example 4

Prove that a two-digit number is divisible by 5 if the last digit is a 0 or a 5.

Solution

To begin, consider a two-digit number, *ab*, where *a* is one of the digits 1 to 9 and *b* is either 0 or 5.

Let *ab* be a two-digit number	Introduce your variables.
$ab = a(10) + b(1)$	
$= a(5)(2) + b(1)$	Using the definition of a two-digit number, $a(10)$, can be rewritten as $a(5)(2)$.
$= 5(2a) + b(1)$	Rearrange to illustrate that $a(10)$ is a multiple of 5.
Now we have two cases: $b = 5$ or $b = 0$.	Divide the scenario into two cases.
Case 1: $b = 5$ Then,	
$ab = 5(2a) + b(1)$	Substitute $b = 5$.
$= 5(2a) + 5(1)$	
$= 5(2a + 1)$	Show that 5 is a factor of *ab*.
Therefore, if the two-digit number, *ab,* ends in 5, then *ab* is divisible by 5 since 5 is a factor.	
Case 2: $b = 0$ Then,	
$ab = 5(2a) + b(1)$	Substitute $b = 0$.
$= 5(2a) + 0(1)$	
$= 5(2a)$	Show that 5 is a factor of *ab*.
Therefore, if the two-digit number, *ab*, ends in 0, then *ab* is divisible by 5 since 5 is a factor.	

In previous mathematics classes, you may have been taught mental math strategies for multiplying. One of those strategies is the doubling-halving strategy. For example, to multiply 15 × 14, you can do the following mentally:

$$\begin{aligned} 15 \times 14 &= (15 \times 2) \times (14 \div 2) \\ &= 30 \times 7 \\ &= 210 \end{aligned}$$

The doubling-halving strategy is particularly effective when you double a number ending in 5 as that produces a number ending in zero. Since multiplying numbers ending in zero is often less complicated, this strategy can make the mental math a little easier.

The doubling-halving strategy may seem logical, but it can also be proven to work for any two numbers.

Example 5

Prove that when finding the product of two numbers, you can always find the answer by doubling one of them and halving the other, then finding the product of those new numbers. In other words, prove the doubling-halving mental math strategy outlined above.

Solution

Let m and n be any two numbers.	Introduce your variables.
The product of the numbers is $m \times n$.	Write an expression for the product.
Half of one number is $\frac{m}{2}$.	Define half of one number.
Double the other number is $2n$.	Define double the other number.
The product is $= \frac{m}{2} \times 2n$	Find the product of the new numbers.
$= \left(\frac{1}{2} \times m\right) \times (2 \times n)$ $= \left(\frac{1}{2} \times 2\right) \times (m \times n)$	Simplify using the associative property for multiplication.
$= (1) \times m \times n$ $= m \times n$	This expression is the product of the original numbers.

Therefore, the product of half one number and double the other is the same as the product of the numbers, regardless of the values of the numbers. You have proven the doubling-halving strategy.

Example 6

Prove that the number trick shown in Lesson 1, Example 5 always ends with the number with which you began.

Number Trick (from Lesson 1, Example 5)

- Choose a number.
- Double the number.
- Subtract 3.
- Add the original number.
- Divide by 3.
- Add 1.

Solution

Use a two-column proof.

Let n be the number.	Choose a number.
$2n$	Double the number.
$2n - 3$	Subtract 3.
$2n - 3 + n$	Add the original number.
$3n - 3$	Simplify (by combining like terms).
$\dfrac{3n-3}{3}$	Divide by 3.
$\dfrac{3n}{3} - \dfrac{3}{3}$	Simplify (by dividing each term by 3).
$n - 1$	
$n - 1 + 1$	Add 1 and simplify.
n	The result is the original number.

Therefore, regardless of the value of the number you started with, when you complete the above number trick, you will always end up with the number you first selected.

Example 7

Use a two-column proof to prove that the solution to $3x - 5 = 16$ is $x = 7$.

Solution:

$3x - 5 = 16$	State the original equation.
$3x - 5 + 5 = 16 + 5$	Add 5 to both sides of the equation.
$3x = 21$	Simplify.
$\frac{3x}{3} = \frac{21}{3}$	Divide both sides of the equation by 3.
$x = 7$	Simplify.

Therefore, the solution to $3x - 5 = 16$ is $x = 7$.

Note: This may not seem like a proof but it is. You are using deductive reasoning to make simpler equations that have the same solution as the original equation. As soon as the answer to the simpler equation is apparent (that is, when you write, $x = 7$), then you know the solution to the more complex equation with which you started.

Be sure to complete the following learning activity to practise what you just learned about deductive reasoning!

Learning Activity 3.2

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Determine the next two terms in this sequence: 1, −1, 1, −1, . . .
2. Determine the next two terms in this sequence: 3, −6, 12, . . .
3. Determine a conjecture to find the next term in this sequence: 32, 16, 8, . . .
4. Find a counter-example to the statement: "All pencils can be sharpened."
5. Solve the equation: $\frac{1}{2}x - 6 = 8$
6. Factor: $3x^2 - 3x$
7. Determine the greatest common factor of $4m^2n$ and $8mn^2$.
8. Convert 14 600 grams to kilograms.

continued

Part B: Deductive Reasoning

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Use the definitions of deductive and inductive reasoning to determine whether each scenario is a representation of deductive or inductive reasoning.
   a) For the pattern 26, 23, 20, 17, 14, . . ., the next term is 11.
   b) Every high school student in Manitoba needs to take Grade 11 History. You are a high school student in Manitoba. Therefore, you need to take Grade 11 History.
   c) The last three times Everett got picked up by the bus, the bus was three minutes late. Everett concludes the bus will be three minutes late today as well.
   d) Awan's mom told him when he was younger not to touch a baby bird if it had fallen from its nest. She told him that the bird's mother would abandon the baby bird when she smelled human scent. In Grade 11 Biology, he read an article written by a biologist proving that birds cannot detect human scent. Awan concluded that his mother was wrong.
2. Use deductive reasoning to come to a conclusion, if possible. If it is not possible to come to a conclusion using deductive reasoning, explain why.
   a) If a team wins four games in the final series of the Stanley Cup playoffs, then they win the Stanley Cup. The Chicago Blackhawks won four games in the Stanley Cup finals. What can you conclude about the Chicago Blackhawks?
   b) Fancy Shoes sells shoes that all cost at least $100. Shelley paid $150 for her pair of shoes. What can you conclude about where Shelley bought her shoes?
   c) All fish breathe using gills. All animals that breathe using their gills can breathe underwater. Snorkel is a fish. What can you conclude about Snorkel?

continued

d) All kangaroos are marsupials. Marsupials are the only animals that have pouches to carry their young.

  i) What can you conclude about the Red Kangaroo?

  ii) What can you conclude about animals that are not marsupials?

3. Prove that when two even integers are multiplied, the product is a multiple of 4.

4. Prove that the sum of five consecutive integers is always a multiple of 5.

5. Prove that when two odd integers are multiplied, the product is odd.

6. Prove that squaring an even integer always gives you an even number.

7. Complete the following subtractions by adding the same value to both numbers to make the subtraction easier to do mentally.

   a) $95 - 67$

   b) $235 - 39$

8. When finding the difference of two numbers, you can create two new numbers by adding the same value to both original numbers. Prove that the difference of the two new numbers will be the same as the difference of the two original numbers. In other words, prove that the mental math strategy you used in the previous question is valid.

9. Prove that a two-digit number is divisible by 2 if the last digit is even.

10. Use deductive reasoning to find the solution to the equation $2(x - 3) + 3 = 5(x - 1)$.

11. Prove that the following number game always gives you the original number.

    - Choose a number.
    - Double it.
    - Subtract 8.
    - Divide by 2.
    - Add 4.

continued

Learning Activity 3.2 (continued)

12. Prove that the following number game always gives an answer of 4.
    - Choose a number.
    - Double it.
    - Add 5.
    - Add the original number.
    - Add 7.
    - Divide by 3.
    - Subtract the original number.

Lesson Summary

In this lesson, you learned that you can use deductive reasoning to prove that conjectures are always true. Deductive reasoning involves reasoning from facts and known properties about numbers to come up with statements that are always true. In the next lesson, you will learn that the only scenarios where deductive reasoning can be faulty involve the use of invalid reasoning, false assumptions, or mathematical errors.

Notes

Assignment 3.1

Inductive and Deductive Reasoning

Total: 35 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking.

Final answers must include units. Answers given without supporting calculations and graphs will not be awarded full marks.

1. Suzie's teacher told her that when you add 2 to an even number, the result is always even.

   a) List three examples that support this conjecture. *(3 marks)*

   b) Is this conjecture reasonable? Explain. *(1 mark)*

Assignment 3.1: Inductive and Deductive Reasoning (continued)

2. Determine a conjecture that could be made about the sum of a multiple of 2 and a multiple of 6. Include examples to verify your conjecture. *(2 marks)*

3. Every morning after the first bell, the principal at Andrew's school plays "O Canada" over the intercom and then reads the announcements for the day. Andrew listens to "O Canada" and the announcements on Monday, Tuesday, and Wednesday during his first class. On Thursday, "O Canada" is not played and there are no announcements during Andrew's first class. Make two conjectures to explain why the national anthem was not played and the announcements were not read. *(2 marks)*

4. Find a counter-example to the conjecture: "All prime numbers are odd." Explain why your counter-example proves the conjecture to be false. *(2 marks)*

Assignment 3.1: Inductive and Deductive Reasoning (continued)

5. If possible, find a counter-example to the conjecture: all fast food is unhealthy. *(1 mark)*

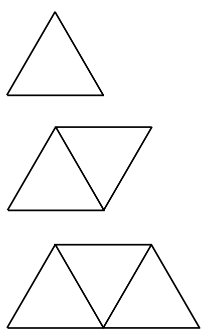
6. Determine whether the following scenarios represent inductive or deductive reasoning. Explain your thinking. *(2 marks)*

   a) Connor's parents have told him that he is not allowed to stay out past 11 p.m. on a school night or he will be grounded. On Monday night, Connor doesn't get home until midnight. He reasons that he is going to be grounded.

   b) Carolyn's five year-old sister notices that their family has a black lab, their neighbour has a black terrier, and she always sees a black pug walking down the street. She states that all dogs must be black.

7. Use inductive reasoning to determine the next three terms in each of the following patterns. *(3 marks)*

   a) 9, 4, 6, 1, 3, . . .

   b) 

   c) 1, 8, 27, 64, . . .

8. Prove that the sum of two odd integers is always even. *(3 marks)*

Assignment 3.1: Inductive and Deductive Reasoning (continued)

9. Prove that a two-digit number is divisible by 10 if the last digit is zero. *(3 marks)*

10. Make a conjecture about the result of the following number trick. *(2 marks)*
    - Choose a number.
    - Divide the number by 2.
    - Add 6.
    - Multiply by 4.
    - Subtract 24.

Assignment 3.1: Inductive and Deductive Reasoning (continued)

11. Prove that the conjecture you made about the number trick in Question 10 is always true. *(3 marks)*

12. Complete the following subtractions mentally by subtracting the number of tens in the second number from the first number and subtracting the number of ones in the second number from that result. *(2 marks)*

    a) $87 - 59$

    b) $56 - 38$

Assignment 3.1: Inductive and Deductive Reasoning (continued)

13. Prove that the mental math strategy you used above to find the difference of the two-digit numbers, *ab* and *cd*, is valid by using deductive reasoning to show that $ab - 10c - d$ is equivalent to the difference of the numbers. *(3 marks)*

14. Use deductive reasoning to find the solutions to the equation $x + 8 = x^2 - x$. *(3 marks)*

Notes

Lesson 3: Invalid Reasoning

Lesson Focus

In this lesson, you will

- ❑ determine if an argument is valid
- ❑ identify the errors in a proof

Lesson Introduction

In order for reasoning to be valid, it needs to be logical. If there is any way that the reasoning is illogical, then the conclusion is invalid. There are many ways that reasoning can be illogical, which you will learn about through this lesson.

Invalid Reasoning

Consider the following statement:

There are tthree errorss in this sentence.

Is this statement valid?

There are two spelling errors in the sentence: “tthree” should be *three* and “errorss” should be *errors*. This means the sentence is false because there are only two errors in the statement.

However, this implies that the statement itself is an error, which brings the total number of errors up to three. Therefore, the statement is true.

A statement cannot be both true and false at the same time. Therefore, this statement is invalid.

There are three types of invalid reasoning that you will learn about in this lesson:

- false assumption
- error in reasoning
- mathematical error

A **false assumption** occurs when you begin a proof with a statement that is not true. For example, if you begin a proof with the statement “1 = 2,” everything that you conclude will be invalid because your premise, or your opening statement, of 1 = 2 is false.

An **error in reasoning** occurs when you use logical principles incorrectly. For example, if a statement is made in the form “If . . . then . . .,” you cannot reason backwards. An example of an error in reasoning is the following:

> *If I have a pet beagle, then I have a dog.*
>
> *Therefore, if I have a dog, then I have a pet beagle.* (reasoning backwards)

You cannot reason backwards in this instance. This would be an error in reasoning.

A **mathematical error** occurs when you make a calculation error, use the rules of algebra incorrectly, or divide by zero in some form. In mathematics, you cannot divide by zero because the result is invalid and cannot be determined.

You may wish to include a summary of these three types of invalid reasoning on your resource sheet.

Example 1

Consider the following argument:

> It is impossible to take Grade 11 Pre-Calculus Mathematics and Grade 11 Applied Mathematics in the same year. Emma is currently taking Grade 11 Applied Mathematics in her first semester. Therefore, she will not be taking Grade 11 Pre-Calculus Mathematics in her second semester.

Explain why the proof above is invalid.

Solution

The previous proof is invalid because it began with a false assumption. The false assumption is that it is impossible to take both Grade 11 Pre-Calculus Mathematics and Grade 11 Applied Mathematics in the same year. This is possible as many students do take more than one Grade 11 mathematics course. Therefore, the conclusion that Emma will not take Grade 11 Pre-Calculus Mathematics in her second semester is invalid. Emma may decide to take Grade 11 Pre-Calculus Mathematics or she may not.

Example 2

Consider the following example of deductive reasoning. Explain why the reasoning is incorrect.

> On Wednesday, the City of Winnipeg has scheduled garbage collection from households in St. Vital. Today, all the household garbage was picked up in St. Vital. Therefore, it is Wednesday.

Solution

This is an example of an error in reasoning. Even though every Wednesday the household garbage is scheduled to be picked up in St. Vital, the statement does not say that household garbage is ONLY picked up on Wednesday. Thus, this week could have an alternate pick-up day due to a holiday or other reasons. Therefore, the conclusion that it is Wednesday is invalid.

Example 3

Owen tried to prove that the following number trick always results in the number 2. Identify and correct the error in Owen's proof.

- Choose a number.
- Triple the number.
- Add 9.
- Divide by 3.
- Subtract the number you started with.
- Subtract 1.

Owen's Proof:

Choose a number.	Let n be a number.
Triple the number.	$3n$
Add 9.	$3n + 9$
Divide by 3.	$n + 9$
Subtract the number you started with.	9
Subtract 1.	8

Solution

The error in Owen's proof occurs in the fourth line when he tries to divide by 3. He forgot to divide the 9 by 3 as well. This is a mathematical error.

His fourth line should be:

Divide by 3.	$\dfrac{3n+9}{3}$
	$\dfrac{3n}{3}+\dfrac{9}{3}$
	$n+3$
Subtract the number you started with.	$n+3-n$
	3
Subtract 1.	$3-1$
	2

Therefore, the result is 2.

Example 4

Jacob claims he can prove that 2 = 1. He provided the following proof:

Let $x = y$. Let x and y be integers.	Introduce your variables.
$x + x = x + y$	As x and y are equal, both sides are equivalent after adding x to both sides.
$2x = x + y$	Simplify the left-hand side.
$2x - 2y = x + y - 2y$	Subtract $2y$ from both sides of the equation.
$2x - 2y = x - y$	Simplify the right-hand side.
$2(x - y) = x - y$	Factor out a 2 from the left-hand side.
$\dfrac{2(x-y)}{x-y} = \dfrac{x-y}{x-y}$	Divide both sides of the equation by $x - y$.
$2(1) = 1$ $2 = 1$	Simplify.

Determine the error in Jacob's proof.

Solution

At first glance, this proof may seem to be mathematically correct. However, there must be something wrong because we know that 2 does not equal 1.

Consider the step where Jacob divides both sides of the equation by $x - y$. In this proof, $x = y$. Therefore:

$x - y$
$= x - x$
$= 0$

Thus, Jacob divided both sides of the equation by zero. This is a mathematical error. As you know, a value cannot be divided by zero, as the result will be undefined. Therefore, Jacob's entire proof is invalid after he divides by zero.

Sometimes, proofs contain *circular reasoning*. Circular reasoning is another type of illogical reasoning that makes a proof invalid. Circular reasoning is a type of reasoning where you assume what you are trying to prove is true before you have actually proven it to be true.

An example of circular reasoning would be the following statement:

Murder is wrong because it is illegal and murder is illegal because it is wrong.

Be careful when you are completing the following learning activity that you are looking for errors in reasoning, mathematical errors, false assumptions, and circular reasoning when determining whether a proof is invalid.

Learning Activity 3.3

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Determine the next two terms in this sequence: 1, 2, 3, 5, . . .
2. Determine the next two terms in this sequence: 8, 4, 2, . . .
3. Find a conjecture to determine the next term in this sequence: 1, 3, 6, 18, 36, . . .
4. Provide a counter-example to the statement: "All animals have legs."
5. Factor: $14x^2 - 4x$
6. Simplify: $2x + 6 - 3(x - 2)$
7. Simplify: $\frac{1}{3} + \frac{2}{11}$
8. Simplify: $(-1)(-1)(2)(-1)(-2)$

continued

Learning Activity 3.3 (continued)

Part B: Invalid Reasoning

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Determine the type of invalid reasoning portrayed in each of the following examples.
   a) Some people who sneeze are sick. Brooklyn is sneezing. Therefore, Brooklyn is sick.
   b) All students graduate from high school in their Grade 12 year. Georgia is a Grade 12 student at Stonewall Collegiate. Therefore, Georgia will be graduating from high school this year.
2. The following brain teaser is a famous puzzle that you may have heard before. It is often referred to as "the missing dollar riddle." Explain how the brain teaser uses invalid reasoning to come to a conclusion.

   Three friends decide to rent a hotel room for a night for $30. They split the cost and pay $10 each. The friends move their luggage up to their room and start unpacking.

   The manager, wanting the friends to come back again, decides to give the friends a $5 discount. He sends the bell boy to give the friends their refund. The bell boy, realizing that $5 is not very easily split among three friends, decides to keep $2 and only give the friends a $3 refund.

   Now, the friends have paid $9 each for the hotel room. In total, the friends paid $27 for the hotel room ($9 for each friend) and the bell boy has $2. This gives a total of $29 for the hotel room. Where did the extra dollar go?

continued

Learning Activity 3.3 (continued)

3. Explain each step of the proof below. Indicate in which step the proof becomes invalid and explain why.

Let $x + y = z$ where x, y, and z are integers.	
$(2x - x) + (2y - y) = (2z - z)$	
$2x - x + 2y - y = 2z - z$	
$2x + 2y - x - y = 2z - z$	
$2x + 2y - 2z - x - y = -z$	
$2x + 2y - 2z = x + y - z$	
$2(x + y - z) = (x + y - z)$	
$\dfrac{2(x + y - z)}{x + y - z} = \dfrac{(x + y - z)}{x + y - z}$	
$2 = 1$	

4. Create an invalid proof that shows 3 = 2. Use Question 3 as a guideline.

5. Explain why the following proof is invalid.

   Step 1: Women earn less money than men for completing the same job.

   Step 2: Ellen Degeneres is a woman.

   Step 3: Therefore, Ellen Degeneres earns less money than men completing the same job.

6. Explain why the following proof is invalid.

   Step 1: All people who have the flu have a fever.

   Step 2: Jayden has a fever.

   Step 3: Therefore, Jayden has the flu.

continued

Learning Activity 3.3 (continued)

7. Explain why the following proof is invalid.

Step 1:	$4 = 4$
Step 2:	$4 = 8 - 4$
Step 3:	$(9 - 5) = 8 - (9 - 5)$
Step 4:	$\dfrac{9-5}{9-5} = 8 - \dfrac{9-5}{9-5}$
Step 5:	$1 = 8 - 1$
Step 6:	$1 = 7$

8. Explain why the following statement is an example of circular reasoning.

   Women can't be cowboys because a cowboy is a man.

9. Determine whether the following argument is valid or invalid. Explain.

   Statement 1: If June 20th is sunny, we will go to the beach.

   Statement 2: We went to the beach on June 20th.

   Statement 3: Therefore, June 20th must have been sunny.

Lesson Summary

In this lesson, you learned all about invalid proofs. There are many different ways a proof can become invalid. Some of the ways that you learned about include false assumptions, errors in reasoning, mathematical errors, and circular arguments.

In the next lesson, you will learn to apply your logical reasoning skills to geometric patterns and problems.

Notes

Lesson 4: Spatial Reasoning

Lesson Focus

In this lesson, you will

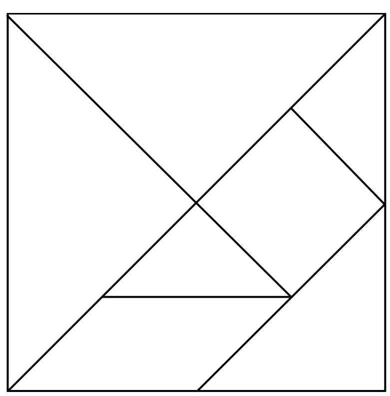
- ❑ use spatial reasoning to solve puzzles and games

Lesson Introduction

Have you ever played Tetris, chess, or checkers, or solved tangram puzzles? All of these puzzles and games use spatial reasoning. Spatial reasoning involves figuring out how shapes fit together by manipulating them in your mind. This is a useful skill when decorating and arranging furniture in a room. Seeing how smaller pieces come together to create a whole is the main application of spatial reasoning.

Tangrams

Tangrams are ancient Chinese puzzles involving seven geometric shapes cut from a square. The seven shapes include two large triangles, one medium triangle, two small triangles, one square, and one parallelogram. A sample tangram template is shown below.



Tangrams are often rearranged to form different shapes and pictures. Consider the following examples.

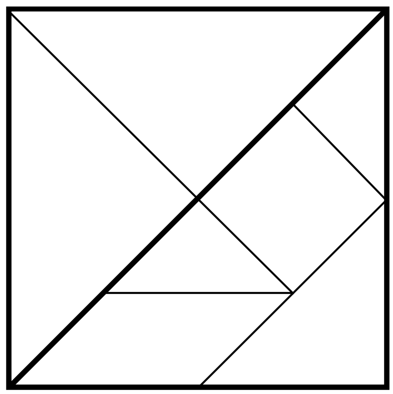
Note: Two tangram templates have been provided at the end of this module to help you with this lesson.

Example 1

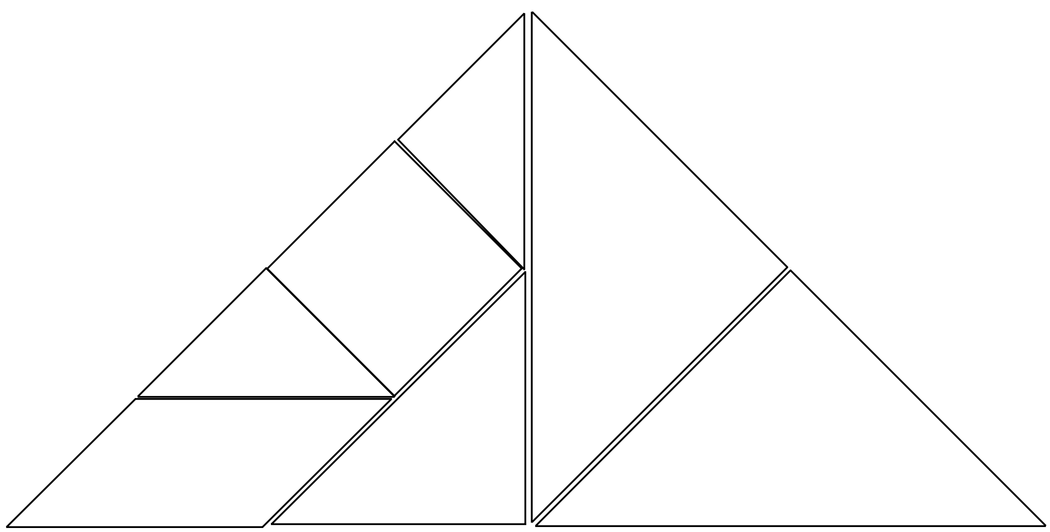
Rearrange all seven tangram components to form one large isosceles triangle.

Solution

In order to solve this tangram puzzle, it might be easiest to look back at the original tangram square. If the original tangram was cut in half diagonally, the halves would be congruent triangles.



If you take these two large triangles and rearrange them, you can form one isosceles right triangle like the completed one below.



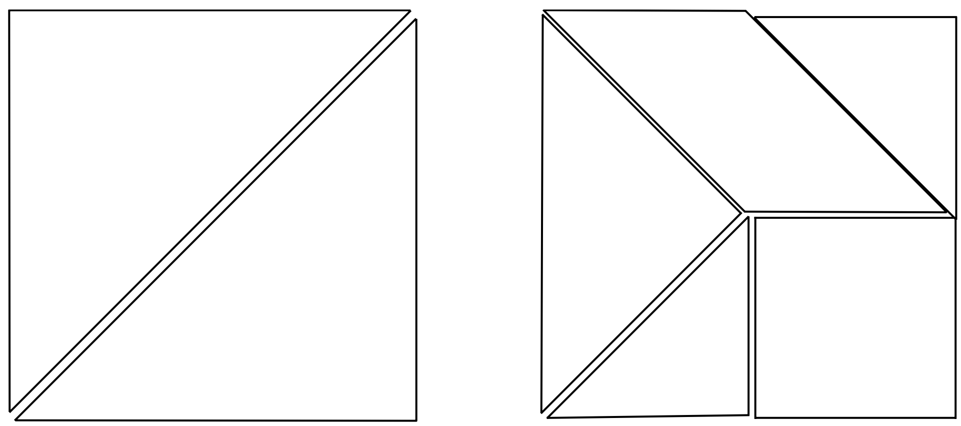
Example 2

Rearrange all seven tangram components to form two squares of equal size.

Solution

First, create one square by using the two large triangles. Remember that any two right triangles of the same size can be fit together to create a square.

Then, you need to use the remaining five pieces to make a square the same size as your first square. It will be helpful if you cut out the tangram template at the end of this module so that you can move the pieces around to see how they fit together to form a square. Take note of the length of the sides of the tangram pieces and which sides are the same length. For example, the bottom of the medium-sized triangle is the same length as the side of the large triangle. These relationships can help you when solving tangram puzzles.



Toothpick Puzzles

Another type of puzzle that uses spatial reasoning is a toothpick puzzle. Toothpick puzzles are also sometimes called matchstick puzzles because a toothpick and a match are relatively the same size.

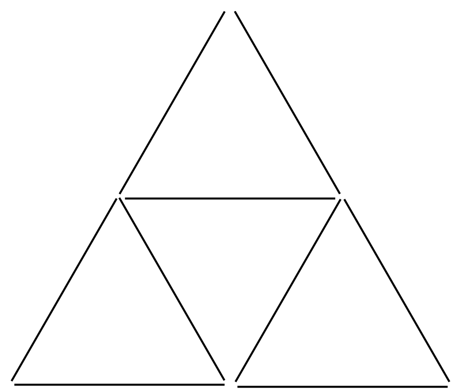
Toothpick puzzles involve shapes made out of toothpicks. Once you are given a shape, you are either asked to remove a certain number of toothpicks or to move a certain number of toothpicks to create a new shape. Consider the following examples.

Example 3

The triangle shown below is made out of nine toothpicks.

a) Remove three toothpicks to leave one triangle.

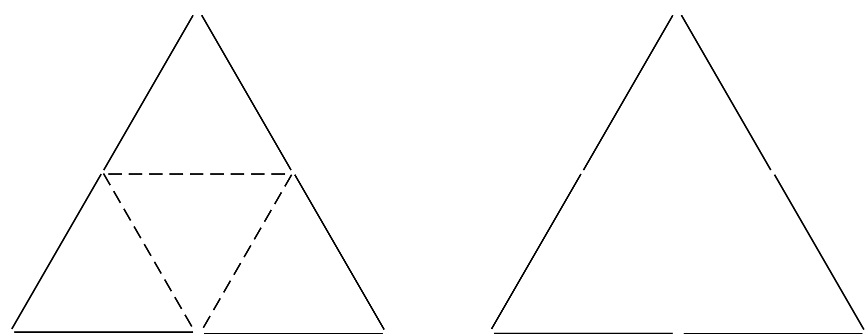
b) Remove two toothpicks to leave three triangles.



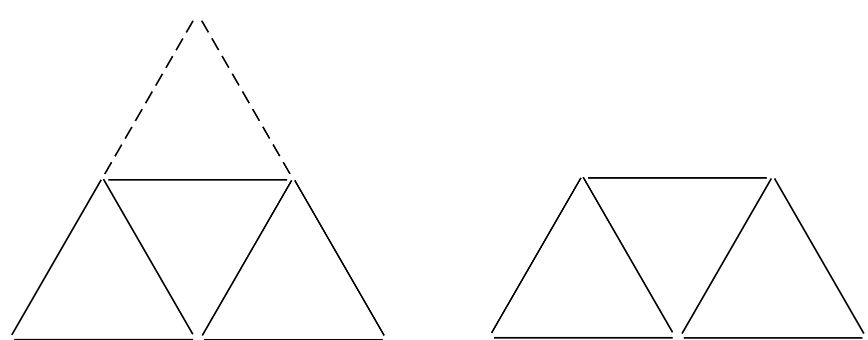
Solution

First, notice that the large triangle is composed of four smaller triangles.

a) If you remove the three interior toothpicks, you will be left with only the larger triangle. This is what you need.



b) If you remove any two toothpicks that form a point on the triangle, you will be left with three triangles. For example, if you remove the top two toothpicks, you will only be left with the bottom row of three triangles. You have solved the puzzle.

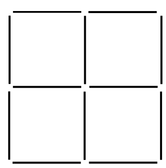


Example 4

Consider the following square made of 12 toothpicks.

a) Remove four toothpicks to leave one square.

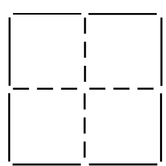
b) Remove four toothpicks to leave two squares.

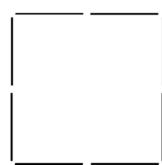


Solution

Again, note that the large square is made up of four smaller squares.

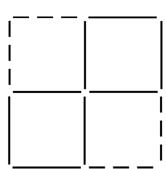
a) If you remove the four interior toothpicks, you will be left with one large square.

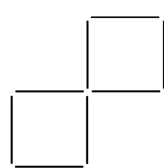




b) If you only want to be left with the top two squares, you would need to remove five toothpicks from the bottom. The same thing would happen if you wanted to leave two squares at the bottom or on the left or right side. But you can only remove four toothpicks, so this won't work.

If you leave two squares that are diagonally across from each other, you need to remove the four toothpicks, as shown below. This is what you want.





The following learning activity offers the opportunity for you to practise using your spatial reasoning skills to solve puzzles, such as the ones taught throughout the lesson. You may want to find some toothpicks to help you solve the toothpick puzzles. You may find this learning activity frustrating at first, as it can be difficult to visualize spatial arrangements and imagine their rearrangement. Keep trying different arrangements; you will get better with practice.

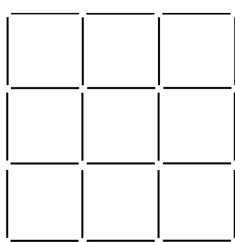
Learning Activity 3.4

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

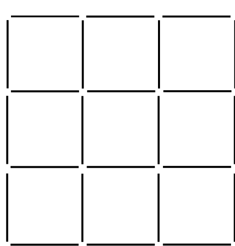
Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Determine the next two terms in this sequence: 1, −2, 4, . . .
2. Determine the next two terms in this sequence: 1, 6, 11, . . .
3. Determine a conjecture to find the next term in this sequence: 1, 2, 5, 10, 17, 26, . . .
4. Find a counter-example to the statement: "All animals have four legs."
5. How many small squares (1×1) can you find in the shape below?



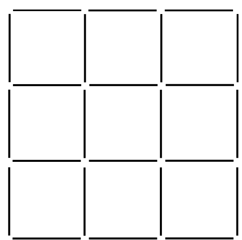
6. How many medium squares (2×2) can you find in the shape below?



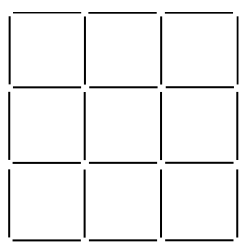
continued

Learning Activity 3.4 (continued)

7. How many large squares (3 × 3) can you find in the shape below?



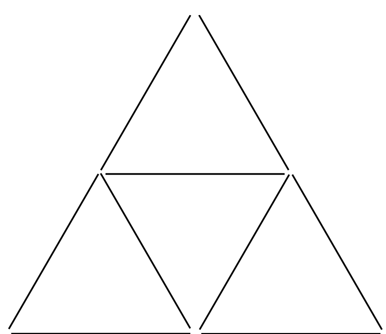
8. What is the total number of squares in the shape below?



Part B: Using Your Spatial Reasoning Skills

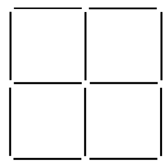
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Use all seven tangram puzzle pieces to create a rectangle.
2. Use all seven tangram components to create a trapezoid.
3. Consider the triangle below made up of nine toothpicks. Move two toothpicks to create four small triangles with no large triangle.

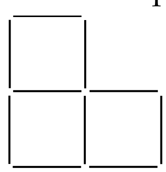


continued

4. Consider the square below made up of 12 toothpicks. Remove two toothpicks to leave two squares.

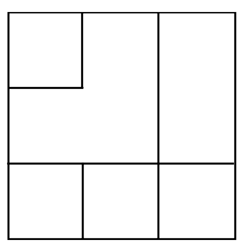


5. Consider the shape below made up of 10 toothpicks.

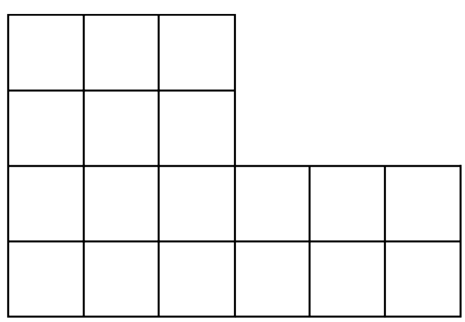


   a) Move two toothpicks to create two squares.
   b) Remove two toothpicks to leave two squares.

6. How many squares can you find in the shape below?



7. Consider the shape below, which is created with toothpicks.



   a) Divide this shape into two equal areas using a border of four toothpicks.
   b) Divide this shape into three equal areas using five toothpicks as borders.

Lesson Summary

In this lesson, you developed your spatial reasoning skills by learning how to manipulate shapes and see the connections between them. You arranged shapes in different patterns by visualizing what would happen if you moved the objects in various ways.

This is the last lesson in the module. Make sure you complete the last assignment for this module that follows this lesson.

Notes

Assignment 3.2

Invalid Proofs and Spatial Reasoning

Total: 23 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking.

Final answers must include units. Answers given without supporting calculations and graphs will not be awarded full marks.

1. Explain why the following proof is invalid. *(2 marks)*

   Step 1: If I win the lottery, I will take my family on a vacation.

   Step 2: I went on vacation with my family.

   Step 3: Therefore, I won the lottery.

2. Explain why the following proof is invalid. *(2 marks)*

   Statement 1: All dogs shed.

   Statement 2: Rusty is a dog.

   Statement 3: Rusty sheds.

Assignment 3.2: Invalid Proofs and Spatial Reasoning (continued)

3. Explain each of the steps in the following proof by completing the chart. Explain why the proof is invalid. *(4 marks)*

Let $a = c$, where a and c are integers.	
$a = c$	
$a + a = c + a$	
$2a = a + c$	
$2a - 2c = a + c - 2c$	
$2a - 2c = a - c$	
$2(a - c) = a - c$	
$\frac{2(a-c)}{a-c} = \frac{a-c}{a-c}$	
$2 = 1$	

Assignment 3.2: Invalid Proofs and Spatial Reasoning (continued)

4. Dustin is trying to prove that the following number trick always ends with the number 2. Explain what type of error he made in his proof and correct the error. *(3 marks)*

n	Choose a number.
$2n$	Double the number.
$2n + 4$	Add 4.
$2n + 2$	Divide by 2.
$n + 2$	Subtract the number you started with.

5. Is the following proof valid? If the proof is invalid, explain why. *(2 marks)*

Step 1: On a school night, I need to go to bed at 10 p.m.

Step 2: It is Sunday night.

Step 3: Therefore, I need to go to bed at 10 p.m.

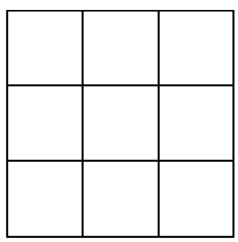
Assignment 3.2: Invalid Proofs and Spatial Reasoning (continued)

6. Use all seven tangram puzzle pieces to create two triangles. *(1 mark)*

Assignment 3.2: Invalid Proofs and Spatial Reasoning (continued)

7. Use all seven tangram components to create one parallelogram. *(3 marks)*

8. Consider the square below made up of 24 toothpicks.

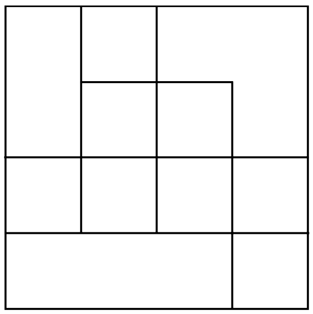


   a) Remove 4 toothpicks to leave 5 squares. *(2 marks)*

   b) Remove 8 toothpicks to leave 5 squares. *(2 marks)*

Assignment 3.2: Invalid Proofs and Spatial Reasoning (continued)

9. How many squares are in this image below? *(2 marks)*



Module 3 Summary

In Module 3, you developed your logical and spatial reasoning skills. First, you learned how to analyze patterns to come up with a conjecture using inductive reasoning. Sometimes you were able to find a counter-example to a pattern that didn't follow the rule.

In order to prove that conjectures are always true, you developed your deductive reasoning skills. You learned how to create two-column proofs, and how to analyze these proofs to see if they were valid. Finally, you learned how to use your spatial reasoning skills to manipulate shapes and arrangements of objects.

In the next module, you will be developing your spatial reasoning skills further by looking at the geometry of angles and triangles.

Submitting Your Assignments

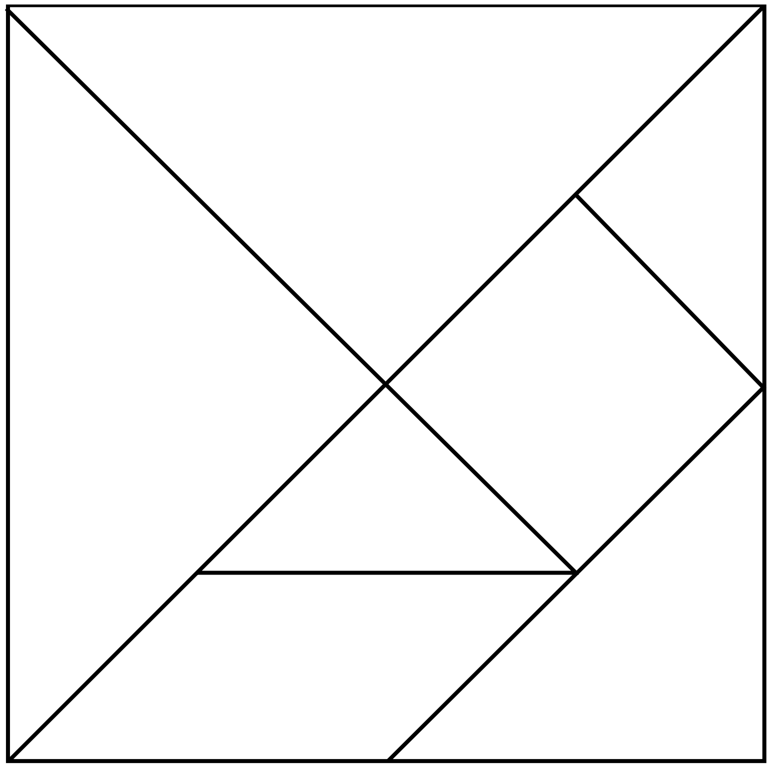
It is now time for you to submit the Module 3 Cover Assignment and Assignments 3.1 and 3.2 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

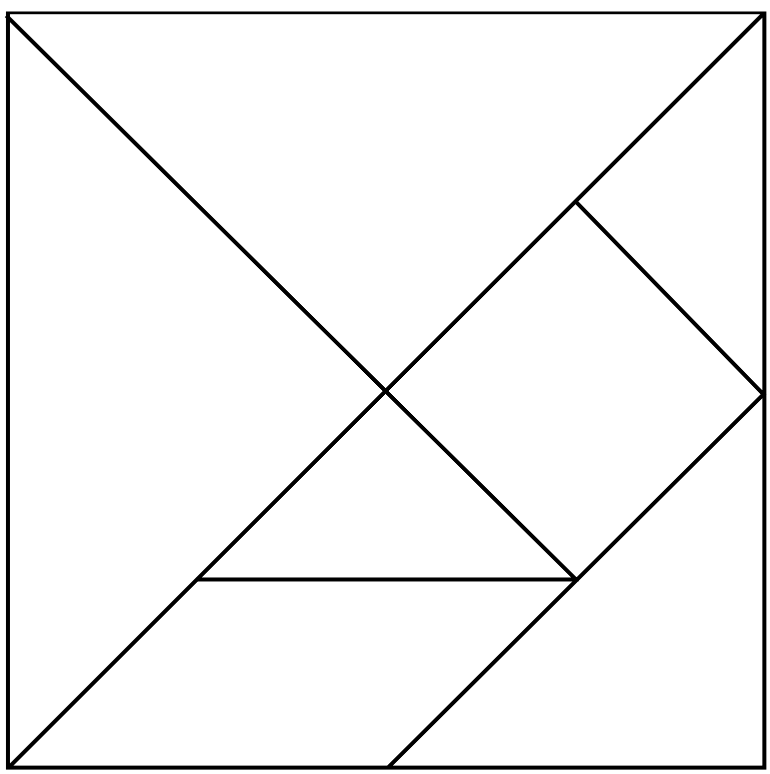
Make sure you have completed all parts of your Module 3 assignments and organize your material in the following order:

- ❑ Module 3 Cover Sheet (found at the end of the course Introduction)
- ❑ Cover Assignment: Logic Puzzles
- ❑ Assignment 3.1: Inductive and Deductive Reasoning
- ❑ Assignment 3.2: Invalid Proofs and Spatial Reasoning

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes





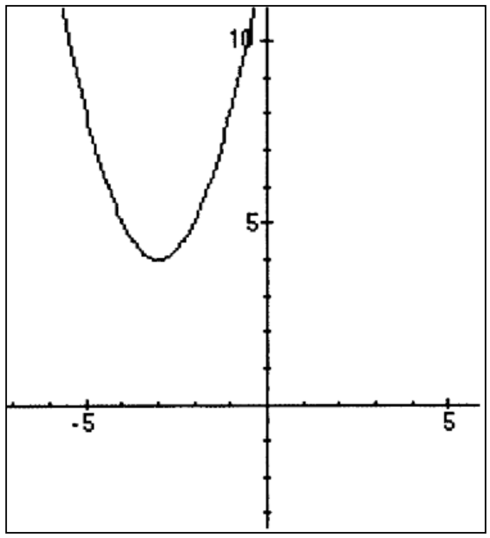
Self-Test

Module 3 – Nonlinear Functions

/70

TEST TIME!

(3 marks) 1. For the given quadratic function graph, state the vertex, axis of symmetry, maximum or minimum value, and the domain and range.



2. Graph the following quadratic functions with your calculator and find the vertex, axis of symmetry, maximum or minimum value, and the domain and range.

(3 marks) a) $y = 2x^2 - 8x + 3$

(3 marks) b) $y = \frac{-2}{3}x^2 + \frac{5}{7}x - 4$

3. Solve by graphing the following applications involving quadratic functions.

   a) The owner of an apple orchard estimates that if 24 trees are planted per acre, then each mature tree will yield 600 apples per year. For each additional tree planted per acre, the number of apples produced by each tree decreases by 12 per year. How many trees should be planted per acre to obtain the most apples? *(5 marks)*

b) A human cannonball is shot from a cannon and follows the path given by

$$h(x) = -\frac{1}{100}x^2 + x + 8$$

where $h(x)$ is the height in feet (above ground) and x is the horizontal distance travelled in feet.

(1 mark) i) Find the maximum height reached by the human cannonball.

(1 mark) ii) What horizontal distance has been travelled when the human cannonball reaches its maximum height?

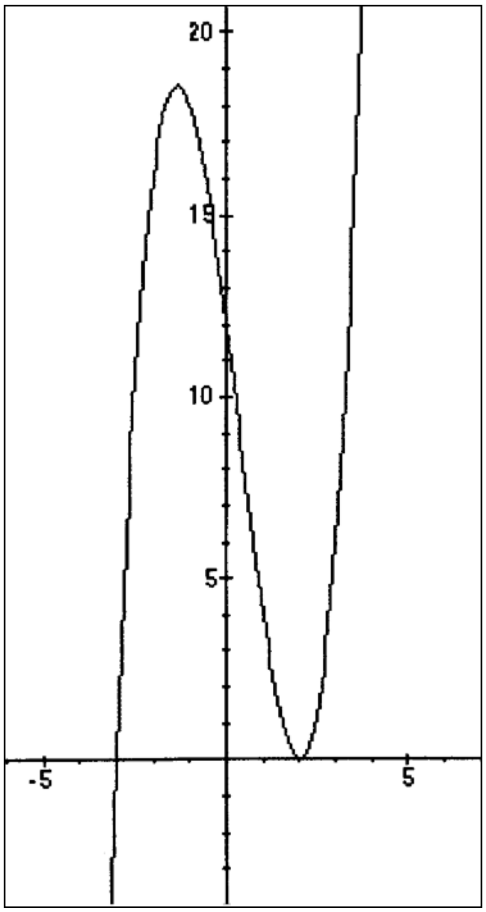
(2 marks) iii) The human cannonball lands in a net that is eight feet above the ground. How many feet must the human cannonball travel horizontally to reach the net?

(1 mark) iv) After travelling 72 feet horizontally, what is the height of the human cannonball?

(5 marks) c) A farmer with 750 feet of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

4. For the graph of the cubic function given, state the maximum and minimum values and the values of x at these points. State the zeros. (Round answers to one decimal place if necessary.)

(4 marks)



5. For the equations of the cubic functions given, state the maximum and minimum values and the values of x at these points. State the zeros. (Round answers to two decimal places if necessary.)

(3 marks) a) $y = x^3 - 3x^2 - 5x - 1$

(3 marks) b) $y = -3x^3$

6. Solve the following cubic function word problems using your graphing calculator.

   a) A box is made from a 30" × 40" piece of cardboard by cutting out squares at each corner and folding up the sides. Find the side length of the square that will maximize the volume. *(5 marks)*

   b) The viscosity (or stickiness) of normal automobile motor oil decreases as its temperature increases. The all-weather motor oils, however, retain a relatively constant viscosity throughout their range of operating temperatures. It is found that the viscosity (V) is related to temperature by *(3 marks)*

$$V = -T^3 + 9T^2 - 24T + 70$$

   where V = viscosity and T = temperature in hundreds of degrees (i.e., T = 1 means a temperature of 100°).

   i) Predict the viscosity at 0°, 500°, and 600°.

   ii) At what temperatures does the viscosity reach a temporary maximum and minimum and what are the values of the viscosity?

   iii) Is there a temperature where the viscosity reaches 0?

7. Graph an exponential function to answer the following questions.

   The formula $A = P(1 + r)^n$ gives the amount, A, to which a principal, P, increases when invested for n years at rate, r, in decimal forms compounded annually.

(2 marks)

   a) How long will it take $5000.00 to amount to $8500.00 if invested at a rate of 6.5% compounded annually?

(2 marks)

   b) How much will an investment of $1000.00 at 5% compounded annually be worth after 12 years?

8. Solve the following systems of equations using your calculator.

   a) $7x - 4y = 26$
      $3x + 4y = -6$ *(2 marks)*

   b) $y = 2x^2 - x - 1$
      $4x + 5y = 37$ *(2 marks)*

9. Solve the following applications by setting up a system of equations or constraint inequalities and using your calculator where necessary to solve the resulting system.

(5 marks)

a) How much sugar should be added to 100 mL of tea to increase its sugar concentration from 7% to 10%?

(5 marks)

b) Flying into a headwind, a 747 jet takes 3.5 hours to travel 1890 km. On the return flight, the 747 jet took three hours under the same weather conditions. Find the speed of

i) the plane in still air

ii) the wind

c) You are a manager of a store that sells home computers. You are getting ready to order next month's stock, and are trying to decide how many of each of two models of monitors to order to obtain a maximum profit. *(10 marks)*

- Model A: your cost is \$250; your profit over cost is \$45.
- Model B: your cost is \$400; your profit over cost is \$50.
- Your combined sales of Models A and B will not exceed 250 units.
- You do not want to spend more than \$70 000 for the total order.

How many of each model should you order?

Grade 11 Applied Mathematics (30S)

Module 4
Geometry of Angles and Triangles

Module 4: Geometry of Angles and Triangles

Introduction

As you work through Module 4, make sure your module resource sheet is complete and up-to-date. You will need it when you start to create your Midterm Examination Resource Sheet.

In this module, you will expand your knowledge of geometry from previous mathematics courses, including your knowledge of the properties of triangles and squares. You will investigate and learn to use the properties of polygons and parallel lines, and their angles. If you look around right now, you should be able to see multiple examples of polygons and of parallel lines (e.g., furniture, paper, walls). These properties are especially useful in construction and building applications. Contractors and architects are examples of professionals who use the properties of parallel lines to ensure their designs and creations are properly constructed. You will be learning more about these properties in this module.

Assignments in Module 4

When you have completed the assignments for Module 4, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Geometry in Tangrams
2	Assignment 4.1	Polygons and Angles
4	Assignment 4.2	Angle Properties of Parallel Lines and Transversals

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, and can be either handwritten or typewritten. Both sides of the sheet may be filled. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as guides.

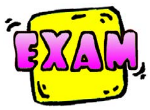
You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 4. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by recording the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1 to 4 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 4.

Resource Sheet for Module 4

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Writing Your Midterm Examination



You will write the midterm examination when you have completed Module 4 of this course. The midterm examination is based on Modules 1 to 4, and is worth 20 percent of your final mark in the course. To do well on the midterm examination, you should review all the work you complete in Modules 1 to 4, including all the learning activities and assignments. You will write the midterm examination under supervision.

Notes

Module 4 Cover Assignment: Geometry in Tangrams

Geometry in Tangrams

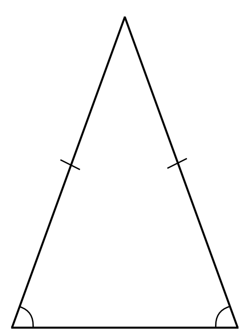
In Module 3, you looked at tangrams and rearranged the seven individual tiles to form various geometric shapes. In this cover assignment, you are going to look at the angle measures and the side lengths of each of the individual tangram tiles.

To accomplish this, you will need to perform the following activities:

- measure angles using a protractor
- determine the perimeter of an object (remember that the perimeter is the distance around an object)
- use properties of isosceles triangles
- use the Pythagorean theorem

Isosceles Triangles

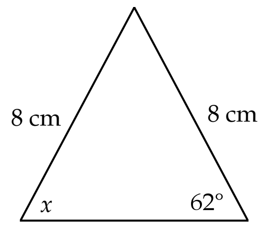
Isosceles triangles are triangles that have two equal sides and two equal angles.



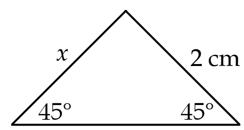
If you know that two sides of a triangle are equal, then the two opposite angles must also be equal. The reverse is also true. If you know that two angles of a triangle are equal, then the two opposite sides must also be equal.

Example 1

a) Determine the measurement of angle x in the triangle below.



b) Determine the measurement of side x in the triangle below.



Solution

a) As the two side lengths both equal 8 cm, both opposite angles must be equal. Therefore, $x = 62°$.

b) Two angles in the triangle equal 45°. Therefore, the opposite sides must also be equal. The missing side, x, must have a length of 2 cm.

Pythagorean Theorem

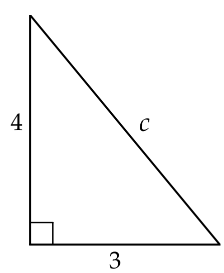
The Pythagorean theorem can be applied to all right triangles. This theorem states that, in a right triangle, $a^2 + b^2 = c^2$, c is the length of the hypotenuse, and a and b are the lengths of the other two sides.

Recall that the hypotenuse is the longest side of the triangle directly across from the right angle.

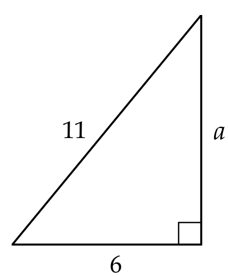
Example 2

Find the missing side lengths in each of the right triangles below. Round to the nearest tenth when necessary.

a)



b)



Solution

a) You are given two side lengths of a right triangle and you want to find the length of the third side. In this situation, you can use the Pythagorean theorem.

$a^2 + b^2 = c^2$

One side has a length of 4 while the other side has a length of 3. It doesn't matter whether you let $a = 4$ or $a = 3$. You can let either side of the triangle, ***not*** the hypotenuse, be a or b.

For this example, let $a = 3$ and $b = 4$. The result is:

$3^2 + 4^2 = c^2$	Square 3 and 4.
$9 + 16 = c^2$	Complete the addition.
$25 = c^2$	Take the square root of both sides.
$5 = c$	

The hypotenuse has a length of 5 units.

b) You are given two side lengths of a right triangle and you are asked to find the third side length. Therefore, you can use the Pythagorean theorem again. However, since one of the side lengths that you are given is the hypotenuse, this time you need to ensure that you substitute 11, the length of the hypotenuse, for c in the equation $a^2 + b^2 = c^2$.

It does not matter whether you let a or b equal 6. For this example, let $b = 6$ and $c = 11$. The result is:

$a^2 + 6^2 = 11^2$	Square 6 and 11.
$a^2 + 36 = 121$	Subtract 36 from both sides of the equation.
$a^2 + 36 - 36 = 121 - 36$	Complete the subtraction.
$a^2 = 85$	Take the square root of both sides.
$a = 9.2$	

The missing side of the triangle has an approximate length of 9.2 units.

Notes

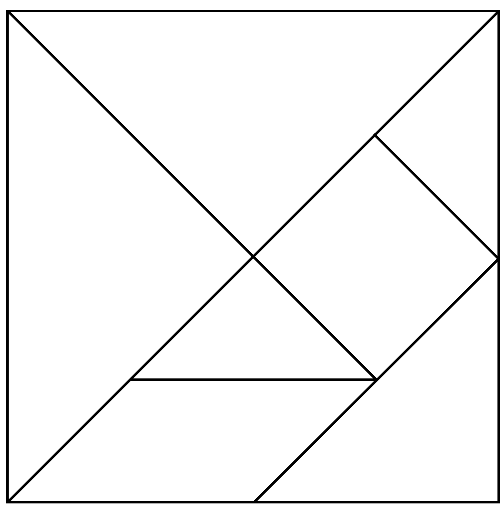
Module 4 Cover Assignment

Geometry of Tangrams

Total: 10 marks

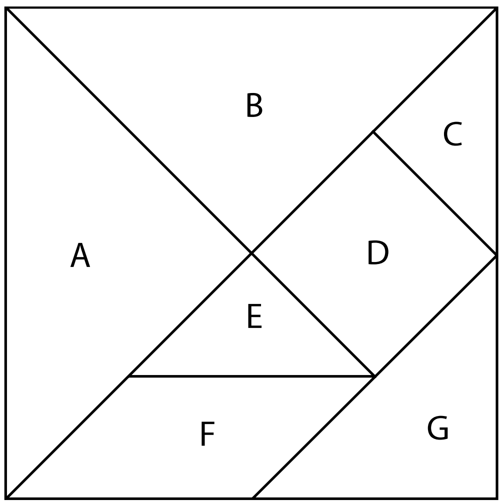
Clearly show the steps in your solution on the question sheets below and submit these pages when you send in your assignments for marking. Answers given without supporting calculations will not be awarded full marks.

1. Find the measurement of **all** interior angles of each tangram piece in the diagram below. Use a protractor. *(3 marks)*



Module 4 Cover Assignment: Geometry of Tangrams (continued)

2. If the perimeter of the small square (the square inside the shape) is 8 cm, determine the lengths of each side of every tangram tile. **Hint:** You will need to use your angle measurements from Question 1 to help you complete this question.
(7 marks)



Lesson 1: Angles and Triangles

Lesson Focus

In this lesson, you will
- ❑ learn about the property of vertically opposite angles
- ❑ apply the properties of angles in triangles

Lesson Introduction

If you watch snowboarding, you will most likely have heard references to terms such as the "backside 180"or "the 1260." These terms all have to do with angles and rotations.

If you rotate completely around once, you will make one circle and end up facing in the same direction as when you started. To rotate in one complete circle, you will have rotated 360°.

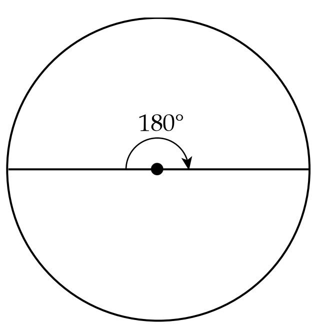
Therefore, the "backside 180" involves only half a rotation. To turn 180°, you will move halfway around a circle, and you will be facing the opposite direction from your starting point. Similarly, "the 1260" represents three and a half rotations ($3.5 \times 360° = 1260°$).

In this lesson, you will investigate the properties of angles and, in particular, how these properties relate to triangles.

Straight Lines and Angles

Think of a straight line.

Now think of this straight line as a diameter of a circle.



There are 360° in a complete circle. Therefore, there are 180° in a semi-circle, or between any two points on a straight line.

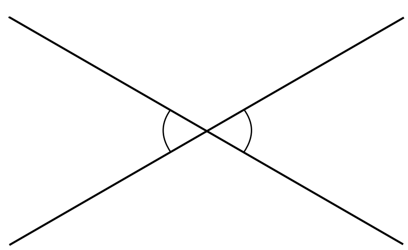
Mathematicians use the term **supplementary** angles to refer to any two angles that can combine to make a straight line since they have a sum of 180°. For example, angles of 60° and 120° are supplementary angles because their sum is 180°, and when placed beside each other they form a straight line. **Two angles whose sum is 180° are called supplementary angles.** You may want to add this information to your resource sheet.

Vertically Opposite Angles

Vertically opposite angles are the pairs of angles formed when two lines intersect. Vertically opposite angles are found on opposite sides of the intersecting point of the two intersecting lines. Unless the lines are perpendicular, one pair of the vertically opposite angles will be obtuse (greater than 90°) and the second pair of vertically opposite angles will be acute (less than 90°).

Knowing that there are 180° on a straight line, the following property of vertically opposite angles becomes apparent using deductive reasoning.

Vertically Opposite Angles Property: When two straight lines intersect, the pairs of angles that are vertically opposite from each other are equal. In the following diagram, the acute vertically opposite angles are shown to be of equal measure. The obtuse pair of vertically opposite angles are also of equal measure.

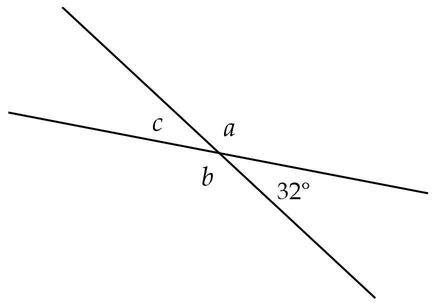


You may want to include the above property on your resource sheet.

Consider the following example.

Example 1

Determine the three missing angles in the diagram below.



Solution

Your first step is to determine either $\angle a$ or $\angle b$. Consider $\angle a$.

Since $\angle a$ and 32° are on a straight line, they are supplementary angles and must have a sum of 180°.

$\angle a + 32° = 180°$

$\angle a = 180° - 32°$

$\angle a = 148°$

Similarly, since $\angle b$ and 32° are on a straight line, they must also have a sum of 180°. Therefore, $\angle b = \angle a = 148°$.

Now consider $\angle c$. $\angle c$ is on a straight line with $\angle a$. Therefore, they are supplementary and must have a sum of 180°.

$\angle c + \angle a = 180°$

$\angle c + 148° = 180°$

$\angle c = 180° - 148°$

$\angle c = 32°$

From this example, you can see that the two angles opposite each other at the intersection point have the same angle measurement. In Learning Activity 4.1, you will use the same deductive reasoning as in the example above to show that this property for vertically opposite angles is true for any angle measures.

Triangles

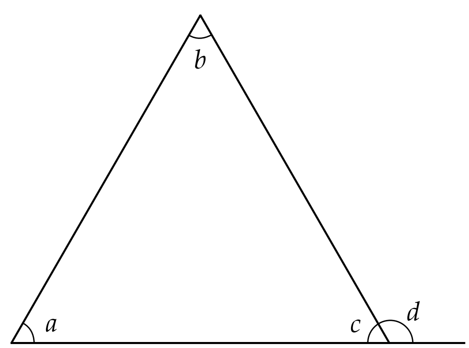
You are already familiar with some properties of triangles. For example, you know the following:

- A triangle is a three-sided shape.
- The sum of the three interior angles of a triangle is always 180°. (In Learning Activity 4.3, you will use the rule that there are 180° in a straight line to prove that the sum of the interior angles of a triangle is always 180°.)

A third property of triangles involves external angles and uses both your knowledge of the number of degrees in a straight line and your knowledge of the interior angles of a triangle.

- In any triangle, an **exterior angle** equals the sum of the two opposite interior angles.

Note: An exterior angle is created when you extend a side of a triangle. In the triangle below, $\angle d$ is an example of an exterior angle.



In this triangle, you know that $\angle a + \angle b + \angle c = 180°$. The exterior angle property tells us that $\angle a + \angle b = \angle d$.

Example 2

Prove that an exterior angle in a triangle is equal to the sum of the two opposite interior angles.

Solution

Use deductive reasoning as you did in Module 3. Two columns can be used to organize the reasons for each statement.

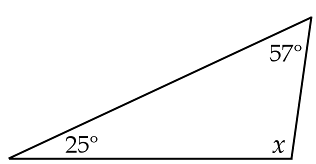
$\angle a + \angle b + \angle c = 180°$	The three interior angles of a triangle have a sum of 180° (accepted truth).
$\angle a + \angle b = 180° - \angle c$	Using algebra, subtract *c* from both sides of the equation.
$\angle c + \angle d = 180°$	Two angles on a straight line have a sum of 180° (accepted truth).
$\angle d = 180° - \angle c$	Using algebra, subtract *c* from both sides of the equation.
$\angle a + \angle b = \angle d$	Equate $\angle a + \angle b$ and $\angle d$, since lines 2 and 4 show they both equal $180° - \angle c$.

Therefore, an exterior angle in a triangle, $\angle d$, is equal to the sum of the two opposite interior angles, $\angle a$ and $\angle b$.

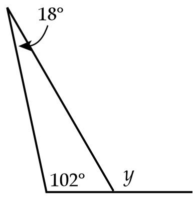
Example 3

Use the triangle properties that you have learned to determine the size of the indicated angles in each of the triangles below.

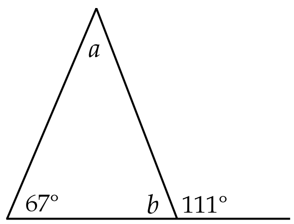
a) Find the missing angle x in the triangle below.



b) Find the missing angle y, an exterior angle to the triangle below.



c) Determine the missing angles a and b in the triangle below.



Solution

a) The three interior angles must have a sum of 180°.

$25° + 57° + x = 180°$	
$82° + x = 180°$	Simplify the left-hand side.
$x = 180° - 82°$	Subtract 82° from both sides.
$x = 98°$	Complete the subtraction.

b) As angle y is an exterior angle, it must be equal to the sum of the two opposite interior angles.

$y = 18° + 102°$	
$y = 120°$	

c) There are two ways to complete this problem. You can either find $\angle a$ first or find $\angle b$ first.

Method 1: Finding $\angle a$ first.

To find $\angle a$, recognize that the exterior angle of 111°, must equal the sum of the two opposite interior angles.

$111° = \angle a + 67°$	
$111° - 67° = \angle a$	Subtract 67° from both sides.
$44° = \angle a$	Complete the subtraction.

Then, to find $\angle b$, remember that all three interior angles must have a sum of 180°.

$\angle a + \angle b + 67° = 180°$	
$44° + \angle b = 180°$	Substitute the known value for $\angle a$.
$111° + \angle b = 180°$	Simplify the left-hand side.
$\angle b = 180° - 111°$	Subtract 111° from both sides.
$\angle b = 69°$	Complete the subtraction.

Method 2: Finding $\angle b$ first.

You can find $\angle b$ by recognizing that it is on a straight line with the 111° angle.

$\angle b + 111° = 180°$	The sum of two angles on a straight line is 180°.
$\angle b = 180° - 111°$	Subtract 111° from both sides.
$\angle b = 69°$	Complete the subtraction.

To find $\angle a$, recognize that all three interior angles of a triangle must have a sum of 180°.

$\angle a + \angle b + 67° = 180°$	
$\angle a + 69° + 67° = 180°$	Substitute the known value for $\angle b$.
$\angle a + 136° = 180°$	Simplify the left-hand side.
$\angle a = 180° - 136°$	Subtract 136° from both sides.
$\angle a = 44°$	Complete the subtraction.

Be sure to practise using the rules you just learned while completing the following learning activity.

Learning Activity 4.1

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Estimate 20% of 900.
2. Evaluate: $\dfrac{(9)(4)}{2} + 5$
3. Factor: $2x^2 + 4x + 4$
4. Solve: $x^2 - 1 = 15$
5. Solve: $\dfrac{1}{3}x - 1 = 5$
6. Jacob is 65 inches tall. Describe his height using feet and inches.
7. Convert 132.3 kilometres to centimetres.
8. Angle A and angle B are supplementary. If $\angle A = 30°$, what is the size of $\angle B$?

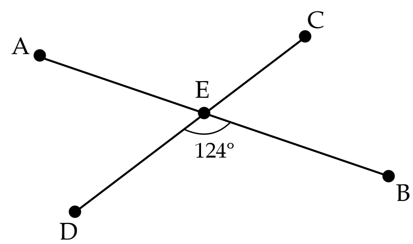
continued

Learning Activity 4.1 (continued)

Part B: Angles and Triangles

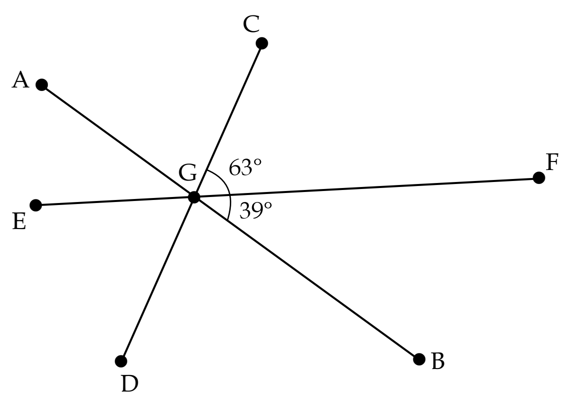
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Using the diagram shown below, determine the following angle measurements.



   a) ∠BED
   b) ∠AED
   c) ∠AEC
   d) ∠BEC

2. Using the diagram shown below, determine the following angle measurements.

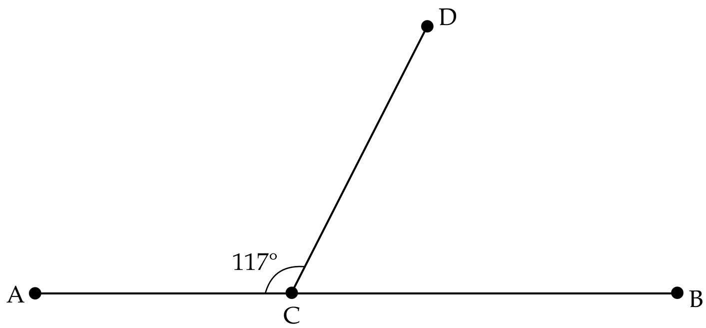


   a) ∠DGB
   b) ∠DGE
   c) ∠AGE
   d) ∠AGC

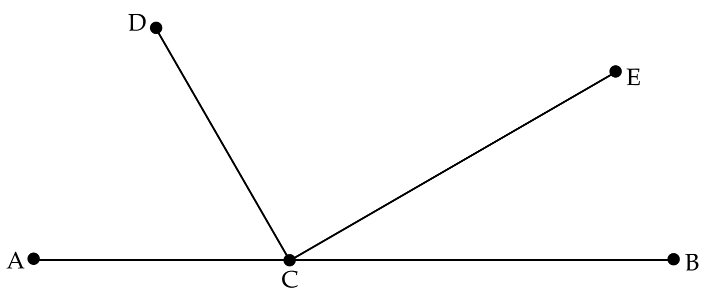
continued

Learning Activity 4.1 (continued)

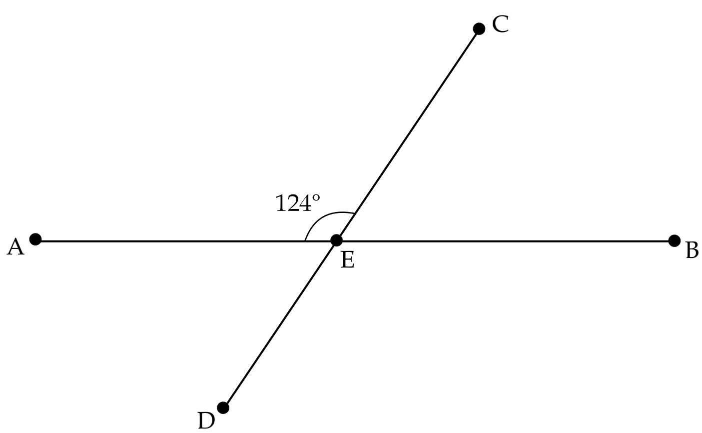
3. If $\angle DCB = 2x - 3°$, determine the value of x.



4. If $\angle ACD = 2x + 30°$ and $\angle BCE = -2x + 60°$, determine the measurement of $\angle DCE$.



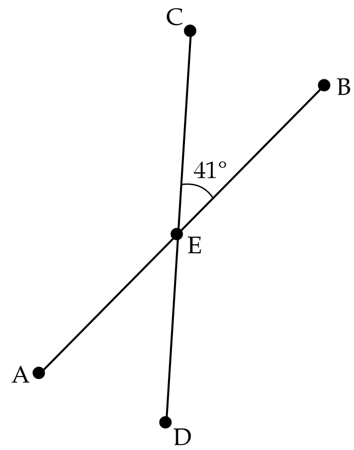
5. Find the measurement of $\angle BED$.



continued

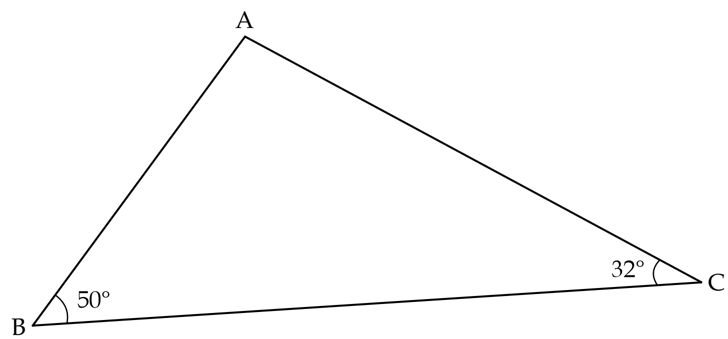
Learning Activity 4.1 (continued)

6. Find the measurement of ∠AED.



7. Prove that when two lines intersect to form vertically opposite angles, they are equal measure.

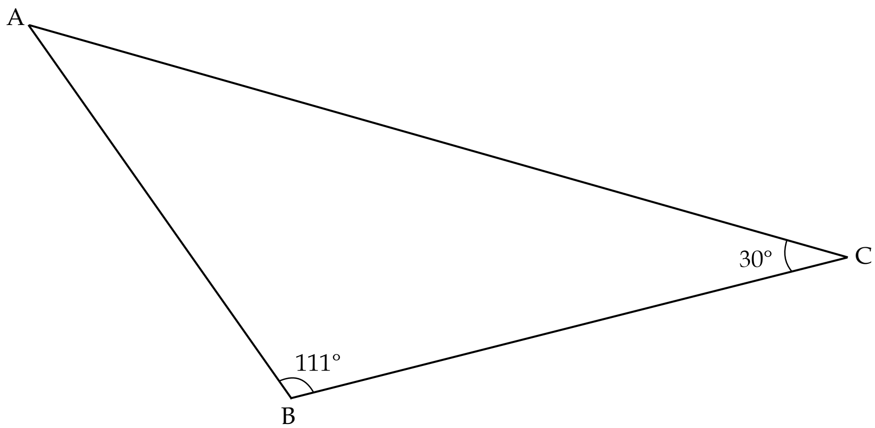
8. Determine the measurement of ∠A in the triangle below.



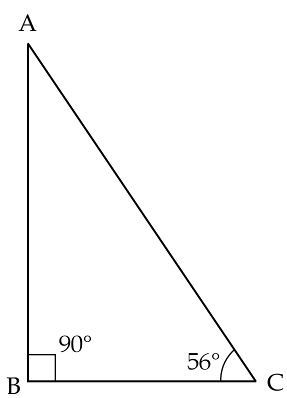
continued

Learning Activity 4.1 (continued)

9. Determine the measurement of ∠A in the triangle below.



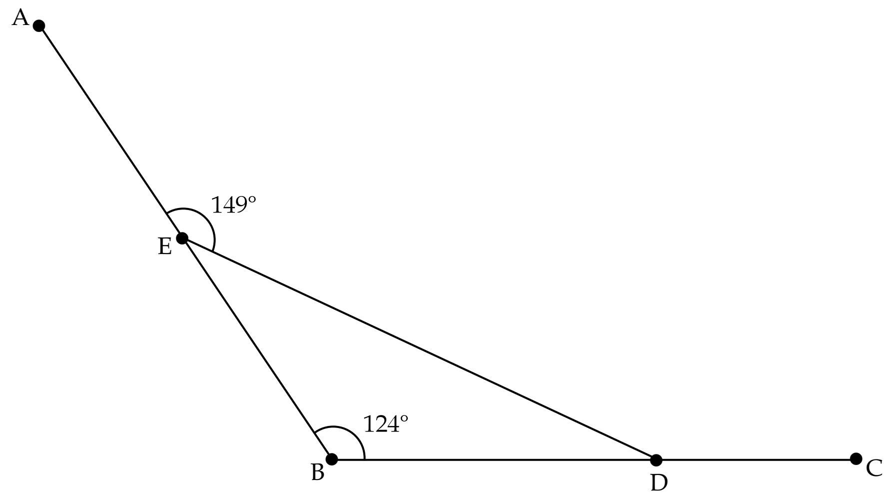
10. Determine the measurement of ∠A in the triangle below.



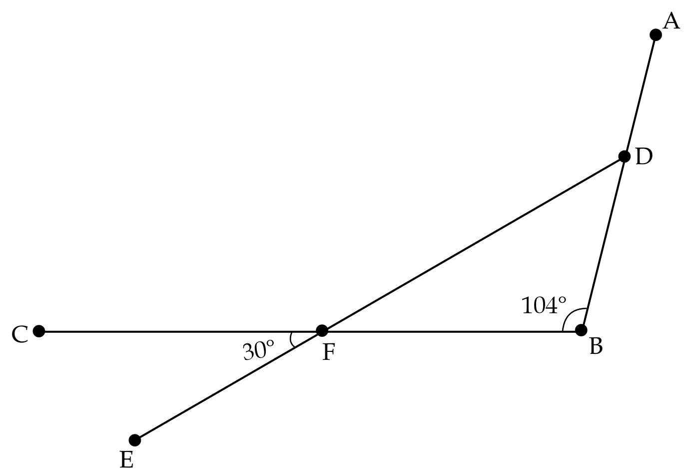
continued

Learning Activity 4.1 (continued)

11. Determine the measurement of exterior ∠CDE in the diagram below.



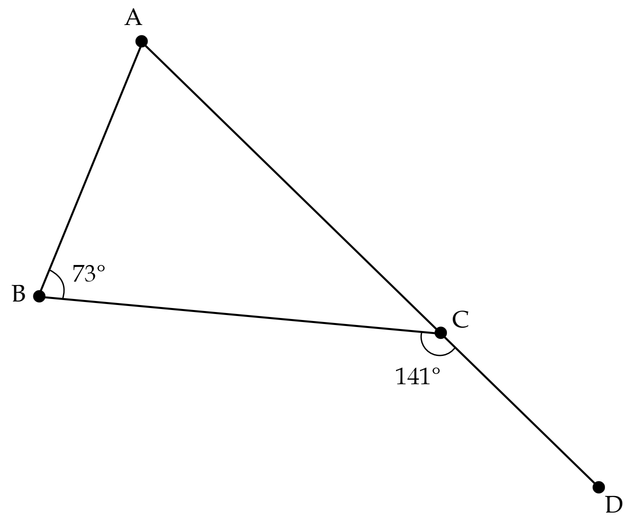
12. Determine the measurement of exterior ∠ADF in the diagram below.



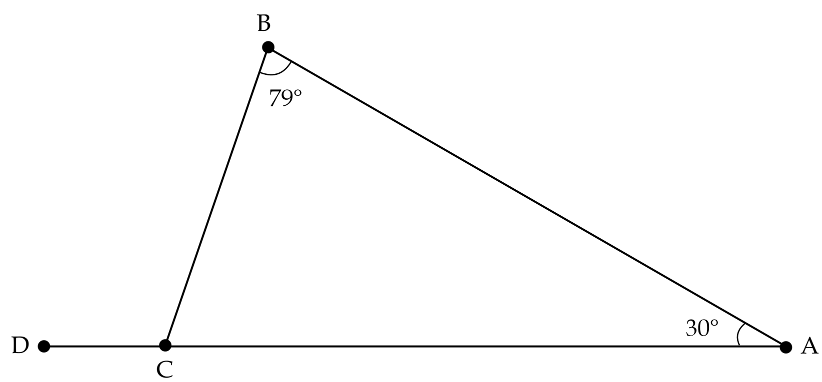
continued

Learning Activity 4.1 (continued)

13. Determine the measurement of ∠CAB in the diagram below.



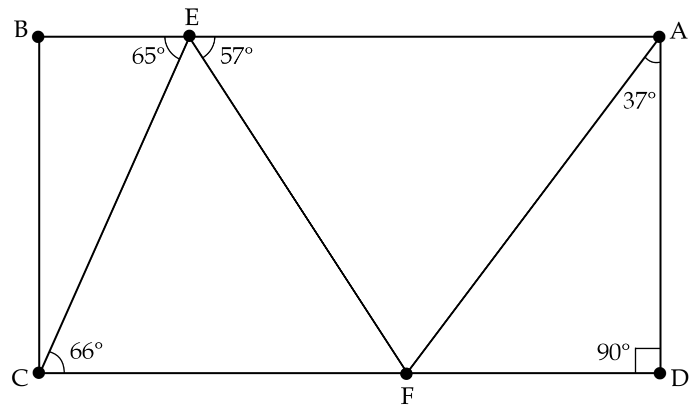
14. Determine the measurement of ∠BCD in the diagram below.



continued

Learning Activity 4.1 (continued)

15. A contractor is designing a truss and has constructed the diagram below. He needs to determine the missing interior angles to complete his diagram. Find the following angles: ∠DFA, ∠CEF, ∠EFC, ∠EFA, and ∠EAF.



Lesson Summary

In this lesson, you learned about properties of vertically opposite angles and supplementary angles. You used the Pythagorean theorem to find the lengths of sides in right triangles. You used the property of triangles that states that all interior angles of a triangle have a sum of 180° to learn about the properties of exterior angles. In Lesson 2, you will discover similar properties in polygons with more than three sides.

Notes

Lesson 2: Angle Properties of Polygons

Lesson Focus

In this lesson, you will

- ❑ develop a formula for finding the sum of the interior angles of a polygon
- ❑ determine the sum of the exterior angles of a polygon
- ❑ use the measurements of the interior and exterior angles of a polygon to solve problems

Lesson Introduction

In the previous lesson, you proved properties involving the interior and exterior angles of triangles. You can use these properties to develop general relationships involving the interior and exterior angles of other polygons.

Polygons and Their Angles

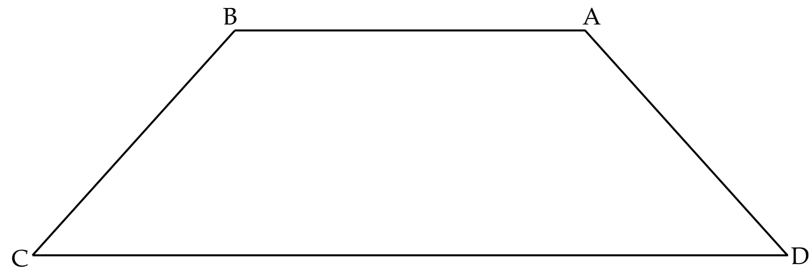
First, you are going to look at the interior angles of polygons. You already know that the sum of the interior angles in any triangle is 180°. You may also know that the sum of the interior angles in any **quadrilateral** (four-sided polygon) is 360°. Now, you will complete an investigation that will show you a pattern of the sum of the interior angles based on the number of sides in any polygon.

To complete this investigation, you will be looking at polygons with four, five, six, and seven sides.

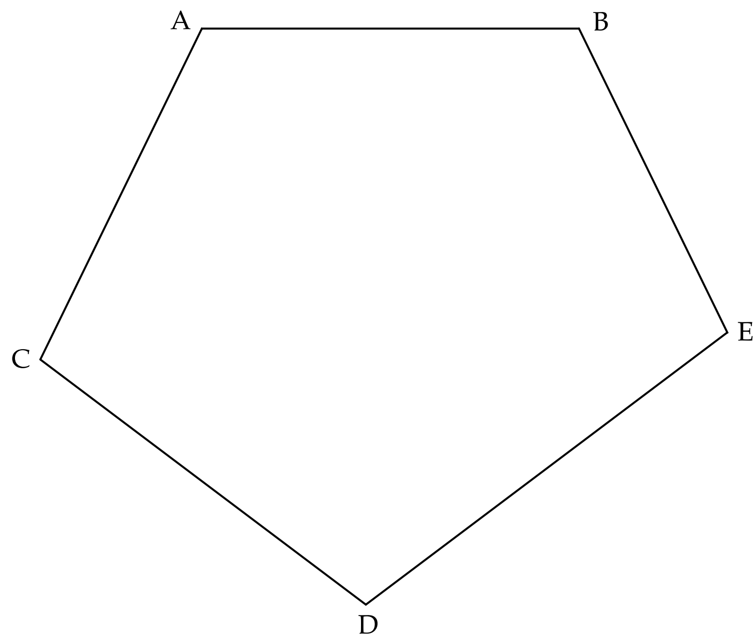
Example 1

Find the sum of the interior angles of the four polygons below by first measuring each interior angle with a protractor and then adding the interior angles in each polygon.

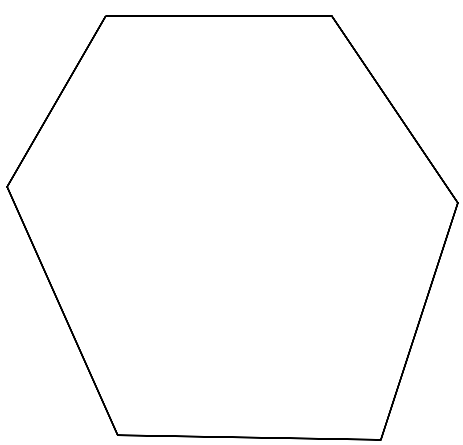
a) Quadrilateral



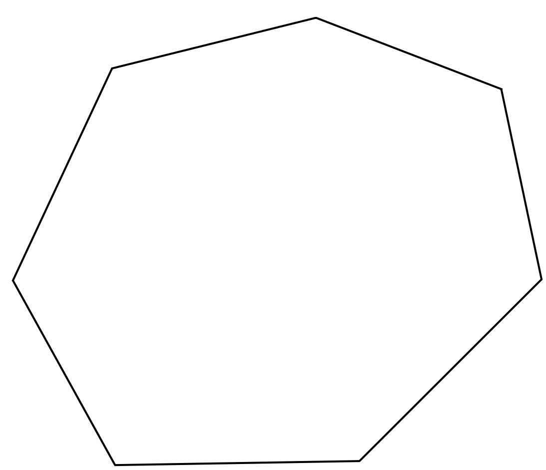
b) Pentagon



c) Hexagon

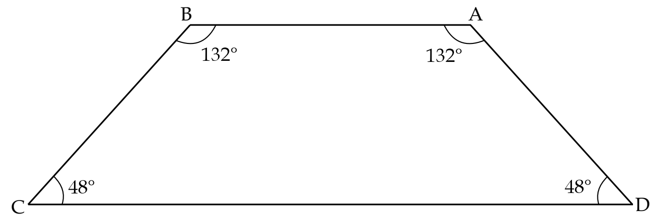


d) Heptagon

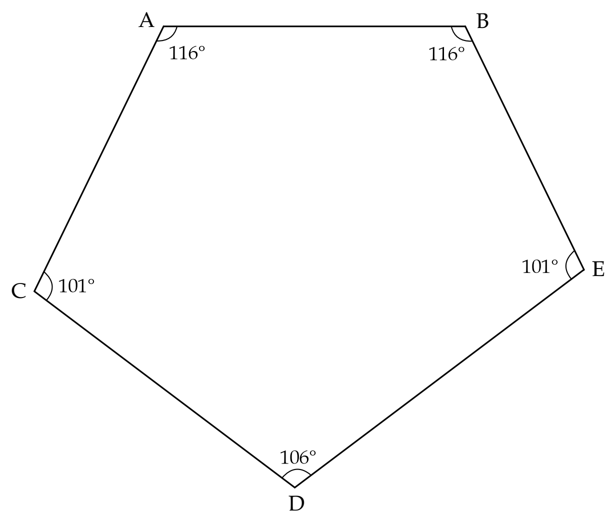


Solution

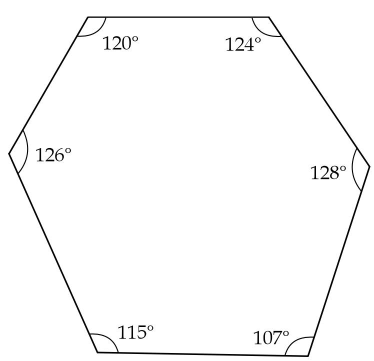
a) The sum of the interior angles is 48° + 132° + 132° + 48° = 360°.



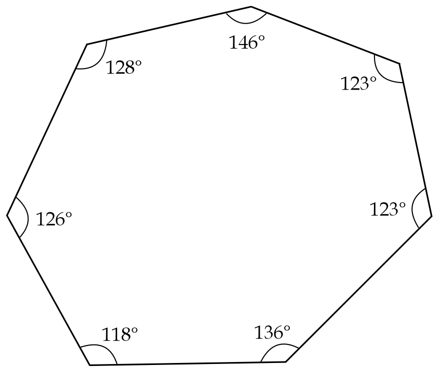
b) The sum of the interior angles is 116° + 116° + 101° + 101° + 106° = 540°.



c) The sum of the interior angles is 120° + 124° + 128° + 107° + 115° + 126° = 720°.



d) The sum of the interior angles is 128° + 146° + 123° + 123° + 136° + 118° + 126° = 900°.



All of the information from Example 1 can be compiled in a chart to help you recognize the pattern.

Polygon	Number of Sides	Sum of Interior Angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
Heptagon	7	900°

Using inductive reasoning skills, can you determine a pattern of how the sum of the interior angles relates to the number of sides in a polygon?

Notice that the sum of the interior angles is always a multiple of 180°. Each time the number of sides of a polygon increases by 1, the sum of the interior angles increases by 180°.

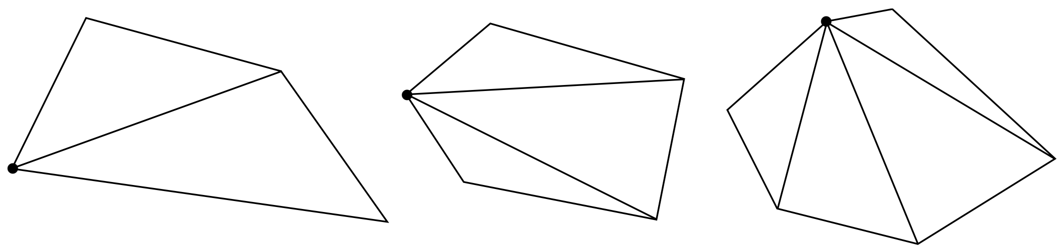
Consider the following pattern:

Triangle	Sum of Interior Angles: 180°(1)	Number of sides: 3
Quadrilateral	Sum of Interior Angles: 180°(2)	Number of sides: 4
Pentagon	Sum of Interior Angles: 180°(3)	Number of sides: 5
Hexagon	Sum of Interior Angles: 180°(4)	Number of sides: 6
Heptagon	Sum of Interior Angles: 180°(5)	Number of sides: 7

Each time, the sum of the interior angles is equal to 180°, multiplied by two less than the number of sides in the polygon. This can be expressed by the following formula:

Sum of the interior angles of a polygon with n sides: $S = (n - 2)(180°)$

It is a common practice in mathematics to recognize a pattern using inductive reasoning and then to use deductive reasoning to justify that the pattern will always work. The pattern above can be justified using deductive reasoning by drawing triangles inside each polygon. Draw the triangles with diagonals all starting at one vertex. When drawn in this fashion, the sum of the interior angles of the polygon is the same as the sum of the interior angles of all of the triangles.



Every quadrilateral has 4 sides and 2 triangles. Every pentagon has 5 sides and 3 triangles. Every hexagon has 6 sides and 4 triangles.

Since the sum of the interior angles of one triangle is 180°, the sum of the interior angles of each polygon is the number of triangles multiplied by 180. The number of triangles is always 2 less than the number of sides of the polygon. Hence the formula $S = (n - 2)(180°)$ is justified using deductive reasoning.

You may wish to add this formula to your resource sheet.

Example 2

Determine the sum of the interior angles of a polygon with the indicated number of sides.

a) 14

b) 20

Solution

a) If a polygon has 14 sides, then using the formula $S = (n - 2)(180°)$, where $n = 14$, you would get:

$S = (14 - 2)(180°)$

$S = 12(180°)$

$S = 2160°$

b) If a polygon has 20 sides, then using the formula $S = (n - 2)(180°)$, where $n = 20$, you would get:

$$S = (20 - 2)(180°)$$
$$S = 18(180°)$$
$$S = 3240°$$

Example 3

Determine the number of sides a polygon has if the sum of the interior angles is 3960°.

Solution

If the sum of the interior angles of a polygon is 3960°, then $S = 3960°$ in the formula $S = (n - 2)(180°)$.

$3960° = (n - 2)(180°)$	
$\dfrac{3960°}{180°} = \dfrac{(n-2)(180°)}{180°}$	Divide both sides of the equation by 180°.
$22 = n - 2$	Simplify.
$22 + 2 = n$	Add 2 to both sides of the equation.
$24 = n$	Complete the addition.

Therefore, if the sum of the interior angles of a polygon is 3960°, the polygon must have 24 sides.

Exterior Angles of Polygons

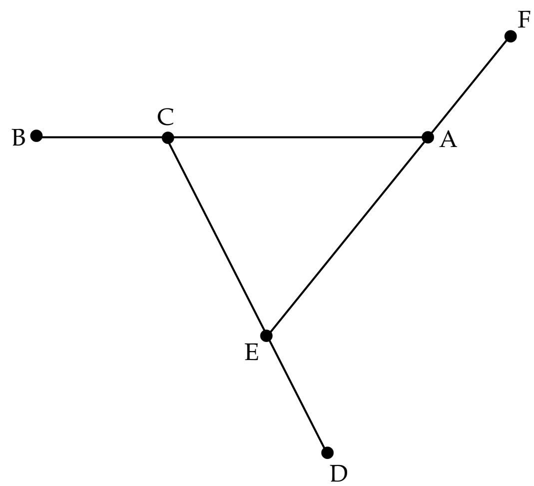
Now that you have developed a formula for the sum of the interior angles of a polygon, you will look at the exterior angles of a polygon.

Consider three shapes: a triangle, a quadrilateral, and a pentagon.

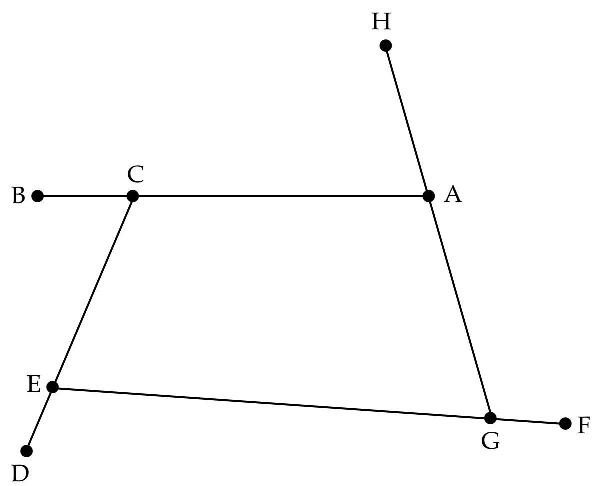
Example 4

Use a protractor to determine the measurement of all exterior angles in each shape shown below. Then, find the sum of the exterior angles for each shape.

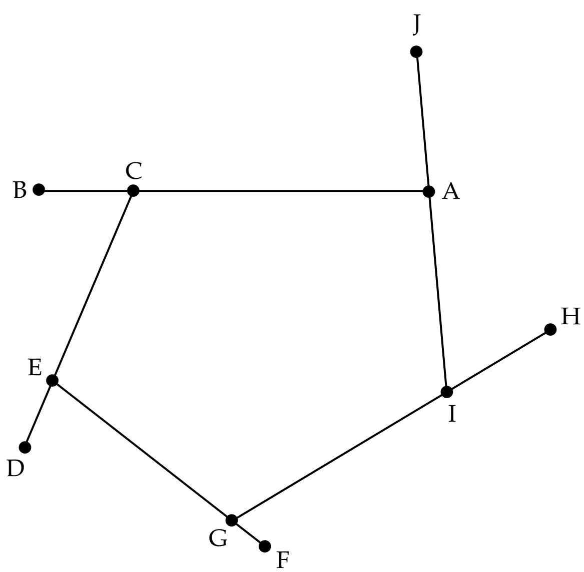
a)



b)

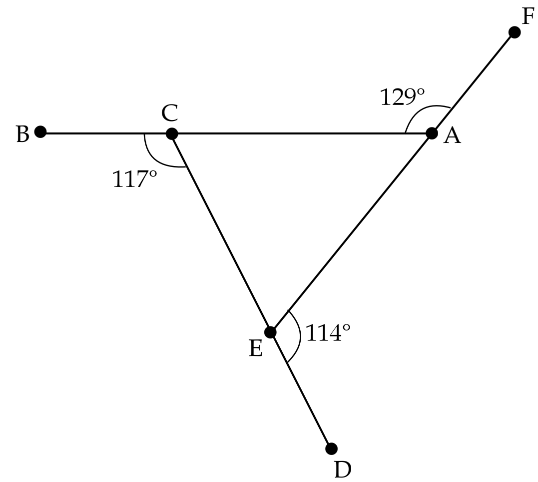


c)

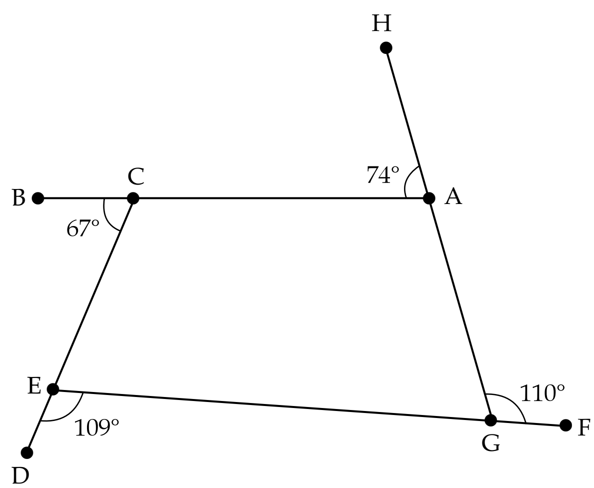


Solution

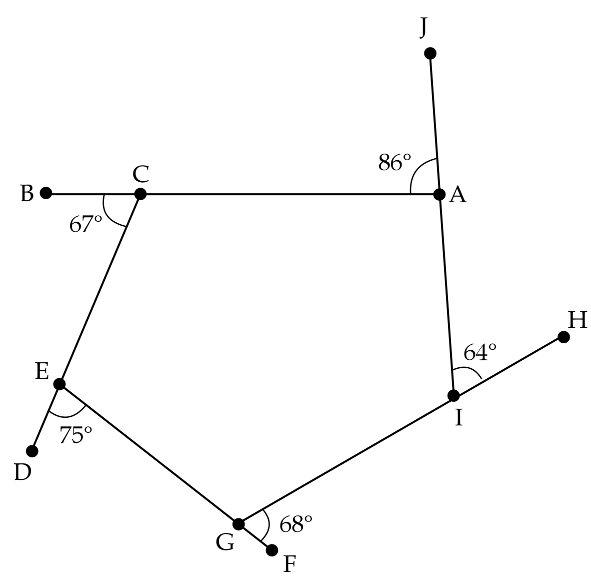
a) The sum of the exterior angles is 117° + 129° + 114° = 360°.



b) The sum of the exterior angles is 67° + 74° + 110° + 109° = 360°.



c) The sum of the exterior angles is 67° + 86° + 64° + 68° + 75° = 360°.

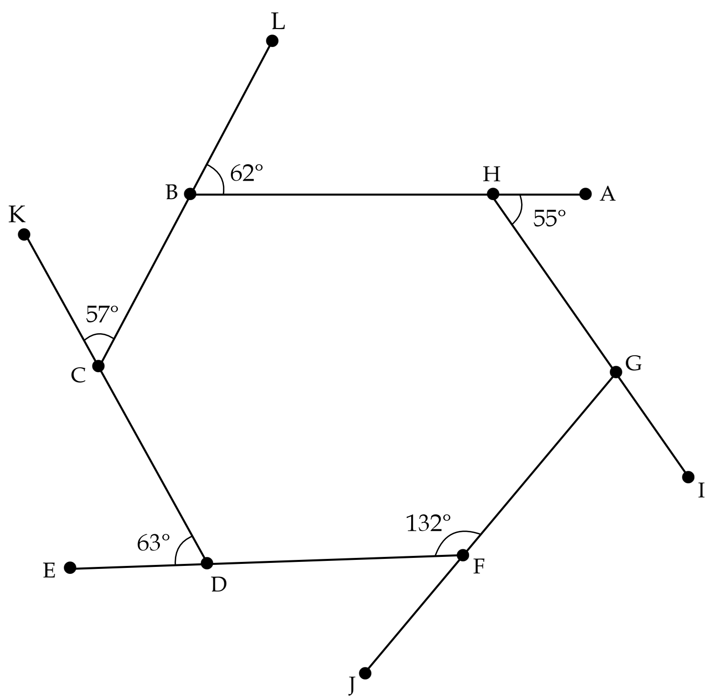


After completing Example 4, can you come up with a conjecture about the sum of the exterior angles of a polygon?

Using inductive reasoning skills, you could have determined that **the sum of the exterior angles of a polygon will always equal 360°.** This is always true. Think of starting at a vertex of one of the polygons and travelling along its edge to the next vertex. Then, turn at an angle equal to the exterior angle to travel along the next edge. When you get back to the original vertex and turn, you will be facing in the same direction as when you started. You will have turned all the way around, or 360°. You may want to add this rule to your resource sheet.

Example 5

Determine the measurement of ∠FGI in the diagram below.



Solution

∠FGI is an exterior angle. In this hexagon, all six exterior angles must have a sum of 360°.

You know the value of four exterior angles: 62°, 55°, 63°, and 57°. The fifth exterior angle, ∠DFJ can be found by using supplementary angles.

$\angle DFJ + 132° = 180°$

$\angle DFJ = 180° - 132°$

$\angle DFJ = 48°$

Now, you can state that:

$62° + 55° + 63° + 57° + 48° + \angle FGI = 360°$	
$285° + \angle FGI = 360°$	Simplify the left-hand side.
$\angle FGI = 360° - 285°$	Subtract 285° from both sides.
$\angle FGI = 75°$	Complete the subtraction.

Make sure you complete the following learning activity to practise the rules of interior and exterior angles of polygons that you just learned in this lesson.

Learning Activity 4.2

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

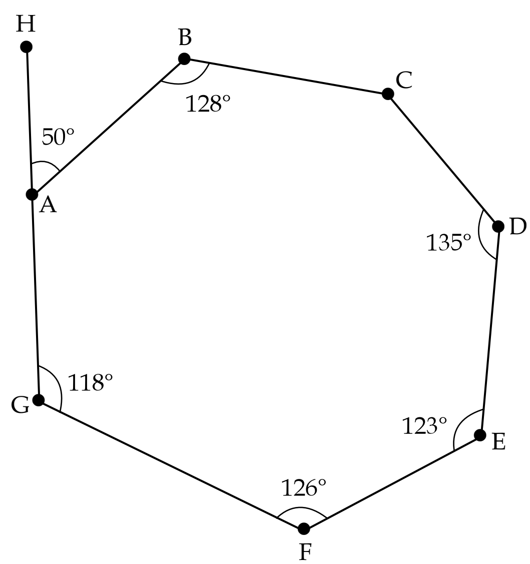
1. Estimate 30% of 1230.
2. Evaluate: $\left[8\left(\frac{1}{2}\right)\right]^2 + 7$
3. If two interior angles of a triangle each measure 70°, what is the measurement of the third angle?
4. Angle A and angle B are vertically opposite angles. If $\angle A = 30°$, what is the size of $\angle B$?
5. Solve for y: $\frac{1}{2}y + 3 = 6$
6. Is 5 a root of the equation $3x + 15 = 0$?
7. Is $0.\overline{6}$ less than, equal to, or greater than $\frac{6}{10}$?
8. Evaluate: $-7 + (-9) - (-2) + 3$

continued

Part B: Interior and Exterior Angles of Polygons

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. What is the sum of the exterior angles in a polygon with 46 sides?
2. The sum of the interior angles of a polygon is 1980°. How many sides does the polygon have?
3. Is it possible for the sum of the interior angles of a polygon to equal 1700°? Explain.
4. If the interior angles of a hexagon are all congruent, what is the size of each interior angle?
5. Determine the size of ∠BCD in the polygon below.



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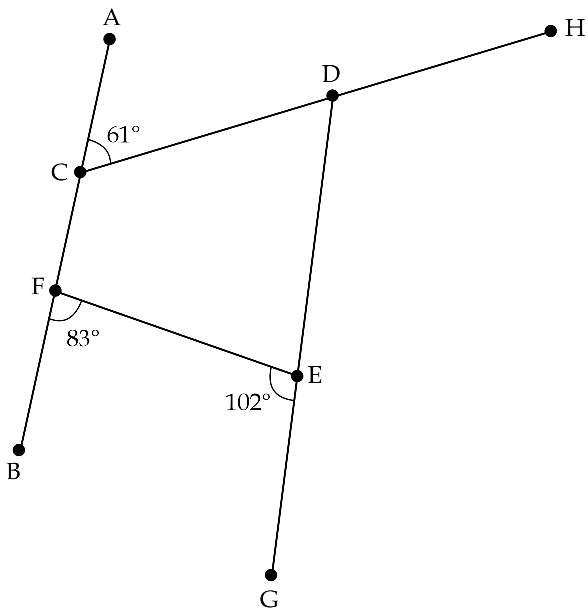
Learning Activity 4.2 (continued)

6. Determine the size of ∠EDH.



7. What is the measurement of each exterior angle in a regular hexagon? **Note:** A regular polygon is a polygon where all of the side lengths and all of the interior angle measurements are equal.

8. Determine the size of each exterior angle of a regular polygon with 15 sides.

9. If each exterior angle of a regular polygon is 45°, how many sides does it have?

Lesson Summary

In this lesson, you found and used the formula for finding the sum of the interior angles of a polygon with three sides or more. You learned that the sum of the exterior angles of any polygon is always 360°. You then used these properties to solve various problems involving polygons. In the next lesson, you will build on this knowledge by studying parallel lines and their properties.

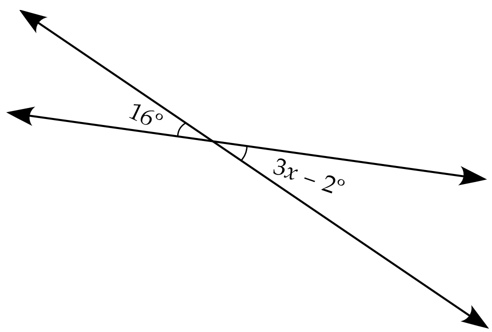
Assignment 4.1

Polygons and Angles

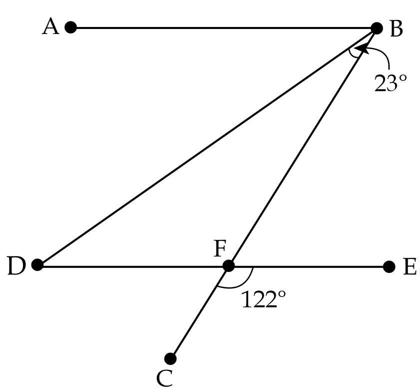
Total: 23 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. Solve for x in the diagram below. *(2 marks)*



2. Consider the following diagram.



a) Determine the size of the following angles. Show the steps in your calculations and the properties used to determine each angle. *(6 marks).*

i) ∠DFB

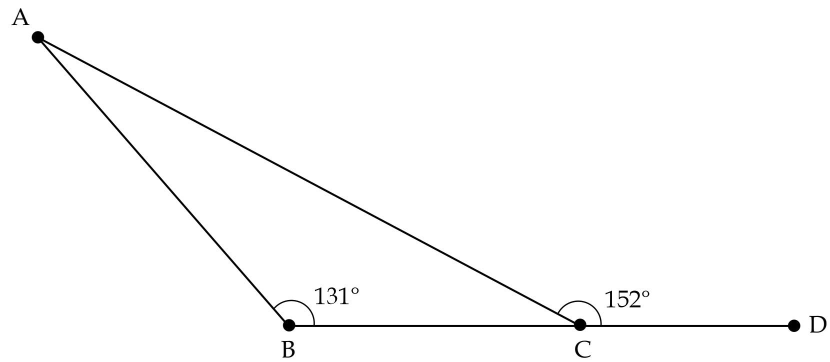
ii) ∠CFD

iii) ∠FDB

3. Janice says it is possible to draw a triangle with two right angles. Do you agree? Explain. *(2 marks)*

Assignment 4.1: Polygons and Angles (continued)

4. Consider the following diagram.



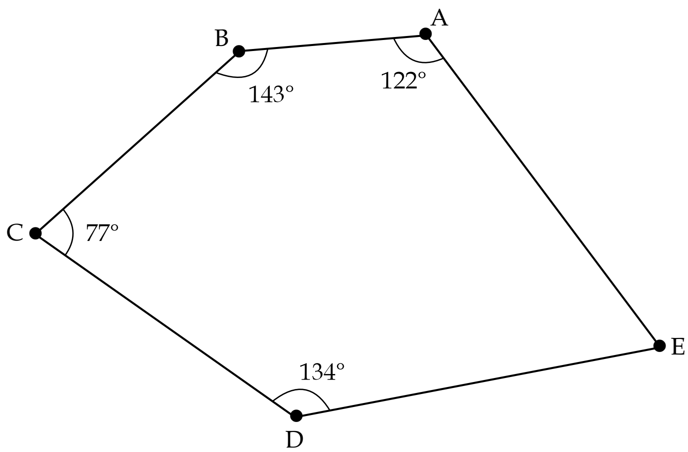
   a) Determine the size of ∠BAC. Show your work. *(2 marks)*

   b) Determine the measurement of ∠ACB. *(1 mark)*

5. If a polygon has 18 sides, what is the sum of the interior angles? *(2 marks)*

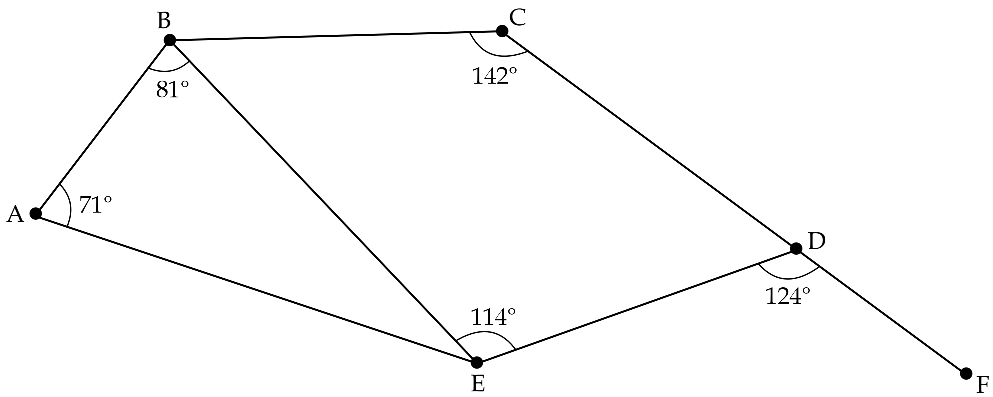
Assignment 4.1: Polygons and Angles (continued)

6. Determine the measurement of the missing interior angle in the polygon below. *(2 marks)*



Assignment 4.1: Polygons and Angles (continued)

7. Consider the following diagram. Determine the indicated angle measurements. Remember to show your work.



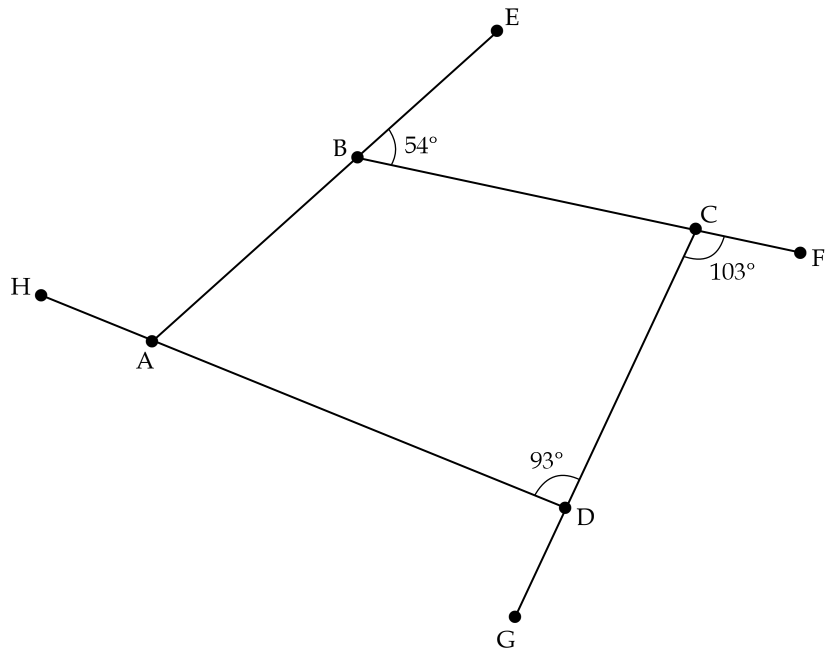
a) ∠AEB *(1 mark)*

b) ∠EDC *(1 mark)*

c) ∠EBC *(1 mark)*

Assignment 4.1: Polygons and Angles (continued)

8. Determine the size of ∠HAB in the diagram below. Show all work. *(3 marks)*



Lesson 3: Exploring and Proving Properties of Parallel Lines and Transversals

Lesson Focus

In this lesson, you will

- ❑ learn about the various properties of parallel lines and transversals including corresponding angles, alternate interior angles, alternate exterior angles, and same-side interior angles
- ❑ use the properties of parallel lines and transversals to prove certain angles are equal

Lesson Introduction

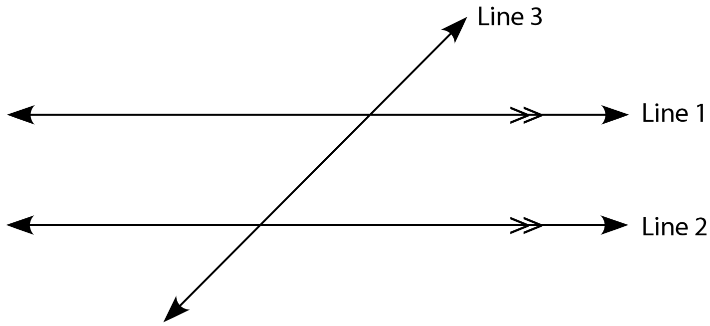
You may not have noticed this, but parallel lines are prevalent everywhere in your everyday life. Fence posts are parallel to each other, the walls of your house are parallel to each other, and parking lots contain multiple sets of parallel lines. What would happen if these objects were not parallel? Some possible scenarios are that houses would be leaning to one side, parking lots would be full of haphazardly parked cars, and construction as we know it would not exist.

Parallel lines have many properties, especially when crossed by a line called a transversal. In this lesson, you will be learning about these properties.

Parallel Lines and Transversals

In Euclidean geometry, which is the geometry that you have been studying in school, **parallel lines** are lines (of infinite length) in the same plane that do not intersect and, as such, are always the same distance away from each other.

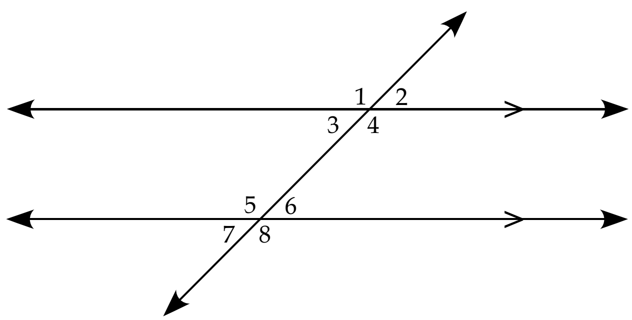
A **transversal** is the name given to a line that crosses through two other lines, all in the same plane (parallel or not).



In the image above, Line 1 and Line 2 are parallel lines. Parallel lines are often indicated by using the notation of matching arrows on each parallel line as shown. Line 3 crosses through both Line 1 and Line 2 and is thus called a transversal.

Consider the eight angles created when the transversal crosses the two parallel lines. These angles have special relationships to one another.

Use a protractor to measure the eight angles below and record their measurements.



$\angle 1$: ______ $\angle 2$: ______ $\angle 3$: ______

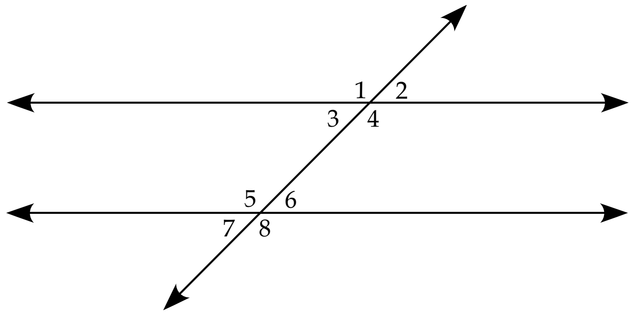
$\angle 4$: ______ $\angle 5$: ______ $\angle 6$: ______

$\angle 7$: ______ $\angle 8$: ______

What do you notice? Which angles are equal?

Since the two lines are parallel, you will have found that angles 1, 4, 5, and 8 have the same measurement. Also, angles 2, 3, 6, and 7 have the same measurement. Throughout this lesson, you will be analyzing and proving the relationships among the sets of angles formed when a transversal crosses parallel lines.

When two parallel lines are crossed by a transversal, four **interior** angles and four **exterior** angles are created. The **interior angles** are the angles created by the transversal that are “inside” or between the parallel lines. In the diagram below, the interior angles are numbered 3, 4, 5, and 6.

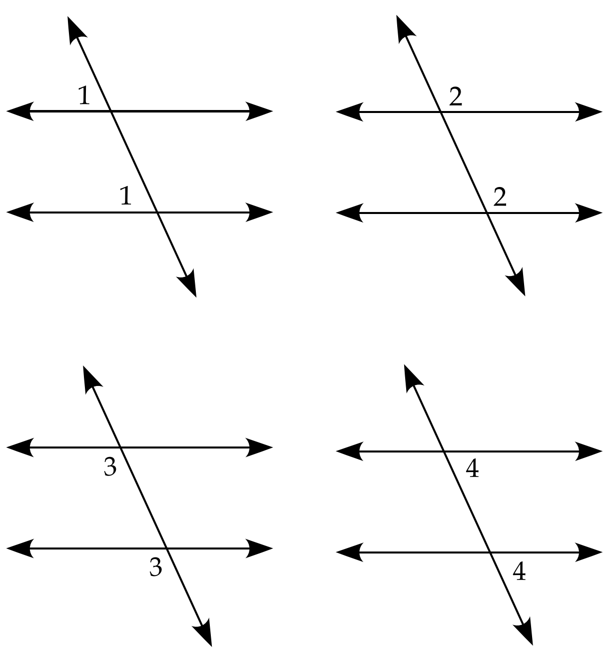


The **exterior angles** are the angles created by the transversal that are “outside” the parallel lines. In the diagram above, the exterior angles are numbered 1, 2, 7, and 8.

Corresponding Angles

When a transversal crosses a pair of parallel lines, the corresponding angles formed by each parallel line and the transversal are equal. This is always true when the lines are parallel and never true when the lines are not parallel.

The four pairs of angles in the diagrams below represent corresponding angles. As you can see from the diagrams below, corresponding angles are on the same side of the transversal and both are either above or below one of the parallel lines. Also, one will be an interior angle and one will be an exterior angle.

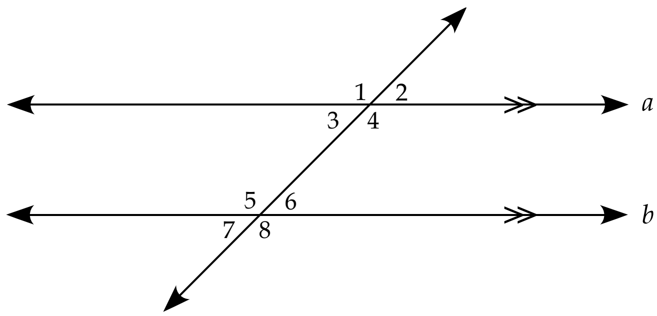


When the lines are parallel, the corresponding angles have the same measurement. This is a property of parallel lines and is, therefore, always true. Therefore, when the lines are parallel, the upper-left angles (number 1s) are the same size, the upper-right angles (number 2s) are the same size, the lower-left angles (number 3s) are the same size, and the lower-right angles (number 4s) are the same size.

You may wish to include the above properties on your resource sheet.

Example 1

Prove $\angle 3 = \angle 6$, given that lines *a* and *b* are parallel.



Solution

To prove $\angle 3 = \angle 6$, you will use deductive reasoning skills. Set up a chart similar to the one below.

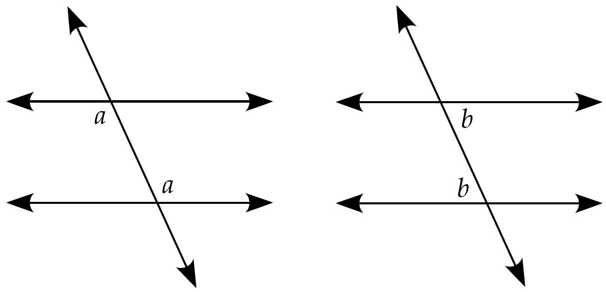
Line *a* is parallel to line *b*.	Given.
$\angle 3 = \angle 2$	Vertically opposite angles are equal.
$\angle 2 = \angle 6$	Corresponding angles of parallel lines are equal.
$\angle 3 = \angle 6$	Both $\angle 3$ and $\angle 6$ are equal to the same thing, $\angle 2$ (called the transitive property).

Note: The transitive property states that if $a = b$ and $b = c$, then a must equal c.

As you will see in the next section, the pair of angles identified in the proof is an example of a pair of **alternate interior angles**. This proof can be adapted to prove that any pair of alternate interior angles formed by a transversal and parallel lines have equal measure.

Alternate Interior Angles

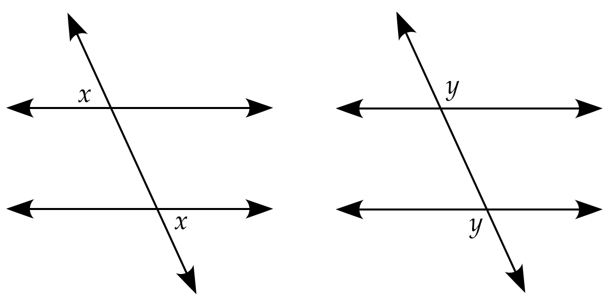
Alternate interior angles are two angles on alternate sides of the transversal on the "inside" of the set of parallel lines. When the lines are parallel, the alternate interior angles will always be equal (as proven in the previous example).



You may want to include this property on your resource sheet.

Alternate Exterior Angles

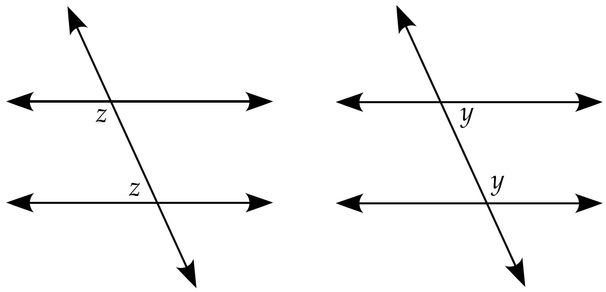
Alternate exterior angles are two angles on alternate sides of the transversal on the "outside" of the set of parallel lines. Alternate exterior angles are always equal. The proof used in Example 1 can be adapted to prove that any pair of alternate exterior angles formed by a transversal and two parallel lines have equal size.



You may want to include this property on your resource sheet.

Same-Side Interior Angles (Consecutive Interior Angles)

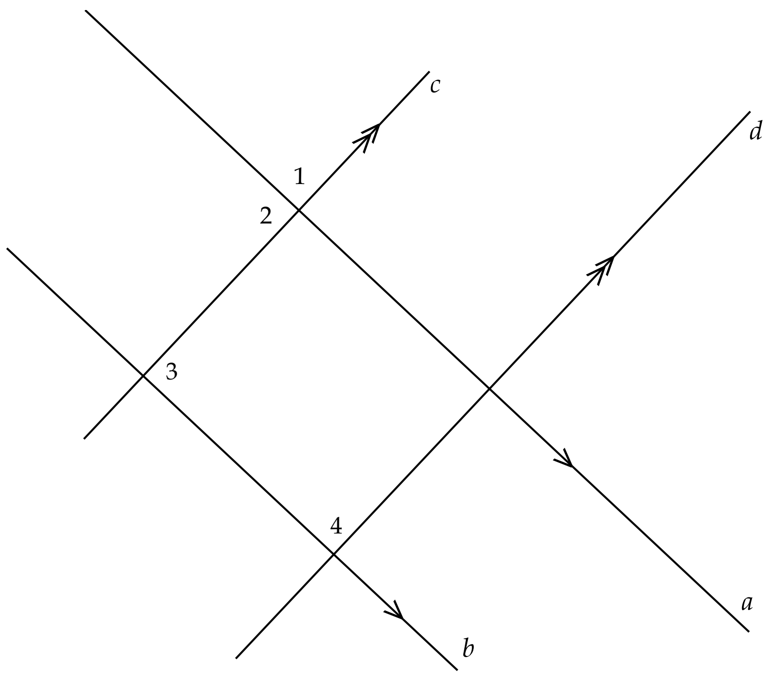
Same-side interior angles, also sometimes called **consecutive interior angles**, are two angles found on the same side of the transversal on the "inside" of the parallel lines. Same-side interior angles are supplementary. In other words, these two angles always add up to 180°.



You may want to add this rule to your resource sheet.

Example 2

Using the following diagram, prove that $\angle 1 = \angle 4$ given that lines c and d are parallel and lines a and b are parallel.



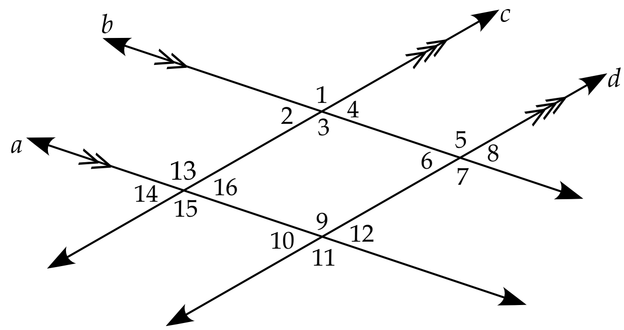
Solution

To prove $\angle 1 = \angle 4$, use deductive reasoning skills. Set up a two-column proof similar to the one below.

Line *a* is parallel to line *b*.	Given.
$\angle 1 + \angle 2 = 180°$	They are supplementary angles.
$\angle 2 = \angle 3$	They are alternate interior angles of parallel lines.
$\angle 1 + \angle 3 = 180°$	Substitute the second equation ($\angle 2 = \angle 3$) into the first equation.
$\angle 1 = 180° - \angle 3$	Rearrange the above equation.
Line *c* is parallel to line *d*.	Given.
$\angle 3 + \angle 4 = 180°$	They are same-side interior angles of parallel lines.
$\angle 4 = 180° - \angle 3$	Rearrange the above equation.
$\angle 1 = \angle 4$	From line 5 and line 8, you can see that $\angle 1$ and $\angle 4$ are equal to the same expression.

Example 3

Consider the diagram below. Lines *a* and *b* are parallel and lines *c* and *d* are parallel. Prove ∠5 = ∠13.



Solution

To prove ∠5 = ∠13, use deductive reasoning skills. Set up a two-column proof similar to the one below.

Line *a* is parallel to line *b*.	Given.
∠5 = ∠9	They are corresponding angles.
Line *c* is parallel to line *d*.	Given.
∠9 = ∠15	They are alternate interior angles.
∠5 = ∠15	The transitive property (both equal ∠9).
∠15 = ∠13	They are vertically opposite angles.
∠5 = ∠13	The transitive property (both equal ∠15).

In the following learning activity, you will practise using and naming these angle properties. Make sure you practise these skills and understand all the properties, as you will be using them again in the next lesson.

Learning Activity 4.3

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Solve: $\frac{3}{5}x = 9$

2. What is 25% of 900?

3. Find the zeros of $(x - 2)(2x + 1) = 0$.

4. Which is the better deal: 20% off \$3.99 or a sale price of \$3.29?

5. Write the fraction $2\frac{2}{5}$ as a decimal.

6. Write the fraction $2\frac{2}{5}$ as a percent.

7. Evaluate: $\left(\frac{8}{3}\right)(27)$

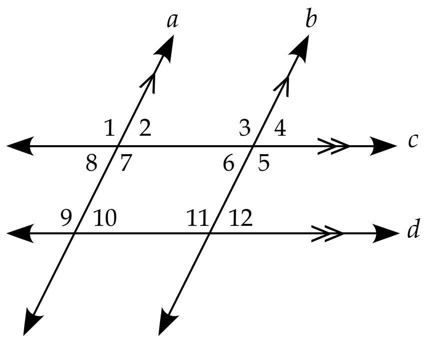
8. You buy a new pair of jeans for \$47.55 including tax. You give the cashier a \$50 bill. How much change should you get back?

continued

Part B: Proving Properties of Parallel Lines and Transversals

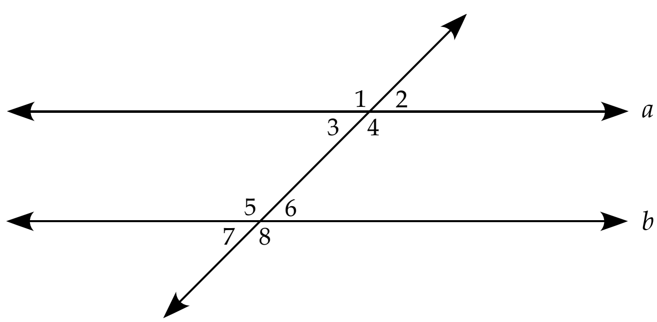
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Consider the diagram below.



   a) What kind of angles are ∠1 and ∠7?
   b) What kind of angles are ∠1 and ∠9?
   c) What kind of angles are ∠8 and ∠10?
   d) What kind of angles are ∠11 and ∠12?
   e) What kind of angles are ∠6 and ∠11?
   f) Label an angle, ∠13, such that it would be an alternate exterior angle with ∠1.
   g) Label an angle, ∠14, such that it would be a corresponding angle with ∠6.

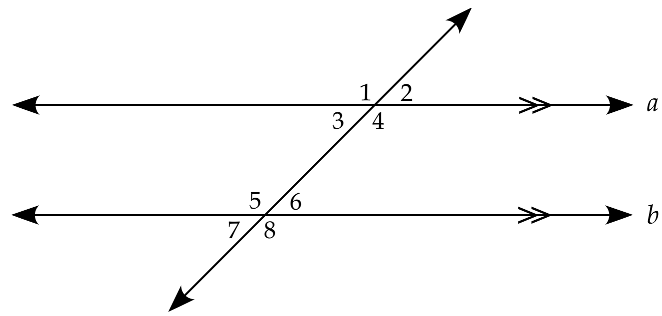
2. Prove ∠1 = ∠8, given that lines *a* and *b* are parallel. In other words, prove that a pair of alternate exterior angles is always equal.



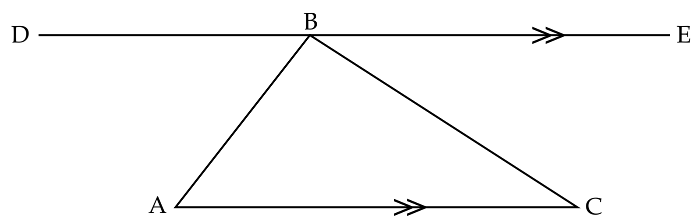
continued

Learning Activity 4.3 (continued)

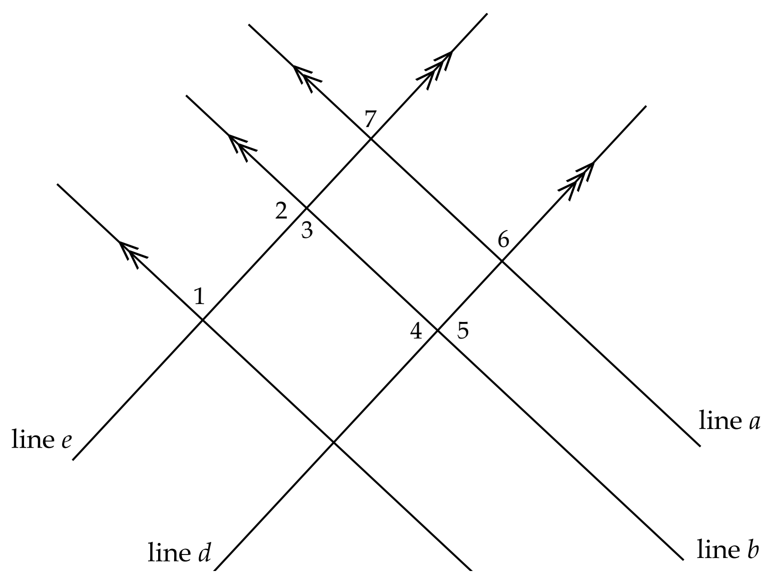
3. Using the following diagram, prove that ∠3 and ∠5 are supplementary angles, given that lines *a* and *b* are parallel. In other words, prove that a pair of same-side interior angles is always supplementary.



4. Using the diagram below, prove that the sum of angles in a triangle equals 180°, where Line DE is drawn through B so that it is parallel to Line AC.



5. Consider the diagram below. Lines *a*, *b*, and *c* are parallel. Lines *d* and *e* are parallel.

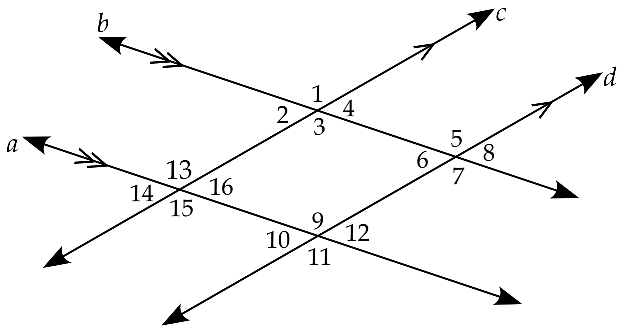


   a) Prove ∠1 = ∠7.
   b) Prove ∠5 is supplementary to ∠3.

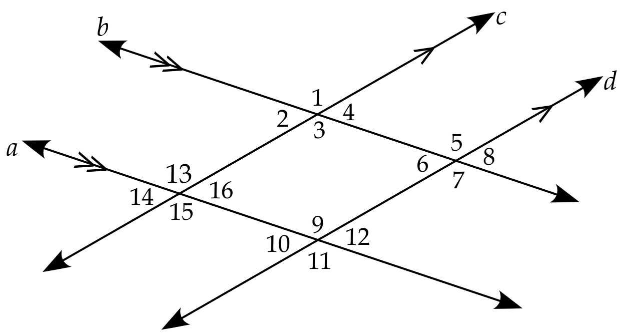
continued

Learning Activity 4.3 (continued)

6. Prove ∠2 = ∠12, given that lines *c* and *d* are parallel and lines *a* and *b* are parallel in the diagram below.



7. Prove ∠6 = ∠14, given that lines *c* and *d* and lines *a* and *b* are parallel in the diagram below.



8. Brad claims that he can prove ∠7 = ∠16 in the diagram above. Find the error in his proof below.

Line *b* is parallel to line *a*.	Given.
∠7 = ∠12	They are alternating interior angles.
Line *c* is parallel to line *d*.	Given.
∠12 = ∠16	They are corresponding angles.
∠7 = ∠16	Use the transitive property.

Lesson Summary

In this lesson, you developed and proved four properties of parallel lines and transversals. In the next lesson, you will continue to work with these properties to analyze the relationships between parallel lines and transversals.

Lesson 4: Applying Properties of Parallel Lines and Transversals

Lesson Focus

In this lesson, you will

- ❑ determine the measurement of angles in diagrams involving parallel lines and transversals
- ❑ identify and correct the error(s) in a solution to a problem involving parallel lines and transversals
- ❑ solve real-life problems involving parallel lines and transversals
- ❑ determine if two lines are parallel by using angle properties

Lesson Introduction

In the last lesson, you discovered and proved four properties of parallel lines and transversals. Throughout this lesson, you will be applying these four properties, along with the properties of angles and polygons that you learned earlier in this module.

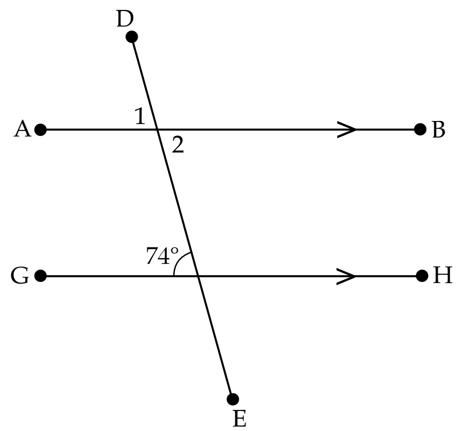
Using the Properties of Parallel Lines and Transversals

Now that you know how to use the four properties of parallel lines and transversals, you are going to combine all your knowledge from this module to find various angle measurements. Consider the following examples.

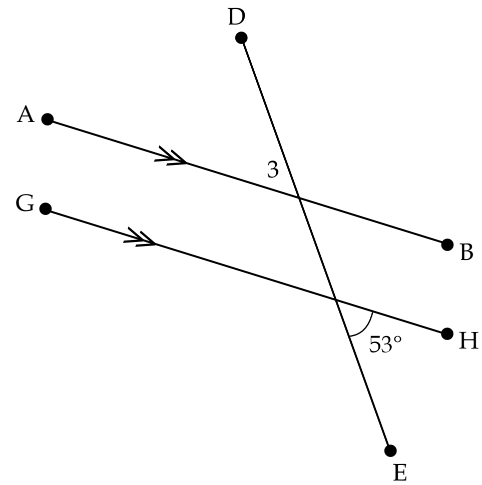
Example 1

Find the indicated angle measurements in each diagram below. In each diagram, lines AB and GH are parallel.

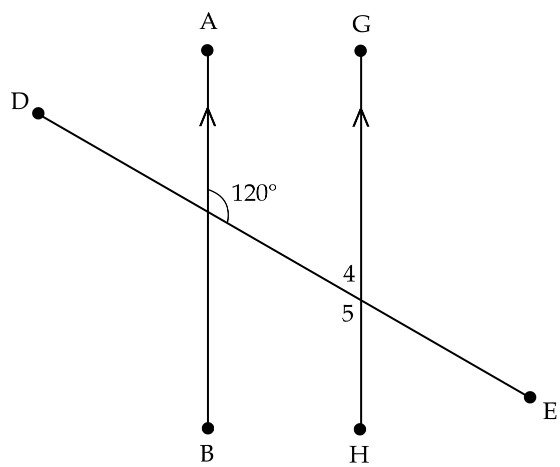
a)



b)



c)



Solution

a) $\angle 1$ is a corresponding angle with the given 74° angle. Therefore, $\angle 1 = 74°$. $\angle 2$ is a vertically opposite angle with $\angle 1$. Therefore, $\angle 1 = \angle 2$ and $\angle 2 = 74°$.

b) $\angle 3$ is an alternate exterior angle with the given 53° angle. Therefore, these angles are equal and $\angle 3 = 53°$.

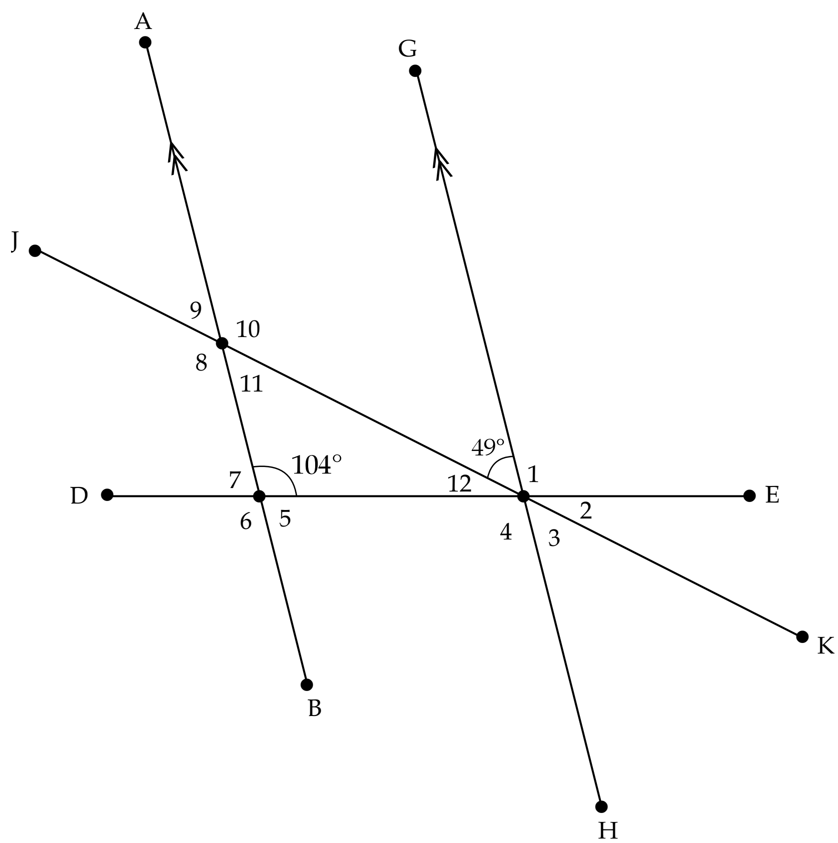
c) $\angle 4$ is a same side interior angle with the given 120° angle. Therefore, these angles are supplementary.

$$\angle 4 + 120° = 180°$$
$$\angle 4 = 180° - 120°$$
$$\angle 4 = 60°$$

$\angle 5$ is an alternate interior angle with the given 120° angle. Therefore, they are equal and $\angle 5 = 120°$.

Example 2

Find all of the missing angles in the diagram below. Lines AB and GH are parallel.



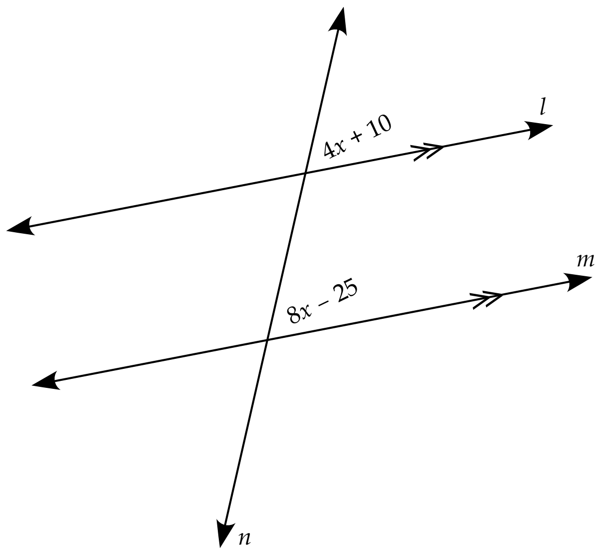
Solution

$\angle 1 = 104°$	$\angle 1$ and the given 104° angle are corresponding angles.
$\angle 4 = 104°$	$\angle 4$ and the given 104° angle are alternate interior angles.
$\angle 3 = 49°$	They are vertically opposite angles.
$49° + \angle 1 + \angle 2 = 180°$	They are supplementary angles.
$49° + 104° + \angle 2 = 180°$	Substitute 104° for $\angle 1$.
$153° + \angle 2 = 180°$	Simplify the left-hand side.
$\angle 2 = 180° - 153°$	Subtract 153° from both sides.
$\angle 2 = 27°$	Simplify the right-hand side.
$\angle 6 = 104°$	$\angle 6$ and the given 104° angle are vertically opposite angles.
$\angle 5 + 104° = 180°$	They are supplementary angles.
$\angle 5 = 180° - 104°$	Subtract 104° from both sides.
$\angle 5 = 76°$	Complete the subtraction.
$\angle 7 = \angle 5$	They are vertically opposite angles.
$\angle 7 = 76°$	
$\angle 12 = \angle 2$	Vertically opposite angles.
$\angle 12 = 27°$	
$\angle 11 = 49°$	Alternate interior angle of transversal JK.
$\angle 9 = \angle 11$	They are vertically opposite angles.
$\angle 9 = 49°$	
$\angle 8 + \angle 11 = 180°$	They are supplementary angles.
$\angle 8 + 49° = 180°$	Substitute in known values.
$\angle 8 = 180° - 49°$	Subtract 49° from both sides.
$\angle 8 = 131°$	Complete the subtraction.
$\angle 10 = \angle 8$	They are vertically opposite angles.
$\angle 10 = 131°$	

Note: There may be variations of logical reasoning that you can use to determine each angle. The results you get should always be the same, but how you get those results may vary. Note the sum of the angles of the interior triangle is 180°, as it should be.

Example 4

Solve for x. Lines l and m are parallel.



Solution

$4x + 10 = 8x - 25$	They are corresponding angles.
$4x - 8x + 10 = 8x - 8x - 25$	Subtract $8x$ from both sides.
$-4x + 10 = -25$	Simplify.
$-4x + 10 - 10 = -25 - 10$	Subtract 10 from both sides.
$-4x = -35$	Simplify.
$x = \dfrac{35}{4}$	Divide both sides of the equation by -4.

Determining if Two Lines are Parallel

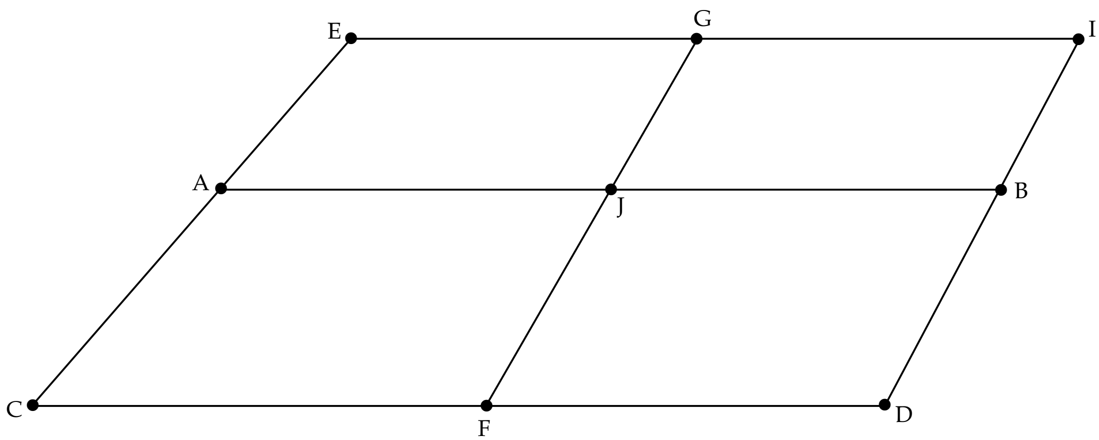
It is often useful to determine whether two lines are parallel. This can be important in construction projects or engineering applications.

If two lines are parallel, a pair of corresponding angles must be equal.

You may want to add this fact to your resource sheet.

Example 5

A contractor designed a modern shelf and drew the following diagram.



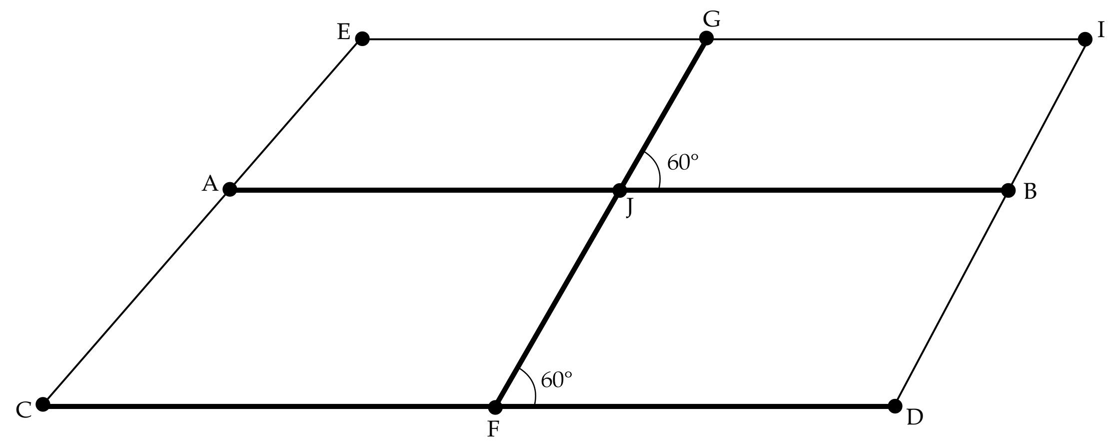
a) Use your protractor to show the contractor that lines AB and CD are parallel.

b) Prove to the contractor that lines GF and ID are not parallel by using your protractor.

Solution

a) To prove that lines AB and CD are parallel, look at a transversal of these lines, GF. Now, pick two corresponding angles in relation to the pair of parallel lines and the transversal. If these lines are parallel, then the measurements of these angles must be equal.

   You can choose to measure ∠GJB and ∠JFD, as in the diagram below.

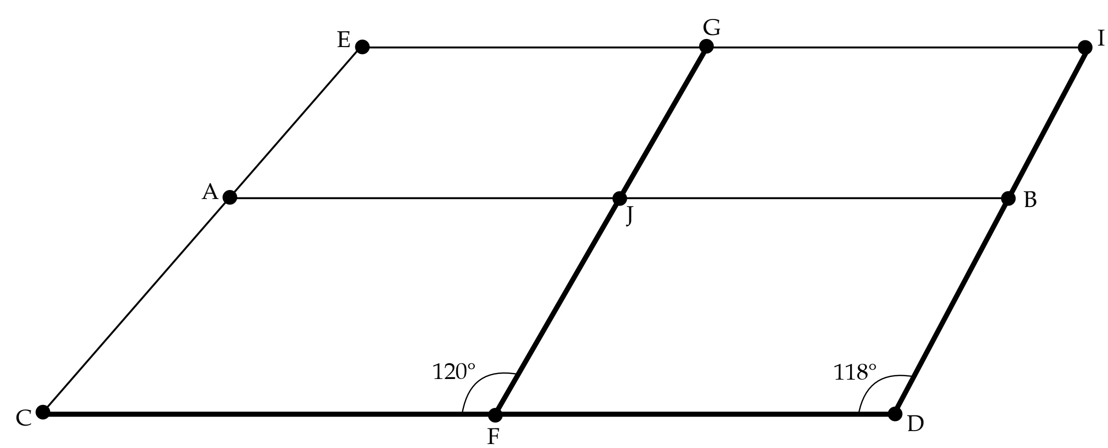


If you measure these angles with a protractor, you will see that they both measure 60°. Therefore, lines AB and CD are parallel.

Alternatively, you could measure the corresponding angles of ∠GJA and ∠JFC and find them both to measure 120°. This would also prove that lines AB and CD are parallel.

b) To prove that lines GF and ID are not parallel, look at a transversal of these lines: CD. Now, pick two corresponding angles in relation to the pair of parallel lines and the transversal. If these lines are not parallel, then the measurements of these angles will not be equal.

You can choose to measure ∠GFC and ∠IDC, as in the diagram below.



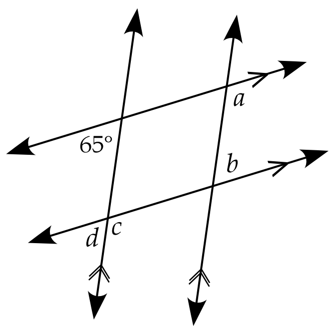
When you measure these angles, you will find they are very close to being the same size, but are not exactly the same. Therefore, lines GF and ID are not parallel.

Correcting Errors in a Solution

Because there are many properties and definitions for parallel lines and the angles they create, these are important topics to be compiled in your resource sheet. It may be helpful to use your resource sheet when completing the examples and learning activity.

Example 6

Correct Jackie's solution to the following problem.



Determine the sizes of $\angle a$ and $\angle b$.

Jackie's solution:

The 65° angle and $\angle b$ are alternate interior angles. Therefore, they are equal.

$\angle b = 65°$

$\angle a$ and $\angle b$ are same-side interior angles. Therefore, they have a sum of 180°.

$\angle a + \angle b = 180°$

$\angle a + 65° = 180°$

$\angle a = 180° - 65°$

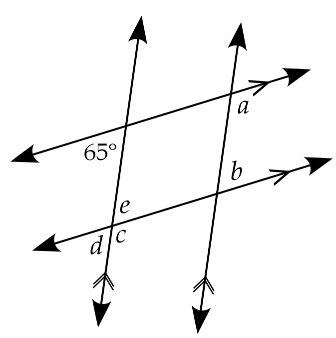
$\angle a = 115°$

Solution

Jackie's solution is incorrect because her first statement contains an error.

The 65° angle and $\angle b$ are not alternate interior angles because they are not using the same transversal.

In order to use the angle properties to get from the 65° angle to $\angle b$, you need to go through an intermediate angle.



Consider $\angle e$. The 65° angle and $\angle e$ are alternate interior angles. Therefore, they are equal.

$\angle e = 65°$

Now, $\angle e$ and $\angle b$ are corresponding angles. Therefore, they are equal.

$\angle e = \angle b$

$\angle b = 65°$

The rest of Jackie's proof showing that $\angle a = 115°$ is now mathematically valid.

Make sure you practise using the angle properties in the following learning activity. It can sometimes be difficult to recognize which angle property you can use in a given situation. The more you practise, the better you will get at recognizing which angle property to use.

Learning Activity 4.4

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Solve: $x^2 - x - 72 = 0$
2. Evaluate: $\dfrac{\sqrt{25 \times 4}}{5}$
3. Add: $\dfrac{1}{5} + \dfrac{2}{25}$
4. You have a budget of $1400 for monthly bills. Your rent is $500, your cellphone is $150, your cable and Internet together cost $150, and your car with insurance costs $400. How much money do you have left for groceries?
5. You put a chicken in the oven at 4:45 p.m. At what time will the chicken be done if it takes 90 minutes to cook?
6. What two numbers have a product of -32 and a sum of 4?

continued

Learning Activity 4.4 (continued)

7. What percent is 17 out of 25?

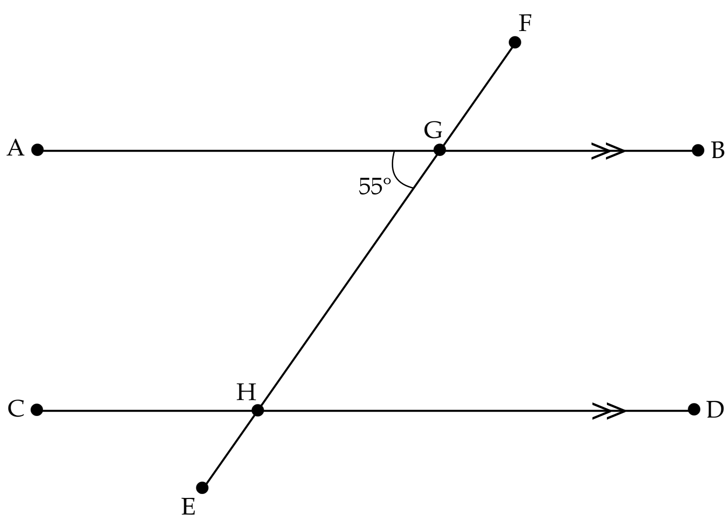
8. Convert the fraction $\frac{25}{4}$ to a mixed fraction.

Part B: Using Properties of Parallel Lines to Find Missing Angle Measurements

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

Note: All proofs in this learning activity will be set up in the format of a two-column proof. When you complete the final assignment for this module, you must record your answers in the format of a two-column proof as well. Practise doing so throughout this learning activity.

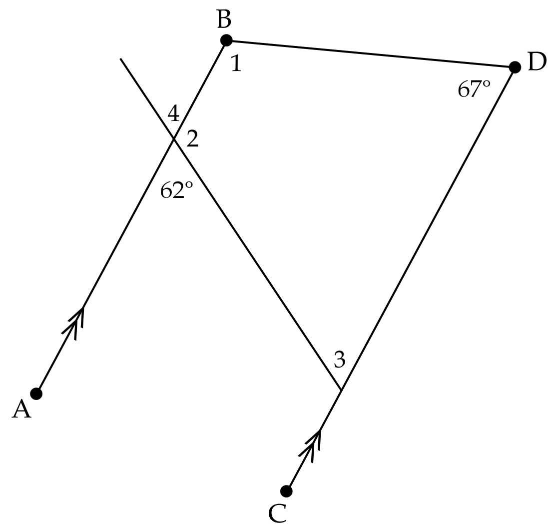
1. Determine the measurement of ∠DHE in the diagram below. The lines AB and CD are parallel. Include your reasoning.



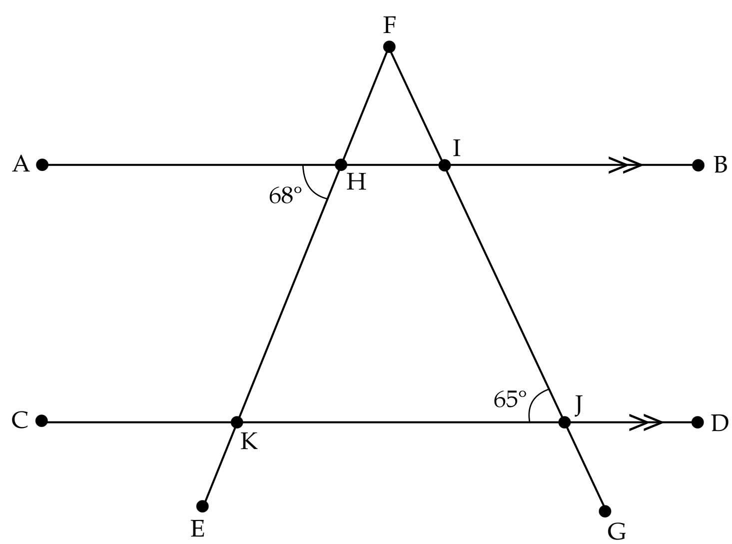
continued

Learning Activity 4.4 (continued)

2. Determine all missing angle measurements in the diagram below, given that AB and CD are parallel lines.



3. Determine the measurement of ∠EFG in the diagram below. The lines AB and CD are parallel. Include your reasoning.

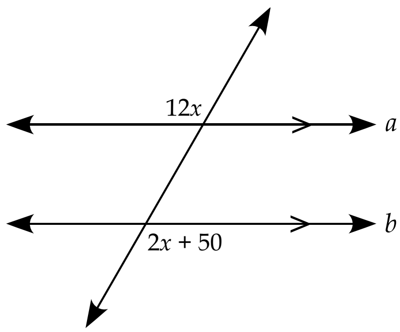


continued

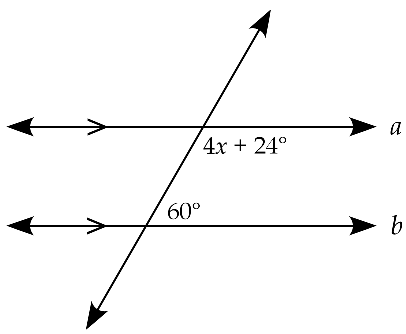
Learning Activity 4.4 (continued)

4. Determine the value for x in each diagram below. Lines a and b are parallel.

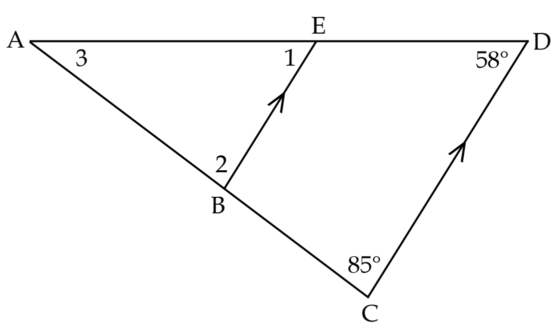
   a)



   b)



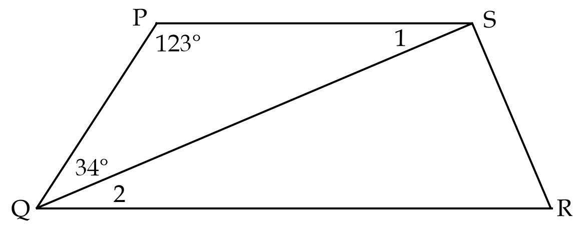
5. Given the following diagram, find the size of angles 1, 2, and 3 if BE is parallel to CD. Explain your thinking.



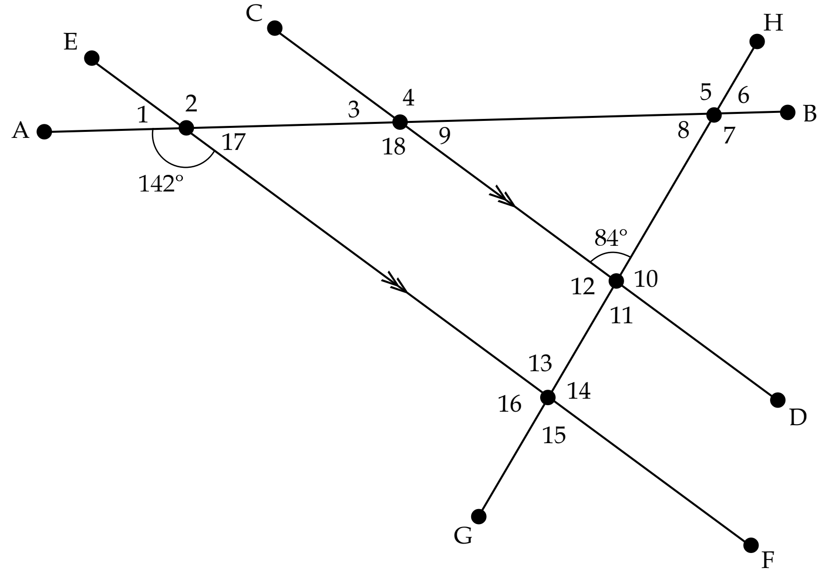
continued

Learning Activity 4.4 (continued)

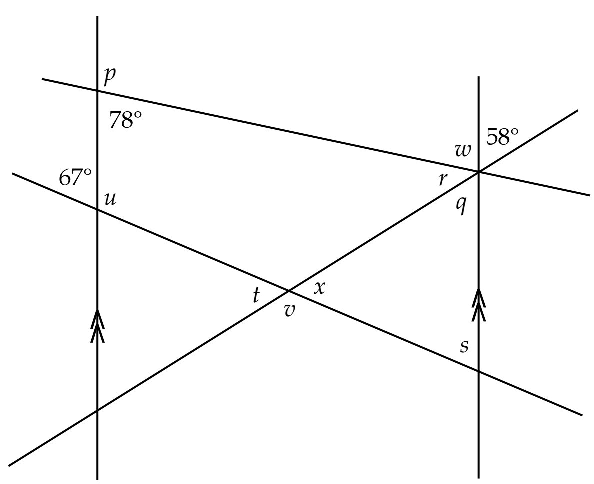
6. In a trapezoid PQRS, PS is parallel to QR. What are the measurements of $\angle 1$ and $\angle 2$?



7. Determine the measurement of all missing angles in the diagram below given that line EF is parallel to line CD.

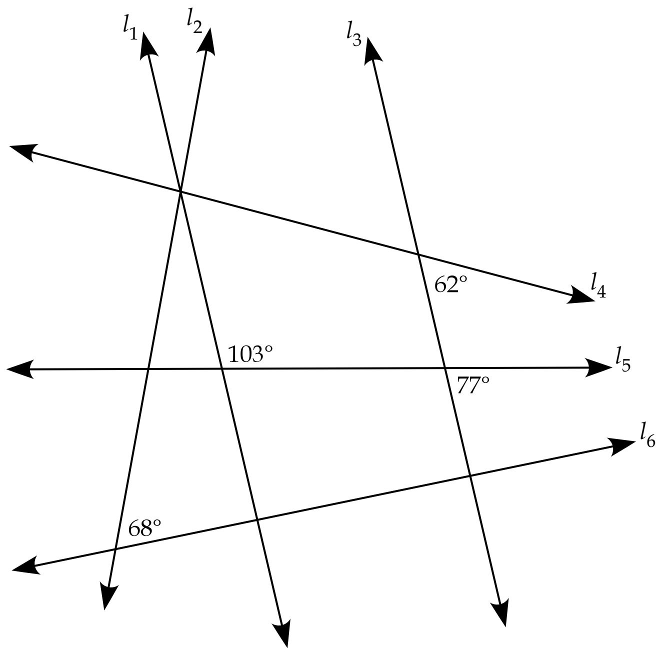


8. Determine the measurement of all missing angles in the diagram below. The vertical lines are parallel.



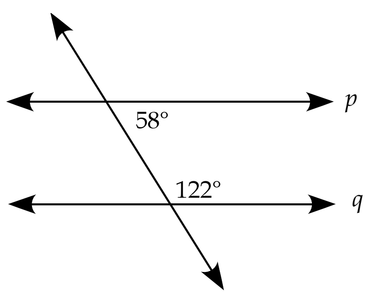
continued

9. Find a pair of parallel lines in the following diagram. Indicate all angles necessary to prove the pair of lines is parallel.

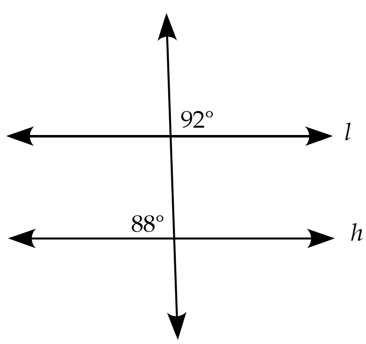


10. Determine if the following sets of lines are parallel.

a)



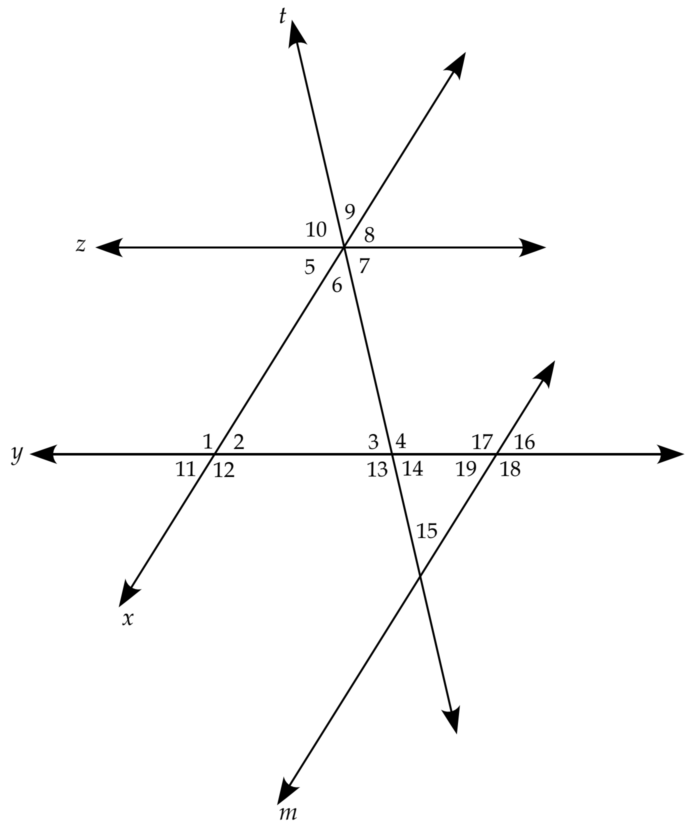
b)



continued

Learning Activity 4.4 (continued)

11. Consider the diagram below. Determine the size of $\angle 3$ if $\angle 2$ is 58° and $\angle 9$ is 45°.



Lesson Summary

In this lesson, you took the four properties of parallel lines and transversals that you discovered in Lesson 3 and used these properties to find missing angles.

Notes

Assignment 4.2

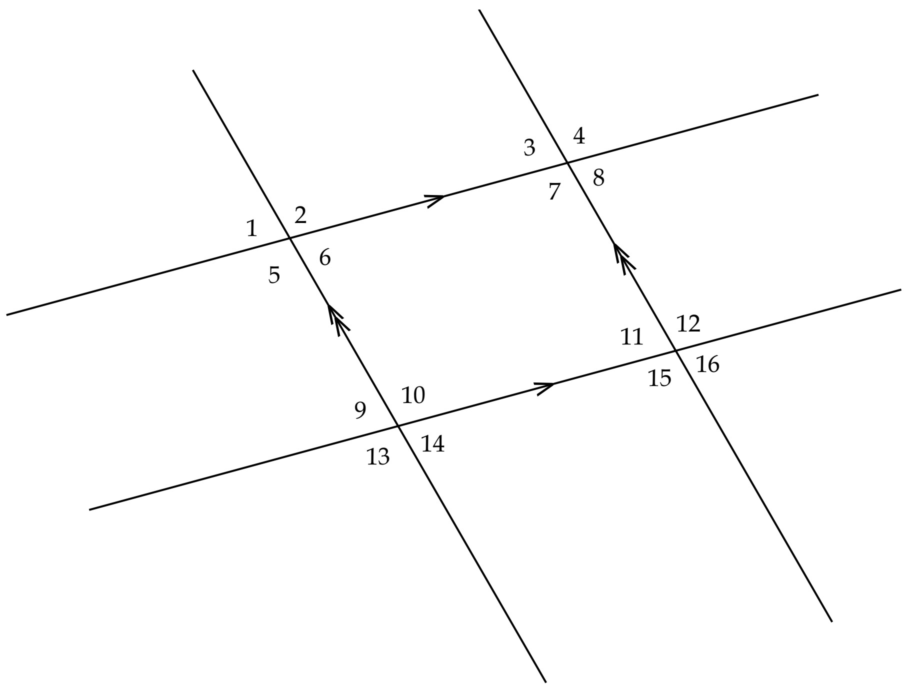
Angle Properties of Parallel Lines and Transversals

Total: 48 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

Note: Whenever possible, solutions for this assignment must be written in the format of a two-column proof.

1. Consider the diagram below.



a) Name one pair of vertically opposite angles. *(1 mark)*

b) Name two sets of alternate interior angles. *(2 marks)*

Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

c) Name one set of alternate exterior angles. *(1 mark)*

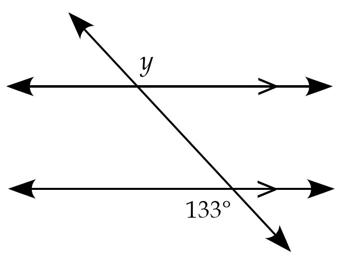
d) Name two sets of same-side interior angles. *(2 marks)*

e) Name an angle that is a corresponding angle to ∠4. *(1 mark)*

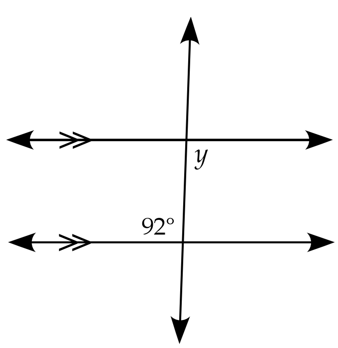
f) Name two angles that are supplementary angles to ∠9. *(2 marks)*

2. For each diagram below, state the size of ∠*y* and the property that allows you to find this measurement. **Note:** Each pair of horizontal lines in the diagrams below is shown to be parallel. *(8 marks)*

a)

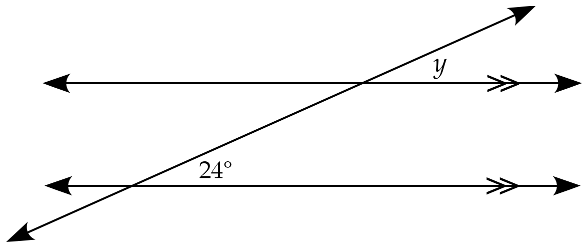


b)

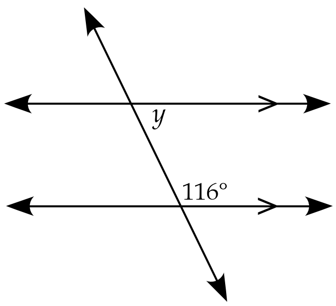


Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

c)

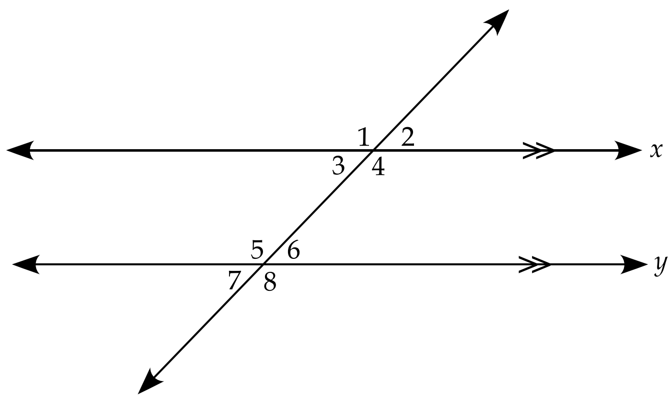


d)

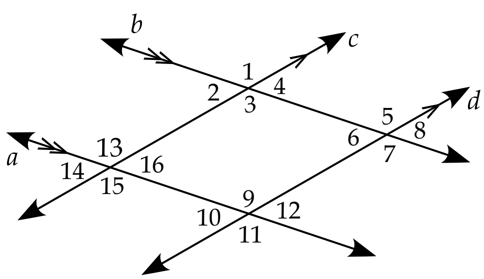


Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

3. Prove ∠4 = ∠5 given that lines *x* and *y* are parallel. In other words, prove the property of alternate interior angles to be true. *(3 marks)*

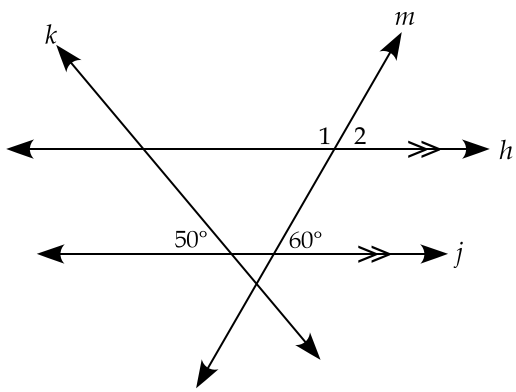


4. Prove ∠9 = ∠3 given that lines *a* and *b* and lines *c* and *d* are parallel. *(3 marks)*

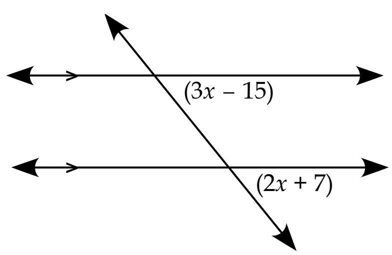


Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

5. Determine the measurement of ∠1 in the diagram below given that lines *h* and *j* are parallel. *(2 marks)*

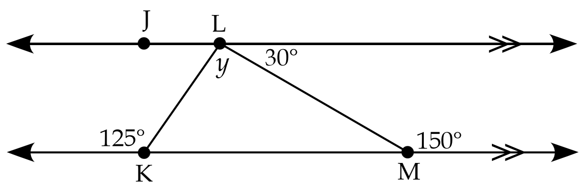


6. Determine the value of *x* in the diagram below. *(2 marks)*



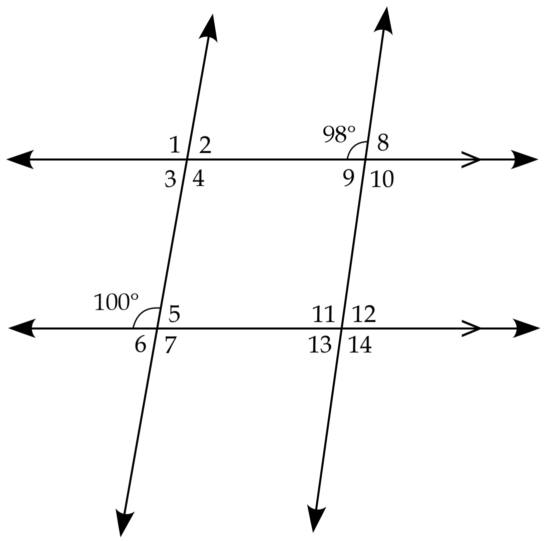
Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

7. Determine the size of $\angle y$ in the diagram below given that lines JL and KM are parallel. *(2 marks)*



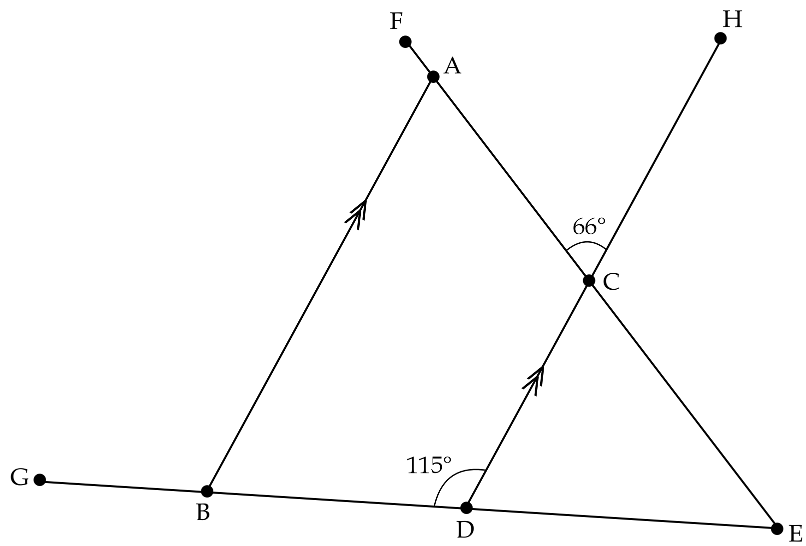
Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

8. Determine all the missing angles in the diagram below, given that the horizontal lines are parallel. *(14 × 0.5 mark each = 7 marks)*



Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

9. In the diagram below, AB is parallel to CD.



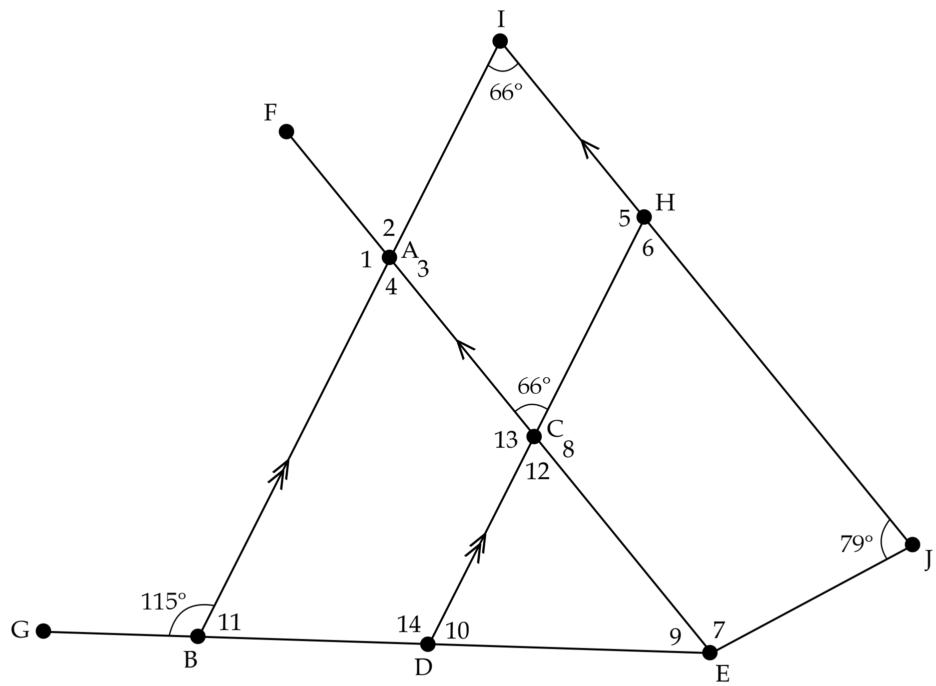
a) Determine the measurement of ∠ABG. *(1 mark)*

b) Determine the measurement of ∠DEC. *(3 marks)*

c) Determine the measurement of ∠CAB. *(1 mark)*

Assignment 4.2: Angle Properties of Parallel Lines and Transversals (continued)

10. Determine all of the numbered angles in the diagram below. The matching arrows indicate parallel lines. *(14 × 0.5 mark each = 7 marks)*



Notes

Module 4 Summary

Congratulations on completing the first half of this course! In this module, you learned about the angle properties of polygons, including their interior and exterior angles. You also learned about four properties of angles that can be applied when a transversal crosses a set of parallel lines. Using these properties, you were able to determine if a pair of lines was parallel, which is often a necessary determination during construction.

