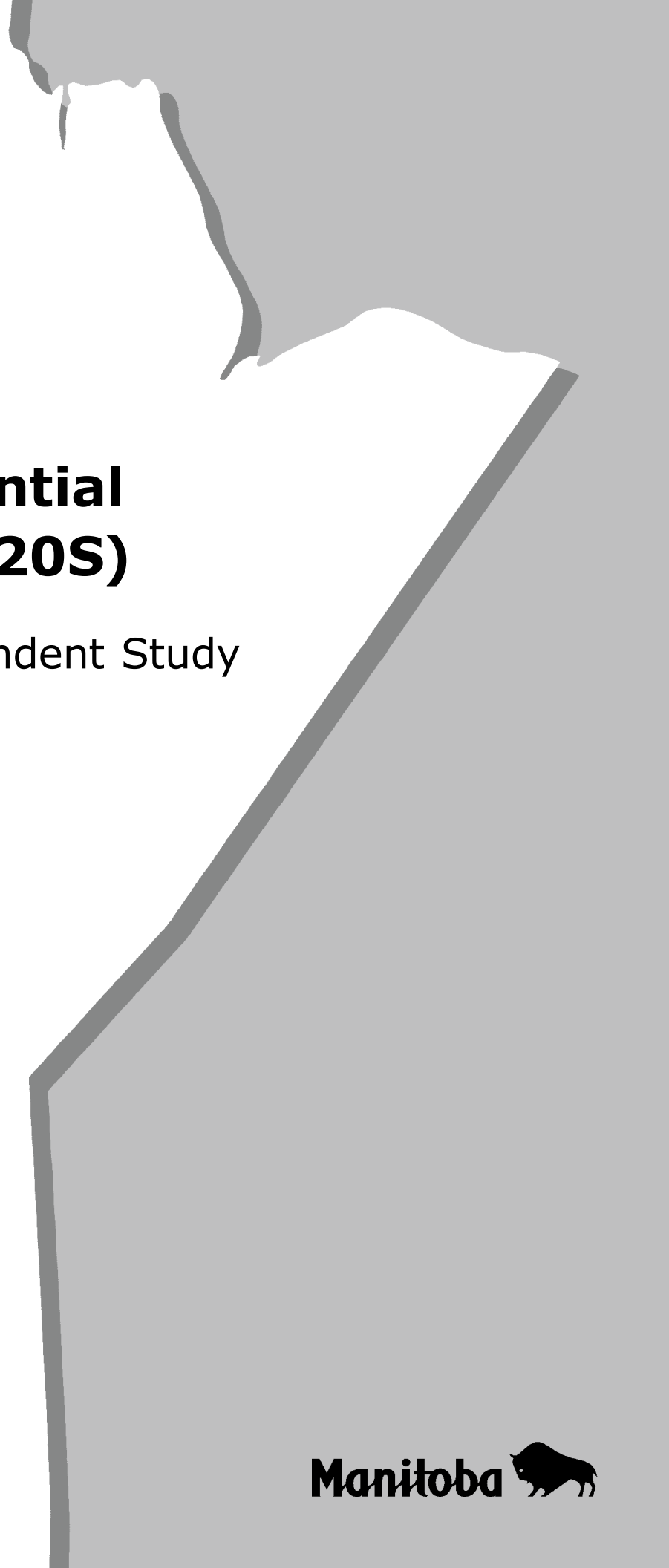
Grade 10 Essential Mathematics (20S)

A Course for Independent Study



Manitoba

Grade 10 Essential Mathematics (20S)

A Course for Independent Study

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CONTENTS

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Grade 10 Essential Mathematics (20S)

Introduction

Introduction

Overview

Welcome to Grade 10 Essential Mathematics! This course is a continuation of the concepts you have been studying in previous years, as well as an introduction to new topics. You will put to use many of the skills that you have already learned to solve problems and do basic arithmetic operations. This course helps you develop the skills, ideas, and confidence that you will need to continue studying math in the future.

As a student enrolled in an independent study course, you have taken on a dual role—that of a student and a teacher. As a student, you are responsible for mastering the lessons and completing the learning activities and assignments. As a teacher, you are responsible for checking your work carefully, noting areas in which you need to improve, and motivating yourself to succeed.

What Will You Learn in This Course?

Problem solving, communication, reasoning, and mental math are some of the themes you will discover in each module. You will engage in a variety of activities that promote the connections between symbolic math ideas and the world around you.

There are four main areas that you will be exploring: Personal Finance, Shapes and Space, Numbers, and Decision Making.

How Is This Course Organized?

The Grade 10 Essential Mathematics course consists of the following eight modules:

- Module 1: Gross Pay, Time Cards, and Percents
- Module 2: Net Pay
- Module 3: Measurement
- Module 4: Geometry
- Module 5: Working with Angles
- Module 6: Consumer Decisions
- Module 7: Trigonometry
- Module 8: Transformations

Each module in this course consists of several lessons, which contain the following components:

- **Lesson Focus:** The Lesson Focus at the beginning of each lesson identifies one or more specific learning outcomes (SLOs) that are addressed in the lesson. The SLOs identify the knowledge and skills you should have achieved by the end of the lesson.
- **Introduction:** Each lesson begins with an explanation of what you will be learning in that lesson.
- **Lesson:** The main body of the lesson is made up of the content that you need to learn. It contains text, explanations, images, diagrams, and completed examples.
- **Learning Activities:** Most lessons include one or more learning activities that will help you learn about the lesson topics and prepare you for the assignments, the midterm examination, and the final examination. Once you complete a learning activity, check your responses against those provided in the Learning Activity Answer Key found at the end of each applicable module. You will not submit the completed learning activities to the Distance Learning Unit.
- **Assignments:** Assignments are found at the end of each lesson that has an assignment. You will mail or electronically submit all your completed assignments to the Distance Learning Unit for assessment at the end of each module. In total, all assignments are worth seventy-five percent (75%) of your final course mark.
- **Summary:** Each lesson ends with a brief review of what you just learned.

This course also includes the following two appendices:

- **Appendix A: Conversions:** The conversion charts found in Appendix A can be used throughout the course.
- **Appendix B: Glossary:** The glossary at the end of the course provides definitions for an alphabetical list of the terms identified throughout the course. You can use the glossary to review terms used in the course.

What Resources Will You Need for This Course?

You do not need a textbook for this course. All the content is provided directly within the course. You will, however, need access to a variety of resources.

The required and optional resources for this course are identified below.

Required Resources

For this course, you will need access to the following resources. If you do not have access to one or more of these resources, contact your tutor/marker.

- **A graphing or scientific calculator:** Use a graphing or scientific calculator as you work through this course. You will also need the calculator for the examination(s).
- **A metric ruler** (15 cm long is fine)
- **An imperial ruler** (6 inches long is fine)
- **A geometry set, including a ruler, a protractor, and a compass:** You **will need** the geometry set for the examinations.

Optional Resources

It would be helpful if you had access to the following resources:

- **A computer with spreadsheet software and graphing capabilities:** Access to spreadsheet software (e.g., Microsoft Excel) and graphing capabilities would help you to present and analyze data graphically.
- **A computer with Internet access:** Some lessons suggest website links as sources of information or for supplementary reference and reading. If you do not have Internet access, you will still be able to complete the course, but you will need to find different ways of accessing information.
- **Photocopier/scanner:** With access to a photocopier/scanner, you could make a copy of your assignments before submitting them so that if your tutor/marker wants to discuss an assignment with you over the phone, each of you will have a copy. It would also allow you to continue studying or to complete further lessons while your original work is with the tutor/marker. Photocopying or scanning your assignments will also ensure that you keep a copy in case the originals are lost.

Who Can Help You with This Course?

Taking an independent study course is different from taking a course in a classroom. Instead of relying on the teacher to tell you to complete a learning activity or an assignment, you must tell yourself to be responsible for your learning and for meeting deadlines. There are, however, two people who can help you be successful in this course: your tutor/marker and your learning partner.

Your Tutor/Marker

Tutor/markers are experienced educators who tutor Independent Study Option (ISO) students and mark assignments and examinations. When you are having difficulty with something in this course, contact your tutor/marker, who is there to help you. Your tutor/marker's name and contact information were sent to you with this course. You can also obtain this information in the learning management system (LMS).

Your Learning Partner

A learning partner is someone **you choose** who will help you learn. It may be someone who knows something about mathematics, but it doesn't have to be. A learning partner could be someone else who is taking this course, a teacher, a parent or guardian, a sibling, a friend, or anybody else who can help you. Most importantly, a learning partner should be someone with whom you feel comfortable and who will support you as you work through this course.

Your learning partner can help you keep on schedule with your coursework, read the course with you, check your work, look at and respond to your learning activities, or help you make sense of assignments. You may even study for your examination(s) with your learning partner. If you and your learning partner are taking the same course, however, your assignment work should not be identical.

How Will You Know How Well You Are Learning?

You will know how well you are learning in this course by how well you complete the learning activities, assignments, and examinations.

Learning Activities

Each learning activity has two parts—Part A has BrainPower questions and Part B has questions related to the content in the lesson

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for you before trying the other questions. Each question should be completed quickly and without using a calculator, and most should be completed without using pencil and paper to write out multiple steps. Some of the questions will relate directly to content of the course. Some of the questions will review content from previous courses—content that you need to be able to answer efficiently.

Being able to do these questions in a few minutes will be helpful to you as you continue with your studies in mathematics. If you are finding it is taking you longer to do the questions, you can try one of the following:

- work with your learning partner to find more efficient strategies for completing the questions
- ask your tutor/marker for help with the questions
- search online for websites that help you practice the computations so you can become more efficient at completing the questions.

None of the assignment questions or examination questions will require you to do the calculations quickly or without a calculator. However, it is for your benefit to complete the questions as they will help you in the course. Also, being able to successfully complete the BrainPower exercises will help build your confidence in mathematics. BrainPower questions are like a warm-up you would do before competing in a sporting event.

Part B: Course Content Questions

One of the easiest and fastest ways to find out how much you have learned is to complete Part B of the learning activities. These have been designed to let you assess yourself by comparing your answers with the answer keys at the end of each module. There is at least one learning activity in each lesson. You will need a notebook or loose-leaf pages to write your answers.

The learning activities in this course will help you to review and practise what you have learned in the lessons. You will **not** submit the completed learning activities to the Distance Learning Unit. Instead, you will complete the learning activities and compare your responses to those provided in the Learning Activity Answer Key found at the end of each module.

Make sure you complete the learning activities. Doing so will not only help you to practise what you have learned, but will also prepare you to complete your assignments and the examination(s) successfully. Many of the questions on the examination(s) will be similar to the questions in the learning activities. Remember that you **will not submit learning activities to the Distance Learning Unit**.

Assignments

Each module in this course contains assignments, which you will complete and submit to the Distance Learning Unit for assessment. The assignments are worth a total of 75% of your final course mark.

There are two types of assignments that you must submit to the Distance Learning Unit. Each module has a cover assignment, which you can complete at any time during the module. Lesson assignments are located throughout the modules, and include questions similar to the questions in the learning activities of previous lessons. The cover assignments and lesson assignments have space provided for you to write your answers on the question sheets. **You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate).**

The tutor/marker will mark your assignments and return them to you. Remember to keep all marked assignments until you have finished the course so that you can use them to study for your examinations.

Resource Sheet

When you write your midterm and final examinations, you will be allowed to take an Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Examination Resource Sheet is not worth any marks.

Creating your own resource sheet is an excellent way to review the course content. It also provides you with a convenient reference and quick summary of the important facts of each module. Each student is asked to complete a resource sheet for each module to help with studying and reviewing.

The lesson summaries throughout the course are written for you to use as a guide, as are the module summaries at the end of each module. Refer to these when you create your own resource sheet. Then, go to Appendix B: Glossary (at the end of the course) to check the information on your resource sheet.

After you complete each module's resource sheet, you should summarize the sheets from all of the modules to prepare your Examination Resource Sheet. The midterm examination is based only on the first four modules of the course, while the final examination is based on Modules 5 to 8.

Midterm and Final Examinations

This course contains a midterm examination and a final examination.

- The midterm examination is based on Modules 1 to 4, and is worth 12.5% of your final mark in this course. You will write the midterm examination when you have completed Module 4.

  As a student, you can use your Midterm Examination Resource Sheet to bring any formulas you have not memorized into the examination with you. You will be required to bring the following supplies to the midterm examination: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Midterm Examination Resource Sheet.

  Formulas are not included with the examination. Be sure to include these on the resource sheet.

- The final examination is based on Modules 5 to 8, and is worth 12.5% of your final mark in this course. You will write the final examination when you have completed Module 8.

  You can use your Final Examination Resource Sheet to bring any formulas you have not memorized into the examination with you. Formulas are not provided on the examination. You will be required to bring the following supplies to the final examination: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Final Examination Resource Sheet.

  Formulas are not included with the examination. Be sure to include these on the resource sheet.

The two examinations are worth a total of 25% of your final course mark. You will write both examinations under supervision.

To do well on each examination, you should review all the work you have completed from the modules, including all learning activities and assignments.

Practice Examinations and Answer Keys

To help you succeed in your examinations, you will have an opportunity to complete a Midterm Practice Examination and a Final Practice Examination. These examinations, along with the answer keys, are found in the learning management system (LMS). If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to obtain a copy of the practice examinations.

These practice examinations are similar to the actual examinations you will be writing. The answer keys enable you to check your answers. This will give you the confidence you need to do well on your examinations.

Requesting Your Examination(s)

You are responsible for making arrangements to have the examinations sent to your proctor from the Distance Learning Unit. Please make arrangements before you finish Module 4 to write the midterm examination. Likewise, you should begin arranging for your final examination before you finish Module 8.

To write your examinations, you need to make the following arrangements:

- **If you are attending school,** your examination will be sent to your school as soon as all the applicable assignments have been submitted. You should make arrangements with your school's ISO school facilitator to determine a date, time, and location to write the examination.

- **If you are not attending school,** check the Examination Request Form for options available to you. Examination Request Forms can be found on the Distance Learning Unit's website, or look for information in the learning management system (LMS). Two weeks before you are ready to write the examination, fill in the Examination Request Form and mail, fax, or email it to

  Distance Learning Unit
  500–555 Main Street
  PO Box 2020
  Winkler MB R6W 4B8
  Fax: 204-325-1719
  Toll-Free Telephone: 1-800-465-9915
  Email: distance.learning@gov.mb.ca

How Much Time Will You Need to Complete This Course?

Learning through independent study has several advantages over learning in the classroom. You are in charge of how you learn and you can choose how quickly you will complete the course. You can complete as many lessons as you wish in a single session. You do not have to wait for your teacher or classmates.

From the date of your registration, you have a maximum of **12 months** to complete this course, but the pace at which you proceed is up to you. Read the following suggestions on how to pace yourself.

Chart A: Semester 1

If you want to start the course in September and complete it in January, you can follow the timeline suggested below.

Module	Completion Date
Module 1	Middle of September
Module 2	End of September
Module 3	Middle of October
Module 4	End of October
Midterm Examination	Beginning of November
Module 5	Middle of November
Module 6	End of November
Module 7	Middle of December
Module 8	Middle of January
Final Examination	End of January

Chart B: Semester 2

If you want to start the course in February and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Module 1	Middle of February
Module 2	End of February
Module 3	Beginning of March
Module 4	Middle of March
Midterm Examination	End of March
Module 5	Beginning of April
Module 6	Middle of April
Module 7	End of April
Module 8	Beginning of May
Final Examination	Middle of May

Chart C: Full School Year (Not Semestered)

If you want to start the course in September and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Module 1	End of September
Module 2	End of October
Module 3	End of November
Module 4	End of December
Midterm Examination	Middle of January
Module 5	Middle of February
Module 6	Middle of March
Module 7	Beginning of April
Module 8	Beginning of May
Final Examination	Middle of May

Timelines

Do not wait until the last minute to complete your work, since your tutor/marker may not be available to mark it immediately. It may take a few weeks for your tutor/marker to assess your work and return it to you or your school.

If you need this course to graduate this school year, all coursework must be received by the Distance Learning Unit on or before the first Friday in May, and all examinations must be received by the Distance Learning Unit on or before the last Friday in May. Any coursework or examinations received after these deadlines may not be processed in time for a June graduation. Assignments or examinations submitted after these recommended deadlines will be processed and marked as they are received.

When and How Will You Submit Completed Assignments?

When to Submit Assignments

While working on this course, you will submit completed assignments to the Distance Learning Unit eight times. The following chart shows you exactly what assignments you will be submitting at the end of each module.

Submission of Assignments

Submission	Assignments You Will Submit
1	**Module 1: Gross Pay, Time Cards, and Percents** Module 1 Cover Sheet Module 1 Cover Assignment: Yearning for Earnings Assignment 1.1: Percents Assignment 1.2: Fractions and Percent Rate of Change Assignment 1.3: Gross Income Assignment 1.4: Overtime Pay Assignment 1.5: Time Cards and Late Penalties Assignment 1.6: Comparisons Assignment 1.7: Job Exploration Project
2	**Module 2: Net Pay** Module 2 Cover Sheet Module 2 Cover Assignment: Pay Yourself First; The Sales Representative Assignment 2.1: Common Deductions Assignment 2.2: CPP Assignment 2.3: EI Assignment 2.4: Income Tax Assignment 2.5: Net Pay Assignment 2.6: Comparisons and Spreadsheets Assignment 2.7: Formulas and Errors

continued

Submission of Assignments (continued)	
Submission	Assignments You Will Submit
3	**Module 3: Measurement** Module 3 Cover Sheet Module 3 Cover Assignment: Following and Writing Instructions Assignment 3.1: Powers of Ten Assignment 3.2: Metric System Assignment 3.3: Imperial System Assignment 3.4: Reading Rulers Assignment 3.5: Conversions Assignment 3.6: Calipers Assignment 3.7: Micrometers Assignment 3.8: Formulas Assignment 3.9: Problems
4	**Module 4: Geometry** Module 4 Cover Sheet Module 4 Cover Assignment: Tower of Hanoi Assignment 4.1: Multiplication Skills Assignment 4.2: Area of Shapes Assignment 4.3: Scale Drawings Assignment 4.4: Area of Odd Shapes Assignment 4.5: Conversions Assignment 4.6: Problems
5	**Module 5: Working with Angles** Module 5 Cover Sheet Module 5 Cover Assignment: Problem Analysis; Analysis of Games and Numbers Assignment 5.1: Identify Angles Assignment 5.2: Estimate Angles Assignment 5.3: Copy and Bisect Angles Assignment 5.4: Lines and Angles Assignment 5.5: Parallel Lines and Angles Assignment 5.6: Perpendicular or Parallel
6	**Module 6: Consumer Decisions** Module 6 Cover Sheet Module 6 Cover Assignment: Pizza Party Assignment 6.1: Unit Pricing Assignment 6.2: Unit Prices Project Assignment 6.3: Percents Assignment 6.4: Percent Increase or Decrease Assignment 6.5: Sales Promotion Assignment 6.6: Currency Exchange

continued

Submission of Assignments (continued)	
Submission	Assignments You Will Submit
7	**Module 7: Trigonometry** Module 7 Cover Sheet Module 7 Cover Assignment: Drawing Similar Figures Assignment 7.1: Ratio and Proportion Assignment 7.2: Similarity Assignment 7.3: Pythagoras and Right Triangles Assignment 7.4: Tangent Ratio Assignment 7.5: Sine Ratio Assignment 7.6: Cosine Ratio, Sine Ratio, and Tangent Ratio Assignment 7.7: Problems
8	**Module 8: Transformations** Module 8 Cover Sheet Module 8 Cover Assignment: Nested Shopping Carts Assignment 8.1: Translations Assignment 8.2: Rotations Assignment 8.3: Reflections Assignment 8.4: Combinations Assignment 8.5: Dilations Assignment 8.6: Applications

How to Submit Assignments

In this course, you have the choice of submitting your assignments either by mail or electronically.

- **Mail:** Each time you **mail** something, you must include the print version of the applicable Cover Sheet (found at the end of this Introduction). Complete the information at the top of each Cover Sheet before submitting it along with your assignments.
- **Electronic submission:** You do not need to include a cover sheet when submitting assignments electronically.

Submitting Your Assignments by Mail

If you choose to mail your completed assignments, please photocopy/scan all the materials first so that you will have a copy of your work in case your package goes missing. You will need to place the applicable module Cover Sheet and assignment(s) in an envelope, and address it to

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Your tutor/marker will mark your work and return it to you by mail.

Submitting Your Assignments Electronically

Assignment submission options vary by course. Sometimes assignments can be submitted electronically and sometimes they must be submitted by mail. Specific instructions on how to submit assignments were sent to you with this course. In addition, this information is available in the learning management system (LMS).

If you are submitting assignments electronically, make sure you have saved copies of them before you send them. That way, you can refer to your assignments when you discuss them with your tutor/marker. Also, if the original hand-in assignments are lost, you are able to resubmit them.

Your tutor/marker will mark your work and return it to you electronically.

The Distance Learning Unit does not provide technical support for hardware-related issues. If troubleshooting is required, consult a professional computer technician.

What Are the Guide Graphics For?

Guide graphics are used throughout this course to identify and guide you in specific tasks. Each graphic has a specific purpose, as described below.

Lesson Introduction: The introduction sets the stage for the lesson. It may draw upon prior knowledge or briefly describe the organization of the lesson. It also lists the learning outcomes for the lesson. Learning outcomes describe what you will learn.

Learning Partner: Ask your learning partner to help you with this task.

Learning Activity: Complete a learning activity. This will help you to review or practise what you have learned and to prepare for an assignment or an examination. You will not submit learning activities to the Distance Learning Unit. Instead, you will compare your responses to those provided in the Learning Activity Answer Key found at the end of the applicable module.

Assignment: Complete an assignment. You will submit your completed assignments to the Distance Learning Unit for assessment at the end of a given module.

Mail or Electronic Submission: Mail or electronically submit your completed assignments to the Distance Learning Unit for assessment at this time.

Phone or Email: Telephone or email your tutor/marker.

Resource Sheet: Indicates material that may be valuable to include on your resource sheet.

Examination: Write your midterm or final examination at this time.

Note: Take note of and remember this important information or reminder.

Remember: If you have questions or need help at any point during this course, contact your tutor/marker or ask your learning partner for help.

Good luck with the course!

Math Goals

In Module 1, the first learning activity involves you having a conversation with your tutor/marker. Having this conversation with your tutor/marker has two important purposes. First, it introduces you to a very valuable resource—your tutor/marker. He or she is available for you to answer questions, explain concepts, and guide you through this course. You can discuss your math learning and progress. Feel free to contact your tutor/marker by phone or email at anytime during this course.

The second important purpose of this assignment is to get you thinking about your math goals. You may have a future career in mind and this course is getting you one step closer to it by completing a prerequisite for a future required course. There may be specific skills or topics you are interested in learning about and they are covered in this course.

If you are unsure of your math goals or why they are important consider that:

- goals give you a sense of direction and purpose in taking this course
- goals help motivate you to learn and do your best, even when it's tough
- when you accomplish your goals, there is a great sense of achievement and success

Good goals need to be realistic, specific, and reflect what is important to you. They should give you direction and take you further down the path from where you have been to where you want to go.

Goals can be long term or short term, but they are the pathway that takes **you** from where you were/are, closer to where you want to go.

Getting Started

Take some time right now to skim through the course material, locate your Cover Sheets, and familiarize yourself with how the course is organized. Get ready to learn!

Grade 10 Essential Mathematics (20S)

Module 1: Gross Pay, Time Cards, and Percents
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 1 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 1: Yearning for Earnings	_____ /12	_____ /12
❑ Assignment 1.1: Percents	_____ /18	_____ /18
❑ Assignment 1.2: Fractions and Percent Rate of Change	_____ /20	_____ /20
❑ Assignment 1.3: Gross Income	_____ /16	_____ /16
❑ Assignment 1.4: Overtime Pay	_____ /17	_____ /17
❑ Assignment 1.5: Time Cards and Late Penalties	_____ /19	_____ /19

continued

Grade 10 Essential Mathematics (20S)

Module 1: Gross Pay, Time Cards, and Percents
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 1 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 1.6: Comparisons	_____ /14	_____ /14
❑ Assignment 1.7: Job Exploration Project	_____ /9	_____ /9
	Total: ____ /125	**Total: ____ /125**

For Tutor/Marker Use
Remarks:

Module 1 Cover Assignment
Rubric for Yearning for Earnings

Score Level	Mathematical Knowledge (Did I know how to do the question?)	Process Knowledge (What strategies and methods did I use to get the answer?)	Communication (Did I communicate my answer clearly?)
4	I got the right answers. I used math terms correctly to show I understand how math works. I made no errors in my calculations.	I found all the important parts of the problems, and I knew how to put them together to solve the problem. I showed all the steps to solve the problems. I showed calculations if I used them in my answers.	I labelled the answers correctly, including a $ sign. I wrote a clear, concise explanation of how I was thinking
3	I made minor errors in my calculations. I used most of the math vocabulary correctly.	I found most of the important parts of the problem. I showed most of the steps I used to solve the problem.	I wrote most of my explanation in a concise manner.
2	I know how to do parts of the problem, but I made major errors in the calculations and got a wrong answer. I gave only part of the answer.	I found some of the important parts of the problem. I showed some of the steps.	I wrote something about my thinking, but it was not the complete answer.
1	I tried to do the problems but I don't understand them.	I couldn't find the important parts of the problem. I showed very few steps that I used to solve the problem. I included unnecessary information.	I wrote something that didn't explain my thinking. I wrote an answer that was not clear.
0	I didn't try to answer the problems.	I didn't show any steps.	I didn't explain anything.
Mark	Math = ____ / *(4 marks)*	Process = ____ / *(4 marks)*	Communication = ____ /*(4 marks)*
Total Mark	Math + Process + Communication = _____		Total Possible: 4 + 4 + 4 = 12

Grade 10 Essential Mathematics (20S)

Module 2: Net Pay
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 2 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 2: Pay Yourself First / The Sales Representative	_____ /12	_____ /12
❑ Assignment 2.1: Common Deductions	_____ /15	_____ /15
❑ Assignment 2.2: CPP	_____ /16	_____ /16
❑ Assignment 2.3: EI	_____ /16	_____ /16
❑ Assignment 2.4: Income Tax	_____ /21	_____ /21
❑ Assignment 2.5: Net Pay	_____ /18	_____ /18

continued

Grade 10 Essential Mathematics (20S)

Module 2: Net Pay
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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555 Main Street
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Mailing Address

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500–555 Main Street
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Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

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City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 2 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 2.6: Comparisons and Spreadsheets	_____ /19	_____ /19
❑ Assignment 2.7: Formulas and Errors	_____ /22	_____ /22
	Total: ____ /139	**Total: ____ /139**
For Tutor/Marker Use		
Remarks:		

Module 2 Cover Assignment Rubric
Pay Yourself First / The Sales Representative

Score Level	Mathematical Knowledge (Did I know how to do the question?)	Process Knowledge (What strategies and methods did I use to get the answer?)	Communication (Did I communicate my answer clearly?)
4	I got the right answers. I used math terms correctly to show I understand how math works. I made no errors in my calculations.	I found all the important parts of the problems, and I knew how to put them together to solve the problem. I showed all the steps to solve the problems. I showed calculations if I used them in my answers.	I labelled the answers correctly. In the drawing I used, I could write an explanation for all of it.
3	I made minor errors in my calculations. I used most of the math vocabulary correctly.	I found most of the important parts of the problem. I showed most of the steps I used to solve the problem.	In the drawing I used, I could write an explanation for most of it.
2	I know how to do parts of the problem, but I made major errors in the calculations and got a wrong answer. I gave only part of the answer.	I found some of the important parts of the problem. I showed some of the steps, but my solution was not clear.	In the drawing I used, I could write an explanation for some of it.
1	I tried to do the problems but I don't understand them.	I couldn't find the important parts of the problem. I showed solutions that are not reasonable. I showed very few steps that I used to solve the problem. I included unnecessary information.	I wrote something that didn't go with my answer. I wrote an answer that was not clear. The drawing I used didn't go with my answer.
0	I didn't try to answer the problems.	I didn't show a method.	I didn't explain anything, either by writing about it or by drawing a picture.
Mark	Math = ____ / *(4 marks)*	Process = ____ / *(4 marks)*	Communication = ____ /*(4 marks)*
Total Mark	Math + Process + Communication = _____		Total Possible: 4 + 4 + 4 = 12

Grade 10 Essential Mathematics (20S)

Module 3: Measurement
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 3 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 3: Following and Writing Instructions	_____ /12	_____ /12
❑ Assignment 3.1: Powers of Ten	_____ /18	_____ /18
❑ Assignment 3.2: Metric System	_____ /16	_____ /16
❑ Assignment 3.3: Imperial System	_____ /16	_____ /16
❑ Assignment 3.4: Reading Rulers	_____ /17	_____ /17
❑ Assignment 3.5: Conversions	_____ /15	_____ /15
❑ Assignment 3.6: Calipers	_____ /12	_____ /12

continued

Grade 10 Essential Mathematics (20S)

Module 3: Measurement
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 3 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 3.7: Micrometers	_____ /16	_____ /16
❑ Assignment 3.8: Formulas	_____ /12	_____ /12
❑ Assignment 3.9: Problems	_____ /15	_____ /15
	Total: ____ /149	**Total: ____ /149**

For Tutor/Marker Use
Remarks:

Module 3 Cover Assignment Rubric
Following and Writing Instructions

Score Level	Following Instructions (Did I know how to do the question?)	Process Knowledge (What strategies and methods did I use to get the answer?)	Writing Instructions (Did I communicate my answer clearly?)
4	I got the right answers.	I found all the important parts of the problems. I knew how to read and follow the instructions to solve the problems.	My third set of instructions was clear and concise. The third diagram matched the original picture. I included all three sets of instructions and pictures.
3	I made minor errors in my answers.	I found most of the important parts of the problem. I knew how to read and follow most of the instructions to solve the problems.	My third set of instructions was mostly clear and concise. The third diagram mostly matched the original picture. I included all three sets of instructions and pictures.
2	I know how to do parts of the problems, but I made major errors in following the instructions and got a wrong answer. I gave only part of the answer.	I found some of the important parts of the problems. I knew how to read and follow some of the instructions to solve the problems.	My third set of instructions was not clear and concise. The third diagram did not match the original picture. I included all three sets of instructions and pictures.
1	I tried to do the problems but I didn't understand them.	I couldn't find many of the important parts of the problems. Mostly, I didn't know how to read and follow the instructions to solve the problems.	I wrote only one set of instructions. The diagram did not match the original picture. I included only one set of instructions and only one picture.
0	I didn't try to answer the problems.	I didn't show a method for solving the problems.	I didn't write any sets of instructions.
Mark	Following Instructions = ____ / *(4 marks)*	Process = ____ / *(4 marks)*	Writing Instructions = ____ / *(4 marks)*
Total Mark	Following Instructions + Process + Writing Instructions = _____		Total Possible: 4 + 4 + 4 = 12

Grade 10 Essential Mathematics (20S)

Module 4: Geometry
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 4 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 4: Tower of Hanoi	_____ /12	_____ /12
❑ Assignment 4.1: Multiplication Skills	_____ /17	_____ /17
❑ Assignment 4.2: Area of Shapes	_____ /17	_____ /17
❑ Assignment 4.3: Scale Drawings	_____ /19	_____ /19
❑ Assignment 4.4: Area of Odd Shapes	_____ /15	_____ /15
❑ Assignment 4.5: Conversions	_____ /17	_____ /17

continued

Grade 10 Essential Mathematics (20S)

Module 4: Geometry
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 4 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 4.6: Problems	_____ /20	_____ /20
	Total: ____ /117	**Total: ____ /117**
For Tutor/Marker Use		
Remarks:		

Grade 10 Essential Mathematics (20S)

Module 5: Working with Angles
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 5 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 5: Problem Analysis / Analysis of Games and Numbers	_____ /21	_____ /21
❑ Assignment 5.1: Identify Angles	_____ /20	_____ /20
❑ Assignment 5.2: Estimate Angles	_____ /20	_____ /20
❑ Assignment 5.3: Copy and Bisect Angles	_____ /20	_____ /20
❑ Assignment 5.4: Lines and Angles	_____ /20	_____ /20
❑ Assignment 5.5: Parallel Lines and Angles	_____ /24	_____ /24

continued

Grade 10 Essential Mathematics (20S)

Module 5: Working with Angles
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 5 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 5.6: Perpendicular or Parallel	_____ /11	_____ /11
	Total: ____ /136	**Total: ____ /136**
For Tutor/Marker Use		
Remarks:		

Grade 10 Essential Mathematics (20S)

Module 6: Consumer Decisions
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 6 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 6: Pizza Party	_____ /12	_____ /12
❑ Assignment 6.1: Unit Pricing	_____ /19	_____ /19
❑ Assignment 6.2: Unit Prices Project	_____ /20	_____ /20
❑ Assignment 6.3: Percents	_____ /19	_____ /19
❑ Assignment 6.4: Percent Increase or Decrease	_____ /21	_____ /21
❑ Assignment 6.5: Sales Promotion	_____ /15	_____ /15

continued

Grade 10 Essential Mathematics (20S)

Module 6: Consumer Decisions
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 6 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 6.6: Currency Exchange	_____ /18	_____ /18
	Total: ___ /124	**Total: ___ /124**

For Tutor/Marker Use
Remarks:

Grade 10 Essential Mathematics (20S)

Module 7: Trigonometry
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 7 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 7: Drawing Similar Figures	_____ /12	_____ /12
❑ Assignment 7.1: Ratio and Proportion	_____ /19	_____ /19
❑ Assignment 7.2: Similarity	_____ /23	_____ /23
❑ Assignment 7.3: Pythagoras and Right Triangles	_____ /21	_____ /21
❑ Assignment 7.4: Tangent Ratio	_____ /21	_____ /21
❑ Assignment 7.5: Sine Ratio	_____ /21	_____ /21

continued

Grade 10 Essential Mathematics (20S)

Module 7: Trigonometry
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 7 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 7.6: Cosine Ratio, Sine Ratio, and Tangent Ratio	_____ /24	_____ /24
❑ Assignment 7.7: Problems	_____ /20	_____ /20
	Total: ____ /161	**Total: ____ /161**

For Tutor/Marker Use
Remarks:

Grade 10 Essential Mathematics (20S)

Module 8: Transformations
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 8 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 8: Nested Shopping Carts	_____ /12	_____ /12
❑ Assignment 8.1: Translations	_____ /22	_____ /22
❑ Assignment 8.2: Rotations	_____ /20	_____ /20
❑ Assignment 8.3: Reflections	_____ /16	_____ /16
❑ Assignment 8.4: Combinations	_____ /19	_____ /19
❑ Assignment 8.5: Dilations	_____ /19	_____ /19

continued

Grade 10 Essential Mathematics (20S)

Module 8: Transformations
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 8 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Assignment 8.6: Applications	_____ /8	_____ /8
	Total: ____ /116	**Total: ____ /116**
For Tutor/Marker Use		
Remarks:		

Module 8 Assignment 8.6
Rubric for Applications Assignment

Score Level	Tessellation	Translation	Reflection	Rotation
2	A tessellation is created, which includes no gaps or overlaps. OR Two pieces of tessellation art are provided with appropriate referencing	A detailed description of the transformation is provided, which includes appropriate mathematical terms	A detailed description of the reflection is provided, which includes appropriate mathematical terms.	A detailed description of the rotation is provided, which includes appropriate mathematical terms.
1	A tessellation is created, which includes some gaps and/or overlaps. OR One piece of tessellation art is provided with appropriate referencing. OR Two pieces of tessellation art are provided without appropriate referencing.	A mathematically unclear or incomplete description of the translation is included.	A mathematically unclear or incomplete description of the reflection is provided.	A mathematically unclear or incomplete description of the rotation is provided.
0	A tessellation is created, which includes many gaps and/or overlaps. OR No pieces of tessellation art are provided. OR One piece of tessellation art is provided without appropriate referencing.	A translation is not included. OR A translation is not described.	A reflection is not included. OR A reflection is not described.	A rotation is not included. OR A rotation is not described.
Mark	Tessellation = ____ / *(2 marks)*	Translation = ____ / *(2 marks)*	Reflection = ____ / *(2 marks)*	Rotation = ____ / *(2 marks)*
Total Mark	Tessellation + Translation + Reflection + Rotation = _____			Total Possible: 2 + 2 + 2 + 2 = 8

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Grade 10 Essential Mathematics (20S)

Module 1
Gross Pay, Time Cards, and Percents

Module 1:
Gross Pay, Time Cards, and Percents

Introduction

In this module, you will learn to calculate your gross earnings for when you start a job. In Lesson 1 and 2 you will review and use your skills with percentages and fractions in solving problems. There are many methods of being paid for your work, many of which will be examined in Lessons 3 and Lesson 4.

Many jobs require workers to "punch" a time card, or more commonly scan the bar code on their name tags. The bar code machine records exactly when the employees actually start work each day, and the length of time they are on the job. You will use time cards to calculate gross pay in Lesson 5. Lesson 6 compares different pay options, and assesses why some might be better than others, depending upon a person's circumstances.

Lesson 7 presents some unique problems related to gross pay. You will be asked to do a small project researching three jobs in your area.

Assignments in Module 1

When you have completed the assignments for Module 1, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Yearning for Earnings
1	Assignment 1.1	Percents
2	Assignment 1.2	Fractions and Percent Rate of Change
3	Assignment 1.3	Gross Income
4	Assignment 1.4	Overtime Pay
5	Assignment 1.5	Time Cards and Late Penalties
6	Assignment 1.6	Comparisons
7	Assignment 1.7	Job Exploration Project

Resource Sheet

When you write your midterm examination, you will be allowed to bring a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Midterm Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 1. You might use your Module 1 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 1, 2, 3, and 4, to prepare your Midterm Examination Resource Sheet. Remember, the midterm examination is based only on the first four modules of the course.

Resource Sheet for Module 1

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Midterm Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Module 1 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work in the spaces provided. The cover assignment can be done at any time while you are completing Module 1. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on the rubric shown on the next page. You can see that the cover assignment is worth a total of 12 marks. Those marks are allotted according to how well you are able to satisfy the requirements explained in the rubric. If you have any questions about the scoring, be sure to contact your tutor/marker for clarification.

Module 1 Cover Assignment
Rubric for Yearning for Earnings

Score Level	Mathematical Knowledge (Did I know how to do the question?)	Process Knowledge (What strategies and methods did I use to get the answer?)	Communication (Did I communicate my answer clearly?)
4	I got the right answers. I used math terms correctly to show I understand how math works. I made no errors in my calculations.	I found all the important parts of the problems, and I knew how to put them together to solve the problem. I showed all the steps to solve the problems. I showed calculations if I used them in my answers.	I labelled the answers correctly, including a $ sign. I wrote a clear, concise explanation of how I was thinking
3	I made minor errors in my calculations. I used most of the math vocabulary correctly.	I found most of the important parts of the problem. I showed most of the steps I used to solve the problem.	I wrote most of my explanation in a concise manner.
2	I know how to do parts of the problem, but I made major errors in the calculations and got a wrong answer. I gave only part of the answer.	I found some of the important parts of the problem. I showed some of the steps.	I wrote something about my thinking, but it was not the complete answer.
1	I tried to do the problems but I don't understand them.	I couldn't find the important parts of the problem. I showed very few steps that I used to solve the problem. I included unnecessary information.	I wrote something that didn't explain my thinking. I wrote an answer that was not clear.
0	I didn't try to answer the problems.	I didn't show any steps.	I didn't explain anything.
Mark	Math = ____ / *(4 marks)*	Process = ____ / *(4 marks)*	Communication = ____ / *(4 marks)*
Total Mark	Math + Process + Communication = _____		Total Possible: 4 + 4 + 4 = 12

Module 1 Cover Assignment

Yearning for Earnings

Total: 12 marks

Suppose you are working and your employer offers you the following two wage options for the month of September.

Option A: $1000 a day

Option B: 1¢ on September 1, and each day after that double the amount of the previous day.

You are to be paid for each day of September.

1. Find your wages for the month of September with Option A.

2. Without doing the actual calculations, estimate your wages for September with Option B.

3. Do the calculations and find your wages for the month with Option B. You can find your wages by completing the table found on the following page for the entire month of September. Fill in the middle column with your daily income and the last column with the total income to date. The table has been completed for the first four days in September.

continued

Module 1 Cover Assignment: Yearnings for Earnings (continued)

Date	Daily Income	Total Income To Date
September 1	$0.01	$0.01
September 2	$0.02	$0.03
September 3	$0.04	$0.07
September 4	$0.08	$0.15
September 5		
September 6		
September 7		
September 8		
September 9		
September 10		
September 11		
September 12		
September 13		
September 14		
September 15		
September 16		
September 17		
September 18		
September 19		
September 20		
September 21		
September 22		
September 23		
September 24		
September 25		
September 26		
September 27		
September 28		
September 29		
September 30		

continued

Module 1 Cover Assignment: Yearnings for Earnings (continued)

4. How does the actual amount of income compare to your estimate? Write an explanation of why this might be so.

5. What pattern do you see in the total daily amounts?

Notes

Learning Activity 1.1

This learning activity is the only one that does not include a BrainPower section, although it does have two parts. Be sure to complete both before starting the lesson.

Part A: Your Tutor/Marker

Fill in the following blanks using information provided with your course:

My tutor/marker's name is _____

I can phone my tutor/marker at 1-8--_____

My tutor/marker's email is _____

Be ready to discuss the following topics and the reasons for your answers with your tutor/marker during your phone conversation. If you'd like, make some notes below before you call in order to help you feel prepared. Feel free to add other questions or comments that you may have as well.

1. I am taking this course by distance education because…

2. What I like about math and can do mathematically is… (favourite topic, skill, where you use math, etc.).

continued

Learning Activity 1.1 (continued)

3. What I dislike about math or have difficulty doing is …

4. Previous math experiences that influence the way I feel about math are…

5. The next math course I would like to take is…

6. What I am hoping this course will help me accomplish and learn for the future is…

continued

Learning Activity 1.1 (continued)

7. I will organize things to help me succeed in this course by…

_____

_____

_____

_____

During your phone conversation, jot down a sentence or two about what you and your tutor/marker talk about, in the spaces above. For example, if you are taking this course because it doesn't fit into your schedule at school or because you travel a lot with your basketball team and this is more convenient, state that in the space below question 1.

Part B: Your Math Pathway

Use the answers to the questions from the conversation with your tutor/marker as a starting point and fill in the following diagram. In the Math History box, jot down point form notes about your prior experience and knowledge about math (Questions 2, 3, and 4). In the Math Destination box, jot down what completing this course will help you accomplish in the future (Questions 5 and 6).

In the second column (Pathway), write down what you will need to do to move down the pathway from your History to your Destination.

Math History	Pathway	Math Destination

For example, if your destination includes needing a 75% in this course so that you can feel confident going into Grade 11 Essential Mathematics, or you need to learn how to make smart consumer decisions, what will help you accomplish this? It may mean figuring out how you best learn and study math. It may mean setting up a schedule so you complete the assignments on time. You may need to find your calculator manual and figure out how to use your calculator, set up regular appointments with your learning partner, research a topic on the Internet, or read a textbook about a certain math concept or skill. Your pathway is unique to you.

As you move through this course and work on achieving your goals, self-assessment becomes important. It is the way for you to determine if you are getting closer to your destination, and if the steps along your pathway are taking you in the right direction. You will need to periodically ask yourself: Am I doing my assignments? Are my note-taking skills improving? How often have I contacted my tutor/marker or worked with my learning partner? Have I found useful homework websites? Is my schedule working? What do I need to change or adjust so I can get to my destination?

Several times during this course, you will go through this cycle of looking at where you have been, where you want to go, and where you currently are. At anytime you may want to revise your goals or set new ones, as you evaluate your own progress and learning.

- Look back/history—reflect on what you know, how far you have come
- Look around/pathway—assess if you are achieving your goals, determine if new learning or understanding has occurred, and check your progress
- Look forward/destination—determine what you want to know, set goals

Each time you go through these steps you will become better at mathematics!

It is important that you keep the diagram of your math pathway handy, as you will revisit it at other points in this course.

Lesson Focus

In this lesson, you will

- ❑ convert percents to decimals
- ❑ convert decimals to percents
- ❑ demonstrate an understanding of calculations using percents

Lesson Introduction

You will use your skills with percents this module. You must have a solid understanding of what percents mean, and how to use them. In this lesson, you will look at how percents are determined. You will also look at how short cuts can be used to simplify converting a percent to a decimal. Employing your skills in solving problems using percents demonstrates your solid understanding.

Decimals and Percents

Percents are everywhere—at school, in the mall, even in the sports pages. You will encounter percents in many situations, and you need to understand how they work.

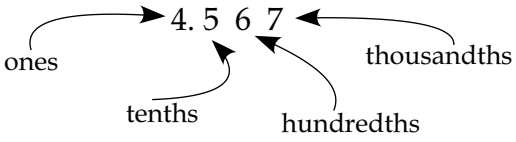
The term "cent" means 100. There are 100 cents in a loonie. There are 100 centimetres in a metre. The term "per" indicates "out of." So, a percent is a value out of 100.

If you scored 65% on a test, that means you earned 65 marks out of a possible 100. When weather forecasters say "there is 100% chance of snow," that indicates there will definitely be snow!

Converting Percents to Decimals

All operations with percents require that you first convert them to decimals.

Recall the place values for decimals.



You may want to include this on your Resource Sheet for this module.

Example 1

Change 42% to a decimal.

Solution:

Since percent means out of 100, then 42% means 42 out of 100. Written as a fraction, 42% is $\frac{42}{100}$ or 42 hundredths. When the fraction is changed to a decimal, the hundredths means there are two decimal places to the right of the decimal point. Thus, 42% is written in decimal form as 0.42.

Example 2

Change 37.5% to a decimal.

Solution:

Given a value of 37.5%, you would write the fraction form of the percent as $\frac{37.5}{100}$. Since there is a decimal in the numerator of the fraction, it must be removed by multiplying the top and the bottom of the fraction by 10.

$$37.5\% = \frac{37.5}{100} = \frac{37.5 \times 10}{100 \times 10} = \frac{375}{1000} = 0.375$$

Thus, 37.5% can be changed to 375 thousandths and is written as 0.375 in decimal form.

Example 3

Change 37.5% to a decimal by using your calculator.

Solution:

You could also use your calculator to do the division for the fraction $\frac{37.5}{100}$.

Just type

37.5	/	100	Enter

You will see the answer of 0.375.

Example 4

Find 26% of 500.

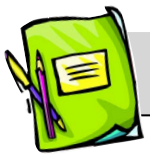
Solution:

To find a percentage of a value, you convert the percent to a decimal, and then multiply.

26% would be written as 0.26 in the equation.

Thus, 26% of 500 would be written as 0.26×500.

Using your calculator, the answer is $0.26 \times 500 = 130$.



Learning Activity 1.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in a few minutes without the use of a calculator or pencil and paper. Especially for the first few times you do these questions, your learning partner can help you figure out strategies to solve these questions in your head.

1. You are going for a walk. You walk north for 5 blocks, turn around, and walk south for 16 blocks. How many blocks are you from where you started? State if you are north or south of where you started.
2. If Evan eats $\frac{3}{5}$ of a pizza and Nick eats $\frac{4}{5}$ of a pizza, how many pizzas do they have to order so that both can eat as much as they like?
3. Write $\frac{6}{2}$ in simplest terms.
4. Knowing 50% of 680 is 340, evaluate 25% of 680.
5. If you buy a shirt for $8 and jeans for $32, how much do you spend all together (before taxes)?

continued

Learning Activity 1.2 (continued)

Part B: Converting Percents to Decimals

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert these percents to decimals.
   a) 70% _____
   b) $12\frac{1}{2}\%$ _____
   c) 6% _____
   d) 125% _____
2. Your friend writes 4% as 0.4 in decimal form. What did he do wrong, and how should 4% be written as a decimal?
3. Find 40% of 600.

Converting Decimals to Percents

The process of converting decimals to percents is the exact reverse of the previous activity.

If you are given a number in decimal form, you can rewrite the decimal as a fraction using the question: "How many hundredths? You don't ask how many tenths or how many thousandths because percent means out of one hundred, not out of ten, or out of a thousand.

Example 1

Change into percents.

a) 0.37

b) 0.8

c) 0.09

Solution:

a) If you were to read 0.37 out loud using place values, and since 0.37 has two decimal places you would say 37 hundredths. As a fraction, this would be $\frac{37}{100}$, which is 37%. Thus, 0.37 = 37%.

b) 0.8 can be rewritten with two decimal places as 0.80. This decimal number is read as 80 hundredths or 80 out of one hundred. In fractional form, 0.80 is $\frac{80}{100}$ or 80%.

c) 0.09 reads as 9 hundredths or $\frac{9}{100}$ which is 9%.

Some values get a little more complicated, but you still read them in terms of "how many hundredths."

Example 2

Change into percents.

a) 1.65

b) 0.047

Solution:

a) 1.65 is read as 1 and $\frac{65}{100}$. Here you have a mixed number that needs to be changed into an improper fraction. Given that $1 = \frac{100}{100}$ and to that you add $\frac{65}{100}$, you get the improper fraction of $\frac{165}{100}$. Now you have a fraction out of one hundred and the percent is 165%. You could also write

$$1.65 = 1 + 0.65 = \frac{100}{100} + \frac{65}{100} = \frac{165}{100} = 165\%$$

b) 0.047 has three decimal places and is read as 47 thousandths or $\frac{47}{1000}$.

To change it to a fraction out of 100 you would need to divide the top and bottom of the fraction by 10.

$$0.047 = \frac{47}{1000} = \frac{47 \div 10}{1000 \div 10} = \frac{4.7}{100}$$

Now you have a fraction out of one hundred, so the percent value is 4.7%.

Learning Activity 1.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in a few minutes without the use of a calculator or pencil and paper. Especially for the first few times you do these questions, your learning partner can help you figure out strategies to solve these questions in your head.

1. Rank the numbers highest to lowest: 0.5, 0.05, 0.3, 0.09, 0.25.
2. Evaluate the following: $2 - 3 + 6 \times 2 - 5 \times 4$.
3. Solve for i: $4i + 3 = 15$.
4. Is an angle that measures 140° acute, right, obtuse, straight, or reflex?
5. Write the next two numbers in the pattern: 1, 2, 4, 8, ___ , ___ .

Part B: Converting Decimals to Percents

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert these decimals to percents.
   a) 0.34 _____
   b) 0.08 _____
   c) 0.065 _____
   d) 2.45 _____
2. Your friend wrote 0.072 as 72%. Identify the mistake she made, and correct it.
3. Find $8\frac{1}{2}\%$ of 900.

Shortcuts

As you were doing these percent questions, you might have noticed a pattern each time you did a conversion.

Example 1

Change each to a percent and comment on the pattern you notice regarding the placement of the decimal point.

a) 0.79

b) 0.045

Solution:

Notice the pattern when you change a decimal to a percent.

a) $0.79 = \frac{79}{100} = 79\%$

b) $0.045 = \frac{45}{1000} = \frac{45 \div 10}{1000 \div 10} = \frac{4.5}{100} = 4.5\%$

The decimal point has been moved two places to the right and a percent sign has been attached.

The shortcut when converting a decimal to a percent follows this pattern. The answer always shows the decimal point moved 2 place values to the right, and a percent sign included at the end.

It may be helpful to include this shortcut on your resource sheet.

Example 2

Change each to a decimal and comment on the pattern you notice regarding the placement of the decimal point.

a) 64%

b) 3.25%

Solution:

Notice the pattern when you change a percent to a decimal.

a) $64\% = \frac{64}{100} = 0.64$

b) $3.25\% = \frac{3.25}{100} = \frac{3.25 \times 100}{100 \times 100} = \frac{325}{10\ 000} = 0.0325$

The decimal point has been moved two places to the left and there is no percent sign.

The shortcut, when converting a percent to a decimal, follows this pattern. The answer always shows the decimal point moved 2 place values to the left and the percent sign is removed.

Once you are comfortable with decimal and percent conversions, you might try using the shortcuts. But be careful, as sometimes using shortcuts can lead to mistakes.

This shortcut would also be helpful to have on your resource sheet.

Students often do not understand the meaning of a percent as being out of 100. For example, 2% is not written as 0.2. Rather, to avoid errors, 2% should first be read as 2 hundredths or $\frac{2}{100}$, and then written as 0.02.

Percents and Sales Tax

In 2010, Manitobans had a Provincial Sales Tax (PST) of 7%, and the federal government Goods and Services Tax (GST) of 5%. These taxes are added to the price of most purchases and services.

Include the two taxes and their percent rates on your resource sheet.

Example 1

You go to a store to purchase some jeans that cost $29. Find the total price including the taxes.

Solution:

GST = 5% of $29.00 = 0.05 × $29 = $1.45

PST = 7% of $29.00 = 0.07 × $29 = $2.03

The taxes are added to the purchase price. The total cost of the jeans includes the taxes.

Total price = $29 + $1.45 + $2.03 = $32.48

Why Do You Pay Tax?

The provincial government uses income or revenue from the PST to fund hospitals, schools, roads, jails, tourism, and many other public works. The federal government uses income or revenue from the GST to fund Canada's military, the RCMP, immigration, prisons, passport distribution, and many other projects.

Lesson Summary

In this lesson, you reviewed skills for converting percents to decimals, and converting decimals to percents. You explained each process and looked for patterns and shortcuts. You applied your skills to the problem of finding the percentage of a number, sales tax, and the total price.

You used several mathematical words, including percent, cent, place value, ones, tenths, hundredths, thousandths, decimal, improper fraction, percentage, PST, and GST.

The next lesson involves conversions with fractions and percents along with finding the "percent rate of change."

Notes

Assignment 1.1

Percents

Total Marks = 18

1. Convert the following percents to decimals. *(6 marks)*

   a) 69% _____ b) 34% _____

   c) 8% _____ d) 6.5% _____

   e) 14.25% _____ f) 120% _____

2. Convert the following decimals to percents. *(6 marks)*

   a) 0.55 _____ b) 1.35 _____

   c) 0.04 _____ d) 0.305 _____

   e) 0.027 _____ f) 0.0875 _____

3. Sally answered this conversion of 0.025 to a percent with 25%. What error did she make, and how should it have been answered? *(3 marks)*

4. The price of an MP3 player is $62 plus GST and PST. Using 5% for GST and 7% for PST, find the total cost of the purchase. *(3 marks)*

Notes

Lesson 2: Fractions and Percent Rate of Change

Lesson Focus

In this lesson, you will

- ❑ convert fractions to decimals and to percents
- ❑ find a percent rate of change
- ❑ solve problems using percent rate of change

Lesson Introduction

In this lesson, you will convert fractions to decimals, then to percents. You will also examine how to find the "percent rate of change." You will work through some problems, comparing discounts on products to see which offers the best discount.

Fractions and Decimals

Examples of using fractions in your daily life are plentiful. Investment choices, mortgages, wage increases, and financing of car loans are situations where you are given fractional values. You must be able to work with these fractions to understand the exact monetary value. One of the ways to make sense of a fraction is to convert the fraction to a decimal.

Fractions to Decimals

The easiest way to convert any fraction to a decimal is to remember the rule: "top divided by bottom." Using your calculator, enter the top value of the fraction (numerator), press the "divide" key, and enter the bottom value (denominator), and press enter.

Example 1

Change the fraction below to a decimal, using your calculator.

$\frac{4}{5}$

Solution:

$\frac{4}{5}$ is entered on the calculator as follows.

4	/	5	Enter

You will see the answer of 0.8.

Example 2

Change the fraction below to a decimal, using your calculator.

$\frac{3}{8}$

Solution:

$\frac{3}{8}$ becomes 0.375.

Example 3

Change the fraction below to a decimal, using your calculator.

$\frac{3}{7}$

Solution:

$\frac{3}{7}$ is shown on the calculator as 0.428 571 428. With this fraction, there appears to be a repeated pattern of 428571 in its decimal form. You can shorten the writing of a repeating decimal by using a bar over the repeating part. Thus, $\frac{3}{7}$ can be written as $0.\overline{428571}$. Or you could also just round off to two decimal places to get 0.43. Rounding off is discussed in the next part of the lesson.

For changing special fractions with a denominator of 2, 4, or 8, to a decimal, leave the complete decimal answer. It might help to memorize the following fractions written as decimals.

You may also include these fractions and decimals on your resource sheet.

Half	Quarters	Eighths
$\frac{1}{2} = 0.5$	$\frac{1}{4} = 0.25$	$\frac{1}{8} = 0.125$
	$\frac{3}{4} = 0.75$	$\frac{3}{8} = 0.375$
		$\frac{5}{8} = 0.625$
		$\frac{7}{8} = 0.875$

Rounding Off

Some decimals are terminating, as in Example 1 and 2, above. Others can be repeating and unending, as in Example 3 above.

The steps for rounding off are as follows:

Step 1 Identify the place value required in the answer.

Step 2 Look at the next number to its right. If that number is less than 5, then do not round. Otherwise, round up.

Rounding off to two decimal places is sufficient unless a question requires something different, or unless you have one of the special fractions with a denominator of 8.

Note: When you round off, you are giving an approximate answer. If you require an exact answer, do not round off.

Add these steps to your resource sheet as a reminder for yourself.

Example 1

Write $\frac{3}{7}$ as a decimal rounded to two decimal places.

Solution:

On the calculator, $\frac{3}{7}$ = 0.428 571 428 and then is rounded to two decimal places to get 0.43. Notice that all numbers after the hundredths place are dropped.

Fractions to Percents

When you convert a fraction to a percent, your answer is to be rounded to the nearest tenth or hundredth in the percent answer.

Example 1

Write $\frac{3}{7}$ as a percent rounded to one decimal place.

Solution:

To change $\frac{3}{7}$ to a percent, you need to write its equivalent decimal form as a fraction out of one hundred. Thus, $\frac{3}{7}$ = 0.428 571 428 would be read as $\frac{42.857\ 142\ 8}{100}$, and would be written as 42.857 142 8%. Now, rounded to one decimal place, you get 42.9%.

However, if you use the shortcut method for changing a decimal to a percent, you could simplify the steps. Just move the decimal place two places to the right and then round off the percent answer to one decimal place.

$$\frac{3}{7} = 0.428\ 571\ 428 = 42.857\ 142\ 8\% = 42.9\%$$

Mixed Numbers to Decimals

When given a mixed number with a whole number part and a fraction part, you only need to convert the fraction part to a decimal. The whole number part remains unchanged and tags along.

Example 1

Change $2\frac{3}{5}$ to a decimal.

Solution:

On your calculator, type $\frac{3}{5}$ to get an answer of 0.6, and no rounding is necessary.

You write $2\frac{3}{5} = 2.6$.

Notice that the whole number part remains the same.

Example 2

Convert $85\frac{1}{8}$ to a decimal.

Solution:

Recognizing the special fraction of $\frac{1}{8}$ and remembering that its decimal form is 0.125, you can immediately write the answer as 85.125.

Otherwise, use your calculator to find $\frac{1}{8}$ = 0.125 and an answer of 85.125.

Notice that you leave the complete decimal because you have a special fraction. You do not round off to two decimal places.

Example 3

Convert $7\frac{1}{3}$ to a decimal.

Solution:

$7\frac{1}{3}$ changes to the decimal form 7.333 333 333 or $7.\overline{3}$. When you round the answer off to two decimal places, the answer is 7.33.

Decimals to Fractions

Sometimes you need to convert decimals to fractions. To accomplish this, you say the decimal as a fraction, according to its place value, and then simplify the fraction into its lowest form.

Example 1

Change 0.85 into a decimal.

Solution:

0.85 is read as 85 hundredths and is written as a fraction as $\frac{85}{100}$.

You could simplify the fraction to lowest terms (no common factor in numerator and denominator).

$$0.85 = \frac{85}{100} = \frac{85 \div 5}{100 \div 5} = \frac{17}{20}$$

Example 2

Change 3.625 into a fraction.

Solution:

Method 1

If you recognize the decimal part as belonging to one of the special fractions, $\frac{5}{8}$, you could immediately write your answer as $3\frac{5}{8}$.

Method 2

You could follow the recommended process described in the lesson.

3.625 is read as a fraction as 3 and $\frac{625}{1000}$. Simplify the fraction part to get the final answer.

$$\frac{625 \div 125}{1000 \div 125} = \frac{5}{8}$$

Therefore, the answer is $3\frac{5}{8}$.

Learning Activity 1.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil in just a few minutes.

1. A school volleyball team wants to practice twice per week. They cannot practice at the school on the weekend (Saturday and Sunday), half the team cannot practice on Monday and Wednesday, and the basketball team uses the gym on Friday. Which days can the team practice?
2. An octave in music includes 8 notes. If you were to go up half an octave, how many notes is that?
3. When in Venice, you notice a great store on the other side of the street. Because the roads are water in Venice, you need to walk to the nearest bridge. The nearest bridge is 6 m away from you, and the 'road' is 2 m wide. How far do you have to walk to get to the store?
4. Write the percent as a decimal: 62%.
5. You are working at the stadium, where there is no electronic till. The customer is buying popcorn for $3.80. If the customer gives you a $5.00 bill, how much change will you give them?

continued

Learning Activity 1.4 (continued)

Part B: Converting among Fractions, Decimals, and Percents

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert these fractions to decimals.

   a) $\frac{5}{8}$ b) $\frac{3}{4}$

   c) $3\frac{7}{8}$ d) $15\frac{2}{3}$

2. Convert these fractions to decimals and to percents to fill in the chart.

Fraction	Decimal	Percent
$\frac{2}{5}$		
$1\frac{7}{8}$		
$\frac{27}{100}$		
$5\frac{6}{7}$		

3. Convert these decimals to fractions. Write your answer in lowest terms.
   a) 0.36
   b) 5.375

Percent Rate of Change

You get a $0.25 increase in your hourly rate at your job. Your friend wants to know what percent change this is in your wage. How do you calculate the answer for him?

The way to calculate a percent rate of change is to apply the following formula.

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original value}} \times 100$$

This formula should be included on your resource sheet.

Dividing the amount of change by the original value will give you a decimal, which is then converted to a percent when you multiply by 100.

In order to calculate this rate, you need to know the original amount, and the amount of change from the original amount. The change could be an increase or a decrease.

Example 1

You are earning $9 per hour, and your boss gives you a $0.25 per hour raise. What is the rate of change, or stated differently, what is the percent rate of increase in your wage?

Solution:

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original value}} \times 100$$

Original amount = $9.00

Change from the original amount = $0.25

$$\text{Percent rate of change} = \frac{0.25}{\$9.00} \times 100 = 2.777\ 777\ 777\% = 2.8\%$$

Example 2

Store A is offering a sale on an HD television. It normally costs $1599 and is on sale for $1399. A competing store, Store B, is selling the same set with a 10% reduction. Which store is offering the better deal?

Solution:

One method of comparing the two offers is to find the percent rate of change for each store.

Store A: $\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original value}} \times 100$

Original value = $1599

Change from the original value = $1599 – $1399 = $200

$\text{Percent rate of change} = \frac{\$200}{\$1599} \times 100 = 12.5\%$

Store A gives us a discount rate of 12.5%

Store B: Gives a discount rate of 10%.

Since both stores offer the same original price for the HD television, then Store A has the better offer.

Lesson Summary

In this lesson, you converted fractions to decimals and decimals to fractions, and you reviewed converting decimals to percents. You learned about special fractions, about rules for rounding off, and about a formula to find the percent rate of change. You applied this formula to solving wage and sales problems. Meanings of the words *numerator*, *denominator*, *top*, and *bottom* were mentioned.

In the next lesson, you calculate gross income.

Assignment 1.2

Fractions and Percent Rate of Change

Total Marks = 20

1. Convert the following fractions to decimals. Round off to two decimal places where appropriate. *(4 marks)*

   a) $\frac{5}{12}$ _____ b) $\frac{5}{8}$ _____

   c) $3\frac{3}{5}$ _____ d) $\frac{1}{15}$ _____

2. Complete the table by converting the fractions to decimals and to percents. Round off appropriately. *(10 marks)*

Fraction	Decimal	Percent
$\frac{3}{4}$		
$\frac{3}{2}$		
$\frac{3}{8}$		
$1\frac{5}{8}$		
$3\frac{5}{6}$		

continued

Assignment 1.2: Fractions and Percent Rate of Change (continued)

3. Convert the following decimals to fractions. *(2 marks)*

   a) 4.875

   b) 0.15

4. Last year you could buy ice cream cones for $1.25. This year, the price is $1.30. Find the percent rate of change in the price of a cone. *(2 marks)*

5. A camp counsellor received a weekly wage of $300 last year. This year she is being paid $350. Find the percent rate of increase in her pay. *(2 marks)*

Lesson 3: Gross Income

Lesson Focus

In this lesson, you will

- ❑ identify various methods of earning income
- ❑ demonstrate an understanding of calculations for gross pay

Lesson Introduction

All of us have to work for a living. Depending on your career choices, you can make money in a variety of ways. This lesson examines the various methods of earning income.

Income

Various methods of earning income include hourly wages, salary, contract, commission, salary plus commission, and piecework. Your total income is called gross pay.

Methods of Earning Income

Wages

Many students find part-time work while they finish school. This provides some income to pay for entertainment or personal items like clothing and cell phone usage. Others work to save money to go on to university or college after graduation from Grade 12. Most commonly, students are paid an hourly wage. The more hours worked, the more income earned. Typically these jobs are found in retail or food service.

Any tips given by satisfied customers in a restaurant are usually shared with the kitchen staff, but the percentage differs for each place of employment.

Many full-time jobs also pay on an hourly basis. Nursing, office jobs, or jobs in the manufacturing industry usually have hourly wages.

In your own words, include the definition of a wage on your resource sheet.

Example 1

Deb works in a restaurant on evenings and weekends. She is paid $10 per hour, plus 90% of all tips received. If she worked 18 hours this week, and received $280 in tips, find her gross pay.

Solution:

Pay for regular hours = $10 × 18 hours = $180

Earnings from tips = $280 × 0.90 = $252

Gross pay = $180 + $252 = $432

Example 2

Carlos works as a nurse, averaging 38.75 hours per week. He is paid $33.50 per hour. Find his gross pay.

Solution:

Gross pay = 38.75 × $33.50 = $1298.13

Example 3

From Example 2 above, if 8 hours Carlos worked were on an evening shift for which he was paid a $2 per hour "shift premium," as extra money for the later hours, find his gross pay.

Solution:

Gross pay without the shift premium from Example 2 above = $1298.13

Shift premium = $2 × 8 = $16.00

Total earnings = $1298.13 + $16.00 = $1314.13

Salary

Teachers, firefighters, police, government employees, and managers are generally paid a salary. Salaried workers receive the same amount of money, regardless of the amount of hours they work. Salaries are generally stated as an annual amount based on how much an employee makes in one year. This amount must be divided by 52 to find the weekly amount, since there are 52 weeks in a year.

Write the definition of a salary, in your own words, on your resource sheet.

Example 1

Mr. McMichael teaches history at the local high school. His annual salary is $68,350.00. Find his weekly gross pay.

Solution:

$$\text{Weekly gross pay} = \frac{\$68,350}{52} = \$1314.42 \text{ per week}$$

If a worker is earning a monthly salary and you need to find the weekly amount of her earnings, multiply the earnings by 12 to find the yearly salary, and then divide by 52 to find the weekly income. You cannot simply divide monthly income by four because most months have four weeks plus a few days.

Example 2

Shannon earns a salary of $4000 per month as a legal secretary. Find her weekly gross income.

Solution:

Annual salary = $4000 × 12 = $48,000

$$\text{Weekly gross income} = \frac{\$48,000}{52} = \$923.08 \text{ per week}$$

Contract

Examples of companies using a contract are companies that do driveway installations, home renovations, property landscaping, or roofing.

Many contractors will have you sign a contract in which you promise to pay them a fixed amount upon completion of the job, regardless of the number of hours needed to complete the job.

Write a description of how a contract payment is used on your resource sheet.

Learning Activity 1.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are going to the store to buy a drink with $2.05 in your pocket. If a drink costs $1.75, will you be able to buy one?
2. Which is larger: 0.76 or 0.07?
3. Solve for g: $3 - g = 15$.
4. Is an angle with a measurement of 86° acute, right, obtuse, straight, or reflex?
5. Complete the pattern: 4, 1, –2, ___ , ___ .

Part B: Gross Pay

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Label the following income situations as hourly wage, salary, or contract.
   a) Enrico is willing to re-roof your house for $3200.
   b) Karen works in a clothing store in the mall on weekends.
   c) Luigi is hired to install paving stones for your driveway and sidewalk.
   d) Henry is a supervisor working for the government.
2. Calculate Joe's gross income if he earns $9.25 per hour at a restaurant and is allowed to keep 65% of the tips he receives. He worked 29 hours and received $370 in tips.
3. Mary receives an annual salary of $56,000 as an office manager. Find her weekly gross pay.
4. Tom is paid a salary of $3000 per month to manage workers at a lawn care business. Find his weekly gross pay.

Commission

Some of the highest paid sales people in Canada are paid strictly on commission where they are paid a percentage of their sales. If you are comfortable talking with people and have a popular product to sell, you can make a nice income on commission. But the reverse can also be true if sales are slow.

People who sell real estate, life insurance, and automobiles are usually paid on commission. However, if the economy is bad, and people are not purchasing as much, commissions are low.

Include a description of commission income on your resource sheet.

Example 1

Charlie is paid $1\frac{1}{2}\%$ commission on house sales. Last week he sold 3 homes, valued at \$95,000, \$130,000, and \$175,500. Find his gross pay for last week.

Solution:

Total sales = \$95,000 + \$130,000 + \$175,500 = \$400,500

Commission rate = $1\frac{1}{2}\%$ = 1.5% = 0.015

Gross pay = commission on sales = 0.015 × \$400,500 = \$6007.50

This is a high weekly salary compared to what most other jobs pay, but remember that if Charlie sold no houses, he would receive no income.

Example 2

Sarah is working in an ice cream store. She is paid strictly on commission at the rate of 22% of all sales she makes. If her sales during one freezing week last February were \$180, find her gross income for that week.

Solution:

Gross income = commission on sales = 0.22 × \$180 = \$39.60.

Example 3

Imagine you have a job in sales and your manager tells you she expects a 10% increase in sales for next year. This year your sales total was $56,000.

To find out your new sales quota, you write $56,000 × 0.10 = $5600. Does this answer make sense?

Solution:

You might think the answer is correct "because the calculator says so," but you would be wrong. It makes no sense to have sales of $56,000 one year, and only $5600 the next.

Upon further examination, you determine that you have found the 10% increase your manager wants, and that this amount should be added to the previous sales amount.

$56,000 + $5600 = $61,600.

This figure makes more sense.

Salary plus Commission

Some businesses, such as big department stores, will pay sales staff a base salary plus a commission. This ensures the staff will at least have some income each week when sales are slow.

Note the difference between a commission and salary plus commission on your resource sheet.

Example 1

Brian is a salesman in a store selling top quality home furnishings. The product is very good, but the price is high and sales are few. To keep sales people on staff, the store owner pays a salary plus a commission. The owner pays Brian a base salary of $450 per week, plus 5% commission on all his sales. If Brian sold $12,275 in furnishings last week, find his gross pay for the week.

Solution:

Base salary = $450

Commission earnings = 0.05 × $12,275 = $613.75

Gross income = $450 + $613.75 = $1063.75

Example 2

Meghan sells appliances for a major department store. She receives a monthly salary of $850 plus 3% commission on her sales. If this week's sales were $6800, find her gross weekly income.

Solution:

Step 1 Convert the base monthly salary to weekly by multiplying by 12 and dividing by 52.

Annual salary = $850 × 12 = $10,200

$$\text{Weekly salary} = \frac{\$10,200}{52} = \$196.15 \text{ per week}$$

Step 2 Calculate the commission.

Commission = 0.03 × $6800 = $204

Step 3 Find the total weekly salary.

Gross pay = $196.15 + $204 = $400.15

Sometimes, students make errors calculating gross pay when given the monthly income. They either forget to convert the monthly salary to annual salary and then to the weekly salary (not reading the questions carefully), or they make errors in the calculation. Always read the questions completely, and remember that monthly to weekly calculations follow the rule: multiply by 12 and then divide by 52.

Piecework

Another method of earning income is through piecework. You are paid for the number of things you make. If you are working in a garment factory, sewing zippers onto jackets, you might get paid an amount for each zipper properly sewn.

Piecework income encourages workers to apply themselves and not waste time. Some companies deduct pay for pieces not assembled properly.

Include your own definition of what constitutes piecework on your resource sheet.

Example 1

Hubert is paid $5 for every bike he assembles and puts on the rack at the local Bike*Mart store. The manager deducts $3.50 off his pay for each bike placed on the rack that is not assembled properly. Calculate Hubert's weekly gross income if he assembled 120 bikes, but had 26 rejected by the manager.

Solution:

Pay for bikes assembled = 120 × $5 = $600

Deduction for bikes improperly assembled = 26 × $3.50 = $91

Gross income = $600 – $91 = $509

Example 2

Shauna has a job assembling phone packages into boxes. She has to insert the phone, the cords, and the power supply. Then the box is closed and sent down the line for shipping. She is paid $0.75 for each box packaged, and has $0.50 deducted for any faulty packaging. Find her weekly gross pay if she packaged 580 boxes, but had 96 returned.

Solution:

Pay for packages completed = 580 × $0.75 = $435

Deduction for faulty packaging = 96 × $0.50 = $48

Gross income = $435 – $48 = $387

Learning Activity 1.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are a very busy person! You have soccer on Sunday, Tuesday, and Thursday nights. You have music lessons on Saturday in the evening. You also have swimming on Monday night. Which nights of the week do you not have any commitments?
2. A basic calculator has 6 rows, and 5 columns of buttons. How many buttons does it have in total?
3. Three students receive their marks for a project. Jane found her mark as a decimal, 0.62; John calculated his mark as a percent, 83%; Jean got $\frac{12}{16}$. Who got the best mark?
4. June is a big birthday month for you. Your brother's is on June 9th, your nephew's is on June 20th, plus Father's Day is in June! You want to spend $30 on each present and you have $85.00 saved up. Will this be possible?
5. If $0.33\overline{3} = \frac{1}{3}$, then what does $0.66\overline{6}$ equal?

continued

Learning Activity 1.6 (continued)

Part B: Types of Income

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Label the following income situations as commission, salary plus commission, or piecework.
   a) Earning $10 for every snow blower assembled
   b) Earning income selling cosmetics by phone
   c) Earning income selling fridges at The Bay
   d) Earning income selling life insurance
   e) Earning income sewing gloves

2. Harry sells cars, earning a $2\frac{3}{4}\%$ commission on each sale. Calculate his gross earnings if his sales totalled $61,300 last week.

3. Patty sells furniture. The store pays her a base salary of $275 per week, plus a $3\frac{1}{2}\%$ commission on her sales. If her sales last week were $4655, find her gross income.

4. Keith has a job where he assembles garden sheds, which are then sold. He is paid $22 for each shed he assembles, but has $10 per shed deducted for any faulty workmanship. If he assembles 42 sheds, but the manager returns 12 of them, find his gross income for the week.

Lesson Summary

In this lesson, you examined various ways of earning gross income including hourly wage, shift premiums, tips, salary, contract work, commissions, salary plus commissions, and piecework. You learned the difference between being paid by the hour and being paid for the job. You learned how to convert a monthly and an annual salary into the equivalent weekly salary.

In the next lesson, you will discuss overtime income, and how it affects calculating gross income.

Assignment 1.3

Gross Income

Total Marks = 16

1. How would an employee working part-time at a fast food restaurant earn income? *(1 mark)*

2. List three jobs where employees would be paid on a salary. *(3 marks)*

3. Find the gross income for someone earning $10.30 per hour if she worked $22\frac{3}{4}$ hours. *(2 marks)*

4. A legal secretary earns $38,500 per year. Find her weekly gross income. *(2 marks)*

continued

Assignment 1.3: Gross Income (continued)

5. A car salesperson is paid $1\frac{1}{2}\%$ commission on all sales. If her sales totalled $74,380, find her gross income. *(2 marks)*

6. An appliance salesperson earns a base salary of $300 per week, plus 4% commission on his sales. If his sales totalled $6845, find his gross earnings. *(3 marks)*

7. You are paid $0.10 per button to sew buttons on shirts. The boss deducts $0.05 for each flaw in your work. If you sewed 6500 buttons last week, but had 800 returned, find your gross earnings. *(3 marks)*

Lesson 4: Overtime Pay

Lesson Focus

In this lesson, you will

- ❑ identify various methods of earning overtime income
- ❑ demonstrate an understanding of calculations for gross pay

Lesson Introduction

Sometimes it is necessary to work more than the usual amount of time. If you work more than eight hours a day, or more than 40 hours per week, this might be considered as overtime. Your gross pay would be calculated as the sum of the regular pay and the overtime pay.

Employees are generally paid more if they have to work past their regular hours. This lesson examines two common methods of calculating overtime.

Note when and how much you can be paid in overtime on your resource sheet.

Overtime

It has been a long day working your eight-hour shift. But when your replacement doesn't show up, the boss asks you to work overtime. You really would rather go home, but you are offered "time and a half" if you stay. This means you will be paid at your regular hourly wage (time) plus an additional half of that hourly rate. For example, instead of earning $10 per hour, you will be paid $15 per hour as overtime. Now working later doesn't seem so bad!

Some people earn "double time" when working on holidays. If your regular wage is $10 per hour, working overtime at double time would give you $20 per hour.

Overtime Policies

Generally, there are three methods of handling overtime hours for workers. Before accepting a job offer with a company, you could ask the owner about the overtime policy. The provincial government has enacted laws to protect workers, and companies must pay overtime for extra hours worked.

Overtime After a 40-Hour Work Week

Perhaps the most common policy is the 40-hour work week. Employees will be paid regular pay for the first 40 hours of work in a week. Any hours worked beyond the 40 hours is considered as overtime, and is paid at time and a half.

Example 1

You are paid $9.75 an hour and time and a half for any hours worked over 40 hours. Calculate your last week's wages from the following time card for last week.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
7	9	9	10	6	4

Solution:

Total hours worked = 7 + 9 + 9 + 10 + 6 + 4 = 45 hours

In total, you worked 45 hours. You are paid 40 hours at the regular hourly rate and time and a half for the extra 5 hours of overtime you have worked.

Regular time = 40 hours × $9.75 = $390

Overtime rate = $9.75 × 1.5 = $14.63

Overtime pay = 5 × $14.63 = $73.15

Gross pay = $390.00 + $73.15 = $463.15

Notice that you calculate the overtime hours paid at "time and a half" by multiplying the wage by 1.5 first and rounding off to the nearest cent. Then you multiply the overtime rate by the number of hours.

Example 2

Your workplace demands that you put in many hours, and you are paid $10.25 per hour. But your employer will pay you time and a half for any hours over 40 in a week, and double time for any hours worked on Sunday. Here is a record of the hours you worked each day. Find your week's gross pay.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
6	10	8	8	9	4	4

Solution:

Hours worked

Total weekday hours from Monday to Saturday
= 6 + 10 + 8 + 8 + 9 + 4 = 45

Regular hours = 40

Overtime hours at time and a half = 5

Sunday hours at double time = 4

Rates

Regular hourly rate = $10.25

Overtime rate on regular hours = 1.5 × $10.25 = $15.38

Double time rate = $10.25 × 2 = $20.50

Gross pay calculations

Regular hours = 40 × $10.25 = $410

Time and one-half overtime pay = 5 × $15.38 = $76.90

Double overtime pay = 4 × $20.50 = $82

Total gross pay = $410 + $76.90 + $82 = $568.90

Overtime After an 8-Hour Day

Some employers will pay overtime on any time worked over eight hours each day. In this situation, you look at the number of hours worked each day, and sort the regular hours from the overtime hours. The overtime rate is usually at time and one-half.

Example 1

Find the number of regular hours and the number of overtime hours for Dan, whose schedule is shown below, if he is paid overtime after an 8-hour day.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
6	9	10.5	7	8	4

Solution:

Regular hours (8 or less each day) = 6 + 8 + 8 + 7 + 8 + 4 = 41 hours

Overtime hours (more than 8 each day) = 0 + 1 + 2.5 + 0 + 0 + 0 = 3.5 hours

Example 2

Your employer pays overtime on any time worked over 8 hours per day, and on any hours worked on Saturday. Your regular wage is $9.85 per hour. Calculate your gross pay given the following schedule.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
9.25	8	10	5	5	3

Solution:

Regular hours (8 or less each day, not including Saturday)
= 8 + 8 + 8 + 5 + 5 = 34 hours

Regular pay = 34 × $9.85 = $334.90

Overtime (over 8 hours per day and on Saturday) = 1.25 + 2 + 3 = 6.25 hours

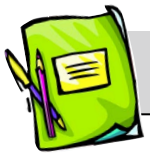
Overtime rate = 1.5 × $9.85 = $14.78

Overtime pay = 6.25 × $14.78 = 6.25 × $14.78 = $92.38

Gross pay = $334.90 + $92.38 = $427.28

Overtime as Time Off for "Banked" Hours

"Banking" hours is a method of recognizing time spent beyond the regular work day and is popular with many people. Instead of being paid overtime, employees can "bank" their extra hours worked, take the time off another day, and still be paid. For example, if you worked 10 hours each day from Monday through Thursday, your boss would allow you to bank the 2 extra hours each day, for a total of 8 hours of "banked" time. You could have the next day off, a Friday, with pay! You would still be paid for the 40 hours at regular pay, and you get to enjoy a three-day weekend.



Learning Activity 1.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A Canadian football field is 100 yards. A scoring drive started on one goal line and went the entire length of the field. On average, each play advanced the ball 5 yards. How many plays were in the scoring drive?
2. Rewrite the following fraction in lowest terms: $\frac{18}{27}$.
3. There is a 50% discount on all candy at the store the day after Hallowe'en. If it cost you $30 to buy candy before Hallowe'en, how much would you spend if you bought the same candy after Hallowe'en?
4. What is the mean of 3, 4, 6, and 7?
5. Jamie is twice as old as Dan. Dan is 3 times as old as Kim. If Kim is 4 years old, how old is Jamie?

continued

Learning Activity 1.7 (continued)

Part B: Gross Pay including Overtime

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given that your employer pays time and one-half for any hours over 40 worked in a week, calculate the gross pay if you worked 43.75 hours at a wage of $11.15 per hour.

2. Given that your employer pays overtime at time and one-half for any hours worked over 8 hours each day, calculate your gross pay if your wage is $9.68 per hour and you worked the following schedule.

Monday	Tuesday	Wednesday	Thursday	Friday
6	5	10.5	7	6

3. Your employer will pay you time and one-half for any hours worked over 40 per week, time and one-half for any hours worked on Saturday, and double time for any hours worked on Sunday. Calculate your gross pay given the hourly wage of $11.35 and the following schedule.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8	9.5	10.25	7	8	6	4

Lesson Summary

In this lesson, you studied two methods by which overtime is paid to employees, after 40 hours per week or after 8 hours per day. Employees could also "bank" paid time off for extra hours worked. You found the time and a half rate and double time rate from the given the hourly rate. You calculated gross pay by adding the pay for overtime hours to the pay for regular hours.

In the next lesson, you will discuss time cards and use them to determine gross income.

Assignment 1.4

Overtime Pay

Total Marks = 17

1. Calculate the gross income for the following employee where the hourly wage is $10.72 and overtime is paid at time and one-half on all hours worked over 40 per week. *(5 marks)*

Monday	Tuesday	Wednesday	Thursday	Friday
8	9.5	8	7.5	9.25

2. Calculate the gross income for the following employee where the hourly wage is $12.34 and overtime is paid at time and one-half on all hours worked over 8 per day and any time on Saturdays. *(5 marks)*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
5	10.25	7	9	5.5	4

continued

Assignment 1.4: Overtime Pay (continued)

3. Calculate the gross income for the following employee if his hourly wage is $11.26 and where overtime is paid at time and one-half on all hours worked over 40 per week and on any Saturday hours. Double time is paid for any hours worked on Sunday. *(7 marks)*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8	10.75	8.5	9	8	4	4.5

Lesson 5: Time Cards and Late Penalties

Lesson Focus

In this lesson, you will

- ❑ learn to analyze and interpret time cards
- ❑ assess any late penalties
- ❑ demonstrate an understanding of calculations for gross pay

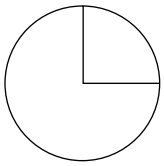
Lesson Introduction

Most businesses that employ many workers will have you "punch in" a card into a time clock, which records the exact time you started your shift, and then "punch out" your card at the end of your shift. Other businesses require that employees wear a name tag, which has a bar code on it. The tag is scanned when you start and when you finish your shift.

This lesson deals with reading time cards with the intent of establishing hours worked and determining gross pay.

Hours at Work

In this day of digital watches, many students have trouble reading a regular analog clock. It is important that you understand that one hour is 60 minutes long. So 15 minutes out of those 60 minutes would constitute a quarter of an hour.



To simplify the process, time worked is rounded to the quarter hour, and expressed in decimal form. So if a worker was on the job for $6\frac{3}{4}$ hours, she would be given 6.75 hours of pay.

Time Cards

Example 1

This is an example of a time card used by accountants in the payroll office to calculate your pay. You are to fill in the card by adding up the regular hours worked, finding any overtime hours, and then calculating the gross pay given the hourly wage for the employee.

Employee: Jim Dandy					Hourly Rate: $9.35		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.			1:00	5:00			
Tues.							
Wed.			1:00	4:30			
Thurs.					5:15	9:30	
Fri.							
Sat.	9:45	12:00	1:30	4:45			
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

Solution:

When you read the time card, you can see that Jim Dandy worked the following hours.

Monday:	1:00 – 5:00		4 hours
Wednesday:	1:00 – 4:30		3.5 hours
Thursday evening:	5:15 – 9:30		4.25 hours
Saturday:	9:45 – 12:00	2.25 hours	
	1:30 – 4:45	3.25 hours,	
Total for Saturday			5.5 hours

These values would be entered in the right-hand column.

Once added together, you record the number of hours in the bottom box, and calculate the gross pay based on the employee's hourly rate.

Employee: Jim Dandy					Hourly Rate: $9.35		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.			1:00	5:00			4
Tues.							
Wed.			1:00	4:30			3.5
Thurs.					5:15	9:30	4.25
Fri.							
Sat.	9:45	12:00	1:30	4:45			5.5
Sun.							
							17.25

	Hours	Rate	Amount
Regular	17.25	$9.35	17.25 x $9.35 = $161.29
Overtime	0		0
Total Hours	17.25	Gross Wages	$161.29

Carefully read the overtime policy in each of the following questions before calculating any overtime earnings.

Example 2

Susan works for a retail store in the mall. The company pays overtime hours over 40 per week at time and one-half. Add up Susan's regular hours, determine the number of overtime hours, and then calculate her gross pay.

Employee: Susan Que					Hourly Rate: $10.28		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	8:00	12:01	1:00	5:05			
Tues.	7:58	12:00	1:03	5:03			
Wed.	7:00	12:05	1:00	5:30	7:00	9:30	
Thurs.	8:04	11:59	12:58	5:00	6:45	9:30	
Fri.	8:00	12:02	12:57	5:04			
Sat.			1:00	5:00			
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

Solution:

As you study Susan's time card, you notice that she may have logged in, or "punched in" a couple of minutes early or even a few minutes late. Most businesses these days allow some flexibility in this area. But if arriving late or leaving early becomes a pattern, your employer will "counsel" you. Continued late arrivals will usually lead to you being dismissed. Later in this lesson, you will use an example of late charges.

After calculating the total hours for each day, rounded to the nearest quarter hour, you carefully add the hours she worked, and fill in the chart. Remember that overtime in this store includes any hours beyond the 40-hour work week.

Monday: 4 + 4 = 8
Tuesday: 4 + 4 = 8
Wednesday: 5 + 4.5 + 2.5 = 12
Thursday: 4 + 4 + 2.75 = 10.75
Friday: 4 + 4 = 8
Saturday: 4

Employee: Susan Que					Hourly Rate: $10.28		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	8:00	12:01	1:00	5:05			8
Tues.	7:58	12:00	1:03	5:03			8
Wed.	7:00	12:05	1:00	5:30	7:00	9:30	12
Thurs.	8:04	11:59	12:58	5:00	6:45	9:30	10.75
Fri.	8:00	12:02	12:57	5:04			8
Sat.			1:00	5:00			4
Sun.							
							50.75

	Hours	Rate	Amount
Regular	40	$10.28	40 x $10.28 = $411.20
Overtime	10.75	$10.28 x 1.5 = $15.42	10.75 x $15.42 = $165.77
Total Hours	50.75	Gross Wages	$411.20 + $165.77 = $576.97

24-Hour Clock

Hospitals, travel agencies, and military offices tend to run on 24-hour clocks instead of the more familiar 12-hour clocks.

The time at 0:00 hours would be midnight, and the day starts from there. 14:30 hours would be 2:30 pm in the afternoon.

The trick is to subtract 12 from the 24-hour clock afternoon and evening times to convert them to a 12-hour clock time.

Example 1

Harry works in a hospital. He is paid overtime at time and one-half for any hours worked over 8 each day from Monday to Friday, and paid double time for any work on Sunday. Use the following time card to calculate Harry's gross pay.

Employee: H. Houdini					Hourly Rate: $17.30		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	05:00	10:15	13:15	17:30			
Tues.	07:30	12:00	13:00	17:30			
Wed.	07:29	12:01	13:02	17:35	19:30	23:15	
Thurs.							
Fri.	07:00	12:00					
Sat.							
Sun.	06:30	12:15					

	Hours	Rate	Amount
Regular			
Overtime x 1.5			
Overtime x 2			
Total Hours		Gross Wages	

Solution:

Carefully note which overtime hours are paid at time and one-half, and which are paid at double time when you complete the bottom of the chart.

Employee: H. Houdini					Hourly Rate: $17.30		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	05:00	10:15	13:15	17:30			9.5
Tues.	07:30	12:00	13:00	17:30			9
Wed.	07:29	12:01	13:02	17:35	19:30	23:15	12.75
Thurs.							
Fri.	07:00	12:00					5
Sat.							
Sun.	06:30	12:15					5.75
							42

	Hours	Rate	Amount
Regular	8+8+8+5 = 29	$17.30	29 x $17.30 = $501.70
Overtime x 1.5	1.5+1+4.75 = 7.25	$17.30 x 1.5 = $25.95	7.25 x $25.95 = $188.14
Overtime x 2	5.75	$17.30 x 2 = $34.60	5.75 x $34.60 = $198.95
Total Hours	42	Gross Wages	$501.70 + $188.14 + $198.95 = $888.79

Learning Activity 1.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. There are 20 cheese sticks in a package. You only eat one each day, and only on weekdays (Monday to Friday). How many weeks will it take you to finish the whole package?
2. Complete the following pattern: 60, 75, ___ , 105, ___ .
3. Is an angle of 235° an acute, an obtuse, a right, a straight, or a reflex angle?
4. Evaluate: $4 - 6 + 2 \times (3 - 8)$.
5. There are 3 pairs of socks in a package. If the whole package costs $6, how much does it cost per pair of socks?

continued

Learning Activity 1.8 (continued)

Part B: Time Cards

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Complete Janice's time card and calculate her gross earnings. She earns $12.56 per hour, and overtime is paid at time and one-half on any hours worked over 40 per week.

Employee: Janice Bird					Hourly Rate: $12.56		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	7:45	12:00	1:00	5:15			
Tues.	8:01	12:03	12:57	5:01			
Wed.	8:00	12:00	1:01	5:27	7:00	9:45	
Thurs.	7:58	11:59	1:01	5:00	7:00	9:15	
Fri.	8:00	12:00	1:01	5:15			
Sat.							
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

continued

Learning Activity 1.8 (continued)

2. Justin is paid $9.95 per hour, and overtime is calculated on any hours over 8 each day, and is paid at time and one-half. Any hours worked on Sunday is paid at double time. Use the time card to calculate his gross earnings.

Employee: Justin					Hourly Rate: $9.95		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	07:00	12:00	13:00	17:30			
Tues.	07:01	12:03	13:00	17:00			
Wed.	07:00	11:30					
Thurs.	07:30	12:00	13:01	18:15			
Fri.							
Sat.							
Sun.			12:00	18:30			

	Hours	Rate	Amount
Regular			
Overtime x 1.5			
Overtime x 2			
Total Hours		Gross Wages	

Late Penalties

Some companies penalize workers for being late or leaving early. The penalty is deducted off the worker's pay for each quarter hour, or part thereof, the worker starts late or leaves early.

Example 1

A worker logging in at 8:04 and leaving at 12:04 arrived 4 minutes late. How much does he lose for being late?

Solution:

Following the usual lateness policy, he would have a quarter-hour deducted from his pay for arriving 4 minutes late in the morning. This means he would only be paid for $3\frac{3}{4}$ hours. No credit is given for working the extra 4 minutes at noon.

Include this late policy on your resource sheet as a reminder.

Example 2

Another employee logged in at 7:58, and left at 11:59. Does he lose pay?

Solution:

Since he left a minute early, he will have a quarter-hour removed from his pay as a penalty for leaving early.

Example 3

This worker logged in at 8:02, and left at 11:57. What pay does he lose?

Solution:

He is penalized a quarter-hour for being late, and another quarter-hour for leaving early. He loses $\frac{1}{2}$ hour of his pay.

Read Your Job Agreement Carefully

Since work environments have different policies regarding overtime pay and late penalties, every worker must be very careful to read the instructions thoroughly before filling in the payroll cards.

Lesson Summary

In this lesson, you learned to calculate gross earnings from time cards where hours were recorded using either a 12-hour clock or a 24-hour clock. Late penalties were discussed.

In the next lesson, you will compare various methods of earning income.

Assignment 1.5

Time Cards and Late Penalties

Total Marks = 19

1. Patty works in a plant that pays overtime at time and one-half for any hours worked over 40 hours per week. Her wage is $13.75 per hour. Use this information to complete her time card. The company does not assess any late penalties. *(5 marks)*

Employee: Patty					Hourly Rate: $13.75		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	8:00	12:01	12:59	4:30			
Tues.	7:58	12:02	1:00	5:03	6:30	10:17	
Wed.	8:01	12:01	12:59	5:15			
Thurs.	7:56	12:06	1:01	5:02			
Fri.	8:00	12:00	1:00	5:32	6:30	10:07	
Sat.							
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

continued

Assignment 1.5: Time Cards and Late Penalties (continued)

2. Keith is employed at a factory, and earns $12.95 per hour. Hours in excess of 8 per day are paid at time and one-half. Any hours worked on Sunday are paid at double time. After 3 lates, Keith gets a "counselling" session with his manager. Fill in the time card to find his gross earnings. Start times are 8:00 am and 12:30 pm except on Thursdays, when it is 1:00 pm. Does Keith need counselling? The company does not assess late penalties. *(7 marks)*

Employee: Keith					Hourly Rate: $12.95		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	8:00	12:00	12:30	5:00			
Tues.	8:01	12:03	12:31	6:15			
Wed.	8:00	11:59	12:29	6:01			
Thurs.			1:00	6:00			
Fri.							
Sat.	8:01	12:00	12:31	6:02			
Sun.					7:00	11:46	

	Hours	Rate	Amount
Regular			
Overtime x 1.5			
Overtime x 2			
Total Hours		Gross Wages	

continued

Assignment 1.5: Time Cards and Late Penalties (continued)

3. Sylvia is employed in a hospital, and earns $27.78 per hour. She is paid double time for any work on Sunday, and time and one-half for any hours over 40 per week. Use the time card to find her gross pay. There are no late penalties. *(7 marks)*

Employee: Sylvia					Hourly Rate: $27.78		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	06:00	11:30	12:30	17:45			
Tues.	06:01	11:32	12:29	18:01			
Wed.	08:00	12:30	13:31	17:00			
Thurs.					18:00	23:59	
Fri.					18:01	24:00	
Sat.							
Sun.	06:00	12:02					

	Hours	Rate	Amount
Regular			
Overtime x 1.5			
Overtime x 2			
Total Hours		Gross Wages	

Notes

Lesson 6: Comparisons

Lesson Focus

In this lesson, you will

- ❑ learn to analyze, compare, and interpret different methods of earning income
- ❑ demonstrate an understanding of calculations for gross pay

Lesson Introduction

What is the best way to earn an income? This lesson compares the various methods discussed in the previous lessons. Using your problems-solving skills, you will examine each situation.

Choosing a Job

As a Grade 10 student, at some point in the near future you will have to decide just what you will do to earn income. This is a challenge for most people. Occupations have advantages and disadvantages. As you advance through your studies, it is helpful to have a goal in mind. Before making a decision, it is a wise idea to look at some considerations. Completing a career preferences quiz will also help guide you toward a good choice.

You should include the different factors that influence job decisions on your resource sheet.

Job Considerations

Everyone has different reasons for entering the career they chose. Before you decide, there are some things that should be considered. Some of these considerations are discussed below.

Leisure Time

If spending time with your friends and going on holidays are priorities for you, then you might want to stay away from careers that demand a lot of evening and weekend hours. Hotel management and work as a chef are two examples of jobs that need this consideration. Most of the customer demand is in the evenings, so these workers must be able to work then.

Level of Income

Usually you will be able to choose from a variety of job offers. If the highest income is your main focus, you may have to sacrifice more of your leisure time to devote the number of hours required to be successful at your job. Employees at manufacturing plants who have been promoted to managers receive a higher income, usually as a salary, rather than an hourly wage. But managers are expected to work between 50 and 60 hours per week. Remember, with a salary you are not paid for any overtime you work.

Also, you will want be sure that you accept the job that is best suited to your interests and abilities.

Opportunity for Advancement

Some people are very careful to select a company in which there is room for advancement. With some experience and a little more training, you could be promoted to a higher level with a higher income.

Job Security

Some people want to select an occupation that has a high level of job security. Occupations that rarely experience layoffs include police work, firefighting, and the military.

Job Satisfaction

This should be a major consideration in selecting a career path. People want to enjoy their work, enjoy going to work, and enjoy the people with whom they work. Sometimes working in a pleasant environment is worth a little less income.

Self-Tests

Many students have difficulty knowing what they want to choose for an occupation or even what they are best suited to do. The federal government offers a Career Navigator to help solve these problems The Career Navigator is a series of short quizzes done online to generate a list of suggested occupations for you, based on how you answer the questions. It is worth the few minutes it takes to complete the questions. You can find the Career Navigator at this website:

www.jobsetc.gc.ca/toolbox/quizzes/quizzes_home.dolang=e

Another very interesting aptitude site sponsored by the federal government is www.jobfutures.ca.

The Canadian government offers support in training, career, and worker information. You can access this support at www.servicecanada.gc.ca or at any Service Canada location.

As an example, you might be considering a job as a sales person. Commissioned sales people can make a lot of money. You must have a product people want, and you must have personality traits that are common with successful sales personnel. You might be an excellent sales person if you have many of the following traits.

- You move around whenever you hear good music being played.
- You have a loud laugh.
- You like to be the centre of attention.
- You are generally cheerful and like to make others happy.
- You can be very persuasive.
- You are a very sociable person.

Job Comparisons

In this section, different job settings are presented. Your task is to compare them, and find the advantages and the disadvantages.

Example 1

Juliette has been offered the manager's position at an ice cream store. The store is located at the entrance to a major city park where thousands of people walk by every day in the summer. The owners have offered her two different packages, and she must make a decision how she will be paid. Option A offers her a flat salary of $45,000. Option B offers her the chance to be paid 9% of the total profits. Last year, after a very warm summer, the store earned profits of $500,000.

a) List some advantages of each option to Juliette.

b) List some disadvantages of each option to Juliette.

Solution:

a) Advantages to each option.

Option A

- Guaranteed income of $45,000.
- No stress regarding ice cream sales.
- She can relax when not at work.

Option B

- There is no limit on the amount of money she could make. Her salary last year was 9% of $500,000 or $45,000. This is the same amount as in Option A, and is dependent on sales of $500,000. If that amount increases this year, she will make more income.
- If she worked harder, she could make more money.
- She may feel that she is in control of her job and her income.

b) Disadvantages to each option.

Option A

- She has no opportunity to increase her income.
- Without any incentive to improve, she might get bored easily.

Option B

- If the weather is poor, sales will drop, and so will her income.
- The job may consume all of her leisure time.
- She will have to hire top salespeople to keep sales growing.

Example 2

Tom and Jerry are friends working at the same electronics store. Tom is paid $19.50 per hour, and works about 30 hours per week. Jerry is paid a base salary of $150 per week, and receives 4% of his sales as a commission. The average amount of weekly sales per salesman is $10,000.

a) Calculate the gross income for Tom and Jerry based on sales of $10,000.

b) Why would Jerry be pleased with his payment option?

c) Why would Tom be pleased with his payment option?

d) Why would the manager want to put Tom on the same plan as Jerry?

Solution:

a) Tom's gross pay = $19.50 × 30 = $585

Jerry's gross pay = $150 + ($10,000 × 0.04) = $150 + $400 = $550

b) Perhaps Jerry likes to sell. If his potential is to sell more than the average, he will earn more income than being on a fixed rate.

c) Without the pressure to sell more products, Tom has less stress at work. Perhaps he doesn't need to put in extra time or to earn the extra income.

d) The manager would hope that Tom would work harder to make more sales.

Learning Activity 1.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is 10% of 500?
2. What is 5% of 500?
3. What is 15% of 500?
4. The distance to the mall from your house is 8 km. Your friend lives half as far away from the mall. What is the distance from your friend's house to the mall?
5. You give the cashier a $10 bill to pay for your lunch. If the total for your lunch is $7.60, how much change will you get back?

Part B: Comparing Payment Options

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Monica has been offered a job selling cosmetics in a retail store. She must choose her salary payment plan. Option A will pay her $12.50 per hour, and she will work a 40-hour week. Option B will pay her 25% commission on all sales.
   a) Find her gross income for Option A.
   b) Under Option B, what level of sales figures must she generate to match Option A?
   c) Give an advantage to each pay option for Monica.
   d) Give a disadvantage to each pay option for Monica.
   e) Why would the store prefer its sales staff to be paid on commission?

continued

Learning Activity 1.9 (continued)

2. Steve works installing drywall. He is paid $22 per hour, and averages about 35 hours per week. The company wants to place him on an annual salary of $41,000.
   a) Find his weekly gross income for each option.
   b) Give an advantage for Steve for each option.
   c) Give a disadvantage for Steve for each option.

Lesson Summary

In this lesson, you discussed several job considerations when deciding the type of occupation you may want to choose. Those considerations were leisure time, level of income, opportunity for advancement, job security, and job satisfaction. You also discussed the importance of participating in an aptitude self-test. You compared various income payment options, and discussed reasons why workers would select one option over another.

The next lesson is a project.

Assignment 1.6

Comparisons

Total Marks = 14

1. Ray is selling electric guitars. Working only in the afternoon, he sells about 12 guitars per week at an average cost of $350 each. At the moment, he receives 15% commission on all sales. His manager wants to place him on an annual salary of $38,000. *(4 marks)*

   a) Find Ray's weekly commission income.

   b) Find Ray's weekly income if he were on salary.

   c) Give a reason why the manager would want to pay more money to have Ray on salary.

   d) Give a reason why Ray would want to remain being paid on commission.

continued

Assignment 1.6: Comparisons (continued)

2. Helen is a very successful real estate salesperson. She sells an average of one house per week, and receives 1½% commission on all sales. The average selling price of her houses is $225,000. Helen estimates she works 60 hours per week. *(5 marks)*

   a) Find Helen's weekly income and her equivalent hourly wage.

   b) Find Helen's annual and monthly income.

   c) If her expenses cost $\frac{1}{2}$% of her sales, find her annual expenses.

   d) Why would she accept a position of branch manager for only $100,000 per year?

   e) Find Helen's annual income after her expenses are deducted and comment on her decision to become a manager.

continued

Assignment 1.6: Comparisons (continued)

3. Paul is quite shy, and is usually a very quiet person. He accepts a job selling cars. Should he accept his pay as a salary or as a commission? Explain your reasoning. *(3 marks)*

4. Why would an employee choose to be paid an hourly wage instead of a salary, even if the salary was slightly more? *(2 marks)*

Notes

Lesson 7: Project

Lesson Focus

In this lesson, you will

❑ make connections in your community with business managers

Lesson Introduction

This lesson asks you to go out into your community and interview three different business managers. You will be researching details about what it would be like to work at each of these businesses. This project will assist you to become acquainted with tools you may use in the future to explore the job market to find an "ideal" job.

Job Project

It is now time for you to do some "field research." By going out and talking with business owners and managers, you will get first-hand information about jobs in your area.

This information will be more meaningful if you attend in person, but it be could be gathered over the phone. And it makes a difference in your mark.

When you are inside the places of business, be sure to look at various things.

- dress code of other employees
- cleanliness of the place
- any obvious workplace hazards
- comfortable environment (lighting, temperature, etc.)

A marking rubric is provided on the next page. Not only will it be useful for your tutor/marker to determine your mark for the project, but also the rubric provides information about what is expected of you as you complete the project.

Assignment 1.7
Job Exploration Project

Score Level	Exploring (Did I interview three business managers?)	Completing (Did I complete three forms correctly?)	Communicating (Did I communicate my answer clearly?)
3	I interviewed three business managers about an hourly paid position at their company. I went to their business to interview them.	I completed all of the research forms from the responses I received from the managers. The information on the forms was accurate and relevant.	I wrote the responses on the forms in a clear, legible, and concise manner. My grammar and spelling were correct.
2	I interviewed two business managers about an hourly paid position at their companies.	I completed most of the questions on the research forms.	I wrote most of the responses on the forms in a clear, legible, and concise manner. My grammar and spelling were mostly correct.
1	I interviewed one business manager about an hourly paid position at her company. I interviewed three managers over the phone, but I did not go to their businesses.	I completed only the first page of each of the research forms.	I wrote the responses on the forms as best as I could as I interviewed the manager. I made some errors, which I did not fix (other than by scribbling over them). I did not check my grammar and spelling.
0	I interviewed no business managers.	None of the research forms were completed.	I didn't write anything on the forms.
Mark	Exploring = ____ / *(3 marks)*	Completing = ____ / *(3 marks)*	Communication = ____ /*(3 marks)*
Total Mark	Exploring + Completing + Communicating = _____		Total Possible: 3 + 3 + 3 = 9

Assignment 1.7

Job Exploration Project

Total Marks = 9

This project requires you to go to three different businesses in your community. For each business, record on the enclosed form the business name, along with the name of the manager with whom you speak.

You are to collect information regarding wages, duties, education, and job openings for a particular job in that business. List any special things you noticed about the workplace. Also, indicate a reason why you would want to work at this type of business and another reason why you would not want to work there. Then describe what you learned from doing this research. The form lists the headings of the type of questions you are to ask.

The three forms are to be submitted to the Distance Learning Unit as part of your assignment package.

Notes

Job Exploration #1

Student's name _____

Date _____

Business name _____ Manager _____

Business address _____ Phone _____

Job name _____ Hourly starting wage _____

Pay range _____ Hours of work per week _____

Shift times _____ Late penalties _____

Education required

Assigned duties

Possibility of finding a job in this career

Job Exploration #1 (continued)

(page 2 of 2)

Special things you noticed about the workplace

Reasons why you would want to work at this business

Reasons why you would not want to work at this business

What you learned from doing this project at this business

Job Exploration #2

Student's name _____

Date _____

Business name _____ Manager _____

Business address _____ Phone _____

Job name _____ Hourly starting wage _____

Pay range _____ Hours of work per week _____

Shift times _____ Late penalties _____

Education required

Assigned duties

Possibility of finding a job in this career

Job Exploration #2 (continued)

Special things you noticed about the workplace

Reasons why you would want to work at this business

Reasons why you would not want to work at this business

What you learned from doing this project at this business

Job Exploration #3

Student's name _____

Date _____

Business name _____ Manager _____

Business address _____ Phone _____

Job name _____ Hourly starting wage _____

Pay range _____ Hours of work per week _____

Shift times _____ Late penalties _____

Education required

Assigned duties

Possibility of finding a job in this career

Job Exploration #3 (continued)

Special things you noticed about the workplace

Reasons why you would want to work at this business

Reasons why you would not want to work at this business

What you learned from doing this project at this business

Module 1 Summary

Congratulations, you have finished the first of the eight modules in the course.

This module explored various ways to earn income, how to calculate that income including overtime, and things to consider when looking for a career.

Vocabulary

Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful.

24-hour clock	ones
banked hours	overtime
commission	percent
contract	percentage
decimal	piecework
denominator (bottom)	place value
double time	PST
gross pay	rounding off
gross earnings	salary
gross income	salary plus commission
GST	shift premium
hourly wage	tens
hundreds	tenths
hundredths	thousands
improper fraction	thousandths
job considerations	time and a half
late penalty	time card
mixed number	tips
numerator (top)	wages

Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

Percentage of a number = percent rate x the number

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original value}} \times 100$$

12/52 Rule Multiply by 12 and divide by 52 to change a monthly salary into a weekly salary.

Submitting Your Assignments

It is now time for you to submit the Module 1 Cover Assignment and Assignments 1.1 to 1.7 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 1 assignments and organize your material in the following order:

Module 1 Cover Sheet

Module 1 Cover Assignment: Yearning for Earnings

Assignment 1.1: Percents

Assignment 1.2: Fractions and Percent Rate of Change

Assignment 1.3: Gross Income

Assignment 1.4: Overtime Pay

Assignment 1.5: Time Cards and Late Penalties

Assignment 1.6: Comparisons

Assignment 1.7: Job Exploration Project

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 10 Essential Mathematics (20S)

Module 1
Gross Pay, Time Cards, and Percents

Learning Activity Answer Keys

Module 1:
Gross Pay, Time Cards, and Percents

Learning Activity 1.1

There is no answer key for this learning activity.

Learning Activity 1.2

Part A: BrainPower

You should be able to complete the following five questions in a few minutes without the use of a calculator or pencil and paper. Especially for the first few times you do these questions, your learning partner can help you figure out strategies to solve these questions in your head.

1. You are going for a walk. You walk north for 5 blocks, turn around, and walk south for 16 blocks. How many blocks are you from where you started? State if you are north or south of where you started.
2. If Evan eats $\frac{3}{5}$ of a pizza and Nick eats $\frac{4}{5}$ of a pizza, how many pizzas do they have to order so that both can eat as much as they like?
3. Write $\frac{6}{2}$ in simplest terms.
4. Knowing 50% of 680 is 340, evaluate 25% of 680.
5. If you buy a shirt for $8 and jeans for $32, how much do you spend all together (before taxes)?

Answers:

1. 11 blocks south (When you turn around and walk south, after 5 blocks you are back where you started. After that you walk 16 — 5 = 11 blocks more.)
2. 2 pizzas $\left(\frac{3}{5}+\frac{4}{5}=\frac{7}{5}\right.$ which is more than one pizza. Since you cannot order part of a pizza, you have to round up to 2 pizzas.)
3. $\frac{6 \div 2}{2 \div 2}=\frac{3}{1}=3$ (You divide top and bottom of the original fraction by a common factor because "in simplest terms" means there is no common factor in both the numerator and denominator.)

4. 170 (Both times you are looking for a percent of the same number. 25% is half of 50%, so half of 340 is $340 \times \frac{1}{2}$ or 170.)

5. $40 ($8 + $32)

Part B: Converting Percents to Decimals

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert these percents to decimals.

   a) 70% *Answer:* $\frac{70}{100} = 0.70$ or 0.7

   b) $12\frac{1}{2}\%$ *Answer:* $\frac{12.5}{100} = \frac{125}{1000} = 0.125$

   c) 6% *Answer:* $\frac{6}{100} = 0.06$

   d) 125% *Answer:* $\frac{125}{100} = 1.25$

2. Your friend writes 4% as 0.4 in decimal form. What did he do wrong, and how should 4% be written as a decimal?

   Answer:

   He thought of 4% as $\frac{4}{10}$ instead of $\frac{4}{100}$.

   4% is $\frac{4}{100}$, and is written as 0.04.

3. Find 40% of 600.

   Answer:

   $40\% = \frac{40}{100} = 0.40$

   $40\% \text{ of } 600 = 0.40 \times 600 = 240$

Learning Activity 1.3

Part A: BrainPower

You should be able to complete the following five questions in a few minutes without the use of a calculator or pencil and paper. Especially for the first few times you do these questions, your learning partner can help you figure out strategies to solve these questions in your head.

1. Rank the numbers highest to lowest: 0.5, 0.05, 0.3, 0.09, 0.25.
2. Evaluate the following: $2 - 3 + 6 \times 2 - 5 \times 4$.
3. Solve for i: $4i + 3 = 15$.
4. Is an angle that measures 140° acute, right, obtuse, straight, or reflex?
5. Write the next two numbers in the pattern: 1, 2, 4, 8, ___ , ___ .

Answers:

1. 0.50, 0.30, 0.25, 0.09, 0.05
2. –9 (Use the order of operations: $2 - 3 + 12 - 20 = -9$.)
3. $i = 3$ (If $4i + 3 = 15$, then $4i = 12$ and $i = 3$.)
4. Obtuse (Acute angles are between 0^o and 90^o; right angles are 90^o; obtuse angles are between 90^o and 180^o; straight angles are 180^o; reflex angles are between 180^o and 360^o.)
5. 16, 32 (Each number is double the previous so 1, $1 \times 2 = 2$, $2 \times 2 = 4$, $4 \times 2 = 8$, $8 \times 2 = 16$, $16 \times 2 = 32$.)

Part B: Converting Decimals to Percents

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert these decimals to percents.

   a) 0.34

   Answer: $0.34 = \frac{34}{100} = 34\%$

   b) 0.08

   Answer: $0.08 = \frac{8}{100} = 8\%$

   c) 0.065

   Answer: $0.065 = \frac{65}{1000} = \frac{65 \div 10}{1000 \div 10} = \frac{6.5}{100} = 6.5\%$ or $6\frac{1}{2}\%$

   d) 2.45

   Answer: $2.45 = 2 + \frac{45}{100} = \frac{200}{100} + \frac{45}{100} = \frac{245}{100} = 245\%$

2. Your friend wrote 0.072 as 72%. Identify the mistake she made, and correct it.

   Answer:

   She read the number as $\frac{72}{100}$ instead of $\frac{72}{1000}$. It should be written as a fraction out of 100. Thus, $\frac{72}{1000} = \frac{72 \div 10}{1000 \div 10} = \frac{7.2}{100} = 7.2\%$.

3. Find $8\frac{1}{2}\%$ of 900.

   Answer:

   First, change $8\frac{1}{2}\%$ to a decimal.

   $8\frac{1}{2}\% = 8.5\% = \frac{8.5}{100} = \frac{8.5 \times 10}{100 \times 10} = \frac{85}{1000} = 0.085.$

   Now multiply $8\frac{1}{2}\%$ and 900 = 0.085 x 900 = 76.5.

Learning Activity 1.4

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. A school volleyball team wants to practice twice per week. They cannot practice at the school on the weekend (Saturday and Sunday), half the team cannot practice on Monday and Wednesday, and the basketball team uses the gym on Friday. Which days can the team practice?
2. An octave in music includes 8 notes. If you were to go up half an octave, how many notes is that?
3. When in Venice, you notice a great store on the other side of the street. Because the roads are water in Venice, you need to walk to the nearest bridge. The nearest bridge is 6 m away from you, and the 'road' is 2 m wide. How far do you have to walk to get to the store?
4. Write the percent as a decimal: 62%.
5. You are working at the stadium, where there is no electronic till. The customer is buying popcorn for $3.80. If the customer gives you a $5.00 bill, how much change will you give them?

Answers:

1. Tuesday and Thursday (Process of elimination: ~~S~~ ~~M~~ T ~~W~~ T ~~F~~ ~~S~~)
2. 4 notes (Half of $8 = \frac{1}{2} \times 8 = 4$.)
3. 14 m (You walk 6 m to the bridge, 2 m across the bridge, and then back 6 m to get to the store, so 6 + 2 + 6 = 14 m.)
4. 0.62
5. $1.20 (The easiest way to make change is to count up from the value owed until you reach the total amount of money you were given. In this case, that would look like: $3.80 + $0.20 = $4.00 and $4.00 + $1.00 = $5.00, so in total you would give the customer back $1.20.)

Part B: Converting among Fractions, Decimals, and Percents

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert these fractions to decimals.

   a) $\frac{5}{8}$

   b) $\frac{3}{4}$

   c) $3\frac{7}{8}$

   d) $15\frac{2}{3}$

   Answers:

   a) 0.625

   b) 0.75

   c) 3.875

   d) 15.666 666 666 = 15.67

2. Convert these fractions to decimals and to percents to fill in the chart.

   Answers:

Fraction	Decimal	Percent
$\frac{2}{5}$	0.4	40%
$1\frac{7}{8}$	1.875	187.5%
$\frac{27}{100}$	0.27	27%
$5\frac{6}{7}$	5.857 142 857 = 5.86	5.857 142 857 = 585.7%

3. Convert these decimals to fractions. Write your answers in lowest terms.

   a) 0.36

   Answer:

   $$0.36 = \frac{36}{100} = \frac{36 \div 4}{100 \div 4} = \frac{9}{25}$$

   b) 5.375

   Answer:

   $$5.375 = 5\frac{3}{8}$$

Learning Activity 1.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are going to the store to buy a drink with $2.05 in your pocket. If a drink costs $1.75, will you be able to buy one?
2. Which is larger: 0.76 or 0.07?
3. Solve for g: $3 - g = 15$.
4. Is an angle with a measurement of 86° acute, right, obtuse, straight, or reflex?
5. Complete the pattern: 4, 1, –2, ___ , ___ .

Answers:

1. Yes (You have more money than you will spend.)
2. 0.76 (In 0.07 the 7 is in the hundredths spot, in 0.76 the 7 is in the tenths spot.)

   or

   (0.76 is read seventy-six hundredths while 0.07 is read seven hundredths. Since 76 > 7, 0.76 > 0.07.)
3. –12 (If $3 - g = 15$, then $3 = 15 + g$ or $-12 = g$.)
4. Acute (86^{o} is less than 90^{o}.)
5. –5, –8 (The pattern is that you subtract 3 each time.)

Part B: Gross Pay

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Label the following income situations as hourly wage, salary, or contract.
   a) Enrico is willing to re-roof your house for $3200.

      Answer: Contract

   b) Karen works in a clothing store in the mall on weekends.

      Answer: Hourly wage

   c) Luigi is hired to install paving stones for your driveway and sidewalk.

      Answer: Contract

   d) Henry is a supervisor working for the government.

      Answer: Salary

2. Calculate Joe's gross income if he earns $9.25 per hour at a restaurant and is allowed to keep 65% of the tips he receives. He worked 29 hours and received $370 in tips.

   Answer:

   Wage = $9.25 × 29 = $268.25

   Earnings from tips = $370 × 0.65 = $240.50

   Gross pay = $268.25 + $240.50 = $508.75

3. Mary receives an annual salary of $56,000 as an office manager. Find her weekly gross pay.

   Answer:

   $$\text{Weekly gross pay} = \frac{\$56,000}{52} = \$1076.92$$

4. Tom is paid a salary of $3000 per month to manage workers at a lawn care business. Find his weekly gross pay.

   Answer:

   Annual salary = $3000 × 12 = $36,000

Weekly gross pay = $\frac{\$36,000}{52} = \$692.31$ per week

Learning Activity 1.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are a very busy person! You have soccer on Sunday, Tuesday, and Thursday nights. You have music lessons on Saturday in the evening. You also have swimming on Monday night. Which nights of the week do you not have any commitments?
2. A basic calculator has 6 rows, and 5 columns of buttons. How many buttons does it have in total?
3. Three students receive their marks for a project. Jane found her mark as a decimal, 0.62; John calculated his mark as a percent, 83%; Jean got $\frac{12}{16}$. Who got the best mark?
4. June is a big birthday month for you. Your brother's is on June 9th, your nephew's is on June 20th, plus Father's Day is in June! You want to spend $30 on each present and you have $85.00 saved up. Will this be possible?
5. If $0.33\overline{3} = \frac{1}{3}$, then what does $0.66\overline{6}$ equal?

Answers:

1. Wednesday and Friday (Process of elimination: ~~S~~ ~~M~~ ~~T~~ W ~~T~~ F ~~S~~)
2. 30 (5 × 6)
3. John (Convert all the marks into one form; e.g., percent, Jane got 0.62 = 62%; John got 83%; Jean got $\frac{12}{16}$, reduced is $\frac{3}{4}, = 75\%.$)
4. No (If you buy 3 gifts for $30 each, the total cost will be 3 x $30 = $90. You have less than $90.)
5. $\frac{2}{3}$ ($0.66\overline{6}$ is double $0.33\overline{3}$, so the fraction should also be doubled—$\frac{1}{3} \times \frac{2}{1} = \frac{2}{3}.$)

Part B: Types of Income

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Label the following income situations as commission, salary plus commission, or piecework.
   a) Earning $10 for every snow blower assembled

      Answer: Piecework

   b) Earning income selling cosmetics by phone

      Answer: Commission

   c) Earning income selling fridges at The Bay

      Answer: Commission or salary plus commission

      (**Note:** Since The Bay is a large department store, the salespeople most likely earn a salary plus commission. However, without this specific information being supplied, either answer is correct.)

   d) Earning income selling life insurance

      Answer: Commission or salary plus commission

   e) Earning income sewing gloves

      Answer: Piecework

2. Harry sells cars, earning a $2\frac{3}{4}\%$ commission on each sale. Calculate his gross earnings if his sales totalled $61,300 last week.

   Answer:

   Commission rate = $2\frac{3}{4}\% = 2.75\% = 0.0275$

   Gross pay = $\$61{,}300 \times 0.0275 = \$1685.75$

3. Patty sells furniture. The store pays her a base salary of $275 per week, plus a $3\frac{1}{2}\%$ commission on her sales. If her sales last week were $4655, find her gross income.

   Answer:

   Commission rate = $3\frac{1}{2}\% = 3.5\% = 0.035$

   Commission earnings = $4655 × 0.035 = $162.93

   Base salary = $275.00

   Total income = $162.93 + $275.00 = $437.93

4. Keith has a job where he assembles garden sheds, which are then sold. He is paid $22 for each shed he assembles, but has $10 per shed deducted for any faulty workmanship. If he assembles 42 sheds, but the manager returns 12 of them, find his gross income for the week.

   Answer:

   Pay for sheds completed = 42 × $22 = $924

   Pay deducted for sheds returned = 12 × $10 = $120

   Gross income = $924 – $120 = $804

Learning Activity 1.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A Canadian football field is 100 yards. A scoring drive started on one goal line and went the entire length of the field. On average, each play advanced the ball 5 yards. How many plays were in the scoring drive?
2. Rewrite the following fraction in lowest terms: $\frac{18}{27}$.
3. There is a 50% discount on all candy at the store the day after Hallowe'en. If it cost you $30 to buy candy before Hallowe'en, how much would you spend if you bought the same candy after Hallowe'en?
4. What is the mean of 3, 4, 6, and 7?
5. Jamie is twice as old as Dan. Dan is 3 times as old as Kim. If Kim is 4 years old, how old is Jamie?

Answers:

1. 22 plays (110 ÷ 5 = 22 or 100 ÷ 5 = 20 and 10 ÷ 5 = 2, so 20 + 2 = 22 plays)
2. $\frac{2}{3}$ (In order to rewrite a fraction in lowest terms, you have to remove all the common factors or the greatest common factor. In this case, the greatest common factor is 9, so we divide the numerator by 9 and the denominator by 9; $\frac{18 \div 9}{27 \div 9} = \frac{2}{3}$.)
3. $15 (50% off is the same thing as half off. Half of 30 or $\frac{1}{2} \times 30 = \$15$.)
4. 5 (Mean is the sum of the numbers, divided by ***how many*** numbers there are. In this case, it is $\frac{3+4+6+7}{4} = \frac{20}{4} = 5$.)
5. Jamie is 24. Dan: 3 × 4 = 12, Jamie: 2 × 12 = 24.

Part B: Gross Pay including Overtime

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given that your employer pays time and one-half for any hours over 40 worked in a week, calculate the gross pay if you worked 43.75 hours at a wage of $11.15 per hour.

   Answer:

   Regular pay = 40 hours × $11.15 = $446

   Overtime rate = 1.5 × $11.15 = $16.73

   Overtime pay = 3.75 × $16.73 = $62.74

   Total gross pay = $446 + $62.74 = $508.74

2. Given that your employer pays overtime at time and one-half for any hours worked over 8 hours each day, calculate your gross pay if your wage is $9.68 per hour and you worked the following schedule.

Monday	Tuesday	Wednesday	Thursday	Friday
6	5	10.5	7	6

   Answer:

   Regular hours = 6 + 5 + 8 + 7 + 6 = 32 hours

   Overtime hours = 0 + 0 + 2.5 + 0 + 0 = 2.5 hours

   Regular pay = 32 × $9.68 = $309.76

   Overtime rate = $9.68 × 1.5 = $14.52

   Overtime hours = 2.5 hours × $14.52 = $36.30

   Total gross income = $309.76 – $36.30 = $346.06

3. Your employer will pay you time and one-half for any hours worked over 40 per week, time and one-half for any hours worked on Saturday, and double time for any hours worked on Sunday. Calculate your gross pay given the hourly wage of $11.35 and the following schedule.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8	9.5	10.25	7	8	6	4

Answer:

Hours

Total hours from Monday to Friday = 8 + 9.5 + 10.25 + 7 + 8 = 42.75 hours

Regular hours = 40 hours

Overtime hours at time and a half = 2.75 hours

Saturday hours at time and a half = 6 hours

Sunday hours at double time = 4 hours

Rates

Regular hourly rate = $11.35

Overtime rate at time and one-half = 1.5 × $11.35 = $17.03

Double time rate = 2 × $11.35 = $22.70

Income

Regular pay = 40 hours × $11.35 = $454.00

Overtime pay at time and one-half = (2.75 + 6) × $17.03 = 8.75 × $17.03 = $149.01

Overtime pay at double time = 4 hours × $22.70 = $90.80

Total gross pay = $454.00 + $149.01 + $90.80 = $693.81

Learning Activity 1.8

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. There are 20 cheese sticks in a package. You only eat one each day, and only on weekdays (Monday to Friday). How many weeks will it take you to finish the whole package?
2. Complete the following pattern: 60, 75, ___ , 105, ___ .
3. Is an angle of 235° an actue, an obtuse, a right, a straight, or a reflex angle?
4. Evaluate: $4 - 6 + 2 \times (3 - 8)$.
5. There are 3 pairs of socks in a package. If the whole package costs $6, how much does it cost per pair of socks?

Answers:

1. 4 weeks (Monday to Friday = 5 days. $20 \div 5 = 4$.)
2. 90, 120 (The pattern is to add 15 to the previous number.)
3. Reflex (235^o is larger than 180^o but smaller than 360^o.)
4. –12 (Use the order of operations: $4 - 6 + 2 \times (-5) = 4 - 6 + (-10) = -12$.)
5. $2 per pair of socks ($6 \div 3 = $2)

Part B: Time Cards

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Complete Janice's time card and calculate her gross earnings. She earns $12.56 per hour, and overtime is paid at time and one-half on any hours worked over 40 per week.

   Answer:

Employee: Janice Bird					Hourly Rate: $12.56		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	7:45	12:00	1:00	5:15			8.5
Tues.	8:01	12:03	12:57	5:01			8
Wed.	8:00	12:00	1:01	5:27	7:00	9:45	11.25
Thurs.	7:58	11:59	1:01	5:00	7:00	9:15	10.25
Fri.	8:00	12:00	1:01	5:15			8.25
Sat.							
Sun.							
							46.25

	Hours	Rate	Amount
Regular	40	$12.56	40 x $12.56 = $502.40
Overtime	6.25	$12.56 x 1.5 = $18.84	6.25 x $18.84 = $117.75
Total Hours	46.25	Gross Wages	$502.40 + $117.75 = $620.15

Note:

Monday: 4.25 + 4.25 = 8.5

Tuesday: 4 + 4 = 8

Wednesday: 4 + 4.5 + 2.75 = 11.25

Thursday: 4 + 4 + 2.25 = 10.25

Friday: 4 + 4.25 = 8.25

2. Justin is paid $9.95 per hour, and overtime is calculated on any hours over 8 each day, and is paid at time and one-half. Any hours worked on Sunday is paid at double time. Use the time card to calculate his gross earnings.

Employee: Justin					Hourly Rate: $9.95		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	07:00	12:00	13:00	17:30			9.5
Tues.	07:01	12:03	13:00	17:00			9
Wed.	07:00	11:30					4.5
Thurs.	07:30	12:00	13:01	18:15			9.75
Fri.							
Sat.							
Sun.			12:00	18:30			6.5
							39.25

	Hours	Rate	Amount
Regular	8 + 8 + 4.5 + 8 = 28.5	$9.95	28.5 x $9.95 = $283.58
Overtime x 1.5	1.5 + 1 + 1.75 = 4.25	$9.95 x 1.5 = $14.93	4.25 x $14.93 = $63.45
Overtime x 2	6.5	$9.95 x 2 = $19.90	6.5 x $19.90 = $129.35
Total Hours	39.25	Gross Wages	$238.58 + $63.45 + $129.35 = $476.38

Note:

Monday: 5 + 4.5 = 9.5

Tuesday: 5 + 4 = 9

Wednesday: 4.5

Thursday: 4.5 + 5.25 = 9.75

Sunday: 6.5

Learning Activity 1.9

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is 10% of 500?
2. What is 5% of 500?
3. What is 15% of 500?
4. The distance to the mall from your house is 8 km. Your friend lives half as far away from the mall. What is the distance from your friend's house to the mall?
5. You give the cashier a $10 bill to pay for your lunch. If the total for your lunch is $7.60, how much change will you get back?

Answers:

1. 50 (To find 10%, move the decimal to the left one place.)
2. 25 (5% is half of 10%.)
3. 75 (15% is the sum of 5% and 10%.)
4. 4 km
5. $2.40 (Remember to count up from the amount due to the amount paid: $7.60 + $0.40 = $8, $8 + $2 = $10.)

Part B: Comparing Payment Options

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Monica has been offered a job selling cosmetics in a retail store. She must choose her salary payment plan. Option A will pay her $12.50 per hour, and she will work a 40-hour week. Option B will pay her 25% commission on all sales.

   a) Find her gross income for Option A.

   Answer:

   $12.50 × 40 = $500

   b) Under Option B, what level of sales figures must she generate to match Option A?

   Answer:

   This problem-solving skill will be practised often in this course.

   Set up a formula relating sales, percent rate of commission, and commission income.

   Sales × 25% = Commission Income

   Then substitute the known values into the formula and solve for the unknown value.

   Sales × 0.25 = $500

   $\frac{S \times 0.25}{0.25} = \frac{500}{0.25}$ (Divide both sides of the equation by 0.25 to isolate "Sales")

   S = 2000

   Monica must generate at least $2000 worth of sales to make as much income on commission as she would on an hourly wage.

   c) Give an advantage to each pay option for Monica.

   Answer:

   Option A: She earns the same pay each week even if the sales are slow.

   Option B: She could make more income if she were good at sales.

d) Give a disadvantage to each pay option for Monica.

Answer:

Option A: There is no incentive to increase pay.

Option B: If the product is not wanted or is over-priced, her income could be low.

e) Why would the store prefer its sales staff to be paid on commission?

Answer:

Such a policy would likely motivate their employees to have increased sales.

2. Steve works installing drywall. He is paid $22 per hour, and averages about 35 hours per week. The company wants to place him on an annual salary of $41,000.

a) Find his weekly gross income for each option.

Answer:

Hourly wage = $22 × 35 = $770 per week

Salary = $\frac{\$41,000}{52} = \$788.46$ per week

b) Give an advantage for Steve for each option.

Answer:

Hourly wage: If business increases, he will make more money.

Salary: If business drops off, he'll still be paid.

c) Give a disadvantage for Steve for each option.

Answer:

Hourly wage: Business could drop off, and he would make much less money.

Salary: He has no control over the number of hours he works, and being on salary, he would not earn overtime pay.

Notes

Grade 10 Essential Mathematics (20S)

Module 2
Net Pay

Module 2:
Net Pay

Introduction

In this module, you will learn to calculate net pay. Gross income is what you earn, but that is not usually the amount you take home. Various deductions are subtracted from your gross pay. This module examines the three mandatory deductions, and teaches you how to calculate them. You will use the CPP, EI, and income tax tables from the Government of Canada as well as formulas to find specific deductions. Your net pay is your gross pay minus the deductions.

Most of your work in this module will involve income earned from weekly pay and weekly deductions.

Assignments in Module 2

When you have completed the assignments for Module 2, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Pay Yourself First/The Sales Representative
1	Assignment 2.1	Common Deductions
2	Assignment 2.2	CPP
3	Assignment 2.3	EI
4	Assignment 2.4	Income Tax
5	Assignment 2.5	Net Pay
6	Assignment 2.6	Comparisons and Spreadsheets
7	Assignment 2.7	Formulas and Errors

Resource Sheet

When you write your midterm examination, you will be allowed to bring a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Midterm Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 2. You might use your Module 2 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 1, 2, 3, and 4, to prepare your Midterm Examination Resource Sheet. Remember, the midterm examination is based only on the first four modules of the course.

Resource Sheet for Module 2

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Midterm Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Module 2 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work in the spaces provided. The cover assignment can be done at any time while you are completing Module 2. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on the rubric shown on the next page. You can see that the cover assignment is worth a total of 12 marks. Those marks are allotted according to how well you are able to satisfy the requirements explained in the rubric. If you have any questions about the scoring, be sure to contact your tutor/marker for clarification.

Module 2 Cover Assignment Rubric
Pay Yourself First / The Sales Representative

Score Level	Mathematical Knowledge (Did I know how to do the question?)	Process Knowledge (What strategies and methods did I use to get the answer?)	Communication (Did I communicate my answer clearly?)
4	I got the right answers. I used math terms correctly to show I understand how math works. I made no errors in my calculations.	I found all the important parts of the problems, and I knew how to put them together to solve the problem. I showed all the steps to solve the problems. I showed calculations if I used them in my answers.	I labelled the answers correctly. In the drawing I used, I could write an explanation for all of it.
3	I made minor errors in my calculations. I used most of the math vocabulary correctly.	I found most of the important parts of the problem. I showed most of the steps I used to solve the problem.	In the drawing I used, I could write an explanation for most of it.
2	I know how to do parts of the problem, but I made major errors in the calculations and got a wrong answer. I gave only part of the answer.	I found some of the important parts of the problem. I showed some of the steps, but my solution was not clear.	In the drawing I used, I could write an explanation for some of it.
1	I tried to do the problems but I don't understand them.	I couldn't find the important parts of the problem. I showed solutions that are not reasonable. I showed very few steps that I used to solve the problem. I included unnecessary information.	I wrote something that didn't go with my answer. I wrote an answer that was not clear. The drawing I used didn't go with my answer.
0	I didn't try to answer the problems.	I didn't show a method.	I didn't explain anything, either by writing about it or by drawing a picture.
Mark	Math = ____ / *(4 marks)*	Process = ____ / *(4 marks)*	Communication = ____ / *(4 marks)*
Total Mark	Math + Process + Communication = _____		Total Possible: 4 + 4 + 4 = 12

Module 2 Cover Assignment

Total: 12 marks

Part A: Pay Yourself First

One of the first rules of planning for your financial well-being is to "pay yourself first." This means that each payday you put a portion of your earnings into a type of savings plan. Many people find once they are in the habit of regularly putting aside some of their earnings, they can still manage to live on the remainder of their income.

You can invest your savings in various ways. In subsequent courses of Essential Mathematics you will learn about such investments. You may already have established a savings plan for yourself.

Financial planners often suggest setting aside at least 10% of your wages from each paycheque.

1. If you saved 10% of your gross weekly wages, how much would you have saved in a year if you earned the following amounts?

   a) $150 a week

   b) $450 a week

   c) $700 a week

continued

Module 2 Cover Assignment: Pay Yourself First (continued)

2. What are two advantages of paying yourself first?

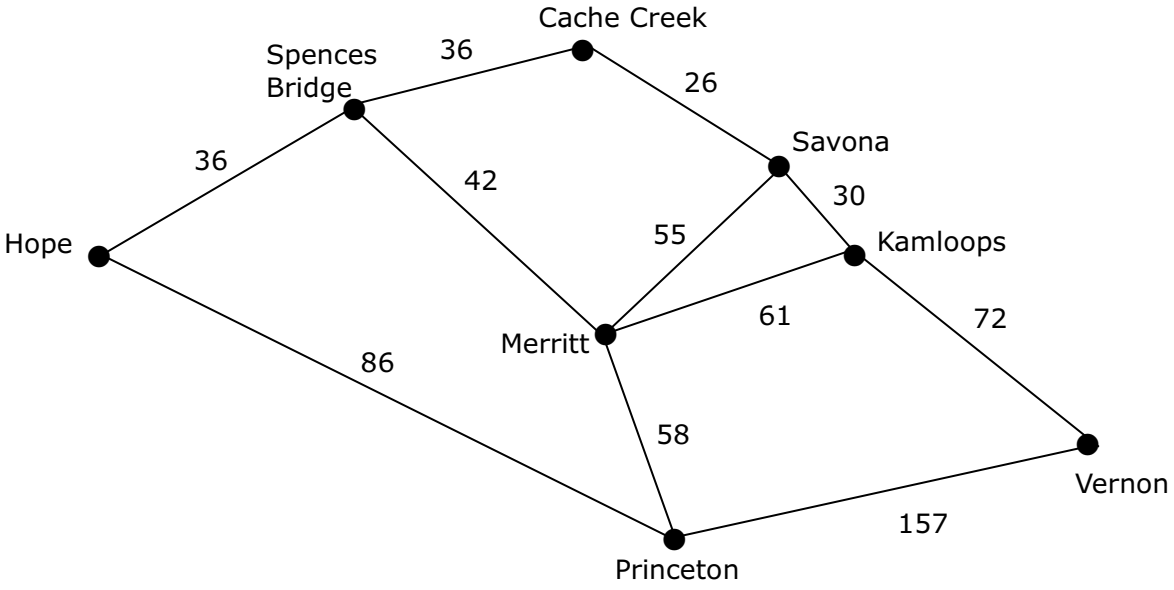
3. What are two disadvantages of paying yourself first?

continued

Part B: The Sales Representative

A travelling sales representative, whose territory includes a part of British Columbia, wishes to travel to each of the towns on the map below. The trip is to begin and end in Spences Bridge. The distances between towns are indicated in kilometres.

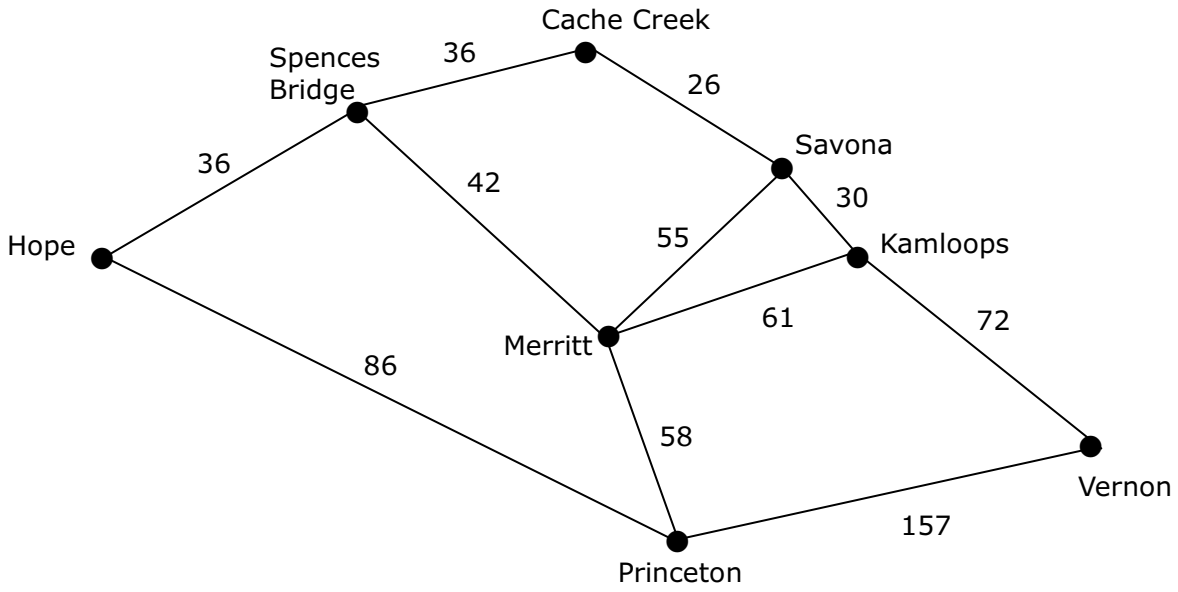
1. Find the route that passes through each town exactly once and satisfies the given condition. Show your route on the map below using a coloured pencil.



continued

Module 2 Cover Assignment: Pay Yourself First (continued)

2. Find the shortest route that includes each town at least once. You can pass through a town more than once. Show your route on the map below using a coloured pencil. Remember to take into account the given condition. Explain why you think this is the shortest route.



3. Calculate the number of kilometres in the shortest route you have drawn in Question 2 above.

Lesson 1: Common Deductions

Lesson Focus

In this lesson, you will

- ❑ explore various deductions from your paycheque
- ❑ find net pay

Lesson Introduction

When you agree to work at a job, you have to expect that certain deductions will be taken off your gross pay. This lesson reviews some common deductions.

Deductions from Pay

Young people who are new to the work world are often surprised at the gap between the gross pay they earn and the actual amount of net pay they take home. Of course, different places of employment have different kinds of deductions. When applying to work somewhere, you should always ask about mandatory and optional deductions from your pay.

Some common deductions include parking, dental plan, medical insurance, social fund, coffee or water fund, and union dues. Depending on where you are employed, these may or may not be mandatory. You also might be offered an opportunity to have RSP (Retirement Savings Plan) or Canada Savings Bond contributions deducted off your gross pay. Not every place of employment has these deductions. Perhaps only a few deductions will apply where you choose to work, but you should check it out before accepting a job there.

The basic three, CPP, EI, and income tax deductions, are deducted from everyone's paycheque and will be examined in further detail in later lessons.

Common Deductions

Union Dues

In some work places, the employees belong to a union. The union negotiates the level of pay, working conditions, holidays, and other benefits. If you accept a job offer at a place where the workers are unionized, you will have union dues subtracted from your gross pay. This would be a condition of employment.

Medical or Dental Plan

It is helpful to belong to a medical or dental plan that covers all or most of your prescription, ambulance, or dental costs, but this coverage is not free. You have to pay to belong to such a plan, and that cost is deducted from your gross pay. The amount of the deduction is dependent on whether you are single or have a family, and on the extent of the coverage. The plans that offer to pay a smaller percentage of your medical or dental costs usually require lower contributions from employees.

Parking

Those parking lots behind places of business are expensive to maintain. Such items as paving, snow removal, and electrical outlets can be quite costly for the business. Many businesses expect a parking fee from their employees. This fee would also be deducted from your gross pay. Usually, the parking stalls which include a plug for your vehicle during the cold winter months are more expensive than the spots without the electrical option. Normally, this deduction is optional, depending on whether or not you need to drive to and from work.

Coffee or Water Fund

Those big bottles of water and the dispenser they sit upon are not free. Somebody has to pay for them. Also, many offices will operate a coffee fund, where employees contribute money to buy the necessary supplies. These deductions are only applied to those workers who drink the water or the coffee. Workers usually contribute a given amount in cash and such a contribution would not be taken off your gross pay. But, it is still a cost of working at that business.

Social Fund

If you are working with a large group of people, chances are very good that one or more of them will be getting married, will have children, or will suffer an illness. In this event, your work's social committee will send flowers or a small gift on behalf of all of the employees. Your contribution to the social fund normally would not be taken off your cheque, but would be given as a cash contribution. This contribution is not mandatory.

Uniforms

In the project in Module 1 you were encouraged to ask about a dress code at the job site. Surprisingly, many workplaces demand that employees wear items like steel-toed boots, or coveralls, or dress pants and shirts of a certain colour. Generally you will have to provide these items at your own expense. Some companies provide you with a uniform, and deduct that cost from your paycheque. These are costs of working there, and must be considered before accepting a job.

Tools

People do not like to share their tools. If you lend your tools, they might get broken or even disappear. If you are looking at working on a job site that uses tools, be sure to check out what tools you are expected to provide. If you need to provide all your tools, purchasing them could be costly at the outset of a new job. Although these costs are not deducted from your gross pay, they are "up-front" costs that will affect how much money you will make working at such a workplace.

Finding Net Pay

Simply stated, net pay is what is left of your gross pay after all the deductions have been subtracted.

Net Pay = Gross Pay — Deductions

To calculate your net pay, or what many call take-home pay, you could follow these steps.

Step 1: Determine your gross pay.

Step 2: Determine your total deductions.

Step 3: Determine your net pay by subtracting the total deductions from your gross pay.

Include the formula for calculating net pay and the steps involved on your resource sheet.

Example 1

Gary works at a bus manufacturing plant. He is paid $13.70 per hour, with time and one-half paid on all hours worked over 40 per week. Last week he worked 43 hours. His deductions include the following.

Union dues	$2.90
Parking	$3.50
CPP	$26.85
EI	$10.55
Federal income tax	$82.45
Provincial income tax	$62

Find Gary's net pay.

Solution:

Step 1: Find Gary's gross pay.

Regular pay = 40 × $13.70 = $548

Overtime rate = $13.70 × 1.5 = $20.55

Overtime pay = 3 × $20.55 × 1.5 = $61.65

Gross pay = $548 + $61.65 = $609.65

Step 2 Find the total of all his deductions.

Deductions = $2.90 + $3.50 + $26.85 + $10.55 + $82.45 + $62 = $188.25

Step 3 Find net pay.

Net pay = gross pay — deductions = $609.65 — $188.25 = $421.40

With all of the deductions, Gary's take-home pay is quite a bit less than what he planned. The CPP, EI, and income tax deductions are determined from the 2009 payroll tables. You will learn to use these tables in later lessons in this module.

Example 2

Laurel accepts a position in a large firm as an executive assistant. She will be paid a monthly salary of $2950. The firm charges her $5.25 per week for parking. She contributes $2 per week to the social fund. Laurel's weekly deductions include CPP $30.37, EI $11.78, federal income tax $92.55, and provincial income tax $70.65. Find Laurel's net pay.

Solution:

Step 1 Find Laurel's gross weekly pay.

Monthly pay = $2950

$$\text{Weekly pay} = \$2950 \times \frac{12}{52} = \$680.77$$

Step 2 Find the total of all her deductions.

Deductions = $5.25 + $2.00 + $30.37 + $11.78 + $92.55 + $70.65 = $212.60

Step 3 Find net pay.

Net pay = gross pay — deductions = $680.77 – $212.60 = $468.17

Learning Activity 2.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. It costs $4.00 for a package of 3 chocolate bars. Geri spends $20 on chocolate bars. How many chocolate bars does she buy?
2. You worked 35 hours this week. Your hourly wage is $10. How much was your gross pay?
3. You are running errands all day. You have to go to the nursery, so you drive 8 km. You then go to the mall, so you drive another 6 km. Finally, you go to the movie store, which is another 3 km, then home—driving 6 km. How far did you drive all together?
4. Write the percent as a decimal: 3.5%.

continued

Leaning Activity 2.1 (continued)

5. Arrange the numbers from largest to smallest: $\frac{1}{2}$, 0.29, $\frac{3}{4}$, 0.65, 0.34.

Part B: Deductions and Net Pay

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Sam works for a seed company that pays him $11.88 per hour. He worked 32 hours last week. His deductions include CPP $15.49, EI $6.58, and income taxes of $50.80 and $38.55. He also contributes $2 per week to the water fund. Find his net pay.

2. Betty is paid a salary of $2775 per month as a clerk. Her weekly deductions include $5 for parking, $12.68 for a dental plan, CPP $28.37, EI $11.08, and income taxes of $86.95 and $65.85. Find her net pay.

3. David sells cars and is paid strictly on a commission of $1\frac{3}{4}\%$ of total sales.
   Last week, he sold two cars worth a total of $42,000. His deductions include CPP $33.05, EI $12.72, and income taxes of $100.35 and $77.40. He contributes $5 per week to the social fund. Find his net pay.

Lesson Summary

This lesson discusses ten types of deductions from your gross pay you could experience when you start work. You wrote a formula for determining net pay, and applied it to problem-solving situations.

In the next lesson, you will look more closely at exactly how the CPP deduction is calculated.

Assignment 2.1

Common Deductions

Total Marks = 15

1. Chad is a real estate salesman, earning $1\frac{1}{2}\%$ commission on all his sales. Last year, he sold 12 houses with a total value of $2,550,000. On average, his weekly deductions were CPP $33.08, EI $12.73, and income taxes of $100.35 and $77.40.

   a) Find Chad's average weekly earnings. *(2 marks)*

   b) Find Chad's weekly net pay. *(3 marks)*

2. Jocelyn earns $17.32 per hour, with any hours over 40 per week being paid at time and one-half. Last week, she worked 47.25 hours. Her deductions include union dues of $2.85, parking $4, CPP $ 40.29, EI $15.24, and income taxes of $129.95 and $94.80.

   a) Find her gross pay. *(3 marks)*

continued

b) Find her net pay. *(2 marks)*

3. Eric receives a basic weekly salary of $225, and 3% commission on all sales. Last week, his sales totalled $10,235. His deductions include $3.50 for the water fund, CPP of $23.01, EI $9.20, and income taxes of $72.10 and $53.70. Eric also contributes $3 per week to the coffee fund, and $1 to the social committee.

   a) Find his gross pay. *(3 marks)*

   b) Find his net pay. *(2 marks)*

Lesson 2: Canada Pension Plan

Lesson Focus

In this lesson, you will

- ❑ understand the history and reason for CPP deductions from gross pay
- ❑ read tables to find CPP deductions
- ❑ use a formula to calculate CPP deductions

Lesson Introduction

This lesson deals with one of the three mandatory deductions from your gross pay. All workers over 18 years of age must pay into this federal plan. This lesson will show you how to calculate the CPP deduction using tables or using a formula.

Canada Pension Plan

The Canada Pension Plan (CPP) is an earnings-related pension program run by the federal government. This national pension plan was initiated in 1966. Everyone across Canada over and including the age of 18 years who earns an income contributes a portion of their earnings to the CPP. This money funds a national retirement fund.

Once you reach retirement age, you are entitled to a monthly pension from the CPP. The amount of your monthly pension is dependent on the contributions you make over your working years. Your employer also makes CPP contributions for all employees.

Using Tables

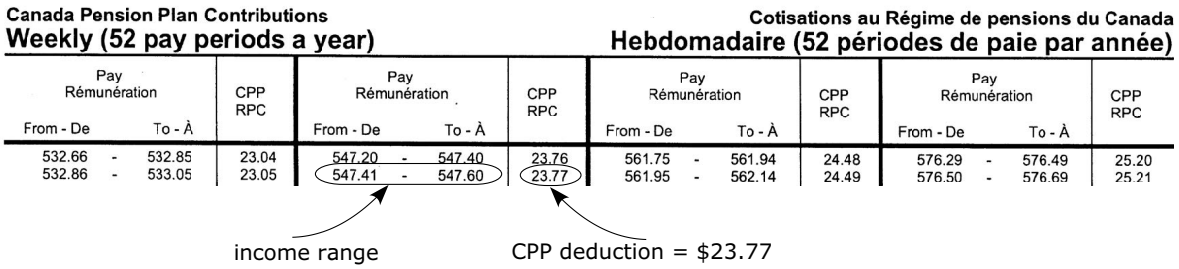
Every year the federal government issues a set of payroll deduction tables to determine the exact CPP contributions you have to make based on your income. The tables you will need for these lessons are shown in the examples, learning activities, assignments, and on the examination.

To use the tables, you find the income level and read the required CPP contribution beside it. All of your calculations will be on *weekly earnings*.

You can access the 2009 rates at the Canada Revenue Agency (CRA) website:

www.cra-arc.gc.ca/tx/bsnss/tpcs/pyrll/t4032/jn09/cpp4032_52pp-RDM-eng.html

Here is a sample from the 2009 CPP tables. If you made $547.52, you look for that amount within the ranges provided. Then you slide to the right, and the CPP contribution you must make is shown.



Canada Pension Plan Contributions
Weekly (52 pay periods a year)

Cotisations au Régime de pensions du Canada
Hebdomadaire (52 périodes de paie par année)

Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC
532.66	532.85	23.04	547.20	547.40	23.76	561.75	561.94	24.48	576.29	576.49	25.20
532.86	533.05	23.05	547.41	547.60	23.77	561.95	562.14	24.49	576.50	576.69	25.21

Example 1

Rudy earned $762.81 last week. Use the tables to find his CPP contribution.

Solution:

Canada Pension Plan Contributions

Weekly (52 pay periods a year)

From	To	CPP	From	To	CPP	From	To	CPP
732.86	733.05	32.95	747.41	747.60	33.67	761.95	762.14	34.39
733.06	733.25	32.96	747.61	747.80	33.68	762.15	762.35	34.40
733.26	733.46	32.97	747.81	748.00	33.69	762.36	762.55	34.41
733.47	733.66	32.98	748.01	748.20	33.70	762.56	762.75	34.42
733.67	733.86	32.99	748.21	748.41	33.71	762.76	762.95	34.43
733.87	734.06	33.00	748.42	748.61	33.72	762.96	763.15	34.44
734.07	734.26	33.01	748.62	748.81	33.73	763.16	763.36	34.45
734.27	734.47	33.02	748.82	749.01	33.74	763-37	763.56	34.46

Rudy would have $34.43 deducted off his gross pay for a CPP contribution.

Learning Activity 2.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What are the factors of 8?
2. You are getting ready for a barbeque you are hosting. It costs $1.50 for a package of 30 plastic cups. How much does each plastic cup cost?
3. At your barbeque you are providing the food, but will have a collection basket for donations to cover the cost. If you buy 2 packages of hamburgers for $12.00 each, and 1 package of chicken burgers for $15.00, how much must you collect to break even?
4. Write the following decimal as a fraction: 0.058.
5. Solve for h: $h + 12 = 32$.

Part B: Using the CPP Table

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the CPP contributions for the following weekly incomes from the CPP table shown in Example 1.

1. $763.20 _____
2. $734.10 _____
3. $747.50 _____
4. $748.08 _____

Using a Formula

You can also use a formula to calculate the CPP contributions. In 2009, the federal government set the CPP contribution at 4.95% of the gross pay to a maximum of $2118.60 per year. Values calculated using the formula are slightly higher than those shown on the tables. However, these values are close enough for the purpose of estimating your CPP deduction.

Include the maximum contribution to CPP on your resource sheet.

Using the formula method can be quicker than looking through the tables. The formula is

CPP = Gross Income × 4.95%

Add this formula to your resource sheet.

Example 1

Find the CPP on a gross income of $562.48, using the percent rate of deduction.

Solution:

To calculate the CPP on a weekly gross income of $562.48, you need only find 4.95% of the gross income. The value of 4.95% written as a decimal is 0.0495.

CPP = $562.48 × 0.0495 = $27.84

Example 2

Natalie worked 43.75 hours at $10.35 per hour. She received overtime at time and one-half on any hours over 40. Find her gross pay and her CPP deduction using the formula.

Solution:

Gross pay:

Regular pay = 40 × $10.35 = $414.00

Overtime rate = $10.35 × 1.5 = $15.53

Overtime pay = $15.53 × 3.75 = $58.24

Gross pay = $414.00 + $58.24 = $472.24

CPP = $472.24 × 0.0495 = $23.38

Learning Activity 2.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You were paid $180 from your part-time job. Your hourly wage is $9. How many hours did you work?
2. Which is smaller: $\frac{4}{5}$ or $\frac{7}{10}$?
3. In the novel trilogy *Lord of the Rings*, there are many rings of power—9 were given to men, 7 to the dwarves, 3 to the elves, and 1 to the evil mastermind. In total, how many rings of power are there?
4. You want to save $12,000 to buy a car 1 year from now. How much do you have to save per month to reach this goal?
5. Rewrite the fraction in lowest terms: $\frac{33}{21}$.

Part B: Using the Formula to Calculate the CPP Deductions

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the formula method to find the CPP deductions for the following gross weekly incomes.

1. a) $680
   b) $285.26
   c) $1285.62
   d) $87.60
2. Use the formula method to calculate Jane's CPP deduction for the year if her annual salary is $56,480.

Lesson Summary

In this lesson, you studied the CPP deduction, and learned two ways to find it.

In the next lesson, you will take a close look at the EI deduction.

Assignment 2.2

CPP

Total Marks = 16

1. Use the 2009 CPP tables shown below to find the CPP deductions for these weekly gross incomes. *(4 marks)*

Canada Pension Plan Contributions

Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC	Pay Rémunération			CPP RPC
From - De		To - Á		From - De		To - Á	
669.02	-	669.21	29.79	683.57	-	683.76	30.51
669.22	-	669.42	29.80	683.77	-	683.96	30.52
669.43	-	669.62	29.81	683.97	-	684.16	30.53
669.63	-	669.82	29.82	684.17	-	684.37	30.54
669.83	-	670.02	29.83	684.38	-	684.57	30.55
670.03	-	670.22	29.84	684.58	-	684.77	30.56
670.23	-	670.43	29.85	684.78	-	684.97	30.57
670.44	-	670.63	29.86	684.98	-	685.17	30.58
670.64	-	670.83	29.87	685.18	-	685.38	30.59

a) $670.00 _____

b) $684.05 _____

c) $670.60 _____

d) $669.69 _____

2. Use the formula method (4.95%) to find the CPP deductions on these gross earnings. *(4 marks)*

a) $258.24 weekly _____

b) $1020.86 weekly _____

c) $96.50 weekly _____

d) $56,000 annually _____

continued

Assignment 2.2: CPP (continued)

3. Shelley has a job where she earns 35% on all cosmetic sales. Last week, she sold $1500 in cosmetics. Find her gross earnings and her CPP deduction using the formula method (4.95%). *(2 marks)*

4. Larry earns $18.58 per hour as a machinist. He is paid overtime at time and one-half on any hours over 8 hours per weekday or any time on Saturday. There are no late penalties. Use the following time card to calculate his gross earnings, and then find his CPP deduction using the formula (4.95%). *(6 marks)*

Employee: Larry					Hourly Rate: $18.58		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	9:00	12:00	1:00	5:00	7:30	9:30	
Tues.	7:29	11:59	12:58	4:30			
Wed.			1:00	5:30	7:30	10:15	
Thurs.	7:30	12:02	1:01	5:30			
Fri.	8:00	12:00	1:02	5:00			
Sat.			1:00	4:45			
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

Lesson 3: Employment Insurance

Lesson Focus

In this lesson, you will

- ❑ understand the history and reason for EI deductions
- ❑ read EI tables to find EI deductions
- ❑ use a formula to calculate EI deductions

Lesson Introduction

The second major deduction from your gross pay is Employment Insurance. This lesson will show you how to calculate the EI deduction using tables or using a formula.

Employment Insurance

EI stands for employment insurance. Basically, if you work a given number of hours and you lose your job, you can draw some funding from EI until you find a new job. This insurance plan is operated by Canada's federal government. The number of hours you must work—usually between 500 and 700 before you qualify for EI support—varies with each region of Canada. Some areas have plenty of employment opportunities, so the standard is set higher. Other areas, which are isolated and without many job opportunities, have a lower minimum standard.

The history of the "Unemployment Insurance" plan dates back to the 1930s when jobs were scarce as a result of the Great Depression. In 1971, the government amended the act to allow benefits for people unable to work because of illness or maternity leave. In 1996, the government changed the name of the plan to "Employment Insurance."

All workers across Canada are required to contribute to this fund.

Using Tables

Every year the federal government issues a set of tables you can use to determine the exact EI contributions you have to make based on your income. To use the tables, you find the income level in the table and match it to the required EI contribution beside it. *All of your calculations will be on weekly earnings.* This is the same process you used to find CPP contributions using the tables.

Example 1

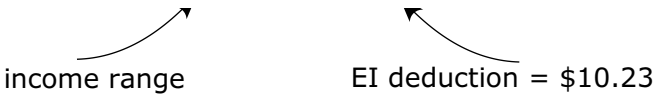
In 2009, you made a gross income of $591.52 in one week. Find your EI deduction using the 2009 EI table shown below.

Solution:

You look for the gross income of $591.52 within the ranges provided. Then you slide to the right, and the EI contribution you must make is shown.

Employment Insurance Premiums

From	To	EI	From	To	EI	From	To	EI
546.54	547.10	9.46	588.16	588.72	10.18	629.77	630.34	10.90
547.11	547.68	9.47	588.73	589.30	10.19	630.35	630.92	10.91
547.69	548.26	9.48	589.31	589.88	10.20	630.93	631.50	10.92
548.27	548.84	9.49	589.89	590.46	10.21	631.51	632.08	10.93
548.84	549.42	9.50	590.47	591.04	10.22	632.09	632.65	10.94
549.43	549.99	9.51	591.05	591.61	10.23	632.66	633.23	10.95



Your EI deduction is $10.23.

Learning Activity 2.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are going to bake a cake for your mom's birthday. Because your family is coming over, you decide to make a double recipe. In the original recipe you need $\frac{1}{2}$ a teaspoon of vanilla. How much vanilla will you need in the double recipe?
2. You are standing on the baseline of a basketball court. The top of the key is 25 feet away from you. The distance to centre court is 47 feet. What is the distance from the top of the key to centre court?
3. You are out for dinner with your best friend for their birthday. The bill comes and you pay for everything. If the total is $35.75 and you leave $40 on the table (including tip), how much are you tipping the server?
4. Is an angle that measures 35° acute, right, obtuse, straight, or reflex?
5. Estimate the CPP if the gross pay is $600.

continued

Learning Activity 2.4 (continued)

Part B: Employment Insurance

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the 2009 EI table to find the contributions for the following weekly incomes.

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á		From - De		To - Á	
499.72	-	500.28	8.65	541.33	-	541.90	9.37
500.29	-	500.86	8.66	541.91	-	542.48	9.38
500.87	-	501.44	8.67	542.49	-	543.06	9.39
501.45	-	502.02	8.68	543.07	-	543.64	9.40
502.03	-	502.60	8.69	543.65	-	544.21	9.41
502.61	-	503.17	8.70	544.22	-	544.79	9.42
503.18	-	503.75	8.71	544.80	-	545.37	9.43
503.76	-	504.33	8.72	545.38	-	545.95	9.44
504.34	-	504.91	8.73	545.96	-	546.53	9.45

1. $544.25 _____
2. $542.85 _____
3. $500.00 _____
4. $503.49 _____

Using a Formula

You can also use a formula to calculate the EI contributions. In 2009, the federal government set the EI contribution at 1.73% of your gross income to a maximum contribution of $731.79 per year. Calculated values are exactly the same as those shown on the tables. Using the formula method can be quicker than looking through the tables. The formula is

EI = Gross Income × 1.73%.

To change 1.73% to a decimal, write 0.0173.

Include the maximum contribution to EI as well as the formula to calculate EI on your resource sheet.

Example 1

If your gross income is $562.48, find your EI deduction using the formula.

Solution:

If your gross income is $562.48, then your EI contribution is $562.48 × 0.0173 = $9.73

Example 2

Natalie worked 43.75 hours at $10.35 per hour. She received overtime at time and one-half on any hours over 40. Find her gross pay and her EI deduction, using the formula.

Solution:

Gross pay

Regular hours = 40 × $10.35 = $414.00

Overtime rate = 1.5 × $10.35 = $15.53

Overtime pay = 3.75 × $15.53 = $58.24

Gross pay = $414.00 + $58.24 = $472.24

EI = $472.24 × 0.0173 = $8.17

Learning Activity 2.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the range of the following numbers: 0.2, 0.6, 0.08, 0.5, 0.03?
2. A dozen muffins costs $8.66. How much would you expect half a dozen muffins to cost?
3. Solve for d: $3d = 9$.
4. Your mom is buying ice cream for your family. The store has tiger, bubblegum, vanilla, and chocolate flavours. Your mom doesn't like bubblegum, your dad doesn't like chocolate, and you don't like vanilla. Which ice cream will your mom buy?
5. You have to walk to school. To get there, you walk 3 blocks north, 2 blocks west, another 6 blocks north, and then 1 block west. Each block is 100 m. How far do you walk?

Part B: Calculating EI Using the Formula

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the formula method to find the EI deductions for the following gross incomes.

1. $680 _____
2. $285.26 _____
3. $1285.62 _____
4. $87.60 _____
5. $56,480 _____

Lesson Summary

In this lesson, you studied the EI deduction, and learned two ways to find it.

In the next, lesson you will take a close look at the income tax deduction.

Notes

Assignment 2.3

EI

Total Marks = 16

1. Use the EI table to find the EI deductions for these weekly gross incomes. *(4 marks)*

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á		From - De		To - Á	
666.19	-	666.76	11.53	707.81	-	708.38	12.25
666.77	-	667.34	11.54	708.39	-	708.95	12.26
667.35	-	667.91	11.55	708.96	-	709.53	12.77
667.92	-	668.49	11.56	709.54	-	710.11	12.28
668.50	-	669.07	11.57	710.12	-	710.69	12.29
669.08	-	669.65	11.58	710.70	-	711.27	12.30
669.66	-	670.23	11.59	711.28	-	711.84	12.31
670.24	-	670.80	11.60	711.85	-	712.42	12.32
670.81	-	671.38	11.61	712.43	-	713.00	12.33

a) $712.85 _____

b) $669.25 _____

c) $708.40 _____

d) $667.04 _____

2. Use the formula method (1.73%) to find the EI deductions on these gross earnings. *(4 marks)*

a) $258.24 weekly _____

b) $1020.86 monthly _____

c) $96.50 weekly _____

d) $56,000 annually _____

continued

Assignment 2.3: EI (continued)

3. Lynne has a job where she earns 38% on all cosmetic sales. Last week, she sold $1800 in cosmetics. Find her gross earnings and the EI deduction using the formula. *(2 marks)*

4. Bill earns $17.58 per hour as a mechanic. He is paid overtime at time and one-half on any hours over 8 hours per day during weekdays. On Saturdays and Sundays, he is paid double time. He is not deducted pay for being late. Use the following time card to calculate his gross earnings, and then find the EI deduction using the formula (1.73%). *(6 marks)*

Employee: Bill					Hourly Rate: $17.58		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	7:30	12:00	1:00	5:00	7:30	9:15	
Tues.	7:29	11:59	12:58	4:30			
Wed.			1:00	5:30	7:00	10:45	
Thurs.	7:30	12:02	1:01	5:32	7:00	9:30	
Fri.	8:00	12:00	1:02	5:00			
Sat.			1:00	5:45			
Sun.							

	Hours	Rate	Amount
Regular			
Overtime (Mon.–Fri.)			
Overtime (Sat.–Sun.)			
Total Hours		Gross Wages	

Lesson 4: Income Tax

Lesson Focus

In this lesson, you will

- ❑ understand the history and reason for income tax deductions from gross pay
- ❑ read tax tables to find income tax deductions
- ❑ use formulas to find income tax deductions

Lesson Introduction

Governments, both provincial and federal, need revenue to build and maintain roads, fund the military, and run hospitals, among other things. Most of this income comes from taxes.

This lesson will show you two methods to calculate the federal and provincial income tax you must pay based on your gross income.

Income Tax

Income tax is exactly what it says, it is a tax on income. The more money you earn, the more income tax you pay.

Governments, both provincial and federal, need revenue to operate. Most of this income comes from income tax. The two previous deductions from gross pay were very specific: CPP for retirement income and EI for income if you are out of work. Both are deposited into special funds.

Income tax revenue goes into a more general fund. The provincial governments get a portion of the income tax deducted from your gross pay. The federal government collects federal and provincial taxes at the same time.

The federal government uses revenue to fund the military, the RCMP, immigration, national parks, and passports. The provincial governments require revenue to fund hospitals, universities, schools, roads, and tourism.

History

There wasn't always a tax on income in Canada. In the early 1900s, the government of the day wanted to encourage immigration, so they never required an income tax. But, when World War I started, the government needed money to outfit the military. So, in 1917, the income tax was introduced as a temporary measure, strictly to fund the costs of being at war. The debt just kept rising, and the government couldn't afford to stop taxing the income of Canadians.

Calculations

In this introductory course, basic calculations using tables and formulas to find the income tax deductions are to be done. There are more detailed rules and regulations as to what is not taxed, like union dues or RSP contributions. Students who are interested in a more in-depth study of tax rules could access the Canada Revenue Agency (CRA) website at www.cra-arc.gc.ca. This website would be very useful if you were planning a career in payroll, accounting, or tax preparation.

The calculations you do will be realistic. Using a CRA online calculator or your company's payroll software will result in similar, but more precise, calculations. The website for the CRA online calculator is www.cra-arc.gc.ca/pdoc/.

Using the Tables

The following is a portion of the January 2009 federal income tax tables.

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)

Pay			Federal claim codes									
			0	1	2	3	4	5	6	7	8	9
From	Less than		deduct									
555	-	563	75.75	46.50	43.75	38.05	32.40	26.70	21.00	15.30	9.65	3.95
563	-	571	76.85	47.70	44.90	39.20	33.50	27.80	22.10	16.45	10.75	5.05
571	-	579	78.00	48.85	46.00	40.30	34.60	28.95	23.25	17.55	11.85	6.20
539	-	587	79.10	49.95	47.10	41.45	35.75	30.05	24.35	18.70	13.00	7.30

The various claims codes, numbered 0 through 9, represent the level of taxation at which you will be assessed. The claim code is established when you first start working at a business. If you have no dependents, you are assessed at the lowest level, claim code 0. If you have dependents, RSP contributions, and education deductions, you will be assigned a higher claim code designation, and will pay less income tax. The claim code designation needs to be known before the tax can be determined.

Example 1

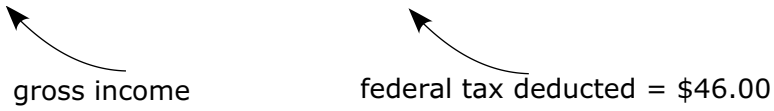
Find the amount of federal income tax deducted if the gross pay is $574.82 and the claim code is 2.

Solution:

Notice the amount of income tax to be deducted is located in the column for claim code 2. The amount of federal tax deducted is $46.00.

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)

Pay			Federal claim codes									
			0	1	2	3	4	5	6	7	8	9
From	Less than		deduct									
555	-	563	75.75	46.50	43.75	38.05	32.40	26.70	21.00	15.30	9.65	3.95
563	-	571	76.85	47.70	44.90	39.20	33.50	27.80	22.10	16.45	10.75	5.05
571	-	579	78.00	48.85	46.00	40.30	34.60	28.95	23.25	17.55	11.85	6.20
539	-	587	79.10	49.95	47.10	41.45	35.75	30.05	24.35	18.70	13.00	7.30



Always try to read the claim code information carefully.

Note that the Manitoba provincial tax tables are read in exactly the same manner.

Example 2

Find the amount of federal income tax payable on earnings of $567.75, claim code 4.

Solution:

Find the line that includes the income, then slide over to the given claim code column.

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)

Pay			Federal claim codes									
			0	1	2	3	4	5	6	7	8	9
From	Less than		deduct									
555	-	563	75.75	46.50	43.75	38.05	32.40	26.70	21.00	15.30	9.65	3.95
563	-	571	76.85	47.70	44.90	39.20	33.50	27.80	22.10	16.45	10.75	5.05
571	-	579	78.00	48.85	46.00	40.30	34.60	28.95	23.25	17.55	11.85	6.20
539	-	587	79.10	49.95	47.10	41.45	35.75	30.05	24.35	18.70	13.00	7.30

This worker would have $33.50 deducted off his gross pay for federal income tax.

Learning Activity 2.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Estimate the EI charged if you have a gross pay of $250.
2. List four different methods of payment for work.
3. You are at the store to buy your mother a present. You have $30 to spend and would like to get her a waffle maker. If it costs $40 and is 20% off, will you be able to get her the waffle maker?
4. I have 9 letters in my name. Is it possible that half of those letters are vowels?
5. What are the first 3 multiples of 7?

continued

Learning Activity 2.6 (continued)

Part B: Income Tax Deductions

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the federal tax deduction tables to find the deductions for the following incomes.

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the provincial table

Pay Rémunération			Federal claim codes/Codes de demande						
			0	1	2	3	4	5	6
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie						
335	-	339	44.65	15.55	12.70	7.30	1.30		
339	-	343	45.20	16.10	13.25	7.55	1.85		
343	-	347	45.80	16.25	13.80	8.10	2.45		
347	-	351	46.35	17.20	14.35	8.65	3.00		
351	-	355	46.90	17.75	14.90	9.25	3.55		
355	-	359	47.45	18.35	15.50	9.80	4.10		
359	-	363	48.00	18.90	16.05	10.35	4.65		
363	-	367	48.60	19.45	16.60	10.90	5.25		
367	-	371	49.15	20.00	17.15	11.45	5.80	.10	
371	-	375	49.70	20.55	17.70	12.05	6.35	.65	
375	-	379	50.25	21.15	18.30	12.60	6.90	1.20	
379	-	383	50.80	21.70	18.85	13.15	7.45	1.80	
383	-	387	51.40	22.25	19.40	13.70	8.00	2.35	
387	-	391	51.95	22.80	19.95	14.25	8.60	2.90	
391	-	395	52.50	23.35	20.50	14.85	9.15	3.45	
395	-	399	53.05	23.95	21.10	15.40	9.70	4.00	
399	-	403	53.60	24.50	21.65	15.95	10.25	4.60	
403	-	407	54.20	25.05	22.20	16.50	10.80	5.15	
407	-	411	54.75	25.60	22.75	17.05	11.40	5.70	
411	-	415	55.30	26.15	23.30	17.65	11.95	6.25	.55

a) Earnings of $386.90, claim code 0

b) Earnings of $410.37, claim code 2

c) Earnings of $410.37, claim code 0

d) Earnings of $350.78, claim code 3

continued

Learning Activity 2.6 (continued)

2. Use the Manitoba provincial tax deductions tables to find the deductions for the same earnings as in #1.

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the federal table

Pay Rémunération			Federal claim codes/Codes de demande						
			0	1	2	3	4	5	6
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie						
333	-	337	34.10	17.25	15.45	11.95	8.45	4.90	1.40
337	-	341	34.55	17.65	15.85	12.35	8.85	5.35	1.80
341	-	345	34.95	18.05	16.30	12.75	9.25	5.75	2.20
345	-	349	35.35	18.45	16.70	13.15	9.65	6.15	2.60
349	-	353	35.75	18.85	17.10	13.55	10.05	6.55	3.00
353	-	357	36.15	19.25	17.50	13.95	10.45	6.95	3.40
357	-	361	36.55	19.65	17.90	14.35	10.85	7.35	3.85
361	-	365	36.95	20.05	18.30	14.80	11.25	7.75	4.25
365	-	369	37.35	10.45	18.70	15.20	11.65	8.15	4.65
369	-	373	37.75	20.85	19.10	15.60	12.05	8.55	5.05
373	-	377	38.15	21.25	19.50	16.00	12.45	8.95	5.45
377	-	381	38.55	21.65	19.90	16.40	12.85	9.35	5.85
381	-	385	38.95	22.05	20.30	16.80	13.30	9.75	6.25
385	-	389	39.35	22.45	20.70	17.20	13.70	10.15	6.65
389	-	393	39.75	22.85	21.10	17.60	14.10	10.55	7.05
393	-	397	40.15	23.30	21.50	18.00	14.50	10.95	7.45
397	-	401	40.55	23.70	21.90	18.40	14.90	11.35	7.85
401	-	405	41.00	24.10	22.30	18.80	15.30	11.80	8.25
405	-	409	41.40	24.50	22.75	19.20	15.70	12.20	8.65
409	-	413	41.80	24.90	23.15	19.60	16.10	12.60	9.05

a) Earnings of $386.90, claim code 0

b) Earnings of $410.37, claim code 2

c) Earnings of $410.37, claim code 0

d) Earnings of $350.78, claim code 3

3. Use your answers for questions 1 and 2 to find the total income tax deducted for each amount of earnings.

a) Earnings of $386.90

b) Earnings of $410.37

c) Earnings of $410.37

d) Earnings of $350.78

When calculating the income tax deduction from gross pay, you must use the federal and the provincial tables to get the exact amount of deductions. Using the formula gives a realistic but not an exact amount.

Using a Formula

When tables are not handy, you could use a formula to estimate the amount of income tax that will be deducted from your gross pay. Your answer will not be as precise as when you use the tables, but the formula method gives a good estimation of what you must pay.

The formula method is also demonstrated because you can closely estimate your income tax deductions when given income levels not represented on the tables provided.

The formula method uses marginal tax rates based on the level of your annual gross income. The federal rates, effective January 1, 2009, are summarized below, rounded to the nearest dollar.

Federal Marginal Rates, January 1, 2009
Any annual income from $0 to $38,832: 15%
plus any annual income from $38,833 to $77,664: 22%
plus any annual income from $77,665 to $126,264: 26%
plus any annual income over $126,264: 29%

Remember that you must also pay income tax to the province. Manitoba's marginal rates for 2009 are listed below, rounded to the nearest dollar.

Manitoba Marginal Rates, 2009
Any annual income from $0 to $31,000: 10.8%
plus any annual income from $31,001 to $67,000: 12.75%
plus any annual income over $67,000: 17.4%

The combined amount of federal and provincial income tax is deducted from your gross pay.

Add to your resource sheet the different income ranges and their respective tax percentages for both federal and provincial taxes.

Example 1

You anticipate earning $15 per hour, and you plan on working 35 hours. Use the marginal rates above to calculate how much will be deducted off your gross pay in federal and provincial income tax each week.

Solution:

Gross pay per week = $15 × 35 = $525

Annual wage = $525 × 52 = $27,300.

Upon examining the federal and provincial tables, your annual income will be within the first level of taxation at both the federal and provincial levels. The federal marginal rate is 15%, and the provincial marginal rate is 10.8%.

Weekly federal tax deduction = $525 × 0.15 = $78.75

Weekly provincial tax deduction = $525 × 0.108 = $56.70

Total income tax deduction = $78.75 + $56.70 = $135.45

Example 2

Mrs. Brown earns $55,000 a year as an administrator. Use the federal and provincial marginal rates to estimate her annual income tax.

Solution:

Since Mrs. Brown's annual income is more than the first level maximum, then several calculations need to be made. You need to consider how much of her income is taxed at each of two levels. Think of filling up the first level of income, and whatever the overflow is goes into the next level of taxation.

Step 1: Federal Income Tax

The first marginal rate is applied to the first $38,832 of her income at 15%.

$38,832 × 0.15 = $5824.80

Now you need to find the amount of her earnings that would be taxed in the second level of taxation. To accomplish this, you subtract the $38,832 from her earnings to find how much of her earnings have "spilled" over into the next marginal rate category.

$55,000 – $38,832 = $16,168

Now you find the tax on this portion of her income at the second-level rate of 22%.

$16,168 × 0.22 = $3556.96

Add this amount to the amount you found in the first level to get the total.

$5824.80 + $3556.96 = $9381.76

Step 2: Provincial Income Tax

The same procedure will apply when calculating the provincial income tax.

The first $31,000 of her income is taxed at 10.8%.

$31,000 × 0.108 = $3348

The remainder is taxed at the next higher marginal rate. To find how much of her income is in the second level, you subtract $31,000 from her total earnings.

$55,000 – $31,000 = $24,000

Now, to this balance, you apply the second level of tax: 12.75%.

$24,000 × 0.1275 = $3060

Finally, you add the two levels of tax together to get a total provincial tax owing.

$3348 + $3060 = $6408

Step 3: Total Income Tax Deduction

You have found the total federal tax and the total provincial tax owing. Add the two taxes together to find what she owes in income taxes for the year.

Total income tax = $9381.76 + $6408 = $15,789.76

Example 3

Using the information from Example 2 above, calculate the percent rate of income tax Mrs. Brown pays altogether. (Hint: The formula for percent rate of change might be found on your resource sheet, since you are keeping it up to date, right?)

Solution:

Total income tax = $15,789.76

Total salary = $55,000

$$\text{Percent rate of tax} = \frac{\text{amount}}{\text{total}} \times 100$$

$$\text{Percent rate of tax} = \frac{\$15,789.76}{\$55,000} \times 100 = 28.7\%$$

Mrs Brown's tax rate is 28.7% of her gross income.

Lesson Summary

In this lesson, you took a close look at the income tax deduction, both federal and provincial. You learned how to read the tax tables and to calculate the tax using marginal rates.

In the next lesson, you find all three deductions—CPP, EI, and income tax—and subtract them from the gross pay to find the net pay.

