Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

A Course for Independent Study

Field Validation Version

Manitoba

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2011, 2019

Manitoba Education

Manitoba Education Cataloguing in Publication Data

Grade 10 introduction to applied and pre-calculus mathematics
(20S) : a course for independent study—Field validation
version

ISBN: 978-0-711-4894-1

1. Mathematics—Study and teaching (Secondary).
2. Calculus—Study and teaching (Secondary).
3. Mathematics—Study and teaching (Secondary)—Manitoba.
4. Calculus—Study and teaching (Secondary)—Manitoba.
5. Mathematics—Programmed instruction.
6. Calculus—Programmed instruction.
7. Distance education—Manitoba.
8. Correspondence schools and courses—Manitoba.
I. Manitoba. Manitoba Education.
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Manitoba Education
Winnipeg, Manitoba, Canada

This resource was published in 2011 and updated in 2019.

Disponible en français.

Available in alternate formats upon request.

Contents

Acknowledgements

Manitoba Education gratefully acknowledges the contributions of the following individuals in the development of *Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S): A Course for Independent Study*.

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Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Introduction

Introduction

Overview

Welcome to *Grade 10 Introduction to Applied and Pre-Calculus Mathematics*! This course is a continuation of the concepts you have been studying in previous years, as well as an introduction to new topics. It lays the foundation for both applied and pre-calculus mathematics courses at the Grade 11 and Grade 12 levels. It also develops the skills, ideas, and confidence that you will need to continue studying math in the future.

As a student enrolled in an independent study course, you have taken on a dual role—that of a student and a teacher. As a student, you are responsible for mastering the lessons and completing the learning activities and assignments. As a teacher, you are responsible for checking your work carefully, noting areas in which you need to improve and motivating yourself to succeed.

What Will You Learn in This Course?

Problem solving, communication, reasoning, and mental math are some of the themes you will discover in each module. You will engage in a variety of activities that promote the connections between symbolic math ideas and the world around you.

There are four main areas that you will be exploring: Number, Patterns and Relations, Shape and Space, and Statistics and Probability.

How Is This Course Organized?

The Grade 10 Introduction to Applied and Pre-Calculus Mathematics course consists of the following eight modules:

- Module 1: Graphs and Relations
- Module 2: Number Sense
- Module 3: Measurement
- Module 4: Trigonometry
- Module 5: Relations and Functions
- Module 6: Polynomials
- Module 7: Coordinate Geometry
- Module 8: Systems of Equations

Each module in this course consists of several lessons, which contain the following components:

- **Lesson Focus:** The Lesson Focus at the beginning of each lesson identifies one or more specific learning outcomes (SLOs) that are addressed in the lesson. The SLOs identify the knowledge and skills you should have achieved by the end of the lesson.
- **Introduction:** Each lesson begins by outlining what you will be learning in that lesson.
- **Lesson:** The main body of the lesson consists of the content and processes that you need to learn. It contains information, explanations, diagrams, and completed examples.
- **Learning Activities:** Each lesson has a learning activity that focuses on the lesson content. Your responses to the questions in the learning activities will help you to practise or review what you have just learned. Once you have completed a learning activity, check your responses with those provided in the Learning Activity Answer Key found at the end of the applicable module. Do not send your learning activities to the Distance Learning Unit for assessment.
- **Assignments:** Assignments are found throughout each module within this course. At the end of each module, you will mail or electronically submit all your completed assignments from that module to the Distance Learning Unit for assessment. All assignments combined will be worth a total of 55 percent of your final mark in this course.
- **Summary:** Each lesson ends with a brief review of what you just learned.

There is also an appendix to the course, which is a glossary of terms and definitions. The online resource can be found in the learning management system (LMS). If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of this online resource.

What Resources Will You Need for This Course?

You do not need a textbook for this course. All of the content is provided directly within the course.

Required Resources

Here is a list of things that you **must** have to complete this course:

- a scientific calculator
- a metric ruler (15 cm long is fine)
- an imperial ruler (6 inches long is fine)
- other measurement tools such as a protractor, metre or yard stick, tape measure
- graph paper
- a notebook or binder so you can keep your completed learning activities together (Learning activities are activities that you complete and check against the answers provided at the end of each module. You do not send them in for assessment.)

Optional Resources

It would be helpful if you had access to the following resources:

- **A computer with spreadsheet and graphing capabilities:** Access to a computer with spreadsheet software and graphing capabilities may be helpful to you for exploration and checking your understanding. However, none of these are required or allowed when writing either your midterm or final examination.
- **A photocopier:** With access to a photocopier/scanner, you could make a copy of your assignments before submitting them so that if your tutor/marker wants to discuss an assignment with you over the phone, each of you will have a copy. It would also allow you to continue studying or to complete further lessons while your original work is with the tutor/marker. Photocopying or scanning your assignments will also ensure that you keep a copy in case the originals are lost.

Who Can Help You with This Course?

Taking an independent study course is different from taking a course in a classroom. Instead of relying on the teacher to tell you to complete a learning activity or an assignment, you must tell yourself to be responsible for your learning and for meeting deadlines. There are, however, two people who can help you be successful in this course: your tutor/marker and your learning partner.

Your Tutor/Marker

Tutor/markers are experienced educators who tutor Independent Study Option (ISO) students and mark assignments and examinations. When you are having difficulty with something in this course, contact your tutor/marker, who is there to help you. Your tutor/marker's name and contact information were sent to you with this course. You can also obtain this information in the learning management system (LMS).

Your Learning Partner

A learning partner is someone **you choose** who will help you learn. It may be someone who knows something about mathematics, but it doesn't have to be. A learning partner could be someone else who is taking this course, a teacher, a parent or guardian, a sibling, a friend, or anybody else who can help you. Most importantly, a learning partner should be someone with whom you feel comfortable and who will support you as you work through this course.

Your learning partner can help you keep on schedule with your coursework, read the course with you, check your work, look at and respond to your learning activities, or help you make sense of assignments. You may even study for your examination(s) with your learning partner. If you and your learning partner are taking the same course, however, your assignment work should not be identical.

One of the best ways that your learning partner can help you is by reviewing your midterm and final practice examinations with you. These are found in the learning management system (LMS), along with their answer keys. Your learning partner can administer your practice examination, check your answers with you, and then help you learn the things that you missed.

How Will You Know How Well You Are Learning?

You will know how well you are learning in this course by how well you complete the learning activities, assignments, and examinations.

Learning Activities

The learning activities in this course will help you to review and practise what you have learned in the lessons. You will not submit the completed learning activities to the Distance Learning Unit. Instead, you will complete the learning activities and compare your responses to those provided in the Learning Activity Answer Key found at the end of each module.

Each learning activity has two parts—Part A has BrainPower questions and Part B has questions related to the content in the lesson.

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for you before trying the other questions. Each question should be completed quickly and without using a calculator, and most should be completed without using pencil and paper to write out the steps. Some of the questions will relate directly to content of the course. Some of the questions will review content from previous courses—content that you need to be able to answer efficiently.

Being able to do these questions in a few minutes will be helpful to you as you continue with your studies in mathematics. If you are finding it is taking you longer to do the questions, you can try one of the following:

- work with your learning partner to find more efficient strategies for completing the questions
- ask your tutor/marker for help with the questions
- search online for websites that help you practice the computations so you can become more efficient at completing the questions

None of the assignment questions or examination questions will require you to do the calculations quickly or without a calculator. However, it is for your benefit to complete the questions as they will help you in the course. Also, being able to successfully complete the BrainPower exercises will help build your confidence in mathematics. BrainPower questions are like a warm-up you would do before competing in a sporting event.

Part B: Course Content Questions

One of the easiest and fastest ways to find out how much you have learned is to complete Part B of the learning activities. These have been designed to let you assess yourself by comparing your answers with the answer keys at the end of each module. There is at least one learning activity in each lesson. You will need a notebook or loose-leaf pages to write your answers.

Make sure you complete the learning activities. Doing so will not only help you to practise what you have learned, but will also prepare you to complete your assignments and the examinations successfully. Many of the questions on the examinations will be similar to the questions in the learning activities. Remember that you **will not submit learning activities to the Distance Learning Unit**.

Assignments

Lesson assignments are located throughout the modules, and include questions similar to the questions in the learning activities of previous lessons. The assignments have space provided for you to write your answers on the question sheets. **You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate).**

Once you have completed all the assignments in a module, you will submit them to the Distance Learning Unit for assessment. The assignments are worth a total of 55 percent of your final course mark. You must complete each assignment in order to receive a final mark in this course. **You will mail or electronically submit these assignments to the Distance Learning Unit along with the appropriate cover page once you complete each module.**

The tutor/marker will mark your assignments and return them to you. Remember to keep all marked assignments until you have finished the course so that you can use them to study for your examinations.

Resource Sheet

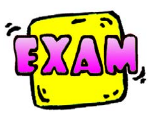
When you write your midterm and final examinations, you will be allowed to take an Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Examination Resource Sheet is not worth any marks.

Creating your own resource sheet is an excellent way to review. It also provides you with a convenient reference and quick summary of the important facts of each module. Each student is asked to complete a resource sheet for each module to help with studying and reviewing.

The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module. Refer to these when you create your own resource sheet. Also, refer to the online glossary found in the learning management system (LMS) to check the information on your resource sheet.

After completing each module's resource sheet, you should summarize the sheets from all of the modules to prepare your Examination Resource Sheet. When preparing your Midterm Examination Resource Sheet, remember that your midterm examination is based on Modules 1 to 4. When preparing your Final Examination Resource Sheet, remember that your final examination is based on the entire course, Modules 1 to 8.

Midterm and Final Examinations



This course contains a midterm examination and a final examination.

- The **midterm examination** is based on Modules 1 to 4, and is worth 20 percent of your final course mark. You will write the midterm examination when you have completed Module 4.
- The **final examination** is cumulative, so it is based on Modules 1 to 8 and is worth 25 percent of your final course mark. The examination content will consist of 20% from Modules 1 to 4 and 80% from Modules 5 to 8. You will write the final examination when you have completed Module 8.

The two examinations are worth a total of 45 percent of your final course mark. You will write both examinations under supervision.

In order to do well on the examinations, you should review all of the work that you have completed from Modules 1 to 4 for your midterm examination and Modules 1 to 8 for your final examination, including all learning activities and assignments. You can use your Examination Resource Sheet to bring any formulas and other important information into the examination with you.

You will be required to bring the following supplies when you write both examinations: pens/pencils (2 or 3 of each), blank paper, a scientific calculator, metric and imperial rulers, a protractor, and your Examination Resource Sheet. Both examinations are 2 hours in duration.

Practice Examinations and Answer Keys

To help you succeed in your examinations, you will have an opportunity to complete a Midterm Practice Examination and a Final Practice Examination. These examinations, along with the answer keys, are found in the learning management system (LMS). If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to obtain a copy of the practice examinations.

These practice examinations are similar to the actual examinations you will be writing. The answer keys enable you to check your answers. This will give you the confidence you need to do well on your examinations.

Requesting Your Examinations

You are responsible for making arrangements to have the examinations sent to your proctor from the Distance Learning Unit. Please make arrangements before you finish Module 4 to write the midterm examination. Likewise, you should begin arranging for your final examination before you finish Module 8.

To write your examinations, you need to make the following arrangements:

- **If you are attending school,** your examination will be sent to your school as soon as all the applicable assignments have been submitted. You should make arrangements with your school's ISO school facilitator to determine a date, time, and location to write the examination.
- **If you are not attending school,** check the Examination Request Form for options available to you. Examination Request Forms can be found on the Distance Learning Unit's website, or look for information in the learning management system (LMS). Two weeks before you are ready to write the examination, fill in the Examination Request Form and mail, fax, or email it to

  Distance Learning Unit
  500–555 Main Street
  PO Box 2020
  Winkler MB R6W 4B8
  Fax: 204-325-1719
  Toll-Free Telephone: 1-800-465-9915
  Email: distance.learning@gov.mb.ca

How Much Time Will You Need to Complete This Course?

Learning through independent study has several advantages over learning in the classroom. You are in charge of how you learn and you can choose how quickly you will complete the course. You can read as many lessons as you wish in a single session. You do not have to wait for your teacher or classmates.

From the date of your registration, you have a maximum of **12 months** to complete the course, but the pace at which you proceed is up to you. Read the following suggestions on how to pace yourself.

Chart A: Semester 1

If you want to start this course in September and complete it in January, you can follow the timeline suggested below.

Module	Completion Date
Module 1	Middle of September
Module 2	End of September
Module 3	Middle of October
Module 4	End of October
Midterm Examination	Beginning of November
Module 5	Middle of November
Module 6	End of November
Module 7	Middle of December
Module 8	Middle of January
Final Examination	End of January

Chart B: Semester 2

If you want to start the course in February and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Module 1	Middle of February
Module 2	End of February
Module 3	Beginning of March
Module 4	Middle of March
Midterm Examination	End of March
Module 5	Beginning of April
Module 6	Middle of April
Module 7	End of April
Module 8	Beginning of May
Final Examination	Middle of May

Chart C: Full School Year (Not Semestered)

If you want to start the course in September and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Module 1	End of September
Module 2	End of October
Module 3	End of November
Module 4	End of December
Midterm Examination	Middle of January
Module 5	Middle of February
Module 6	Middle of March
Module 7	Beginning of April
Module 8	Beginning of May
Final Examination	Middle of May

Timelines

Do not wait until the last minute to complete your work, since your tutor/marker may not be available to mark it immediately. It may take a few weeks for your tutor/marker to assess your work and return it to you or your school.

If you need this course to graduate this school year, all coursework must be received by the Distance Learning Unit on or before the first Friday in May, and all examinations must be received by the Distance Learning Unit on or before the last Friday in May. Any coursework or examinations received after these deadlines may not be processed in time for a June graduation. Assignments or examinations submitted after these recommended deadlines will be processed and marked as they are received.

When and How Will You Submit Completed Assignments?

When to Submit Assignments

While working on this course, you will submit completed assignments to the Distance Learning Unit five times. The following chart shows you exactly what assignments you will be submitting at the end of each module.

Submission of Assignments	
Submission	Assignments You Will Submit
1	**Module 1: Graphs and Relations** Module 1 Cover Sheet Assignment 1.1: Graphing Independent and Dependent Variables Assignment 1.2: Domain and Range Assignment 1.3: Slopes, Intercepts, Domain, and Range Assignment 1.4: What We Can Tell from Slope Assignment 1.5: Slope-y-Intercept Equation
2	**Module 2: Number Sense** Module 2 Cover Sheet Assignment 2.1: Factors and Multiples Assignment 2.2: Perfect Cubes and Squares Assignment 2.3: Rational, Irrational, and Radical Numbers Assignment 2.4: Exponent Laws Review Assignment 2.5: Exponent Laws with Rational and Negative Exponents
3	**Module 3: Quadratic Functions / Module 4: Trigonometry** Module 3/Module 4 Cover Sheet Assignment 3.1: Units, Area, and Volume Assignment 3.2: Measuring with Vernier Calipers and Micrometers Assignment 3.3: Unit Conversions Assignment 3.4: Volume of Prisms and Pyramids Assignment 3.5: Surface Area of Prisms and Pyramids Assignment 3.6: Surface Area and Volume of Spheres, Cylinders, and Cones Assignment 4.1: Tangent Ratios Assignment 4.2: Using Sine, Cosine, and Tangent Assignment 4.3: Inverse Trig Ratios Assignment 4.4: Applying Trig Ratios

continued

Submission of Assignments (continued)	
Submission	Assignments You Will Submit
4	**Module 5: Relations and Functions / Module 6: Polynomials** Module 5/Module 6 Cover Sheet Assignment 5.1: Relations and Functions Assignment 5.2: Domain and Range Notation Assignment 5.3: Functional Notation Assignment 6.1: Describing Polynomials and Multiplying Binomials Assignment 6.2: Multiplying Polynomials Assignment 6.3: Factoring Binomials and Trinomials Assignment 6.4: Factoring Trinomials with $a \in I$ Assignment 6.5: Difference of Squares and Module Review
5	**Module 7: Coordinate Geometry / Module 8: Systems of Equations** Module 7/Module 6 Cover Sheet Assignment 7.1: Distance and Midpoint Assignment 7.2: Linear Relation Formulas Assignment 7.3: Writing Linear Equations Based on Different Information Assignment 7.4: Line of Best Fit and Correlations Assignment 8.1: Solving Systems of Linear Questions Graphically Assignment 8.1: Solving Systems of Equations by Elimination

How to Submit Assignments

In this course, you have the choice of submitting your assignments either by mail or electronically.

- **Mail:** Each time you **mail** something, you must include the print version of the applicable Cover Sheet (found at the end of this Introduction). Complete the information at the top of each Cover Sheet before submitting it along with your assignments.
- **Electronic submission:** You do not need to include a cover sheet when submitting assignments electronically.

Submitting Your Assignments by Mail

If you choose to mail your completed assignments, please photocopy/scan all the materials first so that you will have a copy of your work in case your package goes missing. You will need to place the applicable module Cover Sheet and assignment(s) in an envelope, and address it to

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Your tutor/marker will mark your work and return it to you by mail.

Submitting Your Assignments Electronically

Assignment submission options vary by course. Sometimes assignments can be submitted electronically and sometimes they must be submitted by mail. Specific instructions on how to submit assignments were sent to you with this course. In addition, this information is available in the learning management system (LMS).

If you are submitting assignments electronically, make sure you have saved copies of them before you send them. That way, you can refer to your assignments when you discuss them with your tutor/marker. Also, if the original hand-in assignments are lost, you are able to resubmit them.

Your tutor/marker will mark your work and return it to you electronically.

> The Distance Learning Unit does not provide technical support for hardware-related issues. If troubleshooting is required, consult a professional computer technician.

What Are the Guide Graphics For?

Guide graphics are used throughout this course to identify and guide you in specific tasks. Each graphic has a specific purpose, as described below.

Lesson Introduction: The introduction sets the stage for the lesson. It may draw upon prior knowledge or briefly describe the organization of the lesson. It also lists the learning outcomes for the lesson. Learning outcomes describe what you will learn.

Learning Partner: Ask your learning partner to help you with this task.

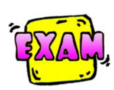
Learning Activity: Complete a learning activity. This will help you to review or practise what you have learned and to prepare for an assignment or an examination. You will not submit learning activities to the Distance Learning Unit. Instead, you will compare your responses to those provided in the Learning Activity Answer Key found at the end of every module.

Assignment: Complete an assignment. You will submit your completed assignments to the Distance Learning Unit for assessment at the end of a given module.

Mail or Electronic Submission: Mail or electronically submit your completed assignments to the Distance Learning Unit for assessment at this time.

Phone Your Tutor/Marker: Telephone your tutor/marker.

Resource Sheet: Indicates material that may be valuable to include on your resource sheet.



Examination: Write your midterm or final examination at this time.

Note: Take note of and remember this important information or reminder.

Applied Mathematics: Indicates mathematical approaches that are related to applied mathematics.

Pre-Calculus Mathematics: Indicates mathematical approaches that are related to pre-calculus mathematics.

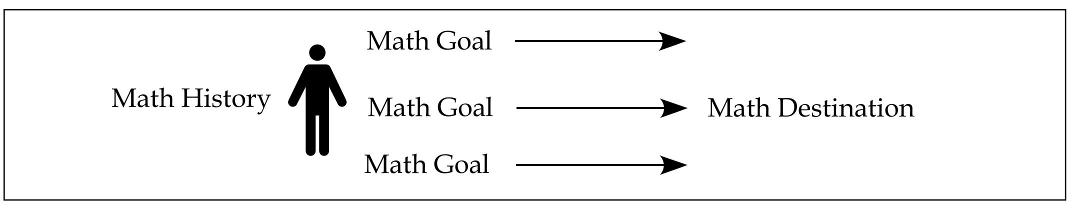
Math Goals

In Module 1, Lesson 1, you will be asked to contact your tutor/marker to discuss your math goals. Having this conversation with your tutor/marker has two important purposes. First, it introduces you to a very valuable resource—your tutor/marker. He or she is available for you to answer questions, explain concepts, and guide you through this course. You can discuss your math learning and progress. Feel free to contact your tutor/marker by phone or email at any time during this course. The second important purpose of this assignment is to get you thinking about your math goals. You may have a future career in mind, and this course is getting you one step closer to it by filling a prerequisite for a future required course.

There may be specific skills or topics you are interested in learning about, and they are covered in this course. If you are unsure of your math goals or why they are important, consider this:

- goals give you a sense of direction and purpose in taking this course
- goals help motivate you to learn and do your best, even when its tough
- when you accomplish your goals, there is a great sense of achievement and success

Good goals need to be realistic and specific, and they should reflect what is important to you. They should give you direction and take you further down the path from where you have been to where you want to go.



From the diagram, you can see that goals can be long-term or short-term, but they are the pathway that takes you from where you were/are, closer to where you want to go.

Getting Started

Take some time right now to skim through the course material, locate your cover sheets, and familiarize yourself with how the course is organized. Get ready to learn!

Remember: If you have questions or need help at any point during this course, contact your tutor/marker or ask your learning partner for help.

Good luck with the course!

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 1: Graphs and Relations
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address
Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address
Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________ Preferred Name: ____________________

Phone: ____________________ Email: ____________________

Mailing Address: ____

City/Town: ____________________ Postal Code: ____________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 1 Assignments	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 1.1: Graphing Independent and Dependent Variables	______ /26	______ /26
❑ Assignment 1.2: Domain and Range	______ /30	______ /30
❑ Assignment 1.3: Slopes, Intercepts, Domain, and Range	______ /29	______ /29
❑ Assignment 1.4: What We Can Tell from Slope	______ /36	______ /36

continued

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Module 1: Graphs and Relations
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________________ Preferred Name: ____________________________

Phone: ________________________________ Email: ________________________________

Mailing Address: ____

City/Town: ____ Postal Code: _______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 1 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	______________ Date Received	______________ Date Received
❑ Assignment 1.5: Slope-y-Intercept Equation	_______ /36	_______ /36
	Total: ____ **/157**	**Total:** ____ **/157**

For Tutor/Marker Use
Remarks:

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 2: Number Sense
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________________ Preferred Name: __________________________

Phone: _______________________________ Email: ___________________________________

Mailing Address: ____

City/Town: ____ Postal Code: _______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 2 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** ______________ Date Received	**Attempt 2** ______________ Date Received
❑ Assignment 2.1: Factors and Multiples	________ /28	________ /28
❑ Assignment 2.2: Perfect Cubes and Squares	________ /23	________ /23
❑ Assignment 2.3: Rational, Irrational, and Radical Numbers	________ /29	________ /29
❑ Assignment 2.4: Exponent Laws Review	________ /25	________ /25

continued

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Module 2: Number Sense
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ___________________________ Preferred Name: _________________________

Phone: ________________________________ Email: __________________________________

Mailing Address: ____

City/Town: ____ Postal Code: ______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 2 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	____________ Date Received	____________ Date Received
❑ Assignment 2.5: Exponent Laws with Rational and Negative Exponents	________ /21	________ /21
	Total: ____ **/126**	**Total:** ____ **/126**

For Tutor/Marker Use
Remarks:

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 3: Measurement / Module 4: Trigonometry
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ______________________ Preferred Name: ______________________

Phone: ______________________ Email: ______________________

Mailing Address: ____

City/Town: ______________________ Postal Code: ____________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 3 and 4 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** __________ Date Received	**Attempt 2** __________ Date Received
❑ Assignment 3.1: Units, Area, and Volume	_______ /25	_______ /25
❑ Assignment 3.2: Measuring with Vernier Calipers and Micrometers	_______ /10	_______ /10
❑ Assignment 3.3: Unit Conversions	_______ /20	_______ /20
❑ Assignment 3.4: Volume of Prisms and Pyramids	_______ /8	_______ /8
❑ Assignment 3.5: Surface Area of Prisms and Pyramids	_______ /25	_______ /25
❑ Assignment 3.6: Surface Area and Volume of Spheres, Cylinders, and Cones	_______ /25	_______ /25

continued

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 3: Measurement / Module 4: Trigonometry
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address

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500–555 Main Street
PO Box 2020
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Phone: ______________________ Email: ______________________

Mailing Address: ____

City/Town: ______________________ Postal Code: __________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 3 and 4 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	__________ Date Received	__________ Date Received
❑ Assignment 4.1: Tangent Ratios	_____ /14	_____ /14
❑ Assignment 4.2: Using, Sine, Cosine, and Tangent	_____ /20	_____ /20
❑ Assignment 4.3: Inverse Trig Ratios	_____ /33	_____ /33
❑ Assignment 4.4: Applying Trig Ratios	_____ /30	_____ /30
	Total: ____ /210	**Total: ____ /210**

For Tutor/Marker Use
Remarks:

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 5: Relations and Functions / Module 6: Polynomials Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address
Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________________ Preferred Name: ____________________________

Phone: ____________________________ Email: ____________________________

Mailing Address: ____

City/Town: ________________________________ Postal Code: ______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 5 and 6 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** ______________ Date Received	**Attempt 2** ______________ Date Received
❑ Assignment 5.1: Relations and Functions	_______ /20	_______ /20
❑ Assignment 5.2: Domain and Range Notation	_______ /30	_______ /30
❑ Assignment 5.3: Functional Notation	_______ /31	_______ /31
❑ Assignment 6.1: Describing Polynomials and Multiplying Binomials	_______ /33	_______ /33
❑ Assignment 6.2: Multiplying Polynomials	_______ /26	_______ /26
❑ Assignment 6.3: Factoring Binomials and Trinomials	_______ /32	_______ /32

continued

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 5: Relations and Functions / Module 6: Polynomials
Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Contact Information

Legal Name: ____________________ Preferred Name: ____________________

Phone: ____________________ Email: ____________________

Mailing Address: ____

City/Town: ____________________ Postal Code: __________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 5 and 6 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 6.4: Factoring Trinomials with $a \in I$	_____ /24	_____ /24
❑ Assignment 6.5: Difference of Squares and Module Review	_____ /40	_____ /40
	Total: ____ /236	**Total: ____ /236**

For Tutor/Marker Use
Remarks:

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 7: Coordinate Geometry / Module 8: Systems of Equations Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________ Preferred Name: ____________________

Phone: ____________________ Email: ____________________

Mailing Address: ____

City/Town: ____________________ Postal Code: __________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 7 and 8 Assignments	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 7.1: Distance and Midpoint	____ /24	____ /24
❑ Assignment 7.2: Linear Relation Formulas	____ /30	____ /30
❑ Assignment 7.3: Writing Linear Equations Based on Different Information	____ /25	____ /25
❑ Assignment 7.4: Line of Best Fit and Correlations	____ /51	____ /51

continued

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 7: Coordinate Geometry / Module 8: Systems of Equations Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Phone: ____________________ Email: ____________________

Mailing Address: ____

City/Town: ____________________ Postal Code: __________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 7 and 8 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	______ Date Received	______ Date Received
❑ Assignment 8.1: Solving Systems of Linear Questions Graphically	______ /30	______ /30
❑ Assignment 8.2: Solving Systems of Equations by Elimination	______ /33	______ /33
	Total: ____ /193	**Total: ____ /193**

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Remarks:

Released 2019

Printed in Canada
Imprimé au Canada

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 1
Graphs and Relations

Module 1: Graphs and Relations

Introduction

This first module forms a foundation for the mathematical concepts you will use in both pre-calculus and applied mathematics in the future, including upcoming modules in this course. It, in turn, is based on ideas and skills developed in previous math courses you have taken. This module will focus on the relationships among data, graphs, and contexts, and use a variety of ways to describe them. Specific attention will be given to linear relations, and their slope, intercepts, domain, and range. You will use words, ordered pairs, tables of values, graphs, and equations as means to describe the characteristics of linear relations.

Assignments in Module 1

When you have completed the assignments for Module 1, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 1.1	Graphing Independent and Dependent Variables
2	Assignment 1.2	Domain and Range
3	Assignment 1.3	Slopes, Intercepts, Domain, and Range
4	Assignment 1.4	What We Can Tell From Slope
5	Assignment 1.5	Slope-y-Intercept Equation

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 1. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1, 2, 3, and 4 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 4.

Resource Sheet for Module 1

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Lesson 1: Graphing Independent and Dependent Variables

Lesson Focus

In this lesson, you will

- ❑ describe the properties of a good graph and create or sketch graphs
- ❑ identify the independent and dependent variables in a graph or context
- ❑ identify continuous data in context
- ❑ describe the relationships between graphs and contexts

Lesson Introduction

If you are given a bunch of numbers in a chart and are expected to understand and make sense of how the numbers are related, it can be a really difficult and confusing task. Sometimes organizing the data by graphing can make it easier to see the relationships. Graphs are visual representations of data, and you can use them to describe or explain situations. In this lesson, you will review what makes a good graph, and create scatterplot graphs to represent situations and display data. Using specific contexts, you will identify continuous data, and the dependent and the independent variables in graphs.

Learning Activity 1.1

This learning activity is the only one that doesn't include a BrainPower section, although it still has two parts.

Part A: Contacting Your Tutor/Marker

Your first task in this course is to contact your tutor/marker by phone (you will have received his/her phone number in the mail with the course), or interview your learning partner.

Be ready to discuss the following topics and the reasons for your answers with your tutor/marker or learning partner. If you like, make some notes below before you call in order to help you feel prepared. Feel free to add any other questions or comments that you may have.

1. I am taking this course by distance education because

    ____

    ____

    ____

    ____

2. What I like about math and can do mathematically is (include favourite topic, skill, where you use math, etc.)

    ____

    ____

    ____

    ____

continued

Learning Activity 1.1 (continued)

3. What I dislike about math or have difficulty doing is

4. Previous math experiences that influence the way I feel about math are

5. The next math course I would like to take is

6. What I am hoping this course will help me accomplish and learn for the future

continued

7. What I am doing/how I will organize things to help me succeed in this course

____

____

____

____

During your phone conversation, jot down a sentence or two about what you and your tutor talk about, in the spaces above. For example, if you are taking this course because it doesn't fit into your schedule at school or because you travel a lot with your basketball team and this is more convenient, state that in the space below question 1.

Part B: Where You Want to Go in Math

Use the answers to the questions from the conversation with your tutor/marker as a starting point and fill in the following diagram. In the Math History box, jot down point-form notes about your prior experience and knowledge about math (questions 2, 3, and 4). In the Math Destination box, jot down what completing this course will help you accomplish in the future (questions 5 and 6).

In between the boxes, write down what you will need to do to move down the pathway from your History to your Destination.

Math History	Pathway	Math Destination

continued

Learning Activity 1.1 (continued)

For example, if your destination includes needing a 75% in this course so that you can feel confident going into Grade 11 Pre-Calculus Mathematics in order to take nursing at college, or you need to learn how to solve equations, what will help you accomplish this? It may mean figuring out how you best learn and study math. It may mean setting up a schedule so you complete the assignments on time. You may need to find your calculator manual and figure out how to use it, set up regular appointments with your learning partner, research a topic on the Internet, or read a textbook about a certain math concept or skill. Your pathway is unique to you.

As you move through this course and work on achieving your goals, self-assessment is important for you to determine whether you are getting closer to your destination. It helps you determine whether the steps along your pathway are taking you in the right direction. You will need to periodically ask yourself: Am I doing my assignments? Are my note-taking skills improving? How often have I contacted my tutor/marker or worked with my learning partner? Have I found useful homework websites? Is my schedule working? What do I need to change or adjust so I can get to my destination?

You will repeatedly go through this cycle of looking at where you have been, where you want to go, and where you currently are. At any time, you may want to revise your goals or set new ones as you evaluate your own progress and learning.

- Look back/history—reflect on what you know, how far you have come.
- Look around/pathway—assess if you are achieving your goals, determine if new learning or understanding has occurred, and check your progress.
- Look forward/destination—determine what you want to know, set goals.

Each time you go through these steps, you will become better at mathematics!

It is important that you keep this diagram handy as you will revisit it at other points in this course.

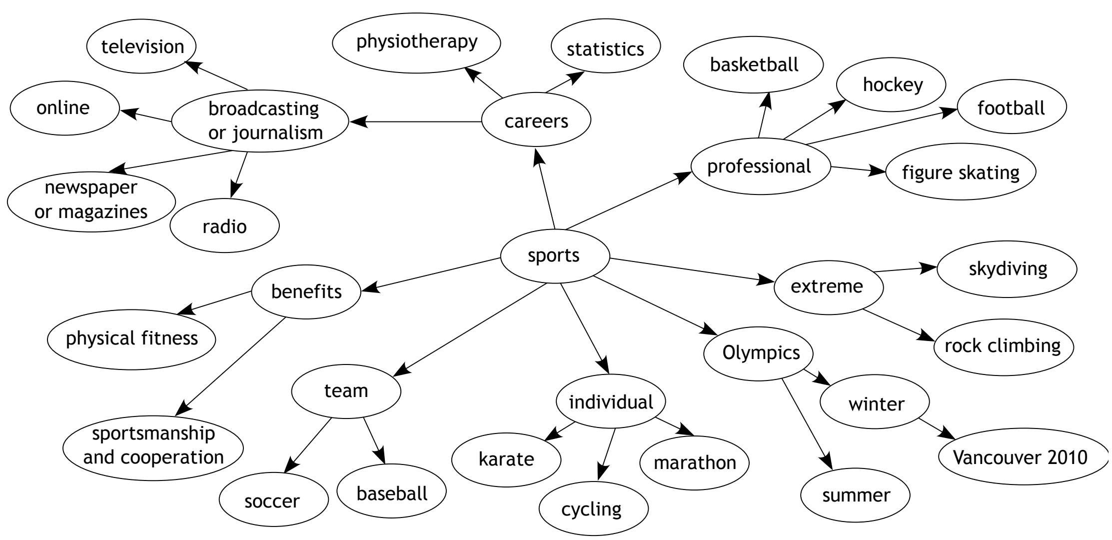
Graphing

Graphs

Think about what you know about graphs and where you have used or seen them.

One way to display what you know about a topic is to create a word web. A word web is a diagram that shows how the different parts or ideas related to a topic are connected. It helps you to think about what you already know and can do, and helps you identify any gaps in your knowledge.

If you are unfamiliar with word webs, they are created by starting with a main concept or topic in the centre of your diagram, and then showing related ideas in connected bubbles around it. They can be drawn by hand or with a computer. To give you an idea of how a word web may be constructed, here is an example of a word web about sports.



Learning Activity 1.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper. The first few times you do these questions, your learning partner can help you figure out strategies to solve them.

1. There are 22 yard markers on a Canadian football field. Each marker represents five yards. How long is a Canadian football field?
2. If Evan eats $\frac{3}{5}$ of a pizza and Nick eats $\frac{4}{5}$ of a pizza, how many pizzas do they have to order so that both can eat as much as they like?
3. Simplify the following fraction to lowest terms: $\frac{18}{27}$.
4. You are working at the stadium where they don't have an electronic till. The customer is buying popcorn for \$3.80. If the customer gives you a \$5.00 bill, how much change will you give them?
5. Rank the numbers from highest to lowest: 0.5, 0.05, 0.3, 0.09, and 0.25.
6. Solve for m: $2 - m = 14$.
7. The distance to the mall from your house is 8 km. Your friend lives half as far away from the mall. What is the distance from your friend's house to the mall?
8. Write the percent as a decimal: 62%.

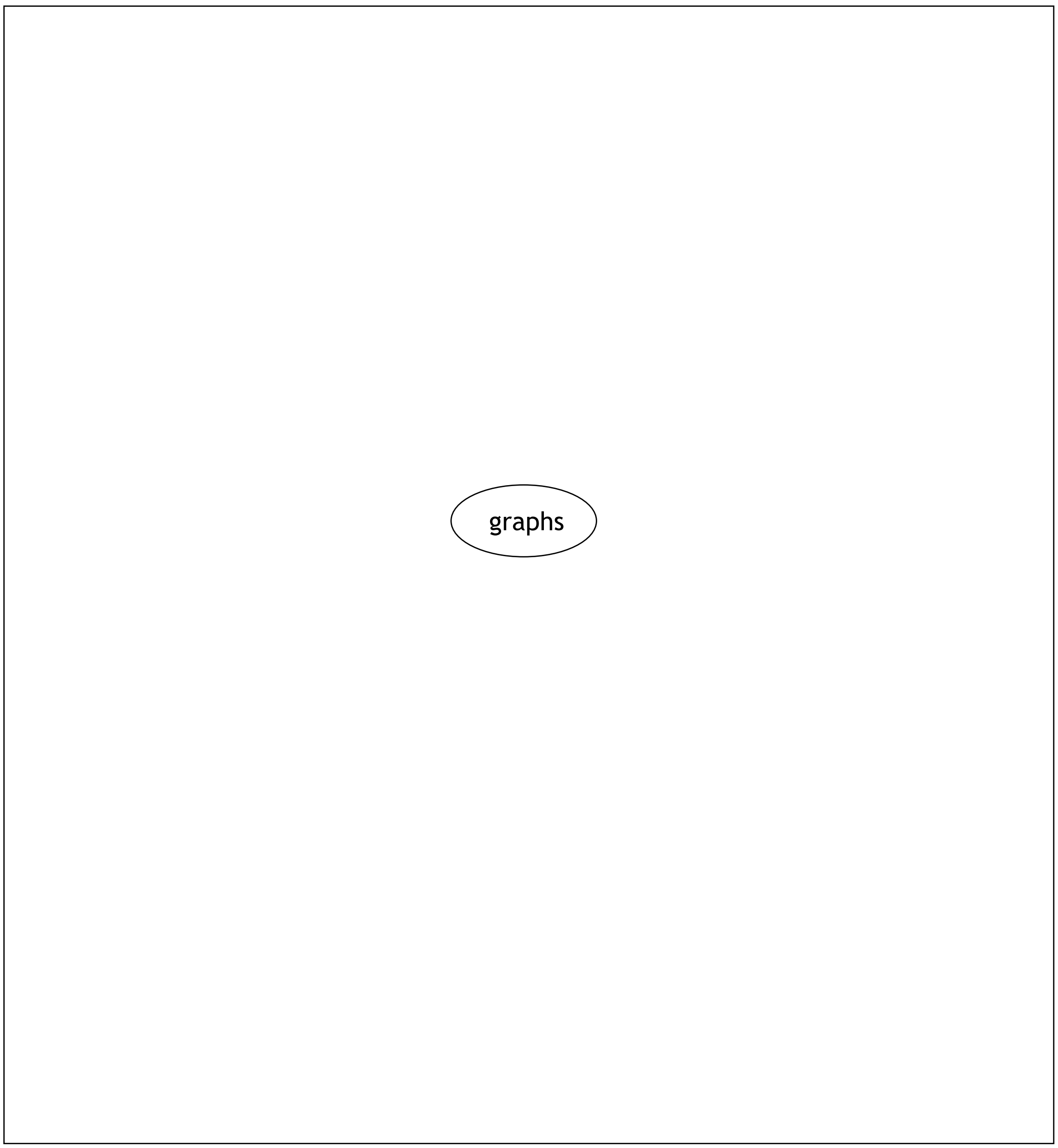
continued

Learning Activity 1.2 (continued)

Part B: Word Web

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

Create a word web showing what you know about graphs. Use bubbles to indicate new ideas or characteristics, and lines to show how they are connected.

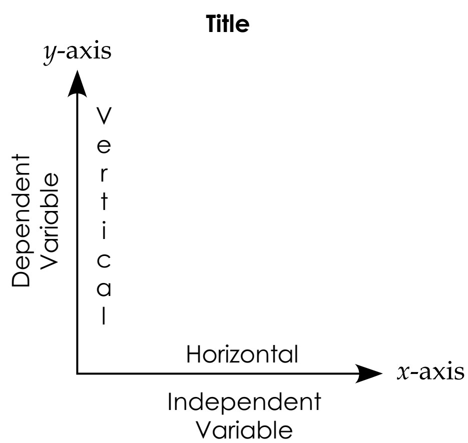


Dependent and Independent Variables

If you have been shopping for a handheld media device that can play music and videos and surf the Internet, you will have noticed that its cost is affected by many factors. You could describe these relationships or patterns using words (verbally or written), equations (theoretically), or with a graph (visual). **A graph is a visual representation used to show a numerical relationship.**

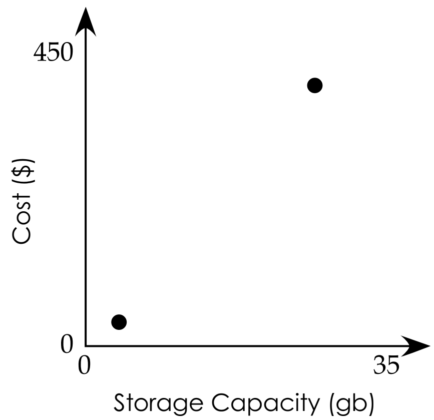
Say you do some online shopping and your comparisons indicate that you could get a device with a capacity between 1 and 32 gigabytes (GB) of memory space at a cost between $55 and $430. There is not enough information given to come up with an equation to describe how cost and capacity are related, but you can describe it with words. Verbally, this relationship can be explained by saying that the cost goes up as the capacity goes up. To visually display the relationship between two variables, you need to first determine which of the two variables being compared depends on, or is affected by, the other variable.

The **dependent** variable is the item that is affected by changes in the other, and it is graphed on the vertical or y-axis. The **independent** variable is the item being compared that is not affected by the other, and it is usually placed along the horizontal or x-axis.



This graphic may be helpful to include on your Resource Sheet.

When buying a media device, the cost generally depends on the size of the storage space or memory capacity. Using the written description given above, the relationship between cost and capacity in a handheld media device may be represented as:



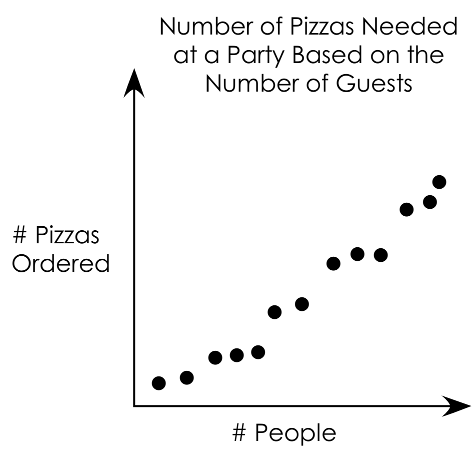
As the capacity goes up, the cost goes up.

Example 1

Determine which variable is dependent and which variable is independent, and sketch a possible graph to describe the relationship between the number of people at a party and the number of pizzas ordered.

Solution:

The number of pizzas ordered depends on the number of people at the party.

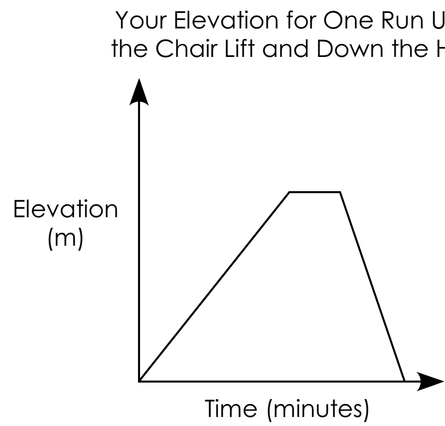


Example 2

Pretend that you are at your favourite ski hill. You take the chairlift to the top, then ski down as quickly as possible. You want to graph the relationship of how high you are on the hill (elevation) compared to the time from when you get on the chairlift until you get back down the hill. Determine which variable is dependent and which is independent, and sketch a possible graph.

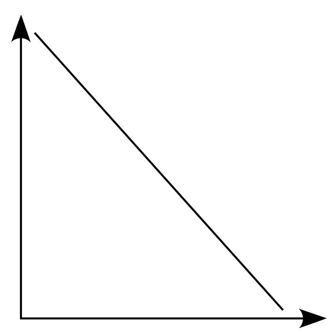
Solution:

Your elevation depends on the time (independent).



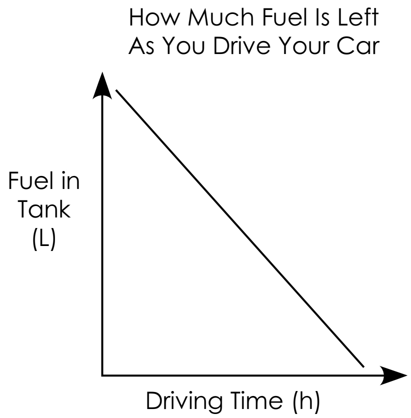
Example 3

Create and explain a situation with a dependent and an independent variable that would fit the following graph. Label the graph with your variables and units.



Solution:

One possible context could be that the number of litres of fuel in a car's tank (dependent) depends on how many hours of driving time (independent) have elapsed since the tank was filled.



Ordered Pairs

An ordered pair (also called a coordinate pair) is a set of two numbers named in a specific order, represented by (x, y). The first number, x, represents the independent variable, graphed along the x-axis, and the second number, y, represents the dependent variable, graphed along the y-axis. When an ordered pair is graphed on a scatterplot, it represents a unique point on the coordinate plane or grid.

Ordered pairs appear in multiple lessons, so you may want to include the definition on your Resource Sheet.

Constructing Graphs from Data

The following data were collected during your comparison shopping for a handheld media device.

Capacity (GB)	1	2	8	16	32
Cost ($)	55	75	170	240	430

Create a scatterplot graph to display the relationship between these variables.

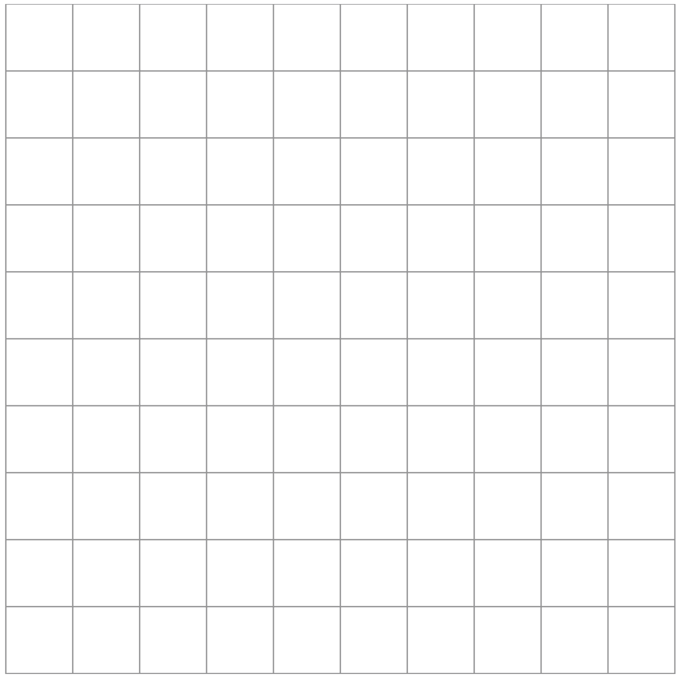
Graphs can be drawn by hand on graph paper, or generated using technology. Spreadsheets or graphing calculators, graphing freeware, or programs like *Graphical Analysis* may be used. In any case, a good graph has the following components:

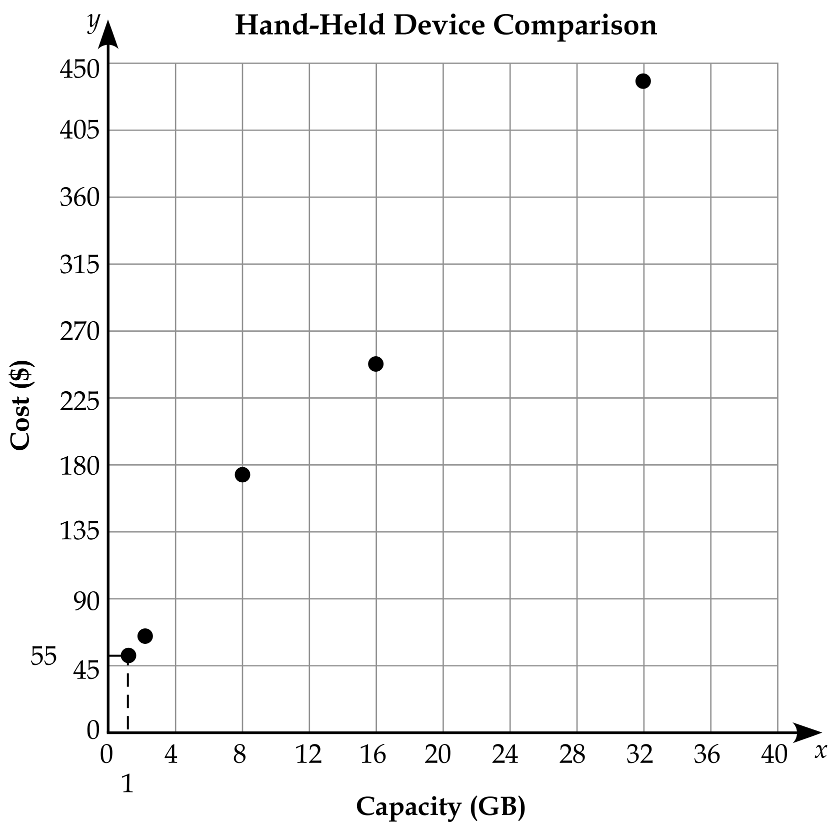
- **Labels:** The x-axis and y-axis have labels to identify the variable and units used.
- **Scale:** Look at the smallest and largest data points given. The values along the axes go slightly beyond these values, and each interval is divided into equal increments. When appropriate, start each scale at zero.
- **Shape and size:** The graph is square, and the data are spread out over most of the space.
- **Title:** A title indicates what the graph is about.

The capacity varies from 1 to 32 GB, so the values along the x-axis (domain) could be from 0 to 40. There are 10 tick marks along the axis, so $\frac{40}{10} = 4$. Use even increments of 4 or 5.

The cost ranges from \$55 to \$430 so you could use values of 0 to 450 along the y-axis $\left(\frac{450}{10} = 45\right)$. Using increments of 45 or 50 would give you a nice square graph, with the data appropriately spread out over the graph area. Remember to include labels, units, and a title.

To plot the data points on the scatterplot, start with one (capacity, cost) pair, like (1, 55). The capacity is the independent variable and so is graphed along the x-axis, while the cost is the dependent variable and is graphed along the y-axis. The pairs are always stated as (x, y). Find where 1 would be along the x-axis and slide up from there until you are at about 55 along the y-axis. Make a mark where these two meet. Continue until you have plotted all the (capacity, cost) pairs.




Hand-Held Device Comparison
y
450
405
360
315
270
225
180
135
90
55
45
0
Cost ($)
0
1
4
8
12
16
20
24
28
32
36
40
x
Capacity (GB)

Continuous Data

This scatterplot has dots representing the cost and capacity of media players. Would it make sense to connect the dots with a line? Think about what the line would represent. Is it possible to purchase a media device with 71.3 GB? Not likely! You can only buy devices with a specific number of GBs, so the data are not continuous. They must be displayed using individual dots.

In the same way, when graphing the number of pizzas ordered for a party like in Example 1, connecting the dots would be inappropriate, as you cannot order partial pizzas, or have half a person attending a party. On the other hand, the graph above indicating the litres of fuel and time spent driving can be represented using a line, because the values along the line are all valid possibilities. You can have fractions of time and partial litres of fuel. These are continuous data—the data points can be connected with a line, and all values along the line are valid or meaningful.

Learning Activity 1.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper. The first few times you do these questions, your learning partner can help you figure out strategies to solve them.

1. What is the range of the following numbers: 2, 6, 4, 8, 7, 13, 11?
2. You are going to the store to buy a drink with $2.05 in your pocket. If a drink costs $1.75, will you be able to buy one?
3. Simplify the fraction $\frac{6}{2}$.
4. Write the ratio as a fraction: 5:2.
5. Solve for a: $9 + a = 13$.
6. Write the next two numbers in the pattern: 1, 2, 4, 8, _____ , _____.

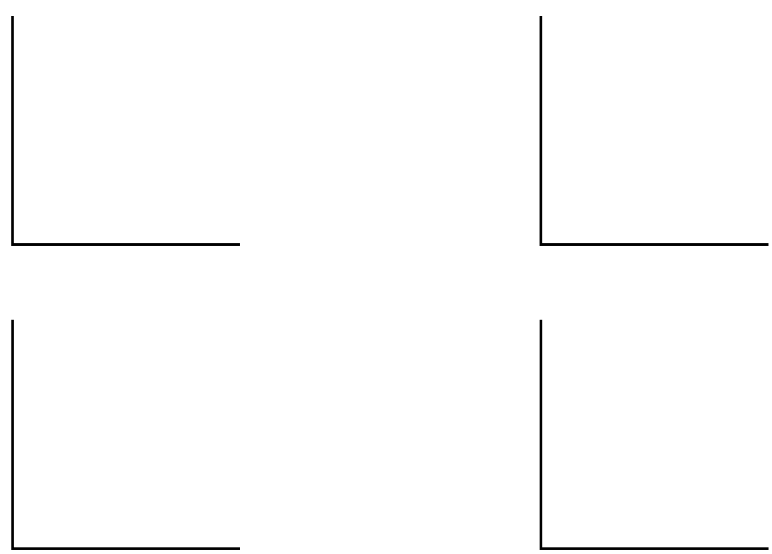
continued

7. You want to bring freezies to your last soccer game of the season. You want to have enough so that each player gets two. If you have 18 people on your team, how many freezies do you need?
8. You are helping your dad build a rectangular deck. If it is 2 m long and 3 m wide, what is the area that it takes up in your yard?

Part B: Independent vs. Dependent Variables and Continuous Data

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

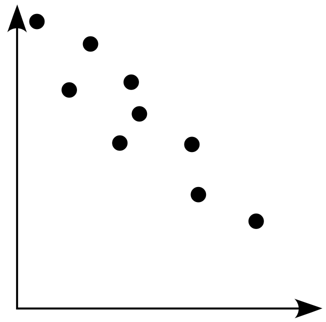
1. State which variable is independent and which variable is dependent in each of the following contexts:
   a) Hours worked in a week with pay of $20 per hour
   b) Final exam mark and average quiz marks for a Grade 10 Math class
   c) Coffee temperature and the time since the cup was poured
   d) Average monthly temperature in Manitoba during the months from January to December
2. Are the situations in question 1 continuous? Explain.
3. Sketch a possible graph based on the contexts given in Question 1. Four graph frames are provided below or you may create your own.



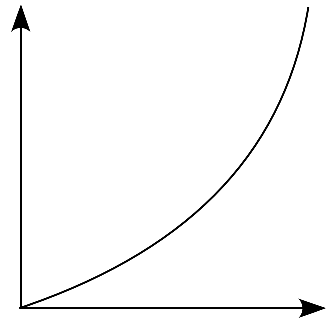
continued

4. Create a possible context that would result in the following graphs. Label each graph with independent and dependent variables, units, appropriate scales (values along the axes), and a title.

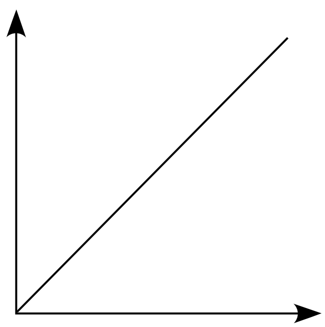
a)



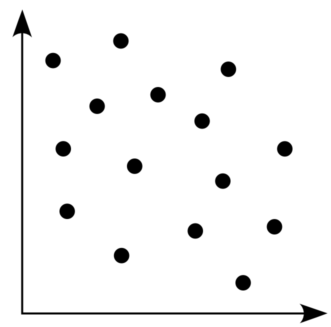
b)



c)



d)



5. Construct a good graph of the following data. It may be done by hand on graph paper or with technology.

A random sample of 11 people was drawn from the population of people between the ages of 30 and 40 who were employed full time in Brandon. The number of years of their schooling and annual income in thousands of dollars was recorded for each of the 11 people. The data are given below:

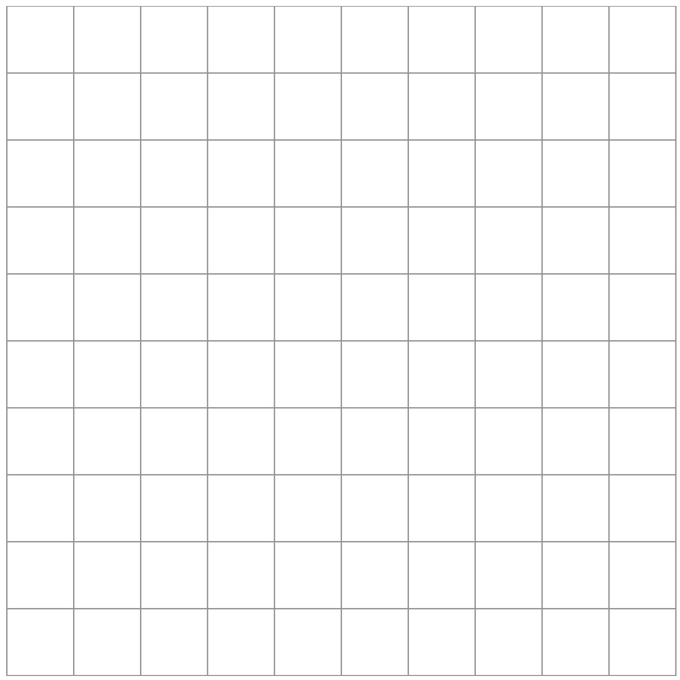
Schooling (years)	10	7	12	11	16	12	18	8	12	14	16
Income ($1000)	32	20	45	43	65	42	75	28	40	60	65

a) Which variable, schooling or income, is the independent variable? Which is the dependent variable?

continued

Learning Activity 1.3 (continued)

b) Graph the data with appropriate scales on the grid below and draw the line of best fit.



c) Are the data continuous?

Lesson Summary

Graphs can help you understand data and situations by creating a visual representation of them. You have learned how to create a good scatterplot graph and how to identify continuous data, dependent variables, and independent variables. In the next lesson, you will build on these concepts and look at what linear graphs are, what restrictions there are on the domain and range of the graph, and find other ways to represent relationships between variables.

Assignment 1.1

Graphing Independent and Dependent Variables

Total Marks = 26

1. Match the following contexts with the appropriate graph. State the independent and dependent variables. Fill in the missing labels, units, and title on each graph.

   a) Number of kilometres driven and the age of a vehicle in years *(5 marks)*

   Independent ________________ Dependent ________________

   b) Elevation of a city above sea level and the average annual temperature *(5 marks)*

   Independent ________________ Dependent ________________

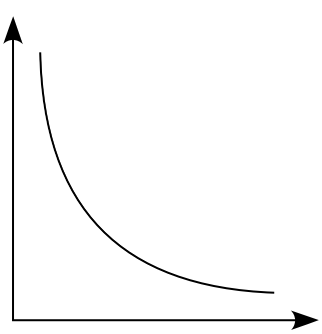
   c) The profit (profit refers to financial gain) of a company and its expenses *(5 marks)*

   Independent ________________ Dependent ________________

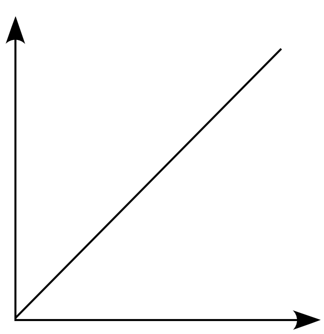
   d) Number of pops per second when making popcorn *(5 marks)*

   Independent ________________ Dependent ________________

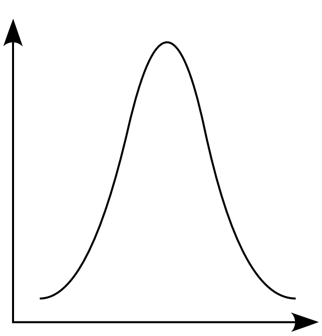
a) _____



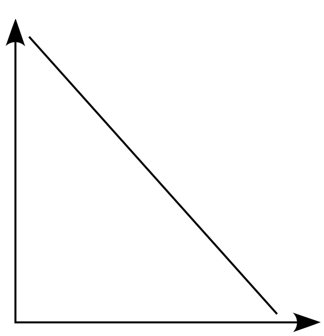
b) _____



c) _____



d) _____



Assignment 1.1: Graphing Independent and Dependent Variables (continued)

2. A ball is dropped from various heights and the height of its first bounce is recorded below. Create a neat, labelled graph to display this relationship. Explain if the data are continuous or not. *(6 marks)*

Drop height (cm)	100	90	80	70	60	50	40	30	20	10
Bounce height (cm)	70	61	52	46	41	32	26	20	14	7

Lesson 2: Domain and Range in Linear Relations

Lesson Focus

In this lesson, you will

- ❑ determine if a context, data in a table of values, ordered pairs, or a graph represent a linear relation
- ❑ graph a linear relation and determine restrictions on the domain and range
- ❑ create corresponding representations of linear relations

Lesson Introduction

In this lesson, you will build on the concepts you learned in the previous lesson and look at what linear graphs are, what restrictions there are on the domain and range of the graph, and find other ways to represent relationships between variables.

Domain and Range of Data

In the last lesson, you described situations visually using scatterplots and words. There are other ways to numerically describe situations. You can use a table of values or ordered pairs.

Table of Values

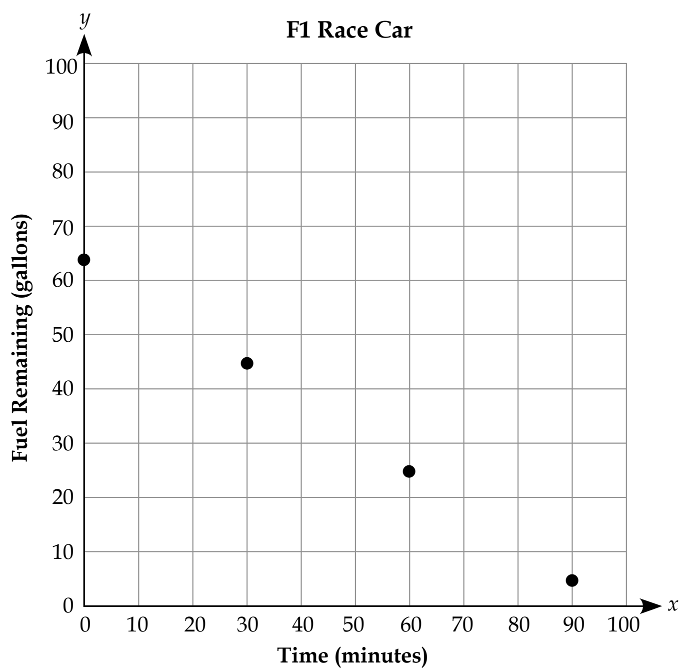
A table of values is an organized list of values that shows the relationship between two variables. It can also be called a T-chart.

Example 1

A Formula 1 race takes about 90 minutes and is around 200 miles long. The cars race at an average of about 140 miles per hour, and get mileage of approximately 3.1 miles per gallon. In a typical race, cars use up to 65 gallons of fuel. The following table of values indicates the number of minutes that have elapsed and the remaining fuel amount in an F1 race car.

Time Elapsed (minutes)	Fuel Remaining (gallons)
0	65
30	45
60	25
90	5

A scatterplot of these data could be displayed as follows:



The fuel remaining depends on the amount of time that has passed. Time is the independent variable because it is not directly affected by the number of gallons remaining.

Example 2

Chocolate milk is available in various-sized containers. In the school cafeteria, you can get a 250 mL carton for $1.25. At a convenience store, a 500 mL carton is $1.75. When grocery shopping, you can buy cartons with 1 L for $2.00, 2 L for $3.25, or a 4 L jug for $5.50.

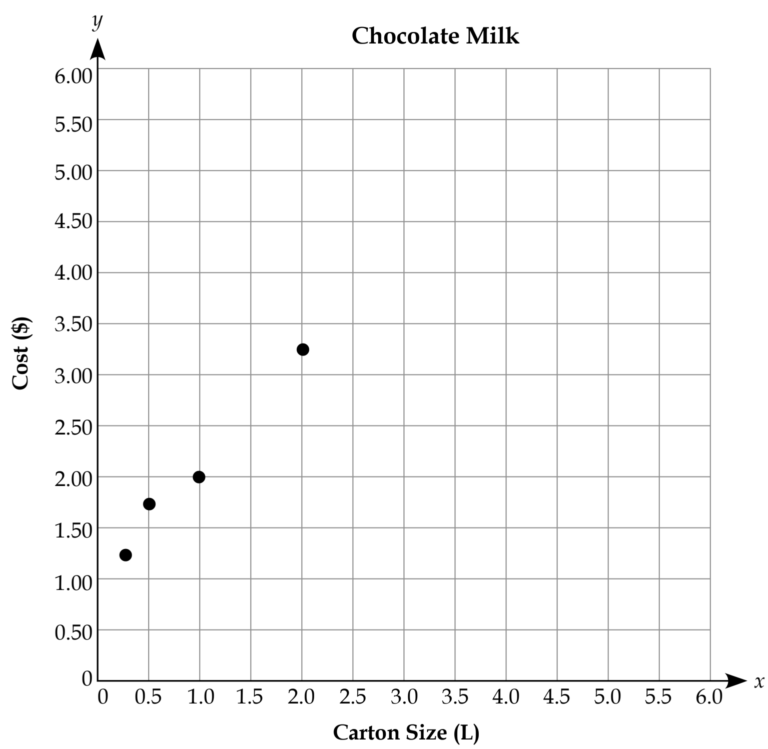
To represent this situation using a graph, you need to first determine the independent and dependent variables. The cost of chocolate milk depends on the size of the carton, so size is independent and graphed along the x-axis. The cost is then the dependent variable, graphed along the y-axis.

The information provided can be written as ordered pairs, (x, y) or (size, cost) coordinate pairs as follows:

(0.250, 1.25), (0.500, 1.75), (1, 2), (2.00, 3.25), (4, 5.50)

Notice that the units, L and $, are not included inside the brackets, and that **all the carton sizes are given in the SAME unit—the mL have been converted to L**.

A scatterplot of these points could be displayed as follows:



Ordered pairs can be organized into a table of values, and the data in a table of values can be written as coordinate pairs! A table of values for the chocolate milk data could look like this:

Size (L)	Cost ($)
0.25	1.25
0.5	1.75
1	2.00
2	3.25
4	5.50

Linear Relations

What are some of the similarities and differences you see in the two scatterplots given in the examples above? Jot down your observations here:

The two graphs are similar in that it would appear that a straight line could be drawn on each graph and the line would pass through, or very close to, all of the data points on that graph. When this occurs, the relation between the two variables is said to be **linear**. The rest of the module explores linear relationships, so it may be helpful to have the definition on your Resource Sheet.

In the first example, this line would go down, or fall, as you moved to the right.

In the second example, this line would go up, or rise, as you moved right.

This is a difference that will be dealt with further in a future lesson.

There are no doubt more similarities and differences in these two examples, but these are the most critical ones for this module.

Using this line, you could make predictions about other points on the graph by examining points that lie on the line between given coordinate points (interpolation) or by examining points that lie on the extension of the line (extrapolation).

Learning Activity 1.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Your mother has invited your entire family over for dinner. As usual, you are complaining because only $\frac{1}{6}$ of them are around your age. If your family has 24 people in it (not including you), how many of them are approximately your age?
2. There are 12 eggs in a dozen. If you are buying four dozen eggs, how many eggs is that?
3. Write the following decimal as a fraction: 0.058.
4. You are craving 5¢ candy. You have $1.43 in your pocket. How much candy can you afford?
5. List the factors of 12.
6. Fill in the missing terms in the following pattern: 0, 3, _____, 9, 12, _____.
7. Circle the independent variable: the height of an airplane compared to the time it takes to land.
8. At the end of the day, a restaurant is left with three different, partially eaten pies. Each pie has $\frac{2}{7}$ left over. How much pie is left over in total?

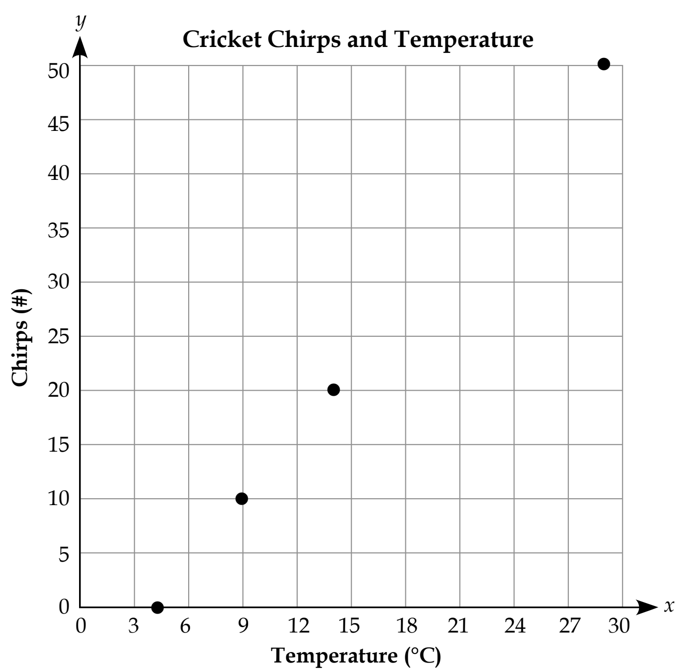
continued

Learning Activity 1.4 (continued)

Part B: Ordered Pairs, Data Tables, and Relationships

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the data given in the table of values in Example 1 to complete the following:
   a) Write the data as ordered pairs.
   b) Write a sentence describing the relationship between the two variables (e.g., As time does 'this', the remaining fuel does 'that').
2. Refer back to the graphs provided or the ones you created in Lesson 2. Which graphs indicate linear relations?
3. In the following scatterplot, a biologist has displayed data relating the temperature to the number of times a cricket chirps.



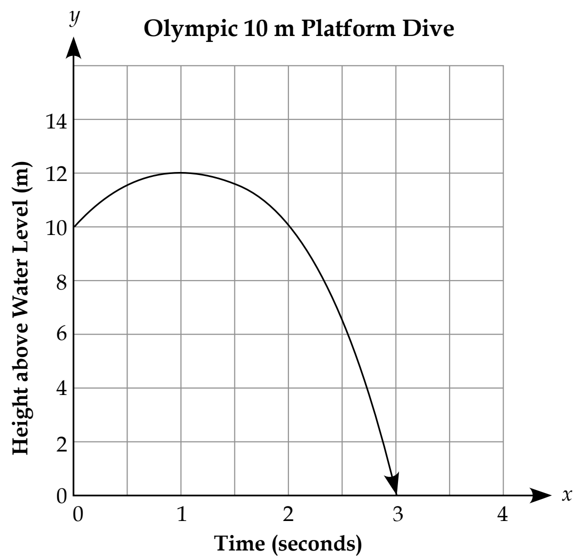
   a) Write the coordinates as ordered pairs.
   b) Create a table of values from the data graphed in this scatterplot.
   c) Write a sentence describing the relationship between these two variables.
   d) Does this graph display a linear relation? Explain.

Restrictions on Domain and Range

A good graph has an appropriate scale along each axis. An appropriate scale would include all the possible or reasonable values for that variable in the given situation.

Example 3

Olympic platform divers jump from a height of 10 m above the level of the water. If you wanted to graph the height of a diver above the water compared to the time of the dive, the graph may look something like this:



The diver jumps up slightly, then dives downwards before entering the water.

The values along the x-axis represent the time in seconds that the diver is in the air. Realistically, a dive may only last a few seconds, so the x-values that are reasonable or valid for this situation may go from 0 to 3 seconds. These values are called the domain. **The domain represents all the x-coordinate values that are possible or reasonable for the independent variable, graphed along the x-axis.**

The values along the y-axis represent the diver's height above the water level. The range of reasonable values for this diver may go as high as 12 m and as low as 0 m. **The range represents all the y-coordinate values that are possible or reasonable for the dependent variable, graphed along the y-axis.**

The scale along the axes should include values slightly above and/or below the domain and the range of the graph. The domain and range refer to the actual possible data values in the context. Most often, you need to use common sense in establishing the domain and range but there are guidelines that can sometimes be followed:

- Very large or very small numbers may be questionable. For example, the size of a house or the speed of a car has limits.
- Negative values may not be possible in linear relations, unless the graph is displaying things like temperatures or financial profit and loss.
- The restrictions on the domain or range of one of the variables may limit or restrict the other variable, because the dependent variable is affected by the independent variable.

Example 4

Biologists wish to compare the length of walleye fish in a particular lake to the age of the fish. They identify fish that have hatched over a number of years and then release them into the lake. Over a period of years, they recapture these fish and measure their lengths. What sort of factors would restrict the domain and range in this situation? What would be reasonable for the domain and range in this context?

The length of the fish is dependent on its age, so age is the independent variable. The domain (possible x-values) is restricted by the age of the fish. The range (y-values) would be restricted by length of a fish. Fish may live up to 15 years, so the domain could be from 0 to 15 years. A fish is tiny when it hatches, and a trophy walleye may be 35–40 inches long, so the range (valid y-values) may be from 0 to 40 inches.

Learning Activity 1.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Simplify the following fraction: $\frac{21}{35}$.
2. Simplify the following fraction: $\frac{12}{20}$.
3. 50% of 680 is 340. 25% of 680 is _____.
4. There is a 50% discount on all candy at the store the day after Hallowe'en. If it cost you $30 to buy candy before Hallowe'en, how much would you spend if you bought the same candy after Hallowe'en?
5. Solve for r: $5 + r = -4$.
6. You are at a doughnut shop and would like to buy doughnuts for your family. There are six people in your family (including you—and you want a doughnut). How much money do you need, if each doughnut costs 60¢?
7. A regular goal in rugby is worth three points. If a team finishes a game with 39 points, how many goals did they score?
8. If it rains 10 mm, how much rain is that in centimetres?

continued

Part B: Domain and Range

Remember, these questions are similar to the ones that will be on your
Remember, these questions are similar to the ones that will be on your
assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You want to rent a banquet room in a restaurant and have dinner with friends to celebrate your birthday. The cost of the meal is $6 per person.
   a) If you were to graph the relation between the cost and the number of people attending, what would be a reasonable domain and range? Explain the restrictions you consider.
   b) Create a possible graph. You may draw it by hand or use technology and print it out.
   c) Is this an example of a linear relation?
   d) Write a sentence describing the relationship between the variables.

Lesson Summary

A linear relationship between variables can be identified in a graph when a straight line goes through or close to all of the coordinate points graphed in a scatterplot. These (x, y) coordinates, or ordered pairs, can also be recorded in a table of values, and their relationship described using words. The domain of a relation is all the possible x-values that are reasonable or valid for that context. The range is all possible y-values that are reasonable or valid for that context. Using common sense will help you to determine restrictions on the domain and range.

In the next lesson, you will learn to identify additional characteristics of linear relations, the slope, and intercepts of the line.

Assignment 1.2

Domain and Range

Total Marks = 30

1. The size of shoe that person requires depends on the size of his or her foot. It is known that a 12-inch foot requires a size 14 shoe, a 10-inch foot requires a size 8 shoe, and an 8 and ½-inch foot requires a size 5 shoe.

   a) Determine which variable—length of foot or shoe size—is the dependent and independent variable in this context. *(1 mark)*

   b) Write ordered pairs to represent these data. *(3 marks)*

   c) Create a table of values from this information. *(2 marks)*

   d) Describe possible restrictions on the domain and range of these data. *(2 marks)*

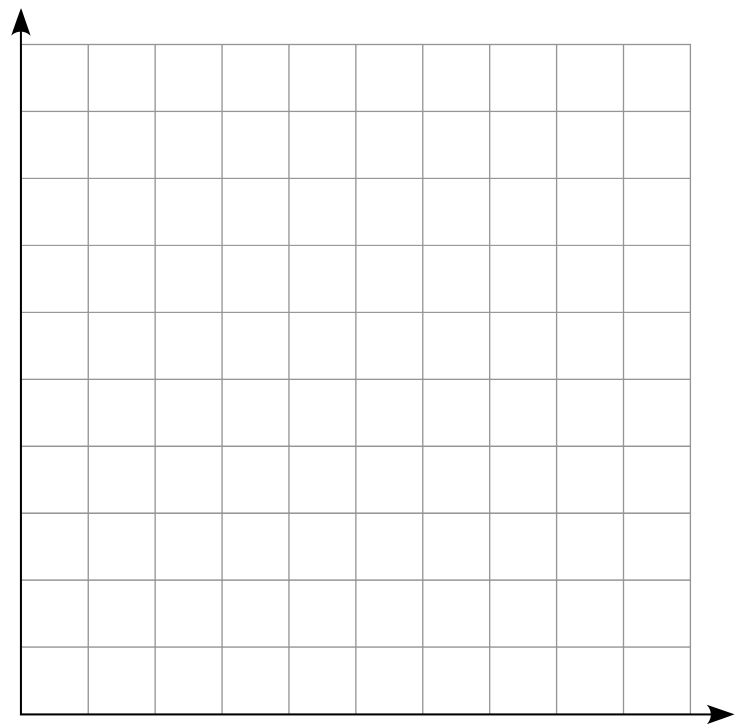
Assignment 1.2: Domain and Range (continued)

e) State the domain and range. *(2 marks)*

____

____

f) Create a graph to display these data. You may use the grid provided below, draw it by hand, or use technology and print it out. *(3 marks)*



g) Do the data from this context represent a linear relation? *(1 mark)*

____

____

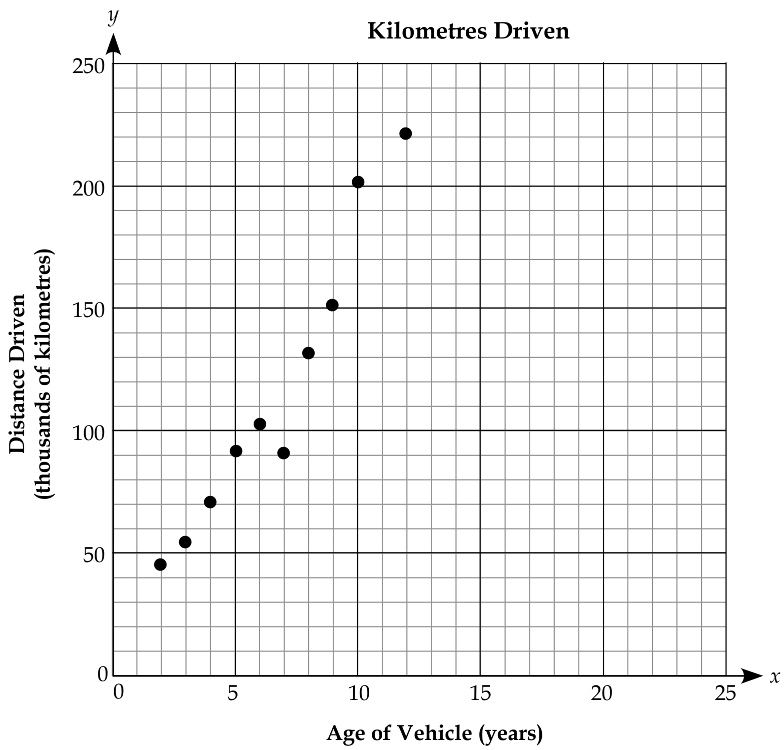
h) Write a sentence describing the relationship between the variables. *(1 mark)*

____

____

Assignment 1.2: Domain and Range (continued)

2. You wish to compare the age of privately owned cars and trucks with the number of kilometres they have been driven. You collect information and create the following graph, based on vehicles of different ages, that have been, for the most part, used continually from the time they were new.



a) Is this a linear relation? *(1 mark)*

b) State three ordered pairs from this graph. *(3 marks)*

c) What restrictions on the domain and range would be reasonable in this case? *(2 marks)*

Assignment 1.2: Domain and Range (continued)

d) State a possible domain and range for this context. *(2 marks)*

e) Are these data continuous? Explain. *(1 mark)*

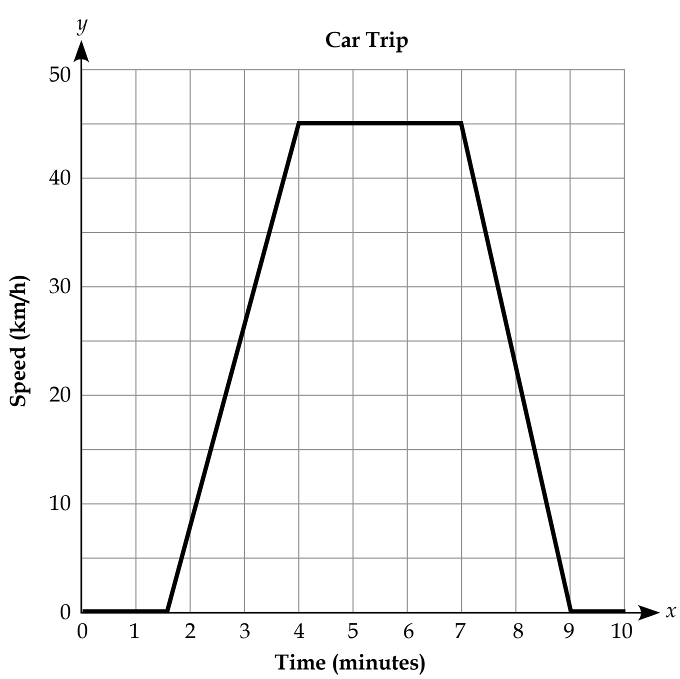
3. Find the matching pairs in the following table of values, ordered pairs, graphs, and contexts. Indicate whether each pair represents a linear relation. Explain why or why not. *(6 marks)*

i)

#	$
2	2
4	3
6	5
8	7

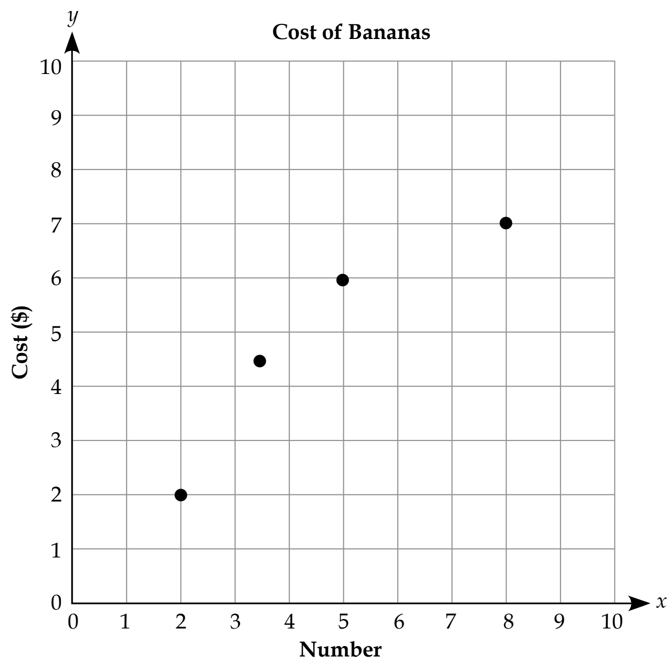
ii) (2, 2), (3.5, 4.5), (5, 6), (8, 7)

iii)



Assignment 1.2: Domain and Range (continued)

iv)



v) A bus at a stop picks up passengers and then drives down the street, where it stops to unload some passengers.

vi) You can buy two chocolate bars for $2, 6 for $5, or 8 for $7.

Notes

Lesson 3: The Slope and Intercepts of a Linear Relation

Lesson Focus

In this lesson, you will

- ❑ explain the slope of a graph as a rate of change
- ❑ determine whether a slope is positive or negative
- ❑ calculate slope
- ❑ determine the x- and y-intercepts of a graph and state them in two ways
- ❑ solve problems involving slope and intercepts

Lesson Introduction

When downhill skiing or snowboarding, slopes may be described as black diamonds or bunny hills. If you are racing downhill, there is a point that defines the starting line, and a point where you cross the finish line.

How steep a line in a graph is drawn can be described numerically. The points where the line crosses the x- and y-axes are called the intercepts. In this lesson, you will learn how to describe the slope of a line and calculate its numerical value. You will also identify the x- and y-intercepts of a line.

What is Slope?

Slope as a Rate of Change

You and a friend are comparing the income from your part-time jobs using a table of values.

Your income compared to the hours worked:

Hours (#)	Income ($)
2	14
5	35
8	56
10	70

Your friend's income and hours worked

Hours (#)	Income ($)
3	33
6	66
7	77
9	99

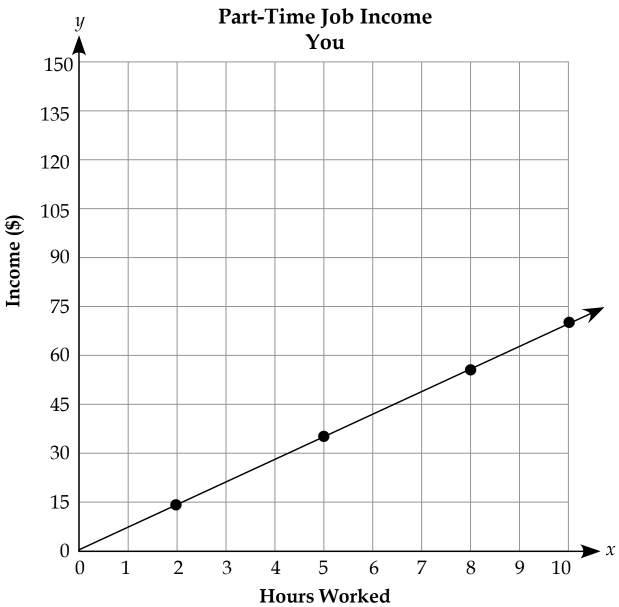
Do these tables reflect linear relations? How can you tell?

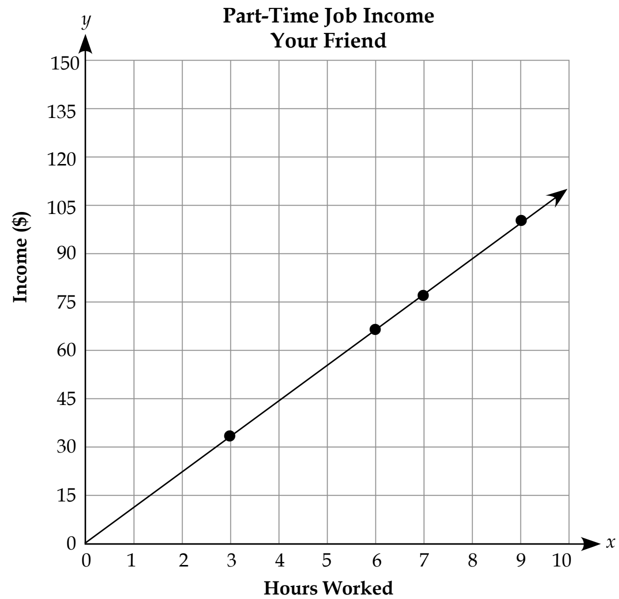
Yes, the data in these tables are linear, because the relationship between the hours and incomes is constant in each of the charts. This is the result of the hourly wages.

Do you earn the same amount per hour? Who earns more? How can you tell?

You earn \$7 per hour, while your friend earns \$11 per hour. This can be determined by finding a pattern between the two columns. The number of hours you work multiplied by 7 gives you the amount of income, while you must multiply your friend's hours by 11 to arrive at her income.

If you were to graph these two linear relations, the data may be displayed as follows:



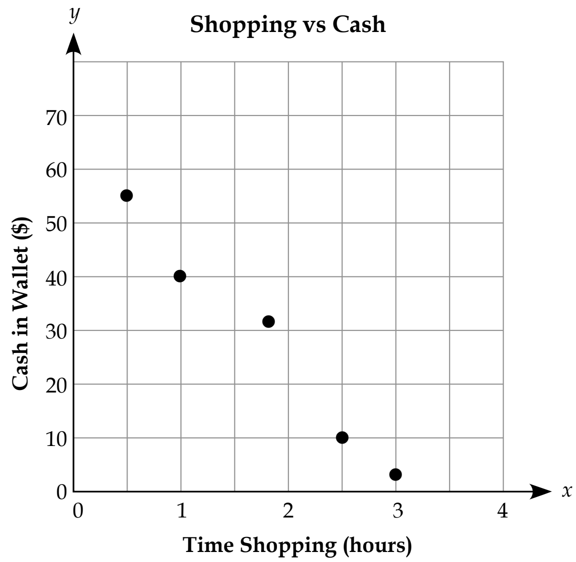


What do you notice about the lines on these two graphs?

The line representing your friend's income rises with a steeper incline, compared to the more gradual incline in the graph of your income. The **slope** of a line is a measure of how steep a line is. **Slope compares how far the line moves vertically (up or down) as it moves horizontally (as it moves to the right).** Because you are now comparing two variables, the slope can be given as a rate of change, or the average change, stated using units like dollars per hour, or km/hr., or m/sec., and so on.

The slope of the line in your graph can be stated as \$7/hr. because the line increases positively by \$7 for each increment of one hour along the *x*-axis.

If you were to take your income in cash with you as you went shopping, the comparison of hours and amount of cash may look like this:



The slope of this linear relation decreases or shows negative growth as you move to the right along the x-axis, but you would not connect the dots since you are not spending money continuously, but at separate stores.

Lines can either have a negative or positive slope. A negative slope decreases as you move to the right and a positive slope increases as you move along the line to the right.

Positive slope Negative slope

Learning Activity 1.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Write the percent as a decimal: 3.5%.
2. Which is larger: 0.76 or 0.07?
3. What is the range of the following numbers? 0.2, 0.6, 0.08, 0.5, 0.03
4. If 3% of 500 is 15, what is 12% of 500?
5. The mat used for floor gymnastics is a square. The length along one side is 40 feet. What is the total area of the mat?
6. Complete the pattern: 9, –7, 5, _____, _____.
7. Solve for v: $9v = 45$.
8. You want to order a pizza, but you only have $15. If a pizza costs $16 but is 10% off, can you afford it (ignore taxes)?

Part B: Rates and Slopes

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Kent is rock climbing up a 400 m high cliff. 45 minutes after beginning his ascent, he has climbed 75 m. After another hour, he is at 175 m.
   a) How many hours of steady climbing will it take him to reach the top? Sketch a graph to help you answer this question.
   b) At what rate is he climbing in m/hr.?

continued

Learning Activity 1.6 (continued)

2. The descent down the cliff only takes Kent two and a half hours to complete.
   a) Create a graph similar in scale to the previous question and sketch this situation. Plot the points that represent his time and location at the top of the cliff and when he reaches the bottom.
   b) Compare the slopes in these two graphs.
   c) What is Kent's rate in this situation?

Calculating Slope

To calculate the rate of change or slope of a relation, determine the change in the y-value between two points, and the change in the x-value between these same two points. The ratio of vertical change/horizontal change is the slope. This is sometimes called $\frac{\text{rise}}{\text{run}}$ ("rise over run").

You will need to be able to calculate slope. It may be helpful to have on your Resource Sheet.

Example 1

Ruby checks the rain gauge in her backyard during a storm. After three hours, the gauge shows there has been 1 mm of rain. At the five-hour mark of the storm, the gauge shows 7 mm of rain. Write the two (x, y) coordinate pairs in this situation, and determine the slope of the line between these points.

difference in y-values (the increase in rainfall) is the vertical change

(3, 1) (5, 7)

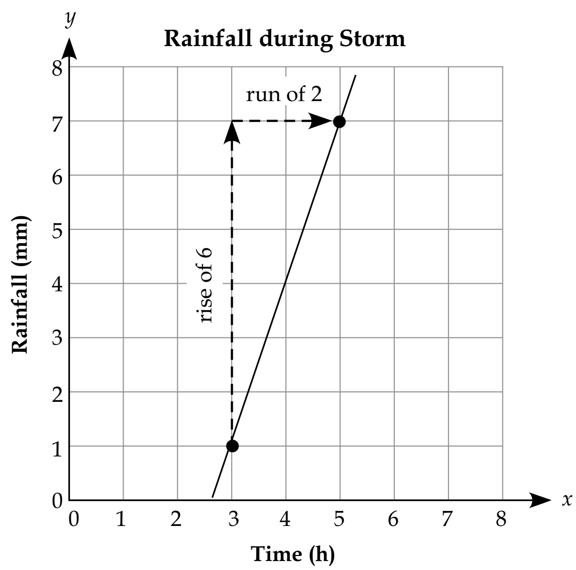
difference in x-values (the time passed) is the horizontal change

From y = 1 to y = 7 is an increase of 6	6 mm of rain has fallen in this interval
From x = 3 to x = 5 is an increase of 2	2 hours has passed

Slope is the ratio of the $\dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{6}{2} = 3.$

The slope is 3. This represents 3 mm/h of average rainfall in this interval. It is an average because after three hours only 1 mm has fallen. Using the slope, after three hours you would expect (3 × 3 = 9) 9 mm of rain to have fallen.

Sketch a graph to visualize the $\dfrac{\text{rise}}{\text{run}}$ of the line.



$$\frac{\text{rise}}{\text{run}} = \frac{6}{2} = 3$$

Example 2

Given the coordinate pairs (4, 5) and (7, 1), calculate the slope of the line between these points.

from y = 5 to y = 1 is a decrease or vertical change of –4

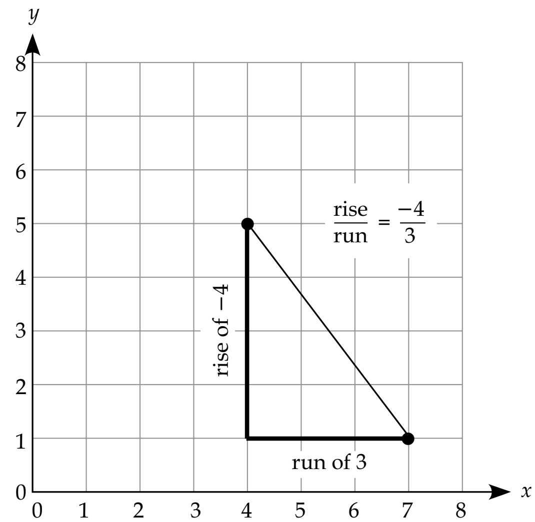
(4, 5) (7, 1)

from x = 4 to x = 7 is an increase or horizontal change of 3

Slope is the ratio of the $\dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{-4}{3}.$

The slope is $\frac{-4}{3}$.

Sketch a graph to visualize the $\frac{\text{rise}}{\text{run}}$ of the line.



$$\frac{\text{rise}}{\text{run}} = \frac{-4}{3}$$

Note: Slope is always read as you move to the right along the x-axis.

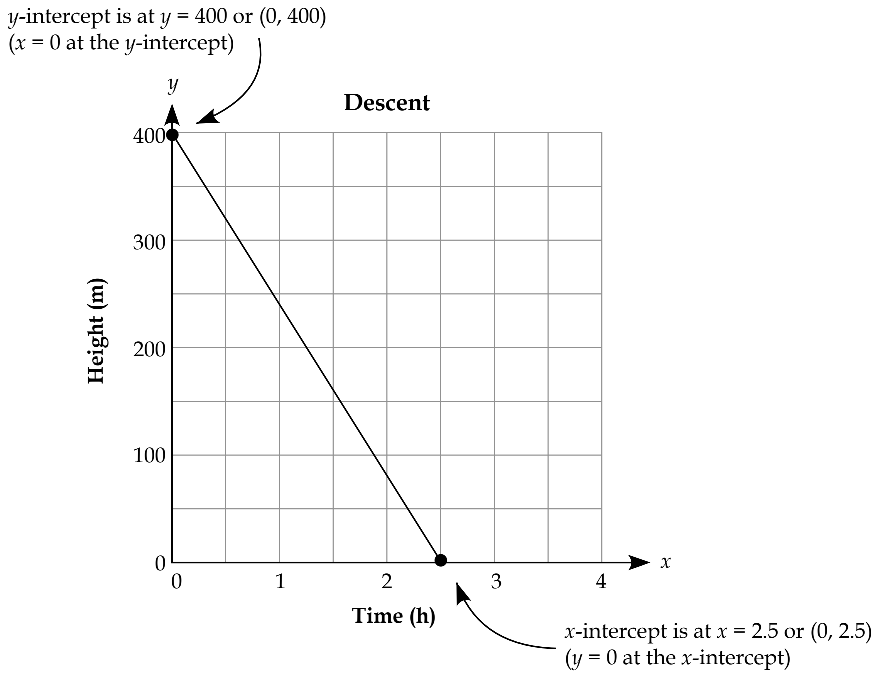
Intercepts

In the last learning activity, you were able to graph a representation of Kent descending the cliff by using two coordinate pairs: (2.5, 0) and (0, 400). What do you notice about these ordered pairs and where they are plotted on the graph?

In each of these ordered pairs, one of the coordinates is a zero. When plotting these points on the graph, one is located on the x-axis and the other one is located along the y-axis. These two critical points are called intercepts.

The x-intercept is the point where the graph crosses the x-axis. The y-coordinate at this point is equal to 0. The x-intercept can be written as an ordered pair; for example, (2.5, 0), or as a value like $x = 2.5$.

The y-intercept is the point where the graph crosses the y-axis. The x-coordinate at this point is equal to 0. The y-intercept can be written as an ordered pair; for example, (0, 400), or as a value like $y = 400$.



Often, the domain and range of a given situation are restricted by the values at the intercepts. In the graph above, the domain would be from 0 to 2.5 and the range would be from 0 to 400.

In the graph representing Kent's climb up the rock cliff, both the x-intercept and the y-intercept occur at the same point: where the x-axis and y-axis intersect. The coordinates of this point are (0, 0). This point has a special name. It is called the origin of the graph.

Learning Activity 1.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

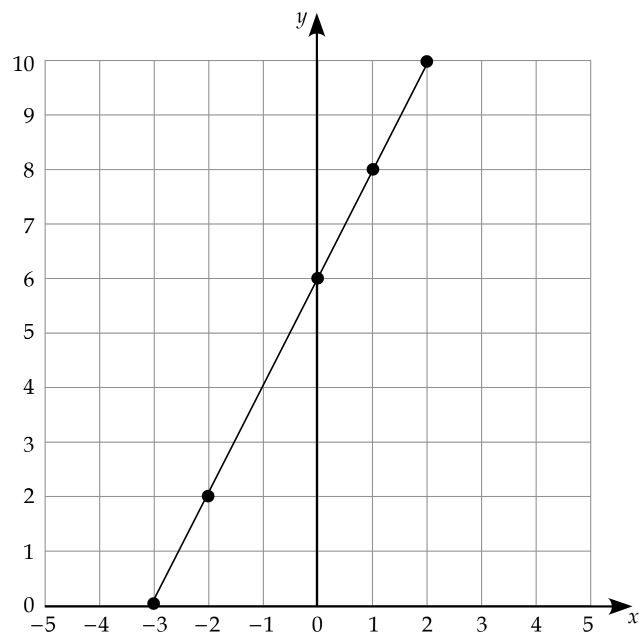
1. Continuous or not: A graph comparing the number of people in a room and the volume of noise.
2. Domain or range: the independent variable has values from 0 to 20.
3. Write the following as an improper fraction: $2\frac{3}{7}$.
4. You are driving to the cabin, which is 104 km from your house. $\frac{1}{8}$ of your drive is in the city. How much of your drive is in the city (in km)?
5. You are at a bake sale. You want to buy a peanut butter cookie. If the whole plate of 16 cookies costs $3.20, how much will you pay for one cookie?
6. Write the following decimal as a percent: 0.84.
7. A marathon is 26.4 miles. A half marathon is how long?
8. Solve for d: $3d = 9$.

continued

Part B: Linear Relations

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the following theoretical graph, indicate:
   a) if the slope is positive or negative
   b) what the rate of change, or slope, is equal to
   c) the value of the x-intercept
   d) the coordinates of the y-intercept
   e) the domain
   f) the range



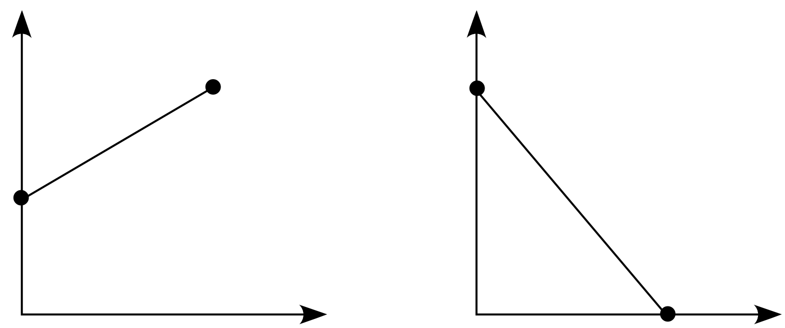
continued

Learning Activity 1.7 (continued)

2. From the first part of the lesson, when you realized how much more income your friend had with her part time job, you found a new position at a pet store. Here, your income is based in part on commission. You earn a percentage of the value of the product you sell. Based on your first three weeks of income, knowing that income depends on sales, you determine the following (sales, income) pairs: (500, 100), (600, 110), and (1000, 150).
   a) Graph these ordered pairs and determine if the relation is linear.
   b) What is the rate of change or slope of the line?
   c) Draw a line to determine the intercepts. What do the intercepts represent?
   d) What are the domain and range in this situation?

How Many Intercepts are Possible?

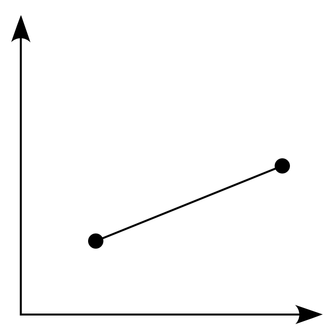
So far you have seen graphs with one or two intercepts in the given context.



one intercept due to restricted domain

two intercepts

Is it possible to have a linear graph with no intercepts? If the domain and range restrict the possible x- and y-values, yes, it is possible to have a graph with no intercepts.

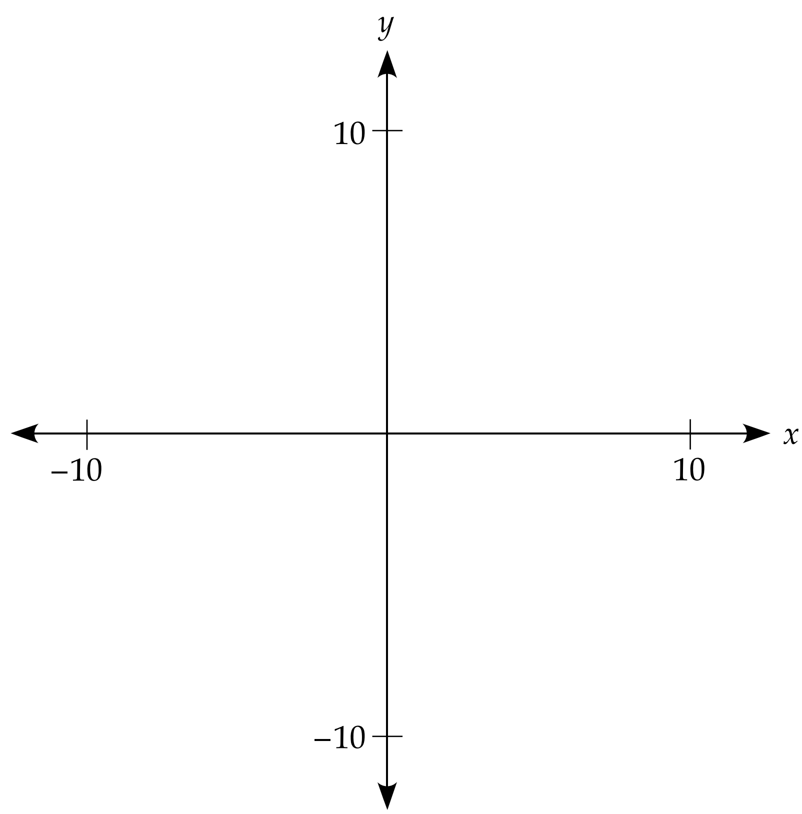


no intercepts due to restrictions on the domain and range

However, consider graphing on the entire Cartesian plane below. If there are no restrictions on the domain and range of a linear relation:

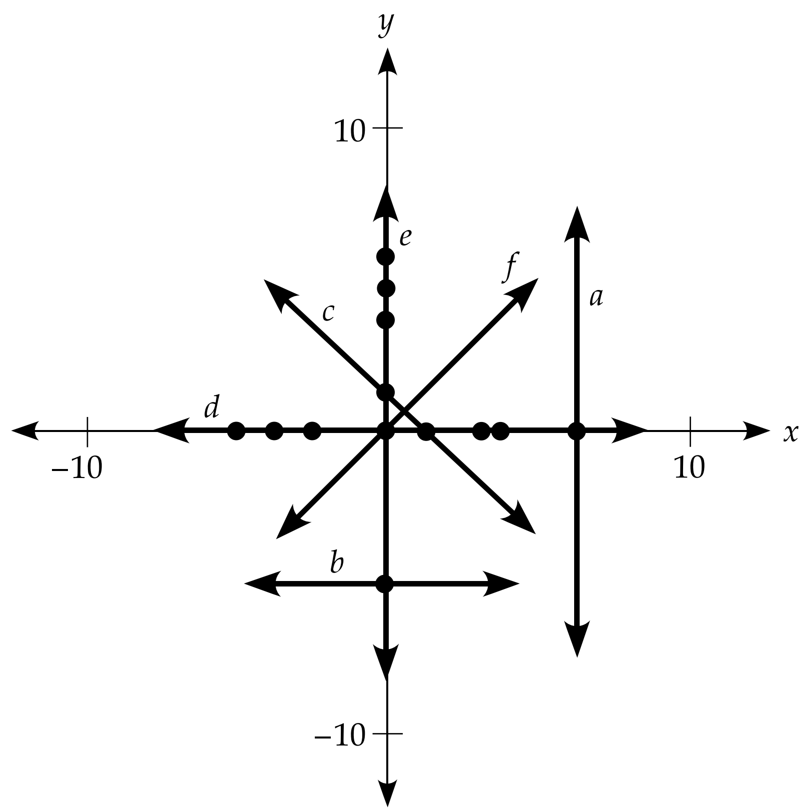
- draw a linear line with no intercepts
- draw a linear line with exactly 1 intercept (either an x-intercept or a y-intercept)
- draw a linear line with exactly 2 intercepts (an x-intercept and a y-intercept)
- draw a linear line that has an infinite number of intercepts (either x-intercepts or y-intercepts)

Cartesian Plane



- It is impossible to draw a linear line with no intercepts if there are no restrictions on the domain and range, and all four quadrants of the Cartesian plane are being used. A straight line that continues indefinitely in both directions will eventually cross one or both axes, as they continue indefinitely as well.
- A line with exactly one intercept must be either a vertical line that intercepts the x-axis in one place, or a horizontal line that intercepts the y-axis at one point (lines a and b on the following graph).

- Any oblique (diagonal) line will have exactly two intercepts: one along the x-axis and one along the y-axis (line c is one possible example of an oblique line).
- A line with an infinite number of x- or y-intercepts must be drawn along one of the axes. A horizontal line drawn on the x-axis intercepts the x-axis at each point along the entire line (line d). A vertical line on the y-axis intercepts the y-axis at each point along the entire line (line e). These lines have an infinite number of intercepts.
- If a line passes through the origin (line f), it appears to have only one intercept. This line is considered to have two intercepts because it passes through both the x-axis and the y-axis.



Lesson Summary

In this lesson, you learned about two characteristics of linear relations—the slope and intercepts. You can now identify whether a slope is positive or negative, and know how to calculate the rate of change using coordinate points along the line or $\frac{\text{rise}}{\text{run}}$. You can identify the intercepts of a line and write them as values or coordinate points. In the next lesson, you will consider the slope of horizontal, vertical, parallel, and perpendicular lines, and use a slope and point to draw lines.

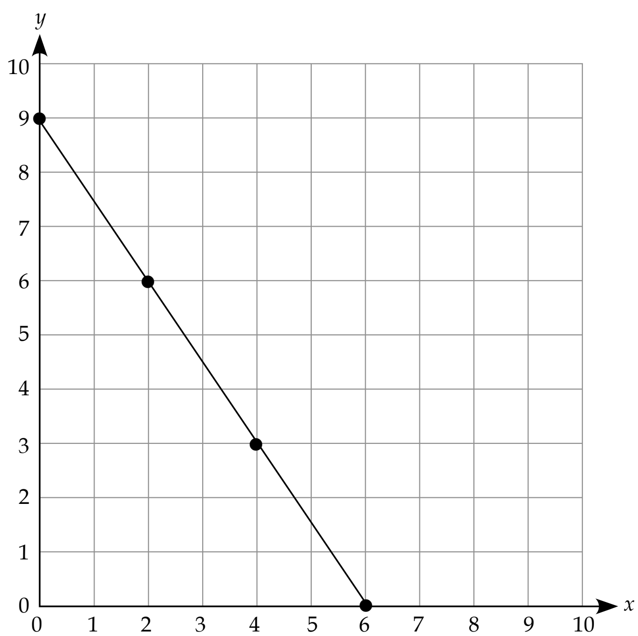
Assignment 1.3

Slopes, Intercepts, Domain, and Range

Total Marks = 29

1. Fill in the required information about the following graphs.

   a) *(6 marks)*



i) Is the slope positive or negative? ________________

ii) Calculate the rate of change (slope). ________________

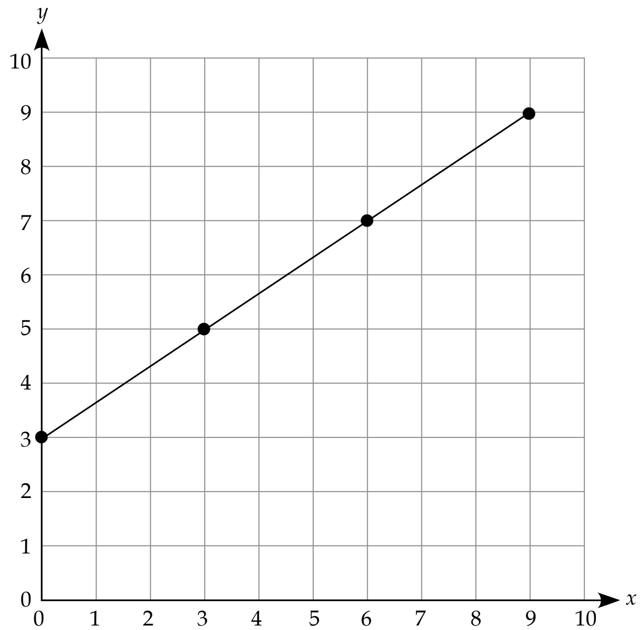
iii) State the value of the x-intercept. ________________

iv) State the value of the y-intercept. ________________

v) State the domain of this linear relation. ________________

vi) State the range of this linear relation. ________________

b) *(6 marks)*



i) Is the slope positive or negative? ____________________

ii) Calculate the $\frac{\text{rise}}{\text{run}}$ (slope). ____________________

iii) State the coordinates of the x-intercept. ____________________

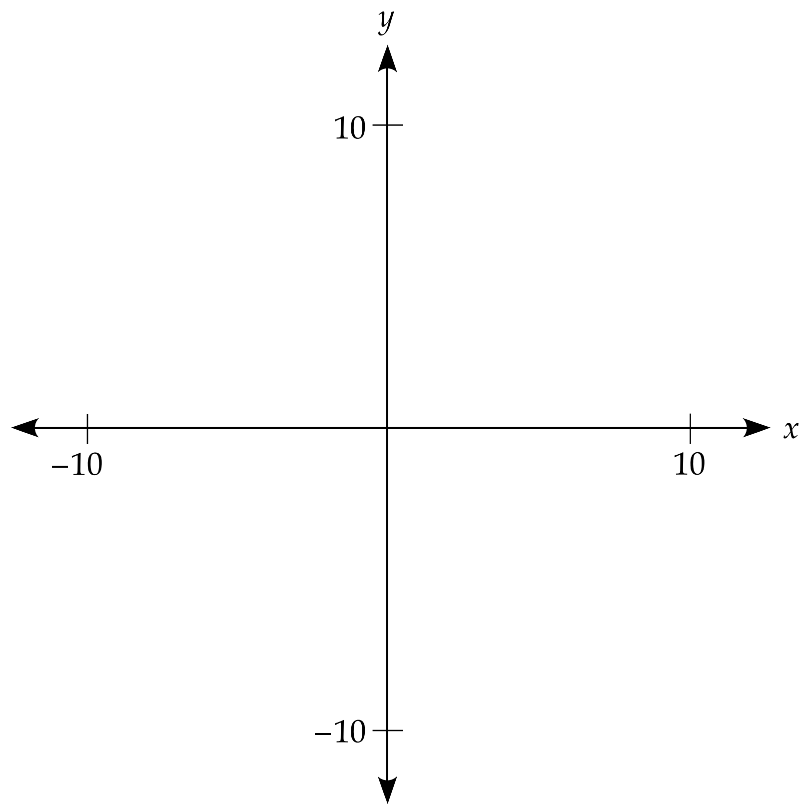
iv) State the coordinates of the y-intercept. ____________________

v) State the domain. ____________________

vi) State the range. ____________________

Assignment 1.3: Slopes, Intercepts, Domain, and Range (continued)

2. On the Cartesian plane below,
   a) sketch a linear relation with exactly one x-intercept, and label it line *a. (1 mark)*
   b) sketch a linear relation with an infinite number of y-intercepts, and label it line b. *(1 mark)*



3. A study shows that a person burns more calories per day if the average daily temperature in degrees Celsius is lower. The study collected the following data:

Temperature (°C)	21	0	10	4	35	–5
Calories Used	3000	3750	3300	3600	2500	4000

a) Create a graph to display these data. You may use the grid below or use graphing technology like *Graphical Analysis* or a spreadsheet. Print your graph and include it with your hand-in work. Include appropriate labels, units, title, and scales. *(5 marks)*

b) Does this represent a linear relation? _______________ *(1 mark)*

c) Use a ruler to draw a line through or as close to as many of the data points as possible. *(1 mark)*

d) Is the slope positive or negative? _______________ *(1 mark)*

Assignment 1.3: Slopes, Intercepts, Domain, and Range (continued)

e) Using your line or two coordinate points from the table of values, calculate the slope of the line. Explain what this represents. *(3 marks)*

f) State the intercepts in two ways (if they exist), and indicate what they represent. *(4 marks)*

Notes

Lesson 4: Calculating Slope

Lesson Focus

In this lesson, you will

❑ calculate the slope of a line segment

❑ explain the meaning of the slope of a horizontal or vertical line

❑ explain why the slope of a line can be determined by using any two points on that line

❑ draw a line, given its slope and a point on the line, and determine another point on that line

❑ generalize and apply a rule for determining whether two lines are parallel or perpendicular

❑ match graphs to their corresponding slopes and intercepts

Lesson Introduction

This lesson applies what you have previously learned about slope and intercepts of linear relations in various contexts to theoretical graphs. In this lesson, you will consider the slope of vertical, horizontal, perpendicular, and parallel lines. You will explain why the slope of any line can be determined by any two points on that line, and draw lines using a point and slope.

The Value of Slope

Slope Formula

In the previous lesson, you developed an understanding of the slope of a line. It represented the amount you were paid per hour, or the speed at which someone climbs a cliff. You used the ratio of the rise (vertical change) and the run (horizontal change) of a line to talk about the rate of change. As well, you determined the difference in the x- and y-values of two coordinate points to determine the mathematical value of the slope.

The letter m is often used to represent the slope of a line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{the difference in } y\text{-values}}{\text{the difference in } x\text{-values}}$$

If you take two coordinate points, and label them using subscripts, calling them Point 1: (x_1, y_1) and Point 2: (x_2, y_2), you can develop a formula for finding the slope of the line between these two points.

$$m = \frac{\text{the difference in } y\text{-values}}{\text{the difference in } x\text{-values}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1

Given the points (4, 2) and (6, 7), determine the slope of the line joining them, and the coordinates of another point on the line.

Solution:

First label the points as (x_1, y_1) and (x_2, y_2).

$$\begin{array}{cccc} (4, & 2) & (6, & 7) \\ \uparrow & \uparrow & \uparrow & \uparrow \\ (x_1, & y_1) & (x_2, & y_2) \end{array}$$

Substitute the values into the formula and solve for the slope.

$$m = \frac{\text{ryse*}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-2}{6-4} = \frac{5}{2}$$

The slope of this line is $\frac{5}{2}$.

From the point (6, 7) you could move up 5 (rise) and right 2 (run) and find another point on this line. The coordinates of this next point would be (6 + 2, 7 + 5) or (8, 12).

*To help you remember which relates to the y-value, you can think of rise as being spelled with a “y”; “ryse”.

Example 2

Calculate the slope of the line joining (3, 9) and (5, 1), and determine the coordinates of another point on the line.

Solution:

(3, 9) (5, 1)

(x_1, y_1) (x_2, y_2)

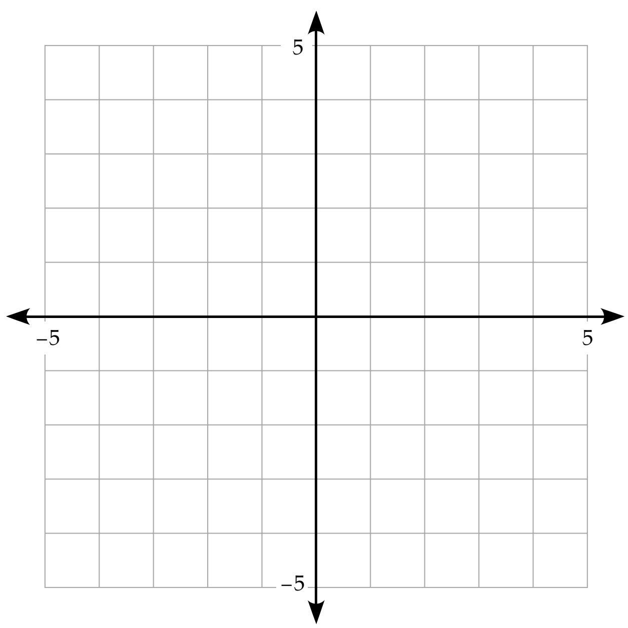
$$m = \frac{y_2 - y_2}{x_2 - x_1} = \frac{1-9}{5-3} = \frac{-8}{2} = \frac{-4}{1}$$

The slope of this line is –4.

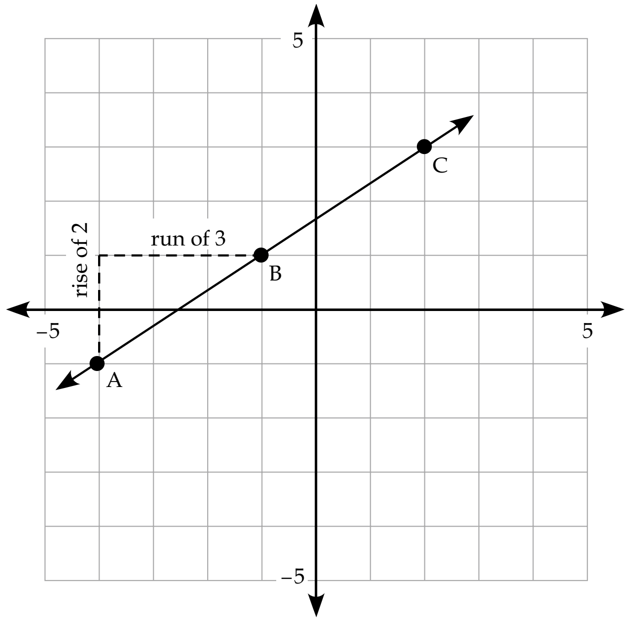
From the point (5, 1), you could move –4 units vertically (or 4 units down) and 1 unit horizontally (or 1 unit right) and find another point on this line. The coordinates of this next point would be (5 + 1, 1 − 4) or (6, −3).

Example 3

The slope of a line is $\frac{2}{3}$. Point A on that line is located at (−4, −1). Draw this line on the grid below.



Solution:



From the point A(−4, −1) you can move 2 units up and 3 units to the right to find the next point on the line B(−1, 1), and then repeat the $\frac{\text{rise}}{\text{run}}$ to find another point C(2, 3). Connect these points with a line.

Note: When using the slope to find another point on the line, you have several options.

- If the slope is positive, you could "rise" up and "run" right or you could "rise" down and "run" left. Note that up is a positive direction and down is a negative direction. Both down and left are negative directions. Since slope is rise divided by run, we can either divide two positives or two negatives to make the slope positive.
- If the slope is negative, you could "rise" up and "run" left or "rise" down and "run" right. Both of these options divide a negative and a positive, making the slope negative.

Example 4

A line passes through the points $(1, y_1)$ and $(4, 5)$ and has a slope of 1. Find the value of y_1 by substituting what you know into the slope formula.

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$1 = \frac{5 - y_1}{4 - 1}$$ Substitute what you know into the formula

$$1 = \frac{5 - y_1}{3}$$ Simplify

$$1(3) = \frac{5 - y_1 \cancel{(3)}}{\cancel{(3)}}$$ Multiply both sides of the formula by 3 to eliminate the denominator

$$3 = 5 - y_1$$

$$3 - 5 = 5 - 5 - y_1$$ Subtract 5 from both sides of the formula to isolate y_1

$$-2 = -y_1$$

$$-2(-1) = -y_1(-1)$$ Multiply each side by -1 to reverse the sign of y_1

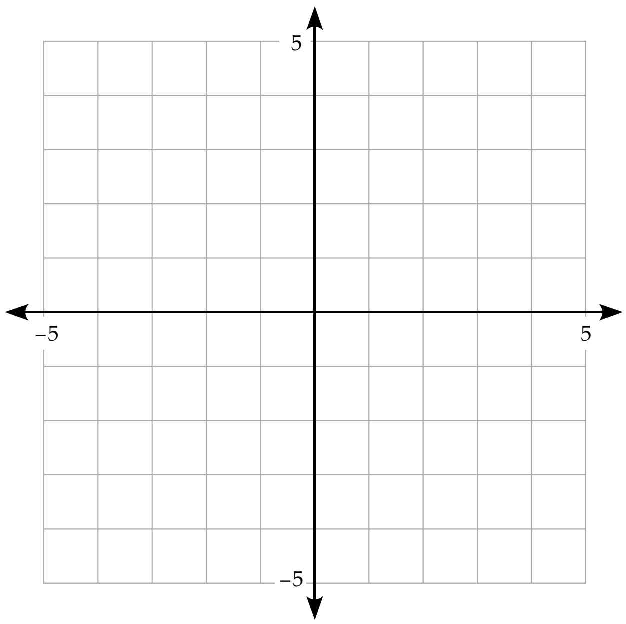
$$2 = y_1$$ The missing y-coordinate is 2

A line with a slope of 1 will pass through the points (1, 2) and (4, 5).

Example 5

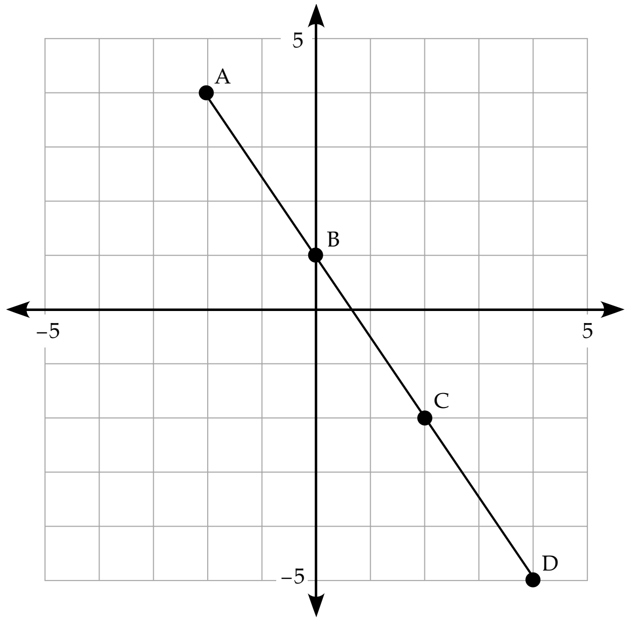
Plot the following points in the grid below and join them with a line.

Point	x	y
A	−2	4
B	0	1
C	2	−2
D	4	−5



Determine the slope of the line between points A and B.
Determine the slope of the line between points B and C.
Determine the slope of the line between points B and D.
What do you notice?

Solution:



Points A, B, C, and D all fall in a linear line.

The slope of the line segment between points A and B is calculated as:

$$\begin{array}{cc} A(-2, & 4) \\ \uparrow & \uparrow \\ (x_1, & y_1) \end{array} \quad \begin{array}{cc} B(0, & 1) \\ \uparrow & \uparrow \\ (x_2, & y_2) \end{array}$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{0-(-2)} = \frac{-3}{2}$$

Slope of line segment BC:

$$\begin{array}{cc} B(0, & 1) \\ \uparrow & \uparrow \\ (x_1, & y_1) \end{array} \quad \begin{array}{cc} C(2, & -2) \\ \uparrow & \uparrow \\ (x_2, & y_2) \end{array}$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{2-0} = \frac{-3}{2}$$

Slope of line segment BD:

$$\begin{array}{cccc} B(0, & 1) & D(4, & -5) \\ \uparrow & \uparrow & \uparrow & \uparrow \\ (x_1, & y_1) & (x_2, & y_2) \end{array}$$

$$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-1}{4-0} = \frac{-6}{4} = \frac{-3}{2}$$

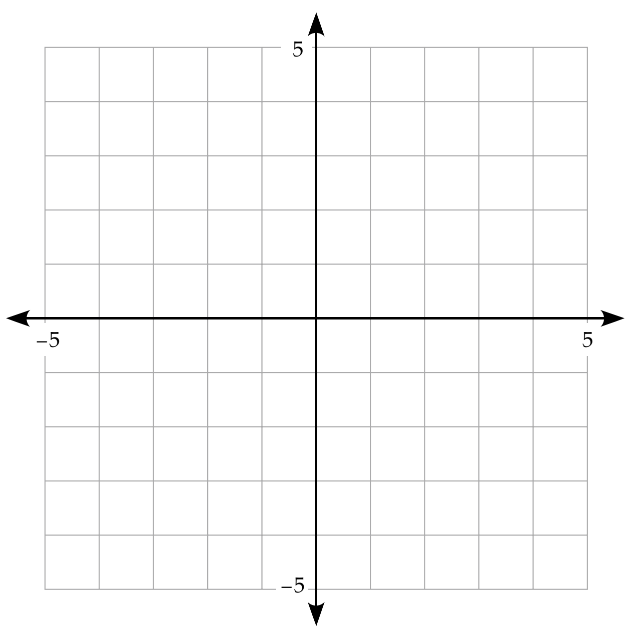
You can choose any two points in a linear relation, calculate the slope of the line that joins them, and you will find that the slope remains the same. **The slope of a linear relation remains constant at all points along the entire line.** When given several points along a line, you can use any two ordered pairs to determine the slope.

The Slope of Horizontal and Vertical Lines

You have discovered that the slope of a line may be a positive or negative whole number or fraction. The slopes of horizontal and vertical lines, however, are a special case.

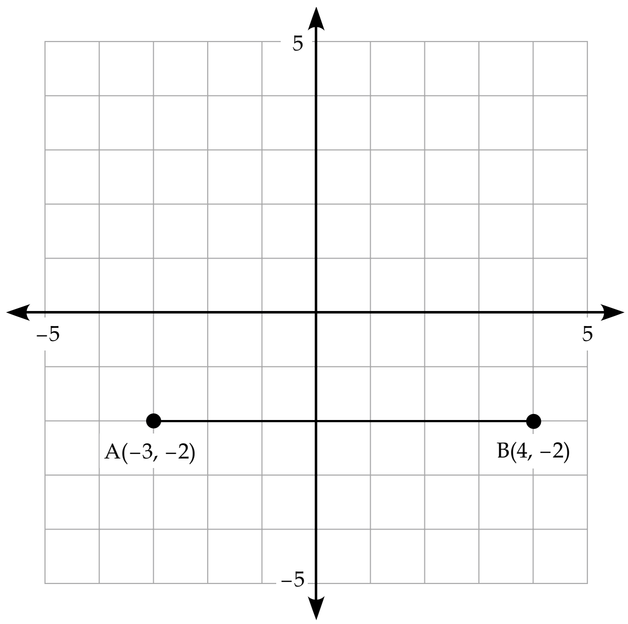
Example 6

Draw a horizontal line on the grid below. Indicate two points on the line and record their coordinates. Use these points to determine the slope of this horizontal line.



Solution:

One possible horizontal line is shown below.



The coordinates of points A and B along this horizontal line are as follows:

A (−3, −2) and B (4, −2)

Using the slope formula,

$$m_{\text{AB}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-2)}{4-(-3)} = \frac{0}{7} = 0$$

The slope of this line is 0.

Draw another horizontal line on the grid, determine two ordered pairs along the line, and calculate the slope.

You will find that **the slope of a horizontal line is always equal to zero**.

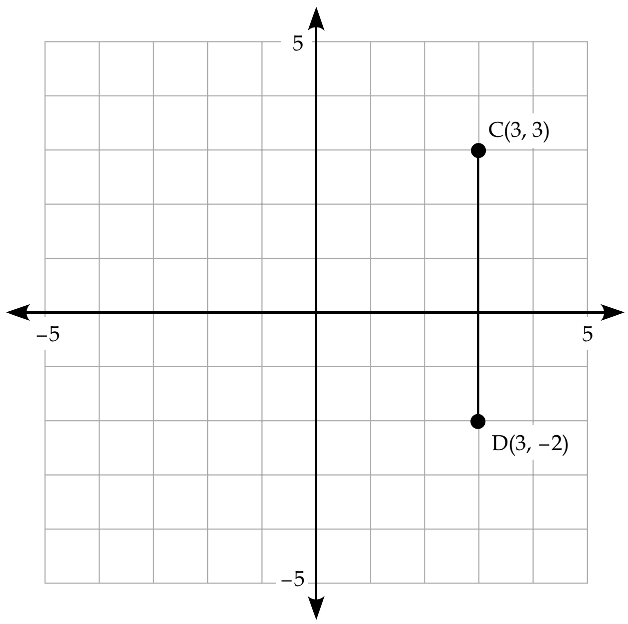
A horizontal line has a rise of zero, and an infinite number of points that could be chosen as its run. As you have experienced before, zero divided by any number will always be zero, so the slope of a horizontal line is 0.