In the next module, you will build on your knowledge from previous mathematics courses to learn more about trigonometry, the study of triangles.

Now that you have completed Module 4, make sure your module resource sheets are complete and up-to-date. You will need them as it is now time to begin to create your Midterm Exam Resource Sheet. It is also time to begin preparing for your midterm examination. Make sure you read and follow the directions below to ensure you are fully prepared for the midterm examination.

Submitting Your Assignments

It is now time for you to submit the Module 4 Cover Assignment and Assignments 4.1 and 4.2 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 4 assignments and organize your material in the following order:

- ❑ Module 4 Cover Sheet (found at the end of the course Introduction)
- ❑ Cover Assignment: Geometry in Tangrams
- ❑ Assignment 4.1: Polygons and Angles
- ❑ Assignment 4.2: Angle Properties of Parallel Lines and Transversals

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Midterm Examination

Congratulations, you have finished Module 4 in the course. The midterm examination is out of 100 marks and worth 20% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 4.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: some pens and/or pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Midterm Examination Resource Sheet.

A maximum of 2.5 hours is available to complete your midterm examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

Graphing technology (either computer software or a graphing calculator) **is required** to complete the examination. Check with your tutor/marker to be sure your graphing technology is appropriate.

At this point you will also have to combine your resource sheets from Modules 1 to 4 onto one 8½″ × 11″ paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you. We suggest that you divide your paper into two quadrants on each side so that each quadrant contains information from one module.

Examination Review

You are now ready to begin preparing for your midterm examination. Please review the content, learning activities, and assignments from Modules 1 to 4.

The midterm practice examination is also an excellent study aid for reviewing Modules 1 to 4.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the midterm examination determines 20% of your final mark in this course and you will have 2.5 hours to complete the examination.

Midterm Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The midterm practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your midterm practice examination, follow these steps:

1. Study for the midterm practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 1 to 4 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 4, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Midterm Examination Resource Sheet. You can use both sides of an 8½” by 11” piece of paper.
6. Bring the following to the midterm practice examination: some pens and/or pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Midterm Examination Resource Sheet.
7. Write your midterm practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don’t check your answers until you have completed the entire examination. Remember that the time allowed for writing the midterm examination is 2.5 hours.
8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn’t need to have on it? Make adjustments to your Midterm Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes

TEST TIME!

Self-Test

Module 4 – Personal Finance I

/64

1. Fill in the blanks.

(3 marks)

a) Name three ways in which a person earns income.

____

____

____

(3 marks)

b) Name three mandatory deductions from gross income that all people have to pay.

____

____

____

(1 mark)

c) State what "Net Pay" is.

____

(2 marks)

d) State the difference between "Selling Price" and "Assessed Value" for a house and property.

____

(1 mark)

e) State the formula used by towns to determine their property tax rate in mills.

Mill Rate = ____

2. Copy the following spreadsheet. Fill in the formulas for all cells used in finding the employees' "net pay". Print the spreadsheet with formulas as well as numerical values. *(10 marks)*

   Note:

   - Time-and-a-half is paid for overtime over 40 hours.
   - Federal income tax is paid at a rate of 25.5% if weekly income is less than or equal to $569.04, 39% if weekly income is over $569.04 but less than or equal to $1138.08, and 43.5% if over $1138.08. Provincial tax is half the federal rates.
   - CPP is paid at a rate of 2.8%.
   - E.I. is paid at a rate of 2.95%.

	A	B	C	D	E
1	EMPLOYEE	TOM	JIM	PAM	JACK
2	RATE	$14.05	$6.10	$9.80	$12.75
3	MONDAY	8	9	6	10
4	TUESDAY	9	4	8	8
5	WEDNESDAY	8	4	10	9
6	THURSDAY	10	8	4	11
7	FRIDAY	12	6	6	7
8	REG HOURS				
9	OT HOURS				
10	GROSS PAY				
11	CPP				
12	EI				
13	INCOME TAX				
14	NET PAY				
15					
16	$0.00				
17	$569.04				
18	$1138.08				
19	$100000.00				
20					

3. **Property Tax**

   a) A town has a projected budget value of $11 500 000 and a total assessed property value of $100 000 000. Find the mill rate for the town. *(1 mark)*

(7 marks)

b) Fill in the missing values in the following Taxation Notice.

PROPERTY DESCRIPTION

ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
585	3	7	2	FF1254	82.5 ft	1

Civic Address

Title or Deed No.	Current Assessment Land	Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port Assessment
B544	6500	65 000	T	?	10	35%	?

←ERRORS AND OMISSIONS EXCEPTED
←ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
←ALL CHEQUES MADE IN CANADIAN FUNDS
←BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
←RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY
71 500

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			?	41.56	?
By-Law No.	Term	Type	Frontage Levy	Mill Rate	Levy
523	98	Sewer and Water	75	0.025	?
633	97	Sidewalk		4.125	?
710	99	Street		7.360	?

EDUCATIONAL TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	?	5.5	?
Provincial Education 2	?	16.4	?

PROVINCIAL TAX CREDITS

(See Manitoba Enclosure For Additional Information)	Assessment	Levy
	Manitoba Resident Homeowner Tax Assistance	$250.00

TOTAL TAXES DUE

Municipal Tax	Education Tax	Total Taxes	Prov. Credits	Net Taxes	Arrears/Credits	Added Taxes	Taxes Due
?	?	?	?	?	?	?	?

4. Find the answers to the following questions regarding foreign exchange. (Use the table provided.)

CANADIAN DOLLARS		
BUYING RATE	FOREIGN CURRENCY	SELLING RATE
1.3424	United States – Dollar	1.3784
2.0473	United Kingdom – Pound	2.1573
1.0309	Australia – Dollar	1.1557
0.2189	Denmark – Kroner	0.2421
0.2652	Finland – Markaa	0.3627
0.2507	France – Franc	0.2744
0.8572	Germany – D. Mark	0.9375
0.162	Hong Kong – Dollar	0.1856
0.7644	Netherlands – Guilder	0.8283
0.1975	Norway – Kroner	0.2184

a) On a trip to the USA the following purchases were made: *(2 marks)*

Sweatshirt	\$19.95
Shoes	\$12.99
Jeans	\$21.99

If there was a sales tax of 9% applied to all purchases, how much would the purchases cost in Canadian funds?

b) A return airplane ticket to England costs \$795 Canadian. A 15% discount is in effect if the ticket is purchased in England. How much would the ticket cost in pounds? *(2 marks)*

(2 marks) 5. Find the answers to the following unit pricing question.

Three brands of cat food were being sold as follows:

4 cans (175 g) for \$1.00
6 cans (300 g) for \$2.39
1 case (12–250 g cans) for \$4.09

Show by unit pricing which is the best buy.

6. Solve the following systems of equations using your calculator.

(2 marks) a) $7x - 3y = 8$
$3x + 5y = -12$

(2 marks) b) $y = x + 1$
$y = x^2 + 3x - 2$

7. Graph the following quadratic function with your calculator and find the vertex, axis of symmetry, maximum or minimum value, and domain and range. *(3 marks)*

   $y = -2x^2 - 16x + 1$

8. For the equation of the cubic function given, state the maximum and minimum values and the values of x at these points. State the zeros. (Round answers to two decimal places if necessary.) *(3 marks)*

   $y = 2x^3 - 5x^2 + 2x$

9. Solve the following applications by setting up a system of equations or constraint inequalities and using your graphing calculator where necessary to solve the resulting system.

(5 marks)

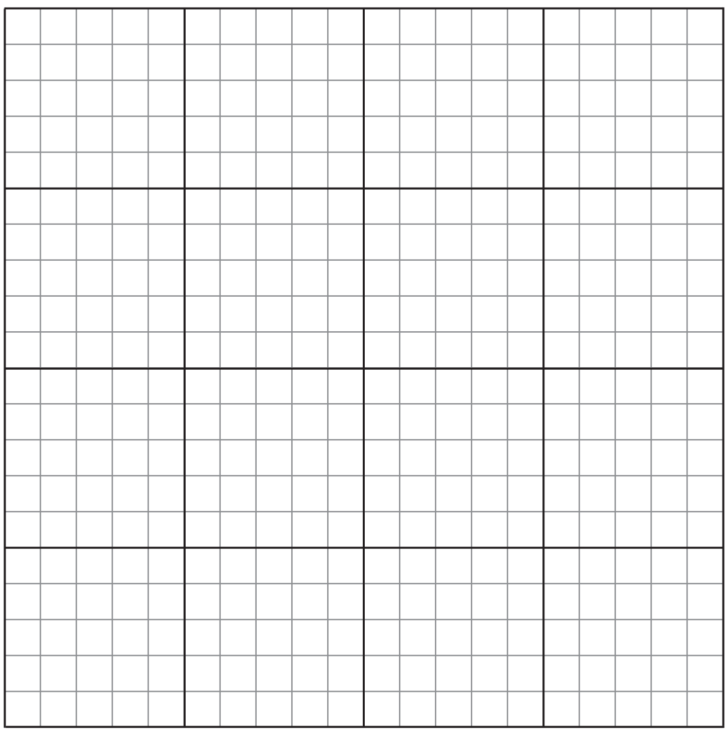
a) Jim is always thinking about food. He had bacon and eggs for breakfast and noticed there were twice as many pieces of bacon as eggs. If he had one more egg, he would have two-thirds the number of pieces of bacon. How many eggs and pieces of bacon were on his plate?

(5 marks)

b) A steamboat with a capacity of 200 passengers is to be chartered for an excursion. The price of one ticket is to be $30.00 if 100 or fewer people buy tickets, but the steamship company agrees to reduce the price of every ticket by 20 cents for each ticket sold in excess of 100. What number of passengers over 100 will provide the steamship company with the largest income?

c) The Econo-Company manufactures two sizes of TV screens, size A and size B. The screens are made by two machines. The old machine makes one size A screen in four minutes and one size B screen in one minute. The new machine makes one size A screen in two minutes and one size B in six minutes. Each machine can only operate two hours a day at most. If the size A screen sells at a profit of \$2 per screen and the size B at a profit of \$3 per screen, how many of each size screen should be manufactured per day to bring in the maximum profit?

(10 marks)



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Grade 11 Applied Mathematics (30S)

Module 5
Trigonometry

Module 5: Trigonometry

Introduction

Trigonometry is defined as the branch of mathematics that studies the relationships among the sides and angles of a triangle. Many professionals use trigonometry in fields such as engineering, surveying, and crime scene analysis. Have you ever noticed how bridges are made up of triangular shapes? Triangles are used in construction because of their inflexible shape. The more triangles included in the construction of a bridge, the stronger the bridge will be.

In previous courses, you learned how to solve for missing angles and side lengths in triangles that contained a right angle. In this module, you will learn how to solve for missing angles and side lengths in triangles that do not contain a right angle. These triangles are called *oblique triangles*. To do this, you will learn about two different methods, the Sine Law and the Cosine Law.

Assignments in Module 5

When you have completed the assignments for Module 5, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Shape and Space
3	Assignment 5.1	Solving Acute Triangles
4	Assignment 5.2	Solving Obtuse Triangles
5	Assignment 5.3	The Ambiguous Case

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, and can be either handwritten or typewritten. Both sides of the sheet may be filled. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 5. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 5 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 5 to 8.

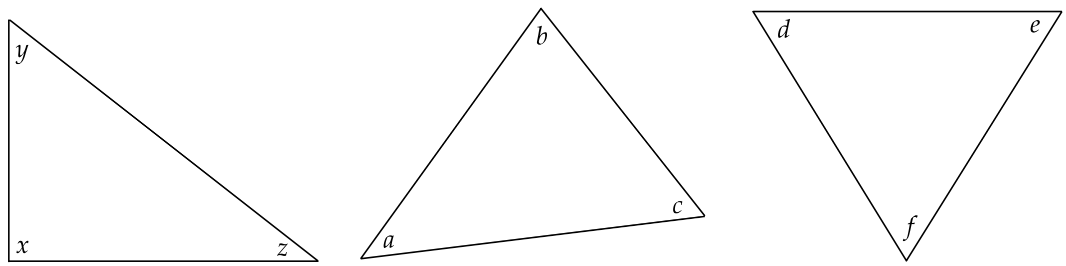
Resource Sheet for Module 5

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Module 5 Cover Assignment: Geometric and Numerical Reasoning

Angles, Lines, and Triangles

In previous courses, and in Module 4, you learned that for every triangle, the three interior angles always have a sum of 180°.

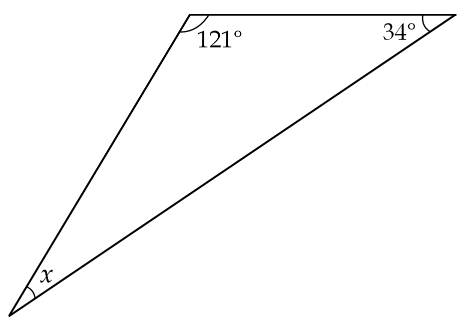


$x + y + z = 180°$ $\quad$ $a + b + c = 180°$ $\quad$ $d + e + f = 180°$

Using this fact, you are able to find the missing angle in a triangle if you are given two angles.

Example 1

Determine the measurement of $\angle x$.



Solution

Since all angles have a sum of 180°:

$121° + 34° + \angle x = 180°$

$\angle x = 180° - 121° - 34°$

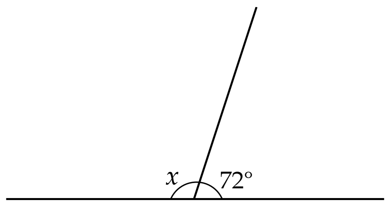
$\angle x = 25°$

You are also familiar with the property of straight lines that states that when two or more angles form a straight line, the angles will have a sum of 180°.

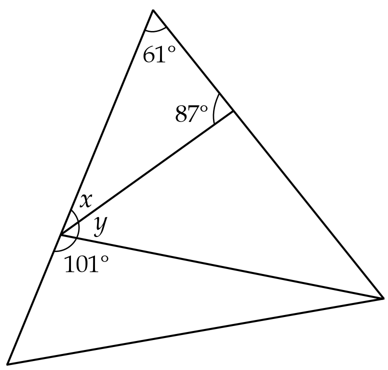
Example 2

Determine the measurement of the indicated angles.

a)



b)



Solution

a) You know that there are 180° in a straight line. Therefore:

$x + 72° = 180°$

$x = 180° - 72°$

$x = 108°$

b) You can first determine $\angle x$ because you know that there are 180° in a triangle.

$\angle x = 180° - 61° - 87°$

$\angle x = 32°$

Now, you can find $\angle y$ by recognizing that $\angle x$, $\angle y$, and the 101° angle need to have a sum of 180°, since they are located on the same side of a straight line.

$\angle x + \angle y + 101° = 180°$

$32° + \angle y + 101° = 180°$

$\angle y = 180° - 101° - 32°$

$\angle y = 47°$

Magic Squares

A magic square is an arrangement of numbers in a square such that the sum of the numbers in any row, column, or diagonal is identical. Consider the completed 3 × 3 magic square below. The numbers 1 to 9 inclusive are used.

			= 15
6	7	2	= 15
1	5	9	= 15
8	3	4	= 15
= 15	= 15	= 15	= 15

As you can see, each of the columns, rows, and diagonals has the same sum, 15.

The value of 15 is called the "magic constant." This constant can be found by adding all of the values used in the magic square and dividing by the number of columns (or rows). In this case, there are 3 columns of numbers and each column must be the same. Thus, divide the total sum of all numbers in the magic square by the number of columns to determine the total for each column. In a 3 × 3 magic square, divide the sum by 3, as shown below.

$$\frac{1+2+3+4+5+6+7+8+9}{3} = \frac{45}{3} = 15$$

Notes

Module 5 Cover Assignment

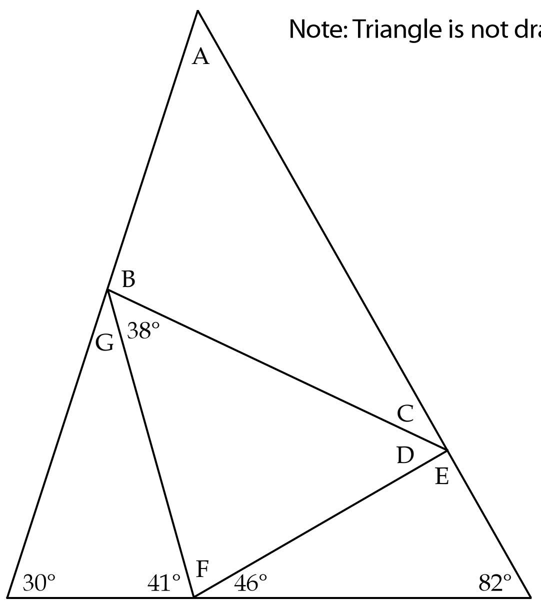
Shape and Space

Total: 10 marks

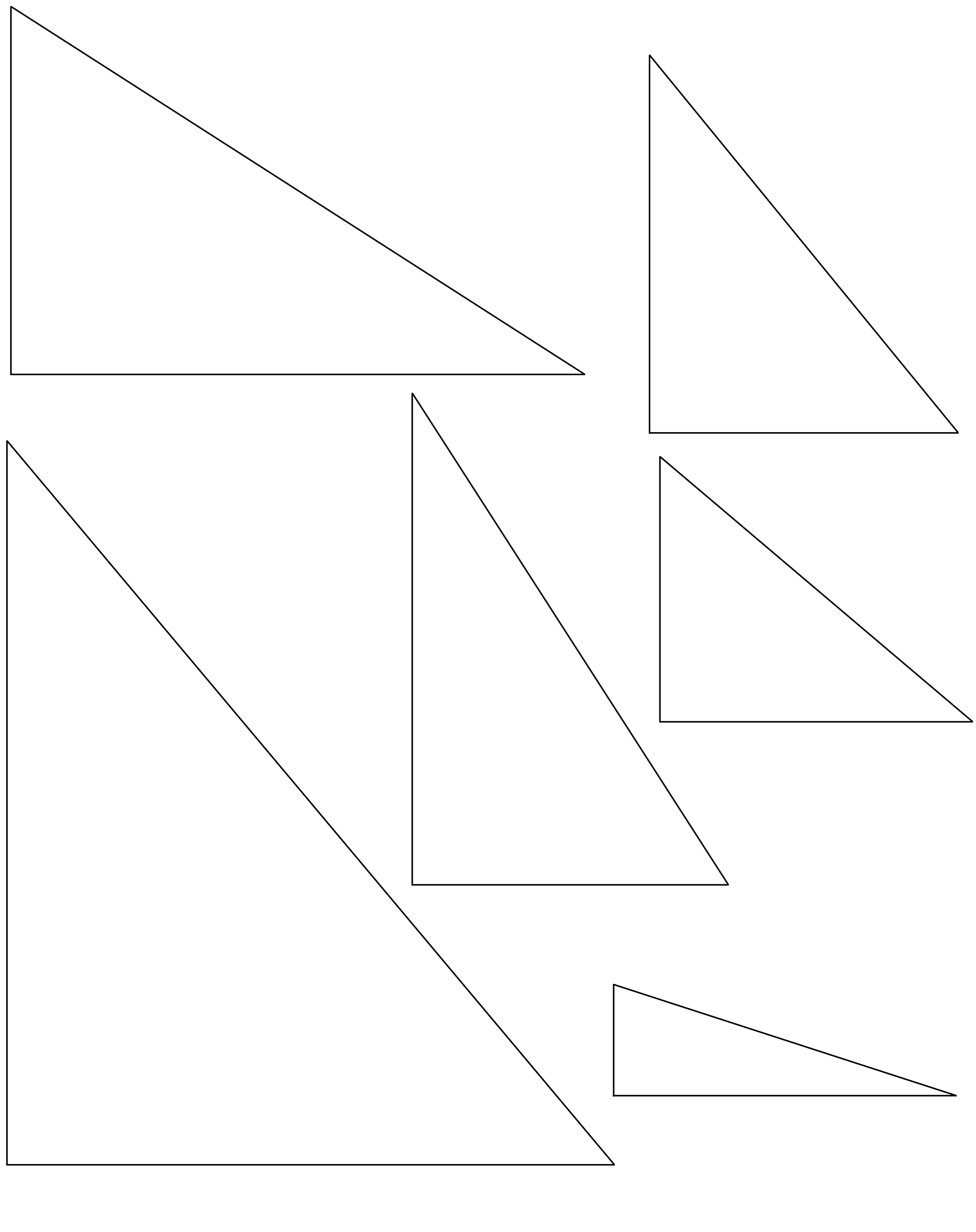
Clearly show the steps in your solution on the question sheets below and submit these pages when you send in your assignments for marking.

1. Determine the measurements of all indicated angles in the triangle below. *(3 marks)*

Note: Triangle is not drawn to scale.



2. Consider the set of six right triangles on the following page. Cut out these six triangles and rearrange the triangles into a square. Once you have discovered the solution, either draw or glue/tape your solution below. *(3 marks)*



3. Consider the incomplete magic square below.

   a) Determine the “magic constant” for this magic square if the numbers from 1 to 16 inclusive are used. *(1 mark)*

   b) Complete the magic square by using the values from 1 to 16 inclusive. *(3 marks)*

16	3		
			8
		7	12
	15		

Notes

Lesson 1: Right-Triangle Trigonometry

Lesson Focus

In this lesson, you will

- ❑ review the Pythagorean theorem and trigonometric ratios
- ❑ observe situations where the Pythagorean theorem does not apply
- ❑ begin to develop a strategy for dealing with oblique triangles

Lesson Introduction

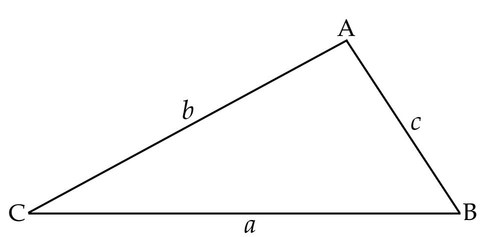
Trigonometry can be used to calculate the side lengths and angle measurements of triangles. In this lesson, you will review some procedures for labelling triangles, as well as for calculating side lengths and angle measurements of right triangles. You will also begin to look at situations where right-triangle trigonometry does not apply.

Triangles

Triangles are polygons that have three sides. The sum of the three interior angles of a triangle is 180°.

Labelling Triangles

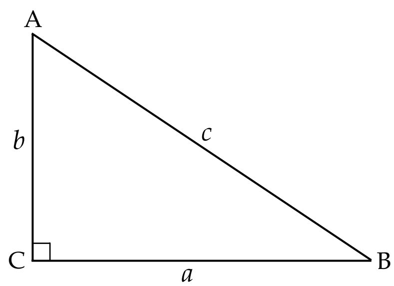
A triangle is a polygon with three vertices and three sides. Each vertex, or angle, is labelled with a capital letter. Each side is labelled with a lower case letter. In a triangle, each angle has exactly one opposite side. For this reason, the side opposite a vertex is labelled with the same letter as the vertex. In the diagram below, side *a* is opposite angle A, side *b* is opposite angle B, and side *c* is opposite angle C.



Types of Triangles

There are four types of triangles that you will use in this module.

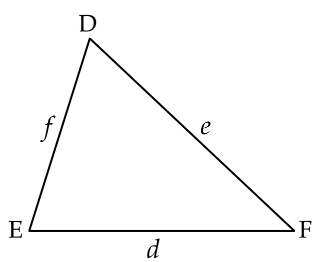
- A **right triangle** (also known as a **right-angled triangle**) has one right angle (90°). The side opposite the right angle is called the *hypotenuse*.



The right angle, ∠C, is always the largest angle in a right triangle. Therefore, the hypotenuse (c), is always the longest side.

- An **oblique triangle** is a triangle that does not contain a right angle. Acute triangles and obtuse triangles are types of oblique triangles.

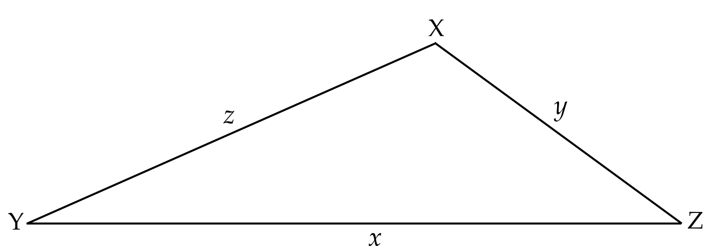
- An **acute triangle** is a triangle where all of the angles are less than 90°.



In ΔDEF,

- ∠D < 90°
- ∠E < 90°
- ∠F < 90°

- An **obtuse triangle** is a triangle where one of the angles is greater than 90°.

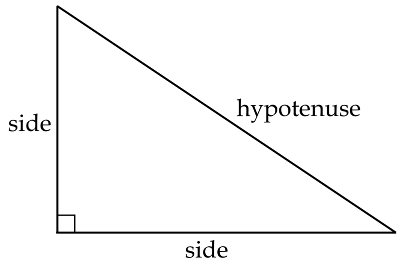


In ΔXYZ,
∠X > 90°.
Therefore, ΔXYZ is an obtuse triangle.

You may wish to include a summary of this information on your resource sheet.

Review of Right-Angled Triangles

In previous courses, you solved right-angled triangles, which are triangles with one 90° angle. Solving a triangle means you are finding all unknown sides and angles.



When you are solving right-angled triangles, your first step is to identify the sides and angles you know, and determine what sides and angles are missing.

- If you have two sides and you are looking for the third side, you can use the Pythagorean theorem, $a^2 + b^2 = c^2$.

  Note: In the formula, c represents the length of the *hypotenuse*.

- If you have two angles and you are looking for the third angle, you can use the fact that the sum of the interior angles in a triangle is equal to 180°.

- If you have two angles (including the 90° angle) and one side, and you are looking for another side length, you can use the **primary trigonometric ratios**.

$$\sin\theta = \frac{\text{side opposite the angle}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{side adjacent to the angle}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{side opposite the angle}}{\text{side adjacent to the angle}}$$

- If you have three sides and you are looking for an angle, use the **inverse trigonometric ratios**.

$$\theta = \sin^{-1}\left(\frac{\text{side opposite the angle}}{\text{hypotenuse}}\right)$$

$$\theta = \cos^{-1}\left(\frac{\text{side adjacent to the angle}}{\text{hypotenuse}}\right)$$

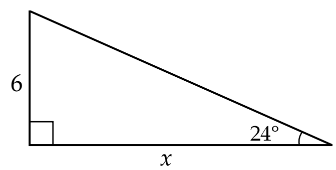
$$\theta = \tan^{-1}\left(\frac{\text{side opposite the angle}}{\text{side adjacent to the angle}}\right)$$

You may want to include a summary of this information on your resource sheet.

Refer back to the Module 4 Cover Assignment if you need help using the Pythagorean theorem.

Example 1

Find the missing side length in the triangle below. Round to **one** decimal place.



Solution

In this example, you are looking for a side length of a right triangle but you are only given one side length instead of two. However, you are given one angle in addition to the right angle. These types of questions can be solved by using the primary trigonometric ratios.

You know that the side length directly opposite the given angle is 6. You are trying to find the side length that is adjacent to the given angle, the side with a length of x. Therefore, you need to use the primary trigonometric ratio, which includes an *opposite* and *adjacent* side length. The tangent ratio is:

$$\tan\theta = \frac{\text{side opposite the angle}}{\text{side adjacent to the angle}}$$

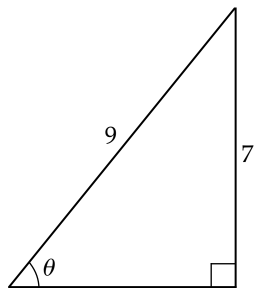
Substitute the values you know into this equation.

$\tan 24° = \frac{6}{x}$	Calculate the tangent ratio using your calculator. Make sure your calculator is in DEGREE (or DEG) mode!
$0.445229 = \frac{6}{x}$	Multiply both sides of the equation by x.
$0.445229x = 6$	Divide both sides of the equation by 0.445229.
$x = \frac{6}{0.445229}$	Complete the division.
$x = 13.5$	

Now that you have reviewed how to find side lengths of right triangles, you can review how to find missing interior angles of right triangles.

Example 2

Determine the size of the missing angle in the triangle below. Round to one decimal place.



Solution

In this example, you are asked to find the size of an angle and you are given two side lengths of a right triangle. To solve this type of problem, you need to use one of the inverse trigonometric ratios.

Because you are given one side opposite the angle (the side with a length of 7 units) and the hypotenuse (the side with a length of 9 units), you should use the inverse sine trigonometric ratio.

$$\theta = \sin^{-1}\left(\frac{\text{side opposite the angle}}{\text{hypotenuse}}\right)$$ Substitute in known values.

$$\theta = \sin^{-1}\left(\frac{7}{9}\right)$$ Use your calculator to determine this ratio.

$$\theta = 51.1°$$

The missing angle measures 51.1°.

Non-Right Angled Triangles

What do you do if you want to find the missing sides or the missing angles in a triangle that is oblique?

Oblique triangles do not have a right angle and, therefore, you cannot use the techniques with the trigonometric ratios that you used with right-angled triangles. Instead, you need to develop new techniques.

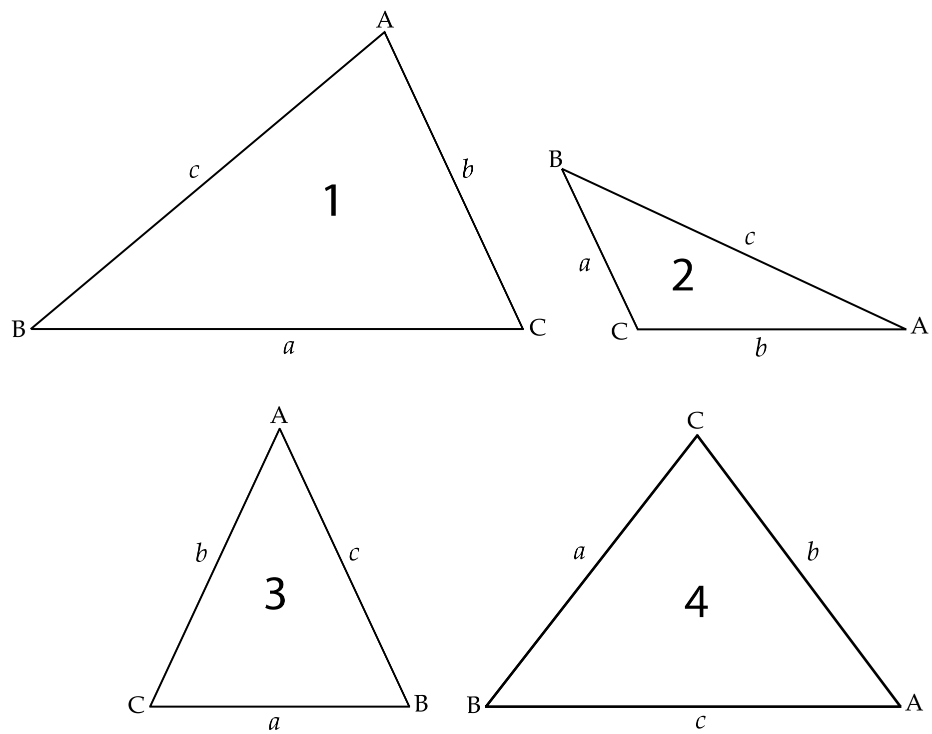
To solve oblique (not right-angled) triangles, you will need different trigonometric tools. One of the trigonometric tools that was developed for oblique triangles is called the **Law of Sines** or the **Sine Law**.

The Law of Sines can be found by observing the patterns obtained from the measurements of the following four triangles. Use a ruler and protractor to measure the sides and angles of each triangle and enter your data on the chart below. Then, use your calculator and the measurements you have taken to complete the last three columns of the chart.

	A	B	C	D	E	F	G	H	I	J
1	Triangle	a	b	c	A	B	C	$\frac{a}{\sin A}$	$\frac{b}{\sin B}$	$\frac{c}{\sin C}$
2	1									
3	2									
4	3									
5	4									

You should notice that the ratios in columns H, I, and J for each triangle are identical or very similar (due to rounding errors). You have discovered the Sine Law! It can be stated as: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Before you continue to the next lesson, make sure you complete Learning Activity 5.1 to remind yourself of right-angled triangle trigonometry.


A
B
C
a
b
c
1
2
3
4

Learning Activity 5.1

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. If the hypotenuse of a right triangle is 10 cm and one side is 8 cm, what is the length of the remaining side?
2. If two angles in a triangle have a sum of 125°, what is the size of the third angle in the triangle?
3. Write as a mixed number: $\frac{23}{6}$
4. Solve for x: $\frac{2}{x} = \frac{7}{3}$
5. Patrick can run 5 kilometres in 30 minutes. How long will it take him to run 8 kilometres?
6. Determine the slope of the line $y = \frac{1}{2}x - 3$.
7. Determine the y-intercept: $y = 3x - 2$
8. Factor: $x^2 - x - 6$

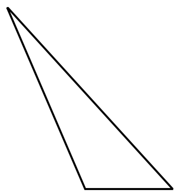
continued

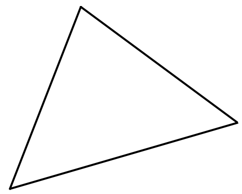
Learning Activity 5.1 (continued)

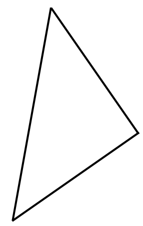
Part B: Review of Right-Triangle Trigonometry

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

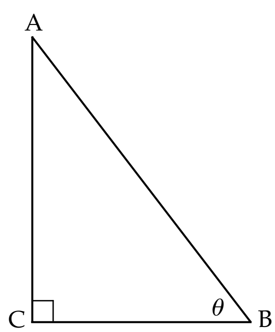
1. Classify the following triangles as right triangles, acute triangles, or obtuse triangles.

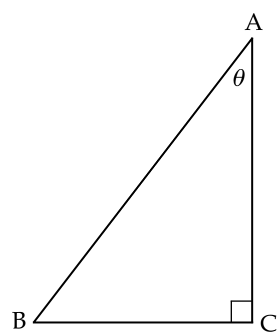
   a) 

   b) 

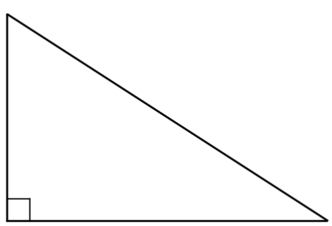
   c) 

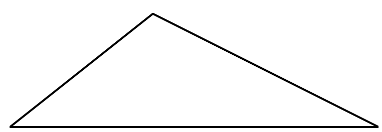
2. Based on the angle, θ, label the sides of each triangle as opposite, adjacent, or hypotenuse.

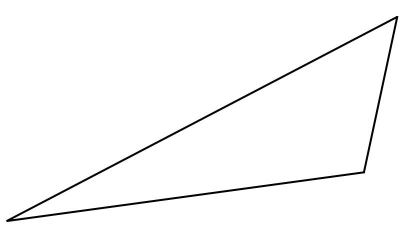
   a) 

   b) 

3. Determine in which of the following triangles you *cannot* use the trigonometric ratios of sin, cos, and tan, and explain why.

   a) 

   b) 

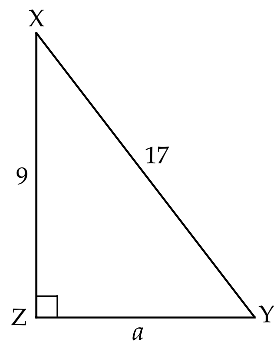
   c) 

continued

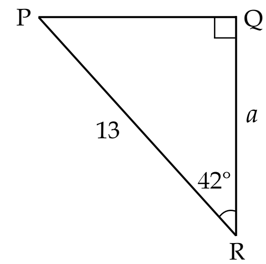
Learning Activity 5.1 (continued)

4. Calculate the value of *a* in the following triangles. Round to one decimal place when necessary.

a)

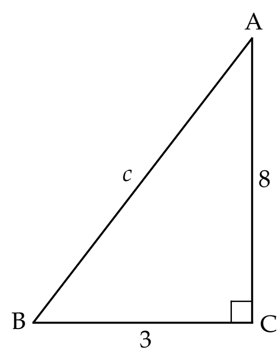


b)

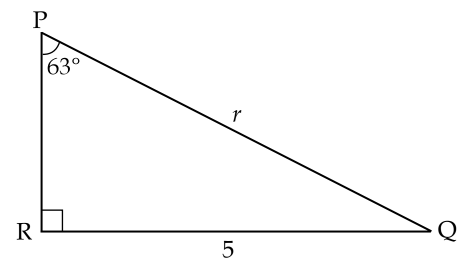


5. Find the length of the hypotenuse in the following right triangles. Round to one decimal place when necessary.

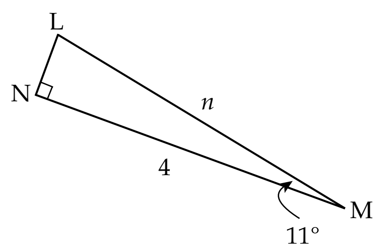
a)



b)



c)

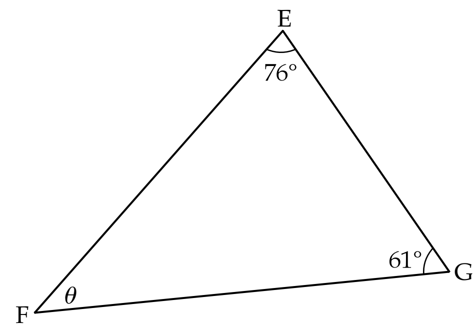


continued

Learning Activity 5.1 (continued)

6. Find the value of θ in the following triangles. Round to one decimal place when necessary.

a)



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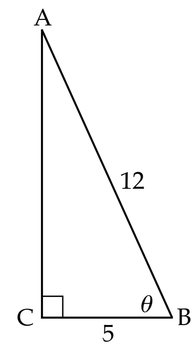

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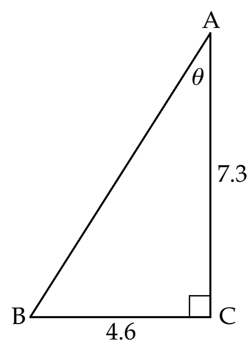


Lesson Summary

In this lesson, you expanded your knowledge of right-triangle trigonometry. You used four methods to solve right triangles including the Pythagorean theorem, the primary trigonometric ratios, the inverse trigonometric ratios, and the fact that the sum of the interior angles in a triangle is 180°.

At the end of the lesson, you were briefly introduced to the Sine Law. You will learn more about the Sine Law in Lesson 2.

Notes

Lesson 2: The Sine Law for Acute Triangles

Lesson Focus

In this lesson, you will

- ❑ learn about the Sine Law
- ❑ identify situations in which the Sine Law can be used
- ❑ learn how to use the Sine Law to solve various types of problems

Lesson Introduction

In Lesson 1, you reviewed how to solve right-angled triangles. However, not every triangle you encounter will be a right-angled triangle. Therefore, it would be helpful to develop strategies to solve triangles that do not include right angles. These types of triangles are also called *oblique triangles*. In this lesson, you will develop one such strategy that involves the ratio between the sides of a triangle and the sine of the opposite angle. This strategy is called the Sine Law.

The Sine Law

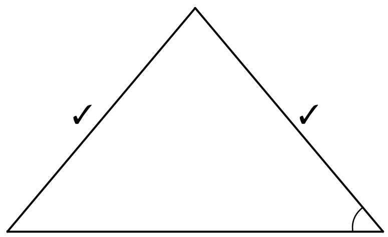
Oblique triangles are triangles that are not right-angled. To solve triangles that are oblique, or non-right angled, you will need to use different formulas and procedures than you have used before. One of the formulas you can use is called the **Sine Law**.

In Lesson 1, you completed an investigation that demonstrated that the ratios $\frac{a}{\sin \text{A}}$, $\frac{b}{\sin \text{B}}$, and $\frac{c}{\sin \text{C}}$ of a triangle are equivalent. This is the Sine Law, which can be written in either of the following two forms:

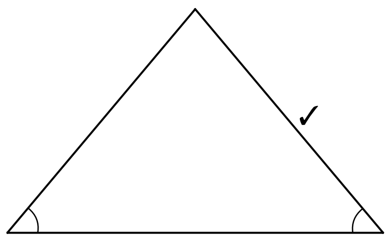
$$\frac{\sin\angle\text{A}}{a} = \frac{\sin\angle\text{B}}{b} = \frac{\sin\angle\text{C}}{c} \quad \text{or} \quad \frac{a}{\sin\angle\text{A}} = \frac{b}{\sin\angle\text{B}} = \frac{c}{\sin\angle\text{C}}$$

The Sine Law can be written in two different ways, as illustrated above, because equal ratios are still equal if you compare their reciprocals. You can use the Sine Law to solve for unknown sides and angles of any oblique triangle when you know a side and the opposite angle (in addition to one other piece of information) as shown below.

- Two side lengths and an angle opposite one of the known sides



- Two angles and any side length (if you know two angles, you can always find the third angle)



When determining the length of a missing side, it is simpler to use the Sine Law formula in the form shown below so that the unknown side will be in the numerator.

$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$

When determining the size of a missing angle, it is simpler to use the Sine Law formula in the form shown below so that the unknown angle will be in the numerator.

$$\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}$$

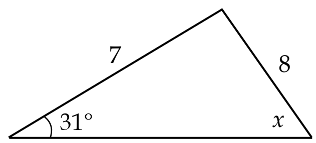
It may be helpful for you to add the Sine Law to your resource sheet. You should also include the situations in which the Sine Law can be used.

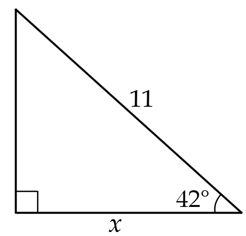
Using the Sine Law

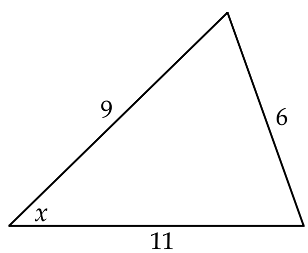
The Sine Law can only be used in two situations, as described previously. The following examples will help you recognize when to use the Sine Law and how to solve problems that require the use of the Sine Law.

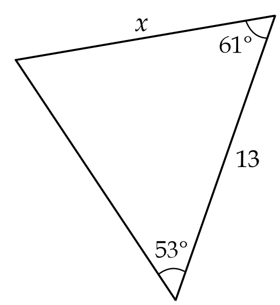
Example 1

In which of the following situations can you use the Sine Law to solve for the indicated angle or side length? Explain.

a) 

b) 

c) 

d) 

Solution

a) You can use the Sine Law to find the size of x in this triangle because you are given two side lengths and an angle directly opposite one of these side lengths (the side with a length of 8 and the 31° angle are directly opposite each other).

b) You could (but you do not need to) use the Sine Law to find the size of x in this triangle because this is a right triangle; you can use right-triangle trigonometric ratios.

c) You cannot use the Sine Law to find the size of x in this triangle because you are given the lengths of the three sides of the triangle. In order to use the Sine Law, you need to know at least one angle that is opposite a known side.

d) You can use the Sine Law to find the size of x in this triangle because:
- you are given the length of one side
- you are given two angles and you can find the third angle opposite the known side by using the fact that the sum of the interior angles in a triangle is 180°

Example 2

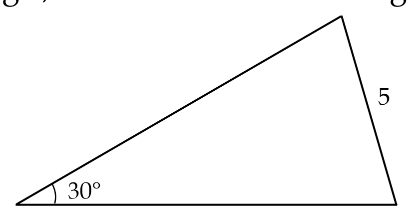
Sketch a triangle that corresponds to each given Sine Law expression.

a) $\dfrac{\sin 30°}{5} = \dfrac{\sin x}{8}$

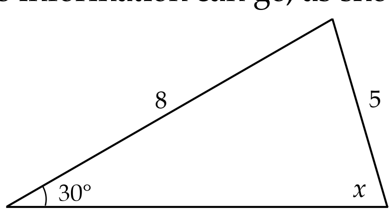
b) $\dfrac{11}{\sin 67°} = \dfrac{x}{\sin 55°}$

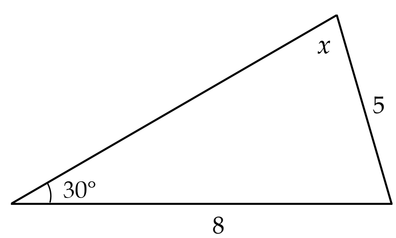
Solution

a) Your first step is to draw an acute triangle with a 30° angle. Opposite this angle, label the side with a length of 5 units.



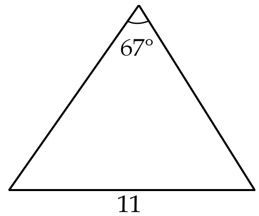
Now, you need to decide where to place the angle x and the side length of 8 units. These need to be directly opposite each other. There are two places this information can go, as shown below.



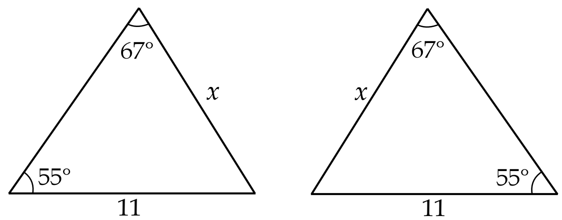


Either one of the above triangles would be a triangle that corresponds to the Sine Law expression of $\dfrac{\sin 30°}{5} = \dfrac{\sin x}{8}$.

b) Your first step is to draw an acute triangle with a 67° angle. Opposite this angle, label the side with a length of 11 units.



Now, you need to decide where to place the side x and the angle of 55°. There are two places this information can go, as shown below.

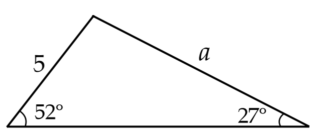


Either one of the above triangles would be a triangle that corresponds to the Sine Law expression of $\dfrac{11}{\sin 67°} = \dfrac{x}{\sin 55°}$.

From Example 2, you can confirm that the corresponding side length and angle measurement in each fraction are *always* directly opposite each other in the sketch of the triangle.

Example 3

Solve for the missing side in each of the following triangles. Round to two decimal places when necessary.

a) 

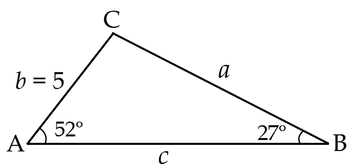
Solution

a) In this triangle, you are given two angles and one side directly opposite one of those angles. You are asked to find the side that is directly opposite the other angle. In this situation, you are able to use the Sine Law.

Because you are looking for a side length, use the version of the Sine Law that has the side lengths in the numerators.

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

You will only use two of the three ratios to create an equation with one unknown. Before you substitute values into the Sine Law, it may be beneficial for you to label the triangle with sides *a*, *b*, and *c*, as well as angles A, B, and C. It does not matter how you label your triangle, as long as each angle is directly opposite the side with the same letter.



Now you can fill in the Sine Law with the information you have.

$$\frac{a}{\sin 52^\circ} = \frac{5}{\sin 27^\circ}$$

Notice that you are using only two of the three ratios.

You can now solve for *a* in two ways.

Method 1

$$\frac{a}{\sin 52^\circ} = \frac{5}{\sin 27^\circ}$$ Multiply both sides of the equation by sin 52°.

$$a = \frac{5 \sin 52^\circ}{\sin 27^\circ}$$ Determine *a* by using your calculator.

$$a = 8.68$$

Note: If you are typing this entire expression into your calculator, make sure you close the brackets around sin (52°), or make sure you press Enter after typing in the numerator of the fraction. Both of these strategies should ensure you get the correct answer of 8.68.

Method 2

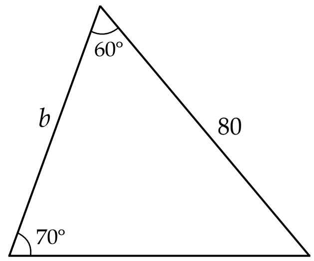
$$\frac{a}{\sin 52^\circ} = \frac{5}{\sin 27^\circ}$$ Evaluate each sine expression.

$$\frac{a}{0.78801} = \frac{5}{0.45399}$$ Multiply both sides of the equation by 0.78801.

$$a = \frac{3.94005}{0.45399}$$ Complete the division.

$$a = 8.68$$

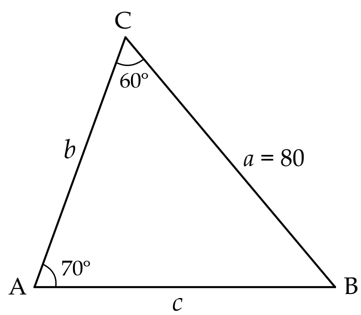
b)



Solution

In this acute triangle, you are given two angles and one side directly opposite one of those angles. In this situation, you are able to use the Sine Law.

First, finish labelling the triangle.



You are being asked to find side b. In order to use the Sine Law, you first need to determine the size of the angle directly opposite b. To do this, subtract the given angles from 180°.

$\angle B = 180° - 70° - 60°$

$\angle B = 50°$

Now you are able to use the Sine Law.

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$$ Substitute in known values for a, ∠A, and ∠B.

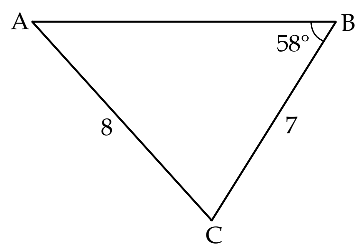
$$\frac{80}{\sin 70°} = \frac{b}{\sin 50°}$$ Multiply both sides of the equation by sin 50°.

$$\frac{80 \sin 50°}{\sin 70°} = b$$ Determine b by using your calculator.

$$b = 65.22$$

Example 4

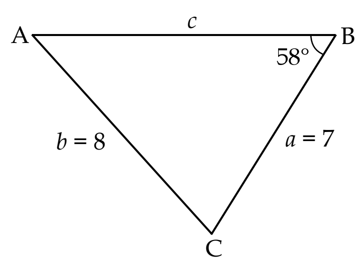
Solve for ∠A in the following triangle. Round to one decimal place.



Solution

You can use the Sine Law in this situation because you are given the lengths of two sides as well as the size of one angle directly opposite one of the given sides.

You may find it helpful to continue labelling your triangle before you calculate the size of the missing angle.



Now you are able to use the Sine Law. However, since you are looking for a missing angle, use the form of the Sine Law that contains the angles in the numerator.

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b}$$ Substitute in known values for a, b, and ∠B.

$$\frac{\sin \angle A}{7} = \frac{\sin 58°}{8}$$ Multiply both sides of the equation by 7.

$$\sin \angle A = \frac{7 \sin 58°}{8}$$ Calculate the ratio using your calculator.

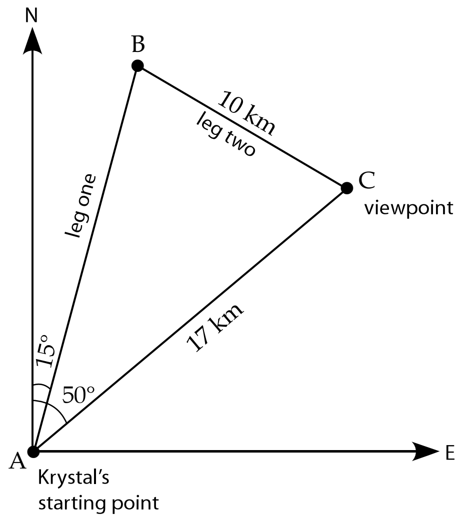
$$\sin \angle A = 0.74204$$ Determine the value of A by taking the inverse sine ($\sin^{-1}$) of both sides of the equation.

$$\angle A = \sin^{-1}(0.74204)$$

$$\angle A = 47.9°$$

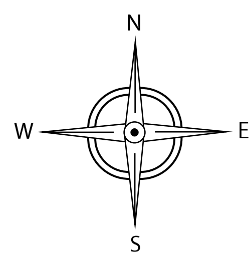
Example 5

Krystal is planning a hiking trip through a forest. Her destination is a viewpoint overlooking a river 17 km away in the direction N 50° E from her starting position. However, in order to arrive safely at the viewpoint, she has to travel around a lake and complete her hike in two legs. To begin the first leg of her trip, she travels N 15° E. If the second leg of her trip is 10 km long, how many kilometres does Krystal hike before arriving at the viewpoint?



Solution

Note: Compass bearings are often used in navigation. N 50° E means start facing north, and turn 50° towards the east. N 15° E means start facing north, and turn 15° towards the east. S 82° W means start facing south, and turn 82° towards the west. Problems involving these types of directions are often solved using trigonometry.



You are given two sides of the triangle and enough information to find one angle in the triangle. To find ∠A you can subtract 15° from 50°.

$$\angle A = 50° - 15° = 35°$$

Now you can set up the Sine Law to solve for ∠B. You cannot solve for the first leg right away because you do not know the size of the angle directly opposite this side, ∠C.

$$\frac{\sin 35}{10} = \frac{\sin \angle B}{17}$$

Multiply both sides of the equation by 17.

$$\frac{17 \sin 35}{10} = \sin \angle B$$

Use your calculator to find the ratio.

$$0.9750799418 = \sin \angle B$$

$$\sin^{-1}(0.9750799418) = \angle B$$

Take the inverse sine of both sides of the equation.

$$\angle B = 77.18206124°$$

Note: Do not round your answer right away. Rounding calculations in the middle of a question can lead to significant errors in your final answer.

To find $\angle C$, subtract $\angle A$ and $\angle B$ from 180°.

$$\angle C = 180° - \angle A - \angle B = 180° - 35° - 77.18206124° = 67.81793876°$$

You are now able to solve for the length of leg one or side c.

$$\frac{10}{\sin 35} = \frac{c}{\sin 67.81793876}$$

Multiply both sides of the equation by sin 67.81793876.

$$c = \frac{10 \sin 67.81793876}{\sin 35}$$

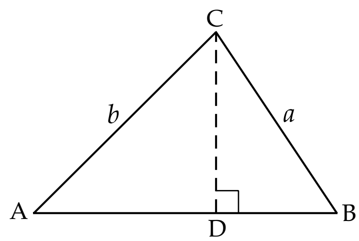
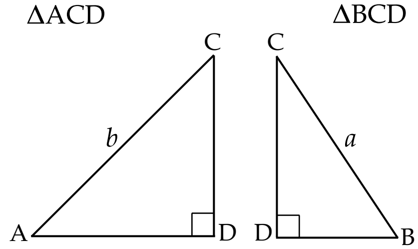
Use your calculator to determine the value of c.

$$c = 16.1$$

The length of Krystal's entire trip was 16.1 km + 10 km = 26.1 km.

Proof of the Sine Law

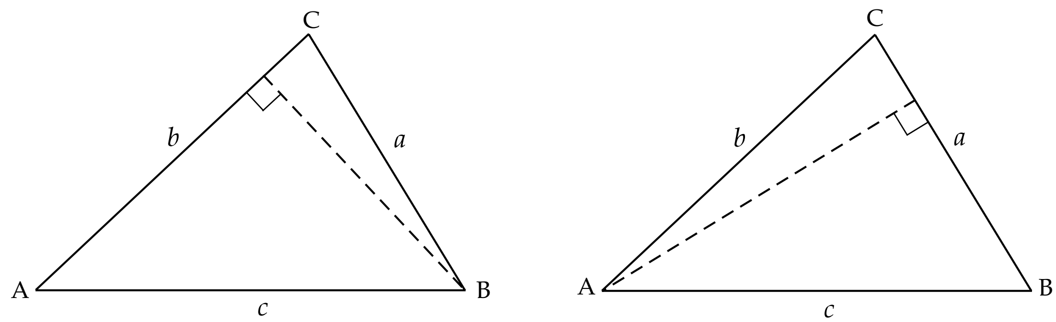
It is possible to prove that the Sine Law always works, as shown below. After examining the steps in this proof, you can use the Sine Law with confidence. To develop the proof of the Sine Law for acute triangles, you can divide an acute triangle into two right triangles, as shown in the diagram below. When you are trying to prove a statement, you should always start with what you know. This is the same type of process as the one you used in Module 4. In this situation, you already know how to solve right triangles. You can set up this proof using two columns, the same method you used in Module 4.

Mathematical Manipulation	Explanation
	Start with any triangle.
ΔACD ΔBCD 	Divide the triangle into 2 right-angled triangles. Draw both right-angled triangles separately.
$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CD}{b}$	Write the sine ratio for ∠A in ΔACD. **Note:** CD is the side length beginning at point C and ending at point D.
$\sin A = \frac{CD}{b}$	Rewrite the formula without the middle step.
$b \sin A = CD$	Multiply both sides of the equation by b.
$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CD}{a}$	Write the sine ratio for ∠B in ΔBCD.
$\sin B = \frac{CD}{a}$	Rewrite the formula without the middle step.
$a \sin B = CD$	Multiply both sides of the equation by a.
$b \sin A = CD$ $a \sin B = CD$	You now have two expressions for CD.
$b \sin A = a \sin B$	Equate these expressions.
$\frac{b \sin A}{ab} = \frac{a \sin B}{ab}$	Divide both sides of the equation by ab.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	Simplify by cancelling common factors.

This is one of the Sine Law ratios.

To determine the other ratio of the Sine Law involving side c and angle C, you need to divide the original triangle ΔACB into two different right-angled triangles.

Note: There are two ways to do this, as illustrated below. You can split the triangle up either way, and finish the proof of the Sine Law.



You will develop the complete proof of the Sine Law in the last question of the following learning activity. This learning activity will also allow you to practice using the Sine Law to solve oblique triangles.

Learning Activity 5.2

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Write in lowest terms: $\dfrac{100}{104}$

2. Write an equivalent fraction to $\dfrac{2}{7}$.

3. Solve for x: $\dfrac{x}{7} = \dfrac{3}{10}$

continued

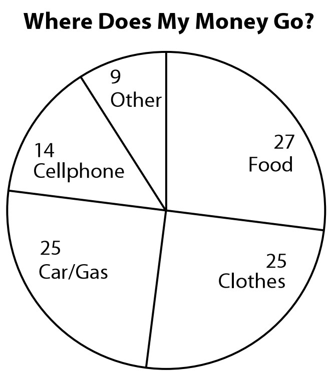
Learning Activity 5.2 (continued)

4. Solve for x: $\dfrac{x-4}{2} = \dfrac{1}{2}$

5. Order from smallest to largest: $\dfrac{7}{10}, \dfrac{3}{4}, \dfrac{5}{8}, \dfrac{7}{9}$

6. Convert 23 mm to cm.

Use the following pie chart to answer questions 7 and 8.



7. Where does this person spend most of his or her money?

8. In which two combined categories does this person spend 50% of his or her money?

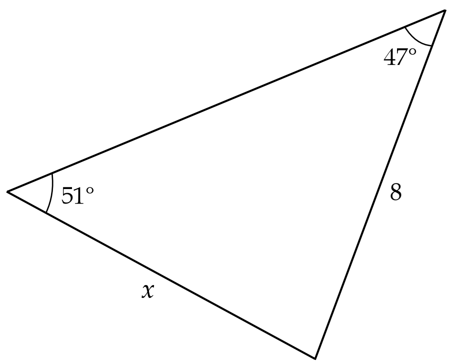
continued

Part B: Using the Sine Law to Solve Acute Triangles

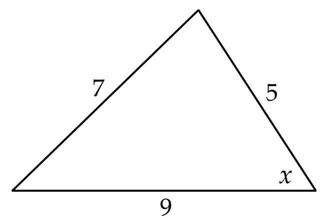
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. State whether you can use the Sine Law to solve for the indicated angle or side in each of the following triangles. Explain how you determined your answer.

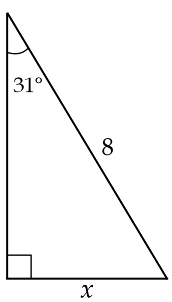
a)



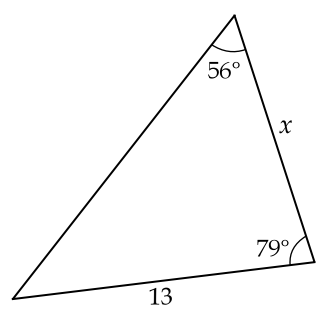
b)



c)



d)



2. Sketch a triangle that corresponds to each of the following Sine Law expressions.

a) $\dfrac{\sin 62^\circ}{6} = \dfrac{\sin 47.4^\circ}{5} = \dfrac{\sin 70.3^\circ}{4.9}$

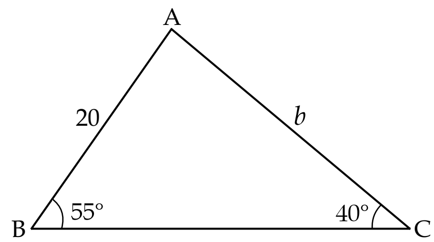
b) $\dfrac{x}{\sin 38^\circ} = \dfrac{9}{\sin 52^\circ}$

continued

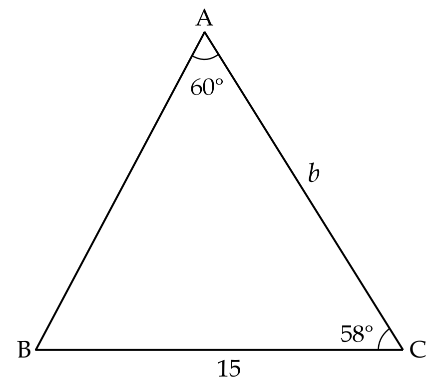
Learning Activity 5.2 (continued)

3. Find the length of side b. Round to one decimal place.

a)

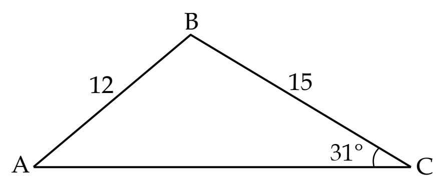


b)

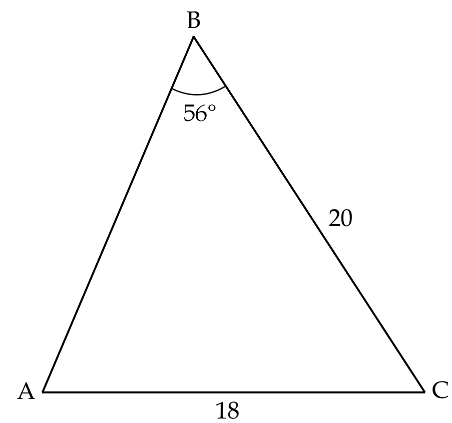


4. Find ∠A. Round to one decimal place.

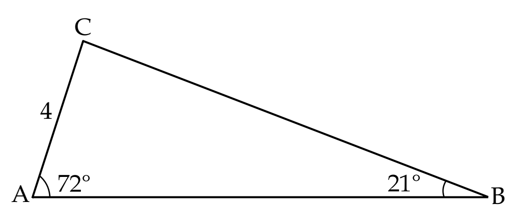
a)



b)



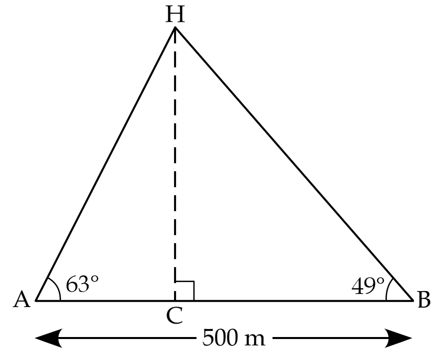
5. Find the lengths of c and a, and the size of ∠C. Round to one decimal place.



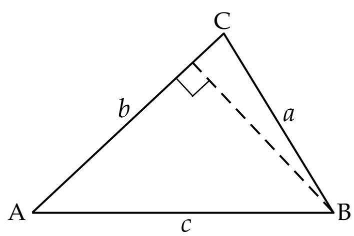
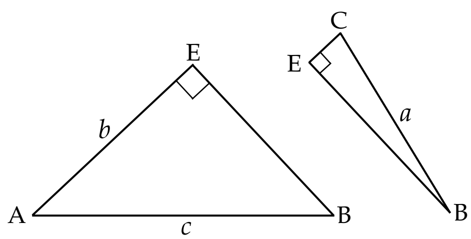
continued

Learning Activity 5.2 (continued)

6. Adrienne and Brody are trying to determine the height of a tower. For Brody, the angle of elevation is 49° and for Adrienne the angle of elevation is 63°. How high is the tower, to the nearest tenth of a metre, if the distance between Adrienne and Brody is 500 metres?



7. Explain each of the steps in the partial proof of the Sine Law below.

Mathematical Manipulation	Explanation
	
	
$\sin A = \frac{EB}{c}$	
$c \sin A = EB$	
$\sin C = \frac{EB}{a}$	
$a \sin C = EB$	
$c \sin A = a \sin C$	
$\frac{\sin A}{a} = \frac{\sin C}{c}$	

Lesson Summary

In this lesson, you used deductive reasoning to prove the Sine Law and used this law to solve for missing side lengths and missing angle measurements in oblique triangles. In the next lesson, you will encounter situations where the Sine Law cannot be used. In those situations, you will discover another relationship among triangle sides and angles that you can use to help you solve triangles. It is called the Cosine Law.

Notes

Lesson 3: The Cosine Law for Acute Triangles

Lesson Focus

In this lesson, you will

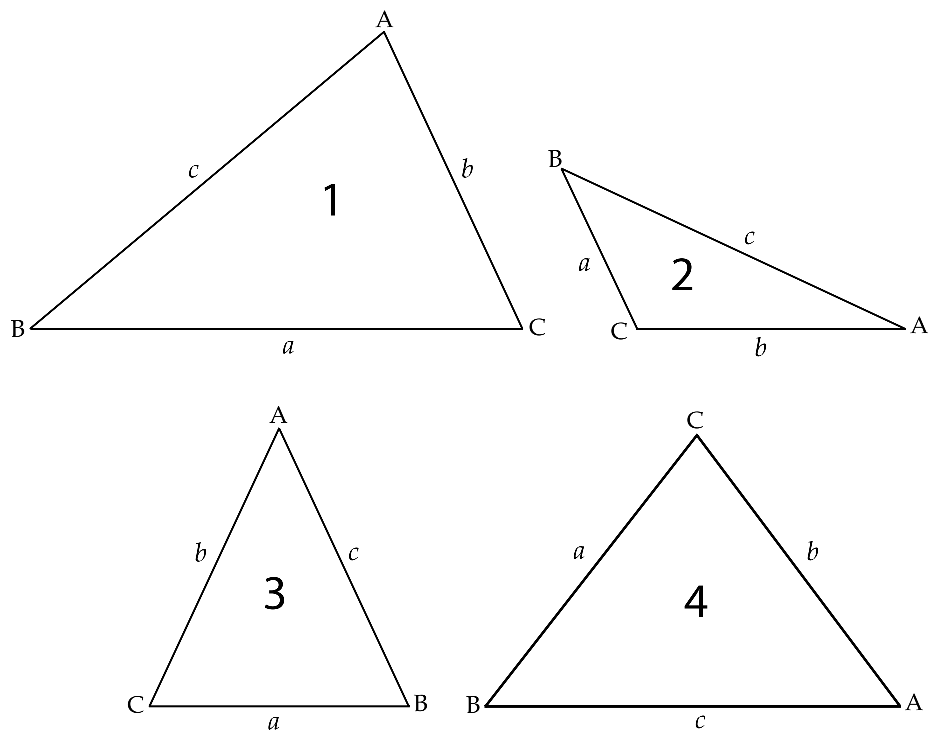
- ❑ learn about the Cosine Law
- ❑ learn to recognize situations where the Cosine Law can be used
- ❑ learn how to use the Cosine Law to solve various types of problems

Lesson Introduction

In Lesson 2, you learned how to solve triangles using the Sine Law. However, as you noticed, the Sine Law does not work in every situation. You discovered certain situations where the Sine Law couldn't be applied. In this lesson, you will learn how the Cosine Law can also be used to solve triangles. Either the Cosine Law or the Sine Law can be used to solve all oblique triangles as long as you are given three pieces of information about a triangle, including at least one side length.

The Cosine Law

The Cosine Law can be observed from the measurements of the following four triangles. These are the same four triangles you measured in Lesson 1. You can use the measurements you recorded earlier to complete the chart found on the following page, or you can use your ruler and protractor to measure each side and angle again. Then, use your calculator to complete the last four columns.



	A	B	C	D	E	F	G	H	I
1	Triangle	a	b	c	$\angle C$	$a^2 + b^2$	c^2	$a^2 + b^2 - c^2$	$2ab \cos \angle C$
2	1								
3	2								
4	3								
5	4								

After completing the calculations for the last four columns in this chart, you will notice that the values in columns H and I for each triangle are identical or, due to measurement accuracy, very similar. This relationship is called the Cosine Law:

$$a^2 + b^2 - c^2 = 2ab \cos \angle C$$

If you add c^2 to both sides and subtract $2ab \cos \angle C$ from both sides, you will arrive at the formula called the Cosine Law:

$$a^2 + b^2 - 2ab \cos \angle C = c^2$$

or

$$c^2 = a^2 + b^2 - 2ab \cos \angle C$$

Notice the similarity between the Cosine Law and the Pythagorean theorem. When angle C is a right angle, the value of $2ab \cos \angle C$ is zero and you are left with $c^2 = a^2 + b^2$, which is only true if C = 90°.

Since there is no right angle in an oblique triangle, there is also no hypotenuse. So the Cosine Law works for any of the three sides of the triangle as long as the angle that is used (it may be called $\angle C$) is the angle opposite the length of the side that appears on the left side of the formula equation (side c). Thus, the Cosine Law can be stated in different ways. The following examples show the same Cosine Law using different arrangements of the variables.

$$c^2 = a^2 + b^2 - 2ab \cos \angle C$$

$$b^2 = a^2 + c^2 - 2ac \cos \angle B$$

$$a^2 = b^2 + c^2 - 2bc \cos \angle A$$

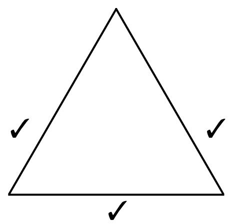
$$p^2 = q^2 + r^2 - 2qr \cos \angle P$$

The Cosine Law requires that the angle on the right of the formula (in capital letter) is the one opposite the side that is on the left of the formula (corresponding small letter).

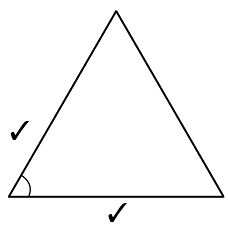
When to Use the Cosine Law

The Cosine Law can be used in two situations. You can use the Cosine Law when you are not given or cannot find both a side and its opposite angle. Use the Cosine Law when you are given the following information about a triangle.

- Three sides of the triangle



- Two sides of a triangle and the angle between those two sides

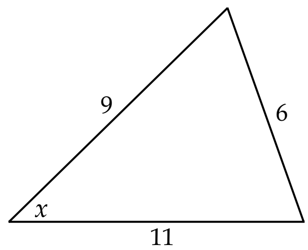


The Cosine Law can be used to solve oblique triangles. Just as the Sine Law could only be used in certain situations, the same is true for the Cosine Law; it can only be used in certain situations as well. It may be helpful for you to add this formula to your resource sheet, as well as the situations where you can use the Cosine Law.

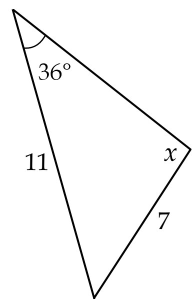
Example 1

In which of the following situations can you use the Cosine Law to solve for the indicated angle or side length? Explain.

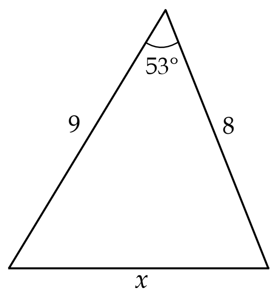
a)



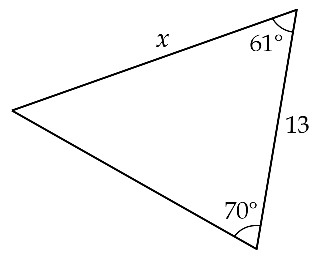
b)



c)



d)



Solution

a) You can solve for the indicated angle in this triangle using the Cosine Law because you are given three sides of the triangle. You could not use the Sine Law.

b) You cannot solve this triangle using the Cosine Law because the given angle is not between the two given side lengths. However, you could solve this triangle using the Sine Law.

c) You can solve this triangle using the Cosine Law because the given angle is between the two given side lengths. Also, the side you are trying to find is directly opposite the given angle. Notice that you could not use the Sine Law because you don't know the measurements for a side length and its opposite angle.

d) You cannot solve this triangle using the Cosine Law. You need to know at least two side lengths to use the Cosine Law. However, you could find the missing angle in this triangle by using the fact that all angles in a triangle add up to 180°. You can then find the missing side by using the Sine Law.

Example 2

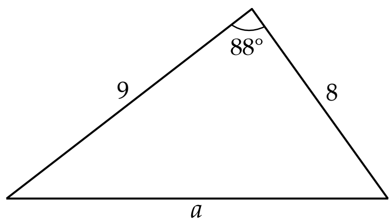
Sketch a triangle that corresponds to each of the following Cosine Law equations.

a) $a^2 = 8^2 + 9^2 - 2(8)(9) \cos 88°$

b) $11^2 = 7^2 + 8^2 - 2(7)(8) \cos \angle A$

Solution

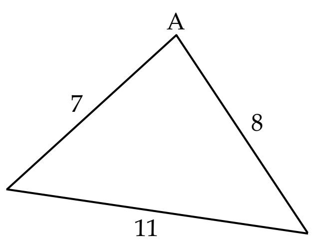
a)



You know that two side lengths of the triangle must be 8 units and 9 units long. You also know that one angle measures 88°.

In this triangle, the angle of 88° must be between the two sides with lengths of 8 and 9. The side with length *a* must be opposite the 88° angle.

b)



You know that the three side lengths of the triangle must be 7 units, 8 units, and 11 units long.

In this triangle, it does not matter where you put each side length. However, ∠A must be directly opposite the side with a length of 11 units.

Solving Triangles Using the Cosine Law

If you are given three sides of an oblique triangle, you can solve the triangle by following these steps:

1. Find the largest angle using the Cosine Law (the largest angle is opposite the largest side).
2. Find the second angle using the Sine Law.
3. Find the third angle using the fact that the sum of angles in a triangle is 180°.

Note

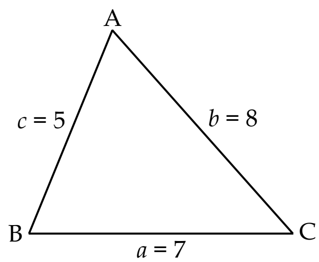
Note: In general, you should round your answers to the nearest tenth (one decimal place) unless stated otherwise in the question.

Example 3

Find the size of the largest angle in triangle ABC, if $a = 7$, $b = 8$, and $c = 5$.

Solution

First, you need to draw this triangle. It does not matter where you put the vertices A, B, and C. However, once these vertices are labelled, you need to put the side length corresponding to each letter directly opposite the corresponding angle.



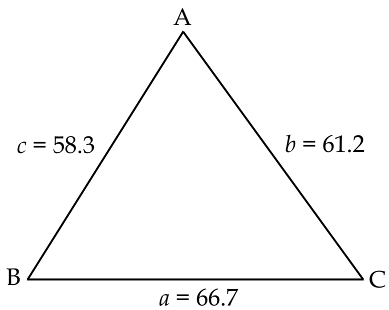
Since $b = 8$ is the longest side, it follows that the angle opposite this, ∠B, is the largest angle.

Use the Cosine Law to determine the size of ∠B.

$b^2 = a^2 + c^2 - 2ac \cos \angle B$	
$8^2 = 7^2 + 5^2 - 2(7)(5) \cos \angle B$	Substitute the known values into the equation.
$64 = 49 + 25 - 70 \cos \angle B$	Evaluate.
$64 - 49 - 25 = -70 \cos \angle B$	Subtract 49 and 25 from both sides.
$-10 = -70 \cos \angle B$	Simplify.
$\dfrac{-10}{-70} = \cos \angle B$	Divide both sides of the equation by -70.
$\angle B = \cos^{-1}\left(\dfrac{10}{70}\right)$	Find the angle using inverse cosine ($\cos^{-1}$) of the ratio.
$\angle B = 81.8°$	

Example 4

Solve for the missing angles in the following triangle.



Solution

Solve for the largest angle that is opposite the largest side. The largest side is side a, so the largest angle is $\angle$A.

Use the Cosine Law that includes $\angle$A.

$$a^2 = b^2 + c^2 - 2bc \cos \angle A$$

Substitute in known values.

$$66.7^2 = 61.2^2 + 58.3^2 - 2(61.2)(58.3) \cos \angle A$$

You can solve for $\angle$A in two ways.

Method 1:

Evaluate each part of the equation and then isolate $\angle$A. **Note:** This is the same method that was used in Example 1.

$$\begin{aligned}
4448.89 &= 3745.44 + 3398.89 - 7135.92 \cos \angle A \\
4448.89 &= 7144.33 - 7135.92 \cos \angle A \\
-2695.44 &= -7135.92 \cos \angle A \\
\frac{-2695.44}{-7135.92} &= \cos \angle A \\
\cos^{-1}\left(\frac{2695.44}{7135.92}\right) &= \angle A \\
67.8° &= \angle A
\end{aligned}$$

Method 2:

Isolate ∠A first and then evaluate the equation.

$$66.7^2 - 61.2^2 - 58.3^2 = -2(61.2)(58.3)\cos \angle A$$

$$\frac{66.7^2 - 61.2^2 - 58.3^2}{-2(61.2)(58.3)} = \cos \angle A$$

$$\cos^{-1}\left(\frac{66.7^2 - 61.2^2 - 58.3^2}{-2(61.2)(58.3)}\right) = \angle A$$

$$67.8° = \angle A$$

Now that you have determined the size of ∠A, you can find the size of the second angle by using the Sine Law.

$$\frac{\sin \angle A}{a} = \frac{\sin \angle C}{c}$$ Substitute the known values into the equation.

$$\frac{\sin 67.8°}{66.7} = \frac{\sin \angle C}{58.3}$$ Multiply both sides by 58.3.

$$\frac{58.3 \sin 67.8°}{66.7} = \sin \angle C$$

$$\sin^{-1}\left(\frac{58.3 \sin 67.8°}{66.7}\right) = \angle C$$ Take the inverse sine ($\sin^{-1}$) of the ratio.

$$\angle C = 54.0°$$

Finally, you can solve for ∠B by using the fact that all three angles in a triangle have a sum of 180°.

$$\angle B = 180° - \angle A - \angle C$$

$$\angle B = 180° - 67.8° - 54.0°$$

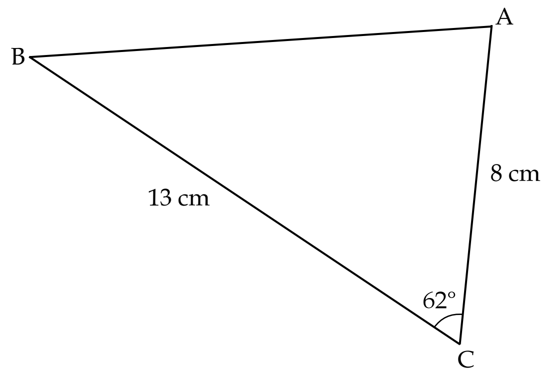
$$\angle B = 58.2°$$

If you are given two sides and one angle of an oblique triangle, you can solve the triangle by following these steps:

- Find the missing side by using the Cosine Law.
- Find the second angle by using the Sine Law (many students find the Sine Law easier to use than the Cosine Law).
- Find the third angle using the fact that the sum of angles in a triangle is 180°.

Example 5

Find the two missing angles and side c in the triangle below.



Solution

First, you need to determine the missing side c in the triangle. Since you don't know the measurement of an angle and its opposite side, you need to use the Cosine Law.

$a = 13$, $b = 8$, and $\angle C = 62°$

$c^2 = a^2 + b^2 - 2(a)(b) \cos C$

$c^2 = 13^2 + 8^2 - 2(13)(8) \cos 62°$ Substitute in known values.

$c^2 = 169 + 64 - 208 \cos 62°$

$c^2 = 233 - 208 \cos 62°$ Simplify.

$c^2 = 135.3499149$

$c = \sqrt{135.3499149}$ Take the square root of both sides of the equation.

$c = 11.6 \text{ cm}$

Note: In step 4, you may find it helpful to first evaluate 208 cos 62° to get 97.65008506. Then complete the subtraction to get 135.3499149. Alternatively, you could do the calculation in Step 3 all at the same time with your calculator.

The next step is to determine the size of the second angle using the Sine Law.

$$\frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

$$\frac{\sin \angle B}{8} = \frac{\sin 62°}{11.6}$$ Substitute in known values.

$$\sin \angle B = \frac{8 \sin 62°}{11.6}$$

$$\angle B = \sin^{-1}\left(\frac{8 \sin 62°}{11.6}\right)$$ Take the inverse sine of both sides of the equation.

$$\angle B = 37.5°$$

Your final step is to find $\angle A$.

$$\angle A = 180° - \angle B - \angle C$$

$$\angle A = 180° - 37.5° - 62°$$

$$\angle A = 80.5°$$

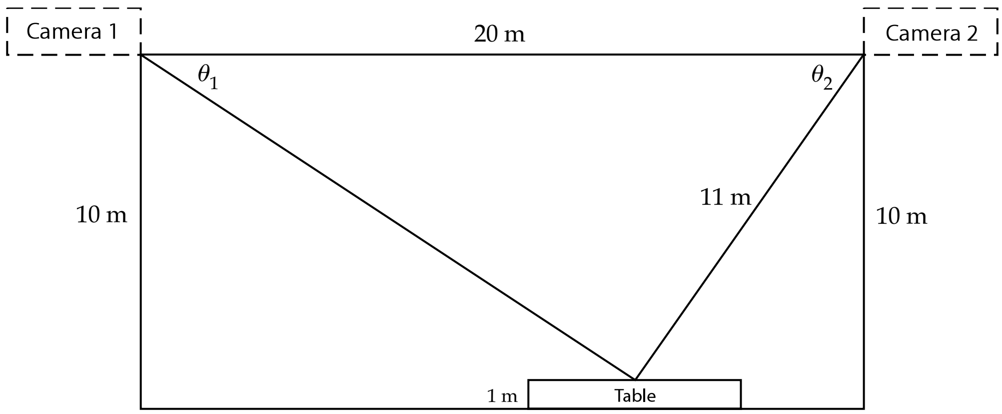
Example 6

In a bank, the room containing the safety deposit boxes needs to have two security cameras installed 20 m apart. Both of these cameras must be focused on the centre of the table located in the middle of the room. The cameras are placed 10 m from the floor on opposite walls and directly across from each other. The centre of the top of the table is 1 m from the floor and 11 m from the camera on the right. Determine the angle of depression at which each camera needs to be placed.

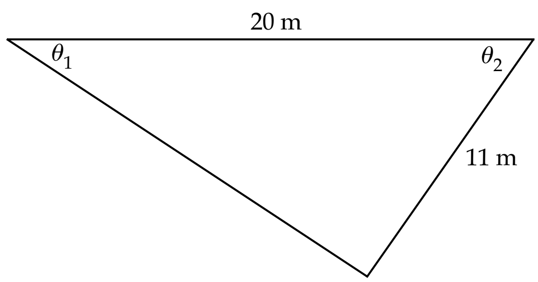
Note: The angle of depression is between a horizontal line and an object. In this situation, the angles of depression are between a horizontal line (the roof) and the tabletop.

Solution

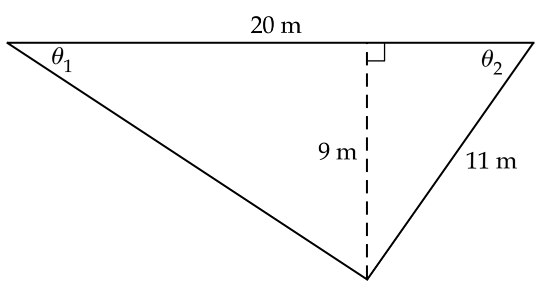
It may be helpful to draw a diagram showing the information you know about this situation.



In the above diagram, you are looking for both θ_1 and θ_2. If you draw the triangle with these angles on its own, you can see more clearly what triangle information you need to find out about the triangle.



Right now, you only have two pieces of information about this triangle. However, you are able to find the height of this triangle, which will be the distance between the table and the height of the cameras. As this table is 1 m high and the wall is 10 m high, the distance between the table and the camera height must be 9 m. This is the height of the triangle.



The large triangle is now divided into two right triangles. You have two pieces of information about the triangle on the right. You have the side opposite the angle (9 m) and the hypotenuse (11 m). Therefore, you can use the sine ratio to solve for the missing angle in this triangle.

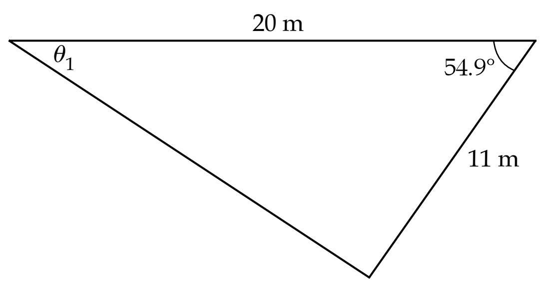
$$\sin \theta_2 = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \theta_2 = \frac{9}{11}$$

$$\theta_2 = \sin^{-1}\left(\frac{9}{11}\right)$$

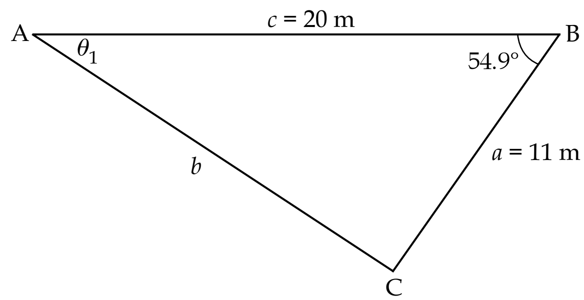
$$\theta_2 = 54.9°$$

You can add this information to your diagram.



Because you have the lengths of two sides and an angle between these two sides, you can use the Cosine Law to find the missing side of this triangle. Once you have determined the length of the missing side, then you can use the Sine Law to determine the size of the angle you need to find, θ_1.

You may find it helpful to label your triangle with vertices A, B, and C, as well as sides a, b, and c.



$$b^2 = a^2 + c^2 - 2ac \cos \angle B$$

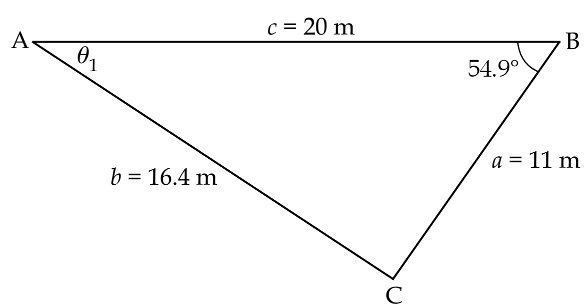
$$b^2 = 11^2 + 20^2 - 2(11)(20) \cos 54.9°$$

$$b^2 = 121 + 400 - 440 \cos 54.9°$$

$$b^2 = 521 - 440 \cos 54.9°$$

$$b = \sqrt{521 - 440 \cos 54.9°}$$

$$b = 16.4 \text{ cm}$$



You can now use the Sine Law to determine the size of θ_1.

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b}$$

$$\frac{\sin \theta_1}{11} = \frac{\sin 54.9°}{16.4}$$

$$\sin \theta_1 = \frac{11 \sin 54.9°}{16.4}$$

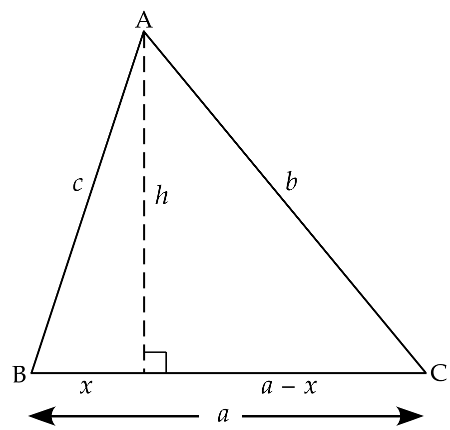
$$\theta_1 = \sin^{-1}\left(\frac{11 \sin 54.9°}{16.4}\right)$$

$$\theta_1 = 33.3°$$

Therefore, the angle of depression of the first camera needs to be 33.3° while the angle of depression of the second camera needs to be 54.9°.

Proof of the Cosine Law

The proof of the Cosine Law involves the use of the Pythagorean theorem as well as the primary trigonometric ratio for cosine.

Mathematical Manipulation	Explanation
	Given any oblique triangle ABC, divide the triangle into two right triangles by drawing a segment for the height, h. Note that side a is divided into two parts—one with length x, and one with length, $a - x$.
$h^2 + x^2 = c^2$	Use the Pythagorean theorem for the right triangle on the left.
$h^2 = c^2 - x^2$	Isolate h^2.
$h^2 + (a - x)^2 = b^2$ $h^2 = b^2 - (a - x)^2$	Use the Pythagorean theorem for the triangle on the right. Isolate for h^2.
$b^2 - (a - x)^2 = c^2 - x^2$	Equate the two expressions for h^2.
$b^2 = c^2 + (a - x)^2 - x^2$	Isolate b^2.
$b^2 = c^2 + (a - x)(a - x) - x^2$	Expand the brackets for $(a - x)^2$.
$b^2 = c^2 + a^2 - 2ax + x^2 - x^2$	Use the distributive property to multiply.
$b^2 = c^2 + a^2 - 2ax$	Simplify.

continued

Mathematical Manipulation	Explanation
	Right now, this equation looks similar to the Cosine Law. However, there is an extra variable, x. To eliminate this variable, substitute in an equivalent expression for x using the primary cosine trigonometric ratio.
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	The cosine ratio for a right-angled triangle.
$\cos \angle B = \dfrac{x}{c}$	Substitute in known values from the triangle on the left. Multiply both sides of the equation by c.
	Substitute this expression in for x into the equation $b^2 = c^2 + a^2 - 2ax$.
$b^2 = c^2 + a^2 - 2ac \cos \angle B$	This is the Cosine Law.

Make sure you complete the following learning activity to practise using the Cosine Law in various situations.

Learning Activity 5.3

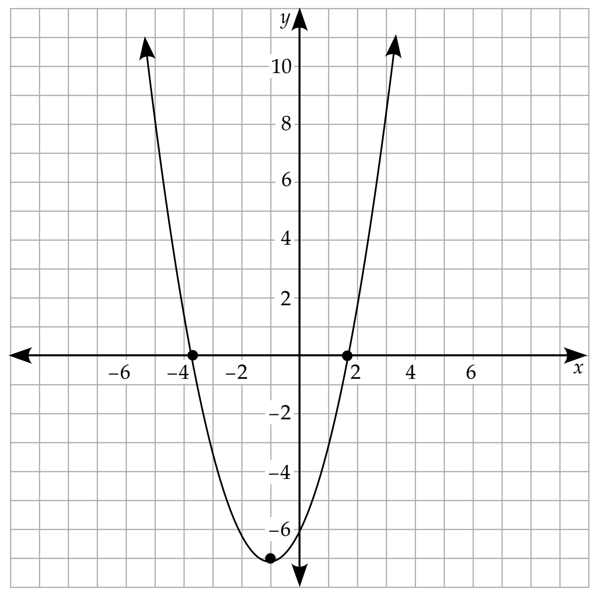
Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

Use the following graph to answer questions 1 to 3.

1. Determine the coordinates of the vertex.
2. Determine the domain.
3. Determine the range.



4. Factor: $x^2 - 12x + 11$
5. Solve for x: $(x - 2)(2x + 1) = 0$
6. What percent of 250 is 50?
7. Solve for x: $\frac{3}{2}x = 4$
8. Which is the better deal, a dozen eggs for $3.49 or eighteen eggs for $4.50?

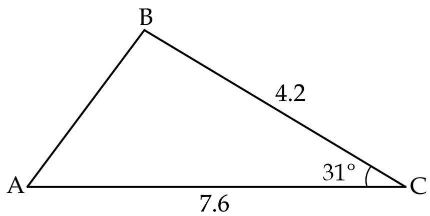
continued

Part B: The Cosine Law

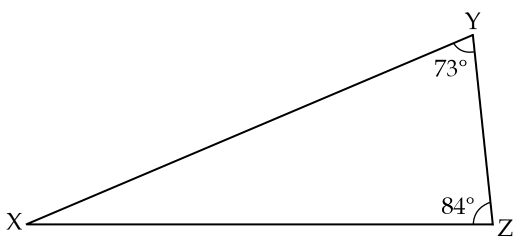
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Which formula should you use to solve the following triangles: the Sine Law or the Cosine Law? Justify your decisions. You do not have to solve the triangles.

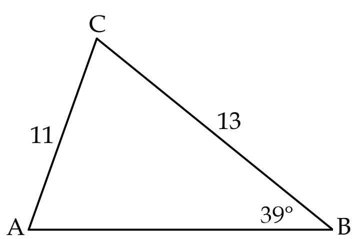
   a)



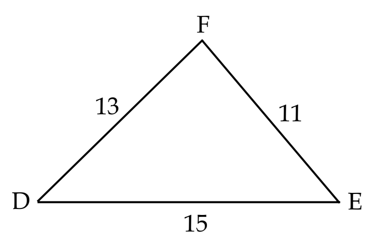
   b)



   c)



   d)



2. Sketch a triangle that corresponds to each of the following Cosine Law expressions.

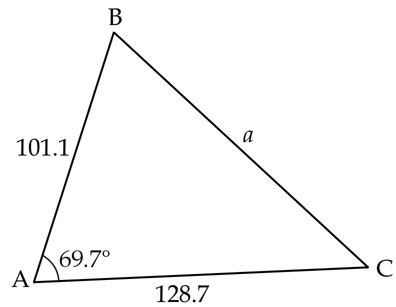
   a) $9^2 = 7^2 + 6^2 - 2(7)(6) \cos 87°$

   b) $a^2 = 3.2^2 + 6.4^2 - 2(3.2)(6.4) \cos 52°$

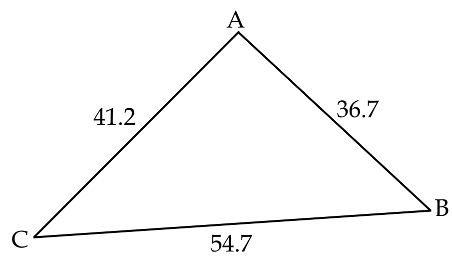
continued

Learning Activity 5.3 (continued)

3. Find the length of side *a*. Round to one decimal place.

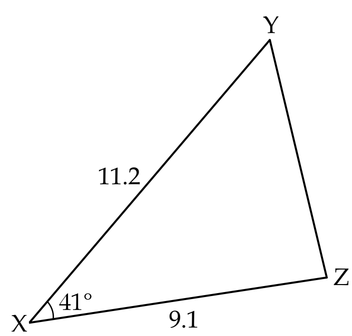


4. Find the size of ∠A. Round to the nearest whole number.

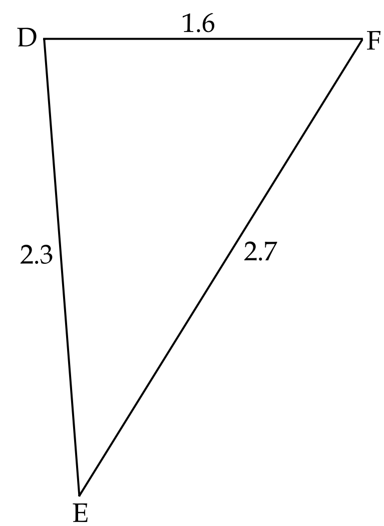


5. Find *all* missing sides and angles in the following triangles. Round your answers to one decimal place.

a)



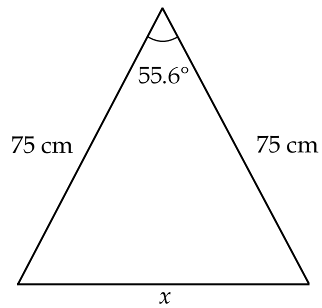
b)



continued

Learning Activity 5.3 (continued)

6. The arms of a “jaws of life” machine used for prying open crushed vehicles in accidents are 75 cm long. The jaws are joined at one end and can open at various angles. If the angle formed by the arms is 55.6°, how far apart are the tips (to the nearest centimetre)?



Lesson Summary

In this lesson, you learned how to use the Cosine Law to solve oblique triangles when the Sine Law could not be used. In the next lesson, you are going to focus on applying the Sine Law and the Cosine Law to solve obtuse triangles.

Assignment 5.1

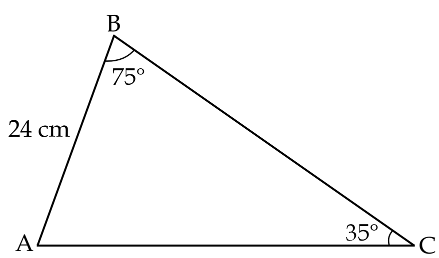
Solving Acute Triangles

Total: 25 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

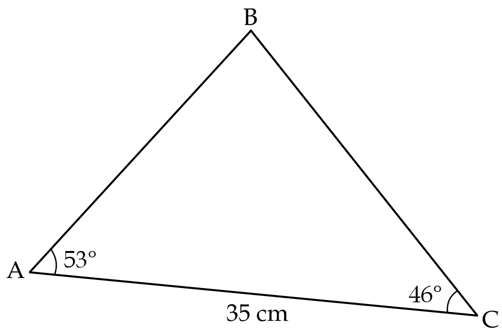
1. Solve for all the missing angles and all the missing sides in each of the triangles below. Round your answers to one decimal place.

   a) *(4 marks)*



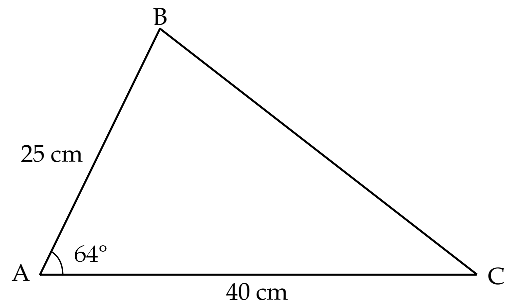
Assignment 5.1: Solving Acute Triangles (continued)

b) *(4 marks)*



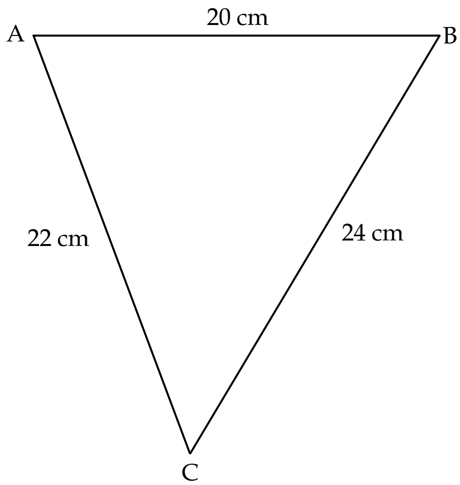
Assignment 5.1: Solving Acute Triangles (continued)

c) *(4 marks)*



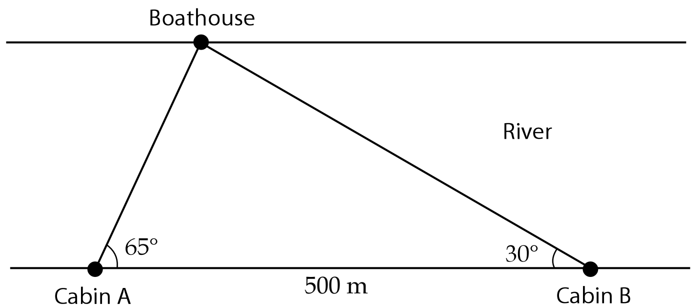
Assignment 5.1: Solving Acute Triangles (continued)

d) *(4 marks)*

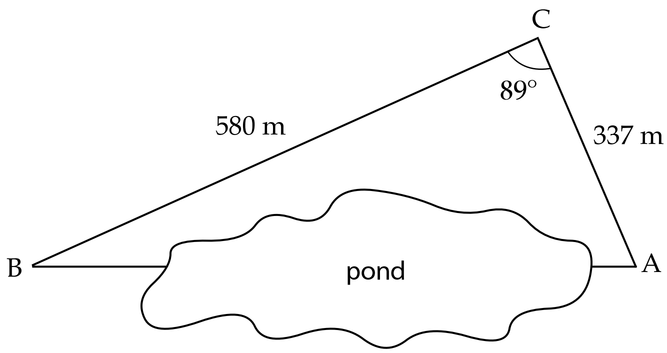


Assignment 5.1: Solving Acute Triangles (continued)

2. Two cabins are located along the bank of a river, on the same side of the river. The cabins are 500 m apart. A boathouse is located on the opposite side of the river, as illustrated in the diagram below. Determine how much further the boathouse is from Cabin B than from Cabin A. Round your answer to the nearest tenth of a metre. *(5 marks)*



3. An electric transmission line is being constructed over a pond. The power line will be supported by posts at points A and B, as shown in the diagram below. A surveyor takes measurements to determine that the distance from point B to point C is 580 m, the distance from point A to point C is 337 m, and ∠BCA is 89°. What is the distance from point A to point B? *(4 marks)*



Lesson 4: Solving Obtuse Triangles

Lesson Focus

In this lesson, you will

- ❑ learn about the sine and cosine ratios of supplementary angles
- ❑ solve obtuse triangles using the Sine Law
- ❑ solve obtuse triangles using the Cosine Law

Lesson Introduction

In the last two lessons, you focused on solving acute triangles by using both the Sine Law and the Cosine Law. What about obtuse triangles? In this lesson, you will use the Cosine Law to solve both acute and obtuse triangles. You will also examine cases where supplementary angles must be used when using the Sine Law to solve obtuse triangles.

Obtuse Triangles

An obtuse angle is an angle between 90° and 180°. Therefore, obtuse triangles are triangles that contain an interior angle between 90° and 180°. To learn about the relationship between acute triangles and obtuse triangles, you need to compare the sine and cosine ratios of angles between 0° and 90° with the sine and cosine ratios of the supplements of these angles.

Supplementary Angles

If you let θ be an angle between 0° and 90°, then $180° - \theta$ will be the supplement of that angle. This angle will be an obtuse angle because the angle will be between 90° and 180°.

First, consider the sine ratio. The table below illustrates the relationship between the sine ratios of acute angles and their corresponding supplementary angles.

θ Acute Angle	$\sin \theta$	$180 - \theta$ Supplementary Angle	$\sin (180 - \theta)$
0°	0	180°	0
30°	0.5	150°	0.5
45°	0.707107	135°	0.707107
60°	0.866025	120°	0.866025
90°	1	90°	1

What do you notice from comparing the $\sin \theta$ column and the $\sin (180° - \theta)$ column?

The ratios in these two columns are identical. In general, $\sin \theta = \sin (180° - \theta)$. In other words, the sine of any angle is the same as the sine of its supplement.

You may wish to add a description of this relationship to your resource sheet.

What about the cosine ratio? The table below illustrates the relationship between the cosine ratios of acute angles and their corresponding supplementary angles.

θ Acute Angle	$\cos \theta$	$180 - \theta$ Supplementary Angle	$\cos (180 - \theta)$
0°	1	180°	–1
30°	0.866025	150°	−0.866025
45°	0.707107	135°	−0.707107
60°	0.5	120°	−0.5
90°	0	90°	0

What do you notice from comparing the $\cos \theta$ column and the $\cos (180° - \theta)$ column?

The ratios in the $\cos (180° - \theta)$ column are the opposite sign of the ratios in the $\cos \theta$ column. In general, $\cos \theta = -\cos (180° - \theta)$. Thus, the cosine of any angle is the same as the cosine of its supplement, but with the opposite sign.

You may wish to add a description of this relationship to your resource sheet.

These conclusions indicate that you need to be careful when you are solving for unknowns in obtuse triangles. If you are using the Sine Law, the inverse sine of a ratio on your calculator will always produce an acute angle. However, there is another angle measurement (its supplement) that has the same sine ratio, since every angle has the same sine as its supplement.

If you are using the Cosine Law, the cosine ratios of an angle and its supplement have the same value but with opposite signs. Thus, an angle determined using the Cosine Law is not ambiguous; its supplement will have a different cosine ratio value.

For these reasons, you are advised to use the Sine Law whenever possible to solve for the acute angles in a triangle and to use the Cosine Law to solve for obtuse angles in a triangle. In the next lesson, you will learn more about situations where there may be more than one possible solution for a triangle.

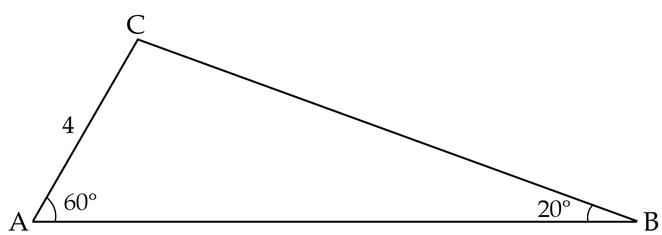
You may wish to include a summary of the above information on your resource sheet.

Solving Obtuse Triangles

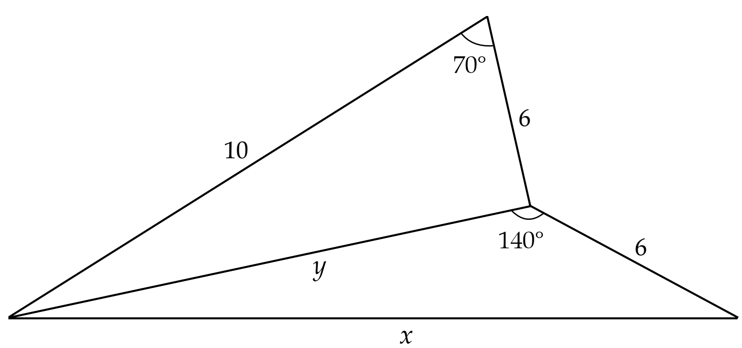
Example 1

Solve for the missing sides in each of the following triangles. Write your answers to the nearest hundredth.

a)



b)



Solution

a) First, solve for the missing angle, $\angle C$.

$$\angle C = 180° - 60° - 20° = 100°$$

Then, solve for side c using the Sine Law.

$$\frac{c}{\sin 100°} = \frac{4}{\sin 20°}$$

$$c = \frac{4 \sin 100°}{\sin 20°}$$ Multiply both sides of the equation by sin 20°.

$$c = 11.52$$ Simplify.

Note: You can use your calculator to find the sine ratio of an angle greater than 90° without any concern. However, when you are finding the *inverse sine* of a ratio, your calculator only shows the angle that is less than 90° and there will be another angle greater than 90° (the supplementary angle) with the same sine ratio value that your calculator doesn't show you.

b) Solve for y in the top triangle using the Cosine Law.

$$y^2 = 10^2 + 6^2 - 2(6)(10) \cos 70°$$

$$y^2 = 136 - 120 \cos 70°$$

$$y = \sqrt{136 - 120 \cos 70°}$$

$$y = 9.74$$

Now, solve for x in the bottom triangle using the Cosine Law.

$$x^2 = 9.74^2 + 6^2 - 2(9.74)(6) \cos 140°$$

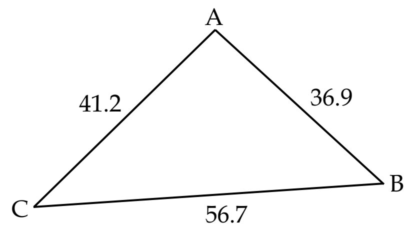
$$x^2 = 130.8676 - 116.88 \cos 140°$$

$$x = \sqrt{130.8676 - 116.88 \cos 140°}$$

$$x = 14.85$$

Example 2

Solve for ∠A. Round your answer to the nearest tenth.



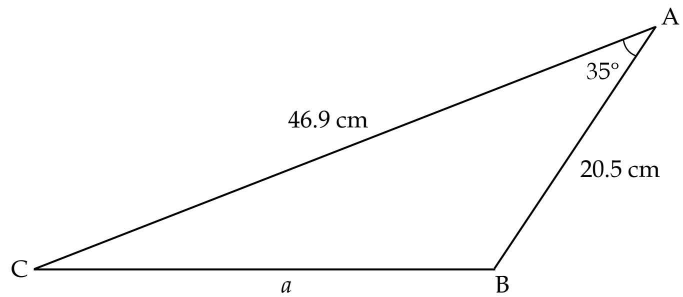
Solution

To solve for ∠A, you need to use the Cosine Law.

$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc \cos \angle A \\
56.7^2 &= 41.2^2 + 36.7^2 - 2(41.2)(36.7) \cos \angle A \\
3214.89 - 1697.44 - 1346.89 &= -3024.08 \cos \angle A \\
\frac{170.56}{-3024.08} &= \cos \angle A \\
\cos^{-1}\left(\frac{170.56}{-3024.08}\right) &= \angle A \\
\angle A &= 93.2°
\end{aligned}
$$

Example 3

Solve for the missing side and the two missing angles in the triangle below. Round your answers to the nearest hundredth.



Solution

Step 1: Solve for side *a* using the Cosine Law.

$$a^2 = b^2 + c^2 - 2bc \cos \angle A$$

$$a^2 = 46.9^2 + 20.5^2 - 2(46.9)(20.5) \cos 35°$$

$$a = \sqrt{2619.86 - 1922.9 \cos 35°}$$

$$a = 32.3$$

Step 2: Solve for the smallest angle, ∠C, using the Sine Law.

$$\frac{\sin \angle C}{c} = \frac{\sin \angle A}{a}$$

$$\frac{\sin \angle C}{20.5} = \frac{\sin 35°}{32.3}$$

$$\angle C = \sin^{-1}\left(\frac{20.5 \sin 35°}{32.3}\right)$$

$$\angle C = 21.3°$$

Step 3: Solve for ∠B:

$$\angle B = 180° - 35° - 21.3° = 123.7°$$

In Step 2, if you had solved for the obtuse angle, ∠B, the $\sin^{-1}$ function on your calculator gives you an acute angle as an answer.

$$\frac{\sin \angle B}{46.9} = \frac{\sin 35°}{32.3}$$

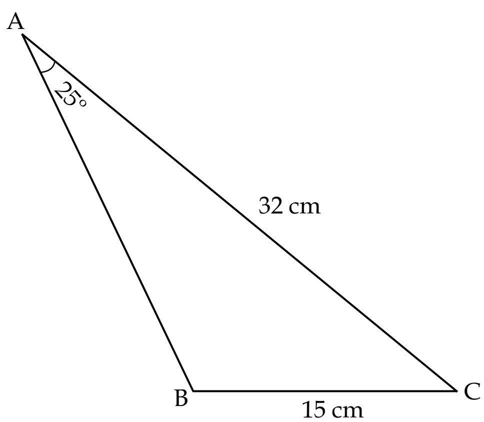
$$\angle B = \sin^{-1}\left(\frac{46.9 \sin 35°}{32.3}\right)$$

$$\angle B = 56.3°$$

However, the actual answer is the supplement, 180° − 56.3° = 123.7°.

Example 4

Determine the size of ∠B. Round your answer to one decimal place.



Solution

You are given two sides of the triangle as well as an angle directly across from one of these sides. In this situation, use the Sine Law.

$$\frac{\sin \angle B}{b} = \frac{\sin \angle A}{a}$$

$$\frac{\sin \angle B}{32} = \frac{\sin 25°}{15}$$

$$\sin \angle B = \frac{32 \sin 25°}{15}$$

$$\angle B = \sin^{-1}\left(\frac{32 \sin 25°}{15}\right)$$

$$\angle B = 64.4°$$

You can see from the diagram that ∠B is the largest angle in the triangle and it is an obtuse angle. However, the calculator found that ∠B measures 64.4°. Whenever you use the sine inverse ($\sin^{-1}$) function you need to consider whether you want the acute angle shown by the calculator or the obtuse supplementary angle that has the same sine ratio. In this instance, you need the *supplement* of this angle since you know this angle must be greater than 90°.

$\angle B = 180° - 64.4°$

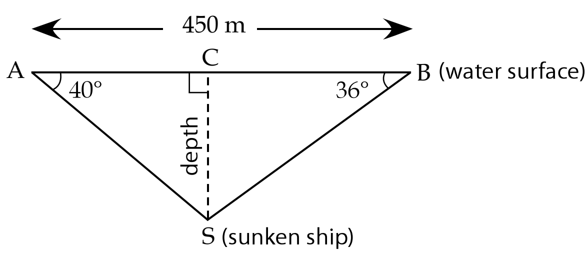
$\angle B = 115.6°$

Whenever you use the inverse sine function of a sine ratio to find the size of an angle in an obtuse triangle, you need to determine whether you are looking for an angle less than 90° or an angle greater than 90°. If you know your angle must be greater than 90°, you need to find the supplement of the angle given by your calculator.

Add this information to your resource sheet for future reference. You will learn more about this type of question in Lesson 5.

Example 5

Two observers on different boats have located a sunken ship using sonar equipment. The sunken ship is in line between the two observers. The angle of depression from Observer A to the sunken ship is 40°, and the angle of depression from Observer B to the sunken ship is 36°. The distance between the observers is 450 metres. How far is the ship below the surface of the water? Round your answer to the nearest metre.



Note that the depth is the perpendicular distance from the sunken ship to the surface of the water.

Solution

First, find ∠ASB by subtracting ∠A and ∠B from 180°.

$$\angle ASB = 180° - \angle A - \angle B = 180° - 40° - 36° = 104°$$

You can now find *b* by using the Sine Law:

$$\frac{b}{\sin 36°} = \frac{450}{\sin 104°}$$

$$b = \frac{450 \sin 36°}{\sin 104°}$$

$$b = 272.6 \text{ m}$$

Now, use right-triangle trigonometry in ΔACS to determine the depth of the ship.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 40° = \frac{\text{CS}}{272.6}$$

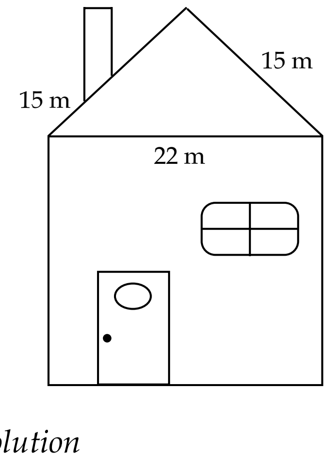
$$\text{CS} = 272.6 \sin 40°$$

$$\text{CS} = 175.2 \text{ m}$$

The depth of the sunken ship is 175 metres.

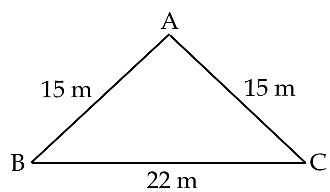
Example 6

Both diagonals of the roof of a house measure 15 metres. The house is also 22 metres across. Determine the angle at which the two sides of the roof meet; that is, determine the angle of the peak of the roof.



Solution

You may find it beneficial to redraw this diagram as a labelled triangle.



You need to determine the size of ∠A. To do this, you can use the Cosine Law.

$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2ab \cos \angle A \\
22^2 &= 15^2 + 15^2 - 2(15)(15) \cos \angle A \\
484 &= 225 + 225 - 450 \cos \angle A \\
484 &= 450 - 450 \cos \angle A \\
\frac{484 - 450}{-450} &= \cos \angle A \\
\cos^{-1}\left(\frac{34}{-450}\right) &= \angle A \\
\angle A &= 94.3^\circ
\end{aligned}
$$

The two sides of the roof meet at an angle of 94.3°.

Be sure to complete the following learning activity. The questions in this learning activity are similar to the questions on the following assignment. Check your answers in the Module 5 answer key to ensure you are completing the questions correctly before you start your assignment.

Learning Activity 5.4

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. If two sides of a right triangle measure 5 cm and 12 cm, what is the length of the hypotenuse?

2. Write as an improper fraction: $3\frac{7}{9}$

3. Solve for x: $\frac{3}{x} + 2 = 4$

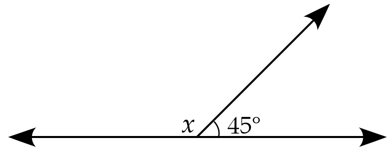
4. 32 is 20% of what number?

5. Convert to a fraction in lowest terms: 0.234

6. Add: $\frac{5}{3} + \frac{3}{2}$

7. Convert 3 km to cm.

8. Determine the size of angle x in the diagram below.



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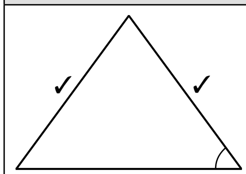
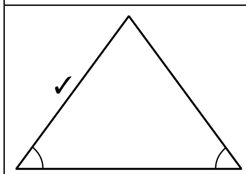
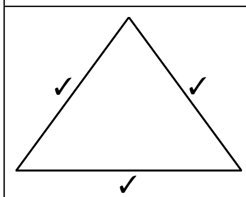
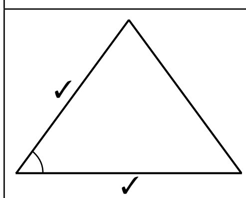
Learning Activity 5.4 (continued)

Part B: Solving Obtuse Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Fill in the following graphic organizer to help you recognize in which situations you need to use the Cosine Law and in which situations you need to use the Sine Law. The first row is provided for you to use as a reference.

Diagram	Information Given	Cosine Law/ Sine Law	Steps for Solving the Triangle
	Two sides of the triangle and one angle of the triangle. One of the given sides is directly opposite the given angle.	Sine Law	■ Find the angle directly opposite the given side using the Sine Law ■ Find the third angle using the 180° rule ■ Find the third side using the Sine Law
			
			
			

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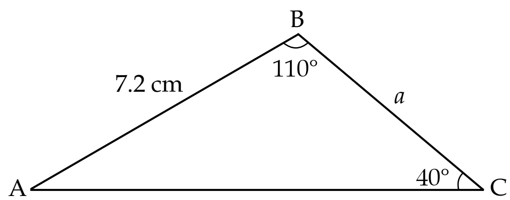
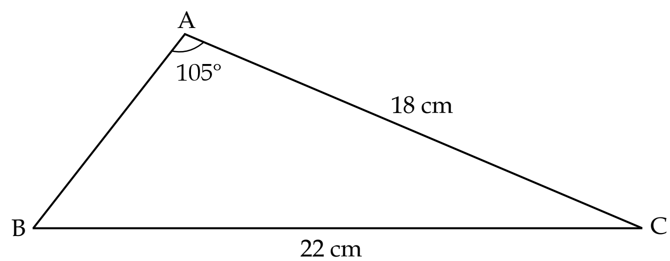
Learning Activity 5.4 (continued)

2. Solve for the indicated measurements. Round your answers to one decimal place.

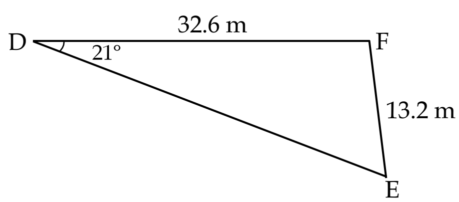
   a) Find the length of side *a*.



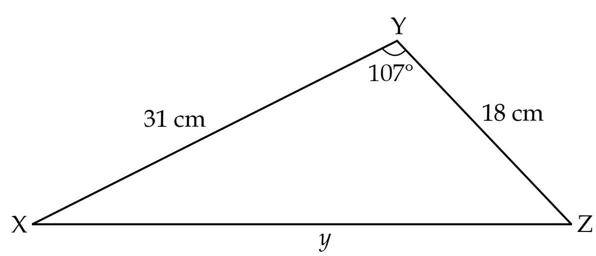
   b) Find the size of ∠C.



   c) Determine the size of ∠E.

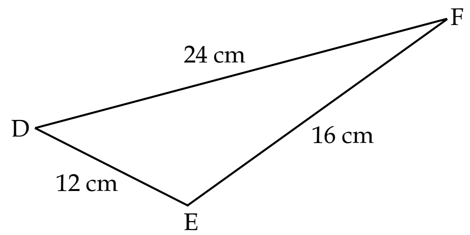


   d) Find the length of side *y*.

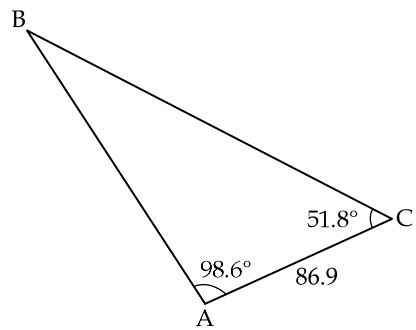


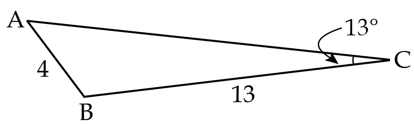
continued

e) Find the size of ∠E.

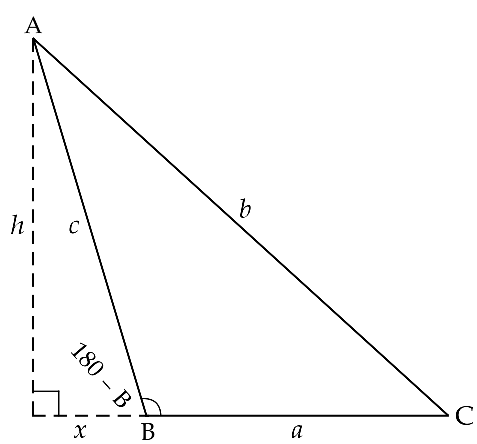


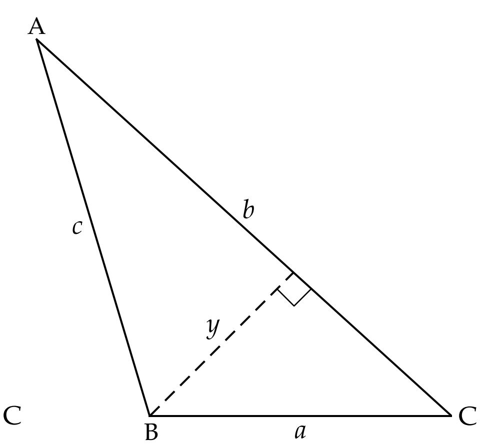
3. Solve the following triangles.

a) 

b) 

4. Explain each of the steps in the following proof of the Sine Law using obtuse triangles.





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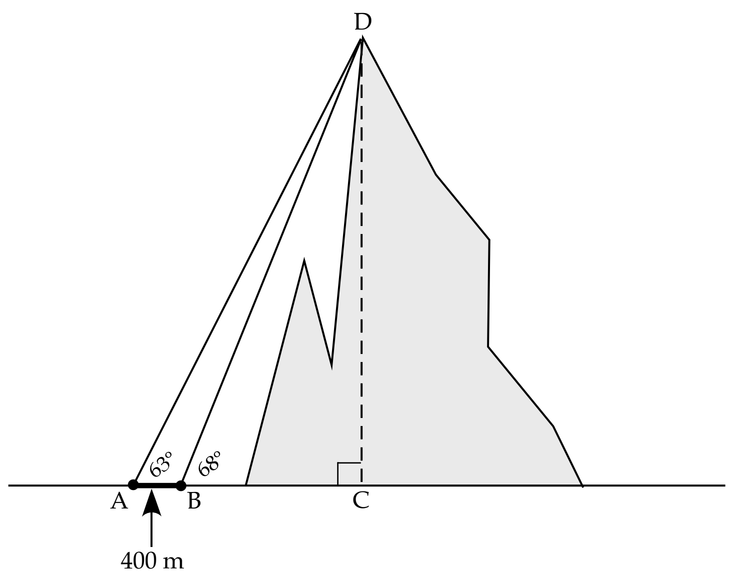
Learning Activity 5.4 (continued)

5. Two hikers are hiking through a mountain trail when they come to a fork in the trail. The first hiker decides to take the trail that heads due east. This trail contains rougher terrain and he is only able to maintain a pace of 4 kph. The second hiker decides to take the trail in a direction of N 25° W. This trail is well groomed and the hiker is able to hike at 4.8 kph. The hikers agree to communicate via Walkie-Talkie after six hours have passed. If the Walkie-Talkies have a range of 45 kilometres, will the hikers be able to communicate?

6. It is difficult to measure the height of a mountain directly. Therefore, surveyors use trigonometry to determine an approximation of the height of a mountain. The surveyors take their measurements and draw the following diagram.



Determine the height of the mountain to the nearest metre.

Lesson Summary

In this lesson, you focused on using the Sine Law and the Cosine Law to solve obtuse triangles. You used the same method to solve obtuse triangles using the Cosine Law as the one you used to solve acute triangles. However, when using the Sine Law with obtuse triangles, you need to consider the supplementary angle when you are taking the inverse sine of a ratio. You will learn more about having more than one possible solution for a triangle in the next lesson.

Notes

Assignment 5.2

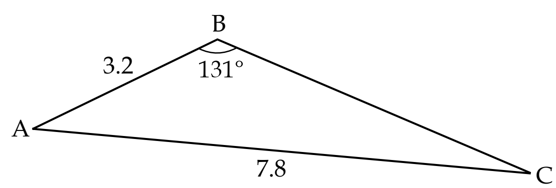
Solving Obtuse Triangles

Total: 36 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

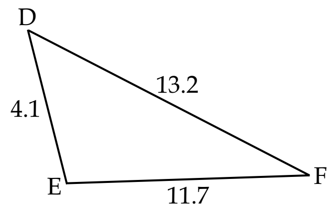
1. Solve for the indicated measurements. Round your answers to one decimal place.

    a) Find the size of ∠A. *(4 marks)*

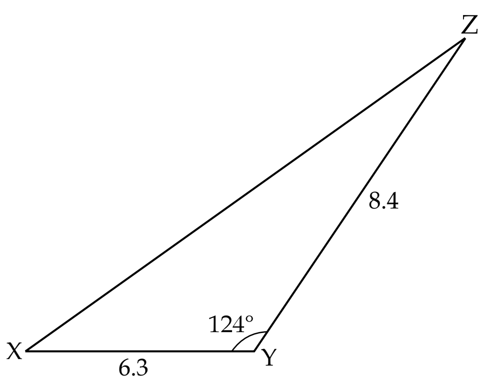


Assignment 5.2: Solving Obtuse Triangles (continued)

b) Find ∠E. *(3 marks)*



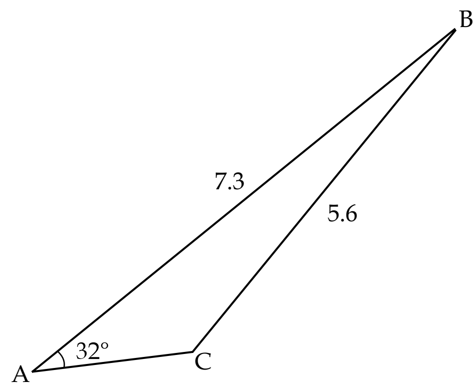
c) Find the length of side *y*. *(3 marks)*



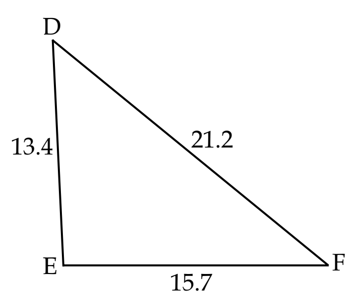
Assignment 5.2: Solving Obtuse Triangles (continued)

2. Solve the following triangles. Round to the nearest tenth.

   a) *(7 marks)*

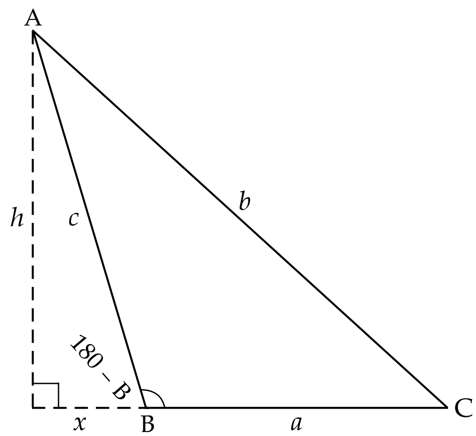


   b) *(7 marks)*



Assignment 5.2: Solving Obtuse Triangles (continued)

3. Explain each of the steps in following proof of the Cosine Law for obtuse triangles. *(5 marks)*

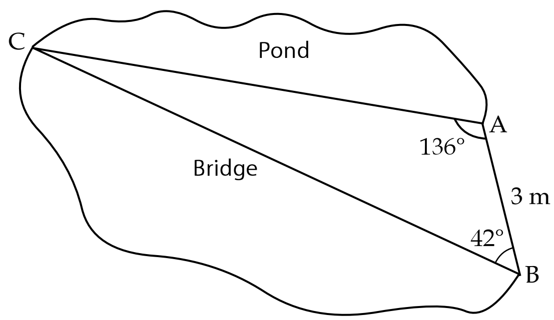


Step	Explanation
Small Triangle	Small Triangle
$\cos(180° - B) = \frac{x}{c}$	
$x = c\cos(180° - \mathrm{B})$	
$x = -c\cos \mathrm{B}$	
$h^2 + x^2 = c^2$	
$h^2 = c^2 - x^2$	
Large Triangle	Large Triangle
$h^2 + (a + x)^2 = b^2$	
$h^2 = b^2 - (a + x)^2$	
$h^2 = b^2 - (a^2 + 2ax + x^2)$	
$h^2 = b^2 - a^2 - 2ax - x^2$	
Both Triangles	Both Triangles
$c^2 - x^2 = b^2 - a^2 - 2ax - x^2$	
$c^2 = b^2 - a^2 - 2ax - x^2 + x^2$	
$c^2 = b^2 - a^2 - 2ax$	
$b^2 = a^2 + c^2 + 2ax$	
$b^2 = a^2 + c^2 + 2a(-c\cos \mathrm{B})$	
$b^2 = a^2 + c^2 - 2ac\cos \mathrm{B}$	

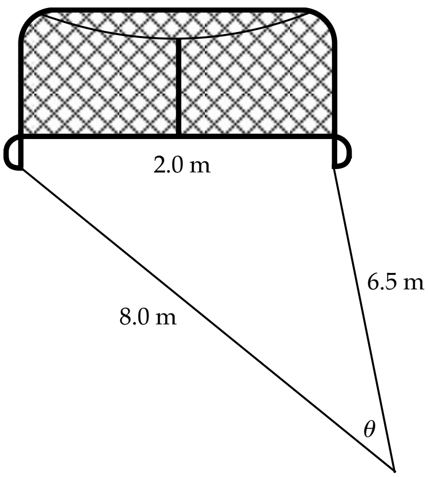
Assignment 5.2: Solving Obtuse Triangles (continued)

4. The manager of a local theme park wants to build a pedestrian bridge across a pond. He has taken two angular measurements, as shown below, and measured the distance between these two angles to be 3 m. How long does the bridge across the pond need to be? Round your answer to the nearest metre. *(4 marks)*

Diagram is not drawn to scale.



5. The posts of a hockey goal are 2.0 m apart. A player attempts to score by shooting the puck along the ice from a point 6.5 m from one post and 8.0 m from the other post, as shown in the diagram below. What is the size of the angle θ that would allow the player to score a goal? *(3 marks)*



Lesson 5: The Ambiguous Case

Lesson Focus

In this lesson, you will

- ❑ recognize when the ambiguous case occurs
- ❑ determine the number of triangles possible with a given set of information

Lesson Introduction

In the last lesson, you discovered that when you use sine inverse ($\sin^{-1}$) to find an angle, the answer shown by your calculator may not always be the correct angle measurement. Sometimes you may want to use its supplement instead. If the triangle is drawn for you, it is easy to tell if the angle should be less than 90° or if the angle should be greater than 90°. However, what happens if the triangle is not drawn for you or if there is not enough information to determine if the angle is acute or obtuse? Then which angle do you want? In this lesson, you will investigate this situation, called the *ambiguous case*.

The Ambiguous Case

The ambiguous case in triangles can occur when you are given the lengths of two sides of the triangle as well as the size of an angle that is opposite one of these sides; that is, you know two side lengths and an angle that is not between these two sides.

The Ambiguous Case for Acute Triangles

There are different solutions for these triangles, depending upon whether the given angle is acute or obtuse. If the given angle is acute, there are four different possible situations.

Example 1

Using a ruler and a protractor, determine how many possible ways there are to construct each triangle. Determine the height of each triangle.

a) ΔABC with $\angle A = 30°$, $a = 3$ cm, and $b = 8$ cm.

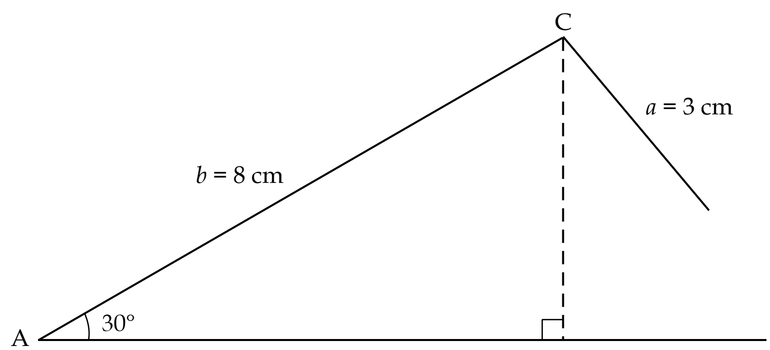
b) ΔABC with $\angle A = 30°$, $a = 4$ cm, and $b = 8$ cm.

c) ΔABC with $\angle A = 30°$, $a = 6$ cm, and $b = 8$ cm.

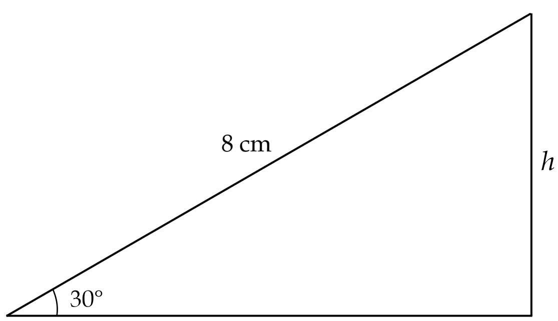
d) ΔABC with $\angle A = 30°$, $a = 10$ cm, and $b = 8$ cm.

Solution

a)



There is no possible way to construct this triangle since side a is too short. You can confirm that the triangle is not possible using the Sine Law.



$$\frac{\sin 30}{3} = \frac{\sin B}{8}$$

$$8\left(\frac{\sin 30}{3}\right) = \sin B$$

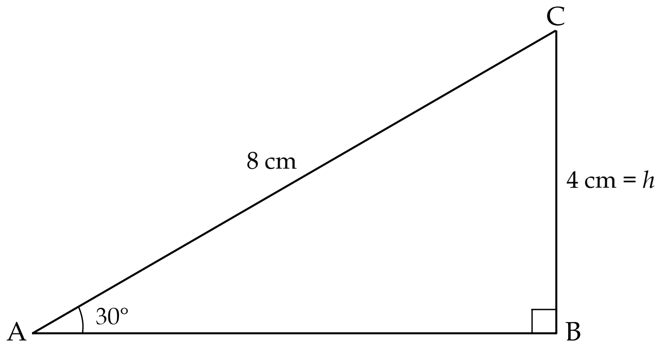
$$\sin B = 1.333$$

$$\angle B = \sin^{-1}(1.333)$$

$$\angle B = \text{ not possible}$$

You get an error message in your calculator since the value of sin B cannot be greater than 1 $\left(\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}}\right.$ and the hypotenuse is longer than the opposite side so the value of sin B must be less than 1). No triangle exists with these measurements.

b)



There is one possible way to construct this triangle since side a is just long enough to touch side AB. Side a is equal to the height of the triangle and a right-angled triangle is the result. Again, using the Sine Law, you can confirm that only one triangle is possible and the triangle must be a right triangle.

$$\frac{\sin 30}{4} = \frac{\sin B}{8}$$

$$8\left(\frac{\sin 30}{4}\right) = \sin B$$

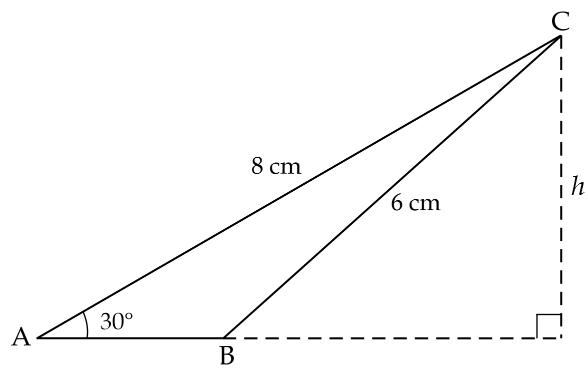
$$\sin B = 1$$

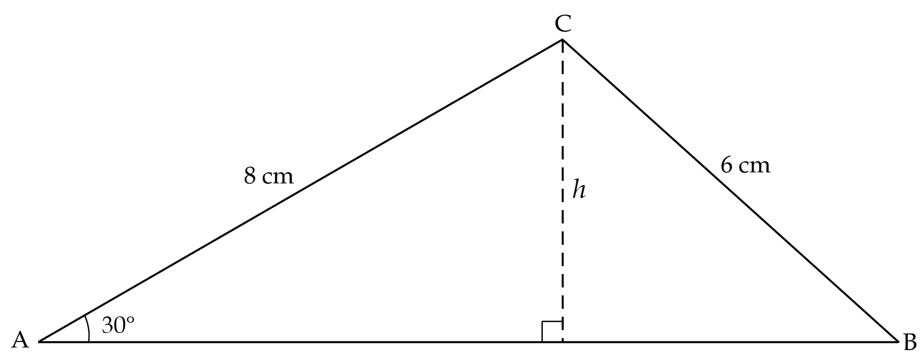
$$\angle B = \sin^{-1}(1)$$

$$\angle B = 90^\circ$$

When you use sine inverse, you should check whether another angle might be possible. The other angle would be the supplement of ∠B. Since the supplement of 90° is 180° − 90° = 90°, only one triangle exists and it is a right triangle.

c)





There are two possible ways to construct this triangle since side *a* is longer than the height of the triangle and shorter than side *b*. Again, using the Sine Law, you can demonstrate that two triangles are possible.

$$\frac{\sin 30}{6} = \frac{\sin \text{B}}{8}$$

$$8\left(\frac{\sin 30}{6}\right) = \sin \text{B}$$

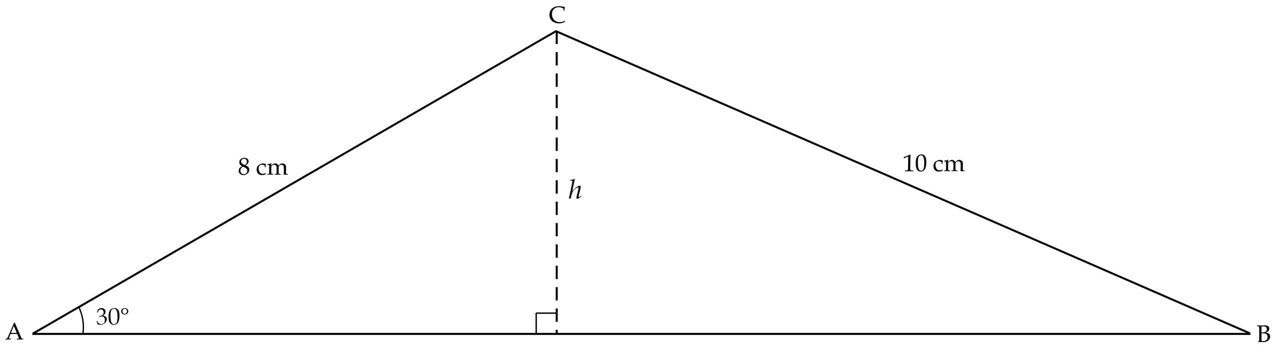
$$\sin \text{B} = 0.6667$$

$$\angle \text{B} = \sin^{-1}(0.6667)$$

$$\angle \text{B} = 41.8^\circ$$

When you use sine inverse, you should check whether another angle might be possible. The other angle would be the supplement of ∠B. The supplement of 41.8° is 180° − 41.8° = 138.2°. Two triangles can be drawn, one with an acute angle B and one with an obtuse angle B.

d)



There is one possible way to construct this triangle since side a is longer than the height of the triangle but is also longer than side b, so it is too long to create a second triangle.

What do you notice about the relationship between a and b as well as the relationship between a and the triangle height (h) in each situation?

ΔABC	Relationship between a and b	Number of Possible Triangles	Relationship between a and h	
$\angle A = 30°$ $a = 3$ cm $b = 8$ cm	a is less than b	0	a is less than height	sin B > 1
$\angle A = 30°$ $a = 4$ cm $b = 8$ cm	a is less than b	1	a is equal to height	sin B = 1
$\angle A = 30°$ $a = 6$ cm $b = 8$ cm	a is less than b	2	a is greater than height	sin B < 1
$\angle A = 30°$ $a = 10$ cm $b = 8$ cm	a is greater than b	1	a is greater than height	sin B < 1

Summary of the Ambiguous Case for Acute Triangles:

- When the side opposite the given acute angle is shorter than the other side length given, there are three situations that could result: it is possible that no triangle could exist or either one or two triangles could be possible. You can determine the number of triangles using the Sine Law.
  - If sin B > 1, then no triangle is possible.
  - If sin B = 1, then the angle is 90° and only one triangle is possible.
  - If sin B < 1, then two triangles are possible.

- When the side opposite the given acute angle is longer than the other side length given, there is one triangle possible. If you use the Sine Law, you will see that sin B < 1, but the side opposite the given angle is too long to create a second triangle.

Include the above information on your resource sheet.

The Ambiguous Case for Obtuse Triangles

If the given angle in a triangle is obtuse, there are two possible situations that could result.

Example 2

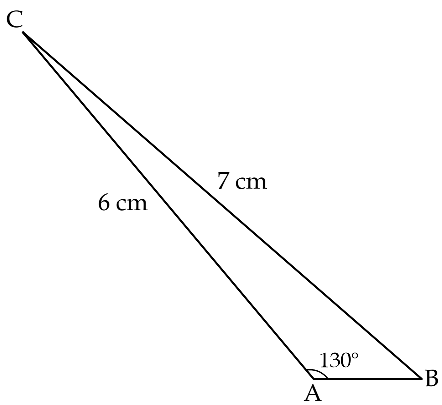
Using a ruler and a protractor, determine how many possible ways there are to construct each triangle.

a) ΔABC with $\angle$A = 130°, a = 7 cm, and b = 6 cm

b) ΔABC with $\angle$A = 130°, a = 7 cm, and b = 8 cm

Solution

a)



There is one triangle possible. You can use the Sine Law to demonstrate that one triangle is possible.

$$\frac{\sin 130}{7} = \frac{\sin B}{6}$$

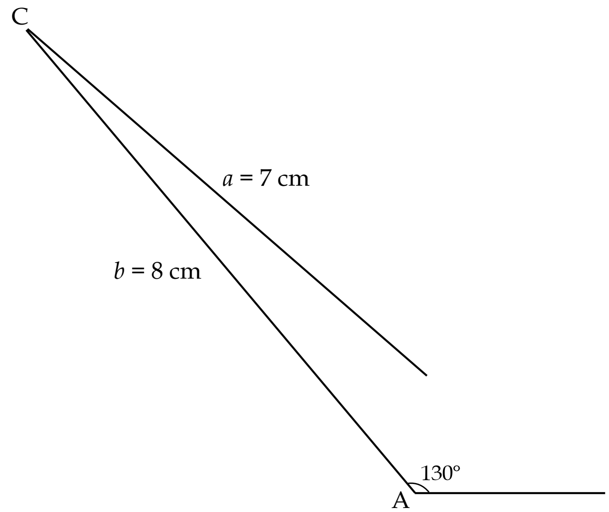
$$6\left(\frac{\sin 130}{7}\right) = \sin B$$

$$\sin B = 0.6566$$

$$\angle B = \sin^{-1}(0.6566)$$

$$\angle B = 41.0^\circ$$

Angle A is 130°, angle B is 41.0°, and angle C is 180° − 130° − 41° = 9.0°.

b) 

There is no triangle possible. You can use the Sine Law to demonstrate that no triangle is possible.

$$\frac{\sin 130}{7} = \frac{\sin B}{8}$$

$$8\left(\frac{\sin 130}{7}\right) = \sin B$$

$$\sin B = 0.8755$$

$$\angle B = \sin^{-1}(0.8755)$$

$$\angle B = 61.1^{\circ}$$

If ∠A is 130° and ∠B is 61.1°, then no triangle is possible since two angles already total more than 180°, and the sum of all interior angles in a triangle is 180°.

What do you notice about the relationship between *a* and *b* in each situation?

∆ABC	**Relationship between *a* and *b***	**Number of Possible Triangles**
∠A = 130° *a* = 7 cm *b* = 6 cm	*a* is greater than *b*	1
∠A = 130° *a* = 7 cm *b* = 8 cm	*a* is less than *b*	0

Summary of the Ambiguous Case for Obtuse Triangles:

- When the side opposite the given obtuse angle is longer than the other side length given, there is one triangle possible.
- When the side opposite the obtuse angle is shorter than the other side length given, no triangle is possible. You already know that the longest side of a triangle must be opposite the largest angle. That is, the longest side must be opposite the obtuse angle; otherwise, the triangle is not possible.

Include the above information on your resource sheet.

Example 3

Determine the number of triangles possible for each situation and justify your answers. You may find it helpful to draw a sketch, showing the information you are given.

a) ΔABC with $\angle$A = 62°, a = 6 cm, and b = 5 cm

b) ΔABC with $\angle$A = 16°, a = 1 cm, and b = 5 cm

c) ΔABC with $\angle$A = 36°, a = 7 cm, and b = 10 cm

d) ΔABC with $\angle$A = 131°, a = 4 cm, and b = 7 cm

e) ΔABC with $\angle$A = 115°, a = 6 cm, and b = 5 cm

f) ΔABC with $\angle$A = 30°, a = 2 cm, and b = 4 cm

Solution

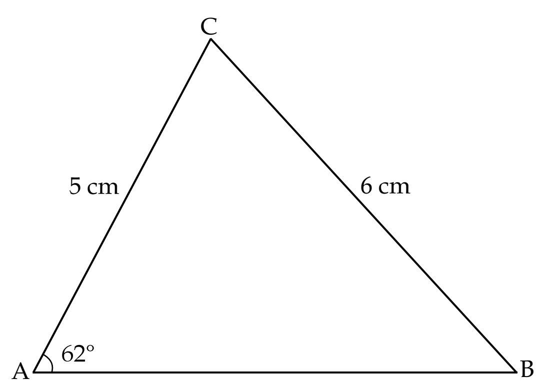
a) Using the Sine Law:

$$\frac{\sin 62}{6} = \frac{\sin B}{5}$$

$$5\left(\frac{\sin 62}{6}\right) = \sin B$$

$$\sin B = 0.7358$$

Since sin B is less than 1, there are two possible values for $\angle$B (acute and obtuse). However, in this triangle, the side opposite the acute angle is greater than the other side length given. Therefore, there is only one triangle possible.



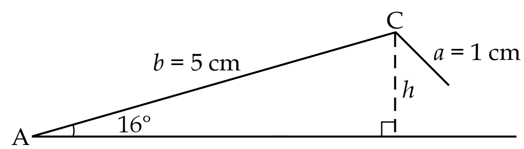
b) Using the Sine Law:

$$\frac{\sin\ 16}{1} = \frac{\sin\ B}{5}$$

$$5\left(\frac{\sin 16}{1}\right) = \sin\ B$$

$$\sin\ B = 1.3782$$

Since the value of sin B is greater than 1, no triangle is possible.



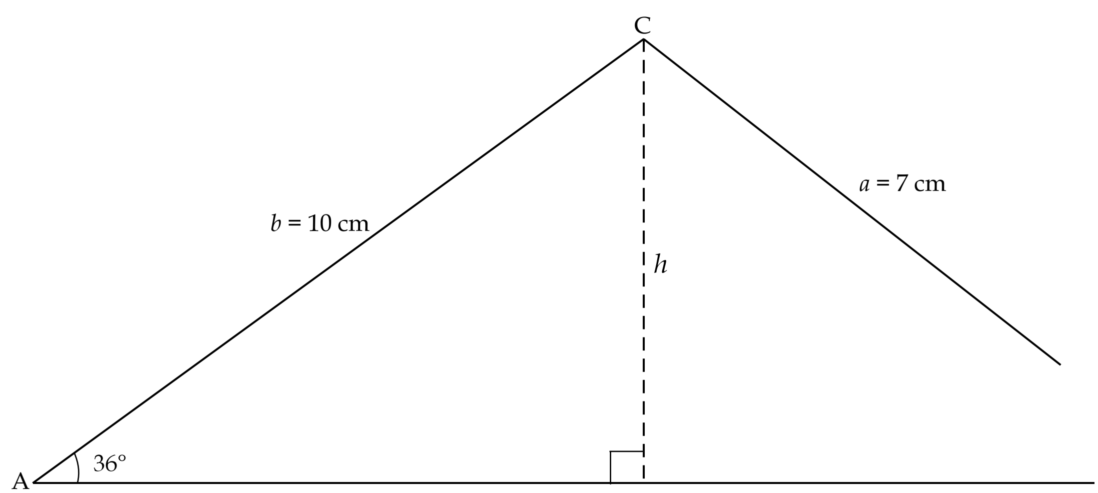
The height of the triangle is longer than the side length of 1, so it is not possible to complete this triangle.

c) Using the Sine Law:

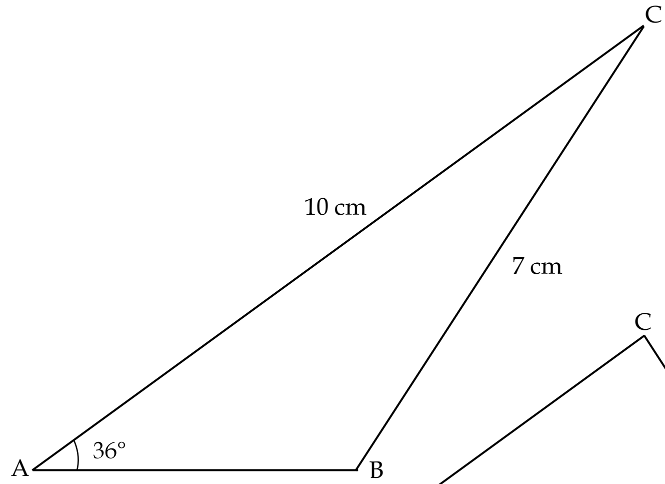
$$\frac{\sin\ 36}{7} = \frac{\sin\ B}{10}$$

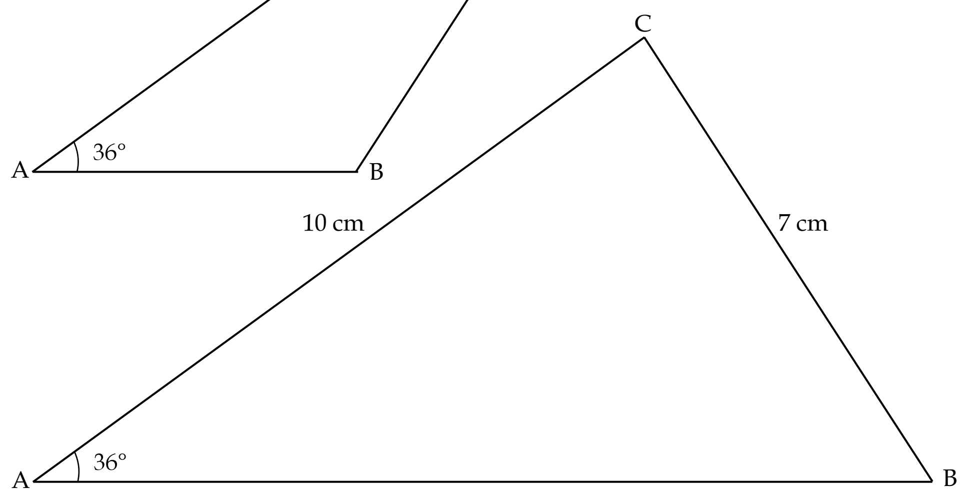
$$10\left(\frac{\sin 36}{7}\right) = \sin\ B$$

$$\sin\ B = 0.8397$$

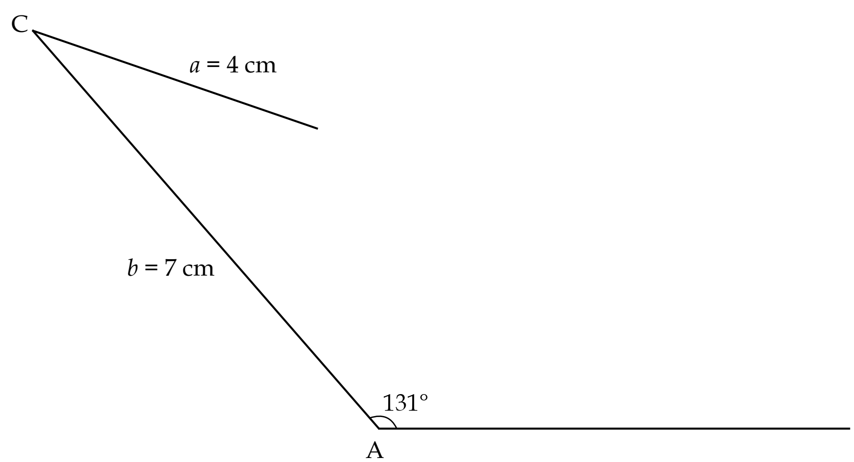


Since the value of sin B is less than 1, there are two possible values for ∠B (acute and obtuse). Furthermore, there are two possible triangles since side *a* is less than side *b*.

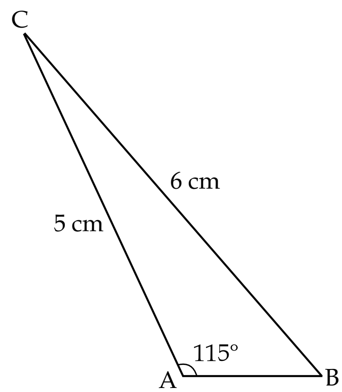




d) Since an obtuse angle is given, at most one triangle is possible. You just need to check that the side opposite the obtuse angle is the longest side. In this triangle, the side opposite the obtuse angle, 4 cm, is shorter than the other side length given, 7 cm. There is no triangle possible.



e) Since an obtuse angle is given, at most one triangle is possible. You just need to check that the side opposite the obtuse angle is the longest side. In this triangle, the side opposite the obtuse angle is longer than the other side length given. There is one triangle possible.

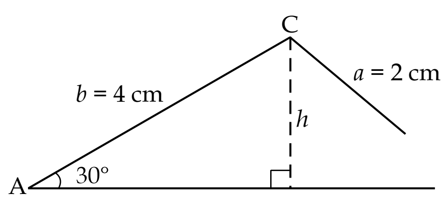


f) Using the Sine Law:

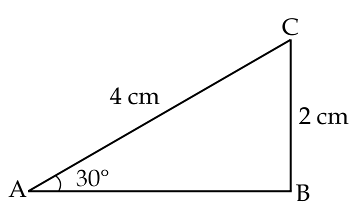
$$\frac{\sin 30}{2} = \frac{\sin B}{4}$$

$$4\left(\frac{\sin 30}{2}\right) = \sin B$$

$$\sin B = 1$$



Since the value of sin B is equal to 1, there is one possible value for ∠B (that is, 90°). Only one right triangle is possible.



Note: In this course you will not be required to solve ambiguous triangles. You will only be required to identify when a triangle shape is ambiguous and indicate how many possible triangles exist.

Make sure you complete the following learning activity to practise identifying how many triangles exist in various ambiguous situations.

Learning Activity 5.5

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Solve for x: $2 = 2x - 7$
2. Solve for x: $(x - 4)(x + 8) = 0$
3. Convert to a decimal: $\frac{7}{20}$
4. Subtract: $\frac{8}{9} - \frac{2}{3}$
5. Evaluate: $2^3 + 3^3$

Use the following data set to answer questions 6 to 8.

1, 1, 2, 3, 3, 3, 4, 4, 5, 5, 6, 11

6. Determine the mean.
7. Determine the median.
8. Determine the mode.

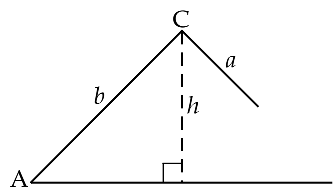
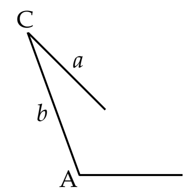
continued

Learning Activity 5.5 (continued)

Part B: The Ambiguous Case

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Complete the following graphic organizer to solidify your knowledge of the number and type of triangles that exist in each situation.

<table>
<tr><th colspan="7">Ambiguous Case
Given: two side lengths and the size of an angle opposite one of these sides</th></tr>
<tr><td rowspan="4"></td><td colspan="4">∠A is acute</td><td colspan="2">∠A is obtuse</td></tr>
<tr><td colspan="4">
</td><td colspan="2">
</td></tr>
<tr><td colspan="3">$a < b$</td><td rowspan="2">$a \geq b$</td><td rowspan="2">$a \leq b$</td><td rowspan="2">$a > b$</td></tr>
<tr><td>$a < h$</td><td>$a = h$</td><td>$a > h$</td></tr>
<tr><td>Sine Law</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Number of Triangle(s)</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Type of Triangle(s)</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

2. Draw a sketch for each triangle. Determine how many triangles are possible for each set of information and explain your reasoning. Do not solve the triangles.

   Note: It is always a good idea to draw the given angle in the lower left corner of the triangle. This will help you recognize whether the situation is ambiguous.

   a) ΔABC with ∠A = 21°, a = 18 cm and, b = 34 cm
   b) ΔXYZ with ∠X = 36°, x = 13 cm and, y = 30 cm
   c) ΔRST with ∠R = 82°, r = 21 cm and, s = 7 cm
   d) ΔDEF with ∠D = 30°, d = 6 cm and, e = 12 cm
   e) ΔMNO with ∠M = 128°, m = 3 cm, and n = 5 cm
   f) ΔABC with ∠A = 133°, a = 18 cm, and b = 9 cm

continued

3. Consider ΔABC where $\angle A = 30°$, and $b = 8$ cm. Find a length for side a such that
   a) no triangle exists.
   b) one right triangle exists.
   c) two triangles exist.
   d) one non-right triangle exists.
4. Consider ΔABC where $\angle A = 125°$, and $b = 8$ cm. Find a length for side a such that
   a) one triangle exists.
   b) no triangle exists.

Lesson Summary

In this lesson, you were introduced to the ambiguous case of the Sine Law. The ambiguous case occurs when you are given the lengths of two sides of a triangle as well as an angle opposite one of these sides. You discovered that there were zero, one, or two possible solutions to a triangle, depending on the information given about the side and angle measurements.

Assignment 5.3

The Ambiguous Case

Total: 17 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. Decide whether there are no solutions, exactly one solution, or two solutions in each situation. You do not need to solve the triangle, but you must justify your choice and include a diagram. *(12 marks)*

   a) How many different triangles can be constructed given that angle A is 35°, its opposite side, *a*, is 5 cm, and another side, *b*, is 10 cm?

   b) How many different triangles can be constructed given that angle A is 120°, its opposite side, *a*, is 16 cm, and another side, *b*, is 10 cm?

Assignment 5.3: The Ambiguous Case (continued)

c) How many different triangles can be constructed given that $\angle C = 60°$, $c = 43$ cm, and $b = 32$?

d) How many different triangles can be constructed given that $\angle A = 18°$, $a = 38$ cm, and $c = 50$ cm?

Assignment 5.3: The Ambiguous Case (continued)

e) How many different triangles can be constructed given that $\angle B = 60°$, $b = 20$ cm, and $c = 15$ cm?

f) How many different triangles can be constructed given that $\angle C = 170°$, $c = 10$ cm, and $b = 15$ cm?

Assignment 5.3: The Ambiguous Case (continued)

2. Consider ΔABC with $\angle A = 36°$ and side $b = 5$ cm. Draw a diagram and determine a measurement for side a such that

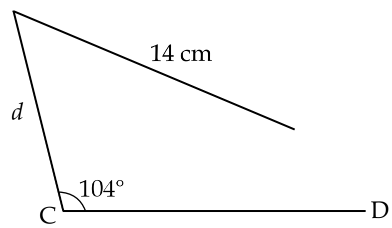
   a) no triangle exists. *(1 mark)*

   b) only one triangle exists. *(1 mark)*

   c) two triangles exist. *(1 mark)*

Assignment 5.3: The Ambiguous Case (continued)

3. Consider ΔCDE with ∠C = 104°, and side c = 14 cm. Determine a measurement for side d such that



a) no triangle exists. *(1 mark)*

b) one triangle exists. *(1 mark)*

Notes

Module 5 Summary

Congratulations, you have finished the first module in the second half of this course.

In this module, you learned about two different techniques for solving oblique triangles—the Sine Law and the Cosine Law. You discovered that these laws could only be used in certain situations but, taken together, these laws can be used to solve any triangle as long as you are given three pieces of information about the triangle, including the length of one side of the triangle.

You also learned that you need to be careful when you are using the Sine Law because of the ambiguous case. The ambiguous case occurs when you are given the lengths of two sides of the triangle, as well as the size of an angle opposite one of those sides. When this happens, there could be one possible triangle, two possible triangles, or no solution.

In the next module, you will be studying statistics and analyzing various representations of data, including numerical and graphical data.

Submitting Your Assignments

It is now time for you to submit the Module 5 Cover Assignment and Assignments 5.1 to 5.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 5 assignments and organize your material in the following order:

- ❑ Module 5 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 5 Cover Assignment: Shape and Space
- ❑ Assignment 5.1: Solving Acute Triangles
- ❑ Assignment 5.2: Solving Obtuse Triangles
- ❑ Assignment 5.3: The Ambiguous Case

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Self-Test

Module 5 – Budgets and Investments

/60

(5 marks)

TEST TIME!

1. Circle the best answer. (There is no penalty for guessing.)

   a) Which of the following is not a type of account offered by banks?

      i) Savings
      ii) Chequing
      iii) Savings/Chequing
      iv) Loans

   b) Which of the following is not a chartered bank?

      i) Bank of Canada
      ii) Royal Bank
      iii) CIBC
      iv) Toronto Dominion

   c) Which of the following does not affect the amount of interest earned by an investment?

      i) Rate
      ii) Bank
      iii) Principal
      iv) Time

   d) What is the amount accumulated if \$20 000 is invested at 8.5% compounded semi-annually for five years?

      i) \$45 219.60
      ii) \$24 627.15
      iii) \$30 324.29
      iv) \$29 604.80

   e) If a man invests \$5000 compounded semi-annually for four years at 6% interest, how much interest does the investment earn?

      i) \$627.54
      ii) \$1333.85
      iii) \$5627.54
      iv) \$6333.85

2. **Bank Records** *(4 marks)*

Write cheques and keep a record of them as well as the deposits and other withdrawals for the following.

Feb 15	Balance Forward	$ 462.30
Feb 18	Cheque to Dandee TV	$ 64.65
Feb 22	Cheque to Mr Ribs	$ 18.56
Feb 25	Paycheque deposit	$1357.45
Feb 28	Service charge	$ 3.56
Mar 1	Loan payment	$ 259.00
Mar 3	Cheque to Modern Food	$ 126.91
Mar 5	Cheque to Mat Hematics (Geometry set)	$ 3.59
Mar 14	Personal withdrawal	$ 212.87
Mar 14	Interest charge	$ 11.52
Mar 25	Paycheque deposit	$1282.39

You may use the current year when dating the cheques.

DATE	NO	PARTICULARS	✓	CHEQUES		DEPOSITS		BALANCE	

CANADIAN BANK

PAY TO THE ORDER OF ____________________ $________

____________________ XX DOLLARS

NO. ________

“ ‘003’”:00167’”010’: ____________________

CANADIAN BANK

PAY TO THE ORDER OF ____________________ $________

____________________ XX DOLLARS

NO. ________

“ ‘003’”:00167’”010’: ____________________

CANADIAN BANK

PAY TO THE ORDER OF ______________________________ $ ________

______________________________ XX DOLLARS

NO. ________

" '003'":00167'"010': ______________________________

CANADIAN BANK

PAY TO THE ORDER OF ______________________________ $ ________

______________________________ XX DOLLARS

NO. ________

" '003'":00167'"010': ______________________________

(4 marks)

3. **Deposit Slip**

   Fill in the following deposit slip according to the information that follows.

   Canadian cash:

   2 Fifty dollar bills
   2 Twenty dollar bills
   8 Ten dollar bills
   4 Five dollar bills
   17 Loonies
   14 Quarters
   18 Dimes
   8 Nickels
   7 Pennies

   Cheques:

   $81.40, $17.53, $56.75, $103.20 all in American funds. (Exchange rate is 1.3748)

CANADIAN BANK

Current Account - Deposit

Date	Account No.	
Account Name Please Print		
x 2		
x 5		
x 10		
x 20		
x 50		
x 100		
Coin		
Total Cash		
Cheques Only		
Total Cheques		
Credit Card		
Subtotal		
Exchange		
Total		

Deposited By

____

4. Prepare bank reconciliations for the following on the accompanying form. *(4 marks)*

   a)

Bank statement balance		$368.32
Chequebook balance		$379.92
Service charge		$ 2.10
Outstanding cheques	#487	$ 18.50
	#489	$ 25.60
	#490	$ 96.40
Deposit not shown on statement		$150.00

(1) CHEQUEBOOK BALANCE . . . . $________

DEDUCT BANK CHARGES

Description	Amount	
Service charge		

(2) TOTAL BANK CHARGES . . . . $________

(3) ADJUSTED CHEQUEBOOK BALANCE . . $________

(4) BANK BALANCE. . . . . $________

OUTSTANDING DEPOSITS

Date	Amount	

(5) TOTAL OUTSTANDING DEPOSITS . . $________

(6) TOTAL . . . . $________

OUTSTANDING CHEQUES

CH. NO.	Amount	

(7) TOTAL OUTSTANDING CHEQUES . . $________

(8) ADJUSTED BANK BALANCE. . . . . . . $________

(6 marks)

b) Chequebook Record

DATE	NO	PARTICULARS	✓	CHEQUES		DEPOSITS		BALANCE	
July 2		Balance Forward						786	57
July 3		Paycheque				452	34	1238	91
July 4	001	Andy's Department Store		57	14			1181	77
July 4		Cash		100	00			1081	77
July 5		Deposit				45	00	1126	77
July 7	002	Nellie's Gas Bar		135	79			990	98
July 9		Deposit				37	25	1028	23
July 9	003	Dandee TV		780	99			247	24
July 12	004	Bargain Store		28	69			218	55
July 12		Deposit				507	88	726	43
July 15	005	Dembinskys		112	65			613	78
July 22	006	Sharkeys		24	70			589	08
July 24	007	Mane Frame		53	25			535	83
July 25	008	Sturleys		237	74			298	09
July 25		Deposit				200	00	498	09

Bank Sstatement

V.R. Student **Bank of Manitoba** **Acct #555**
Box 123, Somewhere, MB

Date	Description	Debits	Credits	Balance
July 3	Balance forward			786.57
July 3			452.34	1238.91
July 4	#001	57.14		1181.77
July 5		100.00		1081.77
July 9	#003	780.99		300.78
July 9			37.25	338.03
July 12			507.88	845.91
July22	#006	24.70		821.21
July 25	Service Charge	10.50		810.71

(1) CHEQUEBOOK BALANCE . . . . $________

DEDUCT BANK CHARGES

Description	Amount	
Service charge		

(2) TOTAL BANK CHARGES . . . . $________

(3) ADJUSTED CHEQUEBOOK BALANCE . . $________

(4) BANK BALANCE . . . . $________

OUTSTANDING DEPOSITS

Date	Amount	

(5) TOTAL OUTSTANDING DEPOSITS . . $________

(6) TOTAL . . . . $________

OUTSTANDING CHEQUES

CH. NO.	Amount	

(7) TOTAL OUTSTANDING CHEQUES . . $________

(8) ADJUSTED BANK BALANCE. . . . . . . $________

(10 marks)

5. Fill in the accompanying Budget Form for the information provided.

   Bob Sampson has a monthly take-home pay of $2135.00. His wife has a monthly net income of $2347.00. The Sampsons have no children.

FIXED expenses for the Sampsons include:

a)	monthly mortgage payment	$ 900.00
b)	monthly car payment	$ 350.00
c)	average monthly telephone bill	$ 35.00
d)	average monthly hydro bill	$ 80.00
e)	average monthly natural gas bill	$ 75.00
f)	yearly car insurance premium	$ 550.00
g)	home is assessed for property tax at $50 000, the mill rate is 70 mills	
h)	home insurance (yearly premium)	$ 300.00
i)	monthly payment into an RRSP	$ 150.00

VARIABLE expenses for the family include:

a)	food (average per month)	$ 450.00
b)	clothing expenses for the year	$1000.00
c)	average car maintenance per year	$ 600.00
d)	gasoline per month	$ 150.00
e)	entertainment per year	$4800.00
f)	yearly vacation	$3000.00
g)	newspapers and periodicals (per year)	$ 350.00
h)	average monthly credit card payment	$ 300.00
i)	Christmas spending per year	$1200.00

Monthly Budget Form

1) Net Income

Primary Annual Income. . . . . . . $______________
Secondary Annual Income . . . . $______________
Other Annual Income . . . . $______________
Total Annual Income . . . . . $______________

Average Monthly Income
1) $______________

2) Savings (10% of Average Monthly Income) 2) $______________

3) Monthly Expenses

Mortgage or Rent . . . . $______________
Car Payment . . . . $______________
Telephone. . . . . $______________
Hydro . . . . $______________
Other Utilities. . . . . $______________
Groceries . . . . $______________
Clothing . . . . $______________
Car Maintenance. . . . . $______________
Gasoline . . . . $______________
Credit Card Debt. . . . . $______________
Entertainment . . . . $______________
Other . . . . $______________
Other . . . . $______________
Other . . . . $______________

Monthly Total

Total Monthly Expenses 3) $______________

4) Annual Expenses: (Monthly contributions required)

Car Insurance. . . . . $______________
Life Insurance . . . . $______________
Property Taxes. . . . . $______________
Home Insurance . . . . $______________
Vacations . . . . $______________
Other . . . . $______________
Other . . . . $______________

Monthly Total

Total Monthly Contributions 4) $______________

Summary

1) Average Monthly Income . . . . 1) $______________
2) Savings . . . . 2) $______________
3) Total Monthly Expenses . . . . 3) $______________
4) Total Monthly Contributions . . 4) $______________
Total amounts (2) + (3) + (4) . . . . 5) $______________
5) Amount Available for Other Savings or Expenditures (Deficit) (1 - 5) . . . . 6) $______________

Note: If the person or family is in a deficit position, the budget needs to be analyzed for possible adjustments.

(7 marks)

6. **Property Tax**

Fill in the missing values in the following Taxation Notice.

PROPERTY DESCRIPTION

ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
514	5	9	2	CR1692	64.5 ft	1

Civic Address

Title or Deed No.	Current Assessment Land	Current Assessment Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port Assessment
B45	17 500	52 000	T	?	10	30%	?

←ERRORS AND OMISSIONS EXCEPTED
←ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
←ALL CHEQUES MADE IN CANADIAN FUNDS
←BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
←RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY
10 000

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			?	52.63	?
By-Law No.	**Term**	**Type**	**Frontage Levy**	**Mill Rate**	**Levy**
74	97	Sewer and Water	65	.032	?
81	97	Sidewalk		2.613	?
112	98	Street		6.75	?

EDUCATIONAL TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	?	6.80	?
Provincial Education 2	?	15.14	?

PROVINCIAL TAX CREDITS

(See Manitoba Enclosure For Additional Information)	Assessment	Levy
	Manitoba Resident Homeowner Tax Assistance	$250.00

TOTAL TAXES DUE

Municipal Tax	Education Tax	Total Taxes	Prov. Credits	Net Taxes	Arrears/Credits	Added Taxes	Taxes Due
?	?	?	?	?	?	?	?

7. Solve the following applications by setting up a system of equations or constraint inequalities and using your calculator where necessary to solve the resulting system.