Assignment 2.4

Income Tax

Total Marks = 21

1. Use the 2009 weekly federal tax deduction tables to find the federal income tax owing on these weekly earnings. *(3 marks)*

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the provincial table

Pay Rémunération			Federal claim codes		
			0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
669	-	677	89.20	30.90	25.25
677	-	685	90.30	32.05	26.35
685	-	693	91.40	33.15	27.45
693	-	701	92.55	34.25	28.60
701	-	709	93.65	35.40	29.70
709	-	717	94.80	36.50	30.80
717	-	725	95.90	37.65	31.95
725	-	733	97.00	38.75	33.05
733	-	741	98.15	39.85	34.20
741	-	749	99.25	41.00	35.30

a) $710, claim code 1 _____

b) $680.25 claim code 0 _____

c) $745.98, claim code 2 _____

continued

Assignment 2.4: Income Tax (continued)

2. Use the 2009 weekly Manitoba provincial tax deduction tables to find the provincial income tax owing on these earnings. *(3 marks)*

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the federal table

Pay Rémunération			Provincial claim codes		
			0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
653	-	661	67.75	50.85	49.10
661	-	669	68.70	51.80	50.05
669	-	677	69.70	52.80	51.05
677	-	685	70.65	53.75	52.00
685	-	693	71.60	54.70	52.95
693	-	701	72.55	55.65	53.90
701	-	709	73.55	56.65	54.85
709	-	717	74.50	57.60	55.85
717	-	725	75.45	58.55	56.80
725	-	733	76.40	59.50	57.75
733	-	741	77.40	60.50	58.70
741	-	749	78.35	61.45	59.70
749	-	757	79.30	62.40	60.65
757	-	765	80.25	63.35	61.60
765	-	773	81.20	64.35	62.55

a) $710, claim code 1 _____

b) $680.25 claim code 0 _____

c) $745.98, claim code 2 _____

3. Use your results from #1 and #2 above to find the combined federal and provincial income tax owing on these weekly earnings. *(6 marks)*

	Federal		Provincial		Total
a) $710, claim code 1	_____	+	_____	=	_____
b) $680.25, claim code 0	_____	+	_____	=	_____
c) $745.98, claim code 2	_____	+	_____	=	_____

continued

Assignment 2.4: Income Tax (continued)

4. Using the federal marginal rate of 15% and the provincial marginal rate of 10.8%, how much income tax would a worker earning $29,500 per year pay? *(3 marks)*

5. Kate works at a credit union and is paid $12.65 per hour. Any hours worked over 40 in a week are paid at time and one-half. She worked 44.5 hours last week. Use the tables to find her CPP, EI, and income tax deductions, and calculate her net pay if her claim code is 0. *(6 marks)*

Canada Pension Plan Contributions

Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC	Pay Rémunération			CPP RPC
From - De		To - Á		From - De		To - Á	
590.84	-	591.03	25.92	605.39	-	605.58	26.65
591.04	-	591.23	25.93	605.59	-	605.78	26.65
591.24	-	591.44	25.94	605.79	-	605.98	26.66
591.45	-	591.64	25.95	605.99	-	606.18	26.67
591.65	-	591.84	25.96	606.19	-	606.39	26.68
591.85	-	592.04	25.97	606.40	-	606.59	26.69
592.05	-	592.24	25.98	606.60	-	606.79	26.70
592.25	-	592.45	25.99	606.80	-	606.99	26.71
592.46	-	592.65	26.00	607.00	-	607.19	26.72

continued

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á	
588.16	-	588.72	10.18
588.73	-	589.30	10.19
589.31	-	589.88	10.20
589.89	-	590.46	10.21
590.47	-	591.04	10.22
591.05	-	591.61	10.23
591.62	-	592.19	10.24
592.20	-	592.77	10.25
592.78	-	593.35	10.26

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Pay Rémunération			Federal claim codes		
			0	1	2
From De		Less than Moins de	Deduct from each pay Retenez sur chaque paie		
555	-	563	75.75	46.60	43.75
563	-	571	76.85	47.70	44.90
571	-	579	78.00	48.85	46.00
579	-	587	79.10	49.95	47.10
587	-	595	80.20	51.10	48.25

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Pay Rémunération			Federal claim codes		
			0	1	2
From De		Less than Moins de	Deduct from each pay Retenez sur chaque paie		
573	-	581	58.50	41.60	39.85
581	-	589	59.30	42.45	40.65
589	-	597	60.15	43.25	41.45
597	-	605	61.00	44.10	42.35
605	-	613	62.00	45.10	43.35

Lesson 5: Net Pay

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of calculations for gross and net pay

Lesson Introduction

Now that you know how to find CPP, EI, and income tax deductions, you can practise solving problems finding net pay.

Gross Pay and Deductions

Once the gross pay is determined, you can add up all the deductions and subtract the total from the gross pay to find the net pay. At times, you will use the percent rate method to calculate CPP (4.95%) and EI (1.73%), and at other times you will be directed to use the tables.

Finding Net Pay

Example 1

Jennifer works part-time at a restaurant, earning $9.75 per hour plus tips. Last week she worked 29 hours and received $82 in tips. Her income tax claim code is 0. Use the tables in this module to determine the tax deductions, and calculate her EI (1.73%) and CPP (4.95%) using the percent rates. Find her net pay. (Do you have the page numbers on your resource sheet for the various tables provided in this module so that you can find them easily?)

Solution:

Gross pay

Wages = 29 × $9.75 = $282.75

Tips = $82.00

Total = $282.75 + $82.00 = $364.75

Deductions

CPP = \$364.75 × 0.0495 = \$18.06

EI = \$364.75 × 0.0173 = \$6.31

Federal income tax = \$48.60

Provincial income tax = \$36.95

Total = \$18.06 + \$6.31 + \$48.60 + \$36.95 = \$109.92

Net pay = \$364.75 – \$109.92 = \$254.83

Note: The majority of restaurants do not add tips to your paycheque, so you are expected to keep track and claim them on your income tax for the year.

Example 2

Shauna works in a retail store, earning \$10.25 per hour. Any work on Saturday or Sunday is paid at time and one-half. Last week, she worked 36 hours during the week and 10 hours on the weekend. Use the percent method to find her CPP and EI, and the marginal rates to find her income tax. Then determine her net pay.

Solution:

Gross pay

Regular pay = 36 × \$10.25 = \$369.00

Overtime rate = 1.5 × \$10.25 = \$15.38

Overtime wage = 10 × \$15.38 = \$153.80

Total = \$369.00 + \$153.80 = \$522.80

Deductions

CPP = \$522.80 × 0.0495 = \$25.88

EI = \$522.80 × 0.0173 = \$9.04

Income tax

Annual income = \$522.80 × 52 = \$27,185.60

Federal tax, first marginal rate, \$522.80 × 0.15 = \$78.42

Provincial income tax, first rate = \$522.80 × 0.108 = \$56.46

Total deductions = \$25.88 + \$9.04 + \$78.42 + \$56.46 = \$169.80

Net pay = \$522.80 – \$169.80 = \$353.00

Learning Activity 2.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You work 9 am to 3 pm on Tuesday, Wednesday, Thursday, Saturday, and Sunday. You work 12 pm to 5 pm on Monday and Friday. How many hours do you work per week?

2. You are craving 5¢ candy. You have $1.43 in your pocket. How much candy can you afford?

3. Evaluate: $\frac{4}{5}+\frac{2}{3}$.

4. You are at a wedding with your family. The 5 of you do not remember what you ordered when they bring out the food. The servers have 2 steak meals, 1 chicken meal, 1 fish meal, and 1 vegetarian meal. You do not like fish. Your sister is a vegetarian. Your mom won't eat red meat. Your brother will only eat chicken. Which meal is yours?

5. Evaluate: $\frac{6}{0}$.

continued

Learning Activity 2.7 (continued)

Part B: Calculating Gross Pay and Net Pay

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the following time card and the income tax tables in the earlier lessons to determine Kyle's net pay. Use the percent rates to find his CPP (4.95%) and EI (1.73%) deductions. Overtime at the rate of time and one-half is paid on any hours over 40. There are no late penalties. Kyle's claim code is 1.

Employee: Kyle					Hourly Rate: $12.80		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	8.01	12:00	1:00	6:30			
Tues.	7:30	12:01	1:00	5:45			
Wed.	8:00	12:00			6:00	10:30	
Thurs.	7:32	11:58	1:00	6:00			
Fri.	8:00	12:00	1:30	5:30			
Sat.			12:00	5:15			
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

2. Use the percent method to find CPP (4.95%) and EI (1.73%) for the following earnings.
   a) $985
   b) $1286.45

continued

Learning Activity 2.7 (continued)

3. José sells insurance, and is paid 3.5% commission on his total sales. He sold 6 policies last week with a total worth of $24,650. Use the tables to find his CPP, EI, and income tax deductions. He also pays $5 per week for parking. Find José's net pay. His claim code is 2.

Canada Pension Plan Contributions
Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC
From - De		To - Á	
861.75	-	861.94	39.33
861.95	-	862.14	39.34
862.15	-	862.35	39.35
862.36	-	862.55	39.36
862.56	-	862.75	39.37
862.76	-	862.95	39.38
862.96	-	863.15	39.39
863.16	-	863.36	39.40
863.37	-	863.56	39.41

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á	
856.68	-	859.24	14.86
859.25	-	859.82	14.87
859.83	-	860.40	14.88
860.41	-	860.98	14.89
860.99	-	861.56	14.90
861.57	-	862.13	14.91
862.14	-	862.71	14.92
862.72	-	863.29	14.93
863.30	-	863.87	14.94

continued

Learning Activity 2.7 (continued)

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the provincial table

Pay Rémunération			Federal claim codes 0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
835	-	843	121.45	92.30	89.90
843	-	851	123.15	94.00	91.20
851	-	859	124.85	95.75	92.90
859	-	867	126.55	97.45	94.60
867	-	875	128.25	99.15	96.30

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the federal table

Pay Rémunération			Federal claim codes 0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
853	-	861	91.90	75.00	73.25
861	-	869	92.85	86.00	74.20
869	-	877	93.85	76.95	75.20
877	-	885	94.80	77.95	75.15
885	-	893	95.80	78.90	77.15

Lesson Summary

In this lesson, you found all the deductions, CPP, EI, and income tax, and subtracted them from the gross pay to find the net pay.

In the next lesson, you will compare tax rates of various provinces.

Assignment 2.5

Net Pay

Total Marks = 18

1. Which two levels of government collect income tax from working people? *(2 marks)*

   a) _____

   b) _____

2. CPP contributions are required from all workers over and including the age of _____. *(1 mark)*

3. Given the following time card, find Marlene's net pay using the income tax tables found earlier in this module and the CPP and EI tables shown on the following page. Overtime is paid after 40 hours/week. Marlene's claim code is 1. *(6 marks)*

Employee: Marlene					Hourly Rate: $12.88		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	8:00	12:00	1:00	5:00			
Tues.			2:15	6:30			
Wed.							
Thurs.	8:00	11:30	1:00	3:45			
Fri.					6:00	10:15	
Sat.	8:00	12:00			6:00	10:30	
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

continued

Assignment 2.5: Net Pay (continued)

Canada Pension Plan Contributions
Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC
From - De		To - Á	
401.75	-	401.94	16.56
401.95	-	402.14	16.57
402.15	-	402.35	16.58
402.36	-	402.55	16.59
402.56	-	402.75	16.60
402.76	-	402.95	16.61
402.96	-	403.15	16.62
403.16	-	403.36	16.63
403.37	-	403.56	16.64

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á	
400.87	-	401.44	6.94
401.45	-	402.02	6.95
402.03	-	402.60	6.96
402.61	-	403.17	6.97
403.18	-	403.75	6.98
403.76	-	404.33	6.99
404.34	-	404.91	7.00
404.92	-	405.49	7.01
405.50	-	406.06	7.02

continued

Assignment 2.5: Net Pay (continued)

4. Jeremy works at a sporting goods store assembling bicycles. He gets paid $10 for each bike he assembles, but loses $6 for every bike that is not put together properly. Last week, he worked 60 hours to assemble 75 bicycles before a big sale. The boss brought eleven bikes back because they were missing parts.

   Calculate Jeremy's net pay, using the 2009 CPP and EI weekly deduction tables shown below. The income tax tables listed earlier in this module can be used. Refer to your resource sheet for the page numbers. Jeremy's claim code is 0. *(5 marks)*

Canada Pension Plan Contributions
Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC
From - De		To - Á	
683.57	-	683.76	30.51
683.77	-	683.96	30.52
683.97	-	684.16	30.53
684.17	-	684.37	30.54
684.38	-	684.57	30.55
684.58	-	684.77	30.56
684.78	-	684.97	30.57
684.98	-	685.17	30.58
685.18	-	685.38	30.59

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á	
681.80	-	682.36	11.80
682.37	-	682.94	11.81
682.95	-	683.52	11.82
683.53	-	684.10	11.83
684.11	-	684.68	11.84
684.69	-	685.26	11.85
685.27	-	685.83	11.86
685.84	-	686.41	11.87
686.42	-	686.99	11.88

Assignment 2.5: Net Pay (continued)

5. Luanne earns $550 per week working at a grocery store. She attempted to calculate her own net pay using the tables with a claim code of 2. The payroll clerk told her she had made a mistake in her calculations. Using the 2009 weekly CPP and EI deduction tables, identify the error she made and correct it. *(4 marks)*

Canada Pension Plan Contributions
Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC
From - De		To - Á	
549.02	-	549.21	23.85
549.22	-	549.42	23.86
549.43	-	549.62	23.87
549.63	-	549.82	23.88
549.83	-	550.02	23.89
550.03	-	550.22	23.90
550.23	-	550.43	23.91
550.44	-	550.63	23.92
550.64	-	550.83	23.93

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á	
546.54	-	547.10	9.46
547.11	-	547.68	9.47
547.69	-	548.26	9.48
548.27	-	548.84	9.49
548.85	-	549.42	9.50
549.43	-	549.99	9.51
550.00	-	550.57	9.52
550.58	-	551.15	9.53
551.16	-	551.73	9.54

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the provincial table

Pay Rémunération			Federal claim codes		
			0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
535	-	539	72.65	43.50	40.70
539	-	543	73.20	44.10	41.25
543	-	547	73.80	44.65	41.80
547	-	551	74.35	45.20	42.35
551	-	555	74.90	45.75	42.90

continued

Assignment 2.5: Net Pay (continued)

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the federal table

Pay Rémunération			Federal claim codes		
			0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
533	-	541	54.50	37.60	35.85
541	-	549	55.30	38.40	36.65
549	-	557	56.10	39.20	37.45
557	-	565	56.90	40.00	38.25
565	-	573	57.70	40.80	39.05

Luanne's Calculations

Gross pay = $550

Deductions

CPP = $23.88

EI = $9.52

Federal income tax = $74.35

Provincial income tax = $56.10

Total = $163.85

Net pay = $386.15

continued

Notes

Lesson 6: Comparisons and Spreadsheets

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of calculations for gross and net pay
- ❑ use technology to investigate changes in income (optional section, not on examination)

Lesson Introduction

Each province has a different provincial tax rate. In this lesson, provincial tax rates are compared. Also, you will discuss the benefits and disadvantages of living in different provinces from a tax point of view. An optional section on spreadsheets is presented. Spreadsheets are used to calculate changes in net pay when gross pay is changed.

Taxes in Different Provinces

People choose to live in regions all across this land for a variety of reasons. Being near your family, having a great job, or loving the outdoors are some reasons why you live where you do. Where would you go if you could live in any province? If you want to live where you will pay the least tax, this lesson will show you how to compare the 2009 tax rates of each of the provinces of Manitoba, Alberta, Ontario, and Nova Scotia.

It is important to remember the population of each area affects the tax rate. Ontario has many millions of Canadians paying income tax, so the provincial government can afford to set slightly lower rates. Smaller provinces that do not have as many contributors may need to have higher rates. Some provinces experience very profitable years and can afford to lower their tax rates. Alberta experienced such a period of high profitability with its oil industry.

Almost everything you buy has PST and GST added to the cost. Different areas of Canada charge different levels of provincial tax. This does affect your cost of living, and should also be a consideration when doing a tax analysis.

Income Threshold Charts

This chart shows the 2009 income tax thresholds for the four provinces you will be comparing: Manitoba, Ontario, Nova Scotia, and Alberta. An income threshold is the same as a marginal rate, which you discussed in a previous lesson.

Refer to this chart to find the percentage amount when solving the problems.

Income Thresholds for January 1, 2009						
Province	Level 1		Level 2		Level 3	
Manitoba	Up to $31,000	10.8%	$31,001 – $67,000	12.75%	$67,001 or more	17.3%
Ontario	Up to $36,848	6.05%	$36,849 – $73,698	9.15%	$73,699 or more	11.16%
Nova Scotia	Up to $29,590	8.79%	$29,591 – $59,180	14.95%	$59,181 or more	16.67%
Alberta	Any income	10%	Any income	10%	Any income	10%

Include this table or the page number on your resource sheet.

Example 1

Jimmy earns $28,750 per year. How much less provincial income tax would he pay if he moved from Manitoba to Ontario?

Solution:

Jimmy's income is less than the maximum for Level 1 in both provinces. Thus, you need only calculate his tax at the one level.

Manitoba tax = $28,750 × 0.108 = $3105.00

Ontario tax = $28,750 × 0.0605 = $1739.38

Difference = $3105.00 – $1739.38 = $1365.62

Jimmy would pay $1365.62 less income tax in Ontario.

Example 2

Allison earns $55,000 as a teacher in Nova Scotia. She wants to move to Alberta. How much less provincial income tax will she pay if she moves?

Solution:

Nova Scotia tax

Since Allison's salary needs to be calculated using two levels, you would proceed as follows.

First threshold amount = maximum for this level = $29,590

First threshold tax = $29,590 × 0.0879 = $2600.96

Second threshold amount = $55,000 – $29590 = $25,410

Second threshold tax = $25,410 × 0.1495 = $3798.80

Total tax = $2600.96 + $3798.80 = $6399.76

Alberta tax

Since there is only one tax rate for all incomes, you just have one calculation.

$55,000 × 0.10 = $5500

Difference = $6399.76 – $5500 = $899.76

Allison would pay $899.76 less provincial income tax in Alberta.

Learning Activity 2.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If 3% of 500 is 15, what is 12% of 500?
2. Write the following as an improper fraction: $2\frac{3}{7}$.
3. If you are $\frac{3}{2}$ taller than your brother and your brother is 4 feet tall, how tall are you?
4. Solve for v: $9v = 63$.
5. Identify the type of angle for an angle that measures 156°.

Part B: Comparing Income Tax

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the income thresholds table to answer the following questions.

1. Which province deducts the most income tax for a person earning more than $75,000?
2. Which province deducts the least income tax for a person earning under $30,000?
3. Which province deducts the least income tax for a person earning more than $75,000?

continued

Learning Activity 2.8 (continued)

4. Leslie works in Manitoba earning $59,000. She is transferred to Nova Scotia. Calculate any change to her provincial income tax deductions.
5. The Province of Ontario deducts 6.05% on income less than $36,848, and almost double that rate on income over $73,698. Explain why this might be important to a young person just entering the working world.
6. Why might a person choose not to move to a different province where the income rates are lower?

Spreadsheets

This section on spreadsheets is optional. No questions on the midterm examination are on spreadsheets.

You can use your skills with spreadsheets and percent rate guidelines to estimate changes in taxation based on changes in earnings. The following spreadsheet shows the formulas to enter into each cell. These formulas are for the Level 1 rates of taxation in Manitoba. When you change the income and "fill down" each formula, the changes can be compared.

Remember, if you want to switch between the "formulas" view and the "calculated values"view, press CTRL ~.

Example 1

Your gross pay is $750 per week. Use the formulas for CPP, EI, federal tax, and provincial tax to find your total deductions and net pay.

Solution:

Formula View

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	$750.00	=A2*0.0495	=A2*0.0173	=A2*0.15	=A2*0.108	=SUM(B2:E2)	=A2–F2
3							
4							

Numbers View

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	$750.00	$37.13	$12.98	$112.50	$81.00	$243.60	$506.40
3							
4							

The total deductions are $243.60. The net pay is $506.40.

Example 2

Your boss has announced that you will receive a 5% increase over your present salary of $750 starting next week. You want to know what the difference will be in your net pay. How would you calculate the difference using a spreadsheet?

Solution:

Step 1 Since your salary has increased by 5%, the new salary will be 105% of the old salary. Using the spreadsheet above as a guide, enter the formula = A2*1.05 into cell A3.

Step 2 Highlight the cells from B2 to G2. Grab the bottom right corner of cell G2, and "fill down." When you fill down, the new values will be calculated and the formulas adjusted appropriately.

Formula View

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	$750.00	=A2∗0.0495	=A2∗0.0173	=A2∗0.15	=A2∗0.108	=SUM(B2:E2)	=A2–F2
3	=A2∗1.05	=A3∗0.0495	=A3∗0.0173	=A3∗0.15	=A3∗0.108	=SUM(B3:E3)	=A3–F3
4							

Numbers View

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	$750.00	$37.13	$12.98	$112.50	$81.00	$243.60	$506.40
3	$787.50	$38.98	$13.62	$118.13	$85.05	$255.78	$531.72
4							

Step 3 Compare your new net pay to the previous net pay.

Difference = $531.72 – $506.40 = $25.32 more

Because formulas were used to calculate your deductions, the net pay increase is the same as the gross pay increase. There may be a slight difference if the tables are used. Check it out. What did you find?

Example 3

You are currently making $10.25 an hour, working 35 hours per week. Your company has offered you a choice of a $0.35 per hour raise or a 4% raise. Use a spreadsheet to determine which option would result in the higher net pay.

Solution:

Step 1 Enter the formulas in the appropriate cells.

Enter the formula = 10.25*35 into cell A2.

Enter the formula = (10.25+0.35)*35 into cell A3

Enter the formula = A2*1.04 into cell A4

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	=10.25∗35	=A2∗0.0495	=A2∗0.0173	=A2∗0.15	=A2∗0.108	=SUM(B2:E2)	=A2−F2
3	=(10.25+0.35)∗35						
4	=A2∗1.04						

Step 2 Fill down the other cells to find the net pay.

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	$358.75	$17.76	$6.21	$53.81	$38.75	$116.52	$242.23
3	$371.00	$18.36	$6.42	$55.65	$40.07	$120.50	$250.50
4	$373.10	$18.47	$6.45	$55.97	$40.29	$121.18	$251.92

Step 3 Make a conclusion

The 4% raise results in a higher net pay. The difference is $251.92 – $250.50 = $1.42 more net pay than the $0.35 per hour option. Over a year, this would be a difference of $1.42 × 52 = $73.84.

Learning Activity 2.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Fill in the blanks for the pattern: –43, –38, –33, ___ , ___ .
2. Your brother takes acting on Monday night and plays football on Tuesdays and Thursdays. You have lacrosse on Wednesday and Thursday, and it is your friend's birthday party on Saturday evening. Your parents have date night every Friday. Will you be able to sit down with your family for dinner this week?
3. You have $4.65. If you buy a package of gum for $2.95, how much money will you have leftover?
4. Which is larger, 0.54 or 39%?
5. You work 5 hours per day, 6 days per week. If your hourly wage is $10, how much do you make biweekly (for two weeks)?

Part B: Calculate Net Pay Using a Spreadsheet

This section on spreadsheets is optional. No questions on the midterm examination are on spreadsheets.

Using a spreadsheet program and the formulas shown in the lesson, solve each of the following changes in income problems.

1. Marley earns $365.25 per week. If she found a new job paying 10% more, calculate the change in her net pay.
2. Daniel earns $12.26 per hour, working 40 hours per week. His boss offered him a $0.45 raise. Find the increase in his total income tax deductions.

Online Calculator

The following link will take you directly to Canada Revenue Agency's online payroll calculator, which uses information from the current time period. Thus, your results will probably be different than the ones in this course, which uses 2009 information.

www.cra-arc.gc.ca/pdoc/

You can enter the name of each different province, and compare the provincial taxes deducted from the same gross pay. You can also access the online calculator through the main web page, looking under business and then payroll. Be sure to set the province of your choice before advancing to the next page, and also to adjust the claim code.

Lesson Summary

In this lesson, you compared provincial income tax thresholds in 4 provinces. You also might have done some calculations on taxes, CPP, and EI using spreadsheets. The federal government website has an online calculator for the same tasks.

The final lesson in the module will deal directly with the formulas used in finding net pay.

Assignment 2.6

Comparisons and Spreadsheets

Total Marks = 19

Use the 2009 Income Threshold chart from this lesson to answer the following questions. (Check your resource sheet for page number references.)

1. Charise earns $30,750 per year in Brampton, Ontario. She and her husband decide to move to Brandon, Manitoba. If she earns exactly the same income, find the change in her provincial income tax. *(5 marks)*

2. James is making $13.50 per hour on shifts of 32 hours per week in Halifax, Nova Scotia. He moves to Alberta where he gets a job in Fort McMurray earning $22.80 per hour working 40 hours per week. How much more is James going to pay per year in provincial income tax? Why should he make this move? *(7 marks)*

continued

Assignment 2.6: Comparisons and Spreadsheets (continued)

3. Roxanne is earning a living in Winnipeg assembling flower arrangements. She is paid $12.50 per arrangement. Roxanne averages 40 arrangements per week. She has found a job in Toronto that pays $13.75 per arrangement. If she moved to Ontario, by how much would her annual net pay be affected? Use percentages for CPP (4.95%) and EI (1.73%), along with the federal and provincial tax thresholds from earlier in the lesson. *(7 marks)*

Lesson 7: Formulas and Errors

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of calculations for gross and net pay
- ❑ solve problems that require manipulation and application of formulas

Lesson Introduction

This lesson shows various income related formulas, and asks that you rearrange them to solve for the unknown quantity. Common errors students make in calculating net pay are brought to your attention. Your task will be to identify and correct the errors in solutions to problems.

Solving an Equation

Just as in previous grades, you need to isolate the variable or the unknown value to solve an equation. Because you are working with equations where both sides are equal, whatever operation you do to one side must be applied equally to the other.

Formula Manipulation

Example 1

A simple income problem would read: "If Jimmy were paid $10 per hour and worked 12 hours, how much money would he earn?"

Solution:

You would likely write the following.

Gross income = hours worked × hourly rate of pay

Gross income = 12 × $10

Gross income = $120

Example 2

Another question using the same formula would be: "If Jimmy were paid $120 and worked 12 hours, find his rate of pay."

Solution:

First, you write the formula.

Gross income = hours worked × hourly rate of pay

You substitute into the equation the known values given in the problem.

$120 = 12 × hourly rate of pay

Let r = hourly rate of pay. The equation can now be written more simply.

$120 = 12$r$

To solve an equation for the unknown quantity, you use inverse operations.

Since the unknown, r, is not yet isolated, you need to eliminate what is still "attached" to it. Since 12 is "attached" to the "r" by multiplication, you need to divide by 12 to eliminate it. You divide because division is the inverse operation of multiplication.

And if you divide on one side of the equation, you must divide on the other side to preserve the equality.

$$\frac{\$120}{12} = \frac{12r}{12}$$

Now simplify both sides of the equation to get the answer.

$10 = 1$r$

Or, more simply, r = $10

The answer to the problem would be: "Jimmy's rate of pay is $10 per hour."

The above two examples demonstrate how to manipulate a formula.

To be successful, using your basic equation-solving skills from earlier grades would be helpful.

Example 3

A worker earned $40,000, and paid $6000 in federal income tax. Find the rate of federal income tax.

Solution:

Method 1

Since rate is a percent, you can set up a proportion where you would solve for the numerator of a fraction whose denominator is 100.

To set up the proportion, perhaps you might think of filling in four slots in a box and then cross multiplying,

$6000	r
$40,000	100

Ensure each column has the same units. Use "*r*" for the unknown rate.

The proportion is written as two equal fractions using the numbers in the same order from the box.

$$\frac{\$6000}{\$40,000} = \frac{r}{100}$$

Now, cross multiply and solve for *r*.

$\$6000 \times 100 = \$40{,}000 \times r$

$\$600{,}000 = \$40{,}000r$

Divide both sides by $40,000 to isolate *r*.

$$\frac{\$600,000}{\$40,000} = \frac{\$40,000r}{\$40,000}$$

Simplify both sides of the equation to get the answer.

$15 = r$

Thus, the federal rate of income tax is 15%.

The following is an alternate way of solving for r using the proportions from Method 1.

Method 2

Use the proportion set up in Method 1:

$$\frac{\$6000}{\$40,000} = \frac{r}{100}$$

Because the ultimate goal is to solve for r, you want to isolate r in the equation because the inverse operation of dividing is multiplying. This means that the 100 needs to be removed. In order to do this, multiply both sides of the equation by 100.

$$100 \times \frac{\$6000}{\$40,000} = \frac{r}{100} \times 100$$

Simplify both sides of the equation to get the answer.

$$\frac{\$600,000}{\$40,000} = r$$

$$15 = r$$

Thus, the federal rate of income tax is 15%.

Method 3

Another way of solving this problem is to write a formula.

Federal tax in dollars = income in dollars × percent rate of tax

You substitute what you know into the formula and solve for the unknown value.

If the worker paid $6000 in tax and his income was $40,000, you can substitute those values into the formula.

$6000 = $40,000 × percent rate of tax

To find the rate, you need to divide both sides of the equation by 40,000.

$$\frac{\$6000}{\$40,000} = \frac{\$40,000 \times \text{rate}}{\$40,000}$$

Simplifying both sides of the equation, you get your answer.

0.15 = rate

As a percent, 0.15 becomes 15%.

Thus, the percent rate of tax is 15%.

Example 4

Karen's new salary is $33,280. Last year, she was earning $32,000. Find the rate of increase in her salary.

Solution:

The amount of Karen's raise = $33,280 – $32,000 = $1280.

You can write a formula to find the percent rate of increase in her salary.

Present salary × percent rate of increase = amount of the raise in dollars

Now, you substitute into the equation the values that you know and solve for what you don't know.

$$\$32{,}000 \times r = \$1280$$

Divide both sides of the equation by $32,000 to isolate the variable, and then simplify to get the final answer.

$$\frac{\$32{,}000 \times r}{\$32{,}000} = \frac{\$1280}{\$32{,}000}$$

$$r = 0.04$$

Thus, Karen's rate of increase in her salary is 0.04 or 4%.

Learning Activity 2.10

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You need to make exact change for a customer at your work. They have given you $60 and their bill is $42.60. How much money will you give them?
2. Write the following percent as a decimal: 46.1%.
3. There are 21 people in your class. You are making cupcakes to bring to school, and the recipe says that it makes 2 dozen cupcakes. Will you have enough cupcakes for your class?
4. Complete the pattern: 24, 28, 32, 36, ___ , ___ .
5. It is 11:30 am right now. You are going out for dinner at 17:00. How long is it until you go out for dinner?

Part B: Working Backwards

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Colin earned $880 last week, and his boss deducted $43.56 for CPP. Colin felt the deduction was too high, so he wanted to check it. What rate did the boss use to calculate the CPP deduction?
2. Denise just moved to Manitoba. She earns $29,850. Her company deducted $3223.80 for provincial income tax. Denise checked on the Canada Revenue Agency's website for the required percentage, and wanted to check her pay stub. What rate did the company use? Were they correct?
3. Bob's deductions for EI for the year were $679.03. He checked the Canada Revenue Agency's website and learned that the rate is 1.73%. Find Bob's annual earnings.

Correcting Errors

It is very common to make calculating errors in math. Many times you have to ask yourself if your answer is reasonable.

Example 1

If the price if a $2 ice cream cone increases by 10% and the clerk charges you $2.02, is that reasonable? Probably not. Can you correct the calculation error?

Solution:

Amount of increase = $2.00 × 0.10 = $0.20

So a 10% increase should have been 20 cents, not just 2 cents.

What did the clerk do in error? She likely changed 10% to its decimal form by writing 0.01 instead of 0.10. Multiplying by 0.01 gives a 2-cent increase.

The error has been identified and corrected.

Example 2

You are in an electronics store shopping for a 42" HD television. The regular price is $1400, and is advertised at 20% off. When you get to the cash register, the bill before taxes is $1372. You are not comfortable with that price, and think the cashier has made a calculation mistake. Can you find the error and correct it?

Solution:

You know that 10% of $1400 is $140, so 20% should be double that, or $280. The sale price should be almost $300 off the regular price, or $1400 – $300 = $1100.

The reduction by the cashier was $1400 – $1372 = $28. The percent rate of discount for $28 is $\frac{\$28}{\$1400} \times 100 = 2\%$.

The cashier made a discount of 2% instead of 20%.

Since the advertised discount of 20% means a reduction in regular price of $280, the sale price should be $1400 – $280 = $1120.

Example 3

Pam calculated her own income taxes last year. She thought she calculated everything properly, yet the Canada Revenue Agency demanded more money. Pam earned $40,000 last year. The combined federal (15%) and provincial (10.8%) rate she used found the following total tax amounts.

Federal tax = $40,000 × 0.15 = $6000

Provincial tax = $40,000 × 0.108 = $4320

Total tax = $6000 + $4320 = $10,320

Identify her error and correct it.

Solution:

Pam used a flat rate for both taxes, instead of applying the threshold percentages.

Since Pam's annual income is $40,000, she needed to use information from two levels to calculate both her federal tax and her provincial tax.

The correct calculations are shown below.

Federal income tax

First threshold level at 15%

Tax = $38,832 at 15% = $38,832 × 0.15 = $5824.80

Second threshold level at 22%

The amount above the first level maximum = $40,000 – $38,832 = $1168

Tax = $1168 × 0.22 = $256.96.

Provincial income tax

First threshold level at 10.8%

Tax = $31,000 at 10.8% = $31,000 × 0.108 = $3348.00

Second threshold level at 12.75%

The amount above the first level maximum is $40,000 – $31,000 = $9000

Tax = $9000 × 0.1275 = $1147.50

Total income tax to be paid = $5824.80 + $256.96 + $3348.00 + $1147.50 = $10,577.26

Lesson Summary

In this lesson, you examined formula manipulations as they relate to income. You developed formulas and solved for unknown values by manipulating the equations. You looked at solutions to problems with errors in them in order to identify and correct the errors.

This concludes the lessons in this module.

Notes

Assignment 2.7

Formulas and Errors

Total Marks = 22

1. A worker earned $52,630 and paid a total of $9736.55 in taxes. Find his percent rate of taxes. *(2 marks)*

2. The formula for finding EI deductions is

   Gross Income × rate = EI deduction

   If the rate this year is 1.73% and the amount of EI collected was $611.56, find the income. *(3 marks)*

continued

Assignment 2.7: Formulas and Errors (continued)

3. Martha is setting her budget for a trip to Saskatchewan. Last year, she saved $350, and she has set a goal to save 10% more this year. Martha calculates she will need to save a total of $353.50 this year. Identify the error Martha made in her calculation and correct it for her. *(3 marks)*

4. John received a raise of $0.56 per hour. He now earns $14.56 per hour. The boss promised him a raise of at least 3.5%. Did the boss keep his word to John? What rate of increase did John actually receive? (*4 marks)*

continued

Assignment 2.7: Formulas and Errors (continued)

5. Use the income threshold rates given earlier in the module to correct the calculations in the problem below. (Use your resource sheet to look up the page number where the information is located.)

   Henrietta is planning a move from Nova Scotia to Alberta. She is earning $41,675, and will earn the same salary in Alberta. When calculating her provincial tax savings, Henrietta followed these steps.

   Nova Scotia $41,675 × 14.95% = $41,675 × 0.1495 = $6230.41

   Alberta $41,675 × 10% = $41,675 × 0.10 = $4167.50

   Tax savings = $6230.41 – $4167.50 = $2062.91

   Her friend checked her calculations and found the savings were much less. Identify the error Henrietta made, and correct it. *(5 marks)*

continued

Assignment 2.7: Formulas and Errors (continued)

6. Bayne calculated his total federal and provincial tax and paid a total of $20,475. He earned $65,000 in Ontario.

   Bayne's calculations

   Federal tax = $65,000 × 22% = $14,300

   Provincial tax = $65,000 × 9.15% = $5947.50

   Total = $14,300 + $5947.50 = $20,247.50

   Bayne was shocked when he received a large refund on his taxes. Identify his mistake, and correct it. *(5 marks)*

Module 2 Summary

In this module, you learned how to calculate net pay from gross pay and deductions. You studied some common deductions and learned how to calculate CPP, EI, and income tax from the tables and by using a formula. You also considered income tax rates for various provinces.

Vocabulary

Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful.

claim code
common deductions
CPP
CRA
deductions
EI
federal income tax
marginal rates
net pay
provincial income tax
spreadsheet

Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

Net pay = Gross Pay – Deductions
CPP = Gross Income × 4.95%
EI = Gross Income × 1.73%
Gross Income = hours worked × hourly rate of pay
Federal income tax = income × percent rate of tax
Increase in salary = present salary × percent rate of increase

Submitting Your Assignments

It is now time for you to submit the Module 2 Cover Assignment and Assignments 2.1 to 2.7 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 2 assignments and organize your material in the following order:

Module 2 Cover Sheet

Module 2 Cover Assignment: Pay Yourself First / The Sales Representative

Assignment 2.1: Common Deductions

Assignment 2.2: CPP

Assignment 2.3: EI

Assignment 2.4: Income Tax

Assignment 2.5: Net Pay

Assignment 2.6: Comparisons and Spreadsheets

Assignment 2.7: Formulas and Errors

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 10 Essential Mathematics (20S)

Module 2
Net Pay

Learning Activity Answer Keys

Module 2:
Net Pay

Learning Activity 2.1

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. It costs $4.00 for a package of 3 chocolate bars. Geri spends $20 on chocolate bars. How many chocolate bars does she buy?
2. You worked 35 hours this week. Your hourly wage is $10. How much was your gross pay?
3. You are running errands all day. You have to go to the nursery, so you drive 8 km. You then go to the mall, so you drive another 6 km. Finally, you go to the movie store, which is another 3 km, then home—driving 6 km. How far did you drive all together?
4. Write the percent as a decimal: 3.5%.
5. Arrange the numbers from largest to smallest: $\frac{1}{2}$, 0.29, $\frac{3}{4}$, 0.65, 0.34.

Answers:

1. 15 chocolate bars ($20 ÷ $4 = 5 packages, 5 × 3 = 15 chocolate bars)
2. $350
3. 23 km (8 + 6 + 3 + 6 = 6 + 6 + 8 + 3 = 12 + 8 + 3 = 20 + 3 = 23)
4. 0.035 (Remember that when you convert from a percent to a decimal that you move the decimal place 2 to the left. If you got 0.35, you only moved the decimal place left once. If you got 35, you moved the decimal to the right instead of the left. Remember that percent means out of 100 so 3.5% = 3.5 ÷ 100.)
5. $\frac{3}{4}$, 0.65, $\frac{1}{2}$, 0.34, 0.29 (Convert all the numbers into one form, e.g., decimals, before putting them in order, so $\frac{1}{2} = 0.5$ and $\frac{3}{4} = 0.75$.)

Part B: CPP Deductions and Net Pay

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Sam works for a seed company that pays him $11.88 per hour. He worked 32 hours last week. His deductions include CPP $15.49, EI $6.58, and income taxes of $50.80 and $38.55. He also contributes $2 per week to the water fund. Find his net pay.

   Answer:

   Step 1 Find Sam's gross pay

   Gross pay = 32 × $11.88 = $380.16

   Step 2 Find the total of all his deductions.

   Deductions = $15.49 + $6.58 + $50.80 + $38.55 + $2.00 = $113.42

   Step 3 Find Sam's net pay

   Net pay = gross pay – deductions = $380.16 – $113.42 = $266.74

2. Betty is paid a salary of $2775 per month as a clerk. Her weekly deductions include $5 for parking, $12.68 for a dental plan, CPP $28.37, EI $11.08, and income taxes of $86.95 and $65.85. Find her net pay.

   Answer:

   Step 1 Find Betty's gross weekly pay.

   Monthly pay = $2775

   $$\text{Weekly pay} = \$2775 \times \frac{12}{52} = \$640.38$$

   Step 2 Find the total of all her deductions.

   Deductions = $5.00 + $12.68 + $28.37 + $11.08 + $86.95 + $65.85 = $209.93

   Step 3 Find net pay.

   Net pay = gross pay – deductions = $640.38 – $209.93 = $430.45

3. David sells cars and is paid strictly on a commission of $1\frac{3}{4}\%$ of total sales.

   Last week, he sold two cars worth a total of $42,000. His deductions include CPP $33.05, EI $12.72, and income taxes of $100.35 and $77.40. He contributes $5 per week to the social fund. Find his net pay.

   Answer:

   Gross pay = $0.0175 \times \$42{,}000 = \$735$

   Deductions = $33.05 + $12.72 + $100.35 + $77.40 +$5.00 = $228.52

   Net pay = $735 – $228.52 = $506.48

Learning Activity 2.2

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What are the factors of 8?
2. You are getting ready for a barbeque you are hosting. It costs $1.50 for a package of 30 plastic cups. How much does each plastic cup cost?
3. At your barbeque you are providing the food, but will have a collection basket for donations to cover the cost. If you buy 2 packages of hamburgers for $12.00 each, and 1 package of chicken burgers for $15.00, how much must you collect to break even?
4. Write the following decimal as a fraction: 0.058.
5. Solve for h: $h + 12 = 32$.

Answers:

1. 1, 2, 4, 8. (Remember, factors are numbers that multiply together to produce a number. In this question, they produce the number 8.)
2. $0.05 (If it helps to change the dollars into cents: $1.50 = 150¢. 150 ÷ 30 = 5¢.)
3. $39 ($2 \times \$12 = \$24 + \$15$)
4. $\frac{58}{1000}$ or $\frac{29}{500}$ (Remember that there are no decimals allowed in fractions. You can figure out what should be in the denominator (10 or 100 or 1000, etc.) by finding the place value of the last digit. In this case, the 8 is the last digit, and it is in the thousandths place.)
5. 20 (32 – 12 = 20)

Part B: Using the CPP Table

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the CPP contributions for the following weekly incomes from the CPP table shown in Example 1.

1. $763.20 *Answer:* $34.45
2. $734.10 *Answer:* $33.01
3. $747.50 *Answer:* $33.67
4. $748.08 *Answer:* $33.70

Learning Activity 2.3

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You were paid $180 from your part-time job. Your hourly wage is $9. How many hours did you work?
2. Which is smaller: $\frac{4}{5}$ or $\frac{7}{10}$?
3. In the novel trilogy *Lord of the Rings*, there are many rings of power—9 were given to men, 7 to the dwarves, 3 to the elves, and 1 to the evil mastermind. In total, how many rings of power are there?
4. You want to save $12,000 to buy a car 1 year from now. How much do you have to save per month to reach this goal?
5. Rewrite the fraction in lowest terms: $\frac{33}{21}$.

Answers:

1. 20 hours ($180 ÷ 9)
2. $\frac{7}{10}$. (Before comparing, change $\frac{4}{5}$ to tenths so the denominators are equal, $\frac{4}{5} = \frac{8}{10}$)
3. 20 rings (9 + 1 + 7 + 3)
4. $1000 ($12 000 ÷ 12)
5. $\frac{11}{7}\left(\frac{33 \div 3}{21 \div 3}\right)$

Part B: Using the Formula to Calculate the CPP Deductions

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the formula method to find the CPP deductions for the following gross weekly incomes.

1. a) $680

   Answer: $680 × 0.0495 = $33.66

   b) $285.26

   Answer: $285.26 × 0.0495 = $14.12

   c) $1285.62

   Answer: $1285.62 × 0.0495 = $63.64

   d) $87.60

   Answer: $87.60 × 0.0495 = $4.34

2. Use the formula method to calculate Jane's CPP deduction for the year if her annual salary is $56,480.

   Answer:

   CPP = $56,480 × 0.0495 = $2795.76

   Since the maximum CPP deduction per year is $2118.60, then Jane's deduction is $2118.60 for the year.

Learning Activity 2.4

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are going to bake a cake for your mom's birthday. Because your family is coming over, you decide to make a double recipe. In the original recipe you need $\frac{1}{2}$ a teaspoon of vanilla. How much vanilla will you need in the double recipe?
2. You are standing on the baseline of a basketball court. The top of the key is 25 feet away from you. The distance to centre court is 47 feet. What is the distance from the top of the key to centre court?
3. You are out for dinner with your best friend for their birthday. The bill comes and you pay for everything. If the total is $35.75 and you leave $40 on the table (including tip), how much are you tipping the server?
4. Is an angle that measures 35° acute, right, obtuse, straight, or reflex?
5. Estimate the CPP if the gross pay is $600.

Answers:

1. 1 teaspoon $\left(\frac{1}{2} \times 2 = \frac{2}{2} = 1\right)$
2. 22 feet (47 feet – 25 or 25 + 20 = 45 + 2 = 47)
3. $4.25 (Count up: $35.75 + $0.25 = $36 and $36 + $4 = $40)
4. Acute (35° is less than 90°)
5. $30 (From the formula, you know that you multiply by 0.0495, which is approximately 0.05, also written as 5%. We know 10% of 600 is $60, so half of that is $30.)

Part B: Employment Insurance

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the 2009 EI table to find the contributions for the following weekly incomes.

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á		From - De		To - Á	
499.72	-	500.28	8.65	541.33	-	541.90	9.37
500.29	-	500.86	8.66	541.91	-	542.48	9.38
500.87	-	501.44	8.67	542.49	-	543.06	9.39
501.45	-	502.02	8.68	543.07	-	543.64	9.40
502.03	-	502.60	8.69	543.65	-	544.21	9.41
502.61	-	503.17	8.70	544.22	-	544.79	9.42
503.18	-	503.75	8.71	544.80	-	545.37	9.43
503.76	-	504.33	8.72	545.38	-	545.95	9.44
504.34	-	504.91	8.73	545.96	-	546.53	9.45

1. $544.25 *Answer:* $9.42
2. $542.85 *Answer:* $9.39
3. $500.00 *Answer:* $8.65
4. $503.49 *Answer:* $8.71

Learning Activity 2.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the range of the following numbers: 0.2, 0.6, 0.08, 0.5, 0.03?
2. A dozen muffins costs $8.66. How much would you expect half a dozen muffins to cost?
3. Solve for d: $3d = 9$.
4. Your mom is buying ice cream for your family. The store has tiger, bubblegum, vanilla, and chocolate flavours. Your mom doesn't like bubblegum, your dad doesn't like chocolate, and you don't like vanilla. Which ice cream will your mom buy?
5. You have to walk to school. To get there, you walk 3 blocks north, 2 blocks west, another 6 blocks north, and then 1 block west. Each block is 100 m. How far do you walk?

Answers:

1. 0.57 (Remember that range is the difference between the highest and lowest values. 0.60 – 0.03 = 0.57.)
2. $4.33 ($8.66 ÷ 2)
3. $d = 3$ (Divide both sides by 3, so $d = 9 \div 3$.)
4. Tiger (Process of elimination: Tiger ~~Bubblegum~~ ~~Vanilla~~ ~~Chocolate)~~
5. 1200 m (You walk 3 + 2 + 6 + 1 = 12 blocks. Each block is 100 m long so 12×100.)

Part B: Calculating EI Using the Formula

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the formula method to find the EI deductions for the following gross incomes.

1. $680 *Answer:* $680 × 0.0173 = $11.76
2. $285.26 *Answer:* $285.26 × 0.0173 = $4.93
3. $1285.62 *Answer:* $1285.62 × 0.0173 = $22.24
4. $87.60 *Answer:* $87.60 × 0.0173 = $1.52
5. $56,480

Answer:

$731.79
By calculating, $56,480 × 0.0173 = $977.10, which is higher than the maximum EI deduction. So the answer is the maximum, $731.79.

Learning Activity 2.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Estimate the EI charged if you have a gross pay of $250.
2. List four different methods of payment for work.
3. You are at the store to buy your mother a present. You have $30 to spend and would like to get her a waffle maker. If it costs $40 and is 20% off, will you be able to get her the waffle maker?
4. I have 9 letters in my name. Is it possible that half of those letters are vowels?
5. What are the first 3 multiples of 7?

Answers:

1. $5 (Based on the formula, you multiply the gross pay by 0.0173 to calculate EI. 0.0173 is approximately 0.02 or 2%. 1% of $250 is $2.50 so 2% of $250 is $5.)
2. Any 4 of the following: piecework, commission, salary, salary plus commission, hourly wage, tips, contract, . . . (Go back and review Module 1, Lesson 3, if you don't remember.)
3. No (20% of $40 = 2 × (10% of $40) = 2 × 4 = $8. $40 – 8 = $32, which is more money than you have.)
4. No (Since 9 ÷ 2 = 4.5, and you cannot have a letter that is only half a vowel, it is not possible. Note: One could argue that 'y' is both a consonant and a vowel, but in this case we are considering it either one or the other.)
5. 7, 14, 21

Part B: Income Tax Deductions

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the federal tax deduction tables to find the deductions for the following incomes.

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the provincial table

Pay Rémunération			Federal claim codes/Codes de demande						
			0	1	2	3	4	5	6
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie						
335	-	339	44.65	15.55	12.70	7.30	1.30		
339	-	343	45.20	16.10	13.25	7.55	1.85		
343	-	347	45.80	16.25	13.80	8.10	2.45		
347	-	351	46.35	17.20	14.35	8.65	3.00		
351	-	355	46.90	17.75	14.90	9.25	3.55		
355	-	359	47.45	18.35	15.50	9.80	4.10		
359	-	363	48.00	18.90	16.05	10.35	4.65		
363	-	367	48.60	19.45	16.60	10.90	5.25		
367	-	371	49.15	20.00	17.15	11.45	5.80	.10	
371	-	375	49.70	20.55	17.70	12.05	6.35	.65	
375	-	379	50.25	21.15	18.30	12.60	6.90	1.20	
379	-	383	50.80	21.70	18.85	13.15	7.45	1.80	
383	-	387	51.40	22.25	19.40	13.70	8.00	2.35	
387	-	391	51.95	22.80	19.95	14.25	8.60	2.90	
391	-	395	52.50	23.35	20.50	14.85	9.15	3.45	
395	-	399	53.05	23.95	21.10	15.40	9.70	4.00	
399	-	403	53.60	24.50	21.65	15.95	10.25	4.60	
403	-	407	54.20	25.05	22.20	16.50	10.80	5.15	
407	-	411	54.75	25.60	22.75	17.05	11.40	5.70	
411	-	415	55.30	26.15	23.30	17.65	11.95	6.25	.55

a) Earnings of $386.90, claim code 0 *Answer:* $51.40

b) Earnings of $410.37, claim code 2 *Answer:* $22.75

c) Earnings of $410.37, claim code 0 *Answer:* $54.75

d) Earnings of $350.78, claim code 3 *Answer:* $8.65

2. Use the Manitoba provincial tax deductions tables to find the deductions for the same earnings as in #1.

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions
in the federal table

Pay Rémunération			Federal claim codes/Codes de demande						
			0	1	2	3	4	5	6
From De		Less than Moins de	Deduct from each pay Retenez sur chaque paie						
333	-	337	34.10	17.25	15.45	11.95	8.45	4.90	1.40
337	-	341	34.55	17.65	15.85	12.35	8.85	5.35	1.80
341	-	345	34.95	18.05	16.30	12.75	9.25	5.75	2.20
345	-	349	35.35	18.45	16.70	13.15	9.65	6.15	2.60
349	-	353	35.75	18.85	17.10	13.55	10.05	6.55	3.00
353	-	357	36.15	19.25	17.50	13.95	10.45	6.95	3.40
357	-	361	36.55	19.65	17.90	14.35	10.85	7.35	3.85
361	-	365	36.95	20.05	18.30	14.80	11.25	7.75	4.25
365	-	369	37.35	10.45	18.70	15.20	11.65	8.15	4.65
369	-	373	37.75	20.85	19.10	15.60	12.05	8.55	5.05
373	-	377	38.15	21.25	19.50	16.00	12.45	8.95	5.45
377	-	381	38.55	21.65	19.90	16.40	12.85	9.35	5.85
381	-	385	38.95	22.05	20.30	16.80	13.30	9.75	6.25
385	-	389	39.35	22.45	20.70	17.20	13.70	10.15	6.65
389	-	393	39.75	22.85	21.10	17.60	14.10	10.55	7.05
393	-	397	40.15	23.30	21.50	18.00	14.50	10.95	7.45
397	-	401	40.55	23.70	21.90	18.40	14.90	11.35	7.85
401	-	405	41.00	24.10	22.30	18.80	15.30	11.80	8.25
405	-	409	41.40	24.50	22.75	19.20	15.70	12.20	8.65
409	-	413	41.80	24.90	23.15	19.60	16.10	12.60	9.05

a) Earnings of $386.90, claim code 0 *Answer:* $39.35

b) Earnings of $410.37, claim code 2 *Answer:* $23.15

c) Earnings of $410.37, claim code 0 *Answer:* $41.80

d) Earnings of $350.78, claim code 3 *Answer:* $13.55

3. Use your answers for questions 1 and 2 to find the total income tax deducted for each amount of earnings.

a) Earnings of $386.90 *Answer:* $51.40 + $39.35 = $90.75

b) Earnings of $410.37 *Answer:* $22.75 + $23.15 = $45.90

c) Earnings of $410.37 *Answer:* $54.75 + $41.80 = $96.55

d) Earnings of $350.78 *Answer:* $8.65 + $13.55 = $22.20

Learning Activity 2.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You work 9 am to 3 pm on Tuesday, Wednesday, Thursday, Saturday, and Sunday. You work 12 pm to 5 pm on Monday and Friday. How many hours do you work per week?
2. You are craving 5¢ candy. You have $1.43 in your pocket. How much candy can you afford?
3. Evaluate: $\frac{4}{5}+\frac{2}{3}$.
4. You are at a wedding with your family. The 5 of you do not remember what you ordered when they bring out the food. The servers have 2 steak meals, 1 chicken meal, 1 fish meal, and 1 vegetarian meal. You do not like fish. Your sister is a vegetarian. Your mom won't eat red meat. Your brother will only eat chicken. Which meal is yours?
5. Evaluate: $\frac{6}{0}$.

Answers:

1. 40 hours (From 9 am to 3 pm is 6 hours. You work for 6 hours, 5 days per week ($6 \times 5 = 30$ hours). From 12 pm to 5 pm is 5 hours. You work for 5 hours, 2 days per week ($5 \times 2 = 10$ hours). $10 + 30 = 40$ hours.)
2. 28 candies (With 100¢ you can get 20 candies. With 43¢, you can get 8 more candies, and you will have 3¢ left over because you cannot buy part of a candy—you will be charged for a whole one.)
3. $\frac{22}{15}$. (You need a common denominator to add fractions together.)

   $$\left(\frac{4}{5}\times\frac{3}{3}\right)+\left(\frac{2}{3}\times\frac{5}{5}\right)=\frac{12}{15}+\frac{10}{15}=\frac{22}{15}$$

   Remember that when you multiply fractions the numerators multiply AND the denominators multiply. When you add fractions, the numerators are added but the denominator stays the same.
4. Steak (Process of elimination:
   STEAK STEAK ~~CHICKEN~~(brother) ~~FISH~~(mom) ~~VEGGIE~~(sister).)
5. Not possible (No number can be divided by zero.)

Part B: Calculating Gross Pay and Net Pay

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the following time card and the income tax tables in the earlier lessons to determine Kyle's net pay. Use the percent rates to find his CPP (4.95%) and EI (1.73%) deductions. Overtime at the rate of time and one-half is paid on any hours over 40. There are no late penalties. Kyle's claim code is 1.

   Answer:

Employee: Kyle					Hourly Rate: $12.80		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	8.01	12:00	1:00	6:30			9.5
Tues.	7:30	12:01	1:00	5:45			9.25
Wed.	8:00	12:00			6:00	10:30	8.5
Thurs.	7:32	11:58	1:00	6:00			9.5
Fri.	8:00	12:00	1:30	5:30			8
Sat.			12:00	5:15			5.25
Sun.							
							50

	Hours	Rate	Amount
Regular	40	$12.80	40 x $12.80 = $512
Overtime	10	1.5 x $12.80 = $19.20	10 x $19.20 = $192
Total Hours	50	Gross Wages	$512 + $192 = $704

Deductions

CPP = \$704 × 0.0495 = \$34.85

EI = \$704 × 0.0173 = \$12.18

Federal income tax = \$35.40

Provincial income tax = \$56.65

Total = \$34.85 + \$12.18 + \$35.40 + \$56.65 = \$139.08

Net pay = \$704 – \$139.08 = \$564.92

2. Use the percent method to find CPP (4.95%) and EI (1.73%) for the following earnings.

   a) \$985

   Answer:

   CPP = \$985 × 0.0495 = \$48.76

   EI = \$985 × 0.0173 = \$17.04

   b) \$1286.45

   Answer:

   CPP = \$1286.45 × 0.0495 = \$63.68

   EI = \$1286.45 × 0.0173 = \$22.26

3. José sells insurance, and is paid 3.5% commission on his total sales. He sold 6 policies last week with a total worth of \$24,650. Use the tables to find his CPP, EI, and income tax deductions. He also pays \$5 per week for parking. Find José's net pay. His claim code is 2.

Canada Pension Plan Contributions
Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC
From - De		To - Á	
861.75	-	861.94	39.33
861.95	-	862.14	39.34
862.15	-	862.35	39.35
862.36	-	862.55	39.36
862.56	-	862.75	39.37
862.76	-	862.95	39.38
862.96	-	863.15	39.39
863.16	-	863.36	39.40
863.37	-	863.56	39.41

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á	
856.68	-	859.24	14.86
859.25	-	859.82	14.87
859.83	-	860.40	14.88
860.41	-	860.98	14.89
860.99	-	861.56	14.90
861.57	-	862.13	14.91
862.14	-	862.71	14.92
862.72	-	863.29	14.93
863.30	-	863.87	14.94

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Pay Rémunération			Federal claim codes		
			0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
835	-	843	121.45	92.30	89.90
843	-	851	123.15	94.00	91.20
851	-	859	124.85	95.75	92.90
859	-	867	126.55	97.45	94.60
867	-	875	128.25	99.15	96.30

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Pay Rémunération			Federal claim codes		
			0	1	2
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie		
853	-	861	91.90	75.00	73.25
861	-	869	92.85	86.00	74.20
869	-	877	93.85	76.95	75.20
877	-	885	94.80	77.95	75.15
885	-	893	95.80	78.90	77.15

Answer:

Gross pay = $24,650 × 0.035 = $862.75

Deductions

CPP = $39.37

EI = $14.93

Federal income tax, claim code 2 = $94.60

Provincial income tax = $74.20

Parking = $5.00

Total = $39.37 + $14.93 + $94.60 + $74.20 + $5.00 = $228.10

Net pay = $862.75 – $228.10 = $634.65

Learning Activity 2.8

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If 3% of 500 is 15, what is 12% of 500?
2. Write the following as an improper fraction: $2\frac{3}{7}$.
3. If you are $\frac{3}{2}$ taller than your brother and your brother is 4 feet tall, how tall are you?
4. Solve for v: $9v = 63$.
5. Identify the type of angle for an angle that measures 156°.

Answers:

1. 60 (12% = 3% × 4, so 15 × 4 = 60)
2. $\frac{17}{7}$ (Remember that an improper fraction has a larger numerator than denominator, $2\frac{3}{7} = 2 + \frac{3}{7} = \frac{14}{7} + \frac{3}{7} = \frac{17}{7}.$)
3. 6 feet $\left(\frac{3}{2} \times \frac{4}{1} = \frac{12}{2} = 6\right)$
4. $v = 7$ (Divide both sides by 9.)
5. Obtuse (between 90° and 180°)

Part B: Comparing Income Tax

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the income thresholds table to answer the following questions.

1. Which province deducts the most income tax for a person earning more than $75,000?

   Answer:

   Manitoba

2. Which province deducts the least income tax for a person earning under $30,000?

   Answer:

   Ontario

3. Which province deducts the least income tax for a person earning more than $75,000?

   Answer:

   Alberta

4. Leslie works in Manitoba earning $59,000. She is transferred to Nova Scotia. Calculate any change to her provincial income tax deductions.

   Answer:

   Manitoba

   First level tax = $31,000 × 0.108 = $3348

   Second level tax = ($59,000 – $31,000) × 0.1275 = $28,000 × 0.1275 = $3570

   Total Manitoba tax = $3348 + $3570 = $6918

   Nova Scotia

   First level tax = $29,590 × 0.0879 = $2600.96

   Second level tax = ($59,000 – $29,590) × 0.1495 = $29,410 × 0.1495 = $4396.80

   Total Nova Scotia tax = $2600.96 + $4396.80 = $6997.76

   Difference = $6997.76 – $6918 = $79.76

   Leslie will pay $79.76 more provincial income tax in Nova Scotia.

5. The Province of Ontario deducts 6.05% on income less than $36,848, and almost double that rate on income over $73,698. Explain why this might be important to a young person just entering the working world.

   Answer:

   A young person entering the work force likely doesn't earn a lot of money, so the lower tax rate allows him to have more take-home pay.

6. Why might a person choose not to move to a different province where the income rates are lower?

   Answer:

   Although the income rates are lower the cost of housing and food may be much higher. For example, in Halifax, Nova Scotia the average cost of a house in 2009, was $234,000, whereas in Fort McMurray, Alberta it was $629,392. To rent a one bedroom apartment the costs were similar, $843 in Halifax and $1853 in Fort McMurray.

Learning Activity 2.9

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Fill in the blanks for the pattern: –43, –38, –33, ___ , ___ .
2. Your brother takes acting on Monday night and plays football on Tuesdays and Thursdays. You have lacrosse on Wednesday and Thursday, and it is your friend's birthday party on Saturday evening. Your parents have date night every Friday. Will you be able to sit down with your family for dinner this week?
3. You have $4.65. If you buy a package of gum for $2.95, how much money will you have leftover?
4. Which is larger, 0.54 or 39%?
5. You work 5 hours per day, 6 days per week. If your hourly wage is $10, how much do you make biweekly (for two weeks)?

Answers:

1. –28, –23 (The pattern is to add 5 each time. –43 + 5 = –38.)
2. Yes, Sunday (Process of elimination: S ~~M~~ ~~T~~ ~~W~~ ~~T~~ ~~F~~ ~~S~~)
3. $1.70 (Count up: 2.95 + 0.05 = 3.00, 3 + 1 = 4, 4 + 0.65 = 4.65
   Total: 0.05 + 1 + 0.65)
4. 0.54 (Convert to one form: 0.54 = 54%, is larger than 39% or 39% = 0.39, is less than 0.54.)
5. $600 (You work 5 × 6 = 30 hours per week, so you make 30 × 10 = $300 per week. For 2 weeks you make $300 × 2 = $600.)

Part B: Calculate Net Pay Using a Spreadsheet

This section on spreadsheets is optional. No questions on the midterm examination are on spreadsheets.

Using a spreadsheet program and the formulas shown in the lesson, solve each of the following changes in income problems.

1. Marley earns $365.25 per week. If she found a new job paying 10% more, calculate the change in her net pay.

   Answer:

   The cell A2 is filled with the original wage. The cell A3 contains a formula to increase Marley's wage by 10%, by multiplying the old wage by 1.10. Then the cells are filled down to calculate the answers.

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	365.25	=A2*0.0495	=A2*0.0173	=A2*0.15	=A2*0.108	=SUM(B2:E2)	=A2–F2
3	=A2*1.1						
4							

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	$365.25	$18.08	$6.32	$54.79	$39.45	$118.63	$246.62
3	$401.78	$19.89	$6.95	$60.27	$43.39	$130.50	$271.28
4							

The difference in the two wages is $271.28 – $246.62 = $24.66.

Marley would receive $24.66 more in net pay.

2. Daniel earns $12.26 per hour, working 40 hours per week. His boss offered him a $0.45 raise. Find the increase in his total income tax deductions.

Answer:

The formulas for the two first cells are as follows.

for A2 enter =12.26*40

for A3 enter =(12.26+0.45)*40

Then fill down the cells to calculate the answers.

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	=12.26*40	=A2*0.0495	=A2*0.0173	=A2*0.15	=A2*0.108	=SUM(B2:E2)	=A2–F2
3	=(12.26+0.45)*40	=A3*0.0495	=A3*0.0173	=A3*0.15	=A3*0.108	=SUM(B3:E3)	=A3–F3
4							

	A	B	C	D	E	F	G
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay
2	$490.40	$24.27	$8.48	$73.56	$52.96	$159.28	$331.12
3	$508.40	$25.17	$8.80	$76.26	$54.91	$165.13	$343.27
4							

You could add an extra column and calculate the total income tax paid and the difference between the two totals.

	A	B	C	D	E	F	G	H
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay	Total Income Tax
2	=12.26*40	=A2*0.0495	=A2*0.0173	=A2*0.15	=A2*0.108	=SUM(B2:E2)	=A2–F2	=SUM(D2:E2)
3	=(12.26+0.45)*40							
4								

When the cells are filled, the difference in income tax can be calculated from the chart.

	A	B	C	D	E	F	G	H
1	Gross Pay	CPP	EI	Federal Tax	Manitoba Tax	Total Deductions	Net Pay	Total Income Tax
2	$490.40	$24.27	$8.48	$73.56	$52.96	$159.28	$331.12	$126.52
3	$508.40	$25.17	$8.80	$76.26	$54.91	$165.13	$343.27	$131.17
4								

Daniel will pay a total federal and provincial income tax of $131.17, an increase of $131.17 – $126.52 = $4.65 over his previous year.

Learning Activity 2.10

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You need to make exact change for a customer at your work. They have given you $60 and their bill is $42.60. How much money will you give them?
2. Write the following percent as a decimal: 46.1%.
3. There are 21 people in your class. You are making cupcakes to bring to school, and the recipe says that it makes 2 dozen cupcakes. Will you have enough cupcakes for your class?
4. Complete the pattern: 24, 28, 32, 36, ___ , ___ .
5. It is 11:30 am right now. You are going out for dinner at 17:00. How long is it until you go out for dinner?

Answers:

1. $17.40 (Count up: $42.60 + 0.40 + 7 + 10 = $60)
2. 0.461
3. Yes (1 dozen = 12, 2 dozen = 2 × 12 = 24 cupcakes)
4. 40, 44 (add 4 each time)
5. 5 hours, 30 min = 5.5 hours (Count up: 11:30 + 30 min = 12:00, 12 + 5 hr = 17:00)

   Note: 5 hr 30 min is not the same as 5.3 hours. This is because there are only 60 minutes in an hour so $\frac{30}{60} = 0.5$.

Part B: Working Backwards

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Colin earned $880 last week, and his boss deducted $43.56 for CPP. Colin felt the deduction was too high, so he wanted to check it. What rate did the boss use to calculate the CPP deduction?

   Answer:

   Gross pay × rate of CPP deduction = CPP deduction in dollars

   $\$880 \times r = \$43.56$

   $$\frac{\$880 \times r}{\$880} = \frac{\$43.56}{\$880}$$

   $r = 0.0495 = 4.95\%$

   The boss used the correct rate.

2. Denise just moved to Manitoba. She earns $29,850. Her company deducted $3223.80 for provincial income tax. Denise checked on the Canada Revenue Agency's website for the required percentage, and wanted to check her pay stub. What rate did the company use? Were they correct?

   Answer:

   Income in dollars × percent rate of tax deduction = amount of dollars of tax

   $\$29{,}850 \times r = \$3223.80$

   $$\frac{\$29{,}850 \times r}{\$29{,}850} = \frac{\$3223.80}{\$29{,}850}$$

   $r = 0.108 = 10.8\%$

   The company used a rate of 10.8%. Yes, they were correct.

3. Bob's deductions for EI for the year were $679.03. He checked the Canada Revenue Agency's website and learned that the rate is 1.73%. Find Bob's annual earnings.

   Answer:

   Income in dollars × percent rate of EI deduction = EI deduction in dollars

   $I \times 0.0173 = \$679.03$

$$\frac{I \times 0.0173}{0.0173} = \frac{\$679.03}{0.0173}$$

I = $39,250.29

Bob's annual earnings are $39,250.29.

Notes

Grade 10 Essential Mathematics (20S)

Module 3
Measurement

Module 3: Measurement

Introduction

In this module, you will learn about measurement. Most countries around the world, including Canada, use the metric system, sometimes referred to as SI (*système international*). This system of measurement includes such units as centimetres, grams, and litres. The United States uses the Imperial system of measurement, which includes such units as miles, feet, ounces, and quarts. This module examines both the metric and imperial systems of measurement, and conversions between the two.

Precise measurements can be made with instruments like calipers or micrometers. This module includes some work with those instruments.

You will need a ruler that shows units in both the metric and imperial systems.

Assignments in Module 3

When you have completed the assignments for Module 3, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Following and Writing Instructions
1	Assignment 3.1	Powers of Ten
2	Assignment 3.2	Metric System
3	Assignment 3.3	Imperial System
4	Assignment 3.4	Reading Rulers
5	Assignment 3.5	Conversions
6	Assignment 3.6	Calipers
7	Assignment 3.7	Micrometers
8	Assignment 3.8	Formulas
9	Assignment 3.9	Problems

Resource Sheet

When you write your midterm examination, you will be allowed to bring a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Midterm Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 3. You might use your Module 3 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 1, 2, 3, and 4, to prepare your Midterm Examination Resource Sheet. Remember, the midterm examination is based only on the first four modules of the course.

Resource Sheet for Module 3

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Midterm Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Module 3 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work in the spaces provided for Exercise A. Use a separate sheet to answer Exercise B. The cover assignment can be done at any time while you are completing Module 3. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on the rubric shown on the next page. You can see that the cover assignment is worth a total of 12 marks. Those marks are allotted according to how well you are able to satisfy the requirements explained in the rubric. If you have any questions about the scoring be sure to contact your tutor/marker for clarification.

Module 3 Cover Assignment Rubric
Following and Writing Instructions

Score Level	Following Instructions (Did I know how to do the question?)	Process Knowledge (What strategies and methods did I use to get the answer?)	Writing Instructions (Did I communicate my answer clearly?)
4	I got the right answers.	I found all the important parts of the problems. I knew how to read and follow the instructions to solve the problems.	My third set of instructions was clear and concise. The third diagram matched the original picture. I included all three sets of instructions and pictures.
3	I made minor errors in my answers.	I found most of the important parts of the problem. I knew how to read and follow most of the instructions to solve the problems.	My third set of instructions was mostly clear and concise. The third diagram mostly matched the original picture. I included all three sets of instructions and pictures.
2	I know how to do parts of the problems, but I made major errors in following the instructions and got a wrong answer. I gave only part of the answer.	I found some of the important parts of the problems. I knew how to read and follow some of the instructions to solve the problems.	My third set of instructions was not clear and concise. The third diagram did not match the original picture. I included all three sets of instructions and pictures.
1	I tried to do the problems but I didn't understand them.	I couldn't find many of the important parts of the problems. Mostly, I didn't know how to read and follow the instructions to solve the problems.	I wrote only one set of instructions. The diagram did not match the original picture. I included only one set of instructions and only one picture.
0	I didn't try to answer the problems.	I didn't show a method for solving the problems.	I didn't write any sets of instructions.
Mark	Following Instructions = ____ / *(4 marks)*	Process = ____ / *(4 marks)*	Writing Instructions = ____ / *(4 marks)*
Total Mark	Following Instructions + Process + Writing Instructions = _____		Total Possible: 4 + 4 + 4 = 12

Module 3 Cover Assignment

Total: 12 marks

Exercise A: Following Instructions

1. Start with the letters of the words USED CAR LOT and follow the instructions below. Print your answers on the line to the right of each instruction. Work in pencil so you can erase any mistakes.

   a) Print the letters USEDCARLOT. _____

   b) Exchange the fourth and eighth letters from the left. _____

   c) Change DOT to DASH. _____

   d) Replace the fourth vowel from the right with an M. _____

   e) Place the last letter between the third and fourth consonants from the left. _____

   f) Replace the second vowel from the left with an F. _____

   g) Exchange the first and third letters from the left. _____

   h) Insert an N between the fourth and fifth letters from the left. _____

   i) Replace the sixth consonant from the left with an E. _____

   j) Place the first consonant from the right in front of the eighth consonant from the left. _____

   k) Place an L at the end. _____

   l) Insert an O between the fourth and fifth letters from the left. _____

   m) Delete the H. _____

   n) Place the third vowel from the left at the end. _____

   o) Place the ninth consonant from the right between the N and the F. _____

   p) Insert an O between the F and the R. _____

   q) Delete the D. _____

   r) Place the seventh consonant from the right at the beginning. _____

continued

Module 3 Cover Assignment (continued)

2. a) Describe how well you were able to follow the instructions.

b) Did you get the solution the first time?

c) What did you learn about yourself as you completed an exercise on reading instructions?

continued

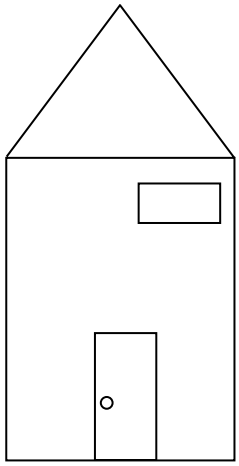
Module 3 Cover Assignment (continued)

Exercise B: Writing Instructions

Do all of Exercise B on a separate page.

Clear instructions can be a challenge to write.

1. Write a set of instructions for drawing the diagram of the front of a house.



2. Test your set of instructions on a friend or family member. Do not tell the person what is being drawn.
3. How accurate is the drawing? Hand in the person's drawing along with your instructions.
4. Since the person was predictably not able to draw the diagram accurately, how might you change your instructions? Write a new set of instructions with the changes. Then test your instructions again. Hand in your second set of instructions and the second diagram.
5. Repeat this process one more time. Make changes to your instructions. Test your instructions on a third person. Did this person draw the diagram correctly? Hand in your third set of instructions and the accompanying diagram.
6. Describe the results when you tested your instructions on these three different people.

Notes

Lesson 1: Powers of Ten

Lesson Focus

In this lesson, you will

- ❑ learn the history of the metric and decimal numbering systems
- ❑ demonstrate an understanding of powers of 10

Lesson Introduction

This lesson takes a quick look at the metric system of measurement, the history of its development, some common units, and the use of decimal points. You will study how the decimal system relates to powers of ten.

Powers of Ten

The system of measurement used extensively in most countries of the world is a version of *système international*, more commonly referred to as the metric system. This system was developed in the late 1700s, with an aim of establishing one common system of measurement for everyone around the world. The metric system is based on the metre. The metre is a fixed unit of length that is one ten-millionth of the distance from the equator to the North Pole, and passing through Paris. The average distance from the earth to the sun is 149 597 870 691 metres.

Base Ten and Decimals

The Base 10 (Decimal) Numbering system was developed by the Indian culture some 5000 years ago. The use of zero by the Hindu mathematicians started some 1500 years ago. The digits you use for the decimal system are the Arabic/Indian digits of 0 through 9.

Given that 10 is the number of fingers and thumbs on a person's hands, it is thought that the decimal system originated using the digits on a person's hands to represent each power of ten.

Moving the Decimal Point

Example 1

You are given the number 3.40.

a) Move the decimal point one place to the right. What number do you get?

b) Move the decimal point two places to the right. What number do you get?

Solution:

a) If you move the decimal point one place to the right, you get 34.0, which is ten times the original value.

b) If you move the decimal two places to the right, you get 340. This is one hundred times the original value.

Each time the decimal point is moved one place to the right, the value of the number increases tenfold.

Example 2

Multiply 76.98 by 10 by moving the decimal point.

Solution:

Since multiplying by 10 moves the decimal point one place to the right, then $76.98 \times 10 = 769.8$.

Notice the decimal has moved one place value to the right, and the result is 10 times the original value.

Include this as a reminder on your resource sheet.

Example 3

You are given the number 12.34.

a) Move the decimal point one place to the left. What number do you get?

b) Move the decimal point two places to the left. What number do you get?

Solution:

a) If you move the decimal point one place value to the left, you get 1.234, which is $\frac{1}{10}$ the value of the original figure. This is the same result as if you had divided by 10.

b) If you move the decimal point 2 units to the left, you get 0.1234, which is $\frac{1}{100}$ the original value. This is the same result as when you divide by 100.

Each time the decimal point is moved one place value to the left, the value of the number decreases tenfold.

Example 4

Divide 856.72 by 100 by moving the decimal point.

Solution:

Since dividing by 100 moves the decimal point two places to the left, then $856.72 \div 100 = 8.5672$.

Notice the decimal point has moved 2 units to the left, and the result is one hundredth the original value.

Make a note of this on your resource sheet as a reminder.

The decimal system is referred to as a "positional numeral system." This system has specific positions for ones, tens, hundreds, etc. Each time the position of the decimal point moves, the value of the number changes by a power of ten.

A zero is used in the decimal system to represent that there is no quantity of that particular place value.

Example 5

Explain the use of the zero in the answer to the multiplication question $4.5 \times 100 = 450$.

Solution:

Notice the zero in the "ones" value in the answer. The zero holds the place, and indicates there are no ones. If the zero is not included, the answer would have been 45.

Learning Activity 3.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are having to buy items at several stores in the mall. You walk 100 m to get to the first store. From there you walk 400 m to the second store. You have to walk 350 m to the third store, then walk 800 m back to your car. How far did you walk?
2. At the end of the day, a restaurant is left with three different, partially eaten pies. Each pie has 2 pieces out of 7 pieces left over. How much pie is left over in total?
3. There is a great sale on clothes; 30% off the marked price. If you are buying a hoodie that is priced $40.00, how much will you save?
4. What is the range of the following numbers: 2, 6, 4, 8, 7, 13, 11?
5. Solve for k: $\frac{k}{8} = 2$.

Part B: Moving the Decimal Point

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Identify the multiple of 10 used to either multiply or divide to find the answer.

1. a) $52.30 \times \_\_\_\_ = 523.0$
   b) $1.4567 \times \_\_\_\_\_ = 145.67$
   c) $0.15 \times \_\_\_\_\_ = 1500.0$

continued

Learning Activity 3.1 (continued)

2. a) 946.3 ÷ _____ = 9.463
   b) 8.77 ÷ _____ = 0.877
   c) 0.25 ÷ _____ = 0.000 25

3. Write the operation of multiplication or division, and the multiple of 10 necessary to reach the answer.
   a) 96.4 _____ = 0.964
   b) 605.25 _____ = 6052.5
   c) 0.1234 _____ = 123.4
   d) 2.3 _____ = 0.0023

Powers of 10

In your Grade 9 Mathematics studies you learned that powers are repeated multiplications. For example, 2^4 would mean $2 \times 2 \times 2 \times 2$. The number, 2, is called the base, whereas, 4 is called the exponent and 2^4 is called a power.

Numbers in the decimal system use powers of 10 as their place values. The base is always 10. Each time you move the decimal point one place value you are either multiplying or dividing by ten.

Example 1

Calculate 56.24×10^2 by moving the decimal point.

Solution:

56.24×10^2 indicates you multiply the number, 56.24 by 10 twice.

Thus, you move the decimal point two places to the right. The answer is $56.24 \times 10^2 = 56.24 \times 10 \times 10 = 5624$.

Example 2

Calculate 0.03874×10^3 by moving the decimal point.

Solution:

0.03874×10^3 indicates you multiply the number, 0.03874, by 10 three times.

Thus, the decimal point is moved 3 units to the right.
$0.03874 \times 10^3 = 0.03874 \times 10 \times 10 \times 10 = 38.74$

What is the pattern? When multiplying by a power of 10 with a positive exponent, the decimal moves to the right the same number of places as the exponent.

When you have a power of 10 with a negative exponent, the decimal will move to the left the same number of places as the exponent.

The notation 10^{-1} means the same as multiplying by $\frac{1}{10}$ or 0.1. A negative exponent does not mean the number is negative. Rather, a negative exponent indicates the operation of "write as a fraction." A negative exponent is just a mathematical notation for writing the reciprocal of the base of the power.

This is a fact that many people forget so you should include it on your resource sheet.

Example 3

Calculate 456.7×10^{-2} by moving the decimal. Explain the process.

Solution:

The exponent on 10^{-2} is negative 2, the decimal point is moved 2 units to the left.

$$456.7 \times 10^{-2} = 4.567.$$

To explain the process, you know that 10^{-2} indicates that you multiply by the fraction, $\frac{1}{10}$ twice.

$$456.7 \times 10^{-2} \text{ means } 456.7 \times \frac{1}{10} \times \frac{1}{10}$$

Instead of using fractions, you could write their decimal equivalent.

$$456.7 \times 10^{-2} = 456.7 \times \frac{1}{10} \times \frac{1}{10} = 456.7 \times 0.1 \times 0.1 = 4.567$$

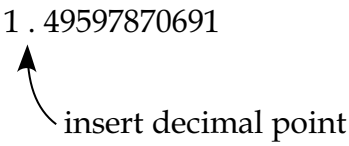
Notice that when the exponent is –2, you are dividing the number by 100 and the decimal point moves 2 places to the left.

Scientific Notation

Scientists are familiar with very large or very small numbers. At the beginning of this lesson, it was mentioned the sun is 149 597 870 691 metres from the earth. That is a large number! A scientist could put this number into scientific notation, using powers of 10.

The rule for scientific notation is that the only number to the left of the decimal point is a value in the ones place. The rest of the numbers are to the right of the decimal point. Also, the number in the ones place cannot be zero.

For the distance of the earth to the sun, you would leave the 1 in the ones place, and then insert a decimal point.



Now, to make the number equal to its original value, you multiply by the power of 10 necessary to move the decimal all the way to the end. You need to move the decimal point 11 places or 10^{11}. The answer would be $1.49597870691 \times 10^{11}$. Now you just need to write the decimal part in metric form. The final answer is $1.495\ 978\ 706\ 91 \times 10^{11}$.

The value is now expressed in "scientific" notation.

To simplify even further, you could round off to two decimal places to get an approximate answer of 1.50×10^{11} metres.

Example 1

The diameter of an atom would be a very small number. In scientific notation, it would be written as 1.0×10^{-14} metres. Change this number to standard form.

Solution:

The negative exponent on 1.0×10^{-14} means you must move the decimal point 14 places to the left to write the number in standard form as 0.000 000 000 000 01 metres.

You cannot just put 14 zeros after the decimal because in the original number, 1.0, the 1 is in front of the decimal. This means that the 1 will take up one of the 14 decimal places.

Example 2

Convert to scientific notation 0.000 000 65 cm.

Solution:

The decimal number 0.000 000 65 converted to scientific notation would be written first as 6.5 with only one number to the left of the decimal point. Then 6.5 is multiplied by a power of 10 that would be the same as moving the decimal point 7 places to the left. The number in scientific notation is 6.5×10^{-7} cm.

Again, you cannot just count the zeros because the 6 must also end up on the left side of the decimal (in the ones position).

Learning Activity 3.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Fill in the missing terms in the following pattern: 0, 3, ___ , 9, 12, ___ .
1. What is 10% of 760?
3. What is 5% of 760?
4. You are paid by commission at work. Your sales for this week were $760. If your commission is 15% of your sales, what is your gross pay?
5. You're missing your cell phone! You know you've left it in one of your pairs of pants, but you're not sure which pair. You wore your jeans on Monday, Wednesday, and Thursday. You wore your sweatpants on Sunday and Tuesday, and you wore your dress pants on Saturday. The last time you remember using your phone was on Wednesday. Which pair of pants should you check first?

continued

Learning Activity 3.2 (continued)

Part B: Scientific Notation

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Without using a calculator, write the answers to the questions below. Use your knowledge of the relationship between the exponent on the base of 10 and the decimal point moving right or left, appropriately.

1. $6535.28 \times 10^{-3} =$ _____
2. $0.056 \times 10^{2} =$ _____
3. $0.004\,89 \times 10^{-3} =$ _____
4. $89.7969 \times 10^{5} =$ _____
5. $1.2345 \times 10^{-4} =$ _____

Reading Calculators

Some calculators will show an "E" on the display to express large numbers. For example, enter into your calculator the following multiplication problem,

$5^{15} \times 3^{12}$. You might have to use the ^ key, the x^y key or the y^x key, depending on the type of calculator you are using, as shown below.

Calculator syntax 1	$5^{15} \times 3^{12} =$ 5^15 × 3^12
Calculator syntax 2	$5^{15} \times 3^{12} =$ 5 y^x 15 × 3 y^x 12
Calculator syntax 3	$5^{15} \times 3^{12} =$ 5 x^y 15 × 3 x^y 12

Chances are your calculator will give an answer of 1.621829224 E 16 or 1.621829224^{16}.

This does not mean "to the exponent 16", but rather $1.621829224 \times 10^{16}$. The "E 16" represents the 16th power of 10.

Notice your calculator automatically places very large or very small numbers in scientific notation.

You could write the answer in metric notation as $1.621\,829\,224 \times 10^{16}$.

Lesson Summary

In this lesson, you identified two measurement systems, imperial and metric, and learned a bit about the history of their development. You practised multiplying a number by a power of 10, by moving the decimal point in the number according to the exponent on the power of 10. Then you learned to write numbers in scientific notation.

In the next lesson, you will apply the names of metric prefixes to each power of 10.

Assignment 3.1

Total: 18 marks

Powers of Ten

1. Indicate by what power of 10 you multiply the first number by to reach the second number. *(4 marks)*

   a) $64.58 \times$ _____ $= 6.458$

   b) $0.12345 \times$ _____ $= 123.45$

   c) $0.286 \times$ _____ $= 0.002\,86$

   d) $6466.32 \times$ _____ $= 646\,632\,000$

2. Write the answer after multiplying the number by the power of 10. *(10 marks)*

   a) $7.65 \times 10^{-3} =$ _____

   b) $0.06086 \times 10^{2} =$ _____

   c) $0.035 \times 10^{-3} =$ _____

   d) $8\,905\,420.6 \times 10^{4} =$ _____

   e) $2.5 \times 10^{2} \times 3.0 \times 10^{3} =$ _____

continued

Assignment 3.1: Powers of Ten (continued)

3. Use your calculator to find the following products. Express your answer using scientific notation, and rounded to 2 decimal places. *(4 marks)*

   a) $6^6 \times 9^8 =$ _____

   b) $8^9 \times 5^{11} =$ _____

   c) $3.5^{-14} \times 4.2^{-8} =$ _____

   d) $12^{-12} \times 11^{-11} =$ _____

Lesson 2: Metric System

Lesson Focus

In this lesson, you will

- ❑ identify the base units of measurement in the metric system
- ❑ determine the relationship among the prefixes
- ❑ identify contexts that involve the metric system

Lesson Introduction

This lesson deals with using a prefix in the naming of each power of 10 and applies this concept to metric measurement.

The Metric System

A sprinter in the Olympic Games runs a 100 m race. The distance from Winnipeg to Regina is 550 km. A fire ant is about 1 cm long. These are all length or distance measurements. To gain an understanding or concept of their length, you must understand the metric prefix and how it relates to a power of 10.

When determining a measurement in the metric system, you must first recognize the type of measurement and the base unit associated with it. Base units could be used for measuring length, area, volume, capacity, mass, density, force, speed, acceleration, energy, temperature, viscosity . . . the list goes on and on.

In this module, you will be discussing length, mass, and capacity. The table shows the three types of measurement, the base unit for each, and their symbols.

Type of Measurement	Base Unit	Symbol
Length	1 metre	1 m
Mass	1 gram	1 g
Capacity	1 litre	1 L

The word *length* is hardly ever misunderstood or misused. However, the other two terms in this table need an explanation.

The first is *mass*. This word is used in this course, rather than weight. Scientifically, there is a difference between weight and mass. Your weight is affected by gravity and so your weight is dependent upon which gravitational field you are in—whether you are on earth or somewhere in space. Your weight on the moon would be about $\frac{1}{6}$ of what it is on earth, and your weight at the Equator is slightly less than your weight at the North Pole. However, your mass on earth is the same as your mass on the moon. Your mass is not affected by gravity.

However, you often say that you are weighing an object to get its mass.

The other notable term in the table above is capacity, which means the amount a container can hold. The unit of measure for capacity is litres.

Sometimes the words *capacity* and *volume* are interchanged. Usually, volume refers to the amount of space an object occupies and is measured in cubic units that are different from the units for capacity.

In this course, you will encounter examples measuring how much a container might hold. In the Grade 11 Essential Mathematics program, the difference between capacity and volume is discussed in more detail.

Prefixes for Larger Numbers

The metric prefixes used for measurements larger than the base unit are shown in the table below.

Prefix	Symbol	Multiplication Factor
mega	M	$10^6 = 1\,000\,000$
kilo	k	$10^3 = 1000$
hecto	h	$10^2 = 100$
deca	da	$10^1 = 10$

Include this table, or at least the page number, on your resource sheet as a reference.

To understand this table, examples are given using the metre as the base unit, although the same prefixes would apply to grams and litres.

1 metre is the base unit for length.

10 metres is called 1 decametre and 100 metres is called 1 hectometre. Thinking in reverse, 1 hectometre is a length of 100 metres.

1000 metres constitute a kilometre.

If you had a pocket full of assorted coins, when you add up the total value, you would say, as an example, you had $3.75, not 375 cents. In mathematics, you wouldn't say that the distance from Winnipeg to Regina is 550 000 metres. People would have trouble understanding just how far that is. In the same way that you simplify fractions to their lowest form, you simplify metric measurements by using prefixes for a larger unit other than the base unit. In this case, 550 000 m would be expressed as 550 km, knowing that each kilometre contains 1000 metres.

Rarely are the prefixes of deca, hecto, or mega used. Even distances in space are expressed in kilometres. An average distance from the centre of the earth to the centre of the moon is 384 400 km. Since 1 megametre is equivalent to 1000 km, you could say the moon is 384 Mm away, but scientists use the more familiar kilometre.

In review, the basic unit is one gram, or one metre, or one litre. The most commonly used prefix is kilo, which represents 1000 of the base units. Values above one are simplified to the higher prefix.

Example

Simplify 8525 m using an appropriate prefix.

Solution:

8525 m would be simplified to 8.525 km or rounded to 8.5 km.

Prefixes for Smaller Numbers

These prefixes form names of numbers that are smaller than the base unit.

Prefix	Symbol	Multiplication Factor	Fraction
deci	d	$10^{-1} = 0.1$	one tenth
centi	c	$10^{-2} = 0.01$	one hundredth
milli	m	$10^{-3} = 0.001$	one thousandth
micro	μ	$10^{-6} = 0.000\ 001$	one millionth
nano	n	$10^{-9} = 0.000\ 000\ 001$	one billionth

Add this table to your resource sheet, or the page number, so that you can find it later.

Have you ever heard the term "nanosecond?" It means one billionth of a second, a very small number indeed! As you can see in the table, the prefixes represent values smaller than the base unit of 1. The negative exponent indicates the decimal point would be moved toward the left.

The base unit is 1 metre or 1 gram or 1 litre. We will consider how these prefixes are used with 1 metre, although these terms apply equally to grams or litres.

Note that the word decimal has "deci" in it, meaning tenth. The word *decimal* is derived from the Latin word *decimus*, which means "tenth."

If you divide 1 metre into 10 equal parts, each part would be a decimetre.

A decimetre is a tenth of a metre. 10 decimetres would equal to one metre.

Dividing the metre into 100 equal parts gives 100 centimetres or 100 cm = 1 m. To remember that the prefix *centi* means $\frac{1}{100}$, you also say there are 100 cents in 1 dollar. And a centipede has 100 legs.

Note that when expressing measurements in metric, you would usually use decimals rather than fractions.

A centimetre is one hundredth of a metre, or 1 cm = 0.01 m.

Your little finger is about 0.05 m long. Instead of saying your finger is $\frac{5}{100}$ of a metre, you say it is 5 cm long. The thickness of a loonie is said to be 1.75 millimetres, rather than $\frac{1.75}{1000}$ metres or 0.0175 m.

In the same way that you simplify measurements larger than 1 base unit to a higher value of prefix, you simplify values less than 1 base unit to the lower-valued prefix.

Example

Simplify each measure using an appropriate prefix.

1. 0.5 m
2. 0.6 cm
3. 0.9 m

Solution:

1. 0.5 m would be more commonly stated as 50 cm.
2. 0.6 cm could be written as 6 mm.
3. 0.9 m would equal 9 decimetres or, more commonly, 90 cm

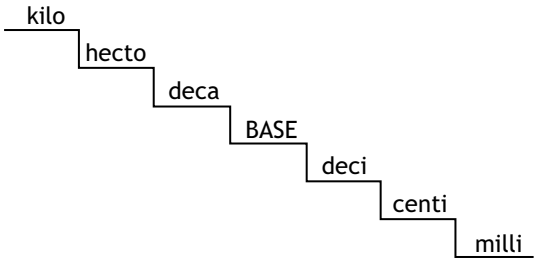
Most Common Metric Units

Length	kilometre (km) metre (m) centimetre (cm) millimetre (mm)
Mass	kilogram (kg) gram (g) milligram (mg)
Capacity	kilolitre (kL) litre (L) millilitre (mL)

Conversions within the Metric System

The metric system units are related to each other in a "base 10" manner. You can convert between metric units by merely moving the decimal point an appropriate number of places. The ladder in the diagram below is useful for conversions within the metric system. As you go down the ladder, the prefix represents a smaller amount.

Metric Prefix Ladder



BASE represents 1 gram or 1 litre or 1 metre.

For example, to convert 5 grams into milligrams, you start on the BASE step of the ladder since grams are the base unit, and move down three the steps to "milli." This represents moving the decimal point three places to the right.

Thus 5 grams = 5.0 grams = 5000 milligrams

Note the page number of this ladder on your resource sheet so that you can refer back to it.

Example 1

Convert 35 centimetres to metres using the ladder method.

Solution:

To convert 35 centimetres to metres, start on the step called "centi" and move up to the required unit of metres. This is 2 steps up. This would indicate moving the decimal point 2 units to the left.

Thus, 35 cm = 35.0 cm = 0.35 m

Learning Activity 3.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You use your left hand to type more than your right when you have your hands on the keyboard properly. Typing the word *factor*, you use your left hand for 5 letters and your right hand for one. Write the fraction that represents how many times you use your right hand in total.
2. Usain Bolt can run 100 m in 10 s. What is his average speed?
3. List the factors of 12.
4. GST (Goods and Services Tax) is 5%. You are buying clothes for your baby cousin (baby clothes only have GST). If the total before tax is $44.00, how much tax are you charged?
5. CPP is supposed to be 4.95% of your gross pay. Your gross pay is $1400 and you are charged $140 for CPP. Are you being charged the correct percent?

Part B: The Metric System

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The base unit for measuring capacity is _____ .
2. The base unit for measuring mass is _____ .
3. The base unit for measuring length is _____ .
4. The word *decimal* is derived from the Latin word *decimus*, which means _____ .
5. Complete the following rule for using the ladder for conversions.

   If you step up, move the decimal point to the _____ . (right or left).

   If you step down move the decimal point to the _____ . (right or left).

continued

Learning Activity 3.3 (continued)

6. Convert the following.
   a) 650 mL = _____ L
   b) 385 m = _____ cm
   c) 8562 mg = _____ g
   d) 6.5 km = _____ m

Appropriate Units According to Context

It is helpful to learn which unit of measurement is most appropriate for a given situation. You wouldn't measure the length of a highway in centimetres, nor would you measure the capacity of a thimble in litres. When matching the unit of measurement with the object to be measured, the following steps could be used. Identify the base unit of metre, gram, or litre. Then consider which prefix would best fit the measurement, and then state a good estimate.

Example 1

Estimate the mass of a wrestler.

Solution:

Step 1: Identify the base unit for mass: gram

Step 2: Consider which the most suitable prefix is: kilo

Step 3: State a good estimate: 80 to 100 kilograms (a kilogram is approximately 2.2 pounds)

Example 2

Estimate how much fluid a soft drink can holds.

Solution:

Step 1: Identify the base unit for capacity: litre

Step 2: Consider which the most suitable prefix is: milli

Step 3: State a good estimate: 300–400 millilitres

Example 3

Estimate the length of your leg.

Solution:

Step 1: Identify the base unit for length: metre

Step 2: Consider which the most suitable prefix is: centi

Step 3: State a good estimate: 85–100 centimetres

Lesson Summary

In this lesson, you examined three common types of measurement in the metric system and the base unit for each of them. You learned 4 prefixes for numbers larger than the base, and 5 prefixes for smaller numbers, and saw how they are related to powers of 10. You listed the 10 most common metric units for the three types of measurement. To make conversions among metric units, you used the ladder method. The three-step process was helpful for deciding the best measurement to use in a particular context.

In the next lesson, you look at the imperial system of measurement used in the United States, and compare it with British and Canadian imperial units.

Notes

Assignment 3.2

Total: 16 marks

Metric System

1. Enter the most appropriate unit of measurement for each situation. *(6 marks)*
   a) the distance from Vancouver to Hong Kong _____
   b) the amount of water a swimming pool can hold _____
   c) the mass of a pinch of salt _____
   d) the mass of a Hummer _____
   e) the amount of fluid a medicine dropper might contain _____
   f) the thickness of a pencil lead _____

2. Convert each measurement to the indicated unit. *(8 marks)*
   a) 9.5 m = _____ cm
   b) 975 mL = _____ L
   c) 85 mg = _____ g
   d) 3.5 L = _____ mL
   e) 48 cm = _____ m
   f) 56 km = _____ mm
   g) 9 kg = _____ mg
   h) 3.88 kL = _____ L

continued

Assignment 3.2: Metric System (continued)

3. Indicate with a checkmark which of the following is NOT measured in metric units. *(2 marks)*

   a) Length of a CFL football field _____

   b) Milk container _____

   c) Tube of toothpaste _____

   d) 2 x 4s at the lumber yard _____

   e) Sandwich meat at a deli _____

Lesson 3: Imperial System

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of three imperial systems
- ❑ compare imperial units for capacity, mass, and length
- ❑ convert within imperial units

Lesson Introduction

The imperial system of measurement is used for some measures in Canada, in Britain, and in the United States. This lesson examines how to use the imperial system.

The Imperial System

How tall are you? If your first thought was, for example, 5 foot 8 inches, then you are thinking in the imperial system. Almost anywhere else in the world, the answer would have been 173 cm.

How much do you weigh? If your answer was, for example, 125 pounds, then again you are thinking in the imperial system. For people elsewhere in the world, the answer would have been that your mass is 57 kg.

Canadians still use imperial measurements in certain situations. When baking, a cup of milk is 8 fluid ounces. When speaking to people who remember the change in Canada from imperial to metric, the imperial system is still very familiar. Another example is the birth of a baby. The hospital measures the length and weight in metric units but birth announcements are usually in pounds, ounces and inches. At auction marts, cattle are sold in pounds while hogs are sold in kilograms.

Because Canadians live so close to the United States and share their culture, entertainment, shopping, sports, and trade agreements, you need to understand their system of measurement.

The speed limit on many Canadian highways is 100 km/h. In the US it is 75 mph, but cars are actually travelling faster on American highways. In Canada, the unit is kilometres per hour and in the US the unit is miles per hour. Once converted to metric units, 75 miles per hour is actually about 121 km/h.

British and US Systems

The "US customary" system of measurement came to the United States from Britain in the 1700s, and used the old British measurement system. After America declared their independence from Britain in 1776, a few of the units were altered. By 1824, the imperial system was adopted in the United States.

The differences are mainly in capacity measurements in fluid ounces with different conversions for the measure of a cup.

This lesson will show you some of these differences, and each question involving capacity units will state exactly which system to use.

British, American, and Canadian Imperial Systems for Capacity and Mass

	British	American	Canadian
cup	10 fl. oz.	8 fl. oz.	8 fl. oz.*
quart	40 fl. oz.	32 fl. oz.	
gallon	160 fl. oz.	128 fl. oz.	160 fl. oz.**
ton	2240 pounds	2000 pounds	2000 pounds
bushels, wet	8 gallons 1280 fl. oz.	8 gallons 1024 fl. oz.	8 gallons 1280 fl. oz.
bushels, dry oats	34 pounds	32 pounds	34 pounds

* Having 8 fl. oz. in a cup means there would be 16 × 8 = 128 fl. oz. in a gallon, but that is not the case in Canada. Be very careful which system is being used for capacity. Using 8 fl. oz. in a cup is a standard measurement in a kitchen.

** Using 160 fl. oz. in a gallon is prescribed by the Canadian government.

Notice in Canada sometimes the measurements agree with the British system and sometimes they agree with the US system. The same words are used but the measures are different.

Generally, liquid measurements in fluid ounces in the US imperial system are roughly 20% smaller than in those in the British system of measurement.

Should you be doing business with a company in Britain, you will want to remember that some of their measurements are different than the Canadian and American versions.

Common Imperial Units

In the metric system, as you discovered in the previous lesson, there are some measurements that are used more often than others. The same pattern applies to the imperial system. It will be those units that are studied in this course.

Most Common Imperial Units	
Length	mile (mi.) yard (yd.) foot (ft. or ') inch (in. or ")
Mass	ton (tn.) pound (lb.) ounce (oz.)
Capacity	fluid ounce (fl. oz.) cup (c.) pint (pt.) quart (qt.) gallon (gal.)

Notice in the list above the word, ounces is used in two different contexts. There are ounces to measure mass and fluid ounces to measure capacity.

Example

How many fluid ounces are in a gallon?

Solution:

From the table above, you can see that the type of system needs to be defined before you can answer the question.

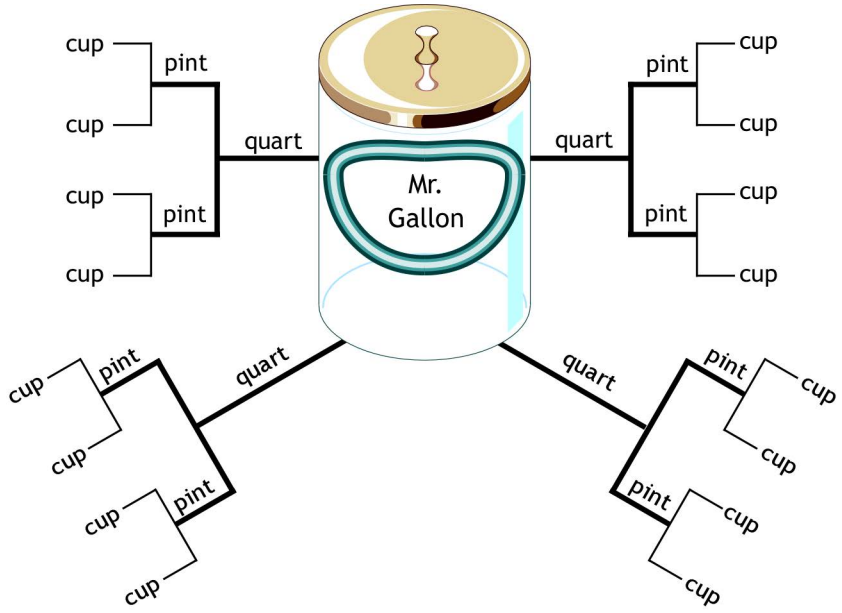
In the British and Canadian systems, there are 160 fluid ounces in a gallon. In the American system, there are 128 fluid ounces in a gallon.

Relationship among Imperial Units

The following table shows how imperial units are related to each other.

Imperial Units Conversion Chart	
Length	1 mile = 1760 yards or 5280 feet 1 yard = 3 feet or 36 inches 1 foot = 12 inches
Mass	1 ton = 2000 pounds (lb.) 1 pound = 16 ounces (oz.)
Capacity	1 gallon = 4 quarts 1 quart = 2 pints 1 pint = 2 cups

The following diagram also helps to remember the conversions among units of capacity.



1 gallon = 4 quarts = 8 pints = 16 cups

Conversions within the Imperial System

When asked to convert one unit to a different unit, you can refer to the relationship chart or to the Mr. Gallon diagram.

Example 1

Convert 5 feet to inches.

Solution:

One method you could use to make conversions is to use proportional thinking.

Set up the relationships in the four slots of the box. Fill the first row with the rule from the conversion table, 1 foot = 12 inches.

1 foot	12 inches

Now fill the second row with the required conversion and use an "x" to represent the unknown amount. Ensure the units in each column are the same. (Note that you could also set up the box by ensuring the units in the rows instead of the columns are the same. To be consistent throughout the course, the column method has been chosen here.)

1 foot	12 inches
5 feet	x inches

Now set up the proportion, cross multiply, and solve for x.

$$\frac{1}{5} = \frac{12}{x}$$

$$(1)(x) = (5)(12)$$

$$x = 60$$

Thus, 5 feet = 60 inches.

Note: Cross multiplying only works in special situations. You must always have an equation that is in the form of two fractions, $\frac{a}{b} = \frac{c}{d}$. Otherwise, cross multiplying just doesn't make sense. Remember, cross multiplying is just a shortcut for removing both denominators in the equation.

Example 2

Fill in the blank, 48 ounces = _____ pounds.

Solution:

Fill in the slots in the box. The appropriate rule from the conversion chart is 1 pound = 16 ounces.

1 pound	16 ounces
x pounds	48 ounces

Notice how each column contains the same units, the first column, pounds, the second column, ounces.

Now set up the proportion, cross multiply, and solve for x.

$$\frac{1}{x} = \frac{16}{48}$$

$$(16)(x) = (48)(1)$$

$$16x = 48$$

Divide both sides of the equation by 16 to isolate x.

$$\frac{16x}{16} = \frac{48}{16}$$

$$x = 3$$

Thus, 48 ounces = 3 pounds.

Example 3

How many cups are in 3 gallons?

Solution:

From the Mr. Gallon diagram, you know that 1 gallon = 16 cups.

Thus, in 3 gallons there are $3 \times 16 = 48$ cups.

Learning Activity 3.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If it rains 10 mm, how much rain is that in centimetres?
2. A marathon is 26.4 miles. A half marathon is how long?
3. Express the fraction as a decimal: $4\frac{3}{5}$.
4. There are two movies coming out on DVD this week that you would like to purchase. Each movie costs $18.99. If you have $35, can you afford to buy both?
5. Penelope is half as tall as Carson. Tara is almost as tall as Penelope. Carson is an inch taller than Zach. Who is the tallest?

Part B: The Imperial System

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Do the basic conversions within the imperial system for each of the questions below.

1. a) 4 feet = _____ inches
   b) 60 inches = _____ feet
   c) 126 inches = _____ feet
   d) $3\frac{1}{2}$ miles = _____ inches

continued

2. a) 3 pounds = _____ ounces
   b) 2 tons = _____ pounds (Canadian)
3. a) 6 quarts = _____ gallons
   b) 3.5 gallons = _____ pints

Appropriate Units in Context

You must be able to use the appropriate unit to estimate the measure of an object in different contexts or situations. If someone tells you he is 68 inches tall, this might not be as effective as him saying he is 5 ft. 8 in. tall. Similarly, a hockey rink hosting an NHL game must be 2400 inches long. This measurement is difficult to understand. However, if you convert 2400 inches to 200 feet, someone might understand this measurement more easily.

When dealing with mass, you might need to round up to the higher unit. Saying your mass is 2160 ounces makes little sense. When you convert those ounces to 135 pounds, you can relate to this measure.

Example 1

Estimate the amount of liquid a large punch bowl might contain.

Solution:

You use the following steps for estimating. This process is somewhat different in the imperial system than it was in the metric system, where you needed to choose the base unit and the prefix. In the imperial system, you need to identify the appropriate unit for length, mass, or capacity, and then state a good estimate.

Step 1: Capacity unit: gallons or cups

Step 2: Estimate: 2 gallons or 32 cups to serve about 24 people

Example 2

Estimate the mass of a house fly.

Solution:

Mass unit: ounces

Estimate: $\frac{1}{100}$ ounce

Example 3

Estimate the distance from Winnipeg to Brandon.

Solution:

Length unit: miles

Estimate: 120 miles

Lesson Summary

This lesson showed the basic units of measurement in the imperial system for length, mass, and capacity. You considered why it is important to learn the imperial system and the difference between the British, Canadian and US Imperial systems. You learned 4 commonly used units for length, 3 for mass, and 5 for capacity. You studied the conversion table and learned the Mr. Gallon diagram relating units in the imperial system. You learned to convert the units within the system by setting up and solving proportion equations. Also, you learned to estimate the correct imperial measurement depending on the context.

The next lesson teaches you to read a ruler using the imperial system, and reviews the basic fraction skills.

Notes

Assignment 3.3

Total: 16 marks

Imperial System

1. Name the most appropriate imperial unit to measure each of the following. *(6 marks)*

   a) width of your hand _____

   b) amount of pop a soft drink can would hold _____

   c) mass of a semi-trailer _____

   d) distance from Winnipeg to Banff _____

   e) amount of water a swimming pool might contain _____

   f) your brother's mass _____

2. Convert the following imperial units as requested. *(8 marks)*

   a) 3 feet = _____ inches

   b) $3\frac{1}{2}$ miles = _____ feet

   c) 25 pounds = _____ ounces

   d) 3.2 tons = _____ pounds

   e) 12 cups = _____ quarts

   f) 8 gallons = _____ quarts

   g) 192 fluid ounces = _____ gallons (American)

   h) 126 720 inches= _____ miles

(continued)

Assignment 3.3: Imperial System (continued)

3. Indicate with a checkmark which items are NOT measured using the imperial system for someone living in Canada. *(2 marks)*

   a) Yards gained in a football game _____

   b) 4′ × 8′ sheet of drywall _____

   c) Gas bought at the service station _____

   d) A milk container from the store _____

   e) A side of beef weighing 350 pounds _____

Lesson 4: Reading Rulers

Lesson Focus

In this lesson, you will

- ❑ use rulers to measure imperial units
- ❑ user rulers to measure metric units

Lesson Introduction

Measurements for length are found by using rulers, whether you are measuring the length of a board, your height, or the width of a garden. Tape measures, yard sticks and rulers are common tools for measuring length. This lesson teaches you how to read rulers accurately, in both imperial and metric units.

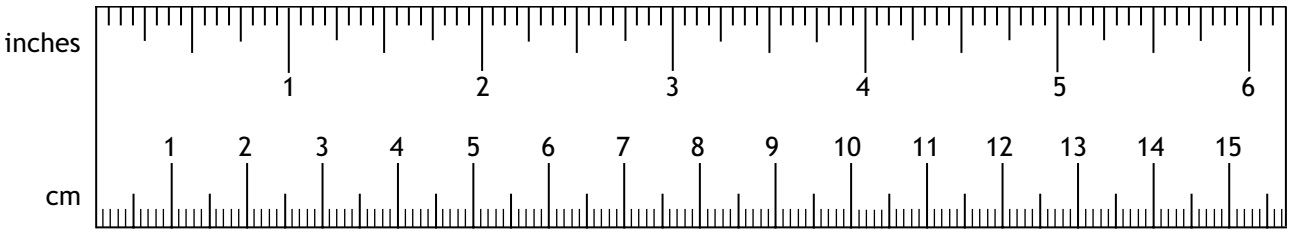
How to Use a Ruler

Accurate readings of measurements are very important. If you are building something, you have to read the measuring tape correctly before you make a cut. A carpenter's mantra is "measure twice, and cut once."

This need for accurate reading applies to baking, sewing, or construction. Adding chemicals to a pool or mixing oil with gas in 2-cycle engines, all demand careful and accurate readings. This lesson will deal primarily with reading measurements using imperial and metric rulers.

Comparing Imperial and Metric Rulers

Compare the increments on the metric and imperial ruler shown below. The most common linear measurements are inches and centimetres.

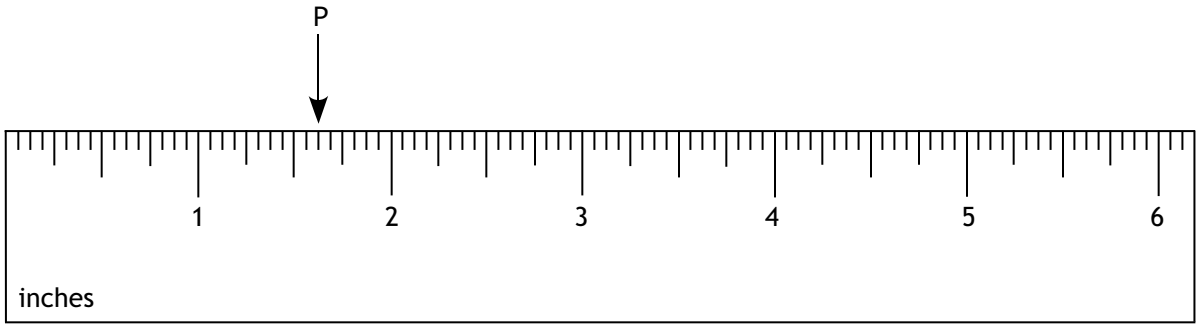


Reading Imperial Rulers

Imperial rulers have inches as units. Each inch on a standard imperial ruler is subdivided into 16 equal parts. The quickest way to read an imperial ruler is to go to the lower inch mark, count the notches above it, write the number of notches as a fraction using 16 as the denominator, and then simplify the fraction, if necessary. Each of those little marks represents $\frac{1}{16}$ of an inch.

Example 1

Find the reading on the ruler at the point P, marked with the arrow.



Solution:

Reading from left to right, point P is past the 1 inch mark, but not past the 2 inch mark.

So you know the reading will be 1 inch, but not 2 inches.

Now, count the little marks past the 1 inch mark. You should get 10 marks.

Since each mark represents $\frac{1}{16}$ of an inch, you have $\frac{10}{16}$ of an inch.

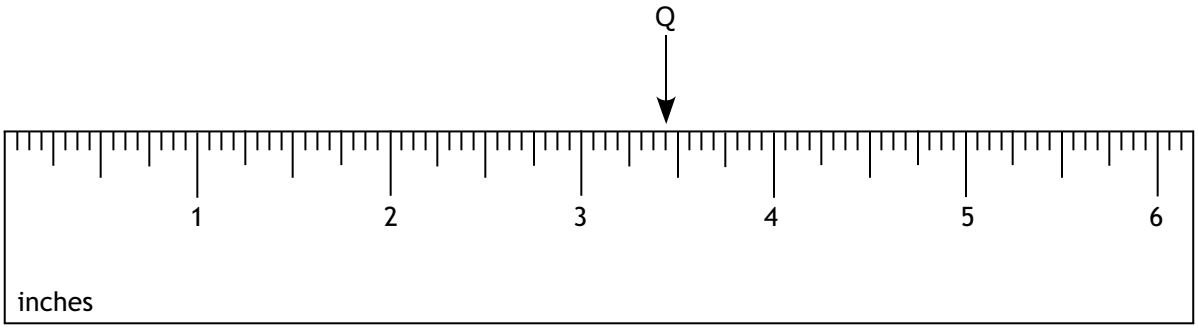
You can reduce $\frac{10}{16}$ to $\frac{5}{8}$ by dividing top and bottom by 2.

The reading on this ruler would be 1 and $\frac{5}{8}$ inches.

This would be written as $1\frac{5}{8}''$.

Example 2

Find the reading on the ruler at the point Q.



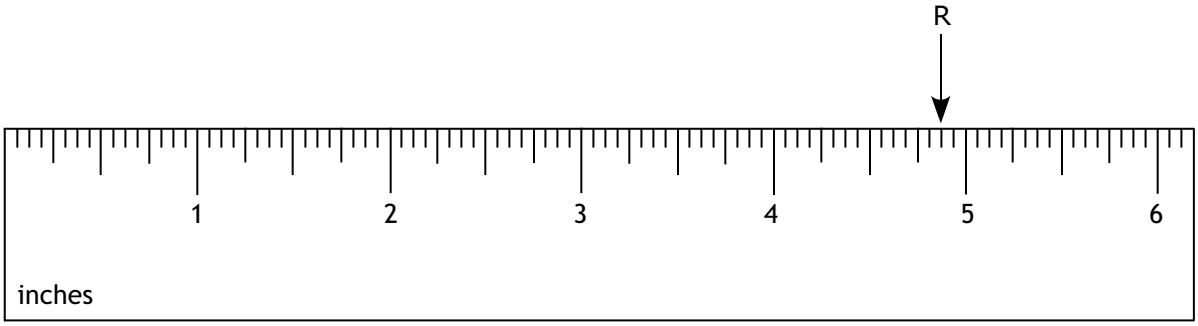
Solution:

In this example, point Q is 7 marks past the 3 inch mark.

The reading is $3\frac{7}{16}$ inches.

Example 3

Find the reading on the ruler at the point R.



Solution:

In this example, the point R is after the 4 inch mark, but not at the 5 inch mark.

Counting the little marks past 3 inches, you get 14. The reading is $4\frac{14}{16}$ inches.

You reduce the fraction by dividing top and bottom by the same number.

$$\frac{14}{16} = \frac{14 \div 2}{16 \div 2} = \frac{7}{8}$$

By reducing the fraction, this can be simplified to $4\frac{7}{8}$ inches.

Learning Activity 3.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If a desk is 90 cm high, how tall is it in metres?
2. Your neighbour just had a baby. The baby is 18" long. How long is the baby in feet?
3. The clock on the wall ticks every second. How many times does it tick in 10 minutes?
4. Write this number in scientific notation: 0.000 243 5.
5. You are at a doughnut shop and would like to buy doughnuts for your family. There are 6 people in your family including you—and you want a doughnut. How much money do you need if each doughnut costs 60¢?

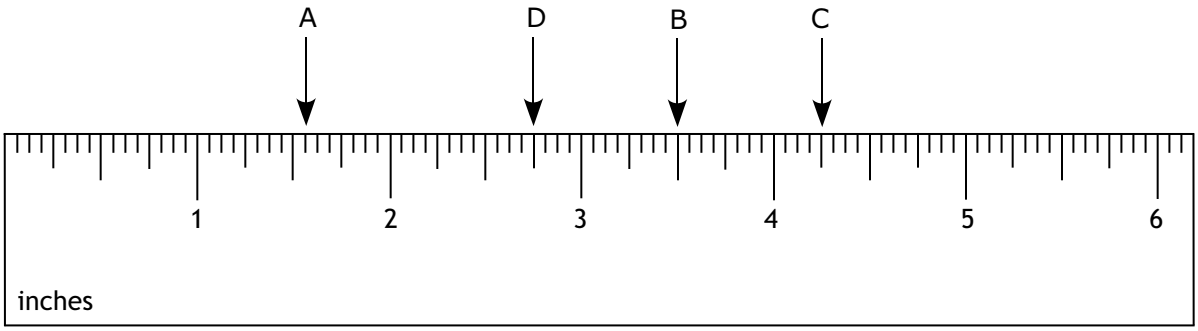
continued

Learning Activity 3.5 (continued)

Part B: Measuring with an Imperial Ruler

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the reading for each point on the imperial ruler.



A _____

B _____

C _____

D _____

2. Use an imperial ruler to draw the length of a line with these measures.
   a) 2″
   b) $5\frac{3}{8}$″
   c) $\frac{9}{16}$″

continued

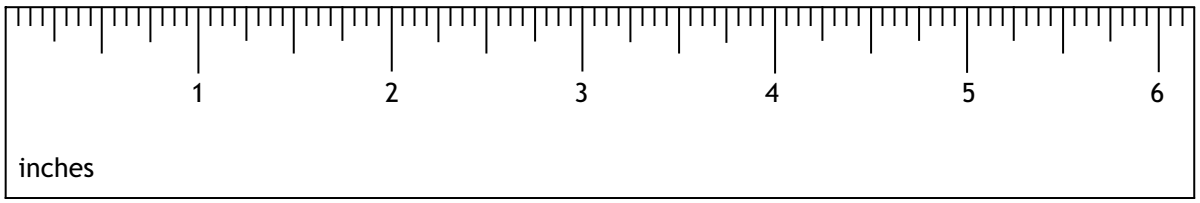
Learning Activity 3.5 (continued)

3. Mark the following points on the blank ruler. Use an arrow to indicate the exact marking.

   a) point G: $3\frac{3}{4}$"

   b) point H: $5\frac{3}{16}$"

   c) point J: $\frac{9}{16}$"

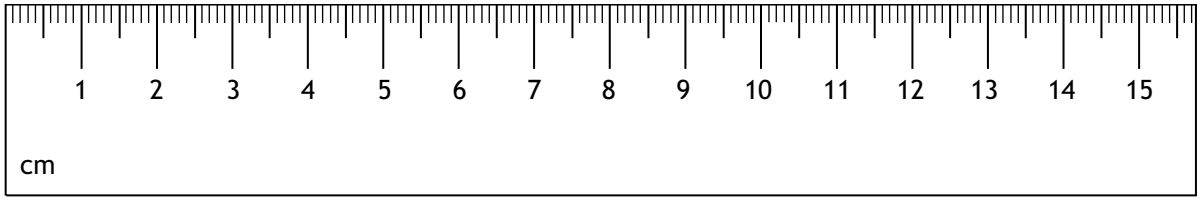


Reading Metric Rulers

The image below is a metric ruler. Count the marks between each number.

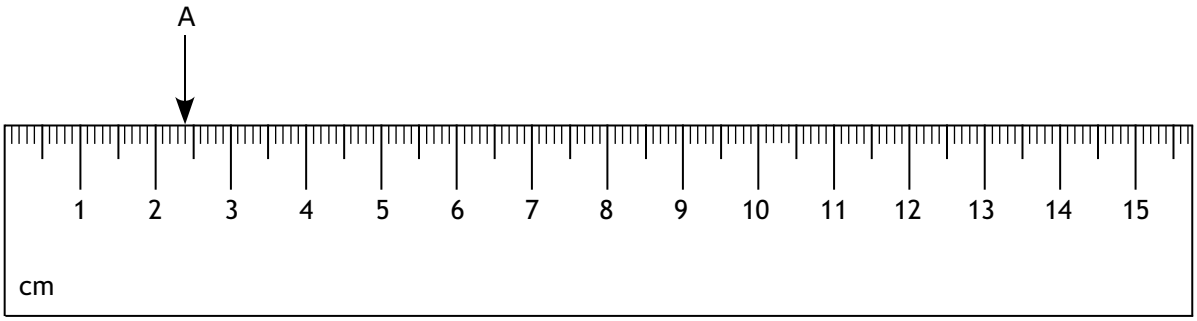
You should count 10. This metric ruler is based on 10 units between each centimetre.

Each mark indicates $\frac{1}{10}$ or 0.1 of a centimetre or 1 millimetre.



Example 1

State the measurement at the point A.



Solution:

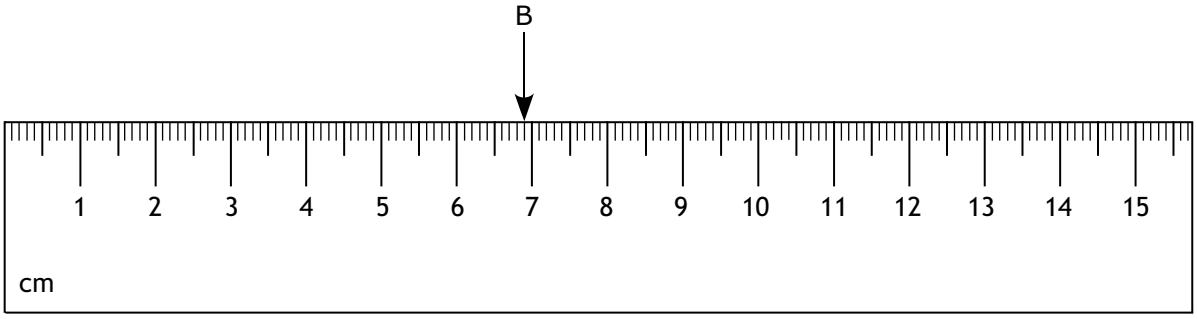
Reading from left to right, point A is past the 2 cm mark, but not past the 3 cm.

The arrow indicates 4 units more than 2 cm.

Since each unit on the metric ruler is 0.1 cm, the total reading would be 2.4 cm.

Example 2

Find the reading on the ruler at the point B.



Solution:

Reading from left to right, point B is past the 6 cm mark, but not past the 7 cm.

The arrow indicates 9 units past 6 cm.

Since each unit on the metric ruler is 0.1 cm, the reading would be 6.9 cm.

Learning Activity 3.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. An anaconda snake can be up to 24 feet in length. How long is that in yards?
2. Solve for m: $2 - m = 14$.
3. Fill in the blanks for the following pattern: 0, 1, 4, 9, ___ , 25, ____ .
4. You want to bring freezies to your last soccer game of the season. You want to have enough so that each player gets two. If you have 17 people on your team, how many freezies do you need?
5. Ryan's usual work week is 38 hours long. This week he worked 46 hours. How many hours of overtime did he work?

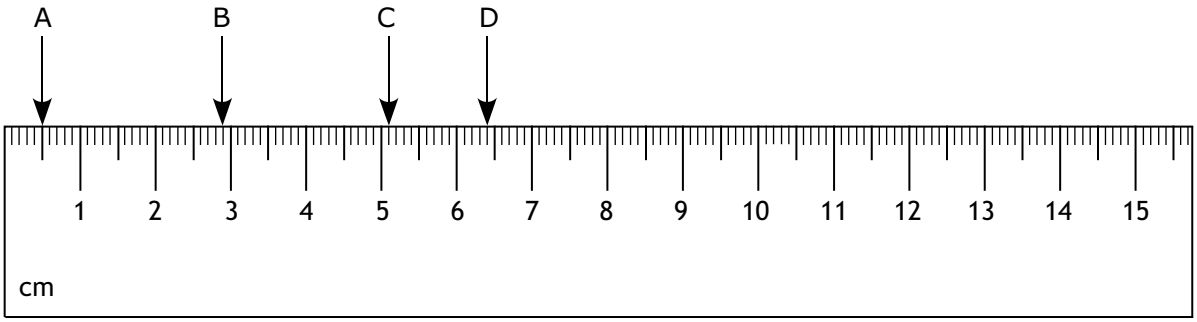
continued

Learning Activity 3.6 (continued)

Part B: Using Metric Rulers

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the reading in centimetres for each point on the metric ruler.



A = _____

B = _____

C = _____

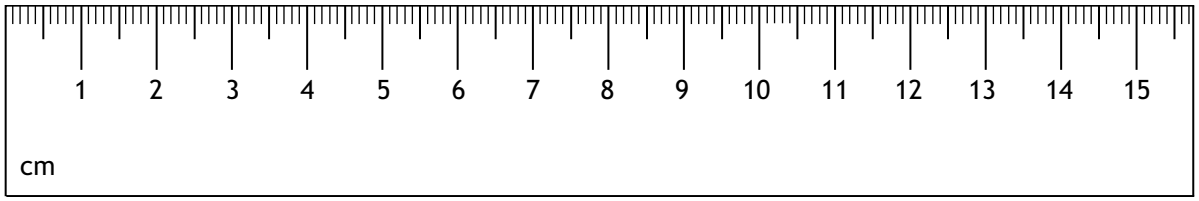
D= _____

2. Use a metric ruler to draw a line, having the length in centimetres.
   a) 5 cm
   b) 7.6 cm
   c) 1.4 cm

continued

Learning Activity 3.6 (continued)

3. Mark the following points on the metric ruler. Use an arrow to indicate the exact marking.
   a) point E at 0.8 cm
   b) point F at 23 mm
   c) point G at 4.8 cm
   d) point H at 6.7 cm



Lesson Summary

In this lesson, you practised reading and drawing lines using an imperial ruler and a metric ruler. The imperial ruler is divided into 16 parts per inch. The metric ruler is divided into 10 parts per centimetre.

The next lesson deals with converting values from one measurement system to the other.

Assignment 3.4

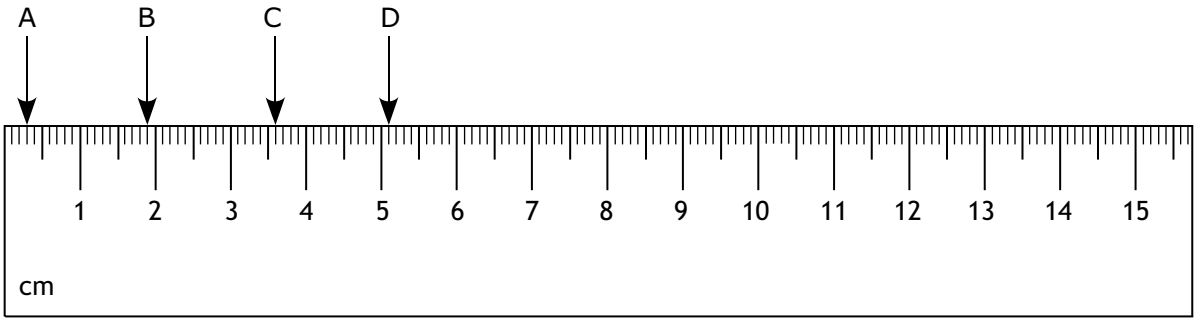
Total: 17 marks

Reading Rulers

1. The smallest unit of measurement on a standard imperial ruler is _____. *(1 mark)*

2. The smallest unit of measurement on a standard metric ruler is _____. *(1 mark)*

3. Write the reading in centimetres for each point on the metric ruler. *(4 marks)*



A = _____

B = _____

C = _____

D = _____

4. Use an imperial ruler to draw a line of the given length. *(3 marks)*

a) $1\frac{7}{8}$"

b) $\frac{11}{16}$"

c) $3\frac{3}{4}$"

continued

Assignment 3.4: Reading Rulers (continued)

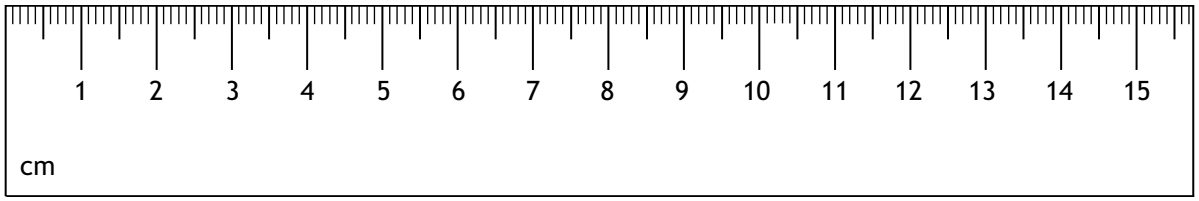
5. Use an arrow to plot these readings on a metric ruler. *(4 marks)*

   a) point S at 6.4 cm

   b) point T at 0.8 cm

   c) point U at 4.5 cm

   d) point V at 3.2 cm



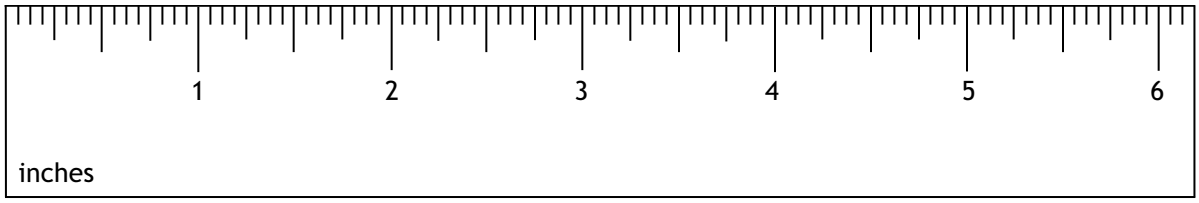
6. Use an arrow to plot these readings on an imperial ruler. *(4 marks)*

   a) point A at $\frac{3}{8}''$

   b) point B at $2\frac{5}{16}''$

   c) point C at $3\frac{1}{4}''$

   d) point D at $4\frac{9}{16}''$



Lesson 5: Conversions

Lesson Focus

In this lesson, you will

- ❑ apply strategies to convert between imperial and metric units
- ❑ provide approximate measures in both systems using referents

Lesson Introduction

This lesson deals with converting basic metric values to imperial values, and basic imperial values to metric readings.

Changing from Metric to Imperial Measurements

Canada did not always use the metric system of measurement. In 1970 the federal government mandated that Canadians would be adopting the metric system. Before that time, imperial measurements were used.

There are still some common measurements in Canada that use imperial units. Some measurements were never changed. Also, you need to understand the imperial system of measurement used in America. The US is by far Canada's largest trading partner and closest neighbour. Many Canadians travel to the US, or buy items from American businesses. Many people from countries outside North America need to adopt the same strategy when dealing with the US.

This lesson identifies some common situations in your daily life where the imperial system is still used. You will learn to convert units of measurement to the metric system.

Some Imperial Measurements Still Used in Canada

In some aspects of your daily life, imperial measurement is still used. A trip to the lumber yard or the hardware store finds you in a world of imperial measurements. Most wrenches are still calibrated in inches, lumber in feet, and paint is still sold in gallon cans. House sizes in Canada are still described in square footage. Golf and football enthusiasts refer to their game in yards. Even marathons in Canada are still 26 miles long. Baseball fields all have imperial dimensions, with the bases being exactly 90 feet apart, and the centre field wall about 400 feet away from home plate. Grocery stores will print both measurements on packages of meat.

In Winnipeg, water bills are calculated in terms of cubic feet. In 2009, water bills were calculated at $3.55 per 100 cubic feet and the sewer rate was $5.26 per 100 cubic feet.

As you can see, you need to have an understanding of both systems of measurement.

Common Metric Measurements

Weather measurements in Canada are in metric units. You measure temperature in degrees Celsius, rain in mm, and snowfall in cm. The winds as well as highway speed limits are measured in km/h. Fuel economy on vehicles is listed as L/100 km.

Grocery items are mainly measured in metric units in Canada. Everything in bottles is measured in mL, and dry goods are measured in grams. Deli meats are usually priced at a cost per 100 grams.

Common Imperial Measurements

If you find yourself in the United States, metric measurements are probably not used. Temperature is recorded in degrees Fahrenheit, and rainfall and snowfall in inches. The speeds on American highways are recorded in miles per hour, and fuel economy in their vehicles is listed as miles per gallon. Grocery items in the US are not often advertised using metric units. The liquid goods are sold in fluid ounces, and the dry goods are sold in ounces or pounds.

Referents

When you are in a situation where you need to make an approximate measurement and don't have the appropriate tools to do so, some things are readily available to help you remember basic measurement guidelines.

- Your thumb is about an inch wide.
- The distance from your nose to your fingertips in an out-stretched arm is about 3 feet or 1 yard.
- A healthy pace is about 1 yard. If you have long legs, a pace is about a metre.
- A metre is about 3 inches more than a yard.
- A coke can holds 355 mL.
- A comfortable room temperature is about 21°C or 70°F.

Examples of Temperatures in Celsius and Fahrenheit

Example	Celsius	Fahrenheit
Boiling point of water	100°C	212°F
Sweaty, hot weather	30°C	86°F
T-shirt and shorts weather	24°C	75°F
Average room temperature	21°C	70°F
Long-sleeve shirt and pants weather	15°C	59°F
Fleece jacket weather	10°C	50°F
Freezing	0°C	32°F
So cold you may want to reconsider going outside, especially with children	–29°C	–20°F

Examples of Mass in Metric or Imperial Units

Example	Grams (g) or Kilograms (kg)	Ounces (oz.) or Pounds (lb.)
Each piece of checked baggage on airplanes is generally charged extra if over a certain amount	23–32 kg	51–70 lb.
Average man's mass	82 kg	180 lb.
Average woman's mass	64 kg	141 lb.
Meat and cheese is weighed per 100 grams in Canada	100 g	3.5 oz.
12 slices of cheese	200 g	7 oz.
Enough sliced meat for about 6 sandwiches	300 g	10.6 oz.

Examples of Capacities in Metric and Imperial

Example	Millilitres	Ounces
Carry-on luggage liquid allowance per container on airplanes	100 mL	3.5 fl. oz.
Can of soda	355 mL	12 fl. oz.

Converting between Systems

You can now practice some examples of converting measurements from one system to another system. Some conversion rules are shown in the table below.

Basic Conversion Formulas between Systems	
Mass	1 kg = 2.2 pounds 454 g = 1 pound 28.4 g = 1 ounce
Capacity	1 gallon (Canadian) = 4.54 L 1 fl. oz. (Canadian) = 28.41 mL 1 gallon (American) = 3.785 L 1 fl. oz. (American) = 29.57 mL
Distance	1 mile = 1.61 km 1 km = 0.62 mi. 1 inch = 2.54 cm 1 metre = 1.094 yards

Include the page number of this table, or the table itself, on your resource sheet.

Example 1

Convert 8 pounds to kilograms.

Solution:

You can use proportional thinking.

Be sure to align the pounds together in one column and the kilograms together in another column.

1 kilogram	2.2 pounds
x kilograms	8 pounds

$$\frac{1}{x} = \frac{2.2}{8}$$

$$(x)(2.2) = (1)(8)$$

$$2.2x = 8$$

$$\frac{2.2x}{2.2} = \frac{8}{2.2}$$

$$x = 3.6$$

Thus, 8 pounds = 3.6 kilograms.

Example 2

In your doctor's office, you step on the scale. It reads 58 kg. Convert this reading to pounds in the imperial system.

Solution:

1 kilogram = 2.2 pounds

Therefore, 58 kilograms = 58 × 2.2 = 127.6 pounds.

Example 3

You know it takes 2 hours to drive to Brandon from Winnipeg. At a normal speed of 100 km/h, the distance is 200 km. How many miles is this?

Solution:

1 kilometre = 0.62 miles

200 kilometres = 200 × 0.62 = 124 miles

Example 4

You purchase 75 litres of fuel for your SUV. Your grandpa wants to know how many gallons this is. What do you tell him?

Solution:

You need to ask him if he means US gallons or Canadian gallons. In either case, proportional reasoning can be used to solve the problem. Both solutions are shown below.

Canadian gallons

1 gallon	4.54 litres
x gallons	75 litres

$$\frac{1}{x} = \frac{4.54}{75}$$
$$(x)(4.54) = (1)(75)$$
$$4.54x = 75$$
$$\frac{4.54x}{4.54} = \frac{75}{4.54}$$
$$x = 16.52$$

Thus, 75 litres = 16.52 gallons (Canadian).

American gallons

1 gallon	3.785 litres
x gallons	75 litres

$$\frac{1}{x} = \frac{3.785}{75}$$
$$(x)(3.785) = (1)(75)$$
$$3.785x = 75$$
$$\frac{3.785x}{3.785} = \frac{75}{3.785}$$
$$x = 19.82$$

Thus, 75 litres = 19.82 gallons (US).

Example 5

The Taxicab Board in Winnipeg has set taxi fees at $3.50 for the first 72.5 metres, plus $0.10 for every 72.5 metres after that, plus $0.10 for every 13.18 seconds of waiting time. GST is included in the fare. How much would you pay for travelling 5 miles to the airport? There was no waiting time.

Solution:

First, change miles to metres.

1 mile = 1.61 kilometres

5 miles = 5 × 1.61 = 8.05 km

1 km = 1000 metres

8.05 km = 8.05 × 1000 = 8050 metres.

The first 72.5 metres cost $3.50.

Remaining metres = 8050 – 72.5 = 7977.5 m

Cost of remaining metres

72.5 m	$0.10
7977.5 m	$$x$

$$\frac{72.5}{7977.5} = \frac{0.10}{x}$$

$$72.5x = (0.10)(7977.5)$$

$$72.5x = 797.75$$

$$\frac{72.5x}{72.5} = \frac{797.75}{72.5}$$

$$x = 11.00$$

The remaining metres cost $11.00.

Thus, the total cost for the ride to the airport is $3.50 + $11.00 = $14.50.

Example 6

If one litre of gas costs $1.00, how much does a gallon of gas cost?

Solution:

1 gallon (Canadian) = 4.54 litres.

1 litre = $1.00

1 gallon (Canadian) = 4.54 litres = 4.54 × 1 = $4.54

In the US, 1 gallon = 3.785 litres

1 gallon (US) = 3.785 × $1.00 = $3.79

Learning Activity 3.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Dino has three times as many sisters and he has brothers. If he has 2 brothers, how many kids are in the family?
2. Eric has a big crush. He sits 6 desks away from his crush. If the desks are each 80 cm wide and there are no spaces in between them, how close does he sit to his crush? Write your answer in metres.
3. Your mother has invited your entire family over for dinner. As usual, you are complaining because only $\frac{1}{6}$ of them are around your age. If your family has 24 people in it (not including you), how many of them are approximately your age?
4. Write the following improper fraction as a mixed fraction: $\frac{29}{9}$.
5. What type of angle has a measure of 270°?

continued

Learning Activity 3.7 (continued)

Part B: Converting between Metric and Imperial

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the basic conversion formulas and proportional thinking to convert these measurements.

1. 6 pounds = _____ kg
2. 65 litres = _____ gallons (Canadian)
3. 10 ounces = _____ g
4. 12 km = _____ miles
5. 12 fl. oz. (Canadian) = _____ mL
6. 26 inches = _____ cm
7. 164 cm = _____ inches
8. 26 miles = _____ km

Interesting Downloads from the Internet

You can confirm for yourself some interesting details about how different measurement systems affect your daily life. For the official measurement system for a country, you can search for their Weights and Measures Act set forth by their government. The Canadian sight is http://laws.justice.gc.ca/PDF/Statute/W/W-6.pdf.

Online Conversions

You can calculate most conversions between the metric and imperial systems by using proportional reasoning. However, you can also access a wide variety of online conversion calculators. Just enter "conversion calculator" into your search engine.

Some examples of such websites are listed here.

- www.worldwidemetric.com/metcal.htm
- www.infoplease.com/pages/unitconversion.html
- www.eustis.army.mil/weather/Weather_Products/wxconversions.htm

Labelling Guide for Processed Fruits and Vegetables

The information shown below is published in the Canadian Food Inspection Agency website at www.inspection.gc.ca/english/fssa/labeti/guide/ch11e.shtml#11.2.

When the quantity is given in both metric and imperial units, the metric units should be displayed first and the two must be grouped together, while leaving sufficient space between them to prevent any confusion.

Indicating the net quantity in brackets is not permitted. All metric and imperial symbols must **not** be followed by a period (e.g., " fl. oz." is not acceptable).

Metric units must be shown in

- millilitres (mL or ml) or litres (L or l) for volume
- grams (g) or kilograms (kg) for weight

Imperial units must be shown in

- fluid ounces (fl oz) for volume
- ounces (oz) and pounds (lb) for weight

For conversion purposes:

- 1 US fl oz = 29.57353 mL
- 1 fl oz (Canadian) = 28.413 mL
- 1 oz = 28.350 g

Example

List one difference between what you have studied and the Canada Food Inspection Agency guidelines.

Solution:

Any of the following are acceptable.

- Use of fl. oz. is not allowed.
- Use of ml for mL is allowed.
- Use of l instead of L for litres is allowed.
- There are three or four decimal places for the conversions instead of just two or one decimal place.

Lesson Summary

In this lesson, you learned some reasons why you need to study the imperial system and where it is still used in Canada. You learned some easy referents to measure objects when you don't have a ruler or other measuring device. You converted values from one measurement system to the other using the conversion table and proportional reasoning. You also looked at some online conversion calculators.

The next lesson will examine tools for making length measurements having more precision than a standard ruler.

Notes

Assignment 3.5

Total: 15 marks

Conversions

1. Which is longer, a mile or a kilometre? _____ *(1 mark)*

2. How many pounds are there in a kilogram? _____ *(1 mark)*

3. A US drink can, which contains 12 fluid ounces, holds about _____mL. *(1 mark)*

4. Convert each mass. Round off to two decimal places. *(4 marks)*

   a) 25 pounds = _____ kg

   b) 600 g = _____ ounces

   c) 2.5 tons = _____ kg

   d) 84 kg = _____ pounds

continued

Assignment 3.5: Conversions (continued)

5. Convert the following distances. Round off to two decimal places. *(4 marks)*

   a) 18 miles = _____ km

   b) 360 km = _____ miles

   c) 38 cm = _____ inches

   d) 29 inches = _____ cm

6. Convert the following Canadian capacities. Round off to two decimal places. *(4 marks)*

   a) 12 gallons = _____ L

   b) 36 fl. oz. = _____ mL

   c) 500 mL = _____ fluid ounces

   d) 75 L = _____ gallons

Lesson 6: Calipers

Lesson Focus

In this lesson, you will

- ❑ find measurements using precision instruments
- ❑ read a metric Vernier caliper

Lesson Introduction

Precision measurement gives more exact readings. This lesson shows you how to read Vernier calipers. It would be very helpful if you had a metric Vernier caliper for this lesson. However, if you don't have one or do not have access to one, you can still do the lesson.

More Precise Measurements

Metric rulers make measurements that are precise to the nearest tenth of a centimetre. Imperial rulers make measurements precise to the nearest 1/16 of an inch. Sometimes you need measurements that are more precise. This lesson will only focus on reading metric calipers that measure length to the nearest hundredth of a centimetre.

Imagine a simple plastic cap that fits on the end of a ball point pen. When the manufacturer makes this cap, he has to use precision measurements. If the inside diameter of the cap is too small, it will not fit. If the diameter is too large, the cap will fall off. The workers must be sure to use exact, precise measurements when manufacturing these caps.

Using a Vernier caliper provides measurements precise to one-tenth of a millimetre.

Precise to What Unit?

If you say the width of your bedroom is 4 m, then the measurement is precise to the nearest metre. If you say the room measures 4.25 m, then it is precise to the nearest cm. This would be especially important if you were cutting carpet to cover the floor. If your measurements were not precise to the nearest cm, you might have a gaping hole along one wall.

Denturists make appliances that fit perfectly inside your mouth to replace missing teeth. The measurements of these dentures must be exact, or you will experience a lot of discomfort. They must be precise to units even smaller than millimetres.

A denturist would use a Vernier calliper to get exact measurements precise to a tenth of a millimetre.

A luthier is an expert craftsman who makes violins. Luthiers need to use exact measurements. If measurements are not precise to the nearest 0.1 of a millimetre, the harmonic series of the instrument is off, and the violin will not sound properly when it is played.

Reading a Vernier Caliper

This Vernier caliper has various parts, including the ones listed below and shown on the diagram on the opposite page.

1. **Outside jaws:** used to take external measures of objects
2. **Inside jaws:** used to take internal measures of objects
3. **Depth probe:** used to measure the depth of objects
4. **Main scale** (cm)
5. **Main scale** (inch)
6. **Vernier scale** (cm)
7. **Vernier scale** (inch)
8. **Retainer:** used to block movable part

Three devices for measuring:

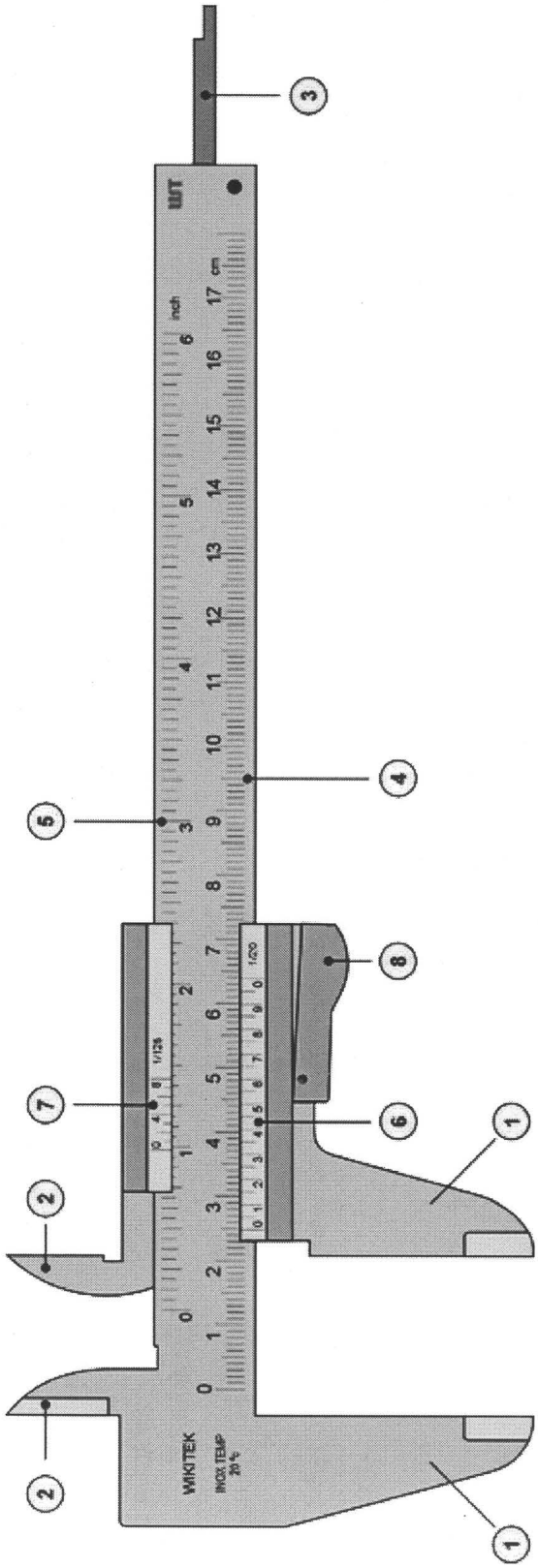
1. The outside jaws are used to measure outer dimensions of objects (e.g., the outer diameter of a pipe).
2. The inside jaws are used to measure the inner dimensions of objects (e.g., the inside diameter of a pipe).
3. The depth gauge is used to measure depth of objects (e.g., the depth of a small container).

Two measurement scales:

1. A fixed or main scale, in both metric and imperial units

2. A moving or sliding Vernier scale, in both metric and imperial units

The fixed scale does not move, and the moving scale is called the Vernier scale.


3
4
5
8
7
6
1
2
2
1
WIKTEK
INOX TEMP
304

In this course, the readings on Vernier calipers calibrated in metric units are studied.

The fixed scale on a Vernier caliper is divided into millimetres, which are 0.1 cm or $\frac{1}{10}$ of a centimetre. The moving Vernier scale represents $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ cm.

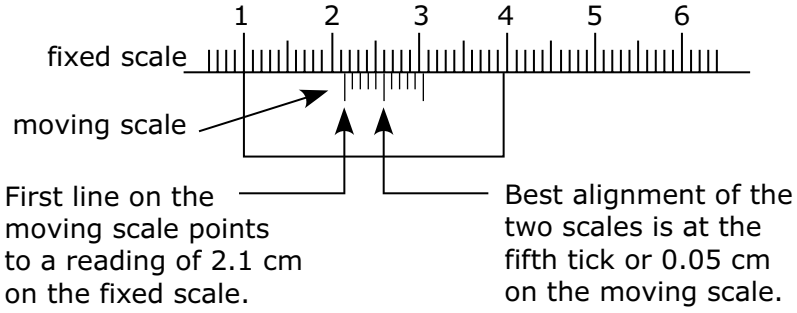
Therefore, measurements taken with a Vernier caliper are precise to the nearest hundredth of a centimetre. Remember that $\frac{1}{100}$ of a centimetre = $\frac{1}{10}$ of a millimetre.

The only possible reading from the Vernier scale is one of the numbers 0.00, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, or 0.09.

The following examples will help you in reading the measurements taken with the Vernier caliper.

Example 1

Read the following Vernier caliper measurement.



Solution:

Step 1: Read the fixed scale, using the first line on the moving scale as a pointer. This line points to a place between 2.1 cm and 2.2 cm. You state the reading from the fixed scale is 2.1 cm, giving you a precision unit to the nearest tenth of a centimetre. You had this much precision with a ruler.

Step 2: Find the line on the moving scale that most closely aligns with a mark on the fixed scale. In this case, the line that best matches is the fifth line on the moving scale. Since the moving scale has 10 divisions, and each division represents 0.01 cm, the reading is 0.05 cm.

Step 3: State the total measurement. In this case, the reading on the caliper = 2.1 cm + 0.05 cm = 2.15 cm. Now you have a measurement that is precise to the nearest hundredth of a centimetre, considerably more precise than with a ruler.

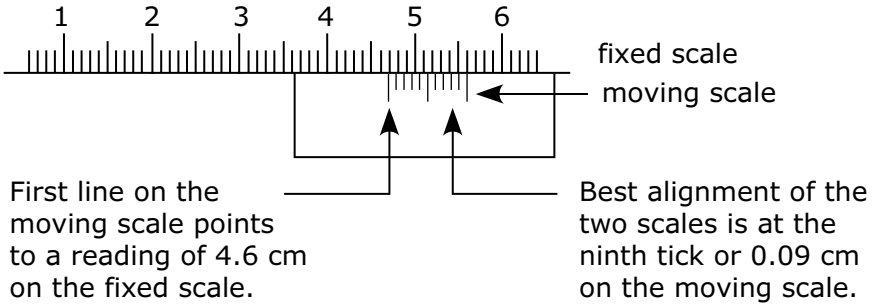
In order to read a Vernier caliper, note the following procedure.

Check where the first line on the moving scale lands. It will point to a place on the fixed scale. This position determines the first digits of the reading.

Examine which line on the moving scale best aligns with a line on the fixed scale. The line on the moving scale determines the last digit.

Example 2

Read the following Vernier caliper measurement.



Solution:

Step 1: The first arrow points to a place on the fixed scale between 4.6 and 4.7. The reading is 4.6 cm.

Step 2: The second arrow points to the alignment of the two scales at the 9th tick mark on the moving scale, giving a reading of 0.09 cm.

Step 3: Therefore, the total reading = 4.6 cm + 0.09 cm = 4.69 cm.

Using Vernier Calipers

Many workers including mechanics, most tradesmen, and farmers use Vernier calipers. If you don't own one or there is not one in your school, see if you can find one in your community and practice measuring objects.

Step 1: Place the object between the main jaws of the caliper.

Step 2: Close the jaws until they just contact the two sides of the object. Read the caliper to find the measurement of the object.

Virtual Calipers

There are many Internet sites that include virtual calipers. You set the width, and read the scale. Then the site tells you if you read it correctly. Enter "Vernier caliper virtual" into your search engine, and try some of them. They are very good practice for reading the Vernier scales.

Learning Activity 3.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You and 6 of your friends are at a fondue restaurant. The bill comes to $42. If you divide the cost evenly, how much will you pay each?
2. Complete the pattern: ___ , ___ , 1, 0.5, 0.25.
3. Solve for a: $18 + a = 23$.
4. A regular goal in rugby is worth 3 points. If a team finishes a game with 39 points, how many goals did they score?
5. You woke up at 8:30 am. You got ready in 45 minutes. You walked to work for 45 min. You worked for 4 hours. You ate lunch for 35 minutes. What time is it now?

continued

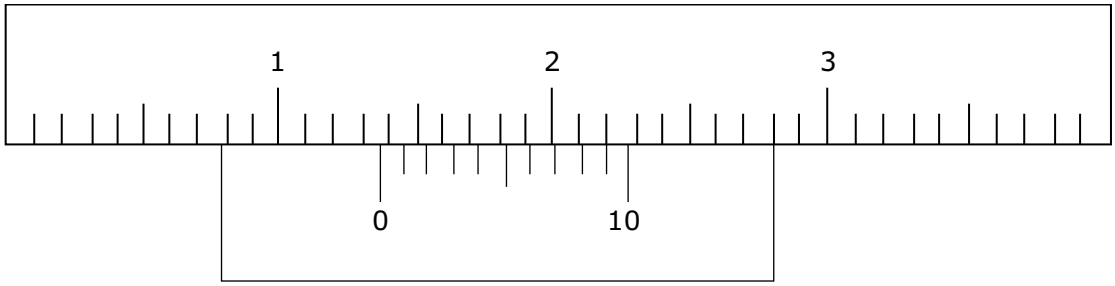
Learning Activity 3.8 (continued)

Part B: Reading Vernier Calipers

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

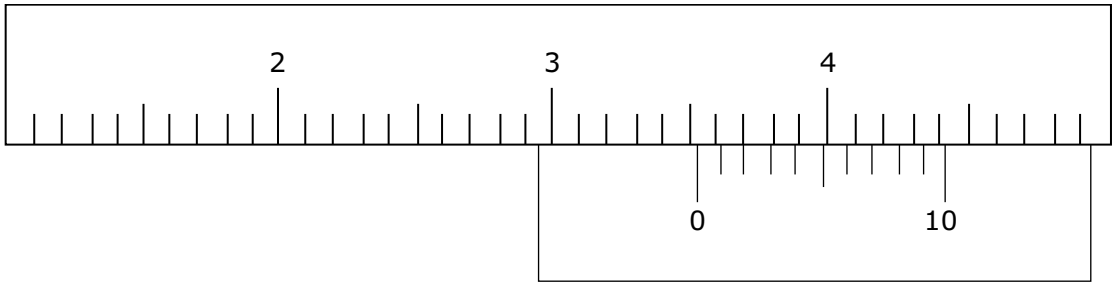
Write down the precision measurements as shown on these Vernier caliper diagrams.

1. Read the following to the nearest hundredth of a centimetre.



Reading = _____

2. Read the following to the nearest hundredth of a centimetre.

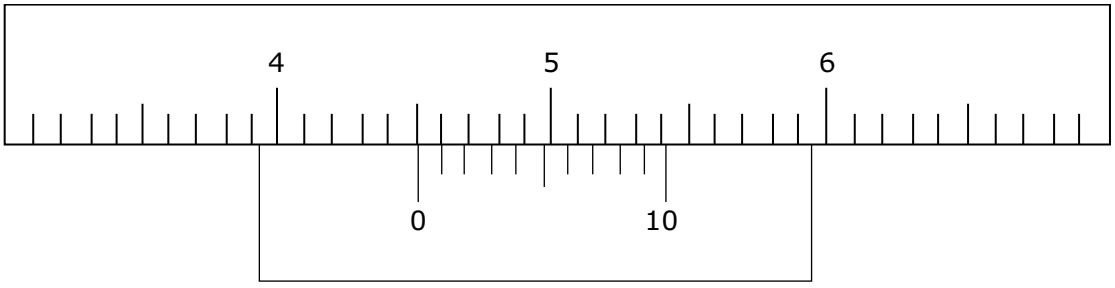


Reading = _____

continued

Learning Activity 3.8 (continued)

3. Read the following to the nearest hundredth of a centimetre.



Reading = _____

Lesson Summary

In this lesson, you learned what a Vernier caliper is, its three measuring devices, its 2 measurement scales, two systems of measurement, and how precise a measurement it can make. You then practiced reading precise measurements using a Vernier caliper diagram.

The next lesson shows you how to use a micrometer to find precise measurements.

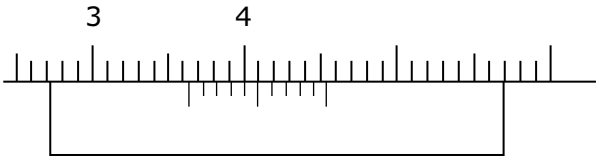
Assignment 3.6

Total: 12 marks

Calipers

1. Read and record the following Vernier caliper measurements. These readings are in metric units. *(6 marks)*

a)



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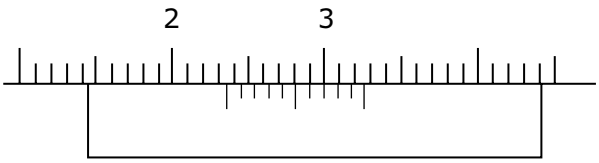
b)



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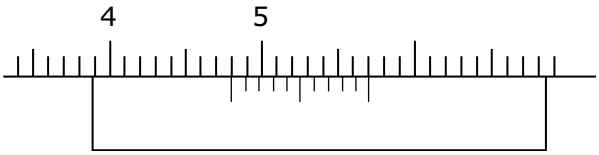
c)



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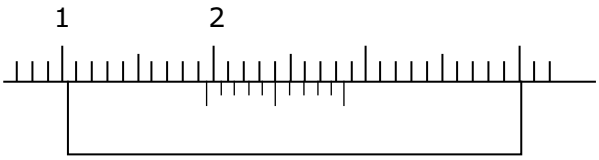
d)



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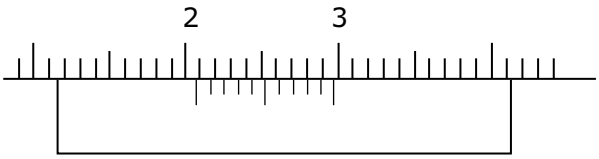
e)



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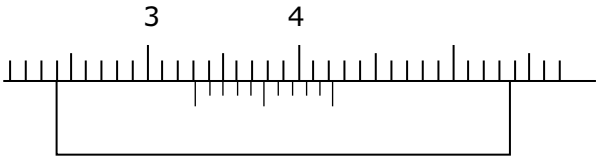
f)



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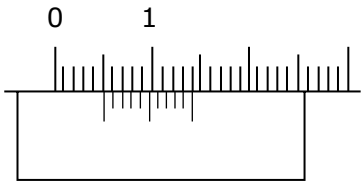
continued

Assignment 3.6: Calipers (continued)

2. Read and record the following Vernier caliper measurements. These readings are in metric units. *(6 marks)*

a) 

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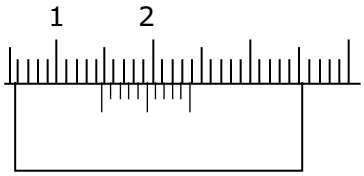
b) 

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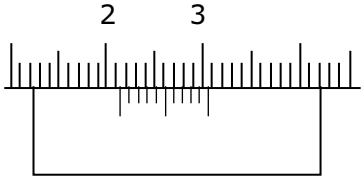
c) 

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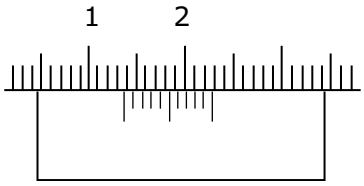
d) 

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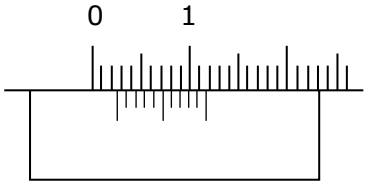
e) 

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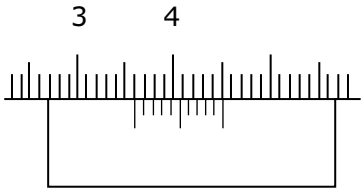
f) 

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LESSON 7: MICROMETERS

Lesson Focus

In this lesson, you will

- ❑ find measurements using precision instruments
- ❑ read a metric micrometer

Lesson Introduction

Using precision measurement tools gives you more exact readings. The previous lesson showed you how to read Vernier calipers. Micrometers measure even more precisely than Vernier calipers. This lesson teaches you how to read measurements on micrometers.

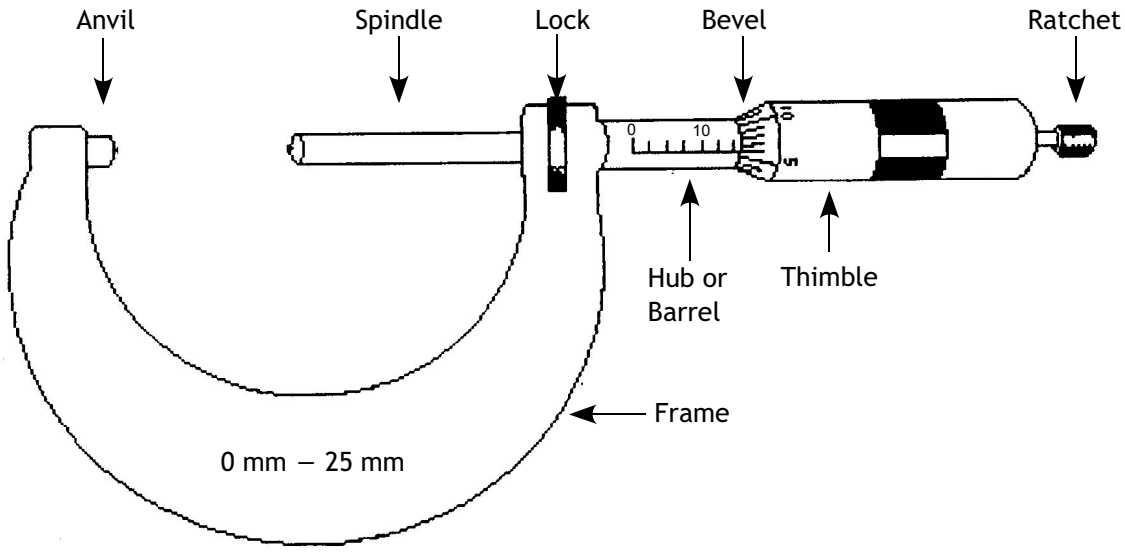
The Micrometer

Reading a Micrometer

Vernier calipers can measure items to the nearest hundredth of a centimetre. Micrometers are even more precise, they measure to the nearest thousandth of a centimetre.

Most micrometers are calibrated to measure items between 0 mm and 25 mm in thickness. Remember that 25 mm is 2.5 cm, or about an inch.

The following diagram shows the key components of a micrometer.



Note the following parts of the instrument.

- The measuring device is called the jaws. The anvil and spindle are used to measure small lengths between 0 mm and 25 mm.
- The two measurement scales, a fixed scale and a moving scale are located on the barrel and the thimble, respectively.
- On any micrometer there is only one system of measurement, either metric or imperial, but not both. To get measurements in both systems you would need two different micrometers.

In this course, the metric micrometer readings are examined.

On a metric micrometer, the measurement scales are as follows:

1. on the barrel

   The fixed scale on the barrel is divided into 25 main divisions. Each main division represents 1 mm.

   Each main division is divided in half, getting 50 sub-divisions in all. Each subdivision represents 0.50 mm or 0.05 cm.

2. on the thimble

   The moving scale on the thimble is divided into 50 divisions. One complete rotation of the thimble represents 0.50 mm. Therefore, each division on the thimble represents $\frac{1}{50}$ of 0.5 mm or 0.01 mm.

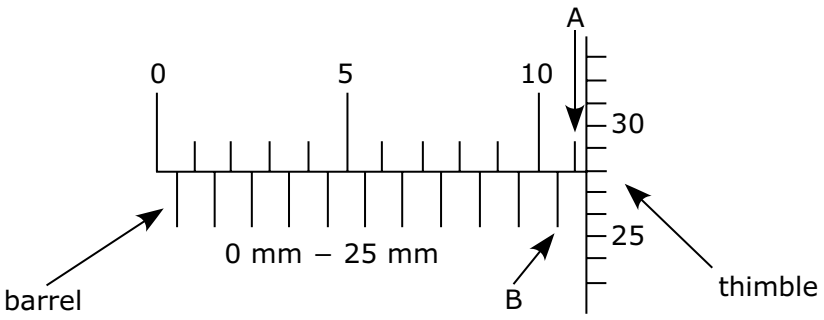
Therefore, measurements taken with a metric micrometer are precise to the nearest hundredth of a millimetre or thousandth of a centimetre.

Using a Micrometer

If you have access to a micrometer, use it to practice measuring the widths of various small objects. Find the thickness of 20 sheets of paper, a hair pin, or even the rim of a coffee mug. Place the object between the jaws, and then rotate the thimble using the ratchet. When the object is secure, and you have heard a few clicks on the ratchet, read the measurement as shown below.

Example 1

Read the following micrometer measurement.



Solution:

Step 1: Read the upper barrel scale in mm. The reading will be a whole number from 0 mm to 24 mm. In this diagram, the measure of the last marking showing on the upper scale, indicated by arrow at A, is 11 mm.

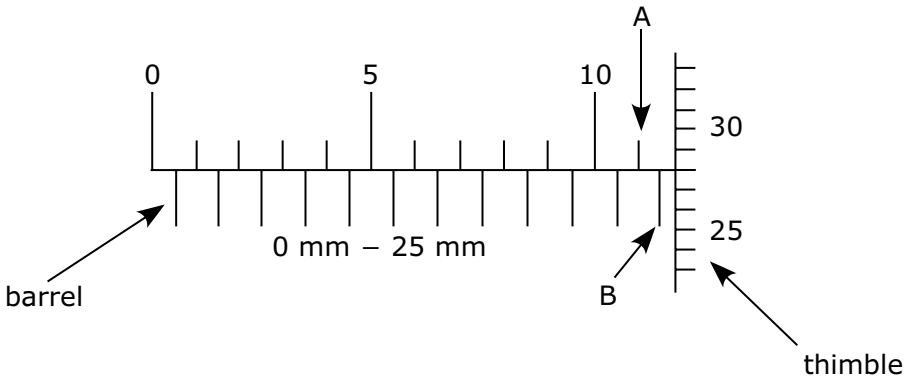
Step 2: Read the lower barrel scale. This scale marks points halfway between each millimetre marking on the top scale. The reading will be either 0.0 mm or 0.5 mm. In this diagram the arrow at B is before the arrow at A so the reading is 0.0 mm. Thus, the final reading will be between 11.00 mm and 11.50 mm.

Step 3: The thimble reading is written as a decimal from 0.00 mm to 0.49 mm. In the diagram, the reading on the thimble is 0.28 mm.

Step 4 Add up the readings from Steps 1, 2 and 3 to arrive at the resulting measurement. The sum is 11 mm + 0.0 mm + 0.28 mm = 11.28 mm.

Example 2

Read the following micrometer measurement.



Solution:

Step 1: The measure of the last marking showing on the upper barrel scale, indicated by arrow A, is 11 mm.

Step 2: The lower barrel scale shows a marking, indicated by arrow B, to the right of arrow A. This means the reading is 0.5 mm. In other words, the reading is more than halfway past 11 mm. The final reading will be between 11.50 mm and 11.99 mm.

Step 3: The thimble reading yields 0.28 mm.

Step 4: The resulting measurement = 11 mm + 0.5 mm + 0.28 mm = 11.78 mm.

Learning Activity 3.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

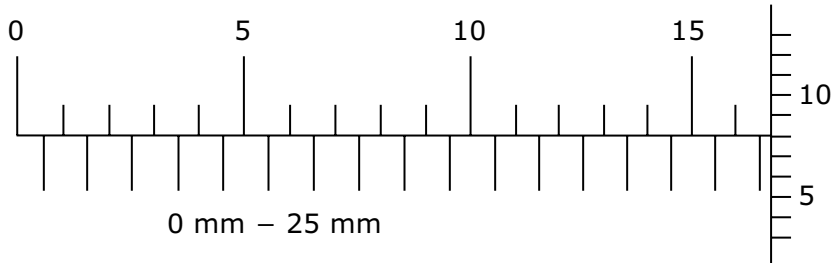
1. There are 8 marbles in a bag. Each marble is red, yellow, or blue. Four of the marbles are red, and one marble is yellow. How many are blue?
2. Solve for r: $5 + r = -4$.
3. Which is larger: 436% or $\frac{18}{5}$?
4. Estimate the taxes, at 12%, of a pair of shoes that cost $74.89.
5. A loonie is approximately 2.5 cm across. Convert to inches.

Part B: Reading a Micrometer

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Read and record these micrometer measurements.

1. Read the following micrometer measurement.



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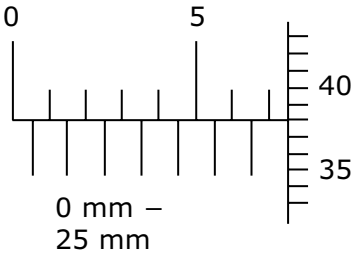
Learning Activity 3.9 (continued)

2. Read the following micrometer measurement.



Lesson Summary

In this lesson, you learned what a micrometer is and the names of its main parts: the jaws, the barrel, and the thimble. You learned how to read the fixed scale and the moving scale to find a measurement precise to the nearest thousandth of a centimetre. If you wanted to make a reading in imperial units you would need a different micrometer. You then practiced reading precise measurements using a micrometer diagram.

The next lesson you will study various formulas in measurement, and will present the formulas in different forms.

Assignment 3.7

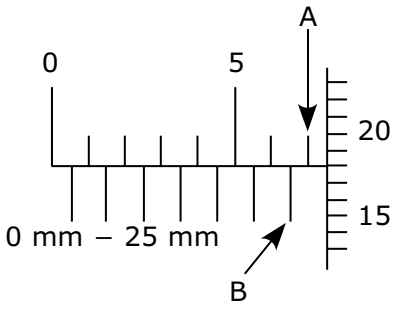
Total: 16 marks

Micrometers

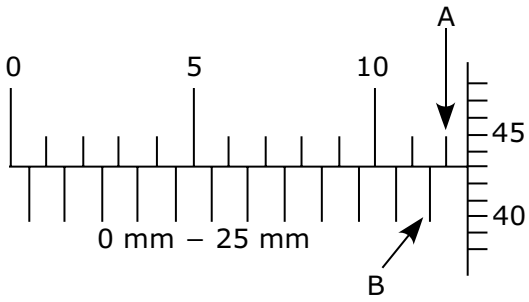
Each question is worth 8 marks, 1 mark for each part.

1. Read and record the following micrometer measurements.

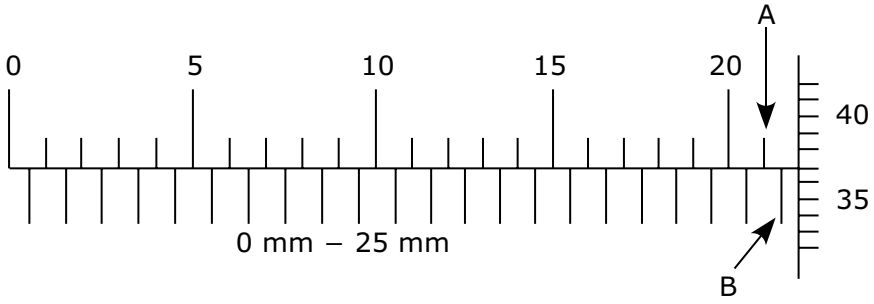
a)



b)



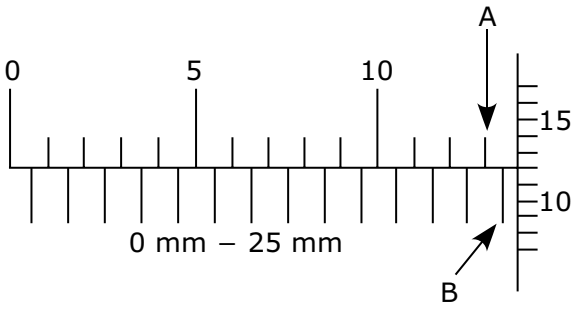
c)



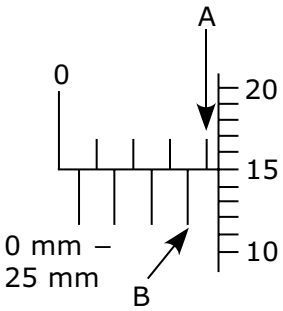
continued

Assignment 3.7: Micrometers (continued)

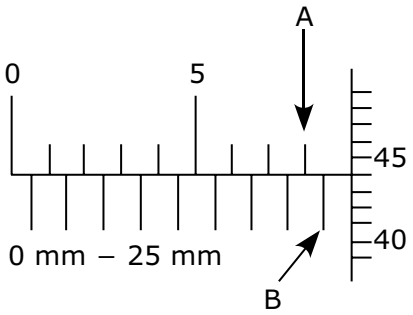
d)



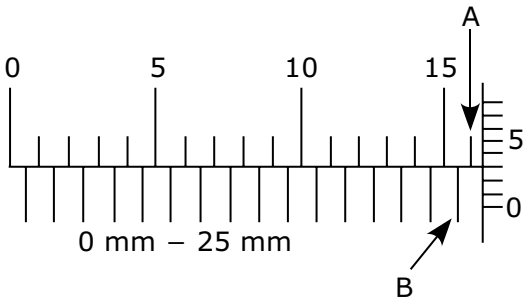
e)



f)



g)



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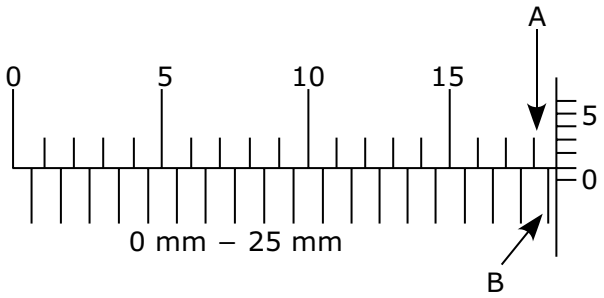
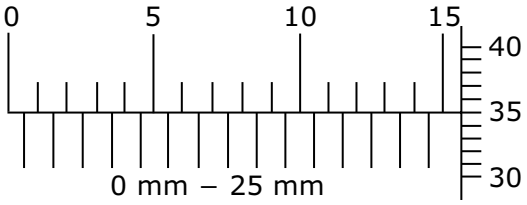
Assignment 3.7: Micrometers (continued)

h)

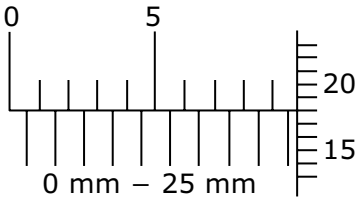


2. Read and record the following micrometer measurements.

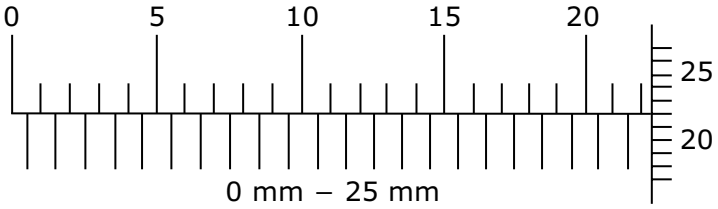
a)



b)



c)



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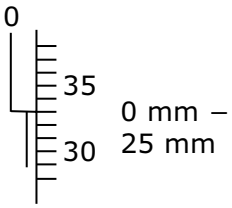

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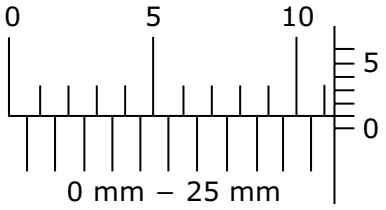
Assignment 3.7: Micrometers (continued)

e)



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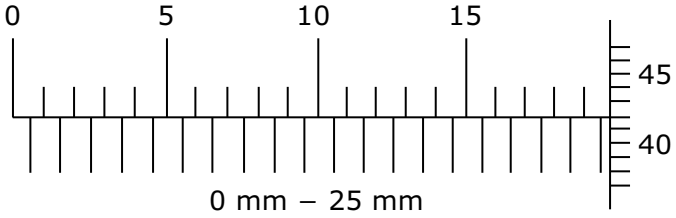
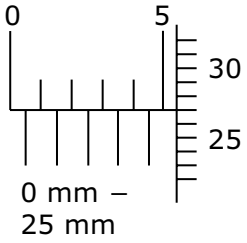


g)



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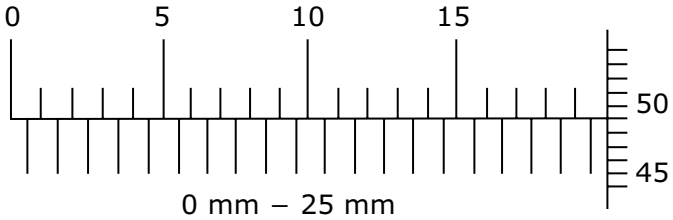


LESSON 8: FORMULAS

Lesson Focus

In this lesson, you will

- ❑ solve problems that require manipulation and application of formulas

Lesson Introduction

You are constantly applying measurement skills to various situations. This lesson shows you some formulas in measurement, and how they can be used in solving problems.

Using a Formula

An alternate method to proportional reasoning is to use formulas. Remember, proportional equation solving requires you have two fractions equal to each other in the format, $\frac{a}{b} = \frac{c}{d}$.

Sometimes formulas need to be "manipulated" to isolate the value you are looking for.

You will need to be careful and apply your basic skills with equation solving using the rule: "what you do to one side you must equally do to the other."

Using Formulas to Solve Problems

Any of the formulas used in the following examples are not to be memorized (unless you want to). These formulas are useful common knowledge, and so will not be given to you on the examination. Your task is to use the formulas correctly to solve problems. You may want to write them on your resource sheet for the examination.

Temperature

Canadians measure temperature in degrees Celsius. Americans measure their temperatures in degrees Fahrenheit. The formula to find degrees Fahrenheit from Celsius degrees is as follows:

$$F = C \times 1.8 + 32$$

This formula allows you to change Celsius temperatures into Fahrenheit temperatures.

Include this formula on your resource sheet.

Example 1

Change 28°C into degrees Fahrenheit.

Solution:

If it is 28°C, the equivalent temperature in Fahrenheit would be found by substituting into the formula and simplifying your answer.

$$\begin{aligned} F &= 28 \times 1.8 + 32 \\ &= 50.4 + 32 \\ &= 82.4 \end{aligned}$$

Thus, 28°C = 82.4°F.

If you are planning a trip into the United States and need to know what temperature to expect, you will need to be able to convert their temperatures into degrees Celsius so that the answer you get is more familiar to you.

The problem arises when you realize the resort where you are going has an average temperature of 104°F, and you have no idea how hot that would be in °C.

Because the formula has addition in it, you cannot use proportions and the box to set up an equation. However, this problem is easily solved using your algebra skills. You can solve for the unknown value by first manipulating the formula and isolating the unknown variable. Or you could write the formula you know, substitute in the values you know, and solve for the unknown value.

Method 1: Manipulating the formula first.

The formula you know is $F = C \times 1.8 + 32$.

You want to isolate C in the formula.

The first step is to subtract 32 from both sides.

$$F - 32 = C \times 1.8 + 32 - 32$$

Simplifying, you get $F - 32 = C \times 1.8$.

Now all that is left to do is remove 1.8. Since C is multiplied by 1.8, you will divide both sides of the equation to get rid of it.

$$\frac{F-32}{1.8} = \frac{C \times 1.8}{1.8}$$

When you simplify this equation, you get

$$\frac{F-32}{1.8} = C$$

Now you substitute into the formula to solve the problem.

Since you know the temperature is 104°F, then the temperature in degrees Celsius is

$$C = \frac{F-32}{1.8} = \frac{104-32}{1.8} = \frac{82}{1.8} = 40$$

Thus, 104°F = 40°C

Method 2: Substitute the values first, then solve.

You start with the formula you know.

$$F = C \times 1.8 + 32$$

The temperature you want to convert to Celsius is $104\ ^0F$, so substitute that value for F into the formula.

$$104 = C \times 1.8 + 32$$

You need to isolate the variable, C.

First, subtract 32 from both sides of the equation and simplify.

$$104 - 32 = C \times 1.8 + 32 - 32$$

$$72 = C \times 1.8$$

Now, to isolate C, you divide both sides by 1.8.

$$\frac{72}{1.8} = \frac{C \times 1.8}{1.8}$$
$$40 = C$$

Thus, 104°F = 40°C.

You have the same answer using either method.

Fuel Economy

Many people consider fuel economy of vehicles when thinking of purchasing a vehicle. Canadians measure fuel economy in litres used per 100 km driven (L/100km).

Example 1

If you drove from Winnipeg to Calgary, a distance of roughly 1400 km, and your car used 150 litres of gas, find the fuel economy for your vehicle.