Example 7

Draw a vertical line on the grid from Example 6 (with your horizontal lines). Indicate two points on the line and record their coordinates. Use these points to determine the slope of this vertical line.

Solution:

One possible vertical line is shown below.



The coordinates of points C and D along this vertical line are as follows:

C (3, 3) and D (3, –2)

Using the slope formula

$$m_{\text{AB}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{3-3} = \frac{-5}{0} = \text{undefined}$$

A vertical line does not have a slope. You cannot divide a number by zero (try it on your calculator—you will get an error message of some sort!). **The slope of a vertical line is undefined.**

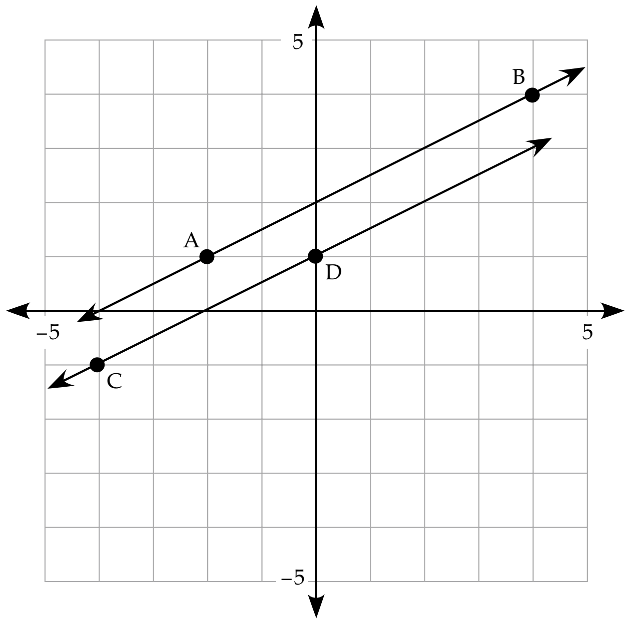
Note that a vertical line has an infinite number of points that could be chosen as its rise but always has a run of zero.

The Slope of Parallel and Perpendicular Lines

Parallel lines are lines that will never intersect, no matter how far they are extended. They will always be the same distance apart.

Example 8

Calculate the slope of lines AB and CD in the diagram below.



A(−2, 1) B(4, 4)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{4-(-2)} = \frac{3}{6} = \frac{1}{2}$$

C(−4, −1) D(0, 1)

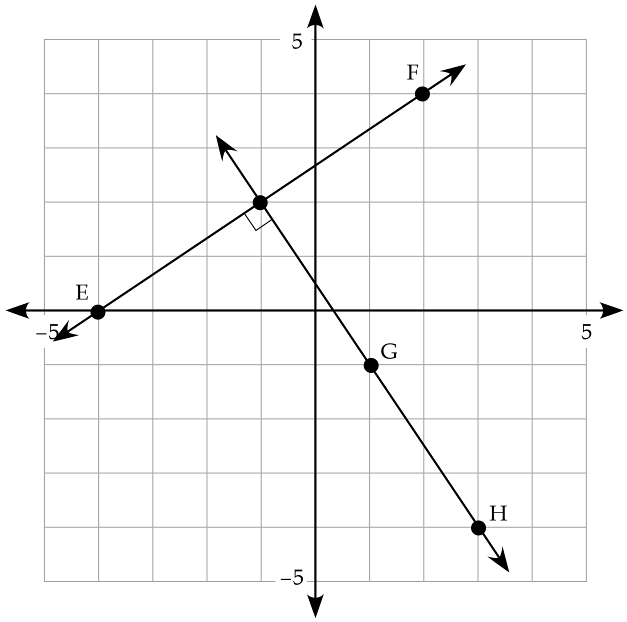
$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-1)}{0-(-4)} = \frac{2}{4} = \frac{1}{2}$$

The slopes of parallel lines will always be the same. If two lines are parallel, they will always have the same slope. Notice that the *y*-intercepts of these two lines are different, but **the slopes of parallel lines are equal**.

Lines are perpendicular if they intersect with each other at a 90° angle.

Example 9

Lines EF and GH in the diagram below are perpendicular. Calculate the slope of lines EF and GH.



E(−4, 0) F(2, 4)

$$m_{EF} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{2-(-4)} = \frac{4}{6} = \frac{2}{3}$$

G(1, −1) H(3, −4)

$$m_{GH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-1)}{3-1} = \frac{-3}{2}$$

What do you notice about the slopes of these lines?

The slope of line EF is positive and the slope of line GH is negative.

The slope of EF has a 2 in the numerator and a 3 in the denominator, and in the slope of GH these numbers are switched around—the 3 is in the numerator and the 2 is in the denominator. $\frac{2}{3}$ and $\frac{-3}{2}$ are called negative reciprocals. **The slopes of perpendicular lines are negative reciprocals.**

Negative reciprocals are values that have their numerators and denominators switched and the sign reversed. The negative reciprocal of $\frac{4}{5}$ is $\frac{-5}{4}$. The negative reciprocal of $-\frac{1}{2}$ is $\frac{2}{1}$ or 2. What is the negative reciprocal of 10? It would be $\frac{-1}{10}$. Remember that when you have a whole number, you can write it as a fraction $\left(10 = \frac{10}{1}\right)$.

The product of a number and its negative reciprocal will always be equal to −1. Try it!

Learning Activity 1.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. You are on your way to Calgary, driving on the Trans-Canada Highway. Your speed is constant at 100 km/h. If you have been driving for 3.3 hours at this speed, how far have you driven?
2. Write the following ratio as a fraction in lowest form: 16 : 12.
3. What is 10% of 500?
4. What is 5% of 500?
5. What is 15% of 500?
6. There are three classes of Grade 10 Science taught in a school by one teacher. Each class has 22 students. If the teacher gives a test to all three classes, how many tests will she have to mark?
7. A student receives a grade of $\frac{18}{20}$ on his test. Estimate the percent value.
8. Jamie is twice as old as Dan. Dan is 3 times as old as Kim. If Kim is 4 years old, how old is Jamie?

continued

Learning Activity 1.8 (continued)

Part B: What Slope Tells Us

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The slope of the line joining points K and M is $\frac{-3}{7}$ $\left(m_{\text{KM}} = \frac{-3}{7}\right)$. Point L is on the same line as points K and M. What is the slope of the line joining points M and L? Explain how you know.
2. (−456, 187) is a point on a line with a slope of $\frac{17}{25}$. Find another point on this line.
3. (5, 12) and (72, y_2) are on a line with a slope of 4. Use the slope formula to find the value of y_2. Show your work.
4. Explain in your own words why the slope of a horizontal line is equal to zero.
5. Explain in your own words how you can use the slope formula to determine if two lines are parallel.

Lesson Summary

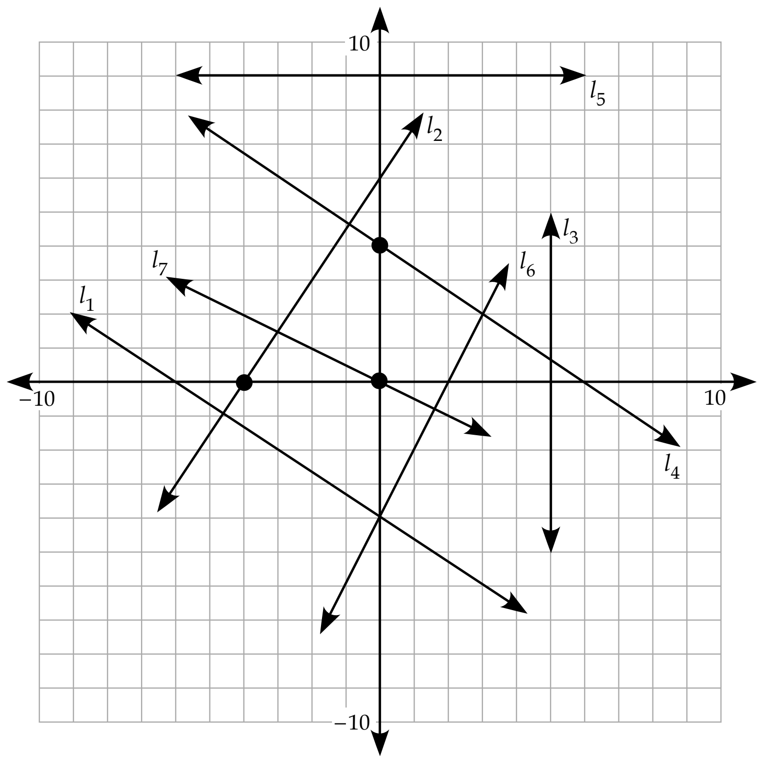
In this lesson, you used the slope formula to calculate the slope of lines and line segments. Using the fact that the slope of a line is constant along its entire length, you were able to graph lines given a slope and a point on that line, and find other points along that line. You investigated and explained the slopes of vertical, horizontal, parallel, and perpendicular lines. In the next lesson, you will describe linear relations using equations and learn two more ways to express the domain and range of a linear relation.

Assignment 1.4

What We Can Tell From Slope

Total Marks = 36

1. Complete the following chart by matching the given values for slopes and intercepts to the lines in the graph below. Some values may be used more than once. Some values may not be used at all. *(21 marks)*



Line	m	x-intercept	y-intercept
l_1			
l_2			
l_3			
l_4			
l_5			
l_6			
l_7			

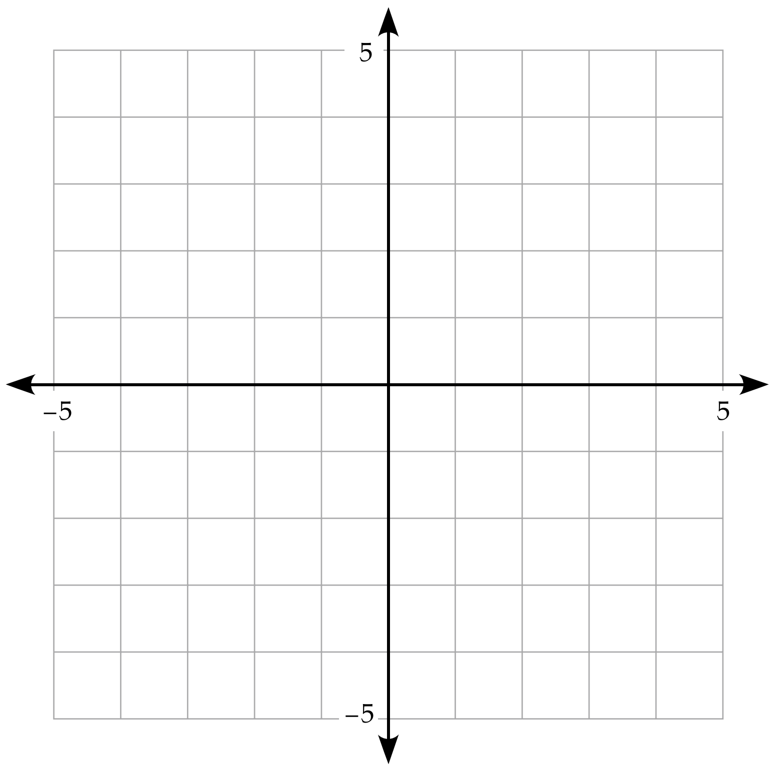
Slopes: $\frac{-3}{2}$, $\frac{-2}{3}$, $\frac{-1}{2}$, 0, undefined, $\frac{2}{3}$, 2, $\frac{3}{2}$

x-intercepts: −6, −4, 0, does not exist, 2, 4, 5, 6

y-intercepts: −6, −5, −4, does not exist, 0, 4, 6, 9

Assignment 1.4: What We Can Tell from Slope (continued)

2. Use the grid below to answer the questions that follow.



a) Draw a line through (−4, −3) with a slope of $\frac{1}{3}$. *(2 marks)*

b) State the coordinates of two other points on this line. *(2 marks)*

____

c) What is the slope between these two new points? Explain how you know this without calculating the slope using the slope formula. *(2 marks)*

____

____

d) Draw a line parallel to the line on the graph. State the y-intercept and how you know the lines are parallel. *(3 marks)*

____

____

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Assignment 1.4: What We Can Tell from Slope (continued)

3. Explain in your own words what the slope of a vertical line means. *(3 marks)*

4. Explain in your own words how you can use the slope formula to determine if two lines are perpendicular. *(3 marks)*

Notes

Lesson 5: The Equation of a Linear Relation

Lesson Focus

In this lesson, you will

- ❑ describe and represent linear relations using an equation
- ❑ determine and explain if an equation represents a linear relation
- ❑ match corresponding representations of linear relations
- ❑ solve problems involving slope and y-intercepts

Lesson Introduction

You have described linear relations using words, ordered pairs, a table of values, and graphs. One more important representation remains—using an equation to describe a linear relation. In this lesson, you will discover what a linear equation consists of and how to determine if an equation represents a linear relation. You will solve contextual problems involving slope and intercepts, and match corresponding representations of linear relations.

Slope in an Equation

The Slope of a Linear Equation

Use words to describe the linear relationship between the x-coordinate and the y-coordinate in the following three examples. Translate this into an equation. Then, determine the slope and the y-intercept for each representation. You may also sketch the graph. Remember that these are examples to show you how to answer this type of question. If you are unsure of how to write the equation of a line, try to reason out how we got it in the solution. Don't forget that if you are stuck, you can call your tutor/marker or ask your learning partner for help.

Example 1

x	y
0	0
2	4
3	6
5	10
9	18

Solution:

Some possible descriptions may be:

The y-value is exactly twice as large as the x-value.
Two times the value of x gives you the value of y.
The y-value is double the x-value.

The equation for this line may be written as $y = 2x$.

The slope of this line can be calculated using any two points. Label them if necessary.

(2, 4) and (3, 6)

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_2}{x_2 - x_1}$$

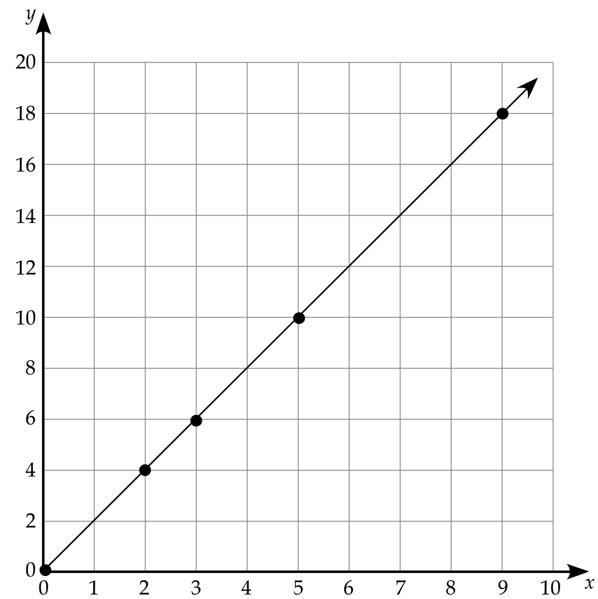
$$m = \frac{6-4}{3-2}$$

$$m = \frac{2}{1}$$

$$m = 2$$

The y-intercept is at the point where $x = 0$. From the table of values, you can determine that the y-intercept is at $y = 0$ or at (0, 0). Remember, this point is also called the origin.

A sketch of this graph may look like this:



The table of values had x-values from 0 to 9. This graph has 10 increments (points along the x-axis), so the scale on this graph starts at 0 and goes up by 1s to 10.

To find this y-scale, find the range of y-values by subtracting the smallest value from the largest and dividing by the number of increments available. Round up to make it easier to graph.

$18 - 0 = 18$

$18 \div 10 = 1.8$

1.8 rounded up to the next whole number is 2.

Use increments of 2 along the y-axis.

Example 2

(0, 0), (3, −9), (5, −15), (8, −24), (10, −30)

Solution:

Some possible descriptions are:

- In each ordered pair, the y-value is negative three times larger than the x-value.
- If you multiply the x-coordinate by −3, you get the value of the y-coordinate.
- The y-value is the opposite of triple the x-value.
- As an equation, this can be written as $y = -3x$.

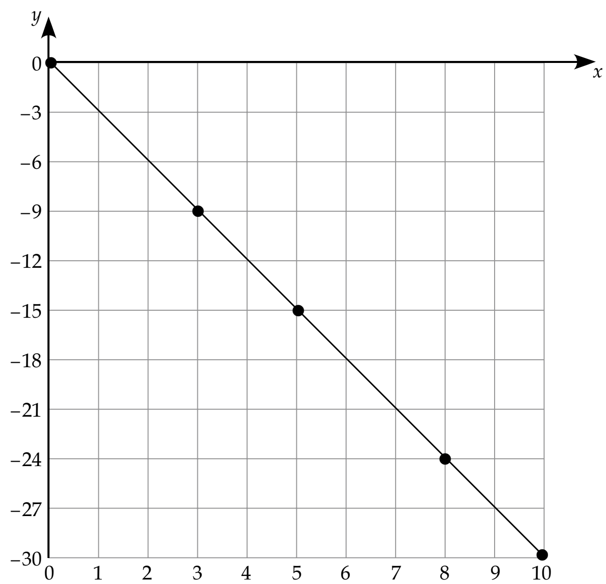
- The slope can be calculated by finding the differences between the y-values and the x-values, and writing them as a ratio of $\frac{\Delta y}{\Delta x}$. The Δ symbol is called *delta* and means change or difference.

$$\begin{array}{r} (5, -15) \\ -(3, -9) \\ \hline 2 \quad -6 \end{array}$$

Organize these values as $\frac{-6}{2} = -3$.

The slope of this linear line is –3.

- The y-intercept is at the point where $x = 0$. From the ordered pairs, you can determine that when $x = 0$, $y = 0$, so the y-intercept is at $y = 0$ or at the origin (0, 0).



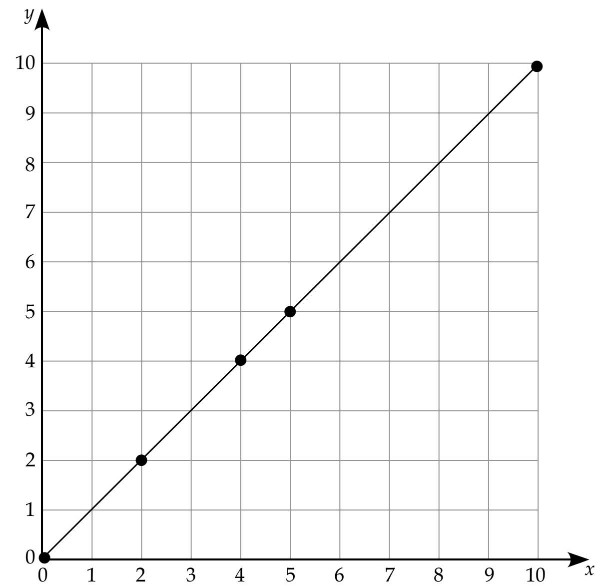
- Notice that the x-axis is along the top of the graph.
- To determine the y-scale, note that the y-values range from −30 to 0.

$0 - (-30) = 30$

$30 \div 10 = 3$

Use increments of 3 along the y-axis, starting at −30.

Example 3



Solution:

You may have noted many things. Here are some possible observations:

- The y-coordinate of each point is the same as the x-coordinate.
- The x-value and the y-value are the same.
- The y-value equals the x-value.
- If you multiply the y-value by 1, it equals the x-value.
- The equation for this line can be written as $y = x$ or $y = 1x$.
- The $\frac{\text{rise}}{\text{run}}$ in this line is $\frac{1}{1} = 1$ because the y-value increases by 1 for each x-value increase of 1.

Similarities and Differences

What are some of the similarities and differences you noticed in the above three examples?

You may have noticed that the y-intercept in each was at the origin, (0, 0).

You may have noticed that two of the linear relations had positive slopes and one had a negative slope.

You may also have noticed that the equations for each of these examples were similar. In each case, you multiplied x and a coefficient to get the corresponding value of y. (The coefficient is the constant value that multiples the variable. In $5x$, the 5 is the coefficient and the x is the variable.)

Did you notice that the slope and the coefficient of x were the same value? In each of these coordinate pairs, the y-value is equal to the slope multiplied by the x-value.

The y-Intercept of a Linear Equation

Use words to describe the linear relationship between the x-coordinate and the y-coordinate in the following three examples. Write this as an equation. Then, determine the slope and the y-intercept for each representation.

Example 4

x	0	2	3	5	9
y	3	5	6	8	12

Solution:

Some possible answers are:

- Each y-value is 3 more than the corresponding x-value.
- If you add 3 to each x-value, you have the y-value.
- Each y-value is 3 greater than the x-value.
- The equation for this linear relation is $y = x + 3$.
- The slope of this line can be calculated using any two points and the slope formula. Choose points (0, 3) and (5, 8).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

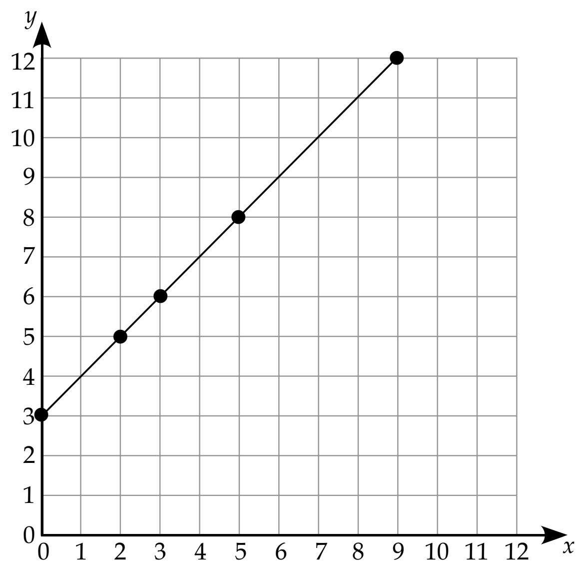
$$m = \frac{8-3}{5-0}$$

$$m = \frac{5}{5}$$

$$m = 1$$

- The slope of this line is 1.
- The y-intercept is at the point where $x = 0$. The point (0, 3) is in the table of values provided, so the y-intercept is at $y = 3$.

- This equation may be graphed as shown below.



Example 5

(0, –5), (3, –2), (5, 0), (10, 5)

Solution:

Some possible answers are:

- Each y-value is 5 less than the corresponding x-value.
- If you subtract 3 from each x-value, you have the y-value.
- The equation for this linear relation is $y = x - 5$.
- The slope of this line can be calculated using any two points and the slope formula. Choose points (0, −5) and (3, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

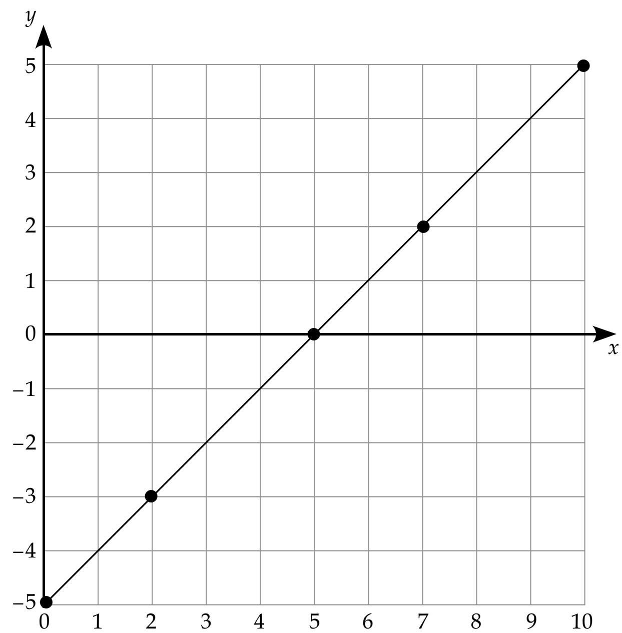
$$m = \frac{-2-(-5)}{3-0}$$

$$m = \frac{3}{3} = 1$$

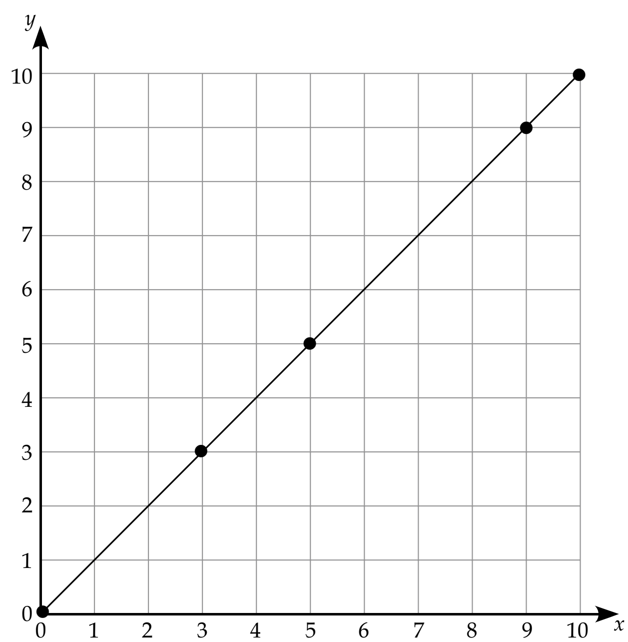
$$m = 1$$

- The slope of this line is 1.
- The y-intercept is at the point where $x = 0$. The point (0, −5) is in the table of values provided, so the y-intercept is at $y = -5$.

■ This equation may be graphed as follows. Notice the y-scale goes from −5 to +5 with the x-axis in the centre of the graph.



Example 6



Solution:

Some possible answers are:

- Some of the points that this line goes through are (0, 0), (2, 2), (9, 9). The x- and y-values in the ordered pairs are the same.
- As an equation, this can be written as $y = x$ or $y = x + 0$.
- The slope of this line is equal to 1.
- The y-intercept of this line is at $y = 0$.

Similarities and Differences

What are some of the similarities and differences you noticed in the above three examples?

You may have noticed that the slope of each of these lines is equal to +1.

You may have noticed that each graph had a different y-intercept. One had a negative y-intercept, one had a positive value, and one was at $y = 0$.

You may also have noticed that the equations for each of these examples were similar. In each case, you found y by combining x with a constant. (To combine means to add a positive or negative number. A constant is a number that does not have a variable. In $4x + 6$, the 6 is a constant. The 4 is a coefficient. This expression means: combine $4x$ and positive 6.)

Did you notice that the constant number was the same as the y-intercept in each case? In each of these coordinate pairs, the y-value is equal to the x-value combined with the y-intercept.

If you combine your observations from these examples, you can come up with a descriptive linear equation like: y = (slope)x + (y-intercept).

Learning Activity 1.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Are the possible values of the dependent variable the domain or the range of the relation?
2. Given the slope, $\frac{6}{11}$, state the rise and the run.
3. What are the factors of 8?
4. What are the first 3 multiples of 7?
5. Write 4^3 in expanded form.
6. Evaluate $\sqrt{25}$.
7. Evaluate 3^2.
8. The distance from the MTS Centre to Polo Park is 5.1 km. On your way to Polo Park from MTS Centre, you decide to stop at a doughnut shop halfway between the two. How far do you have to travel from the doughnut shop to get to Polo Park?

Part B: Graphing Data Points from an Equation

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

This learning activity presents an opportunity for you to assess your own learning. Questions 1–3 are similar, so we recommend that you do Questions 1 and 2. If you want more practice, do Question 3. If you feel that this is enough practice, move on to Question 4.

1. a) Given the equation $y = -2x + 8$, create a table of values with five ordered pairs. Choose any 5 reasonable x-values (start with values between -10 and 10), substitute them into the equation and solve for y. Record the values in a chart.

continued

b) Graph the line using grid paper. Remember to choose an appropriate y-scale for your graph—one that will include all the y-values calculated. To determine what increment to use along the y-axis, calculate the range (subtract smallest y-value from the largest y-value), divide by the number of increments along the y-axis and round up.

c) Calculate the slope of the line.

d) Determine the y-intercept of the line.

e) Verify your answers to (c) and (d) using the equation provided in (a).

2. a) Given the equation $y = \frac{2}{3}x + 3$, create a table of values with 5 ordered pairs. If you have a fraction coefficient, choose x values that are multiples of the denominator so that you can reduce the fraction and eliminate the denominator (start with values like 3, 6…), substitute them into the equation and solve for y. Record the values in a chart.

   b) Graph the line on grid paper.

   c) Calculate the slope of the line.

   d) Determine the y-intercept of the line.

   e) Verify your answers to (c) and (d) using the equation provided in (a).

3. a) Given the equation $y = -5x + 48$, create a table of values with 5 ordered pairs.

   b) Graph the line on grid paper.

   c) Calculate the slope of the line.

   d) Determine the y-intercept of the line.

   e) Verify your answers to (c) and (d) using the equation provided in (a).

4. a) Write the equation of a line with a y-intercept of 2 and a slope of $\frac{4}{5}$.

   b) Sketch the graph. ***Do not*** create a table of values or ordered pairs, except to check your work.

5. a) Given the equation $y = -\frac{4}{5}x + 9$, state the slope and y-intercept without graphing the line or creating a table of values.

   b) Explain how you would know where to draw the line.

The Equation of a Linear Relation

The equation of a linear relation provides information about how x and y values are related in a given context. Using the equation, you can substitute values for x and solve for y and determine (x, y) coordinate pairs. The equation can also tell you at a glance what the slope and y-intercept of the line are. In order to do this, it must be written as $y = mx + b$ where the coefficient m represents the slope and the constant b represents the y-intercept. The x and y represent ordered pairs.

The linear equation when written as $y = mx + b$ is called the slope-y-intercept form of the equation.

Determine if the following equations represent linear relations. Explain why or why not. State the slope and intercept if it is a linear equation.

Equation	Linear or Non-Linear? If not, why?	If so, slope and intercept?
$y = x^2 + 3$	This is not a linear equation because the variable x is squared.	
$y = x$	This is a linear equation.	$m = 1, b = 0$
$y = -4x - 1$	This is a linear equation.	$m = -4, b = -1$
$x = 4y - 80$	This is a linear equation. It can be rearranged to isolate y and fit the form $y = mx + b$. $y = mx + b$ $x = 4y - 80$ $x + 80 = 4y$ $\frac{x+80}{4} = \frac{4y}{4}$ $\frac{1}{4}x + 20 = y$ $y = \frac{1}{4}x + 20$	$m = \frac{1}{4}, b = 20$ This line is perpendicular to the previous example.
$y = x + z - 3$	This is not a linear equation because it has an additional variable.	
$x - y = 3$	This is a linear equation that can be rearranged to fit the slope-y-intercept form of the line. $x - y = 3$ $x - 3 = y$ $y = x - 3$	$m = 1, b = -3$ This line is parallel to the second equation $y = x$.

Example 7

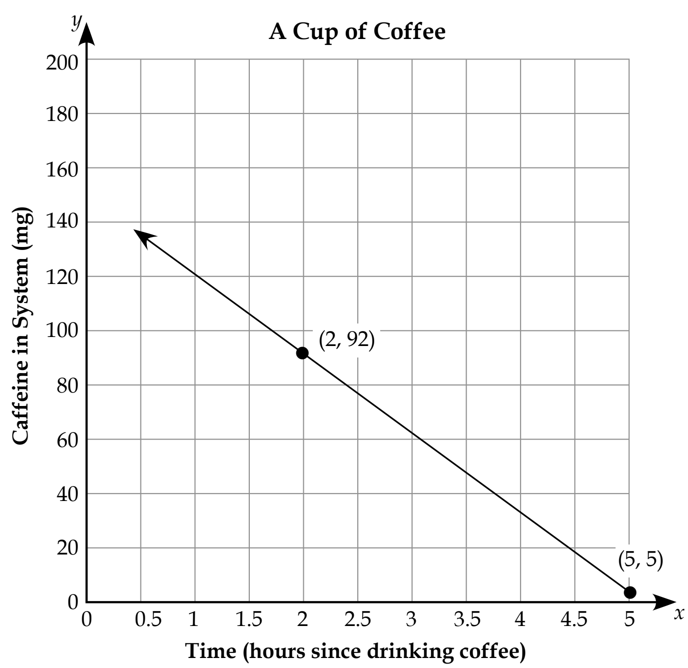
Coffee, tea, and some soft drinks contain caffeine, a stimulant that makes you feel less tired and more alert. The effects of caffeine may be felt as quickly as 15 minutes after consuming it, and may last up to five or six hours. A cup of coffee (250 mL) has about 200 mg of caffeine. The liver is responsible for metabolizing the caffeine in your body. Two hours after having a cup of coffee, there may be about 92 mg of caffeine remaining in your system, while after five hours there may only be approximately 5 mg left.

Plot these points on the grid below and connect them with a linear line. Determine the slope and y-intercept of the line and write an equation to approximately represent this situation. Explain what the slope and y-intercept mean in terms of time and amount of caffeine in your system.

Solution:

The independent variable is time.

time	caffeine left
2 hours	92 mg
5 hours	5 mg



Using the points (2, 92) and (5, 5), calculate the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 92}{5 - 2}$$

$$m = \frac{-87}{3} = -29$$

The slope of this line is −29.

The y-intercept of the line appears to be approximately where $y = 150$. The coordinates of this point can be calculated by substituting the slope and a known point into the equation of the line.

$y = mx + b$ Use the (x, y) coordinate point (2, 92) and the slope, $m = -29$, to solve for b, the y-intercept.

$92 = (-29)(2) + b$

$92 = -58 + b$

$92 + 58 = -58 + 58 + b$

$150 = b$

The y-intercept of this line is at (0, 150).

An equation that represents this situation is $y = -29x + 150$.

In terms of the variables, the y-intercept means that at 0 hours after drinking a cup of coffee, there is about 150 mg of caffeine in your system. The cup of coffee has about 200 mg of caffeine, but several factors may be at work here—you may not be completely finished drinking your cup of coffee at zero hours, or not all of the caffeine may have been absorbed immediately. The slope of –29 means that each hour, your body metabolizes about 29 mg of caffeine.

Lesson Summary

In this lesson, you learned how to represent linear relations using an equation. You incorporated what you learned about calculating the slope and intercepts of a line into the slope-y-intercept form of a linear equation. You can now recognize a linear equation and solve problems about slope and intercepts using it. This course will make extensive use of the linear equation in different modules.

Assignment 1.5

Slope-y-Intercept Equation

Total Marks = 36

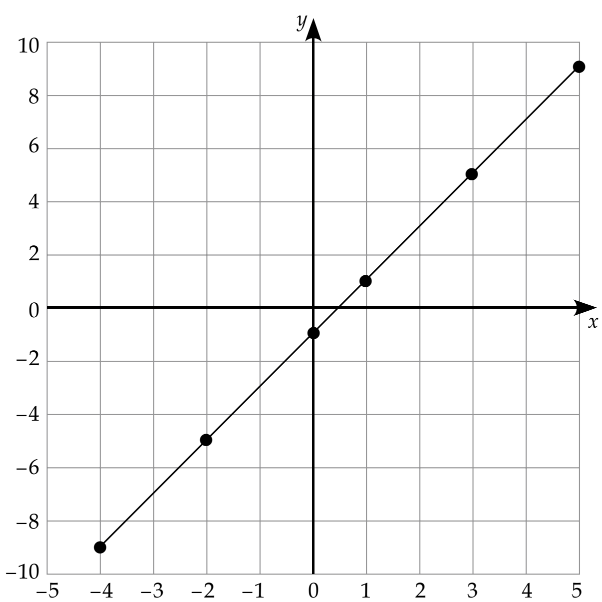
Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

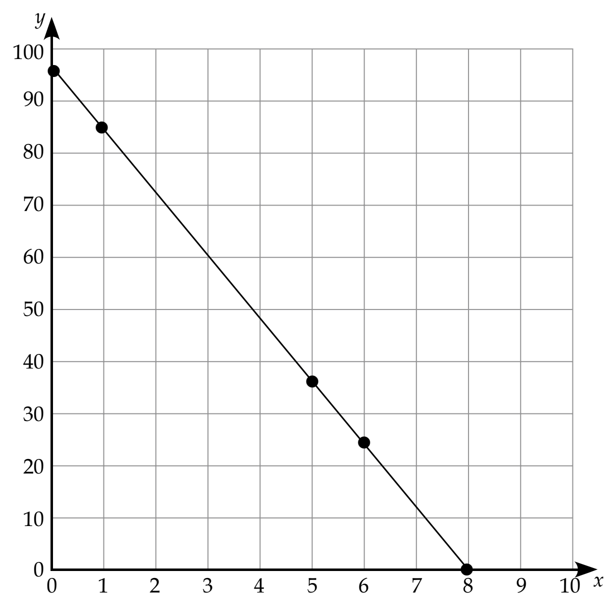
1. a) Match the following data with the correct graph. *(2 marks)*

   i) (5, 36), (8, 0), (6, 24), (0, 96), (1, 84)

   ii)

x	y
–4	–9
–2	–5
0	–1
1	1
3	5
5	9





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b) Write a linear equation in slope-y-intercept form for each graph. Show your calculations. *(6 marks)*

2. Write the slope-y-intercept form of an equation for a line with a slope of $\frac{-1}{41}$ and a y-intercept at 336. *(2 marks)*

3. Explain how you would know where to draw the line $y = \frac{5}{3}x - 9$ without creating a table of values. *(2 marks)*

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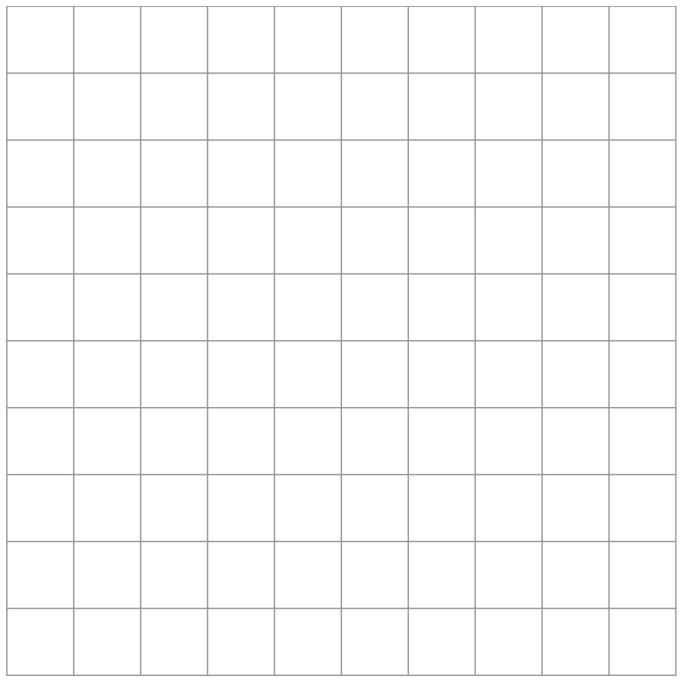
Assignment 1.4: Slope-y-Intercept Equation (continued)

4. At the school cafeteria, you have to wait for five minutes to place your order if there are three people ahead of you in line, and eight minutes if there are five people ahead of you.

   a) Plot these data on a labelled graph, and join the points with a linear line. A grid is provided below or you may create your own. *(5 marks)*



   b) Use your line to determine the slope and y-intercept, and write an equation to describe this situation. *(5 marks)*

   c) Explain what the slope and y-intercept mean in terms of the time and number of people in this scenario. *(4 marks)*

Assignment 1.4: Slope-y-Intercept Equation (continued)

5. Learning Activity 1.2 in Lesson 1 required that you create a word web based on your prior knowledge and experience with graphs. Now that you have completed this module, it's time to review how much you have learned! Find your word web and review it. Now, using what you have learned in this module, modify it in another colour to indicate new learning or create another web reflecting your new knowledge and understanding. As well, in a paragraph, discuss how these two webs are different and why. Discuss how creating these webs may have helped you organize your ideas. Review your learning, or discuss the process you went through to make the webs. Your comments should let your tutor/marker see how and what you are thinking about your learning in this module. You are required to submit both word webs OR your modified original along with the written reflection when you send in your hand-in assignments for marking. *(10 marks)*

Module 1 Summary

Congratulations, you have finished the first of the eight modules in the course.

In this module, you discovered and used concepts that are foundational to pre-calculus and applied mathematics topics. You used graphs, ordered pairs, tables of values, equations, and words to describe the relationships between variables, and the slope, intercepts, and domain and range of relations, specifically linear relations. These skills and concepts will be used throughout this course to develop your mathematical reasoning, problem solving, and communication, while making connections to real-world situations and other math topics.

In the next module, you will explore numbers, including factors, roots, irrational numbers, and powers.

Submitting Your Assignments

It is now time for you to submit Assignments 1.1 to 1.5 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 1 assignments and organize your material in the following order:

- ❑ Module 1 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 1.1: Graphing Independent and Dependent Variables
- ❑ Assignment 1.2: Domain and Range
- ❑ Assignment 1.3: Slopes, Intercepts, Domain, and Range
- ❑ Assignment 1.4: What We Can Tell from Slope
- ❑ Assignment 1.5: Slope-y-Intercept Equation

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 1
Graphs and Relations

Learning Activity Answer Keys

Module 1: Graphs and Relations

Learning Activity 1.1

There is no answer key for this learning activity.

Learning Activity 1.2

Part A: BrainPower

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper. The first few times you do these questions, your learning partner can help you figure out strategies to solve them.

1. There are 22 yard markers on a Canadian football field. Each marker represents five yards. How long is a Canadian football field?
2. If Evan eats $\frac{3}{5}$ of a pizza and Nick eats $\frac{4}{5}$ of a pizza, how many pizzas do they have to order so that both can eat as much as they like?
3. Simplify the following fraction to lowest terms: $\frac{18}{27}$.
4. You are working at the stadium where they don't have an electronic till. The customer is buying popcorn for $3.80. If the customer gives you a $5.00 bill, how much change will you give them?
5. Rank the numbers from highest to lowest: 0.5, 0.05, 0.3, 0.09, and 0.25.
6. Solve for m: $2 - m = 14$.
7. The distance to the mall from your house is 8 km. Your friend lives half as far away from the mall. What is the distance from your friend's house to the mall?
8. Write the percent as a decimal: 62%.

Answers:

1. 110 yards (22 × 5)
2. 2 pizzas $\left(\frac{3}{5}+\frac{4}{5}=\frac{7}{5}=1\frac{2}{5}\text{, so you need }1\frac{2}{5}\text{ pizzas.}\right)$

   You cannot order part of a pizza, so the next whole number is 2.)
3. $\frac{2}{3}$
4. $1.20
5. 0.5, 0.3, 0.25, 0.09, 0.05
6. $m = -12$ ($-m = 14 - 2$)
7. 4 km
8. 0.62

Part B: Word Web

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

Create a word web showing what you know about graphs. Use bubbles to indicate new ideas or characteristics, and lines to show how they are connected.

Answer:

Each word web is unique, but your web about graphs includes some of the following: the purpose, parts, definition, and types of graphs, how they are made, where they are used, the kinds of data they display, and so on. You will come back to this word web at the end of this module.

Learning Activity 1.3

Part A: BrainPower

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper. The first few times you do these questions, your learning partner can help you figure out strategies to solve them.

1. What is the range of the following numbers: 2, 6, 4, 8, 7, 13, 11?
2. You are going to the store to buy a drink with \$2.05 in your pocket. If a drink costs \$1.75, will you be able to buy one?
3. Simplify the fraction $\frac{6}{2}$.
4. Write the ratio as a fraction: 5:2.
5. Solve for a: $9 + a = 13$.
6. Write the next two numbers in the pattern: 1, 2, 4, 8, ____, ____.
7. You want to bring freezies to your last soccer game of the season. You want to have enough so that each player gets two. If you have 18 people on your team, how many freezies do you need?
8. You are helping your dad build a rectangular deck. If it is 2 m long and 3 m wide, what is the area that it takes up in your yard?

Answers:

1. 11 (13 − 2)
2. Yes. (You will get \$2.05 − \$1.75 = \$0.30 in change back.)
3. $\frac{3}{1} = 3$
4. $\frac{5}{2}$
5. $a = 4$
6. 16, 32
7. 36 (2×18)
8. 6 m^2 (2 m $\times$ 3 m)

Part B: Independent vs. Dependent Variables and Continuous Data

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. State which variable is independent and which variable is dependent in each of the following contexts:
   a) Hours worked in a week with pay of $20 per hour
   b) Final exam mark and average quiz marks for a Grade 10 Math class
   c) Coffee temperature and the time since the cup was poured
   d) Average monthly temperature in Manitoba during the months from January to December

   Answers:

	Independent	Dependent
a)	hours	pay
b)	quiz mark	exam mark
c)	time	temperature
d)	month	temperature

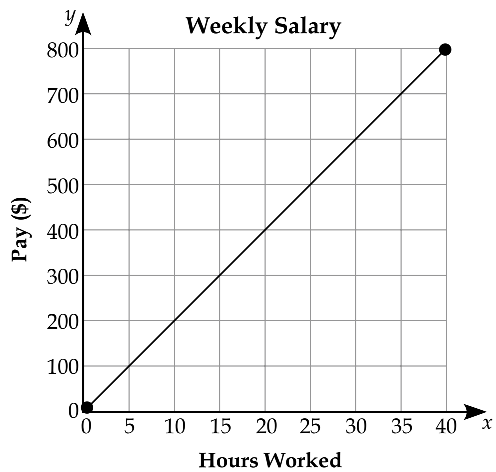
2. Are the situations in Question 1 continuous? Explain.

   Answers:
   a) Continuous—you can get paid in fractions of dollars for partial hours
   b) No—individual quiz and exam marks
   c) Continuous—time and temperature can be meaured in infinite increments
   d) No—there can only be 12 average monthly temperatures in a year

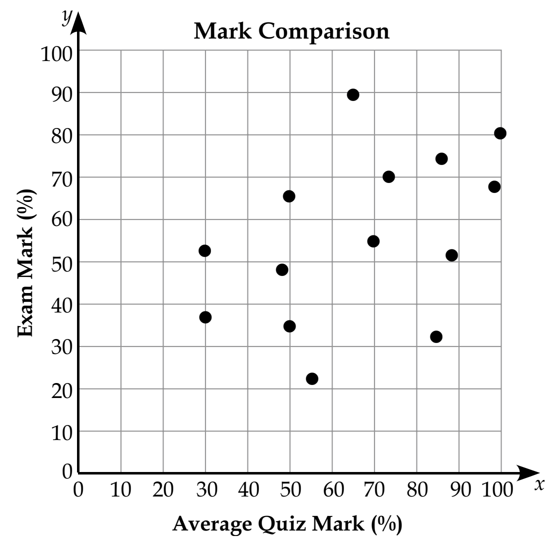
3. Sketch a possible graph based on the contexts given in Question 1.

Answers:

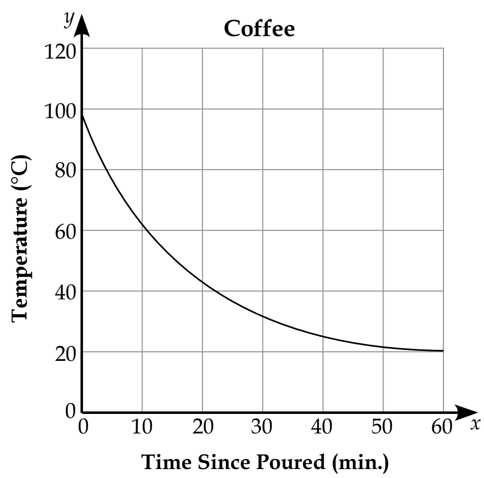
a)



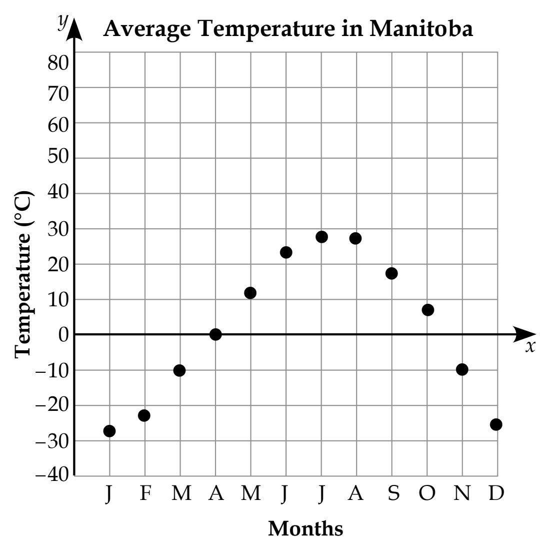
b)



c)



d)

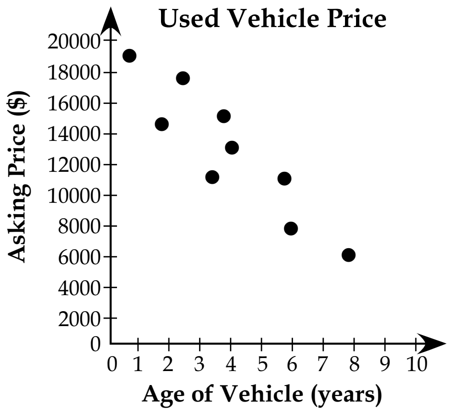


4. Create a possible context that would result in the following graphs. Label each graph with independent and dependent variables, units, appropriate scales (values along the axes), and a title.

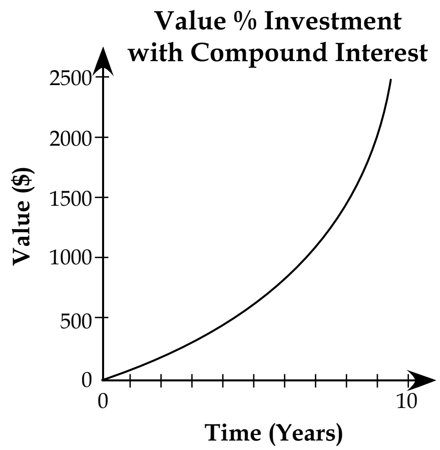
Answers:

Answers may vary. Possible answers include:

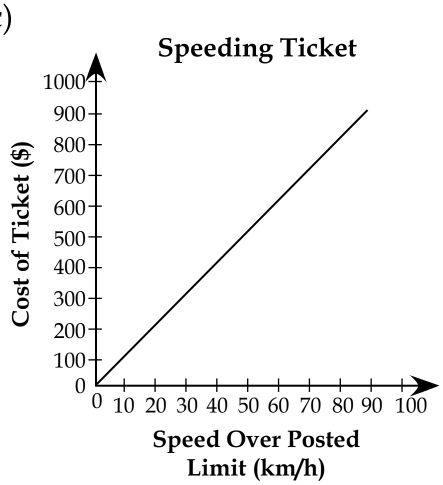
a)



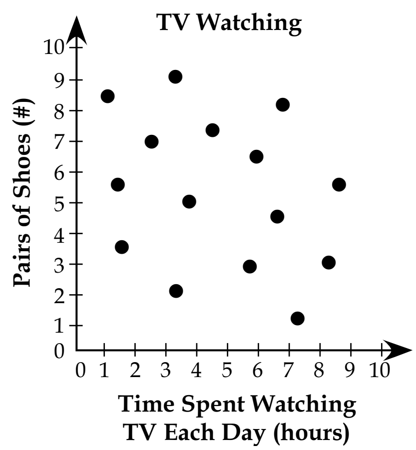
b)



c)



d)



a) The listed asking price for used vehicles in classified ads. Price is dependent on the age of the vehicle.

b) The value of an investment with compound interest. The value depends on how long it has been invested.

c) The cost of a speeding ticket depends on how many km/h you were driving over the posted speed limit.

d) There does not appear to be any relationship between these two variables.

5. Construct a good graph of the following data. It may be done by hand on graph paper or with technology.

   A random sample of 11 people was drawn from the population of people between the ages of 30 and 40 who were employed full time in Brandon. The number of years of their schooling and annual income in thousands of dollars was recorded for each of the 11 people. The data are given below:

Schooling (years)	10	7	12	11	16	12	18	8	12	14	16
Income ($1000)	32	20	45	43	65	42	75	28	40	60	65

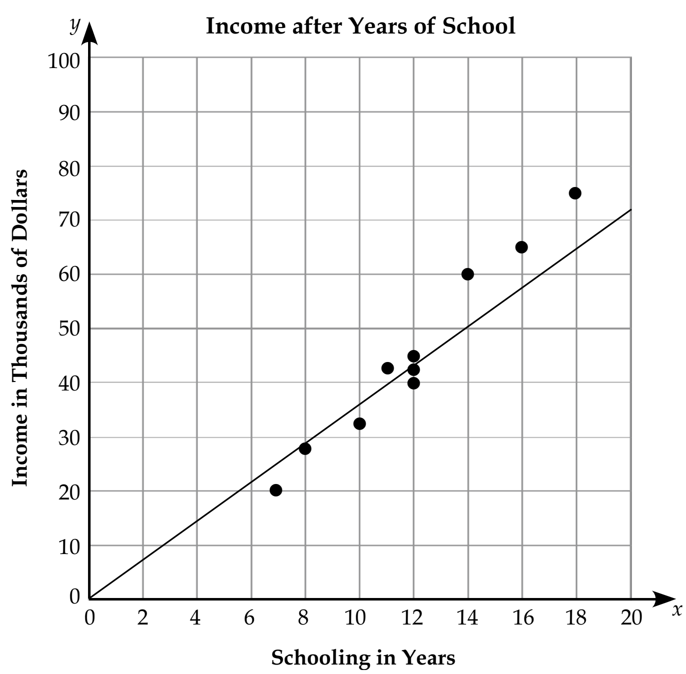
   a) Which variable, schooling or income, is the independent variable? Which is the dependent variable?

      Answer:

      Schooling is the independent variable and Income is the dependent variable.

   b) Graph the data with appropriate scales on the grid below and draw the line of best fit.

      Answer:



Note: Your line of best fit may be slightly different.

Check: Are there approximately the same number of points above the line as below the line?

   c) Is the data continuous?

      Answer:

      The data is not continuous because usually you attend complete, not partial, years of school.

Learning Activity 1.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Your mother has invited your entire family over for dinner. As usual, you are complaining because only $\frac{1}{6}$ of them are around your age. If your family has 24 people in it (not including you), how many of them are approximately your age?
2. There are 12 eggs in a dozen. If you are buying four dozen eggs, how many eggs is that?
3. Write the following decimal as a fraction: 0.058.
4. You are craving 5¢ candy. You have $1.43 in your pocket. How much candy can you afford?
5. List the factors of 12.
6. Fill in the missing terms in the following pattern: 0, 3, ____, 9, 12, ____.
7. Circle the independent variable: the height of an airplane compared to the time it takes to land.
8. At the end of the day, a restaurant is left with three different, partially eaten pies. Each pie has $\frac{2}{7}$ left over. How much pie is left over in total?

Answers:

1. Four people are around your age $\left(24 \times \frac{1}{6}\right)$
2. 48 eggs (12 × 4)
3. $\frac{58}{1000} = \frac{29}{500}$ (either answer is acceptable)
4. 28 (There are 20 nickels in a dollar, and 40 divided by 5 is 8. 20 + 8 = 28.)
5. 1, 2, 3, 4, 6, 12
6. 6, 15
7. The height of the airplane is the independent variable.
8. $\frac{6}{7}\left(3 \times \frac{2}{7}\right)$ of a pie is left, if all the leftover pie was put together

Part B: Ordered Pairs, Data Tables, and Relationships

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the data given in the table of values in Example 1 to complete the following:

   a) Write the data as ordered pairs.

      Answer:

      (0, 65), (30, 45), (60, 25), (90, 5)

   b) Write a sentence describing the relationship between the two variables (e.g., As time does 'this', the remaining fuel does 'that').

      Answer:

      As the time progresses, the amount of fuel remaining decreases in a linear relation.

2. Refer back to the graphs provided or the ones you created in Lesson 2. Which graphs indicate linear relations?

   Answer:

   The following graphs from Lesson 1 indicate linear relations:

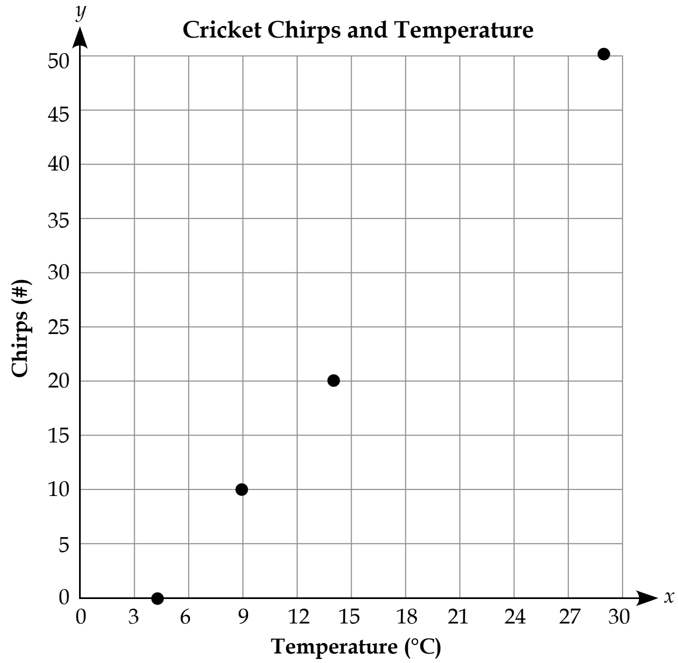
   Example 1, Example 2, and the storage capacity and cost of handheld devices.

   Learning Activity 1.3 #4 graphs (a) and (c), and Learning Activity 1.3 #5.

   Assignment 1.1 #1 graphs (a) and (c), and Assignment 1.1 #2 Ball bounce

   Note: Linear relations can be found in both continuous data as well as data that are not continuous.

3. In the following scatterplot, a biologist has displayed data relating the temperature to the number of times a cricket chirps.



a) Write the coordinates as ordered pairs.

Answer:

(4, 0), (9, 10), (14, 20), (29, 50)

b) Create a table of values from the data graphed in this scatterplot.

Answer:

Temperature (°C)	**Cricket Chirps (#)**
4	0
9	10
14	20
29	50

c) Write a sentence describing the relationship between these two variables.

Answer:

As the temperature increases, the number of times a cricket chirps also increases.

d) Does this graph display a linear relation? Explain.

Answer:

Yes, the relation between temperature and number of chirps appears linear because a straight line could be drawn on the graph and it would go through or near all the data points

Learning Activity 1.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Simplify the following fraction: $\frac{21}{35}$.
2. Simplify the following fraction: $\frac{12}{20}$.
3. 50% of 680 is 340. 25% of 680 is ____.
4. There is a 50% discount on all candy at the store the day after Hallowe'en. If it cost you $30 to buy candy before Hallowe'en, how much would you spend if you bought the same candy after Hallowe'en?
5. Solve for r: $5 + r = -4$.
6. You are at a doughnut shop and would like to buy doughnuts for your family. There are six people in your family (including you—and you want a doughnut). How much money do you need, if each doughnut costs 60¢?
7. A regular goal in rugby is worth three points. If a team finishes a game with 39 points, how many goals did they score?
8. If it rains 10 mm, how much rain is that in centimetres?

Answers:

1. $\frac{3}{5}$ (divide top and bottom by 7)
2. $\frac{3}{5}$ (divide top and bottom by 4)
3. 170 (25% is half of 50% so 25% of 680 must be half of 340)
4. $15.00 (50% of $30 is the same as half of $30)
5. $r = -9$ ($r = -4 - 5$)
6. $6 \times 60¢ = 360¢ = \$3.60$ (in either cents or dollars is acceptable)
7. 13 ($39 \div 3$)
8. 10 mm = 1 cm

Part B: Domain and Range

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You want to rent a banquet room in a restaurant and have dinner with friends to celebrate your birthday. The cost of the meal is \$6 per person.

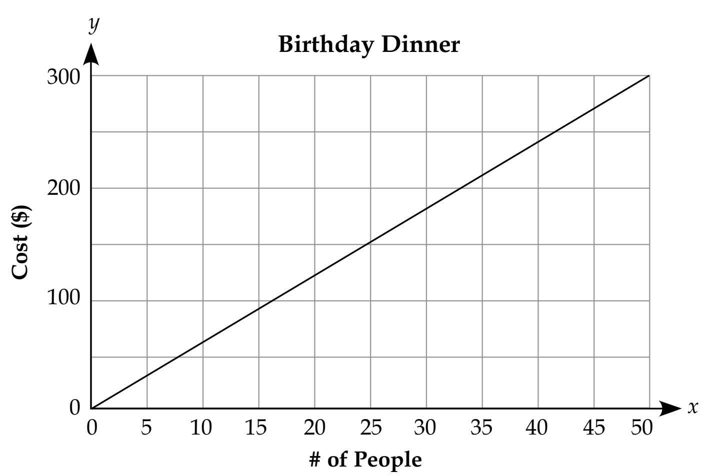
   a) If you were to graph the relation between the cost and the number of people attending, what would be a reasonable domain and range? Explain the restrictions you consider.

   Answer:

   The cost depends on the number of people attending, so the independent variable is the number of people. The domain represents valid x-values of the independent variable. The number of people attending is restricted by the size of the banquet room, and if there is a minimum number of people required to rent the room. The domain could be from 5 to 50 people. Based on the restrictions on the domain, the range, or possible costs, could be from \$30 to \$300, given that a meal is \$6 per person.

   b) Create a possible graph. You may draw it by hand or use technology and print it out.

   Answer:



c) Is this an example of a linear relation?

Answer:

Yes, this is a linear relation.

d) Write a sentence describing the relationship between the variables.

Answer:

As the number of people attending increases, the cost also increases.

Note: Your answer in part (a) and your graph may have been slightly different. That is okay as long as the domain and range you state are reasonable and the values are valid for the restrictions you describe. Your descriptive sentence in part (d) must indicate how the dependent variable changes in relation to changes in the independent variable.

Learning Activity 1.6

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Write the percent as a decimal: 3.5%.
2. Which is larger: 0.76 or 0.07?
3. What is the range of the following numbers? 0.2, 0.6, 0.08, 0.5, 0.03
4. If 3% of 500 is 15, what is 12% of 500?
5. The mat used for floor gymnastics is a square. The length along one side is 40 feet. What is the total area of the mat?
6. Complete the pattern: 9, −7, 5, ____, ____.
7. Solve for v: $9v = 45$.
8. You want to order a pizza, but you only have $15. If a pizza costs $16, but is 10% off, can you afford it? (ignore taxes)

Answers:

1. 0.035
2. 0.76 (0.76 is seventy-six hundredths; 0.07 is seven hundredths)
3. 0.57 (0.6 − 0.03)
4. 60 (3% × 4 = 12%. 15 × 4 = 60. If it helps, think about the minutes in an hour.)
5. 1600 feet (4 × 4 = 16. 40 and 40 have two zeros, so the answer must also have two zeros.)
6. –3, 1 (decreasing odd numbers with alternating signs)
7. $v = 5$ ($45 \div 9 = 5$)
8. Yes (16 × 10% = $1.60. 16 − 1.60 = $14.40 which is less than $15.00)

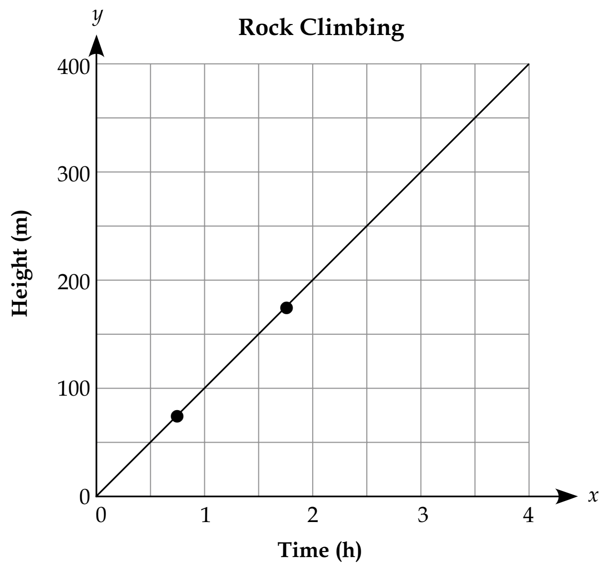
Part B: Rates and Slopes

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Kent is rock climbing up a 400 m high cliff. 45 minutes after beginning his ascent, he has climbed 75 m. After another hour, he is at 175 m.

   a) How many hours of steady climbing will it take him to reach the top? Sketch a graph to help you answer this question.

   Answer:

   From the graph, you can use extrapolation to find how long it takes to go up 400 m. It would take 4 hours.

   b) At what rate is he climbing, in m/h?

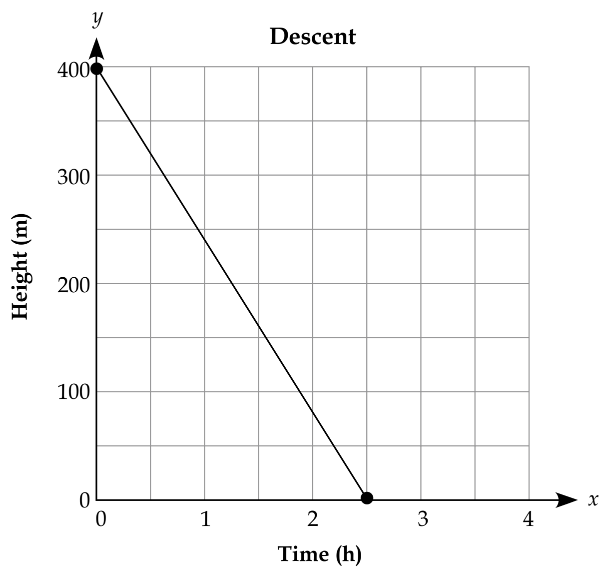
   Answer:

   If it takes him four hours to climb 400 m, he can climb 100 m in one hour. His rate, and the slope of this line, is 100 m/h.

2. The descent down the cliff only takes Kent two and a half hours to complete.

   a) Create a graph similar in scale to the previous question and sketch this situation. Plot the points that represent his time and location at the top of the cliff and when he reaches the bottom.

      Answer:



   b) Compare the slopes in these two graphs.

      Answer:

      The slope in graph #1 is positive and less steep than graph #2. The second slope is negative.

   c) What is Kent's rate in this situation?

      Answer:

      Kent climbs down 400 m in $2\frac{1}{2}$ hours, so he could descend 160 m in one hour (400 $\div$ 2.5). His rate is -160 m/h. The slope is negative.

Learning Activity 1.7

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Continuous or not: A graph comparing the number of people in a room and the volume of noise.
2. Domain or range: the independent variable has values from 0 to 20.
3. Write the following as an improper fraction: $2\frac{3}{7}$.
4. You are driving to the cabin, which is 104 km from your house. $\frac{1}{8}$ of your drive is in the city. How much of your drive is in the city (in km)?
5. You are at a bake sale. You want to buy a peanut butter cookie. If the whole plate of 16 cookies costs \$3.20, how much will you pay for one cookie?
6. Write the following decimal as a percent: 0.84.
7. A marathon is 26.4 miles. A half marathon is how long?
8. Solve for d: $3d = 9$.

Answers:

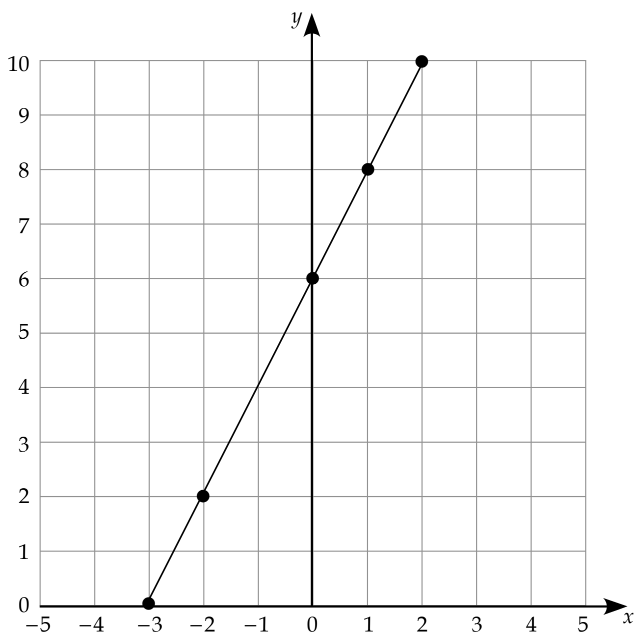
1. Not continuous. You cannot have part of a person.
2. Domain
3. $\frac{17}{7}\left(2\frac{3}{7}=2+\frac{3}{7}=\frac{14}{7}+\frac{3}{7}=\frac{17}{7}\right)$
4. 13 km $\left(104\times\frac{1}{8}\text{ or }\frac{1}{8}\text{ is }\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$

   (So divide by 2 three times: $104 \div 2 = 52$; $52 \div 2 = 26$; $26 \div 2 = 13$)
5. \$0.20 ($32 \div 16 = 2$ so $3.2 \div 16 = \$0.20$)
6. 84%
7. 13.2 miles (26.4 2)
8. $d = 3$ ($9 \div 3$)

Part B: Linear Relations

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the following theoretical graph, indicate:
   a) if the slope is positive or negative
   b) what the rate of change, or slope, is equal to
   c) the value of the x-intercept
   d) the coordinates of the y-intercept
   e) the domain
   f) the range



Answers:

a) The line has a positive slope.

b) If you examine the graph you can see that the y-coordinate of the line increases by 2 for each increase of 1 in the x-coordinate. $\frac{2}{1} = 2,$ so the rate of change is 2. Using the coordinate pairs (–2, 2) and (1, 8), the change in the y-values is 6 and the change in the x-values is 3. $\frac{6}{3} = 2$

c) The value of the x-intercept is -3.

d) The coordinates of the y-intercept is (0, 6).

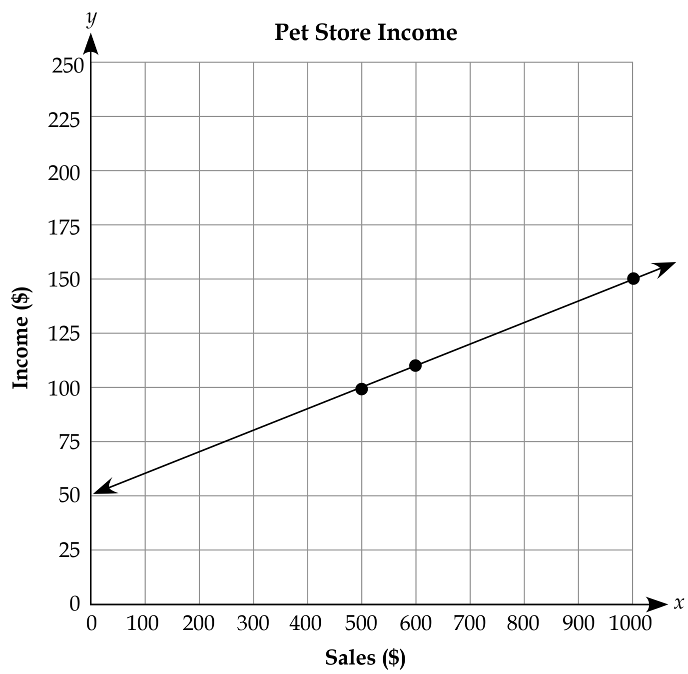
e) The domain of this graph is from -3 to 2.

f) The range is from 0 to 10.

2. From the first part of the lesson, when you realized how much more income your friend had with her part-time job, you found a new position at a pet store. Here, your income is based in part on commission. You earn a percentage of the value of the product you sell. Based on your first three weeks of income, knowing that income depends on sales, you determine the following (sales, income) pairs: (500, 100), (600, 110), and (1000, 150).

   a) Graph these ordered pairs and determine if the relation is linear.

   Answer:



   Yes, the relation is linear.

b) What is the rate of change or slope of the line?

Answer:

The slope is positive. Using the ordered pairs (500, 100), (600, 110), you can see that your income increases by \$10 for an additional sale of \$100. $\frac{10}{100} = 0.10 \text{ or } 10\%$. This represents a commission of 10%. Written as a decimal, the slope is 0.10.

c) Draw a line to determine the intercepts. What do the intercepts represent?

Answer:

There is no x-intercept in this graph. The y-intercept is at $y = 50$ or (0, 50). This represents sales of \$0 and an income of \$50. Your income is based on a commission of 10% as well as a base weekly salary of \$50.

d) What are the domain and range in this situation?

Answer:

Valid x-values in the domain of this relation are from 0 to 1000. The values will never be negative, and if you have a really great week of sales it may be higher, but this seems reasonable for this part-time job situation. Valid y-values range from 50 to 150. You will earn \$50 even if you make no sales, and based on reasonable sales expectations your weekly income could be as high as \$150.

Learning Activity 1.8

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. You are on your way to Calgary, driving on the Trans-Canada Highway. Your speed is constant at 100 km/h. If you have been driving for 3.3 hours at this speed, how far have you driven?
2. Write the following ratio as a fraction in lowest form: 16 : 12.
3. What is 10% of 500?
4. What is 5% of 500?
5. What is 15% of 500?
6. There are three classes of Grade 10 Science taught in a school by one teacher. Each class has 22 students. If the teacher gives a test to all three classes, how many tests will she have to mark?
7. A student receives a grade of $\frac{18}{20}$ on his test. Estimate the percent value.
8. Jamie is twice as old as Dan. Dan is 3 times as old as Kim. If Kim is 4 years old, how old is Jamie?

Answers:

1. 330 km (You will drive 300 km in three hours. If you then drive for another 0.3 hours, you will drive $0.3 \times 100 = 30$ km; $300 + 30 = 330$ km.)
2. $\frac{4}{3}\left(\frac{16}{12} \div \frac{4}{4}\right)$
3. 50 (Move the decimal place to the left once.)
4. 25 (5% is half of 10% so half of 50 is 25.)
5. 75 (10% + 5% = 15% so 50 + 25 = 75)
6. 66 (33×2)
7. 90% (% is out of 100. $20 \times 5 = 100$.
   Method A: $18 \times 5 = 90$.
   Method B: $20 - 18 = 2$, $2 \times 5 = 10$ so $100 - 10 = 90$.
   You could think this through by saying that each mark is worth 5%, so if you are missing 2 marks, you are missing 10%.)
8. 24 (Dan's Age = 3 × Kim's Age = 3 × 4 = 12. Jamie's Age = 2 × Dan's Age = 2 × 12 = 24)

Part B: What Slope Tells Us

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The slope of the line joining points K and M is $\frac{-3}{7}$ $\left(m_{\text{KM}} = \frac{-3}{7}\right)$. Point L is on the same line as points K and M. What is the slope of the line joining points M and L? Explain how you know.

   Answer:

   If $m_{\text{KM}} = \frac{-3}{7}$ then $m_{\text{ML}} = \frac{-3}{7}$. The slope of a linear line is constant. If points K, L, and M are all on the same line, the slope will be the same between all these points.

2. (−456, 187) is a point on a line with a slope of $\frac{17}{25}$. Find another point on this line.

   Answer:

   (−456 + 25, 187 + 17) or (−431, 204) is one other possible point on this line. There are an infinite number of points possible.

3. (5, 12) and (72, y_2) are on a line with a slope of 4. Use the slope formula to find the value of y_2. Show your work.

   Answer:

   (5, 12) (72, y_2) $m = 4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$4 = \frac{y_2 - 12}{72 - 5}$$

$$4 = \frac{y_2 - 12}{67}$$

$$4(67) = \frac{y_2 - 12\cancel{(67)}}{\cancel{(67)}}$$

$$268 = y_2 - 12$$

$$268 + 12 = y_2 - 12 + 12$$

$$280 = y_2$$

   A line with a slope of 4 goes through the points (5, 12) and (72, 280).

4. Explain in your own words why the slope of a horizontal line is equal to zero.

   Answer:

   The coordinate pairs of the points along a horizontal line will all have the same y-value. This means the difference in y-values or rise will always be equal to zero. The run of a horizontal line can be any value, but zero divided by any number is still zero.

5. Explain in your own words how you can use the slope formula to determine if two lines are parallel.

   Answer:

   Two lines are parallel if they have the same slope. For each line, choose any two points on that line and use the slope formula to calculate the slope. If the slopes are the same, the two lines are parallel.

Learning Activity 1.9

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Are the possible values of the dependent variable the domain or the range of the relation?
2. Given the slope, $\frac{6}{11}$, state the rise and the run.
3. What are the factors of 8?
4. What are the first 3 multiples of 7?
5. Write 4^3 in expanded form.
6. Evaluate $\sqrt{25}$.
7. Evaluate 3^2.
8. The distance from the MTS Centre to Polo Park is 5.1 km. On your way to Polo Park from MTS Centre, you decide to stop at a doughnut shop halfway between the two. How far do you have to travel from the doughnut shop to get to Polo Park?

Answers:

1. Range
2. Rise = 6, run = 11
3. 1, 2, 4, 8
4. 7, 14, 21
5. $4 \times 4 \times 4$
6. 5 (Remember that the $\sqrt{\ }$ symbol stands for "square root." This means that you are looking for a number that multiplies with itself to get 25.)
7. 9 (3×3)
8. $\frac{5.1}{2}$ or 2.55 (For the decimal value; $5 \div 2 = 2.5$ and $0.1 \div 2 = 0.05$.

   $2.5 + 0.05 = 2.55$.)

Part B: Graphing Data Points from an Equation

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

This learning activity presents an opportunity for you to assess your own learning. Questions 1–3 are similar, so we recommend that you do Questions 1 and 2. If you want more practice, do Question 3. If you feel that this is enough practice, move on to Question 4.

1. a) Given the equation $y = -2x + 8$, create a table of values with five ordered pairs. Choose any 5 reasonable x-values (start with values between −10 and 10), substitute them into the equation and solve for y. Record the values in a chart.

Answer:

$y = -2x + 8$

Your table of values may have different ordered pairs.

x	y
0	8
2	4
4	0
9	−10
10	−12

Example: Solve for $x = 0$

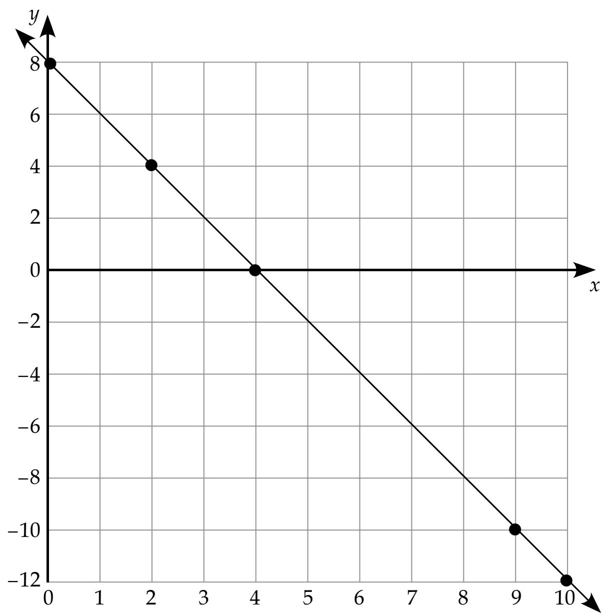
$$y = -2x + 8$$
$$y = -2(0) + 8$$
$$y = 0 + 8$$
$$y = 8$$

Solve for $x = 2$

$$y = -2x + 8$$
$$y = -2(2) + 8$$
$$y = -4 + 8$$
$$y = 4$$

b) Graph the line using grid paper. Remember to choose an appropriate y-scale for your graph, one that will include all the y-values calculated. To determine what increment to use along the y-axis, calculate the range (subtract smallest y-value from the largest y-value), divide by the number of increments along the y-axis and round up.

Answer:



To determine the y-scale:

$8 - (-12) = 20$

$20 \div 10$ increments $= 2$

Begin at −12 and go up to 8 using intervals of 2.

c) Calculate the slope of the line.

Answer:

Choose any two points. (0, 8) and (10, −12)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-12 - 8}{10 - 0}$$

$$m = \frac{-20}{10} = -2$$

d) Determine the y-intercept of the line.

Answer:

When $x = 0$, $y = 8$. The y-intercept is at $y = 8$.

e) Verify your answers to (c) and (d) using the equation provided in (a).

Answer:

The slope was calculated to be −2 and the y-intercept was determined to be at $y = 8$. From the given equation, $y = -2x + 8$, the coefficient of x is the slope, and it is −2 and the constant is the y-intercept. It is 8. My answers are correct.

2. a) Given the equation $y = \frac{2}{3}x + 3$, create a table of values with 5 ordered pairs. If you have a fraction coefficient, choose x values that are multiples of the denominator so that you can reduce the fraction and eliminate the denominator (start with values like 3, 6…), substitute them into the equation and solve for y. Record the values in a chart.

Answer:

$$y = \frac{2}{3}x + 3$$

Possible values for the table:

x	y
0	3
3	5
6	7
9	9
−3	1

Example: Solve for $x = 0$

$$y = \frac{2}{3}(0) + 3$$

$$y = 0 + 3$$

$$y = 3$$

Solve for $x = 6$

$$y = \frac{2}{3}(6) + 3$$

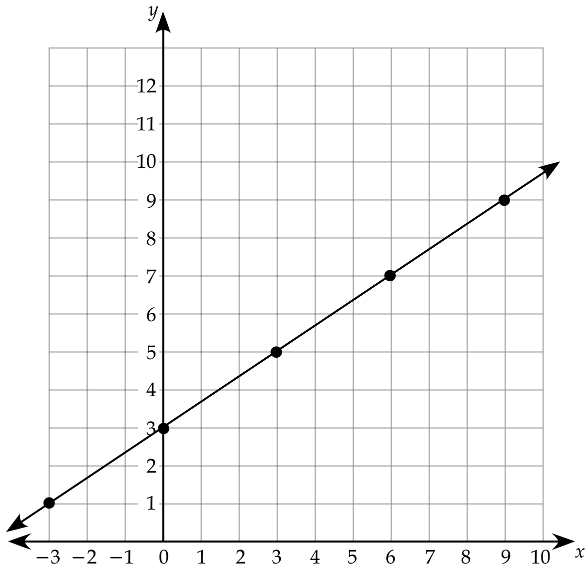
$$y = \frac{2}{\cancel{3}}\cancel{(6)}2 + 3$$

$$y = 4 + 3$$

$$y = 7$$

b) Graph the line on grid paper.

Answer:



c) Calculate the slope of the line.

Answer:

Choose any two points. (0, 3) and (6, 7)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7-3}{6-0}$$

$$m = \frac{4}{6} = \frac{2}{3}$$

d) Determine the y-intercept of the line.

Answer:

The coordinates of the y-intercept are (0, 3).

e) Verify your answers to (c) and (d) using the equation provided in (a).

Answer:

The given equation was $y = \frac{2}{3}x + 3.$ The coefficient of x is the slope and it was calculated to be $\frac{2}{3}.$ The constant is the y-intercept and it was calculated to be at $y = 3$. The answers in (c) and (d) are verified as correct using the equation provided in (a).

3. a) Given the equation $y = -5x + 48$, create a table of values with 5 ordered pairs.

Answer:

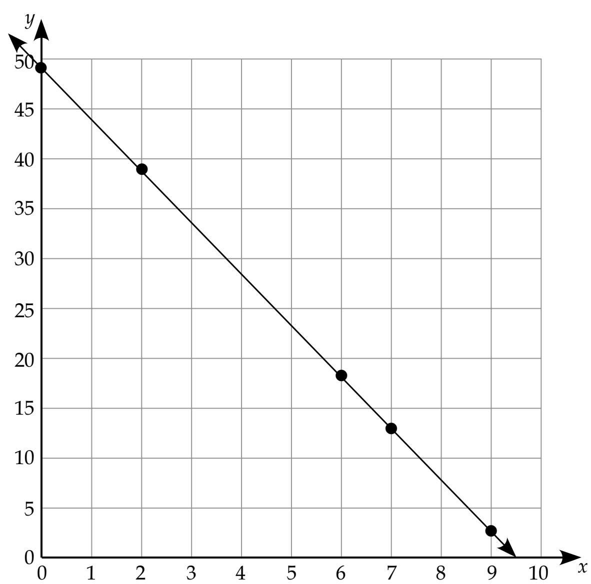
$y = -5x + 48$

Ordered pairs may vary. Show your calculations.

x	y
0	48
2	38
6	18
7	13
9	3

b) Graph the line on grid paper.

Answer:



c) Calculate the slope of the line.

Answer:

Choose any two points. (0, 48) and (2, 38)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{38 - 48}{2 - 0}$$

$$m = \frac{-10}{2} = -5$$

d) Determine the y-intercept of the line.

Answer:

The y-intercept is at $y = 48$.

e) Verify your answers to (c) and (d) using the equation provided in (a).

Answer:

The equation $y = -5x + 48$ verifies that the slope is -5 and the y-intercept is at $y = 48$.

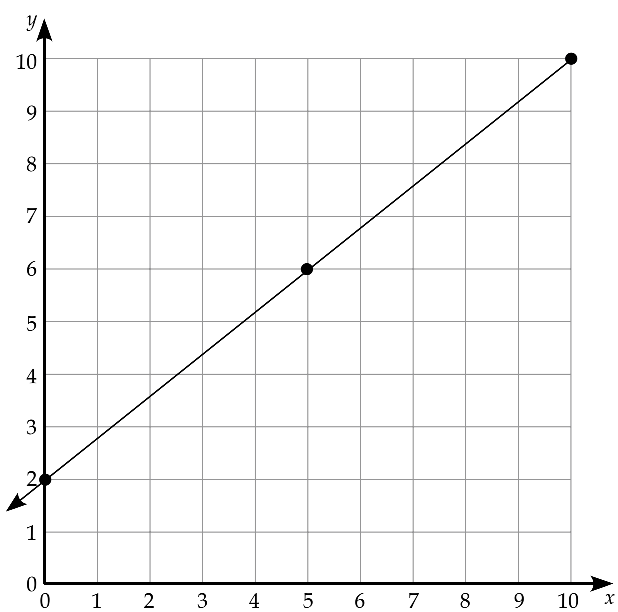
4. a) Write the equation of a line with a y-intercept of 2 and a slope of $\frac{4}{5}$.

Answer:

$$y = \frac{4}{5}x + 2$$

b) Sketch the graph. ***Do not*** create a table of values or ordered pairs, except to check your work.

Answer:



Locate the y-intercept on the graph at (0, 2). From that point, use the slope $\frac{\text{ryse}}{\text{run}} = \frac{4}{5}$ and move up 4 and to the right 5 to the point (0 + 5, 2 + 4) or (5, 6). Repeat to find the next point at (10, 10).

Verify by substituting $x = 5$ and $x = 10$ into the equation $y = \frac{4}{5}x + 2$ and solving for y.

5. a) Given the equation $y = -\frac{4}{5}x + 9$, state the slope and y-intercept without graphing the line or creating a table of values.

Answer:

$$y = -\frac{4}{5}x + 9$$

The slope of this line is $\frac{-4}{5}$ and the y-intercept is at 9.

b) Explain how you would know where to draw the line.

Answer:

I would draw this line by locating the y-intercept at (0, 9) and then using the slope to find another point. I would move down 4 units and 5 units to the right to find another point at (5, 5). I would repeat this to find another point at (10, 1) and then connect the points with a straight line. I could also move 4 units up and 5 units down from (0, 9) to get to (−5, 5).

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 1
Graphs and Relations

Learning Activity Answer Keys

MODULE 1: GRAPHS AND RELATIONS

Learning Activity 1.1

There is no answer key for this learning activity.

Learning Activity 1.2

Part A: BrainPower

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper. The first few times you do these questions, your learning partner can help you figure out strategies to solve them.

1. There are 22 yard markers on a Canadian football field. Each marker represents five yards. How long is a Canadian football field?
2. If Evan eats $\frac{3}{5}$ of a pizza and Nick eats $\frac{4}{5}$ of a pizza, how many pizzas do they have to order so that both can eat as much as they like?
3. Simplify the following fraction to lowest terms: $\frac{18}{27}$.
4. You are working at the stadium where they don't have an electronic till. The customer is buying popcorn for $3.80. If the customer gives you a $5.00 bill, how much change will you give them?
5. Rank the numbers from highest to lowest: 0.5, 0.05, 0.3, 0.09, and 0.25.
6. Solve for m: $2 - m = 14$.
7. The distance to the mall from your house is 8 km. Your friend lives half as far away from the mall. What is the distance from your friend's house to the mall?
8. Write the percent as a decimal: 62%.

Answers:

1. 110 yards (22 × 5)
2. 2 pizzas $\left(\frac{3}{5}+\frac{4}{5}=\frac{7}{5}=1\frac{2}{5}\right.$, so you need $1\frac{2}{5}$ pizzas.$\left.\right)$

   You cannot order part of a pizza, so the next whole number is 2.)
3. $\frac{2}{3}$
4. $1.20
5. 0.5, 0.3, 0.25, 0.09, 0.05
6. $m = -12$ ($-m = 14 - 2$)
7. 4 km
8. 0.62

Part B: Word Web

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

Create a word web showing what you know about graphs. Use bubbles to indicate new ideas or characteristics, and lines to show how they are connected.

Answer:

Each word web is unique, but your web about graphs includes some of the following: the purpose, parts, definition, and types of graphs, how they are made, where they are used, the kinds of data they display, and so on. You will come back to this word web at the end of this module.

Learning Activity 1.3

Part A: BrainPower

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper. The first few times you do these questions, your learning partner can help you figure out strategies to solve them.

1. What is the range of the following numbers: 2, 6, 4, 8, 7, 13, 11?
2. You are going to the store to buy a drink with \$2.05 in your pocket. If a drink costs \$1.75, will you be able to buy one?
3. Simplify the fraction $\frac{6}{2}$.
4. Write the ratio as a fraction: 5:2.
5. Solve for a: $9 + a = 13$.
6. Write the next two numbers in the pattern: 1, 2, 4, 8, ____, ____.
7. You want to bring freezies to your last soccer game of the season. You want to have enough so that each player gets two. If you have 18 people on your team, how many freezies do you need?
8. You are helping your dad build a rectangular deck. If it is 2 m long and 3 m wide, what is the area that it takes up in your yard?

Answers:

1. 11 (13 − 2)
2. Yes. (You will get \$2.05 − \$1.75 = \$0.30 in change back.)
3. $\frac{3}{1} = 3$
4. $\frac{5}{2}$
5. $a = 4$
6. 16, 32
7. 36 (2 × 18)
8. 6 m^2 (2 m × 3 m)

Part B: Independent vs. Dependent Variables and Continuous Data

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. State which variable is independent and which variable is dependent in each of the following contexts:
   a) Hours worked in a week with pay of $20 per hour
   b) Final exam mark and average quiz marks for a Grade 10 Math class
   c) Coffee temperature and the time since the cup was poured
   d) Average monthly temperature in Manitoba during the months from January to December

   Answers:

	Independent	Dependent
a)	hours	pay
b)	quiz mark	exam mark
c)	time	temperature
d)	month	temperature

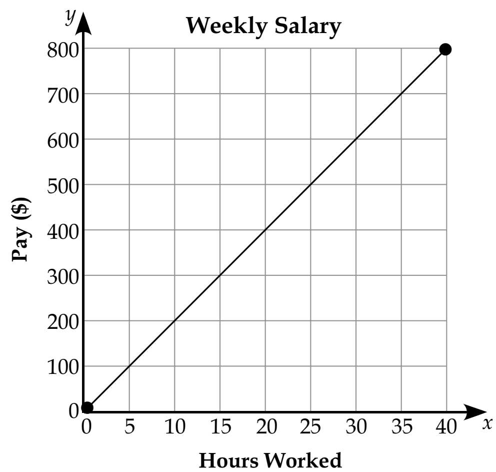
2. Are the situations in Question 1 continuous? Explain.

   Answers:
   a) Continuous—you can get paid in fractions of dollars for partial hours
   b) No—individual quiz and exam marks
   c) Continuous—time and temperature can be meaured in infinite increments
   d) No—there can only be 12 average monthly temperatures in a year

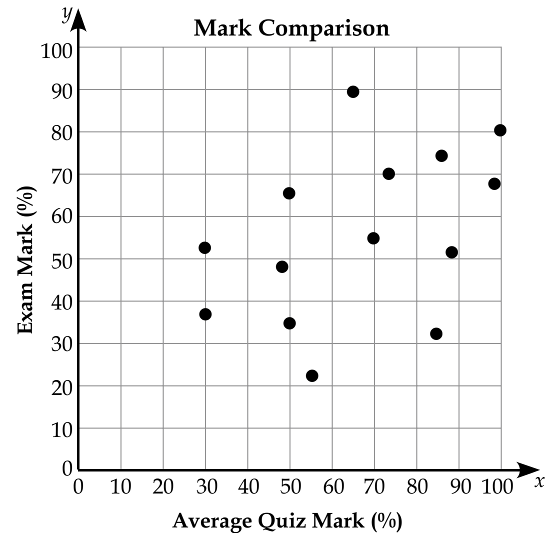
3. Sketch a possible graph based on the contexts given in Question 1.

Answers:

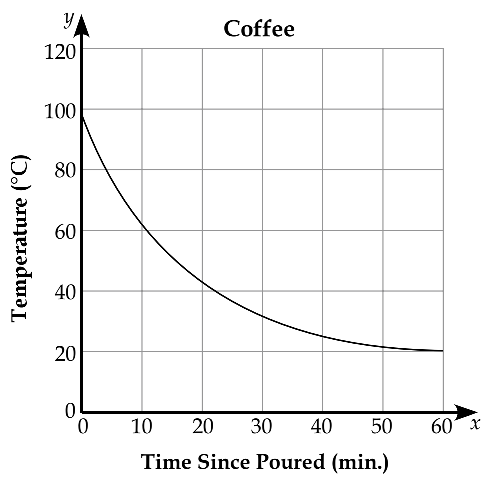
a)



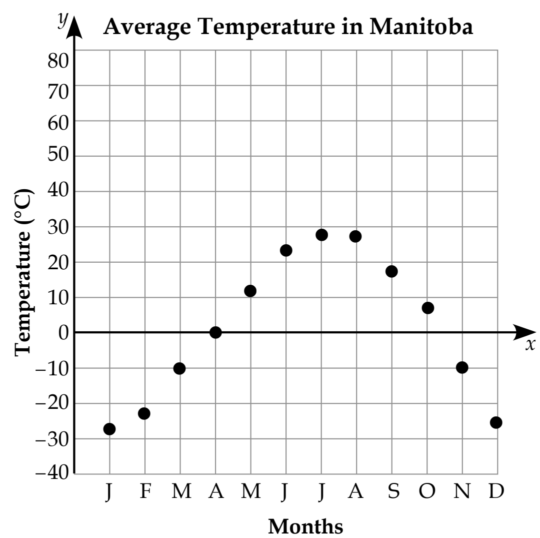
b)



c)



d)

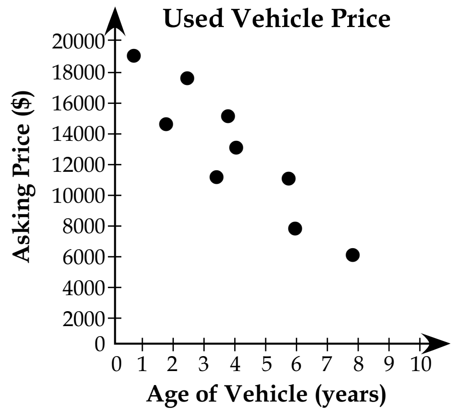


4. Create a possible context that would result in the following graphs. Label each graph with independent and dependent variables, units, appropriate scales (values along the axes), and a title.

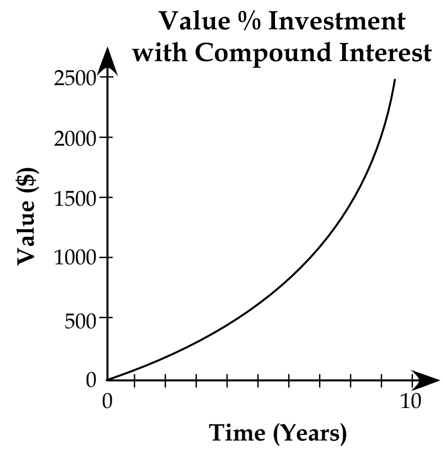
   Answers:

   Answers may vary. Possible answers include:

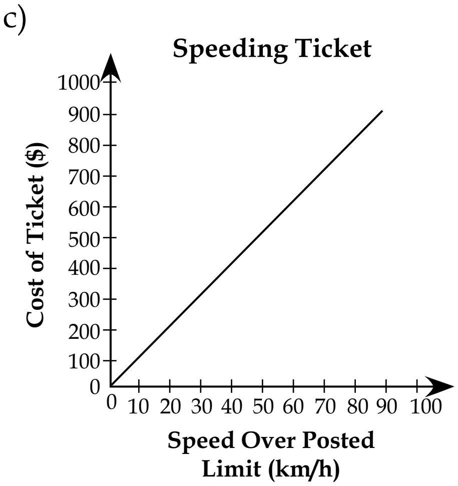
   a)



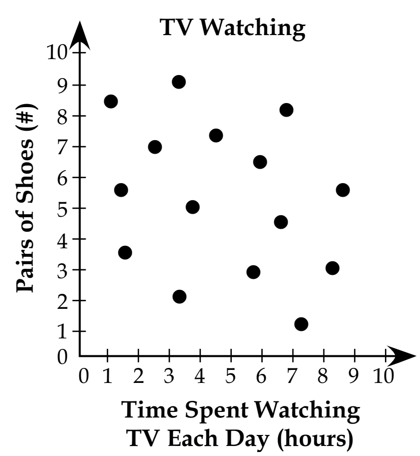
   b)



   c)



   d)



   a) The listed asking price for used vehicles in classified ads. Price is dependent on the age of the vehicle.

   b) The value of an investment with compound interest. The value depends on how long it has been invested.

   c) The cost of a speeding ticket depends on how many km/h you were driving over the posted speed limit.

   d) There does not appear to be any relationship between these two variables.

5. Construct a good graph of the following data. It may be done by hand on graph paper or with technology.

   A random sample of 11 people was drawn from the population of people between the ages of 30 and 40 who were employed full time in Brandon. The number of years of their schooling and annual income in thousands of dollars was recorded for each of the 11 people. The data are given below:

Schooling (years)	10	7	12	11	16	12	18	8	12	14	16
Income ($1000)	32	20	45	43	65	42	75	28	40	60	65

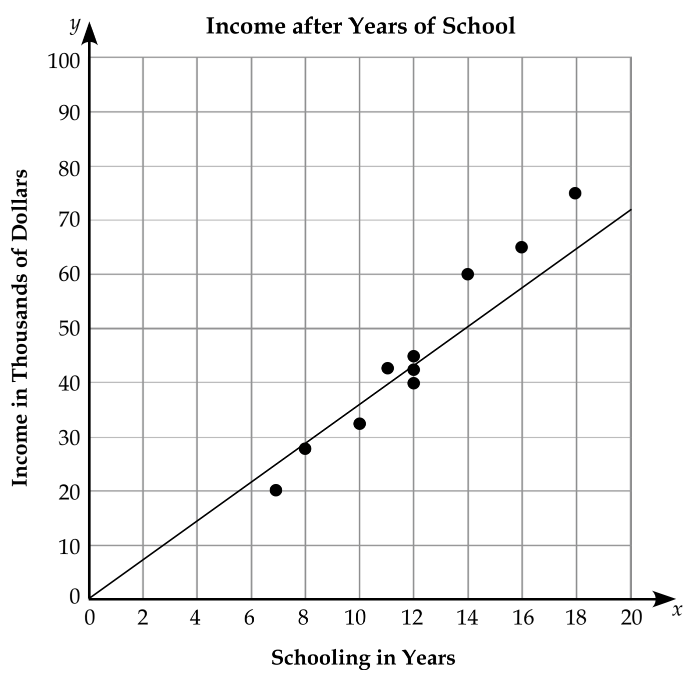
   a) Which variable, schooling or income, is the independent variable? Which is the dependent variable?

   Answer:

   Schooling is the independent variable and Income is the dependent variable.

   b) Graph the data with appropriate scales on the grid below and draw the line of best fit.

   Answer:



Note: Your line of best fit may be slightly different.

Check: Are there approximately the same number of points above the line as below the line?

   c) Is the data continuous?

   Answer:

   The data is not continuous because usually you attend complete, not partial, years of school.

Learning Activity 1.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Your mother has invited your entire family over for dinner. As usual, you are complaining because only $\frac{1}{6}$ of them are around your age. If your family has 24 people in it (not including you), how many of them are approximately your age?
2. There are 12 eggs in a dozen. If you are buying four dozen eggs, how many eggs is that?
3. Write the following decimal as a fraction: 0.058.
4. You are craving 5¢ candy. You have $1.43 in your pocket. How much candy can you afford?
5. List the factors of 12.
6. Fill in the missing terms in the following pattern: 0, 3, ____, 9, 12, ____.
7. Circle the independent variable: the height of an airplane compared to the time it takes to land.
8. At the end of the day, a restaurant is left with three different, partially eaten pies. Each pie has $\frac{2}{7}$ left over. How much pie is left over in total?

Answers:

1. Four people are around your age $\left(24 \times \frac{1}{6}\right)$
2. 48 eggs (12 × 4)
3. $\frac{58}{1000} = \frac{29}{500}$ (either answer is acceptable)
4. 28 (There are 20 nickels in a dollar, and 40 divided by 5 is 8. 20 + 8 = 28.)
5. 1, 2, 3, 4, 6, 12
6. 6, 15
7. The height of the airplane is the independent variable.
8. $\frac{6}{7}\left(3 \times \frac{2}{7}\right)$ of a pie is left, if all the leftover pie was put together

Part B: Ordered Pairs, Data Tables, and Relationships

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the data given in the table of values in Example 1 to complete the following:

   a) Write the data as ordered pairs.

      Answer:

      (0, 65), (30, 45), (60, 25), (90, 5)

   b) Write a sentence describing the relationship between the two variables (e.g., As time does 'this', the remaining fuel does 'that').

      Answer:

      As the time progresses, the amount of fuel remaining decreases in a linear relation.

2. Refer back to the graphs provided or the ones you created in Lesson 2. Which graphs indicate linear relations?

   Answer:

   The following graphs from Lesson 1 indicate linear relations:

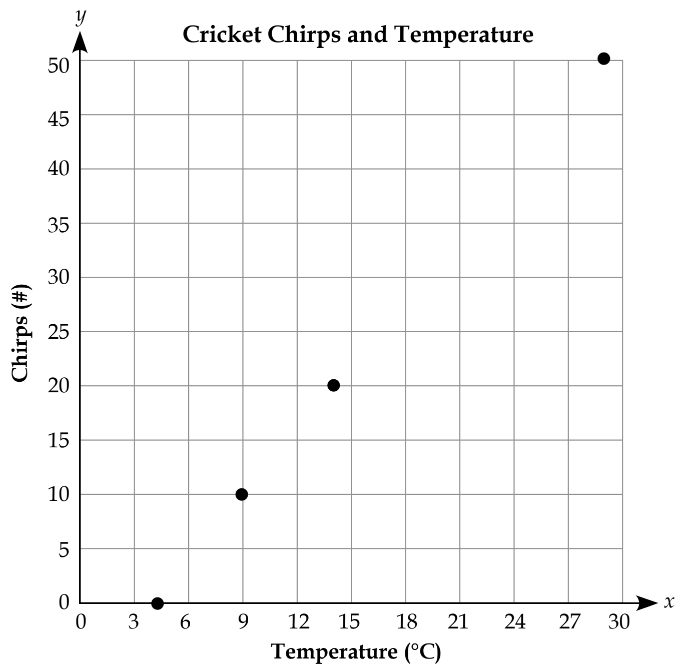
   Example 1, Example 2, and the storage capacity and cost of handheld devices.

   Learning Activity 1.3 #4 graphs (a) and (c), and Learning Activity 1.3 #5.

   Assignment 1.1 #1 graphs (a) and (c), and Assignment 1.1 #2 Ball bounce

   Note: Linear relations can be found in both continuous data as well as data that are not continuous.

3. In the following scatterplot, a biologist has displayed data relating the temperature to the number of times a cricket chirps.



a) Write the coordinates as ordered pairs.

Answer:

(4, 0), (9, 10), (14, 20), (29, 50)

b) Create a table of values from the data graphed in this scatterplot.

Answer:

Temperature (°C)	Cricket Chirps (#)
4	0
9	10
14	20
29	50

c) Write a sentence describing the relationship between these two variables.

Answer:

As the temperature increases, the number of times a cricket chirps also increases.

d) Does this graph display a linear relation? Explain.

Answer:

Yes, the relation between temperature and number of chirps appears linear because a straight line could be drawn on the graph and it would go through or near all the data points

Learning Activity 1.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Simplify the following fraction: $\frac{21}{35}$.
2. Simplify the following fraction: $\frac{12}{20}$.
3. 50% of 680 is 340. 25% of 680 is ____.
4. There is a 50% discount on all candy at the store the day after Hallowe'en. If it cost you $30 to buy candy before Hallowe'en, how much would you spend if you bought the same candy after Hallowe'en?
5. Solve for r: $5 + r = -4$.
6. You are at a doughnut shop and would like to buy doughnuts for your family. There are six people in your family (including you—and you want a doughnut). How much money do you need, if each doughnut costs 60¢?
7. A regular goal in rugby is worth three points. If a team finishes a game with 39 points, how many goals did they score?
8. If it rains 10 mm, how much rain is that in centimetres?

Answers:

1. $\frac{3}{5}$ (divide top and bottom by 7)
2. $\frac{3}{5}$ (divide top and bottom by 4)
3. 170 (25% is half of 50% so 25% of 680 must be half of 340)
4. $15.00 (50% of $30 is the same as half of $30)
5. $r = -9$ ($r = -4 - 5$)
6. $6 \times 60¢ = 360¢ = \$3.60$ (in either cents or dollars is acceptable)
7. 13 ($39 \div 3$)
8. 10 mm = 1 cm

Part B: Domain and Range

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You want to rent a banquet room in a restaurant and have dinner with friends to celebrate your birthday. The cost of the meal is $6 per person.

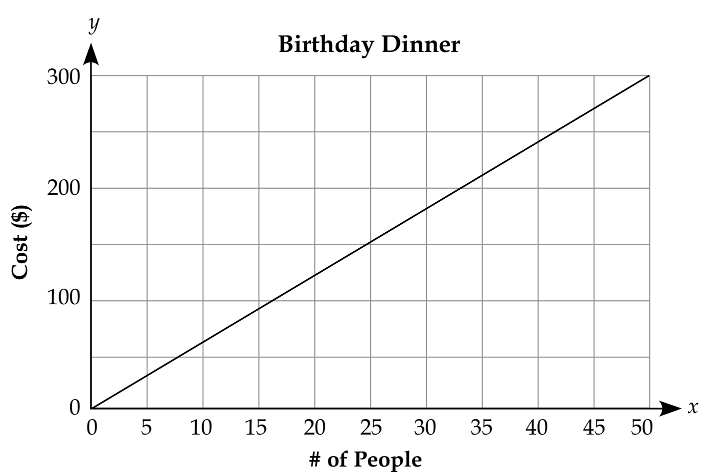
   a) If you were to graph the relation between the cost and the number of people attending, what would be a reasonable domain and range? Explain the restrictions you consider.

   Answer:

   The cost depends on the number of people attending, so the independent variable is the number of people. The domain represents valid x-values of the independent variable. The number of people attending is restricted by the size of the banquet room, and if there is a minimum number of people required to rent the room. The domain could be from 5 to 50 people. Based on the restrictions on the domain, the range, or possible costs, could be from $30 to $300, given that a meal is $6 per person.

   b) Create a possible graph. You may draw it by hand or use technology and print it out.

   Answer:



c) Is this an example of a linear relation?

Answer:

Yes, this is a linear relation.

d) Write a sentence describing the relationship between the variables.

Answer:

As the number of people attending increases, the cost also increases.

Note: Your answer in part (a) and your graph may have been slightly different. That is okay as long as the domain and range you state are reasonable and the values are valid for the restrictions you describe. Your descriptive sentence in part (d) must indicate how the dependent variable changes in relation to changes in the independent variable.

Learning Activity 1.6

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Write the percent as a decimal: 3.5%.
2. Which is larger: 0.76 or 0.07?
3. What is the range of the following numbers? 0.2, 0.6, 0.08, 0.5, 0.03
4. If 3% of 500 is 15, what is 12% of 500?
5. The mat used for floor gymnastics is a square. The length along one side is 40 feet. What is the total area of the mat?
6. Complete the pattern: 9, −7, 5, ____, ____.
7. Solve for v: $9v = 45$.
8. You want to order a pizza, but you only have $15. If a pizza costs $16, but is 10% off, can you afford it? (ignore taxes)

Answers:

1. 0.035
2. 0.76 (0.76 is seventy-six hundredths; 0.07 is seven hundredths)
3. 0.57 (0.6 − 0.03)
4. 60 (3% × 4 = 12%. 15 × 4 = 60. If it helps, think about the minutes in an hour.)
5. 1600 feet (4 × 4 = 16. 40 and 40 have two zeros, so the answer must also have two zeros.)
6. –3, 1 (decreasing odd numbers with alternating signs)
7. $v = 5$ ($45 \div 9 = 5$)
8. Yes (16 × 10% = $1.60. 16 − 1.60 = $14.40 which is less than $15.00)

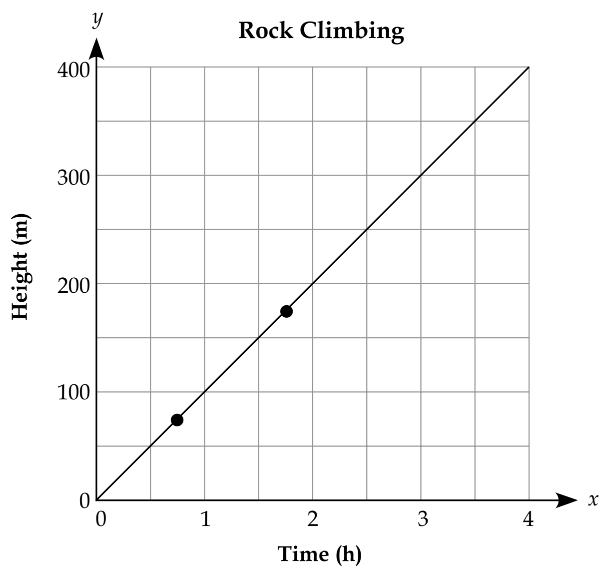
Part B: Rates and Slopes

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Kent is rock climbing up a 400 m high cliff. 45 minutes after beginning his ascent, he has climbed 75 m. After another hour, he is at 175 m.

   a) How many hours of steady climbing will it take him to reach the top? Sketch a graph to help you answer this question.

   Answer:

   From the graph, you can use extrapolation to find how long it takes to go up 400 m. It would take 4 hours.

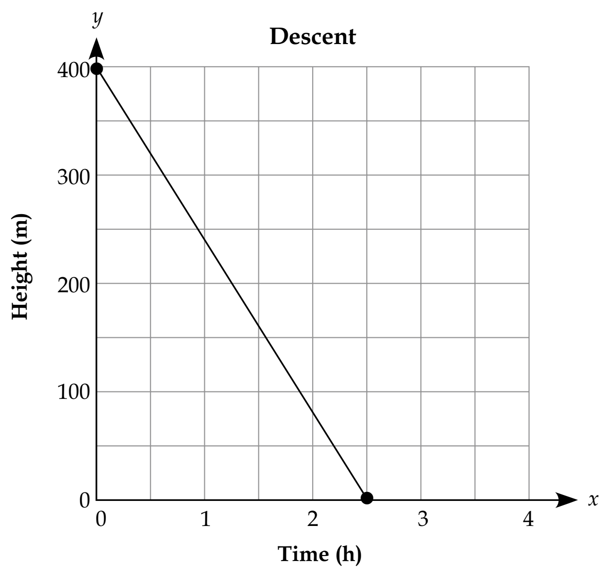
   b) At what rate is he climbing, in m/h?

   Answer:

   If it takes him four hours to climb 400 m, he can climb 100 m in one hour. His rate, and the slope of this line, is 100 m/h.

2. The descent down the cliff only takes Kent two and a half hours to complete.
   a) Create a graph similar in scale to the previous question and sketch this situation. Plot the points that represent his time and location at the top of the cliff and when he reaches the bottom.

      Answer:



   b) Compare the slopes in these two graphs.

      Answer:

      The slope in graph #1 is positive and less steep than graph #2. The second slope is negative.

   c) What is Kent's rate in this situation?

      Answer:

      Kent climbs down 400 m in $2\frac{1}{2}$ hours, so he could descend 160 m in one hour ($400 \div 2.5$). His rate is -160 m/h. The slope is negative.

Learning Activity 1.7

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Continuous or not: A graph comparing the number of people in a room and the volume of noise.
2. Domain or range: the independent variable has values from 0 to 20.
3. Write the following as an improper fraction: $2\frac{3}{7}$.
4. You are driving to the cabin, which is 104 km from your house. $\frac{1}{8}$ of your drive is in the city. How much of your drive is in the city (in km)?
5. You are at a bake sale. You want to buy a peanut butter cookie. If the whole plate of 16 cookies costs $3.20, how much will you pay for one cookie?
6. Write the following decimal as a percent: 0.84.
7. A marathon is 26.4 miles. A half marathon is how long?
8. Solve for d: $3d = 9$.

Answers:

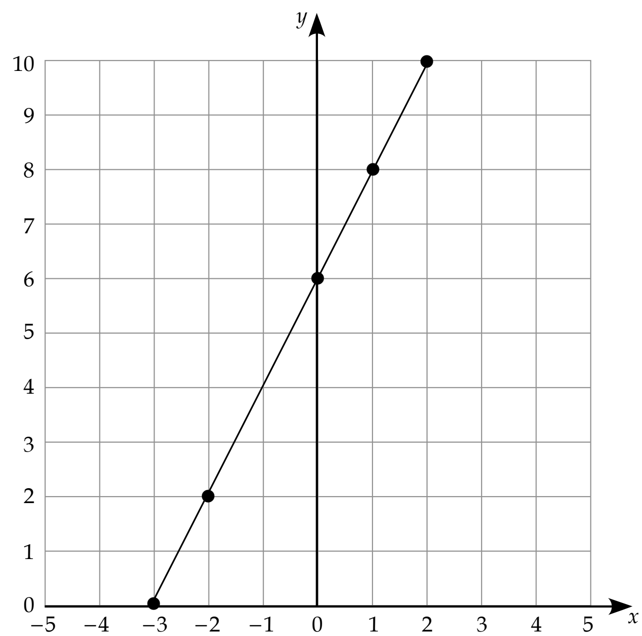
1. Not continuous. You cannot have part of a person.
2. Domain
3. $\frac{17}{7}\left(2\frac{3}{7} = 2 + \frac{3}{7} = \frac{14}{7} + \frac{3}{7} = \frac{17}{7}\right)$
4. 13 km $\left(104 \times \frac{1}{8} \text{ or } \frac{1}{8} \text{ is } \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$

   (So divide by 2 three times: $104 \div 2 = 52$; $52 \div 2 = 26$; $26 \div 2 = 13$)
5. $0.20 ($32 \div 16 = 2$ so $3.2 \div 16 = \$0.20$)
6. 84%
7. 13.2 miles (26.4 2)
8. $d = 3$ ($9 \div 3$)

Part B: Linear Relations

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the following theoretical graph, indicate:
   a) if the slope is positive or negative
   b) what the rate of change, or slope, is equal to
   c) the value of the x-intercept
   d) the coordinates of the y-intercept
   e) the domain
   f) the range



Answers:

a) The line has a positive slope.

b) If you examine the graph you can see that the y-coordinate of the line increases by 2 for each increase of 1 in the x-coordinate. $\frac{2}{1} = 2$, so the rate of change is 2. Using the coordinate pairs (–2, 2) and (1, 8), the change in the y-values is 6 and the change in the x-values is 3. $\frac{6}{3} = 2$

c) The value of the x-intercept is -3.

d) The coordinates of the y-intercept is (0, 6).

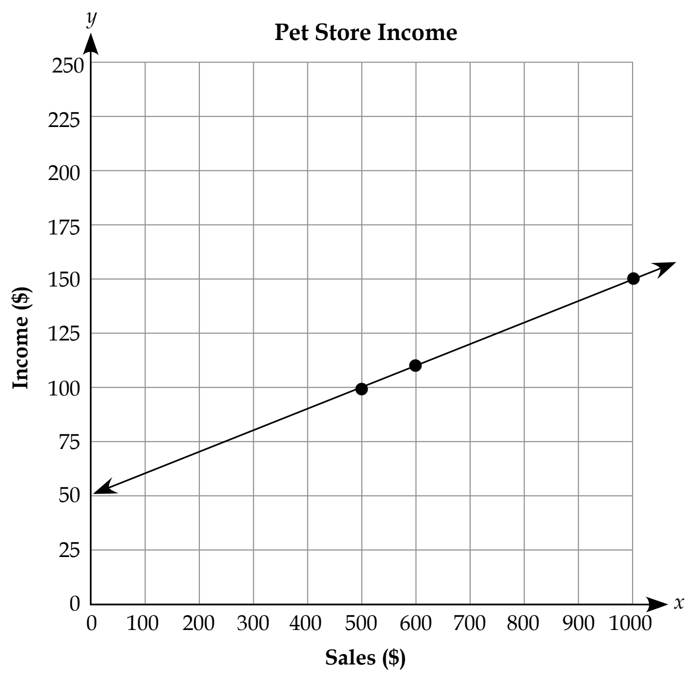
e) The domain of this graph is from -3 to 2.

f) The range is from 0 to 10.

2. From the first part of the lesson, when you realized how much more income your friend had with her part-time job, you found a new position at a pet store. Here, your income is based in part on commission. You earn a percentage of the value of the product you sell. Based on your first three weeks of income, knowing that income depends on sales, you determine the following (sales, income) pairs: (500, 100), (600, 110), and (1000, 150).

    a) Graph these ordered pairs and determine if the relation is linear.

    Answer:



Yes, the relation is linear.

b) What is the rate of change or slope of the line?

Answer:

The slope is positive. Using the ordered pairs (500, 100), (600, 110), you can see that your income increases by \$10 for an additional sale of \$100. $\frac{10}{100} = 0.10 \text{ or } 10\%$. This represents a commission of 10%. Written as a decimal, the slope is 0.10.

c) Draw a line to determine the intercepts. What do the intercepts represent?

Answer:

There is no x-intercept in this graph. The y-intercept is at $y = 50$ or (0, 50). This represents sales of \$0 and an income of \$50. Your income is based on a commission of 10% as well as a base weekly salary of \$50.

d) What are the domain and range in this situation?

Answer:

Valid x-values in the domain of this relation are from 0 to 1000. The values will never be negative, and if you have a really great week of sales it may be higher, but this seems reasonable for this part-time job situation. Valid y-values range from 50 to 150. You will earn \$50 even if you make no sales, and based on reasonable sales expectations your weekly income could be as high as \$150.

Learning Activity 1.8

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. You are on your way to Calgary, driving on the Trans-Canada Highway. Your speed is constant at 100 km/h. If you have been driving for 3.3 hours at this speed, how far have you driven?
2. Write the following ratio as a fraction in lowest form: 16 : 12.
3. What is 10% of 500?
4. What is 5% of 500?
5. What is 15% of 500?
6. There are three classes of Grade 10 Science taught in a school by one teacher. Each class has 22 students. If the teacher gives a test to all three classes, how many tests will she have to mark?
7. A student receives a grade of $\frac{18}{20}$ on his test. Estimate the percent value.
8. Jamie is twice as old as Dan. Dan is 3 times as old as Kim. If Kim is 4 years old, how old is Jamie?

Answers:

1. 330 km (You will drive 300 km in three hours. If you then drive for another 0.3 hours, you will drive $0.3 \times 100 = 30$ km; $300 + 30 = 330$ km.)
2. $\frac{4}{3}\left(\frac{16}{12} \div \frac{4}{4}\right)$
3. 50 (Move the decimal place to the left once.)
4. 25 (5% is half of 10% so half of 50 is 25.)
5. 75 (10% + 5% = 15% so 50 + 25 = 75)
6. 66 (33×2)
7. 90% (% is out of 100. $20 \times 5 = 100$.
   Method A: $18 \times 5 = 90$.
   Method B: $20 - 18 = 2$, $2 \times 5 = 10$ so $100 - 10 = 90$.
   You could think this through by saying that each mark is worth 5%, so if you are missing 2 marks, you are missing 10%.)
8. 24 (Dan's Age = 3 × Kim's Age = 3 × 4 = 12. Jamie's Age = 2 × Dan's Age = 2 × 12 = 24)

Part B: What Slope Tells Us

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The slope of the line joining points K and M is $\frac{-3}{7}$ $\left(m_{\mathrm{KM}} = \frac{-3}{7}\right)$. Point L is on the same line as points K and M. What is the slope of the line joining points M and L? Explain how you know.

   Answer:

   If $m_{\mathrm{KM}} = \frac{-3}{7}$ then $m_{\mathrm{ML}} = \frac{-3}{7}$. The slope of a linear line is constant. If points K, L, and M are all on the same line, the slope will be the same between all these points.

2. (−456, 187) is a point on a line with a slope of $\frac{17}{25}$. Find another point on this line.

   Answer:

   (−456 + 25, 187 + 17) or (−431, 204) is one other possible point on this line. There are an infinite number of points possible.

3. (5, 12) and (72, y_2) are on a line with a slope of 4. Use the slope formula to find the value of y_2. Show your work.

   Answer:

   (5, 12) (72, y_2) $m = 4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$4 = \frac{y_2 - 12}{72 - 5}$$

$$4 = \frac{y_2 - 12}{67}$$

$$4(67) = \frac{y_2 - 12\cancel{(67)}}{\cancel{(67)}}$$

$$268 = y_2 - 12$$

$$268 + 12 = y_2 - 12 + 12$$

$$280 = y_2$$

   A line with a slope of 4 goes through the points (5, 12) and (72, 280).

4. Explain in your own words why the slope of a horizontal line is equal to zero.

   Answer:

   The coordinate pairs of the points along a horizontal line will all have the same y-value. This means the difference in y-values or rise will always be equal to zero. The run of a horizontal line can be any value, but zero divided by any number is still zero.

5. Explain in your own words how you can use the slope formula to determine if two lines are parallel.

   Answer:

   Two lines are parallel if they have the same slope. For each line, choose any two points on that line and use the slope formula to calculate the slope. If the slopes are the same, the two lines are parallel.

Learning Activity 1.9

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Are the possible values of the dependent variable the domain or the range of the relation?
2. Given the slope, $\frac{6}{11}$, state the rise and the run.
3. What are the factors of 8?
4. What are the first 3 multiples of 7?
5. Write 4^3 in expanded form.
6. Evaluate $\sqrt{25}$.
7. Evaluate 3^2.
8. The distance from the MTS Centre to Polo Park is 5.1 km. On your way to Polo Park from MTS Centre, you decide to stop at a doughnut shop halfway between the two. How far do you have to travel from the doughnut shop to get to Polo Park?

Answers:

1. Range
2. Rise = 6, run = 11
3. 1, 2, 4, 8
4. 7, 14, 21
5. $4 \times 4 \times 4$
6. 5 (Remember that the $\sqrt{}$ symbol stands for "square root." This means that you are looking for a number that multiplies with itself to get 25.)
7. 9 (3×3)
8. $\frac{5.1}{2}$ or 2.55 (For the decimal value; $5 \div 2 = 2.5$ and $0.1 \div 2 = 0.05$.

   $2.5 + 0.05 = 2.55$.)

Part B: Graphing Data Points from an Equation

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

This learning activity presents an opportunity for you to assess your own learning. Questions 1–3 are similar, so we recommend that you do Questions 1 and 2. If you want more practice, do Question 3. If you feel that this is enough practice, move on to Question 4.

1. a) Given the equation $y = -2x + 8$, create a table of values with five ordered pairs. Choose any 5 reasonable x-values (start with values between −10 and 10), substitute them into the equation and solve for y. Record the values in a chart.

   Answer:

   $y = -2x + 8$

   Your table of values may have different ordered pairs.

x	y
0	8
2	4
4	0
9	−10
10	−12

Example:

Solve for $x = 0$

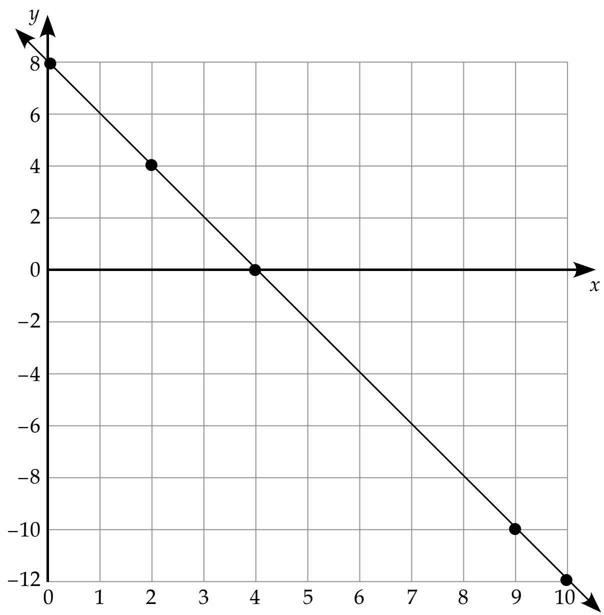
$$y = -2x + 8$$
$$y = -2(0) + 8$$
$$y = 0 + 8$$
$$y = 8$$

Solve for $x = 2$

$$y = -2x + 8$$
$$y = -2(2) + 8$$
$$y = -4 + 8$$
$$y = 4$$

b) Graph the line using grid paper. Remember to choose an appropriate y-scale for your graph, one that will include all the y-values calculated. To determine what increment to use along the y-axis, calculate the range (subtract smallest y-value from the largest y-value), divide by the number of increments along the y-axis and round up.

Answer:



To determine the y-scale:

$8 - (-12) = 20$

$20 \div 10$ increments $= 2$

Begin at −12 and go up to 8 using intervals of 2.

c) Calculate the slope of the line.

Answer:

Choose any two points. (0, 8) and (10, −12)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-12 - 8}{10 - 0}$$

$$m = \frac{-20}{10} = -2$$

d) Determine the y-intercept of the line.

Answer:

When $x = 0$, $y = 8$. The y-intercept is at $y = 8$.

e) Verify your answers to (c) and (d) using the equation provided in (a).

Answer:

The slope was calculated to be −2 and the y-intercept was determined to be at $y = 8$. From the given equation, $y = -2x + 8$, the coefficient of x is the slope, and it is −2 and the constant is the y-intercept. It is 8. My answers are correct.

2. a) Given the equation $y = \frac{2}{3}x + 3$, create a table of values with 5 ordered pairs. If you have a fraction coefficient, choose x values that are multiples of the denominator so that you can reduce the fraction and eliminate the denominator (start with values like 3, 6…), substitute them into the equation and solve for y. Record the values in a chart.

Answer:

$$y = \frac{2}{3}x + 3$$

Possible values for the table:

x	y
0	3
3	5
6	7
9	9
−3	1

Example: Solve for $x = 0$

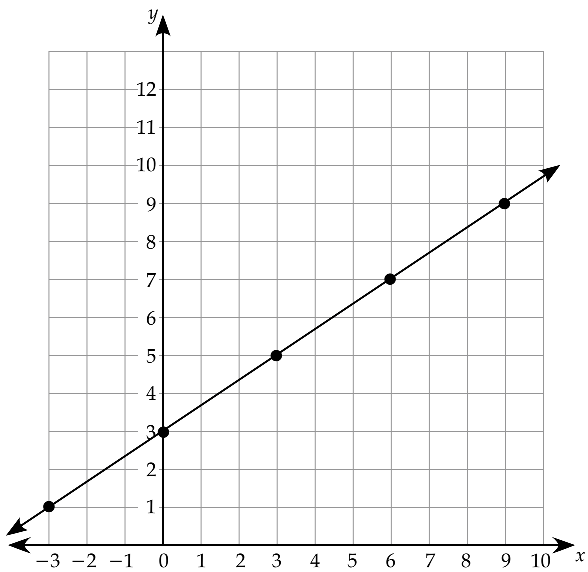
$$y = \frac{2}{3}(0) + 3$$
$$y = 0 + 3$$
$$y = 3$$

Solve for $x = 6$

$$y = \frac{2}{3}(6) + 3$$
$$y = \frac{2}{\cancel{3}}\cancel{(6)}2 + 3$$
$$y = 4 + 3$$
$$y = 7$$

b) Graph the line on grid paper.

Answer:



c) Calculate the slope of the line.

Answer:

Choose any two points. (0, 3) and (6, 7)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7-3}{6-0}$$

$$m = \frac{4}{6} = \frac{2}{3}$$

d) Determine the y-intercept of the line.

Answer:

The coordinates of the y-intercept are (0, 3).

e) Verify your answers to (c) and (d) using the equation provided in (a).

Answer:

The given equation was $y = \frac{2}{3}x + 3$. The coefficient of x is the slope and it was calculated to be $\frac{2}{3}$. The constant is the y-intercept and it was calculated to be at $y = 3$. The answers in (c) and (d) are verified as correct using the equation provided in (a).

3. a) Given the equation $y = -5x + 48$, create a table of values with 5 ordered pairs.

Answer:

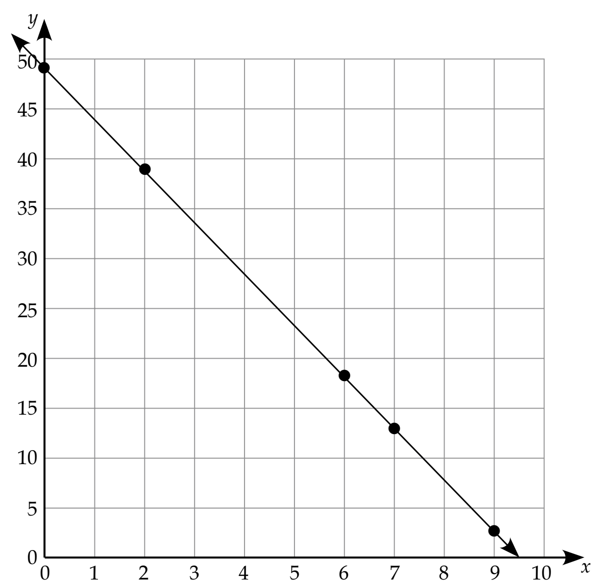
$y = -5x + 48$

Ordered pairs may vary. Show your calculations.

x	y
0	48
2	38
6	18
7	13
9	3

b) Graph the line on grid paper.