(5 marks)

a) Dandelion weed killer sells at \$3.50/L and the crabgrass weed killer sells at \$2.80/L. Kehlin decides to make an all-purpose weed killer by mixing the two types of weed killer and selling it at \$3.08/L. If he makes 50 L of this mixture, how much of each type of weed killer does he use?

b) A man with 160 ft of fencing wishes to fence off an area in the shape of a rectangle. If one side of the area will not require fencing, what should be the dimensions to ensure the largest area possible?

(5 marks)

(10 marks)

c) Two machine operators, Pat and Amy, make bolts and nails. Pat can make 600 bolts and 400 nails per hour. Amy can make 1000 bolts and 400 nails per hour. Pat earns \$15 per hour and Amy earns \$20 per hour. For how many hours should each person work to fill an order for 6000 bolts and 3200 nails at minimum cost?

60

Notes

Grade 11 Applied Mathematics (30S)

Midterm Practice Examination

Grade 11 Applied Mathematics

Midterm Practice Examination

Name: ______________________________________

Student Number: ______________________________

Attending ❑ Non-Attending ❑

Phone Number: _______________________________

Address: ____________________________________

____

____

For Marker's Use Only

Date: ___________________________________

Midterm Mark: _____ /100 = ________ %

Comments:

Instructions

The midterm examination is based on Modules 1 to 4 of the Grade 11 Applied Mathematics course. It is worth 20% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the midterm examination.

Notes

You are allowed to bring the following to the examination: pens/pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the examination. Graphing technology (either computer software or a graphing calculator) **is required** to complete this examination.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer. Final answers without supporting calculations or explanations will **not** be awarded full marks. Indicate equations and/or keystrokes used in calculations.

When using graphing technology, include a screenshot or printout of graphs **or** sketch the image and indicate the window settings (maximum and minimum *x*- and *y*-values), increments, and axis labels, including units.

Name: ______________________________

Answer all questions to the best of your ability. Show all your work.

Module 1: Quadratic Functions *(37 marks)*

1. How many x-intercepts will a quadratic function have if the vertex coordinates are (8, 9) and the function equation is $y = x^2 - 16x + 73$? Explain. *(2 marks)*

2. A quadratic function has vertex coordinates at (3, 0). How many x-intercepts will the quadratic function have? Explain. *(2 marks)*

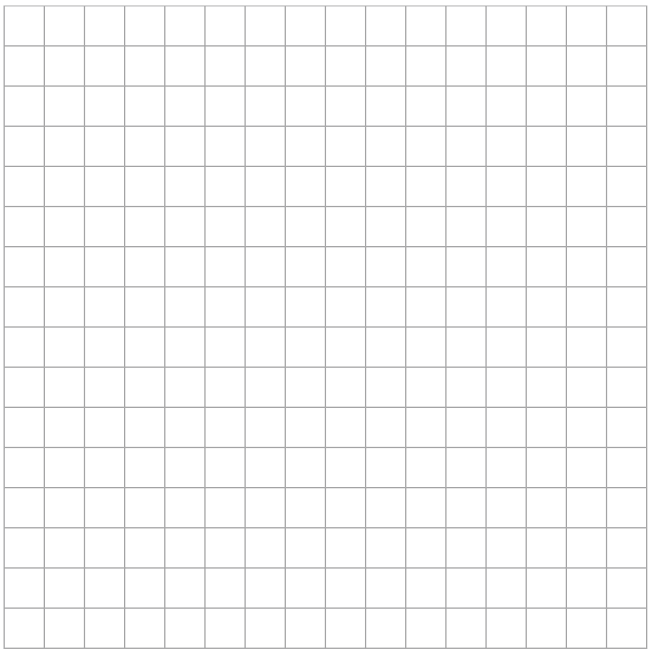
3. Sketch the following features of a quadratic function.

   a) The axis of symmetry equation is $x = 3$.

   b) The coordinates of the vertex are $(3, -3)$.

   c) The x-intercepts are 0 and 6.

   Then sketch the corresponding quadratic function. *(4 marks)*



Name: ____________________________________

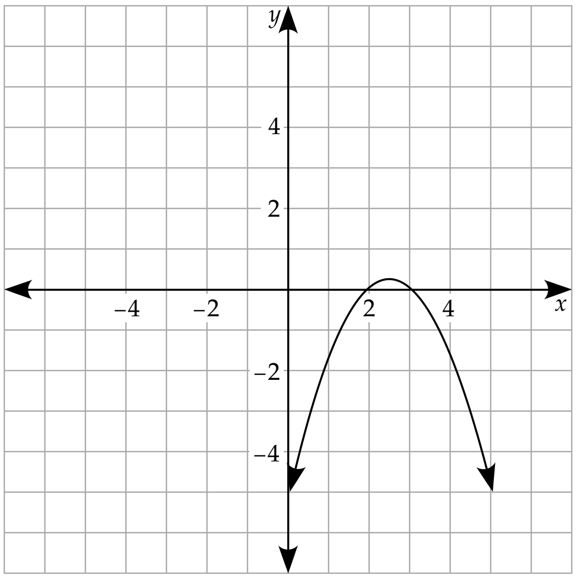
4. The x-intercepts of the quadratic function $y = -1.5x^2 - 2x + 3$ are -2.23 and 0.90.
   a) What are the roots of the equation $-1.5x^2 - 2x + 3 = 0$? *(1 mark)*

   b) What is the relationship between the x-intercepts of a function and the roots of the corresponding equation? *(1 mark)*

5. Use a grapher to determine the x-intercepts, to two decimal places, of the graph of the quadratic function given below. Include a sketch of your graph with intercepts labelled. *(3 marks)*

   $y = -4x^2 + 7x + 7$

6. Write a possible quadratic equation in factored form that has the same x-intercepts and opens in the same direction as the graph below. *(3 marks)*



Name: ____________________________________

7. Draw three separate quadratic functions with the characteristics given. *(3 marks)*

   a) The function has two x-intercepts and opens up.

   b) The function has no x-intercepts and opens up.

   c) The function has one x-intercept and opens down.

a)

b)

c)

8. Consider the quadratic function $y = -\frac{1}{2}x^2 - 5x + 3$, with axis of symmetry equation $x = -5$.

a) Determine the vertex coordinates. *(2 marks)*

b) Determine the maximum and minimum values (if they exist). *(1 mark)*

c) Determine the domain. *(1 mark)*

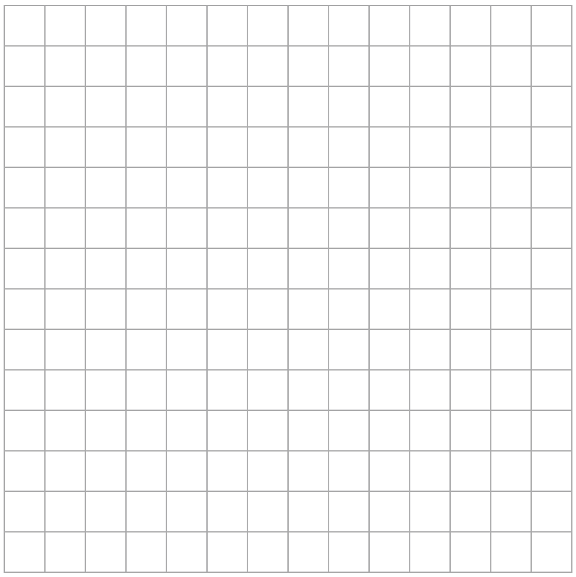
d) Determine the range. *(1 mark)*

Name: ____

9. Create a table of values and sketch a graph of the quadratic function $y = x^2 - 2x + 3$. *(3 marks)*

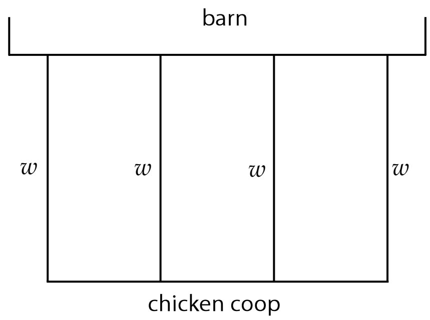
10. Sketch a graph of the following quadratic function using the intercepts. Include the coordinates of the intercepts and vertex on your graph. *(3 marks)*

$y = 3(x - 2)(x)$



Name: ____________________________________

11. An outdoor fenced chicken coop with three sections is to be built attached to a pre-existing barn, as shown. No fence is needed against the barn. Determine the dimensions of the chicken coop with the greatest area that can be enclosed using 650 feet of fencing, by completing the following.



a) If 650 feet of fencing is to be used to create four widths, w, and one length, determine an expression for the length in terms of w. *(1 mark)*

b) Determine a quadratic function model to define area in terms of width. *(2 marks)*

c) Using a graphing utility, find the coordinates of the vertex of the quadratic function and interpret their meaning. *(2 marks)*

d) Determine the dimensions of the pen with the largest area. *(2 marks)*

Module 3: Reasoning to Solve Problems *(28 marks)*

1. Determine a conjecture that could be made about the sum of a multiple of 4 and a multiple of 6. Use at least two examples to help you develop your conjecture. *(3 marks)*

2. Jordyn arrives at her first pre-calculus mathematics class in Grade 11. She realizes that there is only one other girl in the class, and the other members of the class are male. Jordyn then attends her first home economics class and notices that the majority of the class is female. State two possible conjectures Jordyn can make about the gender of students attending pre-calculus and home economics classes. *(2 marks)*

3. If possible, find a counter-example to the following conjectures.

   a) The sum of a multiple of 5 and a multiple of 6 will be an odd number. *(1 mark)*

   b) All cars consume gasoline. *(1 mark)*

Name: ______________________________

4. Consider the following number trick.

   Pick a number.

   Subtract 1.

   Multiply the result by 3.

   Add 12.

   Divide the result by 3.

   Add 5.

   Subtract your original number.

   a) Make a conjecture about the result of the above number trick and provide two examples to support your conjecture. *(2 marks)*

b) Prove that this number trick will always result in the conjecture you made in (a).
(4 marks)

5. Determine whether the following scenarios represent inductive or deductive reasoning.
(2 marks)

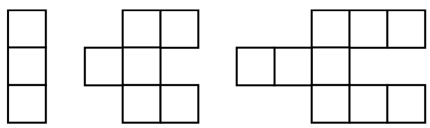
a) Ms Newton told her class that if they receive an A on the final exam, then they will earn a final grade of A in the course. Britney receives an A on the final exam and expects to earn a final grade of A in the course.

b) It has rained for the past three days. Brayden assumes it will rain again tomorrow.

Name: ______________________________

6. Use inductive reasoning to determine the next three terms in each of the following patterns. *(3 marks)*

   a) 1, 2, 3, 2, 4, 6, 4, 8, 12, . . .

   b) 

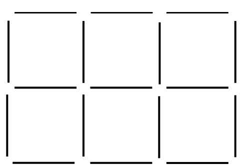
7. Explain why the following proofs are invalid. *(4 marks)*

   a) All parallelograms are quadrilaterals. Figure ABCD is a quadrilateral. Therefore, figure ABCD is a parallelogram.

   b) Dylan is trying to prove that a number trick always results in the value of 5.

Let x be the number.	Pick a number.
$x + 3$	Add 3.
$2x + 6$	Multiply by 2.
$2x + 10$	Add 4.
$2x + 5$	Divide by 2.
$x + 5$	Subtract the original number.

8. Consider the following image made up of 17 toothpicks. Remove 6 toothpicks to leave two squares. *(2 marks)*



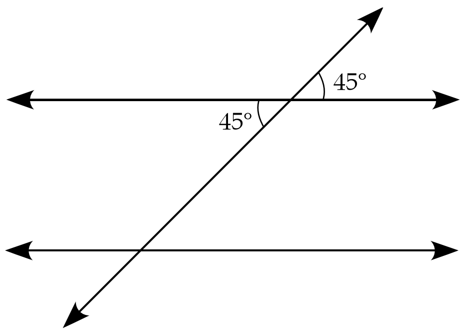
9. Prove that the sum of a multiple of 3 and a multiple of 6 is a multiple of 3. *(4 marks)*

Name: ______________________________

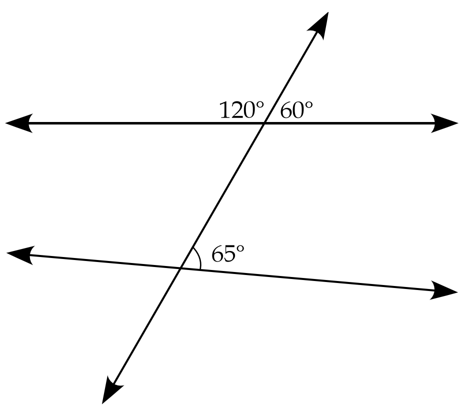
Module 4: Geometry of Angles and Triangles *(35 marks)*

1. Determine if the following sets of lines are parallel. Explain your answers. *(4 marks)*

a)

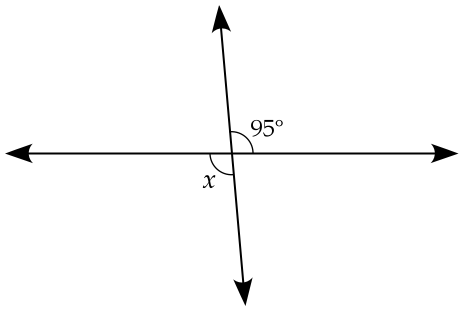


b)

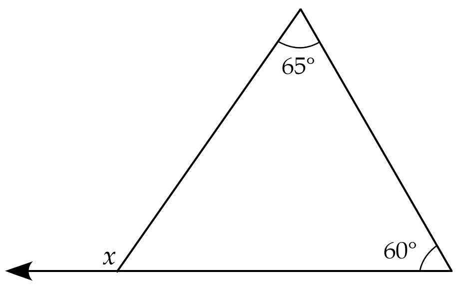


2. Find the indicated angle(s) in each of the diagrams below and state the property or rule you used to determine these angles. *(6 marks)*

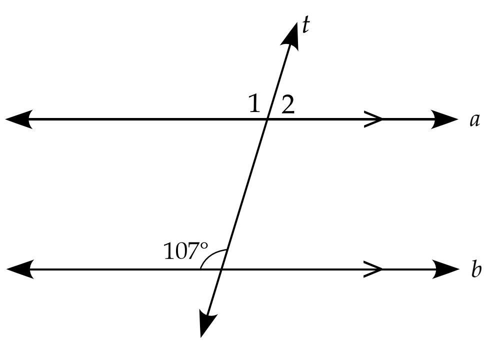
a) *(1 mark)*



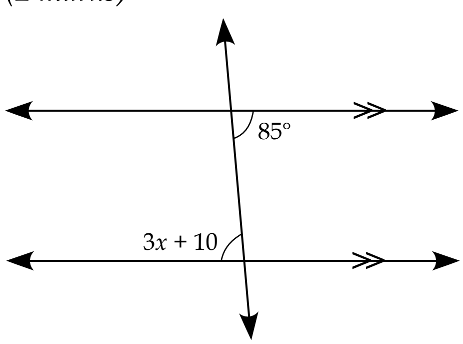
b) *(1 mark)*



c) *(2 marks)*



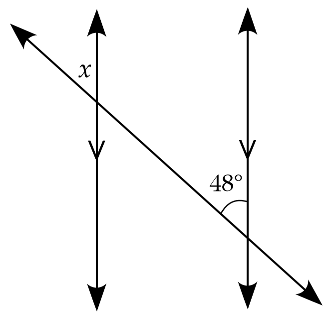
d) *(2 marks)*



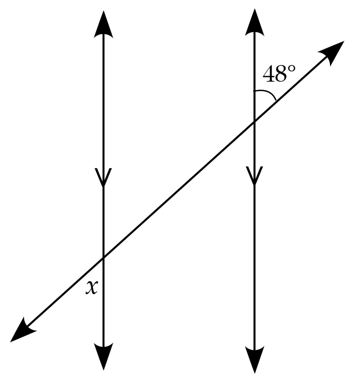
Name: ______________________________________

3. For each diagram below, state the measurement of $\angle x$ and the property that allows you to determine it. *(6 marks)*

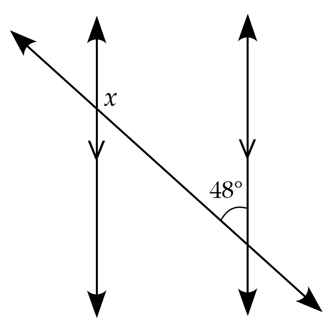
a)



b)



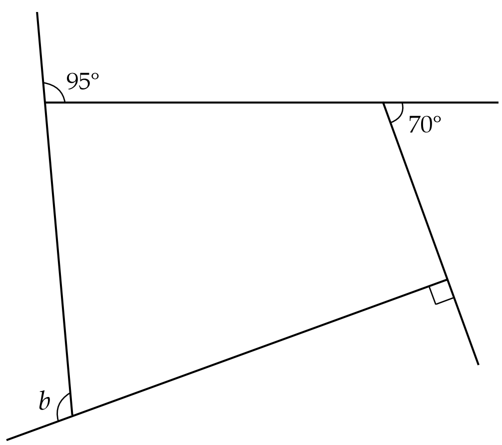
c)



4. If the sum of the interior angles of a polygon is 4680°, how many sides does the polygon have? *(2 marks)*

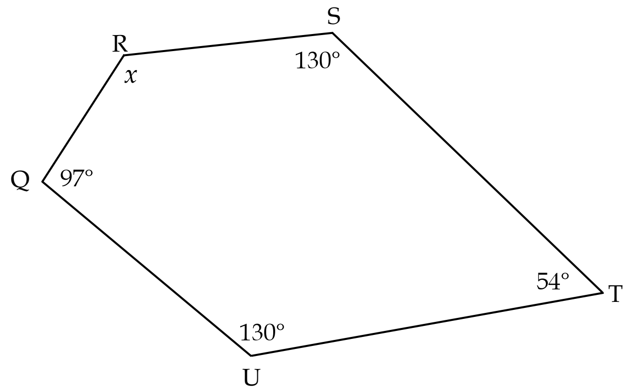
5. If a polygon has 17 sides, what is the sum of the interior angles? *(2 marks)*

6. Determine the size of the missing exterior angle in the polygon below. *(2 marks)*

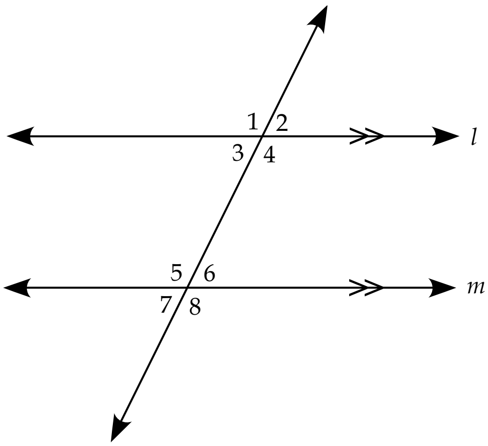


Name: ______________________________

7. Determine the size of the missing interior angle in the polygon below. *(2 marks)*

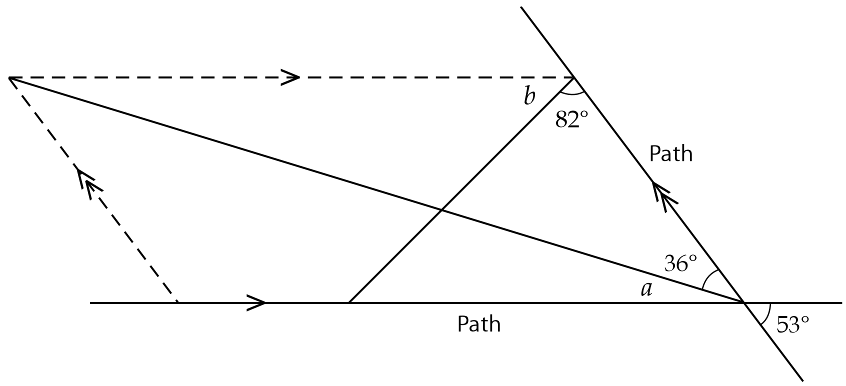


8. Prove that Angle 3 and Angle 5 have a sum of 180° in the diagram below, given that lines *l* and *m* are parallel. In other words, prove that same side interior angles sum to 180°. *(4 marks)*



Name: ______________________________

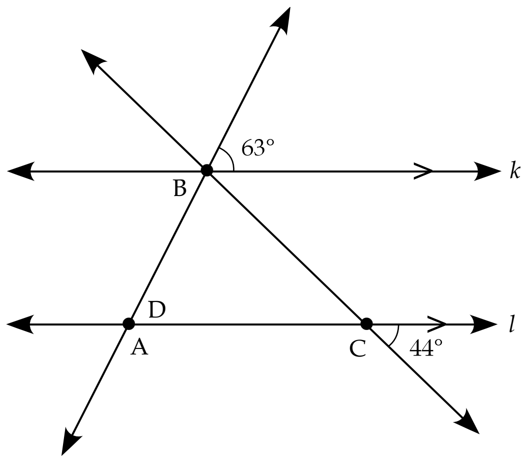
9. A school wants to plant a new flower garden in the shape of a parallelogram, as displayed by the diagram below. The flower garden will be divided into four sections.



a) Explain how the school could find the measurements of all the indicated angles without using a protractor. *(1 mark)*

b) Determine the measurements of angles *a* and *b*. *(2 marks)*

10. Find the value of the angles labelled with a letter in the diagram below and state the property or rule you used to determine these angles. *(4 marks)*



Grade 11 Applied Mathematics (30S)

Module 6
Statistics

Module 6: Statistics

Introduction

The world around us is full of data. Media (television, radio, newspaper, websites, billboards) provide data for a variety of different reasons. Data can be used for reaching out to the public (such as a "quit smoking" campaign), advertising, and numerous other purposes. Once data is collected, it can be analyzed and interpreted. Many types of statistical analysis can be performed to reach conclusions about the material being analyzed.

In this module, you will investigate how conclusions are drawn in statistical studies when data is collected from a number of sources. Since people are bombarded with statistics, whether it is through advertising, governmental concerns, or other sources, it is important to learn how the data is analyzed and what the statistics are actually saying. This gives you a better understanding of the validity and reliability of the conclusions that are drawn from the data. In Module 6, you will review how to calculate measures of central tendency such as the mean, median, and mode. You will also learn about measures of dispersion, including the range and standard deviation.

One important tool for organizing and displaying data is the bell curve, first used in 1835 by Adolphe Quetelet, a Belgian statistician. Quetelet collected and studied data relating to the characteristics of the society around him, from crime and suicide rates to height and weight data. While analyzing his research findings, he realized that when the data was graphed, the graph often took the shape of a bell curve. Since then, the bell curve has been widely used to display statistical data because of its mathematical properties. In this module, you will learn about the relationships that exist among bell curves, standard deviation, and z-scores.

It is important to be aware of accuracy when using statistics. In the final lesson of Module 6, you will learn about confidence intervals, which allow you to determine, with a certain degree of confidence, the accuracy of various statistics.

In this module, you will use technology such as GeoGebra and Microsoft Excel. Feel free to use any technology you wish, including a graphing calculator. If you are unable to figure out how to use a certain piece of technology to compute a certain statistic, a number of Internet sites contain tutorials that can help you. It is your responsibility to learn to use the technology you select.

Assignments in Module 6

When you have completed the assignments for Module 6, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	The Indigenous Identity Population in Canada
2	Assignment 6.1	Measures of Central Tendency and Dispersion
5	Assignment 6.2	*z*-Scores and the Normal Curve
6	Assignment 6.3	Confidence Intervals

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, and can be either handwritten or typewritten. Both sides of the sheet may be filled. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 6. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 5 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 5 to 8.

Resource Sheet for Module 6

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Notes

Module 6 Cover Assignment: The Indigenous Identity Population in Canada

Misleading Statistics

Statistics can be misleading for many reasons, including problems with sampling, unfair polling questions, or misleading interpretations of data.

If the sample population of a survey is too small (e.g. the survey is not available to a large enough population), the results could be inaccurate. Also, if the survey is only administered in one medium (e.g., limited to the Internet), many people could be missed by the survey. Those with no access to the Internet and those who are uncomfortable using the Internet would not be represented by the survey results.

Polling questions can be misleading if they use language that includes double negatives, advanced vocabulary, or guiding statements. Examples include:

- Do you not always purchase organic vegetables whenever they are available?
- Mr. Smith has been convicted of moral turpitude. Should he be impeached?
- How do you feel about unfair laws being passed about cat licensing for law-abiding, peaceful Winnipeg citizens?

Finally, statistics can often be used in the wrong context to misrepresent data. You will discover more about misleading statistics throughout this cover assignment.

Notes

Module 6 Cover Assignment

The Indigenous Identity Population in Canada

Total: 10 marks

Clearly show the steps in your solution on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations and graphs will not be awarded full marks.

1. The following chart displays the population of Indigenous and non-Indigenous people in Canada in 2001 and 2006. The Indigenous population is further broken down into First Nations, Métis, and Inuit subgroups.

Year	Total Population	Indigenous Identity Population	First Nations	Métis	Inuit	Non-Indigenous
2001	29 639 030	976 305	608 850	292 305	45 070	28 662 725
2006	31 241 030	1 172 785	698 025	389 780	50 480	30 068 240

a) Determine the percent increase for each category by completing the following table and using the following formula. The first calculation has been done for you. *(2 marks)*

$$\text{Percent Increase} = \frac{\text{New Population} - \text{Old Population}}{\text{Old Population}} \times 100$$

Year	Total Population	Indigenous Identity Population	First Nations	Métis	Inuit	Non-Indigenous
2001	29 639 030	976 305	608 850	292 305	45 070	28 662 725
2006	31 241 030	1 172 785	698 025	389 780	50 480	30 068 240
Percent Increase	5.4%					

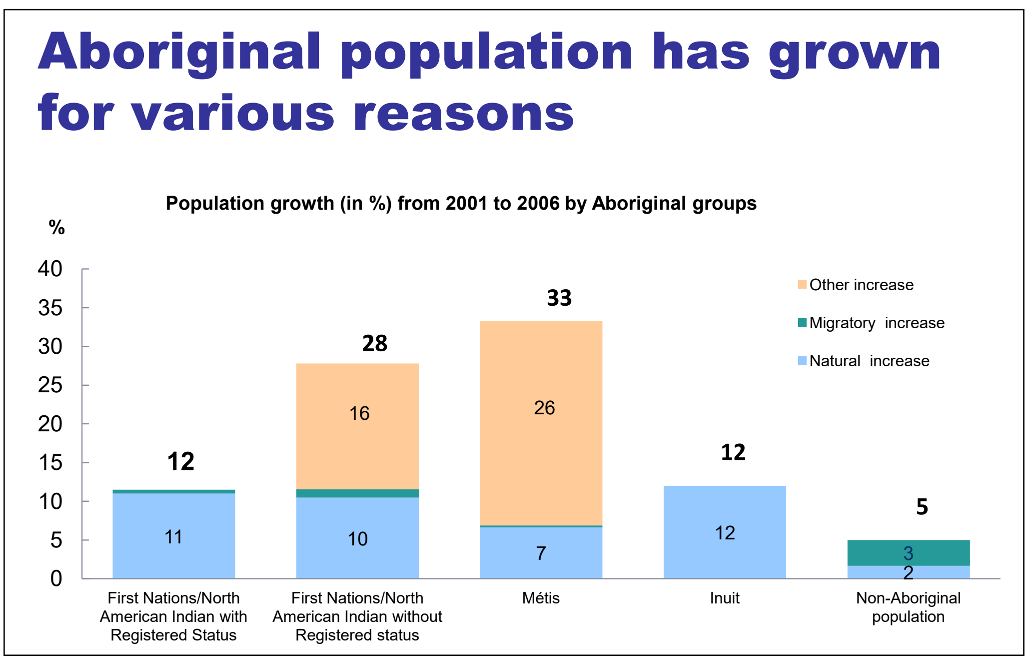
$$\frac{31\ 241\ 030 - 29\ 639\ 030}{29\ 639\ 030} \times 100 = 5.4\%$$

Module 6 Cover Assignment: The Indigenous Identity Population in Canada (continued)

b) Which subgroup of the Indigenous population (First Nations, Métis, or Inuit) has increased by the greatest percentage? Suggest a possible reason to account for this increase. *(1 mark)*

c) How does the percent increase of the non-Indigenous population compare to the percent increase of the Indigenous population? *(1 mark)*

2. There are many reasons why the Indigenous population is increasing at a much faster rate than the non-Indigenous population. These reasons include a higher fertility rate in the Indigenous population and more citizens identifying themselves as Indigenous. Consider the following graph.



Source: *What Statistics Canada Data Tell Us About the Métis Population in Canada*. Senate of Canada: Standing Committee on Aboriginal Peoples. Presentation by Jane Badets, Statistics Canada. March 28, 2012. p. 5.

Module 6 Cover Assignment: The Indigenous Identity Population in Canada (continued)

This graph further divides the First Nations population into those who have Registered Status and those who do not have Registered Status.

a) In the legend on the graph, “Natural increase” refers to births while “Migratory increase” refers to people moving to Canada from other countries. What do you think “Other increase” refers to? *(1 mark)*

b) How could the statement, “The population of Métis peoples in Canada grew by 33% from 2001 to 2006,” be misleading? *(1 mark)*

c) If the only information presented to Canadian citizens about the Indigenous population was the table in Question 1, how could those statistics be misleading? *(1 mark)*

Module 6 Cover Assignment: The Indigenous Identity Population in Canada (continued)

3. The Canadian Indigenous identity population is expected to be between 1.7 million and 2.2 million in 2031. The Canadian Indigenous identity population is expected to be between 4% and 5.3% of the entire Canadian population in 2031.

   a) The Canadian Indigenous identity population in 2006 was 1 172 785. If the Canadian Indigenous identity population grew to be 2.2 million in 2031, what would be the percent increase between 2006 and 2031? *(1 mark)*

   b) If the Canadian Indigenous identity population in 2031 is 2.2 million and this represents 5.3% of the entire Canadian population, how many people are expected to live in Canada in 2031? *(2 marks)*

Lesson 1: Measures of Central Tendency and Histograms

Lesson Focus

In this lesson, you will

- ❑ learn how to calculate the mean, median, mode, and range for a set of raw data
- ❑ learn how to display data using frequency tables and histograms

Lesson Introduction

Imagine you are a statistician and you have just been given a large amount of raw data that you need to analyze. What steps would you take to analyze the data?

In previous courses, you learned how to calculate the mean, median, mode, and range of a data set. These statistical values may be your first step in analyzing the data numerically, but how would you display all of the data values graphically?

One way of displaying large sets of data graphically is to use a histogram, which is a graphical representation of a frequency table. These two types of analysis (histogram and frequency table) allow you to group data into categories, called intervals, so that you can clearly see how your data is organized in relation to each interval.

Measures of Central Tendency

You were introduced to the terms *mean*, *median*, and *mode* in previous mathematics courses. These are all measures of central tendency as they represent central values of the data. They are sometimes referred to as the "average." Together, these three measures of central tendency give a more complete representation of the data and allow you to make conclusions about your data.

Sampling Technology

Whenever a statistical study is required for some topic, data must be collected. The collection method can greatly affect the outcome of the study and, therefore, determine some of the conclusions that may be drawn. It is important that the data is collected in such a way as to have as little bias as possible. The best way is to collect data from every source or every person being considered in the experiment. When this procedure is used, we say that the total **population** has been surveyed.

It can sometimes be difficult, if not impossible, to survey the total population when collecting data. One reason is the number of data sources that you may be considering. Sometimes the number is too large and it may be impossible to contact and collect data from them all. A second reason is that the total population may be constantly changing. For example, if the people of Manitoba were being surveyed for some issue, all Manitobans would make up the population being considered. Due to births and deaths every day, the population would be constantly changing.

Because of the difficulties in surveying a total population, a **sample** is used. The goal is to try to make the sample you choose as close to a perfect representation of the total population as possible. There are no guarantees that this can be achieved, but there are sampling procedures to use that can give you the best chance of reaching the goal. One important procedure in choosing a sample is to make your sample as large as possible. Another procedure in making the sample representative of the population is to give every source of data an equal chance of being selected for data collection. This procedure is called **random sampling**.

Once you have collected your data, it can be used to determine the measures of central tendency.

Mean

The **mean** of a data set can be found by adding each data value together and then dividing by the number of data values in the data set. To help statisticians communicate whether they have used the mean of a sample or a population, they often use two different symbols.

- If you are dealing with a *sample*, the mean is denoted as $\bar{x}$.
- If you are dealing with a *population*, the mean is denoted as μ.

Median

The **median** of a data set is the middle number when the numbers are written in numerical order. If a data set has an even number of values, the median is the mean of the two middle values.

Mode

The **mode** of a data set is the value that appears most frequently. A data set can have no mode, one mode, or more than one mode.

Example 1

The following table displays the number of goals scored in each of the Women's Olympic soccer games in 2012.

3	1	3	7	3	2	2
2	1	4	4	0	1	1

Determine the mean, median, and mode for this set of data.

Solution

To determine the mean, add together each value and divide by the total number of values.

$$\begin{aligned}\bar{x} &= \frac{3+1+3+7+3+2+2+2+1+4+4+0+1+1}{14}\\ &= \frac{34}{14}\\ &= 2.4\end{aligned}$$

The mean number of goals scored in each game was 2.4.

To determine the median, write each of the data values in order from smallest to largest.

0, 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 7

Because there is an even number of values you need to determine the mean of the two middle values. The two middle values are the 7th and 8th values, which are 2 and 2. The mean of 2 and 2 is $\frac{2+2}{2} = 2$. Therefore, the median of this data set is 2.

The mode is the value that occurs the most often. Since 1 occurs most, the mode of this data set is 1.

Measures of Dispersion

Although the mean, median, and mode are very useful in analyzing the centre of a set of data, they paint an incomplete picture of the data. These measures only indicate the centre of the distribution, which is not always adequate since there is no indication of how the data is distributed. In some cases, you may need to analyze how closely the data values cluster around the centre.

The two most common ways to analyze dispersion (or how far away data values are from the centre) involve finding the range and the deviation. The deviation for each piece of data is calculated by subtracting the data value from the mean. The sum of all of the deviations can be calculated and then the average deviation can be found. However, the most common calculation method of deviation is called **standard deviation**. You will learn more about finding the standard deviation in the next lesson.

Range

The **range** of a data set is the difference between the largest value in the distribution and the smallest value.

The range is the simplest measure of variation to calculate since only two numbers are needed to calculate it. It is not commonly used, however, because it does not tell you anything about how the other data values are dispersed or clustered between the smallest and largest values. If there is one extreme value in a distribution, the range will be very large. If you remove the extreme term, the range may become much smaller. Because of the limited information offered by the range calculation, other measures of dispersion such as standard deviation are used.

Example 2

Jolie is trying to find the best way to drive to school in Winnipeg to make her driving time as efficient as possible. During one week she drove to school on Portage Avenue and during a second week she drove on the Perimeter Highway. The number of minutes she needed to drive to school each day are as follows:

- Portage Avenue: 15, 26, 30, 39, 45
- Perimeter Highway: 29, 30, 31, 32, 33

a) Determine the mean, median, and range of each set of data.

b) Determine the distance between each data value, x, and the mean, $\bar{x}$, for both sets of data. Which is the better route for Jolie to take to school?

Solution

a) **Portage Avenue**

Mean: $\dfrac{15 + 26 + 30 + 39 + 45}{5} = \dfrac{155}{5} = 31$ minutes

Median: 30 minutes

Range: $45 - 15 = 30$ minutes

Perimeter Highway

Mean: $\dfrac{29 + 30 + 31 + 32 + 33}{5} = \dfrac{155}{5} = 31$ minutes

Median: 31 minutes

Range: $33 - 29 = 4$ minutes

In each case, the mean and median times that it took Jolie to drive to school were close to 31 minutes. However, when she took Portage Avenue, the range was 30 minutes. When she took the Perimeter Highway, the range was only 4 minutes. This indicates that the Perimeter Highway route is more consistent.

b) Find the difference between each score, x, and the mean, $\bar{x} = 31$.

Portage Avenue

$15 - 31 = -16$

$26 - 31 = -5$

$30 - 31 = -1$

$39 - 31 = 8$

$45 - 31 = 14$

Perimeter Highway

$29 - 31 = -2$

$30 - 31 = -1$

$31 - 31 = 0$

$32 - 31 = 1$

$33 - 31 = 2$

These values would indicate that travelling on the Perimeter Highway is going to produce less variation in travel times and so may be the more reliable route to take.

Note: The calculation you just completed for each data value, $x - \bar{x}$, is called the **deviation**. The standard deviation formula, which you will learn in the next lesson, builds upon this calculation.

Using Technology to Calculate Mean, Median, and Range

It is possible to calculate the mean, median, and range using technology such as a graphing calculator, a spreadsheet program, or software such as GeoGebra. If you have a large amount of data and want to avoid the time-consuming process of ordering each data value or adding all of the data values together, these programs can help you. In each program, you need to know how to enter data values using lists and how to instruct the program to calculate the measures of central tendency or dispersion that you want to find.

Frequency Distributions

Frequency distribution tables and *histograms* are two different formats for viewing large sets of data. Both of these ways of organizing data are called **frequency distributions**. Frequency distribution tables are tables that organize data values into intervals that are usually the same size. Histograms are visual representations of frequency tables. Histograms can be used to analyze whether data follow a normal distribution. You will begin to study the concept of normal distribution in Lesson 3.

Frequency Tables and Histograms

A **frequency distribution table** is a table that organizes raw data (data that has been collected but not processed or organized in any way) into intervals. Tally marks are used to show how many data values fall into each interval. Consider the following example.

Example 3

The number of traffic tickets given at each traffic-camera location in Winnipeg during 2010 is shown in the chart below.

Traffic Tickets in Winnipeg 2010					
0	721	425	118	1302	288
227	596	29	109	1735	620
1020	0	295	2017	169	183
2657	941	300	83	2328	832
2767	302	0	0	1650	22
2203	134	259	409	113	485
275	757	0	243	3464	828
290	1946	1206	490	4573	813
1716	137	227	6121	126	0

Source: http://winnipeg.ca/police/safestreets/docs/2010_photo_enforcement_annual_report.pdf. Accessed August 4, 2016.

a) Create a frequency table that displays this information.

b) Using the frequency table, create a histogram to display this data graphically.

Solution

a) The number of traffic tickets issued at each camera ranges from 0 to 6121. Rather than using individual frequency values, an interval of frequencies is used to group frequencies together. To display this information in a frequency table, you first need to determine the size of each interval.

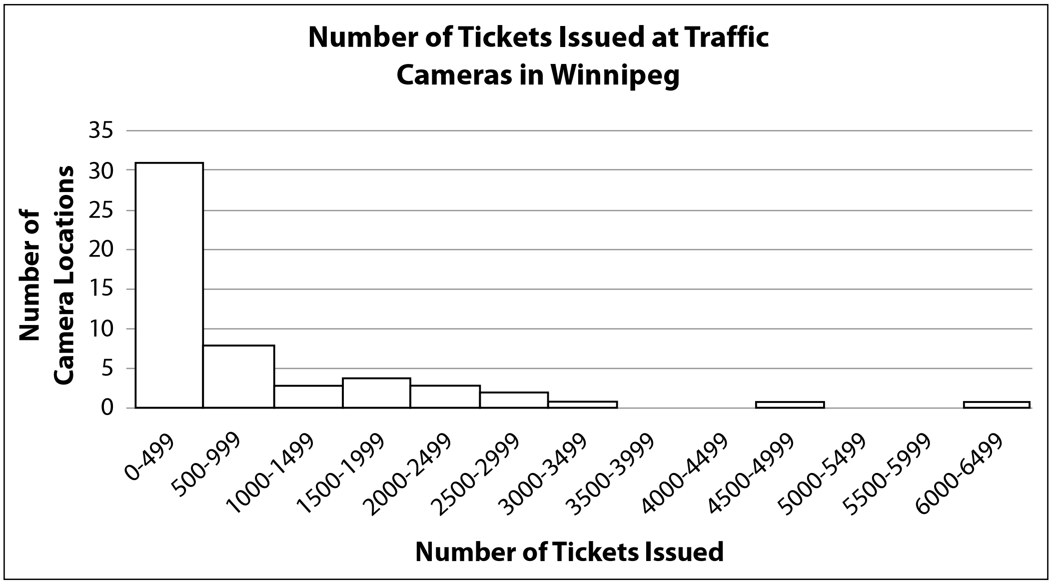
With this data, an appropriate interval size that allows you to display all data values is every 500 tickets. Once you have determined your interval size, create a tally of how many data values reside in each interval. You can then determine the frequency of data values occurring in each interval by determining the number of tally marks you have in each interval.

Number of Tickets Issued	Tally	Frequency (# of camera locations)
0 – 499	~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ |	31
500 – 999	~~||||~~ |||	8
1000 – 1499	|||	3
1500 – 1999	||||	4
2000 – 2499	|||	3
2500 – 2999	||	2
3000 – 3499	|	1
3500 – 3999		0
4000 – 4499		0
4500 – 4999	|	1
5000 – 5499		0
5500 – 5999		0
6000 – 6499	|	1

b) Various technology and software programs can be used to create histograms, such as Microsoft Excel or GeoGebra.

These programs usually require you to enter your raw data and choose your interval sizes. Using this information, the program can create a frequency table and, ultimately, a histogram.

Alternatively, you can also draw a histogram by hand, which is similar to drawing a bar graph. Each interval from the table is displayed as a bar in the histogram. The height of the bar represents the frequency of data values falling within that interval.



By looking at the frequency table and the histogram, it is clear that most traffic cameras issued between 0 and 500 tickets.

Make sure you complete the following Learning Activity to practise what you learned in this lesson. You will need these basic skills in order to understand the rest of the module.

Learning Activity 6.1

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

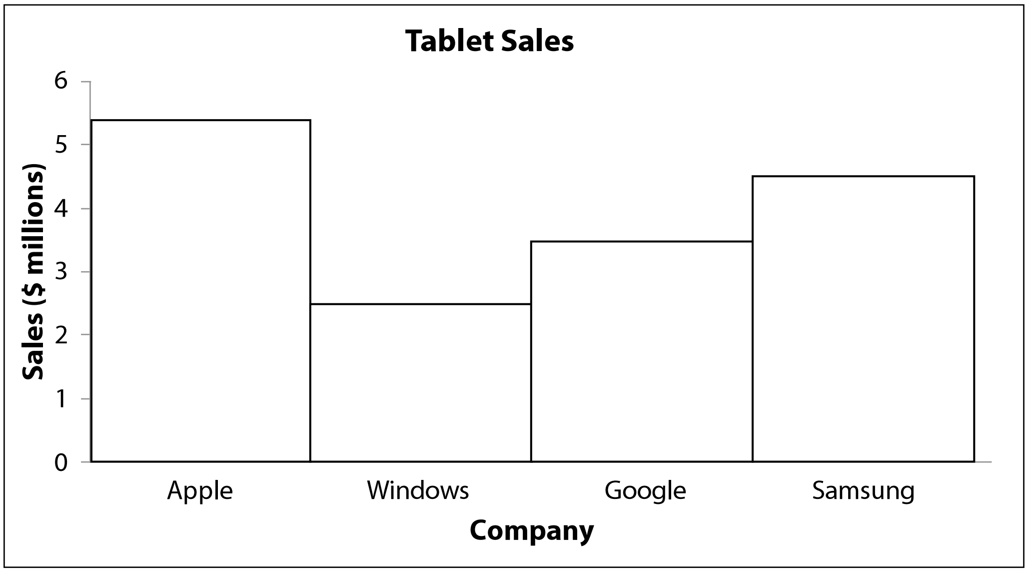
Use the following data set to answer questions 1 to 3.

1, 2, 4, 5, 5, 8, 9, 10, 16, 20

1. Determine the mean.
2. Determine the median.
3. Determine the mode.

continued

Consider the following bar graph to answer questions 4 and 5.



4. Which companies made more than $4 million?
5. Approximately how much more money did Apple make than Windows?
6. 25 percent of what number is 15?
7. Order from smallest to largest: $-\frac{3}{8}, \frac{5}{3}, -\frac{2}{3}, \frac{1}{4}$
8. Solve for y: $-3y \geq 18$

continued

Learning Activity 6.1 (continued)

Part B: Measures of Central Tendency and Frequency Distributions

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. The selling prices of vehicles for one car company are displayed in the chart below.

$14,589	$24,999	$28,999	$24,999	$21,999
$25,999	$26,299	$21,999	$28,999	$22,599
$26,199	$35,999	$22,599	$18,999	$15,999
$26,599	$24,999	$29,999	$25,999	$41,999

   a) Determine the mean, median, mode, and range of the data.
   b) Create a frequency table with six intervals.
   c) Create a histogram to display the data from your frequency table.
   d) What does the histogram tell you about car prices from this dealership?
   e) How could you improve the histogram to show customers more information about the vehicle prices from this dealership?

2. The ages of the players on the Winnipeg Jets hockey team are shown in the table below.

33	21	22	34	21
25	32	23	30	24
29	26	23	22	28
28	32	24	24	28
25	27	29	27	28

   a) Determine the mean, median, mode, and range of the data.
   b) Create a frequency table with eight intervals.
   c) Create a histogram to display the data from your frequency table.
   d) What can you conclude about the information displayed in the frequency table and in the histogram?

continued

Learning Activity 6.1 (continued)

3. The table below displays the number of people in various age groups from the 2011 census in Canada.

Age Groups	Sex		
	Total	**Male**	**Female**
Total—Age groups	33,476,690	16,414,225	17,062,460
0 to 4 years	1,877,095	961,150	915,945
5 to 9 years	1,809,895	925,965	883,935
10 to 14 years	1,920,355	983,990	936,360
15 to 24 years	4,365,585	2,224,625	2,140,965
25 to 34 years	4,332,490	2,136,090	2,196,400
35 to 44 years	4,498,805	2,205,915	2,292,890
45 to 54 years	5,334,100	2,627,740	2,706,355
55 to 64 years	4,393,300	2,149,990	2,243,315
65 to 74 years	2,674,775	1,281,445	1,393,330
75 to 84 years	1,624,770	709,025	915,740
85 years and over	645,510	208,300	437,210
Median age	40.6	39.6	41.5

a) Create a histogram with 11 intervals (one for each of the age groups) that displays the number of people that belong in that age bracket.

b) What does this histogram tell you about the average ages of Canadians?

c) According to the table above, the median age of Canadians in 2011 was 40.6. In South Africa, the median age of the population in 2011 was 25. What do these two statistics tell you about the differences between the population of Canada and the population of South Africa?

continued

Learning Activity 6.1 (continued)

4. Three manufacturers of car batteries all claim that the average life of their batteries, under normal use, is five years. A consumers' group decided to test each manufacturer's claim. The group compiled the following list on the lifespan, in years, of car batteries manufactured by each company:

   Manufacturer A: 0.5, 1.6, 2, 3.5, 4, 4.5, 6, 7, 7.9, 8, 10

   Manufacturer B: 4, 4, 5, 5, 5, 6, 11, 13, 14, 15, 16

   Manufacturer C: 2, 3, 4, 4, 6, 13, 14, 15

   a) Determine the mean of the lifespan of the car batteries manufactured by each company.
   b) Which measure of central tendency was each manufacturer using to support their claim?
   c) From which manufacturer would you buy a car battery? Why?

Lesson Summary

If you have a large set of raw data, it can be difficult to interpret the data. Frequency distributions in the form of frequency tables and histograms are two ways that allow you to organize the data into intervals and analyze how the data is distributed. To help you interpret the data, you can also calculate measures of central tendency and measures of dispersion.

In this lesson, you discovered that the three measures of central tendency (mean, median, and mode), as well as the range—a measure of dispersion—are not always adequate at representing data, since they do not give a complete picture. Sometimes other measures of data such as deviation, or standard deviation, which you will learn about in the next lesson, need to be considered. These allow you to analyze how data values are widely dispersed or tightly gathered around the mean.

Notes

Lesson 2: Standard Deviation

Lesson Focus

In this lesson, you will

- ❑ learn how to calculate the standard deviation of a data set

Lesson Introduction

In the previous lesson, you reviewed how to calculate the mean, median, and mode. These are all measures of central tendency. You also reviewed how to calculate one measure of dispersion—the range. In this lesson, you will learn how to determine the standard deviation of a data set, which is another measure of dispersion.

Standard Deviation

In Lesson 1, you reviewed how to calculate the mean of a data set. Another way to represent the calculation of the mean, μ, of a data set is shown below.

$$\mu = \frac{\sum x}{n}$$

The upper-case greek letter, sigma ($\sum$) is used as a shorthand notation to represent "the sum of" data which, in this case, is the sum of all of the x-values. The variable, n, represents the number of data values (or x-values). This complicated looking formula concisely shows the familiar process of calculating the mean (average) by adding up all the data values and dividing by the number of data values.

The formula for calculating the standard deviation of a data set uses a shorthand notation similar to the one shown above for mean.

Standard Deviation

The **standard deviation** shows dispersion or how "spread out" the data is in relation to the mean.

- If all data values are equal, the standard deviation will be zero since all of the data values will be equal to the mean.
- If data values are close to the mean, or most of the data is relatively close together, the standard deviation will be small.
- If data values are far from the mean, or the data is spread out, the standard deviation will be large.

In a normal distribution, the probability of observing a particular data value is greatest near the mean value. The probability of observing data values that are farther away from the mean decreases as those data values get farther away from the mean. To measure how fast this probability decreases, standard deviation is used. More will be said about the normal distribution in a later lesson.

For example, if you have a large standard deviation, there is a large probability that you will have data values that are far from the mean because the data is very spread out.

On the other hand, if you have a small standard deviation, there is a small probability that you will have data values that are far from the mean because the data is clustered close together.

Standard deviation is denoted by s for a sample and σ (the Greek letter, sigma) for a population. The calculation for s and σ is very similar. There is a slight adjustment when calculating s; however, in this course you will treat the two in the same way.

Even though the standard deviation is more complicated to calculate than the range, it is more commonly used as a measure of dispersion because it takes into consideration all the scores in the data set. The calculation of both the average deviation and the standard deviation involves the measurement of how far each score is from the mean. For each data value, x, the deviation from the mean is calculated as $x - \mu$. However, the average deviation is affected by negative numbers. For example, the values 5 and -5 are far apart, but adding them will give us an average deviation of 0.

The calculation of standard deviation takes care of the negative deviation values by squaring each deviation calculation before adding them. After dividing the sum of the squared x-values by the the number of values, n, to get the mean, the square root is taken to undo the squaring done originally.

The formula to find the standard deviation of a population is shown below.

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{n}}$$

Notice in the formula that the differences between each x-value and the mean value are found. Remember that μ is the symbol you use when you are calculating the mean of a population. This is the same as the deviation calculation you used in the previous lesson.

If each data value is far away from the mean, the standard deviation will be a large number. If each data value is close to the mean, the standard deviation will be a small number.

You should add this formula to your resource sheet.

Note: When calculating standard deviation in this course, you will only be dealing with an entire population rather than a sample of the population. When calculating the standard deviation for a sample of the population, the formula is changed to divide by $n - 1$, rather than dividing by n. Otherwise, the calculation is the same.

Consider the following example from Lesson 1.

Example 1

Jolie is trying to find the best way to drive to school in Winnipeg to make her driving time as effective as possible. During one week she drove to school on Portage Avenue and during a second week she drove on the Perimeter Highway. The number of minutes she needed to drive to school each day are as follows:

- Portage Avenue: 15, 26, 30, 39, 45
- Perimeter Highway: 29, 30, 31, 32, 33

a) Determine the standard deviation of each data set to two decimal places.

Solution

You already determined the deviation of each data value in the last lesson.

Portage Avenue	**Perimeter Highway**
$15 - 31 = -16$	$29 - 31 = -2$
$26 - 31 = -5$	$30 - 31 = -1$
$30 - 31 = -1$	$31 - 31 = 0$
$39 - 31 = 8$	$32 - 31 = 1$
$45 - 31 = 14$	$33 - 31 = 2$

To calculate the standard deviation for each data set, it is best to do the calculation in steps by completing the following table.

Step 1: Calculate the mean (μ).

Step 2: Find the difference between each score and the mean (the deviations, $x - \mu$).

Step 3: Square each of the differences, $\left((x-\mu)^2\right)$.

Step 4: Add each of the squared differences together, $\left(\sum(x-\mu)^2\right)$.

Step 5: Divide the sum by the number of data values, $\left(\frac{\sum(x-\mu)^2}{n}\right)$.

Step 6: Take the square root of the result, $\left(\sqrt{\frac{\sum(x-\mu)^2}{n}}\right)$.

Portage Avenue:

The mean is 31

Data Value, x	Deviation, $x - \mu$	Square of the Deviation $(x-\mu)^2$
15	-16	$(-16)^2 = 256$
26	-5	$(-5)^2 = 25$
30	-1	$(-1)^2 = 1$
39	8	$(8)^2 = 64$
45	14	$(14)^2 = 196$
Sum of the Squares: $\sum(x-\mu)^2$		542

Standard Deviation: $\sqrt{\frac{\sum(x-\mu)^2}{n}} = \sqrt{\frac{542}{5}} = 10.41$

Perimeter Avenue:

The mean is 31

Data Value, x	Deviation, $x - \mu$	Square of the Deviation $(x - \mu)^2$
29	-2	$(-2)^2 = 4$
30	-1	$(-1)^2 = 1$
31	0	$(0)^2 = 0$
32	1	$(1)^2 = 1$
33	2	$(2)^2 = 4$
Sum of the Squares: $\sum(x - \mu)^2$		10

Standard Deviation: $\sqrt{\dfrac{\sum(x-\mu)^2}{n}} = \sqrt{\dfrac{10}{5}} = 1.41$

The standard deviation of the Portage Avenue data set was 10.41 while the standard deviation of the Perimeter Highway data set was 1.41. This indicates that when Jolie takes the Perimeter Highway route, her time varies much less than when she takes the Portage Avenue route.

Example 2

The following are the mathematics test scores of a group of Grade 12 students from Manitoba Rural Collegiate.

32 48 51 54 64 64 78 80 87 92

a) Calculate the mean of this set of test scores.

b) Calculate the standard deviation of this set of test scores.

Solution

a) The formula to calculate the mean is $\mu = \dfrac{\sum x}{n}$.

$$\mu = \frac{32 + 48 + 51 + 54 + 64 + 64 + 78 + 80 + 87 + 92}{10}$$

$$\mu = \frac{650}{10}$$

$$\mu = 65$$

b) To calculate the standard deviation, complete the following table.

Data Value, x	Deviation, $x - \mu$	Square of the Deviation $(x - \mu)^2$
32	$32 - 65 = -33$	$(-33)^2 = 1089$
48	$48 - 65 = -17$	$(-17)^2 = 289$
51	$51 - 65 = -14$	$(-14)^2 = 196$
54	$54 - 65 = -11$	$(-11)^2 = 121$
64	$64 - 65 = -1$	$(-1)^2 = 1$
64	$64 - 65 = -1$	$(-1)^2 = 1$
78	$78 - 65 = 13$	$(13)^2 = 169$
80	$80 - 65 = 15$	$(15)^2 = 225$
87	$87 - 65 = 22$	$(22)^2 = 484$
92	$92 - 65 = 27$	$(27)^2 = 729$
Sum of the Squares: $\sum(x - \mu)^2$		3304

Now, complete the standard deviation calculation by dividing the sum by the number of data values, 10, and taking the square root of the value.

Standard Deviation: $\sqrt{\frac{\sum(x - \mu)^2}{n}} = \sqrt{\frac{3304}{10}} = 18.18$

Example 3

The Grade 12 students at Manitoba Urban Collegiate also wrote the same mathematics test. Their results are as follows.

34 52 60 60 65 65 72 72 76 94

a) Calculate the mean of this set of test scores.

b) Calculate the standard deviation of this set of test scores.

c) Compare the scores of students who attend Manitoba Rural Collegiate with the scores of the students who attend Manitoba Urban Collegiate.

Solution

a) Mean

$$\mu = \frac{34 + 52 + 60 + 60 + 65 + 65 + 72 + 72 + 76 + 94}{10}$$

$$\mu = \frac{650}{10}$$

$$\mu = 65$$

b) Standard Deviation

Data Value, x	Deviation, $x - \mu$	Square of the Deviation $(x - \mu)^2$
34	−31	961
52	−13	169
60	−5	25
60	−5	25
65	0	0
65	0	0
72	7	49
72	7	49
76	11	121
94	29	841
Sum of the Squares: $\sum(x - \mu)^2$		2240

Standard Deviation: $\sqrt{\dfrac{\sum(x-\mu)^2}{n}} = \sqrt{\dfrac{2240}{10}} = 14.97$

c) The standard deviation of the test scores of the two schools is different, even though the mean of both data sets is the same. The standard deviation of the test scores from Manitoba Rural Collegiate is greater than that from the Manitoba Urban Collegiate. Standard deviation is a measure of dispersion. If you examine the test scores from the two high schools, you will find that there is more variation in the scores from Manitoba Rural Collegiate than Manitoba Urban Collegiate.

You can also see this difference in dispersion by drawing histograms. Consider the following example.

Example 4

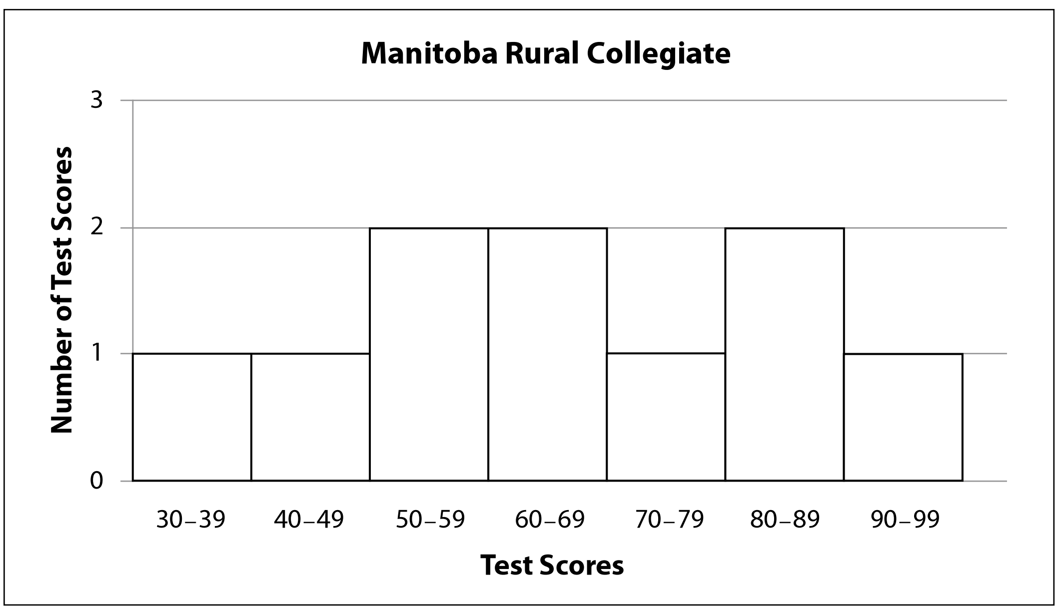
a) Draw a histogram with seven intervals of the test scores from Manitoba Rural Collegiate.

b) Draw a histogram with seven intervals of the test scores from Manitoba Urban Collegiate.

Solution

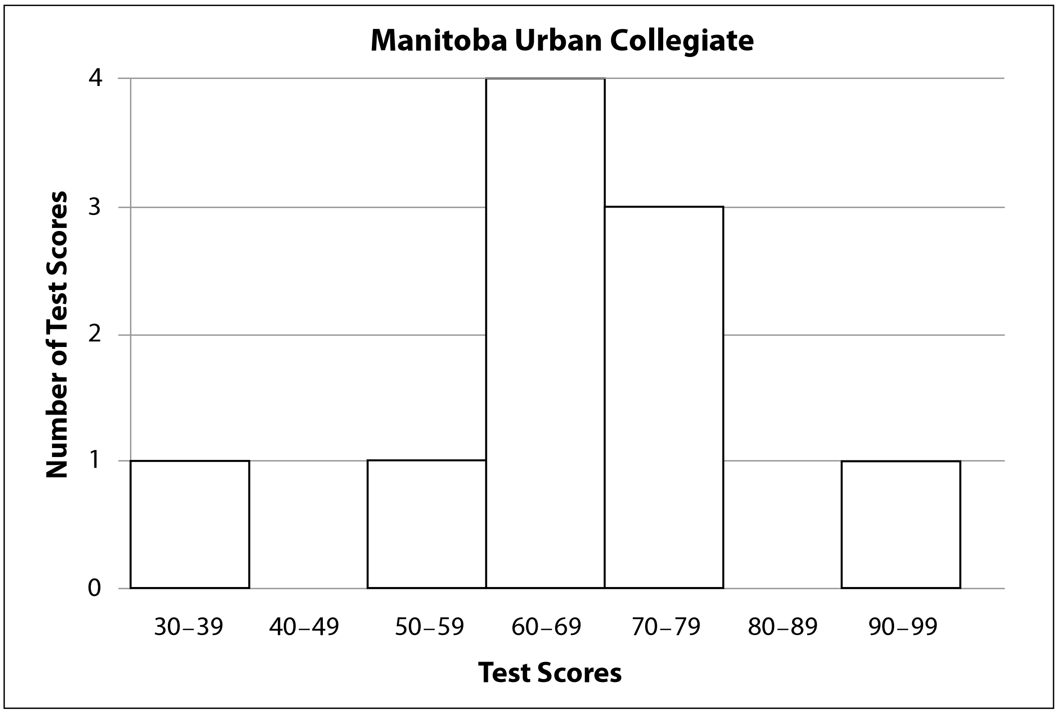
a) The range in the Manitoba Rural Collegiate data is 92 − 32 = 60. Therefore, each interval should contain at least 10 mark values. To make a better comparison between the two histograms, you will use the same intervals for parts (a) and (b)

Marks	Tally	Frequency (# of Scores)
30 – 39	|	1
40 – 49	|	1
50 – 59	||	2
60 – 69	||	2
70 – 79	|	1
80 – 89	||	2
90 – 99	|	1



b) The range in the Manitoba Urban Collegiate data is 94 − 34 = 60. Use the same intervals as part (a).

Marks	Tally	Frequency (# of Scores)
30 – 39	|	1
40 – 49		0
50 – 59	|	1
60 – 69	||||	4
70 – 79	|||	3
80 – 89		0
90 – 99	|	1



You can see from the histograms that the test scores of the students from Manitoba Urban Collegiate are closer to the mean than those from Manitoba Rural Collegiate. The calculated standard deviations indicate this as well. Manitoba Rural Collegiate has a larger standard deviation than Manitoba Urban Collegiate. This means that the test scores for Manitoba Rural Collegiate are more widely dispersed and, on average, are further from the mean than the test scores of Manitoba Urban Collegiate.

Now that you have seen a few examples of calculating and analyzing standard deviation from raw data, consider finding the standard deviation of a data set when you are given a frequency table.

Note: You can use technology to calculate the standard deviation of a data set. Search the Internet for a tutorial or instructions about using your chosen technology to perform these calculations. If you choose to use technology to calculate standard deviation, make sure you get the same answer as displayed in the example below. Also, make sure you show all of the data you typed into the technology you used and what buttons or commands you used to determine the solution.

Example 5

The following table displays the number of calories consumed by the students in a Grade 11 physical education class in a typical day.

Calories Consumed	Number of Students
1000–1300	2
1300–1600	3
1600–1900	5
1900–2200	10
2200–2500	9
2500–2800	1

a) Determine the standard deviation of the data set.

b) A Grade 11 physical education class from a different school collects statistics about their calorie consumption. They determine the mean of their data to be 1990 with a standard deviation of 200. How do the two sets of data compare?

Solution

a) To determine the standard deviation for grouped data, you can use a similar process to what you already know. For grouped data, assume that every data value in the group is the same as the median value for the interval. That is, if the interval is 2800–3000, you can assume that every data value in the interval is 2900.

First, you need to determine the median of each interval. The median of each interval is displayed in the table below.

Calories Consumed	Median of Interval	Number of Students
1000–1300	1150	2
1300–1600	1450	3
1600–1900	1750	5
1900–2200	2050	10

Calories Consumed	Median of Interval	Number of Students
2200–2500	2350	9
2500–2800	2650	1

The median of each interval is a good estimate of each data value in each interval. However, this is an approximation for the actual data.

The data could have been expressed as the following set of numbers:

1150, 1150, 1450, 1450, 1450, 1750, 1750, 1750, 1750, 1750, 2050, 2050, 2050, 2050, 2050, 2050, 2050, 2050, 2050, 2050, 2350, 2350, 2350, 2350, 2350, 2350, 2350, 2350, 2350, 2650

The process for finding the mean and standard deviation for the grouped data is similar to how you find the mean and standard deviation for the data listed as individual values. However, instead of adding up each individual value to find the sum of all data values, you can make the process more efficient since the data comes in groups.

First, find the sum of the data values in each interval by multiplying the frequency (the number of students) by the median of each interval. Then, find the sum of all of these values. This is done in the fourth column below.

The number of data values can be found by adding up the frequencies or the number of students. This is done in the third column below.

Calories Consumed	Median of Interval, x	Number of Students, f	$f \cdot x$
1000–1300	1150	2	(2)(1150) = 2300
1300–1600	1450	3	(3)(1450) = 4350
1600–1900	1750	5	(5)(1750) = 8750
1900–2200	2050	10	(10)(2050) = 20500
2200–2500	2350	9	(9)(2350) = 21150
2500–2800	2650	1	(1)(2650) = 2650
	Total:	30	59700

The mean can then be found by dividing the sum of all the values (the total of the $f \cdot x$ column) by the number of students (the total of the f column).

$$\mu = \frac{59700}{30} = 1990$$

The deviation of each data value from the mean can now be found. You can then square this value. Multiply the result by its frequency since that represents how many times the particular deviation calculation occurs.

Median of Interval, x	Number of Students, f	$f \cdot x$	$(x - \mu)^2$	$f \cdot (x - \mu)^2$
1150	2	(2)(1150) = 2300	$(1150 - 1990)^2 =$ 705600	1411200
1450	3	(3)(1450) = 4350	$(1450 - 1990)^2 =$ 291600	874800
1750	5	(5)(1750) = 8750	$(1750 - 1990)^2 =$ 57600	288000
2050	10	(10)(2050) = 20500	$(2050 - 1990)^2 =$ 3600	36000
2350	9	(9)(2350) = 21150	$(2350 - 1990)^2 =$ 129600	1166400
2650	1	(1)(2650) = 2650	$(2650 - 1990)^2 =$ 435600	435600
			Total:	4212000

You can now find the standard deviation of this data set.

Standard Deviation: $\sqrt{\dfrac{\sum f(x-\mu)^2}{n}} = \sqrt{\dfrac{4212000}{30}} = 374.7$

b) The mean values of these two data sets are identical. However, the standard deviation in each data set is different. In the original Grade 11 physical education class, the standard deviation was approximately 375, while the standard deviation in the second class was 200. This indicates that the original Grade 11 physical education class contained more data values that were further away from the mean. A possible explanation is that more students consumed fewer calories and more students consumed more calories.

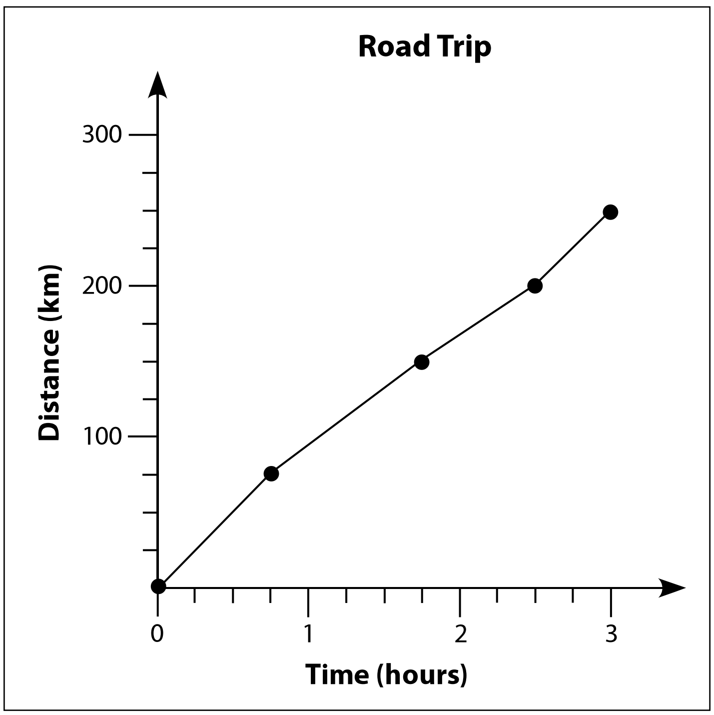
Learning Activity 6.2

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

Consider the following graph of the distance a vehicle travels during a three-hour road trip to answer questions 1 and 2.

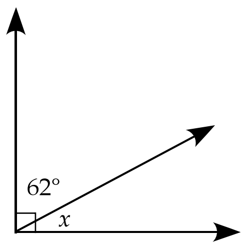


1. How far did the vehicle travel in 2.5 hours?

2. How long did it take the vehicle to travel 75 km?

continued

3. Determine the size of angle x in the diagram below.



4. How many square feet of linoleum are required to cover a kitchen with dimensions 12 ft. by 14 ft.?

5. Solve for x: $(x + 7)(x - 9) = 0$

6. Determine the y-intercept: $y = 2x^2 - 3x + 8$

7. Solve for x: $3x - 12 = 2$

8. Solve for x: $\dfrac{4}{3} = \dfrac{x}{8}$

Part B: Standard Deviation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. The summary statistics of three data sets are listed below.

	Data Set #1	Data Set #2	Data Set #3
Range	20	20	20
Mean	75	75	95
Standard Deviation	5.7	8.9	8.9

a) Compare Data Set #1 with Data Set #2. What do you notice? What can you say about the two data sets?

b) Compare Data Set #2 with Data Set #3. What do you notice? What can you say about the two data sets?

continued

Learning Activity 6.2 (continued)

2. A cell phone company collects data from 10 customers indicating how many text messages they send in a month. The data is displayed below.

   12 138 284 375 513 548 671 743 941 1738

   a) Calculate the mean of this set of data.
   b) Calculate the standard deviation of this set of data.

3. The weights in kilograms of the players on a wrestling team are as follows.

   70 72 75 77 79 79 81 82 82 85 92 98

   a) Determine the mean weight.
   b) Calculate the standard deviation of this set of data.

4. The mean salary of employees in a certain company is $43,000, with a standard deviation of $3500. The company decides to increase the salary of each employee by $1000 during the following year. Find the mean and the standard deviation of the new salaries.

5. Jabar conducted a survey about the social networking habits of his classmates. The data he collected is displayed in the table below.

	Male/Female	**Hours Spent on Twitter Daily**	**Hours Spent on Facebook Daily**
	Male	0.5	0.75
	Female	0	0.25
	Male	0.25	3
	Male	1.5	3.5
	Female	2	1.5
	Male	0	0.5
	Female	0.75	2.75
	Male	1	0
	Male	1	1
	Female	3	1.25
Total Female:	4	5.75	5.75
Total Male:	6	4.25	8.75
Total:	10	10	14.5

continued

a) Determine the standard deviation of the time these students spend on Twitter daily.

b) Determine the standard deviation of the time these students spend on Facebook daily.

c) Compare the standard deviations of the two data sets. What do you notice?

d) Determine the standard deviation of the time the males spend on Twitter daily.

e) Determine the standard deviation of the time the females spend on Twitter daily.

f) Compare the standard deviations of the time males spend on Twitter to the time females spend on Twitter. What do you notice?

Lesson Summary

To determine the dispersion of a data set, or how close together or far apart the data values are, you can determine the mean of the data and find the difference from each data value to the mean, called the deviation from the mean. You can quantify the deviation by finding the average deviation but it is much better to calculate the standard deviation to avoid the effect of negative and positive deviations cancelling each other out. If your data points are close to the mean, your standard deviation will be small. On the other hand, if your data points are farther away from the mean, your standard deviation will be large. The shape of a histogram can give you a good indication of the mean and the deviation from the mean.

For some data sets, more values are found near the mean and fewer values are found as you move away from the mean. A histogram of this data might look like a one-humped camel. Alternatively, the data points may be much higher or much lower than the mean and fewer data points may be found near the mean. A histogram of this data might look like a two-humped camel. Many other histogram shapes are possible when frequency data is plotted. In the next lesson, you will learn about the histogram shape referred to as a “normal distribution.”

Assignment 6.1

Measures of Central Tendency and Dispersion

Total: 18 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. If you use technology, state your keystrokes or values used and your process. Final answers must include units. Answers given without supporting calculations and graphs will not be awarded full marks.

1. The data below displays the number of caffeinated beverages consumed per day for a group of high school students.

0	0	0	1	1	1	2	2	2	2
2	3	3	3	3	4	4	4	5	8

   a) Determine the mean, median, mode, and range of the data. *(4 marks)*

Assignment 6.1: Measures of Central Tendency and Dispersion (continued)

b) Create a frequency table with 5 intervals. *(2 marks)*

c) Create a histogram to display the data from your frequency table. *(2 marks)*

d) What does the histogram tell you about the number of caffeinated beverages consumed by students? *(1 mark)*

Assignment 6.1: Measures of Central Tendency and Dispersion (continued)

2. The following data represent a sample of the pizza delivery times (in minutes) from Pizza Express.

13	19	22	25	27	31	15	20	23	24

a) Calculate the mean of this set of data. *(1 mark)*

b) Calculate the standard deviation of this set of data. *(4 marks)*

Assignment 6.1: Measures of Central Tendency and Dispersion (continued)

c) Another pizza delivery company, Prompt Pizza, has a mean delivery time of 18 minutes and a standard deviation of 10 minutes. Ling lives right between the two pizza companies and is deciding which company to order pizza from based on their delivery times. Ling knows the pizza is equally delicious from each pizza company. Which company is likely to deliver the pizza faster? Explain. *(2 marks)*

3. Mr. Janzen gives a unit test to his Grade 11 Chemistry class. He determines that the mean mark on the test is 62%, with a standard deviation of eight marks. When he returns the tests to his students, a student realizes that one of the questions on the test is incorrect and impossible to solve. As a result, Mr. Janzen decides to add 5 percentage marks to each student's mark. Find the mean and standard deviation of the new set of marks. *(2 marks)*

Lesson 3: The Normal Distribution

Lesson Focus

In this lesson, you will

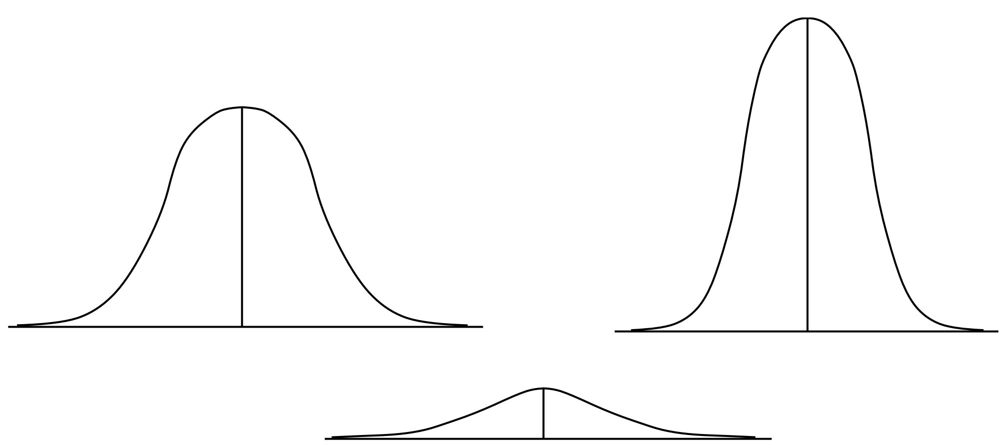
- ❑ learn about the theoretical "normal distribution"
- ❑ learn about the properties of the normal distribution
- ❑ determine how closely a distribution approximates the normal distribution

Lesson Introduction

In Lesson 1, you reviewed how to calculate measures of central tendency. In Lesson 2, you learned how to calculate a measure of dispersion called standard deviation. In this lesson, you will extend your knowledge of these topics by examining their applications with a theoretical distribution called the **normal distribution**. The normal distribution is said to be theoretical because real-life distributions of raw data will not fit the normal distribution perfectly. However, many sets of data create histogram shapes that approximate the normal distribution.

What Is the Normal Distribution?

In the normal distribution, the data or scores are distributed evenly around the mean. If you were to graph the data, it would create a 'bell-shaped' curve such as the ones shown below.



The normal distribution is symmetrical about the mean. The mean occurs at the horizontal centre of the graph, which is indicated by the vertical line in each of the above curves. *Symmetrical about the mean* simply means that half the data is greater than the mean (and to the right on the bell curve) and half the data is less than the mean (and to the left on the bell curve). This also means that the median value is the same as the mean. In a normal distribution, the mean is also the data value that occurs the most frequently. In other words, the mode is also the same as the mean.

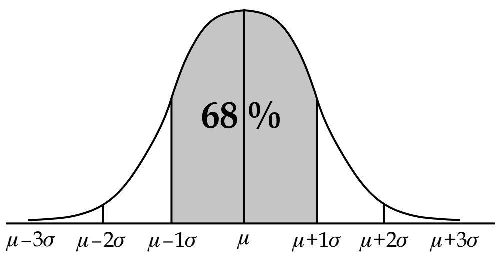
To summarize, in a normal distribution, the mean, median, and mode are all the same value. Similarly, a histogram in the shape of this symmetrical bell curve will have the same value for the mean, median, and the mode.

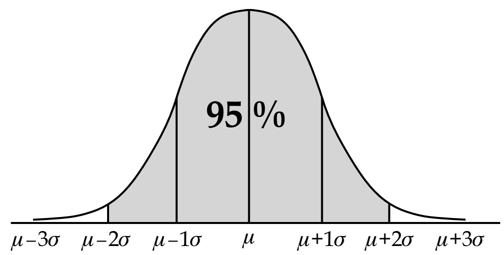
The properties of the normal distribution can be summarized in three categories.

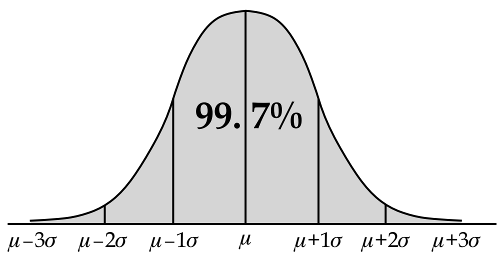
1. **Shape of the Curve**
   - The curve has one "hump."
   - The curve is perfectly symmetrical about the mean. If you were to fold the curve on a line through the mean, the one side of the graph would fall directly on the other side.
   - The mean and standard deviation determine whether the curve is tall and skinny, short and wide, or any variation between.
     - If the standard deviation, σ, is large, the curve will be wider horizontally and shorter vertically.
     - If the standard deviation, σ, is small, the curve will be narrower horizontally and taller vertically.
2. **Mean, Median, and Mode**
   - The mean, the median, and the mode are all the same value
3. **Data Distribution**
   - Almost all data is within three standard deviations of the mean. The probability that a score falls within three standard deviations of the mean is approximately 99.7% or 0.997. Therefore very few of the values are more than 3 standard deviations away from the mean.
   - The probability that a score falls within two standard deviations of the mean is approximately 95% or 0.95. Therefore only 5% of the data values are more than 2 standard deviations away from the mean. Since the curve is symmetrical, there would be 2.5% greater than 2 standard deviations and 2.5% less than −2 standard deviations away from the mean.

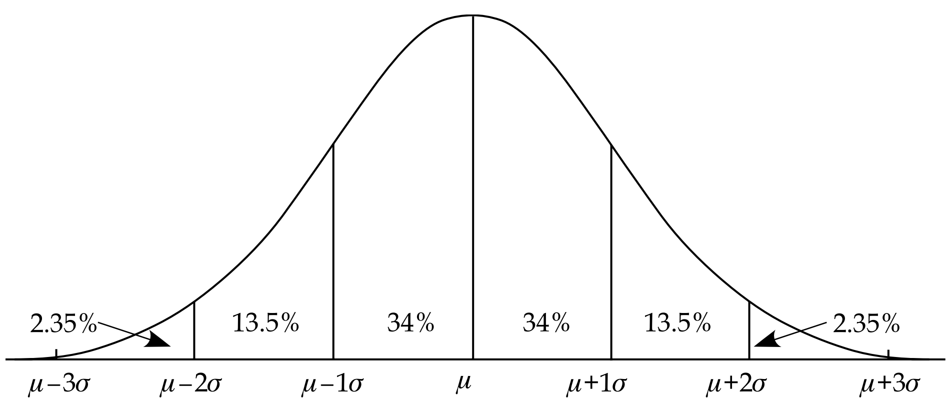
- The probability that a score falls within one standard deviation of the mean on either side is approximately 68% or 0.68. Therefore, the other 32% of the data values are more than 1 standard deviation away from the mean. Since the curve is symmetrical, there would be 16% greater than 1 standard deviation and 16% lower than −1 standard deviation from the mean.
- The total area under the curve is always 1, or 100%. This simply indicates that all of the data is represented by points under the curve.

This can be summarized by the following diagrams:





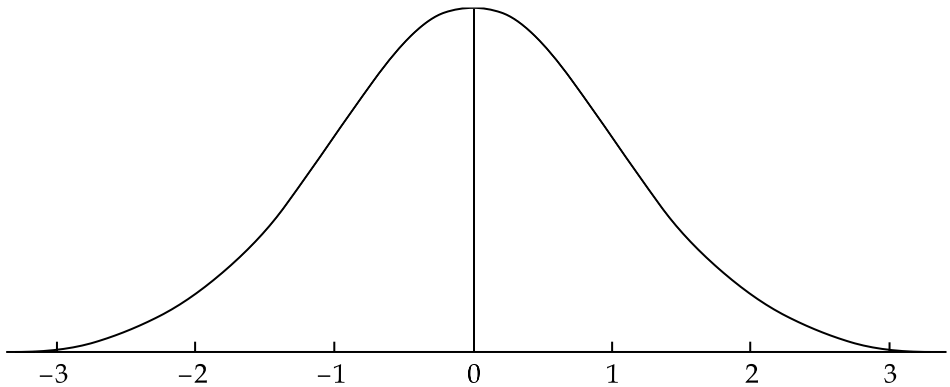




Note

Note: In the above diagrams, μ represents the mean and is the centre of the data. The standard deviation is represented by σ. Therefore, $\mu + 1\sigma$ indicates the point at which data is 1 standard deviation above the mean, and $\mu - 1\sigma$ indicates the point at which data is 1 standard deviation below the mean. Similarly, $\mu + 2\sigma$ indicates the point at which data is 2 standard deviations above the mean.

You may also sometimes encounter the **standard normal distribution**, which is a normal distribution with a mean of 0 and a standard deviation of 1. The diagram of a standard normal distribution is shown below.



When a frequency histogram resembles the bell-shaped curve above, the graph is called a normal curve and its frequency distribution is known as a normal distribution. Many naturally occurring sets of data follow a normal distribution. A few examples of distributions whose histograms approach a normal curve when their samples are taken from large populations are the following:

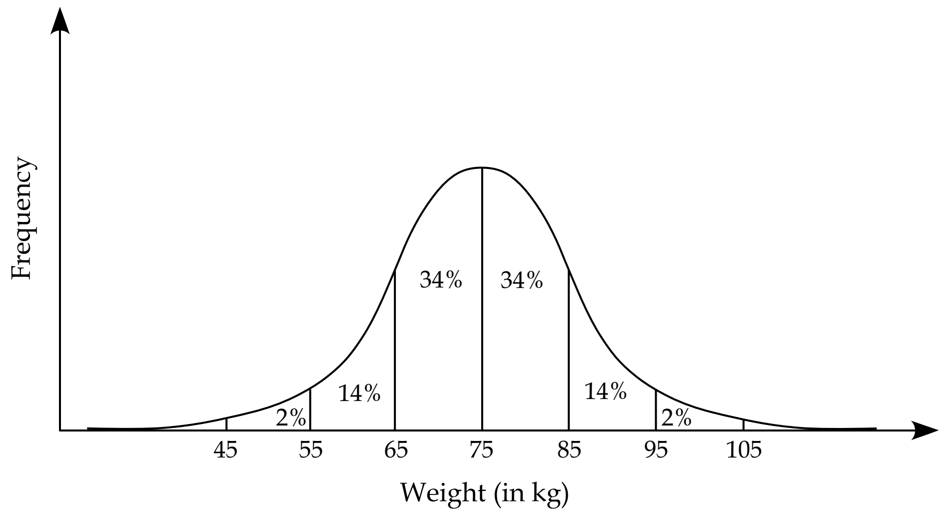
- heights of individuals
- weights of individuals
- IQ scores of individuals
- amount of annual rainfall in a region
- life expectancy of automobiles
- actual mass of material in commercial packaging

One specific example of where the normal distribution is used is when retail stores bring in stock. Imagine that you walked into a men's shoe store and asked to see the entire stock of a certain pair of shoes (assume the store just received the order that morning). If the average size for a man's shoe is 9, there will be more stock of shoes that are size 9 than any other size. For shoe sizes 8 and 10, there will be slightly less stock, but still plenty of shoes. For shoe sizes 6 and 12, there will be significantly less stock. This is because retail stores know that most consumers will wear shoe sizes within a certain range. Therefore, they may decide to only order and stock shoes in the most popular sizes. If a man required a shoe size that was very far from the mean, such as size 15 or size 3, most stores would stock very few or have no shoes in those sizes. It may be difficult, therefore, to find shoes in sizes that are several standard deviations above or below the mean.

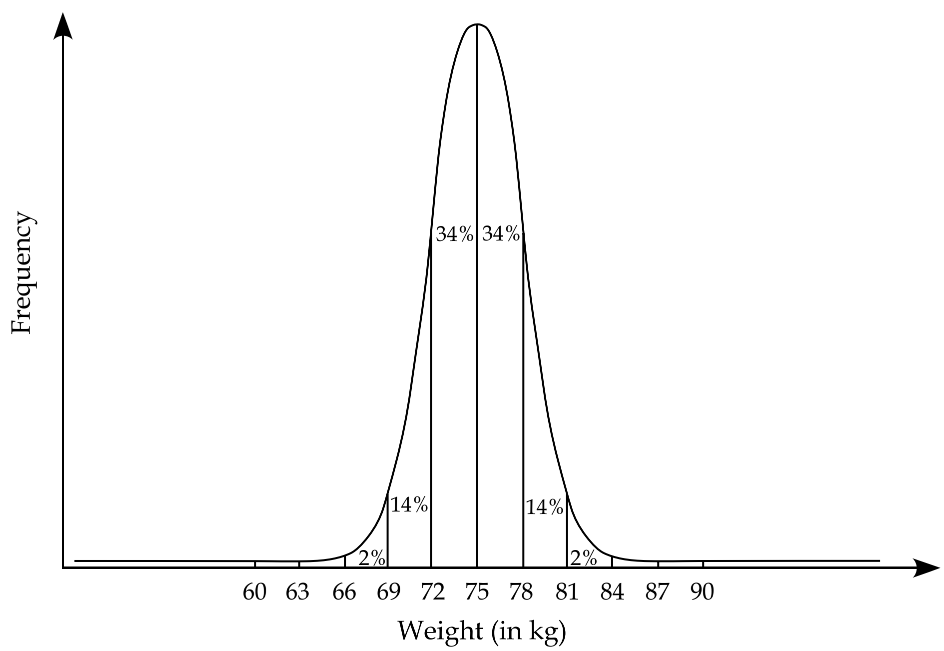
Example 1

a) Curve A and B describe normal distributions. Find the mean and standard deviation of each curve.

b) Describe the similarities and differences between the two curves.



Normal Curve A



Normal Curve B

Solution

a) The mean of curve A is 75 kg.

The standard deviation of curve A is 10 kg. This is because the areas describing one standard deviation (34%) have horizontal sizes of 10 kg.

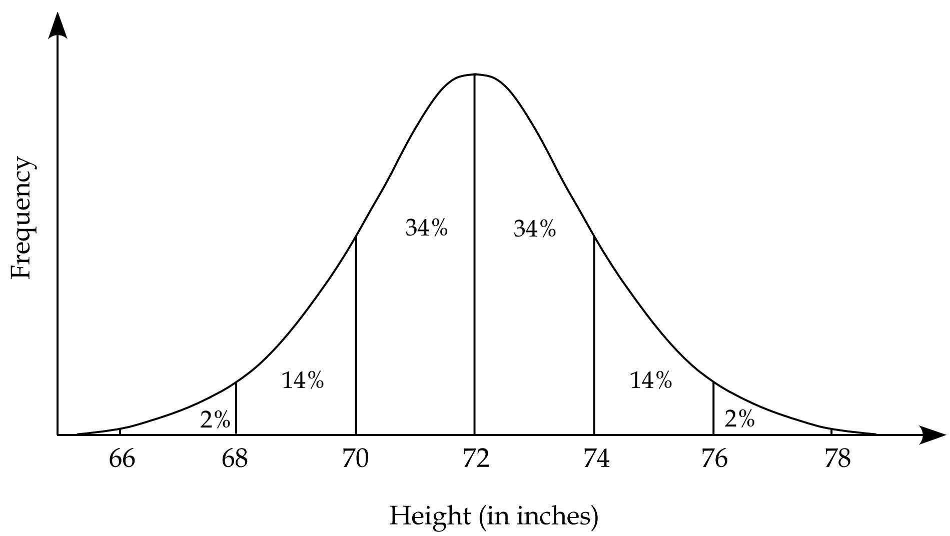
The mean of curve B is also 75 kg.

The standard deviation of curve B is 3 kg. This is because the areas describing one standard deviation (34%) have horizontal sizes of 3 kg.

b) The two curves are similar in that both follow a normal distribution and both have a mean of 75 kg. The two curves are different in that the standard deviation of the first is 10 kg, while the standard deviation of the second is 3 kg. Because the standard deviation of the first curve is greater, the curve is wider and more spread out.

Example 2

There are 80 players in a basketball league. The height of players in the league follows a normal distribution.



a) Find the percent of players that are between 68 in. and 74 in. tall

b) Find the percent of players that are over 74 in. tall.

c) Find the number of players that are over 74 in. tall.

Solution

a) The percent of players that are between 68 in. and 74 in. tall

$= 14\% + 34\% + 34\%$

$= 82\%$

b) The percent of players that are over 74 in. tall

$= 14\% + 2\%$

$= 16\%$

c) There are a number of methods you can use to solve this question. Two of the methods are shown below.

Method 1: Ratio and Proportion

Let x represent the number of players.

$$\frac{16}{100} = \frac{x}{80}$$

$$(16)(80) = (100)(x) \qquad \text{Cross-multiply.}$$

$$1280 = 100x$$

$$\frac{1280}{100} = \frac{100x}{100} \qquad \text{Divide both sides by 100.}$$

$$12.8 = x$$

There are 13 players (rounded to the nearest whole number) who are over 74 in. tall.

Method 2: Percents

To calculate 16% of 80, you multiply $16\% \times 80$.

Number of players $= 16\% \times 80$

$= 0.16 \times 8$

$= 13$ (rounded to the nearest whole number)

Determining Whether Data Follows a Normal Distribution

The characteristics of a normal distribution, along with your knowledge of histograms, can be used to see if a distribution is similar to a normal distribution.

If you want to see how well the data fits a normal distribution, here are some things you can do to test the data.

- Draw a histogram to represent the data. If the overall shape of the histogram resembles a normal curve, you may have a normal distribution.
- Calculate the mean.
  - If half of your data is above the mean and the other half of your data is below the mean, you may have a normal distribution. This would indicate that the median is equal to the mean. If the mode also equals the mean, then the data can likely be modelled with a normal distribution.
- Calculate the standard deviation.
  - If the three following statements are true, use a normal distribution to model the data.
    - 68% of the data is within one standard deviation of the mean.
    - 95% of the data is within two standard deviations of the mean.
    - 99.7% of the data is within three standard deviations of the mean.

Example 3

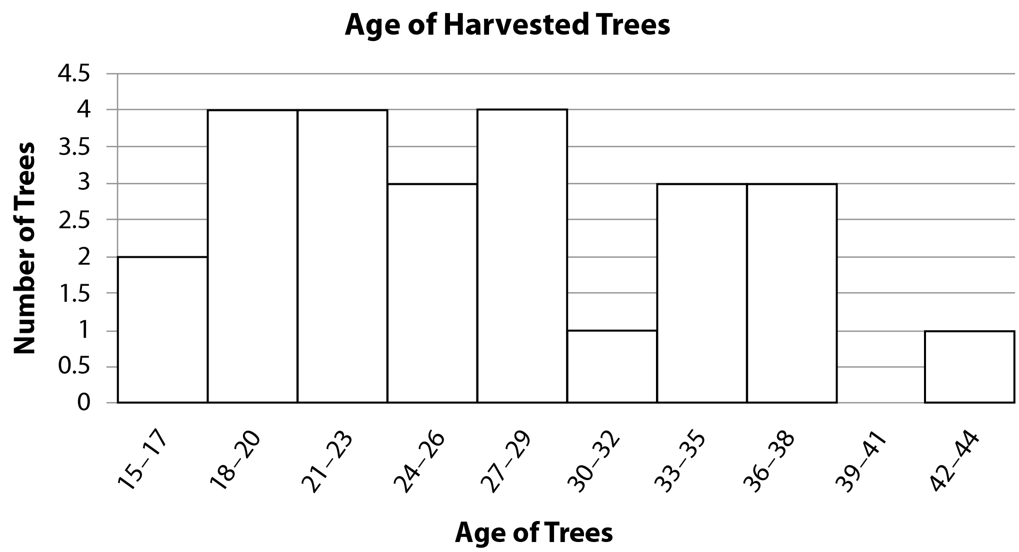
The age in years of the first 25 trees harvested in a new forestry project are shown in the table below.

15	17	18	19	19
20	21	21	22	23
24	25	26	27	28
28	29	32	34	35
35	36	37	38	42

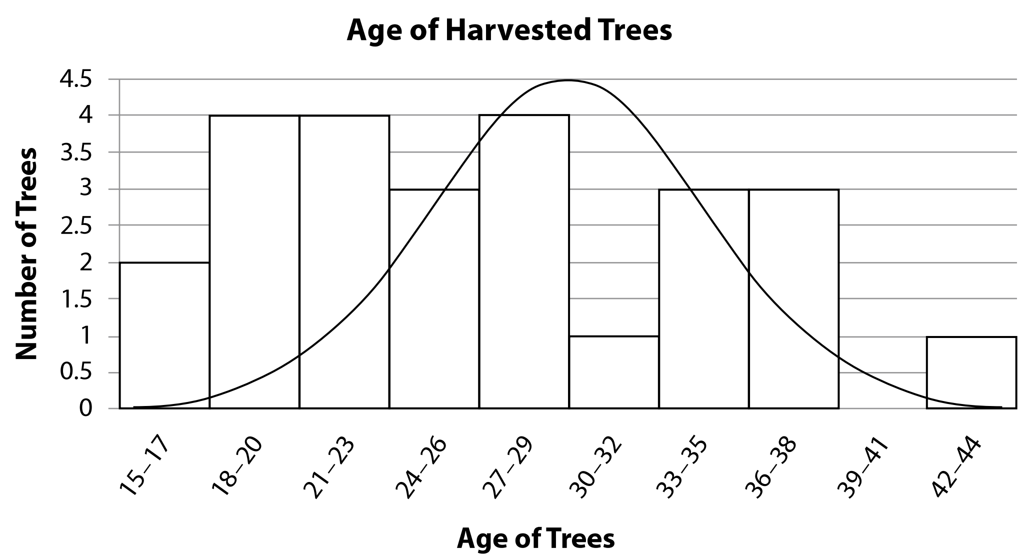
Determine whether this distribution approximates the normal distribution.

Solution

The data graphed in a histogram with intervals of 15 – 17, 18 – 20, 21 – 23, 24 – 26, 27 – 29, 30 – 32, 33 – 35, 36 – 38, 39 – 41, and 42 – 44 is shown below.



You can overlay a normal curve to see if it approximates the shape of the graph.



The shape of the graph does not fit the bell shape of the normal curve. If you want to check further, you could find the mean and the standard deviation to help determine if the data could be modelled by a normal distribution.

Mean = 26.8

Standard deviation = 7.4

Note: The math behind these calculations is not shown since doing those calculations is not the main point of this example. However, you could determine those values yourself for practice.

If you add and subtract one standard deviation to the mean, the results will be 19.4 and 34.2. With rounding, this turns into 20 and 34.

If you count the number of trees that were harvested between the ages of 20 and 34 inclusive, you will find that there are 14 of them or 56%. This means that 56% of the data falls within one standard deviation on each side of the mean. In a normal distribution, this value should be closer to 68%. This confirms that the data is not well represented by a normal distribution.

Example 4

Rose is trying to increase her 100 m race time. She records her last 36 race times below, in seconds.

10.44	10.43	10.06	10.33
10.39	10.15	10.14	10.37
10.14	10.43	10.26	10.58
10.43	10.22	10.59	10.29
10.55	10.52	10.38	10.45
10.36	9.99	10.18	10.25
10.33	10.35	10.57	10.35
10.28	10.20	10.29	10.41
10.50	10.18	10.41	10.27

Determine if Rose's race times resemble a normal distribution by analyzing the percentage of data values that fall within 1, 2, and 3 standard deviations of the mean.

Solution

First, you will need to find the mean and standard deviation. This has been done for you to save you time (feel free to do the work if you need the practice).

The mean is 10.33 seconds.

The standard deviation is 0.15 seconds.

It will also be beneficial if all of your data values are in numerical order.

9.99	10.06	10.14	10.14
10.15	10.18	10.18	10.20
10.22	10.25	10.26	10.27
10.28	10.29	10.29	10.33
10.33	10.35	10.35	10.36
10.37	10.38	10.39	10.41
10.41	10.43	10.43	10.43
10.44	10.45	10.50	10.52
10.55	10.57	10.58	10.59

There are 15 data values below the mean, 2 values identical to the mean, and 19 values above the mean. The data is not perfectly symmetrical but it is close enough to be considered to be modelled by a normal distribution.

To calculate the range for one standard deviation from the mean, subtract one standard deviation from the mean to find the lower boundary. To find the upper boundary, add one standard deviation to the mean.

Lower Boundary:

$$10.33 - 0.15 = 10.18$$

Upper Boundary:

$$10.33 + 0.15 = 10.48$$

The range for one standard deviation from the mean is (10.18, 10.48). There are 25 data values in this range.

This means that 25 of 36 or 69% of the data is within one standard deviation of the mean. A normal distribution would have 68% of the data in this range, so this data is very close to the desired value.

To calculate the range for two standard deviations from the mean, subtract two standard deviations from the mean to find the lower boundary. To find the upper boundary, add two standard deviations to the mean.

Lower Boundary:

$$10.33 - 2(0.15) = 10.03$$

Upper Boundary:

$$10.33 + 2(0.15) = 10.63$$

The range for two standard deviations from the mean would be (10.03, 10.63). There are 35 data values in this range.

This means that 35 of 36 or 97% of data is within two standard deviations from the mean. A normal distribution would have 95% of the data in this range, so this data is very close to the desired value.

To calculate the range for three standard deviations from the mean, subtract three standard deviations from the mean to find the lower boundary. To find the upper boundary, add three standard deviations to the mean.

Lower Boundary:

$10.33 - 3(0.15) = 9.88$

Upper Boundary:

$10.33 + 3(0.15) = 10.78$

The range for three standard deviations from the mean would be (9.88, 10.78). All 36 data values are in this range.

This means that 36 of 36 or 100% of the data are within three standard deviations of the mean. A normal distribution would have 99.7% of the data in this range, so this data is very close to the desired value.

Because the data closely follows the properties of a normal curve, this would signify that the data could be approximated by a normal distribution.

The 68–95–99.7 Rule

If you know that a set of data can be modelled by a normal distribution, then it is possible to calculate the probability of certain events related to your data. This is because the bell curve is also a probability distribution. The probability of obtaining data between two given values is equal to the area under the curve that is between the vertical lines corresponding to the two given values. Consider the following example.

Example 5

Mr. Rampersad is creating a Grade 11 Physics test. From previous years of teaching experience, he knows that the time students take to write the test follows a normal distribution. The mean test-writing time for the physics test is 45 minutes with a standard deviation of 7 minutes.

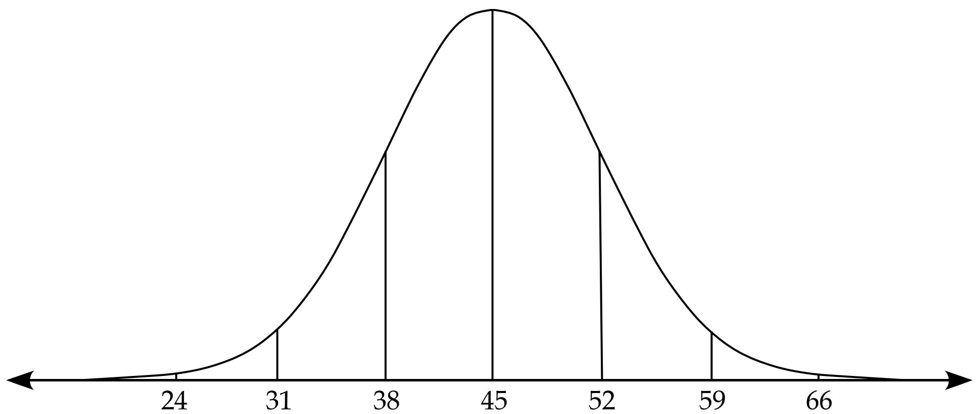
a) What is the probability that any given student will finish the test in 45 minutes or less?

b) What is the probability that any given student will finish the test in 31 minutes or less?

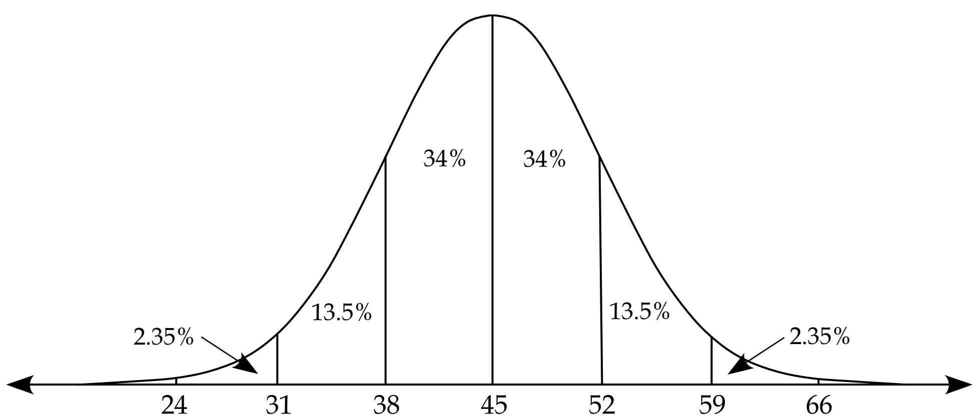
c) What is the probability that any given student will finish the test somewhere between 38 and 59 minutes?

Solution

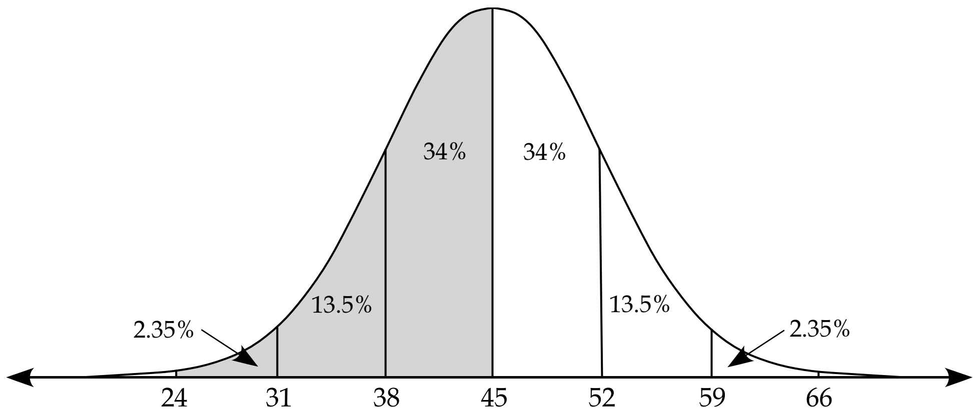
First, draw the normal curve using the mean and standard deviation.
The mean of 45 is located in the centre of the normal curve. The numbers immediately to the left and right of the mean are one standard deviation away ($45 - 7 = 38$ and $45 + 7 = 52$). The next numbers are one more standard deviation (7 less) left or one more standard deviation (7 more) right, so +2 standard deviations is 59, and −2 standard deviations is 31. Continuing this process, you can determine that +3 standard deviations is 66 and −3 standard deviations is 24.



At this point, it may be helpful for you to include the percentage of values (in this case, the percentage of students) that each section of the normal curve represents.

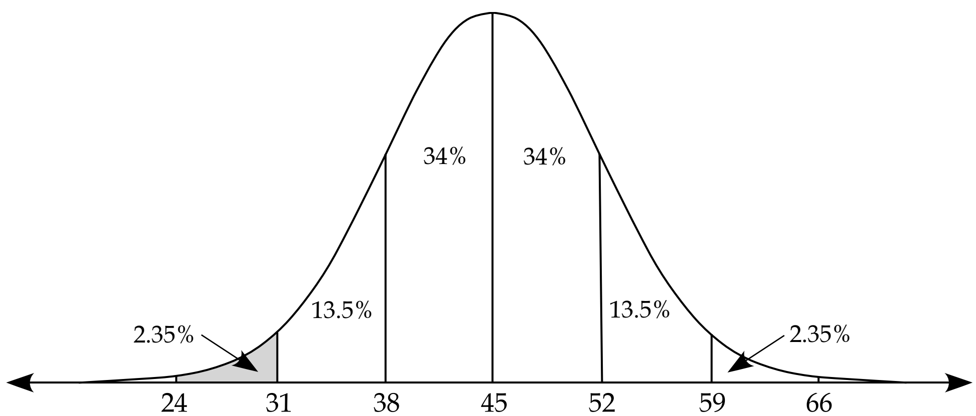


a) To find the probability that any given student will finish the test in 45 minutes or less, shade in the portions of the curve that are to the left of this value.



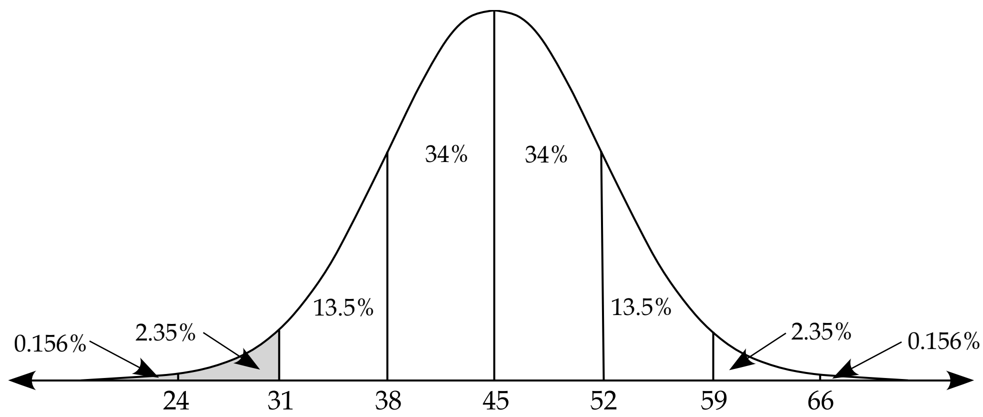
Using the symmetric nature of the normal distribution, there is a 50% chance of any given student finishing the test in 45 minutes or less.

b) To determine the probability of a student finishing the test in 31 minutes or less, shade the portions of the curve that are to the left of this value.



You know that 2.35% of students will finish a test between 24 and 31 minutes. However, how many students will finish the test in less than 24 minutes?

You know that 99.7% of data is within 3 standard deviations of the mean. This means that 0.3% of data is greater than 3 standard deviations away from the mean. In other words, 0.15% of data is more than 3 standard deviations above the mean and 0.15% of data is more than 3 standard deviations below the mean.

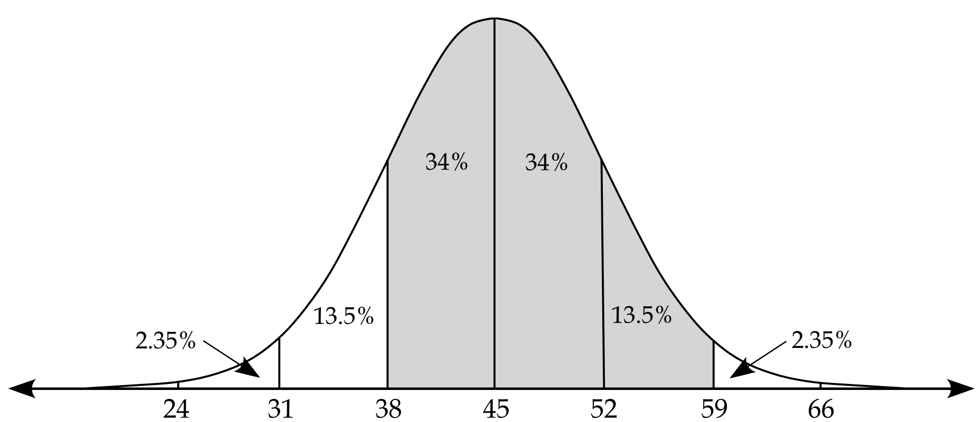


Thus, 0.15% of students will finish the test in less than 24 minutes.

The probability of a student finishing the test in 31 minutes or less is 2.35% + 0.15% = 2.5%.

Alternatively, you could determine the complementary value—that is, the number of students who take 24 or more minutes. Add the probabilities for each region above 31: 13.5% + 34% + 50% = 97.5% That means that 100 − 97.5 = 2.5% take less than 24 minutes.

c) To determine the probability of a student finishing the test between a time of 38 minutes and a time of 59 minutes, shade in the portions of the curve that are between these two values.



To find the probability, simply add the probabilities from each shaded region of the normal curve.

34% + 34% + 13.5% = 81.5%

The probability of a student finishing the test in between 38 and 59 minutes is 81.5%.

Example 6

At the age of 17, the mean height of females is 163 cm, with an approximate standard deviation of 7 cm. The mean height of males is 175 cm, with an approximate standard deviation of 8 cm.

a) Create a normal curve to represent the height of females at age 17.

b) Create a normal curve to represent the height of males at age 17.

c) What is the range of heights for 95% of all 17-year-old females?

d) What is the range of heights for 95% of all 17-year-old males?

e) What do you notice about the range of height for 95% of all 17-year-old females and for all 17-year-old males?

Solution

a) The mean height is 163 cm.

One standard deviation above the mean is $163 + 7 = 170$ cm.

Two standard deviations above the mean is $170 + 7 = 177$ cm.

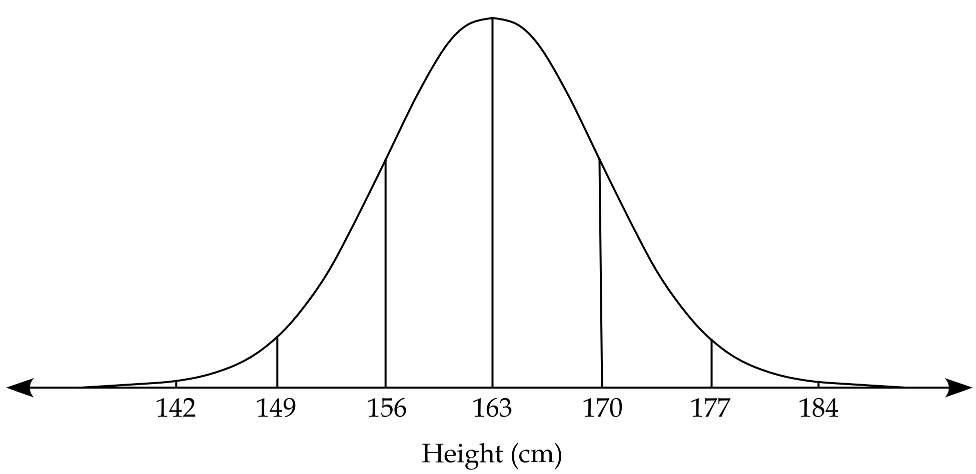
Three standard deviations above the mean is $177 + 7 = 184$ cm.

One standard deviation below the mean is $163 - 7 = 156$ cm.

Two standard deviations below the mean is $156 - 7 = 149$ cm.

Three standard deviations below the mean is $149 - 7 = 142$ cm.

Height of 17-Year-Old Females



b) **Males:**

The mean height is 175 cm.

One standard deviation above the mean is 175 + 8 = 183 cm.

Two standard deviations above the mean is 183 + 8 = 191 cm.

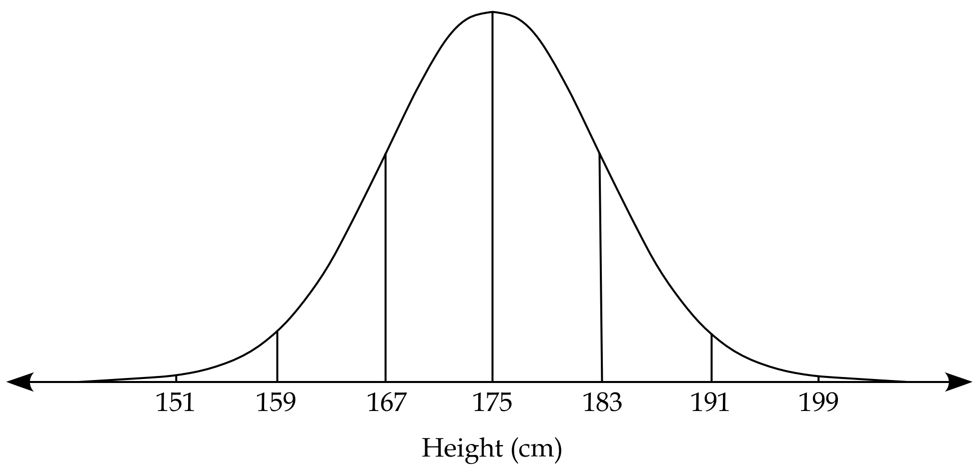
Three standard deviations above the mean is 191 + 8 = 199 cm.

One standard deviation below the mean is 175 − 8 = 167 cm.

Two standard deviations below the mean is 167 − 8 = 159 cm.

Three standard deviations below the mean is 159 − 8 = 151 cm.

Height of 17-Year-Old Males



c) In a normal distribution, 95% of all data values are within 2 standard deviations of the mean. Therefore, the range of heights for 95% of all 17-year old females is from 149 cm to 177 cm.

d) The range of heights for 95% of all 17-year-old males is from 159 cm to 191 cm.

e) The range of heights for males is larger. Also, males generally attain taller heights than females.

Learning Activity 6.3

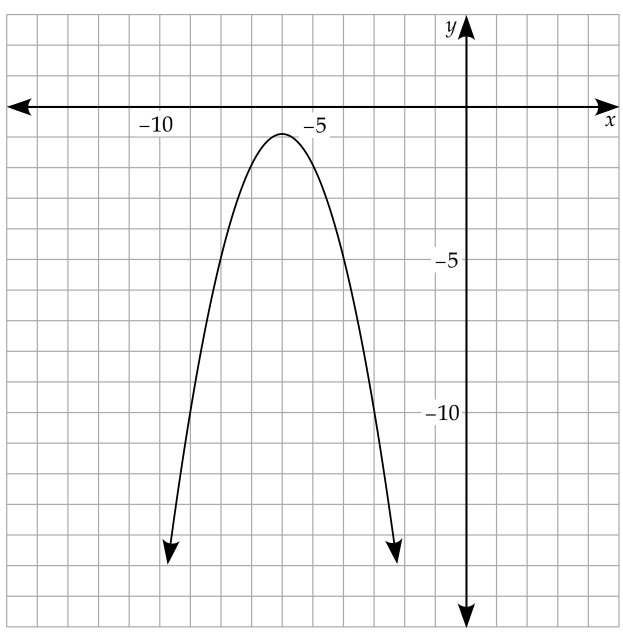
Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. 20 is 25% of what number?
2. What is 75% of 16?
3. Convert to a decimal: $\frac{123}{10000}$

Consider the following graph to answer questions 4 to 6.



4. Determine the coordinates of the vertex.
5. Determine the domain.
6. Determine the range.

continued

Learning Activity 6.3 (continued)

7. If two sides of a right triangle measure 3 cm and 4 cm, what is the length of the hypotenuse?

8. Write in lowest terms: $\frac{21}{49}$

Part B: The Normal Distribution

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Determine if the following set of data resembles a normal distribution by analyzing the percentage of data values that fall within 1, 2, and 3 standard deviations of the mean. The mean of this data set is 35.7 and the standard deviation is 2.

Waist Sizes of Pants Sold by a Men's Clothing Store					
32	32	34	34	34	34
34	34	34	34	36	36
36	36	36	36	36	36
38	38	38	38	40	40

2. Determine how closely the following set of data approximates the normal distribution by drawing a histogram with six intervals.

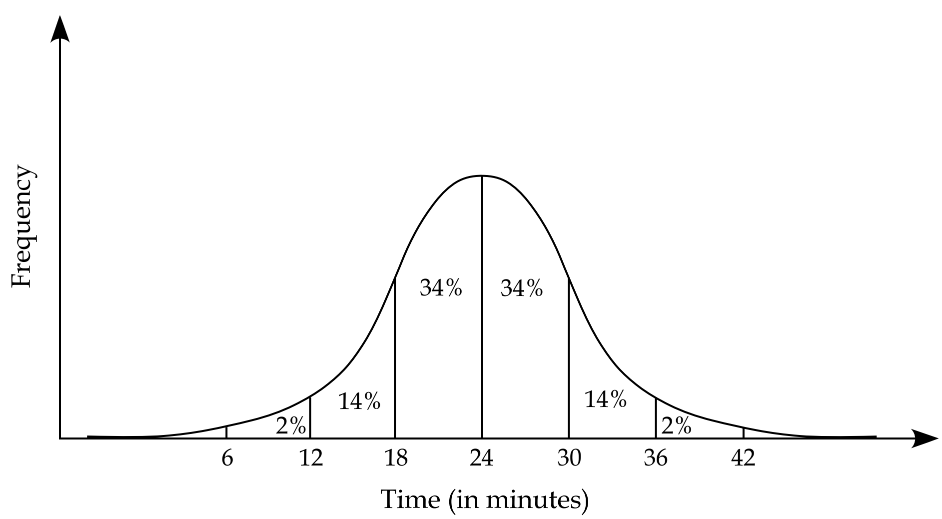
Number of Steak Dinners Served on 50 Consecutive Sundays										
36	38	41	42	42	42	43	43	43	43	44
44	44	44	45	45	45	45	45	46	46	46
47	47	47	47	48	48	48	48	48	49	49
49	49	50	51	52	52	53	54	55	56	57
58	58	61	63	63	65					

continued

Learning Activity 6.3 (continued)

3. Indicate whether the following statements are true or false.
   a) In a normal distribution, approximately 68% of data is within one standard deviation of the mean.
   b) In a normal distribution, the mean is not the median.
   c) The greater the standard deviation, the more flat and spread out the normal curve.
   d) The standard deviation tells us how far the pieces of data in a set are from the mean.
   e) Approximately 2% of data in a normal distribution is between two and three standard deviations of the mean.
   f) A normal distribution with a standard deviation of three units will be taller and skinnier than a normal distribution with a standard deviation of two units.
   g) In a normal distribution, 50% of the data is greater than the mean.
   h) Approximately 28% of data is within two standard deviations of the mean.
   i) Almost all data in a normal distribution falls within three standard deviations of the mean.

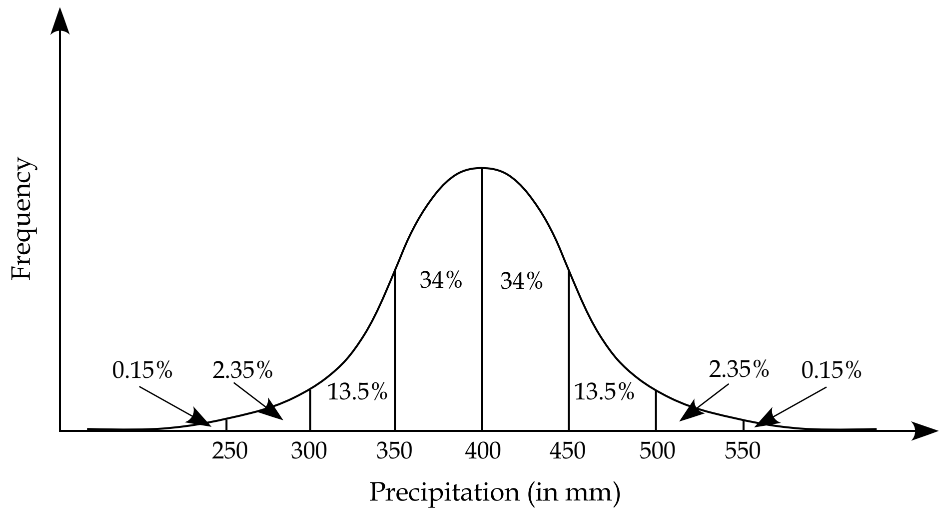
4. The following curve describes a normal distribution. Find its mean and standard deviation.



continued

Learning Activity 6.3 (continued)

5. The amount of annual precipitation in a certain region follows the following normal distribution.



a) Find the mean annual precipitation in the region.

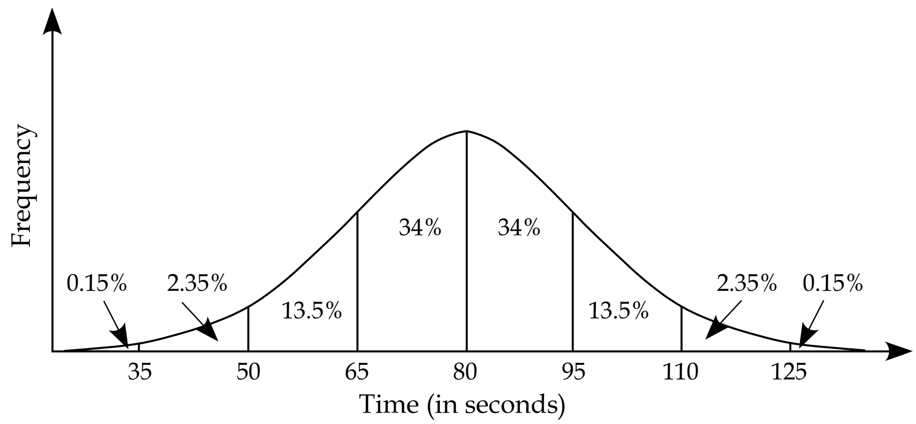
b) Find the percentage of each year in which the annual precipitation is above 300 mm.

c) Find the percentage of each year in which the annual precipitation is between 350 mm and 450 mm.

d) Find the percentage of each year in which the annual precipitation is below 450 mm.

e) What range of annual precipitation should you expect (with 95% accuracy) in any given year?

6. A psychologist gives a puzzle to 200 people to solve. Each person is timed and a graph of the time it takes for everyone to solve the puzzle follows a normal distribution.



   a) Find the number of people who are able to solve the puzzle in less than 80 seconds.
   b) Find the number of people that are able to solve the puzzle in between 65 and 95 seconds.
   c) Is it possible that an individual can solve the problem in less than 35 seconds?

7. The time a skier takes to complete a downhill course follows a normal distribution with a mean of 12.3 minutes and a standard deviation of 0.5 minutes.
   a) Draw a normal curve to represent the data.
   b) Find the percentage of time she skis the course in more than 12.8 minutes.
   c) If she skis the course 20 times in a one-week period, approximately how many times will she complete the course in less than 11.8 minutes?

8. The mass of a jar of a particular brand of coffee follows a normal distribution with a mean of 500 g and a standard deviation of 4 g.
   a) If you purchase 25 jars of coffee a year, how many would have a mass between 496 g and 508 g?
   b) Is it possible to have a jar of coffee with a mass of 517 g? Explain.

Lesson Summary

In this lesson, you learned how creating a histogram from a set of data can help you determine whether the shape of the graph can be modelled by a normal curve. You also learned the percentage of values that fall within certain standard deviations of the mean in a normal distribution. When your data can be modelled by a normal curve or a normal distribution, you are able to make estimates about the percentage of data that will fall in a given range. In the next lesson, you will learn how to determine how many standard deviations a certain value is from the mean.

Notes

Lesson 4: *z*-Scores

Lesson Focus

In this lesson, you will

- ❑ learn about *z*-scores
- ❑ learn how to compare distributions using *z*-scores

Lesson Introduction

You and your friend are both taking Grade 11 Applied Mathematics and decide to compare your latest test scores. You received a mark of 70% on the latest mathematics test given by Mr. Dhillon, while your friend, Riley, received a mark of 85% on the latest mathematics test given by Mr. Hussein. When you review the tests, you conclude that Mr. Dhillon's test was more difficult than the one given by Mr. Hussein. How do you convince your friend that you may have done as well (or even better) than she has?

The answer is *z*-scores. **Z-scores** are *standardized scores* based on the mean and standard deviation of a data set. Therefore, if every student received a high mark on Mr. Hussein's test, Riley's mark of 85% will not look as impressive once you calculate the *z*-score.

Z-Scores

The *z*-score is a **standardized** score that uses the mean and the standard deviation of the distribution of all the scores in the set. The *z*-score transforms any normal curve to the standard normal distribution that has a mean of zero and a standard deviation of 1. This means that normal curves of different shapes can be compared to each other. The *z*-score is a measure of how many standard deviations an individual score is away from the mean in a standard normal distribution. For data modelled by a normal curve, knowing the *z*-score can tell you how likely that particular score is for the population.

Consider a data set where the mean is 50 and the standard deviation is 10. The z-score for a value of 70 is 2 because 70 is two standard deviations above the mean (50 + 10 + 10).

The z-score for 40 is −1 because 40 is one standard deviation below the mean (50 − 10). A z-score can also be a decimal value. For example, a z-score of 1.5 means that the value is between one and two standard deviations above the mean in a standard normal distribution, with a mean of 0 and a standard deviation of 1. The formula of a z-score is included below. You may wish to include the above definition and the formula below on your resource sheet.

Calculating z-Scores

The z-score of any number X in a distribution, where the mean is μ and the standard deviation is σ, can be calculated with this formula:

$$z = \frac{X - \mu}{\sigma}$$

where X = data value

μ = population mean

σ = population standard deviation

The formula for finding the z-score of a sample is identical, except the symbol, $\bar{x}$, is used instead of μ for the mean. In this course, you will only be dealing with z-scores of populations.

The value for σ is always a positive number. However, z-scores can have both positive and negative values, the same as how a standard normal distribution with a mean of zero will have positive and negative data values. The value for z will be a negative number whenever the data value, X, is less than the mean, μ ($X - \mu$ is then a negative number). This means that your z-score will be negative when your data value is less than the mean or to the left of the mean on the normal curve. The value for z will be a positive number whenever the data value, X, is greater than the mean, μ ($X - \mu$ is then a positive number). In this case, your z-score is positive because your data value is above the mean or to the right of the mean on the normal curve. If the z-score is equal to zero, then the value is equivalent to the mean.

Recall that in a normal distribution, the majority (99.7%) of data is within three standard deviations of the mean. Therefore, z-scores are typically between −3 and 3.

Example 1

Paulo has an average long jump distance of 20.2 feet, with a standard deviation of 1.6 feet. At a competition, Paulo jumps a distance of 23.0 feet. What is the *z*-score of this value?

Solution

Use the formula:

$$z = \frac{X - \mu}{\sigma}$$

where X = data value = 23.0 feet

μ = population mean = 20.2 feet

σ = population standard deviation = 1.6 feet

$$z = \frac{23.0 - 20.2}{1.6}$$

$$z = 1.75$$

This *z*-score indicates that Paulo's jump length of 23.0 feet is 1.75 (or almost 2) standard deviations above his mean jump distance of 20.2 feet. This is a good distance for Paulo!

Standardizing Data Using *z*-Scores

In the introduction to this lesson, it was noted that *z*-scores are *standardized scores* based on the mean and standard deviation of a data set. Using *z*-scores, you can compare values from two different sets of data. The two situations in the following example illustrate this concept.

Example 2

a) You received a mark of 70% on the latest mathematics test while Riley received a mark of 85%. The mean from both classes was 65% and the standard deviation from both classes was $\sigma = 8$ (how rare!). Determine which student has performed better.

b) You and Riley both received a mark of 70% on the latest mathematics test. The mean from both classes was 65%. However, the standard deviation from your class was $\sigma = 6$, while the standard deviation from Riley's class was $\sigma = 10$. Determine which student has performed better.

Solution

a) Yourself:

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{70 - 65}{8}$$

$$z = 0.625$$

Riley:

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{85 - 65}{8}$$

$$z = 2.5$$

Intuitively, you know Riley achieved the better relative mark on this test as all other conditions were kept the same. This can be verified by z-scores. Riley's z-score of 2.5 is much higher than your z-score of 0.625. In general, the larger the z-score, the less likely it is that someone will have a higher value.

b) Yourself:

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{70 - 65}{6}$$

$$z = 0.833$$

Riley:

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{70 - 65}{10}$$

$$z = 0.5$$

Both z-scores are positive, indicating that both test scores are above the mean, which you already knew. But how does this relate to the test scores of other students in the class? In your class, there was a smaller standard deviation, which indicates that the marks are clustered close together. In Riley's class, the opposite is true. The standard deviation is larger and thus the test marks are more spread out. When the normal curve is more spread out, it is more likely that a student will score higher than the mean. Therefore, Riley's score of 70 is not as impressive as your score of 70.

Example 3

Consider the situation described in the Introduction to this lesson. You received a mark of 70% on the latest mathematics test given by Mr. Dhillon while your friend, Riley, received a mark of 86% on the latest mathematics test given by Mr. Hussein. The mean mark in Mr. Dhillon's class was μ = 65, with a standard deviation σ = 8. In Mr. Hussein's class, the mean mark was μ = 85, with a standard deviation σ = 4. Using this information, determine which student has performed better.

Solution

Yourself:

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{70 - 65}{8}$$

$$z = 0.625$$

Riley:

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{86 - 85}{4}$$

$$z = 0.25$$

You performed better than Riley on the test. As your z-score is higher, this indicates that you did better in comparison to the other students who wrote the test in your class than Riley did in comparison to the other students who wrote the test in her class.

Calculating Original Scores Using *z*-Scores

Consider a different situation where instead of trying to find the *z*-score, you have the *z*-score and are trying to calculate the corresponding original score. You can either use the *z*-score formula in its standard form, or you can first manipulate the formula to solve for *X*, the original score. The manipulated formula is as follows:

$$z = \frac{X - \mu}{\sigma}$$

$$z\sigma = X - \mu$$

$$z\sigma + \mu = X$$

$$X = z\sigma + \mu$$

You may wish to include one or both formulas on your resource sheet.

Example 4

In a recent survey of the number of hours high school students spend playing video games per week, the mean time was 15 and the standard deviation was 4. If Aaryn had a *z*-score of −1.4, how many hours does she normally spend each week playing video games?

Solution

Method 1: Using the *z*-score formula:

Since $\mu = 15$, $\sigma = 4$, and $z = -1.4$,

$$-1.4 = \frac{X - 15}{4}$$

$$4(-1.4) = X - 15$$

$$-5.6 = X - 15$$

$$-5.6 + 15 = X$$

$$9.4 = X$$

$$X = 9.4$$

Method 2: Using the manipulated *z*-score formula:

Since $\mu = 15$, $\sigma = 4$, and $z = -1.4$,

$$X = (-1.4)(4) + 15$$

$$X = -5.6 + 15$$

$$X = 9.4$$

Aaryn plays video games for approximately 9.4 hours each week. The fact that the z-score is negative indicates that Aaryn plays less than the mean time of 15 hours.

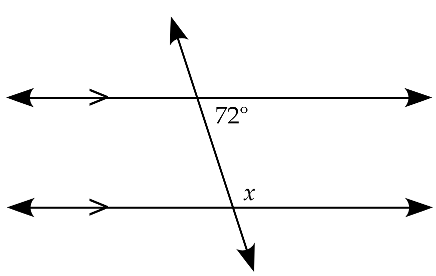
Learning Activity 6.4

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Convert 294.2 m to cm.
2. Factor: $x^2 + 3x + 2$
3. Determine the y-intercept: $\frac{1}{2}x - y = 3$
4. Determine the slope: $y = 3x - 2$
5. Write as a mixed number: $\frac{35}{4}$
6. Multiply: $\frac{3}{4} \cdot \frac{3}{2}$
7. Which is the better deal, two extra large pizzas for \$18.99 or three extra large pizzas for \$24.99?
8. Determine the size of angle x in the diagram below.



continued

Part B: *z*-Scores

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. In a recent weightlifting contest, the mean weight lifted was 250 pounds, with a standard deviation of 20 pounds. Find the z-score for:
   a) Marcos who lifted 290 pounds.
   b) Blaine who lifted 220 pounds
   c) Dawson who lifted 310 pounds
   d) Edwin who lifted 250 pounds.
2. The following z-scores were obtained by five baseball players on their batting averages.

   Davis: 2.63

   Wilson: −1.36

   Johnson: −1.02

   Rodriguez: 1.69

   Martinez: 0.97

   a) Rank these players in terms of their batting averages from highest to lowest.
   b) Which of these players had positive z-scores and scored above the mean?
   c) Which of these players scored below the mean?
3. The manager at a Tim Horton's outlet decided to collect some data about how long each customer had to wait in the drive-through line. The mean waiting time was 1.2 minutes with a standard deviation of 0.4 minutes.
   a) If Thom had a z-score of −0.84, how much time did he spend waiting for his order?
   b) If Evelyn had a z-score of 2.48, how much time did she spend waiting for her order?
   c) If Suzette had a z-score of −2.8, how much time did she spend waiting for her order?

continued

Learning Activity 6.4 (continued)

4. To encourage students to use statistics to analyze their own test scores, Ms. Patterson posted the following information about the marks for the latest test on her bulletin board:

Test Mark	z-Score
55	−2
65	−1
75	0
85	1
95	2

   a) What was the average test mark?

   b) What test score has a z-score of −2.76?

5. Tammy and Wes have both applied to attend the University of Winnipeg. Tammy graduated from Eastern Collegiate with an average of 66%, while Wes graduated from Western Collegiate with an average of 72%. The average of all students graduating from Eastern Collegiate was 84%, with a standard deviation of 6. The average of all students graduating from Western Collegiate was 88%, with a standard deviation of 5.

   a) Assuming the University of Winnipeg uses z-scores based on student averages as the only criterion for accepting students, who is more likely to get accepted into university? Explain your answer.

   b) Suggest one reason why comparing the averages for Tammy and Wes to the averages of their respective schools could be troublesome.

6. The average age of a first-time home buyer is 29 years, with a standard deviation of 3.5 years.

   a) At what age would you like to purchase your own home? Determine the z-score related to this age.

   b) Find the age at which Nicole purchased her first home if her z-score is 2.3.

   c) How does Nicole compare to other first-time home buyers?

continued

7. Gillian is applying for university and is trying to decide which faculty to enter. She decides to take an aptitude test in order to help her select the occupations that would be suitable for her. Her results, as well as the results of the other candidates, are shown below.

Talent	Mean	Standard Deviation	Gillian's Score
Writing	58	3.7	46
Teaching	87	5.8	98
Technology	84	7.9	85
Medicine	37	2.8	44
Law	49	4.7	41

   a) Transform each of Gillian's scores into a z-score.
   b) In which field will she likely have the best chance for success?
   c) In which field will Gillian have the least chance to be successful?

8. Two bands, Unique Express and Restless Zombies, are performing in Portage la Prairie at separate small venues. At their last show, Unique Express sold 78 tickets. The mean attendance at that venue for a show is 60 people, with a standard deviation of 5. On the other hand, Restless Zombies sold 89 tickets to their last show. The mean attendance at that venue for a show is 82 people, with a standard deviation of 3.
   a) Find the z-scores for ticket sales for Unique Express and for Restless Zombies.
   b) Using their z-scores, which band drew a better crowd at their last show?
   c) Another band, The Logarhythms, plays at the same venue as Unique Express but on different nights. On one night, they drew a crowd that produced a z-score of 3.8. How many people attended this show?

Lesson Summary

In this lesson, you learned how to calculate z-scores, which allow you to standardize and compare data from different normally distributed data sets. This allows you to draw conclusions about which scores are comparatively better than others. A positive z-score indicates that a data value is greater than the mean, while a negative z-score indicates that a data value is less than the mean. The z-score value is based on the standard normal distribution where the mean is zero and the standard deviation is one, so the z-score indicates the number of standard deviations from the mean.

In the next lesson, you will be learning more about z-scores and how these scores relate to percentages and the normal distribution.

Notes

Lesson 5: *z*-Scores and the Normal Distribution

Lesson Focus

In this lesson, you will

- ❑ learn how to calculate the percentage of scores that lie between two *z*-scores in a normal distribution
- ❑ learn how to calculate the percentage of scores that lie between two non-standardized scores in a normal distribution

Lesson Introduction

Have you ever heard someone say that a test was being "graded on a curve"? Professors who grade on a curve ensure that only a few students receive As and Fs (the highest and lowest letter grades), while most students receive Bs, Cs, and Ds (the middle-letter grades).

A common translation between letter grades and percentages is shown in the table below.

Grade	%
A^+	90–100
A	80–89
B^+	75–79
B	70–74
C^+	65–69
C	60–65
D	55–59
D^-	50–54
F	0–49

If a university course was graded on a curve, the translation between letter grades and percentages might be shifted. For example, if a professor set an examination that was relatively easy and most students did very well, a grade of 95% may earn an A (instead of an A+), while a grade of 89% may earn a B (instead of an A). Consider a different professor in the same course where students do poorly on the examination. In this case, if the professor decided to grade the examination on a curve, a grade of 80% may earn an A+ (instead of an A), as not many students would have achieved a mark of 80% or higher. Although this sounds like it might happen, it is not common practice for universities to evaluate students' work in this way. Since most classes in a university are relatively small and the students are not randomly selected for classes, it is unlikely that a normal distribution would be an appropriate model for students' marks.

That being said, z-scores could be used to compare student marks between two different classes.

z-Scores and the Normal Distribution

You know that you can express any value in a normal distribution as a z-score. This indicates how many standard deviations away from the mean that particular score lies. You also know the probabilities associated with the normal distribution. Therefore, if you know a z-score, you can calculate the probability of the occurrence of that particular data value.

For example, if a data value is one standard deviation from the mean, then the z-score is either +1 or −1. You know there is a high chance of observing a data value that is between −1 and +1 standard deviations from the mean because 68% of all data values fall in this range.

You can determine even more precise probabilities using technology such as your graphing calculator or GeoGebra. Brief instructions for using GeoGebra will be included in this course. However, you may have to research the technology you choose to help you learn how to perform these operations.

Finding the Probability Associated with z-Scores

Example 1

a) Find the probability that a data value will have a z-score less than or equal to 2.

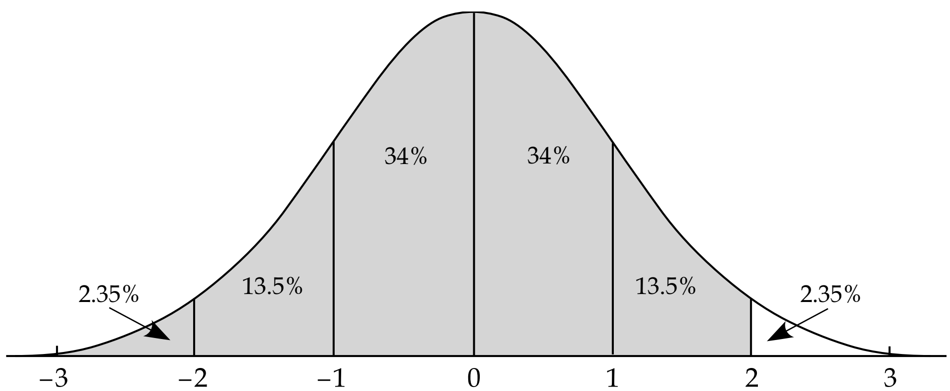
b) Find the probability that a data value will have a z-score greater than −1.

c) Find the probability that a data value will have a z-score between −2 and 1.

Solution

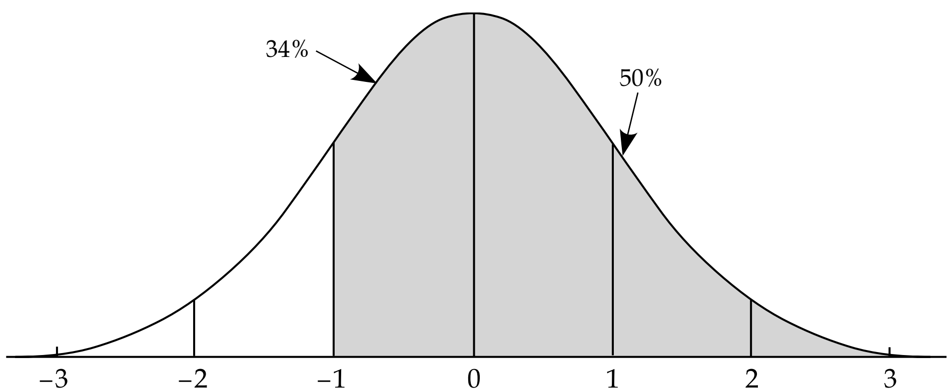
a) A z-score equal to 2 indicates a score that is 2 standard deviations above the mean. Since the mean is 0 and the standard deviation is 1 in a standard normal distribution, 2 standard deviations above the mean is the same as a z-score value of 2.

To determine the probability that a data value will have a z-score less than or equal to 2, simply add the probabilities from each interval below a score of 2.



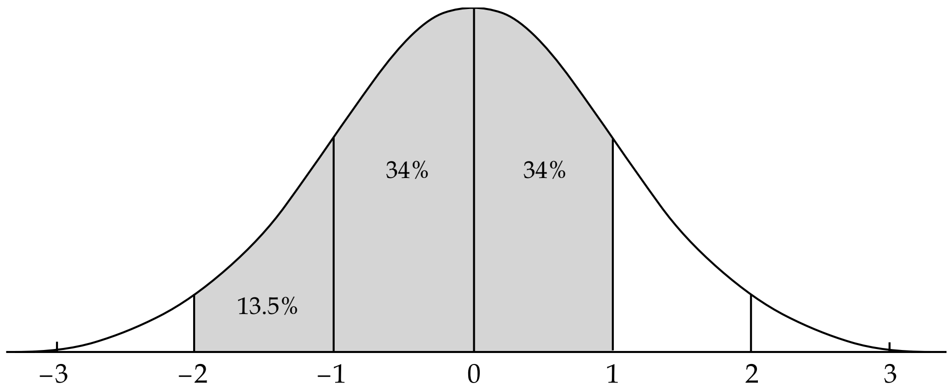
Note: An easy way to do this is to recognize that 50% of data is below the mean, which is located where the standard deviation is 0. Therefore, the probability that a data value will have a z-score less than or equal to 2 is 50% + 34% + 13.5% = 97.5%.

b) To determine the probability that a data value will have a z-score greater than −1, simply add the probabilities from each interval above a score of −1.



Note: The probability that a data value is located above the mean is 50%. Therefore, the probability that a data value will have a z-score greater than −1 is 50% + 34% = 84%.

c) To determine the probability that a data value will have a z-score between −2 and 1, add the probabilities from each interval between these two z-scores.



The probability that a data value will have a z-score between −2 and 1 is 34% + 34% + 13.5% = 81.5%.

Example 2

a) Find the probability that a data value will have a z-score less than 0.78.

b) Find the probability that a data value will have a z-score greater than 1.23.

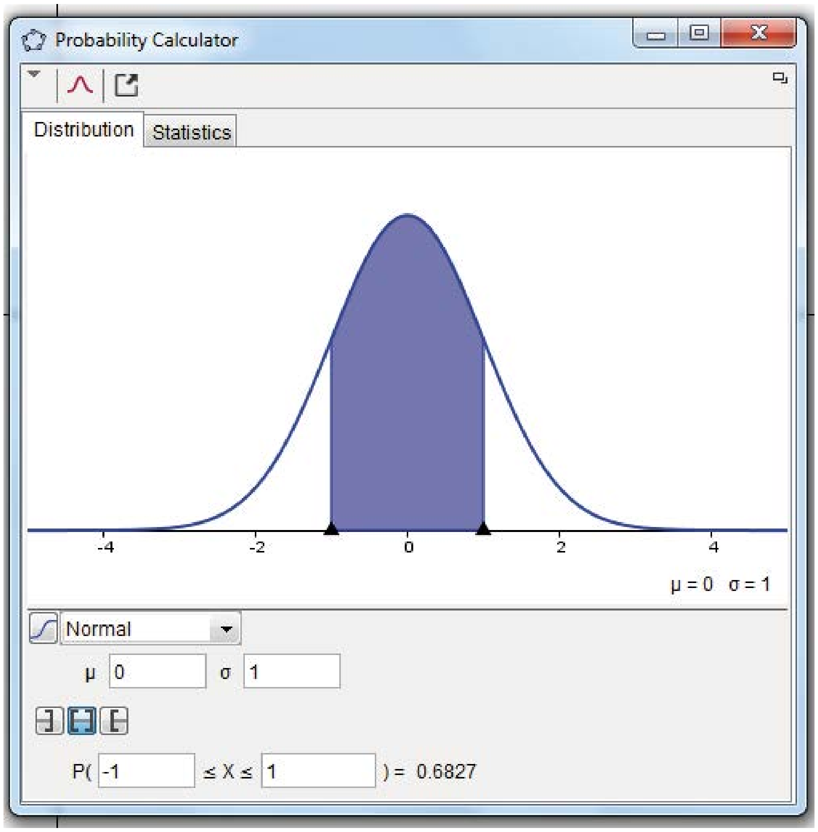
c) Find the probability that a data value will have a z-score between −1.86 and 1.86.

Solution

In this example, you will notice that you do not know the probabilities associated with z-scores or standard deviations that are not whole numbers. Therefore, you need to take a different approach to solving these problems that involves the use of technology.

a) To use GeoGebra, first find the Probability Calculator. The solutions shown here use the app called GeoGebra but you may use another app or a graphing calculator (such as a TI-83 or TI-84, where features on the "DISTR" menu can solve probabilities associated with normal distribution).

Go to the Technology Appendix to find the details for using GeoGebra or the TI-83 or TI-84 graphing calculator for statistical calculations.

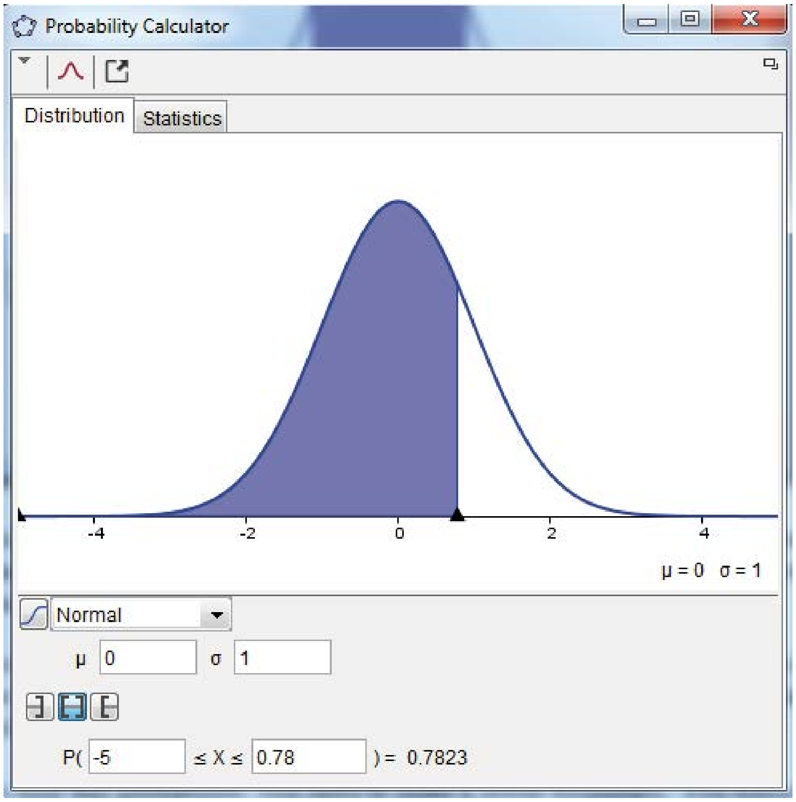


This probability calculator is set to calculate probabilities related to a standard normal distribution. This is exactly what you want to do, so you do not need to change the values of μ or σ.

You do need to indicate between which *z*-scores the probability you are looking for is located. In this example, you want to find the probability that a data value will have a *z*-score smaller than 0.78.

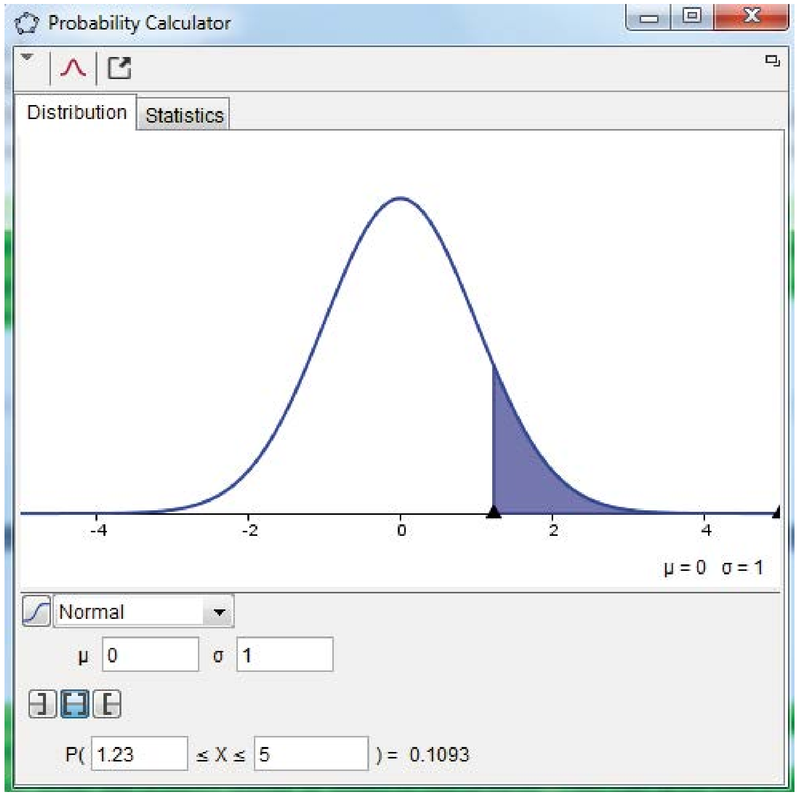
To find this probability, you need to enter a lower boundary. You know that most data is located within 3 standard deviations of the mean. However, just to ensure you include most of the possibilities, set your lower bound to −5, which is 5 standard deviations below the mean.

The upper boundary will be 0.78.



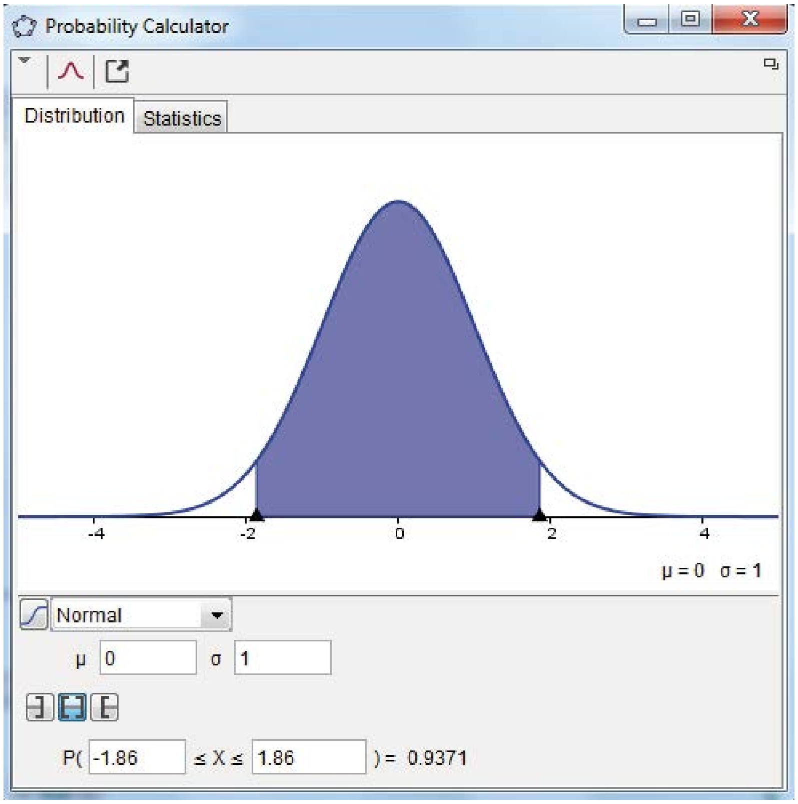
Using the calculator, you will confirm that the probability that a data value will have a *z*-score less than 0.78 is 0.7823.

b) To find the probability that a data value will have a *z*-score greater than 1.23, enter a lower bound of 1.23 and enter an upper bound of 5.



You can confirm that the probability of finding a data value that will have a *z*-score greater than 1.23 is 0.1093.

c) To find the probability that a data value will have a z-score between −1.86 and 1.86, enter a lower bound of −1.86 and an upper bound of 1.86.



You can confirm that the probability of finding a data value with a z-score between −1.86 and 1.86 is 0.9371.

Example 3

a) What is the probability that a score will fall between z-scores of 0.87 and 2.57?

b) What percentage of scores will lie between z-scores of 0.87 and 2.57?

c) Find the area between $z = 0.87$ and $z = 2.57$ in a standard normal distribution.

Solution

a) Using technology, your lower bound will be 0.87 and your upper bound will be 2.57. The probability that a score will fall between these two scores is 0.1871.

Note: Make sure you can get this answer using whichever technology you are using! If not, ask your learning partner for assistance, or research help sites for your technology. For example, if you are using a TI-83 or TI-84, instructions for all statistical operations can be found on the Texas Instruments website.

b) The percentage of scores that lie between z-scores of 0.87 and 2.57 is 0.1871 or 18.71%, which is equivalent to the probability that a score will fall between z-scores of 0.87 and 2.57.

c) The area between $z = 0.87$ and $z = 2.57$ in a standard normal distribution is equivalent to both the probability that a score will fall between these two z-scores and the percentage of scores that lie between these scores. Thus, the area between $z = 0.87$ and $z = 2.57$ on a normal distribution or bell curve is 0.1871.

From Example 3, you will notice that the terms *probability*, *area*, and *percentage* become interchangeable in the context of z-scores and the standard normal distribution.

Finding z-Scores Using Probabilities

In the previous examples, you interpreted the area under the normal curve as a probability or a percentage. This process can also be done in reverse. For example, if you know the probability of an event, you can find the z-score that corresponds to this probability.

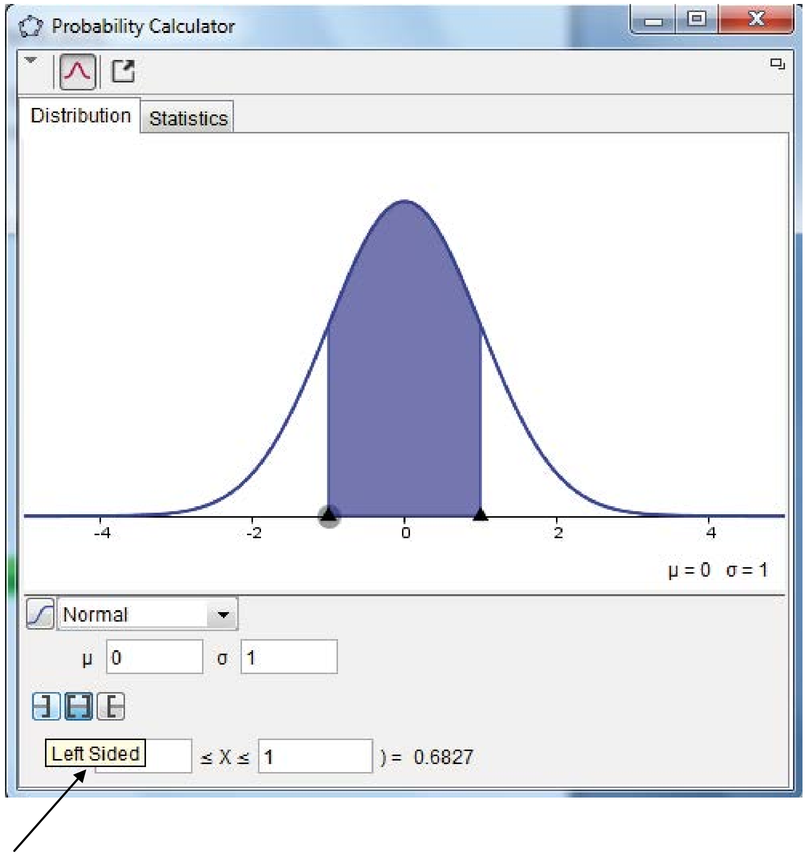
Example 4

If the probability of getting less than a certain z-score is 36.31%, what is the z-score?

Solution

To solve this problem, use the technology of your choice. The following solution involves the use of GeoGebra.

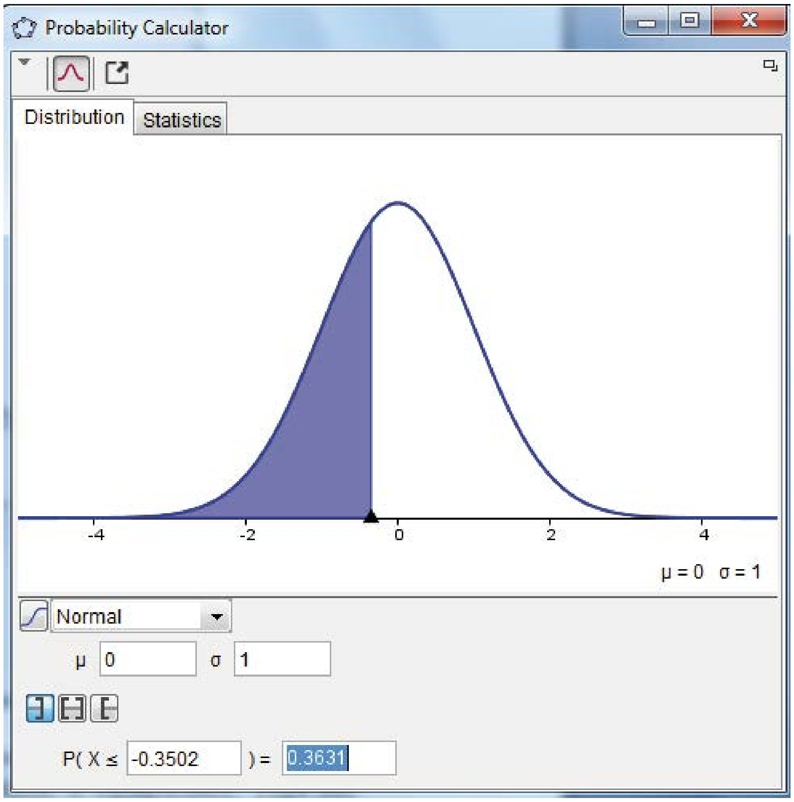
Using GeoGebra, open the Probability Calculator.



Click on the Left Sided button, as you want to find a *z*-score where the probability of finding a value less than this *z*-score is 0.3631.

Note: This is an alternative to entering −5 as your lower bound.

Now, you want the probability to be 0.3631. Therefore, you want the probability, or the second blank, to equal 0.3631. Type this number into the calculator and press Enter.



The calculator will give you a z-score of -0.3502.

Example 5

What z-score is higher than 75% of the data values; that is, 75% of the curve is below it?

Solution

Determine the z-score related to an area of 0.75. Use your technology the same way you did in Example 4. You should find a z-score equal to 0.6745.

Standardizing a Normal Distribution

What happens when you have data that represent a normal distribution but not a standard normal distribution; that is, when each standard deviation from the mean does not have a value of 1, and the mean is not 0? Using a probability calculator, such as the one in GeoGebra, you can standardize these scores by changing the values for μ and σ. Consider the following example.

Example 6

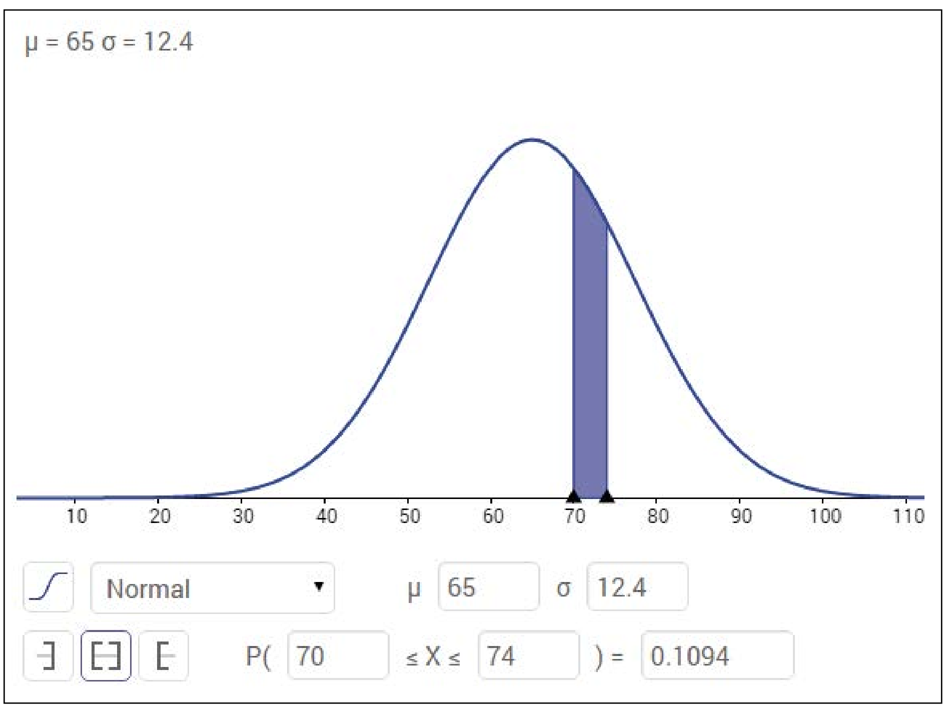
Professor Kerr has 150 students in her psychology course. The mean mark on the final examination was 65%, with a standard deviation of 12.4.

a) What percent of students will have a B or a mark between 70 and 74?

b) Professor Kerr has decided that her examination is too easy. How high should she set the mark for an A^+ (typically between 90 and 100), if she only wants 2% of students to get an A^+?

Solution

a) Using GeoGebra, you want to find the probability (or percent) between the values of 70 and 74. However, since the mean is not 0 and the standard deviation is not 1, you need to change those values. Change the value of μ to 65 and change the value of σ to 12.4.

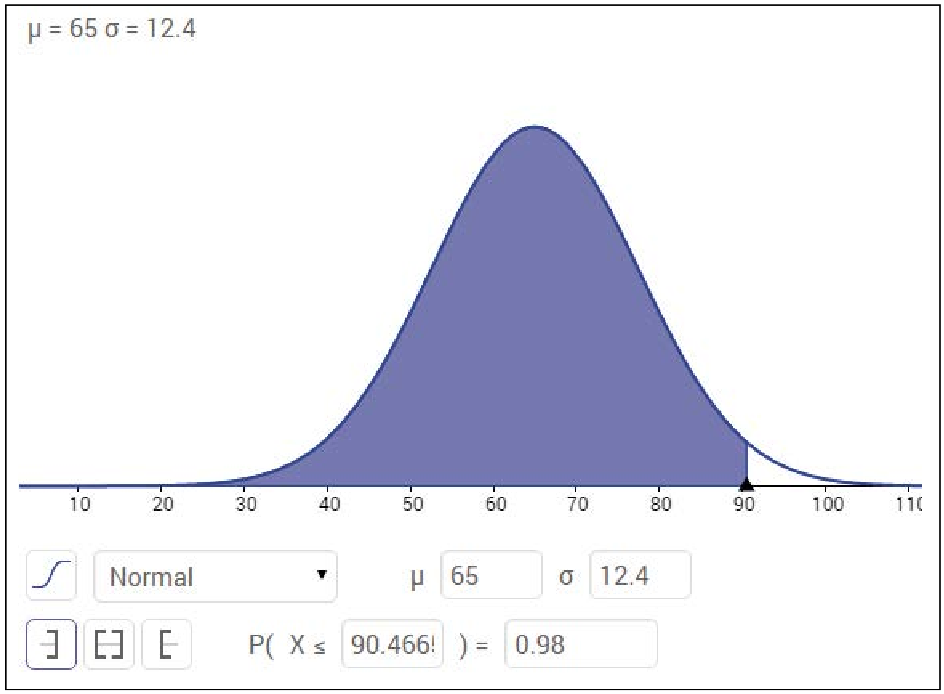


Therefore, 10.94% of students will have a B on their final examination.

b) Professor Kerr only wants 2% of students to get an A$^+$. To determine what mark qualifies for an A$^+$, start at the beginning with what you know.

You know that 2% of 150 students is (0.02)(150) = 3. Therefore Professor Kerr only wants 3 students to get an A$^+$. The rest of the students, which is 98% of the students, do not receive an A$^+$. Therefore, these students receive marks lower than an A$^+$.

Using technology, determine the data value that is greater than 0.98 (98%) of all the values, where the mean is 65 and the standard deviation is 12.4.



Therefore, if Professor Kerr only wants 2% of students to receive an A$^+$, the lowest mark able to receive an A$^+$ is 90.47%.

When are *z*-scores used? They are often used in manufacturing when a company only wants to accept products with certain qualifications. They are also used to determine the length of a warranty based on statistics of how long a device usually lasts.

Consider the following example.

Example 7

A tablet manufacturer knows that the mean lifespan of the devices it markets is 48 months, with a standard deviation of 5.6 months. The manufacturer wants to offer a warranty that ensures it will have to repair less than 5% of all tablets under warranty. What length of warranty should be offered?

Solution

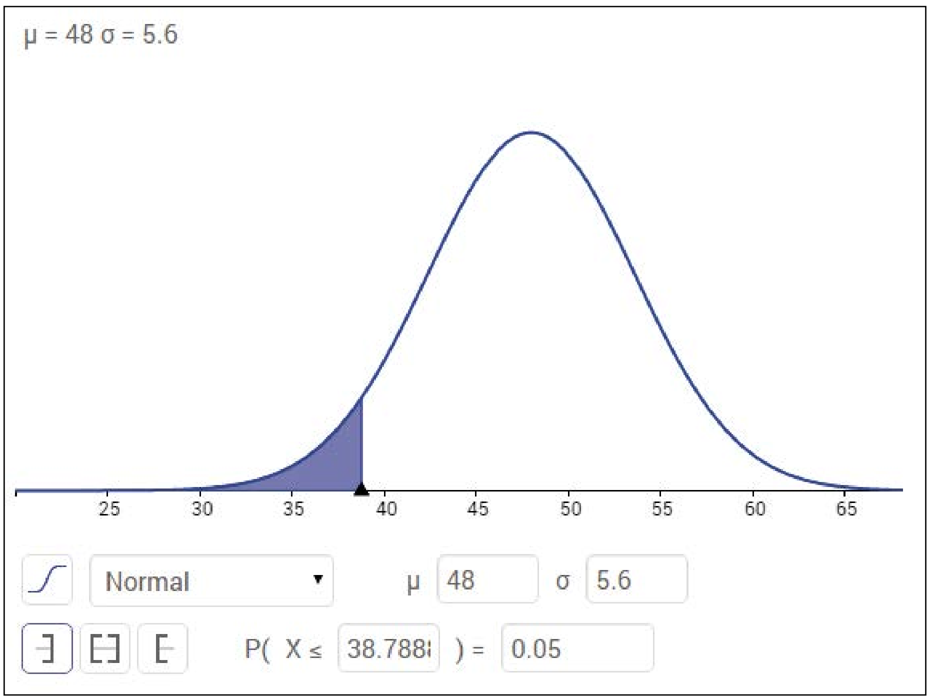
First, determine what you know.

$\mu = 48$

$\sigma = 5.6$

The probability of repair needs to be less than 5%.

Using technology, enter in these values.



Because the tablet manufacturer wants to limit the amount of repairs, you should round down your answer. The tablet manufacturer should offer a warranty of 38 months or just over 3 years.

Make sure you practise what you just learned in the following learning activity. The concepts of *z*-scores, probabilities, and percentages can be confusing when you first start learning them. The more practise you get, the better you will be.

Learning Activity 6.5

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

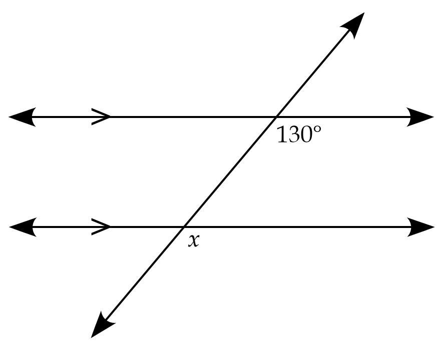
1. Order from smallest to largest: $-0.2, -0.12, -0.340, -0.115$

2. Subtract: $\frac{6}{7} - \frac{3}{4}$

3. Evaluate: $10^2 - 12^2$

4. Solve for x: $2x < 7$

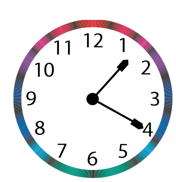
5. Determine the size of angle x in the diagram below.



6. Solve for x: $\frac{2x}{5} = \frac{24}{20}$

7. Complete the pattern: $-\frac{1}{16}, -\frac{1}{4}, -1,$ ____, ____, ____

8. Estimate the angle the hands of a clock make when the time is 1:20.



continued

Learning Activity 6.5 (continued)

Part B: *z*-Scores and Normal Curves

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

Note: When you are using technology for the following questions, the mean must be set to 0 and the standard deviation must be set to 1.

1. In a standard normal distribution, find the area (or probability) that lies
   a) to the left of $z = 0.23$.
   b) between $z = -0.56$ and $z = 0$
   c) to the right of $z = 2.31$
   d) between $z = -1.56$ and $z = 1.69$
   e) between $z = -1.82$ and $z = 0.56$

2. Find the percentage of z-scores in a standard normal distribution that are
   a) above $z = -2.46$
   b) below $z = 0.86$
   c) between $z = 1.24$ and $z = 2.09$

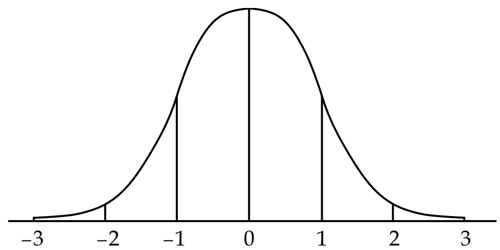
3. Find the z-score in a normal distribution that cuts off the bottom 7% of values.

4. Find z if the area under a standard normal curve
   a) to the right of z is 0.0188
   b) to the left of z is 0.9793

5. Find the percentage of data values in a normal distribution with $\mu = 75$ and $\sigma = 10$ that are
   a) between 60 and 80
   b) greater than 83
   c) less than 72

continued

6. The following is a standard normal curve with various z-scores marked on it. If this curve also represents a normal distribution with $\mu = 24$ and $\sigma = 5$, replace the z-scores with raw scores.



7. It has been found that the weights for newborn babies who are not born prematurely are normally distributed, with a mean of 3486 grams and a standard deviation of 434 grams. If a newborn is selected at random, what is the probability that the infant weighs less than 2724 grams (approximately 6 pounds)?

8. The life of a toaster is found to be normally distributed, with a mean life of 4.7 years and a standard deviation of 0.9 years. The producer of the toasters will replace the toaster free of charge while under warranty. For how many years should the producer guarantee the toasters, if no more than 5% of them are to be replaced?

9. A teacher finds that the results on a final examination are normally distributed, with a mean of 62 and a standard deviation of 12.
   a) If the passing grade is 52, what percentage of the class will fail?
   b) If the teacher wants only 75% of the class to pass, what will be the passing grade?

10. A concession company provides food and beverages at local sports games. The amount of money for sales at these games is normally distributed. The company has determined that the mean sales they make for a game is $8,652. The company has also determined that the standard deviation is $492. Find the probability that the sales for the next game will be between $8000 and $9000.

Lesson Summary

In this lesson, you learned how z-scores were related to the probability of the occurrence of certain data values. You also learned how z-scores are related to the percentage of data values under the normal curve, as well as the area under the normal curve. You were then able to use these probabilities, percentages, and areas to solve problems involving warranty periods and lifespans.

In the next lesson, you will be looking at confidence intervals.

Assignment 6.2

z-Scores and the Normal Curve

Total: 30 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. If you use technology, state your keystrokes or values used and your process. Final answers must include units. Answers given without supporting calculations and graphs will not be awarded full marks.

1. As of 2012, the mean number of water bottles used per person each year is 167. Assume this data is normally distributed, with a standard deviation of 32.
   a) Find the z-score for
      i) Brittany who uses approximately two water bottles every week. *(2 marks)*

      ii) Dominic who purchases a bottle of water every weekday (5 days a week). *(2 marks)*

b) Interpret Brittany's z-score. How does she compare to the rest of the population (write as a percent)? *(1 mark)*

2. The following z-scores were obtained by five individuals based on their IQ scores.

   Brent: 2.42

   Alex: −2.61

   Jordyn: −0.34

   Marika: 1.25

   Ken: 0.83

a) Rank these individuals from lowest IQ to highest IQ. *(1 mark)*

b) Which of these individuals scored above the mean? *(1 mark)*

c) Which individual has the highest IQ? *(1 mark)*

d) Which individual has the lowest IQ? *(1 mark)*

e) The mean for an IQ test is 100, with a standard deviation of 15. Determine the actual scores of Jordyn and Marika. *(2 marks)*

Assignment 6.2: z-Scores and the Normal Curve (continued)

3. In a standard normal distribution, find the area that lies
   a) between $z = 0$ and $z = 1.47$ *(1 mark)*

   b) to the right of $z = 1.93$ *(1 mark)*

   c) to the left of $z = -0.78$ *(1 mark)*

4. Find the percentage of z-scores in a standard normal distribution that are
   a) below $z = -1.51$ *(1 mark)*

   b) above $z = 2.94$ *(1 mark)*

   c) between $z = -1.38$ and $z = 1.09$ *(1 mark)*

Assignment 6.2: z-Scores and the Normal Curve (continued)

5. An automobile tire producer claims that the life of its tires is normally distributed, with mean of 28 000 miles and a standard deviation of 4000 miles. What percentage of tires is expected to last more than 35 000 miles? *(3 marks)*

6. A car company tests the fuel economy of several cars of the same model. The fuel economy is normally distributed. The mean fuel economy is 9.84 L/100 km, with a standard deviation of 0.16 L/100 km.

   a) Find the probability of the car having a fuel economy of 9.80 L/100 km or lower. *(2 marks)*

   b) Find the probability of the car having a fuel economy between 9.65 L/100 km and 9.83 L/100 km. *(3 marks)*

Assignment 6.2: z-Scores and the Normal Curve (continued)

7. The lifetime of a certain model of cell phone is normally distributed. The mean lifetime of the cell phone is 38 months, with a standard deviation of 3.7 months.
   a) What is the probability that a cell phone will last through the entire length of a 3 year contract? *(3 marks)*

   b) The cell phone company wants to create a warranty where only 2% of all cell phones need to be repaired or replaced. How long of a warranty period should the company offer? *(2 marks)*

Notes

LESSON 6: CONFIDENCE INTERVALS

Lesson Focus

In this lesson, you will

- ❑ learn how to determine the confidence interval, margin of error, and confidence level of statistical data
- ❑ learn how confidence intervals and sample sizes are related
- ❑ learn how margins of error and confidence levels are related

Lesson Introduction

You may have listened to the results of an opinion poll and heard the disclaimer: "These results are accurate to within five percentage points, 19 times out of 20." For example, someone might say that a political party is likely to receive 40% of the popular vote and then add a disclaimer such as the one mentioned above.