Solution:

The formula for fuel economy is written below.

$$\text{Fuel Economy} = \frac{\text{number of litres of gas used}}{\text{number of 100 km of distances you have driven}}$$

Notice that the denominator of the formula requires the number of "100 kilometre" amounts you have driven, not just the number of kilometres driven.

There are at least two ways to solve this problem: one is to use the formula, and the other is to use proportions.

Method 1: Using the Formula

Since you are asked to find the fuel economy, you can substitute directly into the formula and then simplify to get the answer.

The number of litres of gas used on the trip is 150 L.

The number of kilometres driven is 1400. Thus, the amount of "100 kilometres" driven is $\frac{1400}{100} = 14$.

The fuel economy = $\frac{150}{14} = 10.7$ L/100 km.

Method 2: Using Proportions.

You are trying to find the number of litres of gas you used in every 100 kilometres on your trip. You could set up a box and then use proportions to solve for your answer. Ensure the same units are in each column.

1400 km	150 litres
100 km	x litres

$$\frac{1400}{100} = \frac{150}{x}$$

$$(x)(1400) = (100)(150)$$

$$1400x = 15\ 000$$

$$\frac{1400x}{1400} = \frac{15\ 000}{1400}$$

$$x = 10.7$$

Thus, your fuel economy is 10.7 litres used per 100 kilometres driven, or 10.7 L/100 km.

Notice you get the same answer in both methods.

Conversions

Example 1

Your mom sends you to the store while you are on a holiday in the US. She instructs you to purchase a 4-litre container of milk. The store only sells milk by the quart or by the pint. How many quarts and pints must you buy to have at least 4 litres of milk?

Solution:

If you buy 4 quarts of milk you will have to convert from quarts to litres.

Note to Students: Have you made a resource sheet for this module? Do you have the conversion charts, or the page reference for the conversion charts on your resource sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

You know that in America, 1 gallon = 3.785 litres, and you know that in 1 gallon there are 4 quarts. Thus, if you buy 4 quarts of milk you will have 3.785 litres of milk. This is not quite enough.

You are missing $4 - 3.785 = 0.2144$ litres.

You would now need to convert pints into litres to decide how many more pints you would need to have 4 litres altogether.

8 pints = 1 gallon = 3.785 litres

$$1 \text{ pint} = \frac{1}{8} \text{ gallons} = \frac{3.785}{8} = 0.47 \text{ litres}$$

Buying 1 pint would be more than enough to make up the difference.

If you bought 4 quarts, and one pint of milk, you would have 3.785 L + 0.47 L = 4.26 L of milk.

Thus, your Mom would have at least 4 L of milk.

Learning Activity 3.10

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The body of a Daddy-Long-Legs spider is about 0.7 cm long. How long is this in mm?
2. Team A has won 5 out of their last 9 games. Team B has won 4 out of their last 7 games. Which team has won a greater percentage of their games?
3. Write as an improper fraction: $1\frac{3}{16}$.
4. You are at a hockey game and would like a snack and a drink. At the counter, you see that popcorn is $3.00, peanuts are $2.25, and hotdogs are $3.75. The drinks are $2.00. If you have $5, what snack(s) can you afford to get if you also buy a drink? (Tax is included in the price.)
5. You earn money through piecework at the factory. Your employer does not have a lot of money so each shirt is worth $2.00 and each pair of pants is worth $3.00. If you make 4 shirts and 3 pants, how much will you be paid (gross)?

continued

Learning Activity 3.10 (continued)

Part B: Conversion Problems

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You observe that the outside temperature in Fargo is 15°F. Find that value in °C.
2. If the temperature in Thompson is 25°C, find that value in °F.
3. The formula for converting kilometres to miles is:

   Number of kilometres × 0.62 = number of miles

   If you drove for $12\frac{3}{4}$ hours at an average speed of 100 km/h, how many miles did you drive?
4. Using the same formula as in question #3, how many kilometres would be equivalent to 500 miles?
5. You, along with 3 friends, step onto an elevator in a store in the US. There is a sign on the wall indicating the maximum mass allowed is 650 pounds. You know that the average mass of a Canadian teenager is 70 kg.
   a) Will your combined mass be within the allowed amount on the elevator?
   b) By how many pounds are you over or under?

Lesson Summary

This lesson dealt with using formulas to solve problems. You looked at various methods including manipulating and solving an equation, using proportions, and converting metric measurement units. If your resource sheet is up to date, you could use it as a reference for a conversion in the problems. You learned how to use a formula for converting temperatures between Fahrenheit and Celsius and for fuel economy.

The last lesson in this module solves problems involving linear measurements.

Notes

Assignment 3.8

Total: 12 marks

Formulas

Solve each of the following problems using a formula or proportions. Show all your work. You may use your resource sheet.

1. Your new car is rated for fuel economy at 11.2 L/100 km. If you went on a road trip and used 470.4 L of gas, how many kilometres did you drive? *(2 marks)*

2. If the formula for converting miles to kilometres is:

   Number of miles × 1.61 = number of kilometres

   How many miles is equivalent to 4200 km? *(3 marks)*

continued

Assignment 3.8: Formulas (continued)

3. The average temperature in San Diego in February is 68°F. Find the equivalent temperature in Celsius degrees. *(3 marks)*

4. The formula for finding your body mass index is

$$\text{BMI} = \frac{\text{your mass in kg}}{(\text{your height in metres})^2}$$

Any rating over 25 is considered overweight. Your doctor told you your BMI reading is 24.5. You know your height is 1.65 m.

a) Use this formula to find your mass in kg. *(3 marks)*

b) What is your mass in pounds? *(1 mark)*

LESSON 9: PROBLEMS

Lesson Focus

In this lesson, you will

❑ solve problems involving linear measurements

Lesson Introduction

Developing problem-solving techniques is a necessary life skill. These problems involve metric and imperial linear estimates and measurements.

Using Formulas to Solve Problems

Estimating Using Referents

Lay your ruler on the table in front of you. Press your thumb down along the imperial scale. Is your thumb "about" an inch wide?

Place your little finger at the 0 mark, and open your hand as wide as it will open. This is called a hand span. Does the thumb come down "about" 9 inches away?

If you have a yard stick or a metre stick handy, lay it along the floor and measure the length of your pace. It should be "about" a yard.

The distance from your nose to the tip of your fingers when your arm is stretched horizontally should be "about" a yard.

You can use these referents to estimate the length of objects. If you remember that a 12" ruler is about 30 cm long, you can use that referent as a guide to estimate in metric units. A metre is "about" the distance from your hip to the floor.

Examples of Using Referents

Finding the Middle

Example 1

Have you ever folded up a tent or a large tarp? But where is the middle?

Solution:

A strategy you could use to find the middle would be to pace off the length of the tarp. Then the middle would be at half of that pace.

Or, you could fold it up so that the edges meet, and where the creases intersect would be the midpoint.

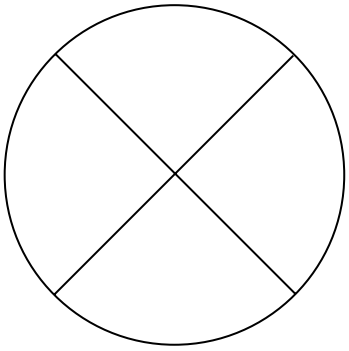
Generally, it is easier to bring the edges in toward the middle, and fold again at that point.

Example 2

How would you find the centre of a circular pond?

Solution:

You could use two sections of garden hose or rope, and lay them along the maximum width of the pond. The two hoses will meet or intersect in the centre.

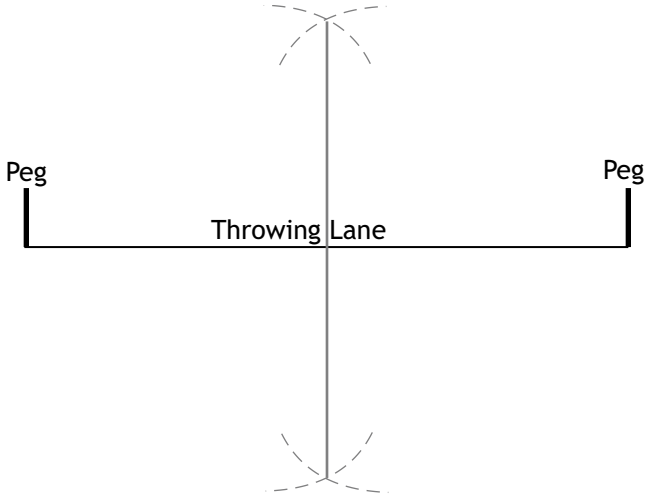


Example 3

Imagine you are setting up a "throwing lane" for a game of horseshoes. You must scratch a line in the dirt at the midpoint, between the two pegs, because every horseshoe must be thrown at least that far. How would you find the middle of the line?

Solution:

Without the benefit of a measuring tape, you could use a long rope, and scratch out the arcs, as shown in the diagram below.



How is this done?

Have a friend hold the rope at one end of the lane where the peg is stuck in the ground.

Using a stick and a section of the rope that is at least half the distance between the pegs, you scratch a circle in the dirt above and below the throwing lane.

Your friend then stands by the other peg, and the same process is repeated. You now will have two points where the arcs meet each other. When you join the two intersecting points, you have a line that intersects the throwing lane at its middle.

Estimating Lengths

Example 1

How tall is the tree in your yard?

Solution:

You could use your own height.

You could estimate how many times taller the tree is than you are and then calculate its height. For example, if you are 180 cm tall and the tree is about three times your height, the tree is $180 \times 3 = 540$ cm or 5.4 m tall.

Example 2

How tall is the office tower near you?

Solution:

You know that one storey in an office building is about 10 feet tall, so you would count the number of floors and multiply by 10 to get the height of the building in feet.

Note that a house is usually about 8 feet tall per story.

Example 3

How far is the place where you live from the road?

Solution:

You could measure how long your pace is exactly, and then pace out the distance from the building to the road. Then multiply the two numbers to get the answer.

Solving Problems

Suitcases on a Plane

When you check your suitcases onto an airline before a flight, the personnel working at the counter have to use estimating skills. The current limit on both major Canadian airlines for the total linear measurement (length + width + height) of your suitcase is 157 cm. Any size exceeding that will either not be allowed, or you might have to pay an extra fee for "over-sized" luggage. Obviously, the airline worker does not measure every suitcase. Quickly developing estimating skills is a job requirement.

Example 1

You drive your grandmother to the airport, and assist her with checking in her luggage. She has a suitcase that measures 90 cm by 50 cm by 20 cm. Does her suitcase satisfy the requirement of no more than 157 cm linear measurement?

Solution:

Linear measurement requirement = L + W + H = 157 cm maximum

Linear measurement = 90 cm + 50 cm + 20 cm = 160 cm

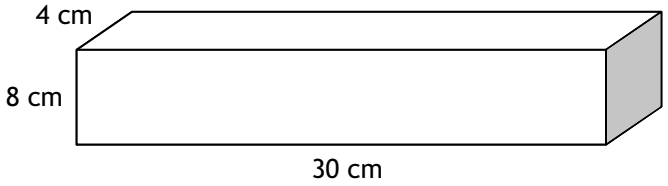
Her suitcase is 3 cm over the limit of 157 cm. She might have to pay extra for over-sized luggage.

Canada Post

Canada Post measures every parcel shipped by its customers. The mass of the parcel is compared to the length and "girth" of the parcel. The girth is determined by measuring the height and width of the parcel and multiplying both by two since there are two sides and a top and bottom. Then, the length is added to the girth. The total cannot exceed 300 cm.

Example 2

Find the girth of the parcel to be shipped. Does this parcel meet the measurement restrictions?



Solution:

Girth = $2w + 2h$

Girth = $2 \times 4 + 2 \times 8$

Girth = 24 cm

Since the length of the parcel is 30 cm, the total length and girth is 24 cm + 30 cm = 54 cm. This parcel does not exceed the maximum of 300 cm allowed.

The smallest size parcel that Canada Post will ship has dimensions of length = 10 cm, width = 7 cm, and height = 0.1 cm.

Circular Pond

The circumference of a circle is the distance around the edge. The diameter of a circle is the line segment through the centre of the circle. The formula for finding circumference is $C = \pi d$, where d is the diameter of the circle and π is approximately 3.14. You could write $C = 3.14d$.

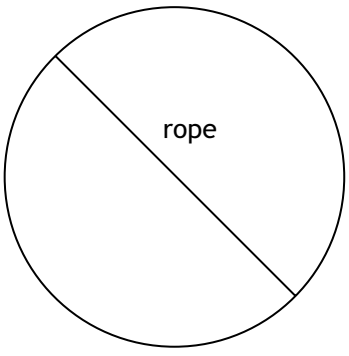
Thus, the circumference of a circle is about 3 times the diameter.

Example 3

You have built a circular pond for your backyard. You want to place a decorative edging around the outside edge of the pond. Explain how you might estimate how much you will need and then find the actual amount.

Solution:

In order to estimate the circumference of the pond to buy decorative edging, you know that the circumference is about triple the diameter. You take a piece of rope or garden hose and stretch it across the maximum width of the pond. Once you have the length, you multiply that measurement by three. This estimates the circumference.



(rope is used to measure the diameter)

If the length of the rope measuring the diameter is 8 m, then the circumference is about $8 \times 3 = 24$ m.

You know that using 3 instead of 3.14 for π will give an estimate that is slightly lower than the true calculation. Your estimate is about 25 m or 26 m of edging.

The actual circumference is $C = \pi d = 3.14 \times 8$ m = 25.12 m.

You would need 26 m of decorative edging.

Learning Activity 3.11

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are at the store to buy your mother a present. You have $30 to spend, and would like to get her a waffle maker. If it costs $40 and is 20% off, will you be able to get her the waffle maker?
2. If 5% of 260 is 13, what is 5% of 520?
3. Evaluate: $3 + 9 - 2 \times 12 \div 3$.
4. According to a study, 75% of class time is used inefficiently. If you have a 60 minute class, how much time is equal to 75%?
5. Convert: 300 m = _____ km.

continued

Learning Activity 3.11 (continued)

Part B: Using Referents

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You are setting up a ball diamond for a quick game of baseball with your friends. You know that first base is 90 feet from home plate. Describe how you would estimate where the base should be set.

2. a) Is a suitcase measuring 85 cm by 45 cm by 30 cm within the linear measurement limits set by an airline?
   b) By how much is it over or under the limit?

3. Describe how you might estimate the length of a pencil.

Lesson Summary

The last lesson in this module dealt with solving problems involving linear measurements. You learned 5 referents for estimating length. The problems involved, among others, the maximum size of a suitcase on a plane, parcel size restrictions at the Post Office, and edging around a circular pond.

Now would be a good time to check your resource sheet for completeness, before you start the assignment.

Assignment 3.9

Total: 15 marks

Problems

1. Manitoba's proposed recreational water quality guideline of 16 micrograms per litre is used to assess water safety in the presence of algae blooms. You have a gallon sample of water and count a total of 68 micrograms. Is the water safe? Show your calculations. *(3 marks)*

2. Estimate in both systems the length of each of the following. *(8 marks)*

	Metric	Imperial
a) tip of your nose to the end of your fingers	_____	_____
b) the length of your index finger	_____	_____
c) the length of a Honda sedan	_____	_____
d) the length of a house fly's wing	_____	_____

continued

Assignment 3.9: Problems (continued)

3. You are planning to fly away on a vacation. The suitcase you have looks a little large. Explain how the airlines measure it, and what their limitations are in terms of luggage size. *(2 marks)*

4. You are given a rectangular sheet of paper. Without the aid of a measuring device, describe how you could find the exact centre of the page. *(2 marks)*

Module 3 Summary

This module was all about measurement, using estimation, metric units, and imperial units. You also learned to convert within each system and between the two systems. You learned how to make more precise measurements using Vernier calipers and micrometers..

Vocabulary

Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful.

barrel of a micrometer	hecto	ounce
base of a power	imperial system	print
base 10	inch	positive exponent
capacity	kilo	pound
centi	length	power
circumference	litre	precise
convert	mass	quart
cup	mega	referent
deca	metre	scientific notation
deci	metric prefix	*système international* (SI)
decimus	metric system	thimble
exponent	micro	ton
fixed or main scale	micrometer	Vernier calipers
fluid ounce	mile	Vernier scale
foot	milli	*yard*
gallon	nano	
gram	negative exponent	

Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

Temperature conversion formula: $F = C \times 1.8 + 32$

Fuel economy: $\dfrac{\text{number of litres of gas used}}{\text{number of 100 km of distances you have driven}}$

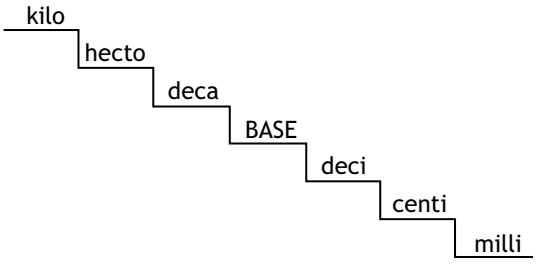
Body-Mass Index: $\text{BMI} = \dfrac{\text{your mass in kg}}{(\text{your height in metres})^2}$

Suitcases on a plane: $l + w + h = 157$ cm maximum

Canada Post maximum: Length + Girth = 300 cm where Girth = $2w + 2h$

Circumference of a circle: $C = 2\pi r$, where $\pi = 3.14$

Metric Prefix Ladder



Conversion Charts

Imperial Units Conversion Chart	
Length	1 mile = 1760 yards or 5280 feet 1 yard = 3 feet or 36 inches 1 foot = 12 inches
Mass	1 ton = 2000 pounds (lb.) 1 pound = 16 ounces (oz.)
Capacity	1 gallon = 4 quarts 1 quart = 2 pints 1 pint = 2 cups

Basic Conversion Formulas between Systems	
Mass	1 kg = 2.2 pounds 454 g = 1 pound 28.4 g = 1 ounce
Capacity	1 gallon (Canadian) = 4.54 L 1 fl. oz. (Canadian) = 28.41 mL 1 gallon (American) = 3.785 L 1 fl. oz. (American) = 29.57 mL
Distance	1 mile = 1.61 km 1 km = 0.62 mi. 1 inch = 2.54 cm 1 metre = 1.094 yards

Submitting Your Assignments

It is now time for you to submit the Module 3 Cover Assignment and Assignments 3.1 to 3.9 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 3 assignments and organize your material in the following order:

Module 3 Cover Sheet

Module 3 Cover Assignment: Following and Writing Instructions

Assignment 3.1: Powers of Ten

Assignment 3.2: Metric System

Assignment 3.3: Imperial System

Assignment 3.4: Reading Rulers

Assignment 3.5: Conversions

Assignment 3.6: Calipers

Assignment 3.7: Micrometers

Assignment 3.8: Formulas

Assignment 3.9: Problems

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 10 Essential Mathematics (20S)

Module 3
Measurement

Learning Activity Answer Keys

MODULE 3:
MEASUREMENT

Learning Activity 3.1

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are having to buy items at several stores in the mall. You walk 100 m to get to the first store. From there you walk 400 m to the second store. You have to walk 350 m to the third store, then walk 800 m back to your car. How far did you walk?
2. At the end of the day, a restaurant is left with three different, partially eaten pies. Each pie has 2 pieces out of 7 pieces left over. How much pie is left over in total?
3. There is a great sale on clothes; 30% off the marked price. If you are buying a hoodie that is priced $40.00, how much will you save?
4. What is the range of the following numbers: 2, 6, 4, 8, 7, 13, 11?
5. Solve for k: $\frac{k}{8} = 2$.

Answers:

1. 1650 m (100 + 400 + 350 + 800)
2. $\frac{6}{7}\left(\frac{2}{7}+\frac{2}{7}+\frac{2}{7} \text{ or } 3 \times \frac{2}{7}\right)$
3. $12 (30% = 3 × 10%; 10% of 40 is 4.00, so 4 × 3 = 12)
4. 11 (Remember, range is the difference between the highest and lowest numbers, so 13 – 2.)
5. k = 16 (Multiply by 8 on both sides.)

Part B: Moving the Decimal Point

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Identify the multiple of 10 used to either multiply or divide to find the answer.

1. a) $52.30 \times \_\_\_\_ = 523.0$

   Answer:

   The decimal point is moved one place to the right. You must multiply by 10 once. The answer is 10.

   b) $1.4567 \times \_\_\_\_\_ = 145.67$

   Answer:

   The decimal point is moved two places to the right. You must multiply by 10 twice. The answer is $10 \times 10 = 100$.

   c) $0.15 \times \_\_\_\_\_ = 1500.0$

   Answer:

   The decimal point is moved four places to the right. You must multiply by 10 four times. The answer is $10 \times 10 \times 10 \times 10 = 10\,000$.

2. a) $946.3 \div \_\_\_\_\_ = 9.463$

   Answer:

   The decimal point is moved two places to the left. You must divide by 10 twice. The answer is $10 \times 10 = 100$.

   b) $8.77 \div \_\_\_\_\_ = 0.877$

   Answer:

   The decimal point is moved one place to the left. You must divide by 10 once. The answer is 10.

   c) $0.25 \div \_\_\_\_\_ = 0.000\,25$

   Answer:

   The decimal point is moved three places to the left. You must divide by 10 three times. The answer is $10 \times 10 \times 10 = 1000$.

3. Write the operation of multiplication or division, and the multiple of 10 necessary to reach the answer.

   a) 96.4 _____ = 0.964

   Answer:

   The decimal point is moved two places to the left. You must divide by 10 twice or 10 × 10 = 100. The answer is ÷ 100.

   b) 605.25 _____ = 6052.5

   Answer:

   The decimal point is moved one place to the right. You must multiply by 10 once. The answer is × 10.

   c) 0.1234 _____ = 123.4

   Answer:

   The decimal point is moved three places to the right. You must multiply by 10 three times or 10 × 10 × 10 = 1000. The answer is × 1000.

   d) 2.3 _____ = 0.0023

   Answer:

   The decimal point is moved three places to the left. You must divide by 10 three times or 10 × 10 × 10 = 1000. Then answer is ÷ 1000.

Learning Activity 3.2

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Fill in the missing terms in the following pattern: 0, 3, ___ , 9, 12, ___ .
2. What is 10% of 760?
3. What is 5% of 760?
4. You are paid by commission at work. Your sales for this week were $760. If your commission is 15% of your sales, what is your gross pay?
5. You're missing your cell phone! You know you've left it in one of your pairs of pants, but you're not sure which pair. You wore your jeans on Monday, Wednesday, and Thursday. You wore your sweatpants on Sunday and Tuesday, and you wore your dress pants on Saturday. The last time you remember using your phone was on Wednesday. Which pair of pants should you check first?

Answers:

1. 6, 15 (The pattern is multiples of 3, or adding 3 each time.)
2. 76 (Move the decimal place left once.)
3. 38 (5% is half of 10% so 76 ÷ 2 = 38)
4. $114 (Use the percentages from above, since 10% + 5% = 15%.)
5. Jeans (You were wearing them the last time you saw it.)

Part B: Scientific Notation

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Without using a calculator, write the answers to the questions below. Use your knowledge of the relationship between the exponent on the base of 10 and the decimal point moving right or left, appropriately.

1. $6535.28 \times 10^{-3} =$ _____

   Answer:

   The exponent is –3 so the decimal point is moved 3 places to the left. You are dividing by 1000, or you are multiplying by $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$. The answer is 6.535 28.

   Notice that the decimal part of the answer is written in metric notation with a space between a group of three digits, starting from the decimal point and working towards the right.

2. $0.056 \times 10^{2} =$ _____

   Answer:

   The exponent is 2 so the decimal point moves 2 places to the right. The answer is 5.6.

3. $0.004\,89 \times 10^{-3} =$ _____

   Answer:

   The exponent is –3 so the decimal point is moved 3 places to the left. The answer is 0.000 004 89.

4. $89.7969 \times 10^{5} =$ _____

   Answer:

   The exponent is 5 so the decimal point moves 5 places to the right. The answer is 8 979 690.

5. $1.2345 \times 10^{-4} =$ _____

   Answer:

   The exponent is –4 so the decimal point is moved 4 places to the left. The answer is 0.000 123 45.

Learning Activity 3.3

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You use your left hand to type more than your right when you have your hands on the keyboard properly. Typing the word *factor*, you use your left hand for 5 letters and your right hand for one. Write the fraction that represents how many times you use your right hand in total.
2. Usain Bolt can run 100 m in 10 s. What is his average speed?
3. List the factors of 12.
4. GST (Goods and Services Tax) is 5%. You are buying clothes for your baby cousin (baby clothes only have GST). If the total before tax is $44.00, how much tax are you charged?
5. CPP is supposed to be 4.95% of your gross pay. Your gross pay is $1400 and you are charged $140 for CPP. Are you being charged the correct percent?

Answers:

1. $\frac{1}{6}$ (You are told you only use your right hand once. The total number of times you press a key is 6—there are 6 letters in factor.)
2. 10 m/s (100 m ÷ 10 s)
3. 1, 2, 3, 4, 6, 12 (Remember, factors multiply together to produce 12.)
4. $2.20 (10% of 44 is 4.40 so 5% is 4.4 ÷ 2 = 2.2)
5. No $\left(\frac{140}{1400} \div \frac{140}{140} = \frac{1}{10} = 10\%, \text{ which is more than } 4.95\%\right)$

Part B: The Metric System

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The base unit for measuring capacity is _____ . *Answer:* litre
2. The base unit for measuring mass is _____ . *Answer:* gram
3. The base unit for measuring length is _____ . *Answer:* metre
4. The word *decimal* is derived from the Latin word *decimus*, which means _____ . *Answer:* tenth
5. Complete the following rule for using the ladder for conversions.

   If you step up, move the decimal point to the _____ . (right or left).

   Answer: left

   If you step down, move the decimal point to the _____ . (right or left).

   Answer: right
6. Convert the following.
   a) 650 mL = _____ L

      Answer:

      On the ladder, you are stepping up three steps from milli to BASE. Move the decimal point three places to the left. The answer is 0.65 L.

   b) 385 m = _____ cm

      Answer:

      On the ladder, you are stepping two steps down from BASE to centi. Move the decimal two places to the right. The answer is 38 500 cm.

   c) 8562 mg = _____ g

      Answer:

      On the ladder, you are stepping 3 steps up from milli to BASE. Move the decimal three places to the left. The answer is 8.562 g.

   d) 6.5 km = _____ m

      Answer:

      On the ladder, you are stepping 3 steps down from kilo to BASE. Move the decimal point 3 places to the right. The answer is 6500 m.

Learning Activity 3.4

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If it rains 10 mm, how much rain is that in centimetres?
2. A marathon is 26.4 miles. A half marathon is how long?
3. Express the fraction as a decimal: $4\frac{3}{5}$.
4. There are two movies coming out on DVD this week that you would like to purchase. Each movie costs $18.99. If you have $35, can you afford to buy both?
5. Penelope is half as tall as Carson. Tara is almost as tall as Penelope. Carson is an inch taller than Zach. Who is the tallest?

Answers:

1. 1 cm (Remember: 1 step up means ÷ 10 or 10 mm = 1 cm)
2. 13.2 miles (26.4 ÷ 2)
3. 4.6 (The 4 is a whole number, so it can be placed ahead of the decimal and $\frac{3}{5} \times \frac{2}{2} = \frac{6}{10} = 0.6.$)
4. No ($18.99 is approximately $19; $19 × 2 = $38)
5. Carson (Penelope is shorter than Carson, Tara is shorter than Penelope, so she has to be shorter than Carson, and Zach is shorter than Carson)

Part B: The Imperial System

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Do the basic conversions within the imperial system for each of the questions below.

1. a) 4 feet = _____ inches

   Answer:

   1 foot = 12 inches

   4 feet = $4 \times 12 = 48$ inches

   b) 60 inches = _____ feet

   Answer:

1 foot	12 inches
x feet	60 inches

$$\frac{1}{x} = \frac{12}{60}$$

$$(x)(12) = (1)(60)$$

$$12x = 60$$

$$\frac{12x}{12} = \frac{60}{12}$$

$$x = 5$$

Thus, 60 inches = 5 feet.

Notice that to obtain the answer, you used division.

c) 126 inches = _____ feet

Answer:

1 foot	12 inches
x feet	126 inches

$$\frac{1}{x} = \frac{12}{126}$$

$$(x)(12) = (1)(126)$$

$$12x = 126$$

$$\frac{12x}{12} = \frac{126}{12}$$

$$x = 10.5$$

Thus, 126 inches = 10.5 feet.

d) $3\frac{1}{2}$ miles = _____ inches

Answer:

You need to convert from miles to feet and then from feet to inches. If there were more rules in the conversion chart, you might have been able to make the conversion in one step.

1 mile = 5280 feet

$3\frac{1}{2}$ miles = 3.5 × 5280 ft. = 18,480 ft.

1 foot = 12 inches

18,480 feet = 18,480 × 12 = 221,760 inches

Notice that in the imperial system commas are used to separate groups of three numbers. In the metric system a space is made between them and no comma is written.

2. a) 3 pounds = _____ ounces

Answer:

1 pound = 16 ounces

3 pounds = 3 × 16 = 48 ounces

b) 2 tons = _____ pounds (Canadian)

Answer:

1 ton = 2000 pounds

2 tons = 2 × 2000 = 4000 pounds

3. a) 6 quarts = _____ gallons

Answer:

1 gallon	4 quarts
x gallons	6 quarts

$$\frac{1}{x} = \frac{4}{6}$$

$$(x)(4) = (1)(6)$$

$$4x = 6$$

$$\frac{4x}{4} = \frac{6}{4}$$

$$x = 1.5$$

Thus, 6 quarts = 1.5 gallons.

b) 3.5 gallons = _____ pints

Answer:

1 gallon = 8 pints (from Mr. Gallon)

3.5 gallons = 3.5 × 8 = 28 pints

Learning Activity 3.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If a desk is 90 cm high, how tall is it in metres?
2. Your neighbour just had a baby. The baby is 18" long. How long is the baby in feet?
3. The clock on the wall ticks every second. How many times does it tick in 10 minutes?
4. Write this number in scientific notation: 0.000 243 5.
5. You are at a doughnut shop and would like to buy doughnuts for your family. There are 6 people in your family including you—and you want a doughnut. How much money do you need if each doughnut costs 60¢?

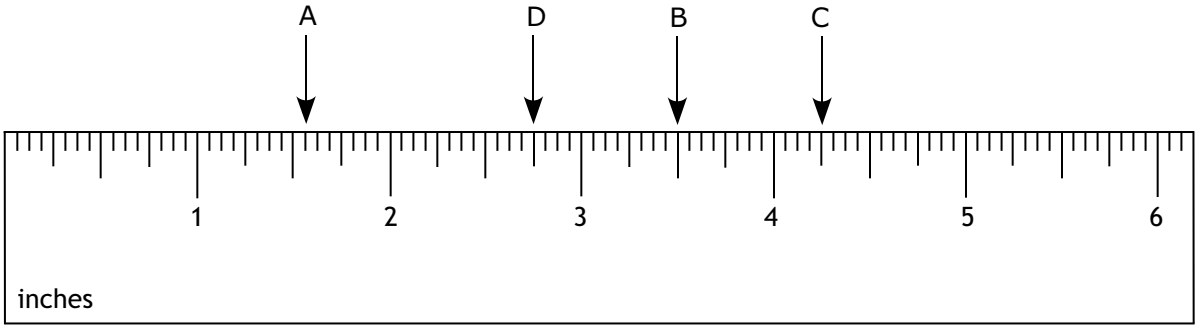
Answers:

1. 0.9 m (Centimetres to metres is 2 steps up so you have to move the decimal 2 places to the left.)
2. 1.5 feet $\left(\frac{18}{12} \div \frac{6}{6} = \frac{3}{2} = 1\frac{1}{2} \text{ feet}\right)$
3. 600 times (There are 60 seconds in a minute; 60 x 10 = 600.)
4. 2.435×10^{-4}
5. \$3.60 (6 × 60¢ = 360¢ or \$3.60)

Part B: Measuring with an Imperial Ruler

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the reading for each point on the imperial ruler.



Answers:

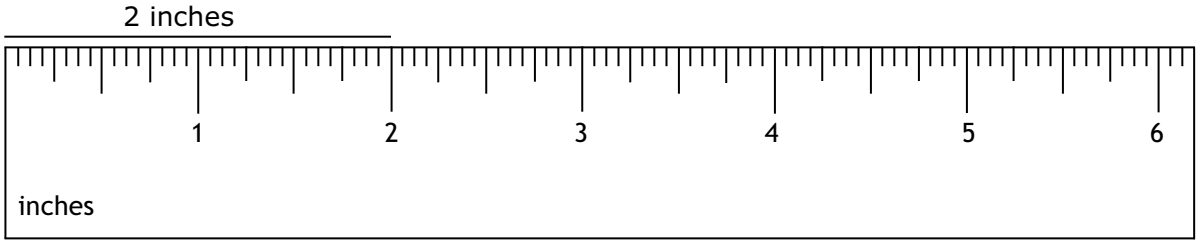
$A = 1\frac{9}{16}''$ $\quad$ $C = 4\frac{1}{4}''$

$B = 3\frac{1}{2}''$ $\quad$ $D = 2\frac{3}{4}''$

2. Use an imperial ruler to draw the length of a line with these measures.

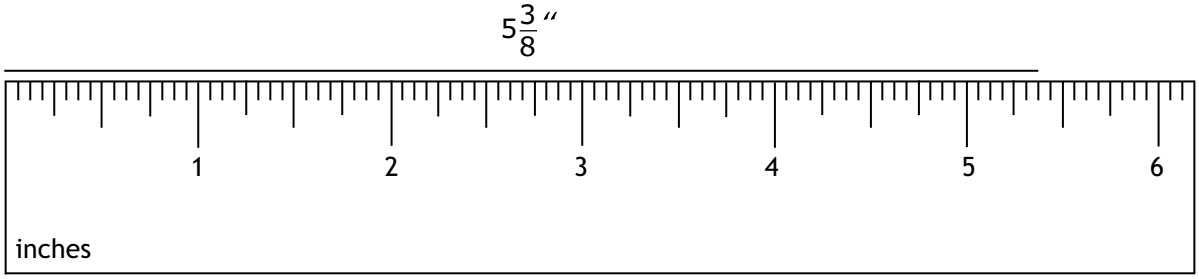
a) 2″

Answer:



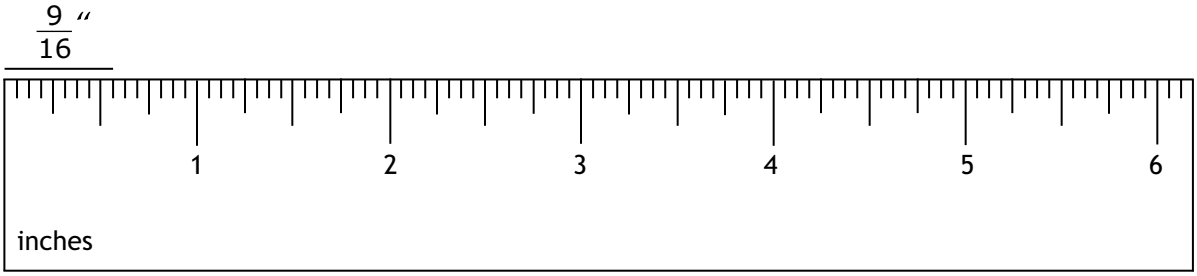
b) $5\frac{3}{8}$″

Answer:



c) $\frac{9}{16}$″

Answer:



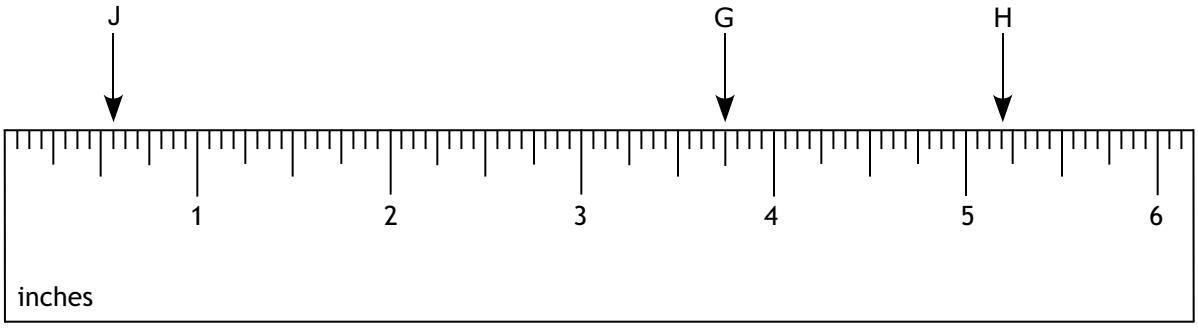
3. Mark the following points on the blank ruler. Use an arrow to indicate the exact marking.

   a) point G: $3\frac{3}{4}$"

   b) point H: $5\frac{3}{16}$"

   c) point J: $\frac{9}{16}$"

   Answer:



Learning Activity 3.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. An anaconda snake can be up to 24 feet in length. How long is that in yards?
2. Solve for m: $2 - m = 14$.
3. Fill in the blanks for the following pattern: 0, 1, 4, 9, ___ , 25, ____ .
4. You want to bring freezies to your last soccer game of the season. You want to have enough so that each player gets two. If you have 17 people on your team, how many freezies do you need?
5. Ryan's usual work week is 38 hours long. This week he worked 46 hours. How many hours of overtime did he work?

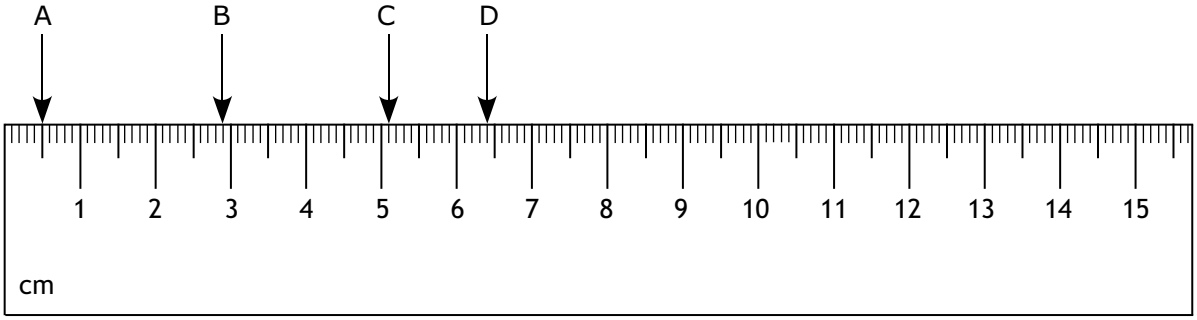
Answers:

1. 8 yards ($24 \div 3$, because there are 3 feet per yard, or 1 yard per 3 feet)
2. $m = -12$ (subtract 2 from both sides: $-m = 14 - 2$; multiply or divide by –1: $m = -12$)
3. 16, 36 (The pattern is to add the next odd number.
   0, 0 + 1, 1 + 3, 4 + 5, 9 + 7, 16 + 9, 25 + 11… or the pattern is the square numbers: $0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, etc.)
4. 34 (17×2)
5. 8 hours ($38 + 2 + 6 = 46$)

Part B: Using Metric Rulers

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the reading in centimetres for each point on the metric ruler.



Answers:

A = 0.5 cm

B = 2.9 cm

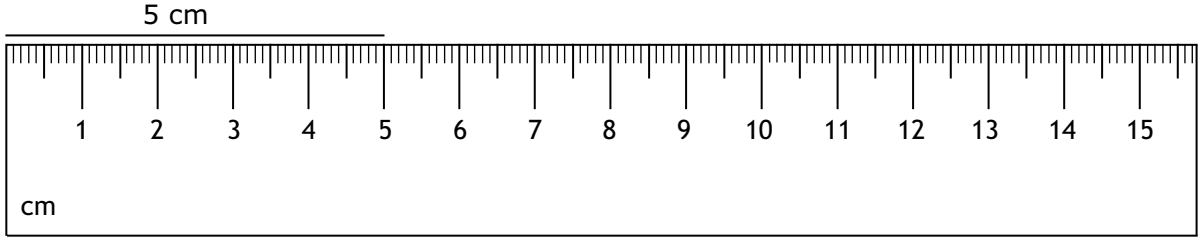
C = 5.1 cm

D= 6.4 cm

2. Use a metric ruler to draw a line, having the length in centimetres.

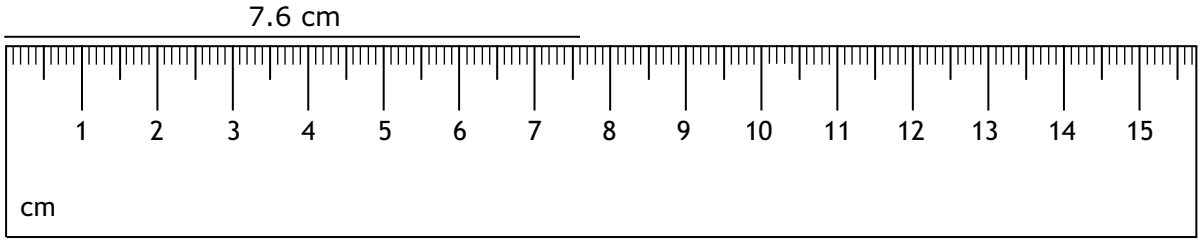
   a) 5 cm

   Answer:



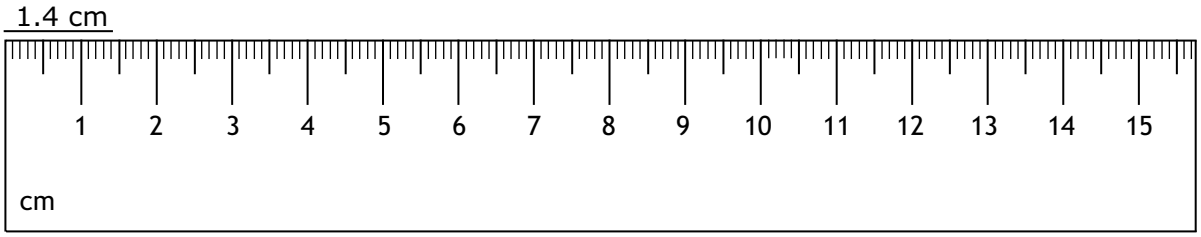
b) 7.6 cm

Answer:

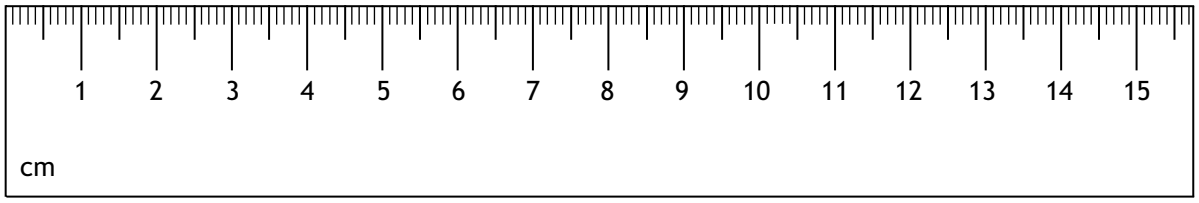


c) 1.4 cm

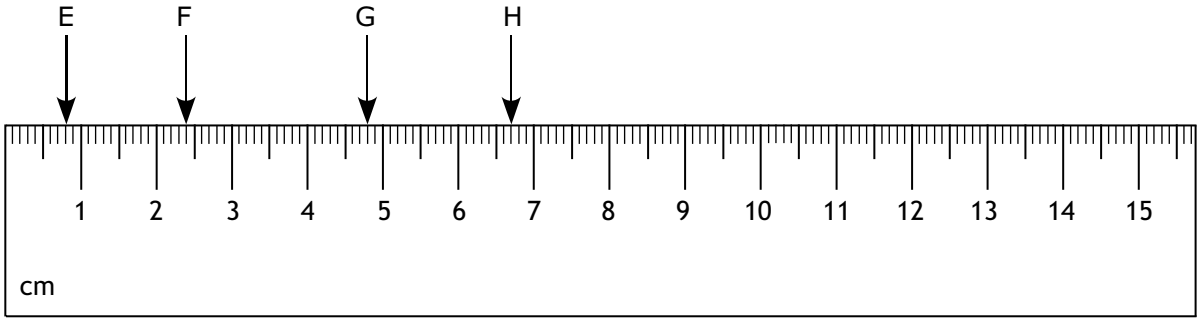
Answer:



3. Mark the following points on the metric ruler. Use an arrow to indicate the exact marking.
   a) point E at 0.8 cm
   b) point F at 23 mm
   c) point G at 4.8 cm
   d) point H at 6.7 cm



Answer:



Learning Activity 3.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Dino has three times as many sisters and he has brothers. If he has 2 brothers, how many kids are in the family?
2. Eric has a big crush. He sits 6 desks away from his crush. If the desks are each 80 cm wide and there are no spaces in between them, how close does he sit to his crush? Write your answer in metres.
3. Your mother has invited your entire family over for dinner. As usual, you are complaining because only $\frac{1}{6}$ of them are around your age. If your family has 24 people in it (not including you), how many of them are approximately your age?
4. Write the following improper fraction as a mixed fraction: $\frac{29}{9}$.
5. What type of angle has a measure of 270°?

Answers:

1. 9 kids (1 Dino, 2 brothers, 3 × 2 = 6 sisters)
2. 4.8 m (6 × 60 = 480 cm, 100 cm = 1 m)
3. $4 \left(\frac{1}{6} \times 24 \right)$
4. $3\frac{2}{9} \left(\frac{29}{9} = \frac{27}{9} + \frac{2}{9} = 3 + \frac{2}{9} \right)$
5. Reflex (larger than 180° but smaller than 360°)

Part B: Converting between Metric and Imperial

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the basic conversion formulas and proportional thinking to convert these measurements.

1. 6 pounds = _____ kg

   Answer:

1 kilogram	2.2 pounds
x kilograms	6 pounds

$$\frac{1}{x} = \frac{2.2}{6}$$

$$(x)(2.2) = (1)(6)$$

$$2.2x = 6$$

$$\frac{2.2x}{2.2} = \frac{6}{2.2}$$

$$x = 2.7$$

   Thus, 6 pounds = 2.7 kilograms.

2. 65 litres = _____ gallons (Canadian)

   Answer:

1 gallon	4.54 litres
x gallons	65 litres

$$\frac{1}{x} = \frac{4.54}{65}$$

$$(x)(4.54) = (1)(65)$$

$$4.54x = 65$$

$$\frac{4.54x}{4.54} = \frac{65}{4.54}$$

$$x = 14.3$$

   Thus, 65 litres = 14.3 gallons.

3. 10 ounces = _____ g

   Answer:

   1 ounce = 28.4 grams

   10 ounces = 10 × 28.4 = 284 grams

4. 12 km = _____ miles

   Answer:

   1 km = 0.62 miles

   12 km = 12 × 0.62 = 7.44 miles

5. 12 fl. oz. (Canadian) = _____ mL

   Answer:

   1 fl. oz. = 28.41 mL

   12 fl. oz. = 12 × 28.41 = 340.92 mL

6. 26 inches = _____ cm

   Answer:

   1 inch = 2.54 cm

   26 inches = 26 × 2.54 = 66.04 cm

7. 164 cm = _____ inches

   Answer:

1 inch	2.54 cm
x inches	164 cm

$$\frac{1}{x} = \frac{2.54}{164}$$

$$(x)(2.54) = (1)(164)$$

$$2.54x = 164$$

$$\frac{2.54x}{1.54} = \frac{164}{2.54}$$

$$x = 64.6$$

   Thus, 164 cm = 64.6 inches.

8. 26 miles = _____ km

   Answer:

   1 mile = 1.61 km

   26 miles = 26 × 1.61 = 41.86 km

Learning Activity 3.8

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You and 6 of your friends are at a fondue restaurant. The bill comes to $42. If you divide the cost evenly, how much will you pay each?
2. Complete the pattern: ___ , ___ , 1, 0.5, 0.25.
3. Solve for a: $18 + a = 23$.
4. A regular goal in rugby is worth 3 points. If a team finishes a game with 39 points, how many goals did they score?
5. You woke up at 8:30 am. You got ready in 45 minutes. You walked to work for 45 min. You worked for 4 hours. You ate lunch for 35 minutes. What time is it now?

Answers:

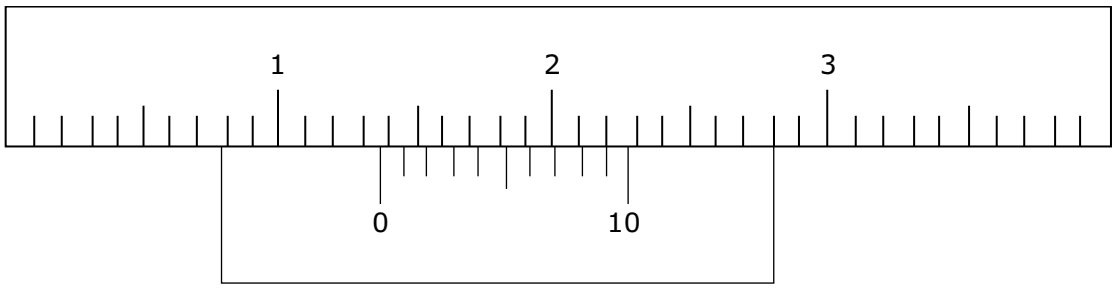
1. $6 each (Including you, 7 people are splitting the bill, $42 \div 7 = 6$.)
2. 4, 2 (The pattern is that each number is half of the one before it so 2 comes before 1 and 4 comes before 2.)
3. $a = 5$ (Subtract 18 from both sides.)
4. 13 ($39 \div 3$)
5. 2:35 pm or 14:35 (8:30 + 45 min. + 45 min. + 4 hr. + 35 min. = 14:35)

Part B: Reading Vernier Calipers

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write down the precision measurements as shown on these Vernier caliper diagrams.

1. Read the following to the nearest hundredth of a centimetre.



Reading = _____

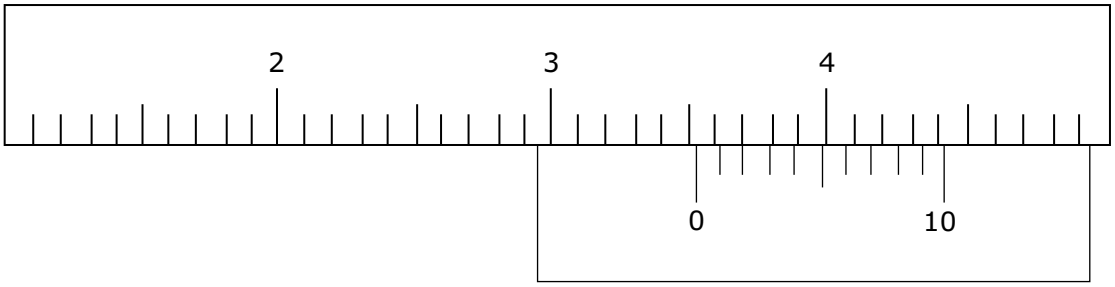
Answer:

The first line of the moving scale, at a reading of 0, points to a space between 1.3 and 1.4 on the fixed scale. Thus, the first reading is 1.3 cm.

The two scales align at the 9th tick on the moving scale, giving a reading of 0.09 cm.

The total reading = 1.3 + 0.09 = 1.39 cm.

2. Read the following to the nearest hundredth of a centimetre.



Reading = _____

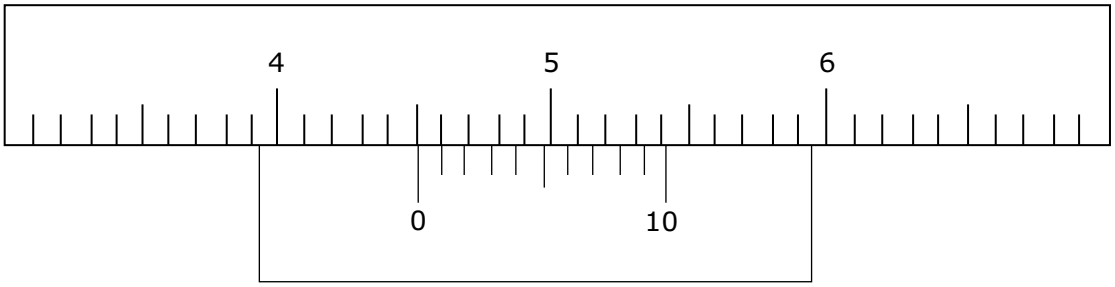
Answer:

The first line of the moving scale, at a reading of 0, points to a space between 3.5 and 3.6. Thus, the first reading is 3.5 cm.

The two scales align at the second tick on the moving scale, giving a reading of 0.02 cm.

The total reading = 3.5 + 0.02 = 3.52 cm.

3. Read the following to the nearest hundredth of a centimetre.



Reading = _____

Answer:

The first line of the moving scale, at a reading of 0, points to a space between 4.5 and 4.6. The first reading is 4.5 cm.

The two scales align at the 0.01 mark.

The total reading = 4.5 + 0.01 = 4.51 cm.

Learning Activity 3.9

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. There are 8 marbles in a bag. Each marble is red, yellow, or blue. Four of the marbles are red, and one marble is yellow. How many are blue?
2. Solve for r: $5 + r = -4$.
3. Which is larger: 436% or $\frac{18}{5}$?
4. Estimate the taxes, at 12%, of a pair of shoes that cost $74.89.
5. A loonie is approximately 2.5 cm across. Convert to inches.

Answers:

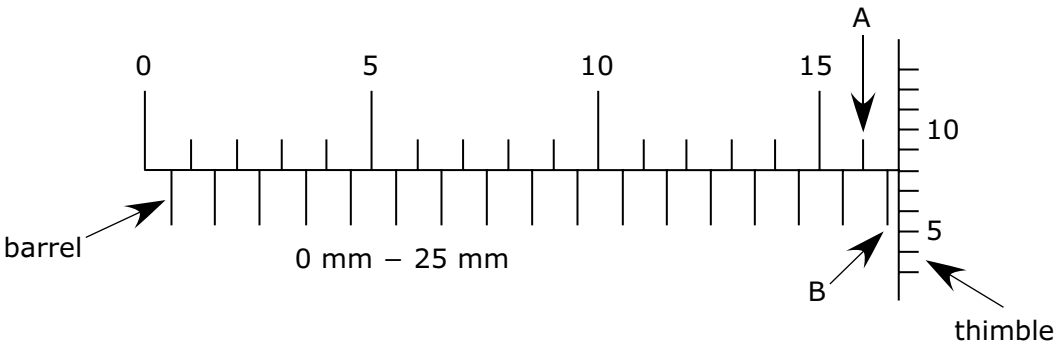
1. There 3 are blue. (8 – 4 – 1)
2. $r = -9$ (Subtract 5 from both sides.)
3. 436% (Convert to the same units: $\frac{18}{5} = \frac{15}{5} + \frac{3}{5} = 3\frac{3}{5} = 3.6 = 360\%.$)
4. Approximately $9 (10% is about 7.50 and 1% is 0.75, so 2% is 1.50; therefore, 12% is 7.50 + 1.50 = 9.00)
5. About 1 inch (1 inch = 2.54 cm)

Part B: Reading a Micrometer

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them..

Read and record these micrometer measurements.

1. Read the following micrometer measurement.



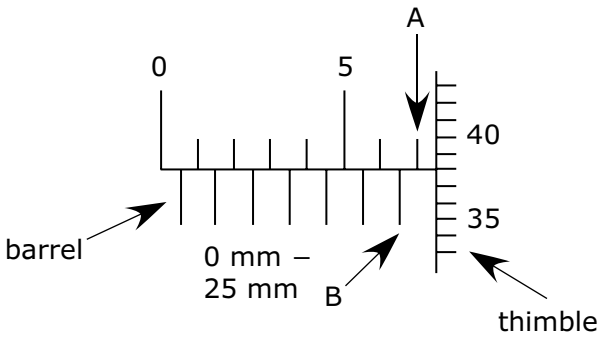
Answer:

Read the number from the upper barrel at A as 16 mm and the lower at B as 0.5 mm. Thus, the total barrel reading is 16.5 mm.

The thimble reading yields 0.08 mm.

The sum and resulting measurement is 16 mm + 0.5 mm + 0.08 mm = 16.58 mm.

2. Read the following micrometer measurement.



Answer:

Read the number from the barrel as 7 mm (Note: the marking indicated by arrow B is to the left of the marking indicated by arrow A).

The thimble reading yields 0.38 mm.

The sum and resulting measurement = 7 mm + 0.38 mm = 7.38 mm.

Learning Activity 3.10

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The body of a Daddy-Long-Legs spider is about 0.7 cm long. How long is this in mm?
2. Team A has won 5 out of their last 9 games. Team B has won 4 out of their last 7 games. Which team has won a greater percentage of their games?
3. Write as an improper fraction: $1\frac{3}{16}$.
4. You are at a hockey game and would like a snack and a drink. At the counter, you see that popcorn is $3.00, peanuts are $2.25, and hotdogs are $3.75. The drinks are $2.00. If you have $5, what snack(s) can you afford to get if you also buy a drink? (Tax is included in the price.)
5. You earn money through piecework at the factory. Your employer does not have a lot of money so each shirt is worth $2.00 and each pair of pants is worth $3.00. If you make 4 shirts and 3 pants, how much will you be paid (gross)?

Answers:

1. 7 mm (10 mm = 1 cm or you go down one step on the prefix ladder)
2. Team B has won a greater percentage. (The fractions for the team are $\frac{5}{9}$ and $\frac{4}{7}$. You need common denominators to compare them, which will be 63. The fractions will be $\frac{5}{9} \times \frac{7}{7} = \frac{35}{63}$ and $\frac{4}{7} \times \frac{9}{9} = \frac{36}{63}$; 36 is larger than 35.)
3. $\frac{19}{16}$ $\left(\text{This is because } 1 = \frac{16}{16} \text{ and you have } 1\frac{3}{16}, \text{ so } \frac{3}{16} + \frac{16}{16} = \frac{19}{16}.\right)$
4. You can get peanuts or popcorn if you get a drink. (A drink is $2, so you have $3 left of your $5 to spend on food.)
5. $17 ((4 × 2) + (3 × 3) = 8 + 9 = 17)

Part B: Conversion Problems

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You observe that the outside temperature in Fargo is 15°F. Find that value in ˚C.

   Answer:

   Formula: $F = C \times 1.8 + 32$

   Rearranged Formula: $\frac{(F-32)}{1.8} = C$

   Substitute: $\frac{(15-32)}{1.8} = C$

   Solve: $-9.4 = C$

   The value in °C is –9.4°C.

2. If the temperature in Thompson is 25°C, find that value in °F.

   Answer:

   Formula: $F = C \times 1.8 + 32$

   Substitute: $F = 25 \times 1.8 + 32$

   Solve: $F = 77$

   The value in °F is 77°F.

3. The formula for converting kilometres to miles is:

   Number of kilometres × 0.62 = number of miles

   If you drove for $12\frac{3}{4}$ hours at an average speed of 100 km/h, how many miles did you drive?

   Answer:

   First find how many kilometres were driven.

   12.75 hours × 100 km in one hour = 1275 km

   Now convert kilometres to miles using the given formula.

   Formula: Number of kilometres × 0.62 = number of miles.

   Substitute: 1275 km × 0.62 = m miles

   Solve: 790.5 = m

   Thus, you drove 790.5 miles.

4. Using the same formula as in question #3, how many kilometres would be equivalent to 500 miles?

   Answer:

   Formula: Number of kilometres × 0.62 = number of miles.

   Substitute: k km × 0.62 = 500 miles.

   Solve: Divide both sides by 0.62 to isolate k.

$$\frac{k \times 0.62}{0.62} = \frac{500}{0.62}$$

$$k = 806.45$$

   Thus, 500 miles is equivalent to 806.45 kilometres.

   Note if the conversion formula is used, the result is 500 × 1.61 = 805 kilometres.

5. You, along with 3 friends, step onto an elevator in a store in the US. There is a sign on the wall indicating the maximum mass allowed is 650 pounds. You know that the average mass of a Canadian teenager is 70 kg.

   a) Will your combined mass be within the allowed amount on the elevator?

      Answer:

      Find the total mass of you and your 3 friends.

      70 kg × 4 = 280 kg

      Convert to pounds.

      1 kg = 2.2 pounds (from your Resource Sheet)

      280 kg × 2.2 = 616 pounds

      Since the elevator maximum is 650 pounds, you will all be safely transported on the elevator.

   b) By how many pounds are you over or under?

      Answer:

      650 – 616 = 34 pounds under the restriction.

Learning Activity 3.11

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are at the store to buy your mother a present. You have $30 to spend, and would like to get her a waffle maker. If it costs $40 and is 20% off, will you be able to get her the waffle maker?
2. If 5% of 260 is 13, what is 5% of 520?
3. Evaluate: $3 + 9 - 2 \times 12 \div 3$.
4. According to a study, 75% of class time is used inefficiently. If you have a 60 minute class, how much time is equal to 75%?
5. Convert: 300 m = _____ km.

Answers:

1. No (20% off $40 = $8, so it will still cost $32.)
2. 26 (520 is double 260, so 5% of 520 is also double.)
3. 4
   (use BEDMAS:
   $3 + 9 - 2 \times 12 \div 3$
   $3 + 9 - 2 \times 4$ divide
   $3 + 9 - 8$ multiply
   4 add and subtract)
4. 45 minutes (75% = $\frac{3}{4}$ and we commonly divide an hour (60 minutes) into fourths, such as "it is quarter past" for 15 minutes past the hour, so three-fourths of an hour is 45 minutes)
5. 0.3 km (1000 m = 1 km or it is 3 steps up to go from metres to kilometres.)

Part B: Using Referents

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You are setting up a ball diamond for a quick game of baseball with your friends. You know that first base is 90 feet from home plate. Describe how you would estimate where the base should be set.

   Answer:

   90′ is 30 yards. If your pace is 1 yard then about 30 paces should be close.

2. a) Is a suitcase measuring 85 cm by 45 cm by 30 cm within the linear measurement limits set by an airline?

      Answer:

      Length + width + height = 85 cm + 45 cm + 30 cm = 160 cm

      The total is more than the limit of 157 cm. It is not within the linear measurement limit.

   b) By how much is it over or under the limit?

      Answer:

      160 cm – 157 cm = 3 cm over the limit

3. Describe how you might estimate the length of a pencil.

   Answer:

   If the pencil is long, a 9″ hand span estimate can be made. If it is short, the inch wide thumb can be used to estimate the length.

Notes

Grade 10 Essential Mathematics (20S)

Module 4
Geometry

Module 4: Geometry

Introduction

In this module, you will learn to calculate the area of various shapes using metric and imperial measurements.

Assignments in Module 4

When you have completed the assignments for Module 4, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Tower of Hanoi
1	Assignment 4.1	Multiplication Skills
2	Assignment 4.2	Area of Shapes
3	Assignment 4.3	Scale Drawings
4	Assignment 4.4	Area of Odd Shapes
5	Assignment 4.5	Conversions
6	Assignment 4.6	Problems

Resource Sheet

When you write your midterm examination, you will be allowed to bring a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Midterm Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 4. You might use your Module 4 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 1, 2, 3, and 4, to prepare your Midterm Examination Resource Sheet. Remember, the midterm examination is based only on the first four modules of the course.

Resource Sheet for Module 4

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Midterm Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Writing Your Midterm Examination

You will write the midterm examination when you have completed Module 4 of this course. The midterm examination is based on Modules 1 to 4, and is worth 12.5 percent of your final mark in the course. To do well on the midterm examination, you should review all the work you complete in Modules 1 to 4, including all the learning activities and assignments. You will write the midterm examination under supervision.

Notes

Module 4 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work on the diagrams provided. The cover assignment can be done at any time while you are completing Module 4. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on whether or not you found a solution, and whether or not your tutor/marker can read your diagrams. The cover assignment is worth a total of 12 marks.

Notes

Module 4 Cover Assignment

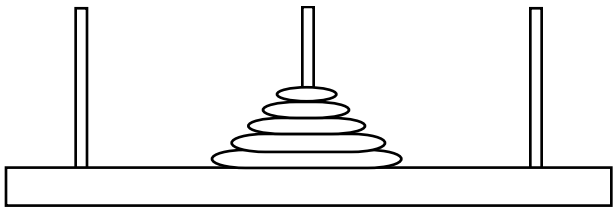
Total: 12 marks

Tower of Hanoi

Background Information

In the city of Hanoi in a temple, there are some monks. They are working on a challenge.

On a base, there are 3 diamond columns. On one of the columns, there are 64 gold discs. The discs each have a different diameter and are stacked so that the largest disc is on the bottom, as in the following diagram.



Tower of Hanoi

The task of the monks is to move the discs to a different column using the following two rules:

1. Only one disc can be moved at a time.
2. A larger disc cannot be placed on top of a smaller disc.

When the task is finished, the world will end.

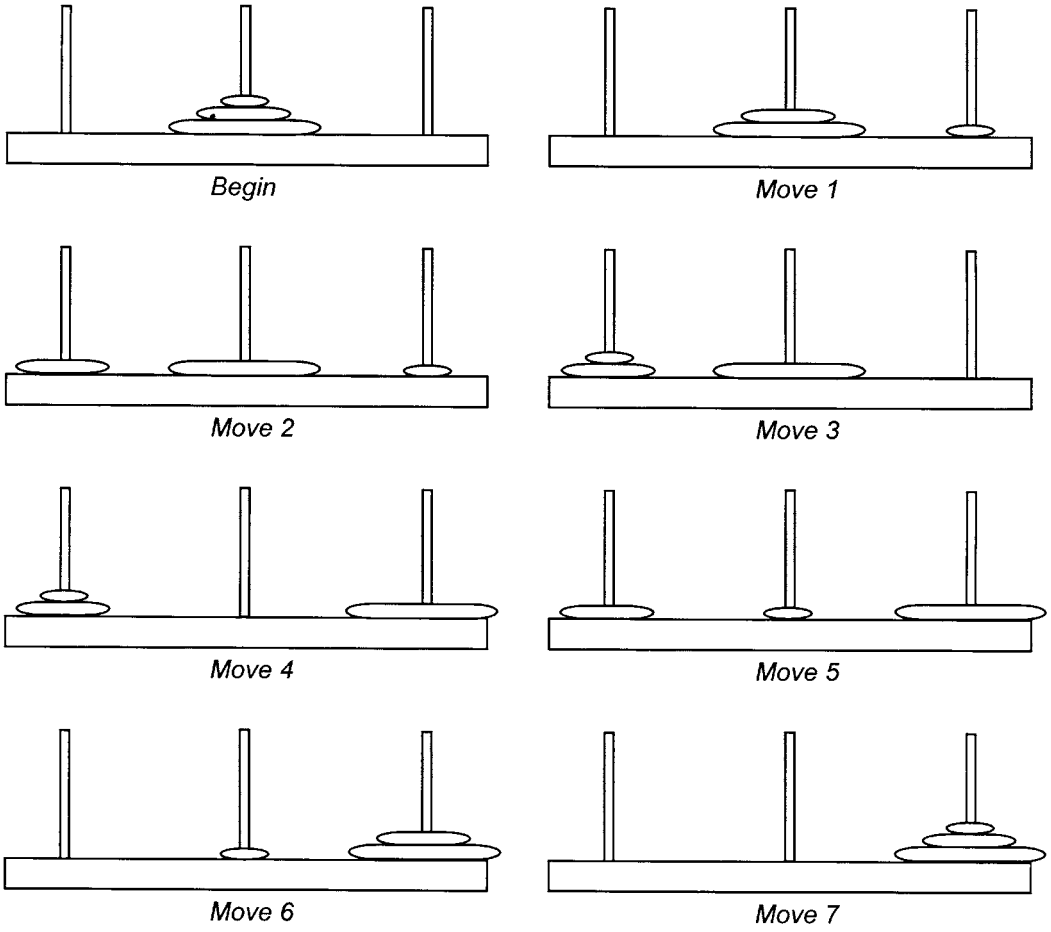
Will it?

Your task is to find the minimum number of moves it will take to move five discs, not 64.

To help you develop a strategy, first try the problem with three disks. The answer is given below.

continued

Module 4 Cover Assignment: Tower of Hanoi (continued)

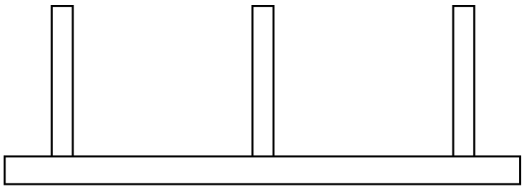


Move Number	Tower 1	Tower 2	Tower 3
Begin		1 2 3	
1		2 3	1
2	2	3	1
3	1 2	3	
4	1 2		3
5	2	1	3
6		1	2 3
7			1 2 3

(continued)

Module 4 Cover Assignment: Tower of Hanoi (continued)

Then use the diagrams below to show your answer for the 5 disks. You probably can answer the question in less than 36 moves. You are to hand in the diagrams for your solution.



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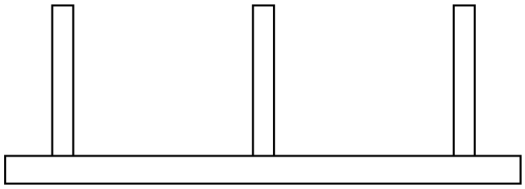

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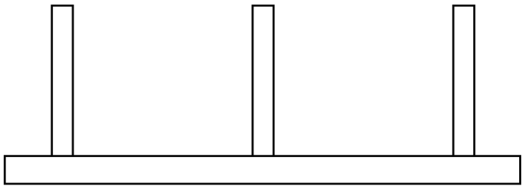

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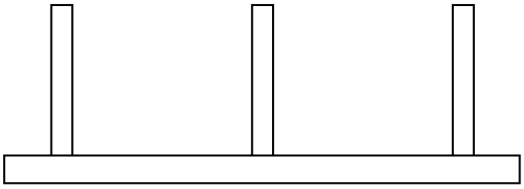

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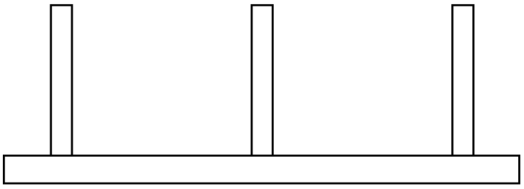

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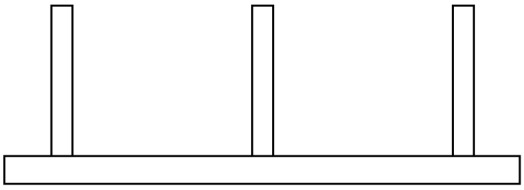

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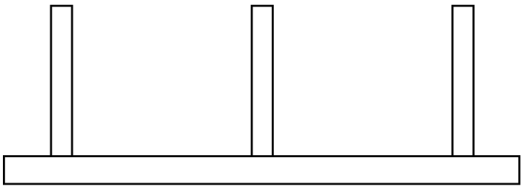

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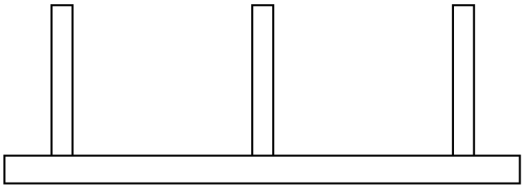

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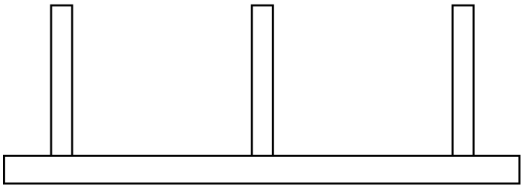

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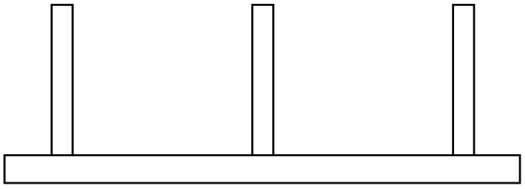

Move 9

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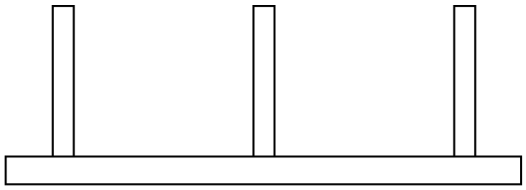
Module 4 Cover Assignment: Tower of Hanoi (continued)



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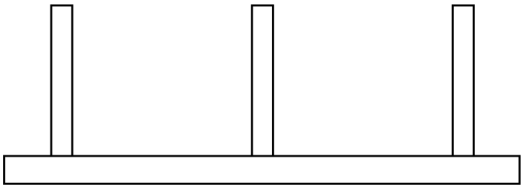

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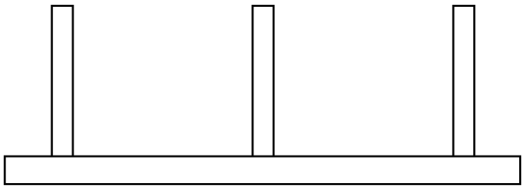

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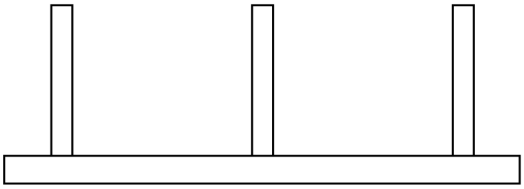

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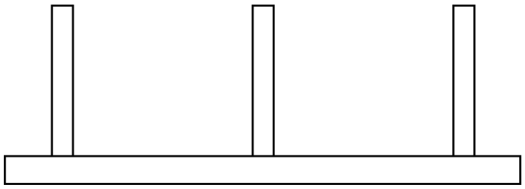

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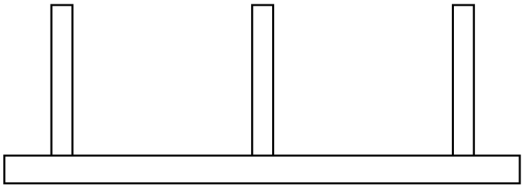

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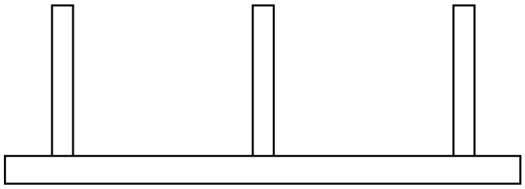

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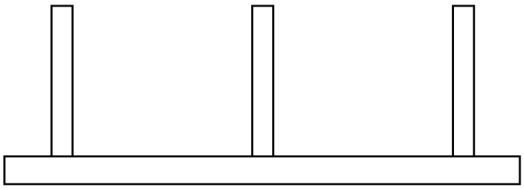

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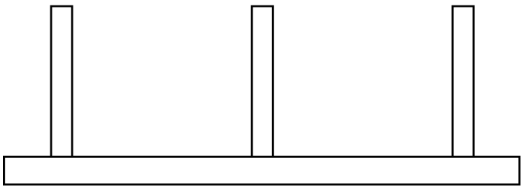

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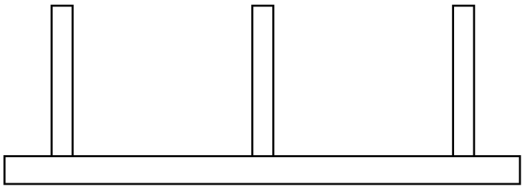

Move 19

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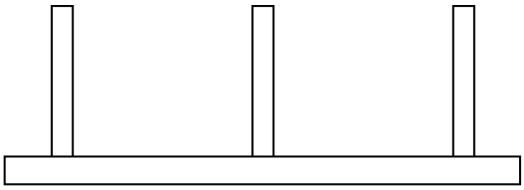
Module 4 Cover Assignment: Tower of Hanoi (continued)



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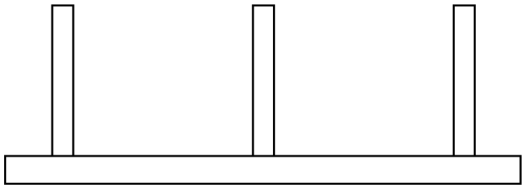

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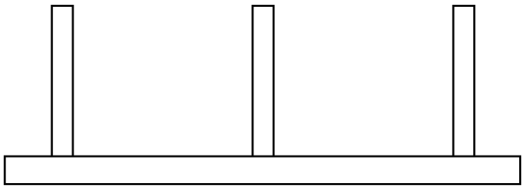

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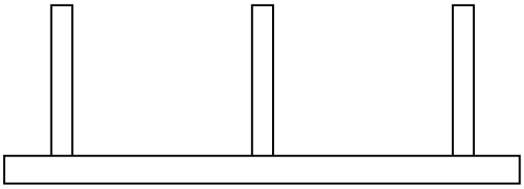

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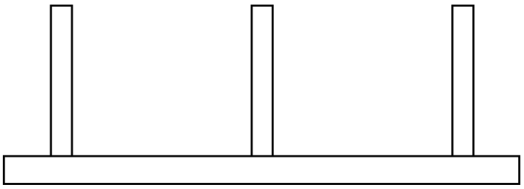

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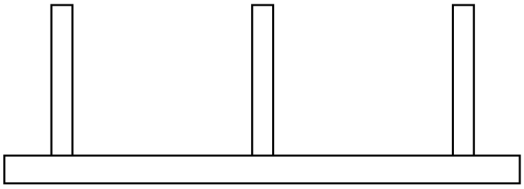

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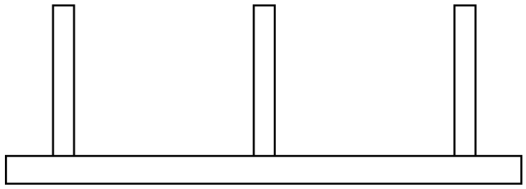

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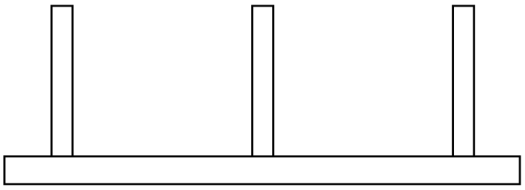

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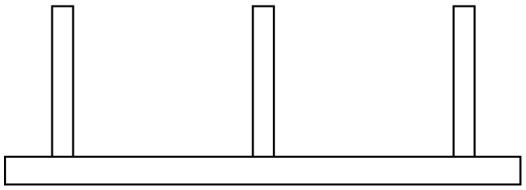

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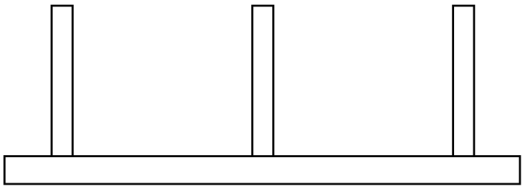

Move 29

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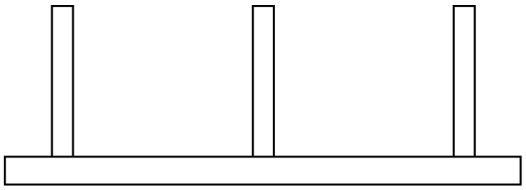
Module 4 Cover Assignment: Tower of Hanoi (continued)



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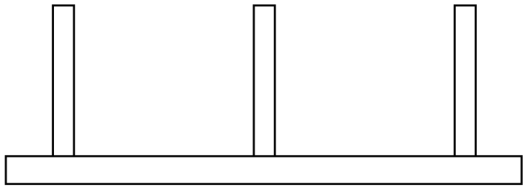

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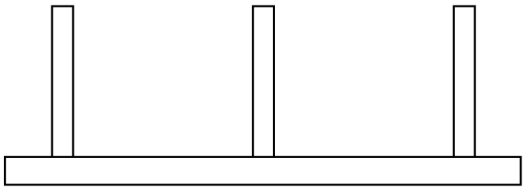

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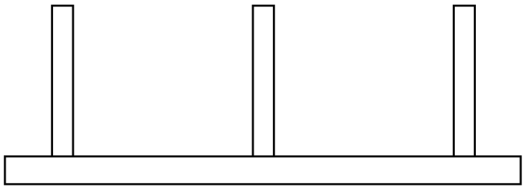

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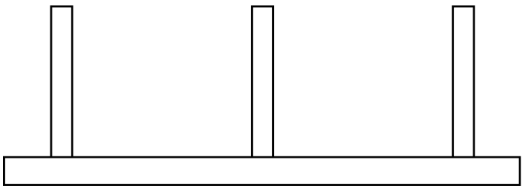

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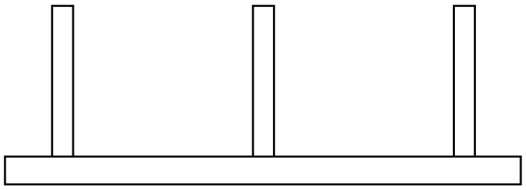


Move 35

continued

Move Number	Tower 1	Tower 2	Tower 3		Move Number	Tower 1	Tower 2	Tower 3
Begin					18			
1					19			
2					20			
3					21			
4					22			
5					23			
6					24			
7					25			
8					26			
9					27			
10					28			
11					29			
12					30			
13					31			
14					32			
15					33			
16					34			
17					35			

Notes

Lesson 1: Multiplication Skills

Lesson Focus

In this lesson, you will

- ❑ practise estimating with fractions and decimals
- ❑ solve problems that include metric and imperial measurements
- ❑ solve problems using multiplication of decimal and fractional measurements

Lesson Introduction

This lesson prepares you for finding areas of shapes. Skills for multiplying decimals and fractions are reviewed, as well as estimating skills.

Using Multiplication

The area of a rectangular shape is generally found by multiplying the length by the width.

A room that measures 9′ by 12′ has a floor space of 9′ × 12′ = 108 ft.^{2}.

However, on many occasions, the measurements are not that easy to multiply. For example, you may have the dimensions of 8.24 m by 12.15 m, or the measurements could be $9\frac{1}{4}'$ by $11\frac{7}{8}'$. How do you calculate the area?

It is necessary that you review your skills with multiplying decimals and fractions, and estimating answers.

Estimating Skills

Usually, you would use a calculator to multiply two numbers. When using technology, you should have some skill with estimating answers. Too often, the wrong key is pressed on the calculator and a wrong answer results. Developing basic estimating skills can save you from making some drastic decisions because of wrongly performed arithmetic operations.

Example 1

Estimate the answer for 2.3 cm × 15.6 cm.

Solution:

Your classmate entered the 2.3 × 15.6 into his calculator, and got an answer of 358.8 cm^2. He insists the answer is correct because "the calculator says so!"

Through estimation, you need to be able to see that his answer is not possible.

If you round the 2.3 cm down to 2 cm and the 15.6 cm up to 16 cm, the answer for 2 cm × 16 cm should be "about" 32 cm^2.

The answer of 358.8 cm^2 that your classmate got is not even close. It is about 10 times too big.

Probably the correct values were not entered on his calculator. You try again.

$$2.3 \text{ cm} \times 15.6 \text{ cm} = 35.88 \text{ cm}^2$$

The digits, or numbers, in his answer were correct. The decimal placement was wrong, making the answer larger than it should have been.

Example 2

You are ordering 8 pizzas for a team party. The price is $16.75 per pizza. The delivery person arrives, and asks for $13.40. Being a very honest person, you do not want to cheat the driver out of the money he deserves! Show how you might explain to him that he is not correct, by using estimating.

Solution:

Using your skills, you round 8 pizzas to 10 pizzas, and $16.75 per pizza to $16.00. You multiply 10 × 16 by moving the decimal point one place to the right, and get an answer of $160. This is not even close to the $13.40 on the bill.

When you type in the amounts on your calculator, you get an exact answer of 8 × $16.75 = $134.

The digits in the delivery person's answer were correct, but the decimal placement was wrong.

Example 3

You are buying a piece of expensive plywood for a cabinet. The measurements are $9\frac{1}{2}'' \times 14\frac{3}{4}''$. Calculate the area of wood you need and then estimate the answer. Was your calculation correct?

Solution:

You calculate that you require 1401.25 in.^{2}. But you remember to "estimate" the answer before you spend all your money on the plywood.

Rounding the $9\frac{1}{2}''$ to 10″ and the $14\frac{3}{4}''$ to 15″, you can find the area.

$$10 \text{ in.} \times 15 \text{ in.} = 150 \text{ in.}^2.$$

Looking at your initial answer of 1401.25 in.^{2}, you know something is not quite right.

The actual area can be calculated by changing the fractions to decimals.

$$\text{Area} = 9\frac{1}{2} \text{ in.} \times 14\frac{3}{4} \text{ in.} = 9.5 \times 14.75 = 140.125 \text{ in.}^2.$$

The original calculation had the correct digits but the decimal place was in the wrong spot.

Multiplying Decimals

Whenever you are doing calculations, you should always estimate what the answer should be, even at a restaurant or grocery store. And when you use your calculator to find the answer, check by estimating to make sure your answer is reasonable.

Example 1

Find the product 3.5×6.4 by first estimating the answer.

Solution:

Using your estimating skills, round the 3.5 to 4, and the 6.4 to 6. Then, 4×6 is 24, so the actual answer should be about 24.

Now use your calculator to find the actual answer.

$$3.5 \times 6.4 = 22.4$$

The value on the calculator is close to your estimate, so you have likely typed the numbers in correctly.

Example 2

Calculate the product, 15.25 × 7.9, by first estimating the answer.

Solution:

Using estimation, the 15.25 rounds to 15, and the 7.9 could be rounded up to 8.

15 × 8 is 120, so the actual answer must be close to that estimate.

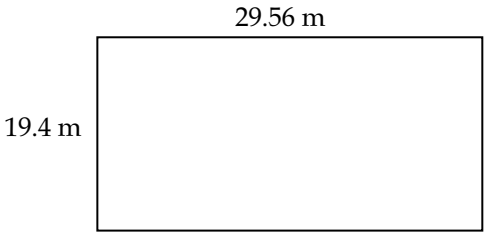
Now use your calculator to find the actual answer.

$$15.25 \times 7.9 = 120.475$$

The value on the calculator is close to your estimate, so you have likely typed the numbers in correctly.

Example 3

Your yard measures 19.4 m by 29.56 m. How many square metres of sod must you order to completely cover the yard? Show your calculations. Estimate the approximate area, and then find the exact answer by using your calculator.



Solution:

Area is found by multiplying the length times the width.

Using estimation and rounding, you could round these values to get an area of 20 m × 30 m = 600 m^2.

Once you enter the numbers into your calculator, you know that the answer should be close to your estimate.

$$\text{Area} = 19.4 \text{ m} \times 29.56 \text{ m} = 573.464 \text{ m}^2$$

The answer of 573.464 is close to 600, so you have likely typed the numbers in correctly.

Don't forget to include the units for area as square units.

Learning Activity 4.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Rewrite the following fraction in lowest terms: $\frac{24}{52}$.
2. You have measured your foot to be 9.5". Is your foot a good referent to approximate a foot in length?
3. You are paid a salary of $960 every two weeks. If you work a total of 96 hours in that time, what is your hourly wage?
4. Solve for p: $p \div 5 = 15$.
5. You want to meet your friends for coffee. You are available from 9 am to 3 pm. Aiden is free from 12 pm until 2 pm. Leah is able to come from 10 am until 1 pm. What time can you meet for coffee?

Part B: Estimating with Decimals

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. How would you estimate this answer before doing the calculation?

   6.8×12.3

2. Is it reasonable for 8 pairs of jeans priced at $29.95 each to cost $2396.00? Show an estimation of how you found "about" what they should cost.
3. Multiply each question, showing an estimate using rounding. Then find the actual answer.
   a) 12×23.65
   b) 5.84×1.2
   c) 1.487×0.12
   d) 0.625×5.6

Multiplying Fractions: Proper Fractions

In the imperial system, the inch is the smallest unit of length. It is divided into even smaller parts. The smallest part of an inch on the standard imperial ruler or tape measure is $\frac{1}{16}$ of an inch.

You will remember that adding and subtracting fractions requires a common denominator. This is not necessary when multiplying or dividing fractions.

There are many ways to multiply fractions. Some methods will be shown here. You should be familiar with all the methods. Unless otherwise stated, however, always choose the method that gives you the correct answer most often.

For proper fractions, you multiply the top numbers, then multiply the bottom numbers, and then simplify.

Example 1

Multiply $\frac{1}{2} \times \frac{3}{4}$.

Solution:

Multiply the tops: $1 \times 3 = 3$

Multiply the bottoms: $2 \times 4 = 8$

Write your answer: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Example 2

Multiply $\frac{3}{4} \times \frac{3}{8}$.

Solution:

Multiply the tops: $3 \times 3 = 9$

Multiply the bottoms: $4 \times 8 = 32$

Write the answer: $\frac{3}{4} \times \frac{3}{8} = \frac{9}{32}$

Example 3

Multiply $5 \times \frac{1}{4}$.

Solution:

In this example, you must remember that 5 is equivalent to, or the same as, $\frac{5}{1}$.

This gives you $5 \times \frac{1}{4} = \frac{5}{1} \times \frac{1}{4} = \frac{5}{4}$.

Multiplying the top numbers and then the bottom numbers gives us $\frac{5}{4}$.

Changing an Improper Fraction To a Mixed Number

In Example 3 above, the answer is an improper fraction because the top number is larger than the bottom number.

There are situations in math when you leave improper fractions in that form. But, for measurement purposes, you usually simplify an improper fraction into a mixed number.

In order to simplify this improper fraction into its mixed number form, you could use the following process.

$\frac{5}{4}$ is the same as $\frac{4}{4} + \frac{1}{4}$.

Since $\frac{4}{4} = 1,$ the answer would be written in simplified form as $1\frac{1}{4}$.

Example

Change $\frac{135}{16}$ into a mixed number.

Solution:

Find the highest multiple of 16 that is smaller than or equal to 135. Since $6 \times 8 = 128$, then 8 is the multiple that is required. The remainder is $135 - 128 = 7$

$$\frac{135}{16} = \frac{128}{16} + \frac{7}{16} = 8\frac{7}{16}$$

Multiplying Mixed Numbers

When you are presented with multiplying mixed numbers, the process becomes more interesting. Four methods are shown below, using grids, using improper fractions, using decimals, and using the fraction key on a calculator.

Multiplying by Using a Grid

One method students like to use to multiply mixed numbers is the "grid" method. You may have used this method when multiplying polynomials in Grade 9, or when multiplying whole numbers in earlier grades.

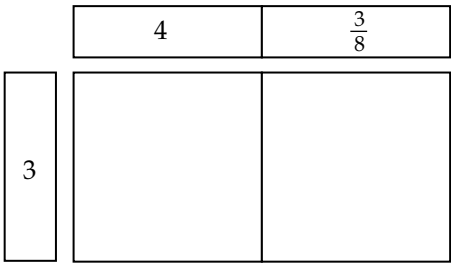
Multiplication is used to find the area of a rectangle when you are given the dimensions of the length and the width. You use a "grid" to break the rectangle into smaller pieces or sections that are easier to work with. The grid method separates the multiplication into steps, and allows you to consider each step individually. When all the multiplication has been completed, you combine your answers to get the solution which is the area of the original rectangle.

Example 1

Multiply $3 \times 4\frac{3}{8}$ using a grid.

Solution:

Use 3 as the length of the rectangle and $4\frac{3}{8}$ as its width. However, since $4\frac{3}{8}$ is a mixed number, separate it into its two parts, the whole number part, 4, and the proper fraction part, $\frac{3}{8}$. The grid is shown below.



Now, find the area of each blank rectangle and then add both areas together to get the total area.

	4	$\frac{3}{8}$
3	$3 \times 4 = 12$	$3 \times \frac{3}{8} = \frac{3}{1} \times \frac{3}{8} = \frac{9}{8}$

The answer would be the sum of the areas = $12 + \frac{9}{8}$.

Since $\frac{9}{8}$ is an improper fraction, it must be changed to a mixed number.

$$\frac{9}{8} = \frac{8}{8} + \frac{1}{8} = 1\frac{1}{8}$$

Thus, $12 + \frac{9}{8} = 12 + 1\frac{1}{8} = 13\frac{1}{8}$.

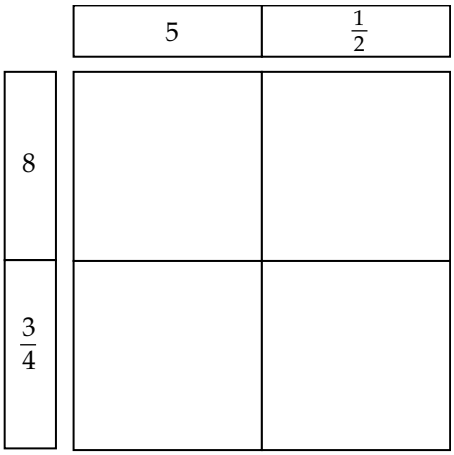
Example 2

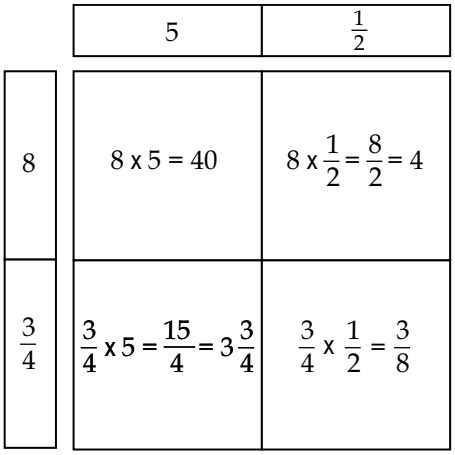
Calculate $8\frac{3}{4} \times 5\frac{1}{2}$ using a grid.

Solution:

Write each mixed number as the length and width of the rectangle, along the sides of the grid. Separate each mixed number into its whole number part and its fraction part. Then find the area of each blank part of the whole rectangle by multiplication. Add the areas together to find the total area and your answer to the question.

$$8\frac{3}{4} \times 5\frac{1}{2}$$





From the smaller rectangles, you have the areas of $40 + 4 + 3\frac{3}{4} + \frac{3}{8}$.

Adding the whole numbers gives you 40 + 4 + 3 = 47.

Adding the fractions $\frac{3}{4} + \frac{3}{8} = \frac{6}{8} + \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$.

Answer = total of all areas = $47 + 1\frac{1}{8} = 48\frac{1}{8}$.

Note that to add $\frac{3}{4} + \frac{3}{8}$, you need to find a common denominator of 8. Then the numerators have to be fixed to get equivalent fractions.

$$\frac{3}{4}+\frac{3}{8}=\frac{3\times 2}{4\times 2}+\frac{3}{8}=\frac{6}{8}+\frac{3}{8}=\frac{9}{8}$$

In this course, the only fractions where you will need to find common denominators will be halves, quarters, eighths, sixteenths, and thirty-seconds. Thus, a common denominator of 32 will work for all fractions. A lower value for the common denominator is easier to work with, but not necessary. In the example above, you could have used 32 as the common denominator.

$$\frac{3}{4}+\frac{3}{8}=\frac{3\times 8}{4\times 8}+\frac{3\times 4}{8\times 4}=\frac{24}{32}+\frac{12}{32}=\frac{36}{32}$$

Now if you simplify your answer, you will be done.

$$\frac{36}{32}=\frac{36\div 4}{32\div 4}=\frac{9}{8}$$

This is the same result as in the example.

Multiplying By Converting To an Improper Fraction

Another method of multiplying fractions involving mixed numbers with a whole number part and a proper fraction part is to create improper fractions, then multiply the top numbers, multiply the bottom numbers, and simplify.

Example 1

Find the answer for $4\frac{1}{2}\times\frac{3}{8}$ by first changing to improper fractions.

Solution:

Change $4\frac{1}{2}$ to an improper fraction.

$$4\frac{1}{2}=4+\frac{1}{2}=\frac{4}{1}+\frac{1}{2}=\frac{4\times 2}{1\times 2}+\frac{1}{2}=\frac{8}{2}+\frac{1}{2}=\frac{9}{2}$$

Now multiply the fractions.

$$4\frac{1}{2}\times\frac{3}{8}=\frac{9}{2}\times\frac{3}{8}$$

Multiply the tops: $9 \times 3 = 27$

Multiply the bottoms: $2 \times 8 = 16$

Write the answer: $4\frac{1}{2} \times \frac{3}{8} = \frac{9}{2} \times \frac{3}{8} = \frac{27}{16}$

Since $\frac{27}{16}$ is an improper fraction, you change it to a mixed number. When simplified, you get $\frac{27}{16} = \frac{16}{16} + \frac{11}{16} = 1\frac{11}{16}$.

The answer is $1\frac{11}{16}$.

The shortcut to creating improper fractions from mixed numbers is to multiply the denominator by the whole number, then add the numerator. This value is the numerator of the improper fraction. The denominator of the fraction remains the same.

$3\frac{1}{2}$ converts to $\frac{2 \times 3 + 1}{2}$ giving you $\frac{6+1}{2}$ or $\frac{7}{2}$.

Example 2

Calculate $3\frac{3}{4} \times 1\frac{5}{8}$.

Solution:

First, change each mixed number into an improper fraction.

$$3\frac{3}{4} = \frac{4 \times 3 + 3}{4} = \frac{12+3}{4} = \frac{15}{4}$$

$$1\frac{5}{8} = \frac{8 \times 1 + 5}{8} = \frac{8+5}{8} = \frac{13}{8}$$

$$3\frac{3}{4} \times 1\frac{5}{8} = \frac{15}{4} \times \frac{13}{8}$$

Multiply the tops: $15 \times 13 = 195$

Multiply the bottoms: $4 \times 8 = 32$

$$3\frac{3}{4} \times 1\frac{5}{8} = \frac{15}{4} \times \frac{13}{8} = \frac{195}{32}$$

To convert this improper fraction to a mixed number, find the largest multiple of 32 less than or equal to 195. Since 32 × 6 = 192, the largest multiple that works is 6. Now you can finish the question.

$$\frac{195}{32} = \frac{192}{32} + \frac{3}{32} = 6 + \frac{3}{32} = 6\frac{3}{32}$$

Multiplying By Converting To a Decimal

Another method of multiplying mixed numbers is to convert the fraction parts to decimals, multiply, and then simplify the answer to its lowest fractional form. This is probably the most complicated method.

It is useful to remember basic fraction to decimal conversions.

Fraction	Decimal
$\frac{1}{8}$	0.125
$\frac{1}{4}$	0.25
$\frac{3}{8}$	0.375
$\frac{1}{2}$	0.5
$\frac{5}{8}$	0.625
$\frac{3}{4}$	0.75
$\frac{7}{8}$	0.875

You can always use your calculator to convert any fraction to a decimal value using the rule, "top divided by bottom." As an example, the fraction, $\frac{1}{5}$ can be written in decimal form using a calculator, by entering 1 ÷ 5 (top divided by bottom). The answer is 0.2. If you were to check your work by changing $\frac{1}{5}$ to a fraction whose denominator is 10, you would write, $\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$.
As a decimal, two-tenths is 0.2, the same answer.

Example 1

Calculate $3\frac{1}{2} \times 4\frac{3}{4}$ by converting to decimals.

Solution:

Using the method of converting to decimals, $3\frac{1}{2} \times 4\frac{3}{4}$ becomes

$3.5 \times 4.75 = 16.625$.

Since you started in the question with fractions, then the answer would be written using fractions, rather than decimals. If you look back at the conversion chart, you see that 0.625 is equivalent to $\frac{5}{8}$.

The answer would be $16\frac{5}{8}$.

Example 2

Calculate $5\frac{3}{4} \times 4\frac{1}{2}$ by changing to decimals.

Solution:

$5\frac{3}{4} \times 4\frac{1}{2} = 5.75 \times 4.5 = 25.875$

If you look back at the conversion chart, you see that 0.875 is equivalent to $\frac{7}{8}$.

The answer is $25.875 = 25\frac{7}{8}$.

Multiplying By Using a Calculator

You can enter in the fractions into the calculator using the fraction button, "a b/c", and the answers will be shown in fractional form. Enter mixed numbers by pressing the fraction key twice. For example, $1\frac{5}{8}$ is entered as

1	a b/c	5	a b/c	8

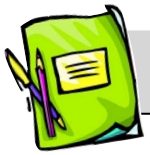
If you use the "shift" or 2nd key, you can change the mixed number into an improper fraction.

1	a b/c	5	a b/c	8	2nd	a b/c

The calculator will show the improper fraction of $\frac{13}{8}$.

For more information, please refer to the user's manual for your calculator.

Remember that you can use any method to multiply fractions. However, it is a very good habit to use estimation to ensure your answers are accurate.



Learning Activity 4.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Estimate the product of 0.524 and 3.57.
2. The last time you counted, you had 54 DVDs. Your house was broken into last night and now you only have 32. How many DVDs were stolen?
3. Complete the pattern: $-1, 2, -3,$ ___ , ___ .
4. A 90° angle is called a _____ angle.
5. What is a logical unit to use when measuring the dimensions of your room?

continued

Learning Activity 4.2 (continued)

Part B: Multiplying Fractions

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Multiply these fractions using "multiply the tops, multiply the bottoms" method.

   a) $\frac{3}{4} \times \frac{5}{8}$

   b) $\frac{1}{2} \times \frac{3}{8}$

2. Use the grid method to multiply these fractions.

   a) $3\frac{1}{2} \times 2\frac{3}{4}$

   b) $4\frac{3}{8} \times 3\frac{1}{4}$

3. Convert these fractions to decimals using the chart or your calculator.

   a) $3\frac{1}{8}$

   b) $50\frac{3}{4}$

4. Convert these mixed numbers to improper fractions, multiply the fractions, and then simplify.

   a) $2\frac{1}{8} \times 3\frac{3}{4}$

   b) $4\frac{1}{2} \times 2\frac{3}{4}$

5. You are one of a group of 8 friends who are evenly splitting $100. Your friend shows you his math.

   $\$100 \times \frac{1}{8} = \$100 \times 0.0125 = \$1.25$

   He hands you $1.25.

   Does this make sense? Try to estimate the correct answer.

Lesson Summary

In this lesson, you used your estimation skills with fractions and decimals. You multiplied decimals using your calculator. You multiplied proper fractions, learned how to convert an improper fraction to a mixed number, and to multiply mixed numbers in 4 different ways. You were given a chart listing common fractions and their decimal equivalents. You applied your skills to solving problems.

In the next lesson, you will examine how to find the area of various shapes.

Notes

Assignment 4.1

Total: 17 marks

Multiplication Skills

1. Your friend gave the following wrong answer. Identify the mistake he made, and correct it. *(2 marks)*

   $5.6 \times 8.25 = 4.62$

2. Show how you would find an estimate, and give an estimate for each question. *(4 marks)*

   a) 6.9×12.23

   b) 14.85×6.17

3. Show how you would estimate the cost of 4 pairs of jeans at $31.50 each. *(2 marks)*

4. Multiply these dimensions to find the area of a rectangle. *(2 marks)*

   a) $2 \text{ m} \times 6.54 \text{ m} =$

   b) $12.6 \text{ cm} \times 18.98 \text{ cm} =$

continued

Assignment 4.1: Multiplication Skills (continued)

5. Multiply these fractional measurements, using the method "multiply the tops, multiply the bottoms." *(2 marks)*

   a) $\frac{7}{8} \times \frac{3}{4}$

   b) $\frac{1}{2} \times \frac{5}{8}$

6. Use the grid method to find the area of the large rectangle and the answer to the multiplication: $4\frac{1}{2} \times 3\frac{5}{8}$. *(5 marks)*

LESSON 2: AREA OF SHAPES

Lesson Focus

In this lesson, you will

❑ solve problems using formulas for area

Lesson Introduction

This lesson teaches you to find the area of various shapes, including rectangles, parallelograms, trapezoids, triangles, and circles.

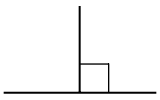
Multiplication and Geometry

In your daily life, you are surrounded with shapes of circles, rectangles, triangles, and others. Whether you are renovating a basement or kitchen, landscaping a yard, painting or shingling a house, or sewing together garments, you have to consider how to find the area of these shapes. The area tells you how much material you need to complete the job, and helps you in establishing the cost of the project. Different formulas are used for different shapes.

When solving problems involving area calculations, it is always a good idea to draw a diagram of the shape and label all the parts for which you know the measures.

In all situations using formulas to find area, you must be sure that all the units of measurement are uniform—that is to say they are the same. They may be given as cm, m, inches, feet, or whatever, but in the calculation of the area, they must all be the same unit.

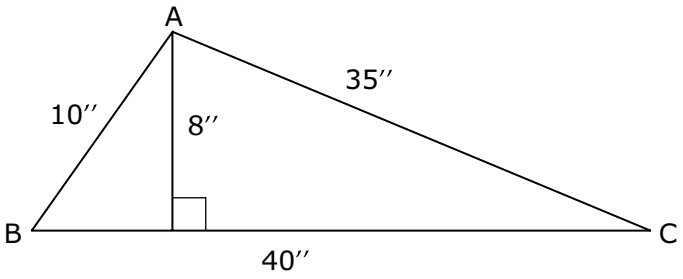
Many formulas refer to the base as b and the height as h. The base and the height always meet each other as perpendicular lines at an angle of 90°.



Always look for this perpendicular connection when substituting values into your formulas for b and h.

Example

Find the area of the triangle.



Solution:

There are 4 measurements given for ΔABC. The formula for area of a triangle is

$$A = \frac{1}{2}bh$$

The only two measurements that share a perpendicular connection are the 40″ and the 8″. So they are the only two measurements that can be entered into the formula to find the area.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(40 \text{ inches} \times 8 \text{ inches})$$

$$A = \frac{1}{2}(320 \text{ sq. in.})$$

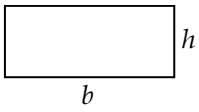
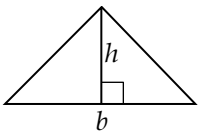
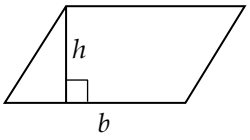
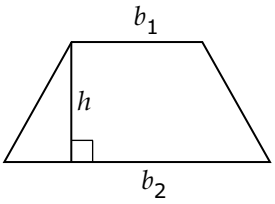
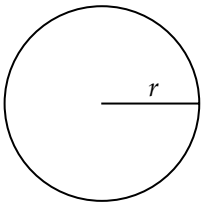
$$A = 160 \text{ sq. in.}$$

As well as triangles, the base–height connection is applicable for area calculations of parallelograms, trapezoids, and rectangles.

Another very important thing to remember is that area units are square units. For example, if you multiply inches by inches, the result is square inches or in^2. The same goes for metres × metres = square metres or m^2. Should you forget to put in the proper units, you are not communicating correctly. If you are a builder, you would need to know what units are used for measuring. (And you will likely not receive full marks on your assignment questions.)

Multiplication notation in mathematics comes in a variety of forms. In arithmetic, for example, if you multiply 3 and 4, you could write the multiplication sign with an × as 3 × 4, or with brackets as (3)(4). In algebra, when letters are used in a formula (e.g., $A = bh$), this notation also indicates multiplication. In other words, $A = bh$ means $A = b \times h$ or $A = (b)(h)$.

In the chart below are listed the formulas for finding the area of various shapes.

Name of Formula	Diagram	Formula
Rectangle		$A = bh$
Triangle		$A = \frac{1}{2}bh$
Parallelogram		$A = bh$
Trapezoid		$A = \frac{1}{2}h(b_1 + b_2)$
Circle Area		$A = \pi r^2$ use $\pi = 3.14$

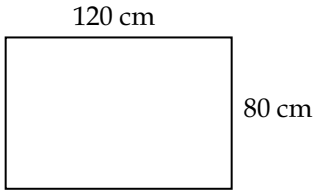
Include the formulas from this table on your resource sheet so that you do not have to flip back and forth throughout the lesson.

Rectangles

Perhaps the easiest and most common of all shapes requiring an area calculation is the rectangle. The area of a rectangle is found by multiplying the length times the width. You must be sure that the units are the same. For example, if you are given a situation where the length is 1.2 m and the width is 80 cm, one of the measurements must be converted to the other. You could write the area as A = 1.2 m × 0.8 m, or A = 120 cm × 80 cm. The two units of measurement must be the same in your calculations for the area.

Example 1

Find the area of the rectangle.



Solution:

The formula for finding area of a rectangle is $A = bh$.

$A = 120 \text{ cm} \times 80 \text{ cm}$

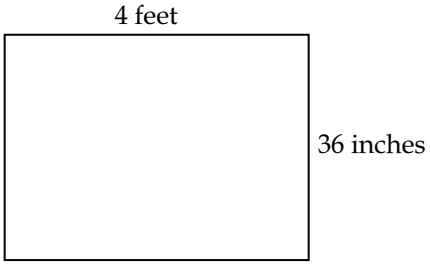
$A = 9600 \text{ cm}^2$

Example 2

The sandbox in your yard measures 4' by 36". Find the area of the lawn it covers.

Solution:

The first step would be to draw a diagram.

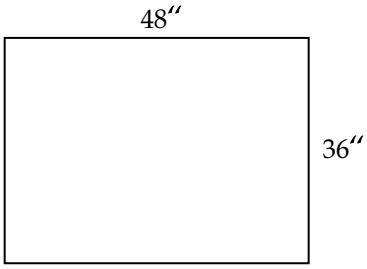


Now as you enter the measurements onto the diagram, you notice they are not in the same units; one is in feet and the other is in inches. You must decide to make them both in inches, or both in feet.

In this example, you could convert 4′ to inches.

Knowing there are 12″ in each foot, then 4′ = 4 × 12 = 48″.

You draw a new diagram with the altered units of measure.



Now calculate the area.

$A = bh$

$= 48″ \times 36″$

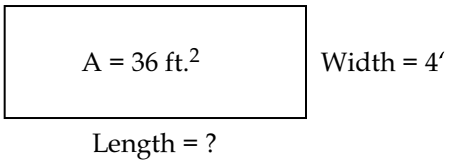
$= 1728 \text{ in.}^2$

Example 3

The store has a carpet remnant that has an area of 36 ft.^{2}, and it is 4′ wide. How long is this piece of carpet?

Solution:

First, you draw a diagram and write information on it.



For rectangles, the area formula of base × height is the same as multiplying the length by the width.

In this question, you know the area, and you are asked to find the length. The formula can be manipulated to isolate the unknown variable.

$A = bh$

$36 \text{ ft.}^2 = (b)(4')$

Isolate the unknown value by dividing both sides by 4′.

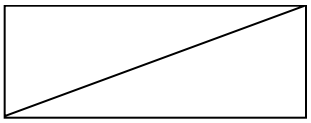
$$\frac{36 \text{ ft.}^2}{4 \text{ ft.}} = \frac{(b)(4')}{4 \text{ ft.}}$$

$$9' = b$$

Thus, the length of the carpet is 9′.

Triangles

A triangle is half of a rectangle. Take any rectangle and fold it along its diagonal line. The result is two equal triangles.

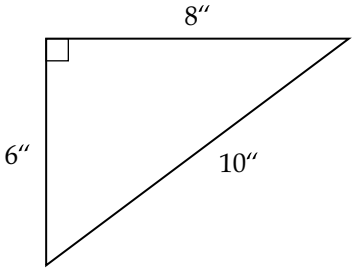


If the formula for the area of a rectangle is $A = bh$, then it makes sense that the formula for the area of a triangle is half of that, or $A = \frac{1}{2}bh$.

Again, you must always remember that the height must meet the base at a right angle.

Example 1

Find the area of the triangle.



Solution:

To find the area of this triangle, you look for the base meeting the height at a right angle. In this example, the only two sides that are perpendicular are the 6″ side and the 8″ side. Those would be the values you substitute into the formula.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(8 \text{ in.})(6 \text{ in.})$$

$$= \frac{1}{2}(48 \text{ in.}^2)$$

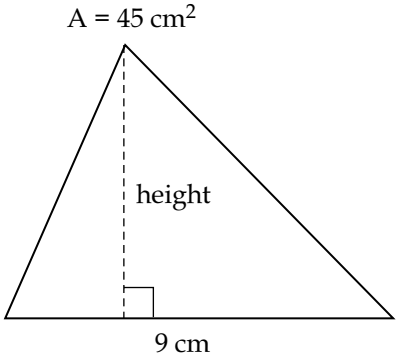
$$= 24 \text{ in.}^2$$

Example 2

If the area of a triangle is 45 cm^2 and the base is 9 cm, find the height.

Solution:

To solve this question, you need to draw the diagram, write the formula, and then substitute in the known values.



$$A = \frac{1}{2}bh$$

$$45 \text{ cm}^2 = \frac{1}{2}(9 \text{ cm})(h)$$

$$45 \text{ cm}^2 = (4.5 \text{ cm})(h)$$

Since h is the unknown, you need to manipulate this formula to isolate the variable h.

$$45 \text{ cm}^2 = (4.5 \text{ cm})(h)$$

$$\frac{45 \text{ cm}^2}{4.5 \text{ cm}} = \frac{(4.5 \text{ cm})(h)}{4.5 \text{ cm}}$$

$$10 \text{ cm} = h$$

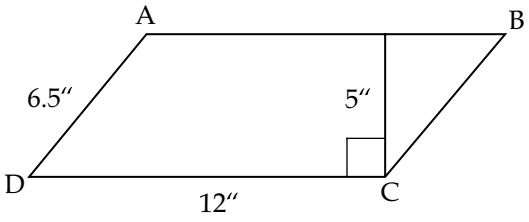
Thus, the height of the triangle is 10 cm.

Parallelograms

A parallelogram does not usually have right-angled corners. The opposite sides are parallel with each other, and are the same length. To find the area of a parallelogram, you need to have the height of the parallelogram meeting the base at a right angle.

Example

Find the area of the parallelogram ABCD.



Solution:

Note that //gm is an abbreviation for parallelogram. Because ABCD is a //gm, you know that its opposite sides are the same length and parallel.

The formula for finding the area of a parallelogram is $A = bh$. The base must meet the height at a right angle.

The two measurements that are perpendicular to each other are the height of 5" and the base of 12".

$A = bh$

$A = (5")(12")$

$A = 60 \text{ in.}^2$

Learning Activity 4.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

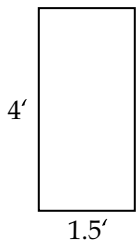
1. You are getting ready to start your first job, and you want to make sure you get there on time. It takes you 20 minutes to ride your bike to your work from your house. You want to get there 15 minutes before your shift starts. It takes you 30 minutes to get ready in the morning. If your shift starts at 10:00 am, what time will you have to wake up?
2. Solve for b: $8 + b - 4 = 16$.
3. Which is larger: 0.66 or $\frac{2}{3}$?
4. If you are looking for a good deal, you found it—bread is 50% off! One loaf usually costs $2.40. How much will you pay with the discount?
5. There is half a pie on the counter. If you eat a quarter of that pie, how much will you have eaten out of the whole pie?

Part B: Areas of Regular Shapes (1)

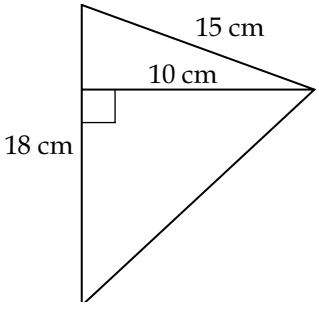
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the area of the following shapes.

   a) rectangle



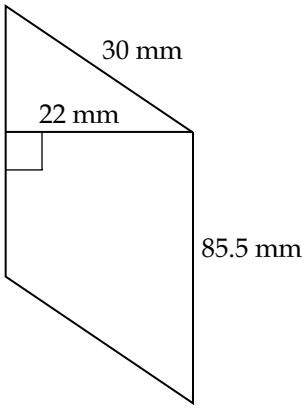
   b) triangle



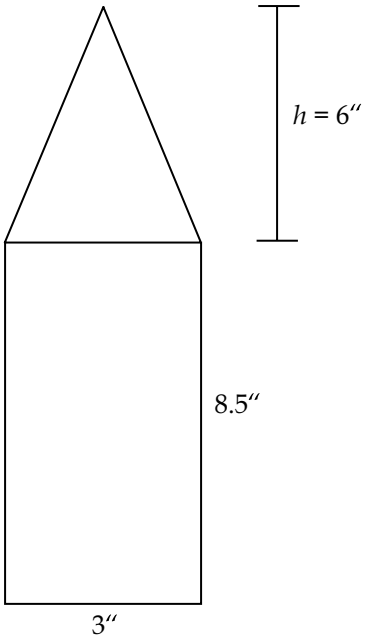
continued

Learning Activity 4.3 (continued)

c) parallelogram

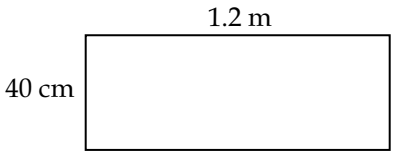


d) combined shapes



2. The area of grass over which you want to place a rectangular fire pit is 6 m^2. If the maximum width of the pit is 2.4 m, find its length.

3. Your friend had this problem marked as incorrect. Identify the error she made, and correct it.



$A = bh$

$= 1.2 \times 40$

$= 48\ m^2$

Circles

The formula for the area of a circle is $A = \pi r^2$.

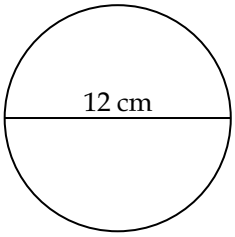
The value of π can be rounded to 3.14.

The radius is half the diameter.

The most common error students make here is to multiply before squaring. You must remember to follow the "order of operations" by performing the exponential operation first, and then multiplying by π.

Example 1

Find the area.



Solution:

This diagram shows a circle with a diameter of 12 cm. The radius would be half of that, or 6 cm. The area can be calculated.

$A = \pi r^2$

$= 3.14\ (6\text{ cm})^2$ (remember to do the squaring first)

$= 3.14\ (36\text{ cm}^2)$

$= 113.04\text{ cm}^2$

Example 2

You have a circular pond to place in your yard. The area of the pond is 12.56 m^2. Find the radius of the pond.

Solution:

You know the area and the formula for area. You substitute the value for the area into the formula and solve for the value of the radius.

$A = \pi r^2$

$12.56\text{ m}^2 = (3.14)\ (r^2)$

You need to isolate the r to find the radius.

You will divide both sides equally by 3.14 to isolate the r^2.

$$\frac{12.56 \text{ m}^2}{3.14} = \frac{(3.14)(r^2)}{3.14}$$

$$4 \text{ m}^2 = r^2$$

Now find the square root of each side to isolate r.

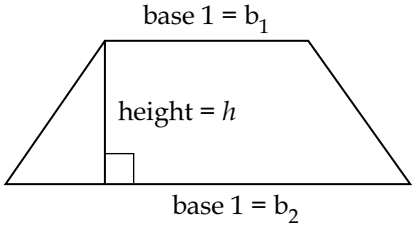
$$\sqrt{4 \text{ m}^2} = \sqrt{r^2}$$

$$2 \text{ m} = r$$

The radius of the pond is 2 m.

Trapezoids

Trapezoids are very interesting shapes. They have at least one set of parallel sides, which are of different lengths.



The trapezoid has two bases that are of different lengths. Also notice the height meets both bases at a right angle.

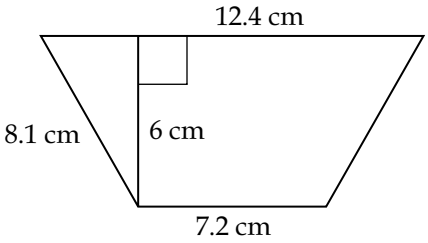
The formula for finding the area of a trapezoid is as follows.

$$A = \frac{1}{2}h(b_1 + b_2)$$

Note that the formula does not use the shorter base, nor the longer base, but rather the average length of the two bases: $\frac{b_1 + b_2}{2}$.

Example

Given the following trapezoid, find the area.



Solution:

You look for the height meeting a base at a right angle. The measurement for the height in this example is 6 cm.

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$= \frac{1}{2}(6 \text{ cm})(12.4 \text{ cm} + 7.2 \text{ cm})$$

$$= \frac{1}{2}(6 \text{ cm})(19.6 \text{ cm})$$

$$= \frac{1}{2}(117.6 \text{ cm}^2)$$

$$= 58.8 \text{ cm}^2$$

Learning Activity 4.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. In Home Economics, the teacher asks your group to make a double recipe of lasagna for the class, while other groups are making different parts of the meal. The recipe for lasagna calls for $\frac{3}{4}$ cup of Parmesan cheese. How much Parmesan cheese will you need if you are making a double recipe?
2. At your dad's office, the carpet is made out of squares. Each square is 2 feet by 2 feet. If your dad's cubicle is 5 squares deep and 5 squares wide, what is the area of your dad's cubicle in feet?
3. To get to Gimli it takes 1.25 hours. If you are travelling at 100 km/h the whole time, how far is Gimli?
4. A ticket to the baseball game is $12.50. You have to pay for parking at the game, which costs $5. Once you are in the ball park, you buy popcorn for $3.00, ice cream for $3.15, and a drink for $2.50. How much did it cost you to go to the baseball game?
5. Put the following numbers in order from smallest to largest: 0.53, 29%, 0.045, 0.13, 78%.

continued

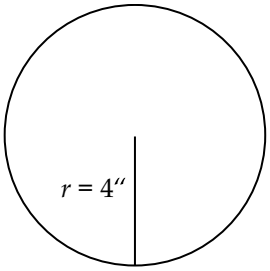
Learning Activity 4.4 (continued)

Part B: Areas of Regular Shapes (2)

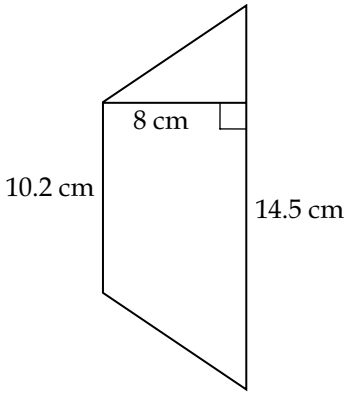
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the area of these shapes.

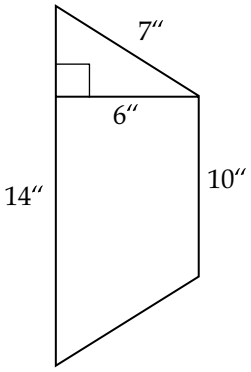
a)



b)

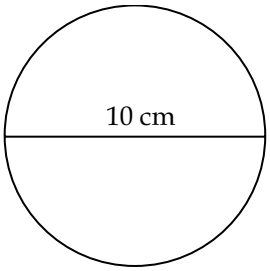


2. Identify the error in this calculation of area, and correct it.



$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(7)(14+10)$$
$$= \frac{1}{2}(7)(24)$$
$$= \frac{1}{2}(168)$$
$$= 84 \text{ in.}^2$$

3. Given a circle with a diameter of 10 cm, find its area.



Lesson Summary

This lesson dealt with finding the area of 5 shapes. A chart listing the formulas for finding the area of each shape was provided. A process for solving area questions included drawing the diagram, marking the measurements onto the diagram, ensuring all the measures were in the same unit, writing the formula, and solving the problem. The base-height connection is important in finding the area of all the shapes, except circles, of course. Your skills were applied to many problem-solving situations.

The next lesson discusses scale drawings.

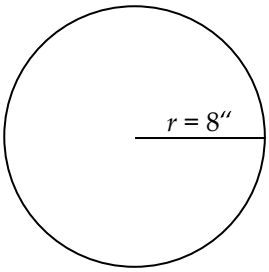
Assignment 4.2

Total: 17 marks

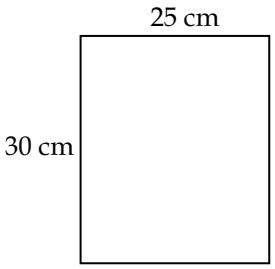
Area of Shapes

1. Find the area of each shape. *(8 marks)*

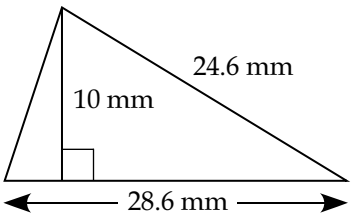
a)



b)



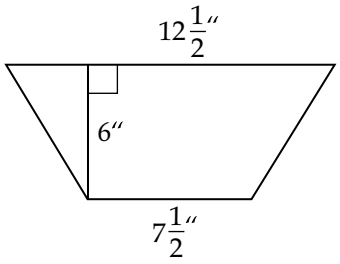
c)



continued

Assignment 4.2: Area of Shapes (continued)

d)



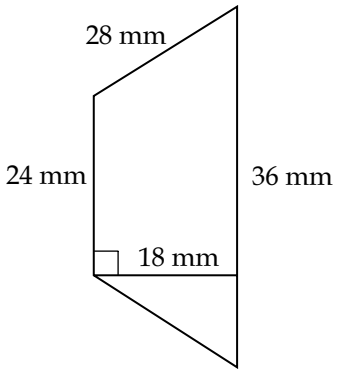
2. A triangular shape has an area of 64 cm^2. If the height is 8 cm, find the base. *(3 marks)*

3. A circular table has an area of 7850 cm^2. Find the radius. *(3 marks)*

continued

Assignment 4.2: Area of Shapes (continued)

4. Your friend had his work marked wrong. Identify the error, and correct it. *(3 marks)*



$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(28\text{ mm})(24\text{ mm} + 36\text{ mm})$$
$$= \frac{1}{2}(28\text{ mm})(60\text{ mm})$$
$$= \frac{1}{2}(1680\text{ mm}^2)$$
$$= 840\text{ mm}^2$$

Notes

Lesson 3: Scale Drawings

Lesson Focus

In this lesson, you will

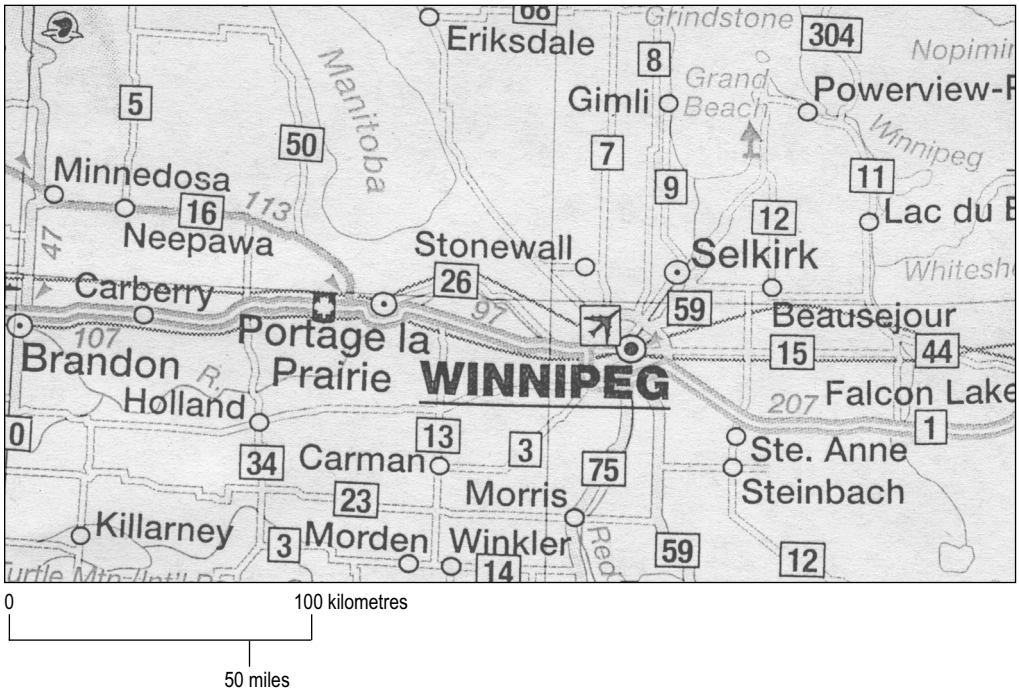
- ❑ solve problems requiring formulas for perimeter and area
- ❑ use scale diagrams to find distance, perimeter, and area

Lesson Introduction

Maps cannot be completely understood without some knowledge of the scale ratio. This lesson will focus on using scale drawings to find perimeter and area.

Drawing "To Scale"

Look closely at this map of the area surrounding Winnipeg. At the bottom left corner is the scale. You can use this scale to measure the distances between points.



Source: MapArt. *Canada Road Atlas*. Oshawa, ON: MapArt Publishing Corportion, 2009. Reproduced in accordance with *Access Copyright Elementary and Secondary Tariff*.

There are actually two scales of measurement on this map: one in metric measure, the other in imperial measure. The top line indicates the length of its line is equivalent to 100 km in actual distance. The bottom scale indicates the length of that line being equivalent to 50 miles in actual distance.

Example 1

Find the distance from Carman to Lac du Bonnet.

Solution:

Measure the length of the top scale as the length of one unit. Then measure the direct distance from Carman to Lac du Bonnet in terms of this unit. You don't need a ruler for this activity. You could just use the side of a piece of paper and mark it accordingly.

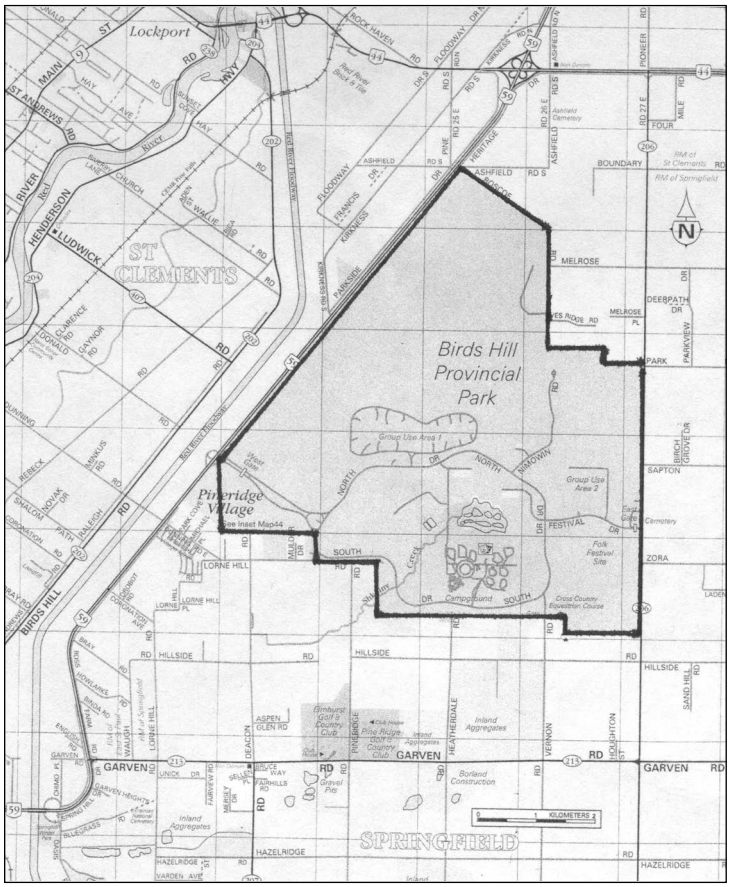
You will find the distance from Carman to Lac du Bonnet is approximately 1.7 units using the top scale as one unit. You read the scale on the map as 1 unit = 100 km, so 1.7 units would represent $1.7 \times 100 = 170$ km in distance if you went directly from Carman to Lac du Bonnet.

Using the bottom scale, the length of the scale is 1 unit and represents 50 miles. Then measure the distance from Carman to Lac du Bonnet using these units. You will see that the unit is repeated about 2 times, and would be a distance of $2 \times 50 = 100$ miles.

If you were driving from Carman to Lac du Bonnet, the highway distance would be approximately 200 km. The distance of 170 km that was measured in this example represents a straight-line distance.

Example 2

This map shows Birds Hill Provincial Park just north of Winnipeg. Find the width and length of the park using the metric scale. Use those measurements to find the area of the park.



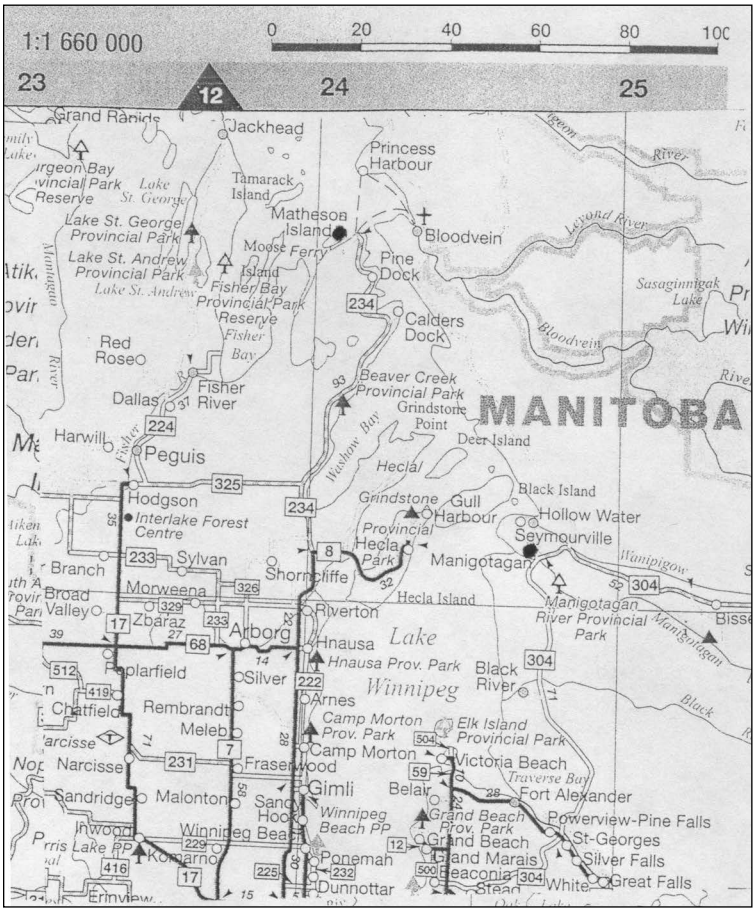
Source: Sherlock Publishing Ltd. *Sherlock's Map of Winnipeg* (9th Edition). Winnipeg, MB: Sherlock Publishing Ltd., 2004. Reproduced in accordance with *Access Copyright Elementary and Secondary School Tariff*.

Solution:

Using the metric scale, the top scale represents 2 km in real distance. Using rough measurements, there are about 3.6 units for width and 4 units for length. The actual distances would be 3.6×2 km = 7.2 km for width and 4×2 km = 8 km for length. To find the area of the park, the length multiplied by the width gives you an estimated area = $7.2 \times 8 = 57.6$ km^2. Manitoba Conservation lists the actual area of Birds Hill Park at 35.1 km^2. One possible explanation for the discrepancy between the two areas is that the perimeter of the park is irregular and measuring at the widest points for width and length will result in an area that is larger than the actual area.

Example 3

Find the actual straight line distance from Manigotagan to Matheson Island using the map below. The scale is in kilometres.



Source: MapArt. *Canada Road Atlas*. Oshawa, ON: MapArt Publishing Corporation, 2009. Reproduced in accordance with *Access Copyright Elementary and Secondary School Tariff.*

Solution:

The scale on this map is slightly different, in that the length of the scale is longer and more measurements are shown. You can hold a string or any object to find the distance between two places. Then lay the string along the scale to find the estimated distance.

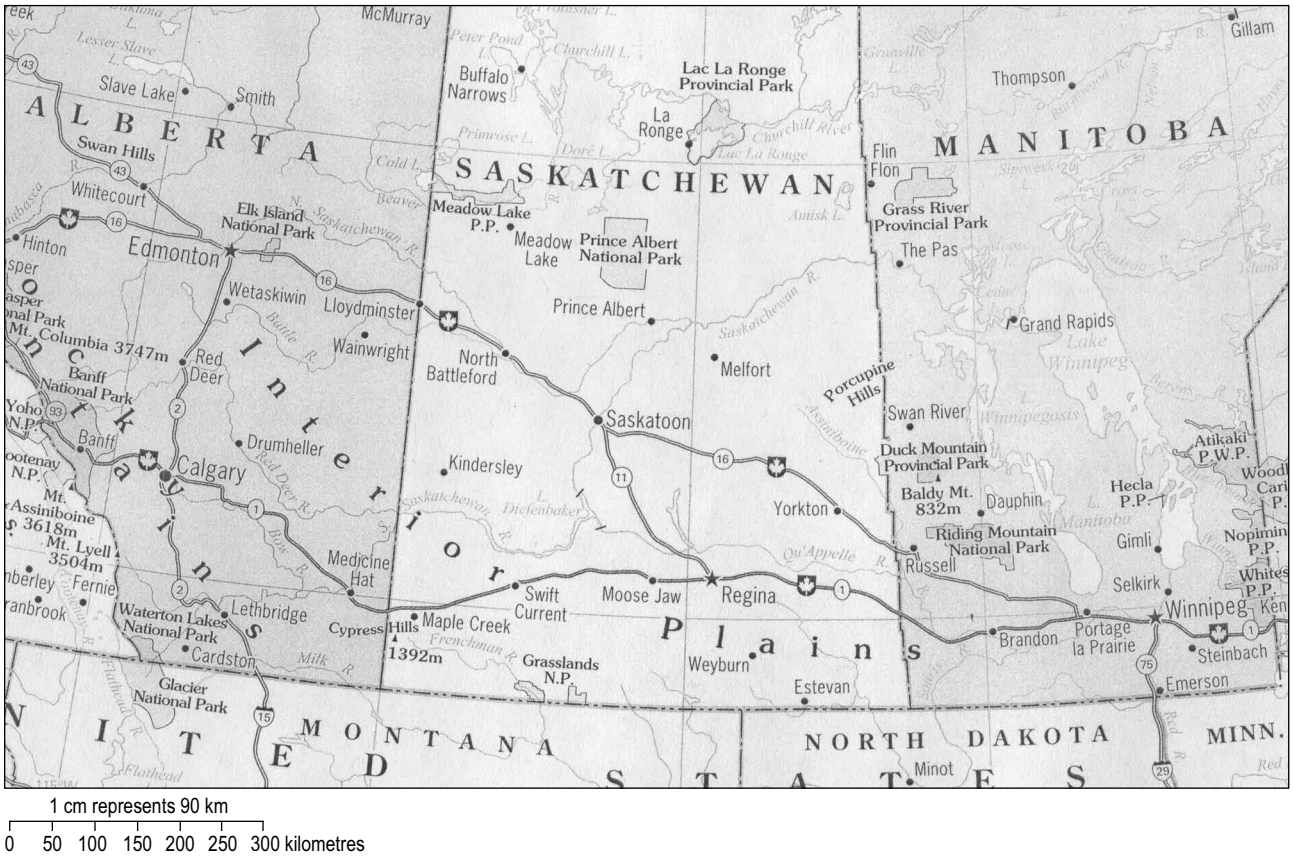
Manigotagan to Matheson Island is about 83 km.

Internet Fun

If you have access to the Internet, you can use "Google maps" to find the distance between places. Try to apply the scale and use ratio and proportion to estimate the distance, and then find the distance given by "Google" to see how close you were. http://maps.google.ca/maps

Example 1

Find the distance from Winnipeg to Calgary.



Source: Nystrom Division of Herff Jones, Inc. *The Nystrom Atlas of Canada and the World*. Chicago, IL: Nydtrom Division of Herff Jones, inc., 2003. Reproduced in accordance with *Access Copyright Elementary and Secondary School Tariff*.

Solution:

This scale shows roughly 1 cm represents 90 km. If you use a ruler and measure the distance from Winnipeg to Calgary, it is about 13.2 cm.

$90 \times 13.2 = 1188$ km from Winnipeg to Calgary

Note: Printing and copying of this diagram will likely change the length of the scale line and the map. If that is the case, just change the measurements in the calculation. You will get approximately the same answer.

Use ratio and proportion to estimate the amount of distance to Calgary.

1 cm	90 km
13.2 cm	x km

Notice how the columns contain the same units.

Now set up the proportion, cross multiply, and solve for x.

$$\frac{1 \text{ cm}}{13.2 \text{ cm}} = \frac{90 \text{ km}}{x \text{ km}}$$

$$(1)(x) = 13.2 \times 90$$

$$x = 1188$$

Thus, the distance from Winnipeg to Calgary is about 1188 km.

The actual distance from Winnipeg to Calgary is listed as 1326 km. Using the scale and measurement, the calculation closely estimated the actual distance.

Deciding on the Scale

If you are making a drawing, the scale that you use must be appropriate to the size of the drawing you want.

Example 1

If you are drawing a map showing the distances from Winnipeg to Regina, then to Saskatoon and back to Winnipeg, you will need a scale that allows the lines to fit within the measurements of your page.

Approximate distances
Winnipeg to Regina = 550 km
Regina to Saskatoon = 260 km
Saskatoon to Winnipeg = 800 km

What scale would you use? Draw a scale diagram to show the map.

Solution:

Perhaps the easiest scale to use for your diagram would be 1 cm : 100 km, so 1 cm on the map represents 100 km actual distance.

Using ratio and proportion, you can quickly determine the length of the lines in your drawing.

Winnipeg to Regina

1 cm: 100 km

x cm: 550 km

The line would be $\frac{550}{100} = 5.5$ cm long.

Regina to Saskatoon

1 cm: 100 km

x cm: 260 km

The line would be $\frac{260}{100} = 2.6$ cm long.

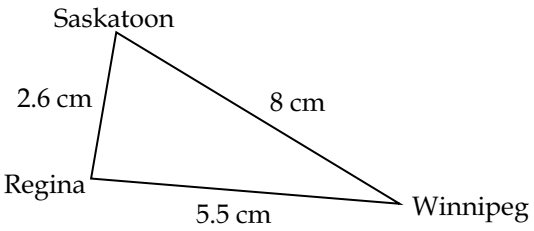
Saskatoon to Winnipeg

1 cm: 100 km

x cm: 800 km

The line would be $\frac{800}{100} = 8$ cm long.

Note: The map shown below is not exact. The distances on your diagram should be exactly correct.



Scale: 1 cm: 100 km

Notice that every scale diagram must include the scale.

Example 2

The Manitoba Legislative Building is 77 m tall. Find an appropriate scale to draw a picture of the building using no more than half a page.

Solution:

The length of a page of note paper is 28 cm. The maximum height of your scale drawing needs to be less than 14 cm. You can use "guess and check" to find the right scale for you.

Here are some sample scales.

1 cm: 1 m and the drawing would be 77 cm high

1 cm: 5 m and the drawing would be $\frac{77}{5} = 15.4$ cm high

1 cm: 10 m and the drawing would be $\frac{77}{10} = 7.7$ cm high

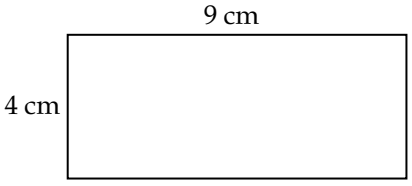
Of the three, the last scale would be the most appropriate. It would fit on the half page.

Perimeter and Area

You can use diagrams drawn to scale to find the actual perimeter and area of a shape.

Example 1

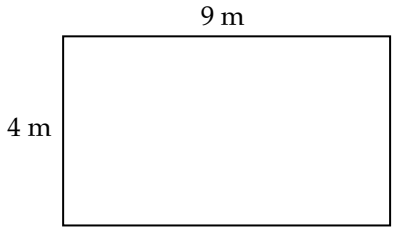
This is a drawing of a garden in your yard. Note the scale on the drawing. Use that scale to determine the actual measurements for perimeter and area of your garden.



Scale: 1 cm: 1 m

Solution:

Since 1 cm represents 1 m, then 4 cm represents 4 m, and 9 cm represents 9 m. A diagram with the actual measurements on it is shown below.

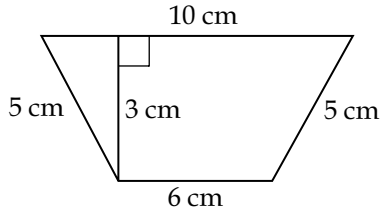


The garden would measure 4 m × 9 m. The perimeter would be two sides of 4 m each, and 2 sides of 9 m each for a total of 4 m + 4 m + 9 m + 9 m = 26 m.

The area would be 4 m × 9 m = 36 m^2.

Example 2

This trapezoid has a scale of 1 cm: 9 cm. Find its perimeter and area.

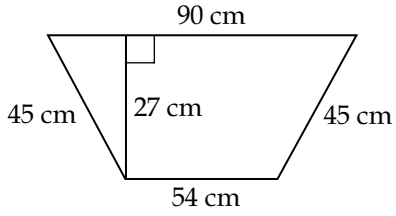


Scale: 1 cm: 9 cm

Solution:

Since the scale is 1 cm: 9 cm, then each measurement on the scale diagram needs to be multiplied by 9 to get the actual measurements.

It is a good idea to redraw your diagram with the actual measurements.



Perimeter: Distance around the outside = 45 cm + 90 cm + 45 cm + 54 cm = 234 cm

Area: $A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(27 \text{ cm})(54 \text{ cm} + 90 \text{ cm})$

$= \frac{1}{2}(27 \text{ cm})(144 \text{ cm})$

$= 1944 \text{ cm}^2$

Note that the answer could be converted into square metres. The number of square metres would be smaller than the number of square centimetres. In the next lesson, you will consider these types of conversions.

Learning Activity 4.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The mat used for floor gymnastics is a square. The length along one side is 40 feet. What is the total area of the mat?
2. A student receives a grade of $\frac{18}{21}$ on his test. Estimate the percent value.
3. Rewrite the following fraction in lowest terms: $\frac{12}{20}$.
4. You want to order takeout, but you only have $15. Your favourite takeout costs $16, but is 10% off. Can you afford it? (Taxes are included in the price.)
5. Your gross pay is $530. Your CPP deduction is $26 and your EI deduction is $10. You get your bonus of $60. How much is your net pay?

continued

Learning Activity 4.5 (continued)

Part B: Ratio and Scale

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the scale to find the actual distance in kilometres to fly from Grand Rapids to Portage la Prairie.

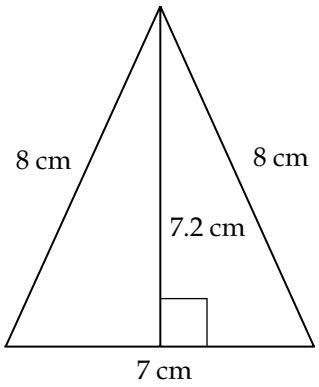
Source: Stanford, Quentin H. (Ed). *Canadian Oxford School Atlas*. Don Mills, ON: Oxford University Press, 2004. Reproduced in accordance with *Access Copyright Elementary and Secondary School Tariff*.

continued

Learning Activity 4.5 (continued)

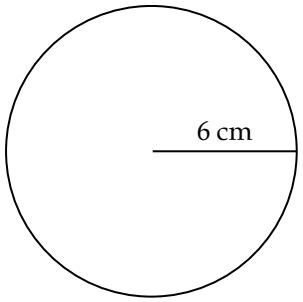
2. Find the actual perimeter and area of these shapes using the scales given in each part.

a)



Scale: 1 cm: 20 cm

b)



Scale: 1 cm: 50 cm

Lesson Summary

This lesson showed you how to use the scale on a drawing or a map to estimate distances and then find the actual perimeter and area. A diagram showing actual measurements was drawn from the scale diagram before solving problems. Also, you practised determining an appropriate scale when making your own scale diagram.

The next lesson deals with using a grid to estimate the area of odd shapes.

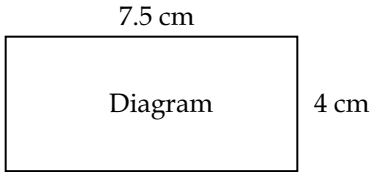
Assignment 4.3

Total: 19 marks

Scale Drawings

1. The scale for a map reads: 1 inch: 50 miles. If the distance between two points on the map measures 3.5", find the actual distance. *(2 marks)*

2. A rectangular garden plot is shown in this diagram with a scale of 1 cm: 75 cm. Find the area and perimeter of the actual garden plot. *(4 marks)*



continued

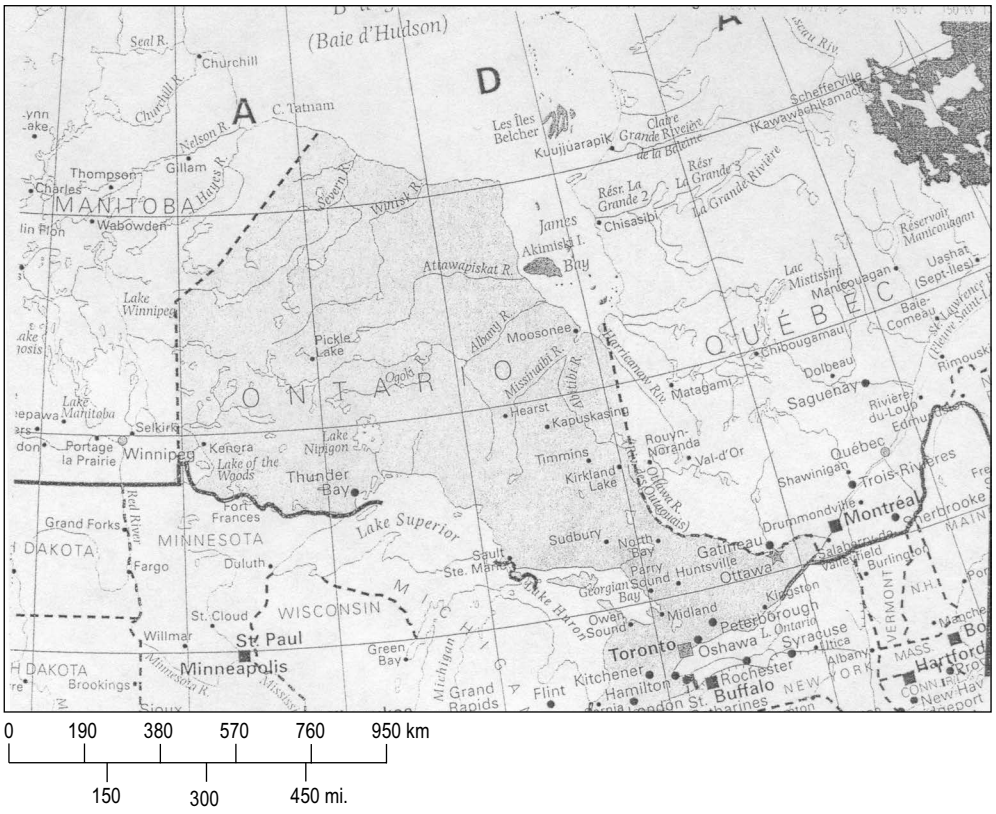
Assignment 4.3: Scale Drawings (continued)

3. You use a piece of string to find the direct distance between two points on a map. The length of the string is 4.8 cm long. The map scale is 2 cm: 100 miles. Find the direct distance between the points. *(2 marks)*

continued

Assignment 4.3: Scale Drawings (continued)

4. Given the following map and its scale at the bottom left corner, estimate the actual straight line distance between Winnipeg and Toronto in both imperial and metric units. *(2 marks)*



Source: Stanford, Quentin H. (Ed). *Canadian Oxford School Atlas*. Don Mills, ON: Oxford University Press, 2004. Reproduced in accordance with *Access Copyright Elementary and Secondary School Tariff*.

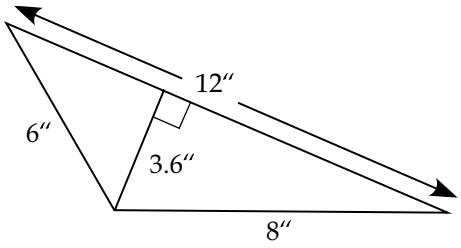
continued

Assignment 4.3: Scale Drawings (continued)

5. Explain why it would be wrong to use a scale of 1 cm: 10 km to draw a map showing the 550 kilometre distance between Winnipeg and Regina. *(1 mark)*

6. Find the actual perimeter and area of each shape drawn to scale. *(8 marks)*

a)



Scale: 1“: 2‘

b)

Scale: 10 mm: 1 m

Lesson 4: Area of Odd Shapes

Lesson Focus

In this lesson, you will

- ❑ solve problems that involve metric and imperial area measurements

Lesson Introduction

Not every shape is easy to measure. This unit shows you how to create a grid and use it to estimate the area of odd shapes.

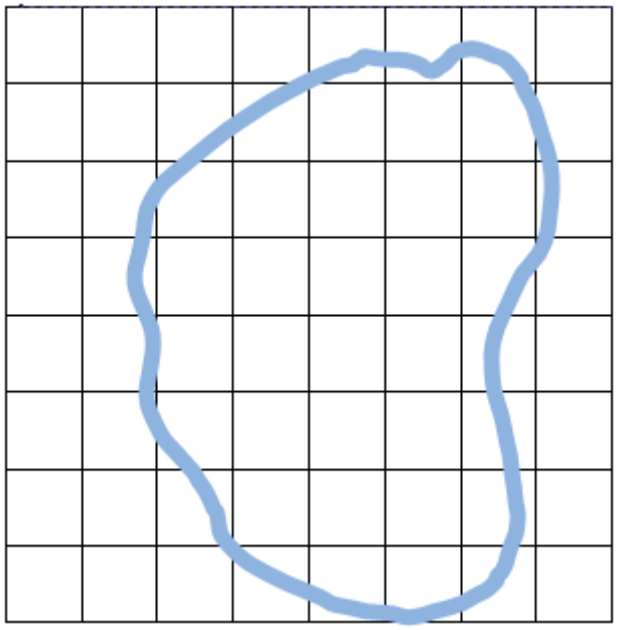
Not All Shapes Are Perfect

You can estimate the area of an odd-shaped lake using a grid where each square has dimensions of 1 unit by 1 unit and an area of $1 \times 1 = 1$ sq. unit. Four different methods are shown below.

Counting Inside

Example

Estimate the actual area of Math Lake using the scale diagram shown below and counting the squares inside the perimeter of the lake.

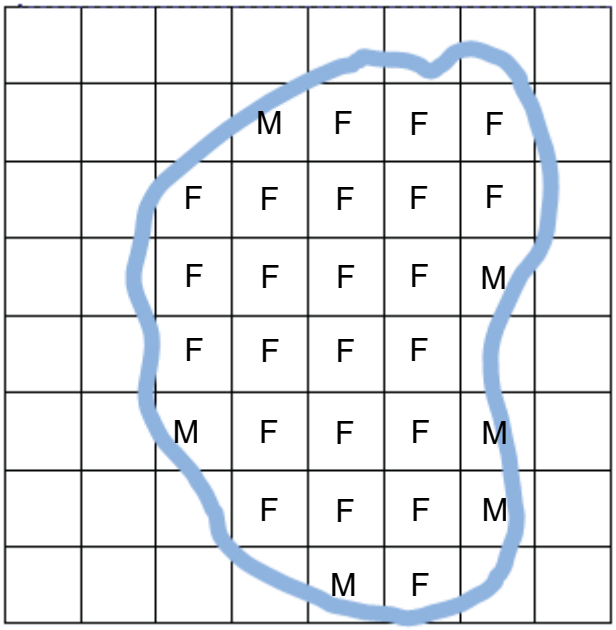


Scale: 1 unit × 1 unit: 500 m × 500 m

Solution:

This is Math Lake, and you want to find the area it covers. Notice that a grid has been placed over the diagram of the lake. The scale shows that 1 unit represents 500 m.

You count the squares that are completely filled (F) by the lake, and the ones partially filled. The partially filled squares can be separated into "more than half" (M) and "less than half" to get a better estimate.



Now count the squares.

Mostly filled squares (represented by F in the diagram) = 23 squares

About half-filled (represented by M in the diagram) = 6 squares

Less than half-filled = 10 squares

Considering two half-filled squares equal a full square, you could consider 6 half-filled squares to equal 3 full squares. The 10 squares less than half-filled could be considered as a quarter of a square each, so this represents $\frac{1}{4} \times 10 = 2.5$ full squares. The total area then would be 23 + 3 + 2.5 = 28.5 squares.

The area of each square = 500 m × 500 m, or 250 000 m^2. Multiply that by 28.5 squares, and you find the lake covers an actual area of 28.5 × 250 000 m^2 = 7 125 000 m^2.

Estimating Length and Width

You could also estimate the area of Math Lake by estimating the length and width of the lake. The length of the lake is about 6.5 squares, or 6.5 × 500 m = 3250 m. The width of the lake averages about 4.5 squares, or 4.5 × 500 m = 2250 m.

Using these estimates to find the actual area, you have an actual area of 3250 m × 2250 m = 7 312 500 m^2.

The two estimates for the area of Math Lake are relatively close.

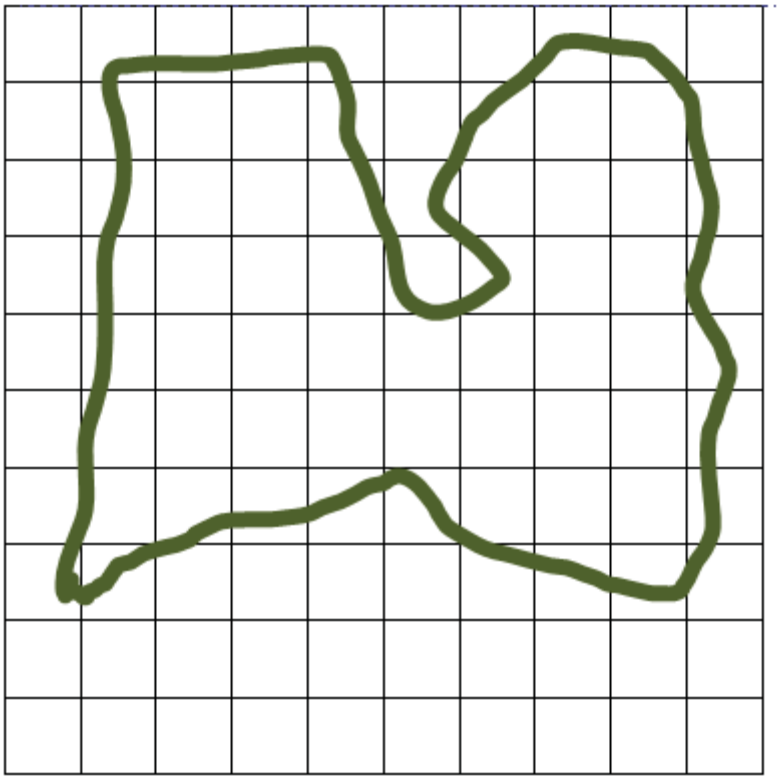
The next lesson teaches you how to convert this answer in square metres into square kilometres so that the resulting number for area is not so large.

Breaking into Smaller Pieces

Sometimes it is helpful to break the scale diagram of an object into smaller pieces and find the area of each piece.

Example

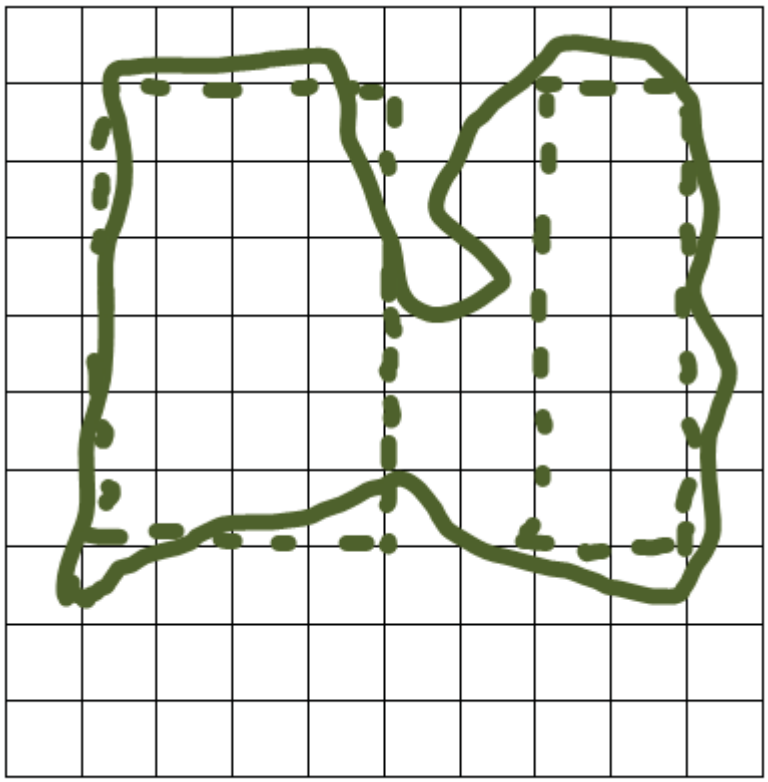
Here is a picture of a forested area. Estimate the actual area of the forest by dividing the forest into known shapes (rectangle, triangle, etc.).



Scale: 1 unit x 1 unit: 150 m x 150 m

Solution:

The picture of the forest seems to lend itself to being divided into three shapes: two rectangles on either side of the middle, and a leftover part between the two.



On the left side of the picture, the rectangular shape roughly measures 4 units by 6 units. When you apply the scale 1 unit: 150 m, the actual measurements are as follows:

Length = $4 \times 150 = 600$ m

Width = $6 \times 150 = 900$ m.

The area of the rectangular piece on the left is 600 m x 900 m = 540 000 m^2.

On the right side, you can see another rectangle imbedded in the diagram. This rectangle measures 2 units x 6 units. The actual dimensions are as follows:

Length = $2 \times 150 = 300$ m

Width = $6 \times 150 = 900$ m

The area for the rectangle on the right = 300 m $\times$ 900 m = 270 000 m^2.

The area in between the rectangles is odd shaped and can be found by counting squares. There are about 8 additional full squares. Since each square has an area of 150 m × 150 m = 22 500 m^2, the 8 additional squares cover an area of 8 × 22 500 m^2 = 180 000 m^2.

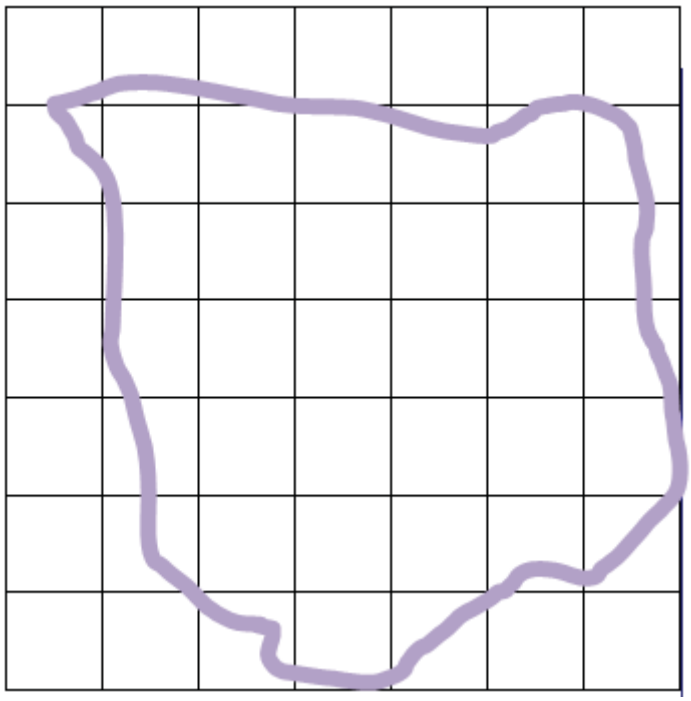
Adding the area of the two rectangular shapes and the additional squares from the middle gives you a total of 540 000 m^2 + 270 000 m^2 + 180 000 m^2 = 990 000 m^2

Subtracting from the Total Area

Another method of using the grid to estimate areas of odd shapes is to find the total area, and subtract the squares with nothing in them.

Example

This diagram shows the amount of damage on the hood of a car. Find its area by counting the empty squares outside of the damaged area and subtracting from the total number of squares.

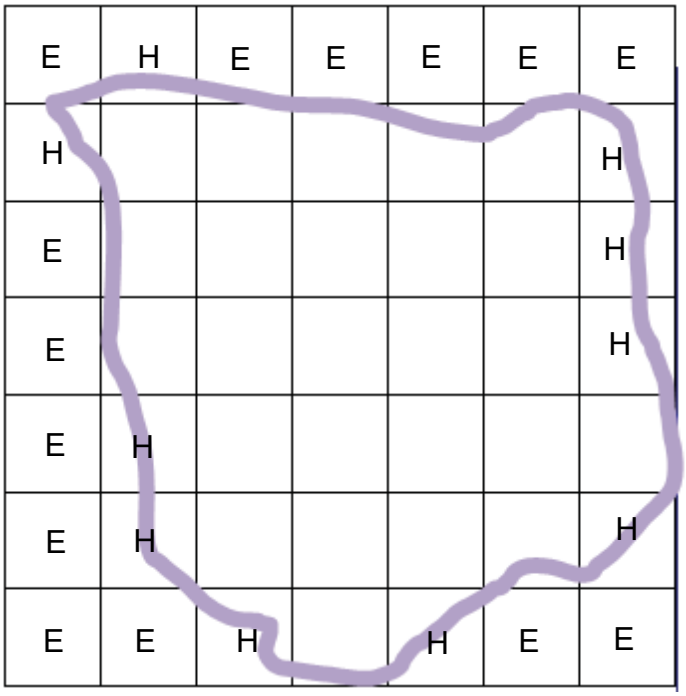


Scale: 1 unit × 1 unit: 6 inches × 6 inches

Solution:

Notice the scale means each square represents 6 inches in actual size. Each square has an actual area of 6 in × 6 in = 36 sq. in.

You count the number of empty squares, half-empty squares, and total number of squares to help you calculate the actual area.



Total number of squares = 7 × 7 = 49 squares.

Empty squares outside of the damage = 14 squares.

Half-empty squares outside of the damage = 10 squares

Total empty squares = $14 + 10 \times \frac{1}{2} = 14 + 5 = 19$ squares

Total number of full squares inside = 49 – 19 = 30 squares.

Since each square represents an area of 36 in.^{2}, an estimate of the actual area would be = 36 in.^{2} × 30 = 1080 in.^{2}.

Learning Activity 4.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Solve for q: $\frac{13}{q} = \frac{1}{3}$.
2. Write the ratio as a fraction: 1: 17.
3. You are buying a hot chocolate. The total is $1.78, but all you have is a $20 bill. How much change will you get back?
4. What bills and coins would you expect to get back if you bought the hot chocolate in Question 3?
5. Cheri works for an hourly wage of $12/hr. This week she worked 42 hours. She is paid time-and-a-half for all hours over 40 hours a week. How much money did she make on overtime?

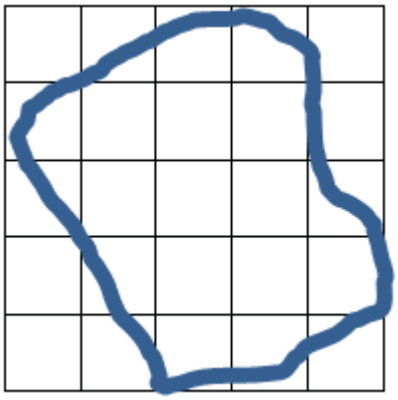
continued

Learning Activity 4.6 (continued)

Part B: Area of Irregular Shapes

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use any of the four methods learned in this lesson to estimate the actual area of the following diagram. Show how you arrived at your answer.

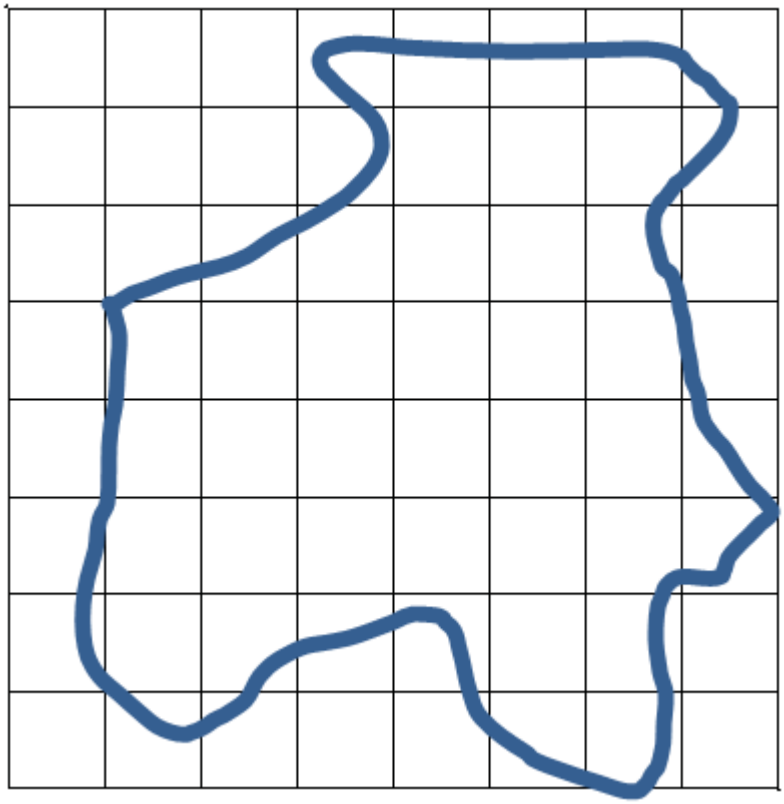


Scale: 1 unit × 1 unit: 2 m × 2 m

continued

Learning Activity 4.6 (continued)

2. Use any of the four methods learned in this lesson to estimate the actual area of the following diagram. Show how you arrived at your answer.



Scale: 1 unit x 1 unit: 3 feet x 3 feet

Lesson Summary

This lesson showed you how to use a grid with an imperial or metric scale to estimate the area of odd shapes in four different ways.

The next lesson teaches you to convert related units in both systems.

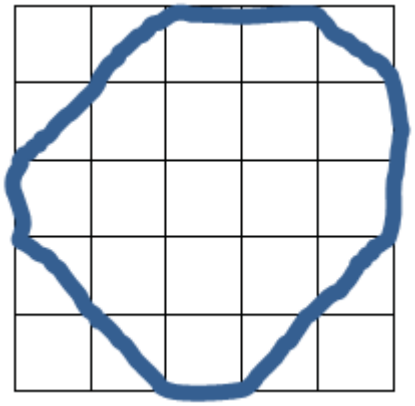
Assignment 4.4

Total: 15 marks

Area of Odd Shapes

1. Estimate the area of each shape. Show your calculations. *(9 marks)*

   a)

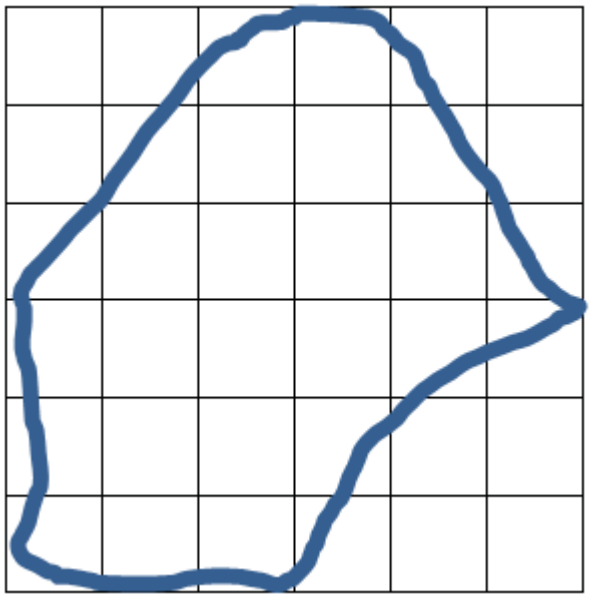


Scale: 1 unit × 1 unit: 50 cm × 50 cm

continued

Assignment 4.4: Area of Odd Shapes (continued)

b)

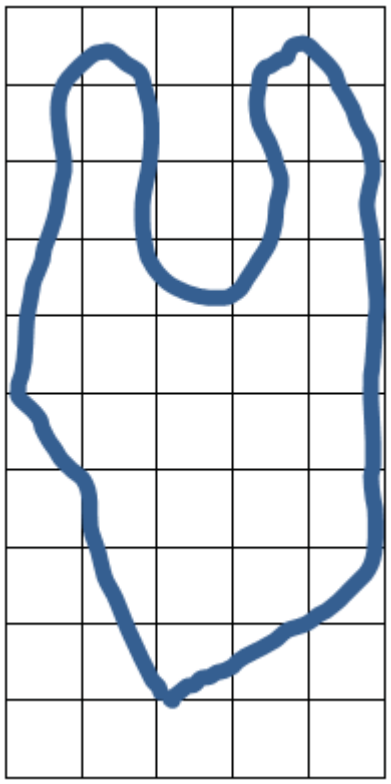


Scale: 1 unit × 1 unit: 1 foot × 1 foot

continued

Assignment 4.4: Area of Odd Shapes (continued)

c)



Scale: 1 unit × 1 unit: 2.5 m × 2.5 m

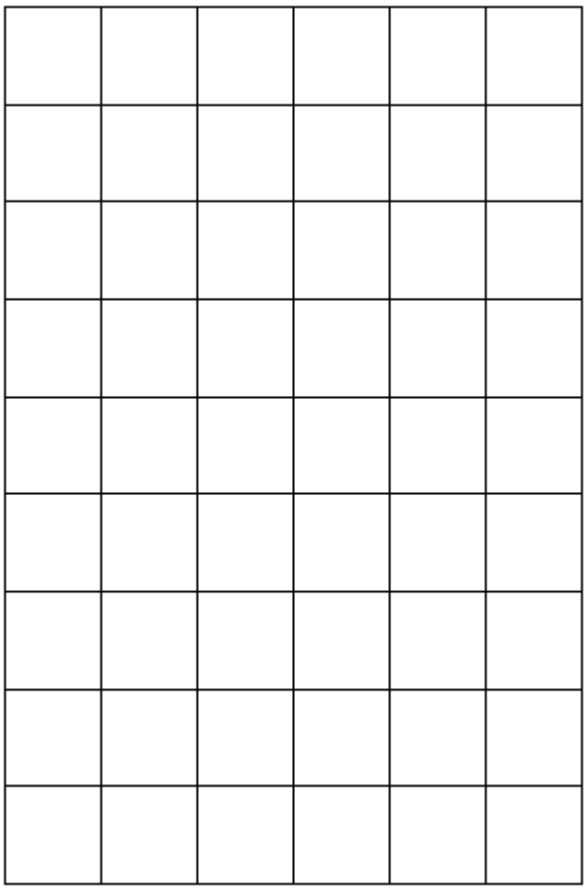
continued

Assignment 4.4: Area of Odd Shapes (continued)

2. You have discovered an area of your lawn that is infested with ants. The lawn measures 60′ by 90′. The area of the infestation is about 2000 ft.^{2}, and roughly shaped like an oval. (). Use the grid given below and a scale of 1 unit × 1 unit: 10 feet × 10 feet. *(6 marks)*

   a) Determine how many squares would be covered.

   b) Draw the area of infestation.



Scale: 1 unit × 1 unit: 10 feet × 10 feet

Lesson 5: Conversions

Lesson Focus

In this lesson, you will

- ❑ solve problems that involve metric and imperial area measurements

Lesson Introduction

This lesson teaches you to simplify square units within each measurement system.

Converting Units

Area Units Conversions

If the area of an object was described to you as being 1296 in.^{2}, you might have some difficulty imagining what it would look like. However, if that measurement were converted to its equivalent of 1 square yard, you would have a better chance of relating to the actual size of the object. Similarly, 10 000 cm^2 sounds like a huge area, but it is only 1 square metre.

You will have increased success with understanding these conversions if you establish a habit of making a diagram to illustrate the conversion.

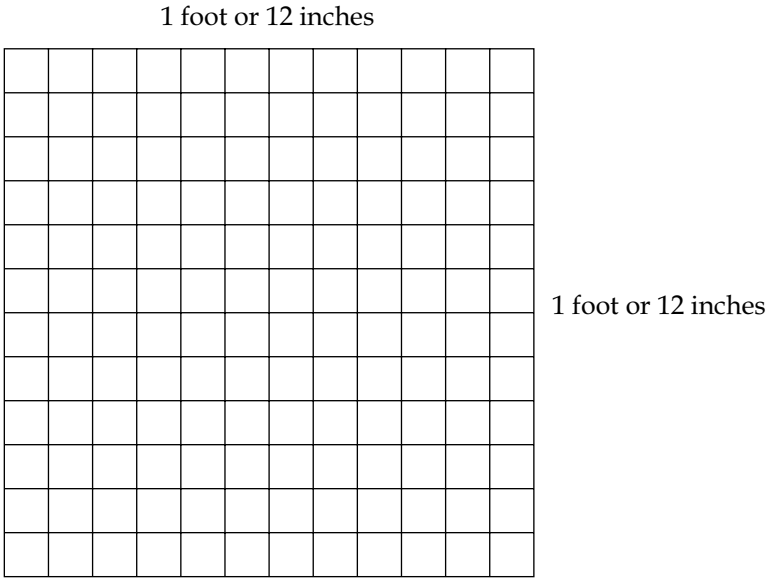
Example 1

Show a diagram to represent the number of 1-inch squares in a square measuring 1 foot by 1 foot.

Solution:

1 foot has 12 inches. A square measuring 1 foot by 1 foot would have 12 inches × 12 inches or 144 squares each having an area of 1 sq. in.

A diagram to show this relationship is shown below.



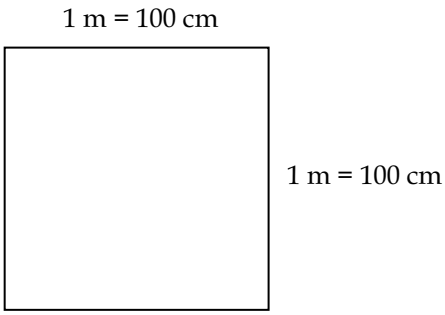
It is easier to visualize that 1 square foot is 144 square inches if you have sketched a quick diagram.

Example 2

Find the number of 1 cm squares in a square whose dimensions are 1 m by 1 m.

Solution:

This question is the same as finding the number of square centimetres in a square metre. To find the area of one square metre in square centimetres, simply sketch a square and label it. You don't really need to insert all the little squares to understand how many there should be.



Clearly, 1 m has 100 cm, so each dimension would be 100 cm. Then, area = length × width gives you 100 cm × 100 cm = 10 000 cm^2.

When converting among square units, you can understand that sketching a quick diagram will save you from making errors.

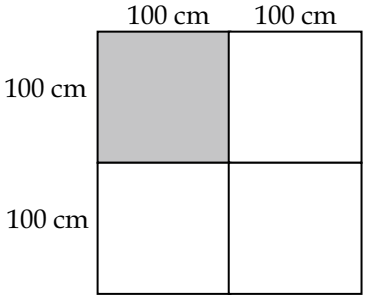
Example 3

If you know there are 10 000 cm^2 in a square 1 m by 1 m, how many square centimetres would there be in a square with its dimensions doubled?

Solution:

Your immediate thought might be to simply double the area of the original square to 20 000 cm^2.

A quick sketch will help to guide your thinking toward the correct answer. Note that when you double both dimensions, both the length and the width are doubled.



The sketch shows a square with dimensions of 200 cm × 200 cm. Its area would be 40 000 cm^2.

If you measure the dimensions in metres instead, then the new square has a length and width of 2 metres each, and its area would be 2 m x 2 m = 4 square metres.

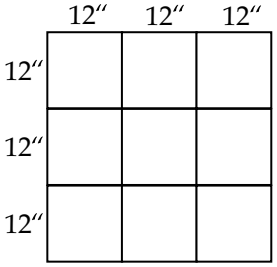
Thus, when the dimensions were doubled, the original area was quadrupled, and the number of square centimetres in the new square is 40 000 cm^2.

Example 4

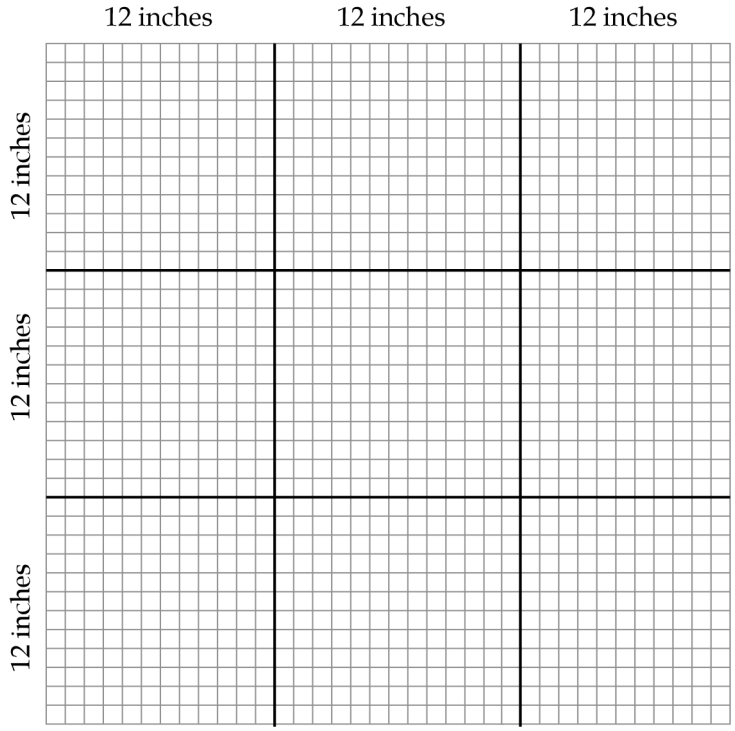
Find how many square inches there are in a square measuring 3 feet by 3 feet.

Solution:

A quick sketch will help find the correct solution.



If you were to draw all the 1 inch squares, you would have quite a few squares, as shown in the diagram below.



This sketch shows a square with dimensions 3 feet × 3 feet and area of 9 square feet.

Since each foot has 12 inches, the total dimensions of each side would be 36 inches.

The total area would be 36" × 36" = 1,296 $in.^2$.

Thus, 9 sq. ft. = 1,296 sq. in.

Since there are 3 feet in a yard, this question is the same as finding the number of square inches in a square yard. In other words,
1 sq. yd. = 1,296 sq. in.