Answer:



c) Calculate the slope of the line.

Answer:

Choose any two points. (0, 48) and (2, 38)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{38 - 48}{2 - 0}$$

$$m = \frac{-10}{2} = -5$$

d) Determine the y-intercept of the line.

Answer:

The y-intercept is at $y = 48$.

e) Verify your answers to (c) and (d) using the equation provided in (a).

Answer:

The equation $y = -5x + 48$ verifies that the slope is -5 and the y-intercept is at $y = 48$.

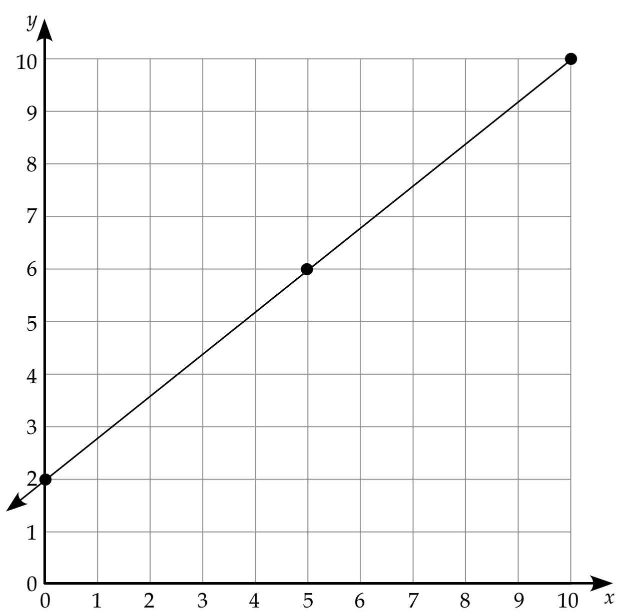
4. a) Write the equation of a line with a y-intercept of 2 and a slope of $\frac{4}{5}$.

Answer:

$$y = \frac{4}{5}x + 2$$

b) Sketch the graph. ***Do not*** create a table of values or ordered pairs, except to check your work.

Answer:



Locate the y-intercept on the graph at (0, 2). From that point, use the slope $\frac{\text{ryse}}{\text{run}} = \frac{4}{5}$ and move up 4 and to the right 5 to the point (0 + 5, 2 + 4) or (5, 6). Repeat to find the next point at (10, 10).

Verify by substituting $x = 5$ and $x = 10$ into the equation $y = \frac{4}{5}x + 2$ and solving for y.

5. a) Given the equation $y = -\frac{4}{5}x + 9$, state the slope and y-intercept without graphing the line or creating a table of values.

   Answer:

   $$y = -\frac{4}{5}x + 9$$

   The slope of this line is $\frac{-4}{5}$ and the y-intercept is at 9.

   b) Explain how you would know where to draw the line.

   Answer:

   I would draw this line by locating the y-intercept at (0, 9) and then using the slope to find another point. I would move down 4 units and 5 units to the right to find another point at (5, 5). I would repeat this to find another point at (10, 1) and then connect the points with a straight line. I could also move 4 units up and 5 units down from (0, 9) to get to (−5, 5).

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 2
Number Sense

Module 2:
Number Sense

Introduction

Number sense, the theme of this module, has less to do with understanding numbers and words, and more to do with making sense of the ideas behind those numbers and words. For example, if you feel positive or negative, it is often a reflection of your attitude, but positive and negative numbers have nothing to do with happiness or sadness. Whether they are positive or negative is just an indicator of where the numbers are located on a number line compared to the zero's position.

This module will explore factors, multiples, and square and cube numbers along with their roots. You will see how the real number system is organized, and determine connections between radical numbers and powers. You will take what you have learned about exponent laws in Grade 9 and apply it to powers with variable bases and rational and negative exponents.

As you work through this module, concentrate on understanding the ideas behind the words and identifying patterns. As you do so, your sense of what numbers mean and what they can do will grow.

Assignments in Module 2

When you have completed the assignments for Module 2, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 2.1	Factors and Multiples
2	Assignment 2.2	Perfect Cubes and Squares
3	Assignment 2.3	Rational, Irrational, and Radical Numbers
4	Assignment 2.4	Exponent Laws Review
5	Assignment 2.5	Exponent Laws with Rational and Negative Exponents

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 2. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1, 2, 3, and 4 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 4.

Resource Sheet for Module 2

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Lesson 1: Factors and Multiples

Lesson Focus

In this lesson, you will

- ❑ determine the prime factors of a whole number
- ❑ explain why 0 and 1 are neither prime nor composite
- ❑ determine the greatest common factor and least/lowest common multiple of numbers in a variety of ways and explain the process
- ❑ solve problems using factors and multiples

Lesson Introduction

In this lesson, you will explore different ways to determine the greatest common factor and the least (or lowest) common multiple of two or more numbers. Special attention will be given to understanding prime factors and the unique status of the numbers 0 and 1.

The Basics of Numbers

Prime and Composite Numbers

In the equation $2 \times 3 = 6$, the 2 and 3 are the **factors** that you multiply to get the **product** of 6. 6 is a **multiple** of both 2 and 3.

A **factor** is a number or expression that is multiplied by another number or expression to get a product.

A **product** is the number that results when two or more factors are multiplied.

A **multiple** is the product of a given number and any other integer (an integer is any positive or negative number, or zero.)

You can arrive at the product of 16 by multiplying the following number pairs or factors:

$16 = 1 \times 16$

$16 = 2 \times 8$

$16 = 4 \times 4 = 4^2$

Writing these numbers in order, you can see that the factors of 16 are 1, 2, 4, 8, and 16. 16 is divisible by all these factors.

Of these numbers, 2 is considered to be a **prime** number, and 4, 8, and 16 are called **composite** numbers.

A **prime** number is an integer greater than 1 that has exactly two different factors: 1 and itself.

A **composite** number is an integer greater than 1 that has more than two factors.

Circle all the prime numbers and put an X through all the composite numbers on this chart:

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49

You should have circled the following prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47

The only factors these numbers have are 1 and itself. These numbers can only be divided evenly by 1 and itself.

Composite numbers are any integers greater than 1 that are not prime. You should have put an X over the following numbers:

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, and 49.

All these numbers have more than two factors. For example, the factors of 9 are 1, 3, and 9. 24 can be divided evenly by 1, 2, 3, 4, 6, 8, 12, and 24.

Did you notice that the numbers 0 and 1 are not in either list? These two numbers are unique, as they are neither prime nor composite. 1 is considered a "unit" (a unit is any quantity used for measurement, similar to a pound or a metre), and has only one factor or divisor. To be prime, a number needs two different factors, and to be composite it needs more than two. Zero, on the other hand, has an infinite number of divisors, as zero can be divided evenly by any value (it would still equal zero), and so it is not prime. Try it out on your calculator. However, you cannot multiply two different non-zero values and have a product of zero, so zero cannot be composite.

Prime Factorization

When you write a number as the product of its prime factors, it is called "prime factorization."

$24 = 4 \times 6$	4 and 6 are factors of 24, but they are composite numbers, which may be factored further.
$24 = 2 \times 2 \times 2 \times 3$	These factors are all prime numbers. This is the prime factorization of 24.

Example 1

Determine the prime factorization of 180.

Solution:

One method is to start by dividing by the smallest possible prime number, and repeat with each new value until the result itself is a prime. 180 is an even number, so divide by 2.

$180 \div 2 = 90$	90 is also divisible by 2, so divide again.
$90 \div 2 = 45$	45 is divisible by 3.
$45 \div 3 = 15$	15 is also divisible by 3.
$15 \div 3 = 5$	5 is a prime number and has no further factors other than 1 and itself.

List the divisors and the final result. These are the prime factors of 180.

$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

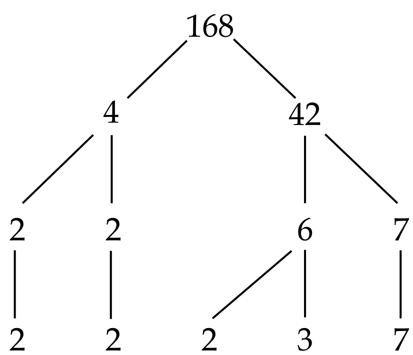
Check your answer by multiplying the factors.

Example 2

Determine the prime factors of 168.

Solution:

Another method is to use a "factor tree" diagram. Use division to find any two factors of the given value. Write them as branches on the factor tree.
$168 \div 4 = 42$



Both of these values are composite. Divide each by a factor.

2 and 7 are prime, but 6 is composite.

These are the prime factors of 168.

There is only one correct prime factorization of a given value, but the factors may be written in any order. Notice that 1 is not used in the prime factorization of numbers.

Greatest Common Factors

If two or more numbers have the same factor, it is called a common factor.

The prime factors of

$30 = 2 \times 3 \times 5$

$18 = 2 \times 3 \times 3$

Notice the common factors of 2 and 3. The product of these common prime factors is called the greatest common factor.

$2 \times 3 = 6$

The greatest common factor (from now on referred to as GCF) is the largest number that divides two or more numbers. The GCF of 30 and 18 is 6, because no larger number divides both 30 and 18. If two numbers have no common prime factor, the GCF is 1.

Example 3

Find the greatest common factor of 60 and 28.

Solution:

First, determine the prime factors of each value. Use any method you like.

a) Division method:

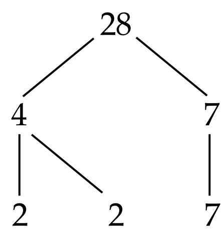
$60 \div 2 = 30$

$30 \div 2 = 15$

$15 \div 3 = 5$

The prime factors of 60 are 2, 2, 3, and 5.

b) Factor Tree method:



The prime factorization of $28 = 2 \times 2 \times 7$.

Compare the prime factors and determine whether there are any common prime factors.

$60 = 2 \times 2 \times 3 \times 5$

$28 = 2 \times 2 \times 7$

Each value has two common 2s. The product of the prime factors is the GCF.

$2 \times 2 = 4$

The greatest common factor of 60 and 28 is 4.

Example 4

Find the greatest common factor of 12, 30, and 42.

Solution:

Using the prime factorization of each value, calculate the product of any common factors.

$$12 = (2) \times 2 \times (3)$$

$$30 = (2) \times (3) \times 5$$

$$42 = (2) \times (3) \times 7$$

The common factors are 2 and 3.

$2 \times 3 = 6$

The GCF of 12, 30, and 42 is 6.

Another way to think about this is that 12, 30, and 42 are all multiples of 6.

$6 \times 2 = 12$

$6 \times 5 = 30$

$6 \times 7 = 42$

Least Common Multiples

If you were to begin listing the multiples of 6 and the multiples of 8, it would look something like this:

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72…

0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96…

Notice the values that are common to both lists: 0, 24, 48.

The least or lowest common multiple (from now on referred to as the LCM) is defined as the smallest *positive* multiple common to a set of numbers. 0 is not considered a positive (or negative) value, so it does not qualify as the LCM of a set of numbers. In the case of 6 and 8, the LCM would be 24. 24 is the smallest number that can be evenly divided by both 6 and 8.

You can use prime factorization to help you find the LCM.

Example 5

Find the least common multiple of 24 and 90.

Solution:

Determine the prime factors of 24 and 90.

$24 = 2 \times 2 \times 2 \times 3$

$90 = 2 \times 3 \times 3 \times 5$

Write the repeated multiplications using exponents, like you learned in Grade 9.

Since $2 \times 2 \times 2$ means multiply 2 by itself three times, this can be written as 2^3.

3×3 means multiply 3 times itself two times, and can be written as 3^2.

5 multiplied by itself only once can be written as 5^1.

So the prime factorization of these numbers can be written as

$24 = 2^3 \times 3^1$

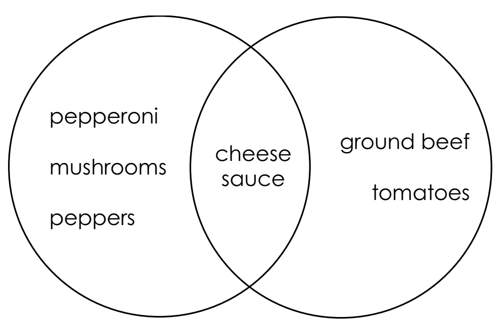
$90 = 2^1 \times 3^2 \times 5^1$

To find the LCM, multiply each prime factor with the highest exponent. The highest exponent on 2 is 3, the highest exponent on 3 is 2, and the highest exponent on 5 is 1.

$2^3 \times 3^2 \times 5^1 = 360$

The lowest common multiple of 24 and 90 is 360.

A Venn diagram can be used to display the prime factors of two numbers and calculate the LCM and GCF. A Venn diagram uses circles to represent sets. Where the circles overlap, the sets share common elements. Suppose you ordered two pizzas—one with pepperoni, mushrooms, peppers, cheese, and sauce, and another with ground beef, tomatoes, cheese, and sauce. A Venn diagram showing the toppings may look like this:



From the diagram, you can tell which toppings both pizzas have (in the overlapping area) and which toppings are unique to each pizza.

Example 6

Create a Venn diagram to illustrate the prime factors of 28 and 36. Find the LCM and GCF of 28 and 36.

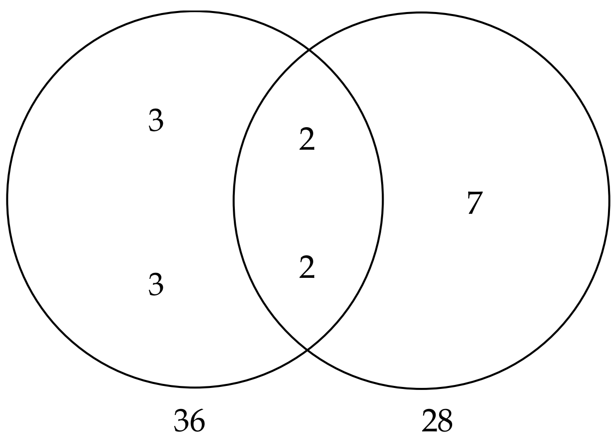
Solution:

Determine the prime factors of each value, and arrange them in the Venn diagram with the common factors written in the overlapping area.

$28 = 2 \times 2 \times 7$

$36 = 2 \times 2 \times 3 \times 3$

The common factors are the two 2s.



Using the Venn diagram, you can find both the LCM and GCF.

If you multiply all the factors in diagram, you have the LCM.

$3 \times 3 \times 2 \times 2 \times 7 = 252$

252 is the smallest number that is divisible by both 36 and 28.

If you find the product of the factors in the overlapping area, you have the GCF.

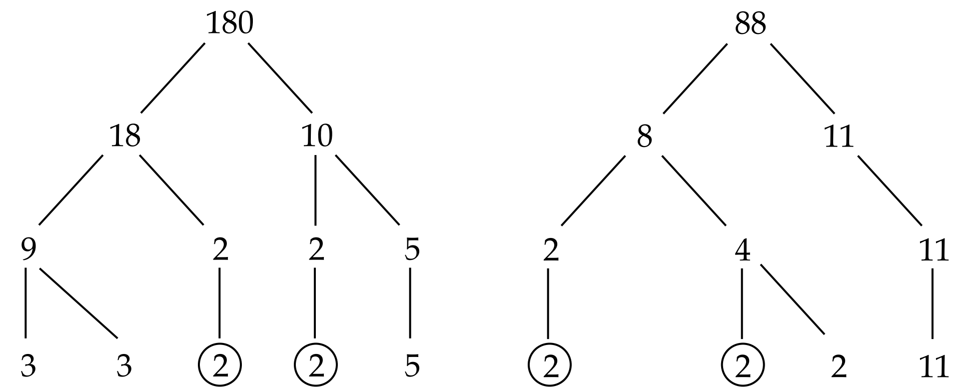
$2 \times 2 = 4$

4 is the largest number by which both 36 and 28 are divisible.

If you know the GCF of two numbers, you can use that to find the LCM of those two numbers.

Example 7

The greatest common factor of 180 and 88 is 4. This is evident from the following factor trees.



Multiply the two original numbers and divide by the GCF and you have the LCM.

$$180 \times 88 = 15840$$

$$15840 \div 4 = 3960$$

The lowest common multiple of 180 and 88 is 3960. Note that this method only works if you are finding the LCM of two numbers.

Learning Activity 2.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. You are at the store to buy your mother a present. You have $30 to spend and would like to get her a waffle maker. If it costs $40 and is 20% off, will you be able to get her the waffle maker?
2. There are three pairs of socks in a package. If the whole package costs $6, how much does it cost per pair of socks?
3. In a football game, the final score is 12 to 28. Since a touchdown without a convert is worth 6 points, is it possible that one team did not get any field goals?
4. It's been raining for the past three hours in Winnipeg. They had 5 mm in the first hour, 2 mm in the next hour, and 5 mm in the last hour. What was the average rainfall per hour?
5. Fill in the blanks for the following pattern: 1, 4, ____, 16, 25, ____.
6. Evaluate 9^2.
7. Rewrite the fraction in simplest terms: $\dfrac{33}{21}$.
8. You have a box with a length of 3 m, a width of 2 m, and a height of 6 m. What is the volume of this box?

continued

Learning Activity 2.1 (continued)

Part B: Factors and Multiples

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. How many factors do prime numbers have? What are they?
2. Fill in the following chart by
   a) listing all the factors of the given composite numbers
   b) determining the number of factors each composite number has
   c) indicating whether the number of factors is an odd or even number
   d) listing the prime factorization of each number

Given Number	(a) List of All Factors of the Composite Number	(b) # of Factors	(c) Odd or Even?	(d) Prime Factors
4	1, 2, 4	3	Odd	$2 \times 2 = 2^2$
6				
8				
9				
12				
16				
21				
24				
25				
27				
36				

   e) Circle the given composite numbers that have an odd number of factors. What pattern do you notice in these numbers? Explain.

3. Determine the prime factors of 210 using a factor tree diagram. Include your diagram.

continued

Learning Activity 2.1 (continued)

4. Determine the prime factors of 84 using the division method. Show your work.
5. List the prime factors of 147. Explain how you found your answer and show your work.
6. Using your answers from questions 3, 4, and 5, state the GCF of 210, 84, and 147. Show your work and explain how you found your answer.
7. State the LCM of 210, 84, and 147. Show your work and explain how you found your answer.
8. In your own words, define what the LCM of a set of numbers is.
9. Draw a Venn diagram to illustrate the prime factors of 45 and 75. Explain how you can use the diagram to calculate the GCF and LCM of 45 and 75. State the GCF and LCM.

Lesson Summary

In this lesson, you learned how to determine the factors and prime factors of numbers, as well as calculate the greatest common factor and least/lowest common multiple of a set of numbers. By defining terms, looking for and describing patterns, and using different ways to represent concepts, you are developing your number sense abilities and learning important math ideas. In the next lesson, you will explore perfect square numbers, perfect cube numbers, and the roots of these numbers.

Assignment 2.1

Factors and Multiples

Total Marks = 28

1. Why are most prime numbers odd? *(2 marks)*

2. a) List the factors of 66. *(2 marks)*

   b) List the prime factors of 66. *(2 marks)*

   c) What is the GCF of 66 and 70? *(2 marks)*

Assignment 2.1: Factors and Multiples (continued)

3. a) Determine the prime factorization of 60 using the division method. Show your work. *(3 marks)*

   b) Determine the prime factors of 20 by drawing a factor tree. *(3 marks)*

   c) What is the GCF of 20 and 60? *(2 marks)*

   d) Rewrite the prime factors of 20 and 60 using exponents to indicate repeated multiplications. *(2 marks)*

   e) What is the LCM of 20 and 60? Show your calculation using the exponents from part (d). *(2 marks)*

Assignment 2.1: Factors and Multiples (continued)

4. In your own words, define the GCF of a set of numbers. *(3 marks)*

5. a) Use a Venn diagram to illustrate the prime factors of 102 and 18. *(3 marks)*

   b) Use the diagram to determine the GCF and LCM of 102 and 18. *(2 marks)*

Notes

Lesson 2: Squares, Cubes, and Roots

Lesson Focus

In this lesson, you will

- ❑ determine whether numbers are perfect squares, perfect cubes, or neither
- ❑ determine the roots of perfect square numbers and perfect square cubes
- ❑ solve problems involving the square roots and cube roots

Lesson Introduction

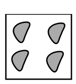
What does the word *perfect* mean to you? In this lesson, you will consider what makes a perfect square number, a perfect cube number, and determine the roots of these numbers.

Powers

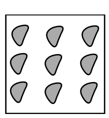
Perfect Square Numbers

In the 6th century B.C.E., Greek mathematicians investigated the special properties of numbers. They discovered that certain numbers could be represented with pebbles arranged in a square.

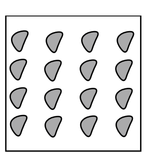
= 1 This square has dimensions of 1 by 1.
Its area would be calculated as $1 \times 1 = 1^2 = 1$ (recall that area of a square = length × width).



= 4 This square has dimensions of 2 by 2.
Its area would be calculated as $2 \times 2 = 2^2 = 4$.



= 9 This square has dimensions of 3 by 3.
Its area would be calculated as $3 \times 3 = 3^2 = 9$.



= 16 This square has dimensions of 4 by 4.
Its area would be calculated as $4 \times 4 = 4^2 = 16$.

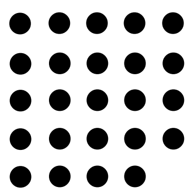
How can you tell if a given number is a perfect square?

Example 1

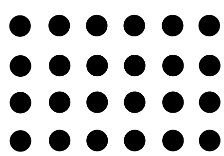
Is 24 a perfect square number?

Solution:

Find some small objects, like bingo chips, pennies, o-shaped cereal, or buttons. Count out 24 and see if you can arrange them in a square. All of the objects must be in straight rows and columns containing the same number of objects (as in the diagram below). If the 24 items form a square with no leftover pieces and no gaps or spaces, then 24 is a perfect square number.



There is one piece missing in this square arrangement, so 24 is not a perfect square number.



You could place the 24 items in this arrangement and not have any gaps, but the dimensions of this shape are 4 by 6. This is a rectangle, but not a square. 24 is not a **perfect square number**.

A **perfect square number** is the product of an integer multiplied by itself.

Remember that an integer can only be a whole number (positive or negative) or zero.

Example 2

Is 25 a perfect square number?

Solution:

Write the prime factors of 25: 5×5 This is a factor multiplied by itself, so 25 is a perfect square number.

Another way to determine whether 25 is a perfect square is to write out its factors. Recall from Lesson 1 that 25 has an odd number of factors. They are 1, 5, and 25. 25 can be written as the product of 5×5. If a number can be written as the product of a factor and itself, it is a perfect square number.

The square numbers are found by multiplying an integer by itself. The following chart shows the square numbers for 1 to 15..

Positive Integers	Perfect Square Numbers
$1 \times 1 = 1^2$	1
$2 \times 2 =$	4
$3 \times 3 =$	9
$4 \times 4 =$	16
$5 \times 5 =$	25
$6 \times 6 =$	36
$7 \times 7 =$	49
$8 \times 8 =$	64
$9 \times 9 =$	81
$10 \times 10 =$	100
$11 \times 11 =$	121
$12 \times 12 =$	144
$13 \times 13 =$	169
$14 \times 14 =$	196
$15 \times 15 =$	225

There are many interesting patterns and characteristics of perfect square numbers. What kinds of patterns do you see?

Did you notice that perfect square numbers never end with 2, 3, 7, or 8?

Perfect square numbers are also the sum of consecutive odd numbers. The chart below shows the sum of the given number of odd numbers, starting with 1.

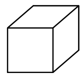
Number of Consecutive Odd Numbers	Sum of Odd Number	Perfect Square Number
1	1 = 1	1
2	1 + 3 = 4	4
3	1 + 3 + 5 =	9
4	1 + 3 + 5 + 7 =	16
5	1 + 3 + 5 + 7 + 9 =	25
6	1 + 3 + 5 + 7 + 9 + 11 =	36
7	1 + 3 + 5 + 7 + 9 + 11 + 13 =	49
8	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 =	64
9	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 =	81
10	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 21 =	100

Perfect Cube Numbers

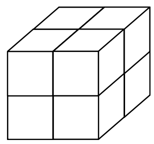
If perfect square numbers can be arranged in a two-dimensional square area and are the product of an integer multiplied by itself, can you guess what perfect cube numbers are?

Perfect cube numbers are found when an integer is multiplied by itself three times.

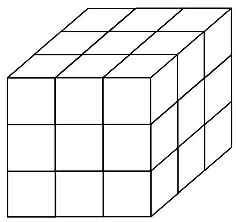
Perfect cube numbers can be visualized as adding the third dimension of depth to a square, or as calculating the volume of an object.



This cube has dimensions of 1 by 1 by 1.
Its volume would be calculated as $1 \times 1 \times 1 = 1^3 = 1$
(recall that volume of a cube = height × width × depth).



This cube has dimensions of 2 by 2 by 2.
Its volume would be calculated as $2 \times 2 \times 2 = 2^3 = 8$.



This cube has dimensions of 3 by 3 by 3.
Its volume would be calculated as $3 \times 3 \times 3 = 3^3 = 27$.

How can you tell if a number is a perfect cube number?

Example 3

Is 40 a perfect cube number?

Solution:

Find 40 small cubes or blocks, and see if you can arrange them into a perfect cube.

A $3 \times 3 \times 3$ cube requires only 27 blocks, so it will have to be larger than that.

You would need $4 \times 4 \times 4 = 64$ blocks to make the next perfect cube, and this is more than 40. 40 is not a perfect cube number.

This reasoning works because integers are whole numbers only.

This method is most effective when you are able to estimate what the root might be, so it is harder to do using larger numbers, as in the next two examples.

Example 4

Is 125 a perfect cube number?

Solution:

Write out the prime factors of 125. Use the division method.

$125 \div 5 = 25$

$25 \div 5 = 5$ 5 is a prime number and cannot be factored further.

The prime factors of $125 = 5 \times 5 \times 5$.

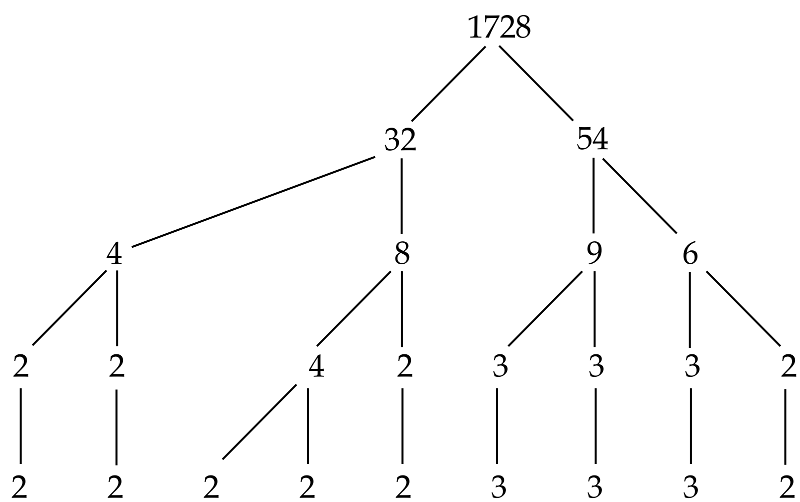
The prime factor is an integer multiplied by itself three times, so 125 is a perfect cube number.

Example 5

Is 1728 a perfect cube number?

Solution:

Write out the prime factorization of 1728 using a factor tree.



There are six 2s and three 3s in the prime factorization. These numbers can be rearranged and grouped in three equal sets: (2 2 3) (2 2 3) (2 2 3) or $12 \times 12 \times 12 = 12^3$. 12 multiplied by itself three times equals 1728, so 1728 is a perfect cube number.

Roots

A number that is multiplied by itself twice to produce a perfect square number is called the **square root** of that number.

If $7 \times 7 = 49$, then 49 is a perfect square number and the square root of 49 is 7.

The number that is multiplied by itself three times to produce a perfect cube number is called the **cube root** of that number.

Square root and cube root are common terms in this lesson as well as the next few, so you may want to include their definitions or examples on your Resource Sheet.

If $6 \times 6 \times 6 = 216$, then 216 is a perfect cube number and the cube root of 216 is 6.

The mathematical symbol used to represent a root is $\sqrt{\ }$.

Generally, this symbol is understood to mean the square root of a number, so when talking about cube roots, an index is used to distinguish it from a square root. A cube root symbol is $\sqrt[3]{\ }$. The number 3 is called the index.

$$\sqrt[3]{216} = 6$$

So far, you have been exploring perfect square numbers and perfect cube numbers with positive integer roots. Roots may be negative integers as well. Consider this:

$$(7)(7) = 49 \text{ and } (-7)(-7) = 49$$

The root of 49 may be +7 or −7.

Can a perfect square number ever be negative?

Any value, whether positive or negative, when multiplied by itself, will result in a positive product.

$$(13)(13) = (13)^2 = 169$$
$$(-13)(-13) = (-13)^2 = 169$$

Notice that the negative is inside the brackets $(-13)^2 = 169$ instead of outside the brackets $-(13)^2 = -169$.

Note: (13)(−13) = −169, but in this case the two factors multiplied were not the same. This could not be written using an exponent of two, and so negative numbers are not perfect square numbers.

Because perfect square numbers are always positive numbers, you cannot take the square root of a negative number.

$\sqrt{-169}$ does not have a real number solution. (In future math courses, you may work with the square roots of a negative number.)

What happens when you cube a negative number?

$$(-3)(-3)(-3) = -27$$

The product of three negative numbers is also a negative number. For this reason, it is possible to find the cube root of a negative number as well as of positive numbers.

$$\sqrt[3]{-27} = -3$$

$$\sqrt[3]{27} = 3$$

Assume that when you are asked to find the square root of a number, it is asking only for the positive root $\left(\sqrt{25} = 5\right)$ from this point forward. The question will otherwise ask you to find all possible roots.

You can determine the square root or cube root of a number using strategies similar to the ones you used when trying to determine whether a number was a perfect square or a perfect cube.

Example 6

What is the square root of 256?

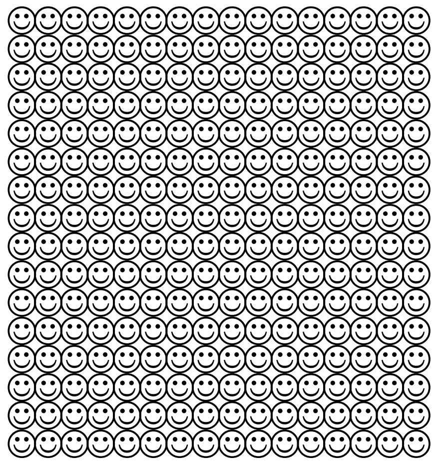
Solution:

If you have 256 small items, you can try to arrange them in a square shape. If it works out perfectly, the dimensions of your square (the length and the width) are the root of 256. It may not be practical to work with that many small items, so drawing a diagram may be easier than constructing a square.

Alternately, you may find the root by finding the factor pairs of 256.

The factor pairs of 256 are 1, 256
2, 128
4, 64
8, 32
16, 16

You will notice that the last factor pair is a number multiplied by itself. This is the square root of 256. If you had drawn the arrangement of 256 small objects into a square area, the dimensions would have been 16 by 16.



Example 7

What is the square root of 1024?

Solution:

Scientific calculators can be used to determine the square root of large numbers when other strategies are inconvenient. Each calculator has unique keystrokes, so you should read the manual that came with your calculator to figure out how your calculator works. Often the $\sqrt{}$ is on the same button as the squared function $\boxed{x^2}$ key and you need to use a second function or shift key to access the square root. Typical keystrokes may include:

$\boxed{1}$ $\boxed{0}$ $\boxed{2}$ $\boxed{4}$ $\boxed{\text{2nd}}$ $\boxed{\sqrt{}}$ $\boxed{=}$

or

$\boxed{\text{2nd}}$ $\boxed{\sqrt{}}$ $\boxed{1}$ $\boxed{0}$ $\boxed{2}$ $\boxed{4}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

Try different keystroke combinations until you find the correct answer. The square root of 1024 is 32. Record the correct keystroke sequence for your calculator below.

$\boxed{}$ $\boxed{}$ $\boxed{}$ $\boxed{}$ $\boxed{}$ $\boxed{}$ $\boxed{}$ $\boxed{}$

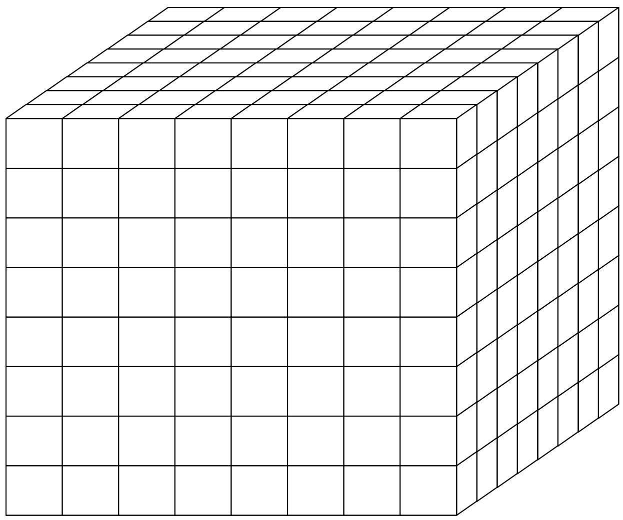
Example 8

What is the cube root of 512?

Solution:

Working with 512 individual blocks to form a cube would be quite a challenge, but you could represent this situation with a drawing.

$8 \times 8 \times 8 = 8^3 = 512$



You may also use a calculator to determine the cube root of a number. Some calculators have buttons with a cube root function $\boxed{\sqrt[3]{x}}$. Other calculators may have a generic root key that looks like $\boxed{\sqrt[x]{y}}$. In this case, you need to identify the index for the calculator. In some graphing calculators, the cube root function may be in a separate math menu. Again, you will have to experiment with your calculator or read the manual that came with it to figure out the unique keystrokes required for your calculator. Some typical keystroke sequences may be as follows:

$\boxed{5}$ $\boxed{1}$ $\boxed{2}$ $\boxed{\text{2nd}}$ $\boxed{\sqrt[3]{\ }}$ $\boxed{=}$

or

$\boxed{\text{2nd}}$ $\boxed{\sqrt[3]{\ }}$ $\boxed{5}$ $\boxed{1}$ $\boxed{2}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

or

$\boxed{3}$ $\boxed{\sqrt[x]{y}}$ $\boxed{5}$ $\boxed{1}$ $\boxed{2}$ $\boxed{=}$

or

$\boxed{\text{MATH}}$ $\boxed{\blacktriangledown}$ $\boxed{\blacktriangledown}$ $\boxed{\blacktriangledown}$ $\boxed{\text{ENTER}}$ $\boxed{5}$ $\boxed{1}$ $\boxed{2}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

(graphing calculator)

Experiment with your calculator until you can be certain of the keystrokes you need to use to find the correct answer $\sqrt[3]{512} = 8$.

Record in the following squares the keystrokes you need to follow to calculate the cube root of 512 on your calculator.

Learning Activity 2.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. What is the GCF of 12 and 18?
2. What is the LCM of 8 and 12?
3. What two numbers have a sum of 5 and a product of 6?
4. Which is the independent variable: as you study more, your test marks increase.
5. If a desk is 90 cm high, how tall is it in metres?
6. You give the cashier a $10 bill to pay for your lunch. If the total for your lunch is $7.60, how much change will you get back?
7. Which is smaller: $\frac{4}{5}$ or $\frac{7}{10}$?
8. A basic calculator has 6 rows and 5 columns of buttons. How many buttons does it have in total?

continued

Learning Activity 2.2 (continued)

Part B: Perfect Cubes and Squares

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Indicate whether the following numbers are perfect cubes, perfect squares, or neither. Support your answer with a diagram and/or an explanation of the process you used.

   a) 169 b) 324 c) −512 d) 111 e) 8 f) 64

2. State **all possible** square roots for the following numbers. Explain your answer.

   a) 144 b) −36

3. State **all possible** cube roots for the following numbers. Explain your answer.

   a) −216 b) 8

4. The year 2011 marked the 144th birthday of Canada. In the Confederation Park garden, Erika wanted to plant 144 flowers to commemorate July 1, 1867. How could she have arranged these flowers to make a square design?

Lesson Summary

In this lesson, you learned about perfect square numbers, perfect cube numbers, square roots, and cube roots. You used a variety of strategies to determine whether numbers are perfect squares or perfect cubes, and used different methods to calculate the roots, including looking at patterns, factors, prime factors, drawing diagrams, or using objects, and using technology. In the next lesson, you will consider different sets of numbers, including rational and irrational numbers, as well as radical numbers.

Assignment 2.2

Perfect Cubes and Squares

Total Marks = 23

1. Complete the following chart, listing the first 10 perfect cube numbers. *(4 marks)*

1^3	2^3	3^3	4^3	5^3	6^3	7^3	8^3	9^3	10^3
1	8								

2. Indicate whether the following numbers are perfect cubes, perfect squares, or neither. Support your answer with a diagram and/or an explanation of the process you used.

   a) 1000 *(2 marks)*

   b) 200 *(2 marks)*

c) 1 *(2 marks)*

d) 196 *(2 marks)*

2. State **all possible** square roots for the following numbers. Explain your answer.

a) 289 *(2 marks)*

____

____

____

b) -9 *(2 marks)*

____

____

____

Assignment 2.2: Perfect Cubes and Squares (continued)

3. State all possible cube roots for the following numbers. Explain your answer.

   a) 216 *(2 marks)*

   ____

   ____

   ____

   b) –343 *(2 marks)*

   ____

   ____

   ____

4. Ryan is organizing the sound system for an outdoor concert that is to be held in a football stadium. There are eight cube-shaped speakers that need to be arranged as compactly as possible, but in such a way that people on all four sides of the field can hear the music. Design a way for Ryan to stack the eight cubes to allow for the best sound. Assume that music only comes out from one side of each speaker. Explain your arrangement and include a diagram. *(3 marks)*

Notes

Lesson 3: Rational, Irrational, and Radical Numbers

Lesson Focus

In this lesson, you will

- ❑ represent the subsets of numbers in the real number system
- ❑ sort, order, and approximate irrational numbers
- ❑ express values as mixed and entire radical numbers

Lesson Introduction

Numbers can belong either to the real or to the imaginary number systems. In this course, you will only work with real numbers, and the study of imaginary numbers will be left to a future mathematics course. This lesson will focus on two subsets of the real number system: rational and irrational numbers. You will sort, order, and approximate irrational numbers, and learn about radical numbers.

Types of Numbers

The Real Number System

The meaning of the word “number” has changed over the course of mathematical history.

Natural Numbers (N)

At first, ”number” meant something you could count, like 15 pebbles or three coins. This set of counting numbers is the simplest and smallest subset of numbers. It begins with 1 and goes on to infinity, symbolized as ∞, and can be represented as N = {1, 2, 3,…}.

Note: the three dots … mean the list keeps on going.

Whole Numbers (W)

It is possible to have no pebbles or coins, and so zero was invented to represent this number. The whole number subset includes zero and all the natural numbers:

W = {0, 1, 2, 3,…}.

Integers (I) or (Z)

Worse than having no money, you could find yourself in a position of owing someone else three coins. This situation could be expressed by assigning a negative value to a counting number like 3. 3 and –3 are opposites. Zero is its own opposite, and so is not considered positive or negative. The subset of integers includes all positive and negative natural numbers and zero:

I = {…–3, –2, –1, 0, 1, 2, 3, …}.

Rational numbers (Q)

A rational number is a number that can be written as a ratio or fraction, like $\frac{a}{b}$, where a and b are integers and $b \neq 0$ (because it's impossible to divide by zero).

Proper fractions, like $\frac{1}{2}$ or $\frac{2}{3}$, are less than 1. Improper fractions are more than 1. For example, $\frac{5}{2}$ is an improper fraction, which may also be written as the mixed numeral $2\frac{1}{2}$. All integers (which include the sets of whole and natural numbers) can be thought of as rational numbers with a denominator of 1, as mentioned when talking about perpendicular slope in Module 1, Lesson 4.

The set of rational numbers is called **Q** because the line in a fraction implies division. The result of division is called a **q**uotient.

Rational numbers may be written in the form of decimals as well. To do so, divide the numerator (top number in the fraction) by the denominator (bottom number in the fraction). You will notice that all decimal equivalents of rational numbers either terminate or repeat.

Terminating Decimals

Rational numbers such as $\frac{3}{4}$ or $\frac{1}{10}$, when written as a decimal, have a finite number of decimal places.

$$\frac{3}{4} = 3 \div 4 = 0.75$$

$$\frac{1}{10} = 1 \div 10 = 0.1$$

$$\frac{-9}{8} = -9 \div 8 = -1.125$$

The decimal expansion of these quotients ends after these digits. No further place values are needed to fully express the exact value of these fractions.

Repeating Decimals

When converting rational numbers such as $\frac{1}{3}$ or $\frac{4}{33}$ to decimal equivalents, the decimal expansion repeats the same digit or pattern of digits indefinitely.

$$\frac{1}{3} = 0.33333333\ldots$$

$$\frac{4}{33} = 0.12121212\ldots$$

Express $\frac{11}{13}$ as a decimal and note the pattern in the digits.

Depending on the size of the screen on your calculator, it may not be evident that the first 6 digits in the decimal expansion are a repeating pattern. Your calculator will round off the number to fit the number of digits it can display, but it doesn't really end there.

$$\frac{11}{13} = 0.846153\overline{846153}\ldots$$

The bar above the last 6 digits indicates that this pattern repeats continually.

Any time you divide two integers, the decimal will either terminate or repeat, but your calculator may not display enough digits to show you the repeating pattern. $\frac{53}{83}$ only repeats after 41 digits.

Irrational Numbers (Q′)

It seems like a rational, logical idea to think that all numbers can be written as a fraction, but it is not so. There are numbers that cannot be written as fractions, and so are not rational numbers. They are called irrational numbers. The symbol Q′ is read as "Q prime" and means "not Q" or, in this case, not rational.

An irrational number cannot be expressed as a ratio of two integers, and if written as decimals the digits do not terminate or repeat in any predictable pattern, no matter how many place values you may determine. Some of the numbers in this subset include $\sqrt{2}$ and π.

π, called pi, is the ratio of a circle's diameter to its circumference. You may know this value as 3.14, but this is simply a decimal approximation of this theoretical calculation. You will never be able to write the exact value of an irrational number using decimals, only an approximation of its value. That is why symbols are used to express the value of an irrational number.

$\sqrt{2}$, the square root of 2, is also an irrational number. Its decimal approximation may be partially written as:

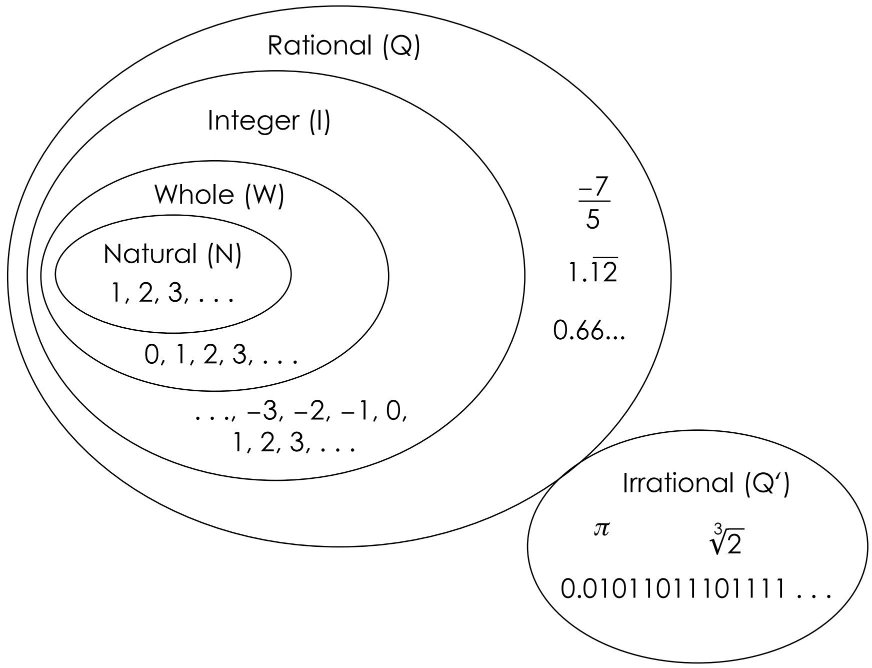
$\sqrt{2} \approx$ 1.4142135623 7309504880 1688724209 6980785696 7187537694 8073176679 7379907324 78462…

The wavy equal sign means "approximately equal to" and the ellipses (the three dots at the end) indicate that it is not possible to express this as an exact decimal value, as the digits continue with no set pattern.

Are all square roots irrational numbers? As you learned in the previous lesson, the square root of a perfect square number is a rational number. $\sqrt{4} = 2$ and so $\sqrt{4}$ is a rational number. So, some square roots are rational.

The real number system includes all rational and irrational numbers. Rational numbers can be categorized into the subsets of natural numbers, whole numbers, and integers. The following diagram illustrates the relationships of these number sets. As you move outwards from the centre set, N, the outer rings include all of the subsets inside it. W includes N, Q includes N, W, and I.

The Real Number System



All real numbers can be placed along a number line. Irrational numbers, as they do not have an exact decimal value, can only be approximated along a number line.

Including the symbols for the different types of numbers on your Resource Sheet may be helpful as a reminder.

Radical Numbers

In the last lesson, you squared and cubed values to get perfect square and perfect cube numbers. You found the square root and cube root of values. You used the $\sqrt{\ }$ symbol to represent square roots, and the $\sqrt[3]{\ }$ symbol to represent cube roots. In this lesson, you determined that the roots of some numbers are irrational numbers, like $\sqrt{2}$ or $\sqrt[3]{15}$, while some roots are rational numbers, like $\sqrt{4}$ or $\sqrt[3]{-27}$. Finding the root of a number is the opposite operation to applying an exponent like 2^2 or $(-3)^3$. Another name for "roots" is "radicals", and the symbol $\sqrt{\ }$ may also be called the radical symbol.

When working with radicals, it is more precise (and more convenient) to use the radical expression $\left(\text{e.g., } \sqrt{2}\right)$, rather than the decimal approximation of 1.414213562.... Decimal values of irrational numbers have been rounded and so are not exact. For this reason, it is important that you learn how to simplify radical expressions.

This section of the lesson can be particularly challenging as it combines ideas learned in the previous two lessons. Make sure you call your tutor/marker or your learning partner if you do not understand something.

Simplifying Radicals

Simplifying a radical expression such as $\sqrt{12}$ does ***not*** mean you find the decimal approximation. It means you write it as another expression with the same exact value, but so that no perfect square factors are left under the radical sign.

Step 1: 12 can be written as the product of 3 and 4.

To simplify $\sqrt{12}$, write it as $\sqrt{4 * 3}$ or $\sqrt{4} * \sqrt{3}$.

Step 2: It has a perfect square factor of 4, whose root is 2.

Rewrite the $\sqrt{4}$ as 2. The final product is an equivalent, simplified expression for $\sqrt{12} = 2\sqrt{3}$.

Step 3: You can check your expression by comparing the decimal approximations.

$\sqrt{12} \approx 3.464101615...$

$2\sqrt{3} \approx 3.464101615...$

The expressions are equivalent.

Example 1

Simplify $\sqrt{50}$.

Solution:

Determine whether the radicand (the number under the radical sign) has any perfect square factors, and write it as the product of this factor pair. Then simplify any perfect square numbers.

$$\begin{aligned}
50 &= 25 * 2 \\
\sqrt{50} &= \sqrt{25 * 2} \\
\sqrt{50} &= \sqrt{25} * \sqrt{2} \\
\sqrt{50} &= 5 * \sqrt{2} && \text{Simplify} \\
\sqrt{50} &= 5\sqrt{2}
\end{aligned}$$

$5\sqrt{2}$ is called a mixed radical in simplest form and $\sqrt{50}$ is called an entire radical.

Example 2

Simplify $\sqrt{1800}$.

Solution:

If the radicand is a large number, simplify it in several steps.

$$\begin{aligned}
1800 &= 100 * 18 \\
\sqrt{1800} &= \sqrt{100} * \sqrt{18} \\
&= 10 * \sqrt{18} && \{18 = 9 * 2\} \\
&= 10 * \sqrt{9} * \sqrt{2} \\
&= 10 * 3 * \sqrt{2} && \text{Simplify} \\
&= 30\sqrt{2}
\end{aligned}$$

If the number under the radical sign cannot be divided by any perfect square number, it is in simplest form.

Example 3

Simplify $\sqrt[3]{54}$.

Solution:

Determine whether 54 has any factors that are perfect cube numbers, and write the factor pair as a product under the radical sign. 54 = 27 × 2 and 27 is a perfect cube number.

$$\sqrt[3]{54} = \sqrt[3]{27 * 2}$$

$$= \sqrt[3]{27} * \sqrt[3]{2}$$

$$= 3 * \sqrt[3]{2} \qquad \text{Simplify}$$

$$= 3\sqrt[3]{2}$$

Example 4

Write $3\sqrt{6}$ as an entire radical.

Solution:

Step 1: Write an equivalent expression for 3 using a radical, $3 = \sqrt{9}$.

Step 2: Substitute that into the given radical and simplify.

$$3\sqrt{6} = \sqrt{9}\sqrt{6}$$

$$= \sqrt{9 * 6}$$

$$= \sqrt{54}$$

Learning Activity 2.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. How much larger is a line length 2 cm than a line length 1 cm?
2. How much larger is the area of a 2-by-2 square than a 1-by-1 square?
3. How much larger is the volume of a 2-by-2-by-2 box than a 1-by-1-by-1 box?
4. What is the GCF of 24 and 28?
5. Rewrite the fraction in simplest terms: $\frac{24}{28}$.
6. There are 3 boys and 2 girls invited to your birthday party. If each boy eats 2 pieces of cake and each girl eats 1 piece, how many pieces of cake will be eaten (not including yours)?
7. If each piece of the cake (from the question above) is $\frac{1}{9}$th of the cake, will there be enough cake for you to have a piece?
8. Evaluate the following: $2 - 3 + 6 \times 2 - 5 \times 4$.

continued

Part B: Rational, Irrational, and Radical Numbers

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Indicate whether the following numbers are rational or irrational, and explain your reasoning.

   a) $\sqrt{5}$

   b) $\sqrt[3]{-3}$

   c) $\sqrt{9}$

   d) 0.25878787...

   e) $\frac{1}{9}$

   f) $\frac{1}{4}$

   g) 0

   h) $\frac{-2}{1}$

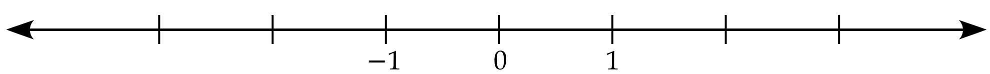
   i) −1.12131415...

   j) $-\frac{12}{4}$

2. Determine the exact or approximate decimal value of the numbers in Question 1. Include as many decimal places as your calculator will display.

3. Indicate in which subset of the real number system each of the rational numbers in Question 1 best fits.

4. Place each number from Question 1 at the appropriate point along the number line.

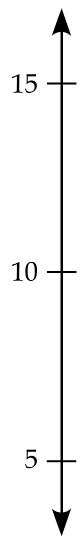


continued

Learning Activity 2.3 (continued)

5. Write the following mixed radicals as entire radicals, and place them in order along the number line.

   $6\sqrt{5}$, $2\sqrt{15}$, $7\sqrt{3}$, $4\sqrt{11}$



Lesson Summary

In this lesson, you learned that numbers can be rational, irrational, or radical, and the subset of the real number system in which these numbers best belong. The real number system was represented with a diagram to show the relationships of the subsets. You approximated the values of irrational numbers and placed them in order along a number line. You simplified radical numbers and wrote them as mixed and whole radicals. The next lesson will focus on the opposite of radicals—exponents—and you will review and expand on the exponent laws you learned in Grade 9 Mathematics.

Notes

Assignment 2.3

Rational, Irrational, and Radical Numbers

Total Marks = 29

1. Indicate whether the following values are rational (Q) or irrational (Q′) numbers. If they are rational, also indicate the subset in which they best belong. *(10 marks)*

   a) The number of NHL teams based in Saskatchewan

   ____

   b) The average temperature during month of January in Manitoba

   ____

   c) $\dfrac{-37}{39}$

   ____

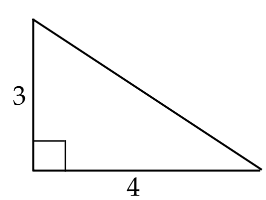
   d) $\sqrt{225}$

   ____

   e) The x-coordinate of the point $(-2, 7)$

   ____

   f) The perimeter of this right triangle



   ____

g) 0.234444444…

h) $\sqrt{56.4}$

i) The amount of profit if an item costs $7.60 to make and sells for $10.99. (Profit in this case refers to the difference between production cost and selling price.)

j) The area of a circle with radius of 4.2 cm. The formula for the area of a circle is $A = \pi r^2$.

2. Numbers are used on a daily basis in newspaper or magazine articles or in TV news reports.

   a) State a reasonable example of a number from each of the four subsets of the rational numbers (N, W, I, Q) that you may read or hear about in a news report. *(4 marks)*

   b) Why are irrational numbers unusual or difficult to find in news reports? *(1 mark)*

Assignment 2.3: Rational, Irrational, and Radical Numbers (continued)

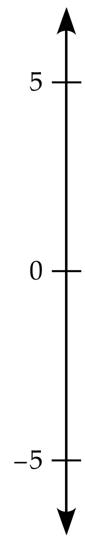
3. State the approximate value of each of the following irrational numbers, and place them in order along the number line. *(8 marks)*

   a) π ______________

   b) $\sqrt[3]{14}$ ______________

   c) $\sqrt{16.25}$ ______________

   d) $-2\sqrt{2}$ ______________



4. What does the index of 3 mean in the radical $\sqrt[3]{64}$? *(1 mark)*

   ____

   ____

   ____

   ____

5. Simplify the following radicals by writing them as mixed radicals. *(5 marks)*

   a) $\sqrt{80}$ ______________

   b) $\sqrt{288}$ ______________

   c) $\sqrt{1224}$ ______________

   d) $\sqrt{300}$ ______________

   e) $\sqrt[3]{-54}$ ______________

Notes

Lesson 4: Exponent Laws 1

Lesson Focus

In this lesson, you will

- ❑ demonstrate an understanding of powers with rational or variable bases and integral exponents (exponents that would fit in the integer subset of rational numbers)
- ❑ apply the following exponents laws: product law, quotient law, power of a power law, power of a product law, and the power of a quotient law

Lesson Introduction

In this lesson, you will apply what you learned about exponent laws in Grade 9 Math to include powers with variable and rational bases.

Combining Terms with Exponents

Powers

You have used exponents as a quick way to express repeated multiplications.

7^3 means 7 multiplied by itself three times.

$7^3 = 7 \times 7 \times 7$

7^3 may also be called a power. The base of the power is the part that gets multiplied by itself (in this case, the 7). It is what the exponent is applied to. The exponent tells you how many times the base is being multiplied by itself (in this case, 3 times).

7^3 can be read as “seven to the third power” or “seven raised to the third power” or “seven to the power of three.” Powers with exponents of 2 or 3 also have special names. A base raised to the power of two is called “squared” and to the power of three is called “cubed.”

The definition of a power x^n, where n is an integer, is:

Keep in mind that the exponent is only applied to the base (whatever number or expression is just to the left of the exponent).

Example 1

$$(-3)^4 = (-3)(-3)(-3)(-3) = 81$$

The base is (–3) because the exponent is just to the right of the bracket.

$$-3^4 = -(3)(3)(3)(3) = -81$$

The base is only the 3.

$$\left(\frac{2}{3}y\right)^5 = \left(\frac{2}{3}y\right)\left(\frac{2}{3}y\right)\left(\frac{2}{3}y\right)\left(\frac{2}{3}y\right)\left(\frac{2}{3}y\right) = \left(\frac{32}{243}y^5\right)$$

When working with powers with numerical bases (e.g., 3^4), it is easier to simplify and use its value of 81 in calculations. However, if you are using powers with a variable base, such as x^5, it is more convenient to use x^5 than $xxxxx$.

In previous courses, you have worked with powers with numerical bases. This lesson will include variable and rational bases. The exponent rules apply in the same way.

The Exponent Laws

You learned these laws in Grade 9 Math. Now we are going apply them to bases other than real numbers, so it is a good idea to review them. You may want to include them in your Resource Sheet as well.

Product Law

When you multiply powers with like bases, add the exponents.

$$\left(2^2\right)\left(2^3\right) = (2)(2)(2)(2)(2) = \left(2^5\right) \qquad (2 + 3 = 5)$$

$$\left(w^5\right)\left(w^9\right) = w^{14}$$

$$\left(x^m\right)\left(x^n\right) = x^{m+n}$$

Quotient Law

When you divide powers with like bases, subtract the exponents.

$$5^8 \div 5^2 = \frac{(5)(5)(5)(5)(5)(5)\cancel{(5)}\,\cancel{(5)}}{\cancel{(5)}\,\cancel{(5)}} = 5^6 \qquad (8-2=6)$$

$$\frac{f^{14}}{f^9} = f^{14-9} = f^5 \text{ where } f \neq 0$$

$$x^m \div x^n = x^{m-n} \text{ where } x \neq 0$$

Power of a Power Law

When a power is raised to a power, multiply the exponents.

$$\left(4^3\right)^2 = \left(4^3\right)\left(4^3\right) = 4^6$$

$$\left(4^3\right)^2 = \left(4^{3\times 2}\right) = 4^6$$

$$\left(y^5\right)^7 = y^{35}$$

$$\left(x^m\right)^n = x^{mn}$$

When variables (such as *m* and *n*) are written beside each other, it is assumed they are being multiplied together.

Power of a Product Law

When the base is a product, apply the exponent to each factor in the base.

$$(3 \times 2)^3$$

$$= 3^3 \times 2^3$$

$$= 27 \times 8$$

$$= 216$$

Check: $\quad x^n = \underbrace{(x)(x)(x)\ldots(x)(x)}_{n \text{ factors}}$

$$(3 * 2)^3 = 6^3 = 216$$

$$\left(8x^3y\right)^2$$

$$= 8^2(x^3)^2y^2$$

$$= 64x^6y^2$$

Check:

$$\left(8x^3y\right)^2 = \left(8x^3y\right)\left(8x^3y\right) = 64x^6y^2$$

$$\left(xy\right)^m = x^my^m$$

Power of a Quotient

When the base is a quotient, apply the exponent to the numerator and denominator.

$$\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{2^3}{5^3}$$

$$\left(\frac{4g}{h^2}\right)^5 = \left(\frac{\left(4g\right)^5}{\left(h^2\right)^5}\right) = \frac{1024g^5}{h^{10}} \text{ where } h \neq 0$$

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m} \text{ where } y \neq 0$$

Learning Activity 2.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Solve for i: $4i + 3 = 15$.
2. You have half an orange in your lunch. You would like to share it with your two good friends. If you cut it up evenly, how much of the original orange will you receive?
3. Would the data in this situation be continuous or discrete?

   "The time that has passed compared to the distance you have run."
4. Evaluate $\sqrt{-64}$.
5. Evaluate $\sqrt[3]{-8}$.
6. Which is larger: 0.54 or 39%?
7. The Blackhawks have won twice as many games as the Maple Leafs. The Maple Leafs have won five fewer games than the Oilers. If the Oilers have won 13 games, how many games have the Blackhawks won?
8. There is a great sale on clothes at 30% off the marked price. If you are buying a hoodie that is priced at $40.00, how much will you save?

continued

Learning Activity 2.4 (continued)

Part B: Power Laws Review

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the base, exponent, coefficient of the variable (where appropriate), and power in each of the following. Write out the power as a repeated multiplication and determine the value, if possible:

Power	Base	Exponent	Coefficient	Expansion	Value
x^2					
$\left(\frac{1}{4}\right)^6$					
$3x^8$					
$-7m^0$					
$(-15p)^3$					

2. Simplify each of the following expressions using the exponent laws. Indicate which exponent law(s) you use. Evaluate where possible. (Simplify means to write the given term or equation in the smallest/simplest form. $4^5 \times 4^2 = 4^7$. Evaluate means to find the value of the given term or equation. $4^5 \times 4^2 = 4^7 = 16384$.)

   a) $(7^4)(7^1)$

   b) $(3x^2)(-2x^3)$

   c) $\dfrac{-8x^4y^5}{2xy^2}$

   d) $-32m^{12} \div 8m^4$

   e) $-\left(\dfrac{5}{3}\right)^2$

   f) $\left(\dfrac{3x^2y}{x^5}\right)^2$

3. Simplify the following expression using exponent laws. Show each step in the solution. Find the value of the expression (evaluate).

$$\left[\left(3^3\right)^2\left(5^3\right)\left(3^2\right)\right]^2$$

continued

Learning Activity 2.4 (continued)

4. Create two expressions that, when simplified using the exponent laws, are equivalent to $\frac{9x^3}{y^2}$. State which laws must be used to simplify your expressions.

Lesson Summary

Exponent laws can be used as shortcuts when simplifying and evaluating expressions that include powers. In this lesson, you applied the exponent laws to variable and rational bases. In the next lesson, you will explore patterns in powers to determine laws that describe how to deal with exponents that are rational or negative.

Notes

Assignment 2.4

Exponent Laws Review

Total Marks = 25

1. Simplify using the exponent laws. Show the steps in your solution. Find the value of the power (evaluate).

   a) $\left(2^3\right)\left(3^2\right)\left(2^5\right)$ *(2 marks)*

   b) $\dfrac{\left(2^5\right)\left(3^4\right)}{3^2}$ *(2 marks)*

   c) $4^3\left(\dfrac{3}{4}\right)^2$ *(2 marks)*

Assignment 2.4: Exponent Laws Review (continued)

2. Simplify. Write each as a single power. Do not evaluate.

   a) $\left(\frac{3}{5}\right)^9\left(\frac{3}{5}\right)^{15}$ *(1 mark)*

   b) $(-Q)^2(-Q)^5(-Q)$ *(1 mark)*

   c) $\frac{(-2)^7}{(-2)^3}$ *(1 mark)*

3. Simplify using the exponent laws. Evaluate where possible.

   a) $\left(m^5\right)^4$ *(1 mark)*

   b) $\left(5r^4\right)^3$ *(1 mark)*

   c) $\left(-2xy\right)^2\left(3y^3\right)$ *(1 mark)*

Assignment 2.4: Exponent Laws Review (continued)

d) $\dfrac{\left(-4n^{3}\right)^{2}}{(3n)^{5}}$ *(1 mark)*

e) $\dfrac{-\left(3b^{3}\right)^{2}\left(-8b^{6}\right)}{-24b}$ *(2 marks)*

f) $\left(\dfrac{9x^{2}}{y}\right)^{2}\left(\dfrac{y}{3x}\right)^{3}$ *(2 marks)*

4. Create four expressions that, when simplified using the exponent laws, are equal to $4x^6$. Each expression should use at least one different exponent law. State which law(s) must be used to simplify each expression. *(8 marks)*

Notes

Lesson 5: Exponent Laws 2

Lesson Focus

In this lesson, you will

- ❑ explain using patterns or exponent laws why $x^{-n} = \dfrac{1}{x^n}$ and $x = 0$
- ❑ explain using patterns why $x^{\frac{1}{n}} = \sqrt[n]{x}$ when $x \neq 0$
- ❑ express powers with rational exponents as radicals and radicals as rational exponents
- ❑ apply the exponent laws to powers with negative and rational exponents
- ❑ solve problems that involve exponent laws or radicals
- ❑ identify errors in the simplification of an expression that involves powers

Lesson Introduction

In this lesson, you will explore patterns as a way to understand and describe negative and rational exponents. You will also extend the exponent laws to include powers with rational and negative exponents.

Exponents That Are Not Natural Numbers

Based on what you learned in Lesson 4, can you expand the powers $9^{\frac{1}{2}}$ and 2^{-2}?

How can you multiply something by itself a negative or fractional number of times?

These two types of exponents present unique opportunities to explore the patterns and ideas behind the concept of powers. Consider the following patterns in negative and rational exponents.

Negative Exponents

Power	Expansion	Value
2^4	(2)(2)(2)(2)	16
2^3	(2)(2)(2)	8
2^2	(2)(2)	4
2^1	(2)	2
2^0		??

What patterns do you notice in the table above?

- Each time, the exponent is one less than the previous.
- The exponent indicates the number of times you multiply the base by itself.
- Each value is half the previous value.

What would happen if these patterns were continued?

Power	Expansion	Value	
2^4	(2)(2)(2)(2)	16	$16 \div 2 = 8$
2^3	(2)(2)(2)	8	$8 \div 2 = 4$
2^2	(2)(2)	4	$4 \div 2 = 2$
2^1	(2)	2	$2 \div 2 = 1$
2^0		1	$1 \div 2 = \frac{1}{2}$
2^{-1}	$\left(\frac{1}{2}\right)$	$\frac{1}{2}$	$\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{2^2}$
2^{-2}	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\frac{1}{2^2}$	$\frac{1}{2^2} \div 2 = \frac{1}{2^2} \times \frac{1}{2} = \frac{1}{2^3}$
2^{-3}	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\frac{1}{2^3}$	$\frac{1}{2^3} \div 2 = \frac{1}{2^3} \times \frac{1}{2} = \frac{1}{2^4}$
2^{-4}	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\frac{1}{2^4}$	Divide previous value by 2 or multiply by $\frac{1}{2}$

Note: Dividing a fraction by two is the same as taking half of it.

If a positive exponent means multiply the base by itself that number of times, a negative exponent means multiply the reciprocal of the base by itself the opposite of that number of times.

For example, 2^{-5} would mean multiply the reciprocal of the base the opposite of -5 times or $2^{-5} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \frac{1}{2^5}$.

> A reciprocal (also called the multiplicative inverse) is one of a pair of numbers that, when multiplied, result in a product of 1. For example, $5 * \frac{1}{5} = 1$, $\frac{3}{4} * \frac{4}{3} = \frac{12}{12} = 1$.
>
> 5 and $\frac{1}{5}$ are reciprocals, $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals. The easiest way to find the reciprocal of a number is to write it as a fraction and then flip it over.

Another way to think about negative exponents is to consider that the negative sign means that the base is on the wrong side of the fraction line and you need to switch the numerator and denominator and then write that with the positive exponent.

$$\left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^5$$

This way of thinking is especially effective when the base is a fraction.

Example 1

Write the following powers with positive exponents.

$p^{-3}, \dfrac{x^4}{y^{-2}}, 2z^{-1}, (3m)^{-2}$

Solution:

$$p^{-3} = \frac{1}{p^3}$$

$$\frac{x^4}{y^{-2}} = x^4y^2$$

$$2z^{-1} = \frac{2}{z}$$

$$(3m)^{-2} = \frac{1}{(3m)^2} = \frac{1}{9m^2}$$

Remember that negative exponents only affect their base.

> The negative exponent law can be expressed in general as:
>
> $$x^{-m} = \left(\frac{1}{x}\right)^m = \frac{1}{x^m}$$

Using the Quotient Law to Explain the Negative Exponent Law

You can use the other exponent laws to prove or explain how and why the negative exponent law operates the way it does.

Consider that $x^5 \div x^3 = x^{5-3} = x^2$.

Writing this another way: $\dfrac{x^5}{x^3} = \dfrac{x^2}{1}$

The quotients (answers) to these two statements are equal. $x^2 = \dfrac{x^2}{1}$

Now switch the order of the dividend (the top value) and divisor (the bottom value):

$$x^3 \div x^5 = x^{3-5} = x^{-2}$$

$$\frac{x^3}{x^5} = \frac{1}{x^2}$$

Therefore,

$$x^{-2} = \frac{1}{x^2}$$

Note: The negative sign in the exponent does not make the result negative. It only means you must take the reciprocal of the base and write it with a positive exponent.

You can also use the quotient law to prove the zero exponent law, which you learned in your Grade 9 Mathematics course. Recall that $x^0 = 1$.

$$2^3 \div 2^3 = 2^{3-3} = 2^0$$

$$\frac{2^3}{2^3} = \frac{8}{8} = 1$$

Therefore,

$$2^0 = 1$$

This reasoning would work with variable bases as well.

$$x^5 \div x^5 = x^{5-5} = x^0$$

$$\frac{x^5}{x^5} = \frac{xxxxx}{xxxxx} = 1$$

Therefore,

$$x^0 = 1$$

Rational Exponents

In Lesson 3, you explored the relationship between radicals and exponents. A radical reverses or ″undoes″ an exponent.

$$4^2 = 16$$

$$\sqrt{16} = 4$$

or

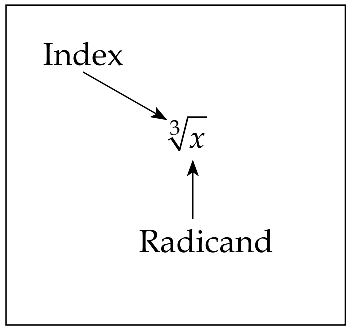
$$\sqrt{4^2} = 4$$

$$2^3 = 8$$

$$\sqrt[3]{8} = 2$$

or

$$\sqrt[3]{2^3} = 2$$



This applies when a variable base is used as well.

$$(h)^2 = (h)(h) = h^2$$

$$\sqrt{h^2} = h$$

According to the product law, when you multiply like bases, you must add the exponents, and when you multiply a factor times itself, you are squaring the factor. The square root of the product is one of the factors. Consider the following table:

Factors Squared	**Square Root**
$\left(x^4\right)\left(x^4\right) = x^8$	$\sqrt[2]{x^8} = x^4$
$\left(x^3\right)\left(x^3\right) = x^6$	$\sqrt[2]{x^6} = x^3$
$\left(x^2\right)\left(x^2\right) = x^4$	$\sqrt[2]{x^4} = x^2$
$\left(x^1\right)\left(x^1\right) = x^2$	$\sqrt[2]{x^2} = x^1$
$\left(x^?\right)\left(x^?\right) = x^1$	$\sqrt[2]{x^1} = x^?$

What patterns do you notice?

- Each factor is multiplied by itself two times and the index is 2.
- The sum of the exponents is exactly two times the exponent of one of the factors.
- The exponent on the square root is exactly half the value of the exponent on the radicand. Since the index is 2, the exponent on the root is equal to the exponent of the radicand divided by the index.

To continue the pattern, what exponent could be used on the factors in the last row to make a sum of 1? Half of 1 is $\frac{1}{2}$.

$\left(x^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}\right)=x^1$	$\sqrt[2]{x^1}=x^{\frac{1}{2}}$

Are there other ways to form a product that equals x^1?

When fractional exponents are an option, there are countless possibilities!

You can cube $x^{\frac{1}{3}}$ and then find the cube root of the product.

$\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{3}}\right)=x^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}=x^{\frac{3}{3}}=x^1$, therefore, $\sqrt[3]{x^1}=x^{\frac{1}{3}}$

This works with integer bases as well.

$\left(7^{\frac{1}{4}}\right)\left(7^{\frac{1}{4}}\right)\left(7^{\frac{1}{4}}\right)\left(7^{\frac{1}{4}}\right)=7^{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}=7^{\frac{4}{4}}=7^1$, therefore, $\sqrt[4]{7^1}=7^{\frac{1}{4}}$

You can now express powers with rational exponents as radicals, and radicals as powers with rational exponents. The denominator of the fractional exponent is equal to the index or root of the radical.

$$x^{\frac{1}{10}} \Leftrightarrow \sqrt[10]{x}$$

x to the power of $\frac{1}{10}$ is the 10th root of x

Rational exponents can be expressed as radicals, and vice versa as $x^{\frac{1}{n}} = \sqrt[n]{x}$, $n \neq 0$.

The exponent laws apply to expressions with rational and negative exponents in the same ways as they apply to powers with natural number exponents. Rules can be combined when simplifying expressions.

Example 2

Evaluate the following expression.

$8^{\frac{-2}{3}}$

Solution:

$8^{\frac{-2}{3}}$ Step 1: Rewrite the exponent as the power of a power.

$=\left(8^{\frac{1}{3}}\right)^{-2}$

$=\left(\sqrt[3]{8}\right)^{-2}$ Step 2: Write the rational exponent as a radical and evaluate.

$=2^{-2}$ Step 3: Take the reciprocal of the base that has a negative exponent and write it as a positive exponent.

$=\dfrac{1}{2^2}$

$=\dfrac{1}{4}$ Step 4: Evaluate.

If you are given a fractional exponent that has not been rewritten in its simplest form, the very first step is to simplify the fraction.

Example 3

Evaluate the following expression.

$$2^{\frac{6}{2}}$$

Solution:

$2^{\frac{6}{2}}$

$= 2^{\frac{3}{1}}$

$= 2^3$

$= 8$

Learning Activity 2.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. How tall is a door in centimetres if it is 2.2 m tall?
2. Is the following number rational or irrational? π
3. Evaluate 4^3.
4. Which costs less: 3 bottles of pop for $3.99, or 1 bottle of pop for $1.50?
5. Which is larger: $\sqrt{121}$ or 4^2?
6. Evaluate $(2x^2)(4y^5)$.
7. The body of a daddy-long-legs spider is about 0.7 cm long. How long is this in mm?

continued

8. In lacrosse, there are 10 players on the field at one time (per team). If you have 16 players on your team and one coach, how many people will be on the bench during the game?

Part B: Exponent Laws with Rational and Negative Exponents

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

This material can be difficult. Remember, you can call your tutor/marker for help if you get stuck or don't understand how to get an answer.

1. Express each radical in exponential form. Simplify using the exponent laws when possible.

   a) $\sqrt{x}$

   b) $\sqrt{dt}$

   c) $\sqrt[7]{g^5}$

   d) $\sqrt[3]{-27x^3y^6}$

2. Express each power as a radical and evaluate when possible.

   a) $125^{\frac{1}{3}}$

   b) $(-243)^{\frac{1}{5}}$

   c) $16^{0.25}$ (**Hint:** Change the decimal into a fraction.)

   d) $4^{\frac{-1}{2}}$

continued

Learning Activity 2.5 (continued)

3. Simplify using the exponent laws. Evaluate when possible. Leave answers with fractional exponents in their simplest form and as an improper fraction when applicable.

   (**Hint:** Change the radical into fractional form.)

   a) $9^{\frac{1}{3}} * 9^{\frac{1}{6}}$

   b) $\dfrac{27^{\frac{1}{2}}}{27^{\frac{1}{6}}}$

   c) $\left(x^4\right)^{\frac{1}{2}}$

   d) $\left(10^2\right)^{\frac{-3}{2}}$

   e) $\dfrac{x^2}{\sqrt[3]{x}}$

   f) $\left(-8x^4y^6\right)^{\frac{1}{3}}$

   g) $\left(\dfrac{9x^3y^6}{4x^1y^0}\right)^{\frac{1}{2}}$

4. Identify the errors a student made in the following problems and correct them. Explain your answer. (Hint: If you can't see the mistake right away, try simplifying the term to the left of the equal sign. What is different about the answer given when compared to the answer you found?)

   a) $\left(\dfrac{2}{5}\right)^{-3} = -\left(\dfrac{2^3}{5^3}\right)$

   b) $3m^{-2} = \dfrac{1}{9m^2}$

Lesson Summary

This lesson used patterns and the exponent laws to explain and explore negative and rational exponents. You learned how to take the reciprocal of a base when it has a negative exponent, and write it with a positive exponent. You expressed radicals as powers with rational exponents and vice versa, and applied the exponent laws to powers with rational and negative exponents.

Notes

Assignment 2.5

Exponent Laws with Rational and Negative Exponents

Total Marks = 21

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and laws on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Express each radical in exponential form. Simplify using the exponent laws when possible.

   a) $\sqrt{k^{\frac{2}{3}}}$ *(1 mark)*

   b) $\sqrt{ab^2}$ *(1 mark)*

   c) $\sqrt[4]{81x^8y^{12}}$ *(1 mark)*

   d) $\sqrt[3]{x}\sqrt{x}$ *(1 mark)*

Assignment 2.5: Exponent Laws with Rational and Negative Exponents (continued)

2. Express each power as a radical.

   a) $9^{\frac{1}{2}}$ *(1 mark)*

   b) $m^{\frac{-1}{2}}$ *(1 mark)*

   c) $3k^{\frac{2}{5}}$ *(1 mark)*

   d) $\left[\left(pq^3\right)^4\right]^{\frac{1}{5}}$ *(1 mark)*

   e) $\dfrac{x^{\frac{2}{3}}y^{\frac{1}{4}}}{x^{\frac{1}{2}}y^{\frac{1}{2}}}$ *(2 marks)*

Assignment 2.5: Exponent Laws with Rational and Negative Exponents (continued)

3. Simplify using the exponent laws. Write the final answer using only positive exponents. Evaluate where possible.

   a) $2x^{-1}$ *(1 mark)*

   b) $\left(5p^{3}\right)^{-2}$ *(1 mark)*

   c) $\left(-32x^{-10}y^{15}\right)^{\frac{1}{5}}$ *(1 mark)*

   d) $\left(\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}x^{\frac{1}{5}}}\right)^{\frac{1}{2}}$ *(2 marks)*

Assignment 2.5: Exponent Laws with Rational and Negative Exponents (continued)

4. Evaluate. Show necessary steps using exponent laws.

   a) $(-125)^{\frac{-2}{3}}$ *(1 mark)*

   b) $\left(2n^{\frac{1}{3}}r^{-2}\right)^{0}$ *(1 mark)*

5. Identify the errors a student made in the following problems and correct them. Explain your answer.

   a) $(7x)^{\frac{1}{3}} = \dfrac{1}{7x^{3}}$ *(2 marks)*

   b) $2^{2} \div 2^{\frac{1}{4}} = 2^{\frac{2}{4}} = 2^{\frac{1}{2}} = \sqrt{2}$ *(2 marks)*

Module 2 Summary

Congratulations! You have finished the second module in the course.

This module was designed to give you opportunities to explore how numbers worked and what they mean in different situations. You explored factors, multiples, and square and cube numbers along with their roots. You organized the real number system and labelled numbers as natural, whole, integers, rational, or irrational. You explored radical numbers and their connection to powers. The exponent laws were applied to powers with variable bases, rational exponents, and negative exponents. Along the way, you simplified, evaluated, created equivalent expressions, and ordered numbers along a number line.

Your understanding of the nature of numbers will continue to expand as you apply this learning in the next module on measurement, in the rest of this course, and with your daily use of numbers, now and in the future.

Submitting Your Assignments

It is now time for you to submit Assignments 2.1 to 2.5 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 2 assignments and organize your material in the following order:

- ❑ Module 2 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 2.1: Factors and Multiples
- ❑ Assignment 2.2: Perfect Cubes and Squares
- ❑ Assignment 2.3: Rational, Irrational, and Radical Numbers
- ❑ Assignment 2.4: Exponent Laws Review
- ❑ Assignment 2.5: Exponent Laws with Rational and Negative Exponents

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 2
Number Sense

Learning Activity Answer Keys

Module 2: Number Sense

Learning Activity 2.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. You are at the store to buy your mother a present. You have \$30 to spend and would like to get her a waffle maker. If it costs \$40 and is 20% off, will you be able to get her the waffle maker?
2. There are three pairs of socks in a package. If the whole package costs \$6, how much does it cost per pair of socks?
3. In a football game, the final score is 12 to 28. Since a touchdown without a convert is worth 6 points, is it possible that one team did not get any field goals?
4. It's been raining for the past three hours in Winnipeg. They had 5 mm in the first hour, 2 mm in the next hour, and 5 mm in the last hour. What was the average rainfall per hour?
5. Fill in the blanks for the following pattern: 1, 4, ____, 16, 25, ____.
6. Evaluate 9^2.
7. Rewrite the fraction in simplest terms: $\frac{33}{21}$.
8. You have a box with a length of 3 m, a width of 2 m, and a height of 6 m. What is the volume of this box?

Answers:

1. No (20% of 40 = 2 × (10% of 40) = 2 × 4 = \$8. You have \$30 but the waffle maker will still cost 40 – 8 = \$32.)
2. \$2 (6 ÷ 3)
3. Yes, it is possible. (12 = 2 × 6, so the losing team could have scored two unconverted touchdowns but no field goals.)

4. 4 mm/h

$$\text{Rate} = \frac{\text{total rainfall}}{\text{total number of hours}}$$

$$= \frac{5+2+5}{3}$$

$$= \frac{12}{3}$$

5. 9, 36 (The pattern contains numbers that are perfect squares. $1^2 = 1$, $2^2 = 4$, $3^3 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$.)

6. 81 (9×9)

7. $\frac{33}{21} \div \frac{3}{3} = \frac{11}{7}$

8. 36 m^3 ($V = l \times w \times h = 3 \times 2 \times 6$)

Part B: Word Web

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. How many factors do prime numbers have? What are they?

   Answer:

   A prime number has exactly two factors: one and itself.

2. Fill in the following chart by
   a) listing all the factors of the given composite numbers
   b) determining the number of factors each composite number has
   c) indicating whether the number of factors is an odd or even number
   d) listing the prime factorization of each number

Answer:

Given Number	(a) List of All Factors of the Composite Number	(b) # of Factors	(c) Odd or Even?	(d) Prime Factors
4	1, 2, 4	3	Odd	$2 \times 2 = 2^2$
6	1, 2, 3, 6	4	Even	3×2
8	1, 2, 4, 8	4	Even	$2 \times 2 \times 2 = 2^3$
9	1, 3, 9	3	Odd	$3 \times 3 = 3^2$
12	1, 2, 3, 4, 6, 12	6	Even	$2 \times 2 \times 3 = 2^2 \times 3$
16	1, 2, 4, 8, 16	5	Odd	$2 \times 2 \times 2 \times 2 = 2^4$
21	1, 3, 7, 21	4	Even	3×7
24	1, 2, 3, 4, 6, 8, 12, 24	8	Even	$2 \times 2 \times 2 \times 3 = 2^3 \times 3$
25	1, 5, 25	3	Odd	$5 \times 5 = 5^2$
27	1, 3, 9, 27	4	Even	$3 \times 3 \times 3 = 3^3$
36	1, 2, 3, 4, 6, 9, 12, 18, 36	9	Odd	$2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

e) Circle the given composite numbers that have an odd number of factors. What pattern do you notice in these numbers? Explain.

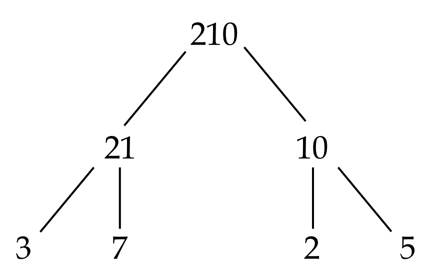
Answer:

All of the perfect square numbers are circled. They have an odd number of factors, because when a number is squared its factor is multiplied by itself, and so it is a single factor and not a factor pair like all others.

3. Determine the prime factors of 210 using a factor tree diagram. Include your diagram.

Answer:

The prime factors of 210 are $3 \times 7 \times 2 \times 5$.



4. Determine the prime factors of 84 using the division method. Show your work.

   Answer:

   84 ÷ 2 = 42

   42 ÷ 2 = 21

   21 ÷ 3 = 7   7 is prime.

   The prime factors of 84 are 2, 2, 3, and 7.

5. List the prime factors of 147. Explain how you found your answer and show your work.

   Answer:

   The prime factors of 147 are 3, 7, and 7. You may have found these by drawing a factor tree, by using division, or by another method.

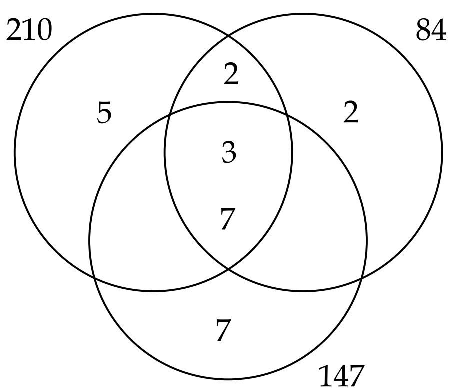
6. Using your answers from questions 3, 4, and 5, state the GCF of 210, 84, and 147. Show your work and explain how you found your answer.

   Answer:

   The prime factors that 210, 84, and 147 have in common are 3 and 7.

   3 × 7 = 21. The GCF of 210, 84, and 147 is 21.

   Alternately, you may have constructed a three-way Venn diagram and multiplied the factors found in the area that overlaps all three circles.



7. State the LCM of 210, 84, and 147. Show your work and explain how you found your answer.

   Answer:

   Rewrite the prime factorization using exponents to represent the repeated multiplications, and find the product of each of the factors with the highest exponent.

   $210 = 2^1 \times 3^1 \times 5^1 \times 7^1$

   $84 = 2^2 \times 3^1 \times 7^1$

   $147 = 3^1 \times 7^2$

   $\text{GCF} = 2^2 \times 3^1 \times 5^1 \times 7^2$

   $\text{GCF} = 2940$

   Alternately, you may have multiplied all the values in the Venn diagram:

   $\text{GCF} = 5 \times 2 \times 3 \times 7 \times 2 \times 7$

   $\text{GCF} = 2940$

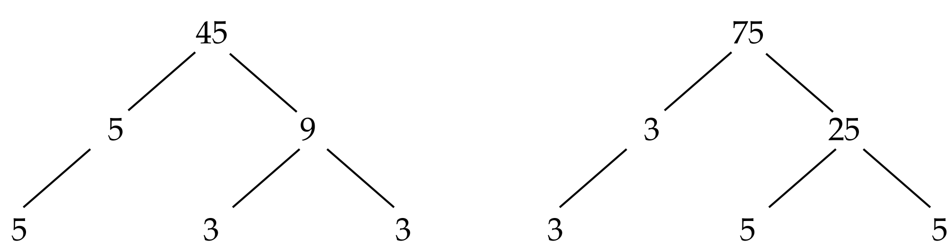
8. In your own words, define what the LCM of a set of numbers is.

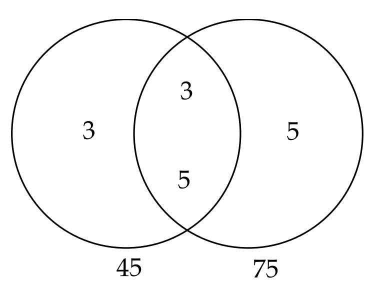
   Answer:

   Your answer should include the idea that the lowest or least common multiple of a set of numbers is the smallest positive integer that can be divided evenly by all the numbers in the set. Alternately, you may say something like the following: the LCM is the smallest common product when each of the numbers in the set are multiplied by the positive integers.

9. Draw a Venn diagram to illustrate the prime factors of 45 and 75. Explain how you can use the diagram to calculate the GCF and LCM of 45 and 75. State the GCF and LCM.

   Answer:





Learning Activity 2.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. What is the GCF of 12 and 18?
2. What is the LCM of 8 and 12?
3. What two numbers have a sum of 5 and a product of 6?
4. Which is the independent variable: as you study more, your test marks increase.
5. If a desk is 90 cm high, how tall is it in metres?
6. You give the cashier a $10 bill to pay for your lunch. If the total for your lunch is $7.60, how much change will you get back?
7. Which is smaller: $\frac{4}{5}$ or $\frac{7}{10}$?
8. A basic calculator has 6 rows and 5 columns of buttons. How many buttons does it have in total?

Answers:

1. 6 (Factors of 12 are 1, 2, 3, 4, 6, 12. Factors of 18 are 1, 2, 3, 6, 9, 18.)
2. 24 (The GCF of 8 and 12 is 4, so 8 × 12 = 96 ÷ 4 = 24 or 8 ÷ 4 = 2 × 12 = 24 or 12 ÷ 4 = 3 × 8 = 24.)
3. 2, 3 (Factor pairs of 6: (1, 6) and (2, 3). 1 + 6 = 7, 2 + 3 = 5.)
4. Time spent studying is the independent variable. If you don't study more, do you expect to improve?
5. 0.90 m (There are 100 cm = 1 metre. $\frac{90}{100} = 0.9.$ Move the decimal to the left two spaces because there are two zeros in 100.)
6. $2.40 change ($7.60 + $0.40 = $8 then $8 + $2 = $10 so 2 + 0.40 = $2.40)
7. $\frac{7}{10}$ $\left(\frac{4}{5} = \frac{8}{10}\right.$, and 8 is larger than 7)
8. 30 buttons (6 × 5)

Part B: Perfect Cubes and Squares

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Indicate whether the following numbers are perfect cubes, perfect squares, or neither. Support your answer with a diagram and/or an explanation of the process you used.

   a) 169 b) 324 c) –512 d) 111 e) 8 f) 64

   Answers:

   a) 169 is a perfect square because 169 items can be arranged in a square with dimension of 13 by 13.

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   b) 324 is a perfect square number. Its factor pairs are:

   1, 324

   2, 162

   3, 108

   4, 81

   6, 54

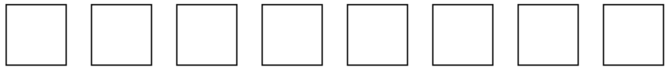
   9, 36

   12, 27

   18, 18

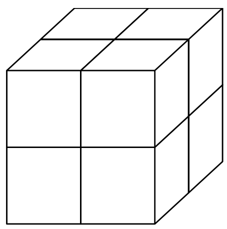
   Because 18 multiplied by itself is equal to 324, $18^2 = 324$ and 324 is a square number.

c) –512 is a perfect cube number with an integer root. Using a calculator, it can be determined that $\sqrt[3]{-512} = -8.$ Indicate the keystrokes entered on your calculator when using this process.



d) 111 is neither a perfect square nor a perfect cube. There is no way to arrange 111 items in a square area or cubic shape without leftover items or gaps in the design. There is no number that can be multiplied by itself two or three times that will result in a product of 111.

e) 8 is a perfect cube number. 8 blocks can be arranged in a 2-by-2-by-2 cube.



f) 64 is both a perfect square number and a perfect cube number: $8^2 = 64$ and $4^3 = 64$. The prime factors of $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$, so it is also a perfect 6th number.

Six 2s can be grouped in two sets of three 2s $(2 \times 2 \times 2)(2 \times 2 \times 2) = (8)(8) = 8^2$, or in three sets of two 2s $(2 \times 2)(2 \times 2)(2 \times 2) = (4)(4)(4) = 4^3$.

There are other numbers that have the unique property of being both perfect square and perfect cube numbers. They are called the perfect 6th powers: 1^6, 2^6, 3^6, 4^6, etc.

2. State **all possible** square roots for the following numbers. Explain your answer.

a) 144 b) −36

Answers:

a) $\sqrt{144} = \pm 12$

(12)(12) = 144 and (−12)(−12) = 144 so the square roots of 144 are positive 12 and negative 12.

b) $\sqrt{-36}$ does not have a real number solution. There is no integer you can multiply by itself to come up with a negative product.

3. State **all possible** cube roots for the following numbers. Explain your answer.

   a) −216 b) 8

   Answers:

   a) $\sqrt[3]{-216} = -6$

   (−6)(−6)(−6) = −216, so the cube root of −216 is −6. +6 is NOT a cube root of −216 because (6)(6)(6) = +216, not −216

   b) $\sqrt[3]{8} = 2$

   (2)(2)(2) = 8 so the cube root of 8 is 2. The prime factors of 8 = 2 × 2 × 2.

4. The year 2011 marked the 144th birthday of Canada. In the Confederation Park garden, Erika wanted to plant 144 flowers to commemorate July 1, 1867. How could she have arranged these flowers to make a square design?

   Answer:

   The square root of 144 is 12, so Erika can plant 12 rows of 12 flowers to make a square design.

Learning Activity 2.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. How much larger is a line length 2 cm than a line length 1 cm?
2. How much larger is the area of a 2-by-2 square than a 1-by-1 square?
3. How much larger is the volume of a 2-by-2-by-2 box than a 1-by-1-by-1 box?
4. What is the GCF of 24 and 28?
5. Rewrite the fraction in simplest terms: $\frac{24}{28}$.
6. There are 3 boys and 2 girls invited to your birthday party. If each boy eats 2 pieces of cake and each girl eats 1 piece, how many pieces of cake will be eaten (not including yours)?
7. If each piece of the cake (from the question above) is $\frac{1}{9}$th of the cake, will there be enough cake for you to have a piece?
8. Evaluate the following: $2 - 3 + 6 \times 2 - 5 \times 4$.

Answers:

1. 1 cm
2. 3 ($2 \times 2 = 4$, $1 \times 1 = 1$, $4 - 1 = 3$)
3. 7 ($2 \times 2 \times 2 = 8$, $1 \times 1 \times 1 = 1$, $8 - 1 = 7$)
4. 4
5. $\frac{6}{7}$ (use the answer from the above question)
6. 8 pieces of cake ($3 \times 2 = 6$, $2 \times 1 = 2$, $6 + 2 = 8$)
7. Yes. If the cake is cut into 9 equal pieces $\left(\frac{1}{9}\text{ths}\right)$, your guests will only eat 8 of the 9.
8. -9 (According to BEDMAS, you do multiplication before addition or subtraction, so
   a) $6 \times 2 = 12$, $5 \times 4 = 20$
   b) $2 - 3 + 12 - 20$
   c) $-1 + 12 - 20$
   d) $11 - 20 = -9$)

Part B: Rational, Irrational, and Radical Numbers

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Indicate whether the following numbers are rational or irrational, and explain your reasoning.

a) $\sqrt{5}$

b) $\sqrt[3]{-3}$

c) $\sqrt{9}$

d) 0.25878787...

e) $\frac{1}{9}$

f) $\frac{1}{4}$

g) 0

h) $\frac{-2}{1}$

i) −1.12131415...

j) $-\frac{12}{4}$

Answers:

Answers for Questions 1, 2, and 3 are shown below.

	Question 1	**Question 2**	**Question 3**
a) $\sqrt{5}$	Irrational: Square root of a non-perfect square value.	2.236067977...	
b) $\sqrt[3]{-3}$	Irrational: Cube root of a non-perfect cube number.	−1.44224957	
c) $\sqrt{9}$	Rational: Square root of a perfect square number.	3	Natural
d) 0.25878787...	Rational: Decimal repeats	0.25878787	Rational
e) $\frac{1}{9}$	Rational: Ratio of two integers	0.1111111 Decimal repeats	Rational
f) $\frac{1}{4}$	Rational: Fraction with integer numerator and denominator	0.25 Decimal terminates	Rational
g) 0	Rational: Can be written as a fraction with an integer denominator; for example, $\frac{0}{1}, \frac{0}{-8}$	0	Whole
h) $\frac{-2}{1}$	Rational: Ratio of integers	−2	Integer
i) −1.12131415...	Irrational: Decimal does not repeat or terminate	−1.12131415	
j) $-\frac{12}{4}$	Rational: Quotient of two integers	−3	Integer

2. Determine the exact or approximate decimal value of the numbers in Question 1. Include as many decimal places as your calculator will display.

   Answer:

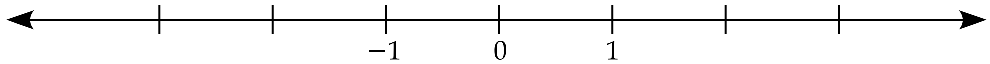
   See Answer for Question 1.

3. Indicate in which subset of the real number system each of the rational numbers in Question 1 best fits.

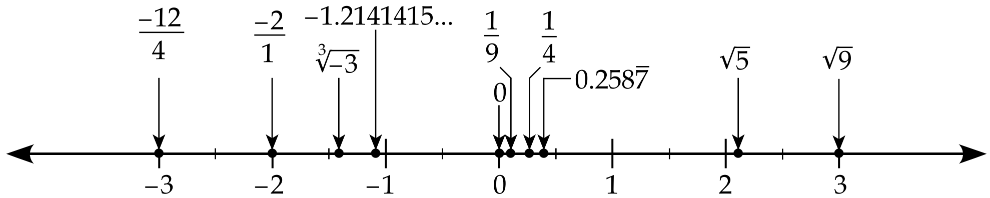
   Answer:

   See Answer for Question 1.

4. Place each number from Question 1 at the appropriate point along the number line.



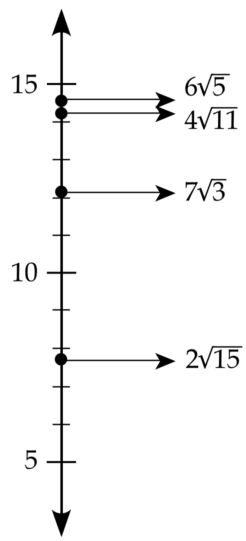
Answer:



5. Write the following mixed radicals as entire radicals, and place them in order along the number line.

$6\sqrt{5}$, $2\sqrt{15}$, $7\sqrt{3}$, $4\sqrt{11}$

Answer:



$$6\sqrt{5} = \sqrt{36} * \sqrt{5} = \sqrt{180} \approx 13.41640786...$$

$$2\sqrt{5} = \sqrt{4} * \sqrt{15} = \sqrt{60} \approx 7.745966692...$$

$$7\sqrt{3} = \sqrt{49} * \sqrt{3} = \sqrt{147} \approx 12.12435565...$$

$$4\sqrt{11} = \sqrt{16} * \sqrt{11} = \sqrt{176} \approx 13.26649916...$$

Learning Activity 2.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Solve for i: $4i + 3 = 15$.
2. You have half an orange in your lunch. You would like to share it with your two good friends. If you cut it up evenly, how much of the original orange will you receive?
3. Would the data in this situation be continuous or discrete?

   "The time that has passed compared to the distance you have run."
4. Evaluate $\sqrt{-64}$.
5. Evaluate $\sqrt[3]{-8}$.
6. Which is larger: 0.54 or 39%?
7. The Blackhawks have won twice as many games as the Maple Leafs. The Maple Leafs have won five fewer games than the Oilers. If the Oilers have won 13 games, how many games have the Blackhawks won?
8. There is a great sale on clothes at 30% off the marked price. If you are buying a hoodie that is priced at $40.00, how much will you save?

Answers:

1. $i = 3$
2. $\frac{1}{6}$th of an orange each $\left(\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}\right)$
3. Continuous. (You can have a fraction of a minute and run part of a mile.)
4. No solution. (You cannot take the square root of a negative number because only $(-8) \times 8 = -64$, but $-8 \neq 8$.)
5. –2 (Because there are three numbers being multiplied together, you can produce a negative number.)
6. 0.54 (If you changed 39% to a decimal = 0.39, which is less than 0.54.)
7. 16 games (The Leafs have won $13 - 5 = 8$ games. The Hawks have won $2 \times 8 = 16$ games.)
8. \$12.00 (10% of \$40 = \$4.00. Since $30 = 3 \times 10$, 30% of \$40 = 3×4 = \$12.00.)

Part B: Power Laws Review

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the base, exponent, coefficient of the variable (where appropriate), and power in each of the following. Write out the power as a repeated multiplication and determine the value, if possible:

   Answer:

Power	Base	Exponent	Coefficient	Expansion	Value
x^2	x	2	1	xx	
$\left(\frac{1}{4}\right)^6$	$\frac{1}{4}$	6		$\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$	$\frac{1}{4096}$
$3x^8$	x	8	3	$3xxxxxxxx$	
$-7m^0$	m	0	-7	$(-7)(1)$	-7
$(-15p)^3$	$-15p$	3	$(-15)^3$	$(-15p)(-15p)(-15p)$	$-3375p^3$

   Note: $(-15p)^3 = (-15p)(-15p)(-15p) = (-15p)^3p^3$, so the coefficient of the variable is -15.

2. Simplify each of the following expressions using the exponent laws. Indicate which exponent law(s) you use. Evaluate where possible. (Simplify means to write the given term or equation in the smallest/simplest form. $4^5 \times 4^2 = 4^7$. Evaluate means to find the value of the given term or equation. $4^5 \times 4^2 = 4^7 = 16384$.)

   a) $\left(7^4\right)\left(7^1\right)$

   b) $\left(3x^2\right)\left(-2x^3\right)$

   c) $\dfrac{-8x^4y^5}{2xy^2}$

   d) $-32m^{12} \div 8m^4$

   e) $-\left(\dfrac{5}{3}\right)^2$

   f) $\left(\dfrac{3x^2y}{x^5}\right)^2$

Answers:

a) $\left(7^4\right)\left(7^1\right)=7^5=16807$ Product Law

b) $\left(3x^2\right)\left(-2x^3\right)=(3)(-2)\left(x^{2+3}\right)=-6x^5$ Product Law

c) $\dfrac{-8x^4y^5}{2xy^2}=-4x^{4-1}y^{5-2}=-4x^3y^3$ Quotient Law

d) $-32m^{12}\div 8m^4=-4m^8$ Quotient Law

e) $-\left(\dfrac{5}{3}\right)^2=-\left(\dfrac{5^2}{3^2}\right)=-\dfrac{25}{9}$ Power of a Quotient Law

f) $\left(\dfrac{3x^4y}{x^5}\right)^2=\dfrac{9x^8y^2}{x^{10}}=9x^{-2}y^2$ Power of a Quotient Law and Power of a Power Law

3. Simplify the following expression using exponent laws. Show each step in the solution. Find the value of the expression (evaluate).

$$\left[\left(3^3\right)^2\left(5^3\right)\left(3^2\right)\right]^2$$

Answer:

$$\left[\left(3^3\right)^2\left(5^3\right)\left(3^2\right)\right]^2=\left[\left(3^6\right)\left(3^2\right)\left(5^3\right)\right]^2$$

Remember that bases must be alike in order to combine them using exponent laws.

$$=\left[\left(3^8\right)\left(5^3\right)\right]^2$$

$$=\left[\left(3^{16}\right)\left(5^6\right)\right]$$

$$=(43046721)(15625)$$

$$=6.726050125\times 10^{11}$$

Exponents are also used to express very large and very small numbers using "scientific notation." This means that the value of the expression is approximately 6.73 multiplied by 10 to the power of 11.

4. Create two expressions that, when simplified using the exponent laws, are equivalent. to $\frac{9x^3}{y^2}$. State which laws must be used to simplify your expressions.

Answer:

You may be as creative as you want when writing equivalent expressions. Two possible solutions are:

$\left(\frac{3x}{y}\right)^2(x)$	This uses the Power of a Quotient Law, and the Product Law.
$\frac{27x^5y^7}{3x^2y^9}$	This expression can be simplified using the Quotient Law.

Learning Activity 2.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. How tall is a door in centimetres if it is 2.2 m tall?
2. Is the following number rational or irrational? π
3. Evaluate 4^3.
4. Which costs less: 3 bottles of pop for $3.99, or 1 bottle of pop for $1.50?
5. Which is larger: $\sqrt{121}$ or 4^2?
6. Evaluate $(2x^2)(4y^5)$.
7. The body of a daddy-long-legs spider is about 0.7 cm long. How long is this in mm?
8. In lacrosse, there are 10 players on the field at one time (per team). If you have 16 players on your team and one coach, how many people will be on the bench during the game?

Answers:

1. 220 cm (There are 100 cm in 1 m, so $2.2 \times 100 = 220$ cm.)
2. Irrational.
3. 64 ($4 \times 4 \times 4$)
4. The 3 bottles would cost less. $\$3.99 \div 3 = \$1.33$ per bottle, which is less than $1.50.
5. 4^2 $\left(\sqrt{121} = 11,\ 4^2 = 16\right)$
6. $8x^2y^5$ ($2 \times 4 = 8$. There is no x in the second term and no y in the first term, so the exponents do not change. You cannot combine the x and y together because they represent different numbers.)
7. 7 mm (10 mm = 1 cm so $0.7 \times 10 = 7$ mm)
8. 7 ($16 - 10 = 6$ players + the coach = 7 people)

Part B: Exponent Laws with Rational and Negative Exponents

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them..

This material can be difficult. Remember, you can call your tutor/marker for help if you get stuck or don't understand how to get an answer.

1. Express each radical in exponential form. Simplify using the exponent laws when possible.

   a) $\sqrt{x}$

   b) $\sqrt{dt}$

   c) $\sqrt[7]{g^5}$

   d) $\sqrt[3]{-27x^3y^6}$

   Answers:

   a) $\sqrt{x} = x^{\frac{1}{2}}$

   b) $\sqrt{dt} = (dt)^{\frac{1}{2}} = d^{\frac{1}{2}}t^{\frac{1}{2}}$

   c) $\sqrt[7]{g^5} = \left(g^5\right)^{\frac{1}{7}} = g^{\frac{5}{7}}$

   d) $$\begin{aligned}\sqrt[3]{-27x^3y^6} &= \left(-27x^3y^6\right)^{\frac{1}{3}} \\ &= (-27)^{\frac{1}{3}}\left(x^3\right)^{\frac{1}{3}}\left(y^6\right)^{\frac{1}{3}} \\ &= -3xy^2\end{aligned}$$

2. Express each power as a radical and evaluate when possible.

a) $125^{\frac{1}{3}}$

b) $(-243)^{\frac{1}{5}}$

c) $16^{0.25}$ (**Hint:** Change the decimal into a fraction.)

d) $4^{\frac{-1}{2}}$

Answers:

a) $125^{\frac{1}{3}}$

$= \sqrt[3]{125}$

$= 5$

b) $(-243)^{\frac{1}{5}}$

$= \sqrt[5]{-243}$

$= -3$

c) $16^{0.25}$

$= 16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

d) $4^{\frac{-1}{2}}$

$= \dfrac{1}{4^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{4}}$

$= \dfrac{1}{2}$

3. Simplify using the exponent laws. Evaluate when possible. Leave answers with fractional exponents in their simplest form and as an improper fraction when applicable.

a) $9^{\frac{1}{3}} * 9^{\frac{1}{6}}$

b) $\dfrac{27^{\frac{1}{2}}}{27^{\frac{1}{6}}}$

c) $\left(x^4\right)^{\frac{1}{2}}$

d) $\left(10^2\right)^{\frac{-3}{2}}$

e) $\dfrac{x^2}{\sqrt[3]{x}}$ (**Hint:** Change the radical into fractional form.)

f) $\left(-8x^4y^6\right)^{\frac{1}{3}}$

g) $\left(\dfrac{9x^3y^6}{4x^1y^0}\right)^{\frac{1}{2}}$

Answers:

a) $9^{\frac{1}{3}} * 9^{\frac{1}{6}}$

$= 9^{\frac{1}{3}+\frac{1}{6}}$

$= 9^{\frac{2}{6}+\frac{1}{6}}$

$= 9^{\frac{3}{6}}$

$= 9^{\frac{1}{2}}$

$= \sqrt{9}$

$= 3$

b) $\dfrac{27^{\frac{1}{2}}}{27^{\frac{1}{6}}}$

$= 27^{\frac{1}{2}-\frac{1}{6}}$

$= 27^{\frac{3}{6}-\frac{1}{6}}$

$= 27^{\frac{2}{6}}$

$= 27^{\frac{1}{3}}$

$= \sqrt[3]{27}$

$= 3$

c) $\left(x^4\right)^{\frac{1}{2}}$

$= x^{\frac{4}{2}}$

$= x^2$

d) $\left(10^2\right)^{\frac{-3}{2}}$

$= \dfrac{1}{\left(10^2\right)^{\frac{3}{2}}}$

$= \dfrac{1}{10^{\frac{6}{2}}}$

$= \dfrac{1}{\left(10^3\right)}$

$= \dfrac{1}{1000}$

e) $\dfrac{x^2}{\sqrt[3]{x}}$

$= \dfrac{x^2}{x^{\frac{1}{3}}}$

$= x^{2-\frac{1}{3}}$

$= x^{\frac{6}{3}-\frac{1}{3}}$

$= x^{\frac{5}{3}}$

$= \sqrt[3]{x^5}$

f) $\left(-8x^4y^6\right)^{\frac{1}{3}}$

$= (-8)^{\frac{1}{3}}\left(x^4\right)^{\frac{1}{3}}\left(y^6\right)^{\frac{1}{3}}$

$= -2x^{\frac{4}{3}}y^2$

g) $\left(\dfrac{9x^3y^6}{4x^1y^0}\right)^{\frac{1}{2}}$

$= \left(\dfrac{9x^2y^6}{4(1)}\right)^{\frac{1}{2}}$

$= \left(\dfrac{9}{4}\right)^{\frac{1}{2}}\left(x^2\right)^{\frac{1}{2}}\left(y^6\right)^{\frac{1}{2}}$

$= \dfrac{3}{2}xy^3$

4. Identify the errors a student made in the following problems and correct them. Explain your answer. (Hint: If you can't see the mistake right away, try simplifying the term to the left of the equal sign. What is different about the answer given when compared to the answer you found?)

   a) $\left(\frac{2}{5}\right)^{-3} = -\left(\frac{2^3}{5^3}\right)$

   b) $3m^{-2} = \frac{1}{9m^2}$

   Answers:

   a) The negative exponent means you should take the reciprocal of the base to the positive power. This student multiplied the power by -1 and then applied the positive exponent to the numerator and denominator. It should be:

      $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5^3}{2^3}\right)$

   b) The student applied the exponent to the coefficient and the base, not just the base. It should be:

      $3m^{-2} = \frac{3}{m^2}$

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 2
Number Sense

Learning Activity Answer Keys

Module 2: Number Sense

Learning Activity 2.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. You are at the store to buy your mother a present. You have \$30 to spend and would like to get her a waffle maker. If it costs \$40 and is 20% off, will you be able to get her the waffle maker?
2. There are three pairs of socks in a package. If the whole package costs \$6, how much does it cost per pair of socks?
3. In a football game, the final score is 12 to 28. Since a touchdown without a convert is worth 6 points, is it possible that one team did not get any field goals?
4. It's been raining for the past three hours in Winnipeg. They had 5 mm in the first hour, 2 mm in the next hour, and 5 mm in the last hour. What was the average rainfall per hour?
5. Fill in the blanks for the following pattern: 1, 4, ____, 16, 25, ____.
6. Evaluate 9^2.
7. Rewrite the fraction in simplest terms: $\frac{33}{21}$.
8. You have a box with a length of 3 m, a width of 2 m, and a height of 6 m. What is the volume of this box?

Answers:

1. No (20% of 40 = 2 × (10% of 40) = 2 × 4 = \$8. You have \$30 but the waffle maker will still cost 40 – 8 = \$32.)
2. \$2 (6 ÷ 3)
3. Yes, it is possible. (12 = 2 × 6, so the losing team could have scored two unconverted touchdowns but no field goals.)

4. 4 mm/h

$$\text{Rate} = \frac{\text{total rainfall}}{\text{total number of hours}}$$
$$= \frac{5+2+5}{3}$$
$$= \frac{12}{3}$$

5. 9, 36 (The pattern contains numbers that are perfect squares. $1^2 = 1$, $2^2 = 4$, $3^3 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$.)

6. 81 (9×9)

7. $\frac{33}{21} \div \frac{3}{3} = \frac{11}{7}$

8. 36 m^3 ($V = l \times w \times h = 3 \times 2 \times 6$)

Part B: Word Web

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. How many factors do prime numbers have? What are they?

   Answer:

   A prime number has exactly two factors: one and itself.

2. Fill in the following chart by
   a) listing all the factors of the given composite numbers
   b) determining the number of factors each composite number has
   c) indicating whether the number of factors is an odd or even number
   d) listing the prime factorization of each number

Answer:

Given Number	(a) List of All Factors of the Composite Number	(b) # of Factors	(c) Odd or Even?	(d) Prime Factors
4	1, 2, 4	3	Odd	$2 \times 2 = 2^2$
6	1, 2, 3, 6	4	Even	3×2
8	1, 2, 4, 8	4	Even	$2 \times 2 \times 2 = 2^3$
9	1, 3, 9	3	Odd	$3 \times 3 = 3^2$
12	1, 2, 3, 4, 6, 12	6	Even	$2 \times 2 \times 3 = 2^2 \times 3$
16	1, 2, 4, 8, 16	5	Odd	$2 \times 2 \times 2 \times 2 = 2^4$
21	1, 3, 7, 21	4	Even	3×7
24	1, 2, 3, 4, 6, 8, 12, 24	8	Even	$2 \times 2 \times 2 \times 3 = 2^3 \times 3$
25	1, 5, 25	3	Odd	$5 \times 5 = 5^2$
27	1, 3, 9, 27	4	Even	$3 \times 3 \times 3 = 3^3$
36	1, 2, 3, 4, 6, 9, 12, 18, 36	9	Odd	$2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

e) Circle the given composite numbers that have an odd number of factors. What pattern do you notice in these numbers? Explain.

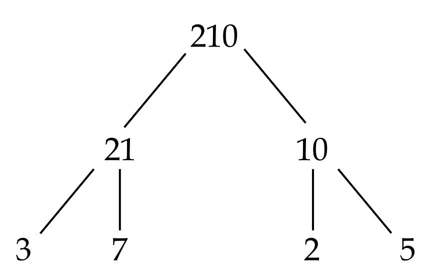
Answer:

All of the perfect square numbers are circled. They have an odd number of factors, because when a number is squared its factor is multiplied by itself, and so it is a single factor and not a factor pair like all others.

3. Determine the prime factors of 210 using a factor tree diagram. Include your diagram.

Answer:

The prime factors of 210 are $3 \times 7 \times 2 \times 5$.



4. Determine the prime factors of 84 using the division method. Show your work.

   Answer:

   $84 \div 2 = 42$

   $42 \div 2 = 21$

   $21 \div 3 = 7$ 7 is prime.

   The prime factors of 84 are 2, 2, 3, and 7.

5. List the prime factors of 147. Explain how you found your answer and show your work.

   Answer:

   The prime factors of 147 are 3, 7, and 7. You may have found these by drawing a factor tree, by using division, or by another method.

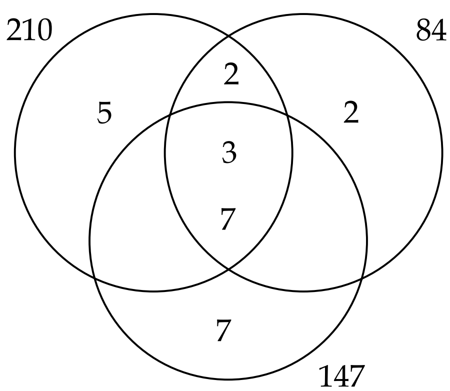
6. Using your answers from questions 3, 4, and 5, state the GCF of 210, 84, and 147. Show your work and explain how you found your answer.

   Answer:

   The prime factors that 210, 84, and 147 have in common are 3 and 7.

   $3 \times 7 = 21$. The GCF of 210, 84, and 147 is 21.

   Alternately, you may have constructed a three-way Venn diagram and multiplied the factors found in the area that overlaps all three circles.



7. State the LCM of 210, 84, and 147. Show your work and explain how you found your answer.

   Answer:

   Rewrite the prime factorization using exponents to represent the repeated multiplications, and find the product of each of the factors with the highest exponent.

   $210 = 2^1 \times 3^1 \times 5^1 \times 7^1$

   $84 = 2^2 \times 3^1 \times 7^1$

   $147 = 3^1 \times 7^2$

   $\text{GCF} = 2^2 \times 3^1 \times 5^1 \times 7^2$

   $\text{GCF} = 2940$

   Alternately, you may have multiplied all the values in the Venn diagram:

   $\text{GCF} = 5 \times 2 \times 3 \times 7 \times 2 \times 7$

   $\text{GCF} = 2940$

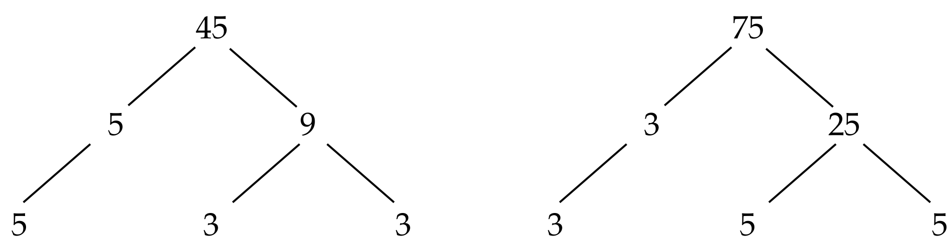
8. In your own words, define what the LCM of a set of numbers is.

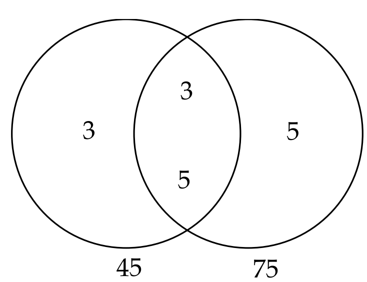
   Answer:

   Your answer should include the idea that the lowest or least common multiple of a set of numbers is the smallest positive integer that can be divided evenly by all the numbers in the set. Alternately, you may say something like the following: the LCM is the smallest common product when each of the numbers in the set are multiplied by the positive integers.

9. Draw a Venn diagram to illustrate the prime factors of 45 and 75. Explain how you can use the diagram to calculate the GCF and LCM of 45 and 75. State the GCF and LCM.

   Answer:





Learning Activity 2.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. What is the GCF of 12 and 18?
2. What is the LCM of 8 and 12?
3. What two numbers have a sum of 5 and a product of 6?
4. Which is the independent variable: as you study more, your test marks increase.
5. If a desk is 90 cm high, how tall is it in metres?
6. You give the cashier a $10 bill to pay for your lunch. If the total for your lunch is $7.60, how much change will you get back?
7. Which is smaller: $\frac{4}{5}$ or $\frac{7}{10}$?
8. A basic calculator has 6 rows and 5 columns of buttons. How many buttons does it have in total?

Answers:

1. 6 (Factors of 12 are 1, 2, 3, 4, 6, 12. Factors of 18 are 1, 2, 3, 6, 9, 18.)
2. 24 (The GCF of 8 and 12 is 4, so $8 \times 12 = 96 \div 4 = 24$ or $8 \div 4 = 2 \times 12 = 24$ or $12 \div 4 = 3 \times 8 = 24$.)
3. 2, 3 (Factor pairs of 6: (1, 6) and (2, 3). $1 + 6 = 7$, $2 + 3 = 5$.)
4. Time spent studying is the independent variable. If you don't study more, do you expect to improve?
5. 0.90 m (There are 100 cm = 1 metre. $\frac{90}{100} = 0.9.$ Move the decimal to the left two spaces because there are two zeros in 100.)
6. $2.40 change ($7.60 + $0.40 = $8 then $8 + $2 = $10 so 2 + 0.40 = $2.40)
7. $\frac{7}{10}$ $\left(\frac{4}{5} = \frac{8}{10}\right.$, and 8 is larger than 7)
8. 30 buttons (6×5)

Part B: Perfect Cubes and Squares

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Indicate whether the following numbers are perfect cubes, perfect squares, or neither. Support your answer with a diagram and/or an explanation of the process you used.

   a) 169 b) 324 c) –512 d) 111 e) 8 f) 64

   Answers:

   a) 169 is a perfect square because 169 items can be arranged in a square with dimension of 13 by 13.

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   b) 324 is a perfect square number. Its factor pairs are:

   1, 324

   2, 162

   3, 108

   4, 81

   6, 54

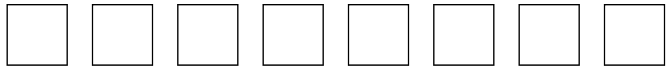
   9, 36

   12, 27

   18, 18

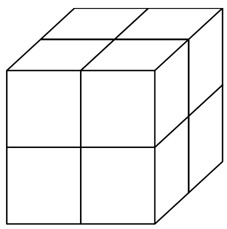
   Because 18 multiplied by itself is equal to 324, $18^2 = 324$ and 324 is a square number.

c) –512 is a perfect cube number with an integer root. Using a calculator, it can be determined that $\sqrt[3]{-512} = -8$. Indicate the keystrokes entered on your calculator when using this process.



d) 111 is neither a perfect square nor a perfect cube. There is no way to arrange 111 items in a square area or cubic shape without leftover items or gaps in the design. There is no number that can be multiplied by itself two or three times that will result in a product of 111.

e) 8 is a perfect cube number. 8 blocks can be arranged in a 2-by-2-by-2 cube.



f) 64 is both a perfect square number and a perfect cube number: $8^2 = 64$ and $4^3 = 64$. The prime factors of $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$, so it is also a perfect 6th number.

Six 2s can be grouped in two sets of three 2s $(2 \times 2 \times 2)(2 \times 2 \times 2) = (8)(8) = 8^2$, or in three sets of two 2s $(2 \times 2)(2 \times 2)(2 \times 2) = (4)(4)(4) = 4^3$.

There are other numbers that have the unique property of being both perfect square and perfect cube numbers. They are called the perfect 6th powers: 1^6, 2^6, 3^6, 4^6, etc.

2. State **all possible** square roots for the following numbers. Explain your answer.

a) 144 b) -36

Answers:

a) $\sqrt{144} = \pm 12$

(12)(12) = 144 and (–12)(–12) = 144 so the square roots of 144 are positive 12 and negative 12.

b) $\sqrt{-36}$ does not have a real number solution. There is no integer you can multiply by itself to come up with a negative product.

3. State **all possible** cube roots for the following numbers. Explain your answer.

   a) −216 b) 8

   Answers:

   a) $\sqrt[3]{-216} = -6$

   (−6)(−6)(−6) = −216, so the cube root of −216 is −6. +6 is NOT a cube root of −216 because (6)(6)(6) = +216, not −216

   b) $\sqrt[3]{8} = 2$

   (2)(2)(2) = 8 so the cube root of 8 is 2. The prime factors of 8 = 2 × 2 × 2.

4. The year 2011 marked the 144th birthday of Canada. In the Confederation Park garden, Erika wanted to plant 144 flowers to commemorate July 1, 1867. How could she have arranged these flowers to make a square design?

   Answer:

   The square root of 144 is 12, so Erika can plant 12 rows of 12 flowers to make a square design.

Learning Activity 2.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. How much larger is a line length 2 cm than a line length 1 cm?
2. How much larger is the area of a 2-by-2 square than a 1-by-1 square?
3. How much larger is the volume of a 2-by-2-by-2 box than a 1-by-1-by-1 box?
4. What is the GCF of 24 and 28?
5. Rewrite the fraction in simplest terms: $\frac{24}{28}$.
6. There are 3 boys and 2 girls invited to your birthday party. If each boy eats 2 pieces of cake and each girl eats 1 piece, how many pieces of cake will be eaten (not including yours)?
7. If each piece of the cake (from the question above) is $\frac{1}{9}$th of the cake, will there be enough cake for you to have a piece?
8. Evaluate the following: $2 - 3 + 6 \times 2 - 5 \times 4$.

Answers:

1. 1 cm
2. 3 ($2 \times 2 = 4, 1 \times 1 = 1, 4 - 1 = 3$)
3. 7 ($2 \times 2 \times 2 = 8, 1 \times 1 \times 1 = 1, 8 - 1 = 7$)
4. 4
5. $\frac{6}{7}$ (use the answer from the above question)
6. 8 pieces of cake ($3 \times 2 = 6, 2 \times 1 = 2, 6 + 2 = 8$)
7. Yes. If the cake is cut into 9 equal pieces $\left(\frac{1}{9}\text{ths}\right)$, your guests will only eat 8 of the 9.
8. -9 (According to BEDMAS, you do multiplication before addition or subtraction, so
   a) $6 \times 2 = 12, 5 \times 4 = 20$
   b) $2 - 3 + 12 - 20$
   c) $-1 + 12 - 20$
   d) $11 - 20 = -9$)

Part B: Rational, Irrational, and Radical Numbers

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Indicate whether the following numbers are rational or irrational, and explain your reasoning.

   a) $\sqrt{5}$

   b) $\sqrt[3]{-3}$

   c) $\sqrt{9}$

   d) 0.25878787...

   e) $\frac{1}{9}$

   f) $\frac{1}{4}$

   g) 0

   h) $\frac{-2}{1}$

   i) −1.12131415...

   j) $-\frac{12}{4}$

Answers:

Answers for Questions 1, 2, and 3 are shown below.

	Question 1	**Question 2**	**Question 3**
a) $\sqrt{5}$	Irrational: Square root of a non-perfect square value.	2.236067977...	
b) $\sqrt[3]{-3}$	Irrational: Cube root of a non-perfect cube number.	−1.44224957	
c) $\sqrt{9}$	Rational: Square root of a perfect square number.	3	Natural
d) 0.25878787...	Rational: Decimal repeats	0.25878787	Rational
e) $\frac{1}{9}$	Rational: Ratio of two integers	0.1111111 Decimal repeats	Rational
f) $\frac{1}{4}$	Rational: Fraction with integer numerator and denominator	0.25 Decimal terminates	Rational
g) 0	Rational: Can be written as a fraction with an integer denominator; for example, $\frac{0}{1}, \frac{0}{-8}$	0	Whole
h) $\frac{-2}{1}$	Rational: Ratio of integers	−2	Integer
i) −1.12131415...	Irrational: Decimal does not repeat or terminate	−1.12131415	
j) $-\frac{12}{4}$	Rational: Quotient of two integers	−3	Integer

2. Determine the exact or approximate decimal value of the numbers in Question 1. Include as many decimal places as your calculator will display.

   Answer:

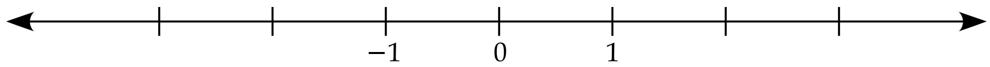
   See Answer for Question 1.

3. Indicate in which subset of the real number system each of the rational numbers in Question 1 best fits.

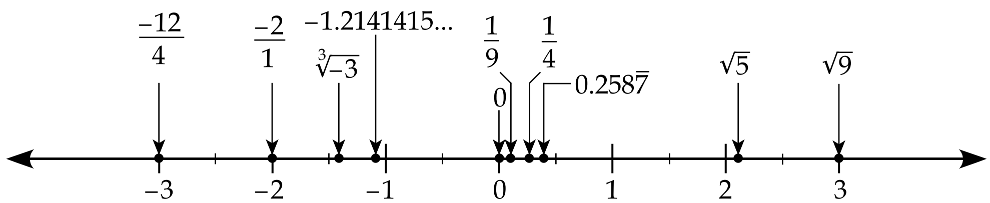
   Answer:

   See Answer for Question 1.

4. Place each number from Question 1 at the appropriate point along the number line.



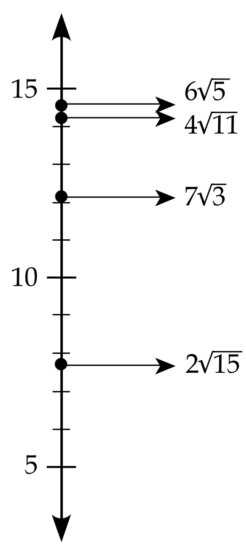
Answer:



5. Write the following mixed radicals as entire radicals, and place them in order along the number line.

$6\sqrt{5}$, $2\sqrt{15}$, $7\sqrt{3}$, $4\sqrt{11}$

Answer:



$$6\sqrt{5} = \sqrt{36} * \sqrt{5} = \sqrt{180} \approx 13.41640786...$$

$$2\sqrt{5} = \sqrt{4} * \sqrt{15} = \sqrt{60} \approx 7.745966692...$$

$$7\sqrt{3} = \sqrt{49} * \sqrt{3} = \sqrt{147} \approx 12.12435565...$$

$$4\sqrt{11} = \sqrt{16} * \sqrt{11} = \sqrt{176} \approx 13.26649916...$$

Learning Activity 2.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. Solve for i: $4i + 3 = 15$.
2. You have half an orange in your lunch. You would like to share it with your two good friends. If you cut it up evenly, how much of the original orange will you receive?
3. Would the data in this situation be continuous or discrete?

   "The time that has passed compared to the distance you have run."
4. Evaluate $\sqrt{-64}$.
5. Evaluate $\sqrt[3]{-8}$.
6. Which is larger: 0.54 or 39%?
7. The Blackhawks have won twice as many games as the Maple Leafs. The Maple Leafs have won five fewer games than the Oilers. If the Oilers have won 13 games, how many games have the Blackhawks won?
8. There is a great sale on clothes at 30% off the marked price. If you are buying a hoodie that is priced at $40.00, how much will you save?

Answers:

1. $i = 3$
2. $\frac{1}{6}$th of an orange each $\left(\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}\right)$
3. Continuous. (You can have a fraction of a minute and run part of a mile.)
4. No solution. (You cannot take the square root of a negative number because only $(-8) \times 8 = -64$, but $-8 \neq 8$.)
5. –2 (Because there are three numbers being multiplied together, you can produce a negative number.)
6. 0.54 (If you changed 39% to a decimal = 0.39, which is less than 0.54.)
7. 16 games (The Leafs have won $13 - 5 = 8$ games. The Hawks have won $2 \times 8 = 16$ games.)
8. $12.00 (10% of $40 = $4.00. Since $30 = 3 \times 10$, 30% of $40 = 3×4 = $12.00.)

Part B: Power Laws Review

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the base, exponent, coefficient of the variable (where appropriate), and power in each of the following. Write out the power as a repeated multiplication and determine the value, if possible:

   Answer:

Power	Base	Exponent	Coefficient	Expansion	Value
x^2	x	2	1	xx	
$\left(\frac{1}{4}\right)^6$	$\frac{1}{4}$	6		$\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$	$\frac{1}{4096}$
$3x^8$	x	8	3	$3xxxxxxxx$	
$-7m^0$	m	0	-7	$(-7)(1)$	-7
$(-15p)^3$	$-15p$	3	$(-15)^3$	$(-15p)(-15p)(-15p)$	$-3375p^3$

   Note: $(-15p)^3 = (-15p)(-15p)(-15p) = (-15p)^3p^3$, so the coefficient of the variable is -15.