This disclaimer makes use of confidence intervals, margins of error, and confidence levels. You will be learning how to interpret these three statistical measures throughout this lesson.

Confidence Intervals

It is often impossible to survey an entire population of people. One reason is that it may be impossible to find everyone, and another reason is that it would take a large amount of time. Therefore, sample populations are often surveyed instead of the entire population.

No matter how large you make your sample or how hard you try to make sure people from every walk of life are represented in your sample, your sample is never going to be perfect. There will always be some error between the results you get from your sample and the results you would get if you surveyed the entire population.

This difference or error is called the **margin of error**. The margin of error is often expressed as plus or minus a percent, such as ±3%.

The **confidence interval** is the range in which the true percentage should lie and is based on the margin of error. If the margin of error is larger, then the confidence interval is larger. Similarly, if the margin of error is smaller, then the confidence interval is smaller.

You know that any sample data is never going to be perfect, but how confident are you that your sample data represents the population?

The amount of certainty that you have in your data is called the **confidence level**. This is the level of certainty with which the estimate is made. The confidence level is usually 90%, 95%, or 99%. A 95% confidence level indicates that 95 times out of 100, a different sample will produce similar results. In other words, there is a 95% chance that this data is highly reliable.

You should add these definitions to your resource sheet.

Example 1

According to Statistics Canada, 43.8% of adult males between the ages of 45 and 64 are overweight. This statistic is accurate to within ±4.6%, 19 times out of 20.

a) Determine the margin of error and explain what this means.

b) Determine the confidence interval and explain what this means.

c) Determine the confidence level and explain what this means.

Solution

a) The margin of error is ±4.6%. This means that the actual percentage could be as much as 4.6% greater or less than the given percentage.

b) The lower boundary in the confidence interval is 43.8% − 4.6% = 39.2%.

The upper boundary in the confidence interval is 43.8% + 4.6% = 48.4%.

Therefore, the confidence interval is between 39.2% and 48.4%.

This interval is where the true percentage of adult males between the ages of 45 and 64 who are overweight is likely to lie.

c) The confidence level is 19 times out of 20, which is equivalent to 95% as $19 \div 20 = 0.95$.

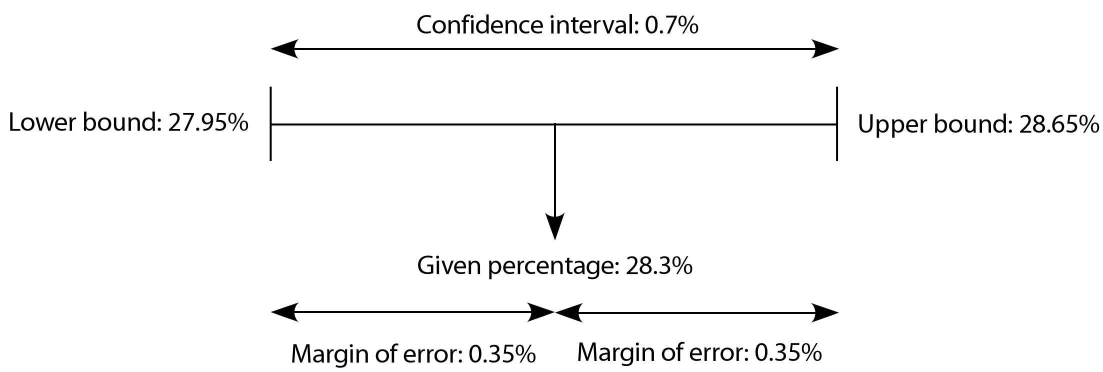
Therefore, if similar surveys were completed, 95% of them would find that between 39.2% and 48.4% of the adult male population is overweight.

Example 2

According to a Canadian Community Health Survey, 28.3% of people aged 18 to 34 are smokers. The accuracy of this data has a range of 0.7 percentage points, 19 times out of 20. If approximately 6.5 million Canadians were between the ages of 18 and 34 in 2005, what is the range of the number of people who smoke?

Solution

If the data is accurate to within 0.7 percentage points, then the confidence interval has a range of 0.7 percentage points. In other words, the margin of error is only half of this amount.



The margin of error is ±0.35%.

With a confidence level of 95%, between 28.3% − 0.35% and 28.3% + 0.35%, or between 27.95% and 28.65% of people in Canada aged 18 to 34 are smokers.

27.95% of 6 500 000 is equal to (6500000)(0.2795) = 1 816 750 people.

28.65% of 6 500 000 is equal to (6500000)(0.2865) = 1 862 250 people.

Therefore, between 1 816 750 and 1 862 250 people aged 18 to 34 in Canada are smokers.

Changes in Sample Size

What do you think should happen to the margin of error and the confidence interval if you increase the size of your sample?

As you would expect, if you increase the size of the sample population involved in your collection of data, the margin of error and, consequently, the confidence interval decreases. This is because as your sample size increases, the more likely your sample is to represent the population as a whole.

Therefore, if you keep the confidence level the same, when you increase your sample size, the confidence interval will decrease.

You may wish to add a summary of the effect of the change in sample size on the confidence interval and margin of error to your resource sheet.

Example 3

A small town is researching whether they should build a new splash pad at the community centre. Two different companies are asked to develop and implement surveys of the population to help the town reach a decision. Their results are shown in the table below, with a 95% confidence level.

Surveying Company	People in Favour	People Against	Sample Size	Margin of Error
Dave's Data Collecting Centre	412	362	774	±3.4%
Sharp Surveys	249	182	431	±5.7%

a) What happens to the margin of error as the sample size changes?

b) What happens to the confidence interval as the sample size changes?

c) Does the town have enough reliable data to decide whether to build a new splash pad? What should the town decide to do? Explain.

Solution

a) From this table, you can confirm that when the sample size is larger, the margin of error is smaller. Alternately, when the sample size is smaller, the margin of error is larger.

b) **Dave's Data Collecting Centre**

First, determine the percentage of people for and against the splash park:

$$\frac{412}{774} = 53.2\% \text{ of people are in favour of the splash pad}$$

$$\frac{362}{774} = 46.8\% \text{ of people are against the splash pad}$$

Therefore, Dave's Data Collecting Centre determined that 53.2% ± 3.4% or 49.8% to 56.6% of people support the splash pad.

Sharp Surveys:

$$\frac{249}{431} = 57.8\% \text{ of people are in favour of the splash pad}$$

$$\frac{182}{431} = 42.2\% \text{ of people are against the splash pad}$$

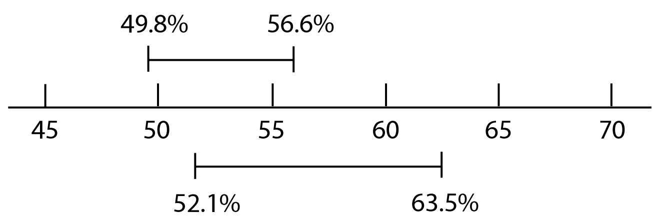
Therefore, Sharp Surveys determined that 57.8% ± 5.7% or 52.1% to 63.5% of people are in support of the splash pad.

As you can see, the confidence interval from Sharp Surveys is much larger than the confidence interval from Dave's Data Collecting Centre.

When the sample size increases, the confidence interval decreases along with the margin of error.

When the sample size decreases, the confidence interval increases along with the margin of error.

c) Dave's Data Collecting Centre determined that 49.8% to 56.6% of people support the splash pad while Sharp Surveys determined that 52.1% to 63.5% of people are in support of the splash pad. These confidence intervals overlap.

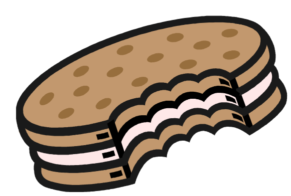


Therefore, it is likely that the true percentage of residents in favour of the splash pad is somewhere between 52.1% and 56.6%. Because these two confidence intervals overlap, you are able to make reliable inferences about the population.

As the interval of 52.1% to 56.6% represents more than half the population, the town should decide to build a new splash pad.

Example 4

An ice cream company specializes in making ice-cream sandwiches. In order to meet production standards, the mass of each ice-cream sandwich must be between 121 g and 127 g. To meet those standards, the manager sets the production line to manufacture ice-cream sandwiches with masses between 122 g and 126 g.



a) Determine the confidence interval the manager has determined to be best for his production line.

b) Determine the margin of error related to this confidence interval.

c) To have a certain confidence level that the ice-cream sandwiches being produced have masses within the required interval, certain sample sizes are needed. This information is shown in the table below.

Confidence Level	Sample Size
90%	421
95%	531
99%	675

Interpret what this table is saying.

d) To have a 99% confidence level that all ice-cream sandwiches have masses within the required interval, how many sample ice-cream sandwiches need to be weighed?

Solution

a) The confidence interval the manager has determined to be best for his production line is 122 g to 126 g.

b) To determine the margin of error, first find the value in the middle of 122 g and 126 g, which is 124 g.

Now, the margin of error is simply the difference between the median value and the upper and lower confidence interval limits, or 2 g. Therefore, the margin of error is ±2 g.

Another way of determining the margin of error is to recognize that it is equivalent to half the size of the confidence interval. As the confidence interval has a range of 4 g, the margin of error is half of 4 g, which is 2 g.

c) The table can be interpreted as follows:

To have a **Confidence Level** *of . . .*	*You need a* **Sample Size** *of . . .*
90%	421
95%	531
99%	675

This means that if the manager wants to be 90% sure that all ice-cream sandwiches have masses in the correct interval, he should sample 421 ice-cream sandwiches to ensure their masses are in the correct interval. Similarly, if the manager wants to be 95% sure that all ice-cream sandwiches have masses in the correct interval, he should sample 531 ice-cream sandwiches to ensure their masses are in the correct interval.

d) To have a 99% confidence level that all ice-cream sandwiches have masses within the required interval, the manager needs to weigh 675 sandwiches and ensure those masses are in the required interval.

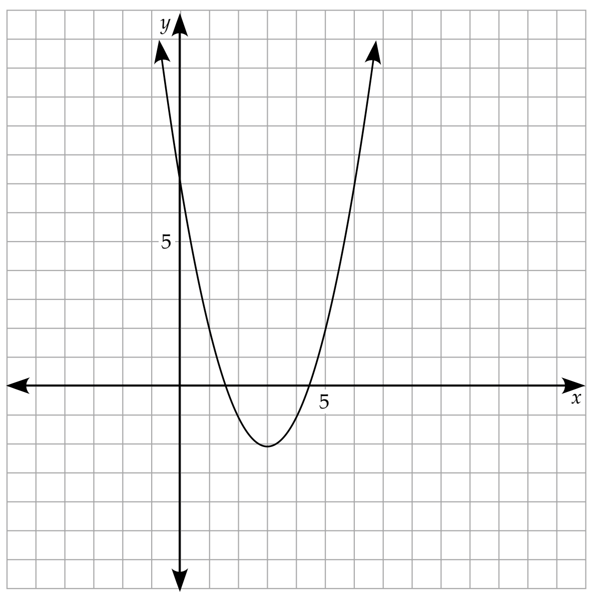
Learning Activity 6.6

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

Use the following graph to answer questions 1 to 3.



1. Determine the minimum value.
2. Determine the domain.
3. Determine the range.

4, Factor: $y^2 + 5y + 6$

5. Solve for x: $3x - 6 = x + 6$

continued

6. Write in lowest terms: $\frac{15}{42}$

7. Write as an equivalent fraction: $\frac{3}{11}$

8. Convert to a fraction in lowest terms: 0.375

Part B: How Confident Are You about Confidence Intervals?

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. A company conducted a study in which they asked consumers if they would use the company's brand of teeth whitening strips again. The results showed that 74% of consumers said yes, with a margin of error of ±5.3%. The study was said to be accurate 9 times out of 10. If all 537 of your Facebook friends tried the whitening strips, determine the range of friends that you can say with 90% confidence would use the whitening strips a second time.

2. A poll of Canadian citizens completed in May 2013 stated that 44% of Canadians would vote for a certain political party in the next federal election. This poll was reported accurate to within ±2%, 95 times out of 100.
   a) Determine the confidence interval.
   b) Determine the confidence level.
   c) What do the results of this poll mean?
   d) If Canada has an approximate population of 35.1 million, what is the range of votes that this party is likely to receive in the next election?

continued

Learning Activity 6.6 (continued)

3. Three different students are surveying the population of a small community. They are trying to determine what type of fundraiser will help them raise the most money. The two fundraiser choices are selling chocolates or magazines. Their results, with a 95% confidence level, are shown in the table below.

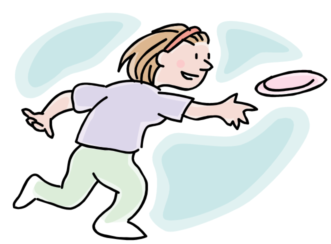
Student Surveyor	People in Favour of Chocolate	People in Favour of Magazines	Sample Size	Margin of Error
Corey	53	135	241	±3.5%
Rachel	24	77	123	±5.2%
Tanner	70	160	265	±3.4%

   a) Determine the confidence interval for the people in favour of selling chocolates, as determined by each student.

   b) Determine the confidence interval for the people in favour of selling magazines, as determined by each student.

   c) Explain which fundraiser would be more profitable for the students.

4. The standard disc in Ultimate Frisbee weighs 175 g. A manufacturer wants to start producing discs that will be as close as possible to the required weight. He sets the production line standards for a disc's weight to be between 174.8 g and 175.2 g.



   a) Calculate the margin of error the manufacturer has determined to be best for his production line.

   b) To determine a confidence level for discs being produced within the required weight interval, certain sample sizes are needed. This information is shown in the table below. Explain the meaning of the values in this table.

Confidence Level	Sample Size
90%	105
95%	214
99%	345

   c) The manufacturer decides to sample 500 discs. She finds that all 500 discs have weights within the required interval. Can she claim that 100% of all discs manufactured have weights between 174.8 g and 175.2 g? Explain.

Lesson Summary

In this lesson, you learned about confidence intervals, confidence levels, and margins of error. The confidence interval is dependent upon the margin of error. When you have a smaller margin of error, the confidence interval is also smaller. Confidence levels are dependent upon sample size. As sample sizes increase, your confidence that the data you gather is representative of the entire population increases and is reflected in the confidence level.

Assignment 6.3

Confidence Intervals

Total: 12 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. If you use technology, state your keystrokes or values used and your process. Final answers must include units. Answers given without supporting calculations and graphs will not be awarded full marks.

1. A poll of Canadian citizens completed in June 2013 stated that 84% of Canadians are proud of Canadian history, in general. This poll was reported accurate to within ±2.5%, 19 times out of 20.
   a) Determine the confidence interval. *(1 mark)*

   b) Determine the confidence level. *(1 mark)*

   c) What do the results of this poll mean? *(1 mark)*

   d) If Canada has a population of approximately 35.1 million, what is the range of people who are generally proud of Canadian history? *(2 marks)*

2. The boss of a small company wants to introduce mandatory casual Fridays, where employers would have to wear casual, informal clothes that day. Before doing so, she wants to get a sense of how many employees would be in favour of this change. To determine how many people she needs to survey, she creates a table to display the various sample sizes and the corresponding confidence levels.

Confidence Level	Sample Size
90%	65
95%	73
99%	85

a) How are confidence levels and sample sizes related? *(1 mark)*

b) How many people does the boss need to survey to have a 95% confidence level with her results? *(1 mark)*

c) She decides to survey 85 people and records that 78 out of the 85 employees surveyed are in favour of mandatory casual Fridays. If the boss's survey has a margin of error of 4.5%, what is the range of people who are likely in favour of mandatory casual Fridays? *(3 marks)*

d) What is the confidence level associated with the boss's survey? *(1 mark)*

e) Should the boss decide to implement casual Fridays? Explain why or why not. *(1 mark)*

Module 6 Summary

In this module, you first reviewed how to calculate measures of central tendency such as the mean, median, and mode. You then extended this knowledge to learn about measures of dispersion including the range and standard deviation.

Using the concept of standard deviation, you were able to understand the normal curve and the relationship between the standard normal curve and z-scores. This allowed you to make interpretations about data represented by normal distributions, and to determine if data values fit a normal curve pattern.

Finally, you were able to interpret the results of various polls using confidence levels, confidence intervals, and margins of error. All of these concepts should allow you to have a deeper understanding of statistics and become more analytical when using or collecting statistical data.

In the next module, you will be learning about scale factors and how they relate to the area of 2-D shapes as well as the surface area and the volume of 3-D objects.

Submitting Your Assignments

It is now time for you to submit the Module 6 Cover Assignment and Assignments 6.1 to 6.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 6 assignments and organize your material in the following order:

❑ Module 6 Cover Sheet (found at the end of the course Introduction)

❑ Module 6 Cover Assignment: The Indigenous Identity Population of Canada

❑ Assignment 6.1: Measures of Central Tendency and Dispersion

❑ Assignment 6.2: z-Scores and the Normal Curve

❑ Assignment 6.3: Confidence Levels

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

TEST TIME!

Self-Test

Module 6 – Precision Measurement

/54

1. Fill in the blanks.

(2 marks)

a) Name two of the four ways to state the tolerance for any measurement.

____

____

(1 mark)

b) State the scale reading for an enlargement that will double the lengths in a diagram of an object.

____

(2 marks)

c) State the maximum, minimum, and tolerance for a measurement reading $56.72^{+0.0025}_{-0.003}$.

____

____

____

(3 marks)

d) For a circle having a radius of 5.1 ± 0.03, find the maximum, minimum, and tolerance for its area if the area of a circle can be found by $A = \pi r^2$.

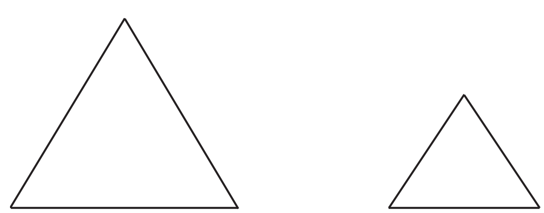
____

____

____

(1 mark)

e) For the given diagrams of triangles, what is the reduction scale?



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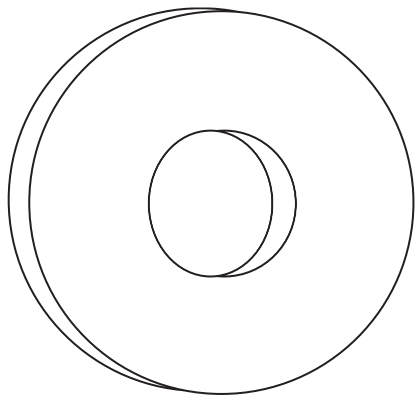
2. **Tolerance**

If the diagram below represents a washer and the radius of the outside circle is 6 ± 0.02 cm while the inside circle, which represents the cut out portion, has a radius of 2 ± 0.01 cm and its depth is $0.5^{+0.03}_{-0.02}$ cm, find the values for the following chart.

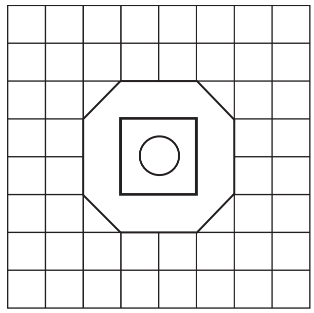


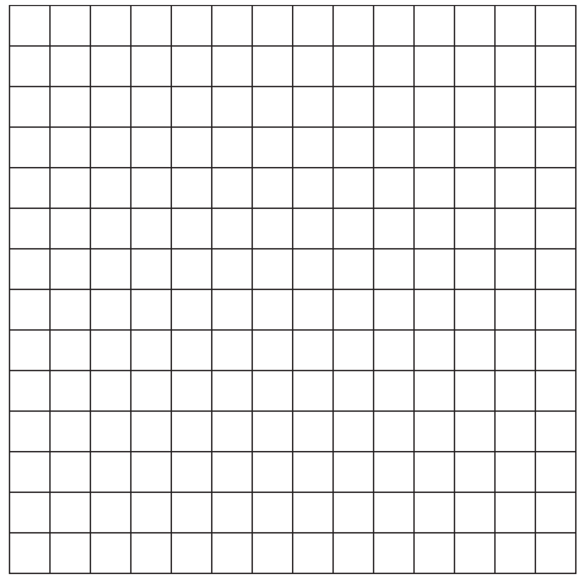
DIMENSION	BASIC	MAXIMUM	MINIMUM	TOLERANCE
Radius of inside of washer				
Radius of outside of washer				
Perimeter of outside of washer				
Area of washer				
Volume of washer				

(2 marks)

3. **Scale Diagrams**

Enlarge the following diagram according to the scale 3:1.





4. **Banking** *(6 marks)*

Prepare a bank reconciliation for the following on the accompanying form.

Chequebook Record

DATE	NO	PARTICULARS	✓	CHEQUES		DEPOSITS		BALANCE	
July 2		Balance Forward						786	57
July 3		Paycheque				452	34	1238	91
July 4	001	Andy's Department Store		57	14			1181	77
July 4		Cash		100	00			1081	77
July 5		Deposit				45	00	1126	77
July 7	002	Coop		135	79			990	98
July 9		Deposit				37	25	1028	23
July 9	003	Dandee TV		780	99			247	24
July 12	004	Bargain Store		28	69			218	55
July 12		Deposit				507	88	726	43
July 15	005	Dembinskys		112	65			613	78
July 22	006	Sharkeys		24	70			589	08
July 24	007	Mane Frame		53	25			535	83
July 25	008	Sturleys		237	74			298	09
July 25		Deposit				200	00	498	09

Bank Statement

V.R. Student **Bank of Manitoba** **Acct #555**
Box 123, Somewhere, Man

Date	Description	Debits	Credits	Balance
July 3	Balance forward			786.57
July 3			452.34	1238.91
July 4	#001	57.14		1181.77
July 5		100.00		1081.77
July 9	#003	780.99		300.78
July 9			37.25	338.03
July 12			507.88	845.91
July22	#006	24.70		821.21
July 25	Service Charge	10.50		810.71

(1) CHEQUE BOOK BALANCE . . . . $________

DEDUCT BANK CHARGES

Description	Amount	
Service charge		

(2) TOTAL BANK CHARGES . . . . $________

(3) ADJUSTED CHEQUE BOOK BALANCE . $________

(4) BANK BALANCE. . . . . $________

OUTSTANDING DEPOSITS

Date	Amount	

(5) TOTAL OUTSTANDING DEPOSITS . . $________

(6) TOTAL . . . . $________

OUTSTANDING CHEQUES

CH. NO.	Amount	

(7) TOTAL OUTSTANDING CHEQUES . . $________

(8) ADJUSTED BANK BALANCE. . . . . . . $________

5. **Property Tax** *(7 marks)*

Fill in the missing values in the following Taxation Notice.

PROPERTY DESCRIPTION

ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
63	5	8	7	CR007	80 ft	1

Civic Address

Title or Deed No.	Current Assessment Land	Current Assessment Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port Assessment
C60	22 000	63 570	T	?	10	35%	?

←ERRORS AND OMISSIONS EXCEPTED
←ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
←ALL CHEQUES MADE IN CANADIAN FUNDS
←BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
←RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY
15 000

MUNICIPAL TAXES

Description	Assessment	Mill Rate	Levy
General Municipal	?	47.32	?

By-Law No.	Term	Type	Frontage Levy	Mill Rate	Levy
62		Sewer and Water	84	.058	?
120		Sidewalk		3.106	?
57		Street		5.715	?

EDUCATIONAL TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	?	5.96	?
Provincial Education 2	?	13.86	?

PROVINCIAL TAX CREDITS

(See Manitoba Enclosure For Additional Information)	Assessment	Levy
	Manitoba Resident Homeowner Tax Assistance	$250.00

TOTAL TAXES DUE

Municipal Tax	Education Tax	Total Taxes	Prov. Credits	Net Taxes	Arrears/Credits	Added Taxes	Taxes Due
?	?	?	?	?	?	?	?

6. Solve the following applications by setting up a system of equations or constraint inequalities and using your calculator where necessary to solve the resulting system.

(5 marks)

a) Premium gasoline sells at 62 cents/L. Regular gasoline sells at 59.5 cents/L. To boost sales, a middle octane gasoline is formed by mixing premium and regular. If 1000 L of this middle octane gasoline is produced and sold at 60.5 cents/L, how much of each type of gasoline was used?

(5 marks)

b) Slacks Incorporated sold 6000 pairs of slacks last month at an average price of $44.00 each. The store is going to increase prices in order to increase profits. Sales forecasts indicate that sales will drop by 200 for every dollar increase in price. On the average the company pays $25.00 for each pair of slacks it sells. What price will maximize profits?

c) A mining company has two mines: Takoma East and Takoma West. One shift of 100 miners at each of the two mines produces lead, uranium, and nickel ores in the following amounts:

(10 marks)

Ore	Takoma East	Takoma West
Lead	2 tonnes	5 tonnes
Uranium	400 tonnes	300 tonnes
Nickel	500 tonnes	500 tonnes

The forecast demand for each of these ores for the next year is:

900	tonnes of Lead
100 000	tonnes of Uranium
150 000	tonnes of Nickel

How many shifts of 100 miners should be worked at each of the mines in the coming year to keep the salary bill to a minimum? (Assume all miners are paid the same.)

Reminder:

When you complete this Self-Test, send it in for marking. Include the Module 6 Cover Sheet.

Send to:

ISO Tutor/
Marker
555 Main St.
Winkler, MB
R6W 1C4

Include Cover Sheet

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Grade 11 Applied Mathematics (30S)

Module 7
Mathematical Models

Module 7: Mathematical Models

Introduction

This module is about solving problems. You will first examine ways in which rates are used to measure such things as speed and money. As well, you will learn how to convert between rates, which is useful knowledge when travelling, as many countries measure distance, velocity, or currency using different units. You will also be introduced to how these rates can be represented in graphical form.

Many problems have more than one possible solution, and in this module you will learn to find the best possible solution to a problem involving linear inequalities. You will learn how linear inequalities share many similarities with linear equations. You will learn how to represent linear inequalities with a special inequality symbol and how to graph linear inequalities—first, by graphing one at a time, and then by graphing multiple inequalities at once. This will prepare you for the final lesson where you combine everything you have learned in order to be able to find the optimal solution to a problem involving linear inequalities.

Problems involving linear inequalities often arise when you are working in situations that involve profit and costs. Businesses usually want to find ways to maximize their profits, which means that they must minimize their costs. You will be able to solve problems such as these after completing this module.

Assignments in Module 7

When you have completed the assignments for Module 7, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Skyscraper Puzzles
2	Assignment 7.1	Interpreting Rates Algebraically and Graphically
5	Assignment 7.2	Linear Inequalities and Systems of Linear Inequalities
6	Assignment 7.3	Optimizing Solutions to Linear Inequalities

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, and can be either handwritten or typewritten. Both sides of the sheet may be filled. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 7. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 5 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 5 to 8.

Resource Sheet for Module 7

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Module 7 Cover Assignment: Skyscraper Puzzles

Skyscraper puzzles are similar to other number logic puzzles such as Sudoku. In a skyscraper puzzle, you are placing "skyscrapers" on a grid. The heights of these skyscrapers are represented by numbers starting with 1. Each increasing number, such as 2, 3, or 4, represents a taller skyscraper.

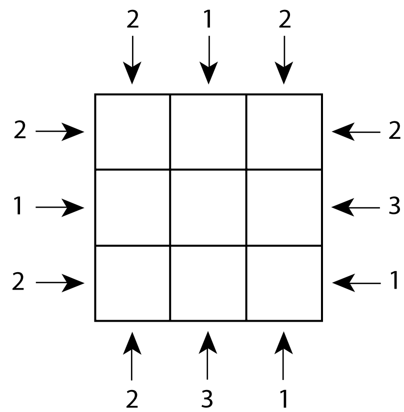
The number clues surrounding the grid represent the number of skyscrapers you can see from that perspective. Therefore, if you have the number "1" as a clue, you can only see one skyscraper. This means that the tallest skyscraper must go first. The reason for this is that the tallest skyscraper blocks the view of all the shorter skyscrapers behind it. The remaining skyscrapers could be in any order behind the tallest skyscraper.

In the course, you will only be dealing with 3 × 3 and 4 × 4 grids. In the 3 × 3 grid, the skyscrapers have heights of 1, 2, and 3. Similarly, in the 4 × 4 grid, the skyscrapers have heights of 1, 2, 3, and 4.

In the 3 × 3 skyscraper grid, each of the numbers 1, 2, and 3 must be used once and only once in each row and column. This is the same rule as in Sudoku. Consider the following examples.

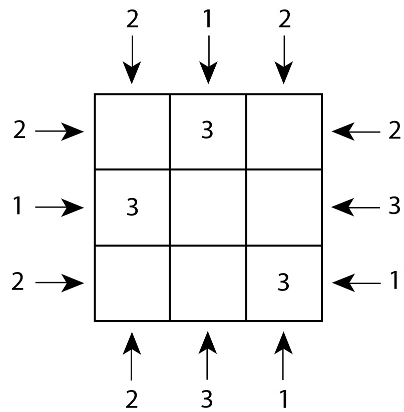
Example 1

Complete the Skyscraper puzzle below by filling in each of the 9 squares with values of 1, 2, and 3, so that the heights of the skyscrapers are determined by the numbers surrounding the grid.

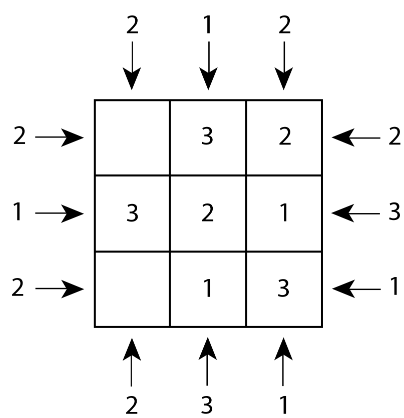


Solution

One strategy is to look for the clues with a value of "1." Since this number tells you that, from that vantage point, you can see only one skyscraper, the first square must contain the tallest skyscraper because it is blocking your view of all the other buildings and you cannot see any skyscrapers behind it. The tallest skyscraper is represented by "3," so these squares of the puzzle can be filled in as follows:

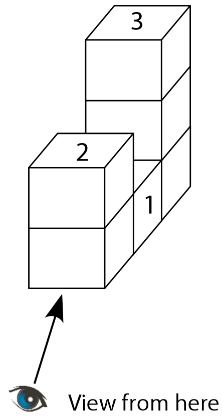


Now, continue with the clues around the outside of the grid that have the largest value in the puzzle. The value "3" indicates that you can see all three skyscrapers from that vantage point, so the smallest skyscraper must be closest and the tallest must be furthest away, and the skyscrapers will be in a line from the lowest to the tallest height. Fill in the puzzle as follows:



Now, you can fill in the rows with the missing numbers. For example, the top right-hand corner needs to be a "2" as there are no 2s in that column. This fits with the top right-hand corner clue of "2" as well. In the row on the right, if you look down, you will have skyscrapers of height 2, then 1, then 3. The skyscraper of height 2 will hide the skyscraper of height 1. However, the skyscraper of height 2 will not hide the skyscraper of height 3. Therefore you can see two skyscrapers—the first skyscraper of height 2 and the last skyscraper of height 3.

Consider the 3-D view below:



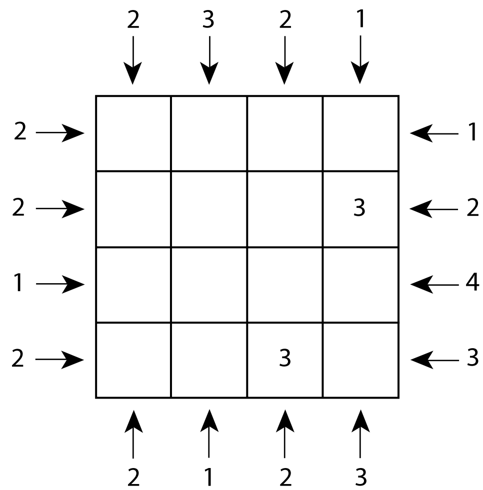
	2	1	2	
2		3	2	2
1	3	2	1	3
2		1	3	1
	2	3	1	

You can now complete the puzzle.

	2	1	2	
2	1	3	2	2
1	3	2	1	3
2	2	1	3	1
	2	3	1	

Example 2

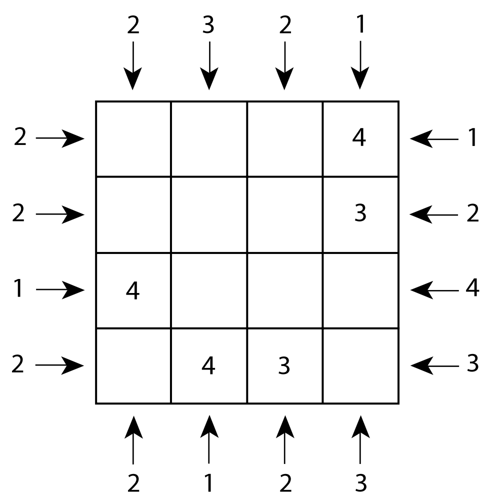
Consider a 4 × 4 puzzle.



Note: In this puzzle, two of the skyscraper heights have been included for you.

Solution

First, look for all the "1" clues. These clues tell you that the tallest skyscraper, which has a height of 4, must be closer to that end of the row or column.



Now, look for all the “4” clues. These clues tell you that the skyscrapers must go in order from lowest to tallest for you to be able to see them all.

	2	3	2	1	
2				4	1
2				3	2
1	4	3	2	1	4
2		4	3		3
	2	1	2	3	

Now, you need to use your logical thinking skills to complete another row. Consider the second column. You need to be able to see 3 skyscrapers from top to bottom. The last 2 skyscrapers have heights of 3 and 4. Therefore, there are only two possible arrangements for the first two skyscrapers:

1 and 2 or

2 and 1

But, if you put skyscraper 1 first and skyscraper 2 second, this will leave you with skyscrapers of heights 1, 2, 3, and 4. In this order, you can see them all. The clue indicates you should only be able to see 3 skyscrapers. Therefore, the skyscrapers must go in the order: 2, 1, 3, 4. In this order, you can see skyscrapers of height 2, 3, and 4.

	2	3	2	1	
2		2		4	1
2		1		3	2
1	4	3	2	1	4
2		4	3		3
	2	1	2	3	

Now, look at the top row where you have a similar scenario. The second box has a skyscraper with height 2. The fourth box has a skyscraper with height 4. Therefore, only 2 arrangements are possible:

1, 2, 3, and 4 or

3, 2, 1, and 4

In the first arrangement, the skyscrapers go from lowest to tallest, meaning you would see all 4 skyscrapers. As the first-row clue indicates, you can only see 2 skyscrapers so this won't work.

In the second arrangement, you would only be able to see 2 skyscrapers: the skyscraper of height 3 (which hides the skyscrapers of heights 1 and 2) and the skyscraper of height 4. This is consistent with the clues given.

	2	3	2	1	
2	3	2	1	4	1
2		1		3	2
1	4	3	2	1	4
2		4	3		3
	2	1	2	3	

You can now complete the rest of the puzzle, since you know that you can only use a number once in each row and in each column.

	2	3	2	1	
2	3	2	1	4	1
2	2	1	4	3	2
1	4	3	2	1	4
2	1	4	3	2	3
	2	1	2	3	

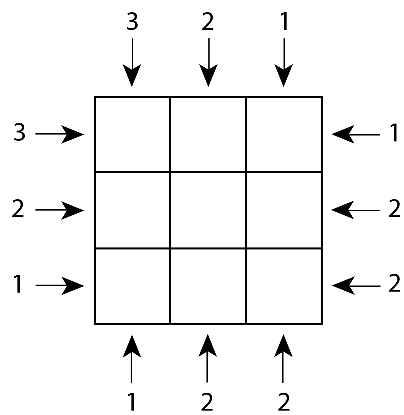
Module 7 Cover Assignment

Skyscraper Puzzles

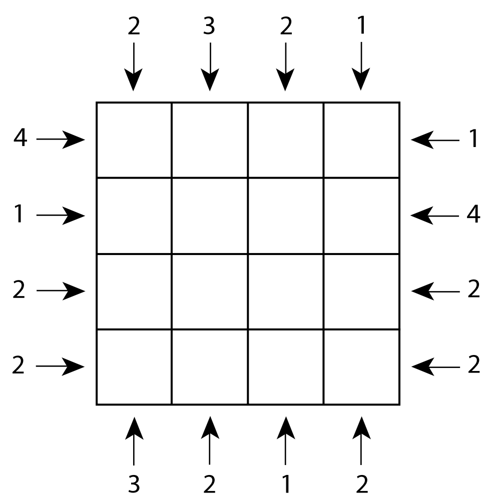
Total: 10 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. Complete the 3×3 skyscraper puzzle below. *(4 marks)*



2. Complete the 4×4 skyscraper puzzle below. *(6 marks)*



Notes

Lesson 1: Rates in Your Life

Lesson Focus

In this lesson, you will
- ❏ learn how to interpret a rate
- ❏ determine the factors that can influence a rate
- ❏ convert between two different rates
- ❏ compare rates

Lesson Introduction

If you have ever shopped at a grocery store, driven a vehicle, or bought gasoline, then you have encountered rates. You would have seen rates such as how much seafood costs per kilogram or how far a vehicle can travel at a certain speed.

In this lesson, you will look at these rates more closely to interpret them. You will also compare different rates, such as comparing speed in kilometres per hour to miles per hour. These are all useful skills that allow you to travel more comfortably and safely in other countries, or to compare the prices of commodities when you shop.

Rates and Unit Rates

In mathematics, a **rate** is a comparison between two quantities that are not measured in the same units. A rate is usually given as a fraction or a decimal. For example:

$1.29/100 g

The two quantities being measured are dollars ($) and grams (g). This rate indicates that this item costs $1.29 for every 100 grams.

A **unit rate** is a specific type of rate between two quantities that you are measuring, where the second quantity is measured as a single unit. For example:

100 km/hr

The two quantities being measured are kilometres (km) and hours (hr.). This rate indicates that the speed is 100 kilometres for every 1 hour. Note that the second quantity, hours, has a measurement of one.

You may wish to include the above definitions on your resource sheet.

Example 1

Describe a scenario that would match each of the rates below. Indicate which of the following rates are unit rates.

a) \$18/hr.

b) 7.1 L/100 km

c) \$22.56/kg

d) 30 mg/kg

Solution

a) This is a unit rate because the rate indicates \$18 for every one hour. A scenario that would match this rate is how much a person is paid per hour.

b) This is not a unit rate because the L are measured in 100 km, and a unit rate would be measured in 1 km. A scenario that would match this rate would be the amount of gas that a car consumes, 7.1 L, to travel 100 kilometres.

c) This is a unit rate because the rate indicates \$22.56 for every one kilogram. A scenario that would match this rate is the cost, \$22.56, for every kilogram of a certain beef product, such as strip loin.

d) This is a unit rate because the rate indicates 30 milligrams for every one kilogram. A scenario that would match this rate is the amount of medicine required, 30 mg, for every kilogram of a patient's body weight.

For each of the scenarios in Example 1, these rates could change for many reasons. For example, a rate of \$18/hour might change if someone had more experience or more education. In that case, the individual might be paid a higher rate, such as \$20/hour or \$25/hour.

Example 2

For each of the scenarios in Example 1, explain what could cause each of the rates to be lower.

Solution

a) This is a rate representing the amount of money a person is paid for every hour of work. If a person were hired for a less demanding job or an entry-level job, such as working at McDonalds, the rate would be lower.

b) This is a rate representing fuel efficiency. This rate would be lower—say, 6.8 L/100 km, if the vehicle were more efficient. In other words, if the vehicle used less gas per kilometre, the rate would be lower.

c) This is a rate representing cost per kilogram of food. If the food were less expensive, such as sandwich meat, then the rate would be lower.

d) This is a rate representing the amount of medicine administered to a patient per kilogram of body weight. If the medicine were stronger, this rate would be lower as less medicine would have the same benefit.

Example 3

Explain what the following rate represents. Then, explain two factors that could influence the rate.

$4.29/sq. ft.

Solution

This rate represents the cost of an item for every square foot. This could represent the cost of hardwood flooring for every square foot purchased.

One factor that could influence this rate is the type of hardwood chosen, since some types of hardwood are more expensive than others. Another factor that could influence this rate is the availability of the desired type of flooring. If a certain type of flooring is in high demand and it is not readily available, then the rate could be higher per square foot.

It is also possible to compare rates to determine which item is the better buy or which vehicle has the faster speed. Consider the following examples.

Example 4

A slow walking speed is 1 m/s. How many kilometres per hour does this represent?

Solution

In this type of scenario, it is a good idea to set the rates up as fractions to compare them.

$$\frac{1\text{ m}}{1\text{ s}} = \frac{?\text{ km}}{?\text{ h}}$$

In this case, you are trying to convert seconds to hours. You know that there are 60 seconds in one minute. There are also 60 minutes in one hour. Therefore, there are 60 × 60 = 3600 seconds in one hour.

You also need to convert metres to kilometres. There are 1000 metres in one kilometre.

To make the conversions from seconds to hours and from metres to kilometres, the values in the fraction must be in the proper position. For example, to convert seconds to hours, you want to cancel out the seconds and only be left with hours. Now, since the term containing the measurement of seconds is in the denominator of the proportion and you need to cancel out the seconds, the conversion factor containing seconds must be in the numerator of the second proportion.

The conversion statement will be:

$$\frac{1\text{ m}}{1\ \cancel{\text{s}}} \times \frac{3600\ \cancel{\text{s}}}{1\text{ h}} = \frac{3600\text{ m}}{1\text{ h}}$$

Note how the units of seconds cancel in the numerator and denominator, and you are left with "m" in the numerator and "h" in the denominator.

Continuing this process, the term containing the units of metres is in the numerator of the proportion, and you want to cancel out metres so that you have only kilometres remaining. Therefore, the conversion factor containing metres must go in the denominator of the proportion.

The conversion statement will be:

$$\frac{3600\ \cancel{\text{m}}}{1\text{ h}} \times \frac{1\text{ km}}{1000\ \cancel{\text{m}}} = \frac{3.6\text{ km}}{1\text{ h}}$$

Note how the units of metres cancel in the numerator and denominator, and you are left with "km" on the top and "h" on the bottom.

Therefore, 1 m/s is equivalent to 3.6 km/h.

Example 5

A cheetah can reach a speed of approximately 60 miles per hour. The average speed for highway travel in Manitoba is 100 kilometres per hour. Compare these two speeds to determine which is faster.

Solution

You can complete this question in one of two ways. You can convert both speeds to either miles/hour or kilometres/hour. In this scenario, convert both speeds to km/h as you are probably more familiar with this system.

The cheetah can run 60 miles per hour. To convert this to kilometres per hour, you need to know how to convert between miles and kilometres.

Since there are 1.6 kilometres in one mile, the conversion calculation would be as follows:

$$\frac{60 \text{ mi.}}{1 \text{ h.}} \times \frac{1.6 \text{ km}}{1 \text{ mi.}} = \frac{96 \text{ km}}{1 \text{ h}}$$

Notice how the units of miles cancel out.

Therefore, a cheetah can run close to 100 kilometres per hour, similar to a car at highway speeds. However, a car at 100 km/h will still be slightly faster than the cheetah at 60 mi./h. or 96 km/h.

Note: You are not expected to memorize the various equivalencies. You may want to record some of the conversions on your resource sheet, such as 1 mile = 1.6 kilometres. You can use measurement conversion tables found on the Internet to find equivalencies you may not know, such as number of pounds in one kilogram.

Example 6

Colby lives in Boissevain and often goes shopping in the United States. Before Thanksgiving dinner, he notices that turkeys are on sale in Boissevain for \$4.46/kg. After crossing the border into the United States, he sees turkeys on sale for \$1.49/lb. Which turkey is the better buy for Colby? (Ignore exchange rates in this scenario. Notice that the abbreviation for pound is lb.).

Solution

Again, you can approach this question in two different ways. You can convert both prices to either dollars per kilogram (\$/kg) or dollars per pound (\$/lb.).

Let's convert both prices to dollars per kilogram. Remember that there are 2.2 pounds in one kilogram. The conversion calculation would be as follows:

$$\frac{\$1.49}{1\text{ lb.}} \times \frac{2.2\text{ lb.}}{1\text{ kg.}} = \frac{\$3.28}{1\text{ kg}}$$

Therefore, a turkey costs $3.28 per kilogram in the United States and $4.46 per kilogram in Canada. The better buy for Colby is to buy the turkey in the United States.

Make sure you complete the following learning activity to practise understanding rates and converting between different types of rates.

Learning Activity 7.1

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Convert 2300 m to km.
2. Convert 36 inches to feet.
3. Convert 532 mm to m.
4. Convert 72 inches to yards.
5. Simplify: $\dfrac{(8+7)(2)}{5}$
6. Are $\dfrac{3}{4}$ and $\dfrac{75}{100}$ equal fractions?
7. Evaluate the expression $x^2 - 2x + 3$ for $x = 3$.
8. Identify the y-intercept for the function $y = -5x + 6$.

continued

Learning Activity 7.1 (continued)

Part B: Rates and Rate Conversion

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Convert 6.7 lb. to kg.
2. Convert 42 m to feet.
3. Convert 72 mi. to km.
4. Describe a scenario that would match each of the rates below. Indicate which rates are unit rates. If the rate is not a unit rate, convert it to a unit rate.
   a) 156 MB/s
   b) 1.61 births/woman
   c) 190 L/4 min.
   d) 180 words/3 min.
5. What would cause each of the rates in question 4 to increase?
6. Explain what the following rate could represent. Then, suggest two factors that could influence the rate.

   $1.25/L
7. The fastest marathon ever run was completed in 2 hours and 3 minutes. A marathon is approximately 42 km. If a marathoner could sustain this pace,
   a) how far could he run in a minute?
   b) how far could he run in 30 minutes?
   c) how far could he run in an hour?
   d) how far could he run in 24 hours?
8. Convert 85 km/h to miles per hour.
9. Convert 60 km/h to metres per second.
10. Which is the better buy: 10.56 $/kg or 3.67 $/lb.?

continued

Learning Activity 7.1 (continued)

11. Cyruss notices that pepperoni is available at the local deli for $1.19/100 g. He is planning to make pepperoni pizzas for a party that he is hosting Friday night and he needs 2.5 kg of pepperoni. How much will Cyruss have to pay for the pepperoni?

12. In November 2015, one Canadian dollar ($1 CAD) was worth $0.75 US.
    a) If a person has $250 US, how much is this worth in Canadian dollars?
    b) If a person has $250 CAD, how much is this worth in US dollars (USD)?

13. Experts have concluded that texting while driving is extremely dangerous. If a driver were to read a text for 7 seconds, he or she would be at great risk. If the driver is travelling at the following speeds, how far will the vehicle have moved during those 7 seconds?
    a) 60 km/h
    b) 80 km/h
    c) 100 km/h

14. Kiera and Cole are hosting a wedding with 180 guests. They plan to order hors d'oeuvres and estimate that each guest will consume four hors d'oeuvres. The hors d'oeuvres come in platters of 25, with a cost of $15 per platter.
    a) How many platters of hors d'oeuvres do they need to order?
    b) How much will this cost?

15. Jacob is a 200-pound man who can burn 380 calories if he runs for 30 minutes. Lexi is a 125-pound woman who can burn 76 calories for every 10 minutes she jumps rope.
    a) How long will Jacob need to exercise to burn 500 calories?
    b) How many calories will Lexi burn in the same amount of time it takes Jacob to burn 500 calories?

Lesson Summary

In Lesson 1, you looked at rates as well as unit rates. You also determined what a rate represented and possible factors that could cause the rate to either increase or decrease. You learned how to make conversions between rates that are expressed in different units. Using this skill, you were able to compare rates to see which was faster, cheaper, or better in some way. In the next lesson, you will look at graphical representations of rates.

LESSON 2: DRAWING GRAPHS TO REPRESENT RATES

Lesson Focus

In this lesson, you will
- ❑ interpret a graph that represents a rate
- ❑ draw a graph to represent a rate
- ❑ explain the relationship between the slope of a graph and a rate

Lesson Introduction

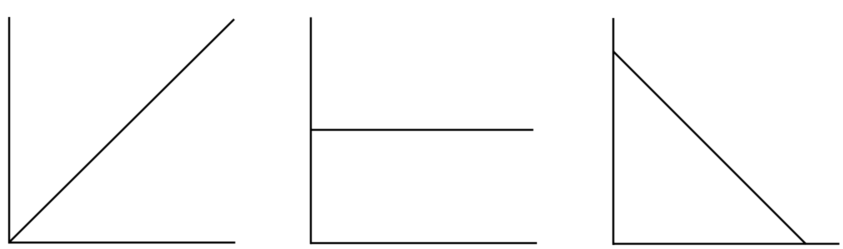
Not all rates stay the same. For example, the price of gasoline is constantly rising and falling. Also, your speed as you drive is constantly increasing or decreasing, depending on factors such as whether you are stopping at a red light or speeding up once the light turns green.

To visualize these changes in rates, mathematicians represent the information graphically. With graphs, it is easy to visualize how rates increase, decrease, or stay the same, simply by analyzing the slope of the graph. When you analyze rates in this way, you can see the bigger picture and can see how the rate is changing over time.

Drawing Graphs to Represent Rates

Consider the three following graphs representing acceleration. Acceleration is the rate at which the speed of a moving object (m/s) changes over time (s). Therefore, the units for acceleration are actually m/s^2. If you take physics, this may look familiar.

Study the three graphs below. Try to interpret what is happening to the rate of acceleration in each of the three graphs.



In the first graph, the line has a positive slope. Therefore, the acceleration is increasing in this graph.

In the second graph, the line has no slope. Therefore, there is no acceleration and the speed stays the same in this graph. In other words, the vehicle is moving at a constant speed.

In the third graph, the line has a negative slope. Therefore, the speed is decreasing and there is deceleration, not acceleration.

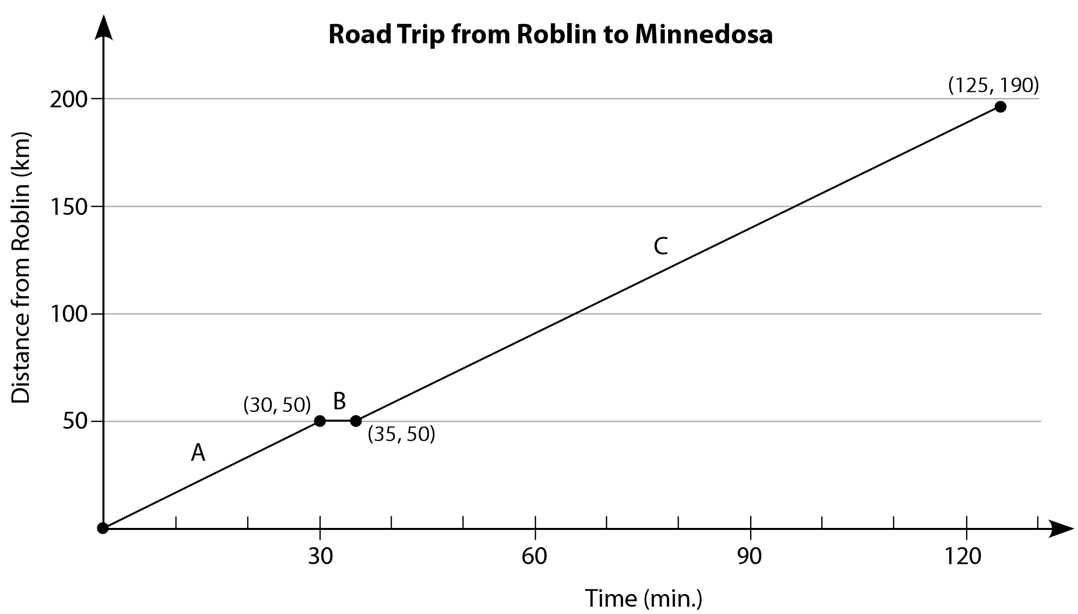
Note: The **slope** of a line measures its steepness. Slope compares how far the line moves vertically in relation to how far the line moves horizontally. Slope is also known as $\frac{\text{rise}}{\text{run}}$.

Consider the following example.

Example 1

Aleasha and her family are travelling from Roblin, Manitoba, to Minnedosa, Manitoba. The total distance is 190 kilometres. Aleasha and her family travel to Russell, which is 50 kilometres away. This portion of the trip takes 30 minutes. Then, they decide to stop for gas, which takes 5 minutes. Aleasha and her family then continue the remaining distance to Minnedosa, which takes them a total of 90 minutes. Draw a graph to represent their trip. Indicate the slope of each segment of their trip and what the slope represents.

Solution



This trip can be divided into three segments (A, B, and C), as displayed on the graph above.

- Section A represents the trip from Roblin to Russell. In this portion of the trip, Aleasha's family travels 50 km in 30 minutes.
- Section B represents the stop in Russell. In this portion of the trip, Aleasha's family does not travel at all for 5 minutes.
- Section C represents the trip from Russell to Minnedosa. In this portion of the trip, Aleasha's family travels the remaining 140 kilometres in 90 minutes.

In this graph, there are three different slopes. Remember that the slope represents the change in the rise (the y-axis) over the change in the run (the x-axis).

In this graph, this would be $\frac{\text{distance}}{\text{time}}$.

The units are $\frac{\text{kilometres}}{\text{hour}}$.

If you look at the units, the slope represents the speed.

In Section A, the speed is 50 km in 30 minutes. This is the same speed as travelling 100 km in 60 minutes or 100 km/h.

In Section B, the speed is 0 km in 5 minutes. This can be represented by 0 km/h.

In Section C, the speed is 140 kilometres in 90 minutes (90 minutes is equivalent to 1.5 hours). As speed is always written in kilometres per hour, you need to convert this speed to the correct units. To do this, you can set up the following proportion.

$$\frac{140 \text{ km}}{1.5 \text{ h}} = \frac{x \text{ km}}{1 \text{ h}}$$

You need to solve for x in this proportion. To do this, you need to isolate x. First, multiply both sides by 1 h to get rid of the denominator on the right-hand side.

$$\frac{140 \text{ km}}{1.5 \text{ h}} \times 1 \text{ h} = \frac{x \text{ km}}{1 \cancel{\text{h}}} \times 1 \cancel{\text{h}}$$

This will result in:

$$\frac{140 \text{ km}}{1.5} = x \text{ km}$$

Note: The units of hours on the left-hand side will cancel.

Now, complete the division on the left-hand side.

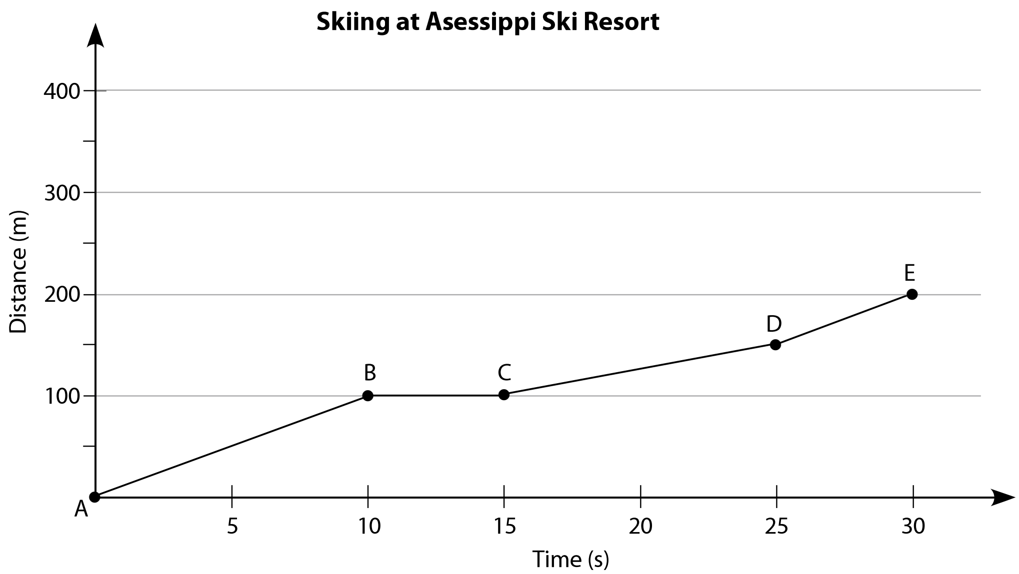
$$93.\overline{3} \text{ km} = x \text{ km}$$

Therefore, the distance travelled in one hour is $93.\overline{3}$ km. This represents a speed, or slope, of $93.\overline{3}$ km/h.

You may also be asked to interpret a graph that represents a rate. Consider the following example.

Example 2

Consider the graph below. This graph represents the distance from the top of a hill as a skier travels down one of the runs at the Asessippi Ski Resort.



a) During which segment is the skier travelling the fastest? How fast is she travelling?

b) During which segment is the skier travelling the slowest? How fast is she travelling?

c) Describe a scenario that could represent this graph.

Solution

a) The skier is travelling the fastest when the slope of the line is the steepest. This occurs during section AB and section DE.

   In section AB, the skier travels 100 m in 10 seconds. Or, the skier is travelling at a rate of 10 m/s.

   In section DE, the skier travels 50 m in 5 seconds. Or, the skier is travelling at a rate of 10 m/s.

b) The skier is travelling the slowest during section BC. During this section, the skier does not move at all for 5 seconds.

c) A scenario that could represent this graph is the following:

   During section AB, the skier travels 100 m down the mountain at a relatively quick pace for 10 seconds. During section BC, the skier comes to a stop and rests for 5 seconds. During section CD, the skier continues at a slower pace, 50 metres per 10 seconds, down the mountain. Finally, during section DE, the skier picks up the pace for the final 5 seconds and travels at a rate of 10 m/s.

Being able to interpret graphs and slopes is an important skill that can be useful in many situations, such as investing in the stock market or interpreting the movement of vehicles or athletes. Be sure to practise these skills in the following learning activity.

Learning Activity 7.2

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Convert 6 feet to inches.
2. Convert 5670 centimetres to metres.
3. Simplify: $4 + (3)(5) - 7$.

continued

4. Are $\frac{2}{10}$ and $\frac{5}{25}$ equal fractions?

5. Evaluate the expression $-2x^2 + 1$ for $x = -2$.

6. Identify the x-intercept for the function $y = -2x + 6$.

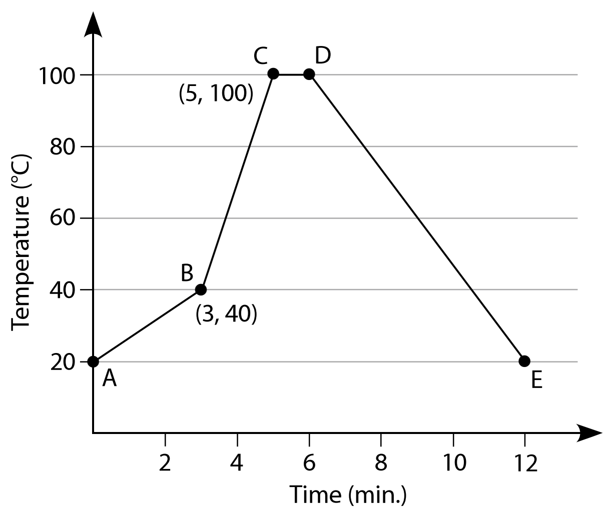
7. If a line segment has a run of 8 and a rise of 13, what is the slope of the line?

8. Is the point (1, 1) on the line $y = -7x + 6$?

Part B: Interpreting Graphs

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. The following graph represents the temperature of water in a kettle over a 15-minute time period.

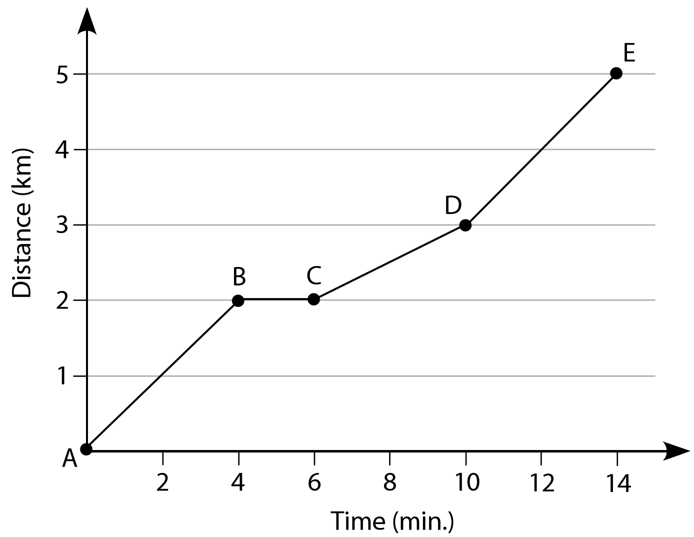


a) What is happening in the first five minutes according to this graph?
b) What is happening in the last six minutes according to this graph?
c) During which period of time does the water in the kettle heat up the fastest? At what rate does this occur?
d) What does a slope of zero mean in this graph?

continued

Learning Activity 7.2 (continued)

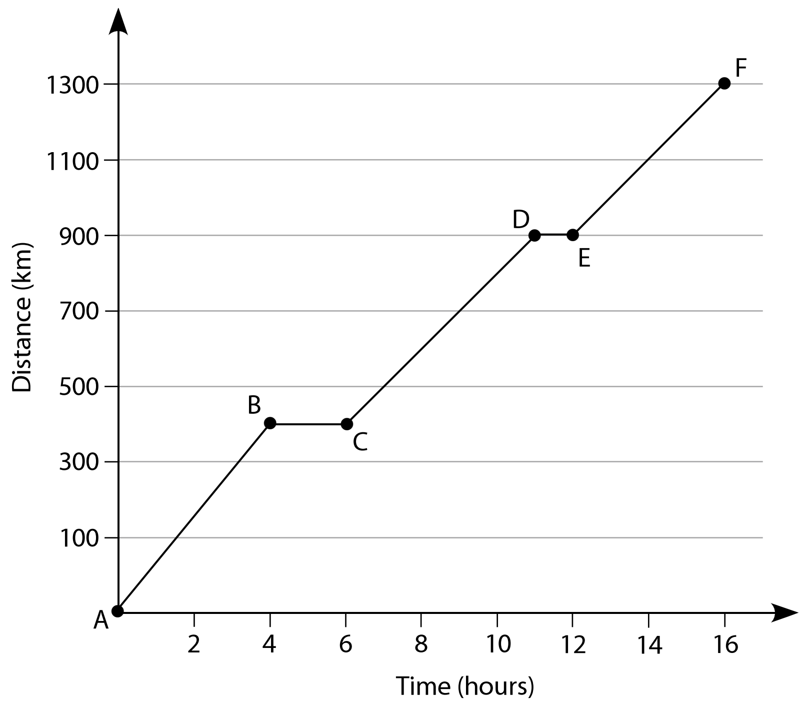
2. The following graph shows Roger's trip as he bikes to school.



   a) How long did it take Roger to cycle to school?
   b) How far is Roger's house from school?
   c) On his way to school, Roger had to stop and wait at a traffic-light-controlled intersection.
      i) Which portion of the graph represents this wait?
      ii) How long did Roger have to wait?
   d) Determine Roger's rate of speed, as a unit rate, for the following line segments:
      i) CD
      ii) DE
   e) What is the difference in Roger's rate of speed between CD and DE? Why might this difference have occurred?

continued

3. A family decides to take a road trip from Brandon, Manitoba to Banff, Alberta. The following graph represents the distance they travelled by car during the road trip.

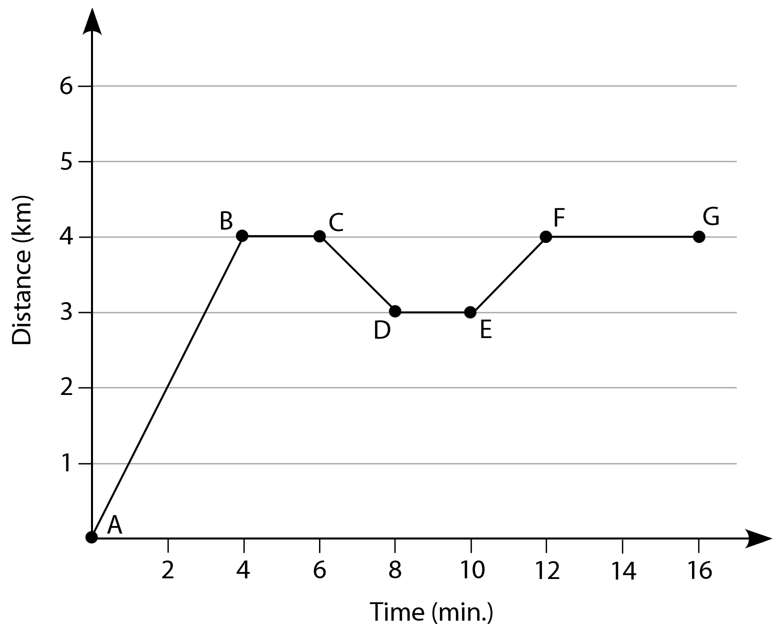


   a) How far is it from Brandon to Banff?
   b) If the family did not make any stops, how long would the road trip have taken?
   c) Did the family travel at the same speed for the entire trip? If so, what was this speed? If not, what were the different speeds that they travelled at during the trip?
   d) Make up a story that represents the family road trip. Make sure you explain each section of the graph.

continued

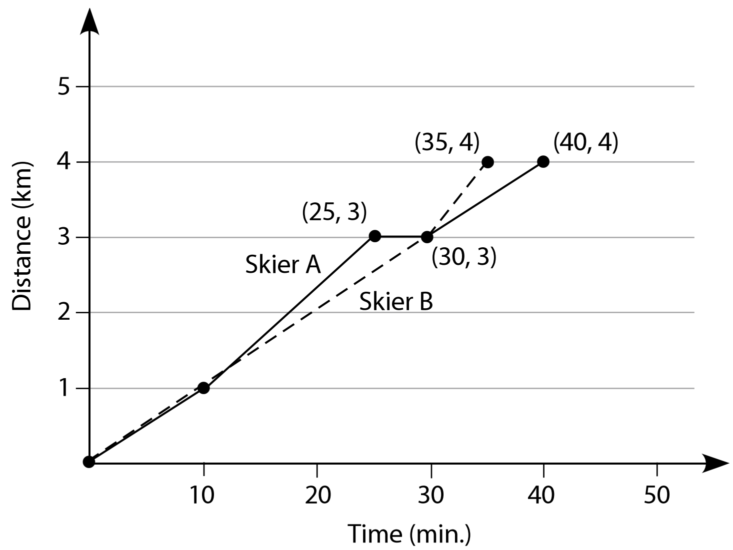
Learning Activity 7.2 (continued)

4. The following graph represents the distance of Jerod's car from his home as he goes for a drive to his friend's house. Use the distance-versus-time graph to make up a story about Jerod's trip.



continued

5. One winter day, two cross-country skiers at the Windsor Park Nordic Centre decided to have a race. The distance of each skier from his or her starting point is represented on the graph below.



   a) Which skier was the fastest?
   b) Which skier travelled at a more consistent pace?
   c) Determine the number of different speeds for Skier A. How is this demonstrated by the slope of the graph?
   d) Over which interval is Skier A travelling the fastest?
   e) Over which interval is Skier B travelling the fastest?
   f) Determine the two different speeds for Skier B in unit rates.

6. Draw a graph to represent the following scenario. Melinda is baking holiday cookies. She needs to preheat the oven to 350° F. The room temperature of Melinda's house is 70° F. She starts preheating the oven at 10 a.m. It takes 10 minutes for the oven to heat. Melinda then puts in two batches of cookies, one right after the other. Each batch of cookies takes 10 minutes to bake. After the cookies are baked, Melinda starts cooking dinner. She needs to increase the oven temperature to 400° F. This takes 5 minutes. Melinda then puts in a casserole that takes 40 minutes to cook. The oven is then turned off and it takes 1 hour to return to room temperature.

continued

Learning Activity 7.2 (continued)

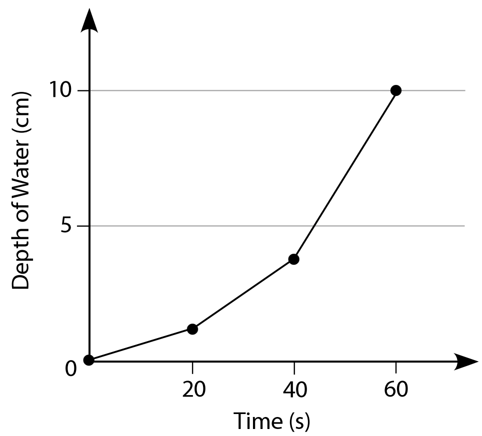
7. Draw a graph to represent the following scenario. Oksana and Rochelle are going for a jog together. They both travel around the same 5-kilometre circular route. However, Oksana starts running counter-clockwise while Rochelle starts running clockwise. In the first 10 minutes, Rochelle runs 2 kilometres. It then takes her the remaining 20 minutes to run 3 kilometres. Oksana runs at 10 km/h and keeps a constant speed.

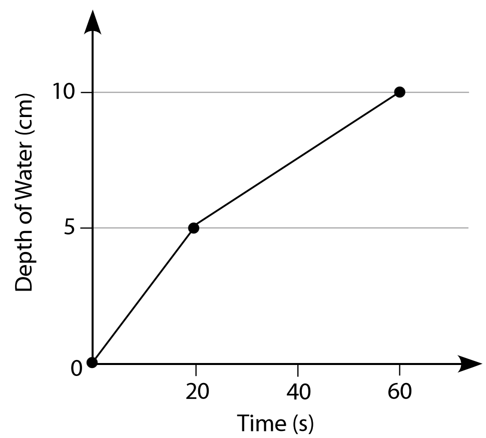
   Label the x-axis as "Time" and the y-axis as "Distance from Start."

   Note: To illustrate that Rochelle and Oksana are running in different directions, let Rochelle start at a distance of 0 km and Oksana start at a distance of 5 km.

   If Oksana runs at 10 km/h, then it will take her half an hour, or 30 minutes, to run 5 km.

8. The following two graphs represent two different cups being filled with water at a constant rate. What can you determine about the shape of the two cups by analyzing the graphs?





Lesson Summary

In this lesson, you first learned how slopes, or rates of change, are connected to graphs. You then used this skill to draw a graph to represent a given scenario. Finally, you were also able to use your knowledge of slope and rates to interpret a graph. In the next lesson, you will be looking at slope in the context of linear functions.

Notes

Assignment 7.1

Interpreting Rates Algebraically and Graphically

Total: 40 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. Describe a scenario that would match each of the rates below. Indicate which of the following rates are unit rates. If each rate is not a unit rate, convert it to a unit rate. *(3 marks)*

   a) 12 L/100 km

   b) 4¢/minute

2. Explain what the following rate represents. Then suggest one factor that could influence the rate. *(2 marks)*

   -2° C/h

Assignment 7.1: Interpreting Rates Algebraically and Graphically (continued)

3. At Food Store, 2 kg of salmon costs $19.89. At Skeeters, 5 kg of salmon costs $45.57. Which rate is greater? Support your answer by calculating and comparing unit rates. *(3 marks)*

4. Convert 72 km/h to miles per hour. *(2 marks)*

5. Convert 50 metres per second to kilometres per hour. *(3 marks)*

Assignment 7.1: Interpreting Rates Algebraically and Graphically (continued)

6. Which is the better buy: \$22.36 per kilogram or \$9.72 per pound? *(2 mark)*

7. In November 2015, one Euro (\$1 EUR) was equal to \$1.52 Canadian (\$1.52 CAD).
   a) If a person has \$250 CAD, how much is this worth in Euros? *(1 mark)*

   b) If a person has \$250 EUR, how much is this worth in Canadian dollars? *(1 mark)*

Assignment 7.1: Interpreting Rates Algebraically and Graphically (continued)

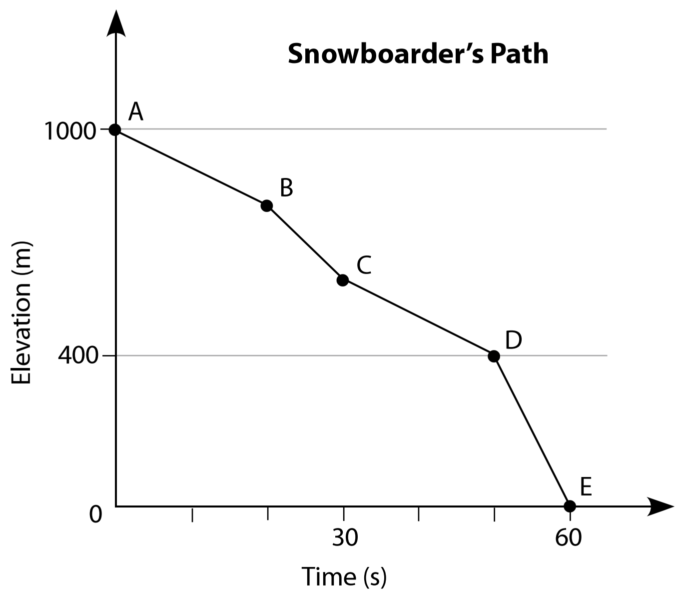
8. Kloe is a 160-pound woman who can burn 115 calories lifting weights for 20 minutes. Jane is a 120-pound woman who can burn 325 calories jogging for 30 minutes. How many calories will Jane burn in the same time it takes Kloe to burn 500 calories? *(4 marks)*

Assignment 7.1: Interpreting Rates Algebraically and Graphically (continued)

9. Draw a graph to represent the following scenario. Wynn wants to go for a 10-km jog. She starts off running at a pace of 10 km/h. She runs at this pace for 30 minutes. Wynn then slows down and runs at a pace of 8 km/h. She runs at this pace for 15 minutes. Wynn then returns to her pace of 10 km/h for the remainder of the 10-km jog. *(6 marks)*

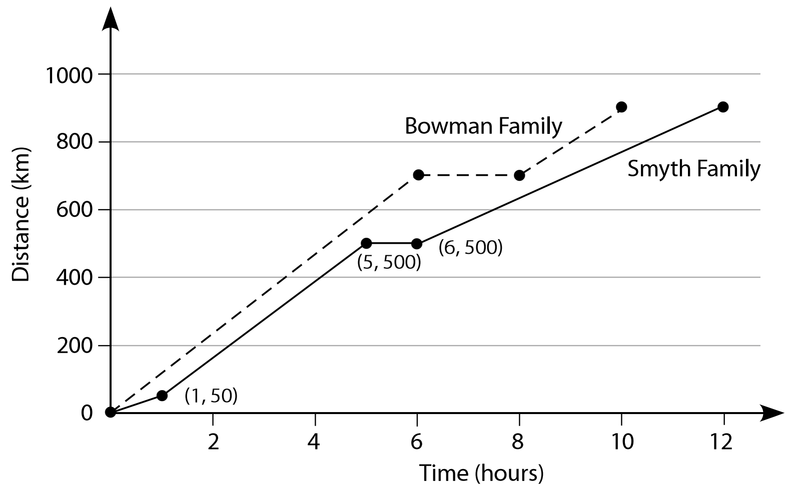
Assignment 7.1: Interpreting Rates Algebraically and Graphically (continued)

10. Consider the graph below illustrating the path a snowboarder takes down a mountain. Construct a story about the trip down the mountain. *(4 marks)*



Assignment 7.1: Interpreting Rates Algebraically and Graphically (continued)

11. The Bowman family and the Smyth family are both travelling 900 kilometres by car. A graph of their trips is shown below.



a) Which family arrived at their destination first? *(1 mark)*

b) How long did the Bowman family stop for lunch? *(1 mark)*

c) How long did the Smyth family stop for lunch? *(1 mark)*

d) What was the Smyth family's speed for the first hour of their trip in kilometres per hour? Suggest a reason to explain why they would be travelling at this speed. *(2 marks)*

e) What was the Bowman family's speed for the first 6 hours of their trip in kilometres per hour? How does this compare to the Bowman family's speed for the last 2 hours of their trip? *(4 marks)*

Notes

Lesson 3: Linear Functions

Lesson Focus

In this lesson, you will
- ❑ review how to graph linear functions
- ❑ review vocabulary terms including *x*-intercept and *y*-intercept
- ❑ review how to solve systems of equations

Lesson Introduction

In the last lesson, you looked at graphs that represented different scenarios, such as skiing down a mountain. These graphs contained multiple line segments to illustrate the changing speeds at which a skier can travel when going down a mountain. In this lesson, you will be looking at graphs consisting of one line called *linear functions*. This lesson includes a review of your knowledge from Grade 10 Introduction to Applied and Pre-Calculus Mathematics, as you will need this knowledge for future lessons.

Linear Functions

A **linear function** is a function that can be written in the form $y = mx + b$. This is also known as the slope-intercept form. A linear function can also be written in the form $Ax + By + C = 0$. Linear functions written in this form are more difficult to graph as it is harder to determine the features of a graph in this form.

In this course, you will be looking at linear functions in the slope-intercept form. If a function is not in slope-intercept form, you can use algebra to manipulate a function to turn it into the slope-intercept form. Many graphing devices require you to do this before you can graph the function. If you are using technology to graph equations, you need to be sure that they are written in this form.

Consider the following example.

Example 1

Write the following functions in the slope-intercept form ($y = mx + b$).

a) $\dfrac{y}{2} + 1 = x$

b) $3x + 2y = 6$

Solution

a) Your first step is to isolate the term containing y on the left-hand side of the equation. To do this, subtract 1 from both sides.

$$\frac{y}{2} + 1 = x$$

$\dfrac{y}{2} + 1 - 1 = x - 1$	Subtract 1 from both sides.
$\dfrac{y}{2} = x - 1$	Simplify.
$y = 2(x - 1)$	Multiply both sides by 2.
$y = 2x - 2$	Simplify.

b) Again, your first step is to isolate the term containing y on the left-hand side of the equation. To do this, subtract $3x$ from both sides.

$$3x + 2y = 6$$

$3x - 3x + 2y = 6 - 3x$	Subtract $3x$ from both sides.
$2y = 6 - 3x$	Simplify.
$2y = -3x + 6$	Rewrite the right-hand side to have the term containing x come first.
$y = -\dfrac{3x}{2} + \dfrac{6}{2}$	Divide both sides by 3.
$y = -\dfrac{3x}{2} + 3$	Simplify.

Once a linear function is in the slope-intercept form, you are ready to graph the function with technology or by using the slope and y-intercept.

Remember that in $y = mx + b$, m represents the slope and b represents the y-intercept.

The slope is the $\frac{\text{rise}}{\text{run}}$ of the graph.

The y-intercept is the point at which the graph crosses the y-axis. This is also the point where $x = 0$.

The x-intercept is the point at which the graph crosses the x-axis. This is also the point where $y = 0$.

You may wish to add the above information to your resource sheet.

Consider the following example.

Example 2

Graph the following functions.

a) $y = \frac{1}{2}x + 4$

b) $y = -3x - 1$

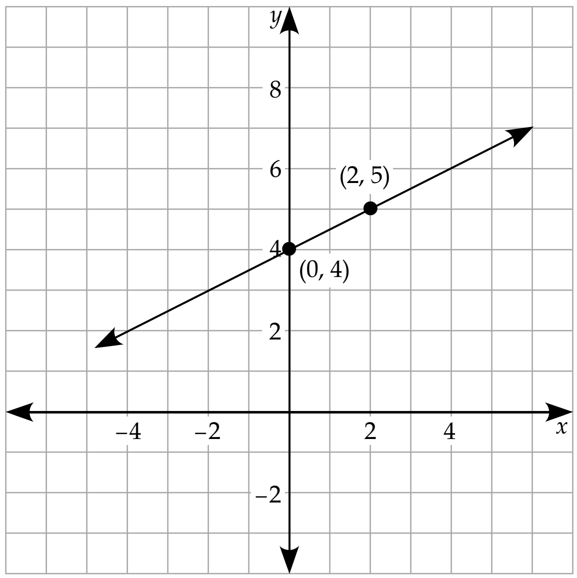
Solution

a) First, identify the slope and the y-intercept.

The slope is $\frac{1}{2}$.

The y-intercept is 4.

To graph this function, you can use technology or you can use the slope and y-intercept. First, plot the y-intercept at 4. Then, use your slope to find additional points. Since the slope represents the rise over run, your graph will be "rising" 1 and "running" 2. This means that from the y-intercept you will go **up 1** and **to the right 2**. You can continue this process by using the slope to find more points. For example, if the slope is $\frac{1}{2}$, that fraction is also equivalent to $\frac{2}{4}$, so you could rise 2 and run 4, or you could use $\frac{3}{6}$ or $\frac{4}{8}$, and many others. You can find other points that have the rise/run of fractions that are equivalent to the rise/run in your original equation.

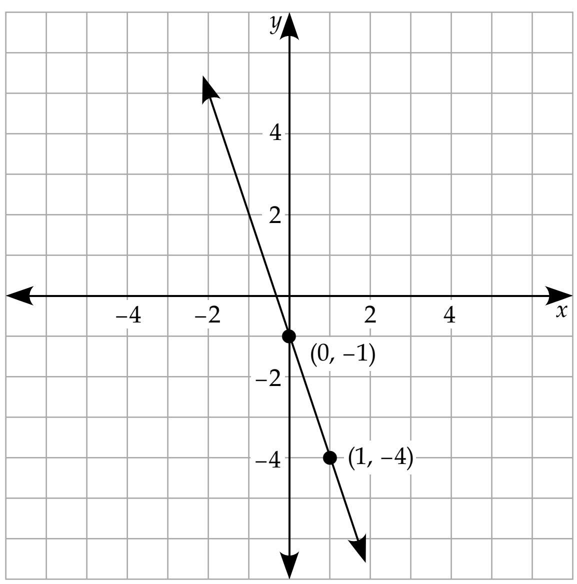


b) First, identify the slope and the y-intercept.

The slope is -3.

The y-intercept is -1.

To graph this function, first plot the y-intercept at -1. Then, use the slope to find additional points. The slope of this graph is -3. This can be written as $-\frac{3}{1}$. When written in this format, it is easier to identify the run of the graph. As the slope is negative, the graph will not be rising but will be falling. This means that the graph will be "falling" 3 and "running" 1. The next point after the y-intercept will be **down 3** and **to the right 1**. You can continue this process by using the slope to find more points.



Now that you know how to graph linear equations, you can practise solving systems of equations. **Systems of equations** are collections of two or more equations. The **solution** to the system of equations is the point at which **all equations intersect**.

You may wish to add the above information to your resource sheet.

Consider the following example.

Example 3

Find the solutions to the following systems of equations.

a) $y = x + 4$
$2y = x + 6$

b) $2y + x = 10$
$3y + 2x = 15$

Solution

a) First, rewrite each equation in the slope-intercept form. The first equation is fine but the second equation needs to be manipulated.

$2y = x + 6$

$y = \frac{x}{2} + \frac{6}{2}$ Divide both sides of the equation by 2.

$y = \frac{1}{2}x + 3$ Simplify.

Now, you can graph each equation, either using technology or using the slope and y-intercept.

Equation 1: $y = x + 4$

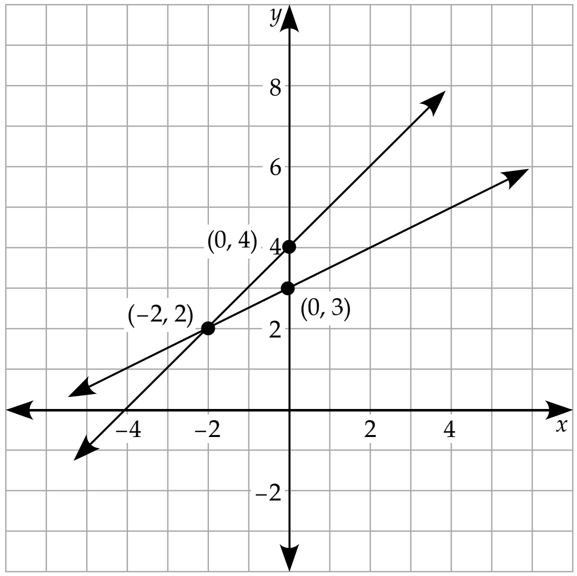
slope: $\frac{1}{1}$

y-intercept: 4

Equation 2: $y = \frac{1}{2}x + 3$

slope: $\frac{1}{2}$

y-intercept: 3



If you graph this system of equations carefully, manually or using technology, you will notice that the two lines intersect at the point (−2, 2). This is the solution to the system of equations.

b) First, rewrite each equation in the slope-intercept form.

Equation 1:

$2y + x = 10$

$2y = 10 - x$	Subtract x from both sides.
$2y = -x + 10$	Rearrange so the x-term comes first on the right-hand side.
$y = -\dfrac{x}{2} + \dfrac{10}{2}$	Divide both sides by 2.
$y = -\dfrac{1}{2}x + 5$	Simplify.

Equation 2:

$3y + 2x = 15$

$3y = 15 - 2x$	Subtract $2x$ from both sides.
$3y = -2x + 15$	Rearrange so the x-term comes first on the right-hand side.
$y = -\dfrac{2}{3}x + 5$	Divide both sides by 3 and simplify.

Now, you can graph each equation.

Equation 1: $y = -\frac{1}{2}x + 5$

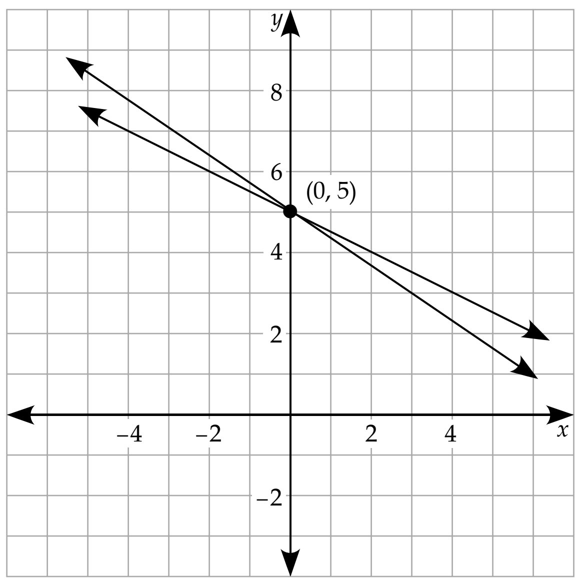
slope: $-\frac{1}{2}$

y-intercept: 5

Equation 2: $y = -\frac{2}{3}x + 5$

slope: $-\frac{2}{3}$

y-intercept: 5



If you graph this system of equations carefully, manually or using technology, you will notice that the two lines intersect at the point (0, 5). This is the solution to the system of equations.

Make sure you practise these skills in the following learning activity. If this lesson did not make sense to you, ask your tutor/marker or your learning partner for help. You will need to have these skills before moving on to the next lesson.

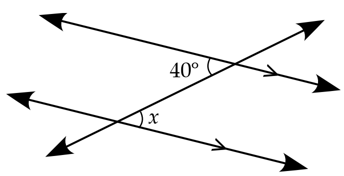
Learning Activity 7.3

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. If 1.6 km = 1 mile, convert 2 miles to kilometres.
2. Convert 855 metres to kilometres.
3. Convert 4 feet 6 inches to inches.
4. What is the size of angle x?



5. Solve for x: $2x + 1 = 3$
6. Simplify: $(9)(0)(8) + 3(7) - 3$
7. Write $\dfrac{60}{66}$ in lowest terms.
8. Evaluate the expression $-3xy + 1$ for $x = -2$ and $y = 6$.

continued

Part B: Graphing Linear Equations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. State the slope and y-intercept of the following lines.
   a) $y = 12x + 19$
   b) $y = -\frac{1}{2}x - 1$
2. Graph the following linear functions. If they are not in the slope-intercept form, rewrite them in the slope-intercept form first.
   a) $y = -\frac{1}{3}x - 6$
   b) $y = 2x - 5$
   c) $4x - 8y = 16$
   d) $\frac{x}{2} + \frac{y}{3} + 1 = 0$
3. Graph the following systems of equations. Determine the solutions.
   a) $x + y = 4$
      $x - y = -2$
   b) $y = -2x + 2$
      $y = x + 5$
   c) $3x - 2y = 4$
      $-2x + 2y = -4$
4. Graph the following systems of equations using technology. Determine the solutions.
   a) $y = 2x - 0.6$
      $y = -\frac{1}{4}x$
   b) $y = 1.25x - 1.45$
      $y = -3.6x + 5.92$

Lesson Summary

In this lesson, you reviewed your linear function knowledge from Grade 10. You reviewed how to graph lines by using the slope and the y-intercept. You also reviewed how to solve systems of equations by graphing more than one line and determining where the lines intersect. In the next lesson, you will be using this knowledge to graph linear inequalities.

Lesson 4: Linear Inequalities

Lesson Focus

In this lesson, you will

- ❑ review how to solve inequalities
- ❑ learn how to graph linear inequalities
- ❑ learn how to interpret the solution region of a linear inequality

Lesson Introduction

In 1903, Max O'Rell wrote a book entitled *Her Royal Highness Woman*. In this book, it recommends that a man should marry a woman who is half his age plus seven. O'Rell's idea has been modified by later writers who suggest that a person should only consider dating someone who is at least half his or her age plus seven. Ignoring the social considerations, let's look at this mathematically. If a woman is 40 years old, she should only consider dating someone who is at least 27 years of age or older ($40 \div 2 + 7$). At least 27 years of age or older could mean that ages of 30, 36, and 44 are acceptable. There is not only one exact age that fits this category but a range of many ages. This is an example of a linear inequality. In this lesson, you will be using the skills you learned in Lesson 3 to graph linear inequalities.

Linear Inequalities

Linear inequalities involve symbols that you are already familiar with, such as $<$, $>$, $\leq$, $\geq$, and $\neq$.

- $<$ represents "less than"
- $>$ represents "greater than"
- $\leq$ represents "less than or equal to"
- $\geq$ represents "greater than or equal to"
- $\neq$ represents "not equal to"

The statement $x > 3$ can be read as "x is greater than 3." The solution to this inequality is all values of x such that x is greater than 3. Values such as 4, 5, 3.5, and 3.01 are solutions to this inequality. However, values such as 3, 2, and 2.99 are not solutions to this inequality because they are not greater than 3. Consider the following example.

Example 1

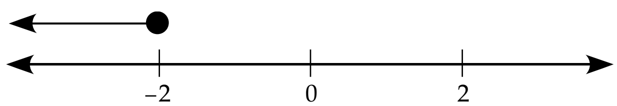
Solve and graph the following inequalities on a number line.

a) $x \leq -2$

b) $-3(x - 2) > x - 6$

Solution

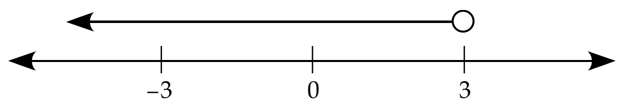
a) The solution to this inequality is all x-values that are less than or equal to -2. To represent this on a number line, use a **closed circle** at point -2. The closed circle represents that the value, -2, is included in the solution. Then, draw a line going to the left to indicate all values less than -2, including -2, are solutions.



b) To solve this inequality, you need to isolate x. To solve a linear inequality, perform the same operations algebraically as you would with an equation. There is, however, one exception due to the nature of negative numbers. When you **multiply or divide by a negative number, the inequality sign is reversed**. **Note:** $3 < 7$ but multiplying both sides by -1 gives $-3 > -7$.

$-3(x-2) > x - 6$	
$-3x + 6 > x - 6$	Multiply to remove the brackets.
$-3x - x + 6 > -6$	Subtract x from both sides.
$-4x + 6 > -6$	Simplify.
$-4x > -6 - 6$	Subtract 6 from both sides.
$-4x > -12$	Simplify.
$-\frac{4x}{-4} < -\frac{12}{-4}$	Divide by -4. When you divide by a negative, the sign flips from "greater than" to "less than."
$x < 3$	Simplify.

To graph this inequality on a number line, use an open circle at 3. An open circle indicates that the value, 3, is not part of the solution. The solution to the inequality would be all numbers that are less than 3 but not including 3, because x is less than but not equal to 3.



In this course, you will be dealing with inequalities with variables x and y in a linear form, such as slope-intercept form. Inequalities are similar to equations, but instead of an equals sign, there is an inequality sign. Also, instead of having only one or a small number of solutions to an equation, inequalities often have infinitely many solutions. You saw this in the examples above.

Consider the following example.

Example 2

For which of the following inequalities is (2, −1) a possible solution?

a) $x + y < 10$

b) $y > 9$

c) $2y - 5 \geq x$

Solution

a) In order to determine if this inequality is true, let $x = 2$ and let $y = -1$. Then substitute these values to see if the statement is true.

$$x + y < 10$$

$$2 + (-1) < 10$$

$$1 < 10$$

This is true. Therefore, the point (2, −1) is a solution to the inequality.

b) In this example, you do not need to use the value for x. The inequality is only dependent upon y. Substitute the value of −1 for y to see if the inequality is true.

$$y > 9$$

$$-1 > 9$$

This is not true. Therefore, the point (2, −1) is not a solution to the inequality.

c) Substitute in the values for x and y to see if the inequality is true.

$$2y - 5 \geq x$$
$$2(-1) - 5 \geq 2$$
$$-2 - 5 \geq 2$$
$$-7 \geq 2$$

This is not true. Therefore, the point (2, −1) is not a possible solution to the inequality.

Graphing a Linear Inequality

When a linear inequality is graphed the line will divide the graph paper or grid into three parts:

a) all points on the boundary line

b) all points above the boundary line

c) all points below the boundary line

The **solution set** to the linear inequality is all points that satisfy the inequality. Depending upon the sign of the inequality, the points that satisfy the inequality may be either above the boundary line or below the boundary line, and the line itself may or may not be a part of the solution set.

The following steps outline the procedure for graphing linear inequalities.

1. Rearrange the inequality so that it is in the slope-intercept form ($y > mx + b$ or $y < mx + b$).

2. Graph the related equation using the slope and the y-intercept; that is, graph it as if it were a linear equation $y = mx + b$, and not an inequality.

3. Decide whether the boundary line is **solid** or **dashed**.

   a) If the inequality is less than or equal to (≤) or greater than or equal to (≥), use a **solid line to represent the boundary line**. The same as for graphing equations, this solid line indicates that all points on the boundary line **are** solutions to the inequality.

   b) If the inequality is less than (<) or greater than (>), **use a dashed line to represent the boundary line**. This dashed line indicates that the points on the boundary line **are not** solutions to the inequality.

4. Determine where your **solution region** lies. The solution region is that part of the graph where all points are solutions to the inequality. To do this, you can use a test point, which can be any point that is not on the boundary line. Often, choosing the point (0, 0) is the simplest, unless the line happens to pass through the origin. Test the point by substituting in the values (0, 0) or other coordinates you have chosen for x and y into your inequality.

   a) If your test point makes the inequality true, then the region of your graph that **contains** the test point is the solution region. Shade this region.

   b) If your test point does not make the inequality true, then the region of your graph that **does not contain** the test point is the solution region. Shade the region that does not contain the test point.

   An alternative to using a test point is to look at the inequality in the slope-intercept form. If the inequality is in the form "$y > \ldots$" or "$y \geq \ldots$" then the solution includes all points where the y-values are greater than or above the line. However, if the inequality is in the form, "$y < \ldots$" or "$y \leq \ldots$," then the solution includes all points where the y-values are less than or below the line. You could decide to use both the test point and the alternate method to verify your solution region.

Note: One of the two regions of your graph represents the solution region. Therefore, one of the two regions of your graph must always be shaded for an inequality in two variables.

Consider the following example.

Example 3

a) Graph the solution region to the inequality $y \geq 2x - 3$.

b) Determine three points that are solutions to this inequality by using your graph.

c) Demonstrate algebraically that the three points you chose are solutions to the inequality.

Solution

a) First, graph the related linear equation using the slope and y-intercept.

Slope: $\frac{2}{1}$

y-intercept: -3

As the inequality symbol includes "equal to," use a **solid line** to represent the boundary line. All points on the boundary line are possible solutions to the inequality.

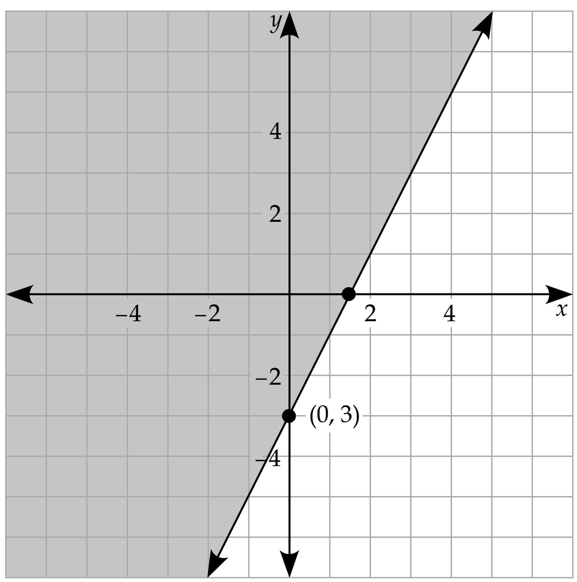
Since the function is written in the form, "$y \geq \ldots$," the solution region is greater than or above the line. Alternatively, choose a test point (0,0). Substitute $x = 0$ and $y = 0$ into the inequality.

$0 \geq 2(0) - 3$

$0 \geq -3$

This is a true statement since zero is greater than or equal to negative three. Therefore, the region of your graph containing (0, 0) is the solution region. This confirms that the region above the line is shaded.

Your graph should look like the graph below.



b) Any point located in the shaded region, or on the boundary line, is a solution to the inequality.

Three possible solutions are (0, −3), (0, 0), (−5, −4).

c) Consider the point (0, −3). Substitute $x = 0$ and $y = -3$ into the inequality $y \geq 2x - 3$ to determine if the inequality is true.

$$-3 \geq 2(0) - 3$$

$$-3 \geq -3$$

Yes, this is true.

Consider the point (0, 0). Substitute $x = 0$ and $y = 0$ into the inequality $y \geq 2x - 3$ to determine if the inequality is true.

$$0 \geq 2(0) - 3$$

$$0 \geq -3$$

Yes, this is true.

Consider the point (−5, −4). Substitute $x = -5$ and $y = -4$ into the inequality $y \geq 2x - 3$ to determine if the inequality is true.

$$-4 \geq 2(-5) - 3$$

$$-4 \geq -10 - 3$$

$$-4 \geq -13$$

Yes, this is true.

Example 4

Graph the solution region for the following linear inequalities.

a) $y < -2x + 4$

b) $y \leq -\dfrac{1}{2}x - 1$

c) $y > -x + 3$

d) $x \geq 4$

e) $y \leq -2$

Solution

a) First, graph the line $y = -2x + 4$ using the slope and y-intercept.

Slope: $\dfrac{-2}{1}$

y-intercept: 4

As the inequality symbol is "less than," use a **dashed line** to represent the boundary line. The points on the boundary line are **not** solutions to the inequality.

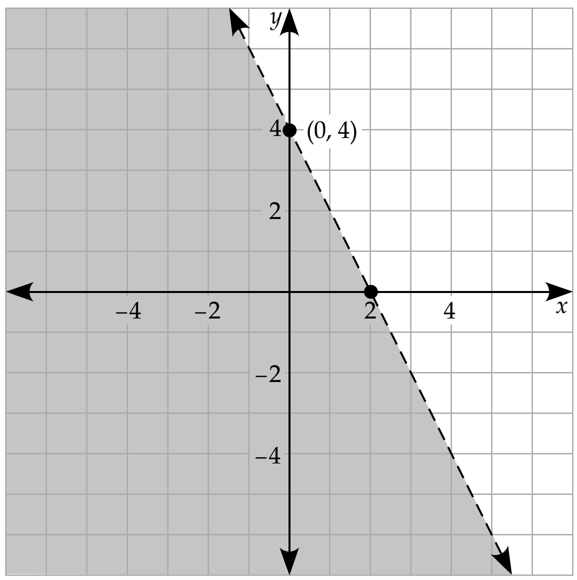
Since the inequality is in the form, "$y < \ldots$," then the solution region is **below the dashed line**. You could check the shaded region by choosing a test point (0, 0). Substitute $x = 0$ and $y = 0$ into the inequality.

$0 < -2(0) + 4$

$0 < 4$

This is true. Therefore, the region of your graph that contains (0, 0) is the solution region. This confirms that the shading is below the line.

Your graph should look like the graph below.



b) First, graph the line $y = -\frac{1}{2}x - 1$ using the slope and y-intercept.

Slope: $-\frac{1}{2}$

y-intercept: -1

As the inequality symbol involves "equal to," use a **solid line** to represent the boundary line. All points on the boundary line are solutions to the inequality.

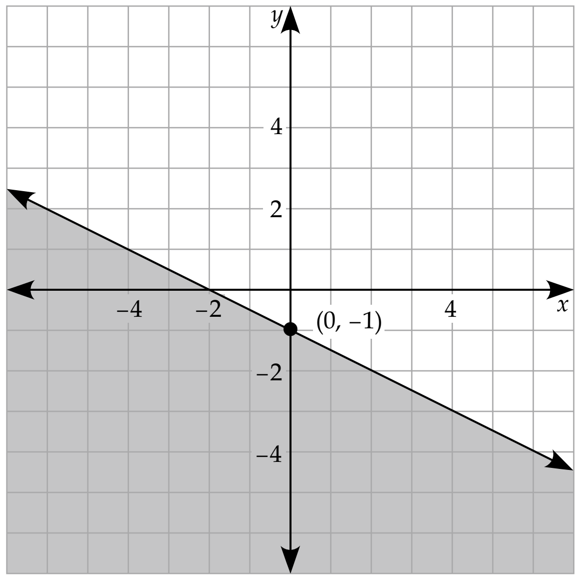
Since the inequality is in the form "$y \leq \ldots$," then the solution region is **below the solid line**. You can check the region by choosing a test point (0, 0). Substitute $x = 0$ and $y = 0$ into the inequality.

$0 \leq -\frac{1}{2}(0) - 1$

$0 \leq -1$

The inequality above is a false statement. Therefore, the region of your graph that does not contain (0, 0) is the solution region. This confirms that the shaded region is below the solid line.

Your graph should look like the graph below:



c) First, graph the line $y = -x + 3$ using the slope and y-intercept.

Slope: $-\frac{1}{1}$

y-intercept: 3

As the inequality symbol is "greater than," use a **dashed line** to represent the boundary line. The points on the boundary line are **not** possible solutions to the inequality.

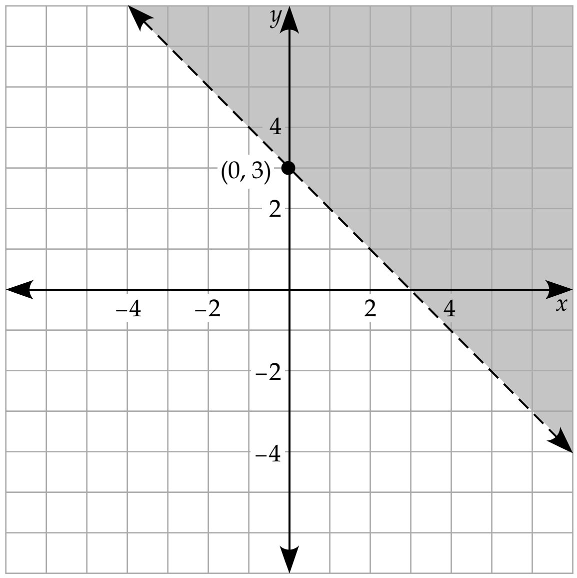
Since the inequality is in the form "$y > \ldots$," then the solution region is **above the dashed line**. You can check the region by choosing a test point (0, 0). Substitute $x = 0$ and $y = 0$ into the inequality.

$0 > -(0) + 3$

$0 > 3$

The inequality statement above is false since zero is not greater than three. Therefore, the region of your graph that does not contain (0, 0) is the solution region. This confirms that the region is above the dashed line.

Your graph should look like the graph below:



d) To graph this inequality, consider the line $x = 4$. This is a vertical line located at $x = 4$.

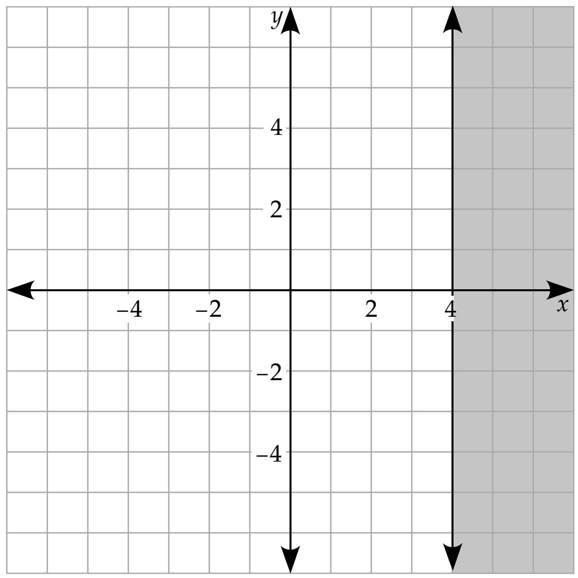
As the inequality sign involves "equal to," the line will be a **solid line**.

Since the inequality is in the form "$x \geq$ . . .," then the solution region is to the **right of the solid line**. Notice that since it is a vertical line, the area that is greater than 4 lies to the right and the area that is less than 4 is to the left. You can check the region by choosing the test point (0, 0).

$0 \geq 4$

The inequality statement above is false since zero is not greater than or equal to 4. Therefore, the region of your graph not containing the point (0, 0) is the solution region. This confirms that the region is to the right of the solid line.

Your graph should look like the graph below:



e) To graph this inequality, consider the line $y = -2$. This is a horizontal line located at $y = -2$.

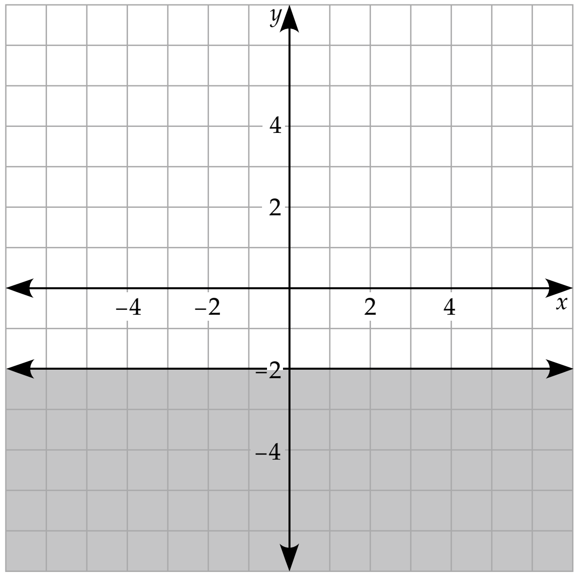
As the inequality sign involves "equal to," the line will be a **solid line**.

Since the inequality is in the form "$y \leq \ldots$," then the solution region is **below the solid line**. You can check the region by choosing the test point (0, 0).

$$0 \leq -2$$

The inequality statement above is false since zero is not less than or equal to −2. Therefore, the region of your graph not containing the point (0, 0) is the solution region. This confirms that the region is below the solid line.

Your graph should look like the graph below:



Linear inequalities are often used to represent scenarios involving multiple items of different weights or different prices. Consider the scenario below.

Example 5

Claude has $50 to spend on birthday presents for his mom and sister at Bath Bubbles Boutique. He wants to buy a combination of bubble bath, which costs $5, and candles, which cost $10.

a) What are the variables in this situation?

b) Write an inequality to represent how many items of bubble bath and candles Claude can buy.

c) Graph the solution region for the linear inequality you created in (b).

d) Determine two combinations of bubble bath and candles that Claude can purchase.

Solution

a) The variables in this situation are the number of bubble bath products purchased and the number of candles purchased.

b) Let x represent the number of bubble bath products purchased.

Let y represent the number of candles purchased.

If bubble bath products cost $5 and candles cost $10, then the total amount Claude spends can be represented as $5x + 10y$.

However, this total must be less than or equal to what Claude has to spend. This gives a linear inequality of $5x + 10y \leq 50$.

c) To graph the inequality, first rewrite the inequality in the slope-intercept form.

$5x + 10y \leq 50$

$10y \leq -5x + 50$	Subtract $5x$ from both sides.
$\dfrac{10y}{10} \leq -\dfrac{5x}{10} + \dfrac{50}{10}$	Divide both sides by 10.
$y \leq -\dfrac{1}{2}x + 5$	Simplify.

Note: The number of bubble bath products and candles purchased cannot be negative because you cannot buy a negative number of products. When you are graphing scenarios such as this, you do not need to include the negative x-axis or the negative y-axis. In other words, you also have the restrictions defined by the inequalities $x \geq 0$ and $y \geq 0$.

To continue graphing:

Slope: $-\dfrac{1}{2}$

y-intercept: 5

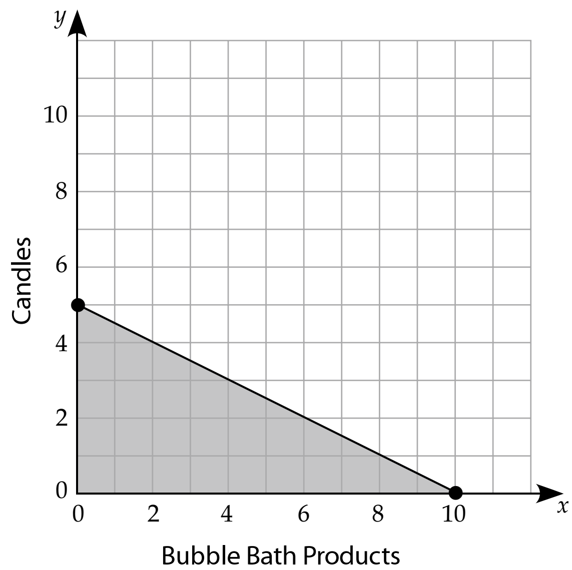
Boundary line: solid ("equal to" is included)

The solution region is below the line since the inequality is of the form "$y \leq \ldots$" This can be confirmed using the test point (0, 0).

$$0 \leq -\frac{1}{2}(0) + 5$$

$$0 \leq 5$$

Since the inequality statement is true for the test point, the region containing (0, 0) is in the solution region. This confirms that the solution region includes all points with whole number values on or below the boundary line.



d) Any set of coordinates in the solution region represents a possible number of bubble bath products and candles that Claude can purchase. The coordinates must be positive whole numbers in this context because you cannot purchase a negative number of bath products and you cannot purchase a fraction of a bath product. Two such points are:

(2, 3): Claude can buy 2 bubble bath products and 3 candles.

(8, 1): Claude can buy 8 bubble bath products and 1 candle.

Graphing linear inequalities can be difficult to understand at first. Try the following learning activity to practise what you have just learned. If you are still having trouble, make sure you contact your tutor/marker for assistance.

Learning Activity 7.4

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

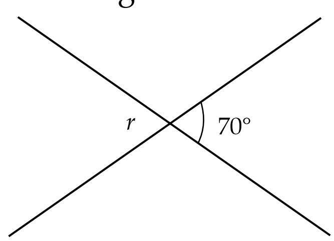
Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Add: $\frac{3}{5} + \frac{2}{7}$

2. Does the parabola $y = -2x^2 + 5x - 6$ have a maximum or minimum?

Use the diagram below to answer questions 3 and 4.



3. What type of angles are displayed?

4. What is the size of angle r?

5. Solve for x: $2x + 1 \geq 0$

6. Is the origin a solution to the inequality $2x + 1 \geq 0$?

7. Identify the slope in the line $y = -\frac{x}{2} + 3$.

8. Evaluate if $x = -15$: $-\frac{2x}{5} + 7$

continued

Part B: Graphing Linear Inequalities

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

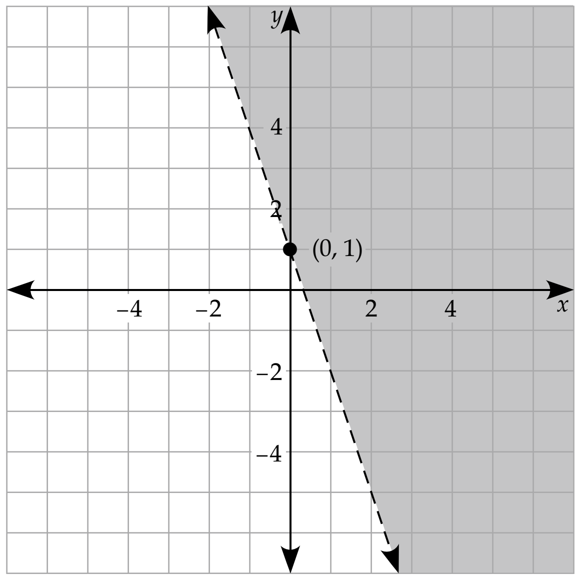
1. Determine whether the point $(-3, 5)$ is in the solution region for the following linear inequalities:
   a) $10y - 12x < 5$
   b) $4x + 2y > -1$
   c) $x + y \geq 2$
2. Graph the solution region for the following linear inequalities:
   a) $x + y \geq 1$
   b) $3y - 6x < -3$
   c) $y < 6$
   d) $x - y \geq -2$
   e) $y \geq -4x + 7$
   f) $x > -1$
3. Consider the inequality $5y - 2x \leq 15$.
   a) Graph the solution region.
   b) Determine three possible coordinates that are solutions to the inequality.

continued

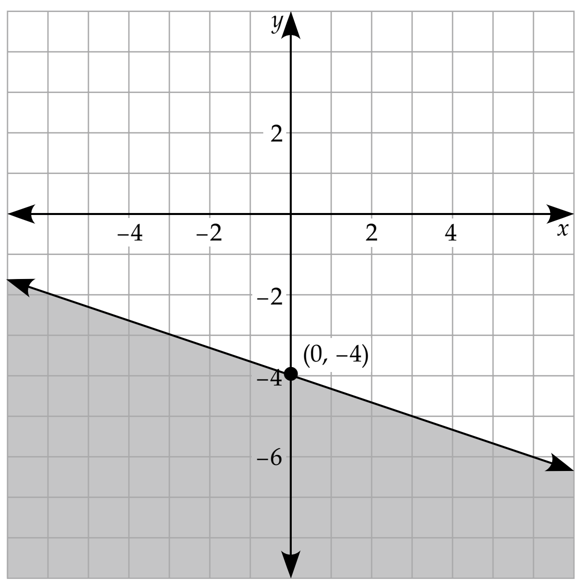
Learning Activity 7.4 (continued)

4. Determine the inequality that represents each graph below:

a)



b)



continued

Learning Activity 7.4 (continued)

5. Garden City Collegiate plans to send a group of students and teachers to a conference about sustainable development. The conference costs $25 for students and $50 for teachers. The school's budget for this conference is $300.
   a) Define the variables in the question.
   b) Write an inequality to represent the number of students and staff Garden City Collegiate can bring to the conference while still staying within the allotted budget.
   c) Graph the solution region to the inequality you created in (b).
   d) List two possible combinations for students and staff who can attend the conference.
   e) For each of the possible combinations you found in (d), determine how much the conference would cost.

6. The student council at Goose Lake High in Roblin is considering different scenarios to raise money for a school dance, which is estimated to cost $501. They know they can sell candy grams and raise $1 for each candy gram sold. They also know they can sell pizza and raise $3 for each pizza combo sold.
   a) Define the variables in the question.
   b) Write an inequality to represent the number of pizza combos and candy grams the student council needs to sell to make at least $501.
   c) Graph the solution region to the inequality you created in (b).
   d) List three possible combinations for candy grams and pizza combos that the student council could sell in order to pay for the dance.

Lesson Summary

In this lesson, you were introduced to the concept of linear inequalities. You expanded your knowledge of graphing lines in the slope-intercept form to graph linear inequalities and their solution sets. The most important difference between a linear equation and a linear inequality is the inequality sign. This illustrates that there are many more solutions for an inequality than for an equation. This is why it is necessary to shade the solution region in a linear inequality. In the next lesson, you will be learning how to graph systems of linear inequalities.

Lesson 5: Graphing Systems of Linear Inequalities

Lesson Focus

In this lesson, you will

- ❑ learn how to solve systems of linear inequalities by graphing the solution region
- ❑ reference the solution region in a system of inequalities to demonstrate solutions to word problems

Lesson Introduction

Graphing systems of linear inequalities is similar to graphing systems of linear equations. It simply involves graphing more than one linear inequality at a time. Situations involving systems of linear inequalities occur when you have more than one restriction on the variables.

Consider a scenario where you are working at a part-time job. The maximum number of hours you can work is 40. However, you need to work at least 15 hours in order to have enough money each month to pay your cell phone bill. This is a system that would contain two inequalities—the maximum number of hours you can work and the minimum number of hours you can work.

Systems of Linear Inequalities

A **system of linear inequalities** occurs when two or more linear inequalities are considered at the same time. The solution is found by graphing the two or more linear inequalities and finding the intersecting solution region. The **solution region** to a system of linear inequalities is the portion of the graph that is a solution to all linear inequalities in the system. For example, if your system contains two linear inequalities, the **solution region** is the area where the shaded regions of the graph of the two linear inequalities overlap.

Graphing Systems of Linear Inequalities

The following examples will help you understand how to graph systems of two, three, or more linear inequalities.

Example 1

Determine the solution region for the following system of linear inequalities:

$$y \le -\frac{3}{2}x - 4$$

$$y > 2x + 3$$

Solution

First, graph the solution region of each linear inequality.

Linear inequality 1:

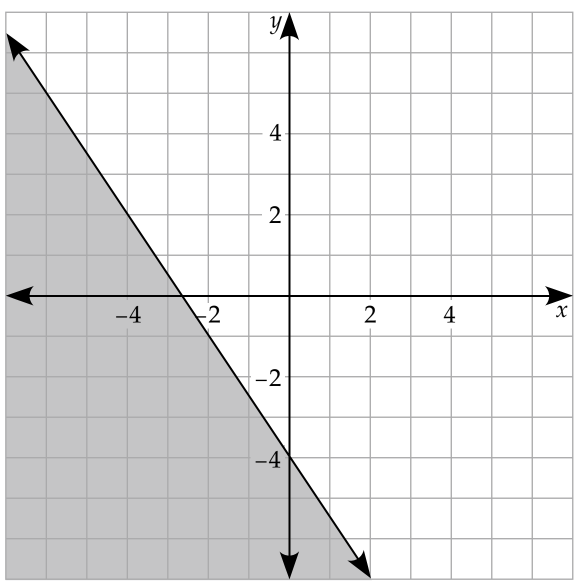
$$y \le -\frac{3}{2}x - 4$$

Slope: $-\frac{3}{2}$

y-intercept: -4

Boundary Line: Solid

Shade below the boundary line since the form is "$y \le \ldots$"



Linear inequality 2:

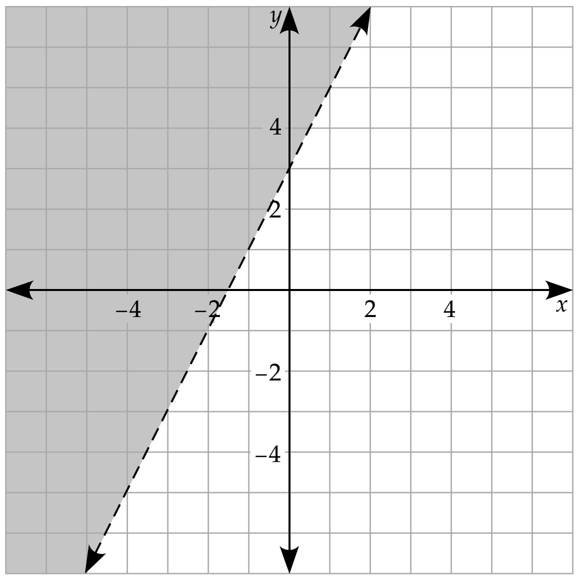
$y > 2x + 3$

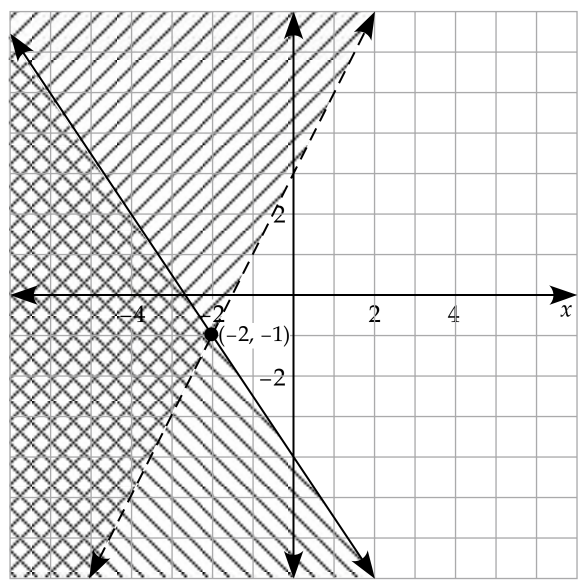
Slope: $\frac{2}{1}$

y-intercept: 3

Boundary line: dashed

Shade above the boundary line since the form is "$y > \ldots$"





When you graph these solutions on the same grid, the solution region will be the region that is shaded twice. This means that the solution region of the system of inequalities is the intersection of the two solution regions of each linear inequality.

The solution region is the left-most region on the graph. This is the only region of your graph that needs to be shaded.

The process for graphing the solution region for a system with three or more linear inequalities is similar to the one you just completed. The only difference is you have additional inequalities to graph.

Example 2

Determine the solution region for the following system of linear inequalities:

$y \leq 2$

$y \leq -x + 5$

$y \geq -x + 2$

$y \geq 0$

Solution

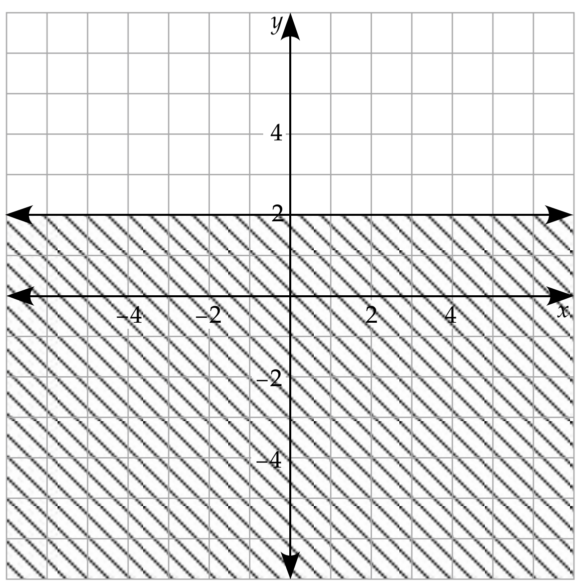
Graph each linear inequality, and then determine the solution regions for all four linear inequalities.

Linear Inequality 1:

$y \leq 2$

This is a horizontal line at $y = 2$. The boundary line is solid.

The solution region is below the line since the form is "$y \leq \ldots$"



Now, graph the second linear inequality on top of the first linear inequality. Use the same set of axes.

Linear Inequality 2: $y \leq -x + 5$

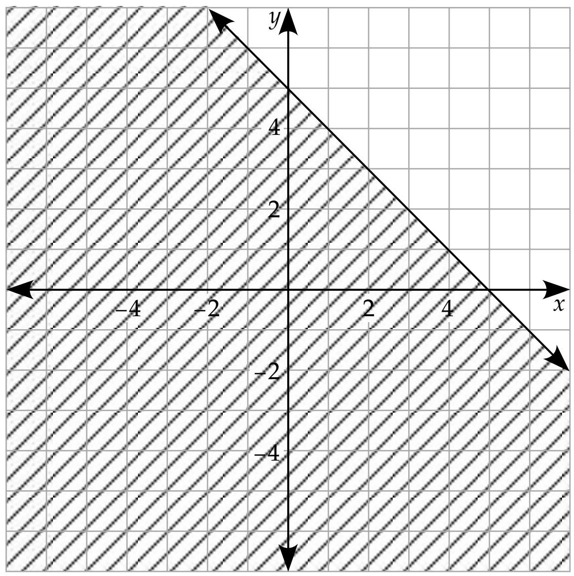
Slope: $-\dfrac{1}{1}$

y-intercept: 5

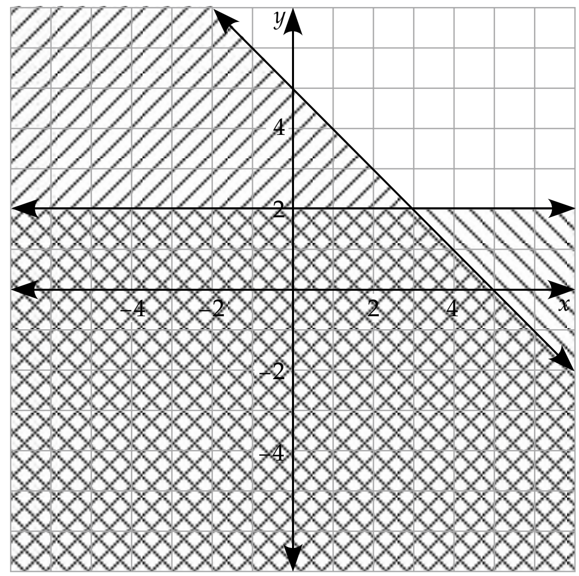
Boundary Line: Solid

Shade below the boundary line since the form is "$y \leq \ldots$"

Solution for $y \leq -x + 5$.



Graph for $y \leq 2$ and $y \leq -x + 5$.



From the graph, you can see that the double-shaded region is the bottom-left region. However, you still need to graph two more linear inequalities.

Linear Inequality 3: $y \geq -x + 2$

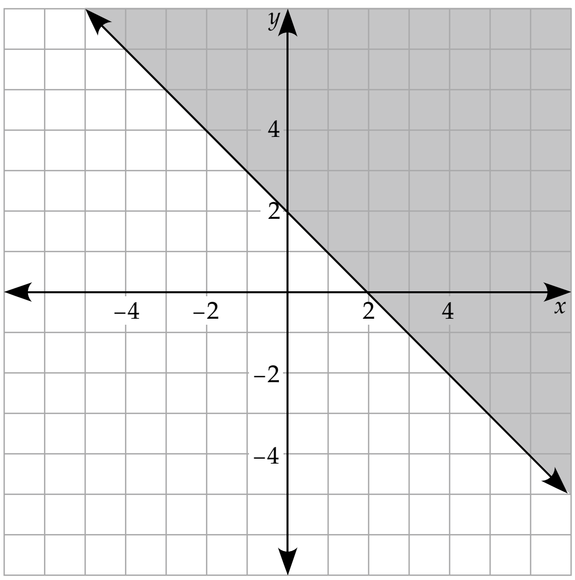
Slope: $-\frac{1}{1}$

y-intercept: 2

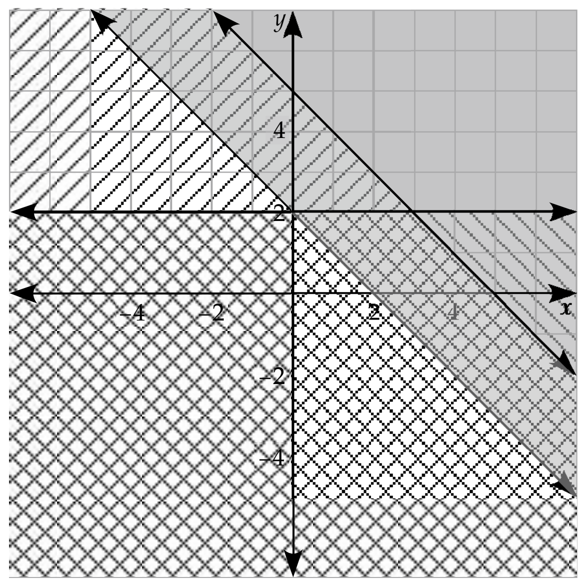
Boundary Line: Solid

The shaded region is above the boundary line since the form is "$y \geq \ldots$"

Solution for $y \geq -x + 2$.



Graph for $y \leq 2$, $y \leq -x + 5$, and $y \geq -x + 2$.



Note: The three shadings give you an idea where the final answer may be. You may find it helpful to shade with different colours to see it more clearly.

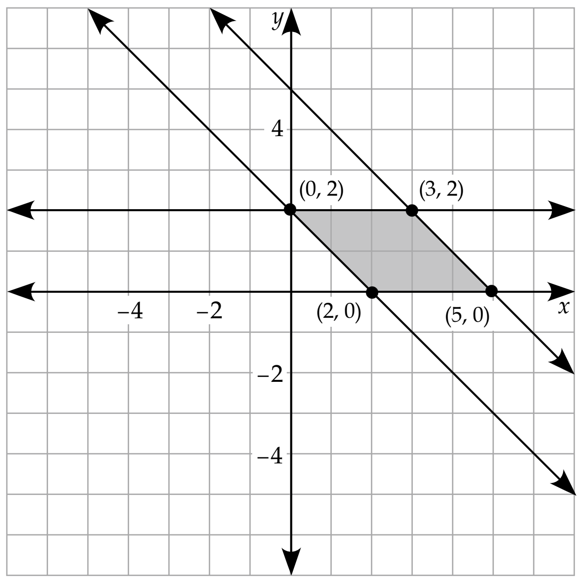
Finally, consider **Linear Inequality 4:** $y \geq 0$

Draw the horizontal boundary line on the x-axis where $y = 0$.

Boundary Line: Solid

The shaded region is above the boundary line since the form is "$y \geq \ldots$"

Consider the graph below. The solution region becomes the region where all four shaded areas overlap, which is inside the parallelogram with coordinates (0, 2), (2, 0), (5, 0), and (3, 2).



Note: To see the final solution more clearly, all shading has been removed except for the overlap area for the four linear inequalities.

Systems of linear inequalities can also be used to help represent scenarios where there are restrictions or limitations, such as the number of hours you can work in a week. Consider the following example.

Example 3

You want you to find a part-time job at a fast-food restaurant. Currently, you have some babysitting employment where you earn \$15 an hour. The fast-food restaurant pays \$11 an hour and requires you to work at least 8 hours a week. Your parents will allow you work a maximum of 20 hours a week. However, to save up for a new car, you need to make at least \$154 a week.

a) Introduce the variables of this question.

b) Write an inequality to represent the number of hours you need to work at the fast-food restaurant.

c) Write an inequality to represent the number of hours you can work in total.

d) Write an inequality to represent the amount of money you need to save per week.

e) State the implied boundaries on the variables.

f) Graph the solution region to the inequalities you created in (b), (c), and (d).

g) Determine two points in the solution region and explain what these points represent.

Solution

a) The variables of this question are:

- The number of hours you work babysitting (x)
- The number of hours you work at a fast-food restaurant (y)

b) You need to work at least 8 hours at the fast-food restaurant. The inequality that represents this scenario is $y \geq 8$.

c) You can work a maximum of 20 hours. The inequality that represents this scenario is $x + y \leq 20$.

d) You need to save \$154. If you make \$15 babysitting and \$11 working at the fast-food restaurant, the inequality that represents how much you need to make is $15x + 11y \geq 154$.

e) You work as a babysitter for a positive number of hours, so $x \geq 0$.

f) You now need to graph the system of linear inequalities:

$$y \geq 8$$
$$x + y \leq 20$$
$$15x + 11y \geq 154$$
$$x \geq 0$$

Linear Inequality 1:

Linear inequality 1 is a horizontal line at $y = 8$. The boundary line is solid. The solution region is above the boundary line since the form is "$y \geq \ldots$"

Linear Inequality 2:

Rearrange into slope-intercept form $y \leq -x + 20$.

The slope is $-\frac{1}{1}$. The y-intercept is 20. The boundary line is solid. The solution region is below the boundary line since the form is "$y \leq \ldots.$" This is confirmed by using the test point (0, 0), which makes the inequality true.

Linear Inequality 3:

Rearrange into slope-intercept form:

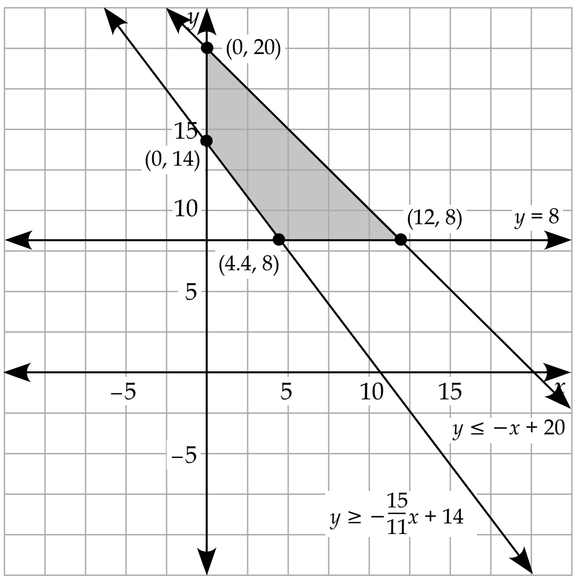
$$11y \geq -15x + 154$$

$$y \geq -\frac{15}{11}x + 14$$

The slope is $-\frac{15}{11}$. The y-intercept is 14. The boundary line is solid. The solution region is above the boundary line since the form is "$y \geq \ldots.$" This is confirmed using the test point (0, 0), which makes a false inequality statement.

Linear Inequality 4:

This is a vertical line at $x = 0$. The boundary line is solid. When dealing with inequalities involving "$x = \ldots$" where the boundary line is vertical, the solution region is to the right or left of the boundary line. In this case, the solution region is to the right of the vertical line since the form is $x \geq \ldots$." You can confirm this by using the test point (1, 15), which makes the inequality, $1 \geq 0$, true.



From the graph above, you can see that the shaded regions all overlap in the quadrilateral with vertices (0, 20), (12, 8), (4.4, 8), and (0, 14). You can find these vertices by graphing each line in a grapher and finding the points of intersection.

g) Two possible points in the solution region are (10, 9.5) and (5, 10). The point (10, 9.5) represents working 10 hours babysitting and 9.5 hours at a fast-food restaurant. The point (5, 10) represents working 5 hours babysitting and 10 hours at a fast-food restaurant.

Make sure you practise what you just learned in the following learning activity.

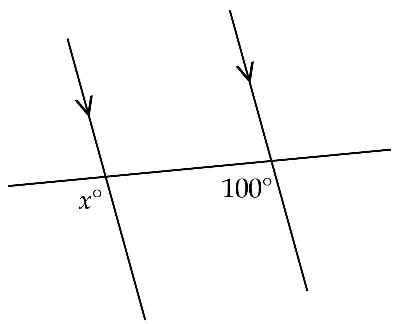
Learning Activity 7.5

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

Use the diagram below to answer questions 1 and 2.



1. What name describes the pair of angles displayed?
2. What is the size of angle x?
3. State the x-intercepts for the function $y = -(x + 2)(x)$.
4. Rewrite $8x + 4y = 16$ in slope-intercept form.
5. Convert 2 yards and 2 feet to feet.
6. Simplify: $(11 - 6)(3)^2 - 1$
7. Solve: $\frac{1}{2}x + 4 = 9$
8. Determine the next three terms in the sequence 1, 3, 6, 8, 11, 13.

continued

Part B: Finding the Solution Region for Systems of Linear Inequalities

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Determine whether (3, −5) is in the solution region for each system of linear inequalities.

   a) $y \le \frac{7}{3}x - 1$

      $2x + 3y \le 0$

   b) $y > -\frac{1}{2}x - 6$

      $y \le 3x - 20$

2. Graph the solution region to each system of linear inequalities.

   a) $x + y > 2$

      $3x + y \ge -3$

   b) $x + 2y \le 4$

      $2x - y > 1$

   c) $y \ge -\frac{2}{7}x + 3$

      $y < \frac{5}{4}x - 1$

continued

Learning Activity 7.5 (continued)

3. Graph the solution region to each system of linear inequalities.

   a) $y \le \frac{5}{2}x - 2$

   $y \ge \frac{1}{2}x + 2$

   $y \le \frac{1}{4}x + 6$

   b) $y < -3$

   $x + 2y < 4$

   $3x + 2y \ge -6$

4. Consider the following system of linear inequalities:

   $y > -\frac{1}{3}x - 3$

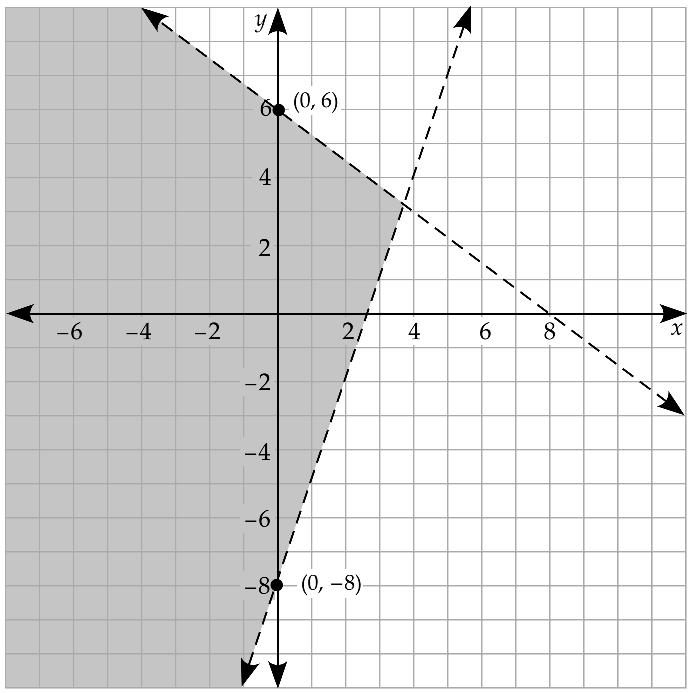
   $2x + y < -3$

   $4x - y \ge -8$

   a) Graph the solution region to the system of linear inequalities.
   b) Use technology to determine the vertices of the solution region.
   c) Determine one point in the solution region that is a solution to the system of linear inequalities.
   d) Verify that this point is a solution to all three linear inequalities.

continued

5. Determine the system of linear inequalities that represents the graph below.



6. Marla is working at two part-time jobs, one as a dog walker and one as a receptionist. She needs to earn \$320 a week in order to pay her bills. However, Marla can only work a maximum of 30 hours a week. She is paid \$12 an hour as a dog walker and \$16 an hour as a receptionist.
   a) Introduce the variables of this question.
   b) State the implied boundaries on the variables.
   c) Write an inequality to represent the number of hours Marla can work in total.
   d) Write an inequality to represent the amount of money Marla needs to make per week.
   e) Graph the solution region to the inequalities you created in (c) and (d).
   f) Determine two points in the solution region and explain what these points represent.
   g) Determine how much Marla would make for each of the points you selected as solutions.

continued

7. A car dealership makes $1000 for every car sold and $3000 for every truck sold. The manager of the car dealership wants to make at least $21,000 a month from these two items. The manager knows that he can sell at least twice as many cars as he can trucks.
   a) Introduce the variables of this question.
   b) State the implied boundaries on the variables.
   c) Write an inequality to represent the amount of money the manager wants to make per month.
   d) Write an inequality to represent the relationship between the number of cars sold and the number of trucks sold.
   e) Graph the solution region to the inequalities you created in (c) and (d).
   f) Determine two points in the solution region and explain what they represent.

8. Between 184 and 256 students and staff are attending a field trip. A school bus holds 40 people and a passenger van holds 8 people. There are only 8 passenger vans and 10 buses available.
   a) Introduce the variables of this question.
   b) State the implied boundaries on the variables.
   c) Write an inequality to represent the total number of vehicles required to carry at least 184 people.
   d) Write an inequality to represent the total number of vehicles required to carry at most 256 people.
   e) Write an inequality to represent the number of passenger vans available.
   f) Write an inequality to represent the number of buses available.
   g) Graph the solution region to the inequalities you created in (c), (d), (e), and (f).
   h) Determine two points in the solution region and explain what they represent.
   i) For each of the two solutions, determine how many passengers will be attending the field trip.

Lesson Summary

In this lesson, you learned how to graph systems of linear inequalities that involved more than one linear inequality. You then learned how to find the region that satisfied all of the linear inequalities in that system. You also began to look at scenarios where systems of linear inequalities may be used. You will continue to look at these scenarios in the next lesson, where you will find the *optimal solution* to a system of linear inequalities.

Assignment 7.2

Linear Inequalities and Systems of Linear Inequalities

Total: 60 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. Determine whether the point $(4, -6)$ is in the solution region for the following linear inequalities: *(2 × 2 marks each = 4 marks)*

   a) $5x - 7y \geq 14$

   b) $y \geq -\frac{7}{2}x + 9$

Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities (continued)

2. Determine whether the point (−2, 7) is in the solution region for the following system of linear inequalities: *(3 marks)*

$$-2x - 4y \leq 5$$

$$\frac{x}{2} + 3y > 5$$

Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities (continued)

3. Graph the solution region for the following linear inequalities, and label the y-intercepts on your graphs. *(4 × 4 marks each = 16 marks)*

   a) $9 + 3y \geq 2x$

   b) $x < -7$

Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities (continued)

c) $4y + 5x \leq 16$

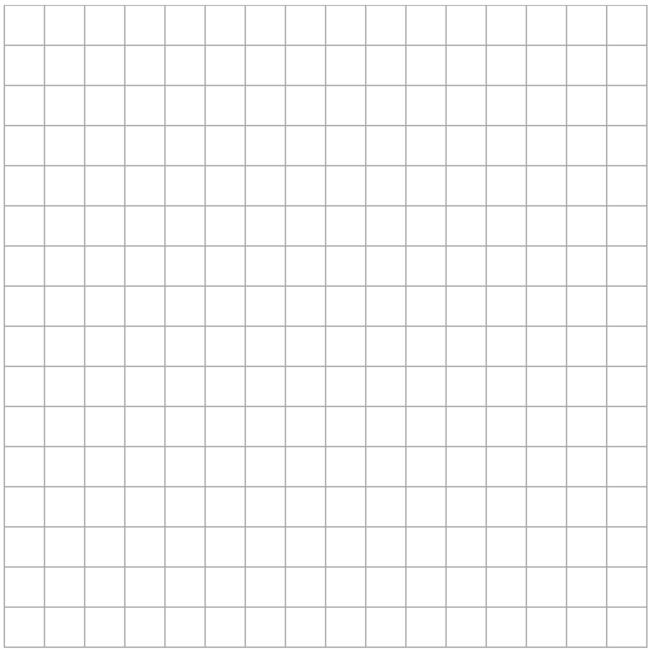
d) $-3x + 2y \geq 12$

Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities (continued)

4. Graph the solution region for the following systems of linear inequalities. Be sure to label the intersection point of the two lines and the y-intercepts.
   (2 × 8 marks each = 16 marks)

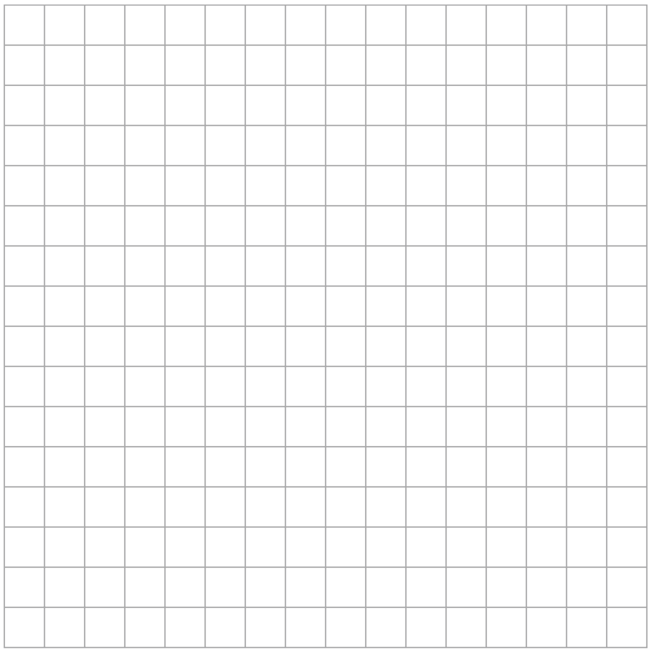
   a) $x + y > -4$

      $2x + y \leq 1$



b) $x + 2y \geq 8$

$5x + 2y < 10$



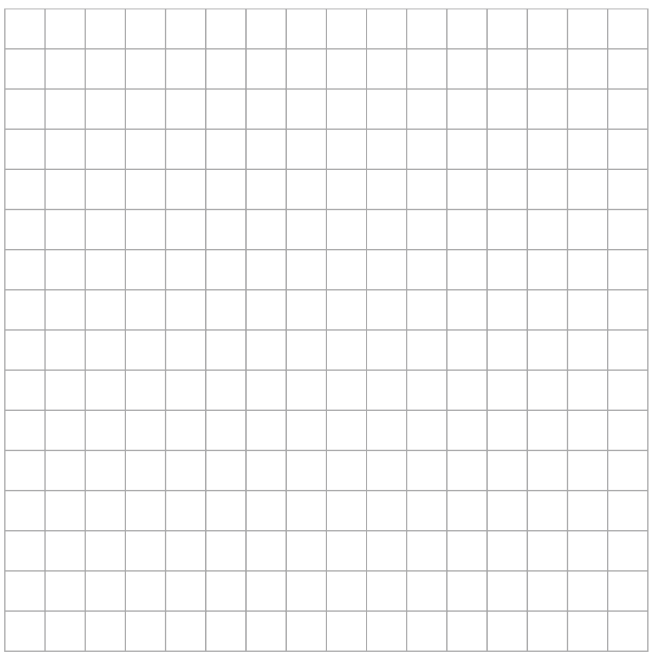
Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities (continued)

5. A clothing store makes a profit of $20 for every sweater they sell and $30 for every pair of jeans they sell. The manager of the clothing store wants to make a profit of at least $600 a day.

   a) What are the variables in this situation? *(2 marks)*

   b) Write an inequality to represent how many sweaters and pairs of jeans the clothing store needs to sell each day to reach the targeted sales goal. *(1 mark)*

   c) Graph the solution region for the linear inequality you created in (b). *(3 marks)*



   d) Determine two combinations of sweaters and jeans that the manager can sell to reach the sales goal. *(2 marks)*

Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities (continued)

6. Brayden is planning an engagement party for his best friend and needs to order a selection of appetizers. He wants to order vegetarian spring rolls and mini chicken pot pies. There are 70 people expected to attend the engagement party and Brayden wants to make sure he has at least two appetizers per person. From previous experience, he knows that at least three chicken pot pies will be consumed for every vegetarian spring roll.

   a) Introduce the variables of this question. *(2 marks)*

   b) State the implied boundaries on the variables. *(1 mark)*

   c) Write an inequality to represent the number of appetizers Brayden needs to order. *(1 mark)*

   d) Write an inequality to represent the relationship between the number of vegetarian spring rolls and the number of mini-chicken pot pies Brayden should order. *(1 mark)*

Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities (continued)

e) Graph the solution region to the inequalities you created in (c) and (d). *(4 marks)*

f) Determine two points in the solution region and explain what these points represent. *(4 marks)*

Notes

Lesson 6: Optimizing Solutions to Linear Inequalities

Lesson Focus

In this lesson, you will

- ❑ learn how to optimize the solution to a system of linear inequalities

Lesson Introduction

In the previous two lessons, you learned how to graph linear inequalities and systems of linear inequalities. You then translated these skills to word problems. You graphed the solution region to a system of linear inequalities that represented the word problem, but what did the solution region actually mean? Was there one point in the solution region that was the "best" solution? In this lesson, you will discover how to *optimize* the solution to systems of linear inequalities. In other words, you will determine the best solution to a system of linear inequalities.

Optimizing Solutions

In this lesson, you will be looking at two types of optimization problems. An **optimization problem** is a problem in which a quantity needs to be maximized or minimized under a set of conditions or constraints. For example, a company may want to *maximize* profit and *minimize* cost.

Each optimization problem has a corresponding **objective function**. This is an equation that represents the relationship between two variables in your system of linear inequalities, as well as the quantity that you are maximizing or minimizing. This function produces values that help you to determine which set of coordinates gives the maximum and the minimum of the objective function.

You will learn a technique called **linear programming** to solve optimization problems. This method involves creating a **feasible region** that defines the possible values for the two variables that fit within the conditions or constraints. This method is an application of the solution to a system of inequalities that you learned in the previous lesson, since the feasible region in linear programming is simply the solution region you were working with when solving a system of inequalities.

As you know, there are often an infinite number of points within the solution region, so finding the point that yields the maximum or minimum value of an objective function can be challenging. The beauty of the linear programming method is that the maximum and minimum values of the objective function are always found at one of the vertices of the feasible region. That fact limits the number of calculations that must be done to determine the point in the feasible region that yields the maximum (or minimum) value of the objective function.

Follow these steps to use the linear programming method:

1. Identify the quantity that you want to maximize or minimize (e.g., profit or area).
2. Introduce your variables that affect that quantity you are maximizing or minimizing.
3. Write a system of linear inequalities to describe all of the conditions of the problem.
4. Graph the system of linear inequalities.
5. Write an objective function to represent the relationship between the two variables in your system of linear inequalities as well as the quantity that you are maximizing or minimizing.
6. Determine the point(s) in your solution region that optimizes your solution.
   a) If you are looking for a maximum: This will be the point in your solution region that gives you the highest output in the objective function.
   b) If you are looking for a minimum: This will be the point in your solution region that gives you the lowest output in the objective function.

Note: The maximum and minimum values for an optimization problem solved in this way are found at one of the **vertices (or corners)** of the solution region.

Consider the following example.

Example 1

A dairy farmer creates two products every day—milk and cheese. The livestock produce 900 litres of milk each day. To produce 1 kilogram of cheese, 5 litres of milk are needed. At the production facilities, it is only possible to produce a maximum of 100 kilograms of cheese each day. The farmer can make $2 for every 1 litre of milk sold and $8 for every kilogram of cheese sold. The farmer wants to know what combinations of milk and cheese will result in the maximum profit and what that profit will be.

a) What quantity does the farmer want to optimize?

b) What are the two variables in this situation?

c) Write a system of linear inequalities to represent these conditions:

   i) the distribution of litres of milk between milk and cheese

   ii) the maximum number of kilograms of cheese that can be made each day

d) What do you know about the restrictions on the domain and range of the variables? Explain.

e) Graph the system of linear inequalities.

f) Write the objective function that represents the quantity the farmer wants to optimize.

g) Determine the vertices of the solution region. These are possible optimal solutions.

h) Determine which coordinates of the vertices will optimize the farmer's profit, and calculate what the maximum profit will be.

Solution

a) The farmer wants to optimize profit.

b) The two variables in this situation are the number of litres of milk the farmer sells and the number of kilograms of cheese the farmer sells. Let x represent the number of litres of milk and let y represent the number of kilograms of cheese.

c) i) The cows produce 900 litres of milk each day and 5 litres of milk are required to make each kilogram of cheese. Therefore, the total number of litres of milk used each day is $x + 5y$. In total, this must be 900 litres or less. This creates the inequality $x + 5y \leq 900$.

   ii) The farmer can only make 100 kilograms of cheese each day. The inequality that represents this is $y \leq 100$.

d) As the domain is the number of litres of milk, this value must be greater than or equal to zero, $x \geq 0$.

Similarly, the range is the number of kilograms of cheese and this value must also be greater than or equal to zero, $y \geq 0$.

It is impossible to produce a negative amount of milk or cheese.

e) The system of linear inequalities that needs to be graphed is:

$x + 5y \leq 900$

$y \leq 100$

$x \geq 0$

$y \geq 0$

In the first linear inequality, $x + 5y \leq 900$, rearrange to slope-intercept form before graphing.

$$5y \leq -x + 900$$

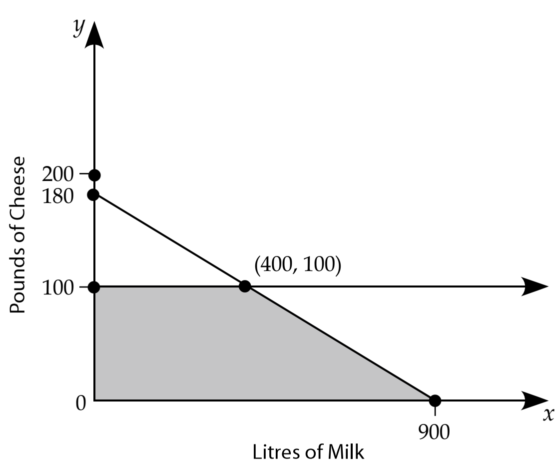
$$y \leq -\frac{1}{5}x + 180$$

The slope of this line is $-\frac{1}{5}$, the y-intercept is 180, and the line is solid.

Shade below the boundary line since the form is "$y \leq \ldots$."

The second linear inequality, $y \leq 100$, is a horizontal line at $y = 100$. Shade below the boundary line since the form is "$y \leq \ldots$."

The last two inequalities demonstrate that the solution to this system must be in the first quadrant.



f) Since you want to optimize profit, the objective function represents how the x- and y-variables are used to calculate the profit the farmer will make. As the farmer makes \$2 per litre of milk sold and \$8 per kilogram of cheese sold, the objective function to calculate the profit, P, is:

$P = 2x + 8y$

g) The maximum and minimum values will occur at one of the vertices. The vertices of the solution region are (0, 100), (400, 100), (900, 0), and (0, 0).

h) To determine which point optimizes the farmer's profit, substitute each set of coordinates into the objective function in (f). The point that creates the smallest value is the minimum profit. The point that creates the largest value is the optimal solution (maximum profit).

(0, 100)
$P = 2x + 8y$
$P = 2(0) + 8(100)$
$P = 0 + 800$
$P = \$800$

(400, 100)
$P = 2(400) + 8(100)$
$P = 800 + 800$
$P = \$1600$

(900, 0)
$P = 2(900) + 8(0)$
$P = 1800 + 0$
$P = \$1800$

(0, 0)
$P = \$0$

The combination of milk and cheese that creates the greatest profit occurs when the farmer only produces 900 litres of milk and no cheese. This will earn the farmer \$1800 per day.

Use the following learning activity to practise what you just learned. These problems add additional steps rather than simply graphing systems of linear inequalities. Thus, you need to practise to make sure you understand how to follow the process of how to optimize the solution to a system of linear inequalities.

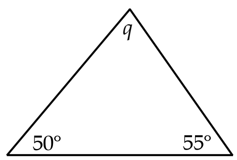
Learning Activity 7.6

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

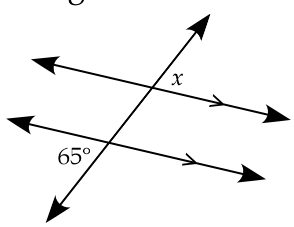
Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Determine the size of angle q in the triangle below.



Use the diagram below to answer questions 2 and 3.



2. What type of angles are displayed?
3. What is the size of angle x?
4. Is it possible to find a counter-example to the statement “numbers are either positive or negative”?
5. If the vertex of a parabola is at the point (3, 0), how many x-intercepts will the parabola have?
6. Convert 356750 centimetres to metres.
7. Evaluate: $\frac{(3-7)^2}{2}$
8. Solve for x: $-2x + 1 \geq 9$

continued

Part B: Optimizing Solutions to Linear Inequalities

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. A ceramics company wants to change its production process in order to maximize its profits. It produces mixing bowls and coffee mugs. Each mixing bowl requires 4 pounds of clay while each coffee mug requires 1 pound of clay. The company makes $5 for every mixing bowl sold and $2 for every coffee mug sold. Each day, the ceramics company receives a shipment of 300 pounds of clay. The factory is open for 10 hours every day. In one hour, 10 mixing bowls can be created or 20 coffee mugs can be created.
   a) What quantity does the ceramics company want to optimize?
   b) What are the two variables in this situation?
   c) Write a system of linear inequalities to represent these conditions:
      i) the distribution of clay between mixing bowls and coffee mugs
      ii) the time available for creating mixing bowls and coffee mugs
   d) What do you know about the restrictions on the domain and range of the variables? Explain.
   e) Graph the system of linear inequalities.
   f) Write the objective function that represents the quantity the ceramics company wants to optimize.
   g) Determine the vertices of the solution region.
   h) Determine which of the vertices optimizes the ceramics company's profit and what the optimum profit is.

continued

2. Eva wants to knit and sell various items as a fundraiser for Winnipeg Harvest. She plans to knit scarves and toques. A scarf takes 3 hours to knit and a toque takes 2 hours to knit. Eva wants to knit for a maximum of 50 hours. Eva knows there is a higher demand for toques than scarves, so she wants to make at least 10 toques. A scarf will make a profit of $10 for Winnipeg Harvest while a toque will make a profit of $7.
   a) What quantity does Eva want to optimize?
   b) What are the two variables in this situation?
   c) Write a system of linear inequalities to represent these conditions:
      i) the total number of hours available for knitting
      ii) the number of toques Eva needs to knit
   d) What do you know about the restrictions on the domain and range of the variables? Explain.
   e) Graph the system of linear inequalities.
   f) Write the objective function that represents the quantity that Eva wants to optimize.
   g) Determine the vertices of the solution region. These are possible optimal solutions.
   h) Determine which of the vertices will optimize Eva's profit, and find the maximum amount of money she will make.

3. A gardener is mixing two types of fertilizer to create an optimum product for the flower pots. One type of fertilizer is called Brand X and the other is called Brand Y. Each application of fertilizer is to use at least 360 grams of phosphorous and at least 180 grams of sulfur. Each unit of Brand X costs $0.80 and has 90 grams of phosphorous and 60 grams of sulfur. Each unit of Brand Y costs $0.50 and has 120 grams of phosphorous and 30 grams of sulfur. How much of each brand should be used to minimize the cost to the gardener?
   a) What quantity does the gardener want to optimize?
   b) What are the two variables in this situation?
   c) Write a system of linear inequalities to represent these conditions:
      i) the requirements for phosphorous
      ii) the requirements for sulfur

continued

d) What do you know about the restrictions on the domain and range of the variables? Explain.

e) Graph the system of linear inequalities.

f) Write the objective function that represents the cost the gardener wants to optimize.

g) Determine the vertices of the solution region. These are possible optimal solutions.

h) Determine which of the vertices will optimize the gardener’s cost so that it will be the lowest possible cost. What will the minimum cost be?

Lesson Summary

In this lesson, you learned how to optimize your solution when graphing systems of linear inequalities, using a process called linear programming. You did this by graphing a system of linear inequalities and creating an objective function to describe the quantity to be optimized. You then found potential optimal solutions, which occur at one of the vertices of your solution. You used your objective function to determine which point was the optimal solution. This is the last lesson in this module. After completing this lesson, you have only one more module to complete in the course.

Notes

Assignment 7.3

Optimizing Solutions to Linear Inequalities

Total: 25 marks

Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. The daycare you work at is raising money for a new play structure. The parents of all the children in the daycare decide to have a bake sale to raise this money. The bake sale will sell cookies and butter tarts. One dozen cookies requires 2 cups of flour and 2 eggs. One dozen butter tarts requires 1.5 cups of flour and 1 egg. The daycare has 15 cups of flour and a dozen eggs in the kitchen. The bake sale will make $9 for each dozen cookies sold and $4 for each dozen butter tarts sold.
   a) What quantity does the daycare want to optimize? *(1 mark)*

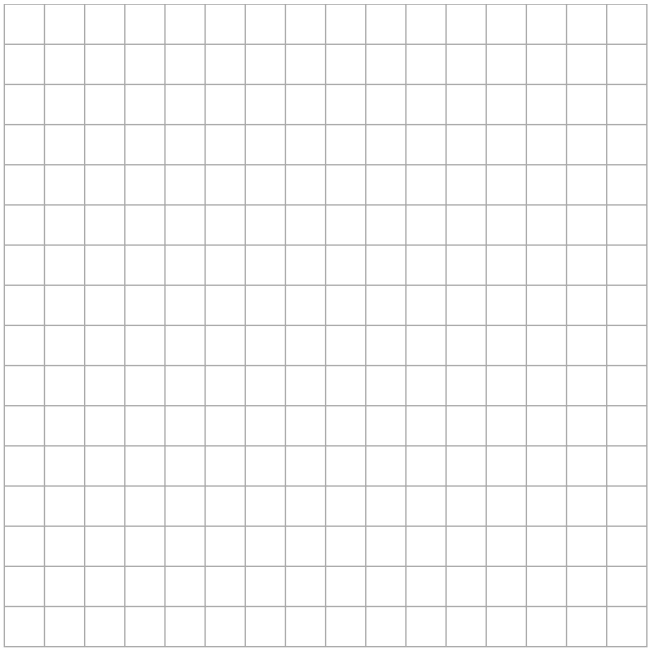
   b) What are the two variables in this situation? *(2 marks)*

   c) Describe the restrictions on the variables. *(1 mark)*

   d) Write a system of linear inequalities to represent these conditions: *(4 marks)*
      i) the total number of cups of flour required

      ii) the total number of eggs required

e) Graph the system of linear inequalities, and label the intersections and y-intercepts. *(6 marks)*



f) Write the objective function that represents the quantity the daycare wants to optimize. *(2 marks)*

g) Determine the vertices of the solution region. These are possible optimal solutions. *(4 marks)*

h) Determine which of the vertices will optimize the daycare's profit and what the maximum profit will be. *(5 marks)*

Notes

Module 7 Summary

In this module, you were first introduced to the concept of rates, such as the cost of an item for each 100 grams of weight or the number of kilometres you travel in one hour. You then applied the concept of rates to graphs and were able to interpret rates based on the slope of the graph. In the next few lessons, you reviewed how to graph linear equations, which enabled you to learn how to graph linear inequalities and then systems of linear inequalities. Finally, you combined all of your knowledge to optimize the solution to a system of linear inequalities.

In the next module, you will be looking at scale factors as well as 2-D and 3-D shapes. You will be looking at scale diagrams of objects as well as their area and volume. You will also be comparing how the area of 2-D shapes and volume of 3-D objects compares to the scale factor used to either enlarge or shrink an object.

Submitting Your Assignments

It is now time for you to submit the Module 7 Cover Assignment and Assignments 7.1 to 7.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 7 assignments and organize your material in the following order:

- ❑ Module 7 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 7 Cover Assignment: Skyscraper Puzzles
- ❑ Assignment 7.1: Interpreting Rates Algebraically and Graphically
- ❑ Assignment 7.2: Linear Inequalities and Systems of Linear Inequalities
- ❑ Assignment 7.3: Optimizing Solutions to Linear Inequalities

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Self-Test

Module 7 – Geometry

/68

(6 marks)

TEST TIME!

1. Define each of the following.

   a) Circle ______________________________

   b) Sector ______________________________

   c) Minor Arc ______________________________

   d) Tangent ______________________________

   e) Central Angle ______________________________

   f) Radius ______________________________

(7 marks)

2. Fill in the blanks.

   a) Two chords are ________________ if their intercepted arcs are the same.

   b) The amount of two-dimensional space covered by a circle is called its ____________.

   c) A ____________________ is perpendicular to a tangent to a circle.

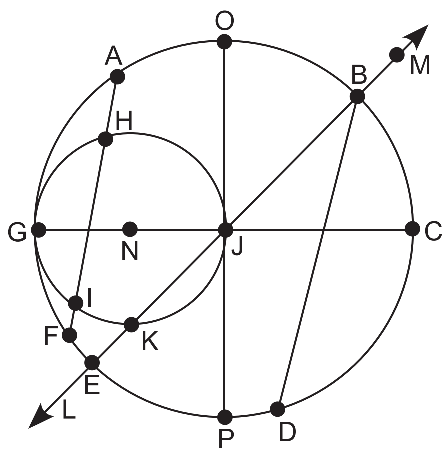
   d) The region bounded by a chord and its intercepted arc is called a(n) ______________.

   e) An angle formed between two chords of a circle sharing a common endpoint is called a(n) __________________.

   f) Any chord that is longer than another is ________________________________.

   g) A region of a circle bounded by two radii and an arc that is less than half the circle is called a(n) ____________________________.

3. *(8 marks)*



For the above diagram:

a) Name two diameters. ____________________

____

b) Name a tangent. ____________________

____

c) Name two chords. ____________________

____

d) Name two central angles. ____________________

____

e) Name two radii. ____________________

____

f) Shade a minor sector and label.

g) Shade a minor segment and label.

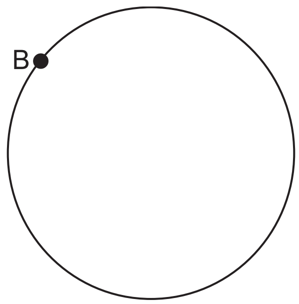
h) Name two inscribed angles. ____________________

____

4. a) For the given circle (using a compass and ruler only):

(1 mark) i) Construct a tangent at B.

(2 marks) ii) Inscribe a regular hexagon in the circle.

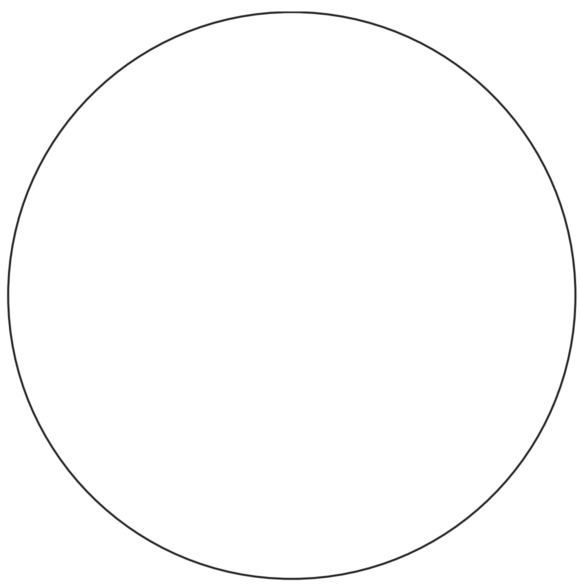


b) For the circle with a radius of 1.5 inches:

(2 marks) i) Inscribe a square in it (using a compass and ruler).

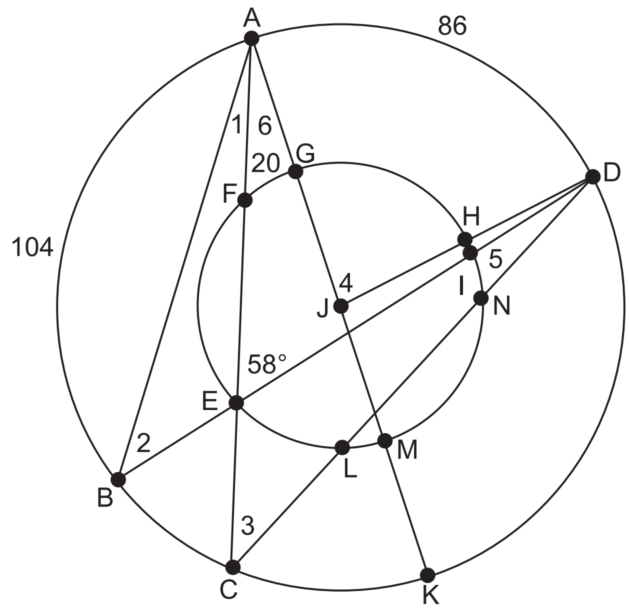
(2 marks) ii) Using the Theorem of Pythagoras, find the length of each side of the square (to two decimal places).

(2 marks) iii) Find the distance from the centre of the circle to one of the sides of the square.



5. Find the measures of the indicated angles or arcs. *(12 marks)*

a)



i) $m\overset{\frown}{GH} =$ ______ vii) $m\angle 1 =$ ______

ii) $m\overset{\frown}{HI} =$ ______ viii) $m\angle 2 =$ ______

iii) $m\overset{\frown}{FGHI} =$ ______ ix) $m\angle 3 =$ ______

iv) $m\overset{\frown}{CK} =$ ______ x) $m\angle 4 =$ ______

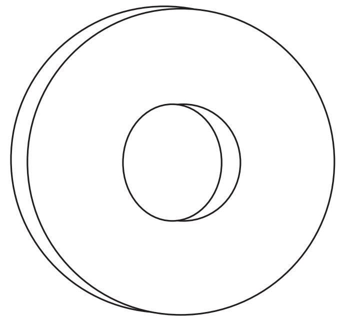
v) $m\overset{\frown}{DK} =$ ______ xi) $m\angle 5 =$ ______

vi) $m\overset{\frown}{ABCKDA} =$ ______ xii) $m\angle 6 =$ ______

b) In the above diagram, if the radius of the outside circle is 8 cm long, and the chords AB and DC are 10 cm and 12 cm, respectively, find the perpendicular distance from the centre to each. *(4 marks)*

(10 marks)

c) If the diagram



represents a washer and the radius of the outside circle is 8 ± 0.03 cm while the inside circle, which represents the cut out portion, has a radius of 4 ± 0.02 cm

and its depth is $1.5^{+0.02}_{-0.03}$ cm, find the values for the following chart.

DIMENSION	BASIC	MAXIMUM	MINIMUM	TOLERANCE
Radius of inside of washer				
Radius of outside of washer				
Perimeter of outside of washer				
Area of washer				
Volume of washer				

(2 marks)

6. Find the answers to the following question regarding foreign exchange. (Use the table supplied.)

CANADIAN DOLLARS		
BUYING RATE	FOREIGN CURRENCY	SELLING RATE
1.3424	United States – Dollar	1.3784
2.0473	United Kingdom – Pound	2.1573
1.0309	Australia – Dollar	1.1557
0.2189	Denmark – Kroner	0.2421
0.2652	Finland – Markaa	0.3627
0.2507	France – Franc	0.2744
0.8572	Germany – D. Mark	0.9375
0.162	Hong Kong – Dollar	0.1856
0.7644	Netherlands – Guilder	0.8283
0.1975	Norway – Kroner	0.2184

Your brother sends you a message from France. He needs 800 francs to pay for a flight to Norway. How much will it cost you in Canadian funds?

7. Solve the following applications using your graphing calculator. (Show the calculator keying sequence used to graph the functions you have found.)

   a) Consider a square piece of cardboard having sides 12 inches in length. Construct a box (without top) from cardboard by cutting out four equal squares from the corners and folding up the sides. What are the lengths of the sides of the box that has the largest volume? *(5 marks)*

   b) A piece of heavy machinery purchased for \$195 000 depreciates in value each year. The value v after n years is given by the formula: *(5 marks)*

$$v = 195000\,(0.85)^n$$

   Find the number of years it would take before the machinery is worth \$78 000.

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Grade 11 Applied Mathematics (30S)

Module 8
Scale Factors for 2-D and 3-D Shapes

Module 8: Scale Factors for 2-D and 3-D Shapes

Introduction

Geometry is a branch of mathematics that deals with the measurement and properties of figures and objects in space. Throughout this course, you have studied various aspects of geometry including angles, triangles, and trigonometry. In this module, you will be studying scale factors and how they relate to both 2-dimensional (2-D) and 3-dimensional (3-D) shapes and objects.

In previous mathematics courses, you studied scale drawings and scale diagrams. Examples of these include blueprints and world maps. In this module, you will be extending this knowledge to scale diagrams that can be used to determine the area of 2-D objects, the surface area of 3-D objects, and the volume of 3-D objects. These calculations are useful for a variety of purposes including determining the size of cities, provinces, and countries, finding the amount of material needed for building an object, and calculating the amount of liquid a container can hold.

You will also study the relationship among scale factors, surface area, and volume. A knowledge of these relationships leads to an understanding of many fascinating facts about the world around you—from why body proportions change over time to why certain animals need to live in the ocean, and much more.

Assignments in Module 8

When you have completed the assignments for Module 8, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Pattern Analysis
2	Assignment 8.1	Finding and Using Scale Factors of 2-D Shapes and 3-D Objects
3	Assignment 8.2	Scale Factors and Areas of 2-D Shapes
5	Assignment 8.3	Scale Factors, Surface Area, and Volume of 3-D Shapes

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, and can be either handwritten or typewritten. Both sides of the sheet may be filled. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

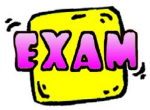
You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 8. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 5 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 5 to 8.

Resource Sheet for Module 8

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Writing Your Final Examination



You will write the final examination when you have completed Module 8 of this course. The final examination is based on Modules 5 to 8, and is worth 25 percent of your final mark in the course. To do well on the final examination, you should review all the work you complete in Modules 5 to 8, including all the learning activities and assignments. You will write the final examination under supervision.

Notes

Module 8 Cover Assignment: Pattern Analysis

Visualization and reasoning are two processes directly related to mathematical thinking. You have been using visualization skills with patterning since you began studying mathematics, doing such things as working with days of a week on a calendar or skip counting by 5s. Often, visual patterns can be analyzed using algebraic thinking. Here is an example:

Question

How many squares would you expect to have in the 11th figure if the pattern below was continued?

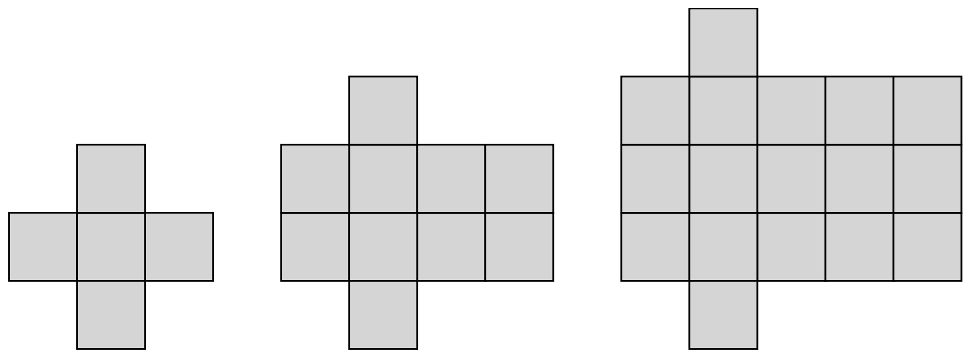


Figure 1 Figure 2 Figure 3

Analysis

Each shape can be divided into a rectangle with two extra squares—one on the top and one on the bottom.

The width of the rectangle increases by 1 unit in each new generation. In each figure, the width of the rectangle is always the same as the figure number. In Figure 1, the rectangle has a width of 1 square; in Figure 2, it has increased to 2 squares; and in Figure 3, it is 3 squares.

Now examine the length of each rectangle to see how it compares to the figure number.

Like the width, the length of each rectangle increases by 1 unit with each figure. However, the length is always 2 units more than the figure number. In Figures 1, 2, and 3, the lengths are 3, 4, and 5 respectively.

This analysis is an example of algebraic thinking. Can you use algebra to calculate the total number of squares required for any figure number?

In the three figures shown above, the total number of squares is length × width, plus two additional squares—one on the top and one on the bottom of the rectangle. Therefore, the number of blocks for figure N is 2 + (width × length) but the width is the same as the figure number, N, and the length is 2 more than the figure number, so

Number = $2 + N(N + 2)$

Now it is easy to answer the question, "How many squares would you expect to have in the 11th figure in this pattern?" The figure number, $N = 11$, so

Number = $2 + 11(11 + 2)$

Number = $2 + 11(13)$

Number = 145 squares for the 11th figure.

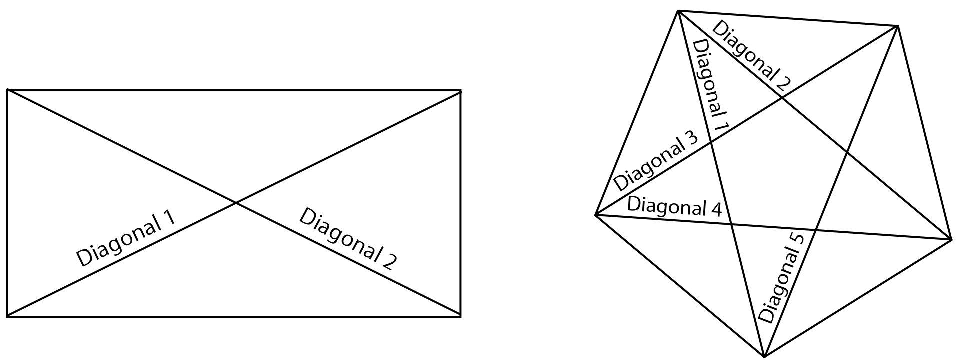
In the following cover assignment, you will work with a variety of patterns that require some algebraic thinking.