Thinking in Reverse

Once you understand that there are 144 in.^{2} in a square foot, or 10 000 cm^2 in a square metre, you can think in reverse to answer a question like the one that follows.

Example 1

Convert 3,888 in.^{2} to square feet.

Solution:

Using proportional thinking, you can set up the relationships as follows.

1 sq. ft.	144 in.^{2}
x sq. ft.	3,888 in.^{2}

$$\frac{1}{x} = \frac{144}{3,888}$$

$$(144)(x) = (1)(3,888)$$

$$144x = 3,888$$

$$\frac{144x}{144} = \frac{3,888}{144}$$

$$x = 27$$

Thus, 3,888 in.^{2} = 27 ft.^{2}

Example 2

A shape has an area of 90 000 cm^2. Convert this measurement to square metres.

Solution:

1 m^2	10 000 cm^2
x m^2.	90 000 cm^2

$$\frac{1}{x} = \frac{10\,000}{90\,000}$$

$$(10\,000)(x) = (1)(90\,000)$$

$$10\,000x = 90\,000$$

$$\frac{10\,000x}{10\,000} = \frac{90\,000}{10\,000}$$

$$x = 9$$

Thus, 90 000 cm^2 = 9 m^2.

It would be helpful to include the conversions from $ft.^2$ to $in.^2$ and from m^2 to cm^2 on your resource sheet.

Example 3

A rectangle measures 145 cm by 60 cm. Find its area in square metres.

Solution:

To find its area in square metres, you can first convert each measurement to metres and then find the area, or you could find the area in square centimetres and then convert the answer to square metres.

Method 1: Convert the dimensions to metres first.

145 cm = 1.45 m

60 cm = 0.6 m.

Now you multiply to find the area.

Area = 1.45 m × 0.6 m = 0.87 m^2

Method 2: First, find the area.

145 cm × 60 cm = 8700 cm^2

Now, convert cm^2 to m^2

Since 10 000 cm^2 = 1 m^2, then 8700 cm^2 will be $\frac{8700}{10\,000}$ or 0.87 m^2.

This is the same answer as you found using the first method. Which method did you like better?

Learning Activity 4.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A 2-door sports car gets 12.2 km per L of gas. A truck gets 7100 m per L of gas. Which is more fuel efficient?
2. 35% of the trees in the forest of 400 have Dutch elm disease. How many trees have this illness?
3. Find the average: 2, 4, 6.
4. A clown needs two balloons to make a flower. If he makes a bouquet of a dozen flowers, how many balloons will he need?
5. You're waiting for your friend at a restaurant and it is 1:30 pm. He finally shows up, and claims he has been sitting at another table for 50 minutes. What time did your friend get to the restaurant?

Part B: Area Conversions (1)

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Sketch a diagram to help find the following values.
   a) The number of square inches in 2 square feet.
   b) The number of square metres in a square kilometre.
   c) The number of square centimetres in 3 square metres.
   d) The number of square inches in 2 square yards.

continued

Learning Activity 4.7 (continued)

2. Convert the following to the units indicated using proportional reasoning.
   a) $9ft.^2$ = _____ $in.^2$
   b) 4 m^2 = _____ cm^2
   c) 90 000 cm^2 = ____ m^2
   d) 576 $in.^2$ = ____ $ft.^2$
3. a) The cost of building a new home is roughly $110 per square foot. You have selected plans for a house that is 1,485 square feet in size. Find the cost to have this new home built.
   b) You only have $150,000 that can be devoted toward the building costs. How large a house in terms of square footage can you consider?

Conversions between Systems

When buying lumber, paint, or floor covering, you often must be able to convert from one system of measurement to the other. The relationships between the units of measure in the two systems are shown in the charts below.

Area Conversion Charts		
Imperial to Metric		
Imperial		Metric
1 $in.^2$		6.4516 cm^2
1 $ft.^2$	144 $in.^2$	0.0929 m^2
1 $yd.^2$	9 $ft.^2$	0.8361 m^2
1 acre	4840 $yd.^2$	4046.9 m^2
1 $mile^2$	640 acres	2.59 km^2
Metric to Imperial		
Metric		Imperial
1 cm^2	100 mm^2	0.1550 $in.^2$
1 m^2	10 000 cm^2	1.1960 $yd.^2$
1 hectare [ha]	10 000 m^2	2.4711 acres
1 km^2	100 ha	0.3861 $mile^2$

Include this table or the page number on your resource sheet so you can refer back to it.

Example 1

The dimensions of your room are 9′ × 12′. When you go to purchase carpet, you realize carpet is sold in units of square metres only. Using the conversion table and proportional reasoning, find the amount of carpet you need in square metres.

Solution:

Area of your room = 9 feet × 12 feet = 108 sq. ft.

1 $ft.^2$ = 0.0929 m^2

108 $ft.^2$ = 108 × 0.0929 m^2 = 10.03 m^2

You might have to purchase 11 m^2, since many stores will not cut less than full units.

Example 2

Using a metric measuring tape, you measure the size of your deck to be 6.4 m × 4.3 m. The lumber yard asks for the measurements in square yards. What is your response?

Solution:

First, find the area in square metres.

Area = 6.4 m × 4.3 m = 27.52 m^2

Now convert to square yards.

1 m^2 = 1.196 $yd.^2$

27.52 m^2 = 1.196 $yd.^2$ × 27.52 = 32.91 $yd.^2$

You will ask for 33 sq. yards of decking.

Example 3

You have decided to cover a bathroom wall with 1 inch square ceramic tile. The wall measures 3 m by 4.1 m. How many tiles must you purchase?

Solution:

Area of bathroom wall = 3 m × 4.1 m = 12.3 m^2

In the conversion table, there is not a direct conversion from m^2 to $in.^2$, only cm^2 to in^2. The m^2 will need to be converted to cm^2.

From the conversion table, 1 m^2 has 10 000 cm^2.

Thus, 12.3 m^2 would have 12.3 × 10 000 cm^2 = 123 000 cm^2.

Now change 123 000 cm^2 to $in.^2$

1 cm^2 = 0.155 $in.^2$

123 000 cm^2 = 123 000 × 0.155 $in.^2$ = 19,065 $in.^2$

You would need to purchase 19 065 of the 1 inch square tiles.

Online Conversion Calculators

There are many helpful Internet sites with conversion calculators. Enter, "area conversion calculator" into your search engine. Then simply select the units you are converting from, and then select the units to which you want to convert, and the online calculator produces the answer.

Learning Activity 4.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Jared lost 55% of his weight when he went on his submarine sandwich diet. He originally weighed 420 pounds. How much does he weigh now?
2. Your suitcase cannot exceed 26 kg when you are going on an international flight. Your cousin weighs around 25 kg. Would they be a good referent?
3. At the gas station, the price posted is $1.20 per L. Your fuel tank holds 40 L. How much is it going to cost to fill up your car?
4. On Monday there are half as many customers at a restaurant as there are on Saturday. If there were 590 customers on Monday, how many customers were there on Saturday?
5. Evaluate: $\frac{2}{7}+\frac{6}{14}$.

continued

Learning Activity 4.8 (continued)

Part B: Area Conversions (2)

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert the following measurements as indicated using the area conversions chart. If you haven't done so already, now might be a good time to update your resource sheet to include the page number where the chart can be located for handy reference.

   Round off your final answer to 2 decimal places.

   a) $3\frac{3}{4}$ yd.^{2} to m^2

   b) 86 cm^2 to in.^{2}

   c) 940 in.^{2} to m^2

   d) 72 square miles to km^2

   e) 3256 cm^2 to ft.^{2}

2. You decide to buy new carpet for your living room. The room measures 4 m by 6.3 m. The store you visit only sells carpet by the square yard. How many square yards of carpet will you need?

Lesson Summary

This lesson showed you how to easily convert square units by drawing a diagram or using a chart. Conversions from metric to imperial or imperial to metric were discussed. A chart was provided for easy reference. Your skills were used to solve area problems.

The next lesson discusses dimension changes and includes solving problems.

Notes

Assignment 4.5

Total: 17 marks

Conversions

Round off your answers to two decimal places.

1. Convert the following area measurements to the units indicated. *(7 marks)*

   a) 288 $in.^2$ = _____ $ft.^2$

   b) 8 km^2 = _____ m^2

   c) 3 $yd.^2$ = _____ $ft.^2$

   d) 2.6 m^2 = _____ cm^2

   e) 4.5 $ft.^2$ = _____ $in.^2$

continued

Assignment 4.5: Conversions (continued)

f) $1.2\ km^2$ = _____ cm^2

g) $5\frac{1}{4}\ yds.^2$ = _____ $in.^2$

2. Sally is building a patio that measures 3.6 m by 5.8 m. She wants to use patio blocks that are 12 inch squares. How many blocks must she purchase? *(3 marks)*

continued

Assignment 4.5: Conversions (continued)

3. Convert the following measurements to the units indicated. *(7 marks)*

   a) 464 cm^2 = _____ $in.^2$

   b) 6.4 $yd.^2$ = _____m^2

   c) 962 km^2 = _____$mile^2$

   d) 645 $in.^2$ = _____cm^2

   e) 5296 $in.^2$ = _____m^2

continued

Assignment 4.5: Conversions (continued)

f) $8.4\ m^2$ = _____$ft.^2$

g) $48\ miles^2$ = _____m^2

LESSON 6: PROBLEMS

Lesson Focus

In this lesson, you will

- ❑ solve problems that involve metric and imperial measurements
- ❑ solve problems that require the manipulation and application of formulas

Lesson Introduction

This is the last lesson before you write the midterm. This lesson presents various problems for you to solve, using skills you have learned throughout the first four modules of the course.

When solving a problem, you could think through the content from all the previous lessons and apply the necessary skills to find your solution.

Problem Solving

Problem-Solving Steps

The ability to solve problems is very necessary. Once you have formulas and charts and the skills to work with them, you learn to use them in the process of solving problems.

When presented with a problem to be solved, the first step is to read the problem carefully, and to be sure you understand the question. Usually, it helps to draw a diagram and label all the parts given in the problem.

The next step is to go through the previous lessons to find the appropriate formula or chart. If you have your resource sheet up to date, you can also use it as a handy reference guide.

Once you have written the formula appropriate to the problem, carefully substitute in the known values into the formula, and find your answer.

The final step in the process of solving problems is to ask yourself whether the answer you found makes sense.

Something as simple as estimating how much wrapping paper is needed to wrap a present demands estimating skills. If you cut off too little paper, the end result is a mess. Cutting off too much paper is wasteful.

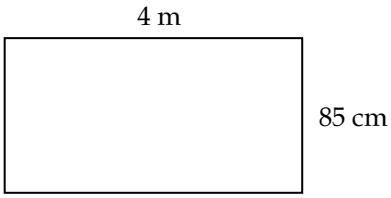
Solving Problems

Example 1

The small area of your yard requiring new sod measures 85 cm by 4 m. When you go to purchase the sod, the garden centre brings out a huge trailer with 340 m^2 of sod stacked on it, and presents you a bill for over $2000. What went wrong?

Solution:

Draw the rectangle with given units marked on it.



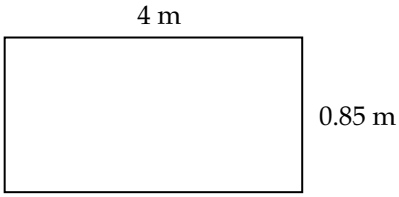
Area = $4 \times 85 = 340$ m^2

Why would this be the wrong answer?

The numbers are correct, but the calculation of the area has an error. When finding area, you must have the same units for the length and the width.

You need to draw a diagram of the rectangle with 85 cm changed to metres.

85 cm = 0.85 m



Now the area can be calculated correctly.

Area = 0.85 m $\times$ 4 m = 3.4 m^2

The charge for this amount of sod is only about $20.00. That makes much more sense to you and your budget.

Example 2

A bag of lawn fertilizer has a label that says there is enough fertilizer to cover 4000 sq. ft. Explain how you might estimate how many bags of fertilizer you will need to buy.

Solution:

First, find out the dimensions of your lawn by pacing out the length and the width.

If your lawn measures 12 paces by 15 paces, then you can estimate the area of your lawn to calculate how many bags of fertilizer to buy.

Your referent for pacing is 1 pace = 3 feet.

12 paces = 12 × 3 = 36 feet

15 paces = 15 × 3 = 45 feet.

Area = length × width = 36 feet × 45 feet

Since you are estimating, you change 36 feet to 40 feet (a bit too high) and 45 feet to 40 feet (bit too low).

An estimate of the area is 40 feet × 40 feet = 1600 square feet.

Since one bag covers 4000 sq. ft., one bag will be more than enough for your lawn.

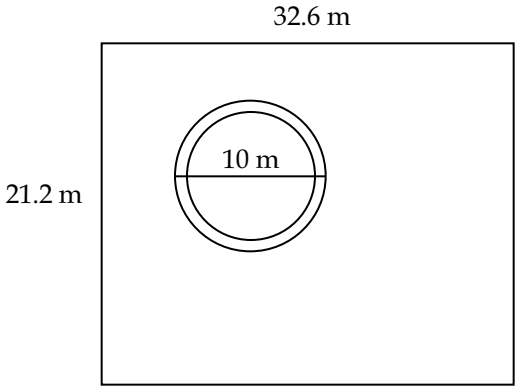
Example 3

You have a circular pool in your rectangular yard. The pool is 8 m across, and you want to install a walkway, whose width is 1 metre, around the pool. If your yard measures 21.2 m by 32.6 m, find the area of the yard requiring sod. Find the total cost of the sod if it costs \$3.25/m^2 plus PST of 7% and GST of 5%.

Solution:

How do you solve this problem? The first step is to draw a diagram.

The diameter of the pool is 8 metres. Once the walkway is added, the diameter will be 1 metre more on each side. The total diameter of the pool and walkway will be 10 metres. A diagram of your yard is shown below.



If you can find the area of the pool and walkway and subtract it from the area of your yard, then you will have the area of the sod you need to buy.

The area of the pool and the walkway require the formula for the area of a circle.

Since the diameter is 10 m the radius is 5 m.

Area of circle: $A = \pi r^2$

$= 3.14\ (5\ \text{m})^2$

$= 78.5\ \text{m}^2$

You need to subtract this amount from the area of the yard. The answer will give you the required area for the sod.

Area of yard: $A = bh$

$= 21.2\ \text{m} \times 32.6\ \text{m}$

$= 691.12\ \text{m}^2$

Area to be covered with sod = Area of your yard minus area of pool and walkway.

Area = $691.12\ \text{m}^2 - 78.5\ \text{m}^2 = 612.62\ \text{m}^2$

Since sod is sold by the square metre, you will need to buy $613\ \text{m}^2$.

Cost of the sod is $\$3.25/\text{m}^2$ plus PST and GST in taxes.

Cost = $613\ \text{m}^2 \times \$3.25 = \$1992.25$

GST = 5% of cost = $0.05 \times \$1992.25 = \$99.61$

PST = 7% of cost = $0.07 \times \$1992.25 = \$139.46$

Total cost of the sod = $\$1992.25 + \$139.46 + \$99.61 = \$2231.32$

Example 4

You measured your rectangular-shaped living room for a new carpet. It measured 9′ by 12′. The bill came to $484.30 before taxes. How much did the store charge per square metre?

Solution:

Area of carpet = length × width

$A = 9' \times 12' = 108$ square feet

To convert to square metres, you need the conversion rule,
1 sq. ft. = 0.0929 m^2.

Thus, 108 sq. ft. = 108 × 0.0929 = 10.0332 m^2.

Note that this number is not rounded. Round off only your final answer in order to get a better approximation in the exact final answer.

The bill was $484.30 for 10.0332 m^2.

You can make a formula and then solve for the answer.

Total cost of carpet = (cost per m^2) × (number of m^2 of carpet)

Substitute the values you know into the formula, and solve for the unknown value.

$\$484.30 = (c) \times (10.0332)$

Divide both sides by 10.033 2 to isolate c.

$$\frac{484.30}{10.0332} = \frac{(c)(10.0332)}{10.0332}$$

$$48.27 = c$$

The store charged $48.27 per square metre.

Example 5

Contractors use estimating skills almost daily as they quote prices on jobs. Many times contractors will just "pace off" the length and width to find area, instead of using a measuring tape. And their estimates are usually very accurate.

A concrete driveway contractor knows that "about" 81 square feet of area will require 1 cubic yard of concrete. If your parking pad has an area of 320 square feet, how many cubic yards of concrete are needed?

Solution:

The contractor knows about $\frac{320}{81} = 4$ cubic yards of concrete are needed.

Learning Activity 4.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Solve for z: $3 \times 7 = z - 4$.
2. Complete the pattern: –1, 0, 1, 0, ___ , ___ , 1.
3. Samantha has twice as many dresses as she has shirts, and three times as many shirts as she has pants. If Samantha has 12 shirts, how many pairs of pants does she have?
4. The radius of a globe is 5 cm. The ratio comparing the size of the globe to the earth is 1 cm: 800 miles, what is the approximate radius of the earth?
5. What is your gross pay if you work for 28 hours at $15 per hour?

Part B: Solving Problems

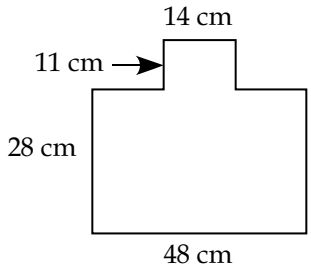
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You are having your rectangular driveway poured with new concrete. The contractor informs you that he will order 6 cubic yards of concrete. He is using his formula that 81 ft.^{2} of driveway requires 1 cubic yard of concrete. You know your driveway is 12 feet wide. How long is it?

continued

Learning Activity 4.9 (continued)

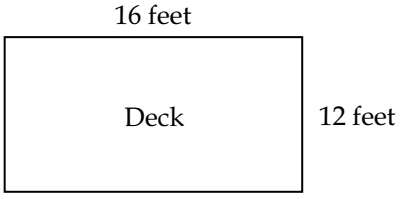
2. A mirror in your house has the shape and measurements shown in the diagram.



   a) If you had a piece of moulding 2 m long, how much of it would be left after you placed moulding all around the mirror?

   b) Find the area of the wall surface that the mirror will cover.

3. You have just built a lovely wooden deck measuring 12′ by 16′. Your neighbour offers you a can of stain. The instructions on the can suggest that the stain will cover 18 m^2. Do you have enough stain to finish the entire deck? Show your calculations.

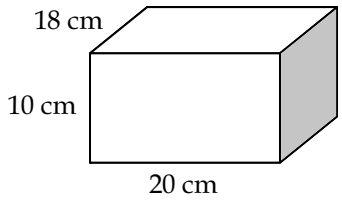


4. Your basement measures 11′ × 24′. You purchased carpet to cover the floor, and the bill came to $950.40, before taxes.

   a) Find the cost per square metre.

   b) Given PST is 7% and GST is 5%, find the total cost.

5. You have a present to wrap and are not sure if you have enough paper. The box measures 20 cm by 18 cm by 10 cm.



   a) Find the total surface area of the box you need to wrap.

   b) The roll of wrapping paper is 80 cm across, and has a length of 30 cm of paper on it. Will you have enough paper?

Lesson Summary

This lesson concluded the first half of your course with a lesson on problem solving. You practiced the skills you have learned. Now would be a good time to complete your resource sheets for each module and then summarize them to make your Midterm Examination Resource Sheet. The only information provided on the examination are the conversion charts and the payroll deduction tables, as shown in the introduction to the course. Any formulas you haven't memorized should be written on your resource sheet for the examination.

Just a reminder to you to make sure you have a geometry set, including a protractor, ruler and a set square, and a scientific calculator available for the second half of the course.

Assignment 4.6

Total: 20 marks

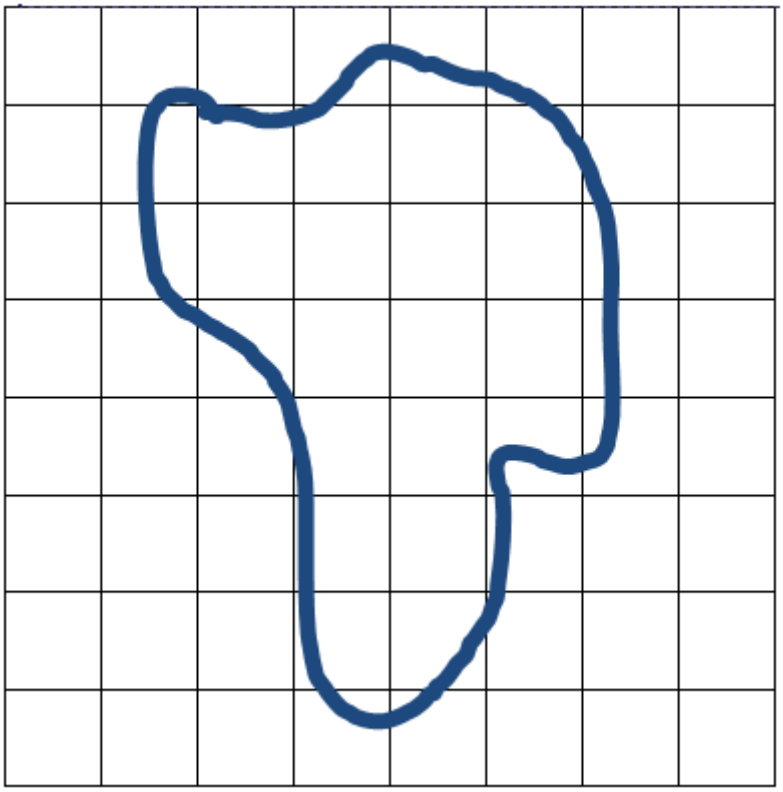
Problems

1. You have a 9′ by 12′ tarp. Will it be large enough to completely cover a circular pond with a radius of 2 m? *(4 marks)*

continued

Assignment 4.6: Problems (continued)

2. A grid is laid over an aerial photograph of a lake. If the scale of the grid is 1 unit × 1 unit: 500 yards × 500 yards, find the area of the lake in square miles. *(4 marks)*



Scale: 1 unit × 1 unit: 500 yards × 500 yards

continued

Assignment 4.6: Problems (continued)

3. You are planning to paint 3 walls in your room. They are 8′ high. Two walls are 10′ wide, and the third wall is $8\frac{3}{4}'$ wide. The window in one of the walls measures 3′ by 2′.

   a) Find the area to be painted. *(3 marks)*

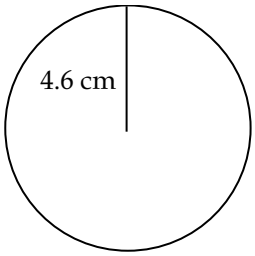
   b) If a can of paint covers 22 m^2, will one can of paint be enough to paint your room? *(2 marks)*

continued

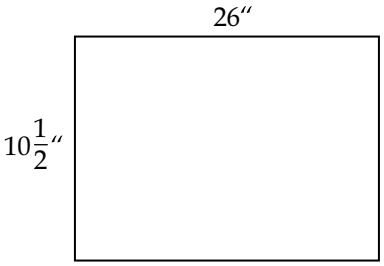
Assignment 4.6: Problems (continued)

4. Find the area of each shape in the units required. Round off your answers to 1 decimal place.

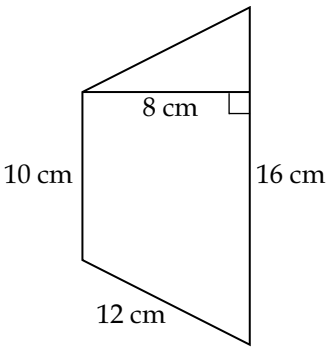
   a) Area in cm^2 *(2 marks)*



   b) Area in $ft.^2$ *(3 marks)*



   c) Area in cm^2 *(2 marks)*



Module 4 Summary

In this module, you practised your multiplication skills with fractions and decimals, and your estimating skills to ensure you had a reasonable answer. You found the area of 5 basic shapes, some shapes that combined the basic shapes, and some irregular shapes. You looked at scale diagrams and also learned how to convert area units. You applied your problem-solving skills to many practical problems.

Congratulations! You have finished the first four of the modules in this course.

Vocabulary

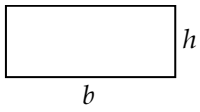
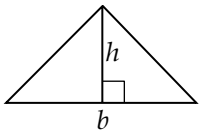
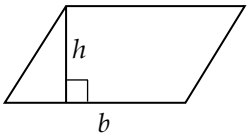
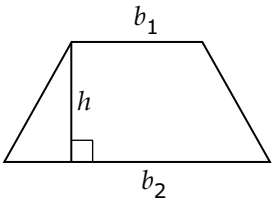
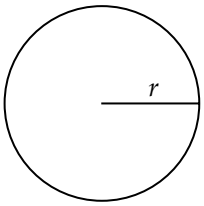
Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful.

accuracy
area of an odd shape
circle
common denominator
diameter
dimensions
estimate
geometry
multiplication grid
oval
parallelogram
perimeter
perpendicular lines
proper fraction
radius
rectangle
right angle
scale drawing
square root
square units
surface area
trapezoid
triangle

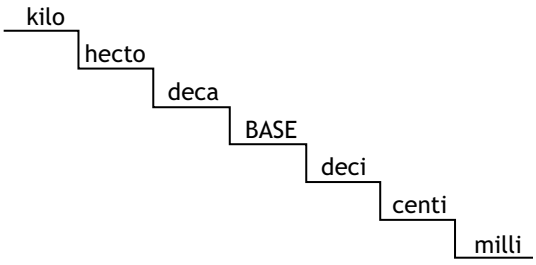
Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

Formula Chart for Area

Name of Formula	Diagram	Formula
Rectangle		$A = bh$
Triangle		$A = \frac{1}{2}bh$
Parallelogram		$A = bh$
Trapezoid		$A = \frac{1}{2}h(b_1 + b_2)$
Circle Area		$A = \pi r^2$ use $\pi = 3.14$

Metric Prefix Ladder



Area Conversion Charts

Imperial to Metric		
Imperial		Metric
1 $in.^2$		6.4516 cm^2
1 $ft.^2$	144 $in.^2$	0.0929 m^2
1 $yd.^2$	9 $ft.^2$	0.8361 m^2
1 acre	4840 $yd.^2$	4046.9 m^2
1 $mile^2$	640 acres	2.59 km^2
Metric to Imperial		
Metric		Imperial
1 cm^2	100 mm^2	0.1550 $in.^2$
1 m^2	10 000 cm^2	1.1960 $yd.^2$
1 hectare [ha]	10 000 m^2	2.4711 acres
1 km^2	100 ha	0.3861 $mile^2$

Submitting Your Assignments

It is now time for you to submit the Module 4 Cover Assignment and Assignments 4.1 to 4.6 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 4 assignments and organize your material in the following order:

Module 4 Cover Sheet

Module 4 Cover Assignment: Tower of Hanoi

Assignment 4.1: Multiplication Skills

Assignment 4.2: Area of Shapes

Assignment 4.3: Scale Drawings

Assignment 4.4: Area of Odd Shapes

Assignment 4.5: Conversions

Assignment 4.6: Problems

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Midterm Examination

Congratulations, you have finished Module 4 in the course. The midterm examination is out of 100 marks and worth 12.5% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 4.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Midterm Examination Resource Sheet. A maximum of 2.5 hours is available to complete your midterm examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point you will also have to combine your resource sheets from Modules 1 to 4 onto one 8½" × 11" paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you. We suggest that you divide your paper into two quadrants on each side so that each quadrant contains information from one module.

This is a sample:

Front

Module 1
Module 2

Back

Module 3
Module 4

Examination Review

You are now ready to begin preparing for your midterm examination. Please review the content, learning activities, and assignments from Modules 1 to 4.

The midterm practice examination is also an excellent study aid for reviewing Modules 1 and 4.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the midterm examination determines 12.5% of your final mark in this course and you will have 2.5 hours to complete the examination.

Midterm Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The midterm practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your midterm practice examination, follow these steps:

1. Study for the midterm practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 1 to 4 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 4, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Midterm Examination Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following to the midterm practice examination: pens/pencils (2 or 3 of each), blank paper, a scientific calculator, and your Midterm Examination Resource Sheet.
7. Write your midterm practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the midterm examination is 2.5 hours.

8. Once you have completed the entire examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Midterm Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes

Grade 10 Essential Mathematics (20S)

Module 4
Geometry

Learning Activity Answer Keys

Module 4:
Geometry

Learning Activity 4.1

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Rewrite the following fraction in lowest terms: $\frac{24}{52}$.
2. You have measured your foot to be 9.5″. Is your foot a good referent to approximate a foot in length?
3. You are paid a salary of $960 every two weeks. If you work a total of 96 hours in that time, what is your hourly wage?
4. Solve for p: $p \div 5 = 15$.
5. You want to meet your friends for coffee. You are available from 9 am to 3 pm. Aiden is free from 12 pm until 2 pm. Leah is able to come from 10 am until 1 pm. What time can you meet for coffee?

Answers:

1. $\frac{6}{13}\left(\frac{24 \div 4}{52 \div 4}\right)$
2. No, it is not close enough to a foot (12″).
3. $10 per hour (960 ÷ 96 = 10)
4. $p = 75$ (Multiply by 5 on both sides.)
5. 12 pm to 1 pm (Process of elimination: ~~9~~ ~~10~~ ~~11~~ 12 1 ~~2~~ ~~3~~)

Part B: Estimating with Decimals

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. How would you estimate this answer before doing the calculation?

   6.8×12.3

   Answer:

   You can round 6.8 up to 7, and 12.3 down to 12. Then multiply 7×12 to get an estimate of 84. The answer should be "about" 84.

2. Is it reasonable for 8 pairs of jeans priced at $29.95 each to cost $2396.00? Show an estimation of how you found "about" what they should cost.

   Answer:

   Round 8 pairs up to 10 pairs, and $29.95 could be rounded to $30.

   The cost should be 10 pairs × $30 or about $300 in total.

   Thus, $2396 is not a reasonable amount.

3. Multiply each question, showing an estimate using rounding. Then find the actual answer.

   a) 12×23.65

   Answer:

   Estimate: $10 \times 24 = 240$ or $12 \times 25 = 400$

   Actual: $12 \times 23.65 = 283.8$

   b) 5.84×1.2

   Answer:

   Estimate: $6 \times 1 = 6$

   Actual: $5.84 \times 1.2 = 7.008$

   c) 1.487×0.12

   Answer:

   Estimate: $1.5 \times 0.1 = 0.15$

   Actual: $1.485 \times 0.12 = 0.178\,44$

   d) 0.625×5.6

   Answer:

   Estimate: $0.5 \times 6 = 3$

   Actual: $0.625 \times 5.6 = 3.5$

Learning Activity 4.2

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Estimate the product of 0.524 and 3.57.
2. The last time you counted, you had 54 DVDs. Your house was broken into last night and now you only have 32. How many DVDs were stolen?
3. Complete the pattern: −1, 2, −3, ___ , ___ .
4. A 90° angle is called a _____ angle.
5. What is a logical unit to use when measuring the dimensions of your room?

Answers:

1. 2 (0.5 × 4)
2. 22 DVDs (54 – 32)
3. 4, –5
4. Right
5. Metres or feet

Part B: Multiplying Fractions

Remember, these questions are similar to the ones that will be on your assignments and midterm exam. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm exam. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Multiply these fractions using "multiply the tops, multiply the bottoms" method.

   a) $\frac{3}{4} \times \frac{5}{8}$

   Answer :

   $$\frac{3}{4} \times \frac{5}{8} = \frac{3 \times 5}{4 \times 8} = \frac{15}{32}$$

b) $\frac{1}{2} \times \frac{3}{8}$

Answer :

$$\frac{1}{2} \times \frac{3}{8} = \frac{1 \times 3}{2 \times 8} = \frac{3}{16}$$

2. Use the grid method to multiply these fractions.

a) $3\frac{1}{2} \times 2\frac{3}{4}$

Answer:

	2	$\frac{3}{4}$
3	3 x 2 = 6	$3 \times \frac{3}{4} = \frac{9}{4}$
$\frac{1}{2}$	$\frac{1}{2}$ x 2 = 1	$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$$\text{Total} = 6 + \frac{9}{4} + 1 + \frac{3}{8}$$

$$= 7 + \frac{9}{4} + \frac{3}{8}$$

$$= 7 + \frac{18}{8} + \frac{3}{8}$$

$$= 7 + \frac{21}{8}$$

$$= 7 + \frac{16}{8} + \frac{5}{8} \left(\text{find the largest multiple of 8 less than or equal to 21}\right)$$

$$= 7 + 2 + \frac{5}{8}$$

$$= 9\frac{5}{8}$$

b) $4\frac{3}{8} \times 3\frac{1}{4}$

Answer:

	3	$\frac{1}{4}$
4	$4 \times 3 = 12$	$4 \times \frac{1}{4} = \frac{4}{1} \times \frac{1}{4} = \frac{4}{4} = 1$
$\frac{3}{8}$	$3 \times \frac{3}{8} = \frac{3}{1} \times \frac{3}{8} = \frac{9}{8}$	$\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$

$$\text{Total of all areas} = 12 + 1 + \frac{9}{8} + \frac{3}{32}$$

$$= 13 + \frac{36}{32} + \frac{3}{32}$$

$$= 13 + \frac{39}{32}$$

$$= 13 + \frac{32}{32} + \frac{7}{32}$$

$$= 13 + 1 + \frac{7}{32}$$

$$= 14\frac{7}{32}$$

3. Convert these fractions to decimals using the chart or your calculator.

a) $3\frac{1}{8}$

b) $50\frac{3}{4}$

Answers:

a) 3.125

b) 50.75

4. Convert these mixed numbers to improper fractions, multiply the fractions, and then simplify.

a) $2\frac{1}{8} \times 3\frac{3}{4}$

Answer :

$$2\frac{1}{8} \times 3\frac{3}{4} = \frac{17}{8} \times \frac{15}{4}$$

$$= \frac{255}{32}$$

$$= \frac{224}{32} + \frac{31}{32}$$ (find the largest multiple of 32 less than or equal to 255)

$$= 7 + \frac{31}{32}$$

$$= 7\frac{31}{32}$$

b) $4\frac{1}{2} \times 2\frac{3}{4}$

Answer :

$$4\frac{1}{2} \times 2\frac{3}{4} = \frac{9}{2} \times \frac{11}{4}$$

$$= \frac{99}{8}$$

$$= \frac{96}{8} + \frac{3}{8}$$ (find the largest multiple of 8 less than or equal to 99)

$$= 12 + \frac{3}{8}$$

$$= 12\frac{3}{8}$$

5. You are one of a group of 8 friends who are evenly splitting $100. Your friend shows you his math.

$$\$100 \times \frac{1}{8} = \$100 \times 0.0125 = \$1.25$$

He hands you $1.25.

Does this make sense? Try to estimate the correct answer.

Answer:

If there were 10 friends, each person would receive $10. There are fewer people, so you should receive a little more than $10. You show your friend that he made an error in converting 1/8 to a decimal, and you correct the error for him.

$$\frac{1}{8} = 0.125$$

$\$100 \times 0.125 = \$12.50$

This answer makes more sense.

Learning Activity 4.3

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are getting ready to start your first job, and you want to make sure you get there on time. It takes you 20 minutes to ride your bike to your work from your house. You want to get there 15 minutes before your shift starts. It takes you 30 minutes to get ready in the morning. If your shift starts at 10:00 am, what time will you have to wake up?
2. Solve for b: $8 + b - 4 = 16$.
3. Which is larger: 0.66 or $\frac{2}{3}$?
4. If you are looking for a good deal, you found it—bread is 50% off! One loaf usually costs $2.40. How much will you pay with the discount?
5. There is half a pie on the counter. If you eat a quarter of that pie, how much will you have eaten out of the whole pie?

Answers:

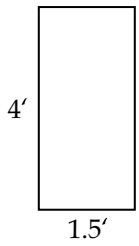
1. 8:55 am (Time needed: 30 + 20 + 15 = 65 min. or 1 hr. 05 min.; 10:00 – 1:05 = 8:55.)
2. $b = 12$ (Add 4 to both sides: $8 + b = 16 + 4$; subtract 8 from both sides: $b = 20 - 8$)
3. $\frac{2}{3}$ (Although they are very similar in decimal form, they are not quite equal; $\frac{2}{3} = 0.666\overline{6}$, or rounded = 0.67.)
4. $1.20 (50% = half so 2.4 ÷ 2 = 1.2; it may help to look at the number as 24, then put the decimal back in after.)
5. $\frac{1}{8}\left(\frac{1}{2} \times \frac{1}{4}\right)$

Part B: Areas of Regular Shapes (1)

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the area of the following shapes.

a) rectangle

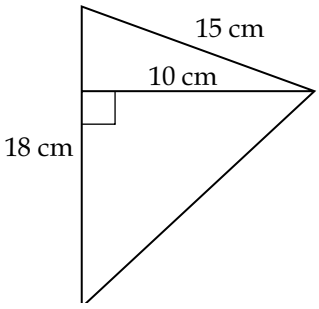


Answer:

$A = bh$

$A = 4' \times 1.5' = 6 \text{ ft.}^2$

b) triangle

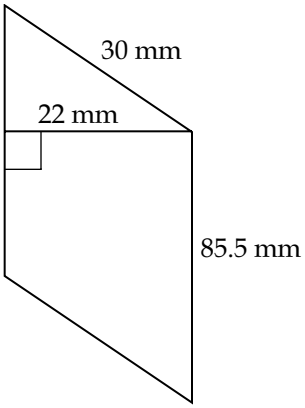


Answer:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10 \text{ cm})(18 \text{ cm}) = 90 \text{ cm}^2$$

c) parallelogram

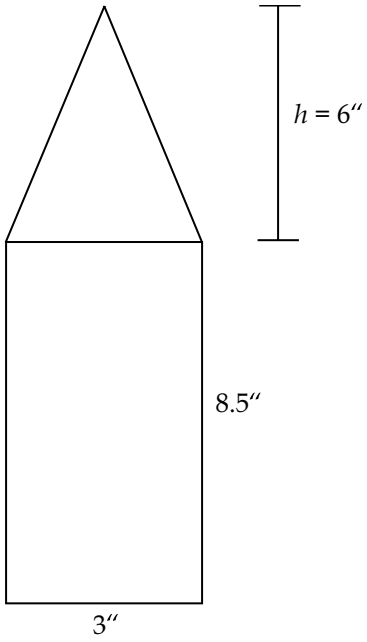


Answer:

Area = base × height

A = 85.5 mm × 22 mm = 1881 mm^2

d) combined shapes



Answer:

Find the area of each shape, and add the areas together.

Rectangle: $A = bh = 3'' \times 8.5'' = 25.5 \text{ in.}^2$

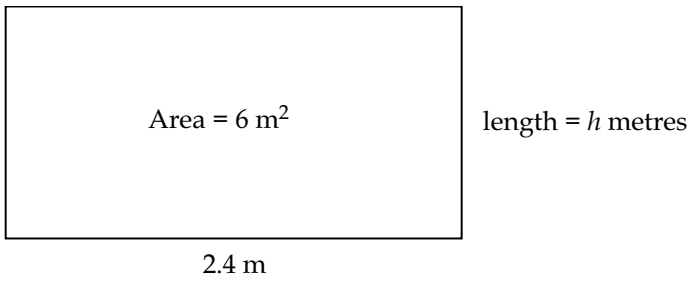
Triangle: $A = \frac{1}{2}bh = \frac{1}{2}(3)''(6)'' = 9 \text{ in.}^2$

Total: $25.5 \text{ in.}^2 + 9 \text{ in.}^2 = 34.5 \text{ in.}^2$

2. The area of grass over which you want to place a rectangular fire pit is 6 m^2. If the maximum width of the pit is 2.4 m, find its length.

Answer:

Draw a diagram.



Write the formula: $A = bh$

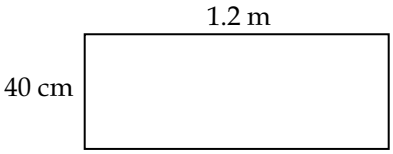
Substitute the known values into the formula: $6\ \text{m}^2 = (2.4\ \text{m})(h)$

Solve for the unknown value: $\dfrac{6\ \text{m}^2}{2.4\ \text{m}} = \dfrac{(2.4\ \text{m})(h)}{2.4\ \text{m}}$

$$2.5\ \text{m} = h$$

The length of the pit is 2.5 m.

3. Your friend had this problem marked as incorrect. Identify the error she made, and correct it.



$A = bh$

$= 1.2 \times 40$

$= 48\ \text{m}^2$

Answer:

Your friend did not make sure the units were the same for both measurements.

If the dimensions are all in centimetres, then the length of 1.2 m becomes 120 cm and the area is $A = 40\ \text{cm} \times 120\ \text{cm} = 4800\ \text{cm}^2$.

If the dimensions are all in metres, then the width of 40 cm becomes 0.4 m and the area is $A = 0.4\ \text{m} \times 1.2\ \text{m} = 0.48\ \text{m}^2$.

Learning Activity 4.4

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. In Home Economics, the teacher asks your group to make a double recipe of lasagna for the class, while other groups are making different parts of the meal. The recipe for lasagna calls for $\frac{3}{4}$ cup of Parmesan cheese. How much Parmesan cheese will you need if you are making a double recipe?
2. At your dad's office, the carpet is made out of squares. Each square is 2 feet by 2 feet. If your dad's cubicle is 5 squares deep and 5 squares wide, what is the area of your dad's cubicle in feet?
3. To get to Gimli it takes 1.25 hours. If you are travelling at 100 km/h the whole time, how far is Gimli?
4. A ticket to the baseball game is $12.50. You have to pay for parking at the game, which costs $5. Once you are in the ball park, you buy popcorn for $3.00, ice cream for $3.15, and a drink for $2.50. How much did it cost you to go to the baseball game?
5. Put the following numbers in order from smallest to largest: 0.53, 29%, 0.045, 0.13, 78%.

Answers:

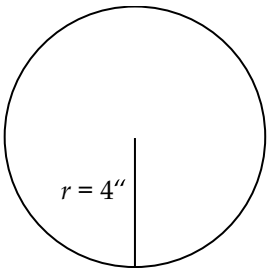
1. $1\frac{1}{2}$ cup $\left(\frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}\right)$
2. 100 feet^2 (5 squares × 2 feet per square = 10 feet for each dimension; 10 × 10 = 100.)
3. 125 km (1.25 × 100 = 125 km)
4. $26.15 (Add all the cents: 50 + 50 + 15 = $1.15; add all the dollars: 12 + 5 + 3 + 3 + 2 + from the cents = $25. Total = 25 + 1.15 = $26.15.)
5. 0.045, 0.13, 29%, 0.53, 78% (Convert the percents to decimals to compare: 29% = 0.29, 78% = 0.78.)

Part B: Areas of Regular Shapes (2)

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the area of these shapes.

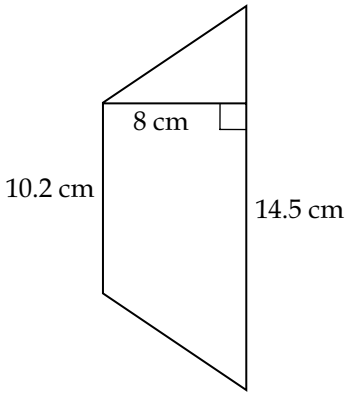
a)



Answer:

$$A = \pi r^2$$
$$= \pi\,(4'')^2$$
$$= 3.14\left(16 \text{ in.}^2\right)$$
$$= 50.24 \text{ in.}^2$$

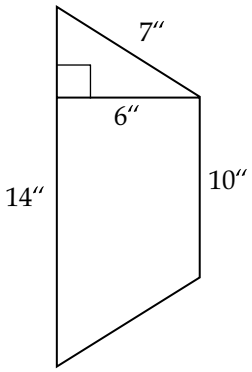
b)



Answer

$$A = \frac{1}{2}h\left(b_1 + b_2\right)$$
$$= \frac{1}{2}\left(8 \text{ cm}\right)\left(10.2 \text{ cm} + 14.5 \text{ cm}\right)$$
$$= \frac{1}{2}\left(8 \text{ cm}\right)\left(24.7 \text{ cm}\right)$$
$$= \frac{1}{2}\left(197.6 \text{ cm}^2\right)$$
$$= 98.8 \text{ cm}^2$$

2. Identify the error in this calculation of area, and correct it.



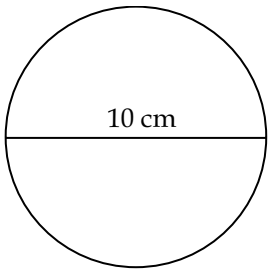
$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(7)(14+10)$$
$$= \frac{1}{2}(7)(24)$$
$$= \frac{1}{2}(168)$$
$$= 84 \text{ in.}^2$$

Answer:

The 7 inch measurement is not the perpendicular height. The 6″ line meets the base at a 90° angle and thus the height is 6″.

$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(6'')(14'' + 10'')$$
$$= \frac{1}{2}(6'')(24'')$$
$$= \frac{1}{2}(144 \text{ in.}^2)$$
$$= 72 \text{ in.}^2$$

3. Given a circle with a diameter of 10 cm, find its area.



Answer:

If the diameter is 10 cm, then the radius is 5 cm.

$A = \pi r^2$

$= 3.14(5 \text{ cm})^2$

$= 3.14 (25 \text{ cm}^2)$

$= 78.5 \text{ cm}^2$

Learning Activity 4.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The mat used for floor gymnastics is a square. The length along one side is 40 feet. What is the total area of the mat?
2. A student receives a grade of $\frac{18}{21}$ on his test. Estimate the percent value.
3. Rewrite the following fraction in lowest terms: $\frac{12}{20}$.
4. You want to order takeout, but you only have $15. Your favourite takeout costs $16, but is 10% off. Can you afford it? (Taxes are included in the price.)
5. Your gross pay is $530. Your CPP deduction is $26 and your EI deduction is $10. You get your bonus of $60. How much is your net pay?

Answers:

1. 1600 feet^2 ($4 \times 4 = 16$ so $40 \times 40 = 1600$)
2. 90% $\left(\frac{18}{21} \text{ is very close to } \frac{18}{20} \times \frac{5}{5} = \frac{90}{100} = 90\%\right)$
3. $\frac{3}{5}\left(\frac{12}{20} \div \frac{4}{4} = \frac{3}{5}\right)$
4. Yes (10% of 16 is 1.6, so the takeout would cost $14.40.)
5. $554 (Total payment: $530 + $60 = $590, total deductions: $26 + $10 = $36, 590 – 36.)

Part B: Ratio and Scale

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the scale to find the actual distance in kilometres to fly from Grand Rapids to Portage la Prairie.

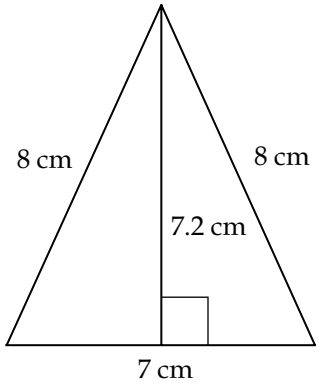
Source: Stanford, Quentin H. (Ed). *Canadian Oxford School Atlas*. Don Mills, ON: Oxford University Press, 2004. Reproduced in accordance with *Access Copyright Elementary and Secondary School Tariff*.

Answer:

The unit length for 100 km on the scale is repeated approximately 3.6 times between Grand Rapids and Portage. The actual distance between the two places would be about 3.6 × 100 km = 360 km, if you traveled in a straight line.

2. Find the actual perimeter and area of these shapes using the scales given in each part.

a)



Scale: 1 cm: 20 cm

Answer:

Change the measurements in the scale diagram to their actual lengths.

Sides

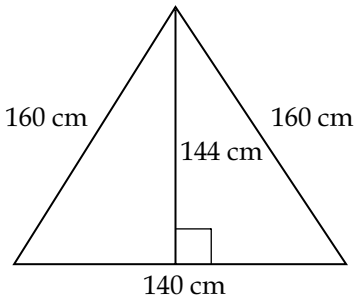
1 cm: 20 cm

7 cm: 7 × 20 cm = 140 cm

8 cm: 8 × 20 cm = 160 cm

Height

7.2 cm: 7.2 × 20 cm = 144 cm



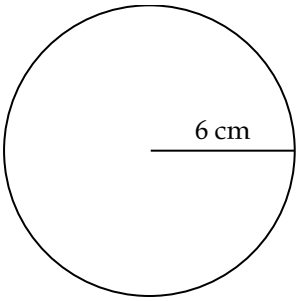
Perimeter = 160 cm + 160 cm + 140 cm = 460 cm

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(140\text{ cm})(144\text{ cm})$$

$$= 10\,080\text{ cm}^2$$

b)



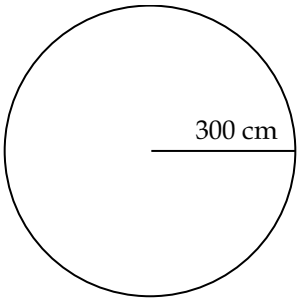
Scale: 1 cm: 50 cm

Answer:

Actual distance:

The scale has 1 cm representing 50 cm.

Thus, 6 cm represents 6 × 50 cm = 300 cm.



Perimeter of a circle is the circumference:

$C = 2\pi r$

$= 2(3.14)(300 \text{ cm})$

$= 1884 \text{ cm}$

Area:

$A = \pi r^2$

$= (3.14)(300 \text{ cm})^2$

$= 282\,600 \text{ cm}^2$

Learning Activity 4.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Solve for q: $\frac{13}{q} = \frac{1}{3}$.
2. Write the ratio as a fraction: 1: 17.
3. You are buying a hot chocolate. The total is $1.78, but all you have is a $20 bill. How much change will you get back?
4. What bills and coins would you expect to get back if you bought the hot chocolate in Question 3?
5. Cheri works for an hourly wage of $12/hr. This week she worked 42 hours. She is paid time-and-a-half for all hours over 40 hours a week. How much money did she make on overtime?

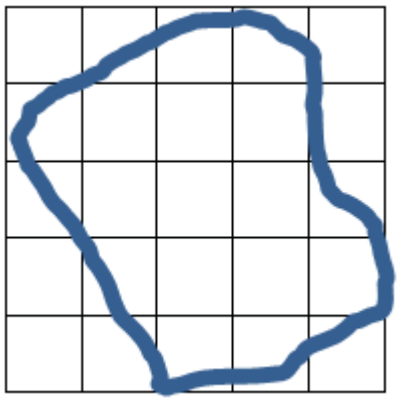
Answers:

1. 39 (Remember how you used proportional thinking in Module 3, Lesson 5: $1 \times q = 13 \times 3$.)
2. $\frac{1}{17}$
3. $18.22 (Count up: $1.78 + 0.02 + [0.20 + 3 + 5 + 10] = $20. You need to add $18.22 to $1.78 to get $20.)
4. 2 pennies, 2 dimes, a loonie, a toonie, a $5 bill, a $10 bill
5. $36 (hourly wage for overtime: 12×1.5 or $18; amount paid for overtime: $18 \times 2 = $36)

Part B: Area of Irregular Shapes

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use any of the four methods learned in this lesson to estimate the actual area of the following diagram. Show how you arrived at your answer.



Scale: 1 unit × 1 unit: 2 m × 2 m

Answer:

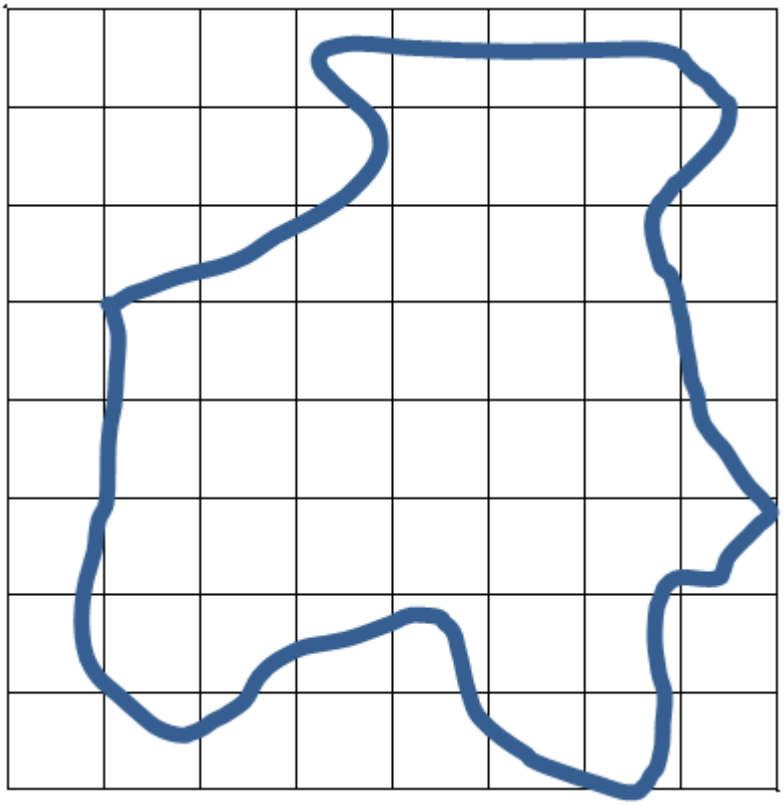
These are estimates, not exact values. Your answers should be reasonably close to these.

There are 25 squares on the grid. Approximately 8 of them are empty squares, so 25 – 8 = 17 are filled squares.

Each square has an area of 2 m × 2 m = 4 m^2.

The area of the actual diagram = 17 × 4 m^2 = 68 m^2.

2. Use any of the four methods learned in this lesson to estimate the actual area of the following diagram. Show how you arrived at your answer.



Scale: 1 unit × 1 unit: 3 feet × 3 feet

Answer:

Remember, these are estimates, not exact values. Your answers should be reasonably close to these.

There are approximately 33 filled squares.

Each square has an area of 3′ × 3′ = 9 ft.^{2}.

The area of the actual diagram = 9 ft.^{2} × 33 = 297 ft.^{2}.

Learning Activity 4.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A 2-door sports car gets 12.2 km per L of gas. A truck gets 7100 m per L of gas. Which is more fuel efficient?
2. 35% of the trees in the forest of 400 have Dutch elm disease. How many trees have this illness?
3. Find the average: 2, 4, 6.
4. A clown needs two balloons to make a flower. If he makes a bouquet of a dozen flowers, how many balloons will he need?
5. You're waiting for your friend at a restaurant and it is 1:30 pm. He finally shows up, and claims he has been sitting at another table for 50 minutes. What time did your friend get to the restaurant?

Answers:

1. The sports car is more fuel efficient. (Convert m into km for the truck: 7100 ÷ 1000 = 7.1 km per L. Then compare. The more distance you get per litre, the more fuel efficient the vehicle is.)
2. 140 trees (10% of 400 = 40 so 30% is 3 × 40 = 120, 5% is half of 10% = 40 ÷ 2 = 20; Total = 120 + 20.)
3. $4 \left(\frac{2+4+6}{3} = \frac{12}{3} = 4 \right)$
4. 24 (A dozen = 12, 2 balloons per one flower so 12 × 2 = 24.)
5. 12:40 pm (Count back: 30 minutes before 1:30 = 1:00, 20 minutes before that was 12:40.)

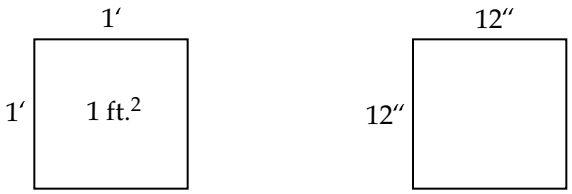
Part B: Area Conversions (1)

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Sketch a diagram to help find the following values.

   a) The number of square inches in 2 square feet.

   Answer:

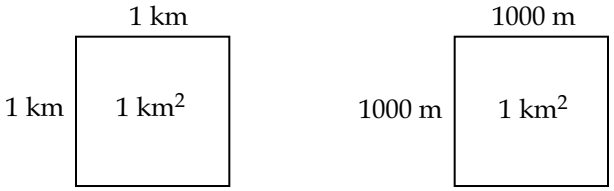


   $1 \text{ ft.}^2 = 12'' \times 12'' = 144 \text{ in.}^2$

   $2 \text{ ft.}^2 = 2 \times 144 \text{ in.}^2 = 288 \text{ in.}^2$

   b) The number of square metres in a square kilometre.

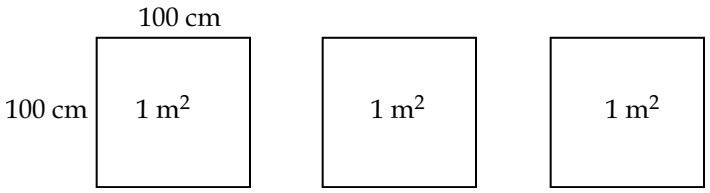
   Answer:



   $1 \text{ km}^2 = 1000 \text{ m} \times 1000 \text{ m} = 1\,000\,000 \text{ m}^2$

   c) The number of square centimetres in 3 square metres.

   Answer:

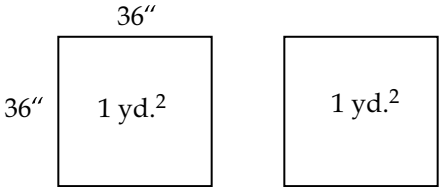


   In one square metre, the area is $1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$.

   In 3 squares, there are $10\,000 \text{ cm}^2 \times 3 = 30\,000 \text{ cm}^2$.

d) The number of square inches in 2 square yards.

Answer:



$36'' \times 36'' = 1{,}296$ in.^{2} in each square yard.

2 yd.^{2} = 2 × 1,296 in.^{2} = 2,592 in.^{2}

2. Convert the following to the units indicated using proportional reasoning.

a) 9ft.^{2} = _____ in.^{2}

Answer:

1 ft.^{2} = 144 in.^{2}

9 ft.^{2} = 9 × 144 in.^{2} = 1,296 in.^{2}

b) 4 m^2 = _____ cm^2

Answer:

1 m^2 = 10 000 cm^2

4 m^2 = 4 × 10 000 cm^2 = 40 000 cm^2

c) 90 000 cm^2 = ____ m^2

Answer:

10 000 cm^2 = 1 m^2

$$90\ 000\ \text{cm}^2 = \frac{90\ 000}{10\ 000\ \text{m}^2} = 9\ \text{m}^2$$

d) 576 in.^{2} = ____ ft.^{2}

Answer:

144 in.^{2} = 1 ft.^{2}

$$576\ \text{in.}^2 = \frac{576}{144\ \text{ft.}^2} = 4\ \text{ft.}^2$$

3. a) The cost of building a new home is roughly $110 per square foot. You have selected plans for a house that is 1,485 square feet in size. Find the cost to have this new home built.

Answer:

1 sq. ft. = $110

1,485 sq. ft. = $110 × 1,485 = $163,350

The cost to have this home built is $163,350.

b) You only have $150,000 that can be devoted toward the building costs. How large a house in terms of square footage can you consider?

Answer:

You could set up a formula, substitute the values you know, and solve for the unknown value.

Formula: Cost = $110 × sq. ft.

Substitute: $150,000 = $110 × ($s$)

Solve:
$$\frac{\$150,000}{\$110} = \frac{\$110 \times s}{\$110}$$

$$1363.6 = s$$

The largest size house you could have built is 1,363.6 square feet.

Learning Activity 4.8

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Jared lost 55% of his weight when he went on his submarine sandwich diet. He originally weighed 420 pounds. How much does he weigh now?
2. Your suitcase cannot exceed 26 kg when you are going on an international flight. Your cousin weighs around 25 kg. Would they be a good referent?
3. At the gas station, the price posted is $1.20 per L. Your fuel tank holds 40 L. How much is it going to cost to fill up your car?
4. On Monday there are half as many customers at a restaurant as there are on Saturday. If there were 590 customers on Monday, how many customers were there on Saturday?
5. Evaluate: $\frac{2}{7}+\frac{6}{14}$.

Answers:

1. 189 lbs. (If he lost 55%, then 45% remains. 10% of 420 = 42 pounds, so 5% is 21. 45% = 4 × 42 + 21 = 168 + 21 = 189.)
2. Yes (Your cousin's weight is close to the weight you want to compare it to.)
3. $48.00 ($1.20 is 120 cents; 120 × 4 = 480. Then you multiply by 10 since you need 40 litres and you get 4800 cents; 4800 cents is 48 dollars.)
4. 1180 customers (590 × 2 = (500 × 2) + (90 × 2) = 1000 + 180 = 1180)
5. $\frac{5}{7}\left(\frac{2}{7}+\frac{6}{14}=\frac{2}{7}+\left(\frac{6\div 2}{14\div 2}\right)=\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}\right)$

Part B: Area Conversions (2)

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert the following measurements as indicated using the area conversions chart. If you haven't done so already, now might be a good time to update your Resource Sheet to include the page number where the chart can be located for handy reference.

   Round off your final answer to 2 decimal places.

   a) $3\frac{3}{4}$ yd.^{2} to m^2

   Answer:

   1 yd.^{2} = 0.8361 m^2

   3.75 yd.^{2} = 3.75 × 0.8361 m^2 = 3.14 m^2

   b) 86 cm^2 to in.^{2}

   Answer:

   1 cm^2 = 0.155 in.^{2}

   86 cm^2 = 86 × 0.155 in.^{2} = 13.33 in.^{2}

   c) 940 in.^{2} to m^2

   Answer:

   This measurement needs to be converted to ft.^{2}, then to m^2.

   144 in.^{2} = 1 ft.^{2}

   $$940 \text{ in.}^2 = \frac{940 \text{ in.}^2}{144 \text{ in.}^2} = 6.5278 \text{ ft.}^2$$

   1 ft.^{2} = 0.0929 m^2

   6.5278 ft.^{2} = 6.5278 × 0.0929 m^2 = 0.61 m^2

   d) 72 square miles to km^2

   Answer:

   1 mile^2 = 2.59 km^2

   72 mile^2 = 72 × 2.59 km^2 = 186.48 km^2

e) 3256 cm^2 to $ft.^2$

Answer:

This measurement needs to be converted to $in.^2$, and then to ft^2.

1 cm^2 = 0.155 $in.^2$

3256 cm^2 = 3256 × 0.155 $in.^2$ = 504.68 $in.^2$

1 $ft.^2$ = 144 in^2

$$504.68 \text{ in.}^2 = \frac{504.68}{144} = 3.5 \text{ ft.}^2$$

2. You decide to buy new carpet for your living room. The room measures 4 m by 6.3 m. The store you visit only sells carpet by the square yard. How many square yards of carpet will you need?

Answer:

Method 1:

1 m = 1.094 yards

4 m × 1.094 = 4.376 yd.

6.3 m × 1.094 = 6.8922 yd.

Find the area

Area = length × width = 4.376 yd. × 6.8922 yd. = 30.16 $yd.^2$

The area of the room in square yards is 30.16 sq. yd.

Method 2:

Find the area in square metres

Area = 4 m × 6.3 m

= 25.2 m^2

Convert to square yards

1 m^2 = 1.196 $yd.^2$

25.2 m^2 = 25.2 × 1.196 = 30.14 $yd.^2$

The answers are different due to rounding off the conversions to only a few decimal places.

Learning Activity 4.9

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Solve for z: $3 \times 7 = z - 4$.
2. Complete the pattern: –1, 0, 1, 0, ___ , ___ , 1.
3. Samantha has twice as many dresses as she has shirts, and three times as many shirts as she has pants. If Samantha has 12 shirts, how many pairs of pants does she have?
4. The radius of a globe is 5 cm. The ratio comparing the size of the globe to the earth is 1 cm: 800 miles, what is the approximate radius of the earth?
5. What is your gross pay if you work for 28 hours at $15 per hour?

Answer:

1. $z = 25$ (multiply $3 \times 7 = 21 = z - 4$; add 4 to both sides $21 + 4 = z$)
2. –1, 0 (The pattern is alternating from –1 to 0 to 1 and back down again.)
3. 4 pairs of pants ($12 \div 3$)
4. 4000 ($5 \times 8 = 40$, add two 0s because it is 800 instead of 8.)
5. \$420 ($28 \times 15 = 28 \times (10 + 5) = (28 \times 10) + (28 \times 5)$, but since 5 is half of 10 $\left(28 \times 10\right) + \left(\frac{28 \times 10}{2}\right) = 280 + 140 = 420$)

Part B: Solving Problems

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You are having your rectangular driveway poured with new concrete. The contractor informs you that he will order 6 cubic yards of concrete. He is using his formula that 81 ft.^{2} of driveway requires 1 cubic yard of concrete. You know your driveway is 12 feet wide. How long is it?

   Answer:

   To solve this problem, you might try a proportion first.

   1 cubic ft. of concrete covers 81 ft.^{2} of driveway.

   6 cubic ft. of concrete covers 6×81 ft.^{2} = 486 ft.^{2} of driveway.

   Now use the area of a rectangle formula to find the length of your driveway, given that the width is 12 feet.

   A = length $\times$ width

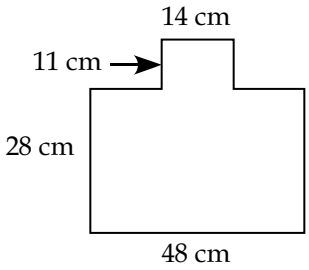
   486 ft.^{2} = length $\times$ 12 feet

   $$\frac{486 \text{ ft.}^2}{12 \text{ ft.}} = \frac{\text{length} \times 12 \text{ ft.}}{12 \text{ ft.}}$$

   $$40.5 \text{ ft.} = \text{length}$$

   Thus, the length of your driveway is 40.5 feet.

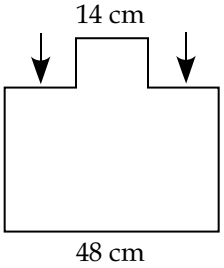
2. A mirror in your house has the shape and measurements shown in the diagram.



a) If you had a piece of moulding 2 m long, how much of it would be left after you placed moulding all around the mirror?

Answer:

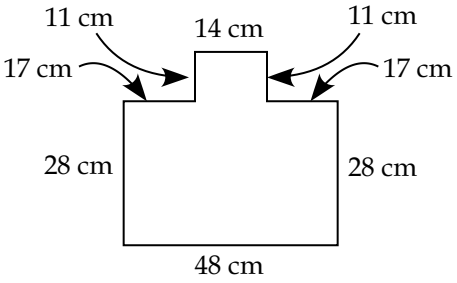
First find the length of the pieces indicated by the arrows in the diagram below.



48 cm – 14 cm = 34 cm

$$\frac{34 \text{ cm}}{2} = 17 \text{ cm each}$$

Label all pieces on the diagram with their measures.



Now find the total length of moulding to go around the outside edge of the mirror.

Perimeter = distance around the edges

= 48 cm + 28 cm + 17 cm + 11 cm + 14 cm + 11 cm + 17 cm + 28 cm

= 174 cm

Now find how much moulding, if any, is left over.

There are 2 m of moulding for the frame.

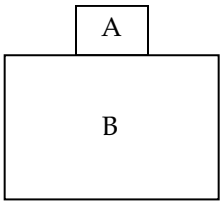
Converting to centimetres, you have 2 m = 200 cm.

Amount left over = 200 cm – 174 m = 26 cm

b) Find the area of the wall surface that the mirror will cover.

Answer:

To find the area of the irregular shaped object, you can separate it into two rectangles and find the area of each one. Then add the areas together to get the total area.



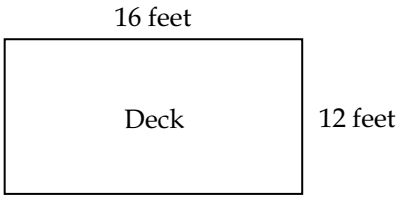
Area of rectangle A = 11 cm × 14 cm = 154 cm^2

Area of rectangle B = 28 cm × 48 cm = 1344 cm^2

Total area = 154 cm^2 + 1344 cm^2 = 1498 cm^2

The mirror covers 1498 cm^2 of wall surface.

3. You have just built a lovely wooden deck measuring 12′ by 16′. Your neighbour offers you a can of stain. The instructions on the can suggest that the stain will cover 18 m^2. Do you have enough stain to finish the entire deck? Show your calculations.



Answer:

Area of deck = 12′ × 16′ = 192 $ft.^2$

Now change the area into metric units.

1 $ft.^2$ = 0.0929 m^2

192 $ft.^2$ = 192 × 0.0929 m^2 = 17.8 m^2

Since the can of stain covers 18 m^2, you will have just enough to cover your deck.

4. Your basement measures 11′ × 24′. You purchased carpet to cover the floor, and the bill came to $950.40, before taxes.

   a) Find the cost per square metre.

   Answer:

   Area = length × width = 11′ × 24′ = 264 square feet.

   Convert to square metres

   1 sq. ft. = 0.0929 m^2

   264 sq. ft. = 264 × 0.0929 = 24.5256 m^2

   $$\text{Cost per } m^2 = \frac{\$950.40}{24.5246} = \$38.75 \text{ per } m^2$$

   b) Given PST is 7% and GST is 5%, find the total cost.

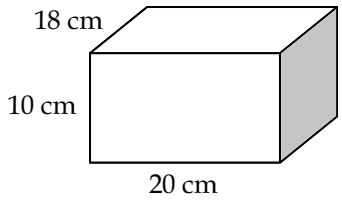
   Answer:

   GST = $950.40 × 5% = $950.40 × 0.05 = $47.52

   PST = $950.40 × 7% = $950.40 × 0.07 = $66.53

   Total cost = $950.40 + $66.53 + $47.52 = $1064.45

5. You have a present to wrap and are not sure if you have enough paper. The box measures 20 cm by 18 cm by 10 cm.



   a) Find the total surface area of the box you need to wrap.

   Answer:

   Area of top and bottom = 20 × 18 × 2 = 720 cm^2

   Area of front and back = 20 × 10 × 2 = 400 cm^2

   Area of left and right sides = 10 × 18 × 2 = 360 cm^2

   Total surface area = 720 cm^2 + 400 cm^2 + 360 cm^2 = 1480 cm^2

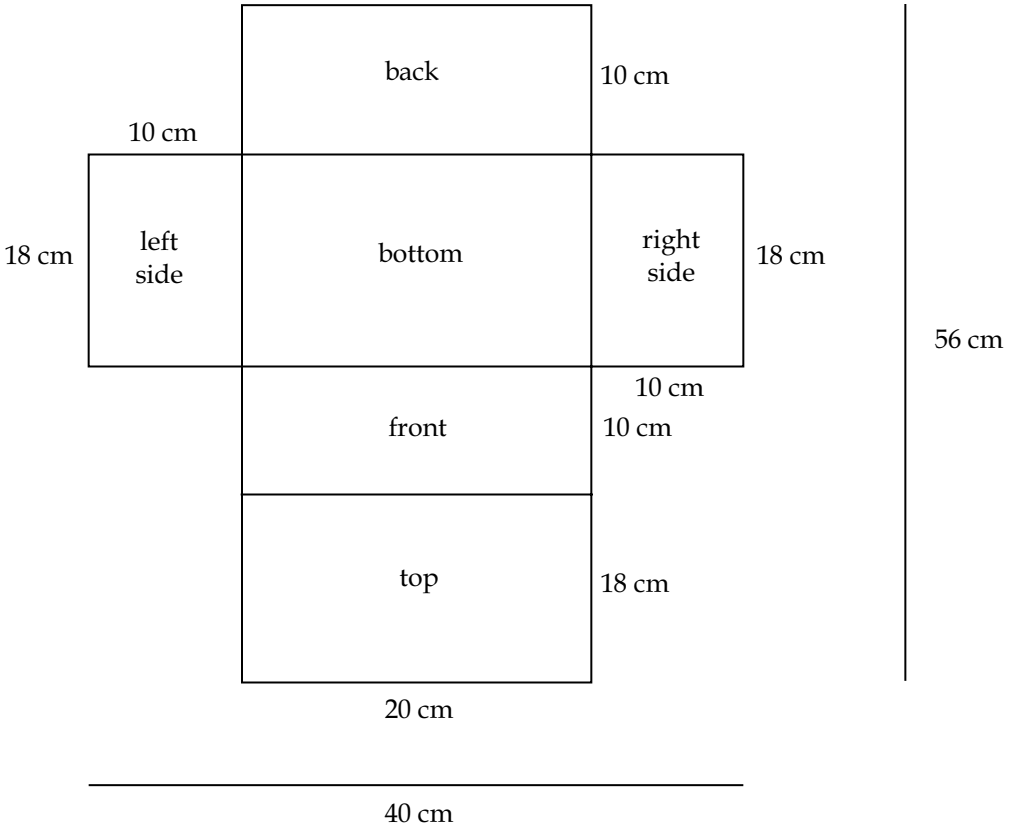
b) The roll of wrapping paper is 80 cm across, and has a length of 30 cm of paper on it. Will you have enough paper?

Answer:

If you find the area of the wrapping paper, you have 80 cm × 30 cm = 2400 cm^2, which is more than the surface area of the box so it seems there may be enough paper.

However, this problem presents an interesting dilemma. By doing the math, an answer is obtained, but in reality it doesn't work the way it is intended to work. If you were to actually wrap the box, you would not have enough paper unless you cut and paste. Here's why.

If you make a diagram of the box with the sides lying flat, then you have a different method of solving this problem.



The width of paper you will need is 10 cm + 20 cm + 10 cm = 40 cm.

The total length is 10 cm + 18 cm + 10 cm + 18 cm = 56 cm.

Thus, a roll of paper 80 cm across and 30 cm wide is not sufficient to wrap your parcel in one piece. Even if you consider the bottom part of the wrapping paper folding up along the sides, and the top folding down to meet it, the pieces will barely touch.

You would need to cut the paper into smaller pieces.

Grade 10 Essential Mathematics (20S)

Module 5
Working with Angles

Module 5:
Working with Angles

Introduction

This module deals with angles—identifying, measuring, drawing, bisecting and copying them. Also, you will work with supplementary and complementary angles, along with finding properties of angles using parallel lines and a transversal.

You will need a protractor, a set square, and a set of compasses for this module.

Assignments in Module 5

When you have completed the assignments for Module 5, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Problem Analysis; Analysis of Games and Numbers
1	Assignment 5.1	Identify Angles
2	Assignment 5.2	Estimate Angles
3	Assignment 5.3	Copy and Bisect Angles
4	Assignment 5.4	Lines and Angles
5	Assignment 5.5	Parallel Lines and Angles
6	Assignment 5.6	Perpendicular or Parallel

Resource Sheet

When you write your final examination you will be allowed to bring a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Final Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 5. You might use your Module 6 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 5, 6, 7, and 8, to prepare your Final Examination Resource Sheet. Remember, the final examination is based on the last four modules of the course.

Resource Sheet for Module 5

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Final Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Module 5 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work in the spaces provided. The cover assignment can be done at any time while you are completing Module 5. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on whether or not you found a solution, and whether or not your tutor/marker can read your diagrams. The cover assignment is worth a total of 21 marks.

Notes

Module 5 Cover Assignment

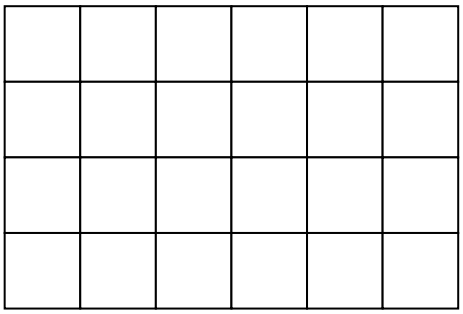
Total: 21 marks

Problem Analysis; Analysis of Games and Numbers

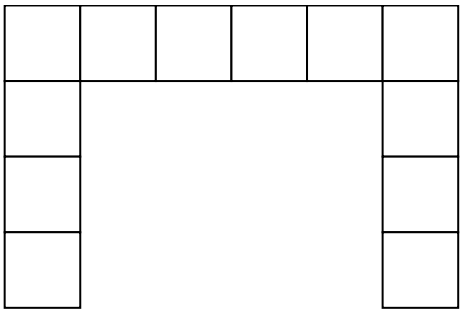
Part A: Problem Analysis

For each of the following problems, you must not only give the answer, but also an explanation of how you obtained that answer.

1. A 4 by 6 rectangle is divided into a number of smaller squares as shown. Find the number of squares of all sizes in this diagram. *(2 marks)*



2. The figure below is formed by 12 squares of the same size. If the area of the figure is 192 cm^2, determine its perimeter. *(2 marks)*

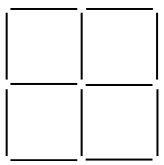


continued

Module 5 Cover Assignment (continued)

3. An open box is constructed by starting with a rectangular sheet of metal 10 cm by 14 cm. Square pieces are cut out of the corners and the resulting flaps are folded upward to make a box. If the box is 2 cm deep, find its volume. *(2 marks)*

4. This large square and four small squares are made using 12 toothpicks. Move three toothpicks to new positions to form only three completed squares. *(2 marks)*



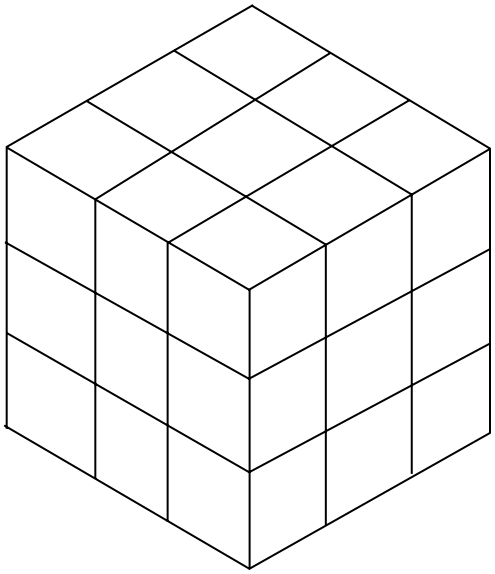
5. What is the greatest number of 2 inch by 3 inch rectangular cards that can be cut from a rectangular sheet that measures 2 feet by 3 feet? *(2 marks)*

continued

Module 5 Cover Assignment (continued)

Part B: Analysis of Games and Numbers

The cube below is 3 cm in each dimension. It is made up of 27 one-centimetre cubes.



The cube is to be painted. Some of the cubes will have paint on three faces, others will have paint on two faces, and others will have paint on just one face. There may be cubes that do not have paint on any faces.

As the size of the cube increases, the numbers above will change.

1. Please complete the chart below. *(5 marks)*

		Number of Cubes Painted On			
Size of Cube	**Total Number of Cubes**	**3 Faces**	**2 Faces**	**1 Face**	**0 Faces**
3 x 3 x 3	27				
4 x 4 x 4					
5 x 5 x 5					
6 x 6 x 6					
10 x 10 x 10					

continued

Module 5 Cover Assignment (continued)

2. Describe any patterns you see in the sets of numbers above. *(5 marks)*

3. If you were told that a cube had a side measure of 1374 cm, describe how you could calculate the numbers to fit the chart above. *(1 mark)*

Lesson 1: Identify Angles

Lesson Focus

In this lesson, you will

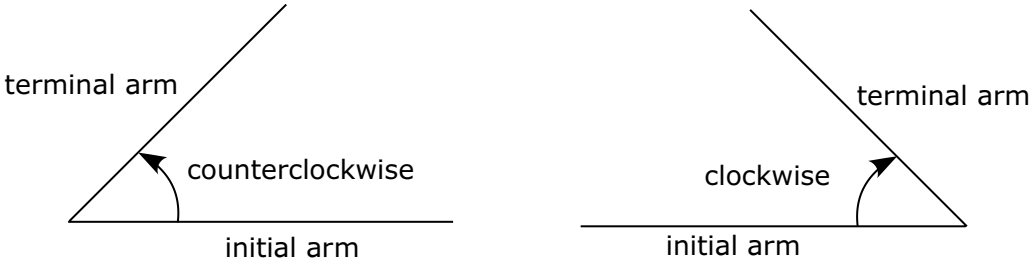
- ❑ draw and describe angles with various measures
- ❑ use a protractor to draw and measure angles

Lesson Introduction

Angles are classified into a few basic groups. This lesson is a refresher of these basic angles. You will construct angles using a protractor.

What Is an Angle?

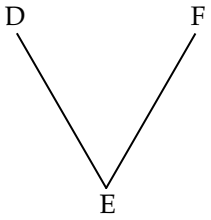
An angle is defined as a shape, formed by two lines coming from a common point, called the vertex. Each line is called an arm of the angle. One of the arms is the initial arm; the other is the terminal arm. The name depends on which direction the angle is opening. If there is no indication of whether the angle opens in a clockwise or in a counterclockwise direction, then no distinction between the two names need be made.



Labelling Angles

Angles have names. The vertex, or the point where the two lines meet, is always in the middle of the angle name.

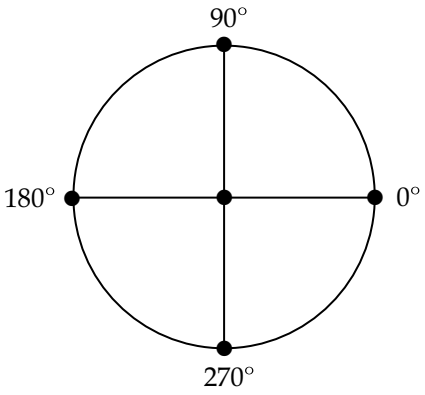
Angle DEF would look like this.



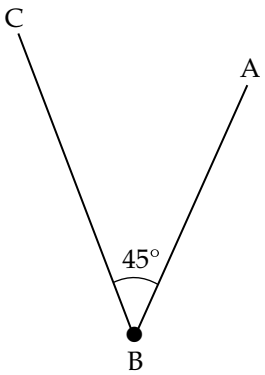
This angle could also be called angle FED. It is the same angle, just named in a different order.

Sometimes angles are named using only the vertex, and in this example you would name the angle as angle E. The symbol for angle is ∠. Thus, there are three ways to name the angle above: ∠FED, ∠DEF, or ∠E

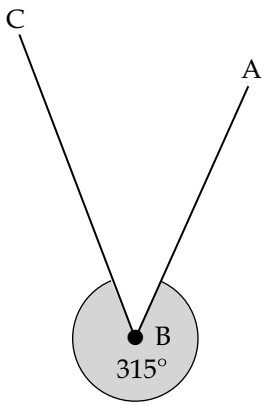
An angle can have two measures. To understand why, you need to know that one complete revolution of the circle is 360°.



In the diagram below, ∠ABC is 45°.



However, the same diagram also gives an angle of 315°. The two angles together make one complete revolution. When you do the math, if one angle is 45°, the other angle must be 360° – 45° = 315°.



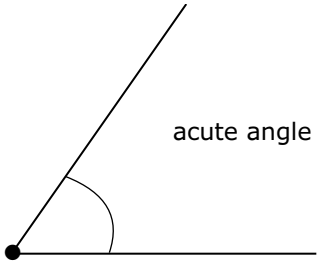
Because there are usually two possible measurements for angles at a vertex, you need to have a method of identifying the intended measurement.

Types of Angles

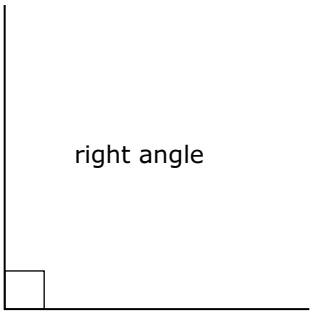
There are five types of angles you will work with. They are named according to their size.

Include the definitions or examples of each type of angle on your resource sheet.

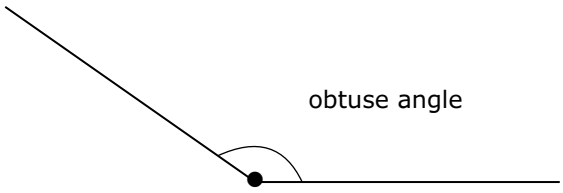
Acute angle: an angle measuring between 0° and 90°.



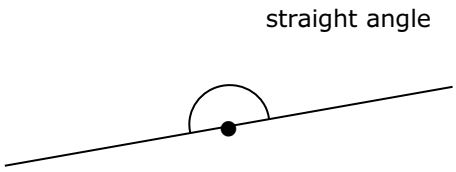
Right angle: measures exactly 90°. Usually a right angle is shown with a small rectangle in the corner, indicating 90°.



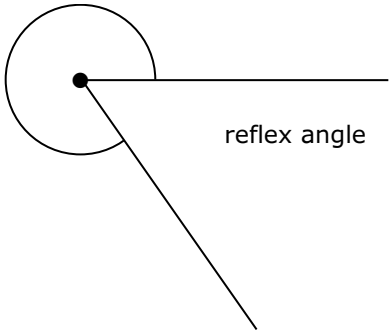
Obtuse angle: measures between 90° and 180°



Straight angle: measures exactly 180°



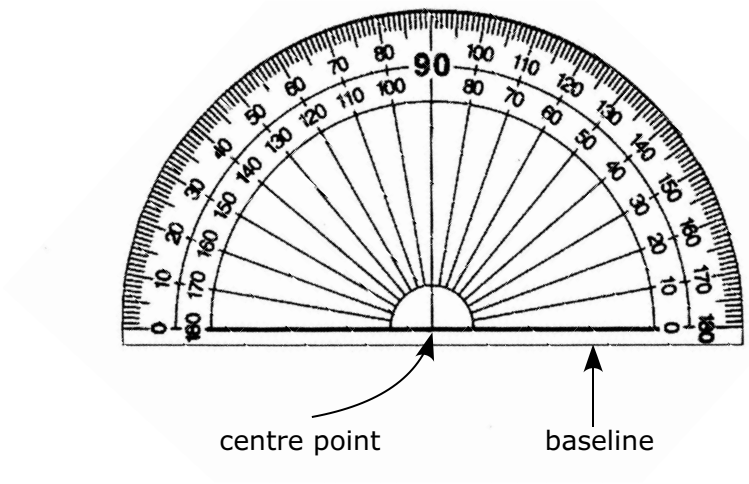
Reflex angle: measures between 180° and 360°



Using a Protractor

How do you use a protractor to measure the size of an angle?

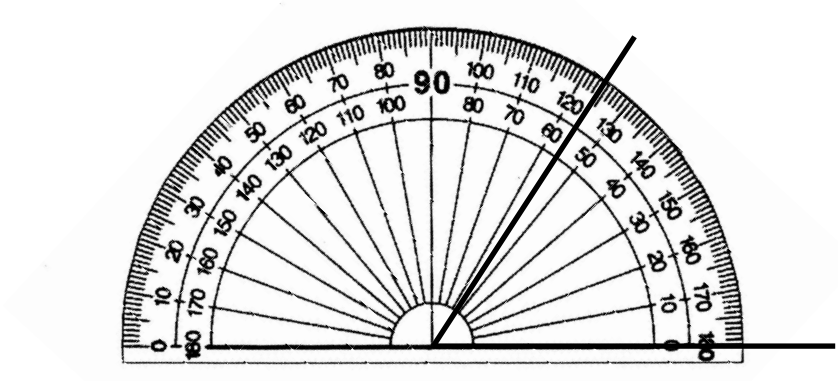
The base line of the protractor at 0° on either side lines up along one of the arms of the angle. The point at the centre of the base of the protractor sits directly over the vertex of the angle.



There are two scales on protractors. One scale has 0° starting on the left side and with increasing measurements in a clockwise direction. The other scale has 0° starting on the right side and with increasing measurements in a counterclockwise direction. You must always be careful to use the scale that is appropriate for the diagram.

In Figure 1 below, a protractor is situated on an angle. Note how the baseline is along the initial arm of the angle, and the centre point of the protractor sits over the vertex of the angle.

Figure 1



Now you can read the measurement for this angle. There are two measurements shown. One is 124° and the other is 56°. Which one is the correct answer?

First, check that the initial arm of the angle lies along the degree measure of 0°, the place from which you start to measure. In Figure 1, 0° is on the inside scale, so that is the scale you use. The correct measurement would be 56°.

You can check your work using your knowledge of angles. You know from the definitions of an acute and obtuse angle, and by looking at the diagram, that this angle you are measuring is less than 90°, not greater than 90°. Thus, if you choose the wrong scale and get an answer of 124°, you would know immediately that this is the wrong answer.

Creating Angles

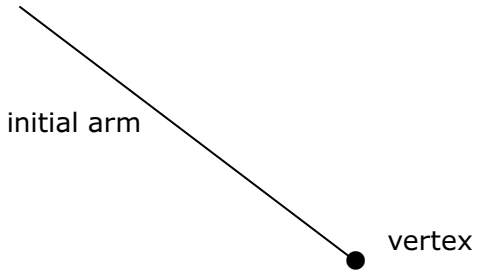
Protractors can also be used to draw angles.

Example

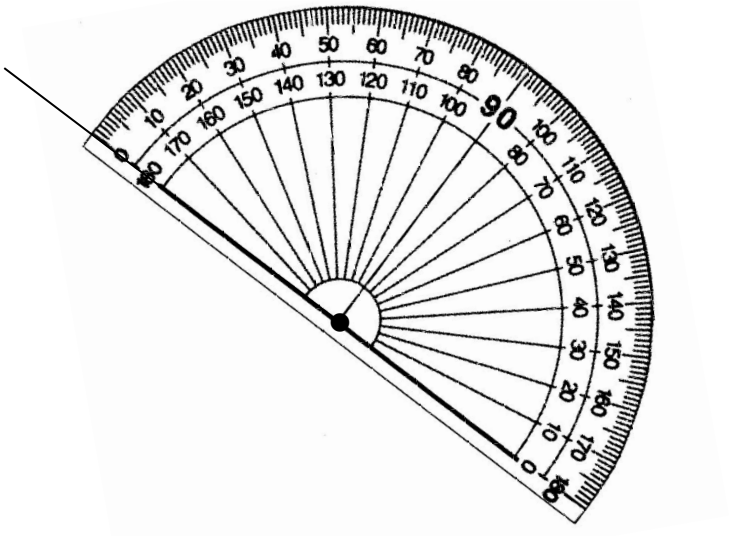
Draw an angle of 110° using you protractor.

Solution:

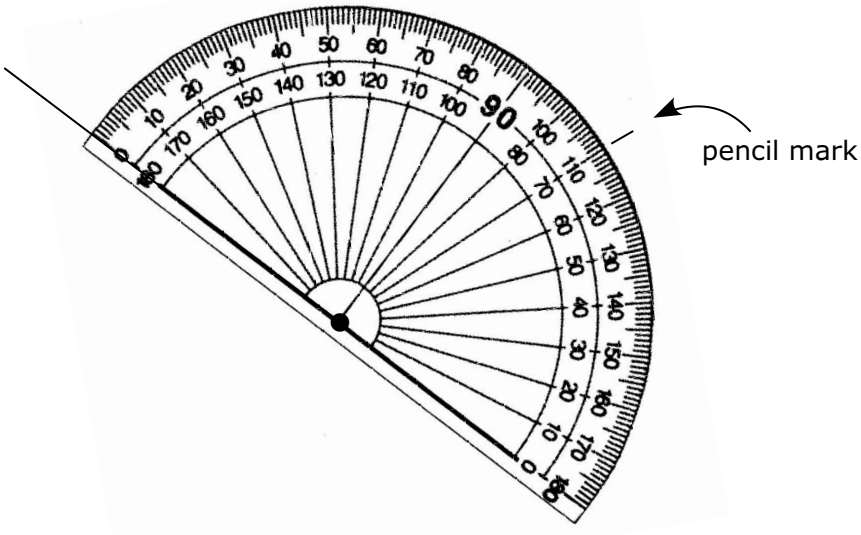
Step 1: Use the straight edge of your ruler to draw a baseline, and draw a small dot on the line where you want the vertex of the angle to be. The dot does not have to be at the end of the line. It can be placed anywhere on the line. This line will be the initial arm of your angle.



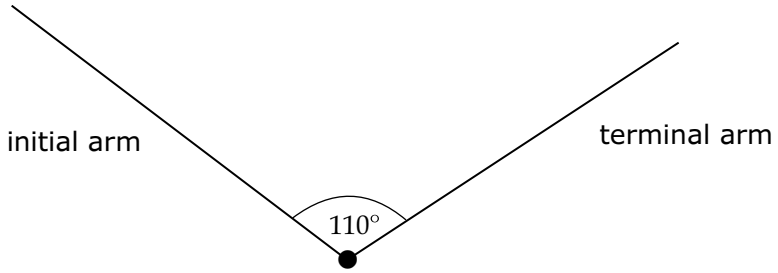
Step 2: Place the centre point of the protractor exactly over the spot where the vertex will be. Also, set the protractor so that the baseline sits directly over the line for the angle.



Step 3: Find which scale starts at 0°, and move in a clockwise direction along the protractor until you locate the 110° mark. Make a little pencil mark at 110°.



Step 4: Remove the protractor, and draw a line joining the mark with the vertex. You have created an angle measuring 110°, measured in a clockwise direction from 0°.

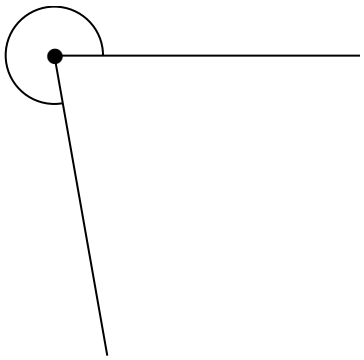


Measuring a Reflex Angle

Since the measure of reflex angles is more than 180°, and since a protractor only measures to 180°, you have to measure a reflex angle in two steps. Knowing that one complete revolution is 360°, you can measure the "other" part of the complete revolution not used by the angle. Then subtract that amount from 360° to find the reflex angle measurement.

Example

Find the measure of the reflex angle shown in the diagram below.



Solution:

Measure the part of one complete revolution that the reflex angle is missing, and subtract that amount from 360° to find the reflex angle measurement.

The missing angle measurement is 80°.

The reflex angle = 360° – 80° = 280°.

Learning Activity 5.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

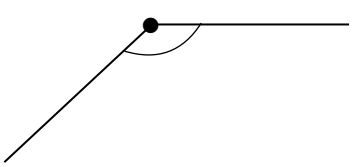
1. How tall is a door in centimetres if it is 2.2 m tall?
2. The ratio comparing the distance on a map to the distance in real life is 1 cm: 1000 m. If the distance from your house to your school on the map is 4 mm, how far do you live from school?
3. If 36% of 500 is 180, what is 18% of 500?
4. Solve for w: $w \div 6 = 2$.
5. The Pythagorean Theorem is $a^2 + b^2 =$ ___ .

Part B: Types of Angles

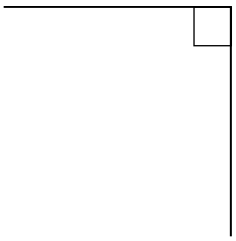
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify each angle as acute, right, obtuse, straight, or reflex.

a)



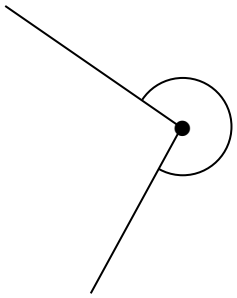
b)



continued

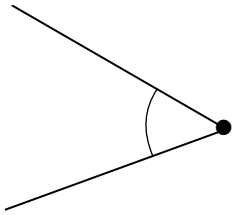
Learning Activity 5.1 (continued)

c)



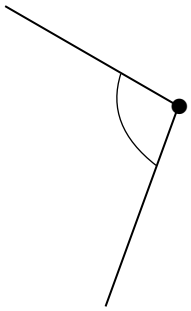
d)

e)

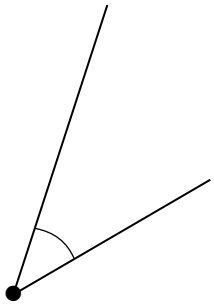


2. Use your protractor to find the measurements of the indicated angles.

a)



b)



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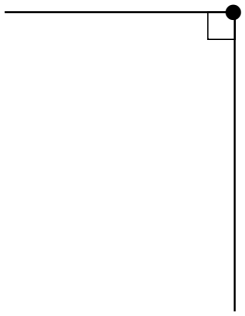

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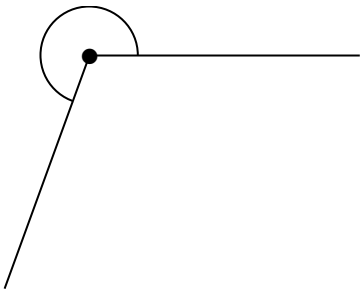

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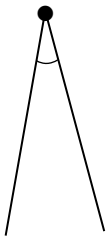


Lesson Summary

This lesson reviewed your skills with creating and measuring angles using a protractor. Also, you labelled angles in one of five categories according to their size.

The next lesson shows you how to estimate angle size.

Notes

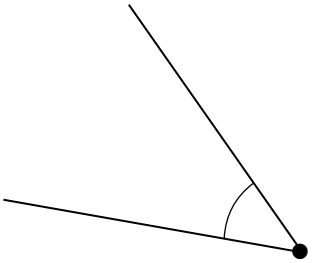
Assignment 5.1

Total: 20 marks

Identify Angles

1. Use your protractor to find the measurements of the following angles. *(5 marks)*

a)



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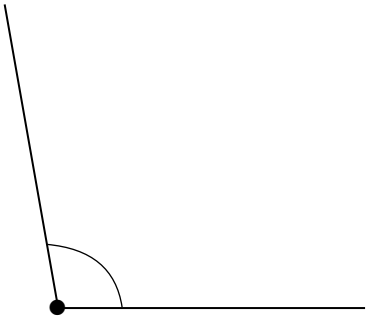
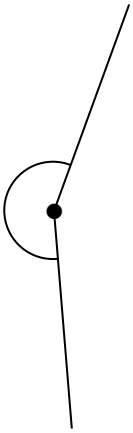
b)



_____

c)



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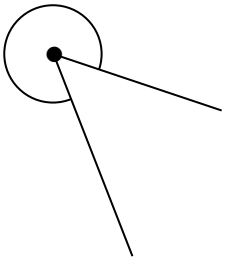
d)

_____

e)



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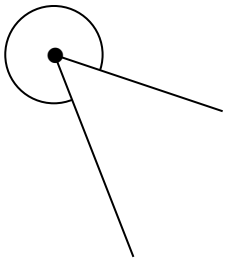
continued

Assignment 5.1: Identify Angles (continued)

2. Identify each angle as acute, right, obtuse, straight, or reflex. *(5 marks)*

a)



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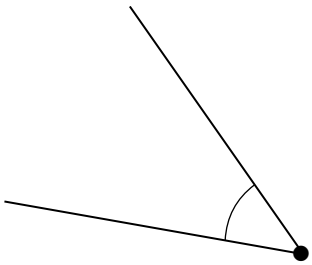
b)

_____

c)



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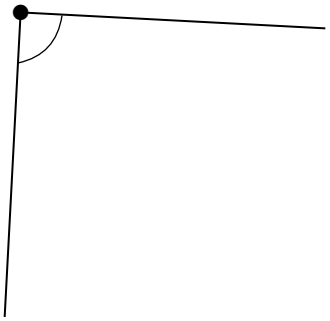
d)



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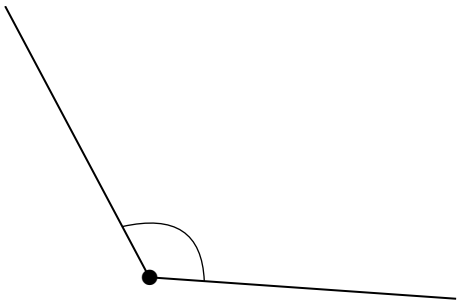
e)



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continued

Assignment 5.1: Identify Angles (continued)

3. Use your protractor to construct the following angles. *(10 marks)*

   a) 115°

   b) 248°

   c) 62°

continued

Assignment 5.1: Identify Angles (continued)

d) 90°

e) 46°

Lesson 2: Estimate Angles

Lesson Focus

In this lesson, you will

- ❑ estimate the measure of a given angle
- ❑ sketch a given angle
- ❑ identify referents for measuring angles
- ❑ solve a problem using angles

Lesson Introduction

Being able to closely estimate an angle's measurement is an important skill to have. This lesson helps you to closely determine an angle's measurement without using a protractor.

When you stand up straight, your body is at a 90° angle to the floor. Probably the room in which you sit at this moment has many examples of right angles: the walls meeting the ceiling, the floor meeting the walls, and the mouldings around the door. Right angles are easily identified.

Similarly, examples of 180° angles, or straight angles, are equally obvious in your life. Any flat surface makes a straight angle.

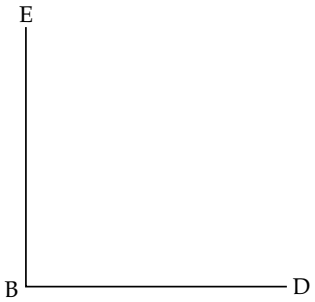
Estimating Angles

Subdividing Right Angles

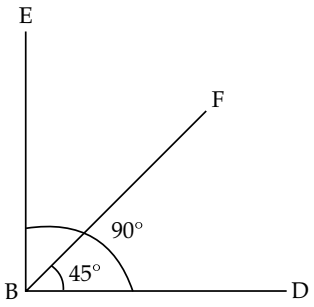
45° angles

If you can identify a 90° angle, you can bisect it, or cut it in half to find two 45° angles. The process is described as follows.

Step 1: Draw a right angle and name it ∠EBD.



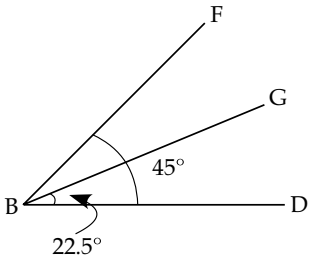
Step 2: Using your protractor, carefully draw a line making an angle of 45° and having the same vertex as EBD. Name this angle, ∠FBD.



You now have bisected a right angle into two equal angles. Each of these angles will be 45°, ∠EBF = ∠DBF and each of them are half of the right angle.

22.5° angles

If you bisect a 45° angle the same way as you bisected the 90° angle, you will have two equal 22.5° angles. ∠FBG and ∠GBD are each 22.5° and ∠FBD = 45°.



Accurate Estimates

Your objective here is to consider how to draw an angle without a protractor knowing the approximate size of the few angles you have drawn so far. Your angles need not have exact measurements, but rather reasonably accurate estimates.

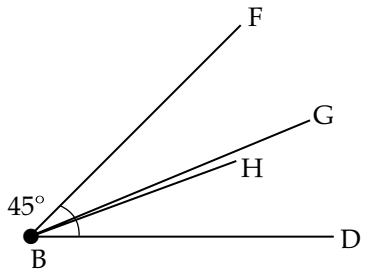
You can easily find angles of 90°, 45°, and 22.5°, as shown above.

Example 1

Draw an angle of 20° without using your protractor.

Solution:

An angle of 20° would be slightly less than the 22.5° angle you found by bisecting the 45° angle.



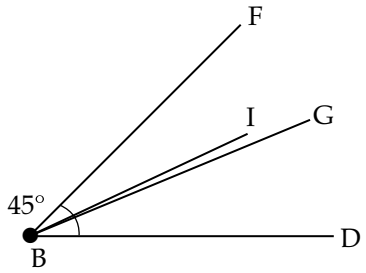
∠HBD = 20° and is slightly less than ∠GBD = 22.5° and ∠FBD = 45°.

Example 2

Draw an angle of 25° without using your protractor.

Solution:

An angle measuring 25° would be slightly more than the 22.5° angle you have drawn by bisecting a 45° angle.



∠IBD is slightly more than ∠GBD = 22.5° found through bisecting 90° and 45° angles.

Learning Activity 5.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

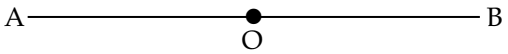
1. Identify the type of angle that has a measure of 315°.
2. Convert 24 inches into feet.
3. You are supposed to be at work for 8:00 every morning. You arrive at work at 8:17 because a train delayed you. If the hour is divided into quarters when determining late penalties, how much time are you penalized?
4. You have received a $50 bill for your birthday. At the music store your total cost is $30.75. How much change will you get back?
5. Your curfew is at 12:00 midnight. On Saturday night you get to the restaurant with your friends at 6:00 pm, and eat dinner for 2 hours. You then walk over to the movie theatre (10 minutes) and wait for the movie to start for 25 minutes.The movie is 2 hours, 45 minutes long. You drive home, which takes 20 minutes. Do you get home before your curfew?

Part B: Sketching Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Do not use your protractor for these questions. Use your skills with subdividing angles.

1. Sketch a 90° angle with vertex at O, using the straight angle, ∠AOB.



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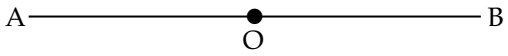
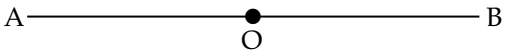
Learning Activity 5.2 (continued)

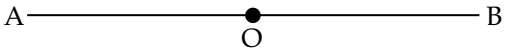
2. Sketch an angle of 45°, having its vertex at O, using the straight angle, ∠AOB.



3. Sketch a 22.5° angle starting with a straight angle, ∠AOB, with the vertex at O.



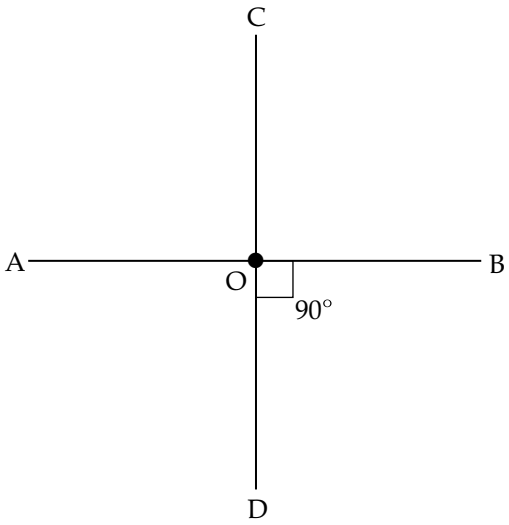
4. Sketch a 15° angle starting from a straight angle, ∠AOB, with the vertex at O.



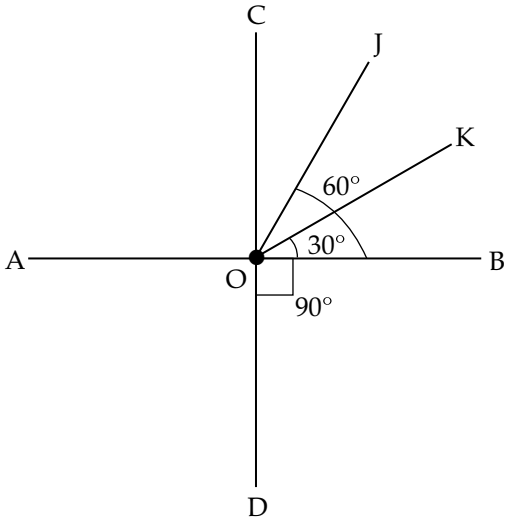
90° — 60° — 30° Angles

In the previous examples, you were bisecting angles into halves. You can sketch a good estimate of a 60° angle and 30° angle by subdividing a 90° angle into thirds.

The basic 90° angle is drawn from straight ∠AOB using O as the vertex.



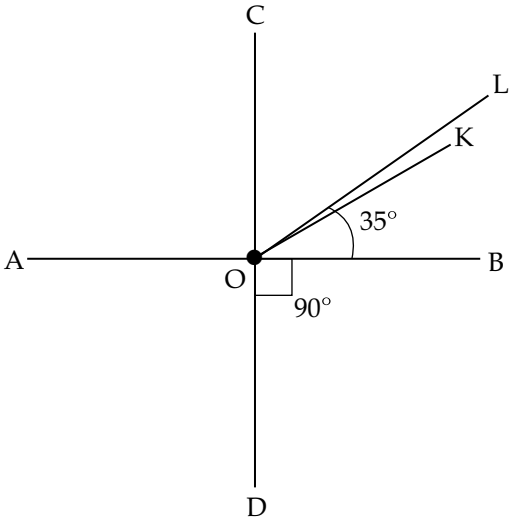
If you divide one of the 90° angles into three equal parts, you will have created three angles of 30° and, by addition, an angle of 60°.



∠KOB is 30°, and ∠JOB is 30° + 30° = 60°.

These angles could have been drawn in the other three 90° angles as well.

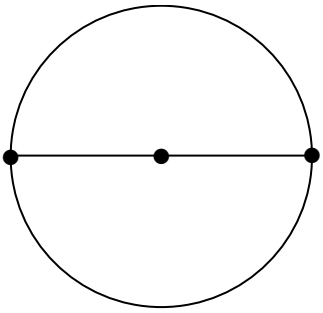
To estimate a 35° angle, you would need an angle slightly more than the 30° angle. In the diagram below, LO is drawn slightly higher than KO. Thus, ∠LOB must be slightly larger than ∠KOB.



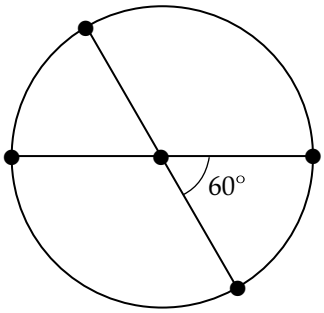
Slicing Pizzas

You manage a pizza–by–the-slice restaurant at the shopping mall. The pizzas you make are circular in shape. Your customers have an option of having the large size or medium size slice. The medium slices are from a pizza cut into 8 pieces, the large from the pizzas cut into 6 pieces.

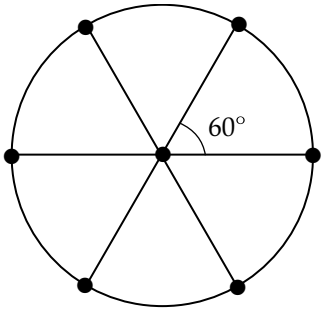
With the pizza cutting tool, you need to cut the pizza through the centre point, and have pieces of equal size. But, before you start cutting through the pizza, you need to have some idea as to what the angles will be. You first cut the pizza in half. Since the line cutting the pizza in half, the diameter, is a straight line, it is an angle of 180°.



Six pieces require that each half-pizza is cut into thirds. Since one-third of 180° is 60°, your job is to visualize where the cuts will be made to make a 60° angle. You hope the pieces you cut are nearly equal in size. Once you start your cut, you carry through into the second half of the pizza.

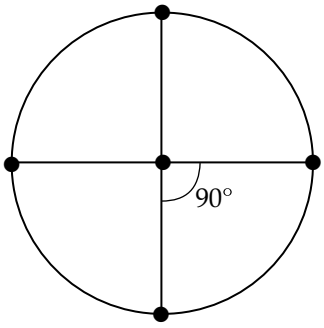


And then the second cut is made.

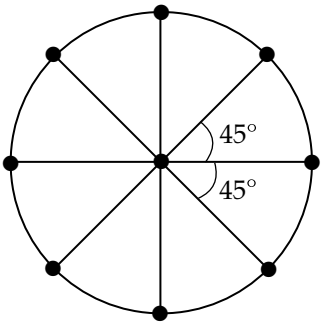


Now you have 6 pieces of equal size, using angles whose measures are approximately 60°.

If your customer prefers a smaller slice of pizza, you cut 8 pieces. You start with halves, and subdivide into quarters.



Now you bisect these quarters into eighths.

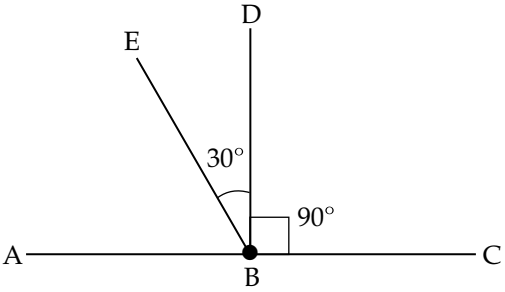


This gives you 8 pieces of equal size, using 45° angles.

Extensions

Example

How can you estimate an angle of 120°?



Solution:

A sketch of an angle of 90° can be made, and then add a 30° angle to it.

∠DBC is 90°

∠EBD is 30°

∠EBC is 90° + 30° = 120°

Learning Activity 5.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You know that your glass holds 1 cup of milk. Would this be a good referent to use to find out how much water your water bottle can hold?
2. The distance from the MTS Centre to Polo Park is 5.1 km. On your way to Polo Park from the MTS Centre you decide to stop at a doughnut shop halfway between the two. How far do you have to travel from the doughnut shop to get to Polo Park?
3. Your gross income is $980. Estimate your CPP (4.95%).
4. Travelling to Calgary, you notice that after 5 hours of driving you have travelled 515 km. How fast are you driving?
5. Solve for r: $\frac{r}{5} = \frac{10}{25}$.

Part B: Sketching Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Do not use your protractor for these questions. Use your skills with subdividing angles.

1. Sketch a 90° angle coming from a straight angle, ∠AOB, with the vertex at O.
2. Sketch a 30° angle coming from a straight angle, ∠AOB, with the vertex at O.
3. Sketch a 60° angle coming from a straight angle, ∠AOB, with the vertex at O.
4. Sketch a 150° angle coming from a straight angle, ∠AOB, with the vertex at O.

Lesson Summary

This lesson taught you how to sketch angles without a protractor by using subdivision of a 180° angle. You learned how to make angles of size 90°, 45°, 22.5°, 30°, and 60° fairly accurately. You applied your estimating skills to a practical problem.

The next lesson has you using your set of compasses to copy an angle, and to bisect an angle.

Notes

Assignment 5.2

Total: 20 marks

Estimate Angles

1. Describe in words and then illustrate how you would use subdivision to estimate an angle of 22.5°. *(4 marks)*

continued

Assignment 5.2: Estimate Angles (continued)

2. Describe in words and illustrate how you would use subdivision to estimate an angle of 30°. *(3 marks)*

3. Describe in words and illustrate how you would use subdivision to estimate an angle of 135°. *(4 marks)*

continued

Assignment 5.2: Estimate Angles (continued)

4. Describe and illustrate how you would cut a circular cheesecake into eight equal pieces. *(4 marks)*

continued

Assignment 5.2: Estimate Angles (continued)

5. Sketch the following angles without using a protractor. *(5 marks)*

a) 90°

b) 45°

c) 30°

d) 120°

e) 22.5°

Lesson 3: Copy and Bisect Angles

Lesson Focus

In this lesson, you will

- ❑ copy angles in a variety of ways
- ❑ bisect angles in a variety of ways

Lesson Introduction

This lesson illustrates a few ways to copy and to bisect angles. You will need your set of compasses for this lesson.

Bisecting Angles

Compass or Compasses?

People have grappled with the problem of whether to say "compass" or "set of compasses" for many years. A compass is an instrument for navigation.

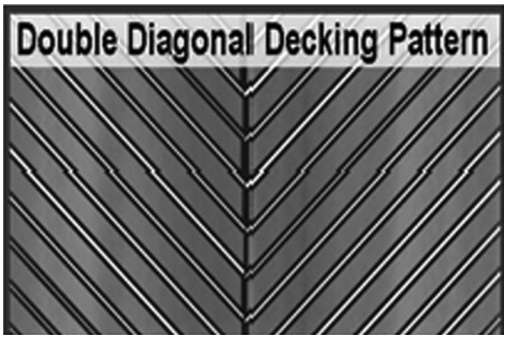
The correct way to use the phrase "set of compasses" is with the instrument used to draw arcs and circles. However, most people commonly use the term compass, when they really mean set of compasses. In this course, the term *compasses* will be used to mean a set of compasses.

How to Copy Angles

There are many ways angles can be copied. Using a set of compasses is only one way among a myriad of other ways.

Overlay Method

Here is an example of a pattern that could be used when building a deck in your yard.



In the diagram, the boards on each side of the vertical middle line are cut at exactly the same angle. Many builders have saws that can be set to cut at a specific angle. Others might use a mitre box for 45° angles.

Once the first angle has been determined, you can simply lay the cut board on top of a new board, trace the line of the cut edge, and then cut the angle for the second board. This ensures the same angle is used each time.

Reflection

Another way to copy angles is to use a Mira, which is an instrument you can find in most math classrooms. This can be held along one arm of an angle so that you can see the reflection of the second arm in the Mira. You can draw the reflection on a piece of paper. The Mira basically reflects the angle over the common arm. A small mirror could be used for the same purpose.

Folding

Actually folding the angle drawn on a piece of paper over one of its sides allows you to copy an angle. The two angles will have a common arm.

Template

Many builders use a template to copy an angle. This is a pre-cut piece with a specific degree measure of an angle. The template is placed on top of the material to be cut. The exact angles are traced around the edges of the template onto the surface to be cut.

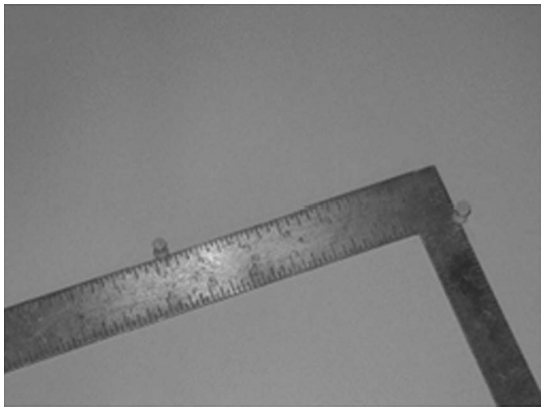
Templates are used frequently when installing kitchen or bathroom sinks. People who sew items like gloves or shoes will frequently use templates to replicate the exact measurements for the pieces they need.

In your geometry set, you have two small versions. They are called set squares. One makes 45° and 90° angles, the other 30°, 60°, and 90°angles.

Tracing

Angles can be copied using onion skin or tracing paper. Another method of tracing is to set the paper with the angle on a writing pad. Pressing heavily, follow the outline of the angle with a pencil. That outline will show as an indentation on the writing pad. Using your pencil, fill in the lines of the indentation, and the angle has been copied.

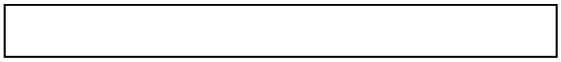
Carpenter's Square

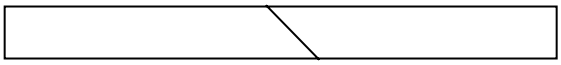


This is a very interesting tool used by carpenters. Carpenter's squares have metric and imperial measurement scales along both sides. The square has many applications. One of them is to copy angles by using the markings along the scales. Note that in your own geometry set, you probably have two set squares, as described earlier, and which can be used like a carpenter's square, as shown in the following example.

Example

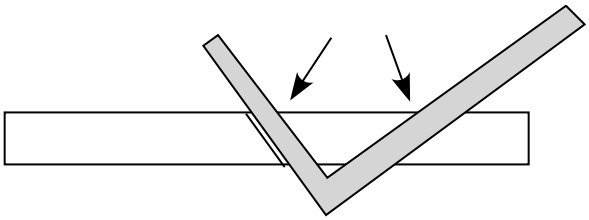
A board is to be cut at a specific angle, as shown in the diagram below. How can you use a carpenter's square to make the correct angle?



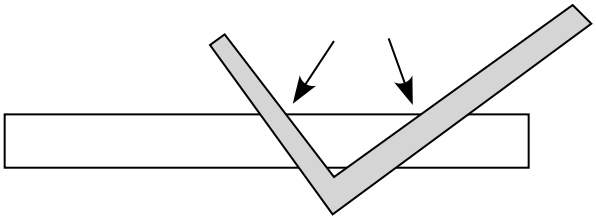


Solution:

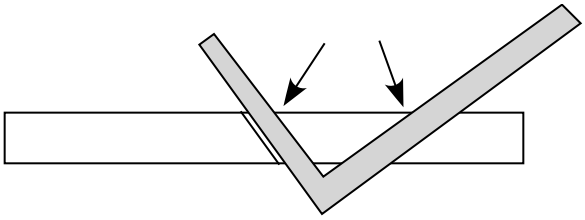
Place one side of the Carpenter's square along the cut line. Record the measurements on both arms of the square. (You could try this with your set square.)



On the board to be cut, place the square so that the top of the board falls exactly along the two measurements.



Draw a line along the edge of the square for your saw cut. This will give you the same angle as the initial cut angle.



Many books are available on the uses of the carpenter's square. Skilled, experienced carpenters use this instrument in many ways.

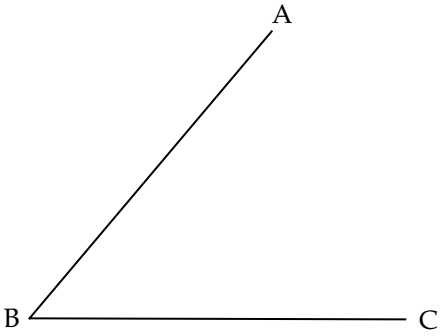
Set of Compasses

Any angle can be replicated using a set of compasses.

As a reminder for yourself, include the following steps or the page number on your resource sheet.

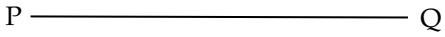
Example

Copy ∠ABC using a set of compasses.

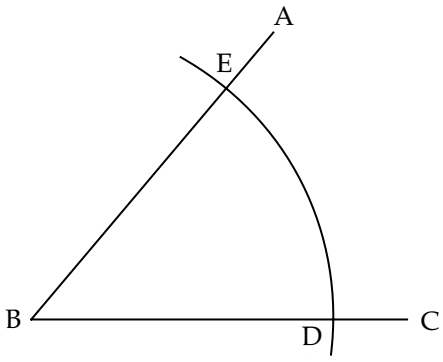


Solution:

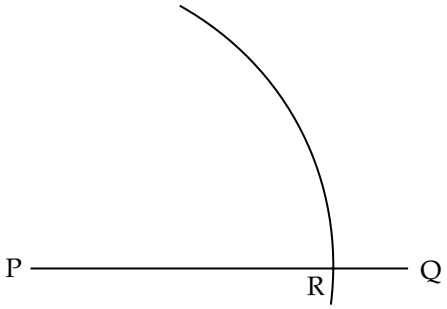
Step 1: Draw a straight line to be used as the initial side of the copied angle, and label it PQ. It does not need to be exactly the same length as BC.



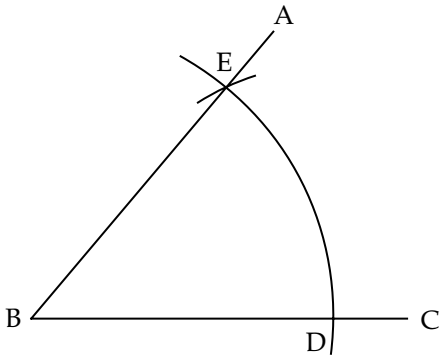
Step 2: Using the vertex of ∠ABC as the centre of a circle, use the compasses to draw an arc on the original angle so that the arc cuts both arms of the angle. Label the place where the arc intersects BC as D, and where it intersects AB as E.



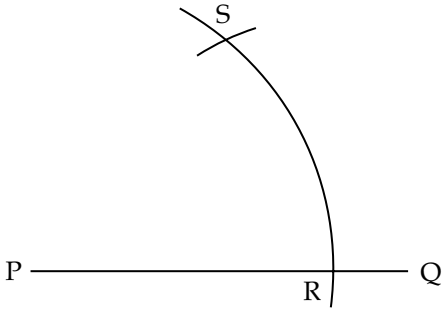
Step 3: Do not adjust your compasses. Instead, use the same measurement on your compasses and the centre at P to draw the same-sized arc intersecting line PQ. Mark the intersection point on PQ as R.



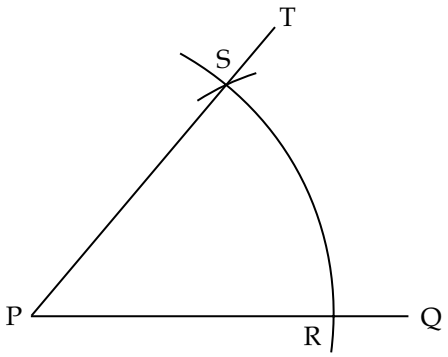
Step 4: Go back to ∠ABC. Set the point of your compasses on D, and adjust the width of your compasses so that the pencil point touches the point E on AB.



Step 5: Without making any adjustments, place the point of the compasses on point R and make an arc to intersect the first arc. Call this point S.



Step 6: Join point P with the point S and extend your line past S. Name the end of this line T. Then, ∠TPQ = ∠ABC, and you have learned how to copy an angle using a set of compasses.



Learning Activity 5.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. An angle that has a measure of 110° is called a _____ angle.
2. Andy has been to the Great Pyramid of Egypt. He bought a miniature version of the pyramid, which was to scale. The ratio comparing the miniature and the real pyramid is 1 cm: 70 royal cubits (an ancient unit of measurement). If the height of the miniature is 4 cm, how tall is the real pyramid?
3. You are out for breakfast with your friends. You order coffee for $2.50, and orange juice for $3.75. For your meal, you order waffles with blueberries for $8.50. You also get a side order of fruit salad that costs $3.25. How much will your breakfast cost before taxes and tip?
4. Approximate the tip if you want it to be about 15% of your bill.
5. For every 8 hours he works, Brian is paid a flat rate of $50 plus 10% of his sales. If, in 8 hours, Brian sells $740 in merchandise, how much money will he make in total?

continued

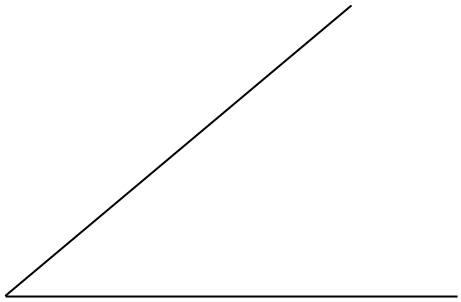
Learning Activity 5.4 (continued)

Part B: Copying Angles

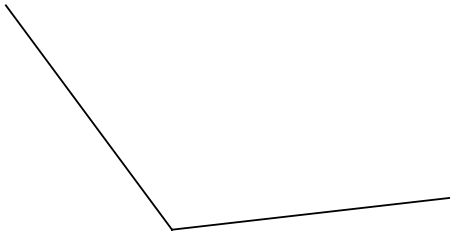
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use your compasses to make a copy of each angle, showing all markings.

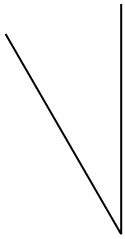
1.



2.

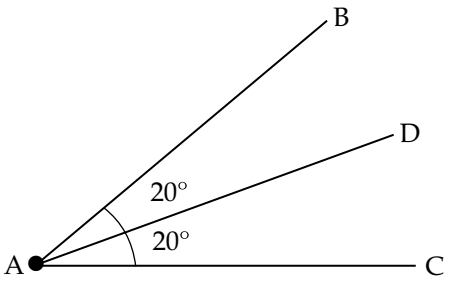


3.



Bisecting an Angle

To bisect something is to divide it into two equal halves. For example, bisecting a 40° angle would make two equal 20° angles.



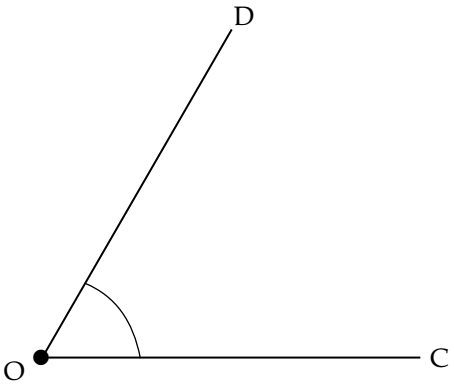
Here, ∠BAC = 40°, DA bisects ∠BAC, and ∠DAC = 20° = ∠BAD.

Protractor Method

Angles can be bisected using a protractor. You measure the original angle, then find the half of that measurement on your protractor and make a mark. Then join that mark with the vertex. You have bisected the angle.

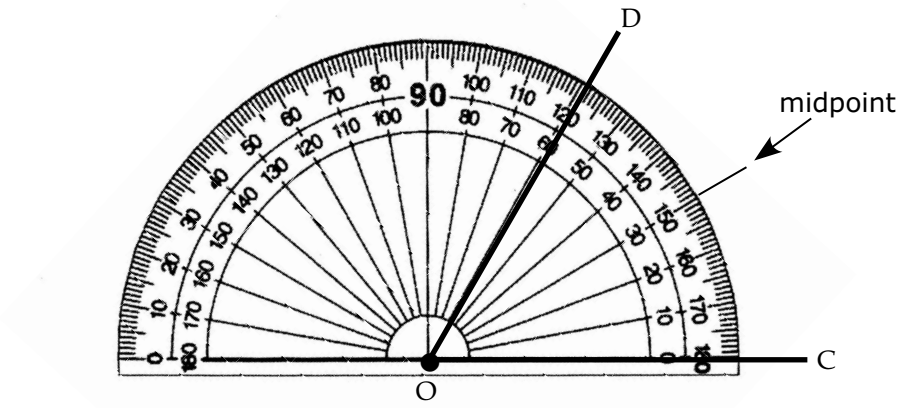
Example

Bisect ∠DOC using your protractor.

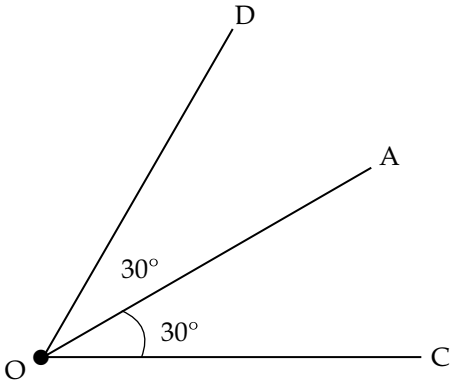


Solution:

Place your protractor's centre point and baseline along the initial arm, OC of ∠DOC. Measure the angle with your protractor to be 60°. Find one-half of the measurement. Use your protractor to make a mark at that measurement of 30°.



Remove the protractor, and draw a line joining the mark with the vertex. This creates the bisector, OA.



AO bisects ∠DOC. The two angles, ∠DOA and ∠AOC, are equal and they both have a measure of 30°.

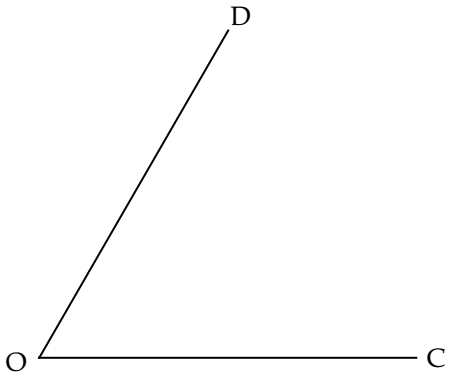
Although this method is perhaps the easiest, it should be noted that mistakes can be made with inaccurate measurements. Students must be sure to read the measurements very carefully.

Folding Method

Draw the angle on a piece of paper. Fold one arm of the angle on top of the other arm, and flatten the seam neatly. When you open the paper, the seam will be the angle bisector.

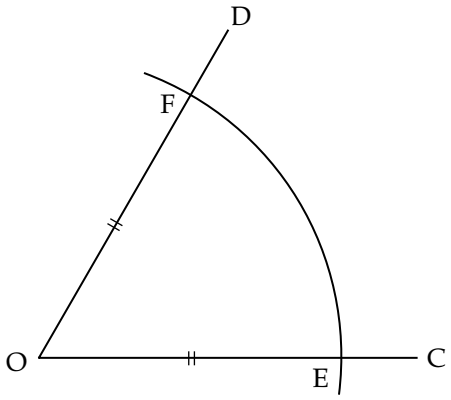
Compasses

Compasses can be used to bisect an angle. The method is shown below for ∠DOC.

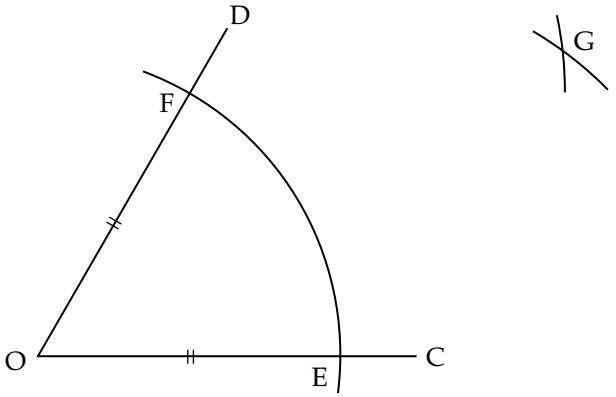


Again, these steps would be a useful reminder to have on your resource sheet.

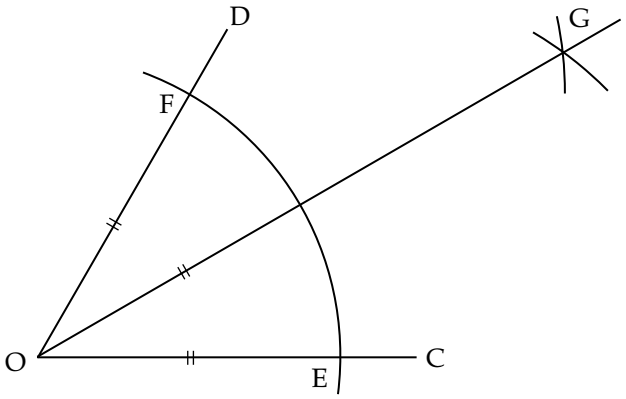
Step 1: Use the compasses to draw an arc using the vertex at O as the centre. The arc must cut both arms of the angle, creating points E and F.



Step 2: Set the width of your compasses to more than half the distance between E and F, and make intersecting arcs from points E and F. Name the point where they intersect as G. Use E and F as the centre and keep the radius the same for both arcs.



Step 3: Draw a line joining the point of intersection of the arcs at G to the vertex at O. This creates the angle bisector OG.



Learning Activity 5.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Rewrite the fraction in lowest terms: $\frac{24}{28}$.
2. Solve for t: $15t - 5 = 40$.
3. You are on your way to Calgary, driving on the Trans-Canada Highway. Your speed is constant at 100 km/h. If you have been driving for 3.3 hours at this speed, how far have you driven?
4. Adrian's time card for this week looks like this: Monday to Friday, 7 am to 1 pm; Saturday and Sunday, 6 am to 10 am. How many hours did he work this week?
5. A garden in your backyard is 2 m long and 1 m deep. What is the area of the garden?

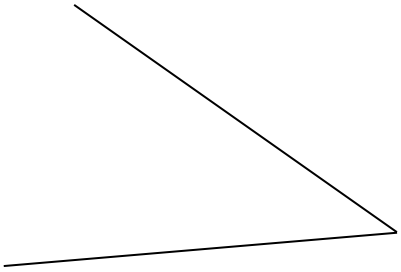
continued

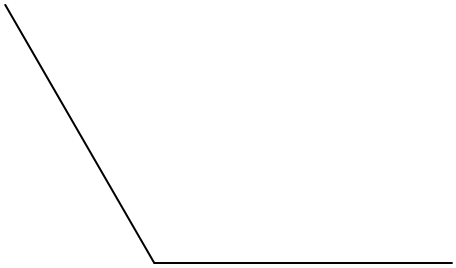
Learning Activity 5.5 (continued)

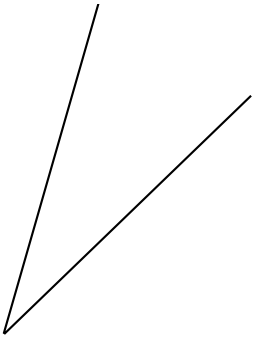
Part B: Bisecting Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use your compasses to construct the bisector of each angle, showing all markings.

1. 

2. 

3. 

Lesson Summary

This lesson showed you how to copy angles, and to bisect angles in a variety of ways. You practised copying an angle or bisecting it using compasses.

The next lesson is about perpendicular and parallel lines, as well as complementary and supplementary angles.

Assignment 5.3

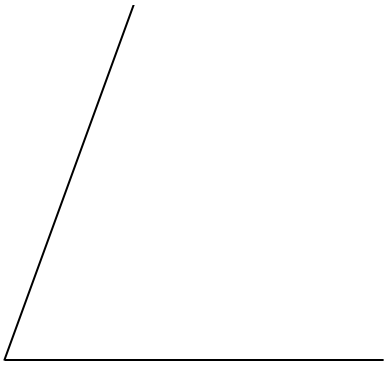
Total: 20 marks

Copy and Bisect Angles

1. List two methods other than using compasses to copy an angle. *(2 marks)*

2. Use your compasses to copy the following angles. Show all your markings. *(9 marks)*

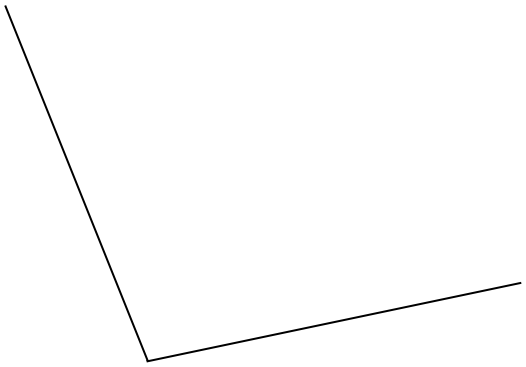
   a)



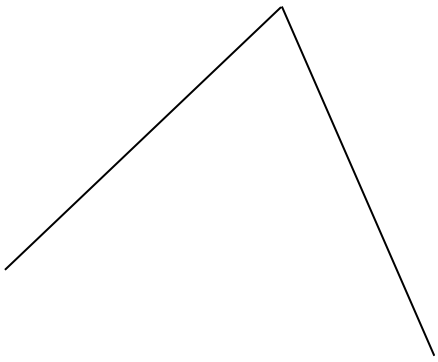
continued

Assignment 5.3: Copy and Bisect Angles (continued)

b)



c)



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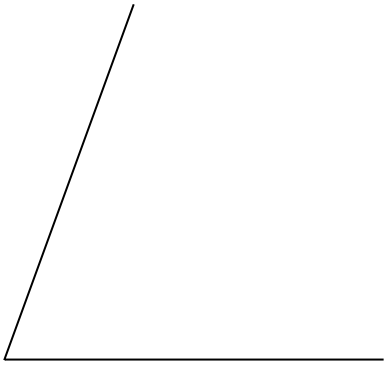
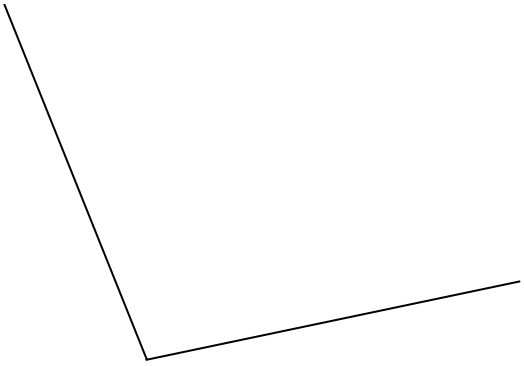
Assignment 5.3: Copy and Bisect Angles (continued)

2. Use your compasses to construct bisectors for each angle. Show all your markings. *(9 marks)*

a)



b)



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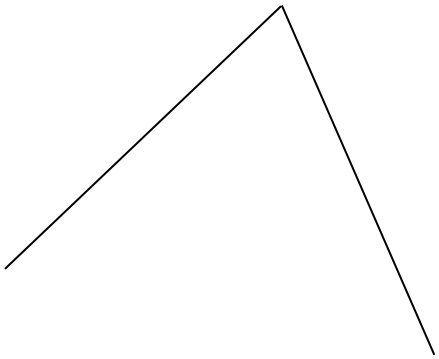
Assignment 5.3: Copy and Bisect Angles (continued)

c)



Lesson 4: Lines and Angles

Lesson Focus

In this lesson, you will

- ❑ identify sets of lines as perpendicular, parallel, or neither
- ❑ illustrate and describe complementary or supplementary angles

Lesson Introduction

Before you study the relationships of angles and parallel lines, you need to be able to identify perpendicular lines and parallel lines. Some angles are related to each other as complementary or supplementary angles.

Lines

Perpendicular Lines

Perpendicular lines are lines that meet at a right angle. A right angle is an angle of 90°. This relationship can be referred to in two ways:

- lines that meet at a right angle are perpendicular
- lines that are perpendicular meet at a right angle

Both statements say the same thing, just in reverse order.

The mathematical symbol you use to show lines are perpendicular to each other when you are writing them is $\perp$.

However, in a diagram, the box in the angle indicates the corner of a rectangle, which by definition is always 90°.

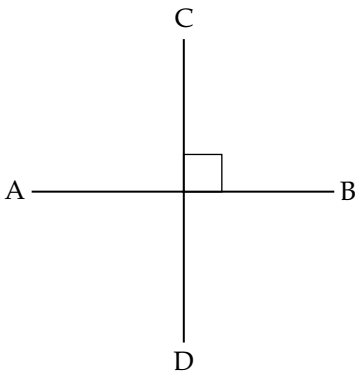
Add this definition and symbol to your resource sheet.

Example 1

Draw a diagram that indicates how line AB meets line CD at a right angle. Write the mathematical symbol for this relationship.

Solution:

The diagram below indicates line AB meets line CD at a right angle, as indicated by the little box.



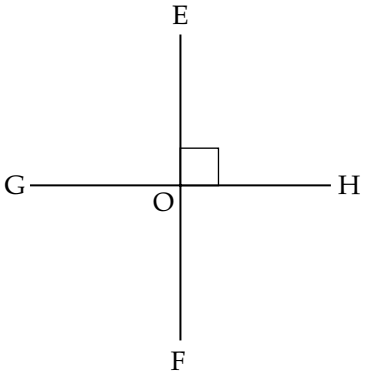
In math symbols, the relationship is written as AB ⊥ CD.

Example 2

If EF ⊥ GH, which are the right angles?

Solution:

Draw line EF perpendicular to line GH, with the two lines intersecting at right angles. Any of the four angles in the diagram are right angles.



∠EOH = 90° is shown in the diagram.

∠EOG and ∠EOH are the two angles sharing a straight line, HOG.

The straight angle, ∠HOG = 180°. If one angle measures 90°, the other must also measure 90°, so ∠EOG = 90°. Similarly, ∠GOF and ∠HOF are each 90°.

The walls in your house are perpendicular to the floor. Your body is perpendicular to the ground when you stand straight. Any two objects that meet at right angles are perpendicular to each other.

Parallel Lines

Parallel lines never meet. They are always the same distance apart. The best example would be railroad tracks. They run for many hundreds of kilometres and always stay exactly the same distance apart.

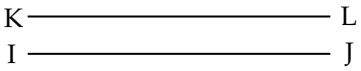
This property of lines can also be stated two ways:

- lines that are parallel are the same distance apart and never meet
- lines that are the same distance apart and never meet are parallel

Include this definition and how you can measure if two lines are parallel (from the example below) on your resource sheet.

Example 1

Is KL parallel to IJ?



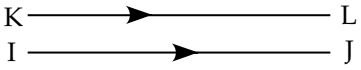
Solution:

To test for parallel, you need to know that the two lines never meet, which would be impossible to determine since lines continue on forever, or you need to know if they are the same distance apart in at least two places. If you use your ruler and find two equal distances, then the lines are parallel.

The mathematical symbol to indicate parallel lines when you are writing is //.

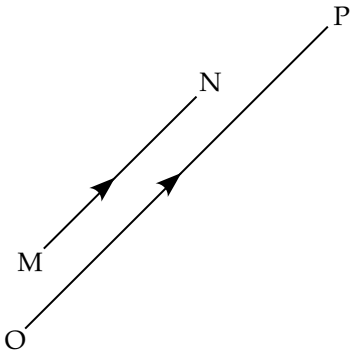
Therefore, you write KL//IJ.

You can also show that two lines are parallel on the diagram by making a set of arrows going the same way. The drawing would be as follows:



Example 2

Are MN and OP parallel?



Solution:

These lines are parallel to each other because the arrows on the lines indicate they are parallel lines. Thus, you can write MN//OP.

Neither Parallel nor Perpendicular Lines

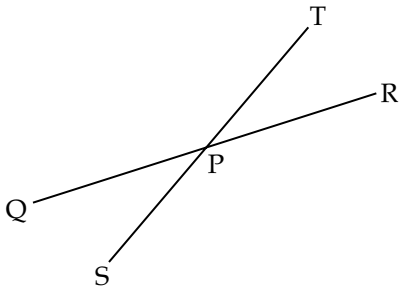
If two lines are not meeting at right angles, they are not perpendicular.

If two lines are not the same distance apart, they are not parallel.

There is no special term for such lines. They are neither parallel nor perpendicular lines. They are just intersecting lines.

Example

Are these lines parallel or perpendicular?



Solution:

Lines QR and ST are neither perpendicular nor parallel. They intersect at one point named P, and they do not intersect to form a right angle.

Learning Activity 5.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

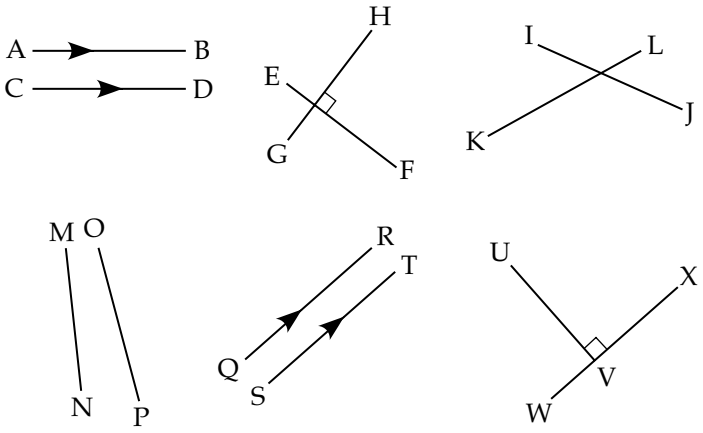
You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Write the following number in scientific notation: 356 300 000 000.
2. Write the following number as a whole number: 45.87×10^{-4}.
3. When you were 3, your brother's age was double yours. How much older is he?
4. Put the following numbers in order from smallest to largest: 43, 12, 3.5×10^{1}, 760%, 21.
5. Your brother asks you to trade him coins for a $5 bill. He gives you 4 nickels, 3 dimes, 5 quarters, and 3 loonies. Is this a fair trade?

Part B: Parallel and Perpendicular Lines

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify any sets of perpendicular lines, and explain why they are perpendicular.



continued

2. From the diagrams in #1 above, identify any sets of parallel lines, and explain why they are parallel.
3. From the diagrams in #1 above, identify any sets of lines that are neither parallel nor perpendicular, and explain why.

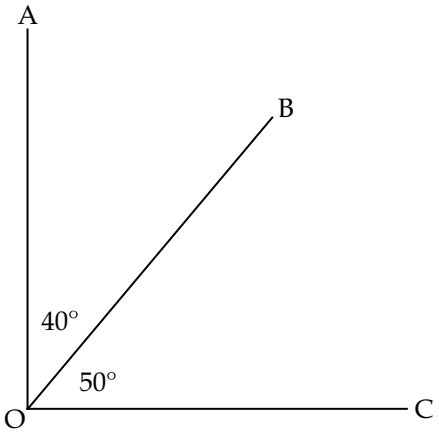
Angles

Complementary Angles

If two angles add to 90°, you say they are complementary to each other. Or you would say one is the complement of the other.

Example 1

Are ∠AOB and ∠BOC complementary?



Solution:

∠AOB = 40°

∠BOC = 50°

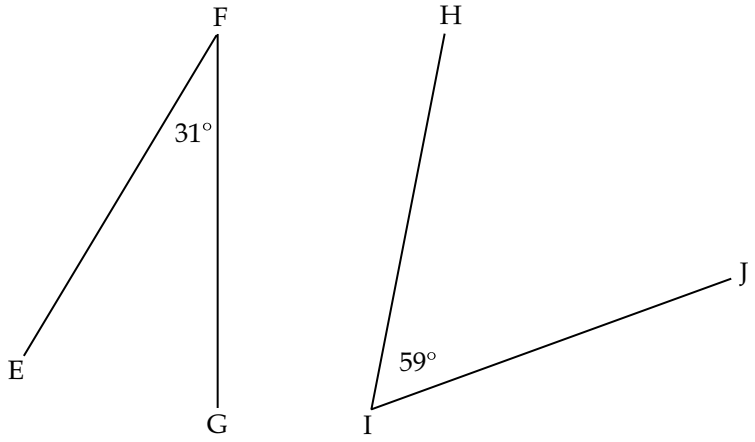
∠AOB + ∠BOC = 40° + 50° = 90°.

The two angles are complementary because their sum is 90°. ∠AOB is the complement of BOC, or ∠BOC is the complement of ∠AOB. Either way is correct.

Sometimes the angles can be separated from each other and still be complementary.

Example 2

Are ∠EFG and ∠HIJ complementary?



Solution:

∠EFG = 31°

∠HIJ = 59°

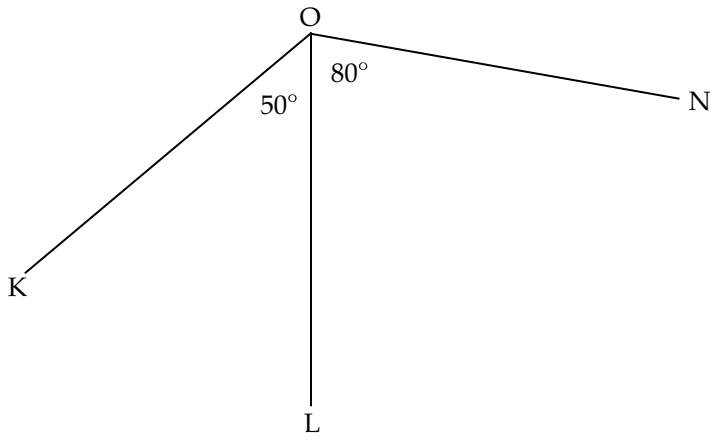
∠EFG + ∠HIJ = 31° + 59° = 90°

Since the two angles add together to make 90°, they are complementary.

∠EFG is the complement of ∠HIJ, or ∠HIJ is the complement of ∠EFG. Either way is correct.

Example 3

Are ∠KOL and ∠NOL complementary?



Solution:

$\angle KOL = 50°$

$\angle LON = 80°$

$\angle KOL + \angle LON = 50° + 80° = 130°$

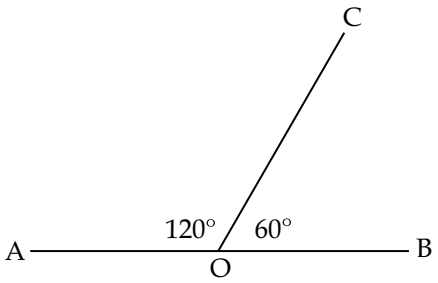
The measures of the two angles do not add up to 90°, so they cannot be complementary.

Supplementary Angles

If two angles added together total 180°, you say one is the supplement of the other.

Example 1

Are $\angle AOC$ and $\angle COB$ supplementary?



Solution:

$\angle AOC = 120°$

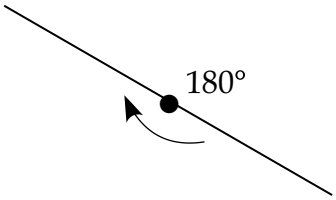
$\angle COB = 60°$

$\angle AOC + \angle COB = 120° + 60° = 180°$

Since the total of the measures of the two angles = 180°, they are supplementary.

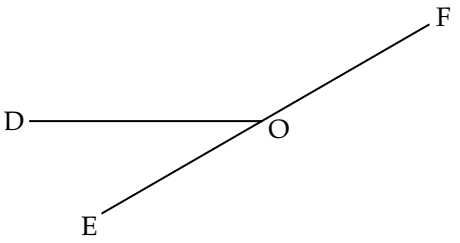
$\angle AOC$ is the supplement of $\angle BOC$ or $\angle BOC$ is the supplement of $\angle AOC$. Either way is correct.

Any two angles sharing a straight line, making a straight angle and having the same vertex, would be supplementary. Straight lines are also straight angles, measuring 180°.



Example 2

Are ∠DOE and ∠DOF supplementary?

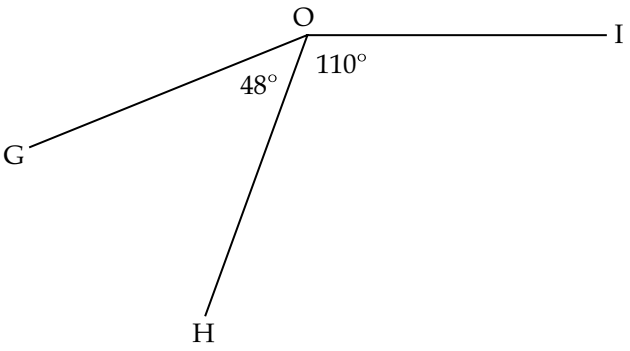


Solution:

Even though you do not know their measurements, ∠DOE and ∠DOF are supplementary. They supplement each other because they share a straight line, making a straight angle of 180° at the common vertex O.

Example 3

Is ∠GOH a supplement of ∠HOI?



Solution:

These two angles do not add together to make 180°, so they are not supplements of each other. Also, ∠GOI is not a straight angle.

Note: To remember whether complementary or supplementary means 90°, just remember that C comes before S in the alphabet, and so 90° comes before 180°.

Learning Activity 5.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Isabella gets paid double-time on statutory holidays. This past week was Labour Day. If she works 8 hours that day at $9/hour, how much will she be paid?
2. Courtney is drawing out her name in art class. She changes the colour of each letter so that the colours go from red to blue to green before starting over again. What colour will the 'y' in her name be?
3. Subtract: $\frac{9}{2} - \frac{5}{2}$.
4. You have 6 blueberries, 4 raspberries, and 8 slices of strawberry in your bowl of cereal. If you get at least one piece of fruit with every mouthful, what is the maximum number of bites it takes to finish your breakfast?
5. The angle at the corner of a picture frame is 90°. It is bisected by a cut, where the two sides have been attached together. What is the measure of the bisected angle?

continued

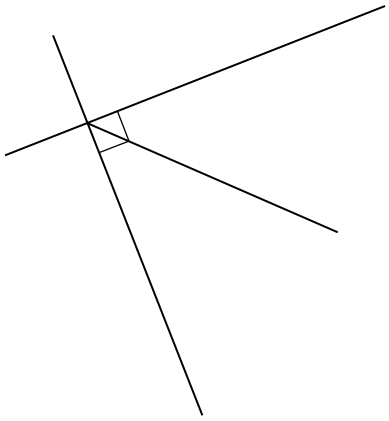
Learning Activity 5.7 (continued)

Part B: Complementary and Supplementary Angles

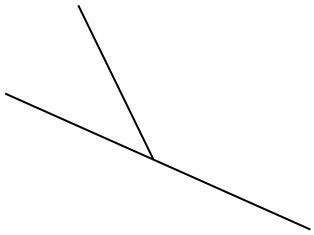
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the following sets of angles to answer the questions below.

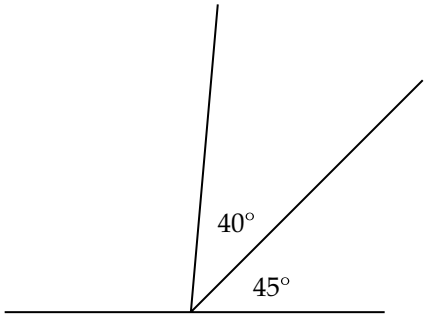
Set A



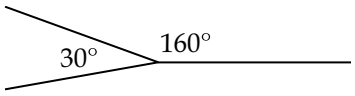
Set B



Set C



Set D



1. Which set(s) of angles illustrates supplementary angles, and why?
2. Which set(s) of angles illustrates complementary angles, and why?
3. Which set(s) of angles illustrates neither supplementary nor complementary angles, and why?
4. Draw an example of complementary angles.
5. Draw an example of supplementary angles.
6. What is the measure of the complement to an angle of 35°?

Lesson Summary

This lesson introduced you to perpendicular and parallel lines. You also investigated complementary and supplementary angles.

The next lesson introduces angle relationships using parallel lines with a transversal.

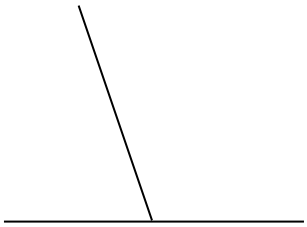
Assignment 5.4

Total: 20 marks

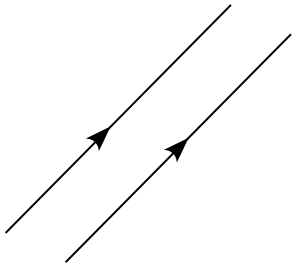
Lines and Angles

Use these sets of lines to answer questions 1 to 3.

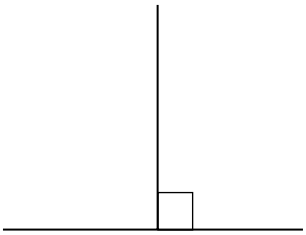
Set A



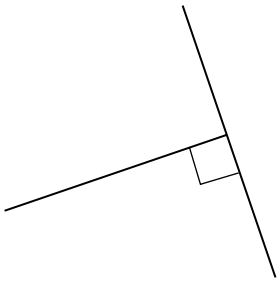
Set B



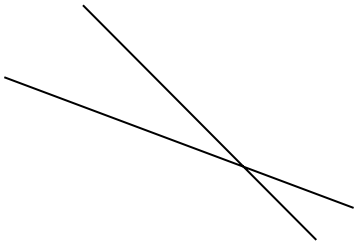
Set C



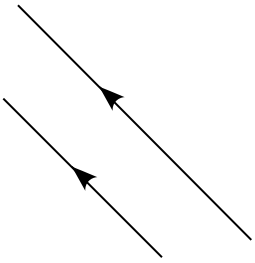
Set D



Set E



Set F



1. Identify which set(s) have perpendicular lines. *(2 marks)*

2. Identify which set(s) have parallel lines. *(2 marks)*

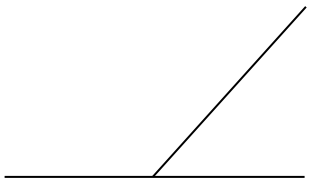
3. Identify which set(s) have neither perpendicular nor parallel lines. *(2 marks)*

continued

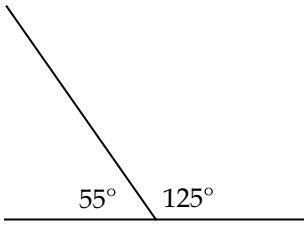
Assignment 5.4: Lines and Angles (continued)

Use these sets of angles to answer questions 4 to 6.

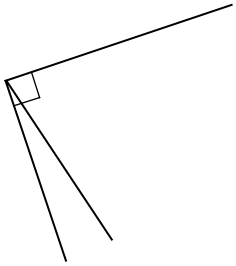
Set A



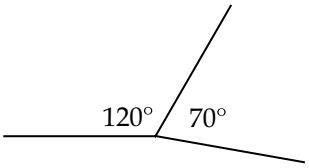
Set B



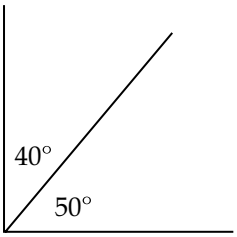
Set C



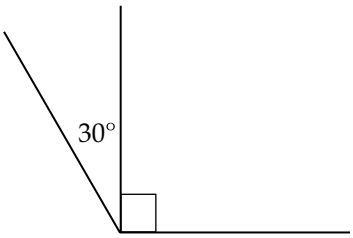
Set D



Set E



Set F



4. Identify which set(s) have neither complementary nor supplementary angles. *(2 marks)*

5. Identify which set(s) have complementary angles. *(2 marks)*

6. Identify which set(s) have supplementary angles. *(2 marks)*

continued

Assignment 5.4: Lines and Angles (continued)

7. Find the complement of each angle. *(4 marks)*

   a) 30° _____

   b) 15° _____

   c) 22° _____

   d) 89° _____

8. Find the supplement of each angle. *(4 marks)*

   a) 90° _____

   b) 115° _____

   c) 75° _____

   d) 162° _____

Notes

Lesson 5: Parallel Lines and Angles

Lesson Focus

In this lesson, you will

- ❑ determine the measures of angles involving parallel lines and a transversal
- ❑ understand the relationships of angles formed by parallel lines and a transversal

Lesson Introduction

Angles and parallel lines have a special relationship. This lesson explores those connections. You will need your protractor for measuring angles.

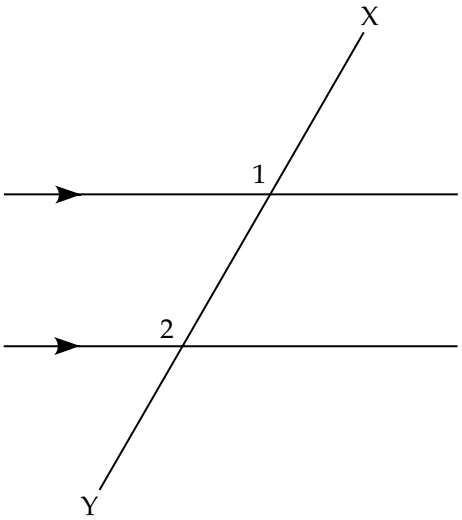
Lines and Angles

Transversal

A transversal is a line that intersects two or more lines. The lines may or may not be parallel.

In Figure 1, XY is a transversal. It creates 8 angles with the two parallel lines.

Figure 1

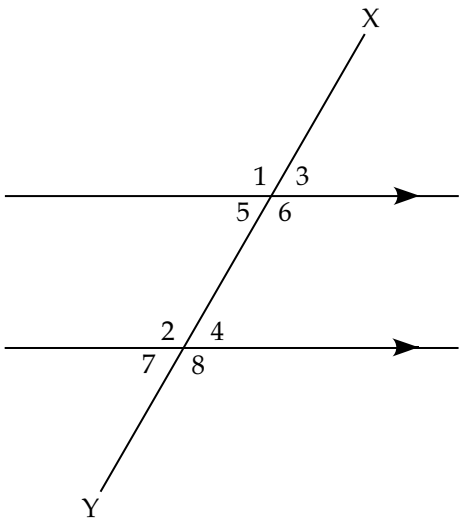


Use your protractor to find the measure ∠1 and ∠2.

You should discover that they have the same measurement.

These angles are called "corresponding" angles because they are similarly situated.

Figure 2



Use your protractor to find any other sets of angles that share the same measurement, and record your results.

You should discover that ∠1, ∠2, ∠6, and ∠8 all have the same measurement, and are all obtuse angles.

Similarly, ∠3, ∠4, ∠5, and ∠7 share identical measurements and are all acute angles.

Corresponding Angles

Using Figure 2, you can see which angles correspond with others where they are similarly situated. Sometimes you refer to corresponding angles using the slide effect. If an angle can slide up or down the transversal and sit exactly on top of another angle, the two angles are said to correspond equally.

Look closely at Figure 2, and notice how the following angles can slide over each other, or correspond:

∠1 and ∠2

∠3 and ∠4

∠5 and ∠7

∠6 and ∠8

Corresponding angles of parallel lines using a transversal are always equal and share the same measurement. If the lines are not parallel, the corresponding angles are not equal.

Stated in reverse, if two angles along a transversal correspond equally, then the lines are parallel. If the corresponding angles are not equal, the lines are not parallel.

You should note this, and Figure 2, on your resource sheet (or record the page number).

Alternate Angles

To alternate is to go back and forth. In geometry, to alternate means you go from one side to the other side of the transversal.

Figure 3

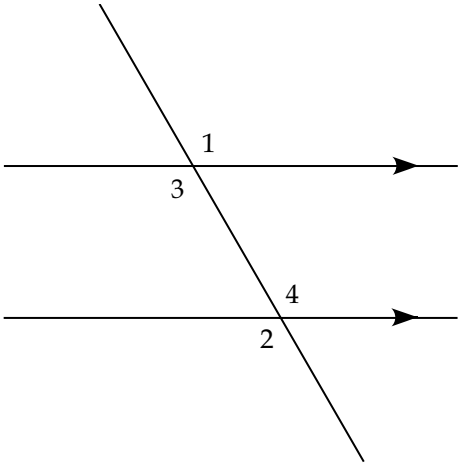


Figure 3 shows ∠1 and ∠2 alternating on the exterior of the parallel lines. They are called exterior alternate angles.

∠3 and ∠4 are inside the parallel lines, and they alternate on either side of the transversal. They are called interior alternate angles.

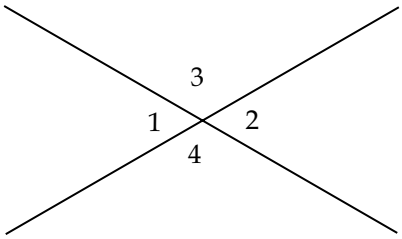
Exterior alternate angles always have the same measurements when the lines are parallel to each other. Likewise, interior alternate angles are always equal when lines are parallel.

Stated in reverse, if the alternate angles across a transversal are equal, then the lines are parallel.

Vertically Opposite Angles

Any time two lines intersect, 4 angles are formed.

Figure 4



Use your protractor to measure ∠1 and ∠2 in the diagram. Did you find that they had the same measurement?

You could also measure ∠3 and ∠4. They too are equal.

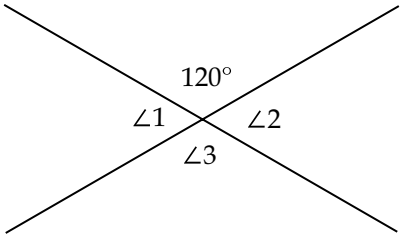
These angles, ∠1 and ∠2, or ∠3 and ∠4, are called vertically opposite angles. Vertically opposite angles are always equal.

Why are Vertically Opposite Angles Equal?

In Figure 4, ∠2 and ∠3 share a line and are supplementary. ∠2 and ∠4 share a line and are supplementary. Since ∠2 + ∠3 = 180° and ∠2 + ∠4 = 180°, then ∠3 and ∠4 should be equal.

Example

Find the measure of ∠1, ∠2, and ∠3.



Solution:

Supplementary angles share a straight line, or 180° angle.

∠1 is supplementary to the angle whose measure is 120°.

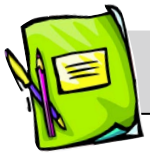
∠1 must be 180° – 120° = 60°.

∠2 is supplementary to the angle whose measure is 120°, so it also must be 180° – 120° = 60°.

Thus, ∠1 and ∠2 are the same measure and are equal. They are equal vertically opposite angles.

∠3 and the 120° angle are vertically opposite each other, and they are equal. The measure of ∠3 is 120°.

Notice that there are two acute angles and two obtuse angles formed when two lines intersect. However, if the lines are perpendicular, all 4 angles will be right angles.



Learning Activity 5.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are facing North. If you turn 360° to your right, which direction are you facing?
2. Because you have a doctor's appointment, you have to leave work at 3:45 instead of 4:30 pm. How much time are you penalized?
3. The size of the park across from your house is about 23 yards across. What is this distance in feet?
4. There are 3 boys and 2 girls invited to your birthday party. If each boy eats 2 pieces of cake and each girl eats 1 piece, how many pieces of cake will be eaten (not including what you eat).
5. If each piece of the cake (from the question above) is $\frac{1}{9}$th of the cake, will there be enough cake for you to have a piece?

continued

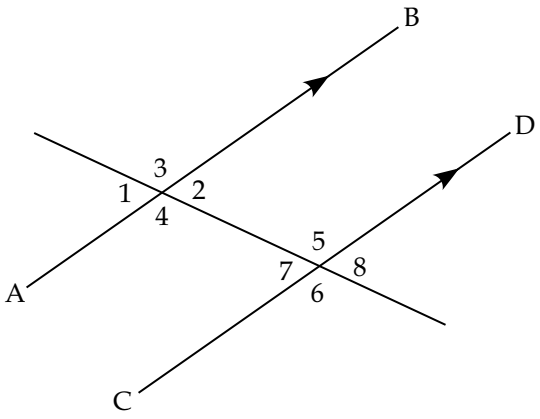
Learning Activity 5.8 (continued)

Part B: Transversal Angles (1)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use Figure 5 to answer the following questions. Given AB//CD.

Figure 5



1. Identify all corresponding angles.
2. Identify all exterior alternate angles.
3. Identify all interior alternate angles.
4. Identify all vertically opposite angles.

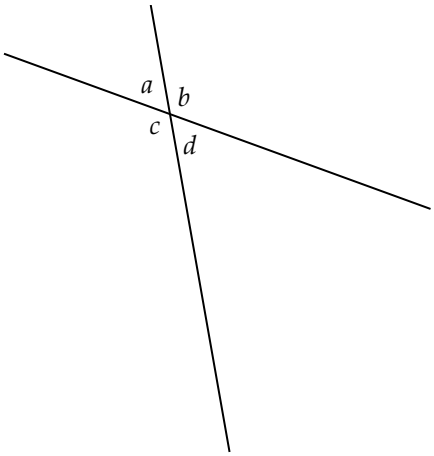
Supplementary Angles

In a previous lesson, you discussed how angles that share a straight line are supplementary, and add together to total 180°.

Example

1. Identify all the supplementary angles in Figure 6.
2. Identify the vertically opposite angles in Figure 6.

Figure 6



Solution:

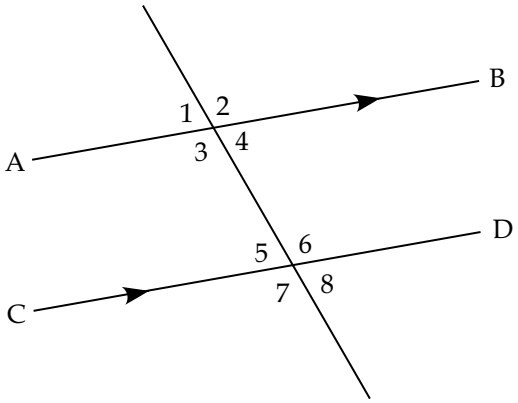
1. Angles *a* and *b* are supplementary.
   Angles *a* and *c* are also supplementary.
   Notice how they share a straight line of 180°.
   Angles *b* and *d* are supplementary.
   Angles *c* and *d* are also supplementary.
2. Angle *a* is vertically opposite *d*. They are equal.
   Angle *b* is vertically opposite *c*, and they are equal.

Finding Measurements without the Protractor

The properties of angles with parallel lines and a transversal can be used to find the measurements of all the angles, if given just one angle.

Example

Given AB//CD and ∠5 = 70°. Find all other angle measurements. Give a reason for each of your answers.



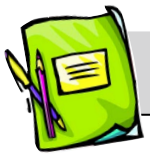
Solution:

∠5 = 70°	given
∠1 = 70°	corresponds with ∠5
∠2 = 110°	supplement to ∠1
∠4 = 70°	vertically opposite to ∠1 or supplement to ∠2 or interior alternate with ∠5
∠3 = 110°	supplement to ∠1 or ∠4 or vertically opposite to ∠2
∠6 = 110°	supplement to ∠5 or interior alternate to ∠3 or corresponding to ∠2
∠8 = 70°	vertically opposite to ∠5 or supplement to ∠6 or corresponding angle with ∠4 or exterior alternate angle to ∠1
∠7 = 110°	exterior alternate angle with ∠2 or corresponding to ∠3 or supplementary to ∠5 or ∠8 or vertically opposite to ∠6

Include the vocabulary used to describe the relationships between angles (corresponds, interior alternate, etc.) on your resource sheet along with their meaning. A diagram would be helpful to include as well, or refer to the diagram you copied from page 74.

There are many possible explanations or reasons you can use to explain why certain angles have the measurements they do. Usually you can use many of the angle properties to justify each measurement. All are correct. Only one is necessary.

Also note that all the angles are either acute or obtuse. All the acute angles are 70°, all the obtuse angles are 110°. You can see by looking at the diagram which are the acute angles and which are the obtuse angles. These size relationships will always be true unless the transversal is perpendicular to the parallel lines, in which case all angles are 90°.



Learning Activity 5.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The Blackhawks have won twice as many games as the Maple Leafs. The Maple Leafs have won 5 fewer games than the Oilers. If the Oilers have won 13 games, how many games have the Blackhawks won?
2. Kaitlin types 50 words per minute. It took her 30 minutes to write her essay for English. Assuming she was typing the whole time, how many words are in her essay?
3. Write 0.456 as a percent.
4. What are complementary angles?
5. How do you calculate net pay?

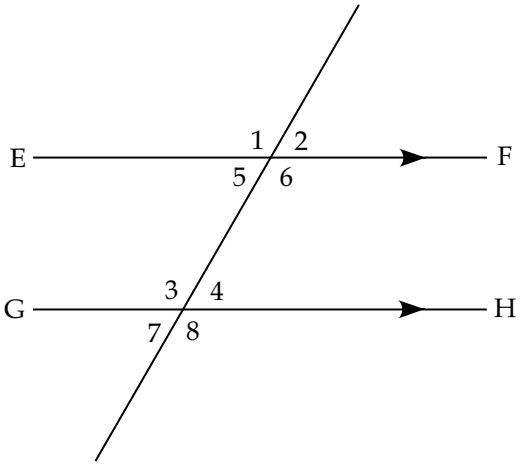
continued

Learning Activity 5.9 (continued)

Part B: Transversal Angles (2)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Given EF //GH and ∠1 = 120°. Use the properties of corresponding angles, alternate angles, supplements, and vertically opposite angles to find the measurements of the angles numbered ∠2 to ∠8. List at least one reason for each statement.



Lesson Summary

This lesson introduced you to some angle properties when working with parallel lines and a transversal. You practised solving angle problems using five different angle relationships.

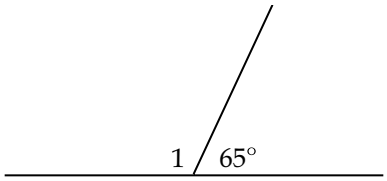
The last lesson in this module asks you to prove lines are perpendicular or parallel, and provides some problem solving.

Assignment 5.5

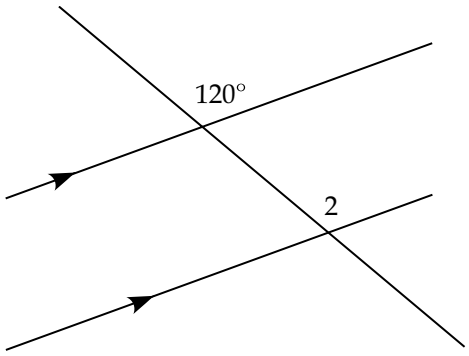
Total: 24 marks

Parallel Lines and Angles

1. Give a reason why $\angle 1 = 115°$ in the diagram below. *(2 marks)*



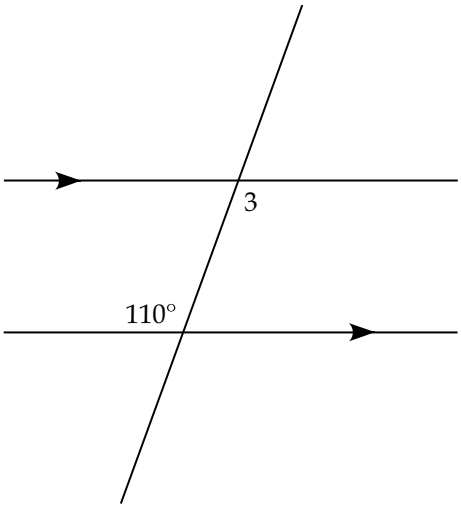
2. Give the property to explain why $\angle 2 = 120°$. *(2 marks)*



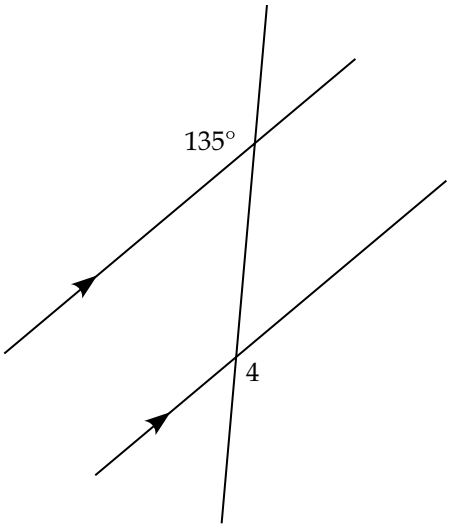
continued

Assignment 5.5: Parallel Lines and Angles (continued)

3. Give a reason why $\angle 3 = 110°$. *(2 marks)*



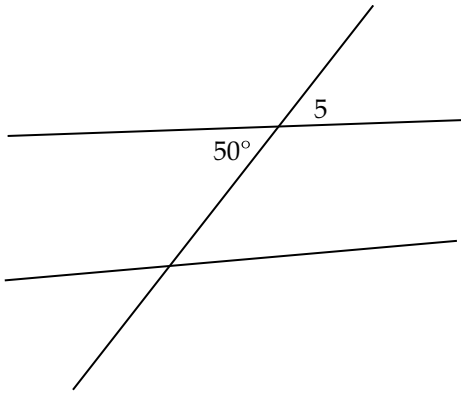
4. Give the property to explain why $\angle 4 = 135°$. *(2 marks)*



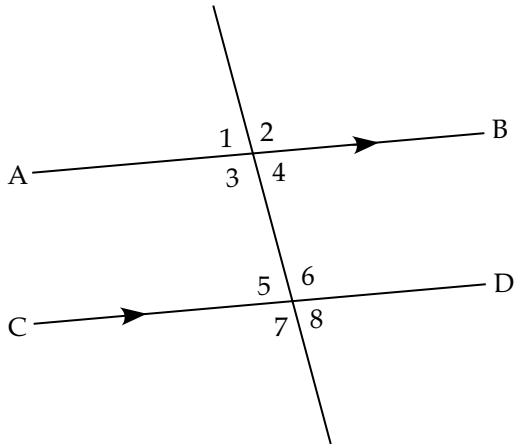
continued

Assignment 5.5: Parallel Lines and Angles (continued)

5. Give a reason why ∠5 = 50°. *(2 marks)*



6. Use angle properties with parallel lines and a transversal to find the measurement for each angle. State a reason for each answer. You are given that AB//CD and ∠5 = 80°. *(14 marks)*



Notes

Lesson 6: Perpendicular or Parallel

Lesson Focus

In this lesson, you will

- ❑ determine if lines are perpendicular or parallel
- ❑ solve a problem using angles on parallel lines and a transversal

Lesson Introduction

How can you determine if sets of lines are perpendicular or parallel to each other?

This lesson explores different strategies for answering this question.

Perpendicular and Parallel Lines

Rarely in mathematics are you allowed to assume things to be true. You need proof. You now have all the necessary skills to establish proof that lines are perpendicular or parallel.

Proving Lines are Perpendicular

The definition of perpendicular is two lines meeting at a right angle. If you can prove that the angle is a 90° angle, then the lines must be perpendicular.

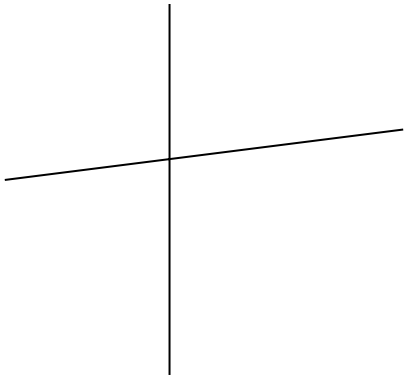
A few of ways to show an angle is a right angle are discussed.

Measurement

One method is to use a protractor and actually measure the angle. If its measurement is 90°, then the lines must be perpendicular.

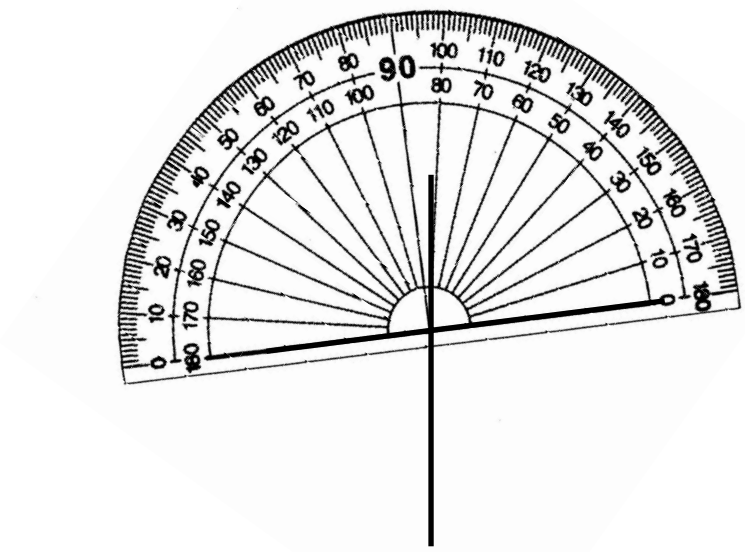
Example

Are these two lines perpendicular?



Solution:

Slide the centre point of the protractor along the baseline, and onto the centre of the line set. Is the angle 90°?



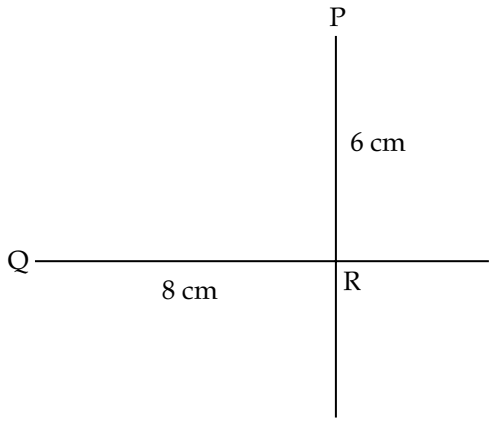
The measurement on the protractor is not 90°, so the lines cannot be perpendicular.

Pythagoras

If you know the lengths of the lines, you could construct a triangle. If the sides satisfy the condition that $a^2 + b^2 = c^2$, then by the Pythagorean theorem, the triangle is a right triangle. You will need a ruler to measure each side. (To learn more about Pythagoras and his theorem, look in the module on trigonometry in Lesson 3.)