2. Simplify each of the following expressions using the exponent laws. Indicate which exponent law(s) you use. Evaluate where possible. (Simplify means to write the given term or equation in the smallest/simplest form. $4^5 \times 4^2 = 4^7$. Evaluate means to find the value of the given term or equation. $4^5 \times 4^2 = 4^7 = 16384$.)

   a) $\left(7^4\right)\left(7^1\right)$

   b) $\left(3x^2\right)\left(-2x^3\right)$

   c) $\dfrac{-8x^4y^5}{2xy^2}$

   d) $-32m^{12} \div 8m^4$

   e) $-\left(\dfrac{5}{3}\right)^2$

   f) $\left(\dfrac{3x^2y}{x^5}\right)^2$

Answers:

a) $(7^4)(7^1) = 7^5 = 16807$ — Product Law

b) $(3x^2)(-2x^3) = (3)(-2)(x^{2+3}) = -6x^5$ — Product Law

c) $\dfrac{-8x^4y^5}{2xy^2} = -4x^{4-1}y^{5-2} = -4x^3y^3$ — Quotient Law

d) $-32m^{12} \div 8m^4 = -4m^8$ — Quotient Law

e) $-\left(\dfrac{5}{3}\right)^2 = -\left(\dfrac{5^2}{3^2}\right) = -\dfrac{25}{9}$ — Power of a Quotient Law

f) $\left(\dfrac{3x^4y}{x^5}\right)^2 = \dfrac{9x^8y^2}{x^{10}} = 9x^{-2}y^2$ — Power of a Quotient Law and Power of a Power Law

3. Simplify the following expression using exponent laws. Show each step in the solution. Find the value of the expression (evaluate).

$$\left[(3^3)^2(5^3)(3^2)\right]^2$$

Answer:

$$\left[(3^3)^2(5^3)(3^2)\right]^2 = \left[(3^6)(3^2)(5^3)\right]^2$$

Remember that bases must be alike in order to combine them using exponent laws.

$$= \left[(3^8)(5^3)\right]^2$$

$$= \left[(3^{16})(5^6)\right]$$

$$= (43046721)(15625)$$

$$= 6.726050125 \times 10^{11}$$

Exponents are also used to express very large and very small numbers using "scientific notation." This means that the value of the expression is approximately 6.73 multiplied by 10 to the power of 11.

4. Create two expressions that, when simplified using the exponent laws, are equivalent. to $\frac{9x^3}{y^2}$. State which laws must be used to simplify your expressions.

Answer:

You may be as creative as you want when writing equivalent expressions. Two possible solutions are:

$\left(\frac{3x}{y}\right)^2(x)$	This uses the Power of a Quotient Law, and the Product Law.
$\frac{27x^5y^7}{3x^2y^9}$	This expression can be simplified using the Quotient Law.

Learning Activity 2.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or paper and pencil.

1. How tall is a door in centimetres if it is 2.2 m tall?
2. Is the following number rational or irrational? π
3. Evaluate 4^3.
4. Which costs less: 3 bottles of pop for $3.99, or 1 bottle of pop for $1.50?
5. Which is larger: $\sqrt{121}$ or 4^2?
6. Evaluate $(2x^2)(4y^5)$.
7. The body of a daddy-long-legs spider is about 0.7 cm long. How long is this in mm?
8. In lacrosse, there are 10 players on the field at one time (per team). If you have 16 players on your team and one coach, how many people will be on the bench during the game?

Answers:

1. 220 cm (There are 100 cm in 1 m, so $2.2 \times 100 = 220$ cm.)
2. Irrational.
3. 64 ($4 \times 4 \times 4$)
4. The 3 bottles would cost less. $\$3.99 \div 3 = \$1.33$ per bottle, which is less than $1.50.
5. 4^2 $\left(\sqrt{121} = 11,\ 4^2 = 16\right)$
6. $8x^2y^5$ ($2 \times 4 = 8$. There is no x in the second term and no y in the first term, so the exponents do not change. You cannot combine the x and y together because they represent different numbers.)
7. 7 mm (10 mm = 1 cm so $0.7 \times 10 = 7$ mm)
8. 7 ($16 - 10 = 6$ players + the coach = 7 people)

Part B: Exponent Laws with Rational and Negative Exponents

Remember, these questions are similar to the ones that will be on your assignments examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them..

This material can be difficult. Remember, you can call your tutor/marker for help if you get stuck or don't understand how to get an answer.

1. Express each radical in exponential form. Simplify using the exponent laws when possible.

   a) $\sqrt{x}$

   b) $\sqrt{dt}$

   c) $\sqrt[7]{g^5}$

   d) $\sqrt[3]{-27x^3y^6}$

   Answers:

   a) $\sqrt{x} = x^{\frac{1}{2}}$

   b) $\sqrt{dt} = (dt)^{\frac{1}{2}} = d^{\frac{1}{2}}t^{\frac{1}{2}}$

   c) $\sqrt[7]{g^5} = \left(g^5\right)^{\frac{1}{7}} = g^{\frac{5}{7}}$

   d) $$\begin{aligned}\sqrt[3]{-27x^3y^6} &= \left(-27x^3y^6\right)^{\frac{1}{3}} \\ &= (-27)^{\frac{1}{3}}\left(x^3\right)^{\frac{1}{3}}\left(y^6\right)^{\frac{1}{3}} \\ &= -3xy^2\end{aligned}$$

2. Express each power as a radical and evaluate when possible.

a) $125^{\frac{1}{3}}$

b) $(-243)^{\frac{1}{5}}$

c) $16^{0.25}$ (**Hint:** Change the decimal into a fraction.)

d) $4^{\frac{-1}{2}}$

Answers:

a) $125^{\frac{1}{3}}$

$= \sqrt[3]{125}$

$= 5$

b) $(-243)^{\frac{1}{5}}$

$= \sqrt[5]{-243}$

$= -3$

c) $16^{0.25}$

$= 16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

d) $4^{\frac{-1}{2}}$

$= \dfrac{1}{4^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{4}}$

$= \dfrac{1}{2}$

3. Simplify using the exponent laws. Evaluate when possible. Leave answers with fractional exponents in their simplest form and as an improper fraction when applicable.

a) $9^{\frac{1}{3}} * 9^{\frac{1}{6}}$

b) $\dfrac{27^{\frac{1}{2}}}{27^{\frac{1}{6}}}$

c) $\left(x^4\right)^{\frac{1}{2}}$

d) $\left(10^2\right)^{\frac{-3}{2}}$

e) $\dfrac{x^2}{\sqrt[3]{x}}$ (**Hint:** Change the radical into fractional form.)

f) $\left(-8x^4y^6\right)^{\frac{1}{3}}$

g) $\left(\dfrac{9x^3y^6}{4x^1y^0}\right)^{\frac{1}{2}}$

Answers:

a) $9^{\frac{1}{3}} * 9^{\frac{1}{6}}$

$= 9^{\frac{1}{3}+\frac{1}{6}}$

$= 9^{\frac{2}{6}+\frac{1}{6}}$

$= 9^{\frac{3}{6}}$

$= 9^{\frac{1}{2}}$

$= \sqrt{9}$

$= 3$

b) $\dfrac{27^{\frac{1}{2}}}{27^{\frac{1}{6}}}$

$= 27^{\frac{1}{2}-\frac{1}{6}}$

$= 27^{\frac{3}{6}-\frac{1}{6}}$

$= 27^{\frac{2}{6}}$

$= 27^{\frac{1}{3}}$

$= \sqrt[3]{27}$

$= 3$

c) $\left(x^4\right)^{\frac{1}{2}}$

$= x^{\frac{4}{2}}$

$= x^2$

d) $\left(10^2\right)^{\frac{-3}{2}}$

$= \dfrac{1}{\left(10^2\right)^{\frac{3}{2}}}$

$= \dfrac{1}{10^{\frac{6}{2}}}$

$= \dfrac{1}{\left(10^3\right)}$

$= \dfrac{1}{1000}$

e) $\dfrac{x^2}{\sqrt[3]{x}}$

$= \dfrac{x^2}{x^{\frac{1}{3}}}$

$= x^{2-\frac{1}{3}}$

$= x^{\frac{6}{3}-\frac{1}{3}}$

$= x^{\frac{5}{3}}$

$= \sqrt[3]{x^5}$

f) $\left(-8x^4y^6\right)^{\frac{1}{3}}$

$= (-8)^{\frac{1}{3}}\left(x^4\right)^{\frac{1}{3}}\left(y^6\right)^{\frac{1}{3}}$

$= -2x^{\frac{4}{3}}y^2$

g) $\left(\dfrac{9x^3y^6}{4x^1y^0}\right)^{\frac{1}{2}}$

$= \left(\dfrac{9x^2y^6}{4(1)}\right)^{\frac{1}{2}}$

$= \left(\dfrac{9}{4}\right)^{\frac{1}{2}}\left(x^2\right)^{\frac{1}{2}}\left(y^6\right)^{\frac{1}{2}}$

$= \dfrac{3}{2}xy^3$

4. Identify the errors a student made in the following problems and correct them. Explain your answer. (Hint: If you can't see the mistake right away, try simplifying the term to the left of the equal sign. What is different about the answer given when compared to the answer you found?)

   a) $\left(\frac{2}{5}\right)^{-3} = -\left(\frac{2^3}{5^3}\right)$

   b) $3m^{-2} = \frac{1}{9m^2}$

   Answers:

   a) The negative exponent means you should take the reciprocal of the base to the positive power. This student multiplied the power by -1 and then applied the positive exponent to the numerator and denominator. It should be:

   $$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5^3}{2^3}\right)$$

   b) The student applied the exponent to the coefficient and the base, not just the base. It should be:

   $$3m^{-2} = \frac{3}{m^2}$$

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 3
Measurement

Module 3: Measurement

Introduction

Measurements of length, mass, volume, and other quantities can be made using a variety of units and systems. This module will showcase the two different systems in use in Canada today: metric and imperial. The metric system, also called SI (based on the French name for the International System of Units), is used in most countries around the world. Since 1982, in Canada, we have used the metric system for measuring. Before that we used the imperial system of measurement. It is still used in the United States today. As both SI and the imperial system have applications in your world today, you will become familiar with and use both systems in this module. You will learn how to convert within and between these systems, and use both SI and imperial units in calculating the surface area and volume of 3-dimensional objects.

You will need a ruler with both centimetres and inches. The use of a measuring tape or metre or yardstick, as well as other measurement tools, would be beneficial

Assignments in Module 3

When you complete Module 4, you will submit your Module 3 assignments, along with your Module 4 assignments, to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 3.1	Units, Area, and Volume
2	Assignment 3.2	Measuring with Vernier Calipers and Micrometers
3	Assignment 3.3	Unit Conversions
4	Assignment 3.4	Volume of Prisms and Pyramids
5	Assignment 3.5	Surface Area of Prisms and Pyramids
6	Assignment 3.6	Surface Area and Volume of Spheres, Cylinders, and Cones

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 3. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1, 2, 3, and 4 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 4.

Resource Sheet for Module 3

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Lesson 1: Linear Measurement

Lesson Focus

In this lesson, you will

- ❑ use estimation and measurement strategies to solve linear measurement questions with SI and imperial units of measure
- ❑ provide and justify your choice of unit measurement, comparing SI and imperial units
- ❑ solve problems using different linear measurement instruments and strategies

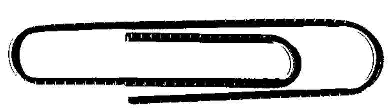
Lesson Introduction

The ability to estimate and measure the size of objects and the distances between places is both a visual and a practical skill. Having a referent, or an object that reminds you of a length, can help you approximate the size, even when you don't have a ruler or measurement tool handy. This lesson will give you the opportunity to practise measuring using traditional tools like rulers, as well as estimate and calculate lengths using referents for both metric and imperial units. You will also use linear measurements to calculate area and volume.

Where Do Measurements Come From?

Referents

A **referent is a known quantity that is used to estimate or compare**. While referents may not make the best foundation for a measurement system, they can be very useful when it comes to understanding, remembering, and comparing units. If an adult's thumb width can be used as a referent for an inch, what could be used to help you estimate or remember the size of a centimetre? Perhaps the width of a child's pinky finger, or the width of a standard-sized paperclip.



Example 1

Think of something you could use as a referent for a metre and a yard. How could this be useful to compare these measures?

Solution:

The doorknob on a standard door is about 1 yard above the floor. A door is typically about 2 m tall. Using a door as a referent, you can see that the knob is just below the middle, so a yard is slightly shorter than a metre.

Example 2

A standard paperclip is about 2 inches long. Explain how you could estimate how wide your desk is using paper clips.

Solution:

A chain of about 12 paper clips would cover the width of an average desk, making it approximately 24 inches wide.

Example 3

Your grandmother has requested you buy her a new table cover after you spilled ink all over her old one. She doesn't have a measuring tape in her house, but you know that your hand span is about 20 cm. How can you determine how much fabric to buy for a new cover?

Solution:

Stretch your fingers out as wide as possible and then place them along the edge of the table. Count how many hand widths it takes to cover the length of the table. Say it takes about seven and a half hand spans along one edge.

$$7.5 \times 20 = 150 \text{ cm.}$$

The fabric for table covers is often sold off a roll, measured by the metre. You would need to buy 1.5 m.

Another way to figure this out would be to consider that there are 100 cm in a metre, and 20 cm in one hand span, so 5 hand spans is about a metre.
$5 + 2.5 = 7.5$ hand spans so you would need one and a half metres.

SI and Imperial Systems of Measurement

The metric system uses prefixes and a decimal structure based on the number 10. When converting to larger or smaller units, factors of 10, 100, 1000, etc. are used, so you can simply move the decimal place rather than perform complex calculations.

Some common prefixes and multipliers are given below.

Name	Symbol	Represents	Multiplier
mega	M	one million	1 000 000
kilo	k	one thousand	1 000
hecto	h	one hundred	100
deca	da	ten	10
base unit		one	1
deci	d	one-tenth	0.1
centi	c	one-hundredth	0.01
milli	m	one-thousandth	0.001
micro	μ	one-millionth	0.000 001

The prefixes of kilo, centi, and milli are used the most frequently. Mega and micro, representing very large and very small values, have useful applications in science and technology. Hecto, deca, and deci are used in specialized situations.

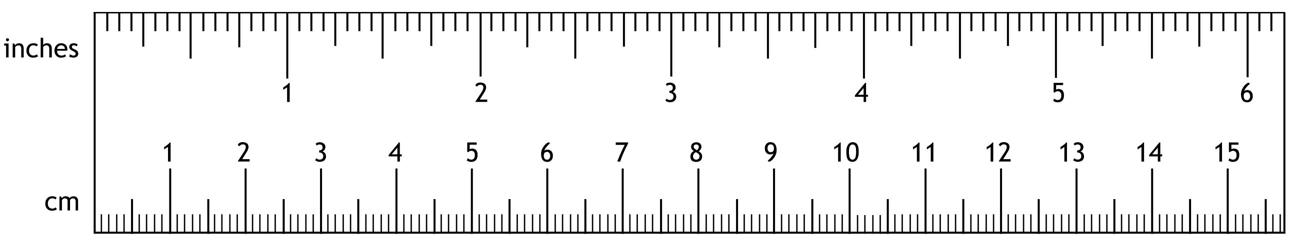
Some of the units and symbols that are used with the metric system include the metre (m), kilogram (kg), and litre (L). A millilitre is $\frac{1}{1000}$th of a litre, or there are 1000 millilitres in a litre. A kilogram is 1000 grams, and a kilometre is 1000 metres.

Historically, the units in the imperial system were convenient referents. The inch was based on a thumb width, the foot was someone's left foot, and 1000 double paces was the basis for the mile. As handy as this may seem, the conversions required in the imperial system are inconvenient and complex, using a variety of factors such as 2, 3, 5.5, 12, 1760…

When you shop in a grocery or hardware store, you may occasionally find both systems being used on packaging labels. The imperial system is still widely used in construction (for example, you may purchase 4′ × 8′ plywood sheets, 2″ × 4″ boards, or 1 pound of nails) and in agriculture (rural roads that mark fields typically form a 1-mile grid). If you travel to the United States, distances are given in miles and temperature readings are in degrees Fahrenheit, rather than kilometres and degrees Celsius. As a result, it's important to understand and be able to use both metric and imperial measurements, and convert within and between the systems.

Linear Measurement in Metric and Imperial Units

Compare the increments on your metric and imperial rulers. The most common linear measurements are inches and centimetres. The centimetre $\left(\frac{1}{100} \text{ of a metre}\right)$ is subdivided into 10 parts, so you can measure to the nearest millimetre $\left(\frac{1}{1000} \text{ of a metre}\right)$. The inch is typically divided into 16 parts, so you can measure to the nearest $\frac{1}{16}$ of an inch.



Example 4

Measure the following lines to the nearest tenth of a centimetre and the nearest $\frac{1}{16}$ of an inch. Record your answers below.

	Metric	Imperial
a) ____	______	______
b) __________	______	______
c) __________________	______	______
d) ____	______	______

Solution:

a) 7.6 cm $\quad 3''$

b) 2.1 cm $\quad \dfrac{13}{16}''$

c) 3.5 cm $\quad 1\dfrac{3}{8}''$

d) 8.4 cm $\quad 3\dfrac{5}{16}''$

Other common linear measurements with their abbreviations and equivalents in both the imperial and the SI systems are listed below.

Imperial

12 inches (12 in. or 12″) = 1 foot (1 ft. or 1′)

36 in. (36″) or 3 ft. (3′) = 1 yard (1 yd.)

5280 ft. (5280′) or 1760 yd. = 1 mile (1 mi.)

Metric

10 millimetres (10 mm) = 1 centimetre (1 cm)

1000 mm or 100 cm = 1 metre (1 m)

1000 m = 1 kilometre (1 km)

This information will be used throughout this module, so it would be useful to have on your Resource Sheet.

The choice of units depends on what is being measured. For long distances, you would use miles or kilometres, while for shorter distances feet or metres may be more appropriate. For small objects or lengths, inches, cm, or mm may be used.

Example 5

Determine the most suitable units in both the imperial and metric systems for measuring the following:

a) your height

b) length and width of your TV screen

c) distance from Winnipeg to Steinbach

d) height of a pop can

e) length of a sheet of curling ice

f) diameter of a dime

Solution:

	Imperial	Metric
a)	feet	centimetres or metres
b)	inches	centimetres
c)	miles	kilometres
d)	inches	centimetres
e)	feet or yards	metres
f)	inches	millimetres

The ability to estimate lengths is very important. It helps determine if your actual measurements and the units you chose to measure in are reasonable.

Example 6

Approximate the size of this paper (length and width) using both systems. Which metric and imperial units would be the most appropriate? Use a referent to help you estimate. Then, measure the page using a ruler and check your approximations.

Solution:

Using the imperial system, inches would be the most appropriate, as both the length and the width of this page are less than one foot long. My pinky finger is about 2 inches long. Using that as a referent, this page is about 4 pinky lengths wide and 5 pinky lengths long. I estimate this page is about 8″ by 10″.

Using metric, centimetres would be the most reasonable choice of units. Millimetres are too small to make a realistic or accurate estimation. If I make an L shape with my thumb and index finger, the diagonal distance between my fingernails is about 15 cm. This page is about one and a half of those lengths wide and about two long. An estimation for the dimensions of this page is 22 cm by 30 cm.

Using a ruler, this page measures $8\frac{1}{2}''$ wide by 11″ long, or 21.5 cm wide by 28 cm long. The estimations using referents were close to these actual dimensions, so the answers are accurate and the units appropriate.

Measurement Strategies

Many different types of devices are available to determine linear measurements. You probably have rulers and tape measures in your home or tool box. Some use laser or electronic means to calculate length. Micrometers and calipers can be used to measure very small objects with great precision and accuracy. Using the appropriate tool makes it easier to measure correctly. But what if you need linear measurements of an object that has irregularly shaped dimensions? A ruler may not be the best tool, or you just may need to get creative in how you use it!

Example 7

You need to determine the circumference of a beach ball, but all you have is a piece of string. How can you do this?

Solution:

Use an established referent like your hand span and determine a length of the string. Say your hand span is 7″. Mark a piece of string that matches your hand span and count how many lengths of string it takes to go around the ball. Then calculate the approximate circumference. If it takes 6 lengths of string to circle the ball, then

$$6 \times 7 = 42 .$$

The circumference of the ball is about 42 inches.

Example 8

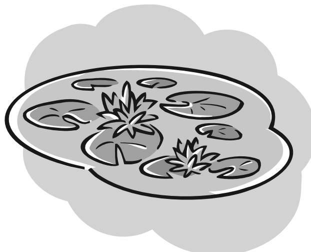
You want to know if you can fit a hot tub that is 2 m by 2 m on your backyard deck. All you have is a piece of paper. How can you estimate the size of the tub?

Solution:

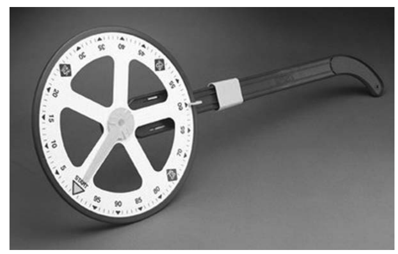
In example 6 above, you estimated and measured the dimensions of a page. The perimeter of a sheet of paper is 99 cm. This is a great referent for a metre length! Choose the spot on the deck you would like to measure, and mark off two lengths and two widths of the paper to estimate 1 m. Repeat to determine approximately 2 m.

Example 9

Your mom wants to plant flowers along the edge of a curved fish pond in the backyard. She needs to know the length of the perimeter. How could you help her?



Solution:



A trundle wheel is a measuring device consisting of a wheel with a handle. As you push the wheel along the ground, it rotates and makes a clicking sound each time it completes one revolution. If the length of the circumference is known, you can estimate the linear length the wheel travels by counting the clicking sounds. Using a trundle wheel, you could determine the perimeter of the pond.

If you do not have a trundle wheel, you could walk around the pond placing your feet heel-to-toe and count the number of steps it takes. Measure the length of your shoe in inches or centimetres and determine the perimeter of the pond.

Alternately, you could measure your stride in metres or feet and count the number of paces it takes you to walk around the pond. The choice of strategy and units would be based on the size of the pond. A large pond would be better measured in larger units like feet, yards, or metres, while a very small pond may be measured in inches or centimetres.

Using Linear Measurements to Determine Area and Volume

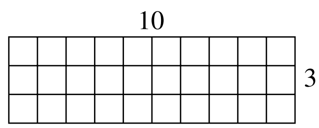
Area

The area of an object is stated in square units. The area of this rectangle is calculated by multiplying the linear measures of its length and width.

$$A = L \times W$$

$$A = 10 \times 3$$

$$A = 30$$



The area of this rectangle is 30 units^2.

Example 10

Refer back to the length and width of the sides of a paper you measured earlier. What is the area of the sheet of paper? State your answer in both cm^2 and square inches.

Solution:

This page measures $8\frac{1}{2}''$ wide by 11″ long or 21.5 cm wide by 28 cm long.

$A = L \times W$ (imperial) or $A = L \times W$ (metric)

$$A = 8\frac{1}{2} \times 11 = 8 \times 11 + \frac{1}{2} \times 11$$

$$A = 88 + \frac{11}{2} = 88 + 5\frac{1}{2} = 93\frac{1}{2}$$

$$A = 21.5 \times 28$$

$$A = 602$$

The area of this page is $93\frac{1}{2}$ in.^{2} or 602 cm^2.

Note: If the measurements are given as fractions, you should give your answer as fraction operations. If the measurements are given as decimals, you should give your answer as a decimal.

Note the following conversions:

Area

Imperial

$12'' \times 12'' = 144$ in.$^2 = 1$ ft.^{2}	(recall $12'' = 1$ ft.)
$3' \times 3' = 9$ ft.$^2 = 1$ yd.^{2}	(recall $3' = 1$ yd.)

Metric

10 mm $\times$ 10 mm = 100 mm^2 = 1 cm^2	(recall 10 mm = 1 cm)
100 cm $\times$ 100 cm = 10 000 cm^2 = 1 m^2	(recall 100 cm = 1 m)

These will be used multiple times throughout this module, so it would be convenient to have them on your Resource Sheet.

Volume

Volume is calculating by including the third dimension of depth, and the answer is stated in cubic units like ft.^{3}, yd.^{3} or cm^3, m^3.

Example 11

Find the volume of a tropical fishtank that measures 4 feet by 1.5 feet by 1.25 feet.

Solution:

$V = L \times W \times D$

$V = 4 \times 1.5 \times 1.25$

$V = 7.5$

The volume of the tank is 7.5 cubic feet.

Note the following conversions:

Volume

Imperial

$12'' \times 12'' \times 12'' = 1728$ in.$^3 = 1$ ft.^{3}

$3' \times 3' \times 3' = 27$ ft.$^3 = 1$ yd.^{3}

Metric

10 mm $\times$ 10 mm $\times$ 10 mm = 1000 mm^3 = 1 cm^3

100 cm $\times$ 100 cm $\times$ 100 cm = 1 000 000 cm^3 = 1 m^3

These conversions will be used several times in this module, so it is a good idea to add them to your Resource Sheet.

Learning Activity 3.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate: $\left(\dfrac{9x^4y^3}{3xy^2}\right)$.
2. Which is the dependent variable: the windchill compared to the time it takes to get frostbite?
3. A school volleyball team wants to practise twice per week. They cannot practise at the school on the weekend (Saturday and Sunday), half the team cannot practise on Monday and Wednesday, and the basketball team uses the gym on Friday. Which days can the team practise?
4. Solve for p: $p \div 15 = 5$.
5. What is the GCF of 19 and 13?
6. What two numbers have a sum of 11 and a product of 18?
7. You want to save up $12 000 to buy a car one year from now. How much do you have to save per month to reach this goal?
8. Three students receive their marks for a project. Jane found her mark as a decimal, 0.62; John calculated his mark as a percent, 83%; Jean got $\dfrac{12}{16}$.

   Who got the best mark?

continued

Learning Activity 3.1 (continued)

Part B: Units

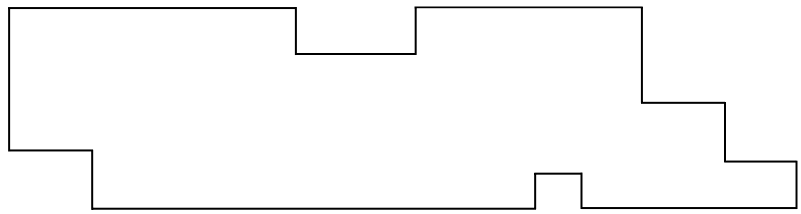
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Provide referents for the following linear measurements.
   a) millimetre
   b) metre
   c) inch
   d) mile
2. Determine a referent based on your height to compare the lengths of a metre and a yard. Use it to estimate the height of a table.
3. Use a referent of your choice to estimate the dimensions of your calculator or a cell phone. Indicate your referent, choice of units, and then measure to verify the accuracy and appropriateness of your measurement and units.
4. Which units from the metric system and from the imperial system would be the most suitable for measuring the following lengths?
   a) Width of a snowboard
   b) Length of a soccer field
   c) Thickness of a coin
   d) Distance a jogger runs every day
5. Estimate the following lengths in both SI and imperial measurements. Check your estimates by measuring.
   a) Length and width of a standard door
   b) Length of a vehicle
   c) Length and width of a two-car driveway
   d) Diameter and thickness of a $2 coin (toonie)

continued

Learning Activity 3.1 (continued)

6. Using an imperial ruler, find the perimeter of the following diagram to the nearest $\frac{1}{16}''$.



7. Your dad bought you a car for your 16th birthday and wants to wrap it with a large ribbon to surprise you. Explain how he could determine the length of ribbon to buy, and which units would be the most appropriate.

8. What would be the most suitable units of measure for the following areas and volumes? Give an answer using both systems.
   a) The volume of water in a thimble
   b) The area of a hockey ice surface
   c) The volume of earth removed for a basement of a house
   d) The area of one side of a coin

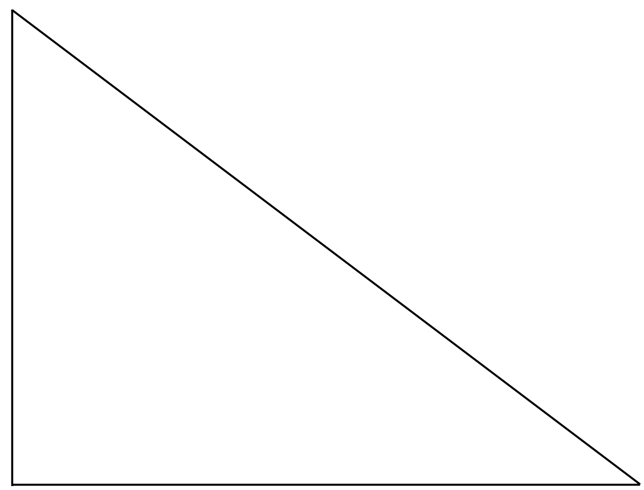
continued

Learning Activity 3.1 (continued)

9. a) Measure the following right triangle to the nearest mm and calculate its area.

   b) Measure the following right triangle in inches and calculate its area.

   Recall: $A_{\Delta} = \dfrac{bh}{2}$



Lesson Summary

You are now a walking ruler! You have established referents for both SI and imperial units that you can use to estimate, compare, and calculate the lengths of objects. You have had practice in measuring using inches and centimetres, and have developed strategies for measuring irregularly shaped objects using measurement tools as well as referents. You have used linear measurements to calculate area and volume. In the next lesson, you will learn how to convert within and between the metric and imperial systems, and what you have learned so far will help you verify the accuracy and appropriateness of your answers. In Lesson 3, you will continue to work with surface area and volume.

Assignment 3.1

Units, Area, and Volume

Total Marks = 25

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Provide referents for the following linear measurements. *(4 marks)*

   a) centimetre ______________________________

   b) kilometre ______________________________

   c) yard ______________________________

   d) foot ______________________________

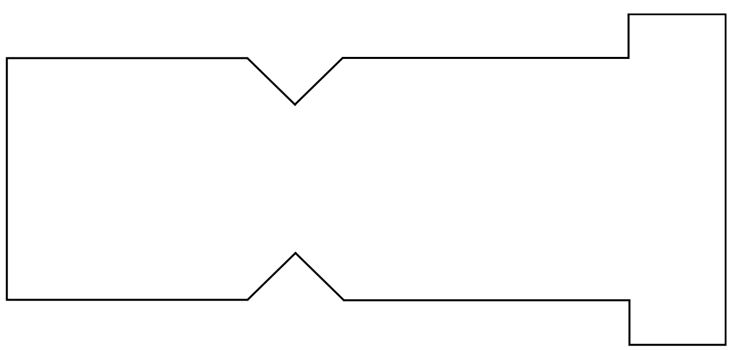
2. Which units from the metric system and which units from the imperial system would be the most suitable for measuring the following? *(4 marks)*

	Metric	Imperial
a) Diameter of a dinner plate	________	________
b) Height of a hot air balloon	________	________
c) Diameter of a diamond	________	________
d) Distance from Brandon to The Pas	________	________

Assignment 3.1: Units, Area, and Volume (continued)

3. Using a measurement instrument like a scale map, pedometer, bicycle odometer, a car trip meter, or GPS, determine a referent for a mile and a kilometre. It may be based on the distance from your house to a landmark like your school, community centre, or a grocery store, or a number of blocks. Compare these two measures. Use it to estimate the distance from your house to a friend's house. Describe the strategy you used to solve this problem. *(10 marks)*

4. Using a metric ruler, find the perimeter of the following diagram to the nearest tenth of a centimetre (millimetre). *(5 marks)*



Assignment 3.1: Units, Area, and Volume (continued)

5. A computer CPU box measures $6\frac{7}{8}''$ by $15\frac{1}{2}''$ by $15\frac{3}{4}''$.

   Determine the volume of the box. *(2 marks)*

Notes

Lesson 2: Calipers and Micrometers

Lesson Focus

In this lesson, you will
- ❑ find measurements using precision instruments
- ❑ read a metric Vernier caliper
- ❑ read a metric micrometer

Note: Although none of the material in this lesson will be on either of the examinations, there is an assignment at the end of the lesson that is to be submitted to the Distance Learning Unit.

Lesson Introduction

Precision measurement gives more exact readings. People who work as carpenters, plumbers, aircraft mechanics, machinists, millwrights, industrial mechanics, mechanical engineers, power electricians, auto mechanics, woodworkers, or electricians use specialized equipment to get very precise measurements. This lesson shows you how to read two of these specialized instruments: Vernier calipers and micrometers. It would be very helpful if you had a metric Vernier caliper or micrometer for this lesson. However, if don't have one, or do not have access to one, you can still do the lesson.

Precision Measurements

Metric rulers make measurements that are precise to the nearest tenth of a centimetre. Imperial rulers make measurements precise to the nearest $\frac{1}{16}$ of an inch. Sometimes you need measurements that are even more precise! This lesson will focus on reading metric calipers, which measure length to the nearest hundredth of a centimetre, and micrometers, which measure to the nearest thousandth of a centimetre.

Imagine a simple plastic cap that fits on the end of a ballpoint pen. When the manufacturer makes this cap, he has to use precision measurements. If the inside diameter of the cap is too small, it will not fit. If the diameter is too large, the cap will fall off. The workers must be sure to use exact, precise measurements when manufacturing these caps.

Precise to What Unit?

If you say the width of your bedroom is 4 m, then the measurement is precise to the nearest metre. If you say the room measures 4.25 m, then it is precise to the nearest cm. This would be especially important if you were cutting carpet to cover the floor. If your measurements were not precise to the nearest cm, you might have a gaping hole along one wall.

A luthier is an expert craftsman who makes violins. Luthiers need to use exact measurements. If measurements are not precise to the nearest 0.1 of a millimetre, the harmonic series of the instrument is off, and the violin will not sound proper when it is played.

Vernier Calipers

This Vernier caliper has various parts, including the ones listed below and shown on the diagram on the opposite page.

1. **Outside jaws:** used to take external measures of objects
2. **Inside jaws:** used to take internal measures of objects
3. **Depth probe:** used to measure the depth of objects
4. **Main scale** (cm)
5. **Main scale** (inch)
6. **Vernier** (cm)
7. **Vernier** (inch)
8. **Retainer:** used to block movable parts

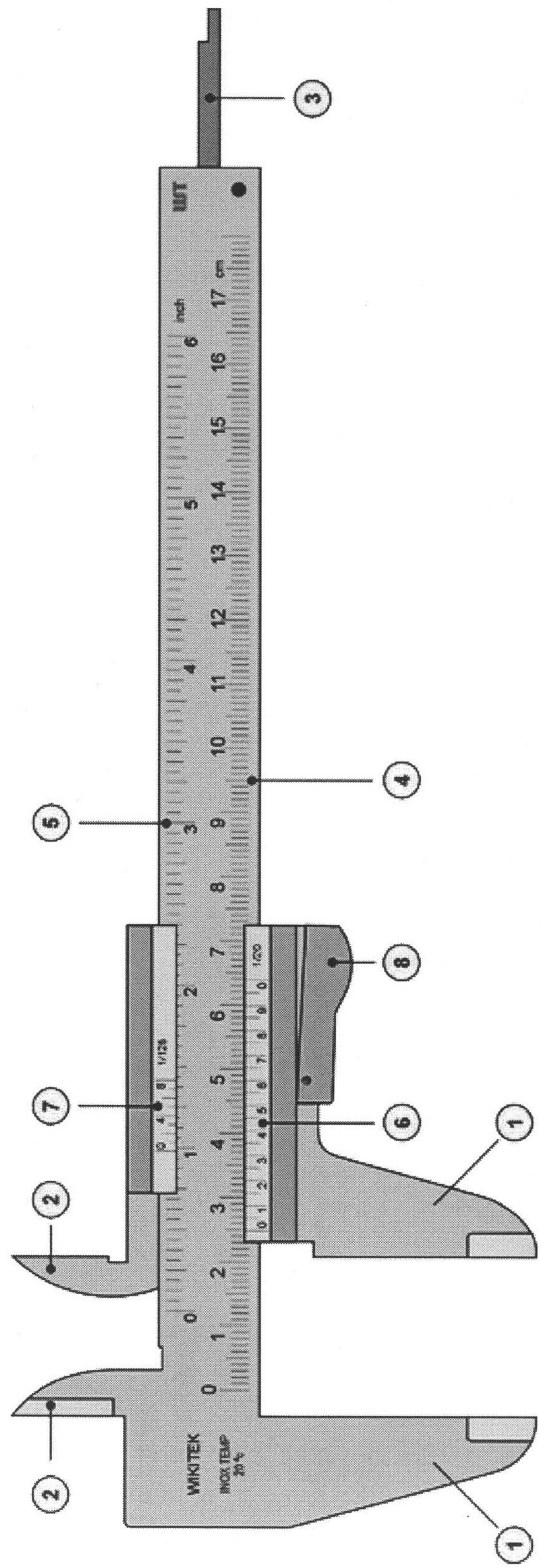
Three devices for measuring:

1. The outside jaws are used to measure outer dimensions of objects (e.g., the outer diameter of a pipe).
2. The inside jaws are used to measure the inner dimensions of objects (e.g., the inside diameter of a pipe).
3. The depth gauge is used to measure depth of objects (e.g., the depth of a small container).

Two measurement scales:

1. A fixed or main scale, in both metric and imperial units
2. A moving or sliding Vernier scale, in both metric and imperial units

The fixed scale does not move, and the moving scale is called the Vernier scale.


WIKITEK
INOX TEMP 20 °c
inch
cm
1/128
1/20

In this course, the readings on Vernier calipers calibrated in metric units are studied.

The fixed scale on a Vernier caliper is divided into millimetres, which are 0.1 cm or $\frac{1}{10}$ of a centimetre. The moving Vernier scale is divided into 0.1 millimetres, which is the same as $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ cm.

Therefore, measurements taken with a Vernier caliper are precise to the nearest hundredth of a centimetre $\left(\frac{1}{100} \text{ of a centimetre} = \frac{1}{10} \text{ of a millimetre}\right)$.

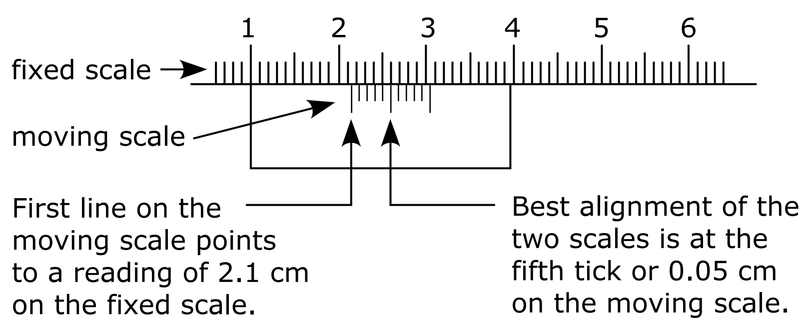
The only possible reading from the Vernier scale is one of the numbers 0.00, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, or 0.09.

Reading Vernier Calipers

The following examples will help you in reading the measurements taken with the Vernier caliper.

Example 1

Read the following Vernier caliper measurement.



Solution:

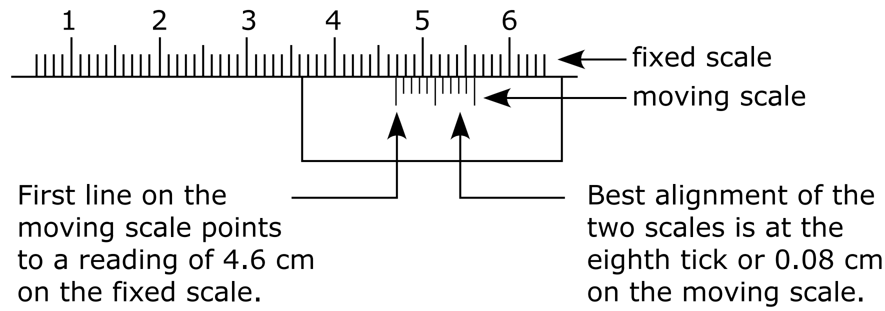
Step 1: Read the fixed scale, using the first line on the moving scale as a pointer. This line points to a place between 2.1 cm and 2.2 cm. You state the reading from the fixed scale as 2.1 cm, giving you a precision unit to the nearest tenth of a centimetre. You had this much precision with a ruler.

Step 2: Find the line on the moving scale that most closely aligns with a mark on the fixed scale. In this case, the line that best matches is the fifth line on the moving scale. Since the moving scale has 10 divisions, and each division represents 0.01 cm, the reading is 0.05 cm.

Step 3: State the total measurement. In this case, the reading on the caliper = 2.1 cm + 0.05 cm = 2.15 cm. Now you have a measurement that is precise to the nearest hundredth of a centimetre—considerably more precise than with a ruler.

Example 2

Read the following Vernier caliper measurement.



Solution:

Step 1: The first arrow points to a place on the fixed scale between 4.6 and 4.7. The reading is 4.6 cm.

Step 2: The second arrow points to the alignment of the two scales at the 8th tick mark on the moving scale, giving a reading of 0.08 cm.

Step 3: Therefore, the total reading = 4.6 cm + 0.08 cm = 4.68 cm.

Using Vernier Calipers and/or Virtual Calipers

If you have access to a Vernier caliper, use it to practise measuring the widths of various small objects. If you don't have access to one, don't worry about it. You can use virtual calipers to practise reading the measurements.

There are many Internet sites that include virtual calipers. You set the width, and read the scale. Then the site tells you if you read it correctly. Enter "Vernier caliper virtual" into your search engine and try some of them. They are very good practice for reading the Vernier scales.

Learning Activity 3.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Convert 24 inches into feet.
2. If the equation of a line is $y = 3x + 4$, what is the y-intercept?
3. Evaluate $(3y^7)^3$.
4. You are at a hockey game and would like a snack and a drink. At the counter, you see that popcorn is $3.00, peanuts are $2.25, and hotdogs are $3.75. The drinks are $2.00. If you have $5, what snack can you afford to get, if you also buy a drink?
5. You are trying to estimate the height of your brother, who is approximately 1′ taller than your sister. Your sister is half a foot taller than you. How tall is your brother if you are 5′ tall?
6. Solve for w: $w \div 6 = 2$.
7. If 5% of 260 is 13, what is 5% of 520?
8. A person gets to choose from two chocolate milkshakes. One milkshake fills the glass $\frac{7}{9}$ full; the other fills the glass $\frac{2}{3}$ full. Which would you prefer?

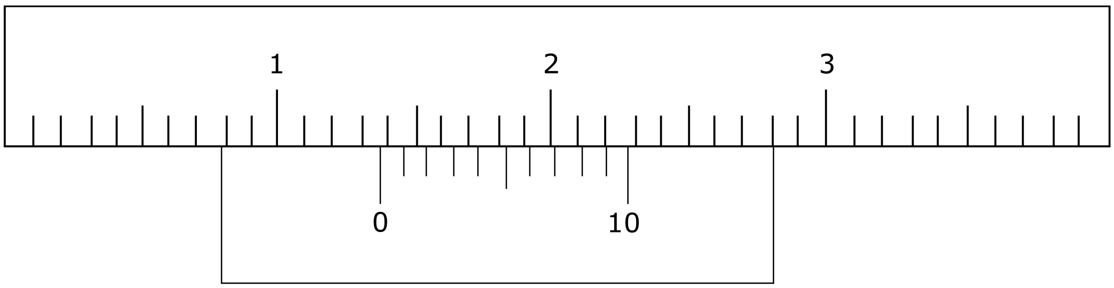
continued

Part B: Vernier Caliper Measurements

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

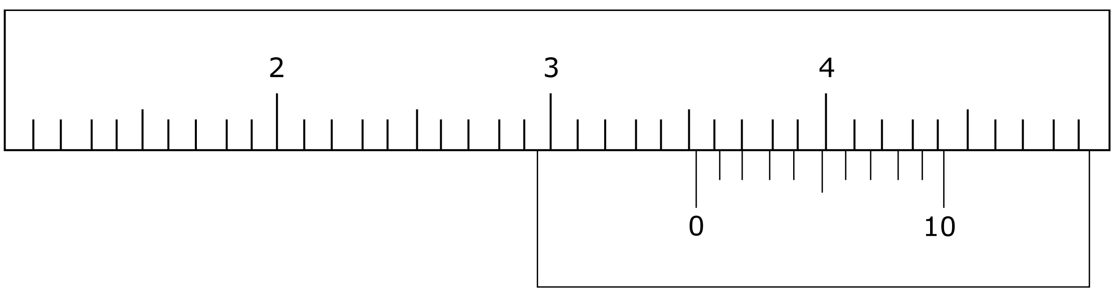
Write down the measurement as shown on these Vernier caliper diagrams. Read the measurements to the nearest hundredth of a centimetre.

1.



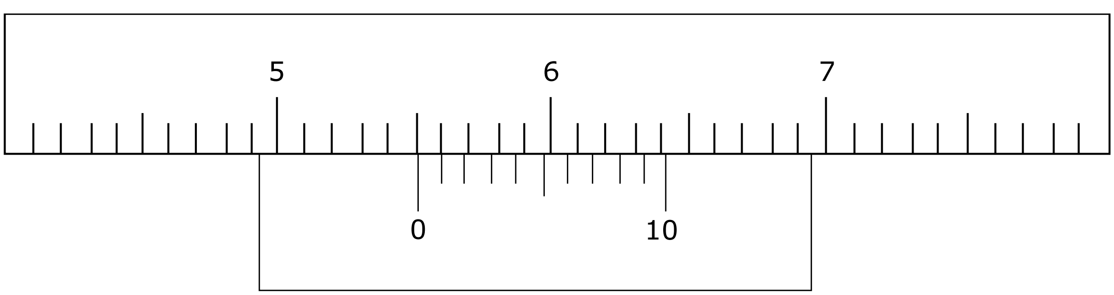
Reading: ________________________

2.



Reading: ________________________

3.

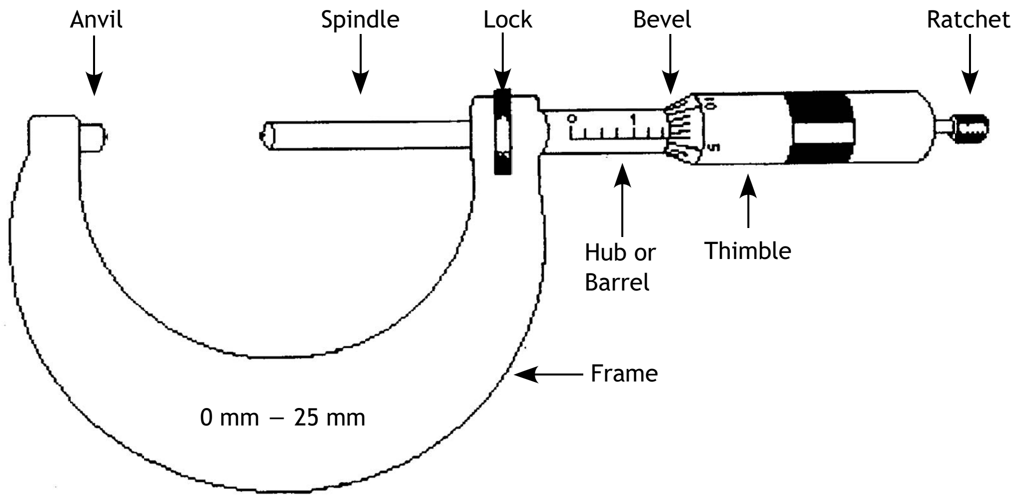


Reading: ________________________

Micrometers

Micrometers are even more precise than vernier calipers! They measure to the nearest thousandth of a centimetre.

The following diagram shows the key components of a micrometer.



Note the following parts of the instrument.

- The measuring device is called the jaws. The anvil and spindle are used to measure small lengths between 0 mm and 25 mm.
- There are two measurement scales. The fixed scale is on the barrel, which has 25 main divisions (1 mm each) and 25 subdivisions (marking 0.5 mm). The moving scale is called the thimble, and has 50 divisions (marking 0.01 mm each). The scale of the thimble makes it possible to measure accurately to the nearest hundredth of a millimetre, or thousandth of a centimetre.
- On any micrometer there is only one system of measurement: either metric or imperial, but not both. To get measurements in both systems, you would need two different micrometers. In the next lesson, you will learn how to convert one unit to another so that you only need one type of measuring tool.

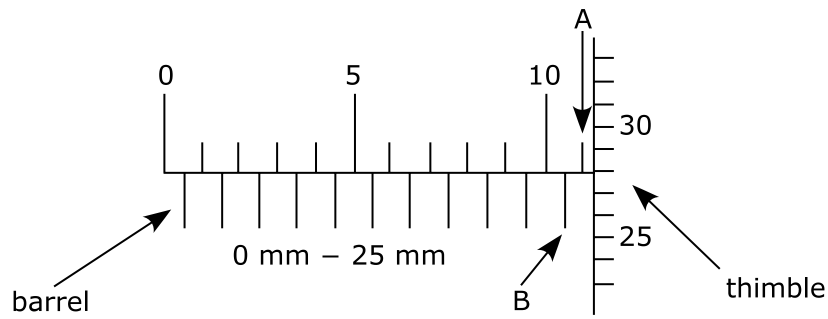
Using Micrometers

If you have access to a micrometer, use it to practice measuring the widths of various small objects. If you don't, don't worry about it—move on to "Reading Micrometers." Find the thickness of 20 sheets of paper, a hair pin, or even the rim of a coffee mug. Place the object between the jaws, and then rotate the thimble using the ratchet. When the object is secure and you have heard a few clicks on the ratchet, read the measurement as shown below.

Reading Micrometers

Example 3

Read the following micrometer measurement.



Solution:

Step 1: Read the upper barrel scale in mm. The reading will be a whole number from 0 mm to 24 mm. In this diagram, the measure of the last marking showing on the upper scale, indicated by arrow at A, is 11 mm.

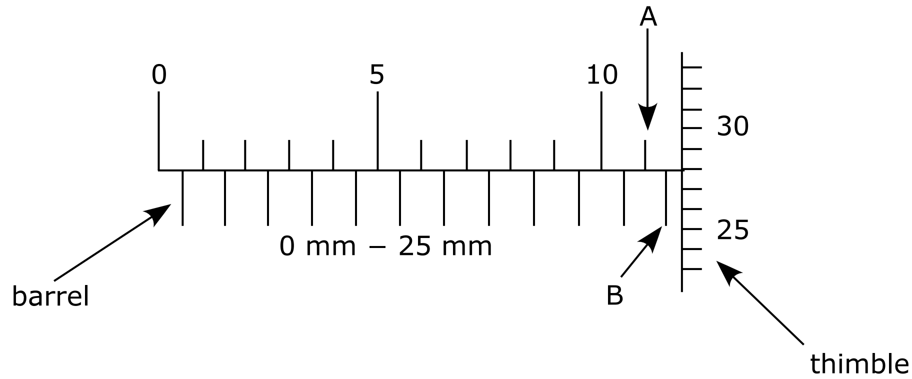
Step 2: Read the lower barrel scale. This scale marks points halfway between each millimetre marking on the top scale. The reading will be either 0.0 mm or 0.5 mm. In this diagram, the arrow at B is before the arrow at A so the reading is 0.0 mm. Thus, the final reading will be between 11.00 mm and 11.50 mm.

Step 3: The thimble reading is written as a decimal from 0.00 mm to 0.49 mm. In the diagram, the reading on the thimble is 0.28 mm.

Step 4: Add up the readings from Steps 1, 2, and 3 to arrive at the resulting measurement. The sum is 11 mm + 0.0 mm + 0.28 mm = 11.28 mm.

Example 4

Read the following micrometer measurement.



Solution:

Step 1: The measure of the last marking showing on the upper barrel scale, indicated by arrow A, is 11 mm.

Step 2: The lower barrel scale shows a marking, indicated by arrow B, to the right of arrow A. This means the reading is 0.5 mm. In other words, the reading is more than halfway past 11 mm. The final reading will be between 11.50 mm and 11.99 mm.

Step 3: The thimble reading yields 0.28 mm.

Step 4: The resulting measurement = 11 mm + 0.5 mm + 0.28 mm = 11.78 mm.

Virtual Micrometers

There are many Internet sites that include virtual micrometers. You set the width and read the scale. Then the site tells you if you read it correctly. Enter "micrometer virtual" into your search engine and try some of them. They provide good practice for reading a micrometer.

Learning Activity 3.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

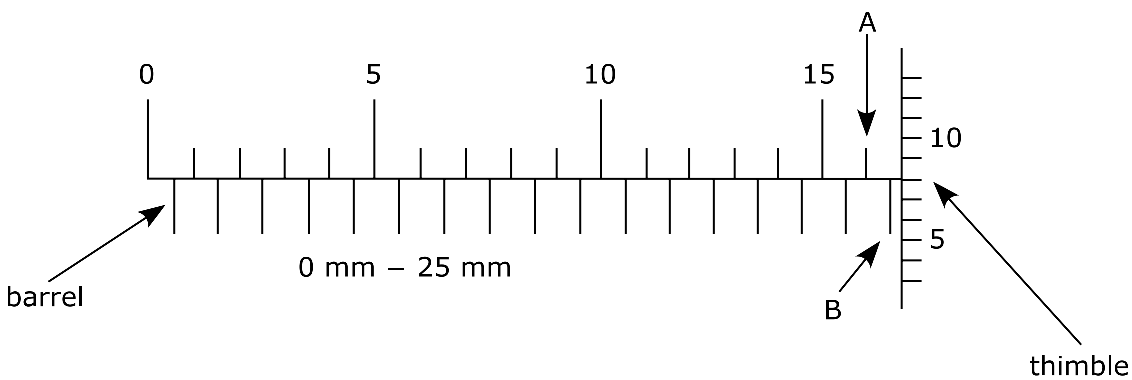
1. Jon and Kate have 8 kids. Each kid shares a bedroom with another sibling. Including Jon and Kate's bedroom, how many bedrooms do they need in their house?
2. If you are looking for a good deal, you found it! Bread is 50% off! One loaf usually costs $2.40. How much will you pay with the discount?
3. Evaluate $2 \times \sqrt[3]{27}$.
4. Evaluate $(4x^3)^{-2}$.
5. If $0.33\overline{3} = \frac{1}{3}$, then what does $0.66\overline{6}$ equal?
6. You are running errands all day. You have to go to the nursery, so you drive 8 km. You then go to the mall, so you drive another 6 km. Finally, you go to the movie store, which is another 3 km, and then 6 km home. How far did you drive altogether?
7. You have measured your foot to be 9.5". Is your foot a good referent to approximate a foot in length?
8. Your friend wants to go to a movie at 9:30 pm. You want to be at the movie 30 minutes early. It takes you 15 minutes to get from your house to the theatre. What time do you have to leave in order to get to the theatre on time?

continued

Part B: Micrometer Measurements

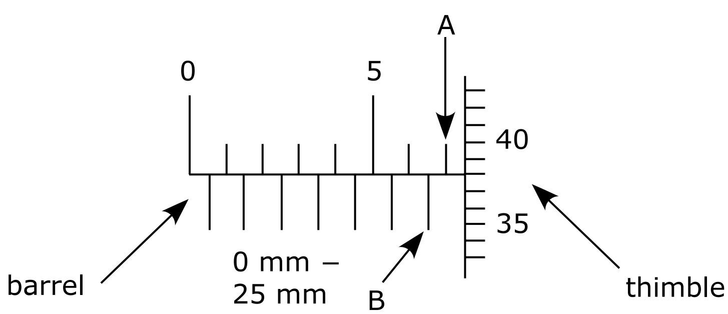
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Read the following micrometer measurement.



Reading: ________________________

2. Read the following micrometer measurement.



Reading: ________________________

Lesson Summary

In this lesson, you learned what a Vernier caliper is, its three measuring devices, its two measurement scales, two systems of measurement, and how precise a measurement it can make. You then practised reading precise measurements using a Vernier caliper diagram. You also learned what a micrometer is, and the names of its parts. You learned how to read the fixed scale and the moving scale to find a measurement precise to the nearest thousandth of a centimetre. You then practised reading precise measurements using a micrometer diagram.

Notes

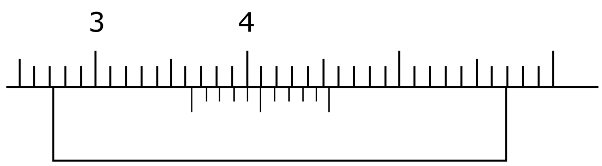
Assignment 3.2

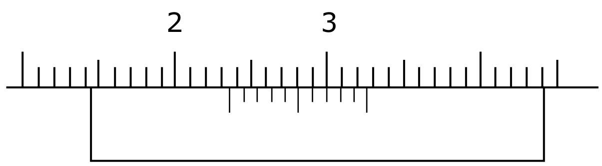
Measuring with Vernier Calipers and Micrometers

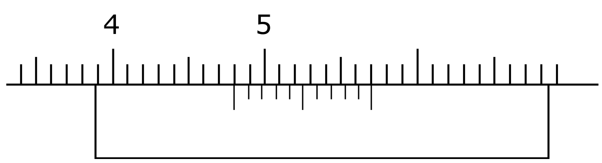
Total Marks = 10

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

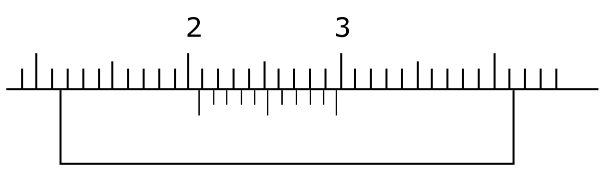
1. Read and record the following Vernier caliper measurements. These readings are in metric units. *(6 marks)*

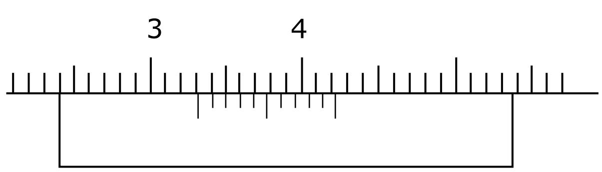
   a)


   b)


   c)


   d)

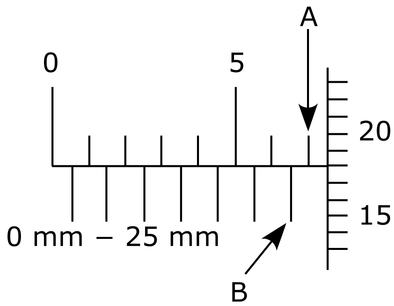

   e)


   f)


Assignment 3.2: Measuring with Vernier Calipers and Micrometers (continued)

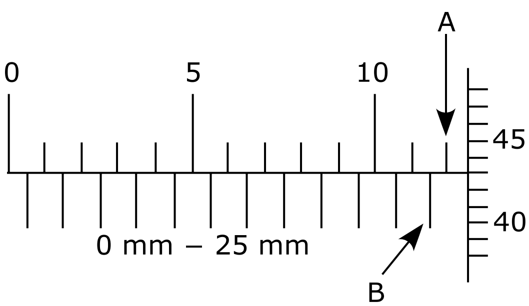
2. Read and record the following micrometer measurements. *(2 marks)*

   a)



   ____

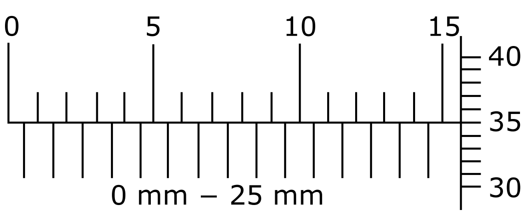
   b)



   ____

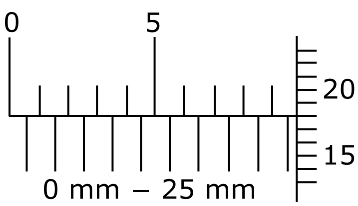
3. Read and record the following micrometer measurements. *(2 marks)*

   a)



   ____

   b)



   ____

Lesson 3: Conversions

Lesson Focus

In this lesson, you will

- ❑ convert measurements within and between SI and imperial systems
- ❑ solve conversion problems and use mental math to justify your answer

Lesson Introduction

On July 23, 1983, shortly after Canada converted to the metric system, Air Canada Flight 143, a Boeing 767 jet, took off from Montreal enroute to Edmonton. About halfway through the flight the jet completely ran out of fuel and made an emergency landing at a former airbase near Gimli, Manitoba. The investigation into the incident revealed that an incorrect conversion factor was used when calculating the amount of fuel needed in the airplane. The mechanics and pilots used an imperial value of 1.77 pounds per litre rather than the necessary metric factor of 0.8 kg/L, resulting in 8703 kg of fuel being added to the tanks, rather than the required 16 131 kg. This was less than half the amount needed! Fortunately, the pilot had experience flying gliders and the co-pilot was familiar with the location of the Gimli airbase. Despite the fact that the abandoned airstrip was being used as a track for go-kart and drag races, and that day the area was filled with cars and spectators, no one was seriously hurt in the incident! (For further information about this incident, see www.wadenelson.com/gimli.html or http://en.wikipedia.org/wiki/Gimli_Glider.)

Changing Units of Measurement

Conversion Ratios within Systems

Sometimes measurements are given in a particular unit and you need to convert the value to another unit in order to use it in further calculations or because using a different unit may be more appropriate. Unit conversions may be done using a ratio and proportional reasoning.

If you wanted to convert from inches to feet, you could set up a ratio using a known conversion factor:

$$\frac{12 \text{ inches}}{1 \text{ foot}}$$

Example 1

A playground slide is six feet long. How many inches is this?

Solution:

Use the conversion ratio of $\frac{12 \text{ inches}}{1 \text{ foot}}$.

$$\frac{12}{1} = \frac{x}{6}$$
$$6(12) = x$$
$$x = 72$$

Note that in the second ratio, you are looking for inches but you know the number of feet.

The slide is 72″ long.

Example 2

Bob is 5′ 10″ tall. How tall is he in inches?

Solution:

We need to convert 5′ to inches. We set up a ratio of inches : feet.

$$\frac{12 \text{ inches}}{1 \text{ foot}}$$

Now set up a proportion.

$$\frac{12}{1} = \frac{x}{5}$$
$$5(12) = x$$
$$x = 60$$

60 + 10 = 70, so Bob is 70 inches tall.

This proportional reasoning works for all conversions within and between systems.

Some equivalents for linear measures in SI and imperial units were given in Lesson 1.

Length

Imperial

12 inches (12 in. or 12″) = 1 foot (1 ft. or 1′)

36 in. (36″) or 3 ft. (3′) = 1 yard (1 yd.)

5280 ft. (5280′) or 1760 yd. = 1 mile (1 mi.)

Metric

10 millimetres (10 mm) = 1 centimetre (1 cm)

1000 mm or 100 cm = 1 metre (1 m)

1000 m = 1 kilometre (1 km)

Equivalents for area and volume:

Area

Imperial

$12'' \times 12'' = 144 \text{ in.}^2 = 1 \text{ ft.}^2$	(recall 12″ = 1 ft.)
$3' \times 3' = 9 \text{ ft.}^2 = 1 \text{ yd.}^2$	(recall 3′ = 1 yd.)

Metric

$10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2 = 1 \text{ cm}^2$	(recall 10 mm = 1 cm)
$100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2 = 1 \text{ m}^2$	(recall 100 cm = 1 m)

Volume

Imperial

$12'' \times 12'' \times 12'' = 1728 \text{ in.}^3 = 1 \text{ ft.}^3$

$3' \times 3' \times 3' = 27 \text{ ft.}^3 = 1 \text{ yd.}^3$

Metric

$10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} = 1000 \text{ mm}^3 = 1 \text{ cm}^3$

$100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} = 1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$

Have you included all of those measurements on your Resource Sheet? If you haven't yet, you should do that now.

Example 3

Nana needs 1.5 cubic feet of topsoil to fill her planters. She buys 2500 cubic inches. Is this enough?

Solution:

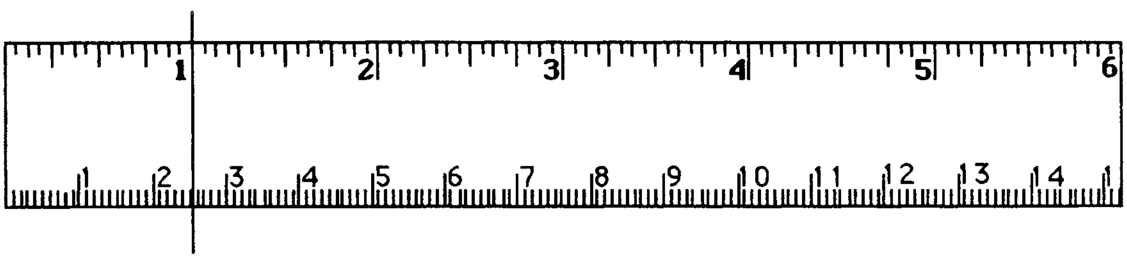
You need to convert 1.5 ft.^{3} to in.^{3} using the conversion ratio of $\frac{1728 \text{ in.}^3}{1 \text{ ft.}^3}$.

$$\frac{1728}{1} = \frac{x}{1.5}$$
$$1.5(1728) = x$$
$$x = 2592$$

She needs 2592 cubic inches of topsoil. It's close, but not quite enough topsoil.

Conversion Ratios between Systems

What about comparisons between the systems? You have used referents to compare centimetres and inches, but how do they match up when placed across from each other on a ruler? Using the illustration below, approximate the numerical relationship between an inch and a centimetre.



From the diagram, 1 inch is approximately 2.5 cm. The answer, correct to 2 decimal places, is exactly 1 inch = 2.54 centimetres.

Example 4

A book shelf is 30 cm deep. Will a textbook that is 12 inches wide fit in the shelf?

Solution:

You could either convert the shelf depth to inches or the book width to centimetres. Just set up the conversion ratio so the **N**ew units are in the **N**umerator.

If you convert the book width to centimetres, use $\dfrac{2.54\text{ cm}}{1''}$ as the conversion ratio.

$$\frac{x}{12} = \frac{2.54}{1}$$
$$x = 30.48$$

The textbook is 30.48 cm wide and will not fit on the shelf, although it nearly fits.

Other comparisons between metric and imperial units could be made. The following conversions are the most commonly used.

Having these conversion units on your Resource Sheet would be convenient.

1 inch = 2.54 cm	1 mm = 0.0394″
1 foot = 30.48 cm	1 cm = 0.394″
1 yard = 0.9144 m	1 m = 39.97″ = 3.28′
1 mile = 1.609 km	1 m = 1.09 yd.
1 gallon = 4.546 litres	1 km = 0.621 mi.
1 pound = 0.454 kg	1 kg = 2.2 lb.
$°F = \frac{9}{5} \times (°C + 32)$	$°C = \frac{5}{9} \times (°F - 32)$

Notice that when comparing the Fahrenheit and Celsius formulas, the numerator and denominator are switched.

In order to avoid an incident similar to the one in Gimli (mentioned at the beginning of the lesson), it should be clear that the gallon listed above is the imperial gallon. In the USA, they use a different gallon, called the US gallon. You are not expected to convert to the US gallon in this course.

Example 5

The repair a mechanic is doing requires a 3 mm drill bit. He has no metric drill bits, but he does have the following imperial sizes: $\frac{1}{16}''$, $\frac{1}{8}''$, and $\frac{3}{16}''$.

Which drill bit would be the best choice?

Solution:

Convert 3 mm to inches using the ratio 1 mm = 0.0394″, putting the new units in the numerator.

$$\frac{x}{3} = \frac{0.0394}{1}$$

$$x = 3 \times 0.0394$$

$$x = 0.1182$$

The drill bit needs to be as close to 0.1182 inches as possible. Use division to convert the fractions to decimals to make it easier to compare.

$$\frac{1}{16}'' = 0.0625''$$

$$\frac{1}{8}'' = 0.125''$$

$$\frac{3}{16}'' = 0.1875''$$

The best choice would be to use the $\frac{1}{8}''$ drill bit, as 0.1182″ is closest to 0.125″.

Example 6

A pool is 7 yd., 1 ft. wide by 9 yd. long. What are these dimensions in metric units? Justify your answer using mental math.

Solution:

When a measurement involves a combination of units, rewrite the values using only one unit and then perform any calculations.

Width:

Write the width using yards only, so convert the 1 foot to yards. The conversion ratio would be $\frac{1\text{ yd.}}{3\text{ ft.}}$.

$$\frac{x}{1} = \frac{1}{3}$$

$$3x = 1$$

$$x = \frac{1}{3} = 0.33\overline{3}$$

$$7 \text{ yd.} + 0.33\overline{3} = 7.33\overline{3} \text{ yards}$$

$$7 \text{ yards, 1 foot } \textit{is } 7.33\overline{3} \text{ yd.}$$

The most appropriate metric unit to convert to would be the metre.
1 yard = 0.9144 m

$$\frac{x}{7.333} = \frac{0.9144}{1}$$

$$x = 7.333 \times 0.9144$$

$$x = 6.7052952 \text{ m}$$

Length:

$$\frac{x}{9} = \frac{0.9144}{1}$$

$$x = 9 \times 0.9144$$

$$x = 8.2296 \text{ m}$$

The metric dimensions of the pool are approximately 6.7 m by 8.2 m

Remember: Use all decimal places in calculations and round the final answer only.

Using mental math to justify the solution, consider that a metre is slightly longer than a yard. Therefore, the lengths in this metric unit should be slightly smaller than when using the corresponding imperial unit. To convert 7 yd., 1 ft × 9 yd. to 6.7 m × 8.2 m appears reasonable.

Example 7

Your favourite burger claims to have a quarter-pound $\left(\frac{1}{4}\text{ lb.}\right)$ of meat. How much would this be in metric units? Justify your answer using mental math.

Solution:

1 pound = 0.454 kg

$$\frac{x}{0.25} = \frac{0.454}{1}$$
$$x = 0.25 \times 0.454$$
$$x = 0.1135 \text{ kg}$$

Using mental math to verify this answer, consider that a pound is about half of a kilogram. One-quarter of a pound would be about one-quarter of half a kilogram, or $\frac{1}{8}$ of a kilogram (0.25 × 0.5 = 0.125). (Think: Half of a quarter is 12.5¢ or $0.125, so the answer 0.1135 kg is a reasonable answer.)

When you are talking about such a small fraction of a kilogram, it may be more meaningful to convert the measurement to grams. The prefix of kilo means one thousand, so there are 1000 grams in a kilogram.

$$0.1135 \text{ kg} * \frac{1000 \text{ g}}{1 \text{ kg}} = 113.5 \text{ g}$$

Learning Activity 3.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. You want to meet your friends for coffee. You are available from 9 am to 3 pm. Aiden is free from 12 pm until 2 pm. Leah is able to come from 10 am until 1 pm. What time can you meet for coffee?
2. Your suitcase cannot exceed 26 kg when you are going on an international flight. Your cousin weighs around 25 kg. Would your cousin be a good referent?
3. What is the LCM of 4, 6, and 8?
4. If the rise of a line is 12 and the run of the same line is 8, what is the slope of the line (in simplest form)?
5. A 2-door sports car gets 12.2 km per L of gas. A truck gets 7100 m per L of gas. Which is more fuel efficient?
6. Complete the pattern: 4, 1, −2, ____, ____.
7. The ratio comparing the distance on a map to the distance in real life is 1 cm: 10 km. If the distance from your house to your school on the map is 4 mm, how far do you live from school?
8. Is 0.2754 a rational or irrational number?

continued

Part B: Imperial-Imperial and Imperial-Metric Conversions

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The owner's manual for a car states that the oil should be changed every 5000 km. How many miles is that? Use mental math to determine if your answer is reasonable.
2. You are listening to an American radio station on your way to Fargo, ND for a weekend. The announcer states that the temperature in Fargo is 18°F. What kind of weather can you expect while there?
3. Convert the following:
   a) 2 m = _______________ mm
   b) 4 ft. = _______________ in.
   c) 6 yd. 2 ft. = _______________ ft.
   d) 6 yd. 2 ft. = _______________ in.
   e) 7500 m = _______________ km
   f) 2 miles = _______________ ft.
   g) 4.7 cm = _______________ mm
   h) 7650 cm = _______________ m
   i) 3520 yd. = _______________ mi.
   j) 720 000 cm = _______________ km
4. Convert the following (Pay close attention to if you are working with volume or area):
   a) Change 7 cm^2 to mm^2.
   b) Change 432 $in.^2$ to $ft.^2$.
   c) Change 3.6 $yd.^2$ to $ft.^2$.
   d) Change 55 000 cm^3 to m^3.

Conversions within the Metric System

Look back at your answers to question 3 in Learning Activity 3.4 above. What do you notice about the conversion within the metric system? One defining characteristic of the metric system is that it is based on multiples of 10. When converting from one SI unit to another, you are simply changing the prefix and moving the decimal place the same number of spaces as there are zeros in the multiple of 10.

$$4.5 \text{ cm} \times \frac{10 \text{ mm}}{1 \text{ cm}} = 45 \text{ mm}$$

$$761 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 7.61 \text{ mm}$$

$$8850 \text{ m} \times \frac{1000 \text{ m}}{1 \text{ km}} = 8.85 \text{ km}$$

$$3.97 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 3970 \text{ g}$$

$$355 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}} = 0.355 \text{ L}$$

If you are converting to a **L**arger unit, the decimal place moves **L**eft, and when you convert to a smaller unit, it slides right by the same number places as there are zeros in the multiple of 10.

In the metric system, you can easily convert units by moving the decimal place and changing the prefixes of the units of length, mass, and volume. What sets the metric system apart from the imperial system, however, is that there are connections between the quantities of volume and mass as well!

If you measure 1 mL of water, its mass will be 1 g and its volume will be 1 cm^3.

Therefore, 1 L of water would have a mass of 1 kg. A cubic metre of water would have a mass of 1 tonne. Try to do that with the imperial system (It isn't quite as simple).

Lesson Summary

Errors in converting within and between SI and imperial units of measure may not always have such dire consequences as in the Gimli Glider incident, but the passengers on that flight must have wished the pilot and crew had paid more attention when doing their math on that fateful day! If they had used the correct conversion ratio and then used mental math to verify the accuracy of their work like you do, this catastrophe may have been avoided. The application of the skills you are learning in these lessons may come in very handy in the future. You will use them in the next lessons, as you solve problems involving surface area and volume of 3-D objects, using SI and imperial units.

Assignment 3.3

Unit Conversions

Total Marks = 20

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Convert the following: *(12 marks)*

   750 mL = ________________ L

   0.35 kg = ________________ g

   77 in. = ________________ cm

   5 gal. = ________________ L

   66 531 ft. = ________________ mi.

   3 yd. 2 in. = ________________ in.

   2 ft.^{3} = ________________ in.^{3}

2. A newborn baby boy weighs 3.5 kg. What is his mass in pounds? Explain how you can use mental math to determine whether your answer is reasonable. *(4 marks)*

Assignment 3.3: Unit Conversions (continued)

3. The summit of Mount Everest is 8840 m above sea level. How many miles is this? *(4 marks)*

Lesson 4: Volume of Prisms and Pyramids

Lesson Focus

In this lesson, you will

- ❑ determine the volume of right prisms and pyramids, using objects or diagrams
- ❑ determine an unknown dimension of a prism or pyramid given its volume
- ❑ describe the relationship between the volume of a pyramid and prism with the same base and height
- ❑ solve contextual problems based on composite 3-D objects (composite objects are objects made up of more than one 3-D shape)

Lesson Introduction

In Lesson 1, you considered how linear measurements could be used to determine the area and volume of 3-dimensional objects (solid objects that have length, width, and height). This lesson will build on that, and you will develop formulas to calculate the volume of prisms and pyramids.

You should add the formulas to your Resource Sheet for volume of a prism, volume of a pyramid, and area of different polygons as you go through this lesson. Make sure when you write down the formulas that you know what each of the symbols stands for, and know what the formula is for.

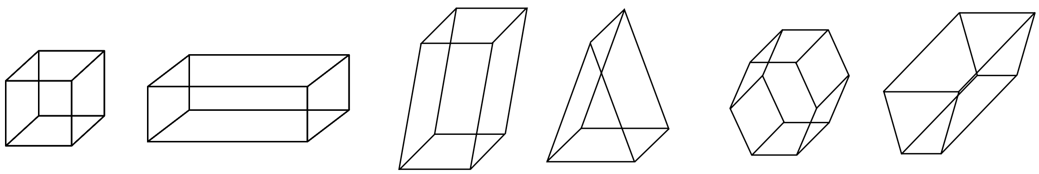
Volume

The volume of your music is measured in decibels, but the mathematical definition of **volume is the amount of space that a 3-D object occupies**. It is the number of cubes (or parts of cubes) it takes to fill an object. It is measured in cubic units like cm^3, $in.^3$, or cubic feet ($ft.^3$) to name a few possibilities.

Volume of Prisms

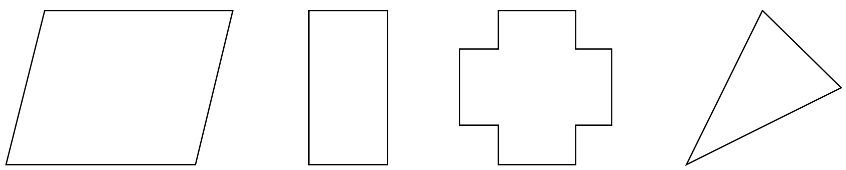
A prism is a three-dimensional figure (solid) that has two of the same sides (faces) that face each other, which are polygons (the bases); the remaining lateral faces are parallelograms. (A parallelogram is a 2-D, 4-sided shape. Its opposite sides are parallel)

Examples:



Note: A polygon is a 2-D or flat figure formed with three or more line segments.

Examples:



This lesson will focus on right prisms, where the base is perpendicular to the remaining faces. It is important to understand that the base is not necessarily the bottom of the prism—it is the shape that is consistent throughout the entire object. In the rectangular prisms pictured above, any pair of parallel sides may be considered the base, but in the triangular prism, the triangle-shaped sides are the "base" and the rectangular sides are the "faces."

In Lesson 1, you calculated the volume of a rectangular solid by multiplying the three dimensions of the object.

$$V = L \times W \times H$$

This would not work for triangular prisms or prisms with different face shapes like the hexagon-shaped prism pictured above.

The definition of a prism is determined by the base shape and the height or length of the faces. The formula should also reflect this.

Consider that in a rectangular prism, the formula to determine the area of the base is

$$A = L \times W.$$

This is then multiplied by the height of the prism to calculate the volume.

$$V = (L \times W) \times H$$

The formula may be rewritten as

$$V = (\text{area of base}) * (\text{height})$$

or

$$V = Bh$$

where B = area of the base and h = the height of the prism.

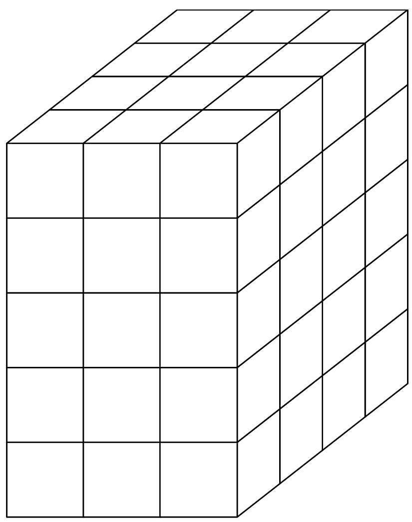
This formula will work for prisms with any shape of polygon base. It would be a useful formula to have on your Resource Sheet.

Example 1

A storage company stacks boxes 3 across, 4 deep, and 5 layers high. What volume of boxes does this represent?

Solution:

Sketch a diagram of this situation.



The area of the base of this stack is 3 by 4 boxes or 12 square boxes.

$$B = 3 \times 4$$

$$B = 12$$

The height of the stack is 5 so the volume can be calculated as

$$V = Bh$$

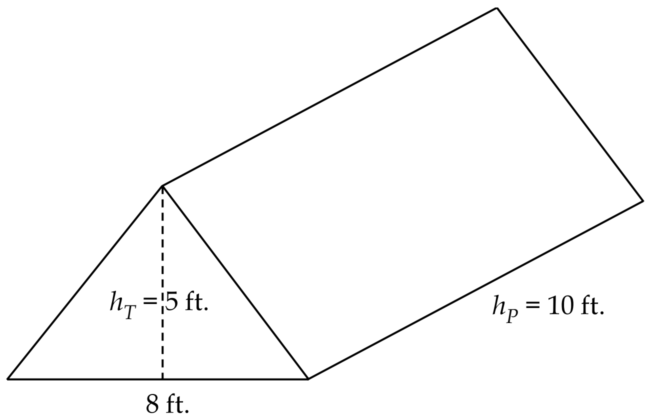
$$V = 12(5)$$

$$V = 60$$

The stack is 60 square boxes.

Example 2

A pup tent has a triangular base that is 8 feet across, 5 feet high, and 10 feet long. Determine the amount of space inside the tent.



Solution:

Even though the triangular shape is on the front side, it is considered the base, as this is the shape that is consistent throughout the length of the tent.

This formula would be useful to have on your Resource Sheet.

The formula for the area of a triangle is

$$A = \frac{bh_T}{2},$$

where h_T is the height of the triangle base.

$$A = \left(\frac{5 * 8}{2}\right)$$

$$A = 20$$

The area of the base is 20 ft.^{2}.

The volume of the tent is then calculated as:

$$V = Bh_P$$

where h_P is the height of the prism

$$V = 20(10)$$

$$V = 200$$

The tent has 200 cubic feet of space inside it, or 200 ft.^{3}.

Note: The term *height* is used in both the calculation of the area of the base and the volume of the prism, but each time it represents a different value. Make sure you use the correct value in each part of the formula.

Example 3

Calculate the volume of a rectangular prism with base dimensions of 10 inches by 12 inches, and a height of 15 inches.

Solution:

$V = Bh$

$V = (10 \times 12)15$

$V = 120 \times 15$

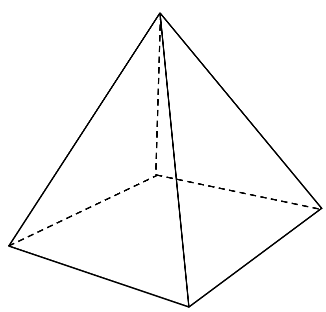
$V = 1800$

The volume of this prism is 1800 cubic inches.

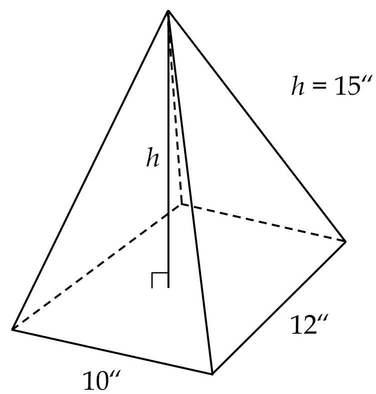
Volume of Pyramids

In a prism, the shape of the base remains consistent throughout the entire height of the object. A pyramid is a figure in which the shape of the base reduces to a single point throughout the height of the figure. **The base of a pyramid is a polygon and its lateral faces are triangles that share a common vertex**.

For example:



A rectangular pyramid with base dimensions of 10 inches by 12 inches and a height of 15 inches has a volume of 600 cubic inches.



Compare that to the previous example, where you calculated the volume of a prism with those exact dimensions. What do you notice?

$$1800 \div 3 = 600$$

The volume of a pyramid is exactly one-third of the volume of a prism when they have the same dimensions. From this, you can conclude that the formula for the volume of a pyramid is

$$V = \frac{1}{3}Bh$$

where B is the area of the base of the pyramid and h is the perpendicular height from the vertex of the pyramid to its base.

This formula would be useful to have on your Resource Sheet.

Example 4

The Great Pyramid of Giza near Cairo, Egypt has been measured to be 755 feet along the edges of its square base and 481 feet high. What volume of rock is in the pyramid? (Ignore the comparatively small volume that was hollowed out for chambers and hallways.)

Solution:

The area of the square base is

$$A = L \times W$$

$$A = 755 \times 755$$

So $B = 755^2$

The volume of a pyramid with a square base is calculated as:

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3} \times \left(755^2\right) \times 481$$

$$V = 91\ 394\ 008.33$$

The volume of rock in the pyramid is over 91 million cubic feet.

This can be written in scientific notation as 9.1394×10^7 cubic feet.

Using the conversion ratio of 1 yd.^{3} = 27 ft.^{3}, the volume can be converted to cubic yards.

$$91\ 394\ 008.33 \text{ ft.}^3 \times \frac{1 \text{ yd.}^3}{27 \text{ ft.}^3} = 3\ 384\ 963.271 \text{ cubic yards or } 3.385 \times 10^6 \text{ yd.}^3.$$

Example 5

A grocery store displays 1 cubic metre of oranges, arranged in a pyramid with a square base, 1.5 m along one side. How high is the display?

Solution:

In this question, you are given the volume and are asked to solve for a missing dimension—in this case, the height of the pyramid.

Write the formula, substituting all known values, and solve for the unknown dimension.

$$V = \frac{1}{3}Bh \text{ where } B = 1.5^2$$

$$1 = \frac{1}{3}\left(1.5^2\right)h$$

$$1 = 0.75h$$

$$h = \frac{1}{0.75}$$

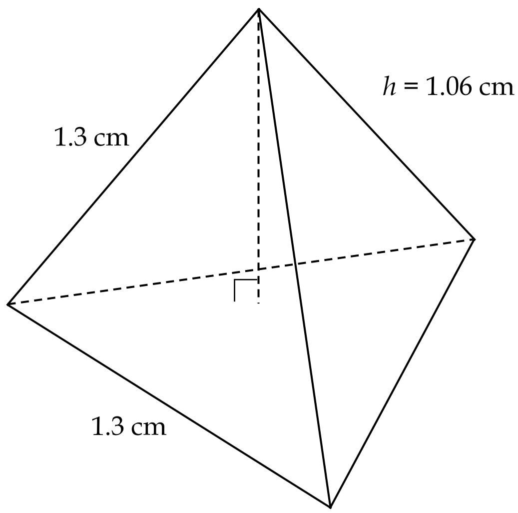
$$h = 1.333$$

The display of oranges is 1.3 m high.

Example 6

Typically, 6-sided dice are used for playing games. It is possible to use a 4-sided die if it is made from a pyramid with equilateral triangles for its base and faces. Sketch a diagram of this die and calculate its volume if the triangles are 1.3 cm along an edge and the height of the pyramid is 1.06 cm.

Solution:



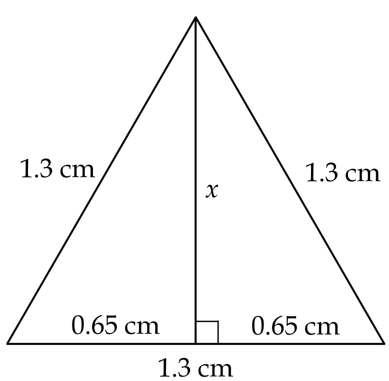
A triangular pyramid with congruent equilateral triangles for each of its faces is called a tetrahedron. The formula for its volume is:

$$V = \frac{1}{3}Bh_P$$

In this case, the base is a triangle, and the formula for the area of a triangle is:

$$A = \frac{bh_T}{2}$$

Consider one face of the tetrahedron and determine its area.



The base of the equilateral triangle is 1.3 cm and its height, x, can be calculated using the Pythagorean Theorem.

$$x^2 + y^2 = z^2$$

$$x^2 + 0.65^2 = 1.3^2$$

$$x^2 = 1.3^2 - 0.65^2$$

$$x^2 = 1.2675$$

$$x = \sqrt{1.2675}$$

$$x = 1.125833025$$

So the area of the base is

$$A = \frac{bh}{2}$$

$$A = \frac{(1.3)(1.125833025)}{2}$$

$$A = 0.7317914662$$

The area of the triangular base is 0.7317914662 cm^2.

Remember to use all decimal places in calculations and only round final answers.

The volume of the die is

$$V = \frac{1}{3}Bh$$

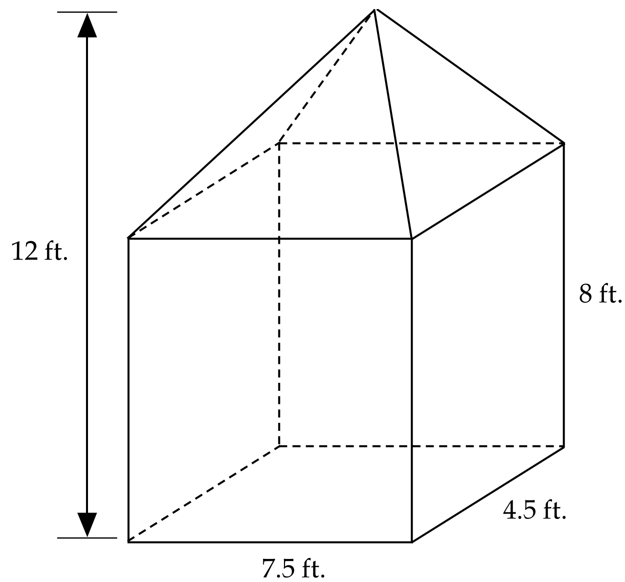
$$V = \frac{1}{3}(0.7317914662)(1.06)$$

$$V = 0.2585663181$$

The volume of the die is about 0.26 cm^3.

Example 7

A tool shed has the following shape. Determine the storage space inside the shed.



Solution:

This object is composed of a rectangular prism and a rectangular pyramid. Calculate the volume of the two spaces and combine them.

Prism

$V = Bh$

The base is rectangular so its area is

$$A = L \times W$$
$$A = 7.5 \times 4.5$$
$$A = 33.75$$

so

$$V = 33.75 \times 8$$
$$V = 270$$

Pyramid

The pyramid has the same shaped base as the prism, so its area is also 33.75 ft.^{2}.

The height of the roof is the difference between the total height and the height of the prism.

The volume of the pyramid is

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}(33.75)(4)$$

$$V = 45$$

Total volume = 270 + 45

Total volume = 315

The shed has a storage space of 315 cubic feet.

Learning Activity 3.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. You are getting ready to start your first job and you want to make sure you get there on time. It takes you 20 minutes to ride your bike to work from your house. You want to get there 15 minutes before your shift starts. It takes you 30 minutes to get ready in the morning. If your shift starts at 10:00 am, what time will you have to wake up?
2. Put the following numbers in order from smallest to largest: 0.53, 29%, 0.045, 0.13, 78%.
3. The Pythagorean Theorem is $a^2 + b^2 =$ _____.
4. A loonie is approximately 2.5 cm across. Convert to inches.

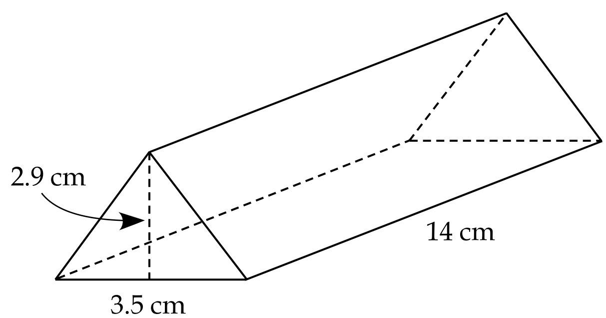
continued

5. Write as a radical: $-6c^{\frac{1}{4}}$.
6. What is the rise of a line if the slope is 2 and the run is 2?
7. What is the GCF of 14 and 18?
8. Rewrite the fraction in simplest form: $\frac{54}{27}$.

Part B: Volume of Prisms and Pyramids

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

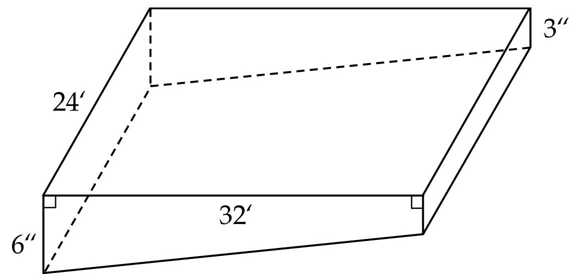
1. A chocolate bar shaped like a triangular prism is 14 cm long. Its triangular base is 3.5 cm long and 2.9 cm high. Calculate the volume of chocolate in the bar.



2. Sketch a rectangular pyramid with a volume of 0.0943 mm^3, and label it with base dimensions of 0.35 mm by 0.47 mm. Determine its height.

continued

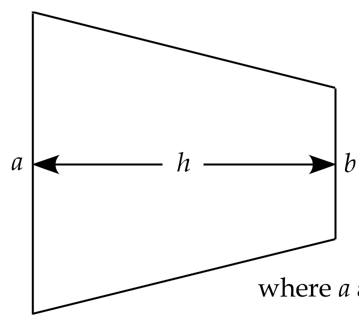
3. A cement floor for a garage has the following trapezoidal shape.



If cement costs $95.75 per cubic yard, how much could the contractor expect to pay for the cement?

Note: Make sure all the measurements are in the same unit before using them in calculations. (Hint: Convert inches to feet.)

The formula for the area of a trapezoid is half the sum of the lengths of the two parallel sides multiplied by the height between them.



where *a* and *b* are parallel

$$A = \frac{1}{2}(a + b)h_T$$

This formula would be useful to have on your Resource Sheet.

Lesson Summary

In this lesson, you used formulas to calculate the volume of a prism or pyramid, and solved for an unknown dimension if the volume was known. You identified the relationship between the volume of a prism and pyramid with the same dimensions. In the next lesson, you will determine the surface area of prisms and pyramids.

Notes

Assignment 3.4

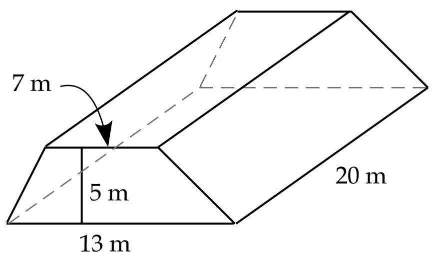
Volume of Prisms and Pyramids

Total Marks = 8

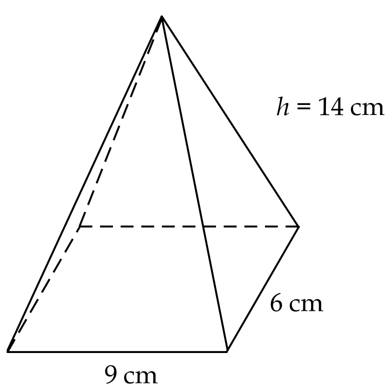
Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Find the volume of each of the following figures.

   a) *(3 marks)*

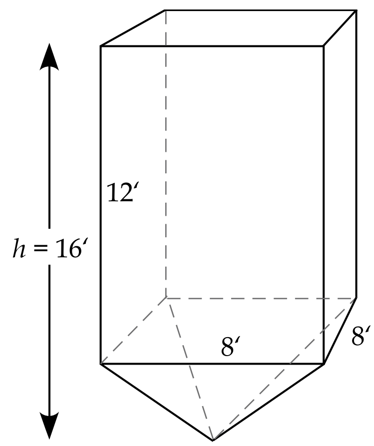


   b) *(2 marks)*



Assignment 3.4: Volume of Prisms and Pyramids (continued)

2. A bin for holding grain has the following shape. Find the volume. *(3 marks)*



Lesson 5: Surface Area of Prisms and Pyramids

Lesson Focus

In this lesson, you will

- ❑ determine the lateral and total surface area of a prism or pyramid using objects or diagrams
- ❑ determine an unknown dimension of a prism or pyramid given its surface
- ❑ solve contextual problems based on composite 3-D objects (objects made up of more than one 3-D shape)

Lesson Introduction

If you have ever wrapped a gift box with paper, you have covered the total surface area of a 3-D object. If you have painted the exterior of a house, you covered its lateral surfaces with colour. This lesson will define total and lateral surface areas, and you will develop formulas to calculate the surface areas of right prisms and right pyramids.

In this lesson, you will be exploring the formulas for lateral surface area and total surface area of both prisms and pyramids. You will want these to be included on your Resource Sheet. Make sure you know what each formula is supposed to be used for (i.e., Is it for the total surface area of a prism or a pyramid?).

Surface Area

The surface area of a 3-D object is the sum of the areas of all its faces. It can be defined in two ways: total surface area or lateral surface area. It is expressed in terms of square units.

Lateral Surface Area

The word *lateral* means sides. Lateral surface area (LSA) refers to the surface area of only the faces. It excludes the base area.

In a pyramid, LSA includes the area of all triangular faces, but not the base.

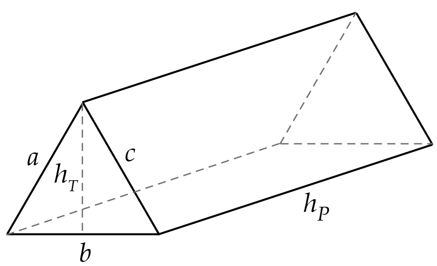
In a prism, LSA includes the area of all the rectangular faces, but not the ends.

Total Surface Area

The total surface area (TSA) of a 3-D object is the sum of the areas of all faces including the ends or base.

Surface Area of Prisms

The lateral surface area of this triangular prism is the sum of the areas of the three rectangular sides. The areas of these sides would be calculated as: *ah*, *bh*, and *ch*, where *h* is the height of the prism.



Lateral Surface Area = sum of areas of faces

$$\text{LSA} = ah + bh + ch$$

If you factor out the common *h* in each of the terms, you are left with

$$\text{LSA} = (a + b + c)h$$

The perimeter of the triangular base in this prism is equal to $a + b + c$, so replace that with *P* in the formula

$$\text{LSA} = Ph$$

where *P* is the perimeter of the base and *h* is the height of the prism.

This formula would be useful to have on your Resource Sheet.

The total surface area of prisms is the lateral surface area plus the area of the two bases. The base of this prism is a triangle. The formula for the area of a triangle, is $\frac{bh_T}{2}$, where *b* is the length of the triangles base and h_T is the perpendicular height of the triangle. If *B* is the area of the base, $B = \frac{bh_T}{2}$.

A prism has two base ends, so add 2*B* to the formula for LSA to give you the TSA of the prism.

$$TSA = LSA + 2B \text{ or}$$

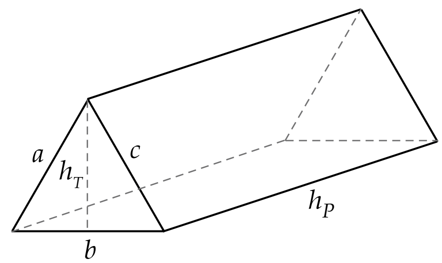
$$TSA = Ph + 2B$$

where P is the perimeter of the base, h is the height of the prism, and B is the area of the base.

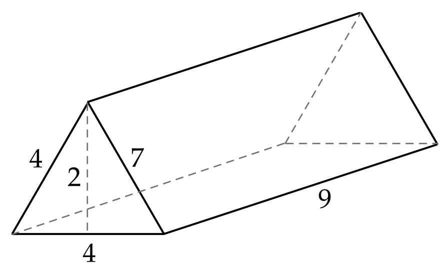
This formula would be useful to have on your Resource Sheet.

Example 1

Using the following diagram, replace the variables with the following values: $a = 4$, $b = 4$, $c = 7$, $h_T = 2$, and $h_P = 9$, and calculate the lateral surface area and the total surface area of the prism.



Solution:



The formula for the lateral surface area of a prism is

$$LSA = Ph,$$

where P is the perimeter of the base and h is the height of the prism.

$$LSA = (4 + 4 + 7) \times 9$$

$$LSA = 15 \times 9$$

$$LSA = 135 \text{ square units}$$

Note: If no units are given, use the generic "unit" in your answer.

The formula for total surface area of a prism is

TSA = $Ph + 2B$,

where P is the perimeter of the base, h is the height of the prism, and B is the area of the base. Use the appropriate formula to calculate the area of the base.

The area of a triangle is calculated as $A = \dfrac{bh}{2}$.

$$\text{TSA} = Ph + 2B$$

$$\text{TSA} = (4+4+7)(9) + 2 \times \frac{(7)(2)}{2}$$

$$\text{TSA} = 135 + 14$$

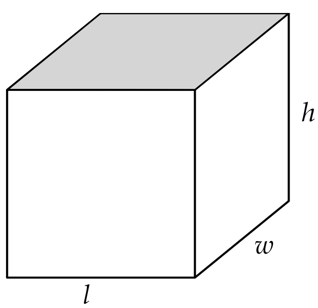
$$\text{TSA} = 149$$

The total surface area of this triangular prism is 149 units^2.

Notice that if no unit of measurement is specified, we write units^2 or units^3 so that it is clear that the measurement is an area or a volume.

Example 2

Given a rectangular prism, explain how the formula for the lateral surface area of a prism applies.



Solution:

The base of this prism is the rectangular sides with the area of $l * w$.

The lateral surface area would be the sum of two sides with area wh and two sides with area lh.

LSA = $2wh + 2lh$

Factor out the common h

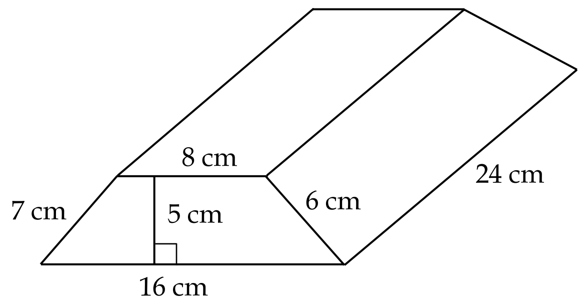
LSA = $(2w + 2l)h$

The perimeter of the base of this prism is calculated as $2w + 2l$, so LSA = Ph where P is the perimeter of the base and h is the height of the prism.

This formula LSA = Ph will work for all right prisms with regular polygon bases.

Example 3

Determine the LSA and TSA of the following trapezoidal prism.



Solution:

LSA = Ph

LSA = $(7 + 8 + 6 + 16) \times 24$

LSA = 888

The lateral surface area is 888 cm^2.

TSA = $Ph + 2B$

The formula to determine the area of a trapezoid is $A = \left[\frac{1}{2}(a+b)h_T\right]$, where a and b are the lengths of the two parallel sides and h_T is the height of the trapezoid.

$$\text{TSA} = (7 + 8 + 6 + 16) \times 24 + 2 \times \left[\frac{1}{2}(8 + 16)5\right]$$

$$\text{TSA} = 888 + 2 \times 60$$

$$\text{TSA} = 888 + 120$$

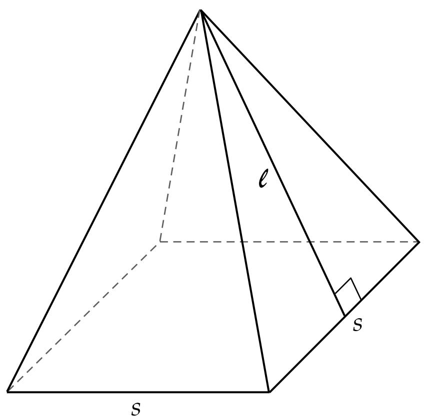
$$\text{TSA} = 1008$$

The total surface area is 1008 cm^2.

Surface Area of Pyramids

The lateral faces of a pyramid are triangles. To calculate the area of these faces you need to know the slant height, ℓ, of the triangles. Remember, the formula for the area of a triangle is $A = \dfrac{bh}{2}$, where h is the height of the triangle.

Square base:



So, the base is s and the height is ℓ.

The formula for area is now:

$$A = \frac{s\ell}{2}$$

The four triangle faces will be congruent (the same).

$$\text{LSA} = 4\left(\frac{s\ell}{2}\right)$$

$$\text{LSA} = \frac{4s\ell}{2}$$ The perimeter, P, of the base of this pyramid is $4s$, so $4s = P$.

$$\text{LSA} = \frac{P\ell}{2}$$

$$\text{LSA} = \frac{1}{2}P\ell$$ Where P is the perimeter of the base and ℓ is the slant height.

This formula would be useful to have on your Resource Sheet.

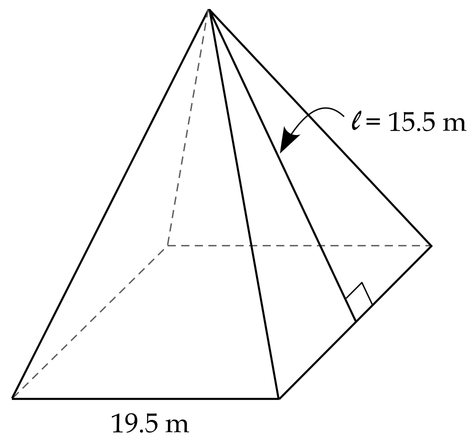
The total surface area will include the square base, where $B = s^2$.

$$\text{TSA} = \frac{1}{2}P\ell + B,$$ where B is the area of the base.

This formula would be useful to have on your Resource Sheet.

Example 4

Determine the lateral surface area and the total surface area of this square pyramid.



Solution:

$$\text{LSA} = \frac{1}{2}P\ell$$

$$\text{LSA} = \frac{1}{2}(19.5 \times 4)(15.5)$$ (Since the base is square, the perimeter is 19.5 × 4.)

$$\text{LSA} = \frac{1}{2}(1209)$$

$$\text{LSA} = 604.5$$

The lateral surface area is 604.5 m^2.

$$\text{TSA} = \frac{1}{2}P\ell + B$$

$$\text{TSA} = 604.5 + 19.5^2$$

$$\text{TSA} = 984.75$$

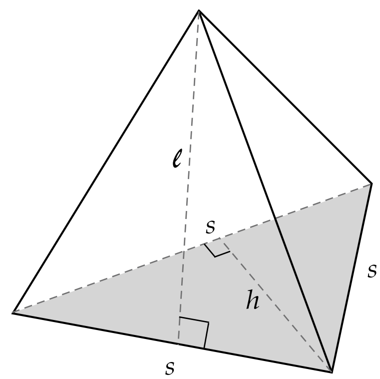
The total surface area of this pyramid is 984.75 m^2.

Example 5

Show how the following formulas apply to a pyramid with a base that is an equilateral triangle.

$$\text{LSA} = \frac{1}{2}P\ell$$

$$\text{TSA} = \frac{1}{2}P\ell + B$$



Solution:

The lateral surface area of this pyramid consists of three congruent triangles. The area of these triangles will be $A = \frac{s\ell}{2}$.

$$\text{LSA} = 3\left(\frac{s\ell}{2}\right)$$

$\text{LSA} = \dfrac{3s\ell}{2}$ The perimeter, P, of the triangular base is $3s$, so $3s = P$.

$$\text{LSA} = \frac{P\ell}{2}$$

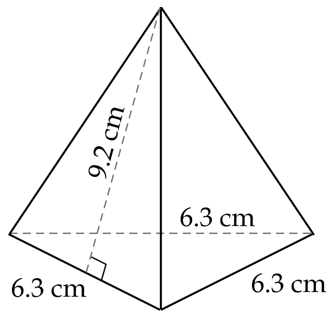
$\text{LSA} = \dfrac{1}{2}P\ell$ P is the perimeter of the base and ℓ is the slant height..

The total surface area includes the area of the base triangle $A = \frac{bh}{2}$.

$\text{TSA} = \dfrac{1}{2}P\ell + B$, where B is the area of the base.

Example 6

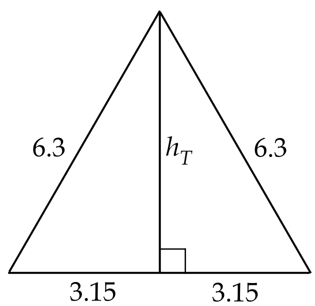
Determine the TSA of this triangular pyramid.



Solution:

$$\text{TSA} = \frac{1}{2}P\ell + B$$

The perimeter of the base is 6.3 × 3 and the slant height is given as 9.2 cm. To determine the area of the base, you need to first calculate the height of the base triangle. The base is an isosceles triangle, so the height can be determined using the Pythagorean Theorem.



$$a^2 + b^2 = c^2$$

$$h_T + 3.15^2 = 6.3^2$$

$$h_T{}^2 = 6.3^2 - 3.15^2$$

$$h_T{}^2 = 28.7675$$

$$h_T = 5.455960044$$

$$\text{TSA} = \frac{1}{2}(3 \times 6.3)(9.2) + \left(\frac{6.3 \times 5.455960044}{2}\right)$$

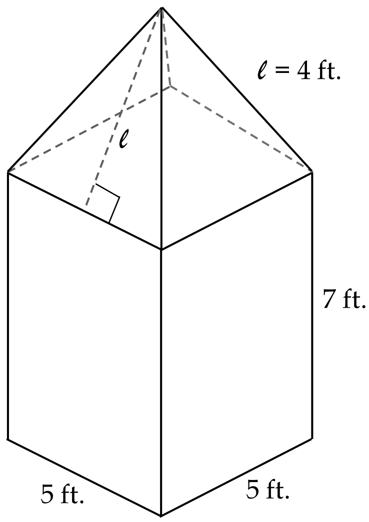
$$\text{TSA} = 86.94 + 17.18627414$$

$$\text{TSA} = 104.1262741$$

The total surface area of this pyramid is approximately 104 cm^2.

Example 7

You have a new treehouse (in the diagram below). You have a 2 L can of paint to finish the wood on the roof and outside walls of a treehouse. If 1 L of paint covers about 10 square yards, will you have enough paint? (Assume no windows or doors have been cut out yet.)



Solution:

You need to determine the lateral surface area of the composite object.

$$\mathrm{LSA}_{\text{pyramid}} = \frac{P\ell}{2}$$

$$\mathrm{LSA} = \frac{(4 * 5)(4)}{2}$$

$$\mathrm{LSA} = 40$$

The lateral surface area of the roof is 40 ft.^{2}.

$$\mathrm{LSA}_{\text{prism}} = Ph$$

$$\mathrm{LSA} = (4 * 5)(7)$$

$$\mathrm{LSA} = 40$$

The lateral surface area of the walls is 140 ft.^{2}.

The combined lateral surface area is 140 + 40 = 180 ft.^{2}.

There are 9 ft.^{2} in 1 yd.^{2}, so 180 ft.^{2} $\times \dfrac{1 \text{ yd.}^2}{9 \text{ ft.}^2}$ = 20 yd.^{2}.

2 L of paint should cover 20 yd.^{2}, so you have just enough paint for the treehouse.

You have used the following formulas in Lessons 4 and 5:

Prism

$$V = Bh$$

$$\text{LSA} = Ph$$

$$\text{TSA} = Ph + 2B$$

Pyramid

$$V = \frac{1}{3}Bh$$

$$\text{LSA} = \frac{1}{2}P\ell$$

$$\text{TSA} = \frac{1}{2}P\ell + B$$

where B is the area of the base
P is the perimeter of the base
h is the height of the prism or pyramid
ℓ is the slant height

Formulas for Area of Polygon Bases

$$\text{Square} = s^2$$

$$\text{Rectangle} = L \times W$$

$$\text{Triangle} = \frac{bh}{2}$$

$$\text{Trapezoid} = \frac{1}{2}(a + b)h$$

$$\text{Parallelogram} = bh$$

It is important that you place these formulas on your Resource Sheet.

Learning Activity 3.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Rewrite the following fraction in simplest form: $\frac{12}{26}$.
2. Rewrite the following fraction in simplest form: $\frac{24}{52}$.
3. Write the following percent as a decimal: 46.1%.
4. There are 8 marbles in a bag. Each marble is either red, yellow, or blue. Four of the marbles are red and one marble is yellow. How many are blue?
5. The side length of a cube is 3 cm. What is the volume?
6. What is the surface area of the same cube?
7. At the store, you compare the price of two hand lotions. The lemon one is $3.00 for a 60 mL bottle. The vanilla one is $6.00 for a 100 mL bottle. Which is the better deal (per mL)?
8. You are a very busy person. You have soccer on Sunday, Tuesday, and Thursday nights. You have music lessons on Saturday in the evening. You also have swimming on Monday night. Which nights of the week do you not have any commitments?

continued

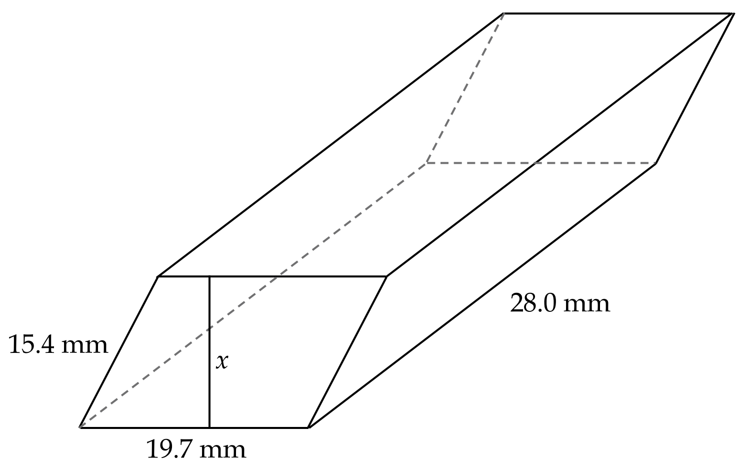
Learning Activity 3.6 (continued)

Part B: Surface Area of Prisms and Pyramids

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The famous Louvre Museum in Paris, France has a square glass pyramid above the main entrance. The pyramid is 35.42 m wide and 21.64 m high with a slant height of 27.96 m. Calculate the lateral surface area of the glass.

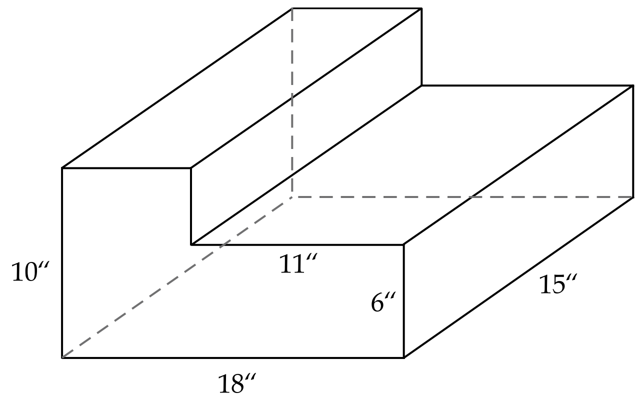
2. The total surface area of this prism, with a parallelogram-shaped base, is 2501.44 mm^2. Determine the height of the parallelogram base.



continued

Learning Activity 3.6 (continued)

3. Find the total surface area of the following 3-D object. State your final answer in scientific notation and also in ft.^{2}.



Lesson Summary

In this lesson, you investigated how to calculate the surface area of prisms and pyramids. You applied the formulas to 3-D objects with a variety of base shapes, and solved for unknown dimensions. In the next lesson, you will explore circular 3-D shapes.

Assignment 3.5

Surface Area of Prisms and Pyramids

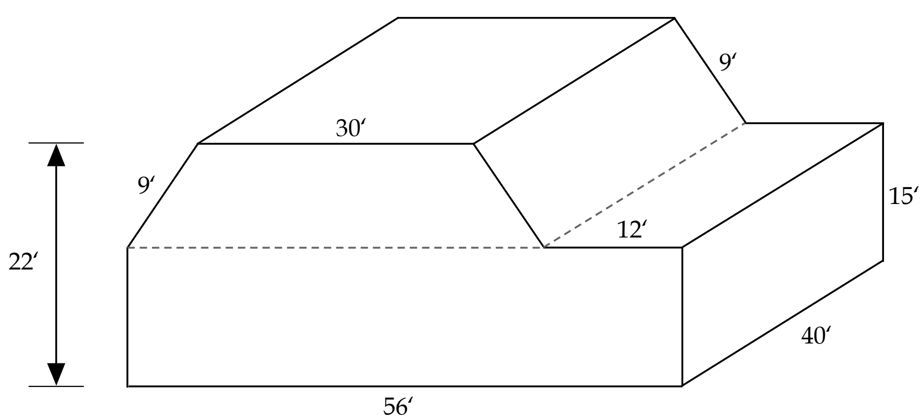
Total Marks = 25

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Find an object shaped like a rectangular prism. It may be something like a die, pencil box, food storage container, tissue box, a textbook, or something larger like a mattress or a cardboard box. Measure the object's dimensions as accurately as possible. Draw a sketch of the object and label the diagram with its measurements, clearly stating the units you used. Determine its volume and total surface area. Include a diagram, state the formulas used, and show your calculations. *(10 marks)*

2. The lateral surface area of a pyramid is 1.111293×10^5 m^2. If its base is an equilateral triangle with a side length of 237 m, determine the slant height. *(5 marks)*

3. A hip-roof barn is built with an attached storage area. Determine the total surface area, including the floor, in square yards if the roof line is a regular trapezoid. *(10 marks)*



Lesson 6: Spheres, Cylinders, and Cones

Lesson Focus

In this lesson, you will

- ❑ determine the surface area and volume of a sphere, cone, and cylinder using objects or diagrams
- ❑ determine an unknown dimension of a circular 3-D object given its surface area or volume
- ❑ solve contextual problems that involve the surface area or volume of a composite object, given its diagram

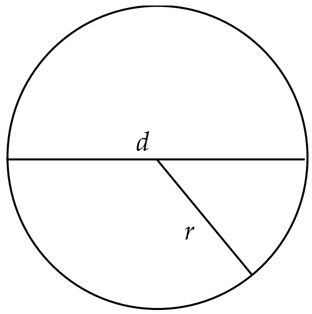
Lesson Introduction

Spheres, cylinders, and cones are three-dimensional objects with a circular shape. They are comparable to prisms and pyramids, but instead of having a polygon base and rectangular or triangular faces, they have circular bases and curved faces. Formulas for circular shapes always include π, called pi (although it isn't quite as tasty as pie). π is the symbol for an irrational number that has a constant value of approximately 3.141592654… It is the ratio of the circumference of a circle to its diameter (circumference is the perimeter of a circle). This lesson will outline the formulas for the surface area and volume of spheres, cylinders, and cones.

Circles in 3-D

Circles

The size of a two-dimensional circle is determined by its radius, r, the distance from the centre of the circle to a point on the circumference, C (the perimeter of the circle).



The circumference is calculated as $C = \pi d$ or $C = 2\pi r$.

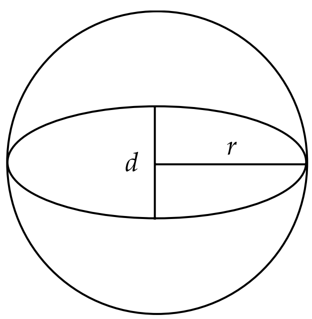
The diameter is two times the length of the radius ($d = 2r$).

The area of a circle is $A = \pi r^2$.

These formulas should be included on your Resource Sheet.

Spheres

A sphere is a 3-D ball-shaped object in which all points are equidistant (the same distance) from the centre.



Surface Area and Volume of a Sphere

The surface area of a sphere is exactly four times larger than the area of the circle that passes through the centre of the circle. Therefore, the surface area of a sphere is stated in square units, and can be calculated as follows:

$$\mathrm{SA} = 4\pi r^2$$

This formula would be useful to have on your Resource Sheet.

The volume of a sphere is stated in cubic units, and the value of the radius in the formula is cubed.

$$V = \frac{4}{3}\pi r^3$$

This formula would be useful to have on your Resource Sheet.

Example 1

Find the surface area of a sphere with a radius of 12 cm.

Solution:

SA = $4\pi r^2$

SA = $4\pi(12)^2$

SA $\approx$ 1810 cm^2

The wavy lines mean that the surface area is approximately equal to 1810 cm^2.

Example 2

Find the volume of a volleyball if its circumference is 26″.

Solution:

First you need to calculate the length of the radius.

$$C = 2\pi r$$

$$26 = 2\pi r$$

$$r = \frac{26}{2\pi}$$

$$r = 4.13802852 \text{ inches}$$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi\left(4.13802852\right)^3$$

$$V = 296.8035205$$

The volume of the volleyball is about 296.8 cubic inches.

If you are using an approximation for the value of π like 3.14 and are rounding values like the length of the radius to 4.1, and then using these in further calculations, your answer will be less accurate:

$$V = \frac{4}{3}\pi r^3$$

$$V = (1.3)(3.14)(4.1)^3$$

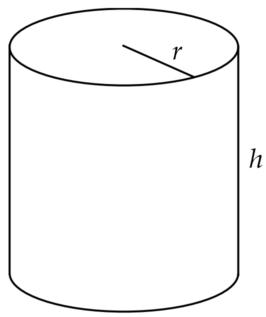
$$V \approx 281 \text{ cubic inches}$$

You should strive to be as accurate as possible, using as many decimal places as possible in your calculations, and only rounding the final answer.

You should be using a scientific calculator. Most of these calculators have a button for π. If there is no button, π may be written above a button, in which case you would press the 2nd function or shift button, then the button with π above it.

Cylinders

A cylinder is made up of one lateral rectangular face that curves around its two parallel and congruent circular ends.



Surface Area and Volume of a Cylinder

The formula for the volume of a cylinder is similar to the formula for the volume of a prism.

$$V = Bh$$

where B is the area of the circular base and h is the height of the cylinder.

This formula should be included on your Resource Sheet.

The lateral surface area includes only the curved wall of the cylinder. If you could cut open this surface and spread it flat, it would be a rectangle. If it helps, think back to previous math courses and draw the net of a cylinder to help you picture its surface area. Alternately, think of a soup can label peeled off in one piece and laid flat.

The area of a rectangle is $A = L \times W$.

In this case, the length is the height of the cylinder and the width is distance around the base, the circumference of the circle. The lateral surface area of a cylinder is calculated as

$$\text{LSA} = (2\pi r)h \text{ or}$$

$$\text{LSA} = Ch$$

where C is the circumference of the circle and h is the height of the cylinder.

This formula would be handy to have on your Resource Sheet.

The total surface area includes the LSA and the area of the two circular ends.

$$\text{TSA} = (2\pi r)h + 2\pi r^2$$

$$\text{TSA} = Ch + 2B$$

This formula would be handy to have on your Resource Sheet.

Example 3

Find the total surface area and volume of a cylinder with a radius of 11 mm and a height of 3 cm.

Solution:

Make sure all the measurements are in the same units before substituting them into the formulas.

$$r = 1.1 \text{ cm}$$

$$h = 3 \text{ cm}$$

$V = Bh$

$V = (\pi \times 1.1^2)(3)$

$V \approx 11.4$

The volume of the cylinder is about 11.4 cm^3.

$\text{TSA} = Ch + 2B$

$\text{TSA} = (2 \times \pi \times 1.1)(3) + 2(\pi \times 1.1^2)$

$\text{TSA} = 20.73451151 + 7.602654222$

$\text{TSA} \approx 28.3$

The total surface area of the cylinder is about 28.3 cm^2.

Example 4

A foam flotation toy used when swimming in a pool is shaped like a long, fat noodle. It has a volume of 733.4 cubic inches. If the circumference of the foam noodle is 12″, determine its length.

Solution:

$$C = 2\pi r$$

$$12 = 2\pi r$$

$$r = \frac{12}{2\pi}$$

$$r = 1.909859317$$

$$V = Bh$$

$$733.4 = \left(\pi * 1.909859317^2\right)h$$

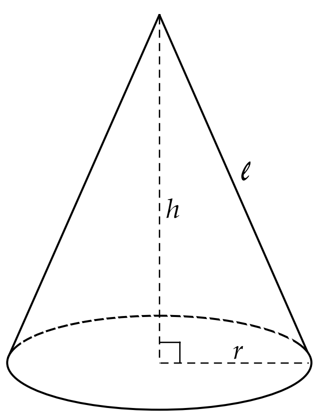
$$h = \frac{733.4}{11.4591559}$$

$$h = 64$$

The pool noodle is 64 inches long.

Cones

A cone is a 3-D object that has a circle base and one curved face that converges to a point, or vertex.



Surface Area and Volume of a Cone

If a cylinder has a volume of 300 cubic units, the cone with the same radius and height will have a volume of 100 cubic units.

$$V_{\text{cylinder}} = Bh$$

$$V_{\text{cone}} = \frac{1}{3}Bh$$

where B = area of the circular base and h is the height of the cone.

This formula would be a good one to have on your Resource Sheet.

The lateral surface area of a cone is found by multiplying the radius and slant height by pi.

$$\text{LSA} = \pi r \ell$$

πr is half of the circumference of a circle, so this can be written as

$$\text{LSA} = \frac{1}{2}C\ell$$

where C is the circumference of the circle and ℓ is the slant height.

Having this formula on your Resource Sheet would be a good idea.

The total surface area would include the area of the base, πr^2.

$$\text{TSA} = \frac{1}{2}C\ell + B$$

This would be a useful formula to have on your Resource Sheet.

Example 5

Find the volume of a carrot that is 9 inches long and 1 inch across at the top.

Solution:

A carrot is approximately cone shaped.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}\left(\pi r^2\right)h$$

$$V = \frac{1}{3}\left(\pi\, 0.5^2\right)9$$

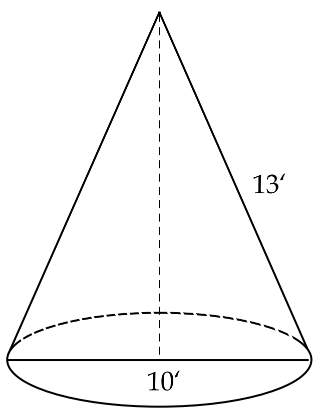
$$V \approx 2.4$$

The volume of this carrot is about 2.4 in.^{3}.

Example 6

Find the total surface area of a cone with a diameter of 10′ and a slant height of 13′. Round your answer to the nearest tenth. Draw a diagram.

Solution:



$$\text{TSA} = \frac{1}{2}C\ell + B$$

$$\text{TSA} = \frac{1}{2}(\pi d)\ell + \pi r^2 \qquad C = \pi d = 2\pi r$$

$$\text{TSA} = \frac{1}{2}(\pi\, 10)(13) + \pi\, 5^2$$

$$\text{TSA} = 204.2035225 + 78.53981634$$

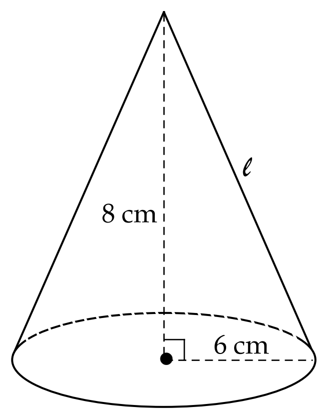
$$\text{TSA} = 282.7433388$$

The total surface area is approximately 282.7 sq. ft.

Example 7

Find the lateral surface area of a cone with a radius of 6 cm and a height of 8 cm. Draw a diagram.

Solution:



In a right cone the height and radius are perpendicular to each other, and form a right triangle with the slant height as the hypotenuse. Use the Pythagorean Theorem to calculate the slant height.

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = \ell^2$$
$$36 + 64 = \ell^2$$
$$\ell^2 = 100$$
$$\ell = 10$$

$$\text{LSA} = \frac{1}{2}C\ell$$
$$\text{LSA} = \frac{1}{2}(2\pi r)\ell$$
$$\text{LSA} = \frac{1}{2}(2\pi 6)10$$
$$\text{LSA} = 188.4955592$$

The lateral surface area of this cone is about 188.5 cm^2.

Learning Activity 3.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate $\sqrt{9 + 16}$.
2. Evaluate $\sqrt{25 + 144}$.
3. The slope of a line is 4. What is the slope of a perpendicular line to this?
4, Terry has a chocolate orange that he wants to eat over one week (Monday to Sunday). He wants to eat the same number of pieces each day. If the orange has 14 pieces, how many pieces will he eat each day?
5. You are out for dinner with your best friend for her birthday. The bill comes and you pay for everything. If the total is $35.75 and you leave $40 on the table (including tip), how much are you tipping the server?
6. Is the following angle acute, obtuse, straight, or reflex: 140^{o}?
7. The volume of a cube is 8 m^3. What are the dimensions of the cube?
8. What two numbers have a product of -10 and a sum of -3?

Part B: Surface Area and Volume of Spheres, Cylinders, and Cones

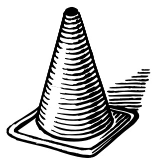
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Describe the relationship between the volumes of a cone and cylinder that have the same radius and height.

continued

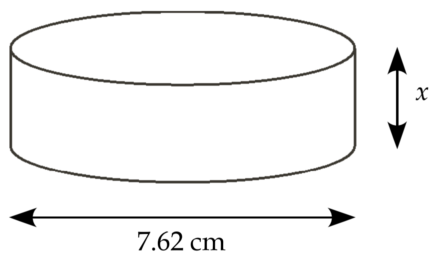
Learning Activity 3.7 (continued)

2. You have been asked to repaint the lateral surface area of cone-shaped pylons used for soccer drills. If they have a radius of 5″ and a height of 12″, calculate the slant height and lateral surface area to be painted.

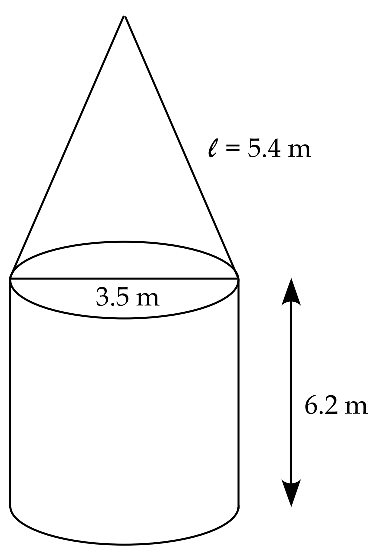


3. Find a sports ball (e.g., a basketball, softball, soccer ball, or tennis ball), and measure its circumference using imperial units. Describe your measurement strategy and use the circumference to determine the ball's radius. Calculate the volume of the ball.

4. A hockey puck has a diameter of 7.62 cm and a volume of 115.83 cm^3. Calculate the thickness of a hockey puck.



5. The turret on a castle is formed by constructing a cone-shaped roof above a cylindrical structure. Determine the lateral surface area of the following turret. Round your final answer to the nearest hundredth of a metre.