Pattern Analysis

Total: 10 marks

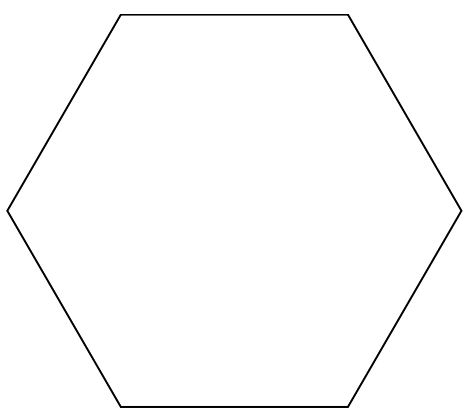
Clearly show the steps in your solutions on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. A diagonal is a line drawn from one vertex inside a shape to another vertex inside that same shape that is not in a direct line with it. A quadrilateral has two diagonals while a pentagon has five diagonals, as shown in the diagrams below.



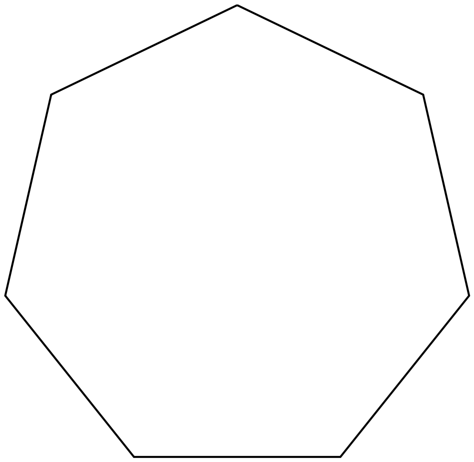
Find the number of diagonals in each of the following figures.

a) a hexagon (6 sides) *(1 mark)*



Module 8 Cover Assignment: Pattern Analysis (continued)

b) a heptagon (7 sides) *(1 mark)*



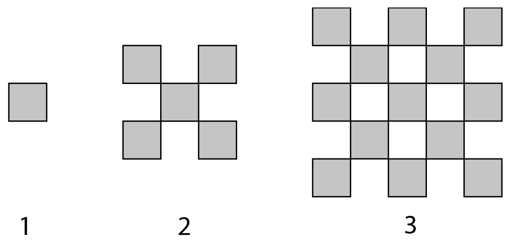
c) Complete the following chart. *(2 marks)*

Shape	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon
Number of Sides							
Number of Diagonals							

d) What is the relationship between the number of diagonals and the number of sides of the figure? *(1 mark)*

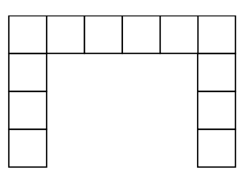
Module 8 Cover Assignment: Pattern Analysis (continued)

2. Consider the pattern below.

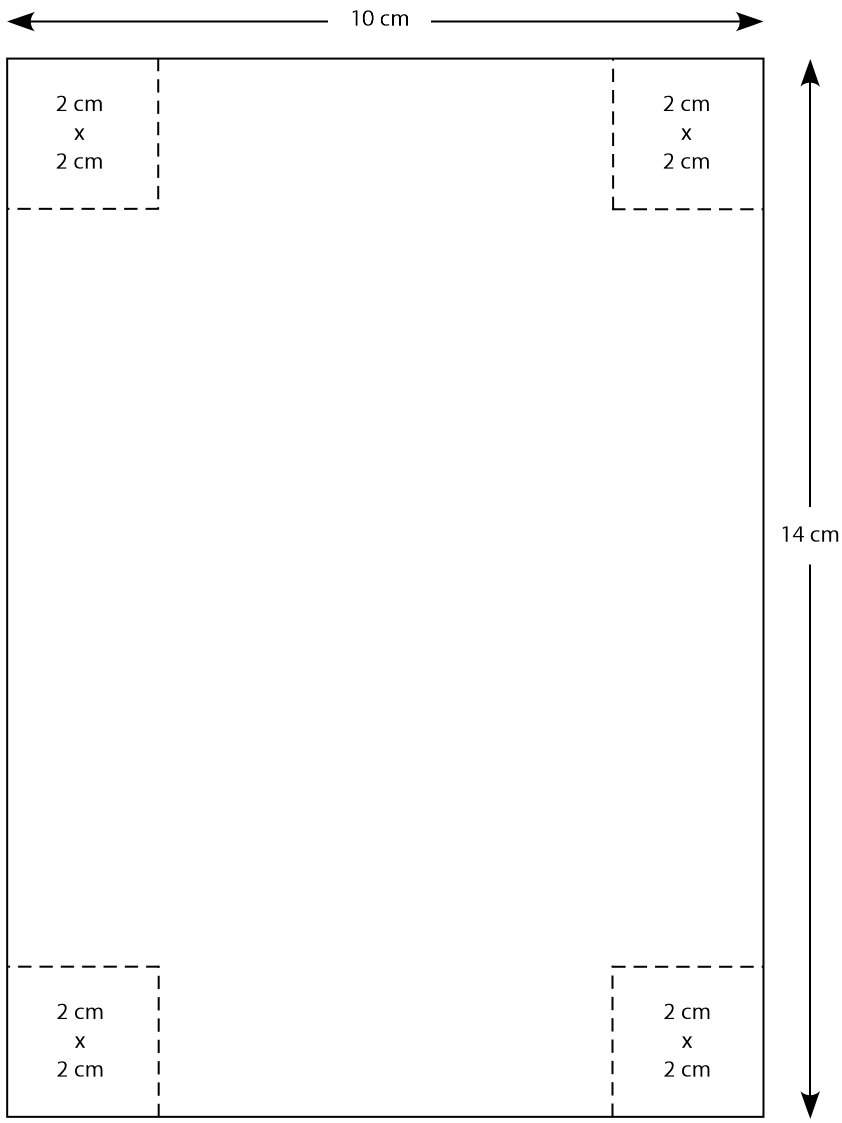


Draw the next two figures in the pattern. *(1 mark)*

3. The figure below is formed by 12 squares of the same size. If the area of the figure is 192 cm^2, determine its perimeter. *(2 marks)*



4. An open box is constructed from a rectangular sheet of metal 10 cm by 14 cm. Square pieces are cut out of the corners, as shown in the diagram below, and the resulting flaps are folded upward to make an open box. If the box is 2 cm deep, find its volume. *(2 marks)*



Lesson 1: Scale Diagrams of 2-D Shapes

Lesson Focus

In this lesson, you will

- ❑ explore where scale diagrams of 2-D shapes are used in your life
- ❑ learn how to draw a scale diagram of a 2-D shape using a scale factor
- ❑ learn how to determine the scale factor of a 2-D drawing
- ❑ learn how to determine an unknown dimension of a 2-D drawing using a scale factor

Lesson Introduction

In Grade 9 Mathematics, you learned how to create scale drawings that were either smaller or larger than the actual object. Sometimes it is necessary to draw a floor plan of a house where the house is much larger than your piece of paper. On the other hand, if you were drawing a diagram of an insect, the diagram might be much larger than the actual insect. These drawings are called **scale diagrams**. In each case, the image and the object will have the same shape but not the same size. The dimensions of the diagram and the object are **proportional**. This means that all measurements for the diagram and the object have the same ratio (for example, each dimension of the actual object is 50 times larger than the drawing).

2-D Shapes

Every scale drawing has a **scale**, which is the ratio of all the dimensions of the drawing to the actual dimensions of the object. The scale of a diagram is usually determined before the diagram is started. The order of the numbers of a scale are written as **drawing dimensions:actual dimensions**. Enlargements can be stated in the form **enlargement factor:1**. For example, a scale of 5:1 on a drawing is an enlargement, since a unit of length on the drawing is the same as five times the same unit of length on the actual object.

Reductions can be stated in the form **1:reduction factor**. A reduction of 1:5 indicates that one unit of length on the drawing corresponds to 5 times the unit of length for the actual object, or the actual object is five times larger than the drawing.

A scale of **1:1** means that the actual object and the diagram are the same size.

Note: A scale drawing is different from a sketch. A sketch is considered a rough or quick drawing where you may attempt to draw it to scale but the shape and size may not be proportional.

A **scale factor** is a number created from the scale ratio. It is usually written as a fraction, a decimal, or a percent.

Take a moment to record some instances where have you have encountered scale diagrams.

____

____

You may have noted examples like using a map of a city or country. You may have encountered scale diagrams when you took a picture of someone or something. This picture represents a scaled-down version of the actual size of the person or object. As well, you may have encountered scale diagrams if you purchased a piece of furniture that comes with instructions and a diagram that demonstrates how to assemble it.

Scale Measurement

Scale drawings involve measuring. Recall the following relationships among units of the metric system. Starting with millimetres (mm), the metric system builds by multiples of 10 (for example, 1 cm equals 10 mm).

	× 10		× 10		× 10		× 10		× 10		× 10	
______ km (kilometres)		______ hm (hectometres)		______ dam (decametres)		______ m (metres)		______ dm (decimetres)		______ cm (centimetres)		______ mm (millimetres)
	÷10		÷10		÷10		÷10		÷10		÷10	

Example 1

Find the missing measurements.

a) 2 km = ______________ m

b) 4 cm = ______________ m

c) 3600 km = ______________ cm

d) $18\frac{1}{2}$ mm = ______________ m

Solution

a) $2 \text{ km} = 2 \cancel{\text{km}} \times \dfrac{1000 \text{ m}}{1 \cancel{\text{km}}} = 2000 \text{ m}$

To change a quantity from kilometres to metres, you multiply by 1000.

b) $4 \text{ cm} = 4 \cancel{\text{cm}} \times \dfrac{1 \text{ m}}{100 \cancel{\text{cm}}} = 0.04 \text{ m}$

To change a quantity from centimetres to metres, you divide by 100.

c) $3600 \text{ km} = 3600 \cancel{\text{km}} \times \dfrac{1000 \cancel{\text{m}}}{1 \cancel{\text{km}}} \times \dfrac{100 \text{ cm}}{1 \text{ m}} = 360\,000\,000 \text{ cm}$

To change a quantity from kilometres to centimetres, you multiply by 100 000.

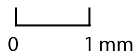
d) $18\frac{1}{2} \text{ mm} = 18.5 \cancel{\text{mm}} \times \dfrac{1 \text{ m}}{1000 \cancel{\text{mm}}} = 0.0185 \text{ m}$

To change a quantity from millimetres to metres, you divide by 1000.

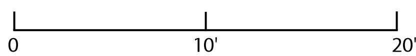
Representing Scales

Scales can be represented in various ways, including ratios, words, fractions, or diagrams. It is important to know that a scale of 1 : 100 does not imply that any specific units must be used. The scale works for all units. For example, it could mean that 1 inch on a diagram represents 100 inches on an actual object, or 1 cm on a diagram represents 100 cm on an actual object. Any unit can be used with a scale.

The following four scales all represent an **enlargement** of an object by a factor of 10.

- Ratio: $10:1$
- Words: 10 mm (1 cm) on the drawing represents 1 mm on the actual object
- Fraction: $\dfrac{10}{1}$
- Diagram: 

The following four scales all represent a **reduction** of an object by a factor of 120.

- Ratio: $1:120$
- Words: 1 inch on the drawing represents 120 inches (10 feet) on the actual object.
- Fraction: $\dfrac{1}{120}$
- Diagram: 

The scale factor, k, of a diagram can be calculated using the following formula:

$$k = \frac{\text{dimension on diagram}}{\text{dimension of actual object}}$$

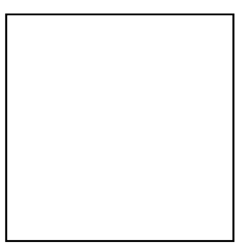
If k is between 0 and 1, this represents a *reduction*.

If k is greater than 1, this represents an *enlargement*.

You may wish to add this information to your resource sheet.

Example 2

A square has a side length of 3 cm. The diagram of the square, shown below, is drawn with a $1:1$ scale.



The diagram is correct. Both the actual square and the diagram have sides of 3 cm.

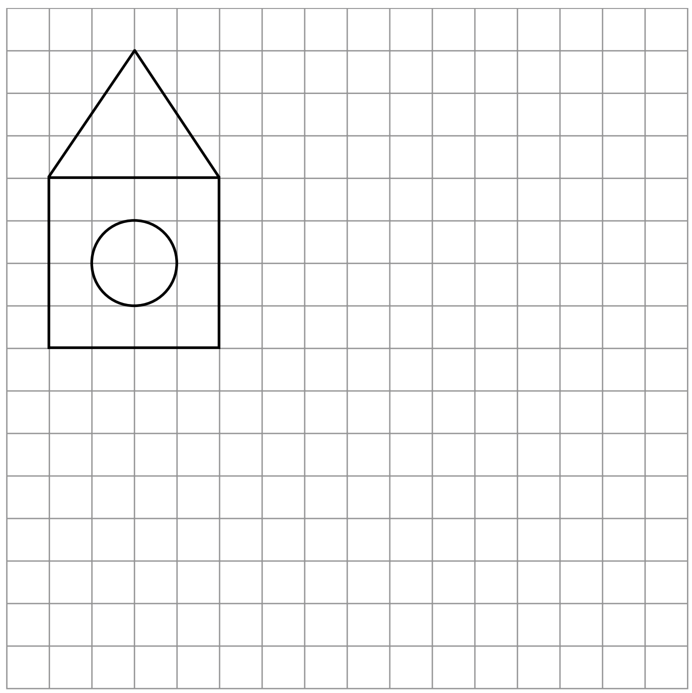
Consider a square with a side length of 30 cm. This figure would not fit on a page this size and, therefore, could only be drawn as a scaled-down diagram. You can use the same diagram above to represent a 30 cm × 30 cm square if you change the scale. Since the side lengths of this square would then be ten times the size of the drawing, the scale will be 1 : 10.

Creating a Scale Diagram

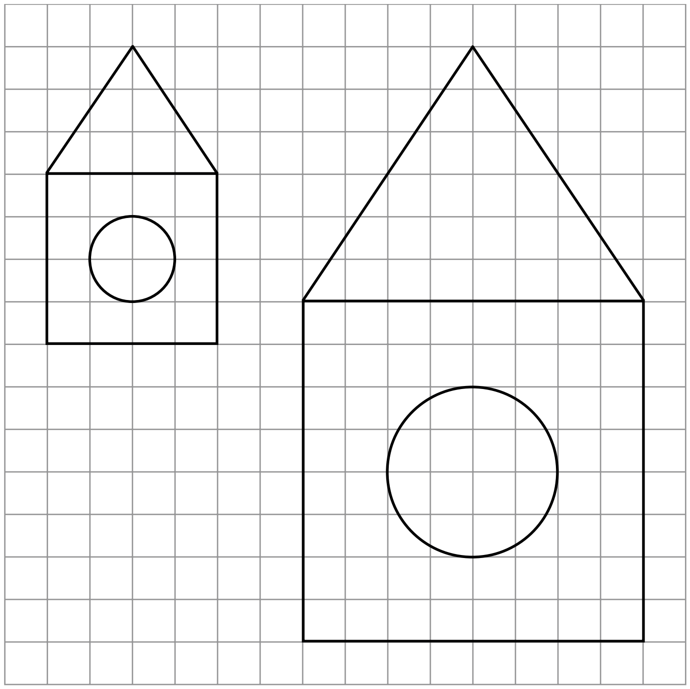
Given the actual dimensions of an object as well as a scale, you can represent the object with a scale drawing. Consider the following examples.

Example 3

Given the following diagram, enlarge it according to the scale 2 : 1.



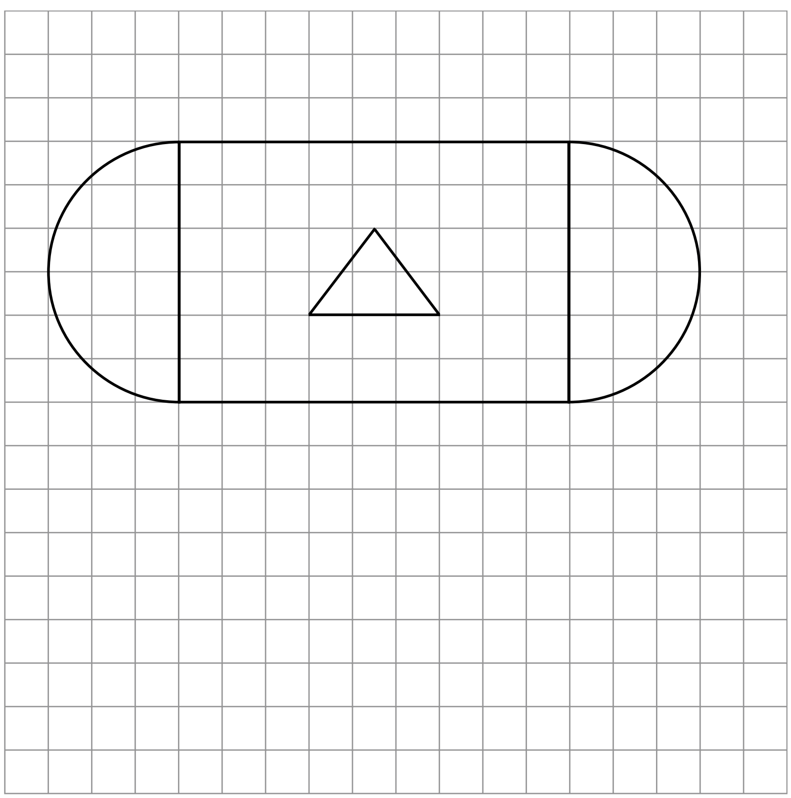
Solution



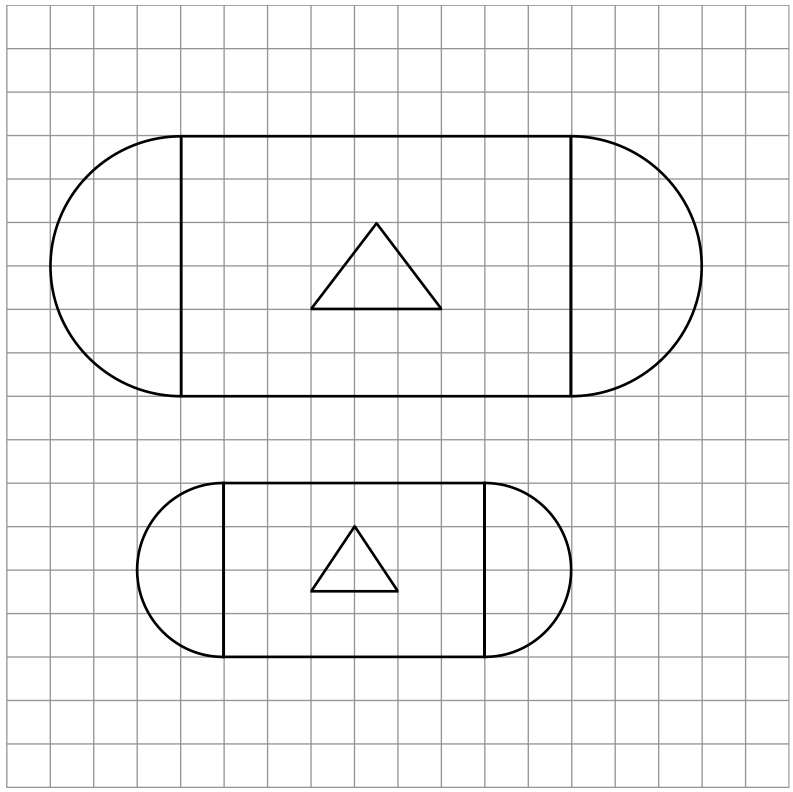
Note: All dimensions are double the original.

Example 4

Reduce the following diagram by 1 : 1.5.



Solution



Note: All dimensions are two-thirds of the original.

Example 5

Using the metric system, the length of a standard Canadian football field is 101 metres and the width is 59 metres. Use a pencil and a ruler to draw a scale diagram of the field using a scale of 1 : 1000.

Solution

The scale works for any units but, if it is helpful, you can substitute specific units into the scale and convert. For example, you could use centimetres so a scale of 1 : 1000 could be thought of as:

- 1 cm (on drawing) = 1000 cm (actual). Now convert 1000 cm to m for convenience.
- 1 cm (on drawing) = 1000 ÷ 100 m (actual). (1 m is the same as 100 cm)
- 1 cm (on drawing) = 10 m (actual).

You could use the scale as 1 : 1000 for any units or work with specific units and a scale of 1 cm : 10 m.

To find the length of the scale drawing, you can set up the following proportion.

	ratio	length
drawing	1 cm	x cm
actual	10 metres	101 metres

Let x be the scale length in cm.

$\frac{1}{10} = \frac{x}{101}$	Set up the proportion from the above table.
$(1)(101) = (x)(10)$	Cross-multiply.
$101 = 10x$	
$\frac{101}{10} = x$	Isolate x by dividing both sides of the equation by 10.
$10.1 = x$	Complete the division.

The length of the drawing of the football field is 10.1 cm.

To find the scale width of the football field, follow the same procedure used to find the length.

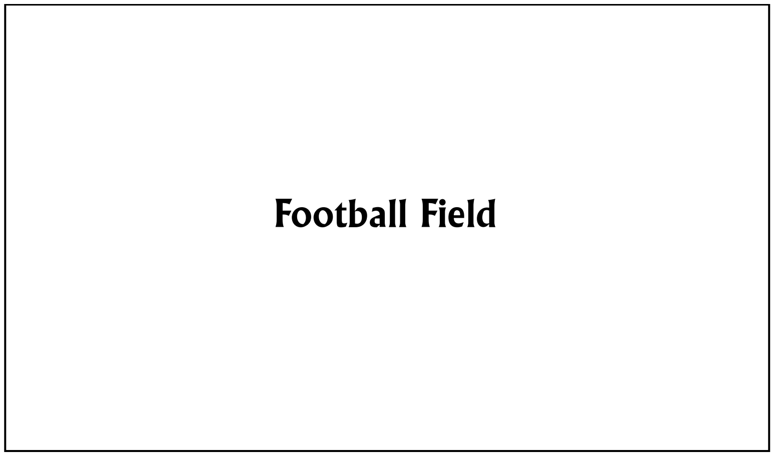
	ratio	width
drawing	1 cm	x cm
actual	10 metres	59 metres

Let x be the scale width in cm.

$\frac{1}{10} = \frac{x}{59}$	Set up the proportion from the above table.
$(1)(59) = (x)(10)$	Cross-multiply.
$59 = 10x$	
$\frac{59}{10} = x$	Isolate x by dividing both sides of the equation by 10.
$5.9 = x$	Complete the division.

The width of the drawing of the football field is 5.9 cm.

The scale drawing of the football field is as follows:



1 cm : 10 m
1 cm : 1000 cm
1 : 1000

You must include the scale on every diagram so that anyone looking at the drawing is able to calculate the actual dimensions of the object by measuring the diagram. You can either write 1 cm : 10 m or 1 : 1000. Your rectangle must be drawn with dimensions 10.1 cm × 5.9 cm. Do not write the drawing dimensions on the drawing; however, it is often helpful, but not necessary, to include the actual dimensions in the drawing.

As you have seen in this example, there are several ways of expressing the scale of a drawing. The most common way of representing the scale is to use a scale without units, such as 1 : 1000. When you express a scale in this way, the scale can then be used with any unit of measurement; you are not limited to only using centimetres or metres.

You may wish to add this statement to your resource sheet for future reference.

Determining Scale Factors

Example 6

The height of the Eiffel Tower is 320 metres. The height of a scale model of the Eiffel Tower is 8 centimetres. Determine the scale factor of the scale model.

Solution

First you need to convert the two given dimensions to the same units of centimetres.

$$320 \text{ metres} = 1\ \cancel{\text{m}} \times \frac{100 \text{ cm}}{1\ \cancel{\text{m}}} = 32\,000 \text{ cm}$$

To determine the scale factor, divide the height of the scale model by the height of the actual Eiffel Tower.

$$k = \frac{\text{height of scale model}}{\text{height of Eiffel Tower}}$$

$$k = \frac{8}{32000}$$

$$k = \frac{1}{4000}$$

Reduce the fraction so that the numerator is equal to one. (Divide the numerator and the denominator by 8.)

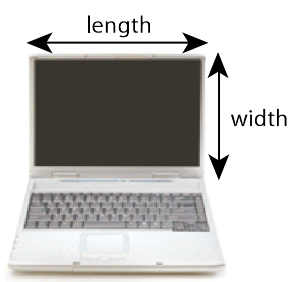
The scale factor of this object is $\frac{1}{4000}$. This scale factor represents a reduction.

The scale can be written $1:4000$.

Finding Missing Dimensions Using Scale Factors

Example 7

The following scale image of a laptop is from an advertisement for an electronics store. The scale is $1:15$. Determine the length and the width of the actual laptop.



Solution

First, use your ruler to measure the dimensions of the laptop in the scale drawing.

In the scale drawing, the length of the laptop is 2.3 cm and the width of the laptop is 1.6 cm.

To find the actual length of the laptop, you could set up a proportion. You could use a table, similar to the one below, to help set up the proportion. Let x represent the actual length of the laptop.

	ratio	length
drawing	1	2.3
actual	15	x

Solve for x using cross-multiplication.

$$\frac{1}{15} = \frac{2.3}{x}$$

$$(1)(x) = (2.3)(15)$$

$$x = 34.5$$

The actual length of the laptop is 34.5 cm.

To find the width of the laptop, follow the same procedure as in finding the length.

Set up the following proportion, letting x represent the actual width of the table.

	ratio	width
drawing	1	1.6
actual	15	x

Cross-multiply.

$$\frac{1}{15} = \frac{1.6}{x}$$

$$(1)(x) = (1.6)(15)$$

$$x = 24$$

The actual width of the laptop is 24 cm.

Notice that with a scale of 1 : 15, all dimensions increased by 15 times.

Learning Activity 8.1

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Solve for x: $\frac{x}{9} = \frac{3}{6}$
2. A recipe calls for $\frac{1}{3}$ cup of chocolate chips for every 2 cups of flour.

   How many cups of chocolate chips should be used for 5 cups of flour?
3. Solve for x: $6x + 2 = 14$
4. Determine the y-intercept: $2y = x^2 + 11x - 2$
5. What percent of 60 is 45?
6. Convert to a decimal: $\frac{12}{1000}$
7. Add: $1\frac{3}{4} + 3\frac{1}{8}$
8. Evaluate: $3^3 - 1^3$

continued

Part B: Scale Diagrams and Scale Factors of 2-D Shapes

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. State the relationship between the actual object and the scale diagram if the following scales are given.
   a) 1 : 3
   b) 4 : 1
   c) 1 : 2.5
   d) 6.5 : 1
   e) 1 cm : 1 kilometre

2. Represent the following scale factors in two other ways.
   a) 1 cm represents 8 m
   b) $\frac{1}{10}$
   c) 3 : 1

3. Complete the following chart.

Length in Drawing (cm)	Actual Length (cm)	Scale
6.8	680	
5.2	10 400	
4.5		1 : 10
	180	1 : 20

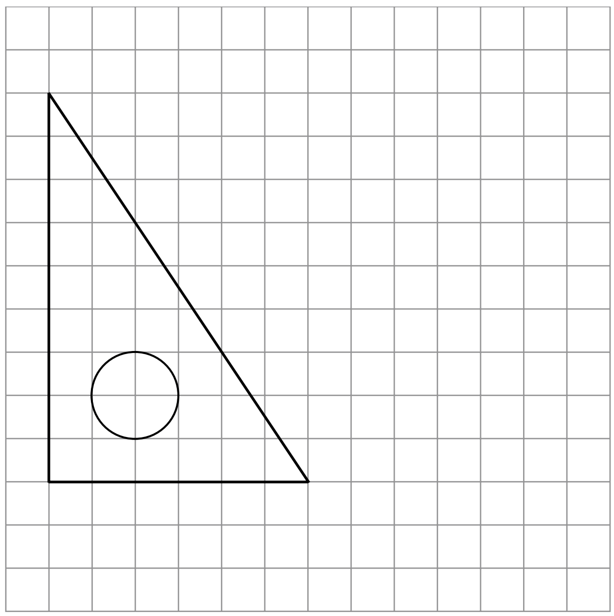
4. A building is 130 m tall. If the height of the building in the scale drawing is 3.25 cm, what scale factor is used in the drawing?

5. A scale drawing of a family room has dimensions of 3.25 cm by 2.75 cm. In the scale drawing, 1 cm represents 400 cm. What are the dimensions of the actual family room?

continued

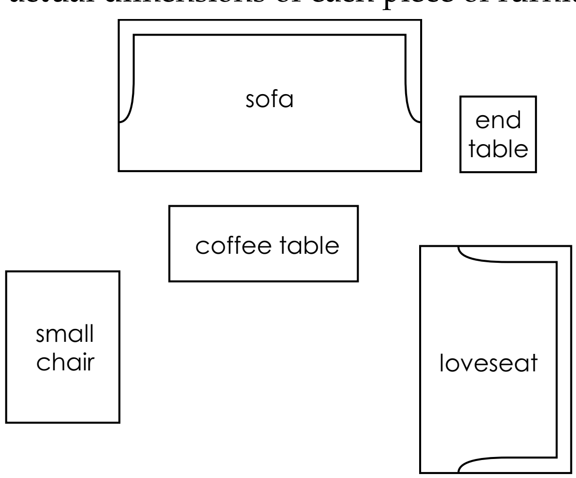
6. A volleyball court measures 5 m by 8 m. If it is drawn with a scale factor of $\frac{1}{300}$, what are the dimensions of the volleyball court in the scale drawing, in centimetres? Round your answers to one decimal place.

7. Using the scale 1 : 3, redraw the diagram below.



8. Represent a rectangular window, measuring 1.6 m by 1.2 m, with a scale drawing. Use a scale factor of $\frac{1}{20}$.

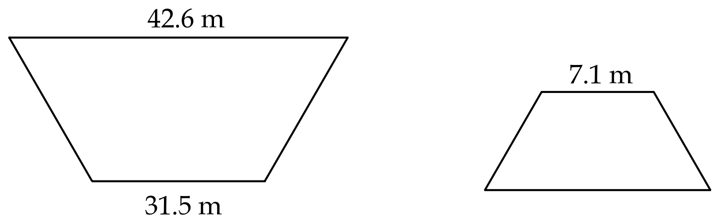
9. The following scale diagram represents an arrangement of furniture in a living room. The drawing uses a scale where 1 cm represents 60 cm. Find the actual dimensions of each piece of furniture in the room in metres.



continued

Learning Activity 8.1 (continued)

10. The following shapes are proportional. Determine the scale factor.



11. Draw a floor plan for a square bathroom with measurements of 10′ × 10′. Include the following items in your floor plan:

    a 3′ × 5′ bathtub

    a 2′ × 5′ sink in cabinet

    a 3′ × 3′ shower

    a 2′ × 2′ toilet

    a 3′ door opening

    Note: Be sure to allow enough clearance for the doorway and walkways around the fixtures.

    State the scale you used.

12. Find a full-length picture of yourself (a picture that includes both your head and your feet). Compare the height of you in the picture to your actual height. What is the scale factor of the picture?

Lesson Summary

In this lesson, you expanded your knowledge of 2-D shapes. You learned how some diagrams represent enlargements of the actual object, such as drawings of cells or insects. You also learned how some diagrams represent reductions of the actual object, such as building plans. When you compared the drawing dimensions to the dimensions of the actual object, you were able to represent the enlargement or reduction with a scale factor. Also, given a scale factor and an object, you were able to draw the enlargement or reduction of the object. In the next lesson, you will begin to look at 3-D objects.

Notes

Lesson 2: Scale Models of 3-D Objects

Lesson Focus

In this lesson, you will

- ❑ explore where scale diagrams or models of 3-D objects are used in your life
- ❑ learn how to determine the scale factor of a 3-D object
- ❑ learn how to determine an unknown dimension of a 3-D object using a scale factor

Lesson Introduction

Have you ever built a scale model of an airplane or vehicle? Have you ever completed a 3-D puzzle of a boat or a building? These are all examples of scale models of 3-D objects.

Scale models are used in situations where it is useful to understand the relationship between the dimensions of an object and what the actual object will look like before it is constructed. For example, architects use scale models to illustrate or promote their designs to various companies or investors.

3-D Objects

Every 3-dimensional scale model has a **scale factor**, similar to every 2-dimensional scale drawing. *All* dimensions of the scale model need to be proportional to the dimensions of the actual object. In other words, *all* dimensions of the scale model need to be either reduced or enlarged by the same factor.

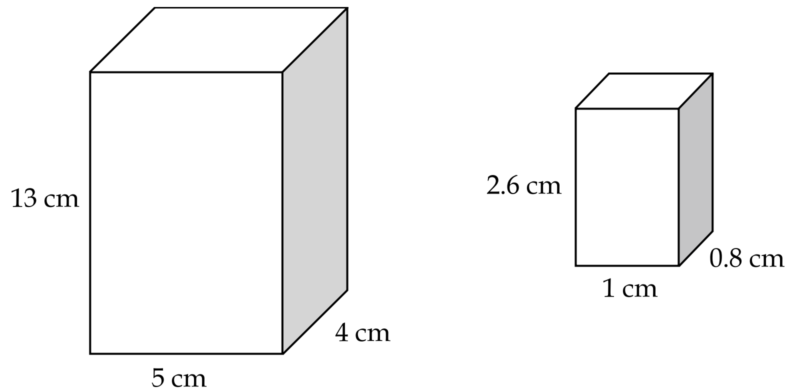
Similar Objects

In previous math courses, you learned about proportional or similar polygons (specifically similar triangles) that have the same shape but different size. All the sides are in proportion to each other and all the angles have exactly the same measurement. Likewise, similar 3-dimensional objects are objects that have the same shape, but not necessarily the same size. In other words, all dimensions of one object are either reduced or enlarged by the same factor to produce the dimensions of the second object.

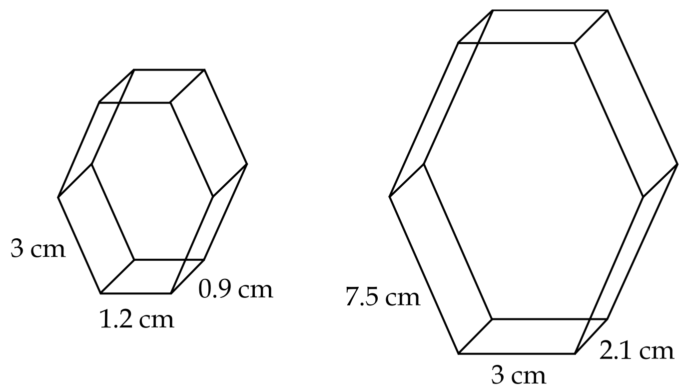
Example 1

Determine if each of the following sets of objects are similar.

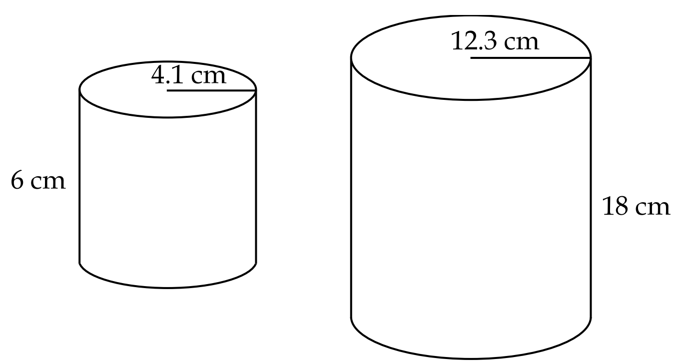
a)



b)



c)



Solution

a) To determine if these two rectangular prisms are similar, you need to check to see if the ratios of all dimensions of both shapes are equivalent. In other words, the ratio of the height, the ratio of the width, and the ratio of the depth on both diagrams of this prism need to be equivalent.

To do this, you need to select one object to represent your "model" object and one to represent the "actual" object. For this example, let the larger rectangular prism represent the model object and let the smaller rectangular prism represent the actual object.

Height:	Width:	Depth:
$k = \frac{\text{model}}{\text{actual}}$	$k = \frac{\text{model}}{\text{actual}}$	$k = \frac{\text{model}}{\text{actual}}$
$k = \frac{13}{2.6}$	$k = \frac{5}{1}$	$k = \frac{4}{0.8}$
$k = 5$	$k = 5$	$k = 5$

The scale factor k is equal to 5 in all three cases. The angles are the same as indicated by the shape. Therefore, these are similar objects.

b) Let the smaller hexagonal prism represent the model object and let the larger hexagonal prism represent the actual object.

Width:	Depth:	Slant Height:
$k = \frac{\text{model}}{\text{actual}}$	$k = \frac{\text{model}}{\text{actual}}$	$k = \frac{\text{model}}{\text{actual}}$
$k = \frac{1.2}{3}$	$k = \frac{0.9}{2.1}$	$k = \frac{3}{7.5}$
$k = 0.4$	$k = 0.429$	$k = 0.4$

The scale factor is not the same for every dimension. Therefore, these two objects are **not** similar.

c) Let the smaller cylinder represent the model object and let the larger cylinder represent the actual object.

Radius:

$$k = \frac{\text{model}}{\text{actual}}$$

$$k = \frac{4.1}{12.3}$$

$$k = \frac{1}{3}$$

Height:

$$k = \frac{\text{model}}{\text{actual}}$$

$$k = \frac{6}{18}$$

$$k = \frac{1}{3}$$

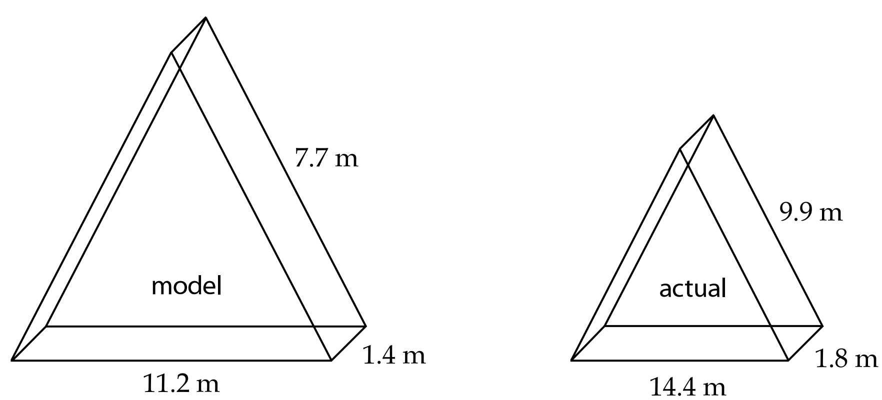
The scale factor is equal to $\frac{1}{3}$ in both dimensions and the angles are the same. Therefore, these two objects are similar.

Determining Scale Factors

To determine the scale factor of an object, you need to compare the dimensions of the actual object to the dimensions of the model object. This is similar to what you did in Lesson 1. Consider the following example.

Example 2

Determine the scale factors for the following set of objects.



Solution

To determine the scale factor for this triangular prism, you need to determine the ratio of the scale model to the actual triangular prism.

$$\frac{\text{bottom of model triangular prism}}{\text{bottom of actual triangular prism}} = \frac{11.2\text{ m}}{14.4\text{ m}} = \frac{7}{9} = 0.778$$

$$\frac{\text{width of model triangular prism}}{\text{width of actual triangular prism}} = \frac{1.4\text{ m}}{1.8\text{ m}} = \frac{7}{9} = 0.778$$

$$\frac{\text{diagonal height of model triangular prism}}{\text{diagonal height of actual triangular prism}} = \frac{7.7\text{ m}}{9.9\text{ m}} = \frac{7}{9} = 0.778$$

The scale factor of this triangular prism is $\frac{7}{9}$ or approximately 0.778.

Example 3

A pan in a child's dollhouse set has a radius of 2 cm, a depth of 0.7 cm, and a handle length of 3 cm. A comparable pan found in a kitchen has a diameter of 30 cm, a depth of 5.25 cm, and a handle length of 22.5 cm. Is the child's toy pan an accurate scale model of the actual pan found in this kitchen?

Solution

If this toy pan is an accurate scale model of an actual pan, then the scale factor of each dimension of the pan should be identical.

$$\frac{\text{radius of toy pan}}{\text{radius of actual pan}} = \frac{2\text{ cm}}{15\text{ cm}} = \frac{2}{15}$$

Note: You must change the given measurement of the diameter of the actual pan to the radius of the actual pan. Alternatively, you could convert both measurements to diameters and then determine the scale factor.

$$\frac{\text{depth of toy pan}}{\text{depth of actual pan}} = \frac{0.7\text{ cm}}{5.25\text{ cm}} = \frac{2}{15}$$

$$\frac{\text{handle of toy pan}}{\text{handle of actual pan}} = \frac{3\text{ cm}}{22.5\text{ cm}} = \frac{2}{15}$$

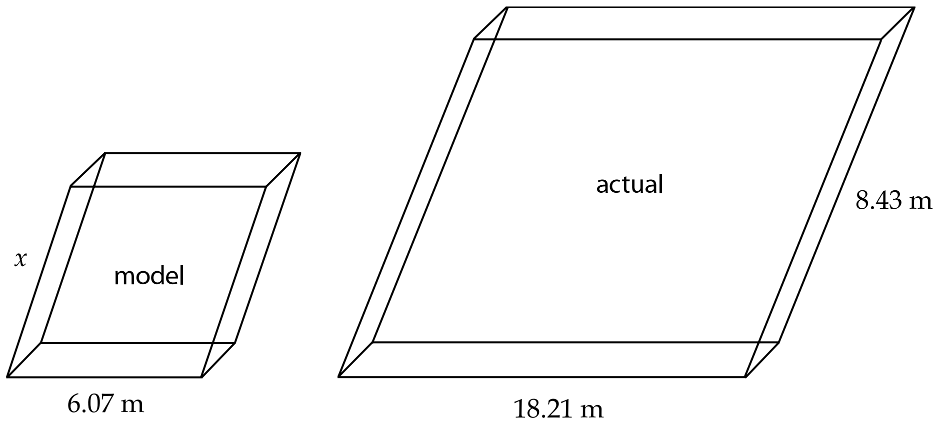
The scale factor of each dimension is identical. Therefore, the toy pan is an accurate scale model of the actual pan found in this kitchen.

Finding Missing Dimensions Using Scale Factors

Using a scale factor of a 3-D object, you can determine a missing dimension of the object itself or the scale model. Consider the following examples.

Example 4

These two shapes are similar. Determine the value of x.



Solution

Note: In order to solve this problem, you need to recognize that the side of the larger object with a length of 8.43 m is proportional to the side of the smaller object with a length of x. The reason for this is that opposite sides of a parallelogram are the same length.

First, find the scale factor relating the actual object to the model object by comparing the length of each object.

$$k = \frac{6.07}{18.21}$$

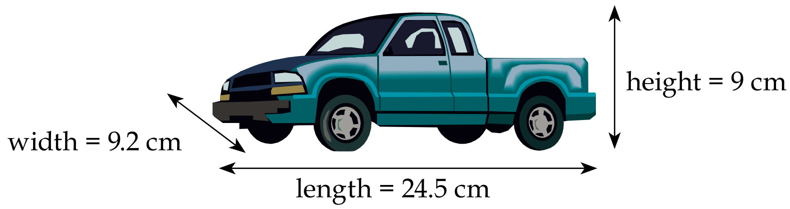
$$k = \frac{1}{3}$$

Then, find the diagonal height of the scale prism by multiplying the diagonal height of the actual prism by the scale factor.

$$\text{Diagonal Height} = (8.43 \text{ m})\left(\frac{1}{3}\right) = 2.81 \text{ m}$$

Example 5

The dimensions of a model truck are shown below. If this truck has been reduced according to a scale factor of $\frac{1}{22}$, what are the actual dimensions of the truck in metres?



Solution

Each of the dimensions of the actual truck has been decreased by a factor of 22 to create the model truck. Therefore, to determine the dimensions of the actual truck, you can multiply each of the dimensions of the model truck by 22.

Actual height of truck:

(9 cm)(22) = 198 cm

198 cm = 1.98 m

Actual length of truck:

(24.5 cm)(22) = 539 cm

539 cm = 5.39 m

Actual width of truck:

(9.2 cm)(22) = 202.4 cm

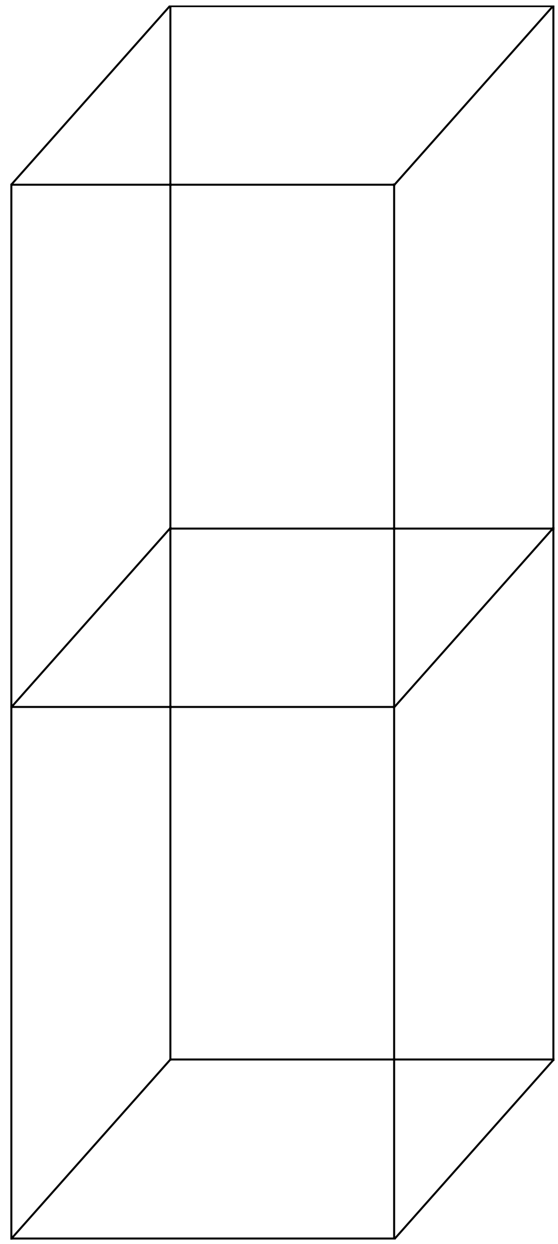
202.4 cm = 2.024 m

Example 6

A mother wants to build a bookcase for her daughter. She sees the following blueprint for a bookcase in a woodworking magazine.

a) If the scale of the drawing is 1 : 9, determine the actual dimensions of the bookcase, in inches.

b) Determine the actual dimensions of each part of the bookcase, including the back.



Scale: 1 : 9

Solution

a) First, you need to measure the width, height, and depth of the bookcase. Since wood is usually purchased in imperial measurements, it will be helpful to measure each of the dimensions in inches.

- The width of the model bookcase is 2 inches.
- The height of the model bookcase is 5.5 inches.
- The depth of the model bookcase is 1.25 inches.

According to the scale, this sketch has been reduced by a factor of 9. To determine the dimensions of the actual bookcase, you can simply multiply each of the measurements by 9.

The actual bookcase:

Width:

9×2 inches = 18 inches

Height:

9×5.5 inches = 49.5 inches

Depth:

9×1.25 inches = 11.25 inches

Top:

18 inches × 11.25 inches

Shelf:

18 inches × 11.25 inches

Bottom:

18 inches × 11.25 inches

Side × 2:

49.5 inches × 11.25 inches

Back:

49.5 inches × 18 inches

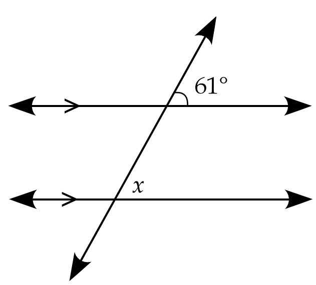
Learning Activity 8.2

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

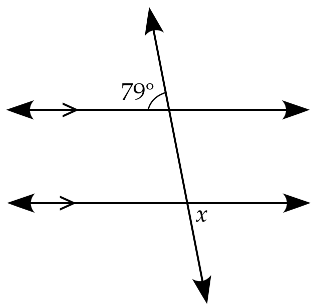
The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Solve for a: $4 - a \leq 9$
2. A normally distributed data set has a mean of 36 and a standard deviation of 5. Determine the two scores between which 68% of the population will fall.
3. Determine the size of angle x in the diagram below.



continued

4. Determine the size of angle x in the diagram below.



5. Which is the better deal if you wish to buy two pairs of jeans?
   a) Buy one pair at $49.99 and get the second pair at half price.
   b) Buy two pairs of jeans for $59.99 and receive a discount of 40% off the total purchase price.

6. What is the volume of a cube with a side length of 2 cm?

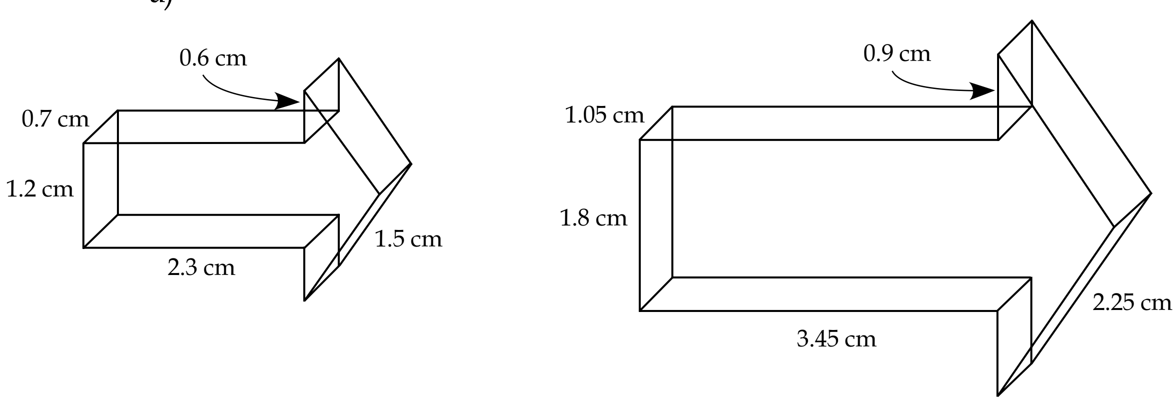
7. Convert 6230 mm to m.

8. Subtract: $2\frac{3}{8} - 1\frac{7}{8}$

Part B: Scale Factors of 3-D Objects

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

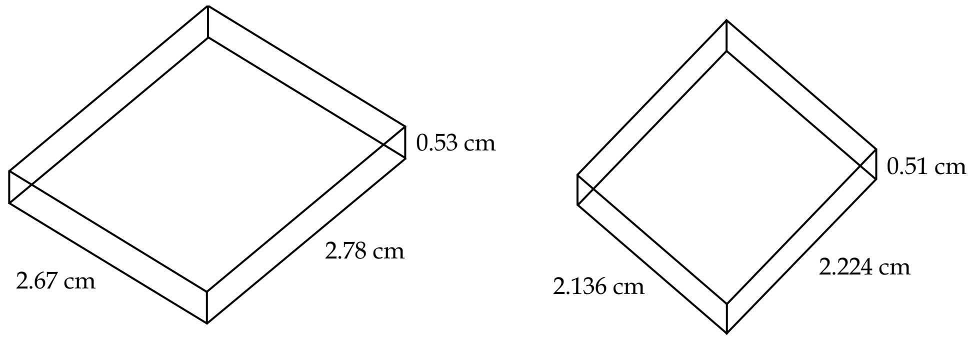
1. Determine if each of the following sets of objects is similar:
   a)



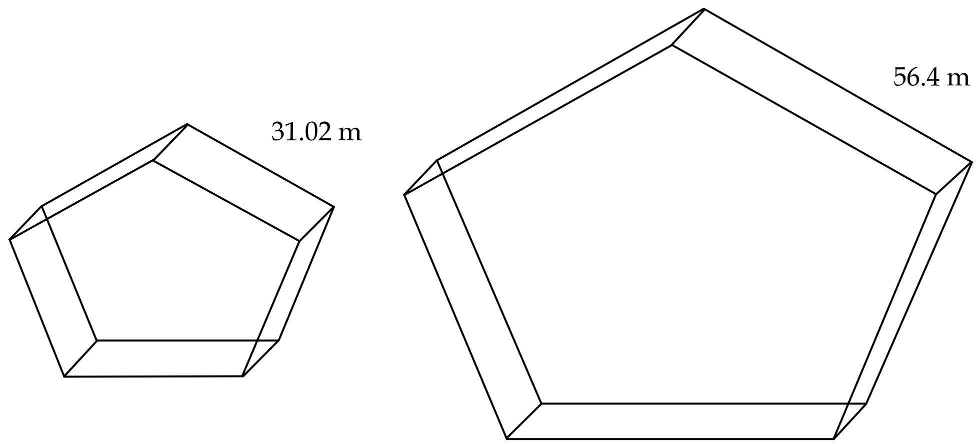
continued

Learning Activity 8.2 (continued)

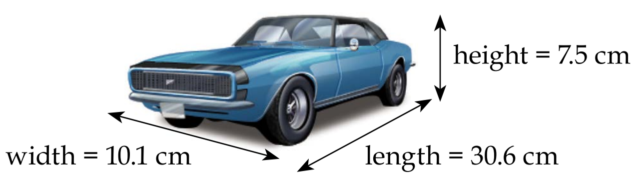
b)



2. Determine the scale factor for the following set of similar objects. Round your answer to two decimal places.

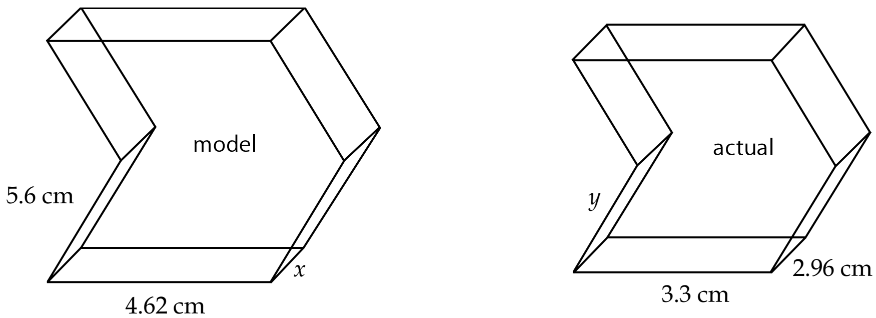


3. The dimensions of a model vehicle are shown below. If this model vehicle was built according to a scale factor of 0.05, determine the dimensions of the actual vehicle.

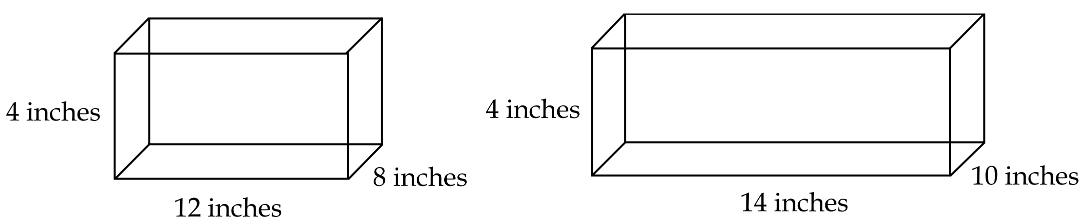


continued

4. The two shapes shown below are similar. Determine the value of x and y. Round your answers to two decimal places when necessary.



5. A company sells two different-sized rectangular cakes. The dimensions of each cake are shown below. Are the cakes similar? Would you expect other cakes that come in different shapes to be similar (i.e., circles)? Why or why not?



6. The actual dimensions of the Great Sphinx of Giza in Egypt are 73.5 m × 20.0 m × 6.0 m. A souvenir shop in Egypt wants to create an accurate model of the Sphinx for tourists to purchase. If they want the Sphinx model to be 10 cm long, what should be the height and the width of the Sphinx model? Round your answers to two decimal places.

7. Find a toy model of an object. Examples could include famous buildings, toy cars, boats, animals, dolls, or other models. Find the dimensions of the actual object. Determine if the toy model is an accurate scale model of the actual object.

Lesson Summary

In this lesson, you expanded your knowledge of finding scale factors of 2-D shapes to finding scale factors of 3-D objects. Using a scale factor for a 3-D drawing, you were able to calculate any missing dimensions of the 3-D object it represents. You were also able to verify if two objects were similar by determining if each dimension of the actual object was proportional to the corresponding dimension of the model object. In the next lesson, you will be looking at the relationship between the scale factor and area of a 2-D shape.

Notes

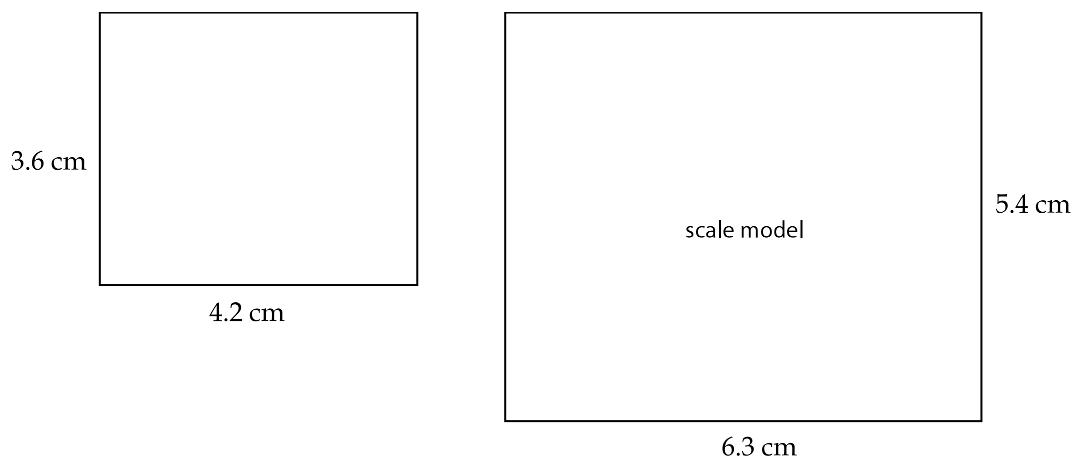
Assignment 8.1

Finding and Using Scale Factors of 2-D Shapes and 3-D Objects

Total: 33 marks

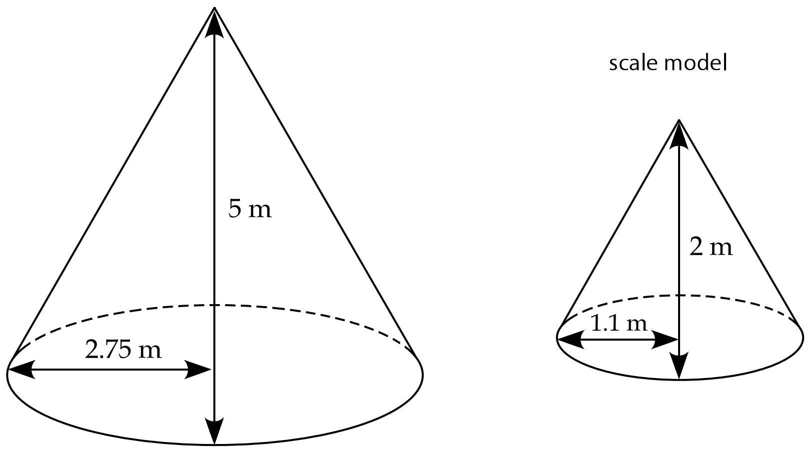
Clearly show the steps in your solution on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. The following shapes are similar. Determine the scale factor. *(3 marks)*



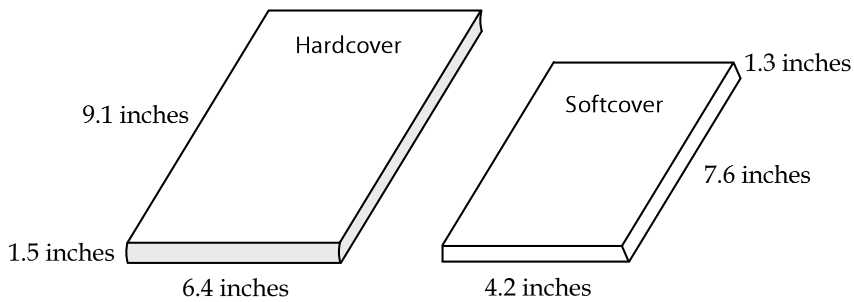
Assignment 8.1: Finding and Using Scale Factors of 2-D Shapes and 3-D Objects (continued)

2. The following objects are similar. Determine the scale factor. *(2 marks)*



Assignment 8.1: Finding and Using Scale Factors of 2-D Shapes and 3-D Objects (continued)

3. A hardcover version and a paperback version of the same novel are shown below. Determine if the books are similar. Explain your reasoning. *(4 marks)*



4. An advertising company wants to enlarge a 21.59 cm × 27.94 cm poster by a scale factor of 4.3. Find the dimensions of the enlarged poster. Round your answers to two decimal places. *(3 marks)*

Assignment 8.1: Finding and Using Scale Factors of 2-D Shapes and 3-D Objects (continued)

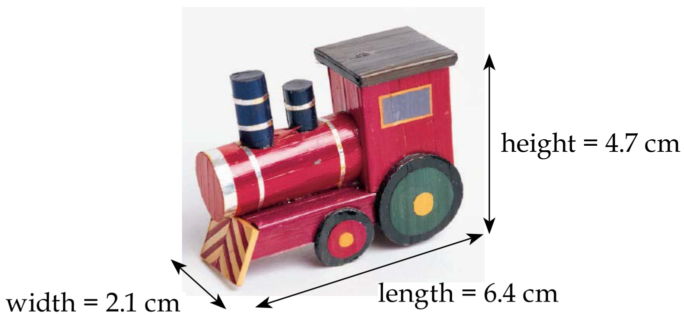
5. A bride wants to create a scale replica of her wedding cake as a memento of her wedding. The cake has two tiers. The larger tier has a height of 11 cm and a radius of 30 cm. The smaller tier has a height of 9 cm and a radius of 20 cm. The bride wants to create a scale model with a scale factor of 0.25. Determine the dimensions of the scale replica of the wedding cake. *(6 marks)*

Assignment 8.1: Finding and Using Scale Factors of 2-D Shapes and 3-D Objects (continued)

6. Represent a round tabletop with a diameter of 1 m by creating a scale drawing. Use a scale of 1 : 25. *(3 marks)*

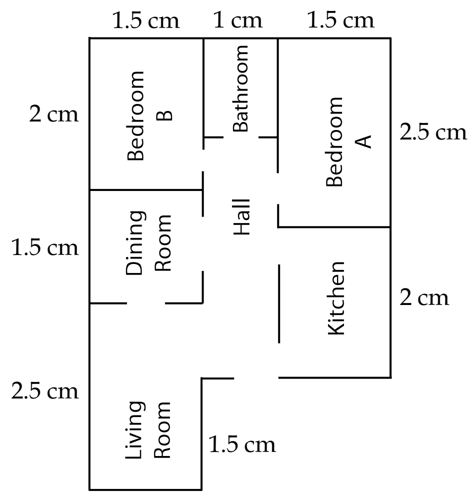
Assignment 8.1: Finding and Using Scale Factors of 2-D Shapes and 3-D Objects (continued)

7. A model train car is shown below. If this train car is drawn to scale and the length of the actual train car is 17.3 m, determine the height and the width of the actual train car. Round your answers to one decimal place. *(6 marks)*



Assignment 8.1: Finding and Using Scale Factors of 2-D Shapes and 3-D Objects (continued)

8. The following floor plan of a house is drawn with a scale of 1 : 250. Find the actual dimensions of bedroom A, bedroom B, the dining room, the kitchen, the living room, and the bathroom. *(6 marks)*



Notes

Lesson 3: Scale Factors and Area of 2-D Shapes

Lesson Focus

In this lesson, you will

- ❑ learn about the relationship between the scale factor and the area of a 2-D shape

Lesson Introduction

If you doubled the length and the width of a rectangle, would you expect the area of the rectangle to double as well? In this lesson, you will investigate this type of relationship to see how the area of a shape is affected when you enlarge a shape by any given scale factor.

Scale Factors and Area in 2-D Shapes

Similar figures have the same shape but are not necessarily the same size. Recall that the scale factor between two objects can be found using the formula $k = \frac{\text{model}}{\text{actual}}$ or k(actual) = model. In other words, you multiply the dimensions of the actual object by the scale factor, k, to determine the dimensions of the object model. To begin investigating how the areas of two similar figures are related, fill in the following charts.

Rectangle	Length (cm)	Width (cm)	Area
A	2	3	
B	4	6	
C	12	18	

Rectangle	Scale Factor	Ratio of Area
B to A		
C to B		
C to A		

See the answers on the following page.

Answers:

Rectangle	Length (cm)	Width (cm)	Area
A	2	3	6 cm^2
B	4	6	24 cm^2
C	12	18	216 cm^2

Rectangle	Scale Factor	Ratio of Area
B to A	2	4
C to B	3	9
C to A	6	36

What is the relationship between the scale factor and the ratio of the areas?

- When the size of the rectangle is doubled, the area of the rectangle is increased 4 times.
- When the size of the rectangle is tripled, the area of the rectangle is increased 9 times.
- When the size of the rectangle is increased by a factor of 6, the area of the rectangle is increased 36 times.

Note:

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

$6^2 = 6 \times 6 = 36$

In general, the ratio of the area of two rectangles is equal to the scale factor of these two rectangles squared.

Is this always true? Consider a different shape, such as a circle. Remember that the area of a circle can be found by using the formula $A = \pi r^2$.

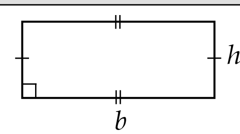
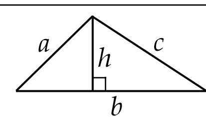
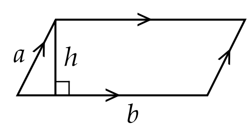
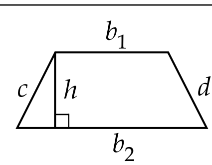
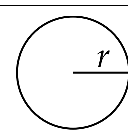
Circle	Radius (cm)	Area
A	2	4π cm^2
B	4	16π cm^2
C	12	144π cm^2

Rectangle	Scale Factor	Ratio of Area
B to A	2	4
C to B	3	9
C to A	6	36

Again, notice that the area of two circles is equal to the scale factor of these two circles squared. This is true for all similar figures. The reason the area is enlarged by the square of the scale factor is that both the dimensions (length and width) are enlarged, so when you multiply the enlarged length by the enlarged width you have multiplied by the scale factor twice.

You may want to include a similar explanation on your resource sheet.

The following chart displays the formulas for finding the area of various 2-D shapes that you have studied in previous courses. If you do not have these formulas memorized already, it would be helpful for you to include them on your resource sheet.

Area Formulas		
Name of Formula	**Diagram**	**Formula**
Rectangle		$A = bh$
Triangle		$A = \frac{1}{2}bh$
Parallelogram		$A = bh$
Trapezoid		$A = \frac{1}{2}h(b_1 + b_2)$
Circle Area		$A = \pi r^2$

Example 1

Parallelogram A has an area of 10.2 cm^2. Find the area of Parallelogram B with a base and a height that are five times larger than that of Parallelogram A.

Solution

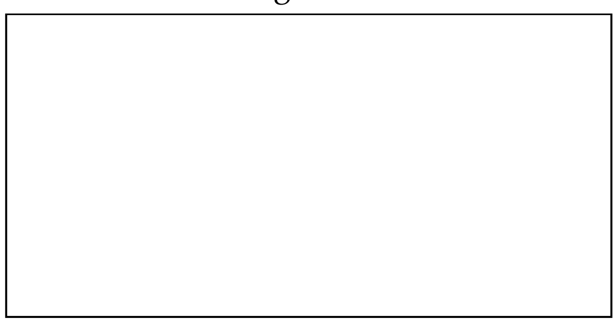
If the base and height of the new Parallelogram B are each five times larger than A, the area of the Parallelogram B is 5^2 or 25 times larger than Parallelogram A.

Therefore, the area of Parallelogram B is 10.2(25) = 255 cm^2.

Example 2

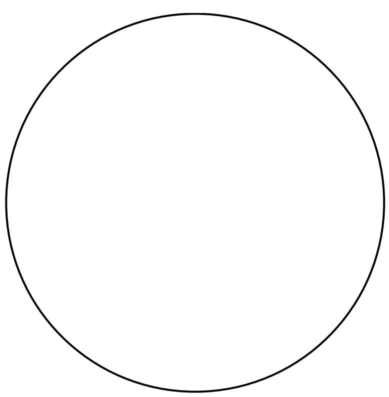
Using the scale diagram of each of the shapes below, determine the area of each of the actual shapes.

a) Cross-section of a trough



Scale: 1 : 5

b) Cross-section of a human hair

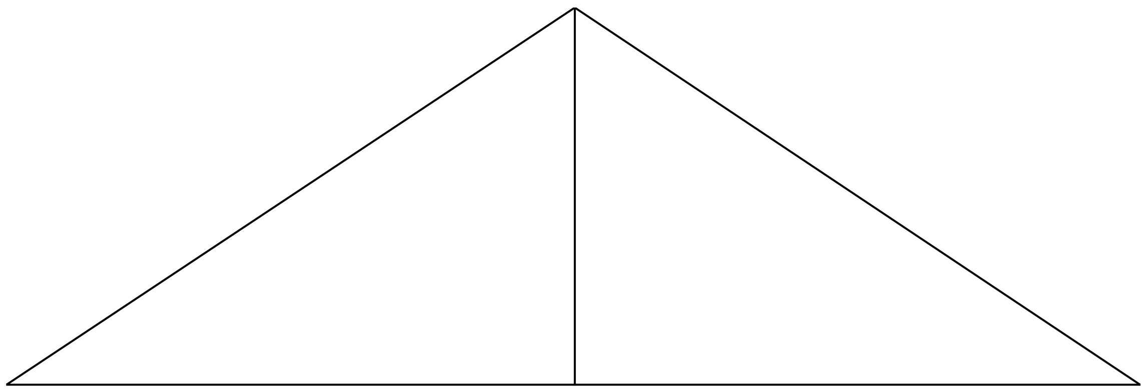


Scale: 10 000 : 1 or 1 : 0.001

c) Cross-section of an eavestrough

Scale: $\frac{1}{4}$

d) Rafter



Scale: 1 : 12

Solution

To solve this problem, you first need to measure the dimensions of each shape.

a) Length = 8 cm

Width = 4 cm

Scale Area = 8(4) cm^2 = 32 cm^2

If the scale is 1 : 5, then the scale factor is $\frac{1}{5}$. This scale represents a reduction. To determine the dimensions of an actual object using a scale factor, use the formula $k = \frac{\text{scale}}{\text{actual}}$ or k(actual) = scale. However, since you are dealing with area in this situation, the scale factor, k, must be squared. This gives:

$$k^2 = \frac{A_{\text{scale}}}{A_{\text{actual}}},\ k^2\left(A_{\text{actual}}\right) = A_{\text{scale}},\ \text{or}\ A_{\text{actual}} = \frac{A_{\text{scale}}}{k^2}$$

$$\text{Actual Area} = \frac{A_{\text{scale}}}{k^2}$$

$$= \frac{\left(32\ \text{cm}^2\right)}{\left(\frac{1}{5}\right)^2} = 32\ \text{cm}^2\left(\frac{5}{1}\right)^2 = \left(32\ \text{cm}^2\right)(25) = 800\ \text{cm}^2$$

b) Diameter = 5 cm

Radius = 2.5 cm

Scale Area = $(2.5)^2\pi$ cm^2 = 6.25π cm^2

$$\text{Actual Area} = \frac{A_{\text{scale}}}{k^2}$$

$$= \frac{\left(6.25\pi \text{ cm}^2\right)}{\left(\frac{10\ 000}{1}\right)^2} = 6.25\pi \text{ cm}^2\left(\frac{1}{10\ 000}\right)^2$$

$$= \frac{6.25\pi \text{ cm}^2}{100\ 000\ 000} = 0.000\ 000\,196 \text{ cm}^2$$

c) Height = 1 cm

B_1 = 2 cm

B_2 = 2.5 cm

$$\text{Scale Area} = \frac{1}{2}(1)(2 + 2.5) \text{ cm}^2 = 2.25 \text{ cm}^2$$

$$\text{Actual Area} = \frac{A_{\text{scale}}}{k^2}$$

$$= \frac{2.25 \text{ cm}^2}{\left(\frac{1}{4}\right)^2} = \left(2.25 \text{ cm}^2\right)\left(\frac{4}{1}\right)^2 = \left(2.25 \text{ cm}^2\right)(16) = 36 \text{ cm}^2$$

d) Height = 5 cm

Base = 15 cm

$$\text{Scale Area} = \frac{1}{2}(15)(5) \text{ cm}^2 = 37.5 \text{ cm}^2$$

$$\text{Actual Area} = \frac{A_{\text{scale}}}{k^2}$$

$$= \frac{37.5 \text{ cm}^2}{\left(\frac{1}{12}\right)^2} = \left(37.5 \text{ cm}^2\right)\left(\frac{12}{1}\right)^2$$

$$= \left(37.5 \text{ cm}^2\right)(144) = 5400 \text{ cm}^2$$

Whether you scale up or down using a particular scale factor to find the area of the scale diagram, you multiply the area of the actual shape by the scale factor squared. This can be represented by the formulas:

$$k^2 = \frac{A_{\text{scale}}}{A_{\text{actual}}}$$

$$k^2 \left(A_{\text{actual}}\right) = A_{\text{scale}}$$

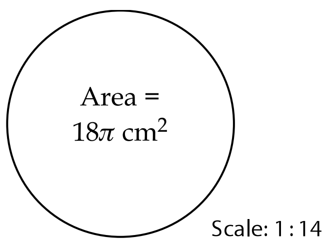
$$A_{\text{actual}} = \frac{A_{\text{scale}}}{k^2}$$

Include these formulas on your resource sheet.

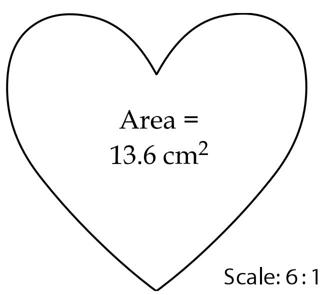
Example 3

Determine the piece of information requested for each set of similar figures below.

a) Determine the actual area of the circle.



b) Find the actual area of the heart.



Solution

a) This scale represents a reduction by a factor of 14. Therefore, the area of the scale model will be 14^2 times smaller than the actual area. To find the actual area, multiply the scale area by 14^2.

Alternatively, use the formula $A_{\text{actual}} = \dfrac{A_{\text{model}}}{k^2}$.

The scale factor is $\dfrac{1}{14}$.

$$\text{A}_{\text{actual}} = \frac{18\pi \text{ cm}^2}{\left(\frac{1}{14}\right)^2} = \left(18\pi \text{ cm}^2\right)\left(14^2\right) = 11\ 083.54 \text{ cm}^2$$

b) The scale represents an enlargement by a factor of 6. Therefore, the area of the scale model will be 6^2 times larger than the actual area. To find the actual area, divide the scale area by 36.

Alternatively, use the formula $A_{\text{actual}} = \dfrac{A_{\text{model}}}{k^2}$.

The scale factor is $\dfrac{6}{1}$ or 6.

$$\text{A}_{\text{actual}} = \frac{13.6 \text{ cm}^2}{(6)^2} = \frac{13.6 \text{ cm}^2}{36} = 0.38 \text{ cm}^2$$

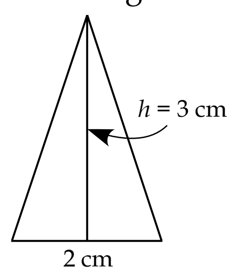
Example 4

Find the information requested for each set of similar figures below.

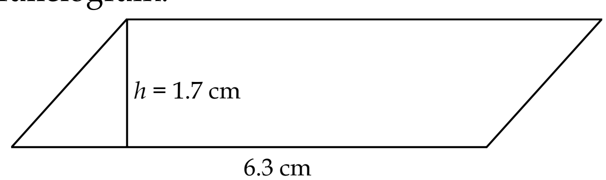
a) Determine the scale factor if the area of a scale diagram is 71 mm^2 and the area of the actual shape is 623 mm^2.

b) Find the scale factor if the area of a scale diagram is 144 m^2 and the area of the actual shape is 4 m^2.

c) The dimensions for the scale model of a triangle are shown below. If the area of the actual triangle is 36 cm^2, determine the dimensions of the actual triangle.



d) The dimensions for a parallelogram are shown below. If the area of the model parallelogram is 27 cm^2, give the dimensions of the model parallelogram.



Solution

a) To find the scale factor, divide the model area by the actual area. Then, take the square root of this value.

$$k^2 = \frac{71\text{ mm}^2}{623\text{ mm}^2}$$

$$k = \sqrt{\frac{71\text{ mm}^2}{623\text{ mm}^2}}$$

$$k = 0.338$$

The scale factor is approximately 0.338.

b) $k^2 = \dfrac{144 \text{ m}^2}{4 \text{ m}^2}$

$$k = \sqrt{\frac{144 \text{ m}^2}{4 \text{ m}^2}}$$

$$k = 6$$

The scale factor is 6.

c) You first need to determine the area of the model triangle. Then, determine the scale factor. Once you have found the scale factor, you can determine the dimensions of the actual triangle.

Step 1: Determine the area of the model triangle.

$$A = \frac{bh}{2}$$

$$A = \frac{3(2)}{2}$$

$$A = 3 \text{ cm}^2$$

Step 2: Find the scale factor.

$$k^2 = \frac{\text{A}_{\text{model}}}{\text{A}_{\text{actual}}}$$

$$k^2 = \frac{3 \text{ cm}^2}{36 \text{ cm}^2}$$

$$k = \sqrt{\frac{3 \text{ cm}^2}{36 \text{ cm}^2}}$$

$$k = 0.289$$

Step 3: Calculate the dimensions of the actual triangle using the scale factor and the dimensions of the model triangle.

Height:

$$k = \frac{\text{model height}}{\text{actual height}}$$

$$0.289 = \frac{3 \text{ cm}}{\text{actual height}}$$

$$\text{actual height} = \frac{3 \text{ cm}}{0.289}$$

$$\text{actual height} = 10.38 \text{ cm}$$

Base:

$$k = \frac{\text{model base}}{\text{actual base}}$$

$$0.289 = \frac{2 \text{ cm}}{\text{actual base}}$$

$$\text{actual base} = \frac{2 \text{ cm}}{0.289}$$

$$\text{actual base} = 6.92 \text{ cm}$$

d) Step 1: Determine the area of the parallelogram.

$$A = bh$$

$$A = (6.3\text{ cm})(1.7\text{ cm})$$

$$A = 10.71\text{ cm}^2$$

Step 2: Find the scale factor.

$$k^2 = \frac{A_{\text{model}}}{A_{\text{actual}}}$$

$$k^2 = \frac{27\text{ cm}^2}{10.71\text{ cm}^2}$$

$$k = \sqrt{\frac{27\text{ cm}^2}{10.71\text{ cm}^2}}$$

$$k = 1.588$$

Step 3: Calculate the dimensions of the model parallelogram.

To determine the dimensions of the model parallelogram, multiply the dimensions of the actual parallelogram by the scale factor.

Height:

model height = k(actual height)
model height = 1.588(1.7 cm)
model height = 2.70 cm

Base:

model base = k(actual base)
model base = 1.588(6.3 cm)
model base = 10.00 cm

Example 5

A typical screen in a movie theatre has an area of 164,736 inches^2. The dimensions of a similar flat-screen television are 13.5 inches × 32 inches.

a) Determine the scale factor relating the flat-screen television to the movie theatre screen.

b) Determine the dimensions of the movie theatre screen.

Solution

a) First, calculate the area of the flat-screen television.

$$A_{\text{flat screen}} = (13.5 \text{ inches})(32 \text{ inches}) = 432 \text{ inches}^2$$

Now, find the scale factor.

$$k^2 = \frac{A_{\text{scale}}}{A_{\text{actual}}}$$

$$k^2 = \frac{164{,}736 \text{ inches}^2}{432 \text{ inches}^2}$$

$$k = \sqrt{\frac{164{,}736 \text{ inches}^2}{432 \text{ inches}^2}}$$

$$k = 19.5$$

The scale factor relating the flat-screen television to the movie theatre screen is 19.5.

b) To find the dimensions of the movie theatre screen, simply multiply each of the dimensions of the flat-screen television by the scale factor 19.5.

Length of movie theatre screen = (32 inches)(19.5) = 624 inches

Width of movie theatre screen = (13.5 inches)(19.5) = 263.25 inches

The dimensions of the movie theatre screen are 624 inches × 263.25 inches.

Learning Activity 8.3

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

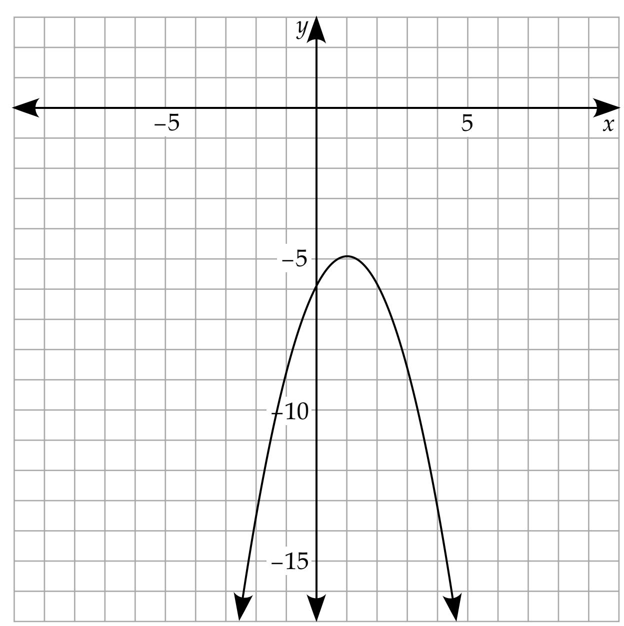
Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Factor: $y^2 - y - 72$
2. Solve for x: $(2x - 4)(x - 2) = 0$
3. Determine the y-intercept: $y = -3x^2 - 2x + 9$
4. Convert to a decimal: $\frac{5}{8}$
5. Order from smallest to largest: 0.1, 0.12, 0.115, 0.125

Use the following graph to answer questions 6 to 8.

6. Determine the maximum value.
7. State the domain.
8. State the range.

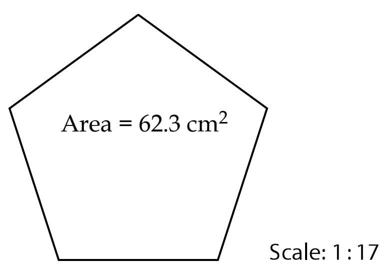


continued

Part B: Scale Factors and Area of 2-D Shapes

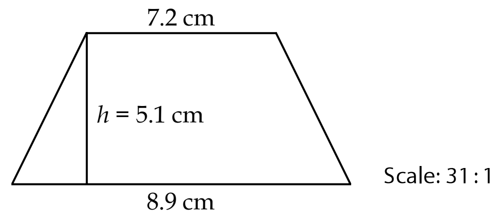
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. A trapezoid has an area of 82 cm^2. Find the area of a trapezoid with dimensions two times as large.
2. You have a square garden with an area of 25 square feet. You want to double the area of your garden for next year. What would be the dimensions of your new garden?
3. Your grandmother is sewing a throw blanket from a pattern. If the dimensions of the blanket on the pattern are 11 cm × 16.5 cm and the scale factor is $\frac{1}{9}$, determine how much fabric she will need to create the blanket.
4. The scale diagram of a house has an area of 1.7 square feet. If the scale diagram is drawn with a 1 : 26 scale, how many square feet does the actual house have?
5. Provide the requested information for each set of similar figures below. Round your answers to three decimal places when necessary.
   a) Determine the actual area of the pentagon represented in the scale diagram shown below.

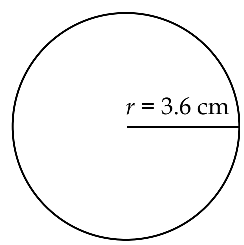


continued

b) Determine the actual area of the trapezoid represented in the scale diagram below.



c) Calculate the radius of the actual circle if the area of the actual circle is 714π cm^2.

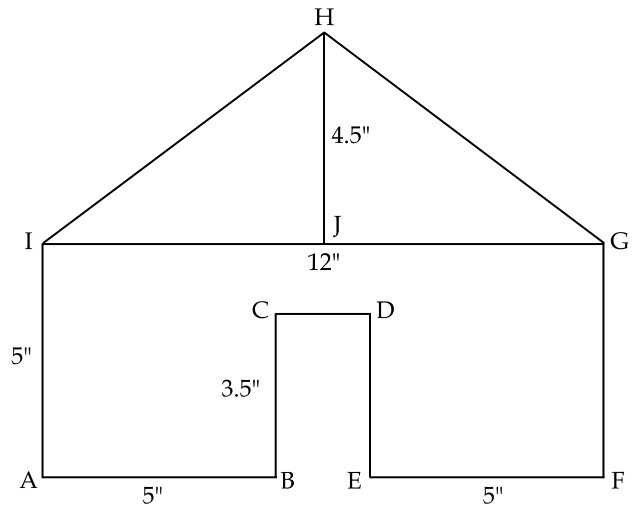


6. A furniture company wants to design a rug with an area of approximately 6 m^2 that is a scale model of a football field. If the dimensions of an actual football field are 101 metres $\times$ 59 metres, what should the dimensions of this rug be? Round your answers to four decimal places.

continued

Learning Activity 8.3 (continued)

7. A scaled reduction of the front of a dollhouse is illustrated as follows:



   Find the following from the drawing:

   a) area of ΔHIG
   b) area of AIGF, without the door
   c) total area of the front of the dollhouse, without the door
   d) area of the door

8. If the scale for the diagram of the dollhouse in question 7 is 1 : 24,

   a) find the actual area of the front of the dollhouse in ft.^{2}, without the door
   b) find the actual area of the door in ft.^{2}

Lesson Summary

In this lesson, you learned how the scale factor of two similar shapes is related to the area of those similar shapes. For example, if you triple the dimensions of a rectangle, the area of the new rectangle will not triple. Instead, the area of the new rectangle will increase by a factor of nine. In other words, the area increases by the square of the scale factor. In the next lesson, you will analyze how the scale factor of two similar shapes is related to the surface area of those shapes.

Assignment 8.2

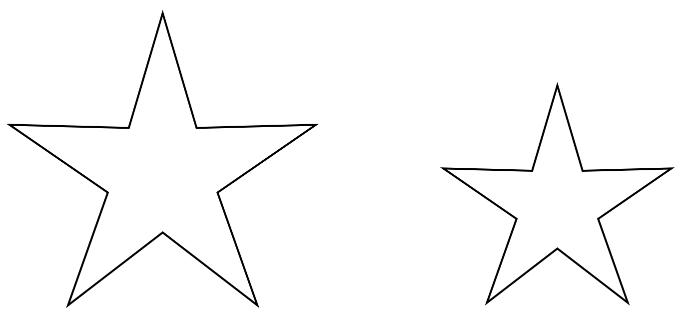
Scale Factors and Areas of 2-D Shapes

Total: 26 marks

Clearly show the steps in your solution on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

1. Determine the requested information for each set of similar figures below. Round your answers to two decimal places when necessary.

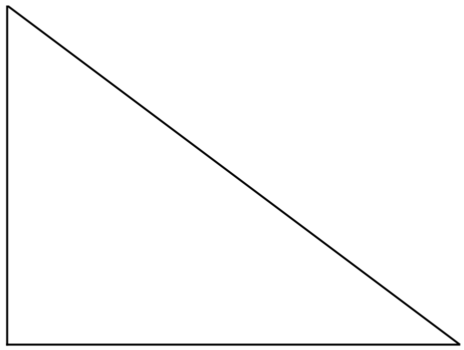
   a) Provide the scale factor relating the two stars below, if the larger star is the scale diagram. *(2 marks)*



Area = 7.1 cm^2 Area = 3.2 cm^2

Assignment 8.2: Scale Factors and Areas of 2-D Shapes (continued)

b) Find the actual area of the triangle represented in the reduction scale diagram shown below, which has been drawn with a scale factor of 0.125. *(3 marks)*



c) A parallelogram with a base of 8.9 m and a height of 4.3 m is reduced according to a scale of 1 : 17. Calculate the area of the model parallelogram. *(3 marks)*

Assignment 8.2: Scale Factors and Areas of 2-D Shapes (continued)

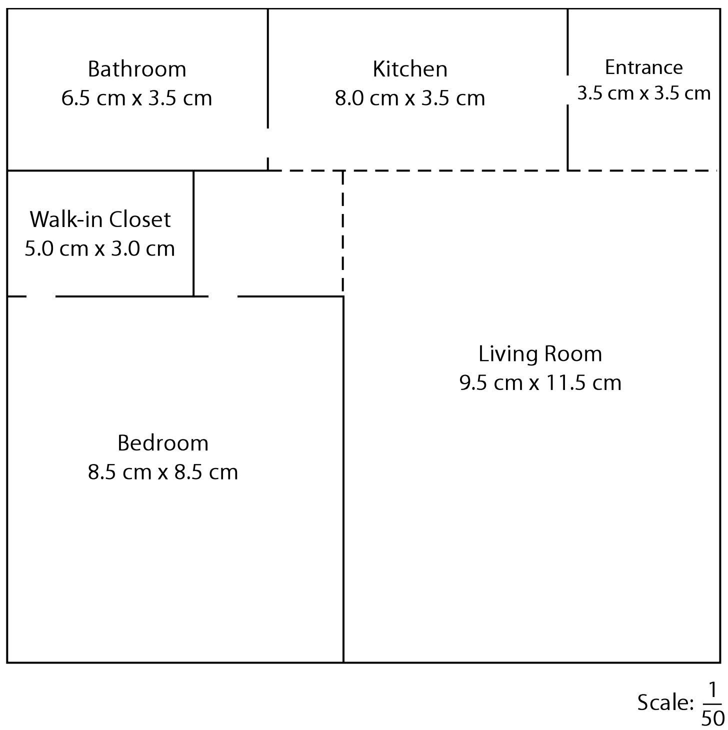
2. Mark wants to create a circular patio that is three-quarters the size of his current patio in terms of area.

   a) If his current patio has a diameter of 8 metres, what would be the diameter of his new patio? Round your answer to two decimal places. *(5 marks)*

   b) Why isn't the diameter of his new patio three-quarters the size of the diameter of his current patio? *(1 mark)*

3. The floor plan for an apartment is shown below. The model measurements for each room have been provided for you.

   Note: 1 m^2 = 10 000 cm^2



   Determine the area, in metres squared, of the following aspects of the apartment. *(12 marks)*

   a) the kitchen

b) the living room

c) the bathroom

Assignment 8.2: Scale Factors and Areas of 2-D Shapes (continued)

d) the bedroom

e) the walk-in closet

Assignment 8.2: Scale Factors and Areas of 2-D Shapes (continued)

f) the entire apartment

Notes

Lesson 4: The Relationship between Scale Factors and Surface Area of 3-D Shapes

Lesson Focus

In this lesson, you will

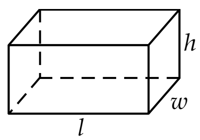
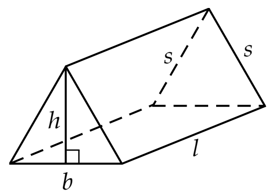
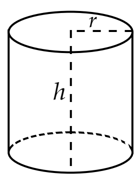
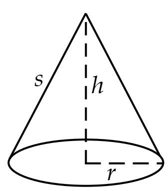
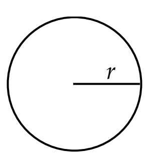
- ❑ learn about the relationship between the scale factor and the surface area of a 3-D object

Lesson Introduction

Surface area and area are similar concepts because they both measure area, but the difference is that surface area is related to 3-D objects while area is related to 2-D shapes. The surface area of a 3-D object is the sum of the areas of all the various 2-D shapes that make up the 3-D object. Does this mean that the surface area is related to the scale factor between two objects in the same way that area is related to the scale factor between two objects? In this lesson, you will find the answer to this question.

Scale Factors and Surface Area of 3-D Shapes

The following surface area formulas can be found in previous mathematics courses. If you are unfamiliar with these formulas, you may want to copy them down on your resource sheet. You will use variations of these formulas throughout this lesson.

Surface Area Formulas		
3-D Object	**Dimensions**	**Formula**
Rectangular Prism 	l = length w = width h = height	$SA = 2(lw + wh + lh)$
Triangular Prism 	b = base h = height l = length s = slant height	$SA = bh + 2ls + lb$
Cylinder 	r = radius h = height	$SA = 2\pi r^2 + 2\pi rh$
Cone 	r = radius h = height s = slant height	$SA = \pi rs + \pi r^2$
Sphere 	r = radius	$SA = 4\pi r^2$

The following chart shows the dimensions of three rectangular prisms and their corresponding surface areas.

Rectangular Prism	Length (cm)	Width (cm)	Height (cm)	Surface Area (cm^2)
A	2	3	1	$2((2)(3) + (3)(1) + (2)(1))$ $= 2(6 + 3 + 2)$ $= 2(11)$ $= 22$
B	4	6	2	$2((4)(6) + (6)(2) + (4)(2))$ $= 2(24 + 12 + 8)$ $= 2(44)$ $= 88$
C	12	18	6	$2((12)(18) + (18)(6) + (12)(6))$ $= 2(216 + 108 + 72)$ $= 2(396)$ $= 792$

To determine how the surface area of an object is related to the scale factor of an object, complete the following chart. Answers are found at the end of the learning activity answer keys.

Rectangular Prism	Scale Factor	Ratio of Surface Area
B to A		
C to B		
C to A		

Based on the information in your chart, the following relationships should be apparent:

- When the size of the dimensions of the rectangular prism doubles, the surface area of the rectangular prism increases four times.
- When the size of the dimensions of the rectangular prism triples, the surface area of the rectangular prism increases 9 times.
- When the size of the dimensions of the rectangular prism increases by a factor of 6, the surface area of the rectangular prism increases 36 times.

In general, the ratio of the surface area of two rectangular prisms is equal to the scale factor of these two rectangular prisms squared.

Is this always true? Consider a different shape, such as a sphere.

Sphere	Radius (cm)	Surface Area (cm^2)
A	2	$4\pi(2)^2 = 16\pi$
B	4	$4\pi(4)^2 = 64\pi$
C	12	$4\pi(12)^2 = 576\pi$

Sphere	Scale Factor	Ratio of Surface Area
B to A	2	4
C to B	3	9
C to A	6	36

Again, you will notice that the ratio of surface area for any two spheres is equal to the scale factor of these two spheres squared. This is true for all similar figures.

How is this relationship similar to the relationship between scale factors and the area of 2-D shapes?

These relationships are identical. This is consistent since the surface area of 3-D objects is really the sum of areas of various 2-D shapes.

In general, to find the surface area of a 3-dimensional model, you can multiply the surface area of the actual shape by the scale factor squared. This can be represented by the following formulas:

$$k^2 = \frac{SA_{\text{model}}}{SA_{\text{actual}}}$$

$$SA_{\text{model}} = k^2(SA_{\text{actual}})$$

$$SA_{\text{actual}} = \frac{SA_{\text{model}}}{k^2}$$

Include these formulas on your resource sheet.

Example 1

A cylinder has a surface area of 61π cm^2. Find the surface area of a cylinder where the radius and height are two times smaller than the original cylinder.

Solution

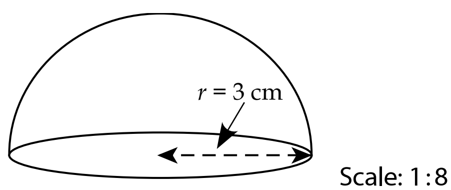
If the radius and height are two times smaller, then the surface area of the cylinder will decrease by a factor of 2^2 or 4.

The surface area of the smaller cylinder will be $\dfrac{61\pi \text{ cm}^2}{4} = 15.25\pi \text{ cm}^2$.

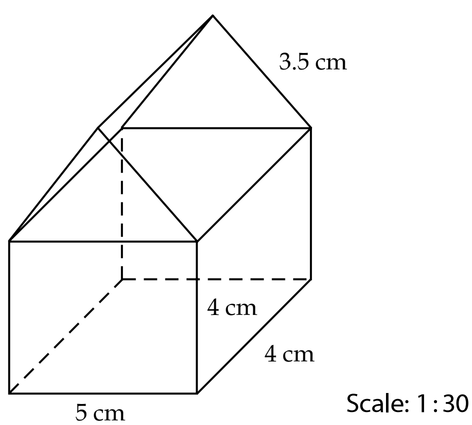
Example 2

Using the scale diagram of each of the objects below, determine the surface area of each of the actual shapes.

a) Hemisphere



b) Doghouse



Solution

a) First, find the surface area of the hemisphere. The surface area of this shape will be half of the surface area of a sphere plus the surface area of the bottom of the hemisphere.

$$\begin{aligned}\text{SA} &= \frac{4\pi r^2}{2} + \pi r^2 \\ &= 2\pi(3)^2 + \pi(3)^2 \text{ cm}^2 \\ &= 18\pi + 9\pi \text{ cm}^2 \\ &= 27\pi \text{ cm}^2\end{aligned}$$

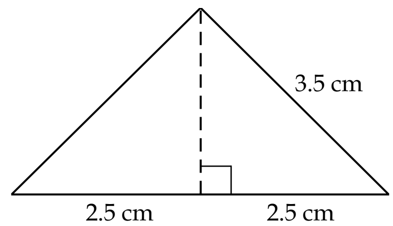
Now, determine the surface area of the actual hemisphere by using the scale factor $\frac{1}{8}$. To find the actual surface area, divide the model surface area by the scale factor squared.

$$\begin{aligned}SA_{\text{actual}} &= \frac{SA_{\text{model}}}{k^2} \\ &= \frac{27\pi \text{ cm}^2}{\left(\frac{1}{8}\right)^2} \\ &= \left(27\pi \text{ cm}^2\right)(64) \\ &= 1728\pi \text{ cm}^2\end{aligned}$$

b) To find the surface area of the doghouse, find the surface area of the bottom rectangular prism without a top. Then, find the surface area of the top triangular prism without a bottom.

$$\begin{aligned}SA_{\text{bottom}} &= 2\left((5)(4) + (4)(4) + (5)(4)\right) - (5)(4) \\ &= 2(20 + 16 + 20) - 20 \\ &= 92 \ cm^2\end{aligned}$$

To find the surface area of the triangular prism, you need to find the height of the prism. You can find this height using the Pythagorean theorem. To create a right triangle, divide the triangular face in half. The base of the right triangle will now measure 2.5 cm.



The Pythagorean theorem is $a^2 + b^2 = c^2$, where c is the length of the hypotenuse of the triangle, or the side directly opposite the right angle of 90°.

In this triangle, c = 3.5 cm and a = 2.5 cm.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (2.5)^2 + b^2 &= (3.5)^2 \\ 6.25 + b^2 &= 12.25 \\ b^2 &= 6 \\ b &= \sqrt{6} \\ b &= 2.45 \text{ cm} \end{aligned}$$

The height of this triangle is approximately 2.45 cm.

Now you can determine the surface area of the top of the doghouse. This will be the total surface area of the triangular prism less the surface area of the bottom of the prism.

You will need to add the areas of the two triangles (front and back) and the areas of the roof (left and right).

$$\begin{aligned} SA_{\text{top}} &= 2\left[\frac{5(2.45)}{2}\right] + 2(4)(3.5) \\ &= 12.25 + 28 \\ &= 40.25 \text{ cm}^2 \end{aligned}$$

The total surface area is:

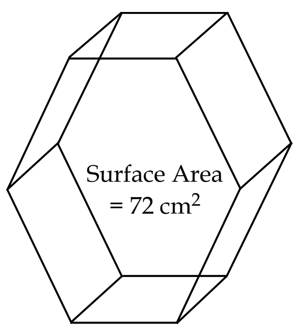
$$\begin{aligned} SA_{\text{total}} &= 92 \text{ cm}^2 + 40.25 \text{ cm}^2 \\ &= 132.25 \text{ cm}^2 \end{aligned}$$

Use the scale factor $\frac{1}{30}$ to determine the actual surface area of the doghouse.

$$\begin{aligned} SA_{\text{actual}} &= \frac{SA_{\text{model}}}{k^2} \\ &= \frac{132.25 \text{ cm}^2}{\left(\frac{1}{30}\right)^2} \\ &= \left(132.25 \text{ cm}^2\right)(900) \\ &= 119\ 025 \text{ cm}^2 \end{aligned}$$

Example 3

Calculate the surface area of the actual object if the scale drawing below is drawn according to a scale of 7 : 1.



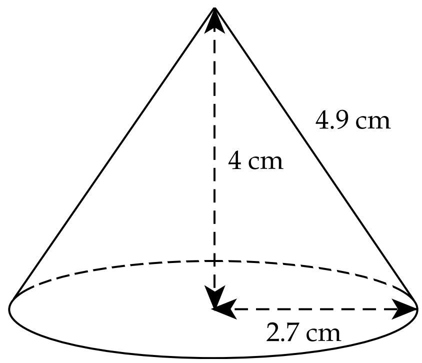
Solution

The scale factor for this shape is $\frac{7}{1}$ or 7. Use this value and the model surface area of 72 cm^2 to determine the actual surface area.

$$\begin{aligned} SA_{\text{actual}} &= \frac{SA_{\text{model}}}{k^2} \\ &= \frac{72 \text{ cm}^2}{(7)^2} \\ &= \frac{72 \text{ cm}^2}{49} \\ &= 1.47 \text{ cm}^2 \end{aligned}$$

Example 4

Given the scale diagram below, find the dimensions of the actual cone. The surface area of the actual cone is $96\pi\ \text{cm}^2$.



Solution

To determine the dimensions of the actual cone, you need to find the scale factor. To find the scale factor, you need to find the surface area of the model cone and relate this value to the surface area of the actual cone.

$$\begin{aligned} SA_{\text{model}} &= \pi(2.7)(4.9) + \pi(2.7)^2 \\ &= 13.23\pi + 7.29\pi \\ &= 20.52\pi\ \text{cm}^2 \end{aligned}$$

Now, determine the scale factor.

$$\begin{aligned} k^2 &= \frac{SA_{\text{model}}}{SA_{\text{actual}}} \\ k^2 &= \frac{20.52\pi\ \text{cm}^2}{96\pi\ \text{cm}^2} \\ k^2 &= 0.21375 \\ k &= 0.46 \end{aligned}$$

The scale factor is 0.46.

To determine the actual dimensions of the cone, divide the model dimensions by the scale factor.

Actual height:

$$\frac{4\ \text{cm}}{0.46} = 8.7\ \text{cm}$$

Actual slant height:

$$\frac{4.9\ \text{cm}}{0.46} = 10.7\ \text{cm}$$

Actual radius:

$$\frac{2.7\ \text{cm}}{0.46} = 5.9\ \text{cm}$$

Example 5

Leandra is a geography student who wants to create a scale model of Earth with a diameter of 30 cm. The actual diameter of Earth is 12 756.2 km.

a) Calculate the scale factor relating Earth to Leandra's globe.

b) Determine the surface area of the globe.

c) Approximately how much larger is the actual surface area of Earth compared to the surface area of the globe? How does this relate to the scale factor between the diameter of the scale globe and the diameter of actual Earth?

d) Why is the approximate surface area you calculated in (c) not equivalent to the actual surface area of Earth?

Solution

a) To determine the scale factor, divide the scale diameter by the actual diameter. However, you must ensure both measurements are in the same units. To do this, convert 12 756.2 kilometres to centimetres by multiplying this value by 100 000.

12 756.2 km = 12 756.2(100 000) cm = 1 275 620 000 cm

$$k = \frac{\text{globe diameter}}{\text{actual diameter}}$$

$$k = \frac{30 \text{ cm}}{1\ 275\ 620\ 000 \text{ cm}}$$

$$k = \frac{3}{127\ 562\ 000}$$

b) $$SA_{\text{globe}} = 4\pi(15)^2$$

$$= 900\pi \text{ cm}^2$$

c) First, determine the surface area of Earth.

$$SA_{\text{Earth}} = 4\pi\left(\frac{1275620000}{2}\right)^2$$

$$SA_{\text{Earth}} = \left(1.63 \times 10^{18}\right)\pi \text{ cm}^2$$

Now, divide this value by the surface area of the globe.

$$\frac{\left(1.63 \times 10^{18}\right)\pi \text{ cm}^2}{900\pi \text{ cm}^2} = 1.81 \times 10^{15}$$

The surface area of Earth is approximately 1.81×10^{15} or 1 810 000 000 000 000 times larger than that of Leandra's globe.

The square root of this value should be approximately equal to how many times larger the diameter of Earth is than the diameter of the globe.

$$\sqrt{1\ 181\ 000\ 000\ 000\ 000} = 42\ 544\ 094.77$$

The diameter of Earth is approximately $\dfrac{1\ 275\ 620\ 000 \text{ cm}}{30 \text{ cm}} =$ 42 520 666.67 times larger than the diameter of the globe.

These two values are extremely similar. The difference can be attributed to rounding errors throughout the various calculations.

d) The approximate surface area of Earth calculated in (c) only accounts for a smooth surface. Earth is full of many valleys, hills, and mountains that do not produce a smooth surface. Therefore, the actual surface area of Earth is much larger than this calculated value.

Learning Activity 8.4

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Write in lowest terms: $\dfrac{39}{81}$

2. Multiply: $\dfrac{3}{7} \cdot \dfrac{9}{11}$

3. Solve for x: $\dfrac{x}{5} = \dfrac{4}{10}$

4. Determine the y-intercept: $-2x + y = -6$

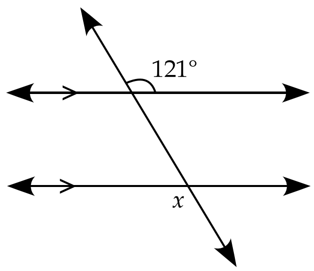
continued

5. Factor: $x^2 + 11x + 18$

6. What is the area of a triangle with a base of 5 inches and a height of 16 inches?

7. Convert 2 940 000 mm to km.

8. Determine the size of angle x in the diagram below.

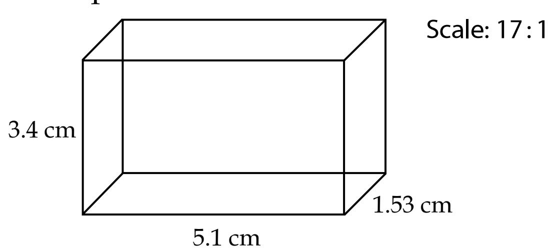


Part B: Scale Factors and Surface Area of 3-D Shapes

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Using the scale diagram of each of the objects below, determine the surface area of each of the actual shapes.

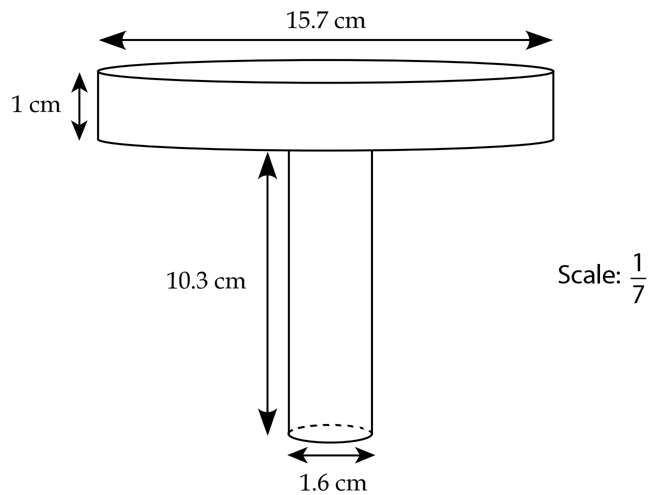
   a) Microchip



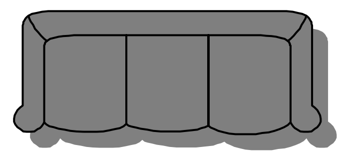
continued

Learning Activity 8.4 (continued)

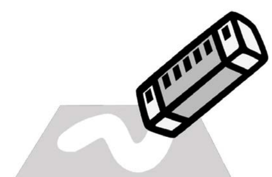
b) Table



2. Kendra is upholstering three rectangular couch cushions. The pattern that she is using calls for 1.7 m^2 of fabric per pillow. However, her couch cushions have dimensions that are 1.5 times as large as those in the pattern. How much fabric should she purchase?



3. A school supplies company is advertising that its new eraser has four times the surface area of the old model (for more mistakes!). If the dimensions of the old eraser were 4 cm × 2 cm × 1 cm, what are the dimensions of the new eraser?



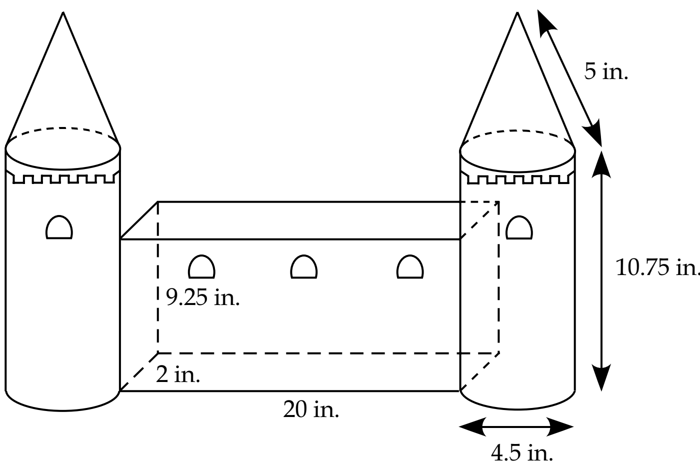
4. A baseball company is creating a souvenir statue of a baseball in memory of Ernie Banks. The diameter of a standard baseball is 76 mm. If they want to make a statue according to a scale of 21 : 1, what will the surface area of the statue of the new baseball be?

continued

5. A typical soup can has a height of 4 inches and a diameter of 2.525 inches. During a promotion, Yummy Soups wants to create a display can that has a surface area of 1000 square inches.
   a) What will be the length of the diameter of the display can?
   b) What will be the height of the display can?
   c) What will be the surface area of the label of the soup can?

6. The surface area of a typical balloon is 400π cm^2. The surface area of a typical hot air balloon is 899 m^2. How many times larger is a hot air balloon than a regular balloon?

7. Jenn is a cake decorator who has been asked to design a cake that is a scale replica of a castle. After examining the dimensions of the actual castle, Jenn has decided to use a scale factor of 1 : 39 to create her cake. She calculated the dimensions of the cake and indicated them on the diagram below.



   a) How much fondant icing will she need to cover the cake?
   b) If Jenn wanted to create a cake with a scale of 1 : 1, how much icing would she require?

Lesson Summary

In this lesson, you learned about the relationship between the surface area of a model object and the surface area of an actual object. These two surface areas are related in the same way that the areas of two similar 2-D shapes are related; that is, the surface area of a model object can be found by multiplying the surface area of the actual object by the scale factor squared. In the next lesson, you will be learning about the relationship between the volumes of two similar 3-D objects.

Notes

Lesson 5: The Relationship between Scale Factors and Volume of 3-D Objects

Lesson Focus

In this lesson, you will

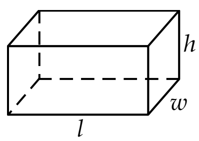
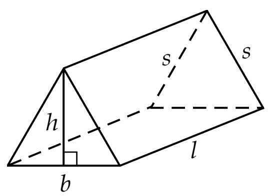
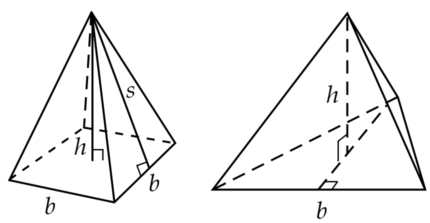
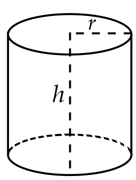
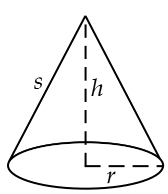
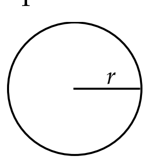
❑ learn about the relationship between the scale factor and the volume of two 3-D shape objects

Lesson Introduction

In the last lesson, you learned about the relationship between scale factor and surface area. The surface areas of two similar 3-dimensional objects are related by the square of the scale factor. How do you think the volumes of two similar objects are related? Do you think the volume increases faster or slower than surface area? In this lesson, you will discover the answers to these questions.

Scale Factors and Volume of 3-D Object

Before you begin analyzing the relationship between the volumes of two similar 3-D objects and the scale factor relating these two objects, it may be helpful to review how to find the volumes of various 3-D objects. You may wish to include some of the formulas from the following chart on your resource sheet.

Volume Formulas		
3-D Object	**Dimensions**	**Formula**
Rectangular Prism 	l = length w = width h = height	$V = lwh$
Triangular Prism 	b = base h = height l = length s = slant height	$V = \left(\frac{bh}{2}\right)l$
General Pyramid 	h = height of the pyramid area of the base = area of the bottom of the pyramid	$V = \left(\frac{1}{3}\right)h\,(\text{area of the base})$
Cylinder 	r = radius h = height	$V = \pi r^2 h$
Cone 	r = radius h = height s = slant height	$V = \left(\frac{1}{3}\right)\pi r^2 h$
Sphere 	r = radius	$V = \left(\frac{4}{3}\right)\pi r^3$

To determine the relationship between volume and scale factor for a cone, use the following information to complete the second chart.

Cone	Radius (cm)	Height (cm)	Volume (cm^3)
A	2	1	$V = \left(\frac{1}{3}\right)\pi(2)^2(1)$ $= \frac{4}{3}\pi$
B	4	2	$V = \left(\frac{1}{3}\right)\pi(4)^2(2)$ $= \frac{32}{3}\pi$
C	12	6	$V = \left(\frac{1}{3}\right)\pi(12)^2(6)$ $= \frac{864}{3}\pi$ $= 288\pi$

Cone	Scale Factor	Ratio of Volume
B to A		
C to B		
C to A		

Answers are found at the end of the learning activity answer keys.

What is the relationship between the scale factor and the ratio of the volumes?

- When the dimensions of the cone are doubled, the volume of the cone is increased 8 times.
- When the dimensions of the cone are tripled, the volume of the cone is increased 27 times.
- When the dimensions of the cone are increased by a factor of 6, the volume of the cone is increased 216 times.

Consider the exponential relationship between these values:

$2^3 = 8$

$3^3 = 27$

$6^3 = 216$

In general, the ratio of the volume of two similar cones is equal to the enlargement scale factor of these two cones cubed.

Is this always true? Consider a different shape, such as a rectangular prism. Instead of determining the volume for various rectangular prisms and their related scale factors, consider the following algebraic comparison.

The formula for the volume of a rectangular prism is $V = lwh$.

Suppose you wanted to increase each of the dimensions of this prism by a factor of k. The new dimensions would be:

Length: kl

Width: kw

Height: kh

The volume of this new rectangular prism would be:

$$
\begin{aligned}
V &= (kl)(kw)(kh) \\
&= k \cdot k \cdot k \cdot (lwh) \\
&= k^3(lwh)
\end{aligned}
$$

The volume of this rectangular prism is equal to the volume of the original rectangular prism multiplied by the scale factor cubed. This relationship is true for the ratio of the volume of all similar figures, since you are enlarging each of the three dimensions (length, width, and height), and that means you are multiplying by the scale factor three times.

In general, to find the volume of the model object, multiply the volume of the actual shape by the scale factor cubed. This can be represented by the following formulas:

$$k^3 = \frac{V_{\text{model}}}{V_{\text{actual}}}$$

$$V_{\text{model}} = k^3\left(V_{\text{actual}}\right)$$

$$V_{\text{actual}} = \frac{V_{\text{model}}}{k^3}$$

Include these formulas on your resource sheet.

Example 1

A cube has a volume of 672 cm^3. Find the volume of a cube whose side lengths are five times smaller than the original cube.

Solution

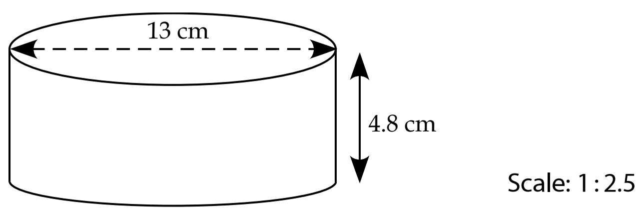
Cubes are always similar shapes since every side is the same length. A cube with side lengths five times smaller than the original cube will have a volume that is five cubed, or 125 times smaller than the volume of the original cube. The new volume will be:

$$\frac{672\text{ cm}^3}{125} = 5.376\text{ cm}^3$$

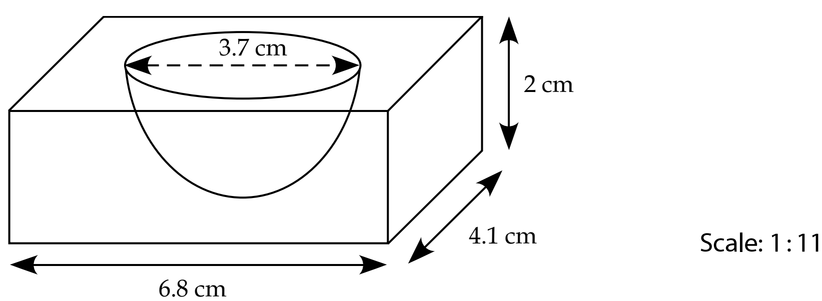
Example 2

Using the scale diagram of each of the models shown below, determine the volume of each of the actual objects.

a) ice cream cake



b) fish tank sink



Solution

a) The volume of the model ice cream cake is:

$$V = \pi r^2 h$$

$$= \pi\left(\frac{13}{2}\right)^2 (4.8)\ \text{cm}^3$$

$$= 637.1\ \text{cm}^3$$

To determine the volume of the actual cake, multiply this value by $(2.5)^3$ or 15.625.

Actual volume:

$$(637.1)(15.625) = 9954.9\ \text{cm}^3$$

You could also complete this problem by using the formula

$V_{\text{actual}} = \dfrac{V_{\text{model}}}{k^3}$.

$$V_{\text{actual}} = \frac{637.1\ \text{cm}^3}{\left(\frac{1}{2.5}\right)^3}$$

$$= 637.1\left(2.5^3\right)$$

$$= (637.1)(15.625)$$

$$= 9954.9\ \text{cm}^3$$

b) The volume of the scale fish tank sink will be the entire volume of the rectangular prism less the volume of the half sphere.

$$V = lwh - \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)\pi r^3$$

$$= (6.8)(4.1)(2) - \left(\frac{4}{6}\right)\pi\left(\frac{3.7}{2}\right)^2\ \text{cm}^3$$

$$= 55.76 - 2.28\pi\ \text{cm}^3$$

$$= 48.6\ \text{cm}^3$$

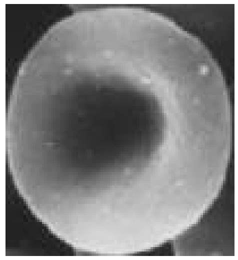
To find the actual volume, multiply the scale volume by 11^3 or 1331.

Actual volume:

$$(48.6)(1331) = 64686.6\ \text{cm}^3$$

Example 3

Determine the volume of an actual red blood cell if the scale drawing below is drawn according to a scale of 4651 : 1.



Volume = 9 mL

Solution

The volume of the scale blood cell is 9 mL. The actual cell is 4651 times smaller than the model cell. To find the volume of the actual cell, divide the volume of the model cell by 4651 cubed.

Actual volume:

$$\frac{9 \text{ mL}}{4651^3} = 0.000\ 000\ 000\ 089 \text{ mL}$$

Note: You could also complete the question by using the formula for determining the actual volume of an object, $V_{\text{actual}} = \dfrac{V_{\text{model}}}{k^3}$.

$$\begin{aligned} V_{\text{actual}} &= \frac{9 \text{ mL}}{\left(\frac{4651}{1}\right)^3} \\ &= \frac{9 \text{ mL}}{4651^3} \\ &= 0.000\ 000\ 000\ 089 \text{ mL} \end{aligned}$$

The Relationship among Scale Factor, Surface Area, and Volume

Consider the following example, which relates the volume of two similar figures to the surface area of these similar figures.

Example 4

Two similar gifts are being wrapped. One gift needs 8496 cm^2 of wrapping paper while the other gift only needs 236 cm^2 of wrapping paper. If the volume of the smaller gift is 240 cm^3, determine the volume of the larger gift.

Solution

To determine the volume of the larger gift, you need to find the scale factor relating the two gifts. To do this, you can use the two surface areas. Recall that you can find the surface area of the model object by multiplying the surface area of the actual object by the scale factor squared. In this example, let the smaller gift be the actual gift.

$$\begin{aligned} k^2 &= \frac{\text{SA}_{\text{model}}}{\text{SA}_{\text{actual}}} \\ k^2 &= \frac{8496\ \text{cm}^2}{236\ \text{cm}^2} \\ k^2 &= 36 \\ k &= 6 \end{aligned}$$

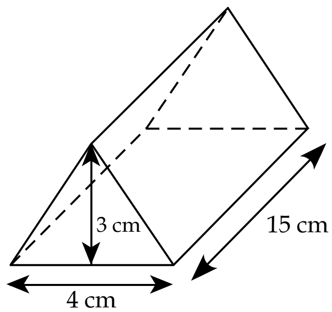
Now, determine the volume of the larger gift by multiplying the volume of the smaller gift by the scale factor cubed.

$$\begin{aligned} V_{\text{model}} &= k^3\left(V_{\text{actual}}\right) \\ &= \left(6^3\right)\left(240\ \text{cm}^3\right) \\ &= 51\ 840\ \text{cm}^3 \end{aligned}$$

The volume of the larger gift will be 51 840 cm^3.

Example 5

The dimensions of an actual chocolate bar are shown below. This particular chocolate bar company wants to create a special edition chocolate bar with a volume of 1000 cm^3. What should be the dimensions of the special edition chocolate bar?



Solution

To determine the dimensions of the new chocolate bar, you first need to find the scale factor that relates the two chocolate bars. You can find this value by determining the volume of the actual chocolate bar and relating that volume to the volume of the special edition chocolate bar.

$$\begin{aligned} V_{\text{actual}} &= \left(\frac{bh}{2}\right)l \\ &= \frac{(4)(3)}{2}(15) \\ &= 90 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} k^3 &= \frac{V_{\text{model}}}{V_{\text{actual}}} \\ &= \frac{1000 \text{ cm}^3}{90 \text{ cm}^3} \\ &= 11.11 \\ k &= \sqrt[3]{11.11} \\ k &= 2.23 \end{aligned}$$

To find the dimensions of the special edition chocolate bar, multiply each dimension of the actual chocolate bar by the scale factor.

Dimensions of the special edition chocolate bar:

Base: (4 cm)(2.23) = 8.92 cm

Height: (3 cm)(2.23) = 6.69 cm

Length: (15 cm)(2.23) = 33.45 cm

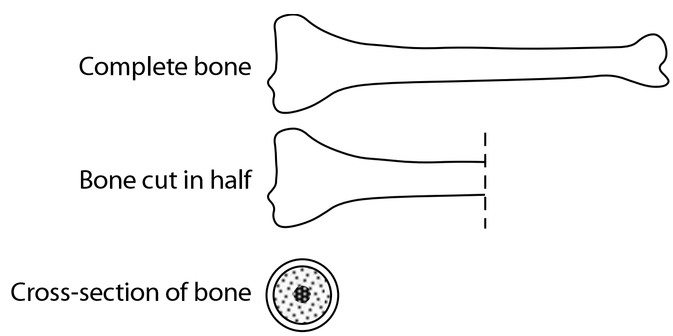
As a way to check if your dimensions are correct, you can find the volume of the new chocolate bar to see if it is close to 1000 cm^3.

$$\begin{aligned} V_{\text{special edition chocolate bar}} &= \left(\frac{bh}{2}\right)l \\ &= \frac{(8.92)(6.69)}{2}(33.45) \\ &= 998.06\ \text{cm}^3 \end{aligned}$$

This is close to 1000 cm^3 but not exact due to rounding errors. The dimensions you calculated are accurate.

Surface Area and Volume Ratios in the World

The ratio of surface area and volume is important for all living things. For example, the strength of bone is proportional to the cross-sectional area of the bone. If you cut a bone in half and looked at the new surface, the strength of the bone would be proportional to the area of the surface you are looking at.

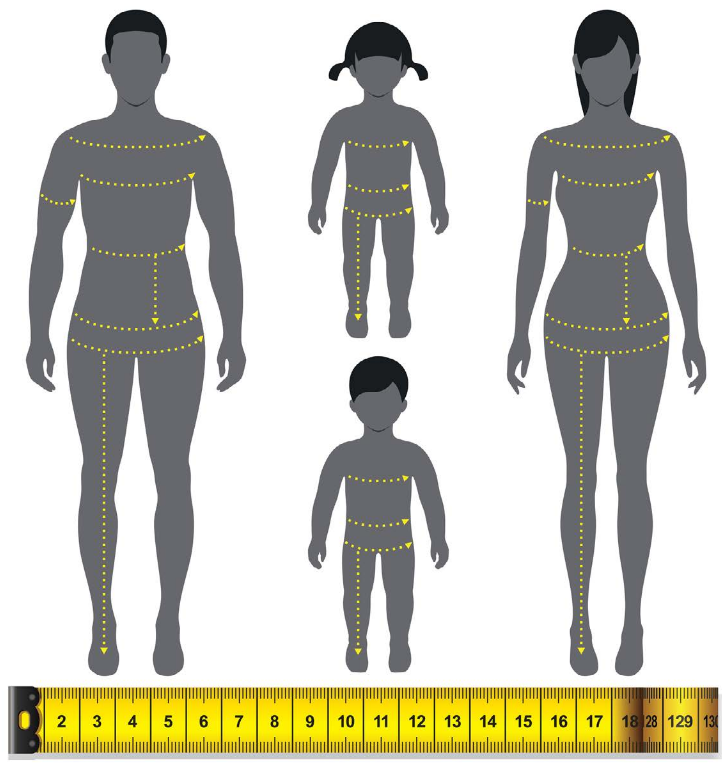


Consider a child that is 3 feet tall and has similar proportions to a 6-foot-tall adult.

Note that the adult is *double* the height of the child, which creates a scale factor of 2.

The bones of this adult would be twice as tall, twice as wide, and twice as thick as those of the child. As the strength of bone is proportional to the cross-sectional *area* of these bones, the adult's bones would be able to support 2^2 or 4 times the weight of the bones of the child.

However, since you are considering what is inside the body, the mass of an object (including children and adults) is related to the volume of the object. Therefore, the adult would weigh 2^3 or 8 times as much as the young child. This creates a situation where the bones of the adult must support twice as much (8 ÷ 4) weight as those of a child. This is why it is important that the proportions of a person change as he or she grows from child to adult. You can see the proportions in the image below.



Example 6

Why must a whale, the largest mammal in the world, live in the ocean?

Solution

If a whale lived on land, with such a large volume, it would be crushed by its own weight without the support of bones with a very large cross-sectional area. As whales are supported by water in the ocean, they can reach larger sizes than would be possible on land. In this case, the support of the water makes up for the lack of bone support in the whale.

Learning Activity 8.5

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. Write as an improper fraction: $2\frac{4}{5}$

2. Solve for x: $\frac{2}{x} = \frac{7}{3}$

3. Determine the slope: $-2x + y = -6$

4. Add: $\frac{3}{4} + \frac{6}{7}$

5. Convert 23 km to m.

6. Convert 0.2 km to mm.

7. Solve for x: $-\frac{1}{2}x \leq 6$

8. A normally distributed data set has a mean of 42 and a standard deviation of 7. Determine the two scores between which 68% of the population will fall.

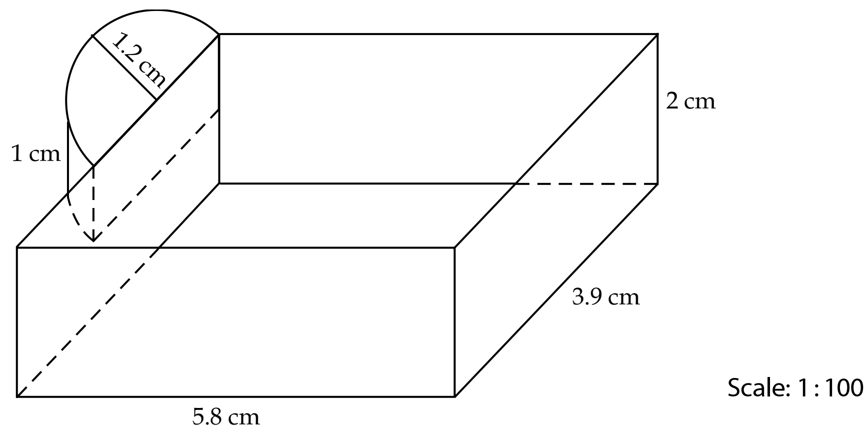
continued

Part B: Scale Factors and Volume of 3-D Shapes

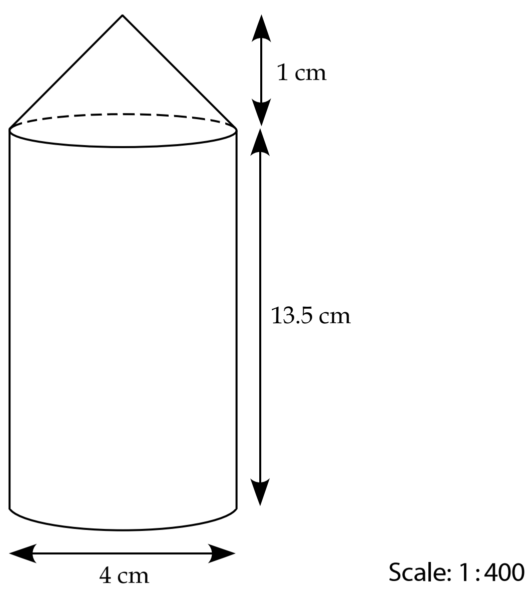
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you were not able to answer them correctly, you need to go back to the lesson and learn the necessary concepts.

1. Using the scale diagram of each of the objects below, determine the volume of each of the actual shapes.

   a) pool



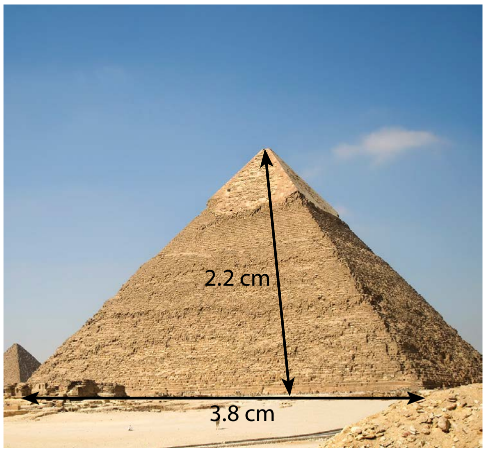
   b) grain bin



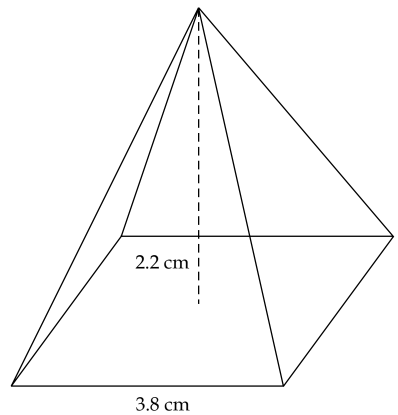
continued

2. The diameter of Earth is about 3.7 times that of the moon. How many times bigger is the volume of Earth than the volume of the moon?

3. Find the volume of sand used to create the square Egyptian pyramid given the dimensions shown for the image and the scale.



Scale: 1:5250



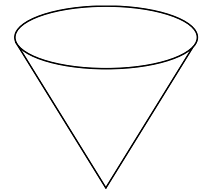
4. The diameter of Earth is approximately 12 756 kilometres. The total amount of water on Earth is approximately 1.4 billion km^3. The amount of freshwater on Earth is approximately 35 million km^3. The amount of usable freshwater (omitting water that is inaccessible in areas such as swamps or glaciers) is approximately 200 000 km^3.
   a) What is the volume of Earth?
   b) How many times larger is the volume of Earth than
      i) the total amount of water on Earth?
      ii) the amount of freshwater on Earth?
      iii) the amount of usable freshwater on Earth?
   c) If you could create a sphere of each of the following bodies of water, how long would the diameter of each be?
      i) the total amount of water on Earth
      ii) the amount of freshwater on Earth
      iii) the amount of usable freshwater on Earth
   d) What do these comparisons tell you about the amount of usable freshwater on Earth?

continued

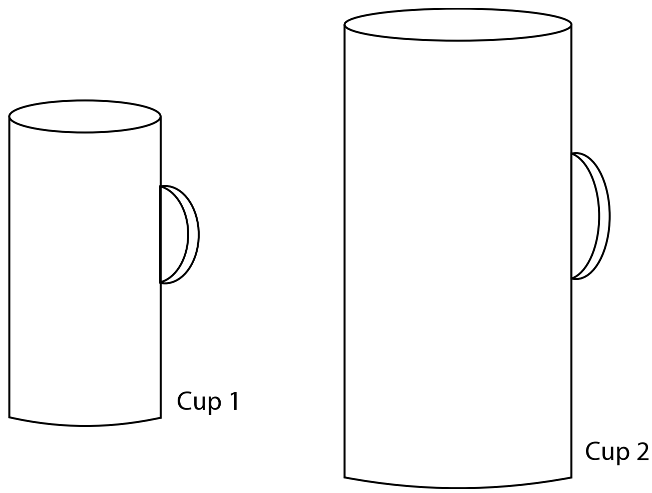
Learning Activity 8.5 (continued)

5. The world's largest cup of tea has a height of 10 feet and a diameter of 8 feet. If a typical tea cup has dimensions that are 45 times smaller than the dimensions of the world's largest cup of tea, approximately how many times more tea will the world's largest cup of tea hold than a typical cup of tea?

6. A restaurant uses cone-shaped cups for iced beverages that are 9 cm high and 7 cm across. If the restaurant wanted to double the amount of beverage that each cup could hold, what would be the approximate dimensions of the new cone-shaped cups?



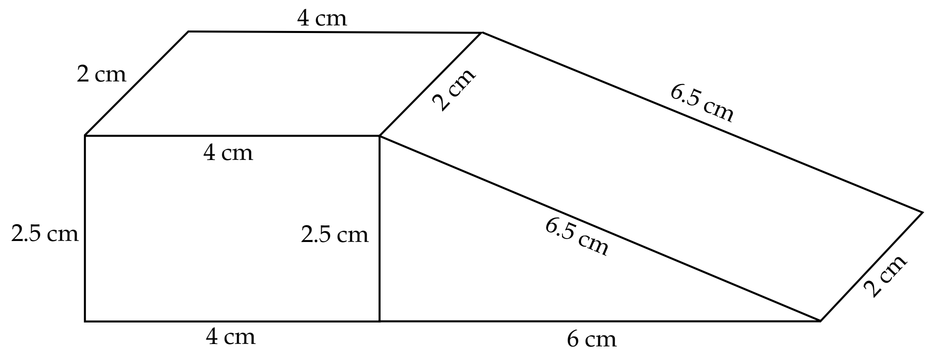
7. Consider the two similar cups shown below.



   a) Without measuring, estimate how much larger the volume of Cup 2 is than Cup 1.
   b) Measure the diameter and height of each cup to determine the respective volume of each.
   c) How much larger is the volume of Cup 2 than Cup 1?
   d) Compare your estimate with your answer in (c). If your estimate was not accurate, explain why it was higher or lower than the actual difference in volume.

continued

8 A company designs and manufactures skateboard ramps for skate parks. One design is shown below.



a) Find all of the measurements of the actual ramp. The scale is 1 : 20.

b) If the ramps are to be made from solid concrete, find the amount of concrete that goes into making one of these ramps. Write your answer in m^3.

c) If concrete costs $110 per cubic metre, how much would it cost to construct 10 ramps?

9. A typical male gorilla can reach a maximum height of 5.9 feet and weigh up to 400 pounds. King Kong, a fictional monster gorilla, was scaled after a typical gorilla but with heights that varied from between 18 feet to 70 feet.

a) Determine the scale factor relating the typical male gorilla to the 18-foot-tall King Kong.

b) How many times would the surface area of the 18-foot-tall King Kong increase? List some factors that are influenced by the surface area scale factor of a gorilla's body.

c) How many times would the volume of an 18-foot-tall King Kong increase? List some factors that are influenced by the volume scale factor of a gorilla's body.

d) Do you think King Kong would be able to support his weight? Justify your answer with reference to your calculations in parts (b) and (c) above.

continued

Learning Activity 8.5 (continued)

10. Two similar birdhouses are being constructed. The volume of the smaller birdhouse is 978 cm^3, while the volume of the larger birdhouse is 5713 cm^3. If the smaller birdhouse needs paint to cover a surface area of 846 cm^2, what amount of paint will be needed to paint the larger birdhouse?

Lesson Summary

In this lesson, you learned how the volume of two similar objects is related to the scale factor between these two objects. If the dimensions of an object increase by a scale factor, then the volume of the object will increase by the scale factor cubed.

Notes

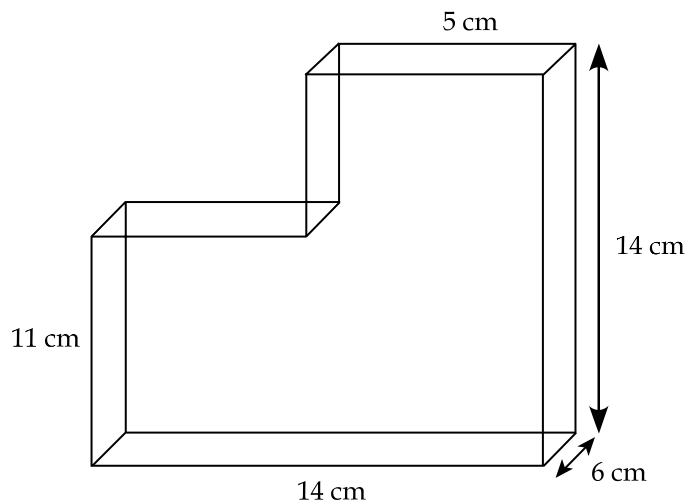
Assignment 8.3

Scale Factors, Surface Area, and Volume of 3-D Shapes

Total: 41 marks

Clearly show the steps in your solution on the question sheets below and submit these pages when you send in your assignments for marking. Final answers must include units. Answers given without supporting calculations will not be awarded full marks.

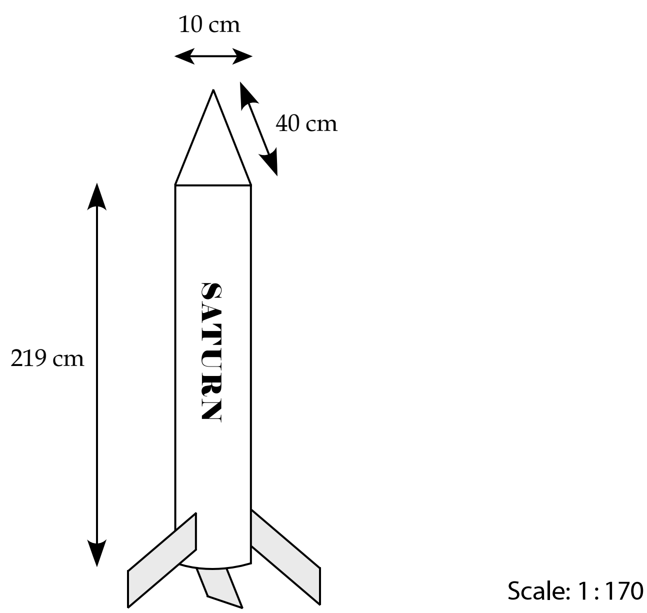
1. Using the scale diagram of the fish tank below, find the surface area of the tank and the volume of water it will hold. *(8 marks)*



Scale: 1 : 5

Assignment 8.3: Scale Factors, Surface Area, and Volume of 3-D Shapes (continued)

2. A model rocket is a replica of the Saturn Space Rocket. The dimensions of the model are shown in the image below.



   a) Determine the surface area of the model rocket, excluding the three wings at the base. *(3 marks)*

   b) Determine how much larger the surface area of the Saturn Space Rocket was, compared to the replica. *(1 mark)*

Assignment 8.3: Scale Factors, Surface Area, and Volume of 3-D Shapes (continued)

3. A paint roller has a diameter of 7.5 cm and a length of 20 cm. To paint his room faster, Jehu wants to find a paint roller with triple the surface area.

   a) Find the surface area of the new paint roller. *(3 marks)*

   b) Find the scale factor relating the dimensions of the old paint roller to the new paint roller. *(2 marks)*

   c) Calculate the dimensions of the new paint roller. *(2 marks)*

Assignment 8.3: Scale Factors, Surface Area, and Volume of 3-D Shapes (continued)

4. Saifullah is comparing two similar oranges in a bowl of fruit. He estimates that the smaller orange has a diameter of 6 cm while the larger orange has a diameter of 10 cm.

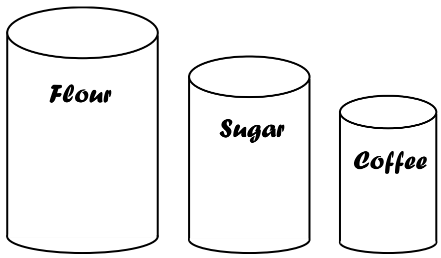
   a) How much more orange would Saifullah get to eat if he chooses the larger orange? *(3 marks)*

   b) How much more peel would Saifullah have to discard if he chooses the larger orange? *(2 marks)*

5. Sam is making two pies for a bake sale. The first pie has a diameter of 8 inches while the second pie has a diameter of 12 inches. Sam knows that she will need approximately 2.25 cups of pie filling for the 8-inch pie. Determine approximately how much filling Sam will need for the 12-inch pie. *(3 marks)*

Assignment 8.3: Scale Factors, Surface Area, and Volume of 3-D Shapes (continued)

6. Three similar storage containers are made to hold flour, sugar, and coffee. Each container holds half as much as the previous container.



If the flour container is 30 cm tall and has a radius of 7 cm, determine

a) the volume of the sugar container. *(2 marks)*

b) the dimensions of the sugar container. *(3 marks)*

Assignment 8.3: Scale Factors, Surface Area, and Volume of 3-D Shapes (continued)

c) the dimensions of the coffee container. *(3 marks)*

d) How are the surface area of the flour container and the coffee container related? *(2 marks)*

Assignment 8.3: Scale Factors, Surface Area, and Volume of 3-D Shapes (continued)

7. Two similar bottles of shampoo are being developed. The original bottle of shampoo has a volume of 591 cm^3 and a surface area of 452 cm^2. If the shampoo company wants the volume of the new bottle of shampoo to be 825 cm^3, what will be the surface area of the new bottle of shampoo? *(4 marks)*

Notes

Module 8 Summary

In this module, you used scale diagrams and scale factors to determine the area of 2-D objects, surface area of 3-D objects, and volume of 3-D objects. You also studied the relationship between scale factors and both surface area and volume. In doing so, you were able to determine the surface area of an object if you were given the surface area of a similar object as well as the scale factor. Both the areas and surface areas of similar shapes are related by the square of the scale factor of their dimensions.

Similarly, you learned that you are able to determine the volume of an object if you are given the volume of a similar object as well as the scale factor. The volumes of two similar shapes are related by the cube of the scale factor of their dimensions. This is similar to how volume is measured in three dimensions.

Congratulations on completing the last module in the course! It is now time to prepare for your final examination.

Submitting Your Assignments

It is now time for you to submit the Module 8 Cover Assignment and Assignments 8.1 to 8.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 8 assignments and organize your material in the following order:

- ❑ Module 8 Cover Sheet (found at the end of the course Introduction):
- ❑ Module 8 Cover Assignment: Pattern Analysis
- ❑ Assignment 8.1: Finding and Using Scale Factors of 2-D Shapes and 3-D Objects
- ❑ Assignment 8.2: Scale Factors and Areas of 2-D Shapes
- ❑ Assignment 8.3: Scale Factors, Surface Area, and the Volume of 3-D Shapes

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Final Examination

Congratulations, you have finished Module 8 in the course. The final examination is out of 100 marks and worth 25% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 5 to 8.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: some pens and/or pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Final Examination Resource Sheet.

A maximum of 2.5 hours is available to complete your final examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

Graphing technology (either computer software or a graphing calculator) **is required** to complete the examination. Check with your tutor/marker to be sure your graphing technology is appropriate.

At this point you will also have to combine your resource sheets from Modules 5 to 8 onto one 8½″ × 11″ paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you. We suggest that you divide your paper into two quadrants on each side so that each quadrant contains information from one module.

Examination Review

You are now ready to begin preparing for your final examination. Please review the content, learning activities, and assignments from Modules 5 to 8.

The final practice examination is also an excellent study aid for reviewing Modules 5 to 8.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the final examination determines 25% of your final mark in this course and you will have 2.5 hours to complete the examination.

Midterm Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The final practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your final practice examination, follow these steps:

1. Study for the final practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 5 to 8 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 5 to 8, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Final Examination Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following to the final practice examination: some pens and/or pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Final Examination Resource Sheet.
7. Write your final practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the midterm examination is 2.5 hours.
8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Final Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes

Self-Test

Module 8 – Data Management and Analysis

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TEST TIME!

(10 marks)

1. Multiple Choice. Circle the best answer. There is no penalty for guessing.

   a) Which of the following data is qualitative?

      i) Number of children
      ii) Religion
      iii) Average on a test
      iv) Distance between towns

   b) Which of the following data is quantitative and continuous?

      i) Colour of house
      ii) Number of pages in a book
      iii) Person's weight
      iv) Number of cars owned

   c) A person who works 41 hours in one week at a rate of \$10.00 per hour and is paid time-and-a-half for overtime will have a weekly gross income of

      i) \$410.00
      ii) \$415.00
      iii) \$51.00
      iv) \$615.00

   d) If a town has a need for \$2 000 000 to be raised by property taxes and has total assessed property values of \$18 500 000, the mill rate would be

      i) 9.25
      ii) 10.8
      iii) 108.1
      iv) 9250

   e) If a square has a side length of $8^{+.02}_{-.03}$, the maximum value its area can be is

      i) 64.3204
      ii) 8.02
      iii) 63.5209
      iv) 0.7995

f) For a measurement given as $7.5 \pm .02$, the tolerance would be

i) 7.52
ii) 0.04
iii) 7.48
iv) 7.5

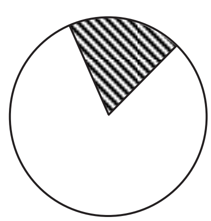
g) A line segment joining any point on a circle to its centre is called

i) Radius
ii) Chord
iii) Arc
iv) Secant

h) An angle formed between two chords of a circle sharing a common endpoint is called

i) Central
ii) Inscribed
iii) Supplementary
iv) Reflex

i) The shaded region of the following circle is called



i) Minor sector
ii) Major sector
iii) Minor segment
iv) Major segment

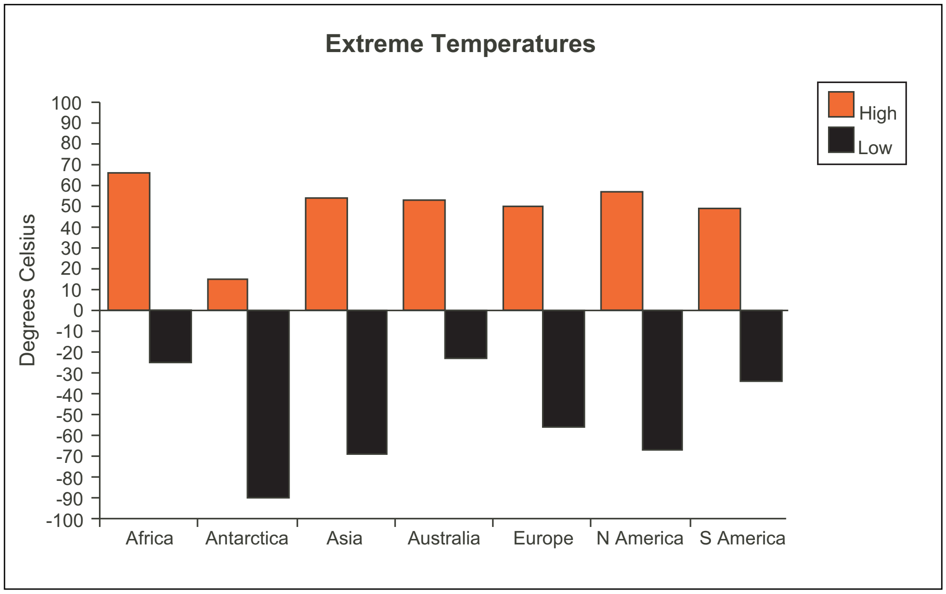
j) Which of the following is not necessary for a cheque to be legal?

i) Signature
ii) Date
iii) Particulars about what the cheque is for
iv) Amount in numbers and words

(4 marks)

2. Create or interpret the graphs in the following questions.

   a) The following graph indicates the recorded extreme temperatures for each of the seven continents. (Degrees Celsius)



i) Which continent has the highest recorded temperature?

ii) Which continent has the lowest recorded temperature?

iii) Where in North America would the lowest recorded temperature probably have been recorded?

iv) Which continent has the least variation in temperature (Highest – Lowest) and what would that value be approximately?

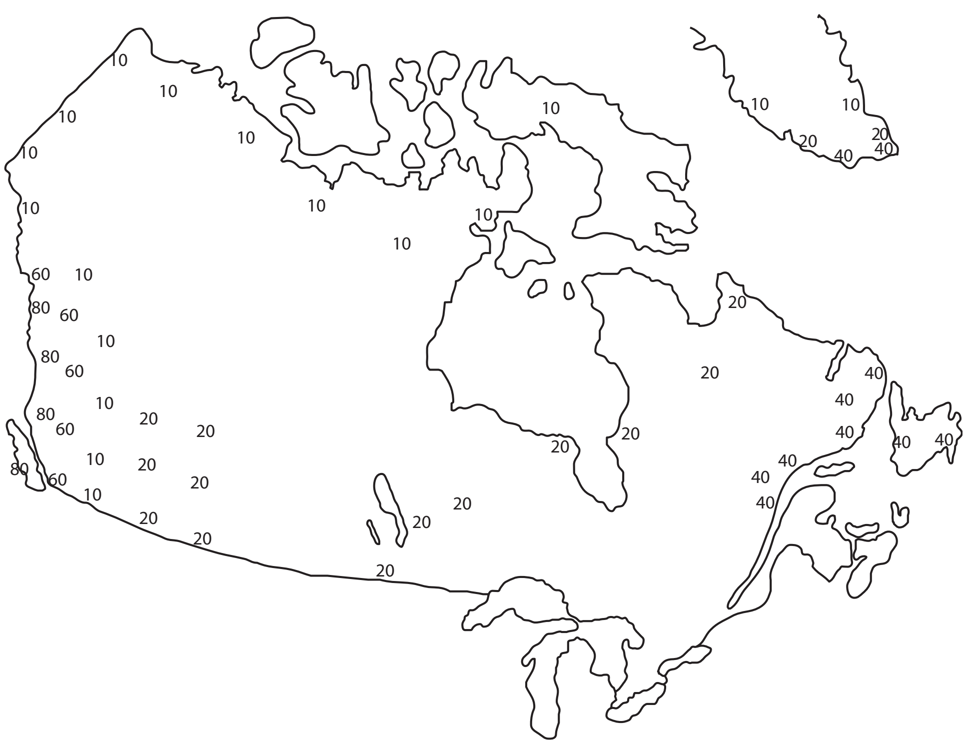
b) A student records the number of minutes spent doing homework and watching TV over a 10-day period. The results are in the following table:

	A	B	C
1	DAY	HOMEWORK	TV
2	Day 1	4	78
3	Day 2	10	30
4	Day 3	40	15
5	Day 4	11	72
6	Day 5	46	30
7	Day 6	46	90
8	Day 7	23	40
9	Day 8	28	56
10	Day 9	57	35
11	Day 10	52	95

i) Create a graph that will show the data accurately. *(2 marks)*

ii) Modify the graph to indicate that the difference in time spent doing homework and watching TV is not significantly noticeable. *(1 mark)*

c) The following map shows average annual precipitation in inches for the various regions of Canada. *(3 marks)*



i) Draw the contour lines for the map to better indicate the regions of precipitation.

ii) From the graph, determine what the average annual precipitation would be for most of the prairie provinces.

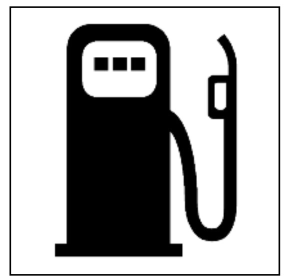
iii) Which province of Canada would receive the most precipitation?

(2 marks)

d) What would the following glyphs indicate?

i)

ii)



e) A baseball scout rates three players A, B, and C on a scale of 1–10 on the following four baseball skills: Throwing, Running, Fielding, and Hitting. The data collected are displayed in the following table:

	A	B	C	D
1	SKILL	Player A	Player B	Player C
2	Throwing	8	6	10
3	Running	7	5	3
4	Fielding	9	8	8
5	Hitting	4	10	9

(4 marks)

i) Construct a circle-and-ray glyph (to scale) to report on the three players.

ii) Which player would you rank as the best prospect overall? State reasons for your choice. *(1 mark)*

3. Solve the following applications using your graphing calculator. (Show the calculator keying sequence used to graph the functions you have found.) *(5 marks)*

   a) A total of \$9000 is invested in two funds paying 5% and 6% annual interest. The combined interest is \$510. How much of the \$9000 should be invested in each fund?

   b) Frank wishes to use 30 metres of fence to enclose all sides of his rectangular garden. What will be the dimensions of the largest possible garden? *(5 marks)*

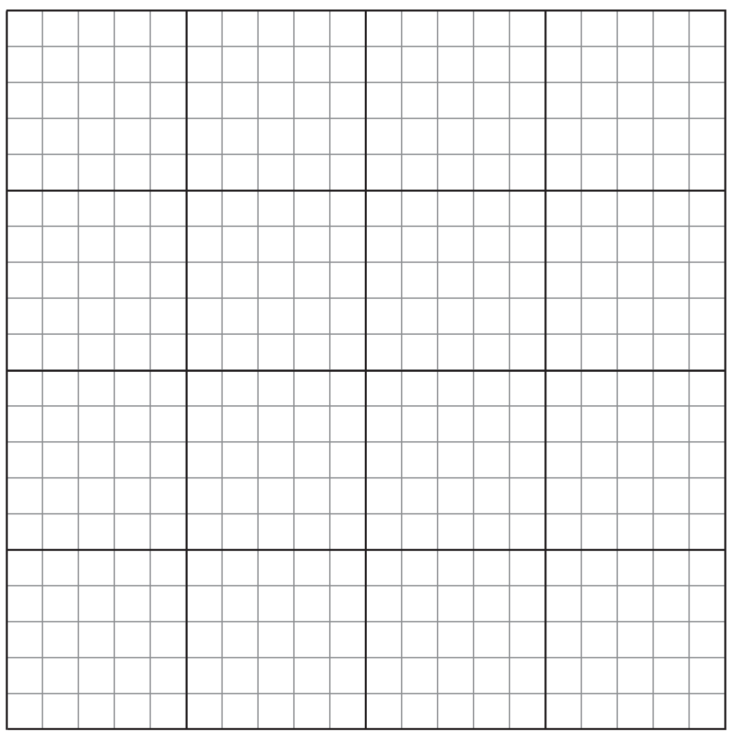
(10 marks)

4. Solve the following linear programming application.

   You are hunting for a special health drink that you read about, but you can't find one with the recommended amounts of protein and carbohydrates. You find two drinks that have the right ingredients but the wrong proportions. You wonder how much of each to drink and still keep your cost down.

   – Recommended Minimum Daily Amount: 3 cups for protein, 5 cups for carbohydrates.
   – Blend A: 25% protein; 75% carbohydrate; cost is \$0.25 per cup
   – Blend B: 50% protein; 50% carbohydrate; cost is \$0.20 per cup

   Use linear programming to find how many cups of each blend you could drink to meet your minimum daily requirement with a minimum cost.



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Notes

Grade 11 Applied Mathematics (30S)

Final Practice Examination

GRADE 11 APPLIED MATHEMATICS

Final Practice Examination

Name: ______________________________________

Student Number: ______________________________

Attending ❑ Non-Attending ❑

Phone Number: ________________________________

Address: _____________________________________

____

____

For Marker's Use Only

Date: ___________________________________

Final Mark: _________ /100 = _________ %

Comments:

Instructions

The final examination is based on Modules 5 to 8 of the Grade 11 Applied Mathematics course. It is worth 25% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the final examination.

Notes

You are allowed to bring the following to the examination: pens/pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Final Exam Resource Sheet. Your Final Exam Resource Sheet must be handed in with the examination. Graphing technology (either computer software or a graphing calculator) **is required** to complete this examination.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer. Final answers without supporting calculations or explanations will **not** be awarded full marks. Indicate equations and/or keystrokes used in calculations.

When using graphing technology, include a screenshot or printout of graphs **or** sketch the image and indicate the window settings (maximum and minimum *x*- and *y*-values), increments, and axis labels, including units.

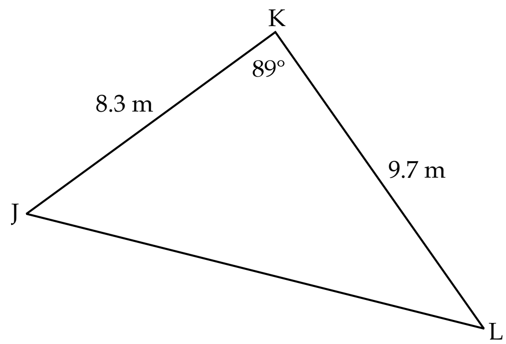
Name: ____

Answer all questions to the best of your ability. Show all your work.

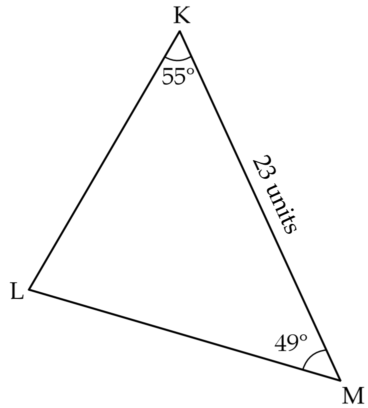
Module 5: Trigonometry *(25 marks)*

1. Solve for all the missing angles and all the missing sides in the triangles below. Round your answers to one decimal place.

   a) *(6 marks)*

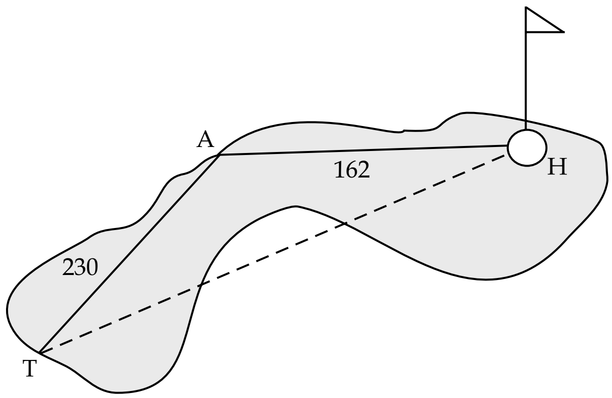


b) *(5 marks)*

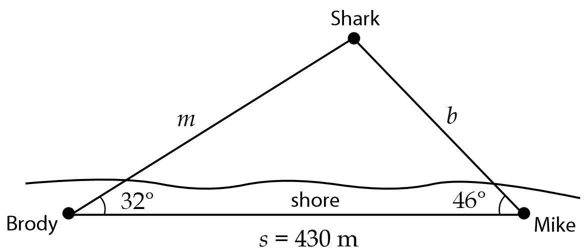


Name: ____

2. A golf hole has a dogleg as shown in the diagram below. What is the angle at the dogleg (TAH), if the distance from the tee, T, to the hole, H, is 362 m? *(3 marks)*



3. Brody and Mike are two harbour masters who are tracking the position of a great white shark. The harbour masters are located in buildings on the ocean shore that are 430 m apart. Brody's line of sight to the shark makes an angle of 32° with his line of sight to Mike. Mike's line of sight to the shark makes an angle of 46° with his line of sight to Brody. Determine the distance of the shark from each harbour master. Round your answers to the nearest metre. *(5 marks)*



Name: ____________________________________

4. Given points A, B, and C, determine measurements for angle A, side *a*, and side *b*, so that each of the following situations is created. *(3 × 2 marks each = 6 marks)*

   a) no triangle is possible

b) two triangles are possible

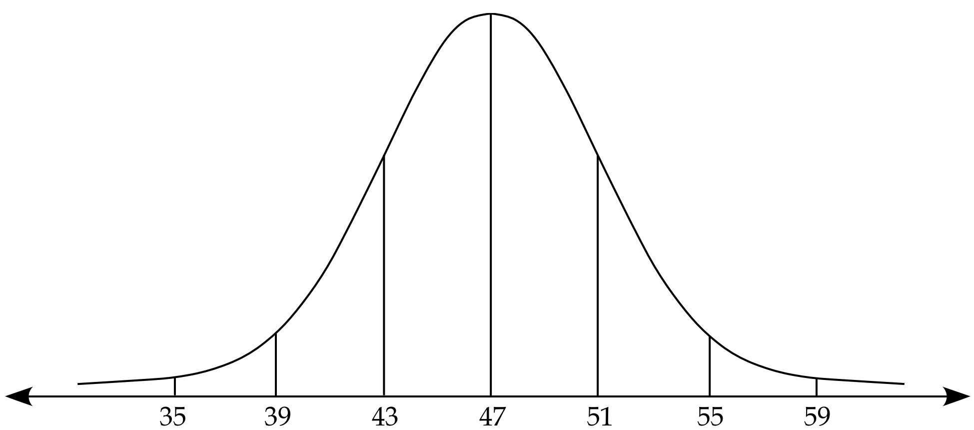
Name: ____

c) one right triangle is possible

Module 6: Statistics *(23 marks)*

1. State two properties that apply to all normal distributions. *(2 marks)*

2. Determine the mean and standard deviation of the following normal curve. *(2 marks)*



Name: ____________________

3. The following data represents a sample of the waiting times at a dentist's office. These times are expressed in minutes.

12	19	2	13	21	23	18	13	20
23	26	7	10	8	16	12	17	21

a) Determine the mean and standard deviation for the data. Round to one decimal place. *(2 marks)*

b) Calculate the percentage, to one decimal place, of the waiting times that fall within ± 1 standard deviation of the mean. *(2 marks)*

c) Verify if the waiting times resemble a normal distribution. Justify your answer.
(3 marks)

Name: ____

4. The lifetime of a microwave is normally distributed with a mean of 4.7 years and a standard deviation of 0.4 years.

   What percent of microwaves will last at least 5 years? *(2 marks)*

5. For a university calculus course, the professor decided that only 5% of students would fail. Assume the marks are normally distributed.

   a) What *z*-score represents this value? *(1 mark)*

   b) If the mean mark was 71.4 and the standard deviation was 14.3, what is the lowest mark that would represent a passing mark? *(2 marks)*

   c) If the mean was 60 and the standard deviation was 16.2, will you pass with a mark of 45? Justify your answer using mathematical calculations. *(2 marks)*

Name: ____

6. Based on survey results, 61 out of 100 people who go to a car dealership end up purchasing a vehicle. These results are accurate to within $\pm 5\%$, 19 times out of 20.

   a) Determine the confidence level. *(1 mark)*

   b) State the margin of error. *(1 mark)*

   c) What is the confidence interval? *(1 mark)*

7. Explain how the margin of error affects the confidence interval. *(1 mark)*

8. In a study of the water quality of Manitoba lakes, pollutants are measured in parts per million (ppm). The average amount of pollutants found in the lakes for a particular region of Manitoba is 0.88 ppm, with a standard deviation of 0.27. The data follows a normal distribution. The measured pollutants at Green Lake are 1.21 ppm. Determine the equivalent z-score. *(1 mark)*

Module 7: Mathematical Models *(27 marks)*

1. A delivery truck is bringing goods to a store 180 km away. For the first half hour, the driver maintains a speed of 60 km/h. The driver then accelerates to 100 km/h for the remainder of the trip. Create a graph to represent this trip. *(3 marks)*

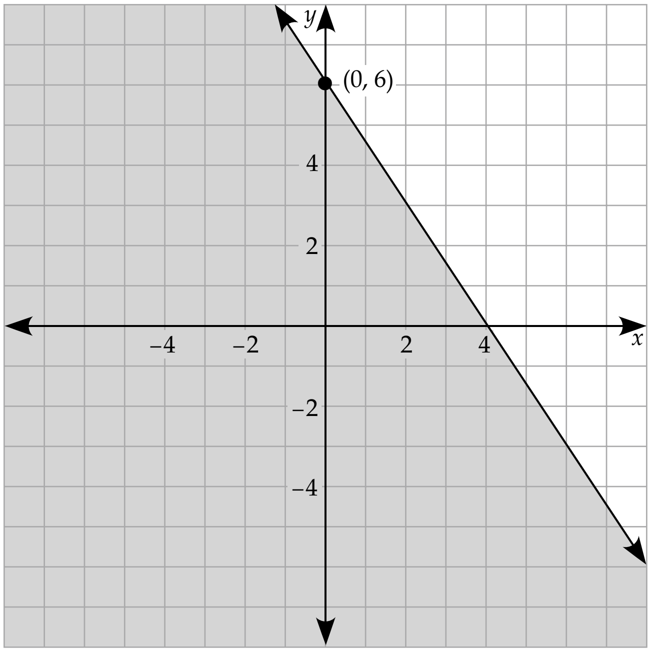
Name: ____________________

2. Determine a point that is in the solution region for the following system of linear inequalities. Prove that your point is in the solution region. *(3 marks)*

$$y \le -\frac{1}{2}x + 2$$

$$y \ge -2x + 5$$

3. Write the linear inequality that is represented in the graph below. *(3 marks)*

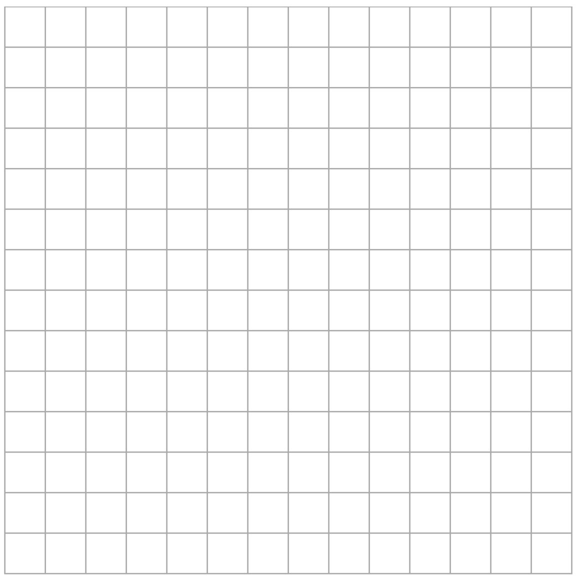


Name: ______________________________

4. Sketch the following system of inequalities. Shade the region of overlap. *(5 marks)*

$4x + 3y \geq 9$

$-y < -2x + 3$



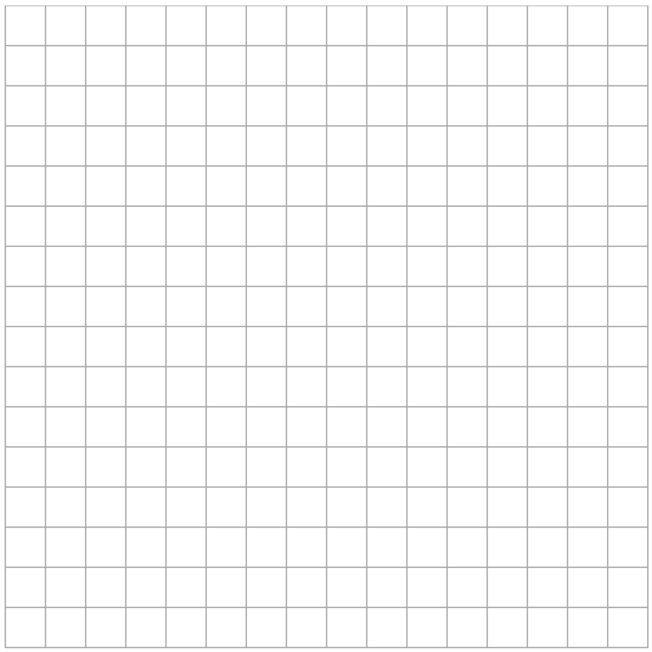
5. You have \$200 to spend on clothing. Shirts costs \$20 and pants cost \$35. Determine how many shirts and pants you can purchase.
   a) Define the two variables used in this scenario. *(1 mark)*

   b) What are the restrictions on these variables? Explain. *(2 marks)*

   c) Write an inequality to represent how many shirts and how many pants you can purchase. *(1 mark)*

Name: ____________________________________

d) Graph the inequality you created in (b) and (c), and label the vertices of the solution region. *(3 marks)*



e) If you buy 4 shirts, how many pairs of pants can you possibly buy? *(1 mark)*

f) If you buy 3 pairs of pants, how many shirts can you possibly buy? *(1 mark)*

6. A food truck sells pizza and chicken wings. Each day, they sell at least three times as many orders of pizza as they do chicken wings. The food truck only has room to hold 42 orders of pizza and 30 orders of chicken wings. Pizzas are sold for $10 each and chicken wings are sold for $8 each. Determine the number of pizza and chicken wings that would have to be sold to maximize the amount of money the food truck receives in sales in one day.

   a) Write the equation of the objective function. *(1 mark)*

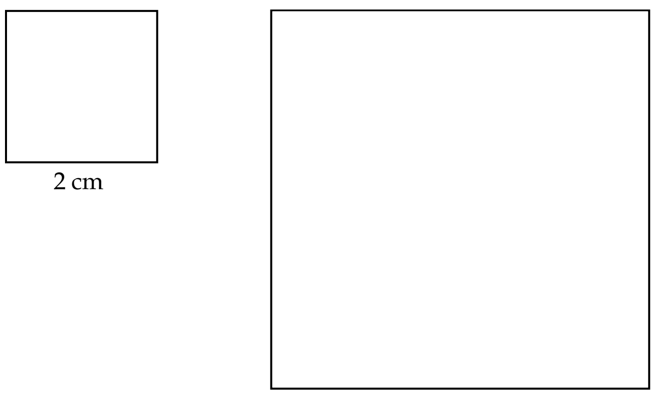
   b) The owner of the food truck graphs this inequality and finds the vertices of the solution region to be (0, 0), (42, 14), and (42, 0). How many of each type of food should be sold to maximize profits? Verify your solution by testing all vertices. *(3 marks)*

Name: ______________________________

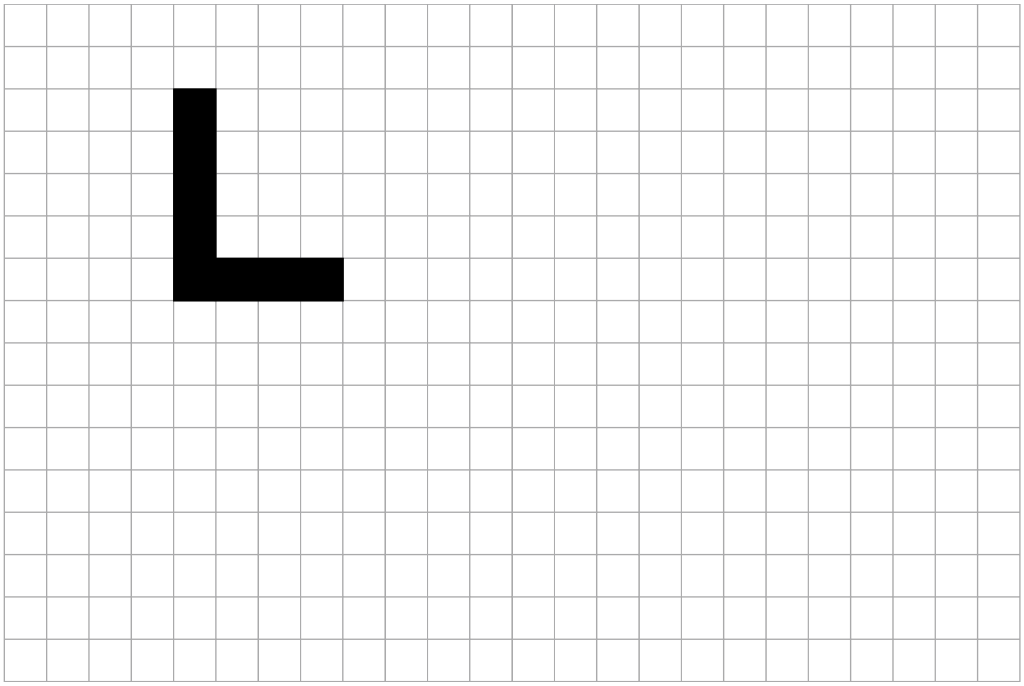
Module 8: Scale Factors for 2-D and 3-D Shapes *(25 marks)*

1. Provide an example in your life of when you would use a model of a 2-D shape. *(1 mark)*

2. The following squares are similar. Determine the scale factor if the larger square is the scale model and the area of the larger square is 25 cm^2. *(3 marks)*

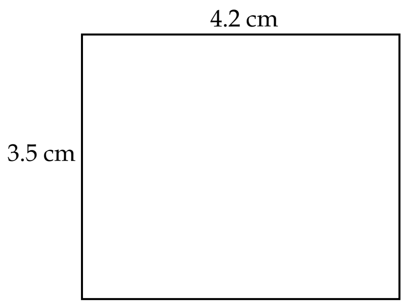


3. Draw a shape similar to the one below, using a scale of 2:1. *(2 marks)*



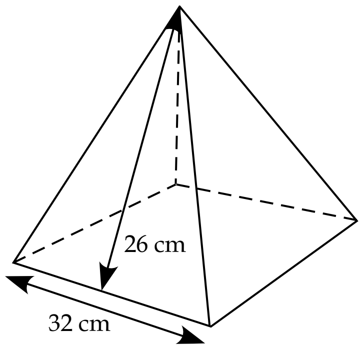
Name: ______________________________

4. a) The floor of a room is drawn to scale as shown in the diagram below, using a reduction factor of 100. Calculate the area of the actual room that the diagram represents. *(3 marks)*



b) A doorway in the actual room is 0.9 m wide. Draw the doorway on the diagram above, using the reduction scale factor of 100. *(1 mark)*

5. The dimensions for a right square pyramid are shown below. A scale model of this pyramid is being created, with a surface area of 4023 cm^2. Calculate the dimensions of the model pyramid. *(5 marks)*

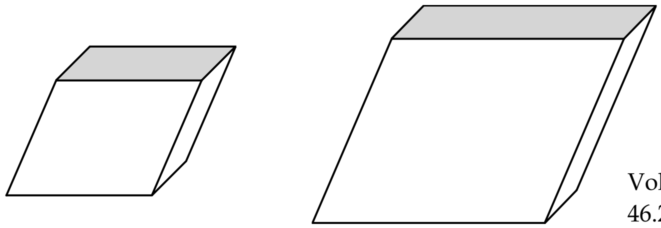


Name: ______________________________________

6. Determine the scale factor if the volume of a scale diagram is 21 mm^3 and the volume of the actual shape is 68 cm^3. *(3 marks)*

7. a) Find the scale factor relating the two objects below if the larger prism is the scale diagram. *(2 marks)*

Volume = 12.9 cm^3

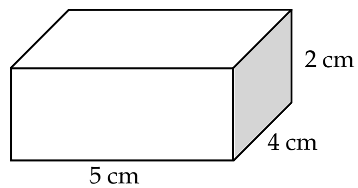


Volume = 46.2 cm^3

b) If the surface area of the top of the small prism is 3.9 cm^2, what is the surface area of the top of the large prism? *(2 marks)*

Name: ______________________________

8. Determine the surface area of the scale model of the rectangular prism given the dimensions of the actual prism below. The scale factor is $\frac{7}{2}$. *(2 marks)*



9. How many times would the volume of a milk container increase if all of the dimensions were doubled? *(1 mark)*

Grade 11 Applied Mathematics (30S)

Technology Appendix

Grade 11 Applied Mathematics Technology Appendix

The Technology Appendix provides basic information to help you learn how to use certain technology software and applications to answer the questions in this course.