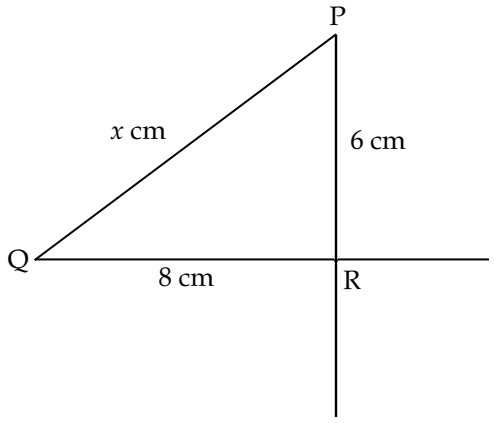
Example

Two measurements for the lines are shown in the diagram. PR = 6 cm and QR = 8 cm. Are the lines perpendicular?



Solution:

Construct a line joining the endpoints of the two lines whose measurements are known. Join Q and P.



Apply the Pythagorean theorem to ΔPQR to determine if the sum of the squares of the sides equals the square of the hypotenuse.

$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = x^2$$

$$36 + 64 = x^2$$

$$100 = x^2$$

$$10 = x$$

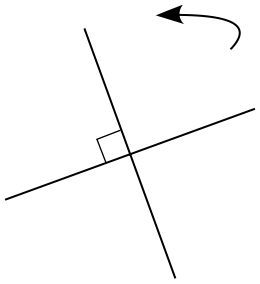
Measure the length of the hypotenuse. If it is 10 cm, then you can conclude that the measurements satisfy the Pythagorean theorem, and ΔPQR is a right triangle with the hypotenuse, PQ, being the longest side. The right angle must be at R.

If x is not of length 10 cm, then there is no right angle at R.

Rotation Symmetry

Another way to determine if two lines are perpendicular is to use rotation symmetry. In Grade 9 you might have studied rotation symmetry. In this course, rotations are discussed more fully in the module on transformations.

Consider the diagram below. If you rotate the lines 90° counterclockwise about the vertex, the lines should fall perfectly on top of each other on all sides. You could do this rotation four times, and each rotation of 90° would have the lines sitting on top of each other.



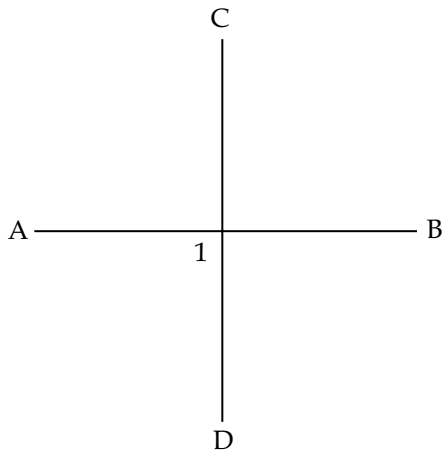
Therefore, if you are given two lines and you don't know whether they are perpendicular or not, you could try rotating them 90°. If the lines sit perfectly atop each other after the rotation, then the lines must be perpendicular.

Definition

The easiest method is probably to use the definition of perpendicular. If you are told the angle is a right angle of 90°, then, by definition, the lines are perpendicular.

Example

Are AB and CD perpendicular to each other when ∠1 = 90°?



Solution:

You are given ∠1 = 90° and are asked to prove AB ⊥ CD. Since the lines intersect at an angle of 90°, then all other angles around the centre must also be 90°. By definition, AB ⊥ CD.

Proving Lines are Parallel

Lines are parallel if they are exactly the same distance apart and never meet. Since it is impossible to prove that two lines never meet, you will either measure the distance between the lines or use the angle properties to prove a set of lines is parallel.

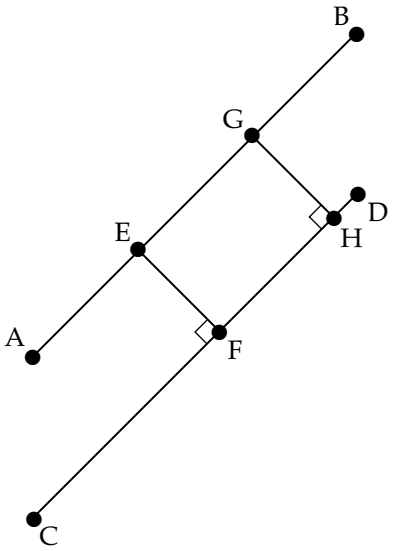
Measurement

You could use your ruler and measure the distance between two lines at various locations. If they prove to be the same distance apart every time, then the lines would be parallel.

You want to find the shortest distance between the lines for each measurement.

Example

Are AB and CD parallel to each other?



Solution:

Using a ruler, you measure EF and GH. You ensure that both EF and GH are perpendicular to AB and CD. This way, you know you are measuring the shortest distance between the two lines.

Note that you could use the set square in your geometry set to measure the perpendicular distances without having to draw the lines EF and GH.

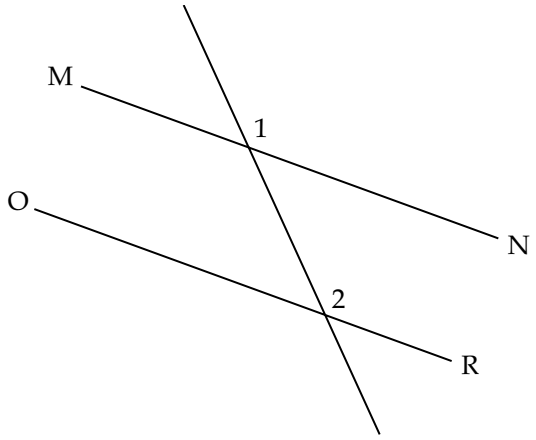
You find measurements for EF and GH are the same. The lines AB and CD are the same distance apart. You can say that line AB is parallel to line CD by definition, and you can write AB//CD.

Corresponding Angles

You learned in an earlier lesson that if two corresponding angles along a transversal are equal, the lines must be parallel.

Example

Are MN and OR parallel to each other when you are given $\angle 1 = \angle 2$?



Solution:

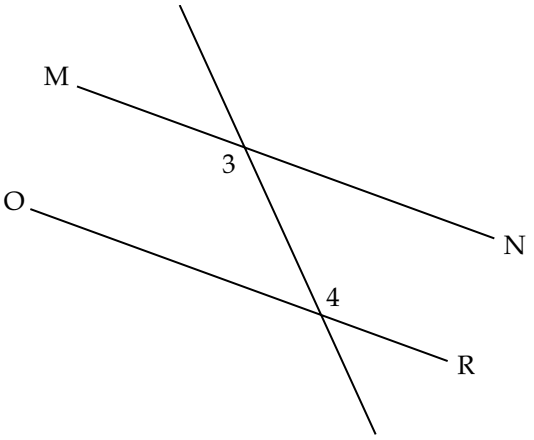
Since you are given $\angle 1 = \angle 2$, and since these angles are corresponding angles, then the lines must be parallel to each other. You can write MN //OR.

Alternate Angles

If two angles, either interior alternate or exterior alternate situated across a transversal, are equal, then the lines must be parallel.

Example 1

Are MN and OR parallel to each other if $\angle 3 = \angle 4$?

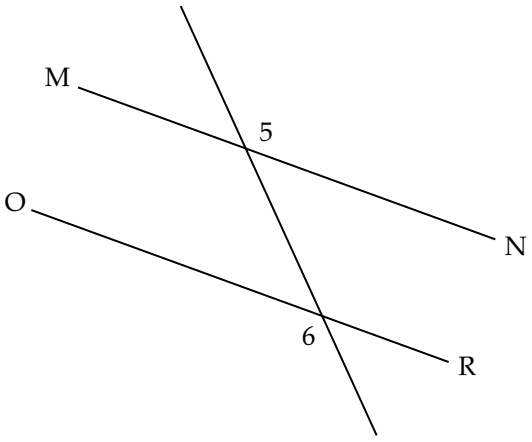


Solution:

Since you are given $\angle 3 = \angle 4$ and these angles are interior alternate angles along the transversal, then MN//OR.

Example 2

Given $\angle 5 = \angle 6$, are MN and OR parallel to each other?



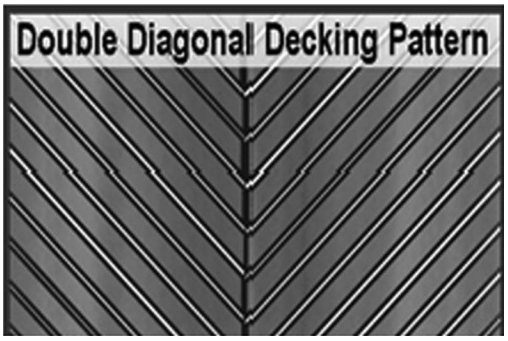
Solution:

Since $\angle 5$ and $\angle 6$ are exterior alternate angles along the transversal, and since they are equal, then MN//OR.

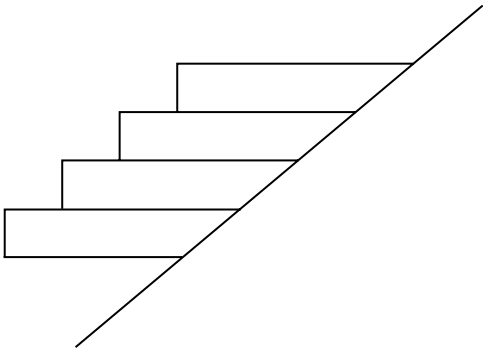
Application Problem

Example

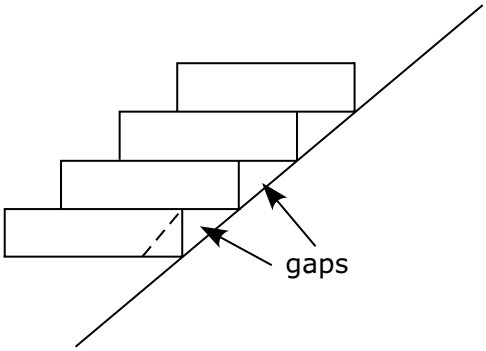
You are planning to build a wooden deck, and the boards must be cut to fit snugly along one edge. You have decided to place the boards using a diagonal pattern. Recall the diagram from Lesson 3 where you were looking at slanting the boards towards each other.



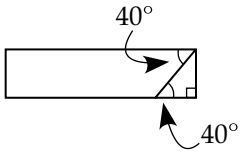
You now have the problem of fitting the boards on a slant against the top side of the deck. You want the top of the deck to look like this.



But before you cut the boards, there are gaps.



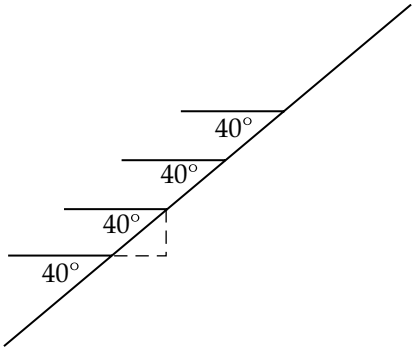
Each board must be cut on an angle to eliminate the gaps. Using a scrap piece of wood and trial and error to find the exact angle, you determine the angle of the cut to be 40°.



1. Will you need to determine the angle of the cut for each board or just the first board?
2. What will be the angle of each cut?
3. How do you know this to be true?

Solution:

1. Because the boards are parallel, and the corresponding angles slide along a transversal, the angles are all the same. The cut angle will be the same for all the boards.



2. Each cut will be 40° since that is the measure of the first angle you have determined.
3. Since you have parallel lines and a transversal, the corresponding angles must be the same.

All the boards fit nicely along the edge. They all have the same angle. You will have filled in the gaps.

Learning Activity 5.10

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

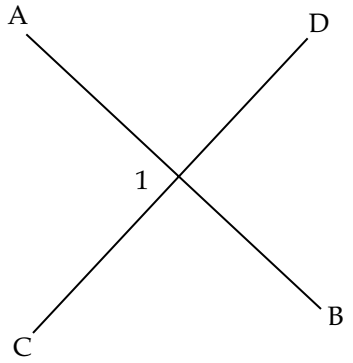
You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What are 'supplementary angles'?
2. In Winnipeg, Fort Street and Garry Street are parallel to each other. Will these two streets ever cross?
3. In Winnipeg, Main Street and Broadway are perpendicular to each other. Will these two streets ever cross?
4. Your bed is 2 m long. If you are 180 cm tall, will you fit on your bed?
5. It is 11:00 am right now. You work until 12:00 noon, go for lunch for 1 hour, then go back to work for another 3 hours. It takes you an hour to drive home. The clock in your house reads 18:00. Is this correct?

Part B: Transversal Angles (3)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given $\angle 1 = 90°$. Why is AB $\perp$ CD? Explain your reasoning.



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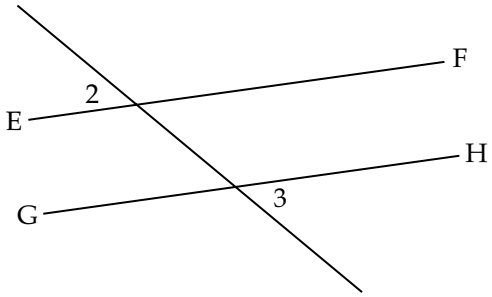
Learning Activity 5.10 (continued)

2. Given $\angle 2 = \angle 3$, why is EF//GH? Explain your reasoning.



Lesson Summary

In this lesson, you learned four ways to show that two lines would be perpendicular to each other and two ways to show two lines are parallel to each other. You applied your knowledge to solving a problem involving building a deck.

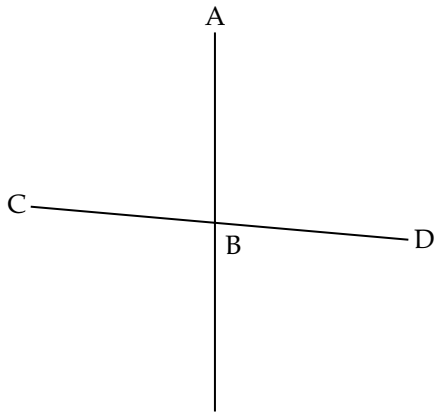
This completes your studies with angles and lines.

Assignment 5.6

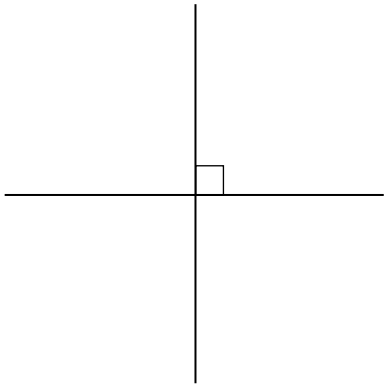
Total: 11 marks

Perpendicular or Parallel

1. Use your protractor to determine if these lines are perpendicular. State the measurement you found, and whether or not the lines are perpendicular. *(3 marks)*



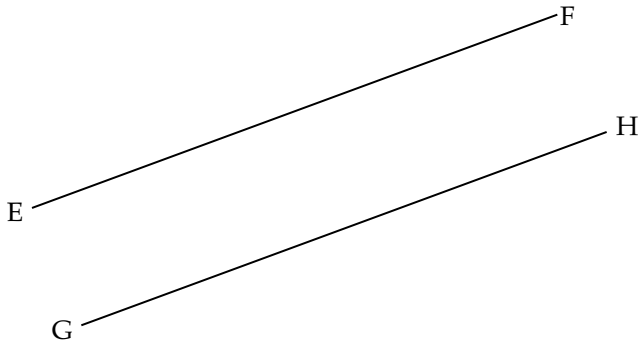
2. Explain how you know these lines are perpendicular. *(2 marks)*



continued

Assignment 5.6: Perpendicular or Parallel (continued)

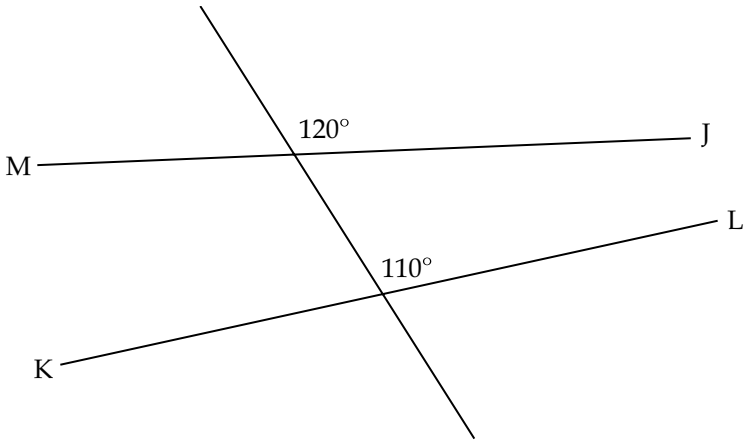
3. Use your ruler or set square to determine whether or not these lines are parallel. Explain your answer. *(2 marks)*



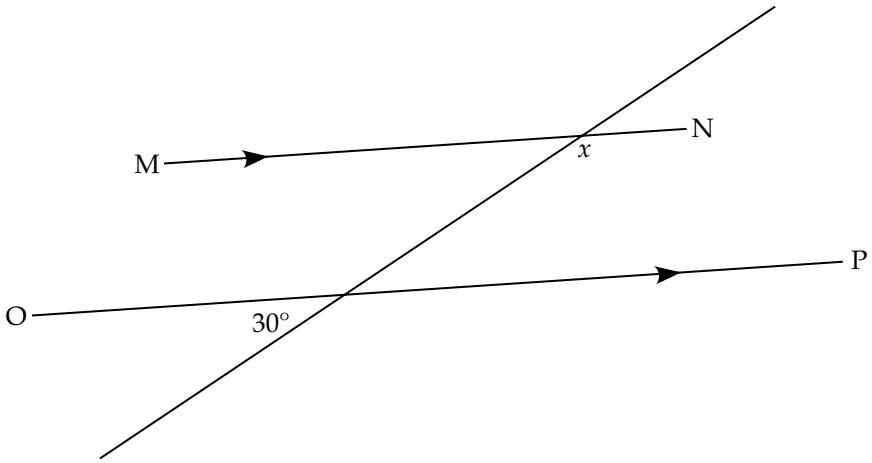
continued

Assignment 5.6: Perpendicular or Parallel (continued)

4. Is MJ//KL? Explain why or why not. *(2 marks)*



5. You are making a quilt and have to cut pieces to fit a pattern. The angle of the piece you need to cut is indicated by x in the diagram below. Determine the angle of the cut you must make, and the reasoning behind your determination. You are given MN//OP. *(2 marks)*



Notes

Module 5 Summary

In this module, you identified, bisected, and copied angles. You learned the names of different types of angles and several relationships among them, especially with parallel lines. You learned to estimate the size of an angle. You learned how to tell if two lines are perpendicular or are parallel.

Congratulations! You have finished the first five of the modules in this course, only three more to go.

Vocabulary

Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful.

acute angle
alternate angles
angle
arc
bisect
carpenter's square
centre
clockwise (CW)
compasses
complementary angles
corresponding angles
counterclockwise (CCW)
degrees
initial arm
intersection point
obtuse angle
overlay
parallel lines
perpendicular lines
protractor
Pythagorean Theorem
reflection
reflex angle
revolution
right angle
set square
straight angle
supplementary angles
template
terminal arm
tracing
transversal
vertex
vertically opposite angles

Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

Reflex angle = 360° – inside angle

Pythagoras Theorem: $a^2 + b^2 = c^2$

Submitting Your Assignments

It is now time for you to submit the Module 5 Cover Assignment and Assignments 5.1 to 5.6 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 5 assignments and organize your material in the following order:

Module 5 Cover Sheet

Module 5 Cover Assignment: Problem Analysis / Analysis of Games and Numbers

Assignment 5.1: Identify Angles

Assignment 5.2: Estimate Angles

Assignment 5.3: Copy and Bisect Angles

Assignment 5.4: Lines and Angles

Assignment 5.5: Parallel Lines and Angles

Assignment 5.6: Perpendicular or Parallel

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 10 Essential Mathematics (20S)

Module 5
Working with Angles

Learning Activity Answer Keys

MODULE 5:
WORKING WITH ANGLES

Learning Activity 5.1

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. How tall is a door in centimetres if it is 2.2 m tall?
2. The ratio comparing the distance on a map to the distance in real life is 1 cm: 1000 m. If the distance from your house to your school on the map is 4 mm, how far do you live from school?
3. If 36% of 500 is 180, what is 18% of 500?
4. Solve for w: $w \div 6 = 2$.
5. The Pythagorean Theorem is $a^2 + b^2 =$ ___ .

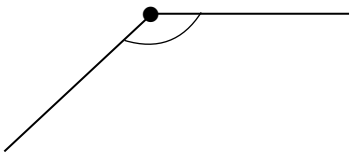
Answers:

1. 220 cm (100 cm = 1 m or two steps up from cm to m)
2. 400 m (convert 4 mm to cm to get 0.4 cm [one step down, divide by 10]; using the given ratio, $0.4 \times 1000 = 400$)
3. 90 (18% is half of 36% so $180 \div 2$)
4. $w = 12$ (Multiply both sides by 6: $2 \times 6 = 12$)
5. c^2 (Remember from Grade 8, c is the side across from the right angle.)

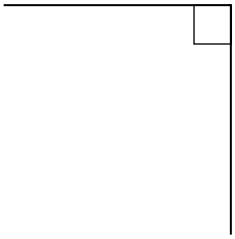
Part B: Estimating with Decimals

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

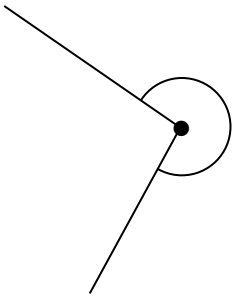
1. Identify each angle as acute, right, obtuse, straight, or reflex.

a) 

Answer: Obtuse

b) 

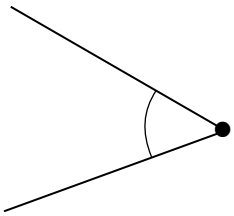
Answer: Right

c) 

Answer: Reflex

d) 

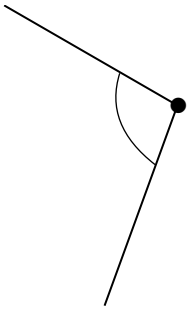
Answer: Straight

e) 

Answer: Acute

2. Use your protractor to find the measurements of the indicated angles.

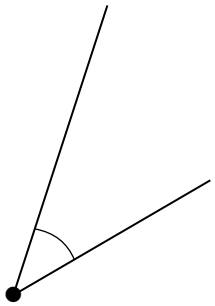
a)



Answer: 100°

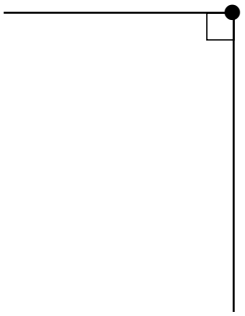
Note to students: The measurement may be slightly different than your answer because of changes in the size of the angle due to printing. You should have an answer that is ±3°.

b)



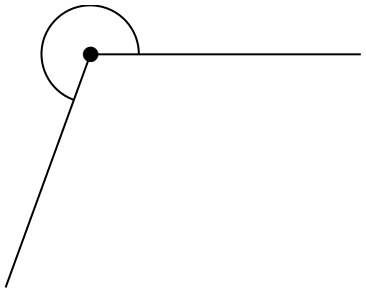
Answer: 42°

c)



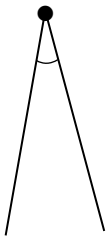
Answer: 90°

d)



Answer: Reflex angle = 360° – 110° = 250°

e)



Answer: 25°

Learning Activity 5.2

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Identify the type of angle that has a measure of 315°.
2. Convert 24 inches into feet.
3. You are supposed to be at work for 8:00 every morning. You arrive at work at 8:17 because a train delayed you. If the hour is divided into quarters when determining late penalties, how much time are you penalized?
4. You have received a $50 bill for your birthday. At the music store your total cost is $30.75. How much change will you get back?
5. Your curfew is at 12:00 midnight. On Saturday night you get to the restaurant with your friends at 6:00 pm, and eat dinner for 2 hours. You then walk over to the movie theatre (10 minutes) and wait for the movie to start for 25 minutes. The movie is 2 hours, 45 minutes long. You drive home, which takes 20 minutes. Do you get home before your curfew?

Answers:

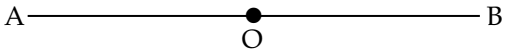
1. Reflex (more than 180° but less than 360°)
2. 2′ (1 foot = 12 inches, 24 ÷ 12 = 2)
3. 30 minutes (When assessed late penalties, the hour is divided into quarters—0:00, 0:15, 0:30, 0:45. Since 8:17 is passed the 0:15 quarter you are then penalized 0:30 or 30 minutes.)
4. $19.25 (Count up: 30.75 + 0.25 + 4 + 5 + 10 = $50)
5. Yes (Add the minutes: 10 + 25 + 45 + 20 = 100 minutes = 1 hour, 40 minutes; count up from 6:00, counting the minutes separate from the hours: 6:00 + 2:00 + 2:00 + 1:40 = 11:40 pm)

Part B: Sketching Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

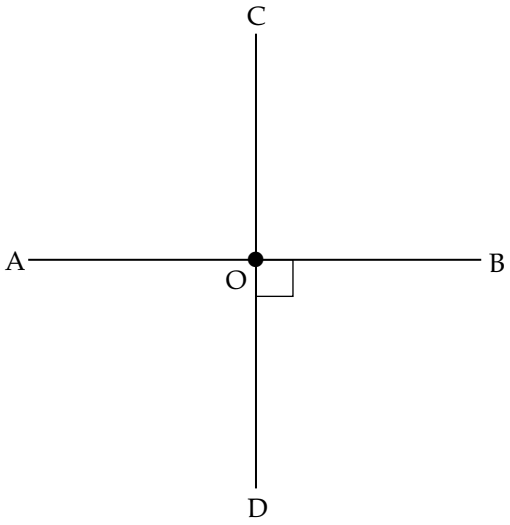
Do not use your protractor for these questions. Use your skills with subdividing angles.

1. Sketch a 90° angle with vertex at O, using the straight angle, ∠AOB.



Answer:

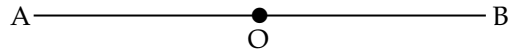
There are two possibilities, either above or below the line. Either one is correct.



OC and OD both meet line AOB at 90°.

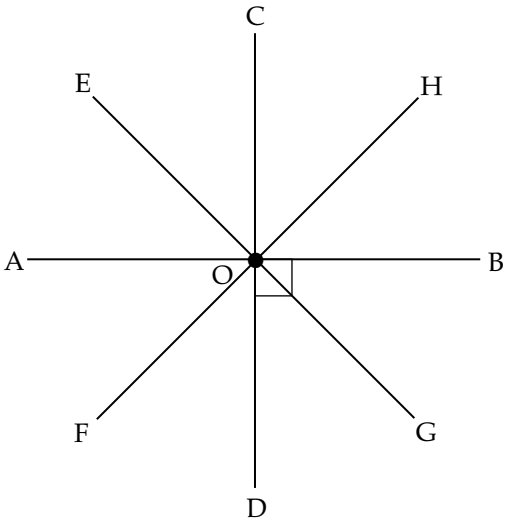
There are four right angles in the diagram, ∠COB, ∠BOD, ∠DOA, and ∠AOC.

2. Sketch an angle of 45°, having its vertex at O, using the straight angle, ∠AOB.



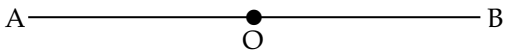
Answer:

There are four possibilities for drawing the right angle first. Then each right angle can be bisected. Any one of the eight angles is correct.



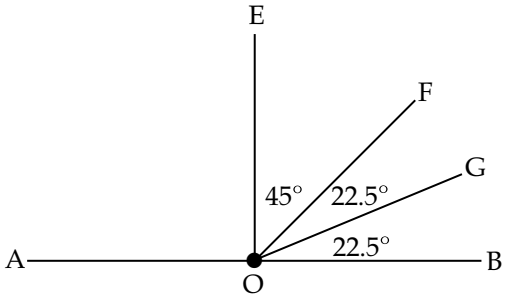
The lines, OE, OF, OG, and OH all bisect a 90° angle, and create 45° angles. The angles measuring 45° are ∠AOF, ∠FOD, ∠DOG, ∠GOB, ∠BOH, ∠HOC, ∠COE, and ∠EOA.

3. Sketch a 22.5° angle starting with a straight angle, ∠AOB, with the vertex at O.

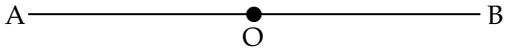


Answer:

First draw the right angle, and then bisect it twice to get an angle of 22.5°.

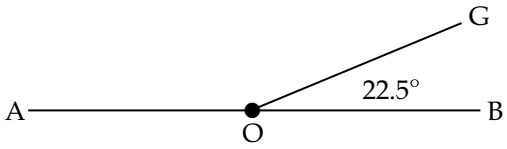


4. Sketch a 15° angle starting from a straight angle, ∠AOB, with the vertex at O.

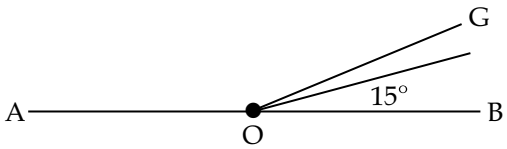


Answer:

Once you have drawn the angle of 22.5° you know that an angle of 15° is less than this, and a little bit more than half of it.



∠HOB = 15°



Learning Activity 5.3

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You know that your glass holds 1 cup of milk. Would this be a good referent to use to find out how much water your water bottle can hold?
2. The distance from the MTS Centre to Polo Park is 5.1 km. On your way to Polo Park from the MTS Centre you decide to stop at a doughnut shop halfway between the two. How far do you have to travel from the doughnut shop to get to Polo Park?
3. Your gross income is $980. Estimate your CPP (4.95%).
4. Travelling to Calgary, you notice that after 5 hours of driving you have travelled 515 km. How fast are you driving?
5. Solve for r: $\frac{r}{5} = \frac{10}{25}$.

Answers:

1. Yes, but it would not be very accurate. A water bottle can hold more than one cup of water, but remember that if it holds a fraction of a cup, you cannot be sure just how much is in it.
2. 2.55 mi. $\left(\frac{1}{2} \text{ of } 5.1 = \left(\frac{1}{2} \text{ of } 5\right) + \left(\frac{1}{2} \text{ of } 0.1\right) = 2.5 + 0.05\right)$
3. $49 (Round the CPP to 5%. 10% of 980 = 98, since 5 is half of 10, half of 98 = 49)
4. 103 km/hr.
   $\left(\frac{515 \text{ km}}{5 \text{ hr.}} = \frac{500 \text{ km} + 15 \text{ km}}{5 \text{ km}} = \frac{500 \text{ km}}{5 \text{ hr.}} + \frac{15 \text{ km}}{5 \text{ hr.}} = 100 + 3 \text{ km/hr.}\right)$
5. 2 (Multiply both sides by 5.)

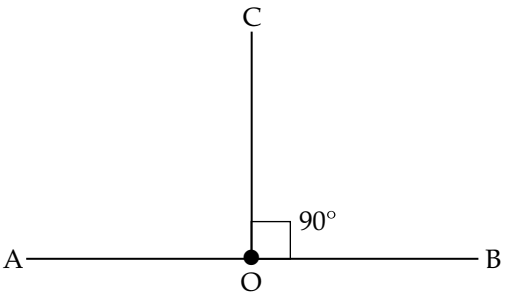
Part B: Sketching Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Do not use your protractor for these questions. Use your skills with subdividing angles.

1. Sketch a 90° angle coming from a straight angle, ∠AOB, with the vertex at O.

   Answer:

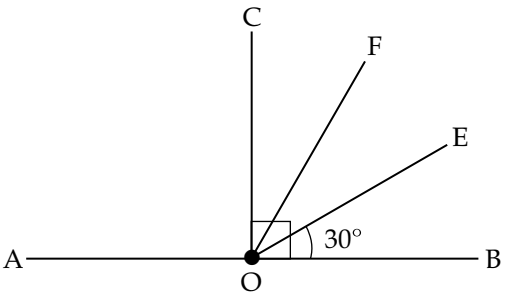


   You could also draw the angles from ∠AOB beneath the line.

2. Sketch a 30° angle coming from a straight angle, ∠AOB, with the vertex at O.

   Answer:

   First make a right angle and then subdivide it into thirds.

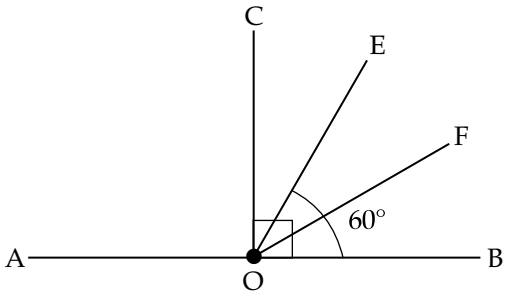


   ∠EOB, ∠FOE, and ∠COF are each 30°.

3. Sketch a 60° angle coming from a straight angle, ∠AOB, with the vertex at O.

   Answer:

   Sketch a right angle, and then divide it into thirds.



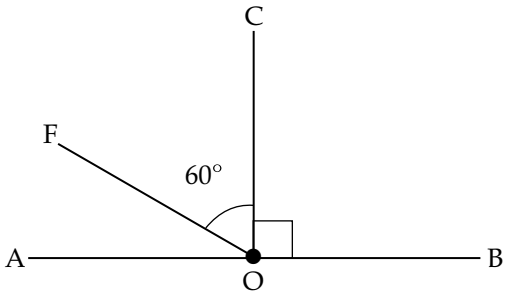
   ∠FOB, ∠EOF, and ∠EOC = 30°

   ∠EOB = ∠EOF + ∠FOB = 30° + 30° = 60°

4. Sketch a 150° angle coming from a straight angle, ∠AOB, with the vertex at O.

   Answer:

   Draw a 90° angle, and then add a 60° angle to it to make 150°.



   ∠COB = 90°

   ∠FOC = 60°

   ∠BOF = 90° + 60° = 150°

Learning Activity 5.4

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. An angle that has a measure of 110° is called a _____ angle.
2. Andy has been to the Great Pyramid of Egypt. He bought a miniature version of the pyramid, which was to scale. The ratio comparing the miniature and the real pyramid is 1 cm: 70 royal cubits (an ancient unit of measurement). If the height of the miniature is 4 cm, how tall is the real pyramid?
3. You are out for breakfast with your friends. You order coffee for $2.50, and orange juice for $3.75. For your meal, you order waffles with blueberries for $8.50. You also get a side order of fruit salad that costs $3.25. How much will your breakfast cost before taxes and tip?
4. Approximate the tip if you want it to be about 15% of your bill.
5. For every 8 hours he works, Brian is paid a flat rate of $50 plus 10% of his sales. If, in 8 hours, Brian sells $740 in merchandise, how much money will he make in total?

Answers:

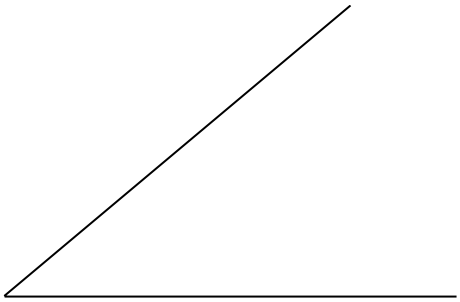
1. Obtuse (between 90° and 180°)
2. 280 royal cubits (4 × 7 = 28, add a zero because it is 70, not just 7. In more familiar units, the Great Pyramid is approximately 481 feet tall, or 146 m.)
3. $18 (add all the cents together: 50 + 75 + 50 + 25 = 200¢ = $2.00; add the dollars: 2 + 3 + 8 + 3 + 2 = $18)
4. $2.70 (10% of 18 = 1.8, so 5% of 18 = 0.9; total = 1.8+0.9)
5. $124 ($50 + 10% of 740 = 50 + 74 = $124)

Part B: Copying Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use your compasses to make a copy of each angle, showing all markings.

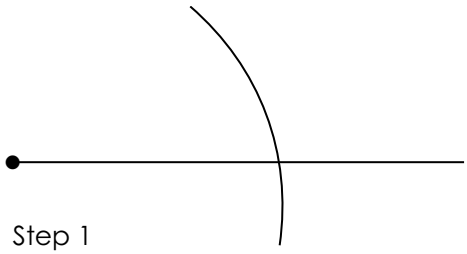
1.



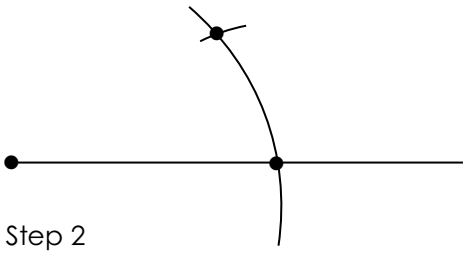
Use your protractor to measure the original angles, and your copies. They should read the same measurement as the original.

Notice the copied angle does not necessarily need to be placed the same way as the original angle.

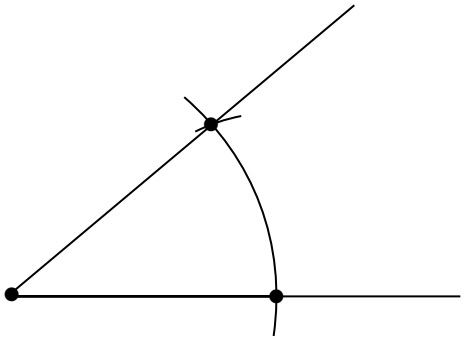
Answer:



Step 1

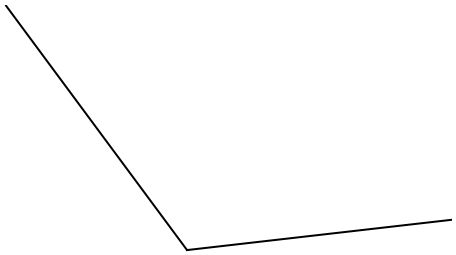


Step 2

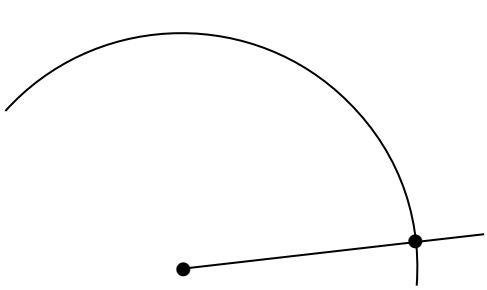


Step 3

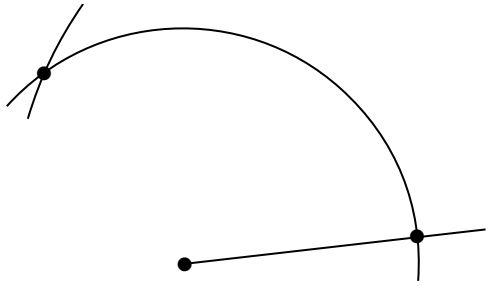
2.



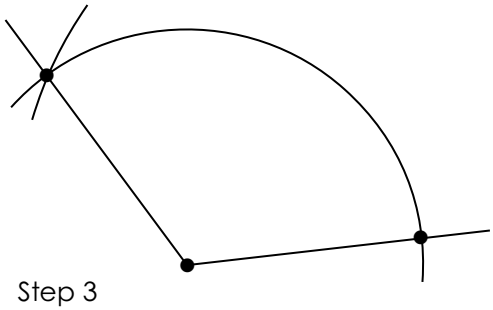
Answer:



Step 1

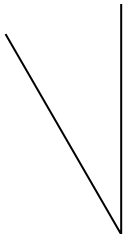


Step 2

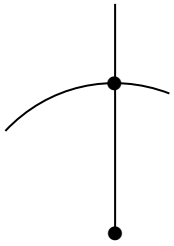


Step 3

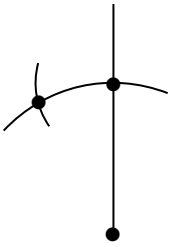
3.



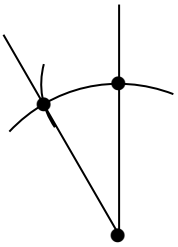
Answer:



Step 1



Step 2



Step 3

Learning Activity 5.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Rewrite the fraction in lowest terms: $\frac{24}{28}$.
2. Solve for t: $15t - 5 = 40$.
3. You are on your way to Calgary, driving on the Trans-Canada Highway. Your speed is constant at 100 km/h. If you have been driving for 3.3 hours at this speed, how far have you driven?
4. Adrian's time card for this week looks like this: Monday to Friday, 7 am to 1 pm; Saturday and Sunday, 6 am to 10 am. How many hours did he work this week?
5. A garden in your backyard is 2 m long and 1 m deep. What is the area of the garden?

Answers:

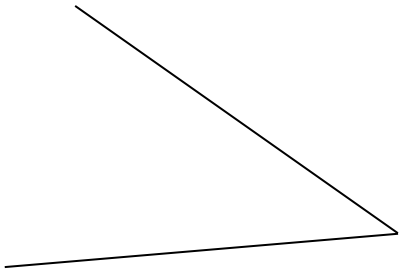
1. $\frac{6}{7}\left(\frac{24 \div 4}{28 \div 4} = \frac{6}{7}\right)$
2. $t = 3$ (add 5 to each side: $15t = 40 + 5$; divide both sides by 15: $t = 45 \div 15$)
3. 330 km (3.3 × 100 = move the decimal right two places: 330 km)
4. 38 hours (Monday to Friday he works for 6 hours, 5 × 6 = 30 hours; Saturday and Sunday he works for 4 hours, 2 × 4 = 8 hours)
5. 2 m^2 (A = length times width)

Part B: Bisecting Angles

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

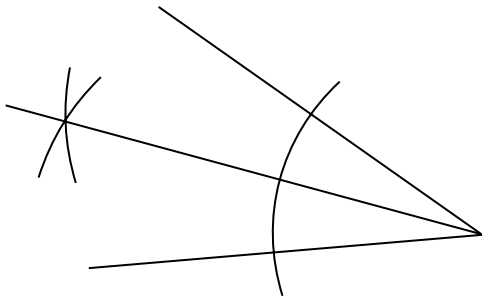
Use your compasses to construct the bisector of each angle, showing all markings.

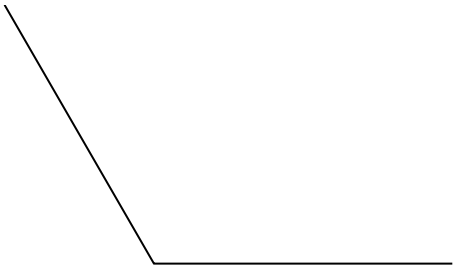
1.



Answer:

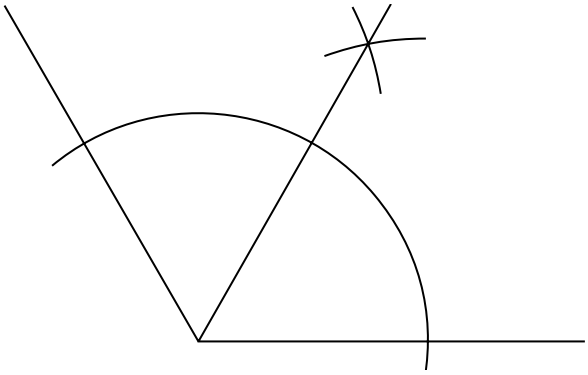
Use your protractor to check that each angle is bisected into two equal parts.

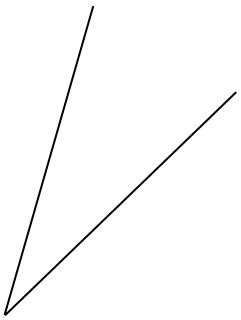


2. 

Answer:

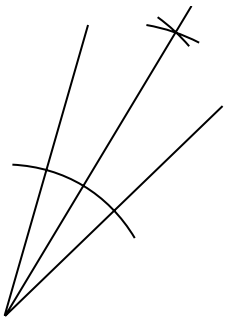
Use your protractor to check that each angle is bisected into two equal parts.



3. 

Answer:

Use your protractor to check that each angle is bisected into two equal parts.



Learning Activity 5.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Write the following number in scientific notation: 356 300 000 000.
2. Write the following number as a whole number: 45.87×10^{-4}.
3. When you were 3, your brother's age was double yours. How much older is he?
4. Put the following numbers in order from smallest to largest: 43, 12, 3.5×10^{1}, 760%, 21.
5. Your brother asks you to trade him coins for a $5 bill. He gives you 4 nickels, 3 dimes, 5 quarters, and 3 loonies. Is this a fair trade?

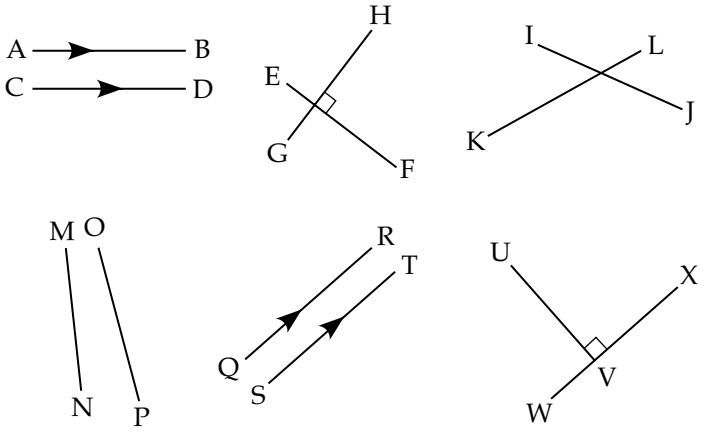
Answers:

1. 3.563×10^{11}
2. 0.004 587
3. 3 years (When you were 3, he was $3 \times 2 = 6$ so $6 - 3 = 3$ years older.)
4. 760%, 12, 21, 3.5×10^{1}, 43 (Convert them all into one form: 760% = 7.6 and $3.5 \times 10^{1} = 35$.)
5. No (4 nickels = $0.20, 3 dimes = $0.30, 5 quarters = $1.25, 3 loonies = $3.00: total = $4.75)

Part B: Parallel and Perpendicular Lines

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify any sets of perpendicular lines, and explain why they are perpendicular.



Answer:

HG ⊥ EF and UV ⊥ WX because they meet at 90° angles.

2. From the diagrams in #1 above, identify any sets of parallel lines, and explain why they are parallel.

   Answer:

   AB//CD and QR//ST because they are the same distance apart and they are marked as being parallel.

3. From the diagrams in #1 above, identify any sets of lines that are neither parallel nor perpendicular, and explain why.

   Answer:

   There are two sets of lines, IJ and KL, and MN and OP that are neither perpendicular nor parallel because they do not have any marking on them to indicate perpendicular or parallel.

Learning Activity 5.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Isabella gets paid double-time on statutory holidays. This past week was Labour Day. If she works 8 hours that day at $9/hour, how much will she be paid?
2. Courtney is drawing out her name in art class. She changes the colour of each letter so that the colours go from red to blue to green before starting over again. What colour will the 'y' in her name be?
3. Subtract: $\frac{9}{2} - \frac{5}{2}$.
4. You have 6 blueberries, 4 raspberries, and 8 slices of strawberry in your bowl of cereal. If you get at least one piece of fruit with every mouthful, what is the maximum number of bites it takes to finish your breakfast?
5. The angle at the corner of a picture frame is 90°. It is bisected by a cut, where the two sides have been attached together. What is the measure of the bisected angle?

Answers:

1. $144 (On a usual day she would be paid 8 × 9 = 72, but because it is double-time, this number is doubled: 72 × 2 = 144.)
2. Blue

C	O	U	R	T	N	E	Y
red	blue	green	red	blue	green	red	blue

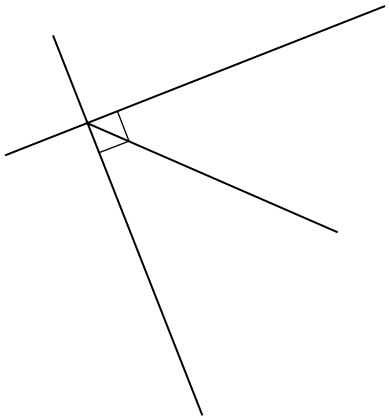
3. $\frac{4}{2} = 2$ (Remember that subtracting fractions is similar to adding fractions. The denominators should be the same, and the numerators are subtracted.)
4. 18 (6 + 4 + 8)
5. 45° (90° ÷ 2)

Part B: Complementary and Supplementary Angles

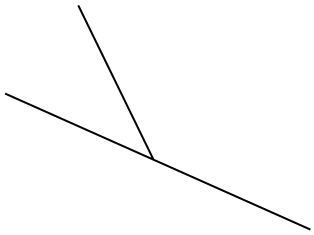
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the following sets of angles to answer the questions below.

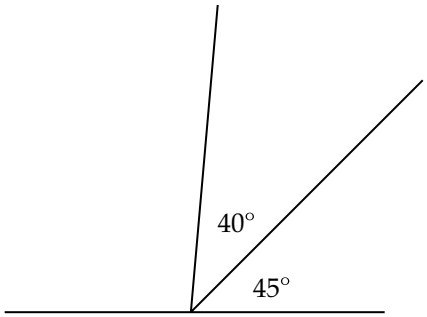
Set A



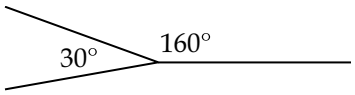
Set B



Set C



Set D



1. Which set(s) of angles illustrates supplementary angles, and why?

   Answer:

   Set B because the two angles together make a straight angle.

   Set C because all 3 angles together make a straight line. Or you could say there are two supplementary angles, one of which has a measure of 40° + 45° = 85°, the other a measure of 180° – 85° = 95°.

2. Which set(s) of angles illustrates complementary angles, and why?

   Answer:

   Set A because two angles together make a right angle.

3. Which set(s) of angles illustrates neither supplementary nor complementary angles, and why?

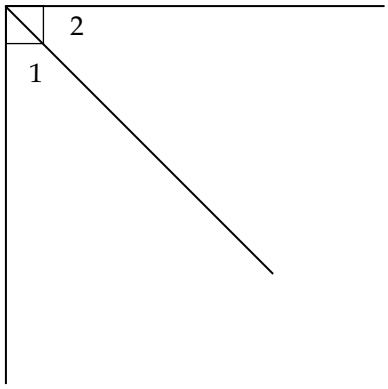
   Answer:

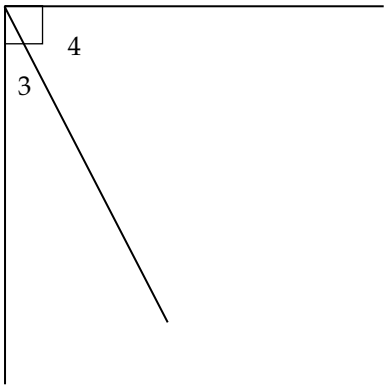
   Set D is the only answer because it is the only set where there are neither right angles nor straight angles.

4. Draw an example of complementary angles.

   Answer:

   Any two angles which add together to total 90° are complementary. The two angles need not have the same vertex, nor do they need to be equal.



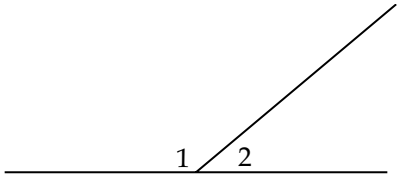


   ∠1 and ∠2 are complementary. ∠3 and ∠4 are complementary.

5. Draw an example of supplementary angles.

   Answer:

   Any two angles which add together to total 180° are supplementary. They need not have the same vertex.



   ∠1 and ∠2 are supplementary angles.

6. What is the measure of the complement to an angle of 35°?

   Answer:

   Complementary angles have a sum of 90° when they are added together. Thus, the complement of 35° is 90° – 35° = 55°.

Learning Activity 5.8

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are facing North. If you turn 360° to your right, which direction are you facing?
2. Because you have a doctor's appointment, you have to leave work at 3:45 instead of 4:30 pm. How much time are you penalized?
3. The size of the park across from your house is about 23 yards across. What is this distance in feet?
4. There are 3 boys and 2 girls invited to your birthday party. If each boy eats 2 pieces of cake and each girl eats 1 piece, how many pieces of cake will be eaten (not including what you eat).
5. If each piece of the cake (from the question above) is $\frac{1}{9}$th of the cake, will there be enough cake for you to have a piece?

Answers:

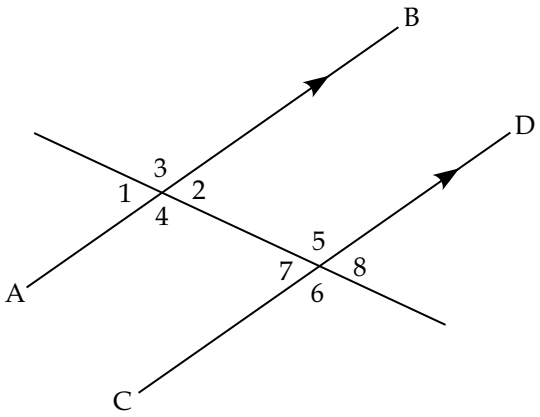
1. North (There are 360° in one revolution.)
2. 45 minutes (Count up to find out how early you left: 3:45 + 0:15 + 0:30 = 4:30)
   (4:00)
3. 69 feet (1 yard = 3 feet so 3 × 23 = 69)
4. 8 pieces ((3 × 2) + (2 × 1) = 6 + 2)
5. Yes $\left(8 \times \frac{1}{9} = \frac{8}{9}\right.$ are already accounted for, so there is $\frac{1}{9}$ left for you, which is one piece.)

Part B: Transversal Angles (1)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use Figure 5 to answer the following questions. Given AB//CD.

Figure 5



1. Identify all corresponding angles.

   Answer:

   ∠1 and ∠7

   ∠2 and ∠8

   ∠3 and ∠5

   ∠4 and ∠6

2. Identify all exterior alternate angles.

   Answer:

   ∠1 and ∠8

   ∠3 and ∠6

3. Identify all interior alternate angles.

   Answer:

   ∠2 and ∠7

   ∠4 and ∠5

4. Identify all vertically opposite angles.

   Answer:

   $\angle 1$ and $\angle 2$

   $\angle 3$ and $\angle 4$

   $\angle 5$ and $\angle 6$

   $\angle 7$ and $\angle 8$

Learning Activity 5.9

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The Blackhawks have won twice as many games as the Maple Leafs. The Maple Leafs have won 5 fewer games than the Oilers. If the Oilers have won 13 games, how many games have the Blackhawks won?
2. Kaitlin types 50 words per minute. It took her 30 minutes to write her essay for English. Assuming she was typing the whole time, how many words are in her essay?
3. Write 0.456 as a percent.
4. What are complementary angles?
5. How do you calculate net pay?

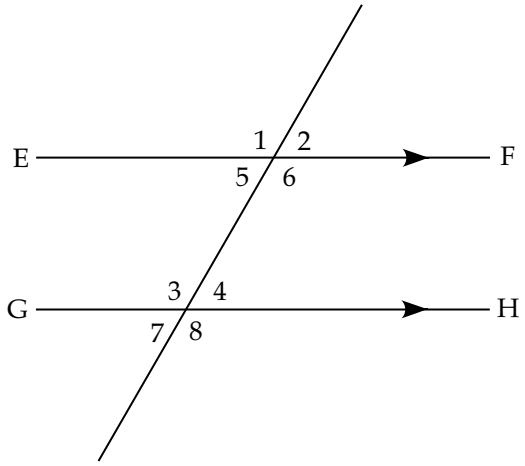
Answers:

1. 16 games (Oilers = 13; Leafs = 13 – 5 = 8; Blackhawks = 2 × 8 = 16)
2. 1500 words (5 × 3 = 15, add 2 zeros because it is 30 instead of 3 and 50 instead of 5)
3. 45.6%
4. Two angles that when added together equal 90^{o}.
5. Gross pay (including overtime, bonuses, and vacation pay) minus deductions.

Part B: Transversal Angles (2)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Given EF //GH and ∠1 = 120°. Use the properties of corresponding angles, alternate angles, supplements, and vertically opposite angles to find the measurements of the angles numbered ∠2 to ∠8. List at least one reason for each statement.



Answers:

You may have used other reasons than the ones given below. This list represents some of the possible answers.

∠2 = 60°	supplement to ∠1
∠3 = 120°	corresponding to ∠1
∠4 = 60°	corresponding to ∠2
∠5 = 60°	vertically opposite to ∠2 or interior alternate with ∠4
∠6 = 120°	interior alternate with ∠3 or supplement to ∠2
∠7 = 60°	corresponding to ∠5 or vertically opposite to ∠4
∠8 = 120°	corresponds with ∠6

All the acute angles are 60°, all the obtuse angles are 120°.

If you used different reasons than the ones given, talk to your learning partner or tutor/marker to make sure your reasons are accurate.

Learning Activity 5.10

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What are 'supplementary angles'?
2. In Winnipeg, Fort Street and Garry Street are parallel to each other. Will these two streets ever cross?
3. In Winnipeg, Main Street and Broadway are perpendicular to each other. Will these two streets ever cross?
4. Your bed is 2 m long. If you are 180 cm tall, will you fit on your bed?
5. It is 11:00 am right now. You work until 12:00, noon go for lunch for 1 hour, then go back to work for another 3 hours. It takes you an hour to drive home. The clock in your house reads 18:00. Is this correct?

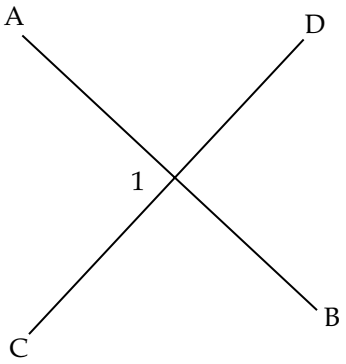
Answers:

1. Two angles that when added together total 180^{o}.
2. No (Parallel lines never cross.)
3. Yes (Perpendicular lines meet (and cross) at a 90° angle.)
4. Yes (180 cm = 1.8 m)
5. No (Count up: 12 + 1 + 3 + 1 = 17:00 or 5:00 pm)

Part B: Transversal Angles (3)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given ∠1 = 90°. Why is AB ⊥ CD? Explain your reasoning.

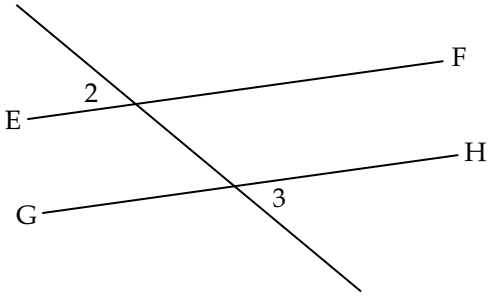


Answer:

AB ⊥ CD because ∠1 = 90°

Two lines that meet at a right angle are perpendicular to each other.

2. Given ∠2 = ∠3, why is EF//GH? Explain your reasoning.



Answer:

EF // GH because ∠2 and ∠3 are equal exterior alternate angles along a transversal.

If alternating angles along a transversal are equal, then the lines are parallel.

Notes

Grade 10 Essential Mathematics (20S)

Module 6
Consumer Decisions

Module 6:
Consumer Decisions

Introduction

In this module, you will learn to calculate the unit price of items. You will also examine various sales promotion techniques, and determine the real cost of these promotions. The third lesson revisits the determination of percent discounts. Also, in this module, you will use various methods to determine foreign currency exchanges.

Assignments in Module 6

When you have completed the assignments for Module 6, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Pizza Party
1	Assignment 6.1	Unit Pricing
2	Assignment 6.2	Unit Prices Project
3	Assignment 6.3	Percents
4	Assignment 6.4	Percent Increase or Decrease
5	Assignment 6.5	Sales Promotion
6	Assignment 6.6	Currency Exchange

Resource Sheet

When you write your final examination you will be allowed to bring a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Final Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 6. You might use your Module 6 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 5, 6, 7, and 8, to prepare your Final Examination Resource Sheet. Remember, the final examination is based on the last four modules of the course.

Resource Sheet for Module 6

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Final Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Module 6 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work in the spaces provided. The cover assignment can be done at any time while you are completing Module 6. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on whether or not you found a solution, and whether or not your tutor/marker can read your diagrams. The cover assignment is worth a total of 12 marks.

Notes

Total: 12 marks

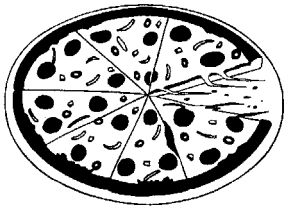
Pizza Party

In your town, there are 3 pizza places. You are having a party for 25 friends where pizzas will be served. The costs for the pizzas are as follows:

Presto Pizza	
	14″
Basic cheese pizza	12.49
Toppings (each)	1.25

Pizzas with Pizzazz	
	15″
Basic cheese pizza plus one topping	14.55
Extra toppings (each)	2.20

Pizza (Pizza)	
	14″
Basic cheese pizza plus one topping	13.99
Extra toppings (each)	1.50



continued

Module 6 Cover Assignment: Pizza Party (continued)

1. When you buy pizza, what factors do you consider before making your decision? *(3 marks)*

   _____

   _____

   _____

   _____

   _____

2. What is the most important factor when you decide on what pizza to order? *(1 mark)*

   _____

   _____

3. Find the area of a 14″ pizza using the formula for the area of a circle ($A = \pi r^2$). Note that 14″ refers to the diameter of the pizza. Show your calculations. Use $\pi = 3.14$. *(2 marks)*

4. Find the area of a 15″ pizza. Note that 15″ refers to the diameter of the pizza. Show your calculations. *(1 mark)*

continued

Module 6 Cover Assignment: Pizza Party (continued)

5. Find the cost of 1 square inch of pizza with one topping from each of the pizza places. Round off your answer to 4 decimal places. Do not include PST or GST. *(3 marks)*

   a) Presto Pizza

   b) Pizzas with Pizzazz

   c) Pizza (Pizza)

6. If you had to buy your pizza from the three pizza places in this cover assignment, from which pizza place would you order? Explain. *(2 marks)*

Notes

Lesson 1: Unit Pricing

Lesson Focus

In this lesson, you will

- ❑ solve problems that involve unit pricing

Lesson Introduction

Consumers buy things. We buy things. Some items consumers purchase can be less expensive than others, based on quality, size, location, and amount of available money. This lesson examines the various considerations regarding what consumers decide to buy.

Being Cost-Conscious

Improving Your Buying Decisions

Part B of the following learning activity should be looked at as a self-test for you to rate your "Buyer Behaviour."

Learning Activity 6.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A checkerboard has 8 squares along its length and 8 squares along its width. How many squares all together are on the board?
2. The squares on a checkerboard alternate between black and white. How many squares are black?
3. You are buying coffee for your family of 6. Each coffee costs $1.50. How much will it cost in total?
4. The shortest sentence that contains all the letters of the alphabet is "The quick brown fox jumps over the lazy dog." Write a fraction comparing the number of words with 'o' to the total number of words in the sentence.
5. If 10% of 45 is 4.5, what is 20% of 90?

continued

Learning Activity 6.1 (continued)

Part B: Buyer Behaviour Self-Test

Directions: Use the scale below as you answer each item.

4 Always
3 Often
2 Seldom
1 Never

	1. I use ads to get information about products and services.
	2. I find information about products and services from places other than ads.
	3. I compare prices and quality of different brands at different stores before buying.
	4. I read labels and guarantees on purchases, and follow their directions.
	5. I look over, try on, try out, and ask questions about products before buying.
	6. I follow a list when shopping.
	7. I let manufacturing companies and businesses know what I like and dislike about their products and services.
	8. I plan in advance what products and services to buy.
	9. I consider my needs and wants as well as price when deciding what to buy.

Add up your scores for each item. Rate yourself according to this scale.

28 – 36 Super shopper
19 – 27 Careful consumer
0 – 18 Poorly prepared purchaser
0 – 9 You need help

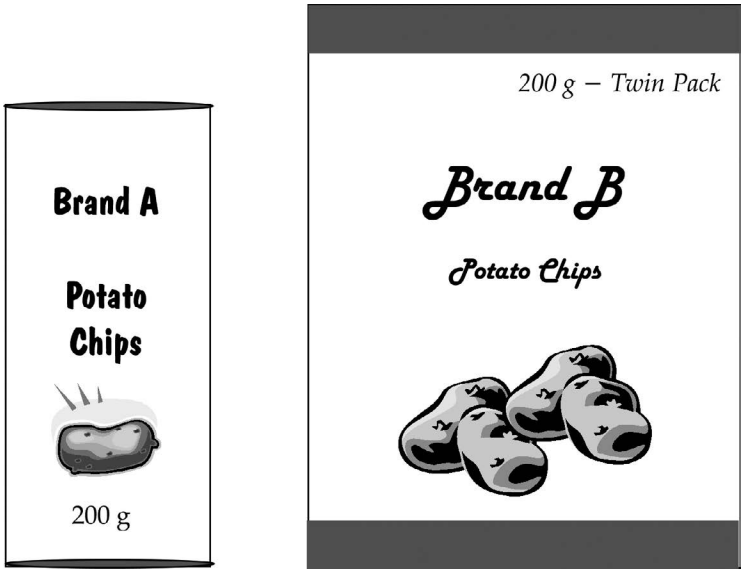
Source: Florida Board of Regents. *Buying and Selling.* Tallahassee, FL: Florida Board of Regents, 1976. Reproduced in accordance with *Access Copyright Elementary and Secondary School Tariff.*

By completing this self-test, you now will have more information about what kind of consumer you are according to its rating scale. You might need to make a few changes in how you are making your consumer decisions. Most people could use some help in this area.

Packaging

Many of your consumer decisions are based on what you see on the outside of the product—the packaging. Good packaging protects the product and makes it easier to handle. It also helps sell the product.

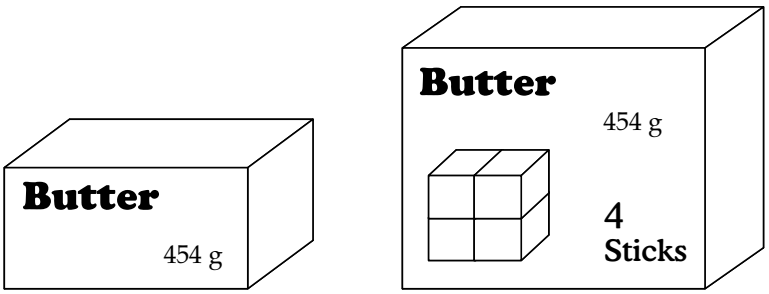
The shape of a package is one way products are made to look appealing. However, the shape of a package can also be deceptive. Look at the following two packages. Which one looks like it contains more?



If you look at the labels, you realize that both boxes of potato chips have the same mass of 200 g. At first glance, it may appear that Brand B has more product than Brand A.

The chips in Brand A are packed one on top of the other, while those in Brand B are loosely placed in a bag. Many boxes of breakfast cereal can be compared the same way.

This shows the same amount of butter, packaged in two ways.



Some people might think that the larger package holds more of the product. The amount is the same in both brands.

Labels

Labels contain information and protect consumers. Both food and clothing are required to have certain information listed on their labels.

Most food labels must include the following items in French and English:

- name of the product
- net weight or liquid capacity of the product in metric units
- name and address of the manufacturer or distributor
- ingredients, including all additives and preservatives, listed in order of weight from most to least
- description of the special nature of the product
- nutrition information—serving size, calories per serving, grams of protein and carbohydrates per serving, as well as important vitamin and mineral content

Clothing labels often list the following items:

- name of the manufacturer or distributor
- size
- material the item is made of
- washing instructions

Comparing Prices

One suggestion for you when considering the purchase of an expensive product, such as a computer, HD television, or even a car: you might spend more time researching your possible choices.

Price is an important factor in purchasing a product. You want to make sure you get the best possible price when making a purchase. If a product is available at more than one store, you should compare the prices. Often one store will match the price of its competitors.

Some products are available in different sizes. Groceries, for example, are often available in a variety of sizes. You can determine which size is a better buy by comparing the unit price of each. Usually you spend less per unit when buying the larger size, since there is less packaging cost.

Many grocery stores will post the unit price on the shelves, especially for liquid products.

Example

Which is the best buy for 1 litre of milk: (a), (b), or (c)?

a) 1 L container of milk for $1.95

b) 2 L container of milk for $3.85

b) 4 L container of milk for $7.50

Solution:

To compare these prices, you first find the unit price, based on a common unit.

You decide the unit. It could be price per litre or price per 100 mL. The key is to convert each item to the same unit, and find each price per unit.

In this example, the unit price has been found for 1 litre of milk. You find the price per litre for each.

a) The unit price for one litre is already given: unit price = $1.95 per litre.

b) When the unit price may not be so obvious, you can use proportional thinking as one method to get the answer.

	litres	cost
given amount	2 L	$3.85
unit amount	1 L	$$x$

$$\frac{2\text{ L}}{1\text{ L}} = \frac{\$3.85}{\$x}$$

$$(2\text{ L})(\$x) = (1\text{ L})(\$3.85)$$

You could write this equation more simply as $2x = 3.85$

Now divide both sides of the equation by 2 in order to isolate x.

$$\frac{2x}{2} = \frac{3.85}{2}$$

$$x = 1.93$$

The unit price of a 2 L container of milk is \$1.93 per litre.

c)

	litres	cost in \$
given amount	4 L	\$7.50
unit amount	1 L	$\$x$

$$\frac{4\text{ L}}{1\text{ L}} = \frac{\$7.50}{\$x}$$

$$4x = 7.50$$

$$\frac{4x}{4} = \frac{7.50}{4}$$

$$x = 1.88$$

The unit price of a 4 L container of milk is \$1.88 per litre.

Therefore, if you are only looking at the cost of the milk, then the cheapest, best buy is the 4 L size for \$1.88 per litre.

In the above example, you can notice that instead of setting up the proportion you might have just divided. If you know what 2 L of milk costs, then divide by 2 to find the cost of 1 L. If you know what 4 L of milk costs, then divide by 4 to find the cost of 1 litre.

Reasons to Not Pick Best Price

Perhaps you do not need 4 litres of milk, or maybe you do not have adequate storage for a large container. Should the milk be spoiled before you use it all, then the lowest price is not in your best interest.

Another consideration is distance to the store. Perhaps a supermarket across town has a better price, but you would spend more in gas than you would save on buying a cheaper product. Small corner stores typically charge a little more for their products, but they are very convenient.

Another reason consumers purchase smaller items even though they might have a higher unit price is the amount of available money. Perhaps they just do not have enough cash at that moment to afford the larger item.

Unit Pricing

Example

A store offers one brand of rice in 3 different sizes: 500 g for $2.30, 1.5 kg for $6.50, and 3.2 kg for $13.50. Find the unit price per 500 g for each size. Which size has the best price?

Solution:

You can use the proportion method to find each unit price. Once you decide on an appropriate unit, set up the proportions.

In this example, the unit size is chosen as 500 g since the first price is in that amount, but you could use 100 g or 1000 g instead. Whatever unit you choose is fine, as long as you are consistent with it throughout your calculations.

If you have your resource sheets available from the first half of the course, you could refer to the metric prefix ladder to convert kilograms to grams. Or you may recall from the module on measurement the relationship between kilograms and grams.

1 kg = 1000 g

1.5 kg = 1.5 × 1000 = 1500 g

3.2 kg = 3.2 × 1000 = 3200 g

Now you can find the unit prices.

First size: 500 g for \$2.30 is already the unit price.

Second size: 1.5 kg for \$6.50 or 1500 g for \$6.50

	grams	cost in \$
given amount	1500 g	\$6.50
unit amount	500 g	\$$x$

Method 1

$$\frac{1500\text{ g}}{500\text{ g}} = \frac{\$6.50}{\$x}$$

$$1500x = (500)(6.50)$$

$$1500x = 3250$$

$$\frac{1500x}{1500} = \frac{3250}{1500}$$

$$x = 2.17$$

Method 2

Notice in the ratio $\frac{1500}{500}$ the numerator is 3 times as big as the denominator. Therefore, to solve for x in the denominator of the second ratio, you could just divide \$6.50 by 3 to get \$2.17.

The unit price is \$2.17 for 500 g.

Third size: 3.2 kg for \$13.50 or 3200 g for \$13.50

	grams	cost in \$
given amount	3200 g	\$13.50
unit amount	500 g	\$$x$

$$\frac{3200\text{ g}}{500\text{ g}} = \frac{\$13.50}{\$x}$$

$$3200x = (500)(13.50)$$

$$3200x = 6750$$

$$\frac{3200x}{3200} = \frac{6750}{3200}$$

$$x = 2.11$$

The unit price is \$2.11 for 500 g.

Comparing the three unit prices, \$2.30, \$2.17, and \$2.11, the third size has the least cost.

The 3.2 kg package would be the best buy for \$2.11 per 500 g.

Learning Activity 6.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the complementary angle of 79°?
2. You run every second day. You run 3.5 miles on Tuesday, 4 miles on Thursday, and 4.5 miles on Saturday. Which days will you run next week?
3. How far will you run each of those days if you continue the pattern?
4. Solve for e: $\frac{1}{2} = \frac{4}{e}$.
5. Your work day is supposed to be from 8:30 to 4:30. You arrive at work at 7:54, and leave at 4:23. What is the late penalty for your workday?

Part B: Unit Pricing

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the unit price for each item, as indicated.
   a) 355 mL soda pop costing $1.25, unit is 100 mL
   b) 3.5 kg sugar costs $5.30, unit is 1 kg
   c) 1.5 kg sliced meat costs $11.25, unit is 100 g
2. Use unit pricing to compare the two products and find the best buy.
   a) 6 cans of size 355 mL of diet soda for $2.85 or a 2 L bottle costing $2.10. Use a unit of 100 mL.
   b) 1 kg jar peanut butter for $3.95 or 5 kg jar for $18.95. The unit is 1 kg.
3. Give two reasons why someone might not purchase a larger item, knowing the unit price is lower.

Lesson Summary

In this lesson, you determined unit prices of goods, and compared different sizes of containers according to their unit prices. You learned about your consumer buyer behaviour, and not to be distracted by packaging. Reading labels is a helpful way to shop wisely. Reasons for choosing other than the cheapest product were given.

The next lesson requires that you go to a store and complete a data collection project.

Notes

Assignment 6.1

Total: 19 marks

Unit Pricing

1. Find the unit price for each item, as indicated. Show your calculations for full marks. *(8 marks)*

   a) 2 L milk costs $3.25; unit is 100 mL.

   b) 1.5 kg meat costs $12.75; unit is 100 g.

continued

Assignment 6.1: Unit Pricing (continued)

c) 22 kg potatoes for $10.99; unit is 500 g.

d) 130 mL toothpaste costs $1.78; unit is 50 mL.

continued

Assignment 6.1: Unit Pricing (continued)

2. Use unit pricing to compare prices and find the best buy. Show your work, and state which one is the better buy. *(9 marks)*

   a) 3 gallons of oil for $19.89 or 1 quart of oil for $1.79. Use 1 quart as the unit. Note that there are 4 quarts in a gallon.

   b) 10 pounds of flour for $11.85 or 2 pounds for $2.98. Use 1 pound as the unit.

continued

Assignment 6.1: Unit Pricing (continued)

c) 12 cans of soda with 355 mL per can for $4.80 or two 2 L bottles for $4.15. Use a unit of 100 mL.

3. Give two reasons why someone would knowingly pay more for a smaller container of an item, knowing the unit price is cheaper for a larger quantity. *(2 marks)*

Lesson 2: Unit Prices Project

Lesson Introduction

This lesson requires that you actually go to your family grocery store and collect data on 10 specific items.

It has been shown that when students physically touch things, hold and examine articles, and record their findings, they learn at a much deeper level.

This project asks you to find 9 items on your grocer's shelves. Each item should come in 2 different sizes.

The attached form "Unit Prices" is to be completed and submitted with your module assignments. It will have a point value of 20 marks. You will be graded on whether your form is complete, and whether it can be read by your marker.

You are to note the prices of 9 different grocery items, each in two sizes and find the unit price for each size. If you need to do any calculations for the unit prices, show your work on a separate sheet of paper and hand it in with your work sheet.

Be sure to write in the name of the store you attended, the address, and the date.

Lesson Summary

This lesson sent you to your grocery store to collect data on ten grocery items, and find their unit prices.

The next lesson discusses percent discounts, and using them to find sale prices.

Assignment 6.2

Total: 20 marks

Unit Prices Project

Take the form found on the following page to your grocery store, record the data, and find the unit price for the ten items listed. If any of the items is not available in two sizes, substitute it with another item that is available in two sizes. Also, it is ideal that the two items be the same brand.

continued

Notes

Assignment 6.2: Unit Prices Project (continued)

Unit Prices (Hand-in)							
Item	Brand	Smaller Container			Larger Container		
		Weight or Volume	Cost	Unit Price	Weight or Volume	Cost	Unit Price
Milk							
Breakfast Cereal							
Ketchup							
Can of Tomato Sauce							
Sugar							
Potato Chips							
Peanut Butter							
Shampoo							
Toothpaste							

Name of grocery store: _____

Address of grocery store: _____

Date of unit pricing: _____

Lesson 3: Percents

Lesson Focus

In this lesson, you will

- ❑ determine the sale price of an item given the percent discount

Lesson Introduction

This lesson shows you how to determine the sale price of an item, given the percent rate of discount and how to estimate with percents.

Percent Discounts

Percent values are regularly used in advertisements. Stores have sales and advertise certain percent points off the regular price. Some stores even allow you to scratch a card, with varying percent discount values offered.

Other stores rarely have sales. They advertise that they have lower prices. You must be able to determine and compare sale prices from the percent discount listed. Only then will you know which store is offering the best deal.

Estimating

Everyone needs to be able to estimate or use their mental math skills to determine values. When you don't have a calculator handy, estimating skills become necessary.

If you are in a crowded store with customers grabbing items off the shelves, you need to be able to quickly determine the actual sale price of items.

Estimating 10%

Perhaps the easiest percent calculation one can perform mentally is 10%.

10% is $\frac{10}{100}$, and can be written as 0.1 as a decimal numeral. To find 10% of a value, you slide the decimal point one unit to the left.

Example 1

If a television that normally sells for $1299 is on sale at 10% off, how much is the discount?

Solution:

Mentally visualize the $1299. Move the decimal one place to the left, and you have $129.90. That is the amount you will save.

Example 2

Try the following questions without using your calculator.

Find 10% of:

a) $6565 _____

b) $169.85 _____

c) $1250.50 _____

d) $75825 _____

Solution:

A little practice and you will have quickly mastered this skill. Just move the decimal place to the left one place value.

a) $6565 becomes $656.50

b) $169.85 becomes $16.985 or $16.99

c) $1250.50 becomes $125.050 or $125.05

d) $7825 becomes $782.50

Estimating 5%

Estimating 5% of a value involves finding 10% and using half of that value.

Example

Estimate 5% of $6428.

Solution:

To estimate 5% of $6428, round off to $6400, find 10% by moving the decimal one place to the left, giving you $640. Now take half of that value, getting an answer of $320.

Estimating 20%

Estimating 20% of a value also involves first finding 10%. Then you double that amount, since 20% is twice 10%.

Example

Estimate 20% of $832.

Solution:

To find 20% of 832, round 832 to 800, move the decimal one unit to the left to find 10%, getting an answer of $80. Then you double that value, since 20% is double 10%, and get $160. This will be a little lower than the actual answer because you rounded down.

Estimating 3%

These smaller percentages, like 3%, get a little trickier to estimate. You must find 1% of the given value by moving the decimal over to the left 2 units. 1% means $\frac{1}{100}$ or 0.01 as a decimal numeral. Once you have that value, you multiply it by 3 since 3% is 3 times 1%.

Example

Estimate 3% of $843.

Solution:

To estimate the value of 3% of $843 round $843 to $800, move the decimal to the left 2 units, getting an answer of $8. Now multiply this value by 3, getting an estimate of $24. This answer will be a little lower than the actual answer because you rounded down.

Learning Activity 6.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Linden loves milkshakes. He gets to choose from two chocolate milkshakes. One fills the glass $\frac{7}{9}$ full. The other fills the glass $\frac{2}{3}$ full. Which will he choose?
2. You are at a bake sale. You want to buy a peanut butter cookie. If the whole plate of 16 cookies costs $3.20, how much will you pay for one cookie?
3. Think of a clock. How many degrees does the minute hand move between 17:30 and 18:00?
4. Vertically opposite angles are _____.
5. Writers are paid by piecework. Jill Jackson is paid $80 for every children's story she writes and $50 for every poem she writes. Over the past 6 months she has written 4 children's stories and 6 poems. How much money was she paid?

Part B: Calculating 10%

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the amount of the percentage without using a calculator.

1. 10% of $567 _____
2. 10% of $628.09 _____
3. 10% of $6.75 _____

continued

Learning Activity 6.3 (continued)

4. 5% of $8422 _____
5. 5% of $468.80 _____
6. 20% of $364.20 _____
7. 1% of $56.52 _____
8. 2% of $6259 _____

Finding the Sale Price

Finding the sale price involves your applying the percent rate to the full price to find the amount of the discount. Then you subtract the discount amount from the regular price to get the sale price.

If a flyer from an electronics store advertises a sale of 25% off, you could convert the 25% to a decimal 0.25. This value is multiplied by the regular price to find the amount of the discount. Your answer is subtracted from the regular price to find the sale price.

Example 1

A GPS navigation unit is regularly priced at $299.95. The store has it advertised at 25% off.

a) Find the sale price.

b) Use your estimating skills to check if your answer is reasonable.

Solution:

a) Regular price = $299.95

Discount = $299.95 × 0.25 = $74.99

Sale price = $299.95 – $74.99 = $224.96

b) To estimate the calculation first, the full price of $299.95 is rounded off to $300.

The discount = 25% or one-quarter of the full price.

$300 divided by 4 gives you $75.

Sale price = full price – discount = $300 – $75 = $225.

The answer of $224.96 is reasonable.

Example 2

A shoe store is having a sale offering a 30% discount on all purchases. The shoes you like have a regular price of $89.75.

a) Find the sale price.

b) Use your estimating skills to check if this answer is reasonable.

Solution:

a) Regular price = $89.75

Discount = $89.75 × 0.30 = $26.93

Sale price = $89.75 – 26.93 = $62.82

b) To check the answer by estimating, you first round off the full price of $89.75 up to $90.

The discount of 30% is 3 units of 10% each.

To find 10%, you move the decimal one unit to the left, getting $9.

Since there are 3 of these units, the discount is $9 × 3 = $27.

The sale price = full price – discount = $90 – $27 = $90 – $30 + $3 = $60 + $3 = $63

The answer of a sale price of $62.82 is reasonable.

Add the Taxes

You cannot escape paying the sales taxes. When finding the total cost of purchases, the 7% PST and 5% GST must be added to the sale price.

Example

You are interested in buying a used car at a dealership. To make the sale, the manager offers a 10% discount. The car is regularly priced at $9875. Find the sale price and the total price when taxes are added.

Solution:

Regular price: $9875

Discount: $9875 × 0.10 = $987.50

Sale price: $9875 – $987.50 = $8887.50

GST = $8887.50 × 0.05 = $444.38

PST = $8887.50 × 0.07 = $622.13

Total = $8887.50 + $444.38 + $622.13 = $9954.01

Shortcut

The following shortcut might save you a little time when finding the sale price without having to calculate the discount first.

If an item is discounted, for example, by 10%, then the sale price you are paying is 90% of the regular price. Instead of calculating the 10% discount and then subtracting it from the regular price, you can just find 90% of the regular price. In one step, you can find the sale price.

A 25% discount means you are paying 100% – 25% = 75% of the regular price.
A 15% discount means you are paying 100% – 15% = 85% of the regular price.

The percent rate of the discount is subtracted from 100%, and that is the percent rate you use to find the sale price directly from the full price.

Example 1

The garden centre has shrubs on sale with a 15% discount. You select some roses regularly priced at $28.50. Find the sale price, and then the total price including taxes.

Solution:

A 15% discount means you are paying 100% – 15% = 85% of the regular price.

Sale price: $28.50 × 0.85 = $24.23

GST = $24.23 × 0.05 = $1.21

PST = $24.23 × 0.07 = $1.70

Total = $24.23 + $1.21 + $1.70 = $27.14

Example 2

You are deciding whether or not to purchase a bedroom suite. To help you decide to buy, the salesman offers a 10% discount. The furniture is regularly priced at $2899. Find the sale price and the total price including taxes.

Solution:

A 10% discount means you are paying 100% – 10% = 90% of the regular price.

Sale price: $2899 × 0.9 = $2609.10

GST = $2609.10 × 0.05 = $130.46

PST = $2609.10 × 0.07 = $182.64

Total = $2609.10 + $130.46 + $182.64 = $2922.20

Learning Activity 6.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Find 10% of 430.
2. Calculate 20 + 80.
3. Calculate 5 × 30.
4. You have $10.00. You spent $2.60 on candy. How much do you have left?
5. Which is larger: $\frac{1}{9}$ or $\frac{1}{11}$?

Part B: Percent Discount (1)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the sale price of each item before the taxes.

1. Regularly priced at $56.95 with 15% off
2. Regularly priced at $424.68 with 7.5% off
3. Regularly priced at $38.62 with 5% off
4. Regularly priced at $2688.46 with 26.4% off

Lesson Summary

This lesson showed you how to find the sale price of an item given the percent rate of discount. You learned how to estimate the percent of a number, to add the taxes to the sale price, and to find the sale price using a shortcut if you don't need to find the amount of discount.

Lesson 4 further explores percent operations to find increases and decreases.

Assignment 6.3

Total: 19 marks

Percents

1. A hardware store is having a sale, offering 20% off the purchase of any tools. You decide to buy an air compressor regularly priced at $387.75. Find the sale price. *(2 marks)*

2. Last month you spent $350 on entertainment. You have decided to reduce your spending by 10%. How much money would you have for spending on entertainment this month? *(2 marks)*

3. The car of your dreams is priced at $12,750. The saleswoman has agreed to reduce the price 12%. Find the sale price and the total price including taxes. *(4 marks)*

continued

Assignment 6.3: Percents (continued)

4. Store A has official footballs regularly priced at $78.50. Store B has the same footballs priced at $85, but they are on sale with a 10% discount. Which store has the better deal, and by how much, before taxes? *(3 marks)*

5. You are shopping for a flat screen television. Luckily, the two stores in your area are having sales. The same model of television is available at both stores. Store A has it priced at $899 with a 10% discount. Store B has the television priced at $935, but is offering a 15% discount. Find the sale price for each store, and then find the total cost, including the taxes, at each store. Where do you buy your TV? *(8 marks)*

Lesson 4: Percent Increase or Decrease

Lesson Focus

In this lesson, you will

- ❑ determine the percent increase or decrease of an item

Lesson Introduction

This lesson will focus on how to determine the percent increase or decrease of an item.

Finding the Percent Discount

Wise shoppers know the regular price of items that interest them. They wait for a sale to be announced, and then save by purchasing the item at that time.

Percent Rate of Change

Example 1

Let's say a bicycle you want is regularly priced at $589.95, and is available at two stores for the same regular price. Store A is selling the bike for $499.95, while Store B has a sale offering a 15% discount. The time has come to make a decision, but you want to be sure you get the best price.

a) What percent rate of decrease did Store A apply to the price?

b) What percent rate of decrease did Store B apply to the price?

c) What is the better price?

Solution:

a) To calculate any percent rate of change, either an increase or a decrease, you always use the following formula.

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original amount}} \times 100$$

Store A dropped the price by \$589.95 – \$499.95 = \$90. To find the percent rate of discount, substitute \$90 into the formula as the amount of change. The original amount is the regular price of \$589.95.

$$\text{Percent rate of decrease} = \frac{\text{amount of decrease}}{\text{original amount}} \times 100$$

$$\text{Percent rate of decrease} = \frac{\$90}{\$589.95} \times 100 = 15.3\%$$

The percent rate of the decrease in the price at Store A is 15.3%.

b) Store B's percent rate of decrease is given in the problem to be 15%.

c) Unfortunately for Store B, Store A has a higher percent rate of decrease and offers the better price.

Example 2

Your boss is offering you a 35-cent raise in your hourly wage. Your current hourly wage is \$12.50. The local newspaper reports that most workers are receiving a raise of 3% this year. Is the offer from your boss the same as what others are offering?

Solution:

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original amount}} \times 100$$

Change in hourly wage = \$0.35

Original wage = \$12.50

$$\text{Percent rate of increase} = \frac{\$0.35}{\$12.50} \times 100 = 2.8\%$$

You have found that your raise is only 2.8% and is lower than what others are offering.

Example 3

Your business operates on a slim profit margin and is affected by rises in fuel costs. The price of gas went from 91.9¢ to 95.9¢ per litre. If there is more than a 4% increase in fuel costs, you have to raise your prices to your customers. Find the percent increase in fuel.

Solution:

Amount of change = 95.9¢ – 91.9¢ = 4¢

Original amount = 91.9¢

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original amount}} \times 100$$

$$\text{Percent rate of increase} = \frac{4¢}{91.9¢} \times 100 = 4.4\%$$

The percent increase is 4.4%. You will have to raise your prices accordingly.

Example 4

You have saved all the money given to you on birthdays and family gatherings, and have it invested in an RSP (registered savings plan) with a interest rate that fluctuates up and down with the stock markets. The bank statement from 3 months ago showed you had $2650 in your account. The new statement shows your balance at $2498. Find the percent rate of decrease.

Solution:

Amount of change = $2650 – $2498 = $152

Original amount = $2650

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original amount}} \times 100$$

$$\text{Percent rate of decrease} = \frac{\$152}{\$2650} \times 100 = 5.7\%$$

Your investment decreased by 5.7% over the 3 month period.

Solving for the Original Amount

You can now practice your solving skills using the percent rate formula. You can find the original amount given the new amount, and the percent increase or decrease.

Example 1

Consider the price of fruit in your local grocery store. This year, bananas cost $0.89 per pound. This includes a 7% increase due to higher transportation costs. Find their original cost.

Solution:

To find the original cost, you must first realize that the new price includes 100% of the old price, plus the 7% increase. This gives you 107% of the original cost.

Using proportional reasoning, you can determine the original price.

	cost per pound	percent
new price	$0.89	107%
original price	$$x$	100%

$$\frac{\$0.89}{\$x} = \frac{107\%}{100\%}$$

$$107x = (0.89)(100)$$

$$107x = 89$$

$$\frac{107x}{107} = \frac{89}{107}$$

$$x = 0.83$$

The original price of the bananas was $0.83 per pound before the increase.

Example 2

The price of a new car has dropped this year. Now you can buy the car of your liking for $16,750. The dealer tells you this represents a 10% drop in prices. Find the original price last year.

Solution:

If the price dropped 10%, then this new value represents 90% of the original price. Notice in the previous example the price was increasing, so you added the percent increase to get the new price. Here, the price is dropping, so you subtract the percent decrease to get the new price.

Use proportional reasoning to find the original price.

	cost of the car	percent
new price	$16,750	90%
original price	$\$x$	100%

$$\frac{\$16,750}{\$x} = \frac{90\%}{100\%}$$

$$90x = 16\ 750 \times 100$$

$$90x = 1\ 675\ 000$$

$$\frac{90x}{90} = \frac{1\ 675\ 000}{90}$$

$$x = \$18,611.11$$

Last year, the car was priced at $18,611.11 before the 10% discount.

Learning Activity 6.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are making banana bread but only need to make half the recipe. The recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar will you use to make half the recipe?
2. One-quarter of the Grade 10 students in a school are going to the divisional track meet. There are 120 Grade 10 students in the school. How many are going to the track meet?
3. At a car wash, Doug is paid $5 per car he washes. Doug washes 20 cars during his shift. How much money will he get paid?
4. Which of the following fractions are equivalent?

   $\frac{4}{8}$ $\frac{5}{12}$ $\frac{7}{14}$ $\frac{9}{16}$
5. If 100 cm of snow is equivalent to 50 mm of water, 150 cm of snow is equivalent to how much water?

continued

Learning Activity 6.5 (continued)

Part B: Percent Discount (2)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the percent rate of increase or decrease for each situation. Show your work and then fill in the missing numbers in the table. Round off your answers to two decimal places.

	Original	New Amount	Change	Percent
1	$36.50	$39.00		
2	$280.50		$15.75 increase	
3		$864.60	$96.40 decrease	
4	$7.85			5.35% increase
5	$26.86		$4.82 decrease	
6		$13.75		4% increase

Lesson Summary

This lesson was about finding the percent rate of increase or decrease by using the formula for percent rate of change. This is the same formula you have used in an earlier module. You also learned how to solve for an unknown value in the formula when you are given the percent rate.

The next lesson explores different sales promotions that stores offer, and how to be buyer-conscious about which is the better deal.

Notes

Assignment 6.4

Total: 21 marks

Percent Increase or Decrease

1. Find the percent rate of increase or decrease, and fill in all the blanks. Round off to two decimal places. *(10 marks)*

Original	New Amount	Change	Percent
$644.25	$658.60		
$89.75	$86.34		
$3758	$3689		
$2.78	$3.02		
$25.68	$24.90		

2. You receive a raise of $0.25 per hour at your job. Your present wage is $11.20 per hour. Find the percent rate of increase. *(2 marks)*

continued

Assignment 6.4: Percent Increase or Decrease (continued)

3. A DVD you like is on sale for $13.95. The regular price for this movie is $16.49, Find the percent rate of decrease. *(3 marks)*

4. You want to buy a Boogie Board to have at the lake. Two stores have the board you like, both equally priced at $274.99. Store A has them on sale for $249.99, and Store B is offering a 10% discount. Which store has the higher percent rate of decrease? *(4 marks)*

continued

Assignment 6.4: Percent Increase or Decrease (continued)

5. The price of tomatoes is now $2.05 per pound including an increase of 5%. Find the original cost. *(2 marks)*

Notes

Lesson 5: Sales Promotions

Lesson Focus

In this lesson, you will

- ❑ compare different sales promotion techniques

Lesson Introduction

This lesson examines the various options stores give consumers to purchase items on credit.

Sales

Stores want to sell their products. They offer sales promotions to entice you into their store and encourage you to buy from them. Have you ever gone to buy something at the store, and end up getting two because there is a special on, like 2 for $10? This type of offer is quite common, so you need to be able to think critically about what the benefits of the sale actually are.

In this lesson, we have used the GST and PST from when this course was written, so they will be 5% and 7% respectively. Remember that although this may not be accurate, it is more important to understand the calculations, and the numbers just allow you to complete the calculations.

Which Offer is the "Better Buy"?

As mentioned above, there are sales that state 2 for $10 or buy one, get one-half off. It is important to be able to recognize which is the "better buy" because you want to get the best deal possible. This is similar to unit pricing, except instead of looking for 'per kg' or 'per mL,' you want to figure out the cost per item.

Package Deals

One of the most frequently used sales techniques is to package or bundle items in order to sell selected items. These usually take the form of "5 for $25" so that if you buy a certain number of items you will pay a set price. Usually this ensures that your 'per item' cost is less than if you paid for each item individually, but this is not guaranteed.

Example 1

Ackerly is at the video store, where they are having a massive sale—all the previously viewed DVDs are 2 for $20. Ackerly finds two movies that he would like to buy. One is $8.99 and the other is $9.99. Is the 2 for $20 a better deal than buying the DVDs at their regular price?

Solution:

2 for $20	**Regular Price**
The total cost of the two DVDs before tax would be $20.	The total cost of the two DVDs before tax would be 8.99 + 9.99 = $18.98.

It would cost less to buy the DVDs at their regular price, so the sale price is not a better buy.

Example 2

Sacha is at the store looking for sandals. Each pair costs $14.99, so Sacha wants to buy 3 pairs. At another store, the same sandals are on sale—3 for $30. Which is the better buy?

Solution:

Method 1

Store 1	**Store 2**
The total cost for 3 pairs would be $14.99 x 3 = $44.97 before taxes.	The total cost for 3 pairs would be $30 before taxes.

Store 2 has the better deal.

Another method of finding out which is the better deal is to calculate how much it would cost per pair at store 2, and compare the prices.

Method 2

Store 1	**Store 2**
The cost for 1 pair is $14.99 before taxes.	The cost for 1 pair of sandals would be 30 ÷ 3 = $10.00, not including taxes.

Store 2 has the better deal.

Notice that both methods still reach the same conclusion. Sacha should buy sandals at store 2.

We do not include tax in these calculations because if something costs less before tax, it will cost less after taxes are added.

Remember that when you are tempted buy a 'package deal,' the store wants you to buy more. Ask yourself, do I really need all of this? Did Sacha really need 3 pairs of sandals? Who knows, but perhaps it was because of the deal that he felt it was necessary.

Comparing % Discounts and $ Discounts

Another way that stores encourage customers to choose them over a competitor is to offer a similar discount, but to advertise it differently. While one grocery store may offer 25¢ off per pound of apples, another may offer 10% off. You need to be able to recognize which is the better deal.

Example 1

Araluen is looking at the flyers for two different nurseries. At Ambercrombie, snapdragon seeds are 15% off. At St. Paul's, snapdragon seeds are 50¢ off per packet. Both stores usually sell snapdragon seeds for $4.00. Which is the better deal?

Before you look at the solution—which nursery do you think is offering the better deal?

Solution:

Method 1

Ambercrombie	**St. Paul's**
15% off of $4.00 0.15 × 4 = $0.60 off	$0.50 off of $4.00

Ambercrombie has the better deal.

Method 2

Ambercrombie	**St. Paul's**
15% discount seeds are 85% of original price $4 \times 0.85 = \$3.40$	\$0.50 off Cost = 4 – 0.5 = \$3.50 for the seeds

Ambercrombie has the better deal.

Again, both methods get you to the same answer, so it is your decision which one you will use.

While looking at flyers or comparing prices between stores, it is also important to read the 'fine print'. Some stores may advertise a sale 'per item' while others may offer a discount that applies to your whole bill.

Example 2

Fruitfest market is having a sale on all of their fruit at \$0.80 off per pound of fruit. Healthy Treats market is also having a sale on fruit at \$5.00 off every purchase. Both stores charge \$2.50 per pound of fruit.

a) Corowa is buying fruit for her family. If she buys 5 lb. of fruit, which store should she go to?

b) Dorak is buying fruit to make a fruit salad for the kids at the daycare. If he needs 10 lb. of fruit, which store should he go to?

Predict which store you think each person should go to based on savings?

Solutions:

a) Corowa

Fruitfest	**Healthy Treats**
Cost per pound with discount = 2.50 – 0.80 = 1.70 Total cost = $1.70 \times 5 = \$8.50$	Cost before discount = $5 \times 2.50 = 12.50$ Cost after discount = 12.50 – 5.00 = \$7.50

Corowa should buy her fruit from Healthy Treats market.

b) Dorak

Fruitfest	**Healthy Treats**
Cost per pound with discount = 2.50 – 0.80 = $1.70 Total cost = 1.70 × 10 = $17.00	Cost before discount = 10 × 2.50 = $25.00 Cost after discount = 25.00 – 5.00 = $20.00

Dorak should go to Fruitfest to buy the fruit for the daycare.

As you can see, in this type of situation it is especially important to consider which deal is the 'better buy' because it varies from person to person. It depends on how much you are going to buy. For this reason, it is your situation that dictates which store you should shop at.

Paying the Taxes

Often, stores that sell 'big ticket' items such as furniture, appliances, and tools will advertise, "For one day only, we will pay the taxes on any purchase made". This sale is popular because, although taxes are not much when the cost is small. For example, 5% and 7% of $50 is only $2.50 and $3.50. The amount of tax paid on expensive items is far greater. For example, the total amount of taxes that you would pay on a fridge for $2000 would be $240!

So we'll ask this question:

Which do you predict is the better deal, 12% off the original price and you pay the taxes OR paying regular price but the store pays the taxes?

Example 1

Alecta is buying a car. The sedan that she wants costs $18 000. Prairie Range has a promotion where the dealership will pay the taxes. Mountain Lot, in an attempt to compete with Prairie Range, is offering 12% off before taxes. Which is the better buy?

Solution:

Prairie Range	**Mountain Lot**
Total cost = $18 000 The dealer pays the taxes.	Cost with discount, before taxes C = (1.00 – 0.12) × 18 000 C = 0.88 × 18 000 = $15 840 GST = 0.05 × 15 840 = $792 PST = 0.07 × 15 840 = $1108.80 Cost including taxes Total = 15 840 + 792 + 1108.80 Total = $17 740.80

The better buy is Mountain Lot.

The reason that the 12% the original price is always the better deal involves how you are finding 12%. The amount of discount is 12% of the full price whereas the 12% in taxes is calculated on the sale price. Thus, **you take off more than you add on, so it will always be the better buy to get 12% off the original price instead of not paying the taxes.**

You have found that 12% off the original price when you pay taxes is better than paying the regular price when the store pays the taxes. Now we will investigate whether 10% or 15% makes the deal worthwhile.

Example 2

Kastor needs a new washer and dryer set, and the set he wants will cost $1500. Store A department store has a promotion on—they will pay the taxes. Store B has a 10% off appliances sale. Store C has a 15% off sale on all appliances. Which is the best buy?

Solution:

Store A	**Store B**	**Store C**
Total cost = $1500	Cost with discount, no taxes C = (1.00 – 0.10) × 1500 = $1350 Taxes: PST = 0.07 × 1350 = $94.50 GST = 0.05 × 1350 = $67.50 Total including taxes Total = 1350 + 94.50 + 67.50 Total = $1512	Cost with discount before taxes C = (1.00 – 0.15) × 1500 = $1275 Taxes: PST = 0.07 × 1275 = $89.25 GST = 0.05 × 1275 = $63.75 Total including taxes Total = 1275 + 89.25 + 63.75 Total = $1428

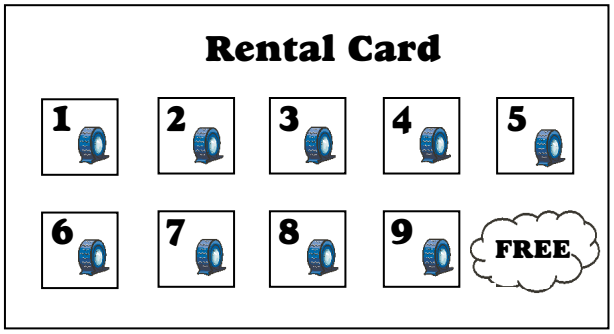
The best buy is at Store C.

You might also notice that with only 10% off, choosing the store that will pay the taxes is a better deal.

Loyalty Cards

You have learned about sales that stores offer, how they encourage people to come to their store, and how they use these sales to compete with other stores. You may ask, "So then what? Do people always go to the store for the best deal?" You can probably answer these questions yourself. Do you always go to the same store to buy shoes or do you compare prices and go to the store with the best deal? If you always go to the same store, why do you go there?

More and more frequently, stores are beginning to offer loyalty cards to customers who purchase something in their store. Loyalty cards provide extra discounts or allow you to collect points.



Stores offer these cards to customers in order to encourage people to come back to their stores instead of going somewhere else. In some stores you have to purchase the card, and as a reward you will get a discount on your first purchase. This includes department store credit cards. Perhaps you have heard the cashier ask if you are a member, and if you say no, they often will proceed to give a description of all the benefits of having the card. The following example will illustrate the type of offers a store may have if you join their loyalty program.

Example 1

Matran is buying some clothes at CP Jenni's. The cashier tells Matran that if he signs up for their credit card today at a cost of $25, he will get 20% off of the purchase he is making. Plus he will get 5% off of all future purchases he makes at the store for the next year. The total bill for Matran is $80 excluding taxes. Should he get the credit card, if he does not usually shop at CP Jenni's?

Solution:

Your automatic answer may have been "yes, he gets 20% off his purchase". Unfortunately this may not be financially wise because he has to pay $25 extra for the credit card.

Without Credit Card	With Credit Card
Matran's bill would be $80 before taxes	Cost of purchase including discount = (1.00 – 0.20) × 80 = $64 Cost including price of credit card = $64 + 25 = $89

Based on these calculations, if Matran doesn't go to the store for the rest of the year, he should not get the credit card because he will pay more if he gets the credit card than if he doesn't.

The Cost of Travel

You may have thought, "Why wouldn't Matran start going to CP Jenni's more often to take advantage of the credit card and discounts?" Since Matran is fictional, we may never know the answer, but in reality there are other factors that influence your decision to pursue a sale.

If you drive across the city to save $1 on an item, you may not actually save that much because you are using gas. When the gas you use to get to a farther store costs more than the amount that you are saving, you are better off to shop at the closer store.

In addition to the cost of gas, you should ask yourself, "How much is my time worth?" Is it worth your time to drive half an hour in both directions just so that you get the best deal on an item? Perhaps not.

Larger items such as furniture and appliances may warrant a farther trip than groceries because they cost more and so discounts are also worth more. You could keep in mind that although you may be saving the discount, you are paying with your time and gas.

Now that you are aware of many types of sales promotions and techniques that may be used, complete the following learning activity to apply the knowledge you've acquired.

Learning Activity 6.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are visiting your grandparents in Toronto with your two brothers. Your parents send a cheque for the three of you to split. If the cheque is for $150, how much will each of you get?
2. Solve for n: $4n - 3 = 2 + 19$.
3. At the store, you compare the price of two hand lotions. The lemon lotion is $3.00 for a 60 mL bottle. The vanilla lotion is $6.00 for a 100 mL bottle. Which is the better deal (per mL)?
4. What is the supplementary angle of 68°?
5. Which metric unit would be the best to describe the distance from Canada to Mexico?

Part B: Sales Promotions

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. BargainBin sells used CDs for $9.95 each. Today, they also have a sale of any 3 CDs for $20.
   a) Which is a better deal per CD?
   b) Lucas can only find one CD that he really wants. Should he still buy 3 CDs?
2. Sonja needs a new pair of skates. The skates Sonja wants cost $250. Sports Unlimited has hockey skates for sale at $25 off. Ice Athletics has all hockey skates on sale for 15% off. Which is the better buy?

continued

Learning Activity 6.6 (continued)

3. Ludovic would like to get a laptop with the money he earned at his summer job. He would like a Pear laptop, which costs $1200. The Pear Store has a promotion to pay the taxes for any computer or laptop purchased. Finest Purchase has a sale on all electronics at 15% off before taxes. Where should Ludovic purchase his laptop? Try to use what you learned in this lesson to decide without doing any calculations; then check your answer with calculations.

4. Imagine that you are a store manager at the mall. You are putting on a sale and you have to decide how to attract more customers.

   OPTION A: You make signs advertising "30% off all merchandise."

   OPTION B: You mark down each item of clothing so the tag has the old price and the sale price. For example, a sweater that was $30.00 is now $21.00.

   State which option you think would be more enticing for the customer and justify your answer with 2 reasons for your decision.

Lesson Summary

In this lesson, you examined different sales promotions and tactics used by stores, and calculated the actual costs of these promotions. Remember that the stores want you to spend your money, so it is your responsibility to yourself to decide what is most cost-effective for you.

The next lesson will delve into the world of currency. You will learn about exchange rates and how to convert between currencies.

Assignment 6.5

Total: 15 marks

Sales Promotion

1. Sorhab has 4 CDs that he would like to buy. OSN sells CDs for $9.50 each. Music Etc. is selling CDs 2 for $20. Both store prices include taxes.

   a) Which is the better buy? *(3 marks)*

   b) If Sorhab only has $35, can he afford to buy 4 CDs? Explain your answer. *(1 mark)*

continued

Assignment 6.5: Sales Promotion (continued)

2. Salem is going out to buy a calculator for school. At HaloKart, they are selling calculators at 10% off. Aim has their calculators on sale for $5 off.

   a) Which is the better buy if the calculator that Salem needs is $36.85? By how much? *(4 marks)*

   b) Imagine that you are in Salem's position. You live across the street from HaloKart, which you can walk to. The drive to Aim is 30 minutes and will cost $2.50 in gas. Which store would you go to, based on the total cost? *(2 marks)*

continued

Assignment 6.5: Sales Promotion (continued)

3. Charlie and Jordan need a dining room set for their new house. The set they want is $1000. Janet's Furniture Store will pay the taxes for any furniture bought this weekend. Parker's Department Store is selling the same dining room set at $100 off.

   a) Where should they buy their dining room set from? *(3 marks)*

   b) Charlie and Jordan are having family over for a fancy dinner next weekend. Janet's Furniture does not have the set they want in stock, and it will take 4 weeks to get it in. Parker's will be able to deliver in 3 days from the day they buy it. Which store should they buy from? Why? *(2 marks)*

Notes

Lesson 6: Currency Exchange

Lesson Focus

In this lesson, you will

- ❑ solve problems using currency exchange
- ❑ convert between Canadian and foreign currencies

Lesson Introduction

Canadians live in a "global economy" where we trade goods with countries all around the globe. But how do you compare prices in two different currencies—theirs and ours?

Each country has its own currency, or money. This lesson shows you how to determine how much Canadian money is worth in a foreign currency. You will study various foreign currency exchanges.

Currency Exchange

When you travel to foreign countries or buy items from them online, you have to use their money, or currency. For example, if you bought a camera online from someone in the United States, they would have to be paid in American dollars. You would have to convert your Canadian currency to US dollars, and send that amount to the seller.

Buy-Sell Rates

You can obtain buy and sell rates for currencies for most countries at your local bank or credit union. If you use your credit card to buy or sell currencies, the bank will convert the currencies on your bill.

The banks or credit unions will charge you a fee for providing this service. For example, in February 2009 the bank would sell you one American dollar for $1.2634. If you brought that dollar back to the bank, they would buy it back for $1.2106. The difference of roughly 5 cents per dollar is the bank's fee for providing foreign currency.

Example

You are travelling to Minneapolis for a family holiday and will need some American money. You decide to get $50 US for some shopping. When you go to the bank, they SELL you the US money at $1.2634 per dollar.

a) How much will this cost you in Canadian dollars?

b) If the trip was cancelled due to bad weather, you need to return to the bank to convert the US money back to Canadian funds. The bank will BUY it back at $1.2106. How much money in Canadian dollars will the bank return to you?

Solution:

a) Since you want to buy American dollars, the bank sells them to you at a rate of $1.2634 Canadian for $1 US.

Sell Rate

$1 US costs $1.2634 Canadian

$50 US costs $50 \times 1.2634 = \$63.17$ Canadian

Your cost is $63.17 Canadian.

b) The bank buys the US dollars from you.

Buy Rate

$1 US = $1.2106 Canadian

$50 US = $50 \times 1.2106 = \$60.53$.

The difference is $63.17 – $60.53 = $2.64, which is the bank's profit.

It is important for you to understand that the buy-sell rates are based on what the bank decides, considering the daily market for buying and selling money. Thus, whenever you read a buy-sell currency chart, remember it is made from the bank's point of view, not the customer's. The bank will sell you currency, and the bank will buy your currency back at a lower rate.

Market Fluctuations

Currency rates on the money markets change every hour that the stock exchanges are open. Consequently, every major bank or credit union has a currency converter on its website to show you the up-to-date information. In 2008 the Canadian dollar was worth more than the American dollar. By early 2009, the US dollar had risen about 25% compared to the Canadian currency. There have been times in recent history where the US dollar was worth $1.65 Canadian. At that time, a flight costing $1000 US would have cost you $1650 in Canadian dollars.

Euro

Many countries across Europe have united in using one currency—the Euro. As of February 2009, the names of these countries are listed below.

Austria
Belgium
Cyprus (since 1 January 2008)
Finland
France
Germany
Greece
Ireland
Italy
Luxembourg
Malta (since 1 January 2008)
The Netherlands
Portugal
Slovakia (from 1 January 2009)
Slovenia (since 1 January 2007)
Spain

Table

The table on the following page shows the Bank of Canada's currency rates as of February 2009. Each value represents the Canadian dollar amount for buying or selling 1 unit in the listed currencies.

You can use these values for your exercises in this lesson. If you are actually going to exchange some money or buy an item from another country, check the current rates through your bank or credit union before making any purchasing decisions.

Remember, when you bring foreign currency into your bank to convert to Canadian dollars, they are buying that currency from you. If you want some foreign currency, the bank is selling it to you. Always ask yourself "what is the bank doing." That rule will help you select either the BUY or SELL rate.

Currency Converter as of February 2009		
	Bank's Selling Rate	**Bank's Buying Rate**
Canadian Dollar ($)	1	1
Australian Dollar ($)	0.843	0.7832
British Pound (£)	1.8197	1.7397
Danish Krone (kr)	0.2219	0.2049
Euro (€)	1.6345	1.5475
Hong Kong Dollar ($)	0.1643	0.1549
Japanese Yen (¥)	0.013797	0.013109
Mexican Peso ($)	0.0909	0.0794
New Zealand Dollar ($)	0.6701	0.6241
Norwegian Krone (kr)	0.1891	0.1746
Saudi Riyal (SAR)	0.3458	0.3138
Singapore Dollar ($)	0.8487	0.7917
Swedish Krona (kr)	0.1522	0.1426
Swiss Franc (CHF)	1.0934	1.0354
American Dollar ($)	1.2634	1.2106

www.bankofcanada.ca/en/rates/exchange.html. Reprinted with permission.

Note the page number or include the table on your resource sheet.

Example 1

Use the currency table to convert 800 New Zealand dollars to Canadian dollars.

Solution:

You have 800 New Zealand dollars, so the bank is buying them from you. Use the Bank's Buying Rate. From the table, the rate is 0.6241.

$$\therefore 800 \times 0.6241 = \$499.28$$

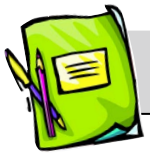
Example 2

Use the currency table to find the cost in Canadian dollars of obtaining 10 000 Norwegian kroner from the bank.

Solution:

You want to buy 10 000 Norwegian kroner from the bank, so the bank is selling to you. Use the Bank's Selling Rate. From the table, the rate is 0.1891.

$$\therefore 10\,000 \times 0.1891 = \$1891$$



Learning Activity 6.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the area of a triangle with a base of 2 m and a height of 5 m?
2. You work 40 hours per week and are paid $15 per hour. What is your gross pay for the year?
3. A map of Winnipeg is drawn to scale so that 1 cm = 5 km. If Winnipeg is 25 km from east to west, what will be the distance on the map?
4. You changed your cell phone plan and saved $12 a month. How much do you save in one year?
5. You want to watch a movie that is 145 minutes long but you have to leave the house in 2 hours. Do you have enough time to watch the movie before leaving the house?

continued

Learning Activity 6.7 (continud)

Part B: Bank Currency Exchange

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the currency table to convert the following currencies to Canadian dollars. In other words, how much money (in Canadian dollars) would you get for the following currencies? Show your calculations.
   a) 5000 Euros _____
   b) 1500 Swiss Francs _____
   c) 30 000 Japanese Yen _____
   d) 85 000 Euros _____
   e) 2500 British Pounds _____
2. Use the currency table to find the cost in Canadian dollars of obtaining the following currencies from the bank. Show your calculations.
   a) 3250 Australian dollars _____
   b) $750 US dollars _____
   c) 975 Danish Kroner _____
   d) 8500 Euros _____
   e) 1 000 000 Mexican Pesos _____

Proportional Reasoning

You can solve any currency problem using proportions. The currency conversion table shows you the actual cost of each currency in Canadian dollars. But, there are situations where the table cannot help you solve the problem directly. In these cases you can use a proportion.

Example 1

If you have $300 Canadian, how many US dollars can you buy?

Solution:

Proportional reasoning is the method to use. Be careful to label the columns and rows of the box so that the data is placed in the proper spots.

Since you need to acquire some US dollars, the bank will sell them to you. You use the SELL rate.

	Canadian	American
SELL rate	\$1.2634	\$1
your money	\$300	\$$x$

$$\frac{\$1.2634}{\$300} = \frac{\$1}{\$x}$$

$$1.2634x = 300$$

$$\frac{1.2634x}{1.2634} = \frac{300}{1.2634}$$

$$x = 237.45$$

Therefore, \$300 of your Canadian money will buy you \$237.45 in US funds.

Example 2

You are following an online auction selling a fancy pair of runners you like. The seller is in the US, so you will have to pay him in US funds if you win the auction. You have a total of \$50 Canadian available to you. At what point (in US funds) must you stop bidding due to lack of funds?

Solution:

	Canadian	American
SELL rate	\$1.2634	\$1
your money	\$50	\$$x$

$$\frac{\$1.2634}{\$50} = \frac{\$1}{\$x}$$

$$1.2634x = 50$$

$$\frac{1.2634x}{1.2634} = \frac{50}{1.2634}$$

$$x = 39.58$$

You have a total of \$39.58 US for this purchase.

Estimating

If you go on a trip outside of Canada and you have purchased items on your trip, you will have to make a declaration to Canadian Customs when you re-enter Canada. If you were outside the country for at least 48 hours, you are allowed to bring back up to $400 Canadian in goods without paying extra duty charges. If you were outside the country for at least seven days, you are allowed to bring back up to $750 Canadian in goods without paying extra duty charges.

Example 1

You travel to Europe for a month, with the Euro being about $1.63 Canadian.

a) Find the amount of goods in Euros you can declare.

b) Make an estimate to check your calculations.

Solution:

a)

	Canadian	Euros
SELL rate	$1.63	€1
your money	$750	€x

$$\frac{\$1.63}{\$750} = \frac{1 \text{ Euro}}{x \text{ Euros}}$$

$$1.63x = 750$$

$$\frac{1.63x}{1.63} = \frac{750}{1.63}$$

$$x = 460.12$$

You have a total of 460.12 Euro to spend on goods you want to bring back to Canada without having to pay extra duty charges.

b) You could round the sell rate of $1.63 to $1.60 and your answer in part (a) to $460. When you multiply them together, your answer should be about $750.

$1.6 \times 460 = 736$

Example 2

If you travel to the US for a weekend, when the American dollar is worth about $1.25 Canadian, how much can you bring back to Canada in American dollars? Estimate your answer.

Solution:

If you were to buy $100 US worth of goods, this would be 100 x 1.25 = $125 Canadian,

$200 worth, would be 200 × 1.25 = $250 CAN, and

$300 worth would be 300 × 1.25 = $375 CAN.

You could buy approximately $300 US worth of goods to bring home.

The exact answer is $\frac{\$400}{\$1.25} = \$320$ US.

Example 3

Should you be out of the country for at least 7 days, the tax-free limit you can bring home is $750 Canadian. Estimate the amount of US funds, using the SELL rate of $1.25 CAN you can bring back to Canada.

Solution:

You can use arbitrary examples to find the closest value to $750 Canadian. The multiplication is done in two steps and the results added together to get the final amount.

First Guess	**Second Guess**
$500 × $1.25	$600 × $1.25
$500 × 1 = $500	$600 × 1 = 600
$500 × 0.25 = $125	$600 × 0.25 = $150
Total = $500 × 1.25 = $500 + $125 = $625	Total = $600 × 1.25 = $600 + $150 = $750

You can purchase up to $600 US in goods before having to pay additional duty charges.

Learning Activity 6.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If you have 5 quarters, 2 dimes, and 1 nickel, how much money do you have?
2. How many minutes in 2.25 hours?
3. Emma is 60 inches tall. What is her height in feet?
4. Which costs less per bottle: 3 bottles of pop for $3.99 or 1 bottle of pop for $1.50?
5. Write two equivalent fractions to $\frac{12}{14}$.

Part B: Applying Currency Exchange

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Nicole is planning a trip to Great Britain and Germany. She estimates she will need 500 British pounds and 2000 Euros. How much in Canadian funds will she require to purchase these foreign currencies?
2. Nathan wants to travel to Hong Kong. The trip will cost $8000 Hong Kong dollars. He has $1350 in his savings account. Can Nathan afford to go on this trip?
3. Breanne acquires $800 US for a trip to Florida. The trip was cancelled, and she returned the US money to the bank for Canadian funds. How much money did she lose in this transaction?
4. Kyle's uncle in New Zealand sent him a graduation present. The cheque was for $2500 in New Zealand dollars. When he takes it to his bank, how much money will he receive in Canadian funds?

Lesson Summary

This lesson helped you learn how to convert currencies from other countries into Canadian funds and vice versa. You determined how to calculate the bank costs for such transactions.

This concludes your work in Module 6.

Notes

Assignment 6.6

Total: 18 marks

Currency Exchange

1. Convert the following to Canadian funds from the given currency. Show your calculations. *(4 marks)*

   a) 12 500 Hong Kong Dollars _____

   b) 6894 Mexican Pesos _____

   c) 962 British Pounds _____

   d) 125 US dollars _____

2. Determine the cost in Canadian funds to acquire these currencies. Show your calculations. *(4 marks)*

   a) 1600 Euros _____

   b) 68 950 Japanese Yen _____

   c) 1200 Swiss Francs _____

   d) 10 000 Mexican Pesos _____

continued

Assignment 6.6: Currency Exchange (continued)

3. Kristen has $500 in her bank account. How much American money can she acquire? *(3 marks)*

4. Anya received a cheque from her grandmother in England in the amount of 75 pounds. Find its value in Canadian funds. *(2 marks)*

continued

Assignment 6.6: Currency Exchange (continued)

5. Shay went on a trip to Europe, taking 3500 Euros with him. He also took 1000 Danish Kroner. Upon his return, he converted 460 Euros and 120 Kroner back into Canadian funds. Find the total cost of his trip. *(5 marks)*

Notes

Module 6 Summary

This module contained important information for you, as a consumer, to become a wiser shopper. Unit pricing, costs of sales promotions, estimating percentages, comparing discounts and sale prices, and converting currencies are all important skills that were discussed. The formula for finding percent rate of change was used, and a shortcut for finding the sales price from the full price was shown. Many applications to solving problems using proportions and solving for the unknown were learned.

Vocabulary

Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful.

bank's buy rate
bank's sell rate
buy now-pay later
cash price
consumer
currency exchange
estimating percent
foreign currency
full price
purchase price
sale price
sales promotion
unit price
foreign currency
full price

Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

Sale price = Full price – discount

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original amount}} \times 100$$

Submitting Your Assignments

It is now time for you to submit the Module 6 Cover Assignment and Assignments 6.1 to 6.6 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 6 assignments and organize your material in the following order:

Module 6 Cover Sheet

Module 6 Cover Assignment: Pizza Party

Assignment 6.1: Unit Pricing

Assignment 6.2: Unit Prices Project

Assignment 6.3: Percents

Assignment 6.4: Percent Increase or Decrease

Assignment 6.5: Sales Promotion

Assignment 6.6: Currency Exchange

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 10 Essential Mathematics (20S)

Module 6
Consumer Decisions

Learning Activity Answer Keys

Module 6:
Consumer Decisions

Learning Activity 6.1

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A checkerboard has 8 squares along its length and 8 squares along its width. How many squares all together are on the board?
2. The squares on a checkerboard alternate between black and white. How many squares are black?
3. You are buying coffee for your family of 6. Each coffee costs $1.50. How much will it cost in total?
4. The shortest sentence that contains all the letters of the alphabet is "The quick brown fox jumps over the lazy dog." Write a fraction comparing the number of words with 'o' to the total number of words in the sentence.
5. If 10% of 45 is 4.5, what is 20% of 90?

Answers:

1. 64 (8 × 8)
2. 32 (One row would be either BWBWBWBW or WBWBWBWB; both have 4 black and 4 white squares, so half the board is black squares, 64 ÷ 2 = 32.)
3. $9 (Option A: 6 × 1.5; Option B: 1.5 × 2 = $3, 6 ÷ 2 = 3, 3 × $3 = 9. Although option A looks shorter, it is sometimes harder to do without writing it out.)
4. $\frac{4}{9}$ (Number of words with 'o': brown, fox, over, dog is 4. Total words: 9.)
5. 18 (20% is double 10% and 90 is double 45 so 4.5 doubled, then doubled again is 4.5 × 2 × 2 = 18)

Part B: Buyer Behaviour Self-Test

Directions: Use the scale below as you answer each item.

4 Always
3 Often
2 Seldom
1 Never

	1. I use ads to get information about products and services.
	2. I find information about products and services from places other than ads.
	3. I compare prices and quality of different brands at different stores before buying.
	4. I read labels and guarantees on purchases, and follow their directions.
	5. I look over, try on, try out, and ask questions about products before buying.
	6. I follow a list when shopping.
	7. I let manufacturing companies and businesses know what I like and dislike about their products and services.
	8. I plan in advance what products and services to buy.
	9. I consider my needs and wants as well as price when deciding what to buy.

Add up your scores for each item. Rate yourself according to this scale.

28 – 36 Super shopper
19 – 27 Careful consumer
0 – 18 Poorly prepared purchaser
0 – 9 You need help

Learning Activity 6.2

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the complementary angle of 79°?
2. You run every second day. You run 3.5 miles on Tuesday, 4 miles on Thursday, and 4.5 miles on Saturday. Which days will you run next week?
3. How far will you run each of those days if you continue the pattern?
4. Solve for e: $\frac{1}{2} = \frac{4}{e}$.
5. Your work day is supposed to be from 8:30 to 4:30. You arrive at work at 7:54, and leave at 4:23. What is the late penalty for your workday?

Answers:

1. 11^{o} (90 – 79 = 11 or count up: 79 + 1 + 10. Remember that complementary angles add up to 90°)
2. Monday, Wednesday, Friday
3. 5 miles, 5.5 miles, 6 miles
4. $e = 8$ (cross-multiply, so $e = 2 \times 4 = 8$)
5. 15 min. (You are not late this morning, but your work does not pay you for getting there early. You leave 7 min. early so you are docked a quarter of an hour, 15 minutes.)

Part B: Unit Pricing

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the unit price for each item, as indicated.

   a) 355 mL soda pop costing $1.25, unit is 100 mL

   Answer:

	millilitres	cost in $
given amount	355 mL	$1.25
unit amount	100 mL	$$x$

$$\frac{355}{100} = \frac{1.25}{x}$$

$$355x = (1.25)(100)$$

$$355x = 125$$

$$\frac{355x}{355} = \frac{125}{355}$$

$$x = 0.35$$

The unit price is $0.35 for 100 mL.

   b) 3.5 kg sugar costs $5.30, unit is 1 kg

   Answer:

	kg	cost in $
given amount	3.5 kg	$5.30
unit amount	1 g	$$x$

$$\frac{3.5}{1} = \frac{5.30}{x}$$

$$3.5x = 5.30$$

$$\frac{3.5x}{3.5} = \frac{5.30}{3.5}$$

$$x = 1.51$$

The unit price is $1.51 for 1 kg.

c) 1.5 kg sliced meat costs \$11.25, unit is 100 g

Answer:

First convert 1.5 kg to grams.

1.5 kg = 1000 × 1.5 = 1500 g

	grams	cost in \$
given amount	1500 kg	\$11.25
unit amount	100 g	\$$x$

Method 1

$$\frac{1500\text{ g}}{100\text{ g}} = \frac{\$11.25}{\$x}$$

$$1500x = (100)(11.25)$$

$$1500x = 1125$$

$$\frac{1500x}{1500} = \frac{1125}{1500}$$

$$x = 0.75$$

Method 2

Notice in the ratio, $\frac{1500}{100}$ the numerator is 15 times as big as the denominator. Therefore, to solve for x in the denominator of the second ratio, you could just divide \$11.25 by 15 to get \$0.75.

The unit price is \$0.75 for 100 g.

2. Use unit pricing to compare the two products and find the best buy.

a) 6 cans of size 355 mL of diet soda for \$2.85 or a 2 L bottle costing \$2.10. Use a unit of 100 mL.

Answer:

First size

Find total millilitres for 6 cans.

355 mL × 6 = 2130 mL

	mL	cost in \$
given amount	2130 mL	\$2.85
unit amount	100 mL	\$$x$

$$\frac{2130}{100} = \frac{2.85}{x}$$

$$2130x = (100)(2.85)$$

$$2130x = 285$$

$$\frac{2130x}{2130} = \frac{285}{2130}$$

$$x = 0.1338$$

The unit price is \$0.13 for 100 mL.

Second size

Find total millilitres for 2 litres.

2 L = 1000 mL × 2 = 2000 mL

	mL	cost in \$
given amount	2000 mL	\$2.10
unit amount	100 mL	\$$x$

$$\frac{2000}{100} = \frac{2.10}{x}$$

$$2000x = (100)(2.10)$$

$$2000x = 210$$

$$\frac{2000x}{2000} = \frac{210}{2000}$$

$$x = 0.105$$

The unit price is \$0.11 for 100 mL.

Comparing the two unit prices, \$0.13 and \$0.11, the second size has a smaller cost.

The 2 L bottle is the better buy.

b) 1 kg jar peanut butter for $3.95 or 5 kg jar for $18.95. The unit is 1 kg.

Answer:

First size

The unit price is already given as 1 kg costs $ $3.95.

Second size

	kg	cost in $
given amount	5 kg	$18.95
unit amount	1 kg	$$x$

Method 1

$$\frac{5\text{ kg}}{1\text{ kg}} = \frac{\$18.95}{\$x}$$

$$5x = 18.95$$

$$\frac{5x}{5} = \frac{18.95}{5}$$

$$x = 3.79$$

Method 2

Notice in the ratio, $\frac{5}{1}$ the numerator is 5 times as big as the denominator. Therefore, to solve for x in the denominator of the second ratio, you could just divide $18.95 by 5 to get $3.79.

The unit price is $3.79 for 1 kg.

Comparing the two unit prices, $3.95 and $3.79, the second size has a smaller cost.

The 5 kg container is cheaper.

3. Give two reasons why someone might not purchase a larger item, knowing the unit price is lower.

Answer:

Any two of the following:

- not enough cash at the moment
- can't use or don't need a large quantity
- lack of storage space
- inconvenient to travel to the larger store

Learning Activity 6.3

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Linden loves milkshakes. He gets to choose from two chocolate milkshakes. One fills the glass $\frac{7}{9}$ full. The other fills the glass $\frac{2}{3}$ full. Which will he choose?
2. You are at a bake sale. You want to buy a peanut butter cookie. If the whole plate of 16 cookies costs $3.20, how much will you pay for one cookie?
3. Think of a clock. How many degrees does the minute hand move between 17:30 and 18:00?
4. Vertically opposite angles are _____.
5. Writers are paid by piecework. Jill Jackson is paid $80 for every children's story she writes and $50 for every poem she writes. Over the past 6 months she has written 4 children's stories and 6 poems. How much money was she paid?

Answers:

1. $\frac{7}{9}$ (Make the fractions have a common denominator so you can compare them: $\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$.)
2. $0.20 (Convert from dollars to cents. This is helpful when the amount you are dividing by is greater than the dollar amount. $3.20 = 320¢; $\frac{320}{16} = 20¢$)
3. 180° (When the minute hand is at 30 min., it is pointing directly downward. When the minute hand is at 0 min., it is pointing directly upward. These two points form a straight angle, which is 180°.)
4. Equal
5. $620 (books: 4 × $80 = $320, poems: 6 × $50 = $300)

Part B: Calculating 10%

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the amount of the percentage without using a calculator.

1. 10% of $567 _____

   Answer:

   Move the decimal place 1 place to the left to get $56.70.

2. 10% of $628.09 _____

   Answer:

   Move the decimal place 1 place to the left to get $62.809 or $62.81.

3. 10% of $6.75 _____

   Answer:

   Move the decimal place 1 place to the left to get $0.675 or $0.68.

4. 5% of $8422 _____

   Answer:

   For 10%, move the decimal place 1 place to the left to get $842.20. Then divide the answer by 2 to get $421.10.

5. 5% of $468.80 _____

   Answer:

   Move the decimal place 1 place to the left to get $46.88. Then divide the answer by 2 to get $23.44.

6. 20% of $364.20 _____

   Answer:

   Move the decimal place 1 place to the left to get $36.42. Then multiply the answer by 2 to get $72.84.

7. 1% of $56.52 _____

   Answer:

   Move the decimal place 2 places to the left to get $0.5652 or $0.57.

8. 2% of $6259 _____

   Answer:

   Move the decimal place 2 places to the left to get $62.59. Then multiply by 2 to get $125.18.

Learning Activity 6.4

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Find 10% of 430.
2. Calculate 20 + 80.
3. Calculate 5×30.
4. You have $10.00. You spent $2.60 on candy. How much do you have left?
5. Which is larger: $\frac{1}{9}$ or $\frac{1}{11}$?

Answers:

1. 43
2. 100 (It may help to think about 2 + 8.)
3. 150 (It may help to think about 5×3.)
4. $7.40 (Count up.)
5. $\frac{1}{9}$ (When you divide something into 9 groups and divide the same thing into 11 groups, each group of the 9 is larger than each group of the 11. Think of slicing a pizza into 9 pieces or 11 pieces. Which size of pizza slice would be bigger?)

Part B: Percent Discount (1)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the sale price of each item before the taxes.

1. Regularly priced at $56.95 with 15% off

   Answer:

   100% – 15% = 85%

   $56.95 × 0.85 = $48.41

2. Regularly priced at $424.68 with 7.5% off

   Answer:

   100% – 7.5% = 92.5%

   $424.68 × 0.925 = $392.83

3. Regularly priced at $38.62 with 5% off

   Answer:

   100% – 5% = 95%

   $38.62 × 0.95 = $36.69

4. Regularly priced at $2688.46 with 26.4% off

   Answer:

   100% – 26.4% = 73.6%

   $2688.46 × 0.736 = $1978.71

Learning Activity 6.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are making banana bread but only need to make half the recipe. The recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar will you use to make half the recipe?
2. One-quarter of the Grade 10 students in a school are going to the divisional track meet. There are 120 Grade 10 students in the school. How many are going to the track meet?
3. At a car wash, Doug is paid $5 per car he washes. Doug washes 20 cars during his shift. How much money will he get paid?
4. Which of the following fractions are equivalent?

   $\frac{4}{8}$ $\frac{5}{12}$ $\frac{7}{14}$ $\frac{9}{16}$

5. If 100 cm of snow is equivalent to 50 mm of water, 150 cm of snow is equivalent to how much water?

Answers:

1. $\frac{1}{4}$ cup (Half of a half = $\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$)
2. 30 Grade 10 students $\left(120 \times \frac{1}{4} = \frac{120}{4} = 30\right)$
3. $100 (5 × 20. It may help to think of 5 × 2.)
4. $\frac{4}{8}$ and $\frac{7}{14}$ (Both of these simplify to $\frac{1}{2}$: $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$ and $\frac{7 \div 7}{14 \div 7} = \frac{1}{2}$; $\frac{5}{12}$ is less than $\frac{1}{2}$ and $\frac{9}{16}$ is greater than $\frac{1}{2}$.)
5. 75 mm water (If you compare $\frac{50 \text{ mm} \div 50}{100 \text{ cm} \div 50} = \frac{1}{2}$, the unit changes from cm to mm, and the amount changes by half. Half of 150 = 150 ÷ 2 = 75.)

Part B: Percent Discount (2)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Find the percent rate of increase or decrease for each situation. Show your work and then fill in the missing numbers in the table. Round off your answers to two decimal places.

	Original	New Amount	Change	Percent
1	$36.50	$39.00		
2	$280.50		$15.75 increase	
3		$864.60	$96.40 decrease	
4	$7.85			5.35% increase
5	$26.86		$4.82 decrease	
6		$13.75		4% increase

Answer:

The completed table is shown below the calculations.

1. Change = $39 − $36.50 = $2.50

$$\text{Percent} = \frac{\$2.50}{\$36.50} \times 100 = 6.85\%$$

2. New amount = $280.50 + $15.75 = $ 296.25

$$\text{Percent} = \frac{\$15.75}{\$280.50} \times 100 = 5.61\%$$

3. Original amount = $864.60 + $96.40 = $961

$$\text{Percent} = \frac{\$96.40}{\$961} \times 100 = 10.03\%$$

4. Amount of change in value = percent rate of change × the original amount

   Amount of change in value = 0.0535 × $7.85 = $0.42

   New amount = $7.85 + $0.42 = $8.27

5. New amount = \$26.86 − \$4.82 = \$22.04

$$\text{Percent} = \frac{\$4.82}{\$26.86} \times 100 = 17.94\%$$

6. The new amount is a 4% over the original amount. Thus, \$13.75 is 104% of the original.

	cost	percent
new price	\$13.75	104%
original price	\$$x$	100%

$$\frac{\$13.75}{\$x} = \frac{104\%}{100\%}$$

$$104x = 1375$$

$$\frac{104x}{104} = \frac{1375}{104}$$

$$x = 13.22$$

The original price amount is \$13.22.

The change is \$13.75 − \$13.22 = \$0.53.

	Original	New Amount	Change	Percent
1	\$36.50	\$39.00	\$2.50	6.85%
2	\$280.50	\$296.25	\$15.75 increase	5.61%
3	\$961	\$864.60	\$96.40 decrease	10.03%
4	\$7.85	\$8.27	\$0.42	5.35% increase
5	\$26.86	\$22.04	\$4.82 decrease	17.94%
6	\$13.22	\$13.75	\$0.53	4% increase

Learning Activity 6.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are visiting your grandparents in Toronto with your two brothers. Your parents send a cheque for the three of you to split. If the cheque is for $150, how much will each of you get?
2. Solve for n: $4n - 3 = 2 + 19$.
3. At the store, you compare the price of two hand lotions. The lemon lotion is $3.00 for a 60 mL bottle. The vanilla lotion is $6.00 for a 100 mL bottle. Which is the better deal (per mL)?
4. What is the supplementary angle of 68°?
5. Which metric unit would be the best to describe the distance from Canada to Mexico?

Answers:

1. $50 (150 ÷ 3)
2. $n = 6$ (combine 2 and 19: $4n - 3 = 21$; add 3 to both sides: $4n = 21 + 3$; divide both sides by 4: $n = 24 \div 4$)
3. The lemon lotion is the better deal. ($3 = 300¢, so $\frac{300}{60} = 50¢/\text{mL}$.

   $6 = 600¢, so $\frac{600}{100} = 60¢/\text{mL}$.)
4. 112° (Count up: 68 + 2 + 30 + 80 = 180)
5. kilometres (Anything smaller would result in a huge number. Anything larger may reduce the size of the number, but because we are not used to working with these units, the number would not provide much meaning.)

Part B: Sales Promotions

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. BargainBin sells used CDs for $9.95 each. Today, they also have a sale of any 3 CDs for $20.

   a) Which is a better deal per CD?

   Answer:

Regular Price	On Sale
$9.95 per CD	$20 for 3 CDs = 20 ÷ 3 = $6.67 per CD

   The sale is a better price per CD.

   b) Lucas can only find one CD that he really wants. Should he still buy 3 CDs?

   Answer:

   No, Lucas should only buy the one CD. If he bought another 2 CDs, he would be wasting $10 on CDs that he did not want and will probably never listen to.

2. Sonja needs a new pair of skates. The skates Sonja wants cost $250. Sports Unlimited has hockey skates for sale at $25 off. Ice Athletics has all hockey skates on sale for 15% off. Which is the better buy?

   Answer:

Sports Unlimited	Ice Athletics
amount of discount: $25	Amount of discount: 15% of 250 = 0.15 × 250 = $37.50

   Sonja should buy the skates at Ice Athletics.

3. Ludovic would like to get a laptop with the money he earned at his summer job. He would like a Pear laptop, which costs $1200. The Pear Store has a promotion to pay the taxes for any computer or laptop purchased. Finest Purchase has a sale on all electronics at 15% off before taxes. Where should Ludovic purchase his laptop? Try to use what you learned in this lesson to decide without doing any calculations; then check your answer with calculations.

   Answer:

   From the examples in this lesson we know that 15% is always the better deal because, although you pay 12% in taxes, the total cost will still be less than the regular price without taxes.

   The Pear Store

   total cost = $1200

   Finest Purchase

   cost before taxes: $(1.00 - 0.15) \times 1200 = 0.85 \times 1200 = \$1020$

   cost with PST and GST: $1020 \times 1.12 = \$1142.40$

4. Imagine that you are a store manager at the mall. You are putting on a sale and you have to decide how to attract more customers.

   OPTION A: You make signs advertising "30% off all merchandise."

   OPTION B: You mark down each item of clothing so the tag has the old price and the sale price. For example, a sweater that was $30.00 is now $21.00.

   State which option you think would be more enticing for the customer and justify your answer with 2 reasons for your decision.

   Answer:

   Answers may vary. Here are some possibilities.

   Option A

   - signs can be eye-catching
   - more people could see the signs
   - 30% off sounds like a large portion of the original price you do not have to pay

   Option B

   - customers will be able to see how much they are saving, dollar wise, on the price tag
   - some items can be marked down by more than 30%

Learning Activity 6.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the area of a triangle with a base of 2 m and a height of 5 m?
2. You work 40 hours per week and are paid $15 per hour. What is your gross pay for the year?
3. A map of Winnipeg is drawn to scale so that 1 cm = 5 km. If Winnipeg is 25 km from east to west, what will be the distance on the map?
4. You changed your cell phone plan and saved $12 a month. How much do you save in one year?
5. You want to watch a movie that is 145 minutes long but you have to leave the house in 2 hours. Do you have enough time to watch the movie before leaving the house?

Answers:

1. 5 m^2 $\left(A = \frac{1}{2}bh;\ \text{half of 2 is 1, so the area is 5 } m^2.\right)$
2. $31,300 (4 × 15 = 60 so you earn $600 per week. There are 52 weeks per year so you earn 600 × (50 + 2) = (600 × 50) + (600 × 2) = 30000 + 1200 or $31,200 per year.)
3. 5 cm (1 cm = 5 km so 25 km = $\frac{25}{5}$ = 5 cm)
4. $144 (12 × 12 = $144 in a year)
5. No (2 hours = 60 × 2 = 120 minutes which is less than the length of the movie)

Part B: Bank Currency Exchange

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the currency table to convert the following currencies to Canadian dollars. In other words, how much money (in Canadian dollars) would you get for the following currencies? Show your calculations.
   a) 5000 Euros — *Answer:* 5000 × 1.5475 = $7737.50
   b) 1500 Swiss Francs — *Answer:* 1500 × 1.0354 = $1553.10
   c) 30 000 Japanese Yen — *Answer:* 30 000 × 0.013109 = $393.27
   d) 85 000 Euros — *Answer:* 85 000 × 1.5475 = $131,537.50
   e) 2500 British Pounds — *Answer:* 2500 × 1.7397 = $4349.25

2. Use the currency table to find the cost in Canadian dollars of obtaining the following currencies from the bank. Show your calculations.
   a) 3250 Australian dollars — *Answer:* 3250 × 0.843 = $2739.75
   b) $750 US dollars — *Answer:* 750 × 1.2634 = $947.55
   c) 975 Danish Kroner — *Answer:* 975 × 0.2219 = $216.35
   d) 8500 Euros — *Answer:* 8500 × 1.6345 = $13,893.25
   e) 1 000 000 Mexican Pesos — *Answer:* 1 000 000 × 0.0909 = $90,900

Learning Activity 6.8

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If you have 5 quarters, 2 dimes, and 1 nickel, how much money do you have?
2. How many minutes in 2.25 hours?
3. Emma is 60 inches tall. What is her height in feet?
4. Which costs less per bottle: 3 bottles of pop for $3.99 or 1 bottle of pop for $1.50?
5. Write two equivalent fractions to $\frac{12}{14}$.

Answers:

1. $1.50 (5 quarters = $1.25, 2 dimes = $0.20, 1 nickel = $0.05; total = $1.50)
2. 135 minutes (2 hours = 120 minutes, 0.25 hours = 0.25 × 60 = 15 minutes; total = 135)
3. 5 feet (There are 12 inches in a foot so 60 ÷ 12 = 5; therefore, Emma is 5 feet tall.)
4. 3 bottles for $3.99 (3.99 ÷ 3 = $1.33 per bottle is less than $1.50)
5. $\frac{6}{7}, \frac{18}{21}$ (There are many answers. The most simplified form of this fraction is $\frac{6}{7}$, so any multiple of this fraction would be fine; $\left(\frac{18}{21}, \frac{24}{28}, \frac{30}{35}, \text{etc.}\right)$.

Part B: Applying Currency Exchange

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Nicole is planning a trip to Great Britain and Germany. She estimates she will need 500 British pounds and 2000 Euros. How much in Canadian funds will she require to purchase these foreign currencies?

   Answer:

   The bank is selling Nicole the currency.

   1 pound = \$1.8197 Canadian

   500 pounds = 500 × 1.8197 = \$909.85 CAN

   1 Euro = \$1.6345 Canadian

   2000 Euros = 2000 × 1.6345 = \$3269 CAN

   Total = \$909.85 + \$3269 = \$ 4178.85 CAN

2. Nathan wants to travel to Hong Kong. The trip will cost \$8000 Hong Kong dollars. He has \$1350 in his savings account. Can Nathan afford to go on this trip?

   Answer:

	Canadian	Hong Kong \$
SELL rate	\$0.1643	\$1
your money	\$1350	\$$x$

$$\frac{\$0.1643}{\$1350} = \frac{\$1}{\$x}$$

$$0.1643x = 1350$$

$$\frac{0.1643x}{0.1643} = \frac{1350}{0.1643}$$

$$x = 8216.68$$

   Nathan will have HK \$8216.68, which is more than his costs of \$8000. He can afford to take this trip.

3. Breanne acquires $800 US for a trip to Florida. The trip was cancelled, and she returned the US money to the bank for Canadian funds. How much money did she lose in this transaction?

   Answer:

   The bank sells Breanne the US currency.

   $1 US = $1.2634 Can

   $800 US = 800 × 1.2634 = $1010.72 CAN

   The bank buys back her US money.

   $1 US = $1.2106 CAN

   $800 US = 800 × 1.2106 = $968.48 CAN

   Bank fees = $1010.72 – $968.48 = $42.24

4. Kyle's uncle in New Zealand sent him a graduation present. The cheque was for $2500 in New Zealand dollars. When he takes it to his bank, how much money will he receive in Canadian funds?

   Answer:

   The bank is buying the NZ currency.

   $1 NZ = $0.6241 CAN

   $2500 NZ = 2500 × 0.6241 = $1560.25 CAN

Grade 10 Essential Mathematics (20S)

Module 7
Trigonometry

Module 7: Trigonometry

Introduction

This module deals with ratio and proportion, similarity, right triangles, Pythagorean Theorem, and the three basic trigonometric ratios. You will be solving problems using the Pythagorean relation and the three basic trigonometric ratios.

Trigonometry is based on the relationship between the measure of the angles and the lengths of the sides of a right triangle. These skills are used regularly in occupations such as surveying, carpentry, aviation, and astronomy.

Assignments in Module 7

When you have completed the assignments for Module 7, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Drawing Similar Figures
1	Assignment 7.1	Ratio and Proportion
2	Assignment 7.2	Similarity
3	Assignment 7.3	Pythagoras and Right Triangles
4	Assignment 7.4	Tangent Ratio
5	Assignment 7.5	Sine Ratio
6	Assignment 7.6	Cosine Ratio, Sine Ratio, and Tangent Ratio
7	Assignment 7.7	Problems

DPSU 2018–04

Resource Sheet

When you write your final exam you will be allowed to bring a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Final Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 7. You might use your Module 7 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 5, 6, 7, and 8, to prepare your Final Examination Resource Sheet. Remember, the final examination is based on the last four modules of the course.

Resource Sheet for Module 7

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Final Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Module 7 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work in the spaces provided. The cover assignment can be done at any time while you are completing Module 7. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on whether or not you found a solution, and whether or not your tutor/marker can read your diagrams. The cover assignment is worth a total of 12 marks.

Notes

Module 7 Cover Assignment

Total: 12 marks

Drawing Similar Figures

Similar figures are figures that have the same shape but are not necessarily the same size. When you take a figure and enlarge or reduce it so that its shape remains the same, the new figure is similar to the original one. The amount by which you have enlarged or reduced it is known as the scale factor.

One way you can draw similar figures is by using a grid:

1. Draw a grid of squares on top of the original figure.
2. Draw a second grid of squares either larger, smaller, or the same size as the first grid.
3. Copy the original figure square by square from the first grid onto the second grid. The more carefully you draw the figure, the more similar the shapes of the two figures will be.

This technique is demonstrated on the following page.

Each part is worth 4 marks for a total of 12 marks. Your mark will be determined on how well you have drawn the required figures.

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Module 7 Cover Assignment: Drawing Similar Figures (continued)

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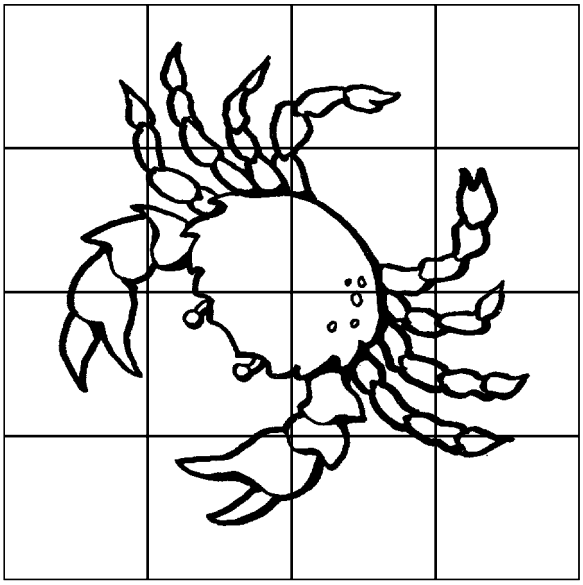

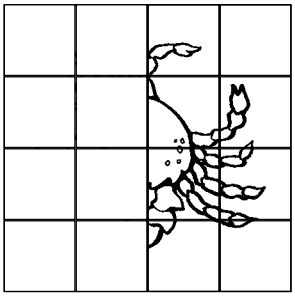

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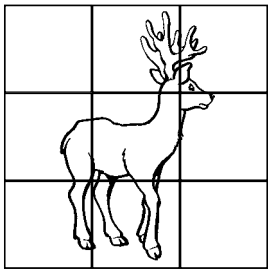

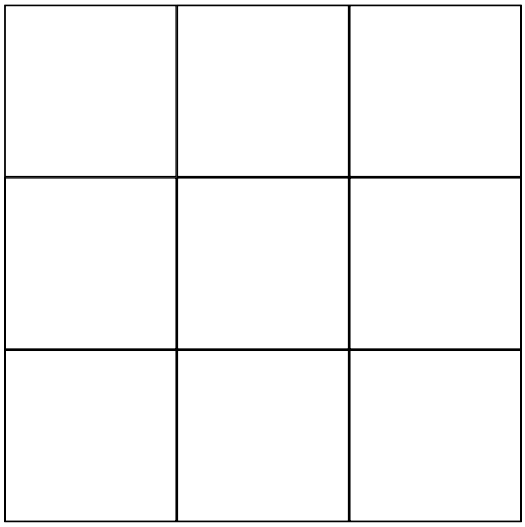

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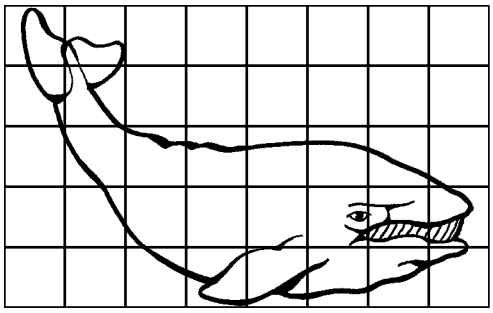

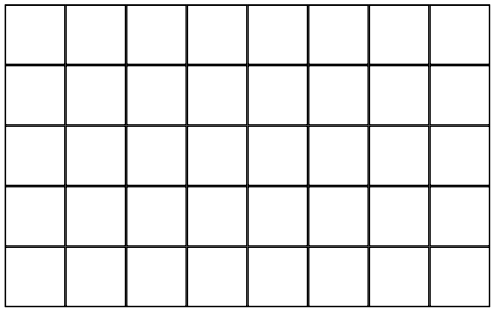

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Module 7 Cover Assignment: Drawing Similar Figures (continued)

In case you need extra paper.

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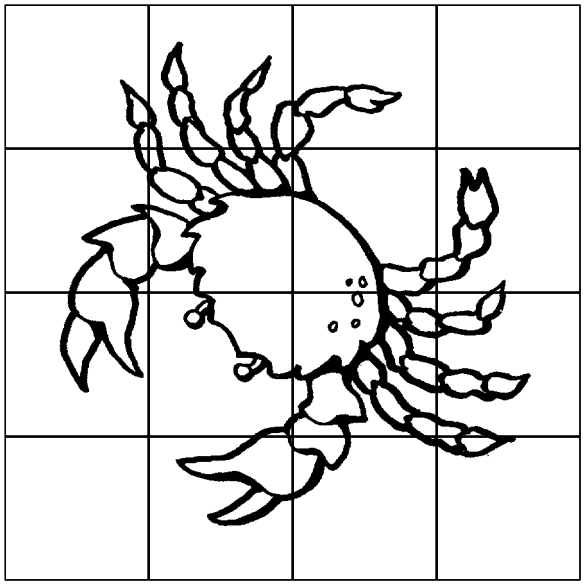

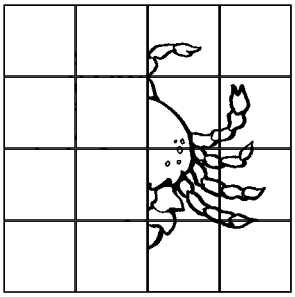

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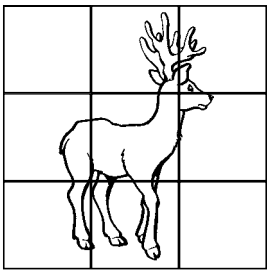

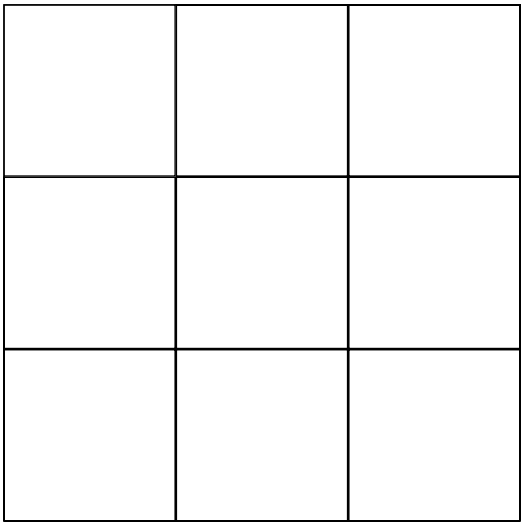

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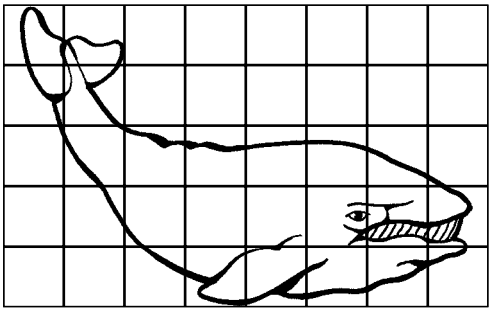
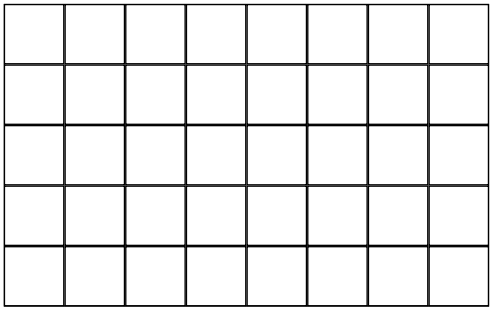




Notes

Lesson 1: Ratio and Proportion

Lesson Focus

In this lesson, you will

- ❑ review the concept of equivalent ratios
- ❑ use equivalent ratios to solve for missing terms

Lesson Introduction

Over the years, as well as in previous modules in this course, you have used proportional thinking in much of your math studies. You will continue using those skills when working with trigonometric ratios.

What Is a Ratio?

Ratios

Consider the following statements.

1. The ratio of students in Grade 10 compared to all high school students is 2 to 7.
2. The ratio of goals scored to goals attempted is 1 to 15.
3. The ratio of homes built this year compared to homes built last year is 4 to 3.

All of the above statements involve comparisons. In mathematics, a comparison is expressed as a ratio. A ratio is a comparison of one quantity to another. Ratios have many practical applications and occur frequently in everyday life.

A ratio can be expressed as a fraction. The numerator and denominator of the fraction are called the terms of the ratio.

Example 1

Write the comparisons in the statements listed above as fractions.

Solution:

1. $\frac{2}{7}$
2. $\frac{1}{15}$
3. $\frac{4}{3}$

Proportions

When you write the ratio 3 to 9 and the ratio 2 to 6 as fractions, you can see that they both are equal to the ratio of 1 to 3, when they are simplified. These two ratios are equivalent.

$$\frac{3}{9} = \frac{2}{6} = \frac{1}{3}$$

When you write two ratios equal to each other, the statement is called a proportion.

The following examples show how to find a missing term in a proportion.

Example 1

Solve for x in the proportion, $\frac{x}{30} = \frac{10}{6}$.

Solution:

By observation, you can see that the bottom term of the first ratio is 5 times bigger than the bottom term of the second ratio. Since the two ratios are equal, then the top term in each fraction must satisfy the same rule. In other words, x must be 5 times bigger than 10. You could show the mathematics as follows.

$$\frac{x}{30} = \frac{10}{6}$$

$$\frac{x}{30} = \frac{10 \times 5}{6 \times 5}$$

$$\frac{x}{30} = \frac{50}{30}$$

Thus, $x = 50$.

Example 2

Solve for x in the proportion, $\frac{3}{7} = \frac{x}{12}$, and round off your answer to the nearest tenth.

Solution:

By observation, you can see that you cannot easily use the same method as you did in Example 1 above. However, since you know how to cross multiply, you could use that method here.

$$\frac{3}{7} = \frac{x}{12}$$

$$(7)(x) = (3)(12)$$

$$7x = 36$$

$$\frac{7x}{7} = \frac{36}{7}$$

$$x = 5.1$$

Example 3

Solve for x in the proportion, $\frac{4}{12}=\frac{5}{x}$, and round off your answer to the nearest tenth.

Solution:

As in Example 2, you could cross-multiply to solve for x.

$$\frac{4}{12}=\frac{5}{x}$$
$$(4)(x)=(5)(12)$$
$$4x=60$$
$$\frac{4x}{4}=\frac{60}{4}$$
$$x=15$$

Because the fraction, $\frac{4}{12}$, can be simplified, there is an alternate method that you can use to solve this question.

$$\frac{4}{12}=\frac{5}{x}$$

However, we can simplify $\frac{4}{12}$ to $\frac{1}{3}$, so

$$\frac{4}{12}=\frac{1}{3}=\frac{5}{x}$$
$$\frac{1}{3}=\frac{5}{x}$$
$$\frac{1\times 5}{3\times 5}=\frac{5}{x}$$
$$\frac{5}{15}=\frac{5}{x}$$
$$15=x$$

Example 4

The ratio of the width of a television screen to its height is 4 to 3. What is the height of a screen with a width of 32 cm?

Solution:

You could let x represent the height of the television screen. Express the two ratios as fractions. You could use a "box" like you used in earlier modules to set up the proportion. This time, however, you might want to be more specific by naming the rows and columns. This might help to avoid errors.

The ratio of width to height is 4 to 3.

The ratio for the actual amounts is 32 to x.

	width	height
ratio	4	3
actual	32 cm	x cm

Notice how the units of measure are also included where possible.

Now, you write the proportion and solve for x.

$$\frac{4}{32\text{ cm}} = \frac{3}{x\text{ cm}}$$

$$(4)(x\text{ cm}) = (3)(32\text{ cm})$$

$$4x\text{ cm} = 96\text{ cm}$$

$$\frac{4x\text{ cm}}{4} = \frac{96\text{ cm}}{4}$$

$$x\text{ cm} = 24\text{ cm}$$

The television screen is 24 cm high.

The method used in Example 3 above is a very useful one for solving problems involving ratios. Since comparisons can be used in your daily life, you can use this method whenever you need to find the missing term in a proportion. This method can be used in geometry, changing currency, price comparisons, and sports.

Percents

This method of setting up a proportion and solving can also be used in problems involving percent. Since percent is a ratio comparing a quantity to 100, you can answer questions with percents by setting up a proportion.

Example 1

What percent is 2500 out of 4000? Solve using a proportion.

Solution:

Percent means out of 100. You could let x represent the numerator of the fraction for the percent. The two equivalent ratios can be written as follows.

2500 to 4000
x to 100

	quantity	out of
actual	2500	4000
percent	x	100

Now set up the proportion and solve for x.

$$\frac{2500}{x} = \frac{4000}{100}$$

$$(4000)(x) = (2500)(100)$$

$$4000x = 250\ 000$$

$$\frac{4000x}{4000} = \frac{250\ 000}{4000}$$

$$x = 62.5$$

Since x represents the numerator of the fraction whose denominator is 100, you can write $\frac{62.5}{100}$ or 62.5%.

Thus, 2500 out of 4000 is the same as 62.5%.

Example 2

Gene Seller works at The Jean Warehouse and earns $10.00 an hour. After 8 months, his employer gives him a 15% raise. What is his new hourly rate?

Solution:

Notice that x represents the quantity of the increase in his salary and not the new salary, because 15% represents the increase in salary not the new salary. However, once you find the amount of increase, you can easily find the new salary by adding.

The ratios could be entered into the "box" as follows.

	quantity	out of
actual	$\$x$	\$10.00
percent	15	100

Now set up the proportion and solve for x.

$$\frac{\$x}{15} = \frac{\$10.00}{100}$$

$$(100)(\$x) = (15)(\$10.00)$$

$$\$100x = \$150$$

$$\frac{\$100x}{100} = \frac{\$150}{100}$$

$$x = \$1.50$$

Gene's salary increases by $1.50.

His new salary will be $10.00 + $1.50 = $11.50.

Learning Activity 7.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You saved $104 in one year by putting aside the same amount each week. How much did you save each week?
2. You know that 30×4 is 120. What is 60×4?
3. Convert 6 litres to millilitres.
4. Is $\frac{4}{5}+\frac{3}{7}$ greater or less than 1?
5. You started your math homework at 3:30 pm and finished at 4:25 pm. How many minutes did you spend on your math homework?

Part B: Proportions and Ratios

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing term in each of the following proportions.
   a) $\frac{3}{5}=\frac{x}{20}$
   b) $\frac{5}{7}=\frac{25}{x}$
   c) $\frac{30}{x}=\frac{18}{3}$
2. Find what percent is 3200 out of 5000.
3. A flag has a length to width ratio of 2: 1. If the width is 30 cm, find the length.

Check your answers. If you got the wrong answer for a question, go back and review the example in the lesson. You may also want to include the example, or the question from the learning activity, on your resource sheet.

Lesson Summary

This lesson reviewed skills with solving for unknowns using proportional thinking. Ratios, proportions, equivalent fractions, and terms of a proportion were discussed in the lesson. Two methods were shown how to solve proportions. Many problem-solving situations, including questions with percents, were noted.

The next lesson introduces similarity in shapes.

Notes

Assignment 7.1

Total: 19 marks

Ratio and Proportion

1. Find the missing term in each of the following. If necessary, round off your answer to the nearest tenth. *(4 marks)*

   a) $\frac{x}{7} = \frac{4}{35}$

   b) $\frac{3}{18} = \frac{25}{x}$

continued

c) $\frac{3.2}{x} = \frac{16}{4}$

d) $\frac{x}{7} = \frac{16}{56}$

2. The ratio of the length of the Canadian flag to its width is 2 to 1. What is the length if the width is 24 cm? *(3 marks)*

continued

Assignment 7.1: Ratio and Proportion (continued)

3. At the Winter Olympics in Albertville, France, the ratio of Canada's medals to Austria's medals was 1 to 3. Austria won 21 medals. How many medals did Canada win? *(3 marks)*

4. If the human body burns 420 kilojoules for every 10 minutes of swimming, calculate how many kilojoules the body burns in 1 hour of swimming. *(3 marks)*

continued

Assignment 7.1: Ratio and Proportion (continued)

5. A recipe calls for $3\frac{1}{2}$ cups of flour to 1 cup of shortening. How many cups of shortening are required if $10\frac{1}{2}$ cups of flour are used? *(3 marks)*

6. The ratio of the mass of a heart to the mass of the human body is 1: 200. If a person has a mass of 68 kg, what is the mass of his or her heart? *(3 marks)*

Lesson 2: Similarity

Lesson Focus

In this lesson, you will

❑ understand similarity in triangles

Lesson Introduction

You will need your protractor and ruler for this lesson.

Similarity

Similarity Definition

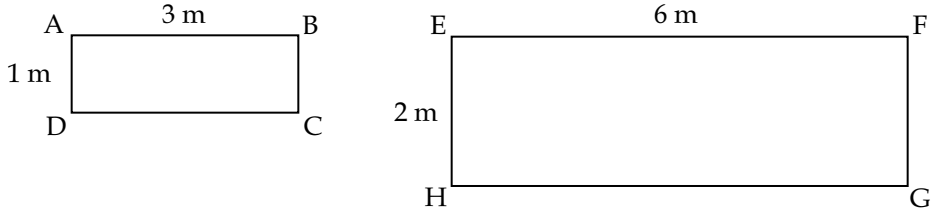
The definition of similarity states that two shapes are similar if their corresponding angles have the same measure and their corresponding sides are proportional.

Add this definition to your resource sheet.

What does this mean? What are corresponding angles? Corresponding sides? Are corresponding angles for similar figures the same as corresponding angles for parallel lines?

Example 1

Which are the corresponding sides in these two rectangles?



Solution:

Side AB on the first rectangle corresponds with side EF on the second rectangle (or with side GH). They are all in the horizontal position. Their ratio is 3 m to 6 m, which becomes 1 to 2 when it is simplified.

For the vertical sides, notice that EH is twice the length of AD. Since the corresponding sides are in the same ratio as the horizontal sides, the sides of the rectangle are said to be proportional.

Example 2

Are the two rectangles in Example 1 above similar?

Solution:

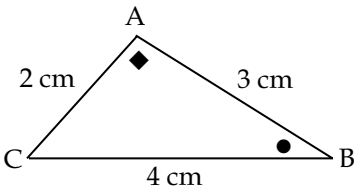
The angles are all the same and are each 90°. The sides are proportional, as shown in Example 1 above. Both conditions of the definition of similarity are satisfied. Thus, rectangle ABCD is "similar" to EFGH. The symbol used in math to show similarity is ~. You write the similarity relationship for the two rectangles as follows.

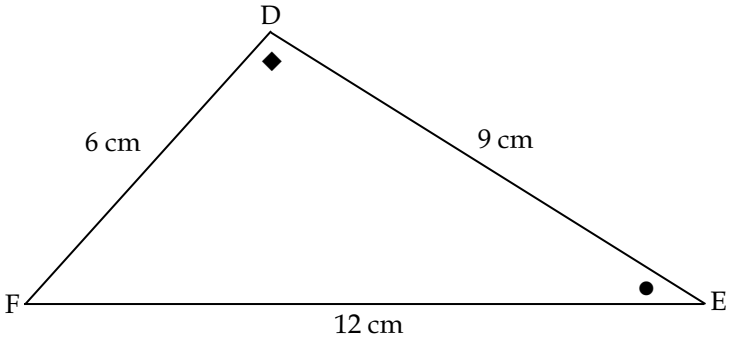
rectangle ABCD ~ rectangle EFGH.

In triangles, the same rules apply. The corresponding sides must be proportional, and the corresponding angles must have the same measure. This next example shows you how to state which are the corresponding sides and angles in triangles.

Example 3

Which are the corresponding angles and corresponding sides in these two triangles? Are the triangles similar?





Solution:

Notice the little marks in the angles—a diamond and a dot. They help you identify which angles correspond, and they indicate equality. The angle with the diamond symbol in ΔABC is equal to the corresponding angle with the same symbol in ΔDEF. In other words, $\angle A = \angle D$. Any symbol can be used to indicate equality. In later examples, a more mathematical way of showing equal corresponding angles is given.

You would write the three sets of corresponding equal angles as: $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

Notice that if two sets of corresponding angles in two triangles are equal, then the third set will also be equal. Why would that be? The answer to this question is coming up later in the lesson. Watch for it!

Also note that corresponding angles in similar figures are not the same as corresponding angles for parallel lines. The same word has two completely different applications.

You must also identify the corresponding sides and then see if they are all in the same ratio. If they are, then the corresponding sides are said to be proportional.

The corresponding sides are opposite the corresponding angles. For example, the side opposite $\angle A$ is BC, and the side opposite $\angle D$ is EF. Since $\angle A$ and $\angle D$ are corresponding angles, then BC and EF are corresponding sides.

To sort out all the information, you could make a chart where the columns list the sets of corresponding angles and the sides opposite these equal angles.

corresponding angles	◆ $\angle A = \angle D$	● $\angle B = \angle E$	$\angle C = \angle F$
	sides opposite each angle	sides opposite each angle	third sides of the triangles
smaller triangle	BC = 4 cm	AC = 2 cm	AB = 3 cm
bigger triangle	EF = 12 cm	DF = 6 cm	DE = 9 cm

From the columns in the chart, you can easily set up the ratios for the corresponding sides.

$$\frac{BC}{EF} : \frac{AC}{DF} : \frac{AB}{DE}$$

To confirm that the triangles are similar, you now need only to show that their sides are in the same ratio.

In other words, does $\frac{BC}{EF} = \frac{AC}{DF} = \frac{AB}{DE}$?

Substitute the values for each side into the ratios.

Does $\frac{4\text{ cm}}{12\text{ cm}} = \frac{2\text{ cm}}{6\text{ cm}} = \frac{3\text{ cm}}{9\text{ cm}}$?

Each ratio simplifies to $\frac{1}{3}$. Therefore, yes, the sides are proportional.

Since the corresponding angles are equal and the corresponding sides are proportional, you can say that $\Delta ABC \sim \Delta DEF$.

Notice that when the similarity relationship is written for triangles, the equal corresponding angles are in the same relative position. In this example, A is written first for ΔABC and D is written first for ΔDEF. This indicates that $\angle A = \angle D$. The same thing is true for $\angle B$ and $\angle E$ in second position, and then $\angle C$ and $\angle F$ in the last position. Notice the chart also shows the correct order for the angles. If you interchange the columns of the chart, the similarity relationship would be written differently, but the equal angles would still be in the same relative position. For example, you could write $\Delta ACB \sim \Delta DFE$.

Triangles can be similar, even if the triangles are not drawn in exactly the same way. The following example shows the second triangle rotated.

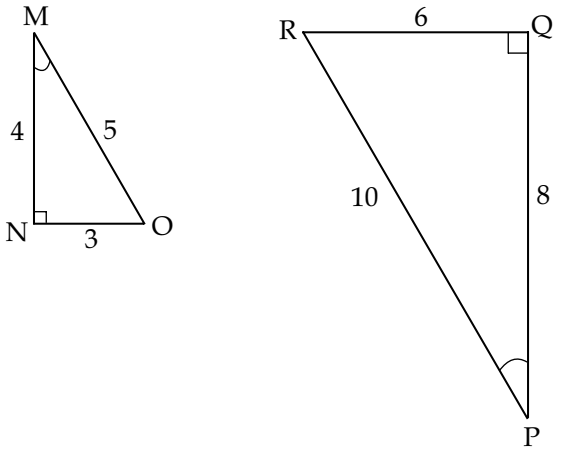
Example 4

a) List the corresponding equal angles.

b) Write the ratios for the corresponding sides.

c) Are the triangles similar?

d) If the triangles are similar, write the correspondence between the two triangles.



Solution:

a) From the markings in the two triangles, the corresponding angles are as follows.

$\angle M = \angle P$ (both angles have the same markings)

$\angle N = \angle Q$ (both are right angles)

$\angle O = \angle R$ (both are unmarked angles and the third set of angles in the triangle)

b) To find the corresponding sides and ratios, you could make a chart so that the corresponding equal angles are listed in each column and the sides opposite these equal angles are shown.

	$\angle M = \angle P$	$\angle N = \angle Q = 90°$	$\angle O = \angle R$
	sides opposite each angle	sides opposite each angle	third sides of the triangles
first triangle	ON = 3	OM = 5	MN = 4
second triangle	RQ = 6	PR = 10	PQ = 8

Now you can easily write the ratios from the columns as $\frac{ON}{RQ} : \frac{OM}{PR} : \frac{MN}{PQ}$.

By substituting the values for the sides into the ratios, you can see if the sides are proportional.

$\frac{3}{6} : \frac{5}{10} : \frac{4}{8}$

$\frac{1}{2} : \frac{1}{2} : \frac{1}{2}$

Since the ratios all simplify to the same value of $\frac{1}{2}$, the sides are proportional.

c) Since the corresponding angles are equal and the corresponding sides are proportional, the two triangles are similar.

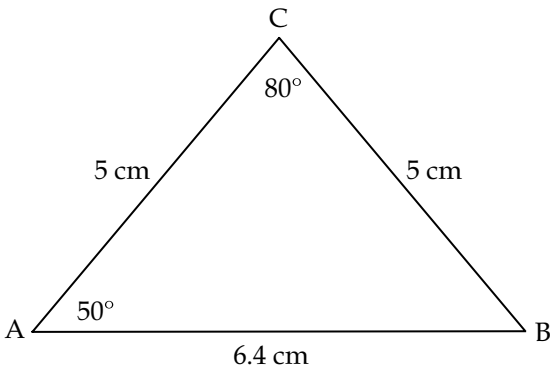
d) The order of the letters for the similarity relationship can be written using the first row of the chart, where the equal angles are listed: $\Delta MNO \sim \Delta PQR$ (These letters are in alphabetical order, but that has nothing to do with the similarity relationship.)

Using your Ruler and Protractor

Given one triangle, you can construct a similar triangle using a ruler and a protractor.

Example

Construct a triangle similar to ΔABC and which is 1.5 times bigger than ΔABC. Note that ΔABC is not drawn to scale in the diagram below.



Solution:

Call the new triangle ΔNEW, and write the relationship as $\Delta ABC \sim \Delta NEW$

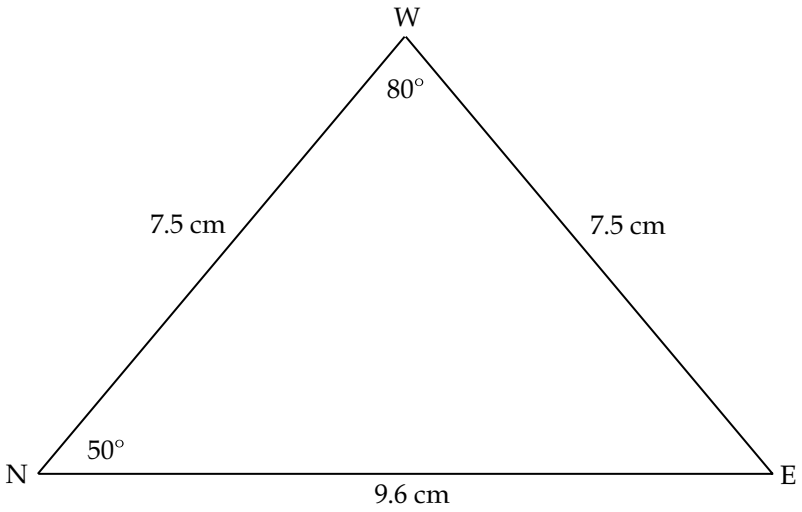
The corresponding angles in the new triangle will be equal to those in the original triangle. $\angle A = \angle N = 50°$, $\angle B = \angle E$, and $\angle C = \angle W = 80°$. The sides in ΔNEW are 1.5 times bigger than the sides of ΔABC.

$5 \text{ cm} \times 1.5 = 7.5 \text{ cm}$
$6.4 \text{ cm} \times 1.5 = 9.6 \text{ cm}$

The sides of ΔNEW are 9.6 cm, 7.5 cm, and 7.5 cm.

Use your ruler to draw the base corresponding to AB, and call it NE. Its length is 9.6 cm. Use your protractor to draw the angle of 50° at N. Measure the length of the side of the angle as 7.5 cm. This side would be named NW. At W, make an angle of 80°. If you are relatively accurate in your drawings, the side of the angle you have drawn should meet the base at E. Its length should be 7.5 cm.

Your diagram would look like the one below. However, this diagram is only a sketch. Its measurements are only approximate. Your diagram should have exact measurements.

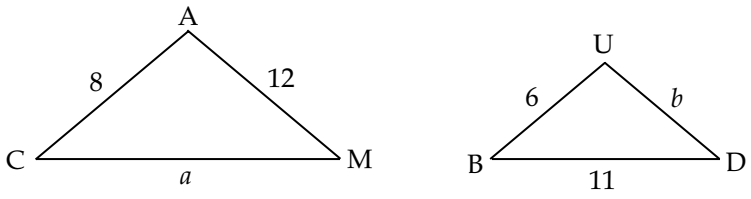


Finding Missing Values

You can now apply your understanding of similarity to find missing sides or angles.

Example

Given that $\Delta CAM \sim \Delta BUD$, find the missing values, a and b.



Solution:

Notice first of all that the side opposite ∠A is called "*a*" and the side opposite ∠B is called "*b*". This is a common notation with triangles. The lower case letter is used for the length of the side, whereas the upper case letter is used for the vertex of the angle and for the vertex of the triangle. Using this notation makes the chart easier to set up.

You could set up the chart to determine the corresponding sides, given the similarity relationship ΔCAM ~ ΔBUD. Because of the position in the writing of the similarity relationship, ∠C and ∠B are written first so ∠C = ∠B. ∠A and ∠U are written second, so ∠A = ∠U. And finally, ∠M and ∠D are written last so ∠M = ∠D.

	∠C = ∠B	∠A = ∠U	∠M = ∠D
	sides opposite each angle	sides opposite each angle	sides opposite each angle
ΔCAM	$c = 12$	$a = ?$	$m = 8$
ΔBUD	$b = ?$	$u = 11$	$d = 6$

To find side "*b*", set up a proportion using *b* and another ratio where both the top and bottom are known values. In this case, you would use the ratio from column 3 because both *m* and *d* are known values.

$$\frac{c}{b} = \frac{m}{d}$$

Substitute the known values and solve for *b*.

$$\frac{12}{b} = \frac{8}{6}$$
$$8b = (12)(6)$$
$$8b = 72$$
$$\frac{8b}{8} = \frac{72}{8}$$
$$b = 9$$

To find side "*a*", set up a proportion using the corresponding sides where one ratio is completely known. Since you now know both values for column 1, you could use the data from either column 1 or column 3. The better choice, however, is to use column 3, since those values were given in the question.

$$\frac{a}{u} = \frac{m}{d}$$

Substitute the known values and solve for a.

$$\frac{a}{11} = \frac{8}{6}$$
$$6a = (8)(11)$$
$$6a = 88$$
$$\frac{6a}{6} = \frac{88}{6}$$
$$a = 14.7$$

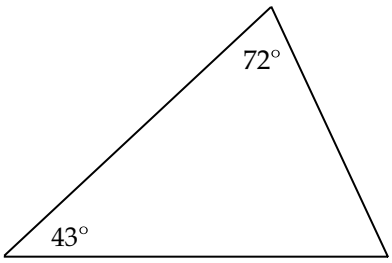
The value for a is 14.7 and for b is 9.

Sum of the Angles in a Triangle

The measures of the three angles of any triangle always add up to total 180°. If you know the measures of two of the angles, the third can be easily found.

Example

Find the measure of the third angle in the following triangle.



Solution:

To find the measure of the third angle, you add the two known angles and subtract that sum from 180°.

Third angle = 180° – (72° + 43°)
= 180° – 115°
= 65°

The third angle has a measure of 65°.

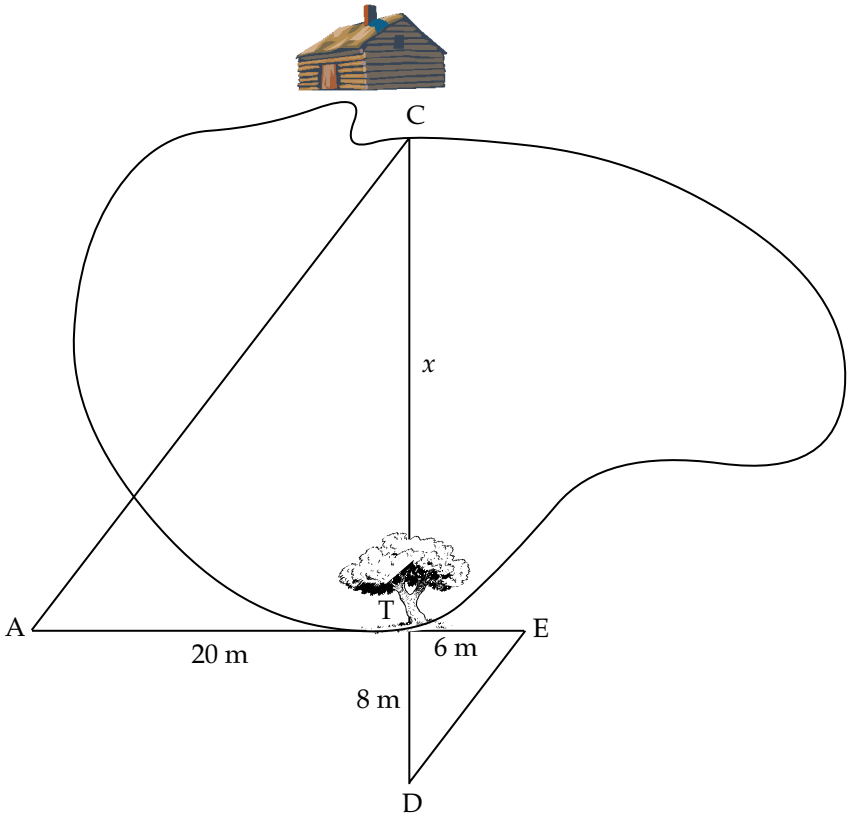
Perhaps this rule now can answer the question posed to you earlier about why the third sets of angles in a pair of triangles, where the first two sets are equal, must always have the third set of angles equal.

Similar Right Triangles

You can apply your knowledge of similar triangles to solving problems involving right triangles.

Example 1

This is a diagram of a lake with a cabin on the far side of the lake. Your goal is to find the distance across the lake from the tree to the cabin. How will you accomplish this task?