Lesson Summary

The 3-D objects used in this lesson all had some sort of circular shape. Note that the formulas to determine the surface area and volume of the cones and cylinders had similarities to the formulas to determine the surface area and volume for the prisms and pyramids. These similarities will make them easier to remember and apply.

Assignment 3.6

Surface Area and Volume of Spheres, Cylinders, and Cones

Total Marks = 25

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Assume a soda pop can is a perfect cylinder. It has a circumference of $8\frac{1}{4}$ inches and a height of $4\frac{13}{16}$ inches. Calculate the lateral surface area of a pop can. *(2 marks)*

2. A sphere has a surface area of 18.1 square units. Determine its diameter. *(3 marks)*

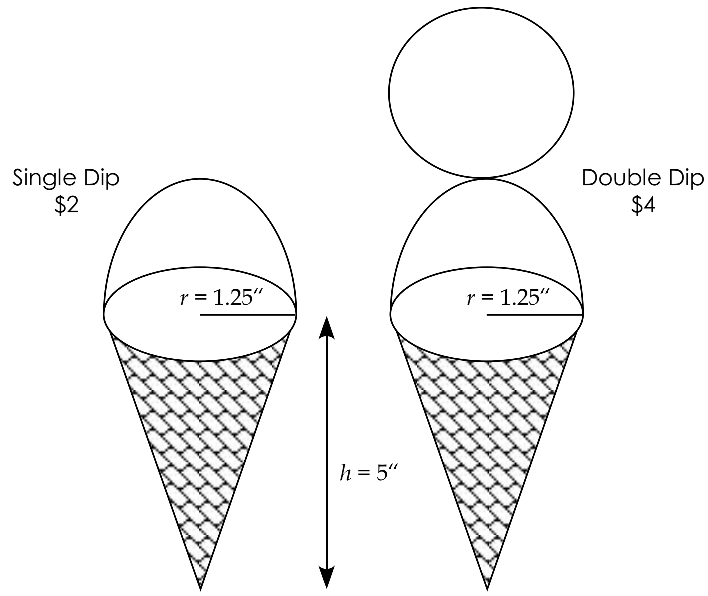
Assignment 3.6: Surface Area and Volume of Spheres,Cylinders, and Cones (continued)

3. Determine the total surface area of the Canadian \$2 coin by measuring the diameter and thickness of a toonie to the nearest mm. Assume the coin is a regular cylinder. Write your answer in scientific notation. *(7 marks)*

4. Calculate the volume of a cone with a diameter of 18 cm and a slant height of 41 cm. Include a diagram. Round your final answer to the nearest tenth of a centimetre. *(6 marks)*

Assignment 3.6: Surface Area and Volume of Spheres,Cylinders, and Cones (continued)

5. An ice cream cone has a radius of 1.25″ and a height of 5″. If you buy a single-dip cone for $2, the cone is filled completely with ice cream and there is a half-sphere shaped scoop above the cone. If you order the double-dip for $4, they put an extra sphere-shaped scoop of ice cream on top. Which is the better deal? Explain. *(7 marks)*



Notes

Module 3 Summary

Congratulations! You have finished the third module in the course. This unit incorporated both the metric and imperial systems of measurement. You estimated, used referents, developed creative measurement strategies, and practised making accurate measurement using a variety of units. You converted units within and between these two systems using conversion ratios, and used mental math to verify the accuracy and appropriateness of your work. You also solved problems involving the volume and surface area of three-dimensional objects.

Many different formulas and conversion values were used in this module. Ensure that all formulas and conversion values are on your Resource Sheet. You will be expected to know how to use the formulas and conversions correctly.

You should note that although the material that you learned in this module is used in future pre-calculus and applied courses, the application of measurement is explored more in depth in applied mathematics.

The next module is also part of the space and shape/measurement strand of mathematics. In it, you will develop and apply the trigonometric ratios of sine, cosine, and tangent to solve problems in right triangles.

Submitting Your Assignments

You will not submit your Module 3 assignments to the Distance Learning Unit at this time. Instead, you will submit them, along with the Module 4 assignments, when you have completed Module 4.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 3
Measurement

Learning Activity Answer Keys

Module 3: Measurement

Learning Activity 3.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate: $\left(\dfrac{9x^4y^3}{3xy^2}\right)$.
2. Which is the dependent variable: the windchill compared to the time it takes to get frostbite?
3. A school volleyball team wants to practise twice per week. They cannot practise at the school on the weekend (Saturday and Sunday), half the team cannot prastice on Monday and Wednesday, and the basketball team uses the gym on Friday. Which days can the team practise?
4. Solve for p: $p \div 15 = 5$.
5. What is the GCF of 19 and 13?
6. What two numbers have a sum of 11 and a product of 18?
7. You want to save up $12 000 to buy a car one year from now. How much do you have to save per month to reach this goal?
8. Three students receive their marks for a project. Jane found her mark as a decimal, 0.62; John calculated his mark as a percent, 83%; Jean got $\dfrac{12}{16}$.

   Who got the best mark?

Answers:

1. $3x^3y$
2. Time it takes to get frostbite.
3. Tuesday and Thursday
4. $p = 75$ ($p = 15 \div 5$)
5. 1 (19 and 13 are both prime numbers)
6. 2, 9 (The factor of pairs of 18 are (1, 18), (2, 9), (3. 6). Only 2 + 9 = 11.)
7. $1000 ($12, 000 ÷ 12)
8. John got the best mark. If you put them all into one form (e.g., percent), you can compare the marks. Jane: 0.62 = 62%, John: 83%, Jean: $\dfrac{12}{16}$ = 75%.

Part B: Units

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Provide referents for the following linear measurements.
   a) millimetre
   b) metre
   c) inch
   d) mile

   Answer:

   Answers will vary. Some possible answers are:
   a) diameter of wire used in a standard paper clip, thickness of a dime, thickness of a credit card
   b) two steps or one pace, distance from floor to average woman's waist, perimeter of a sheet of printer paper
   c) diameter of a $1 coin (loonie), width of an average adult thumb
   d) distance a fit person can run in 7 minutes, distance between country roads, about 15–20 city blocks (depending on how the streets are organized, this may be more or less)

2. Determine a referent based on your height to compare the lengths of a metre and a yard. Use it to estimate the height of a table.

   Answer:

   Your answer will vary, based on your height.

   A possible solution: a person who is about 5′ 5″ or 165 cm tall would measure about 1 m from the floor to their waist, while 1 yard would be from the floor to their hip. Based on this comparison, a yard is just slightly shorter than a metre.

   Standing next to the table, it may reach about one hand span below a person's hip or one and a half hand spans below a person's waist. If their hand span was about 8 inches or 20 cm, the height of the table could be estimated at about 8 inches less than a yard

   Since 1 yd. = 36″, $36 - 8 = 28$

   The table is about 28 inches high.

Using SI, one and a half hand spans below their waist:

$100 - (1.5 \times 20)$

$100 - 30 = 70$

The table is about 70 cm high.

3. Use a referent of your choice to estimate the dimensions of your calculator or a cell phone. Indicate your referent, choice of units, and then measure to verify the accuracy and appropriateness of your measurement and units.

Answer:

Answers will vary depending on the choice of referent and unit. Check your answer with your learning partner.

A possible solution: A $1 coin has a diameter of about 1″. A graphing calculator is about 3 loonies wide by 7 loonies long. Using a ruler, the dimensions are determined to be $3\frac{1}{4}$ inches by $7\frac{1}{8}$ inches. The units and measurements are accurate and appropriate.

A permanent marker has a diameter of about 1 cm, and its cap is about 5 cm long. The calculator is about 8 marker widths across and about three and a half cap lengths long. Using this referent, the calculator is about 8 cm wide and $3.5 \times 5 = 17.5$ cm long. Using a metric ruler, the dimensions of the graphing calculator are determined to be about 8 cm by 18 cm. The estimate is close to the actual measurement.

4. Which units from the metric system and from the imperial system would be the most suitable for measuring the following lengths?

a) Width of a snowboard

b) Length of a soccer field

c) Thickness of a coin

d) Distance a jogger runs every day

Answers:

a) Metric: centimetres	Imperial: inches
b) Metric: metres	Imperial: yards
c) Metric: millimetres	Imperial: inches
d) Metric: kilometres	Imperial: miles

5. Estimate the following lengths in both SI and imperial measurements. Check your estimates by measuring.
   a) Length and width of a standard door
   b) Length of a vehicle
   c) Length and width of a two-car driveway
   d) Diameter and thickness of a $2 coin (toonie)

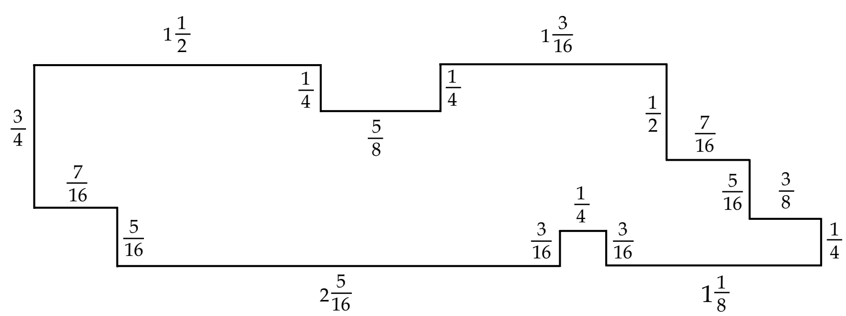
Answers:

Answers will vary. Check your answer with your learning partner.

	Imperial	SI
a)	$2'8'' \times 6'8''$	81 cm × 203 cm
b)	$15'$ to $20'$	4.5 m to 6 m
c)	$16\frac{1}{4}' \times 37\frac{1}{2}'$	4.95 m × 11.43 m
d)	$1\frac{1}{8}'' \times \frac{1}{16}''$	2.7 cm × 1.5 mm

6. Using an imperial ruler, find the perimeter of the following diagram to the nearest $\frac{1}{16}''$.

Answer:



The perimeter is $11\frac{7}{16}$ inches long.

(Answers from $11\frac{1}{8}$ to $11\frac{3}{4}$ are acceptable because of inaccuracies in measurement that occur as a result of a slight shrinkage of images in the photocopying process.)

7. Your dad bought you a car for your 16th birthday and wants to wrap it with a large ribbon to surprise you. Explain how he could determine the length of ribbon to buy, and which units would be the most appropriate.

Answer:

Your dad may choose to use a flexible measuring tape that can go under and around the car. If he did not have a flexible measuring tape, he may estimate the number of paces long and wide the car is, and calculate the length of one of his paces. He would have to add additional length to account for the height of the car, and include the amount needed if he wanted to add a bow on top.

Feet, yards, or metres would be the most appropriate units to use, as inches or centimetres would result in a very large value for the answer. Ribbon is typically sold at a fabric store by the metre, or on a roll with both metres and yards given.

8. What would be the most suitable units of measure for the following areas and volumes? Give an answer using both systems.
   a) The volume of water in a thimble
   b) The area of a hockey ice surface
   c) The volume of earth removed for a basement of a house
   d) The area of one side of a coin

   Answers:

   a) mm^3 or cm^3 or $in.^3$
   b) m^2 or $yd.^2$
   c) m^3 or $yd.^3$
   d) mm^2 or cm^2 or $in.^2$

9. a) Measure the following right triangle to the nearest mm and calculate its area.

Answer:

$$A_{\Delta} = \frac{bh}{2}$$

$$A = (83 \times 63) \div 2$$

$$A = 5229 \div 2$$

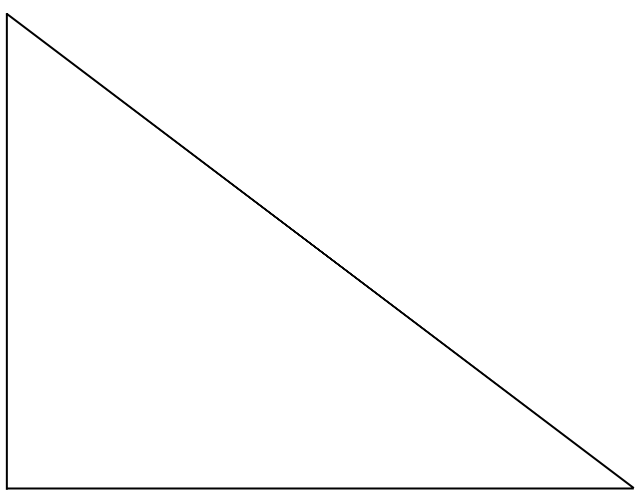
$$A = 2614.5$$

(Answers may be in the 2542 cm^2 to 2688 cm^2 range because of inaccuracies in measurement that occur as a result of a slight shrinkage of images in the photocopying process.)

The area of the triangle is 2614.5 mm^2. Since 1 cm^2 = 100 mm^2, this could be stated as 26.145 cm^2.

b) Measure the following right triangle in inches and calculate its area.

Recall: $A_{\Delta} = \frac{bh}{2}$



Answer:

$$A_{\Delta} = \frac{bh}{2}$$

$$A = \left(2\frac{1}{2} \times 3\frac{1}{4}\right) \div 2$$

$$A = \left(2 \times 3 + 2 \times \frac{1}{4} + \frac{1}{2} \times 3 + \frac{1}{2} \times \frac{1}{4}\right) \div 2$$

$$A = \left(8\frac{1}{8}\right) \div 2$$

$$A = 8\frac{1}{8} \times \frac{1}{2}$$

$$A = 4\frac{1}{16}$$

The area of the triangle is $4\frac{1}{16}$ in.^{2}.

(Answers may be in the 4 in.^{2} to $4\frac{1}{2}$ in.^{2} range due to a slight shrinkage of images in the photocopying process.

Learning Activity 3.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Convert 24 inches into feet.
2. If the equation of a line is $y = 3x + 4$, what is the y-intercept?
3. Evaluate $(3y^7)^3$.
4. You are at a hockey game and would like a snack and a drink. At the counter, you see that popcorn is \$3.00, peanuts are \$2.25, and hotdogs are \$3.75. The drinks are \$2.00. If you have \$5, what snack can you afford to get, if you also buy a drink?
5. You are trying to estimate the height of your brother, who is approximately 1′ taller than your sister. Your sister is half a foot taller than you. How tall is your brother if you are 5′ tall?
6. Solve for w: $w \div 6 = 2$.
7. If 5% of 260 is 13, what is 5% of 520?
8. A person gets to choose from two chocolate milkshakes. One milkshake fills the glass $\frac{7}{9}$ full; the other fills the glass $\frac{2}{3}$ full. Which would you prefer?

Answers:

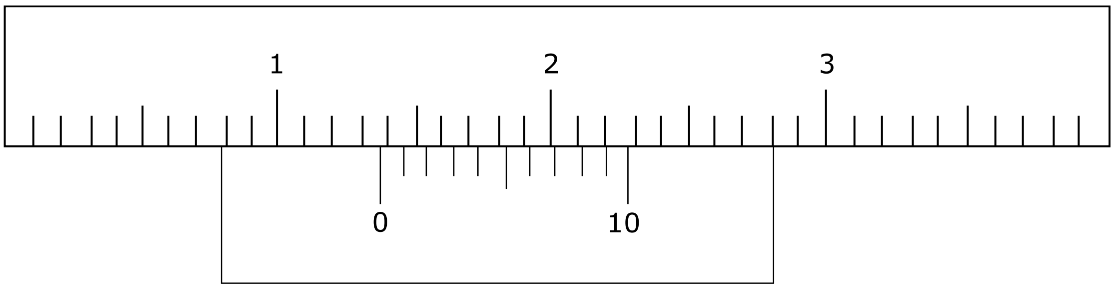
1. 2 feet (12 inches = 1 foot, so $24 \div 12 = 2$ feet)
2. $y = 4$ (The y-intercept is when $x = 0$, so $y = 3(0) + 4 = 4$.)
3. $27y^{21}$ (This is the Power of a Power Law, $(3y^7)^3 = 3^3(y^7)^3$)
4. You can have peanuts or popcorn. (\$2.25 + \$2.00 (peanuts and a drink) = \$4.25; \$3.00 + \$2.00 (popcorn and drink) = \$5.00.)
5. 6.5′ (5′ + 0.5′ = 5.5′ (your sister's height), 5.5′ + 1′ = 6.5′)
6. $w = 12$ ($w = 2 \times 6$)
7. 26 (520 is double 260, so 5% of 520 is double 5% of 260)
8. $\frac{7}{9}$ of a full glass $\left(\frac{2}{3} \times \frac{3}{3} = \frac{6}{9} \text{ which is less}\right)$

Part B: Vernier Caliper Measurements

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write down the measurement as shown on these Vernier caliper diagrams. Read the measurements to the nearest hundredth of a centimetre.

1.



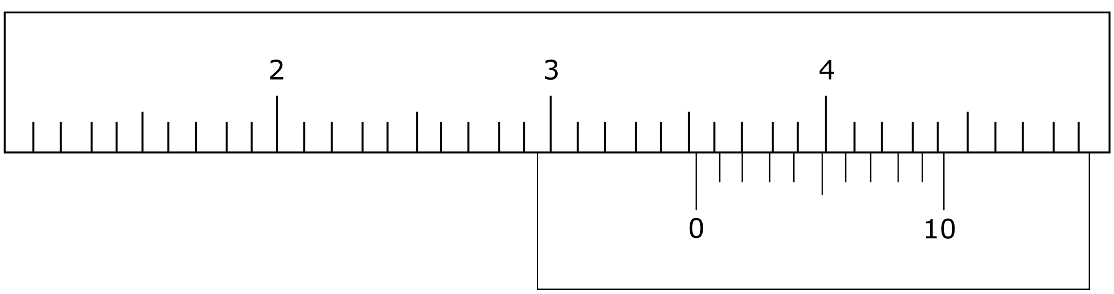
Answer:

The first line of the moving scale, at a reading of 0, points to a space between 1.3 and 1.4 on the fixed scale. Thus, the first reading is 1.3 cm.

The two scales align at the 9th tick on the moving scale, giving a reading of 0.09 cm.

The total reading = 1.3 + 0.09 = 1.39 cm.

2.



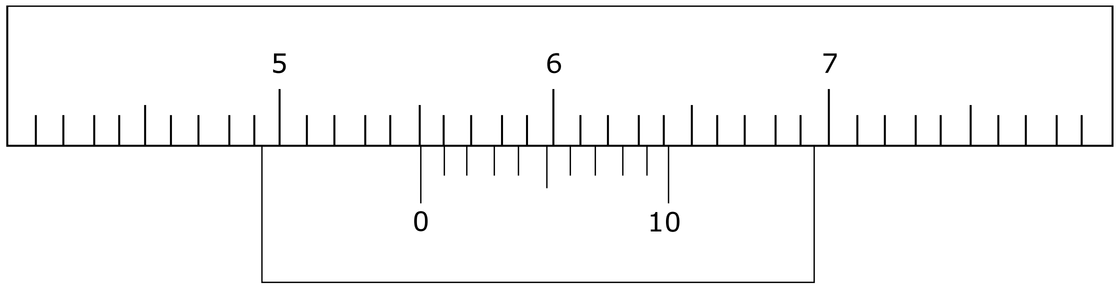
Answer:

The first line of the moving scale, at a reading of 0, points to a space between 3.5 and 3.6. Thus, the first reading is 3.5 cm.

The two scales align at the second tick on the moving scale, giving a reading of 0.02 cm.

The total reading = 3.5 + 0.02 = 3.52 cm.

3.



Answer:

The first line of the moving scale, at a reading of 0, points to a space between 5.5 and 5.6. The first reading is 5.5 cm.

The two scales align at the first tick, giving a reading of 0.01 cm.

The total reading = 5.5 + 0.01 = 5.51 cm.

Learning Activity 3.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Jon and Kate have 8 kids. Each kid shares a bedroom with another sibling. Including Jon and Kate's bedroom, how many bedrooms do they need in their house?
2. If you are looking for a good deal, you found it! Bread is 50% off! One loaf usually costs \$2.40. How much will you pay with the discount?
3. Evaluate $2 \times \sqrt[3]{27}$.
4. Evaluate $(4x^3)^{-2}$.
5. If $0.33\bar{3} = \frac{1}{3}$, then what does $0.66\bar{6}$ equal?
6. You are running errands all day. You have to go to the nursery, so you drive 8 km. You then go to the mall, so you drive another 6 km. Finally, you go to the movie store, which is another 3 km, and then 6 km home. How far did you drive altogether?
7. You have measured your foot to be 9.5". Is your foot a good referent to approximate a foot in length?
8. Your friend wants to go to a movie at 9:30 pm. You want to be at the movie 30 minutes early. It takes you 15 minutes to get from your house to the theatre. What time do you have to leave in order to get to the theatre on time?

Answers:

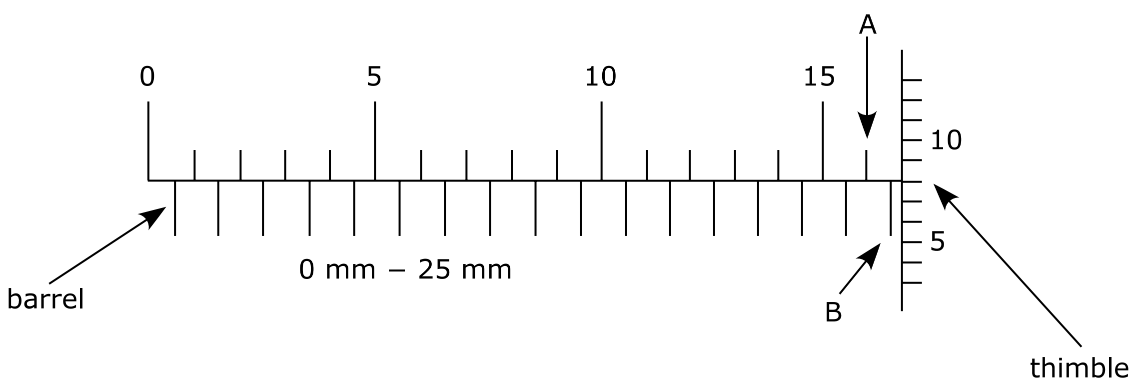
1. 5 bedrooms (If there are 8 kids and two per bedroom, then you need 8 ÷ 2 = 4 bedrooms for the kids + 1 for Jon and Kate = 5 bedrooms.)
2. \$1.20 (50% of \$2.40 = 2.4 ÷ 2 = \$1.20)
3. $6 \left(\sqrt[3]{27} = 3;\ 3 \times 2 = 6\right)$
4. $\frac{1}{16x^6} \left(\left(4x^3\right)^{-2} = \frac{1}{\left(4x^3\right)^2} = \frac{1}{16x^6}\right)$
5. $\frac{2}{3} \left(\text{Since } 0.66\bar{6} \text{ is double } 0.33\bar{3},\ \frac{1}{3} \times 2 = \frac{2}{3}.\right)$
6. 23 km (8 + 6 + 3 + 6 = 23 km)

7. No (A foot in length is 12”, so your foot is too small to be a referent.)
8. 8:45 pm (30 minutes before 9:30 is 9:00 pm. 15 minutes before that is 8:45 pm.)

Part B: Micrometer Measurements

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Read the following micrometer measurement.



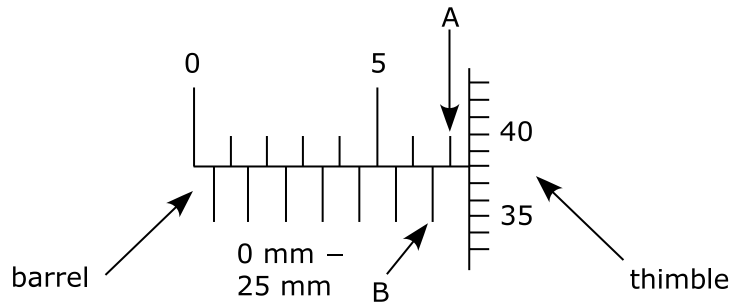
Answer:

Read the number from the upper barrel as 16 mm and the lower as 0.5 mm. Thus, the total barrel reading is 16.5 mm.

The thimble reading yields 0.08 mm.

The sum and resulting measurement is 16 mm + 0.5 mm + 0.08 mm = 16.58 mm.

2. Read the following micrometer measurement.



Answer:

Read the number from the barrel as 7 mm. (Note: The marking indicated by arrow B is to the left of the marking indicated by arrow A.)

The thimble reading yields 0.38 mm.

The sum and resulting measurement = 7 mm + 0.37 mm = 7.38 mm.

Learning Activity 3.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. You want to meet your friends for coffee. You are available from 9 am to 3 pm. Aiden is free from 12 pm until 2 pm. Leah is able to come from 10 am until 1 pm. What time can you meet for coffee?
2. Your suitcase cannot exceed 26 kg when you are going on an international flight. Your cousin weighs around 25 kg. Would your cousin be a good referent?
3. What is the LCM of 4, 6, and 8?
4. If the rise of a line is 12 and the run of the same line is 8, what is the slope of the line (in simplest form)?
5. A 2-door sports car gets 12.2 km per L of gas. A truck gets 7100 m per L of gas. Which is more fuel efficient?
6. Complete the pattern: 4, 1, –2, ____, ____.
7. The ratio comparing the distance on a map to the distance in real life is 1 cm: 10 km. If the distance from your house to your school on the map is 4 mm, how far do you live from school?
8. Is 0.2754 a rational or irrational number?

Answers:

1. From 12 pm until 1 pm (The earliest all three people can get together is 12 pm, because Aiden is busy before that. The meeting must end at 1 pm because Leah is not free after that.)
2. Yes. (Your cousin's weight is very close to 26 kg.)
3. 24 (The LCM of 4 and 6 is 12 [(4 × 6) ÷ 2], but this is not a multiple of 8. The next lowest common multiple is 4 and 6 is 24, which is also a multiple of 8.)
4. $\frac{3}{2}\left(\frac{12}{8} \div \frac{4}{4}\right)$
5. The sports car is more fuel efficient. (1000 m = 1 km, so 7100 m = 7.1 km/L)
6. −5, −8 (The pattern is to subtract three from the previous term.)
7. 4 km (10 mm = 1 cm so 4 mm = 0.4 cm. Since 1 cm: 10 km, 0.4 × 10 = 4 km.)
8. Rational (The decimal terminates.)

Part B: Imperial-Imperial and Imperial-Metric Conversions

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The owner's manual for a car states that the oil should be changed every 5000 km. How many miles is that? Use mental math to determine if your answer is reasonable.

   Answer:

   1 km = 0.621 mi.

   $$\frac{x}{5000} = \frac{0.621}{1}$$

   $$x = 5000 \times 0.621$$

   $$x = 3105$$

   You would need to change the oil every 3105 miles. Considering that there is just over 1.5 km per mile, you can use mental math to approximate $1.5 \times 3000 = 4500$ (think 3000 plus half of 3000) to check your answer. Since you are estimating with slightly smaller values, 4500 is close enough to 5000 to mentally verify the reasonableness of the answer.

2. You are listening to an American radio station on your way to Fargo, ND for a weekend. The announcer states that the temperature in Fargo is 18°F. What kind of weather can you expect while there?

   Answer:

   $$°\mathrm{C} = \frac{5}{9} \times (°\mathrm{F} - 32)$$

   $$°\mathrm{C} = \frac{5}{9} \times (18 - 32)$$

   $$°\mathrm{C} = \frac{5}{9} \times (-14)$$

   $$°\mathrm{C} = -7.7\ldots$$

   The temperature in Fargo is about −7.8°C. You could expect it to be cold and possibly snowy.

3. Convert the following:

a) 2 m = ________________ mm
b) 4 ft. = ________________ in.
c) 6 yd. 2 ft. = ________________ ft.
d) 6 yd. 2 ft. = ________________ in.
e) 7500 m = ________________ km
f) 2 miles = ________________ ft.
g) 4.7 cm = ________________ mm
h) 7650 cm = ________________ m
i) 3520 yd. = ________________ mi.
j) 720 000 cm = ________________ km

Answers:

a) $\dfrac{x}{2} = \dfrac{1000}{1}$

$x = 2 \times 1000 = 2000 \text{ mm}$

b) $\dfrac{x}{4} = \dfrac{12}{1}$

$x = 4 \times 12 = 48''$

c) $\dfrac{x}{6} = \dfrac{3}{1}$

$x = 18 \text{ ft.}$

$18 + 2 = 20'$

d) $\dfrac{x}{6} = \dfrac{36}{1}$

$x = 6 \times 36 = 216 \text{ in.}$

$\dfrac{y}{2} = \dfrac{12}{1}$

$y = 2 \times 12 = 24''$

$216 + 24 = 240''$

e) $\dfrac{x}{7500} = \dfrac{1}{1000}$

$x = \dfrac{7500}{1000} = 7.5 \text{ km}$

f) $\dfrac{x}{2} = \dfrac{5280}{1}$

$x = 2 \times 5280 = 10\ 560 \text{ ft.}$

g) $\dfrac{x}{4.7} = \dfrac{10}{1}$

$x = 4.7 \times 10 = 47 \text{ mm}$

h) $\dfrac{x}{7650} = \dfrac{1}{100}$

$x = \dfrac{7650}{100} = 76.5 \text{ m}$

i) $\dfrac{x}{3520} = \dfrac{1}{1760}$

$x = \dfrac{3520}{1760} = 2 \text{ mi.}$

j) $\dfrac{x}{720\ 000} = \dfrac{1}{100\ 000}$

$x = \dfrac{720\ 000}{100\ 000} = 7.2 \text{ km}$

4. Convert the following (Pay close attention to whether you are working with volume or area):
   a) Change 7 cm^2 to mm^2.
   b) Change 432 $in.^2$ to $ft.^2$.
   c) Change 3.6 $yd.^2$ to $ft.^2$.
   d) Change 55 000 cm^3 to m^3.

   Answers:

   a) $$\frac{x\ \text{mm}^2}{7\ \text{cm}^2} = \frac{100\ \text{mm}^2}{1\ \text{cm}^2}$$

   $$x = 7 \times 100 = 700\ \text{mm}^2$$

   b) $$\frac{x\ \text{ft.}^2}{432\ \text{in.}^2} = \frac{1\ \text{ft.}^2}{144\ \text{in.}^2}$$

   $$x = \frac{432}{144} = 3\ \text{ft.}^2$$

   c) $$\frac{x\ \text{ft.}^2}{3.6\ \text{yd.}^2} = \frac{9\ \text{ft.}^2}{1\ \text{yd.}^2}$$

   $$x = 3.6 \times 9 = 32.4\ \text{ft.}^2$$

   d) $$\frac{x\ \text{m}^3}{55\ 000\ \text{cm}^3} = \frac{1\ \text{m}^3}{1\ 000\ 000\ \text{cm}^3}$$

   $$x = \frac{55\ 000}{1\ 000\ 000} = 0.055\ \text{m}^3$$

Learning Activity 3.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. You are getting ready to start your first job and you want to make sure you get there on time. It takes you 20 minutes to ride your bike to work from your house. You want to get there 15 minutes before your shift starts. It takes you 30 minutes to get ready in the morning. If your shift starts at 10:00 am, what time will you have to wake up?
2. Put the following numbers in order from smallest to largest: 0.53, 29%, 0.045, 0.13, 78%.
3. The Pythagorean Theorem is $a^2 + b^2 =$ ____.
4. A loonie is approximately 2.5 cm across. Convert to inches.
5. Write as a radical: $-6c^{\frac{1}{4}}$.
6. What is the rise of a line if the slope is 2 and the run is 2?
7. What is the GCF of 14 and 18?
8. Rewrite the fraction in simplest form: $\dfrac{54}{27}$.

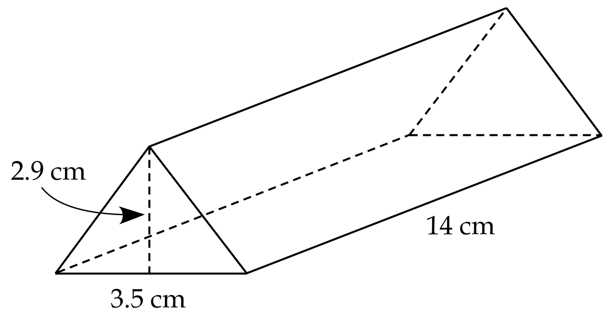
Answers:

1. 8:55 am (15 min. before 10:00 = 9:45 am; 20 min. before 9:45 = 9:25 am, 30 min. before 9:25 = 8:55 am)
2. 0.045, 0.13, 29%, 0.53, 78%
3. $a^2 + b^2 = c^2$, where c is the length of the hypotenuse
4. Approximately 1 inch
5. $-6\sqrt[4]{c}$
6. $4 \left(\text{slope} = \dfrac{\text{rise}}{\text{run}} \text{ so } 2 = \dfrac{\text{rise}}{2}\text{; rise} = 2 \times 2 = 4\right)$
7. 2
8. $2 \left(\dfrac{54}{27} \div \dfrac{27}{27} = \dfrac{2}{1}\right)$

Part B: Volume of Prisms and Pyramids

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. A chocolate bar shaped like a triangular prism is 14 cm long. Its triangular base is 3.5 cm long and 2.9 cm high. Calculate the volume of chocolate in the bar.



Answer:

$V = Bh$

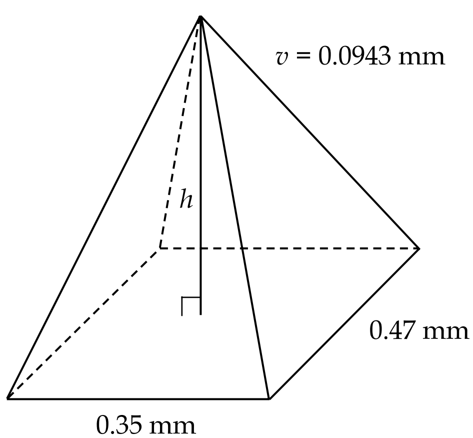
$$V = \frac{(3.5 \times 2.9)}{2} \times 14$$

$V = 71.05$

There is 71.05 cm^3 of chocolate in the bar.

2. Sketch a rectangular pyramid with a volume of 0.0943 mm^3, and label it with base dimensions of 0.35 mm by 0.47 mm. Determine its height.

Answer:



$$V = \frac{1}{3}Bh$$

$$0.0943 = \frac{1}{3}(0.35 \times 0.47) \times h$$

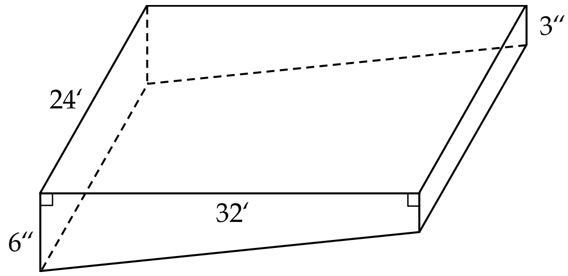
$$0.0943 = 0.05483h$$

$$h = \frac{0.0943}{0.05483}$$

$$h = 1.71986139$$

The height of the pyramid is about 1.72 mm.

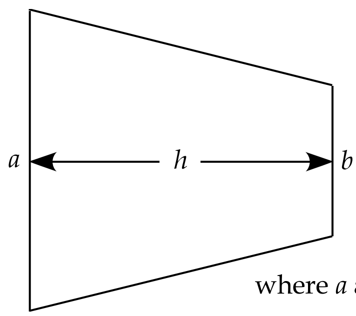
3. A cement floor for a garage has the following trapezoidal shape.



If cement costs \$95.75 per cubic yard, how much could the contractor expect to pay for the cement?

Note: Make sure all the measurements are in the same unit before using them in calculations. (Hint: Convert inches to feet.)

The formula for the area of a trapezoid is half the sum of the lengths of the two parallel sides multiplied by the height between them.



where *a* and *b* are parallel

$$A = \frac{1}{2}(a+b)h_T$$

Answer:

$$V = Bh$$

$$V = \left(\frac{1}{2}(a+b)h_1\right)h_2$$

h_1 is the height of the trapezoid base.
h_2 is the height of the prism.

$$V = \left(\frac{1}{2}(0.25+0.5)(32)\right)(24)$$

Convert inches to feet.

$$V = 288 \text{ ft.}^3$$

$$288 \text{ ft.}^3 \times \frac{1 \text{ yd.}^3}{27 \text{ ft.}^3} = 10.66667 \text{ yd.}^3$$

Convert to cubic yards using conversion ratio.

Cost = (price per cubic yard)(volume in cubic yards)
Cost = 95.75 × 10.66667
Cost = 1021.333653
The contractor could expect to pay around \$1021.33 for the cement.

Learning Activity 3.6

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Rewrite the following fraction in simplest form: $\frac{12}{26}$.
2. Rewrite the following fraction in simplest form: $\frac{24}{52}$.
3. Write the following percent as a decimal: 46.1%.
4. There are 8 marbles in a bag. Each marble is either red, yellow, or blue. Four of the marbles are red and one marble is yellow. How many are blue?
5. The side length of a cube is 3 cm. What is the volume?
6. What is the surface area of the same cube?
7. At the store, you compare the price of two hand lotions. The lemon one is $3.00 for a 60 mL bottle. The vanilla one is $6.00 for a 100 mL bottle. Which is the better deal (per mL)?
8. You are a very busy person. You have soccer on Sunday, Tuesday, and Thursday nights. You have music lessons on Saturday in the evening. You also have swimming on Monday night. Which nights of the week do you not have any commitments?

Answers:

1. $\frac{6}{13}$
2. $\frac{6}{13}$
3. 0.461
4. 3 blue marbles $(8 - (4 + 1))$
5. 27 cm^3 ($V = Bh$; $B = s \times s$, h (cube) $= s$, so $V = s^3$)
6. 54 cm^2 (For a cube, any side can be the base because all faces are the same. Since there are 6 faces on a cube, TSA = $6B = 6(s \times s)$.)
7. $3.00 for a 60 mL bottle is the better deal. (3.00 ÷ 60 = $0.05 per mL, 6.00 ÷ 100 = $0.06 per mL)
8. Wednesday and Friday

Part B: Surface Area of Prisms and Pyramids

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The famous Louvre Museum in Paris, France has a square glass pyramid above the main entrance. The pyramid is 35.42 m wide and 21.64 m high with a slant height of 27.96 m. Calculate the lateral surface area of the glass.

Answer:

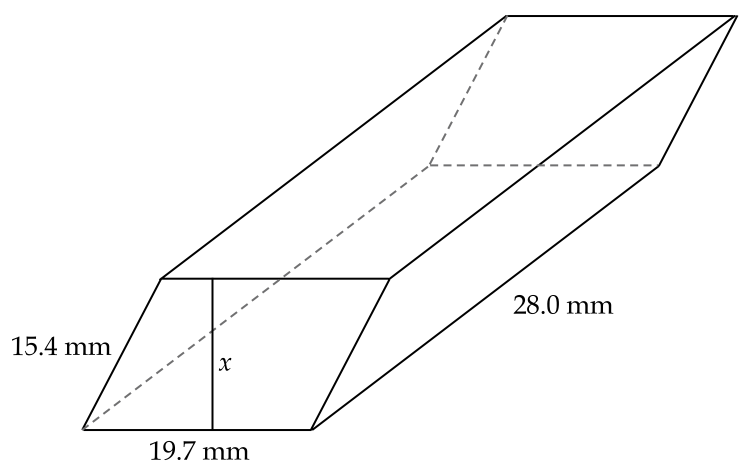
$$\text{LSA} = \frac{1}{2}P\ell$$

$$\text{LSA} = \frac{1}{2}(4 \times 35.42)(27.96)$$

$$\text{LSA} = 1980.6864$$

The lateral surface area of glass if 1980.6864 m^2.

2. The total surface area of this prism, with a parallelogram-shaped base, is 2501.44 mm^2. Determine the height of the parallelogram base.



Answer:

$$\text{TSA} = Ph + 2B$$

$$2501.44 = \left[(2 \times 15.4) + (2 \times 19.7)\right](28.0) + 2(19.7)(x)$$

$$2501.44 = 1965.6 + 39.4x$$

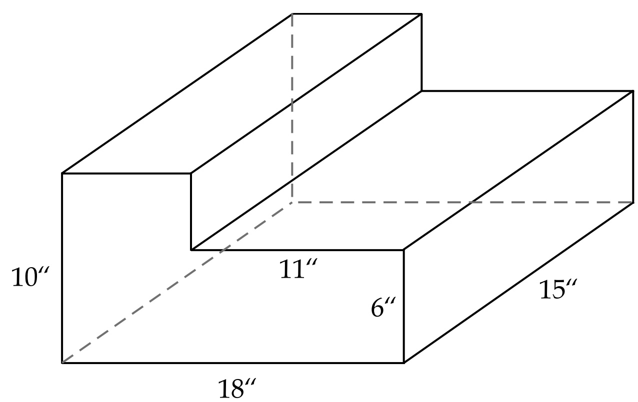
$$2501.44 - 1965.6 = 39.4x$$

$$\frac{535.84}{39.4} = \frac{39.4x}{39.4}$$

$$x = 13.6$$

The height of the parallelogram is 13.6 mm.

3. Find the total surface area of the following 3-D object. State your final answer in scientific notation and also in ft.^{2}.



Answer:

The shape of the L-shaped base is consistent throughout the entire length of the object, so this is a prism. The total surface area of a prism is TSA = $Ph + 2B$.

You need to find the missing dimensions. On the top left of the L-shape, you know it is 18 − 11 or 7″ long. The vertical line on the top is 10 − 6 or 4″ long.

TSA = (10 + 7 + 4 + 11 + 6 + 18)(15) + 2((7 × 4)+(6 × 18))

TSA = 56 × 15 + 2 × 136

TSA = 840 + 272

TSA = 1112

The total surface area is 1112 in.^{2} or 1.112×10^3 square inches.

$$1112 \text{ in.}^2 \times \frac{1 \text{ ft.}^2}{144 \text{ in.}^2} = 7.72 \text{ ft.}^2$$

The total surface area is about 7.72 square feet.

Learning Activity 3.7

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate $\sqrt{9 + 16}$.
2. Evaluate $\sqrt{25 + 144}$.
3. The slope of a line is 4. What is the slope of a perpendicular line to this?
4, Terry has a chocolate orange that he wants to eat over one week (Monday to Sunday). He wants to eat the same number of pieces each day. If the orange has 14 pieces, how many pieces will he eat each day?
5. You are out for dinner with your best friend for her birthday. The bill comes and you pay for everything. If the total is \$35.75 and you leave \$40 on the table (including tip), how much are you tipping the server?
6. Is the following angle acute, obtuse, straight, or reflex: 140°?
7. The volume of a cube is 8 m^3. What are the dimensions of the cube?
8. What two numbers have a product of −10 and a sum of −3?

Answers:

1. 5 $\left(\sqrt{9 + 16} = \sqrt{25}\right)$
2. 13 $\left(\sqrt{25 + 144} = \sqrt{169}\right)$
3. $-\frac{1}{4}$ (Perpendicular slope is the negative reciprocal.)
4. 2 pieces (14 ÷ 7)
5. \$4.25 tip (35.75 + 0.25 = \$36, \$36 + \$4 = \$40, so \$4 + 0.25 = \$4.25)
6. Obtuse (Acute angles are between 0° and 90°; obtuse is between 90° and 180°; straight angle is 180°; reflex angle is between 180° and 360°.)
7. 2 m (All dimensions of a cube are equal, so $\sqrt[3]{8} = 2$.)
8. −5, 2 (The factors of 10 are (1, 10) and (2, 5), so for −10, a negative would be in front of one number in the pair. 5 − 2 = 3, −5 + 2 = −3.)

Part B: Surface Area and Volume of Spheres, Cylinders, and Cones

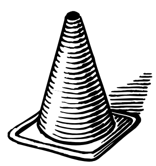
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Describe the relationship between the volumes of a cone and cylinder that have the same radius and height.

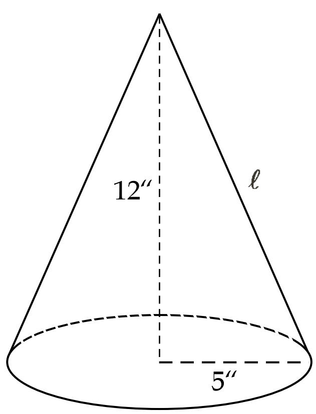
   Answer:

   The volume of a cone that has the same radius and height as a cylinder will be one-third of the cylinder. Alternately, the volume of a cylinder with the same height and radius as a cone will be three times larger than the volume of the cone.

2. You have been asked to repaint the lateral surface area of cone-shaped pylons used for soccer drills. If they have a radius of 5″ and a height of 12″, calculate the slant height and lateral surface area to be painted.



Answer:



$$r^2 + h^2 = \ell^2$$ where r is the radius, h is the height, and ℓ is the slant height

$$5^2 + 12^2 = \ell^2$$

$$25 + 144 = \ell^2$$

$$169 = \ell^2$$

$$\ell = 13$$

$$C = 2\pi r$$

$$C = 2\pi(5)$$

$$C = 31.41592654$$

$$\text{LSA} = \frac{1}{2}C\ell$$

$$\text{LSA} = \frac{1}{2}(31.41592654)(13)$$

$$\text{LSA} = 204.2035225$$

The lateral surface of the cone that needs to be painted is about 204 in.^{2}.

3. Find a sports ball (e.g., a basketball, softball, soccer ball, or tennis ball), and measure its circumference using imperial units. Describe your measurement strategy and use the circumference to determine the ball's radius. Calculate the volume of the ball.

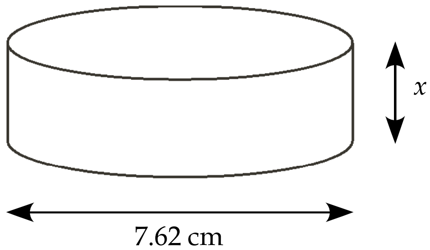
   Answer:

   To determine the circumference of a sports ball, you may have used a flexible measuring tape or wrapped a string around the widest part of the ball and then used a ruler to measure the length of string. You may have used another creative way. Using the formula $C = 2\pi r$ or $r = \frac{C}{2\pi}$, solve for the length of the radius. Substitute that value into the formula $V = \frac{4}{3}\pi r^3$, and solve for the volume of the ball.

   Compare your answer with the possible solutions below:

Type of Ball	Circumference	Radius	Volume
Soccer ball	27.5 inches	4.377″	351.252 in.^{3}
Basketball	29.5 inches	4.695″	433.506 in.^{3}
Baseball	9 inches	1.432″	12.300 in.^{3}
Softball	12 inches	1.910″	29.187 in.^{3}
Tennis ball	8 inches	1.273″	8.641 in.^{3}

4. A hockey puck has a diameter of 7.62 cm and a volume of 115.83 cm^3. Calculate the thickness of a hockey puck.



Answer:

$$d = 2r$$

$$r = \frac{d}{2}$$

$$r = \frac{7.62}{2}$$

$$r = 3.81$$

$$V = Bh$$

$$V = (\pi r^2)h$$

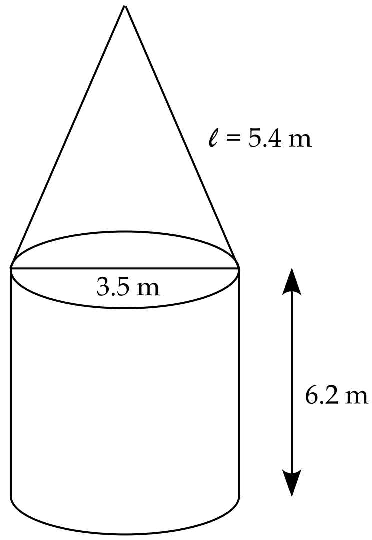
$$115.83 = \pi (3.81^2)x$$

$$x = \frac{115.83}{\pi \times 14.5161}$$

$$x = 2.539926986$$

The thickness of a hockey puck is 2.54 cm, which is equivalent to 1″. It would make a great referent!

5. The turret on a castle is formed by constructing a cone-shaped roof above a cylindrical structure. Determine the lateral surface area of the following turret. Round your final answer to the nearest hundredth of a metre.



Answer:

This object is composed of a cylinder and a cone. Find the sum of the two lateral surface areas to determine the lateral surface area of this object.

$$\text{LSA} = \text{LSA}_{\text{cone}} + \text{LSA}_{\text{cylinder}}$$

$$\text{LSA} = \frac{1}{2}C\ell + Ch \quad (\text{where } C = \pi d)$$

$$C = \pi(3.5)$$

$$C = 10.99557429$$

$$\text{LSA} = \frac{1}{2}(10.99557429)(5.4) + (10.99557429)(6.2)$$

$$\text{LSA} = 29.68805058 + 68.17256058$$

$$\text{LSA} \approx 97.86$$

The lateral surface area of this turret is about 97.86 m^2.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 3
Measurement

Learning Activity Answer Keys

Module 3: Measurement

Learning Activity 3.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate: $\left(\dfrac{9x^4y^3}{3xy^2}\right)$.
2. Which is the dependent variable: the windchill compared to the time it takes to get frostbite?
3. A school volleyball team wants to practise twice per week. They cannot practise at the school on the weekend (Saturday and Sunday), half the team cannot prastice on Monday and Wednesday, and the basketball team uses the gym on Friday. Which days can the team practise?
4. Solve for p: $p \div 15 = 5$.
5. What is the GCF of 19 and 13?
6. What two numbers have a sum of 11 and a product of 18?
7. You want to save up $12 000 to buy a car one year from now. How much do you have to save per month to reach this goal?
8. Three students receive their marks for a project. Jane found her mark as a decimal, 0.62; John calculated his mark as a percent, 83%; Jean got $\dfrac{12}{16}$.

   Who got the best mark?

Answers:

1. $3x^3y$
2. Time it takes to get frostbite.
3. Tuesday and Thursday
4. $p = 75$ ($p = 15 \div 5$)
5. 1 (19 and 13 are both prime numbers)
6. 2, 9 (The factor of pairs of 18 are (1, 18), (2, 9), (3. 6). Only 2 + 9 = 11.)
7. $1000 ($12, 000 ÷ 12)
8. John got the best mark. If you put them all into one form (e.g., percent), you can compare the marks. Jane: 0.62 = 62%, John: 83%, Jean: $\dfrac{12}{16}$ = 75%.

Part B: Units

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Provide referents for the following linear measurements.

   a) millimetre

   b) metre

   c) inch

   d) mile

   Answer:

   Answers will vary. Some possible answers are:

   a) diameter of wire used in a standard paper clip, thickness of a dime, thickness of a credit card

   b) two steps or one pace, distance from floor to average woman's waist, perimeter of a sheet of printer paper

   c) diameter of a $1 coin (loonie), width of an average adult thumb

   d) distance a fit person can run in 7 minutes, distance between country roads, about 15–20 city blocks (depending on how the streets are organized, this may be more or less)

2. Determine a referent based on your height to compare the lengths of a metre and a yard. Use it to estimate the height of a table.

   Answer:

   Your answer will vary, based on your height.

   A possible solution: a person who is about 5′ 5″ or 165 cm tall would measure about 1 m from the floor to their waist, while 1 yard would be from the floor to their hip. Based on this comparison, a yard is just slightly shorter than a metre.

   Standing next to the table, it may reach about one hand span below a person's hip or one and a half hand spans below a person's waist. If their hand span was about 8 inches or 20 cm, the height of the table could be estimated at about 8 inches less than a yard

   Since 1 yd. = 36″, 36 − 8 = 28

   The table is about 28 inches high.

Using SI, one and a half hand spans below their waist:

$100 - (1.5 \times 20)$

$100 - 30 = 70$

The table is about 70 cm high.

3. Use a referent of your choice to estimate the dimensions of your calculator or a cell phone. Indicate your referent, choice of units, and then measure to verify the accuracy and appropriateness of your measurement and units.

Answer:

Answers will vary depending on the choice of referent and unit. Check your answer with your learning partner.

A possible solution: A $1 coin has a diameter of about 1″. A graphing calculator is about 3 loonies wide by 7 loonies long. Using a ruler, the dimensions are determined to be $3\frac{1}{4}$ inches by $7\frac{1}{8}$ inches. The units and measurements are accurate and appropriate.

A permanent marker has a diameter of about 1 cm, and its cap is about 5 cm long. The calculator is about 8 marker widths across and about three and a half cap lengths long. Using this referent, the calculator is about 8 cm wide and $3.5 \times 5 = 17.5$ cm long. Using a metric ruler, the dimensions of the graphing calculator are determined to be about 8 cm by 18 cm. The estimate is close to the actual measurement.

4. Which units from the metric system and from the imperial system would be the most suitable for measuring the following lengths?

a) Width of a snowboard

b) Length of a soccer field

c) Thickness of a coin

d) Distance a jogger runs every day

Answers:

a)	Metric: centimetres	Imperial: inches
b)	Metric: metres	Imperial: yards
c)	Metric: millimetres	Imperial: inches
d)	Metric: kilometres	Imperial: miles

5. Estimate the following lengths in both SI and imperial measurements. Check your estimates by measuring.
   a) Length and width of a standard door
   b) Length of a vehicle
   c) Length and width of a two-car driveway
   d) Diameter and thickness of a $2 coin (toonie)

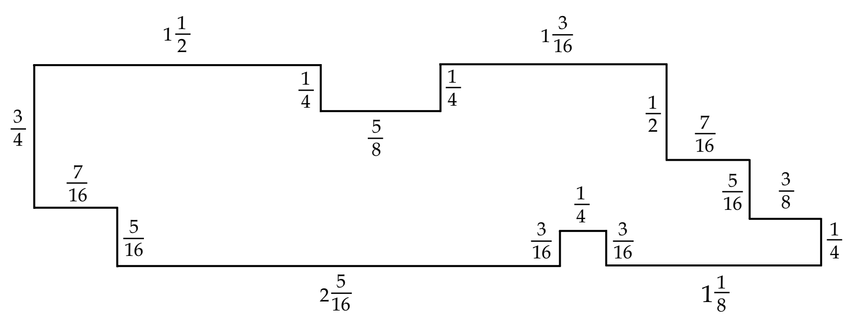
Answers:

Answers will vary. Check your answer with your learning partner.

	Imperial	SI
a)	$2'8'' \times 6'8''$	81 cm × 203 cm
b)	$15'$ to $20'$	4.5 m to 6 m
c)	$16\frac{1}{4}' \times 37\frac{1}{2}'$	4.95 m × 11.43 m
d)	$1\frac{1}{8}'' \times \frac{1}{16}''$	2.7 cm × 1.5 mm

6. Using an imperial ruler, find the perimeter of the following diagram to the nearest $\frac{1}{16}''$.

Answer:



The perimeter is $11\frac{7}{16}$ inches long.

(Answers from $11\frac{1}{8}$ to $11\frac{3}{4}$ are acceptable because of inaccuracies in measurement that occur as a result of a slight shrinkage of images in the photocopying process.)

7. Your dad bought you a car for your 16th birthday and wants to wrap it with a large ribbon to surprise you. Explain how he could determine the length of ribbon to buy, and which units would be the most appropriate.

Answer:

Your dad may choose to use a flexible measuring tape that can go under and around the car. If he did not have a flexible measuring tape, he may estimate the number of paces long and wide the car is, and calculate the length of one of his paces. He would have to add additional length to account for the height of the car, and include the amount needed if he wanted to add a bow on top.

Feet, yards, or metres would be the most appropriate units to use, as inches or centimetres would result in a very large value for the answer. Ribbon is typically sold at a fabric store by the metre, or on a roll with both metres and yards given.

8. What would be the most suitable units of measure for the following areas and volumes? Give an answer using both systems.
   a) The volume of water in a thimble
   b) The area of a hockey ice surface
   c) The volume of earth removed for a basement of a house
   d) The area of one side of a coin

   Answers:

   a) mm^3 or cm^3 or $in.^3$
   b) m^2 or $yd.^2$
   c) m^3 or $yd.^3$
   d) mm^2 or cm^2 or $in.^2$

9. a) Measure the following right triangle to the nearest mm and calculate its area.

Answer:

$$A_\Delta = \frac{bh}{2}$$

$$A = (83 \times 63) \div 2$$

$$A = 5229 \div 2$$

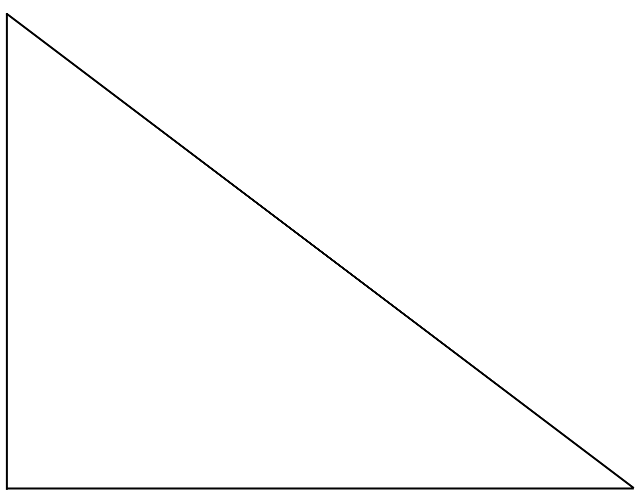
$$A = 2614.5$$

(Answers may be in the 2542 cm^2 to 2688 cm^2 range because of inaccuracies in measurement that occur as a result of a slight shrinkage of images in the photocopying process.)

The area of the triangle is 2614.5 mm^2. Since 1 cm^2 = 100 mm^2, this could be stated as 26.145 cm^2.

b) Measure the following right triangle in inches and calculate its area.

Recall: $A_\Delta = \frac{bh}{2}$



Answer:

$$A_\Delta = \frac{bh}{2}$$

$$A = \left(2\frac{1}{2} \times 3\frac{1}{4}\right) \div 2$$

$$A = \left(2 \times 3 + 2 \times \frac{1}{4} + \frac{1}{2} \times 3 + \frac{1}{2} \times \frac{1}{4}\right) \div 2$$

$$A = \left(8\frac{1}{8}\right) \div 2$$

$$A = 8\frac{1}{8} \times \frac{1}{2}$$

$$A = 4\frac{1}{16}$$

The area of the triangle is $4\frac{1}{16}$ in.^{2}.

(Answers may be in the 4 in.^{2} to $4\frac{1}{2}$ in.^{2} range due to a slight shrinkage of images in the photocopying process.

Learning Activity 3.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Convert 24 inches into feet.
2. If the equation of a line is $y = 3x + 4$, what is the y-intercept?
3. Evaluate $(3y^7)^3$.
4. You are at a hockey game and would like a snack and a drink. At the counter, you see that popcorn is \$3.00, peanuts are \$2.25, and hotdogs are \$3.75. The drinks are \$2.00. If you have \$5, what snack can you afford to get, if you also buy a drink?
5. You are trying to estimate the height of your brother, who is approximately 1′ taller than your sister. Your sister is half a foot taller than you. How tall is your brother if you are 5′ tall?
6. Solve for w: $w \div 6 = 2$.
7. If 5% of 260 is 13, what is 5% of 520?
8. A person gets to choose from two chocolate milkshakes. One milkshake fills the glass $\frac{7}{9}$ full; the other fills the glass $\frac{2}{3}$ full. Which would you prefer?

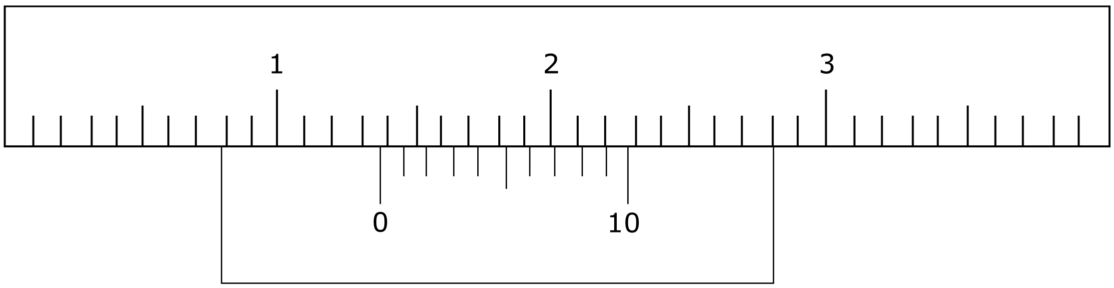
Answers:

1. 2 feet (12 inches = 1 foot, so $24 \div 12 = 2$ feet)
2. $y = 4$ (The y-intercept is when $x = 0$, so $y = 3(0) + 4 = 4$.)
3. $27y^{21}$ (This is the Power of a Power Law, $(3y^7)^3 = 3^3(y^7)^3$)
4. You can have peanuts or popcorn. (\$2.25 + \$2.00 (peanuts and a drink) = \$4.25; \$3.00 + \$2.00 (popcorn and drink) = \$5.00.)
5. 6.5′ (5′ + 0.5′ = 5.5′ (your sister’s height), 5.5′ + 1′ = 6.5′)
6. $w = 12$ ($w = 2 \times 6$)
7. 26 (520 is double 260, so 5% of 520 is double 5% of 260)
8. $\frac{7}{9}$ of a full glass $\left(\frac{2}{3} \times \frac{3}{3} = \frac{6}{9} \text{ which is less}\right)$

Part B: Vernier Caliper Measurements

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write down the measurement as shown on these Vernier caliper diagrams. Read the measurements to the nearest hundredth of a centimetre.

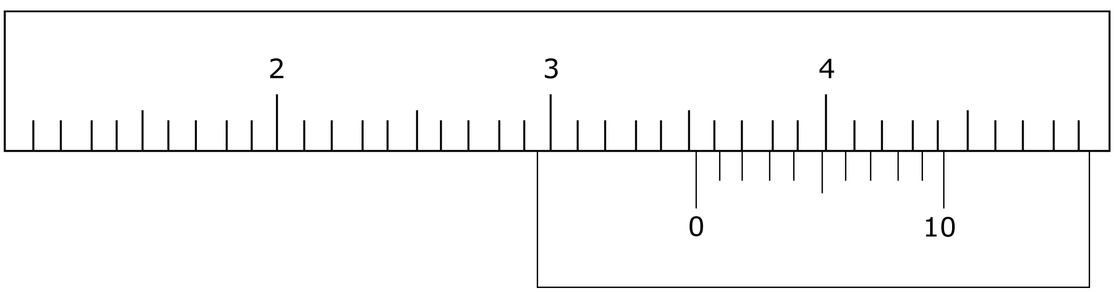
1. 

Answer:

The first line of the moving scale, at a reading of 0, points to a space between 1.3 and 1.4 on the fixed scale. Thus, the first reading is 1.3 cm.

The two scales align at the 9th tick on the moving scale, giving a reading of 0.09 cm.

The total reading = 1.3 + 0.09 = 1.39 cm.

2. 

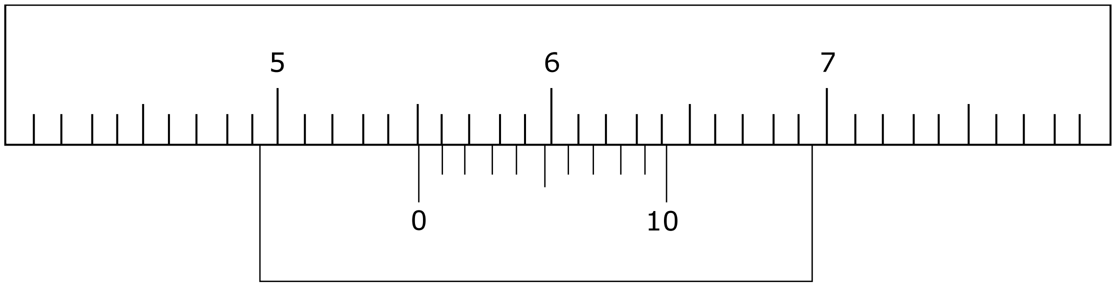
Answer:

The first line of the moving scale, at a reading of 0, points to a space between 3.5 and 3.6. Thus, the first reading is 3.5 cm.

The two scales align at the second tick on the moving scale, giving a reading of 0.02 cm.

The total reading = 3.5 + 0.02 = 3.52 cm.

3.



Answer:

The first line of the moving scale, at a reading of 0, points to a space between 5.5 and 5.6. The first reading is 5.5 cm.

The two scales align at the first tick, giving a reading of 0.01 cm.

The total reading = 5.5 + 0.01 = 5.51 cm.

Learning Activity 3.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Jon and Kate have 8 kids. Each kid shares a bedroom with another sibling. Including Jon and Kate's bedroom, how many bedrooms do they need in their house?
2. If you are looking for a good deal, you found it! Bread is 50% off! One loaf usually costs $2.40. How much will you pay with the discount?
3. Evaluate $2 \times \sqrt[3]{27}$.
4. Evaluate $(4x^3)^{-2}$.
5. If $0.33\bar{3} = \dfrac{1}{3}$, then what does $0.66\bar{6}$ equal?
6. You are running errands all day. You have to go to the nursery, so you drive 8 km. You then go to the mall, so you drive another 6 km. Finally, you go to the movie store, which is another 3 km, and then 6 km home. How far did you drive altogether?
7. You have measured your foot to be 9.5". Is your foot a good referent to approximate a foot in length?
8. Your friend wants to go to a movie at 9:30 pm. You want to be at the movie 30 minutes early. It takes you 15 minutes to get from your house to the theatre. What time do you have to leave in order to get to the theatre on time?

Answers:

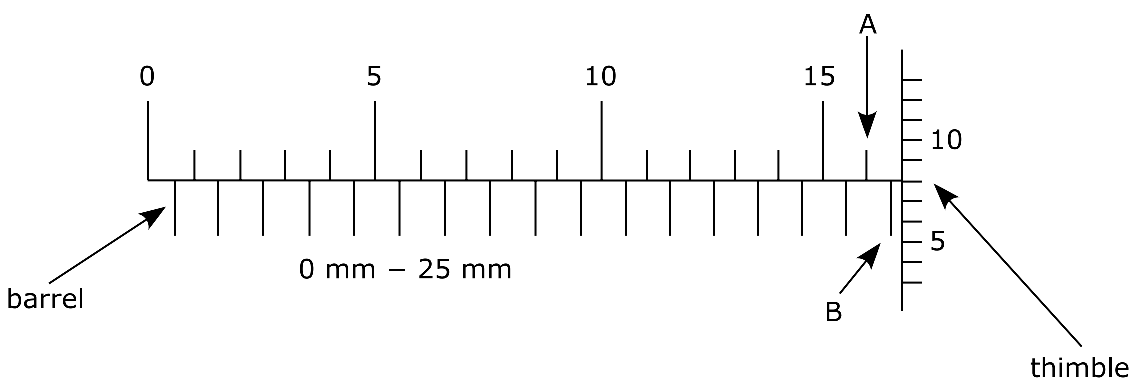
1. 5 bedrooms (If there are 8 kids and two per bedroom, then you need 8 ÷ 2 = 4 bedrooms for the kids + 1 for Jon and Kate = 5 bedrooms.)
2. $1.20 (50% of $2.40 = 2.4 ÷ 2 = $1.20)
3. $6 \left(\sqrt[3]{27} = 3;\ 3 \times 2 = 6\right)$
4. $\dfrac{1}{16x^6} \left(\left(4x^3\right)^{-2} = \dfrac{1}{\left(4x^3\right)^2} = \dfrac{1}{16x^6} \right)$
5. $\dfrac{2}{3} \left(\text{Since } 0.66\bar{6} \text{ is double } 0.33\bar{3},\ \dfrac{1}{3} \times 2 = \dfrac{2}{3}. \right)$
6. 23 km (8 + 6 + 3 + 6 = 23 km)

7. No (A foot in length is 12”, so your foot is too small to be a referent.)
8. 8:45 pm (30 minutes before 9:30 is 9:00 pm. 15 minutes before that is 8:45 pm.)

Part B: Micrometer Measurements

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Read the following micrometer measurement.



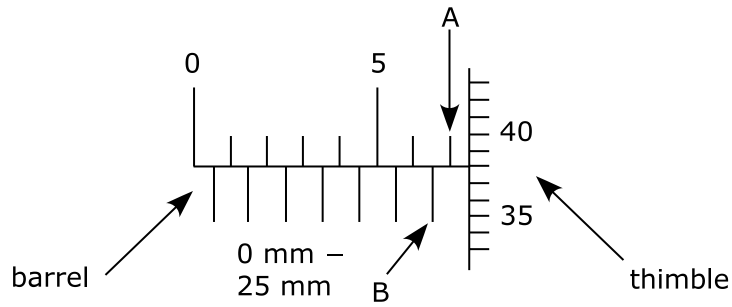
Answer:

Read the number from the upper barrel as 16 mm and the lower as 0.5 mm. Thus, the total barrel reading is 16.5 mm.

The thimble reading yields 0.08 mm.

The sum and resulting measurement is 16 mm + 0.5 mm + 0.08 mm = 16.58 mm.

2. Read the following micrometer measurement.



Answer:

Read the number from the barrel as 7 mm. (Note: The marking indicated by arrow B is to the left of the marking indicated by arrow A.)

The thimble reading yields 0.38 mm.

The sum and resulting measurement = 7 mm + 0.37 mm = 7.38 mm.

Learning Activity 3.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. You want to meet your friends for coffee. You are available from 9 am to 3 pm. Aiden is free from 12 pm until 2 pm. Leah is able to come from 10 am until 1 pm. What time can you meet for coffee?
2. Your suitcase cannot exceed 26 kg when you are going on an international flight. Your cousin weighs around 25 kg. Would your cousin be a good referent?
3. What is the LCM of 4, 6, and 8?
4. If the rise of a line is 12 and the run of the same line is 8, what is the slope of the line (in simplest form)?
5. A 2-door sports car gets 12.2 km per L of gas. A truck gets 7100 m per L of gas. Which is more fuel efficient?
6. Complete the pattern: 4, 1, –2, ____, ____.
7. The ratio comparing the distance on a map to the distance in real life is 1 cm: 10 km. If the distance from your house to your school on the map is 4 mm, how far do you live from school?
8. Is 0.2754 a rational or irrational number?

Answers:

1. From 12 pm until 1 pm (The earliest all three people can get together is 12 pm, because Aiden is busy before that. The meeting must end at 1 pm because Leah is not free after that.)
2. Yes. (Your cousin's weight is very close to 26 kg.)
3. 24 (The LCM of 4 and 6 is 12 [(4 × 6) ÷ 2], but this is not a multiple of 8. The next lowest common multiple is 4 and 6 is 24, which is also a multiple of 8.)
4. $\frac{3}{2}\left(\frac{12}{8} \div \frac{4}{4}\right)$
5. The sports car is more fuel efficient. (1000 m = 1 km, so 7100 m = 7.1 km/L)
6. −5, −8 (The pattern is to subtract three from the previous term.)
7. 4 km (10 mm = 1 cm so 4 mm = 0.4 cm. Since 1 cm: 10 km, 0.4 × 10 = 4 km.)
8. Rational (The decimal terminates.)

Part B: Imperial-Imperial and Imperial-Metric Conversions

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The owner's manual for a car states that the oil should be changed every 5000 km. How many miles is that? Use mental math to determine if your answer is reasonable.

   Answer:

   1 km = 0.621 mi.

   $$\frac{x}{5000} = \frac{0.621}{1}$$

   $$x = 5000 \times 0.621$$

   $$x = 3105$$

   You would need to change the oil every 3105 miles. Considering that there is just over 1.5 km per mile, you can use mental math to approximate $1.5 \times 3000 = 4500$ (think 3000 plus half of 3000) to check your answer. Since you are estimating with slightly smaller values, 4500 is close enough to 5000 to mentally verify the reasonableness of the answer.

2. You are listening to an American radio station on your way to Fargo, ND for a weekend. The announcer states that the temperature in Fargo is 18°F. What kind of weather can you expect while there?

   Answer:

   $$°\mathrm{C} = \frac{5}{9} \times (°\mathrm{F} - 32)$$

   $$°\mathrm{C} = \frac{5}{9} \times (18 - 32)$$

   $$°\mathrm{C} = \frac{5}{9} \times (-14)$$

   $$°\mathrm{C} = -7.7\ldots$$

   The temperature in Fargo is about −7.8°C. You could expect it to be cold and possibly snowy.

3. Convert the following:

a) 2 m = _______________ mm
b) 4 ft. = _______________ in.
c) 6 yd. 2 ft. = _______________ ft.
d) 6 yd. 2 ft. = _______________ in.
e) 7500 m = _______________ km
f) 2 miles = _______________ ft.
g) 4.7 cm = _______________ mm
h) 7650 cm = _______________ m
i) 3520 yd. = _______________ mi.
j) 720 000 cm = _______________ km

Answers:

a) $\frac{x}{2} = \frac{1000}{1}$

$x = 2 \times 1000 = 2000 \text{ mm}$

b) $\frac{x}{4} = \frac{12}{1}$

$x = 4 \times 12 = 48''$

c) $\frac{x}{6} = \frac{3}{1}$

$x = 18 \text{ ft.}$

$18 + 2 = 20'$

d) $\frac{x}{6} = \frac{36}{1}$

$x = 6 \times 36 = 216 \text{ in.}$

$\frac{y}{2} = \frac{12}{1}$

$y = 2 \times 12 = 24''$

$216 + 24 = 240''$

e) $\frac{x}{7500} = \frac{1}{1000}$

$x = \frac{7500}{1000} = 7.5 \text{ km}$

f) $\frac{x}{2} = \frac{5280}{1}$

$x = 2 \times 5280 = 10\ 560 \text{ ft.}$

g) $\frac{x}{4.7} = \frac{10}{1}$

$x = 4.7 \times 10 = 47 \text{ mm}$

h) $\frac{x}{7650} = \frac{1}{100}$

$x = \frac{7650}{100} = 76.5 \text{ m}$

i) $\frac{x}{3520} = \frac{1}{1760}$

$x = \frac{3520}{1760} = 2 \text{ mi.}$

j) $\frac{x}{720\ 000} = \frac{1}{100\ 000}$

$x = \frac{720\ 000}{100\ 000} = 7.2 \text{ km}$

4. Convert the following (Pay close attention to whether you are working with volume or area):
   a) Change 7 cm^2 to mm^2.
   b) Change 432 in.^{2} to ft.^{2}.
   c) Change 3.6 yd.^{2} to ft.^{2}.
   d) Change 55 000 cm^3 to m^3.

   Answers:

   a) $$\frac{x\text{ mm}^2}{7\text{ cm}^2} = \frac{100\text{ mm}^2}{1\text{ cm}^2}$$

   $$x = 7 \times 100 = 700\text{ mm}^2$$

   b) $$\frac{x\text{ ft.}^2}{432\text{ in.}^2} = \frac{1\text{ ft.}^2}{144\text{ in.}^2}$$

   $$x = \frac{432}{144} = 3\text{ ft.}^2$$

   c) $$\frac{x\text{ ft.}^2}{3.6\text{ yd.}^2} = \frac{9\text{ ft.}^2}{1\text{ yd.}^2}$$

   $$x = 3.6 \times 9 = 32.4\text{ ft.}^2$$

   d) $$\frac{x\text{ m}^3}{55\ 000\text{ cm}^3} = \frac{1\text{ m}^3}{1\ 000\ 000\text{ cm}^3}$$

   $$x = \frac{55\ 000}{1\ 000\ 000} = 0.055\text{ m}^3$$

Learning Activity 3.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. You are getting ready to start your first job and you want to make sure you get there on time. It takes you 20 minutes to ride your bike to work from your house. You want to get there 15 minutes before your shift starts. It takes you 30 minutes to get ready in the morning. If your shift starts at 10:00 am, what time will you have to wake up?
2. Put the following numbers in order from smallest to largest: 0.53, 29%, 0.045, 0.13, 78%.
3. The Pythagorean Theorem is $a^2 + b^2 =$ ____.
4. A loonie is approximately 2.5 cm across. Convert to inches.
5. Write as a radical: $-6c^{\frac{1}{4}}$.
6. What is the rise of a line if the slope is 2 and the run is 2?
7. What is the GCF of 14 and 18?
8. Rewrite the fraction in simplest form: $\frac{54}{27}$.

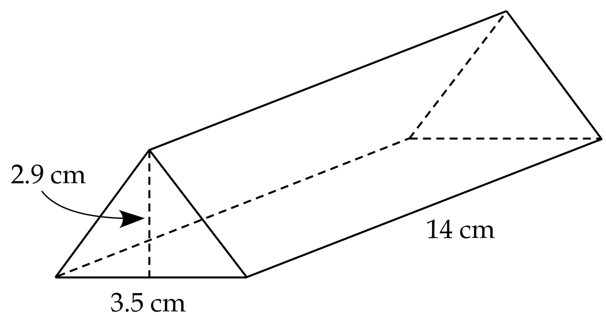
Answers:

1. 8:55 am (15 min. before 10:00 = 9:45 am; 20 min. before 9:45 = 9:25 am, 30 min. before 9:25 = 8:55 am)
2. 0.045, 0.13, 29%, 0.53, 78%
3. $a^2 + b^2 = c^2$, where c is the length of the hypotenuse
4. Approximately 1 inch
5. $-6\sqrt[4]{c}$
6. $4 \left(\text{slope} = \frac{\text{rise}}{\text{run}} \text{ so } 2 = \frac{\text{rise}}{2}\text{; rise} = 2 \times 2 = 4\right)$
7. 2
8. $2 \left(\frac{54}{27} \div \frac{27}{27} = \frac{2}{1}\right)$

Part B: Volume of Prisms and Pyramids

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. A chocolate bar shaped like a triangular prism is 14 cm long. Its triangular base is 3.5 cm long and 2.9 cm high. Calculate the volume of chocolate in the bar.



Answer:

$V = Bh$

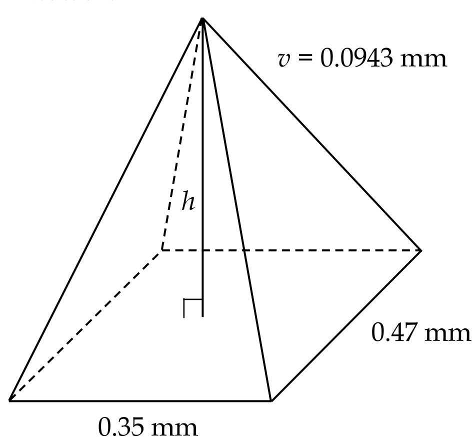
$$V = \frac{(3.5 \times 2.9)}{2} \times 14$$

$V = 71.05$

There is 71.05 cm^3 of chocolate in the bar.

2. Sketch a rectangular pyramid with a volume of 0.0943 mm^3, and label it with base dimensions of 0.35 mm by 0.47 mm. Determine its height.

Answer:



$$V = \frac{1}{3}Bh$$

$$0.0943 = \frac{1}{3}(0.35 \times 0.47) \times h$$

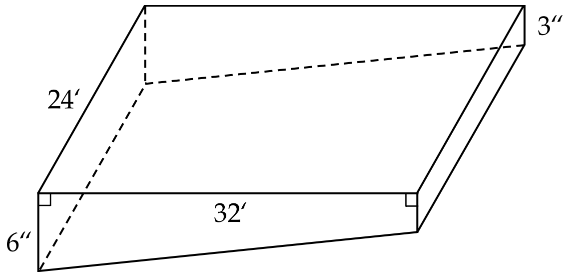
$$0.0943 = 0.05483h$$

$$h = \frac{0.0943}{0.05483}$$

$$h = 1.71986139$$

The height of the pyramid is about 1.72 mm.

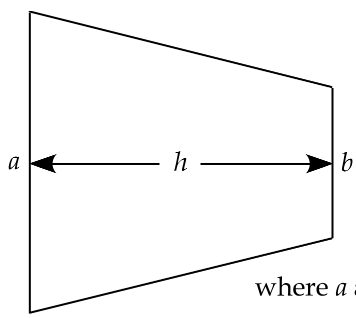
3. A cement floor for a garage has the following trapezoidal shape.



If cement costs $95.75 per cubic yard, how much could the contractor expect to pay for the cement?

Note: Make sure all the measurements are in the same unit before using them in calculations. (Hint: Convert inches to feet.)

The formula for the area of a trapezoid is half the sum of the lengths of the two parallel sides multiplied by the height between them.



where *a* and *b* are parallel

$$A = \frac{1}{2}(a+b)h_T$$

Answer:

$$V = Bh$$

$$V = \left(\frac{1}{2}(a+b)h_1\right)h_2$$

h_1 is the height of the trapezoid base.
h_2 is the height of the prism.

$$V = \left(\frac{1}{2}(0.25+0.5)(32)\right)(24)$$

Convert inches to feet.

$$V = 288 \text{ ft.}^3$$

$$288 \text{ ft.}^3 \times \frac{1 \text{ yd.}^3}{27 \text{ ft.}^3} = 10.66667 \text{ yd.}^3$$

Convert to cubic yards using conversion ratio.

Cost = (price per cubic yard)(volume in cubic yards)
Cost = 95.75 × 10.66667
Cost = 1021.333653
The contractor could expect to pay around $1021.33 for the cement.

Learning Activity 3.6

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Rewrite the following fraction in simplest form: $\frac{12}{26}$.
2. Rewrite the following fraction in simplest form: $\frac{24}{52}$.
3. Write the following percent as a decimal: 46.1%.
4. There are 8 marbles in a bag. Each marble is either red, yellow, or blue. Four of the marbles are red and one marble is yellow. How many are blue?
5. The side length of a cube is 3 cm. What is the volume?
6. What is the surface area of the same cube?
7. At the store, you compare the price of two hand lotions. The lemon one is \$3.00 for a 60 mL bottle. The vanilla one is \$6.00 for a 100 mL bottle. Which is the better deal (per mL)?
8. You are a very busy person. You have soccer on Sunday, Tuesday, and Thursday nights. You have music lessons on Saturday in the evening. You also have swimming on Monday night. Which nights of the week do you not have any commitments?

Answers:

1. $\frac{6}{13}$
2. $\frac{6}{13}$
3. 0.461
4. 3 blue marbles $(8 - (4 + 1))$
5. 27 cm^3 ($V = Bh$; $B = s \times s$, h (cube) $= s$, so $V = s^3$)
6. 54 cm^2 (For a cube, any side can be the base because all faces are the same. Since there are 6 faces on a cube, TSA = $6B = 6(s \times s)$.)
7. \$3.00 for a 60 mL bottle is the better deal. (3.00 ÷ 60 = \$0.05 per mL, 6.00 ÷ 100 = \$0.06 per mL)
8. Wednesday and Friday

Part B: Surface Area of Prisms and Pyramids

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The famous Louvre Museum in Paris, France has a square glass pyramid above the main entrance. The pyramid is 35.42 m wide and 21.64 m high with a slant height of 27.96 m. Calculate the lateral surface area of the glass.

Answer:

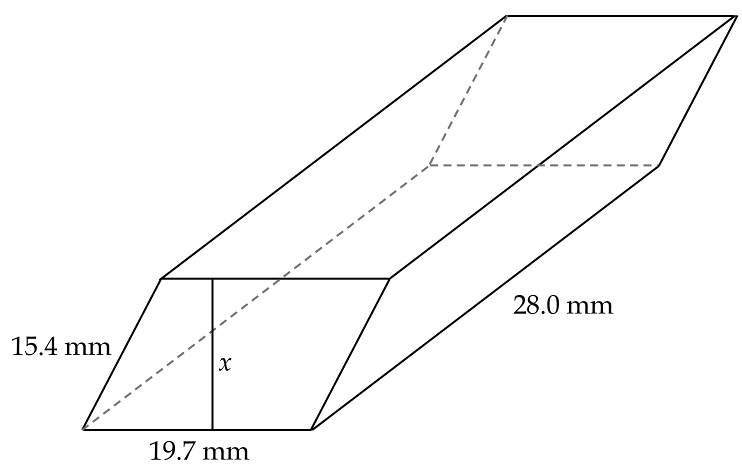
$$\text{LSA} = \frac{1}{2}P\ell$$

$$\text{LSA} = \frac{1}{2}(4 \times 35.42)(27.96)$$

$$\text{LSA} = 1980.6864$$

The lateral surface area of glass if 1980.6864 m^2.

2. The total surface area of this prism, with a parallelogram-shaped base, is 2501.44 mm^2. Determine the height of the parallelogram base.

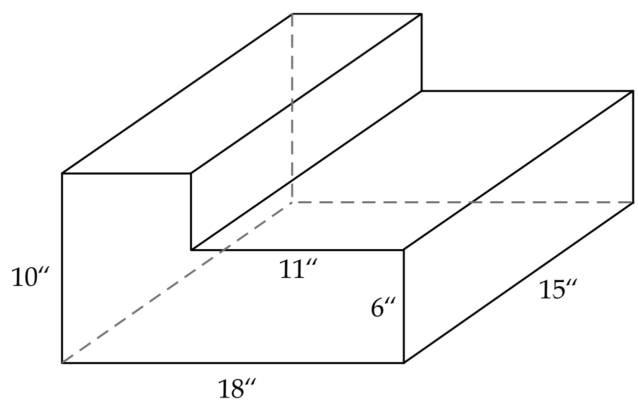


Answer:

$$\begin{aligned}
\text{TSA} &= Ph + 2B \\
2501.44 &= \left[(2 \times 15.4) + (2 \times 19.7)\right](28.0) + 2(19.7)(x) \\
2501.44 &= 1965.6 + 39.4x \\
2501.44 - 1965.6 &= 39.4x \\
\frac{535.84}{39.4} &= \frac{39.4x}{39.4} \\
x &= 13.6
\end{aligned}$$

The height of the parallelogram is 13.6 mm.

3. Find the total surface area of the following 3-D object. State your final answer in scientific notation and also in ft.^{2}.



Answer:

The shape of the L-shaped base is consistent throughout the entire length of the object, so this is a prism. The total surface area of a prism is TSA = $Ph + 2B$.

You need to find the missing dimensions. On the top left of the L-shape, you know it is 18 − 11 or 7″ long. The vertical line on the top is 10 − 6 or 4″ long.

$$\text{TSA} = (10 + 7 + 4 + 11 + 6 + 18)(15) + 2((7 \times 4)+(6 \times 18))$$

$$\text{TSA} = 56 \times 15 + 2 \times 136$$

$$\text{TSA} = 840 + 272$$

$$\text{TSA} = 1112$$

The total surface area is 1112 in.^{2} or 1.112×10^3 square inches.

$$1112 \text{ in.}^2 \times \frac{1 \text{ ft.}^2}{144 \text{ in.}^2} = 7.72 \text{ ft.}^2$$

The total surface area is about 7.72 square feet.

Learning Activity 3.7

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate $\sqrt{9+16}$.
2. Evaluate $\sqrt{25+144}$.
3. The slope of a line is 4. What is the slope of a perpendicular line to this?
4, Terry has a chocolate orange that he wants to eat over one week (Monday to Sunday). He wants to eat the same number of pieces each day. If the orange has 14 pieces, how many pieces will he eat each day?
5. You are out for dinner with your best friend for her birthday. The bill comes and you pay for everything. If the total is \$35.75 and you leave \$40 on the table (including tip), how much are you tipping the server?
6. Is the following angle acute, obtuse, straight, or reflex: 140º?
7. The volume of a cube is 8 m^3. What are the dimensions of the cube?
8. What two numbers have a product of −10 and a sum of −3?

Answers:

1. 5 $\left(\sqrt{9+16}=\sqrt{25}\right)$
2. 13 $\left(\sqrt{25+144}=\sqrt{169}\right)$
3. $-\dfrac{1}{4}$ (Perpendicular slope is the negative reciprocal.)
4. 2 pieces (14 ÷ 7)
5. \$4.25 tip (35.75 + 0.25 = \$36, \$36 + \$4 = \$40, so \$4 + 0.25 = \$4.25)
6. Obtuse (Acute angles are between 0° and 90°; obtuse is between 90° and 180°; straight angle is 180°; reflex angle is between 180° and 360°.)
7. 2 m (All dimensions of a cube are equal, so $\sqrt[3]{8}=2$.)
8. −5, 2 (The factors of 10 are (1, 10) and (2, 5), so for −10, a negative would be in front of one number in the pair. 5 − 2 = 3, −5 + 2 = −3.)

Part B: Surface Area and Volume of Spheres, Cylinders, and Cones

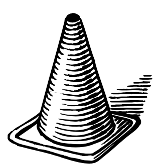
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Describe the relationship between the volumes of a cone and cylinder that have the same radius and height.

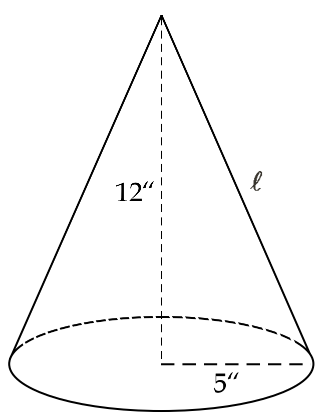
   Answer:

   The volume of a cone that has the same radius and height as a cylinder will be one-third of the cylinder. Alternately, the volume of a cylinder with the same height and radius as a cone will be three times larger than the volume of the cone.

2. You have been asked to repaint the lateral surface area of cone-shaped pylons used for soccer drills. If they have a radius of 5″ and a height of 12″, calculate the slant height and lateral surface area to be painted.



Answer:



$$r^2 + h^2 = \ell^2$$

where r is the radius, h is the height, and ℓ is the slant height

$$5^2 + 12^2 = \ell^2$$

$$25 + 144 = \ell^2$$

$$169 = \ell^2$$

$$\ell = 13$$

$$C = 2\pi r$$

$$C = 2\pi(5)$$

$$C = 31.41592654$$

$$\text{LSA} = \frac{1}{2}C\ell$$

$$\text{LSA} = \frac{1}{2}(31.41592654)(13)$$

$$\text{LSA} = 204.2035225$$

The lateral surface of the cone that needs to be painted is about 204 in.^{2}.

3. Find a sports ball (e.g., a basketball, softball, soccer ball, or tennis ball), and measure its circumference using imperial units. Describe your measurement strategy and use the circumference to determine the ball's radius. Calculate the volume of the ball.

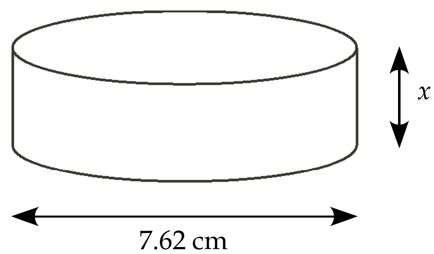
   Answer:

   To determine the circumference of a sports ball, you may have used a flexible measuring tape or wrapped a string around the widest part of the ball and then used a ruler to measure the length of string. You may have used another creative way. Using the formula $C = 2\pi r$ or $r = \frac{C}{2\pi}$, solve for the length of the radius. Substitute that value into the formula $V = \frac{4}{3}\pi r^3$, and solve for the volume of the ball.

   Compare your answer with the possible solutions below:

Type of Ball	Circumference	Radius	Volume
Soccer ball	27.5 inches	4.377″	351.252 in.^{3}
Basketball	29.5 inches	4.695″	433.506 in.^{3}
Baseball	9 inches	1.432″	12.300 in.^{3}
Softball	12 inches	1.910″	29.187 in.^{3}
Tennis ball	8 inches	1.273″	8.641 in.^{3}

4. A hockey puck has a diameter of 7.62 cm and a volume of 115.83 cm^3. Calculate the thickness of a hockey puck.



Answer:

$$d = 2r$$

$$r = \frac{d}{2}$$

$$r = \frac{7.62}{2}$$

$$r = 3.81$$

$$V = Bh$$

$$V = \left(\pi r^2\right)h$$

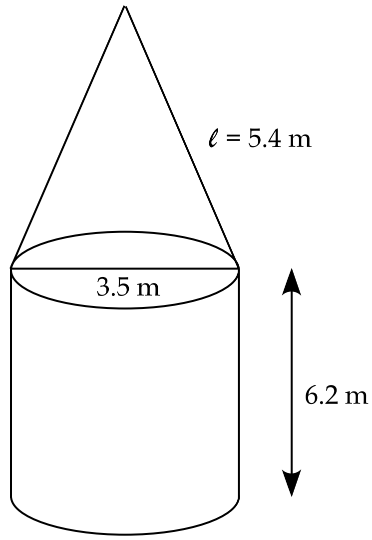
$$115.83 = \pi\left(3.81^2\right)x$$

$$x = \frac{115.83}{\pi \times 14.5161}$$

$$x = 2.539926986$$

The thickness of a hockey puck is 2.54 cm, which is equivalent to 1”. It would make a great referent!

5. The turret on a castle is formed by constructing a cone-shaped roof above a cylindrical structure. Determine the lateral surface area of the following turret. Round your final answer to the nearest hundredth of a metre.



Answer:

This object is composed of a cylinder and a cone. Find the sum of the two lateral surface areas to determine the lateral surface area of this object.

$$\text{LSA} = \text{LSA}_{\text{cone}} + \text{LSA}_{\text{cylinder}}$$

$$\text{LSA} = \frac{1}{2}C\ell + Ch \quad (\text{where } C = \pi d)$$

$$C = \pi(3.5)$$

$$C = 10.99557429$$

$$\text{LSA} = \frac{1}{2}(10.99557429)(5.4) + (10.99557429)(6.2)$$

$$\text{LSA} = 29.68805058 + 68.17256058$$

$$\text{LSA} \approx 97.86$$

The lateral surface area of this turret is about 97.86 m^2.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 4
Trigonometry

Module 4: Trigonometry

Introduction

The word trigonometry comes from two Greek words: *Trigonon* meaning triangle and *Metria* meaning to measure. Trigonometry (or "trig") is the study of the relationships between the side lengths and angle measures in triangles. It is useful when you need to determine lengths and angles that are difficult to measure physically. For example, Greek mathematicians used trig to study the distances between celestial bodies.

This module will introduce you to the three primary trigonometric ratios, named the sine, cosine, and tangent ratios. You will use them to solve for side lengths and angles in triangles. For this module, you will need a protractor and measuring devices such as a metre stick, yard stick, or measuring tape.

Assignments in Module 4

When you have completed the assignments for Module 4, submit your completed assignments for Module 3 and Module 4 to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 4.1	Tangent Ratio
2	Assignment 4.2	Using Sine and Cosine
3	Assignment 4.3	Inverse Trig Ratios
4	Assignment 4.4	Applying Trig Ratios

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 4. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1, 2, 3, and 4 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 4.

Resource Sheet for Module 4

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Writing Your Midterm Examination

You will write the midterm examination when you have completed Module 4 of this course. The midterm examination is based on Modules 1 to 4, and is worth 20 percent of your final mark in the course. To do well on the midterm examination, you should review all the work you complete in Modules 1 to 4, including all the learning activities and assignments. You will write the midterm examination under supervision.

Notes

Lesson 1: The Tangent Ratio

Lesson Focus

In this lesson, you will

- ❑ review the Pythagorean Theorem
- ❑ identify the names of the sides of a right triangle, in relation to a given acute angle
- ❑ discover the primary trigonmetric ratio of tangent and use it to solve triangles

Lesson Introduction

This lesson will review the Pythagorean Theorem and introduce you to the first of the trigonometric ratios—tangent. You will use both to solve for side lengths in right triangles.

Review of Triangles

Types of Triangles

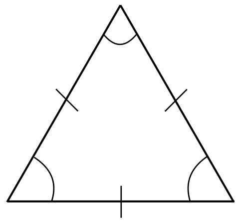
A triangle is a polygon with three sides and three angles. While the sum of the angles in a triangle is always 180°, there are different types of triangles with unique geometric characteristics:

Congruent triangles: Two or more triangles having the same length or measure.

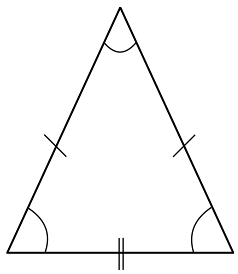
Similar triangles: Two or more triangles having the same shape, but not necessarily the same size.

The following terms will be used throughout this module. It would be helpful to include their definitions on your Resource Sheet.

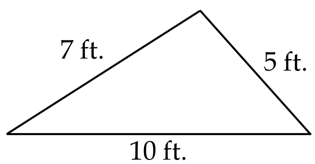
Equilateral triangle: Three congruent sides and three congruent angles.



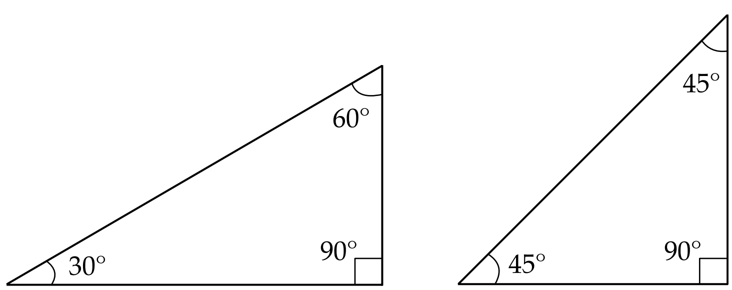
Isosceles triangle: At least two congruent sides and two congruent angles.



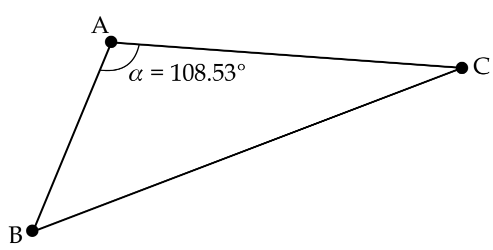
Scalene triangle: No congruent sides and no congruent angles.



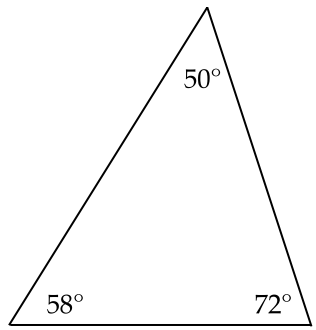
Right triangle: Contains one right angle (measures exactly 90°).



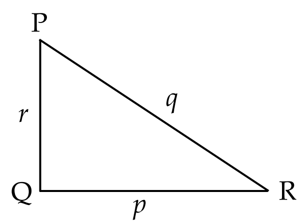
Obtuse triangle: Contains one obtuse angle (measures more than 90° but less than 180°).



Acute triangle: All three angles are acute (measure more than 0° but less than 90°).

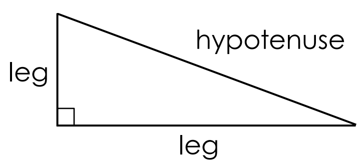


Note: Upper case letters are often used to label the vertices (angles) of a triangle. The same lower case letter is used to label the side opposite the vertex.

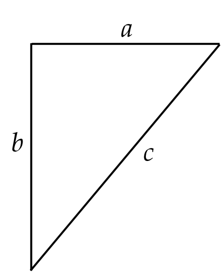


Right Triangles

This module will focus on right triangles. The longest side in a right triangle is always opposite the right angle, and it is called the hypotenuse. To find the hypotenuse, first find the right angle and then go straight across. The two sides that form the right angle are called the legs.

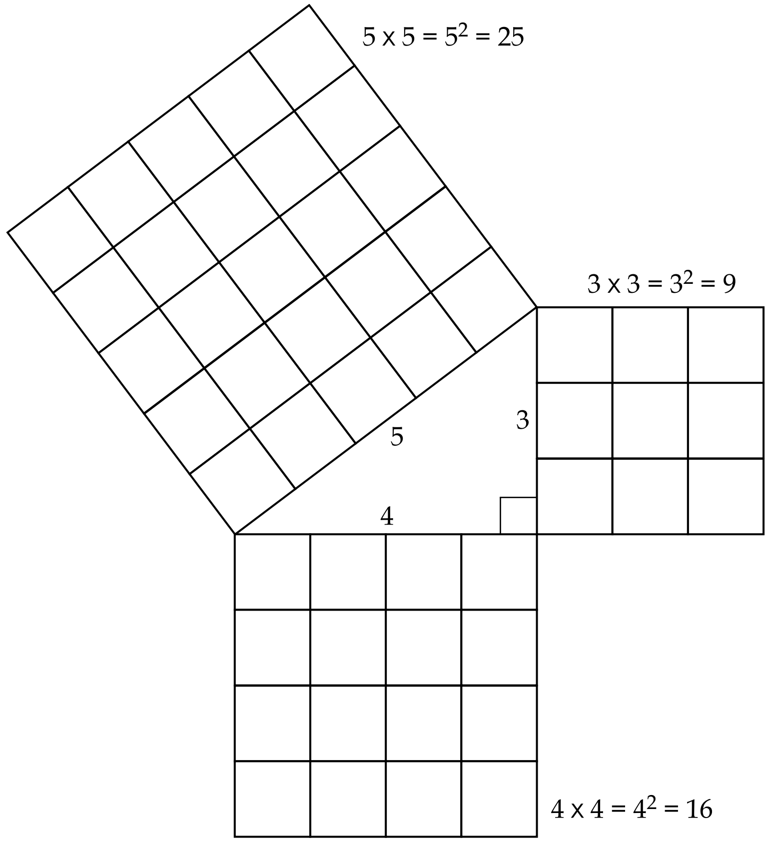


In previous math courses, you studied the Pythagorean Theorem. The Pythagorean Theorem states that in any right triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse. If you label the legs a and b and the hypotenuse c, then $a^2 + b^2 = c^2$.



It would be a good idea to include the Pythagorean Theorem and diagram on your Resource Sheet.

This can be illustrated by drawing a square along each of the sides of the triangle and comparing their areas.

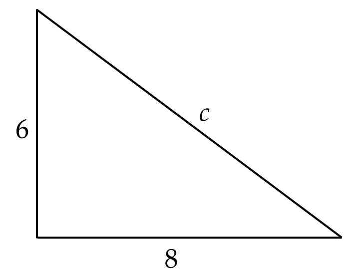


In this triangle a = 3, b = 4 and c = 5.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \\ 25 &= 25 \end{aligned}$$

Example 1

Determine the length of the hypotenuse.



Solution:

$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

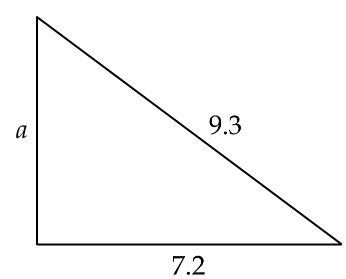
$$c = \sqrt{100}$$

$$c = 10$$

The length of the hypotenuse is 10 units.

Example 2

Determine the length of the missing side.



Solution:

$$a^2 + b^2 = c^2$$

$$a^2 + 7.2^2 = 9.3^2$$

$$a^2 = 9.3^2 - 7.2^2$$

$$a^2 = 86.49 - 51.84$$

$$a^2 = 34.65$$

$$a = \sqrt{34.65} = 5.886425$$

The length of side a is approximately 5.9 units.

Naming the Legs in a Right Triangle

The two legs in a right triangle may be labelled with different names. Unlike the hypotenuse, which is always directly across from the right angle, the legs are named in relation to one of the other two angles, and they may be exchanged, depending on which angle is being referred to.

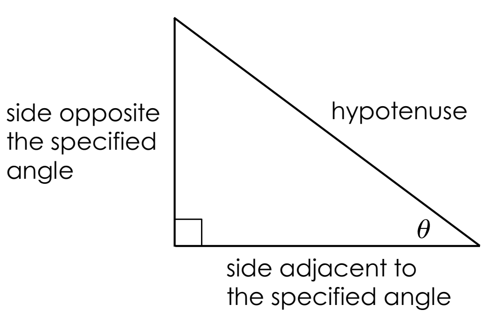
The leg that meets the hypotenuse to create the specified angle is called the adjacent side. The leg that is across from the specified angle is called the opposite side.

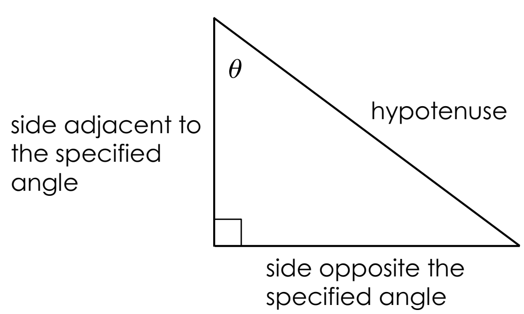
Tricks to remember these names:

- Adjacent means "beside," and the adjacent side is beside the specified angle.
- Opposite is sometimes used to describe objects that are facing (across from) each other. The opposite side is across from the specified angle

The specified angle can be identified by a symbol like θ (called theta), x, or by a given value. The specified angle will never be the right angle.

You may want to include these diagrams on your Resource Sheet.

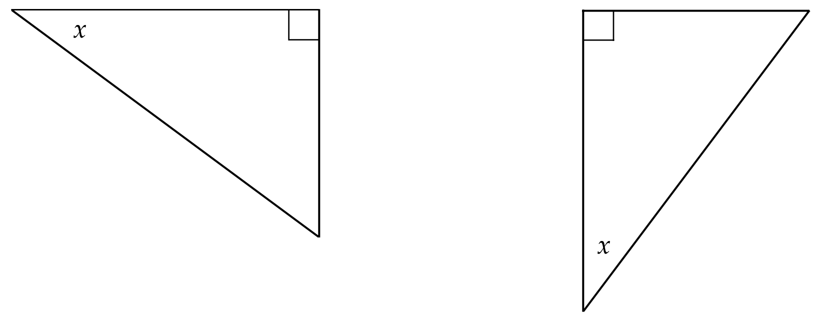




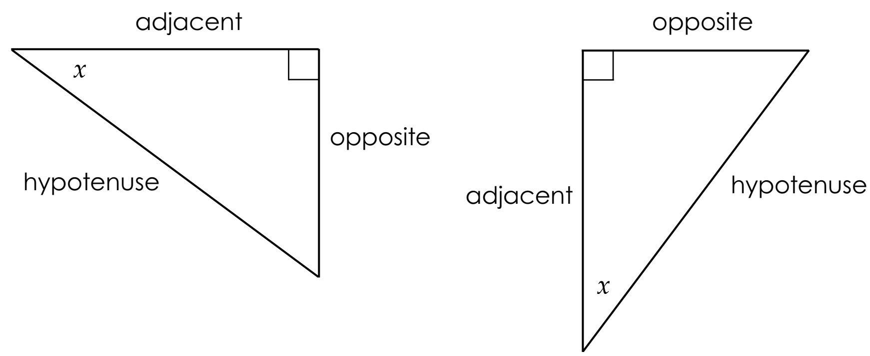
The hypotenuse will always be opposite the right angle, the adjacent side will form the specified angle with the hypotenuse, and the opposite side will be directly across from the specified angle.

Example 3

Label the sides in these right triangles, given the specified angle, x.

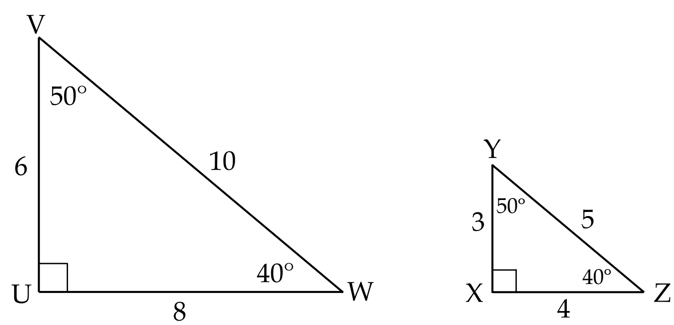


Solution:



Similar Right Triangles

Two triangles are said to be similar if they have the same shape but are a different size. This means similar triangles have equal corresponding angles and proportional corresponding sides.



$\Delta UVW \sim \Delta XYZ$

Triangle UVW is similar to triangle XYZ. The angles are exactly the same in these two triangles, and the side lengths are proportional, with a ratio of 2:1.

Similar right triangles can be used to explore the ratios of side lengths.

Learning Activity 4.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. The circumference of a circle is 32π. What is the radius of the circle?
2. Rewrite the following fraction in simplest terms: $\frac{12}{45}$.
3. What is the average of 3, 4, 6, and 7?
4. Fill in the blanks for the pattern: −43, −38, −33, ____, ____.
5. The equation of a line is $y = 6x - 2$. What is the y-intercept?
6. There are two movies coming out on DVD this week that you would like to purchase. Each movie costs $18.99. If you have $35, can you afford to buy both?
7. Is the number 0 rational or irrational?
8. The volume of a cylinder is 12 cm^3. What is the volume of a cone with the same base and height?

continued

Learning Activity 4.1 (continued)

Part B: Right Triangles and Similar Triangles

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

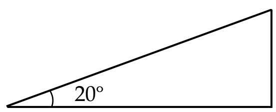
Consider the following similar triangles, each with the measure of a given angle specified.

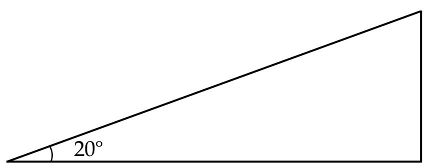
Label the hypotenuse, opposite, and adjacent side in each triangle.

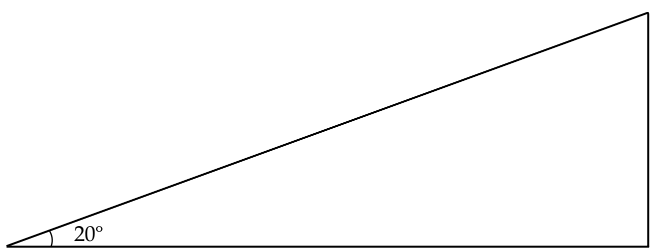
Using a metric ruler, measure the lengths of the opposite and adjacent sides in each, and record your measurements in the chart below to the nearest tenth of a cm.

Write the ratio of the lengths as indicated in the fourth column.

Calculate the value of the ratio to 2 decimal places using a calculator.

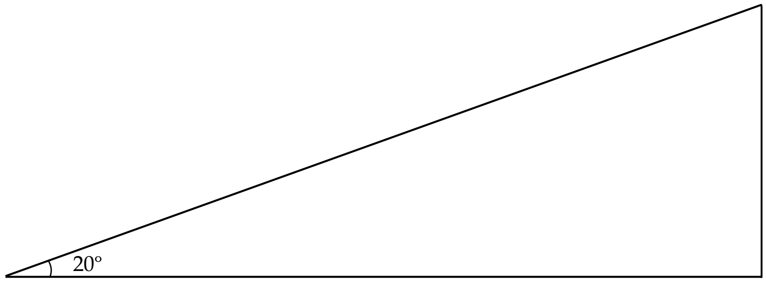






continued

Learning Activity 4.1 (continued)



Triangle	Opposite	Adjacent	$\frac{\text{Opposite}}{\text{Adjacent}}$	Calculated Value of Ratio
1				
2				
3				
4				

The Tangent Ratio

Did you notice that the values of the ratios were almost exactly the same for each triangle?

In relation to a 20° angle, the ratio of the opposite and adjacent side lengths in a right triangle will always be 0.36 to 1 or $\frac{0.36}{1}$.

This ratio of side lengths has a special name. It is called the tangent ratio.

Your calculator must always be "in the DEG" mode, or sometimes "D" depending on the calculator. If it is in the "RAD" or "GRAD" mode, none of your answers will be correct. Always check that your calculator shows D or DEG mode.

Your calculator most likely uses the abbreviation TAN for the tangent ratio. Find the TAN button on your calculator and use it to determine the value of the tangent of 20.

Check your calculator manual for the correct order of keystrokes. You may have to enter

[TAN] [20] [=] or [20] [TAN]

or [TAN] [20] [)] [ENTER]

tan 20° = 0.36397

Write the keystrokes you need to use with your calculator.

This is what you calculated when measuring the ratio of the lengths of the opposite and adjacent sides in the similar right triangles above.

The tangent ratio is $\frac{\text{opposite}}{\text{adjacent}}$ and in any right triangle, the ratio of side lengths will be consistent with the size of the specified angle.

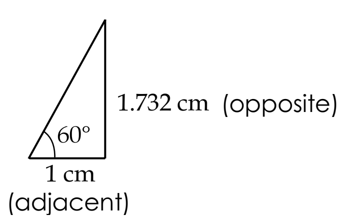
Example 4

Determine the tangent of 60°, and explain what this means using a diagram.

Solution:

Using a calculator, the tangent of 60° is found to be 1.732.

Since the tangent ratio is $\frac{\text{opposite}}{\text{adjacent}}$, tan 60° = 1.732 means that in a right triangle with a 60° angle, the ratio of the side lengths opposite and adjacent to the 60° angle will be approximately $\frac{1.732}{1}$. This can be illustrated with a right triangle as follows:

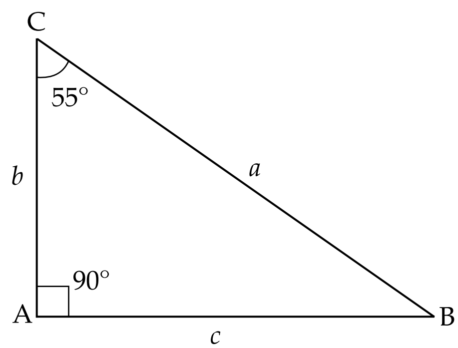


Using the Tangent Ratio to Solve Right Triangles

When using the Pythagorean Theorem, you need to know two side lengths in order to solve for the third side length. With the tangent ratio, you can solve for the length of one leg when you know one angle and the length of the other leg, either opposite or adjacent to that angle.

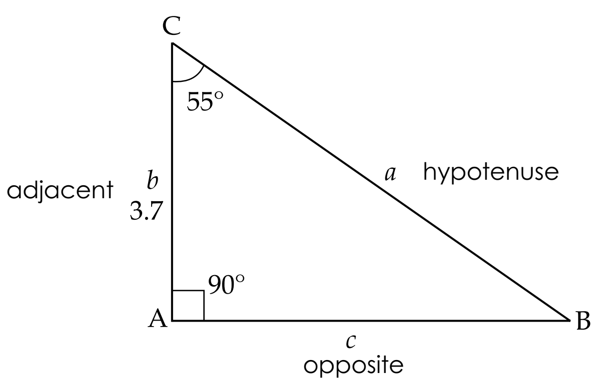
Example 5

Determine the length of side c if b is 3.7 cm.



Solution:

Label the sides of the triangle.



You know the length of the side adjacent to the 55° angle is 3.7 cm and want to find the opposite side length. The tangent ratio is $\frac{\text{opposite}}{\text{adjacent}}$.

$\tan 55° = \dfrac{c}{3.7}$ Step 1: Set up the ratio.

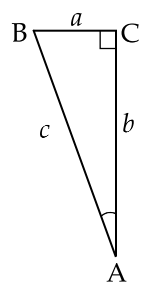
$1.428 = \dfrac{c}{3.7}$ Step 2: Use your calculator to determine tan 55°.

$(3.7)1.428 = \dfrac{c}{3.7}(3.7)$ Step 3: Isolate the variable by eliminating the denominator. Multiply both sides of the equation by 3.7.

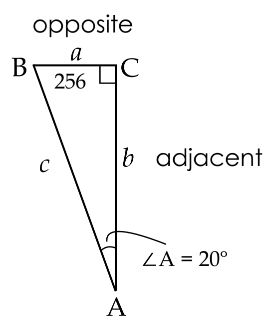
$5.2836 = c$ Step 4: The length of side c is approximately 5.2 cm.

Example 6

Determine the length of side b if $\angle A = 20°$, $\overline{BC} = 256$.



Solution:



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$ Set up the ratio.

$$\tan 20^\circ = \frac{256}{b}$$ Substitute known values. Note that the unknown variable is in the denominator.

$$(b)\tan 20^\circ = \frac{256}{b}(b)$$

$$\frac{\tan 20^\circ b}{\tan 20^\circ} = \frac{256}{\tan 20^\circ}$$ Isolate the variable by multiplying both sides of the equation by b and then dividing both sides by tan 20°.

$$b = \frac{256}{\tan 20^\circ}$$ Notice that you have simply switched the trig function (in this case tan 20°) and the variable, b.

$$b = \frac{256}{0.36397}$$

$$b = 703.35$$ Solve for the unknown variable.

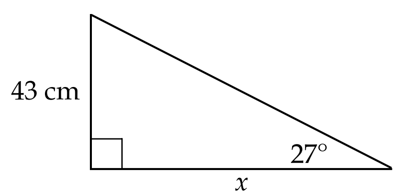
The length of side b is approximately 703.4 units.

Example 7

In a right triangle, the side opposite a 27° angle is 43 cm. Solve the triangle (find the values for all 3 sides and all 3 angles). Include a diagram.

Solution:

To solve a right triangle means to determine all three side lengths and two angle measures (you already know that one angle equals 90°). A diagram of this triangle may look like this:



First solve for the length of the unknown leg.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 27° = \frac{43}{x}$$

$$(x)\tan 27° = \frac{43}{x}(x)$$

$$\frac{(x)\tan 27°}{\tan 27°} = \frac{43}{\tan 27°}$$

$$x = \frac{43}{\tan 27°}$$

$$x = 84.39225174$$

The length of the adjacent side is approximately 84.4 cm long.

If you know the lengths of two sides in a right triangle, you can solve for the third side using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$43^2 + 84.39225174^2 = c^2$$

$$8971.052153 = c^2$$

$$c = \sqrt{8971.052153}$$

$$c = 94.71563838$$

The length of the hypotenuse is approximately 94.7 cm.

If you know the measure of one angle in a right triangle, you can determine the measure of the third angle, as the sum of the angles will always be 180°.

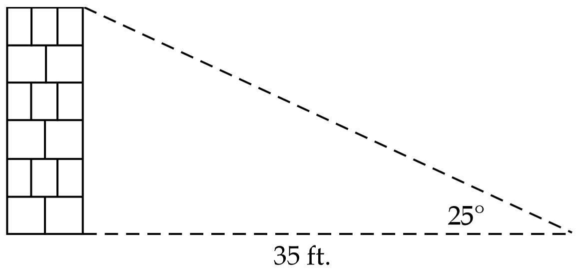
$$180 - 90 - 27 = 63$$

Alternately, since you know that the right angle is 90° and the remaining two angles must add up to 90°, simply calculate 90 − 27 = 63.

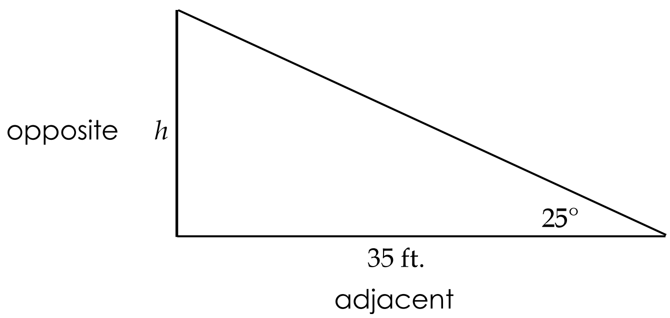
The measure of the third angle is 63°.

Example 8

What is the height of the building in the diagram below?



Solution:



$$\tan 25^\circ = \frac{h}{35}$$

$$h = 35 \tan 25^\circ$$

$$h = 16.3 \text{ ft.}$$

Learning Activity 4.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate: 8^2.
2. Evaluate: $\sqrt{100-64}$.
3. The garden in your backyard has an area of 1 yard^2. Convert this area to feet^2.
4. What is the LCM of 10 and 7?
5. You are standing on the baseline of a basketball court. The top of the key is 25 feet away from you. The distance to centre court is 47 feet. What is the distance from the top of the key to the centre court?
6. GST (Goods and Services Tax) is 5%. You are buying clothes for your baby cousin (baby clothes only have GST). If the total before tax is $44.00, how much tax are you charged?
7. Is this data continuous or discrete?

   The number of red lights you get stopped at compared to the distance you drive in the city.
8. Solve for t: $15t - 5 = 40$.

continued

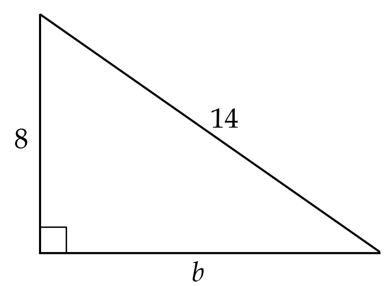
Learning Activity 4.2 (continued)

Part B: Tangent Ratio

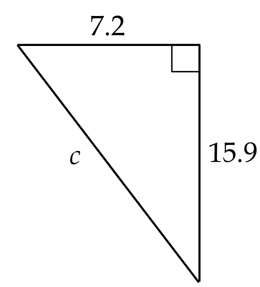
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the Pythagorean Theorem to solve for the missing side lengths in the following triangles.

1. a)



b)



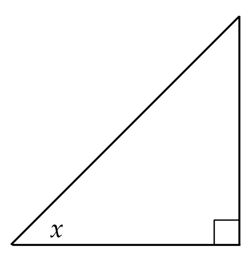
3. Can the lengths listed below be the sides of a right triangle? Explain. (Remember that the hypotenuse is always the longest side of a right triangle.)

   a) 3, 4, 5

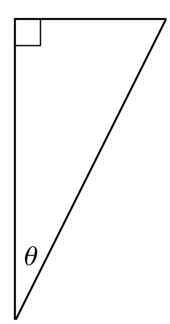
   b) 6, 11, 13

3. Label the sides of the following triangles in relation to the specified angle.

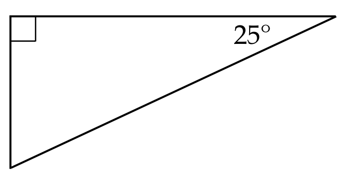
a)



b)



c)

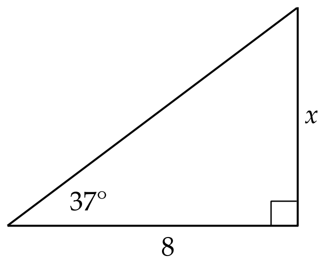


continued

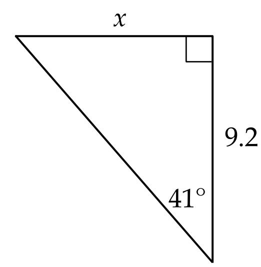
Learning Activity 4.2 (continued)

4. Calculate tan 15° to 3 decimal places, and explain what it means using a diagram.
5. Use the tangent ratio to solve for the missing side length x in each triangle.

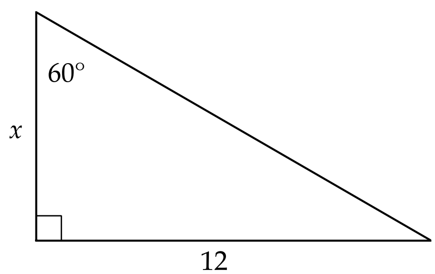
a)



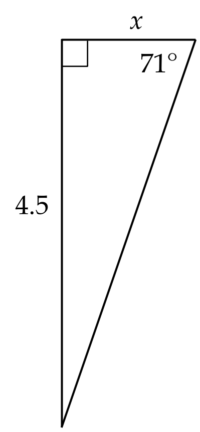
b)



c)



d)



6. The foot of a 13 m ladder is 5 m from the base of a tall building. How far up the building does the ladder reach? Draw a diagram.

Lesson Summary

In this lesson, you reviewed how to solve for an unknown side length in a right triangle, using the Pythagorean Theorem. You learned how to label the hypotenuse and the opposite and adjacent sides in relation to a specified angle. You saw how the ratio of the opposite and adjacent side lengths in similar triangles is defined as the tangent ratio, and how trigonometry can be used to help solve for the sides and angles in right triangles.

In the next lesson, you will explore the sine and cosine ratios.

Notes

Assignment 4.1

Tangent Ratio

Total Marks = 14

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions, formulas, and diagrams on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

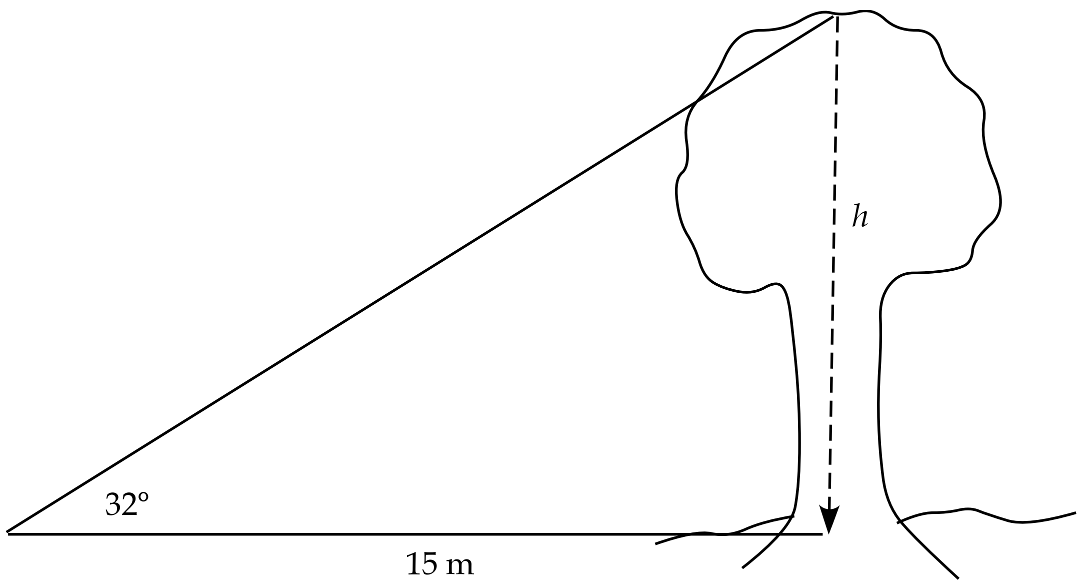
1. The foot of a 1.7 m ladder is 0.8 m from the base of a house. How far up the wall does the ladder reach? Draw a diagram and show your calculations. *(2 marks)*

2. Calculate tan 56° to the nearest tenth, and explain what it means using a diagram. *(3 marks)*

Assignment 4.1: Tangent Ratios (continued)

3. Why is tan 45° = 1? Explain using a diagram. (Hint: You may want to draw this triangle using a protractor and ruler.) *(2 marks)*

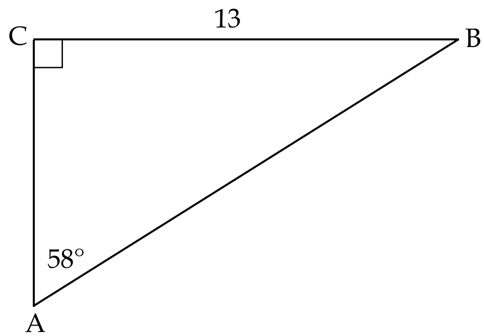
4. What is the height of the tree in the diagram below? *(2 marks)*



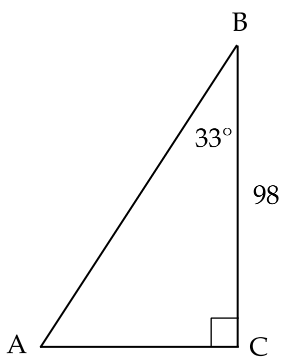
Assignment 4.1: Tangent Ratios (continued)

5. Solve the following triangles.

   a) *(2 marks)*



   b) *(3 marks)*



Notes

Lesson 2: The Sine and Cosine Ratios

Lesson Focus

In this lesson, you will

- ❑ define the sine and cosine ratios and use them to solve right triangles
- ❑ solve problems involving angles of elevation and depression

Lesson Introduction

In Lesson 1, you defined the tangent ratio as the ratio of the opposite and adjacent side lengths in a right triangle. This lesson will identify and use the sine (abbreviated as sin but still pronounced “sine”) and cosine (or cos) ratios to solve triangle problems, including triangles with angles of elevation and depression.

Sine and Cosine Ratios

The sine and cosine ratios are calculated using the length of the hypotenuse and either the opposite or adjacent side of a right triangle. Complete the following learning activity to determine which sides are used for each ratio.

Learning Activity 4.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Is an angle of 86° acute, obtuse, straight, or reflex?
2. Write the following improper fraction as a mixed fraction: $\frac{29}{9}$.
3. You woke up at 8:30 am. You got ready in 45 minutes. You walked to work for 45 minutes. You worked for 4 hours. You ate lunch for 35 minutes. What time is it now?
4. What two numbers have a product of 16 and a sum of 8?
5. What two numbers have a product of -16 and a sum of 0?
6. A ticket to the baseball game is \$12.50. You have to pay for parking at the game, which costs \$5. Once you are in the ballpark, you buy popcorn for \$3.00, ice cream for \$3.15, and a drink for \$2.50. How much did cost you to go to the baseball game?
7. You are going for a walk. You walk north for 5 blocks, turn around, and walk south for 16 blocks. How many blocks are you from where you started? State if you are north or south.
8. Evaluate: $\frac{3}{15} \div \frac{1}{5}$.

continued

Learning Activity 4.3 (continued)

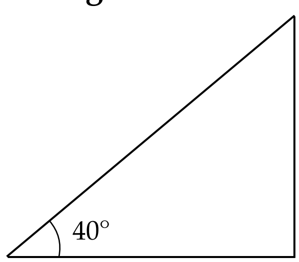
Part B: Exploring Sine and Cosine

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

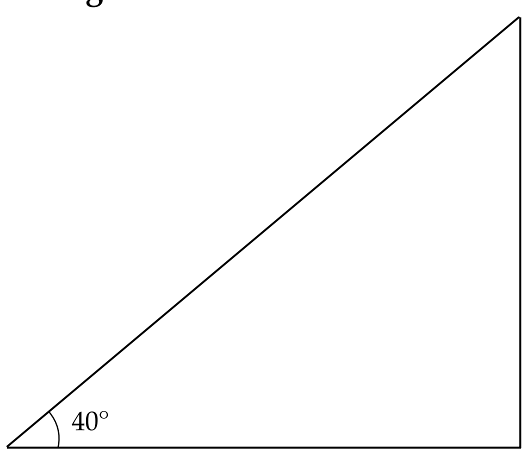
In each of the following similar right triangles, label the sides as hypotenuse, opposite, or adjacent in relation to the specified 40° angle. Measure each side to the nearest tenth of a centimetre, and complete the following chart.