You are **not expected** to use these applications, nor are you limited to the ones indicated below. There are many technology options available for use. These are simply examples and hints on how to use a variety of applications.

You **are expected** to use technology in this course. You may choose whatever calculator, software, apps, programs, or online applications that are available to you, and which meet the expected outcomes for this course. You are expected to read the handbook or access the online help provided with your choice of technology in order to learn how to effectively use it to fulfill the requirements of this course.

Technology is constantly evolving, and it is understood that after this course is printed, changes will be made to the technology applications explained below. It is your responsibility to learn how to use the upgraded versions or to find alternate applications if the ones outlined below cease to be available.

This appendix only provides basic keystrokes and examples of how to graph equations, find the coordinates of the vertex, find the x-intercepts, y-intercept, and solve for points on the line, as well as intersection points, given more than one graph. Statistical Calculations involving a normal distribution and z-scores are shown using the TI-83 graphing calculator and GeoGebra. Applications beyond what is outlined here are possible, and you are encouraged to explore and use the technology of your choice to its fullest potential.

On the midterm and final examinations, you will be expected to have access to technology. You must indicate which application or software you are using and show your work by including a printout of the screen, sketching the screen, or indicating the input and output values.

Check with your tutor/marker to be sure the graphing technology you are using will be appropriate for assignments and examinations.

There are many different websites and apps available for use. Some are free and others must be purchased. Some places to explore include:

- GeoGebra www.geogebra.org
- Winplot math.exeter.edu/rparris/winplot.html
- Meta Calculator www.meta-calculator.com/
- khan academy www.khanacademy.org/
- Graphmatica www.graphmatica.com/
- purple math www.purplemath.com/
- Texas Instruments TI-83 Plus graphing calculator online emulator—you can search for “TI-83 Flash Debugger” to find an online emulator, license requirements, and instructions for downloading.
- Texas Instruments TI-83/TI-84 Plus Graphing Calculator

Graphing Technology (TI-83 or TI-84)

The following instructions show you how to graph on a TI-83 or TI-84 graphing calculator.

Graph an Equation

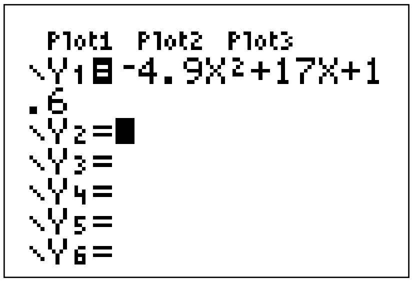
Graph the polynomial function $h = -4.9r^2 + 17t + 1.6$.

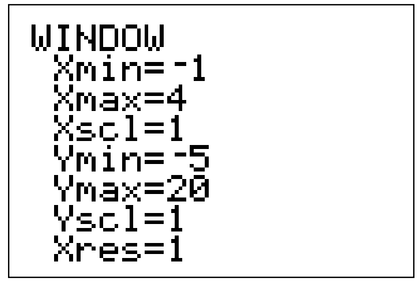
To access the equation editor, press $\boxed{\text{Y=}}$.

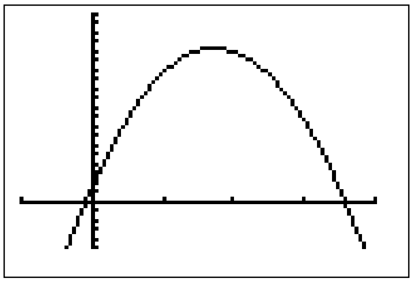
Enter the equation using $\boxed{\text{X, T, }\theta\text{, }n}$ for the variable and $\boxed{x^2}$ for the power of 2.

Press $\boxed{\text{WINDOW}}$ to set the maximum and minimum values of x and y, and the increments for the scale along the axes.

To view the graph, press GRAPH .

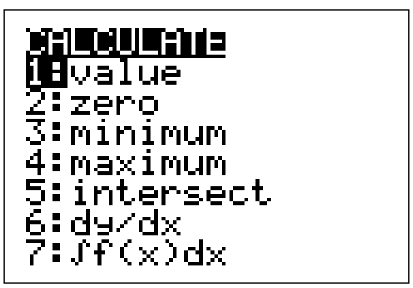


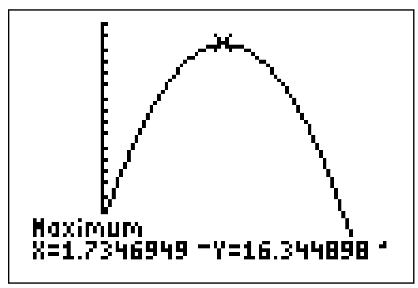




Determine the coordinates of the vertex.

In this case, the vertex is at a maximum *y*-value.





2nd CALC Maximum (above the TRACE key)

Using the blue arrow keys, move the cursor just to the left of the vertex and press ENTER .

Move the cursor just to the right of the vertex and press ENTER .

Move the cursor as close to the maximum point as possible and press ENTER .

Find the *x*-intercepts.

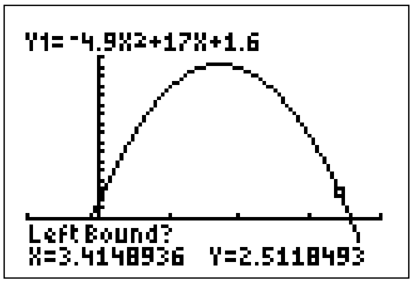
On the TI-83, the keystrokes are:

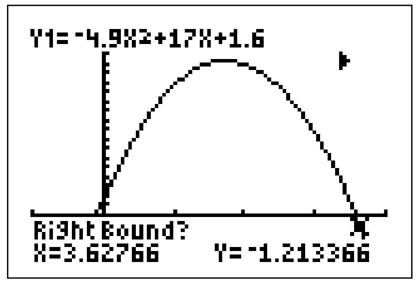
2nd CALC Zero (above the TRACE key)

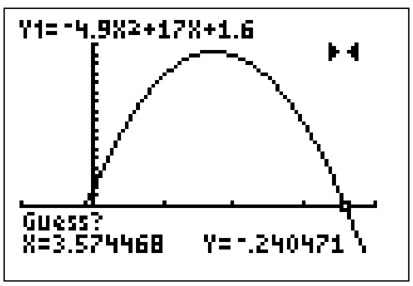
The line is read from left to right so to determine the value of the *x*-intercept near the right side of the screen, use the blue arrow keys to move the cursor just above the intercept on the right and press ENTER.

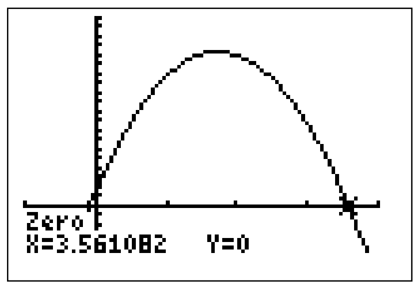
Move the cursor just below the intercept and press ENTER.

Move the cursor reasonably close to the intercept and press ENTER.









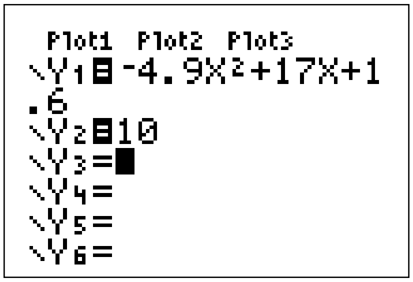
The line of a graph is read from left to right, so the left and right bounds may be above or below the *x*-axis, depending on the end behaviour of the graph.

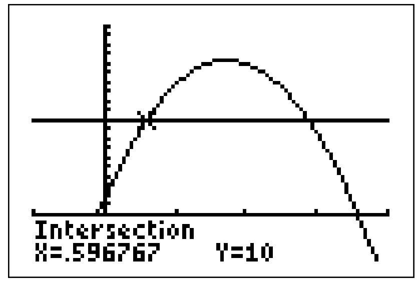
Find the point of intersection between two graphs.

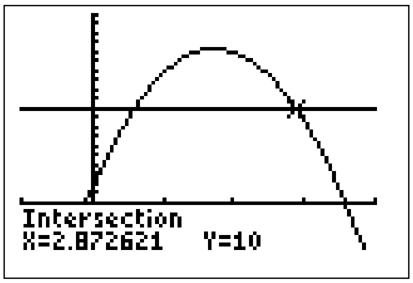
Enter the two equations into the equation editor and view the graph.

On the graphing calculator press 2nd CALC Intersect (above the TRACE key).

Move the cursor along each line as close to the point of intersection as possible. Press ENTER each time. Repeat for the other point of intersection.



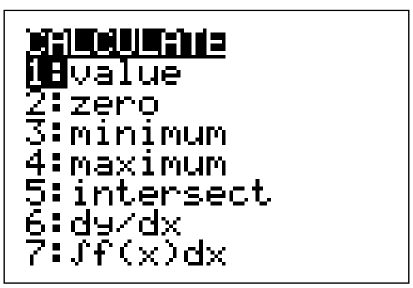


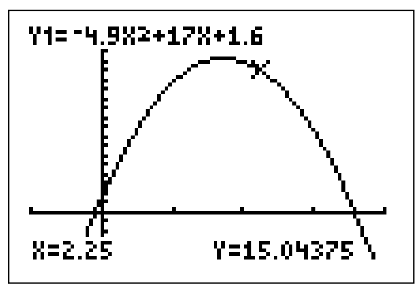


Finding a *y*-value for a given *x*-value:

Press $\boxed{\text{2nd}}$ $\boxed{\text{CALC}}$ $\boxed{\text{Value}}$ (above the TRACE key)

To find the *y*-value of a given *x*-coordinate (e.g., $x = 2.25$), type the number 2.25 and then $\boxed{\text{ENTER}}$.

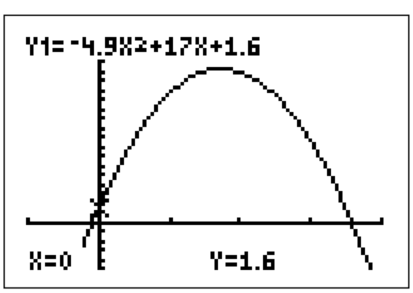




Find the *y*-intercept.

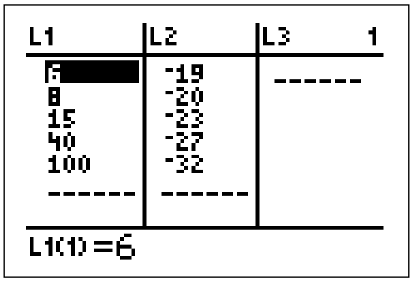
The *y*-intercept is found where $x = 0$.

On the graphing calculator, press $\boxed{\text{2nd}}$ $\boxed{\text{CALC}}$ $\boxed{\text{Value}}$ and type the value of 0. Press $\boxed{\text{ENTER}}$.



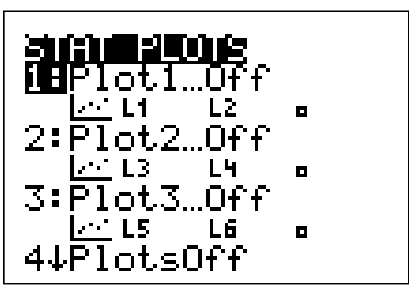
Plotting Points in a Scatterplot

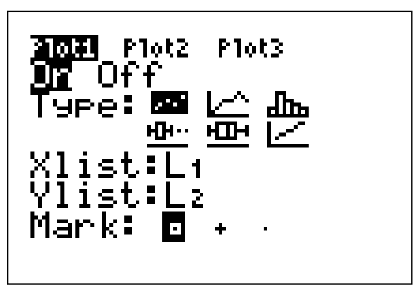
Press STAT EDIT to access the Lists. To clear an existing list of values, move the cursor to the heading L1, press CLEAR and the blue down arrow, ▼. Do not press DELETE. Under L1 for list 1, enter the values for the independent variable. Enter the dependent data in L2. Make sure that you have exactly the same number of values in each list.

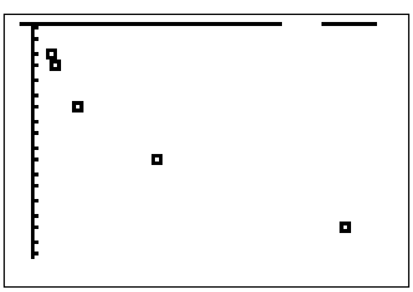


To activate the scatterplot, press 2nd STAT PLOT (above the Y= key). You may see that the plots are off.

To turn one of the plots on, press 1 and toggle the plot On by pressing ENTER. Make sure the first graph type is highlighted and the Xlist is L_1 and the Ylist is L_2. These can be changed by pressing 2nd 1 or 2nd 2. Select the large square point marker. To fit the window to the data, press ZOOM 9. Make sure no previously graphed equations in the Y= register or STAT PLOTs are toggled on. You may change the WINDOW and press GRAPH to display the data if you prefer a different window setting.







Determine a Regression Equation

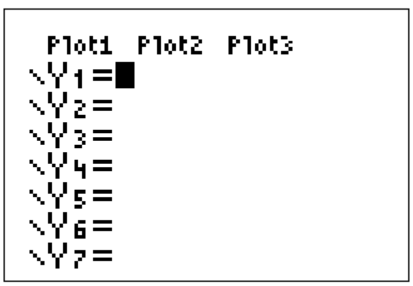
The equation of a line or curve of best fit is also called a regression equation. It models or represents the relationship between the variables and can be used to estimate values. You can use the TI-83 graphing calculator to find different types of regression equations. You can also use a computer app, such as GeoGebra.

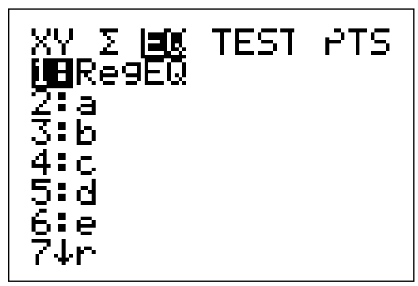
STAT Use the blue arrow to the right ▶ to select CALC and notice the list of possible regression equations. To determine a quadratic regression equation, select 5, the QuadReg.

QuadReg shows up on your home screen. Identify which lists of values you want the regression equation calculated on by pressing 2nd 1 , 2nd 2 ENTER for L_1 and L_2. The comma is above the 7 key.

You can manually enter the regression equation into the Y= register or copy and paste it using the variable memory.

Pressing Y= clears all existing equations. Position cursor next to \Y1=.

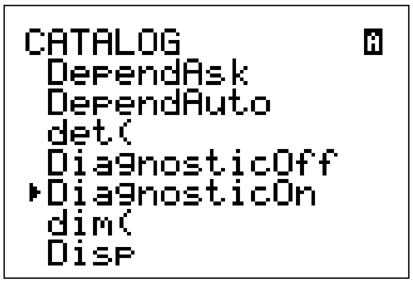


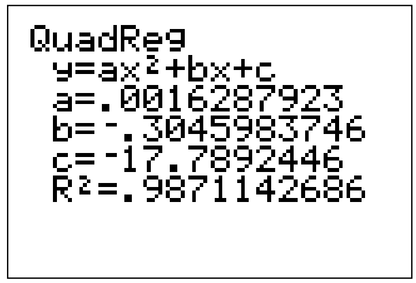


VARS 5 ▶ ▶ to highlight EQ ENTER and GRAPH.

The Coefficient of Determination, R^2 Value

To turn on the diagnostic function, press [2nd] [0] to access the catalogue. Notice the A in the top right corner indicating the ALPHA register is activated. Press [x^{-1}] (it has a green D above it) to get to the D section of the alphabetical listing and use the blue arrow keys to scroll down until you see the triangle indicator pointing to DiagnosticOn. Press [ENTER] and then [ENTER] again.



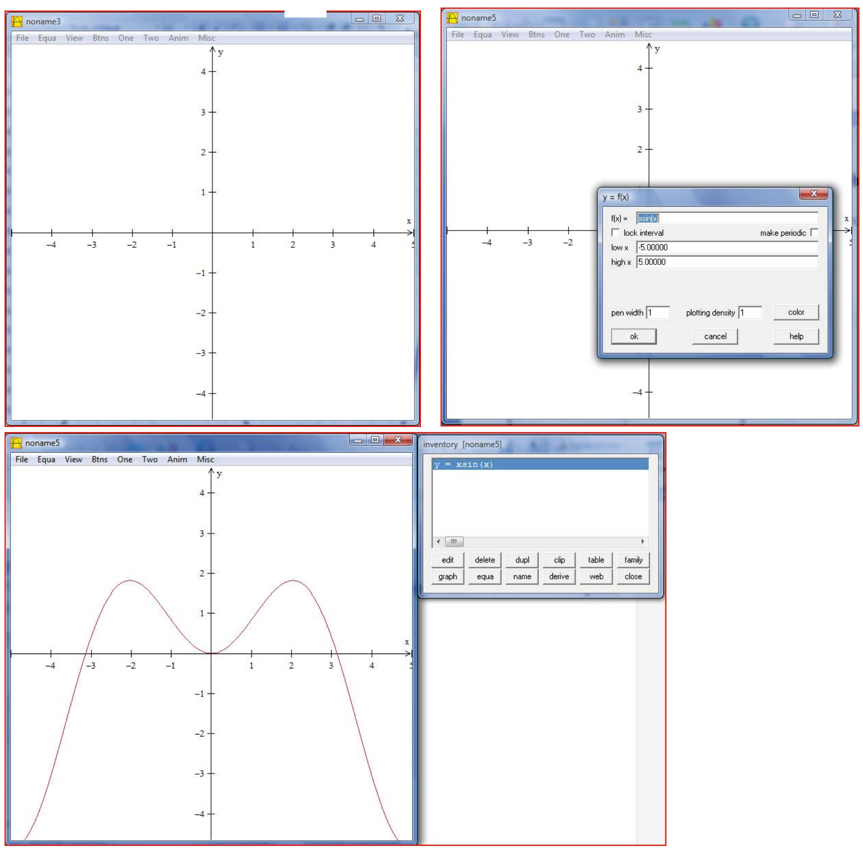


Graphing Technology (Winplot)

This program can be downloaded for free and is useful for graphing and analyzing functions.

Graph an Equation

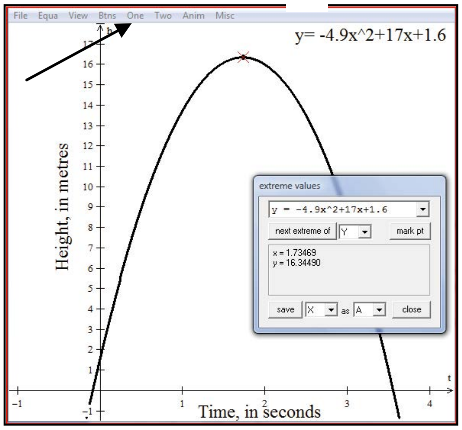
Click WINDOW and 2-dim with Use defaults selected. Select the Equa menu tab and choose Explicit. Enter the equation, specify the max and min x-values, and click OK. The Inventory window allows you to edit the equation.

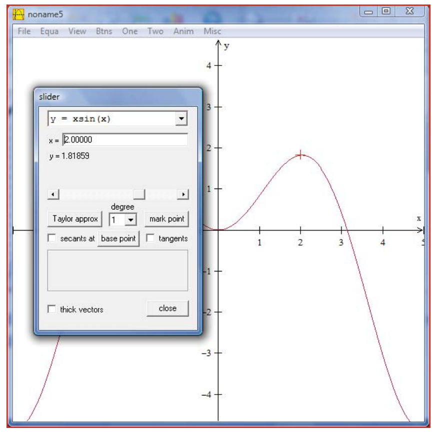


Use the View menu tab to adjust the window, show the grid, and add labels.

Find Values (and Vertex)

To find values of coordinate points along the graph, choose the One tab on the top menu. Select Extremes ... to locate the minimum and maximum *y*-values, such as the vertex.

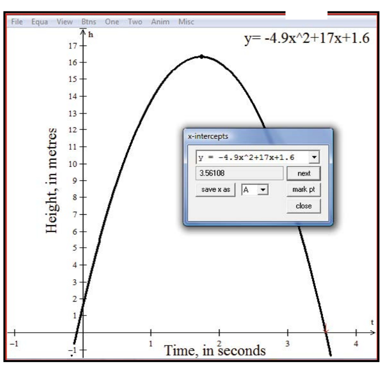




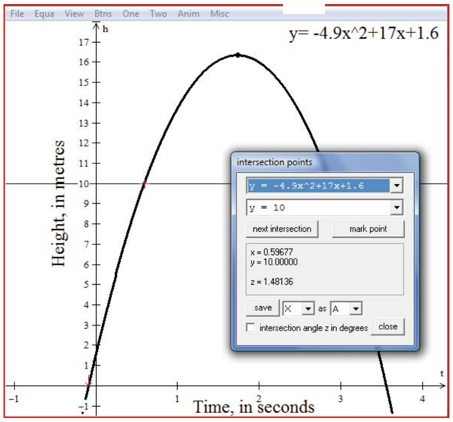
To find the *y*-value of a coordinate when the *x*-value is known, select the One tab and choose Slider. Enter the known value for *x* and press return.

The *x*-intercepts can be found by selecting the One tab and the Zeros ... function.

The first intercept in this example is stated is at −.09169. Select Next and the value of the second intercept is given as 3.56108.



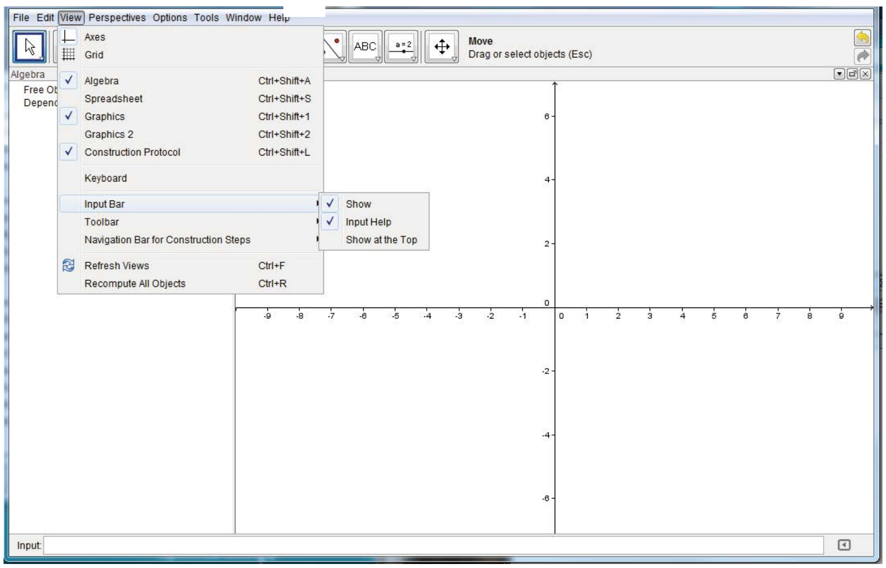
To find a point of intersection between two graphs, use the `Two` tab on the menu bar and select `Intersections ...` The first intersection of the lines is at 0.59677 and the second is at 2.87262.



Graphing Technology (GeoGebra)

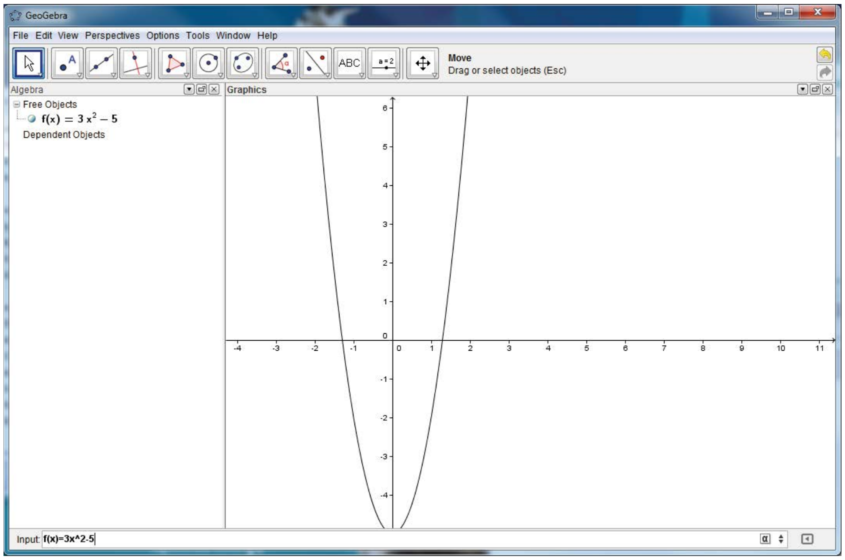
This program can be downloaded at www.geogebra.org. It can be used for graphing and analyzing functions and finding regression equations given a set of points. You can find detailed instructions for GeoGebra on its website (www.geogebra.org).

The basic structure of the GeoGebra window is shown in the image below. Under the Menu bar at the top, you can see (in the image below) the Algebra window on the left, the Graphics window on the right, and the Input Bar at the bottom. If you don't see one of the windows or the input bar, you can change that by using the View drop-down menu.



Graphing a Function

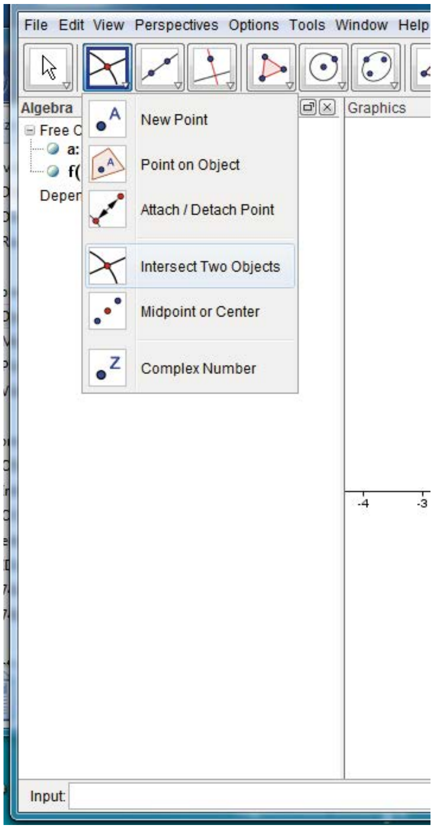
You can enter a function to graph using the Input Bar by typing a function equation such as $f(x) = x * \sin(x)$ or $y = 3x \wedge 2 - 5$, as shown in the image below. Note the "^" symbol is used to indicate an exponent is to follow.

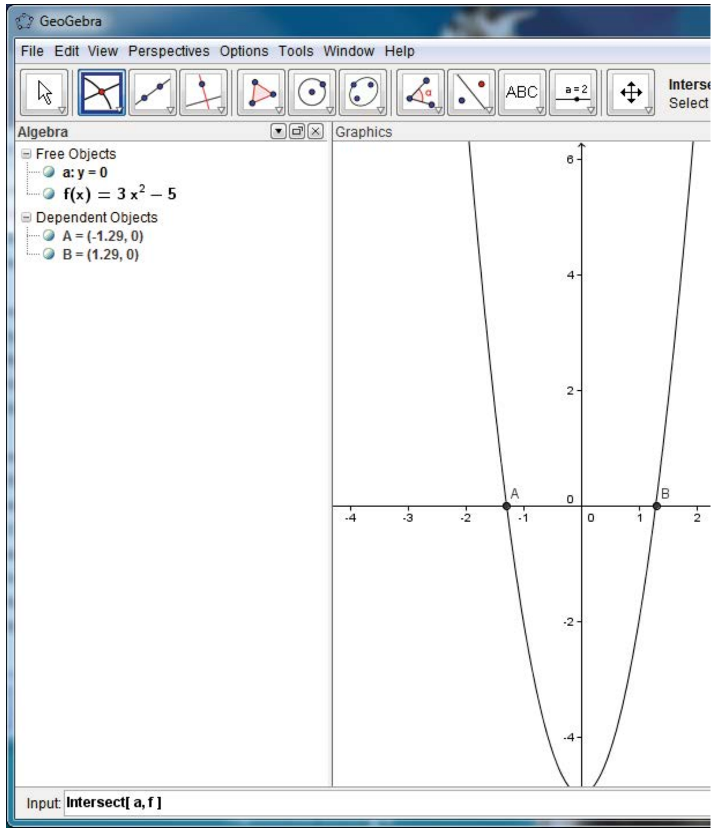


Finding *x*-intercepts (and Other Function Intersections)

After the function is graphed, you can find the x-intercepts by graphing the function $y = 0$, which is given the function name, "*a*" by the GeoGebra program. Then choose the option Intersect Two Objects from the menu bar. Select $y = 0$ and $f(x) = 3x^2 - 5$ as your two objects to intersect. See the windows shown below. As an alternative to using the menu bar, you can enter the command directly into the Input Bar. As you begin to type "intersect," you will notice after entering the first few letters, "inters," that a list of options becomes available. One of the options is Intersect[<Object>, <Object>]. Press Enter on that option to have it displayed in the Input Bar and then replace one <Object> with the name *a* and the other <Object> with the name *f* (different letters may be assigned in your window). In the Input Bar, you will see Intersect[a,f]. Press enter and you will see the window as shown on the right in the following illustration.

The x-intercepts are plotted in the graphics window and their values are listed in the algebra window. In this case, one x-intercept is -1.29 and the other x-intercept is 1.29.



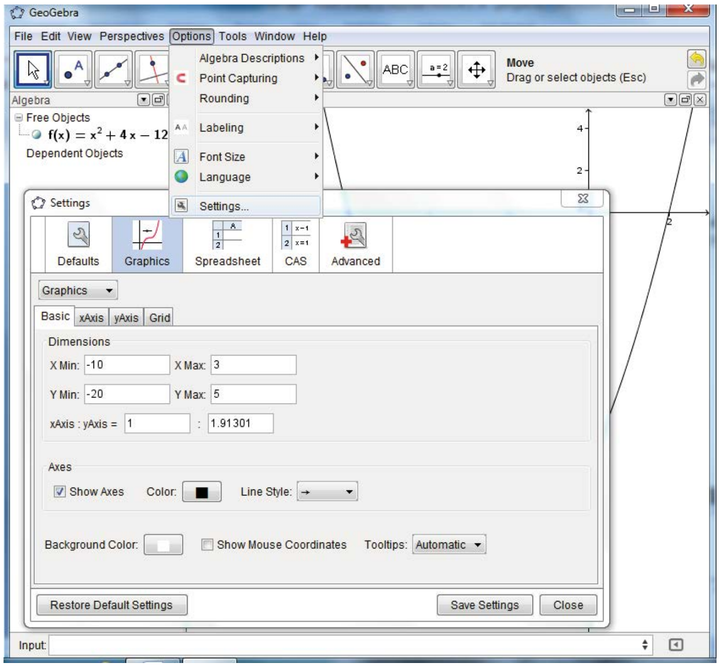


Finding the y-intercept

The process is the same as when finding *x*-intercepts. After the function is graphed, you can find the *y*-intercept by graphing the relation $x = 0$, which will be given a function name (maybe "*b*") by the GeoGebra program. Find the *y*-intercept by using Intersect Two Objects from the menu and selecting the function under investigation and the vertical line, $x = 0$.

Changing the Graphics Window Settings

Under the Options menu, select Settings to open a window. Choose the Graphics settings. Enter the desired values for X Min and X Max and for Y Min and Y Max (e.g., 5). The values chosen for the window shown are XMin = –10, XMax = 3, YMin = –20 and YMax = 5.



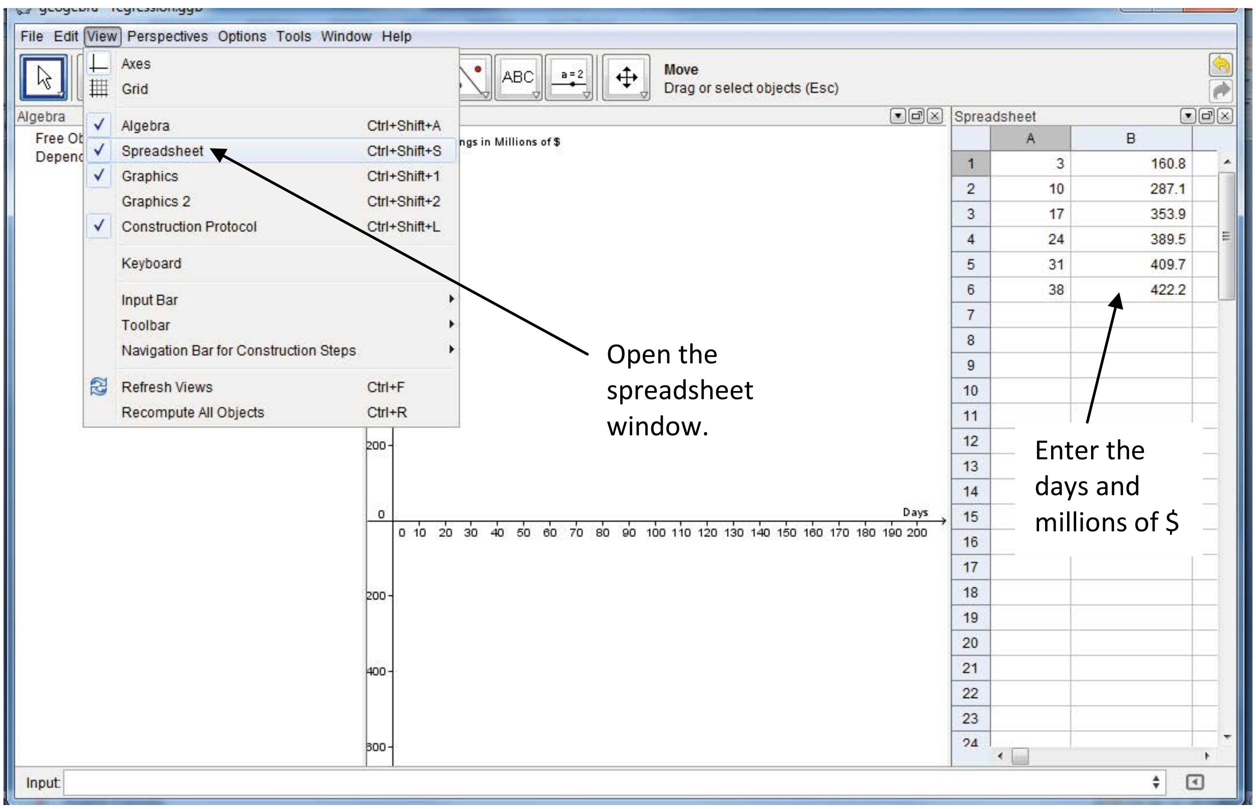
Note: You can also change the min and max axis values directly on the graphics window by selecting the Move Graphics View arrows and then clicking on one axis at a time and dragging it toward or away from the origin (0, 0).

Finding a Regression Equation using GeoGebra

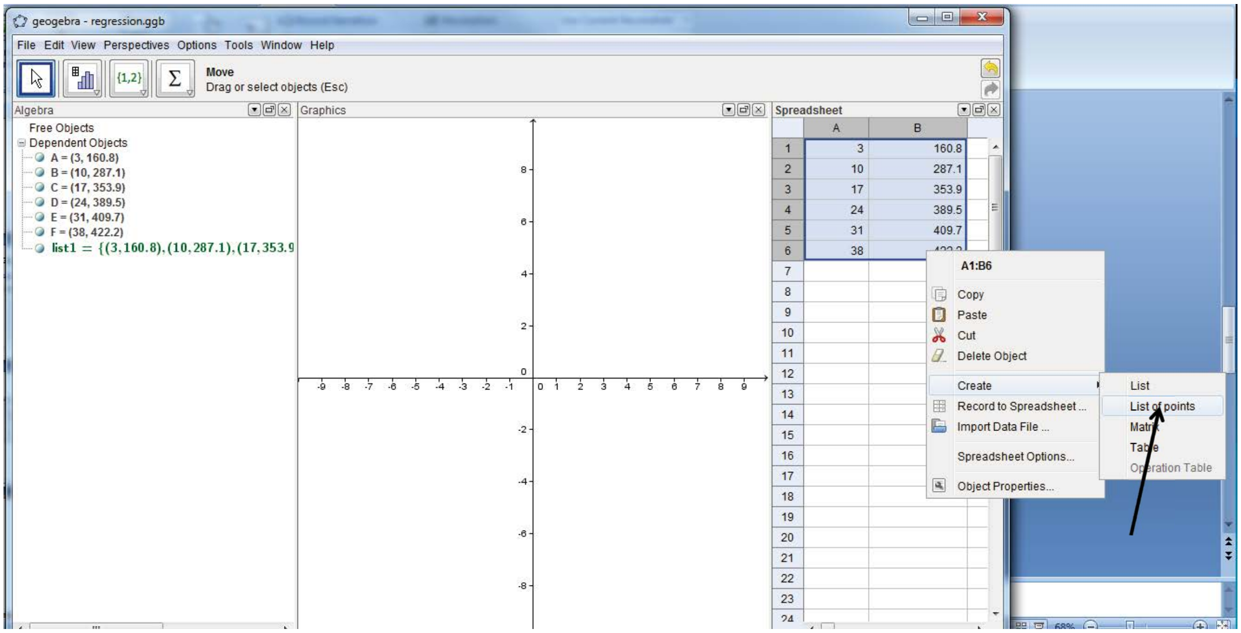
You can use GeoGebra to enter data points and determine a line of best fit through those data points. The details for using the program can be found at www.geogebra.org. As well, the example below shows the required steps. The example uses the data in the table below, which shows gross earnings at the box office for a movie.

Days	3	10	17	24	31	38
Gross (million $)	160.8	287.1	353.9	389.5	409.7	422.2

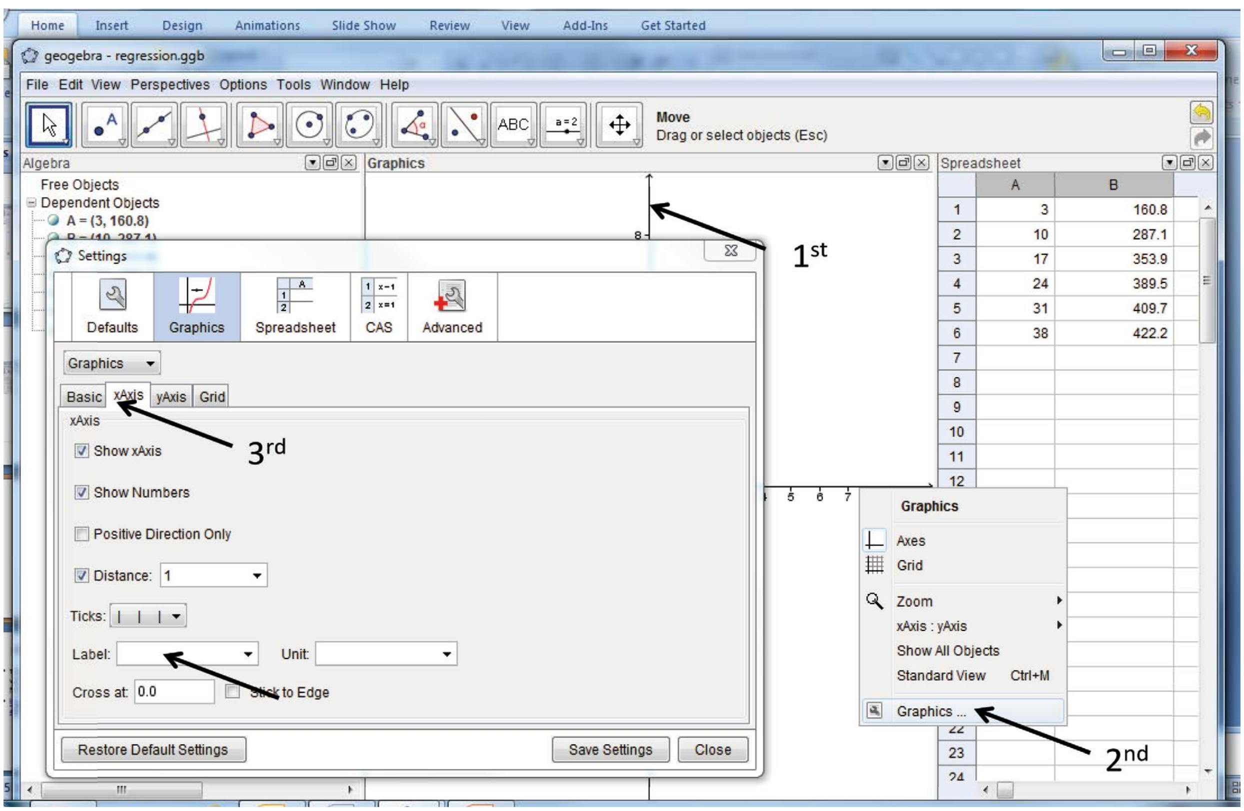
Open GeoGebra and go to the View menu. Select the Algebra window, the Graphics window, and the Spreadsheet window, and enter the data in two columns of the Spreadsheet window as shown below.



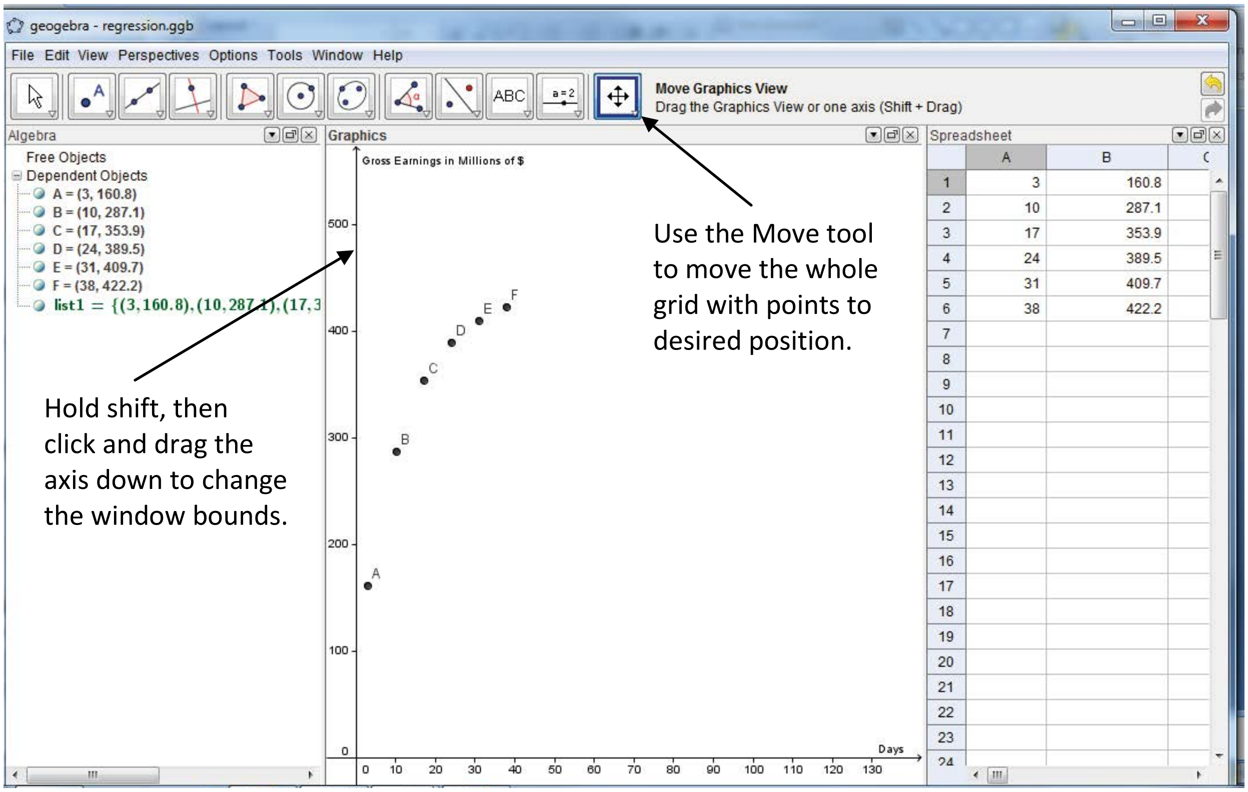
Create a list of coordinates by clicking and dragging to select the desired spreadsheet cells, and then right click to create the List of points. All of the points and the list appear in the Algebra window as shown below.



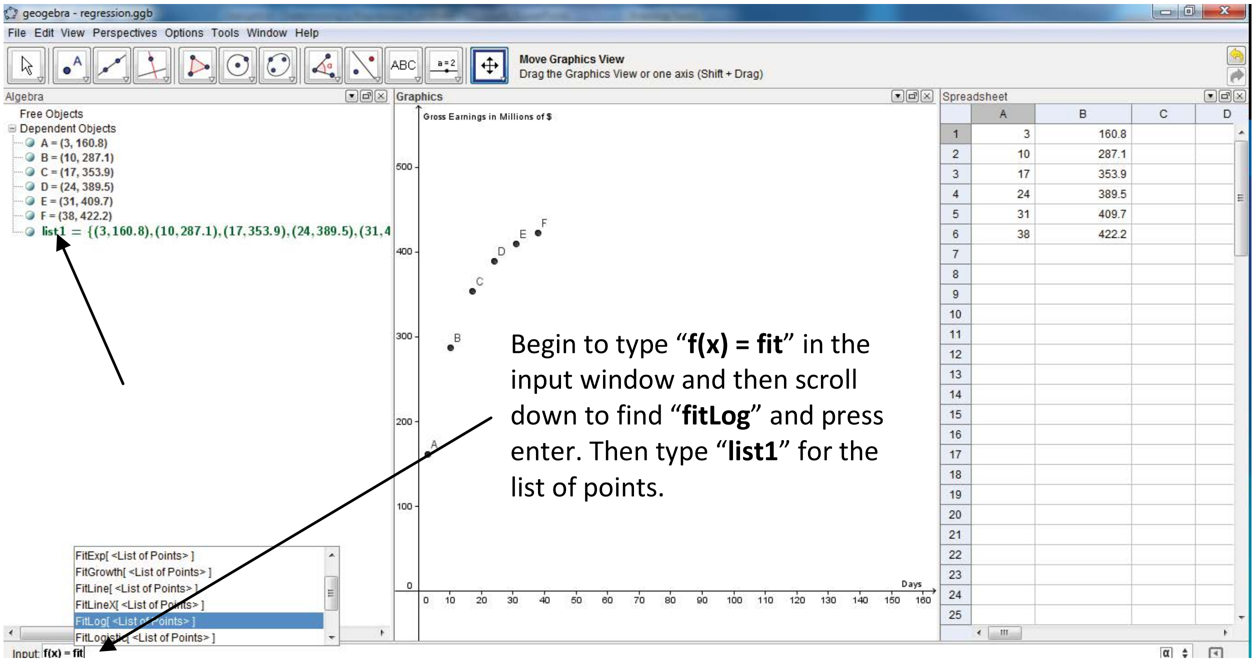
You have the flexibility to change the Graphics window details. Right click on one of the axes (1st) to open the Graphics window (2nd). Set the basic *x*-axis and *y*-axis parameters that you want (3rd).



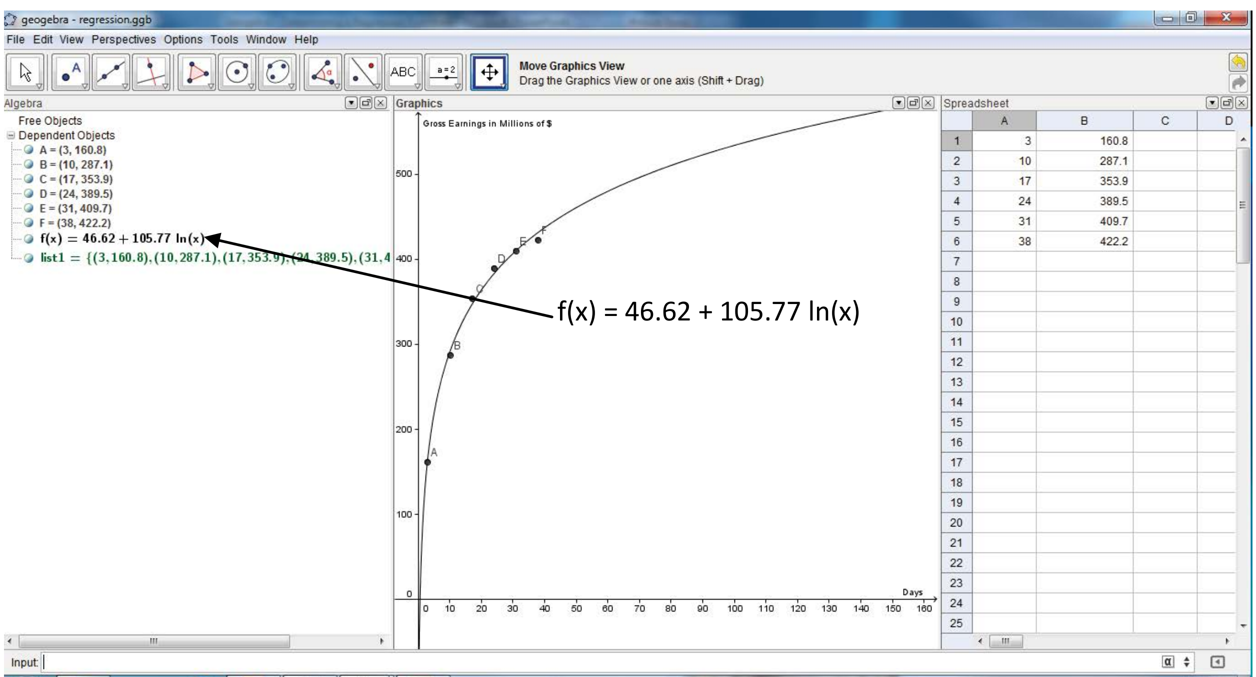
You can adjust the grid in the window by selecting the Move tool as shown. Then adjust the viewing window by holding shift, then click and drag the y-axis to change the window bounds. Do the same with the x-axis.



You can try a variety of function types. This example shows the use of regression to find the best fit of a Log function to the list of data points.



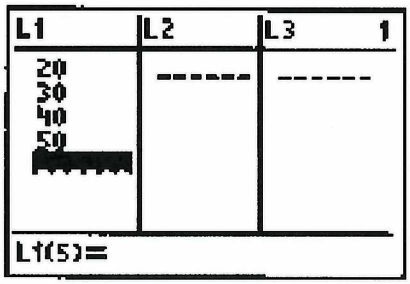
The regression function is graphed in the Graphics window and the equation is displayed in the Algebra window.



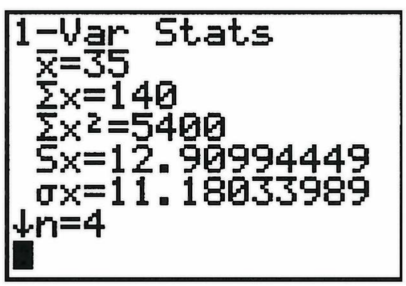
By specifying a list of points, you can find the best fit of other function types using the input line (for example, fitExp, fitLine, fitLog, fitPoly, or fitSin). When you use fitPoly, you must also specify the degree of the polygon (2 for quadratic, 3 for cubic, etc), along with the list of points.

Statistical Calculations (TI-83 or TI-84)

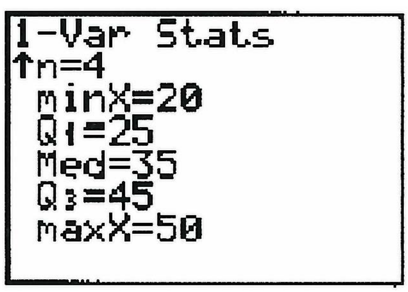
The TI-83 or TI-84 graphing calculator can be used to do statistical calculations. You can enter data into lists by pressing the STAT button and choosing the **EDIT** option as described previously in this appendix when entering data for a scatterplot. For example, enter the data into list 1 as follows:



Once the data is entered into the list you can display the one-variable statistics information by pressing STAT again and selecting the CALC menu. Choose the 1-Var-Stats option and the following information will be displayed:

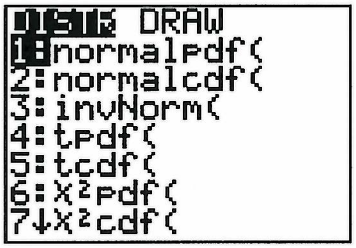


After scrolling down:

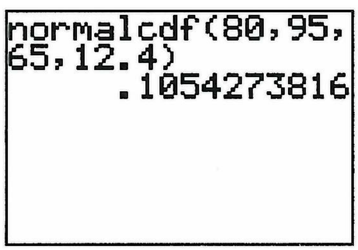


This information indicates, among other things, that the mean is 35, standard deviation, σx, is 11.18, and the median is 35.

To do some calculations involving a normal distribution you can press the 2nd DISTR function and your screen will display:

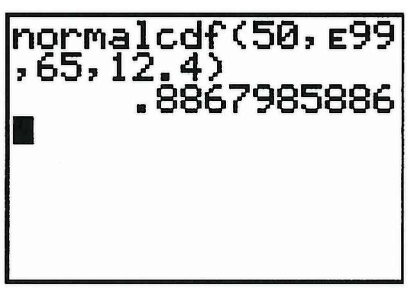


You can use the normal cumulative density function by selecting normalcdf(from the menu. It is used to calculate the probability of scores lying between a lower and higher value of a normal distribution. You can enter the necessary information in the following order: lower score, higher score, mean, standard deviation. For example, to find the probability that a data value is between 80 and 95, given that the mean of the data is 65 and the standard deviation is 12.4, enter the following:



The result means that 10.5% of normally distributed data values will lie between 80 and 95.

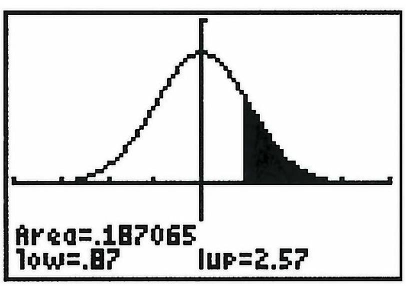
If you want to find the probability of a value being above 50 in the same data set, you need to set the higher score to the maximum calculator value (1×10^{99}) by pressing 2nd EE followed by 99. The following display shows that 88.7% of the data values will be above 50:



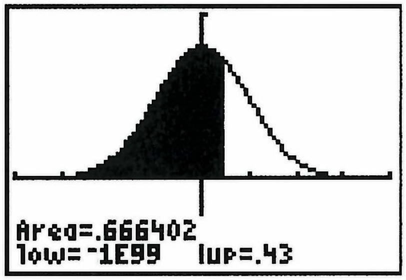
If you want to work with z-scores, you can use the normalcdf(function. A z-score is a probability representing a normal distribution of data with a mean of 0 and a standard deviation of 1. With the normalcdf(function, if no mean and standard deviation are specified, it is assumed that the mean is 0 and the standard deviation is 1. To find the probability of data lying between a z-score of 0.87 and 2.57, you can enter the following:

You can graph the area of concern using the ShadeNorm(function, which is located under 2nd DISTR and the Draw menu heading. Since the function creates a drawing (like a graph), you may want to select ClrDraw to remove old graphs first. You also may want to ensure that Y= and StatPlot graphs are off. You can set Window variable so that the distribution fits the screen. You could use xMin=−3.5, xMax=3.5, yMin=−0.5, yMax=0.5.

To calculate the probability and see a graph of the normal distribution with shading between two z-scores, such as 0.87 and 2.57, enter the command ShadeNorm(0.87, 2.57). The following graph will be displayed:

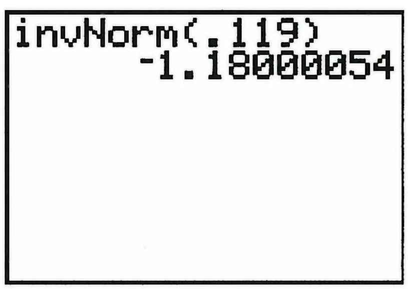


The probability of getting a z-score less than 0.43 in a standard normal distribution can be calculated and graphed using the command ShadeNorm(–1E99, 0.43). The following graph shows that the probability of 0.6664 will be displayed:



Sometimes you may have a need to do the inverse operation that does the process in reverse; that is, if you know the probability of an event, you can determine the corresponding z-score. You use the invNorm(function to determine the z-score associated with a probability. This function of the calculator will always give a z-score that reflects an area on the normal curve less than the z-score desired. If you want the z-score from which the area or probability is less than a given value, then use the invNorm(function directly. If you want the z-score from which the area or probability is greater than a given value, then use the negative of the invNorm function.

For example, if the probability of getting less than a certain z-score is 11.90%, find the z-score using:



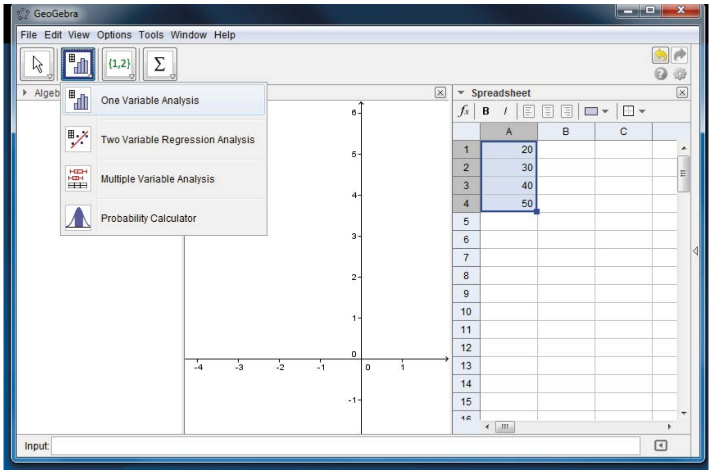
Thus, the probability of getting a z-score less than –1.18 is 0.1190.

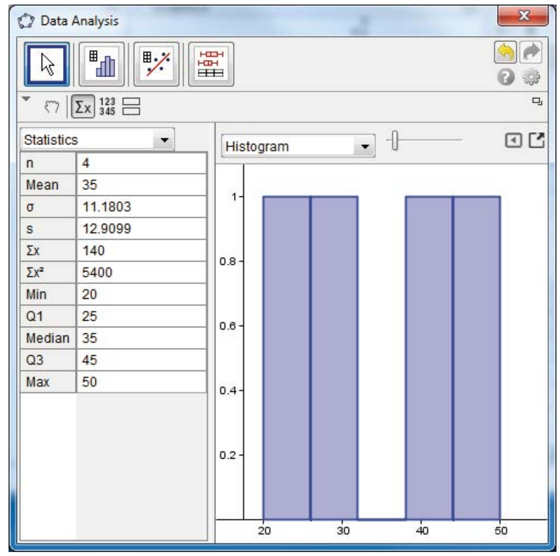
As another example, if the probability of getting a z-score larger than a certain z-score is 0.0129, find the z-score using:

Since the probability represents the area of the normal curve greater than the z-score, the negative sign is required to return a positive z-score of 2.229.

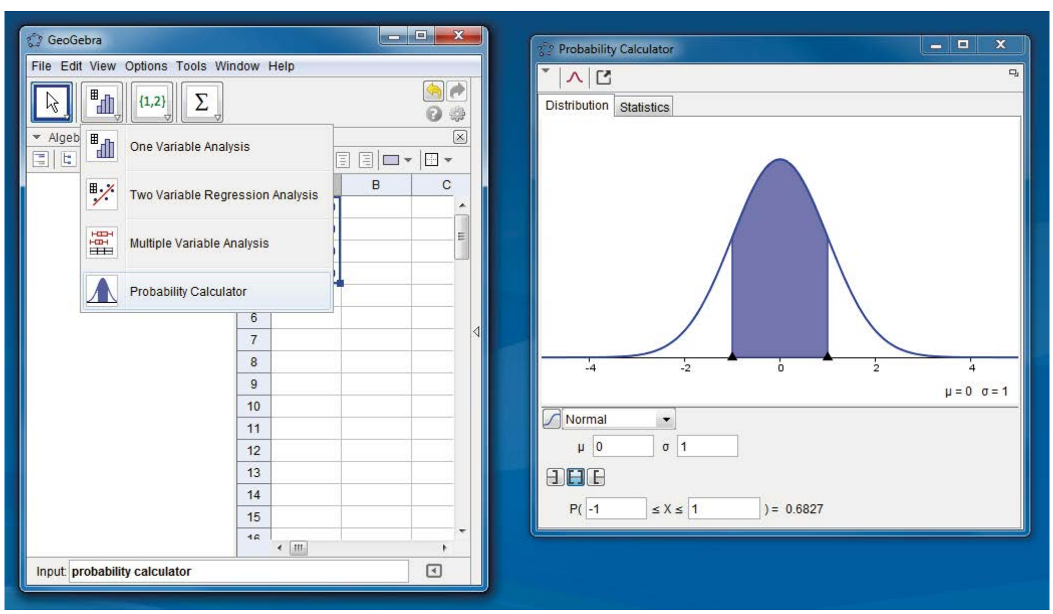
Statistical Calculations (GeoGebra)

The GeoGebra app can be used to do statistical calculations. You can enter data into a table using the spreadsheet view and then select One Variable Analysis to display the related stats as shown.

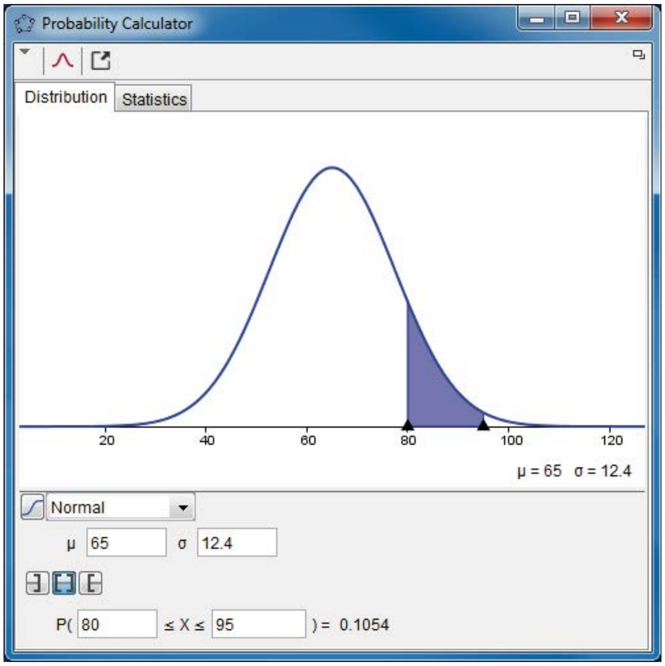




To do calculations with a normal curve, you can select GeoGebra's Probability Calculator from the drop-down menu. The initial image below shows the Normal Distribution. Given the mean of 0 and standard deviation of 1, the range values of −1 to 1 represent the z-score boundary values. The probability of 0.6827 corresponds to an area under the normal curve of 0.6827 or approximately 68%.

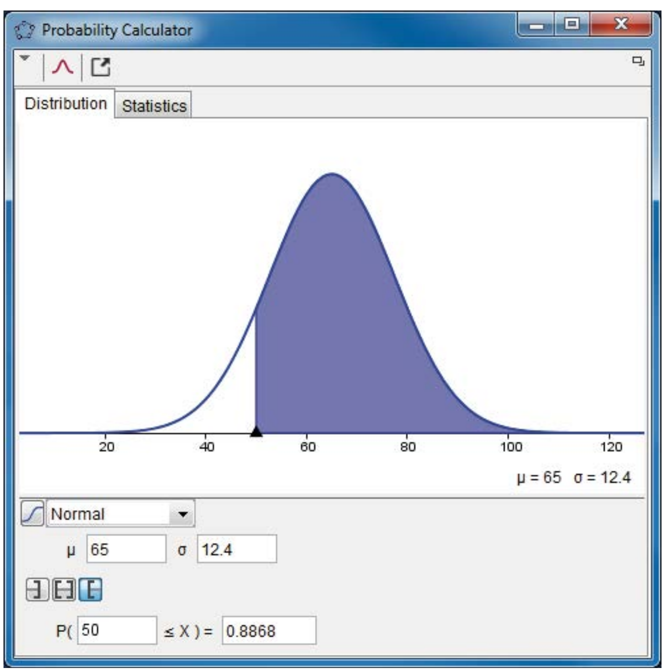


To calculate probabilities corresponding to a set of data that is normally distributed, you can enter the necessary information into the Probability Calculator. For example, to find the probability that a data value is between 80 and 95, given that the mean of the data is 65 and the standard deviation is 12.4, enter the following values into the Probability Calculator:



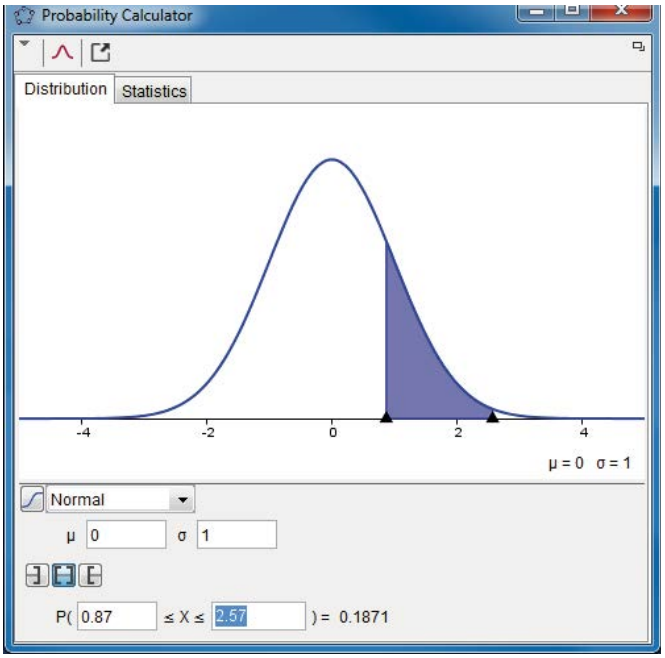
The result means that 10.5% of normally distributed data values will lie between 80 and 95.

If you want to find the probability of a value being above 50 in the same data set, you need to select the "[" option. Stating that 50 ≤ X is the same as saying X ≥ 50.



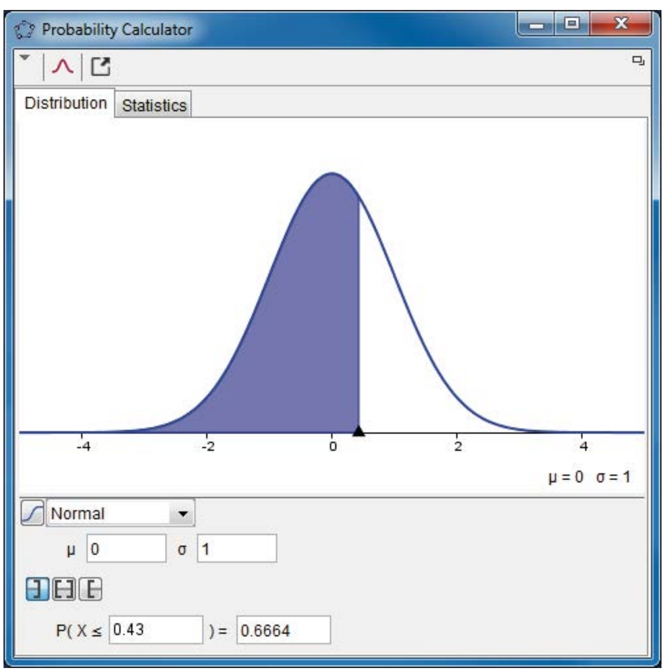
The image shows that 88.7% of the data values will be at or above 50.

If you want to work with z-scores, you can use the Probability Calculator. A z-score is a probability representing a normal distribution of data with a mean of 0 and a standard deviation of 1. To find the probability of data lying between a z-score of 0.87 and 2.57, you can choose the "[ ]" option and enter the following:



The image shows that 18.7% of the data values will be between z-scores of 0.87 and 2.57.

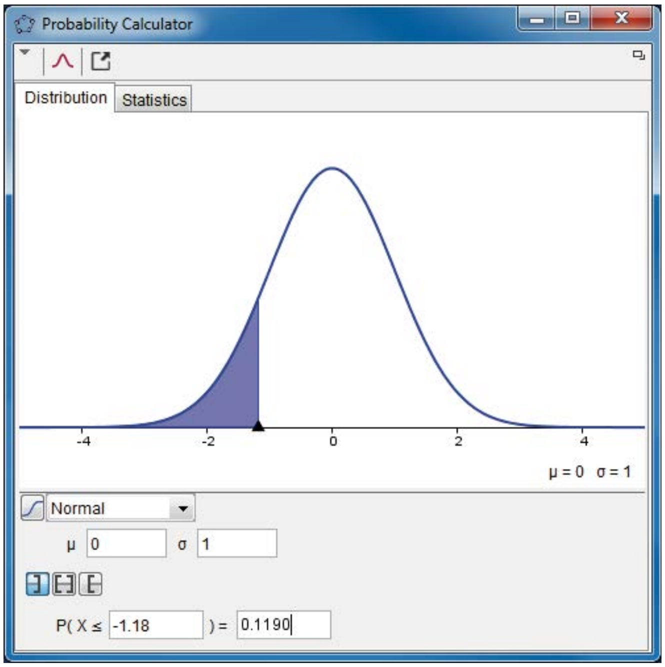
The probability of getting a z-score less than 0.43 in a standard normal distribution can be calculated and graphed. Select the "]" option. The following graph shows that the probability of 0.6664 will be displayed:



The image shows that 66.6% of the data values will be at or below a z-score of 0.43.

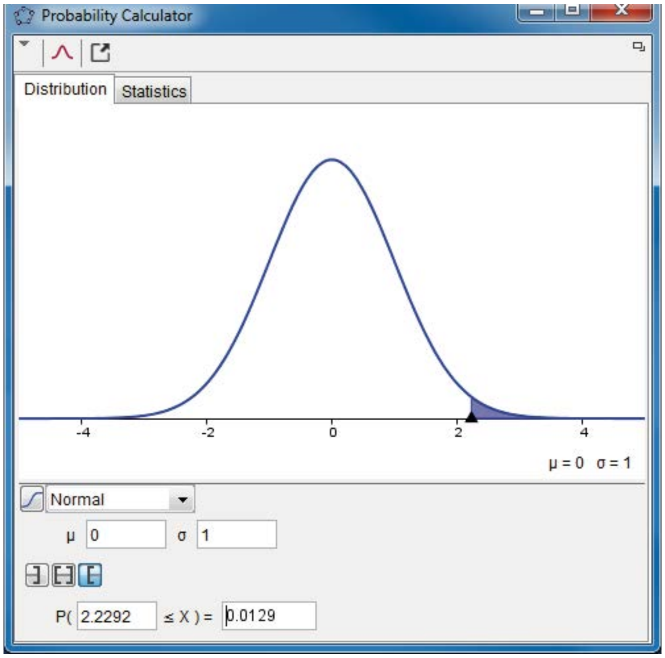
Sometimes you may have a need to do the inverse operation that does the process in reverse; that is, if you know the probability of an event, you can determine the corresponding *z*-score. This is done in the same way by entering the known values into the Probability Calculator.

For example, if the probability of getting less than a certain *z*-score is 11.90%, find the *z*-score. Enter the value of the probability as 0.1190 using the "]" option and press enter to find the *z*-score.



As shown, the probability of getting a *z*-score less than −1.18 is 0.1190.

As another example, if the probability of getting a *z*-score larger than a certain *z*-score is 0.0129, find the *z*-score by entering the probability using the "[" option.



As shown, the probability of getting a *z*-score greater than 2.229 is 0.0129.

Notes

GRADE 11 APPLIED MATHEMATICS

Module 1
Answer Key

Module 1, Lesson 1, Answer Key

Review of Simple Equation-Solving Techniques

Assignment

1\. a)

$$3x-4=12$$

$$3x-4+4=12+4 \qquad \text{Add 4 to both sides.}$$

$$3x=16$$

$$x=\frac{16}{3} \qquad \text{Divide both sides by 3.}$$

b)

$$\frac{2}{3}x+\frac{5}{8}=-6$$

$$\frac{2}{3}x+\frac{5}{8}-\frac{5}{8}=-6-\frac{5}{8} \qquad \text{Subtract } \frac{5}{8} \text{ from both sides.}$$

$$\frac{2}{3}x=\frac{-48}{8}-\frac{5}{8}$$

$$\frac{2}{3}x=\frac{-53}{8}$$

$$x=\frac{-53}{8}\cdot\frac{3}{2} \qquad \text{Divide on both sides by } \frac{2}{3} \text{ or multiply by } \frac{3}{2}.$$

$$x=\frac{-159}{16}$$

OR

$$\frac{2}{3}x+\frac{5}{8}=-6 \qquad \text{Multiply by 24.}$$

$$(24)\left(\frac{2x}{3}\right)+24\left(\frac{5}{8}\right)=24(-6)$$

$$16x+15=-144 \qquad \text{Subtract 15 from both sides.}$$

$$16x=-159 \qquad \text{Divide both sides by 16.}$$

$$x=\frac{-159}{16}$$

c) $4(3x-5)=7(x+1)$

$12x-20=7x+7$	Distribute multiplication over brackets.
$12x-7x=7+20$	Subtract $7x$ from both sides.
$5x=27$	
$x=\dfrac{27}{5}$	Divide both sides by 5.

d) $\dfrac{7x-3}{4}=\dfrac{5x+4}{9}$

$63x-27=20x+16$	Cross multiply.
$63x-20x=16+27$	Subtract $20x$ from both sides.
$43x=43$	
$x=\dfrac{43}{43}$	Divide both sides by 43.
$x=1$	

e) $\dfrac{5x-7}{6x+5}=\dfrac{-3}{4}$

$20x-28=-18x-15$	
$20x+18x=-15+28$	Cross multiply.
$38x=13$	Add $18x$ to both sides.
$x=\dfrac{13}{38}$	Divide both sides by 38.

f) $\frac{-7}{5}x+9=6$

$\frac{-7}{5}x+9-9=6-9$ Subtract 9 from both sides.

$\frac{-7}{5}x=-3$ Divide both sides by $\frac{-7}{5}$

$x=-3\cdot\frac{-5}{7}$ or multiply by $\frac{-5}{7}$.

$x=\frac{15}{7}$

g) $\frac{2(3x-4)}{7}=\frac{5(2x+3)}{6}$

$12(3x-4)=35(2x+3)$

$36x-48=70x+105$ Cross multiply.

$36x-70x=105+48$ Subtract $70x$ from both sides.

$-34x=153$

$x=\frac{-153}{34}$ Divide both sides by 34.

2. a) Quadratic
   b) Exponential
   c) Cubic
   d) Trigonometric
   e) Logarithmic
   f) Trigonometric

3. 2(a)—Quadratic, therefore, maximum number of solutions is two.

   2(c)—Cubic, therefore, maximum number of solutions is three.

Notes

Module 1, Lesson 2, Answer Key

Using a Graphing Calculator to Solve Equations

Assignment

1. $\frac{7}{3}x - \frac{5}{8} = 0$

   [Y=] [7] [÷] [3] [X,T,θ,n] [–] [5] [÷] [8] [GRAPH]

   [2nd] [CALC] [2]

   Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

   Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

   $x_1 = 0.27$ (to two decimal places)

2. $3(x + 3) = 5(–2x + 1)$
   $3(x + 3) – 5(–2x + 1) = 0$
   [Y=] [3] [(] [X,T,θ,n] [+] [3] [)] [–] [5] [(] [–2] [X,T,θ,n] [+] [1] [)] [GRAPH]

   [2nd] [CALC] [2]

   Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

   Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

   $x_1 = –0.31$ (to two decimal places)

3. $3^{2x-3} = 5$
$3^{2x-3} - 5 = 0$
[Y=] [3] [^] [(] [2] [X,T,θ,n] [–] [3] [)] [–] [5] [GRAPH]

[2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x_1 = 2.23$ (to two decimal places)

4. $2x^2 - x - 6 = 0$
[Y=] [2] [X,T,θ,n] [^] [2] [–] [X,T,θ,n] [–] [6] [GRAPH]

[2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x_1 = -1.50$ $x_2 = 2.00$ (to two decimal places)

5. $x^2 = -5x + 1$
$x^2 + 5x - 1 = 0$
[Y=] [X,T,θ,n] [^] [2] [+] [5] [X,T,θ,n] [–] [1] [GRAPH]

[2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x_1 = -5.19$ $x_2 = .19$ (to two decimal places)

6. $4\sin x + 3 = 5$ (between 0º and 180º)
   $4\sin x + 3 - 5 = 0$
   $4\sin x - 2 = 0$

   Be sure that the calculator is set in degree mode for angle measurement. Press [MODE] and highlight degree in the Radian Degree line.

   Press [ZOOM] [7] to set the window for Trigonometric equations.

   [Y=] [4] [sin] [X,T,θ,*n*] [)] [–] [2] [GRAPH]

   [2nd] [CALC] [2]

   Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set. (Note: You are only interested in zeroes between 0º and 180º.)

   Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

   $x_1 = 30^\circ$ $x_2 = 150^\circ$

7. $2\cos x = \frac{3}{5}$ (between 0° and 180°)

   $2\cos x - \frac{3}{5} = 0$

   Be sure that the calculator is set in degree mode for angle measurement. Press [MODE] and highlight degree in the Radian Degree line.

   Press [ZOOM] [7] to set the window for Trigonometric equations.

   [Y=] [2] [cos] [X,T,θ,*n*] [)] [–] [3] [÷] [5] [GRAPH]

   [2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set. (Note: You are only interested in zeroes between 0º and 180º.)

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x_1 = 72.5^\circ$

8. $3\log x - 1 = 2$
   $3\log x - 1 - 2 = 0$
   $3\log x - 3 = 0$
   [Y=] [3] [LOG] [X,T,θ,n] [)] [–] [3] [GRAPH]

   [2nd] [CALC] [2]

   Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

   Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

   $x_1 = 10$

9. $2^{3x} = 7^{x+2}$
   $2^{3x} - 7^{x+2} = 0$
   [Y=] [2] [^] [(] [3] [X,T,θ,n] [)] [–] [7] [^] [(] [X,T,θ,n] [+] [2] [)] [GRAPH]

   [2nd] [CALC] [2]

   Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

   Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

   $x_1 = 29.15$

10. $7\cos x - 5 = 0$ (0º to 360º)

Be sure that the calculator is set in degree mode for angle measurement. Press [MODE] and highlight degree in the Radian Degree line.

Press [ZOOM] [7] to set the window for Trigonometric equations.

[Y=] [7] [cos] [X,T,θ,n] [)] [–] [5] [GRAPH]

[2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set. (Note: You are only interested in zeros between 0º and 360º.)

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x_1 = 44.4^\circ$ $x_2 = 315.6^\circ$

Notes

Module 1, Lesson 3, Answer Key

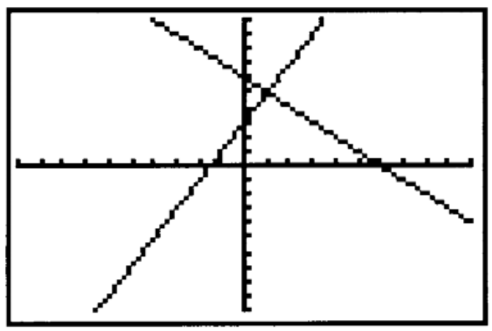
Solving Systems of Linear Equations Graphically

1. $y = 2x + 3$
   $x + y = 6$

   Solve the two equations for y so that they may be graphed.

   $y = 2x + 3$
   $y = -x + 6$

   Graph the two equations on the same grid.
   [Y=] [2] [X,T,θ,*n*] [+] [3] [ENTER] [(–)] [X,T,θ,*n*] [+] [6] [GRAPH]

   Find where the graphs meet by pressing [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. Press [ENTER] when prompted by the calculator for what the first and second equations are and when prompted for a guess. Since the only two curves you will have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

   $x = 1$ $y = 5$ Point of intersection is (1, 5)

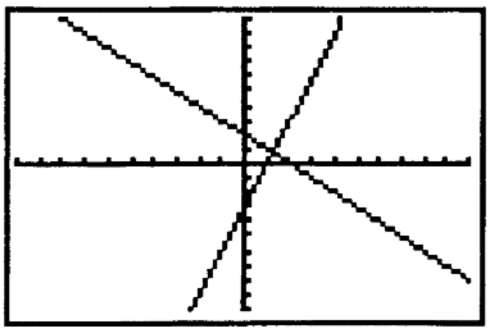
2. $x = 2 - y$
   $3x + 2 = y + 5$

   Solve the two equations for y so that they may be graphed.

   $y = -x + 2$
   $y = 3x - 3$

   Graph the two equations on the same grid.

   [Y=] [(–)] [X,T,θ,n] [+] [2] [ENTER] [3] [X,T,θ,n] [–] [3] [GRAPH]



   Find where the graphs meet by pressing [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER] (See Question 1)

   $x = 1.25$ $y = 0.75$ Point of intersection is (1.25, 0.75)

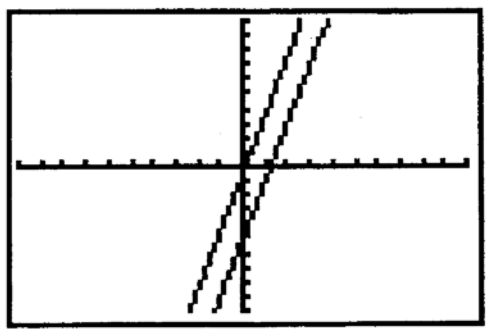
3. $4x - y = 5$
   $2y = 8x - 1$

   Solve the two equations for y so that they may be graphed.

   $y = 4x - 5$

   $y = 4x - \frac{1}{2}$

   Graph the two equations on the same grid.
   [Y=] [4] [X,T,θ,n] [–] [5] [ENTER] [4] [X,T,θ,n] [–] [1] [÷] [2] [GRAPH]



   Find where the graphs meet by pressing [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER] (See Question 1).

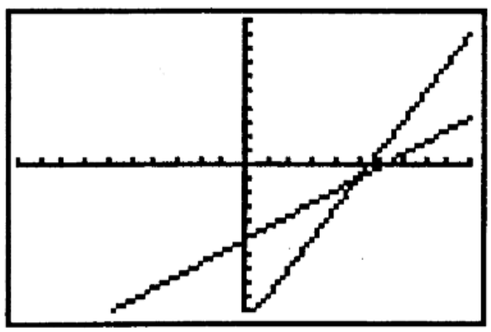
   The calculator will indicate ERROR.
   Since the lines run parallel, the system is inconsistent.

4. $5x - 6y = 31$
   $6x - 3y = 33$

   Solve the two equations for y so that they may be graphed.

   $$y = \frac{5}{6}x - \frac{31}{6}$$
   $$y = 2x - 11$$

   Graph the two equations on the same grid.
   [Y=] [5] [÷] [6] [X,T,θ,n] [–] [31] [÷] [6] [ENTER] [2] [X,T,θ,n] [–] [11] [GRAPH]



   Find where the graphs meet by pressing [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER] (See Question 1).

   Solution

   $x = 5$ $y = -1$ Point of intersection is $(5, -1)$

5. $y = 7x - 3$
$14x - 2y = 6$

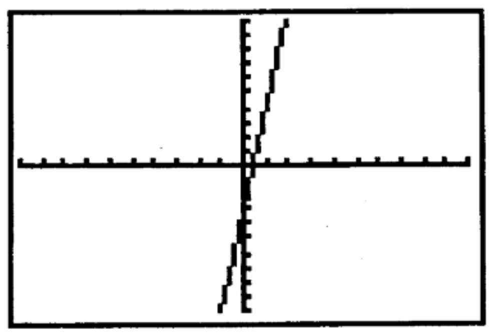
Solve the two equations for y so that they may be graphed.

$y = 7x - 3$
$y = 7x - 3$

These two lines are identical.

Graph the two equations on the same grid.

[Y=] [7] [X,T,θ,n] [–] [3] [ENTER] [7] [X,T,θ,n] [–] [3] [GRAPH]



Only one line appears.
Solution consists of all points on the line.

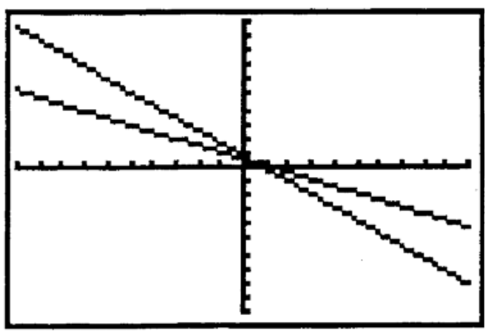
Since the graphs are the same line, the system is called dependent.

6. $8x + 9y = 7$
   $10x + 21y = 12$

   Solve the two equations for y so that they may be graphed.

   $$y = \frac{-8}{9}x + \frac{7}{9}$$
   $$y = -\frac{10}{21}x + \frac{12}{21}$$

   Graph the two equations on the same grid.
   [Y=] [(–)] [8] [÷] [9] [X,T,θ,*n*] [+] [7] [÷] [9] [ENTER] [(–)] [10] [÷] [21] [X,T,θ,*n*] [+] [12] [÷] [21] [GRAPH]



   Find where the graphs meet by pressing [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER] (See Question 1).

   Solution

   $x = 0.5$ $y = 0.33$ The point of intersection is (0.5, 0.33).

Module 1, Lesson 4, Answer Key

Solving Systems of Linear Equations Algebraically

Assignment

Your solutions may vary from the ones supplied here, but the final answers are correct regardless of the method used.

1. Addition/Subtraction Method

a) $5x + 2y = 11$
$3x + 4y = 1$

$$\begin{array}{r} 10x + 4y = 22 \\ \underline{3x + 4y = 1} \\ 7x = 21 \end{array}$$

Multiply first equation by 2.

Subtract the equations.

$$\frac{7x}{7} = \frac{21}{7}$$

Divide both sides by 7.

$\boldsymbol{x = 3}$

$$5(3) + 2y = 11$$
$$15 + 2y = 11$$

Substitute $x = 3$ into first equation.

$$15 - 15 + 2y = 11 - 15$$
$$2y = -4$$

Subtract 15 from both sides.

$$\frac{2y}{2} = \frac{-4}{2}$$

Divide both sides by 2.

$\boldsymbol{y = -2}$ **Answer: (3, –2)**

b) $2x + y = 3$
$3x - y = 2$

$$\begin{array}{r} 2x + y = 3 \\ \underline{3x - y = 2} \\ 5x = 5 \end{array}$$ Add the equations.

$$\frac{5x}{5} = \frac{5}{5}$$ Divide both sides by 5.

$\boldsymbol{x = 1}$

$2(1) + y = 3$ Substitute $x = 1$ into first equation.

$2 + y = 3$
$2 + y - 2 = 3 - 2$ Subtract 2 from both sides.

$\boldsymbol{y = 1}$ **Answer: (1, 1)**

c) 3x + 2y = 11
4x – 3y = 9

$$\begin{aligned} 12x + 8y &= 44 \\ 12x - 9y &= 27 \\ \hline 17y &= 17 \end{aligned}$$

Multiply first equation by 4.
Multiply second equation by 3.
Subtract the equations.

$$\frac{17y}{17} = \frac{17}{17}$$

Divide both sides by 17.

$\boldsymbol{y = 1}$

$$3x + 2(1) = 11$$
$$3x + 2 = 11$$

Substitute $y = 1$ into first equation.

$$3x + 2 - 2 = 11 - 2$$
$$3x = 9$$

Subtract 2 from both sides.

$$\frac{3x}{3} = \frac{9}{3}$$

Divide both sides by 3.

$\boldsymbol{x = 3}$

Answer: (3, 1)

d) $7x + 3y = 27$
$2x + 5y = 16$

$$\begin{array}{r} 14x + 6y = 54 \\ 14x + 35y = 112 \\ \hline -29y = -58 \end{array}$$

Multiply first equation by 2.
Multiply second equation by 7.
Subtract the equations.

$$\frac{-29y}{-29} = \frac{-58}{-29}$$

Divide both sides by –29.

***y* = 2**

$$7x + 3(2) = 27$$
$$7x + 6 = 27$$

Substitute $y = 2$ into first equation.

$$7x + 6 - 6 = 27 - 6$$
$$7x = 21$$

Subtract 6 from both sides.

$$\frac{7x}{7} = \frac{21}{7}$$

Divide both sides by 7.

***x* = 3**

Answer: (3, 2)

2. Substitution Method

a)
$$5x - y = 8$$
$$3x + 2y = 10$$

$$5x - 5x - y = 8 - 5x$$
$$-y = 8 - 5x$$
Subtract $5x$ from both sides of first equation.

$$y = 5x - 8$$
Divide by –1.

$$3x + 2(5x - 8) = 10$$
$$3x + 10x - 16 = 10$$
Substitute $5x - 8$ for y in other equation.

$$13x - 16 + 16 = 10 + 16$$
$$13x = 26$$
Add 16 to both sides.

$$\frac{13x}{13} = \frac{26}{13}$$
Divide by 13.

$\boldsymbol{x = 2}$

$$5(2) - y = 8$$
Substitute $x = 2$ into first equation.

$$10 - 10 - y = 8 - 10$$
$$-y = -2$$
Subtract 10 from both sides.

$$\frac{-y}{-1} = \frac{-2}{-1}$$
Divide both sides by –1.

$\boldsymbol{y = 2}$ **Answer: (2, 2)**

b) $2x - 3y = -1$

$5x - 2y = 14$

$2x - 2x - 3y = -1 - 2x$ $-3y = -1 - 2x$	Subtract 2*x* from both sides of first equation.
$y = \dfrac{2x+1}{3}$	Divide by –3.
$5x - 2\left(\dfrac{2x+1}{3}\right) = 14$	Substitute $\left(\dfrac{2x+1}{3}\right)$ for *y* in other equation.
$15x - 4x - 2 = 42$	Multiply all terms by 3 and use distributive law.
$11x - 2 + 2 = 42 + 2$ $11x = 44$	Add 2 to both sides.
$\dfrac{11x}{11} = \dfrac{44}{11}$	Divide by 11.
$\boldsymbol{x = 4}$	
$2(4) - 3y = -1$	Substitute $x = 4$ into first equation.
$8 - 8 - 3y = -1 - 8$ $-3y = -9$	Subtract 8 from both sides.
$\dfrac{-3y}{-3} = \dfrac{-9}{-3}$	Divide both sides by –3.
$\boldsymbol{y = 3}$	**Answer: (4, 3)**

c) $2x + y = 8$

$x + y = 5$

$2x - 2x + y = 8 - 2x$ Subtract $2x$ from both sides of first equation.

$y = 8 - 2x$

$x + 8 - 2x = 5$ Substitute $8 - 2x$ for y in other equation.

$x - 2x + 8 = 5$

$-x + 8 - 8 = 5 - 8$ Subtract 8 from both sides.

$-x = -3$

$\frac{-x}{-1} = \frac{-3}{-1}$ Divide by -1.

$\boldsymbol{x = 3}$

$2(3) + y = 8$ Substitute $x = 3$ into first equation.

$6 - 6 + y = 8 - 6$ Subtract 6 from both sides.

$\boldsymbol{y = 2}$ **Answer: (3, 2)**

d) $3x+5y=37$
$2x-3y=12$

$3x-3x+5y=37-3x$ Subtract $3x$ from both sides of
$5y=37-3x$ first equation.

$y=\dfrac{37-3x}{5}$ Divide by 5.

$2x-3\left(\dfrac{37-3x}{5}\right)=12$ Substitute $\left(\dfrac{37-3x}{5}\right)$ for y in other equation.

$10x-111+9x=60$ Multiply all terms by 5 and use distributive law.

$19x-111+111=60+111$ Add 111 to both sides.
$19x=171$

$\dfrac{19x}{19}=\dfrac{171}{19}$ Divide by 19.

$\boldsymbol{x=9}$

$3(9)+5y=37$ Substitute $x = 9$ into first equation.
$27-27+5y=37-27$ Subtract 27 from both sides.
$5y=10$

$\dfrac{5y}{5}=\dfrac{10}{5}$ Divide both sides by 5.

$\boldsymbol{y=2}$ **Answer: (9, 2)**

Module 1, Lesson 5, Answer Key

Applications of Systems of Linear Equations

Assignment

1. Let x = smaller number
   Let y = larger number

   $x + y = 2x + 7$ Sum of the numbers is seven more than twice the smaller number.

   $\frac{1}{2}y = x - 3$ One half the larger number is three less than the smaller number.

   Solve the two equations for y so that they may be graphed.

   $y = x + 7$
   $y = 2x - 6$

   Graph the two equations on the same grid.
   [Y=] [X,T,θ,n] [+] [7] [ENTER] [2] [X,T,θ,n] [–] [6] [GRAPH]

   Zoom out to see point of intersection. Find where the graphs meet by pressing [2nd] [CALC] [5]. [ENTER] [ENTER] [ENTER].

   Solution

   $x = 13$ $y = 20$

   The two numbers are 13 and 20.

2. Let x = smaller number
   Let y = larger number

   $x + \frac{1}{2}y = 11$ Sum of the smaller number and one half the larger is 11.

   $\frac{1}{4}y - x = -1$ One quarter the larger number minus the smaller results in –1.

   Solve the two equations for y so that they may be graphed.

   $y = -2x + 22$
   $y = 4x - 4$

Graph the two equations on the same grid.
[Y=] [(–)] [2] [X,T,θ,n] [+] [22] [ENTER] [4] [X,T,θ,n] [–] [4] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5]. [ENTER] [ENTER] [ENTER].

Solution

$x = 4.33$ $y = 13.33$

The two numbers are 4.33 and 13.33.

3. Let Jane be x years old
   Let her father be y years old

$y = 4x$	Father is four times as old as Jane.
$y - 5 = 9(x - 5)$	Five years ago Father was nine times as old as Jane.

   Solve the two equations for y so that they may be graphed.

   $y = 4x$
   $y = 9x - 40$

   Graph the two equations on the same grid.
   [Y=] [4] [X,T,θ,n] [ENTER] [9] [X,T,θ,n] [–] [40] [GRAPH]

   Find where the graphs meet by pressing [2nd] [CALC] [5]. [ENTER] [ENTER] [ENTER].

   Solution

   $x = 8$ $y = 32$

   Jane is eight years old and her father is 32 years of age.

4. Let x = the number of nickels
   Let y = the number of quarters

$x + y = 31$	Total number of coins is 31.
$5x + 25y = 375$	Number of cents in 'x' nickels plus number of cents in 'y' quarters is 375.

Solve the two equations for y so that they may be graphed.

$$y = x + 31$$

$$y = -\frac{1}{5}x + 15$$

Graph the two equations on the same grid.
[Y=] [(–)] [X,T,θ,n] [+] [31] [ENTER] [(–)] [1] [÷] [5][X,T,θ,n] [+] [15] [GRAPH]

Zoom out to see point of intersection. Find where the graphs meet by pressing [2nd] [CALC] [5]. [ENTER] [ENTER] [ENTER].

Solution

$x = 20$ $y = 11$

Pete will have 20 nickels.

5. Let $\$x$ = amount invested at $3\frac{1}{4}\%$

Let $\$y$ = amount invested at $4\frac{1}{2}\%$

$x + y = 12\ 000$	Total investment is \$12 000.
$0.0325x + 0.045y = 490$	Total interest earned is \$490.
	Note: $3\frac{1}{4}\% = 0.0325$

Solve the two equations for y so that they may be graphed.

$$y = x + 12\ 000$$

$$y = -\frac{0.0325}{0.045}x + \frac{490}{0.045}$$

Graph the two equations on the same grid.
[Y=] [(–)] [X,T,θ,n] [+] [12000] [ENTER] [(–)] [0.0325] [÷] [0.045] [X,T,θ,n] [+] [490] [÷] [0.045] [GRAPH]

To see the point of intersection, change the WINDOW to:

Xmin = 0
Xmax = 12000
Xscl = 1000
Ymin = 0
Ymax = 12000
Yscl = 1000

Find where the graphs meet by pressing [2nd] [CALC] [5].
[ENTER] [ENTER] [ENTER].

Solution

$x = 4000$ $y = 8000$

He will invest \$4000 at $3\frac{1}{4}$% and \$8000 at $4\frac{1}{2}$%.

6. Let $\$x$ = amount invested at 9%
   Let $\$y$ = amount invested at 6%

$x + y = 1800$	Total investment is \$1800.
$0.09x = 0.06y + 102$	Interest earned on 9% investment is \$102 more than interest earned on the 6% investment.

Solve the two equations for y so that they may be graphed.

$$y = -x + 1800$$

$$y = \frac{0.09}{0.06}x - \frac{102}{0.06}$$

Graph the two equations on the same grid.
[Y=] [(–)] [X,T,θ,n] [+] [1800] [ENTER] [0.09] [÷] [0.06] [X,T,θ,n] [–] [102] [÷] [0.06] [GRAPH]

To see the point of intersection, change the WINDOW to:

Xmin = 0
Xmax = 1800
Xscl = 200
Ymin = 0
Ymax = 1800
Yscl = 200

Find where the graphs meet by pressing [2nd] [CALC] [5].
[ENTER] [ENTER] [ENTER].

Solution

$x = 1400$ $y = 400$

She invested \$1400 at 9% and \$400 at 6%.

7. Let x = number of litres of 23-cent gas
   Let y = number of litres of 25-cent gas

$x + y = 10\ 000$	Tank holds 10 000 L.
$0.23x + 0.25y = 0.242(10\ 000)$	Total cost of 10 000 L.

   Solve the two equations for y so that they may be graphed.

   $$y = -x + 10\ 000$$

   $$y = -\frac{0.23}{0.25}x + \frac{0.242(10\ 000)}{0.25}$$

   Graph the two equations on the same grid.
   [Y=] [(–)] [X,T,θ,n] [+] [10000] [ENTER] [(–)] [.23] [÷] [.25] [X,T,θ,n] [+] [.242] [x] [10000] [÷] [.25] [GRAPH]

   To see the point of intersection, change the WINDOW to:

   Xmin = 0
   Xmax = 10000
   Xscl = 1000
   Ymin = 0
   Ymax = 10000
   Yscl = 1000

   Find where the graphs meet by pressing [2nd] [CALC] [5]. [ENTER] [ENTER] [ENTER].

   Solution

   $x = 4000 \quad y = 6000$

   They will mix 4000 L of 23-cent gas with 6000 L of 25-cent gas.

8. Let x = number of kg of 90% iron ore
   Let y = number of kg of 60% iron ore

$x + y = 150$	Order is for 150 kg.
$0.90x + 0.60y = 0.80(150)$	Total 80% alloy.

   Solve the two equations for y so that they may be graphed.

   $$y = -x + 150$$

   $$y = -\frac{0.90}{0.60}x + \frac{0.80(150)}{0.60}$$

Graph the two equations on the same grid.
[Y=] [(–)] [X,T,θ,n] [+] [150] [ENTER] [(–)] [0.90] [÷] [0.60]
[X,T,θ,n] [+] [0.80] [x] [150] [÷] [0.60] [GRAPH]

To see the point of intersection, change the WINDOW to:

Xmin = 0
Xmax = 150
Xscl = 10
Ymin = 0
Ymax = 150
Yscl = 10

Find where the graphs meet by pressing [2nd] [CALC] [5].
[ENTER] [ENTER] [ENTER].

Solution

$x = 100 \quad y = 50$

They will mix 100 kg of 90% iron ore with 50 kg of 60% iron ore.

9. Let x mph = speed of the plane in still air
Let y mph = speed of the wind

$x + y = 300$ With the wind, it goes 2200 mi in $\frac{22}{3}$ h.

$$\text{Speed} = 2200 \div 7\frac{1}{3} = 2200 \div \frac{22}{3}$$
$$= 2200 \times \frac{3}{22} = 300 \text{ mph}$$

$x - y = 220$ Against the wind, it goes 2200 mi in 10 h.

$$\text{Speed} = \frac{2200}{10} = 220 \text{ mph}$$

Solve the two equations for y so that they may be graphed.

$y = -x + 300$
$y = x - 220$

Graph the two equations on the same grid.
[Y=] [(–)] [X,T,θ,n] [+] [300] [ENTER] [X,T,θ,n] [–] [220] [GRAPH]

Adjust your WINDOW to see the point of intersection.

Find where the graphs meet by pressing [2nd] [CALC] [5].
[ENTER] [ENTER] [ENTER].

Solution

$x = 260$ $y = 40$

The plane travelled 260 mph in still air and the speed of the wind was 40 mph.

10. Let the length of one of the congruent sides be x inches.
Let the length of the third side (base) be y inches.

$2x + y = 9$	Perimeter of the triangle is nine inches.
$x = 13y$	The length of one of the congruent sides is 13 times the length of the third side.

Solve the two equations for y so that they may be graphed.

$$y = -2x + 9$$
$$y = \frac{1}{13}x$$

Graph the two equations on the same grid.
[Y=] [(–)] [2] [X,T,θ,n] [+] [9] [ENTER] [1] [÷] [13] [X,T,θ,n] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5]. [ENTER] [ENTER] [ENTER].

Solution

$x = 4.33$ $y = 0.33$

The sides will be 4.33 inches, 4.33 inches, 0.33 inches.

Notes

Module 1, Lesson 6, Answer Key

Solving Systems Involving Nonlinear Equations and Their Applications

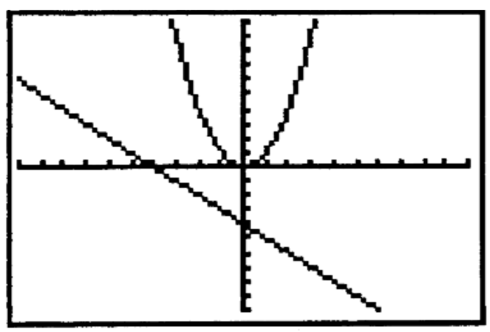
Assignment

1. $y = x^2$
   $x + y = -4$

   Solve the two equations for y so that they may be graphed.

   $y = x^2$
   $y = -x - 4$

   Graph the two equations on the same grid.
   [Y=] [X,T,θ,*n*] [^] [2] [ENTER] [(–)] [X,T,θ,*n*] [–] [4] [GRAPH]

   Find where the graphs meet by pressing [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER].

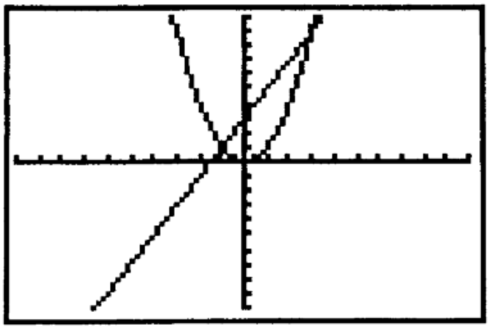
   This gives an ERROR message because the two graphs do not meet and the solution is the empty set.

2. $y = x^2$
   $y - 2x = 3$

   Solve the two equations for y so that they may be graphed.

   $y = x^2$
   $y = 2x + 3$

Graph the two equations on the same grid.
[Y=] [X,T,θ,*n*] [^] [2] [ENTER] [2] [X,T,θ,*n*] [+] [3] [GRAPH]



Use the cursor button [◀] and [▶] to place cursor close to one of the intersection points. Press [ENTER] [ENTER] [ENTER]. Repeat to find second point of intersection.

Solutions

$x_1 = -1$ $\quad$ $y_1 = 1$
$x_2 = 3$ $\quad$ $y_2 = 9$ $\quad$ or $\quad$ (–1, 1) and (3, 9)

3. $x^2 + 8xy = 4$
$x = y$

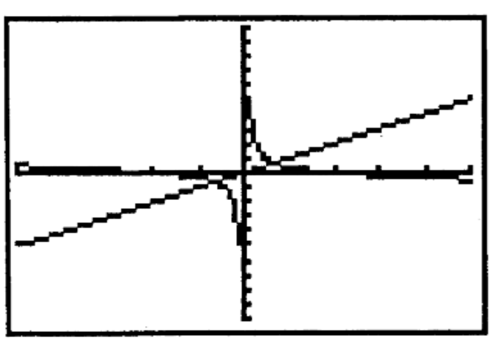
Solve the two equations for y so that they may be graphed.

$$y = \frac{4 - x^2}{8x}$$

$$y = x$$

Use a window with Xmin = –5, Xmax = 5.

Graph the two equations on the same grid.
[Y=] [(] [4] [–] [X,T,θ,*n*] [^] [2] [)] [÷] [(] [8] [X,T,θ,*n*] [)] [ENTER]
[X,T,θ,*n*] [GRAPH]



Find where the graphs meet by pressing TRACE, and moving the cursor closer to a point of intersection. Press [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. Repeat for other point of intersection.

Solutions

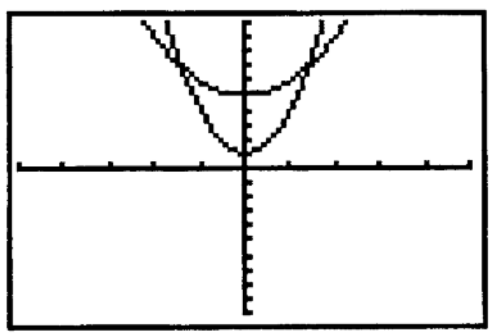
$x_1 = -0.67$	$y_1 = -0.67$	or	(–0.67, –0.67)
$x_2 = 0.67$	$y_2 = 0.67$	and	(0.67, 0.67)

4. $y = 3x^2 + 1$
   $y = x^2 + 5$

These are already in the correct format for entering in the graphing calculator.

Use a window with Xmin = –5, Xmax = 5.

Graph the two equations on the same grid.
[Y=] [3] [X,T,θ,*n*] [^] [2] [+] [1] [ENTER] [X,T,θ,*n*] [^] [2] [+] [5] [GRAPH]



Find where the graphs meet by pressing TRACE, and moving the cursor closer to a point of intersection. Press [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. Repeat for the other point of intersection.

Solution

$x_1 = 1.41$	$y_1 = 7$	or	(1.41, 7)
$x_2 = -1.41$	$y_2 = 7$	and	(–1.41, 7)

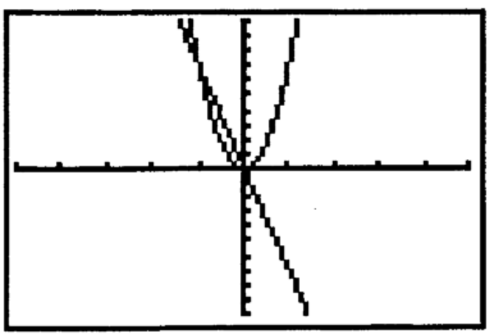
5. $y + 7x = 0$
   $7x^2 = y$

   Solve the two equations for y so that they may be graphed.

   $y = -7x$
   $y = 7x^2$

   Use a window with Xmin = –5, Xmax = 5.

   Graph the two equations on the same grid.
   [Y=] [(–)] [7] [X,T,θ,n] [ENTER] [7] [X,T,θ,n] [^] [2] [GRAPH]



   Find where the graphs meet by pressing TRACE, and moving the cursor closer to a point of intersection. Press [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. Repeat for the other point of intersection.

   Solution

   $x_1 = 0$ $y_1 = 0$ or (0, 0) and (–1, 7)
   $x_2 = -1$ $y_2 = 7$

6. Let x = first number
   Let y = second number

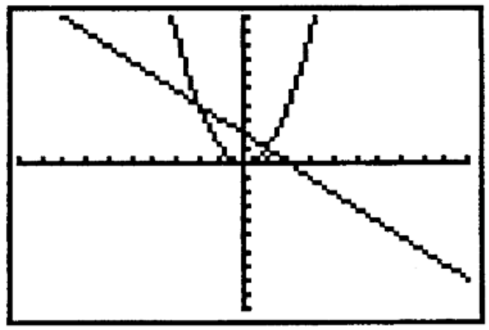
   $y = x^2$ Square of one number is equal to the other.

   $x + y = 2$ Sum of the numbers is 2.

   Solve the two equations for y so that they may be graphed.

   $y = x^2$
   $y = -x + 2$

Graph the two equations on the same grid.
[Y=] [X,T,θ,n] [^] [2] [ENTER] [–] [X,T,θ,n] [+] [2] [GRAPH]



Find where the graphs meet by pressing TRACE, and moving the cursor closer to a point of intersection. Press [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. Repeat for other point of intersection.

Solutions

$x_1 = 1$ $y_1 = 1$
$x_2 = -2$ $y_2 = 4$

The two numbers are 1 and 1, or –2 and 4.

7. Let x = first number
Let y = second number

$x + y = 1\frac{7}{15}$ Sum of the numbers is $1\frac{7}{15}$.

$\frac{1}{x} \cdot \frac{1}{y} = 1\frac{7}{8}$ Product of their reciprocals is $1\frac{7}{8}$.

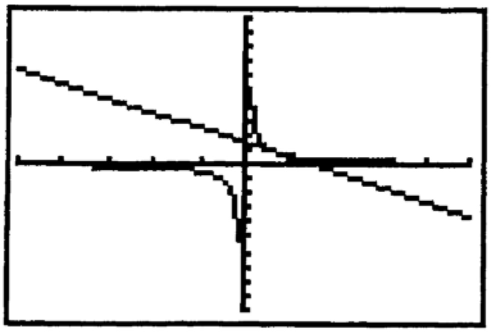
Solve the two equations for y so that they may be graphed.

$y = -x + \frac{22}{15}$

$y = \frac{8}{15x}$ Note: $\frac{1}{xy} = \frac{15}{8}$, $15xy = 8$, $y = \frac{8}{15x}$

Use a window with Xmin = –5, Xmax = 5.

Graph the two equations on the same grid.
Press [Y=] [(–)][X,T,θ,n] [+] [22] [÷] [15] [ENTER] [8] [÷] [(] [15] [X,T,θ,n] [)] [GRAPH.]



Find where the graphs meet by pressing TRACE, and moving the cursor closer to a point of intersection. Press [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. Repeat for other points of intersection.

Solution

$x_1 = 0.67$ $y_1 = 0.80$
$x_2 = 0.80$ $y_2 = 0.67$

The two numbers are 0.67 and 0.80.

8. Let x = first number
Let y = second number

$x \cdot y = 8$ Product of two numbers is 8.

$\frac{1}{x} + \frac{1}{y} = \frac{3}{4}$ Sum of their reciprocals is $\frac{3}{4}$.

Solve the two equations for y so that they may be graphed.

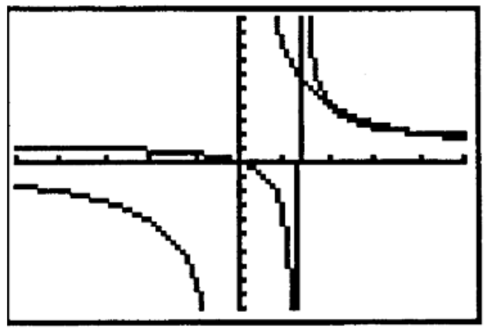
$y = \frac{8}{x}$

$y = \frac{4x}{3x-4}$

Note:
$$\frac{y+x}{xy} = \frac{3}{4}$$
$$4y + 4x = 3xy$$
$$4y - 3xy = -4x$$
$$y(4-3x) = -4x$$
$$y = \frac{-4x}{4-3x} = \frac{4x}{3x-4}$$

Use a window with Xmin = –5, Xmax = 5.

Graph the two equations on the same grid.
[Y=] [8] [÷] [X,T,θ,*n*] [ENTER] [4] [X,T,θ,*n*] [÷] [(] [3] [X,T,θ,*n*] [–] [4] [)] [GRAPH]



Find where the graphs meet by pressing TRACE, and moving the cursor closer to a point of intersection. Press [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. Repeat for other points of intersection.

Solution

$x_1 = 2 \quad y_1 = 4$

The two numbers are 2 and 4.

Notes

Self-Test Answer Key

Module 1 – Graphing and Systems of Equations

/64

In some solutions you may find that your answers differ by a small amount. This may be caused by rounding too soon or, sometimes, by using 3.14 instead of π in formulas.

(10 marks)

1. a) $\frac{4}{3}(2x-5)=6(x+1)$

   Rearrange to give $\frac{4}{3}(2x-5)-6(x+1)=0$

   Graph $\frac{4}{3}(2x-5)-6(x+1)$.

   Keying sequence:
   [Y=] [4] [÷] [3] [(] [2] [X,T,θ,*n*] [–] [5] [)] [–] [6] [(] [X,T,θ,*n*] [+] [1] [)] [GRAPH]

   To find the zero of the function, use the keying sequence:
   [2nd] [CALC] [2]

   Then set left and right bounds by moving the cursor to the left and right of the point(s) where the graph crosses the x-axis, and pressing [ENTER] after each is set. Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

   The calculator indicates
   $x = –3.8$ $y = 0$
   Solution of the original equation is $x = –3.8$

   b) $3x^2 = 8x – 5$

   Rearrange to give $3x^2 – 8x + 5 = 0$

   Graph $y = 3x^2 – 8x + 5$

   Keying sequence:
   [Y=] [3] [X,T,θ,*n*] [^] [2] [–] [8] [X,T,θ,*n*] [+] [5] [GRAPH]

   Find the zeros by using the keying sequence:
   [2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the points where the graph crosses the x-axis, and pressing [ENTER] after each is set. Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x_1 = 1 \quad x_2 = 1.7$

c) $7\sin x + 2 = 5 \quad (0^\circ - 180^\circ)$

Rearrange to give $7\sin x + 2 - 5 = 0$

Graph $y = 7\sin x - 3$

Be sure that the calculator is set in degree mode for angle measurement. Press [MODE] and highlight degree in the Radian Degree line. Press [ZOOM] [7] to set the window for Trigonometric equations.

Keying sequence:
[Y=] [7] [sin] [X,T,θ,n] [)] [–] [3] [GRAPH]

To find zeros, use the keying sequence:
[2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph crosses the x-axis, and pressing [ENTER] after each is set. (Note: You are only interested in zeroes between 0º and 180º.)

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x_1 = 25.4^\circ \quad x_2 = 154.6^\circ$

d) $8^{2x+1} = 9$

Rearrange to give $8^{2x+1} - 9 = 0$

Graph $y = 8^{2x+1} - 9$

Keying sequence:
[Y=] [8] [^] [(] [2] [X,T,θ,n] [+] [1] [)] [–] [9] [GRAPH]

To find zeros, use the keying sequence:
[2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x = 0.03$

e) $7\log x - 5 = 0$

Graph $y = 7\log x - 5$

Keying sequence:
[Y=] [7] [LOG] [X,T,θ,n] [)] [–] [5] [GRAPH]

Find zero function using keying sequence:
[2nd] [CALC] [2]

Set left and right bounds by moving the cursor to the left and right of the point(s) where the graph hits the x-axis, and pressing [ENTER] after each is set.

Press [ENTER] when the calculator prompts you for a guess, since the calculator will provide a guess for you that will be accurate enough to complete the task here.

$x = 5.2$

(8 marks) 2. a) $2x + 3y = 6$
$3x - 2y = 7$

$6x + 9y = 18$	Multiply first equation by 3.
$6x - 4y = 14$	Multiply second equation by 2.
$13y = 4$	Subtract the equations.
$\frac{13y}{13} = \frac{4}{13}$	Divide both sides by 13.

$$\boldsymbol{y = \frac{4}{13}}$$

$2x+3\left(\frac{4}{13}\right)=6$	Substitute $x = \frac{4}{13}$ into first equation.
$2x+\frac{12}{13}=6$	
$2x+\frac{12}{13}-\frac{12}{13}=6-\frac{12}{13}$	Subtract $\frac{12}{13}$ from both sides.
$2x=\frac{66}{13}$	
$x=\frac{66}{13}\div 2$	Divide both sides by 2.
$\boldsymbol{x=\frac{33}{13}}$	

Solution: $\boldsymbol{\frac{33}{13}, \frac{4}{13}}$

b)
$$3x = 2y + 7$$
$$5y = 2x - 12$$

$$3x - 2y = 7$$
$$-2x + 5y = -12$$

$6x - 4y = 14$	Multiply first equation by 2.
$-6x + 15y = -36$	Multiply second equation by 3.
$11y = -22$	Add the equations.
$\frac{11y}{11} = \frac{-22}{11}$	Divide both sides by 11.
$\boldsymbol{y = -2}$	
$3x - 2(-2) = 7$	Substitute $y = -2$ into first equation.
$3x + 4 = 7$	
$3x + 4 - 4 = 7 - 4$	Subtract 4 from each side.
$3x = 3$	
$\frac{3x}{3} = \frac{3}{3}$	Divide both sides by 3.
$\boldsymbol{x = 1}$	

Solution: (1, –2)

(8 marks)

3. a)

$$3x-2y=6$$
$$x+2y=-2$$

$$x+2y-2y=-2-2y$$
$$x=2-2y$$

Subtract $2y$ from both sides of second equation.

$$3(-2-2y)-2y=6$$
$$-6-6y-2y=6$$

Substitute $-2-2y$ for x in other equation.

$$-8y-6+6=6+6$$
$$-8y=12$$

Add 6 to both sides.

$$\frac{-8y}{-8}=\frac{12}{8}$$

Divide by –8.

$$\boldsymbol{y=\frac{-3}{2}}$$

$$x+2\left(\frac{-3}{2}\right)=-2$$

Substitute $y=\frac{-3}{2}$ into second equation.

$$x-3+3=-2+3$$

Add 3 to both sides

$$\boldsymbol{x=1}$$

Solution: $1, \frac{3}{2}$

b)

$$7x+3y=27$$
$$2x+5y=16$$

$$7x-7x+3y=27-7x$$
$$3y=27-7x$$

Subtract $7x$ from both sides of first equation.

$$y=\frac{27-7x}{3}$$

Divide by 3.

$$2x+5\left(\frac{27-7x}{3}\right)=16$$

Substitute $\left(\frac{27-7x}{3}\right)$ for y in other equation.

$6x+135-35x=48$ Multiply all terms by 3.

$-29x+135-135=48-135$ Subtract 135 from both sides.

$-29x=-87$

$\frac{-29x}{-29}=\frac{-87}{-29}$ Divide by –29.

$\boldsymbol{x = 3}$

$7(3)+3y=27$ Substitute $x = 3$ into first equation.

$21-21+3y=27-21$ Subtract 21 from both sides.

$3y=6$

$\frac{3y}{3}=\frac{6}{3}$ Divide both sides by 3.

$\boldsymbol{y = 2}$

4. a) $-2x + 3y = 6$ *(8 marks)*

$6x + 5y = 30$

Solve the two equations for y so that they may be graphed.

$y=\frac{2}{3}x+2$

$y=\frac{-6}{5}x+6$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [2] [÷] [3] [X,T,θ,n] [+] [2] [ENTER] [(–)] [6] [÷] [5]
[X,T,θ,n] [+] [6] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 2.14 \quad y = 3.43$

b) $\frac{x}{5} + \frac{y}{6} = 3$

$\frac{x}{10} - \frac{y}{3} = -1$

Solve the two equations for y so that they may be graphed.

$y = \frac{-6}{5}x + 18$

$y = \frac{3}{10}x + 3$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [6] [÷] [5] [X,T,θ,n] [+] [18] [ENTER] [3] [÷] [10] [X,T,θ,n] [+] [3] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 10 \quad y = 6$

c) $x^2 - 5x - y + 4 = 0$
$x - 4y = 1$

Solve the two equations for y so that they may be graphed.

$y = x^2 - 5x + 4$

$y = \frac{1}{4}x - \frac{1}{4}$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [X,T,θ,n] [^] [2] [–] [5] [X,T,θ,n] [+] [4] [ENTER] [1] [÷] [4] [X,T,θ,n] [–] [1] [÷] [4] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for what the first and second curves are and when prompted for a guess. Since the only two curves you will have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

Repeat to find second point of intersection. You may need to use the [◀] or [▶] key to set close to intersection point.

$x_1 = 1$ $y_1 = 0$
$x_2 = 4.25$ $y_2 = 0.8125$

d) $3xy + 6y = 4$
$2y - 3x - 4 = 0$

Solve the two equations for y so that they may be graphed.

$$y = \frac{4}{3x+6}$$
$$y = \frac{3x+4}{2}$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [4] [÷] [(] [3] [X,T,θ,n] [+] [6] [)] [ENTER] [(] [3] [X,T,θ,n] [+] [4] [)] [÷] [2] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for what the first and second curves are and when prompted for a guess. Since the only two curves you will have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x_1 = –0.7$ $y_1 = 1$
$x_2 = –2.7$ $y_2 = –2$

(5 marks)

5. a) Let $\$x$ = be amount of money invested at 10%
Let $\$y$ = be amount of money invested at 12%

$x + y = 660$	Total investment is \$660.
$0.10x = 0.12y$	Interest earned on 10% investment is the same as the 12% investment.

Solve the two equations for y so that they may be graphed.

$$y = -x + 660$$

$$y = \frac{0.1}{0.12}x$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [X,T,θ,n] [+] [660] [ENTER] [0.1] [÷] [0.12] [X,T,θ,n] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 360 \quad y = 300$

She will invest \$360 at 10% and \$300 at 12%.

(5 marks)

b) Let x = the number of dozen at 54 cents
Let y = the number of dozen at 60 cents

$54x + 60y = 2010$	Paid \$20.10 for all the oranges.
$75(x + y) = 2625$	Sold all the oranges for 75 cents a dozen. Total received = \$20.10 + \$6.15

Solve the two equations for y so that they may be graphed.

$$y = \frac{-54}{60}x + \frac{2010}{60}$$

$$y = -x + \frac{2625}{75}$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [54] [÷] [60] [X,T,θ,n] [+] [2010] [÷] [60] [ENTER]
[(–)] [X,T,θ,n] [+] [2625] [÷] [75] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 15 \quad y = 20$

The grocer bought 15 dozen 54-cent oranges and 20 dozen 60-cent oranges.

c) Let x kg be quantity of the 10% mixture *(5 marks)*
Let y kg be quantity of the 30% mixture

$x + y = 10$	10 kg of mixture to be made.
$0.1x + 0.3y = 10(0.24)$	10% of the first mixture and 30% of the second mixture gives 24% of the total 10 kg of mixture.

Solve the two equations for y so that they may be graphed.

$$y = -x + 10$$

$$y = -\frac{1}{3}x + 8$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [1] [÷] [3] [X,T,θ,n] [+] [8] [ENTER] [(–)] [X,T,θ,n] [+]
[10] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 3 \quad y = 7$

They will mix 3 kg of 10% mixture with 7 kg of 30% mixture.

(5 marks)

d) Let x = first number
Let y = second number

$\frac{x+y}{2} = 11$ One-half their sum is 11.

$\frac{x-y}{3} = 2$ One-third their difference is 2.

Solve the two equations for y so that they may be graphed.

$y = -x + 22$
$y = x - 6$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [X,T,θ,n] [+] [22] [ENTER] [X,T,θ,n] [–] [6] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 14 \quad y = 8$

The two numbers are 14 and 8.

e) Let speed of the plane in still air be x km per hour
Let speed of the wind be y km per hour *(5 marks)*

$x + y = 240$ With the wind it goes 480 km in 2 h.

$x - y = 160$ Against the wind it goes 480 km in 3 h.

Solve the two equations for y so that they may be graphed.

$y = -x + 240$

$y = x - 160$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [X,T,θ,n] [+] [240] [ENTER] [X,T,θ,n] [–] [160] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 200$ $y = 40$

The plane will travel at 200 km per hour in still air and the speed of the wind is 40 km per hour.

f) Let x = first number
Let y = second number *(5 marks)*

$x \cdot y = 100$ Product of two numbers is 100.

$x - y = 15$ Difference in the numbers is 15.

Solve the two equations for y so that they may be graphed.

$$y = \frac{100}{x}$$

$$y = x - 15$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [100] [÷] [X,T,θ,n] [ENTER] [X,T,θ,n] [–] [15] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x_1 = 20$ $y_1 = 5$
$x_2 = -5$ $y_2 = -20$

The two numbers are 20 and 5 or –5 and –20.

Notes

GRADE 11
APPLIED MATHEMATICS

Module 2
Answer Key

Module 2, Lesson 1, Answer Key

Graphing Linear Inequalities

Required regions are shaded.

1. $5x - 2y \geq 4$

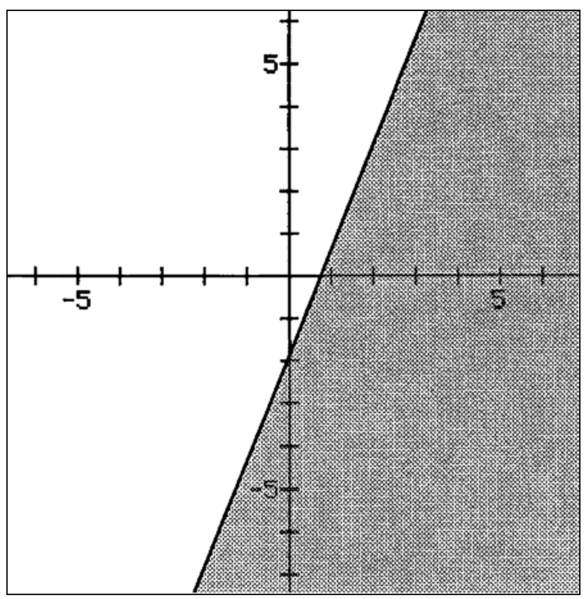
   $5x - 2y = 4$ Related equation.

x	0	$\frac{4}{5}$
y	–2	0

   Find the intercepts.

   Test point (0, 0)
   $5x - 2y$
   $= 5(0) - 2(0) = 0$
   0 is not greater than 4.

   Therefore, (0, 0) is **not** in the required region.
   Line is solid because inequality here is $\geq$.



   Shade the side of the line that does not contain (0, 0) because (0, 0) does not satisfy the original inequality.

2. $2x + 7y < 6$

$2x + 7y = 6$ Related equation.

x	0	3
y	$\frac{6}{7}$	0

Find the intercepts.

Test point (0, 0)

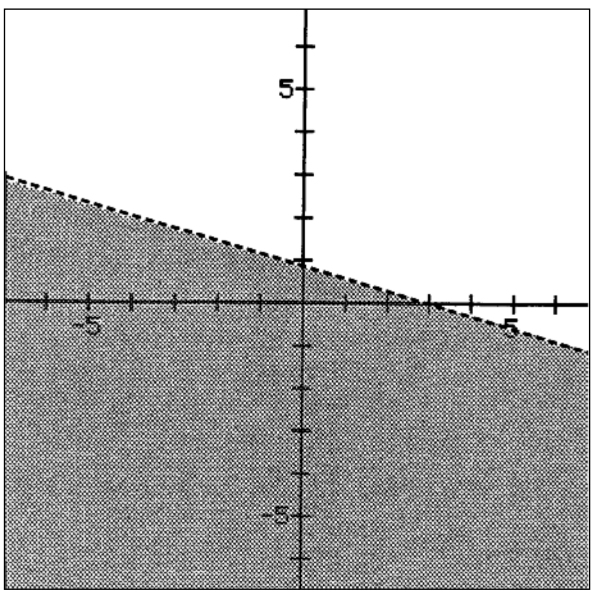
$2x + 7y$

$= 2(0) + 7(0)$

$= 0$

$0 < 6$

Therefore, (0, 0) lies in required region.
Line is dotted because inequality is <.



Shade the side of the line that contains (0, 0) because (0, 0) satisfies the original inequality.

3. $5y \le 3x + 1$

$5y = 3x + 1$ Related equation.

x	0	$\frac{-1}{3}$
y	$\frac{1}{5}$	0

Find the intercepts.

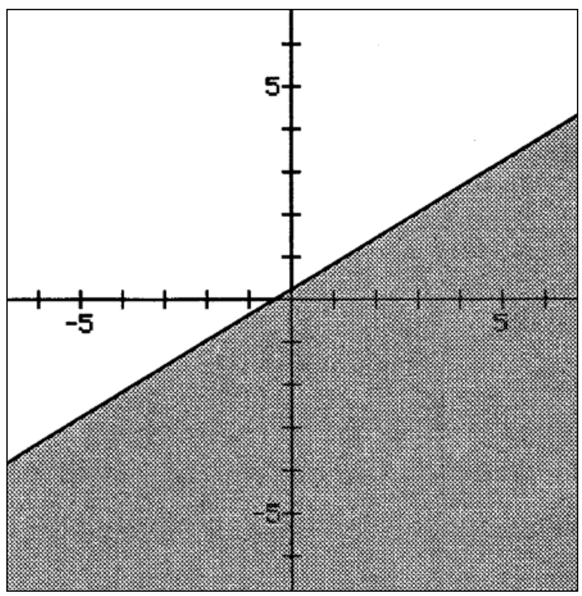
Test point (0, 0)

$5y = 5(0) = 0$

$3x + 1 = 3(0) + 1 = 1$

$0 < 1$

Therefore, inequality is satisfied by (0, 0).
(0, 0) lies in required region.
Line is solid because inequality is ≤.



Shade the side of the line that contains (0, 0) because (0, 0) does satisfy the original inequality.

4. $x \geq 6y + 5$

$x = 6y + 5$ Related equation.

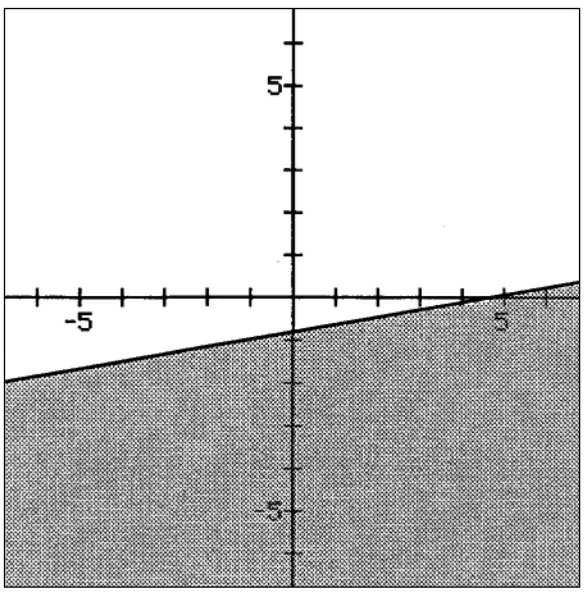
x	0	5
y	$\frac{-5}{6}$	0

Find the intercepts.

Check (0, 0)

$$x = 0$$
$$6y + 5 = 6(0) + 5$$
$$= 5$$
$$0 < 5$$

Therefore, inequality is **not** satisfied and (0, 0) does not lie in the required region.



Shade the side of the line that does not contain (0, 0) because (0, 0) does **not** work in the original inequality.

5. $2x + 3y > 0$

$2x + 3y = 0$ Related equation.

x	0	3
y	0	–2

Find the intercepts and one other point (3, –2) since the two intercepts are the same point.

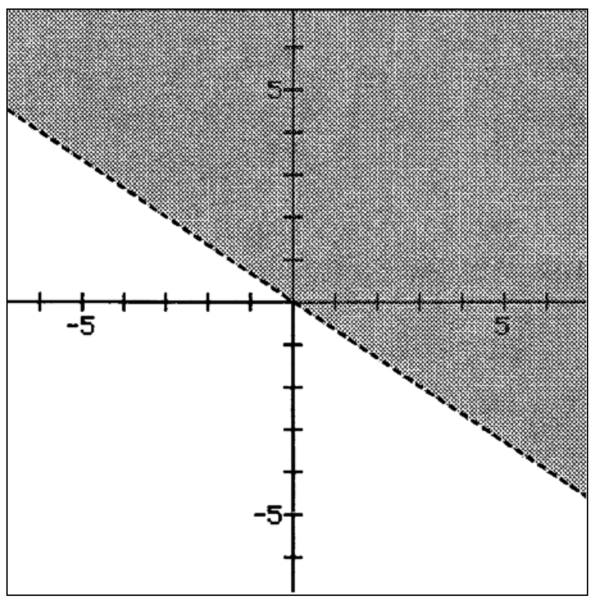
Test point (0, –1)

$2x + 3y$

$= 2(0) + 3(–1) = –3$

$–3 < 0$

Therefore, (0, –1) does **not** satisfy the given inequality.
(0, –1) is not in the required region.
Line is dotted because inequality is >.



Shade the side of the line that does not contain (0, –1) because (0, –1) does not satisfy the original inequality.

6. $7x \le y + 4$

$7x = y + 4$ Related equation.

x	0	$\frac{4}{7}$
y	–4	0

Find the intercepts.

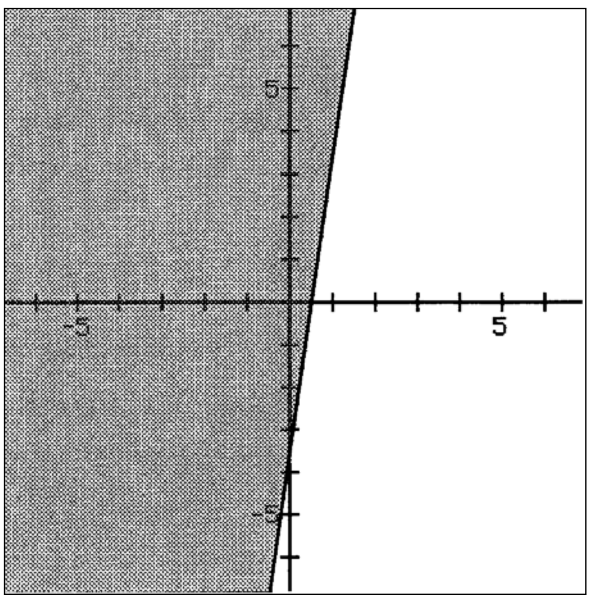
Test point (0, 0)

$7x = 7(0) = 0$

$y + 4 = (0) + 4 = 4$

$0 < 4$

Therefore, (0, 0) satisfies inequality.
(0, 0) is in required region.
Line is solid because inequality is ≤.



Shade the side of the line that contains (0, 0) because (0, 0) does satisfy the original inequality.

7. $x - y > 6$

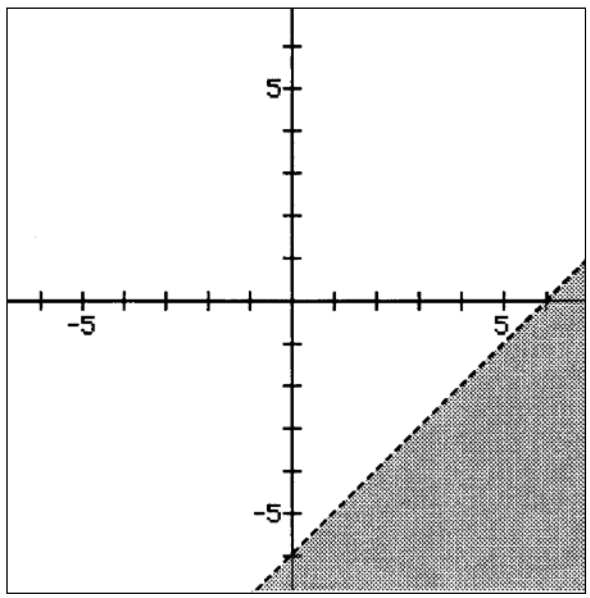
$x - y = 6$ Related equation.

x	0	6
y	–6	0

Find the intercepts.

Test point (0, 0)
$x - y = 0 - 0 = 0$
$0 < 6$

Therefore, (0, 0) does **not** lie in the required region.
Line is dotted because inequality is >.



Shade the side of the line that does not contain (0, 0) because (0, 0) does not satisfy the original inequality.

8. $4 \le 2x + 3y$

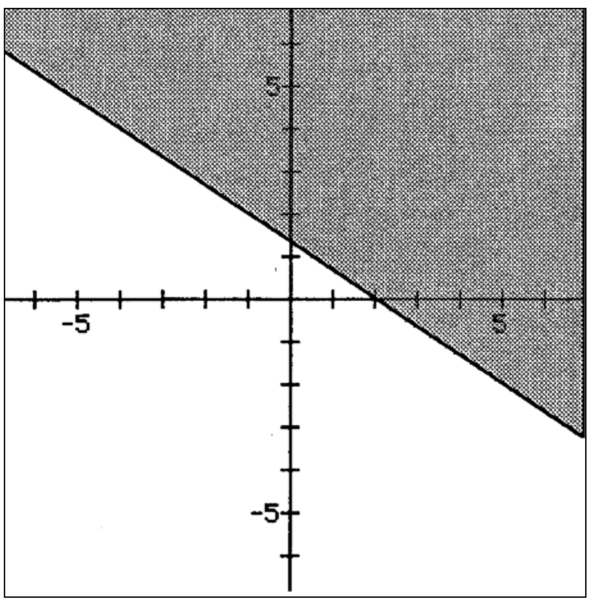
$4 = 2x + 3y$ Related equation.

x	0	2
y	$\frac{4}{3}$	0

Find the intercepts.

Test point (0, 0)
$2x + 3y$
$= 2(0) + 3(0)$
$= 0$
4 is **not** less than or equal to 0.

Therefore (0, 0) does **not** lie in required region.
Line is solid because inequality is $\le$.

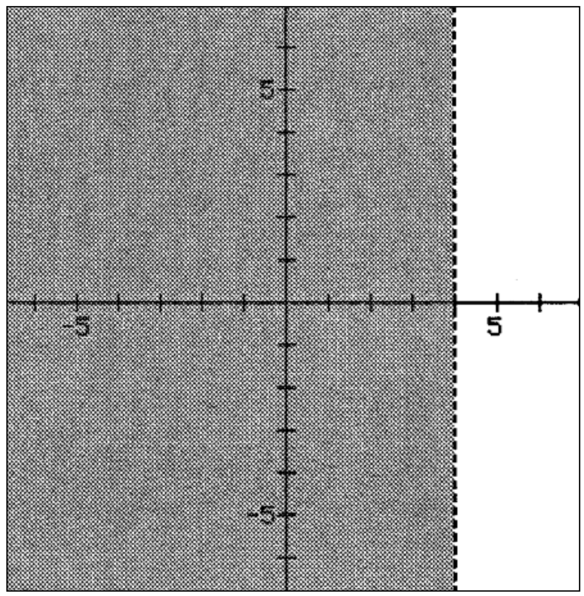


Shade the side of the line that does not contain (0, 0) because (0, 0) works in the original inequality.

9. $x < 4$

$x = 4$ Related line.

All points to the left of this line have x-coordinates less than 4.

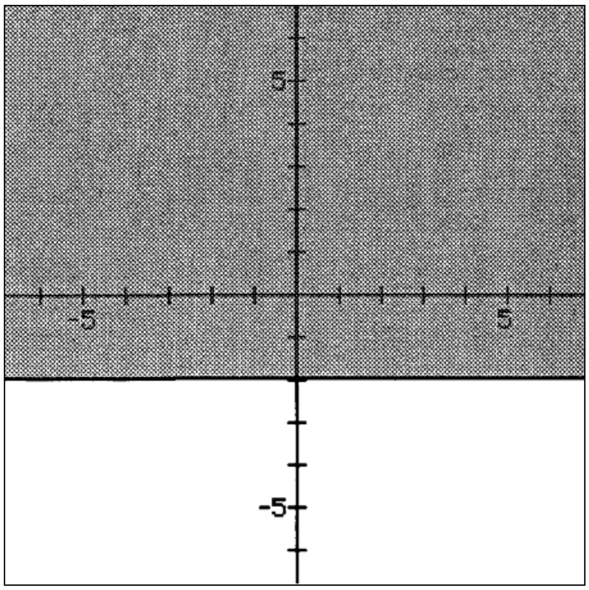


Required region is shaded and does not include the points of the line.

10. $y \geq -2$

    $y = -2$ Related line.

    All points above line have y-coordinates greater than -2. All points on line have y-coordinates equal to -2.



Required region is shaded and includes the points of the line.

Module 2, Lesson 2, Answer Key
Graphing Systems of Linear Inequalities

Note: You may use the intercept method or the graphing calculator to draw the lines. The intercepts and the functions are given in the solutions.

1. $2x - y > 2$ $\quad 3x + y > 1$

$2x - y = 2$ $\quad 3x + y = 1$ Related equations.

x	0	1
y	–2	0

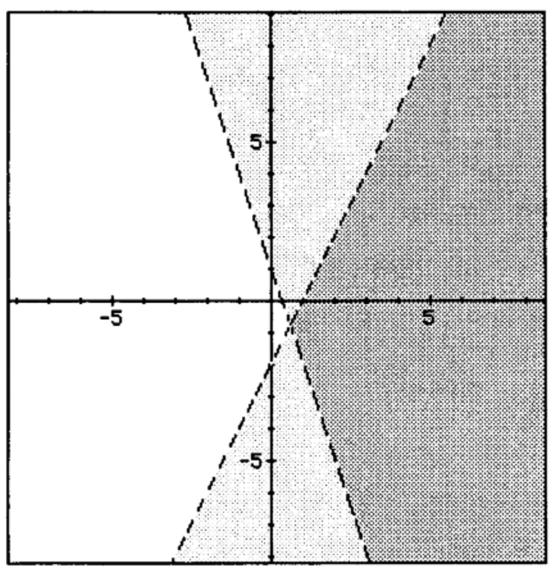
x	0	$\frac{1}{3}$
y	1	0

Find the intercepts.

For graphing calculator, equations are

$y = 2x - 2$ and
$y = -3x + 1$

Shade the required areas.
Test points (0, 0) in each inequality.



Boundary lines are dotted because inequalities are >. The solution is given where the two regions overlap.

2. $\dfrac{2x + y}{3} \le 1 \qquad 2x + y \ge 0$

$\dfrac{2x + y}{3} = 1 \qquad 2x + y = 0$ Related equations.

x	0	$\frac{3}{2}$
y	3	0

x	0	1
y	0	–2

Find the intercepts or other graphing points.

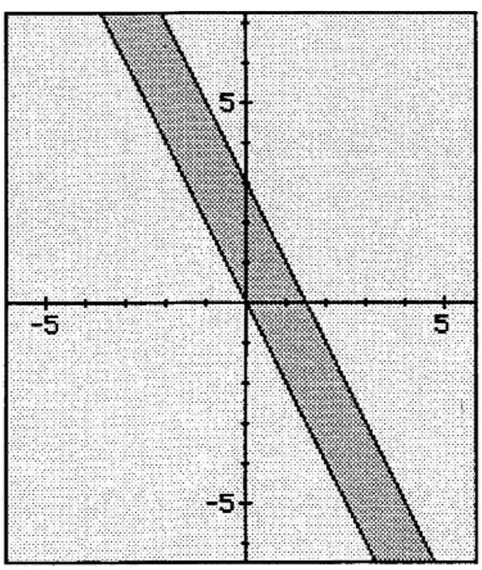
For graphing calculator, equations are
$y = –2x + 3$ and
$y = –2x$

Shade the required areas. The solution area is where they overlap. Test points to determine regions for each inequality. Use (0, 0) for first inequality.

$\dfrac{2x + y}{3} = \dfrac{2(0) + (0)}{3} = 0$

$0 < 1$

Therefore, (0, 0) lies in region determined by that inequality.



Use (1, 0) for second inequality.
$2x + y = 2(1) + 0 = 2$
$2 > 0$

Therefore, (1, 0) lies in that region.

3. $4x - 3y \le 10$ $2x + 5y > 6$

$4x - 3y = 10$ $2x + 5y = 6$ Related equations.

x	0	$\frac{5}{2}$
y	$\frac{-10}{3}$	0

x	0	3
y	$\frac{6}{5}$	0

Find the intercepts.

For graphing calculator, equations are

$y = \frac{4}{3}x - \frac{10}{3}$ and

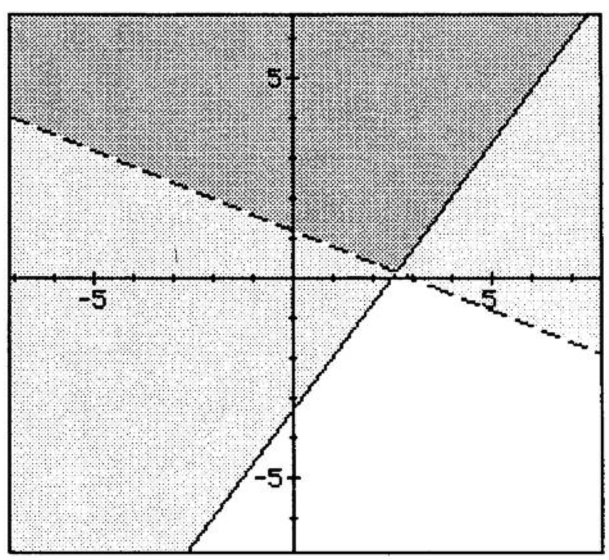
$y = -\frac{2}{5}x + \frac{6}{5}$

Test with (0, 0).
For first inequality
$4x - 3y = 4(0) - 3(0) = 0$
$0 < 10$

Therefore (0, 0) is in region.

For second inequality
$2x + 5y = 2(0) + 5(0) = 0$
$0 < 0$

Therefore (0, 0) does **not** satisfy inequality and does **not** lie in region.



Shade the regions. The final solution is the part where the shadings overlap (the darkest region).

4. $7x - 3y < 27$ $4x - 5y > 8$

$7x - 3y = 27$ $4x - 5y = 8$ Related equations.

x	0	$\frac{27}{2}$
y	–9	0

x	0	2
y	$\frac{-8}{5}$	0

Find the intercepts.

For graphing calculator, equations are

$y = \frac{7}{3}x - 9$ and

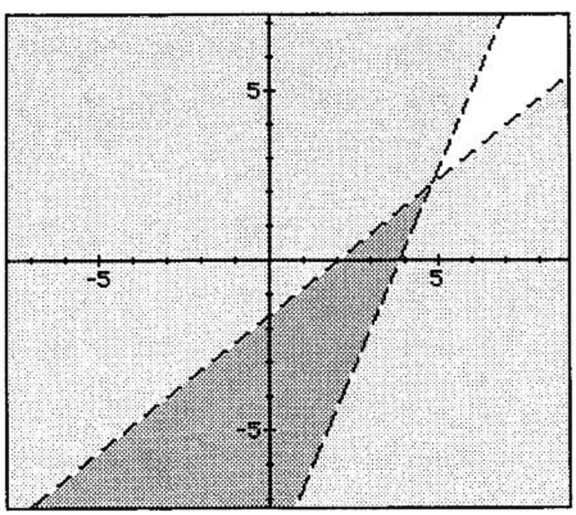
$y = \frac{4}{5}x - \frac{8}{5}$

Test with (0, 0).
For first inequality
$7x - 3y = 7(0) - 3(0) = 0$
$0 < 27$

Therefore, (0, 0) lies in required region.

For second inequality
$4x - 5y = 4(0) - 5(0) = 0$
$0 < 8$

Therefore, (0, 0) does **not** satisfy the inequality and does **not** lie in required region.



Shade the regions. The final solution is the part where they overlap (the darkest region).

5. $6x - 5y < 30$ $2x + 3y \geq 1$

$6x - 5y = 30$ $2x + 3y = 1$ Related equations.

x	0	5
y	–6	0

x	0	$\frac{1}{2}$
y	$\frac{1}{3}$	0

Find the intercepts.

For graphing calculator, equations are

$y = \frac{6}{5}x - 6$ and

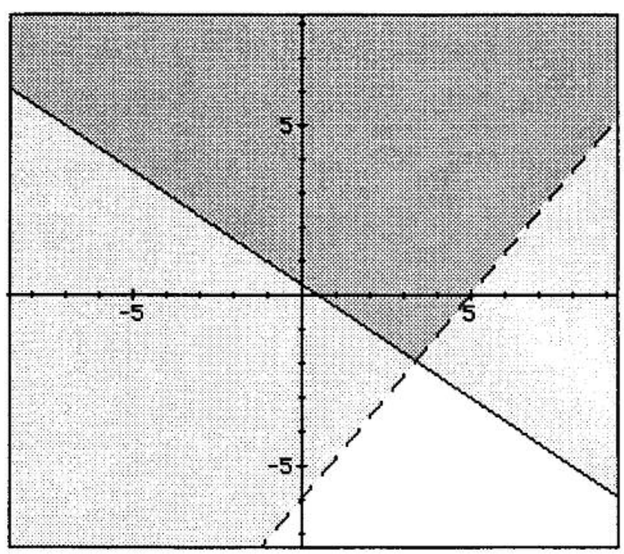
$y = \frac{2}{3}x + \frac{1}{3}$

Test (0, 0)
For first inequality
$6x - 5y = 6(0) - 5(0) = 0$
$0 < 30$

Therefore, (0, 0) lies in required region.

For second inequality
$2x + 3y = 2(0) - 3(0) = 0$
$0 < 1$

Therefore, (0, 0) does **not** satisfy inequality and does **not** lie in region.



Shade the regions. The final solution is the part where they overlap (the darkest region).

6. $x - 2y \leq 7$ $\qquad 8x + 3y \geq 6$

$x - 2y = 7$ $\qquad 8x + 3y = 6$ Related equations.

x	0	7
y	$\frac{-7}{2}$	0

x	0	$\frac{3}{4}$
y	2	0

Find the intercepts.

For graphing calculator, equations are

$y = \frac{1}{2}x - \frac{7}{2}$ and

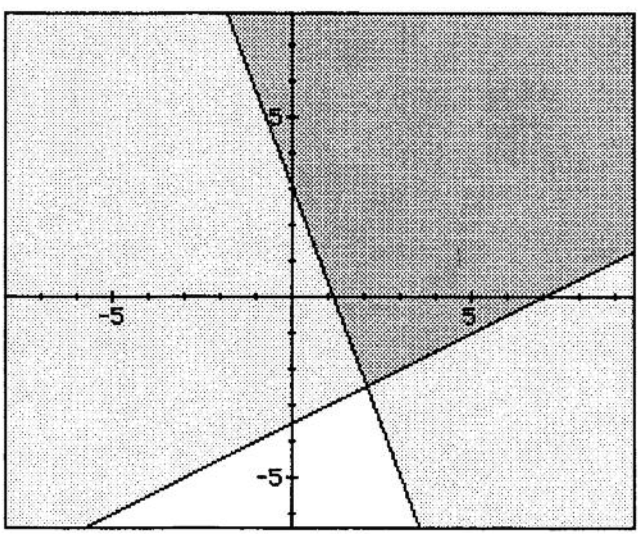
$y = \frac{8}{3}x + 2$

Test (0, 0)
For first inequality
$x - 2y = 0 - 2(0) = 0$
$0 < 7$

Therefore, (0, 0) lies in required region.

For second inequality
$8x + 3y = 8(0) + 3(0) = 0$
$0 < 6$

Therefore, (0, 0) does **not** satisfy inequality and does **not** lie in region.



Shade the regions. The final solution is the part where they overlap (the darkest region).

7. Line 1:

   Points are (0, 4) and (4, 0).

   Slope = $\dfrac{4-0}{0-4} = -1$

   y-intercept = 4
   Equation is: $y = -x + 4$ or $x + y = 4$.
   (0, 0) lies in required region.
   For (0, 0) $x + y = 0 + 0 = 0$, $0 < 4$.

   Therefore, inequality is $x + y < 4$.
   Line is solid, therefore, required inequality is $x + y \leq 4$.

   Line 2:

   Points are (0, –4) and (3, 5).

   Slope = $\dfrac{-4-5}{0-3} = 3$

   y-intercept = –4
   Equation is: $y = 3x - 4$ or $3x - y = 4$.
   For (0, 0) $3x - y = 3(0) - 0 = 0$, $0 < 4$.

   Therefore, $3x - y < 4$.
   Line is solid, therefore, required inequality is $3x - y \leq 4$.
   System is $3x - y \leq 4$
   $x + y \leq 4$

8. Line 1:

   Vertical line: $x = -1$, dotted.
   Inequality required: $x > -1$

   Line 2:

   Points are (4, 0), (0, 2).

   Slope = $\dfrac{0-2}{4-0} = -\dfrac{1}{2}$

   y-intercept = 2

   Therefore, line is $y = -\dfrac{1}{2}x + 2$ or $x + 2y = 4$.

   (0, 0) lies in region.
   For (0, 0) $x + 2y = 0 + 2(0) = 0$, $0 < 4$.

   Therefore, inequality is $x + 2y < 4$.
   Line is solid, therefore, required inequality is $x + 2y \leq 4$.
   System is $x > -1$
   $x + 2y \leq 4$

Notes

Module 2, Lesson 3, Answer Key

Determining the Corner Points of a Feasible Region

The solutions show the lines. The only region shaded is the feasible region which is where all the separate regions overlap. Show only this final region. You may use either the intercept method or the graphing calculator to draw the lines. Intercepts are shown as well as the equations for the graphing calculator.

1. $x \geq 0$ (right of y-axis shaded) $y \geq 0$ (above x-axis shaded) $3x + 4y \leq 12$

   $3x + 4y = 12$ Related equation

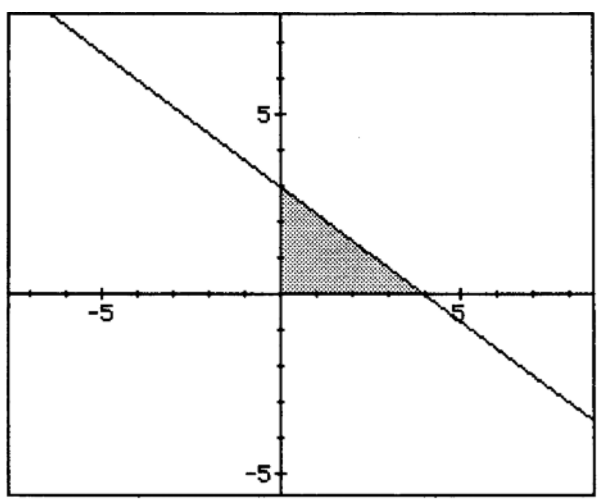
x	0	4
y	3	0

   Intercept method

   Equation for graphing calculator

   $y = -\frac{3}{4}x + 3$

   Draw lines and shade **only** the feasible region.

   The shaded area is the feasible region and has corner points of: (0, 0), (0, 3), and (4, 0).

2. $x \geq -2$ $y \leq 6$ $x \leq 3$ $x - 2y \leq -2$

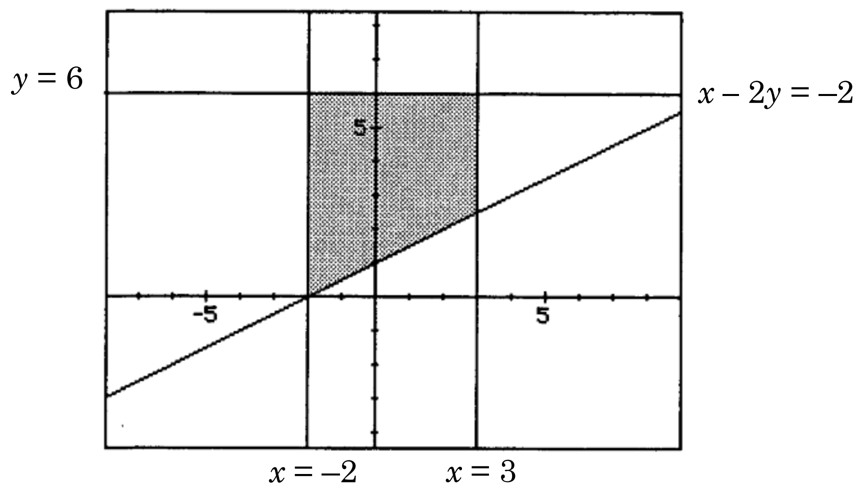
$x - 2y = -2$ Related line

x	0	–2
y	1	0

Intercept method

Equation for graphing calculator

$y = \frac{1}{2}x + 1$



Corner points where:

a) $x = -2$ crosses $y = 6$ (–2, 6)
b) $x = -2$ crosses $x - 2y = -2$ (–2, 0)
c) $x = 3$ crosses $y = 6$ (3, 6)
d) $x = 3$ crosses $x - 2y = -2$

Solve
$$3 - 2y = -2$$
$$-2y = -5$$
$$y = 2\frac{1}{2} \qquad \left(3,\ 2\frac{1}{2}\right)$$

The shaded region is the feasible region and has corner points of (–2, 0), (–2, 6), (3, $2\frac{1}{2}$), and (3, 6).

3. $x \geq 0$ $y \geq 0$ $2x + y \leq 6$ $x + 2y \leq 6$

$x = 0$ $y = 0$ $2x + y = 6$ $x + 2y = 6$ Related equations

x	0	3
y	6	0

x	0	6
y	3	0

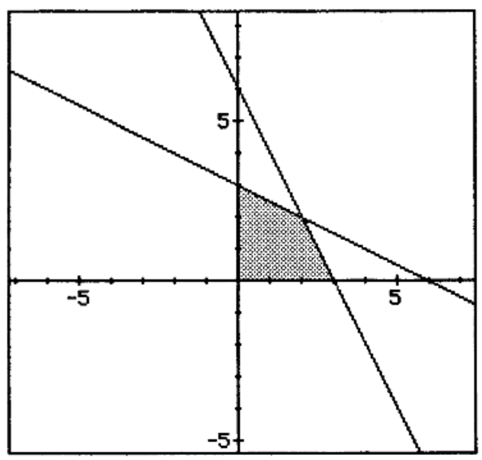
Intercept method

Equations for graphing calculator

$y = -2x + 6$

$y = -\frac{1}{2}x + 3$

Keying sequence for graphing calculator to graph lines and find point of intersection: [Y=] [(–)] [2] [X,T,θ,n] [+] [6] [ENTER] [(–)] [1] [÷] [2] [X,T,θ,n] [+] [3] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]



Shade **only** the feasible region.

Corner points:

a) $x = 0$ crosses $y = 0$ (0, 0)

b) $x = 0$ crosses $x + 2y = 6$

$0 + 2y = 6$

$y = 3$ (0, 3)

c) $y = 0$ crosses $2x + y = 6$

$2x = 6$

$x = 3$ (3, 0)

d) $2x + y = 6$ crosses $x + 2y = 6$

Use add/subtract method

$2x + 4y = 12$	Multiply second equation by 2.
$-3y = -6$	Subtract.
$y = 2$	
$2x + 2 = 6$	Substitute.
$2x = 4$	
$x = 2$	(2, 2)

The shaded region is the feasible region and has corner points of: (0, 0), (0, 3), (3, 0), and (2, 2).

4. $y \le x + 2$ $y \ge x - 2$ $-2 \le y$ $y \le 3$

$y = x + 2$ $y = x - 2$ $y = -2$ $y = 3$ Related equations

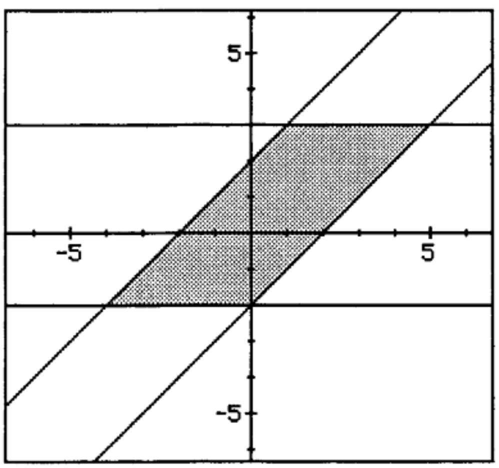
x	0	–2
y	2	0

x	0	2
y	–2	0

Intercept method

Equations for graphing calculator

$y = x + 2$
$y = x - 2$



Corner points:

a) $y = x + 2$ crosses $y = -2$
where $-2 = x + 2$, $x = -4$ (–4, –2)

b) $y = x + 2$ crosses $y = 3$
where $3 = x + 2$, $x = 1$ (1, 3)

c) $y = x - 2$ crosses $y = -2$
where $-2 = x - 2$, $x = 0$ (0, –2)

d) $y = x - 2$ crosses $y = 3$
where $3 = x - 2$, $x = 5$ (5, 3)

The shaded region is the feasible region and has corner points of: (5, 3), (0, –2), (1, 3), and (–4, –2).

5. $x \geq 0$ $y \geq 0$ $x + y \geq 6$ $2x + y \geq 8$ $2x + 3y \geq 14$

Related equations

$x = 0$ $y = 0$ $x + y = 6$ $2x + y = 8$ $2x + 3y = 14$

Intercept method

x	0	6
y	6	0

x	0	4
y	8	0

x	0	7
y	$\frac{14}{3}$	0

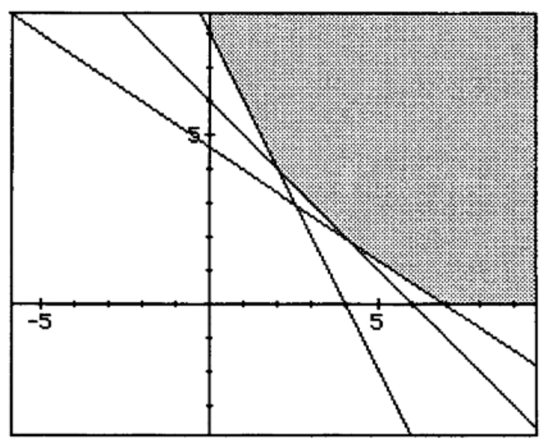
Equations for graphing calculator

$y = -x + 6$

$y = -2x + 8$

$$y = -\frac{2}{3}x + \frac{14}{3}$$

Shade **only** the feasible region.



$2x + 3y = 14$

$2x + y = 8$ $x + y = 6$

Corner points:

a) $x = 0$ crosses $2x + y = 8$
b) $y = 0$ crosses $2x + 3y = 14$
c) $x + y = 6$ crosses $2x + 3y = 14$
d) $x + y = 6$ crosses $2x + y = 8$

The points (7, 0) and (0, 8) can be seen easily on the graph. For the other points, use the graphing calculator.

Finding corner points using the graphing calculator. (Clear all functions from [Y=] list.)

To find where $y = -x + 6$ crosses $y = -2x + 8$.

Keying sequence: [Y=] [(–)] [X,T,θ,n] [+] [6] [ENTER] [(–)] [2] [X,T,θ,n] [+] [8] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (2, 4).

To find where $y = -x + 6$ crosses $y = -\frac{2}{3}x + \frac{14}{3}$

Keying sequence: [Y=] [(–)] [X,T,θ,n] [+] [6] [ENTER] [(–)] [2] [÷] [3] [X,T,θ,n] [+] [14] [÷] [3] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (4, 2).

The unshaded region is the feasible region and has corner points of: (7, 0), (0, 8), (2, 4), and (4, 2).

Module 2, Lesson 4, Answer Key

Finding Maximum and Minimum Values from Corner Points

1. a) $x \geq 0$ $y \geq 0$ $3x + 2y \geq 6$ $2x + 3y \geq 6$

$x = 0$ $y = 0$ $3x + 2y = 6$ $2x + 3y = 6$ Related equations

x	0	2
y	3	0

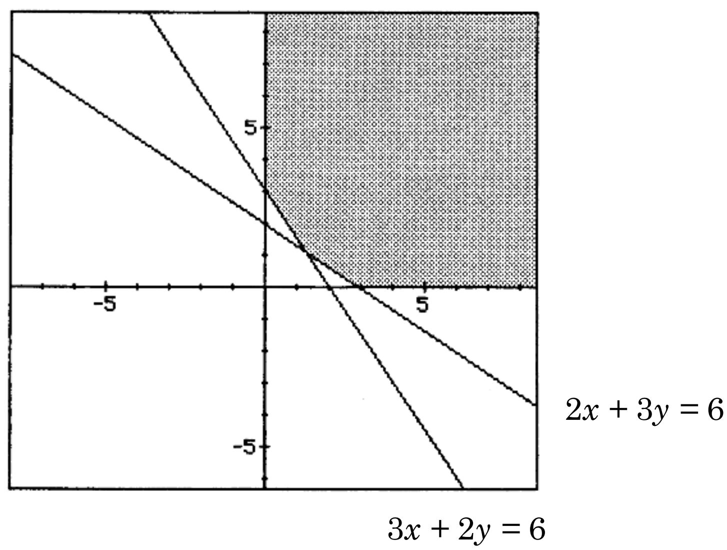
x	0	3
y	2	0

Intercept method

Equations for graphing calculator

$$y = -\frac{3}{2}x + 3$$

$$y = -\frac{2}{3}x + 2$$



Corner points where:

i) $x = 0$ crosses $3x + 2y = 6$ (0, 3)
ii) $y = 0$ crosses $2x + 3y = 6$ (3, 0)
iii) $3x + 2y = 6$ crosses $2x + 3y = 6$

The corner points (3, 0) and (0, 3) are found from the graph, but the corner point (1.2, 1.2) is not easily found from the graph. To find corner points of this nature you can graph the two equations that create the point and find the intersection using your TI-83 calculator in the same manner as you did in Module 1, Lesson 3 (Graphing and Systems of Equations).

Keying sequence: [Y=] [(–)] [3] [÷] [2] [X,T,θ,n] [+] [3] [ENTER] [(–)] [2] [÷] [3] [X,T,θ,n] [+] [2] [GRAPH] [2nd] (CALC) [5] [ENTER] [ENTER] [ENTER]

The shaded region is the feasible region having corner points of: (3, 0), (0, 3), and (1.2, 1.2).

b) Expression $M = 4x + y$
Substitute each corner point into the given expression.
Find the minimum value.

For (3, 0) $M = 4x + y = 4(3) + 0 = 12$
For (0, 3) $M = 4x + y = 4(0) + 3 = 3$
For (1.2, 1.2) $M = 4x + y = 4(1.2) + 1.2 = 6$

Minimum value = 3 and occurs at (0, 3).

2. a. $x + y \le 4$ $x + 4y \ge 7$ $-x + 2y \le 5$

$x + y = 4$ $x + 4y = 7$ $-x + 2y = 5$ Related equations

x	0	4
y	4	0

x	0	7
y	$\frac{7}{4}$	0

x	0	–5
y	$\frac{5}{2}$	0

Intercept method

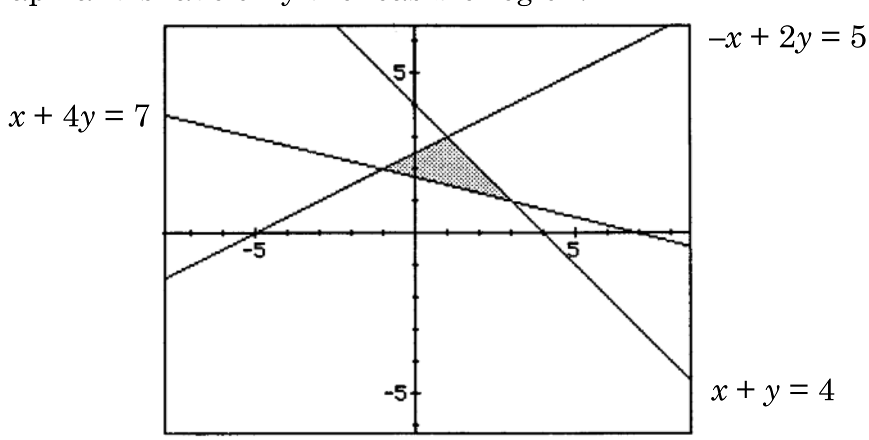
Equations for graphing calculator

$y = -x + 4$

$y = -\frac{1}{4}x + \frac{7}{4}$

$y = \frac{1}{2}x + \frac{5}{2}$

Find the region representing each inequality. Draw the graph and shade only the feasible region.



Point of Intersection of $y = -x + 4$ and $y = -\frac{1}{4}x + \frac{7}{4}$:

Keying sequences for finding points from the graphing calculator: [Y=] [(–)] [X,T,θ,n] [+] [4] [ENTER] [(–)] [X,T,θ,n] [÷] [4] [+] [7] [÷] [4] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives the point (3, 1).

Point of Intersection of $y = -x + 4$ and $y = \frac{1}{2}x + \frac{5}{2}$:

[Y=] [(–)] [X,T,θ,n] [+] [4] [ENTER] [X,T,θ,n] [÷] [2] [+] [5] [÷] [2] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives the point (1, 3).

Point of Intersection of $y = \frac{1}{2}x + \frac{5}{2}$ and $y = -\frac{1}{4}x + \frac{7}{4}$:

[Y=] [X,T,θ,n] [÷] [2] [+] [5] [÷] [2] [ENTER] [(–)] [X,T,θ,n] [÷] [4] [+] [7] [÷] [4] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives the point (–1, 2).

The shaded region is the feasible region and has corner points of: (–1, 2), (1, 3), and (3, 1).

b) Substitute each corner point into the given expression to be maximized and minimized and find the maximum and minimum values.

For (3, 1) $M = 3x + 2y = 3(3) + 2(1) = 11$
For (1, 3) $M = 3x + 2y = 3(1) + 2(3) = 3 + 6 = 9$
For (–1, 2) $M = 3x + 2y = 3(-1) + 2(2) = -3 + 4 = 1$

Minimum value = 1 and occurs at (–1, 2).
Maximum value = 11 and occurs at (3, 1).

3. a) $x + y \le 0$ $-x + 3y + 6 \ge 0$ $x + 2y - 3 \le 0$ $2x + y + 9 \ge 0$

Related equations

$x + y = 0$ $-x + 3y + 6 = 0$ $x + 2y - 3 = 0$ $2x + y + 9 = 0$

Intercept method

x	0	1
y	0	–1

x	0	6
y	–2	0

x	0	3
y	$\frac{3}{2}$	0

x	0	$\frac{-9}{2}$
y	–9	0

Equations for graphing calculator

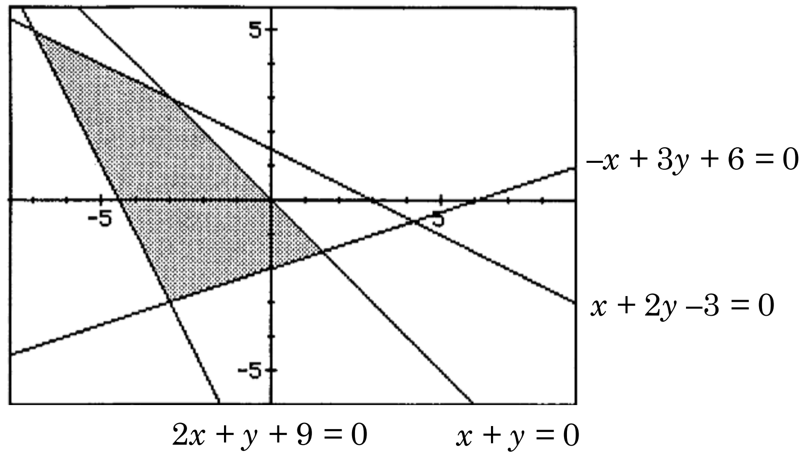
$y = -x$

$y = \frac{x}{3} - 2$

$y = -\frac{x}{2} + \frac{3}{2}$

$y = -2x - 9$

Draw the lines, find the regions, and shade only the feasible region.



Corner points of the feasible region:

i) $x + y = 0$ crosses $-x + 3y + 6 = 0$

Keying sequence: [Y=] [(–)] [X,T,θ,n] [ENTER] [X,T,θ,n] [÷] [3] [–] [2] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (1.5, –1.5).

ii) $2x + y + 9 = 0$ crosses $-x + 3y + 6 = 0$

Keying sequence: [Y=] [(–)] [2] [X,T,θ,n] [–] [9] [ENTER] [X,T,θ,n] [÷] [3] [–] [2] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (–3, –3).

iii) $2x + y + 9 = 0$ crosses $x + 2y - 3 = 0$

Keying sequence: [Y=] [(–)] [2] [X,T,θ,n] [–] [9] [ENTER] [(–)] [X,T,θ,n] [÷] [2] [+] [3] [÷] [2] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (–7, 5).

iv) $x + 2y - 3 = 0$ crosses $x + y = 0$

Keying sequence: [Y=] [(–)] [X,T,θ,n] [ENTER] [(–)] [X,T,θ,n] [÷] [2] [+] [3] [÷] [2] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (–3, 3).

The shaded region is the feasible region having corner points of: (–7, 5), (–3, –3), (–3, 3), and (1.5, –1.5).

b) Substitute each corner point into the given expression to be maximized or minimized and find the maximum or minimum value.

For (–7, 5) $M = 3x + 2y - 1 = 3(-7) + 2(5) - 1 = -12$
For (–3, –3) $M = 3x + 2y - 1 = 3(-3) + 2(-3) - 1 = -16$
For (–3, 3) $M = 3x + 2y - 1 = 3(-3) + 2(3) - 1 = -4$
For (1.5, –1.5) $M = 3x + 2y - 1 = 3(1.5) + 2(-1.5) - 1 = 0.5$

Minimum value = –16 and occurs at (–3, –3).
Maximum value = 0.5 and occurs at (1.5, –1.5).

4. $x \geq 0$ $y \geq 0$ $x + 4y \leq 41$ $y \geq 2x - 10$ $7x + 64 \geq 11y$

Related equations

$x = 0$ $y = 0$ $x + 4y = 41$ $y = 2x - 10$ $7x + 64 = 11y$

Intercept method

x	0	41
y	$\frac{41}{4}$	0

x	0	5
y	–10	0

x	0	$\frac{-64}{7}$
y	$\frac{64}{11}$	0

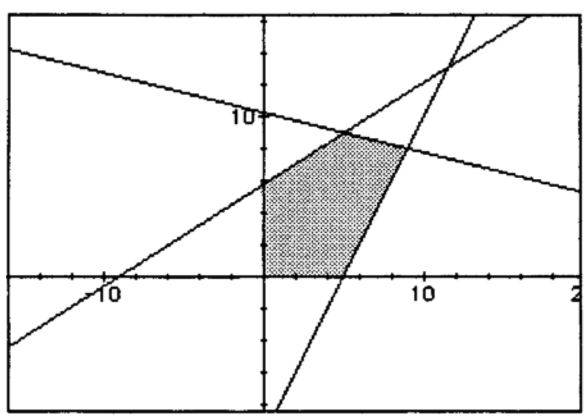
Equations for graphing calculator

$$y = -\frac{1}{4}x + \frac{41}{4}$$

$$y = 2x - 10$$

$$y = \frac{7}{11}x + \frac{64}{11}$$

Draw the lines, shade only the feasible region.



$x + 4y = 41$

$7x + 64 = 11y$

$y = 2x - 10$

Corner points of the feasible region:

$x = 0$ and $y = 0$ cross at (0, 0)
$y = 0$ and $y = 2x - 10$ cross at (5, 0)

$x = 0$ and $7x + 64 = 11y$ cross at $\left(0, \dfrac{64}{11}\right)$

$x + 4y = 41$ crosses $y = 2x - 10$

Keying sequence: [Y=] [(–)] [X,T,θ,n] [÷] [4] [+] [41] [÷] [4] [ENTER] [2] [X,T,θ,n] [–] [10] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (9, 8).

$x + 4y = 41$ crosses $7x + 64 = 11y$

Keying sequence: [Y=] [(–)] [X,T,θ,n] [÷] [4] [+] [41] [÷] [4] [ENTER] [7] [÷] [11] [X,T,θ,n] [+] [64] [÷] [11] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (5, 9).

Equations for Graphing Calculator

$y = -\dfrac{1}{4}x + \dfrac{41}{4}$

$y = 2x - 10$

$y = \dfrac{7}{11}x + \dfrac{64}{11}$

The shaded region is the feasible region having corner points of: $(0, 0)$, $\left(0, \dfrac{64}{11}\right)$, $(5, 0)$, $(5, 9)$, and $(9, 8)$.