Solution:

You know that if you can measure off some distances to form two similar right triangles, you can use proportions to find the distance across the lake. You would want to make right triangles because you would know that at least one set of corresponding angles would be equal. You would need a method of measuring angles to find the other set.

You measure 20 m from the tree to a marker, A, for the large triangle, and 8 m to a marker, D, for the smaller triangle.

Since you need one more set of equal corresponding angles, you keep ∠C and ∠D the same and measure off 6 m for a leg of the smaller triangle to the point E.

The first step in the calculation is to identify the corresponding sides from the equal angles.

	∠ATC = ∠DTE = 90°	∠C = ∠D	∠A = ∠E
large triangle		c = 20 m	$a = x$ m
small triangle		d = 6 m	e = 8 m

Now you set up the proportions and solve.

$$\frac{20}{6} = \frac{x}{8}$$

$$6x = (20)(8)$$

$$6x = 160$$

$$\frac{6x}{6} = \frac{160}{6}$$

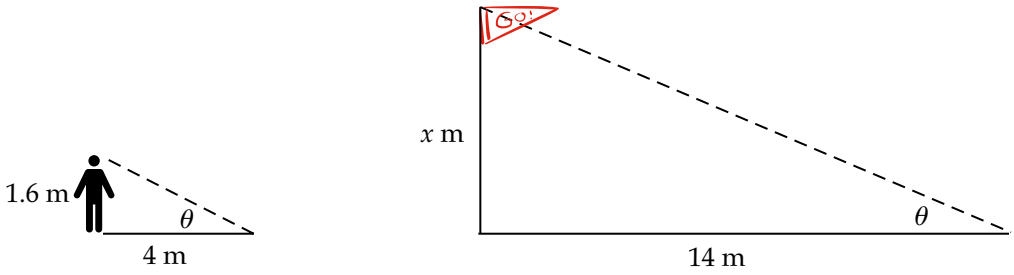
$$x = 26.7 \text{ m}$$

The distance across the lake is 26.7 m.

Example 2

A man is 1.6 m in height. Behind him the sun casts his shadow of length 4 m. He is standing beside a flagpole, which has a shadow along the ground of 14 m in length.

Use similar right triangles to find the height of the flagpole.



Solution:

Identify the corresponding sides and set up the proportions.

You can assume that both the man and the flagpole make a right angle with the ground. That gives one set of equal angles. The other corresponding angles are the angles formed between the sun's rays and the ground. They will be equal in both triangles. Thus, the third set of angles will be equal.

Now you can set up the corresponding sides to solve the proportion.

	90° angles	angle of the sun $= \theta$	third set of angles
triangle with the man		1.6 m	4 m
triangle with the flagpole		x m	14 m

$$\frac{1.6 \text{ m}}{x \text{ m}} = \frac{4 \text{ m}}{14 \text{ m}}$$

$$4x = (14)(1.6)$$

$$4x = 22.4$$

$$\frac{4x}{4} = \frac{22.4}{4}$$

$$x = 5.6$$

The height of the flagpole is 5.6 m.

Learning Activity 7.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

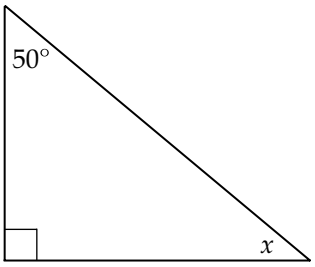
1. Is the answer to –25 + 16 positive or negative?
2. A shirt is 20% off of its regular price of $40.00. How much will you pay for it on sale?
3. A 500 g package of chocolate chips is $6.00. How much do the chocolate chips cost per gram?
4. You work 4.5 hour shifts 4 times per week. You are paid $10 per hour. How much do you make per week?
5. Calculate: 78 + 12.

Part B: Similarity

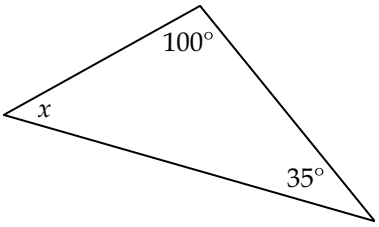
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the measure of the missing angles in each triangle. Remember, the sum of all three angles in a triangle is 180°.

a)

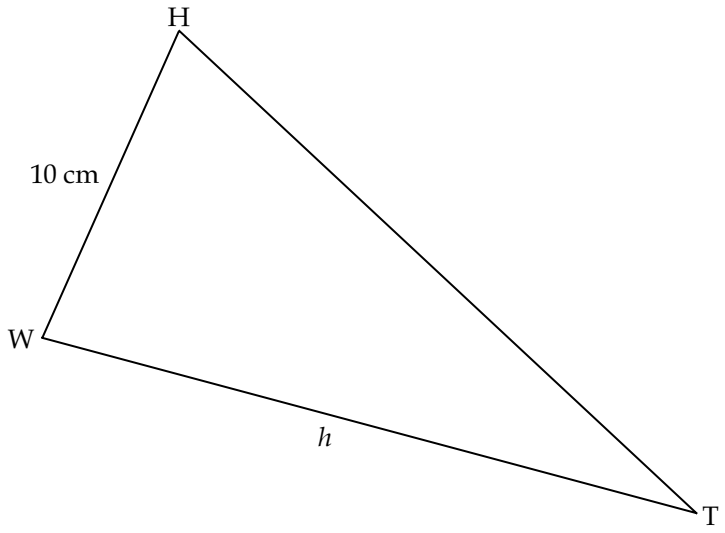


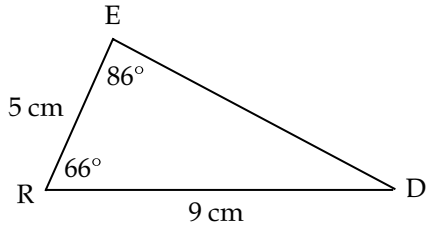
b)



continued

2. Given ΔRED ~ ΔWHT, find the length of h and the measure of ∠T.





3. A girl's height is 1.4 m. She casts a shadow of 3.8 m in the sun. She stands near a building that casts a shadow of 45.6 m. Use similar right trianges to find the height of the building.

Lesson Summary

This lesson dealt with similarity of triangles, and how to find the corresponding angles and the corresponding sides. It also showed how to determine if the corresponding sides were proportional. A chart organizing the information was helpful. The symbol showing the similarity relationship was given, and how to write the correct order of the letters was determined from the chart. These skills were then applied to solving for missing values for the lengths of the sides or the measures of the angles. A rule for the sum of the angles in a triangle was given.

The next lesson focuses on right angle triangles and the Pythagorean relation.

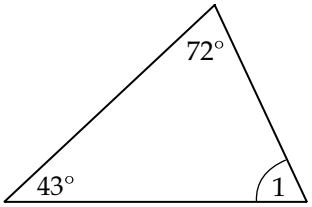
Assignment 7.2

Total: 23 marks

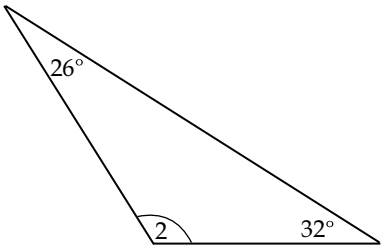
Similarity

1. Using the fact that the sum of the measures of a triangle is 180°, find the measure of the missing angles of the following triangles. *(5 marks)*

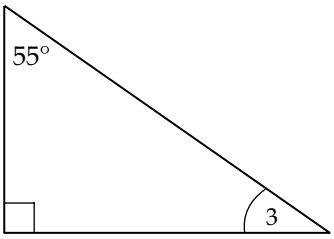
a)



b)



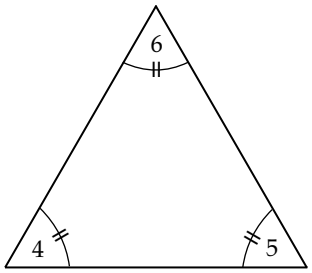
c)



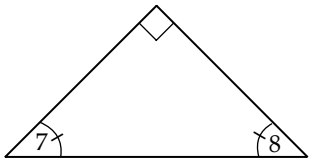
continued

Assignment 7.2: Similarity (continued)

d)



e)



2. Draw each triangle exactly, using a ruler and a protractor. For each triangle, label the vertices with upper case letters and the sides with lower case letters. *(5 marks)*

   a) ΔDEF with $\angle D = 65°$, $\angle E = 50°$, and $f = 4$ cm

continued

Assignment 7.2: Similarity (continued)

b) ΔJKT with $\angle T = 90°$, $k = 4$ cm, and $j = 6.1$ cm

c) ΔMNP with $n = 4$ cm, $\angle P = 58°$, and $m = 7$ cm

d) ΔQRS with $\angle Q = 60°$, $s = 6$ cm, and $s = q$

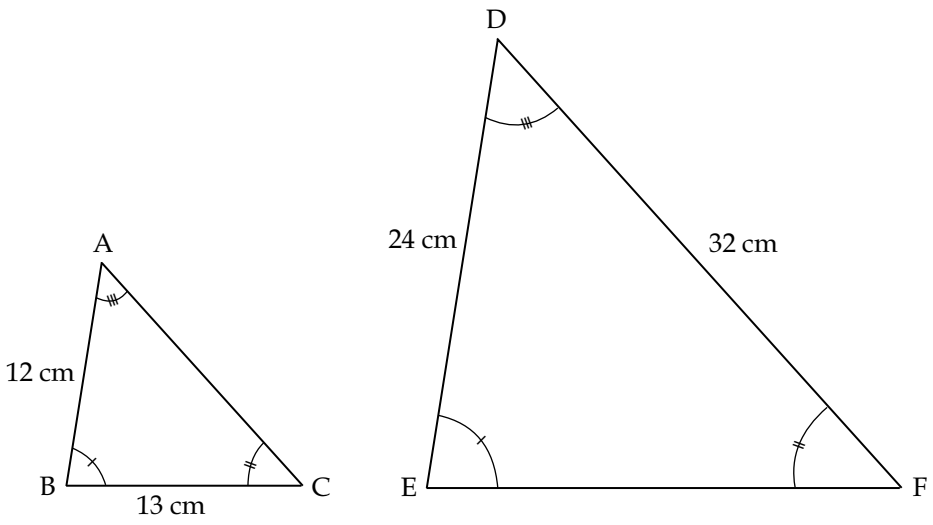
e) ΔUVW with $v = 3$ cm, $w = 4$ cm, and $u = 5$ cm

continued

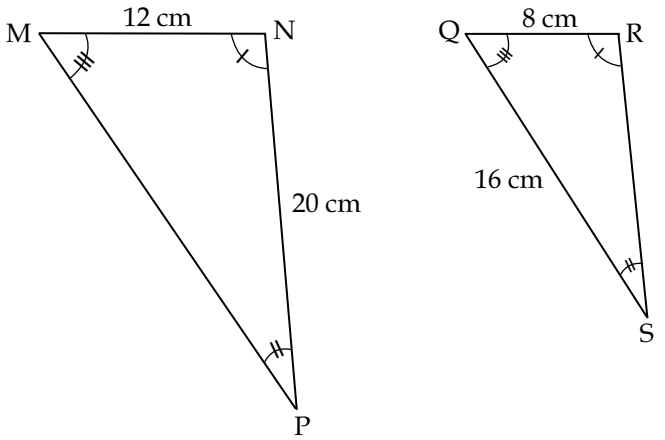
Assignment 7.2: Similarity (continued)

3. Find the missing sides of the following triangles using equivalent ratios. *(8 marks)*

a)



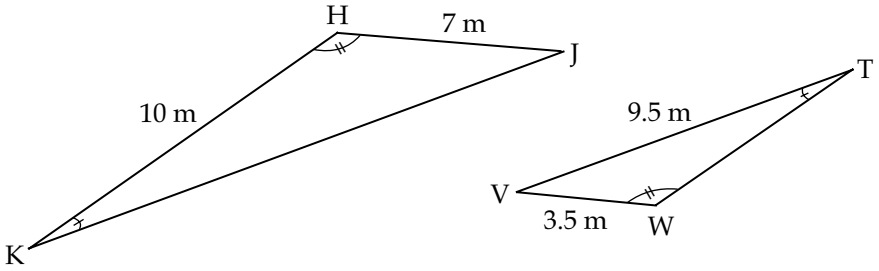
b)



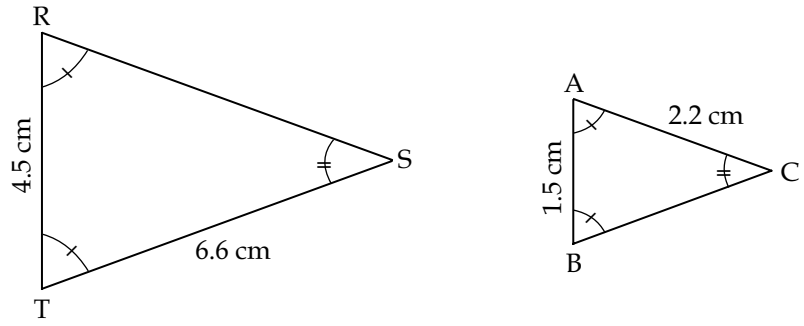
continued

Assignment 7.2: Similarity (continued)

c)



d)



continued

Assignment 7.2: Similarity (continued)

4. Find the length of the missing side in each of the following pairs of similar triangles. Make sketches of the triangles first. Your diagrams need not be drawn exactly but they are part of your solution. *(3 marks)*

   a) $\Delta HJK \sim \Delta UVW$. If $j = 17$ mm, $k = 20$ mm, and $v = 30$ mm, find UV.

   b) $\Delta ABC \sim \Delta DEF$. $\angle A = 30°$, $\angle B = \angle E = 90°$. If $a = 1.2$ m, $b = 2.4$ m, and $e = 1.8$ m, find EF.

continued

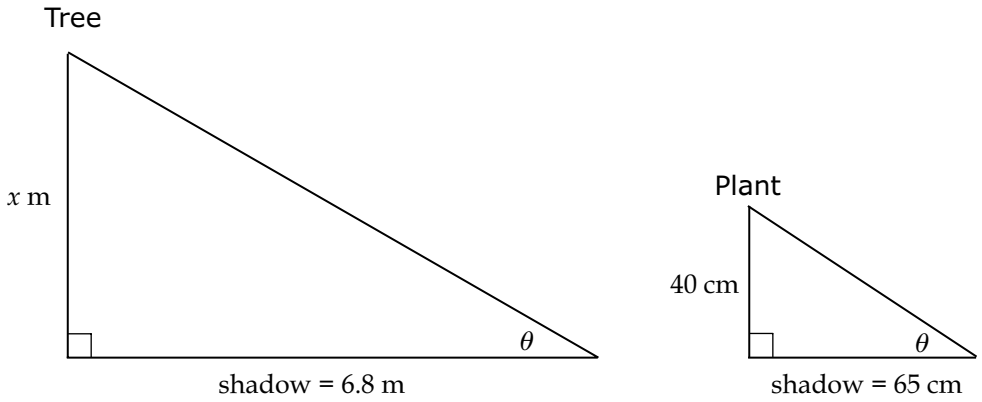
Assignment 7.2: Similarity (continued)

c) Given ΔMNP and ΔTVW. $\angle M = \angle T = 90°$, $\angle P = 39°$, and $\angle V = 51°$. If $m = 86$ cm, $p = 64$ cm, and $w = 118$ cm, find VW.

continued

Assignment 7.2: Similarity (continued)

5. The length of a shadow cast by a tree is 6.8 m. At exactly the same time, the length of a shadow cast by a 40 cm tall plant is 65 cm. How tall is the tree? *(2 marks)*



Lesson 3: Pythagoras and Right Triangles

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of the Pythagorean theorem

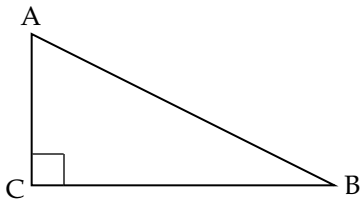
Lesson Introduction

The triangles you study in this course are primarily right triangles.

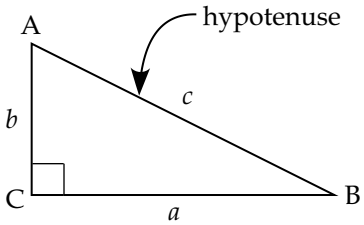
In this lesson, you will work with right triangles as a starting place to lead into the study of trigonometry.

Using the Pythagorean Theorem

Right triangles are triangles that have a 90° angle, indicated by a small box in the corner.



The "hypotenuse" is the longest side opposite the right angle. The shorter two sides are called the "legs" of the right triangle. As before, the sides are labelled in lower case letters according to the angles opposite them.

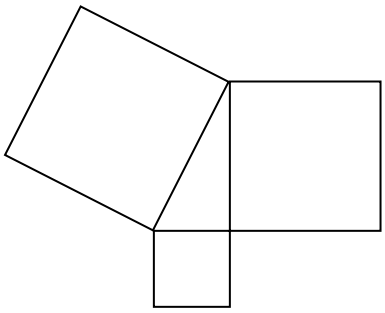


Include these definitions and the diagram on your resource sheet.

Pythagorean Theorem

Pythagoras lived in Greece around 500 BCE. He was a brilliant mathematician, musician, and physicist. He saw mathematical precision in music and astronomy. He believed that music and the orbits of planets around stars could all be explained by mathematical patterns and equations. One discovery credited to and named for him is the Pythagorean theorem.

The Pythagorean Theorem: The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).



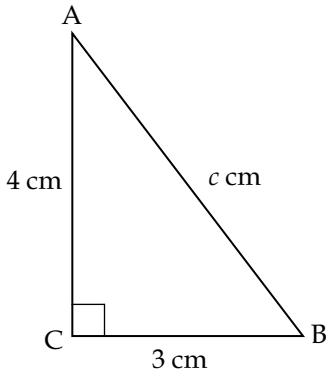
More commonly, you would say the sum of the squares of the two sides equals the square of the hypotenuse. In mathematical notation, the Pythagorean theorem is written as $a^2 + b^2 = c^2$.

This theorem only applies to right triangles.

Add the Pythagorean Theorem to your resource sheet.

Example 1

Given the following right triangle, find the length of the hypotenuse.



Solution:

To use the Pythagorean theorem, you first write the formula.

$$a^2 + b^2 = c^2$$

The most common mistake students make is to substitute the wrong value for the hypotenuse into this equation. Remember the hypotenuse is always the largest of the lengths of the sides of a right triangle and is always the side opposite the right angle.

The most important step at this point is to substitute the value of the hypotenuse into the equation.

$$a^2 + b^2 = c^2$$

hypotenuse value = c (unknown value in this example)

Now substitute in the other values.

$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$

You have found the value for c^2, but you want the value for c. To find this, you need to find the square root of each side, since the inverse operation of squaring is square rooting. The square root of c^2 is c.

$$\sqrt{25} = \sqrt{c^2}$$
$$5 = c$$

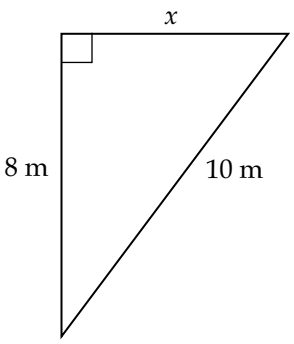
The length of the hypotenuse is 5 cm.

This right triangle is commonly known as the 3-4-5 right triangle, and is used in the trades to measure right angles by measuring the lengths of the sides of the triangle.

Note: In order to take the square root of a number, you may need to use the 2nd button on your calculator, then press the x^2 button. Make sure you know how to do this on your calculator, as each calculator is different. If you need help, ask your learning partner.

Example 2

Find the value of x.



Solution:

To find the value of x, you first write the formula.

$$a^2 + b^2 = c^2$$

Then you substitute the value of the hypotenuse in place of the c.

$$a^2 + b^2 = 10^2$$

Then you substitute in the other known or unknown values for the sides.

$$8^2 + x^2 = 10^2$$

Then you simplify the equation, and isolate the term with the variable.

$$64 + x^2 = 100$$
$$64 + x^2 - 64 = 100 - 64$$
$$x^2 = 36$$

Find the square root of both sides.

$$\sqrt{x^2} = \sqrt{36}$$
$$x = 6$$

The value for x is 6 m.

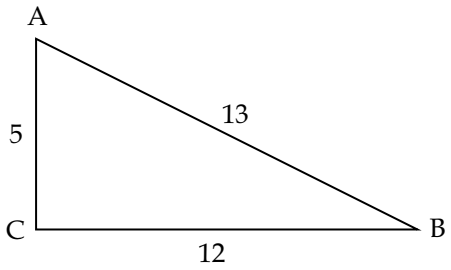
Proving an Angle in a Triangle is 90°

The formula for the Pythagorean theorem works for right triangles. What if you knew that the formula works for some triangle, but didn't know if the triangle was right-angled? Pythagoras was able to determine that this law also is true. Just ask a carpenter or builder. Very often the law is used to ensure that two walls in the structure that is being built are at right angles to each other. How does this work?

You can use the same equation to prove whether or not a triangle has a right angle. Substitute values into the formula for *a*, *b*, and *c*, and simplify. If the left-hand side of the equation equals the right-hand side, then by the Pythagorean rule the triangle must be a right triangle.

Example 1

Given ΔABC, by using the Pythagorean relation, determine if it is a right triangle.



Solution:

Step 1: Write the formula.

$a^2 + b^2 = c^2$

Step 2: Substitute the value for the hypotenuse in place of the *c*. The hypotenuse is the longest side, and in this case is 13.

$a^2 + b^2 = 13^2$

Step 3: Substitute the other values for *a* and *b*. The order is irrelevant.

$5^2 + 12^2 = 13^2$

Step 4: Simplify each side separately.

$25 + 144 = 169$

$169 = 169$

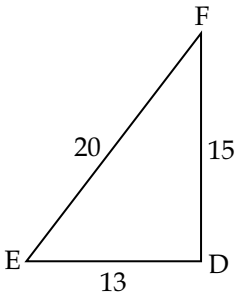
Step 5: Make a conclusion. If the left-hand side of the equation equals the right-hand side, then you can conclude the triangle is a right triangle.

In this example, both sides are equal. This shows that the sum of the squares of the sides does equal the square of the hypotenuse, so the triangle, by Pythagoras Theorem, must be a right triangle. Also, since the longest side is opposite the biggest angle in any triangle, then the right angle must be at C.

Note that this is the method a carpenter uses to square up a 90° corner. Two lengths are marked along the walls (for example, 5 inches and 12 inches), and then the hypotenuse is measured. If it is not the correct amount (13 inches), the walls are nudged over until the measure is correct. (In Lesson 6 of the module on angles, this method is explained in detail.)

Example 2

Given ΔDEF, use the Pythagorean relation to prove it is or it is not a right triangle.



Solution:

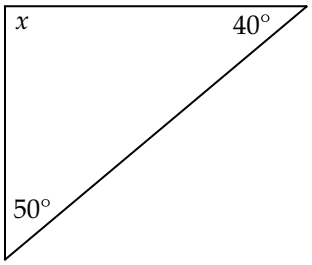
Formula:	$e^2 + f^2 = d^2$
	Does $13^2 + 15^2$ equal 20^2?
Using sides:	$13^2 + 15^2 = 169 + 225 = 394$
Using hypotenuse:	$20^2 = 400$
	$394 \neq 400$
Conclusion:	The sum of the squares of the sides does not equal the square of the hypotenuse. Therefore ΔDEF is not a right angle triangle.

Adding the Angles

The three angles in any triangle always add together to make a sum of 180^0. Applying this rule to determine if a triangle is a right triangle, if two angles in a triangle add up to 90°, then the third angle must be 90°. And the triangle must be a right triangle.

Example 1

Is this a right triangle?



Solution:

Since all three angles must total 180°, then angle at x can be found.

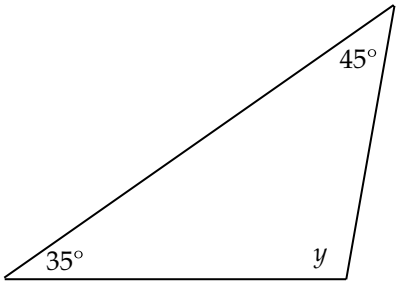
$$x = 180° - (50° + 40°) = 180° - 90° = 90°$$

Because x has a measure of 90°, the triangle is a right triangle.

To shorten your work, you could just add the two given angles together. If the answer is 90°, then the triangle is a right triangle. In the example above, 40° + 50° = 90°, and thus there is a right triangle.

Example 2

Is this a right triangle?



Solution:

45° + 35° = 80°

The angle at y must be 100°.

Thus, the triangle is not a right triangle.

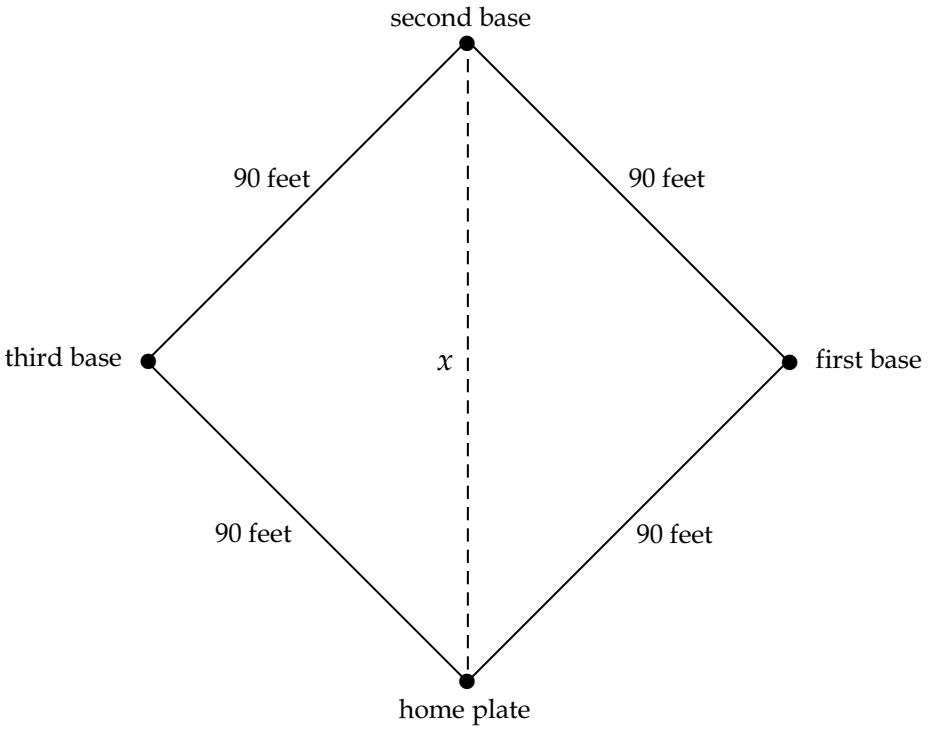
Applying the Pythagorean Theorem to Solving Problems

Identify a triangle having a right angle and identify where the hypotenuse would be.

Then substitute the known values into the formula and solve for the unknown value.

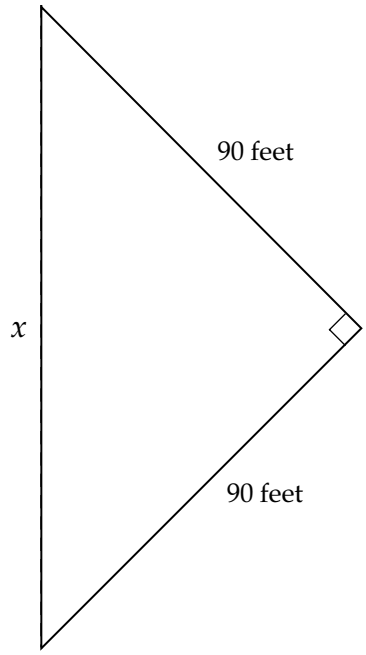
Example 1

The diagram below shows a baseball diamond. How far must the catcher throw the ball from home plate for the ball to reach second base?



Solution:

The right angle is at first base (or third base). You are asked to find the distance, x, which will be the hypotenuse of the right triangle. The legs are each 90 feet. (Baseball has not changed to metric.)



Formula: $a^2 + b^2 = c^2$

Substitution: $90^2 + 90^2 = x^2$

Solve: $8100 + 8100 = x^2$

$$16\ 200 = x^2$$

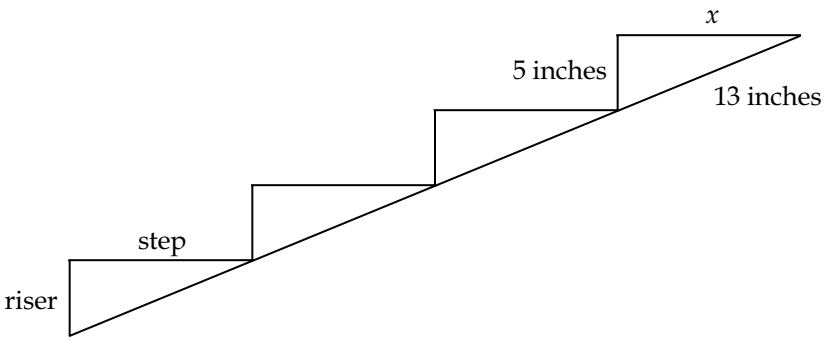
$$\sqrt{16\ 200} = \sqrt{x^2}$$

$$127.3 = x$$

The catcher must make a throw of 127.3 feet for a ball to travel from home plate to second base.

Example 2

You are building a set of stairs. The riser will be 5 inches, and the diagonal support board is 13 inches long for each step. How wide will the step be?



Solution:

Given that each step forms a right triangle, you can use the Pythagorean relation to solve the problem.

Formula: $a^2 + b^2 = c^2$

Substitution: $a^2 + b^2 = 13^2$

$5^2 + b^2 = 13^2$

Solve:

$$25 + b^2 = 169$$

$$25 + b^2 - 25 = 169 - 25$$

$$b^2 = 144$$

$$\sqrt{b^2} = \sqrt{144}$$

$$b = 12$$

Each step will be 12 inches wide.

Learning Activity 7.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Two angles in a triangle are 30° and 65°. What is the third angle?
2. Calculate: $\frac{5}{3} \times \frac{4}{5}$.
3. Alana, Meghan, Jen, and Kristen are at a restaurant. Alana doesn't want to sit beside Meghan. Jen wants to sit across from Meghan. If there are two seats on both sides of the table, who is sitting beside Alana?
4. To pay for your movie ticket, you give the cashier $15. The cost of the ticket is $13.25. The cashier gives you back a loonie, a quarter and 3 dimes. Is this the correct change?
5. Is $\frac{8}{5}$ closer to 1 or 2?

continued

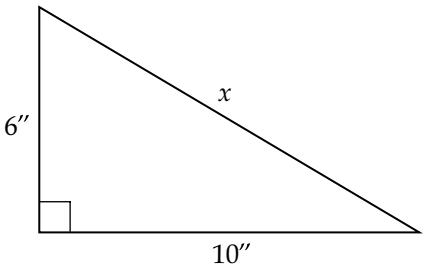
Learning Activity 7.3 (continued)

Part B: Pythagorean Theorem

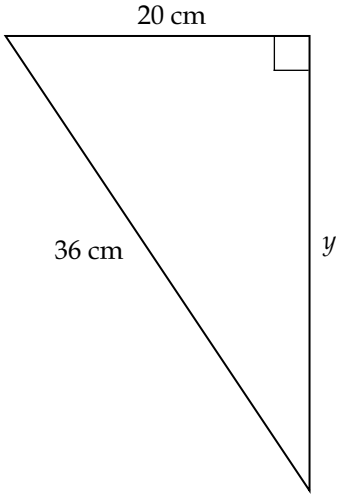
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing sides. Round off to one decimal place.

a)

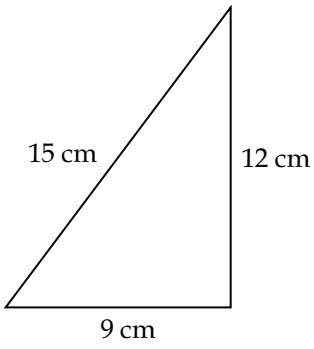


b)

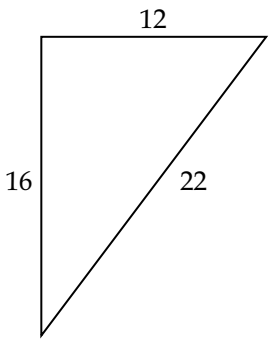


2. Use the Pythagorean theorem to prove whether or not these are right triangles.

a)



b)



3. You are building a shed out of some scrap lumber. The longest piece of wood you have for a diagonal rafter is 16 feet. You want the height of the roof to be 8 feet up. Use the Pythagorean formula to find the maximum width from the centre beam to the edge. Round to one decimal place.

Lesson Summary

This lesson focused on the Pythagorean theorem and using similar right triangles to solve problems. You learned to identify the hypotenuse first before substituting the values for the legs of the triangle into the formula. You also learned the Pythagorean Theorem could be used to check if a triangle is a right triangle. You applied the sum of the angles in a triangle rule to check whether or not a triangle is a right triangle if you are given two angles in the triangle.

The next lesson introduces you to trigonometric ratios.

Notes

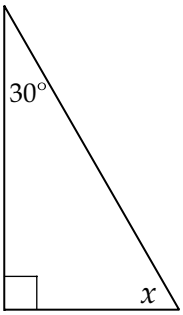
Assignment 7.3

Total: 21 marks

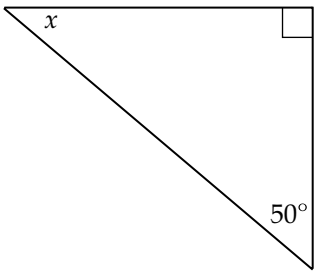
Pythagoras and Right Triangles

1. Find the missing angle in each triangle. *(2 marks)*

a)



b)

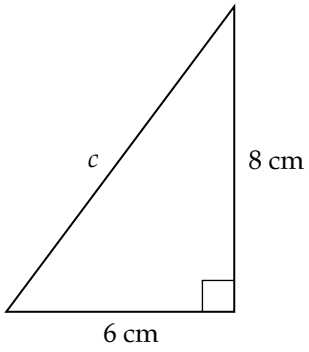


continued

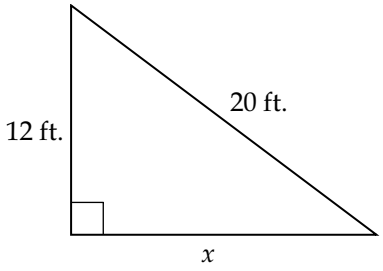
Assignment 7.3: Pythagoras and Right Triangles (continued)

2. Use the Pythagorean relation to find the missing side in each triangle. Show your calculations for full marks. Round to one decimal place. *(9 marks)*

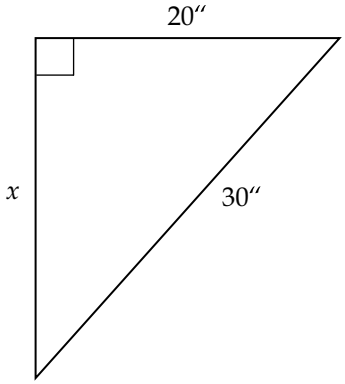
a)



b)



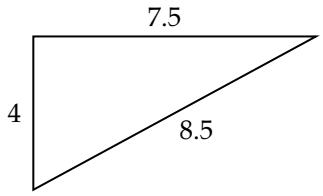
c)



continued

Assignment 7.3: Pythagoras and Right Triangles (continued)

3. Use the Pythagorean theorem to prove that the given triangle is a right triangle. *(3 marks)*



4. Given a right triangle with hypotenuse c and legs a and b. Round to one decimal place. *(4 marks)*

   a) Find c if $a = 5$ and $b = 12$

continued

b) Find a if $b = 8$ and $c = 12$.

5. A rectangle has base 6 and height 10. What is the length of the diagonal to the nearest tenth? *(3 marks)*

Lesson 4: Tangent Ratio

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of the tangent ratio

Lesson Introduction

Now your attention will be directed to finding sides and angles of right triangles without using similarity. You will learn about the ancient skills of trigonometry.

Triangle Ratios

Now that you have practised working with similarity and right triangles, you can learn more ways of finding the length of sides and the measures of angles. You do not need to use rulers and protractors, but instead you will use trigonometric ratios.

Identifying Sides Relative to a Given Angle

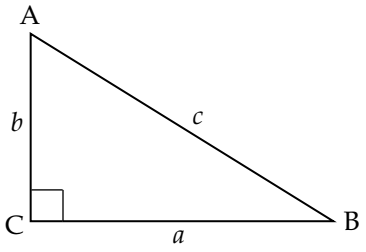
To use trigonometry ratios, you must be able to identify the sides of the triangle from the viewpoint of a designated angle within the triangle. You use three terms, and you must fully understand how to identify them to be successful in this lesson. The right angle is not one of the designated angles; just one of the two smaller angles can be one.

Given ΔABC with sides a, b, and c, the three terms are as follows.

- *hypotenuse:* the side opposite the right angle
- *opposite:* the side opposite the featured angle
- *adjacent:* the side next to, or beside, the featured angle

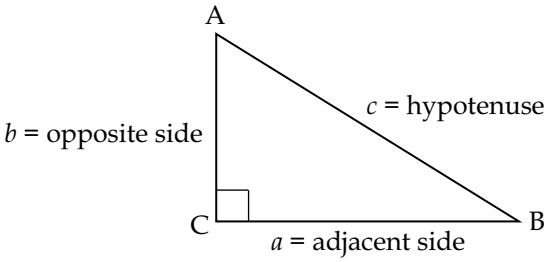
Example 1

Choose ∠B as the designated angle. Identify the hypotenuse, the opposite side, and the adjacent side with respect to ∠B.



Solution:

Name the hypotenuse first and then the opposite side. Then, the third side has to be the adjacent side.



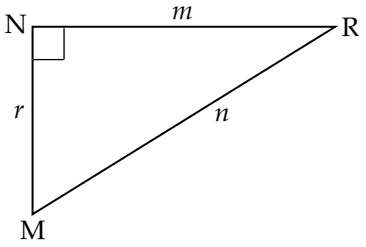
Hypotenuse = c

Opposite side = b

Adjacent = a

Example 2

Identify the following sides in ΔMNR.



a) relative to ∠R.

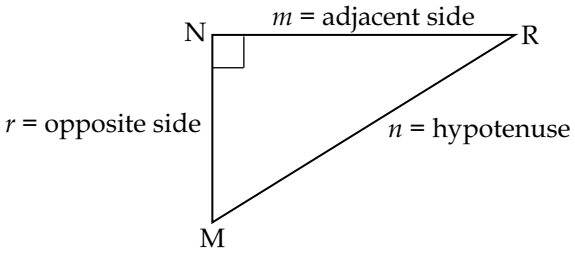
opposite: _____ adjacent: _____ hypotenuse: _____

b) relative to ∠M.

opposite: _____ adjacent: _____ hypotenuse: _____

Solution:

a)

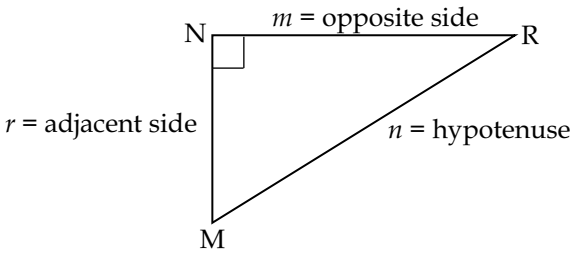


The hypotenuse, opposite the right angle, is n.

The side opposite $\angle R$ is r.

The side adjacent to $\angle R$ is m.

b)



The hypotenuse, opposite the right angle, is n.

The side opposite $\angle M$ is m.

The side adjacent to $\angle M$ is r.

Notice that the hypotenuse is the same no matter which angle is chosen as the designated angle. However, the opposite side and the adjacent side interchange, depending on which angle is featured.

As you work with the trigonometry ratios, you need to identify the sides relative to the chosen angle to solve trigonometric problems.

Learning Activity 7.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The angle between the two equal sides of an isosceles triangle is 80°. What are the values of the other two angles?
2. You can buy a 400 g package of chocolate eggs for $8.00, or you can pay $0.10 per gram. Which is the better buy?
3. 1 out of every 25 vehicles that you see on your way home from the cabin is a truck. Express this as a ratio.
4. Is the answer to $-14 - 29$ positive or negative?
5. Solve for a: $3a = \frac{9}{2}$.

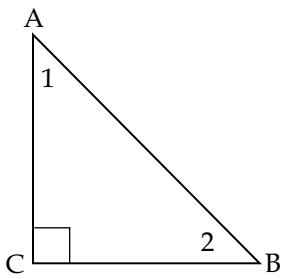
continued

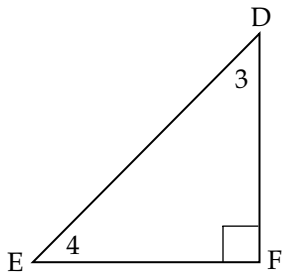
Learning Activity 7.4 (continued)

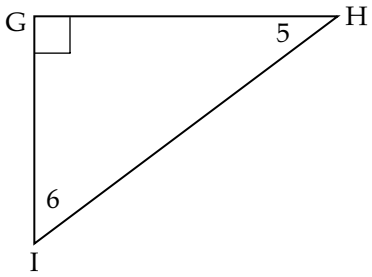
Part B: Opposite and Adjacent Sides

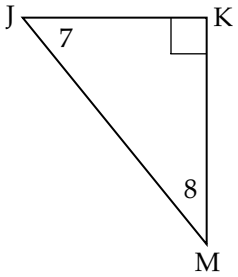
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Given the following right triangles, identify the sides as listed.





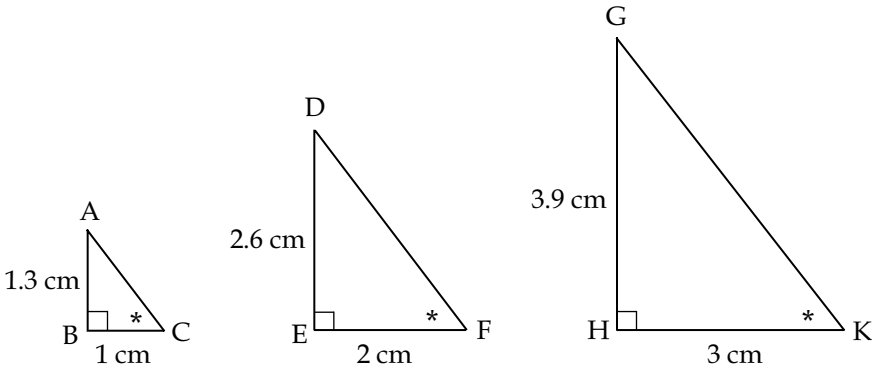




from ∠1	opposite:	_____	from ∠2	opposite:	_____
	adjacent:	_____		adjacent:	_____
	hypotenuse:	_____		hypotenuse:	_____
from ∠3	opposite:	_____	from ∠4	opposite:	_____
	adjacent:	_____		adjacent:	_____
	hypotenuse:	_____		hypotenuse:	_____
from ∠5	opposite:	_____	from ∠6	opposite:	_____
	adjacent:	_____		adjacent:	_____
	hypotenuse:	_____		hypotenuse:	_____
from ∠7	opposite:	_____	from ∠8	opposite:	_____
	adjacent:	_____		adjacent:	_____
	hypotenuse:	_____		hypotenuse:	_____

Ratios in Similar Right Triangles

Here are three similar right triangles, $\Delta ABC \sim \Delta DEF \sim \Delta GHK$.

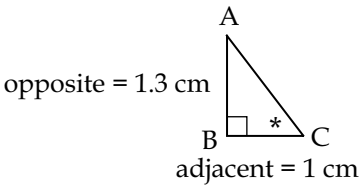


Because the triangles are similar, the angles indicated by $*$ will be equal for all triangles.

Using these corresponding angles as the designated angles, you set up the ratio of $\frac{\text{opposite}}{\text{adjacent}}$ for each triangle.

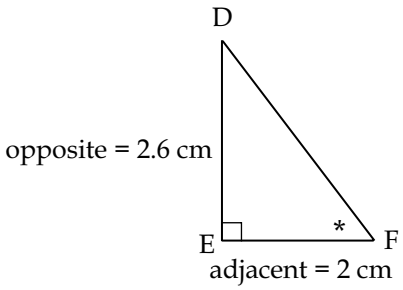
This ratio is the called the "tangent" ratio in trigonometry. Notice that the designated angle is always written beside the word "tangent." Notice also, that the ratio is simplified wherever possible and that tangent is abbreviated as "tan."

ΔABC



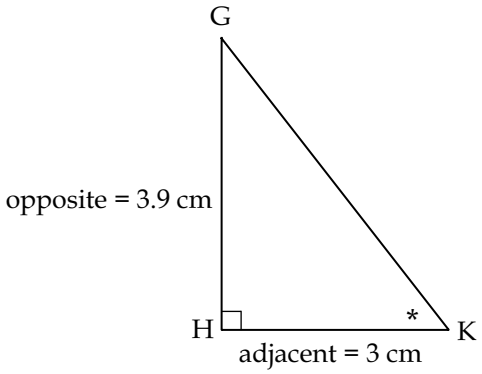
$$\tan \angle C = \frac{\text{opposite}}{\text{adjacent}} = \frac{1.3}{1} = 1.3$$

ΔDEF



$$\tan \angle F = \frac{\text{opposite}}{\text{adjacent}} = \frac{2.6}{2} = \frac{1.3}{1} = 1.3$$

ΔGHK



$$\tan \angle K = \frac{\text{opposite}}{\text{adjacent}} = \frac{3.9}{3} = \frac{1.3}{1} = 1.3$$

Scientific Calculator

Instead of using similar triangles with proportional corresponding sides, you now can use the tangent ratio to solve for missing sides in a triangle.

You will need a scientific calculator to perform trigonometric calculations. A scientific calculator has the tangent ratio key, abbreviated as "tan".

Angles can be measured in degrees, radians, or gradients. In this course, angles are measured only in degrees. Therefore, you must ensure that your calculator is always making trigonometric calculations using degrees.

Your calculator must always be in the "DEG" mode, or sometimes "D," depending on the calculator. If it is in the "RAD" or "GRAD" mode, none of your answers will be correct. Always check that your calculator shows D or deg mode.

It can be difficult to see at times, but the mode should be displayed somewhere on the screen. In this example "DEG" is showing, and that is the correct mode for this work.

If your calculator is not showing degrees, it can be changed by using the mode key, or the DRG key, or sometimes by turning the calculator off and then on again, depending on which calculator you have.

Finding the Tangent of an Angle

Example 1

Find the tangent of a 60° angle.

Solution:

The key syntax you use depends upon your calculator.

Method 1

Ensure your calculator is in degree mode.

Press the "tan" key, then 60.

You may need to press "Enter," or you may not need to.

Method 2

Some calculators may require you to enter the angle first, and then press the tan key. You may need to press "Enter" to get the answer.

If 1.732050808 appears on the display for your calculator, you did it right.

Example 2

Find the tangent of 45°.

Solution:

tan 45° = 1

Example 3

Find the tangent ratio for each of these angles. Round off to 5 decimal places.

a) tan 50° = _____ b) tan 22° = _____

c) tan 38° = _____ d) tan 16.8° = _____

Solution:

a) tan 50° = 1.19175 b) tan 22° = 0.40403

c) tan 38° = 0.78129 d) tan 16.8° = 0.30192

Inverse Tangent Process

Now that you understand how your calculator finds the tangent ratio of angles, you can do the entire process in reverse. You can find the angle given its tangent ratio. You need to use the $\tan^{-1}$ key. This key is called "tan inverse."

To find the angle measurement given the tangent ratio, you need to find the key on your calculator that will access those small letters above the "tan" key. You may have to experiment with your calculator to find the right combination of key strokes. Usually you press SHIFT or 2nd and the tan key.

Example 1

The tangent ratio for an angle is 1.191 753 593. Find the angle.

Solution:

Method 1

Press 2nd or "INV," then "tan," then "1.191753593," and then "Enter."

Method 2

Some calculators require that you enter the numbers first, and then press inverse tan.

Find the combination of key strokes that works on your calculator. You may want to write down the steps on your resource sheet.

When you find the way your calculator works, the answer will be 50°.

A useful symbol for indicating an designated angle in a triangle is the Greek symbol called "theta". The symbol is θ.

Example 2

Find each angle θ, given $\tan \theta$. Use the inv tan method you learned in Example 1 above. Round off your answer to the nearest degree.

a) $\tan \theta = 0.70021$

b) $\tan \theta = 2.35585$

c) $\tan \theta = 0.14054$

Solution:

a) $\tan \theta = 0.70021$

$\theta = \tan^{-1} (0.70021)$

$\theta = 35°$

b) $\tan \theta = 2.35585$

$\theta = \tan^{-1} (2.35585)$

$\theta = 67°$

c) $\tan \theta = 0.14054$

$\theta = \tan^{-1} (0.14054)$

$\theta = 8°$

To remember whether to press tan or inv tan, just remember that when you are finding the angle, it is as if you are solving. Therefore, you need to use the inverse tangent operation.

Tangent Ratio

With practice, you can understand how to identify the opposite and adjacent sides of an angle.

You can also understand how to use your scientific calculator to find the tangent and its inverse.

Remember that the tangent ratio is $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$.

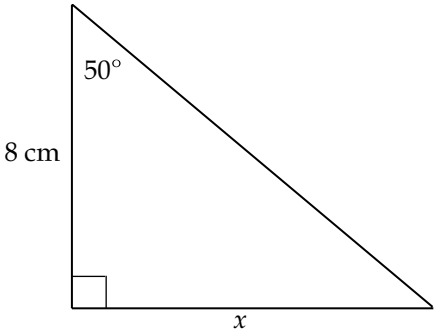
If you apply these new skills to solving a right triangle, you can find the sides and the angles.

Finding Values for Sides

When you use the ratio $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$, there are three values, θ, opposite and adjacent that need to be considered. You would need to know any two of the values and then you can substitute these two known values into the formula to find the unknown value.

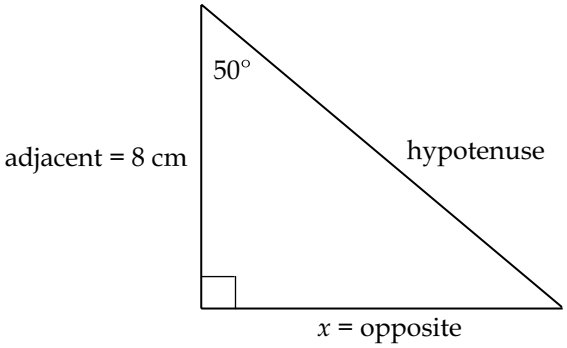
Example 1

Solve for the length of the side opposite the given angle to the nearest tenth.



Solution:

Before solving for an unknown, it is helpful to mark the diagram and to fill in a chart of the values as follows.



designated angle = 50°

side opposite designated angle = x

side adjacent to designated angle = 8 cm

This list could be abbreviated considerably.

$\theta = 50°$

opp = x

adj = 8 cm

You are asked to find the length of the opposite side. To accomplish this, you set up the tan ratio and substitute into it the values you know.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 50° = \frac{x}{8}$$

To solve for x, you must isolate it by eliminating 8. Since you are dividing by 8, you can eliminate 8 using the inverse operation of multiplying by 8. And, if you multiply by 8 on one side, you must multiply by 8 on the other side.

$$\tan 50° \times 8 = \frac{(x)(8)}{8}$$

This notation is rather awkward.

You might be finding the tangent $50° \times 8$, which is tan 400°. That would give you a wrong answer.

You are advised to write 8 in front of tan 50° instead of after it in order to avoid mistakes.

Writing any of the following notations is better than the one above.

$$8 \times \tan 50° = \frac{x}{8} \times 8 \text{ or}$$

$$8 \tan 50° = \frac{x}{8} \times 8 \text{ or}$$

$$8 \tan 50° = \frac{x}{8}(8)$$

In any case, you can simplify and get your answer.

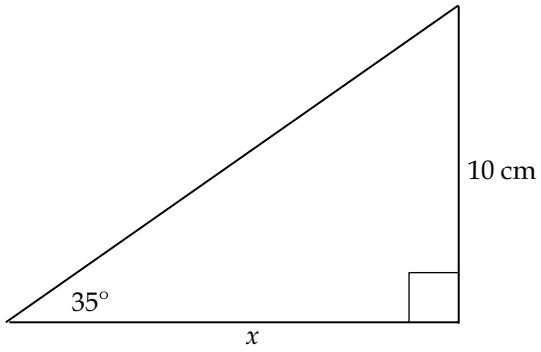
$$8 \tan 50° = x$$

$$9.5 = x$$

The side opposite the angle is 9.5 cm long.

Example 2

Find the side adjacent to the given angle to the nearest tenth.



Solution:

Step 1: Make the list.

$\theta = 35°$
opp = 10 cm
adj = x

Step 2: Write the formula.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

Step 3: Substitute the values that you know into the formula.

$$\tan 35° = \frac{10}{x}$$

Step 4: Solve for the unknown value.

To solve for x, you must isolate it. This requires one additional step, as compared to the process in Example 1 above. Since you are dividing by x, you must multiply both sides equally by x to get it out of the denominator.

$$x \tan 35° = \frac{10}{x} \times x$$

$$x \tan 35° = 10$$

Now to isolate the variable, you divide both sides of the equation equally by tan 35°.

$$\frac{x \tan 35°}{\tan 35°} = \frac{10}{\tan 35°}$$

$$x = 14.3$$

Step 5: Write the answer.

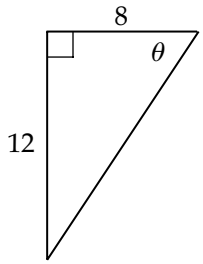
The side adjacent to the given angle is 14.3 cm.

Compare Example 1 and Example 2. You notice that in Example 1, you multiplied by tan 50° to get the answer for x. In Example 2, you divided by tan 35° to get the answer for x. Keep this pattern in mind as you practise more questions in the Learning Activity. Perhaps you will be able to find a short cut.

Finding Values for Angles

Example

Find the measure of θ to the nearest tenth.



Solution:

Step 1: Make the list.

$\theta = ?$

opp = 12

adj = 8

Step 2: Write the formula.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

Step 3: Substitute.

$$\tan\theta = \frac{12}{8}$$

Step 4: Solve.

$$\theta = \tan^{-1}\left(\frac{12}{8}\right)$$

$$\theta = 56.3°$$

Step 5: Write the answer.

The measure of angle θ is 56.3°.

When entering the information into your calculator, using the brackets is crucial. Otherwise, your calculator follows the BEDMAS rules and will give you an incorrect answer. Also, make sure you press ENTER in the final step of your solving process.

Learning Activity 7.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The side length of a square is 5 cm. What is the area of the square?
2. You work at 4:00, but you have some errands to do before your shift. Driving to the grocery store and buying food for the next week takes 50 minutes. Driving home and putting the groceries away takes 30 minutes. You then clean your bedroom, which takes 1 hr. 30 min. You want to eat before work, which takes you 45 minutes to prepare and eat. Your commute to work is
   15 minutes long. If it is 1:00 right now, will you be able to do everything you want to before work?
3. Convert to a decimal: 0.25%.
4. Evaluate: $\frac{3}{4} \times \frac{5}{7}$.
5. Janelle works Monday to Friday, and is paid $80 per day. How much money does she make in two weeks?

continued

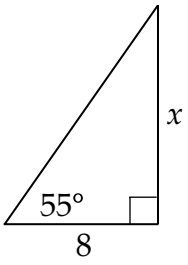
Learning Activity 7.5 (continued)

Part B: Tangent Ratio

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing side using the tangent ratio. Round off to two decimal places.

a)



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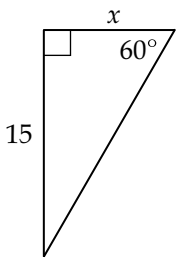
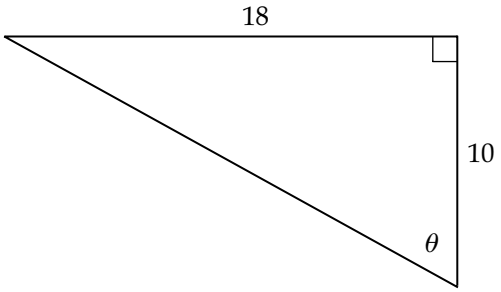
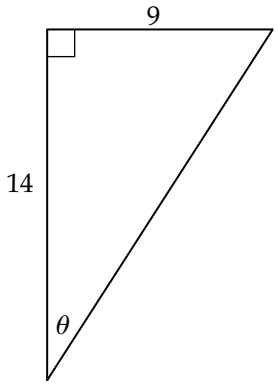


2. Find the value of θ using the tangent ratio and its inverse. Round off to two decimal places.

a)



b)



Lesson Summary

This lesson introduced you to the tangent ratio in trigonometry. You learned how to identify the opposite, adjacent, and hypotenuse as sides of a right triangle respective to a designated acute angle. You learned that for each measure of an angle in a right triangle, there is only one answer for the tangent ratio, no matter what the size of the sides. You learned the process for finding sides of a triangle using the tangent of the designated angle. You also learned how to find an angle using the tangent ratio if you were given the length of the opposite side and the adjacent side. On your calculator, you learned the correct keys to press to find a missing side or a missing angle.

The next lesson introduces you to the sine ratio.

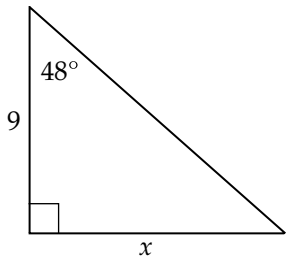
Assignment 7.4

Total: 21 marks

Tangent Ratio

1. Use the tangent ratio to find the missing side. Round off your answer to two decimal places. Show your work for full marks. *(12 marks)*

a)



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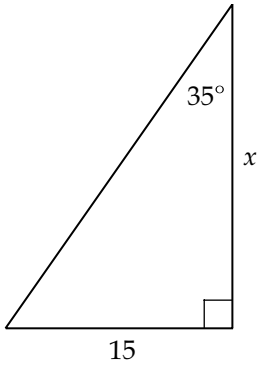
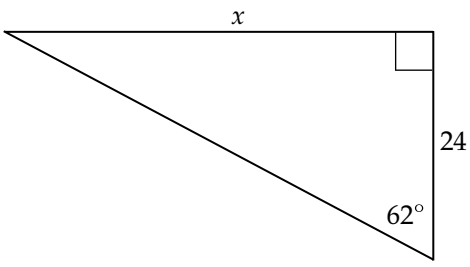
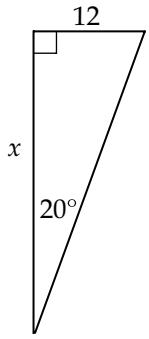


c)



d)

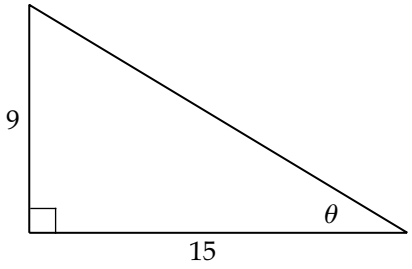


continued

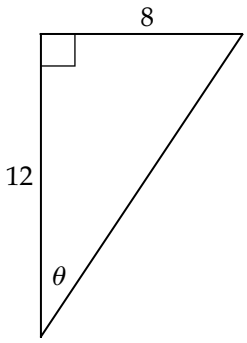
Assignment 7.4: Tangent Ratio (continued)

2. Use the inverse tangent to find the missing angle. Show your work for full marks. *(9 marks)*

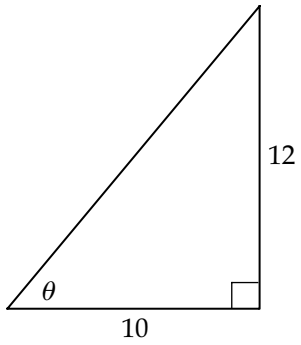
a)



b)



c)



Lesson 5: Sine Ratio

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of sine ratio

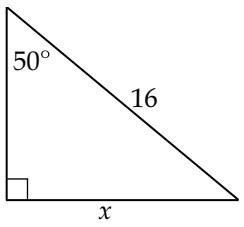
Lesson Introduction

There are three primary trigonometry ratios. You learned the tangent ratio in the last lesson. This lesson introduces you to the sine ratio.

Sine Ratio

The previous lesson worked exclusively with the tangent ratio. The sine ratio is very similar to the tangent ratio, except you now use the hypotenuse and the opposite side after designating a specific acute angle.

When you have or are looking for values for the opposite side and the hypotenuse, you use the sine ratio.

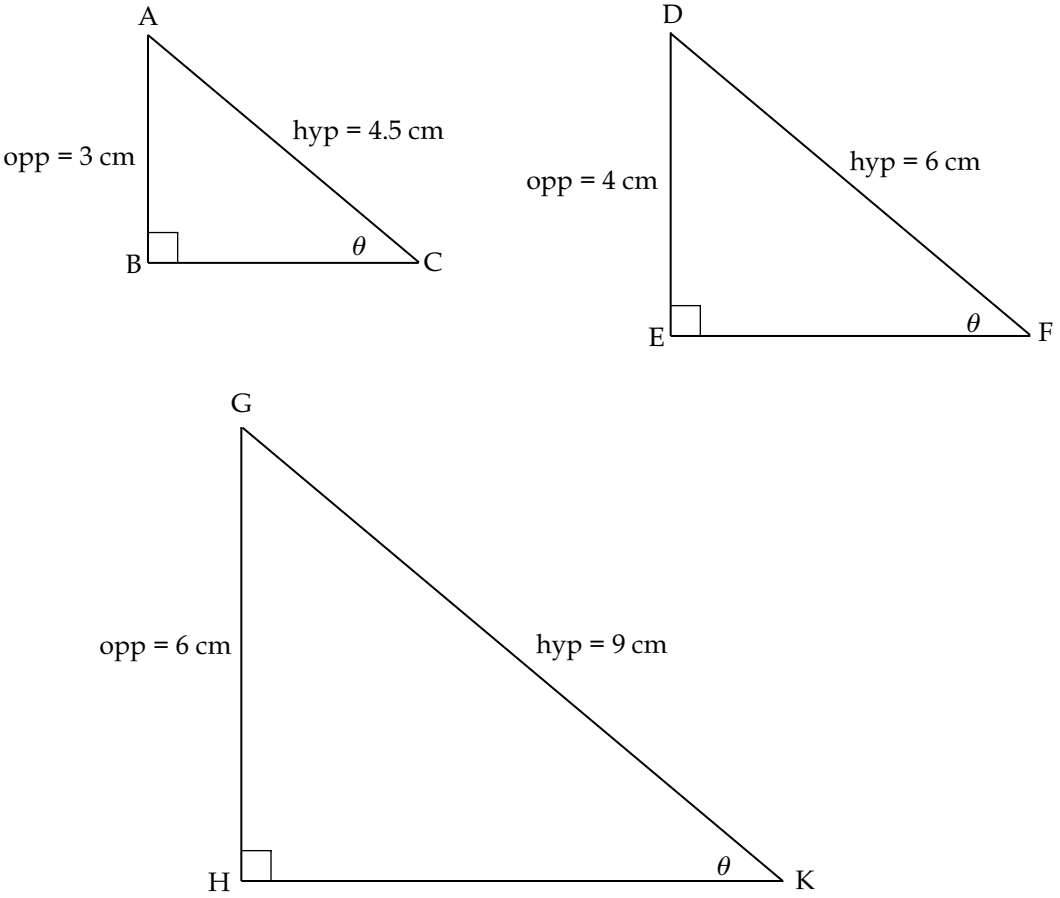


Notice that the x-side is opposite the given angle, and the side with a value of 16 is on the hypotenuse, the side opposite the right angle. The hypotenuse is always opposite the right angle. When you are presented with a triangle using the opposite side and the hypotenuse, you use the sine ratio. The tangent ratio is used when you have triangles with values on the opposite and adjacent sides.

Your scientific calculator will show the sine key with the abbreviation "sin."

Ratios in Similar Triangles

Here are three similar right triangles, $\Delta ABC \sim \Delta DEF \sim \Delta GHK$.

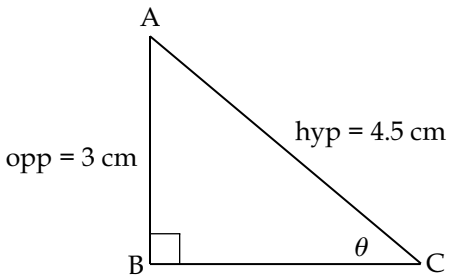


Because the triangles are similar, the angles indicated by θ will be the same for each triangle.

Using θ as the designated angle, set up a ratio of $\frac{\text{opposite}}{\text{hypotenuse}}$ for each triangle.

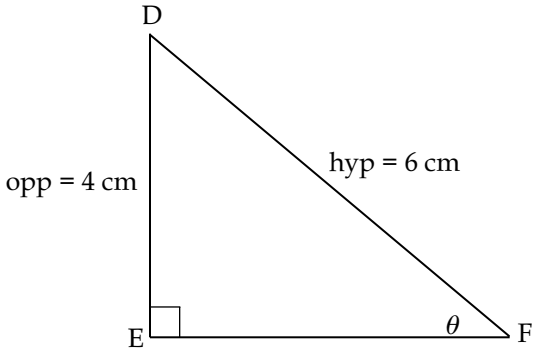
This is the "sine" ratio.

ΔABC



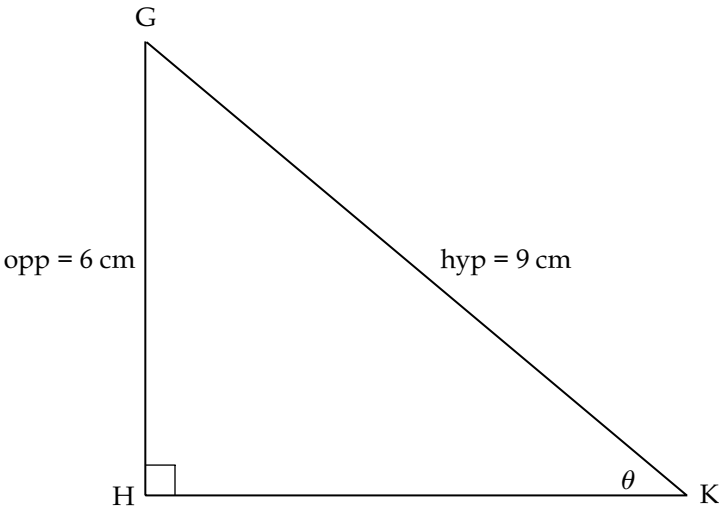
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{4.5} = 0.66666667$$

ΔDEF



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{6} = 0.66666667$$

ΔGHK



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{9} = 0.66666667$$

Note that for the same measure of an angle in the three similar triangles, the sine ratios all have the same value of 0.66666667. In other words, if you have the measure of a given acute angle in a right triangle, then its sine will be the same no matter what the size of the triangle.

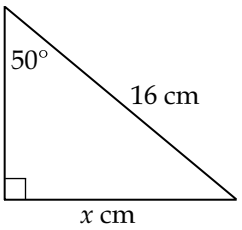
In the next few pages, you will be solving right triangles as you did with the tangent ratio. Here, you would know to use the sine ratio because the three terms in the ratio are θ, opposite, and hypotenuse. It is these terms that distinguish whether you use the tangent ratio or the sine ratio. Remember the tangent ratio needs the adjacent side not the hypotenuse.

When you use the sine ratio, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, you can substitute two known values into the formula. Then you can solve for the unknown value. The process is the same as the one you used in the previous lesson for the tangent ratio. Use the steps, multiply when the x is on the top, divide when x is on the bottom, and use inverse sine when solving for an angle.

Finding Values for Sides

Example 1

Solve for x.



Solution:

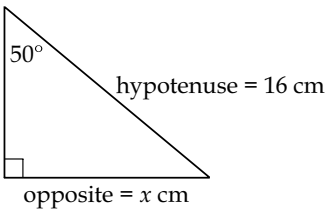
Step 1: Make the list.

Before solving for an unknown, it is helpful to mark the diagram and to list the values as follows.

the designated angle is $\theta = 50°$

the side opposite the designated angle is x cm

the hypotenuse is 16 cm



In abbreviated form, the list would be written as follows.

$\theta = 50°$

opp = x cm

hyp = 16 cm

Step 2: Write the formula.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

Step 3: Substitute.

$$\sin 50° = \frac{x}{16}$$

Step 4: Solve.

$$16 \sin 50° = \frac{x}{16}(16)$$

$$16 \sin 50° = x$$

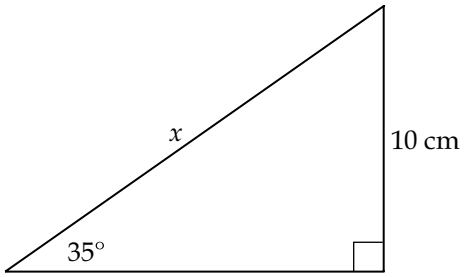
$$12.26 = x$$

Step 5: Write the answer.

The side opposite the angle is 12.26 cm long.

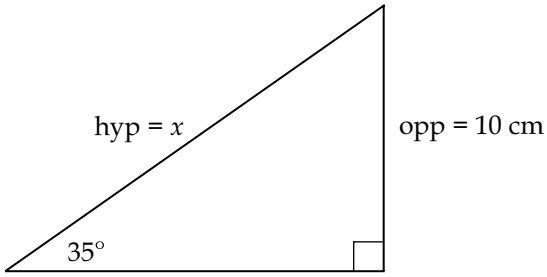
Example 2

Solve for x.



Solution:

Step 1: Make the list and mark the diagram.



$\theta = 35°$

opp = 10 cm

hyp = x cm

Step 2: Write the formula.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

Step 3: Substitute.

$$\sin 35° = \frac{10}{x}$$

Step 4: Solve.

$$x \sin 35° = \frac{10(x)}{x}$$

$$x \sin 35° = 10$$

$$\frac{x \sin 35°}{\sin 35°} = \frac{10}{\sin 35°}$$

$$x = \frac{10}{\sin 35°}$$

$$x = 17.43$$

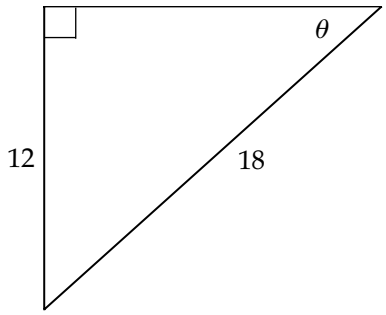
Step 5: Write the answer.

The hypotenuse is 17.43 cm.

Finding Values for Angles

Example 1

Find the angle θ.



Solution:

θ = ?

opp = 12

hyp = 18

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

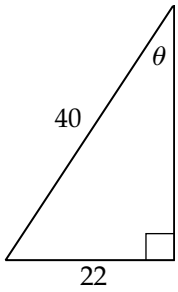
$$\sin\theta = \frac{12}{18}$$

$$\theta = \sin^{-1}\left(\frac{12}{18}\right)$$ (Remember to use the brackets on your calculator.)

$$\theta = 41.81°$$

Example 2

Find angle θ.



Solution:

θ = ?

opp = 22

hyp = 40

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin\theta = \frac{22}{40}$$

$$\theta = \sin^{-1}\left(\frac{22}{40}\right)$$

$$\theta = 33.37°$$

Learning Activity 7.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

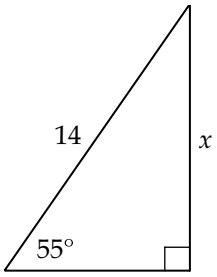
1. A CFL end zone is 60 feet deep. The field is 65 yards wide. What is the area of the end zone in yards?
2. Is the answer to –16 + (–15) positive or negative?
3. You are buying a DVD player for $100. If the in-store warranty is 6.5% of the cost, how much will you pay for the warranty?
4. Write two equivalent fractions to $\frac{15}{9}$.
5. The scale of a diagram is 1 cm : 50 cm. If the diagram is 10 cm long, what is the length in reality?

Part B: Sine Ratio

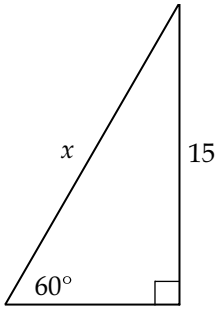
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing side using the sine ratio.

a)



b)



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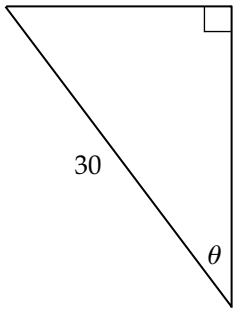
Learning Activity 7.6 (continued)

2. Find angle θ using the inverse sine.

a)



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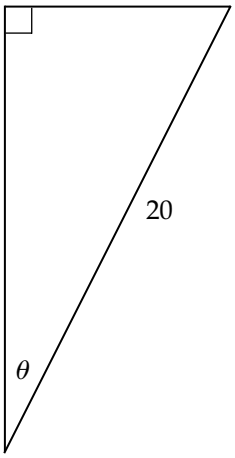


Lesson Summary

This lesson introduced you to the sine ratio in trigonometry. You learned that for each measure of an acute angle in a right triangle, there is only one answer for the sine ratio, no matter what the size of the sides. You learned the process for finding sides of a triangle using the sine of the designated angle. You also learned how to find an angle using the sine ratio if you were given the length of the opposite side and the hypotenuse.

The next lesson introduces you to cosine ratio, and asks you to use all three primary ratios to find unknown values for sides and angles.

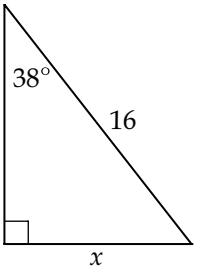
Assignment 7.5

Total: 21 marks

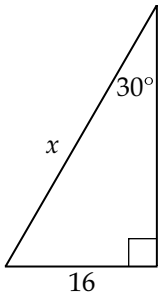
Sine Ratio

1. Use the sine ratio to find the missing side. Round off your answer to two decimal places, if necessary. Show your work for full marks. *(12 marks)*

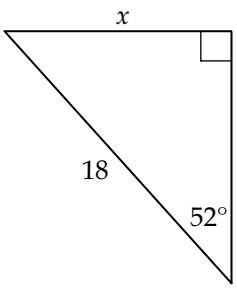
a)



b)



c)



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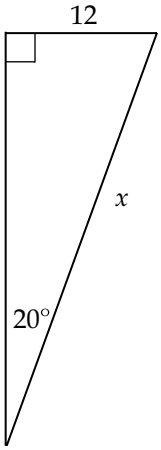

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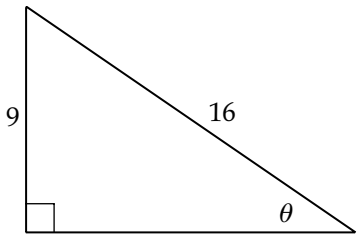
Assignment 7.5: Sine Ratio (continued)

2. Use the sine ratio and its inverse to find the missing angle. Round off to two decimal places. Show your work for full marks. *(9 marks)*

a)



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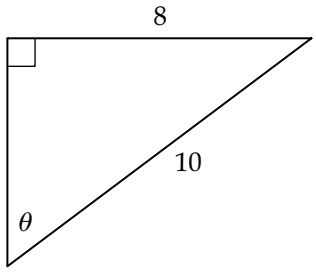
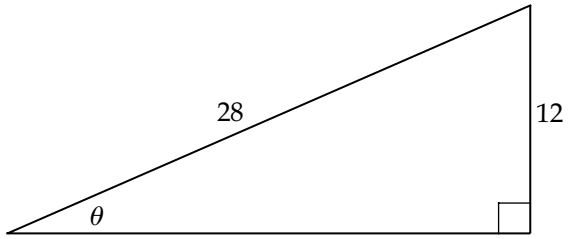


c)



Lesson 6: Cosine Ratio

Lesson Focus

In this lesson, you will

❑ demonstrate an understanding of primary trigonometric ratios

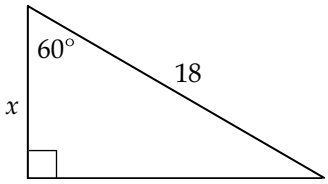
Lesson Introduction

The third basic trigonometry ratio is the cosine. This lesson introduces the cosine ratio, and asks you to use all three trig ratios to find unknown values for sides and angles.

Cosine Ratio and Selecting a Ratio

In the previous lessons, you worked exclusively with the sine ratio or the tangent ratio. The cosine ratio is very similar to the tangent and sine ratios, except you now use the hypotenuse and the adjacent sides.

When you have or are looking for values for the adjacent side and the hypotenuse, you use the cosine ratio.



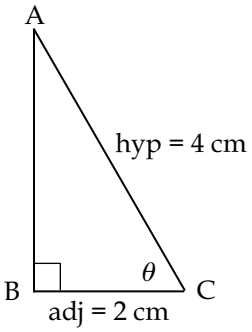
Notice that the x-side is adjacent to the given angle and the side with a value of 18 is on the hypotenuse. When you are presented with a triangle using the adjacent side and the hypotenuse, you use the cosine ratio. Remember that the tangent ratio is used when you have triangles with values on the opposite and adjacent sides, and the sine ratio is used when you have the opposite side and the hypotenuse.

Your scientific calculator will show the cosine key with the abbreviation "cos."

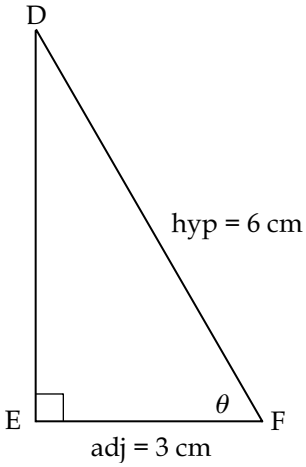
Cosine Ratios in Similar Triangles

Here are three similar right triangles, $\Delta ABC \sim \Delta DEF \sim \Delta GHK$.

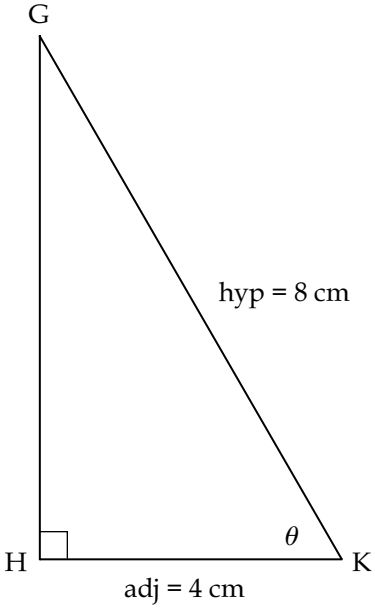
ΔABC



ΔDEF



ΔGHK

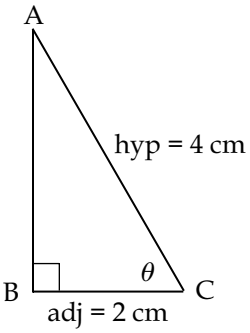


Because the triangles are similar, the indicated angles will be the same for each triangle.

Using θ as the designated angle, set up the ratio of $\frac{\text{adjacent}}{\text{hypotenuse}}$ for each triangle.

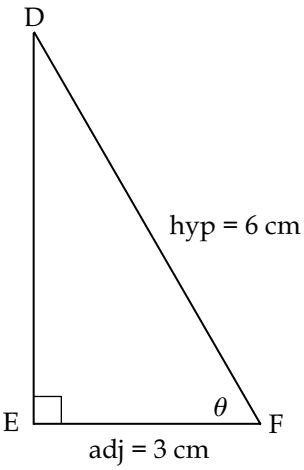
This is the "cosine" ratio.

ΔABC



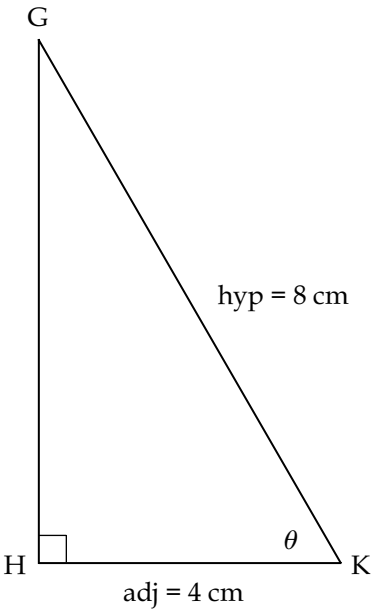
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2}{4} = 0.5$$

ΔDEF



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{6} = 0.5$$

ΔGHK



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{8} = 0.5$$

Note that for the same measure of an angle in the three similar triangles, the cosine ratios all have the same value of 0.5. In other words, if you have the measure of a given acute angle in a right triangle, then its cosine will be the same no matter what the size of the triangle.

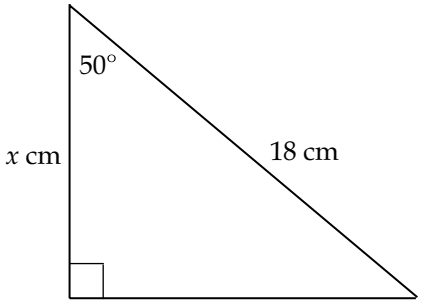
In the next few pages, you will be solving right triangles as you did with the tangent and sine ratio. Here, you would know to use the cosine ratio because the three terms in the ratio are θ, adjacent, and hypotenuse. It is these terms that distinguish whether you use the tangent, the sine, or the cosine ratio. Note that cosine is abbreviated as "cos."

When you use the cosine ratio, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, you can substitute two known values into the formula. Then you can solve for the unknown value. The process is the same as the one you used in the previous lessons for the tangent and sine ratios. Use the steps, multiply when the x is on the top, divide when x is on the bottom, and use inverse sine when solving for an angle.

Finding Values for Sides

Example 1

Solve for x.



Solution:

Step 1: Identify the sides and make the list.

$\theta = 50°$

adj = x cm

hyp = 18 cm

Step 2: Write the formula.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

Step 3: Substitute.

$$\cos 50° = \frac{x}{18}$$

Step 4: Solve.

$$18\cos 50° = \frac{x(18)}{18}$$

$$18\cos 50° = x$$

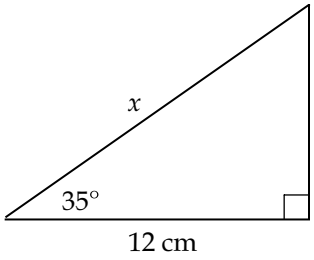
$$11.57 = x$$

Step 5: Write the answer.

The side adjacent to the angle is 11.57 cm long.

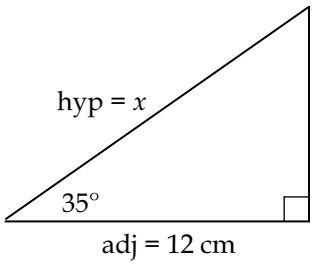
Example 2

Find the length of the hypotenuse.



Solution:

Step 1: Make the list and identify the sides on the diagram.



$\theta = 35°$

adj = 12 cm

hyp = x cm

Step 2: Write the formula.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

Step 3: Substitute and solve.

$$\cos 35° = \frac{12}{x}$$

$$x \cos 35° = \frac{12(x)}{x}$$

$$x \cos 35° = 12$$

$$\frac{x \cos 35°}{\cos 35°} = \frac{12}{\cos 35°}$$

$$14.65 = x$$

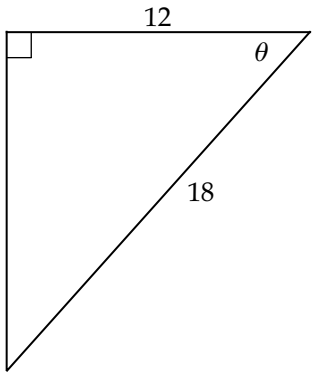
Step 4: Write the answer.

The hypotenuse is 14.65 cm long.

Finding Values for Angles

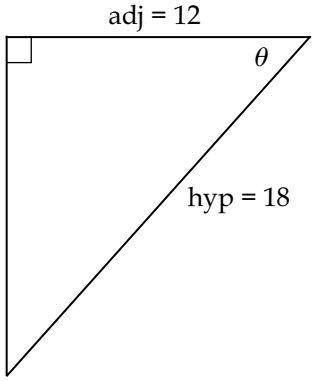
Example 1

You use your skills with finding the inverse of the cosine ratio to find the angle, θ.



Solution:

Step 1: Make the list and identify the sides on the diagram.



θ = ?

adj = 12

hyp = 18

Step 2: Write the formula.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

Step 3: Substitute.

$$\cos\theta = \frac{12}{18}$$

Step 4: Solve.

$$\theta = \cos^{-1}\left(\frac{12}{18}\right)$$

$$\theta = 48.19°$$

Step 5: Write the answer.

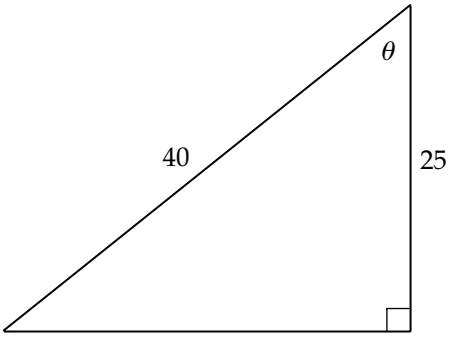
The angle θ has a measure of 48.19°.

Example 2

You use your skills with finding the inverse of the cosine ratio to find the angle θ.

Solution:

Step 1: Make the list and identify the sides on the diagram.



θ = ?

adj = 25

hyp = 40

Step 2: Write the formula.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

Step 3: Substitute.

$$\cos\theta = \frac{25}{40}$$

Step 4: Solve.

$$\theta = \cos^{-1}\left(\frac{25}{40}\right)$$

$$\theta = 51.32°$$

Step 5: Write the answer.

The angle θ has a measure of 51.32°.

Have you added the three trigonometric formulas to your resource sheet? If you haven't done so yet, add them now.

Learning Activity 7.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

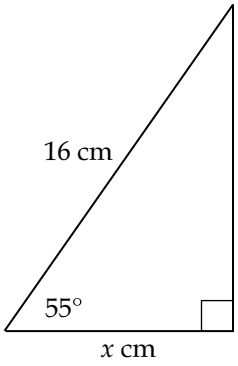
1. If two of the angles in a triangle are 45° and 105°, what is the third angle?
2. What is the value for $3x - 3$ if $x = 4$?
3. Is the answer to $-5 - (-12)$ positive or negative?
4. It is really hot outside so you are very thirsty. In your fridge, there is orange juice with pulp, apple juice, water, and milk. You are lactose-intolerant. You do not like water, nor do you like pulp. What will you drink?
5. Convert 34 km into cm.

Part B: Cosine Ratio

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing side using the cosine ratio. Round off to two decimal places.

a)



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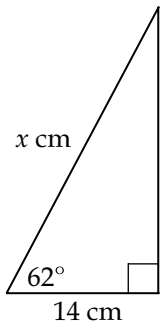

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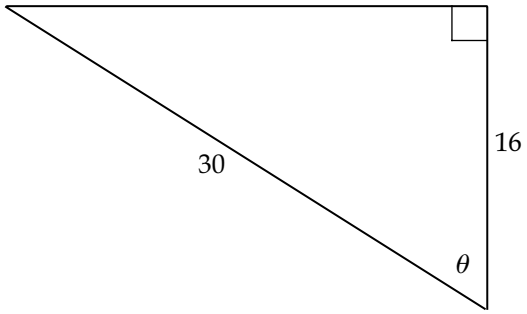
Learning Activity 7.7 (continued)

2. Find angle θ using the cosine ratio and its inverse.

a)



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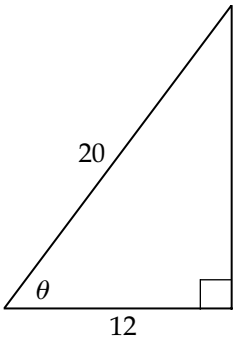


SOH CAH TOA

Three ratios, each with a different arrangement of sides, can be difficult to remember. A "memory hook" you can use is SOH CAH TOA. The letters in each group tell you which ratio to use, depending on which sides are involved in the question.

$$\text{SOH represents Sin } \theta = \frac{\text{Opp}}{\text{Hyp}}$$

$$\text{CAH represents Cos } \theta = \frac{\text{Adj}}{\text{Hyp}}$$

$$\text{TOA represents Tan } \theta = \frac{\text{Opp}}{\text{Adj}}$$

You are encouraged to use SOH CAH TOA for every question when you need to decide which ratio you need. Write SOH CAH TOA in the margin of your papers while working on trigonometric questions. It really will help you make more accurate calculations.

Using the list to help identify each side is also helpful.

θ =

Opposite =

Adjacent =

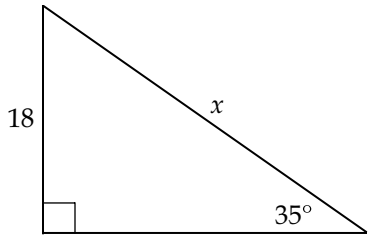
Hypotenuse =

Selecting the Ratio

When you are presented with a right triangle to solve, you need to decide which of the three ratios you can use to solve for the unknown.

Example 1

Solve for x.



Solution:

Step 1: Make the list.

This time, make a complete list of all the sides.

$\theta = 35°$

opp = 18

adj =

hyp = x

Step 2: Write the formula.

Since you have the side opposite the angle, and the hypotenuse is the required side, check SOH CAH TOA to see which ratio uses opposite and hypotenuse. That would be SOH, the sine ratio, since SOH is Sine = Opposite over Hypotenuse.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

Step 3: Substitute.

$$\sin 35° = \frac{18}{x}$$

Step 4: Solve.

$$x \sin 35° = 18$$

$$x = \frac{18}{\sin 35°}$$

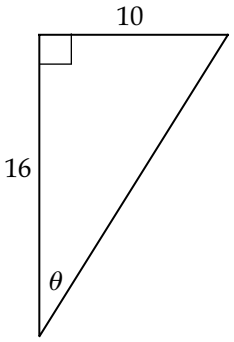
$$x = 31.38$$

Step 5: Write the answer.

The value of x is 31.38.

Example 2

Find the measure of θ.



Solution:

Step 1: Make the list.

θ = ?

opp =10

adj = 16

hyp =

Step 2: Write the formula.

Since you are given the values for opposite and adjacent, then upon checking SOH CAH TOA, you realize you can use the tangent ratio.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

Step 3: Substitute.

$$\tan\theta = \frac{10}{16}$$

Step 4: Solve.

$$\theta = \tan^{-1}\left(\frac{10}{16}\right)$$

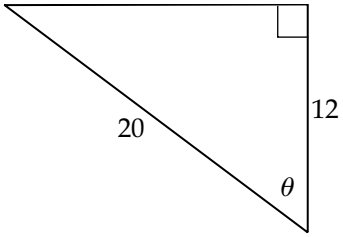
$$\theta = 32.01°$$

Step 5: Write the answer.

The angle θ has a measure of 32.01°.

Example 3

Find the measure of θ.



Solution:

Step 1: Make the list.

$\theta = ?$

opp =

adj = 12

hyp = 20

Step 2: Write the formula.

Since you are given the values for adjacent and hypotenuse, then upon checking SOH CAH TOA, you realize you can use the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

Step 3: Substitute.

$$\cos\theta = \frac{12}{20}$$

Step 4: Solve.

$$\theta = \cos^{-1}\left(\frac{12}{20}\right)$$

$$\theta = 53.13°$$

Step 5: Write the answer.

The angle θ has a measure of 53.13°.

Learning Activity 7.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. It costs $4 for a bag of mini-donuts at a football game. It costs $5 for a hotdog and $3 for a bottle of pop. If you buy one bag of mini-donuts, 3 hotdogs, and 2 bottles of pop, how much will you spend on food at the game?
2. Evaluate: $\frac{5}{6} - \frac{1}{3}$.
3. The three sides of a right angle triangle are 6.5, 9.7, 7.2. Which is the hypotenuse?
4. Is the answer to 10 – 14 positive or negative?
5. You are supposed to work from 9 am to 5 pm. You arrive at work at 8:55 am, and leave at 4:50 pm. How much time does your work pay you for?

continued

Learning Activity 7.8 (continued)

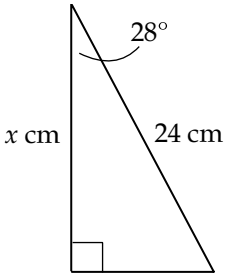
Part B: Trigonometry Ratios (all 3)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Use one of the three primary trigonometry ratios to solve for each unknown. Round off to two decimal places.

a)



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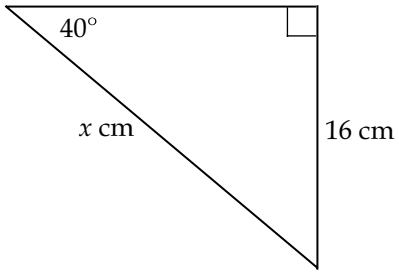

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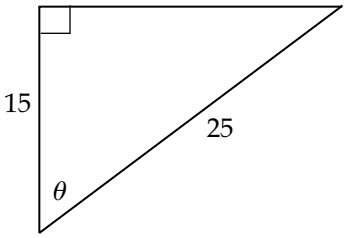

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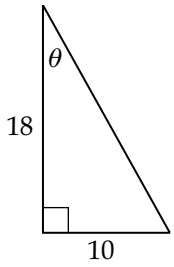


Lesson Summary

This lesson introduced the cosine ratio. Also, you were shown how to decide which of the three primary trigonometry ratios to use when solving for unknowns using a handy memory device.

The next lesson applies your skills with trigonometry ratios in solving problems.

Notes

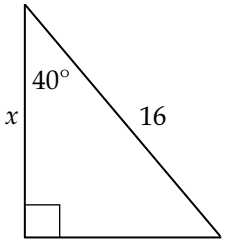
Assignment 7.6

Total: 24 marks

Cosine Ratio, Sine Ratio, and Tangent Ratio

1. Use trigonometry ratios to find the indicated side. Round off to two decimal places. Show your work for full marks. *(12 marks)*

a)



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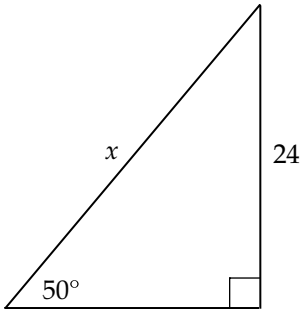

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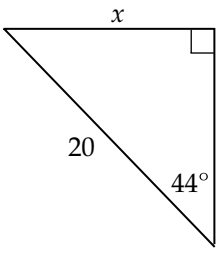

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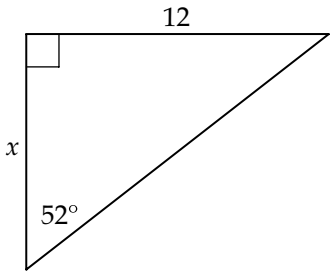

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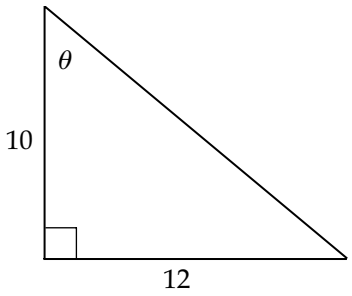
Assignment 7.6: Cosine Ratio, Sine Ratio, and Tangent Ratio (continued)

2. Use trigonometry ratios and their inverses to find the indicated angles. Round off your answer to two decimal places. Show your work for full marks. *(12 marks)*

a)



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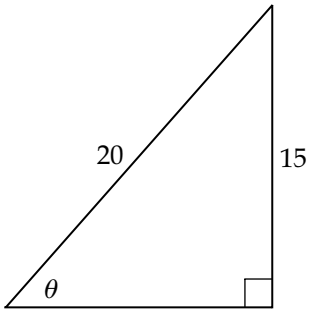

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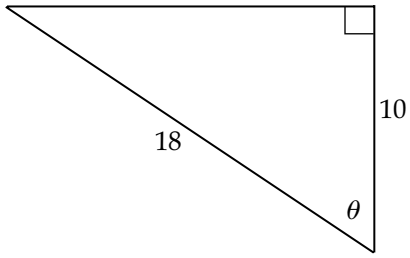
c)



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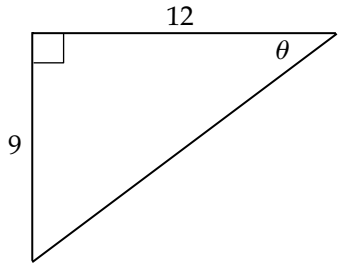


Notes

Lesson 7: Problems

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of primary trigonometric ratios
- ❑ consider whether or not an answer is reasonable

Lesson Introduction

In this lesson, you apply your understanding of trigonometric ratios to solve problems.

Lesson Content

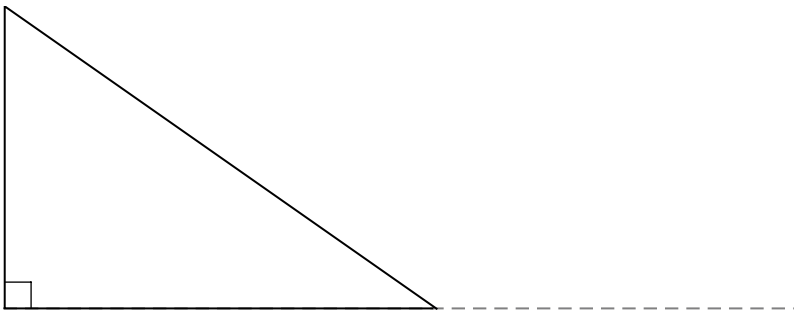
The ability to use your skills with trigonometry ratios in solving problems is the goal. You must be sure that you understand the question being asked, draw an accurate diagram, select the appropriate trigonometric ratio, and calculate the answer. Finally, you want to write a sentence answering the question at the end of each problem.

Example 1

Skiers ski down a slope that is inclined at a 35° angle to the horizontal at ground level. If the skiers reach the ground level after travelling a distance of 325.6 m, how high is the ski slope where they began their run?

Solution:

Since the problems in this lesson involve right triangles, it would be a safe start for you to draw a sketch of a right triangle.

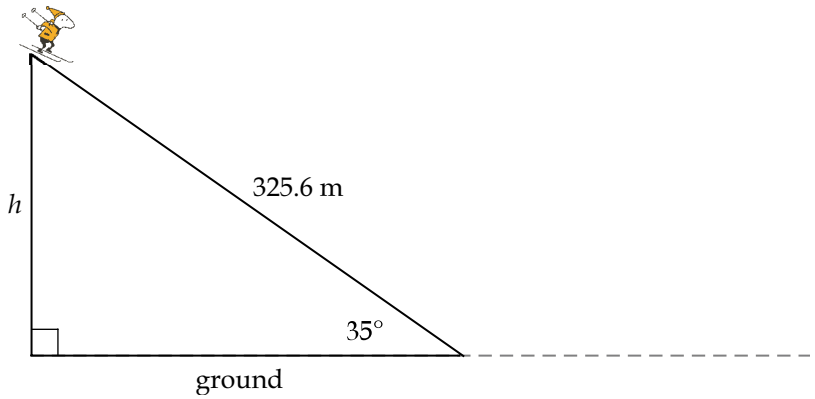


The challenge in trig problems is to draw the diagram, and label it correctly.

Write the information on the diagram that you are given in the problems. You know the angle of the slope with the ground is 35°, and you know the skiers travel down the slope for a distance of 325.6 m.

The triangle becomes your starting point for solving the question.

The question asks us to find the height of the ski slope, so we'll label that with a variable h.



Now that you have a labelled diagram, you look to see which sides you have or need to find, and select the appropriate ratio.

$\theta = 35°$

opp = h

adj = the ground

hyp = 325.6 m

This diagram has a side opposite the angle and the hypotenuse. Thinking of SOH CAH TOA, you see that the ratio involving the O and the H is the sine ratio.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 35° = \frac{h}{325.6}$$

$$325.6 \sin 35° = h$$

$$186.76 = h$$

The height of ski slope is 186.76 m above the ground.

Example 2

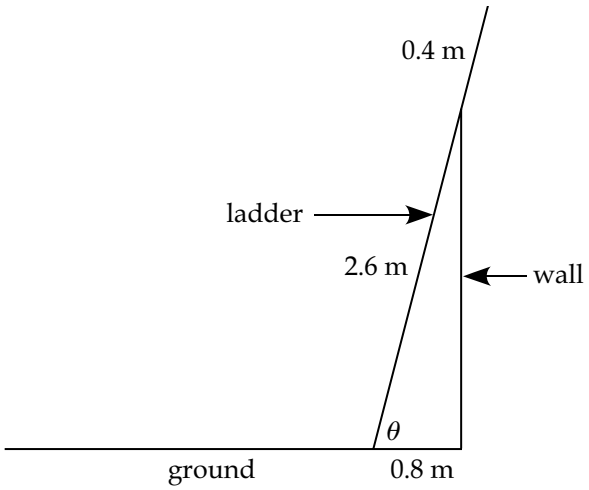
A 3 m ladder rests against the side of a garage. It overextends the top of the garage by 0.4 m. The bottom of the ladder is 0.8 m away from the wall.

a) Find the angle that the ladder forms with the ground.

b) Manitoba Workplace Safety and Health recommends a ladder have a minimum cosine ratio of 1 : 4. This means the ladder is to be 1 m away from the wall for every 4 metres of length of the ladder. Does the way the ladder is placed meet the safety standards?

Solution:

a) This question is a bit tricky. The ladder totals 3 m in length. If 0.4 metres of the ladder are above the garage, then 3 – 0.4 = 2.6 m would be the length of ladder from the ground to the top of the garage.



θ = ?

opp = the wall

adj = 0.8 m on the ground from the garage to the foot of the ladder

hyp = 2.6 m

The labelled diagram and the list show that you have an adjacent side and the hypotenuse. Thinking of SOH CAH TOA, the ratio that uses the Adjacent and the Hypotenuse is the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos\theta = \frac{0.8}{2.6}$$

$$\theta = \cos^{-1}\left(\frac{0.8}{2.6}\right) = 72.08°$$

The ladder meets the ground at a 72.08° angle.

b) To answer this part of the question, you could compare cosine values or you could compare angles.

Method 1: Compare cosines.

The cosine of the angle in this problem is $\frac{0.8}{2.6} = 0.31.$ The recommended value is 1:4 or 0.25. This ladder needs to be placed closer to the wall to make a bigger angle.

Method 2: Compare angles.

To find the recommended angle, you would solve the equation as follows.

$$\cos\theta = \frac{1}{4}$$

$$\theta = \cos^{-1}\left(\frac{1}{4}\right)$$

$$\theta = 75.5°$$

Since the ladder makes a angle of 72.08° with the ground, the angle is too small. The ladder needs to be adjusted to meet safety standards.

Is This Answer Reasonable?

As you progress with your skills in mathematics, you should be able to determine whether an answer is reasonable. This would be an application of using your estimating skills.

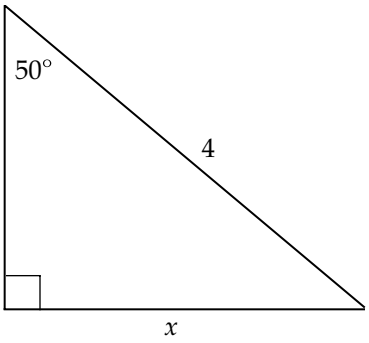
When working with right triangles, the hypotenuse is always the longest side because its square is the sum of the squares of the other two sides. Also, the hypotenuse is the biggest side of the triangle because it is opposite the biggest angle in the triangle. The other two angles in a right triangle are acute angles and are smaller than 90°, and the sides opposite them will be smaller than the hypotenuse.

If you encounter an answer that does not show this, then the answer is not reasonable. Also, if the sides are smaller numbers and you get a value for the long side to be a huge number, that would not be reasonable either.

Often, the wrong button is pressed on the calculator. Next time you buy something where a calculator was used to find the total bill, check your bill. Were the correct entries made for the cost of the items you purchased?

Or, if using a calculator new to you, you might enter the key strokes in the wrong order. You need to pause and ask yourself if the answer you find is reasonable.

Example 1



When asked to find the value of side x, you identify the sine ratio as the one to be used. You write the following answer.

$$\sin 50° = \frac{x}{4}$$

$$\sin 50° \times 4 = x$$

$$-0.34202 = x$$

How could this make sense?

Solution:

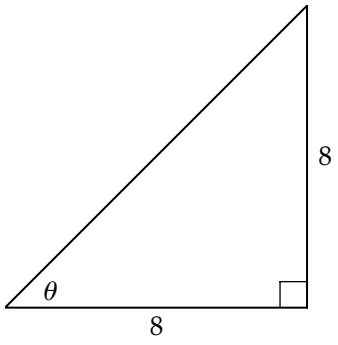
A side cannot be a negative value for its length. And since the hypotenuse is 4, the value for x must be less than 4.

This should indicate to you to look more closely at the calculation.

Perhaps if had you written it as 4 sin 50° instead of sin 50° × 4, then you might have found the correct value of 3.06. This answer would be more reasonable. It is a positive value less than 4.

Example 2

Find the angle θ.



Solution:

You identify the need to use the tangent ratio.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{8}{8}$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 90°$$

You know that a 90° angle does not look like θ in the diagram. Also, this triangle already has a 90° angle. This answer is not reasonable. What could be wrong?

Looking more closely, the $\sin^{-1}$ key was pressed instead of the $\tan^{-1}$ key. The correct answer would be 45°.

Learning Activity 7.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Each time you get paid, you save 30% of your paycheque. This week your paycheque was $400. How much will you save?
2. Solve for j: $\frac{5}{8} = \frac{j}{12}$.
3. When Brendan was born, his brother was 6. Brendan is now 15. How old is his brother?
4. The area taken up by a building is 3200 m^2. If the building is 40 m deep, how long is it?
5. The exchange rate from Canadian dollars to Thai baht is 1:37. If you have $1000 for your trip to Thailand, how many baht will you have?

continued

Learning Activity 7.9 (continued)

Part B: Applying Trigonometry Ratios

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Clearly show all your work as you solve these problems. To be a master problem solver, you could

- draw a right triangle
- label the triangle using the given information and the required value
- write the appropriate trigonometry ratio
- show your substitutions
- solve for the unknown value
- write a sentence answering the question
- think about whether your answer is reasonable

Round off your answers to two decimal places.

1. A kite is attached to a 50 m long string. The string is all out, and it meets the ground at a 48° angle. Find how high the kite is above the ground.
2. A ship is nearing the rocks. The captain can see the top of the cliff at an angle of 28° with the horizon. His charts tell him that the cliff is 175 m above sea level. How far is the ship from hitting the rocks?
3. A tree is 15.6 m tall and has fallen against your cottage, which is 7 m tall. Find the angle at which the tree meets the cottage.

Lesson Summary

This concludes your work in the trigonometry module. In this lesson, you have learned a process for presenting a solution to a trigonometric problem. You have practiced your skills in setting up and solving the problem. You checked for how reasonable your answer seems to you.

Assignment 7.7

Total: 20 marks

Problems

Clearly show all your work as you solve these problems. You need to include the following items as part of your solution.

- draw a right triangle
- label the triangle
- write the trigonometry ratio
- show your substitutions and steps for solving
- write a sentence answering the question
- write a sentence stating why you think your answer is reasonable, except for #6

Round off your answers to two decimal places.

1. A ramp is 10 m long and reaches 5 m up a wall. Find the angle at which the ramp meets the ground. (This angle between the horizontal and the hypotenuse is called the angle of inclination.) *(4 marks)*

continued

Assignment 7.7: Problems (continued)

2. A missile is fired from the ground to an enemy aircraft. The angle of inclination is 25°, and the aircraft is 5000′ above ground. How far must the missile travel to hit the aircraft? *(4 marks)*

continued

Assignment 7.7: Problems (continued)

3. You are building rafters for a roof on a shed. The height of the rafter will be 9′, and the diagonal beam will be 16′. Find the angle where the beam meets the top of the roof. *(3 marks)*

continued

Assignment 7.7: Problems (continued)

4. A staircase rises off the floor at an incline of 32°. It meets the wall at a height of 6 m. How far away from the wall is the foot of the staircase? *(3 marks)*

continued

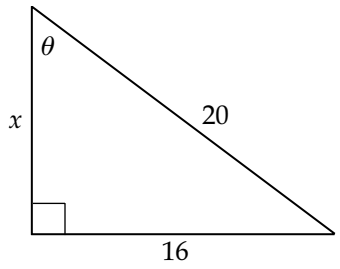
Assignment 7.7: Problems (continued)

5. A 45 m long wire is used to anchor a cell phone tower to the ground. The tower is 28 m tall. Find the angle at which the wire meets the tower. *(3 marks)*

continued

Assignment 7.7: Problems (continued)

6. You were asked to find θ in this triangle. The answer you found is 36.9°.



a) Is this answer reasonable? Explain your answer. Try to find an explanation that doesn't include redoing the question. *(2 marks)*

b) If the answer is not reasonable, redo the question and find the correct answer. *(1 mark)*

Module 7 Summary

Congratulations! You have finished seven modules in this course. And what a module it was! You have learned similar triangles and trigonometry, and you have applied your problem-solving skills to questions involving right triangles. Good work!

Vocabulary

Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination, but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful..

adjacent side
angle of inclination
corresponding sides
corresponding angles for similar shapes
cosine ratio
cosine inverse
correspondence for similar triangles
designated angle
diagonal
equivalent fractions
hypotenuse
opposite side
proportion
proportional sides
Pythagorean Theorem
ratio
scale factor
similar rectangles
similar triangles
sine ratio
sine inverse
tangent
tangent inverse
theta (θ)
trigonometry

Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

Sum of the angles in a triangle = 180°

SOH CAH TOA

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

Submitting Your Assignments

It is now time for you to submit the Module 7 Cover Assignment and Assignments 7.1 to 7.7 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 7 assignments and organize your material in the following order:

Module 7 Cover Sheet

Module 7 Cover Assignment: Drawing Similar Figures

Assignment 7.1: Ratio and Proportion

Assignment 7.2: Similarity

Assignment 7.3: Pythagoras and Right Triangles

Assignment 7.4: Tangent Ratio

Assignment 7.5: Sine Ratio

Assignment 7.6: Cosine Ratio, Sine Ratio, and Tangent Ratio

Assignment 7.7: Problems

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 10 Essential Mathematics (20S)

Module 7
Trigonometry

Learning Activity Answer Keys

MODULE 7:
TRIGONOMETRY

Learning Activity 7.1

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You saved $104 in one year by putting aside the same amount each week. How much did you save each week?
2. You know that 30×4 is 120. What is 60×4?
3. Convert 6 litres to millilitres.
4. Is $\frac{4}{5}+\frac{3}{7}$ greater or less than 1?
5. You started your math homework at 3:30 pm and finished at 4:25 pm. How many minutes did you spend on your math homework?

Answers:

1. $2 (There are 52 weeks in a year, so $104 $\div$ 52 = $2/week.)
2. 240 (Since 60 is double 30, $120 \times 2 = 240$.)
3. 6000 mL (There are 1000 mL in l L, so 6 L = 6000 mL.)
4. greater than 1 $\left(\frac{4}{5}+\frac{3}{7}=\frac{28}{35}+\frac{15}{35}=\frac{43}{35}\right.$, so the sum is greater than 1)
5. 55 minutes (3:30 to 4:30 is 1 hour or 60 minutes. 3:30 to 4:25 is 5 minutes less, so it is 60 – 5 = 55 minutes)

Part B: Proportions and Ratios

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing term in each of the following proportions.

   a) $\frac{3}{5} = \frac{x}{20}$

   b) $\frac{5}{7} = \frac{25}{x}$

   c) $\frac{30}{x} = \frac{18}{3}$

   Answers:

   a) By observation, you can see that the denominator of the second ratio is 4 times bigger than the denominator of the first ratio.

   $$\frac{3}{5} = \frac{x}{20}$$

   $$\frac{3 \times 4}{5 \times 4} = \frac{x}{20}$$

   $$\frac{12}{20} = \frac{x}{20}$$

   $$x = 12$$

   The bottoms of each ratio are the same. Thus, the tops must be the same.

   b) By observation, you can see that the numerator of the second ratio is 5 times bigger than the numerator of the first ratio.

   $$\frac{5 \times 5}{7 \times 5} = \frac{25}{x}$$

   $$\frac{25}{35} = \frac{25}{x}$$

   Since the numerators are the same, the denominators must be the same and x must be 35.

c) This proportion doesn't show an easy way to multiply to get the answer. Instead, you could use cross multiplication.

$$\frac{30}{x} = \frac{18}{3}$$
$$(18)(x) = (30)(3)$$
$$18x = 90$$
$$\frac{18x}{18} = \frac{90}{18}$$
$$x = 5$$

2. Find what percent is 3200 out of 5000.

Answer:

First, organize the information in the box.

	quantity	out of
actual	3200	5000
percent	x	100

Now, set up the proportion and solve for x.

$$\frac{3200}{x} = \frac{5000}{100}$$
$$(5000)(x) = (3200)(100)$$
$$5000x = 320\ 000$$
$$\frac{5000x}{5000} = \frac{320\ 000}{5000}$$
$$x = 64$$

Since x represents the numerator of the fraction whose denominator is 100, you can write $\frac{64}{100}$ or 64%.

Thus, 3200 out of 5000 is the same as 64%.

3. A flag has a length to width ratio of 2: 1. If the width is 30 cm, find the length.

Answer:

To organize the information, use a box.

	length	width
ratio	2	1
actual	x cm	30 cm

Now you set up the proportion and solve for x.

$$\frac{2}{x \text{ cm}} = \frac{1}{30 \text{ cm}}$$

$$(1)(x \text{ cm}) = (2)(30 \text{ cm})$$

$$x \text{ cm} = 60 \text{ cm}$$

The length of the flag is 60 cm.

Learning Activity 7.2

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Is the answer to –25 + 16 positive or negative?
2. A shirt is 20% off of its regular price of $40.00. How much will you pay for it on sale?
3. A 500 g package of chocolate chips is $6.00. How much do the chocolate chips cost per gram?
4. You work 4.5 hour shifts 4 times per week. You are paid $10 per hour. How much do you make per week?
5. Calculate: 78 + 12.

Answers:

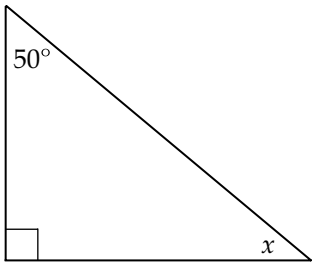
1. negative (Because 25 is larger than 16 and 25 is negative, the answer is negative.)
2. $32.00 (You will be paying 100 – 20 = 80% of the original price. 10% of $40 = 4, so 80% = $32.00.)
3. 1.2¢ ($6 = 600¢ so $600 \div 500 = \frac{6}{5}$¢ or 1.2¢.)
4. $180 (Your hours per week are 4.5 × 4 = 18 hours. $10 × 18 = $180 per week.)
5. 90

Part B: Similarity

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the measure of the missing angles in each triangle. Remember, the sum of all three angles in a triangle is 180°.

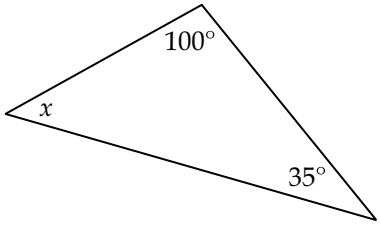
   a)



   Answer:

   $x = 180° - (50° + 90°) = 180° - 140° = 40°$

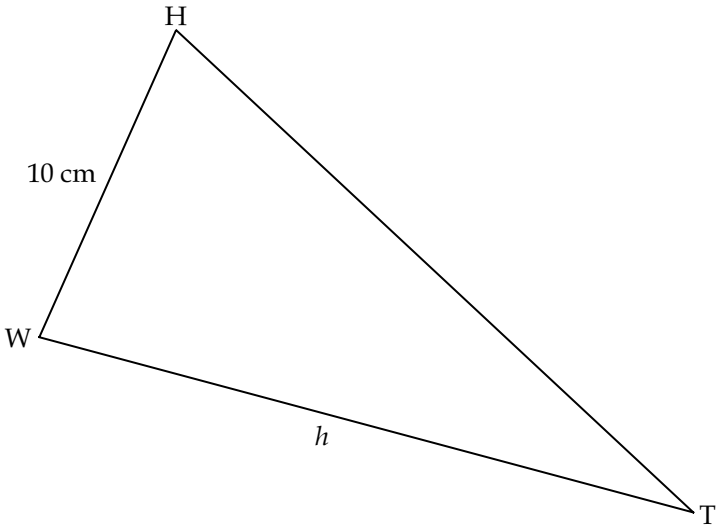
   b)

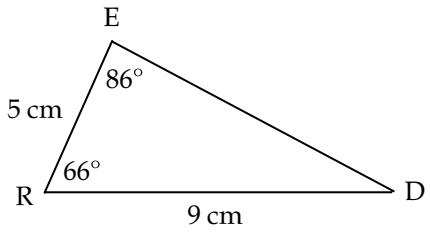


   Answer:

   $x = 180° - (100° + 35°) = 180° - 135° = 45°$

2. Given ΔRED ~ ΔWHT, find the length of h and the measure of ∠T.





Answer:

From the similarity relationship ΔRED ~ ΔWHT, the chart can be made.

	∠R = ∠W = 66°	∠E = ∠H = 86°	∠D = ∠T = ?
	sides opposite each angle	sides opposite each angle	sides opposite each angle
ΔRED	r = ?	e = 9 cm	d = 5 cm
ΔWHT	w = ?	h = ?	t = 10 cm

To find side "h," set up a proportion using the corresponding sides where one ratio is completely known.

$$\frac{e}{h}=\frac{d}{t}$$

Substitute the known values and solve for h.

$$\frac{9}{h} = \frac{5}{10}$$

$$5h = (9)(10)$$

$$5h = 90$$

$$\frac{5h}{5} = \frac{90}{5}$$

$$h = 18$$

The length of h is 18 cm.

To find the measure of $\angle T$, use the fact that the sum of the angles in a triangle = 180°.

In ΔWHT, you have the following information.

Since $\angle W$ corresponds to $\angle R$, and $\angle R = 66°$, then $\angle W = 66°$.

Since $\angle H$ corresponds to $\angle E$, and $\angle E = 86°$, then $\angle H = 86°$.

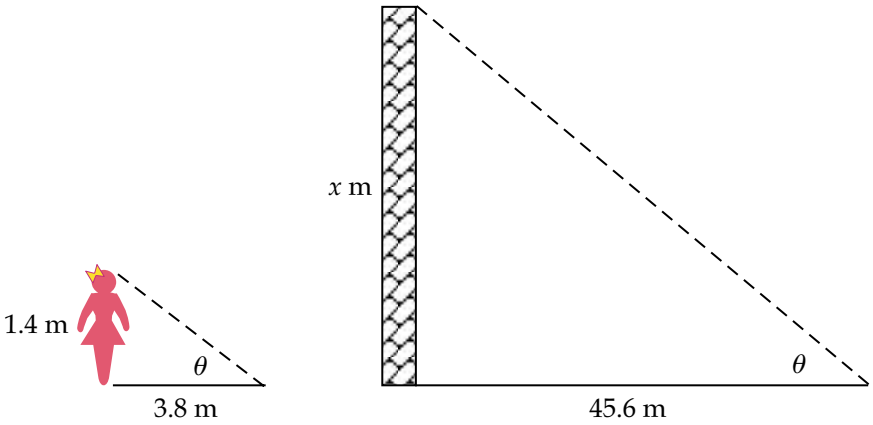
Now you can find $\angle T$.

$\angle T = 180° - (66° + 86°) = 180° - 152° = 28°$

The length of h is 18 cm. The measure of $\angle T$ is 28°.

3. A girl's height is 1.4 m. She casts a shadow of 3.8 m in the sun. She stands near a building that casts a shadow of 45.6 m. Use similar right triangles to find the height of the building.

Answer:



You can assume that both the girl and the flagpole make a right angle with the ground. That gives one set of equal angles. The other corresponding angles are the angles at the bottom of the triangles. They give the angle of the sun's rays and will be equal in both triangles. Thus, the third set of angles will be equal.

Now you can set up the proportions.

	90° angles	angle of the sun	third set
triangle with the girl		1.4 m	3.8 m
triangle with the building		x m	45.6 m

$$\frac{1.4\text{ m}}{x\text{ m}} = \frac{3.8\text{ m}}{45.6\text{ m}}$$

$$3.8x = (45.6)(1.4)$$

$$3.8x = 63.84$$

$$\frac{3.8x}{3.8} = \frac{63.84}{3.8}$$

$$x = 16.8$$

The height of the building is 16.8 m.

Learning Activity 7.3

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Two angles in a triangle are 30° and 65°. What is the third angle?
2. Calculate: $\frac{5}{3} \times \frac{4}{5}$.
3. Alana, Meghan, Jen, and Kristen are at a restaurant. Alana doesn't want to sit beside Meghan. Jen wants to sit across from Meghan. If there are two seats on both sides of the table, who is sitting beside Alana?
4. To pay for your movie ticket, you give the cashier $15. The cost of the ticket is $13.25. The cashier gives you back a loonie, a quarter and 3 dimes. Is this the correct change?
5. Is $\frac{8}{5}$ closer to 1 or 2?

Answers:

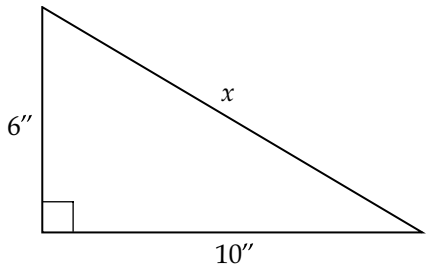
1. 85° (180° – (65° + 30°) = 85°)
2. $1\frac{1}{3}\left(\frac{5}{3} \times \frac{4}{5} = \frac{5 \times 4}{3 \times 5} = \frac{4}{3} \times \frac{5}{5} = \frac{4}{3} \times 1 = \frac{4}{3} = 1\frac{1}{3}\right)$
3. Jen (Because there are only two sides of the table being used, if Alana and Meghan aren't beside each other, then Meghan is on the other side of the table. Since Jen wants to sit across from Meghan, she must be sitting beside Alana.)
4. No (1 loonie = $1, 1 quarter = $0.25, 3 dimes = $0.30. Total = $1.55; you should have received $1.75.
5. 2 (It may be easier to tell in decimal form, so $\frac{8}{5} = 1\frac{3}{5} = 1\frac{6}{10} = 1.6$, which is closer to 2.)

Part B: Pythagorean Theorem

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing sides. Round off to one decimal place.

a)



Answer:

$a^2 + b^2 = c^2$

The hypotenuse is the unknown value of x.

$a^2 + b^2 = x^2$

The legs are 6″ and 10″.

$$6^2 + 10^2 = x^2$$

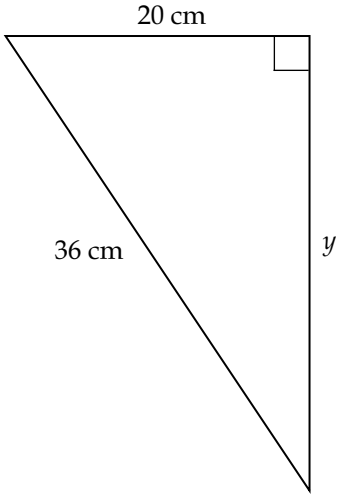
$$36 + 100 = x^2$$

$$136 = x^2$$

$$\sqrt{136} = \sqrt{x^2}$$

$$11.7'' = x$$

b)



Answer:

$a^2 + b^2 = c^2$

The hypotenuse is 36 cm.

$a^2 + b^2 = 36^2$

The legs are 20 cm and y cm.

$$y^2 + 20^2 = 36^2$$

$$y^2 + 400 = 1296$$

$$y^2 + 400 - 400 = 1296 - 400$$

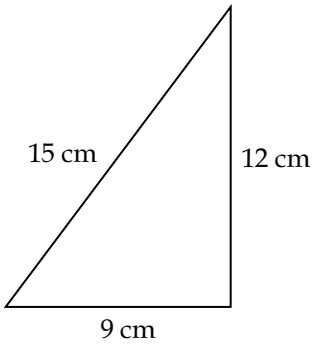
$$y^2 = 896$$

$$\sqrt{y^2} = \sqrt{896}$$

$$y = 29.9 \text{ cm}$$

2. Use the Pythagorean theorem to prove whether or not these are right triangles.

a)



Answer:

$a^2 + b^2 = c^2$

The longest side is 15 cm.

$a^2 + b^2 = 15^2$

The two legs are 9 cm and 12 cm.

$9^2 + 12^2 = 15^2$

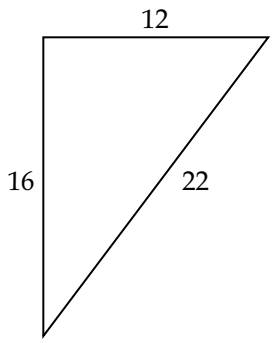
$81 + 144 = 225$

$225 = 225$

The left-hand side of the equation equals its right-hand side.

Therefore, it is a right triangle.

b)



Answer:

$a^2 + b^2 = c^2$

The longest side is 22 cm.

$a^2 + b^2 = 22^2$

The two legs are 12 cm and 16 cm.

$12^2 + 16^2 = 22^2$

$144 + 256 = 484$

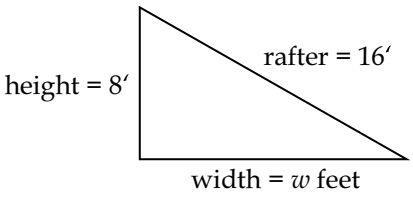
$400 \neq 484$

The sides of the equation are not equal.

Therefore, it is not a right triangle.

3. You are building a shed out of some scrap lumber. The longest piece of wood you have for a diagonal rafter is 16 feet. You want the height of the roof to be 8 feet up. Use the Pythagorean formula to find the maximum width from the centre beam to the edge. Round to one decimal place.

Answer:



$a^2 + b^2 = c^2$

The hypotenuse is 16′. The legs are 8′ and w feet.

$$w^2 + 8^2 = 16^2$$

$$w^2 + 64 = 256$$

$$w^2 + 64 - 64 = 256 - 64$$

$$w^2 = 192$$

$$\sqrt{w^2} = \sqrt{192}$$

$$w = 13.9'$$

The maximum width from the centre beam to the edge is 13.9 feet.

Learning Activity 7.4

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The angle between the two equal sides of an isosceles triangle is 80°. What are the values of the other two angles?
2. You can buy a 400 g package of chocolate eggs for $8.00, or you can pay $0.10 per gram. Which is the better buy?
3. 1 out of every 25 vehicles that you see on your way home from the cabin is a truck. Express this as a ratio.
4. Is the answer to –14 – 29 positive or negative?
5. Solve for a: $3a = \frac{9}{2}$.

Answers:

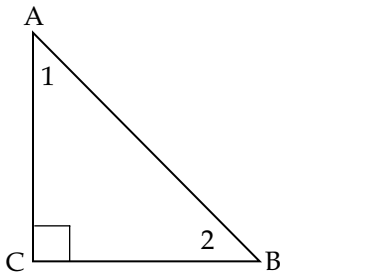
1. 50° (The angles formed between one of the equal sides and the base of the triangle are equal so 180° – 80° = 100° is the sum of the two angles. Each angle is 100 ÷ 2 = 50°.)
2. package of eggs ($8 = 800¢, $\frac{800}{400} = 2¢/\text{g}.$)
3. 1 : 25
4. negative (–14 is negative. Subtracting 29 will make the number more negative (move to the left on a number line), so the final answer will also be negative.)
5. $1\frac{1}{2}$ (Divide by 3 on both sides: $a = \frac{9}{2} \div 3 = \frac{9}{2} \times \frac{1}{3} = \frac{9}{2 \times 3} = \frac{9}{6}$.

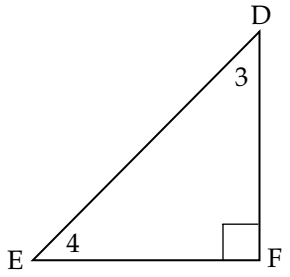
   Simplify the fraction: $\frac{9 \div 3}{6 \div 3} = \frac{3}{2}$.)

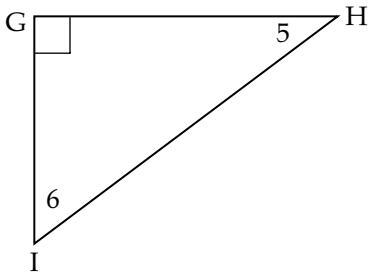
Part B: Opposite and Adjacent Sides

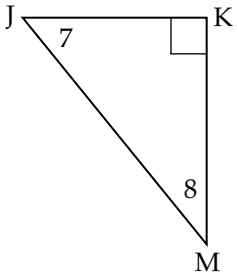
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Given the following right triangles, identify the sides as listed.









from ∠1	opposite:	*a*	from ∠2	opposite:	*b*
	adjacent:	*b*		adjacent:	*a*
	hypotenuse:	*c*		hypotenuse:	*c*
from ∠3	opposite:	*d*	from ∠4	opposite:	*e*
	adjacent:	*e*		adjacent:	*d*
	hypotenuse:	*f*		hypotenuse:	*f*
from ∠5	opposite:	*h*	from ∠6	opposite:	*i*
	adjacent:	*i*		adjacent:	*h*
	hypotenuse:	*g*		hypotenuse:	*g*
from ∠7	opposite:	*j*	from ∠8	opposite:	*m*
	adjacent:	*m*		adjacent:	*j*
	hypotenuse:	*k*		hypotenuse:	*k*

Learning Activity 7.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The side length of a square is 5 cm. What is the area of the square?
2. You work at 4:00, but you have some errands to do before your shift. Driving to the grocery store and buying food for the next week takes 50 minutes. Driving home and putting the groceries away takes 30 minutes. You then clean your bedroom, which takes 1 hr. 30 min. You want to eat before work, which takes you 45 minutes to prepare and eat. Your commute to work is 15 minutes long. If it is 1:00 right now, will you be able to do everything you want to before work?
3. Convert to a decimal: 0.25%.
4. Evaluate: $\frac{3}{4} \times \frac{5}{7}$.
5. Janelle works Monday to Friday, and is paid $80 per day. How much money does she make in two weeks?

Answers:

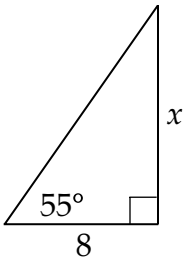
1. 25 cm^2 (area = length × width = 5 × 5 = 25)
2. No (Find out how long it will take you to do everything: 0:50 + 0:30 + 1:30 + 0:45 + 0:15 = 3:50. 1:00 + 3:50 = 4:50. You would not have enough time.)
3. 0.0025
4. $\frac{15}{28}$ $\left(\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}\right)$
5. $800 (In 1 week she works 5 days, so in two weeks she works 10 days. 10 × 80 = $800)

Part B: Tangent Ratio

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing side using the tangent ratio. Round off to two decimal places.

a)



Answer:

$$\theta = 55°$$

$$\text{opp} = x$$

$$\text{adj} = 8$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

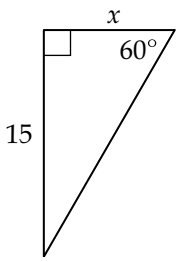
$$\tan 55° = \frac{x}{8}$$

$$8 \times \tan 55° = \frac{x(8)}{8}$$

$$8 \tan 55° = x$$

$$11.43 = x$$

b)



Answer:

$$\theta = 60°$$

$$\text{opp} = 15$$

$$\text{adj} = x$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 60° = \frac{15}{x}$$

$$x \tan 60° = \frac{15(x)}{x}$$

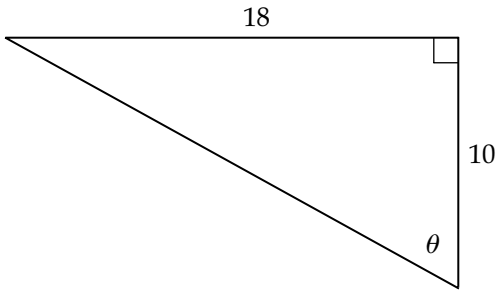
$$x \tan 60° = 15$$

$$\frac{x \tan 60°}{\tan 60°} = \frac{15}{\tan 60°}$$

$$x = 8.66$$

2. Find the value of θ using the tangent ratio and its inverse. Round off to two decimal places.

a)



Answer:

$\theta = ?$

$\text{opp} = 18$

$\text{adj} = 10$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

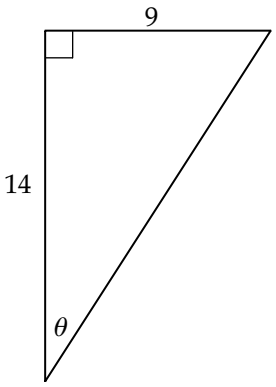
$\tan\theta = \dfrac{18}{10}$

$\theta = \tan^{-1}\left(\dfrac{18}{10}\right)$

$\theta = 60.95°$

Notice that you use the inverse tan to find the measure of the angle.

b)



Answer:

$\theta = ?$

$\text{opp} = 9$

$\text{adj} = 14$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan\theta = \dfrac{9}{14}$

$\theta = \tan^{-1}\left(\dfrac{9}{14}\right)$

$\theta = 32.74°$

Learning Activity 7.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A CFL end zone is 60 feet deep. The field is 65 yards wide. What is the area of the end zone in yards?
2. Is the answer to –16 + (–15) positive or negative?
3. You are buying a DVD player for $100. If the in-store warranty is 6.5% of the cost, how much will you pay for the warranty?
4. Write two equivalent fractions to $\frac{15}{9}$.
5. The scale of a diagram is 1 cm: 50 cm. If the diagram is 10 cm long, what is the length in reality?

Answers:

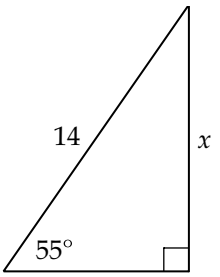
1. 1300 sq. yd. (Convert 60 feet to yards: 60 ÷ 3 = 20 yards.
   20 × 65 = 1300 sq. yd.)
2. negative (Although you are adding, the number being added is negative, so you move left on the number line.)
3. $6.50 (100 × 0.065)
4. $\frac{5}{3}, \frac{10}{6}$ (There are many answers, $\frac{15}{9} = \frac{15 \div 3}{9 \div 3} = \frac{5}{3}$; other examples are: $\frac{10}{6}, \frac{30}{18}, \frac{20}{12}, \frac{100}{60}$, etc.)
5. 500 cm or 5 m (50 × 10 = 500 cm or 5 m long)

Part B: Sine Ratio

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing side using the sine ratio.

a)



Answer:

$$\theta = 55°$$

$$\text{opp} = x$$

$$\text{hyp} = 14$$

$$\sin 55° = \frac{\text{opp}}{\text{hyp}}$$

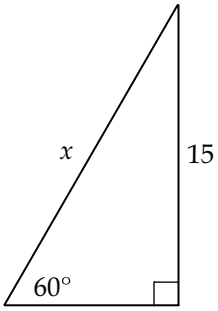
$$\sin 55° = \frac{x}{14}$$

$$14 \sin 55° = \frac{x(14)}{14}$$

$$14 \sin 55° = x$$

$$11.47 = x$$

b)



Answer:

$$\theta = 60°$$

$$\text{opp} = 15$$

$$\text{hyp} = x$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 60° = \frac{15}{x}$$

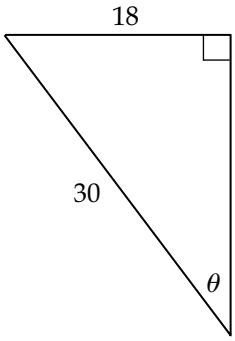
$$x \sin 60° = \frac{15(x)}{x}$$

$$\frac{x \sin 60°}{\sin 60°} = \frac{15}{\sin 60°}$$

$$x = 17.32$$

2. Find angle θ using the inverse sine.

a)



Answer:

$\theta = ?$

$\text{opp} = 18$

$\text{hyp} = 30$

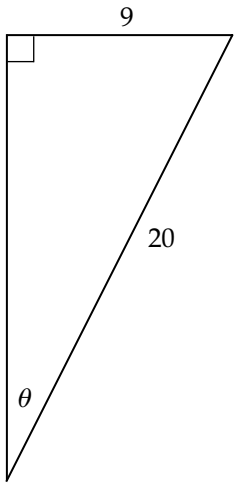
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

$\sin\theta = \dfrac{18}{30}$

$\theta = \sin^{-1}\left(\dfrac{18}{30}\right)$

$\theta = 36.87°$

b)



Answer:

$\theta = ?$

$\text{opp} = 9$

$\text{hyp} = 20$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

$\sin\theta = \dfrac{9}{20}$

$\theta = \sin^{-1}\left(\dfrac{9}{20}\right)$

$\theta = 26.74°$

Learning Activity 7.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. If two of the angles in a triangle are 45° and 105°, what is the third angle?
2. What is the value for $3x - 3$ if $x = 4$?
3. Is the answer to –5 – (–12) positive or negative?
4. It is really hot outside so you are very thirsty. In your fridge, there is orange juice with pulp, apple juice, water, and milk. You are lactose-intolerant. You do not like water, nor do you like pulp. What will you drink?
5. Convert 34 km into cm.

Answers:

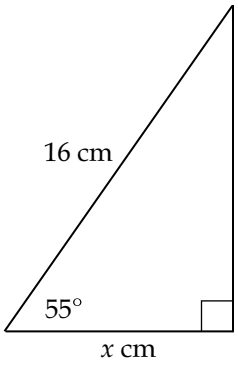
1. 30° (180° – (45° + 105°) = 30°)
2. 9 (3(4) – 3 = 12 – 3 = 9)
3. positive (Although you are subtracting 12 (you would expect to move left on the number line), the 12 is negative so you move the opposite direction (right on the number line). Since 12 is larger than 5, you will move into the positive numbers.)
4. apple juice (~~OJ (pulp)~~ AJ ~~water~~ ~~milk~~)
5. 3 400 000 cm (34 km = 34000 m = 3 400 000 cm)

Part B: Cosine Ratio

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the missing side using the cosine ratio. Round off to two decimal places.

a)



Answer:

$$\theta = 55°$$

$$\text{adj} = x \text{ cm}$$

$$\text{hyp} = 16 \text{ cm}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 55° = \frac{x}{16}$$

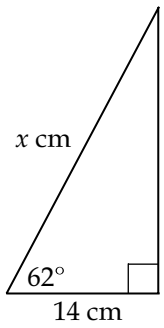
$$16 \cos 55° = \frac{(x)(16)}{16}$$

$$16 \cos 55° = x$$

$$9.18 = x$$

The value of x is 9.18 cm.

b)



Answer:

$$\theta = 62°$$

$$\text{adj} = 14 \text{ cm}$$

$$\text{hyp} = x \text{ cm}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 62° = \frac{14}{x}$$

$$x \cos 62° = \frac{14(x)}{x}$$

$$x \cos 62° = 14$$

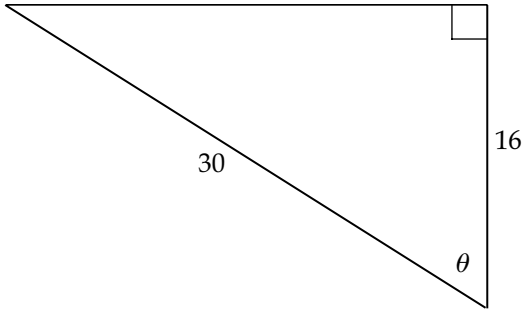
$$\frac{x \cos 62°}{\cos 62°} = \frac{14}{\cos 62°}$$

$$29.82 = x$$

The hypotenuse is 29.82 cm long.

2. Find angle θ using the cosine ratio and its inverse.

a)



Answer:

$\theta = ?$

$\text{adj} = 16$

$\text{hyp} = 30$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

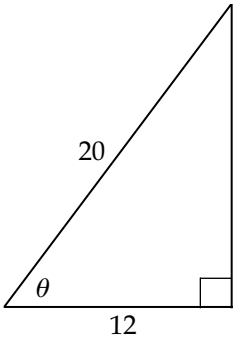
$\cos\theta = \dfrac{16}{30}$

$\theta = \cos^{-1}\left(\dfrac{16}{30}\right)$

$\theta = 57.77°$

The angle θ has a measure of 57.77°.

b)



Answer:

$\theta = ?$

$\text{adj} = 12$

$\text{hyp} = 20$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\cos\theta = \dfrac{12}{20}$

$\theta = \cos^{-1}\left(\dfrac{12}{20}\right)$

$\theta = 53.13°$

The angle θ has a measure of 53.13°.

Learning Activity 7.8

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. It costs $4 for a bag of mini-donuts at a football game. It costs $5 for a hotdog and $3 for a bottle of pop. If you buy one bag of mini-donuts, 3 hotdogs, and 2 bottles of pop, how much will you spend on food at the game?
2. Evaluate: $\frac{5}{6} - \frac{1}{3}$.
3. The three sides of a right angle triangle are 6.5, 9.7, 7.2. Which is the hypotenuse?
4. Is the answer to 10 – 14 positive or negative?
5. You are supposed to work from 9 am to 5 pm. You arrive at work at 8:55 am, and leave at 4:50 pm. How much time does your work pay you for?

Answers:

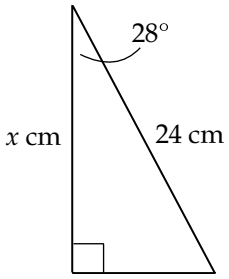
1. $25 (4 + (3 × 5) + (2 × 3) = 4 + 15 + 6 = $25)
2. $\frac{1}{2}\left(\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}\right)$
3. 9.7 (The hypotenuse is always the longest.)
4. negative (Since you are subtracting you move left on the number line. Since 14 is larger than 10, you will pass zero and become negative.)
5. $7\frac{3}{4}$ hours (Arriving early does not count toward your daily hours. Leaving 10 minutes early means you are docked 15 minutes pay, based on the 15 minute rule.)

Part B: Trigonometry Ratios (all 3)

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use one of the three primary trigonometry ratios to solve for each unknown. Round off to two decimal places.

a)



Answer:

$\theta = 28°$
opposite = ?
adjacent = x cm
hypotenuse = 24 cm

Since you have adjacent and hypotenuse, then SOH CAH TOA says, use cosine.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

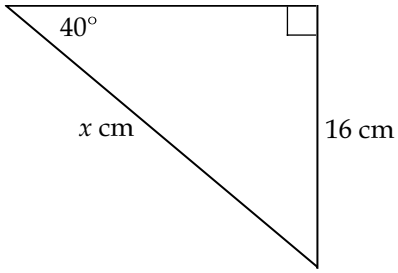
$$\cos 28° = \frac{x}{24}$$

$$24\cos 28° = \frac{(x)(24)}{24}$$

$$24\cos 28° = x$$

$$21.19 = x$$

The value of x is 21.19 cm.

b) 

Answer:

$\theta = 40°$
opposite = 16
adjacent = ?
hypotenuse = x cm

Since you have opposite and hypotenuse, then SOH CAH TOA says use sine.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 40° = \frac{16}{x}$$

$$x \sin 40° = \frac{16(x)}{x}$$

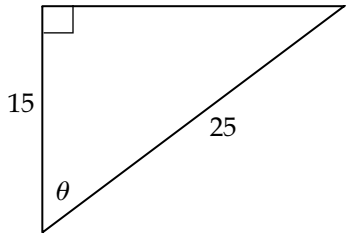
$$x \sin 40° = 16$$

$$\frac{x \sin 40°}{\sin 40°} = \frac{16}{\sin 40°}$$

$$x = 24.89$$

The hypotenuse is 24.89 cm long.

c)



Answer:

θ = ?
opposite = ?
adjacent = 15
hypotenuse = 25

Since you are given the values for adjacent and hypotenuse, then upon checking SOH CAH TOA, you realize you can use the cosine ratio.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

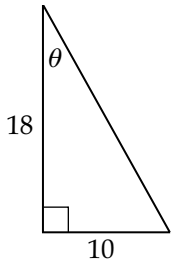
$$\cos\theta = \frac{15}{25}$$

$$\theta = \cos^{-1}\left(\frac{15}{25}\right)$$

$$\theta = 53.13°$$

The angle θ has a measure of 53.13°.

d)



Answer:

θ = ?
opposite = 10
adjacent = 18
hypotenuse = ?

Since you are given the values for opposite and adjacent, then upon checking SOH CAH TOA, you realize you can use the tangent ratio.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{10}{18}$$

$$\theta = \tan^{-1}\left(\frac{10}{18}\right)$$

$$\theta = 29.05°$$

The angle θ has a measure of 29.05°.

Learning Activity 7.9

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Each time you get paid, you save 30% of your paycheque. This week your paycheque was $400. How much will you save?
2. Solve for j: $\frac{5}{8} = \frac{j}{12}$.
3. When Brendan was born, his brother was 6. Brendan is now 15. How old is his brother?
4. The area taken up by a building is 3200 m^2. If the building is 40 m deep, how long is it?
5. The exchange rate from Canadian dollars to Thai baht is 1:37. If you have $1000 for your trip to Thailand, how many baht will you have?

Answers:

1. $120 (10% of $400 = 40 so 30% is (3 × 40) $120 saved.)
2. 7.5 (Multiply both sides by 12, then simplify the fraction:

$$j = \frac{5 \times 12}{8} = \frac{60 \div 4}{8 \div 4} = \frac{15}{2} = 7.5.\Big)$$

3. 21 (15 + 6)
4. 80 m (3200 ÷ 40)
5. 37000 baht (1000 × 37)

Part B: Applying Trigonometry Ratios

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

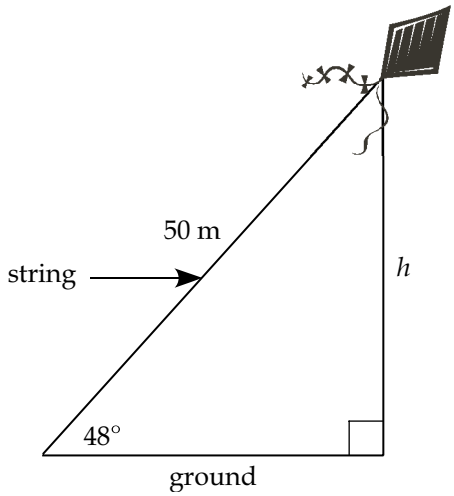
Clearly show all your work as you solve these problems. To be a master problem solver, you could

- draw a right triangle
- label the triangle using the given information and the required value
- write the appropriate trigonometry ratio
- show your substitutions
- solve for the unknown value
- write a sentence answering the question
- think about whether your answer is reasonable

Round off your answers to two decimal places.

1. A kite is attached to a 50 m long string. The string is all out, and it meets the ground at a 48° angle. Find how high the kite is above the ground.

   Answer:



   $\theta = 48°$

   opp = height of the kite = ?

   adj = the ground

   hyp = 50 m

   Using SOH CAH TOA, the ratio you need is one involving opposite and hypotenuse. You choose the sine ratio.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 48° = \frac{h}{50}$$

$$50 \sin 48° = h$$

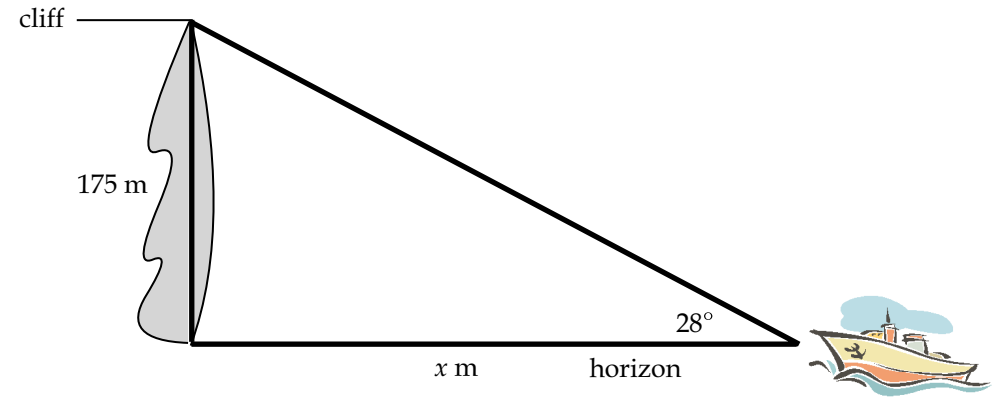
$$37.16 = h$$

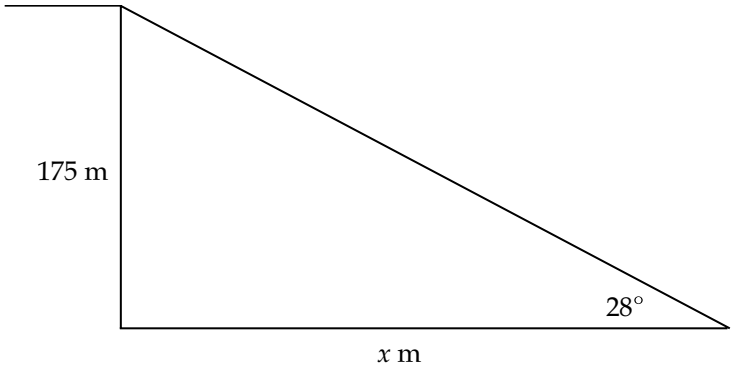
   The kite is 37.16 m above the ground.

   This answer seems reasonable since the value is close to 50 m and less than the value for the hypotenuse.

2. A ship is nearing the rocks. The captain can see the top of the cliff at an angle of 28° with the horizon. His charts tell him that the cliff is 175 m above sea level. How far is the ship from hitting the rocks?

Answer:





$\theta = 28°$

opp = 175 m

adj = x m

hyp = ?

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 28° = \frac{175}{x}$$

$$x = \frac{175}{\tan 28°}$$

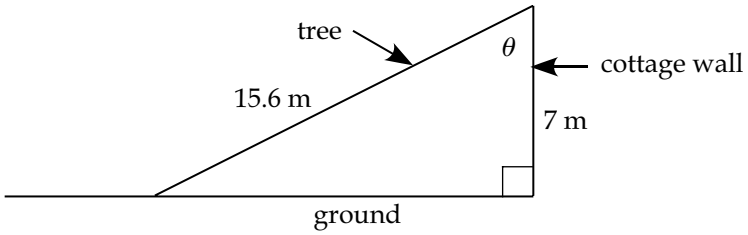
$$x = 329.13$$

The ship is 329.13 m away from the rocks.

The answer seems reasonable since the angle is quite small, only 28°, so the horizontal distance would be much longer than the height of the triangle.

3. A tree is 15.6 m tall and has fallen against your cottage, which is 7 m tall. Find the angle at which the tree meets the cottage.

Answer:



θ = the angle between the tree and the cottage = ?

opp = the ground

adj = the cottage wall = 7 m

hyp = the tree = 15.6 m

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos\theta = \frac{7}{15.6}$$

$$\theta = \cos^{-1}\left(\frac{7}{15.6}\right) = 63.34°$$

The tree meets the cottage at an angle of 63.34°.

To check if the answer is reasonable, you might just redo the question, paying particular notice to the ratio and your calculations.

Grade 10 Essential Mathematics (20S)

Module 8
Transformations

Module 8:
Transformations

Introduction

A transformation involves movement of a shape from one position to another, usually on a page of paper. You will study 2-dimensional shapes as you slide, rotate, and reflect them while maintaining the exact size of the shape. This module also deals with dilating shapes, where the shape becomes larger or smaller while the measurements remain proportional and the shapes similar. You will apply your knowledge of transformations to various designs and tessellations.

Assignments in Module 8

When you have completed the assignments for Module 8, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Nested Shopping Carts
1	Assignment 8.1	Translations
2	Assignment 8.2	Rotations
3	Assignment 8.3	Reflections
4	Assignment 8.4	Combinations
5	Assignment 8.5	Dilations
6	Assignment 8.6	Applications

Resource Sheet

When you write your final examination you will be allowed to bring a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Final Examination Resource Sheet is not worth any marks.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. You are asked to complete a resource sheet for each module to help with your studying and reviewing. The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module.

In an attempt to prepare yourself for making such a sheet, a list of instructions is provided below for you to complete as you work through Module 8. You might use your Module 8 resource sheet for mathematics terms, formulas, sample questions, or a list of places where you often make mistakes. You might write out what you need or you might refer to page numbers in the lessons to be especially reviewed when studying for the examination.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from Modules 5, 6, 7, and 8, to prepare your Final Examination Resource Sheet. Remember, the final examination is based on the last four modules of the course.

Resource Sheet for Module 8

1. List the math terms that are introduced in each lesson.
2. List any formulas stated in each lesson.
3. What strategies for making calculations were discussed in each lesson?
4. What questions need to be copied onto your resource sheet as being representative of the questions in each lesson?
5. What questions were the most difficult? List page numbers on your module resource sheet so that you can redo these questions before the examination. If any of these problems are "sticklers," you could then write the problems and solutions on your Final Examination Resource Sheet so that you have them with you during the examination.
6. What other reminders do you need to make to yourself to help you prepare for the examination?

Writing Your Final Examination

You will write the final examination when you have completed Module 8 of this course. The final examination is based on Modules 5 to 8, and is worth 12.5 percent of your final mark in the course. To do well on the final examination, you should review all the work you complete in Modules 5 to 8, including all the learning activities and assignments. You will write the final examination under supervision.

Notes

Module 8 Cover Assignment

Instructions for Cover Assignment

Students are to do all the work in the spaces provided. The cover assignment can be done at any time while you are completing Module 8. However, when you are finished the assignment, you are to send it to the Distance Learning Unit along with the other assignments for this module.

Your evaluation for the assignment is based on whether or not you found a solution, and whether or not your tutor/marker can read your diagrams, or not. The cover assignment is worth a total of 12 marks.

Notes

Module 8 Cover Assignment

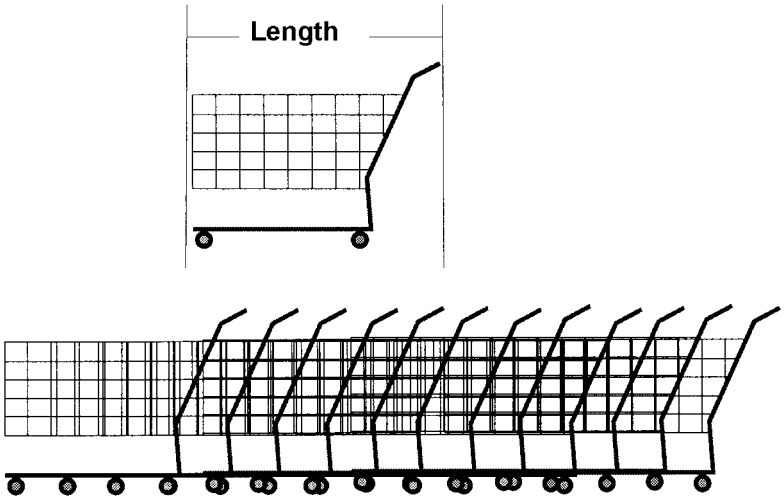
Total: 12 marks

Nested Shopping Carts

In this task, you are asked to analyze a real situation involving shopping carts, construct a simple function that models this situation, and use the formula to answer questions about the situation.

The diagram below shows a drawing of a single shopping cart. It also shows a drawing of 12 shopping carts that have been nested together.

The drawings are $\frac{1}{24}$ of the real size.



1. What is the length in centimetres of 1 full-size shopping cart? *(2 marks)*

continued

Module 8 Cover Assignment: Nested Shopping Carts (continued)

2. a) When they are "nested," by how much distance does each shopping cart stick out beyond the next one in the line? *(2 marks)*

   b) Show in a rough sketch of nested carts to what this distance refers. *(2 marks)*

3. What would be the total length of a row of 20 nested carts? *(3 marks)*

4. How many nested carts could fit in a space 10 metres long? *(3 marks)*

Lesson 1: Translations

Lesson Focus

In this lesson, you will

- ❑ identify the translation that was performed
- ❑ draw the image of a shape resulting from a given translation
- ❑ create, analyze, and describe designs using translations

Lesson Introduction

This lesson investigates the properties of translations. A translation moves the shape from its original position on a page to a new position by a "slide" to the new location, keeping its configuration by not rotating it. The slide can be horizontal or vertical. There are three notations for showing the translation.

Add the description of a translation to your resource sheet. This will help you to remember what each type of transformation looks like.

What is a Translation?

Any flat object that has no height, like a shape drawn on paper, has two dimensions—length and width. You call such objects 2-dimensional shapes or 2-D shapes for short.

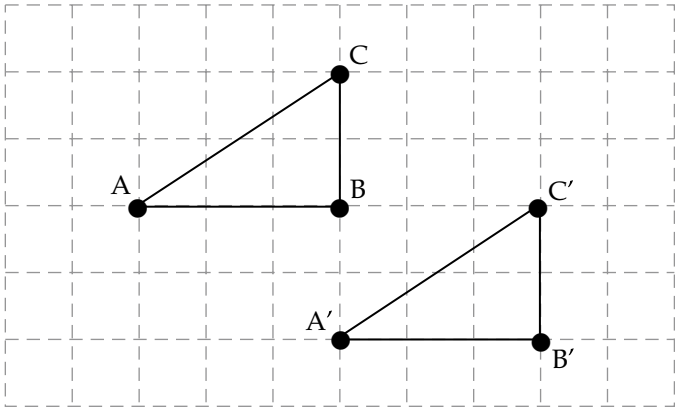
When these shapes are moved to somewhere else on the page, an "image" of the original shape is created. If the shape has not been rotated in any way, its image will have exactly the same configuration as the original shape. The only difference will be its position on the page.

A translation is one of three transformations that keep the size of the image the same as the original size. To translate a shape, you "slide" it somewhere on the page.

Example 1

What translation has been performed on the original triangle, ΔABC to get its image, ΔA′B′C′?

Note: A′ is read as A prime.



Solution:

Notice the triangle shape, ΔABC. Then notice the image of the shape, ΔA′B′C′ after it has been translated, or moved.

Focus on one of the points, for example, point A. Now locate where its image, A′ is placed. Notice that A has been moved to the right 3 units, and down 2 units to the point, A′.

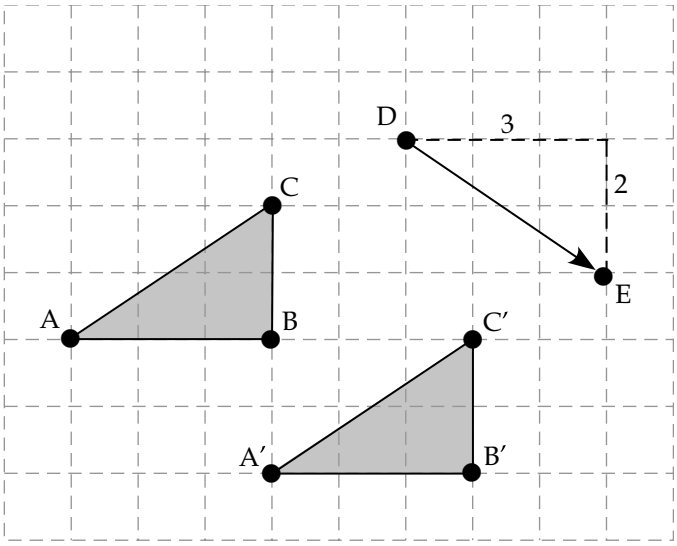
If you focus on point B or point C you will find each of them has been moved in the same direction. This would make sense, since the points are all attached as sides of the triangle. If one point moves, they all move.

This shape, ΔABC was translated 3 units to the right, and 2 units down.

Instructions

The above example gives written instructions, using words to indicate where the shape is to be translated.

Another method of giving directions is by using a visual instruction. An arrow or a vector indicates the direction of the change and the overall change. The arrow is written on the diagram.

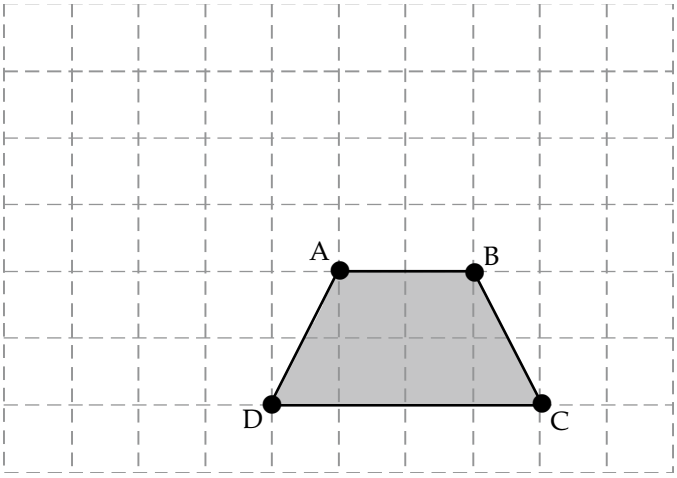


Notice how the arrow pointing from D to E represents movement from point D to point E, as 3 units to the right and 2 units down.

A third method of showing the instructions for the translation is the rule method. Sliding the shape to the right 3 units and down 2 units would be written in rule format as [R3, D2].

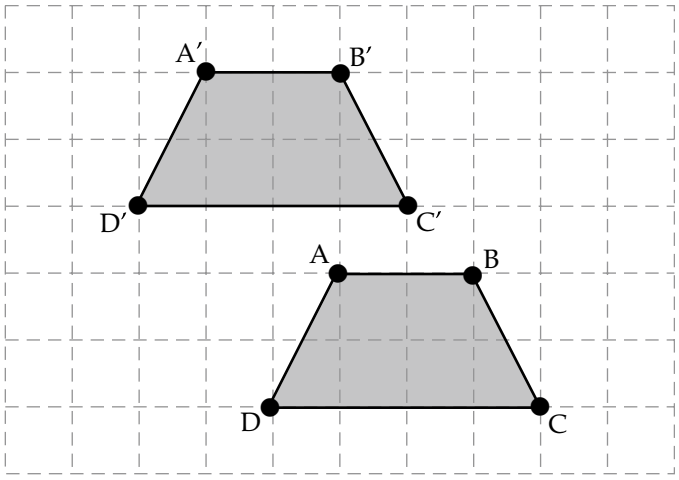
Example 2

Illustrate the translation of the following shape using the rule [L2, U3].



Solution:

Pick any point on the shape and slide it according to the instruction, [L2, U3]. This means left 2 units and up 3 units. The other points on the shape have exactly the same shift in position. The image of the trapezoid, ABCD is the trapezoid, A'B'C'D'. The image has exactly the same size and is in a different position than the original. Notice that you would want to name the image correctly.



Learning Activity 8.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the formula for tangent?
2. The sides of a right triangle are 85, 36, 77. What are the two possible tangent ratios of this triangle?
3. A circle is divided into 360°. How many degrees are in a semicircle (half-circle)?
4. You are paid $15 per hour. If you work 20 hours per week, how much money will you earn in a week?

continued

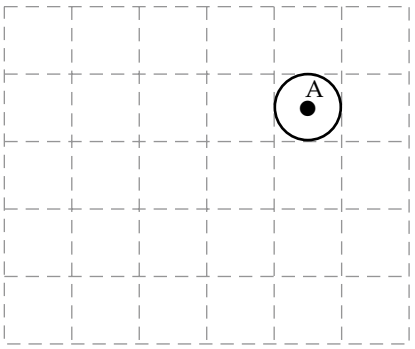
Learning Activity 8.1 (continued)

5. You would like to get your poster framed. The area of the poster is 4500 cm^2. If the height of the poster is 0.3 m, what is the length in centimetres?

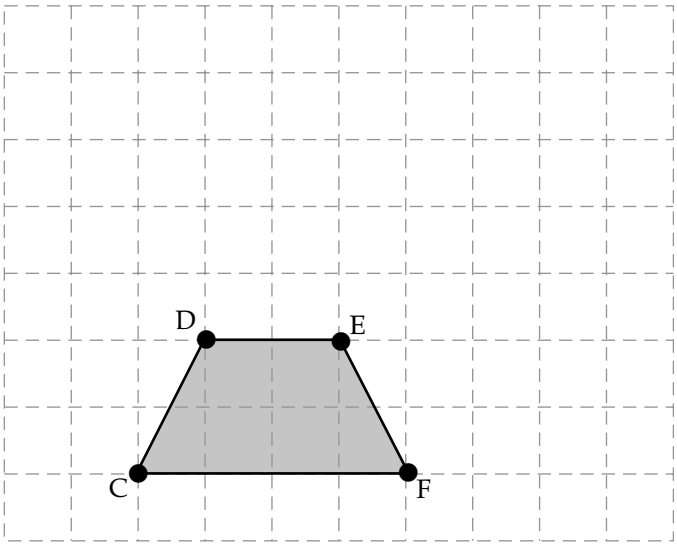
Part B: Single Translations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Draw the translations as instructed.
   a) Slide the circle with centre at A, 3 units left and 2 units down.



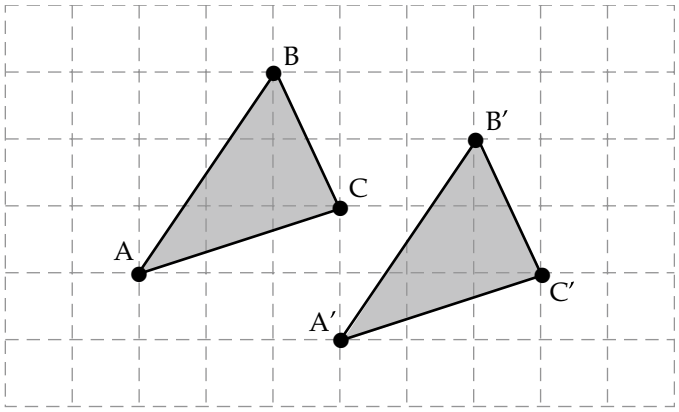
   b) Slide the trapezoid using the rule [R2, U4].



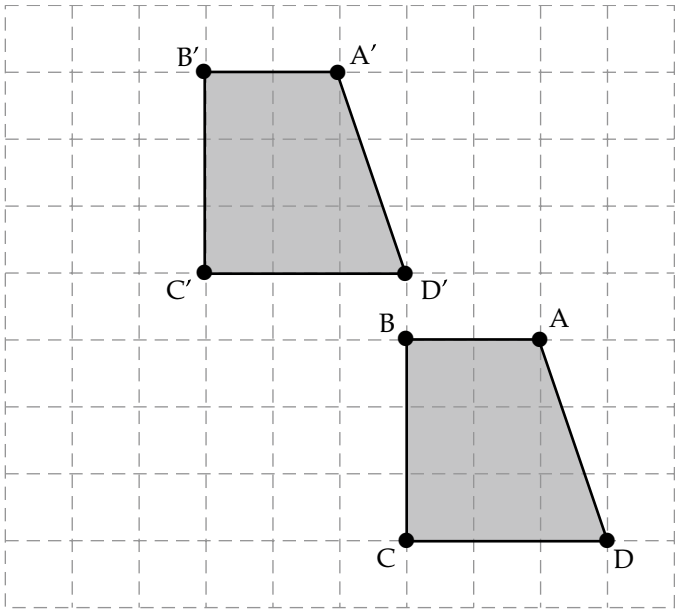
continued

Learning Activity 8.1 (continued)

2. Given the translated shape, identify the translation
   a) by writing the instructions in words.



   b) by writing the rule.

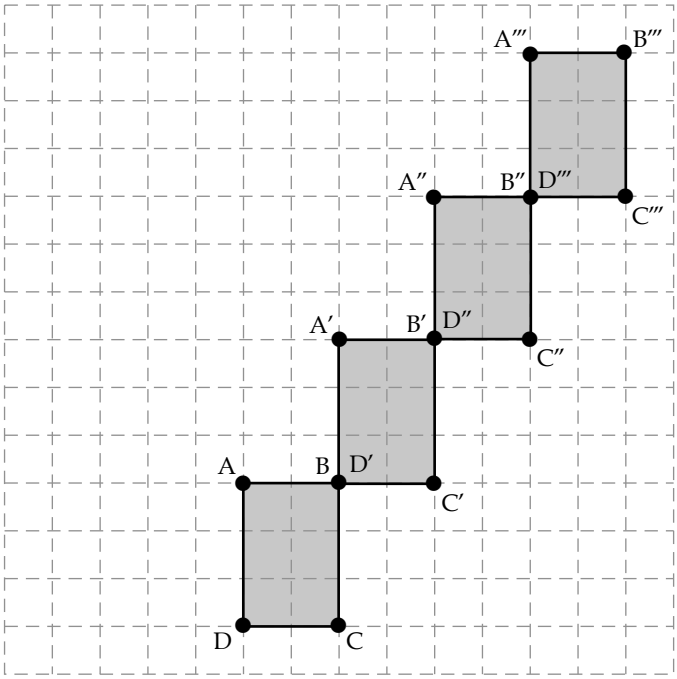


Creating Shapes

You can use translations to create shapes.

Example 1

Identify the translation used to create this shape. The original shape is on the bottom left.

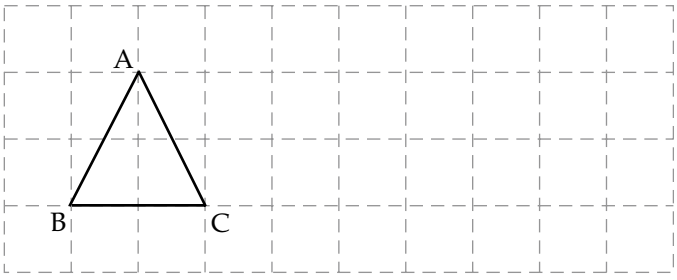


Solution:

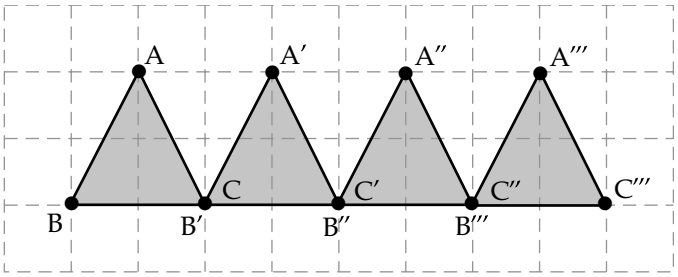
Each time the shape is redrawn, it has a translation of 2 units to the right and 3 units up. The slide is repeated 3 times altogether. In other words, there are three iterations of the translation.

Example 2

Draw the shape on the left three more times, using the rule [R2, D0]



Solution:



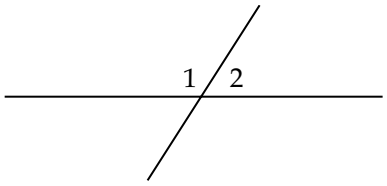
Learning Activity 8.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are buying a t-shirt for $14.83. If you pay with a $20 bill, how much change will you receive?
2. A right triangle has the sides 8, 15, 17. What are the two possible sine ratios?
3. Evaluate: 90×3.
4. Your paycheque was $500. Estimate your CPP (4.95%).
5. Consecutive angles (beside each other) of a transversal are equal, complementary, or supplementary.



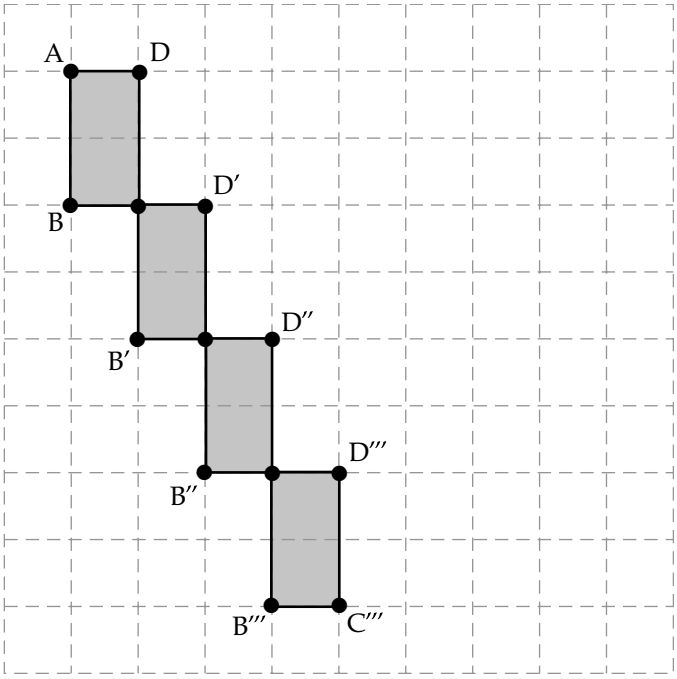
continued

Learning Activity 8.2 (continued)

Part B: Multiple Translations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the following succession of translations, identify the pattern used.



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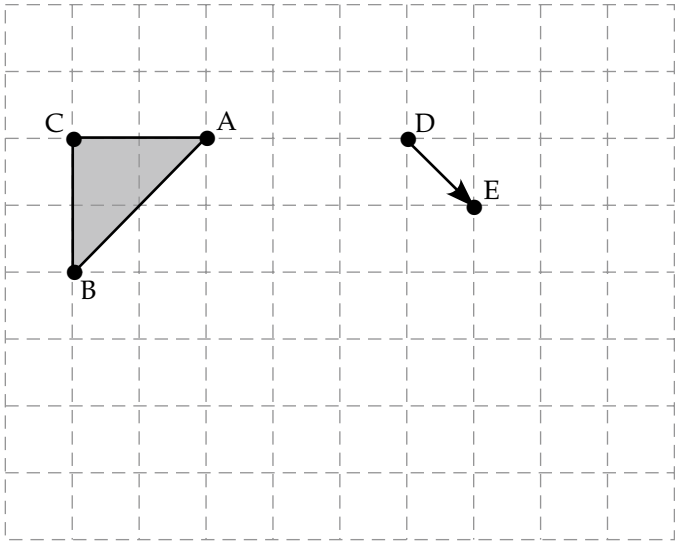
Learning Activity 8.2 (continued)

2. Given the shape and the instruction in a vector, draw 3 iterations (3 images) of the original shape.



Lesson Summary

This lesson introduced you to translations, the first of four basic transformations. You learned three ways to state the instructions for the translation of a shape from its original position to its image. You practised making translations using graph paper. You learned how to name the image using the same letters as on the original. You learned how to make several iterations of a shape using one set of instructions.

The next lesson discusses the second transformation, a rotations of shapes.

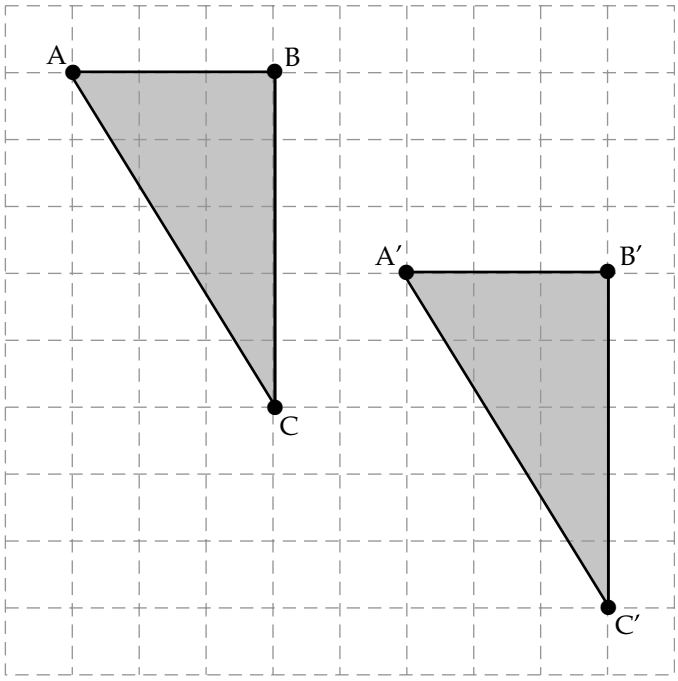
Assignment 8.1

Total: 22 marks

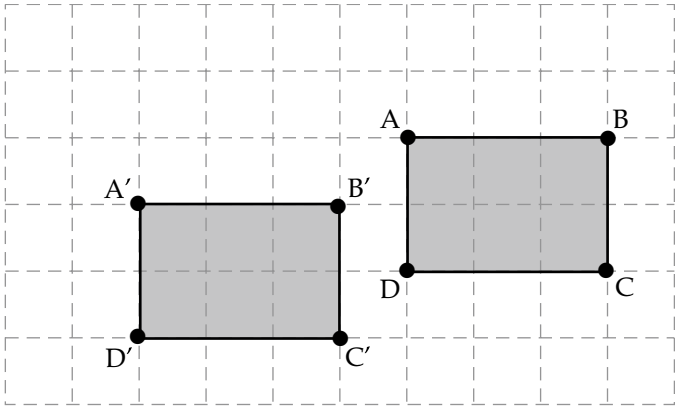
Translations

1. Identify the pattern used in the following translations, either in words or as a rule. *(6 marks)*

a)



b)



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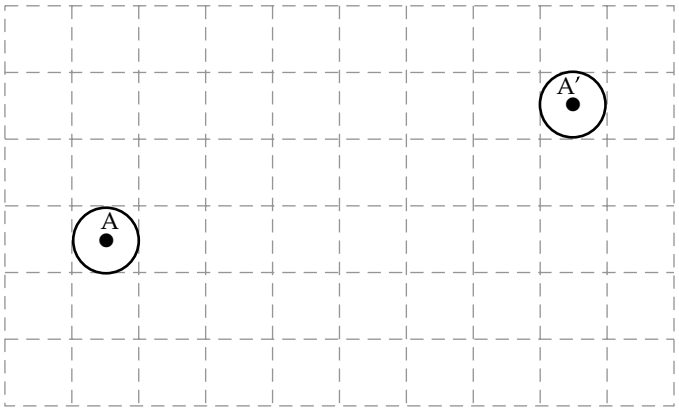
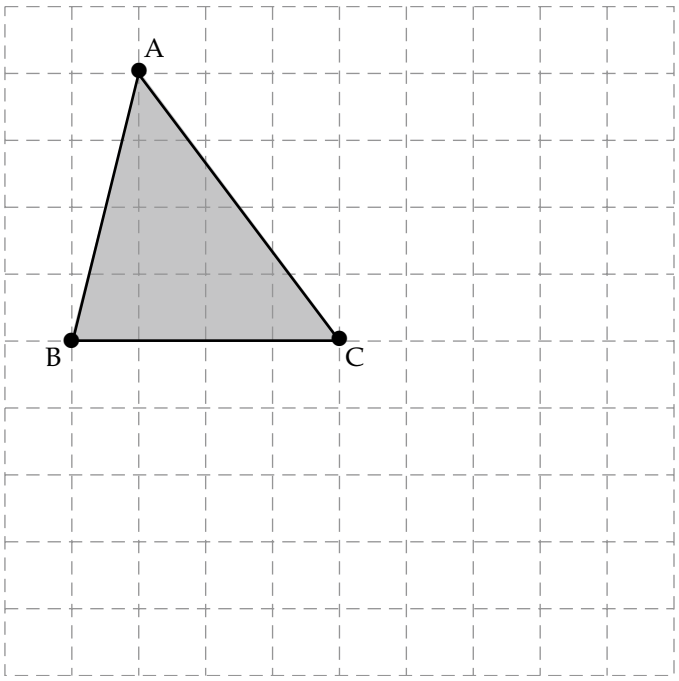
Assignment 8.1: Translations (continued)

c)



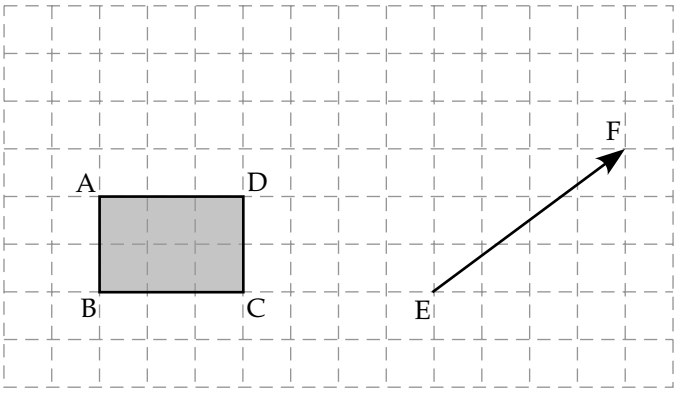
2. Given the shape and the translation instructions, draw the translation. *(6 marks)*

   a) [R4, D3]

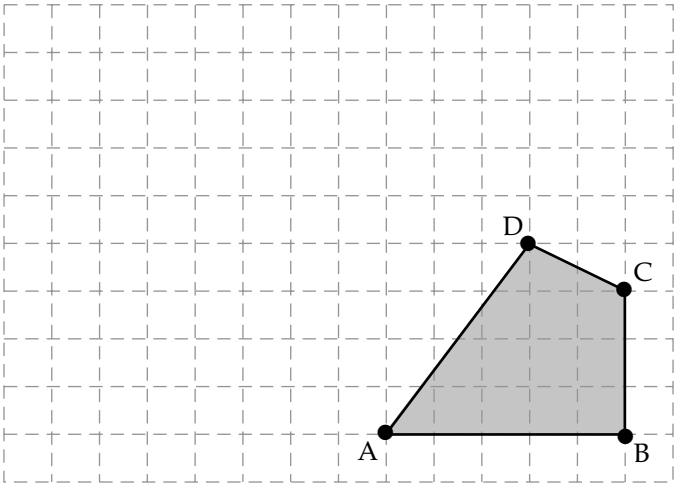


continued

b)

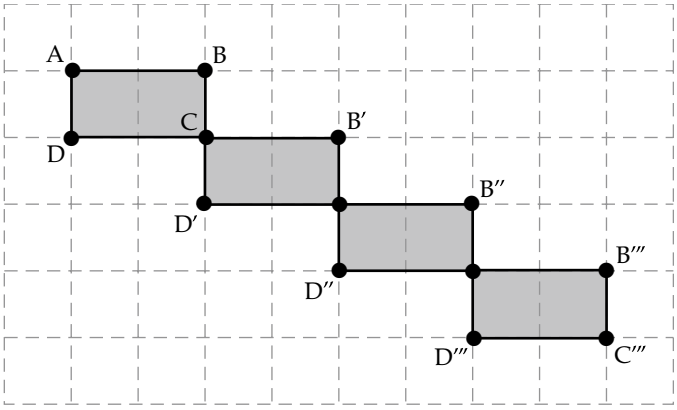


c) 5 units to the left and 3 units up



3. Given the following designs, identify the translation rules that apply. *(4 marks)*

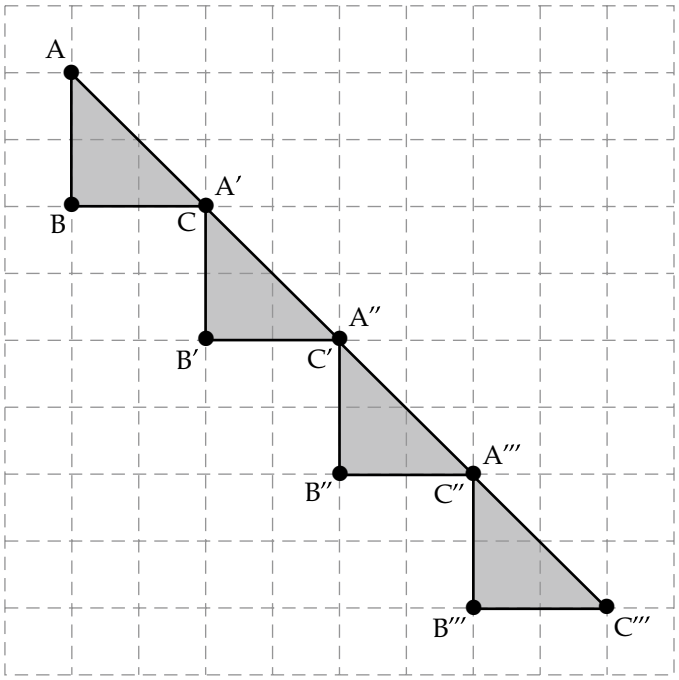
a)



continued

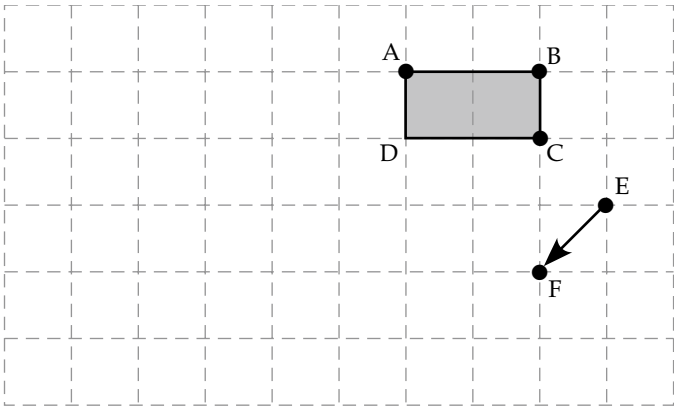
Assignment 8.1: Translations (continued)

b)



4. Given the shape and the translation rule, draw 3 iterations of each on the graph paper provided. *(6 marks)*

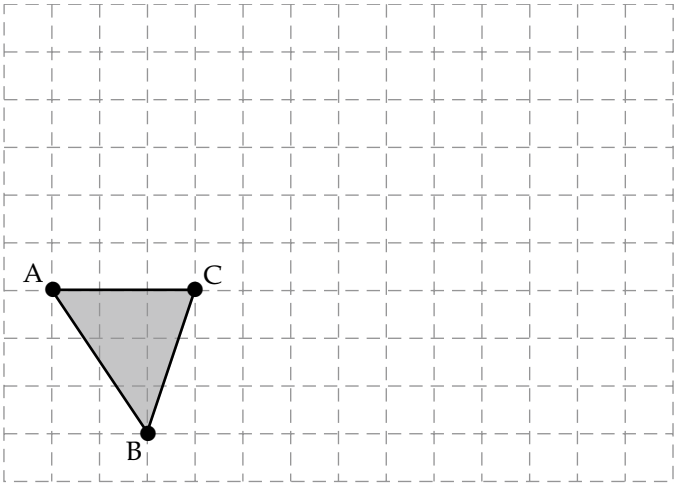
a)



continued

Assignment 8.1: Translations (continued)

b) 3 units to the right, 1 unit up



Notes

Lesson 2: Rotations

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of rotations

Lesson Introduction

A rotation involves a turn. It can be about the centre of the object, about a point on the object, or about a point outside the object. This lesson examines various rotations of shapes.