Triangle	Opposite	Adjacent	Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	Value of Ratio	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	Value of Ratio
1							
2							
3							

Triangle 1:



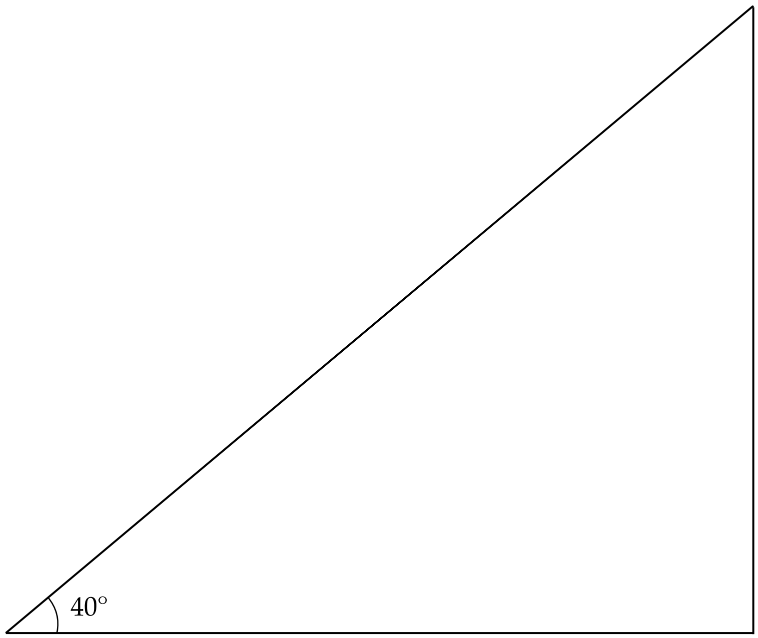
Triangle 2



continued

Learning Activity 4.3 (continued)

Triangle 3



Using your calculator, determine the value of the following:

Sin 40° = ____________________ Cos 40° = ____________________

Based on your measurements and calculations, what can you conclude about the trigonometric ratios of sine and cosine? Write each as the ratio of the appropriate side lengths:

Sine = ______________________ Cosine = ______________________

Therefore, the primary trigonometric ratios of sine and cosine are defined as the following ratios of side lengths:

$$\text{Sine} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{Cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

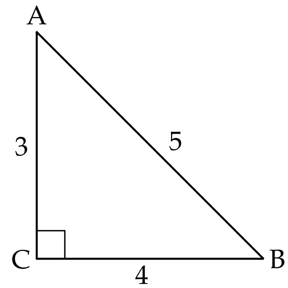
$$\text{Recall that Tangent} = \frac{\text{opposite}}{\text{adjacent}}.$$

It is a good idea to put these formulas on your Resource Sheet.

An easy way to remember these three trig ratios is to use the first letter of each word: $s\frac{o}{h}\ c\frac{a}{h}\ t\frac{o}{a}$ or simply SOH CAH TOA.

Example 1

Find the sine, cosine, and tangent of angle A.



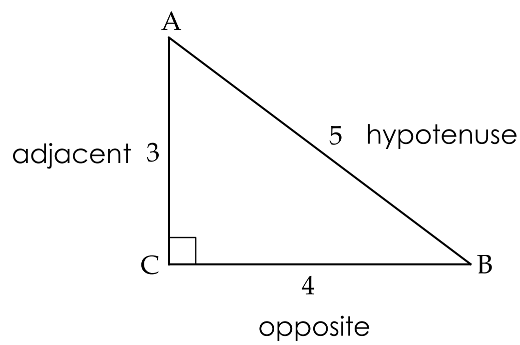
Solution:

Label the sides in relation to angle A.

$$\sin \angle A = \frac{\text{opposite}}{\text{hypotenuse}} \text{ or } \frac{4}{5}$$

$$\cos \angle A = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ or } \frac{3}{5}$$

$$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}} \text{ or } \frac{4}{3}$$



Solving Triangles using the Sine and Cosine Ratios

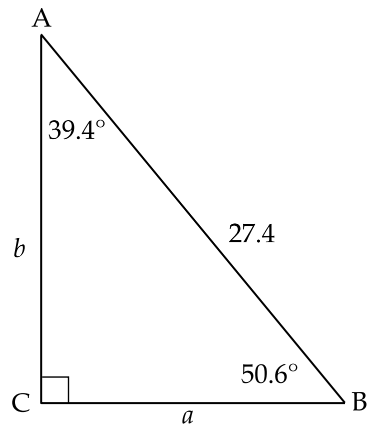
The trigonometric ratios of sine, cosine, and tangent are useful for solving for the side lengths in right triangles. But how do you decide which ratio to use? It all depends on which side length is known and which you want to solve for. Make a sketch or label the diagram with the information you know, label the sides, and then determine which ratio is most appropriate.

Example 2

Given the triangle ABC with ∠C = 90°, c = 27.4, and ∠B = 50.6°, solve the triangle.

Solution:

Begin by making a sketch and labelling the triangle. Calculate the measure of the third angle.



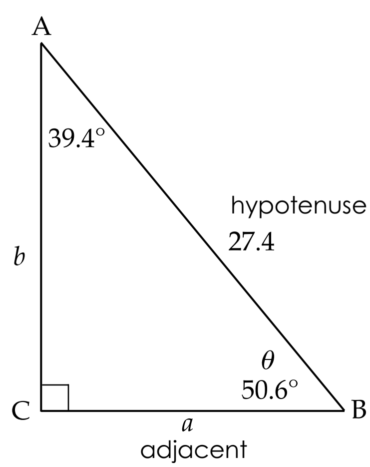
$$90 - 50.6 = 39.4$$
$$\angle A = 39.4°$$

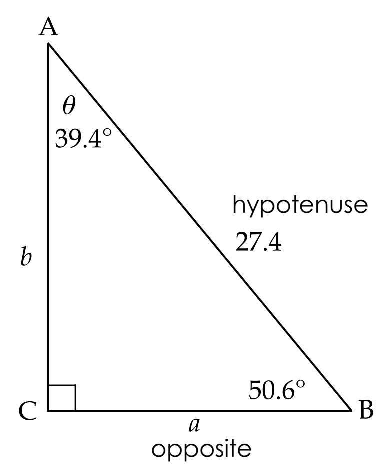
Choose which side length you want to solve for first and which angle you want to use. That will determine which ratio is appropriate.

If you choose to solve for side a using ∠B, then side a is the adjacent side and you must use $\frac{\text{adjacent}}{\text{hypotenuse}}$, the cosine ratio, because the length of the hypotenuse is given.

If you choose to solve for side a using ∠A, then side a is the opposite side and you must use $\frac{\text{opposite}}{\text{hypotenuse}}$, the sine ratio.

Either choice is correct!





Using a, $\angle$B,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 50.6° = \frac{a}{27.4}$$

$$(27.4)\cos 50.6 = \frac{a\cancel{(27.4)}}{\cancel{(27.4)}}$$

$$a = (27.4)(\cos 50.6)$$

$$a = 17.4$$

Using a, $\angle$A,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 39.4 = \frac{a}{27.4}$$

$$a = (\sin 39.4)(27.4)$$

$$a = 17.4$$

Note that both answers are the same.

If you know two sides in a right triangle, you can choose to use a trigonometric ratio to solve for the third side, or use the Pythagorean Theorem.

Using the Pythagorean Theorem to solve for side b:

$$a^2 + b^2 = c^2$$

$$17.4^2 + b^2 = 27.4^2$$

$$b^2 = 27.4^2 - 17.4^2$$

$$b^2 = 488$$

$$b = \sqrt{488}$$

$$b = 21.16601049$$

Using the trigonometric ratios to solve for side b, any of the following would be correct:

Using $\angle$A, b is adjacent, so you may use cosine or tangent.

$$\cos 39.4 = \frac{b}{27.4}$$

$$b = 21.2$$

or

$$\tan 39.4 = \frac{17.4}{b}$$

$$b = 21.2$$

Using ∠B, b is the opposite side, so you may choose to use sine or tangent.

$$\sin 50.6 = \frac{b}{27.4}$$

$$b = 21.2$$

or

$$\tan 50.6 = \frac{b}{17.4}$$

$$b = 21.2$$

You can see that each of the trigonometric ratios results in approximately the same answer.

∠A = 39.4° ∠B = 50.6° ∠C = 90°

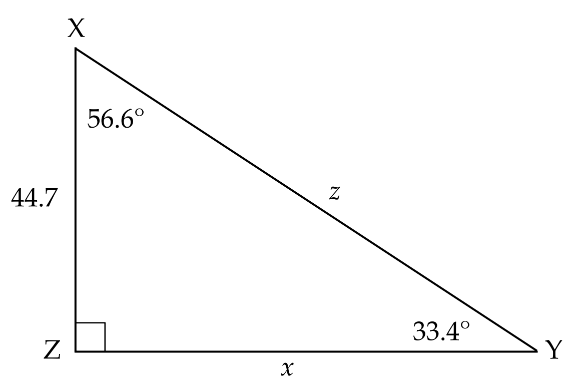
a = 17.4 b = 21.2 c = 27.4

One way to check if this answer is reasonable is to compare the sides and opposite angles. In the list above, the angles are listed in increasing order—angle A is the smallest angle, angle C is the largest angle. Note the shortest side is a, which is opposite the smallest angle. Side c is the largest side and is opposite the largest angle. This should be true for all triangles.

Example 3

Given triangle XYZ with ∠Z = 90°, y = 44.7, ∠X = 56.6°, solve the triangle.

Solution:



90 – 56.6 = 33.4
∠Y = 33.4°

One possible way to begin would be to use ∠X and the cosine ratio to solve for the hypotenuse.

$$\cos X = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 56.6 = \frac{44.7}{z}$$

$$z = \frac{44.7}{\cos 56.6}$$

$$z = 81.2$$

Then you could use the Pythagorean Theorem to solve for x.

$$x^2 + y^2 = z^2$$

$$x^2 + 44.7^2 = 81.2^2$$

$$x^2 = 81.2^2 - 44.7^2$$

$$x^2 = 4595.35$$

$$x = \sqrt{4595.35}$$

$$x = 67.78901091$$

∠X = 56.6°	∠Y = 33.4°	∠Z = 90°
x = 67.8	y = 44.7	z = 81.2

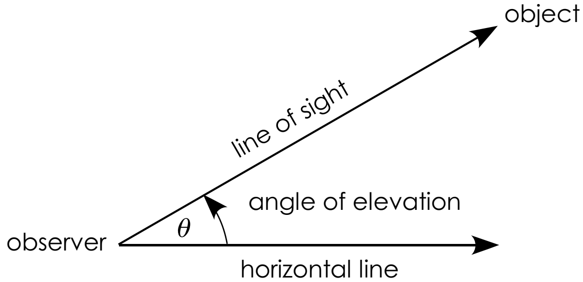
Experiment with the other trigonometric ratios to find different ways to solve this triangle. Notice that the shortest side is opposite the smallest angle and the longest side (the hypotenuse) is opposite the largest angle (the right angle).

Angles of Elevation and Depression

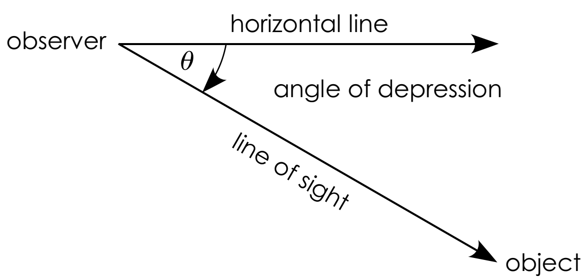
When you are looking around, your line of sight is typically along an imaginary horizontal line.

Observer ————→ horizontal line of sight

When you look up to see an object above you, an **angle of elevation** is created between the horizontal line and your elevated line of sight.

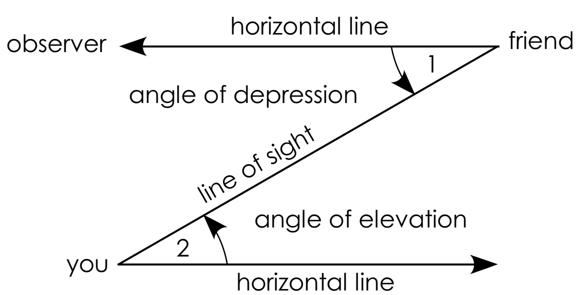


Similarly, if you look down, an **angle of depression** is formed between the horizontal line and your downward line of sight.



It may be handy to have these two definitions on your Resource Sheet.

If you look up at a friend who is looking down at you, your angle of elevation will be the same as her angle of depression, as the horizontal lines are parallel.



$\angle 1 = \angle 2$

These congruent angles are called **alternate interior angles**, as they are on alternate (opposite) sides of a line that cuts diagonally through parallel lines.

Angles of elevation and depression can be used when solving triangle problems. Follow the same steps as in solving other triangle questions:

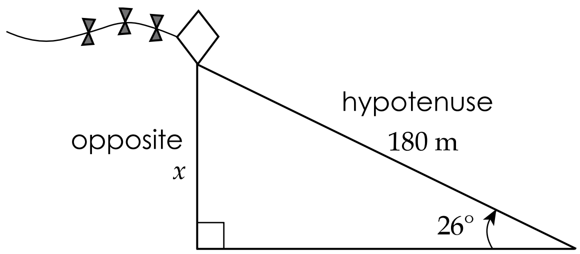
1. Draw a diagram of the situation
2. Label the known and unknown values in the diagram
3. Solve the triangle for unknown values—show your work
4. State an answer for the problem—include units

If you are having a hard time remembering these steps, you can include them on your Resource Sheet.

Example 4

How high is a kite if the string is 180 m long and it makes a 26° angle of elevation with the ground?

Solution:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 26° = \frac{x}{180}$$

$$x = (\sin 26°)(180)$$

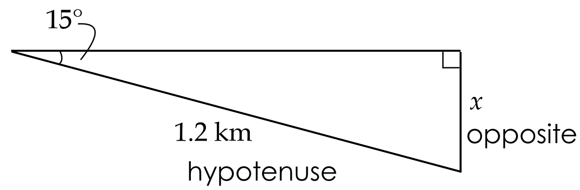
$$x = 78.90680642$$

The kite is approximately 78.9 m above the ground.

Example 5

A mine shaft has an inclined shaft that enters the ground at an angle of depression of 15°. If a miner travels 1.2 km down the incline, how far below the surface of the ground is he?

Solution:



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 15° = \frac{x}{1.2}$$

$$x = \sin 15°(1.2)$$

$$x = 0.3105828541$$

The miner is about 0.31 km or 310 m below the ground surface.

Learning Activity 4.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Solve for g: $3 - g = 15$
2. What two numbers have a product of -27 and a sum of 6?
3. Your mom is buying ice cream for your family. The store has tiger, bubblegum, vanilla, and chocolate flavours. Your mom doesn't like bubblegum, your dad doesn't like chocolate, and you don't like vanilla. Which ice cream will your mom buy?
4. What is the formula for tangent?
5. Arrange the numbers from largest to smallest: $\frac{1}{2}$, 0.29, $\frac{3}{4}$, 0.65, 0.34.
6. Evaluate: $\sqrt[3]{125}$.
7. An octave in music includes 8 notes. If you were to go up half an octave, how many notes is that?
8. When in Venice, you notice a great store on the other side of the street. Because the roads are water in Venice, you need to walk to the nearest bridge. The nearest bridge is 6 m away from you, and the 'road' is 2 m wide. How far do you have to walk to get to the store?

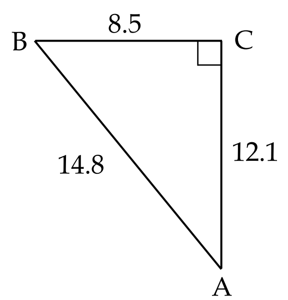
continued

Learning Activity 4.4 (continued)

Part B: Applying Sine and Cosine

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

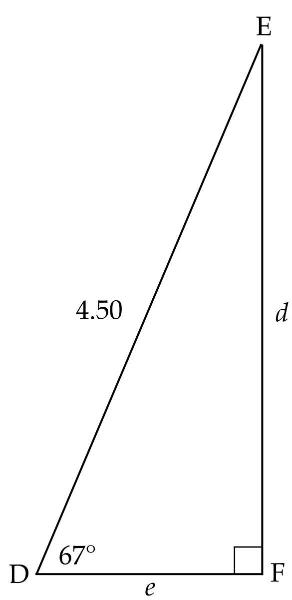
1. Write the ratio for the sine, cosine, and tangent of angle A.



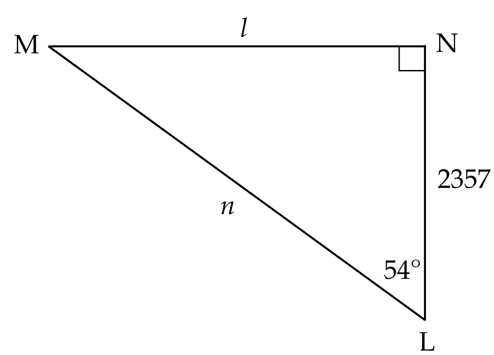
2. Explain what cos 35° means. Include a diagram.

3. Solve the following triangles. Round your answers to the nearest tenth.

a)



b)



4. A right triangle has angles of 36° and 54°. Find the length of the shortest leg if the hypotenuse is 44 cm.

5. A brace cable supporting a streetlight pole is attached at the top of the pole, 35 feet above the ground. How long must the cable be if it makes a 68° angle with the ground?

Lesson Summary

In this lesson, you used the trigonometric ratios of sine and cosine to solve right triangle problems. Some questions involved the angles of elevation and depression. In the next lesson, you will use the inverse trigonometric ratios of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ to solve for unknown angles in right triangles.

Notes

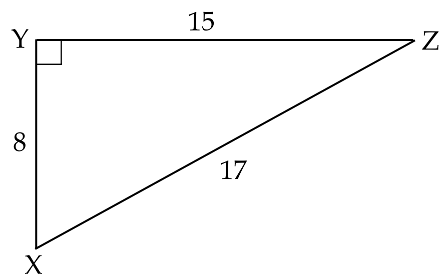
Assignment 4.2

Using Sine, Cosine, and Tangent

Total Marks = 20

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions, formulas, and diagrams on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Write the ratio for the sine, cosine, and tangent of angle Z. *(3 marks)*

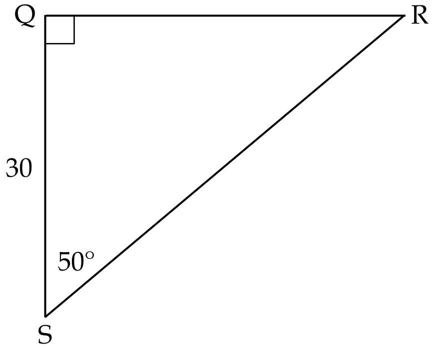


2. Explain what sin 71° means. *(2 marks)*

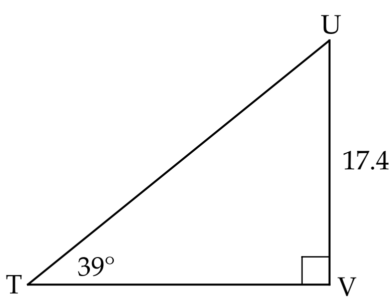
Assignment 4.2: Using Sine, Cosine, and Tangent (continued)

3. Solve the following triangles.

   a) *(4 marks)*



   b) *(4 marks)*



Assignment 4.2: Using Sine, Cosine, and Tangent (continued)

4. Find the length of the longest leg in a right triangle with angles of 68°, 22°, and a hypotenuse of 5.9 m. *(2 marks)*

5. Create a unique word problem that involves an angle of elevation or an angle of depression, and requires you to use sine, cosine, or tangent to solve for an unknown side length. Provide a diagram and solution. *(5 marks)*

Notes

Lesson 3: Solving for Angles

Lesson Focus

In this lesson, you will

- ❑ use the inverse trigonometric ratios of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ to solve for angle measures in right triangles

Lesson Introduction

Since the sum of the measures of the angles in a triangle is 180°, and a right angle is 90°, when you know one angle in a right triangle, you can find the measure of the third angle by subtracting from 90°. But what happens if you don't know the measure of either angle in a right triangle? Another application of the three primary trigonometric ratios is to use them to solve for angles in right triangles, using the inverse ratios of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

What About When You Don't Know the Angle?

The Inverse Trigonometric Ratios

So far, you have used the trigonometric ratios of sine, cosine, and tangent to solve for side lengths in right triangles when an angle is given. The opposite can also be done. If you know the ratio of side lengths, you can calculate the measure of the angle using the "inverse trigonometric" ratios.

The inverse trigonometric ratios of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ are typically found on the same calculator buttons as sin, cos, and tan, but often can only be accessed by first pressing the 2nd, shift, or inverse (INV) button.

Example 1

Find the measure of ∠A if sin A = 0.6691.

Solution:

sin A = 0.6691 means that the ratio of the opposite and hypotenuse lengths is $\frac{0.6691}{1}$. Only one angle measure between 0° and 90° will result in this value.

To determine the measure of the angle, calculate $\sin^{-1}(0.6691)$. Refer to the calculator's manual if you are unsure of the keystrokes required. You may have to input

0.6691 then INV and SIN or 2nd SIN 0.6691) ENTER or = .

Write the keystrokes your calculator uses to find the answer. Don't forget to make sure your calculator is in the degree mode.

If sin A = 0.6691 then ∠A = $\sin^{-1}(0.6691)$ = 41.99764032.

The angle that results in the opposite and hypotenuse lengths having a ratio of 0.6691 is approximately 42°.

You may want to record the keystrokes on your Resource Sheet.

Example 2

Find the measure of ∠X if tan X = 3.2.

Solution:

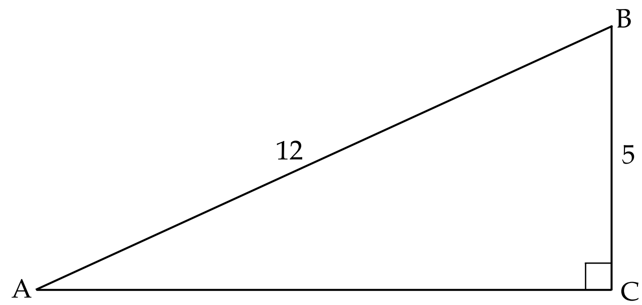
tan X = 3.2

∠X = $\tan^{-1}$ (3.2)

∠X = 72.6°

Example 3

Find the measure of $\angle B$ in the diagram shown below.



Solution:

In relation to the specified angle, you know the lengths of the hypotenuse and adjacent sides. Use the inverse cosine ratio to solve for angle B.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{5}{12}$$

$$\cos B = 0.41\overline{6}$$

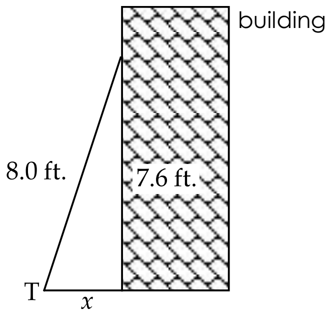
$$\angle B = \cos^{-1}\left(0.41\overline{6}\right)$$

$$\angle B = 65.4^\circ$$

Example 4

An 8-foot ladder leaning against a building reaches 7.6 feet up the side of the wall. Find the measure of the angle the ladder makes with the ground. How far is the foot of the ladder from the building?

Solution:



$$\sin T = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin T = \frac{7.6}{8.0}$$

$$\sin T = 0.95$$

$$\angle T = \sin^{-1}(0.95)$$

$$\angle T = 71.80512766^\circ$$

The ladder makes a 71.8° angle with the ground.

$$x^2 + y^2 = z^2$$

$$x^2 + 7.6^2 = 8.0^2$$

$$x^2 = 64 - 57.76$$

$$x^2 = 6.24$$

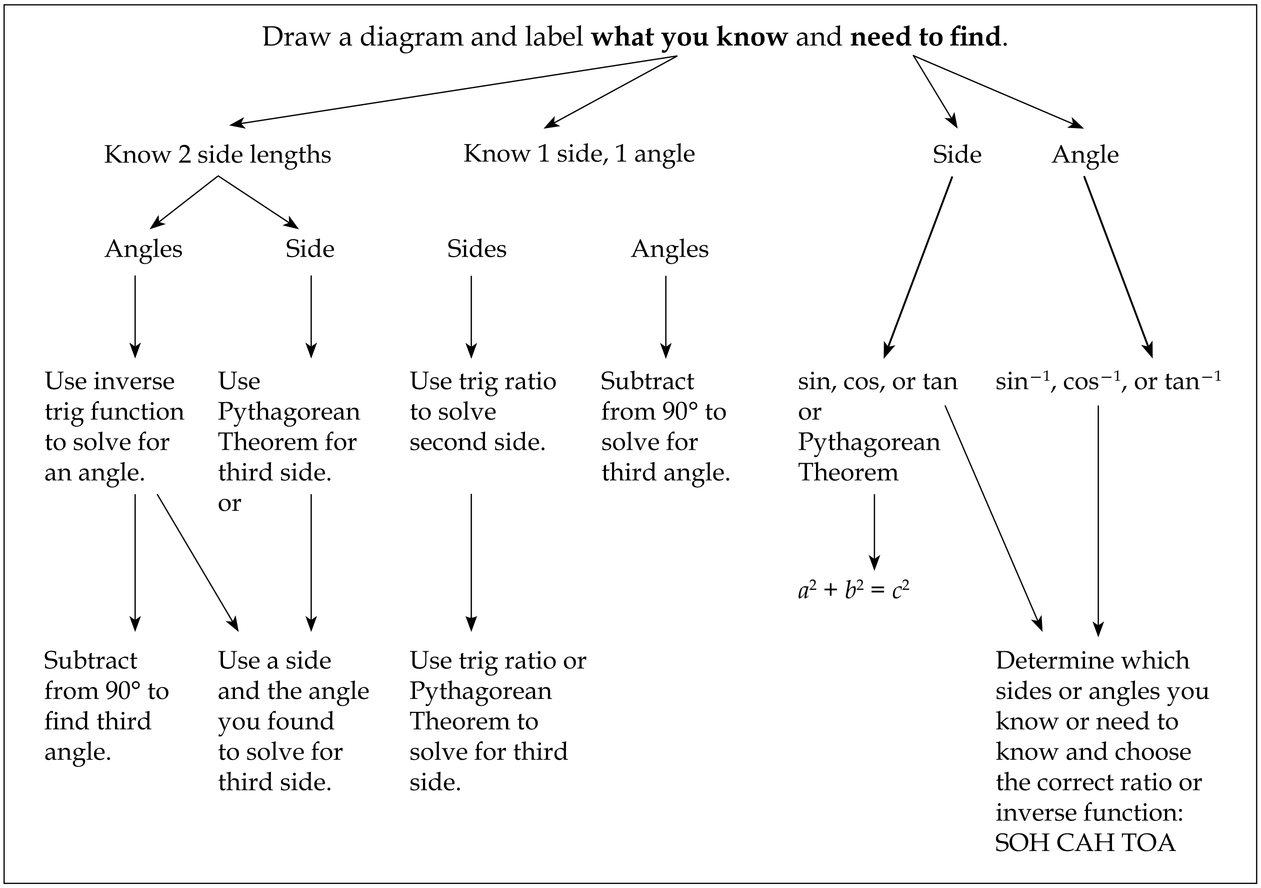
$$x = \sqrt{6.24}$$

$$x = 2.497999199$$

The foot of the ladder is about 2.5 feet from the building.

Strategy Map

Sometimes it can be hard to know where to start and which trigonometric ratio to use when solving right triangles. Here are some hints to get you started!



This diagram is very helpful for solving a triangle. It may be useful to have it (at least in point form) on your Resource Sheet.

Learning Activity 4.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. In the novel trilogy *Lord of the Rings*, there are many rings of power. Nine were given to men, seven to the dwarves, three to the elves, and one to the evil mastermind. In total, how many rings of power are there?
2. Evaluate: $x^{\frac{-1}{6}}$.
3. What would be a good range for a graph comparing a woman's age to her weight?
4. A checkerboard has 8 squares along its length and 8 squares along its width. How many squares altogether are on the board?
5. The squares on a checkerboard alternate between black and white. How many squares are black?
6. Solve for h: $h + 12 = 32$.
7. What is the formula for cosine?
8. What two numbers have a product of 21 and a sum of 22?

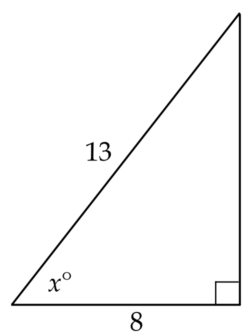
continued

Learning Activity 4.5 (continued)

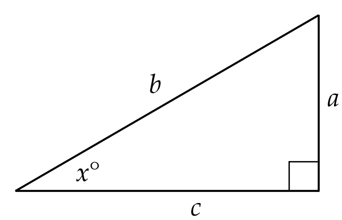
Part B: Inverse Trig Ratios

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the measure of the angle whose trigonometric ratio is given below. Round your answer to the nearest tenth of a degree.
   a) cos B = 0.4556
   b) sin Y = 0.5
   c) tan P = 6.78
   d) cos T = 0.0013
2. Find x to the nearest degree.



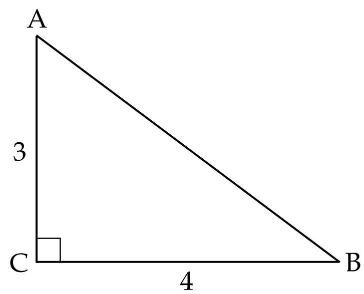
3. Find x to the nearest degree if:
   a) $a = 5$ and $b = 10$
   b) $b = 20$ and $c = 4$
   c) $a = 4$ and $c = 2$



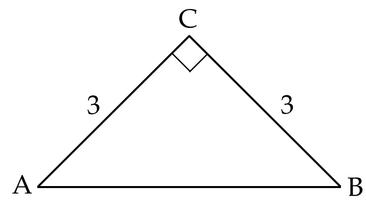
continued

4. You are given 3 measures in each of the following triangles. Find the value of the three missing measure in each triangle. Round your answer to 2 decimal places.

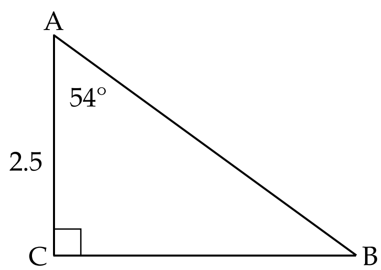
a)



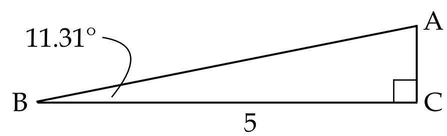
b)



c)



d)



5. Find the measure of the angle between the diagonal and the shorter side of a rectangle that is 20 cm long and 8 cm wide.

Lesson Summary

In this lesson, you learned to drive in "trigonometric reverse"—you used the inverse trigonometric ratios to find the angle associated with the given ratio of side lengths. Now you have the skills needed to solve right triangles. You will apply these skills to problem-solving situations in the next lesson.

Assignment 4.3

Inverse Trig Ratios

Total Marks = 33

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions, formulas, and diagrams on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Find the measure of the angle whose trigonometric ratio is given below. Round your answer to the nearest degree. *(4 marks)*

   a) sin Q = 0.9848

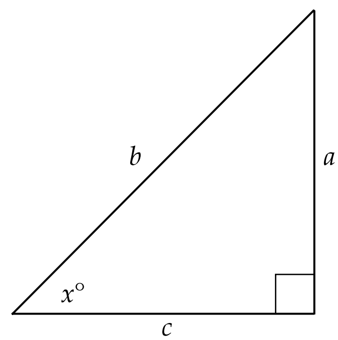
   b) tan H = 2.29

   c) cos X = 0.9354

   d) sin K = 0.1650

Assignment 4.3: Inverse Trig Ratios (continued)

2. Find x to the nearest degree if:



a) $a = 7$ and $b = 10$ *(2 marks)*

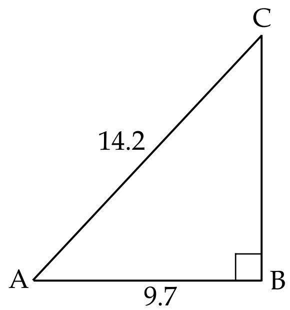
b) $a = 10$ and $c = 1\ 0$ *(2 marks)*

c) $b = 12$ and $c = 6$ *(2 marks)*

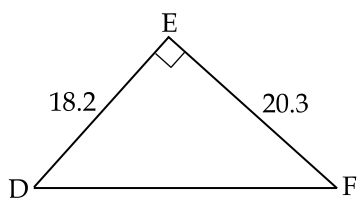
Assignment 4.3: Inverse Trig Ratios (continued)

3. Solve the following triangles. Show all your work. Organize your answers neatly.

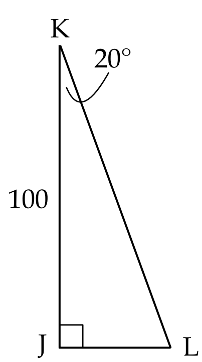
   a) *(5 marks)*



   b) *(5 marks)*



   c) *(5 marks)*



Assignment 4.3: Inverse Trig Ratios (continued)

4. Find the measure of the angle formed by a diagonal and the longer side of a rectangle that is 12 m long and 9 m wide. Draw a diagram. *(3 marks)*

5. A friend has missed class and needs an explanation of how to solve for angles in a right triangle. In your own words, explain how to do this and what the inverse trigonometric ratios mean. *(5 marks)*

Lesson 4: Solving Right Triangles

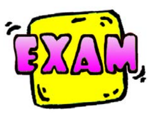
Lesson Focus

In this lesson, you will

- ❑ solve one and two-right triangle questions using the trigonometric ratios, inverse trigonometric ratios, and the Pythagorean Theorem

Lesson Introduction

This lesson will provide lots of opportunities for you to practice your triangle-solving skills, and to apply them to unique situations. You will solve a variety of triangle questions, including ones that require the use of measurement instruments like a metre stick or a clinometer (used to measure angle of elevation).



Now is the time to sign up to write your midterm exam. You should register for your exam before you complete the last assignment of this module.

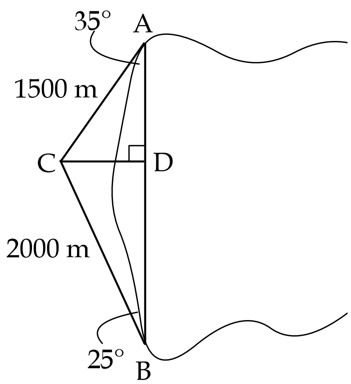
Solving Right Triangle Problems

Two-Triangle Questions

Occasionally, you will need to create a diagram consisting of two right triangles in order to solve a problem.

Example 1

From two points A and B on the opposite sides of a bay, the distance to a point C were measured and found to be 1500 m and 2000 m, respectively.



If $\angle A = 35°$ and $\angle B = 25°$, find the distance AB across the bay.

Solution:

Draw a perpendicular line from C to BA. Label the intersection point D.

$$\cos 35° = \frac{AD}{1500} \qquad \cos 25° = \frac{BD}{2000}$$

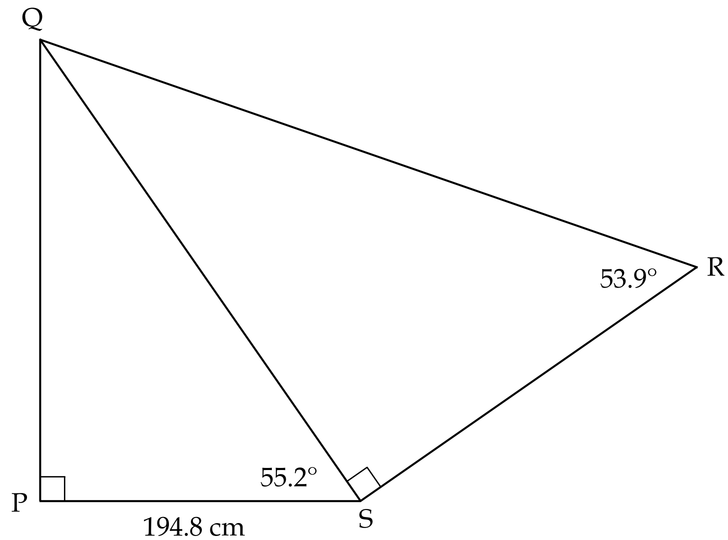
$$AD = 1228.7 \text{ m} \qquad BD = 1812.6 \text{ m}$$

The distance AB across the bay = 1228.7 + 1812.6

= 3041.3 m.

Example 2

Find the length of QR, given the following diagram.



Solution:

There are no known side lengths in ΔQRS, but side QS is part of both ΔQRS and ΔQSP. Find the length of side QS and use it to solve for the length of QR.

$$\cos 55.2° = \frac{194.8}{QS}$$

$$QS = \frac{194.8}{\cos 55.2°}$$

$$QS = 341.3270878$$

Note: Do not round values that will be used in further calculations. Round final answers only.

$$\sin 53.9° = \frac{341.3270878}{QR}$$

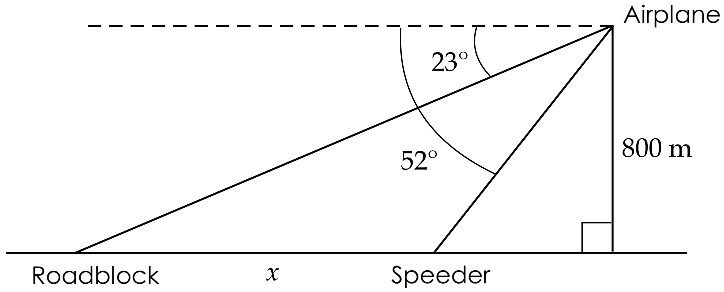
$$QR = \frac{341.3270878}{\sin 53.9°}$$

$$QR = 422.4398035$$

The length of QR is approximately 422.4 cm.

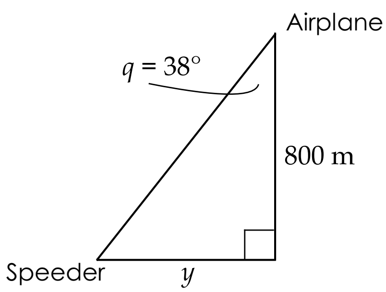
Example 3

A police airplane, flying at an altitude of 800 m, spots a speeding vehicle at an angle of depression of 52°. If a road block is set up along the same highway at an angle of depression of 23°, find the distance the vehicle is from the road block (to the nearest hundredth of a kilometre).



Solution:

The horizontal distance from the airplane to the speeder can be found using a right-angled triangle, as follows:



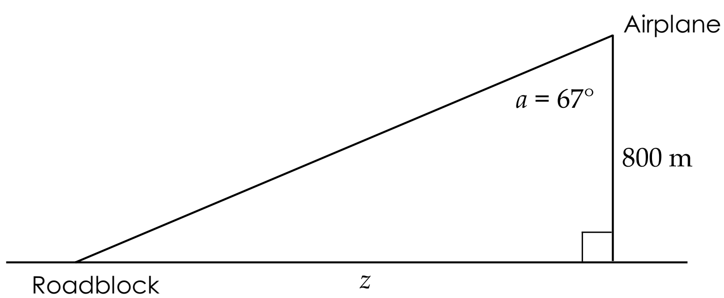
$$\tan 38° = \frac{y}{800}$$

$$y = 800 \tan 38°$$

$$y = 625.0 \text{ m}$$

Note: $q = 90 - 52$
$q = 38°$

Then the horizontal distance from the airplane to the roadblock can be found using this right-angled triangle:



$$\tan 67° = \frac{z}{800}$$

$$z = 800 \tan 67°$$

$$z = 1884.7 \text{ m}$$

Note: $a = 90 - 23$
$a = 67°$

The distance x can then be found:

$$x = z - y$$

$$x = 1884.7 - 625.0$$

$$x = 1259.7 \text{ m}$$

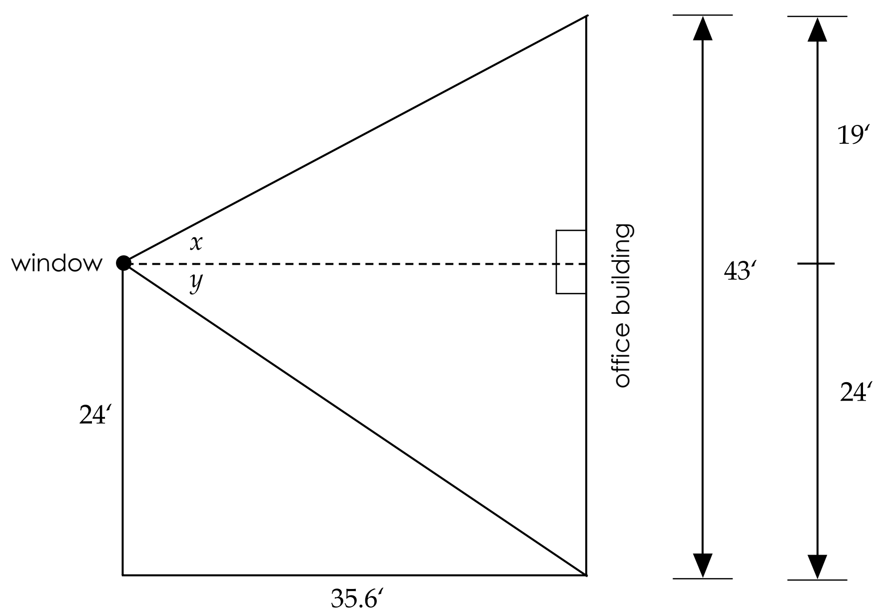
$$x = 1.26 \text{ km}$$

Therefore, the distance between the vehicle and the roadblock is 1.26 km.

Example 4

You are looking out of your apartment window 24 feet above the ground at an office building 35.6 feet across the street. The height of the office building is 43 feet. Find the angle of elevation to the top of the building and the angle of depression to the bottom of the building.

Solution:



angle of depression

$$\tan Y = \frac{24}{35.6}$$

$$\angle Y = \tan^{-1}\left(\frac{24}{35.6}\right)$$

$$\angle Y = 34°$$

angle of elevation

$$\tan X = \frac{19}{35.6}$$

$$\angle X = \tan^{-1}\left(\frac{19}{35.6}\right)$$

$$\angle X = 28°$$

Learning Activity 4.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. The surface area of a globe is 100π cm^2. What is the radius of the globe?
2. If the ratio comparing the size of the globe to the Earth is 1 cm : 800 miles, what is the approximate radius of the Earth (use the answer from above)?
3. If you buy a shirt for $8 and jeans for $32, how much do you spend altogether?
4. If 20% of your class of 20 students has already done the social studies project, how many people are done their project?
5. What is the formula for sine?
6. The volume of a pyramid is 5 m^3. What is the volume of a prism with the same base and height?
7. Is the following data continuous or discrete?

   The amount of money in your bank account over a period of time.
8. Is the number 1 prime, composite, or neither?

continued

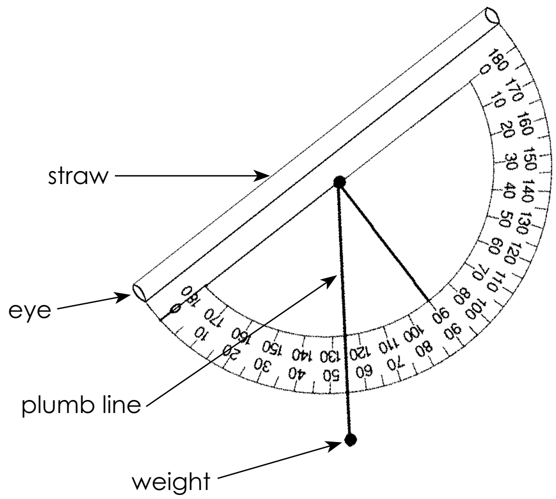
Part B: Applying Trig Ratios

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your assignments and exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. In ΔABC, $\angle$A = 90°, a = 15 m, and c = 10 m. Find the measures of the other two angles.
2. In ΔPQR, $\angle$R = 90°, $\angle$Q = 40°, and p = 30. Find the lengths of the other two sides.
3. A hot-air balloon, advertising for a real estate company, is 250 m above ground level. The angle of depression from the balloon to the landing area is 30°. What is the distance along the ground from a point beneath the balloon to the landing area?
4. A surveyor uses a transit (a device used to measure angles) to determine that the angle of elevation from the transit to the top of a building is 34°. The horizontal distance from the top of the transit to the building is 34 m, and the height of the transit is 1.9 m. How high is the building?
5. A tree 2.50 m tall casts a shadow 4.36 m long. Calculate the angle of elevation from the ground to the sun to the nearest degree.
6. From the top of a 100 m tall fire tower, a fire ranger observes two fires: one at an angle of depression of 5°, and the other at an angle of depression of 2°. Assuming that the fires and the tower are in a straight line, how far apart are the fires if they are
   a) on the same side of the tower?
   b) on opposite sides of the tower?
7. From a point 133 m away from the centre of the base of the Manitoba Legislative Building, the angle of elevation to the top of the torch of the Golden Boy is 30°. From the same point, the angle of elevation to his feet is 24°. Find the height of the Golden Boy.

continued

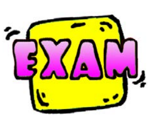
8. A clinometer is a device used to measure angles of elevation or depression. You can construct a clinometer by taping a sighting straw to the straight edge of a plastic protractor. Create a plumb line from string or fish line with a small weight at one end (e.g., a small metal nut or a paper clip). Attach the string so that it swings freely from the centre of the horizontal zero line along the straight edge (see diagram). Holding the straw horizontal, the plumb line should pass over the 90° mark.



Using your clinometer, an imperial measure device (such as a tape measure or yardstick) and trigonometry, show by the use of a sketch how you would verify that a basketball hoop is 10 feet off the floor of a gymnasium.

Lesson Summary

This lesson pulled together the concepts you learned in Lessons 1, 2, and 3. You practised solving triangles using angles of elevation and depression, as well as two-triangle diagrams. You were given an opportunity to apply indirect measurement using a clinometer, and measuring tools to find the height of tall objects. If you have any questions, please make sure to contact your tutor/marker to clarify these concepts before your exam.



Be sure that you have made arrangements to write your midterm examination before you complete the last assignment of this module.

Assignment 4.4

Applying Trig Ratios

Total Marks = 30

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions, formulas, and diagrams on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

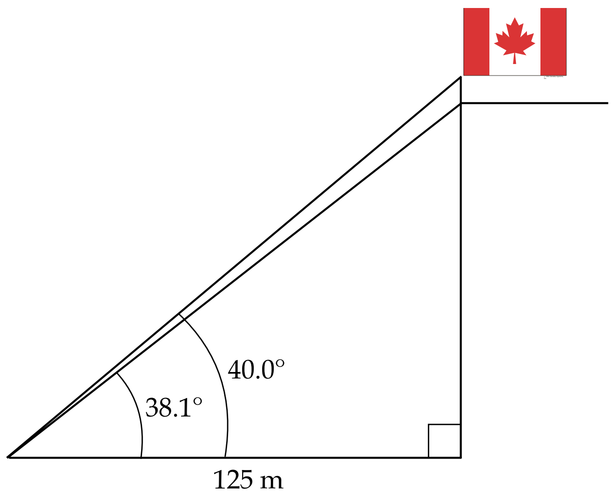
1. The side lengths in a triangle are 7, 9, and 12 cm. Use the Pythagorean Theorem to verify whether this is a right triangle. Show your work. *(2 marks)*

2. A skier travels 600 m down the ski run to reach the bottom. If the vertical drop is 165 m, find the angle of elevation of the ski slope to the nearest degree. Include a labelled diagram. *(3 marks)*

Assignment 4.4: Applying Trig Ratios (continued)

3. You are standing 45 m from the Gateway Arch in St. Louis, Missouri. The angle of elevation to the top of the arch is 78°. What is the height of the arch to the nearest tenth of a metre? *(2 marks)*

4. From a point 125 m from the foot of a building, the angles of elevation of the top and bottom of the flagpole are 40.0° and 38.1° respectively. The flagpole is set on the roof of the building.

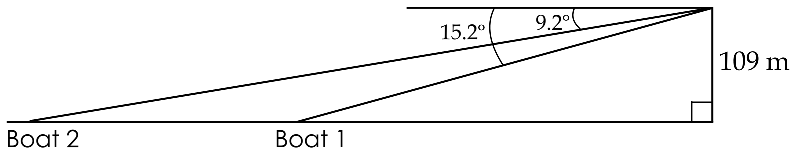


Calculate

a) the height of the building. *(2 marks)*

b) the height of the flagpole. *(2 marks)*

5. From the top of a cliff 109 m high, the angles of depression of two small boats on the water are 9.2° and 15.2°.



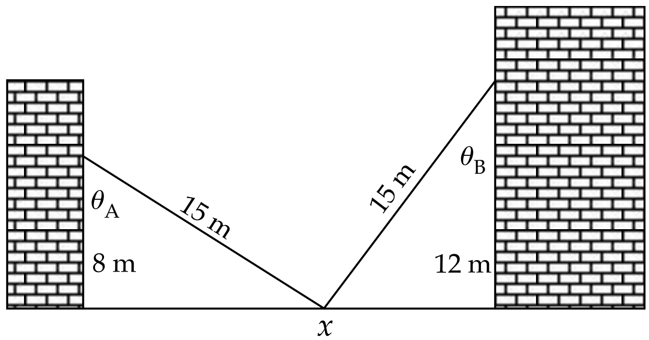
Calculate the distance

a) from the foot of the cliff to the closer boat. *(2 marks)*

b) between the boats if they are sighted in the same direction. *(2 marks)*

Assignment 4.4: Applying Trig Ratios (continued)

6. A ladder 15 m long is placed on a driveway between two buildings so that it reaches 12 m up one building. If it is turned over, its foot being held in position, it will reach 8 m up the other building.

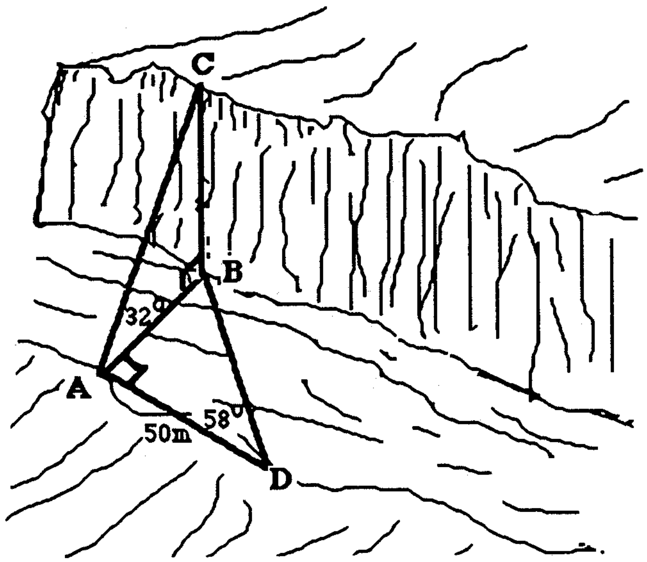


a) How wide is the driveway from building to building? *(3 marks)*

b) What angles (θ_A and θ_B) does the ladder make with each wall? *(4 marks)*

Assignment 4.4: Applying Trig Ratios (continued)

7. A surveyor wishes to find the height BC of an inaccessible cliff. To do this, she sets up her transit at A, and measures ∠CAB = 32°. She then lays off a baseline AD so that ∠BAD = 90° and AD = 50 m. She measures ∠ADB = 58°. Calculate the height of the cliff. *(3 marks)*



Assignment 4.4: Applying Trig Ratios (continued)

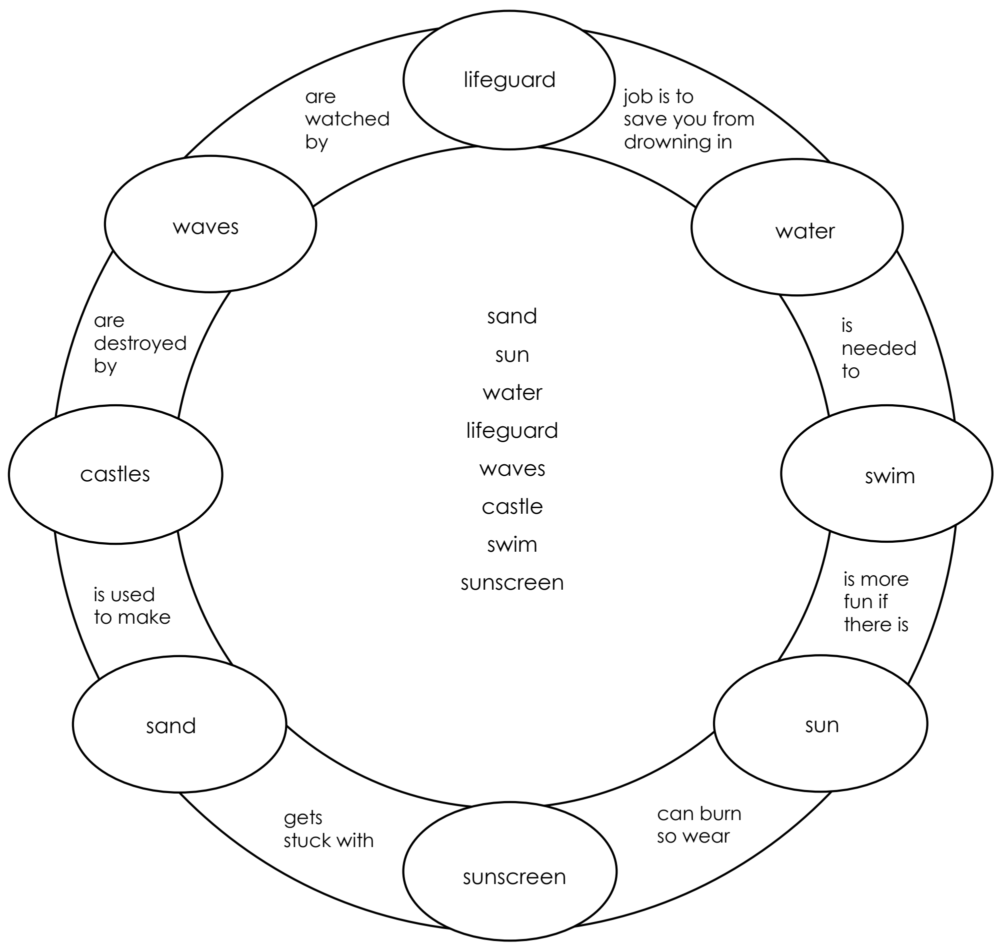
8. A Word Wheel is an organizational tool that can be used to show how concepts are related to each other. The 8 words from the centre of the wheel must be placed in any order in the small circles around the rim. The space between the circles on the rim is for you to write an explanation of how the two adjoining concepts are related. There is no one correct order or answer.

   An example of a Word Wheel showing relationships between things found at a beach is shown below.

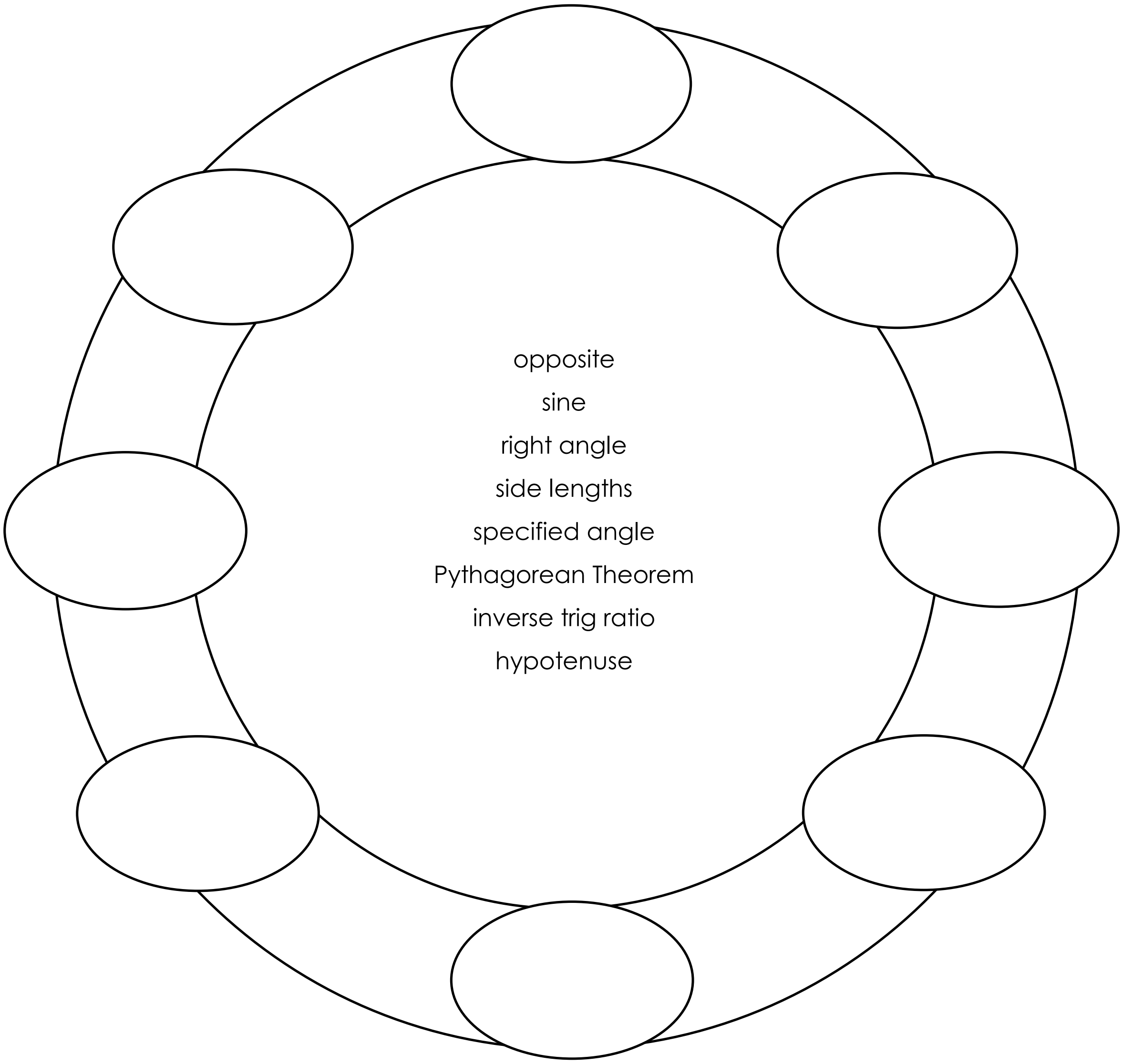
   Complete the trigonometry Word Wheel found on the following page with your own arrangement and descriptions of relationships. *(5 marks)*

   Sample:

Beach Word Wheel



Trigonometry Word Wheel



Marking Guide:

4 marks for appropriate connections between words *(8 x 1/2 = 4 marks)*

1 mark for using all 8 words *(1 mark)*

Module 4 Summary

Congratulations! You have finished the fourth module in the course.

Trigonometry is a branch of mathematics that has lots of practical applications. It can be used for indirect measurement of angles and lengths, and in future math courses you will find many more uses and extensions of these concepts.

This module had examples of a new organizational tool—a Word Wheel. This tool can be applied to other concepts and courses, and hopefully you will find it useful for studying, now and in the future.

As you have reached the halfway mark in this course, now would be a good time to look back at the goals you set for yourself when you began this course. Are you getting closer to your math destination? Have your goals, or where you want this course to take you, changed? Is the pathway you outlined for yourself still practical and efficient? Self-assessment is a critical component of success in math!

Recall the steps outlined when you set your goals:

- Look back: Reflect on what you know and how far you have come.
- Look forward: Determine what you want to know and set goals.
- Look around: Assess whether you are achieving your goals, determine if new learning or understanding has occurred, and check your progress.

Look back at what you wrote in the chart in Module 1, Lesson 1, and make any changes you feel are necessary (revise your short- or long-term goals or change steps in your pathway to achieving your goals). Record your new pathway and destination in the following chart, and, in the space below it, explain why you feel these changes are appropriate.

Math History	Pathway	Math Destination

Reflections: Comment on the changes in your pathway or destination, describe progress you have seen since the beginning of this course, and reflect on your steps along the pathway.

Submitting Your Assignments

It is now time for you to submit Assignments 3.1 to 3.6 from Module 3 and Assignments 4.1 to 4.4 from Module 4 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 3 and Module 4 assignments and organize your material in the following order:

Cover Sheet for Modules 3 and 4 (found at the end of the Introduction)

Assignment 3.1 Units, Area, and Volume

Assignment 3.2 Measuring with Vernier Calipers and Micrometers

Assignment 3.3 Unit Conversions

Assignment 3.4 Volume of Prisms and Pyramids

Assignment 3.5 Surface Area of Prisms and Pyramids

Assignment 3.6 Surface Area and Volume of Spheres, Cylinders, and Cones

Assignment 4.1 Tangent Ratio

Assignment 4.2 Using Sine, Cosine, and Tangent

Assignment 4.3 Inverse Trig Ratios

Assignment 4.4 Applying Trig Ratios

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Midterm Examination

Congratulations, you have finished Module 4 in the course. The midterm examination is out of 100 marks and worth 20% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 4.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens/pencils (2 or 3 of each), blank paper, metric and imperial rulers, a protractor, a scientific calculator, and your Examination Resource Sheet. A maximum of 2 hours is available to complete your midterm examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point you will also have to combine your resource sheets from Modules 1 to 4 onto one 8½″ × 11″ paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you. We suggest that you divide your paper into two quadrants on each side so that each quadrant contains information from one module.

Examination Review

You are now ready to begin preparing for your midterm examination. Please review the content, learning activities, and assignments from Modules 1 to 4.

The midterm practice examination is also an excellent study aid for reviewing Modules 1 to 4.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the midterm examination determines 20% of your final mark in this course and you will have 2 hours to complete the examination.

Midterm Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The midterm practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your midterm practice examination, follow these steps:

1. Study for the midterm practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 1 to 4 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 4, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Midterm Examination Resource Sheet. You can use both sides of an 8½″ by 11″ piece of paper.
6. Bring the following to the midterm practice examination: pens/pencils (2 or 3 of each), blank paper, metric and imperial rulers, a protractor, a scientific, and your Midterm Examination Resource Sheet.
7. Write your midterm practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the midterm examination is 2 hours.
8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Midterm Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 4
Trigonometry

Learning Activity Answer Keys

Module 4: Trigonometry

Learning Activity 4.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. The circumference of a circle is 32π. What is the radius of the circle?
2. Rewrite the following fraction in simplest terms: $\frac{12}{45}$.
3. What is the average of 3, 4, 6, and 7?
4. Fill in the blanks for the pattern: −43, −38, −33, ____, ____.
5. The equation of a line is $y = 6x - 2$. What is the y-intercept?
6. There are two movies coming out on DVD this week that you would like to purchase. Each movie costs $18.99. If you have $35, can you afford to buy both?
7. Is the number 0 rational or irrational?
8. The volume of a cylinder is 12 cm^3. What is the volume of a cone with the same base and height?

Answers:

1. 16 ($C = 2\pi r$ or $32 = 2r$, $r = 32 \div 2$)
2. $\frac{4}{15}$
3. $5 \left(\frac{3+4+6+7}{4}\right)$
4. −28, −23 (Add 5 each time.)
5. −2 ($y = 6(0) - 2$)
6. No (18.99 is approximately 19, $19 \times 2 = \$38$)
7. Rational (0 can be written as $\frac{0}{a}$ where a is an integer, so it is rational)
8. 4 cm^3 (The volume of a cone is one-third the volume of a cylinder with the same dimensions, $(12 \div 3) = 4$ cm^3.)

Part B: Right Triangles and Similar Triangles

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

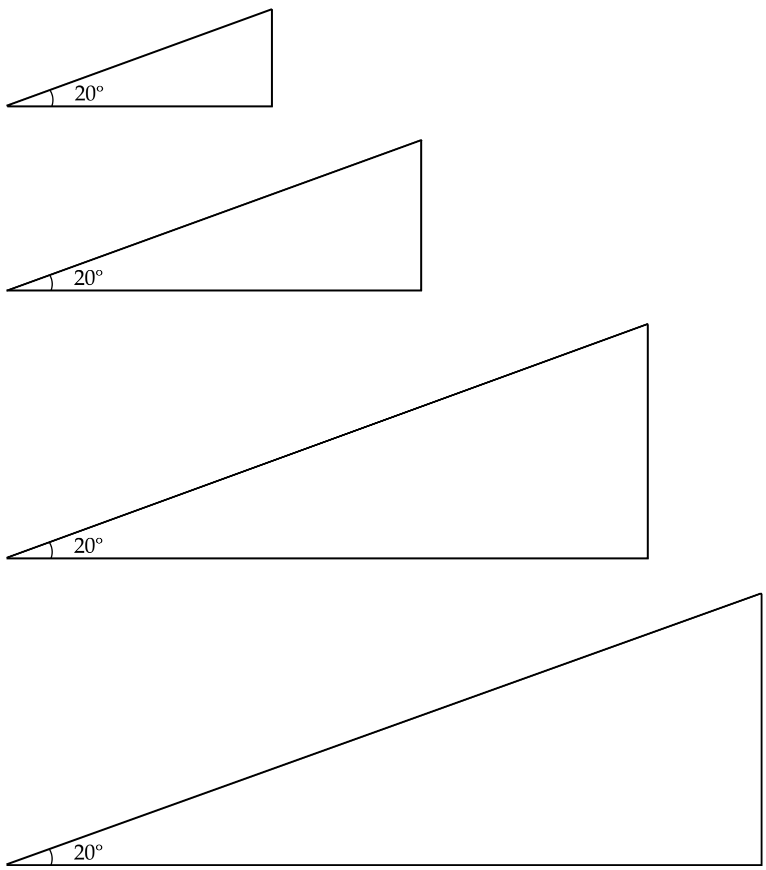
Consider the following similar triangles, each with the measure of a given angle specified.

Label the hypotenuse, opposite, and adjacent side in each triangle.

Using a metric ruler, measure the lengths of the opposite and adjacent sides in each, and record your measurements in the chart below to the nearest tenth of a cm.

Write the ratio of the lengths as indicated in the 4th column.

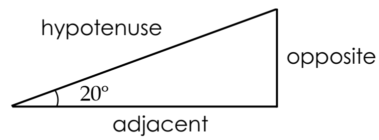
Calculate the value of the ratio to 2 decimal places using a calculator.



Triangle	Opposite	Adjacent	$\frac{\text{Opposite}}{\text{Adjacent}}$	Calculated Value of Ratio
1				
2				
3				
4				

Answer:

In each triangle, the sides should be labelled as:



Triangle	Opposite	Adjacent	$\frac{\text{Opposite}}{\text{Adjacent}}$	Calculated Value of Ratio
1	1.3	3.5	$\frac{1.3}{3.5}$	0.37
2	2.0	5.5	$\frac{2.0}{5.5}$	0.36
3	3.1	8.5	$\frac{3.1}{8.5}$	0.36
4	3.6	10.0	$\frac{3.6}{10.0}$	0.36

Learning Activity 4.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate: 8^2.
2. Evaluate: $\sqrt{100-64}$.
3. The garden in your backyard has an area of 1 yard^2. Convert this area to feet^2.
4. What is the LCM of 10 and 7?
5. You are standing on the baseline of a basketball court. The top of the key is 25 feet away from you. The distance to centre court is 47 feet. What is the distance from the top of the key to the centre court?
6. GST (Goods and Services Tax) is 5%. You are buying clothes for your baby cousin (baby clothes only have GST). If the total before tax is $44.00, how much tax are you charged?
7. Is this data continuous or discrete?

   The number of red lights you get stopped at compared to the distance you drive in the city.
8. Solve for t: $15t - 5 = 40$.

Answers:

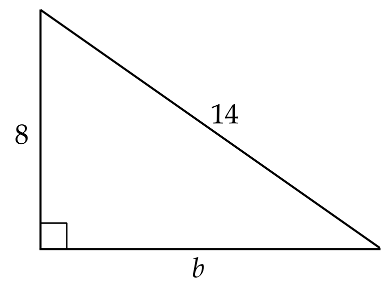
1. 64
2. $6 \left(\sqrt{100-64} = \sqrt{36}\right)$
3. 9 feet^2 (1 yard = 3 feet, so 1 yard^2 = (3 feet)^2 = 9 feet^2)
4. 70 (Since 10 and 7 have no common factors (other than 1), their first common multiple will be 10×7.)
5. 22 feet ($47 - 2 = 45$ feet, $45 - 20 = 25$ feet, so $2 + 20 = 22$ feet)
6. $2.20 (10% of $44.00 = $4.00 (move the decimal one place to the left), 5% is half of 10% so $4.4 \div 2$ = $2.20)
7. Discrete (You cannot partially be stopped by a red light.)
8. $t = 3$ ($t = (40 + 5) \div 15$)

Part B: Tangent Ratio

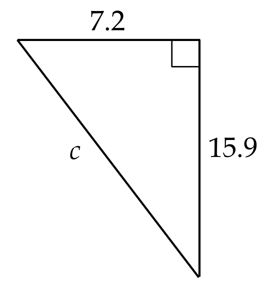
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the Pythagorean Theorem to solve for the missing side lengths in the following triangles.

1. a)



b)



Answers:

a)

$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 14^2$$
$$b^2 = 14^2 - 8^2$$
$$b^2 = 196 - 64$$
$$b^2 = 132$$
$$b = \sqrt{132}$$
$$b = 11.5$$

b)

$$a^2 + b^2 = c^2$$
$$7^2 + 15.9^2 = c^2$$
$$51.84 + 252.81 = c^2$$
$$304.65 = c^2$$
$$c = \sqrt{304.65}$$
$$c = 17.5$$

2. Can the lengths listed below be the sides of a right triangle? Explain. (Remember that the hypotenuse is always the longest side of a right triangle.)

a) 3, 4, 5

b) 6, 11, 13

Answers:

a) Yes

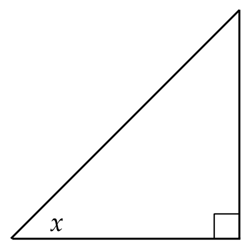
$a^2 + b^2$		c^2
$3^2 + 4^2$		5^2
$9 + 16$		25
25	$=$	25

b) No

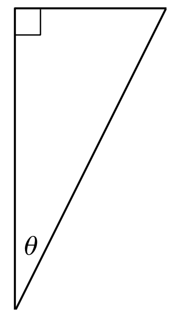
$a^2 + b^2$		c^2
$6^2 + 11^2$		13^2
$36 + 144$		169
157	$\neq$	169

3. Label the sides of the following triangles in relation to the specified angle.

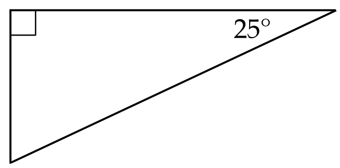
a)



b)

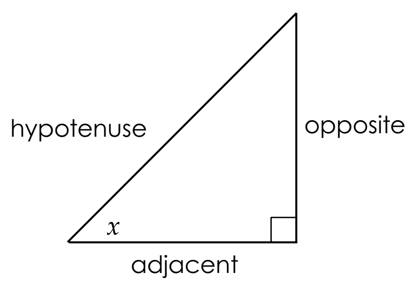


c)

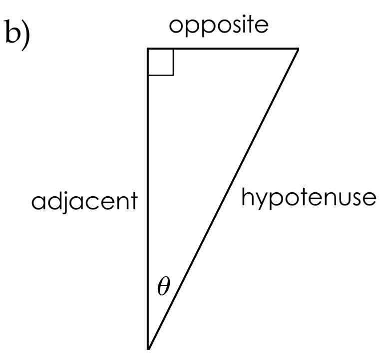


Answers:

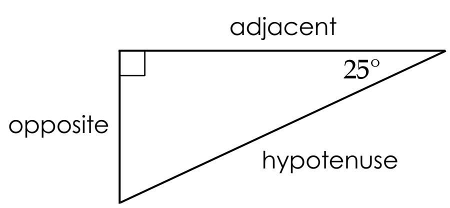
a)



b)



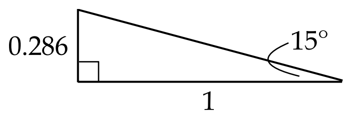
c)



4. Calculate tan 15° to 3 decimal places, and explain what it means using a diagram.

   Answer:

   tan 15° = 0.268

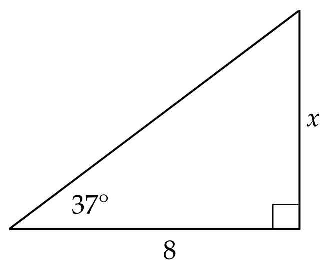
   

   The tangent ratio is $\dfrac{\text{opposite}}{\text{adjacent}}$.

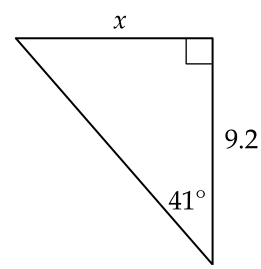
   This means it is a right-angled triangle where the ratio of the opposite side to the adjacent side is 0.286 to 1.

5. Use the tangent ratio to solve for the missing side length x in each triangle.

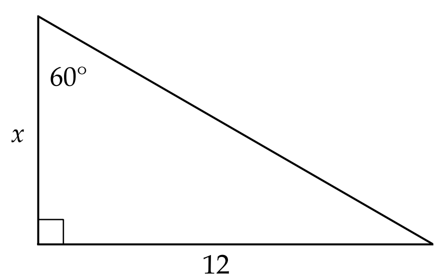
   a)

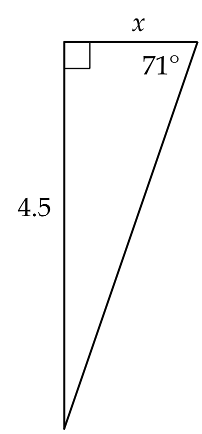
   b)

   c)

   d)

   Answers:

   a)
   $$\tan 37° = \frac{x}{8}$$
   $$(8)\tan 37° = \frac{x}{\cancel{8}}\cancel{(8)}$$
   $$x = 6.0$$

   b)
   $$\tan 41° = \frac{x}{92}$$
   $$(92)\tan 41° = \frac{x}{\cancel{92}}\cancel{(92)}$$
   $$x = 80.0$$

c) $\tan 60° = \dfrac{12}{x}$

$$x = \frac{12}{\tan 60°}$$

$$x = 6.9$$

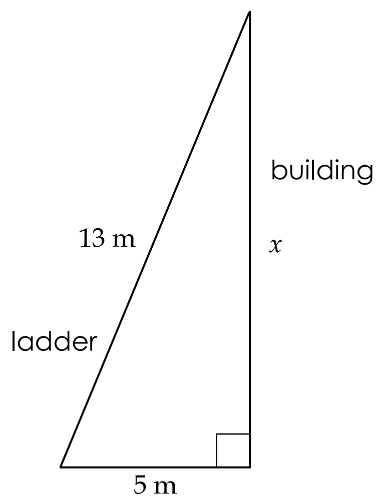
d) $\tan 71° = \dfrac{4.5}{x}$

$$x = \frac{4.5}{\tan 71°}$$

$$x = 1.5$$

6. The foot of a 13 m ladder is 5 m from the base of a tall building. How far up the building does the ladder reach? Draw a diagram.

Answer:



$$a^2 + b^2 = c^2$$
$$5^2 + x^2 = 13^2$$
$$13^2 - 5^2 = x^2$$
$$169 - 25 = x^2$$
$$x^2 = 144$$
$$x = \sqrt{144}$$
$$x = 12$$

The ladder reaches 12 m up the building.

Learning Activity 4.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Is an angle of 86° acute, obtuse, straight, or reflex?
2. Write the following improper fraction as a mixed fraction: $\frac{29}{9}$.
3. You woke up at 8:30 am. You got ready in 45 minutes. You walked to work for 45 minutes. You worked for 4 hours. You ate lunch for 35 minutes. What time is it now?
4. What two numbers have a product of 16 and a sum of 8?
5. What two numbers have a product of −16 and a sum of 0?
6. A ticket to the baseball game is $12.50. You have to pay for parking at the game, which costs $5. Once you are in the ballpark, you buy popcorn for $3.00, ice cream for $3.15, and a drink for $2.50. How much did cost you to go to the baseball game?
7. You are going for a walk. You walk north for 5 blocks, turn around, and walk south for 16 blocks. How many blocks are you from where you started? State if you are north or south.
8. Evaluate: $\frac{3}{15} \div \frac{1}{5}$.

Answers:

1. Acute (The angle is less than 90°.)
2. $3\frac{2}{9}$ (9 × 3 = 27, 29 – 27 = 2, so there are three 9s in 29, with 2 left over)
3. 2:35 pm (8:30 + 0:45 = 9:15 am, 9:15 + 0:45 = 10:00 am, 10:00 + 4:00 = 14:00 (subtract 12 hours to get the same in am/pm form) = 2:00 pm. 2:00 + 0:35 = 2:35 pm)
4. 4, 4 (The factor pairs of 16 are (1, 16), (2, 8), (4, 4). Only 4 + 4 = 8.)
5. 4, –4 (From the above question, only 4 − 4 = 0.)
6. $26.15 (Add all the dollars (12 + 5 + 3 + 3 + 2 = 25), add all the cents (0.50 + 0.15 + 0.50 = 1.15). Add the dollars and cents (25 + 1.15 = $26.15).)
7. 11 blocks south (Think about being on the origin of a graph. You move positive 5, then negative 16, so 5 − 16 = −11.)
8. 1 $\left(\frac{3}{15} \times \frac{5}{1} = \frac{15}{15}\right)$

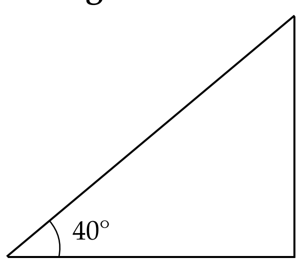
Part B: Exploring Sine and Cosine

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

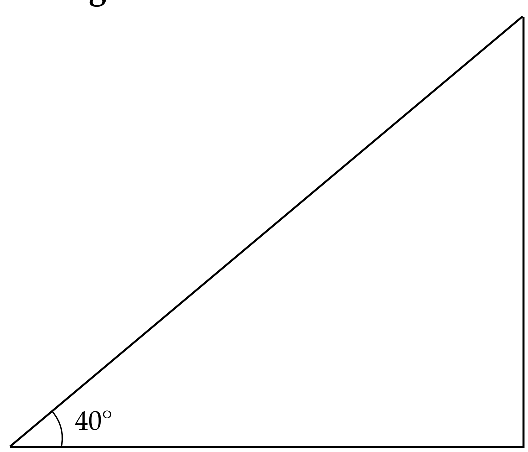
In each of the following similar right triangles, label the sides as hypotenuse, opposite, or adjacent in relation to the specified 40° angle. Measure each side to the nearest tenth of a centimetre, and complete the following chart.

Triangle	Opposite	Adjacent	Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	Value of Ratio	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	Value of Ratio
1							
2							
3							

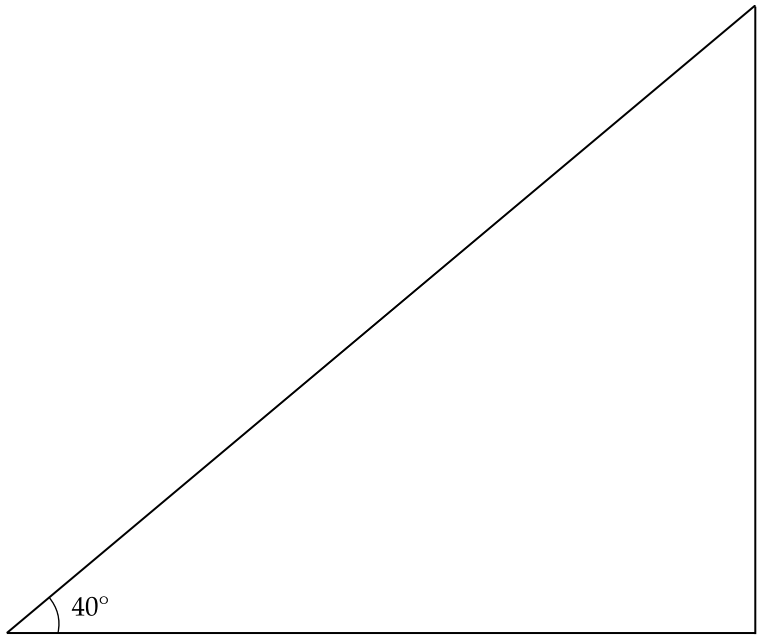
Triangle 1:



Triangle 2



Triangle 3



Using your calculator, determine the value of the following:

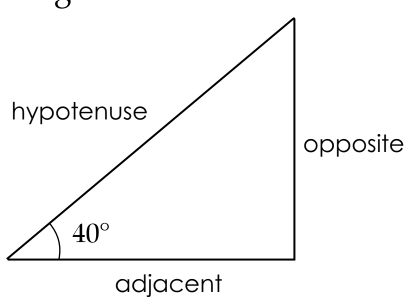
Sin 40° = ____________________ Cos 40° = _____________________

Based on your measurements and calculations, what can you conclude about the trigonometric ratios of sine and cosine? Write each as the ratio of the appropriate side lengths:

Sine = ______________________ Cosine = ______________________

Answers:

Each triangle should be labelled as:



Triangle	Opposite	Adjacent	Hypotenuse	$\dfrac{\text{Opposite}}{\text{Hypotenuse}}$	Value of Ratio	$\dfrac{\text{Adjacent}}{\text{Hypotenuse}}$	Value of Ratio
1	3.2	3.8	5.0	$\dfrac{3.2}{5.0}$	0.64	$\dfrac{3.8}{5.0}$	0.76
2	5.7	6.8	8.9	$\dfrac{5.7}{8.9}$	0.64	$\dfrac{6.8}{8.9}$	0.76
3	8.3	9.9	12.9	$\dfrac{8.3}{12.9}$	0.64	$\dfrac{9.9}{12.9}$	0.76

Using your calculator, determine the value of the following:

Sin 40° = 0.6427876097 Cos 40° = 0.7660444431

Learning Activity 4.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Solve for g: $3 - g = 15$
2. What two numbers have a product of −27 and a sum of 6?
3. Your mom is buying ice cream for your family. The store has tiger, bubblegum, vanilla, and chocolate flavours. Your mom doesn't like bubblegum, your dad doesn't like chocolate, and you don't like vanilla. Which ice cream will your mom buy?
4. What is the formula for tangent?
5. Arrange the numbers from largest to smallest: $\frac{1}{2}$, 0.29, $\frac{3}{4}$, 0.65, 0.34.
6. Evaluate: $\sqrt[3]{125}$.
7. An octave in music includes 8 notes. If you were to go up half an octave, how many notes is that?
8. When in Venice, you notice a great store on the other side of the street. Because the roads are water in Venice, you need to walk to the nearest bridge. The nearest bridge is 6 m away from you, and the 'road' is 2 m wide. How far do you have to walk to get to the store?

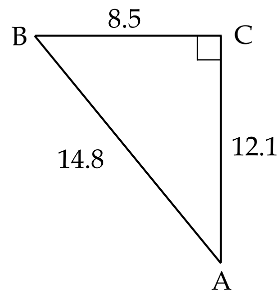
Answers:

1. −12 ($-g = 15 - 3$, but the g is still negative so you need to multiply both sides by −1.)
2. 9, −3 (The factor pairs of 27 are (1, 27) and (3, 9). The signs of the factors have to be one positive and one negative in order to produce −27. 9 − 3 = 6 so the 3 is negative.)
3. Tiger ice cream
4. $\tan \theta = \dfrac{\text{opposite}}{\text{adjacent}}$
5. $\frac{3}{4}$, 0.65, $\frac{1}{2}$, 0.34, 0.29
6. 5
7. 4 (8 ÷ 2)
8. 14 m (You walk 6 m on the bridge, 2 m across, then 6 m back to the store so 6 + 2 + 6.)

Part B: Applying Sine and Cosine

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the ratio for the sine, cosine, and tangent of angle A.



Answer:

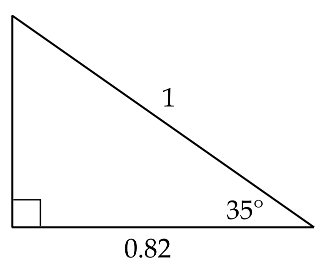
$$\sin A = \frac{8.5}{14.8}, \cos A = \frac{12.1}{14.8}, \tan A = \frac{8.5}{12.1}$$

2. Explain what cos 35° means. Include a diagram.

Answer:

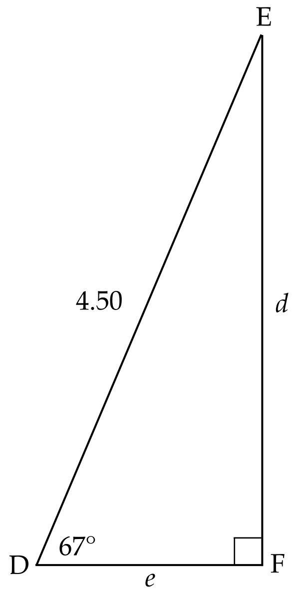
cos 35° = 0.8191520443

The cosine ratio is $\frac{\text{adjacent}}{\text{hypotenuse}}$. This means that in a right triangle, the leg adjacent to the 35° angle will be approximately 0.82 units if the hypotenuse is 1 unit. The ratio of the lengths of the $\frac{\text{adjacent}}{\text{hypotenuse}}$ sides will be $\frac{0.82}{1}$.

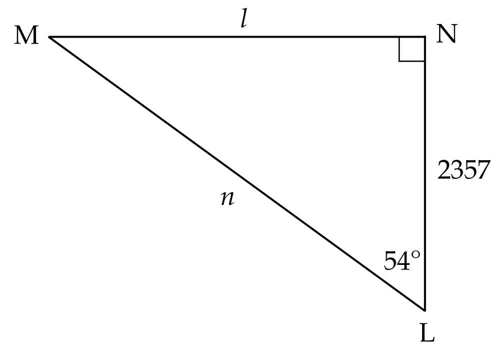


3. Solve the following triangles. Round your answers to the nearest tenth.

a)



b)



Answers:

a)

$$\angle E = 90 - 67 = 23°$$

$$\sin\ 67° = \frac{d}{4.5}$$

$$d = 4.142271841$$

$$\sin\ 67° = \frac{e}{4.5}$$

$$e = 1.758290078$$

$\angle D = 67°$	$\angle E = 23°$	$\angle F = 90°$
$d = 4.14$	$e = 1.76$	$f = 4.50$

b)

$$\angle M = 90 - 54 = 36°$$

$$\cos\ 54° = \frac{2357}{n}$$

$$n = \frac{2357}{\cos\ 54°}$$

$$n = 4009.967911$$

$$\cos\ 54° = \frac{l}{2357}$$

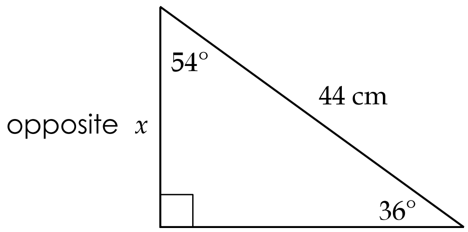
$$l = 3244.132187$$

$\angle L = 54°$	$\angle M = 36°$	$\angle N = 90°$
$l = 3244$	$m = 2357$	$n = 4010$

4. A right triangle has angles of 36° and 54°. Find the length of the shortest leg if the hypotenuse is 44 cm.

Answer:

The shortest leg will be opposite the smallest angle.

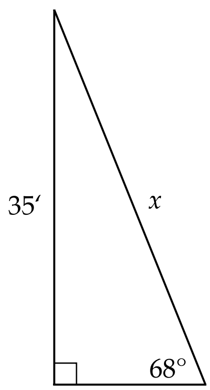


$$\sin 36° = \frac{x}{44}$$
$$x = \sin 36°(44)$$
$$x = 25.8625511$$

The shortest leg is about 25.9 cm long.

5. A brace cable supporting a streetlight pole is attached at the top of the pole, 35 feet above the ground. How long must the cable be if it makes a 68° angle with the ground?

Answer:



$$\sin 68° = \frac{35}{x}$$
$$(x)\sin 68° = 35$$
$$x = \frac{35}{\sin 68°}$$
$$x = 37.74871599$$

The cable must be 37.7 feet long.

Learning Activity 4.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. In the novel trilogy *Lord of the Rings*, there are many rings of power. Nine were given to men, seven to the dwarves, three to the elves, and one to the evil mastermind. In total, how many rings of power are there?
2. Evaluate: $x^{\frac{-1}{6}}$.
3. What would be a good range for a graph comparing a woman's age to her weight?
4. A checkerboard has 8 squares along its length and 8 squares along its width. How many squares altogether are on the board?
5. The squares on a checkerboard alternate between black and white. How many squares are black?
6. Solve for h: $h + 12 = 32$.
7. What is the formula for cosine?
8. What two numbers have a product of 21 and a sum of 22?

Answers:

1. 20 ($9 + 1 = 10$, $7 + 3 = 10$, $10 + 10$)
2. $\frac{1}{\sqrt[6]{x}}$
3. 4 to 400 pounds (Weight is the dependent variable, so the range is the weight. When you are born you do not weigh 0 pounds, and the majority of women do not exceed 400 pounds.)
4. 64 (8×8)
5. 32 ($64 \div 2$)
6. $h = 20$
7. $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
8. 1, 21 (The only other factor pair is (3, 7), which cannot add up to 22.)

Part B: Inverse Trig Ratios

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the measure of the angle whose trigonometric ratio is given below. Round your answer to the nearest tenth of a degree.
   a) cos B = 0.4556
   b) sin Y = 0.5
   c) tan P = 6.78
   d) cos T = 0.0013

   Answers:

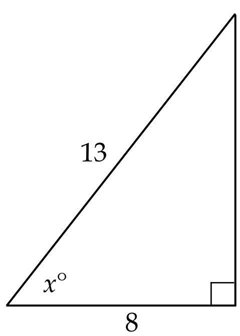
   a) $\cos B = 0.4556$
   $$\angle B = \cos^{-1}(0.4556)$$
   $$\angle B = 62.9°$$

   b) $\sin Y = 0.5$
   $$\angle Y = \sin^{-1}(0.5)$$
   $$\angle Y = 30°$$

   c) $\tan P = 6.78$
   $$\angle P = \tan^{-1}(6.78)$$
   $$\angle P = 81.6°$$

   d) $\cos T = 0.0013$
   $$\angle T = \cos^{-1}(0.0013)$$
   $$\angle T = 89.9°$$

2. Find x to the nearest degree.



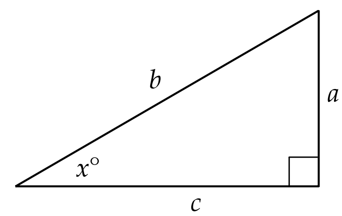
   Answer:
   $$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$$
   $$\cos x = \frac{8}{13}$$
   $$x = \cos^{-1}(0.6153846154)$$
   $$x = 52°$$

3. Find x to the nearest degree if:

   a) $a = 5$ and $b = 10$

   b) $b = 20$ and $c = 4$

   c) $a = 4$ and $c = 2$



   Answers:

   a) $\sin x = \frac{5}{10}$

   $x = \sin^{-1}\left(\frac{1}{2}\right)$

   $x = 30°$

   b) $\cos x = \frac{4}{20}$

   $x = \cos^{-1}(0.2)$

   $x = 78°$

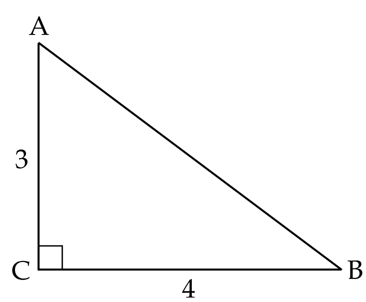
   c) $\tan x = \frac{4}{2}$

   $x = \tan^{-1}(2)$

   $x = 63°$

4. You are given 3 measures in each of the following triangles. Find the value of the three missing measure in each triangle. Round your answer to 2 decimal places.

   a)



   Answer:

   Given: $b = 3$, $a = 4$, $\angle C = 90°$

   Use the Pythagorean Theorem

   $c^2 = a^2 + b^2$

   $c^2 = 3^2 + 4^2$

   $c^2 = 9 + 16$

   $c^2 = 25$

   $c = \pm 5$

   but c is a length, so $c = 5$

   $\tan B = \frac{3}{4}$

   $B = \tan^{-1}\left(\frac{3}{4}\right)$

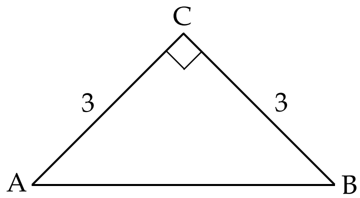
   $B = 36.87°$

   $\tan A = \frac{4}{3}$

   $A = \tan^{-1}\left(\frac{4}{3}\right)$

   $A = 53.13°$

b)



Answer:

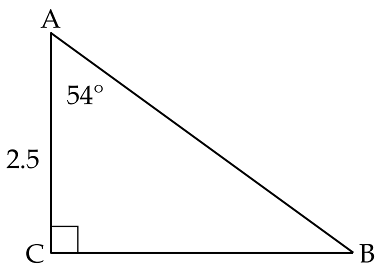
Given: a = 3, b = 3, C = 90°

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 3^2$$
$$c^2 = 9 + 9$$
$$c^2 = 18$$
$$c = 4.24$$

$$\tan A = \frac{3}{3}$$
$$= 1$$
$$A = \tan^{-1}(1)$$
$$= 45°$$

$$\tan B = \frac{3}{3}$$
$$= 1$$
$$B = \tan^{-1}(1)$$
$$= 45°$$

c)



Answer:

Given: b = 2.5, ∠A = 56°, ∠C = 90°

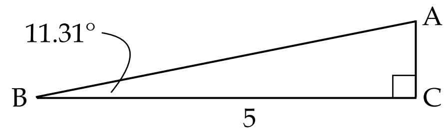
∠B = 180° − 90° – 54°

= 36°

$$\cos 54° = \frac{2.5}{c}$$
$$c = \frac{2.5}{\cos 54°}$$
$$c = 4.25$$

$$\tan 54° = \frac{a}{2.5}$$
$$a = \tan 54° \cdot 2.5$$
$$a = 3.44$$

d)



Answer:

Given: $a = 5$, $\angle B = 11.31°$, $\angle C = 90°$

$\angle A = 180° - 90° - 11.31°$

$= 78.69°$

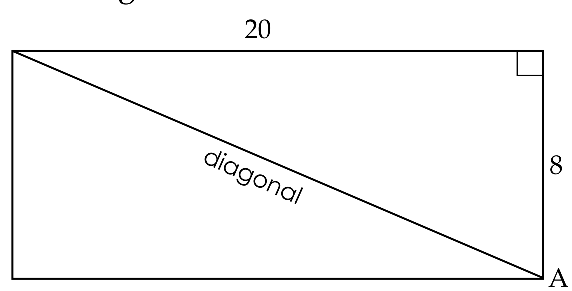
$$\tan 11.31° = \frac{b}{5} \qquad b = 1.00$$

$$\cos 11.31° = \frac{5}{c} \qquad c = \frac{5}{\cos 11.31°} \qquad c = 5.10$$

5. Find the measure of the angle between the diagonal and the shorter side of a rectangle that is 20 cm long and 8 cm wide.

   Answer:

   Draw a diagram.



The 20 cm side is opposite the desired angle and the 8 cm side is adjacent to the desired side.

$$\tan A = \frac{20}{8} = 2.5$$

Use the $\tan^{-1}$ function to find the angle associated with this ratio.

$A = \tan^{-1} 2.5$

$\angle A = 68.2°$

Learning Activity 4.6

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. The surface area of a globe is 100π cm^2. What is the radius of the globe?
2. If the ratio comparing the size of the globe to the Earth is 1 cm: 800 miles, what is the approximate radius of the Earth (use the answer from above)?
3. If you buy a shirt for \$8 and jeans for \$32, how much do you spend altogether?
4. If 20% of your class of 20 students has already done the social studies project, how many people are done their project?
5. What is the formula for sine?
6. The volume of a pyramid is 5 m^3. What is the volume of a prism with the same base and height?
7. Is the following data continuous or discrete?

   The amount of money in your bank account over a period of time.
8. Is the number 1 prime, composite, or neither?

Answer:

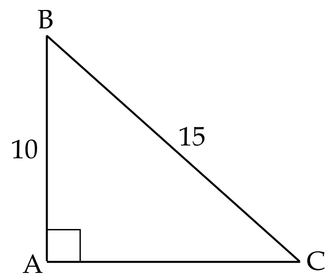
1. 5 cm ($SA_{sphere} = 4\pi r^2$ so $100 \div 4 = 25$, $\sqrt{25} = 5$)
2. 4000 miles (5 cm $\times$ 800)
3. \$40
4. 4 people (10% of 20 = 2, 10% $\times$ 2 = 2 $\times$ 2 = 4)
5. $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
6. 15 m^3 (A prism has 3 times the volume of a pyramid with the same dimensions, so 5 $\times$ 3 = 15.)
7. Discrete (You do not deposit money as a continuous stream, but in specific amounts.)
8. Neither

Part B: Applying Trig Ratios

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. In ΔABC, ∠A = 90°, a = 15 m, and c = 10 m. Find the measures of the other two angles.

   Answer:

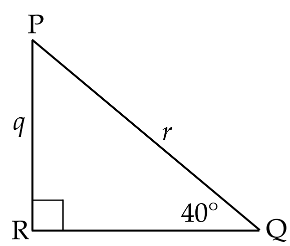


$$\sin \angle C = \frac{10}{15} = 0.\overline{66}$$
$$\angle C = 41.8°$$

$$\therefore \angle B = 180° - (90° + 41.8°)$$
$$\angle B = 48.2°$$

2. In ΔPQR, ∠R = 90°, ∠Q = 40°, and p = 30. Find the lengths of the other two sides.

   Answer:

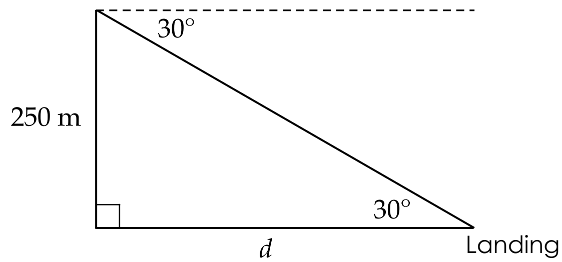


$$\tan 40° = \frac{q}{30}$$
$$30 \tan 40° = q$$
$$q = 30(0.8391) = 25.2$$

$$\cos 40° = \frac{30}{r}$$
$$r = \frac{30}{\cos 40°}$$
$$r = \frac{30}{0.766} = 39.2$$

3. A hot-air balloon, advertising for a real estate company, is 250 m above ground level. The angle of depression from the balloon to the landing area is 30°. What is the distance along the ground from a point beneath the balloon to the landing area?

   Answer:

   $$\tan 30° = \frac{250}{d}$$

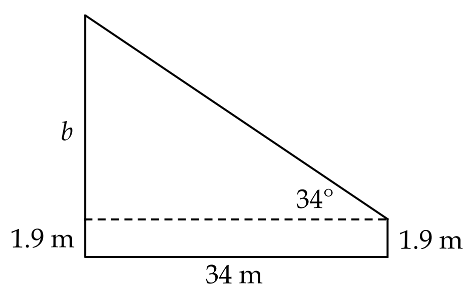
   $$d = \frac{250}{\tan 30°} = \frac{250}{0.57735}$$

   $$d = 433.0127019$$

   The distance from the point beneath the balloon to the landing area is approximately 433 m.

4. A surveyor uses a transit (a device used to measure angles) to determine that the angle of elevation from the transit to the top of a building is 34°. The horizontal distance from the top of the transit to the building is 34 m, and the height of the transit is 1.9 m. How high is the building?

   Answer:

   $$\tan 34° = \frac{b}{34}$$

   $$34 \tan 34° = b$$

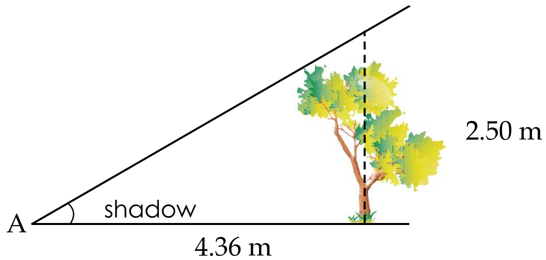
   $$34(0.6745) = b$$

   $$22.9 = b$$

   The building is 22.9 m + 1.9 m = 24.8 m high.

5. A tree 2.50 m tall casts a shadow 4.36 m long. Calculate the angle of elevation from the ground to the sun to the nearest degree.

   Answer:



$$\tan \angle A = \frac{2.50}{4.36}$$

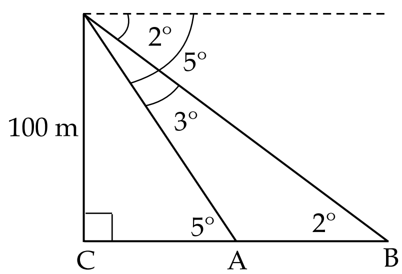
$$\angle A = \tan^{-1}\left(\frac{2.50}{4.36}\right)$$

$$\angle A = 30°$$

6. From the top of a 100 m tall fire tower, a fire ranger observes two fires: one at an angle of depression of 5°, and the other at an angle of depression of 2°. Assuming that the fires and the tower are in a straight line, how far apart are the fires if they are

   a) on the same side of the tower?

      Answer:



$$\tan 2° = \frac{100}{BC}$$

$$BC = \frac{100}{\tan 2°} = \frac{100}{0.03492}$$

$$BC = 2863.63$$

$$\tan 5° = \frac{100}{AC}$$

$$AC = \frac{100}{\tan 5°} = \frac{100}{0.8749}$$

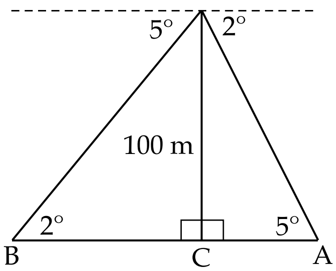
$$AC = 1143.01$$

Therefore, the distance between the fires is

$$AB = 2863.63 - 1143.01 = 1720.62 \text{ m}.$$

b) on opposite sides of the tower?

Answer:



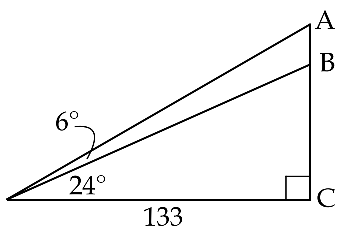
As in part (a), BC = 2863.63 and AC = 1143.01.

∴ AB = 2863.63 + 1143.01 = 4006.64 m.

If they are on opposite sides, the fires are 4006.63 m apart.

7. From a point 133 m away from the centre of the base of the Manitoba Legislative Building, the angle of elevation to the top of the torch of the Golden Boy is 30°. From the same point, the angle of elevation to his feet is 24°. Find the height of the Golden Boy.

Answer:

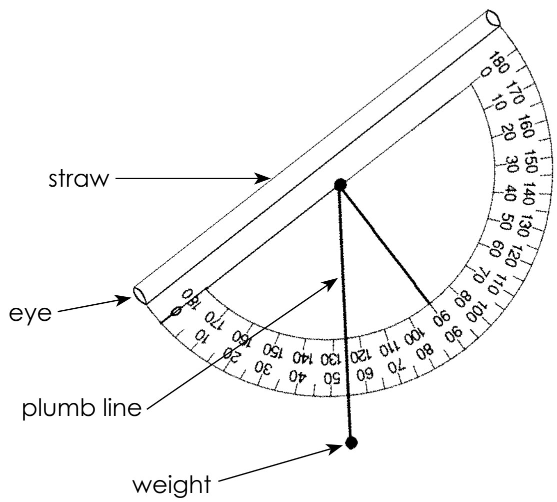


$$\tan 24° = \frac{BC}{133}$$
$$BC = 133 \tan 24°$$
$$BC = 133(0.44523)$$
$$BC = 59.2$$

$$\tan 30° = \frac{AC}{133}$$
$$AC = 133 \tan 30°$$
$$AC = 133(0.57735)$$
$$AC = 76.8$$

The height of the Golden Boy is 76.8 − 59.2 = 17.6 m.

8. A clinometer is a device used to measure angles of elevation or depression. You can construct a clinometer by taping a sighting straw to the straight edge of a plastic protractor. Create a plumb line from string or fish line with a small weight at one end (e.g., a small metal nut or a paper clip). Attach the string so that it swings freely from the centre of the horizontal zero line along the straight edge (see diagram). Holding the straw horizontal, the plumb line should pass over the 90° mark.



Using your clinometer, an imperial measure device (such as a tape measure or yardstick) and trigonometry, show by the use of a sketch how you would verify that a basketball hoop is 10 feet off the floor of a gymnasium.

Answer:

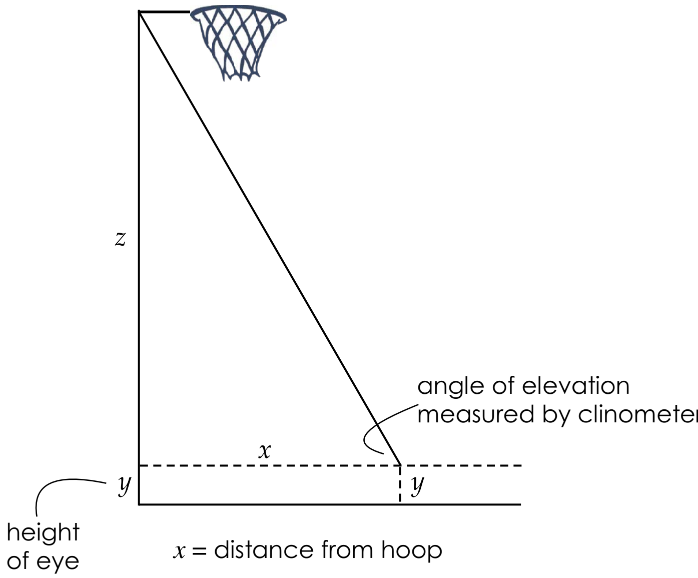
Measure the distance from the floor to your eye (y).

Walk away from a point directly below the basketball hoop and measure the distance you are away from that point (x).

Look through the straw of the clinometer until the basketball hoop is sighted.

Note the angle the plumb line is away from the 90° mark on the protractor (e.g., if the plumb line reads 60°, then the angle is 90° − 60° = 30°). This angle is the angle of elevation of the basketball hoop from the horizontal line at eye level.

The following diagram will indicate the resulting right-angled triangle to be measured.



Using trigonometry to find z, $z + y$ should be 10 feet.

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 4
Trigonometry

Learning Activity Answer Keys

Module 4: Trigonometry

Learning Activity 4.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. The circumference of a circle is 32π. What is the radius of the circle?
2. Rewrite the following fraction in simplest terms: $\frac{12}{45}$.
3. What is the average of 3, 4, 6, and 7?
4. Fill in the blanks for the pattern: −43, −38, −33, ____, ____.
5. The equation of a line is $y = 6x - 2$. What is the y-intercept?
6. There are two movies coming out on DVD this week that you would like to purchase. Each movie costs $18.99. If you have $35, can you afford to buy both?
7. Is the number 0 rational or irrational?
8. The volume of a cylinder is 12 cm^3. What is the volume of a cone with the same base and height?

Answers:

1. 16 ($C = 2\pi r$ or $32 = 2r$, $r = 32 \div 2$)
2. $\frac{4}{15}$
3. $5 \left(\frac{3+4+6+7}{4}\right)$
4. −28, −23 (Add 5 each time.)
5. −2 ($y = 6(0) - 2$)
6. No (18.99 is approximately 19, $19 \times 2 = \$38$)
7. Rational (0 can be written as $\frac{0}{a}$ where a is an integer, so it is rational)
8. 4 cm^3 (The volume of a cone is one-third the volume of a cylinder with the same dimensions, $(12 \div 3) = 4$ cm^3.)

Part B: Right Triangles and Similar Triangles

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

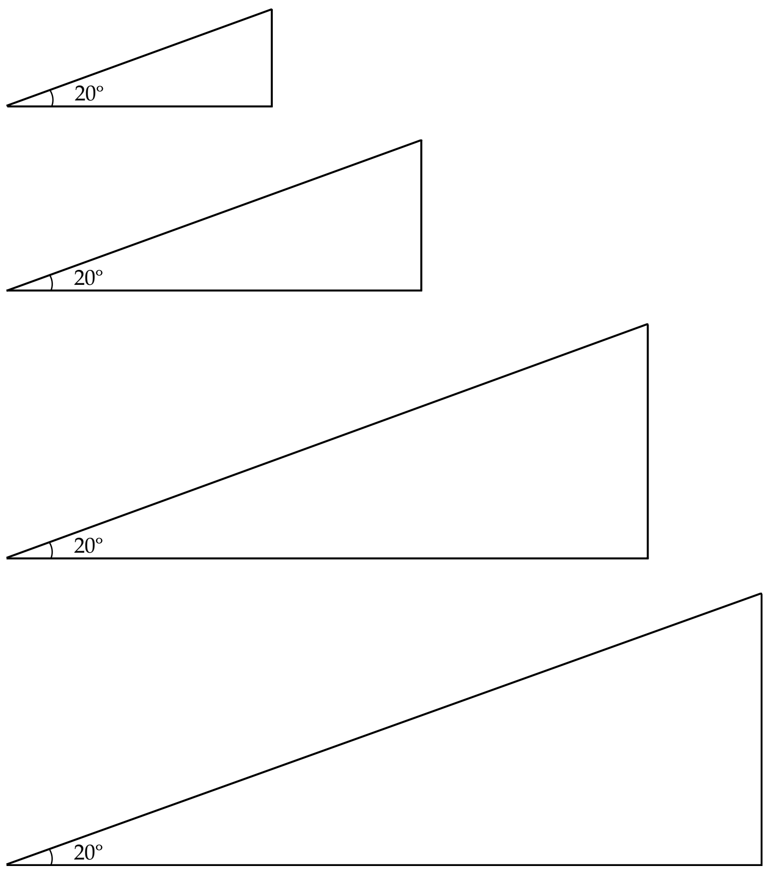
Consider the following similar triangles, each with the measure of a given angle specified.

Label the hypotenuse, opposite, and adjacent side in each triangle.

Using a metric ruler, measure the lengths of the opposite and adjacent sides in each, and record your measurements in the chart below to the nearest tenth of a cm.

Write the ratio of the lengths as indicated in the 4th column.

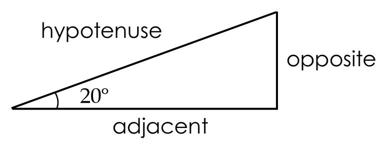
Calculate the value of the ratio to 2 decimal places using a calculator.



Triangle	Opposite	Adjacent	$\frac{\text{Opposite}}{\text{Adjacent}}$	Calculated Value of Ratio
1				
2				
3				
4				

Answer:

In each triangle, the sides should be labelled as:



Triangle	Opposite	Adjacent	$\frac{\text{Opposite}}{\text{Adjacent}}$	Calculated Value of Ratio
1	1.3	3.5	$\frac{1.3}{3.5}$	0.37
2	2.0	5.5	$\frac{2.0}{5.5}$	0.36
3	3.1	8.5	$\frac{3.1}{8.5}$	0.36
4	3.6	10.0	$\frac{3.6}{10.0}$	0.36

Learning Activity 4.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Evaluate: 8^2.
2. Evaluate: $\sqrt{100-64}$.
3. The garden in your backyard has an area of 1 yard^2. Convert this area to feet^2.
4. What is the LCM of 10 and 7?
5. You are standing on the baseline of a basketball court. The top of the key is 25 feet away from you. The distance to centre court is 47 feet. What is the distance from the top of the key to the centre court?
6. GST (Goods and Services Tax) is 5%. You are buying clothes for your baby cousin (baby clothes only have GST). If the total before tax is $44.00, how much tax are you charged?
7. Is this data continuous or discrete?

   The number of red lights you get stopped at compared to the distance you drive in the city.
8. Solve for t: $15t - 5 = 40$.

Answers:

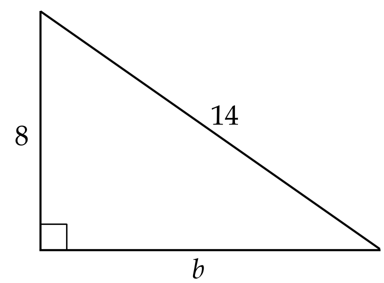
1. 64
2. $6 \left(\sqrt{100-64} = \sqrt{36}\right)$
3. 9 feet^2 (1 yard = 3 feet, so 1 yard^2 = (3 feet)^2 = 9 feet^2)
4. 70 (Since 10 and 7 have no common factors (other than 1), their first common multiple will be 10×7.)
5. 22 feet ($47 - 2 = 45$ feet, $45 - 20 = 25$ feet, so $2 + 20 = 22$ feet)
6. $2.20 (10% of $44.00 = $4.00 (move the decimal one place to the left), 5% is half of 10% so $4.4 \div 2$ = $2.20)
7. Discrete (You cannot partially be stopped by a red light.)
8. $t = 3$ ($t = (40 + 5) \div 15$)

Part B: Tangent Ratio

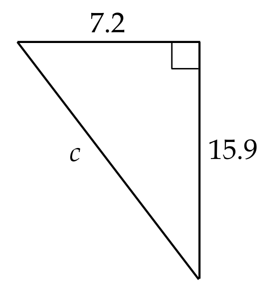
Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the Pythagorean Theorem to solve for the missing side lengths in the following triangles.

1. a)



b)



Answers:

a)

$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 14^2$$
$$b^2 = 14^2 - 8^2$$
$$b^2 = 196 - 64$$
$$b^2 = 132$$
$$b = \sqrt{132}$$
$$b = 11.5$$

b)

$$a^2 + b^2 = c^2$$
$$7^2 + 15.9^2 = c^2$$
$$51.84 + 252.81 = c^2$$
$$304.65 = c^2$$
$$c = \sqrt{304.65}$$
$$c = 17.5$$

2. Can the lengths listed below be the sides of a right triangle? Explain. (Remember that the hypotenuse is always the longest side of a right triangle.)

a) 3, 4, 5

b) 6, 11, 13

Answers:

a) Yes

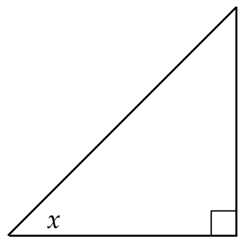
$a^2 + b^2$		c^2
$3^2 + 4^2$		5^2
$9 + 16$		25
25	$=$	25

b) No

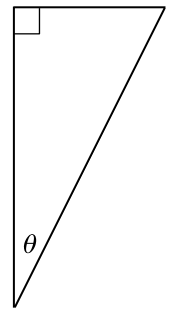
$a^2 + b^2$		c^2
$6^2 + 11^2$		13^2
$36 + 144$		169
157	$\neq$	169

3. Label the sides of the following triangles in relation to the specified angle.

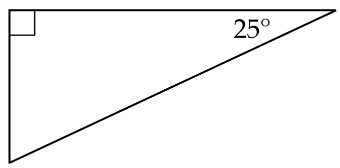
a)



b)

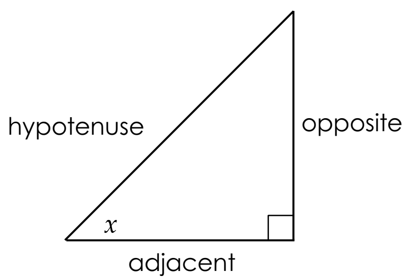


c)



Answers:

a)



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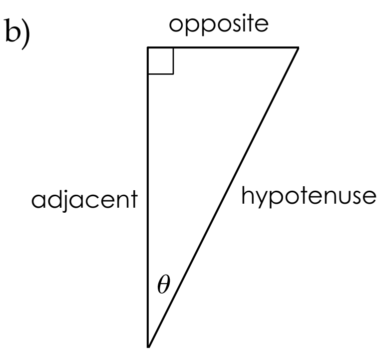
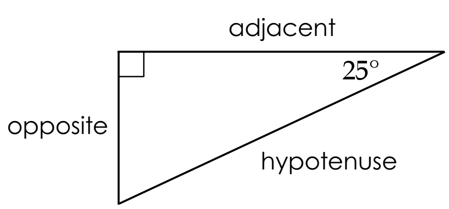


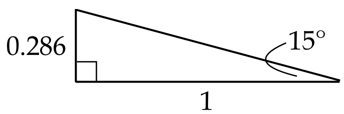
c)



4. Calculate tan 15° to 3 decimal places, and explain what it means using a diagram.

Answer:

tan 15° = 0.268

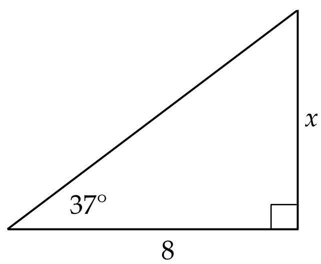


The tangent ratio is $\dfrac{\text{opposite}}{\text{adjacent}}$.

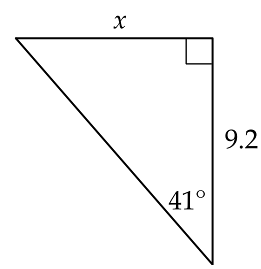
This means it is a right-angled triangle where the ratio of the opposite side to the adjacent side is 0.286 to 1.

5. Use the tangent ratio to solve for the missing side length x in each triangle.

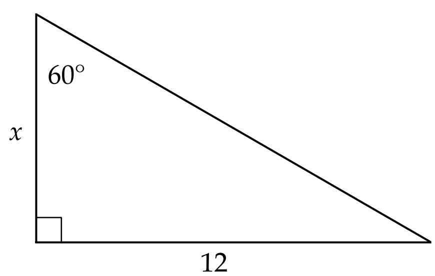
a)



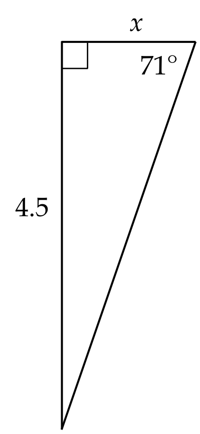
b)



c)



d)



Answers:

a)

$$\tan 37° = \frac{x}{8}$$

$$(8)\tan 37° = \frac{x}{\cancel{8}}\cancel{(8)}$$

$$x = 6.0$$

b)

$$\tan 41° = \frac{x}{92}$$

$$(92)\tan 41° = \frac{x}{\cancel{92}}\cancel{(92)}$$

$$x = 80.0$$

c) $\tan 60° = \dfrac{12}{x}$

$$x = \frac{12}{\tan 60°}$$

$$x = 6.9$$

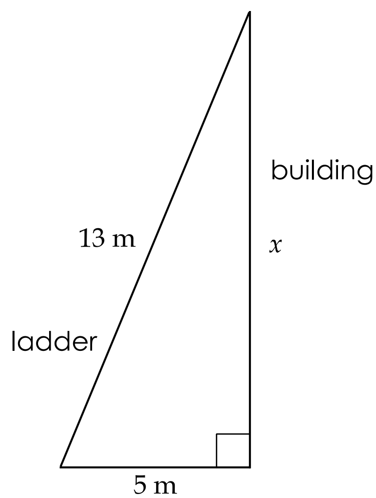
d) $\tan 71° = \dfrac{4.5}{x}$

$$x = \frac{4.5}{\tan 71°}$$

$$x = 1.5$$

6. The foot of a 13 m ladder is 5 m from the base of a tall building. How far up the building does the ladder reach? Draw a diagram.

Answer:



$$a^2 + b^2 = c^2$$
$$5^2 + x^2 = 13^2$$
$$13^2 - 5^2 = x^2$$
$$169 - 25 = x^2$$
$$x^2 = 144$$
$$x = \sqrt{144}$$
$$x = 12$$

The ladder reaches 12 m up the building.

Learning Activity 4.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Is an angle of 86° acute, obtuse, straight, or reflex?
2. Write the following improper fraction as a mixed fraction: $\frac{29}{9}$.
3. You woke up at 8:30 am. You got ready in 45 minutes. You walked to work for 45 minutes. You worked for 4 hours. You ate lunch for 35 minutes. What time is it now?
4. What two numbers have a product of 16 and a sum of 8?
5. What two numbers have a product of −16 and a sum of 0?
6. A ticket to the baseball game is $12.50. You have to pay for parking at the game, which costs $5. Once you are in the ballpark, you buy popcorn for $3.00, ice cream for $3.15, and a drink for $2.50. How much did cost you to go to the baseball game?
7. You are going for a walk. You walk north for 5 blocks, turn around, and walk south for 16 blocks. How many blocks are you from where you started? State if you are north or south.
8. Evaluate: $\frac{3}{15} \div \frac{1}{5}$.

Answers:

1. Acute (The angle is less than 90°.)
2. $3\frac{2}{9}$ (9 × 3 = 27, 29 – 27 = 2, so there are three 9s in 29, with 2 left over)
3. 2:35 pm (8:30 + 0:45 = 9:15 am, 9:15 + 0:45 = 10:00 am, 10:00 + 4:00 = 14:00 (subtract 12 hours to get the same in am/pm form) = 2:00 pm. 2:00 + 0:35 = 2:35 pm)
4. 4, 4 (The factor pairs of 16 are (1, 16), (2, 8), (4, 4). Only 4 + 4 = 8.)
5. 4, –4 (From the above question, only 4 − 4 = 0.)
6. $26.15 (Add all the dollars (12 + 5 + 3 + 3 + 2 = 25), add all the cents (0.50 + 0.15 + 0.50 = 1.15). Add the dollars and cents (25 + 1.15 = $26.15).)
7. 11 blocks south (Think about being on the origin of a graph. You move positive 5, then negative 16, so 5 − 16 = −11.)
8. $1 \left(\frac{3}{15} \times \frac{5}{1} = \frac{15}{15}\right)$

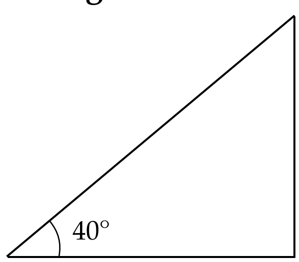
Part B: Exploring Sine and Cosine

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

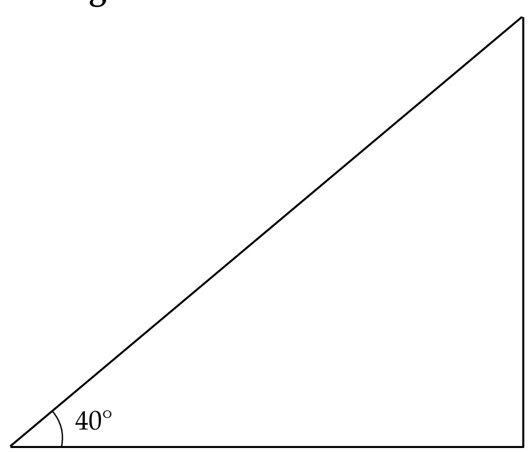
In each of the following similar right triangles, label the sides as hypotenuse, opposite, or adjacent in relation to the specified 40° angle. Measure each side to the nearest tenth of a centimetre, and complete the following chart.

Triangle	Opposite	Adjacent	Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	Value of Ratio	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	Value of Ratio
1							
2							
3							

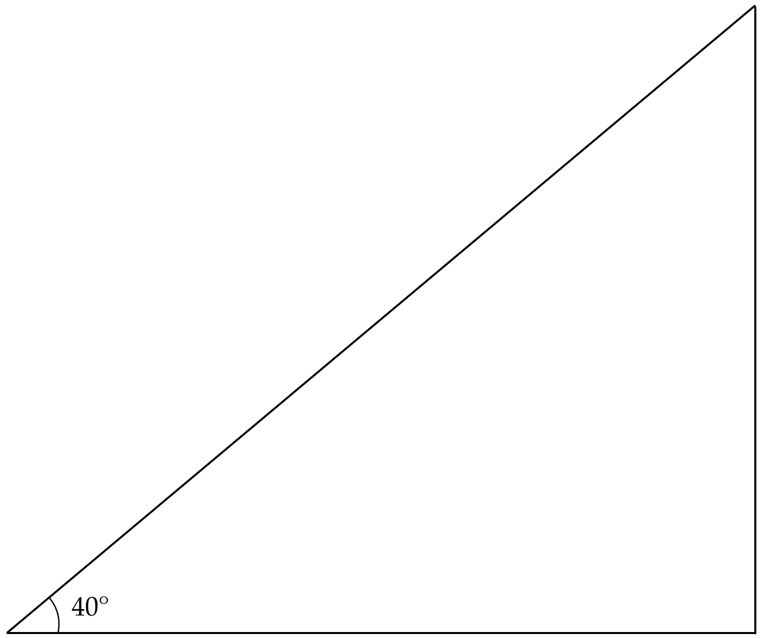
Triangle 1:



Triangle 2



Triangle 3



Using your calculator, determine the value of the following:

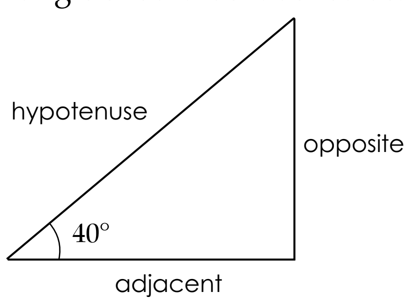
Sin 40° = ____________________ Cos 40° = _____________________

Based on your measurements and calculations, what can you conclude about the trigonometric ratios of sine and cosine? Write each as the ratio of the appropriate side lengths:

Sine = ______________________ Cosine = ______________________

Answers:

Each triangle should be labelled as:



Triangle	Opposite	Adjacent	Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	Value of Ratio	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	Value of Ratio
1	3.2	3.8	5.0	$\frac{3.2}{5.0}$	0.64	$\frac{3.8}{5.0}$	0.76
2	5.7	6.8	8.9	$\frac{5.7}{8.9}$	0.64	$\frac{6.8}{8.9}$	0.76
3	8.3	9.9	12.9	$\frac{8.3}{12.9}$	0.64	$\frac{9.9}{12.9}$	0.76

Using your calculator, determine the value of the following:

Sin 40° = 0.6427876097 Cos 40° = 0.7660444431

Learning Activity 4.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Solve for g: $3 - g = 15$
2. What two numbers have a product of −27 and a sum of 6?
3. Your mom is buying ice cream for your family. The store has tiger, bubblegum, vanilla, and chocolate flavours. Your mom doesn't like bubblegum, your dad doesn't like chocolate, and you don't like vanilla. Which ice cream will your mom buy?
4. What is the formula for tangent?
5. Arrange the numbers from largest to smallest: $\frac{1}{2}$, 0.29, $\frac{3}{4}$, 0.65, 0.34.
6. Evaluate: $\sqrt[3]{125}$.
7. An octave in music includes 8 notes. If you were to go up half an octave, how many notes is that?
8. When in Venice, you notice a great store on the other side of the street. Because the roads are water in Venice, you need to walk to the nearest bridge. The nearest bridge is 6 m away from you, and the 'road' is 2 m wide. How far do you have to walk to get to the store?

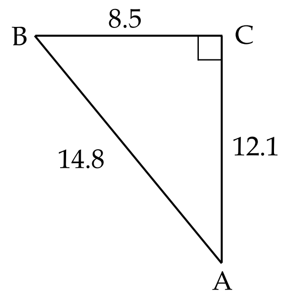
Answers:

1. −12 ($-g = 15 - 3$, but the g is still negative so you need to multiply both sides by −1.)
2. 9, −3 (The factor pairs of 27 are (1, 27) and (3, 9). The signs of the factors have to be one positive and one negative in order to produce −27. 9 − 3 = 6 so the 3 is negative.)
3. Tiger ice cream
4. $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
5. $\frac{3}{4}$, 0.65, $\frac{1}{2}$, 0.34, 0.29
6. 5
7. 4 (8 ÷ 2)
8. 14 m (You walk 6 m on the bridge, 2 m across, then 6 m back to the store so 6 + 2 + 6.)

Part B: Applying Sine and Cosine

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the ratio for the sine, cosine, and tangent of angle A.



Answer:

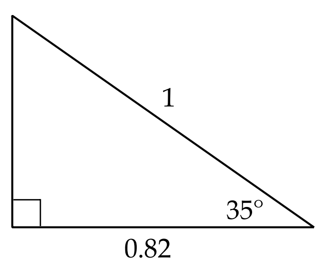
$$\sin A = \frac{8.5}{14.8}, \cos A = \frac{12.1}{14.8}, \tan A = \frac{8.5}{12.1}$$

2. Explain what cos 35° means. Include a diagram.

Answer:

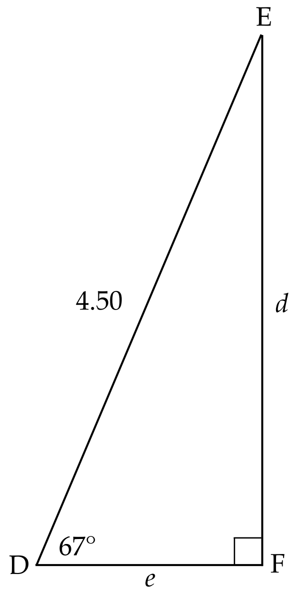
$\cos 35° = 0.8191520443$

The cosine ratio is $\frac{\text{adjacent}}{\text{hypotenuse}}$. This means that in a right triangle, the leg adjacent to the 35° angle will be approximately 0.82 units if the hypotenuse is 1 unit. The ratio of the lengths of the $\frac{\text{adjacent}}{\text{hypotenuse}}$ sides will be $\frac{0.82}{1}$.