Substitute each corner point into the given expression to be maximized and find the maximum value.

For (0, 0) $Q = 3x + 5y = 3(0) + 5(0) = 0$
For $\left(0, \dfrac{64}{11}\right)$ $Q = 3x + 5y = 3(0) + 5\left(\dfrac{64}{11}\right) = 29.09$
For (5, 0) $Q = 3x + 5y = 3(5) + 5(0) = 15$
For (5, 9) $Q = 3x + 5y = 3(5) + 5(9) = 60$
For (9, 8) $Q = 3x + 5y = 3(9) + 5(8) = 67$

Maximum value = 67 and occurs at (9, 8).

5. x ³ 0 y ³ 0 $2x + y \le 260$ $x + 5y \le 400$

$x = 0$ $y = 0$ $2x + y = 260$ $x + 5y = 400$ Related equations

x	0	130
y	260	0

x	0	400
y	80	0

Intercept method

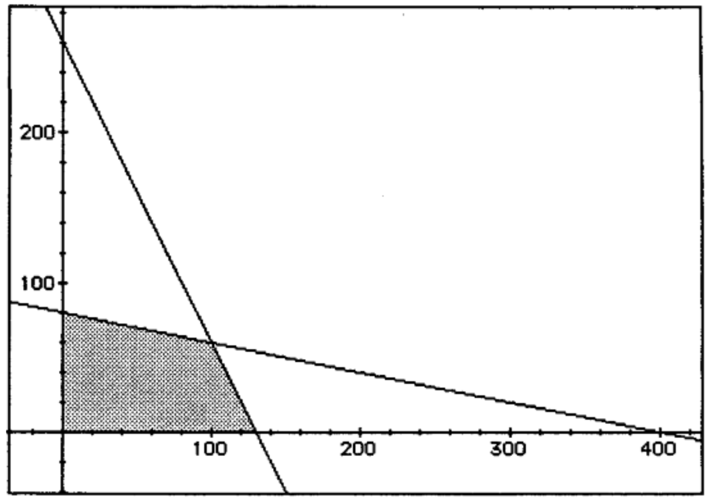
Equations for graphing calculator

$y = -2x + 260$

$y = -\dfrac{1}{5}x + 80$

Shade only the feasible region.

$x + 5y = 400$



$2x + y = 260$

Corner points of the feasible region:

$x = 0$ crosses $y = 0$ at (0, 0)
$y = 0$ crosses $2x + y = 260$ at (130, 0)
$x = 0$ crosses x + 5y = 400 at (0, 80)
2x + y = 260 crosses $x + 5y = 400$
Set window with x from –150 to 150
y from – 100 to 100

Keying sequence: [Y=] [(–)] [2] [X,T,θ,n] [+] [260] [ENTER] [(–)] [1] [÷] [5] [X,T,θ,n] [+] [80] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives (100, 60).

The shaded region is the feasible region having corner points of: (0, 0), (0, 80), (130, 0), and (100, 60).

Substitute each corner point into the given expression to be maximized and find the maximum value.

For (0, 0) E = $5x + 4y = 5(0) + 4(0) = 0$
For (0, 80) E = $5x + 4y = 5(0) + 4(80) = 320$
For (130, 0) E = $5x + 4y = 5(130) + 4(0) = 650$
For (100, 60) E = $5x + 4y = 5(100) + 4(60) = 740$

Maximum value = 740 and occurs at (100, 60).

Equations for Graphing Calculator

$y = -2x + 260$

$y = -\frac{1}{5}x + 80$

Notes

Module 2, Project, Answer Key

Linear Programming

1. Introductory statement

   In this project the process of finding a region indicated by a set of inequalities is extended to use quadratic functions.

2. Description of Method

   Functions used are:

   $y = -3x + 4$ $\quad\quad$ $y = x^2$

   $y = -x^2 + 8$

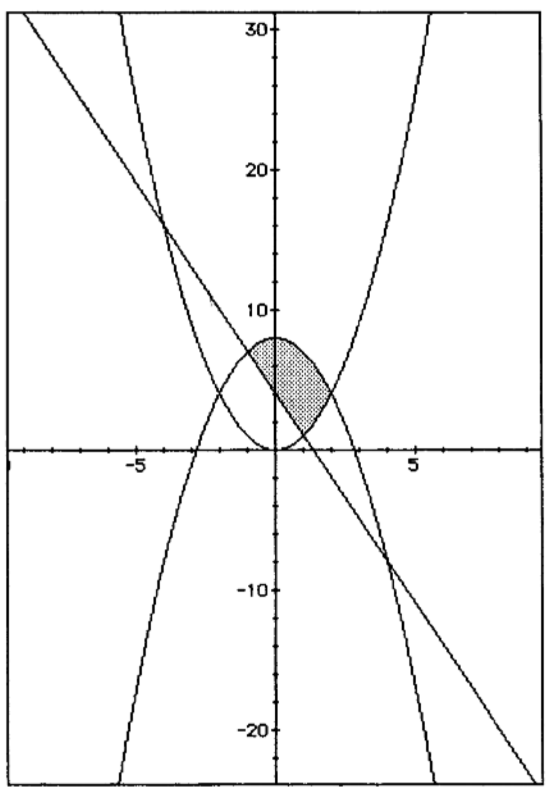
   Points tested to determine region.

3. Mathematical Solution

   $3x + y \geq 4$

   $y \geq x^2$

   $x^2 + y \leq 8$



4. Concluding statement

   Corner points of the shaded feasible region are:

   (2, 4), (1, 1), and (–1, 7)

   The linear programming method can be used with quadratic functions.

5. Overall Organization

   Note: In this problem, parts 2 and 3 may be combined.

Module 2, Lesson 5, Answer Key

Application of Linear Programming

1. Let \$P = profit
   Let x = number of pairs of tennis shoes repaired
   Let y = number of pairs of jogging shoes repaired

$P = 6.00x + 10.00y$	Since each pair of tennis shoes has a profit of \$6.00 and each pair of jogging shoes has a profit of \$10.00.

Constraints

Stripping: $32x + 16y \le 960$	Since each pair of tennis shoes takes 32 min on the stripping machine, each pair of jogging shoes takes 16 min on the same machine, and only 16 h or 960 min is available on the stripping machine. (2 people x 8 h)
Resewing: $24x + 32y \le 960$	Since each pair of tennis shoes takes 24 min on the resewing machine, each pair of jogging shoes takes 32 min on the same machine, and only 16 h or 960 min is available on the resewing machine. (2 people x 8 h)
$x \ge 0$	Since there cannot be a negative number of tennis shoes.
$y \ge 0$	Since there cannot be a negative number of jogging shoes.

Graph the constraints:

$x \geq 0 \quad y \geq 0 \quad 32x + 16y \leq 960 \quad 24x + 32y \leq 960$

$x = 0 \quad y = 0 \quad 32x + 16y = 960 \quad 24x + 32y = 960$ Related equations

x	0	30
y	60	0

x	0	40
y	30	0

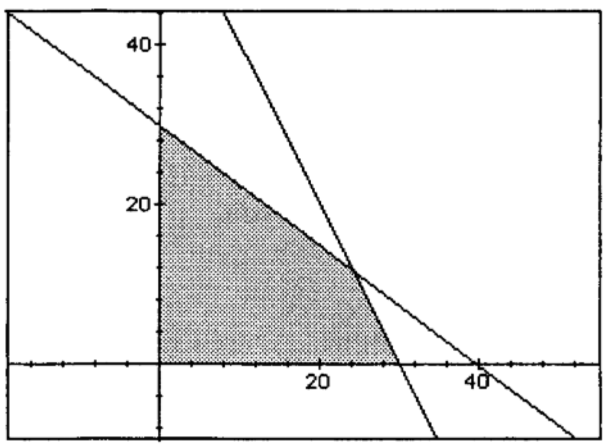
Intercept method

Equations for graphing calculator

$y = -2x + 60$

$y = -\frac{3}{4}x + 30$

Shade only the feasible region



$32x + 16y = 960 \qquad 24x + 32y = 960$

Corner points of the feasible region:

$x = 0$ and $y = 0$ cross at (0, 0)
$x = 0$ crosses $24x + 32y = 960$ at (0, 30)
$y = 0$ crosses $32x + 16y = 960$ at (30, 0)
$32x + 16y = 960$ crosses $24x + 32y = 960$

Keying sequence: [Y=] [(–)] [2] [X,T,θ,n] [+] [60] [ENTER] [(–)] [3] [÷] [4] [X,T,θ,n] [+] [30] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives the point (24, 12).

The shaded region is the feasible region having corner points of: (0, 0), (0, 30), (30, 0), and (24, 12).

Substitute each corner point into the profit function $P = 6.00x + 10.00y$, and find the maximum value.

For (0, 0) $P = 6x + 10y = 6(0) + 10(0) = 0$
For (0, 30) $P = 6x + 10y = 6(0) + 10(30) = 300$
For (30, 0) $P = 6x + 10y = 6(30) + 10(0) = 180$
For (24, 12) $P = 6x + 10y = 6(24) + 10(12) = 264$

Maximum profit = \$300 is made when zero pairs of tennis shoes and 30 pairs of jogging shoes are repaired.

2. Let \$C = cost
Let x = number of kg of Husky feed
Let y = number of kg of Vibrant feed

$C = 25.00x + 32.00y$

Constraints

Iron:	$0.05x + 0.02y \geq 1.9$	Since the amount of iron in the Husky feed is 5% and the amount of iron in Vibrant feed is 2%.
Riboflavin:	$0.02x + 0.03y \geq 1.2$	Since the amount of riboflavin in the Husky feed is 2% and the amount of riboflavin in Vibrant feed is 3%.
	$x \geq 0$	Since there can only be a positive number or 0 kg of Husky.
	$y \geq 0$	Since there can only be a positive number or 0 kg of Vibrant.

Graph the constraints:

$x \geq 0$ $y \geq 0$ $0.05x + 0.02y \geq 1.9$ $0.02x + 0.03y \geq 1.2$

Related equations: $x = 0$ $y = 0$ $0.05x + 0.02y = 1.9$ $0.02x + 0.03y = 1.2$

Intercept method

x	0	38
y	95	0

x	0	60
y	40	0

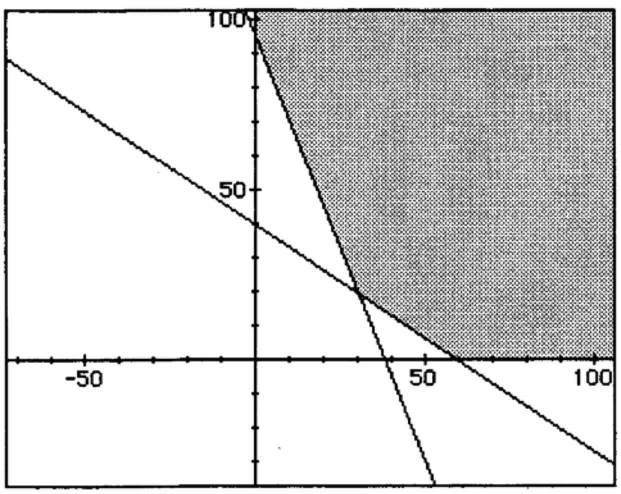
Equations for graphing calculator

$$y = -\frac{0.05}{0.02}x + \frac{1.9}{0.02}$$

$$y = -\frac{0.02}{0.03}x + \frac{1.2}{0.03}$$

Note: In graph, window shows x from –75 to 75
y from –100 to 100.

Draw the lines and shade **only** the feasible regions.



$0.02x + 0.03y = 1.2$

$0.05x + 0.02y = 1.9$

Corner points of the feasible region:

$0.05x + 0.02y = 1.9$ crosses x = 0 at (0, 95)
$0.02x + 0.03y = 1.2$ crosses y = 0 at (60, 0)
$0.05x + 0.03y = 1.9$ crosses $0.02x + 0.03y = 1.2$

Keying sequence: [Y=] [(–)] [0.05] [÷] [0.02] [X,T,θ,n] [+] [1.9] [÷] [0.02] [ENTER] [(–)] [0.02] [÷] [0.03] [X,T,θ,n] [+] [1.2] [÷] [0.03] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives the point (30, 20).

The corner points are (0, 95), (60, 0), and (30, 20).

Substitute each corner point into the cost function C = $25.00x + 32.00y$ and find the minimum value.

For (0, 95) C = $25x + 32y$ = 25(0) + 32(95) = 3040
For (60, 0) C = $25x + 32y$ = 25(60) + 32(0) = 1500
For (30, 20) C = $25x + 32y$ = 25(30) + 32(20) = 1390

Minimum cost = $1390 brought about by 30 kg of Husky feed and 20 kg of Vibrant feed being purchased. The 30 kg of Husky feed costs $750.00; the 20 kg of Vibrant feed costs $640.00.

3. Let \$P = profit
Let x = number of 2-bulb bedroom lamps
Let y = number of 4-bulb living room lamps

$P = 20.00x + 35.00y$

Constraints

Bulbs: $2x + 4y \leq 480$	Since each bedroom lamp takes two bulbs, each living room lamp takes four bulbs, and the number available each day is 480.
Shades: $x + y \leq 180$	Since each two-bulb lamp, each four-bulb lamp takes one shade, and the number available each day is 180.
$x \geq 0$	Since there cannot be a negative number of bedroom lamps.
$y \geq 0$	Since there cannot be a negative number of living room lamps.

Graph the constraints:

$x \geq 0 \quad y \geq 0 \quad 2x + 4y \leq 480 \quad x + y \leq 180$

$x = 0 \quad y = 0 \quad 2x + 4y = 480 \quad x + y = 180$ Related equation

x	0	240
y	120	0

x	0	180
y	180	0

Intercept method

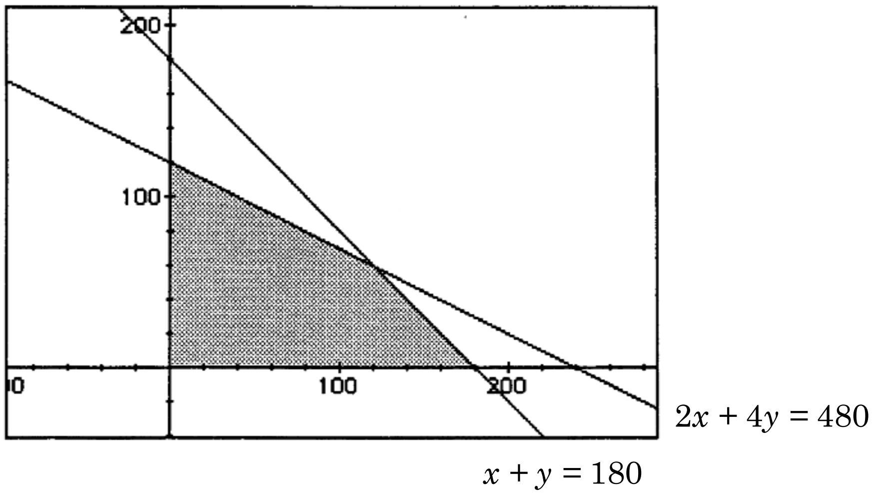
Equations for graphing calculator

$$y = -\frac{2}{4}x + \frac{480}{4}$$

$$y = -x + 180$$

Window: x from –10 to 250
y from –10 to 200

Draw lines and shade the feasible region.



Corner points of the feasible region:

$x = 0$ crosses $y = 0$ at (0, 0)
$x = 0$ crosses $2x + 4y = 480$ at (0, 120)
$y = 0$ crosses $x + y = 180$ at (180, 0)

Use the graphing calculator to find the intersection of $x + y = 180$ and $2x + 4y = 480$.

Keying sequence: [Y=] [(–)] [X,T,θ,*n*] [+] [180] [ENTER] [(–)] [2] [÷] [4] [X,T,θ,*n*] [+] [480] [÷] [4] [GRAPH] [2nd] (CALC) [5] [ENTER] [ENTER] [ENTER]. This gives the point (120, 60).

The shaded region is the feasible region and has corner points (0, 0), (0, 120), (180, 0), (120, 60).

Substitute each corner point into the profit function $P = 20.00x + 35.00y$ and find the maximum value.

For (0, 0) $P = 20x + 35y = 20(0) + 35(0) = 0$
For (0, 120) $P = 20x + 35y = 20(0) + 35(120) = 4200$
For (180, 0) $P = 20x + 35y = 20(180) + 35(0) = 3600$
For (120, 60) $P = 20x + 35y = 20(120) + 35(60) = 4500$

Maximum profit = \$4500 where 120 two-bulb lamps and 60 four-bulb lamps are made.

4. Let \$P = profit
   Let x = number of self-winding watches made
   Let y = number of automatic watches made

   $P = 25.00x + 18.00y$

   Constraints

Machine time: $1.5x + 0.5y \le 3$	Since each self-winding watch takes 1.5 h on the machine, each automatic watch takes 0.5 h on the same machine, and there are only 3 h available on the machine.
Jeweller time: $x + 2y \le 7$	Since each self-winding watch takes 1 h of jeweller time, each automatic watch takes 2 h of jeweller time, and there are only 7 h available for jeweller time.
$x \ge 0$	Since there can only be a positive number of or 0 self-winding watches made.
$y \ge 0$	Since there can only be a positive number of or 0 automatic watches made.

   Graph the constraints:

   $x \ge 0$ $y \ge 0$ $1.5x + 0.5y \le 3$ $x + 2y \le 7$

   $x = 0$ $y = 0$ $1.5x + 0.5y = 3$ $x + 2y = 7$ Related equation

x	0	2
y	6	0

x	0	7
y	3.5	0

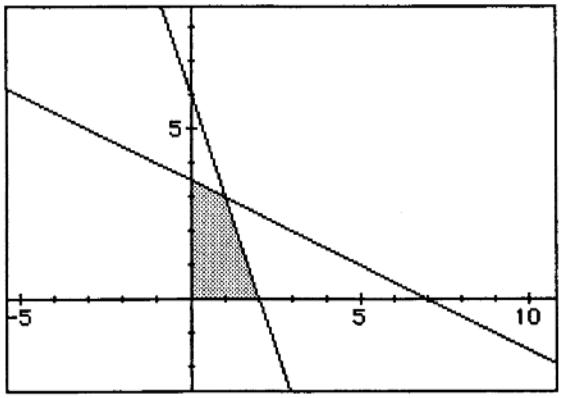
   Intercept method

   Equations for graphing calculator

   $$y = -\frac{1.5}{0.5}x + \frac{3}{0.5}$$

   $$y = -\frac{1}{2}x + \frac{7}{2}$$

Draw lines and shade feasible regions.



$x + 2y = 7$

$1.5x + 0.5y = 3$

Corner points of the feasible region:

$x = 0$ crosses $y = 0$ at (0, 0)
$x = 0$ crosses $x + 2y = 7$ at (0, 3.5)
$y = 0$ crosses $1.5x + 0.5y = 3$ at (2, 0)
$1.5x + 0.5y = 3$ crosses $x + 2y = 7$

Keying sequence: [Y=] [(–)] [1.5] [÷] [0.5] [X,T,θ,n] [+] [3] [÷] [0.5] [ENTER] [(–)] [1] [÷] [2] [X,T,θ,n] [+] [7] [÷] [2] [GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]. This gives the point (1, 3).

The shaded region is the feasible region and has corner points (0, 0), (0, 3.5), (2, 0), and (1, 3).

Substitute each corner point into the profit function P = $25.00x + 18.00y$ and find the maximum value.

For (0, 0) P = $25x + 18y = 25(0) + 18(0) = 0$
For (0, 3.5) P = $25x + 18y = 25(0) + 18(3.5) = 63$
For (2, 0) P = $25x + 18y = 25(2) + 18(0) = 50$
For (1, 3) P = $25x + 18y = 25(1) + 18(3) = 79$

Maximum profit = $79 when one self-winding watch and three automatic watches are made.

5. Let \$C = cost
   Let x = the number of hours John should work
   Let y = the number of hours John's son should work

   $C = 24.00x + 8.00y$

   Constraints

Toasters:	$4x + 6y \geq 50$	Since John can service four toasters in one hour (15 min each), his son can service six toasters in one hour (10 min each), and they must service at least 50 toasters in order to fill the order.
Kettles:	$5x + 3y \geq 40$	Since John can service five kettles in one hour (12 min each), his son can service three kettles in one hour (20 min each), and they must service at least 40 kettles in order to fill the order.
	$x \geq 0$	Since there can only be a positive number of or 0 toasters serviced.
	$y \geq 0$	Since there can only be a positive number of or 0 kettles serviced.

   Graph the constraints:

   $x \geq 0$ $\quad y \geq 0$ $\quad 4x + 6y \geq 50$ $\quad 5x + 3y \geq 40$

   $x = 0$ $\quad y = 0$ $\quad 4x + 6y = 50$ $\quad 5x + 3y = 40$ Related equations

x	0	$\frac{50}{4}$
y	$\frac{50}{6}$	0

x	0	8
y	$\frac{40}{3}$	0

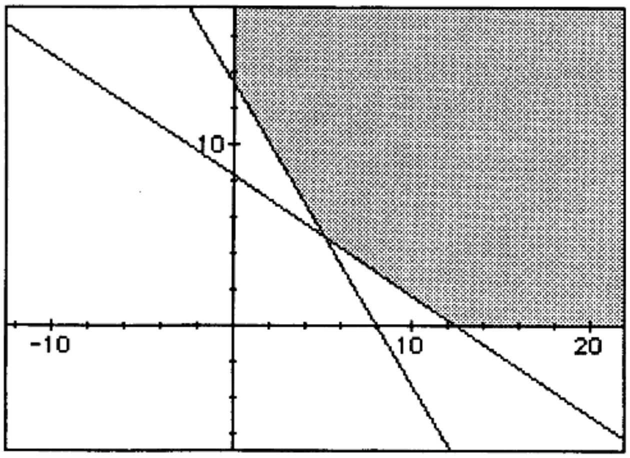
   Intercept method

   Equations for graphing calculator

   $$y = -\frac{4}{6}x + \frac{50}{6}$$

   $$y = -\frac{5}{3}x + \frac{40}{3}$$

Draw lines and shade the feasible area



Corner points of the feasible region:

$x = 0$ crosses $5x + 3y = 40$ at (0, 13.3)
$y = 0$ crosses $4x + 6y = 50$ at (12.5, 0)
$4x + 6y = 50$ crosses $5x + 3y = 40$

Find point using graphing calculator.

Keying sequence: [Y=] [(–)] [4] [÷] [6] [X,T,θ,*n*] [+] [50] [÷] [6] [ENTER] [(–)] [5] [÷] [3] [X,T,θ,*n*] [+] [40] [÷] [3] [GRAPH] [2nd] (CALC) [5] [ENTER] [ENTER] [ENTER]. This gives the point (5, 5).

The shaded region is the feasible region and has corner points of: (0, 13.3), (12.5, 0), and (5, 5).

Substitute each corner point into the cost function $C = 24.00x + 8.00y$ and find the minimum value.

For (0, 13.3) $C = 12x + 8y = 12(0) + 8(13.3) = 106.4$
For (12.5, 0) $C = 12x + 8y = 12(12.5) + 8(0) = 150$
For (5, 5) $C = 12x + 8y = 12(5) + 8(5) = 100$

Note: If you use $\frac{40}{3}$ instead of 13.3 you get 106.67.

Minimum cost = $100 when each works five hours.

6. Let \$C = cost
   Let x = number of hours George should work
   Let y = number of hours Janet should work

   $C = 6.50x + 8.00y$

   Constraints

Rake handles:	$6x + 10y \geq 60$	Since George can make 6 rake handles in 1 h and Janet can make 10 rake handles in 1 h and they must make at least 60 rake handles in order to fill the order.
Shovel handles:	$4x + 4y \geq 32$	Since George can make 4 shovel handles in 1 h and Janet can make 4 shovel handles in 1 h and they must make at least 32 shovel handles in order to fill the order.
	$x \geq 0$	Since there can only be a positive number of or 0 rake handles made.
	$y \geq 0$	Since there can only be a positive number of or 0 shovel handles made.

   Graph the constraints:

   $x \geq 0 \quad y \geq 0 \quad 6x + 10y \geq 60 \quad 4x + 4y \geq 32$

   $x = 0 \quad y = 0 \quad 6x + 10y = 60 \quad 4x + 4y = 32$ Related equations

x	0	10
y	6	0

x	0	8
y	8	0

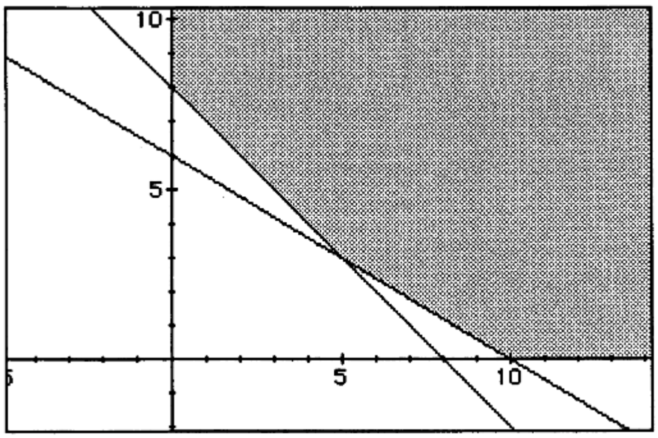
   Intercept method

   Equations for graphing calculator

   $y = -\frac{6}{10}x + \frac{60}{10}$ or $y = -\frac{3}{5}x + 6$

   $y = -\frac{4}{4}x + \frac{32}{4}$ or $y = -x + 8$

Draw lines and shade feasible region.



$4x + 4y = 32 \quad 6x + 10y = 60$

Corner points of the feasible region:

4x + 4y = 32 crosses x = 0 at (0, 8)
6x + 10y = 60 crosses y = 0 at (10, 0)
6x + 10y = 60 crosses 4x + 4y = 32

Use graphing calculator to find this point.

Keying sequence: [Y=] [(–)] [6] [÷] [10] [X,T,θ,n] [+] [60] [÷] [10] [ENTER] [(–)] [4] [÷] [4] [X,T,θ,n] [+] [32] [÷] [4] [GRAPH] [2nd] (CALC) [5] [ENTER] [ENTER] [ENTER]. This gives the point (5, 3).

The shaded region is the feasible region having corner points of: (0, 8), (10, 0), and (5, 3).

Substitute each corner point into the cost function C = 6.50x + 8.00y and find the minimum value.

For (0, 8) C = 6.5x + 8y = 6.5(0) + 8(8) = 64
For (10, 0) C = 6.5x + 8y = 6.5(10) + 8(0) = 65
For (5, 3) C = 6.5x + 8y = 6.5(5) + 8(3) = 56.5

Minimum cost = $56.50 when George works five hours and Janet works three hours.

7. Let \$C = cost
   Let x = number of hours Steve should work
   Let y = number of hours Sandra should work

   $C = 6.00x + 5.00y$

   Constraints

Tennis racquets:	$4x + 2y \geq 24$	Since Steve can put grips on 4 tennis racquets in 1 h, Sandra can put grips on 2 tennis racquets in 1 h, and they must make at least 24 tennis racquets in order to fill the order for the day.
Squash racquets:	$2x + 3y \geq 20$	Since Steve can put grips on 2 squash racquets in 1 h, Sandra can put grips on 3 squash racquets in 1 h, and they must make at least 20 squash racquets in order to fill the order for the day.
	$x \geq 0$	Since there can only be a positive number of tennis racquets made (including 0).
	$y \geq 0$	Since there can only be a positive number of squash racquets made (including 0).

   Graph the constraints:

   $x \geq 0$ $y \geq 0$ $4x + 2y \geq 24$ $2x + 3y \geq 20$

   $x = 0$ $y = 0$ $4x + 2y = 24$ $2x + 3y = 20$ Related equations

x	0	6
y	12	0

x	0	10
y	$\frac{20}{3}$	0

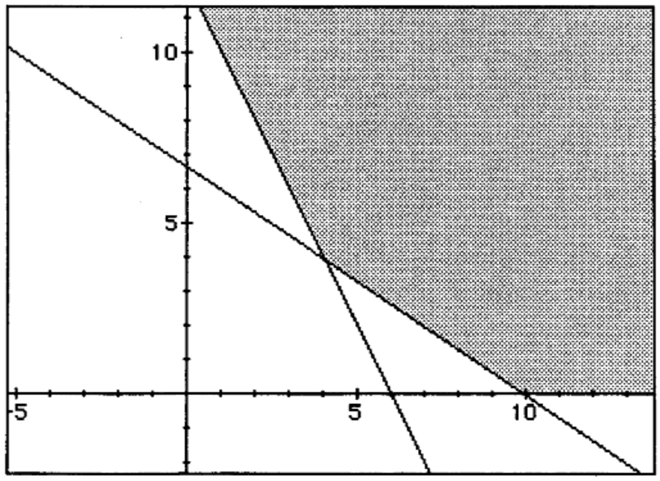
   Intercept method

   Equations for graphing calculator

   $y = -\dfrac{4}{2}x + \dfrac{24}{2}$ or $y = -2x + 12$

   $y = -\dfrac{2}{3}x + \dfrac{20}{3}$

Draw lines and shade feasible region.



$4x + 2y = 24$ $2x + 3y = 20$

Corner points of the feasible region:

$x = 0$ crosses $4x + 2y = 24$ at (0, 12)
$y = 0$ crosses $2x + 3y = 20$ at (10, 0)
$4x + 2y = 24$ crosses $2x + 3y = 20$

Use the graphing calculator to find this point.

Keying sequence: [Y=] [(–)] [4] [÷] [2] [X,T,θ,n] [+] [24] [÷] [2] [ENTER] [(–)] [2] [÷] [3] [X,T,θ,n] [+] [20] [÷] [3] [GRAPH] [2nd] (CALC) [5] [ENTER] [ENTER] [ENTER]. This gives the point (4, 4).

The shaded region is the feasible region having corner points of: (0, 12), (10, 0), and (4, 4).

Substitute each corner point into the cost function C = $6.00x + 5.00y$ and find the minimum value.

For (0, 12) $C = 6x + 5y = 6(0) + 5(12) = 60$
For (10, 0) $C = 6x + 5y = 6(10) + 5(0) = 60$
For (4, 4) $C = 6x + 5y = 6(4) + 5(4) = 44$

Minimum cost = \$44.00 when Steve works four hours and Sandra works four hours.

a) Steve and Sandra should each work four hours.
b) 24 tennis racquets and 20 squash racquets will have grips put on them.

8. Let \$P = profit
Let x = number of acres of corn planted
Let y = number of acres of soybeans planted

$P = 24.00x + 48.00y$

Constraints

Expenses:	$6x + 12y \le 2400$	Since each acre of corn planted costs \$6.00, each acre of soybeans planted costs \$12.00, and there is only \$2400.00 available for the two crops.
Storage:	$12x + 16y \le 3600$	Since each acre of corn planted takes 12 bushels of storage, each acre of soybeans planted takes 16 bushels of storage, and there is storage available for only 3600 bushels of the crops.
	$x \ge 0$	Since there cannot be a negative number of acres of corn planted.
	$y \ge 0$	Since there cannot be a negative number of acres of soybeans planted.

Graph the constraints:

$x \ge 0$	$y \ge 0$	$6x + 12y \le 2400$	$12x + 16y \le 3600$	Related
$x = 0$	$y = 0$	$6x + 12y = 2400$	$12x + 16y = 3600$	equations

x	0	400
y	200	0

x	0	300
y	225	0

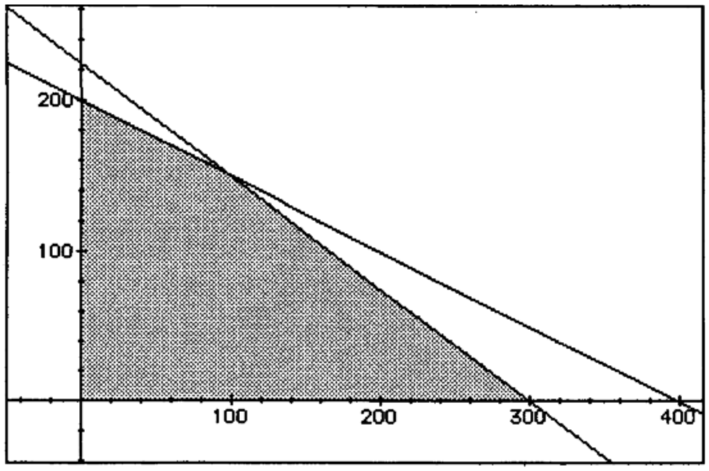
Intercept method

Equations for graphing calculator

$$y = -\frac{6}{12}x + \frac{2400}{12} \quad \text{or } y = -\frac{1}{2}x + 200$$

$$y = -\frac{12}{16}x + \frac{3600}{16} \quad \text{or } y = -\frac{3}{4}x + 225$$

Draw the lines and shade only the feasible region.



$6x + 12y = 2400$

$12x + 16y = 3600$

Corner points of the feasible region:

$x = 0$ crosses $y = 0$ at (0, 0)
$x = 0$ crosses $6x + 12y = 2400$ at (0, 200)
$y = 0$ crosses $12x + 16y = 3600$ at (300,0)
$6x + 12y = 2400$ crosses $12x + 16y = 3600$

Use graphing calculator to find this point.

Keying sequence:

[Y=] [(–)] [6] [÷] [12] [X,T,θ,n] [+] [2400] [÷] [12] [ENTER] [(–)] [12] [÷] [16] [X,T,θ,n] [+] [3600] [÷] [16] [GRAPH] [2nd] (CALC) [5] [ENTER] [ENTER] [ENTER]. This gives the point (100, 150).

The shaded region is the feasible region and has corner points of: (0, 0), (0, 200), (300, 0), and (100, 150).

Substitute each corner point into the profit function
$P = 24.00x + 48.00y$ and find the maximum value.

For (0, 0) $P = 24x + 48y = 24(0) + 48(0) = 0$
For (0, 200) $P = 24x + 48y = 24(0) + 48(200) = 9600$
For (300, 0) $P = 24x + 48y = 24(300) + 48(0) = 7200$
For (100, 150) $P = 24x + 48y = 24(100) + 48(150) = 9600$

Maximum profit = $9600.00 brought about by either planting zero acres of corn and 200 acres of soybeans or 100 acres of corn and 150 acres of soybeans.

GRADE 11 APPLIED MATHEMATICS

Module 3
Answer Key

Module 3, Lesson 1, Answer Key

Quadratic Functions and Their Properties

You may have chosen a window that is slightly different from the one shown here.

Any window that shows the important features of the graph is satisfactory.

1. a) Vertex (2,1)
Axis of symmetry: $x = 2$
Maximum value: $y = 1$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \leq 1\}$
Zeros: $x_1 = 1$, $x_2 = 3$
y-intercept = –3

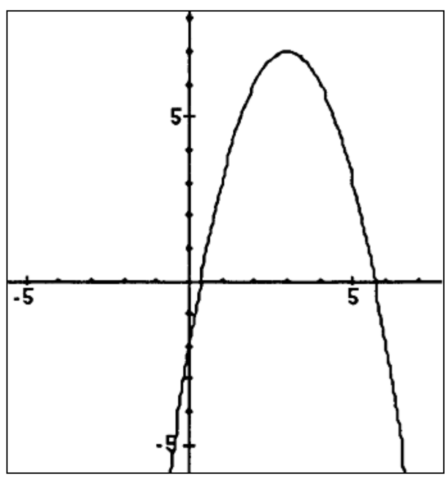
   b) Vertex (–1,3)
Axis of symmetry: $x = -1$
Minimum value: $y = 3$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \geq 3\}$
Zeros: none
y-intercept = 5

2. a) $y = -x^2 + 6x - 2$

Keying sequence:

[Y=] [(–)] [X,T,θ,n] [x^2] [+] [6] [X,T,θ,n] [–] [2]

Graph



Properties

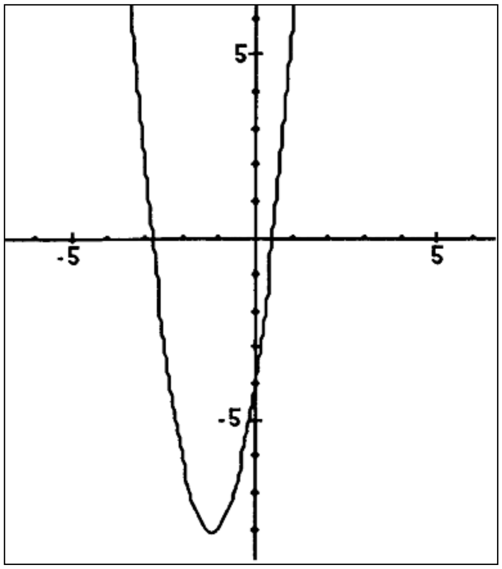
Vertex (3, 7)
Axis of symmetry: $x = 3$
Maximum value: $y = 7$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \leq 7\}$
Zeros: $x_1 = 5.65$, $x_2 = 0.35$
y-intercept = –2

b) $y = 3x^2 + 7x - 4$

Keying sequence:

[Y=] [3] [X,T,θ,n] [x^2] [+] [7] [X,T,θ,n] [–] [4]

Graph



Properties

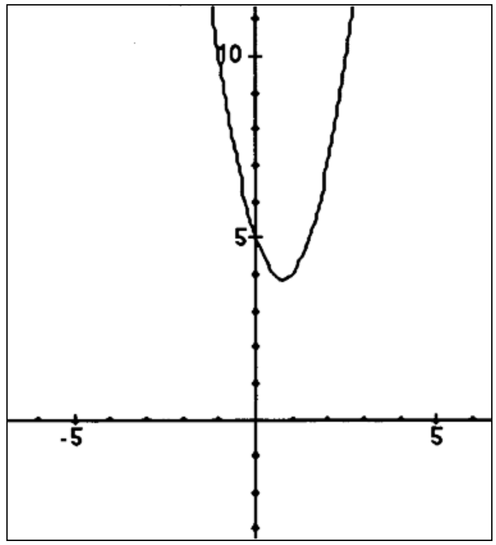
Vertex (–1.17, –8.08)
Axis of symmetry: $x = -1.17$
Minimum value: $y = -8.08$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \geq -8.08\}$
Zeros: $x_1 = -2.81$, $x_2 = 0.47$
y-intercept = –4

c) $y = 2x^2 - 3x + 5$

Keying sequence:

[Y=] [2] [X,T,θ,n] [x^2] [–] [3] [X,T,θ,n] [+] [5]

Graph



Properties

Vertex (0.75, 3.88)
Axis of symmetry: $x = 0.75$
Minimum value: $y = 3.88$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \geq 3.88\}$
Zeros: None
y-intercept = 5

3. It will determine whether the graph opens up or down, whether it has a maximum or a minimum value, and whether the function's range will be $y \leq$ some value or $y \geq$ some value.

Notes

Module 3, Lesson 2, Answer Key

Application of Quadratic Functions

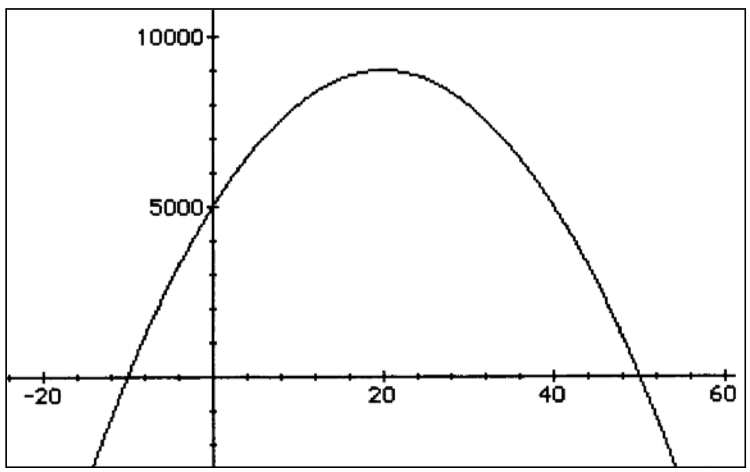
1. Let $\$y$ = gross monthly income
   Let x = the number of 10-cent decreases in subscription rate

   Gross income = number of subscribers x rate per subscription
   $y = (1000 + 100x)(5.00 - 0.10x)$
   $y = 5000 - 100x + 500x - 10x^2$
   $y = -10x^2 + 400x + 5000$

   Graph the function.
   Set window with x from –60 to + 60
   y from –1000 to 10 000

   Keying sequence:

   [Y=] [(–)] [10] [X,T,θ,*n*] [x^2] [+] [400] [X,T,θ,*n*] [+] [5000]

   Find the maximum value using the keying sequence:

   [2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (20, 9000).

   This indicates that the maximum Gross Income will be \$9000.00 brought about by twenty \$0.10 decreases in subscription rate to bring the rate down to \$(5.00 – 20(0.10)) = \$3.00.

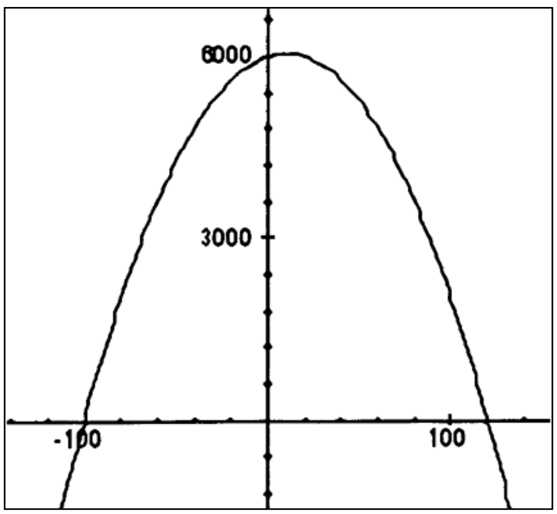
2. Let $\$y$ = income
   Let x = the number of additional passengers over 100

   Income = number of passengers x rate per ticket
   $y = (100 + x)(60.00 - 0.50x)$
   $y = 6000 - 50x + 60x - 0.5x^2$
   $y = -0.5x^2 + 10x + 6000$

   Graph the function.
   Set the window with x from –150 to 150
   y from –1000 to 7000

   Keying sequence:

   [Y=] [(–)] [0.5] [X,T,θ,*n*] [x^2] [+] [10] [X,T,θ,*n*] [+] [6000]

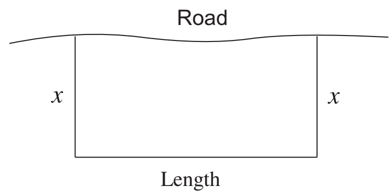


   Find the maximum value using the keying sequence:

   [2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (10, 6050).

   This indicates that the maximum income will be $6050.00 when there are 10 additional passengers, or a total of 110 passengers.

3. Let area of the parking lot be y yards^2
   Let the width of the parking lot be x yards

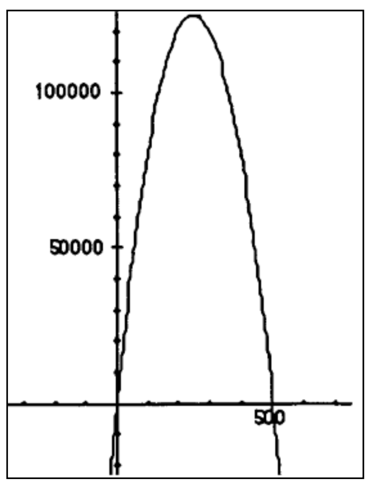


The length of the lot will be $1000 - 2x$ yards, since the length of the wire available for the three sides of the lot is 1000 yards.

Area = Length x Width
$y = (1000 - 2x)(x)$
$y = 1000x - 2x^2$
$y = -2x^2 + 1000x$

Graph the function.
Set the window with x from –100 to 600
y from –1000 to 150 000

Keying sequence:
[Y=] [(–)] [2] [X,T,θ,n] [x^2] [+] [1000] [X,T,θ,n]

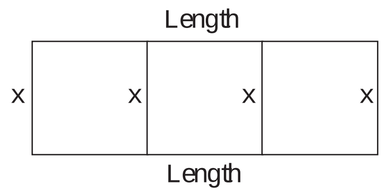


Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (250, 125 000).

This indicates that the maximum area will be 125 000 yards^2 when the width is 250 yards and the length is $1000 - 2(250) = 500$ yards.

4. Let the area be y yards^2
   Let the width of the lot be x yards



Then the length of the lot will be $\dfrac{660 - 4x}{2}$ yards because 660 yards of fencing are available.

Note: There are four "widths" of x yards and two "lengths".

Area = Length x Width

$$y = \left(\frac{660 - 4x}{2}\right)(x)$$

$$y = (330 - 2x)(x)$$

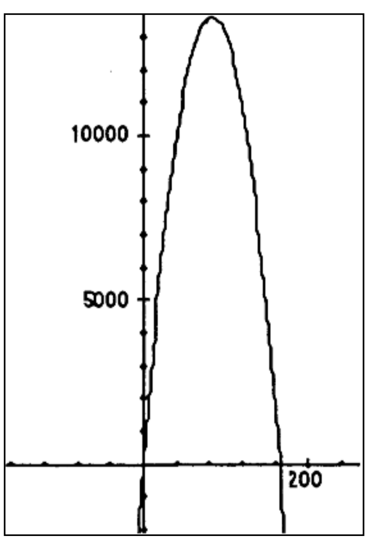
$$y = 330x - 2x^2$$

$$y = -2x^2 + 330x$$

Graph the function.
Set the window with x from –50 to 200
y from –1000 to 15 000

Keying sequence:

[Y=] [(–)] [2] [X,T,θ,n] [x^2] [+] [330] [X,T,θ,n]



Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (82.5, 13 612.5).

This indicates that the maximum area will be 13 612.5 yards^2 when the width is 82.5 yards and the length is $(330 - 2x)$ = 165 yards.

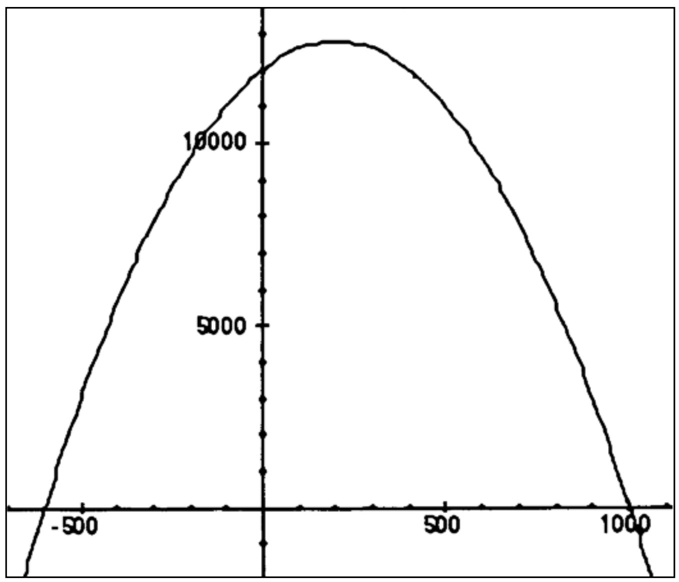
5. Let $\$y$ = profit
   Let x = the number of additional articles over 600

   Profit = number of articles x profit per article
   $y = (600 + x)(20.00 - 0.02x)$
   $y = 12\,000 - 12x + 20x - 0.02x^2$
   $y = -.02x^2 + 8x + 12\,000$

   Graph the function.
   Set the window with x from –600 to 1200
   y from –1000 to 15 000

   Keying sequence:

   [Y=] [(–)] [0.02] [X,T,θ,n] [x^2] [+] [8] [X,T,θ,n] [+] [12000]



   Find the maximum value using the keying sequence:

   [2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (200, 12 800).

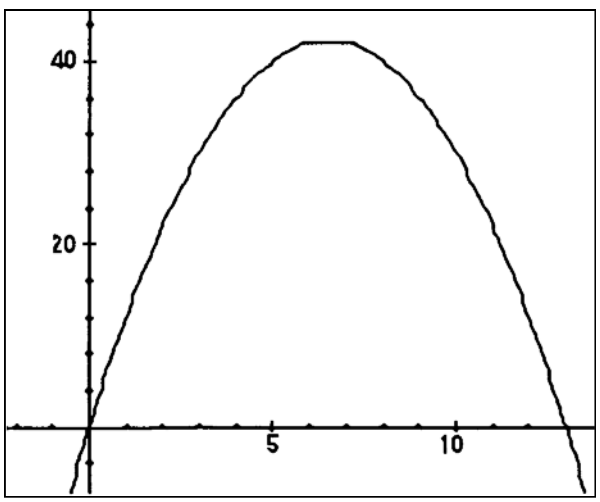
   This indicates that the maximum profit will be \$12 800.00 when 200 additional articles are made.

6. Let y = product
   Let x = the first of the numbers
   $13 - x$ = the second number since the sum of the two numbers is 13

   Product = first number x second number
   $y = (x)(13 - x)$
   $y = 13x - x^2$
   $y = -x^2 + 13x$

   Graph the function.
   Set the window with x from –5 to 15
   y from –5 to 50

   Keying sequence:
   [Y=] [(–)] [X,T,θ,n] [x^2] [+] [13] [X,T,θ,n]



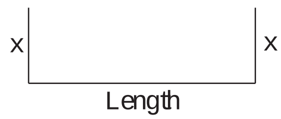
   Find the maximum value using the keying sequence:
   [2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (6.5, 42.25).

   This indicates that the maximum product will be 42.25 when the two numbers are 6.5 and 6.5.

7. Since the greatest capacity for the rain gutter is reached by the largest area of the cross section of the end of the gutter:

   Let y = area
   Let x inches = the width of the gutter

   Then the length of the gutter will be $12 - 2x$ inches, since the sum of the two widths and length of the cross section is 12 inches.



   Area = Length x Width

   $y = (12 - 2x)(x)$

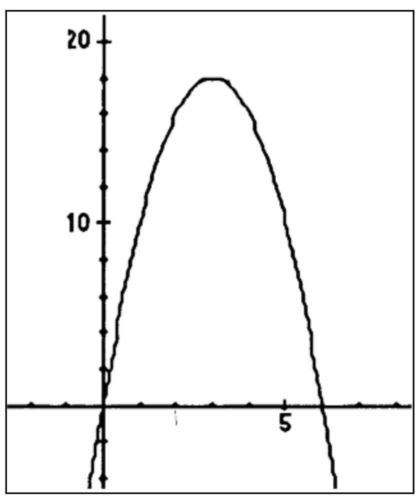
   $y = 12x - 2x^2$

   $y = -2x^2 + 12x$

   Graph the function.
   Set the window with x from –5 to 10
   y from –5 to 20

   Keying sequence:

   [Y=] [(–)] [2] [X,T,θ,n] [x^2] [+] [12] [X,T,θ,n]



   Find the maximum value using the keying sequence:

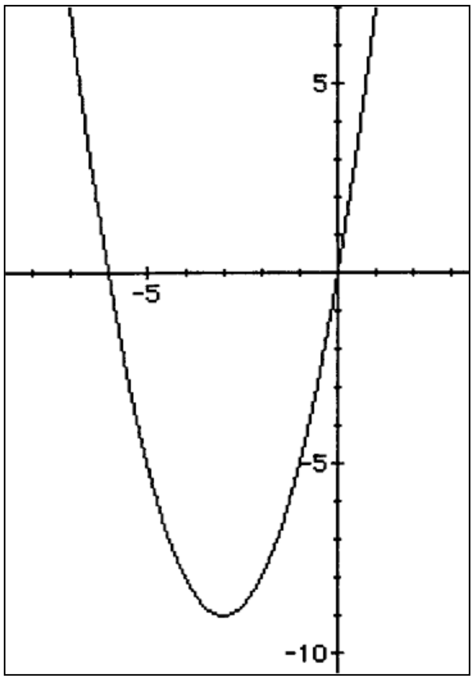
   [2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (3, 18).

   This indicates that the maximum area will be 18 in.^{2} brought about by a width of 3 inches and a length of $(12 - 2x) = 6$ inches.

8. Let the numbers be x and $x + 6$
   Product = $y = x(x + 6)$

   Keying sequence:

   [Y=] [X,T,θ,n] [(] [X,T,θ,n] [+] [6] [)]



   Minimum point is (–3, –9).
   Minimum value of the product is –9 and occurs when x = –3.
   Numbers are –3 and –3 + 6 = 3.

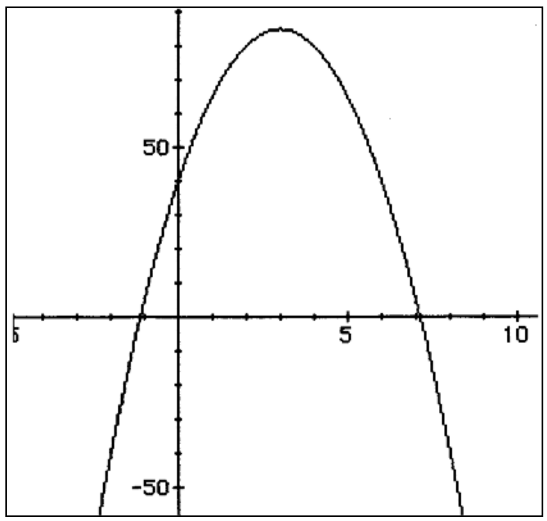
9. The quadratic function is given.

   Keying sequence:

   [Y=] [40] [+] [30] [X,T,θ,n] [–] [5] [X,T,θ,n] [x^2]

   To set window:

   Press [WINDOW]
   Xmin = –10
   Xmax = 10
   Ymin = 0
   Ymax = 100

   Press [2nd] [CALC] [4] and set Lower Bound = 0 and Upper Bound = 5 to obtain the maximum point (3, 85).

   a) Maximum height above the ground is 85 feet.

      Need to find the time (x) when the height (y) is 40, since the ball is thrown vertically upwards from the roof of a building 40 metres high.

      Use TRACE to find what x is when y = 40.

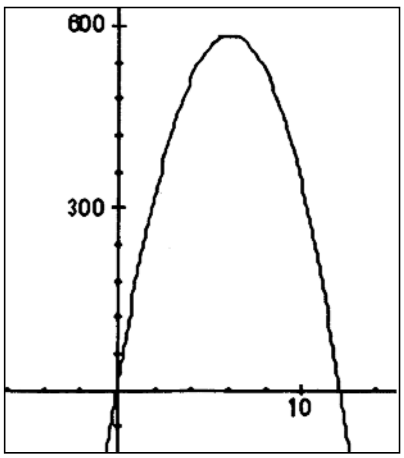
   b) When x = 6, y = 40. This means that after six seconds, the ball hits the roof again.

10. $h(t) = 6 + 192t - 16t^2$
$h(t) = -16t^2 + 192t + 6$

Graph the function.
Set the window with x from –5 to 15
y from –100 to 600

Keying sequence:

[Y=] [(–)] [16] [X,T,θ,n] [x^2] [+] [192] [X,T,θ,n] [+] [6]



Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the value (6, 582).

This indicates that the maximum height the object will reach will be 582 ft. after six seconds.

Find how many seconds it will take for the object to reach 518 ft. by tracing the graph until you hit y = 518 and read the value of x. Note: You will not necessarily hit 518 exactly. Use [2nd] [CALC] [1] and try x-values.

The object will reach a height of 518 ft. after four seconds. Look at the graph to see that the symmetry of the graph indicates that the height is 518 ft. for a second time after eight seconds.

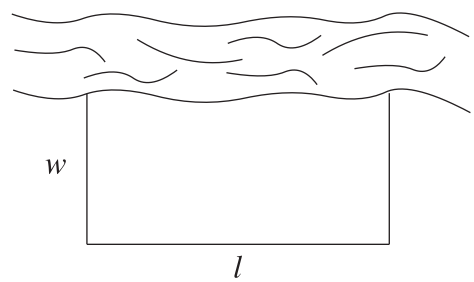
Module 3, Project, Answer Key

Nonlinear Functions

This project involves finding the dimensions of a field that give the greatest area and use only limited amount of fencing.

(*1 mark*) Solution of problem, diagrams, graph, description of method

(*6 marks*) Length of fencing available = 1000 yards.



Let the length be l yards and the area be A yards^2. Ordered pairs will be given by (l, A).

Here are some:

a) Length = 6 yards Area = $(6)\dfrac{(1000-6)}{2}$ = 2982 yards^2
Ordered pair (6, 2982)

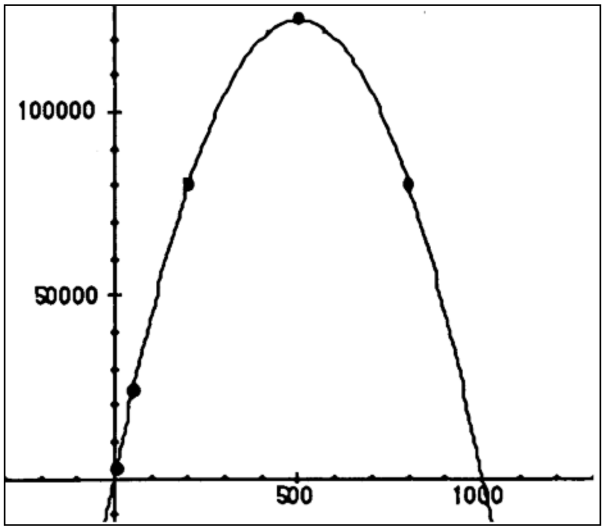
Length = 50 yards Area = $(50)\dfrac{(1000-50)}{2}$ = 23 750 yards^2
Ordered pair (50, 23 750)

Length = 200 yards Area = $(200)\dfrac{(1000-200)}{2}$ = 80 000 yards^2
Ordered pair (200, 80 000)

Length = 500 yards Area = $(500)\dfrac{(1000-500)}{2}$ = 125 000 yards^2
Ordered pair (500, 125 000)

Length = 800 yards Area = $(800)\dfrac{(1000-800)}{2}$ = 80 000 yards^2
Ordered pair (800, 80 000)

b)



c) Function given is
$y = -0.5x^2 + 500x$

d) Set window to show Xmin = –100
Xmax = 1100
Xscl = 100
Ymin = –5000
Ymax = 130 000

Find maximum point by using [2nd] [CALC] [4: maximum] and setting suitable Left and Right bounds or by tracing the graph. Maximum point is (500, 125 000).

The greatest area is 125 000 yards^2 and is obtained when the length is 500 yards and the width is 250 yards. (*1 mark*)

Overall organization. (*2 marks*)

Module 3, Lesson 3, Answer Key

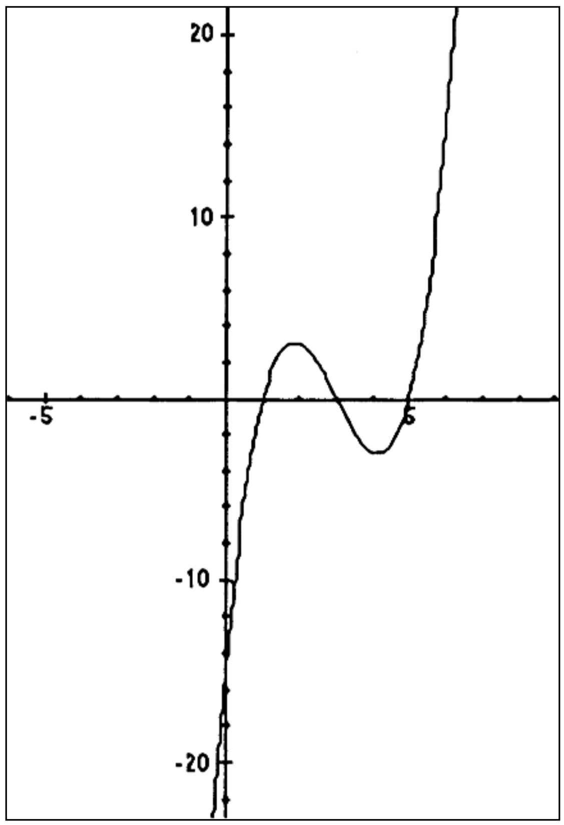
Cubic Functions and Their Properties

1. $f(x) = x^3 - 9\boldsymbol{x}^2 + 23x - 15$

   Keying sequence:

   [Y=] [X,T,θ,n] [^] [3] [–] [9] [X,T,θ,n] [x^2] [+] [23] [X,T,θ,n] [–] [15]

   Graph

   Properties

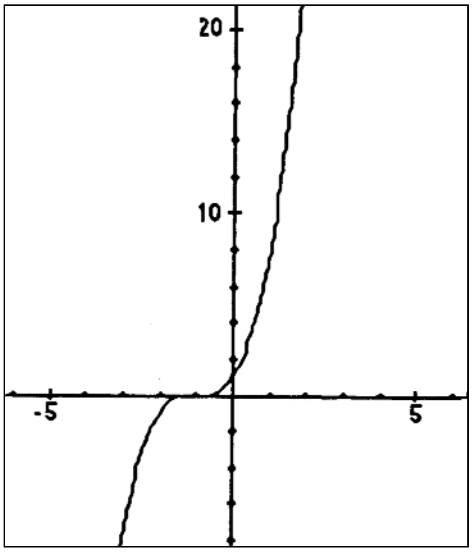
   Minimum point (4.15, –3.08)
   Maximum point (1.85, 3.08)
   Minimum value: $y = -3.08$
   Maximum value: $y = 3.08$
   Domain: $\{x \mid x \in \Re\}$
   Range: $\{y \mid y \in \Re\}$
   Zeros: $x_1 = 1$, $x_2 = 3$, $x_3 = 5$
   y-intercept: $y = -15$

2. $f(x) = x^3 + 3x^2 + 3x + 1$

   Keying sequence:

   [Y=] [X,T,θ,n] [^] [3] [+] [3] [X,T,θ,n] [x^2] [+] [3] [X,T,θ,n] [+] [1]

   Graph



   Properties

   Minimum value: None
   Maximum value: None
   Domain: $\{x \mid x \in \Re\}$
   Range: $\{y \mid y \in \Re\}$
   Zeros: $x_1 = -1$
   y-intercept: $y = 1$

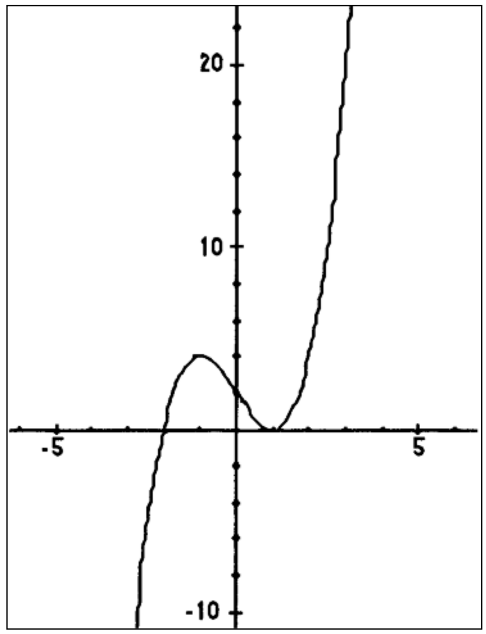
   Note: There are no turning points here.

3. $y = (x - 1)2\ (x + 2)$

Keying sequence:

[Y=] [(] [X,T,θ,n] [–] [1] [)] [^] [2] [x] [(] [X,T,θ,n] [+] [2] [)]

Graph



Default window is satisfactory.

In the diagram, the window is set to have

Xmin = –5
Xmax = 5
Ymin = –10
Ymax = 25

Properties

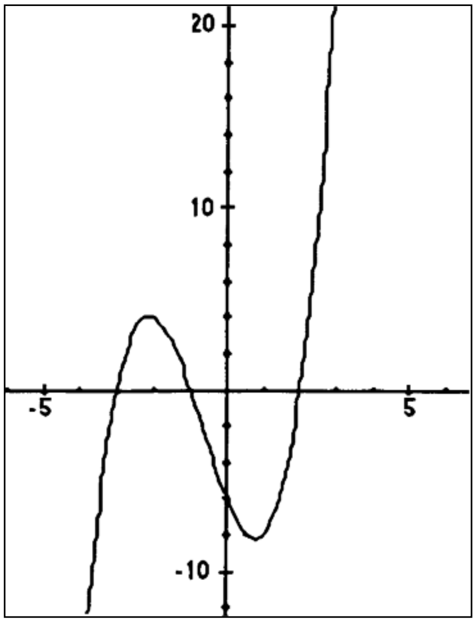
Minimum point: (1, 0)
Maximum point: (–1, 4)
Minimum value: $y = 0$
Maximum value: $y = 4$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \in \Re\}$
Zeros: $x_1 = -2$, $x_2 = 1$
y-intercept: $y = 2$

4. $y = (x + 3)(x - 2)(x + 1)$

   Keying sequence:

   [Y=] [(] [X,T,θ,n] [+] [3] [)] [x] [(] [X,T,θ,n] [–] [2] [)] [x] [(] [X,T,θ,n] [+] [1] [)]

   Graph



   Properties

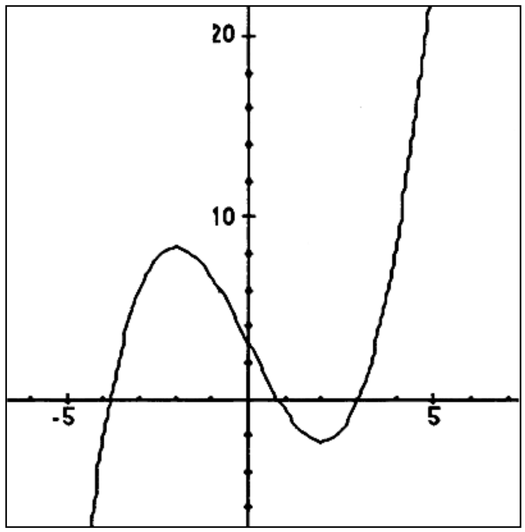
   Minimum point: (0.79, –8.21)
   Maximum point: (–2.12, 4.06)
   Minimum value: $y = -8.21$
   Maximum value: $y = 4.06$
   Domain: $\{x \mid x \in \Re\}$
   Range: $\{y \mid y \in \Re\}$
   Zeros: $x_1 = -3$, $x_2 = 2$, $x_3 = -1$
   y-intercept: $y = -6$

5. $y = \frac{1}{3}x^3 - 4x + 3$

Keying sequence:

[Y=] [1] [÷] [3] [X,T,θ,n] [^] [3] [–] [4] [X,T,θ,n] [+] [3]

Graph



Properties

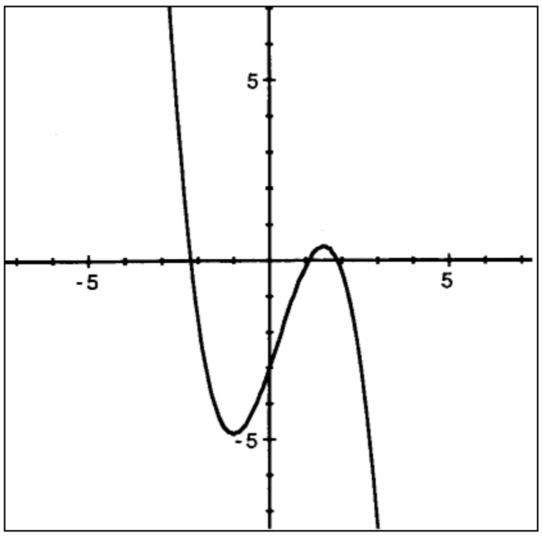
Minimum point: (2, –2.33)
Maximum point: (–2, 8.33)
Minimum value: $y = -2.33$
Maximum value: $y = 8.33$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \in \Re\}$
Zeros: $x_1 = -3.79$, $x_2 = 0.79$, $x_3 = 3$
y-intercept: $y = 3$

6. $y = -\frac{2}{3}x^3 + \frac{1}{2}x^2 + 3x - 3$

Keying sequence:

[Y=] [(–)] [2] [÷] [3] [X,T,θ,n] [^] [3] [+] [1] [÷] [2] [X,T,θ,n] [x^2] [+] [3] [X,T,θ,n] [–] [3]

Graph



Properties

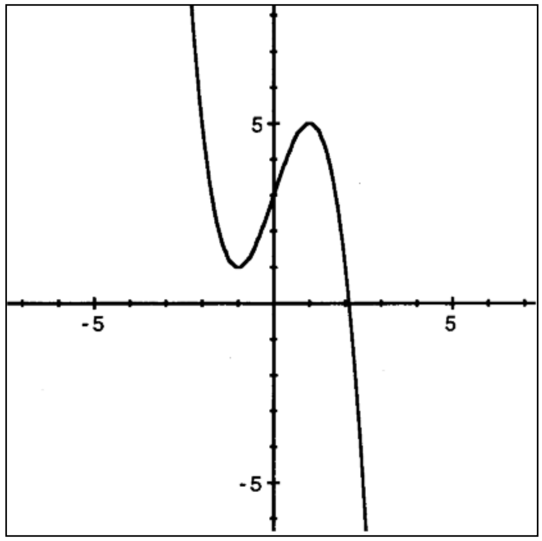
Minimum point: (–1, –4.83)
Maximum point: (1.50, 0.375)
Minimum value: $y = -4.83$
Maximum value: $y = 0.375$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \in \Re\}$
Zeros: $x_1 = -2.21$, $x_2 = 1.09$, $x_3 = 1.87$
y-intercept: $y = -3$

7. $y = -x^3 + 3x + 3$

Keying sequence:

[Y=] [(–)] [X,T,θ,n] [^] [3] [+] [3] [X,T,θ,n] [+] [3]

Graph



Properties

Minimum point: (–1, 1)
Maximum point: (1, 5)
Minimum value: $y = 1$
Maximum value: $y = 5$
Domain: $\{x \mid x \in \Re\}$
Range: $\{y \mid y \in \Re\}$
Zeros: $x = 2.1$
y-intercept: $y = 3$

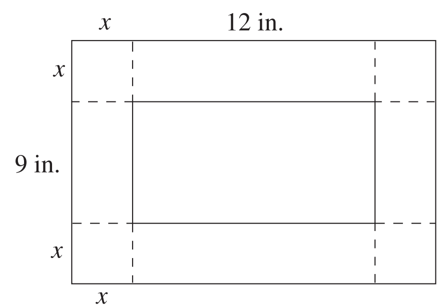
Notes

Module 3, Lesson 4, Answer Key

Applications of Cubic Functions

1. Let length of square to be cut out be x in.
   Let volume of tray be V in.^{3}

   V = l x w x d
   V = $(12 - x)(9 - 2x)(x)$ (see diagram)
   V = $(108 - 24x - 9x + 2x^2)(x)$
   V = $108x - 33x^2 + 2x^3$
   V = $2x^3 - 33x^2 + 108x$

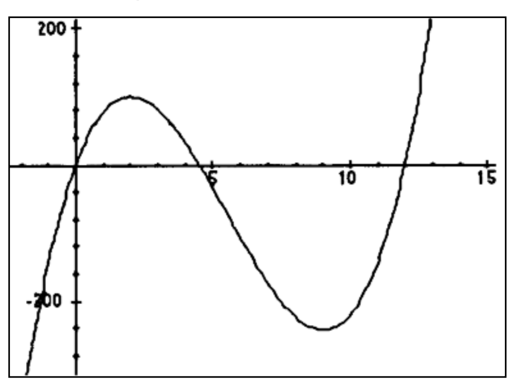


Keying sequence:

[Y=] [2] [X,T,θ,n] [^] [3] [–] [33] [X,T,θ,n] [x^2] [+] [108] [X,T,θ,n]

Note: Press [WINDOW]
Set Xmin = –5
Xmax = 15
Ymin = –300
Ymax = 300

Graphing:



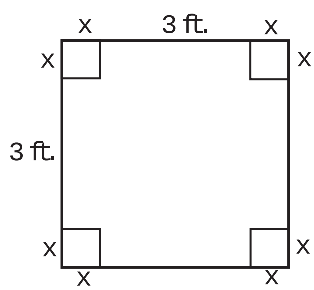
Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate Left and Right bounds, and then pressing [ENTER] to find the maximum point (2,100).

This indicates that the maximum volume will be 100 in.^{3} when a square of side 2 in. is cut out of each of the two corners.

2. Let side length of square to be cut out be x ft.
   Let volume of box be V ft.^{3}

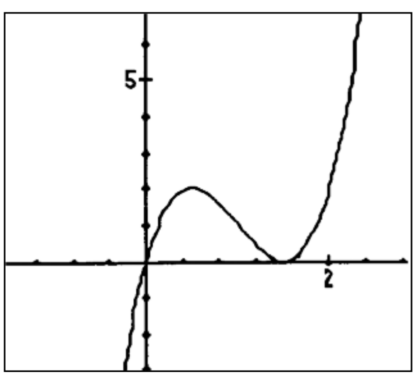
   $V = l \times w \times d$
   $V = (3 - 2x)(3 - 2x)(x)$ (see diagram)
   $V = (9 - 6x - 6x + 4x^2)(x)$
   $V = 9x - 12x^2 + 4x^3$
   $V = 4x^3 - 12x^2 + 9x$



Keying sequence:

[Y=] [4] [X,T,θ,n] [^] [3] [–] [12] [X,T,θ,n] [x^2] [+] [9] [X,T,θ,n]

Graphing:

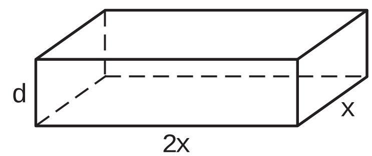


Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate Left and Right bounds, and then pressing [ENTER] to find the maximum point (0.5, 2).

This indicates that the maximum volume will be 2 ft.^{3} when a square of side 0.5 ft. is cut out of each corner.

3. Let width of the box be x in.
   Then length of the box is $2x$ in.
   Let the depth be d in.



Area of bottom = area of top = $(2x)(x)$ in.^{2}
$= 2x2$ in.^{2}

Area of sides = $xd + xd + 2xd + 2xd$ in.^{2}
$= 6\ xd$ in.^{2}

Surface area = $6x \times d + 2(2x^2)$ in.^{2}
$54 = 6x \times d + 4x^2$

$$d = \frac{54 - 4x^2}{6x}$$

Let volume of box be V in.^{3}
V = length x width x depth

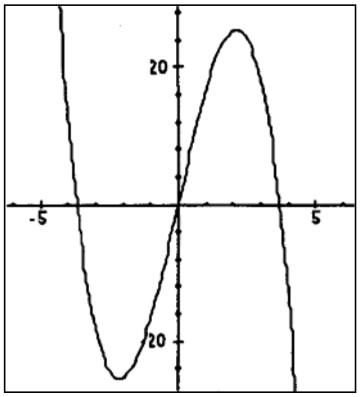
$$V = (2x)(x)\left(\frac{54-4x^2}{6x}\right)$$

$$V = 2x^2\left(\frac{54-4x^2}{6x}\right) = \frac{108x-8x^3}{6} = 18x - \frac{4}{3}x^3 = -\frac{4}{3}x^3 + 18x$$

Keying sequence:

[Y=] [(–)] [4] [÷] [X,T,θ,n] [^] [3] [+] [18] [X,T,θ,n]

Graphing:



Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate Left and Right bounds, and then pressing [ENTER] to find the value (2.12, 25.46).

This indicates that the maximum volume will be 25.46 in.^{3}.

Width = 2.12 in.

Length = 4.24 in.

Depth = 2.83 in. $\left(\text{Substitute } x = 2.12 \text{ into } \dfrac{54-4x^2}{6x}\right)$

4. Let radius of the can be x in.
   Let height of the can be h in.
   Let volume of the can be V in.^{3}

$$\text{Because } V_{\text{cylinder}} = \pi r^2 h$$

$$V_{\text{cylinder}} = \pi x^2 h$$

$$\text{But, } SA_{\text{open cylinder}} = \pi r^2 + 2\pi r h$$

$$\text{Cost of materials for the open cylinder} = (0.05)\pi x^2 + (0.03)2\pi x h$$

$$7.35 = 0.05\pi x^2 + 0.06\pi x h$$

$$\text{And, } h = \frac{7.35 - 0.05\pi x^2}{0.06\pi x}$$

$$\text{Therefore, } V_{\text{cylinder}} = \quad x^2\left(\frac{7.35-0.05\ \ x^2}{0.06\ \ x}\right)$$

$$V_{\text{cylinder}} = \frac{7.35\ \ x^2 - 0.05\ \ ^2x^4}{0.06\ \ x}$$

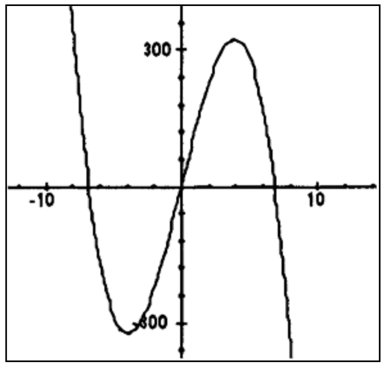
$$V_{\text{cylinder}} = 122.5x - 2.62x^3$$

$$V_{\text{cylinder}} = -2.62x^3 + 122.5x$$

Keying sequence:

[Y=] [(–)] [2.62] [X,T,θ,n] [^] [3] [+] [122.5] [X,T,θ,n]

Graphing:

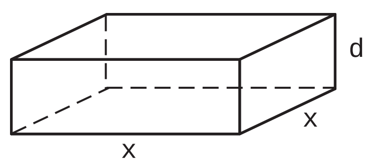


Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate Left and Right bounds, and then pressing [ENTER] to find the value (3.95, 322.4).

This indicates that the maximum volume will be 322.4 in.^{3}. The radius of the bottom is 3.95 in and the height of the cylinder is $\frac{7.35 - 0.05\pi(3.95)^2}{0.06\pi(3.95)} = 6.58$ in.

5. Let width of the box be x metres
Then length of the box is x metres
Let depth of box be d metres



The total length of all edges of the box = $4d + 8x$ metres. (See diagram.)

Therefore, $6 = 4d + 8x$

$$d = \frac{6 - 8x}{4}$$

Let V metres = volume of box

Volume = length x width x depth

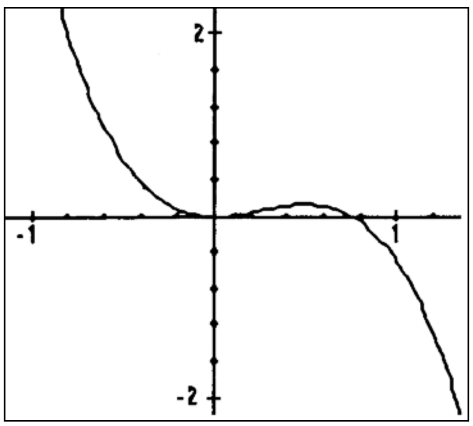
$$V = (x)(x)\left(\frac{6-8x}{4}\right)$$

$$V = x^2\left(\frac{6-8x}{4}\right) = \left(\frac{6x^2-8x^3}{4}\right) = \frac{3}{2}x^2 - 2x^3 = -2x^3 + \frac{3}{2}x^2$$

Keying sequence:

[Y=] [(–)] [2] [X,T,θ,*n*] [^] [3] [+] [3] [÷] [2] [X,T,θ,*n*] [x^2]

Graphing:



Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate Left and Right bounds, and then pressing [ENTER] to find the maximum point (0.5, 0.125).

This indicates that the maximum volume will be 0.125 m^3 when $x = 0.5$.
Therefore, width and length are each 0.5 m.

Depth = $\frac{6 - 8(0.5)}{4}$ = 0.5 m

Therefore, all twelve pieces are 0.5 metres long.

6. Let radius of the can be x inches
   Let height of the can be h inches
   Let volume of the can be V inches^3

$$SA_{\text{closed cylinder}} = 2\pi r^2 + 2\pi rh$$

Cost of materials for the closed cylinder = $(0.06)2\pi x^2 + (0.03)2\pi xh$

$$1.44 = 0.12\ \ x^2 + 0.06\ \ xh$$

$$\text{And, } h = \frac{1.44 - 0.12\ \ x^2}{0.06\ \ x}$$

$$\text{Because } V_{\text{cylinder}} = \ \ r^2 h$$

$$V_{\text{cylinder}} = \ \ x^2 h$$

$$\text{Therefore, } V_{\text{cylinder}} = \ \ x^2\left(\frac{1.44 - 0.12\ \ x^2}{0.06\ \ x}\right)$$

$$V_{\text{cylinder}} = \frac{1.44\ \ x^2 - 0.12\ \ ^2x^4}{0.06\ \ x}$$

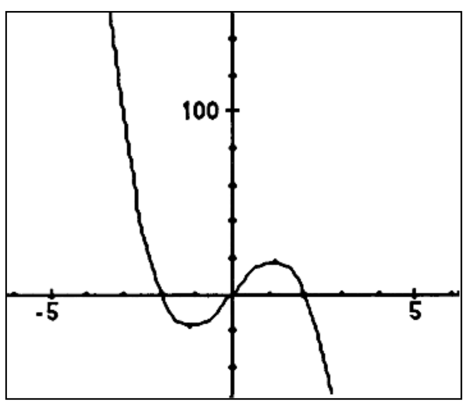
$$V_{\text{cylinder}} = 24x - 6.28x^3$$

$$V_{\text{cylinder}} = -6.28x^3 + 24x$$

Keying sequence:

[Y=] [(–)] [6.28] [X,T,θ,n] [^] [3] [+] [24] [X,T,θ,n]

Graphing:



Find the maximum value using the keying sequence:

[2nd] [CALC] [4], setting appropriate Left and Right bounds, and then pressing [ENTER] to find the maximum point (1.13, 18.06).

This indicates that the maximum volume will be 18.06 in.^{3} and radius = 1.13 in. and height = $\dfrac{1.44 - 0.12\pi(1.13)^2}{0.06\pi(1.13)}$ = 4.5 in.

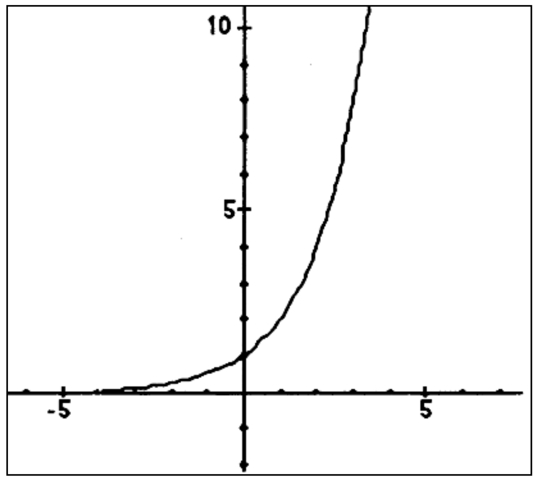
Notes

Module 3, Lesson 5, Answer Key

Exponential Functions and Their Properties

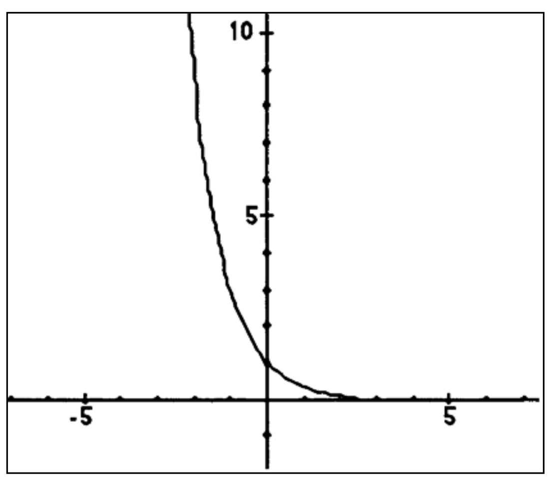
1. a) $y = 2x$

   Keying sequence: [Y=] [2] [^] [X,T,θ,n]
   Press [WINDOW]
   Set Ymin = –2



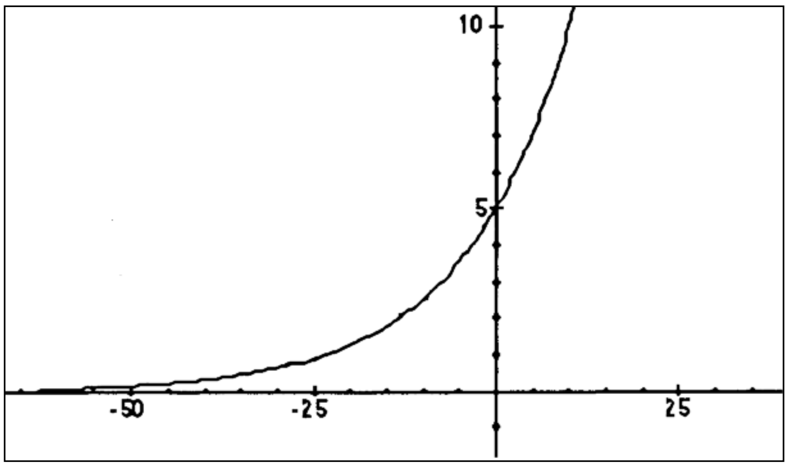
   b) $y = \left(\frac{1}{3}\right)^x$

   Keying sequence: [Y=] [(] [1] [÷] [3] [)] [^] [X,T,θ,n]
   Press [WINDOW]
   Set Ymin = –2



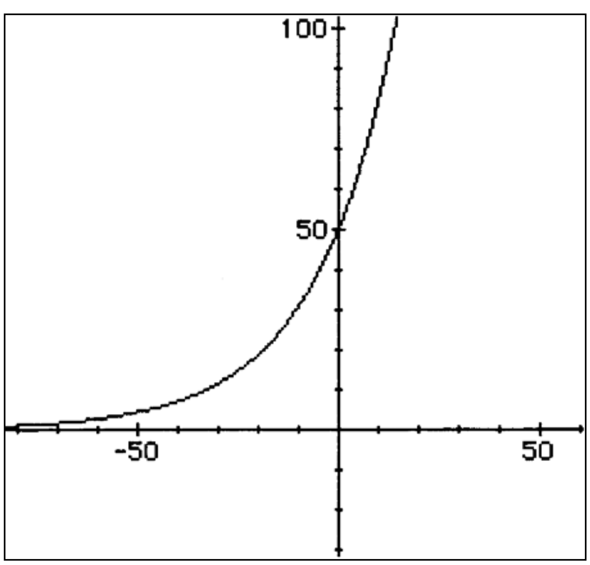
c) $y = 5 \cdot (10)^{0.03x}$

Keying sequence: [Y=] [5] [x] [10] [^] [(] [0.03] [X,T,θ,n] [)]
Press [WINDOW]
Set Xmin = –60
Xmax = 30
Ymin = –2
Ymax = 15



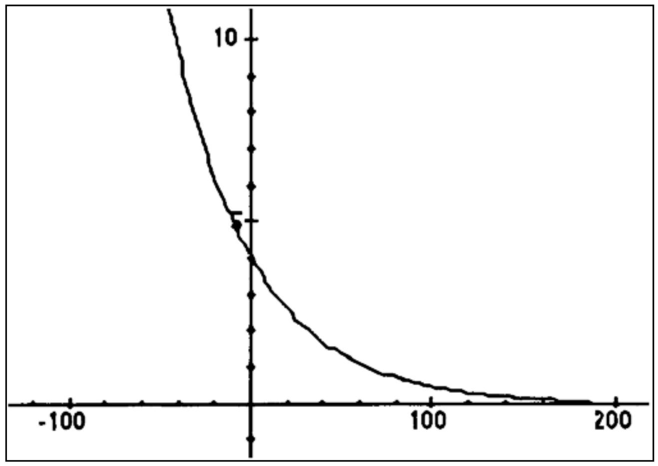
d) $y = 50 \cdot (1.05)x$

Keying sequence: [Y=] [50] [x] [1.05] [^] [X,T,θ,n]
Press [WINDOW]
Set Xmin = –100
Xmax = 50
Ymin = –10
Ymax = 100



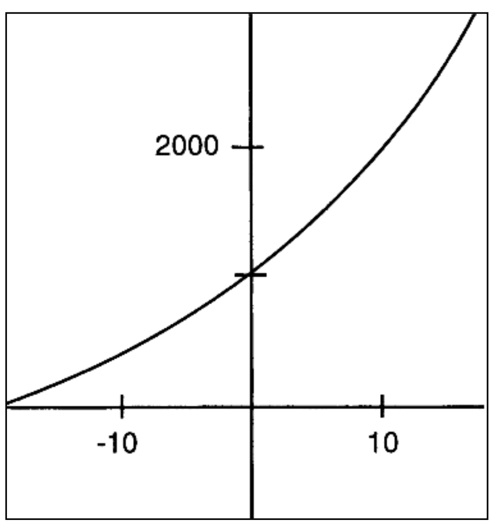
e) $y = 4 \cdot (3)^{-0.02x}$

Keying sequence: [Y=] [4] [x] [3] [^] [(] [(–)] [0.02] [X,T,θ,n] [)]
Press [WINDOW]
Set Xmin = –100
Xmax = 100
Ymin = –2
Ymax = 10



f) $y = 1000 \cdot (1.075)x$

Keying sequence: [Y=] [1000] [x] [1.075] [^] [X,T,θ,n]
Press [WINDOW]
Set Xmin = –20
Xmax = 20
Ymin = –100
Ymax = 2000

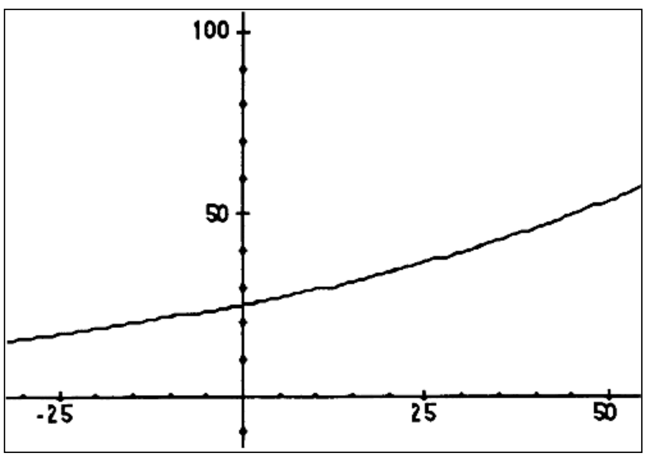


2. For any exponential function of the form $y = a \cdot bx$, if the base, b, is less than 1 but larger than 0, or if the exponent has a negative number associated with it, the function will decrease. If $b > 1$ **and** the exponent has a positive number associated with it, the function will increase.

3. a) Current population = 25 000 000
   Therefore $N_0 = 25$
   $N(t) = 25(10)^{0.0066t}$

   Graph the function:

   Keying sequence: [Y=] [25] [x] [10] [^] [(] [0.0066] [X,T,θ,n] [)]
   Press [WINDOW]
   Set Xmin = –25
   Xmax = 50
   Ymin = –10
   Ymax = 100



   To find the population two years from now:

   Keying sequence: [2nd] [CALC] [1] then press [2] [ENTER]. This gives $y = 25.77$. This indicates that in two years from the present, the population will be 25.77 million, or 25 770 000.

   b) To find the population 12 years from now:

   Keying sequence: [2nd] [CALC] [1] then press [12] [ENTER]. This gives $y = 30.00$. This indicates that in 12 years from the present, the population will be 30.00 million, or 30 000 000.

c) To find when the population will be 36 000 000:

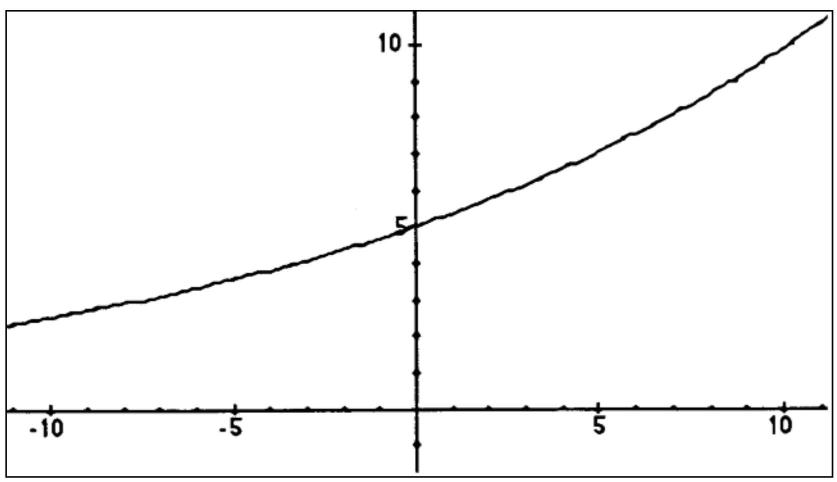
Keying sequence: [TRACE] and then use [▶] or [◀] until the coordinates show $y = 36.00$. You will probably be able to get something close to this, e.g., $x = 23.7$, $y = 35.82$.

Now use keying sequence [2nd] [CALC] [1] [24] to show that $y = 36.00$ when $x = 24$. The population is expected to reach 36 million 24 years from now.

4. For this situation, the investment is 5 thousands of dollars. The rate in decimal form is 0.07. The value of the investment after x years will be given in thousands of dollars by the function $y = 5(1.07)^x$.

Graph the function:

Keying sequence: [Y=] [5] [x] [1.07] [^] [X,T,θ,n]
Press [WINDOW]
Set Xmin = –10
Xmax = 10
Ymin = 0
Ymax = 10



a) To find the value after five years:

Keying sequence: [2nd] [CALC] [1] [5] [ENTER]. This gives $x = 5$, $y = 7.012759$. This indicates that after five years the value is 7.012759 thousands of dollars or $7012.76.

b) To find the time after which the investment reaches $8000.00, i.e., $y = 8$:

Use [TRACE] and the cursors until you reach the point where $y = 8$, or very close to $y = 8$, e.g., $x = 7.021$, $y = 8.0405$. To get a value for y closer to 8.000, you may use [2nd] [CALC] [1] and then enter a slightly smaller value for x, e.g., $x = 7$ or $x = 6.9$. The closer solution is $x = 6.95$. After 6.95 years, the value of the investment is very close to $8000.

c) To find the time taken for the investment to double in value, $100 000, you need to find the time after which $y = 10$. Repeat the procedure of 4b using [TRACE] and [2nd] [CALC] [1] to get $y = 10$. This occurs when $x \approx 10.25$. After 10.25 years, the investment doubles in value.

5. Function giving quantity remaining after t days is $y = 200(0.75)t$.

Keying sequence: [Y=] [200] [x] [0.75] [^] [X,T,θ,n]
Press [WINDOW]
Set Xmin = 0
Xmax = 5
Ymin = 0
Ymax = 200

a) Find amount after three days.
Keying sequence: [2nd] [CALC] [1] [3] [ENTER]
This gives $y = 84.375$.
Amount after three days = 84.375 g

b) Find time to decrease to 60 g.
Press [TRACE] [▶] until $y = 60$.
This process gives $x \approx 4.15$ $y = 60.62$
and $x \approx 4.20$ $y = 59.71$
The amount reaches 60 g after about 4.18 days.

c) For half-life, find time to decrease to 100 g.
Press [TRACE] and use cursor until $y = 100$.
This occurs when $x \approx 2.4$. The half-life is 2.4 days.

6. a) Half-life of strontium is 25 years, therefore

$M(t) = M_0 \cdot 2^{\frac{-t}{25}}$, $M_0 = 100$

Time	**Mass**
0	100 g
25	50 g
50	25 g

Mass after 50 years is 25 g.

b) Half-life of polonium is 140 days, therefore

$M(t) = M_0 \cdot 2^{\frac{-t}{140}}$, $M_0 = 1$

Time	**Mass**
0	1 g
140	0.5 g
280	0.25 g

At this point 0.75 g has been lost.
Therefore, time taken is 280 days.

c) 320 g decays to 10 g in 40 days.
Let half-life be x days, therefore

$M(t) = M_0 \cdot 2^{\frac{-t}{x}}$, $M_0 = 320$

Time	**Mass**
0	320 g
x	160 g
$2x$	80 g
$3x$	40 g
$4x$	20 g
$5x$	10 g

Hence, $5x = 40$ $\quad x = 8$
Half-life is eight days.

(Note: Half-life of radioactive iodine is actually 8.1 days.)

Notes

Self-Test Answer Key

Module 3 – Nonlinear Functions

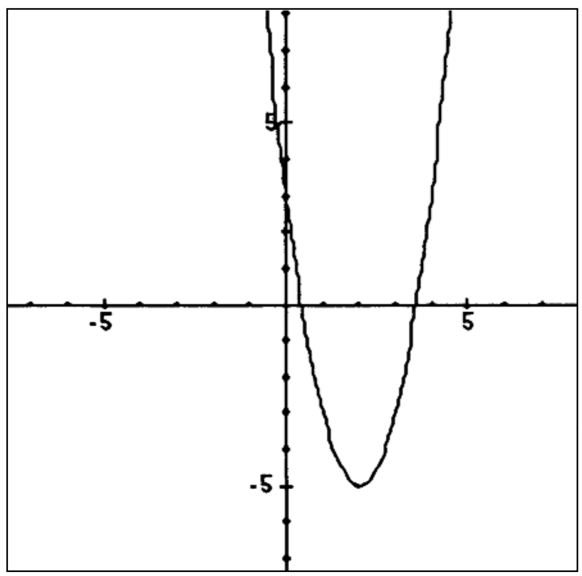
(3 marks)

1. Vertex (–3, 4)
   Axis of symmetry $x = -3$
   Minimum Value $y = 4$
   Domain $\{x \mid x \varepsilon \text{R}\}$
   Range $\{y \mid y \geq 4\}$

(3 marks)

2. a) $y = 2x^2 - 8x + 3$

   Keying sequence:
   [Y=] [2] [X,T,θ,n] [x^2] [–] [8] [X,T,θ,n] [+] [3]



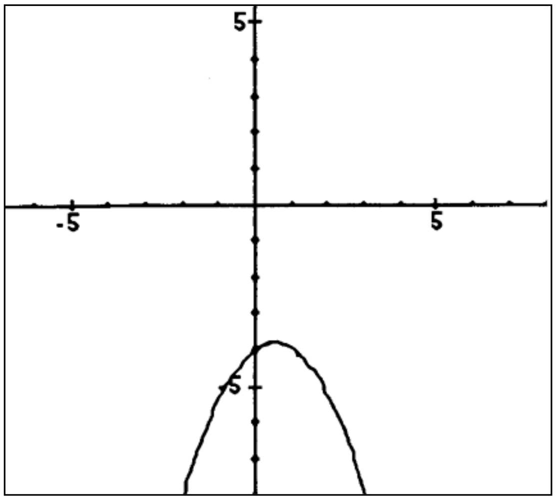
Vertex (2, –5)
Axis of symmetry $x = 2$
Minimum Value $y = -5$
Domain $\{x \mid x \varepsilon \text{R}\}$
Range $\{y \mid y \geq -5\}$

Note: Find the minimum point using the keying sequence: [2nd] [CALC] [3], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (2, –5).

b) $y = \frac{-2}{3}x^2 + \frac{5}{7}x - 4$ *(3 marks)*

Keying sequence:

[Y=] [(–)] [2] [÷] [3] [X,T,θ,*n*] [x^2] [+] [5] [÷] [7] [X,T,θ,*n*] [–] [4]



Vertex	(0.54, –3.81)
Axis of symmetry	$x = 0.54$
Maximum Value	$y = –3.81$
Domain	$\{x \mid x \in R\}$
Range	$\{y \mid y \le –3.81\}$

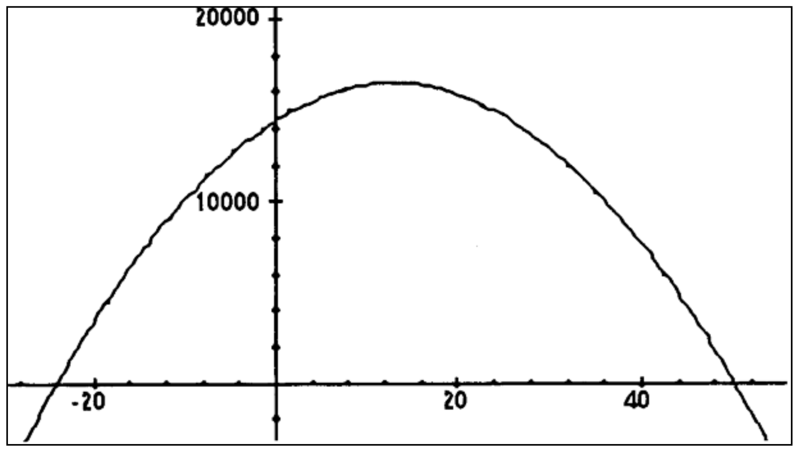
Note: Find the maximum point using the keying sequence: [2nd] [CALC] [4], setting left and right bounds, and then pressing [ENTER] to find the point (0.54, –3.81).

(5 marks)

3. a) Let y = yield of apples per tree
Let x = the number of additional trees per acre
Yield = number of trees x number of apples per tree
$y = (24 + x)(600 - 12x)$
$y = 14\,400 - 288x + 600x - 12x^2$
$y = -12x^2 + 312x + 14\,400$

Graph the function.

Keying sequence:
[Y=] [(–)] [12] [X,T,θ,n] [x^2] [+] [312] [X,T,θ,n] [+] [14 400]



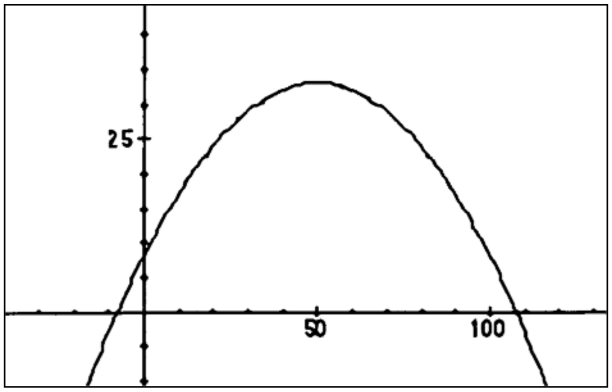
Find the maximum point using the keying sequence:

[2nd] [CALC] [4], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (13, 16 428).

This indicates that the maximum yield of apples will be 16 428 brought about by 13 additional trees per acre.

b) Graph the function.

Keying sequence:
[Y=] [(–)] [1] [÷] [100] [X,T,θ,n] [x^2] [+] [X,T,θ,n] [+] [8]



Reset window to show graph. Use the [TRACE] [◀] or [▶] to find suitable values for the window.

Note: If you are not sure about how to reset the window, look at the information in Module 3, Lessons 3, 4, and 5 from the *Senior 2 Applied Mathematics* course.

i) Find the maximum point using the keying sequence: [2nd] [CALC] [4], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (50, 33). *(1 mark)*

The maximum height reached is 33 ft.

ii) The horizontal distance travelled to reach its maximum height is 50 ft. *(1 mark)*

iii) The human cannonball travels 100 ft horizontally to reach the net. This is found by tracing the function until y is approximately 8. *(2 marks)*

At $x = 99.25$, $y = 8.73$

At $x = 100.63$, $y = 7.35$

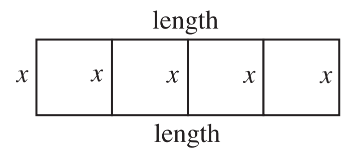
Continue to use [TRACE] and [ZOOM] [2] to reach a value for y that is closer to 8 or guess and enter $x = 100$ [100] [ENTER] to show $y = 8$.

iv) After travelling 72 ft horizontally, the human cannonball is about 28 ft above the ground. *(1 mark)*

Use [2nd] [CALC] [1] and [72] [ENTER] to give $y = 28$

(5 marks)

c) Let area = y square feet
Let the width of the lot be x feet.



Then the length of the lot will be $\frac{750 - 5x}{2}$ feet since the total length of five widths and two lengths of the lot is 750 feet.

Area = Length × Width

$$y = \left(\frac{750 - 5x}{2}\right)(x)$$

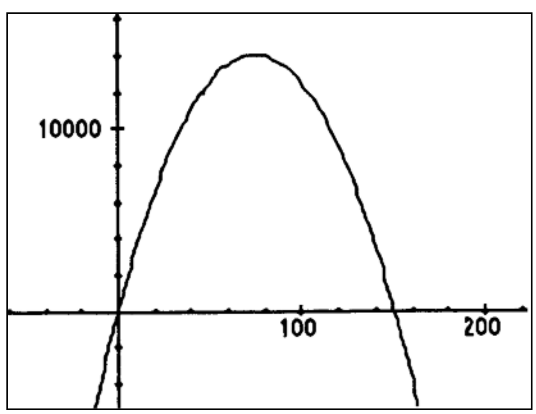
$$y = \left(375 - \frac{5}{2}x\right)(x)$$

$$y = 375x - \frac{5}{2}x^2$$

$$y = -\frac{5}{2}x^2 + 375x$$

Graph the function.

Keying sequence:
[Y=] [(–)] [5] [÷] [2] [X,T,θ,n] [x^2] [+] [375] [X,T,θ,n]



Reset the window as necessary

Find the maximum point using the keying sequence:

[2nd] [CALC] [4], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (75, 14 062.5).

This indicates that the maximum area will be 14 062.5 ft.^{2} brought about by a width of 75 ft. and a length of $\left(375 - \frac{5}{2}x\right) = 187.5$ ft.

4. Maximum value $y \approx 18.5$ where $x \approx -1.3$ *(4 marks)*

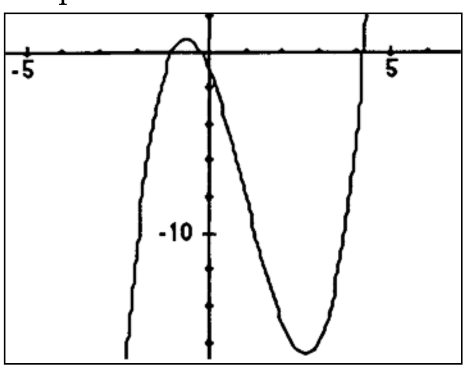
Minimum value $y = 0$ where $x = 2$

Zeros $x_1 = -3$ and $x_2 = 2$

5. a) $y = x^3 - 3x^2 - 5x - 1$ *(3 marks)*

Keying sequence:
[Y=] [X,T,θ,n] [^] [3] [–] [3] [X,T,θ,n] [x^2] [–] [5] [X,T,θ,n] [–] [1]

Graph:



Reset the window

Properties

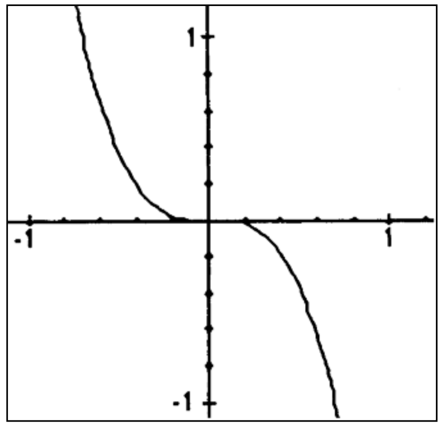
Use [2nd] [CALC] [4] and [2nd] [CALC] [3] to find maximum and minimum values.

Maximum value $y = 0.71$ where $x = -0.63$
Minimum value $y = -16.71$ where $x = 2.63$
Zeros $x_1 = -1$, $x_2 = -0.24$, $x_3 = 4.24$

b) $y = -3x^3$ *(3 marks)*

Keying sequence:
[Y=] [(–)] [3] [X,T,θ,n] [^] [3]

Graph:



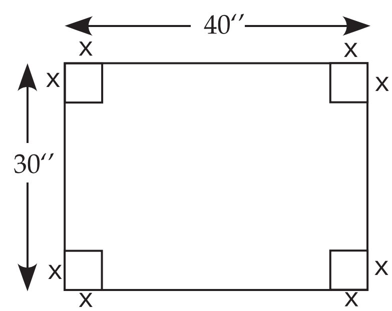
Reset the window.

Properties

Use the graph to see that:
No maximum point exists and
No minimum point exists
Zeros $x_1 = 0$

(5 marks)

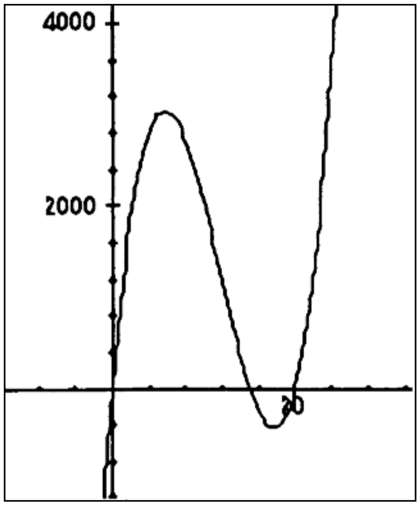
6. a) Let side length of square to be cut out be x inches
Let volume of box be V cubic inches



$V = l \times w \times d$
$V = (40 - 2x)(30 - 2x)(x)$ (see diagram)
$V = (1200 - 80x - 60x + 4x^2)(x)$
$V = 1200x - 140x^2 + 4x^3$
$V = 4x^3 - 140x^2 + 1200x$

Keying sequence:
[Y=] [4][X,T,θ,n] [^] [3] [–] [140] [X,T,θ,n] [x^2] [+] [1200] [X,T,θ,n]

Graph:



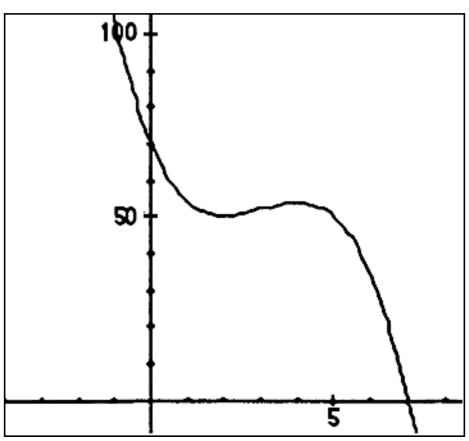
Find the maximum point using the keying sequence:

[2nd] [CALC] [4], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (5.66, 3032.3).

This indicates that the maximum volume will be 3032.3 in^3 when a 5.66 inch × 5.66 inch square is cut out of the four corners.

b) $V = -T^3 + 9T^2 - 24T + 70$ *(3 marks)*
Graph:

Keying sequence:
[Y=] [(–)] [X,T,θ,n] [^] [3] [+] [9] [X,T,θ,n] [x^2] [–] [24] [X,T,θ,n] [+] [70]



Reset window.

i) Find the values of y when $x = 0$ by using the keying sequence [2nd] [CALC] [1] [0] to give the point (0,70). Repeat using [2nd] [CALC] [1] [5] and then [2nd] [CALC] [1] [6] to find (5, 50) and (6, 34).

When temperature is 0°,
Viscosity = 70

When temperature is 500°,
Viscosity = 50

When temperature is 600°,
Viscosity = 34.

Recall: T = 1 means T = 100°

ii) Find the maximum point using the keying sequence:

[2nd] [CALC] [4], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (4, 54). The maximum viscosity is 54 when the temperature is 400°.

Find the minimum point using the keying sequence:

[2nd] [CALC] [3], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (2, 50). The minimum viscosity is 50 when the temperature is 200°.

iii) Use the keying sequence [2nd] [CALC] [2], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (7, 0). Viscosity is 0 when temperature reaches 700°.

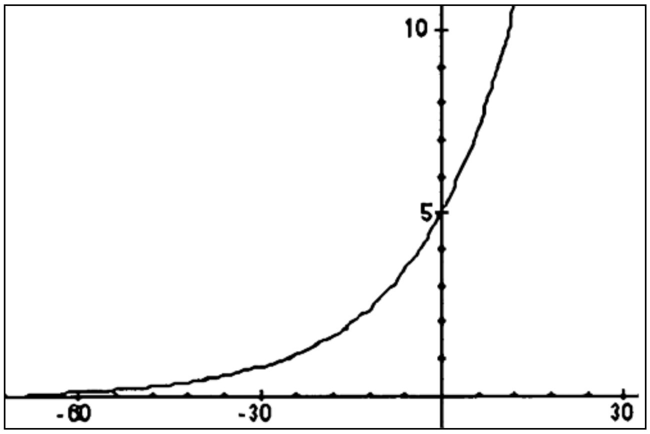
(2 marks) 7. a) $A = 5000\,(1 + 1.065)^n$

Graph the function $y = 5(1.065)^x$

Note: y will give the amount in thousands of dollars.

Graphing:

Keying sequence:
[Y=] [5] [x] [1.065] [^] [X,T,θ,n]



Note: y scale is in thousands of dollars.

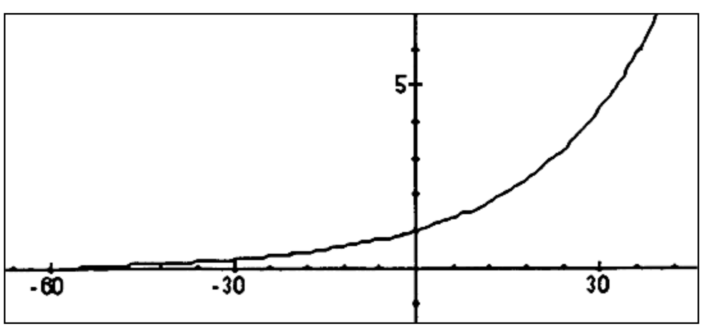
Use [TRACE] [◀] or [▶] to find a point where $y \approx 8.5$. x will be close to 8.4. Check by using [8.4] [ENTER] to show $y = 8.5$. This indicates that the amount will reach $8500 after 8.4 years.

7. b) A = 1000 $(1 + 0.05)^n$ *(2 marks)*
Graph the function $y = 1(1.05)^x$

y will give the amount in thousands of dollars.

Keying sequence:
[Y=] [1] [x] [1.05] [^] [X,T,θ,n]

Graph



Note: y scale is in thousands of dollars.

Keying sequence:
[2nd] [CALC] [1] [12] gives the point (12, 1.79585) which means that the amount will be $1795.85 after 12 years.

(2 marks)

8. a) $7x - 4y = 26$
$3x + 4y = -6$

Solve the two equations for y so they may be graphed.

$$y = \frac{7}{4}x - \frac{13}{2}$$

$$y = -\frac{3}{4}x - \frac{3}{2}$$

Graph the two equations on the same grid.

[Y=] [7] [÷] [4] [X,T,θ,n] [–] [13] [÷] [2] [ENTER] [(–)] [3] [÷] [4] [X,T,θ,n] [–] [3] [÷] [2] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 2$ $y = -3$

(2 marks)

b) $y = 2x^2 - x - 1$
$4x + 5y = 37$

Solve the two equations for y so they may be graphed.

$$y = 2x^2 - x - 1$$

$$y = \frac{-4}{5}x + \frac{37}{5}$$

Graph the two equations on the same grid.

[Y=] [2] [X,T,θ,n] [^] [2] [–] [X,T,θ,n] [–] [1] [ENTER] [(–)] [4] [÷] [5] [X,T,θ,n] [+] [37] [÷] [5] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x_1 = -2$ $y_1 = 9$
$x_2 = 2.1$ $y_2 = 5.72$

9. a) Let amount of sugar to be added be x mL *(5 marks)*
Let total amount of solution be y mL

$y = 100 + x$	Total solution is 100 plus the amount of sugar added.
$0.1y = 0.07(100) + x$	10% of the total mixture is sugar, while 7% of the original 100 mL added to sugar x gives the same amount.

Solve the two equations for y so that they may be graphed.

$y = x + 100$
$y = 10x + 70$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [X,T,θ,n] [+] [100] [ENTER] [10] [X,T,θ,n] [+] [70] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 3.3$ $y = 103.3$

Mix 3.3 mL of sugar with the 100 mL of 7% solution to make 103.3 mL of 10% solution.

b) Let speed of the jet in still air be x km per hour (kph). *(5 marks)*
Let speed of the wind be y km per hour (kph).

$x + y = 630$	With the wind it goes 1890 km in 3 hours. $1890 \div 3 = 630$.
$x - y = 540$	Against the wind it goes 1890 km in 3.5 hours. $1890 \div 3.5 = 540$.

Solve the two equations for y so that they may be graphed.

$y = -x + 630$
$y = x - 540$

Graph the two equations on the same grid.

[Y=] [(–)] [X,T,θ,n] [+] [630] [ENTER] [X,T,θ,n] [–] [540] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 585 \qquad y = 45$

The jet will travel at 585 kph in still air and the speed of the wind is 45 kph.

(10 marks)

c) Let profit be $P
Let x = number of Model A monitors to order
Let y = number of Model B monitors to order

$P = 45.00x + 50.00y$

Constraints

$x + y \leq 250$	Since the total number of monitors ordered cannot be more than 250.
$250x + 400y \leq 70\ 000$	Since maximum order is $70 000.00.
$x \geq 0$	There cannot be a negative number of monitors.
$y \geq 0$	There cannot be a negative number of monitors.

Graph the constraints:

Related equations

$x \geq 0 \qquad y \geq 0 \qquad x + y \leq 250 \qquad 250x + 400y \leq 70\ 000$

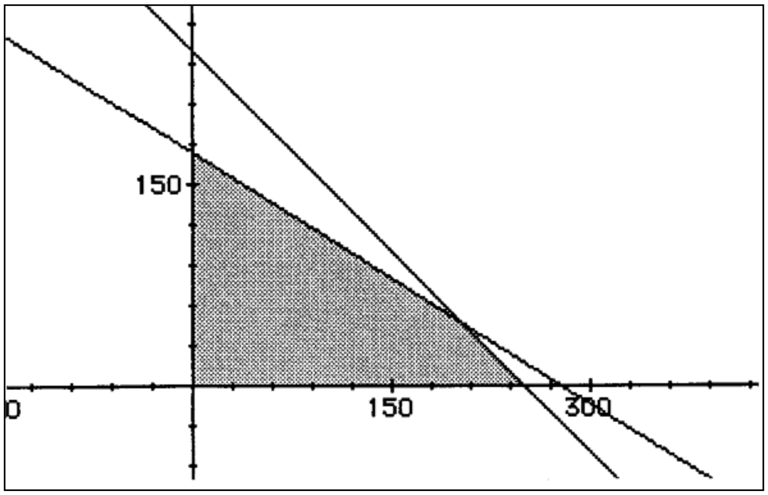
$x = 0 \qquad y = 0 \qquad x + y = 250 \qquad 250x + 400y = 70\ 000$

Intercept method

x	0	250
y	250	0

x	0	280
y	175	0

Check inequalities with point (0, 0) to determine the feasible region. Shade the feasible region.



The shaded region is the feasible region having corner points of: (0, 0), (0, 175), (250, 0), and (200, 50), all locatable from the graph or by finding the points of intersection algebraically or on the graphing calculator.

Substitute each corner point into the profit function and find the maximum value.

For (0, 0): P = $45x + 50y$ = 45(0) + 50(0) = 0
For (0, 175): P = $45x + 50y$ = 45(0) + 50(175) = 8750
For (250, 0): P = $45x + 50y$ = 45(250) + 50(0) = 11 250
For (200, 50): P = $45x + 50y$ = 45(200) + 50(50) = 11 500

Maximum profit = $11 500.00, from an order for 200 Model A monitors and 50 Model B monitors.

GRADE 11
APPLIED MATHEMATICS

Module 4
Answer Key

Module 4, Lesson 1, Answer Key

Earning an Income

1. With calculations:

	A	B	C	D	E	F
1	Employee	Rate	Tip %	Hours	Meals sold	Weekly Earnings
2	Bill	$7.40	10.00%	30	$1750.00	$397.00

With formulas revealed:

	A	B	C	D	E	F
1	Employee	Rate	Tip %	Hours	Meals sold	Weekly Earnings
2	Bill	$7.40	10.00%	30	$1750.00	=B2 * D2 + C2 * E2

2. With calculations:

	A	B	C	D	E	F	G
1	EMPLOYEE	RATE	REG HRS	O.T. HRS	REG PAY	O.T. PAY	GROSS PAY
2	Sheet Metal Worker	$10.78	38	0	$409.64	$0.00	$409.64
3	Mechanic	$9.75	40	6	$390.00	$87.75	$477.75
4	Inspector	$10.35	40	8	$414.00	$124.20	$538.20
5	Machinist	$12.95	40	2.25	$518.00	$43.71	$561.71
6	Legal Secretary	$10.45	40	15	$418.00	$235.12	$653.12
7	Accountant	$12.75	25	3.5	$318.75	$66.94	$385.69
8	Graphic Artist	$8.93	40	6.75	$357.20	$90.42	$447.62
9	Baker	$11.39	40	5.25	$455.60	$89.70	$545.30

With formulas revealed:

	A	B	C	D	E	F	G
1	EMPLOYEE	RATE	REG HRS	O.T. HRS	REG PAY	O.T. PAY	GROSS PAY
2	Sheet Metal Worker	$10.78	38	0	=B2 * C2	=B2 * 1.5 * D2	=E2 + F2
3	Mechanic	$9.75	40	6	=B3 * C3	=B3 * 1.5 * D3	=E3 + F3
4	Inspector	$10.35	40	8	=B4 * C4	=B4 * 1.5 * D4	=E4 + F4
5	Machinist	$12.95	40	2.25	=B5 * C5	=B5 * 1.5 * D5	=E5 + F5
6	Legal Secretary	$10.45	40	15	=B6 * C6	=B6 * 1.5 * D6	=E6 + F6
7	Accountant	$12.75	25	3.5	=B7 * C7	=B7 * 1.5 * D7	=E7 + F7
8	Graphic Artist	$8.93	40	6.75	=B8 * C8	=B8 * 1.5 * D8	=E8 + F8
9	Baker	$11.39	40	5.25	=B9 * C9	=B9 * 1.5 * D9	=E9 + F9

3. With calculations:

	A	B	C	D	E	F
1	NAME	RATE	HOURS	TIP AMOUNT	TIP %	GROSS PAY
2	Josh	$5.75	40	$250.00	100.00%	$480.00
3	Celeste	$7.80	25	$115.00	100.00%	$310.00
4	George	$6.15	20	$82.00	80.00%	$188.60
5	Vanessa	$6.40	24	$104.00	75.00%	$231.60
6	Danny	$8.35	40	$212.00	85.00%	$514.20
7	Louise	$7.90	16.5	$48.00	95.00%	$175.95
8	Leslie	$6.10	34.5	$143.00	90.00%	$339.15

With formulas revealed:

	A	B	C	D	E	F
1	NAME	RATE	HOURS	TIP AMOUNT	TIP %	GROSS PAY
2	Josh	$5.75	40	$250.00	100.00%	=B2 * C2 + D2 * E2
3	Celeste	$7.80	25	$115.00	100.00%	=B3 * C3 + D3 * E3
4	George	$6.15	20	$82.00	80.00%	=B4 * C4 + D4 * E4
5	Vanessa	$6.40	24	$104.00	75.00%	=B5 * C5 + D5 * E5
6	Danny	$8.35	40	$212.00	85.00%	=B6 * C6 + D6 * E6
7	Louise	$7.90	16.5	$48.00	95.00%	=B7 * C7 + D7 * E7
8	Leslie	$6.10	34.5	$143.00	90.00%	=B8 * C8 + D8 * E8

4. With calculations:

	A	B	C	D	E	F
1	EMPLOYEE	RATE	HOURS	REG PAY	OT PAY	GROSS PAY
2	Electrician	$21.35	42	$854.00	$64.05	$918.05
3	Bookkeeper	$15.27	36	$549.72	$0.00	$549.72
4	Cab Driver	$11.35	62	$454.00	$374.55	$828.55

With formulas revealed:

	A	B	C	D	E	F
1	EMPLOYEE	RATE	HOURS	REG PAY	OT PAY	GROSS PAY
2	Electrician	$21.35	42	=B2*IF(C2<=40, C2, 40)	=B2*1.5*IF(C2<=40,0, (C2–40))	=D2+E2
3	Bookkeeper	$15.27	36	=B3*IF(C3<=40, C3, 40)	=B3*1.5*IF(C3<=40,0, (C3–40))	=D3+E3
4	Cab Driver	$11.35	62	=B4*IF(C4<=40, C4, 40)	=B4*1.5*IF(C4<=40,0, (C4–40))	=D4+E4

Module 4, Lesson 2, Answer Key

Gross Pay and Net Income

1. With calculations:

	A	B	C	D	E	F
1	GROSS PAY	EI	C.P.P.	INCOME TAX	OTHER DEDUCTIONS	NET PAY
2	$567.89	$16.75	$15.90	$144.81	0	$390.42
3	$907.78	$26.78	$25.42	$354.03	0	$501.55
4	$1250.00	$36.88	$35.00	$543.75	0	$634.38
5	$257.31	$7.59	$7.20	$65.61	0	$176.90
6						
7	0	0.255				
8	569.04	0.39				
9	1138.08	0.435				
10	100000					

With formulas revealed:

	A	B	C	D	E	F
1	GROSS PAY	EI	C.P.P.	INCOME TAX	OTHER	NET PAY
2	$567.89	=0.295*A2	=0.028*A2	=A2*(LOOKUP(A2,A7..A10,B7..B10))	0	=A2–B2–C2–D2
3	$907.78	=0.295*A3	=0.028*A3	=A3*(LOOKUP(A3,A7..A10,B7..B10))	0	=A3–B3–C3–D3
4	$1250.00	=0.295*A4	=0.028*A4	=A4*(LOOKUP(A4,A7..A10,B7..B10))	0	=A4–B4–C4–D4
5	$257.31	=0.295*A5	=0.028*A5	=A5*(LOOKUP(A5,A7..A10,B7..B10))	0	=A5–B5–C5–D5
6						
7	0	0.255				
8	569.04	0.39				
9	1138.08	0.435				
10	100000					

Note: Final values may differ slightly due to rounding off.

2. With calculations:

	A	B	C	D	E	F
1	**EMPLOYEE**	**BOB**	**BILL**	**ALICE**	**CAROL**	**TED**
2	RATE	$17.54	$14.65	$18.75	$11.60	$9.45
3	MON	8	8	9.5	9.5	8
4	TUES	8	8	10	8	8
5	WED	9	8	8	10	9
6	THURS	8	8	12	10	8
7	FRI	6	8	8.5	7	8
8	REG HOURS	39	40	40	40	40
9	OT HOURS	0	0	8	4.5	1
10	GROSS PAY	$684.06	$586.00	$975.00	$542.30	$392.18
11	E.I.	$20.18	$17.29	$28.76	$16.00	$11.57
12	C.P.P.	$19.15	$16.41	$27.30	$15.18	$10.98
13	INCOME TAX	$266.78	$228.54	$380.25	$138.29	$100.00
14	NET PAY	$377.94	$323.76	$538.69	$372.83	$269.62
15						
16	0	0.255				
17	569.04	0.39				
18	1138.08	0.435				
19	100000					

With formulas revealed:

	A	B
1	**EMPLOYEE**	**BOB**
2	RATE	$17.54
3	MON	8
4	TUES	8
5	WED	9
6	THURS	8
7	FRI	6
8	REG HOURS	=IF(SUM(B3. . B7)<=40,SUM(B3. . B7),40)
9	OT HOURS	=IF(SUM(B3. .B7)<=40,0,(SUM(B3. .B7)–40))
10	GROSS PAY	=B8 * B2 + B9 * B2 * 1.5
11	E.I.	=0.0295 * B10
12	C.P.P.	=0.028 * B10
13	INCOME TAX	=B10 * (LOOKUP(B10,A16. .A19,B16. .B19))
14	NET PAY	=B10 – B11 – B12 – B13
15		
16	0	0.255
17	569.04	0.39
18	1138.08	0.435
19	100000	

Continued:

	A	B
1	**EMPLOYEE**	**BILL**
2	RATE	$14.65
3	MON	8
4	TUES	8
5	WED	8
6	THURS	8
7	FRI	8
8	REG HOURS	=IF(SUM(C3. . C7)<=40,SUM(C3. . C7),40)
9	OT HOURS	=IF(SUM(C3. .C7)<=40,0,(SUM(C3. .C7)–40))
10	GROSS PAY	=C8 * C2 + C9 * C2 * 1.5
11	E.I.	=0.0295 * C10
12	C.P.P.	=0.028 * C10
13	INCOME TAX	=C10 * (LOOKUP(C10,A16. .A19,B16. .B19))
14	NET PAY	=C10 – C11 – C12 – C13
15		
16	0	
17	569.04	
18	1138.08	
19	100000	

Continued:

	A	B
1	**EMPLOYEE**	**ALICE**
2	RATE	$18.75
3	MON	9.5
4	TUES	10
5	WED	8
6	THURS	12
7	FRI	8.5
8	REG HOURS	=IF(SUM(D3. . D7)<=40,SUM(D3. . D7),40)
9	OT HOURS	=IF(SUM(D3. .D7)<=40,0,(SUM(D3. .D7)–40))
10	GROSS PAY	=D8 * D2 + D9 * D2 * 1.5
11	E.I.	=0.0295 * D10
12	C.P.P.	=0.028 * D10
13	INCOME TAX	=D10 * (LOOKUP(D10,A16. .A19,B16. .B19))
14	NET PAY	=D10 – D11 – D12 – D13
15		
16	0	
17	569.04	
18	1138.08	
19	100000	

Continued:

	A	B
1	**EMPLOYEE**	**CAROL**
2	RATE	$11.60
3	MON	9.5
4	TUES	8
5	WED	10
6	THURS	10
7	FRI	7
8	REG HOURS	=IF(SUM(E3. . E7)<=40,SUM(E3. . E7),40)
9	OT HOURS	=IF(SUM(E3. .E7)<=40,0,(SUM(E3. .E7)–40))
10	GROSS PAY	=E8 * E2 + E9 * E2 * 1.5
11	E.I.	=0.0295 * E10
12	C.P.P.	=0.028 * E10
13	INCOME TAX	=E10 * (LOOKUP(E10,A16. .A19,B16. .B19))
14	NET PAY	=E10 – E11 – E12 – E13
15		
16	0	
17	569.04	
18	1138.08	
19	100000	

Continued:

	A	B
1	**EMPLOYEE**	**TED**
2	RATE	$9.45
3	MON	8
4	TUES	8
5	WED	9
6	THURS	8
7	FRI	8
8	REG HOURS	=IF(SUM(F3. . F7)<=40,SUM(F3. . F7),40)
9	OT HOURS	=IF(SUM(F3. .F7)<=40,0,(SUM(F3. .F7)–40))
10	GROSS PAY	=F8 * F2 + F9 * F2 * 1.5
11	E.I.	=0.0295 * F10
12	C.P.P.	=0.028 * F10
13	INCOME TAX	=F10 * (LOOKUP(F10,A16. .A19,B16. .B19))
14	NET PAY	=F10 – F11 – F12 – F13
15		
16	0	
17	569.04	
18	1138.08	
19	100000	

3. With calculations:

	A	B	C	D	E	F
1	**EMPLOYEE**	**BOB**	**BILL**	**ALICE**	**CAROL**	**TED**
2	RATE	$17.54	$14.65	$18.75	$11.60	$9.45
3	NEW RATE	$18.59	$15.53	$19.88	$12.30	$10.02
4	MON	8	8	9.5	9.5	8
5	TUES	8	8	10	8	8
6	WED	9	8	8	10	9
7	THURS	8	8	12	10	8
8	FRI	6	8	8.5	7	8
9	REG HOURS	39	40	40	40	40
10	OT HOURS	0	0	8	4.5	1
11	GROSS PAY	$725.10	$621.16	$1033.50	$574.84	$415.71
12	E.I.	$21.39	$18.32	$30.49	$16.96	$12.26
13	C.P.P.	$20.30	$17.39	$28.94	$16.10	$11.64
14	INCOME TAX	$282.79	$242.25	$403.06	$224.19	$106.00
15	NET PAY	$400.62	$343.19	$571.01	$317.60	$285.80
16						
17	0	0.255				
18	569.04	0.39				
19	1138.08	0.435				

Because you will be filling right for these formula changes, only those for Bob will be shown here.

With formulas revealed:

	A	B
1	**EMPLOYEE**	**BOB**
2	RATE	$17.54
3	NEW RATE	=B2 * 1.06
4	MON	8
5	TUES	8
6	WED	9
7	THURS	8
8	FRI	6
9	REG HOURS	=IF(SUM(B4. . B8)<=40,SUM(B4. . B8),40)
10	OT HOURS	=IF(SUM(B4. .B8)<=40,0,(SUM(B4. .B8)–40))
11	GROSS PAY	=B9 * B3 + B10 * B3 * 1.5
12	E.I.	=0.0295 * B11
13	C.P.P.	=0.028 * B11
14	INCOME TAX	=B11 * (LOOKUP(B11,A17. .A20,B17. .B20))
15	NET PAY	=B11 – B12 – B13 – B14
16		
17	0	0.255
18	569.04	0.39
19	1138.08	0.435

Notes

Module 4, Lesson 3, Answer Key

Analyzing Property Tax Notices

1. a) Mill rate = $\frac{75\ 000\ 000}{830\ 000\ 000}$ x 1000 = 90.36

   b) Tax rate = $\frac{75\ 000\ 000}{830\ 000\ 000}$ x 100 = 9.04 cents on the dollar

2.

	A	B	C
1	ASSESSMENT	MILL RATE	TAXES DUE
2	15980	49.7	$794.21
3	28750	75.5	$2170.62
4	45900	28.74	$1319.17
5	76000	65.8	$5000.80

3. Assessed value = $40 000
   Rate of assessment = 65%
   Total portion assessment = ($40 000)(0.65) = $26 000
   Mill rate = 56 mills
   General tax = ($26 000)(56) ÷ 1000 = $1456.00
   Improvement tax = $98.00
   Total tax = $1456.00 + $98.00 = $1554.00

4. Assessed value = $45 000
   Rate of assessment = 100%
   Total portion assessment = ($45 000)(1.00) = $45 000
   Mill rate = 55 mills
   General tax = ($45 000)(55) ÷ 1000 = $2475.00
   Education tax = ($45 000)(17.3) ÷ 1000 = $778.50
   Hospital tax = ($45 000)(5.1) ÷ 1000 = $229.50
   Total tax = $2475.00 + $778.50 + $229.50 = $3483.00

5. Assessed value = $35 000
   Rate of assessment = 30%
   Total portion assessment = ($35 000)(0.30) = $10 500
   Mill rate = 58.9 mills
   General tax = ($10 500)(58.9) ÷ 1000 = $618.45
   Improvement tax = (21.7)($2.79 + $1.25) = $87.67
   Total tax = $618.45 + $87.67 = $706.12

6. Assessed value = \$65 000
   Rate of assessment = 35%
   Total portion assessment = (\$65 000)(0.35) = \$22 750
   Mill rate = 56.8 mills
   General tax = (\$22 750)(56.8) ÷ 1000 = \$1292.20
   Education tax = (\$22 750)(18.5) ÷ 1000 = \$420.88
   Improvement tax = (\$18 000)(9.3) ÷ 1000 = \$167.40
   Provincial tax credit = \$250.00
   Total tax = \$1292.20 + \$420.88 + \$167.40 – \$250.00 = \$1630.48

7.

PROPERTY DESCRIPTION

ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
F546	8	9	9	CR1094	25.5 m	1

Civic Address

Title or Deed No.	Current Assessment Land	Current Assessment Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port Assessment
254	17 500	68 200	T	85 700	10	45%	38 565

←ERRORS AND OMISSIONS EXCEPTED
←ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
←ALL CHEQUES MADE IN CANADIAN FUNDS
←BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
←RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY
38 565

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			\$38 565	43.8	\$1689.15
By-Law No.	Term	Type	Frontage Levy	Mill Rate	Levy
487	96	Sidewalk	85	.268	\$95.34
1235	97	Streets		2.54	\$97.96
568	97	Water		3.8	\$146.55

EDUCATIONAL TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	38 565	5.6	\$215.96
Provincial Education 2	38 565	12.8	\$493.63

PROVINCIAL TAX CREDITS

(See Manitoba Enclosure For Additional Information)	Assessment	Levy
	Manitoba Resident Homeowner Tax Assistance	\$250.00

TOTAL TAXES DUE

Municipal Tax	Education Tax	Total Taxes	Prov. Credits	Net Taxes	Arrears/Credits	Added Taxes	Taxes Due
\$2029.00	\$709.59	\$2738.59	\$250	\$2488.59	0	0	\$2488.59

8. With calculations:

	A	B	C	D	E	F
1			ASSESSMENTS			
2	CURRENT ASSESSMENT		TOTAL ASSESSMENT	PORTION	TOTAL PORTION ASSESSMENT	LOCAL IMPROVEMENT
3	LAND	BUILDINGS		%		
4	$17500	$68200.00	$85700.00	0.45	$38565.00	$38565.00
5						
6			MUNICIPAL TAXES			
7		FRONTAGE	ASSESSMENT	MILL RATE	LEVY	
8	GENERAL		$38565.00	43.8	$1689.15	
9	IMPROVEMENT 1	$85.00	$38565.00	0.268	$95.34	
10	IMPROVEMENT 2		$38565.00	2.54	$97.96	
11	IMPROVEMENT 3		$38565.00	3.8	$146.55	
12						
13			EDUCATION TAXES			
14		EDUCATION 1	$38565.00	5.6	$215.96	
15		EDUCATION 2	$38565.00	12.8	$493.63	
16						
17			PROVINCIAL CREDITS			
18			$250.00			
19						
20			SUMMARY OF TAXES			
21	MUNICIPAL		TOTAL TAXES	PROV. CREDITS	NET TAXES	TAXES DUE
22	$2028.98	$709.60	$2738.58	$250.00	$2488.58	$2488.58
23	ARREARS		ADD. TAXES			

Note: Discrepancies of one or two cents may occur depending on how the numbers are formatted.

With formulas revealed:

	A	B	C	D	E	F
1			ASSESSMENTS			
2	CURRENT ASSESSMENT		TOTAL ASSESSMENT	PORTION	TOTAL PORTION ASSESSMENT	LOCAL IMPROVEMENT
3	LAND	BUILDINGS		%		
4	$17500.00	$68200.00	=A4+B4	0.45	=C4 * D4	=E4
5						
6			MUNICIPAL TAXES			
7		FRONTAGE	ASSESSMENT	MILL RATE	LEVY	
8	GENERAL		=E4	43.8	=C8*D8/1000	
9	IMPROVEMENT 1	$85.00	=F4	0.268	=B9+C9*D9/1000	
10	IMPROVEMENT 2		=F4	2.54	=B10+C10*D10/1000	
11	IMPROVEMENT 3		=F4	3.8	=B11+C11*D11/1000	
12						
13			EDUCATION TAXES			
14		EDUCATION 1	=C8	5.6	=C14*D14/1000	
15		EDUCATION 2	=C8	12.8	=C15*D15/1000	
16						
17			PROVINCIAL CREDITS			
18			$250.00			
19						
20			SUMMARY OF TAXES			
21	MUNICIPAL		TOTAL TAXES	PROV. CREDITS	NET TAXES	TAXES DUE
22	=SUM(E8. .E11)	=SUM(E14. .E15)	=A22+B22	=C18	=C22–D22	=E22+B23+D23
23	ARREARS		ADD. TAXES			

9. a)

	A	B	C	D	E	F
1			ASSESSMENTS			
2	CURRENT ASSESSMENT		TOTAL ASSESSMENT	PORTION	TOTAL PORTION ASSESSMEMT	LOCAL IMPROVEMENT
3	LAND	BUILDINGS		%		
4	$23000.00	$58200.00	$81000.00	0.3	$24300.00	$24000.00
5						
6			MUNICIPAL TAXES			
7		FRONTAGE	ASSESSMENT	MILL RATE	LEVY	
8	GENERAL		$24300.00	52.7	$1280.61	
9	IMPROVEMENT 1	$95.00	$24000.00	1.1	$121.40	
10	IMPROVEMENT 2		$24000.00	3.5	$84.00	
11	IMPROVEMENT 3		$24000.00	4.7	$112.80	
12						
13			EDUCATION TAXES			
14		EDUCATION 1	$24300.00	3.8	$92.34	
15		EDUCATION 2	$24300.00	15.6	$379.08	
16						
17			PROVINCIAL CREDITS			
18			$250.00			
19						
20			SUMMARY OF TAXES			
21	MUNICIPAL		TOTAL TAXES	PROV. CREDITS	NET TAXES	TAXES DUE
22	$1598.81	$471.42	$2070.23	$250.00	$1820.23	$1820.23
23	ARREARS		ADD. TAXES			

b)

	A	B	C	D	E	F
1			ASSESSMENTS			
2	CURRENT ASSESSMENT		TOTAL ASSESSMENT	PORTION	TOTAL PORTION ASSESSMENT	LOCAL IMPROVEMENT
3	LAND	BUILDINGS		%		
4	$23000.00	$58200.00	$81000.00	0.3	$24300.00	$24000.00
5						
6			MUNICIPAL TAXES			
7		FRONTAGE	ASSESSMENT	MILL RATE	LEVY	
8	GENERAL		$24300.00	63.5	$1543.05	
9	IMPROVEMENT 1	$95.00	$24000.00	1.1	$121.40	
10	IMPROVEMENT 2		$24000.00	3.5	$84.00	
11	IMPROVEMENT 3		$24000.00	4.7	$112.80	
12						
13			EDUCATION TAXES			
14		EDUCATION 1	$24300.00	3.8	$92.34	
15		EDUCATION 2	$24300.00	15.6	$379.08	
16						
17			PROVINCIAL CREDITS			
18			$250.00			
19						
20			SUMMARY OF TAXES			
21	MUNICIPAL		TOTAL TAXES	PROV. CREDITS	NET TAXES	TAXES DUE
22	$1861.25	$471.42	$2332.67	$250.00	$2082.67	$2082.67
23	ARREARS		ADD. TAXES			

It would increase taxes by $262.44.

c)

	A	B	C	D	E	F
1			ASSESSMENTS			
2	CURRENT ASSESSMENT		TOTAL ASSESSMENT	PORTION	TOTAL PORTION ASSESSMENT	LOCAL IMPROVEMENT
3	LAND	BUILDINGS		%		
4	$23000.00	$58000.00	$81000.00	0.26	$21060.00	$24000.00
5						
6			MUNICIPAL TAXES			
7		FRONTAGE	ASSESSMENT	MILL RATE	LEVY	
8	GENERAL		$21060.00	52.7	$1109.86	
9	IMPROVEMENT 1	$95.00	$24000.00	1.1	$121.40	
10	IMPROVEMENT 2		$24000.00	3.5	$84.00	
11	IMPROVEMENT 3		$24000.00	4.7	$112.80	
12						
13			EDUCATION TAXES			
14		EDUCATION 1	$21060.00	3.8	$80.03	
15		EDUCATION 2	$21060.00	15.6	$328.54	
16						
17			PROVINCIAL CREDITS			
18			$250.00			
19						
20			SUMMARY OF TAXES			
21	MUNICIPAL		TOTAL TAXES	PROV. CREDITS	NET TAXES	TAXES DUE
22	$1428.06	$408.57	$1836.63	$250.00	$1586.63	$1586.63
23	ARREARS		ADD. TAXES			

It would decrease taxes by $233.60.

d)

	A	B	C	D	E	F
1			ASSESSMENTS			
2	CURRENT ASSESSMENT		TOTAL ASSESSMENT	PORTION	TOTAL PORTION ASSESSMENT	LOCAL IMPROVEMENT
3	LAND	BUILDINGS		%		
4	$24000.00	$65000.00	$89000.00	0.3	$26700.00	$24000.00
5						
6			MUNICIPAL TAXES			
7		FRONTAGE	ASSESSMENT	MILL RATE	LEVY	
8	GENERAL		$26700.00	46.9	$1252.23	
9	IMPROVEMENT 1	$95.00	$24000.00	1.1	$121.40	
10	IMPROVEMENT 2		$24000.00	3.5	$84.00	
11	IMPROVEMENT 3		$24000.00	4.7	$112.80	
12						
13			EDUCATION TAXES			
14		EDUCATION 1	$26700.00	3.8	$101.46	
15		EDUCATION 2	$26700.00	15.6	$416.52	
16						
17			PROVINCIAL CREDITS			
18			$250.00			
19						
20			SUMMARY OF TAXES			
21	MUNICIPAL		TOTAL TAXES	PROV. CREDITS	NET TAXES	TAXES DUE
22	$1570.43	$517.98	$2088.41	$250.00	$1838.41	$1838.41
23	ARREARS		ADD. TAXES			

It would increase taxes by $18.18.

Module 4, Lesson 4, Answer Key

Unit Costs and Comparison Buying

1. a). Sugar — Cost per kilogram

2 kg for \$2.69	\$2.69 ÷ 2 = \$1.345
4 kg for \$4.99	\$4.99 ÷ 4 = \$1.248
10 kg for \$10.99	\$10.99 ÷ 10 = \$1.10

b) Flour — Cost per kilogram

2 kg for \$4.49	\$4.49 ÷ 2 = \$2.245
1 kg for \$2.29	\$2.29 ÷ 1 = \$2.29
10 kg for \$9.99	\$9.99 ÷ 10 = \$1.00

c) Cans of beans — Cost per millilitre

398 mL for \$1.29	\$1.29 ÷ 398 = \$0.0032
540 mL for \$1.69	\$1.69 ÷ 540 = \$0.0031
796 mL for \$2.29	\$2.29 ÷ 796 = \$0.0029
1.36 L for \$3.39	\$3.39 ÷ 1360 = \$0.0025

d) Toilet tissue — Cost per roll

4 rolls of 2-ply for \$1.99	\$1.99 ÷ 4 = \$0.498
8 rolls of 3-ply for \$4.49	\$4.49 ÷ 8 = \$0.561
24 rolls of 3-ply for \$12.45	\$12.45 ÷ 24 = \$0.519

e) Soft Drinks — Cost per millilitre

24-355-mL cans for \$10.75	\$10.75 ÷ 8520 = \$0.00126
12-355-mL cans for \$5.89	\$5.89 ÷ 4260 = \$0.00138
600-mL bottle for \$0.99	\$0.99 ÷ 600 = \$0.00165
1-L bottle for \$1.49	\$1.49 ÷ 1000 = \$0.00149
2-L bottle for \$2.19	\$2.19 ÷ 2000 = \$0.00110

Other factors besides price that may come into play in making the purchases are the amount required, the convenience of a particular size, storage space, size of family/group, quality, etc.

2. Nails

Nails	Cost per pound
Common \$4.95 for 5 lb	\$4.95 ÷ 5 = \$0.99
Coated \$0.89 for 1 lb	\$0.89
Shingle \$2.89 for 1 kg	\$2.89 ÷ 2.2 = \$1.314
Galvanized \$2.70 for 2 lb	\$2.70 ÷ 2 = \$1.35
Spiral \$2.10 for 1 kg	\$2.10 ÷ 2.2 = \$0.955
Ring \$7.50 for 4 kg	\$7.50 ÷ 8.8 = \$0.852

Other factors might be the number of nails actually required, where the nails will be set, and what they will be used for. Nails that are set outdoors, for example, must be galvanized to prevent rust.

3. Motor oil

Motor oil	Cost per L
1-L for \$1.49	\$1.49
4-L for \$5.29	\$5.29 ÷ 4 = \$1.32

You may buy oil by the litre if you are using it to top up a motor, whereas you may buy the 4-litre container if you are going to change the oil in a motor.

4. a) Cost per millilitre

   Smaller: \$1.32 ÷ 284 = 0.4648¢
   Larger: \$2.09 ÷ 540 = 0.3870¢

   b) Cost per 100 mL

   Smaller: 46.48¢
   Larger: 38.70¢

5. Price of 284-mL size = 34.9 ÷ 100 x 284 = 99¢

   Price of 540-mL size = 35.0 ÷ 100 x 540 = \$1.89

Module 4, Lesson 5, Answer Key

Foreign Exchange

1. £500 x $2.2902 = CAN $1145.10.
   1000 marks x $0.7934 = CAN $793.40 for a total of $1938.50.

2. U.S. $429.99 costs $429.99 x CAN $1.4168 = CAN $609.21
   Total cost = $609.21 + $35.00 = CAN $644.21.

3. $500.00 Australian will return $500 x $0.9956 = $497.80 Canadian.

4. You will be able to deposit 5000 –1800 = 3200 markkas, which is equivalent to 3200 x $0.2411 = CAN $771.52 each week into your Canadian account.

5. Cost to you: $700 x $1.4168 = CAN $991.76
   Return of U.S. $700.00 : $700 x $1.3633 = CAN $954.31
   Overall cost to you: $991.76 – $954.31 = $37.45

6. Cost of jeans in Canadian dollars = 25 x $2.2902 = $57.26
   This seems to be a good deal, if you need the jeans.

7. Cost of jacket in Austria = 3500 schillings
   This would cost **you** 3500 x $0.11226 = CAN $392.91.
   This is not much different from the cost in Canada.

Notes

Self-Test Answer Key

Module 4 – Personal Finance I

(3 marks) 1. a) Any three of: Salary
Wage
Contract
Commission
Self employed

(3 marks) b) Income tax
Canada pension
Employment insurance

(1 mark) c) Net Pay is pay left after deductions.

(2 marks) d) Selling price is the market value of the property, and the assessed value of the property is a figure used for property taxation purposes.

(1 mark) e) $\dfrac{\text{Funds needed in budget}}{\text{Total property value of the town}} \times 1000$

(10 marks) 2. With numbers:

	A	B	C	D	E
1	EMPLOYEE	TOM	JIM	PAM	JACK
2	RATE	$14.05	$6.10	$9.80	$12.75
3	MON	8	9	6	10
4	TUES	9	4	8	8
5	WED	8	4	10	9
6	THU	10	8	4	11
7	FRI	12	6	6	7
8	REG HOURS	40	31	34	40
9	OT HRS	7	0	0	5
10	GROSS PAY	$709.52	$189.10	$333.20	$605.62
11	E.I.	$20.93	$5.58	$9.83	$17.87
12	C.P.P.	$19.87	$5.29	$9.33	$16.96
13	INC TAX	$276.71	$48.22	$84.97	$236.19
14	NET PAY	$392.01	$130.01	$229.07	$334.61
15					
16	0	0.255			
17	569.04	0.39			
18	1138.08	0.435			
19	100000				
20					

and with formulas:

	B	C
1	EMPLOYEE	TOM
2	RATE	$14.05
3	MON	8
4	TUE	9
5	WED	8
6	THU	10
7	FRI	12
8	REG HRS	=IF(SUM(B3..B7)<=40,SUM(B3..B7),40)
9	OT HRS	=IF(SUM(B3..B7)<=40,0,(SUM(B3..B7)–40))
10	GROSS PAY	=B8*B2+B9*B2*1.5
11	E.I.	=0.0295*B10
12	C.P.P.	=0.028*B10
13	INC TAX	=B10*(LOOKUP(B10,A16..A19,B16..B19))
14	NET PAY	=B10–B11–B12–B13
15		
16	0	0.255
17	569.04	0.39
18	1138.08	0.435
19	100000	
20		

Continued:

	C
1	JIM
2	$6.10
3	9
4	4
5	4
6	8
7	6
8	=IF(SUM(C3..C7)<=40,SUM(C3..C7),40)
9	=IF(SUM(C3..C7)<=40,0,(SUM(C3..C7)–40))
10	=C8*C2+C9*C2*1.5
11	=0.0295*C10
12	=0.028*C10
13	=C10*(LOOKUP(C10,A16..A19,B16..B19))
14	=C10–C11–C12–C13
15	
16	

Continued:

	C
1	PAM
2	$9.80
3	6
4	8
5	10
6	4
7	6
8	=IF(SUM(D3..D7)<=40,SUM(D3..D7),40)
9	=IF(SUM(D3..D7)<=40,0,(SUM(D3..D7)–40))
10	=D8*D2+D9*D2*1.5
11	=0.0295*D10
12	=0.028*D10
13	=D10*(LOOKUP(D10,A16..A19,B16..B19))
14	=D10–D11–D12–D13
15	
16	

Continued:

	C
1	JACK
2	$12.75
3	10
4	8
5	9
6	11
7	7
8	=IF(SUM(E3..E7)<=40,SUM(E3..E7),40)
9	=IF(SUM(E3..E7)<=40,0,(SUM(E3..E7)–40))
10	=E8*E2+E9*E2*1.5
11	=0.0295*E10
12	=0.028*E10
13	=E10*(LOOKUP(E10,A16..A19,B16..B19))
14	=E10–E11–E12–E13
15	
16	

3. a) Mill rate $\dfrac{11\ 500\ 000}{100\ 000\ 000} \times 1000 = 115$ mills *(1 mark)*

b) *(7 marks)*

PROPERTY DESCRIPTION

ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
585	3	7	2	FF1254	82.5 ft	1

Civic Address

Title or Deed No.	Current Assessment Land	Current Assessment Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port Assessment
B544	6500	65 000	T	71 500	10	35%	25 025

←ERRORS AND OMISSIONS EXCEPTED
←ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
←ALL CHEQUES MADE IN CANADIAN FUNDS
←BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
←RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY
71 500

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			25 025	41.56	1040.04
By-Law No.	**Term**	**Type**	**Frontage Levy**	**Mill Rate**	**Levy**
523	98	Sewer and Water	75	0.025	76.79
633	97	Sidewalk		4.125	294.94
710	99	Street		7.360	526.24

EDUCATIONAL TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	25 025	5.5	137.64
Provincial Education 2	25 025	16.4	410.41

PROVINCIAL TAX CREDITS

	Assessment	Levy
(See Manitoba Enclosure For Additional Information)	Manitoba Resident Homeowner Tax Assistance	$250.00

TOTAL TAXES DUE

Municipal Tax	Education Tax	Total Taxes	Prov. Credits	Net Taxes	Arrears/Credits	Added Taxes	Taxes Due
1938.01	548.05	2486.06	250.00	2236.06	0	0	2236.06

4. a) Cost = $(19.95 + 12.99 + 21.99) × 1.09 U.S. *(2 marks)*
= $59.8737 U.S.
= $59.8737 U.S. × 1.3784 CAN
= $82.53 CAN

b) Cost = 0.85 × $795 CAN *(2 marks)*
= $675.75 CAN
$= \dfrac{,675.75}{2.1573} = ,313.24$

5. Prices per gram. *(2 marks)*

100¢ ÷ (4 × 175) = 0.142857 cents per gram

239¢ ÷ (6 × 300) = 0.132778 cents per gram

409¢ ÷ (12 × 250) = 0.136333 cents per gram

6 cans (300 g) for $2.39 is the best buy based only on price.

(2 marks)

6. a) $7x - 3y = 8$
$3x + 5y = -12$

Solve the two equations for y so that they may be graphed.

$$y = \frac{7}{3}x - \frac{8}{3}$$
$$y = \frac{-3}{5}x - \frac{12}{5}$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [7] [÷] [3] [X,T,θ,n] [–] [8] [÷] [3] [ENTER] [(–)] [3] [÷] [5] [X,T,θ,n] [–] [12] [÷] [5] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 0.09$ $y = -2.45$

(2 marks)

b) $y = x + 1$
$y = x^2 + 3x - 2$

Solve the two equations for y so that they may be graphed.

$y = x^2 + 3x - 2$
$y = x + 1$

Graph the two equations on the same grid or screen.

Keying sequence:
[Y=] [X,T,θ,n] [^] [2] [+] [3] [X,T,θ,n] [–] [2] [ENTER] [X,T,θ,n] [+] [1] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

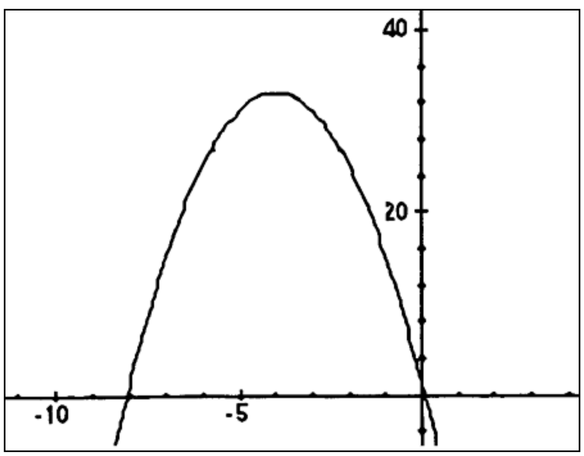
$x_1 = -3 \quad y_1 = -2$

$x_2 = 1 \quad y_2 = 2$

7. $y = -2x^2 - 16x + 1$ *(3 marks)*

Keying sequence:

[Y=] [(–)] [2] [X,T,θ,n] [x^2] [–] [16] [X,T,θ,n] [+] [1]



Reset window

Vertex	(–4, 33)
Axis of symmetry	$x = -4$
Maximum Value	$y = 33$
Domain	$\{x \mid x \varepsilon \mathrm{R}\}$
Range	$\{y \mid y \leq 33\}$

Note: Find the maximum value using the keying sequence: [2nd] [CALC] [4], setting left and right bounds, and then pressing [ENTER] to find the point (–4, 33).

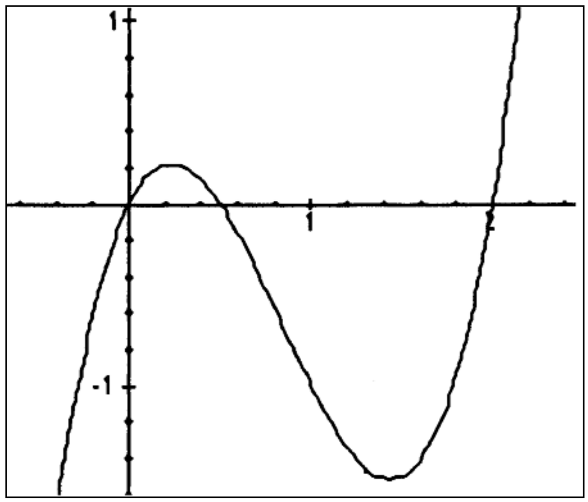
(3 marks)

8. $y = 2x^3 - 5x^2 + 2x$

Keying sequence:

[Y=] [2] [X,T,θ,n] [^] [3] [–] [5] [X,T,θ,n] [x^2] [+] [2] [X,T,θ,n]

Graph:



Reset window

Properties

Use [2nd] [CALC] [4] and [2nd] [CALC] [3]

Maximum value $y = 0.22$ where $x = 0.23$
Minimum value $y = -1.52$ where $x = 1.43$
Zeros $x_1 = 0$, $x_2 = 0.5$, $x_3 = 2$

(5 marks)

9. a) Let x = number of eggs on his plate
Let y = number of pieces of bacon on his plate

$y = 2x$	There were twice as many pieces of bacon as eggs.
$x + 1 = \frac{2}{3}y$	If he had one more egg he would have had $\frac{2}{3}$ the number of pieces of bacon.

Solve the two equations for y so that they may be graphed.

$y = 2x$

$y = \frac{3}{2}(x + 1)$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [2] [X,T,θ,*n*] [ENTER] [3] [÷] [2] [(] [X,T,θ,*n*] [+] [1] [)]
[GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 3 \quad y = 6$

OR

You solve the system using the substitution method

$y = 2x$, therefore,

$$\begin{aligned} x+1 &= \frac{2}{3}(2x) \\ 3x+3 &= 4x \\ 3 &= x \\ y &= 2(3) = 6 \end{aligned}$$

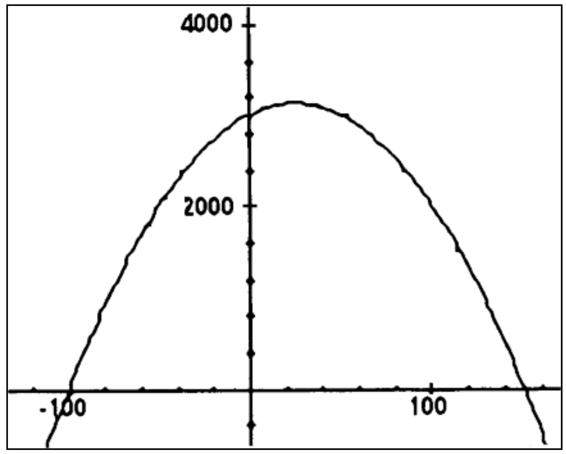
There were three eggs and six pieces of bacon on Jim's plate.

b) Let $\$y$ = income *(5 marks)*
Let x = the number of additional tickets over 100

Profit = number of tickets x income per ticket
$y = (100 + x)(30.00 - 0.20x)$
$y = 3000 - 20x + 30x - 0.20x^2$
$y = -0.20x^2 + 10x + 3000$

Graph the function.

Keying sequence
[Y=] [(–)] [0.2] [X,T,θ,*n*] [x^2] [+] [10] [X,T,θ,*n*] [+] [3000]



Find the maximum value using the program:

[2nd] [CALC] [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the point (25, 3125).

This indicates that the maximum income will be $3125.00 brought about by selling 25 additional tickets.

(10 marks)

c) Let profit be $P
Let x = number of size A screens made
Let y = number of size B screens made

$P = 2.00x + 3.00y$

Constraints

Old Machine time:	$4x + y \leq 120$	Since each size A screen takes 4 minutes on the old machine, size B screen takes 1 minute on the machine, and only 2 hours are available on the machine.
New Machine time:	$2x + 6y \leq 120$	Since each size A screen takes 2 minutes on the new machine, size B screen takes 6 minutes on the machine, and only 2 hours are available on the machine.

$x \geq 0$	There cannot be a negative number of A screens.
$y \geq 0$	There cannot be a negative number of B screens.

Graph the constraints:

$x \geq 0$	$y \geq 0$	$4x + y \leq 120$	$2x + 6y \leq 12$
$x = 0$	$y = 0$	$4x + y = 120$	$2x + 6y = 120$

Related equations

x	0	30
y	120	0

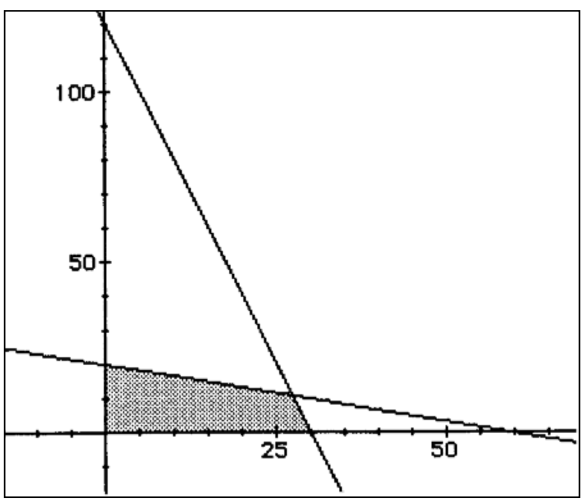
x	0	60
y	20	0

Intercept method

Test inequalities to find feasible region and shade that region. Equations for calculator.

$$y = -4x + 120$$

$$y = -\frac{1}{3}x + 20$$



The shaded region is the feasible region having corner points of: (0,0), (0, 20), and (30, 0), all able to be found from the graph, but the corner point (27.27, 10.9) is not easily found from the graph. To find corner points of this nature you can graph the two equations that create the point and find the intersection using your TI-83 calculator as you did in Lesson 3 of Module 2 (Systems of Equations).

Keying sequence:
[Y=] [(–)] [4] [X,T,θ,n] [+] [120] [ENTER] [(–)] [1] [÷] [3] [X,T,θ,n] [+] [20] GRAPH] [2nd] [CALC] [5] [ENTER] [ENTER] [ENTER]

This gives (27.27, 10.9) but you have to make a whole number of screens, therefore use (27, 11).

Substitute each corner point into the profit function and find the maximum value.

For (0, 0): P = $2x + 3y = 2(0) + 3(0) = 0$
For (30, 0): P = $2x + 3y = 2(30) + 3(0) = 60$
For (0, 20): P = $2x + 3y = 2(0) + 3(20) = 60$
For (27, 11): P = $2x + 3y = 2(27) + 3(11) = 87$

Maximum profit = \$87.00 brought about by making 27 size A screens and 11 size B screens.

Notes

Grade 11 Applied Mathematics (30S)

Midterm Practice Examination Answer Key

GRADE 11 APPLIED MATHEMATICS

Midterm Practice Examination
Answer Key

Name: ______________________________________

Student Number: ______________________________

Attending ❑ Non-Attending ❑

Phone Number: _______________________________

Address: ____________________________________

____

____

For Marker's Use Only

Date: ______________________________________

Midterm Mark: ______/100 = ________ %

Comments:

Instructions

The midterm examination is based on Modules 1 to 4 of the Grade 11 Applied Mathematics course. It is worth 20% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the midterm examination.

Notes

You are allowed to bring the following to the examination: pens/pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the examination. Graphing technology (either computer software or a graphing calculator) **is required** to complete this examination.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer. Final answers without supporting calculations or explanations will **not** be awarded full marks. Indicate equations and/or keystrokes used in calculations.

When using graphing technology, include a screenshot or printout of graphs **or** sketch the image and indicate the window settings (maximum and minimum *x*- and *y*-values), increments, and axis labels, including units.

Name: ____

Answer all questions to the best of your ability. Show all your work.

Module 1: Quadratic Functions *(37 marks)*

1. How many x-intercepts will a quadratic function have if the vertex coordinates are (8, 9) and the function equation is $y = x^2 - 16x + 73$? Explain. *(2 marks)*
   (Module 1, Lesson 3)

   Answer:

   The vertex of this function is above the x-axis. Also, the quadratic function opens up. Therefore, there will be no x-intercepts.

2. A quadratic function has vertex coordinates at (3, 0). How many x-intercepts will the quadratic function have? Explain. *(2 marks)* (Module 1, Lesson 3)

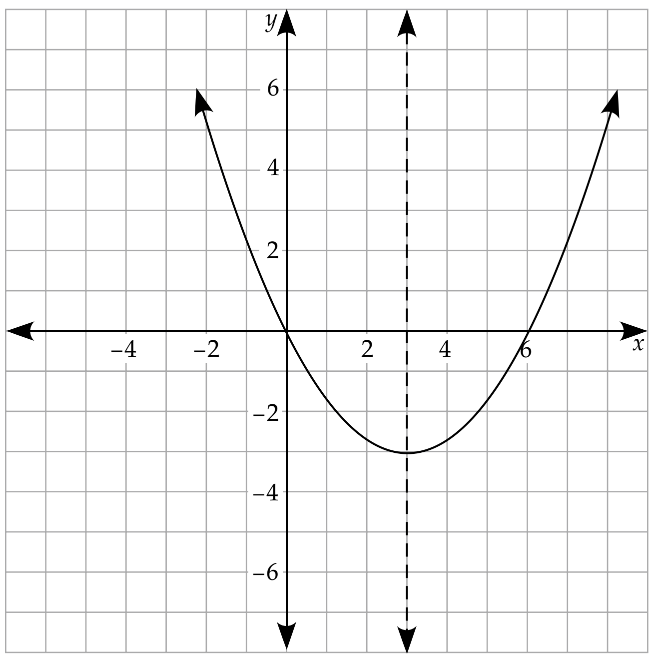
   Answer:

   The vertex is on the x-axis. Therefore, the quadratic function will have one x-intercept.

3. Sketch the following features of a quadratic function.
   a) The axis of symmetry equation is $x = 3$.
   b) The coordinates of the vertex are $(3, -3)$.
   c) The x-intercepts are 0 and 6.

   Then sketch the corresponding quadratic function. *(4 marks)* (Module 1, Lesson 1)

   Answer:



(1 mark for sketch of axis of symmetry)
(1 mark for vertex)
(1 mark for x-intercepts)
(1 mark for sketch of parabola)

Name: ______________________________

4. The x-intercepts of the quadratic function $y = -1.5x^2 - 2x + 3$ are -2.23 and 0.90. (Module 1, Lesson 2)

   a) What are the roots of the equation $-1.5x^2 - 2x + 3 = 0$? *(1 mark)*

      Answer:

      $x = -2.23$ and $x = 0.90$

   b) What is the relationship between the x-intercepts of a function and the roots of the corresponding equation? *(1 mark)*

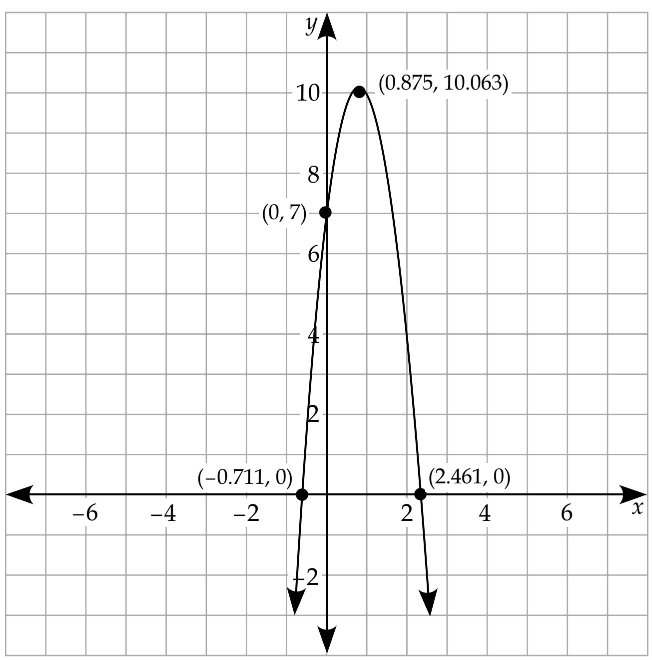
      Answer:

      The x-intercepts of a function have the same value as the roots of the corresponding equation.

5. Use a grapher to determine the x-intercepts, to two decimal places, of the graph of the quadratic function given below. Include a sketch of your graph with intercepts labelled. *(3 marks)* (Module 1, Lesson 2)

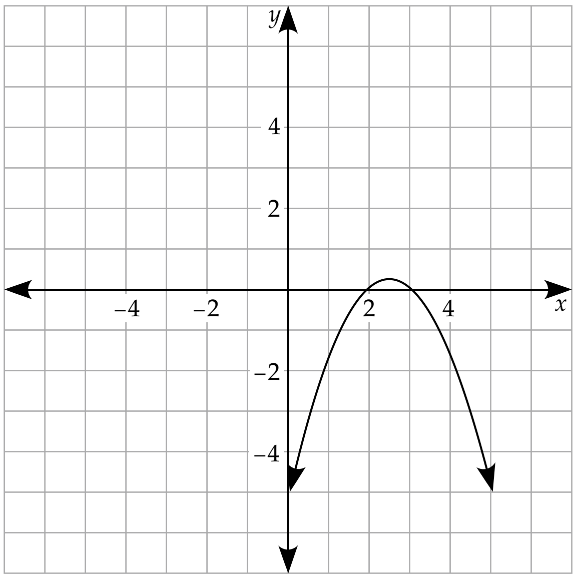
   $y = -4x^2 + 7x + 7$

   Answer:



The x-intercepts are -0.71 and 2.46.

6. Write a possible quadratic equation in factored form that has the same x-intercepts and opens in the same direction as the graph below. *(3 marks)* (Module 1, Lesson 2)



Answer:

The x-intercepts of the function are $x = 2$ and $x = 3$. This gives a possible quadratic equation of $y = -(x - 2)(x - 3)$.

Name: ________________________________

7. Draw three separate quadratic functions with the characteristics given. *(3 marks)* (Module 1, Lesson 1)

   a) The function has two x-intercepts and opens up.

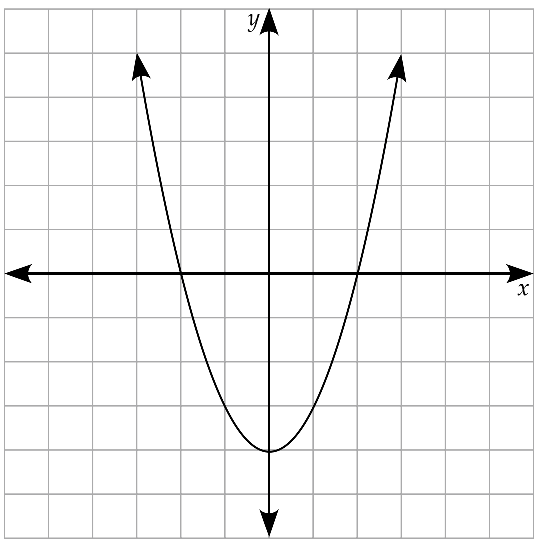
   b) The function has no x-intercepts and opens up.

   c) The function has one x-intercept and opens down.

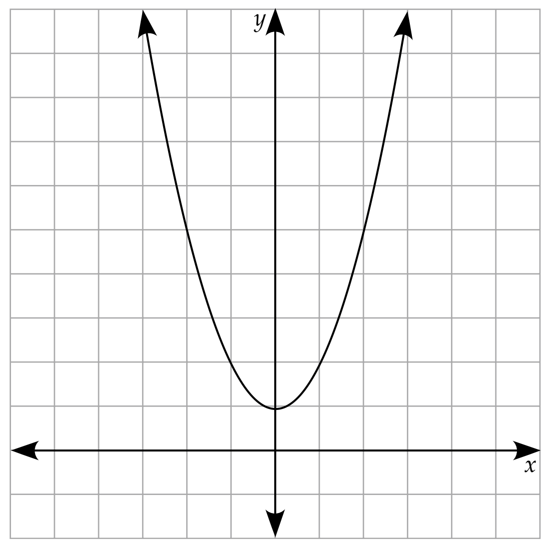
   Answer:

   (The graphs may vary from the function curves shown.)

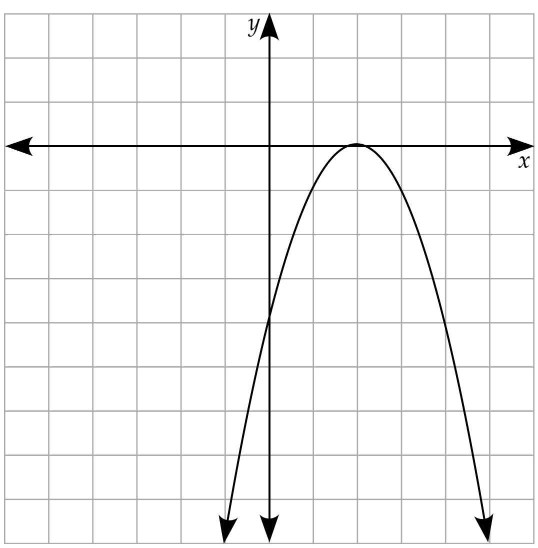
   a)



   b)



   c)



8. Consider the quadratic function $y = -\frac{1}{2}x^2 - 5x + 3$, with axis of symmetry equation $x = -5$. (Module 1, Lesson 3)

a) Determine the vertex coordinates. *(2 marks)*

Answer:

The y-value of the vertex occurs at the centre of the parabola, when $x = -5$.

$$y = -\frac{1}{2}(-5)^2 - 5(-5) + 3 = -\frac{25}{2} + 25 + 3 = 15.5$$

The vertex is $(-5, 15.5)$.

b) Determine the maximum and minimum values (if they exist). *(1 mark)*

Answer:

This quadratic function has a maximum value of 15.5.