What is a Rotation?

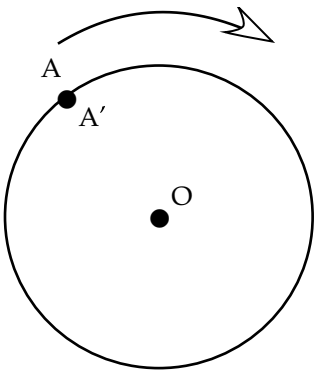
A rotation transformation moves a shape from its original position and in a circular motion about a fixed point. To find a rotation transformation, you need to know the centre of rotation, the amount of rotation, and in which direction the turn is made—clockwise or counterclockwise.

In your own words, include the description of a rotation on your resource sheet.

Rotation around the Centre of the Shape

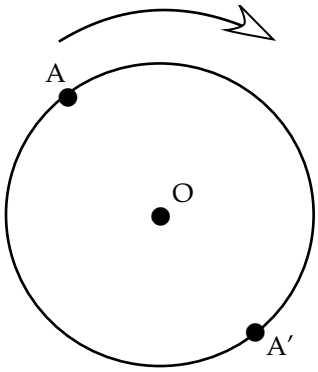
To understand rotation transformations, try to first understand rotating a circle around its centre. One complete rotation of a circle is 360°.

If you turn the circle one complete revolution so that point A returns to the place where it started, that rotation is 360°. The image of A would be A′ and both would be located at the same point.

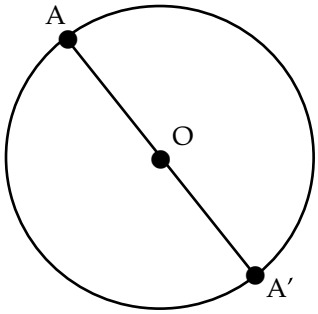


The arrow in the diagram above shows a turn in the clockwise direction. However, you can also make rotations in the counterclockwise direction.

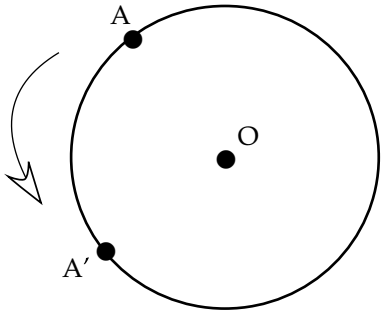
In the diagram below, half turn from A to A′ would be a rotation of 180° around the centre. This circle was rotated about the centre 180° in a clockwise direction. The image point for A is A′.



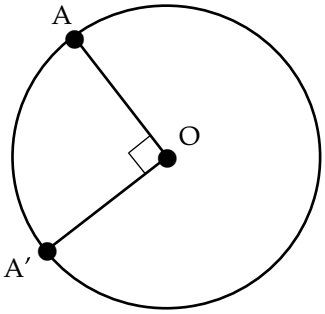
Notice that if you were to draw ∠AOA′ by joining AO and A′O, you would have a straight angle whose measure is 180°, the amount of the rotation. The vertex of the angle is the centre of rotation.



You could rotate the circle 90° in a counter-clockwise direction, and point A would have its image at A′, as shown in the diagram below.



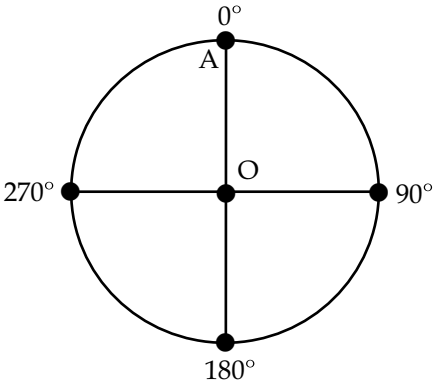
Notice that if you were to draw ∠AOA′ by drawing AO and A′O and measure the resulting angle, you would have a right angle—the amount of the rotation.



This lesson does not require exact measurements, but you must be able to make a reasonable approximation for the outcome of a rotation.

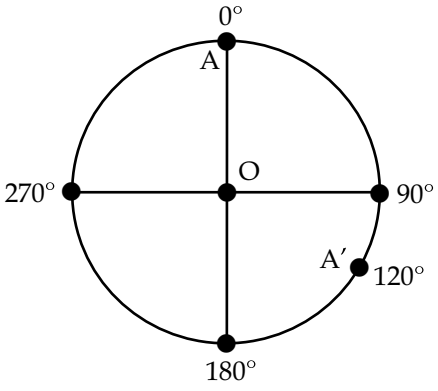
Example 1

A circle is divided into quarters, with point A at the top. If you were to rotate this circle 120° clockwise about its centre, where would the image of A be located?

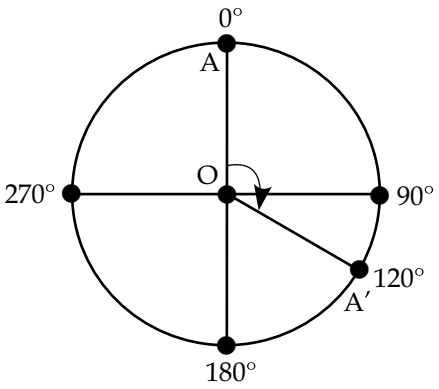


Solution:

Since 120° is 30° more than 90°, the image point would fall between the 90° and the 180°, and about one-third of the distance away from 90°.



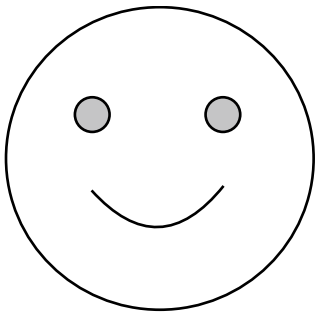
You might practise your drawing skills on a separate piece of paper. Try making this diagram without using your geometry set, and then measure your angles with your protractor. How close were you? Note that ∠AOA′ is 120°, the angle of rotation, O is the centre of rotation, and the direction is clockwise.



By doing a bit of practice now, the rest of the lessons in this module will be easier.

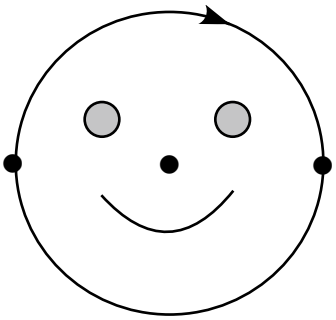
Example 2

Rotate this shape 180° clockwise about the centre of the circle.

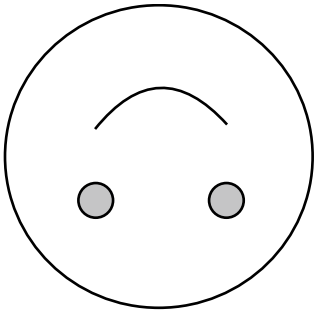


Solution:

In this situation, the centre remains where it is as the centre of the shape and the centre of rotation. The entire shape is turned 180° around the centre of rotation. The nose is the centre of the circle, and the ear at point A will become the other ear at A′. And ∠AOA′ will be 180°.



The eyes are now at the bottom, and the smile turns into a frown.

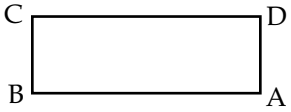


Rotating about a Fixed Point on the Shape

A fixed point on the perimeter of the shape can be the centre of the rotation. It is also the centre of an imaginary circle that you are rotating, just like you practised in the examples above.

Example 2

Rotate this shape 90° clockwise about point A.



Solution:

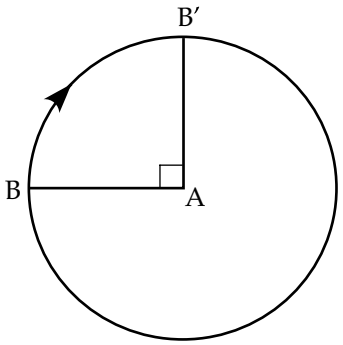
The method for solving this rotation shown below is just one of many ways of looking at the problem. This is a general method and is given as an option. You may be able to see the answer right away without going through all the following steps. They are here for reference if you get stuck on any of the later problems.

A rotation about a fixed point on the perimeter of a shape means that point is its own image. This means that the point A stays where it is, and the rest of the shape is rotated about that point. This also means A is the centre of an imaginary circle of rotation.

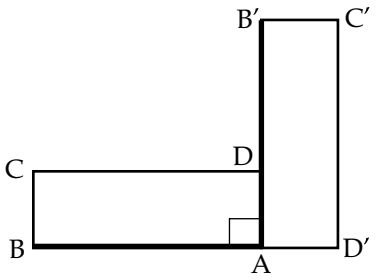
If you were to focus on rotating the point B, for example, then imagine B as a point on the circumference of your imaginary circle. Its image will be still on the circumference on the circle after rotating the imaginary circle 90° clockwise about its centre A.

Also note that since you are rotating point B, then the distance from A to B is the radius of your imaginary circle. And the distance AB will be the same as the length of AB′—both are equal radii of the imaginary circle. In a rotation transformation, this will always be true for each point you choose to rotate.

Here is a diagram of the imaginary circle showing how point B rotates around the circumference of the circle to its image point at B′, sweeping out an angle of 90° in the clockwise direction.

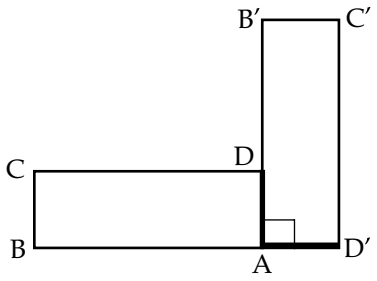


Since the rest of the rectangle is attached, it is rotated along with B. The result of the rotation is shown in the diagram below.

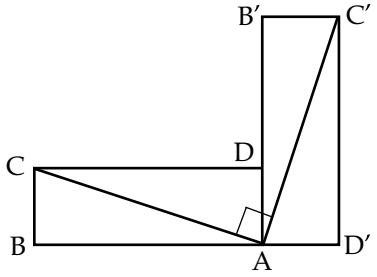


Notice that ∠BAB′ = 90°.

This time, focus on point D instead. The centre of rotation is fixed at A. The radius of the imaginary circle this time is AD. Note that AD is a vertical line. If you rotate a vertical line 90°, it becomes a horizontal line. Also notice that AD is the height of the original rectangle, so upon rotation its image will be the base of the rectangle in its new position. Note that ∠DAD′ = 90°.



Now consider point C. Of course, its image can be easily determined because it is attached to the rest of the rectangle you are rotating. Would it be true that ∠CAC′ is also 90°? Join AC and AC′ and confirm your answer.

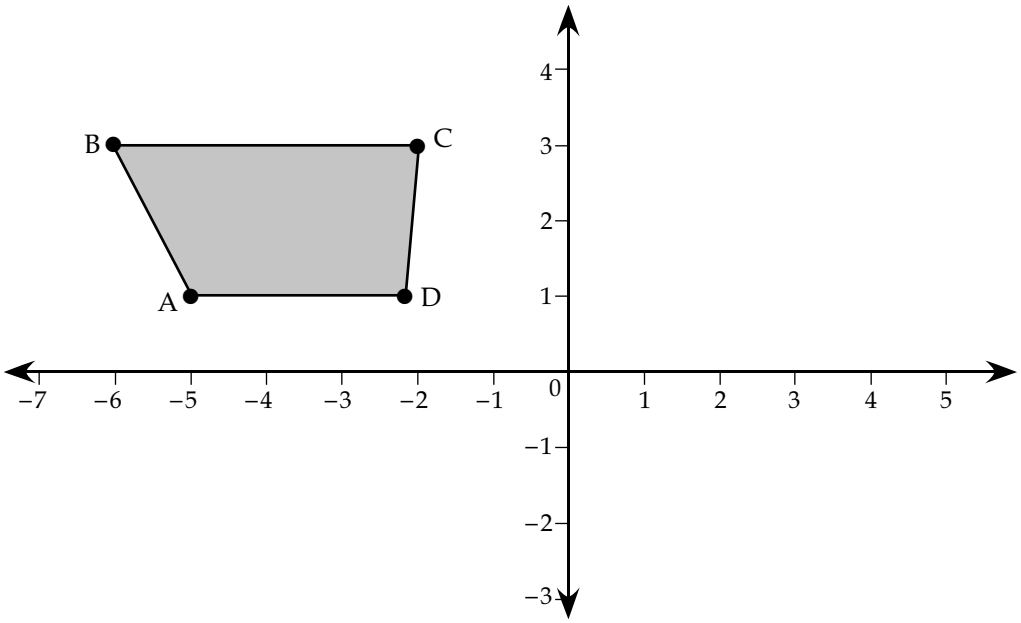


Is the length of AC the same as the length of AC′? Yes, because they are the diagonals of the same rectangle.

You can try this rotation on your own in a different way. Cut out two paper rectangles. Lay them exactly one on top of the other, as shown in the original question. Put your finger firmly on the bottom-right corner at point A and rotate the top rectangle 90° in a clockwise direction. Your result should look just like the diagrams above.

Example 3

Rotate this shape 270° counter-clockwise about point C. Make your diagram without using your tools from your geometry set. Then check your work with a ruler and protractor. Take your time to be as accurate as you can be with your estimates.



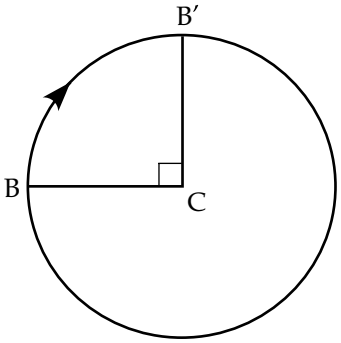
Solution:

Point C stays where it is. It is the centre of rotation. Its image is C′ and C = C′. The other three points will be considered separately. If you can find the image reasonably accurately without going through all these steps, that's great. Carry on!

The entire shape is to rotate 270° counter-clockwise. Remember, a 270° rotation is three-quarters around a circle. Such a rotation would be the same as rotating the shape 90° in a clockwise direction, which will be done in this solution.

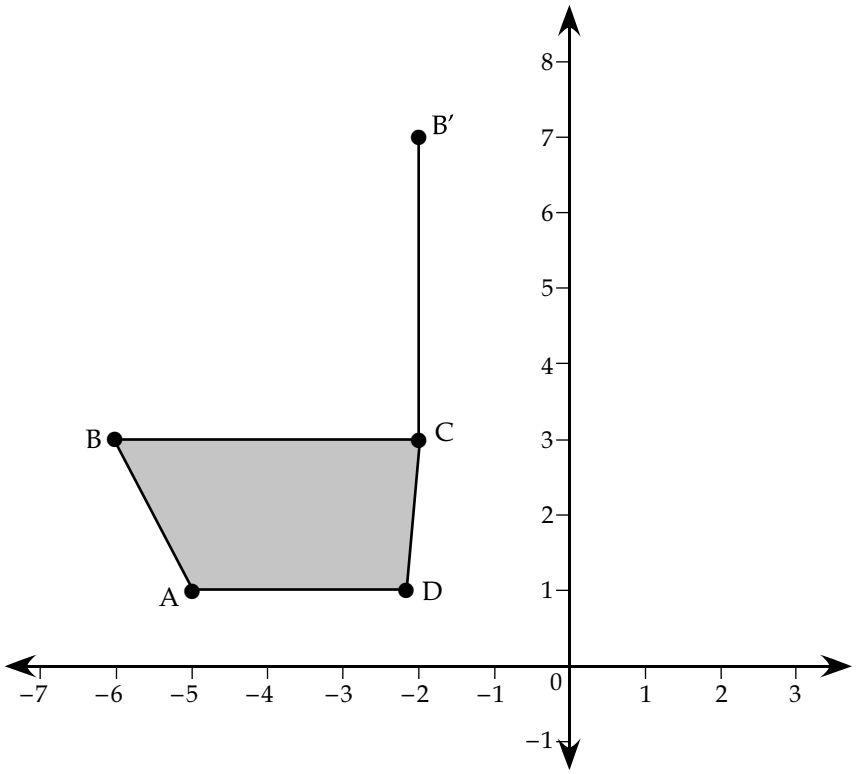
Try first focusing on the easiest point to rotate, point B. This is the easiest place to start because BC is horizontal. A 90° rotation will result in a vertical line. The explanation is written below.

Imagine a circle with centre at C and radius BC. Rotate BC 90° clockwise. ∠BCB′ will be 90° and B will be located straight north of C at a distance equal to BC. The circle shown below would be the imaginary circle of rotation for the point B.



By counting the squares on the graph, you can measure the length of BC exactly.

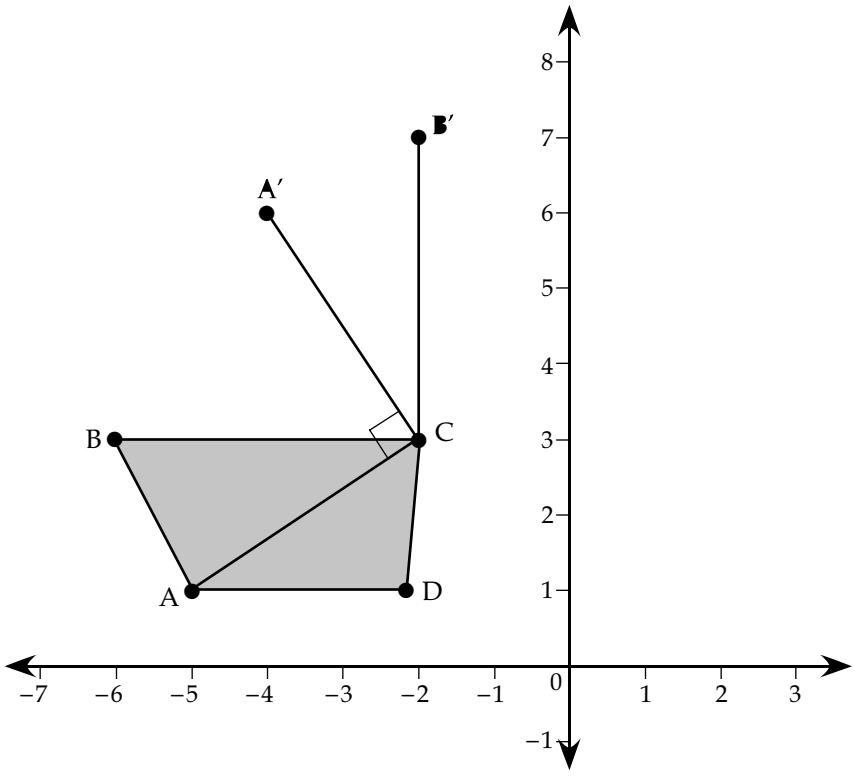
B is at (–6, 3) and C is at (–2, 3) so BC is 4 units in length. The image point B′ must be 4 units straight up from C and located at the point (–2, 7) on the graph.



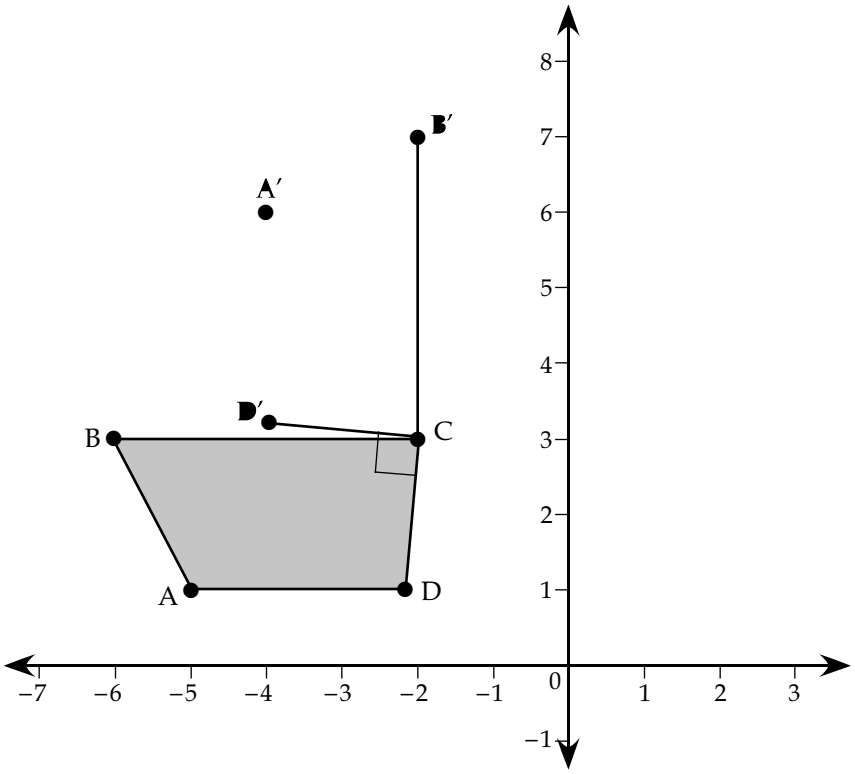
Now your task is to follow the same process with the other two points, A and D. They are not quite as easy. You will need to approximate the length of AC and CD, and you will need to estimate a 90° angle.

To begin, focus on point A. The imaginary circle of rotation has its centre at C and its radius is from C to A. Imagine making an angle from A to C to A′ whose measure is 90° clockwise.

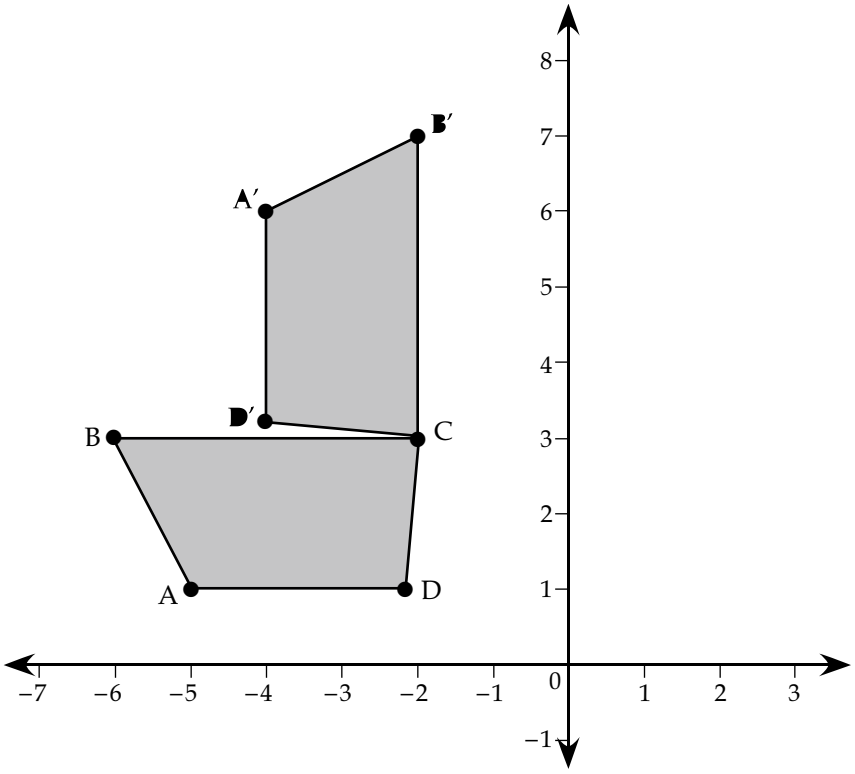
You will need to guess at how long CA is and then use an equal amount for CA′. The 90° angle of rotation from A to A′ is shown in the diagram for you to be able to visualize it.



Now try finding the image of the last point. Imagine the circle with centre at C and radius CD. Make a rotation of 90° around the circle in the clockwise direction, and place your result at the image point D′. Imagine making ∠DCD′ = 90°. Thus, D′ is just above the line from C to B.



Now you can join the image points to make the shape in its required rotated position.



Rotating about a Point Not on the Shape

The centre of the rotation can be anywhere. You have practised rotations when the centre is on the shape or when it is the centre of the shape. Now you can consider a rotation if the centre of rotation is away from the shape.

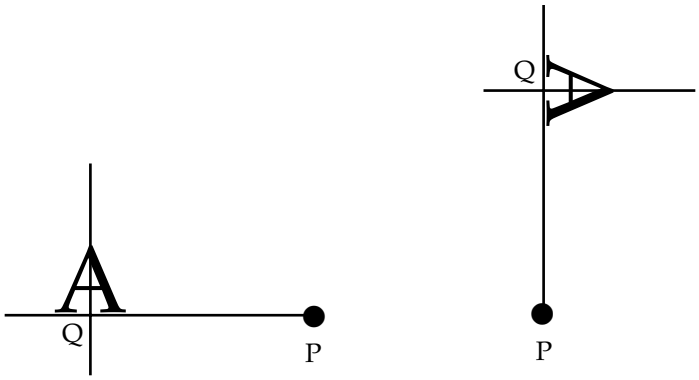
Example 1

Rotate the letter A 90° clockwise about the point P.

Solution:

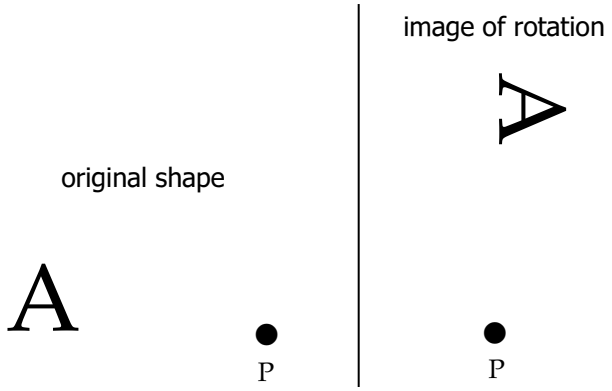
P is the fixed point and is the centre of rotation.

Draw a set of vertical and horizontal lines on the diagram, and mark point Q at their intersection. The radius will be the distance from P to Q.



Since PQ is horizontal, its image after a rotation of 90° will be a vertical line. The top of the letter rotates 90°, and the rest of the letter follows the same pattern.

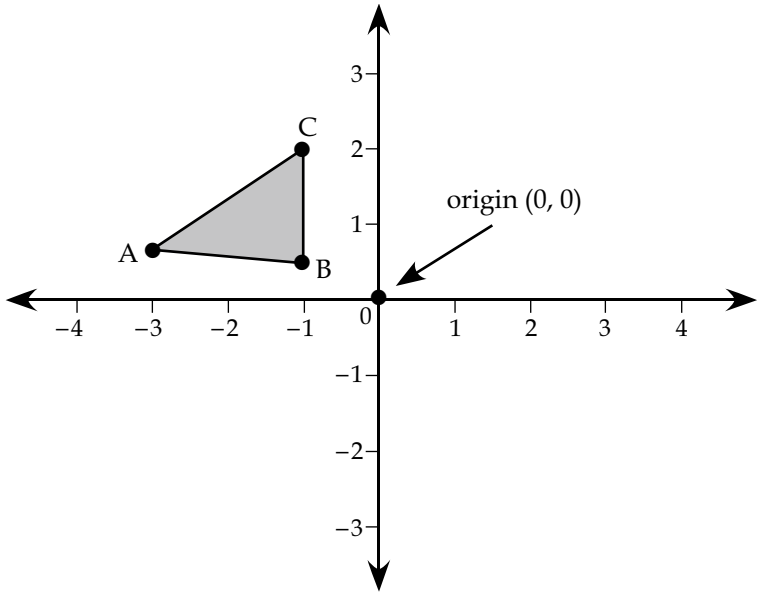
The rotation without the lines and symbols would be shown as follows.



Note that if you were not given the fixed point of rotation, but rather just the direction, the result would be an A lying on its side, as shown below.

Example 2

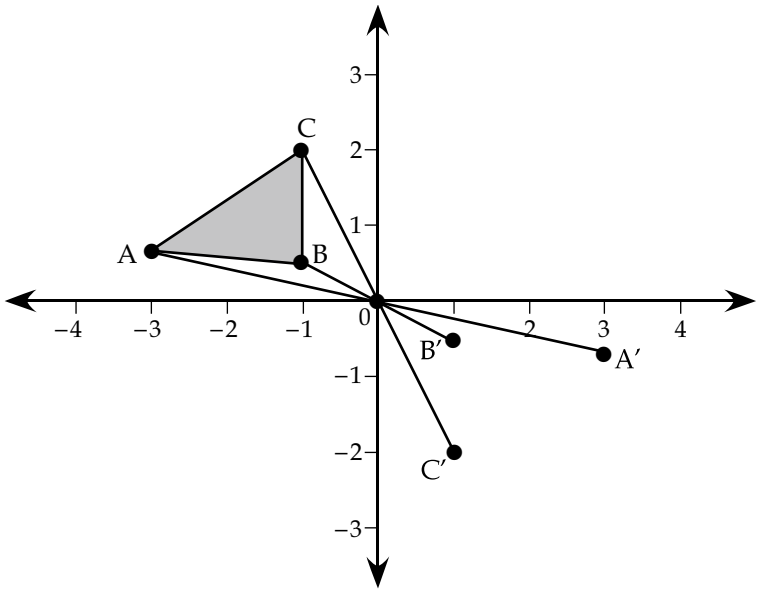
Rotate ΔABC 180° about the origin (0, 0).



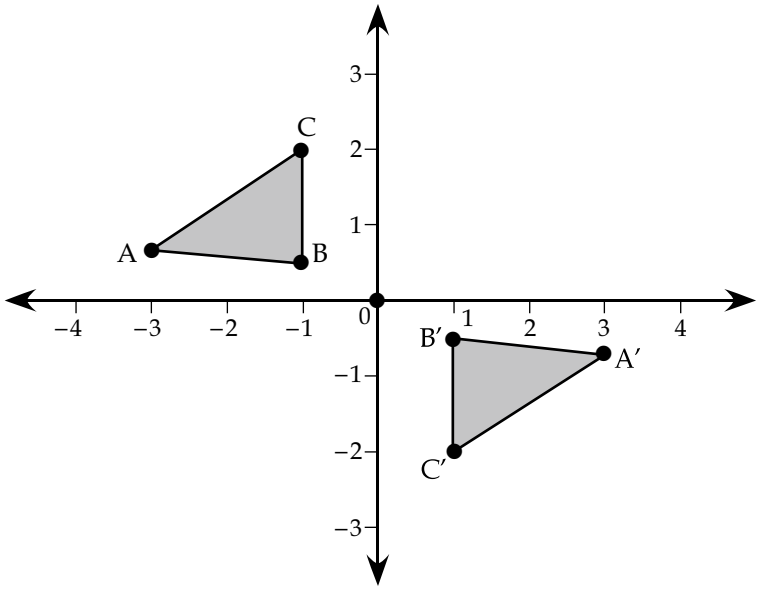
Solution:

The centre of rotation is the origin and B will become the first point to be considered. The radius to the circle of rotation becomes OB and ∠BOB′ is a straight angle. Thus, B′ is located in Quadrant IV and OB = OB′

In a similar fashion, a straight angle can be drawn from both points A and C to form ∠AOA′ and ∠COC′. In every case, O will be the midpoint of the lines, AOA′, BOB′ and COC′, and AO = AO′, and CO = CO′.



Now you can join the points A′, B′, and C′ to make the required image of the original triangle.



Learning Activity 8.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

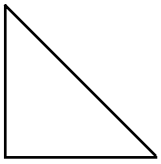
1. You get home from school at 4:15. You want to go for a run for 1 hour, then shower and get dressed, which takes about 35 minutes. You eat dinner for 30 minutes with your family. You need to do your laundry, which will take a total of 2 hours and 15 minutes. How much time do you have to do homework if you go to bed at 10:00 pm?
2. There are 20 players on your rugby team. Seven of them are missing the game for a school concert. What percent of your team is at the game?
3. Is the answer to -11×-13 positive or negative?
4. If you translate a point 4 units to the left then 3 to the right, how far is the new point from the original point and in what direction?
5. Your hourly wage is $8.00. You are paid time-and-a-half for overtime. If you work 4 hours of overtime, how much will you be paid for it?

Part B: Rotations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

Draw the rotation for each shape, as indicated.

1. 180° clockwise



continued

2. 90° counter-clockwise

3. 270° clockwise

Create a Shape

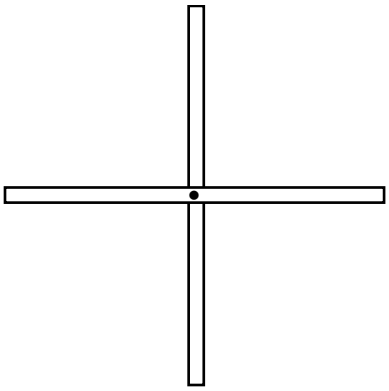
You can use successive rotations of one shape to create a design.

Example 1

Make a design by rotating the flat rectangle shown below. Rotate this shape 90° clockwise about the centre, then 45° clockwise, and 45° counter-clockwise. Draw a new rectangle every time you rotate it. Keep each image in the same diagram.

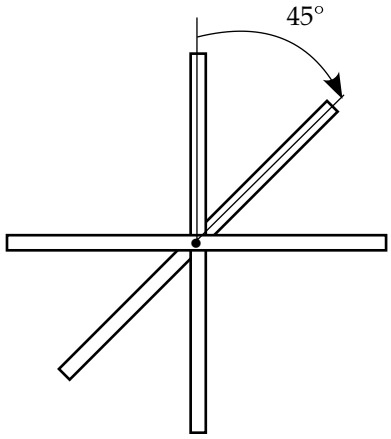
Solution:

First, rotate the rectangle 90° clockwise about the centre so that you now have two rectangles in the design.

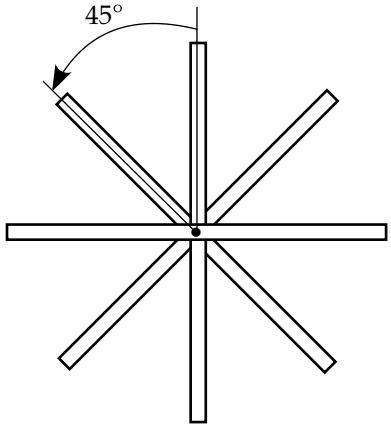


Then think of rotating the original rectangle 45° clockwise about the centre.

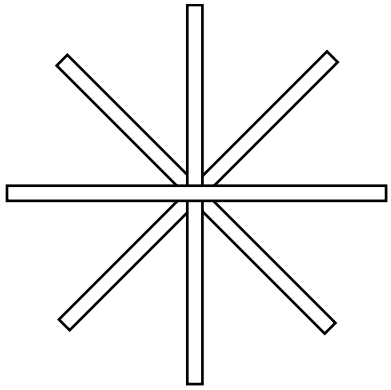
A 45° angle is one-half of a 90° angle. Think of rotating the rectangle clockwise so that it lays halfway between the vertical rectangle and the horizontal rectangle that you have already drawn.



The last rectangle is to be rotated 45° in a counter-clockwise direction around the centre point.

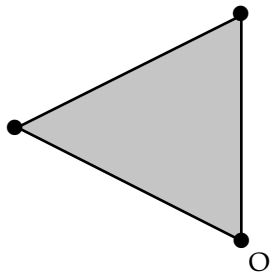


The finished design is shown below.



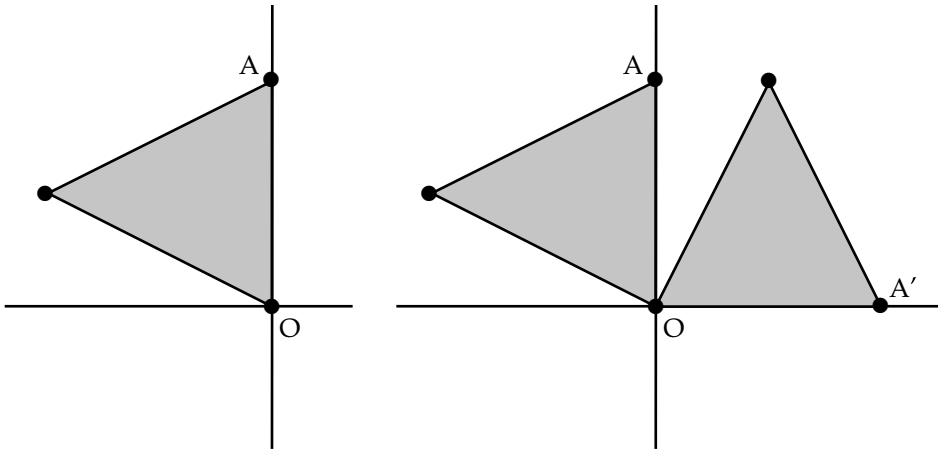
Example 2

Rotate the triangle 3 times about point O and 90° clockwise each time.

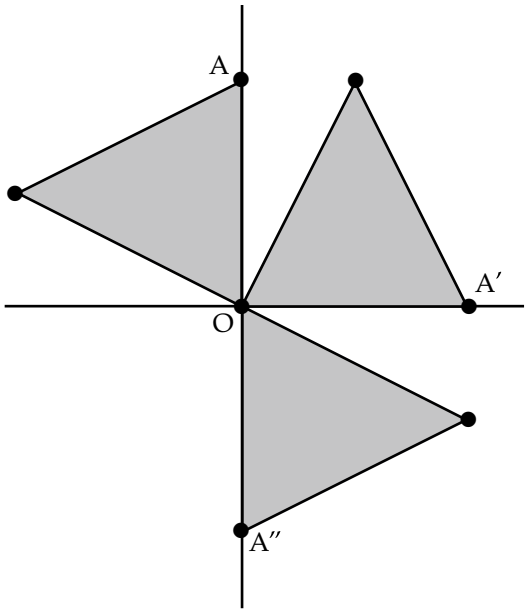


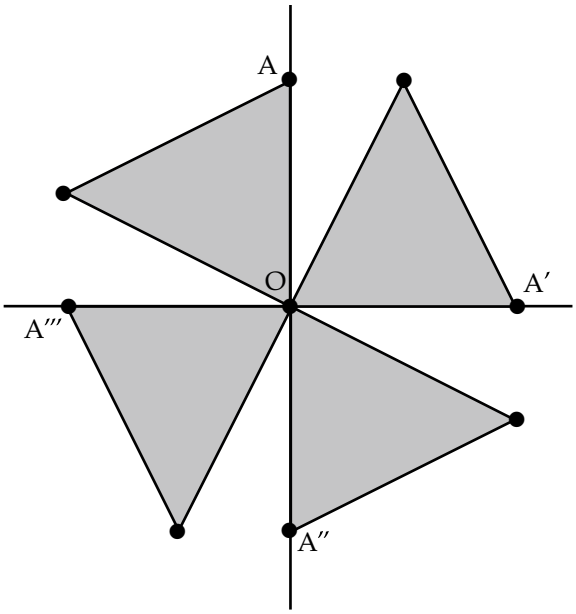
Solution:

Start by making a vertical and a horizontal axis through O, and then rotate the triangle 90° clockwise. The vertical line, OA, becomes the horizontal line OA′ and $\angle AOA' = 90°$.

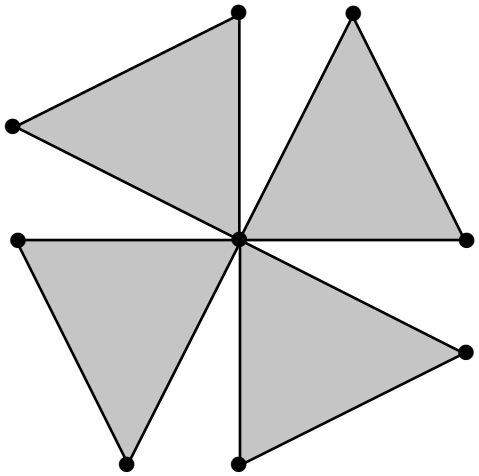


Rotate the first image 90° clockwise on the same diagram, and then repeat the process one more time. OA′ rotates to OA″. Then OA″ rotates to OA‴. The rest of the triangle rotates as the side OA and its images rotate.





The finished design without the axes and the labels is shown below.



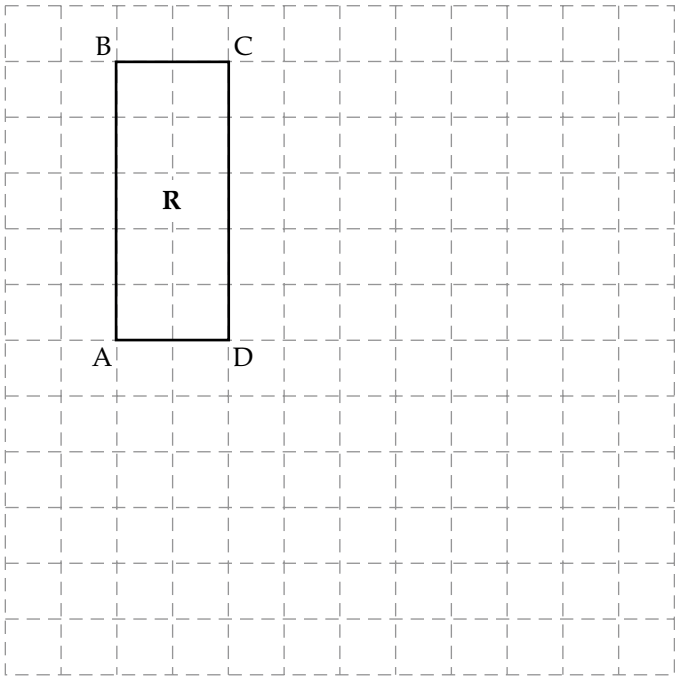
The more practicing you do, the more creative you can become and the more interesting the designs you create.

Combining Translations and Rotations

You can combine translating and rotating shapes to make interesting designs.

Example 1

Start with rectangle R, as shown in the diagram below.

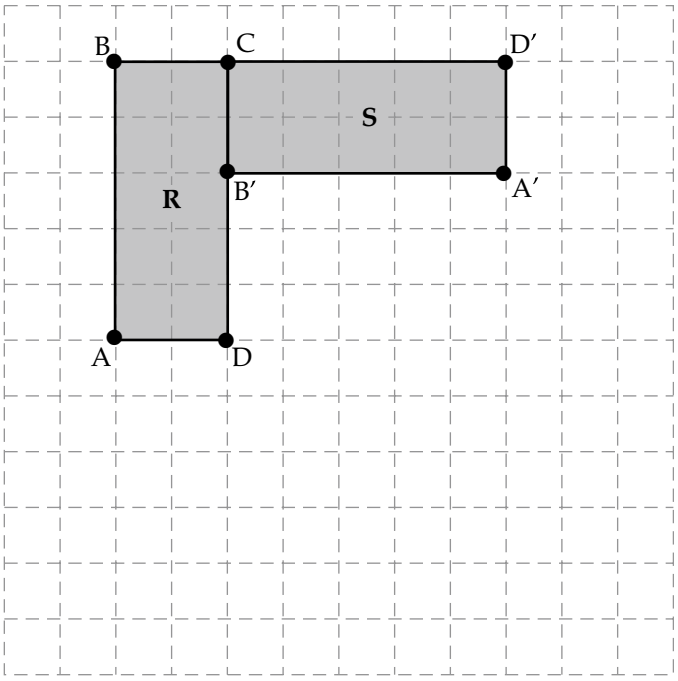


Make a design as described in the steps below. Try drawing the design yourself on the graph paper before you look at the solution.

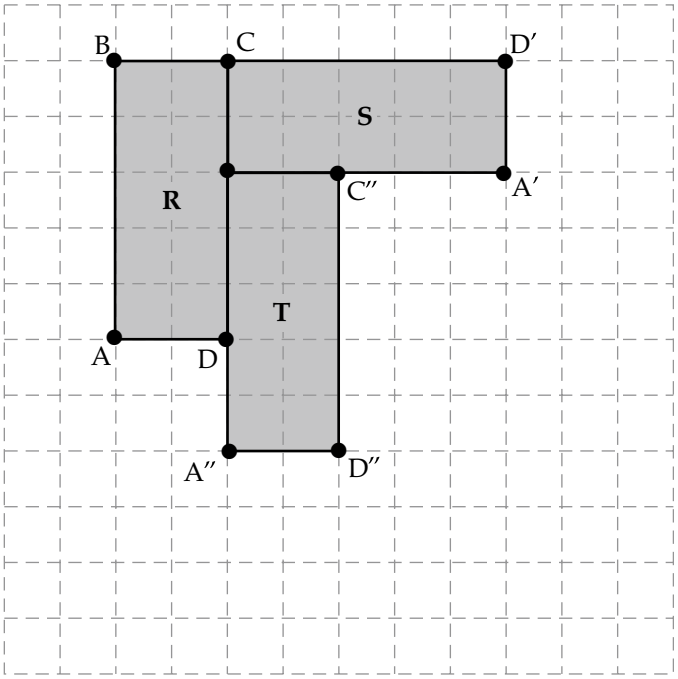
1. Rotate rectangle R 90° counter-clockwise about point C to create rectangle S.
2. Using rectangle R, translate it according to the rule [R2, D2] to create rectangle T.
3. Continue the same pattern, rotating rectangle T 90° counter-clockwise about point C″, creating rectangle U.
4. Translate rectangle T using the rule, [R2, D2], creating rectangle V.

Solution:

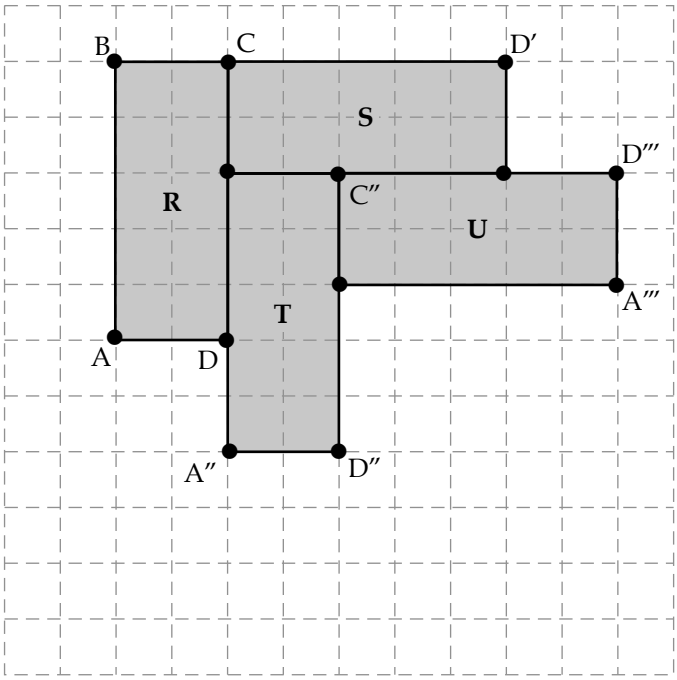
1. Rotate rectangle R 90° counter-clockwise about point C. This creates rectangle S. Notice ∠DCD′ = 90° and CD = CD′. The rest of the rectangle tags along.



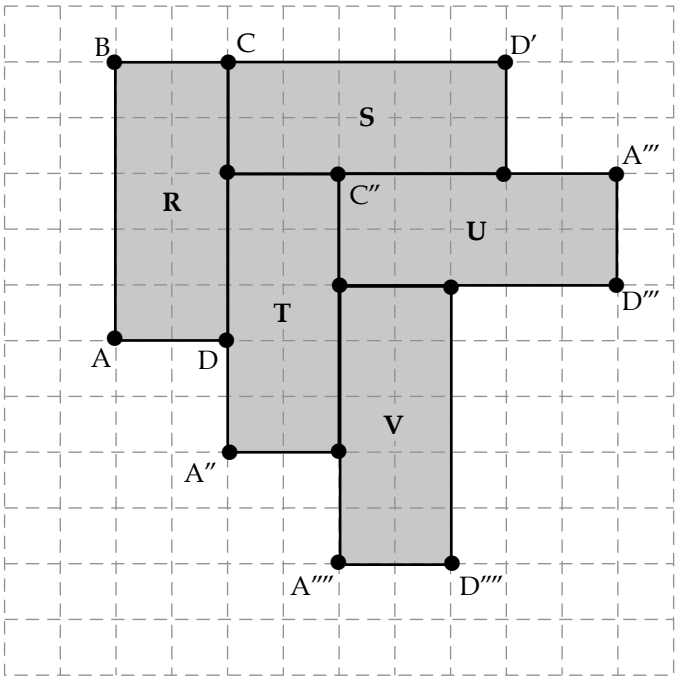
2. Translating rectangle R according to the rule [R2, D2] creates a second image of the rectangle R. The image is named rectangle T.



3. Rectangle T is rotated 90° counter-clockwise about point C″, creating a third image whose name is rectangle U.



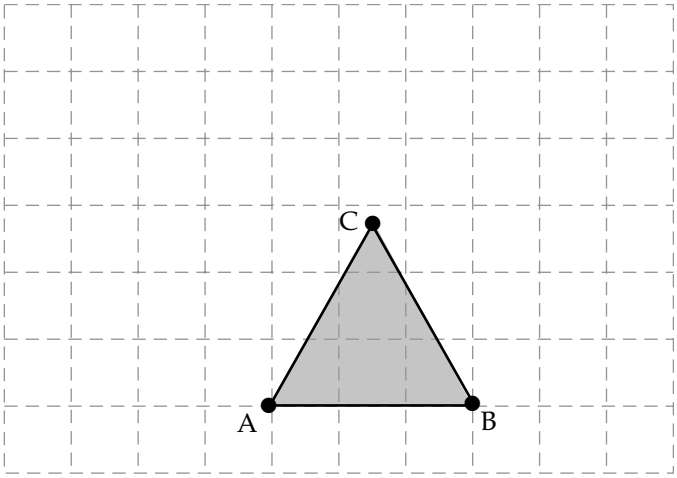
4. The last step is to translate rectangle T using the rule [R2, D2] to create the fourth image identified as rectangle V.



This pattern could continue to create a shape of any size. Using translations and rotations, interesting shapes can be created.

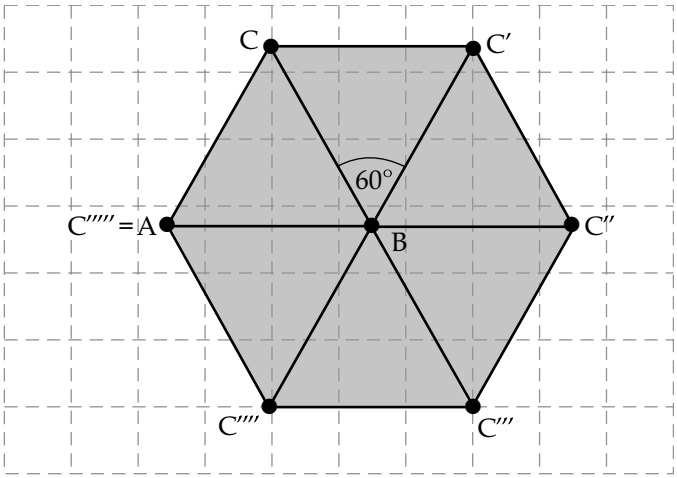
Example 2

Given this equilateral triangle ΔABC, can you do a series of five 60° rotations clockwise about point B to create a hexagon?



Solution:

The image of point C after 5 rotations is the point A.



You can cut various shapes out of paper or cardboard, and create your own artistic designs. The shapes do not have to be polygons with straight sides. Any shape that is repeated can be used to make a design.

Lesson Summary

This lesson dealt with rotating images and creating shapes using a succession of rotations and translations. You learned the definition of a rotation, the centre of the rotation, and the direction of rotation—clockwise or counter-clockwise. You learned there are three types of rotations depending upon the location of the centre of rotation.

The next lesson is all about reflections.

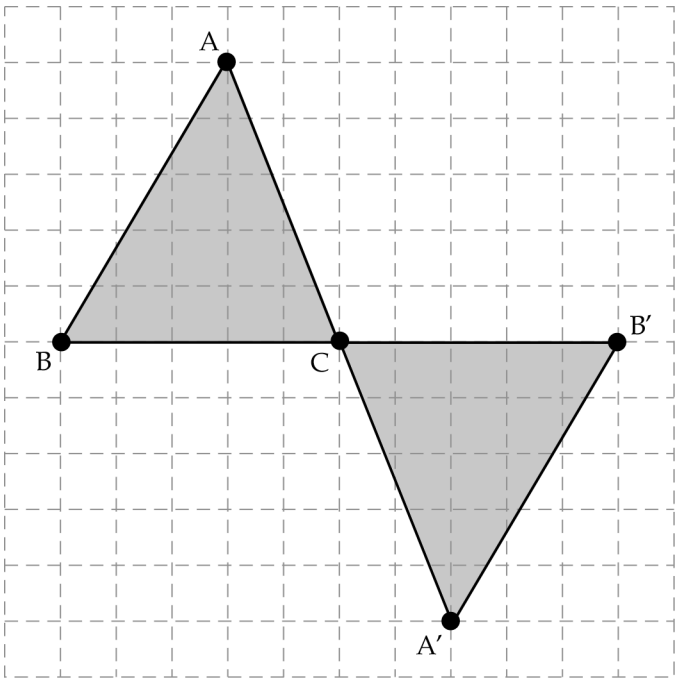
Assignment 8.2

Total: 20 marks

Rotations

1. Identify the centre of rotation, the degree of rotation as 90°, 180°, or 270°, and the direction as clockwise or counter-clockwise. *(8 marks)*

   a)



Centre of rotation _____

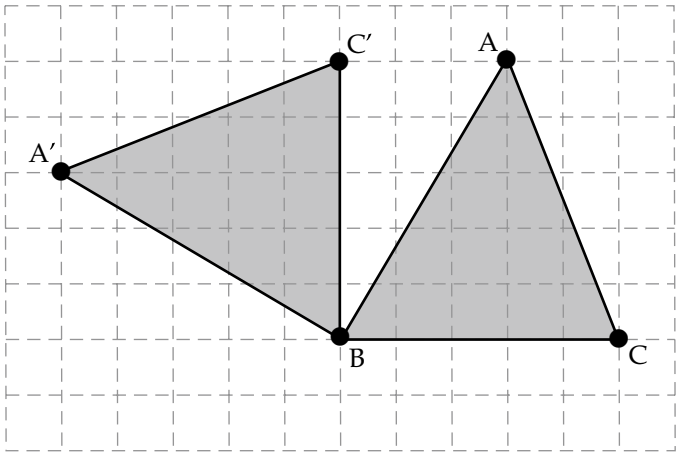
Angle _____

Direction _____

continued

Assignment 8.2: Rotations (continued)

b)

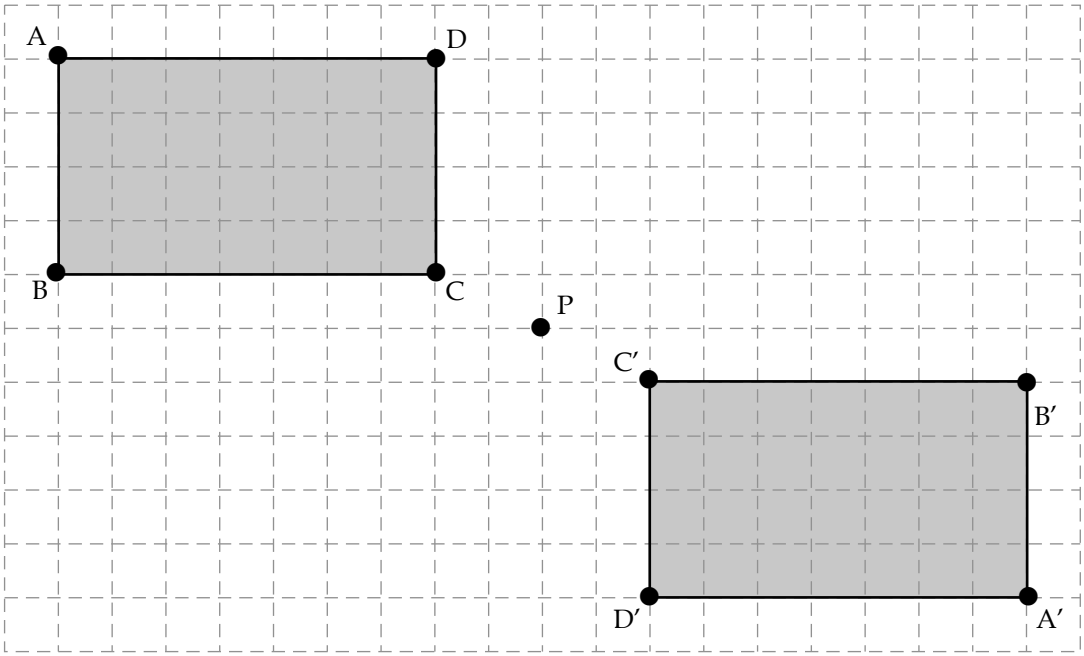


Centre of rotation _____

Angle _____

Direction

c)



Centre of rotation _____

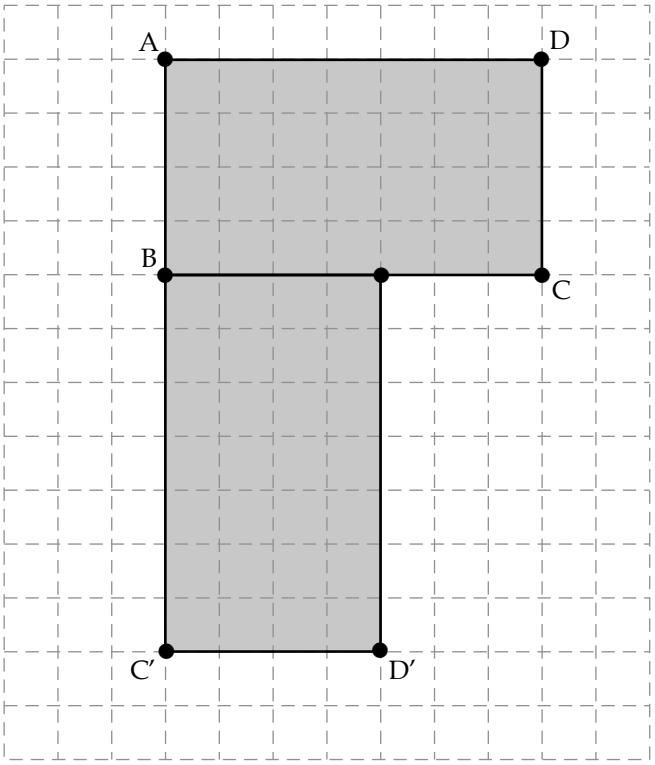
Angle _____

Direction _____

continued

Assignment 8.2: Rotations (continued)

d)



Centre of rotation _____

Angle _____

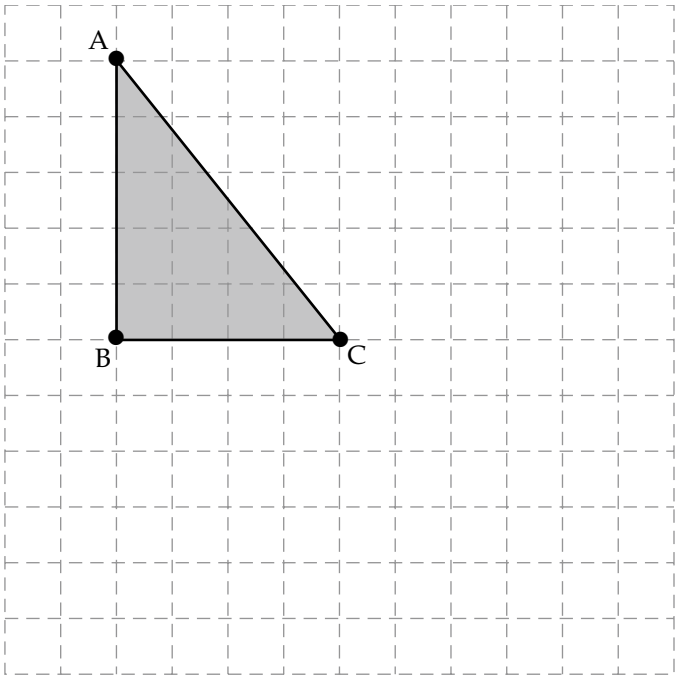
Direction _____

continued

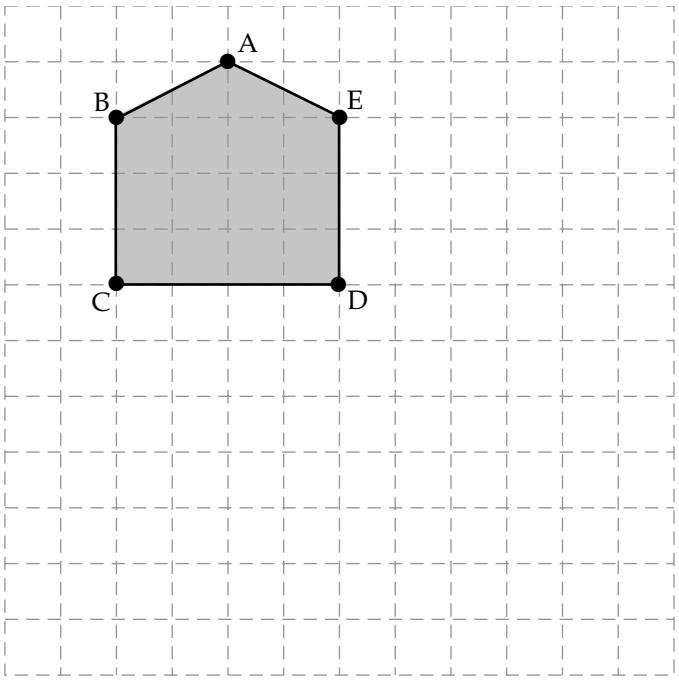
Assignment 8.2: Rotations (continued)

2. Draw the rotation transformation as indicated. *(9 marks)*

   a) 90° counter-clockwise from C



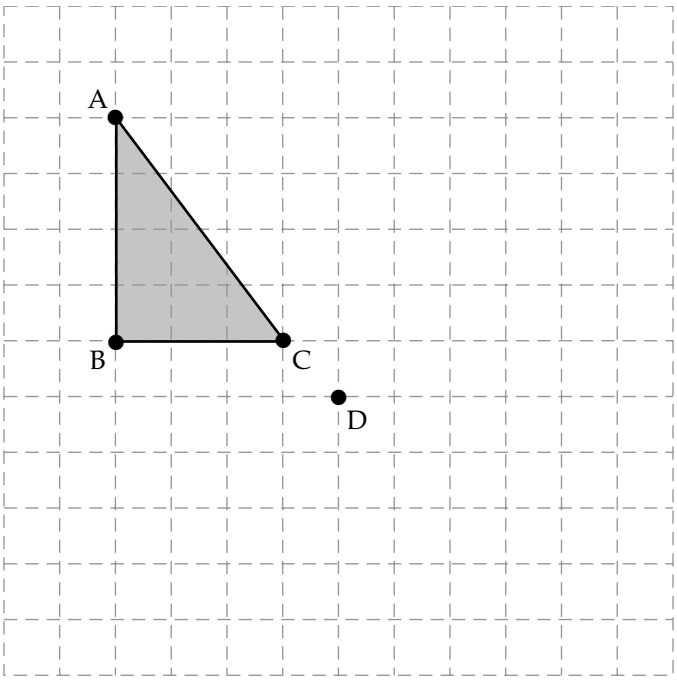
   b) 180° clockwise about point D



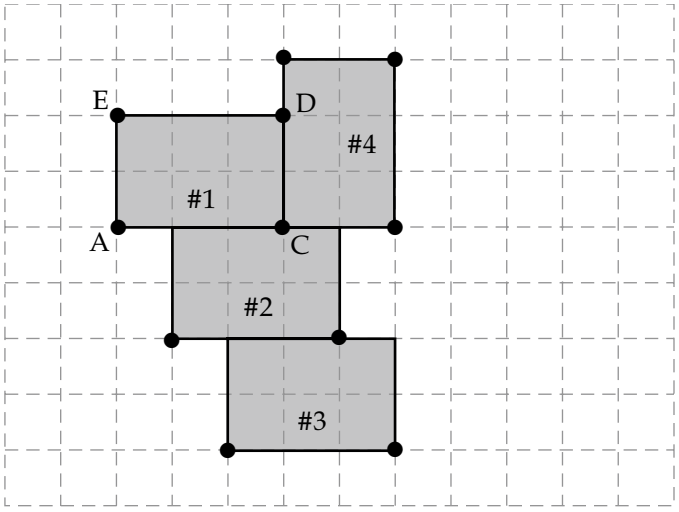
continued

Assignment 8.2: Rotations (continued)

c) 270° counter-clockwise from point D



3. Give one possible combination of rotations and/or translations that could create this shape. *(3 marks)*.



Notes

Lesson 3: Reflections

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of reflections

Lesson Introduction

The third transformation is the reflection.

What is a Reflection Transformation?

When you look into a mirror, you see a reflected image. The mirror's surface is the line of reflection. The distance you are away from the mirror will be the same as the distance the reflected image is from the mirror.

When you reflect a shape over a line, the image will be the same distance from the line as the shape. To do a reflection, the line of reflection needs to be given.

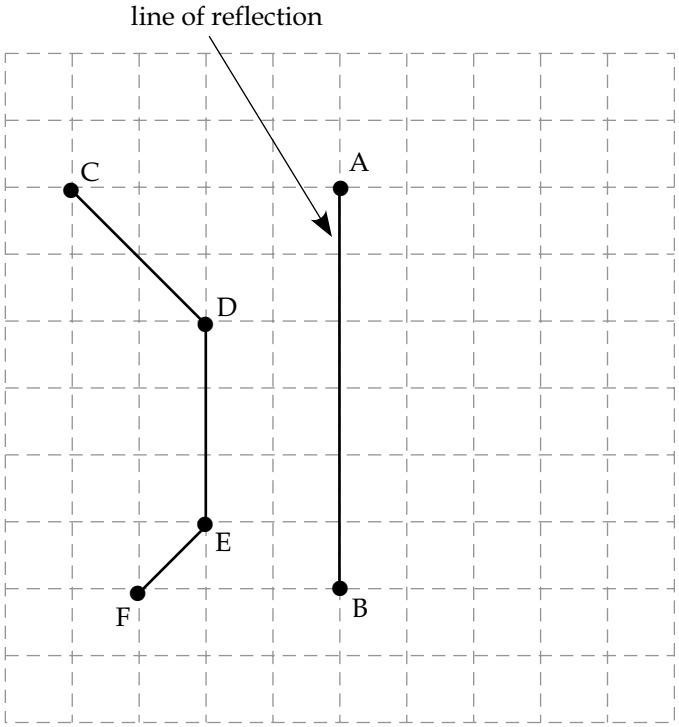
Another name for a reflection over a horizontal line is a flip, and over a vertical line is a flop.

In your own words, add a description of what a reflection is to your resource sheet.

Lines of Reflection

Example 1

Using the line of reflection shown in the diagram, find the image of CDEF.



Solution:

Each point on the reflected image of the shape will be equally distant from the line of reflection as the original. Remember distance from a point to a line means the perpendicular distance from the point to the line.

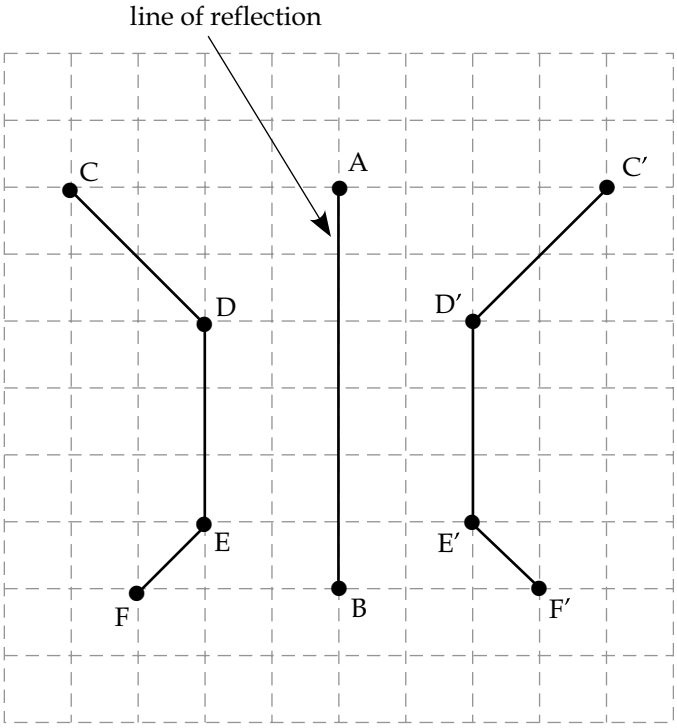
C is 4 units left of AB, so C′ will be 4 units right of AB.

D is 2 units left of AB, so D′ will be 2 units right of AB.

E is 2 units left of AB, so E′ will be 2 units right of AB.

F is 3 units left of AB, so F′ will be 3 units right of AB.

The reflection transformation is shown in the diagram below.



Example 2

Reflect the letter, P, over the given horizontal line.

P

Solution:

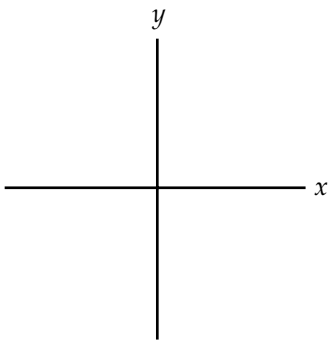
The bottom of the letter is closest to the line, so the bottom of the letter in the image will be closest to the line. Each part of the reflected image is the same distance from the line as in the original.

P

Ь

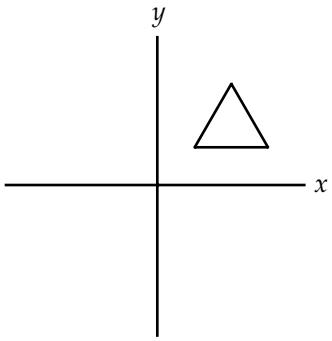
Axes

Some problems ask that you reflect a shape over the x-axis or the y-axis. You need to remember that the x-axis is always the horizontal axis and the y-axis is the vertical axis.



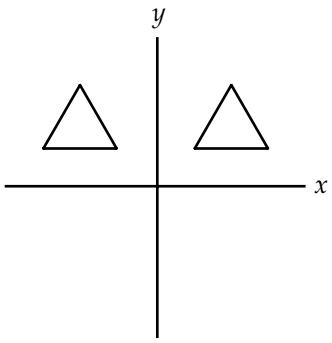
Example 1

Reflect the given shape over the y-axis.



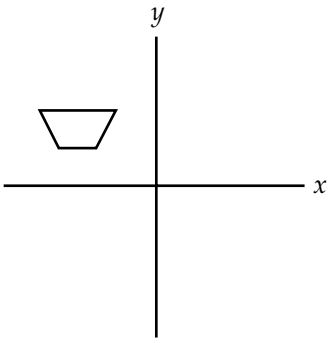
Solution:

Since the y-axis is the vertical axis, the shape is reflected to the left. It is always helpful to label the axes.



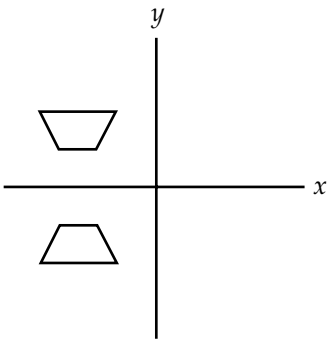
Example 2

Reflect the shape over the x-axis.



Solution:

The longer side is the furthest away from the x-axis. In the reflected image, it must also be the furthest away.



Learning Activity 8.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A right triangle has sides 0.21, 0.20, 0.29. Write the ratios for cosine of this triangle.
2. Evaluate if $w = 5$: $2w - 7$.
3. What are the factors of 28?
4. Dylan is 6′ 2″ tall. How tall is he in inches?
5. The following table shows how many hours you watch on TV each night.

Day of the Week	Monday	Tuesday	Wednesday	Thursday	Friday
Number of Hours	1	2	1.5	2	0.5

On average, how much time do you spend watching TV each day?

continued

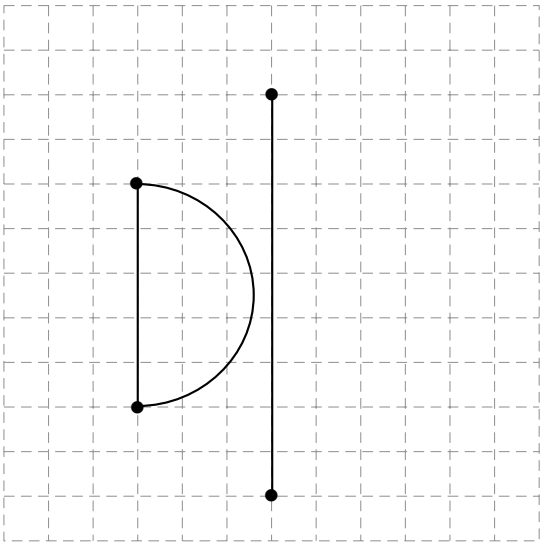
Learning Activity 8.4 (continued)

Part B: Reflections

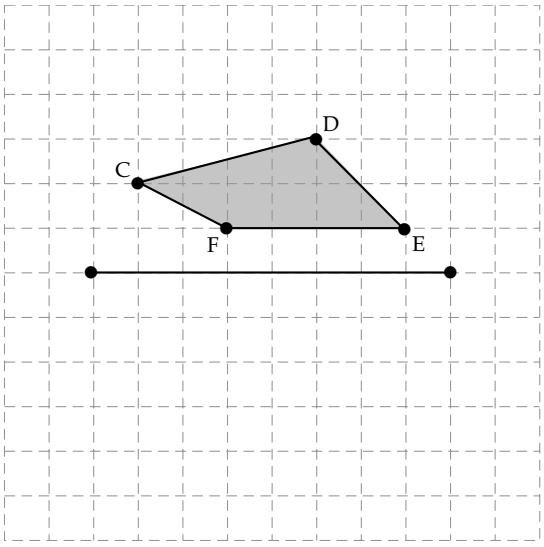
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Reflect the shape over the line of reflection given in each diagram, and draw the reflection.

a)



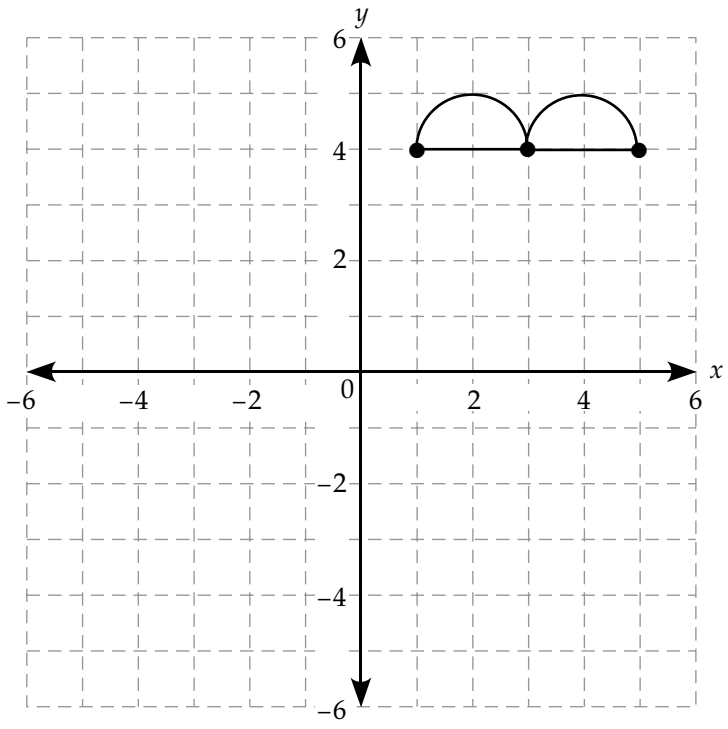
b)



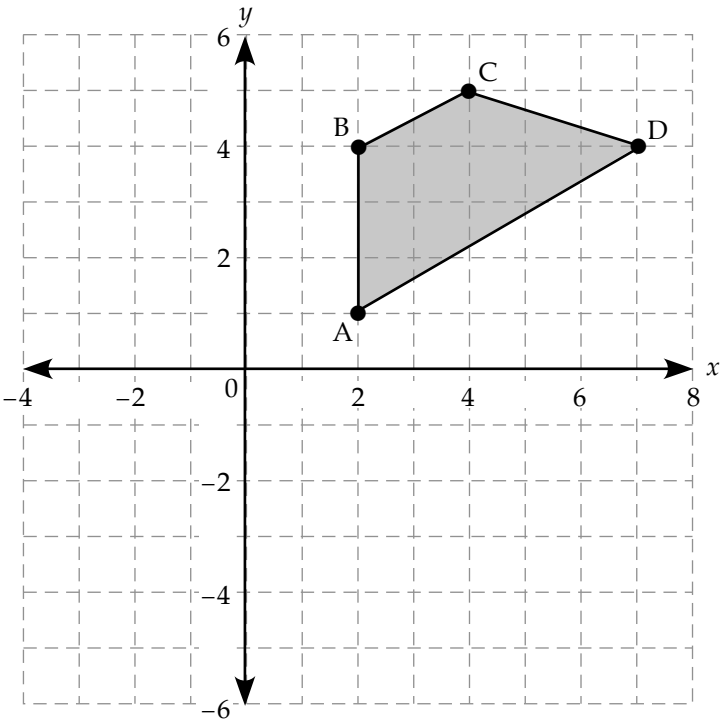
continued

Learning Activity 8.4 (continued)

2. a) Reflect this shape over the y–axis.



b) Reflect this shape over the x-axis.



Diagonal Lines

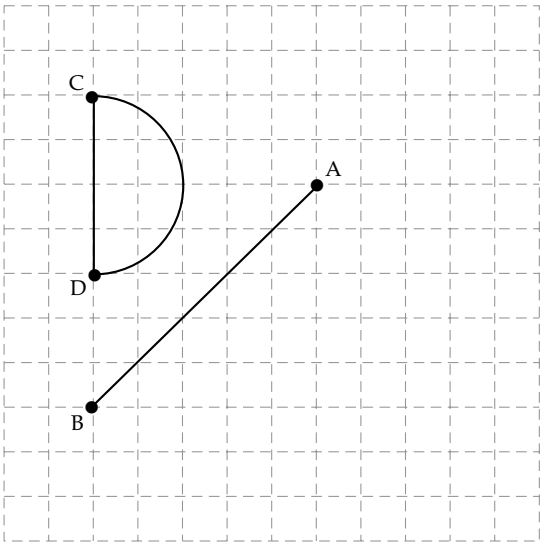
Reflecting a shape over a diagonal line is a little more complicated. In any reflection, all points on the image must be the same distance from the line as the original. The shortest distance from a point to a line is the perpendicular distance. When the line joining a point on the original shape is joined to its image, that line will be perpendicular to the line of reflection. To determine exactly where the image point is placed, you need to construct a line perpendicular to the line of reflection from a point on the original shape. The reflected point will be the exact same distance away from the line of reflection as the original point.

Many of the methods for finding the reflection transformation are discussed below.

Perpendicular Lines

Example

Reflect the letter D over the diagonal line of reflection, AB. Use perpendicular lines to determine the image points.



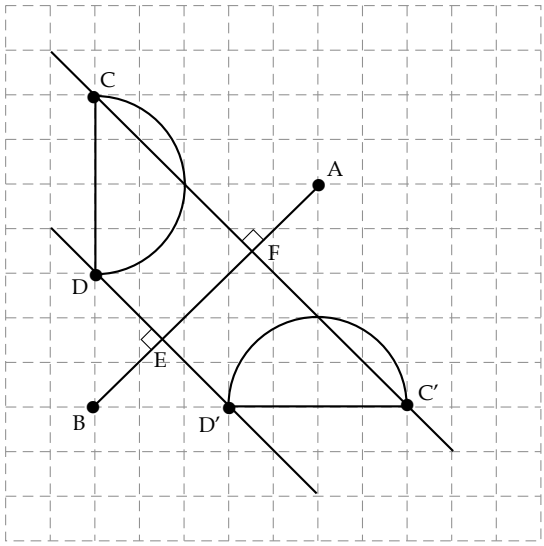
Solution:

Use the diagram drawn below to follow these steps.

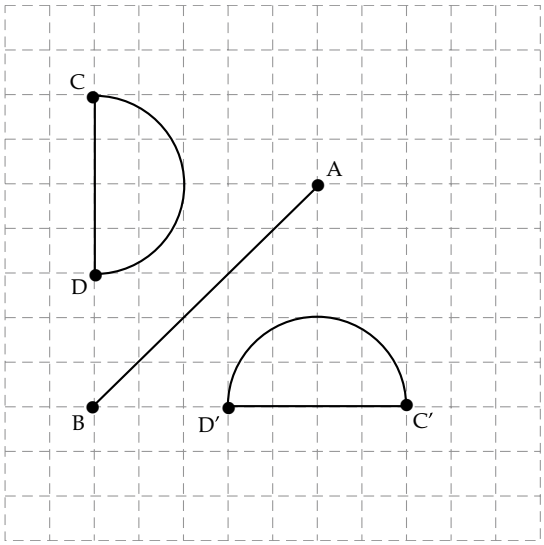
From D, draw a line perpendicular to AB and extend it past AB. Measure the distance from D to AB along the perpendicular line. Then use that distance to mark the point D′ on the perpendicular and on the other side of AB. Follow the same process to locate C′.

The lines CC′ and DD′ will be perpendicular to AB. Also, DE = ED′ and CF = FC′.

The maximum width of the letter is 2 units in the original position, so it must have a height of 2 units in its image.



The reflection transformation without the lines is shown below.

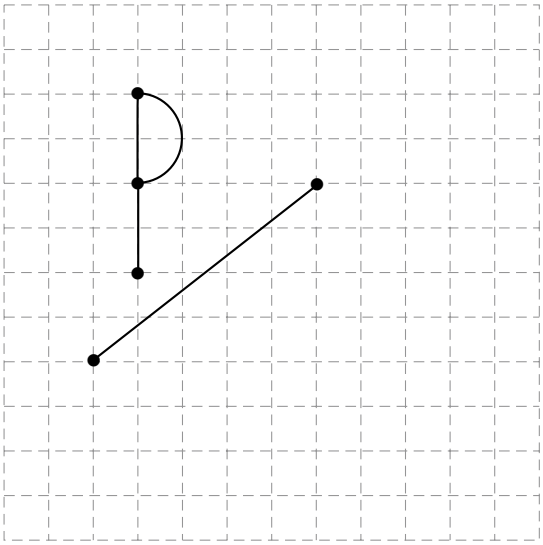


Paper Folding

Perhaps an easier method would be to fold the paper along the line of reflection, and trace the shape.

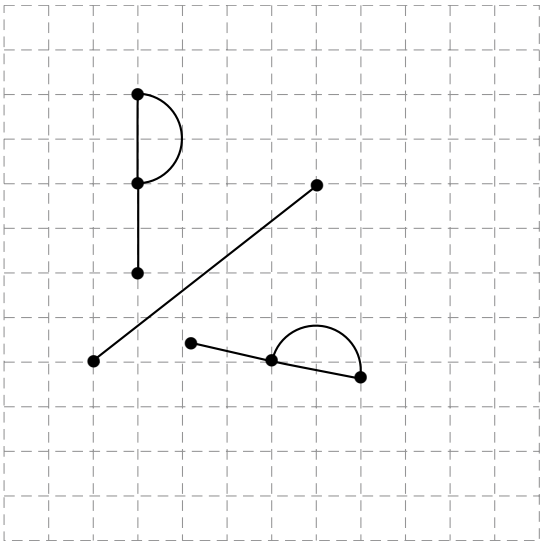
Example

Copy this shape and the line of reflection onto a piece of paper and fold the bottom part of the graph paper back along the line of reflection. Then trace your answer and unfold the paper.



Solution:

The reflection transformation is shown below.



Using a Mira

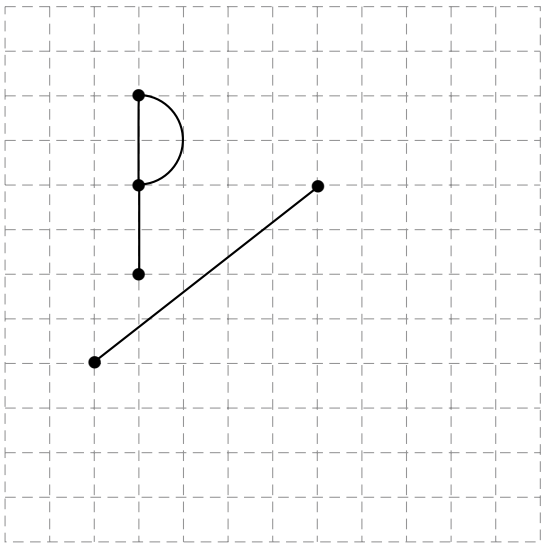
If you have a small mirror or perhaps a mira, placing it along the line of reflection will show you where the image is to be placed.

Repositioning the Page

Yet another method to simplify things is to turn the page until the line of reflection is horizontal. Then it is easier to see how the various points for the image align with the original shape.

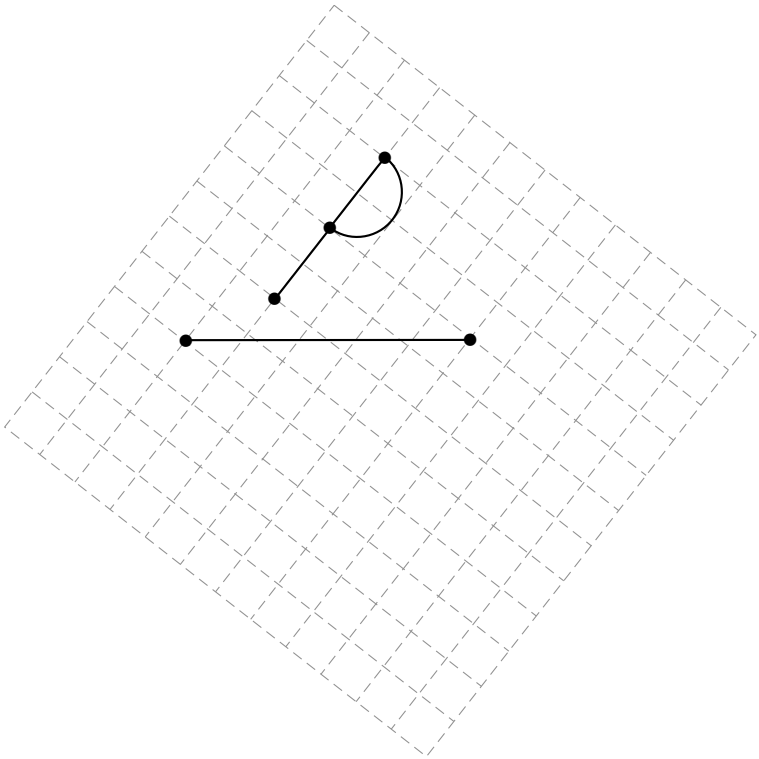
Example

Find the reflection of the letter P by first repositioning your page.

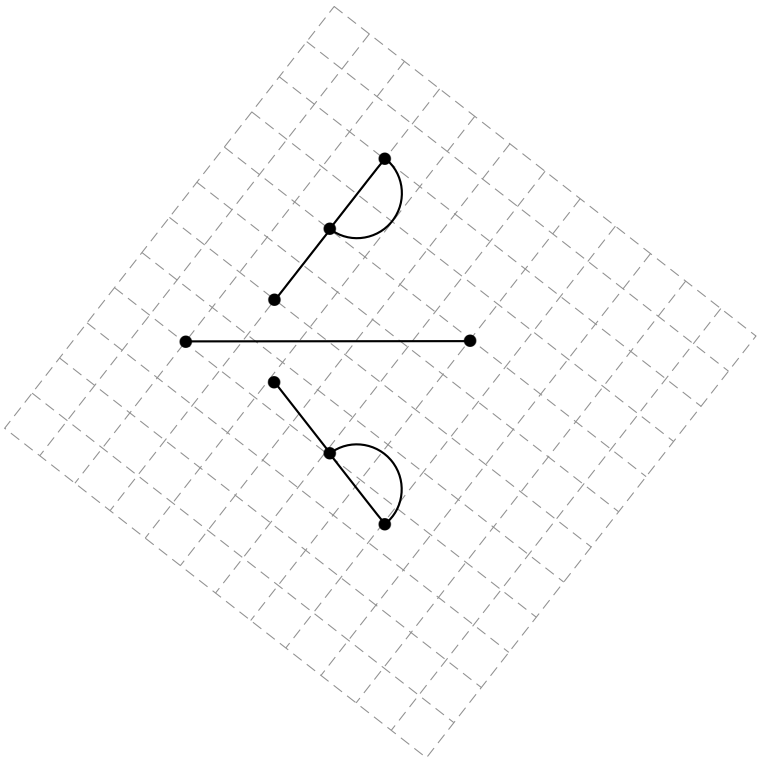


Solution:

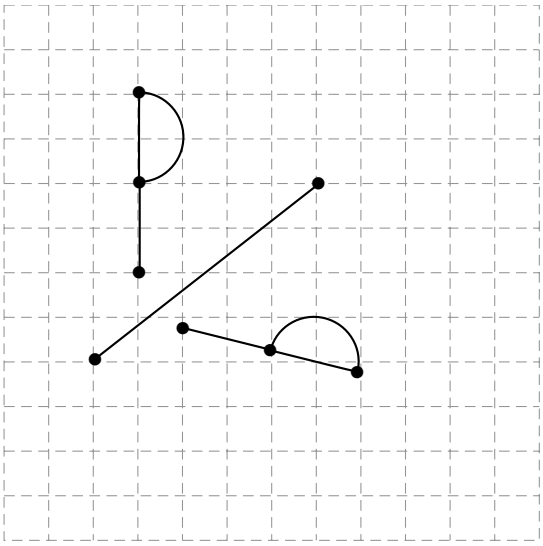
Rotate your page so that the line of reflection is horizontal.



Reflect the letter over the line of reflection to get the required image.



Then rotate your page back to its original position. The line of reflection will be diagonal again. The required reflection transformation is shown below.



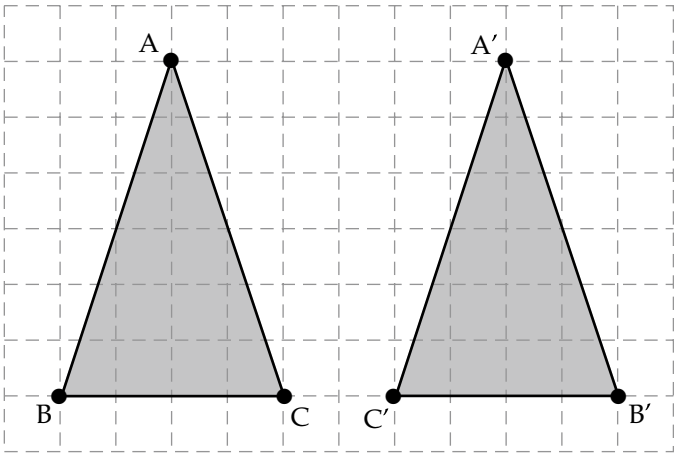
Finding the Line of Reflection

If you are given the shape and its reflected image but not the line of reflection, then you can use your knowledge of reflection transformations to find the missing line.

You know that any point on the reflected image is the same distance from the line of reflection as the original point is. The line of reflection must be midway between the original and its image.

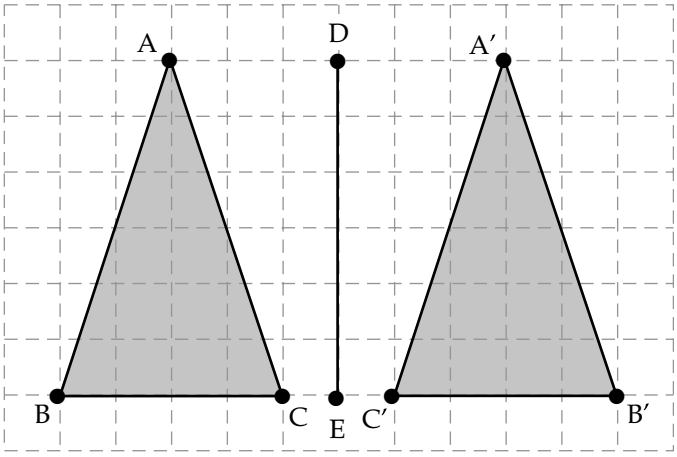
Example 1

Given the original shape and its reflected image, draw the line of reflection.



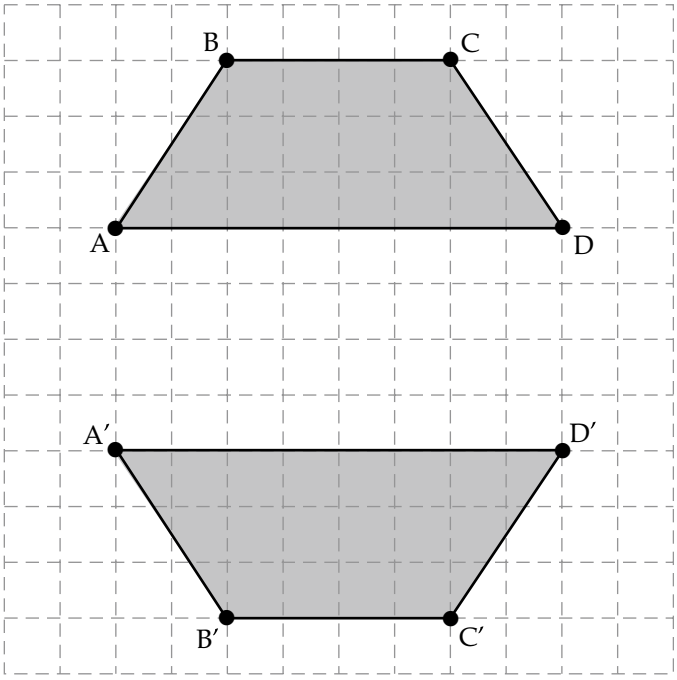
Solution:

The distance between the C and C′ is 2 squares. The line of reflection would have to be equally distant from each point, so it would be 1 square to the right of point C or 1 square to the left of C′. You also know that the line of reflection must be perpendicular to the line CC′. The required line of reflection is DE.



Example 2

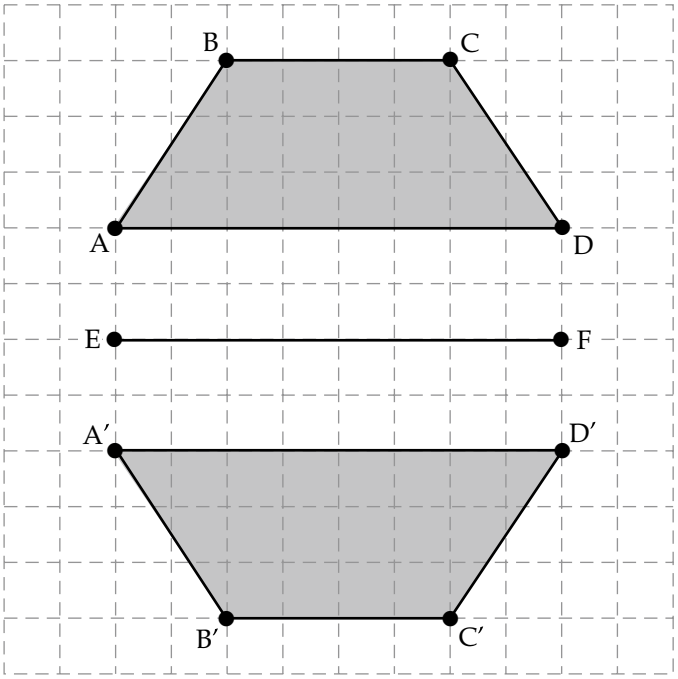
Given the original shape and its reflected image, draw the line of reflection.



Solution:

The reflected image is below the original shape, and both AD and A'D' are horizontal. Thus, the line of reflection must be a horizontal line midway between the two of them.

The distance between the AD and A'D' is 4 squares, so the line of reflection must be half of that. It must be 2 squares away from each line. The line of reflection is EF, as shown at the right.



Lesson Summary

This lesson was about repositioning shapes using a reflection transformation. The line of reflection can be horizontal, vertical, or diagonal. The reflection line needs to be given before the transformation can be drawn. You learned four ways to make a reflection if the line is on the diagonal. Also, you practised locating the line of reflection if the shape and its image are given.

The next lesson deals with line symmetry and combining transformations.

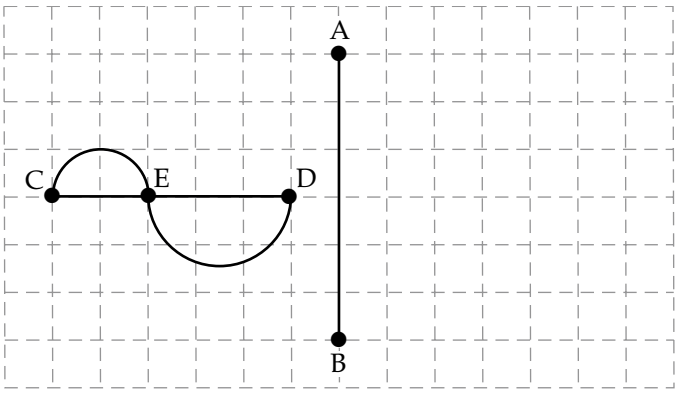
Assignment 8.3

Total: 16 marks

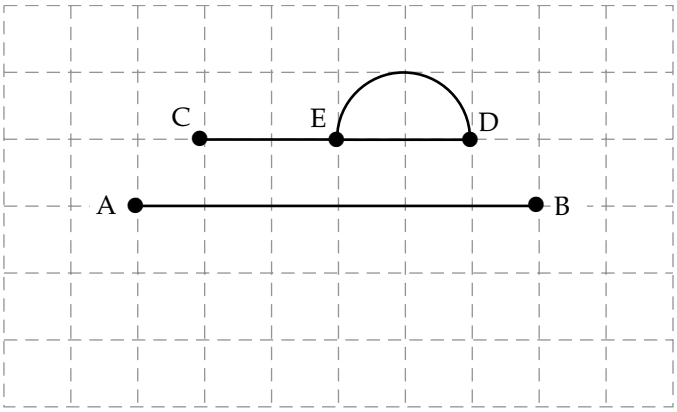
Reflections

1. Draw the reflected image. *(8 marks)*

   a) Over the line AB



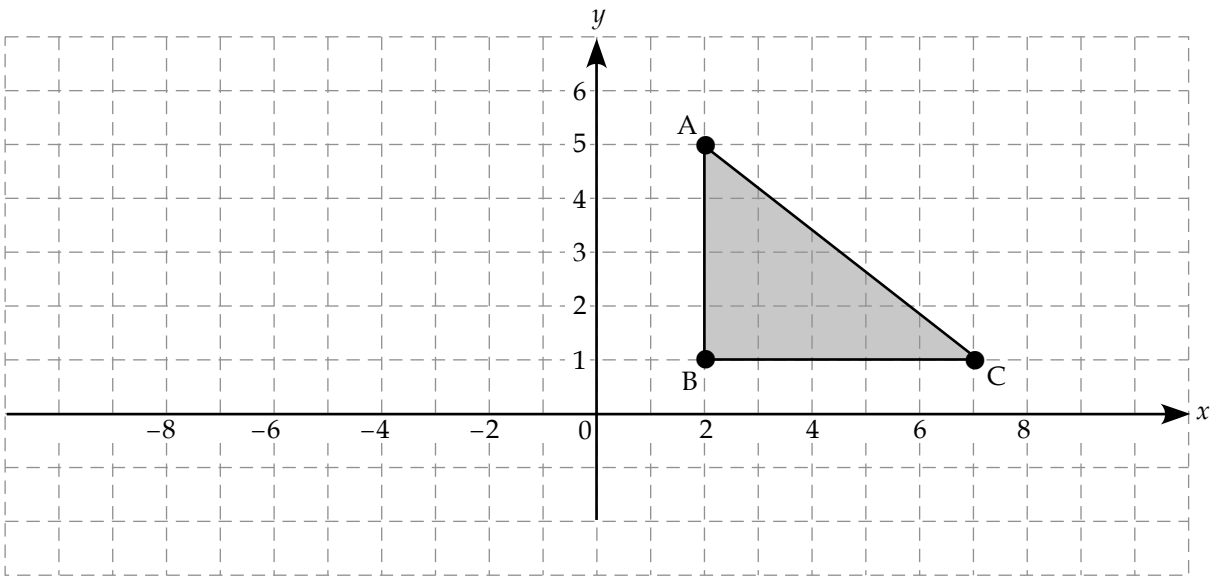
   b) Over the line AB



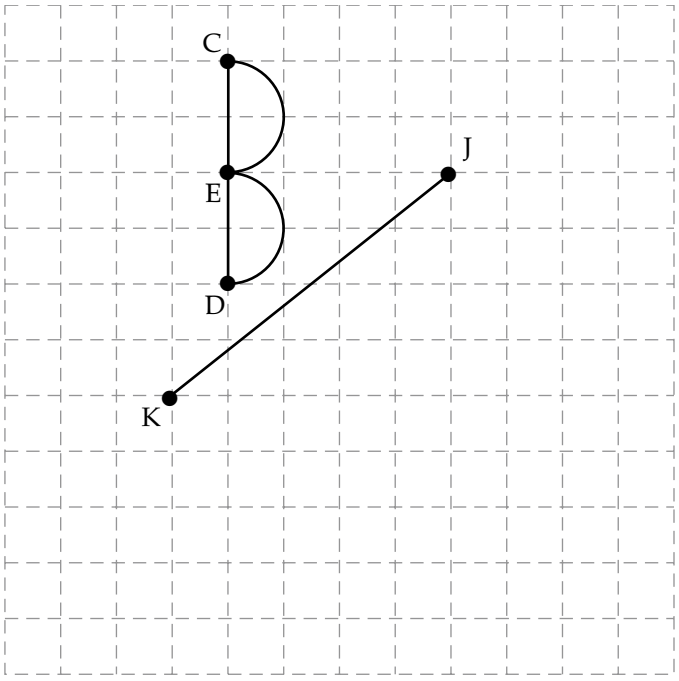
continued

Assignment 8.3: Reflections (continued)

c) Over the y-axis



d) Over the diagonal line JK

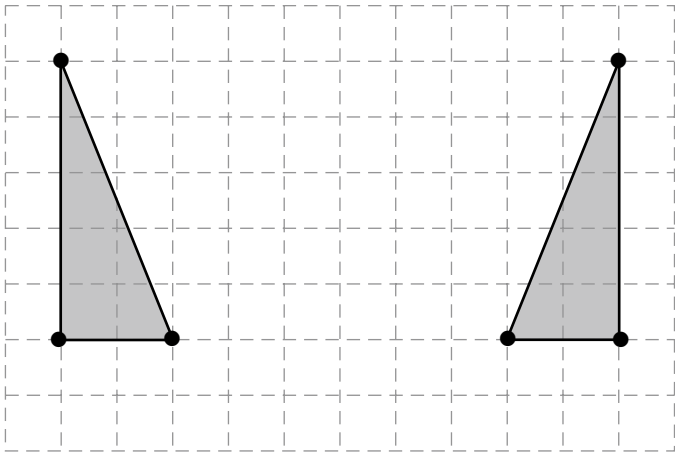


continued

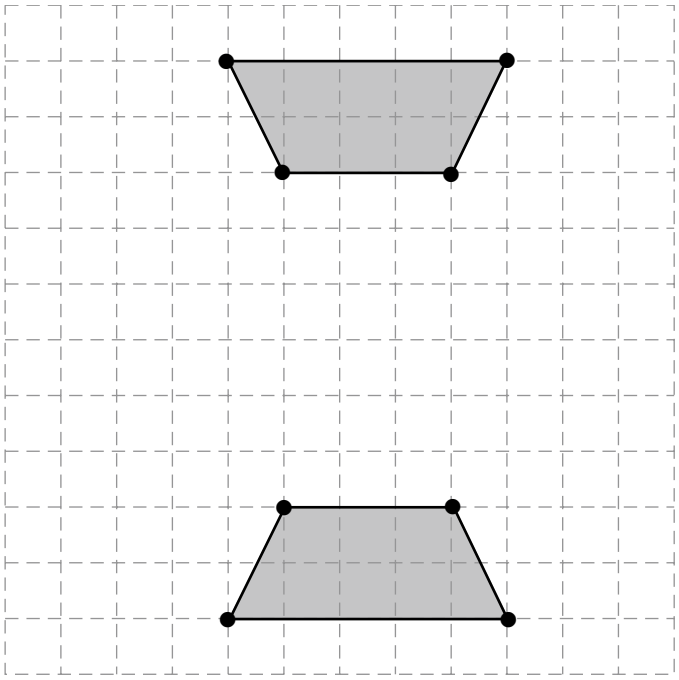
Assignment 8.3: Reflections (continued)

2. Draw the line of reflection. *(4 marks)*

a)



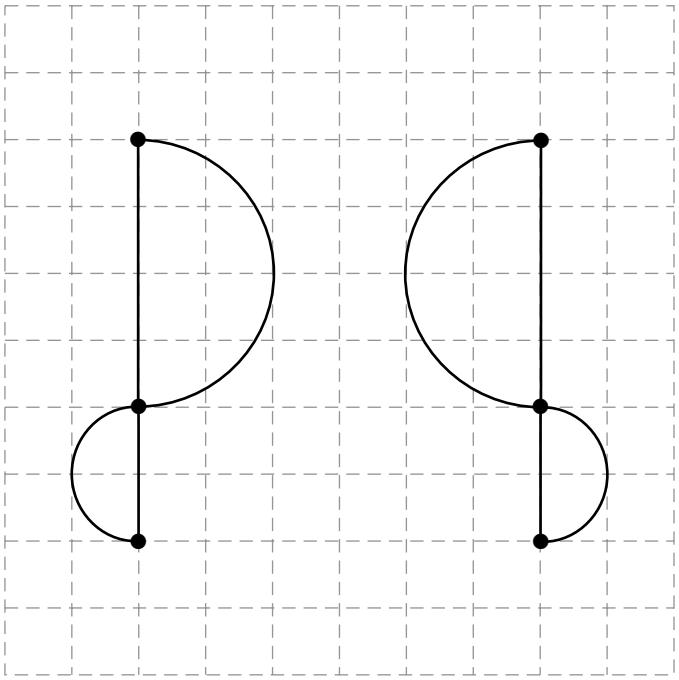
b)



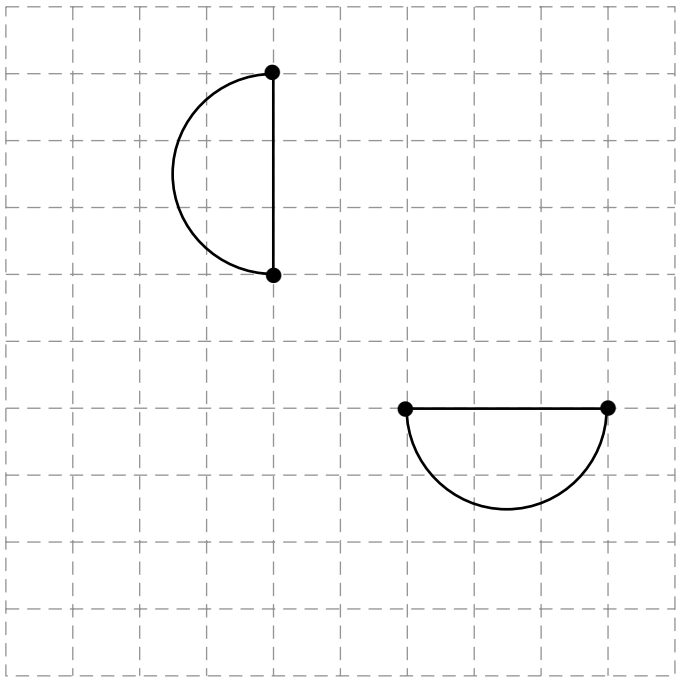
continued

Assignment 8.3: Reflections (continued)

c)



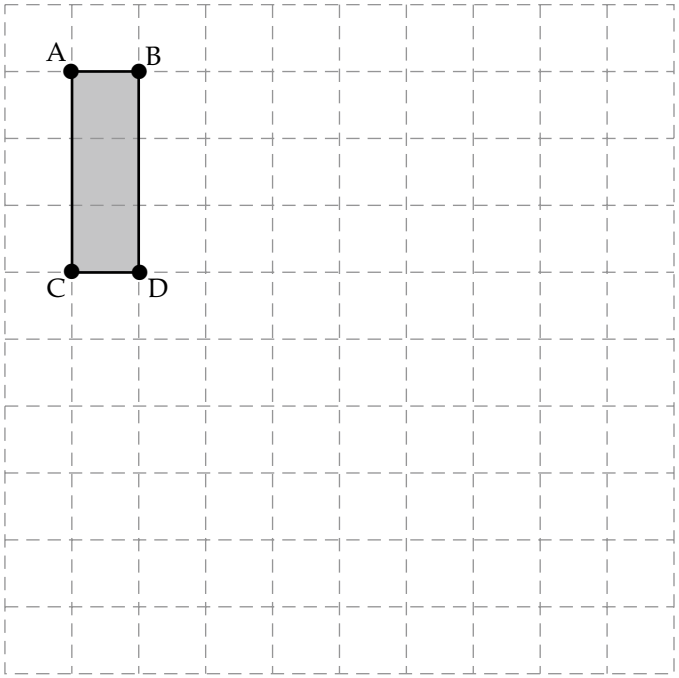
d)



continued

Assignment 8.3: Reflections (continued)

3. Translate the shape according to the rule [R2, D3], and then reflect that image over a vertical line that is 2 squares to its right. *(4 marks)*



Notes

Lesson 4: Combinations

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of transformations
- ❑ use combinations of transformations to create designs

Lesson Introduction

This lesson shows how various transformations can be combined to create designs.

Symmetry vs Reflection

Line Symmetry vs Line of Reflection

In Grade 9, you were introduced to line symmetry. A shape is said to have line symmetry if there is a fold line where the shape can be folded in half over it, and the image copies the original half. Generally, a line of symmetry divides a given shape into two equal halves. The line of symmetry always passes through the shape itself.

Example 1

Where is the line of symmetry in the Canadian flag?

Solution:

The Canadian flag has only one line of symmetry, a vertical line down the centre of the flag. This is also a line of reflection, since the image in the reflection is an exact copy of the original half. There is no distance between the original half and the line of reflection.

Also note you are reflecting only half of the shape, not the whole shape. However, the original half and its image together make the whole shape.

Example 2

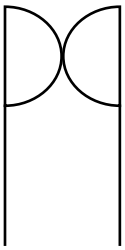
Does the letter P have line symmetry? Is there a point where you could draw a line of reflection, and the image reflected would copy the original?

Solution:

No such line can be found, so this shape would not have any line symmetry.

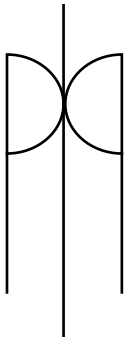
Example 3

Where is the line of symmetry?



Solution:

There would be a vertical line of symmetry through the middle of the diagram.



Combinations of Transformations

Given a basic shape and a series of transformations, translation, rotation or reflection, you can draw the resulting image.

To help visualize the transformations, you may want to trace the shape on a piece of paper, cut it out, and use that shape to slide, rotate, and reflect as required.

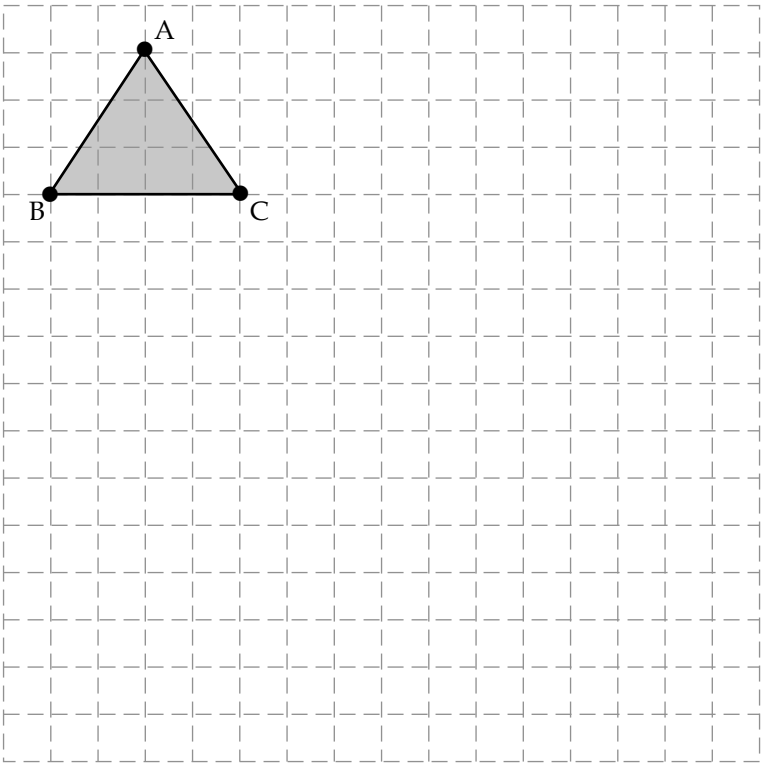
Example 1

Perform the following transformations in the order listed on $\triangle ABC$. Each successive transformation is performed on the previous image. Draw the resulting images. Try finding the result first without looking at the solution.

Step 1: Rotate 90° clockwise about point C.

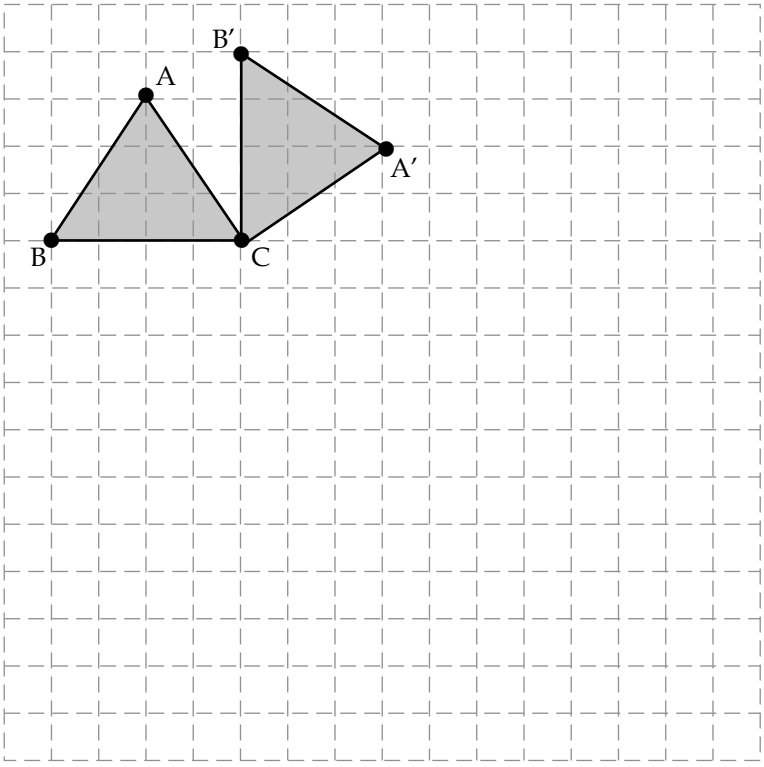
Step 2: Translate the image using the rule [R2, D1].

Step 3: Reflect the image over a horizontal line 2 squares below the shape's lowest point.



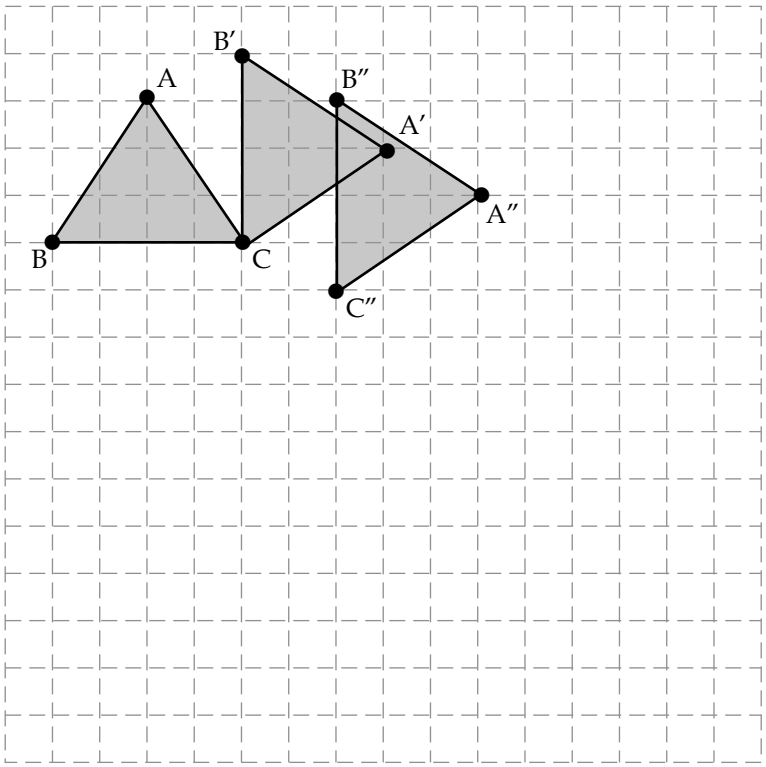
Solution:

Step 1: Rotate 90° clockwise about point C.



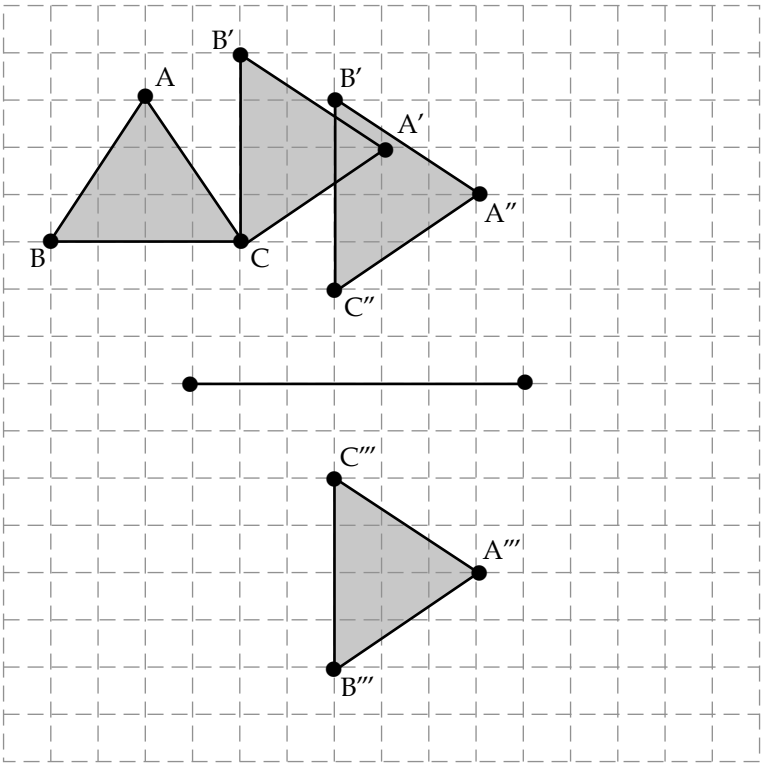
In the diagram ensure that ∠BCB′ = 90°. The rest of the triangle tags along with it. Then you can check ∠ACA′ to make sure it also is 90°.

Step 2 Translate the image using the rule [R2, D1].



Step 3: Reflect the image over a horizontal line 2 squares below the shape's lowest point.

The complete solution is shown below. Notice that C″C‴ is perpendicular to the line of reflection. Also note that C″ is 2 units above the reflection line and C‴ is 2 units below the reflection line. The same process can be used for finding the image of B″ and A″.



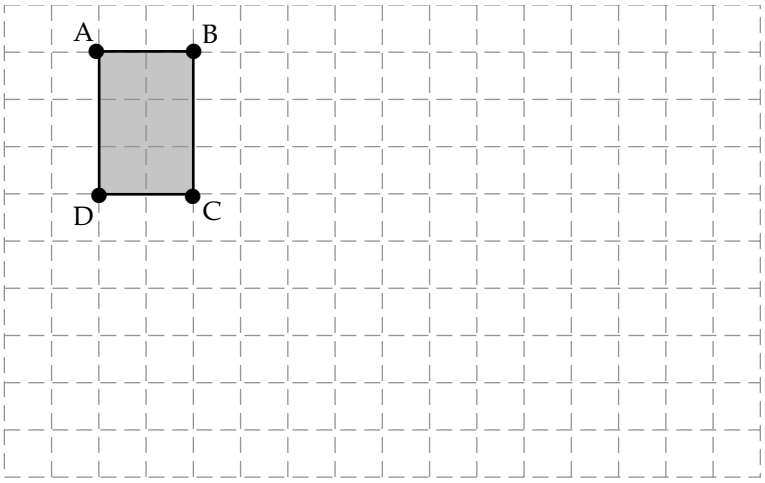
Example 2

Perform the following transformations, in the order listed, on rectangle ABCD. Each successive transformation is performed on the previous image. Try finding the result first without looking at the solution.

Step 1: Slide [R2, D3].

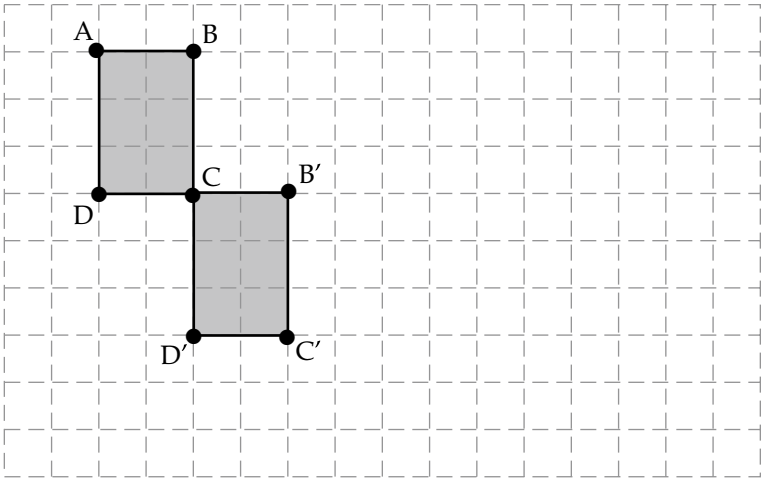
Step 2: Rotate about point B′ 90° clockwise.

Step 3: Reflect over a vertical line 3 units to the right of the previous image.

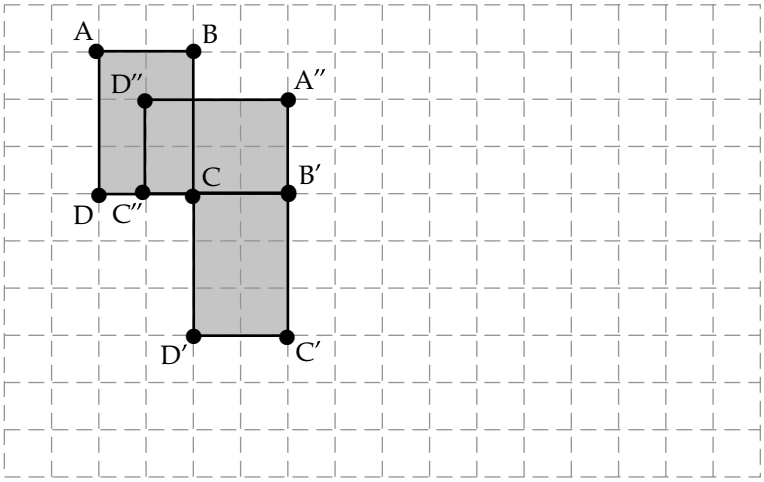


Solution:

Step 1: Slide [R2, D3].

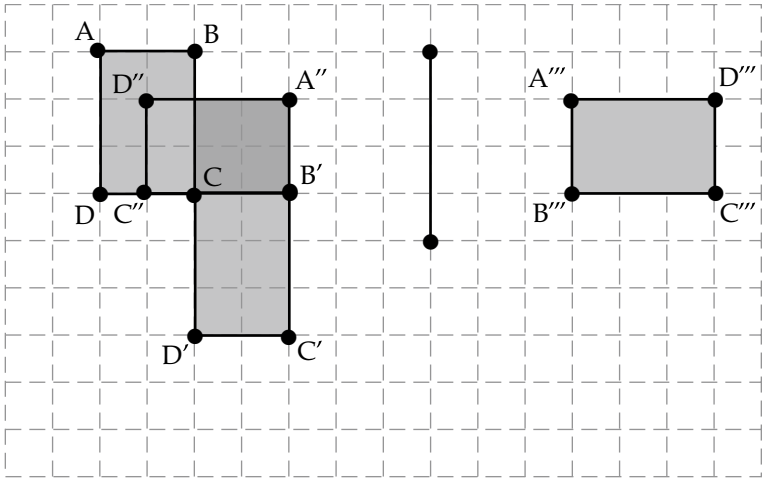


Step 2: Rotate about point B′ 90° clockwise.



Step 3: Reflect over a vertical line 3 units to the right of the previous image.

The final result is shown below. Notice that you are asked to reflect only the previous image, not all the rectangles.



Learning Activity 8.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Audrey reads 2 pages per minute. The latest book she is reading is 720 pages long. How long will it take her to read it?
2. What is the complementary angle to 41°?
4. You work 6.5 hours per day, 6 days per week. How many hours per week do you work?
4. Your younger brother tells you that he is 100 cm tall. Estimate how tall he is in inches (1 inch = 2.54 cm)
5. You are given $3.32 change for buying breakfast at Moonbucks. If you gave the cashier a $10 bill and your total was $6.78, did you get the correct change?

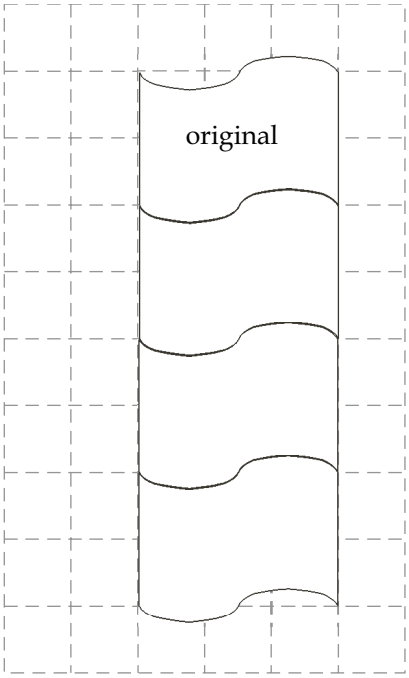
continued

Learning Activity 8.5 (continued)

Part B: Multiple Transformations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

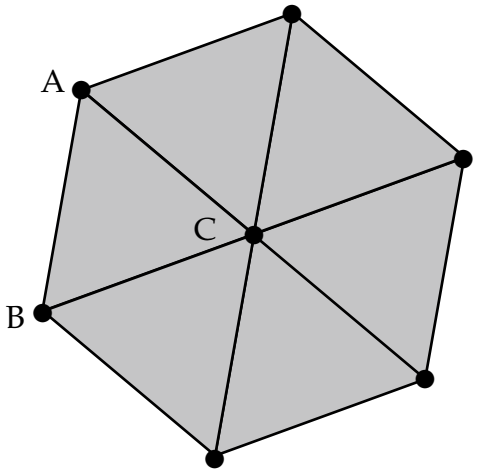
1. Identify the series of transformations applied to the original shape to create the larger shape. Each successive transformation is performed on the previous image.



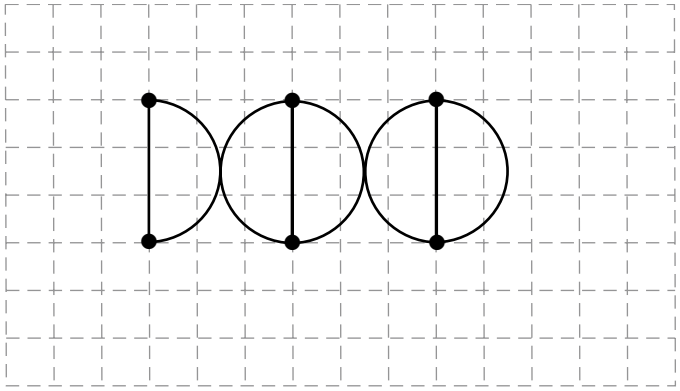
continued

Learning Activity 8.5 (continued)

2. Identify the transformations applied to ΔABC to form this six-sided shape. ΔABC is an equilateral triangle with all sides equal and all angles equal to 60°. Each successive transformation is performed on the previous image.



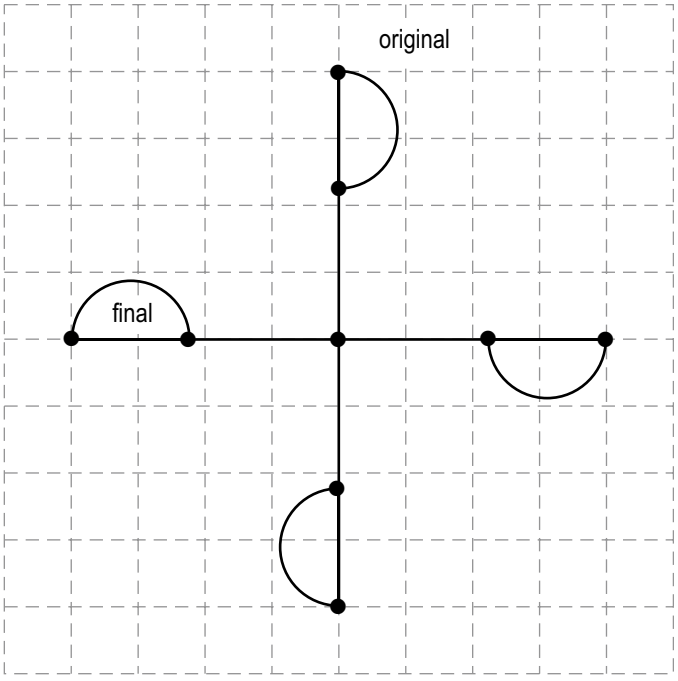
3. Identify the transformations to move the position of the original shape, the letter D on the left, to have the result on the right side at the end. Each successive transformation is performed on the previous image.



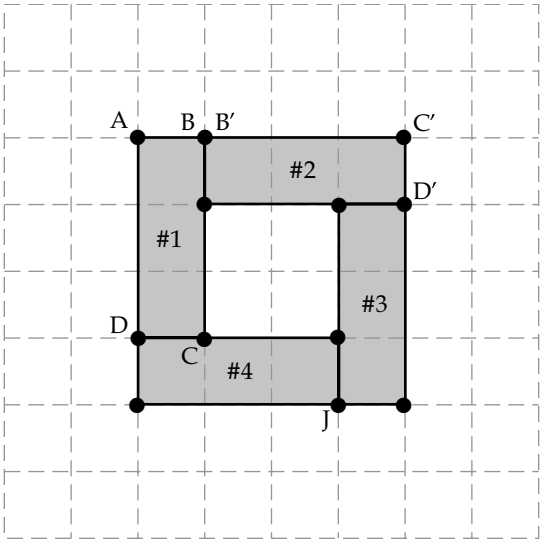
continued

Learning Activity 8.5 (continued)

4. Identify the series of transformations to the original shape. Each successive transformation is performed on the previous image.



5. Identify the transformations done in the order shown in the diagram, starting with rectangle ABCD, and resulting in rectangle #4. Each successive transformation is performed on the previous image.



Lesson Summary

This lesson focused on lines of symmetry and combining a series of transformations to a shape where each successive transformation was performed to the previous image.

If you are having a hard time remembering which way is left and which is right, one way to check is to make an L with your thumb and pointer finger. The one that looks like a real L is the left, the backwards L is the right. Alternately, you may want to mark the left corner of your resource sheet with an L and the right corner with an R so that you can refer back to it if you get confused. If you label your resource sheet, make sure you ask your learning partner to make sure that you have labelled it correctly.

The next lesson is about the fourth transformation, dilation.

Assignment 8.4

Total: 19 marks

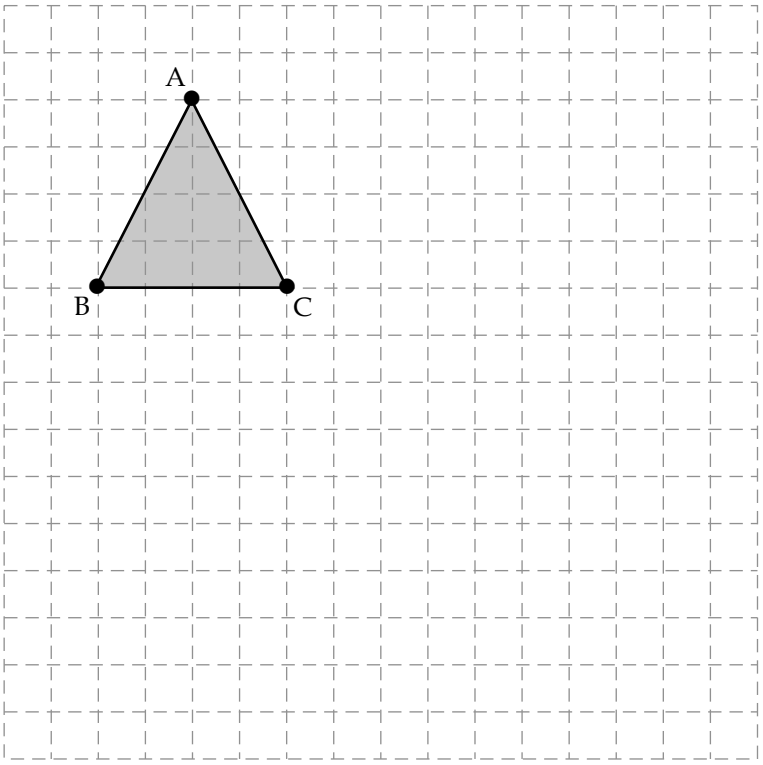
Combinations

1. Draw the listed combination of transformations for each shape. Each successive transformation is performed on the previous image. *(9 marks)*

   a) Step 1: Rotate 180° about point C.

   Step 2: Reflect over a horizontal line touching point A′.

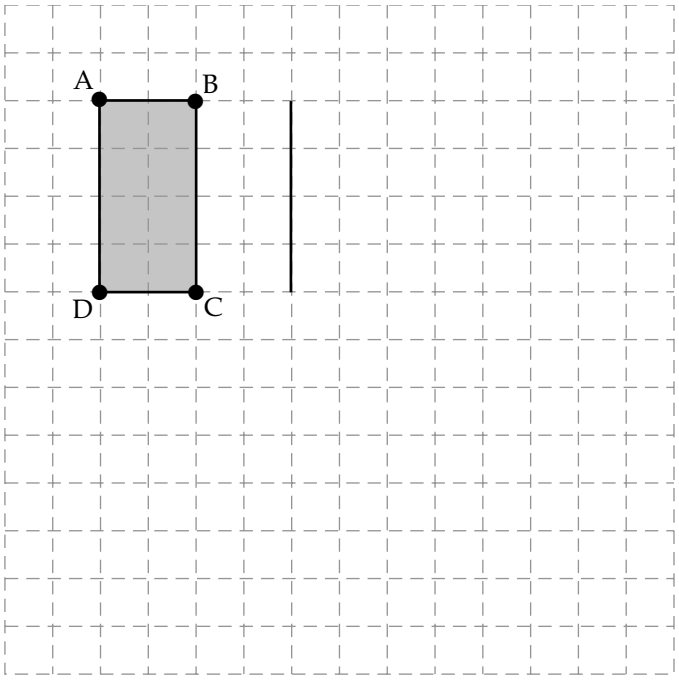
   Step 3: Translate [L3, U2].



continued

Assignment 8.4: Combinations (continued)

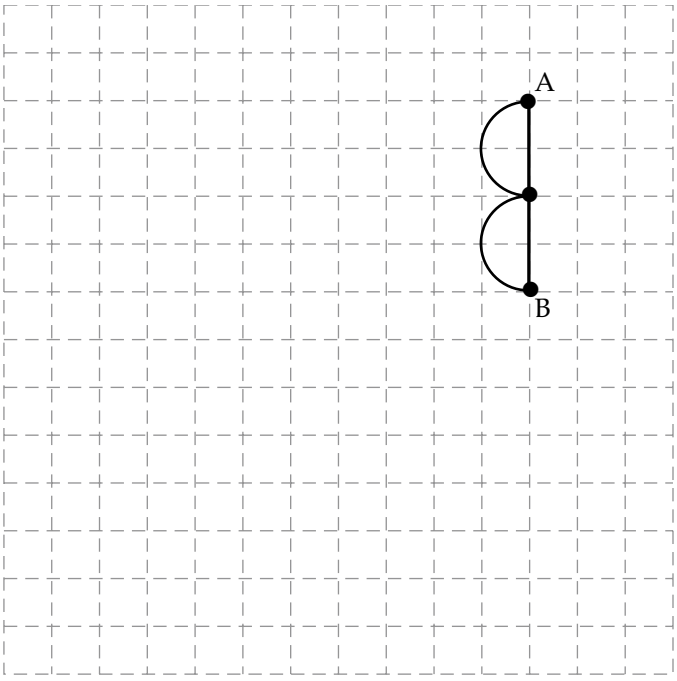
b) Step 1: Reflect the rectangle over the line of reflection.
Step 2: Rotate the image 90° counter-clockwise about point D′.
Step 3: Translate the image [L4, D1].



continued

Assignment 8.4: Combinations (continued)

c) Step 1: Rotate shape 180° about point B
Step 2: Reflect over the vertical line segment B'A'
Step 3: Translate [L4, U4]

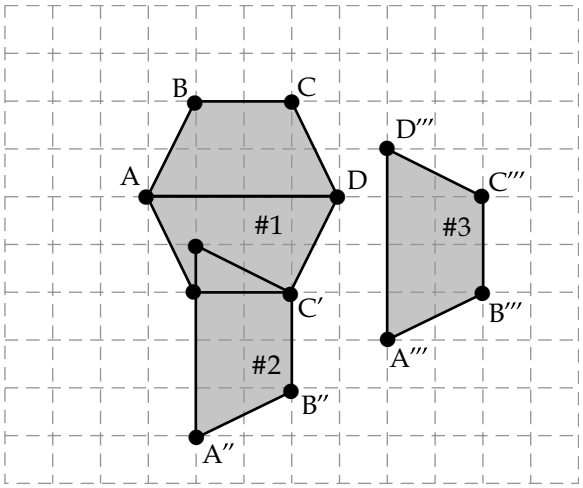


continued

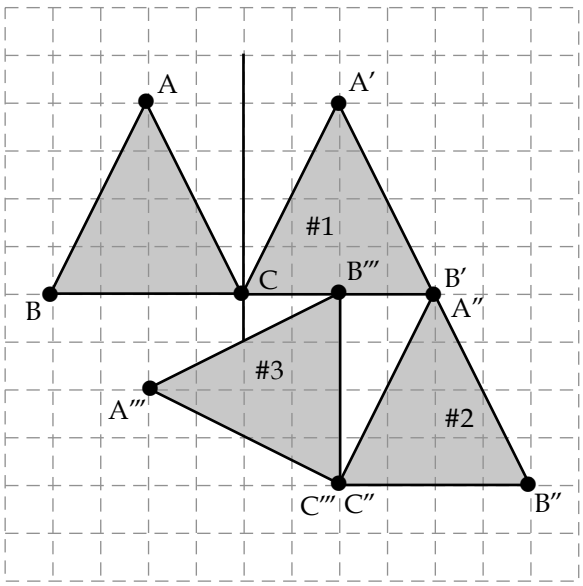
Assignment 8.4: Combinations (continued)

2. Identify the combined transformations that occurred to get the result from the original diagram.

   a) The original shape is trapezoid ABCD. *(3 marks)*



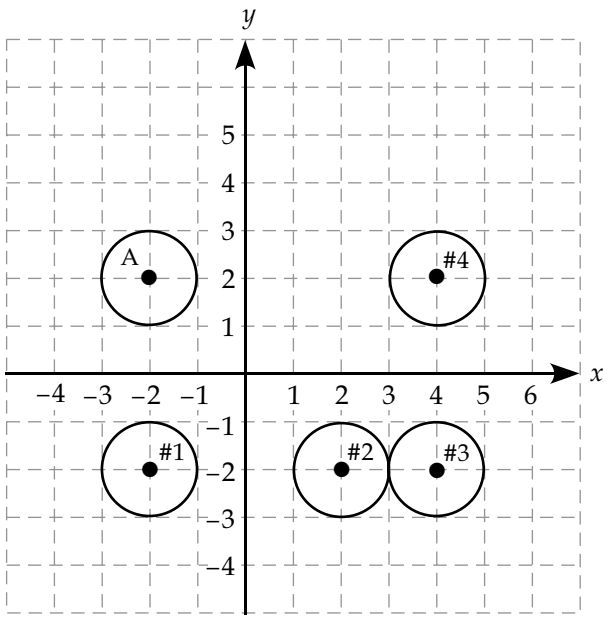
   b) The original shape is ΔABC. *(3 marks)*



continued

Assignment 8.4: Combinations (continued)

c) The original is circle A. *(4 marks)*



Notes

Lesson 5: Dilations

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of dilations
- ❑ determine whether an image is a dilation of another shape

Lesson Introduction

This lesson uses the properties of similarity to show the transformation called dilation.

What is Dilation?

Dilation is a similarity transformation in which a figure is enlarged or reduced using a scale factor other than zero. The centre of the dilation remains fixed where it is for the original shape and its image. You may have heard this word before when talking about your eyes. Your pupil (the black part in the middle) becomes larger in the dark in order to allow more light into your eye, and it becomes smaller in the light so that your eye isn't damaged.

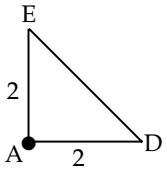
Add your own description of dilation to your resource sheet.

Centre

The centre of dilation is not necessarily in the middle of the dilation. The centre of the dilation is the name given to the point shared by both the original shape and the dilation.

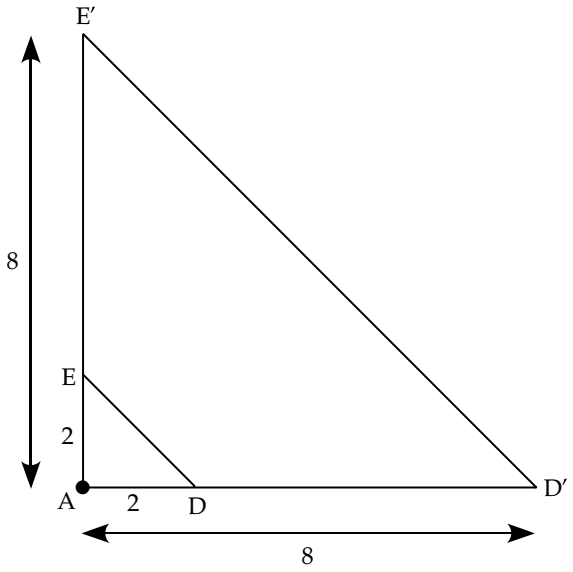
Example 1

Enlarge ΔADE to ΔABC using a scale factor 4. Point A is the centre of dilation.



Solution:

Since two sides of ΔADE are each 2 units, and since the scale factor is 4, multiply 2 × 4 = 8 to get the length of two sides of ΔAD′E′. Since A is the centre of dilation, the sides AD and AE are extended to a length of 8 units. The hypotenuse of the triangle just tags along with the rest of the triangle. The result of the dilation is shown in the diagram below.



Looking closely, side AD′ is 4 times the length of side AD and side AE′ is 4 times the length of side AE.

If the shape is truly similar, then the hypotenuse, E′D′ will be proportionately the same as ED. In other words 4(ED) = E′D′. (Notice also, that the hypotenuse for the image is parallel to the hypotenuse in the original. ∠E′D′A = ∠EDA and they are corresponding angles for parallel lines.)

You can say ΔAD′E′ is similar to ΔADE. The concept of similar triangles was discussed earlier in the course.

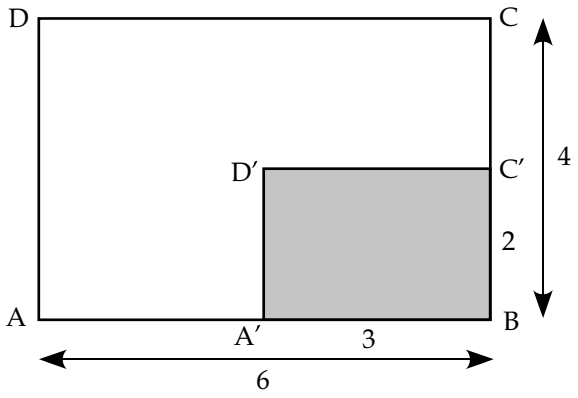
The new terminology in this lesson is dilation. This transformation states that ΔAD′E′ is a dilation of ΔADE by a scale factor of 4.

Dilation of a shape is a transformation using similarity and proportions. You need to be given the scale factor and the centre of dilation.

Dilation can also make a shape smaller.

Example 2

Given rectangle ABCD, determine if the image is dilation. If so, state the centre of dilation and the scale factor.



Solution:

The original rectangle, ABCD has length BA = 6 units. The corresponding side on the image is BA′, and has a length of 3 units. This means the scale factor for these two sides is $\frac{3}{6}$ or $\frac{1}{2}$.

The width of the original rectangle is BC and has a length of 4 units. The corresponding side of the image is BC′, and has a length of 2 units. This means the scale factor for these two sides is $\frac{2}{4}$ or $\frac{1}{2}$.

Because two sets of corresponding sides have the same scale factor, you can state the shape and its image are similar. You can also state that the image is a dilation of the original shape, by a scale factor of $\frac{1}{2}$ and the centre of dilation is point B.

Notice that the scale factor is the ratio of $\frac{\text{image of length}}{\text{original length}}$. Sometimes this scale factor is greater than one; sometimes it is less than one.

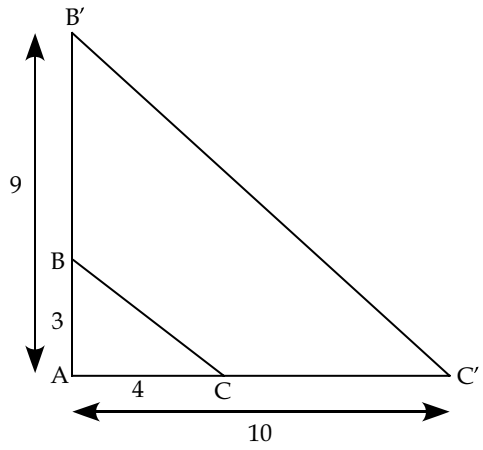
When the scale factor of dilation is greater than 1, the dilation is an enlargement. When the scale factor of dilation is less than 1, the dilation is a reduction.

Example 3

Determine if the image, ΔAB′C′, is an enlargement dilation of the original shape, ΔABC.

Solution:

Given ΔABC and image ΔAB′C′ with point A being the centre of dilation as shown in the diagram.



If the corresponding sides are proportional, then the triangles are similar and the image would be a dilation of the original. Since A is the centre of dilation, the similarity relationship would be ΔABC ~ ΔAB′C′.

To check for proportionality, you could set up the ratios of corresponding sides.

ΔABC	AB = 3	BC = ?	AC = 4
ΔAB′C′	AB′ = 9	B′C′ = ?	AC′ = 10

In this diagram, if two sets of sides are proportional, the third set will be also.

Are the ratios equal?

$$\frac{3}{9} = \frac{4}{10}$$

When you cross multiply, how does (3)(10) compare to (4)(9)?

Since (3)(10) = 30 and (4)(9) = 36, these ratios are not equal and the sides are not proportional. This is not a dilation.

Another way to look for similarity of triangles or proportional sides is to find the scale factor.

Side AB has been multiplied by $\frac{9}{3}$ or 3. AB′ is 3 times bigger than AB. The scale factor is 3.

Side AC has been multiplied by $\frac{10}{4}$ or 2.5. AC″ is 2.5 times bigger than AC. The scale factor is 2.5.

Since these scale factors are not equal, the sides are not proportional and the triangles are not similar.

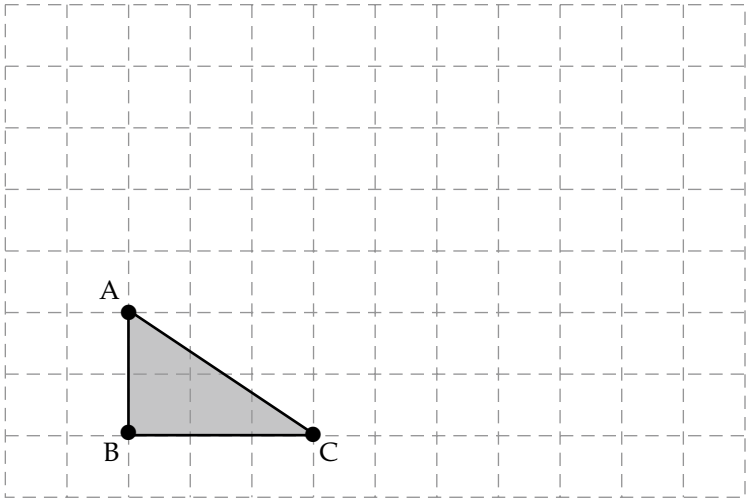
ΔAB′C′ cannot be a dilation of ΔABC because the sides are not proportional.

Drawing a Dilation

Now you can use your understanding of scale factors to draw a dilation of a shape. Just apply the scale factor to each side of the original shape and keep the centre of the dilation fixed.

Example 1

a) Draw a dilation of ΔABC with a scale factor of 3, using B as the centre of dilation.



Solution:

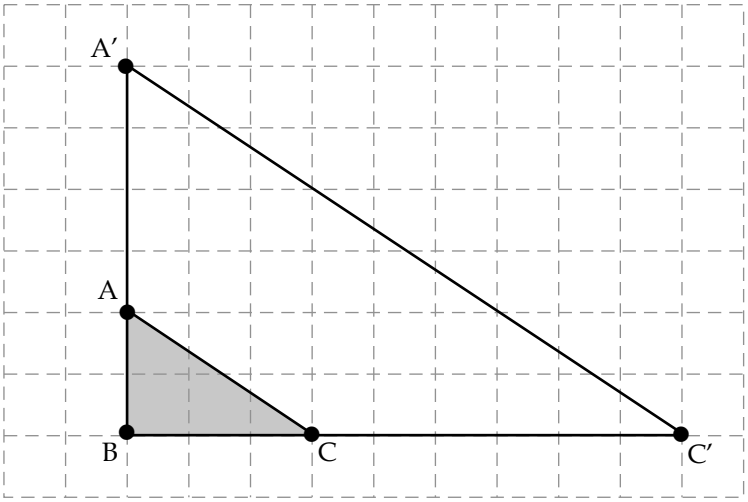
The centre of dilation is B. Thus, extend sides BA and BC by a scale factor of 3 to get BA′ and BC′.

From the graph paper, BA = 2 units and BC = 3 units.

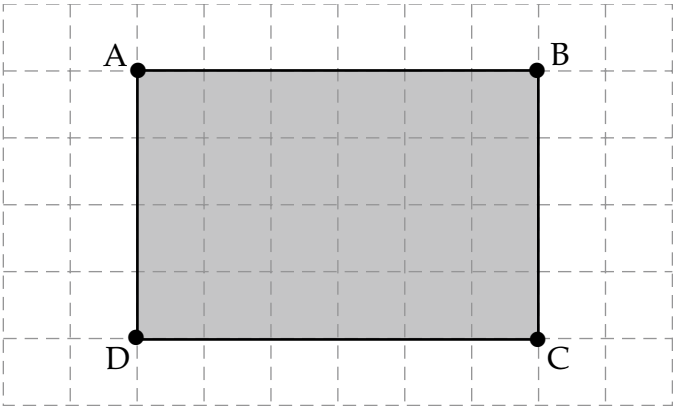
Thus, BA′ = 3 × 2 = 6 units and BC′ = 3 × 3 = 9 units.

Join A′ and C′ to complete the dilation.

ΔA′BC′ is a dilation of ΔABC because the sides are proportional.



b) Draw a dilation of the rectangle ABCD with a scale factor of 0.5, using B as the centre.



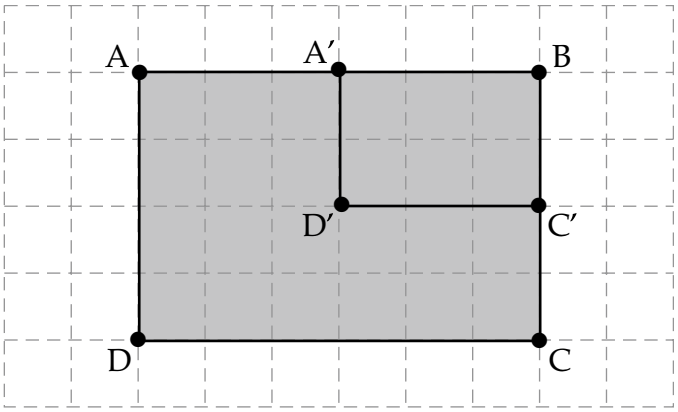
Solution:

Since B is the centre of dilation, then the sides BA and BC need to be changed by a factor of 0.5. From the graph paper, BA = 6 units and BC = 4 units.

Thus, BA′ = 6 × 0.5 = 3 units and BC′ = 4 × 0.5 = 2 units.

Mark the two points, A′ and C′, draw the image rectangle, and name its fourth point D′.

Rectangle A′BC′D′ is a dilation of rectangle ABCD because the sides are proportional.



Learning Activity 8.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. There is a big sale at your favourite store. Everything is 30% off. Your bill before the discount is $200. How much will you pay after the discount?
2. Solve for b: $\frac{b}{8} = \frac{6}{20}$.
3. There are 3 teams in a tournament. Team A wins against Team B. Team B loses to team C. Team C loses to Team A. Does any team win both their games?
4. The map of your mom's office has a scale of 1 cm: 3 m. Your mom's office is
   1 cm by 1 cm. What is the area of your mom's office in real life?
5. Rotation 90° clockwise is the same as rotating counter-clockwise ____°?

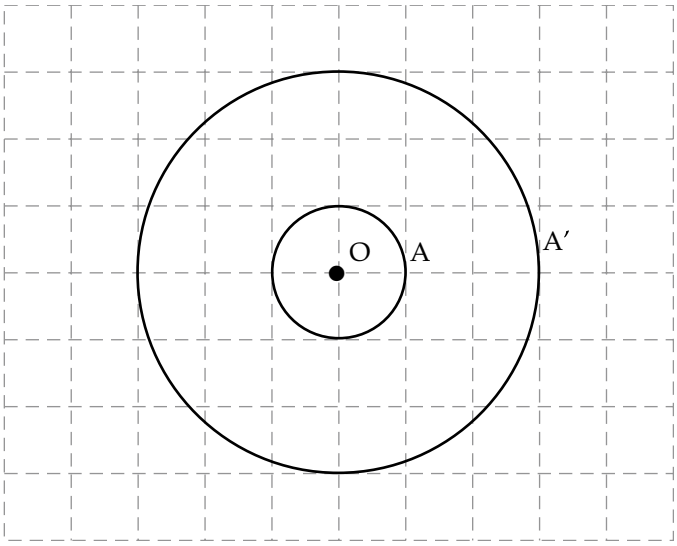
continued

Learning Activity 8.6 (continued)

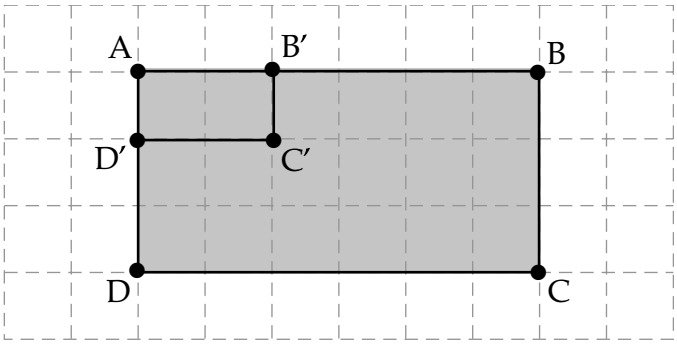
Part B: Dilation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Determine the scale factor of each dilation.
   a) Circle A′ is a dilation of circle A with the common centre at O.

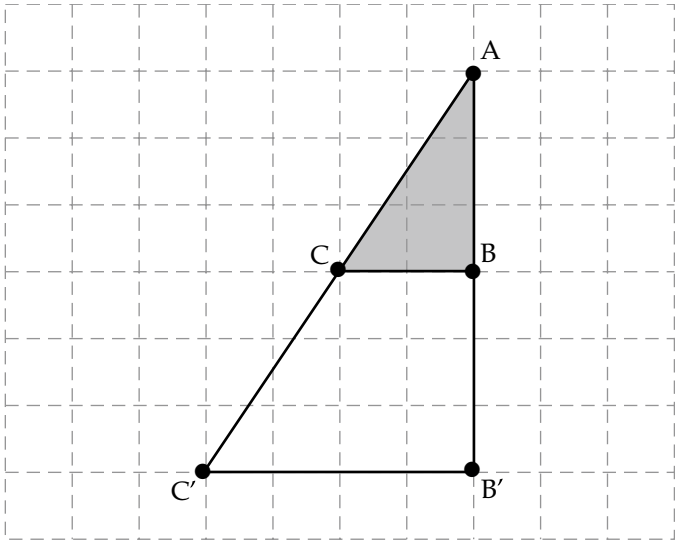


   b) Rectangle AB′C′D′ is a dilation of rectangle ABCD with point A as the centre of dilation.



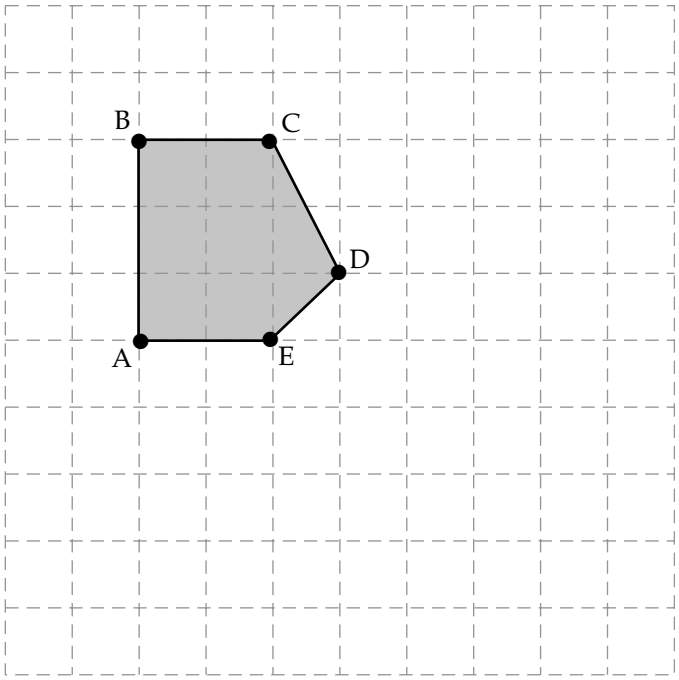
continued

c) $\Delta AB'C'$ is a dilation of ΔABC with point A as the centre of dilation.



2. Draw the dilation given the scale factor and centre of dilation.

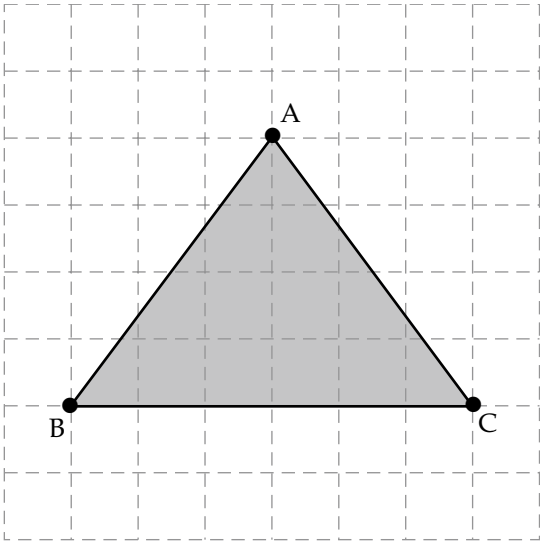
a) Scale factor 2 using point B as the centre of dilation.



continued

Learning Activity 8.6 (continued)

b) Scale factor 0.5 using point B as the centre of dilation.



Lesson Summary

This lesson demonstrated the dilation transformation. You learned new terminology, such as centre of dilation, scale factor, enlargement, and reduction. You reviewed similar triangles and proportions. You drew dilations and determined scale factors from the dilations. You learned a formula for calculating scale factor, and you learned how to determine if the image of a shape was actually a dilation.

The next lesson in this unit is about identifying and describing transformations.

Notes

Assignment 8.5

Total: 19 marks

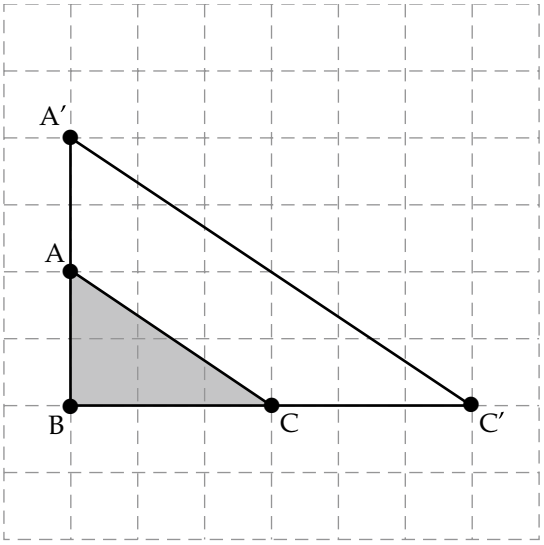
Dilations

1. An image is only a true dilation of a shape if the corresponding sides are _____. *(1 mark)*

2. An enlargement dilation would have a scale factor greater than _____. *(1 mark)*

3. Determine the scale factor and the centre of dilation of the given shape and its dilation. *(8 marks)*

   a) Given ΔABC as the original shape



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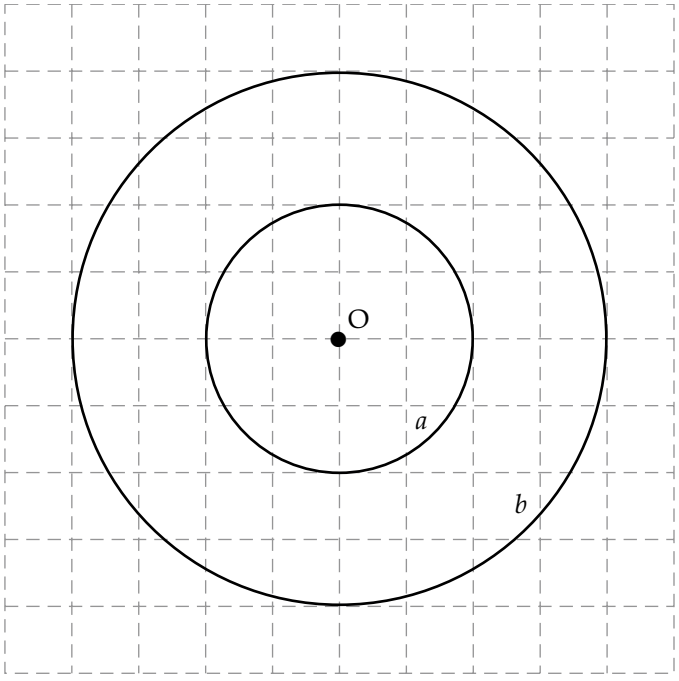
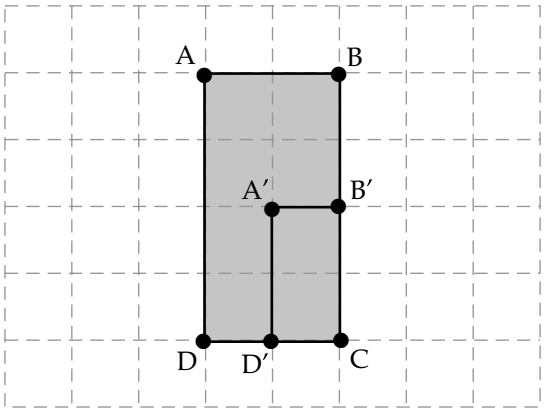
Assignment 8.5: Dilations (continued)

b) Given Circle *b* as the original shape



c) Given rectangle ABCD as the original shape



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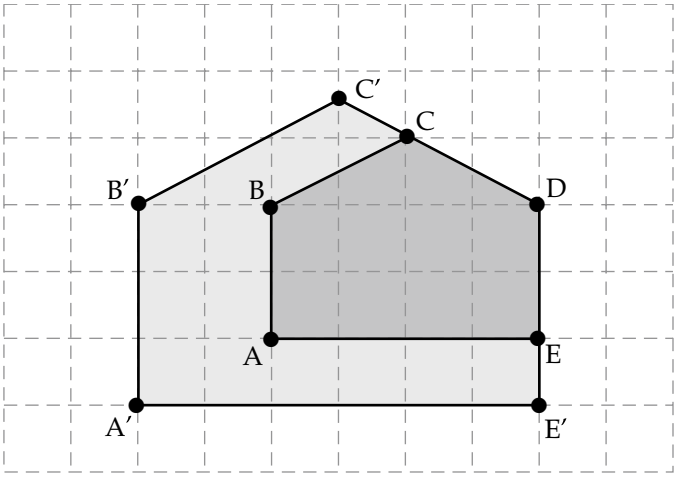
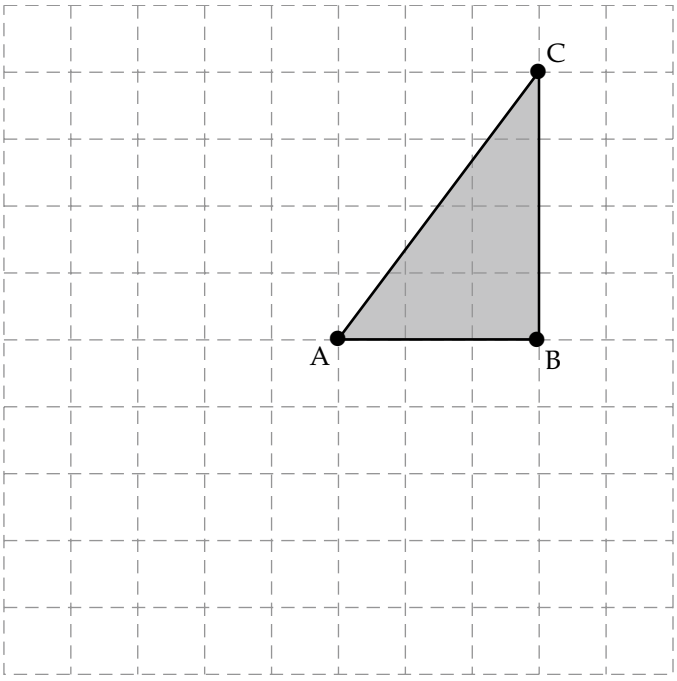
Assignment 8.5: Dilations (continued)

d) Given pentagon ABCDE as the original shape



4. Given the original shape, scale factor, and centre of dilation, draw the dilation. *(9 marks)*

a) Scale factor 2, centre of dilation C



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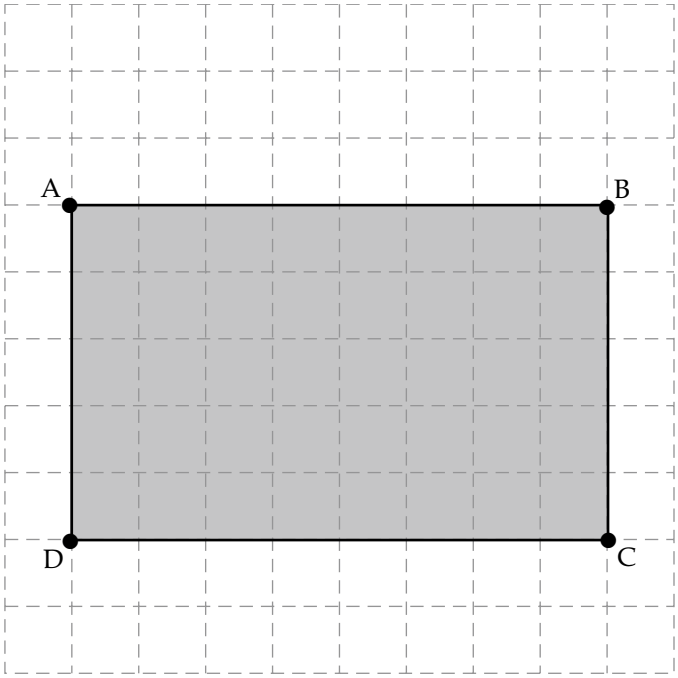
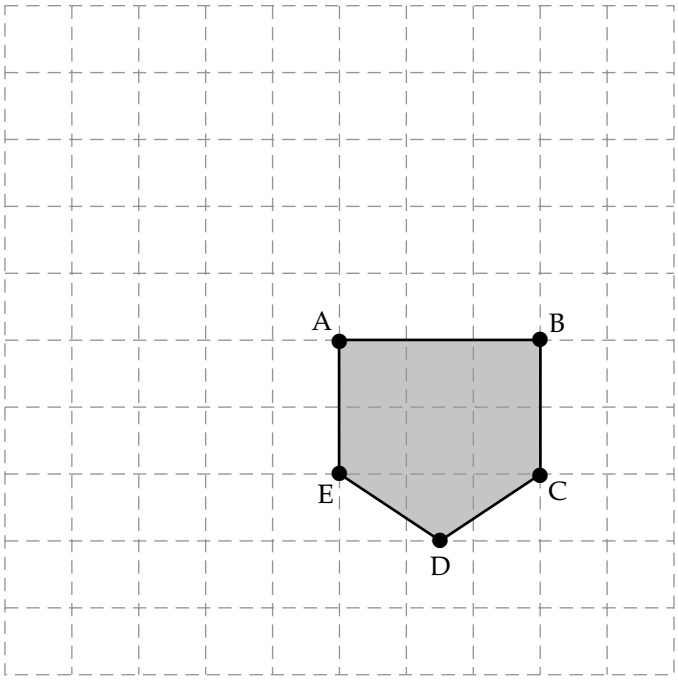
Assignment 8.5: Dilations (continued)

b) Scale factor 0.5, centre of dilation D



c) Scale factor 2, centre of dilation C



LESSON 6: APPLICATIONS

Lesson Focus

In this lesson, you will

❑ identify and describe applications of transformations

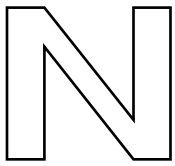
Lesson Introduction

This lesson is about how to make patterns from transformations.

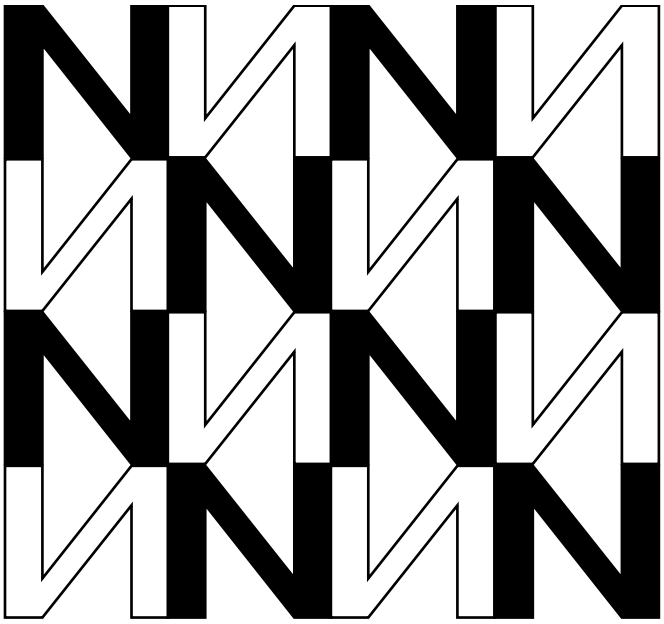
Using Transformations

Tessellation

If you take a basic shape and apply various transformations to it so that a larger shape is created with no overlaps and no gaps, you call this larger shape a tessellation.



For example, the letter N is repeated several times in the diagram below. When you examine this design, you can identify translations, rotations, and reflections. However, there are gaps so this is not a true tessellation.



Art and Tessellations

Many art creations have tessellations.

Example 1

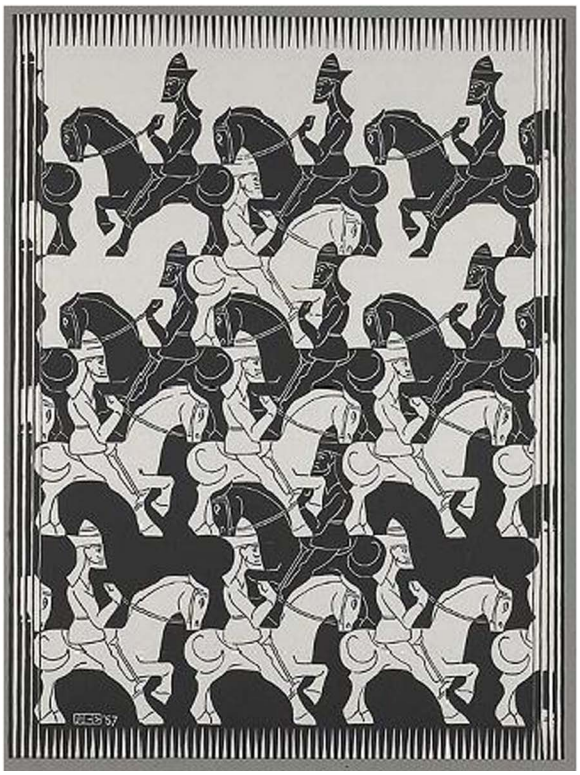
The same basic shape, a cat, is tessellated repeatedly to create a design. Can you identify each of the three transformations that are present in the design?

Solution:

Translation and rotation are present, but not reflection. You can check by looking at the faces. If you look at it "right side" up, the black eye is always on the left. If it were reflected, the black eye would flip to the right.

Example 2

The Dutch artist, M. C. Escher created this painting in 1957. He was inspired to make drawings using tessellations after studying the tile designs in a castle in Spain.



Look closely at the drawing. There are no gaps or overlaps. You should be able to identify translations and reflections.

Ambulances

Have you ever noticed how the word "AMBULANCE" painted on the very front of an ambulance is written backwards?

It is painted this way so that car drivers in front, when looking in their rear view mirrors, can see the word "AMBULANCE." The drivers in front of the ambulance are reading the reflection of the word and can quickly move to the right side of the road and stop until the ambulance passes by.

If you have a mirror handy, hold it up in front of this word, and see the word reflected so that you can read it.

Quilting

There are some wonderfully creative designs in quilts. Some quilters take a lot of time to plan out the overall design. Many use tessellations in their patterns.

Example 1

Look closely at all the little triangles that have been rotated around the circumference of an oval. Each oval with its triangles has been translated. Can you imagine the lines of reflection where one shape is reflected onto another?

Example 2

This is a basic design for a quilt, yet it includes translations and reflections.

Construction

The easiest place to find tessellations in the construction industry is on the wall of your school. Even the ceiling tiles are tessellations.

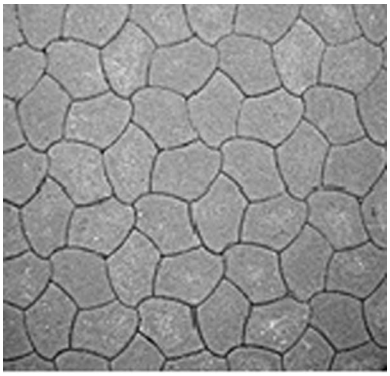
Example 1

Cinder blocks or bricks are used to build many schools. Take a closer look at the picture of the brick wall in the diagram below.

Place a mirror along any of the vertical joints in this picture of the wall, and the reflection sits directly over the original. Also, you can see translations here, where each brick is moved up and over the same amount each time. Cinder block and brick walls constructed this way have more strength and stability.

Example 2

Another place in the construction world where tessellations are found is in paving stone patterns for driveways or patios. In the diagram below, each stone is the same shape. The stones are turned, slid, and flipped to create a tessellation.

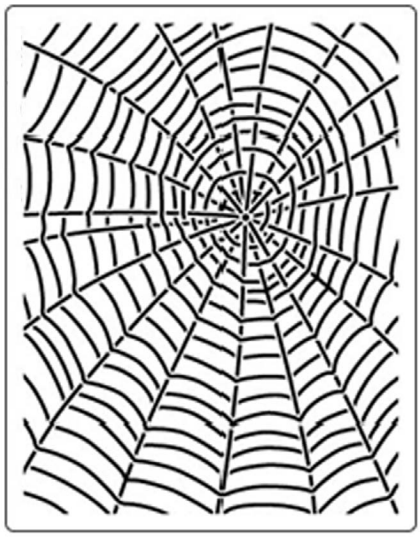


Nature

Many patterns and artistic inspiration come from nature.