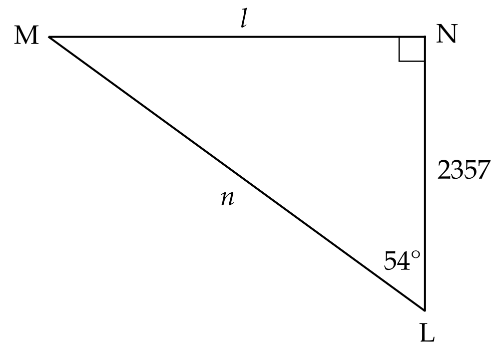


3. Solve the following triangles. Round your answers to the nearest tenth.

a)



b)



Answers:

a) $\angle E = 90 - 67 = 23°$

$$\sin 67° = \frac{d}{4.5}$$
$$d = 4.142271841$$

$$\sin 67° = \frac{e}{4.5}$$
$$e = 1.758290078$$

$\angle D = 67°$ $\quad\angle E = 23°$ $\quad\angle F = 90°$

$d = 4.14$ $\quad e = 1.76$ $\quad f = 4.50$

b) $\angle M = 90 - 54 = 36°$

$$\cos 54° = \frac{2357}{n}$$
$$n = \frac{2357}{\cos 54°}$$
$$n = 4009.967911$$

$$\cos 54° = \frac{l}{2357}$$
$$l = 3244.132187$$

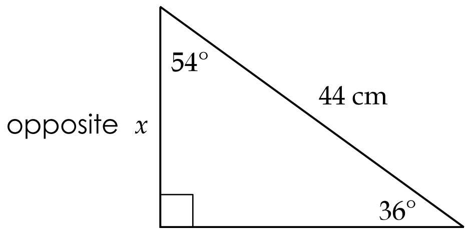
$\angle L = 54°$ $\quad\angle M = 36°$ $\quad\angle N = 90°$

$l = 3244$ $\quad m = 2357$ $\quad n = 4010$

4. A right triangle has angles of 36° and 54°. Find the length of the shortest leg if the hypotenuse is 44 cm.

Answer:

The shortest leg will be opposite the smallest angle.

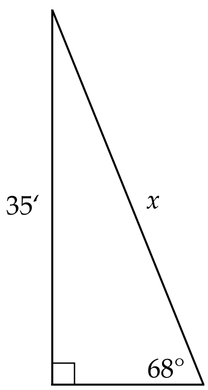


$$\sin 36° = \frac{x}{44}$$
$$x = \sin 36°(44)$$
$$x = 25.8625511$$

The shortest leg is about 25.9 cm long.

5. A brace cable supporting a streetlight pole is attached at the top of the pole, 35 feet above the ground. How long must the cable be if it makes a 68° angle with the ground?

Answer:



$$\sin 68° = \frac{35}{x}$$
$$(x)\sin 68° = 35$$
$$x = \frac{35}{\sin 68°}$$
$$x = 37.74871599$$

The cable must be 37.7 feet long.

Learning Activity 4.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. In the novel trilogy *Lord of the Rings*, there are many rings of power. Nine were given to men, seven to the dwarves, three to the elves, and one to the evil mastermind. In total, how many rings of power are there?
2. Evaluate: $x^{\frac{-1}{6}}$.
3. What would be a good range for a graph comparing a woman's age to her weight?
4. A checkerboard has 8 squares along its length and 8 squares along its width. How many squares altogether are on the board?
5. The squares on a checkerboard alternate between black and white. How many squares are black?
6. Solve for h: $h + 12 = 32$.
7. What is the formula for cosine?
8. What two numbers have a product of 21 and a sum of 22?

Answers:

1. 20 ($9 + 1 = 10$, $7 + 3 = 10$, $10 + 10$)
2. $\dfrac{1}{\sqrt[6]{x}}$
3. 4 to 400 pounds (Weight is the dependent variable, so the range is the weight. When you are born you do not weigh 0 pounds, and the majority of women do not exceed 400 pounds.)
4. 64 (8×8)
5. 32 ($64 \div 2$)
6. $h = 20$
7. $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
8. 1, 21 (The only other factor pair is (3, 7), which cannot add up to 22.)

Part B: Inverse Trig Ratios

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the measure of the angle whose trigonometric ratio is given below. Round your answer to the nearest tenth of a degree.
   a) $\cos B = 0.4556$
   b) $\sin Y = 0.5$
   c) $\tan P = 6.78$
   d) $\cos T = 0.0013$

   Answers:

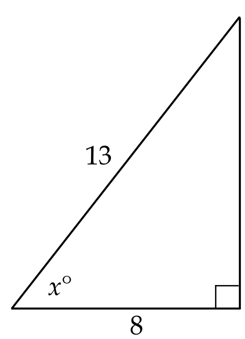
   a) $\cos\ B = 0.4556$
   $$\angle B = \cos^{-1}(0.4556)$$
   $$\angle B = 62.9°$$

   b) $\sin\ Y = 0.5$
   $$\angle Y = \sin^{-1}(0.5)$$
   $$\angle Y = 30°$$

   c) $\tan\ P = 6.78$
   $$\angle P = \tan^{-1}(6.78)$$
   $$\angle P = 81.6°$$

   d) $\cos\ T = 0.0013$
   $$\angle T = \cos^{-1}(0.0013)$$
   $$\angle T = 89.9°$$

2. Find x to the nearest degree.



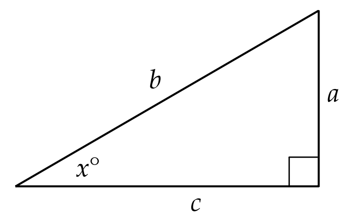
   Answer:
   $$\cos\ x = \frac{\text{adjacent}}{\text{hypotenuse}}$$
   $$\cos\ x = \frac{8}{13}$$
   $$x = \cos^{-1}(0.6153846154)$$
   $$x = 52°$$

3. Find x to the nearest degree if:

   a) $a = 5$ and $b = 10$

   b) $b = 20$ and $c = 4$

   c) $a = 4$ and $c = 2$



   Answers:

   a) $\sin x = \frac{5}{10}$

   $x = \sin^{-1}\left(\frac{1}{2}\right)$

   $x = 30°$

   b) $\cos x = \frac{4}{20}$

   $x = \cos^{-1}(0.2)$

   $x = 78°$

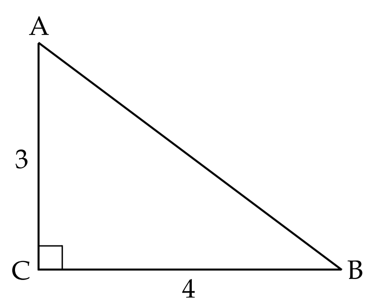
   c) $\tan x = \frac{4}{2}$

   $x = \tan^{-1}(2)$

   $x = 63°$

4. You are given 3 measures in each of the following triangles. Find the value of the three missing measure in each triangle. Round your answer to 2 decimal places.

   a)



   Answer:

   Given: $b = 3$, $a = 4$, $\angle C = 90°$

   Use the Pythagorean Theorem

   $c^2 = a^2 + b^2$

   $c^2 = 3^2 + 4^2$

   $c^2 = 9 + 16$

   $c^2 = 25$

   $c = \pm 5$

   but c is a length, so $c = 5$

   $\tan \text{B} = \frac{3}{4}$

   $\text{B} = \tan^{-1}\left(\frac{3}{4}\right)$

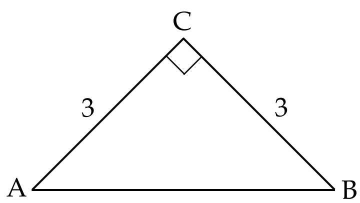
   $\text{B} = 36.87°$

   $\tan \text{A} = \frac{4}{3}$

   $\text{A} = \tan^{-1}\left(\frac{4}{3}\right)$

   $\text{A} = 53.13°$

b)



Answer:

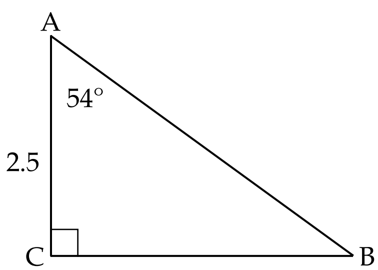
Given: $a = 3$, $b = 3$, $C = 90°$

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 3^2$$
$$c^2 = 9 + 9$$
$$c^2 = 18$$
$$c = 4.24$$

$$\tan A = \frac{3}{3}$$
$$= 1$$
$$A = \tan^{-1}(1)$$
$$= 45°$$

$$\tan B = \frac{3}{3}$$
$$= 1$$
$$B = \tan^{-1}(1)$$
$$= 45°$$

c)



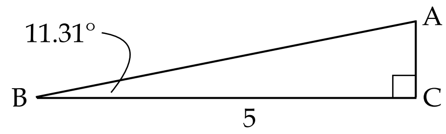
Answer:

Given: $b = 2.5$, $\angle A = 56°$, $\angle C = 90°$

$$\angle B = 180° - 90° - 54°$$
$$= 36°$$

$$\cos 54° = \frac{2.5}{c}$$
$$c = \frac{2.5}{\cos 54°}$$
$$c = 4.25$$

$$\tan 54° = \frac{a}{2.5}$$
$$a = \tan 54° \cdot 2.5$$
$$a = 3.44$$

d) 

Answer:

Given: $a = 5$, $\angle B = 11.31°$, $\angle C = 90°$

$\angle A = 180° - 90° - 11.31°$

$= 78.69°$

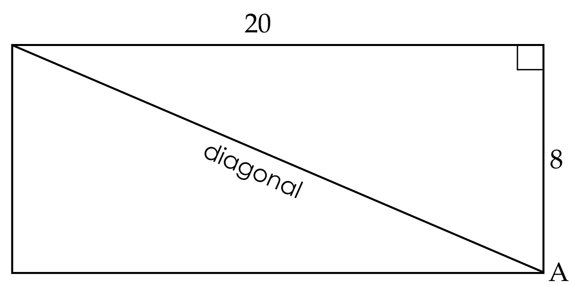
$$\tan 11.31° = \frac{b}{5}$$
$$b = 1.00$$

$$\cos 11.31° = \frac{5}{c}$$
$$c = \frac{5}{\cos 11.31°}$$
$$c = 5.10$$

5. Find the measure of the angle between the diagonal and the shorter side of a rectangle that is 20 cm long and 8 cm wide.

   Answer:

   Draw a diagram.



The 20 cm side is opposite the desired angle and the 8 cm side is adjacent to the desired side.

$$\tan A = \frac{20}{8} = 2.5$$

Use the $\tan^{-1}$ function to find the angle associated with this ratio.

$A = \tan^{-1} 2.5$

$\angle A = 68.2°$

Learning Activity 4.6

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. The surface area of a globe is 100π cm^2. What is the radius of the globe?
2. If the ratio comparing the size of the globe to the Earth is 1 cm: 800 miles, what is the approximate radius of the Earth (use the answer from above)?
3. If you buy a shirt for $8 and jeans for $32, how much do you spend altogether?
4. If 20% of your class of 20 students has already done the social studies project, how many people are done their project?
5. What is the formula for sine?
6. The volume of a pyramid is 5 m^3. What is the volume of a prism with the same base and height?
7. Is the following data continuous or discrete?

   The amount of money in your bank account over a period of time.
8. Is the number 1 prime, composite, or neither?

Answer:

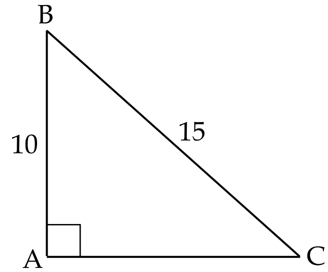
1. 5 cm ($SA_{\text{sphere}} = 4\pi r^2$ so $100 \div 4 = 25$, $\sqrt{25} = 5$)
2. 4000 miles (5 cm $\times$ 800)
3. $40
4. 4 people (10% of 20 = 2, 10% $\times 2 = 2 \times 2 = 4$)
5. $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
6. 15 m^3 (A prism has 3 times the volume of a pyramid with the same dimensions, so $5 \times 3 = 15$.)
7. Discrete (You do not deposit money as a continuous stream, but in specific amounts.)
8. Neither

Part B: Applying Trig Ratios

Remember, these questions are similar to the ones that will be on your assignments and examinations. So, if you were able to answer them correctly, you are likely to do well on your assignments and examinations. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. In ΔABC, ∠A = 90°, a = 15 m, and c = 10 m. Find the measures of the other two angles.

   Answer:

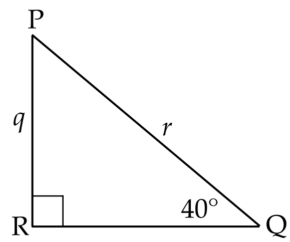


$$\sin \angle C = \frac{10}{15} = 0.\overline{66}$$
$$\angle C = 41.8°$$

$$\therefore \angle B = 180° - (90° + 41.8°)$$
$$\angle B = 48.2°$$

2. In ΔPQR, ∠R = 90°, ∠Q = 40°, and p = 30. Find the lengths of the other two sides.

   Answer:

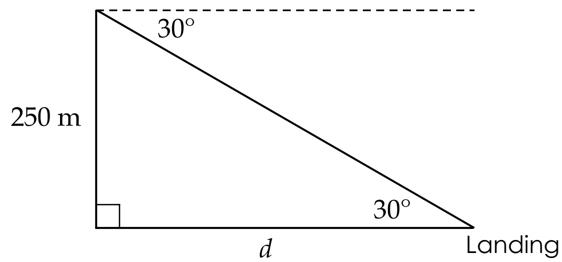


$$\tan 40° = \frac{q}{30}$$
$$30 \tan 40° = q$$
$$q = 30(0.8391) = 25.2$$

$$\cos 40° = \frac{30}{r}$$
$$r = \frac{30}{\cos 40°}$$
$$r = \frac{30}{0.766} = 39.2$$

3. A hot-air balloon, advertising for a real estate company, is 250 m above ground level. The angle of depression from the balloon to the landing area is 30°. What is the distance along the ground from a point beneath the balloon to the landing area?

Answer:



$$\tan 30° = \frac{250}{d}$$

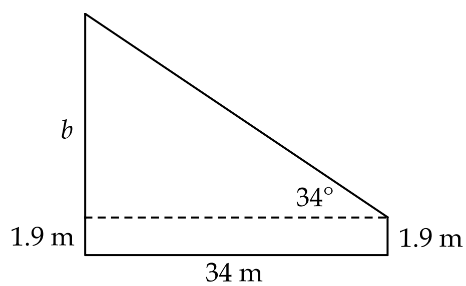
$$d = \frac{250}{\tan 30°} = \frac{250}{0.57735}$$

$$d = 433.0127019$$

The distance from the point beneath the balloon to the landing area is approximately 433 m.

4. A surveyor uses a transit (a device used to measure angles) to determine that the angle of elevation from the transit to the top of a building is 34°. The horizontal distance from the top of the transit to the building is 34 m, and the height of the transit is 1.9 m. How high is the building?

Answer:



$$\tan 34° = \frac{b}{34}$$

$$34 \tan 34° = b$$

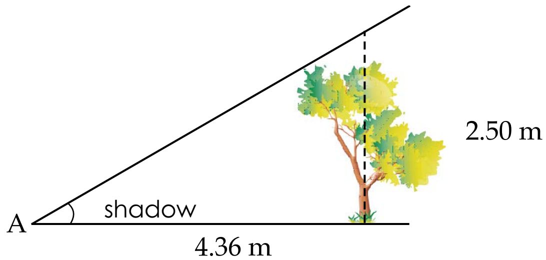
$$34(0.6745) = b$$

$$22.9 = b$$

The building is 22.9 m + 1.9 m = 24.8 m high.

5. A tree 2.50 m tall casts a shadow 4.36 m long. Calculate the angle of elevation from the ground to the sun to the nearest degree.

Answer:



$$\tan \angle A = \frac{2.50}{4.36}$$

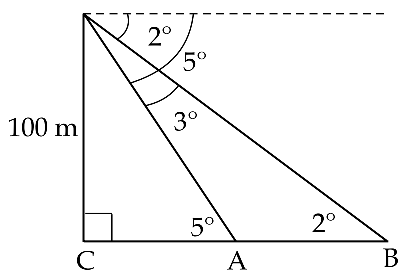
$$\angle A = \tan^{-1}\left(\frac{2.50}{4.36}\right)$$

$$\angle A = 30^\circ$$

6. From the top of a 100 m tall fire tower, a fire ranger observes two fires: one at an angle of depression of 5°, and the other at an angle of depression of 2°. Assuming that the fires and the tower are in a straight line, how far apart are the fires if they are

a) on the same side of the tower?

Answer:



$$\tan 2^\circ = \frac{100}{BC}$$

$$BC = \frac{100}{\tan 2^\circ} = \frac{100}{0.03492}$$

$$BC = 2863.63$$

$$\tan 5^\circ = \frac{100}{AC}$$

$$AC = \frac{100}{\tan 5^\circ} = \frac{100}{0.8749}$$

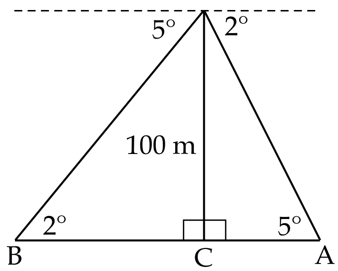
$$AC = 1143.01$$

Therefore, the distance between the fires is

AB = 2863.63 − 1143.01 = 1720.62 m.

b) on opposite sides of the tower?

Answer:



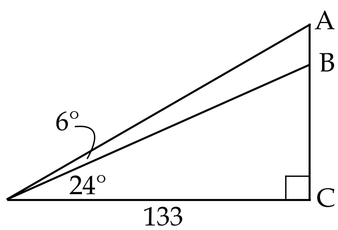
As in part (a), BC = 2863.63 and AC = 1143.01.

$\therefore$ AB = 2863.63 + 1143.01 = 4006.64 m.

If they are on opposite sides, the fires are 4006.63 m apart.

7. From a point 133 m away from the centre of the base of the Manitoba Legislative Building, the angle of elevation to the top of the torch of the Golden Boy is 30°. From the same point, the angle of elevation to his feet is 24°. Find the height of the Golden Boy.

Answer:

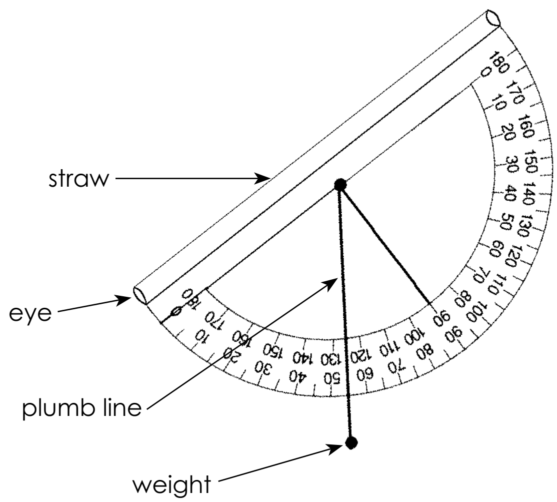


$$\tan 24° = \frac{BC}{133}$$
$$BC = 133 \tan 24°$$
$$BC = 133(0.44523)$$
$$BC = 59.2$$

$$\tan 30° = \frac{AC}{133}$$
$$AC = 133 \tan 30°$$
$$AC = 133(0.57735)$$
$$AC = 76.8$$

The height of the Golden Boy is 76.8 − 59.2 = 17.6 m.

8. A clinometer is a device used to measure angles of elevation or depression. You can construct a clinometer by taping a sighting straw to the straight edge of a plastic protractor. Create a plumb line from string or fish line with a small weight at one end (e.g., a small metal nut or a paper clip). Attach the string so that it swings freely from the centre of the horizontal zero line along the straight edge (see diagram). Holding the straw horizontal, the plumb line should pass over the 90° mark.



Using your clinometer, an imperial measure device (such as a tape measure or yardstick) and trigonometry, show by the use of a sketch how you would verify that a basketball hoop is 10 feet off the floor of a gymnasium.

Answer:

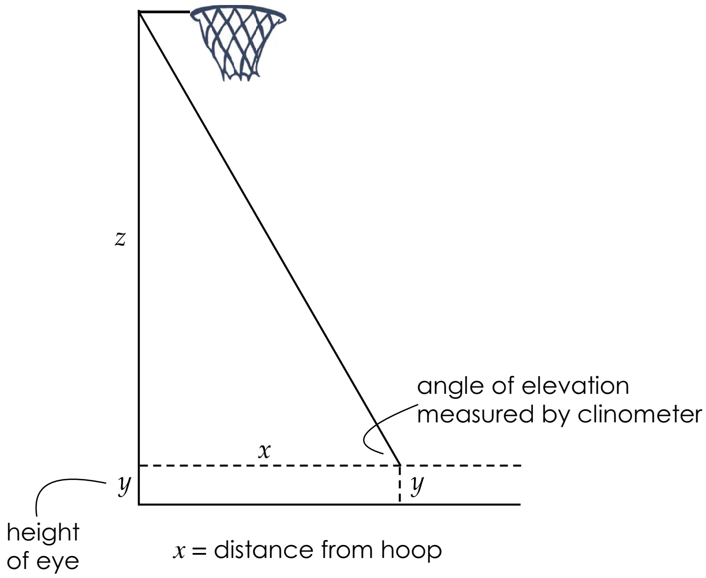
Measure the distance from the floor to your eye (y).

Walk away from a point directly below the basketball hoop and measure the distance you are away from that point (x).

Look through the straw of the clinometer until the basketball hoop is sighted.

Note the angle the plumb line is away from the 90° mark on the protractor (e.g., if the plumb line reads 60°, then the angle is 90° − 60° = 30°). This angle is the angle of elevation of the basketball hoop from the horizontal line at eye level.

The following diagram will indicate the resulting right-angled triangle to be measured.



Using trigonometry to find z, $z + y$ should be 10 feet.

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 5
Relations and Functions

Module 5: Relations and Functions

Before We Get Started

Before you begin working through the next part of the course, now would be a good time to look at the Resource Sheet that you used for your midterm. Answer the following questions in the space below, so that when you are making your final exam Resource Sheet, you will make changes that you think will be helpful.

- During the midterm exam, did you wish you had included more information on your Resource Sheet?
- Was there any information on your Resource Sheet that you did not need?
- Was there any information on your Resource Sheet that you did not remember or didn't understand when you looked at it during the midterm exam?

Introduction

You began this course back in Module 1 by exploring graphs and relations. You looked at the relationships between sets of data and described their characteristics (including slope, intercepts, domain, and range) using graphs, words, equations, and ordered pairs. This module will build on that knowledge, focusing on a special relation called a "function." You will learn how to express domain and range using different types of notation, and use functional notation to express linear equations. As well, you will review graphing linear functions

Assignments in Module 5

When you complete Module 6, you will submit your Module 5 assignments, along with your Module 6 assignments, to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 5.1	Relations and Functions
2	Assignment 5.2	Domain and Range Notation
3	Assignment 5.3	Functional Notation

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions that follows to help you with preparing your resource sheet for the material in Module 5. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 1 to 8.

Resource Sheet for Module 5

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Lesson 1: Functions

Lesson Focus

In this lesson, you will

- ❑ consider domain and range as the input and output of a relation
- ❑ represent relations as ordered pairs, a table of values or mapping, a graph, or with a rule or equation
- ❑ determine if a relation represents a function

Lesson Introduction

In Module 1, you initially identified a linear relationship in data by graphing ordered pairs on a scatterplot and checking to see if they could be joined by a straight line. By now, you can recognize a linear relation from its equation, and from it determine the slope and intercepts of the line. This lesson will concentrate on a specific type of relation—a function.

What Is a Function?

Representing Relations

A relation is any set of ordered pairs (x, y) describing the relationship between two variables. The ordered pair is named in a specific order. The possible values of the first variable, x, are called the elements of the domain of the relation. The values for the second variable, y, determine the range of the relation.

Having the definition of relation on your Resource Sheet will probably be helpful.

Example 1

State the domain and range of the following relation.

$A = \{(1,2)\ (3,4)\ (5,6)\ (7,8)\}$

Solution:

The domain is all possible x-values. $D = \{1, 3, 5, 7\}$

The range is all valid y-values. $R = \{2, 4, 6, 8\}$

A relation may also be written in a table of values, as an equation or rule or it may be graphed.

Example 2

This type of question would be seen in an applied mathematics course, but the methods demonstrated are used in both pre-calculus and applied mathematics.

The provincial sales tax in Manitoba is 7% of the list price. Write a relation to illustrate the relationship between the list price and the cost of the item to the consumer. State the domain and range of this relation.

Solution:

Some ordered pairs for this relation are (\$10.00, \$10.70), (\$20.00, \$21.40), (\$0.70, \$0.75). The possible list price and cost pairs are endless. To simplify, you could specify the relationship using a formula with variables to represent the relationship. The cost is the list price plus 7% of the list price.

$$C = L + 0.07L$$

or

$$C = 1.07L$$

Where C is the cost to the consumer (price after tax) and L is the list price.

The domain of this relation is the set of all possible list prices, and the range is the resulting cost to the consumer when tax is added to the list price. These values will be a subset of positive real numbers.

From the above example it is obvious that the domain (the list price of an item) affects the range (the total cost). You can think of it in terms of input and output. The x-value that is put into the relation determines the y-value that comes out. Organized as a table of values, it may look like this:

Input	Output
L	*C*
$10.00	$10.70
$20.00	$21.40
$0.70	$0.75

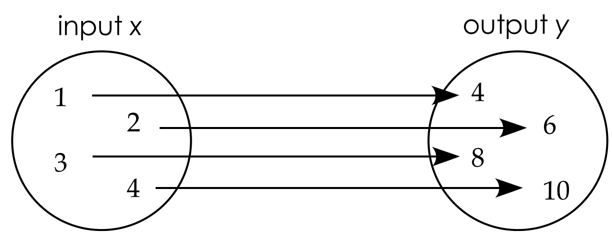
Example 3

Given the following ordered pairs, state the domain and range of the relation.

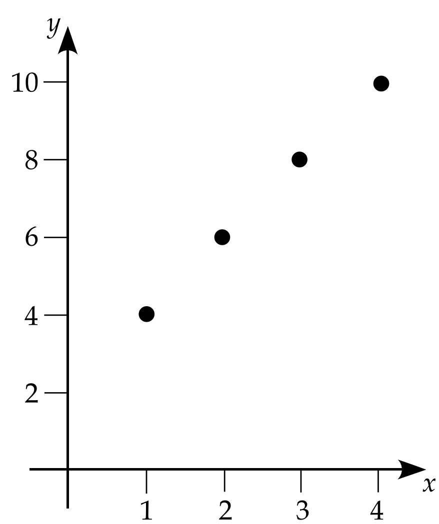
{(1,4) (2,6) (3,8) (4,10)}

Solution:

Functions may also be represented by mapping. Think of mapping as a type of table of values that has arrows showing which input results in a given output.



This relation could also be represented by a graph.

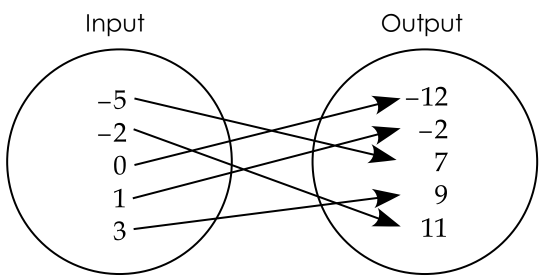


If you wanted to write this as an equation or rule, it could be expressed as $y = 2x + 2$, where $x \in \{1, 2, 3, 4\}$. (The symbol $\in$ means element, so "$x \in \{1, 2, 3, 4\}$" is read as "x is an element of 1, 2, 3, 4" or "x can be 1, 2, 3, 4".)

The second part of the statement reads as "where x is an element of the set $\{1, 2, 3, 4\}$." When you use a rule or equation to represent a function, you must state the domain to indicate which numbers are permissible as values of x in the formula. In this case, only the four natural numbers are part of the domain, as given in the mapping above.

Functions

Consider the relation represented by the following mapping diagram. The arrow connects each domain element with its respective range element. In this example, you can trace only one arrow from each input to exactly one output.



This is a special type of relation called a **function**.

A function is a relation, a set of ordered pairs (x, y), such that for each value of x there is exactly one value of y.

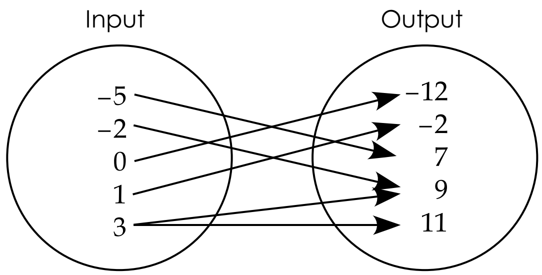
Note that all functions are relations, but only the relations that fit the above definition are considered to be functions.

This information about functions would be useful to have on your Resource Sheet.

Example 4

Determine whether the following three mappings represent functions or non-functions.

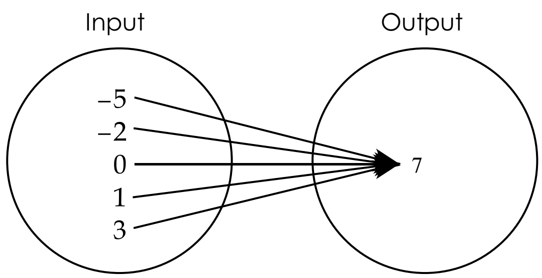
a)



Solution:

This represents a relation but not a function. The input of 3 has two arrows indicating two possible outputs. In a function, each element in the domain must have only one possible range value. This relation is a non-function.

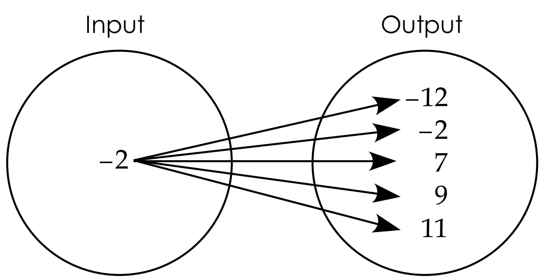
b)



Solution:

Each input in this relation has one and only one output. Even though the value of the range is the same for each element in the domain, it is both a relation and a function.

c)



Solution:

The input has multiple outputs, so while it is a relation, it is a non-function.

Example 5

Determine if the following ordered pairs represent functions or relations.

a) (0, 1) (1, 2) (2, 3) (3, 4) (4, 5)

b) (2, −3) (2, 5) (2, 0) (2, 1) (2, −2)

c) (−−4, −1) (3, −1) (0, −1) (−1, −1) (5, −1)

d) (5, 3) (4, 3) (−3, 3) (0, 5) (−3, −1)

e) (−2, 4) (−1, 1) (0, 0) (1, 1) (2, 4)

Solution:

a) This is a function and a relation because each x-value has only one possible y-value.

b) This is only a relation because the input of 2 has a variety of possible outputs.

c) This is a function and a relation because each element in the domain has only one possible value for its range.

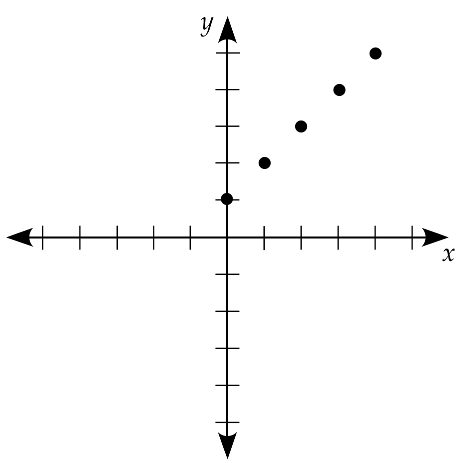
d) This relation is a non-function. The input of −3 has two possible outputs: 3 and −1.

e) This represents a relation and a function. The value of 1 is the output in two ordered pairs (−1, 1) and (1, 1), but each input has only one possible output so it still fits the definition of a function.

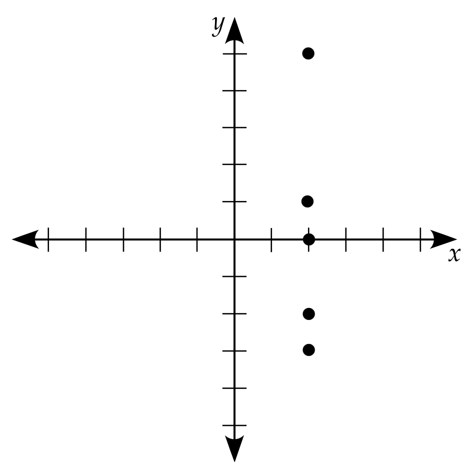
When you are given a mapping or the ordered pairs that represent a relation, you can graph the coordinates and examine what the visual representation of a relation or function may look like.

Here are the graphs of the ordered pairs from Example 5 above.

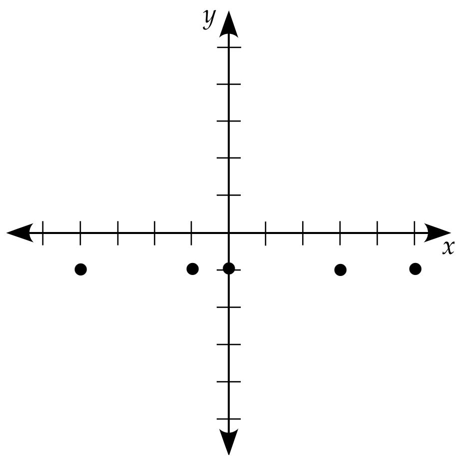
a) Function



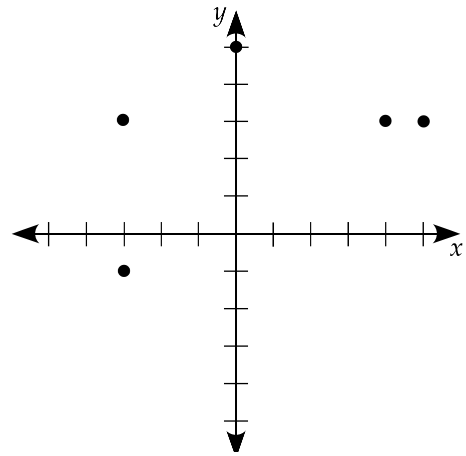
b) Relation



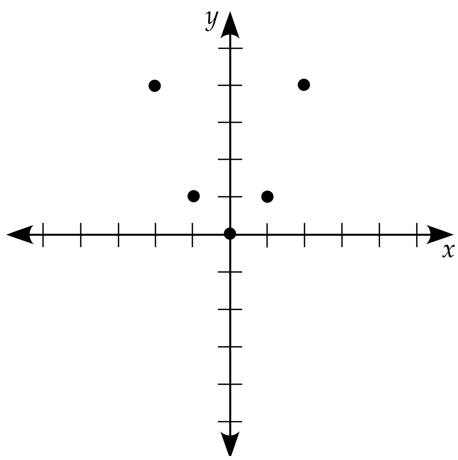
c) Function



d) Relation



c) Function



What do you notice about the graphs that are only relations?

Two or more of the points in the relations can be connected with a vertical line.

If a relation is represented graphically, you can determine whether or not it is a function by using the **vertical line test**.

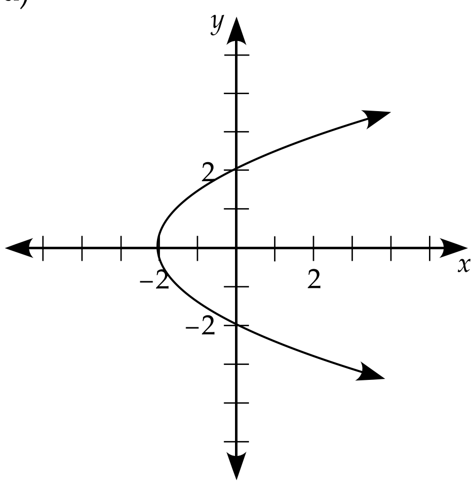
Vertical line test: If a vertical line can be drawn anywhere on the graph and intersect the relation in more than one point, then the graph does **not** represent a function.

It is important that you know how to do the vertical line test, so it may be helpful to have this description on your Resource Sheet.

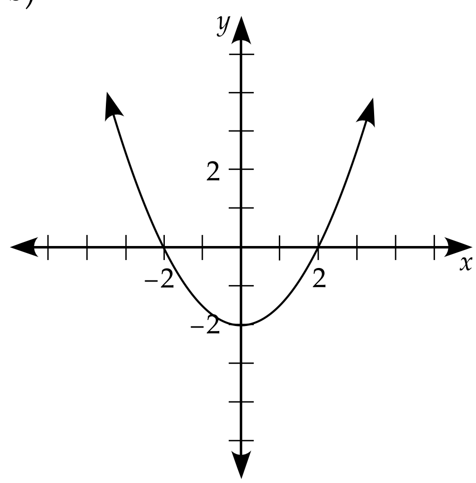
Example 6

Identify which of these graphed equations represents a function.

a)

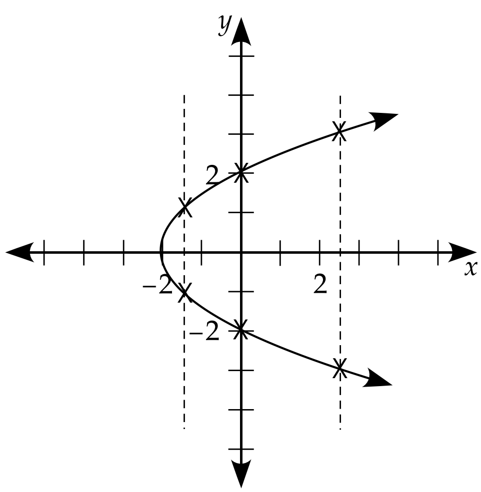


b)



Solution:

Graph (a) is not a function because a vertical line with the equation $x = k$ where $k > -2$ will cross the graph in two places.



For example, the vertical line $x = 0$ (along the y-axis) will intersect with the graph at (0, 2) and (0, −2). These ordered pairs show that the input of 0 has two possible outputs, and the relation is not a function.

Graph (b) represents a function because a vertical line, no matter where it is drawn on the graph, will only ever cross the graph once. A vertical line will never intersect graph (b) in more than one point.

Learning Activity 5.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Which is the independent variable? Your distance from your house compared to how long you've been walking.
2. Simplify: $\left(4x^4\right)^{\frac{3}{2}}$.
3. The slope of a line is −3. What is the slope of a line parallel to this one?
4. Your brother takes acting on Monday night, and plays football on Tuesdays and Thursdays. You have lacrosse on Wednesday and Thursday, and it is your friend's birthday party on Saturday evening. Your parents have a date night every Friday. Will you be able to sit down with your family for dinner this week?
5. Solve for b: $8 + b - 4 = 16$.
6. You are going to bake a cake for your mom's birthday. Because your family is coming over, you decide to make a double recipe. In the original recipe, you need half a teaspoon of vanilla. How much vanilla will you need in the double recipe?
7. Kaitlin types 50 words per minute. It took her 30 minutes to write her essay for English. Assuming she was typing the whole time, how many words are in her essay?
8. When you were 3, your brother's age was double yours. How much older is he?

continued

Learning Activity 5.1 (continued)

Part B: Relations and Functions

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

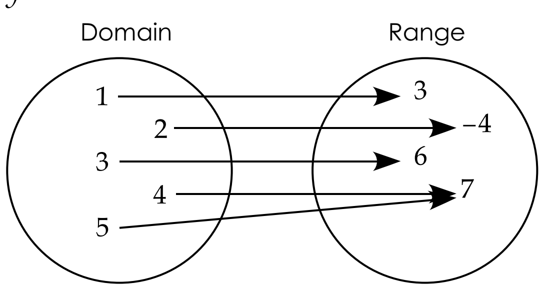
1. In your own words, explain the following terms (based on how they are used in this module).
   a) domain
   b) relation

   You may want to include definitions for these items as well as for range and function on your Resource Sheet.

2. What is the rule that you use to determine whether a relation (mapped or in a table) represents a function?

3. In your own words, explain what the vertical line test is used for and how it works.

4. Determine whether the following relations are functions. Justify your answer.
   a) A = {(1, 2), (1, 7), (1, 8), (1, 9), (1, 10)}
   b) $y = -x + 3$
   c)



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Learning Activity 5.1 (continued)

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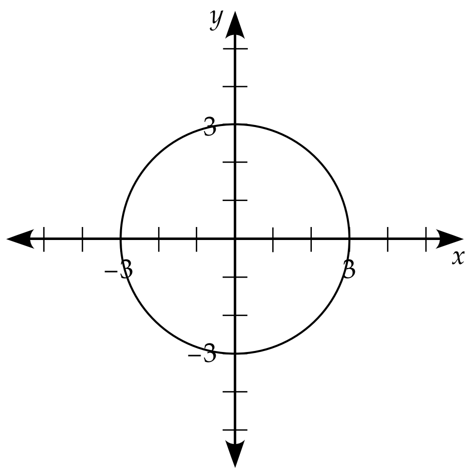

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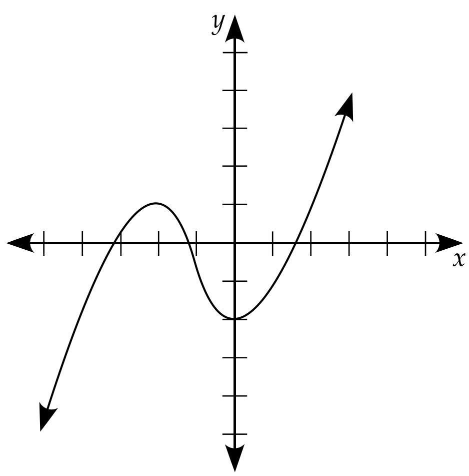

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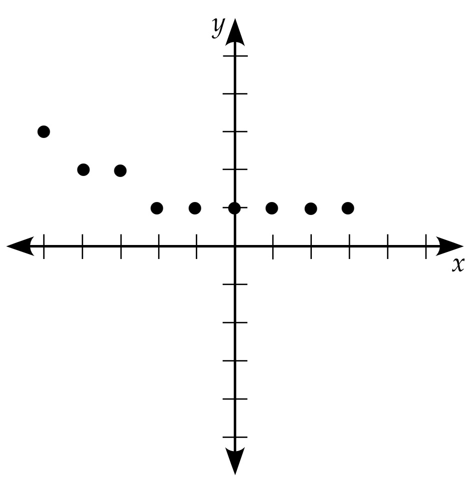

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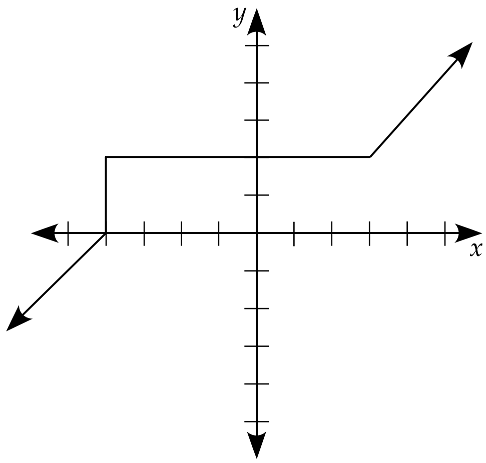


5. The physical education teacher calls out the first name of 6 students in a class (Aiden, Brady, Chad, Chad, Devon, Everett) and numbers them 1 through 6.

   If this information were written as ordered pairs (number, name), would this represent a function or a non-function? Explain.

Lesson Summary

In this lesson, you represented relations as ordered pairs in a table of values or by mapping, as well as with equations or rules and graphs. You determined which relations are functions by examining their inputs (domain) and outputs (range), and by performing the vertical line test on the graphs of equations. In the next lesson, you will use a variety of ways to express the domain and range of a relation.

Assignment 5.1

Relations and Functions

Total Marks = 20

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. In your own words, explain the following terms (based on how they are used in this module).

   a) Range *(1 mark)*

   b) Function *(1 mark)*

2. What is the rule that you use to determine if a set of ordered pairs represents a function or just a relation? *(1 mark)*

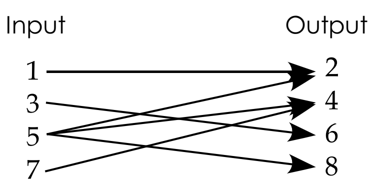
Assignment 5.1: Relations and Functions (continued)

3. Explain, using examples, why all functions are relations but not all relations are functions. *(3 marks)*

4. State if the following mappings, ordered pairs, and graphs represent relations or functions. Justify your answer. *(2 marks each × 7 = 14 marks)*

   a) B = {(2, 1), (7, 1), (3, 1), (4, 1)}

   b) $y = 4x - 2$

   c) 

Assignment 5.1: Relations and Functions (continued)

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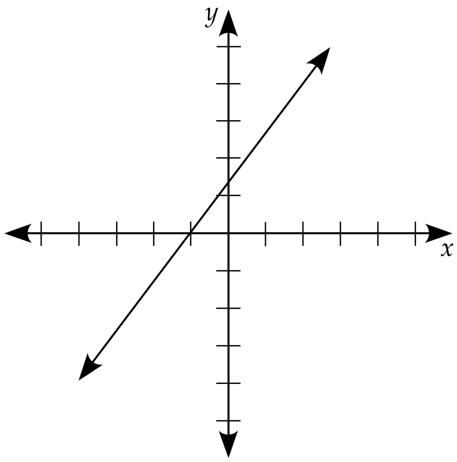

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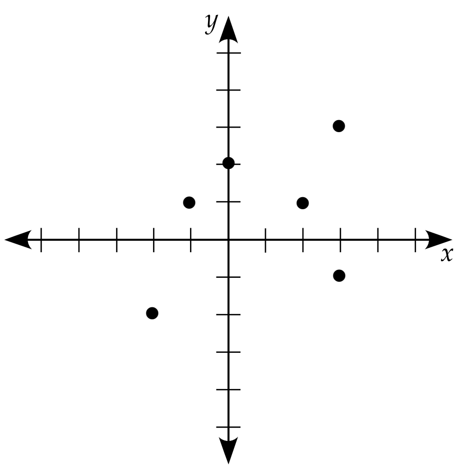
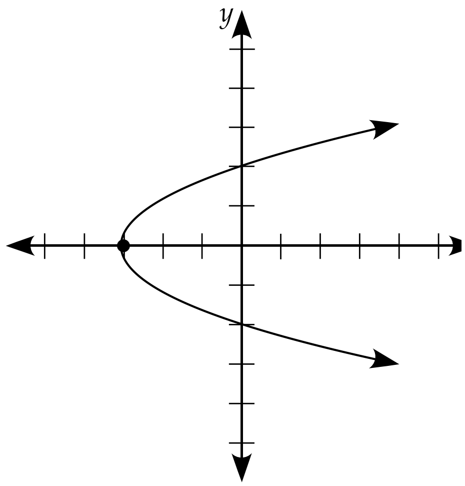
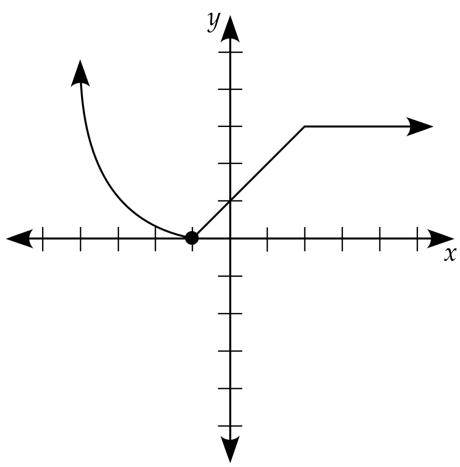


Assignment 5.1: Relations and Functions (continued)

f) 

g) 

Lesson 2: Domain and Range

Lesson Focus

In this lesson, you will

- ❑ express the domain and range of a relation using words or lists of values
- ❑ express the domain and range of a relation using set notation and interval notation

Lesson Introduction

The domain of a function is made up of all valid input values and its range is the resulting output values. In an ordered pair (x, y), the domain is x and the range is y. This lesson will introduce you to other ways of expressing the domain and range of a relation.

Domain and Range of Data

Fisheries and Oceans Canada predicts the heights and times of the high and low tides along the Bay of Fundy, New Brunswick. At Hopewell Cape, low tide is between 2.5 and 4.0 metres high. High tide ranges from 9.8 to 11.9 metres. In the Bay of Fundy it takes on average 6 hours and 13 minutes for the tide to either come in or go out. If you were to plot the time, x and depth, y of the water during one high/low cycle, the **domain** would begin at 0 hours and end about 12.5 hours later. The **range** of depths would be from 2.5 to 11.9 m.

In a context like this, where time and depth can be measured in infinite (limitless) increments, using words to describe the domain and range is possible. Look back at the above paragraph and pick out the words like *between* ____ *and* ____ or *from* ____ *to* ____ or *begins at* ____ and *ends at* ____. These phrases describe domain and range.

If you are given a table of values, ordered pairs, or a graph with only specific points, you can list the finite (limited) points of the domain and range, grouping the individual numbers within brackets.

Example 1

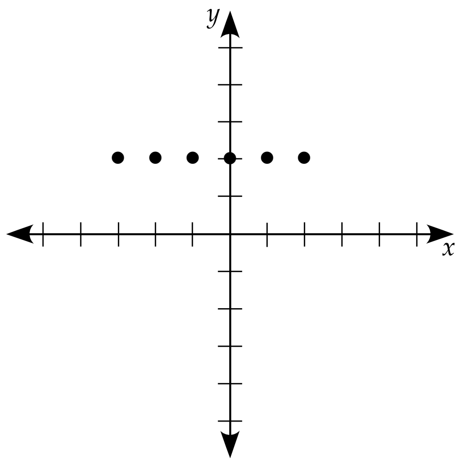
State the domain and range of the given relations. Indicate whether it is a function or a non-function.

a)

X	−3	−2	−1	0	1	2	3
Y	10	5	2	1	2	5	10

b) (2, −1) (3, −2) (3, −3) (4, −4) (4, −5)

c)



Solutions:

a) Function

D: {−3, −2, −1, 0, 1, 2, 3} List the individual points in the domain and range.

R: {10, 5, 2, 1} Omit any repetition.

b) Non-function (relation)

D: {2, 3, 4} The domain is all x-values.

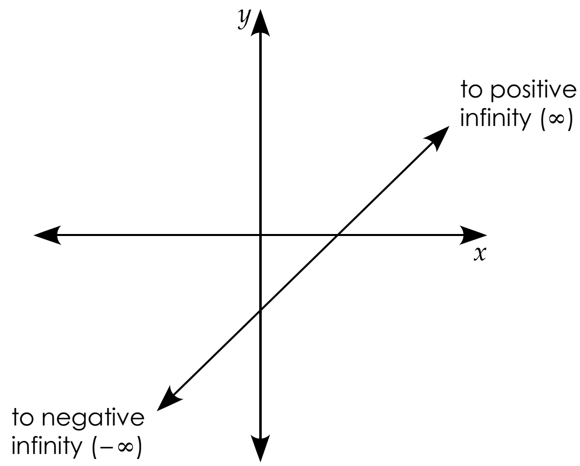
R: {−1, −2, −3, −4, –5} The range is all y-values.

c) Function

D: {−3, −2, −1, 0, 1, 2} Use the coordinate points on the grid.

R: {2}

These kinds of lists of the domain and range values only work if you are given a finite number of points. It would not work for a graph where the line includes all the points along the line, and the line has arrows at the ends, indicating the line continues on to negative and positive infinity.



It would not work if the relation were given as a rule or equation, as it is impossible to write a list of all possible values of x and y in these cases. Another method is needed in these situations.

Set and Interval Notation for Domain and Range

This lesson will introduce you to two styles of notation commonly used to communicate the domain and range of relations. They both say the same thing, but express it in a different way.

Set Notation

Set notation describes which values are included in the domain and range using symbols:

less than:	$<$
greater than:	$>$
less than or equal to:	$\leq$
greater than or equal to:	$\geq$
such that:	$\mid$
is an element of:	$\in$
the real number system:	$\Re$

Do you think you can remember all of these? If not, you will want to include them on your Resource Sheet.

When reading graphs, always read the values from negative to positive. Read x-values across from left to right and y-values from the bottom up.

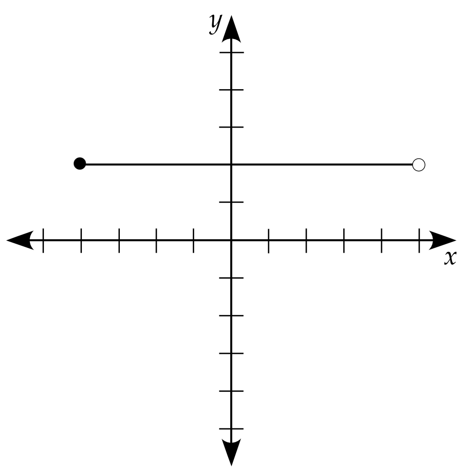
When points on a graph are indicated with solid dots (•), it implies the point is included. If points are hollow (◦), the graph goes up to that point but it is not included in the graph. A line with an arrow indicates the graph includes all points along the line, and it continues on indefinitely in the direction(s) indicated.

If you do not think you will remember what these points mean, you should include them on your Resource Sheet.

Example 2

State the domain and range of the following graphs in set notation.

a)



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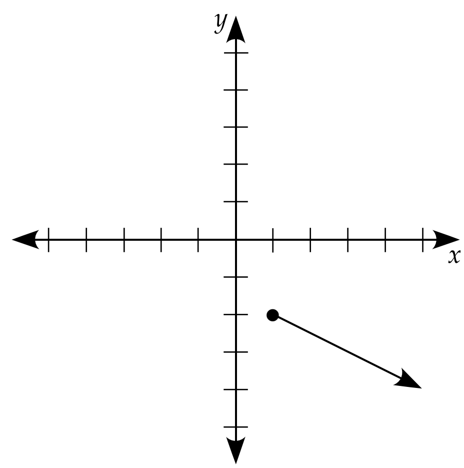

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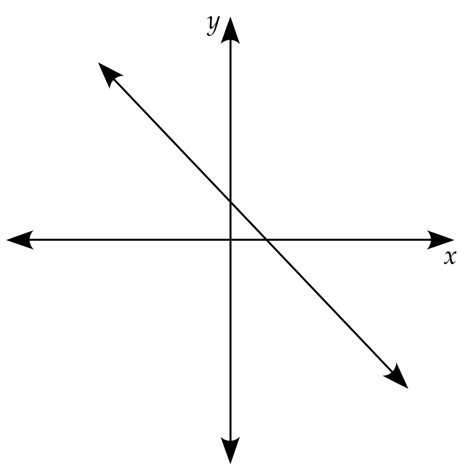


Solution:

a) D: $\{x \mid -4 \le x < 5, x \in \Re\}$ This sentence is read as "x is any value such that -4 is less than or equal to x and x is less than 5 where x is an element of the real number system." Because the dot at $(-4, 2)$ is solid, -4 is included in the domain, but the hollow point at $(5, 2)$ means the graph goes up to 5 but it is not included.

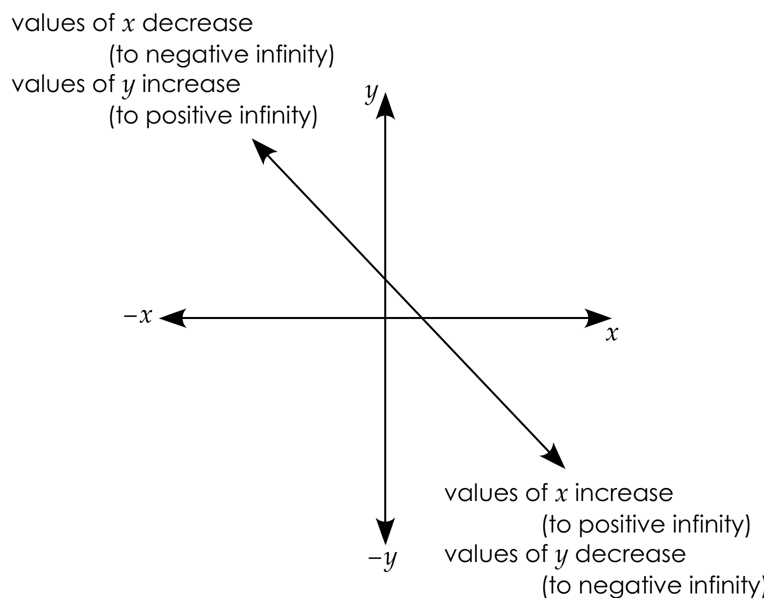
R: $\{y \mid y = 2, y \in \Re\}$ The only valid value of y is 2.

b) The point $(1, -2)$ is included in the graph, and the arrow indicates that all x values greater than that (as the graph moves to the right) and all values of y less than that are included, from negative infinity. Remember to read the graph from negative to positive. (For domain, that is left to right; for range, that is down to up.)

D: $\{x \mid x \ge 1, x \in \Re\}$

R: $\{y \mid y \le -2, y \in \Re\}$

c) This linear function continues on indefinitely in all directions. Its x-values would increase positively in value as the line moves down and negatively as the line moves up. The y-values would increase in value as the line moves up and decrease in value as the line goes down.



Basically, all real values of x and y are valid. This can be written symbolically as:

D: $\{x \mid x \in \Re\}$

R: $\{y \mid y \in \Re\}$

This will be true of all linear functions with no restrictions that are not horizontal lines. Vertical lines are linear relations but are not linear functions.

Interval Notation

Interval notation describes the restrictions on the domain and range using different types of brackets:

starts at but does not include:	(
goes up to but does not include:	)
starts at and includes:	[
goes up to and includes:	]
the positive and negative infinity symbols:	∞ and $-\infty$

These symbols are important to know, so they should be included on your Resource Sheet.

Because infinity is a concept and not an actual value that you can include, the round brackets will always be used with the symbols ∞ and $-\infty$.

The different types of brackets can be mixed.

Summary of Symbols

does not include: $\circ$ > < ( )

includes: • $\geq$ $\leq$ [ ]

Example 3

Refer back to the same three graphs given in Example 2 above, and write the domain and range using interval notation.

Solution:

a) D: $[-4, 5)$ This means that valid values of x include -4 and move in the positive direction up to, but not including, 5.

R: [2] 2 is the only valid output in this function

Note: Generally, interval notation would not be used to describe a range that only has one value. Set notation or describing the range in words would be more appropriate.

b) D: $[1, \infty)$ x may have any value beginning with 1 and moving to infinity.

R: $(-\infty, -2]$ y begins at negative infinity and goes up to and includes -2.

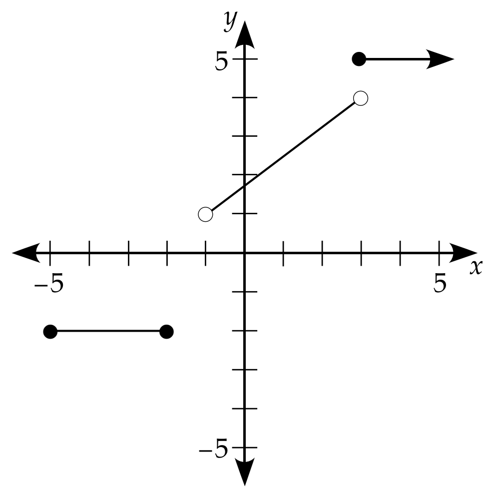
c) D: $(-\infty, \infty)$ The domain of this function is from negative infinity to positive infinity. It can be any real value.

R: $(-\infty, \infty)$ The range is any real value.

The functions you have considered in the above examples have been linear. They could have been described using an equation in the form $y = mx + b$. Functions are not restricted to linear equations. In fact, a function may be a composition of several different equations, depending on what the input is. This is called a piecewise graph. Different parts have different equations.

Example 4

State the domain and range of this piecewise function in both set and interval notation.



Solution:

There are 3 distinct parts to this graph. From $-5 \leq x \leq -2$ y is equal to -2. From $-1 < x < 3$ y ranges in value from 1 to 4. If $x \geq 3$, $y = 5$. There is no valid input for values of x less than -5 or between -2 and -1, so there are no output values here either. Notice the points used at $x = 3$. One is solid, which means 3 is included, and one is hollow, so the graph goes up to that point but does not include 3. In this way, the graph is still a function because a vertical line could be drawn anywhere, including at $x = 3$, and it would still only intersect the graph at one point.

This can be expressed in set notation as

$$\text{D: } \{x \mid -5 \leq x \leq -2 \cup x > -1, x \in \Re\}$$

The ∪ symbol stands for union, and it means that the first statement AND the second statement are part of the domain.

$$\text{R: } \{y \mid y = -2 \cup 1 < y < 4 \cup y = 5, y \in \Re\}$$

In interval notation

$$\text{D: } [-5, -2] \cup (-1, \infty)$$

$$\text{R: } [-2] \cup (1, 4) \cup [5]$$

Note: Again, interval notation is not usually used when the range only includes one output value.

Learning Activity 5.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Solve for z: $3 \times 7 = z - 4$.
2. Complete the following pattern: 60, 75, ____, 105, ____.
3. What two numbers have a product of 32 and a sum of 18?
4. Is $\sqrt{72}$ a rational or irrational number?
5. Your cousin has twice as many stuffed animals as she has dolls. She has half as many movies as she has stuffed animals. How many dolls does your cousin have if she has 4 movies?
6. Carrie keeps her shoes in their boxes so that they don't get ruined. In her closet, she has 5 piles of boxes and each pile is 4 boxes high. How many pairs of shoes does Carrie have?
7. You are in a sandbox that takes up an area of 4 m^2. Convert to centimetres.
8. Express the fraction as a decimal: $4\frac{3}{5}$.

Part B: Domain and Range Notation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Describe the domain and range of the following situation in words:

   A 4 L pail is being filled with water from a tap that flows 125 mL per second.

continued

Learning Activity 5.2 (continued)

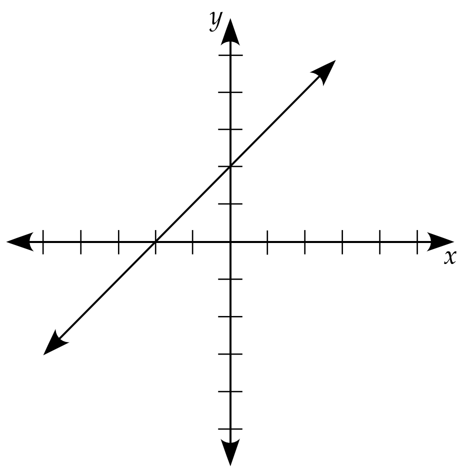
2. Describe the domain and range of the following situation using lists of inputs and outputs.

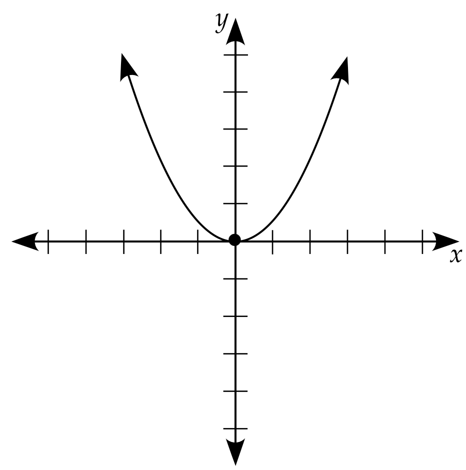
   Pizzas can be ordered in small 9 in., medium 14 in., or large 20 in. sizes and cut into 6, 8, or 12 pieces, respectively.

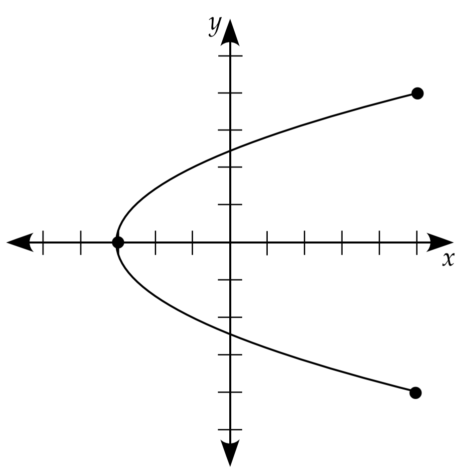
3. Write a possible domain and range of the following in set notation. Explain your answer.

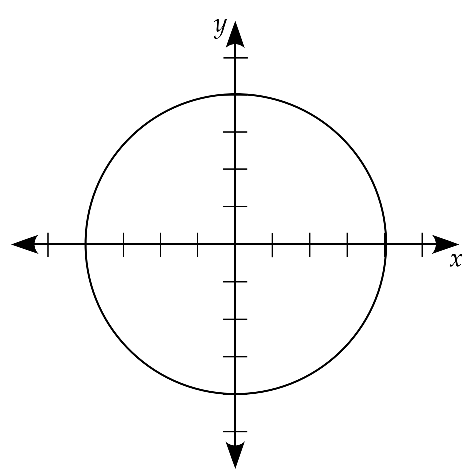
   There are 350 students in your school and orders for yearbooks are being taken by the printing company.

4. Express the domain and range of the following graphs of equations using both set and interval notation.

a) 

b) 

c) 

d) 

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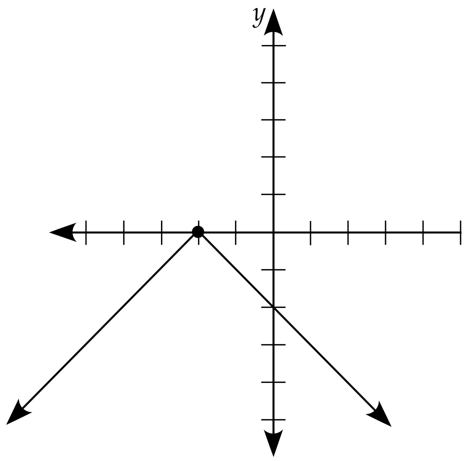

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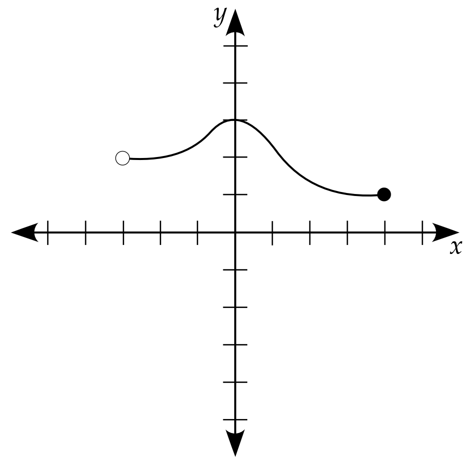


Lesson Summary

Being able to communicate in a variety of ways is a critical life skill. In mathematics, it is important to be confident in your abilities to say the same thing in different ways, as well as to be able to understand when things are written in different notations. This lesson explained how to express domain and range using words, lists, set notation, and interval notation. The next lesson will describe how to express the equations of linear functions in a new way—using functional notation.

Assignment 5.2

Domain and Range Notation

Total Marks = 30

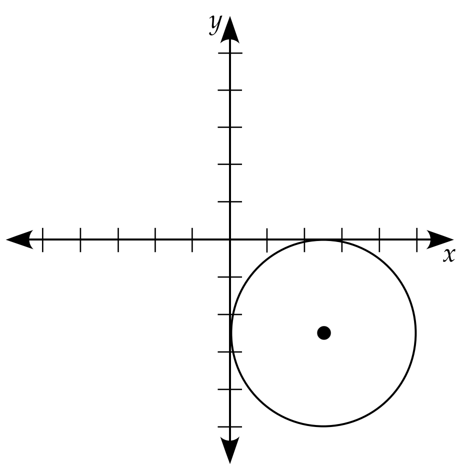
Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. On a package of candy-coated chocolates, it states the box contains 147 grams. In reality, the mass can vary between 145 g and 150 g. You may find between 63 and 69 pieces of candy in any given box. Explain which variable represents the domain and range, and state them as lists of values. *(3 marks)*

2. Use words to describe a reasonable domain and range for the number of people painting a house and the time required to finish the job. Explain your answer. *(3 marks)*

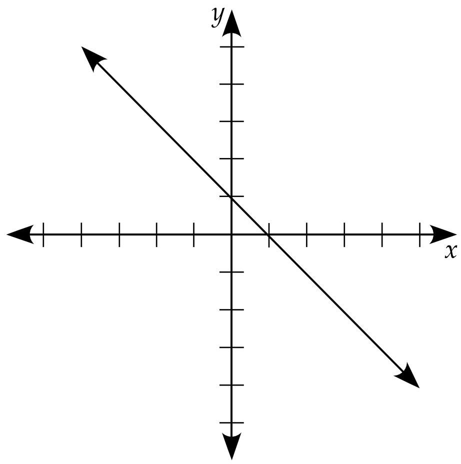
3. Express the domain and range of the following graphs of equations in both set and interval notation. *(4 marks each × 6 = 24 marks)*

a)



Assignment 5.2: Domain and Range Notation (continued)

b)



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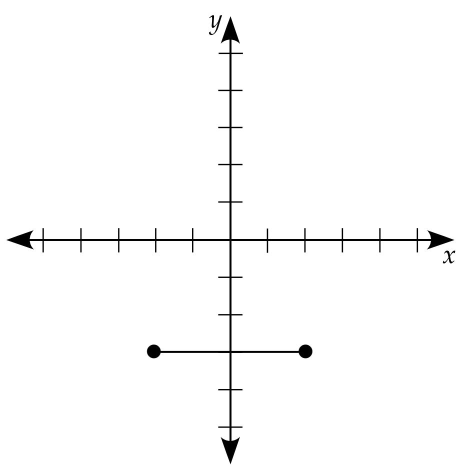

Assignment 5.2: Domain and Range Notation (continued)

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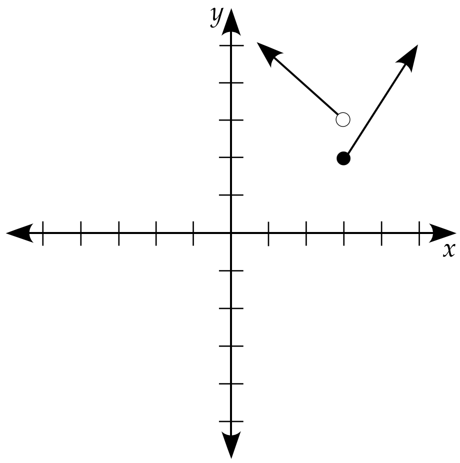

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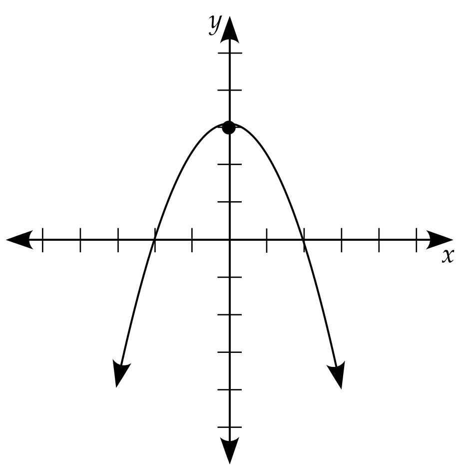

Assignment 5.2: Domain and Range Notation (continued)

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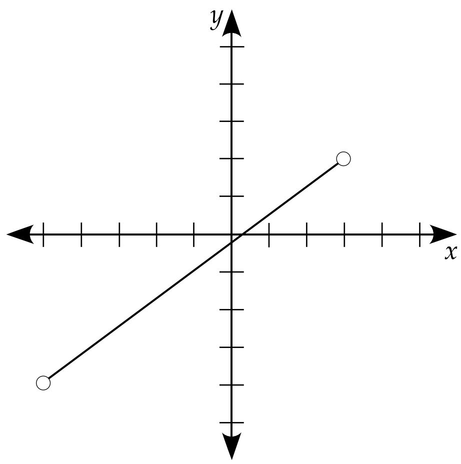


Notes

LESSON 3: GRAPHING FUNCTIONS IN FUNCTIONAL NOTATION

Lesson Focus

In this lesson, you will

- ❑ express the equation of a linear function in two variables using functional notation
- ❑ express a function given in functional notation as an equation of a linear function
- ❑ determine the range value, given a domain value for a linear function
- ❑ determine the domain value, given a range value for a linear function
- ❑ sketch the graph of a linear function expressed in functional notation

Lesson Introduction

Functional notation is just another way of expressing something you already know! You can write and use linear equations in the form of $y = mx + b$, as well as $ax + by + c = 0$. Writing a linear equation in functional notation is sort of like writing your name in cursive and printing it in block letters. They mean the same thing, but they look different. Similarly, when you graph a linear equation as compared to graphing a function in functional notation, you are doing basically the same thing. This lesson will highlight similarities in how you express and graph linear equations using functional notation.

Graphing a Relation

Using Rules to Express the Relationship between *x* and *y*

Rules or equations are one way to describe how inputs and outputs of a function are related.

Example 1

Given the following ordered pairs, determine the rule or equation that describes how x and y are related. Write it in the form $y = mx + b$.

$(-2, -6)\ (-1, -5)\ (0, -4)\ (1, -3)\ (2, -2)\ (3, -1)$

Solution:

Step 1: Consider what operation(s) you would have to perform on x to result in the given value of y. In this case, each output is 4 less than the input.

Step 2: Write the possible rule as an equation. So the rule could be written as $y = x - 4$.

Step 3: Check this rule by substituting some of the points to see if they make true statements

$(-1, -5)$		
y		x –– 4
-5		$-1 - 4$
-5	$=$	-5

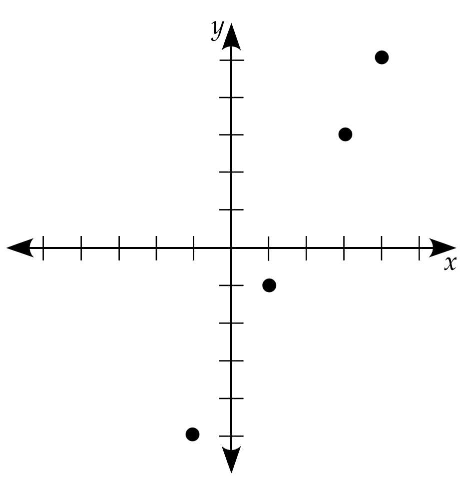
This is a true statement

$(3, -1)$		
y		$x - 4$
-1		$3 - 4$
-1	$=$	-1

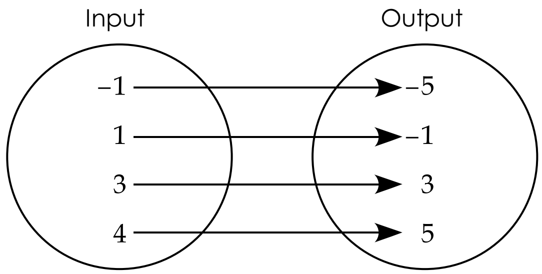
This is a true statement

Try another one.

Rules or equations can also be determined if you are given a table of values, a graph, or a mapping diagram.



If you find it more challenging to determine the equation from a graph, write the coordinates of the points in a table of values or mapping diagram. This may help you see the relationship between input and output more clearly.



Try a variety of operations or combinations of operations to determine the rule. In this case, the function f, $y = 2x - 3$ works to map the inputs onto each of the given outputs.

Using Functional Notation

Another way to indicate that −1 is mapped onto −5 by the rule or function f (given above) is to use functional notation.

$$f(-1) = -5$$

This means when you input a –1 into the function f, the output is −5.

$$f(4) = 5$$

This means that if you substitute the number 4 into the rule f in the place of x, the answer is 5. So the general notation is $f(x) = y$.

You have done this kind of substituting and solving before. You are just using a different notation now.

This notation is only used for relations that are functions.

Often, the relation given by a rule such as $y = 2x - 3$ is written as $f(x) = 2x - 3$ to indicate that the relation is a function.

Example 2

Write the following relations in functional notation. Use the letter name indicated for the function.

a) $y = 3x + 7$ — label the function g

b) $y = -\frac{1}{2}x - 44$ — label the function h

c) $a = -5b$ — label the function k

d) $5x + 2y - 8 = 0$ — label the function P

Solution:

a) $g(x) = 3x + 7$

This is read as "the g of x is equal to three x plus 7."

b) $h(x) = -\frac{1}{2}x - 44$

c) $k(b) = -5b$

Since the variable in the rule was given as b, this function is read as "the k of b is equal to negative five times b."

d) $5x + 2y - 8 = 0$

First, rewrite the equation in slope-intercept form.

$$-2y = 5x - 8$$

Solve for y.

$$\frac{-2y}{-2} = \frac{5x}{-2} - \frac{8}{-2}$$

Next write the equation in functional notation by replacing y with $P(x)$.

$$y = \frac{-5x}{2} + 4$$

$$P(x) = \frac{-5}{2}x + 4$$

If you are told that $t(x) = -6x + 3$ and $t(3) = -15$, this means 3 was used as the input into the function t and the output was -15.

This can be demonstrated by showing the substitution.

$$t(x) = -6x + 3$$
$$t(3) = -6(3) + 3$$
$$t(3) = -18 + 3$$
$$t(3) = -15$$

The coordinate pair $(3, -15)$ belongs to the function t.

Example 3

Given the function $g(x) = 3x + 7$, determine the value of $g(1)$.

Solution:

$g(x) = 3x + 7$

$g(1) = 3(1) + 7$ Step 1: Use brackets to substitute the given input into the function.

$g(1) = 3 + 7$ Step 2: Evaluate.

$g(1) = 10$ Step 3: The answer is the related output.

(1, 10) is a coordinate point belonging to the function g.

Example 4

Use functional notation to complete the following table of values for the function $M(n) = 5n + 2$.

n	$M(n)$
-4	
-2	
0	
	17
	32

Solution:

$M(n) = 5n + 2$ $M(-4) = 5(-4) + 2$ $M(-4) = -20 + 2$ $M(-4) = -18$	$M(n) = 5n + 2$ $M(-2) = 5(-2) + 2$ $M(-2) = -10 + 2$ $M(-2) = -8$	$M(n) = 5n + 2$ $M(0) = 5(0) + 2$ $M(0) = 0 + 2$ $M(0) = 2$	$M(n) = 5n + 2$ $17 = 5(n) + 2$ $17 - 2 = 5n$ $15 = 5n$ $n = 3$	$M(n) = 5n + 2$ $32 = 5(n) + 2$ $32 - 2 = 5n$ $30 = 5n$ $n = 6$

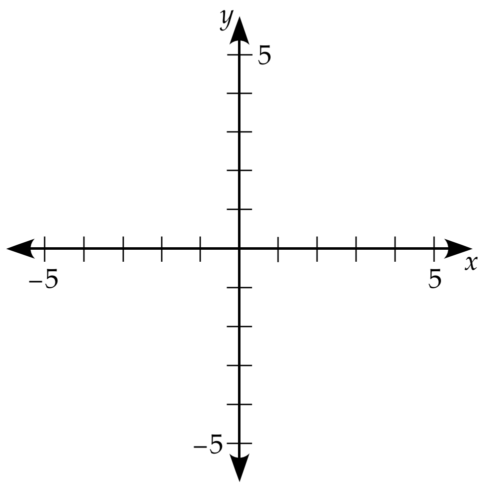
n	$M(n)$
-4	-18
-2	-8
0	2
3	17
6	32

Graphing Linear Functions

Back in Module 1, you graphed linear equations using coordinate points, the slope, and/or intercepts of the line and parallel or perpendicular lines. All these skills apply to graphing linear functions.

Example 5

Write the function $Q(x) = \frac{-3}{2}x + 4$ as a linear equation in two variables and graph it on the axes below. State the slope, and x- and y-intercepts.



Solution:

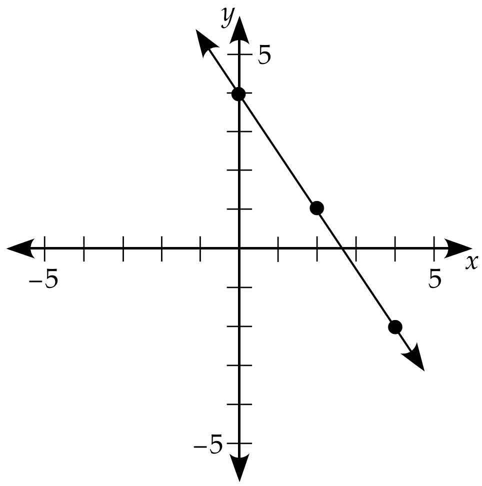
$$Q(x) = \frac{-3}{2}x + 4$$
$$y = \frac{-3}{2}x + 4$$

To graph the linear equation, determine the slope, $\frac{-3}{2}$, and y-intercept, 4, from the equation and use that to plot points on the graph.

The x-intercept is found where $y = 0$.

$$0 = \frac{-3}{2}x + 4$$
$$-4 = \frac{-3}{2}x$$
$$\frac{-8}{-3} = x$$

The x-intercept is found at $\frac{8}{3}$, which is equal to $2\frac{2}{3}$.



Example 6

The graph of a linear function has a slope of $\frac{1}{2}$ and passes through the point (16, –21). Write the equation of this line in functional notation. Find the x-intercept and sketch the graph.

Solution:

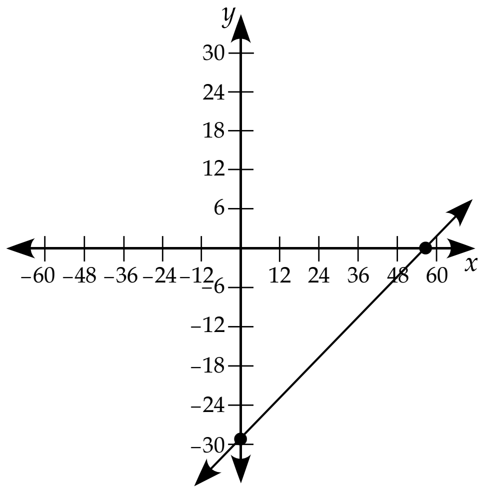
A linear function can be written as

$f(x) = mx + b$ where $x = 16$, $f(x) = -21$ and $m = \frac{1}{2}$. Solve for b.

$$-21 = \frac{1}{2}(16) + b$$
$$-21 = 8 + b$$
$$b = -29$$
$$f(x) = \frac{1}{2}x - 29$$

The x-intercept is where $f(x) = 0$

$$0 = \frac{1}{2}x - 29$$
$$29 = \frac{1}{2}x$$
$$x = 58$$



Adjust the scales on the axes to accommodate the values you need to display on the graph.

Learning Activity 5.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Write the equation as a function: $y = 3x + 5$.
2. Is this relation a function: {(0,1),(1,2), (3,1), (4, 2)}?
3. A square prism has a width of 5 cm and a length of 8 cm. What is the volume?
4. Expand: $5\left(x^6\right)^{\frac{-2}{3}}$.
5. If 36% of 500 is 180, what is 18% of 500?
6. Solve for k: $\frac{6}{k} = 2$.
7. What two numbers have a product of −72 and a sum of 1?
8. What two numbers have a product of −36 and a sum of 0?

continued

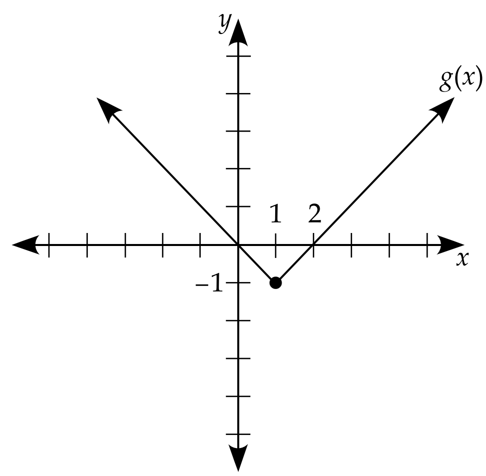
Learning Activity 5.3 (continued)

Part B: Functional Notation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the function $f(x) = 4 - x$, find the value of
   a) $f(1)$
   b) $f(2)$
   c) $f(4)$
   d) $f(-3)$

2. Given a graph of the function g, find the value of
   a) $g(0)$
   b) $g(1)$
   c) $g(2)$
   d) $g(3)$

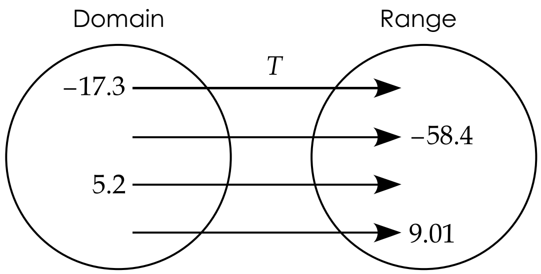


3. Given the function $g = \{(2, 3), (5, -2), (7, 8), (-1, 4)\}$, what is the value of
   a) $g(2)$
   b) $g(5)$
   c) $g(-1)$

4. The x-intercept of a linear equation is where $y = 0$, and the y-intercept is where $x = 0$. Write the linear equation $-9x - y + 17 = 0$ in functional notation, and determine the coordinates of the intercepts. Show your work in functional notation.

continued

5. Complete the following mapping diagram for the function $T(d) = 8.1d - 5.9$. Show your work in functional notation. Write your answers as rational numbers.



6. Sketch a graph of $T(x) = \frac{-5}{3}x + 2.$ State the slope, x- and y-intercepts and the domain and range in interval notation.

7. A gondola ride along the canals in Venice costs $128/hour before sunset. After sunset you must add $50 to the total cost you would pay before sunset. Write two linear equations in functional notation to express
   a) the cost of a gondola ride in daylight hours
   b) the cost of a gondola ride after sunset

Lesson Summary

Functional notation is another way of expressing a relation, which indicates that the equation represents a function. You can use it to show how you solve for intercepts or when substituting values of inputs or outputs to determine the domain or range. Graphing functions uses the same skill set as graphing linear equations.

Assignment 5.3

Functional Notation

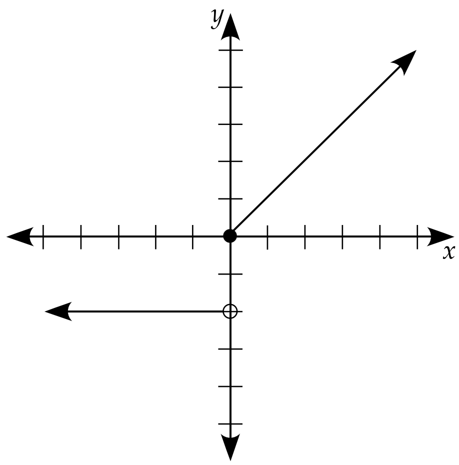
Total Marks = 31

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Given the following 4 functions,

   $f(x) = 3x - 2$

   $g(x)$ represented by the graph



   $h(x) = \{(1, 2), (4, -1), (9, 10), (25, -2)\}$

   $k(x) = 8x$

   find the value of

   a) $f(-9) =$ *(1 mark)*

   b) $g(-3) =$ *(1 mark)*

c) $h(9) =$ *(1 mark)*

---

d) $k(-22) =$ *(1 mark)*

---

e) $f\left(\dfrac{1}{4}\right) =$ *(1 mark)*

---

f) $g(0) =$ *(1 mark)*

---

2. Write the linear equation $4x - 9y = -45$ in functional notation, and determine the x- and y-intercepts. Show your work using functional notation. *(6 marks)*

Assignment 5.3: Functional Notation (continued)

3. Write the following function as a linear equation. *(1 mark)*

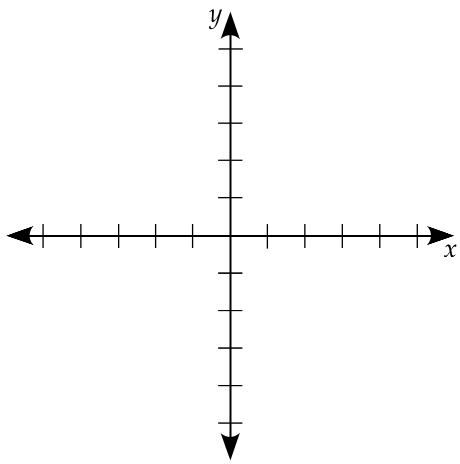
   $B(x) = 5998x - 7563$

4. An element in the range of the function $R(z) = 1 - 3z$ is 52. Find the input associated with this value. *(2 marks)*

Assignment 5.3: Functional Notation (continued)

5. Sketch the graph of the following function: $Q(x) = -\frac{6}{5}x + 16$.

   State the x- and y-intercepts and the domain and range in set notation. *(6 marks)*



Assignment 5.3: Functional Notation (continued)

7. Complete the following Alike and Different Frame to summarize what you have learned about how relations and functions are the same or different. List the characteristics of relations and functions you learned in this module in the appropriate box, and include diagrams and a summary sentence. *(10 marks)*

   Marking Guide: 2 marks per section of the frame

<table>
<tr><th colspan="2">Alike and Different Frame</th></tr>
<tr><td>How are relations and functions ALIKE?
(List characteristics they share or ways they can both be expressed.)</td><td>Picture it.
(Illustrate a way that relations and functions are similar.)</td></tr>
<tr><td>How are relations and functions DIFFERENT?
(How are functions and relations unique.)</td><td>Picture it.
(Illustrate how functions and relations are different.)</td></tr>
<tr><td colspan="2">Summary
(Write a sentence to explain the most important concept you learned about functions and relations.)</td></tr>
</table>

Notes

Module 5 Summary

Congratulations! You have finished the fifth module in the course.

Functions and relations share common characteristics, but certain things set them apart from each other. In this module, you learned why only some relations are functions, but all functions are relations and how to identify them when given ordered pairs, graphs, mapping diagrams, or rules. You expressed the domain and range of functions and relations using words, lists of values, and using set and interval notation. You expressed linear equations in functional notation and vice versa, and used functional notation to solve for values in the domain and range of functions. Using skills and information from Module 1, you applied what you know about graphing linear equations and graphed functions given in functional notation.

In the next module, you will continue to explore and explain relationships: the relationship between different representations of multiplication, and the relationship between multiplying and factoring polynomials. As in the last module, concepts and skills learned earlier in this course will be used and expanded in Module 6 to develop new ideas and abilities.

Submitting Your Assignments

You will not submit your Module 5 assignments to the Distance Learning Unit at this time. Instead, you will submit them, along with the Module 6 assignments, when you have completed Module 6.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 5
Relations and Functions

Learning Activity Answer Keys

Module 5: Relations and Functions

Learning Activity 5.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Which is the independent variable? Your distance from your house compared to how long you've been walking.
2. Simplify: $\left(4x^4\right)^{\frac{3}{2}}$.
3. The slope of a line is −3. What is the slope of a line parallel to this one?
4. Your brother takes acting on Monday night, and plays football on Tuesdays and Thursdays. You have lacrosse on Wednesday and Thursday, and it is your friend's birthday party on Saturday evening. Your parents have a date night every Friday. Will you be able to sit down with your family for dinner this week?
5. Solve for b: $8 + b - 4 = 16$.
6. You are going to bake a cake for your mom's birthday. Because your family is coming over, you decide to make a double recipe. In the original recipe, you need half a teaspoon of vanilla. How much vanilla will you need in the double recipe?
7. Kaitlin types 50 words per minute. It took her 30 minutes to write her essay for English. Assuming she was typing the whole time, how many words are in her essay?
8. When you were 3, your brother's age was double yours. How much older is he?

Answers:

1. How long you've been walking.
2. $8x^6 \left(\left(4x^4\right)^{\frac{3}{2}} = \left(\sqrt{4x^4}\right)^3 = \left(2x^2\right)^3 = 8x^6\right)$
3. −3
4. Yes, on Sunday.
5. 12 ($b = 16 - 8 + 4$)

6. 1 teaspoon $\left(\frac{1}{2} \times 2\right)$

7. 1500 words (50×30)

8. 3 years ($3 \times 2 = 6$, $6 - 3 = 3$)

Part B: Relations and Functions

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. In your own words, explain the following terms (based on how they are used in this module).
   a) domain
   b) relation

   You may want to include definitions for these items as well as for range and function on your Resource Sheet.

   Answers:

   Answers may vary but should explain as indicated below.

   a) The domain of a relation is all possible inputs. It is all starting points or the x-values of the ordered pairs (x, y).
   b) A relation describes how two variables (x, y) are related. If two quantities are related in such a way that a given value in one quantity determines the value of the second quantity, the mathematical model is called a relation.

2. What is the rule that you use to determine whether a relation (mapped or in a table) represents a function?

   Answer:

   A table of values or mapping represents a function if each input has only one possible output. There will be only one arrow from each element in the domain connecting it to a single possible range value.

3. In your own words, explain what the vertical line test is used for and how it works.

   Answer:

   The vertical line test is used to determine whether or not the graph of an equation represents a function. If a vertical line crosses the graph in more than one point, it is not the graph of a function.

4. Determine whether the following relations are functions. Justify your answer.

   a) A = {(1, 2), (1, 7), (1, 8), (1, 9), (1, 10)}

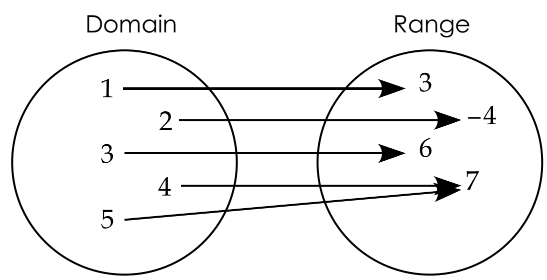
      Answer:

      Non-function—the input of 1 has multiple possible outputs.

   b) $y = -x + 3$

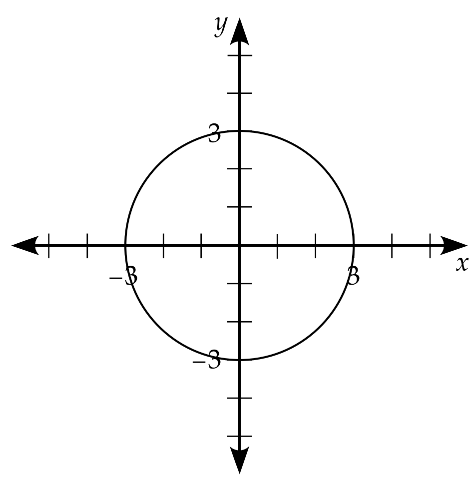
      Answer:

      Function—the equation is a linear function with a slope of −1. It is not a vertical line.

   c) 

      Answer:

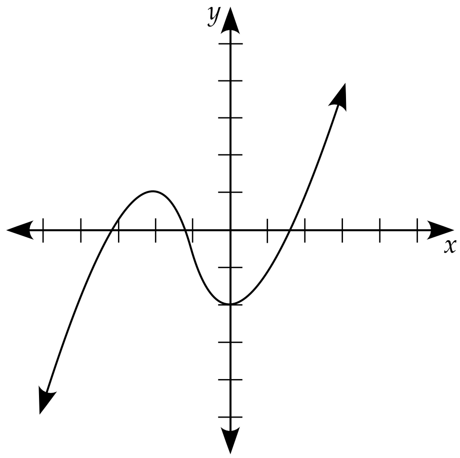
      Function—each element in the domain has exactly one possible value for range.

   d) 

      Answer:

      Non-function—a vertical line drawn on this graph will pass through more than one point.

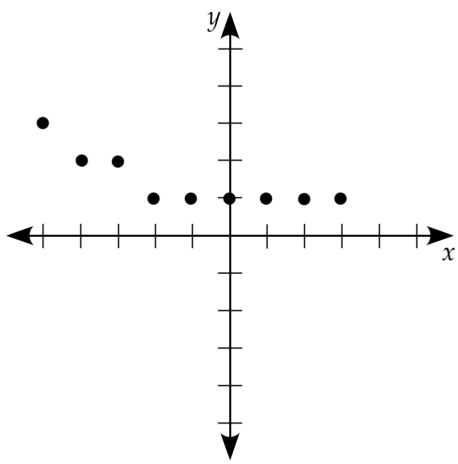
e)



Answer:

Function—a vertical line drawn on this graph will only ever pass through one point.

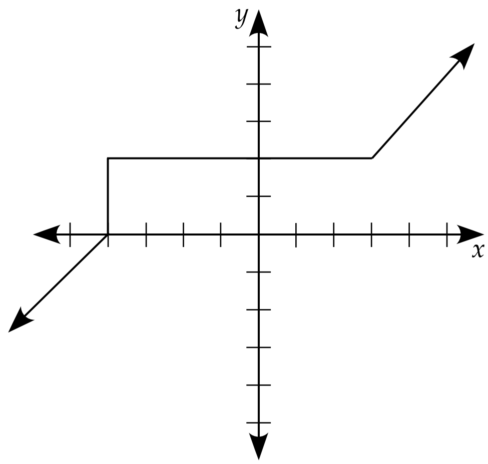
f)



Answer:

Function—a vertical line drawn on this graph will only ever pass through one point.

g)



Answer:

Non-function—a portion of this graph is a vertical line, so it would not pass the vertical line test. A vertical line drawn at $x = -4$ would pass through more than one point of the graph.

5. The physical education teacher calls out the first name of 6 students in a class (Aiden, Brady, Chad, Chad, Devon, Everett) and numbers them 1 through 6.

   If this information were written as ordered pairs (number, name), would this represent a function or a non-function? Explain.

   Answer:

   While it is possible that you may have two or more students with the same first name in a class, each number would only be used once. Written so that the input is *number* and the output is *name*, this would be a function because each input would only have one output.

Learning Activity 5.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Solve for z: $3 \times 7 = z - 4$.
2. Complete the following pattern: 60, 75, ____, 105, ____.
3. What two numbers have a product of 32 and a sum of 18?
4. Is $\sqrt{72}$ a rational or irrational number?
5. Your cousin has twice as many stuffed animals as she has dolls. She has half as many movies as she has stuffed animals. How many dolls does your cousin have if she has 4 movies?
6. Carrie keeps her shoes in their boxes so that they don't get ruined. In her closet, she has 5 piles of boxes and each pile is 4 boxes high. How many pairs of shoes does Carrie have?
7. You are in a sandbox that takes up an area of 4 m^2. Convert to centimetres.
8. Express the fraction as a decimal: $4\frac{3}{5}$.

Answers:

1. $z = 25$ ($21 = z - 4$; $21 + 4 = z$)
2. 90, 120 (pattern is to add 15 each time)
3. 16, 2 (The factor pairs of 32 are (1, 32), (2, 16), (4, 8). $16 + 2 = 18$)
4. Irrational
5. 4 dolls (Because she has double the amount of stuffed animals as she does both dolls and movies, she must have the same number of dolls and movies.)
6. 20 pairs of shoes (5×4)
7. 40000 cm^2 (There are 100 cm in 1 m, so $(1\text{ m})^2 = 1\text{ m}^2 = (100\text{ cm})^2 = 10000\text{ cm}^2$. $10000 \times 4 = 40000\text{ cm}^2$)
8. 4.6 (The four stays on the left side of the decimal because it is a whole number. $\frac{3}{5}$ is equivalent to $\frac{6}{10}$ or 0.6.)

Part B: Domain and Range Notation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Describe the domain and range of the following situation in words:

   A 4 L pail is being filled with water from a tap that flows 125 mL per second.

   Answer:

   There are 1000 mL in a litre, so 4000 mL divided by 125 mL/sec is 32 seconds. The volume depends on the time, so time is the independent variable, x, and volume is y. The domain (valid x inputs) for this function would be from 0 to 32 seconds. The range would be from 0 to 4000 mL or 0 to 4 L

2. Describe the domain and range of the following situation using lists of inputs and outputs.

   Pizzas can be ordered in small 9 in., medium 14 in., or large 20 in. sizes and cut into 6, 8, or 12 pieces, respectively.

   Answer:

   D: {9, 14, 20}

   R: {6, 8, 12}

3. Write a possible domain and range of the following in set notation. Explain your answer.

   There are 350 students in your school and orders for yearbooks are being taken by the printing company.

   Answer:

   The number of yearbooks, y, ordered from a printing company depends on the number of students, x, in a school. It isn't likely that everyone will buy one, but it's unlikely that anybody will order more than one. A possible domain and range may be:

   D: $\{x \mid x \leq 350, x \in W\}$

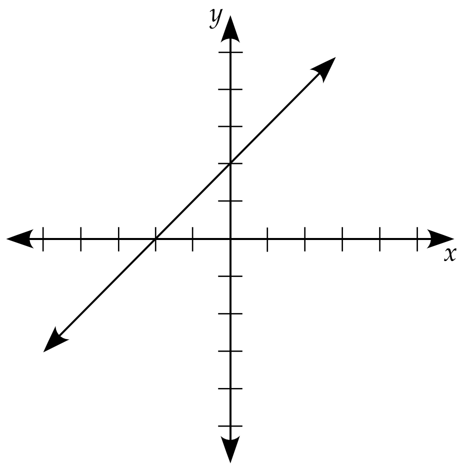
   R: $\{y \mid y \leq 350, y \in W\}$

   The valid input and output values would be part of the whole number system, W, as you cannot have negative or partial values of people or books.

4. Express the domain and range of the following graphs of equations using both set and interval notation.

Answers:

a)



Set Notation:

D: $\{x \mid x \in \Re\}$

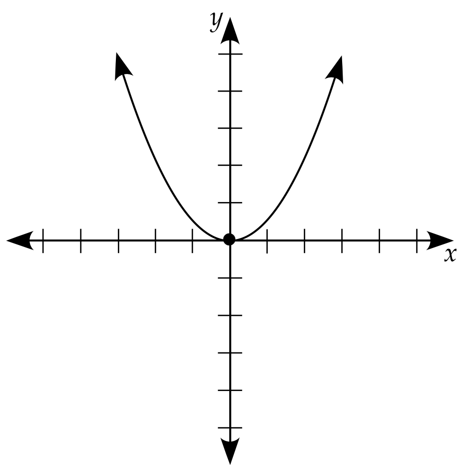
R: $\{y \mid y \in \Re\}$

Interval Notation:

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

b)



Set Notation:

D: $\{x \mid x \in \Re\}$

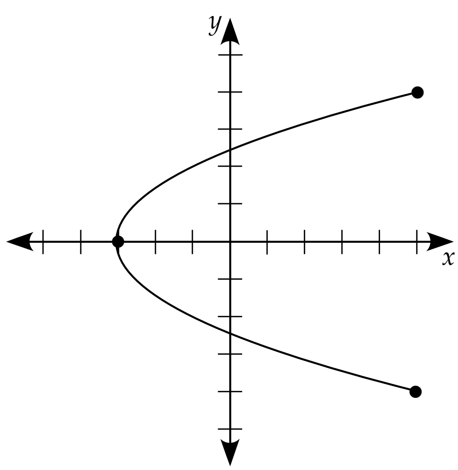
R: $\{y \mid y \geq 0, y \in \Re\}$

Interval Notation:

D: $(-\infty, \infty)$

R: $[0, \infty)$

c)



Set Notation:

D: $\{x \mid -3 \leq x \leq 5, x \in \Re\}$

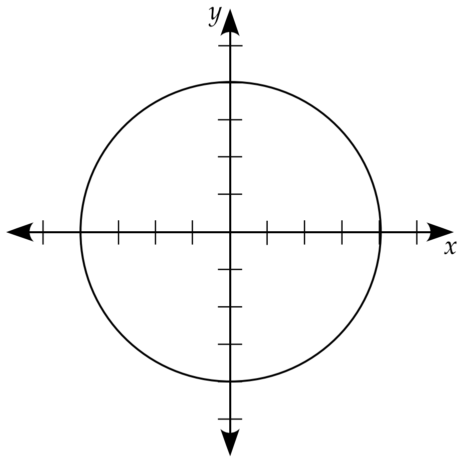
R: $\{y \mid -4 \leq y \leq 4, y \in \Re\}$

Interval Notation:

D: $[-3, 5]$

R: $[-4, 4]$

d)



Set Notation:

D: $\{x \mid -4 \le x \le 4, x \in \Re\}$

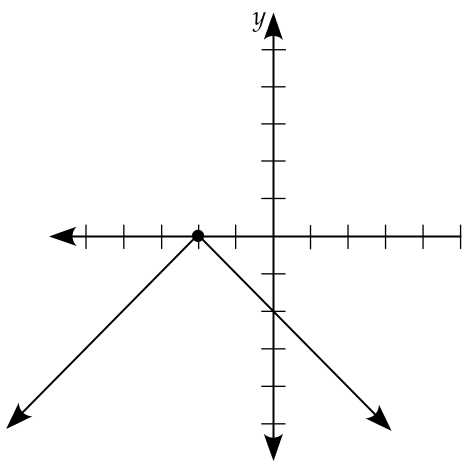
R: $\{y \mid -4 \le y \le 4, y \in \Re\}$

Interval Notation:

D: $[-4, 4]$

R: $[-4, 4]$

e)



Set Notation:

D: $\{x \mid x \in \Re\}$

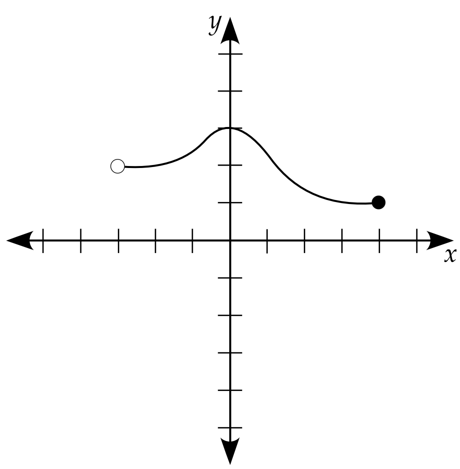
R: $\{y \mid y \le 0, y \in \Re\}$

Interval Notation:

D: $(-\infty, \infty)$

R: $(-\infty, 0]$

f)



Set Notation:

D: $\{x \mid -3 < x \le 4, x \in \Re\}$

R: $\{y \mid 1 \le y \le 3, y \in \Re\}$

Interval Notation:

D: $(-3, 4]$

R: $[1, 3]$

Learning Activity 5.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Write the equation as a function: $y = 3x + 5$.
2. Is this relation a function: {(0,1),(1,2), (3,1), (4, 2)}?
3. A square prism has a width of 5 cm and a length of 8 cm. What is the volume?
4. Expand: $5\left(x^6\right)^{\frac{-2}{3}}$.
5. If 36% of 500 is 180, what is 18% of 500?
6. Solve for k: $\frac{6}{k} = 2$.
7. What two numbers have a product of −72 and a sum of 1?
8. What two numbers have a product of −36 and a sum of 0?

Answers:

1. $f(x) = 3x + 5$
2. Yes, the relation is a function. (No x-value appears more than once.)
3. 200 ($V_{(\text{prism})} = Bh = (5^2) \times 8$)
4. $\frac{5}{x^4}$ $\left(5\left(x^6\right)^{\frac{-2}{3}} = \frac{5}{\sqrt[3]{\left(x^6\right)^2}}\right)$
5. 90
6. $k = 3$
7. 9, −8
8. 6, −6

Part B: Functional Notation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the function $f(x) = 4 - x$, find the value of

   a) $f(1)$

   b) $f(2)$

   c) $f(4)$

   d) $f(-3)$

   Answers:

   a) $f(1) = 4 - (1) = 3$

   b) $f(2) = 4 - (2) = 2$

   c) $f(4) = 4 - (4) = 0$

   d) $f(-3) = 4 - (-3) = 7$

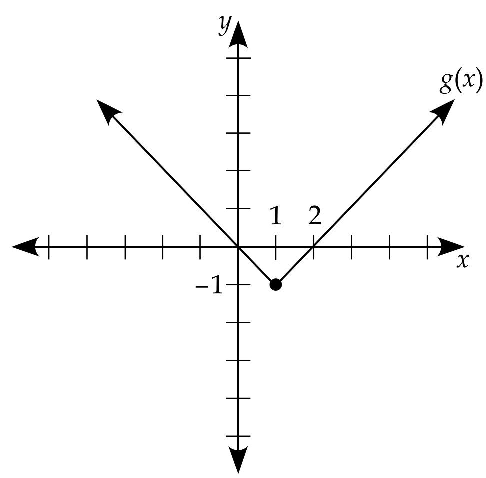
2. Given a graph of the function g, find the value of

   a) $g(0)$

   b) $g(1)$

   c) $g(2)$

   d) $g(3)$



   Answers:

   a) At the point where $x = 0$, $y = 0$, the graph passes through the origin. $g(0) = 0$

   b) $g(1) = -1$

   c) $g(2) = 0$

   d) $g(3) = 1$

3. Given the function $g = \{(2, 3), (5, -2), (7, 8), (-1, 4)\}$, what is the value of
   a) $g(2)$
   b) $g(5)$
   c) $g(-1)$

   Answers:

   a) $g(2) = 3$
   b) $g(5) = -2$
   c) $g(-1) = 4$

4. The x-intercept of a linear equation is where $y = 0$, and the y-intercept is where $x = 0$. Write the linear equation $-9x - y + 17 = 0$ in functional notation, and determine the coordinates of the intercepts. Show your work in functional notation.

   Answer:

   Linear equation in functional notation:

   $$-9x - y + 17 = 0$$
   $$-9x + 17 = y$$
   $$f(x) = -9x + 17$$

   y-intercept is where $x = 0$

   $$f(0) = -9(0) + 17$$
   $$f(0) = 17$$

   The y-intercept is at 17

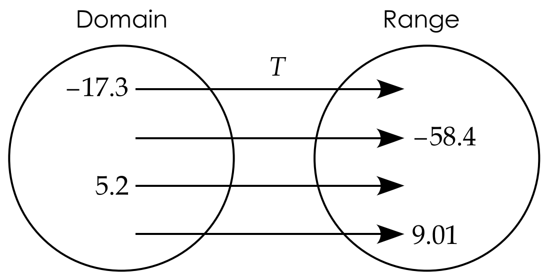
   x-intercept is where $y = 0$

   $$f(x) = -9x + 17$$
   $$0 = -9x + 17$$
   $$-17 = -9x$$
   $$x = \frac{17}{9}$$

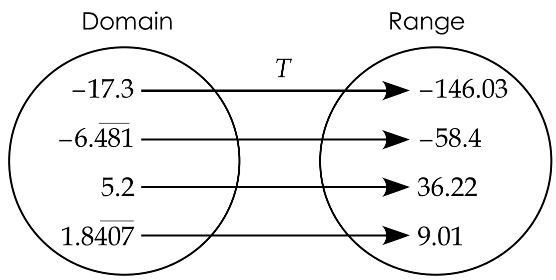
   The x-intercept is at $\frac{17}{9}$.

   You may graph this function on a graphing calculator to verify your answers.

5. Complete the following mapping diagram for the function $T(d) = 8.1d - 5.9$. Show your work in functional notation.



Answer:



$$\begin{aligned} T(d) &= 8.1d - 5.9 \\ T(-17.3) &= 8.1(-17.3) - 5.9 \\ T(-17.3) &= -146.03 \end{aligned}$$

(decimal terminates or may be written as $-146\frac{3}{100}$)

$$\begin{aligned} T(d) &= 8.1d - 5.9 \\ -58.4 &= 8.1d - 5.9 \\ -58.4 + 5.9 &= 8.1d \\ -52.5 &= 8.1d \\ d &= -6.481481481\ldots \end{aligned}$$

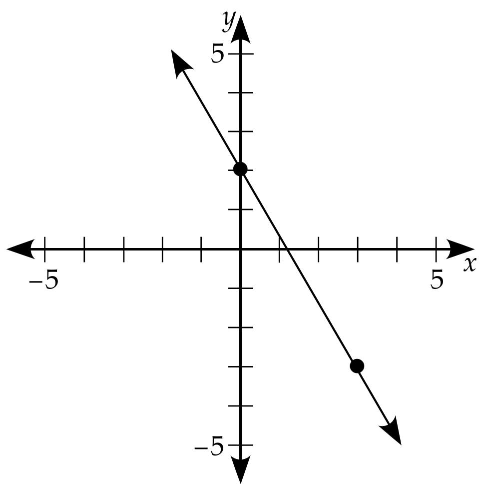
(decimal repeats)

$$\begin{aligned} T(d) &= 8.1d - 5.9 \\ T(5.2) &= 8.1(5.2) - 5.9 \\ T(5.2) &= 36.22 \end{aligned}$$

$$
\begin{aligned}
T(d) &= 8.1d - 5.9 \\
9.01 &= 8.1d - 5.9 \\
9.01 + 5.9 &= 8.1d \\
14.91 &= 8.1d \\
d &= 1.8407407407\ldots
\end{aligned}
$$

6. Sketch a graph of $T(x) = \frac{-5}{3}x + 2$. State the slope, x- and y-intercepts and the domain and range in interval notation.

Answer:



$\text{slope} = \frac{-5}{3}$

y-intercept = 2

x-intercept is where $T(x) = 0$

$$
\begin{aligned}
0 &= \frac{-5}{3}x + 2 \\
-2 &= \frac{-5}{3}x \\
\frac{-6}{-5} &= x
\end{aligned}
$$

The x-intercept is at $\frac{6}{5} = 1.2$.

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

7. A gondola ride along the canals in Venice costs \$128/hour before sunset. After sunset you must add \$50 to the total cost you would pay before sunset. Write two linear equations in function notation to express
   a) the cost of a gondola ride in daylight hours
   b) the cost of a gondola ride after sunset

   Answers:
   a) Daylight costs: $C(h) = 128h$
   b) After sunset: $C(h) = 128h + 50$

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 5
Relations and Functions

Learning Activity Answer Keys

Module 5: Relations and Functions

Learning Activity 5.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Which is the independent variable? Your distance from your house compared to how long you've been walking.
2. Simplify: $\left(4x^4\right)^{\frac{3}{2}}$.
3. The slope of a line is −3. What is the slope of a line parallel to this one?
4. Your brother takes acting on Monday night, and plays football on Tuesdays and Thursdays. You have lacrosse on Wednesday and Thursday, and it is your friend's birthday party on Saturday evening. Your parents have a date night every Friday. Will you be able to sit down with your family for dinner this week?
5. Solve for b: $8 + b - 4 = 16$.
6. You are going to bake a cake for your mom's birthday. Because your family is coming over, you decide to make a double recipe. In the original recipe, you need half a teaspoon of vanilla. How much vanilla will you need in the double recipe?
7. Kaitlin types 50 words per minute. It took her 30 minutes to write her essay for English. Assuming she was typing the whole time, how many words are in her essay?
8. When you were 3, your brother's age was double yours. How much older is he?

Answers:

1. How long you've been walking.
2. $8x^6 \left(\left(4x^4\right)^{\frac{3}{2}} = \left(\sqrt{4x^4}\right)^3 = \left(2x^2\right)^3 = 8x^6 \right)$
3. −3
4. Yes, on Sunday.
5. 12 ($b = 16 - 8 + 4$)

6. 1 teaspoon $\left(\frac{1}{2} \times 2\right)$
7. 1500 words (50 × 30)
8. 3 years (3 × 2 = 6, 6 − 3 = 3)

Part B: Relations and Functions

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. In your own words, explain the following terms (based on how they are used in this module).
   a) domain
   b) relation

   You may want to include definitions for these items as well as for range and function on your Resource Sheet.

   Answers:

   Answers may vary but should explain as indicated below.
   a) The domain of a relation is all possible inputs. It is all starting points or the x-values of the ordered pairs (x, y).
   b) A relation describes how two variables (x, y) are related. If two quantities are related in such a way that a given value in one quantity determines the value of the second quantity, the mathematical model is called a relation.

2. What is the rule that you use to determine whether a relation (mapped or in a table) represents a function?

   Answer:

   A table of values or mapping represents a function if each input has only one possible output. There will be only one arrow from each element in the domain connecting it to a single possible range value.

3. In your own words, explain what the vertical line test is used for and how it works.

   Answer:

   The vertical line test is used to determine whether or not the graph of an equation represents a function. If a vertical line crosses the graph in more than one point, it is not the graph of a function.

4. Determine whether the following relations are functions. Justify your answer.

   a) A = {(1, 2), (1, 7), (1, 8), (1, 9), (1, 10)}

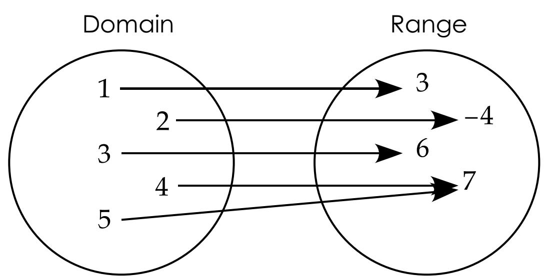
      Answer:

      Non-function—the input of 1 has multiple possible outputs.

   b) $y = -x + 3$

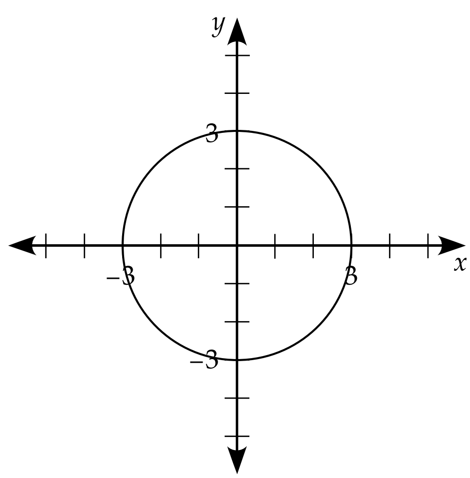
      Answer:

      Function—the equation is a linear function with a slope of −1. It is not a vertical line.

   c) 

      Answer:

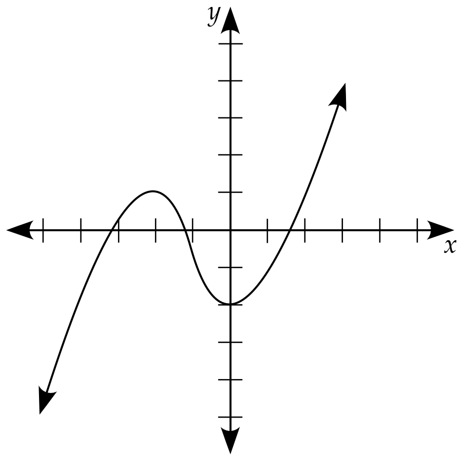
      Function—each element in the domain has exactly one possible value for range.

   d) 

      Answer:

      Non-function—a vertical line drawn on this graph will pass through more than one point.

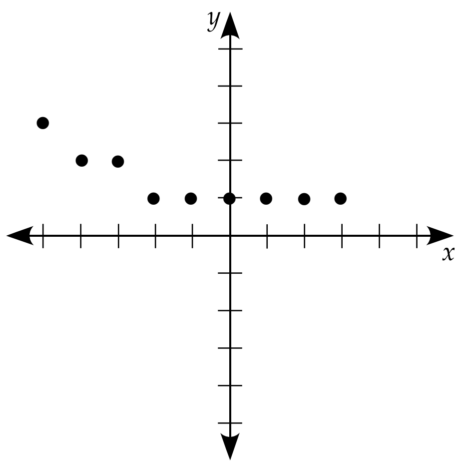
e)



Answer:

Function—a vertical line drawn on this graph will only ever pass through one point.

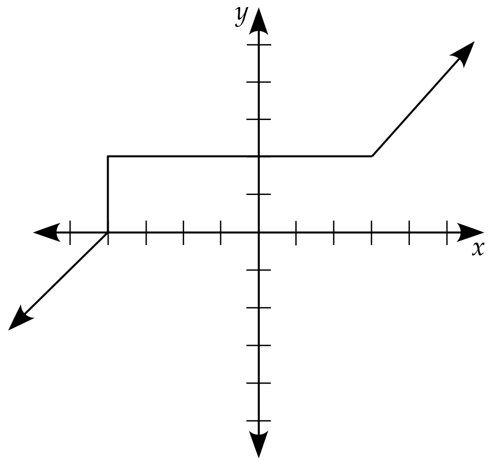
f)



Answer:

Function—a vertical line drawn on this graph will only ever pass through one point.

g)



Answer:

Non-function—a portion of this graph is a vertical line, so it would not pass the vertical line test. A vertical line drawn at $x = -4$ would pass through more than one point of the graph.

5. The physical education teacher calls out the first name of 6 students in a class (Aiden, Brady, Chad, Chad, Devon, Everett) and numbers them 1 through 6.

   If this information were written as ordered pairs (number, name), would this represent a function or a non-function? Explain.

   Answer:

   While it is possible that you may have two or more students with the same first name in a class, each number would only be used once. Written so that the input is *number* and the output is *name*, this would be a function because each input would only have one output.

Learning Activity 5.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Solve for z: $3 \times 7 = z - 4$.
2. Complete the following pattern: 60, 75, ____, 105, ____.
3. What two numbers have a product of 32 and a sum of 18?
4. Is $\sqrt{72}$ a rational or irrational number?
5. Your cousin has twice as many stuffed animals as she has dolls. She has half as many movies as she has stuffed animals. How many dolls does your cousin have if she has 4 movies?
6. Carrie keeps her shoes in their boxes so that they don't get ruined. In her closet, she has 5 piles of boxes and each pile is 4 boxes high. How many pairs of shoes does Carrie have?
7. You are in a sandbox that takes up an area of 4 m^2. Convert to centimetres.
8. Express the fraction as a decimal: $4\frac{3}{5}$.

Answers:

1. $z = 25$ ($21 = z - 4$; $21 + 4 = z$)
2. 90, 120 (pattern is to add 15 each time)
3. 16, 2 (The factor pairs of 32 are (1, 32), (2, 16), (4, 8). 16 + 2 = 18)
4. Irrational
5. 4 dolls (Because she has double the amount of stuffed animals as she does both dolls and movies, she must have the same number of dolls and movies.)
6. 20 pairs of shoes (5×4)
7. 40000 cm^2 (There are 100 cm in 1 m, so $(1\text{ m})^2 = 1\text{ m}^2 = (100\text{ cm})^2 = 10000\text{ cm}^2$. $10000 \times 4 = 40000\text{ cm}^2$)
8. 4.6 (The four stays on the left side of the decimal because it is a whole number. $\frac{3}{5}$ is equivalent to $\frac{6}{10}$ or 0.6.)

Part B: Domain and Range Notation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Describe the domain and range of the following situation in words:

    A 4 L pail is being filled with water from a tap that flows 125 mL per second.

    Answer:

    There are 1000 mL in a litre, so 4000 mL divided by 125 mL/sec is 32 seconds. The volume depends on the time, so time is the independent variable, x, and volume is y. The domain (valid x inputs) for this function would be from 0 to 32 seconds. The range would be from 0 to 4000 mL or 0 to 4 L

2. Describe the domain and range of the following situation using lists of inputs and outputs.

    Pizzas can be ordered in small 9 in., medium 14 in., or large 20 in. sizes and cut into 6, 8, or 12 pieces, respectively.

    Answer:

    D: {9, 14, 20}

    R: {6, 8, 12}

3. Write a possible domain and range of the following in set notation. Explain your answer.

    There are 350 students in your school and orders for yearbooks are being taken by the printing company.

    Answer:

    The number of yearbooks, y, ordered from a printing company depends on the number of students, x, in a school. It isn't likely that everyone will buy one, but it's unlikely that anybody will order more than one. A possible domain and range may be:

    D: $\{x \mid x \leq 350, x \in W\}$

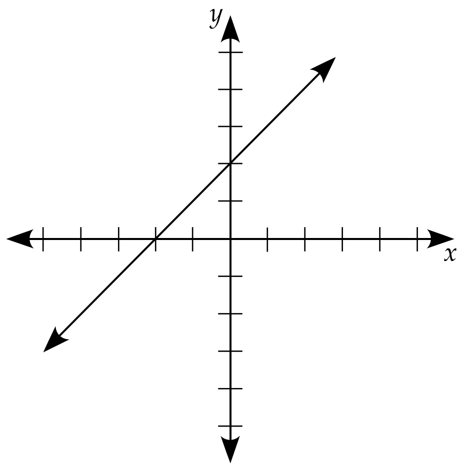
    R: $\{y \mid y \leq 350, y \in W\}$

    The valid input and output values would be part of the whole number system, W, as you cannot have negative or partial values of people or books.

4. Express the domain and range of the following graphs of equations using both set and interval notation.

Answers:

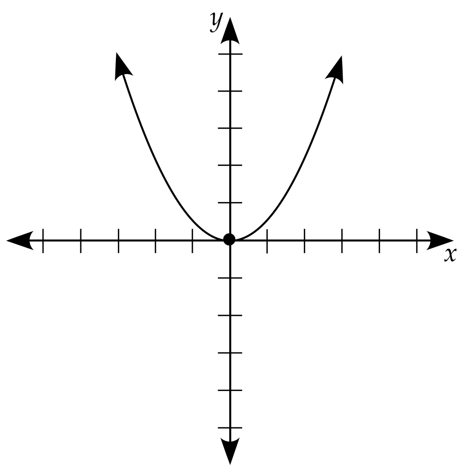
a)



Set Notation:
D: $\{x \mid x \in \Re\}$
R: $\{y \mid y \in \Re\}$

Interval Notation:
D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

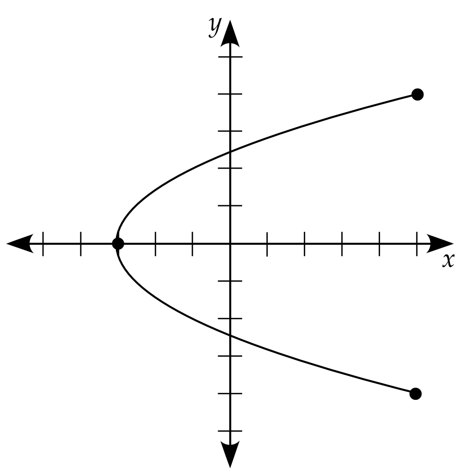
b)



Set Notation:
D: $\{x \mid x \in \Re\}$
R: $\{y \mid y \geq 0, y \in \Re\}$

Interval Notation:
D: $(-\infty, \infty)$
R: $[0, \infty)$

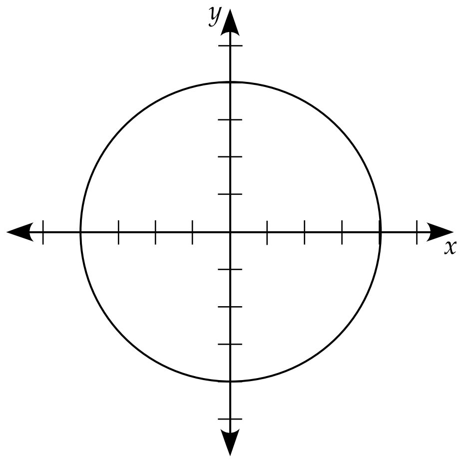
c)



Set Notation:
D: $\{x \mid -3 \leq x \leq 5, x \in \Re\}$
R: $\{y \mid -4 \leq y \leq 4, y \in \Re\}$

Interval Notation:
D: $[-3, 5]$
R: $[-4, 4]$

d)



Set Notation:

D: $\{x \mid -4 \le x \le 4, x \in \Re\}$

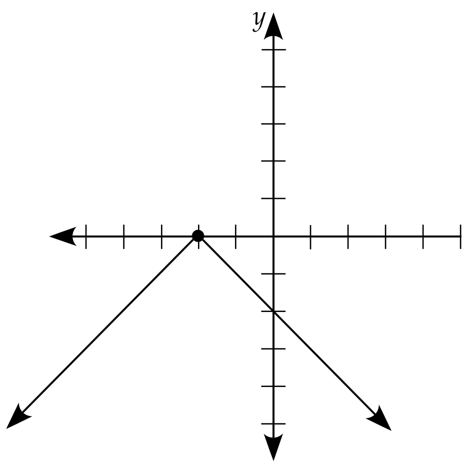
R: $\{y \mid -4 \le y \le 4, y \in \Re\}$

Interval Notation:

D: $[-4, 4]$

R: $[-4, 4]$

e)



Set Notation:

D: $\{x \mid x \in \Re\}$

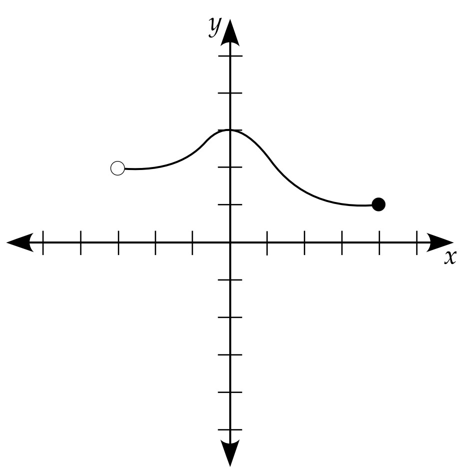
R: $\{y \mid y \le 0, y \in \Re\}$

Interval Notation:

D: $(-\infty, \infty)$

R: $(-\infty, 0]$

f)



Set Notation:

D: $\{x \mid -3 < x \le 4, x \in \Re\}$

R: $\{y \mid 1 \le y \le 3, y \in \Re\}$

Interval Notation:

D: $(-3, 4]$

R: $[1, 3]$

Learning Activity 5.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Write the equation as a function: $y = 3x + 5$.
2. Is this relation a function: {(0,1),(1,2), (3,1), (4, 2)}?
3. A square prism has a width of 5 cm and a length of 8 cm. What is the volume?
4. Expand: $5\left(x^6\right)^{\frac{-2}{3}}$.
5. If 36% of 500 is 180, what is 18% of 500?
6. Solve for k: $\frac{6}{k} = 2$.
7. What two numbers have a product of −72 and a sum of 1?
8. What two numbers have a product of −36 and a sum of 0?