There is no minimum.

c) Determine the domain. *(1 mark)*

Answer:

$(-\infty, \infty)$ or $\{x \in \Re\}$

d) Determine the range. *(1 mark)*

Answer:

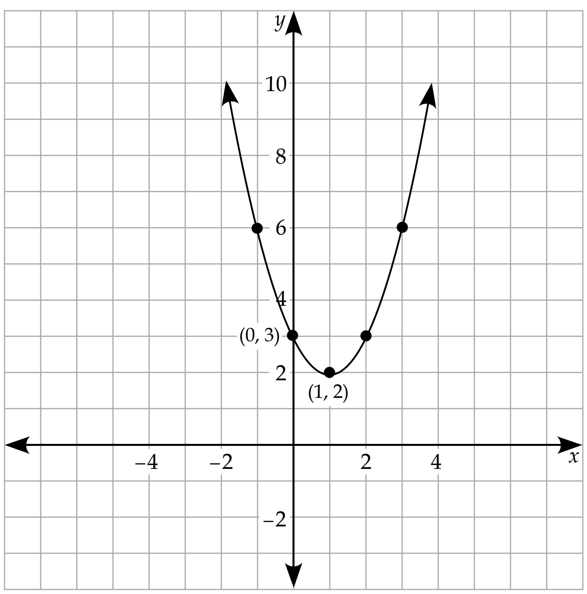
$(-\infty, 15.5]$ or $\{x \leq 15.5\}$

Name: ____

9. Create a table of values and sketch a graph of the quadratic function $y = x^2 - 2x + 3$. *(3 marks)* (Module 1, Lesson 4)

Answer:

x	y
-3	18
-2	11
-1	6
0	3
1	2
2	3
3	6

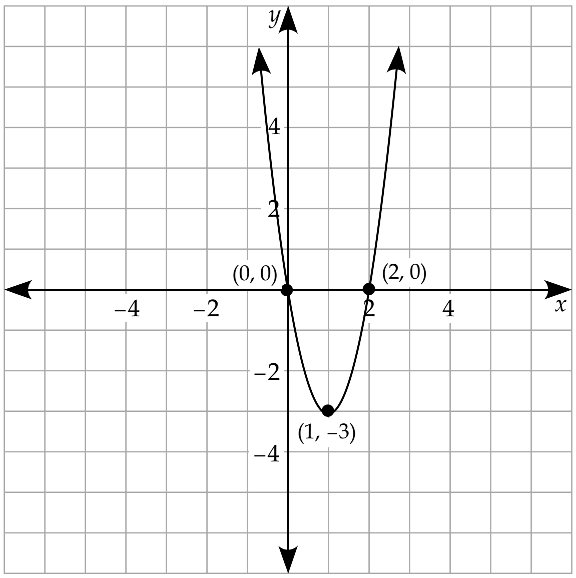


10. Sketch a graph of the following quadratic function using the intercepts. Include the coordinates of the intercepts and vertex on your graph. *(3 marks)* (Module 1, Lesson 4)

$y = 3(x - 2)(x)$

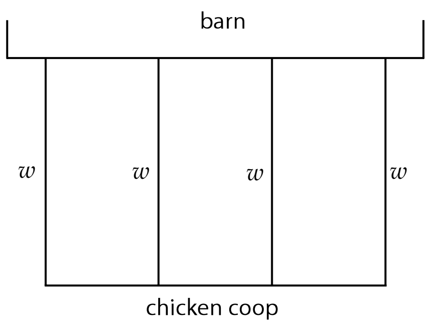
Answer:

The x-intercepts are 2 and 0. The y-intercept is $y = 3(-2)(0)$, which is 0. The function opens up. The axis of symmetry is at $x = 1$. The vertex is at $(1, -3)$.



Name: ____________________________________

11. An outdoor fenced chicken coop with three sections is to be built attached to a pre-existing barn, as shown. No fence is needed against the barn. Determine the dimensions of the chicken coop with the greatest area that can be enclosed using 650 feet of fencing, by completing the following. (Module 1, Lesson 5)



a) If 650 feet of fencing is to be used to create four widths, w, and one length, determine an expression for the length in terms of w. *(1 mark)*

Answer:

Length is $650 - 4w$.

b) Determine a quadratic function model to define area in terms of width. *(2 marks)*

Answer:

Area $= f(w) = (650 - 4w)(w) = -4w^2 + 650w$

c) Using a graphing utility, find the coordinates of the vertex of the quadratic function and interpret their meaning. *(2 marks)*

Answer:

The vertex is (81.25, 26 406). When the width of the pen is 81.25 feet, the area is a maximum of 26 406 square feet.

d) Determine the dimensions of the pen with the largest area. *(2 marks)*

Answer:

Length: $650 - 4w = 650 - 4(81.25) = 650 - 325 = 325$ feet

Width: 81.25 feet

Dimensions: 81.25 feet × 325 feet

Module 3: Reasoning to Solve Problems *(28 marks)*

1. Determine a conjecture that could be made about the sum of a multiple of 4 and a multiple of 6. Use at least two examples to help you develop your conjecture. *(3 marks)* (Module 3, Lesson 1)

   Possible Answer:

   Use 8 (a multiple of 4) and 12 (a multiple of 6). Their sum is 20.

   Use 12 (a multiple of 4) and 18 (a multiple of 6). Their sum is 30.

   A possible conjecture is:

   "The sum of a multiple of 4 and a multiple of 6 is always even."

2. Jordyn arrives at her first pre-calculus mathematics class in Grade 11. She realizes that there is only one other girl in the class, and the other members of the class are male. Jordyn then attends her first home economics class and notices that the majority of the class is female. State two possible conjectures Jordyn can make about the gender of students attending pre-calculus and home economics classes. *(2 marks)* (Module 3, Lesson 1)

   Possible Answer:

   Jordyn can make the conjectures that mainly males take pre-calculus mathematics and that mainly females take home economics classes.

3. If possible, find a counter-example to the following conjectures. (Module 3, Lesson 1)

   a) The sum of a multiple of 5 and a multiple of 6 will be an odd number. *(1 mark)*

      Possible Answer:

      Choose 10, a multiple of 5, and 12, a multiple of 6. Their sum is 22—an even number.

   b) All cars consume gasoline. *(1 mark)*

      Possible Answer:

      Electric cars do not consume gasoline.

Name: ____

4. Consider the following number trick.

   Pick a number.

   Subtract 1.

   Multiply the result by 3.

   Add 12.

   Divide the result by 3.

   Add 5.

   Subtract your original number.

   a) Make a conjecture about the result of the above number trick and provide two examples to support your conjecture. *(2 marks)* (Module 3, Lesson 1)

   Answer:

   Students must use two examples.

   Example: Pick 4.

   4 subtract 1 is 3.

   3 multiplied by 3 is 9.

   9 add 12 is 21.

   21 divided by 3 is 7.

   7 plus 5 is 12.

   12 subtract 4 is 8.

   The conjecture is:

   "The result will always be 8."

b) Prove that this number trick will always result in the conjecture you made in (a). *(4 marks)* (Module 3, Lesson 2)

Answer:

Let x be the number.	Pick a number.
$x - 1$	Subtract 1.
$3(x - 1)$	Multiply the result by 3.
$3x - 3$	Simplify.
$3x - 3 + 12$	Add 12.
$3x + 9$	Simplify.
$\frac{3x + 9}{3}$	Divide the result by 3.
$x + 3$	Simplify.
$x + 3 + 5$	Add 5.
$x + 8$	Simplify.
$x + 8 - x$	Subtract the original number.
8	Simplify.

Therefore, the result of the number trick will always be 8.

5. Determine whether the following scenarios represent inductive or deductive reasoning. *(2 marks)* (Module 3, Lesson 2)

a) Ms Newton told her class that if they receive an A on the final exam, then they will earn a final grade of A in the course. Britney receives an A on the final exam and expects to earn a final grade of A in the course.

Answer:

This is deductive reasoning.

b) It has rained for the past three days. Brayden assumes it will rain again tomorrow.

Answer:

This is inductive reasoning.

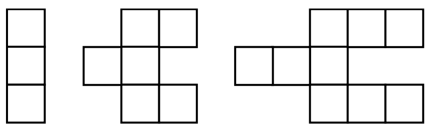
Name: ____

6. Use inductive reasoning to determine the next three terms in each of the following patterns. *(3 marks)* (Module 3, Lesson 1)

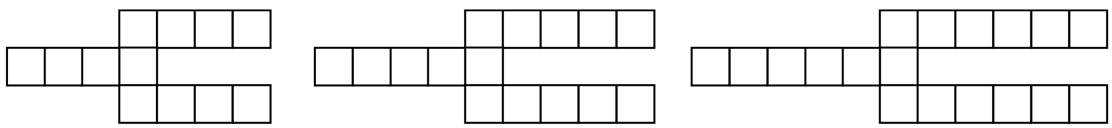
   a) 1, 2, 3, 2, 4, 6, 4, 8, 12, . . .

   Answer:

   The next three terms are 8, 16, and 24.

   b) 

   Answer:

7. Explain why the following proofs are invalid. *(4 marks)* (Module 3, Lesson 3)

   a) All parallelograms are quadrilaterals. Figure ABCD is a quadrilateral. Therefore, figure ABCD is a parallelogram.

   Answer:

   This is an error in reasoning. Not all quadrilaterals are parallelograms.

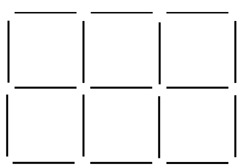
   b) Dylan is trying to prove that a number trick always results in the value of 5.

Let x be the number.	Pick a number.
$x + 3$	Add 3.
$2x + 6$	Multiply by 2.
$2x + 10$	Add 4.
$2x + 5$	Divide by 2.
$x + 5$	Subtract the original number.

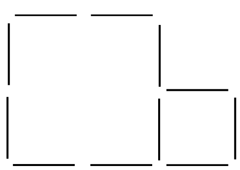
   Answer:

   This proof contains a mathematical error in step 5 as Dylan did not divide every term by 2.

8. Consider the following image made up of 17 toothpicks. Remove 6 toothpicks to leave two squares. *(2 marks)* (Module 3, Lesson 4)



Answer:



9. Prove that the sum of a multiple of 3 and a multiple of 6 is a multiple of 3. *(4 marks)* (Module 3, Lesson 2)

Answer:

Let $3a$ be a multiple of 3. Let $6b$ be a multiple of 6 where a and b are integers.	Introduce your variables.
$3a + 6b$	State their sum.
$3a + 3(2b)$	Factor out a 3 from $6b$.
$3(a + 2b)$	Factor out a 3.

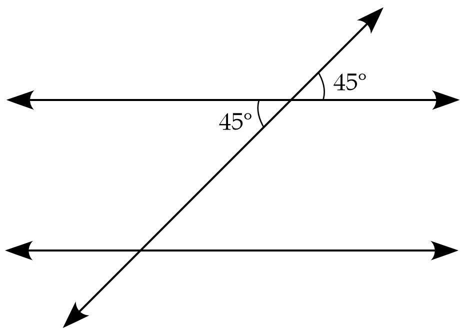
Therefore, the result is a multiple of 3.

Name: ______________________________

Module 4: Geometry of Angles and Triangles *(35 marks)*

1. Determine if the following sets of lines are parallel. Explain your answers. *(4 marks)* (Module 4, Lesson 4)

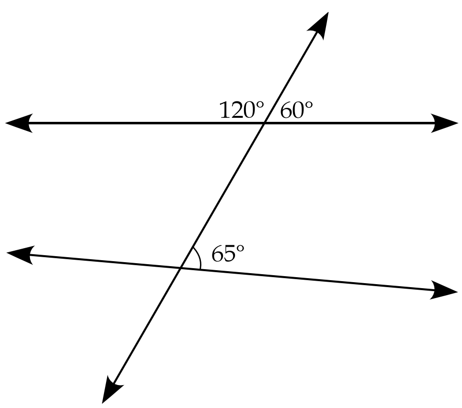
   a)



Answer:

You do not have enough information to determine if the lines are parallel. These 45° angles are vertically opposite angles, which are always equal, so the lines may or may not be parallel.

   b)

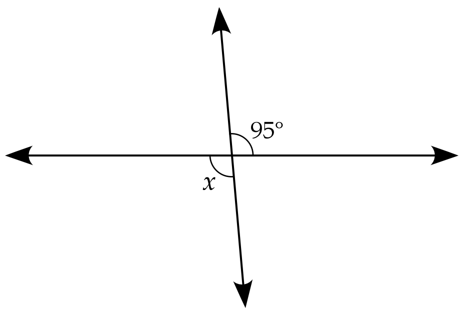


Answer:

These lines are not parallel because the 60° angle and the 65° angle are corresponding angles, which would be equal if the lines were parallel.

2. Find the indicated angle(s) in each of the diagrams below and state the property or rule you used to determine these angles. *(6 marks)*

a) *(1 mark)*

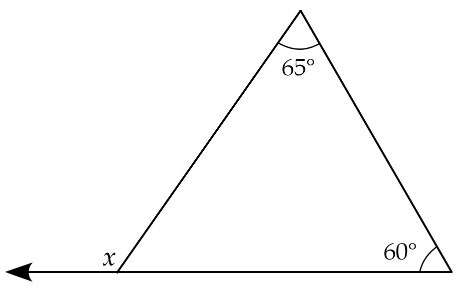


Answer: (Module 4, Lesson 1)

$y = 95°$

(Vertically opposite angles are equal)

b) *(1 mark)*



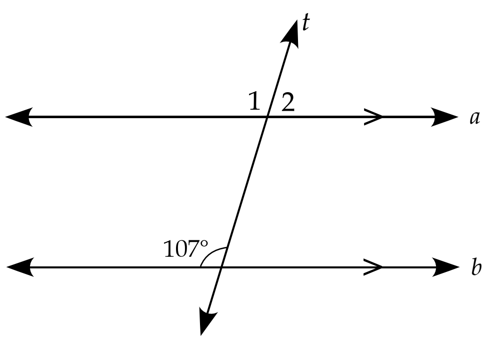
Answer: (Module 4, Lesson 1)

$\angle x = 65° + 60°$

$\angle x = 125°$

(Exterior angle = sum of opposite interior angles)

c) *(2 marks)*

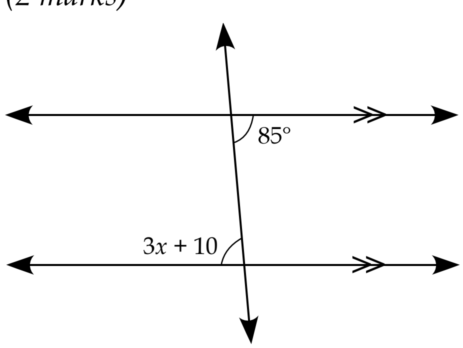


Answer: (Module 4, Lesson 3)

$\angle 1 = 107°$ (Corresponding angles are equal)

$\angle 2 = 73°$ (Supplementary angle with angle 1)

d) *(2 marks)*



Answer: (Module 4, Lesson 3)

$3x + 10 = 85°$ (Alternate interior angles are equal)

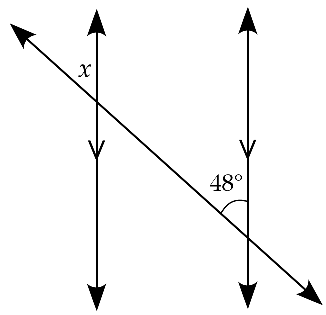
$3x = 75°$

$x = 25°$

Name: ____

3. For each diagram below, state the measurement of $\angle x$ and the property that allows you to determine it. *(6 marks)* (Module 4, Lesson 3)

a)

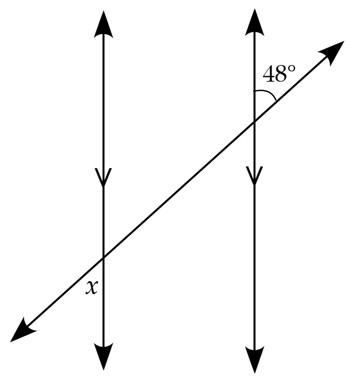


Answer:

$\angle x = 48°$

Property: Corresponding angles are equal

b)

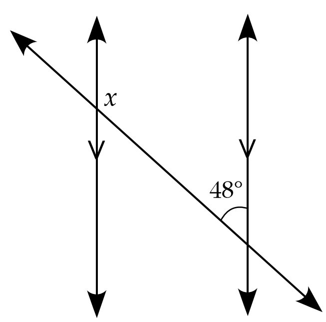


Answer:

$\angle x = 48°$

Property: Alternate exterior angles are equal

c)



Answer:

$\angle x = 132°$

Property: Same-side interior angles are supplementary

4. If the sum of the interior angles of a polygon is 4680°, how many sides does the polygon have? *(2 marks)* (Module 4, Lesson 2)

   Answer:

   $4680 = 180(n - 2)$

   $26 = n - 2$

   $28 = n$

   Therefore, the polygon has 28 sides.

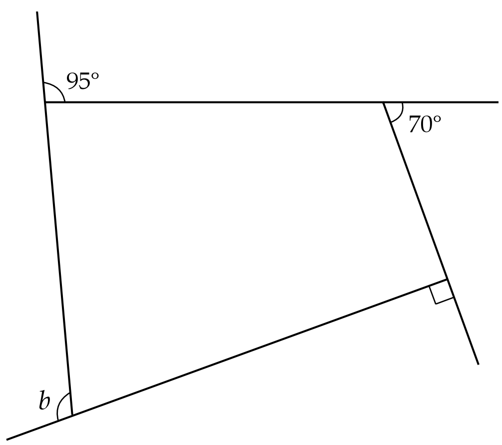
5. If a polygon has 17 sides, what is the sum of the interior angles? *(2 marks)* (Module 4, Lesson 2)

   Answer:

   $S = 180(17 - 2)$

   $S = 2700°$

6. Determine the size of the missing exterior angle in the polygon below. *(2 marks)* (Module 4, Lesson 2)



   Answer:

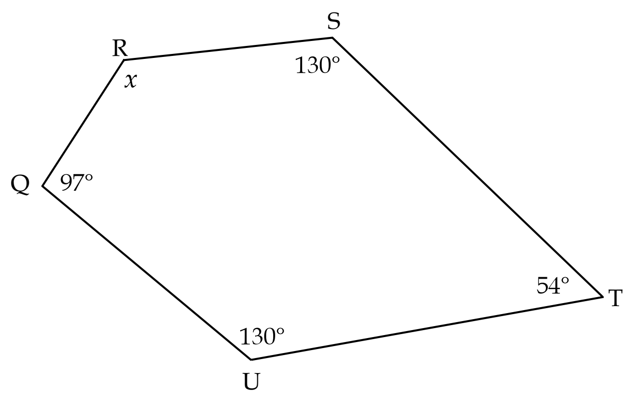
   $b + 95° + 70° + 90° = 360°$

   $b + 255° = 360°$

   $b = 105°$

Name: ______________________________________

7. Determine the size of the missing interior angle in the polygon below. *(2 marks)* (Module 4, Lesson 2)



Answer:

Sum of the interior angles in a polygon with 5 sides:

$S = 180(5 - 2)$

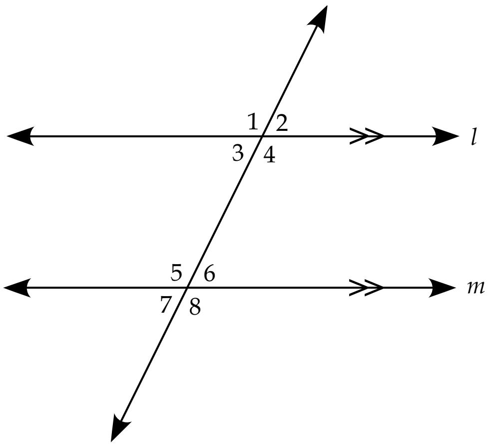
$S = 540°$

Missing Angle:

$x = 540° - 130° - 54° - 130° - 98°$

$x = 129°$

8. Prove that Angle 3 and Angle 5 have a sum of 180° in the diagram below, given that lines *l* and *m* are parallel. In other words, prove that same side interior angles sum to 180°. *(4 marks)* (Module 4, Lesson 3)

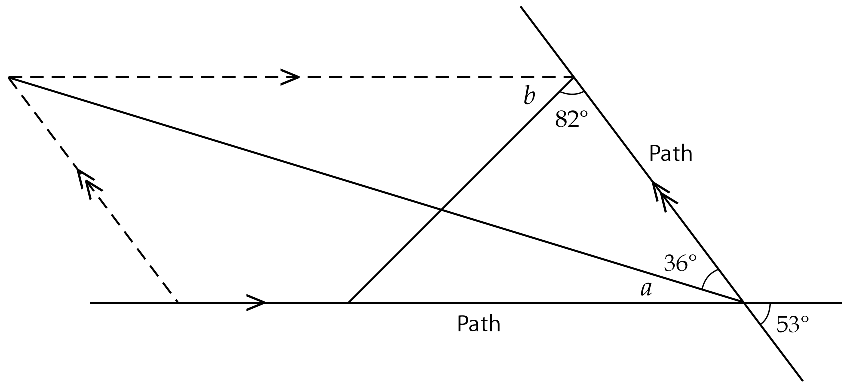


Answer:

Line *l* is parallel to line *m*.	Given.
$\angle 1 + \angle 3 = 180°$	They are supplementary angles.
$\angle 1 = \angle 5$	They are corresponding angles.
$\angle 5 + \angle 3 = 180°$	Substitute same values.

Name: ____

9. A school wants to plant a new flower garden in the shape of a parallelogram, as displayed by the diagram below. The flower garden will be divided into four sections.



a) Explain how the school could find the measurements of all the indicated angles without using a protractor. *(1 mark)* (Module 4, Lesson 4)

Answer:

The school can use properties of parallel lines and transversals to find $\angle a$, namely the property of supplementary angles or vertically opposite angles. The school can also use the property of same side interior angles to determine $\angle b$.

b) Determine the measurements of angles *a* and *b*. *(2 marks)* (Module 4, Lesson 4)

Answer:

The 36° angle and $\angle a$ combined are vertically opposite to the 53° angle. Therefore, they must add up to 53°.

$\angle a = 17°$

The 36° angle and $\angle a$ combined are same side interior angles with the combined 82° angle and $\angle b$. Therefore, these four angles must add up to 180°.

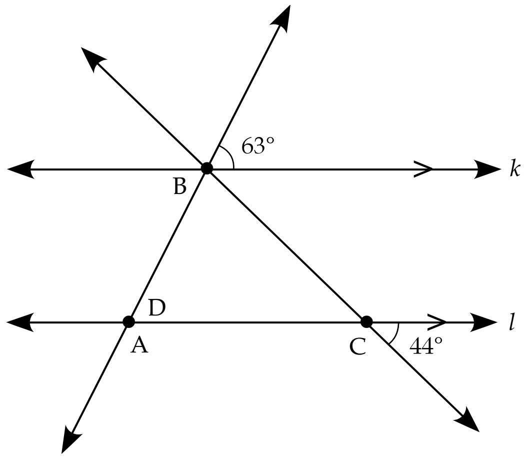
$\angle b + 82° + \angle a + 36° = 180°$

$\angle b + 82° + 17° + 36° = 180°$

$\angle b + 135° = 180°$

$\angle b = 45°$

10. Find the value of the angles labelled with a letter in the diagram below and state the property or rule you used to determine these angles. *(4 marks)* (Module 4, Lesson 4)



Answer:

$\angle B = 63°$	Vertically opposite to given angle.
$\angle C = 180° - 44°$	∠C and the given 44° angle are supplementary angles.
$\angle C = 136°$	
$\angle D = 63°$	Corresponding angle with given 63° angle.
$\angle A = 180° - 63°$	∠A and ∠D are supplementary angles.
$\angle A = 117°$	

GRADE 11
APPLIED MATHEMATICS

Module 5
Answer Key

Module 5, Lesson 1, Answer Key

Types of Accounts and the Operation of a Chequing Account

1. a)

DATE	NO	PARTICULARS	✓	CHEQUES		DEPOSITS		BALANCE	
Nov 7		Balance Forward						856	47
Nov 8	001	Manitoba Electric Company		57	80			798	67
Nov 8	002	Telephone Company		32	10			766	57
Nov 9		Paycheque				457	31	1223	88
Nov 15	003	Cable TV		18	78			1205	10
Nov 20	ATM	Cash		150	00			1055	10
Nov 23		Paycheque				380	92	1436	02

CANADIAN
BANK

November 8, (current year)

PAY TO THE
ORDER OF

Manitoba Electric Company $ *57.80*

Fifty - seven --80 DOLLARS
XX

NO. *001*

“ ‘003’”:00167’”010’:

Signature

CANADIAN
BANK

November 8, (current year)

PAY TO THE
ORDER OF

Telephone Company $ 32.10

Thirty-two --10 DOLLARS
XX

NO. 002

" '003'":00167'"010':

Signature

CANADIAN
BANK

November 15, (current year)

PAY TO THE
ORDER OF

Cable TV $ 18.78

Eighteen --78 DOLLARS
XX

NO. 003

" '003'":00167'"010':

Signature

b)

DATE	NO	PARTICULARS	✓	CHEQUES		DEPOSITS		BALANCE	
Jan 1		Balance Forward						412	85
Jan 3		Service Charge		1	83			411	02
Jan 8	004	Wholesale Outlet		218	56			192	46
Jan 8	ATM	Superstore		106	18			86	28
Jan 8	005	Hardware Store		58	40			27	88
Jan 17		Paycheque				265	40	293	28
Jan 20	006	Cathy's Clothing		32	12			261	16

CANADIAN
BANK

January 8, (current year)

PAY TO THE
ORDER OF Wholesale Outlet $ 218.56

Two hundred eighteen------------------------------------56/XX DOLLARS

NO. 004

" '003'':00167''010':

Signature

CANADIAN
BANK

January 8, (current year)

PAY TO THE
ORDER OF Hardware Store $ 58.40

Fifty-eight--40/XX DOLLARS

NO. 005

" '003'':00167''010':

Signature

CANADIAN
BANK

January 20, (current year)

PAY TO THE
ORDER OF

Cathy's Clothing $ 32.12

Thirty-two---$\frac{12}{XX}$ DOLLARS

NO. 006

" '003'":00167'"010':

Signature

c)

DATE	NO	PARTICULARS	✓	CHEQUES		DEPOSITS		BALANCE	
Mar 15		Balance Forward						256	08
Mar 17		Interest				3	12	259	20
Mar 20	ATM	Electronics		123	49			135	71
Mar 25		Paycheque				1726	41	1862	12
Mar 27	007	City of Portage		22	45			1839	67
Mar 27	008	Gas Canada		215	70			1623	97
Mar 30	009	ABC Grocery		189	67			1434	30
Mar 31		Loan Payment		416	00			1018	30

CANADIAN
BANK

March 27, (current year)

PAY TO THE
ORDER OF

City of Portage $ 22.45

Twenty-two---$\frac{45}{XX}$ DOLLARS

NO. 007

" '003'":00167'"010':

Signature

CANADIAN BANK

March 27, (current year)

PAY TO THE ORDER OF

Gas Canada $ 215.70

Two hundred fifteen-- 70/XX DOLLARS

NO. 008

" '003'":00167'"010':

Signature

CANADIAN BANK

March 30, (current year)

PAY TO THE ORDER OF

ABC Grocery $ 189.67

One hundred eighty-nine---------------------------------- 67/XX DOLLARS

NO. 009

" '003'":00167'"010':

Signature

2. a)

CANADIAN BANK

Current Account - Deposit

Date Nov 4	Account No. 001	
Account Name: Bill's Account (Please Print)		
x 2		
15 x 5	75	00
27 x 10	270	00
3 x 20	60	00
x 50		
x 100		
Coin	90	09
Total Cash	495	09
Cheques Only	24 31 8 49 76 47	47 37 19 95 80 89
Total Cheques	238	67
Credit Card		
Subtotal	733	76
Exchange		
Total	733	76

Deposited By *Bill*

b)

CANADIAN BANK

Current Account - Deposit

Date May 15	Account No. 456	
Account Name: Val's Account (Please Print)		
x 2		
11 x 5	55	00
28 x 10	280	00
4 x 20	80	00
x 50		
x 100		
Coin	72	89
Total Cash	487	89
Cheques Only	24 £ 86 USA 37	32 00 83
Total Cheques	*24	32
Credit Card		
Subtotal	512	21
Exchange	239	93
Total	752	14

Deposited By *Val*

£86 = 86 x Canadian \$2.1902
= Canadian \$188.36

U.S. \$37.83 = 37.83 x Canadian \$1.3633
= Canadian \$51.57

Total exchange = \$239.93

*Canadian cheque only

Module 5, Lesson 2, Answer Key

Account Reconciliation

1.

(1) CHEQUEBOOK BALANCE . . . . $ 171.88

DEDUCT ACCOUNT CHARGES

Description	Amount	
Service Charge	1	26

(2) TOTAL ACCOUNT CHARGES . . . . $ 1.26

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 170.62

(4) ACCOUNT BALANCE. . . . . $ 211.24

OUTSTANDING DEPOSITS

Date	Amount	
	50	00

(5) TOTAL OUTSTANDING DEPOSITS. . $ 50.00

(6) TOTAL. . . . . $ 261.24

OUTSTANDING CHEQUES

CH. NO.	Amount	
15	37	95
19	8	27
22	6	90
23	37	50

(7) TOTAL OUTSTANDING CHEQUES . . $ 90.62

(8) ADJUSTED ACCOUNT BALANCE. . . $ 170.62

2.

(1) CHEQUEBOOK BALANCE . . . . $ 348.68

DEDUCT ACCOUNT CHARGES

Description	Amount	
Service Charge	1	68

(2) TOTAL ACCOUNT CHARGES . . . . $ 1.68

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 347.00

(4) ACCOUNT BALANCE. . . . . $ 234.47

OUTSTANDING DEPOSITS

Date	Amount	
	190	00

(5) TOTAL OUTSTANDING DEPOSITS . . $ 190.00

(6) TOTAL. . . . . $ 424.47

OUTSTANDING CHEQUES

CH. NO.	Amount	
180	35	84
188	3	00
190	17	88
194	20	75

(7) TOTAL OUTSTANDING CHEQUES . . $ 77.47

(8) ADJUSTED ACCOUNT BALANCE. . . $ 347.00

3.

(1) CHEQUEBOOK BALANCE . . . . $ 1650.51

DEDUCT ACCOUNT CHARGES

Description	Amount	
Service Charge	1	26

(2) TOTAL ACCOUNT CHARGES . . . . $ 1.26

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 1649.25

(4) ACCOUNT BALANCE. . . . . $ 1661.75

OUTSTANDING DEPOSITS

Date	Amount	

(5) TOTAL OUTSTANDING DEPOSITS . . $ Ø

(6) TOTAL. . . . . $ 1661.75

OUTSTANDING CHEQUES

CH. NO.	Amount	
44	3	50
46	9	00

(7) TOTAL OUTSTANDING CHEQUES . . $ 12.50

(8) ADJUSTED ACCOUNT BALANCE. . . $ 1649.25

4.

(1) CHEQUEBOOK BALANCE . . . . $ 439.82

DEDUCT ACCOUNT CHARGES

Description	Amount	
Service Charge	1	12

(2) TOTAL ACCOUNT CHARGES . . . . $ 1.12

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 438.70

(4) ACCOUNT BALANCE. . . . . $ 365.00

OUTSTANDING DEPOSITS

Date	Amount	
	100	00

(5) TOTAL OUTSTANDING DEPOSITS . . $ 100.00

(6) TOTAL. . . . . $ 465.00

OUTSTANDING CHEQUES

CH. NO.	Amount	
211	7	00
213	7	30
217	6	80
218	5	20

(7) TOTAL OUTSTANDING CHEQUES . . $ 26.30

(8) ADJUSTED ACCOUNT BALANCE. . . $ 438.70

5.

(1) CHEQUEBOOK BALANCE . . . . $ 1765.42

DEDUCT ACCOUNT CHARGES

Description	Amount	
Service Charge	10	00
	12	00

(2) TOTAL ACCOUNT CHARGES . . . . $ 22.00

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 1743.42

(4) ACCOUNT BALANCE. . . . . $ 596.12

OUTSTANDING DEPOSITS

Date	Amount	
	1485	00

(5) TOTAL OUTSTANDING DEPOSITS . . $ 1485.00

(6) TOTAL. . . . . $ 2081.12

OUTSTANDING CHEQUES

CH. NO.	Amount	
45	35	80
47	76	10
50	225	80

(7) TOTAL OUTSTANDING CHEQUES . . $ 337.70

(8) ADJUSTED ACCOUNT BALANCE. . . $ 1743.42

6.

(1) CHEQUEBOOK BALANCE . . . . $ 185.86

DEDUCT ACCOUNT CHARGES

Description	Amount	
Service Charge	6	60

(2) TOTAL ACCOUNT CHARGES . . . . $ 6.60

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 179.26

(4) ACCOUNT BALANCE. . . . . $ 149.24

OUTSTANDING DEPOSITS

Date	Amount	
	27	70
	12	30

(5) TOTAL OUTSTANDING DEPOSITS . . $ 40.00

(6) TOTAL . . . . $ 189.24

OUTSTANDING CHEQUES

CH. NO.	Amount	
132	9	98

(7) TOTAL OUTSTANDING CHEQUES . . $ 9.98

(8) ADJUSTED ACCOUNT BALANCE. . . $ 179.26

7.

(1) CHEQUEBOOK BALANCE . . . . $ 6547.65

DEDUCT ACCOUNT CHARGES

Description	Amount	
Service Charge	16	00

(2) TOTAL ACCOUNT CHARGES . . . . $ 16.00

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 6531.65

(4) ACCOUNT BALANCE. . . . . $ 7232.43

OUTSTANDING DEPOSITS

Date	Amount	

(5) TOTAL OUTSTANDING DEPOSITS . . $_______

(6) TOTAL . . . . $ 7232.43

OUTSTANDING CHEQUES

CH. NO.	Amount		CH. NO.	Amount	
784	58	17	791	182	13
785	73	27	792	16	18
789	72	83	Mar 29	152	03
790	146	17			

(7) TOTAL OUTSTANDING CHEQUES . . $ 700.78

(8) ADJUSTED ACCOUNT BALANCE. . . $ 6531.65

Module 5, Lesson 3, Answer Key

Investments and Compound Interest

1. $I = p \cdot r \cdot t$
   $I = 1900(0.075)(3)$
   Interest earned = \$427.50

2. $I = p \cdot r \cdot t$
   $I = \$2500(0.0625)(0.75)$
   Interest earned = \$117.19

3. a) $I = p \cdot r \cdot t$
   $I = \$5000(0.062)(0.25)$
   Interest earned on the three month certificate = \$77.50

   b) \$77.50 – \$68.75 = \$8.75
   The three month certificate earns \$8.75 more

4. For the first 6 months:
   $I = p \cdot r \cdot t$
   I = \$6000(0.072)(0.5)
   Interest earned= \$216.00

   For the second 6 months:
   $I = p \cdot r \cdot t$
   I = \$6216(0.072)(0.5)
   Interest earned = \$223.78

   Total interest earned: \$216.00 + \$223.78 = \$439.78

5. a)
   $$A = P\left(1 + \frac{r}{s}\right)^{n \cdot s}$$
   $$A = \$6000\left(1 + \frac{0.072}{2}\right)^{(1)(2)}$$
   $$A = \$6439.78$$

   Interest earned = \$6439.78 – \$6000.00 = \$439.78

   b) They are the same.

   c) The first method of investing requires that the money be reinvested a second time. The second method requires that you invest the money only once.

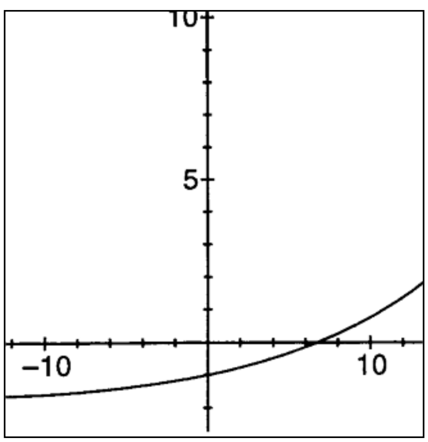
6.
$$A = P\left(1+\frac{r}{s}\right)^{n \cdot s}$$
$$6000 = P\left(1+\frac{0.0575}{12}\right)^{(3)(12)}$$
$$P = \frac{6000}{\left(1+\frac{0.0575}{12}\right)^{(3)(12)}}$$

Original investment = \$5051.43

7.
$$A = P\left(1+r\right)^{n}$$
$$9000 = 4500\left(1+0.085\right)^{n}$$
$$2 = 1.085^{n}$$

This is equivalent to $0 = 1.085^{n} - 2$

To find n, graph the function $y = 1.085^{x} - 2$ with the Keying sequence: [Y=] [1.085] [^] [X,T,θ,n] [–] [2] [GRAPH]



Using the program [2nd] [CALC] [2] and setting appropriate left and right bounds, find the zero for the function. This gives x = 8.5. This indicates that in 8.5 years the investment will double itself.

8.
$$I = p \cdot r \cdot t$$
$$I = 450\left(0.175\right)\left(\frac{1}{12}\right)$$
$$\text{Interest} = \$6.56$$

Note: $p = 650 - 200 = \$450$ and $t = \frac{1}{12}$ of a year

Module 5, Lesson 4, Answer Key

Budgeting

1.	Food	\$27.85, \$17.15 Total for 3 months = \$45.00 Monthly average = \$45.00 ÷ 3 = \$15.00
	Transportation	\$50.00, \$21.95, \$35.00, \$43.00 Total for 3 months = \$149.95 Monthly average = \$149.95 ÷ 3 = \$49.98
	Entertainment	\$8.00, \$6.50, \$5.00, \$2.00, \$8.50, \$12.50, \$10.00, \$13.00 Total for 3 months = \$65.50 Monthly average = \$65.50 ÷ 3 = \$21.83
	Clothes	\$80.35, \$34.75 Total for 3 months = \$115.10 Monthly average = \$115.10 ÷ 3 = \$38.37
	Miscellaneous	\$5.89, \$3.25, \$13.49, \$2.43, \$3.25, \$19.72 Total for 3 months = \$48.03 Monthly average = \$48.03 ÷ 3 = \$16.01

2. Rent, insurance, and loans are fixed.
   Clothing, donations, supplies, and food are variable.

3. Unexpected car repairs, medical expenses, and an appliance breakdown are examples of unexpected expenses. Bonus salary or wage increases, lottery winnings, and unexpected inheritances are examples of unexpected income.

4.

Monthly Budget Form

1) Net Income

Primary Annual Income	$ 31 680.00	
Secondary Annual Income	$ 29 400.00	Average
Other Annual Income Family allowance	$ 0.00	Monthly
Total Annual Income	$ 61 080.00	Income
		1) $ 5090.00
2) Savings (10% of Average Monthly Income)		2) $ 509.00

3) Monthly Expenses

Mortgage or Rent	$ 900.00	
Car Payment	$ 550.00	
Telephone	$ 155.00	
Hydro	$ 115.00	
Other Utilities Natural gas	$ 95.00	
Groceries	$ 475.00	
Clothing 1125 ÷ 12	$ 93.75	
Car Maintenance 400 ÷ 12	$ 33.33	
Gasoline	$ 275.00	
Credit Card Debt	$ 225.00	
Entertainment 5200 ÷ 12	$ 433.33	
Other Newspapers etc. 150 ÷ 12	$ 12.50	Monthly
Other RRSP	$ 300.00	Total
Other	$	
Total Monthly Expenses		3) $ 3662.91

4) Annual Expenses (Monthly payments required)

Car Insurance 725 ÷ 12	$ 60.42	
Life Insurance	$ 0.00	
Property Taxes 5850 ÷ 12	$ 487.50	
Home Insurance 475 ÷ 12	$ 39.58	
Vacations 6500 ÷ 12	$ 541.67	Monthly
Other Gifts etc. 900 ÷ 12	$ 75.00	Total
Other	$	
Total Monthly Payments		4) $ 1204.17

Summary

1) Average Monthly Income		1) $ 5090.00
2) Savings	2) $ 509.00	
3) Total Monthly Expenses	3) $ 3662.91	
4) Total Monthly Payments	4) $ 1204.17	
Total amounts (2) + (3) + (4)		5) $ 5376.08
5) Amount Available for Other Savings or Expenditures (Deficit) (1 - 5)		6) $ (286.08)

Note: If the person or family is in a deficit position, the budget needs to be analyzed for possible adjustments.

5. **Monthly Budget Form**

1) Net Income

Primary Annual Income	$ 14 820.00	
Secondary Annual Income	$ 27 820.00	Average Monthly Income
Other Annual Income Family allowance	$ 564.00	
Total Annual Income	$ 43 204.00	1) $ 3600.33

2) Savings (10% of Average Monthly Income) 2) $ 360.03

3) Monthly Expenses

Mortgage or Rent	$ 680.00	
Car Payment	$ 295.00	
Telephone	$ 85.30	
Hydro	$	
Other Utilities	$ 168.90	
Groceries	$ 574.00	
Clothing 885 ÷ 12	$ 73.75	
Car Maintenance	$ 55.00	
Gasoline	$ 125.00	
Credit Card Debt	$ 200.00	
Entertainment 1850 ÷ 12	$ 154.17	
Other Babysitting 780 ÷ 12	$ 65.00	Monthly
Other Loan payment	$ 312.00	Total
Other Newspapers etc. 158 ÷ 12	$ 13.17	
Total Monthly Expenses		3) $ 2801.29

4) Annual Expenses (Monthly payments required)

Car Insurance 587 ÷ 12	$ 48.92	
Life Insurance	$ 0.00	
Property Taxes 4070 ÷ 12	$ 339.17	
Home Insurance 296 ÷ 12	$ 24.67	
Vacations 4260 ÷ 12	$ 355.00	Monthly
Other Gifts etc. 780 ÷ 12	$ 65.00	Total
Other	$	
Total Monthly Payments		4) $ 832.76

Summary

1) Average Monthly Income		1) $ 3600.33
2) Savings	2) $ 360.03	
3) Total Monthly Expenses	3) $ 2801.29	
4) Total Monthly Payments	4) $ 832.76	
Total amounts (2) + (3) + (4)		5) $ 3994.08
5) Amount Available for Other Savings or Expenditures (Deficit) (1 - 5)		6) $ (393.75)

Note: If the person or family is in a deficit position, the budget needs to be analyzed for possible adjustments.

Notes

Self-Test Answer Key

Module 5 – Budgets and Investments

/60

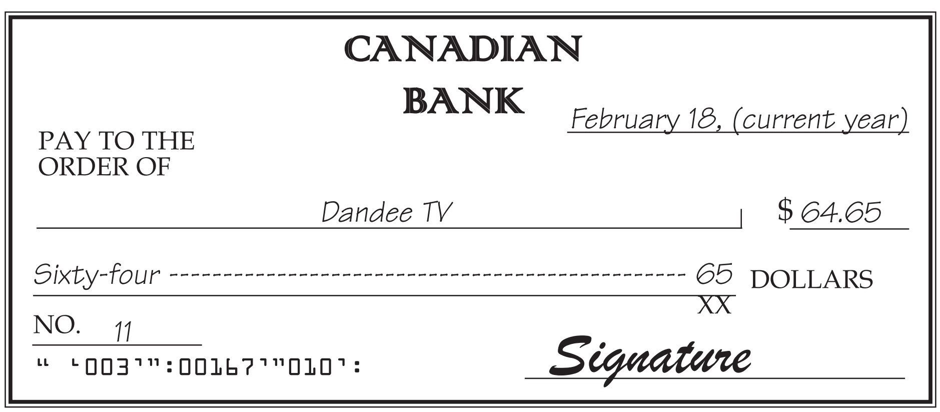
(5 marks)

1. a) iv
   b) i
   c) ii
   d) iii
   e) ii

(4 marks)

2.

DATE	NO	PARTICULARS	✓	CHEQUES		DEPOSITS		BALANCE	
Feb 15		Balance Forward						462	30
Feb 18	11	Dandee TV		64	65			397	65
Feb 22	12	Mr. Ribs		18	56			379	09
Feb 25		Paycheque				1357	45	1736	54
Feb 28		Service Charge		3	56			1732	98
Mar 1		Loan Payment		259	00			1473	98
Mar 3	13	Modern Foods		126	91			1347	07
Mar 5	14	Mat Hematics		3	59			1343	48
Mar 14		Cash		212	87			1130	61
Mar 14		Interest Charge		11	52			1119	09
Mar 25		Paycheque				1282	39	2401	48



CANADIAN BANK

February 22, (current year)

PAY TO THE ORDER OF Mr. Ribs $ 18.56

Eighteen -- 56/XX DOLLARS

NO. 12

" '003'":00167'"010':

Signature

CANADIAN BANK

March 3, (current year)

PAY TO THE ORDER OF Modern Foods $ 126.91

One hundred twenty six ------------------------------------91/XX DOLLARS

NO. 13

" '003'":00167'"010':

Signature

CANADIAN BANK

March 5, (current year)

PAY TO THE ORDER OF Mat Hematics $ 3.59

Three ---59/XX DOLLARS

NO. 14

" '003'":00167'"010':

Signature

(4 marks)

3.

CANADIAN BANK

Current Account - Deposit

Date Jan 16/99	Account No. 10256

Account Name

Signature

Please Print

4 x 5	20	00
8 x 10	80	00
2 x 20	40	00
2 x 50	100	00
x 100		
Coin	22	77
Total Cash	262	77
Cheques Only	81 17 56 103	40 53 75 20
Total Cheques	258	88
Credit Card		
Subtotal	521	65
Exchange	97	03
Total	618	68

Deposited By

Signature

4. a) *(4 marks)*

(1) CHEQUEBOOK BALANCE . . . . $ 379.92

DEDUCT BANK CHARGES

Description	Amount	
Service charge	2	10

(2) TOTAL BANK CHARGES . . . . $ 2.10

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 377.82

(4) BANK BALANCE. . . . . $ 368.32

OUTSTANDING DEPOSITS

Date	Amount	
	150	00

(5) TOTAL OUTSTANDING DEPOSITS . . $ 150.00

(6) TOTAL . . . . $ 518.32

OUTSTANDING CHEQUES

CH. NO.	Amount	
487	18	50
489	25	60
490	96	40

(7) TOTAL OUTSTANDING CHEQUES . . $ 140.50

(8) ADJUSTED BANK BALANCE. . . . . . . $ 377.82

b) *(6 marks)*

(1) CHEQUEBOOK BALANCE . . . . $ 498.09

DEDUCT BANK CHARGES

Description	Amount	
Service charge	10	50

(2) TOTAL BANK CHARGES . . . . $ 10.50

(3) ADJUSTED CHEQUEBOOK BALANCE . . $ 487.59

(4) BANK BALANCE. . . . . $ 810.71

OUTSTANDING DEPOSITS

Date	Amount	
	45	00
	200	00

(5) TOTAL OUTSTANDING DEPOSITS . . $ 245.00

(6) TOTAL . . . . $ 1055.71

OUTSTANDING CHEQUES

CH. NO.	Amount	
002	135	79
004	28	69
005	112	65
007	53	25

008 237.74

(7) TOTAL OUTSTANDING CHEQUES . . $ 568.12

(8) ADJUSTED BANK BALANCE. . . . . . . $ 487.59

(10 marks)

5. **Monthly Budget Form**

1) Net Income

Primary Annual Income	$ 28 164.00	
Secondary Annual Income	$ 25 620.00	Average Monthly Income
Other Annual Income	$ 0.00	
Total Annual Income	$ 53 784.00	
		1) $ 4482.00
2) Savings (10% of Average Monthly Income)		2) $ 448.20

3) Monthly Expenses

Mortgage or Rent	$ 900.00	
Car Payment	$ 350.00	
Telephone	$ 35.00	
Hydro	$ 80.00	
Other Utilities . . . Natural gas	$ 75.00	
Groceries	$ 450.00	
Clothing . . . . . . 1000 ÷ 12	$ 83.33	
Car Maintenance . . 600 ÷ 12	$ 50.00	
Gasoline	$ 150.00	
Credit Card Debt	$ 300.00	
Entertainment . . . 4800 ÷ 12	$ 400.00	
Other . . . . RRSP	$ 150.00	Monthly
Other . . Newspapers etc. 350 ÷ 12	$ 29.17	Total
Other	$	
Total Monthly Expenses		3) $ 3052.50

4) Annual Expenses (Monthly payments required)

Car Insurance . . . . 550 ÷ 12	$ 45.83	
Life Insurance	$ 0.00	
Property Taxes . . . . 3500 ÷ 12	$ 291.67	
Home Insurance . . . 300 ÷ 12	$ 25.00	
Vacations . . . . . 3000 ÷ 12	$ 250.00	Monthly
Other . . . Gifts etc. 1200 ÷ 12	$ 100.00	Total
Other	$	
Total Monthly Payments		4) $ 712.50

Summary

1) Average Monthly Income		1) $ 4482.00
2) Savings	2) $ 448.20	
3) Total Monthly Expenses	3) $ 3052.50	
4) Total Monthly Payments	4) $ 712.50	
Total amounts (2) + (3) + (4)		5) $ 4213.20
5) Amount Available for Other Savings or Expenditures (Deficit) (1 – 5)		6) $ 268.80

Note: If the person or family is in a deficit position, the budget needs to be analyzed for possible adjustments.

6. *(7 marks)*

	A	B	C	D	E	F
1			ASSESSMENTS			
2	CURRENT ASSESSMENT		TOTAL ASSESSMENT	PORTION	TOTAL PORTION ASSESSMENT	LOCAL IMPROVEMENT
3	LAND	BUILDINGS		% IN DEC. FORM		
4	$17500.00	$52000.00	$69500.00	0.3	$20850.00	$10000.00
5						
6			MUNICIPAL TAXES			
7		FRONTAGE	ASSESSMENT	MILL RATE	LEVY	
8	GENERAL		$20850.00	52.63	$1097.34	
9	IMPROVEMENT 1	$65.00	$10000.00	0.032	$65.32	
10	IMPROVEMENT 2		$10000.00	2.613	$26.13	
11	IMPROVEMENT 3		$10000.00	6.75	$67.50	
12						
13			EDUCATION TAXES			
14		EDUCATION 1	$20850.00	6.8	$141.78	
15		EDUCATION 2	$20850.00	15.14	$315.67	
16						
17			PROVINCIAL CREDITS			
18			$250.00			
19						
20			SUMMARY OF TAXES			
21	MUNICIPAL	EDUCATION	TOTAL TAXES	PROV. CREDITS	NET TAXES	TAXES DUE
22	$1256.29	$457.45	$1713.73	$250.00	$1463.73	$1463.73
23	ARREARS	$0.00	ADD. TAXES	$0.00		

* Final answers may differ by a few cents due to rounding.

(5 marks)

7. a) Let x = the number of litres of dandelion killer
Let y = the number of litres of crabgrass killer

$x + y = 50$	50 litres are made.
$3.50x + 2.80y = 3.08(50)$	The 50 litres of the mix is worth \$3.08/litre.

Solve the two equations for y so that they may be graphed.

$$y = -x + 50$$
$$y = \frac{-3.5}{2.8}x + \frac{154}{2.8}$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [3.5] [÷] [2.8] [X,T,θ,*n*] [+] [154] [÷] [2.8] [ENTER]
[(–)] [X,T,θ,*n*] [+] [50] [GRAPH]

Find where the graphs meet by pressing [2nd] [CALC] [5].

Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 20 \quad y = 30$

Kehlin will mix 20 litres of dandelion killer with 30 litres of crabgrass killer.

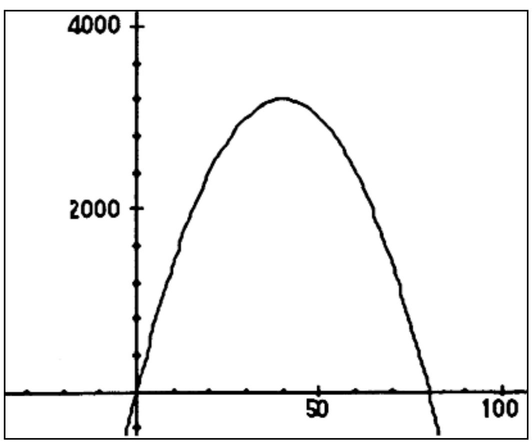
b) Let area be y ft.^{2} *(5 marks)*
Let the width of the rectangle be x ft.

Then the length of the lot will be $160 - 2x$ ft. since the sum of the two widths and the length of the lot is 160 ft.

Area = Length x Width
$y = (160 - 2x)(x)$
$y = 160x - 2x^2$
$y = -2x^2 + 160x$

Graph the function.

Keying sequence:
[Y=] [(–)] [2] [X,T,θ,n] [x^2] [+] [160] [X,T,θ,n] [ENTER] [GRAPH]



Reset window

Find the maximum value using the program:

[2nd] [CALC] [4], setting appropriate left and right bounds, and then pressing [ENTER] to find the point (40, 3200).

This indicates that the maximum area will be 3200 ft.^{2} brought about by a width of 40 ft. and a length of $(160 - 2x) = 80$ ft.

(10 marks)

c) Let \$C = cost
Let x = number of hours Pat should work
Let y = number of hours Amy should work

C = \$15.00x + \$20.00y

Constraints

Bolts: $600x + 1000y \geq 6000$	Since Pat can make 600 bolts in one hour and Amy can make 1000 bolts in one hour and they must make at least 6000 bolts in order to fill the order.
Nails: $400x + 400y \geq 3200$	Since Pat can make 400 nails in one hour and Amy can make 400 nails in one hour and they must make at least 3200 nails in order to fill the order.
$x \geq 0$	There cannot be a negative umber of hours.
$y \geq 0$	There cannot be a negative number of hours.

Graph the constraints:

$x \geq 0$ $y \geq 0$ $600x + 1000y \geq 6000$ $400x + 400y \geq 3200$

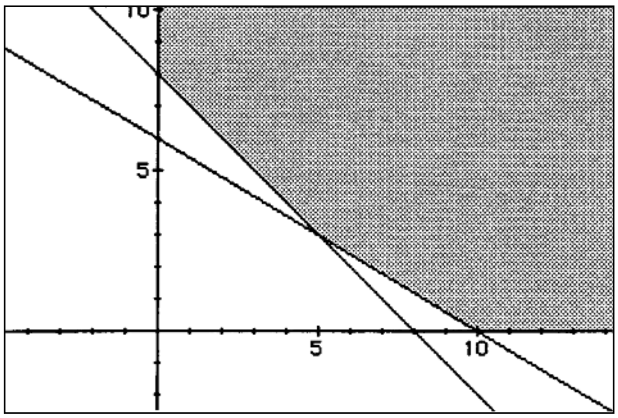
$x = 0$ $y = 0$ $600x + 1000y = 6000$ $400x + 400y = 3200$

Related equations

x	0	10
y	6	0

x	0	8
y	8	0

Intercept method



Equations for calculator:

$$y = \frac{-3}{5}x + 6$$

$$y = -x + 8$$

The shaded region is the feasible region having corner points of: (0, 8), (10, 0), and (5, 3). All may be found from the graph.

Substitute each corner point into the cost function and find the minimum value.

For (0, 8): $C = 15x + 20y = 15(0) + 20(8) = 160$
For (10, 0): $C = 15x + 20y = 15(10) + 20(0) = 150$
For (5, 3): $C = 15x + 20y = 15(5) + 20(3) = 135$

Minimum cost = $135.00 brought about by Pat working five hours and Amy working three hours.

60

GRADE 11
APPLIED MATHEMATICS

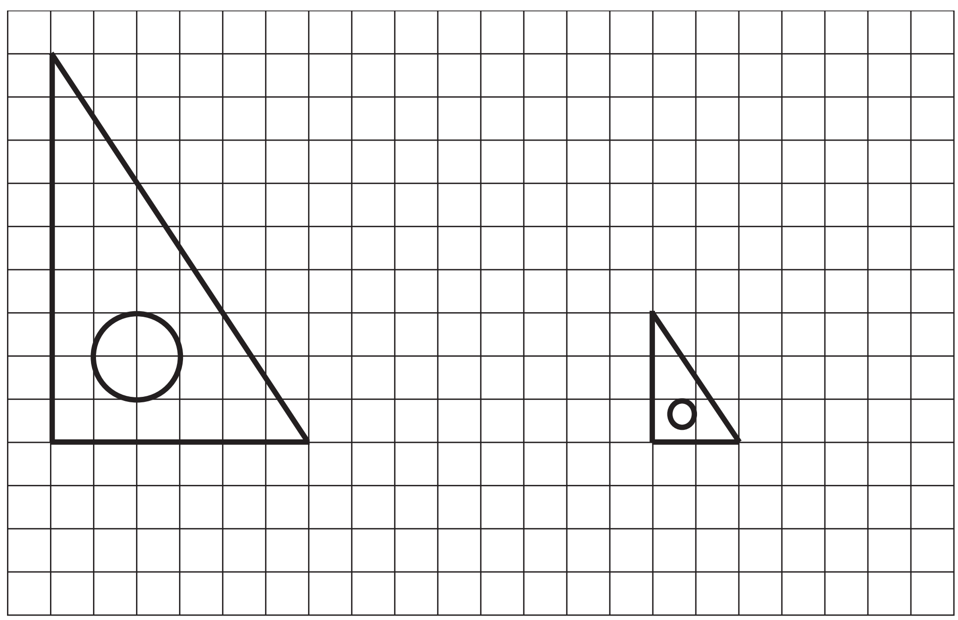
Module 6
Answer Key

Module 6, Lesson 1, Answer Key

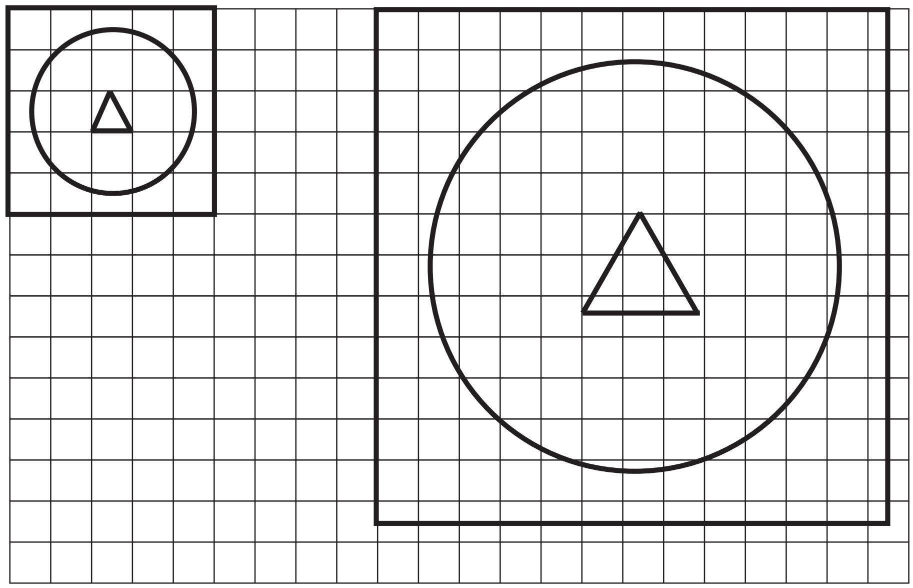
Scale Diagrams

1. State the relationship between the actual object and the scale diagram if the following scales are given:
   a) The diagram is a reduction by a factor of 3.
   b) The diagram is an enlargement by a factor of 4.
   c) The diagram is a reduction by a factor of 2.5.
   d) The diagram is an enlargement by a factor of 6.5.
   e) The diagram is an enlargement by a factor of 144. (12" = 1').
   f) The diagram is a reduction by a factor of 3 168 000.
   1 mile = 63 360 inches
   50 miles = 3 168 000 inches.

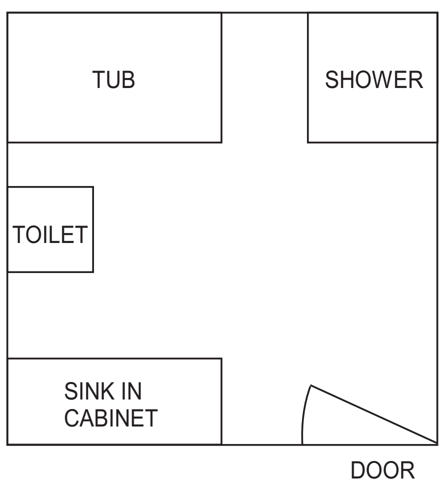
2. a) Scale 1:3



b) Scale 2.5:1



3. One possible floor plan could be as follows.
   (There are many more.)



Scale: 1" : 4'

Module 6, Lesson 2, Answer Key

Tolerance Levels

Object 1

DIMENSION	BASIC SIZE	TOLERANCE	UPPER LIMIT	LOWER LIMIT
A	90	5	90	85
B	32	0.2	32.1	31.9
C	25	3	28	25
D	60	0.04	60.02	59.98
E	6	0.04	6.02	5.98
F	33	8.2	33.1	24.9
G	8	0.2	8.1	7.9
H	28	0.6	28.3	27.7
I	20	0.8	20.4	19.6

Note: The basic size of F is found by taking A – B – C. The upper limit of F is found by taking the upper limit of A minus the lower limit of B minus the lower limit of C, while the lower limit of F is found by taking the lower limit of A minus the upper limit of B minus the upper limit of C.

The basic size of U is found by taking T – I. The upper limit of U is found by taking the upper limit of T minus the lower limit of I, while the lower limit of U is found by taking the lower limit of T minus the upper limit of I.

Tolerance = Upper limit – Lower limit.

Object 2

DIMENSION	BASIC SIZE	TOLERANCE	UPPER LIMIT	LOWER LIMIT
G	75	0.76	75	72.24
H	50	1	50.5	49.5
J	20	0.02	20	19.98
K	12.48	0.04	12.50	12.46
L	25	1.76	25.50	23.74

Note: The upper limit of L is found by taking the upper limit of G minus the lower limit of H, while the lower limit of L is found by taking the lower limit of G minus the upper limit of H.

Object 3

DIMENSION	BASIC SIZE	TOLERANCE	UPPER LIMIT	LOWER LIMIT
M	90	3	91.5	88.5
N	20	0.5	20.25	19.75
P	15	0.5	15.25	14.75
Q	55	1	55.50	54.50
R	70	0.5	70.25	69.75
S	10	0.02	10.02	10

Note: The upper limit of Q is found by taking the upper limit of R minus the lower limit of P, and the lower limit of Q is found by taking the lower limit of R minus the upper limit of P.

Module 6, Lesson 3, Answer Key

Effects of Tolerance on Area and Volume

OBJECT	MEASUREMENT	BASIC MEASUREMENT	MINIMUM LIMIT	MAXIMUM LIMIT	TOLERANCE
1	Area of one circle	28.26	28.07	28.45	0.38
1	Area of non-shaded	2263.48	2070.80	2293.36	222.56
1	Volume of object	6790.44	6191.69	6903.01	711.32
2	Area of total figure	1312.00	1284.81	1328.75	43.94
2	Volume of object	6560.00	6385.51	6643.75	258.24
3	Area of one circle	78.50	78.50	78.81	0.31
3	Area of non-shaded	1643.00	1590.26	1695.88	105.62
3	Volume of object	7064.90	6806.31	7326.20	519.89

See next page for detailed calculations. Use $\pi = 3.14$.

Calculated by:

BASIC MEASUREMENT	MINIMUM LIMIT	MAXIMUM LIMIT
OBJECT 1 $A = \pi r^2$ $A = \pi(3)^2$ A = 28.26 units^2	$A = \pi r^2$ $A = \pi(2.99)^2$ A = 28.07 units^2	$A = \pi r^2$ $A = \pi(3.01)^2$ A = 28.45 units^2
$A = A_{\text{Complete Figure}} - 2 \cdot A_{\text{One Circle}}$ $A = (A \cdot T - C \cdot I) - 2(28.26)$ $A = ((90)(28) - (25)(8)) - 2(28.26)$ A = 2263.48 sq. units	$A = A_{\text{Complete Figure}} - 2 \cdot A_{\text{One Circle}}$ $A = (A_{\text{Min}} \cdot T_{\text{Min}} - C_{\text{Max}} \cdot I_{\text{Max}}) - 2(28.45)$ $A = ((85)(27.7) - (28)(8.1)) - 2(28.45)$ A = 2070.8 sq. units	$A = A_{\text{Complete Figure}} - 2 \cdot A_{\text{One Circle}}$ $A = (A_{\text{Max}} \cdot T_{\text{Max}} - C_{\text{Min}} \cdot I_{\text{Min}}) - 2(28.07)$ $A = ((90)(28.3) - (25)(7.9)) - 2(28.07)$ A = 2293.36 sq. units
V = A(Thickness) V = (2263.48)(3) V = 6790.44 cu. units	V = A(Thickness) V = (2070.8)(2.99) V = 6191.69 cu. units	V = A(Thickness) V = (2293.36)(3.01) V = 6903.01 cu. units
OBJECT 2 $A = (H \cdot J + K \cdot L)$ A = (50)(20) + (12.48)(25) A = 1312.0 sq. units	$A = (H_{\text{Min}} \cdot J_{\text{Min}} + K_{\text{Min}} \cdot L_{\text{Min}})$ A = (49.5)(19.98) + (12.46)(23.74) A = 1284.81 sq. units	$A = (H_{\text{Max}} \cdot J_{\text{Max}} + K_{\text{Max}} \cdot L_{\text{Max}})$ A = (50.5)(20) + (12.50)(25.5) A = 1328.75 sq. units
V = A(Thickness) V = (1312.0)(5) V = 6560.0 cu. units	V = A(Thickness) V = (1284.81)(4.97) V = 6385.61 cu. units	V = A(Thickness) V = (1328.75)(5) V = 6643.75 cu. units
OBJECT 3 $A = \pi r^2$ $A = \pi(5)^2$ A = 78.5 sq. units	$A = \pi r^2$ $A = \pi(5)^2$ A = 78.5 sq. units	$A = \pi r^2$ $A = \pi(5.01)^2$ A = 78.81 sq. units
$A = A_{\text{Complete Figure}} - 2 \cdot A_{\text{One Circle}}$ $A = (M \cdot N) - 2(78.5)$ A = (90)(20) – 2(78.5) A = 1643.0 sq. units	$A = A_{\text{Complete Figure}} - 2 \cdot A_{\text{One Circle}}$ $A = (M_{\text{Min}} \cdot N_{\text{Min}}) - 2(78.81)$ A = (88.5)(2019.75) – 2(78.5) A = 1590.26 sq. units	$A = A_{\text{Complete Figure}} - 2 \cdot A_{\text{One Circle}}$ $A = (M_{\text{Max}} \cdot N_{\text{Max}}) - 2(78.5)$ A = (91.5)(20.25) – 2(78.5) A = 1695.88 sq. units
V = A(Thickness) V = (1643.0)(4.30) V = 7064.9 cu. units	V = A(Thickness) V = (1590.26)(4.28) V = 6806.31 cu. units	V = A(Thickness) V = (1695.88)(4.32) V = 7326.20 cu. units

Tolerance = Maximum limit – Minimum limit.

Module 6, Project, Answer Key

Precision Measurement

(1 mark)

1. Introductory statement (something like this)

   This is an investigation of the variation in diameter of electrical wire at different places along the wire.

(2 marks)

2. Chart of measured amounts.

   Results will depend on gauge of wire used.

Sample	Measured Amount in cm
1	
2	
3	
4	
5	

Note: Micrometer gives measurement to 0.01 mm which is the same as 0.001 cm.

(3 marks)

3. Spreadsheet to calculate volumes.

   Formula for volume is $V = \pi r^2 h$ where r is the radius of the wire and h is the length.

	A	B
1	Diameter in cm	Volume of one metre in cubic cm
2		= π * A1 * A1 x 100
3		= π * A2 * A2 x 100
4		= π * A3 * A3 x 100
5		= π * A4 * A4 x 100
6		= π * A5 * A5 x 100
7		

Formula for B1 = π * A1 * A1 x 100

Fill down for B2–B5

4. Tolerance for diameters

(1 mark) Tolerance = Largest Diameter – Smallest Diameter.

Tolerance for volumes = Largest Volume – Smallest Volume.

5. What does “gauge” of wire mean? *(3 marks)*

   The “gauge” of wire is a number that measures the thickness of the wire. The numbers go from 1 to 22.

Gauge 1	is thick wire used when there is a lot of current. Wire carrying electricity from the street to a house may be gauge 1.
Gauge 4	wire into house
Gauge 8–10	stoves and air conditioners
Gauge 14	lights
Gauge 18	telephone
Gauge 22	is the thinnest wire used for electrical purposes

6. Overall impression *(1 mark)*

GRADE 11 APPLIED MATHEMATICS

Module 7
Answer Key

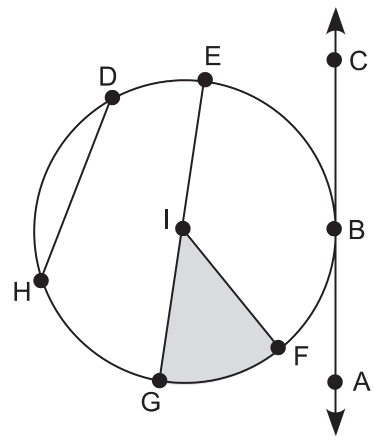
Module 7, Lesson 1, Answer Key

Geometric Definitions Regarding Circles

1. a) $\overline{EI}$, $\overline{IF}$, $\overline{IG}$ b) $\overline{EG}$

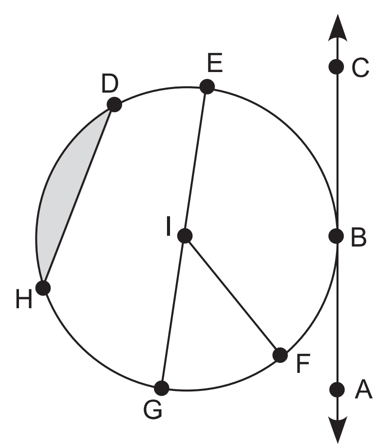
c)

A



d)

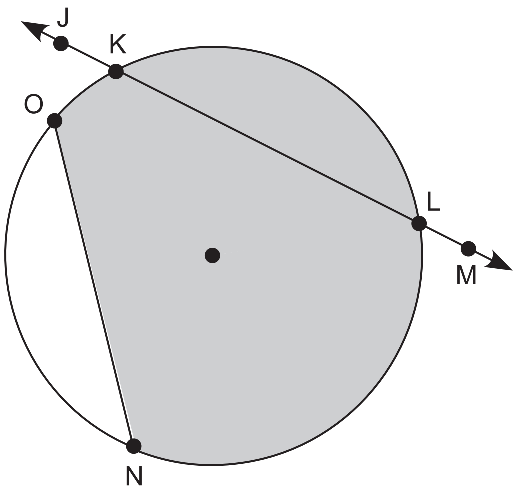
A



e) $\overset{\frown}{EDG}$, $\overset{\frown}{EBG}$

2. a)

B



b) Any two from
$\overset{\frown}{OK}$, $\overset{\frown}{KL}$, $\overset{\frown}{ON}$, $\overset{\frown}{LN}$, $\overset{\frown}{OL}$, $\overset{\frown}{KN}$

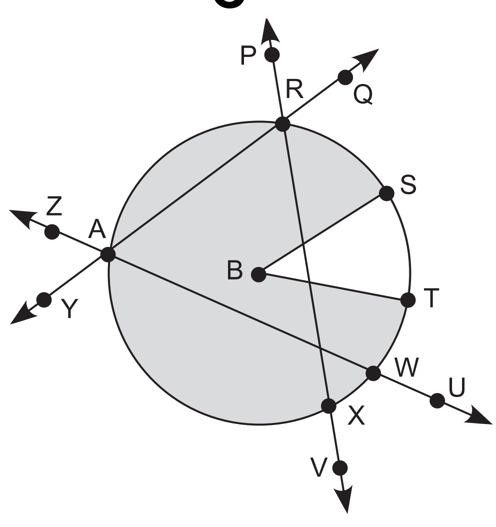
c) Any two from
$\overset{\frown}{ONL}$, $\overset{\frown}{KLN}$, $\overset{\frown}{KOL}$

3. a) $\angle SB'T$

b) $\angle RA'W$, $\angle A'RX$

c)

C



4. a) $\overline{RA'}$, $\overline{RX}$, $\overline{A'W}$, $\overline{KL}$, $\overline{ON}$, $\overline{DH}$, $\overline{EG}$

b) $\overleftrightarrow{JM}$, $\overleftrightarrow{QY}$, $\overleftrightarrow{ZU}$, $\overleftrightarrow{PV}$

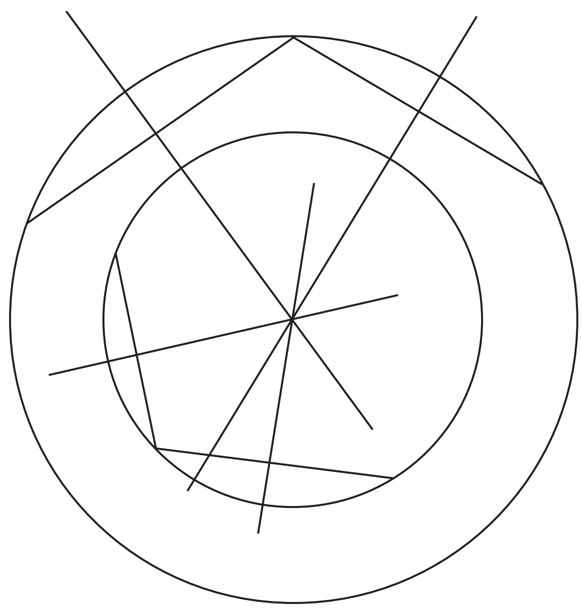
c) $\overleftrightarrow{AC}$

Module 7, Lesson 2, Answer Key

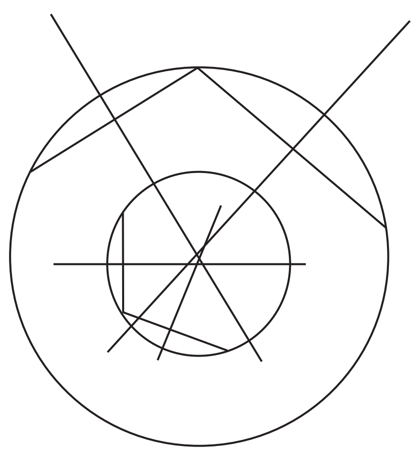
Relationships Between the Parts of a Circle and Other Geometric Shapes

1. Draw two non-parallel chords in the circle, bisect each one, and the two bisectors of the chords will meet at the centre of the circle.

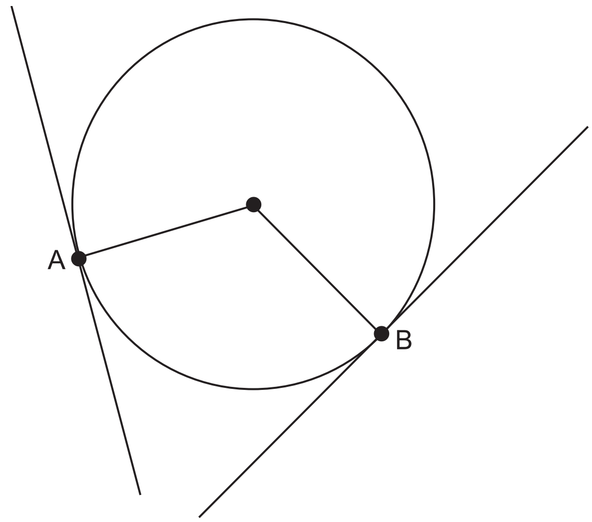
2. a) Yes, the circles are concentric because their centres are the same point.



   b) No, they are not concentric, because their centres are not the same point.

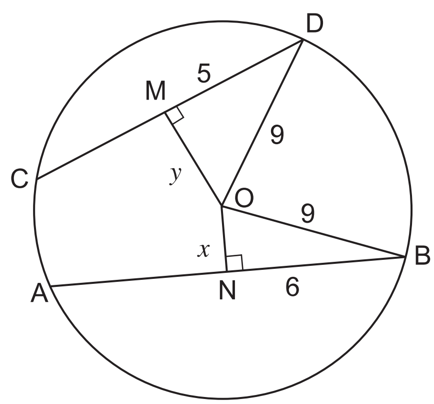


3. Draw radii at the two points A and B. Using your protractor draw lines at right angles to these radii.



4. $\overline{AB}$, because it is longer.

5.



M is the midpoint of $\overline{CD}$
$\angle DMO = 90°$

N is the midpoint of $\overline{AB}$
$\angle BNO = 90°$

Using the Pythagorean theorem,

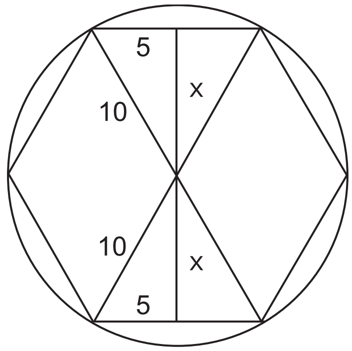
$$c^2 = a^2 + b^2$$
$$9^2 = 5^2 + y^2$$
$$81 = 25 + y^2$$
$$y^2 = 56$$
$$y = 7.48$$

$$c^2 = a^2 + b^2$$
$$9^2 = 6^2 + x^2$$
$$81 = 36 + x^2$$
$$x^2 = 45$$
$$x = 6.71$$

Distance from centre to $\overline{CD} \approx 7.48$ cm

Distance from centre to $\overline{AB} \approx 6.71$ cm

6. Length of each side = 10 cm

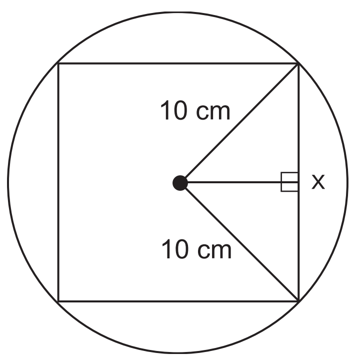


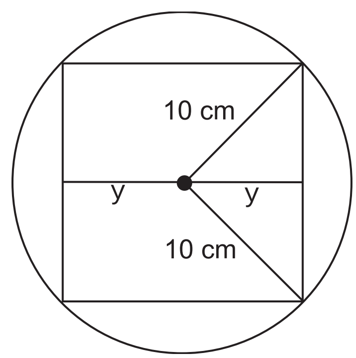
Let x cm be perpendicular distance from the centre to one side.

$$c^2 = a^2 + b^2$$
$$10^2 = 5^2 + x^2$$
$$100 = 25 + x^2$$
$$75 = x^2$$
$$x \approx 8.66$$

Distance from one side to the opposite side = $2x$ cm $\approx 2(8.66)$ cm ≈ 17.32 cm.

7.





Let length of side of square be x cm

$$c^2 = a^2 + b^2$$
$$x^2 = 10^2 + 10^2$$
$$x^2 = 100 + 100$$
$$x^2 = 200$$
$$x = 14.14$$

Length of side of square ≈ 14.14 cm

Let distance from centre to side of square be y cm

Sides of square are all the same length. Distance between opposite sides $= 2y$ cm = side of square

$$2y = 14.14$$
$$y = 7.07$$

Distance ≈ 7.07 cm

Notes

Module 7, Lesson 3, Answer Key

Properties of Angles with Respect to Circular Systems

1.

$\angle ABC = 40°$	Δ CAB is isosceles
$\angle BDC = 27°$	Δ CBD is isosceles
$\angle BCD = 126°$	Angles of a triangle add to 180°
$\angle BCA = 100°$	Angles of a triangle add to 180°
$\angle ACD = 134°$	360°– Sum of $\angle BCA$ and $\angle BCD$
$\angle ABD = 67°$	Sum of $\angle ABC$ and $\angle CBD$

2. a) Because they intercept the same arcs:

$\angle LPU$ and $\angle LMU$
$\angle MPU$ and $\angle MLU$
$\angle PUM$ and $\angle PLM$
$\angle LUP$ and $\angle LMP$

b)

$\angle PUM = 32°$	
$\angle LPU = 47°$	
$\angle LSP = 101°$	Angles of a triangle add to 180°
$\angle MSU = 101°$	Angles of a triangle add to 180°
$\angle LSU = 79°$	Supplement of $\angle MSU$
$\angle PSM = 79°$	Supplement of $\angle MSU$

You would need one more angle to find the rest of the angles in the diagram ($\angle PML$, $\angle LUP$, $\angle MPU$, and $\angle MLU$ are missing).

c)

$\angle LUP = 53°$	Intercepts the same arc as $\angle PML$
$\angle MPU = 45°$	Intercepts the same arc as $\angle MLU$

These are not possible angles as given because the angles in Δ PSM do not add to 180°.

d)

$\angle PUM = 32°$	
$\angle LPU = 47°$	
$\angle LSP = 101°$	Angles of a triangle add to 180°
$\angle MSU = 101°$	Angles of a triangle add to 180°
$\angle LSU = 79°$	Supplement of $\angle MSU$
$\angle PSM = 79°$	Supplement of $\angle MSU$
$\angle MPU = 45°$	Intercepts the same arc as $\angle MLU$
$\angle PML = 56°$	Angles of a triangle add to 180°
$\angle LUP = 56°$	Angles of a triangle add to 180°

No. $\overparen{MP}$ and $\overparen{PL}$ add to only 176° (using inscribed angles ∠PML and ∠PLM), while $\overparen{LU}$ and $\overparen{UM}$ add to 184° (using inscribed angles ∠LPU and ∠MLU. For $\overline{ML}$ to be a diameter, the two sums need to be 180°.

3. a) $\overparen{GH} = \overparen{AD} = 84^\circ$

   b) $\overparen{HI} = \overparen{FGI} - \overparen{FG} - \overparen{GH}$
   $= 112^\circ - 20^\circ - 84^\circ$
   $= 8^\circ$

   c) $\overparen{FGI} = 2(\angle FEI) = 112^\circ$

   d) $\overparen{CK} = 52^\circ = 2(\angle 6)$

   e) $\overparen{DK} = 180^\circ - 84^\circ = 96^\circ$

   f) $\overparen{ABA} = 360^\circ$ (complete circle)

   g) $\angle 1 = \angle 5 = 14^\circ$

   h) $\angle 2 = \frac{1}{2}\angle 4 = 42^\circ$

   i) $\angle 3 = \frac{1}{2}\angle 4 = 42^\circ$

   j) $\angle 4 = \overparen{AD} = 84^\circ$

   k) $\angle 5 = 180^\circ - (42^\circ + 124^\circ)$
   $= 14^\circ$ (from ΔDEC)

   l) $\angle 6 = \angle BAK - \angle 1$
   $= 40^\circ - 14^\circ = 26^\circ$

4. Let distance to $\overline{AB} = x$
   $c^2 = a^2 + b^2$
   $6^2 = x^2 + 4^2$
   $x^2 = 36 - 16$
   $x^2 = 20$
   $x \approx 4.47$
   Distance to $\overline{AB} \approx 4.47$ cm

   Let distance to $\overline{DC} = y$
   $c^2 = a^2 + b^2$
   $6^2 = y^2 + 5^2$
   $y^2 = 36 - 25$
   $y^2 = 11$
   $y \approx 3.32$
   Distance to $\overline{DC} \approx 3.32$ cm

5. a) $\angle AOP = 70^\circ$
   b) $\angle OAC = 90^\circ$
   c) $\angle AEO = 20^\circ$
   d) $\angle ACP = 90^\circ$
   e) $\angle PCE = 90^\circ$
   f) $\angle PDB = 90^\circ$
   g) $\overline{BF} = 70^\circ$
   h) $\overparen{DGC} = 220^\circ$

Module 7, Lesson 4, Answer Key

Applications of the Properties of Circles and Polygons

1. a) Polygon with 14 sides splits into 12 triangles.
   Sum of angles of 12 triangles = 12 x 180° = 2160°.

   b) Polygon with 16 sides splits into 14 triangles.
   Sum of angles of 14 triangles = 14 x 180º = 2520°.

2. a) 102 sides, therefore, 100 triangles.

   b) Multiply number of triangles by 180° to give the total number of degrees.

   c) Sum of angles = 100 x 180° = 18 000°.

3. If there are n sides, the sum of the angles = $(n - 2) \times 180°$.

4. $(n - 2)\ 180° = 4500°$

$$n - 2 = \frac{4500}{180}$$

$$n - 2 = 25$$

$$n = 27$$

   Polygon has 27 sides.

5. The sum of the angles of any polygon is divisible by 180.
   4000 is not divisible by 180.

   Therefore, 4000° is not the sum of the angles of any polygon,

   OR

   using the algebraic method as in the previous question, the equation to be solved is

$$(n - 2)\ 180° = 4000°$$

$$n - 2 = \frac{4000}{180}$$

   But this does not give a whole number for the value of n and so 4000° is not the sum of the angles of any polygon.

6. a) Sum of the interior angles of a pentagon is 540°.
Sum of the interior angle and the exterior angle at any vertex is 180°.

Therefore, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + 540°$
$= 5(180°) = 900°$.

Therefore, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$
$= 900° - 540° = 360°$.

b) It does not matter if the pentagon is not regular because the total of the exterior angles is still 360°. For a hexagon, the sum of the exterior angles + the sum of the interior angles = 6(180°) = 1080°.

Therefore, the sum of the exterior angles + 720° = 1080°.
The sum of the exterior angles = 1080° – 720° = 360°.
For a decagon, the sum of the exterior angles is 1800° – 1440° = 360°.
For every polygon, the sum of the exterior angles is 360°.

7. a) $\angle BCD = \frac{1}{2}\overset{\frown}{DAB}$ Inscribed angle $= \frac{1}{2}$ intercepted arc.

$$= \frac{1}{2}\left(\overset{\frown}{DA} + \overset{\frown}{AB}\right)$$

$$= \frac{1}{2}\left(44^\circ + 104^\circ\right) = \frac{1}{2}\left(148^\circ\right) = 74^\circ$$

$\angle BAD = 180° - 74° = 106°$ because $\angle BCD$ and $\angle BAD$ are opposite angles of a cyclic quadrilateral.

b) $$\overset{\frown}{BC} = \frac{1}{2}\left(360^\circ - 148^\circ\right) = \frac{1}{2}\left(212^\circ\right) = 106^\circ$$

$$\angle ABC = \frac{1}{2}\left(\overset{\frown}{AD} + \overset{\frown}{DC}\right) = \frac{1}{2}\left(44^\circ + 106^\circ\right) = \frac{1}{2}\left(150^\circ\right) = 75^\circ$$

8. AC is a diameter. $\overset{\frown}{ABC} = \overset{\frown}{ADC} = 180^{o}$ Semicircles

Therefore, $\angle ABC = \frac{1}{2}\left(180^{o}\right) = 90^{o}$.

$\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{ABC} = 180^{o}$

$\overset{\frown}{AB} = 2\ \overset{\frown}{BC}$ Given in problem

Therefore, $2\ \overset{\frown}{BC} + \overset{\frown}{BC} = 180^{o}$.

Therefore, $3\ \overset{\frown}{BC} = 180^{o}$.

Therefore, $\overset{\frown}{BC} = 60^{o}$.

$\angle BAC = \frac{1}{2}\overset{\frown}{BC} = \frac{1}{2}\left(60^{o}\right) = 30^{o}$ Inscribed angle

Similarly for $\angle CAD$

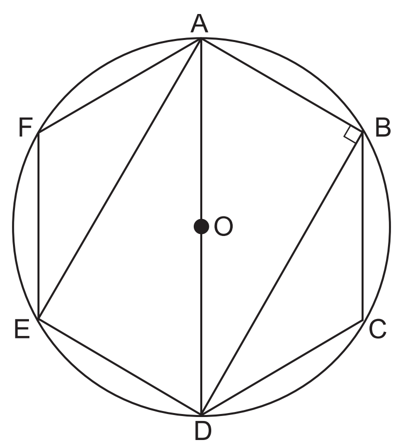
$\therefore \angle BAD = 60^{o}$

$\overset{\frown}{AB} = 2\overset{\frown}{BC} = 2\left(60^{o}\right) = 120^{o}$ Given in problem

$\angle ACD = \frac{1}{2}\overset{\frown}{AD} = \frac{1}{2}\left(120^{o}\right) = 60^{o}$

9. a) Diameter of circle = AD = 20 cm, therefore, radius = 10 cm.

Length of side of Hexagon:
The hexagon can be split into six triangles (ΔODC, ΔOBC etc.). The six angles at point O have a sum of 360°, therefore, each angle = 60° (e.g., $\angle$DOC = 60°).
Since OD = OC = 10 cm, $\angle$ODC = $\angle$OCD = 60°, and ΔODC is equilateral, therefore, CD = 10cm.
AD is a diameter. Therefore, $\angle$ABD = 90°.
Distance between opposite sides AB and ED is BD.

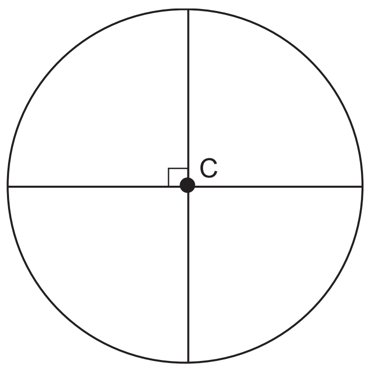


Let BD = x cm.
AD = 20 cm.
AB = 10 cm. Using Pythagorean Theorem in ΔABD,
$x^2 + 10^2 = 20^2$
$x^2 + 100 = 400$
$x^2 = 400 - 100 = 300$
$x = 17.32$

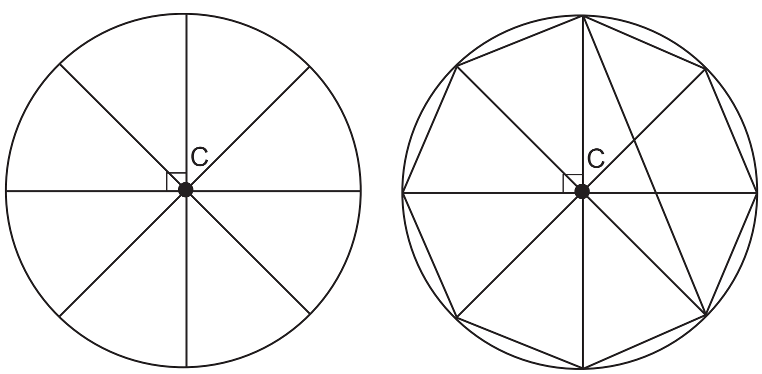
Distance between opposite sides AB and ED = 17.32 cm.

b) This problem may be done using trigonometry, but the solution is more complicated than for the hexagon. Here is a construction method using ruler, compass, and protractor.

Step 1. Draw a circle with radius 10 cm. Call the centre C. Draw a diameter through C. Use a ruler and a protractor to draw a second diameter through C, perpendicular to the first one. Your diagram should look something like this one. Note that this one is not to scale.



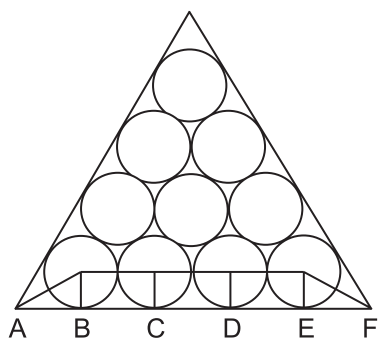
Step 2. Bisect the angles between where the diameters cross and mark the points where they meet the circle. Join the eight points to form a regular octagon.



Step 3. Measure the length of a side.
Note that the diagrams here are not the correct size. The side should be about 7.65 cm long.
Measure the distance between two opposite sides.
This should be about 18.47 cm.

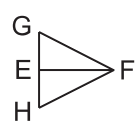
10. To find the length of the side, consider AF.

AF = AB + BC + CD + DE + EF
BC = CD = DE = 2 in. BA = EF. These are calculated by applying Pythagoras' Theorem to ΔGEF. ΔGEF is half an equilateral triangle with GH = 2 in. Therefore, GF = 2 in.



Let EF = x in.
Then, $x^2 + 1^2 = 2^2$.
$x^2 = 4 - 1 = 3$
$x = 1.73$
AF = AB + BC + CD + DE + EF
= 1.73 + 2 + 2 + 2 + 1.73 in.
= 9.46 in.

The length of the side of the equilateral triangle is 9.46 in.



Notes

Self-Test Answer Key

Module 7 – Geometry

$\overline{}$ 68

(6 marks)

1. a) *Circle:* The set of all points equidistant from a given point.

   b) *Sector:* A region of a circle bounded by a central angle and the arc it cuts off.

   c) *Minor Arc:* Any part of a circle that is less than half.

   d) *Tangent:* A line that intersects with a circle at only one point

   e) *Central Angle:* An angle formed between two radii of a circle having its vertex at the centre.

   f) *Radius:* A line segment joining the centre of a circle to any point on the circle.

(7 marks)

2. a) congruent or the same length
   b) area
   c) radius
   d) segment
   e) inscribed angle
   f) closer to the centre
   g) minor sector

(8 marks)

3. a) Any two of: $\overline{GC}$, $\overline{EB}$, $\overline{GJ}$, $\overline{OP}$

   b) $\overleftrightarrow{OP}$

   c) Any two of: $\overline{GC}$, $\overline{EB}$, $\overline{GJ}$, $\overline{OP}$, $\overline{DB}$, $\overline{AF}$, $\overline{KJ}$, $\overline{HI}$

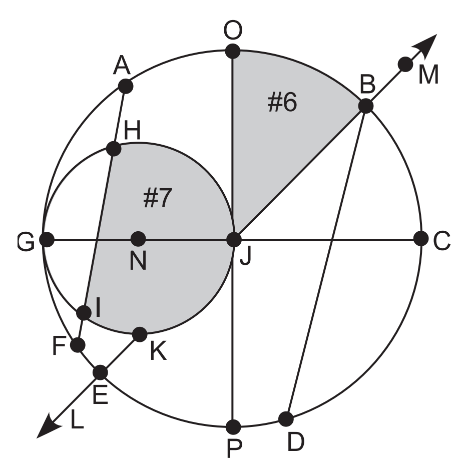
   d) Any two of: ∠OJC, ∠CJP, ∠OJB, ∠BJC, ∠EJP, ∠EJG, ∠OJG, ∠BJG, ∠CJE, ∠BJP

   e) Any two of: $\overline{NG}$, $\overline{NJ}$, $\overline{JG}$, $\overline{JE}$, $\overline{JP}$, $\overline{JC}$, $\overline{JB}$, $\overline{OJ}$

   f) See diagram. Others: Sector BJC, Sector EJP, etc.

   g) See diagram.

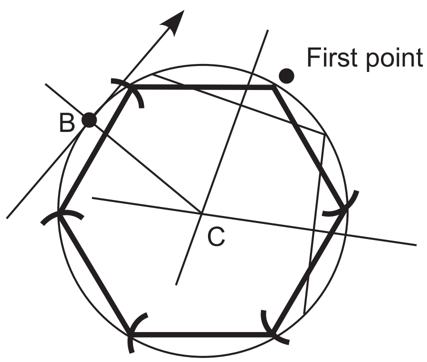
   h) ∠EBD, ∠GJK



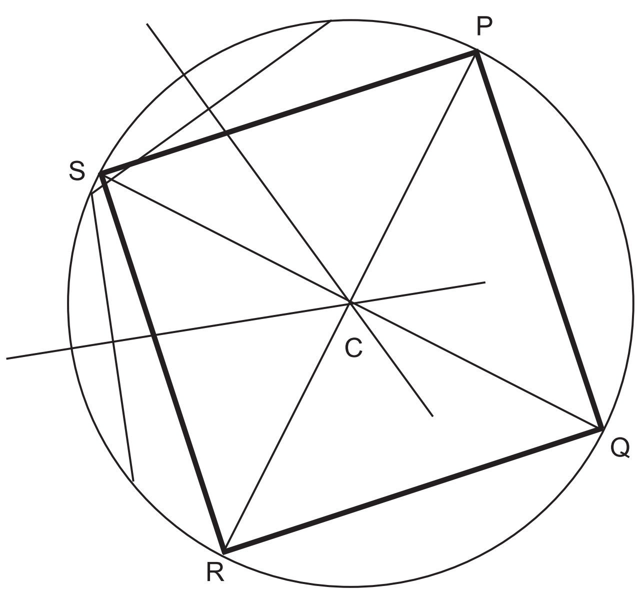
4. a) i) Find the centre, C, by drawing any two chords that are not parallel and their right bisectors. They cross at the centre C. *(1 mark)*

Draw $\overleftrightarrow{CB}$ and construct a line through B, perpendicular to $\overleftrightarrow{CB}$. This line is the required tangent.

ii) Use the compass with radius the same as in the given circle, BC. Use any point of the circle as the first point and step off five more points on the circle. Join to form hexagon. *(2 marks)*

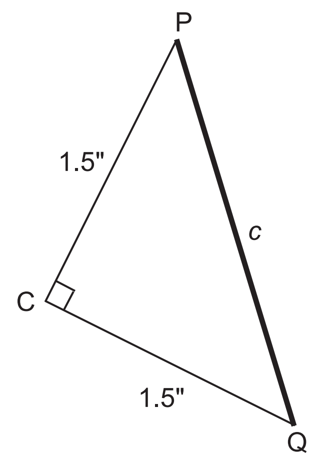


b) i) Find the centre of the circle, draw a diameter for the circle, draw another diameter at right angles to the first, join the four resulting points to create the square, PQRS. *(2 marks)*



(2 marks)

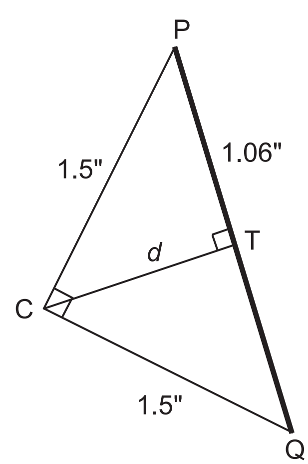
ii) Using one of the triangles formed and the Theorem of Pythagoras, find 'c'.



$c^2 = 1.5^2 + 1.5^2$
$c^2 = 2.25 + 2.25$
$c^2 = 4.5$
$c = 2.12$
Length of side of square is 2.12 in.

(2 marks)

iii) Using one of the triangles as follows and the Theorem of Pythagoras, find 'd'.

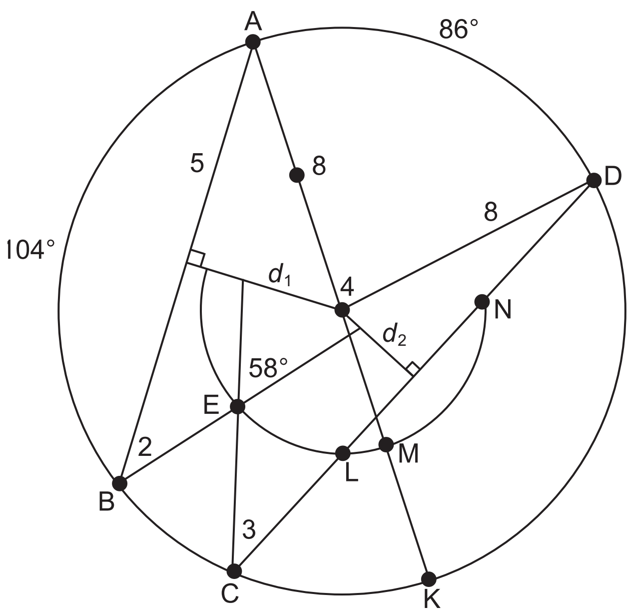


$1.5^2 = 1.06^2 + d^2$
$d^2 = 1.1264$
$d = 1.06$
Distance from centre to a side of the square is 1.06 in.

5. a) *(12 marks)*

i)	$m\overset{\frown}{GH} = 86^\circ$	vii)	$m\angle 1 = 15^\circ$
ii)	$m\overset{\frown}{HI} = 10^\circ$	viii)	$m\angle 2 = 43^\circ$
iii)	$m\overset{\frown}{FGHI} = 116^\circ$	ix)	$m\angle 3 = 43^\circ$
iv)	$m\overset{\frown}{CK} = 46^\circ$	x)	$m\angle 4 = 86^\circ$
v)	$m\overset{\frown}{DK} = 94^\circ$	xi)	$m\angle 5 = 15^\circ$
vi)	$m\overset{\frown}{ABCKDA} = 360^\circ$	xii)	$m\angle 6 = 23^\circ$

b) *(4 marks)*



$8^2 = 5^2 + d_1^{\,2}$
$d_1^{\,2} = 64 - 25$
$d_1 = 6.24$
Distance from centre to $\overline{AB}$ is 6.24 cm.

$8^2 = 6^2 + d_2^{\,2}$
$d_2^{\,2} = 64 - 36$
$d_2 = 5.29$
Distance from centre to $\overline{DC}$ is 5.29 cm.

(10 marks)

c)

DIMENSION	BASIC	MAXIMUM	MINIMUM	TOLERANCE
Radius of inside of washer	4	4.02	3.98	0.04
Radius of outside of washer	8	8.03	7.97	0.06
Perimeter of outside of washer	50.27	50.454	50.077	0.377
Area of washer	150.80	152.809	148.787	4.022
Volume of washer	226.19	232.269	218.717	13.552

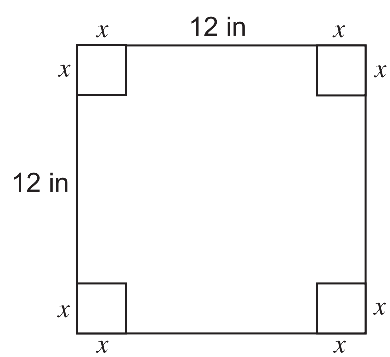
If you use 3.14 for π the values for perimeter, area, and volume will be slightly smaller.

(2 marks)

6. 800 Francs
= \$800 x 0.2744 Canadian
= \$219.52 Canadian

(5 marks)

7. a) Let side length of square to be cut out be x inches.
Let volume of box be V in^3.



$V = l \times w \times d$

$V = (12 - 2x)(12 - 2x)\,x$ (see diagram)

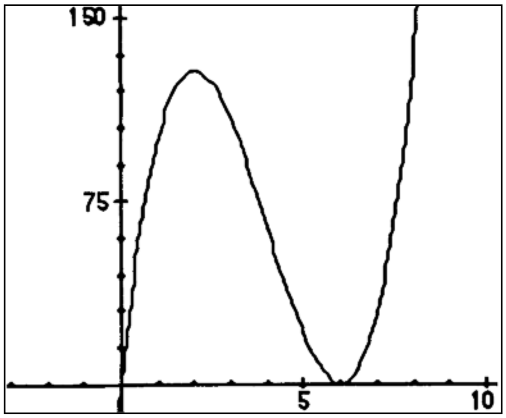
$V = (144 - 24x - 24x + 4x^2)(x)$

$V = 144x - 48x^2 + 4x^3$

$V = 4x^3 - 48x^2 + 144x$

Keying sequence:
[Y=] [4] [X,T,θ,n] [^] [3] [–] [48] [X,T,θ,n] [x^2] [+] [144] [X,T,θ,n]

Graph:



Reset window

Find the maximum value using the keying sequence:

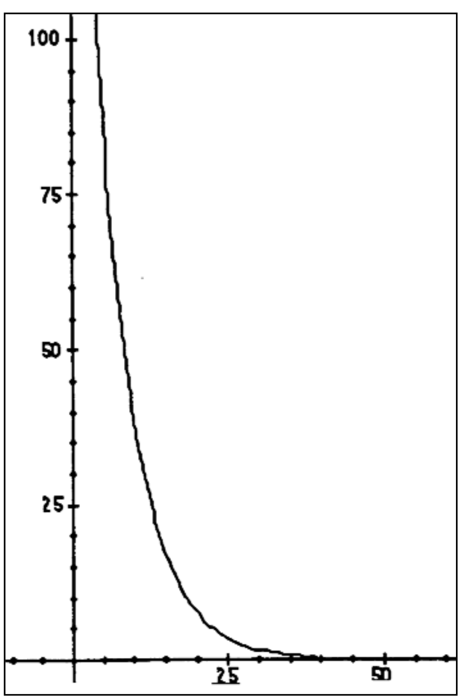
[2nd] [CALC] [4], setting appropriate Left and Right bounds, and then pressing [ENTER] to find the point (2, 128).

This indicates that the maximum volume will be 128 in^3 brought about by a 2″ x 2″ square being cut out of each of the four corners.

b) Rewrite the function as $V = 195\ (0.85)^n$ *(5 marks)*
V will be given in thousands of dollars.

Graphing the function with:

Keying sequence:
[Y=] [195] [x] [0.85] [^] [X,T,θ,n]



Reset window

Use the [TRACE] feature to trace the function until you get close to $y = 78$ ($x = 5.638$ gives $y = 77.996$). At this point you can enter other x-values to get a closer approximation. The machinery is worth $78 000.00 after approximately 5.6 years.

Notes

GRADE 11
APPLIED MATHEMATICS

Module 8
Answer Key

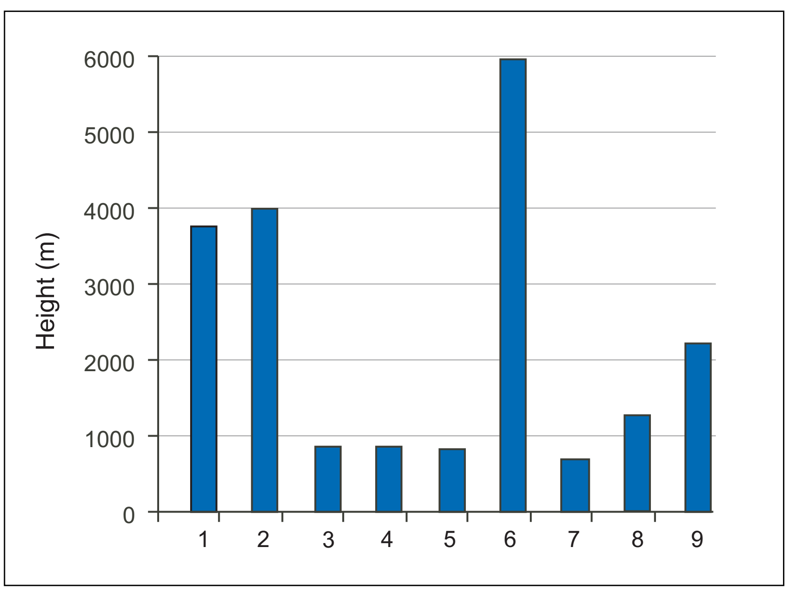
Module 8, Lesson 1, Answer Key

Displaying and Interpreting Discrete and Continuous Data Using Spreadsheet Graphs

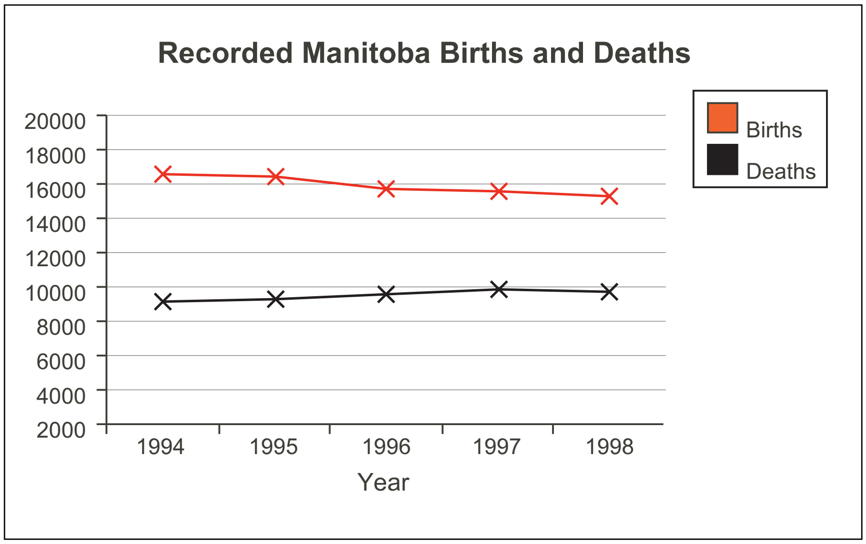
1. a) Quantitative – Discrete
   b) Qualitative
   c) Quantitative – Discrete
   d) Quantitative – Continuous
   e) Qualitative
   f) Quantitative – Discrete
   g) Quantitative – Continuous
   h) Qualitative

2. a)

	A	B
1	Name	Height (m)
2	1. Columbia, Alberta	3747
3	2. Mt. Waddington, B.C.	4016
4	3. Mt. Baldy, Manitoba	832
5	4. Carleton, N.B.	820
6	5. Lewis Hills, Nfld.	814
7	6. Mt. Logan, Yukon	5951
8	7. Ogidaki, Ontario	665
9	8. Jacques Cartier, Quebec	1268
10	9. Golden Hinde, B.C.	2200

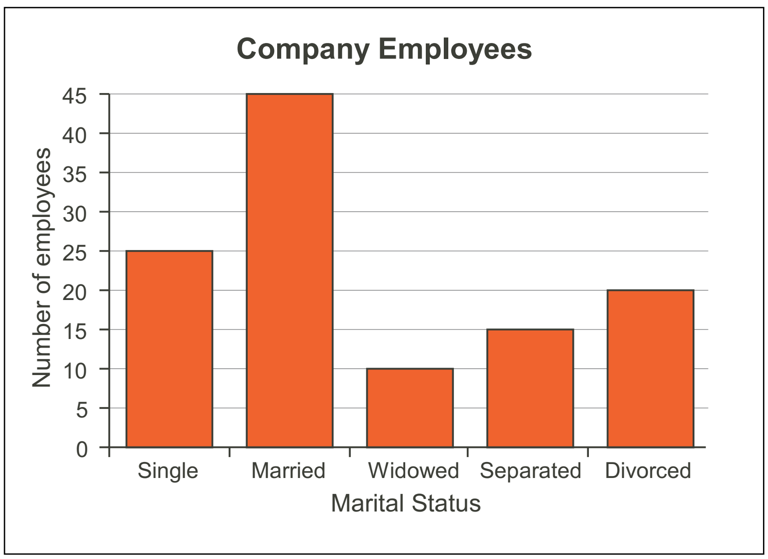


b)



Recorded Manitoba Births and Deaths
Births
Deaths
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
1994
1995
1996
1997
1998
Year

c)



Company Employees
Number of employees
45
40
35
30
25
20
15
10
5
0
Single
Married
Widowed
Separated
Divorced
Marital Status

3. a) 75 mm

   b) July, June, and September in that order

   c) Winter – November, 75 mm; December, 77 mm; January, 68 mm; February, 56 mm; March, 71 mm; April, 67 mm.
   Total Snow = 414 mm

   Summer – May, 72 mm; June, 87 mm; July, 90 mm; August, 75 mm; September, 79 mm; October, 68 mm.
   Total Rain = 471 mm

   This community gets more rain.

   Note: All of these figures are approximate.

4. a) $3900.00

   b) $6000.00

   c) March

   d) December – Possibly due to the Christmas sales

   e) Total sales = $11 800.00 ($6000 + $5800)

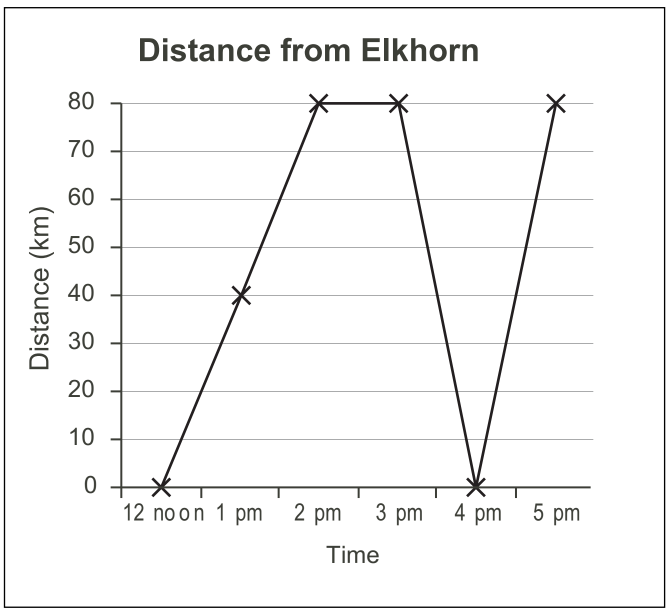
   Total commission $(11 800)(0.10) = $1180

   Each person receives $\dfrac{\$1180}{2} = \$590.$

5. a) The data are measured data which theoretically can take any real numbers as values and are measured with precision determined by the measuring intrument. Points between the actual data points may be used to estimate distances from Elkhorn at intermediate times.

b)

	A	B
1	Time	Distance from Elkhorn
2	12 noon	0
3	1 pm	40
4	2 pm	80
5	3 pm	80
6	4 pm	0
7	5 pm	80



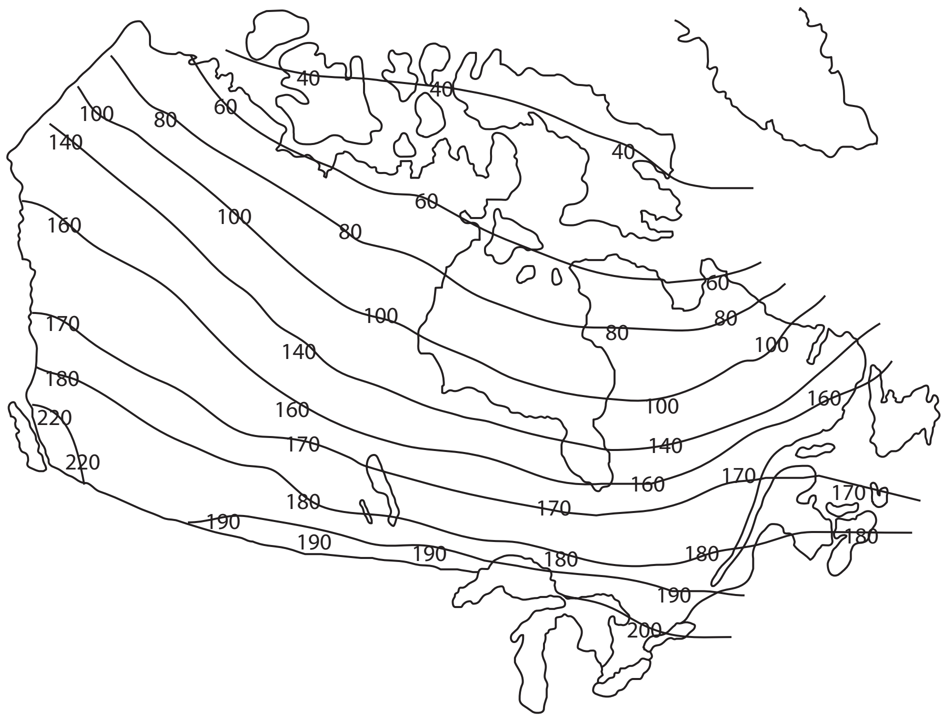
c) Distance at 1:30 p.m. is approximately 60 km, and at 4:45 p.m. is approximately 60 km.

d) The vehicle is 30 km from Elkhorn at about 12:45 p.m., at 3:30 p.m., and at 4:30 p.m. Assume the vehicle travels at a uniform speed while moving.

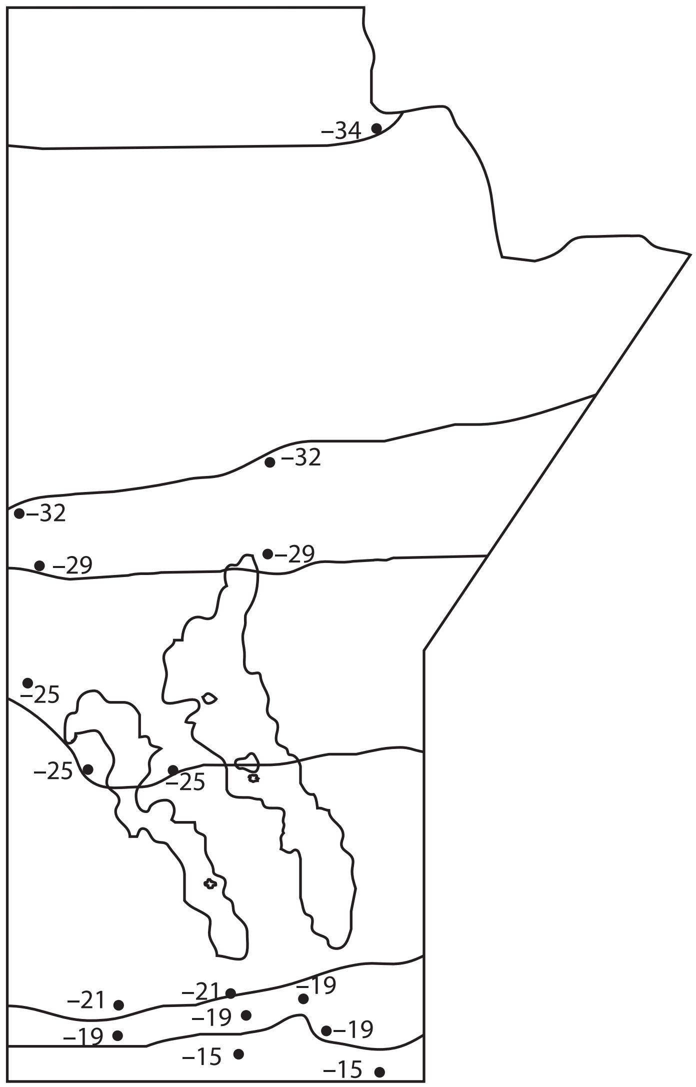
Module 8, Lesson 2, Answer Key

Displaying and Interpreting Data Using Contour Lines

1.



2.



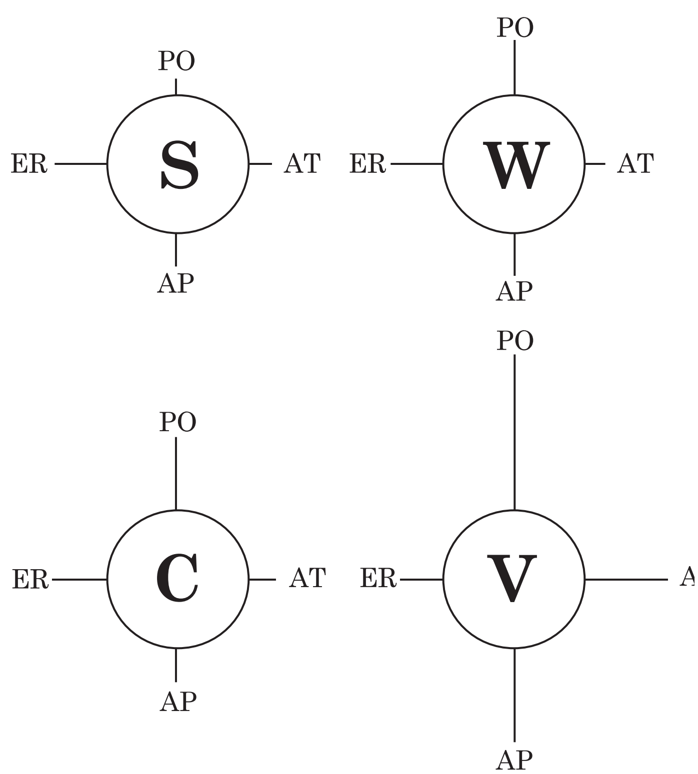
Module 8, Lesson 3, Answer Key

Interpreting and Drawing Glyphs

1. a) Do not pass
   b) No smoking
   c) Ticket sales for the 1998 – 1999 World Junior Hockey Championships on Tuesday, December 22 was 140 174. This is approaching the world record ticket sales of 146 852.

2. a) Comfort – Brand C
   Horsepower – Brand B
   Gas Mileage – Brand B
   Affordability – Brand A
   b) Answers will vary but could be Brand B because it has the most power with the best gas mileage.

3. PO – population
   AT – average temperature
   AP – average precipitation
   ER – employment rate

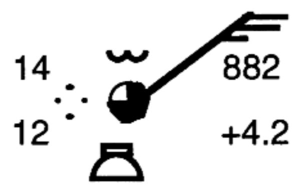


Scale:

Population:	1 cm = 1 million
Average temperature:	1 cm = 10°C
Average precipitation:	1 cm = 1000 mm
Employment rate:	1 cm = 100%

4. a) Cirro-cumulus
   b) Swelling cumulus
   c) 1005.6 mb
   d) Up 3 mb in the past three hours
   e) From the northeast, between 32 and 41 km per hour
   f) Overcast
   g) –4°C
   h) –6°C
   i) Steady light snow

5.

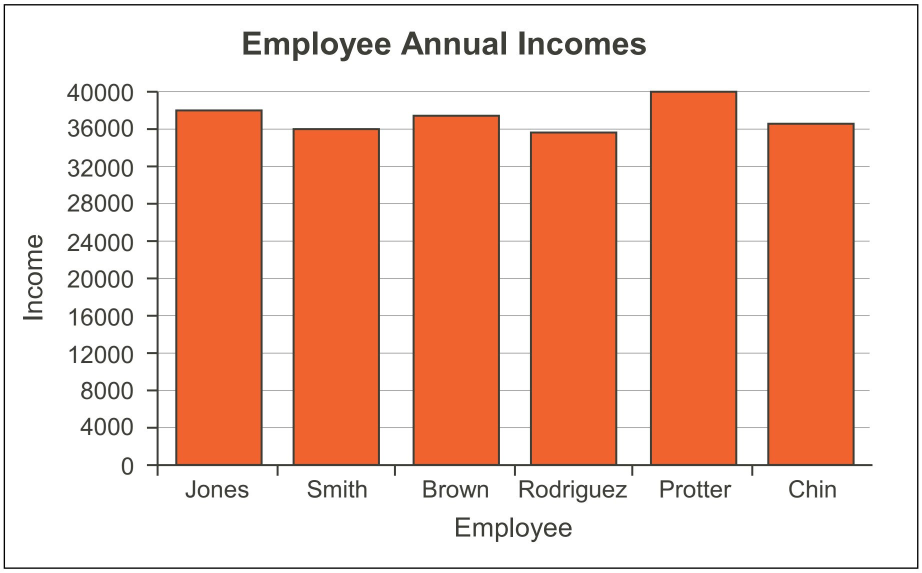


Module 8, Lesson 4, Answer Key

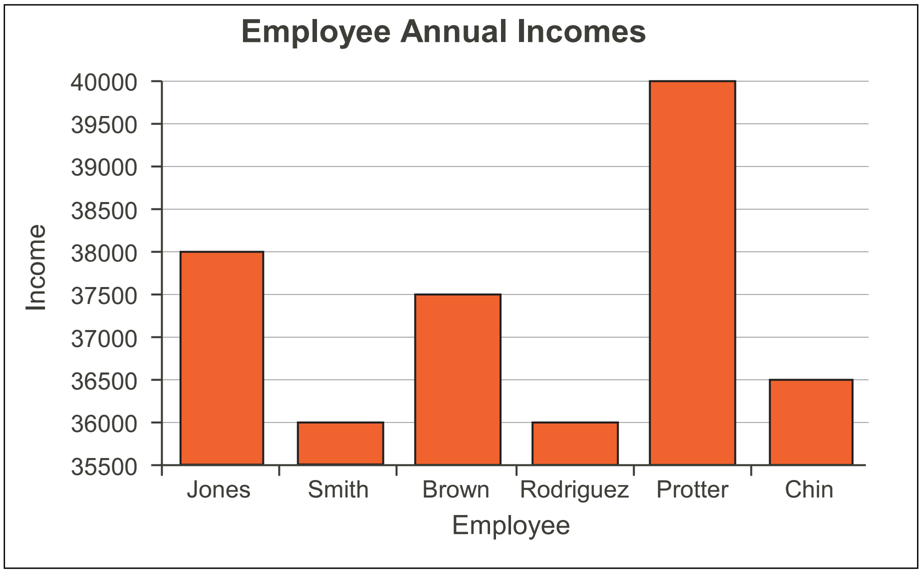
Altering the Display of Data for Special Effects

1. The survey was conducted at a location where baseball is popular. Therefore, it seems reasonable that the people there would watch a baseball-related event. The survey should be a more random sample of Manitobans or Winnipegers in order to get more accurate information.

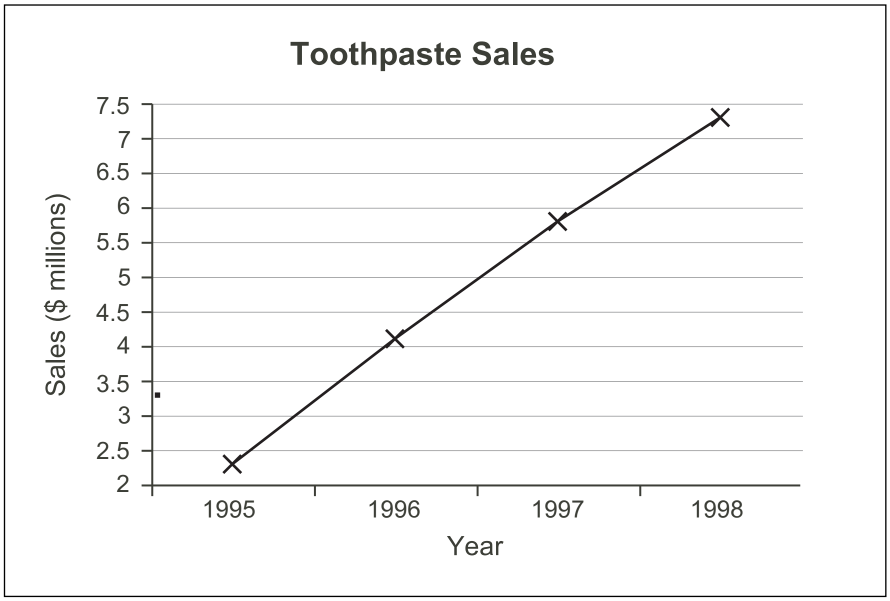
2. a)



b)



3. a)


Toothpaste Sales
Sales ($ millions)
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1995
1996
1997
1998
Year

b)


Toothpaste Sales
Sales ($ millions)
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1998
1997
1996
1995
Year

Self-Test Answer Key

Module 8 – Data Management and Analysis

/ 47

(10 marks)

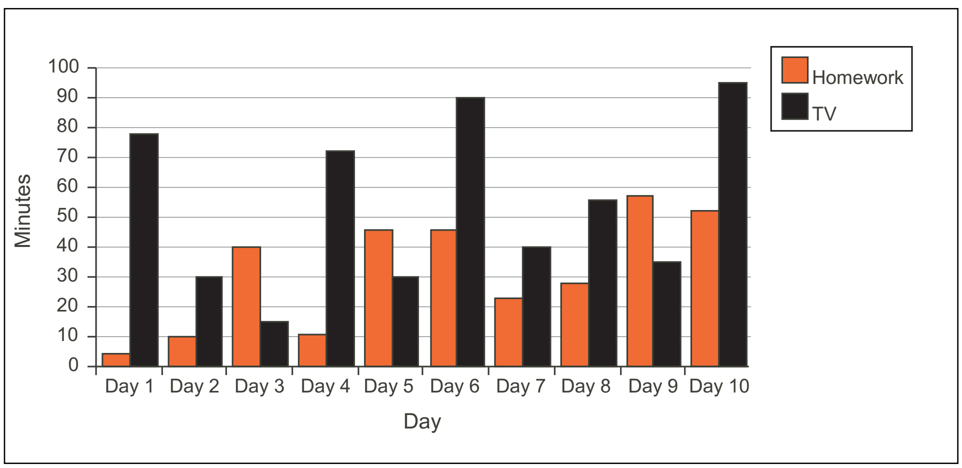
1. a) ii.
   b) iii.
   c) ii.
   d) iii.
   e) i.
   f) ii.
   g) i.
   h) ii.
   i) i.
   j) iii.

(4 marks)

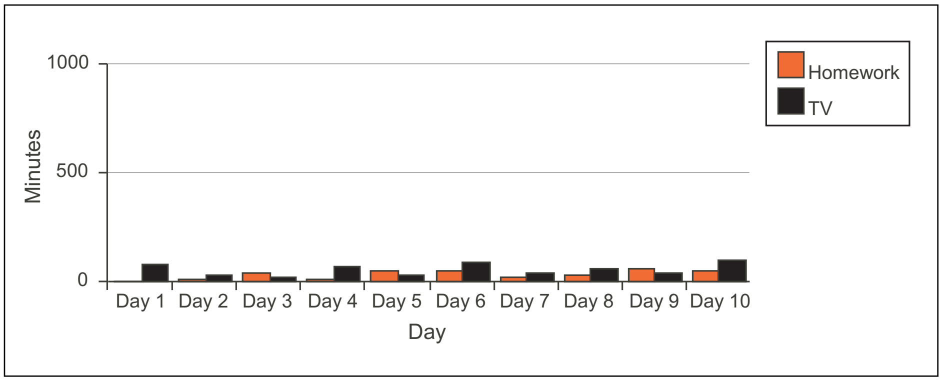
2. a) i) Africa
   ii) Antarctica
   iii) Northern Canada or Alaska
   iv) Australia – Approximately 75°
   (54° – (–21°) = 54° + 21° = 75°)

(2 marks)

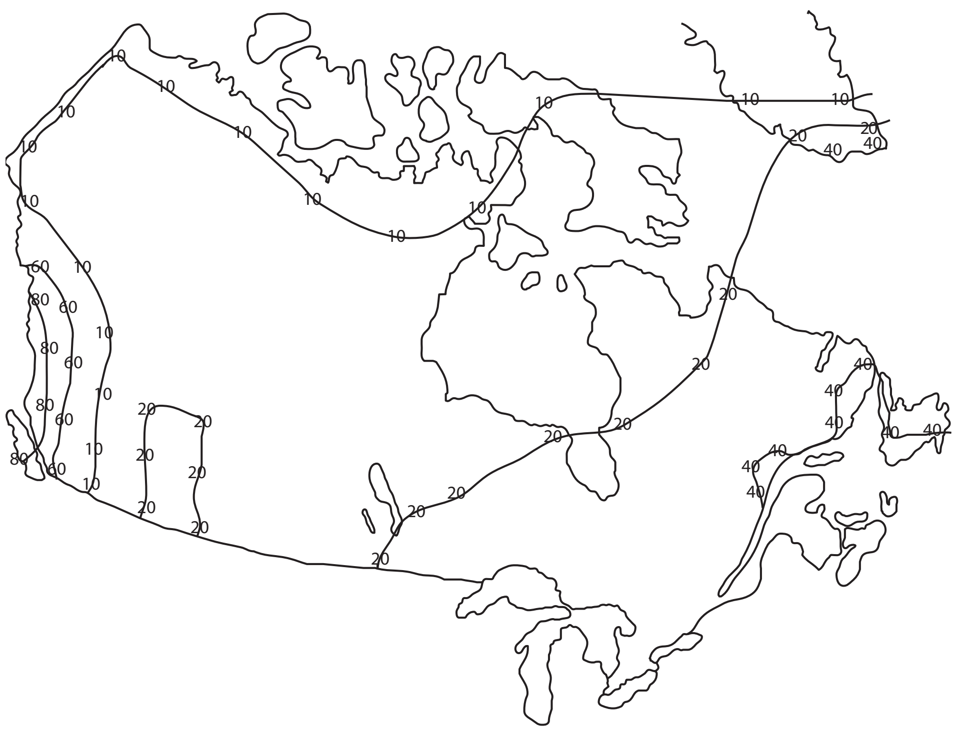
b) i)



ii) By changing the scale to a much larger scale, the following graph would indicate a smaller difference in time studying as opposed to watching TV. *(1 mark)*



c) i) *(3 marks)*



ii) Average annual precipitation for most of the prairie provinces would be between 10" and 20".

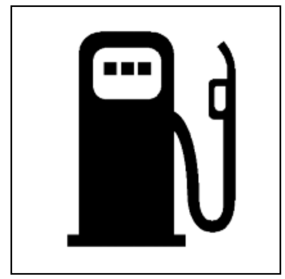
iii) British Columbia

(2 marks)

d) What would the following glyphs indicate?

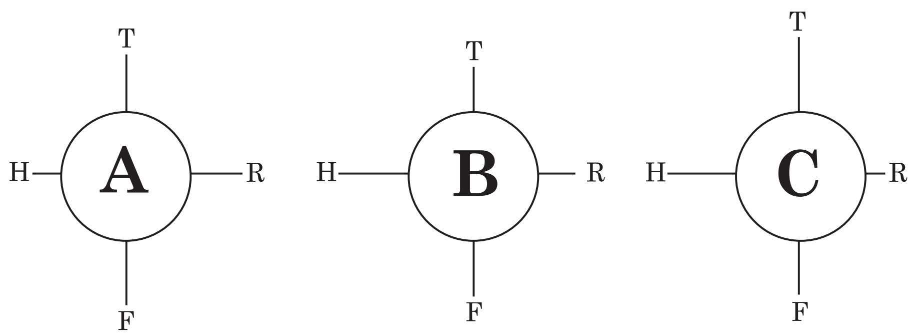
i) Two-way traffic

ii) Automobile fuel available



(4 marks)

e. i)



(1 mark)

ii) It would depend on what type of position that a player plays for a team, but if you add all rankings for all three players, the best overall would be Player C.

Player A	28
Player B	29
Player C	30

3. a) Let \$$x$ be amount of money invested at 5% *(5 marks)*
Let \$$y$ be amount of money invested at 6%

$x + y = 9000$ Total investment is \$9000.
$0.05x + 0.06y = 510$ Total interest earned is \$510.

Solve the two equations for y so that they may be graphed.

$y = -x + 9000$

$$y = \frac{-0.05}{0.06}x + \frac{510}{0.06}$$

Graph the two equations on the same grid.

Keying sequence:
[Y=] [(–)] [X,T,θ,n] [+] [9000] [ENTER] [(–)] [0.05] [÷] [0.06] [X,T,θ,n] [+] [510] [÷] [0.06] [GRAPH] Reset window.

Find where the graphs meet by pressing [2nd] (CALC) [5].

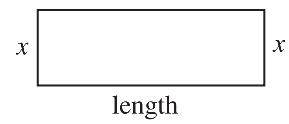
Press [ENTER] when prompted by the calculator for the first and second curves and when prompted for a guess. Since the only two curves you have graphed are the two in the system, and since the calculator will make a good enough guess for what you require here, this will perform the task at hand.

$x = 3000$ $y = 6000$

He will invest \$3000 at 5% and \$6000 at 6%.

(5 marks)

b) Let area be y metres^2
Let the width of the lot be y metres
Then the length of the lot will be $\dfrac{30-2x}{2}$ metres since the perimeter of two widths and two lengths of the lot is 30 metres.

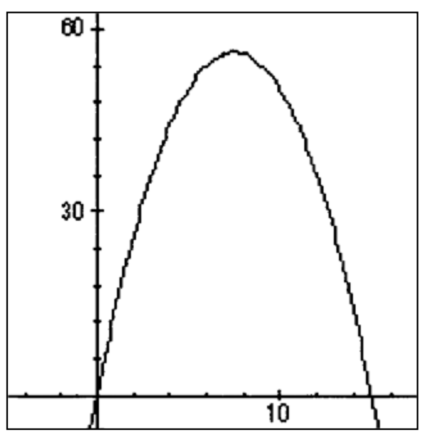


Area = Length × Width

$$y=\left(\frac{30-2x}{2}\right)(x)$$
$$y=(15-x)(x)$$
$$y=15x-x^2$$
$$y=-x^2+15x$$

Graph the function:

Keying sequence:
[Y=] [(–)] [X,T,θ,n] [x^2] [+] [15] [X,T,θ,n]



Reset window

Find the maximum point using the program:

[2nd] (CALC) [4], setting appropriate upper and lower bounds, and then pressing [ENTER] to find the point (7.5, 56.25).

This indicates that the maximum area will be 56.25 m^2 brought about by a width of 7.5 m and a length of $(15 - x)$ = 7.5 m.

4. Let \$C = cost
Let x = number of cups of Blend A
Let y = number of cups of Blend B

$C = 0.25x + 0.20y$ — Since the cost per cup of Blend A is 25 cents and the cost per cup of Blend B is 20 cents.

(10 marks)

Constraints

Protein:	$0.25x + 0.50y \geq 3$	Since the amount of protein in Blend A is 25% and the amount of protein in Blend B is 50%.
Carbohydrate:	$0.75x + 0.50y \geq 5$	Since the amount of carbohydrate in Blend A is 75% and the amount of carbohydrate in Blend B is 50%.
	$x \geq 0$	There cannot be a negative number of cups of Blend A.
	$y \geq 0$	There cannot be a negative number of cups of Blend B.

Graph the constraints:

Related lines

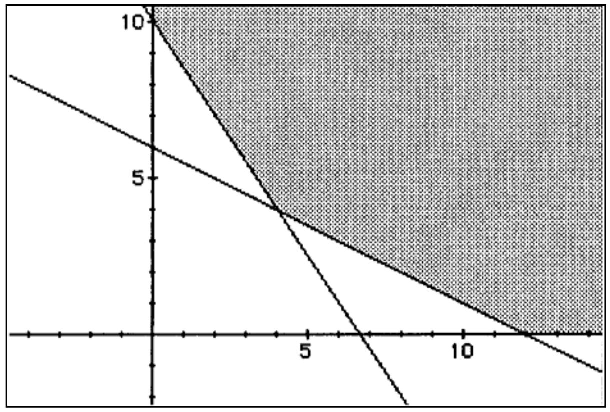
$x \geq 0$ $y \geq 0$ $0.25x + 0.5y \geq 3$ $0.75x + 0.50y \geq 5$

$x = 0$ $y = 0$ $0.25x + 0.5y = 3$ $0.75x + 0.50y = 5$

Intercept method

x	0	12
y	6	0

x	0	$\frac{20}{3}$
y	8	0



Test inequalities to determine the feasible region.

The shaded region is the feasible region having corner points of: (0, 10), (12, 0), and (4, 4). All may be found from the graph.

Substitute each corner point into the cost function and find the minimum value.

For (0, 10): C = $0.25x + 0.20y = 0.25(0) + 0.20(10) = 2.00$

For (4, 4): C = $0.25x + 0.20y = 0.25(4) + 0.20(4) = 1.80$

For (12, 0): C = $0.25x + 0.20y = 0.25(12) + 0.20(0) = 3.00$

Minimum cost = \$1.80 brought about by four cups of Blend A and four cups of Blend B being purchased.

Notes

Grade 11 Applied Mathematics (30S)

Final Practice Examination
Answer Key

GRADE 11 APPLIED MATHEMATICS

Final Practice Examination Answer Key

Name: ______________________________________

Student Number: _____________________________

Attending ❑ Non-Attending ❑

Phone Number: ______________________________

Address: ___________________________________

____

____

For Marker's Use Only

Date: __________________________________

Final Mark: ________ /100 = ________ %

Comments:

Instructions

The final examination is based on Modules 5 to 8 of the Grade 11 Applied Mathematics course. It is worth 25% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the final examination.

Notes

You are allowed to bring the following to the examination: pens/pencils (2 or 3 of each), metric and imperial rulers, a graphing and/or scientific calculator, and your Final Exam Resource Sheet. Your Final Exam Resource Sheet must be handed in with the examination. Graphing technology (either computer software or a graphing calculator) **is required** to complete this examination.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer. Final answers without supporting calculations or explanations will **not** be awarded full marks. Indicate equations and/or keystrokes used in calculations.

When using graphing technology, include a screenshot or printout of graphs **or** sketch the image and indicate the window settings (maximum and minimum *x*- and *y*-values), increments, and axis labels, including units.

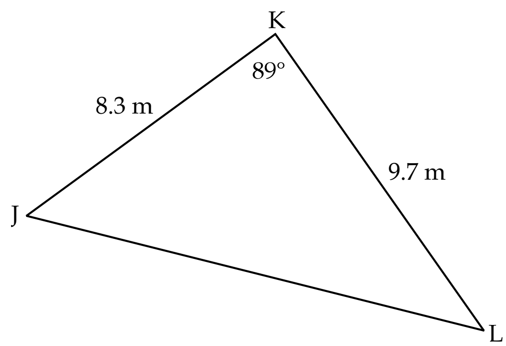
Name: ______________________________

Answer all questions to the best of your ability. Show all your work.

Module 5: Trigonometry *(25 marks)*

1. Solve for all the missing angles and all the missing sides in the triangles below. Round your answers to one decimal place.

   a) *(6 marks)* (Module 5, Lesson 3)



Answer:

First, find the size of k, using the Cosine Law.

$$k^2 = j^2 + l^2 - 2(j)(l)\cos\angle\mathrm{K}$$

$$k^2 = 9.7^2 + 8.3^2 - 2(9.7)(8.3)\cos 89^\circ$$

$$k^2 = 162.98 - 161.02\cos 89^\circ$$

$$k = \sqrt{162.98 - 161.02\cos 89^\circ}$$

$$k = 12.7\text{ m}$$

Now, find $\angle$L using the Sine Law.

$$\frac{\sin\angle\mathrm{L}}{l} = \frac{\sin\angle\mathrm{K}}{k}$$

$$\frac{\sin\angle\mathrm{L}}{8.3} = \frac{\sin 89^\circ}{12.7}$$

$$\angle\mathrm{L} = \sin^{-1}\left(\frac{8.3\sin 89^\circ}{12.7}\right)$$

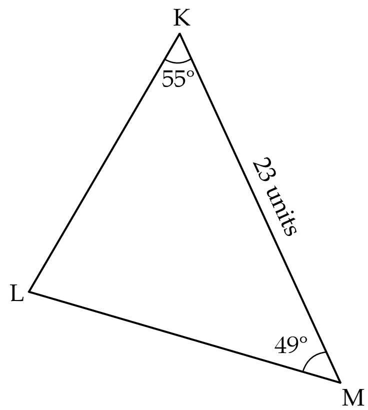
$$\angle\mathrm{L} = 40.8^\circ$$

Finally, find $\angle$J using the 180° rule.

$$\angle\mathrm{J} = 180^\circ - 89^\circ - 40.8^\circ$$

$$\angle\mathrm{J} = 50.2^\circ$$

b) *(5 marks)* (Module 5, Lesson 2)



Answer:

First, find ∠L using the 180° rule.

$$180° - 55° - 49° = 76°$$

Now use the Sine Law to find k.

$$\frac{k}{\sin 55°} = \frac{23}{\sin 76°}$$

$$k = 19.4 \text{ units}$$

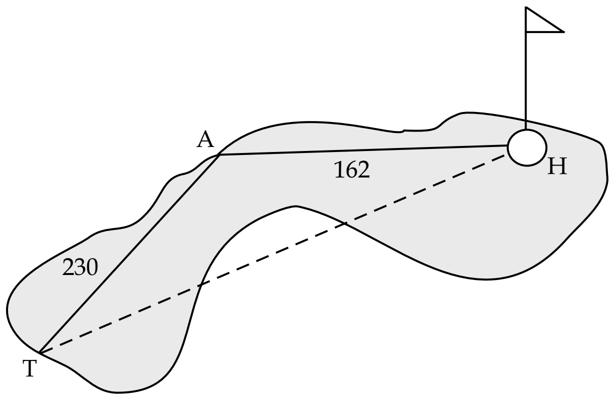
Now use the Sine Law again to find m.

$$\frac{m}{\sin 49°} = \frac{23}{\sin 76°}$$

$$m = 17.9 \text{ units}$$

Name: ___________________________________

2. A golf hole has a dogleg as shown in the diagram below. What is the angle at the dogleg (TAH), if the distance from the tee, T, to the hole, H, is 362 m? *(3 marks)*
(Module 5, Lesson 3)



Answer:

To find this angle, use the Cosine Law.

$$a^2 = t^2 + h^2 - 2(t)(h)\cos\angle A$$

$$362^2 = 162^2 + 230^2 - 2(162)(230)\cos\angle A$$

$$131044 = 79144 - 74520\cos\angle A$$

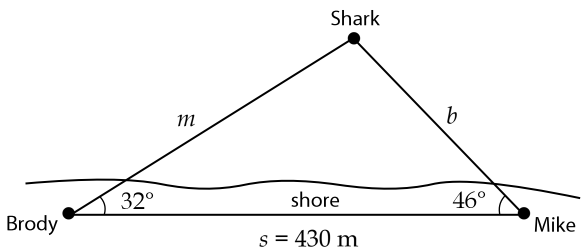
$$51900 = -74520\cos\angle A$$

$$\angle A = \cos^{-1}\left(-\frac{51900}{74520}\right)$$

$$\angle A = 134.1^\circ$$

The angle at the dogleg is 134.1°.

3. Brody and Mike are two harbour masters who are tracking the position of a great white shark. The harbour masters are located in buildings on the ocean shore that are 430 m apart. Brody's line of sight to the shark makes an angle of 32° with his line of sight to Mike. Mike's line of sight to the shark makes an angle of 46° with his line of sight to Brody. Determine the distance of the shark from each harbour master. Round your answers to the nearest metre. *(5 marks)* (Module 5, Lesson 4)



Answer:

To solve this problem, first find the missing angle in the diagram of the triangle above, using the 180° rule.

$$\angle S = 180° - 32° - 46°$$

$$\angle S = 102°$$

Now, you can determine the distance between the shark and Mike using the Sine Law.

$$\frac{b}{\sin \angle B} = \frac{s}{\sin \angle S}$$

$$b = \frac{430 \sin 32°}{\sin 102°}$$

$$b = 233 \text{ m}$$

The shark and Mike are 233 m apart.

Finally, calculate the distance between the shark and Brody using the Sine Law.

$$\frac{m}{\sin \angle M} = \frac{s}{\sin \angle S}$$

$$m = \frac{430 \sin 46°}{\sin 102°}$$

$$m = 316 \text{ m}$$

The shark and Brody are 316 m apart.

4. Given points A, B, and C, determine measurements for angle A, side *a*, and side *b*, so that each of the following situations is created. *(3 × 2 marks each = 6 marks)*
(Module 5, Lesson 5)

a) no triangle is possible

Answer:

Note: Other solutions are possible.

In ΔABC, let ∠A = 32°, *a* = 1 m, and *b* = 3.5 m.

To show there is no triangle possible, you can use the Sine Law.

$$\frac{\sin 32°}{1} = \frac{\sin B}{3.5}$$

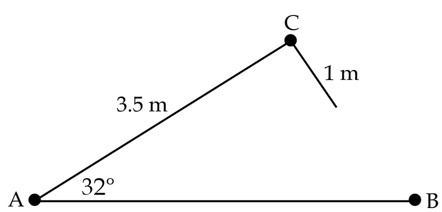
$$3.5\left(\frac{\sin 32°}{1}\right) = \sin B$$

$$\sin B = 1.854$$

$$\angle B = \sin^{-1}(1.854)$$

$$\angle B = \text{ not possible}$$

As the value of sin B > 1, it is not possible to complete this triangle.



b) two triangles are possible

Answer:

In ΔABC, let $\angle A = 42°$, $a = 6$ m, and $b = 7$ m.

To show there are two solutions, you can use the Sine Law.

$$\frac{\sin 42°}{6} = \frac{\sin B}{7}$$

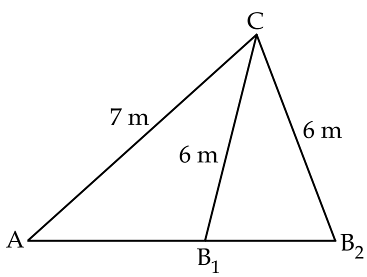
$$7\left(\frac{\sin 42°}{6}\right) = \sin B$$

$$\sin B = 0.7806$$

$$\angle B = \sin^{-1}(0.7806)$$

$$\angle B = 51.3°$$

As the value of $\sin B < 1$, and side a is shorter than side b, it is possible to create two triangles.



Name: ____________________________________

c) one right triangle is possible

Answer:

In ΔABC, let ∠A = 30°, a = 3.5 m, and b = 7 m.

To show that only one right triangle is possible, you can use the Sine Law.

$$\frac{\sin 30}{3.5} = \frac{\sin B}{7}$$

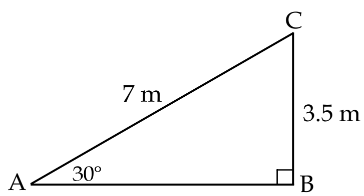
$$7\left(\frac{\sin 30°}{3.5}\right) = \sin B$$

$$\sin B = 1$$

$$\angle B = \sin^{-1}(1)$$

$$\angle B = 90°$$

As the value of sin B = 1, only one triangle is possible and it is a right triangle.



Module 6: Statistics *(23 marks)*

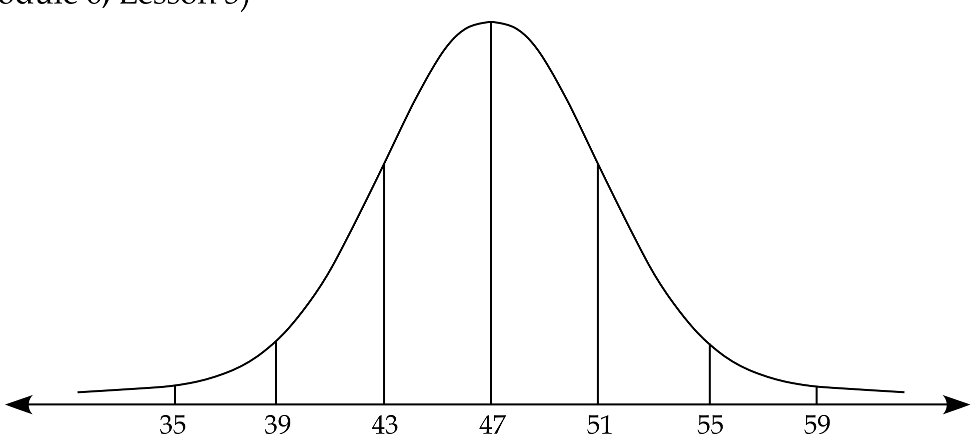
1. State two properties that apply to all normal distributions. *(2 marks)* (Module 6, Lesson 3)

   Answer:

   Any two of the following:

   - The curve is perfectly symmetrical about the mean. The two halves of the curve are a mirror image of each other around the mean.
   - The standard deviation determines whether the curve is tall and skinny or short and wide or any variation between.
     - If the standard deviation, σ, is large, the curve will be wider horizontally and shorter vertically.
     - If the standard deviation, σ, is small, the curve will be narrower horizontally and taller vertically.
   - The mean, the median and the mode are all the same value.
   - Almost all data, 99.7%, is within 3 standard deviations of the mean.
   - The probability that a score falls within one standard deviation of the mean on either side is approximately 68% or 0.68.
   - The probability that a score falls within two standard deviations of the mean is approximately 95% or 0.95.
   - The probability that a score falls within three standard deviations of the mean is approximately 99.7% or 0.997.
   - The total area under the curve is always 1, or 100%. This simply indicates that all of the data is represented by points under the curve.

2. Determine the mean and standard deviation of the following normal curve. *(2 marks)* (Module 6, Lesson 3)



   Answer:

   The mean is 47.

   The standard deviation is 4.

Name: ______________________________

3. The following data represents a sample of the waiting times at a dentist's office. These times are expressed in minutes.

12	19	2	13	21	23	18	13	20
23	26	7	10	8	16	12	17	21

a) Determine the mean and standard deviation for the data. Round to one decimal place. *(2 marks)* (Module 6, Lesson 2)

Answer:

Mean: 15.6

Standard Deviation: 6.4

b) Calculate the percentage, to one decimal place, of the waiting times that fall within ±1 standard deviation of the mean. *(2 marks)* (Module 6, Lesson 3)

Answer:

One standard deviation less than the mean is 9.2. One standard deviation more than the mean is 22.0.

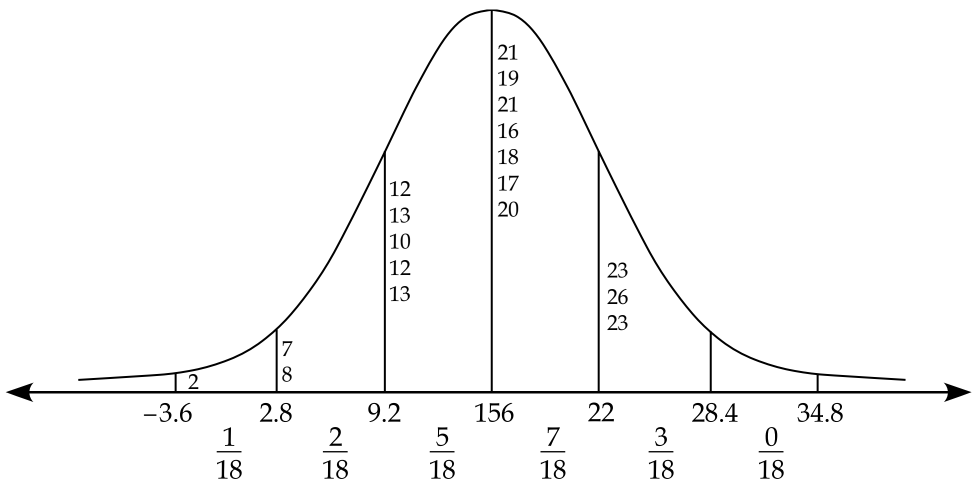
The data values that fall in this range are 12, 19, 13, 10, 21, 16, 18, 12, 13, 17, 20, and 21.

There are 12 out of 18 values in this range. This is equal to 67%.

c) Verify if the waiting times resemble a normal distribution. Justify your answer. *(3 marks)* (Module 6, Lesson 3)

Answer:

The percentage of data that falls within one standard deviation of the mean is close to what it should be. Therefore, we should sketch a normal curve to help determine if the data resembles a normal distribution.



±1 standard deviation: $\dfrac{12}{18} = 66.7\%$

±2 standard deviations: $\dfrac{17}{18} = 94.4\%$

±3 standard deviations: $\dfrac{18}{18} = 100\%$

This data is close to approximating a normal curve, but not exact. The percentages of data that should fall in each category in the normal distribution are similar to what they should be. The percentage of data within one standard deviation of the mean is close to 68%. the percentage of data that is within two standard deviations of the mean is close to 95%. All of the data is within three standard deviations of the mean, which is close to 99.7%. Therefore, this data does approximate a normal distribution.

Name: ____

4. The lifetime of a microwave is normally distributed with a mean of 4.7 years and a standard deviation of 0.4 years. (Module 6, Lesson 5)

   What percent of microwaves will last at least 5 years? *(2 marks)*

   Answer:

   The mean is 4.7.

   The standard deviation is 0.4.

   The lower bound is 5.

   Using a probability calculator, the percent of microwaves that will last at least 5 years is 22.66%.

5. For a university calculus course, the professor decided that only 5% of students would fail. Assume the marks are normally distributed. (Module 6, Lesson 5)

   a) What z-score represents this value? *(1 mark)*

      Answer:

      Using a probability calculator:

         The mean is 0.

         The standard deviation is 1.

         The percent is 0.05.

         The z-score at which 5% of data is below that value is -1.6449.

         5% of the data will have a z-score of -1.6449 or lower.

   b) If the mean mark was 71.4 and the standard deviation was 14.3, what is the lowest mark that would represent a passing mark? *(2 marks)*

      Answer:

      $$-1.6449 = \frac{X - 71.4}{14.3}$$

      The lowest passing mark is 48.

   c) If the mean was 60 and the standard deviation was 16.2, will you pass with a mark of 45? Justify your answer using mathematical calculations. *(2 marks)*

      Answer:

      $$z = \frac{45 - 60}{16.2} = -0.9259$$

      This is a z-score that is above the lowest z-score of -1.6449. Therefore, you will pass.

Name: ___________________________________

6. Based on survey results, 61 out of 100 people who go to a car dealership end up purchasing a vehicle. These results are accurate to within ±5%, 19 times out of 20. (Module 6, Lesson 6)

   a) Determine the confidence level. *(1 mark)*

      Answer:

      $$\frac{19}{20} = 95\%$$

   b) State the margin of error. *(1 mark)*

      Answer:

      ±5%

   c) What is the confidence interval? *(1 mark)*

      Answer:

      56% − 66%

7. Explain how the margin of error affects the confidence interval. *(1 mark)* (Module 6, Lesson 6)

   Answer:

   The smaller the margin of error, the smaller the confidence interval. The larger the margin of error, the larger the confidence interval.

8. In a study of the water quality of Manitoba lakes, pollutants are measured in parts per million (ppm). The average amount of pollutants found in the lakes for a particular region of Manitoba is 0.88 ppm, with a standard deviation of 0.27. The data follows a normal distribution. The measured pollutants at Green Lake are 1.21 ppm. Determine the equivalent *z*-score. *(1 mark)* (Module 6, Lesson 4)

   Answer:

   $$z = \frac{1.21 - 0.88}{0.27}$$

   $$z = 1.22$$

Module 7: Mathematical Models *(27 marks)*

1. A delivery truck is bringing goods to a store 180 km away. For the first half hour, the driver maintains a speed of 60 km/h. The driver then accelerates to 100 km/h for the remainder of the trip. Create a graph to represent this trip. *(3 marks)* (Module 7, Lesson 2)

   Answer:

Note:

1 mark for each correct slope

1 mark for correct axes

Name: ____________________________________

2. Determine a point that is in the solution region for the following system of linear inequalities. Prove that your point is in the solution region. *(3 marks)*
(Module 7, Lesson 5)

$$y \le -\frac{1}{2}x + 2$$

$$y \ge -2x + 5$$

Answer:

A possible point is (3, 0).

For the first inequality:

$$0 \le -\frac{1}{2}(3) + 2$$

$$0 \le -1.5 + 2$$

$$0 \le 0.5$$

For the second inequality:

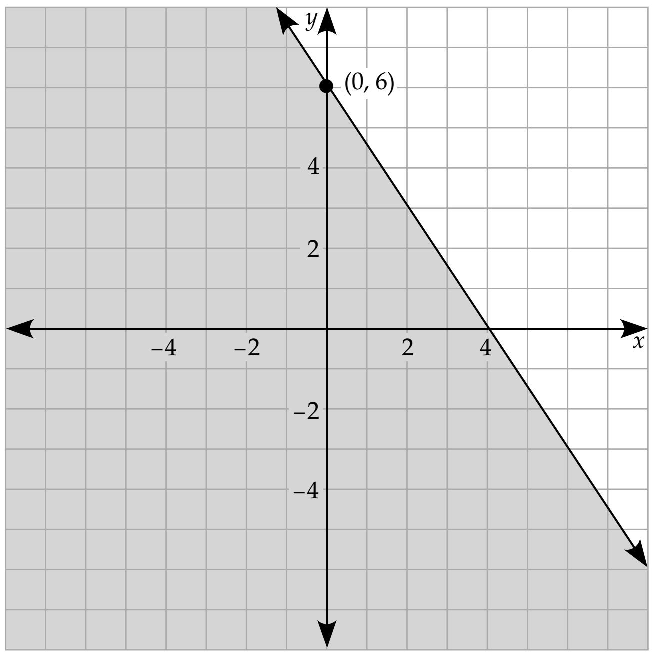
$$0 \ge -2(3) + 5$$

$$0 \ge -6 + 5$$

$$0 \ge -1$$

Since both statements are true, the point (3, 0) is in the solution region.

3. Write the linear inequality that is represented in the graph below. *(3 marks)*
(Module 7, Lesson 4)



Answer:

y-intercept: 6

Slope: $\frac{-3}{2}$

Therefore, the equation of the boundary line is $y = \frac{-3}{2}x + 6$.

Boundary line: solid

Therefore, the inequality is either $\leq$ or $\geq$.

Since the graph is shaded below the line, the inequality is $y \leq \frac{-3}{2}x + 6$.

Note:

1 mark for the correct slope

1 mark for the correct y-intercept

1 mark for the correct inequality sign

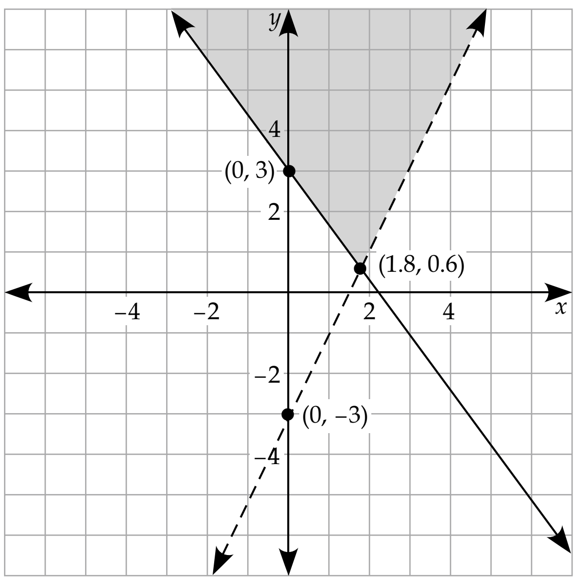
Name: ____

4. Sketch the following system of inequalities. Shade the region of overlap. *(5 marks)* (Module 7, Lesson 5)

   $4x + 3y \geq 9$

   $-y < -2x + 3$

   Answer:



$4x + 3y \geq 9$	$-y < -2x + 3$
$3y \geq -4x + 9$	$y > 2x - 3$
$y \geq \frac{-4}{3}x + 3$	Slope: $\frac{2}{1}$
Slope: $\frac{-4}{3}$	y-intercept: -3
y-intercept: 3	
Boundary line: solid Shade above	Boundary line: dashed Shade above

Point of intersection: (1.8, 0.6)
(found using technology)

Note:

1 mark for each correct line (half-mark for slope and half-mark for y-intercept) × *2 lines*

1 mark for each correct solid or dashed line × *2 lines*

1 mark for the correctly shaded region

5. You have \$200 to spend on clothing. Shirts costs \$20 and pants cost \$35. Determine how many shirts and pants you can purchase. (Module 7, Lesson 4)

   a) Define the two variables used in this scenario. *(1 mark)*

      Answer:

      Let x represent the number of shirts purchased.

      Let y represent the number of pants purchased.

   b) What are the restrictions on these variables? Explain. *(2 marks)*

      Answer:

      $x \geq 0$

      $y \geq 0$

      You cannot buy a negative number of shirts and pants.

   c) Write an inequality to represent how many shirts and how many pants you can purchase. *(1 mark)*

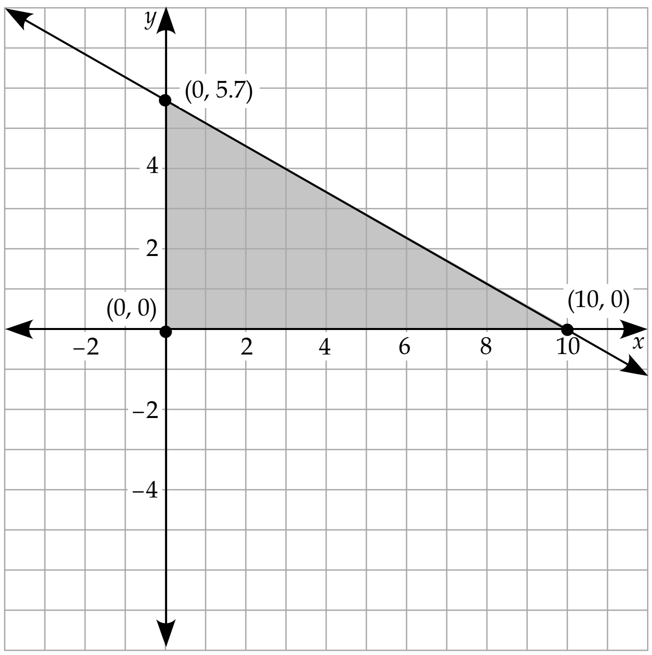
      Answer:

      $20x + 35y \leq 200$

Name: ____

d) Graph the inequality you created in (b) and (c), and label the vertices of the solution region. *(3 marks)*

Answer:



$$20x + 35y \leq 200$$

$$35y \leq -20x + 200$$

$$y \leq \frac{-20}{35}x + \frac{200}{35}$$

$$y \leq \frac{-4}{7}x + \frac{40}{7}$$

y-intercept: $\frac{40}{7}$ or approximately 5.7

e) If you buy 4 shirts, how many pairs of pants can you possibly buy? *(1 mark)*

Answer:

You can buy 1, 2, or 3 pairs of pants.

f) If you buy 3 pairs of pants, how many shirts can you possibly buy? *(1 mark)*

Answer:

You can buy 1, 2, 3, or 4 shirts.

6. A food truck sells pizza and chicken wings. Each day, they sell at least three times as many orders of pizza as they do chicken wings. The food truck only has room to hold 42 orders of pizza and 30 orders of chicken wings. Pizzas are sold for $10 each and chicken wings are sold for $8 each. Determine the number of pizza and chicken wings that would have to be sold to maximize the amount of money the food truck receives in sales in one day. (Module 7, Lesson 6)

   a) Write the equation of the objective function. *(1 mark)*

   Answer:

   $P = 10x + 8y$

   b) The owner of the food truck graphs this inequality and finds the vertices of the solution region to be (0, 0), (42, 14), and (42, 0). How many of each type of food should be sold to maximize profits? Verify your solution by testing all vertices. *(3 marks)*

   Answer:

   Check the vertices:

   $P = 10x + 8y$

   (0, 0)

   $P = \$0$

   (42, 14)

   $P = \$532$

   (42, 0)

   $P = \$420$

   The greatest profit comes from selling 42 orders of pizza and 14 orders of chicken wings.

Name: ____________________

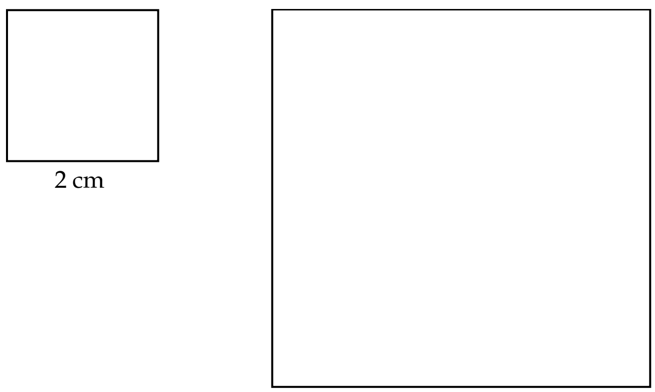
Module 8: Scale Factors for 2-D and 3-D Shapes *(25 marks)*

1. Provide an example in your life of when you would use a model of a 2-D shape. *(1 mark)* (Module 8, Lesson 1)

   Answer:

   Examples may vary. Possible answers could be drawing of a room floor plan or using a map.

2. The following squares are similar. Determine the scale factor if the larger square is the scale model and the area of the larger square is 25 cm^2. *(3 marks)* (Module 8, Lesson 2)

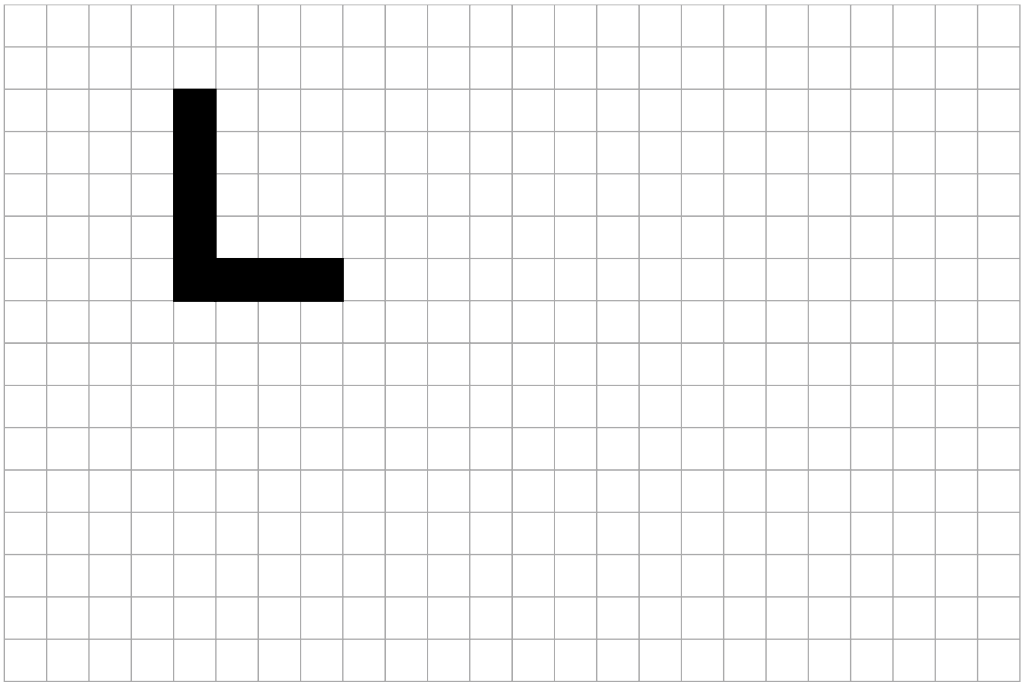


   Answer:

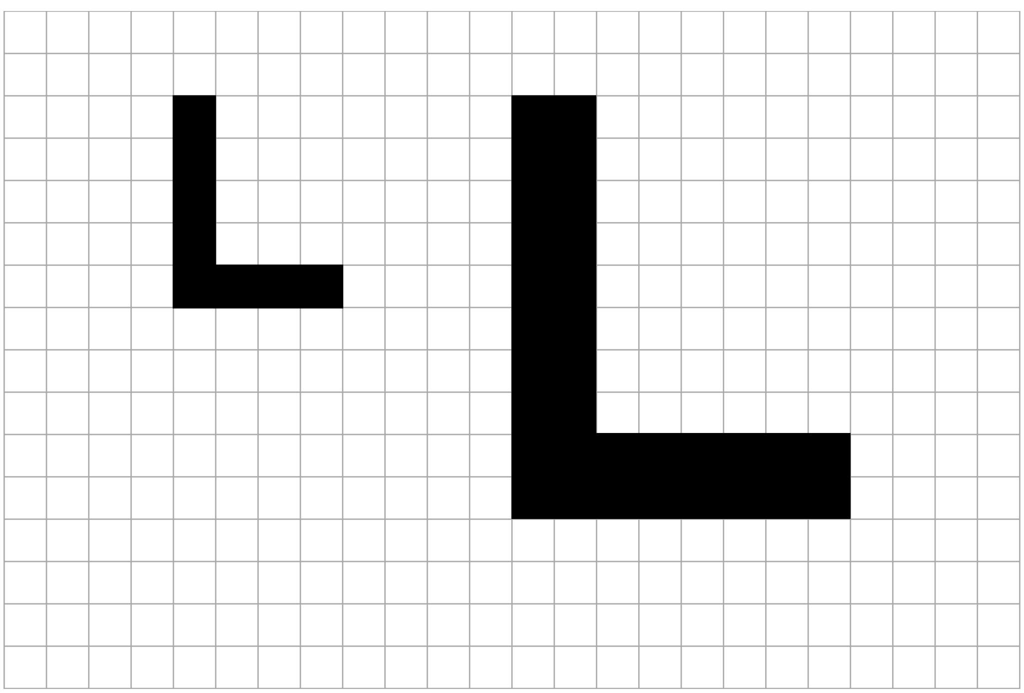
   The side length of the large square must be 5 cm. Then:

$$\text{scale factor} = \frac{\text{length of scale square}}{\text{length of actual square}} = \frac{5}{2} = 2.5$$

3. Draw a shape similar to the one below, using a scale of 2 : 1. *(2 marks)*
(Module 8, Lesson 1)

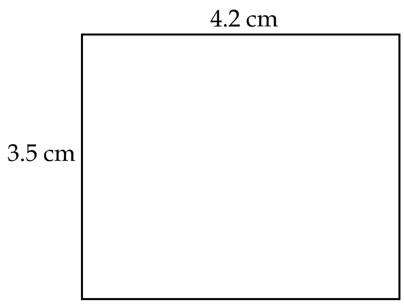


Answer:



Name: ____________________________________

4. a) The floor of a room is drawn to scale as shown in the diagram below, using a reduction factor of 100. Calculate the area of the actual room that the diagram represents. *(3 marks)* (Module 8, Lesson 3)



Answer:

Area Scale = 3.5 cm × 4.2 cm

= 14.7 cm^2

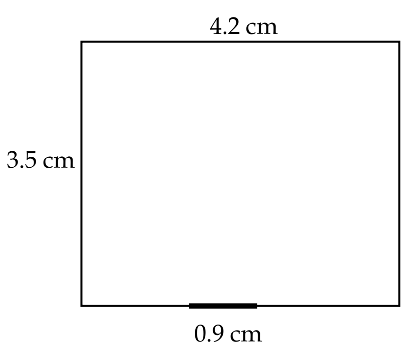
$$R^2 = \frac{\text{Area Scale}}{\text{Area Actual}}$$

$$\frac{1}{100^2} = \frac{14.7 \text{ cm}^2}{\text{Area Actual}}$$

$$\text{Actual} = 147\ 000 \text{ cm}^2$$

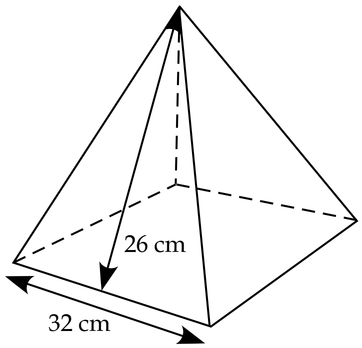
b) A doorway in the actual room is 0.9 m wide. Draw the doorway on the diagram above, using the reduction scale factor of 100. *(1 mark)* (Module 8, Lesson 1)

Answer:



Note: The door could be anywhere on the rectangle but needs to be exactly 0.9 cm.

5. The dimensions for a right square pyramid are shown below. A scale model of this pyramid is being created, with a surface area of 4023 cm^2. Calculate the dimensions of the model pyramid. *(5 marks)* (Module 8, Lesson 4)



Answer:

First, determine the surface area of the actual pyramid. Then, find the scale factor relating the two surface areas. Finally, calculate the dimensions of the scale pyramid.

To find the surface area of a square pyramid, first recognize that all four triangular faces are the same. Therefore, multiply the formula for the area of a triangle by 4. Then, determine the area of the square base.

$$\begin{aligned} SA_{\text{actual}} &= 4\left(\frac{1}{2}\right)bh + b^2 \\ &= (4)\left(\frac{1}{2}\right)(32)(26) + (32)(32) \\ &= 1664 + 1024 \\ &= 2688 \text{ cm}^2 \end{aligned}$$

Now find the scale factor.

$$k^2 = \frac{SA_{\text{scale}}}{SA_{\text{actual}}} = \frac{4023}{2688} = 1.5$$

$$k = 1.2$$

Now, multiply the dimensions of the actual pyramid by the scale factor to determine the dimensions of the scale pyramid.

Scale height: 26(1.2) = 31.2 cm

Scale base: 32(1.2) = 38.4 cm

Name: ____

6. Determine the scale factor if the volume of a scale diagram is 21 mm^3 and the volume of the actual shape is 68 cm^3. *(3 marks)* (Module 8, Lesson 5)

Answer:

You first need to convert both units to mm^3. Therefore, 68 cm^3 is $68 \times 10 \times 10 \times 10 =$ 68 000 mm^3. You can then calculate the scale factor relating the two diagrams.

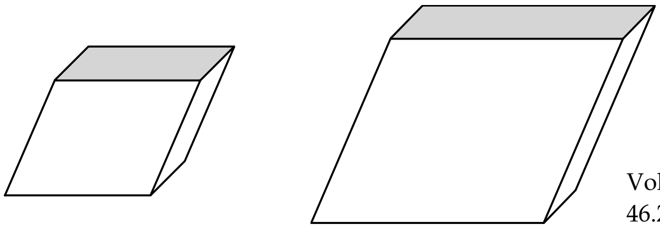
$$k^3 = \frac{21\text{ mm}^2}{68\ 000\text{ mm}^2}$$

$$= \frac{21}{68\ 000}$$

$$k = \sqrt[3]{\frac{21}{68\ 000}}$$

$$k = 0.07$$

7. a) Find the scale factor relating the two objects below if the larger prism is the scale diagram. *(2 marks)* (Module 8, Lesson 5)



Answer:

$$k^3 = \frac{V_{\text{scale}}}{V_{\text{actual}}} = \frac{46.2}{12.9}$$

$$k = 1.53$$

b) If the surface area of the top of the small prism is 3.9 cm^2, what is the surface area of the top of the large prism? *(2 marks)* (Module 8, Lesson 4)

Answer:

$$k = 1.53$$

$$k^2 = \frac{A_{\text{scale}}}{A_{\text{actual}}}$$

$$1.53^2 = \frac{A_{\text{scale}}}{3.9}$$

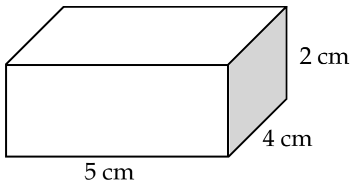
$$9.1 = A_{\text{scale}}$$

The surface area is 9.1 cm^2.

Name: ___________________________________

8. Determine the surface area of the scale model of the rectangular prism given the dimensions of the actual prism below. The scale factor is $\frac{7}{2}$. *(2 marks)*

   (Module 8, Lesson 4)



   Answer:

   $SA_{\text{actual}} = 5(4)(2) + 5(2)(2) + 4(2)(2) = 40 + 20 + 16 = 76\ \text{cm}^2$

   $SA_{\text{scale}} = \left(\frac{7}{2}\right)^2 (76) = \left(\frac{49}{4}\right)(76) = 931\ \text{cm}^2$

9. How many times would the volume of a milk container increase if all of the dimensions were doubled? *(1 mark)* (Module 8, Lesson 5)

   Answer:

   The volume would increase by a factor of 8 (2^3).