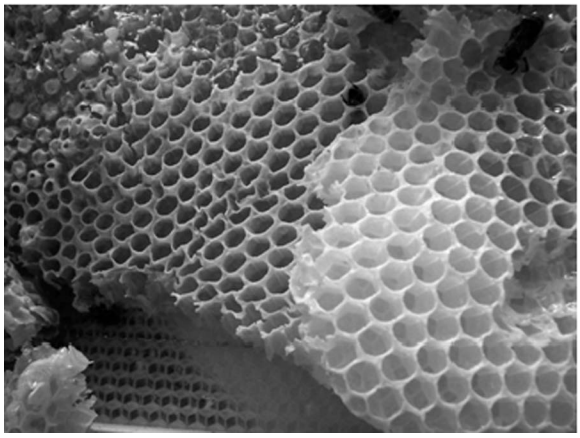
Example 1

Study this picture of a spider web. Can you see any transformations? The same shape is definitely repeated. One could argue that each is a dilation of the previous shape.



Example 2

Observe this picture of a honeycomb from a bee hive. All three transformations are represented here.



Learning Activity 8.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The dimensions of Edward's yard are 20 m by 60 m. How much fence will Edward need if he is building a fence around his yard?
2. You take approximately 5 minutes to answer long-answer questions and 1 minute to answer short-answer questions. Your class is 60 minutes long. Do you have enough time to write a test with 15 short answer questions and 8 long-answer questions?
3. You are filling up your scooter with gas that costs $1.00 per litre. If your scooter's gas tank holds 8 L and you've still got 1.5 L in the tank, how much will it cost you to fill up?
4. In order for a rotation to appear exactly the same as the original, how much must you rotate the original object (in degrees)?
5. Evaluate: $\frac{4}{7} \times \frac{8}{3}$.

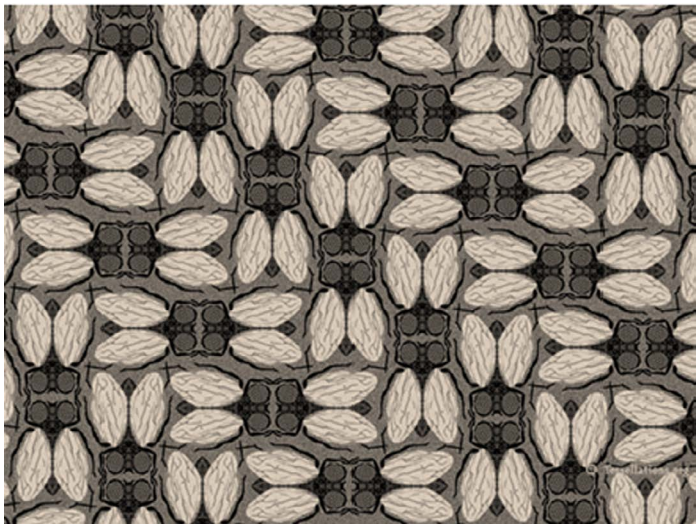
continued

Learning Activity 8.7 (continued)

Part B: Applying Transformations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

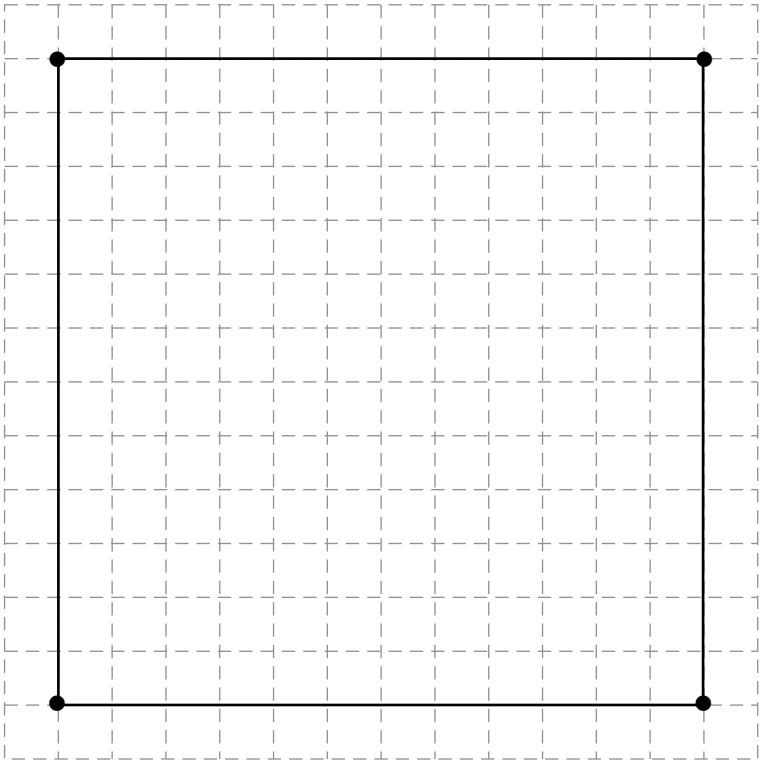
1. Here is a design using repeated pictures of a fly. Circle and label an example of each transformation, translation, rotation, and reflection.



continued

Learning Activity 8.7 (continued)

2. You have been hired to design a decorative stone pattern for an area measuring 2 feet by 2 feet. The rectangular stones you will use measure 2″ by 6″. You are not allowed to have more than 3 stones positioned the same way at a time. Draw a layout of how you will arrange the stones, and count the total number of stones you will use. Each square represents 2″. Use the graph paper below.



Lesson Summary

This concludes your work with transformations. In this lesson, you studied many applications of transformations and practised identifying them and drawing designs.

Notes

Assignment 8.6

Total: 8 marks

Applications

This assignment is a project.

You are to complete only one of the following questions and submit it with your assignments. You will find the marking rubric on the following page.

1. Create a tessellation in any manner including drawing, cutting and pasting, or using technology. Write an explanation describing your tessellation in terms of translations, rotations, and reflections.
2. Find at least two pieces of art in print or online that are tessellations, and explain them in terms of translations, rotations, and reflections. Include a photocopy of the diagram with your answer. Do not tear the original out of a book.

Module 8 Assignment 8.6
Rubric for Applications Assignment

Score Level	Tessellation	Translation	Reflection	Rotation
2	A tessellation is created, which includes no gaps or overlaps. OR Two pieces of tessellation art are provided with appropriate referencing	A detailed description of the transformation is provided, which includes appropriate mathematical terms	A detailed description of the reflection is provided, which includes appropriate mathematical terms.	A detailed description of the rotation is provided, which includes appropriate mathematical terms.
1	A tessellation is created, which includes some gaps and/or overlaps. OR One piece of tessellation art is provided with appropriate referencing. OR Two pieces of tessellation art are provided without appropriate referencing.	A mathematically unclear or incomplete description of the translation is included.	A mathematically unclear or incomplete description of the reflection is provided.	A mathematically unclear or incomplete description of the rotation is provided.
0	A tessellation is created, which includes many gaps and/or overlaps. OR No pieces of tessellation art are provided. OR One piece of tessellation art is provided without appropriate referencing.	A translation is not included. OR A translation is not described.	A reflection is not included. OR A reflection is not described.	A rotation is not included. OR A rotation is not described.
Mark	Tessellation = ____ / *(2 marks)*	Translation = ____ / *(2 marks)*	Reflection = ____ / *(2 marks)*	Rotation = ____ / *(2 marks)*
Total Mark	Tessellation + Translation + Reflection + Rotation = _____			Total Possible: 2 + 2 + 2 + 2 = 8

Module 8 Summary

In this module, you learned five different transformations. You learned how to identify them, to draw them, and to use them to make patterns. You also looked at various applications in art, nature, and construction.

Congratulations! You have finished all the modules.

Vocabulary

Here is a list of math words that were used in this module. Students are not being asked to write a definition of the words on the examination, but you need to know the meanings of them in order to complete the questions. Perhaps making a note on your resource sheet for each of the words you don't understand would be helpful.

2-D	origin
centre of dilation	quadrant
centre of rotation	reduction
diagonal line	reflection
dilation	rotation
enlargement	slide
flip	tessellation
flop	transformation
horizontal line	translation
image	turn
image notation	vector
iteration	vertical line
line of reflection	x-axis
line of symmetry	y-axis

Remember that a glossary is provided in Appendix B found after Module 8.

Formulas

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

Submitting Your Assignments

It is now time for you to submit the Module 8 Cover Assignment and Assignments 8.1 to 8.6 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 8 assignments and organize your material in the following order:

Module 8 Cover Sheet

Module 8 Cover Assignment: Nested Shopping Carts

Assignment 8.1: Translations

Assignment 8.2: Rotations

Assignment 8.3: Reflections

Assignment 8.4: Combinations

Assignment 8.5: Dilations

Assignment 8.6: Applications

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Final Examination

Congratulations, you have finished Module 8 in the course. The final examination is out of 100 marks and worth 12.5% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 5 to 8.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Final Examination Resource Sheet. A maximum of 2.5 hours is available to complete your final examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point you will also have to combine your resource sheets from Modules 5 to 8 onto one 8½" × 11" paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you. We suggest that you divide your paper into two quadrants on each side so that each quadrant contains information from one module.

This is a sample:

Front

Module 5
Module 6

Back

Module 7
Module 8

Examination Review

You are now ready to begin preparing for your final examination. Please review the content, learning activities, and assignments from Modules 5 to 8.

The final practice examination is also an excellent study aid for reviewing Modules 5 to 8.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the final examination determines 12.5% of your final mark in this course and you will have 2.5 hours to complete the examination.

Final Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The final practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your final practice examination, follow these steps:

1. Study for the practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 5 to 8 that you found most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 5 to 8, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Final Examination Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following things to the final practice examination: pens and/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Final Examination Resource Sheet.

7. Write your final practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the examination is 2.5 hours.

8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.

9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Final Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes

Grade 10 Essential Mathematics (20S)

Module 8
Transformations

Learning Activity Answer Keys

MODULE 8:
TRANSFORMATIONS

Learning Activity 8.1

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. What is the formula for tangent?
2. The sides of a right triangle are 85, 36, 77. What are the two possible tangent ratios of this triangle?
3. A circle is divided into 360°. How many degrees are in a semicircle (half-circle)?
4. You are paid $15 per hour. If you work 20 hours per week, how much money will you earn in a week?
5. You would like to get your poster framed. The area of the poster is 4500 cm^2. If the height of the poster is 0.3 m, what is the length in centimetres?

Answers:

1. $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

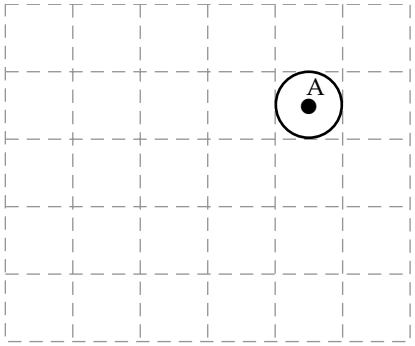
2. $\dfrac{36}{77}$ or $\dfrac{77}{36}$ (The two legs are 36 and 77 (since 85 is the hypotenuse). There are two acute angles that we could be 'looking' from, so one is the opposite and the other is adjacent, depending upon which angle is used. The tangent ratios could be $\dfrac{36}{77}$ or $\dfrac{77}{36}$.)

3. 180° (360° ÷ 2 = 180°)
4. $300 (15 × $20)
5. 150 cm (4500 ÷ 30 = 150 cm; 0.3 m = 30 cm)

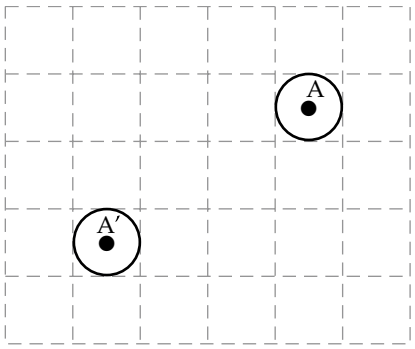
Part B: Single Translations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

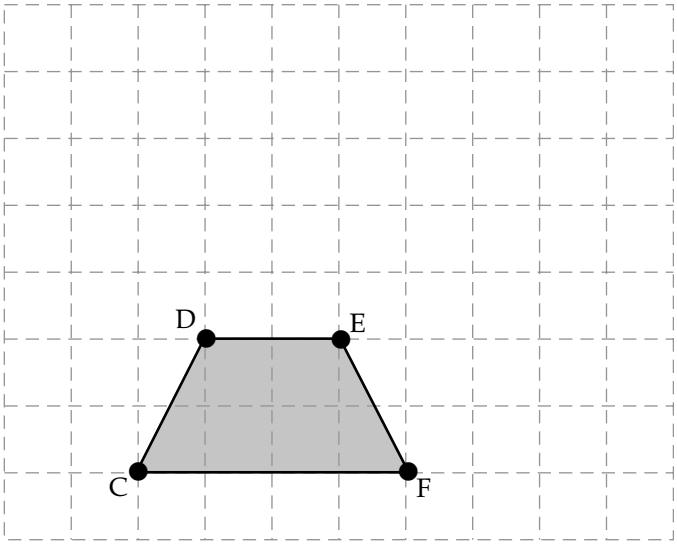
1. Draw the translations as instructed.
   a) Slide the circle with centre at A, 3 units left and 2 units down.



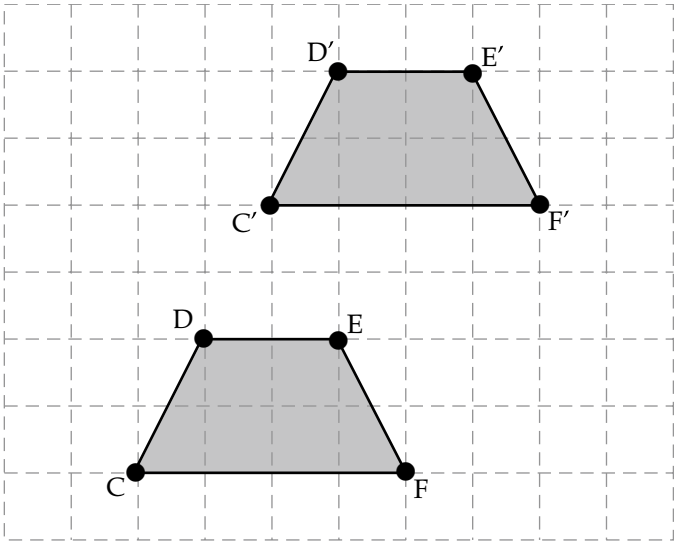
Answer:



b) Slide the trapezoid using the rule [R2, U4].

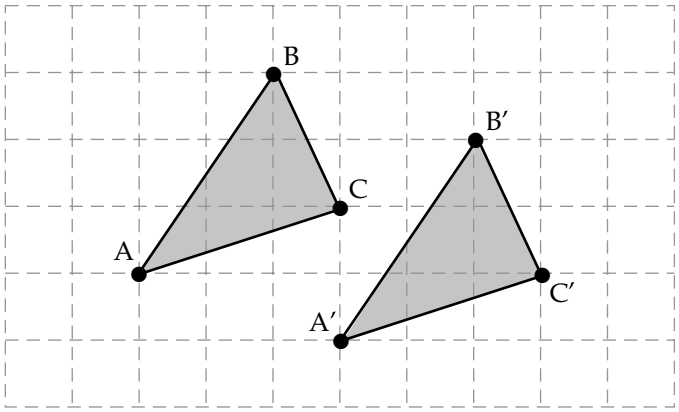


Answer:



2. Given the translated shape, identify the translation

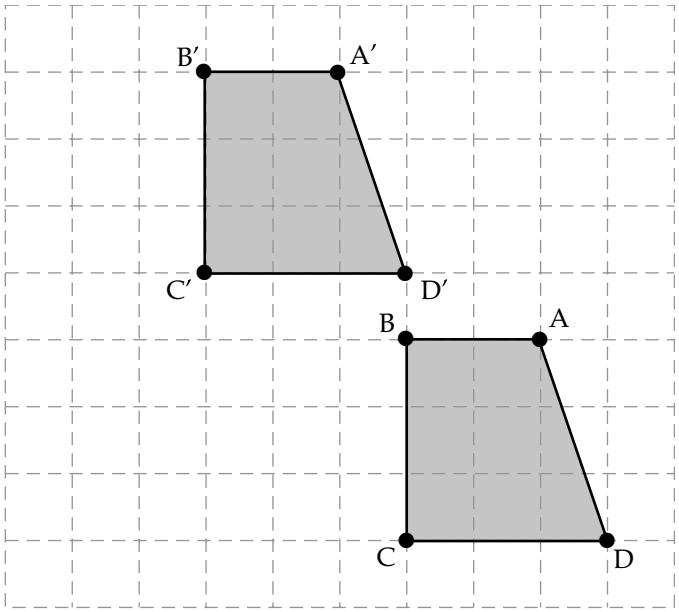
   a) by writing the instructions in words.



   Answer:

   3 units to the right, 1 unit down

   b) by writing the rule.



   Answer:

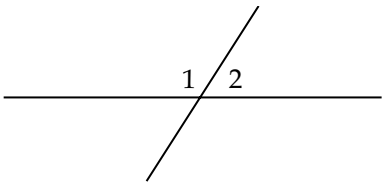
   [L3, U4]

Learning Activity 8.2

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You are buying a t-shirt for $14.83. If you pay with a $20 bill, how much change will you receive?
2. A right triangle has the sides 8, 15, 17. What are the two possible sine ratios?
3. Evaluate: 90×3.
4. Your paycheque was $500. Estimate your CPP (4.95%).
5. Consecutive angles (beside each other) of a transversal are equal, complementary, or supplementary.



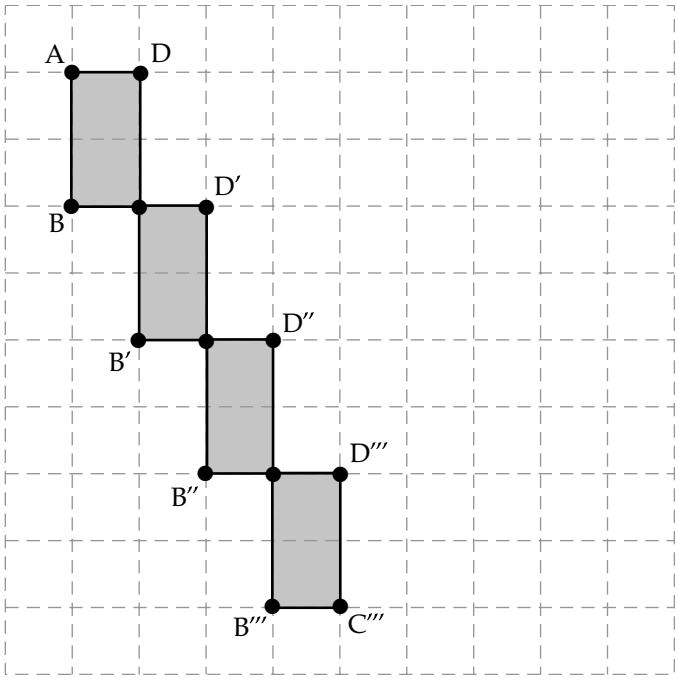
Answers:

1. $5.17 ($14.83 + $0.07 + $0.10 + $5 = $20)
2. $\frac{8}{17}, \frac{15}{17}$ $\left(\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}\text{; the two legs are 8 and 15, and the hypotenuse is 17, so the ratios are } \frac{8}{17} \text{ and } \frac{15}{17}.\right)$
3. 270
4. $25 ($4.95 \approx 5\%$; 10% of $500 = $50, so 5% is $25)
5. supplementary

Part B: Multiple Translations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

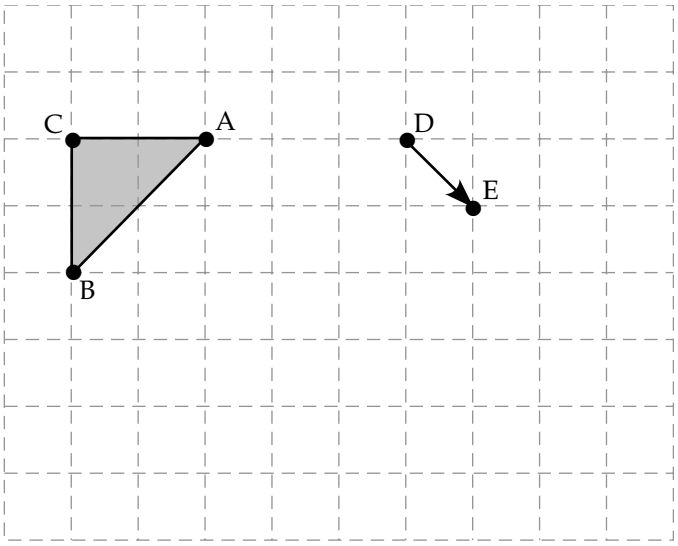
1. Given the following succession of translations, identify the pattern used.



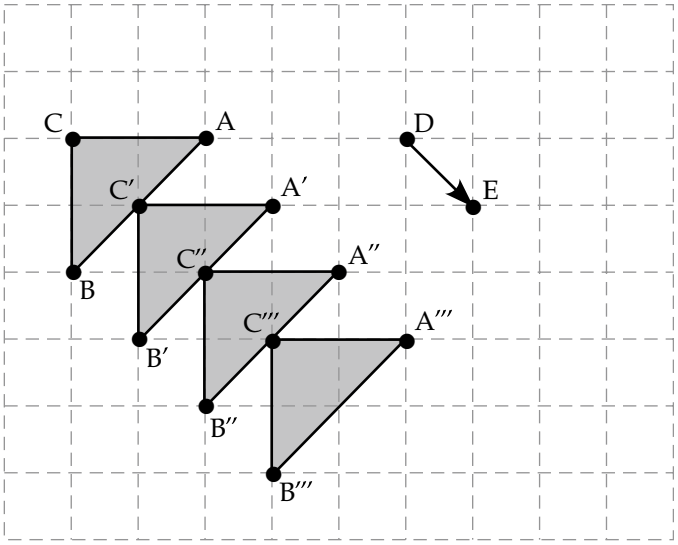
Answer:

[R1, D2] with three iterations

2. Given the shape, and the instruction in a vector, draw 3 iterations (3 images) of the original shape.



Answer:



Learning Activity 8.3

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. You get home from school at 4:15. You want to go for a run for 1 hour, then shower and get dressed, which takes about 35 minutes. You eat dinner for 30 minutes with your family. You need to do your laundry, which will take a total of 2 hours and 15 minutes. How much time do you have to do homework if you go to bed at 10:00 pm?
2. There are 20 players on your rugby team. Seven of them are missing the game for a school concert. What percent of your team is at the game?
3. Is the answer to -11×-13 positive or negative?
4. If you translate a point 4 units to the left then 3 to the right, how far is the new point from the original point and in what direction?
5. Your hourly wage is $8.00. You are paid time-and-a-half for overtime. If you work 4 hours of overtime, how much will you be paid for it?

Answers:

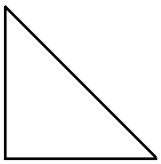
1. 1 hour, 25 minutes (The time between when you get home and when you go to bed is 5 hours, 45 minutes. The time you will spend not doing your homework is 4 hours, 20 minutes (1:00 + 0:35 + 0:30 + 2:15), so you have 1 hour, 25 minutes to do your homework (assuming that you don't multi-task and start your laundry before you go for a run, or you don't do your homework while the washing machine is running).
2. 65% (20 – 7 = 13 players so $\frac{13 \times 5}{20 \times 5} = \frac{65}{100} = 65\%$)
3. positive (Negative multiplied by a negative is positive.)
4. 1 unit left (Think of a translation on a grid.)
5. \$48 (Overtime hourly pay is $\frac{3}{2} \times 8 = \frac{24}{2} = \$12/\text{hour}$. He makes \$12 × 4 = \$48.)

Part B: Rotations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

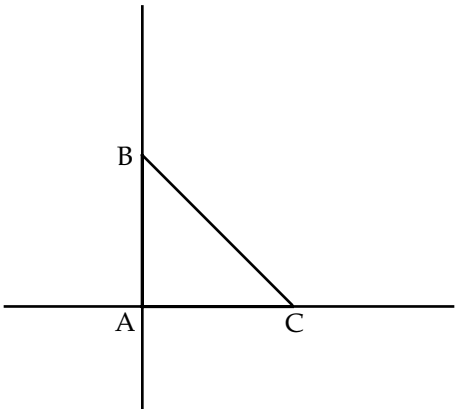
Draw the rotation for each shape, as indicated.

1. 180° clockwise



Answer:

To draw the rotation, you may want to first make horizontal and vertical lines. This is just one method of arriving at the answer.

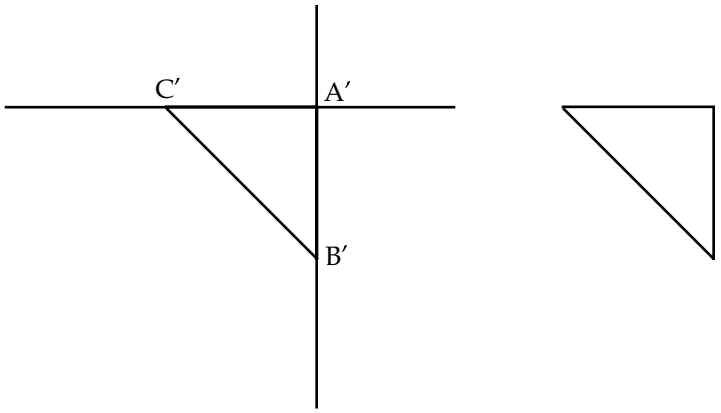


To rotate clockwise 180°, use A as the centre of rotation.

Line AB rotates to line AB′ with B′ on the vertical line below A. The distance from A to B′ will equal the distance from A to B.

Line AC rotates to line AC′ with C′ on the horizontal line to the left of A. The distance from A to C′ will equal the distance from A to C.

The result is shown below with the lines and then without the lines.



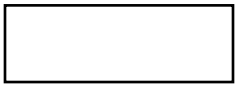
2. 90° counter-clockwise

P

Answer:

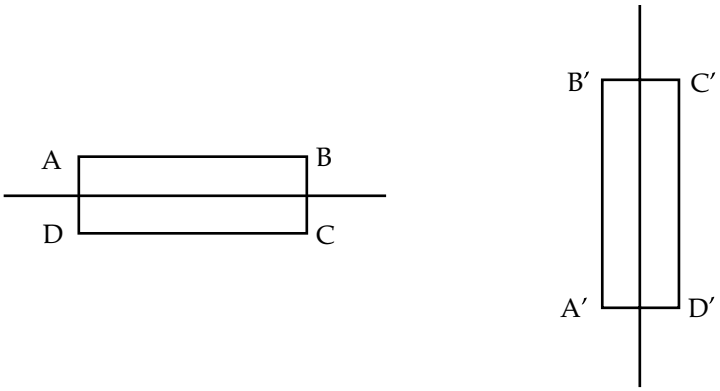
Since no centre of rotation is given, just imagine a vertical line through P that is being rotated 90° to the left. The vertical line becomes a horizontal line and the letter ends up on its back.

3. 270° clockwise

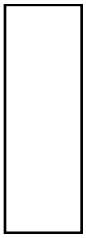


Answer:

No centre of rotation is given. Thus, imagine a horizontal line through the middle of the rectangle. The rotation is 270° clockwise, which is the same as 90° counter-clockwise. Thus, the image of the horizontal line will be a vertical line. The image of the rectangle can be drawn.



The required image is shown below without the lines.



Learning Activity 8.4

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. A right triangle has sides 0.21, 0.20, 0.29. Write the ratios for cosine of this triangle.
2. Evaluate if $w = 5$: $2w - 7$.
3. What are the factors of 28?
4. Dylan is 6′ 2″ tall. How tall is he in inches?
5. The following table shows how many hours you watch on TV each night.

Day of the Week	Monday	Tuesday	Wednesday	Thursday	Friday
Number of Hours	1	2	1.5	2	0.5

On average, how much time do you spend watching TV each day?

Answers:

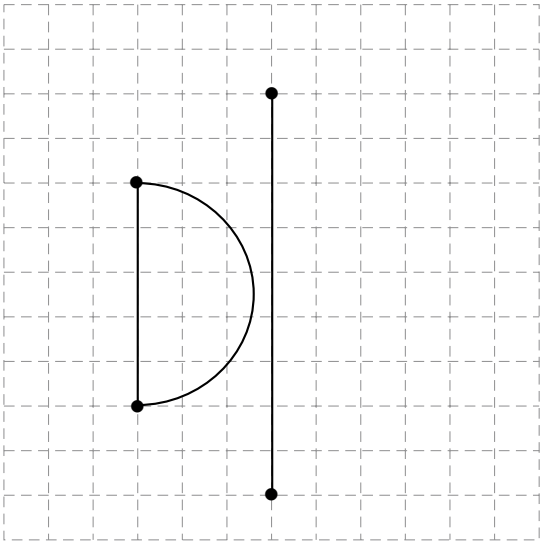
1. $\frac{21}{29}$ and $\frac{20}{29}$ (29 is the hypotenuse, so it is in the denominator for both. The decimal places are removed to make equivalent fractions by multiplying the top and bottom by 100.)
2. 3 (2(5) – 7 = 10 – 7 = 3)
3. 1, 2, 4, 7, 14, 28
4. 74 inches ((6 × 12) + 2 = 72 + 2 = 74″)
5. 1.4 hours $\left(\frac{1+2+1.5+2+0.5}{5} = \frac{7}{5}\right.$ hours or $1\frac{2}{5}$ hours or 1.4 hours or 1 hour, 24 minutes because 0.4 × 60 minutes = 24 minutes)

Part B: Reflections

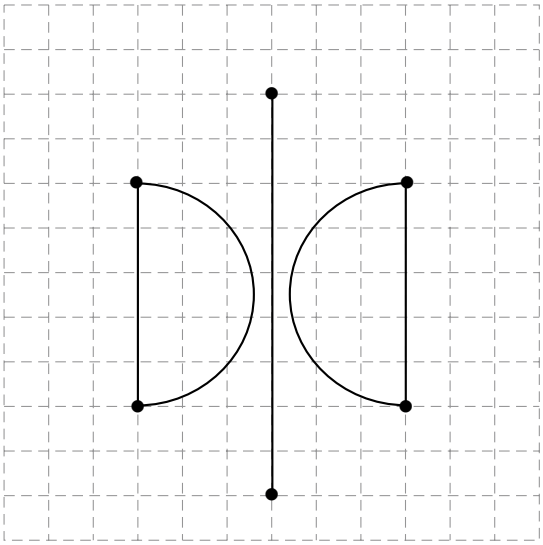
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Reflect the shape over the line of reflection given in each diagram, and draw the reflection.

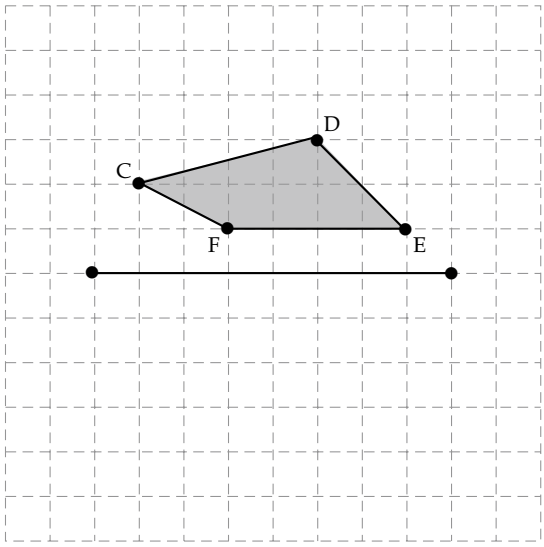
   a)



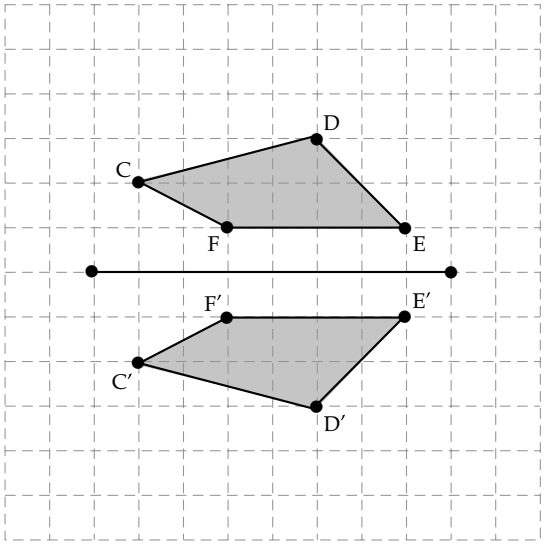
   Answer:



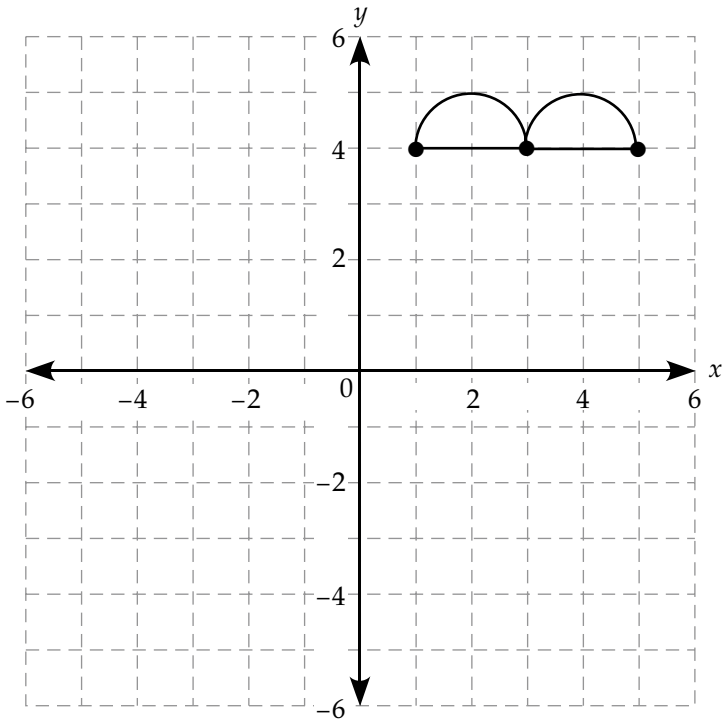
b)



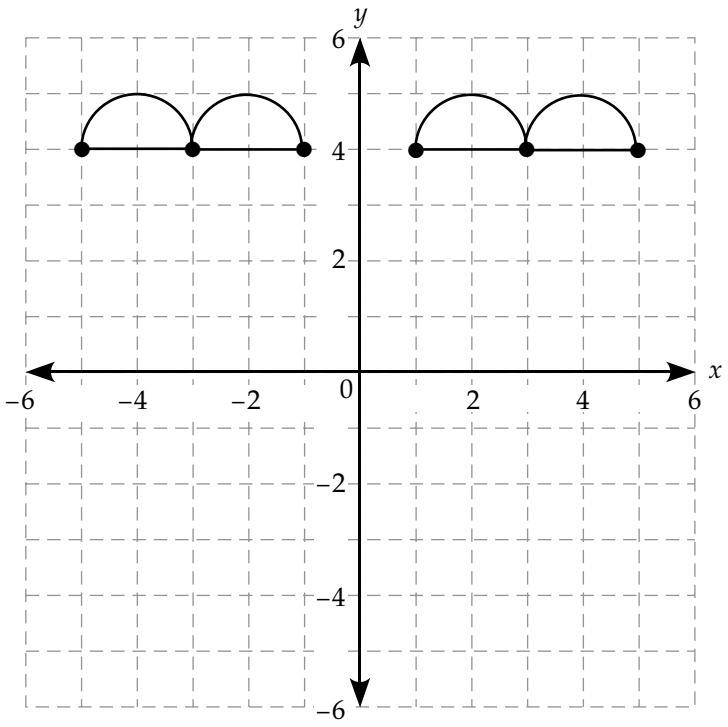
Answer:



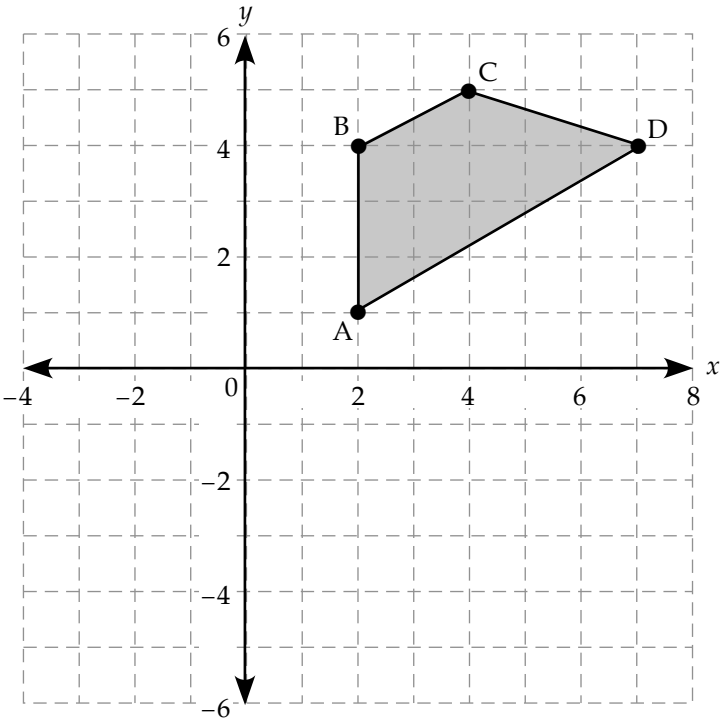
2. a) Reflect this shape over the y–axis.



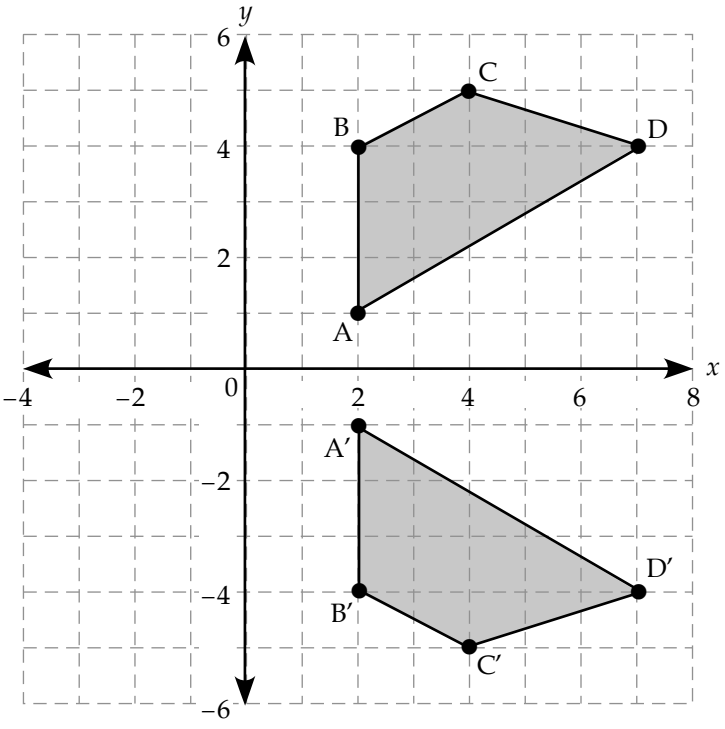
Answer:



b) Reflect this shape over the x-axis.



Answer:



Learning Activity 8.5

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. Audrey reads 2 pages per minute. The latest book she is reading is 720 pages long. How long will it take her to read it?
2. What is the complementary angle to 41°?
4. You work 6.5 hours per day, 6 days per week. How many hours per week do you work?
4. Your younger brother tells you that he is 100 cm tall. Estimate how tall he is in inches (1 inch = 2.54 cm)
5. You are given $3.32 change for buying breakfast at Moonbucks. If you gave the cashier a $10 bill and your total was $6.78, did you get the correct change?

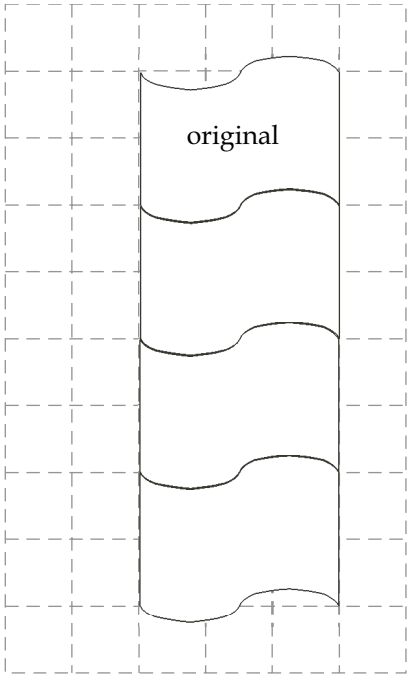
Answers:

1. 6 hours (720 ÷ 3 = 360 minutes or 6 hours)
2. 49° (Complementary angles add up to 90°, so 90 – 41 = 49°.)
3. 39 hours (6.5 × 6 = (6 × 6) + (0.5 × 6) = 36 + 3 = 39 hours)
4. 40 inches (Since 1 inch = 2.54 cm, then 2 inches is approximately 5 cm, and 4 inches is 10 cm. Thus, $\frac{100}{10} \times 4 = 10 \times 4 = 40$ inches.)
5. No, you received too much. (6.78 + 0.02 + 0.20 + 3 = 10 so you should have gotten $3.22 instead)

Part B: Multiple Transformations

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

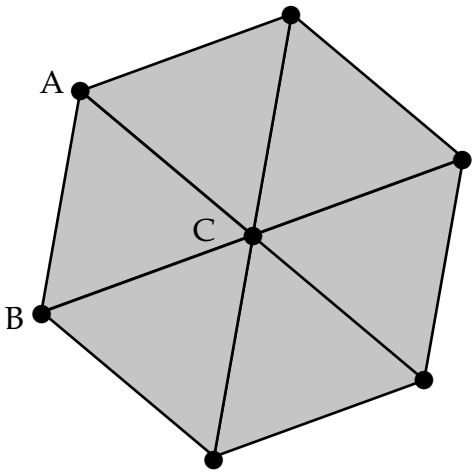
1. Identify the series of transformations applied to the original shape to create the larger shape. Each successive transformation is performed on the previous image.



Answer:

For each iteration, translate the shape down 2 units. Do 3 iterations.

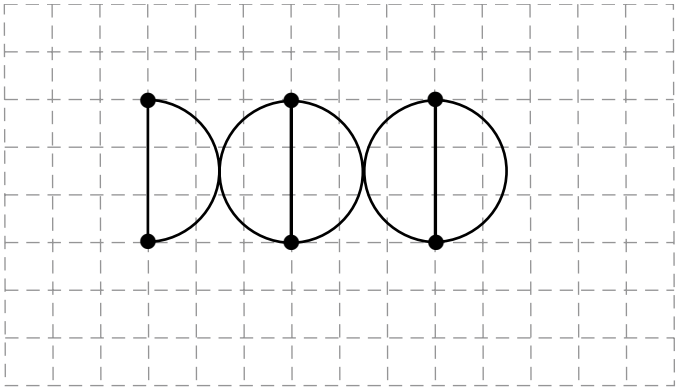
2. Identify the transformations applied to ΔABC to form this six-sided shape. ΔABC is an equilateral triangle with all sides equal and all angles equal to 60°. Each successive transformation is performed on the previous image.



Answer:

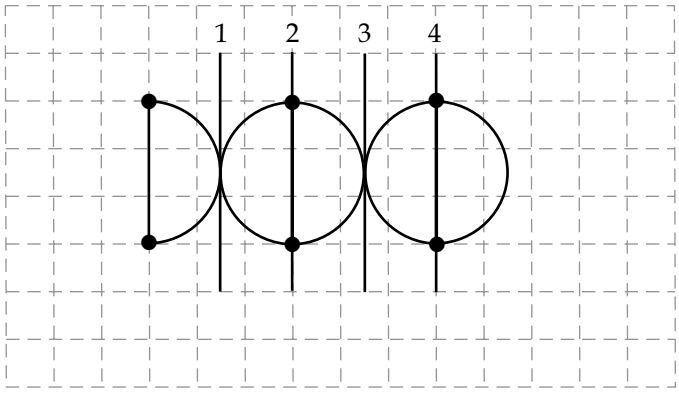
Rotate ΔABC 60° either clockwise or counter-clockwise about point C, and repeat 4 more times.

3. Identify the transformations to move the position of the original shape, the letter D on the left, to have the result on the right side at the end. Each successive transformation is performed on the previous image.

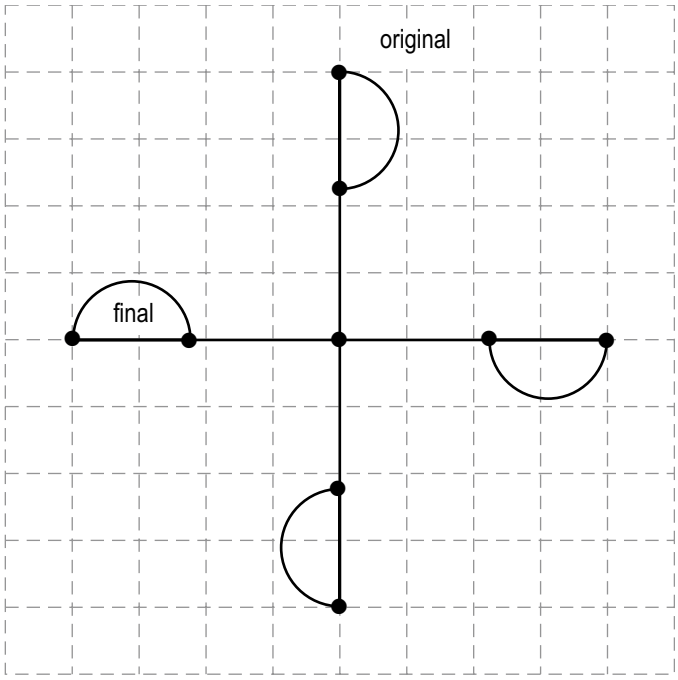


Answer:

Reflect over a vertical line 4 times. The vertical reflection line is shifted $1\frac{1}{2}$ units to the right each time.



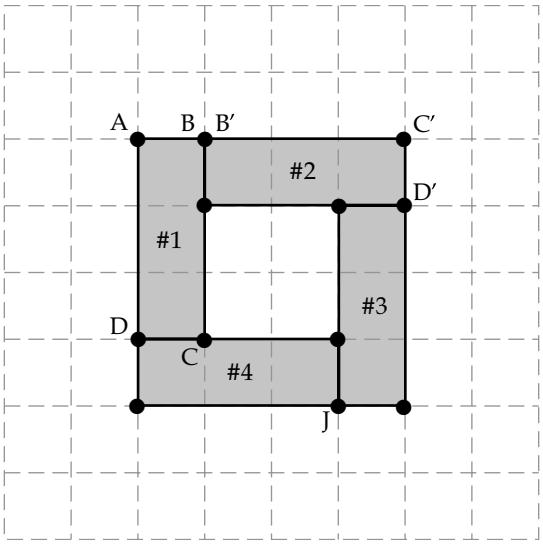
4. Identify the series of transformations to the original shape. Each successive transformation is performed on the previous image.



Answer:

Rotate the letter P 90° clockwise 3 times.

5. Identify the transformations done in the order shown in the diagram, starting with rectangle ABCD, and resulting in rectangle #4. Each successive transformation is performed on the previous image.



Answer:

Step 1: Rotate rectangle #1 90° counter-clockwise about point B to get rectangle #2.

Step 2: Rotate rectangle #2 90° counter-clockwise about point D′ to get rectangle #3.

Step 3: Rotate #3 90° counter-clockwise about point J to get rectangle #4.

Learning Activity 8.6

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. There is a big sale at your favourite store. Everything is 30% off. Your bill before the discount is $200. How much will you pay after the discount?
2. Solve for b: $\frac{b}{8} = \frac{6}{20}$.
3. There are 3 teams in a tournament. Team A wins against Team B. Team B loses to team C. Team C loses to Team A. Does any team win both their games?
4. The map of your mom's office has a scale of 1 cm: 3 m. Your mom's office is 1 cm by 1 cm. What is the area of your mom's office in real life?
5. Rotation 90° clockwise is the same as rotating counter-clockwise _____°?

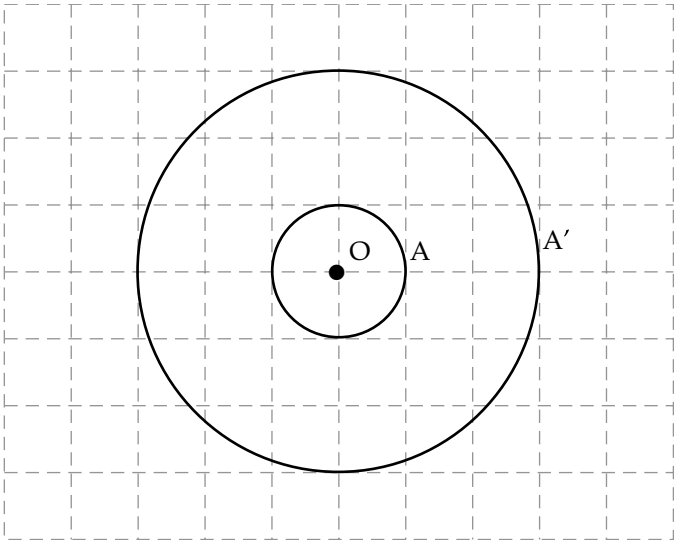
Answers:

1. $140 (It is 30% off, so you are paying 70% of the original price. 70% of $100 = $70, so 70% of $200 is $140.)
2. 2.4 $\left(b = \frac{6 \times 8}{20} = \frac{48 \div 4}{20 \div 4} = \frac{12}{5} = 2.4 \right)$
3. Team A. (Team B loses both games, Team C wins one and loses one.)
4. 9 m^2 (1 cm by 1 cm converts to 3 m by 3 m, making the area 9 m^2)
5. 270° (360° – 90° = 270°)

Part B: Dilation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Determine the scale factor of each dilation.
   a) Circle A′ is a dilation of circle A with the common centre at O.



Answer:

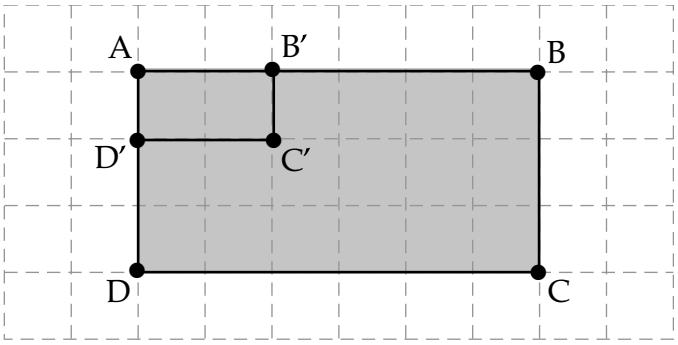
The radius of circle O = 1 unit.

The radius of circle A = 3 units

Both circles have the same centre of dilation.

The scale factor is $\frac{3}{1}$ or 3.

b) Rectangle AB′C′D′ is a dilation of rectangle ABCD with point A as the centre of dilation.



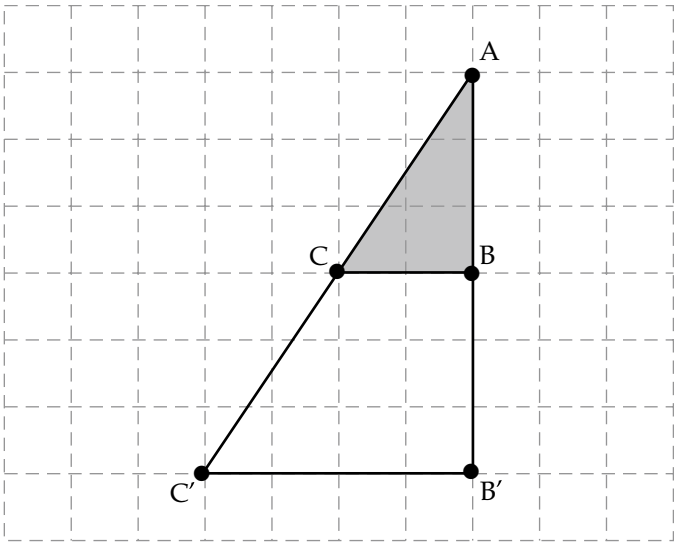
Answer:

AB = 6 units

AB′ = 2 units

The scale factor is $\frac{2}{6}$ or $\frac{1}{3}$.

c) ΔAB′C′ is a dilation of ΔABC with point A as the centre of dilation.



Answer:

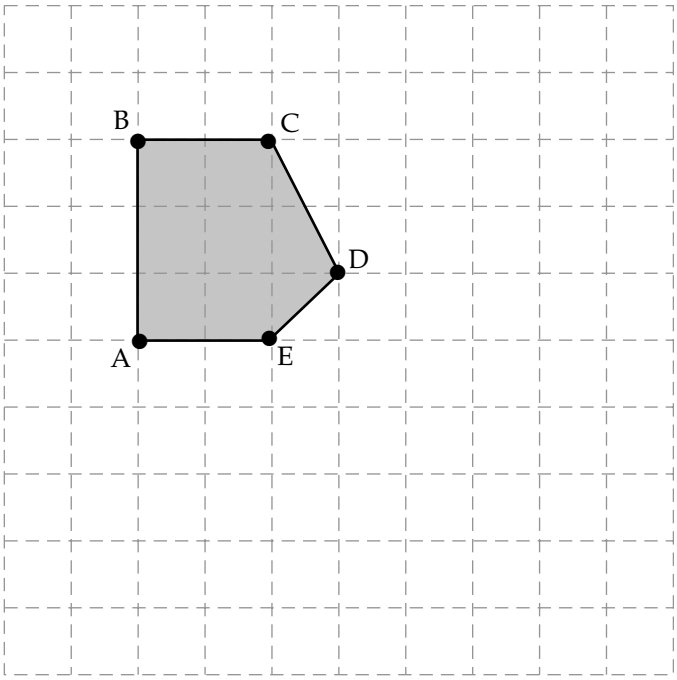
AB = 3 units

AB′ = 6 units

The scale factor is $\frac{6}{3} = 2$.

2. Draw the dilation given the scale factor and centre of dilation.

   a) Scale factor 2 using point B as the centre of dilation.



Answer:

Each side of the original must be multiplied by 2 to find the length of the corresponding side in the image.

First find as many vertical and horizontal distances as you can.

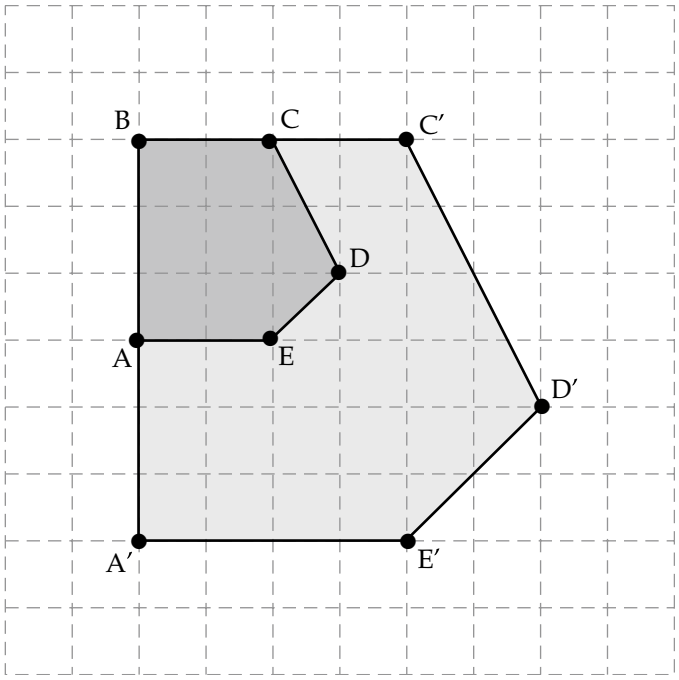
BC = 2, BC′ = 4

BA = 3, BA′ = 6

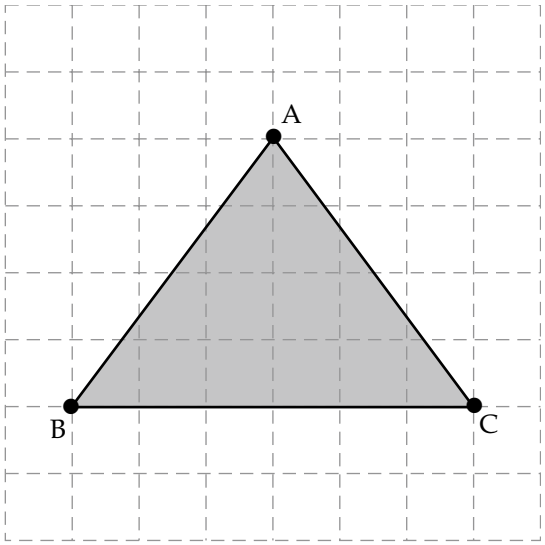
AE = 2, A′E′ = 4

The distance of D from AB = 3, thus in the image the corresponding distance of D′ from A′B must be 6. Notice also that CD will be parallel to C′D′ and DE is parallel to D′E′. Also, D is [R1, U1] from E so D′ is [R2, U2] from E′.

Now you can draw the image correctly by marking the points A′, C′, D′, and E′ on the graph and joining the points.



b) Scale factor 0.5 using point B as the centre of dilation.

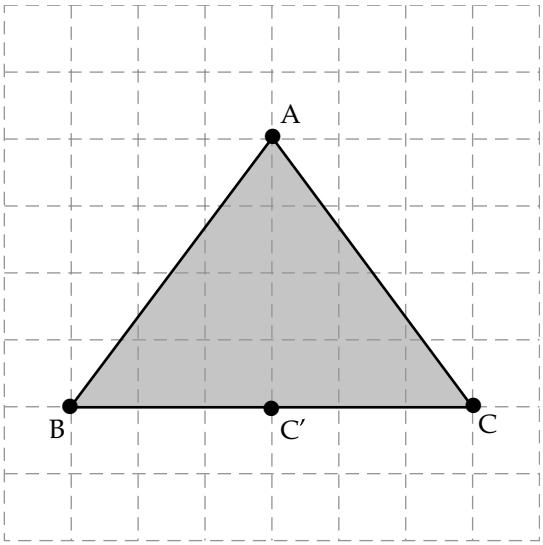


Answer:

BC = 6

BC′ = 6 × 0.5 = 3

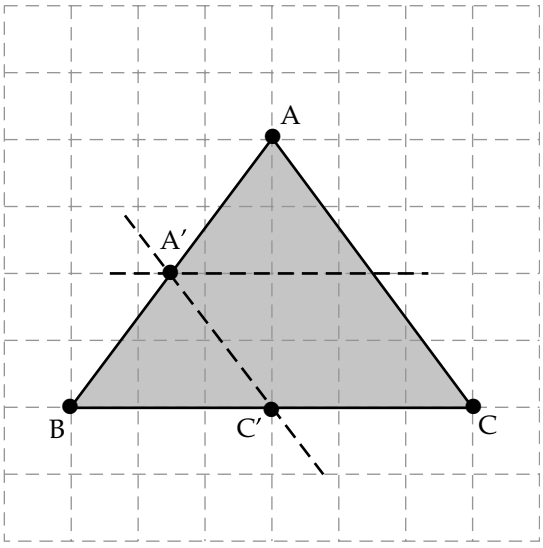
You can mark C′ on your diagram, but how will you find A′?



There are a few ways; here are two of them.

The height from BC to A is 4 units, so the height from BC to A′ must be 2 units. The point A′ is located on the line 2 units above BC and where that line intersects AB.

Another method would be to draw a line parallel to AC starting from C′. Where it intersects AB would be the point A′.



Learning Activity 8.7

Part A: BrainPower

You should be able to complete the following five questions without using a calculator or paper and pencil.

1. The dimensions of Edward's yard are 20 m by 60 m. How much fence will Edward need if he is building a fence around his yard?
2. You take approximately 5 minutes to answer long-answer questions and 1 minute to answer short-answer questions. Your class is 60 minutes long. Do you have enough time to write a test with 15 short answer questions and 8 long-answer questions?
3. You are filling up your scooter with gas that costs $1.00 per litre. If your scooter's gas tank holds 8 L and you've still got 1.5 L in the tank, how much will it cost you to fill up?
4. In order for a rotation to appear exactly the same as the original, how much must you rotate the original object (in degrees)?
5. Evaluate: $\frac{4}{7} \times \frac{8}{3}$.

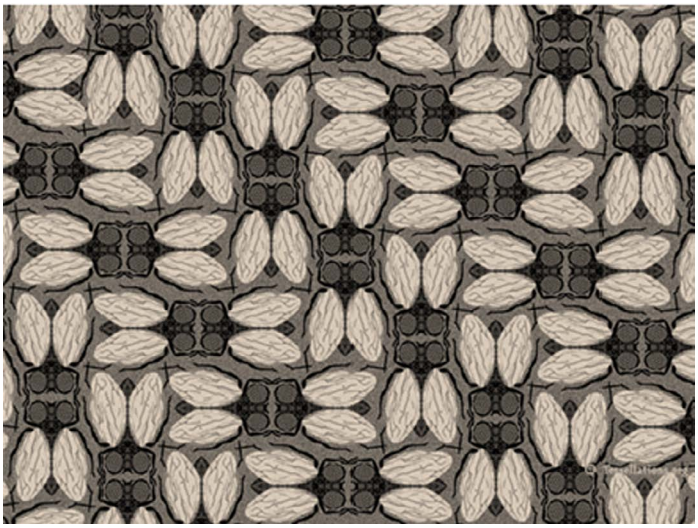
Answers:

1. 160 m (Find the perimeter: (2 × 20) + (2 × 60) = 160 m of fence)
2. Yes (15 short answer = 15 minutes, 8 long answer = 8 × 5 = 40 minutes; Total: 55 minutes, so you do have enough time)
3. $6.50 (You need 6.5 L, so 6.5 × 1.00)
4. 360°
5. $\frac{32}{21}$ $\left(\frac{4}{7} \times \frac{8}{3} = \frac{4 \times 8}{7 \times 3} = \frac{32}{21}\right)$

Part B: Applying Transformations

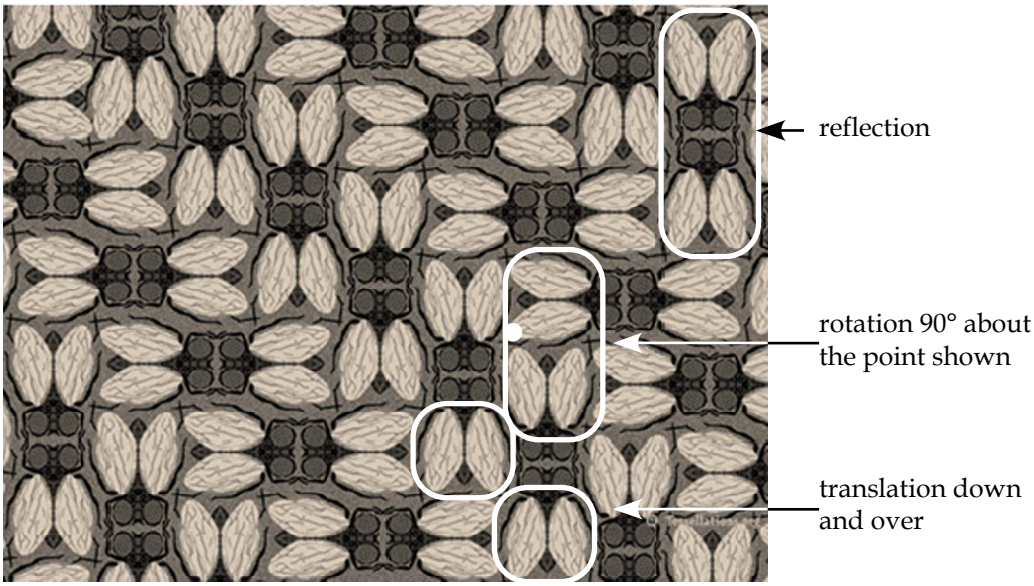
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and final examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Here is a design using repeated pictures of a fly. Circle and label an example of each transformation, translation, rotation, and reflection.

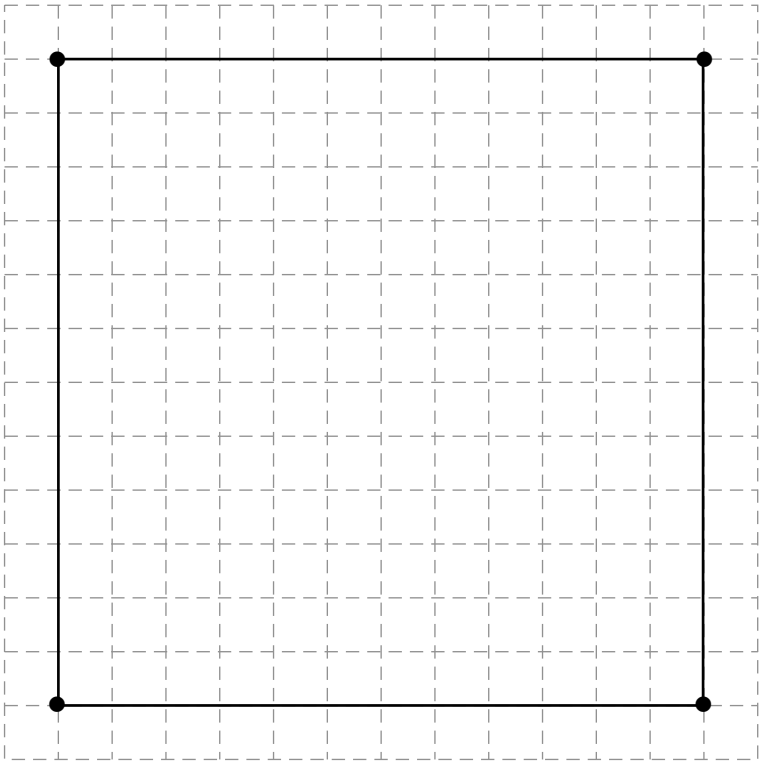


Answer:

An example of each type of transformation is circled in the diagram below.

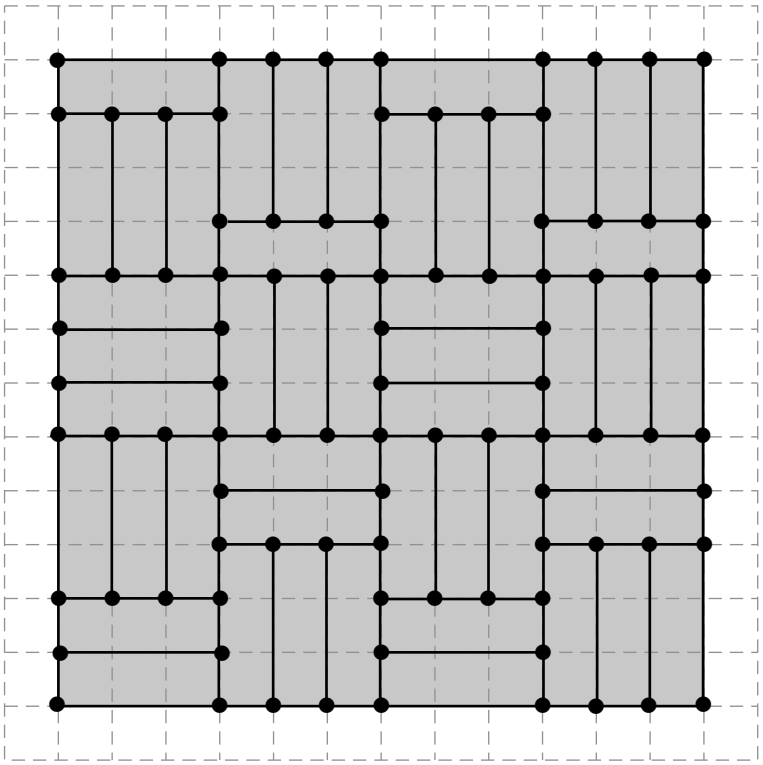


2. You have been hired to design a decorative stone pattern for an area measuring 2 feet by 2 feet. The rectangular stones you will use measure 2″ by 6″. You are not allowed to have more than 3 stones positioned the same way at a time. Draw a layout of how you will arrange the stones, and count the total number of stones you will use. Each square represents 2″. Use the graph paper below.



Answer:

The number of answers to this problem is almost unlimited. You will need to design a pattern that fills the grid, and satisfies the condition of no more than 3 stones in one pattern. One examples follows. It has a total of 48 tiles.



Notes

Grade 10 Essential Mathematics (20S)

Appendices

Appendix A: Conversions
Appendix B: Glossary

Appendix A: Conversions

Basic Conversion Formulas between Systems	
Mass	1 kg = 2.2 pounds 454 g = 1 pound 28.4 g = 1 ounce
Capacity	1 gallon (Canadian) = 4.54 L 1 fl. oz. (Canadian) = 28.41 mL 1 gallon (American) = 3.785 L 1 fl. oz. (American) = 29.57 mL
Distance	1 mile = 1.61 km 1 km = 0.62 mi. 1 inch = 2.54 cm 1 metre = 1.094 yards

Imperial Units Conversion Chart	
Length	1 mile = 1760 yards or 5280 feet 1 yard = 3 feet or 36 inches 1 foot = 12 inches
Mass	1 ton = 2000 pounds (lb.) 1 pound = 16 ounces (oz.)
Capacity	1 gallon = 4 quarts 1 quart = 2 pints 1 pint = 2 cups

Area Conversion Charts

Imperial to Metric		
Imperial		Metric
1 $in.^2$		6.4516 cm^2
1 $ft.^2$	144 $in.^2$	0.0929 m^2
1 $yd.^2$	9 $ft.^2$	0.8361 m^2
1 acre	4840 $yd.^2$	4046.9 m^2
1 $mile^2$	640 acres	2.59 km^2
Metric to Imperial		
Metric		Imperial
1 cm^2	100 mm^2	0.1550 $in.^2$
1 m^2	10 000 cm^2	1.1960 $yd.^2$
1 hectare [ha]	10 000 m^2	2.4711 acres
1 km^2	100 ha	0.3861 $mile^2$

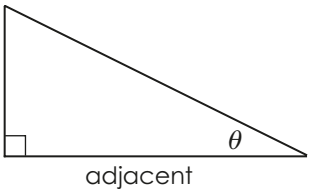
Appendix B: Glossary

accuracy
How close a measurement or calculation is to the actual value.

acute angle
An angle less than 90°.

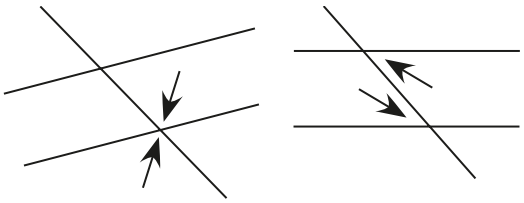
adjacent side
The side of a right triangle beside the angle (θ°) and not the hypotenuse.

Example



alternate angles
A pair of angles on opposite sides of a transversal, both angles must be between the two parallel lines or both must be outside the parallel lines.

Example



area of an odd shape
The space taken up by a 2-D object.

base of a power
The number being multiplied together with itself in a power (4 is the base in 4^3).

BEDMAS (Brackets, Exponents, Division, Multiplication, Addition, Subtraction)
Division and multiplication (and addition and subtraction) are to be completed in the order in which they appear from left to right in the expression or equation (see order of operations).

bisect
To cut into two equal parts.

Canada Revenue Agency (CRA)
The federal government agency that is in charge of taxes.

Canadian Pension Plan (CPP)
A pension savings plan run by the government where you contribute based on the amount that you earn.

capacity
The amount a container can hold; similar to volume.

centi (c)
A metric prefix; multiplication factor = $10^{-2} = 0.01$.

circle
A shape with 1 edge (circumference) that curves around a centre point.

circumference
The distance around the edge of a circle (also known as the perimeter).
$C = \pi d$

commission
Amount you are paid based on a percentage of your sales.

common denominator
Two or more fractions that have the same number in the denominator.

complementary angles
Two angles that add up to 90°.

contract

An agreement that is signed by both parties; usually used in construction to set the fee no matter how much time the job takes.

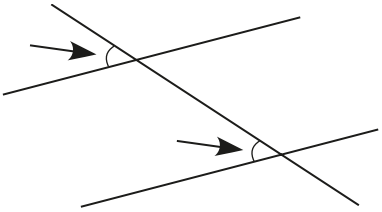
convert

To change the form but not the amount of a measurement or value.

corresponding angles for two parallel lines

Angles formed by a transversal and two parallel lines that are equal; these angles line up when you 'slide' one along the transversal to the other. They are both on the same side of the transversal. One is interior, the other is exterior.

Example



cosine ratio

The ratio relating the adjacent side and the hypotenuse to the angle (θ^o).

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

currency exchange

To trade 1 type of money (e.g., Canadian) for an 'equivalent' value of another type of money (e.g., American).

deci (d)

A metric prefix; multiplication factor = $10^{-1} = 0.1$.

decimal

A fractional number written in base ten form; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number).

deduction

Money removed from your gross pay before you receive it.

deca (da)

A metric prefix; multiplication factor = $10^1 = 10$.

denominator

The number below the line in a fraction that can state the total number of items, or the number of equal pieces that something is divided into.

Example

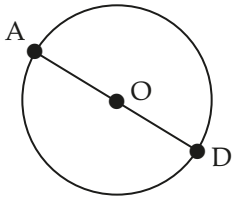
$\frac{2}{3}$ ← denominator

diameter

A line segment that passes through the centre of the circle.

Example

AD is the diameter and O is the centre.



dimensions

Measurements of a figure (length, width, height, radius, etc.).

discount

The amount by which an item's cost is reduced.

Employment Insurance (EI)

While you work, part of your gross pay is put toward this insurance; if you become unemployed, you have the right to apply to CRA to receive some compensation.

estimate

To find the approximate value.

evaluate

Find the value of an expression.

exponent
The number of times a number is multiplied together in a power (3 is the exponent in 4^3).

geometry
Math that involves the position, size, and shape of figures.

Goods and Services Tax (GST)
Tax set by and paid to the federal government.

gross pay
How much you are paid before any taxes or deductions are removed.

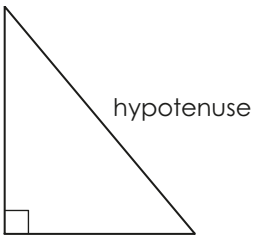
hecto (h)
A metric prefix; multiplication factor = $10^2 = 100$.

hundreds place
The place value located three places to the left of the decimal point in a number; a digit in the hundreds place has a value of 100 times the value of the digit.

hundredths place
The place value located two places to the right of the decimal point in a number; a digit in the hundredths place has a value of $\frac{1}{100}$ the value of the digit.

hypotenuse
The side of a right triangle across from the right angle; the longest side of a right triangle.

Example



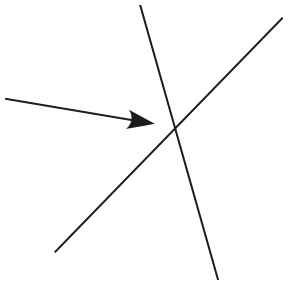
imperial system
The system of measurement used in the US, and sometimes still in Canada and Britain; includes feet, yards, pounds, gallons, and quarts.

improper fraction
A fraction that is larger than 1; the numerator is larger than the denominator $\left(\text{e.g., } \frac{9}{4}\right)$.

income tax
Tax charged on your income by both the provincial and federal governments.

intersection point
The point where two lines cross

Example



kilo (k)
A metric prefix; multiplication factor = $10^3 = 1\ 000$.

late penalty
If you arrive late at work, some work places will reduce your pay to reflect the amount of time worked.

marginal rates
Tax rates that are based on how much money you make.

mega (M)
A metric prefix; multiplication factor = $10^6 = 1\ 000\ 000$.

metric system
See Systeme Internationale.

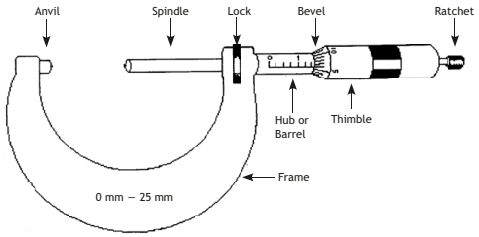
micro (μ)

A metric prefix; multiplication factor = $10^{-6} = 0.000\ 001$.

micrometer

An instrument used to make precise measurements to the nearest thousandth of a centimetre.

Example



milli (m)

A metric prefix; multiplication factor = $10^{-3} = 0.001$.

mixed number

A number larger than 1, written as a whole number and a proper fraction $\left(\text{e.g., } 4\frac{5}{6}\right)$.

nano (n)

A metric prefix; multiplication factor = $10^{-9} = 0.000\ 000\ 001$.

net pay

The actual amount you are paid.

Net pay = gross pay - deductions

numerator

The number above the line in a fraction that states the number of parts being considered.

Example

$\frac{2}{3}$ ← numerator

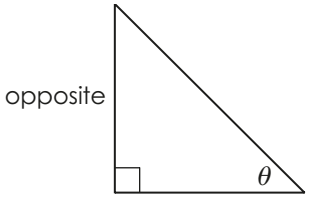
obtuse angle

An angle that is between 90° and 180°.

opposite side

The side of a triangle opposite the angle (θ^o) and not the hypotenuse.

Example



order of operations (BEDMAS)

A specified sequence in which mathematical operations are expected to be performed. An arithmetic expression is evaluated by following these ordered steps:

1. Simplify within grouping symbols such as parentheses or **B**rackets, starting with the innermost grouping.
2. Apply **E**xponents—powers and roots.
3. Perform all **D**ivision and **M**ultiplication in order from left to right.
4. Perform all **A**dditions and **S**ubtractions in order from left to right.

overtime

Time you are required to work beyond your usual amount of time. Two common ways to calculate overtime are:

- hours over 8 hours per day
- hours over 40 hours per week

Other ways are possible.

parallel lines

Lines that never cross and are always the same distance apart. They do not get closer together or farther apart.

parallelogram

A 4-sided shape with parallel opposite sides, and 4-angles that do not have to be 90°.

Example

percent (%)

A number expressed in relation to 100; represented by the symbol % (e.g., 40 parts out of 100 is 40%).

percent discount

The amount off the original price is written as a percent of the original price; the same as the percent rate of change.

percent rate of change

A ratio that compares two values times 100.

$$\text{Percent rate of change (\%)} = \frac{\text{amount of change}}{\text{original amount}} \times 100\%$$

perimeter

The distance around the outside of a shape.

perpendicular lines

Two lines that meet at 90°.

piecework

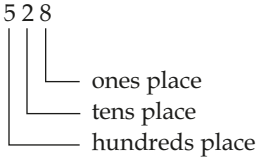
You are paid based on the number of items you make.

place value

The value of a digit in a number based on its position.

Example

In the number 528, the 5 has a value of 5 hundreds (or 500), the 2 has a value of 2 tens (or 20), and the 8 has a value of 8 ones (or 8).



power

The product of a number multiplied with itself several times ($3 \times 3 \times 3 = 27$ is described as '3 to the power of 3' or 'the third power of 3').

precise

The exact measurement.

proper fraction

A fraction whose value is less than 1; the numerator is less than the denominator, e.g., $\frac{5}{8}$.

proportion

Two equivalent ratios.

provincial sales tax (PST)

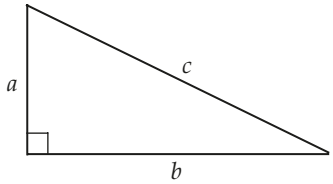
Tax set by and paid to the provincial government.

Pythagorean Theorem

The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c) of a right triangle.

$a^2 + b^2 = c^2$

Example

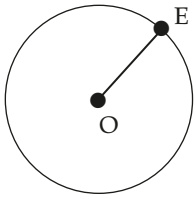


radius

A line from the centre of a circle to the edge (circumference) of the circle; half the diameter.

Example

EO is the radius



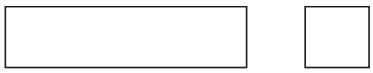
ratio

A comparison of two like numbers or quantities and written as a fraction.

rectangle

A 4-sided shape that has 4 right angles (90°), and opposite sides are congruent (equal).

Examples



referent

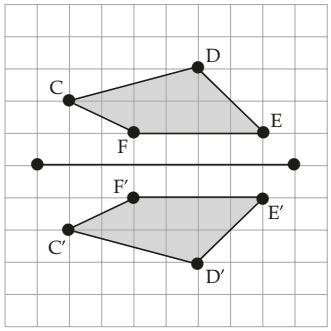
An object that can be used to estimate a measurement.

reflection

To 'flip' an image or object over a line of reflection to produce a mirror image; all the new points are the same distance from the line as the matching point in the original image.

Example

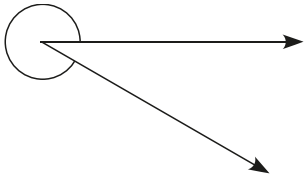
C'D'E'F' is the reflection of CDEF.



reflex angle

An angle between 180° and 360°.

Example



revolution

One whole rotation around the circle (360°).

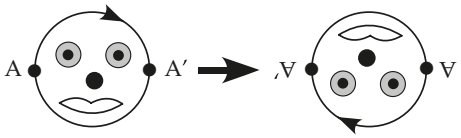
right angle

An angle that is 90°.

rotation

To move a shape from its original position in a circular motion about a fixed point; the order of points does not change – if you follow the perimeter clockwise in the original and the order is AA', the clockwise order will be AA' on the rotation if you still start on point A.

Example



salary

How much someone is paid, no matter how many hours of work they do.

salary plus commission

You are paid a flat rate (which is usually quite low) plus a percent of your sales.

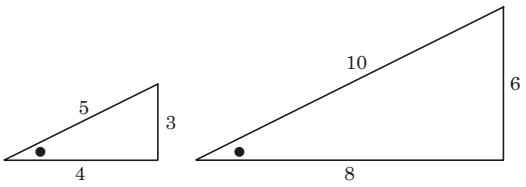
scale drawing

A drawing of an object that is larger or smaller than the actual object, but the two are similar.

similar shapes

Two shapes whose corresponding angles are equal and corresponding sides are proportional.

Example



sine ratio

The ratio relating the opposite side and the hypotenuse to the angle (θ^o).

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

square root of a number

A number when multiplied by itself, produces the given number; square root is the inverse operation of squaring.

straight angle

An angle of 180°.

sum of angles in a triangle

The sum of the angles in a triangle is 180° (angle 1 + angle 2 + angle 3 = 180°).

supplementary angles

Two angles that add up to 180°.

système international (SI)

Measurement system based on the multiples of 10; commonly used throughout the world; also known as the metric system.

tangent ratio

The ratio relating the opposite and adjacent sides to the angle (θ^o).

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

tens place

The place value located two places to the left of the decimal point; a digit in the tens place has a value of 10 times the value of the digit.

tenths place

The place value located one place to the right of the decimal point; a digit in the tenths place has a value of $\frac{1}{10}$ the value of the digit.

thousands place

The place value located four places to the left of the decimal point in a number; a digit in the thousands place has a value of 1000 times the value of the digit.

thousandths place

The place value located three places to the right of the decimal point; a digit in the thousandths place has a value of $\frac{1}{1000}$ the value of the digit.

transformation

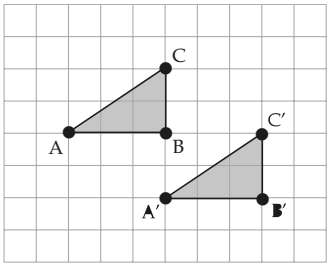
The result of a change (position, rotation, reflection) made to an image or object.

translation

Movement of an image so that its position is changed but it is not rotated or reflected; every point of the image is moved the same distance, in the same direction.

Example

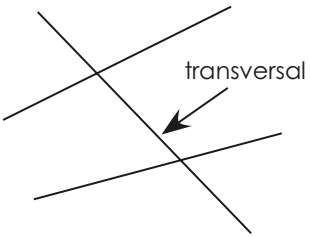
A′B′C′ is a translation of ABC.



transversal

A line that intersects (crosses) two or more lines (which do not have to be parallel).

Example



trapezoid

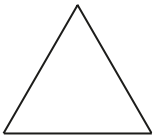
A 4-sided shape with 2 parallel sides.

Example

triangle

A 3-sided object with 3 angles; sides and angles can but don't have to be equal.

Example



trigonometry

The study of triangles and triangular calculations.

unit price

The cost per unit of an item (e.g., it is 99.4¢ per L of gas).

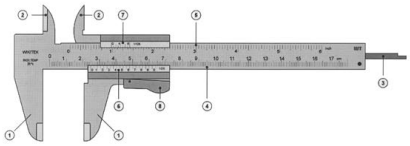
variable

A letter or symbol that represents an unknown value (x, y, n, θ).

Vernier caliper

An instrument used to make precise measurements to the nearest hundredth of a centimetre.

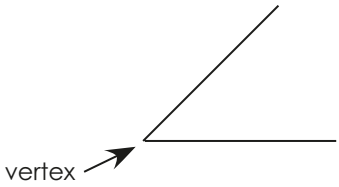
Example



vertex

The point where two lines meet to make an angle.

Example



wages

What you are paid for work you do.

GRADE 10 ESSENTIAL MATHEMATICS (20S)

Midterm Practice Exam

GRADE 10 ESSENTIAL MATHEMATICS

Midterm Practice Examination

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____ /100 = _____ %

Comments:

Instructions

The midterm examination will be weighted as follows:

Modules 1–4	100%

The format of the examination will be as follows:

Part A: Multiple Choice	9 marks
Part B: Long Answer	91 marks

Time allowed: 2.5 hours

Note: You are allowed to bring the following to the exam: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a ruler, a protractor, a compass, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module work that is submitted for marking. **Diagrams may not be drawn to scale.**

Part A: Multiple Choice (*9 x 1 = 9 marks*)

Circle the letter of the correct answer for each question.

1. A best estimate of the amount of liquid a soda can holds is
   a) 350 litres
   b) 35 gallons
   c) 350 mL
   d) 350 fl. oz.

2. CPP is known as
   a) income from the federal government when between jobs
   b) retirement income from the federal government
   c) amount deducted from gross earnings
   d) a savings plan

3. Overtime pay may be defined as
   a) earnings after deductions have been taken off
   b) earnings before any deductions have been taken off
   c) earnings for more than 8 hours per day
   d) income from the government when you are laid off from your job

4. Deductions are defined as
   a) earnings after deductions have been taken off
   b) earnings before deductions have been taken off
   c) earnings more than 8 hours per day
   d) amount that gross pay is reduced by

5. Workers are paid by piecework when
   a) they work more than 40 hours per week
   b) they work on weekends
   c) they are paid based on the number of items they assemble
   d) they work shift work

6. Workers who are paid by a commission are likely
   a) teenagers working part-time in fast food restaurants
   b) teachers, managers, government workers
   c) car salespeople, realtors, insurance salesmen
   d) commissionaires and security people

7. The heaviest measurement is
   a) kilogram
   b) pound
   c) ounce
   d) gram

8. The smallest unit of measurement on a standard imperial ruler is
   a) $\frac{1}{4}''$
   b) $\frac{1}{2}''$
   c) $\frac{1}{8}''$
   d) $\frac{1}{16}''$

9. A square measuring one yard by one yard has an area of
   a) 144 sq. ft
   b) 9 sq. ft.
   c) 3 sq. ft.
   d) 36 square inches

Part B: Long Answer (*91 marks*)

Write your answers in the space provided.

1. Nancy is paid overtime on any hours worked over 8 per day. Using this time card, identify how many regular hours and how many overtime hours she worked last week. *(2 marks)*

Monday	Tuesday	Wednesday	Thursday	Friday
6	11.5	7.5	9.25	9.5

Regular: _____

Overtime: _____

2. Identify four methods you studied by which workers can earn income. *(4 marks)*

_____

_____

_____

_____

3. Explain why a salesman might prefer to be paid by commission. Also, describe what would be needed for the salesman to succeed. *(2 marks)*

_____

_____

_____

_____

_____

_____

_____

4. Jon offers to totally landscape his aunt's yard for $2000. This is an example of earning income by what method? *(1 mark)*

_____

5. Fill in the time card to find the gross pay. Overtime is paid at time and one-half on any hours over 40 hours per week. There are no late penalties. *(7 marks)*

Employee: Scott					Hourly Rate: $11.75		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	07:30	12:00	1:00	5:00	7:30	9:45	
Tues.	07:29	11:59	12:58	4:30			
Wed.			1:00	5:30	7:30	10:15	
Thurs.	07:30	12:02	1:01	5:32	7:00	10:30	
Fri.	08:00	12:00	1:02	5:00			
Sat.			1:00	4:45			
Sun.							

	Hours	Rate	Amount
Regular			
Overtime			
Total Hours		Gross Wages	

6. List six possible deductions one might expect from their gross earnings. *(6 marks)*

_____

_____

_____

_____

_____

_____

7. Jessica earns $11.44 per hour. She works 35 hours. Use the payroll deduction tables to find her net pay. Her claim code is 0. *(7 marks)*

Canada Pension Plan Contributions

Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC	Pay Rémunération			CPP RPC
From - De		To - Á		From - De		To - Á	
399.93	-	400.12	16.47	438.11	-	438.31	18.36
400.13	-	400.33	16.48	438.32	-	438.51	18.37
400.34	-	400.53	16.49	438.52	-	438.71	18.38
400.54	-	400.73	16.50	438.72	-	438.91	18.39
400.74	-	400.93	16.51	438.92	-	439.11	18.40
400.94	-	401.13	16.52	439.12	-	439.32	18.41
401.14	-	401.34	16.53	439.33	-	439.52	18.42
401.35	-	401.54	16.54	439.53	-	439.72	18.43
401.55	-	401.74	16.55	439.73	-	439.92	18.44

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á		From - De		To - Á	
395.67	-	396.24	6.85	437.29	-	437.86	7.57
396.25	-	396.82	6.86	437.87	-	438.43	7.58
396.83	-	397.39	6.87	438.44	-	439.01	7.59
397.40	-	397.97	6.88	439.02	-	439.59	7.60
397.98	-	398.55	6.89	439.60	-	440.17	7.61
398.56	-	399.13	6.90	440.18	-	440.75	7.62
399.14	-	399.71	6.91	440.76	-	441.32	7.63
399.72	-	400.28	6.92	441.33	-	441.90	7.64
400.29	-	400.66	6.93	441.91	-	442.48	7.65

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Pay Rémunération			Federal claim codes/ Codes de demande			
			0	1	2	3
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie			
399	-	403	53.60	24.50	21.65	15.95
403	-	407	54.20	25.05	22.20	16.50
407	-	411	54.75	25.60	22.75	17.05
411	-	415	55.30	26.15	23.30	17.65
415	-	419	55.85	26.75	23.90	18.20
419	-	423	56.40	27.30	24.45	18.75
423	-	427	57.00	27.85	25.00	19.30
427	-	431	57.55	28.40	25.55	19.85
431	-	435	58.10	28.95	26.10	20.45
435	-	439	58.65	29.50	26.70	21.00
439	-	443	59.20	30.10	27.25	21.55

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Pay Rémunération			Provincial claim codes/ Codes de demande			
			0	1	2	3
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie			
397	-	401	40.55	23.70	21.90	18.40
401	-	405	41.00	24.10	22.30	18.80
405	-	409	41.40	24.50	22.75	19.20
409	-	413	41.80	24.90	23.15	19.60
413	-	417	42.20	25.30	23.55	20.00
417	-	421	42.60	25.70	23.95	20.40
421	-	425	43.00	26.10	24.35	20.80
425	-	429	43.40	26.50	24.75	21.25
429	-	433	43.80	26.90	25.15	21.65
433	-	437	44.20	27.30	25.55	22.05
437	-	441	44.60	27.70	25.95	22.45

Gross earnings: _____

CPP: _____

EI: _____

Federal income tax, claim code 0: _____

Manitoba income tax, claim code 0: _____

Total deductions: _____

Net pay: _____

8. Joey works at a sporting goods store assembling bicycles. He gets paid $12 for each bike he assembles, but loses $8 for every bike that is not put together properly. Last week he worked 60 hours to assemble 65 bicycles before a big sale. The boss brought 11 bikes back because they were missing parts. Calculate Joey's income earned. *(1 mark)*

9 How much more federal income tax will you pay if you get a $1 per hour raise, assuming you earn $10 per hour, work 40 hours per week, and use claim code 2. *(1 mark)*

10. Jamie earns a salary of $40,040.00 per year. She pays $10 for union dues and $15 for parking. Use 4.95% for CPP, 1.73% for EI, and 29% for the combined federal and provincial income taxes to find her net weekly pay. *(7 marks)*

Gross weekly earnings: _____

CPP: _____

EI: _____

Income tax: _____

Other deductions: _____

Total deductions: _____

Net pay: _____

11. Fred is paid a salary of $80,000 per year. Find his annual provincial income taxes using the marginal rates tables shown below. *(5 marks)*

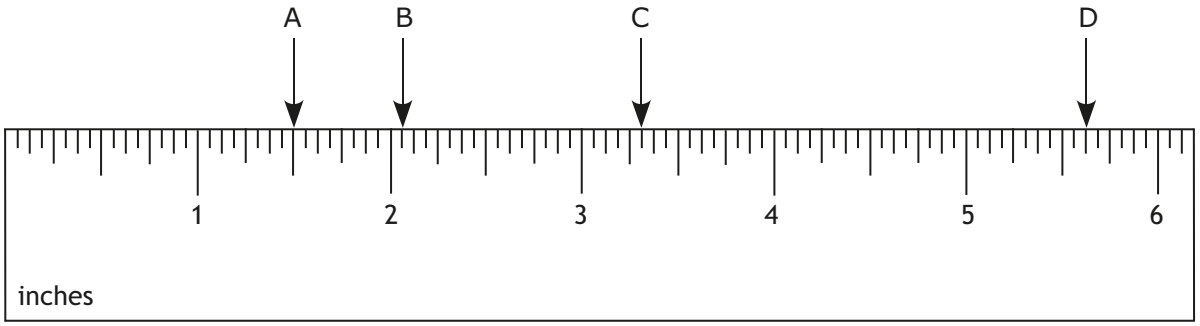
Manitoba Marginal Rates

Any annual income from $0 to $31,000: 10.8%

plus any annual income from $31,001 to $67,000: 12.75%

plus any annual income from $67,000: 17.4%

12. Find the readings on this imperial ruler. *(4 marks)*



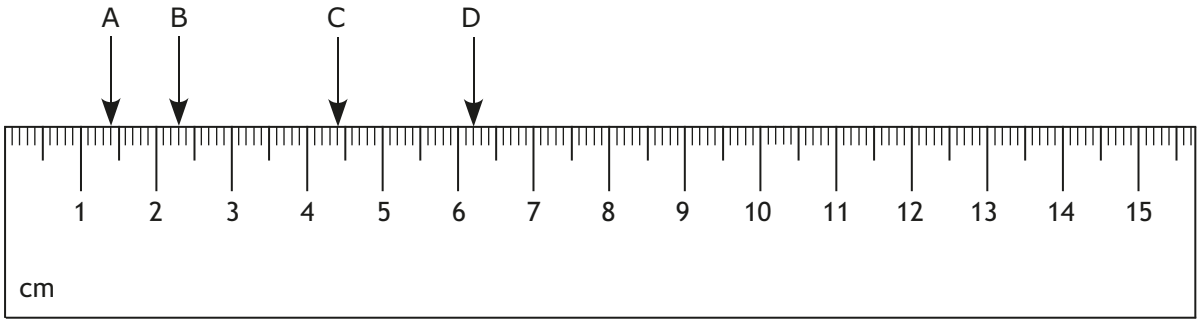
A _____

B _____

C _____

D _____

13. Record the readings in centimetres on this metric ruler. *(4 marks)*



A _____

B _____

C _____

D _____

14. Given that the temperature is 110°F, find the equivalent Celsius temperature by using the formula. Show your work. *(3 marks)*

15. Use the conversion table to convert the following. Round off to two decimal places. Show your work. *(4 marks)*

Basic Conversion Formulas between Systems	
Mass	1 kg = 2.2 pounds 454 g = 1 pound 28.4 grams = 1 ounce
Capacity	1 gallon (Canadian) = 4.54 L 1 fl. oz. (Canadian) = 28.41 mL 1 gallon (American) = 3.785 L 1 fl. oz. (American) = 29.57 mL
Distance	1 mile = 1.61 km 1 km = 0.62 mi. 1 inch = 2.54 cm 1 metre = 1.094 yards

a) 9 km = _____ miles

b) 25 fl. oz. = _____ mL (Canadian)

c) 150 pounds = _____ kg

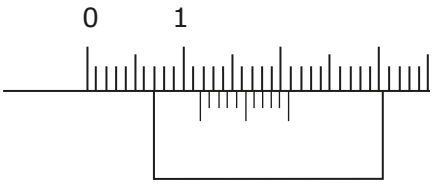
d) 350 cm = _____ inches

16. The distance from Winnipeg to Minneapolis is about 500 miles. How far is the distance in kilometres? The conversion rules are given in #15 above. *(2 marks)*

17. Record the metric Vernier caliper readings. *(2 marks)*

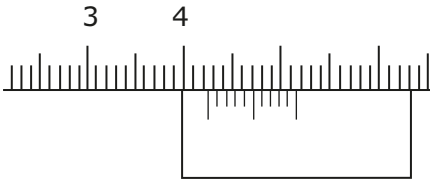
Note: Your answer will be marked correct if it is ±0.02 from the given answer.

a)



Reading = _____

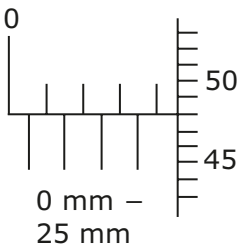
b)



Reading = _____

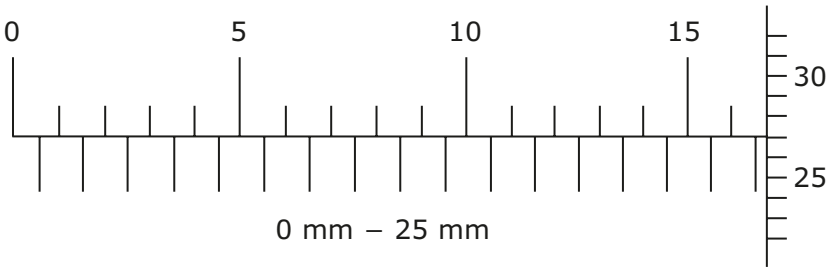
18. Record the micrometer readings. *(2 marks)*

a)



Reading = _____

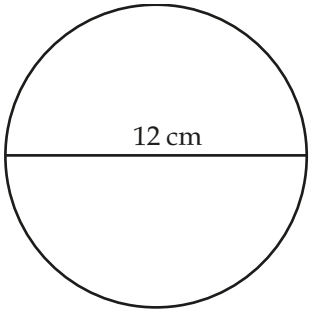
b)



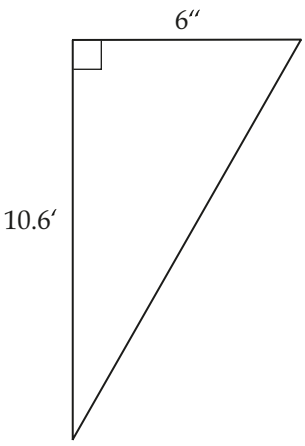
Reading = _____

19. Find the area of each shape. *(9 marks)*

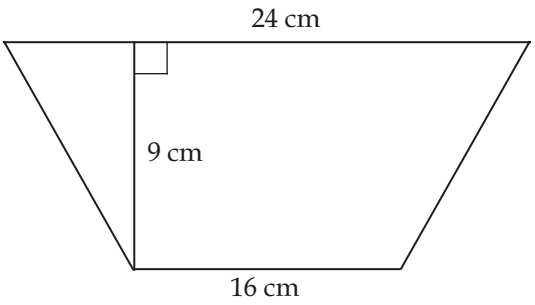
a)



b)



c)



20. You have enough sod to cover 300 m^2 of your lawn. Knowing the length of the lawn is 40 m, find the width. *(2 marks)*

21. You use a piece of string to find the direct distance between two points on a map. The length of the string is 4.6 cm long. The map scale is 1 cm: 200 miles. Find the direct distance between the points. *(2 marks)*

22. Convert the following measurements as indicated using the area conversions chart. Round off to two decimal places. *(9 marks)*

Imperial		Metric
1 $in.^2$		6.4516 cm^2
1 $ft.^2$	144 $in.^2$	0.0929 m^2
l $yd.^2$	9 $ft.^2$	0.8361 m^2
1 acre	4840 $yd.^2$	4046.9 m^2
1 $mile^2$	640 acres	2.59 km^2

Metric		Imperial
1 cm^2	100 mm^2	0.1550 $in.^2$
1 m^2	10 000 cm^2	1.1960 $yd.^2$
1 hectare [ha]	10 000 m^2	2.4711 acres
1 km^2	100 ha	0.3861 $mile^2$

a) $4\frac{1}{4}$ yd.^{2} to m^2 *(1 mark)*

b) 100 cm^2 to in.^{2} *(1 mark)*

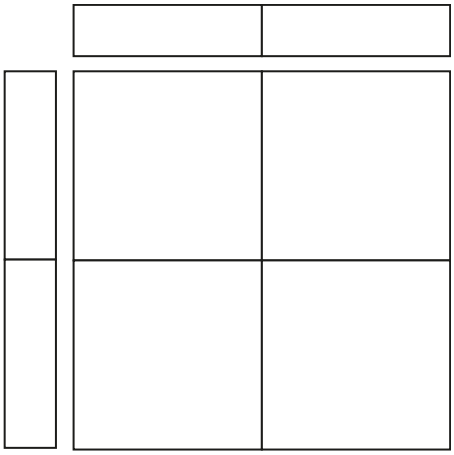
c) 640 in.^{2} to m^2 *(2 marks)*

d) 102 square miles to km^2 *(1 mark)*

e) 4000 cm^2 to $ft.^2$ *(4 marks)*

23. Multiply these fractions using the grid method. Write your answer as a mixed number. *(2 marks)*

$$1\frac{3}{4} \times 7\frac{1}{2}$$



24. The cost of the new football stadium is predicted to be $137,500,000. Write this number in scientific notation. *(1 mark)*

25. Madeline works in a bakery. She makes a minimum wage of $9.25 per hour but has been offered a rase to $10.05. What percent rate of increase is this? Round off to two decimal places. *(2 marks)*

Remember: Attach your Midterm Exam Resource Sheet to your exam paper and submit it along with your exam.

Notes

GRADE 10 ESSENTIAL MATHEMATICS (20S)

Midterm Practice Exam
Answer Key

GRADE 10 ESSENTIAL MATHEMATICS

Midterm Practice Examination
Answer Key

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____ /100 = _____ %

Comments:

Instructions

The midterm examination will be weighted as follows:

Modules 1–4	100%

The format of the examination will be as follows:

Part A: Multiple Choice	9 marks
Part B: Long Answer	91 marks

Time allowed: 2.5 hours

Note: You are allowed to bring the following to the exam: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a ruler, a protractor, a compass, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module work that is submitted for marking. **Diagrams may not be drawn to scale.**

Part A: Multiple Choice (*9 x 1 = 9 marks*)

Circle the letter of the correct answer for each question.

1. A best estimate of the amount of liquid a soda can holds is
   a) 350 litres
   b) 35 gallons
   c) 350 mL
   d) 350 fl. oz.

   Module 3, Lesson 5

2. CPP is known as
   a) income from the federal government when between jobs
   b) retirement income from the federal government
   c) amount deducted from gross earnings
   d) a savings plan

   Module 2, Lesson 2

3. Overtime pay may be defined as
   a) earnings after deductions have been taken off
   b) earnings before any deductions have been taken off
   c) earnings for more than 8 hours per day
   d) income from the government when you are laid off from your job

   Module 1, Lesson 4

4. Deductions are defined as
   a) earnings after deductions have been taken off
   b) earnings before deductions have been taken off
   c) earnings more than 8 hours per day
   d) amount that gross pay is reduced by

   Module 2, Lesson 1

5. Workers are paid by piecework when
   a) they work more than 40 hours per week
   b) they work on weekends
   c) they are paid based on the number of items they assemble
   d) they work shift work

   Module 1, Lesson 3

6. Workers who are paid by a commission are likely
   a) teenagers working part-time in fast food restaurants
   b) teachers, managers, government workers
   c) car salespeople, realtors, insurance salesmen
   d) commissionaires and security people — Module 1, Lesson 3

7. The heaviest measurement is
   a) kilogram
   b) pound
   c) ounce
   d) gram — Module 3, Lesson 5

8. The smallest unit of measurement on a standard imperial ruler is
   a) $\frac{1}{4}''$
   b) $\frac{1}{2}''$
   c) $\frac{1}{8}''$
   d) $\frac{1}{16}''$ — Module 3, Lesson 4

9. A square measuring one yard by one yard has an area of
   a) 144 sq. ft
   b) 9 sq. ft.
   c) 3 sq. ft.
   d) 36 square inches — Module 4, Lesson 5

Part B: Long Answer (*91 marks*)

Write your answers in the space provided.

1. Nancy is paid overtime on any hours worked over 8 per day. Using this time card, identify how many regular hours and how many overtime hours she worked last week. *(2 marks)* (Module 1, Lesson 4)

Monday	Tuesday	Wednesday	Thursday	Friday
6	11.5	7.5	9.25	9.5

Regular: *Answer:* 6 + 8 + 7.5 + 8 + 8 = 37.5 hours

Overtime: *Answer:* 3.5 + 1.25 + 1.5 = 6.25 hours

2. Identify four methods you studied by which workers can earn income. *(4 marks)* (Module 1, Lesson 3)

   Answer:

   Any four: salary, hourly wage, commission, contract, piecework, salary plus commission

3. Explain why a salesman might prefer to be paid by commission. Also, describe what would be needed for the salesman to succeed. *(2 marks)* (Module 1, Lesson 3)

   Answer:

   Various answers are acceptable.

   Unlimited income if sales are good, control of his/her own income.

   He/she will need to be open with people, to be personable, to sell a good product people want, to be where the customers are.

4. Jon offers to totally landscape his aunt's yard for $2000. This is an example of earning income by what method? *(1 mark)* (Module 1, Lesson 3)

   Answer: contract

5. Fill in the time card to find the gross pay. Overtime is paid at time and one-half on any hours over 40 hours per week. There are no late penalties. *(7 marks)* (Module 1, Lesson 5)

Employee: Scott					Hourly Rate: $11.75		
Day	Morning In	Out	Afternoon In	Out	Evening In	Out	Total
Mon.	07:30	12:00	1:00	5:00	7:30	9:45	10.75
Tues.	07:29	11:59	12:58	4:30			8
Wed.			1:00	5:30	7:30	10:15	7.25
Thurs.	07:30	12:02	1:01	5:32	7:00	10:30	12.5
Fri.	08:00	12:00	1:02	5:00			8
Sat.			1:00	4:45			3.75
Sun.							
							50.25

	Hours	Rate	Amount
Regular	40	$11.75	40 x $11.75 = $470.00
Overtime	10.25	$11.75 x 1.5 = $17.63	10.25 x $17.63 = $180.71
Total Hours	50.25	Gross Wages	$470 + $180.71 = $650.71

6. List six possible deductions one might expect from their gross earnings. *(6 marks)* (Module 2, Lesson 1)

   Answer:

   Any six of: dental plan, social fund, coffee, water, parking, CPP, EI, federal and provincial income tax, union dues, RSP

7. Jessica earns $11.44 per hour. She works 35 hours. Use the payroll deduction tables to find her net pay. Her claim code is 0. *(7 marks)* (Module 2, Lesson 5)

Canada Pension Plan Contributions

Weekly (52 pay periods a year)

Pay Rémunération			CPP RPC	Pay Rémunération			CPP RPC
From - De		To - Á		From - De		To - Á	
399.93	-	400.12	16.47	438.11	-	438.31	18.36
400.13	-	400.33	16.48	438.32	-	438.51	18.37
400.34	-	400.53	16.49	438.52	-	438.71	18.38
400.54	-	400.73	16.50	438.72	-	438.91	18.39
400.74	-	400.93	16.51	438.92	-	439.11	18.40
400.94	-	401.13	16.52	439.12	-	439.32	18.41
401.14	-	401.34	16.53	439.33	-	439.52	18.42
401.35	-	401.54	16.54	439.53	-	439.72	18.43
401.55	-	401.74	16.55	439.73	-	439.92	18.44

Employment Insurance Premiums

Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable			EI premium Cotisation d'AE
From - De		To - Á		From - De		To - Á	
395.67	-	396.24	6.85	437.29	-	437.86	7.57
396.25	-	396.82	6.86	437.87	-	438.43	7.58
396.83	-	397.39	6.87	438.44	-	439.01	7.59
397.40	-	397.97	6.88	439.02	-	439.59	7.60
397.98	-	398.55	6.89	439.60	-	440.17	7.61
398.56	-	399.13	6.90	440.18	-	440.75	7.62
399.14	-	399.71	6.91	440.76	-	441.32	7.63
399.72	-	400.28	6.92	441.33	-	441.90	7.64
400.29	-	400.66	6.93	441.91	-	442.48	7.65

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Pay Rémunération			Federal claim codes/ Codes de demande			
			0	1	2	3
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie			
399	-	403	53.60	24.50	21.65	15.95
403	-	407	54.20	25.05	22.20	16.50
407	-	411	54.75	25.60	22.75	17.05
411	-	415	55.30	26.15	23.30	17.65
415	-	419	55.85	26.75	23.90	18.20
419	-	423	56.40	27.30	24.45	18.75
423	-	427	57.00	27.85	25.00	19.30
427	-	431	57.55	28.40	25.55	19.85
431	-	435	58.10	28.95	26.10	20.45
435	-	439	58.65	29.50	26.70	21.00
439	-	443	59.20	30.10	27.25	21.55

Manitoba provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Pay Rémunération			Provincial claim codes/ Codes de demande			
			0	1	2	3
From De	Less than Moins de		Deduct from each pay Retenez sur chaque paie			
397	-	401	40.55	23.70	21.90	18.40
401	-	405	41.00	24.10	22.30	18.80
405	-	409	41.40	24.50	22.75	19.20
409	-	413	41.80	24.90	23.15	19.60
413	-	417	42.20	25.30	23.55	20.00
417	-	421	42.60	25.70	23.95	20.40
421	-	425	43.00	26.10	24.35	20.80
425	-	429	43.40	26.50	24.75	21.25
429	-	433	43.80	26.90	25.15	21.65
433	-	437	44.20	27.30	25.55	22.05
437	-	441	44.60	27.70	25.95	22.45

Gross earnings: *Answer:* \$11.44 × 35 = \$400.40

CPP: *Answer:* $16.49

EI: *Answer:* $6.93

Federal income tax, claim code 0: *Answer:* $53.60

Manitoba income tax, claim code 0: *Answer:* $40.55

Total deductions: *Answer:* $16.49 + $6.93 + $53.60 + $40.55 = $117.57

Net pay: *Answer:* $400.40 – $117.57 = $282.83

8. Joey works at a sporting goods store assembling bicycles. He gets paid $12 for each bike he assembles, but loses $8 for every bike that is not put together properly. Last week he worked 60 hours to assemble 65 bicycles before a big sale. The boss brought 11 bikes back because they were missing parts. Calculate Joey's income earned. *(1 mark)*
(Module 1, Lesson 3)

   Answer:

   Earnings for bikes assembled = $12 × 65 = $780

   Earnings lost for improper assembly = $8 × 11 = $88

   Total earnings = $780 – $88 = $692

9 How much more federal income tax will you pay if you get a $1 per hour raise, assuming you earn $10 per hour, work 40 hours per week, and use claim code 2. *(1 mark)* (Module 2, Lesson 4)

Answer:

Income tax at $10/hour = 10 × 40 = $400

Federal tax = $21.65

Income at $11/hour = 11 × 40 = $440

Federal tax = $27.25

Extra tax to be paid = $27.25 – $21.65 = $5.60

10. Jamie earns a salary of $40,040.00 per year. She pays $10 for union dues and $15 for parking. Use 4.95% for CPP, 1.73% for EI, and 29% for the combined federal and provincial income taxes to find her net weekly pay. *(7 marks)* (Module 2, Lesson 5)

Gross weekly earnings: *Answer:* $40,040/52 = $770.00

CPP: *Answer:* $770 × 0.0495 = $38.12

EI: *Answer:* $770 × 0.0173 = $13.32

Income tax: *Answer:* $770 × 0.29 = $223.30

Other deductions: *Answer:* $10 + $15 = $25

Total deductions: *Answer:* $38.12 + $38.32 + $223.30 + $25 = $299.74

Net pay: *Answer:* $770 – $299.74 = $470.26

11. Fred is paid a salary of $80,000 per year. Find his annual provincial income taxes using the marginal rates tables shown below. *(5 marks)* (Module 2, Lesson 4)

Manitoba Marginal Rates

Any annual income from $0 to $31,000: 10.8%

plus any annual income from $31,001 to $67,000: 12.75%

plus any annual income from $67,000: 17.4%

Answer:

Since Fred's income spills over into the second and third marginal levels, he will pay the maximum amounts in the first and second levels, and the spillover for the third level.

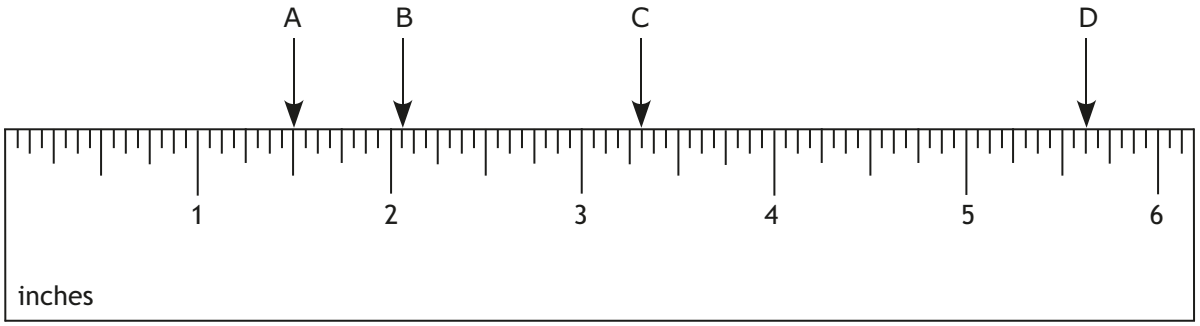
First level, income tax = $31,000 × 0.108 = $3348

Second level, income tax = ($67,000 – $31,000) × 0.1275 = $36,000 × 0.1275 = $4590

Third level, income tax = ($80,000 – $67,000) × 0.174 = $13,000 × 0.174 = $2262

Total provincial tax = $3348 + $4590 + $2262 = $10,200.

12. Find the readings on this imperial ruler. *(4 marks)* (Module 3, Lesson 4)



Answers:

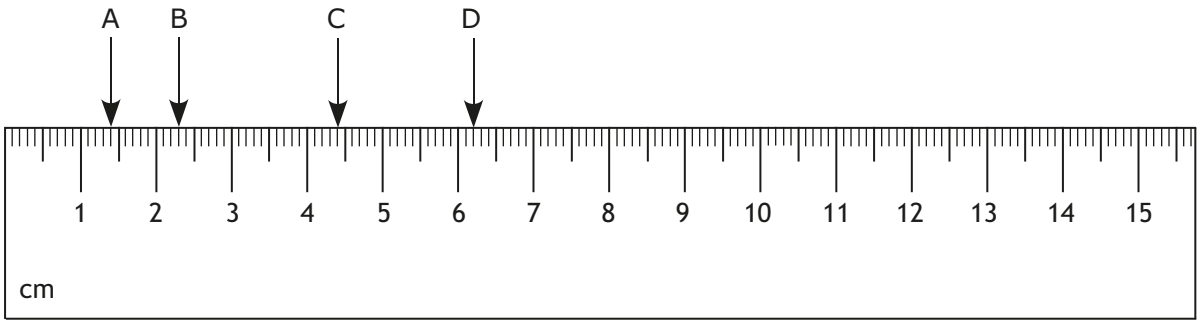
A $1\frac{1}{2}''$

B $2\frac{1}{16}''$

C $3\frac{5}{16}''$

D $5\frac{5}{8}''$

13. Record the readings in centimetres on this metric ruler. *(4 marks)* (Module 3, Lesson 4)



Answers:

A 1.4 cm

B 2.3 cm

C 4.4 cm

D 6.2 cm

14. Given that the temperature is 110°F, find the equivalent Celsius temperature by using the formula. Show your work. *(3 marks)* (Module 3, Lesson 8)

Answer:

Formula: $F = C \times 1.8 + 32$

Substitute: $110 = C \times 1.8 + 32$

Solve: $110 - 32 = C \times 1.8$

$$78 = 1.8C$$

$$\frac{78}{1.8} = \frac{1.8C}{1.8}$$

$$43.33 = C$$

The Celsius temperature is 43.33°C.

15. Use the conversion table to convert the following. Round off to two decimal places. Show your work. *(4 marks)* (Module 3, Lesson 5)

Basic Conversion Formulas between Systems	
Mass	1 kg = 2.2 pounds 454 g = 1 pound 28.4 grams = 1 ounce
Capacity	1 gallon (Canadian) = 4.54 L 1 fl. oz. (Canadian) = 28.41 mL 1 gallon (American) = 3.785 L 1 fl. oz. (American) = 29.57 mL
Distance	1 mile = 1.61 km 1 km = 0.62 mi. 1 inch = 2.54 cm 1 metre = 1.094 yards

a) 9 km = _____ miles

Answer:

1 km = 0.62 miles

9 km = 9 × 0.62 = 5.58 miles

b) 25 fl. oz. = _____ mL (Canadian)

Answer:

1 fl. oz. = 28.41 mL

25 fl. oz. = 25 × 28.41 = 710.25 mL

c) 150 pounds = _____ kg

Answer:

1 kg	2.2 pounds
x kg	150 pounds

$$\frac{1}{x} = \frac{2.2}{150}$$

$$(x)(2.2) = (1)(150)$$

$$2.2x = 150$$

$$\frac{2.2x}{2.2} = \frac{150}{2.2}$$

$$x = 68.18$$

Thus, 150 pounds = 68.18 kg.

d) 350 cm = _____ inches

Answer:

1 inch	2.54 cm
x inches	350 cm

$$\frac{1}{x} = \frac{2.54}{350}$$

$$(x)(2.54) = (1)(350$$

$$2.54x = 350$$

$$\frac{2.54x}{2.54} = \frac{350}{2.54}$$

$$x = 137.80$$

Thus, 350 cm = 137.80 inches.

16. The distance from Winnipeg to Minneapolis is about 500 miles. How far is the distance in kilometres? The conversion rules are given in #15 above. *(2 marks)*
(Module 3, Lesson 5)

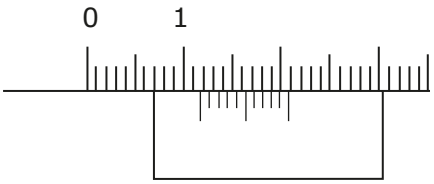
Answer:

1 mile = 1.61 km

500 miles = 500 × 1.61 = 805 km

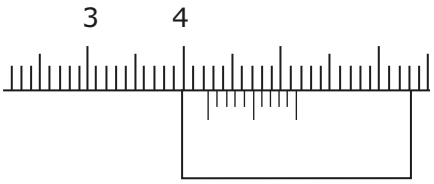
17. Record the metric Vernier caliper readings. *(2 marks)* (Module 3, Lesson 6)
Note: Your answer will be marked correct if it is ±0.02 from the given answer.

a)



Answer:
Reading = 1.18 cm

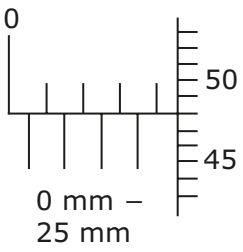
b)



Answer:
Reading = 4.26 cm

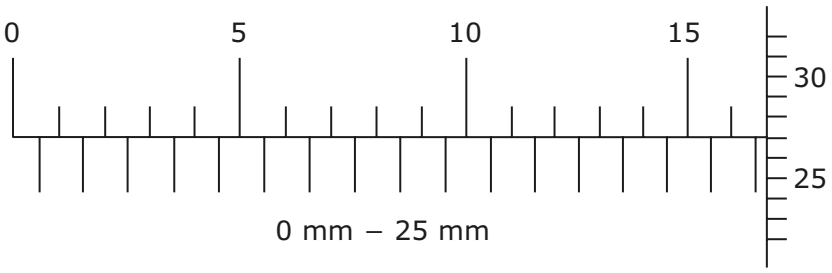
18. Record the micrometer readings. *(2 marks)* (Module 3, Lesson 7)

a)



Answer:
Reading = 4.48 mm

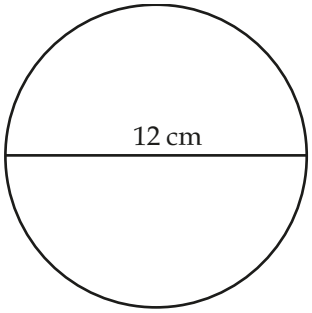
b)



Answer:
Reading = 16.77 mm

19. Find the area of each shape. *(9 marks)* (Module 4, Lesson 2)

a)



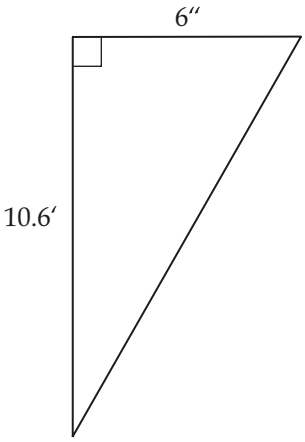
Answer:

$A = \pi r^2$

$A = 3.14(6\text{ cm})^2$

$A = 113.04\text{ cm}^2$

b)



Answer:

First, change the units to feet.

1 foot = 12 inches

$$6\text{ inches} = \frac{6}{12} = 0.5\text{ feet}$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}\left(0.5' \times 10.6'\right)$$

$$A = \frac{1}{2}\left(5.3'\right)$$

$$A = 2.65\text{ sq. ft.}$$

If you changed to the same units of inches, the solution would be as follows:

1 foot = 12 inches

10.6 feet = 10.6 × 12 = 127.2 inches

$$A = \frac{1}{2}bh$$

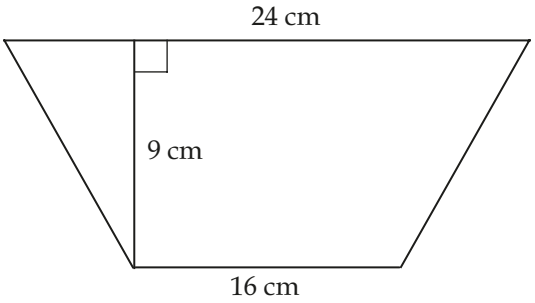
$$A = \frac{1}{2}(127.2'' \times 6'')$$

$$A = \frac{1}{2}(763.2'')$$

$$A = 381.6 \text{ sq. in.}$$

Either answer is acceptable.

c)



Answer:

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$A = \frac{1}{2}(9 \text{ cm})(16 \text{ cm} + 24 \text{ cm})$$

$$A = 180 \text{ cm}^2$$

20. You have enough sod to cover 300 m^2 of your lawn. Knowing the length of the lawn is 40 m, find the width. *(2 marks)* (Module 4, Lesson 2)

Answer:

Formula: $A = lw$

Substitute: $300 = 40w$

Solve $\frac{300}{40} = \frac{40w}{40}$

$7.5 = w$

The width of your lawn is 7.5 m.

21. You use a piece of string to find the direct distance between two points on a map. The length of the string is 4.6 cm long. The map scale is 1 cm: 200 miles. Find the direct distance between the points. *(2 marks)* (Module 4, Lesson 3)

Answer:

1 cm: 200 miles

4.6 cm = 4.6 × 200 = 920 miles

22. Convert the following measurements as indicated using the area conversions chart. Round off to two decimal places. *(9 marks)* (Module 4, Lesson 5)

Imperial		Metric
1 $in.^2$		6.4516 cm^2
1 $ft.^2$	144 $in.^2$	0.0929 m^2
l $yd.^2$	9 $ft.^2$	0.8361 m^2
1 acre	4840 $yd.^2$	4046.9 m^2
1 $mile^2$	640 acres	2.59 km^2

Metric		Imperial
1 cm^2	100 mm^2	0.1550 $in.^2$
1 m^2	10 000 cm^2	1.1960 $yd.^2$
1 hectare [ha]	10 000 m^2	2.4711 acres
1 km^2	100 ha	0.3861 $mile^2$

a) $4\frac{1}{4}$ yd.^{2} to m^2 *(1 mark)*

Answer:

1 yd.^{2} = 0.8361 m^2

4.25 yd.^{2} = 4.25 × 0.8361 = 3.55 m^2

b) 100 cm^2 to in.^{2} *(1 mark)*

Answer:

1 cm^2 = 0.155 in.^{2}

100 cm^2 = 100 × 0.155 = 15.5 in.^{2}

c) 640 in.^{2} to m^2 *(2 marks)*

Answer:

144 in.^{2}	0.0929 m^2
640 in.^{2}	x m^2

$$\frac{144}{640} = \frac{0.0929}{x}$$

$$144x = (640)(0.0929)$$

$$144x = 59.456$$

$$\frac{144x}{144} = \frac{59.456}{144}$$

$$x = 0.41$$

Thus, 640 in.^{2} = 0.41 m^2.

d) 102 square miles to km^2 *(1 mark)*

Answer:

1 $mile^2$ = 2.59 km^2

102 $mile^2$ = 102 × 2.59 = 264.18 km^2

e) 4000 cm^2 to $ft.^2$ *(4 marks)*

Answer:

1 cm^2 = 0.155 $in.^2$

4000 cm^2 = 4000 × 0.155 = 620 $in.^2$

1 $ft.^2$ = 144 $in.^2$

x sq. ft. = 620 sq. in.

$$\frac{1}{x} = \frac{144}{620}$$

$$144x = 620$$

$$\frac{144x}{144} = \frac{620}{144}$$

$$x - 4.31$$

Thus, 4000 cm^2 = 4.31 sq. ft.

23. Multiply these fractions using the grid method. Write your answer as a mixed number. *(2 marks)* (Module 4, Lesson 1)

$$1\frac{3}{4} \times 7\frac{1}{2}$$

Answer:

	7	$\frac{1}{2}$
1	$1 \times 7 = 7$	$1 \times \frac{1}{2} = \frac{1}{2}$
$\frac{3}{4}$	$\frac{3}{4} \times 7 = \frac{21}{4}$	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

$$1\frac{3}{4} \times 7\frac{1}{2} = 7 + \frac{1}{2} + \frac{21}{4} + \frac{3}{8} = 7 + \frac{4}{8} + \frac{42}{8} + \frac{3}{8} = 7 + \frac{49}{8} = 7 + 6\frac{1}{8} = 13\frac{1}{8}$$

24. The cost of the new football stadium is predicted to be $137,500,000. Write this number in scientific notation. *(1 mark)* (Module 3, Lesson 1)

Answer:

$\$1.375 \times 10^8$

25. Madeline works in a bakery. She makes a minimum wage of $9.25 per hour but has been offered a raise to $10.05. What percent rate of increase is this? Round off to two decimal places. *(2 marks)* (Module 1, Lesson 2)

Answer:

$$\text{Percent rate of increase} = \frac{\text{amount of change}}{\text{original value}} \times 100$$

$$\text{Percent increase in wage} = \frac{(\$10.05 - \$9.25)}{\$9.25} \times 100 = \frac{\$0.80}{\$9.25} \times 100 = 8.65\%$$

Remember: Attach your Midterm Exam Resource Sheet to your exam paper and submit it along with your exam.

Notes

GRADE 10 ESSENTIAL MATHEMATICS (20S)

Final Practice Exam

GRADE 10 ESSENTIAL MATHEMATICS

Final Practice Examination

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____ /100 = _____ %

Comments:

Instructions

The final examination will be weighted as follows:

Modules 5–8	100%

The format of the examination will be as follows:

Part A: Multiple Choice	10 marks
Part B: Long Answer	90 marks

Time allowed: 2.5 hours

Note: You are allowed to bring the following to the exam: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a ruler, a protractor, a compass, and your Final Exam Resource Sheet. Your Final Exam Resource Sheet must be handed in with the exam. **Diagrams may not be drawn to scale.**

Part A: Multiple Choice (*10 x 1 = 10 marks*)

Circle the letter of the correct answer for each question.

1. Cos 55° =
   a) 1.42814
   b) 0.57358
   c) 0.81915
   d) 0.022126

2. If cos θ = 0.6691306, then θ =
   a) 48°
   b) 42°
   c) 33.8°
   d) 22.9°

3. If $\Delta ABC \sim \Delta PQR$, then
   a) AB = PQ
   b) $\angle A = \angle P$
   c) $a = b$
   d) $c = r$

4. The cost of 4 L of milk is $7.35. The unit cost per litre is
   a) $3.68
   b) $1.84
   c) $2.45
   d) $0.735

5. A 10% discount off an item priced at $30.00 would save you
   a) $3.00
   b) $27.00
   c) $10.00
   d) $1.00

6. The currency widely used across Europe is
   a) US dollar
   b) Japanese yen
   c) Euro
   d) Peso

7. A reflection of a shape is
   a) larger than the original
   b) smaller than the original
   c) distorted
   d) a mirror image

8. A rotation of a shape is
   a) a slide
   b) a translation
   c) a turn
   d) a mirror image

9. Which statement is true about angles with parallel lines with a transversal?
   a) All angles are 60°
   b) Interior alternate angles are complementary
   c) Corresponding angles are supplementary
   d) Exterior alternate angles are congruent

10. If two lines are perpendicular, they
   a) meet at a right angle
   b) are always the same distance apart
   c) never meet
   d) are supplementary

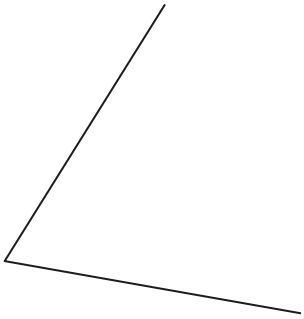
Part B: Long Answer (*90 marks*)

Write your answers in the space provided.

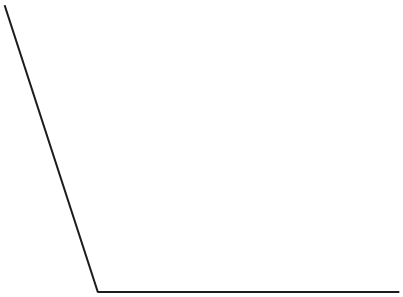
1. Explain what a right angle is. *(1 mark)*

2. Draw an example of two angles that are supplementary but do not have the same vertex. *(1 mark)*

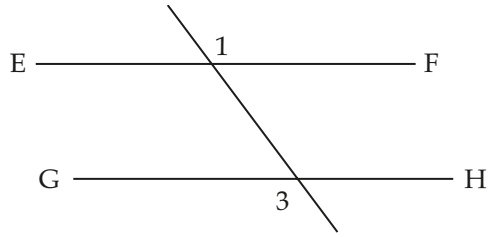
3. Use any method to copy the angle shown below, and describe the process you used. *(4 marks)*



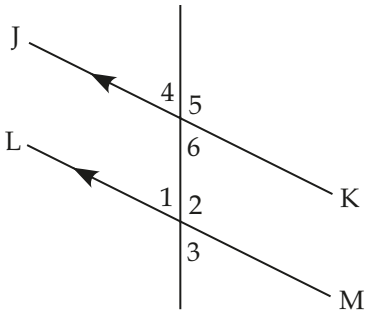
4. Use any method to bisect this angle, and describe the process you used. *(4 marks)*



5. Given ∠1 and ∠3 are congruent, explain why EF // GH. *(2 marks)*



6. Identify any four pairs of equal angles and give a reason why they are equal. You are given JK//LM. *(4 marks)*



7. Your new wage is $10.90 per hour. You were making $10.75 per hour. Find the percent rate of change. *(3 marks)*

8. Your job in the restaurant pays you $11.32 per hour. Your boss offers you either a 3% raise, or $0.35 per hour raise. Which is the better offer? Explain your reasoning. *(3 marks)*

9. Apple juice is sold in packages of various sizes.

   a) Find the unit price per 100 mL to determine the best buy. *(3 marks)*

   i) 350 mL for $1.05

   ii) 1 L for $3.15

   iii) 2 L for $5.95

   b) Give one reason why a person might want a smaller size, knowing it costs more. *(1 mark)*

10. Dane needs a new washer and dryer set, and the set he wants will cost $1345. Southdale Department Store has a promotion on where the store will pay the taxes. Charles' is having a 10% off all appliances sale. The Lighte has a 15% off sale on all appliances.

    a) Which is the best buy? *(4 marks)*

    b) State two other types of promotions that stores use. *(2 marks)*

    c) State two reasons why you would choose to go to a store with more expensive prices. *(2 marks)*

11. Griffen was planning a trip to Mexico and the southern United States. He budgeted for 18 200 pesos and $850 American dollars.

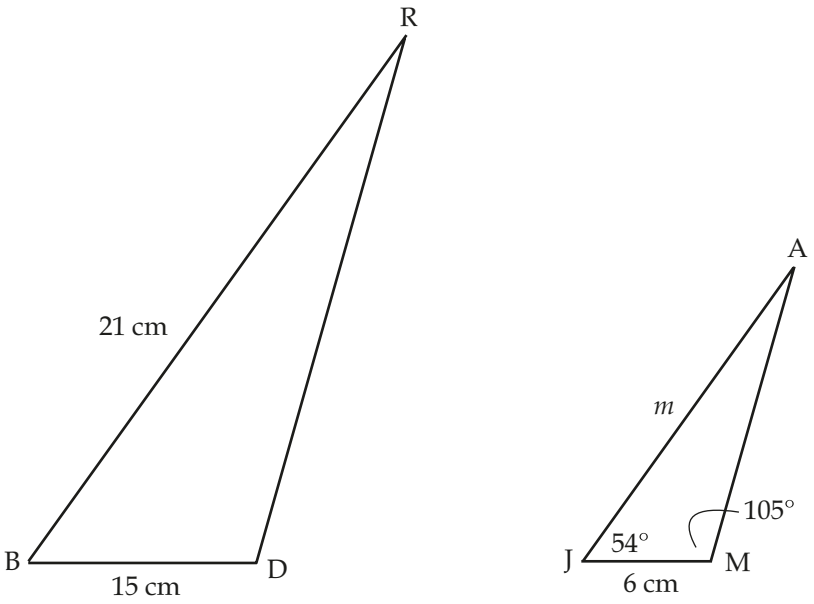
    a) Use the Currency Converter Chart to find the cost of obtaining these currencies from a bank. *(3 marks)*

Currency Converter as of February, 2009		
	Bank's Selling Rate	**Bank's Buying Rate**
Canadian Dollar	1	1
Mexican Peso	0.0909	00.0794
American Dollar	1.2634	1.2106

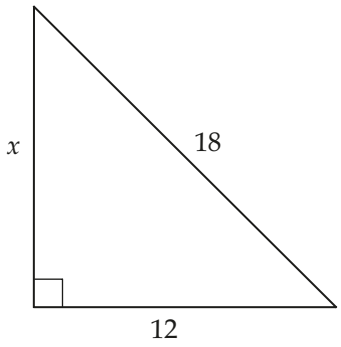
    b) Upon his return, Griffen had 1425 pesos and $52 American dollars to return to the bank. How much did he receive for this exchange? *(3 marks)*

12. You have $730 Canadian for a trip to Grand Forks. How much US currency can you get from the bank? Use the Currency Converter chart in #11 above. *(2 marks)*

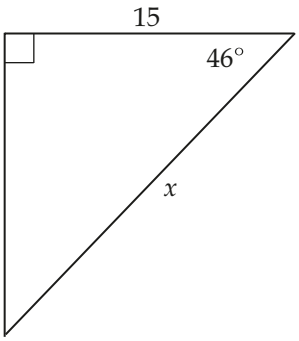
13. Given two similar triangles, ΔBRD ~ ΔJAM, as shown in the diagram below, find the length of *m* and the measure of ∠R. *(4 marks)*



14. Find the length of the missing side. Round off to two decimal places. *(3 marks)*

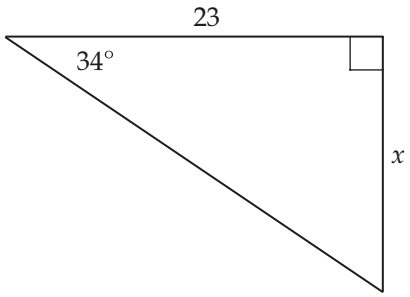


15. Determine the length of the missing side using a trigonometric ratio. Round off to two decimal places. *(3 marks)*



16. A cell phone tower is held in place by a cable 20 m long. The base of the cable is 14 m away from the base of the tower. Determine the height of the tower. *(4 marks)*

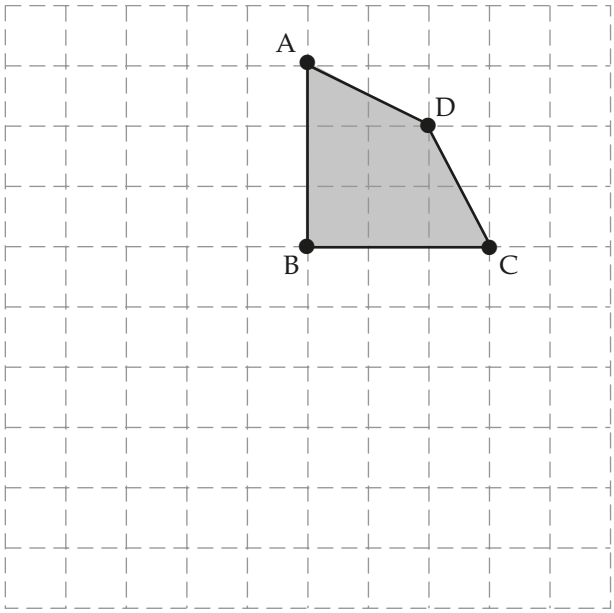
17. Find the missing side. Round off to two decimal places. *(3 marks)*



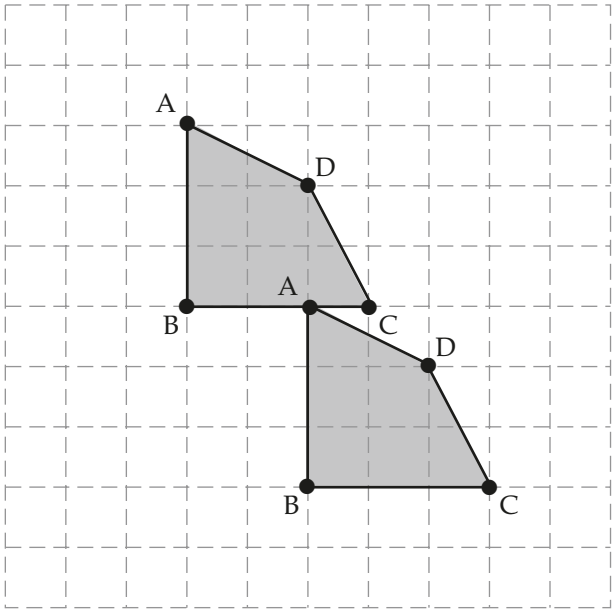
18. A hot air balloon is attached to a 25 m rope. The balloon is 15 m above the ground. Find the angle at which the rope is inclined to the ground. Draw the diagram, set up the appropriate trig ratio, and solve for θ. Round off to two decimal places. *(5 marks)*

19. A meteor is racing toward Earth and will soon enter the atmosphere. The meteor is above the top of a hill 22 miles away from where you are. You look upward 16 miles to see the meteor. Find the angle between the ground and your line of sight as you look towards the meteor. Draw the diagram, set up the appropriate trig ratio, and solve for θ. *(5 marks)*

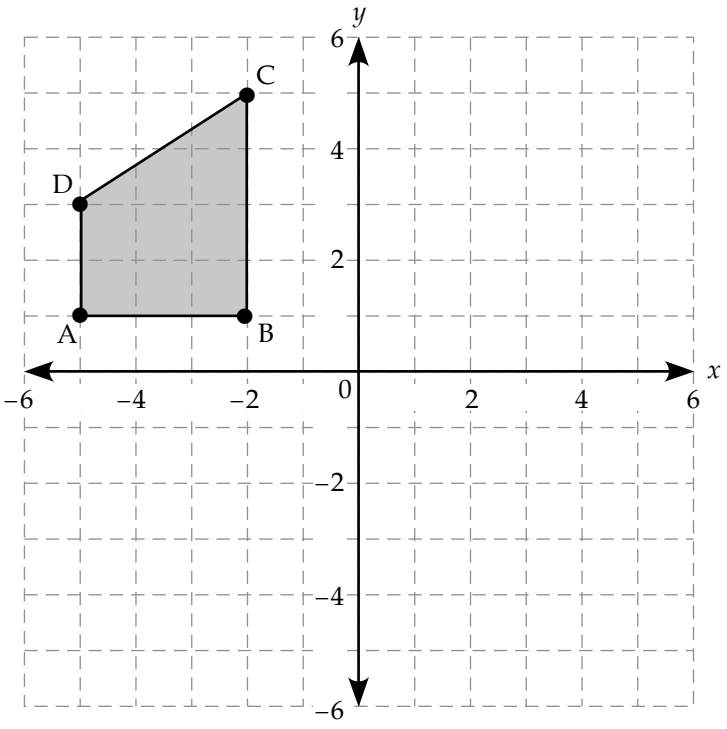
20. Draw the translation of the following shape, applying the rule [L2, D4], and label the points of the image. *(4 marks)*



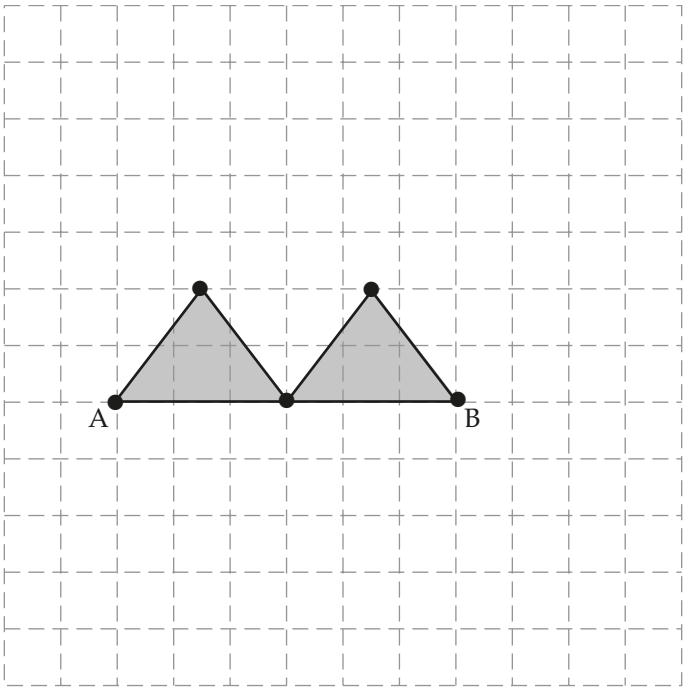
21. Identify the translation used. Write your answer in words. *(2 marks)*



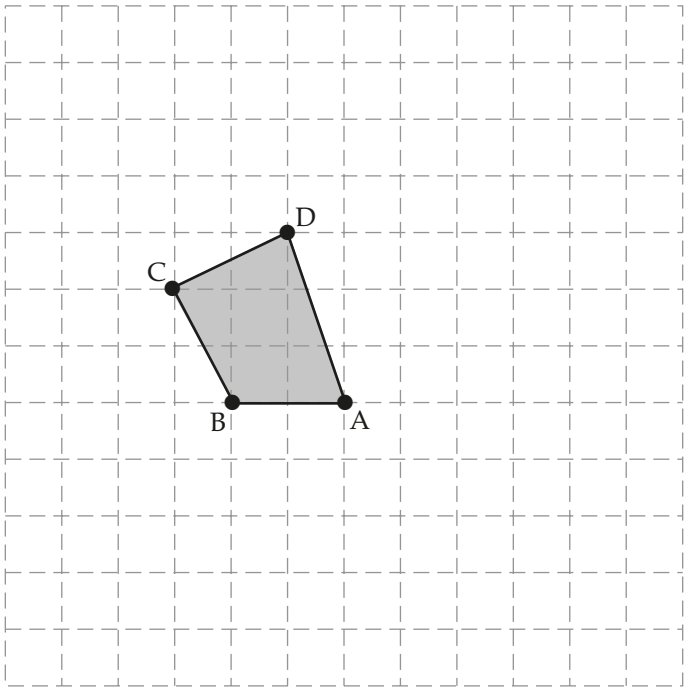
22. Draw the reflection of this shape over the x-axis, and label the points in the image. *(4 marks)*



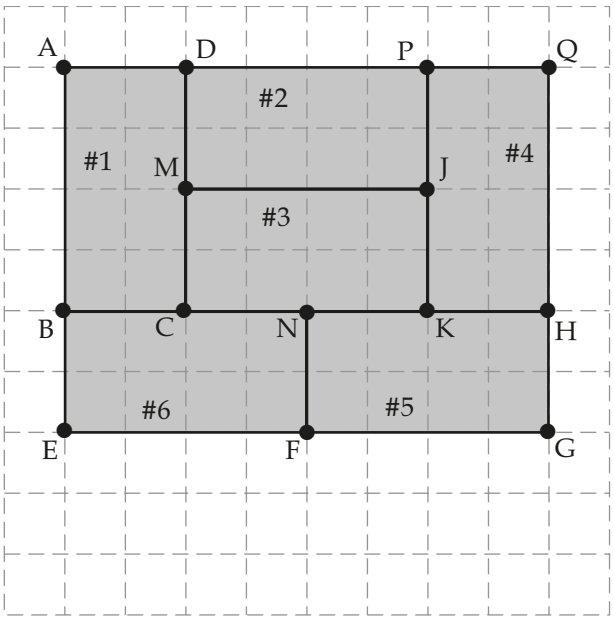
23. Rotate the shape 90° clockwise about point B, and draw the rotation. *(3 marks)*



24. Draw a dilation of this shape using a scale factor of 2, with point B being the centre of the dilation. *(4 marks)*



25. Identify a possible pattern of transformations used to create this shape, starting with rectangle ABCD. Each successive transformation is performed to the previous image. *(4 marks)*



Remember: Attach your Final Exam Resource Sheet to your exam paper and submit it along with your exam.

GRADE 10 ESSENTIAL MATHEMATICS (20S)

Final Practice Exam
Answer Key

Final Practice Examination
Answer Key

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____/100 = _____ %

Comments:

Instructions

The final examination will be weighted as follows:

Modules 5–8	100%

The format of the examination will be as follows:

Part A: Multiple Choice	10 marks
Part B: Long Answer	90 marks

Time allowed: 2.5 hours

Note: You are allowed to bring the following to the exam: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a ruler, a protractor, a compass, and your Final Exam Resource Sheet. Your Final Exam Resource Sheet must be handed in with the exam. **Diagrams may not be drawn to scale.**

Part A: Multiple Choice (*10 x 1 = 10 marks*)

Circle the letter of the correct answer for each question.

1. Cos 55° =
   a) 1.42814
   b) 0.57358
   c) 0.81915
   d) 0.022126

   Module 7, Lesson 6

2. If cos θ = 0.6691306, then θ =
   a) 48°
   b) 42°
   c) 33.8°
   d) 22.9°

   Module 7, Lesson 6

3. If $\Delta ABC \sim \Delta PQR$, then
   a) AB = PQ
   b) $\angle A = \angle P$
   c) $a = b$
   d) $c = r$

   Module 7, Lesson 2

   Note: AB corresponds to PQ but is not equal to it. Side *c* corresponds to side *r* but is not equal to it.

4. The cost of 4 L of milk is $7.35. The unit cost per litre is
   a) $3.68
   b) $1.84
   c) $2.45
   d) $0.735

   Module 6, Lesson 1

5. A 10% discount off an item priced at $30.00 would save you
   a) $3.00
   b) $27.00
   c) $10.00
   d) $1.00

   Module 6, Lesson 3

6. The currency widely used across Europe is
   a) US dollar
   b) Japanese yen
   c) Euro
   d) Peso — Module 6, Lesson 6

7. A reflection of a shape is
   a) larger than the original
   b) smaller than the original
   c) distorted
   d) a mirror image — Module 8, Lesson 3

8. A rotation of a shape is
   a) a slide
   b) a translation
   c) a turn
   d) a mirror image — Module 8, Lesson 2

9. Which statement is true about angles with parallel lines with a transversal?
   a) All angles are 60°
   b) Interior alternate angles are complementary
   c) Corresponding angles are supplementary
   d) Exterior alternate angles are congruent — Module 5, Lesson 5

10. If two lines are perpendicular, they
   a) meet at a right angle
   b) are always the same distance apart
   c) never meet
   d) are supplementary — Module 5, Lesson 4

Part B: Long Answer (*90 marks*)

Write your answers in the space provided.

1. Explain what a right angle is. *(1 mark)* Module 5, Lesson 1

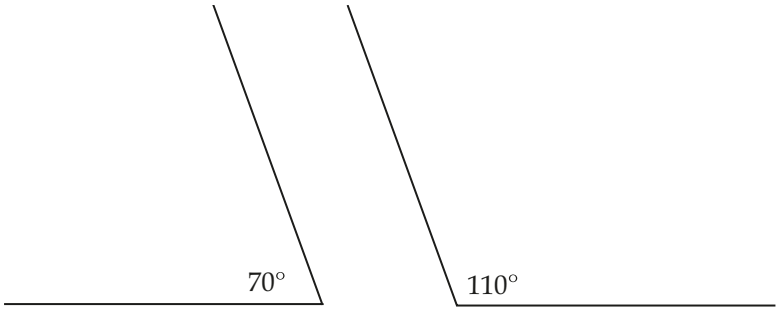
   Answer:

   An angle whose measure is exactly 90°.

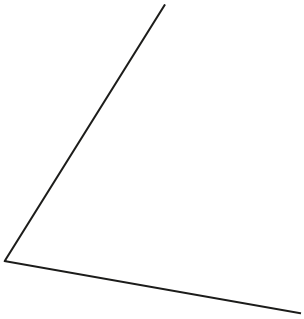
2. Draw an example of two angles that are supplementary but do not have the same vertex. *(1 mark)* Module 5, Lesson 4

   Answer:

   Any two angles whose sum is 180° are acceptable.



3. Use any method to copy the angle shown below, and describe the process you used. *(4 marks)* Module 5, Lesson 3



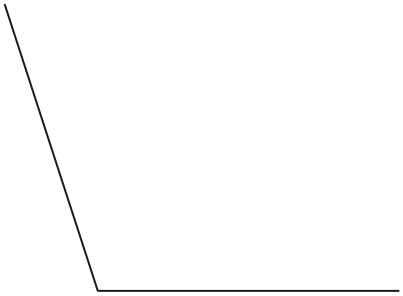
Answer:

Answers may vary.

Angle must be exact within 3° and approximate size is 68°.

Descriptions could include protractor, folding, tracing, or compass.

4. Use any method to bisect this angle, and describe the process you used. *(4 marks)* Module 5, Lesson 3

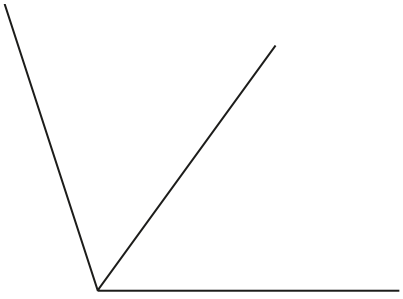


Answer:

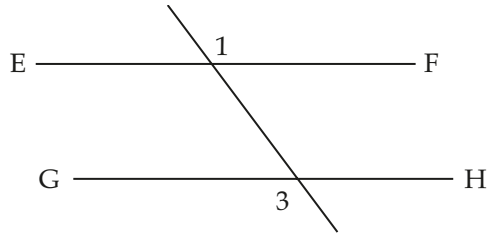
Answers may vary.

Angle must be bisected within 3° and each angle is approximately 54°.

Descriptions could include protractor, folding, tracing, or compass.



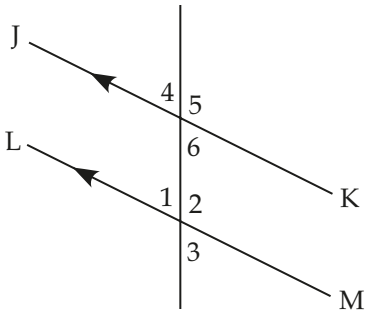
5. Given ∠1 and ∠3 are congruent, explain why EF // GH. *(2 marks)* Module 5, Lesson 6



Answer:

∠1 and ∠3 are exterior alternate angles. Since they are equal, then the lines must be parallel.

6. Identify any four pairs of equal angles and give a reason why they are equal. You are given JK//LM. *(4 marks)* Module 5, Lesson 5



Answer:

Answers may vary. Any four of the following:

∠1 = ∠3	vertically opposite angles
∠1 = ∠4	corresponding angles
∠1 = ∠6	interior alternate angles
∠2 = ∠5	corresponding angles
∠3 = ∠4	exterior alternate angles
∠3 = ∠6	corresponding angles
∠4 = ∠6	vertically opposite angles

7. Your new wage is $10.90 per hour. You were making $10.75 per hour. Find the percent rate of change. *(3 marks)* Module 6, Lesson 4

Answer:

$$\text{Percent rate of change} = \frac{\text{amount of change}}{\text{original value}} \times 100$$

$$\text{Percent rate of change} = \frac{0.15}{10.75} \times 100 = 1.4\%$$

8. Your job in the restaurant pays you $11.32 per hour. Your boss offers you either a 3% raise, or $0.35 per hour raise. Which is the better offer? Explain your reasoning. *(3 marks)* Module 6, Lesson 4

Answer:

Percent raise

$11.32 × 1.03 = $11.66

Money raise

$11.32 + 0.35 = $11.67

The $0.35 increase is better.

Students may choose to convert the % increase into dollars.

0.03 × $11.32 = $0.34

9. Apple juice is sold in packages of various sizes.

a) Find the unit price per 100 mL to determine the best buy. *(3 marks)*

i) 350 mL for $1.05

ii) 1 L for $3.15

iii) 2 L for $5.95 Module 6, Lesson 1

Answer:

i) 350 mL for $1.05

350 mL	$1.05
100 mL	$$x$

$$\frac{350}{100} = \frac{1.05}{x}$$

$$350x = 105$$

$$\frac{350x}{350} = \frac{105}{350}$$

$$x = 0.3$$

Therefore, unit price = $0.30 per 100 mL.

ii) 1 L for $3.15

1 litre = 1000 mL

1000 mL	$3.15
100 mL	$$x$

$$\frac{1000}{100} = \frac{3.15}{x}$$

$$1000x = 315$$

$$\frac{1000x}{1000} = \frac{315}{1000}$$

$$x = 0.315$$

Therefore, unit price = $0.32 per 100 mL.

iii) 2 L for $5.95

1 litre = 1000 mL

2 L = 2 × 1000 = 2000 mL

2000 mL	$5.95
100 mL	$\$x$

$$\frac{2000}{100} = \frac{5.95}{x}$$

$$2000x = 595$$

$$\frac{2000x}{2000} = \frac{595}{2000}$$

$$x = 0.2975$$

Therefore, unit price = $0.30 per 100 mL.

Either the 350 mL or the 2 L size would be the better buy, although the 2 L is $0.0025 cheaper if you use all the decimal points.

b) Give one reason why a person might want a smaller size, knowing it costs more. *(1 mark)* Module 6, Lesson 1

Answer:

Answers will vary.

Not enough money, not that thirsty, larger size might spoil, no storage room, distance from store

10. Dane needs a new washer and dryer set, and the set he wants will cost $1345. Southdale Department Store has a promotion on where the store will pay the taxes. Charles' is having a 10% off all appliances sale. The Lighte has a 15% off sale on all appliances. Module 6, Lesson 5

a) Which is the best buy? *(4 marks)*

Answer:

Southdale

Total Cost: $1345

Charles'

Cost with Discount, No Taxes: $(1 - 0.10) \times 1345 = \$1210.50$

PST: $0.07 \times 1210.50 = \$84.74$

GST: $0.05 \times 1210.50 = \$60.53$

Total Including Taxes: $\$1350 + \$84.74 + \$60.53 = \$1355.77$

The Lighte

15% off: $(1 - 0.15) \times \$1345 = \$11.43.25$

PST: $0.07 \times \$1143.25 = \$80.03$

GST: $0.05 \times \$1143.25 = \$57.17$

Total: $\$1143.25 + \$80.03 + \$57.17 = \$1280.45$

b) State two other types of promotions that stores use. *(2 marks)*

Answer:

Answers may vary. Responses may include the following:

- no taxes
- loyalty cards
- % discounts
- $ discounts
- package deals

c) State two reasons why you would choose to go to a store with more expensive prices. *(2 marks)*

Answer:

Answers may vary. Responses may include:

- time to get to the store
- cost of gas/amount of gas to get there
- rewards/loyalty cards

11. Griffen was planning a trip to Mexico and the southern United States. He budgeted for 18 200 pesos and $850 American dollars.

a) Use the Currency Converter Chart to find the cost of obtaining these currencies from a bank. *(3 marks)* Module 6, Lesson 6

Currency Converter as of February, 2009		
	Bank's Selling Rate	**Bank's Buying Rate**
Canadian Dollar	1	1
Mexican Peso	0.0909	00.0794
American Dollar	1.2634	1.2106

Answer:

Griffin is buying currency from the bank, so he needs to use the bank's selling rates.

Pesos

1 Peso = $1.6345 Canadian

18 200 pesos = 18 200 × $0.0909 = $1654.38

US dollars

1 US dollar = $1.2634 Canadian

850 US dollars = 850 × $1.2634 = $1073.89

Total = $1654.38 + $1073.89 = $2727.27

b) Upon his return, Griffen had 1425 pesos and $52 American dollars to return to the bank. How much did he receive for this exchange? *(3 marks)* Module 6, Lesson 6

Answer:

Griffin is selling currency to the bank, so he needs to use the bank's buying rates.

Euro

1 Peso = $0.0794 Canadian

1425 pesos = 1425 × $0.0794 = $113.15

US dollars

1 US dollar = $1.2106 Canadian

52 US dollars = 52 × $1.2106 = $62.95

Total refund = $113.15 + $62.95 = $176.10

12. You have $730 Canadian for a trip to Grand Forks. How much US currency can you get from the bank? Use the Currency Converter chart in #11 above. *(2 marks)*
Module 6, Lesson 6

Answer:

$1.2634 Canadian	$1 US
$730 Canadian	$\$x$ US

$$\frac{1.2634}{730} = \frac{1}{x}$$

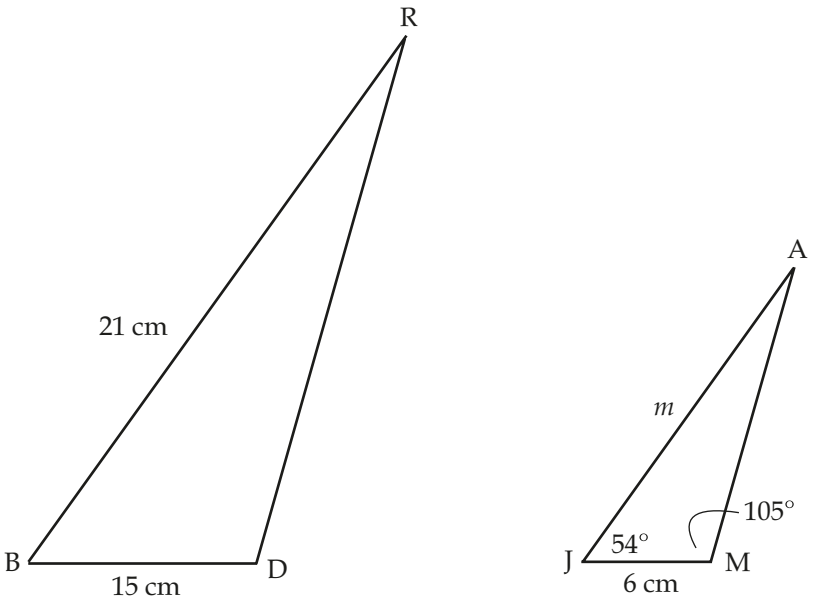
$$1.2634x = 730$$

$$\frac{1.2634x}{1.2634} = \frac{730}{1.2634}$$

$$x = 577.81$$

Therefore, you can buy $577.81 US from the bank.

13. Given two similar triangles, ΔBRD ~ ΔJAM, as shown in the diagram below, find the length of m and the measure of ∠R. *(4 marks)* Module 7, Lesson 2



Answer:

From the similarity relationship, ΔBRD ~ ΔJAM, the chart can be made.

	∠B = ∠J = 54°	∠R = ∠A = ?	∠D = ∠M = 105°
ΔBRD	b = ?	r = 15 cm	d = 21 cm
ΔJAM	j = ?	a = 6 cm	m = ?

$$\frac{r}{a} = \frac{d}{m}$$

$$\frac{15}{6} = \frac{21}{m}$$

$$15m = (6)(21)$$

$$15m = 126$$

$$\frac{15m}{15} = \frac{126}{15}$$

$$m = 8.4$$

The length of m is 8.4 cm.

To find the measure of ∠R, use the fact that the sum of the angles in a triangle = 180°.

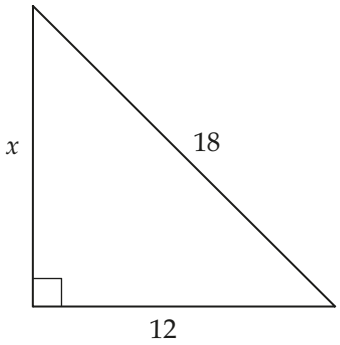
Since ΔBRD ~ ΔJAM, then ∠B = ∠J = 54° and ∠D = ∠M = 105°.

Thus, ∠R = 180° – (54° + 105°) = 180° – 159° = 21°.

The length of m is 8.4 cm.

The measure of ∠R is 21°.

14. Find the length of the missing side. Round off to two decimal places. *(3 marks)* Module 7, Lesson 3



Answer:

$$a^2 + b^2 = c^2$$

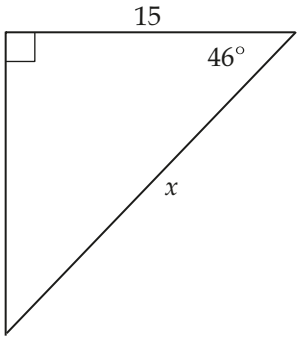
$$x^2 + 12^2 = 18^2$$

$$x^2 + 144 = 324$$

$$x^2 = 180$$

$$x = 13.42$$

15. Determine the length of the missing side using a trigonometric ratio. Round off to two decimal places. *(3 marks)* Module 6, Lesson 6



Answer:

$$\cos 46° = \frac{15}{x}$$

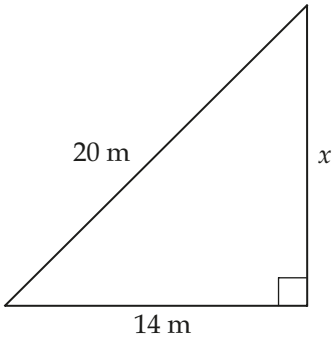
$$x = \frac{15}{\cos 46°}$$

$$x = 21.59$$

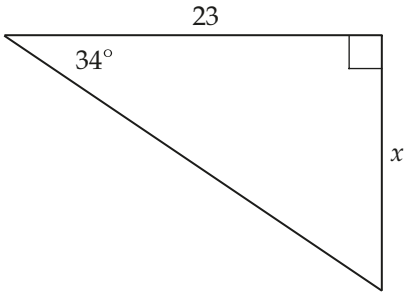
16. A cell phone tower is held in place by a cable 20 m long. The base of the cable is 14 m away from the base of the tower. Determine the height of the tower. *(4 marks)*
Module 7, Lesson 3

Answer:

$$a^2 + b^2 = c^2$$
$$x^2 + 14^2 = 20^2$$
$$x^2 + 196 = 400$$
$$x^2 = 204$$
$$x = 14.3 \text{ m}$$



17. Find the missing side. Round off to two decimal places. *(3 marks)*
Module 7, Lesson 4



Answer:

$$\tan 34° = \frac{x}{23}$$
$$x = 23 \tan 34°$$
$$x = 15.5$$

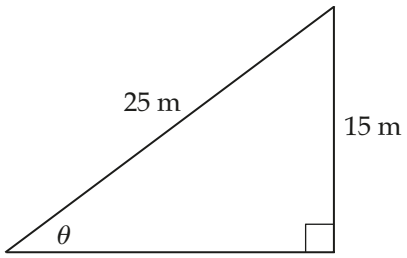
18. A hot air balloon is attached to a 25 m rope. The balloon is 15 m above the ground. Find the angle at which the rope is inclined to the ground. Draw the diagram, set up the appropriate trig ratio, and solve for θ. Round off to two decimal places. *(5 marks)* Module 7, Lesson 7

Answer:

$$\sin\theta = \frac{15}{25}$$

$$\theta = \sin^{-1}\left(\frac{15}{25}\right)$$

$$\theta = 36.87°$$



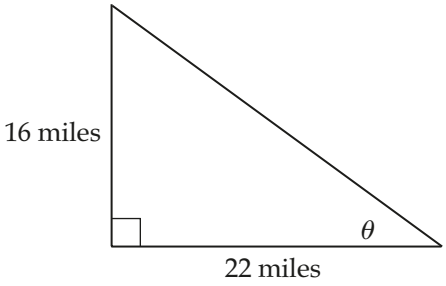
19. A meteor is racing toward Earth and will soon enter the atmosphere. The meteor is above the top of a hill 22 miles away from where you are. You look upward 16 miles to see the meteor. Find the angle between the ground and your line of sight as you look towards the meteor. Draw the diagram, set up the appropriate trig ratio, and solve for θ. *(5 marks)* Module 7, Lesson 7

Answer:

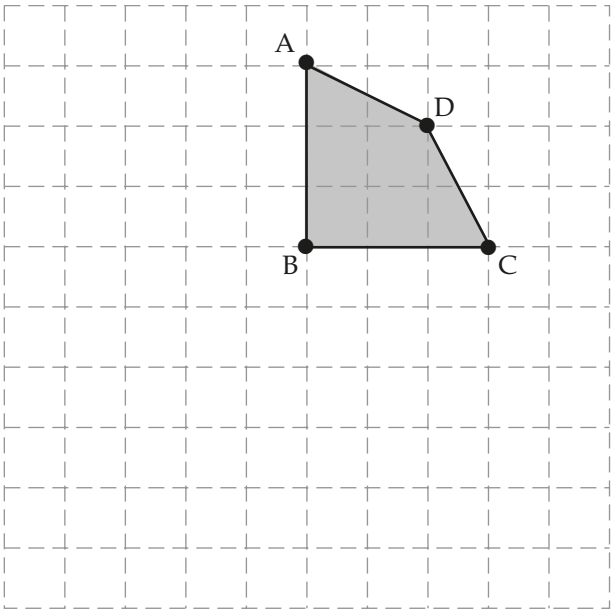
$$\tan\theta = \frac{16}{22}$$

$$\theta = \tan^{-1}\left(\frac{16}{22}\right)$$

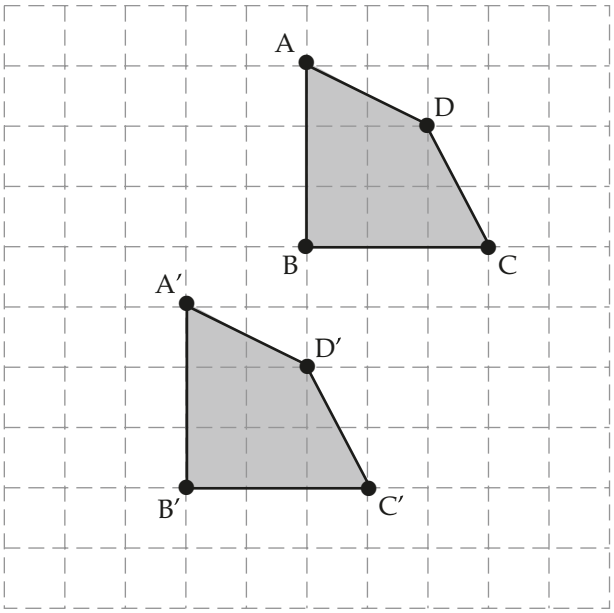
$$\theta = 36°$$



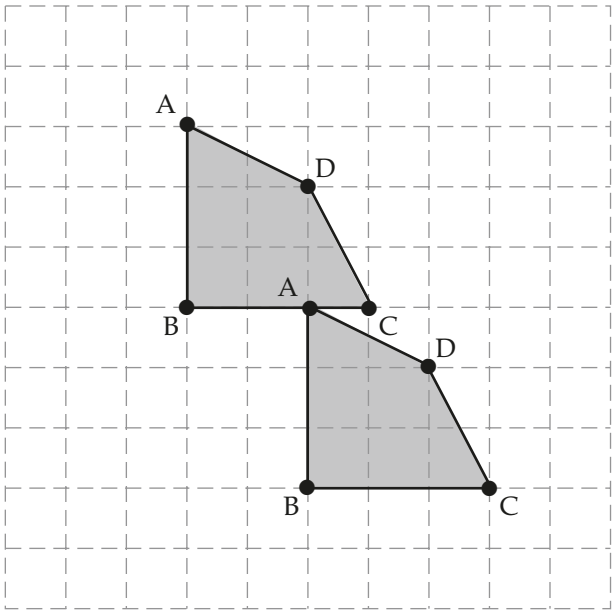
20. Draw the translation of the following shape, applying the rule [L2, D4], and label the points of the image. *(4 marks)* Module 8, Lesson 1



Answer:



21. Identify the translation used. Write your answer in words. *(2 marks)*
Module 8, Lesson 1

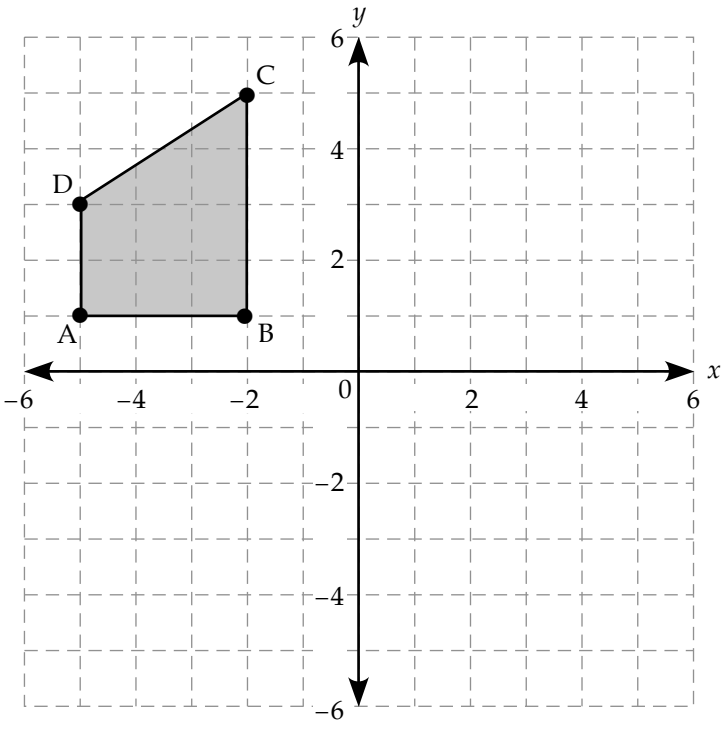


Answer:

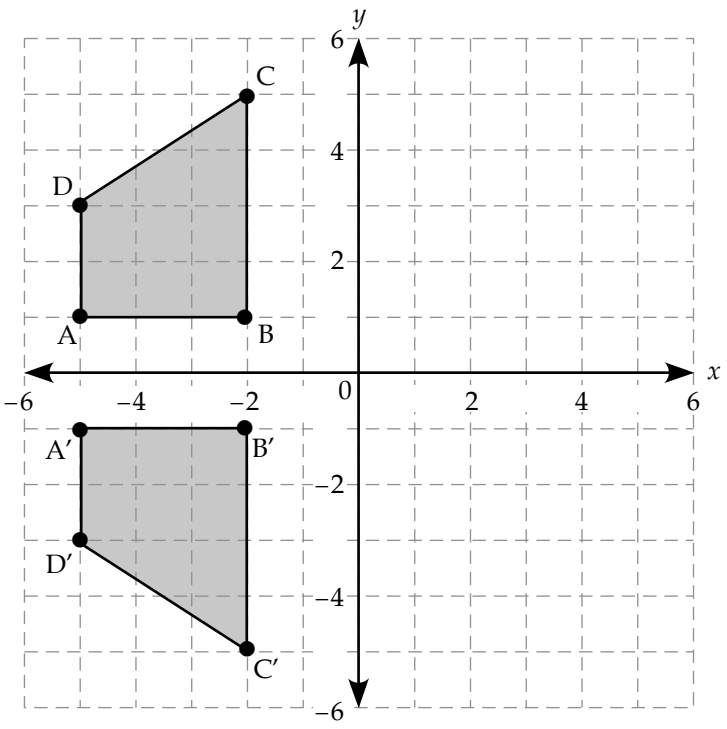
Rule: [R2, D3]

Words: Right 2 units, down 3 units

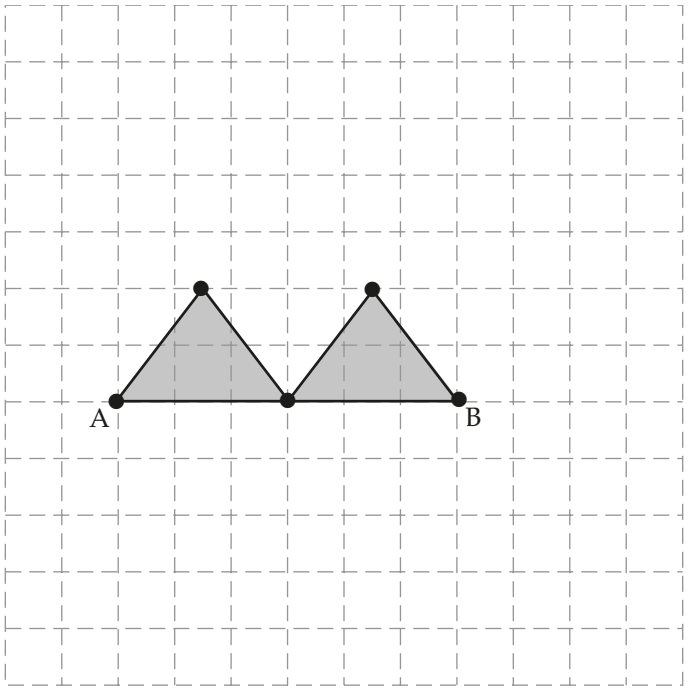
22. Draw the reflection of this shape over the x-axis, and label the points in the image. *(4 marks)* Module 8, Lesson 3



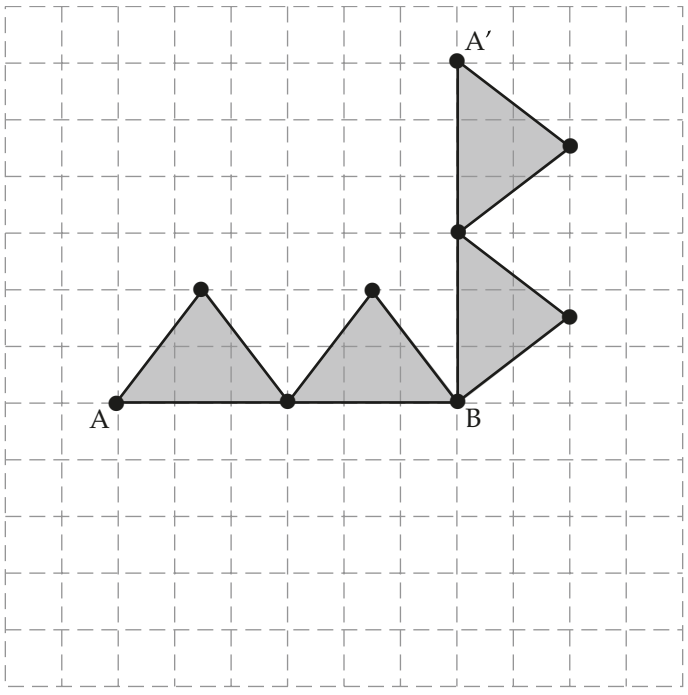
Answer:



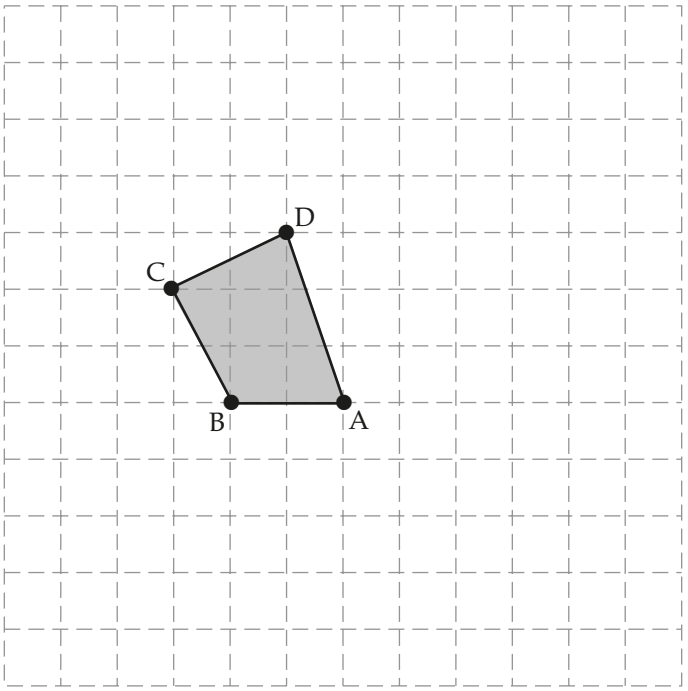
23. Rotate the shape 90° clockwise about point B, and draw the rotation. *(3 marks)*
Module 8, Lesson 2



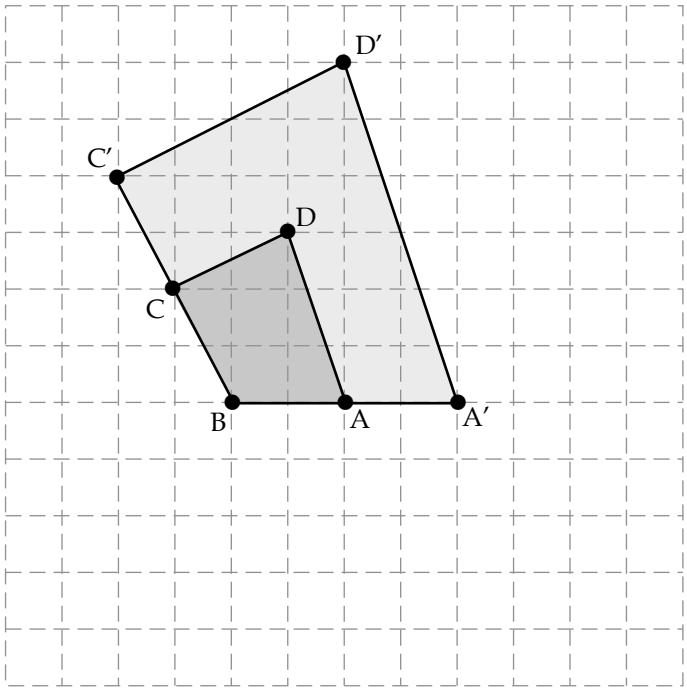
Answer:



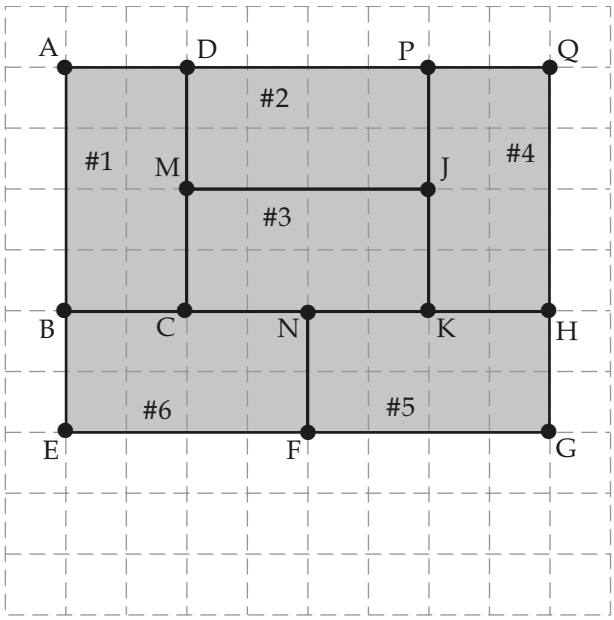
24. Draw a dilation of this shape using a scale factor of 2, with point B being the centre of the dilation. *(4 marks)* Module 8, Lesson 5



Answer:



25. Identify a possible pattern of transformations used to create this shape, starting with rectangle ABCD. Each successive transformation is performed to the previous image. *(4 marks)* Module 8, Lesson 4



Answer:

Answers will vary, any methods that would work are acceptable.

Rotate #1 90° CCW about point D, to make #2.

Translate #2 [R0, D2] to make #3.

Rotate #3 90° CW about point K to make #4.

Rotate #4 90° CCW about H to make #5.

Translate #5 [R4, D0] to make #6.

Remember: Attach your Final Exam Resource Sheet to your exam paper and submit it along with your exam.

Notes