Answers:

1. $f(x) = 3x + 5$
2. Yes, the relation is a function. (No x-value appears more than once.)
3. 200 ($V_{(prism)} = Bh = (5^2) \times 8$)
4. $\frac{5}{x^4} \left(5\left(x^6\right)^{\frac{-2}{3}} = \frac{5}{\sqrt[3]{\left(x^6\right)^2}} \right)$
5. 90
6. $k = 3$
7. 9, −8
8. 6, −6

Part B: Functional Notation

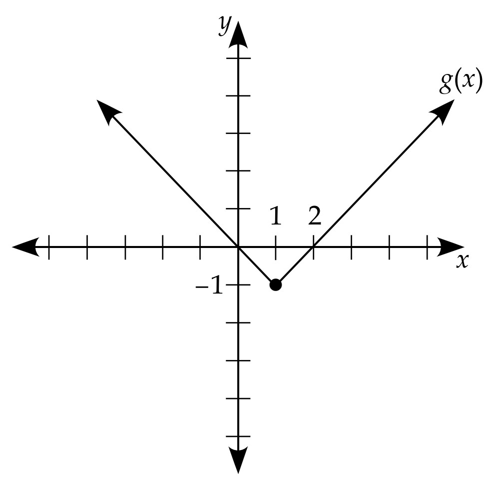
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the function $f(x) = 4 - x$, find the value of
   a) $f(1)$
   b) $f(2)$
   c) $f(4)$
   d) $f(-3)$

   Answers:

   a) $f(1) = 4 - (1) = 3$
   b) $f(2) = 4 - (2) = 2$
   c) $f(4) = 4 - (4) = 0$
   d) $f(-3) = 4 - (-3) = 7$

2. Given a graph of the function g, find the value of
   a) $g(0)$
   b) $g(1)$
   c) $g(2)$
   d) $g(3)$



   Answers:

   a) At the point where $x = 0$, $y = 0$, the graph passes through the origin. $g(0) = 0$
   b) $g(1) = -1$
   c) $g(2) = 0$
   d) $g(3) = 1$

3. Given the function $g = \{(2, 3), (5, -2), (7, 8), (-1, 4)\}$, what is the value of
   a) $g(2)$
   b) $g(5)$
   c) $g(-1)$

   Answers:

   a) $g(2) = 3$
   b) $g(5) = -2$
   c) $g(-1) = 4$

4. The x-intercept of a linear equation is where $y = 0$, and the y-intercept is where $x = 0$. Write the linear equation $-9x - y + 17 = 0$ in functional notation, and determine the coordinates of the intercepts. Show your work in functional notation.

   Answer:

   Linear equation in functional notation:

$$
\begin{aligned}
-9x - y + 17 &= 0 \\
-9x + 17 &= y \\
f(x) &= -9x + 17
\end{aligned}
$$

   y-intercept is where $x = 0$

$$
\begin{aligned}
f(0) &= -9(0) + 17 \\
f(0) &= 17
\end{aligned}
$$

   The y-intercept is at 17

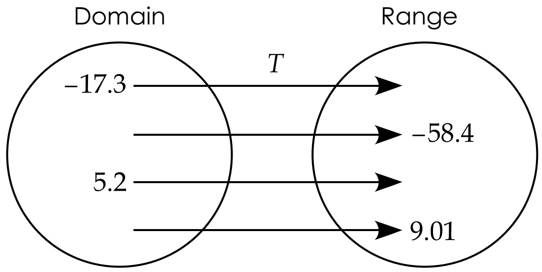
   x-intercept is where $y = 0$

$$
\begin{aligned}
f(x) &= -9x + 17 \\
0 &= -9x + 17 \\
-17 &= -9x \\
x &= \frac{17}{9}
\end{aligned}
$$

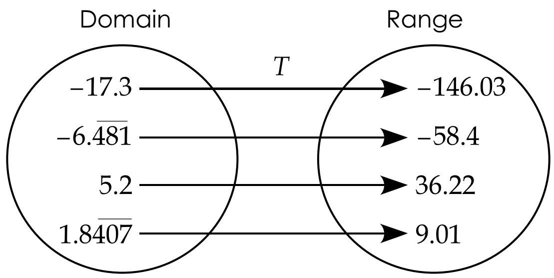
   The x-intercept is at $\frac{17}{9}$.

   You may graph this function on a graphing calculator to verify your answers.

5. Complete the following mapping diagram for the function $T(d) = 8.1d - 5.9$. Show your work in functional notation.



Answer:



$$T(d) = 8.1d - 5.9$$
$$T(-17.3) = 8.1(-17.3) - 5.9$$
$$T(-17.3) = -146.03$$

(decimal terminates or may be written as $-146\frac{3}{100}$)

$$T(d) = 8.1d - 5.9$$
$$-58.4 = 8.1d - 5.9$$
$$-58.4 + 5.9 = 8.1d$$
$$-52.5 = 8.1d$$
$$d = -6.481481481\ldots$$

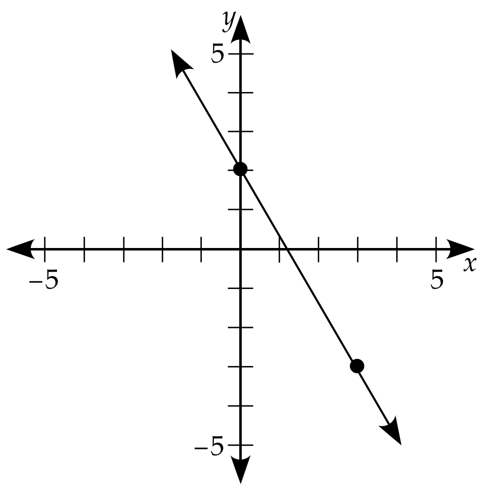
(decimal repeats)

$$T(d) = 8.1d - 5.9$$
$$T(5.2) = 8.1(5.2) - 5.9$$
$$T(5.2) = 36.22$$

$$
\begin{aligned}
T(d) &= 8.1d - 5.9 \\
9.01 &= 8.1d - 5.9 \\
9.01 + 5.9 &= 8.1d \\
14.91 &= 8.1d \\
d &= 1.8407407407\ldots
\end{aligned}
$$

6. Sketch a graph of $T(x) = \dfrac{-5}{3}x + 2$. State the slope, x- and y-intercepts and the domain and range in interval notation.

Answer:



$$\text{slope} = \frac{-5}{3}$$

y-intercept = 2

x-intercept is where $T(x) = 0$

$$
\begin{aligned}
0 &= \frac{-5}{3}x + 2 \\
-2 &= \frac{-5}{3}x \\
\frac{-6}{-5} &= x
\end{aligned}
$$

The x-intercept is at $\dfrac{6}{5} = 1.2$.

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

7. A gondola ride along the canals in Venice costs \$128/hour before sunset. After sunset you must add \$50 to the total cost you would pay before sunset. Write two linear equations in function notation to express

   a) the cost of a gondola ride in daylight hours

   b) the cost of a gondola ride after sunset

   Answers:

   a) Daylight costs: $C(h) = 128h$

   b) After sunset: $C(h) = 128h + 50$

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 6
Polynomials

Module 6:
Polynomials

Introduction

This module is a continuation of some of the math topics that you have been learning about since the early years of your education: combining values, recognizing patterns, following rules, and using "hands-on" tools to help you understand and express theoretical concepts. All these ideas and more come together when you work with polynomials.

More specifically, this module will apply these concepts as you explore the relationship between multiplying and factoring in the context of polynomials.

Assignments in Module 6

When you have completed the assignments for Module 6, submit your completed assignments for Module 5 and Module 6 to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 6.1	Describing Polynomials and Multiplying Binomials
2	Assignment 6.2	Multiplying Polynomials
3	Assignment 6.3	Factoring Binomials and Trinomials
4	Assignment 6.4	Factoring Trinomials When $a \in I$
5	Assignment 6.5	Difference of Squares and Module Review

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions that follows to help you with preparing your resource sheet for the material in Module 6. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 1 to 8.

Resource Sheet for Module 6

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Lesson 1: Multiplying Polynomials using Tiles

Lesson Focus

In this lesson, you will

- ❑ model the multiplication of two binomial expressions concretely or pictorially and record the process symbolically
- ❑ relate the multiplication of two binomial expressions to area

Lesson Introduction

This lesson will review some concepts and definitions from previous math courses in order to help you set the stage for multiplying and factoring polynomials. You will use tiles to represent binomial multiplication, and simplify expressions by combining like terms.

Polynomials

By now, you are familiar with mathematical expressions that have exponents and variables like $3x^2$ or $-\frac{1}{2}y^2$. These individual expressions are called terms. If you combine several of them together in an algebraic expression or mathematical sentence using addition or subtraction signs, you have created a **polynomial**.

Polynomial: a mathematical expression with one or more terms.

The polynomial $-5x^2 + 4x - 2$ has three terms. The variable is represented by the letter x. It stands for an unknown number. Notice the exponents in the terms. They are given in **descending order of power**. The first term has x to the power of 2, the second term is understood to be x to the power of 1, and the third term could have x^0, which is always equal to 1. When **a term is written without a variable, it is called a constant** because, no matter what, a constant will always have the same value. Negative 2 is a constant in the above polynomial. When a term has both a number and a variable, the number is called a coefficient. The coefficient tells you how many times to multiply that variable. The polynomial above has coefficients of -5 and $+4$.

If two terms have the exact same variables and degree (the same powers or exponents), they are called *like terms*.

$17r^3t$ and $-4r^3t$ are like terms.

Polynomials can be named in different ways, depending on the number of terms they have or by their degree.

"Poly" means "many," and so a polynomial refers to a math expression with one or more terms. Polynomial expressions with 1, 2, or 3 terms have special names:

- **Monomial** means 1 term.
- **Binomial** refers to an expression with 2 terms.
- **Trinomial** is an expression with 3 terms.

Expressions with more than 3 terms are simply called polynomials.

The degree of the polynomial is defined as the highest exponent in the leading term of the polynomial, when terms are written in descending order.

Degree	Name
1	constant
2	quadratic
3	cubic
4	quartic
5	quintic

The names of polynomials and their degree would be handy to have on your Resource Sheet.

Example 1

Fill in the chart. State the number of terms and polynomial name, degree and degree name, variables, coefficients, and constants in the following polynomials.

Polynomial	No. of Terms	Polynomial Name	Degree	Degree Name	Variables	Coefficients	Constants
x^2							
$-4y^3$							
$5x^2 - 1$							
$8r^2 - 4r + 2$							
$9m^4 + m^2 - 3$							
$-6r^5 + 2r^3 - k - 10$							

Solution:

Polynomial	No. of Terms	Polynomial Name	Degree	Degree Name	Vari-ables	Coefficients	Constants
x^2	1	Monomial	2	Quadratic	x	1	none
$-4y^3$	1	Monomial	3	Cubic	y	-4	none
$5x^2 - 1$	2	Binomial	2	Quadratic	x	5	-1
$8r^2 - 4r + 2$	3	Trinomial	2	Quadratic	r	$8, -4$	2
$9m^4 + m^2 - 3$	3	Trinomial	4	Quartic	m	$9, 1$	-3
$-6r^5 + 2r^3 - k - 10$	4	Polynomial	5	Quintic	r, k	$-6, 2, -1$	-10

Operations on Polynomials

In your Grade 9 Math course, you simplified polynomials by adding (combining like terms) and subtracting (adding the opposite), as well as by multiplying and dividing (by applying distribution and exponent laws). Go back and review your notes or find a reputable website to refresh your memory if needed. Look back at Module 2 in this course to review the exponent laws.

Remember that **to simplify means to write another expression with the exact same value**.

To do this, you must apply the **correct order of operations (BEDMAS)**:

- **Brackets:** perform any operations enclosed in brackets first
- **Exponents:** Remember that exponents apply only to what they are connected to, either a base or a bracket.
- **Division and Multiplication:** in the order they appear from left to right
- **Addition and Subtraction:** in the order they appear from left to right

Example 2

Simplify each of the expressions below.

a) $(3x-3y)+(4y-2x)$

b) $(4m^2-2m-4)-(-3m^2-2m+5)$

c) $2x(x+5x^3)$

d) $\dfrac{8x^4y^3}{-2x^3y}$

Solution:

a) $(3x-3y)+(4y-2x)$

$=(3x-2x)+(-3y+4y)$ Combine *like* terms.

$=x+y$

b) $(4m^2-2m-4)-(-3m^2-2m+5)$

$=4m^2-2m-4+3m^2+2m-5$ Add the *opposite* of the second polynomial.

$=7m^2-9$ Combine like terms.

c) $2x(x+5x^3)$

$=2x^2+10x^4$

Distribute the $2x$ over the terms in $(x+5x^3)$. When multiplying powers with the same base, add the exponents

d) $\dfrac{8x^4y^3}{-2x^3y}$

$=-4xy^2$

When dividing powers with the same base, subtract the exponents

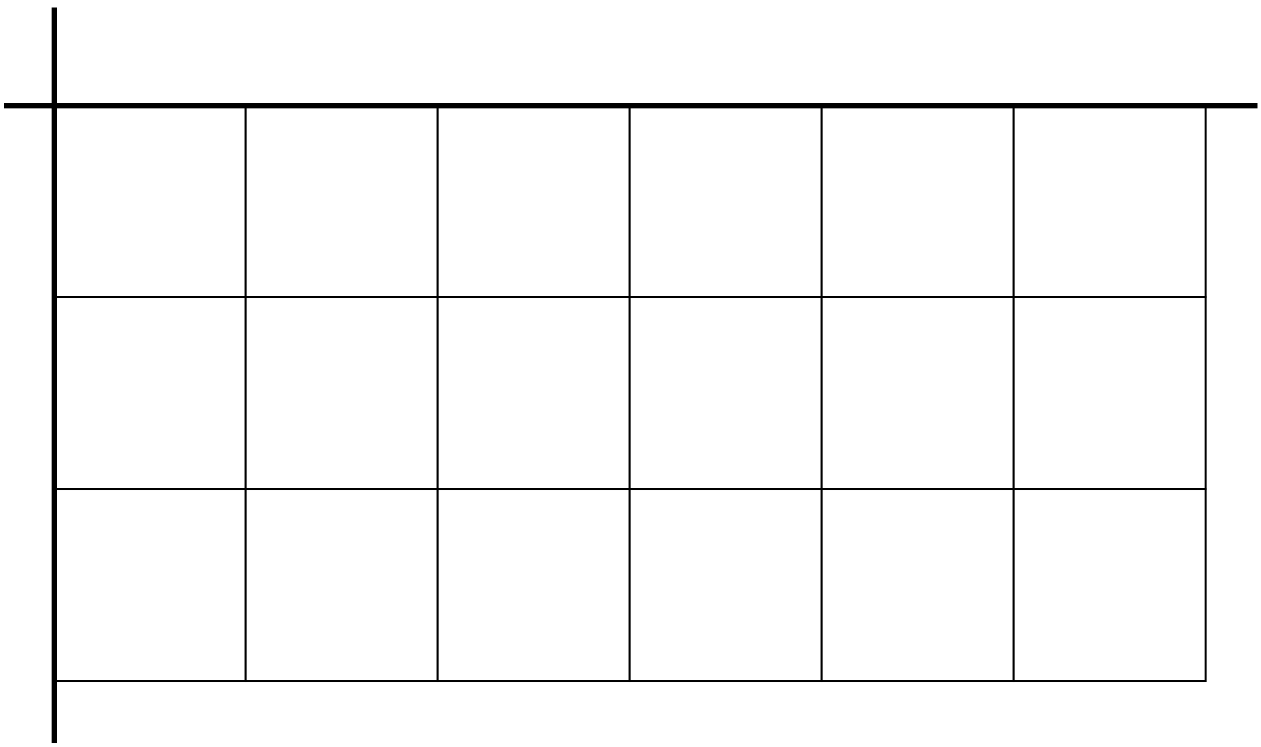
Multiplication as Area

Example 3

Kathy wants to buy an area carpet for her living room. It needs to be 3 feet by 6 feet. How many square feet of carpet must she buy?

Solution:

You can draw a diagram to represent this situation and use it to identify the solution. If you use a scale of 1 inch = 1 foot and draw a 3-by-6 rectangle, it is obvious from the diagram that she will need to buy 18 square feet of carpet. Count the squares!

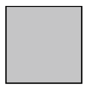


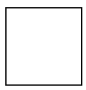
3 feet by 6 feet means $3 \times 6 = 18$ square feet.

This area model can be applied to multiplying polynomials as well.

Representing Polynomials

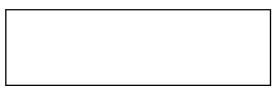
In previous math courses, you would have used objects or diagrams of squares and rectangles to represent values and variables. Often a coloured square tile is used to represent the value of 1 and a white square tile represents negative 1.


$= 1$

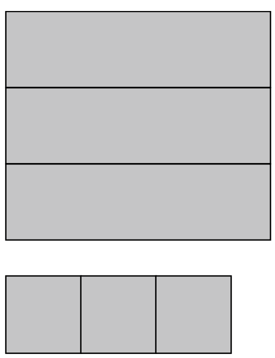

$= -1$

A rectangle with the same height as the square but of unknown length is used to represent the variable x. Again, no colour means negative.


$= x$


$= -x$

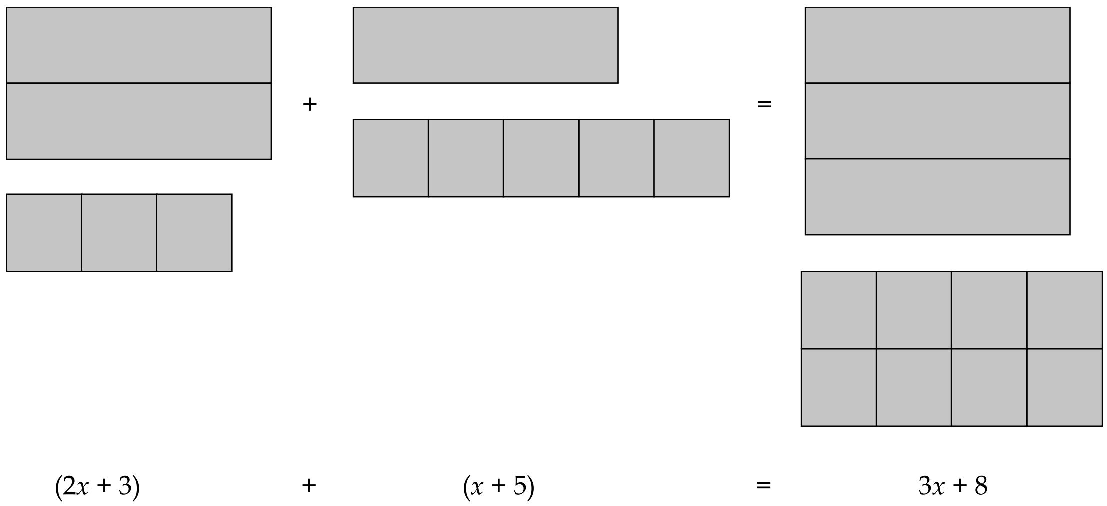
$3x + 4$ could be illustrated using tiles as



Using Tiles to Illustrate Polynomial Addition

Tiles can be used to represent the addition of polynomials.

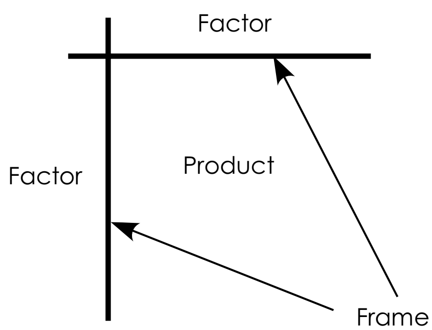
$(2x + 3) + (x + 5)$ can be illustrated as

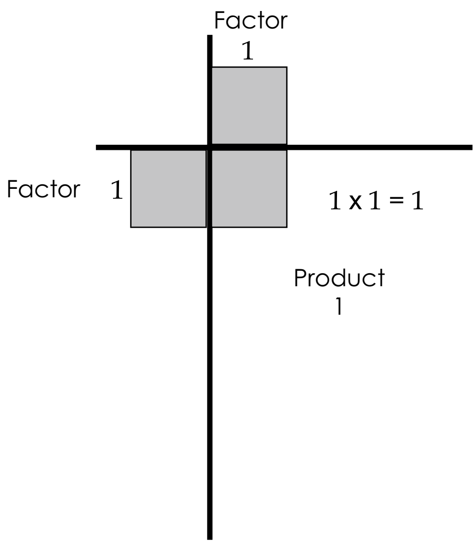


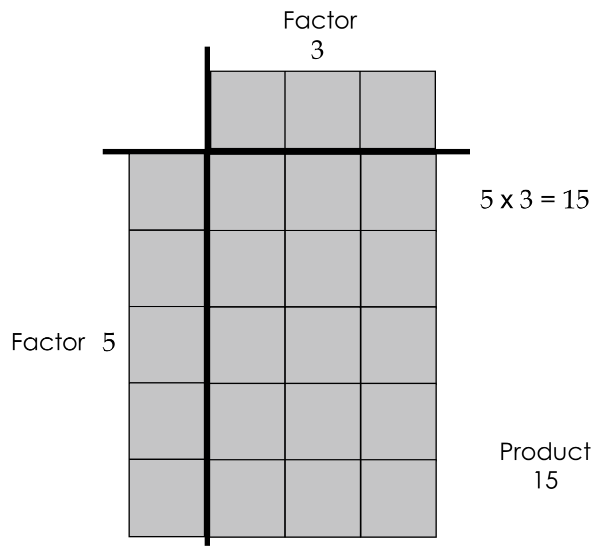
Notice that only like terms are combined.

Using Tiles to Illustrate Polynomial Multiplication

Applying a strategy similar to what you did to solve the rug question above, tiles illustrate the product of two factors using a multiplication frame. The factors (what you are multiplying) go along the top and side, and the product or answer is the rectangular area inside the frame.

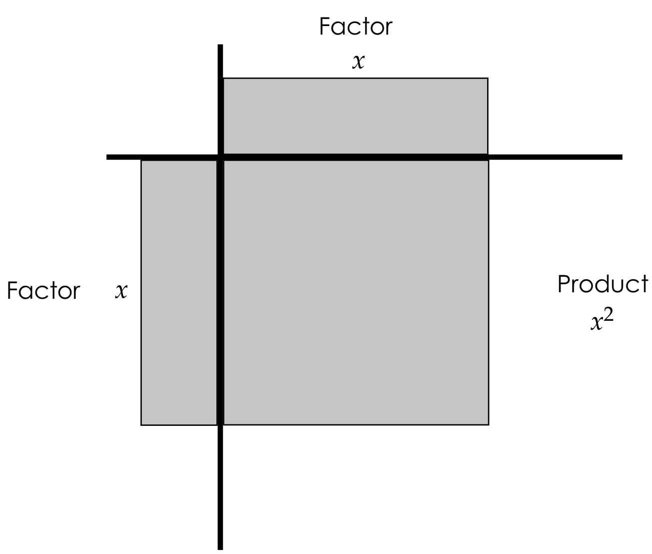






When multiplying factors, fill in the entire rectangular area inside the frame beneath and beside the factors.

The product of $x \times x$ is illustrated as follows:



The resulting shaded square tile represents x^2.

Tiles are a great way to illustrate polynomial expressions, but there are some limitations when using tiles to model the concepts of multiplication and factoring with negative values. In this course, tiles will be used to represent positive quantities and another area model will be used to illustrate operations involving negatives.

You can create your own set of tiles by cutting out the template pieces provided at the end of this lesson. You may also explore online algebra tiles. Type "algebra tile applets" into your Internet search engine and see what is available.

Example 4

Use tiles to help you find the products of the polynomials.

a) $(2x + 3)(x + 2)$

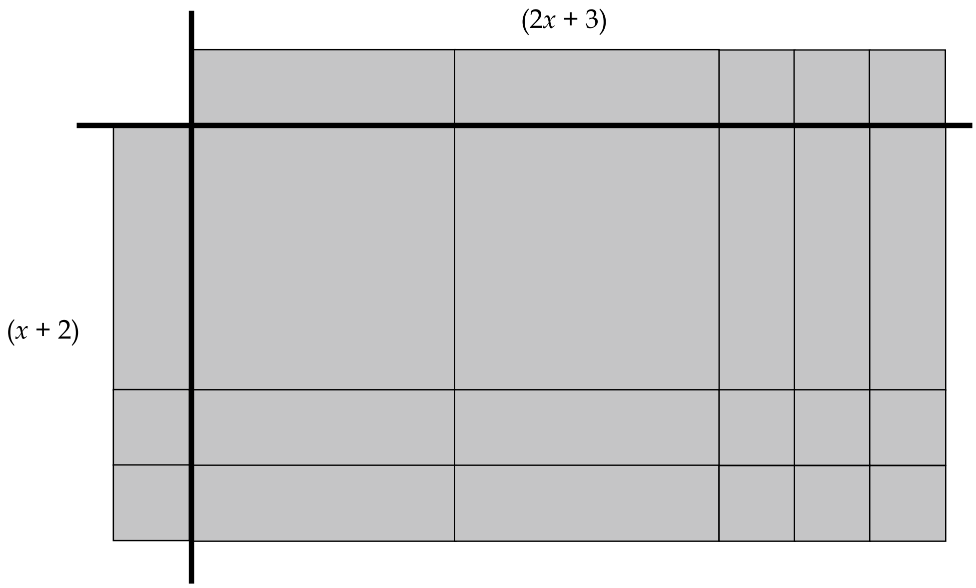
b) $(x + 4)(3x + 1)$

c) $(x + 5)(2x + 2)$

d) $(3x + 2)(x + 2)$

Solutions:

a) $(2x + 3)(x + 2)$



Step 1: Factors can be put along either the top or the side, in any order, because multiplication is commutative. That is, $(2x + 3)(x + 2)$ is the same as $(x + 2)\,(2x + 3)$.

Step 2: When filling in the product area, carry the dimensions of the factors down and across, and use the largest tiles possible.

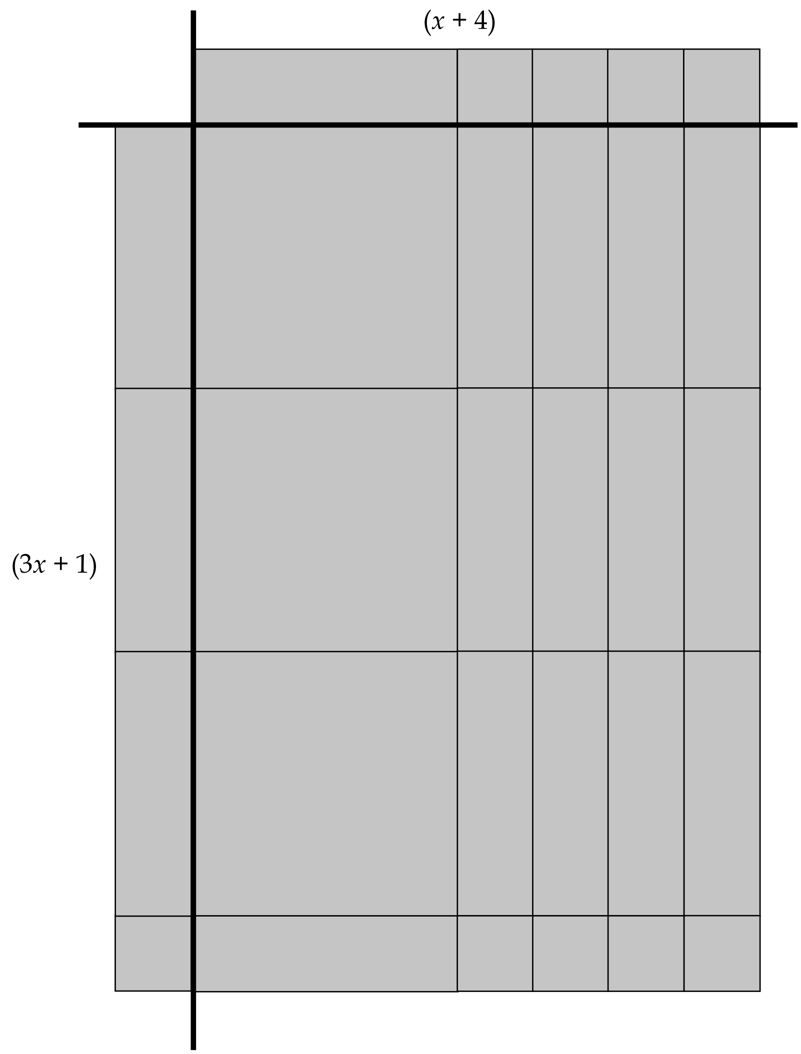
$(2x + 3)(x + 2)$

$= 2x^2 + 3x + 4x + 6$ Step 3: List all tiles.

$= 2x^2 + 7x + 6$ Step 4: Combine like terms.

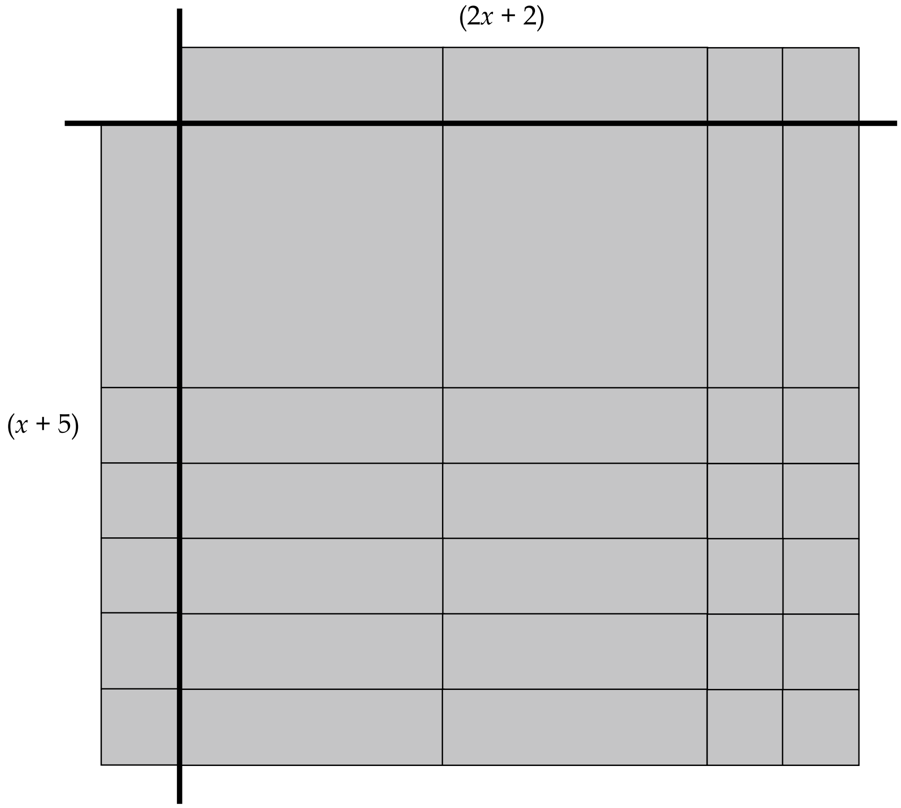
You may want to have these steps recorded on your Resource Sheet.

b) $(x + 4)(3x + 1)$



$(x + 4)(3x + 1)$
$= 3x^2 + 12x + 1x + 4$
$= 3x^2 + 13x + 4$

c) $(x + 5)(2x + 2)$

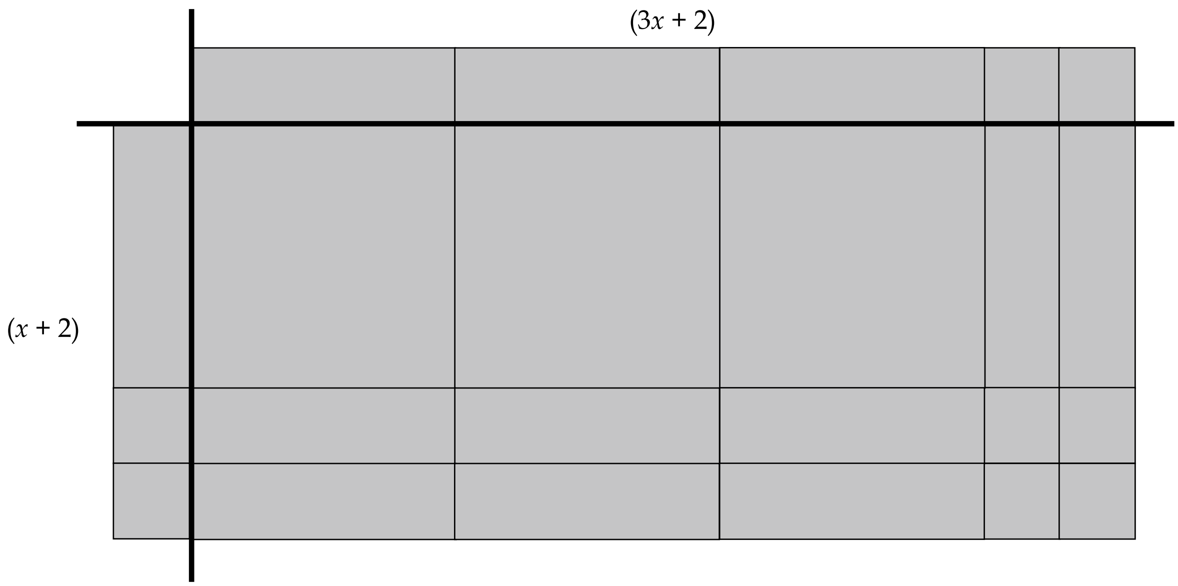


$(2x + 2)(x + 5)$

$= 2x^2 + 2x + 10x + 10$

$= 2x^2 + 12x + 10$

d) $(3x + 2)(x + 2)$



$(x + 2)(3x + 2)$

$= 3x^2 + 2x + 6x + 4$

$= 3x^2 + 8x + 4$

Learning Activity 6.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Write the following equation as a function: $x + y = 76$.
2. If you are $\frac{3}{2}$ taller than your brother, and your brother is 4 feet tall, how tall are you?
3. The sides of a right triangle are 3, 5, 4. How long is the hypotenuse?
4. Is this relation a function: {(0,1), (3,6), (4, 8), (0, 10)}?
5. There are 120 employees at your work. Your boss says that three-quarters of the staff are coming to the meeting on Saturday morning. How many people will be attending the meeting?
6. Usain Bolt can run 100 m in 10 s. What is his average speed?
7. You need to make exact change for a customer at your work. They have given you \$60 and their bill is \$42.60. How much money will you give them?
8. Evaluate: $\left(6^2\right)^{\frac{1}{2}}$.

continued

Learning Activity 6.1 (continued)

Part B: Binomial Multiplication

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write a quadratic trinomial of degree 2 with the variable m, coefficients –3 and 1 and a constant of 5.
2. Write 3 like terms that are not exactly the same.
3. Illustrate the following products using tiles, and write your solution steps to show how it may be simplified.
   a) $(x + 5)(3x + 2)$
   b) $(2x + 1)(x + 4)$
   c) $(4x + 1)(x + 6)$
   d) $(x + 3)(2x + 3)$

Lesson Summary

This lesson reviewed definitions and concepts related to polynomials. You identified the number of terms, degree, variables, coefficients, and constants in polynomials. Using a tile diagram, you illustrated the product of two polynomial factors, and showed the solution steps to indicate how it may be simplified. The next lesson will continue with the multiplication of polynomials, looking at strategies to use when working with negative values, pictorially as well as symbolically.

Assignment 6.1

Describing Polynomials and Multiplying Binomials

Total Marks = 33

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Fill in the blanks in the following chart. *(1/2 mark each = 17 marks)*

Polynomial	No. of Terms	Polynomial Name	Degree	Degree Name	Variables	Coefficients	Constants
$2y^3$							
$-19m^4 - 10$							
$7x^2 + 7y - 7$							
		Trinomial	2		k	6, −1	9
	5			Quartic	a	−1, 2, −3, 4	−5
$6p^3 + 2q^2 + r - 3$							

Assignment 6.1: Describing Polynomials and Multiplying Binomials (continued)

2. Illustrate the following products using tiles, and write your solution steps to show how it may be simplified.

   a) $(2x + 3)(x + 2)$ *(4 marks)*

Assignment 6.1: Describing Polynomials and Multiplying Binomials (continued)

b) $(3x + 1)(2x + 4)$ *(4 marks)*

Assignment 6.1: Describing Polynomials and Multiplying Binomials (continued)

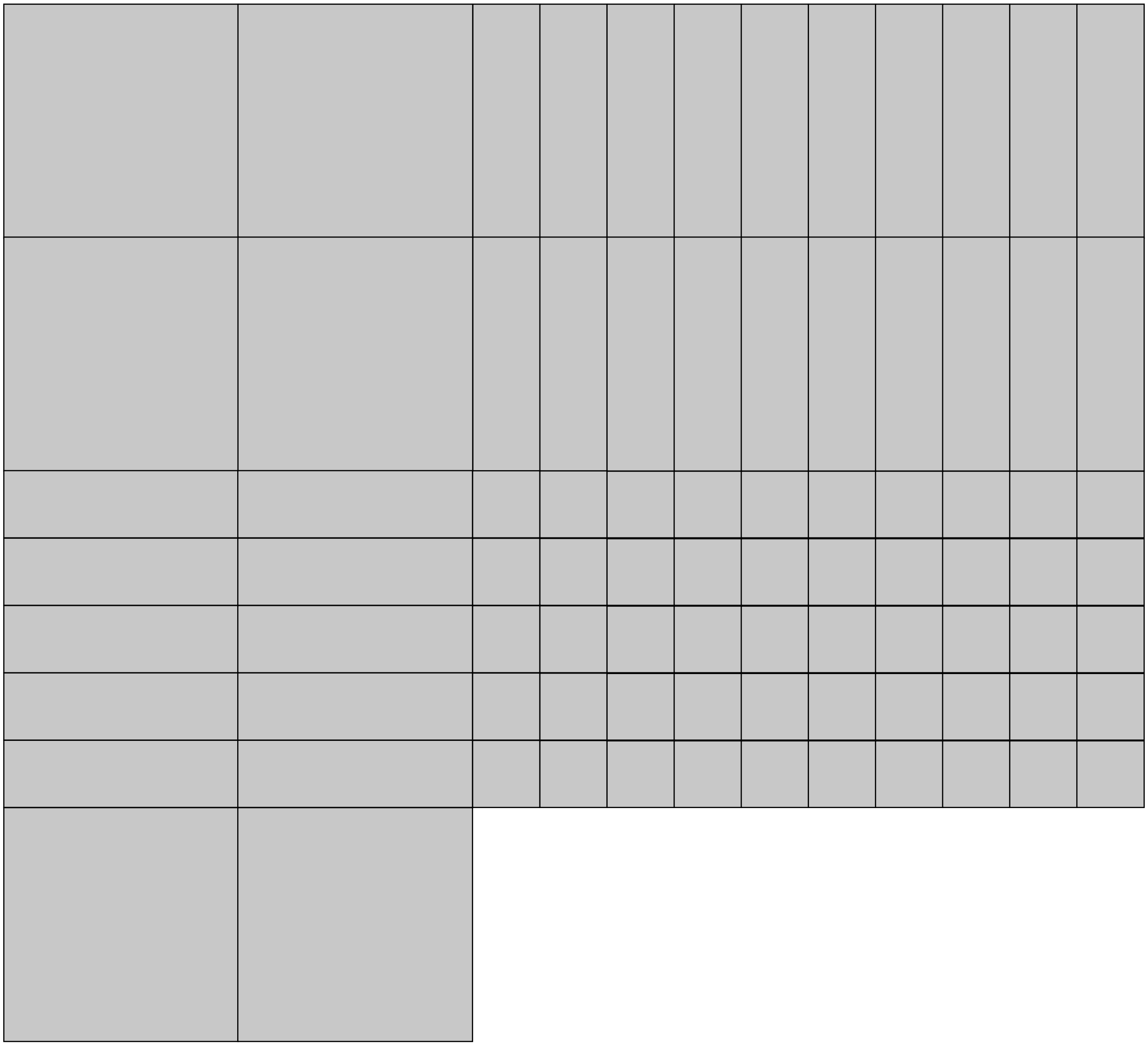
c) $(x + 5)(2x + 4)$ *(4 marks)*

Assignment 6.1: Describing Polynomials and Multiplying Binomials (continued)

d) $(3x + 2)(x + 3)$ *(4 marks)*

Notes

Algebra Tile Template



Cut out the squares and rectangles above. Here are their values.

Small Square:	1×1 dimensions = 1
Rectangle:	$1 \times x$ dimensions = x
Large Square:	$x \times x$ dimensions = x^2

Lesson 2: Multiplying Polynomials

Lesson Focus

In this lesson, you will

- ❑ relate the multiplication of two binomial expressions to an area model
- ❑ multiply two polynomials symbolically and simplify the product by combining like terms
- ❑ generalize and explain a strategy for multiplying polynomial expressions
- ❑ verify a polynomial product by substituting numbers for the variables
- ❑ generalize and explain a strategy for the multiplication of polynomials
- ❑ identify and explain errors in a solution for a polynomial multiplication

Lesson Introduction

Using tiles is a great visual way to represent a theoretical concept, but they are not necessarily quick or convenient to use. The purpose of using tiles is to help you see and understand how to multiply polynomials. The next step is then to apply that understanding to a symbolic situation. This lesson will help you take what you learned from working with the tiles and use it to develop a strategy that can be applied to multiplying polynomial expressions.

Multiplying Binomials

Using an Area Model to Represent Binomial Multiplication

If you were asked to multiply 12×34 without using a calculator, what strategy would you use? Do it!

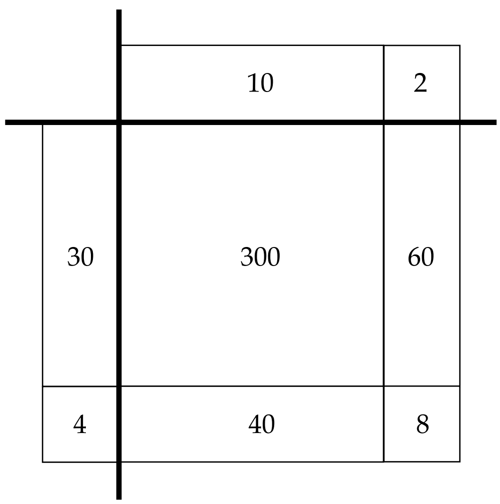
Chances are you grabbed a pen and wrote it as

$$\begin{array}{r} 12 \\ \underline{\times\ 34} \end{array}$$

and began to multiply the 4 and 2, then the 4 and 1, wrote a 0 under the 8 and then multiplied the 3 and 2 and the 3 and 1. Did you come up with 408 when you combined the two rows? If so, you correctly applied a distributive strategy—multiplying the 4 by both digits in 12 and then multiplying 3 by both digits in 12. The zero you wrote under the 8 was there to indicate you were actually multiplying the 12 by 30, not just 3. To simplify your answer, you then combined the 48 and 360 by adding the 8 and 0 in the ones column. In the tens column, you were adding 40 and 60 so you carried the 1 to the hundreds column and wrote a zero. Then you added 100 and 300 to get a total of 408.

This is similar to the strategy you likely used when multiplying polynomial tiles and writing your solution steps in the last lesson.

Visualize it using an area model like this:



Note that the size of the boxes along the frame differentiates between the tens and the ones, but the size doesn't reflect the quantity in any way.

Notice that the area model gives the same partial products that you got using the other method: 300 + 60 + 40 + 8 = 408.

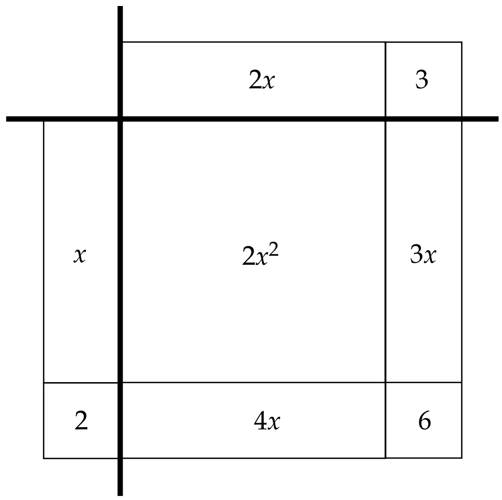
We can use this method with polynomials as well.

Would an example of using the area model be helpful for you to have on your Resource Sheet?

Example 1

Illustrate the product of $(2x + 3)(x + 2)$ using an area model. Simplify the solution.

Solution:



$(2x + 3)(x + 2)$

$= 2x^2 + 4x + 3x + 6$

$= 2x^2 + 7x + 6$

Note that the size of the boxes along the frame differentiates between the terms with variables and the constants, but the size doesn't reflect the quantity or value in any way.

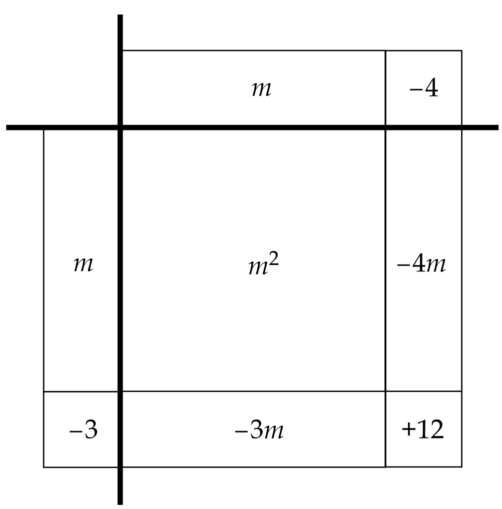
The nice thing about using this area model to illustrate polynomial multiplication is that you can easily adapt it to represent different variables, and can include negative values.

Example 2

Illustrate $(m - 4)(m - 3)$ using the area model and find the product.

Solution:

$(m - 4)(m - 3)$



$(m - 4)(m - 3)$

$= m^2 - 3m - 4m + 12$

$= m^2 - 7m + 12$

Remember to apply what you know about positive and negative values when multiplying and simplifying.

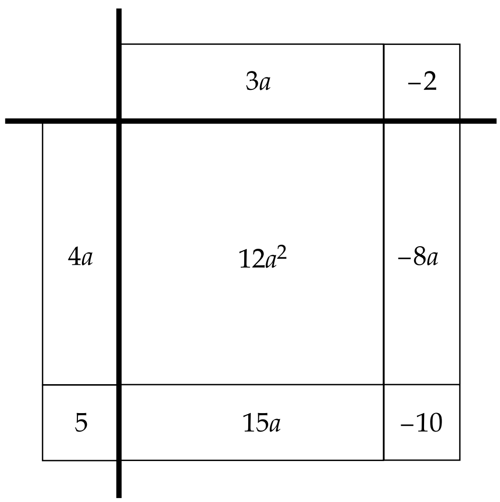
$$(+)(+) = (+)$$
$$(-)(-) = (+)$$
$$(+)(-) = (-)$$
$$(-)(+) = (-)$$

Example 3

Illustrate $(3a - 2)(4a + 5)$ using the area model and find the product.

Solution:

$(3a - 2)(4a + 5)$



$$\begin{aligned}&(3a - 2)(4a + 5)\\ &= 12a^2 + 15a - 8a - 10\\ &= 12a^2 + 7a - 10\end{aligned}$$

Remember to multiply the coefficients as well as the variables.

$$(3a)(4a) = 12a^2$$

When simplifying, combine like terms with both positive and negative coefficients. Consider which you have more of, and use that sign to show how many.

$$(15) + (-8) = +7$$

If you combine fifteen positives and eight negatives, you have more positives than negatives. How many more? Seven.

Binomial Multiplication

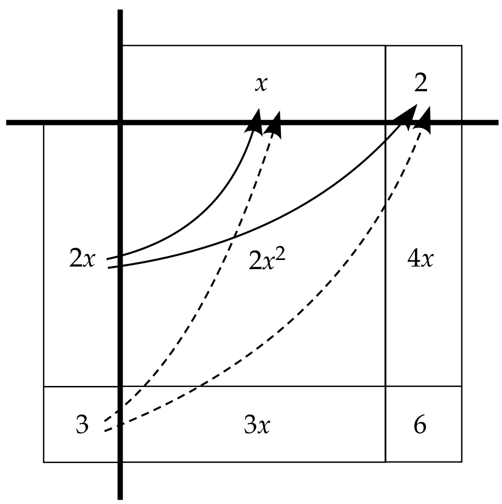
The purpose of tile diagrams and the area model is to give you a concrete or pictorial way to see how binomial multiplication works. As with any other tool or strategy, you can use it as long as it is helpful and convenient to do so. The goal is to use the strategies until you understand how and why the concepts they illustrate work, and then move on to do the math symbolically without them. Eventually, being able to multiply binomials without a diagram will allow you to work more quickly.

These area models are a great way to demonstrate the symbolic process of the distributive property, used when multiplying binomials.

Example 4

Multiply $(2x + 3)$ $(x + 2)$ using the area model.

Solution:



$$(2x + 3)(x + 2)$$
$$= 2x^2 + 4x + 3x + 6$$
$$= 2x^2 + 7x + 6$$

The arrows show that the two terms in the factor along the vertical edge are being multiplied with each of the terms along the horizontal edge.

This can also be illustrated using the arrows and the two binomials.

The product simplifies the same way. Just combine the like terms.

Given the multiplication expression of $(2x + 3)$ $(x + 2)$, the two terms in the first binomial were multiplied by, or distributed to, the two terms in the second binomial.

$$(2x + 3)(x + 2) = 2x(x + 2) + 3(x + 2) \qquad \text{(The distributive property)}$$

The Distributive Property of Multiplication and FOIL

The **distributive property of multiplication** states that: $a \times (b + c) = ab + ac$.

$$a(b + c) = ab + ac$$

If you substitute values in for the variables a, b, and c, you can see that this is true.

Including the definition of the distributive property of multiplication on your Resource Sheet would be a good idea.

For example, if $a = 2$, $b = 5$ and $c = 3$

$a \times (b + c)$	$= \ ab + ac$
$2(5 + 3)$	$2 \times 5 + 2 \times 3$
$2(8)$	$10 + 6$
16	16

The left-hand side of the equation is equal to the right-hand side, so the distributive property is a true statement.

When this property is applied to binomial multiplication, you have to repeat the steps, distributing or multiplying both terms in the first binomial by both terms in the second binomial. Always simplify by combining the like terms if possible.

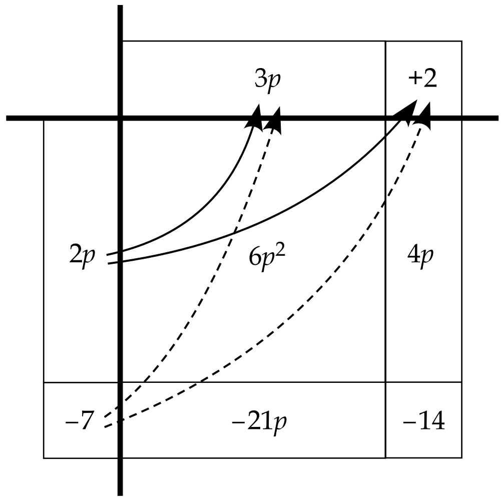
$$(a + b)(c + d) = ac + ad + bc + bd$$

Example 5

Multiply $(2p - 7)(3p + 2)$. Illustrate the product using an area model, and show how the distributive property is applied.

Solution:

$(2p - 7)(3p + 2)$



$(2p - 7)(3p + 2)$

$= 6p^2 + 4p - 21p - 14$

$= 6p^2 - 17p - 14$

An acronym that can be used to help you remember the order of distribution in binomial multiplication is **FOIL**, which stands for

First

Outside

Inside

Last

It may be helpful to include this acronym on your Resource Sheet.

Example 6

Multiply $(4x - 1)(x + 3)$ using the distributive property or FOIL, and then show how the result matches the diagram from an area model.

Solution:

Multiply the *first* term in each binomial

$(4x - 1)(x + 3) = 4x^2$

Multiply the two *outside* terms

$(4x - 1)(x + 3) = 12x$

Multiply the two *inside* terms

$$(4x - 1)(x + 3) = -x$$

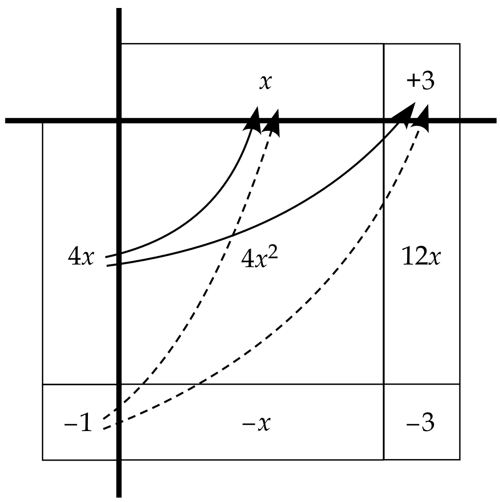
Multiply the *last* term in each binomial

$$(4x - 1)(x + 3) = -3$$

Then simplify the product by combining like terms

$$4x^2 + 12x - x - 3 = 4x^2 + 11x - 3$$

This can be represented graphically using the area model.



$$\begin{aligned}&(4x - 1)(x + 3)\\&= 4x^2 + 12x - x - 3\\&= 4x^2 + 11x - 3\end{aligned}$$

When you compare the products inside the area model to the products from each step above, you notice they are the same.

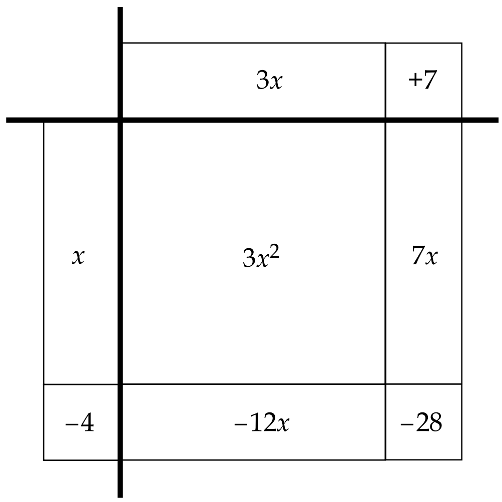
Example 7

Multiply $(x - 4)(3x + 7)$, using FOIL.

Illustrate the product using an area model. Show the steps symbolically and simplify the answer.

Solution:

Area Model:



Using FOIL:

First: $(x)(3x) = 3x^2$
Outside: $(x)(7) = 7x$
Inside: $(-4)(3x) = -12x$
Last: $(-4)(7) = -28$
Simplifying: $3x^2 - 5x - 28$

$(x - 4)(3x + 7)$

$= 3x^2 + 7x - 12x - 28$
$= 3x^2 - 5x - 28$

Symbolically:

$(x - 4)(3x + 7)$

Example 8

Solve $(x - 5)^2$.

Solution:

Be careful that you are not fooled by the exponent of 2. In Module 2, you were asked to simplify questions that looked similar to this but were actually different!

Module 2 $(5x)^2$	Module 6 $(x - 5)^2$
The 2 as the exponent tells us to multiply the base by itself so	
$= (5x)(5x)$	$(x - 5)(x - 5)$

In Module 2, this is the same as distributing the 2 inside the brackets (just combine like terms.

Distribute: $= 5^2x^2$
$= 25x^2$

Multiply: $= 5 \times x \times 5 \times x$
$= 25x^2$

In this example, that will not get you the same result

Distribute: $= (x^2 - 5^2)$ $\qquad$ Multiply: $= x^2 - 5x - 5x + 25$

$= x^2 - 25$ $\qquad$ $= x^2 - 10x + 25$

These are not the same!

If we let $x = 1$ to verify:

$= (1)^2 - 25$ $\qquad$ $= (1)^2 - 10(1) + 25$

$= -24$ $\qquad$ $= 1 - 10 + 25 = 16$

You get the correct answer only when you multiply, because that is the definition of an exponent (multiply the base together "x" times, depending on the number).

Therefore, $(x - 5)^2 = x^2 - 10x + 25$

You will want to make a note of this on your Resource Sheet, especially since most students automatically do what they did in Module 2.

While the area model can be adapted to represent any polynomial * polynomial product, FOIL as a strategy for multiplying works only with binomial × binomial factors. For other polynomial multiplication questions, apply the same idea using the distributive property. Multiply each term in the first polynomial by each term in the second polynomial. Simplify your answer.

Example 9

Multiply $(x - y)(x^2 + 3xy - y^2)$

Solution:

$(x - y)(x^2 + 3xy - y^2)$

Apply the first term in the binomial to each term in the trinomial, then distribute the second term in the binomial to all the terms in the trinomial. Simplify.

like terms

$= x^3 + 3x^2y - xy^2 - x^2y - 3xy^2 + y^3$

like terms

$= x^3 + 2x^2y - 4xy^2 + y^3$

Make sure the terms in your final answer are arranged alphabetically and in descending order of power of the first variable (in this case, x).

	x^2	$3xy$	$-y^2$
x	x^3	$3x^2y$	$-xy^2$
$-y$	$-x^2y$	$-3xy^2$	$+y^3$

$x^3 + 3x^2y - x^2y - xy^2 - 3xy^2 + y^2 = x^3 + 2x^2y - 4xy^2 + y^3$

Tiles, area model, distributive property, FOIL—what should you use and when?

You can use whatever strategy you feel most comfortable with, as long as it fits the question. Tiles only work when all terms are positive; FOIL only works when you are multiplying two binomials. The area model works any time you are multiplying two expressions, and the distributive property can be applied to any polynomial multiplication. In any case, always simplify your final answer by combining like terms.

If you are having a hard time remembering which strategy to use where, you may want to include this information on your Resource Sheet (in point form).

Verifying Your Solutions

To check your solution to a binomial multiplication question, you can substitute a value for the variable into the original question and into your solution to determine if they agree.

Example 10

Verify that $(2x + 3)(x + 2)$ is equal to $2x^2 + 7x + 6$ by checking a value of x.

Solution:

Choose a value for x that is easy to work with and substitute it into the expressions.

$x = 1$

$$(2(1) + 3)(1 + 2)$$
$$= (2 + 3)(3)$$
$$= 5 \times 3$$
$$= 15$$

Compare that to $2x^2 + 7x + 6$ with the same value of x.

$$2(1)^2 + 7(1) + 6$$
$$= 2 + 7 + 6$$
$$= 15$$

$15 = 15$

The solutions agree.

Learning Activity 6.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. At your dad's office they have carpet made out of squares. Each square is 2 feet by 2 feet. If your dad's cubicle is 5 squares deep and 5 squares wide, what is the area of your dad's cubicle in feet?
2. Evaluate: $\sqrt{2^{-4}}$.
3. 1 yard^3 = ________ feet^3.
4. What is the domain of the function $f(x) = x + 4$?
5. It costs $4.00 for a package of 3 chocolate bars. Geri spends $20 on chocolate bars. How many chocolate bars does she buy?
6. I have 9 letters in my name. Is it possible that half of those letters are vowels?
7. What is the LCM of 3 and 5?
8. Solve: $\frac{6}{5} + \frac{2}{3}$.

Part B: Multiplying Polynomials

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Simplify each product. Show the steps pictorially and symbolically.
   a) $(5x + 3)(x + 2)$
   b) $(4h - 5)(-3h + 7)$
   c) $(x^2 - x)(x + 2)$
   d) $(x + y)(x^2 + 2x - 1)$

continued

2. Simplify each product and verify your solution.
   a) $(2x - 1)^2$
   b) $(x + 3)^3$ (Hint: Write out the multiplication like you did in (a), then multiply only two polynomials together at a time)

3. Simplify $(x + y + z)^2$.

4. Rita simplified the product of $(x + 4)(x - 2)$ as $x^2 - 8$.

   Identify and explain the error she made, and show how to correct it.

5. Multiply the following polynomials.

   a) $\left(2x^2y\right)\left(3xy^2\right)$
   b) $\left(\dfrac{-2}{3}a^3b\right)\left(-6ab^3\right)$
   c) $\left(3x^2\right)\left(4x^3\right)\left(5x^4\right)$
   d) $2x(x+1)$
   e) $\left(-2x^2\right)\left(x^3+3x^2-x\right)$
   f) $\left(-3-5p+9p^2\right)(-2p)$
   g) $(x+1)(x+2)$
   h) $(2x-4)\left(3x^2+x-2\right)$
   i) $(2x-3y)(3x+y)$
   j) $(x-2y)\left(x^2+xy-4y^2\right)$
   k) $(3x-2)^2$
   l) $(a+b-c)(a-b+c)$
   m) $(x+3)\left(x^2-3x+9\right)$
   n) $\left(1-2x+x^2\right)(1+3x)$

Lesson Summary

This lesson provided opportunities for you to take what you know about multiplying polynomials using tiles and 2-digit multiplication and apply it to multiplying binomials and expressions with more than 2 terms. You used the area model and the distributive property to multiply polynomials and simplify the solutions by combining like terms. You learned how to verify if your answers are correct, and practised identifying errors and correcting them.

In the next lesson, you will look at the opposite of multiplying and learn how to find the factors of a polynomial, given the product. In order to do so efficiently, knowing how to multiply binomials is a critical skill. The best way to get good at anything is to practise!

For that reason, additional binomial multiplication questions are included here for you to practice. You are not required to send these questions to the Distance Learning Unit. The answers are provided in the Learning Activity Keys for you to check and correct your own work.

Notes

Practice Questions Set A

1. $(x + 6)(x + 4)$
2. $(x - 1)(x + 7)$
3. $(x - 10)(x + 6)$
4. $(x + 2)(x + 3)$
5. $(x - 4)(x + 1)$
6. $(x - 1)(x - 5)$
7. $(x - 3)(x - 1)$
8. $x(x - 4)$
9. $(x - 3)(x + 2)$
10. $(x - 5)(x + 3)$
11. $(2x - 1)(x + 1)$
12. $(x - 7)(x - 3)$
13. $(x + 6)(x + 8)$
14. $(x + 9)(x + 6)$
15. $(x + 9)(x + 3)$
16. $(x + 9)(x - 3)$
17. $(x + 5)(x + 3)$
18. $(x + 5)(x - 3)$
19. $(2x + 1)(x + 3)$
20. $(2x + 1)(x - 3)$

Practice Questions Set B

1. $(x + 2)(x + 3)$
2. $(x - 2)(x - 3)$
3. $(x - 2)(x + 3)$
4. $(x + 2)(x - 3)$
5. $(x + 1)(x + 6)$
6. $(x - 1)(x - 6)$
7. $(x + 1)(x - 6)$
8. $(x - 1)(x + 6)$
9. $(x + 3)(x + 4)$
10. $(x - 3)(x - 4)$
11. $(x - 3)(x + 4)$
12. $(x + 3)(x - 4)$
13. $(x + 1)(x + 12)$
14. $(x - 1)(x - 12)$
15. $(x + 1)(x - 12)$
16. $(x - 1)(x + 12)$
17. $(x + 2)(x + 6)$
18. $(x - 2)(x - 6)$
19. $(x - 2)(x + 6)$
20. $(x + 2)(x - 6)$

Notes

Assignment 6.2

Multiplying Polynomials

Total Marks = 26

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Simplify each product. Show your steps.

   a) $(x - 5)(x + 9)$ *(2 marks)*

   b) $(6y - 7)(4y + 8)$ *(2 marks)*

Assignment 6.2: Multiplying Polynomials (continued)

c) $(x^2 + 3)(x^2 + 2)$ *(2 marks)*

d) $(m + n)(3m^2 - 5mn - 2n^2)$ *(2 marks)*

Assignment 6.2: Multiplying Polynomials (continued)

2. Simplify and verify.

   a) $(-x^2 + x)(7x^2 + 3x - 9)$ *(4 marks)*

   b) $(x - 10)^2$ *(4 marks)*

3. Ron stated the answer to $(x + 3)^2$ as $x^2 + 9$. Identify and explain his error, and show how to correct it. *(4 marks)*

(Solution Box for Question 4)

Write the letters in the appropriate box below to show the order of the pieces in the completed puzzle.

Assignment 6.2: Multiplying Polynomials (continued)

4. These 12 pieces of a Binomial Multiplication Puzzle fit together in a 4 × 3 arrangement. Cut out each piece, solve the binomial multiplications, and arrange the pieces so each product matches up next to its factors. Write the order of the letters on the pieces of the completed puzzle on the blank frame provided on the previous page. **Note:** The questions/answers along the outside edge of the completed puzzle will not match up with a solution. There is more than one correct arrangement. You just need to find one. *(6 marks – 1/2 mark each)*

$x^2 - 4$ $(x^2 + 3)(x^2 + 4)$ **S** $x^2 + 10x + 24$ $x^2 + 5x + 4$	$x^2 + 2xy + y^2$ $(x + 6)(x + 4)$ **B** $x^2 + 9x + 18$ $x^2 - 100$	$(x - 9)(x + 9)$ $(x + 3)(x + 6)$ **F** $x^2 + 8x + 15$ $x^2 + 4x + 4$
$(x - 11)(x + 11)$ $(x + 6)(x + 3)$ **W** $x^4 + 7x^2 + 12$ $(x + 6)(x + 8)$	$x^2 + 5x + 4$ $(x + 5)(x + 3)$ **Q** $x^2 + 10x + 21$ $(x - 2)(x + 3)$	$x^2 + 10x + 25$ $(x + 7)(x + 3)$ **P** $x^2 + 9x + 18$ $(x^2 + 3)(x^2 + 3)$
$x^2 + 14x + 48$ $(x - 3)(x + 3)$ **H** $(x - 12)(x + 12)$ $(x + 4)(x + 1)$	$(x + 4)(x + 1)$ $x^2 - 144$ **K** $x^2 + 15x + 54$ $(x + 5)^2$	$x^4 + 6x^2 + 9$ $(x + 6)(x + 5)$ **T** $x^2 + 10x + 27$ $(x - 2)(x + 2)$
$x^2 + x - 6$ $(x + 8)(x + 3)$ **M** $x^2 + 11x + 30$ $x^2 - 121$	$(x - 10)(x + 10)$ $(x + 9)(x + 6)$ **J** $x^2 - 9$ $x^2 - 81$	$(x + 2)(x + 2)$ $(x + 3)(x + 9)$ **V** $x^2 + 11x + 24$ $(x + y)^2$

Notes

Lesson 3: Factoring Polynomials

Lesson Focus

In this lesson, you will

- ❑ determine the common factors in the terms of a polynomial
- ❑ factor trinomials and record the process pictorially and symbolically
- ❑ verify the factors of a polynomial by multiplying
- ❑ express a polynomial in factored form, as the product of its factors
- ❑ identify and explain errors in the factorization of polynomials

Lesson Introduction

When you learn to drive a car, it's important that you figure out how to drive in reverse. Knowing how to multiply polynomials is like driving forward, and factoring is like driving backward. This lesson will look at how tiles illustrate the factoring of polynomials. You will use tiles to find common factors and factor trinomials, and write out the solutions symbolically.

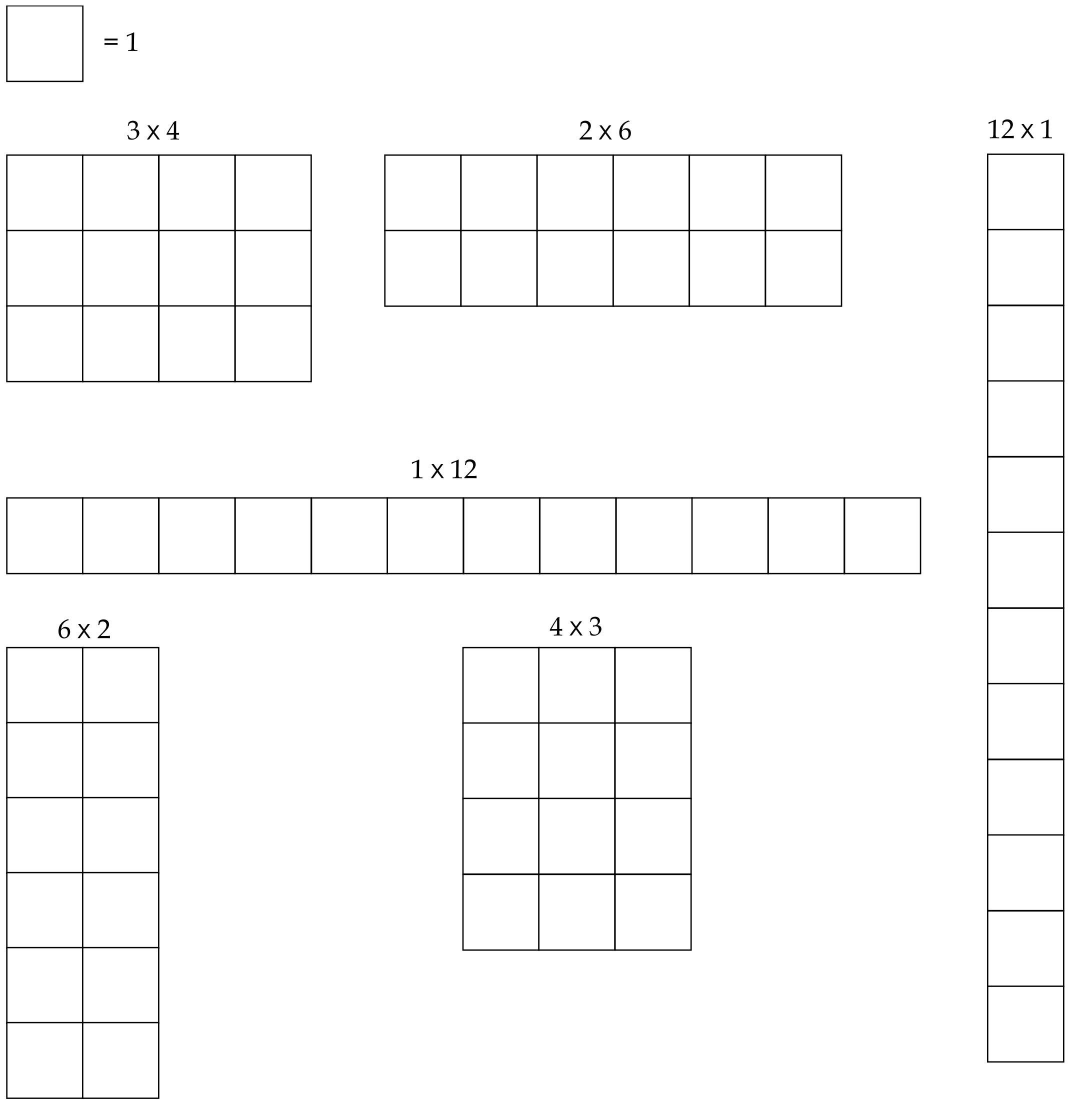
How Do You Factor?

In the last two lessons you have used the term *factor* to indicate the two (or more) expressions you multiply together to find the 'product' or answer. In this lesson, you will start with the product and try to find the factors that would multiply together to produce that product.

Before you continue, we would like to remind you that your tutor/marker is available to help you, should you need it. A large number of students struggle with factoring, so if you find it confusing you are not alone. Also, working with your learning partner can be quite helpful, so ask him or her to sit down with you and work through some examples if you need to.

Factoring with Tiles

If you are given 12 1-tiles, you can arrange them in a rectangular array in several different ways.



The dimensions of the rectangles, when multiplied, give you the area, which is 12 square units.

$3 \times 4 = 12$

$2 \times 6 = 12$

$1 \times 12 = 12$

Each of these is a factor pair that multiplies to give you the product of 12.

You will notice that some arrangements and dimensions in the diagrams are the same.

12×1 and 1×12 are the same array, only rotated.

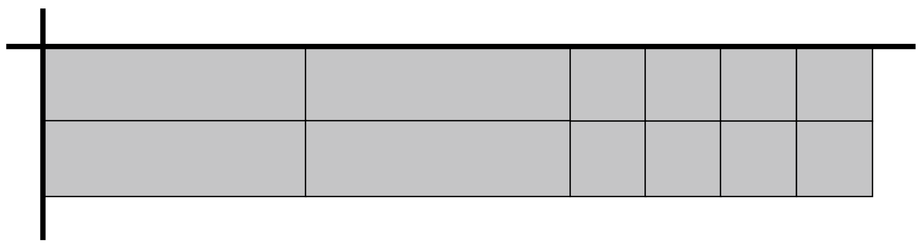
Some arrangements are a closer approximation to a square shape, while others have more extreme rectangular dimensions.

The same idea applies to tile representations of polynomials.

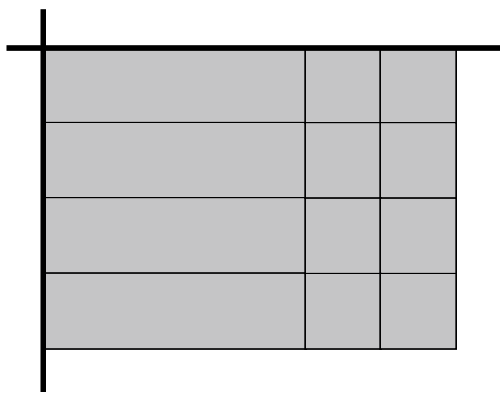
Example 1

Arrange $4x + 8$ tiles in a rectangle and determine its factor pairs.

Solution:

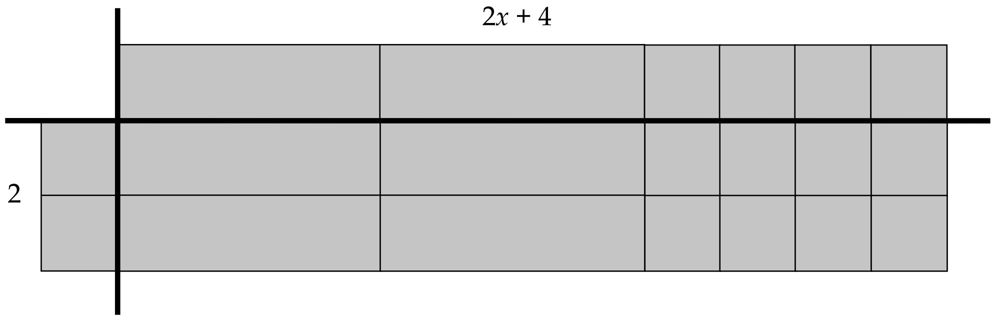


or

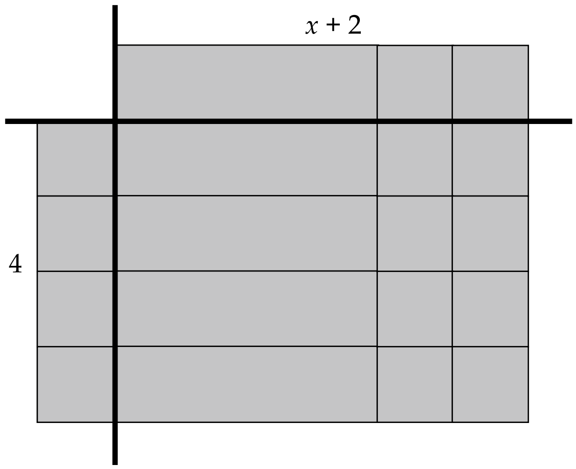


There are two possible rectangular arrangements of these tiles. Notice that the similar shaped tiles are grouped together when possible.

Now, determine which tiles can be placed along the frame edges to result in that product or area.



or



The factor pairs that result in the product of $4x + 8$ are

$(2)(2x + 4)$ or $(4)(x + 2)$

Use the distributive property of multiplication to check that the factors are correct.

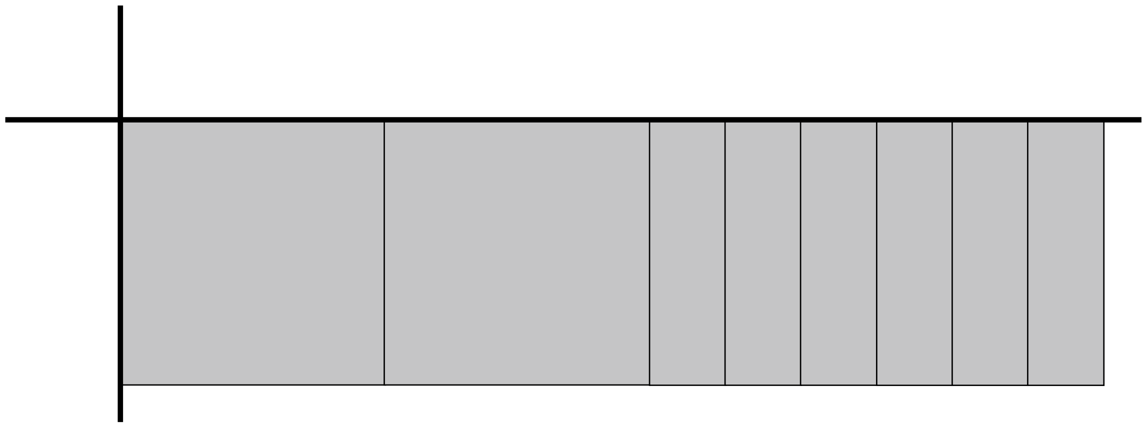
$(2)(2x + 4) = 4x + 8$

$(4)(x + 2) = 4x + 8$

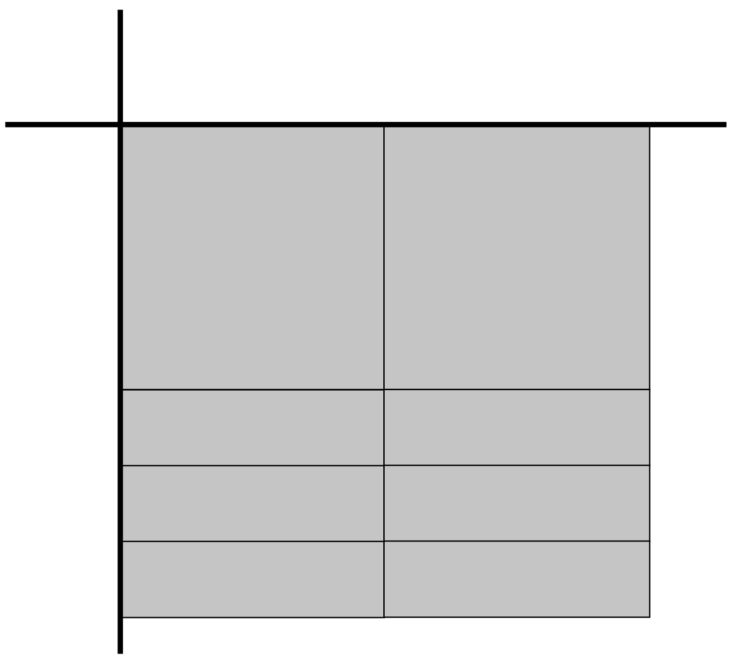
Example 2

Arrange $2x^2 + 6x$ tiles in a rectangular array and determine its factors.

Solution:

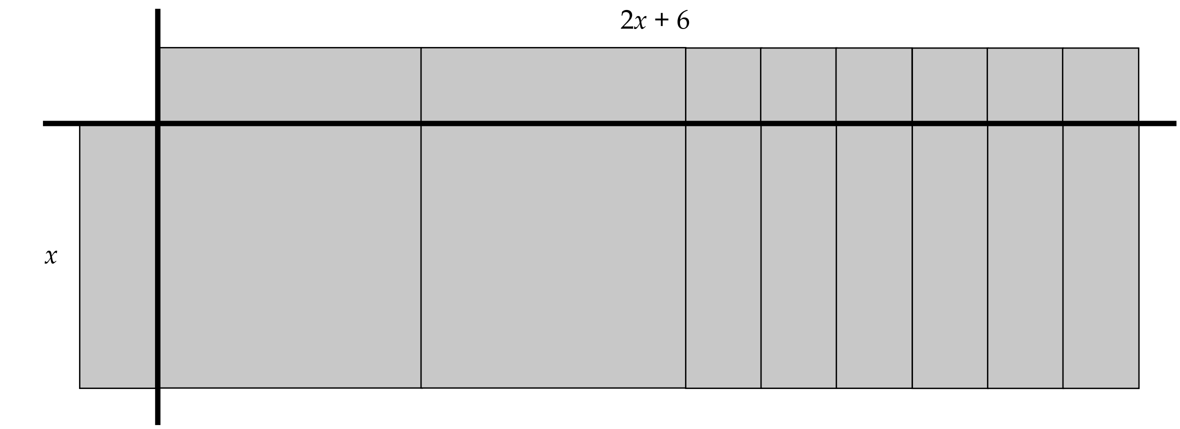


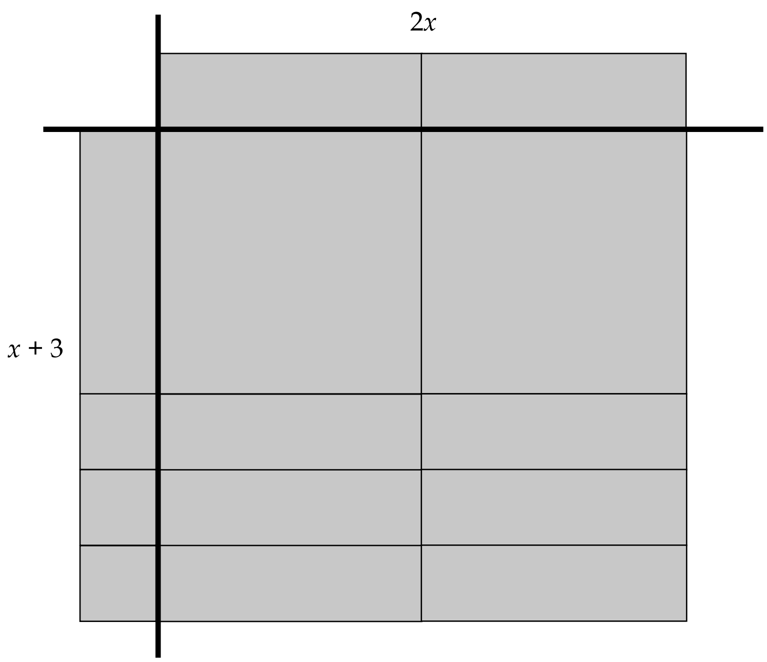
or



Two possible unique arrangements are shown.

Now, determine which tiles can be placed along the frame to match the area shown.





$(x)(2x + 6)$ and $(2x)(x + 3)$ are both factor pairs that when multiplied, produce the product of $2x^2 + 6x$.

Is one arrangement better than the other?

Both are correct, but the arrangement that more closely resembles a square is "better" because it identifies "common factors" of the two terms in the binomial ($2x$ is a factor of both $2x^2$ and $6x$).

Common Factors

Take a look at the two terms in the binomial $2x^2 + 6x$.

What do they have in common? Both have the variable x and both coefficients are even numbers, so they are divisible by 2. The second tile arrangement isolates these common factors, $2x$, along one edge of the frame.

$$(2x)(x + 3) = 2x^2 + 6x$$

$2x$ is the greatest common factor of the two terms in the polynomial. $(x + 3)$ is what you must multiply $(2x)$ by to get the product of $2x^2 + 6x$.

Example 3

Determine the greatest common factor in the terms of the binomial $12x^2 + 9x$ using tiles, and state your solution steps symbolically.

Solution:

From the example above, you know that 12 square tiles can be arranged in

12×1

6×2 or

4×3 arrays.

Somehow, you must also arrange 9 x-tiles alongside it to make a rectangle. 9 is a multiple of 3, so the 4 by 3 arrangement is best.

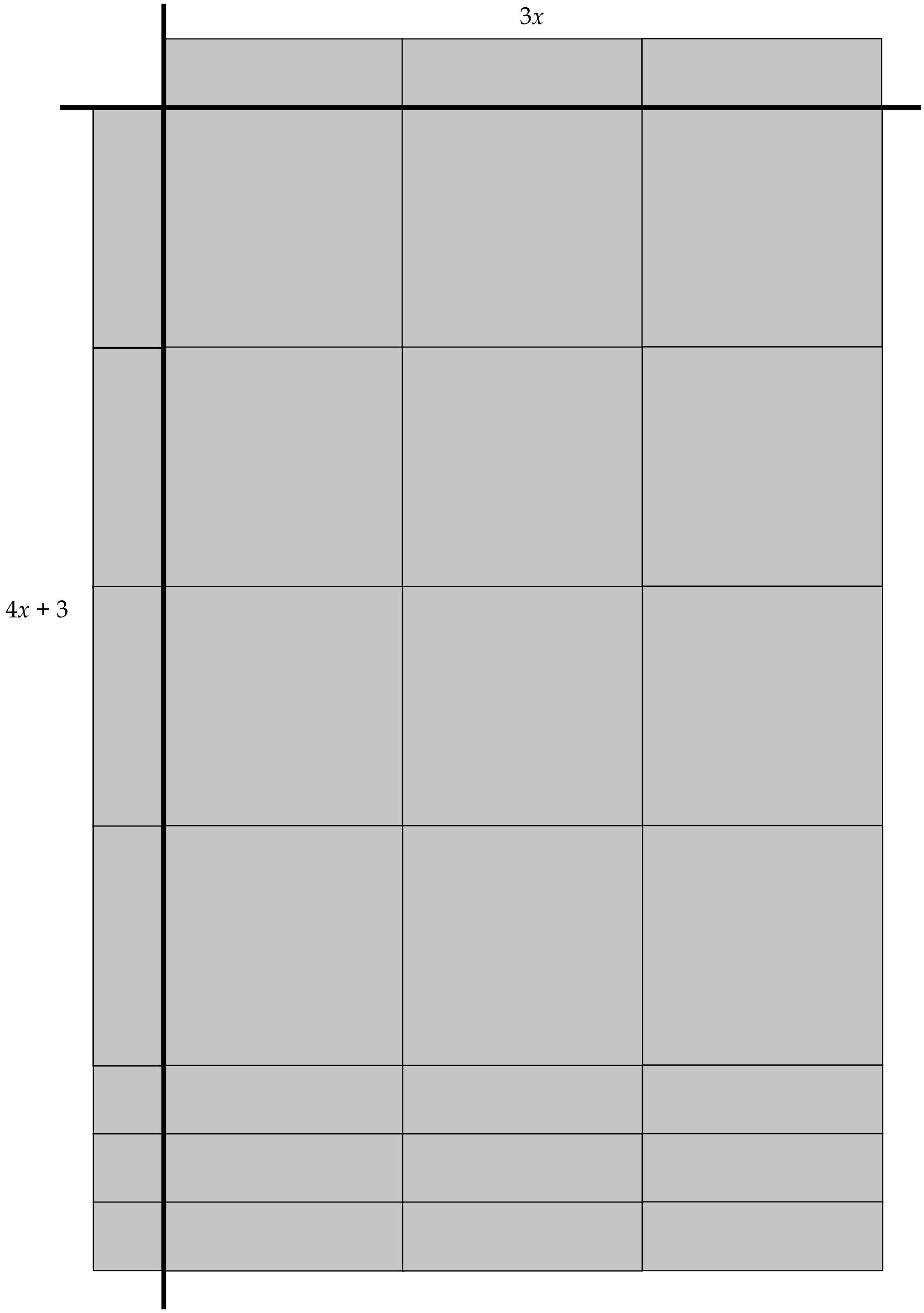
This arrangement is as close to a square as possible (see following page). Therefore, the greatest common factor has been isolated.

Look back at the terms in the binomial.

$$12x^2 + 9x$$

They both have the variable x and both 12 and 9 are divisible by 3. The common factor in these terms is $3x$.

$$(3x)(4x - 3) = 12x^2 - 9x$$



3x
4x + 3

Example 4

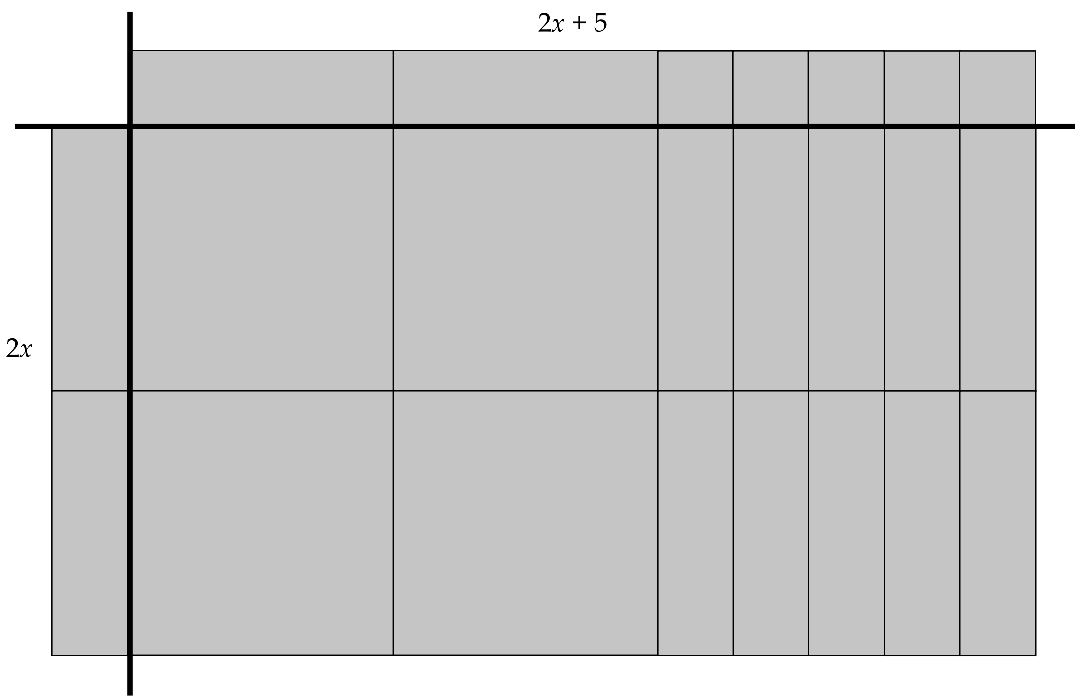
Factor the binomial. Use tiles to support your answer. Verify your answer using distributive multiplication.

$4x^2 + 10x$

Solution:

The terms in the binomial have a common factor of $2x$. The factors of this binomial are $(2x)(2x + 5)$.

This arrangement is as close to a square as possible.



Check: $2x(2x + 5) = 4x^2 + 10x$

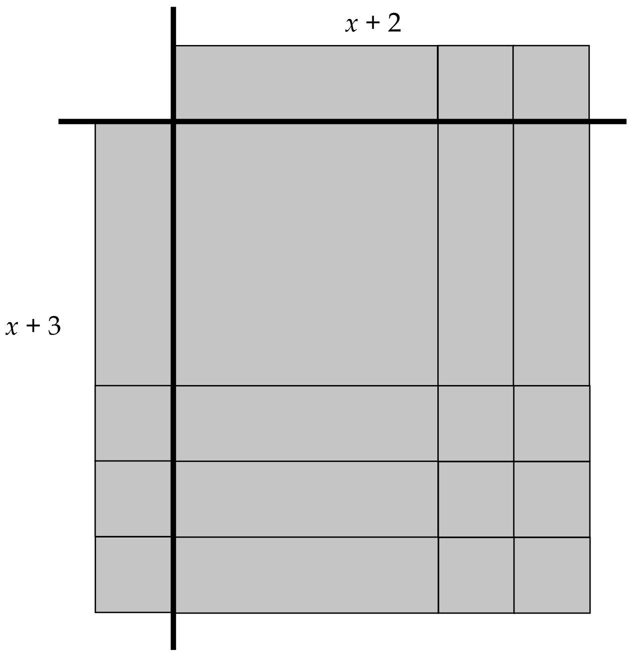
Factoring Trinomials

Example 5

Arrange tiles in a rectangle to represent the trinomial $x^2 + 5x + 6$ and state its factors. Verify using multiplication.

Solution:

The first step is to check for common factors. As the terms in this trinomial have no common factors, you will have to play with the tiles to try and find a rectangular arrangement that works.



Verify:

$$(x + 2)(x + 3)$$
$$= x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

The factors of $x^2 + 5x + 6$ are the binomials $(x + 2)$ and $(x + 3)$.

Example 6

Factor $x^2 + 8x + 12$ and verify your answer.

Solution:

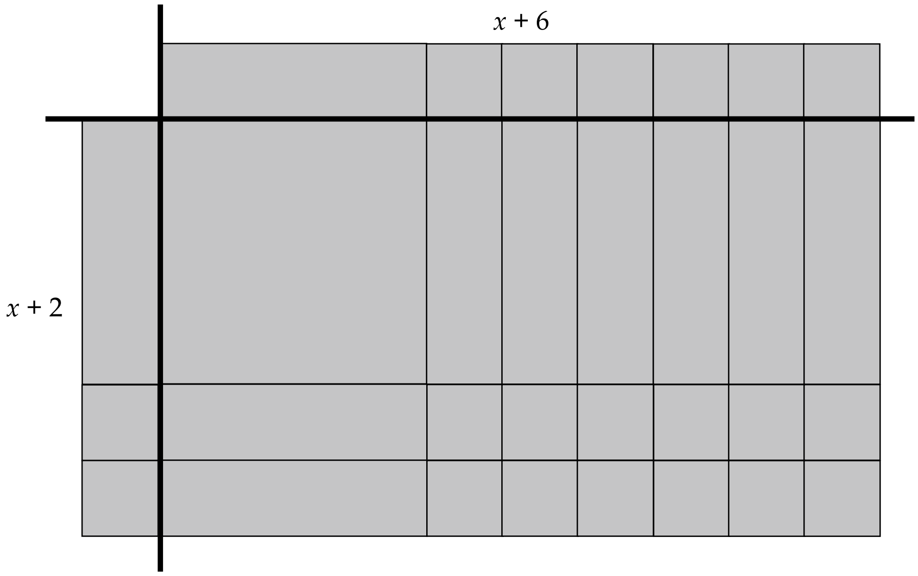
This trinomial has many more tiles to arrange! Is there a shortcut to help you decide which way will work? Notice in example 5 that the five x-tiles are arranged so that there are 2 x-tiles placed vertically and 3 placed horizontally. This is a good way to arrange the 5 tiles because 2 and 3 give you a sum of 5 (the coefficient of x in the trinomial) and a product of 6 (the constant in the trinomial and the number of 1-tiles you need). Does this pattern work for 8 and 12? Can you think of two numbers that have a sum of 8 and a product of 12? List the factor pairs for the product of 12.

$1 \times 12 = 12$

$2 \times 6 = 12$

$3 \times 4 = 12$

2 and 6 add up to 8 and give you a product of 12. Use that to help you arrange the tiles.



Verify:

$(x + 6)(x + 2)$

$= x^2 + 2x + 6x + 12$

$= x^2 + 8x + 12$

$x^2 + 8x + 12 = (x + 6)(x + 2)$

Note: Do you recognize this type of question from the BrainPower questions at the start of each learning activity?

Example 7

Factor $x^2 + 8x + 15$ and check your answer using multiplication.

Solution:

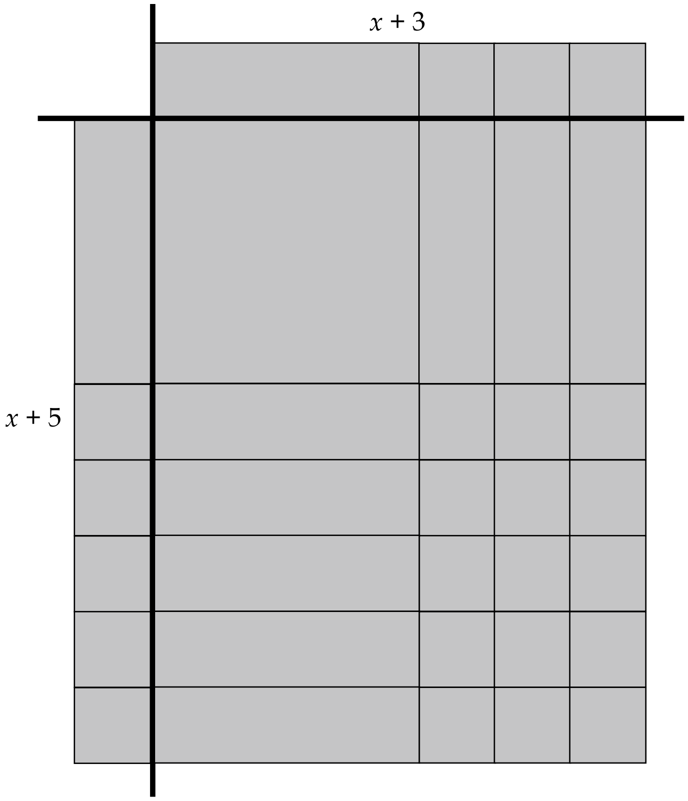
What factor pair of 15 will give you a sum of 8?

$1 \times 15 = 15$

$3 \times 5 = 15$

From these factor pairs of 15, 3 and 5 give you a sum of 8.

The tiles can be arranged in this way:



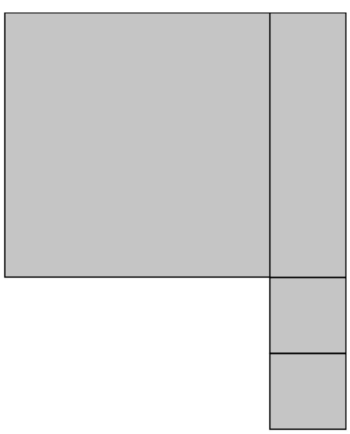
Verify:
$(x + 3)(x + 5)$
$= x^2 + 5x + 3x + 15$
$= x^2 + 8x + 15$

$x^2 + 8x + 15 = (x + 3)(x + 5)$

Example 8

Factor $x^2 + x - 2$, using tiles to illustrate your answer.

Solution:



It's impossible to make a rectangle given only the tiles stated in the terms of this trinomial!

So is this trinomial unfactorable? No, but the usefulness of tiles is limited to expressions with only positive coefficients and constants.

You can, however, still use the algebraic strategy from above.

Is there a factor pair of −2 that gives you a sum of +1? To get a negative product, you will need one positive and one negative factor.

$$-1 \times 2 = -2$$

$$1 \times -2 = -2$$

The sum of (−1) and (2) is 1, so that factor pair should work.

$$(x - 1)(x + 2)$$

Apply FOIL, the distributive property, or the area model to check the factors.

FOIL Method:

$(x - 1)(x + 2)$

F O I L

$= x^2 + 2x - x - 2$

$= x^2 + x - 2$

Distributive Property

$x(x + 2) - 1(x + 2)$

$x^2 + 2x - x - 2$

$x^2 + x - 2$

Area Model:

	x	-1
x	x^2	$-x$
2	$2x$	-2

Example 9

Factor $x^2 - 2x - 8$

Solution:

This trinomial has negative values, so tile diagrams are not appropriate.

List the factor pairs of -8 and see if any result in a sum of -2.

The factor pairs of 8 are

1, 8

2, 4

To get a negative product, you will need to multiply one positive and one negative factor.

$-1, +8$

$+1, -8$

$-2, +4$

$+2, -4$

The factor pair of $+2$ and -4 give you a product of -8 and a sum of -2.

The factors of $x^2 - 2x - 8$ are $(x + 2)(x - 4)$.

Verify this with the distributive property or by using the area model.

$(x + 2)(x - 4)$
$= x^2 - 4x + 2x - 8$
$= x^2 - 2x - 8$

or

	x	2
x	x^2	$2x$
-4	$-4x$	-8

Learning Activity 6.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Samantha has twice as many dresses as she has shirts and three times as many shirts as she has pants. If Samantha has 12 shirts, how many pairs of pants does she have?
2. Multiply: $(x + 2)(x + 3)$.
3. Write the following as a function: $y - x = 25$.
4. Convert: 1 foot^2 = ________ inches^2.
5. The slope of a line is $\frac{4}{5}$. One point on the line is (2, 3). What are the coordinates of another point on this line?
6. Heather is packing her suitcase for Europe. If the dimensions of the suitcase are 100 cm by 80 cm by 25 cm, what is the volume of the suitcase in metres?
7. Team A has won 5 out of their last 9 games. Team B has won 4 out of their last 7 games. Which team has won a greater percentage of their games?
8. You and your friend want to split the bill for dinner. The total is $35.00 for the meal. How much will you pay for dinner?

continued

Learning Activity 6.3 (continued)

Part B: Factoring Binomials and Trinomials

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Sketch a rectangular arrangement of tiles to represent the following expressions. Use the tiles to determine the factors of the polynomial. Verify your answer by multiplying the factors.

   a) $8x^2 + 12x$
   b) $2x^2 + 6x$
   c) $12x^2 + 3x$
   d) $x^2 + x$
   e) $x^2 + 12x + 36$
   f) $x^2 + 7x + 10$
   g) $x^2 + 7x + 12$
   h) $x^2 + 7x + 6$

2. Complete the following chart to identify patterns in factoring the given trinomials.

Trinomial	Coefficient of x^2	Coefficient of x	Constant	Binomial Factors	Sum of Constants in Binomials	Product of Constants in Binomials
$x^2 + 5x + 6$	1	5	6	$(x + 2)(x + 3)$	$2 + 3 = 5$	$(2)(3) = 6$
$x^2 + 8x + 12$	1		12		$2 + 6 = 8$	
$x^2 + 8x + 15$				$(x + 5)(x + 3)$		
$x^2 + x - 2$						$(2)(-1) = -2$
$x^2 - 2x - 8$		-2			$(-4) + 2 = -2$	

continued

Learning Activity 6.3 (continued)

3. a) Given a trinomial in the form $ax^2 + bx + c$, where a is the coefficient of x^2, b is the coefficient of x, and c is a constant, complete the following chart.

Trinomial $ax^2 + bx + c$	***a***	***b***	***c***	**List all factor pairs of *c*. State what signs are needed to produce the sign of the product**	**State factor pair of *c* that gives you the sum of *b*, including the signs**	**State the binomial factors of the trinomial**
$x^2 + 4x - 21$	1	4	−21	1, 21, 3, 7 signs: (+) (−)	−3, 7	$(x - 3)(x + 7)$
$x^2 + 9x + 20$						
$x^2 + 2x - 48$						
$x^2 - 11x + 28$						

b) Write a summary statement describing the relationship between the constants in the binomial factors and b and c in the trinomial (the coefficient of x and the constant in the trinomial).

4. a) Verify the binomial factors in the last column of the chart in 3(a) above by applying the distributive property of multiplication to each.

b) Explain, using examples, the relationship between multiplication and factoring.

5. Factor the polynomials. Verify your answer using distributive multiplication.

a) $x^2 - 7x - 18$

b) $a^2 - 10a - 11$

c) $x^2 - 13x + 40$

d) $y^2 + 13y + 36$

e) $2x^2 + 8x - 64$

HINT: Find a common factor for all the coefficients and constants, then try to factor the trinomial that doesn't include the common factor (e.g., $3x^2 + 6x - 9 = (3)(x^2 + 2x - 3)\ldots$).

6. Identify and explain errors in the following factorizations. Show the correct solution.

a) $x^2 - 5x - 6 = (x - 3)(x + 2)$

b) $18y^2 - 12y = 2(9y^2 - 6y)$

c) $3x^2 - 3x - 6 = (3x - 6)(x + 1)$

d) $x^2 + 20x + 9 = (x + 4)(x + 5)$

Learning Activity 6.3 (continued)

7. Factor the given expressions.

   Note: If you are feeling confident that you are able to factor binomials and trinomials, only do (a) to (d) and (i) to (l). If you want more practice, work through (a) to (p). If you are struggling with these concepts, do not forget that you can contact your tutor/marker, and you can ask your learning partner a question or work with him or her.

a) $12m - 24p$	b) $a - ar^3y$
c) $2a^2 - 12ab + 14ac$	d) $6x^2 - 18z^6y - 6ax^3z$
e) $3r^2 - 15rh$	f) $4n^3 - 4n^2$
g) $32x^2y + 4x^3y$	h) $3mn + 6n^2m^2$
i) $x^2 - 7x + 12$	j) $x^2 - 10x - 24$
k) $x^2 + 25x + 24$	l) $x^2 - 4x - 12$
m) $x^2 + x - 72$	n) $c^2 - 4c - 12$
o) $4 - 5c + c^2$	p) $x^2 - x - 6$

Lesson Summary

This lesson illustrated how to factor out common factors in polynomials, as well as how to factor trinomials pictorially and symbolically.

You learned that factoring is the reverse of the distributive property (multiplying polynomials).

The trinomials you factored in this lesson were in the form $ax^2 + bx + c$ where $a = 1$. The next lesson will use tiles to illustrate a process to factor trinomials where a is an integer ($a \neq 0$), as well as the following special case in factoring polynomials: perfect square trinomials.

Just like tiles were used as a tool in the first lesson to help you learn how to multiply polynomials, hopefully the use of tiles in this lesson helped you to identify patterns in factoring polynomials. The tiles are a valuable visual strategy for learning how to factor. One limitation of tiles, however, is that they can only be used to illustrate expressions with all positive terms; another is that they can be time consuming to draw.

With each new application of tiles, use them as long as they are helpful to you. Once you are confident in your ability to apply the strategies to factoring symbolically, and you can verify your answers without using the tiles, feel free to move on without them.

Assignment 6.3

Factoring Binomials and Trinomials

Total Marks = 32

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Sketch an arrangement of tiles to illustrate the factors of the following polynomials. Verify by multiplying the factors.

   a) $x^2 + 11x + 10$ *(3 marks)*

   b) $2x^2 + 14x$ *(3 marks)*

Assignment 6.3: Factoring Binomials and Trinomials (continued)

2. Determine the greatest common factor in the terms of the given expression, and write each polynomial in factored form.

   a) $5x + 20$ *(1 mark)*

   b) $17x^3 - 51x^2$ *(1 mark)*

   c) $30x^2y - 24xy^3$ *(2 marks)*

   d) $\frac{1}{2}ax + \frac{1}{2}bx + \frac{1}{2}cx$ *(2 marks)*

Assignment 6.3: Factoring Binomials and Trinomials (continued)

3. Factor the trinomials, and write each as the product of its factors.

   a) $x^2 - 10x - 24$ *(2 marks)*

   b) $x^2 - 9x + 18$ *(2 marks)*

   c) $x^2 + 14x + 45$ *(2 marks)*

   d) $x^2 + 6x - 40$ *(2 marks)*

Assignment 6.3: Factoring Binomials and Trinomials (continued)

4. Factor completely (that is, remove common factors first).

   a) $x^3 + x^2 - 12x$ *(2 marks)*

   b) $2x^2 + 4x - 30$ *(2 marks)*

   c) $3x^3 + 21x^2 + 36x$ *(2 marks)*

5. Describe how finding the factor pairs of the constant in a trinomial can help you factor the trinomial. *(2 marks)*

Assignment 6.3: Factoring Binomials and Trinomials (continued)

6. Identify and describe the errors. Show the correct factors.

   a) $x^2 + 3x - 28 = (x - 7)(x + 4)$ *(2 marks)*

   b) $x^2 - 3x + 2 = (x - 1)(x + 3)$ *(2 marks)*

Notes

Lesson 4: Factoring Trinomials

Lesson Focus

In this lesson, you will

- ❑ factor trinomials in the form $ax^2 + bx + c$, where $a \in \mathrm{I}$, $a \neq 0$ and record the process pictorially and symbolically
- ❑ identify and factor perfect square trinomials
- ❑ verify the factors of a polynomial by multiplying
- ❑ express a polynomial in factored form as the product of its factors
- ❑ identify and explain errors in the factorization of polynomials

Lesson Introduction

Learning new skills in mathematics often takes what you already know and builds on it. This lesson will do just that. You can factor polynomials with common factors and trinomials with a leading coefficient of 1. This lesson will take the next step and you will factor trinomials with common factors as well as leading coefficients that are integers. You will use tiles to identify patterns and trends, and then expand your strategies to include steps to help you factor a variety of polynomials.

Factoring Trinomials

Before you continue, we would like to remind you that your tutor/marker is available to give you help, should you need it. A large number of students struggle with factoring, so if you find it confusing you are not alone. Also, working with your learning partner can be quite helpful, so ask him or her to sit down with you and work through some examples if you need the practice.

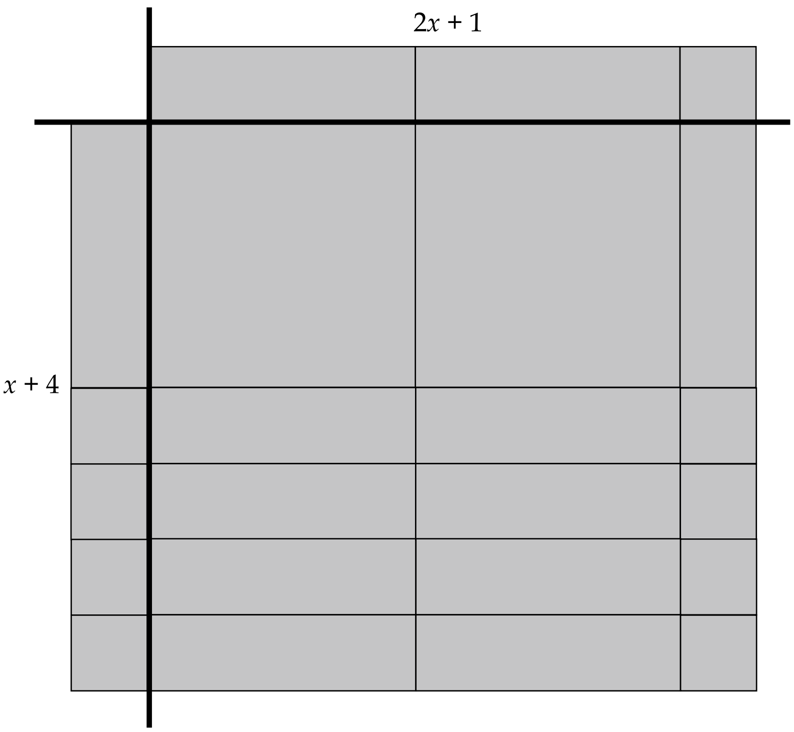
Factoring Trinomials

Look back over the tile diagrams in the previous lesson illustrating trinomial factoring. Each of them has only one x^2 tile. They are all in the form $ax^2 + bx + c$ where $a = 1$. Factoring trinomials where a is an integer ($a \neq 0$) is also possible.

Example 1

Arrange tiles in a rectangle to represent the trinomial $2x^2 + 9x + 4$ and verify its factors.

Solution:



Verify the factors using the distributive property.

$(2x + 1)(x + 4)$

$= 2x^2 + 8x + x + 4$

$= 2x^2 + 9x + 4$

Example 2

Factor $3x^2 + 11x + 6$ and verify the answer with multiplication.

Solution:

You are looking for two binomials that will multiply to give you the product of $3x^2 + 11x + 6$. The only factors of $3x^2$ are $(3x)(x)$ so those will be the first terms in the binomials.

$(3x + \quad)(x + \quad)$

Using the strategy from the last lesson and listing the factor pairs of 6 gives you 1, 6 or 2, 3. Unfortunately, neither of these pairs gives you a sum of 11, so a different strategy may have to be used. You could try a guess-and-check approach and apply the distributive property to each of the following possibilities to see which one works:

$(3x + \boldsymbol{1})(x + \boldsymbol{6})$
$= 3x^2 + 18x + x + 6$
$= 3x^2 + 19x + 6$

$(3x + \boldsymbol{6})(x + \boldsymbol{1})$
$= 3x^2 + 3x + 6x + 6$
$= 3x^2 + 9x + 6$

$(3x + \boldsymbol{3})(x + \boldsymbol{2})$
$= 3x^2 + 6x + 3x + 6$
$= 3x^2 + 9x + 6$

$(3x + \boldsymbol{2})(x + \boldsymbol{3})$
$= 3x^2 + 9x + 2x + 6$
$= 3x^2 + 11x + 6$

The factors $(3x + 2)(x + 3)$ give you the product $3x^2 + 11x + 6$.

You have found the correct answer, but this strategy could become very time consuming if you were asked to factor something like $12x^2 - 11x - 15$. There would be 24 different possibilities, given the factor pairs of 12 and -15! There must be a more convenient way!

The strategy from the last lesson considered only trinomials where the coefficient of x^2 was equal to 1. However, it can be modified to apply to all trinomials, regardless of the coefficient of x^2 by following these steps.

Example 3

Factor $2x^2 + 7x + 3$.

Solution:

Step 1: Find the product of the coefficient of x^2 and the constant in the polynomial.

$$\overbrace{2x^2 + 7x + 3}^{(2)(3) = 6}$$

The product is 6.

Step 2: List the factor pairs of that product.

$6 = 1, 6$

$\quad 2, 3$

Step 3: Determine which factor pair gives you the sum equal to the coefficient of x.

$$2x^2 + 7x + 3$$

$$\uparrow$$

$$1 + 6 = 7$$

Use 1, 6.

Step 4: Replace the middle term in the trinomial with two terms equivalent to it, using the factor pair as coefficients.

$$\begin{aligned} &2x^2 + 7x + 3 \\ &= 2x^2 + \underbrace{1x + 6x}_{1x+6x=7x} + 3 \end{aligned}$$

Step 5: Consider just the first two terms of the polynomial, and determine if they have any common factors. Factor it out.

$$= \underbrace{2x^2 + 1x}_{\substack{\text{common factor} \\ \text{of } x}} + 6x + 3$$

$$= x(2x + 1) + 6x + 3$$

Step 6: Consider just the last two terms in the polynomial, and determine if they have any common factors. Factor it out.

$$= x(2x + 1) + \underbrace{6x + 3}_{\substack{\text{common factor} \\ \text{of 3}}}$$

$$= x(2x + 1) + 3(2x + 1)$$

Step 7: Notice that the expressions in the brackets are the same. They are common factors in this binomial. Factor them out.

common factor

$$= x(2x + 1) + 3(2x + 1)$$

$$= (2x + 1)(x + 3)$$

common factor

Some similar common factor examples:

$5xy + 9yz = y(5x + 9z)$

$ax + bx = x(a + b)$

$k(m + n) + j(m + n) = (m + n)(k + j)$

Step 8: Verify the factors by applying the distributive property.

$(x + 3)(2x + 1)$ or $x(2x + 1) + 3(2x + 1)$

$= 2x^2 + x + 6x + 3$ Compare this to Step 6.

$= 2x^2 + 7x + 3$

These steps would be helpful to have on your Resource Sheet.

Example 4

Factor the trinomials. Express it as the product of its factors.

a) $3x^2 + 10x + 3$

b) $2x^2 - 7x + 5$

c) $2x^2 + x - 6$

Solution:

a) $3x^2 + 10x + 3$

Factor pairs of 9: 1, 9
3, 3

$$\overbrace{3x^2 + 10x + 3}^{(3)(3)=9}$$

$$1 + 9 = 10$$

$$\underset{\text{sum of 10}}{\uparrow}$$

$$= 3x^2 + \underbrace{1x + 9x}_{=10x} + 3$$

Find common factors.

$$= \underbrace{3x^2 + 1x}_{\substack{\text{common factor} \\ \text{of } x}} + \underbrace{9x + 3}_{\substack{\text{common factor} \\ \text{of 3}}}$$

$$= x(3x + 1) + 3(3x + 1)$$

$$= (3x + 1)(x + 3)$$

Verify.

$$(3x + 1)(x + 3)$$

$$= 3x^2 + 9x + x + 3$$

$$= 3x^2 + 10x + 3$$

Therefore, $3x^2 + 10x + 3 = (3x + 1)(x + 3)$.

b) $\overbrace{2x^2 - 7x + 5}^{(2)(5)=10}$

$\uparrow$

-7

$= 2x^2 \underbrace{-2x - 5x}_{= -7x} + 5$

$= \underbrace{2x^2 - 2x}_{\text{common factor of } 2x} \quad \underbrace{-}_{\text{match sign}} \quad \underbrace{5x + 5}_{\text{common factor of } 5}$

$= 2x(x - 1) - 5(x - 1)$

$= (x - 1)(2x - 5)$

Factor pairs of 10: 1, 10
2, 5

Sum: -7

To get a positive product and a negative sum, both factors must be negative.

$10 = (-2)(-5)$

$-7 = (-2) + (-5)$

Make sure to distribute the negative sign correctly.

Verify.

$(x-1)(2x-5)$

$= 2x^2 - 5x - 2x + 5$

$= 2x^2 - 7x + 5$

Therefore, $2x^2 - 7x + 5 = (x - 1)(2x - 5)$.

c) $\overbrace{2x^2 + x - 6}^{(2)(-6) = -12}$

$\uparrow$

$+1$

To get a negative product, one factor must be positive and one must be negative.

factor pairs of 12: 1, 12
2, 6
3, 4

product of $-12 = (-3)(4)$

sum of $+1 = (-3) + (4)$

$= 2x^2 - 3x + 4x - 6$

$= \underbrace{2x^2 - 3x}_{\text{common factor of } x} + \underbrace{4x - 6}_{\text{common factor of } 2}$

$= x(2x - 3) + 2(2x - 3)$

$= (2x - 3)(x + 2)$

or

$= 2x^2 + 4x - 3x - 6$

$= \underbrace{2x^2 + 4x}_{\text{common factor of } 2x} + \underbrace{3x - 6}_{\text{common factor of } 3}$

$= 2x(x + 2) - 3(x + 2)$

$= (x + 2)(2x - 3)$

Notice that the order you replace the terms in the trinomial does not matter. The factors are the same.

Note: Watch the signs when factoring out what is common and check your factoring by using the distributive property.

Verify.

$(2x-3)(x+2)$

$=2x^2+4x-3x-6$

$=2x^2+x-6$

Therefore, $2x^2 + x - 6 = (2x - 3)(x + 2)$.

Perfect Square Trinomials

Example 5

Factor the following trinomials and verify each answer with a tile diagram.

a) $4x^2 + 12x + 9$

b) $x^2 + 10x + 25$

Solution:

a) $4x^2 + 12x + 9$

$(4)(9) = 36$

Factor pairs of 36: 1, 36
2, 18
3, 12
4, 9
6, 6

To get a positive product of 36 and a positive sum of 12, the factors must both be positive.

$36 = (6)(6)$

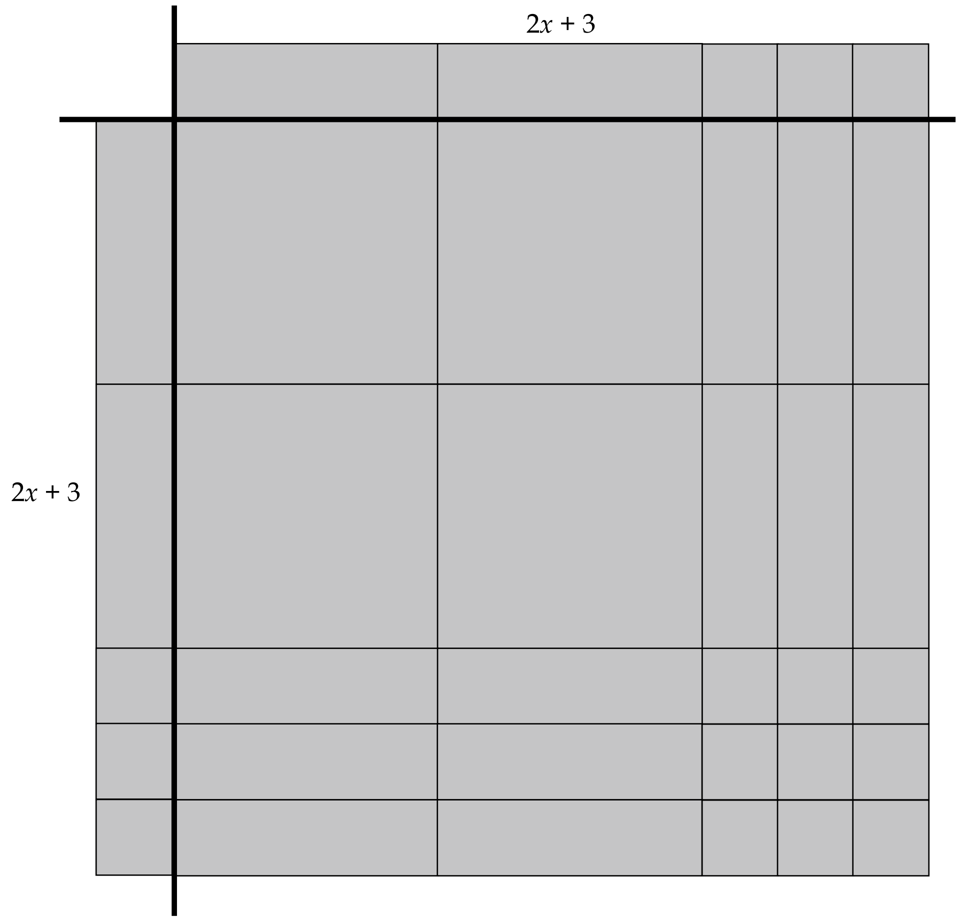
$12 = (6) + (6)$

$4x^2 + 12x + 9$

$= 4x^2 + 6x + 6x + 9$

$= 2x(2x + 3) + 3(2x + 3)$

$= (2x + 3)(2x + 3)$



The tile diagram verifies the factors.

b) $x^2 + 10x + 25$ (1)(25) = 25

Factor pairs of 25: 1, 25

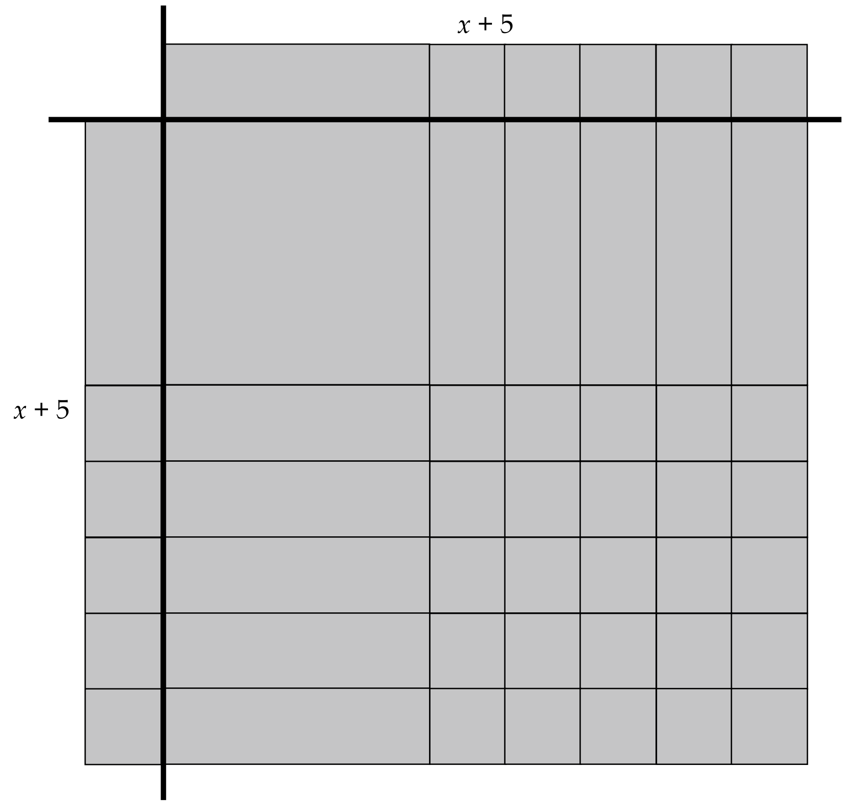
5, 5

sum: 10 = 5 + 5

$= x^2 + 5x + 5x + 25$

$= x(x + 5) + 5(x + 5)$

$= (x + 5)(x + 5)$



What do you notice about the two tile diagrams above?

The product areas are squares.

What do you notice about the two factors in each example?

They are the same.

These trinomials could be written as the product of their factors like this:

$$4x^2 + 12x + 9 = (2x + 3)(2x + 3) = (2x + 3)^2$$

$$x^2 + 10x + 25 = (x + 5)(x + 5) = (x + 5)^2$$

What is significant about the values of the coefficients of x^2 and the constants?

They are perfect square numbers: (1, 4, 9, and 25).

As well, the product of the coefficient of x^2 and the constant is a perfect square number: ($4 \times 9 = 36$ and $1 \times 25 = 25$).

What connection is there between the products above and the coefficient of x in the trinomial?

The coefficient of x is double the square root of the product (coefficient of x^2 times the constant).

$$4x^2 + 12x + 9 \qquad (4)(9) = 36 \qquad \sqrt{36} = \pm 6 \qquad 2 \times (6) = 12$$
$$x^2 + 10x + 25 \qquad (1)(25) = 25 \qquad \sqrt{25} = \pm 5 \qquad 2 \times (5) = 10$$

What connections do you notice between the coefficients and constants in the trinomials and the terms in the binomial factors?

$$4x^2 + 12x + 9 = (2x + 3)^2$$
$$x^2 + 10x + 25 = (x + 5)^2$$

The terms in the binomial factors are the square roots of the first and last terms in the trinomials, and the sign in the binomial matches the sign of the middle term in the trinomial!

$$4x^2 + 12x + 9$$
$$\sqrt{4x^2} = 2x$$
$$\sqrt{9} = 3$$

Middle term is positive so

$$4x^2 + 12x + 9 = (2x + 3)^2$$

$$x^2 + 10x + 25$$
$$\sqrt{x^2} = x$$
$$\sqrt{25} = 5$$

Middle term is positive so

$$x^2 + 10x + 25 = (x + 5)^2$$

Trinomials that follow these patterns are called perfect square trinomials.

Example 6

Factor the following perfect square trinomials. Verify by multiplying the factors.

a) $x^2 - 8x + 16$

b) $9x^2 + 6x + 1$

c) $4x^2 - 12x + 9$

Solution:

a) Determine the square root of the first and last term in the trinomial, and use the sign of the second term.

$x^2 - 8x + 16 = (x - 4)^2$

Verify

$(x - 4)(x - 4)$

$= x^2 - 4x - 4x + 16$

$= x^2 - 8x + 16$

b) Determine the square root of the first and last term in the trinomial, and use the sign of the second term.

$9x^2 + 6x + 1 = (3x + 1)^2$

Verify

$(3x + 1)(3x + 1)$

$= 9x^2 + 3x + 3x + 1$

$= 9x^2 + 6x + 1$

c) Determine the square root of the first and last term in the trinomial, and use the sign of the second term.

$4x^2 - 12x + 9 = (2x - 3)^2$

Verify

$(2x - 3)(2x - 3)$

$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$

Example 7

What coefficient of x would make this a perfect square trinomial?

$$x^2 - \underline{\quad\quad} x + 4$$

State the factors of this perfect square trinomial and verify your answer.

Solution:

$(1)(4) = 4$	Find the product of the coefficient of x^2 and the constant.
$2\sqrt{4} = 4$	Calculate twice the square root of that product.
$x^2 - 4x + 4$	A perfect square trinomial.
$= (x - 2)^2$	Its factors.
$(x - 2)(x - 2)$	Verify by multiplying.
$= x^2 - 2x - 2x + 4$	
$= x^2 - 4x + 4$	A perfect square trinomial.

Learning Activity 6.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. What two numbers have a product of -8 and a sum of -2?
2. There are 20 cheese sticks in a package. You only eat one each day, and only on weekdays (Monday to Friday). How many weeks will it take you to finish the whole package?
3. Solve: $4 - 6 + 2 \times (3 - 8)$.
4. You are playing baseball on a co-ed team. There are 16 people on your team. If you need $\frac{1}{4}$ of the team to be girls, how many have to be girls?
5. Multiply: $(x + 5)(x - 9)$.

continued

6. The last time you counted, you had 54 DVDs. Your house was broken into last night and now you only have 32. How many DVDs were stolen?
7. Complete the pattern: –1, 0, ______, 0, ______, 0, 1.
8. You use your left hand to type more than your right when you have your hands on the keyboard properly. Typing the word *factor*, you use your left hand for 5 letters and your right hand for one. Write the fraction that represents how many times you use your right hand in total.

Part B: Factoring Trinomials when $a \in I$

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Factor completely.
   a) $2x^2 + 11x + 15$
   b) $4x^2 - 21x + 5$
   c) $5x^2 - 4x - 12$
   d) $6x^2 + 5x - 6$
   e) $2x^3 + x^2 - 15x$
   f) $2x^3 - 22x^2 + 36x$
   g) $-12x^2 + 26x + 10$
2. Factor each expression. (If you are feeling confident in your ability to factor trinomials with $a \in I$, you do not have to do these questions. If you want more practice, work through as many as you need.)
   a) $2x^2 + 5x + 3$
   b) $5x^2 + 6x + 1$
   c) $5a^2 - 16a + 3$
   d) $3y^2 + 4y + 1$
   e) $24x^2 + 2x - 1$
   f) $6y^2 + 20 + 23y$
   g) $10 + y - 2y^2$
   h) $60y^2 - 27y - 60$
   i) $15x^2 + 37x + 20$
   j) $15a^2 + 8a - 12$
3. For what integral values (whole numbers) of k can $4x^2 + kx + 3$ be factored? Write out all possible trinomials as a product of its factors.
4. Fill in the space so that each trinomial is a perfect square trinomial.
   a) $4x^2 +$ _______ $+ 4$
   b) $25x^2 +$ _______ $+ 9$
   c) $x^2 + 14\,x +$ _______

continued

Learning Activity 6.4 (continued)

5. Factor each perfect square trinomial.
   a) $x^2 - 8x + 16$
   b) $4x^2 - 4x + 1$

6. Identify and explain the errors in the factorization, and state the correct solution.

   $x^2 - 8x + 16 = (x + 4)^2$

Lesson Summary

The trinomial factoring strategies you learned in Lesson 3 were adapted to work with trinomials with a leading coefficient other than 1. You factored trinomials in which you had to first remove a common factor. Perfect square trinomials were identified—both pictorially, with a tile diagram that was a square, and by characteristics in the coefficients and constants. You learned how to factor perfect square trinomials.

In the next lesson, you will factor polynomials that represent a "difference of squares."

Notes

Assignment 6.4

Factoring Trinomials when $a \in \mathrm{I}$

Total Marks = 24

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Factor completely.

   a) $2x^2 - 3x - 14$ *(2 marks)*

   b) $3x^2 + 16x + 21$ *(2 marks)*

   c) $6x^2 + x - 12$ *(2 marks)*

   d) $-6x^2 + 8x - 2$ *(2 marks)*

Assignment 6.4: Factoring Trinomials with a ∈ I (continued)

2. For what integral values of k can $4x^2 + kx - 3$ be factored? *(2 marks)*

3. a) Explain why $x^2 - 2x + 1$ is a perfect square trinomial. *(2 marks)*

   b) Fill in the space so that the trinomial is a perfect square trinomial. *(2 marks)*

   $64y^2 -$ _______ $+ 1$

Assignment 6.4: Factoring Trinomials with a ∈ I (continued)

4. Factor completely.

   a) $x^2 + 2x + 1$ *(2 marks)*

   b) $9x^2 + 12x + 4$ *(2 marks)*

   c) $x^2 - 18x + 81$ *(2 marks)*

5. Identify and explain the errors in the following factorization and correct them. *(4 marks)*

$$
\begin{aligned}
&-12x^2 + 26x + 10 \\
&= -2(6x^2 + 13x - 5) \\
&= -2(6x^2 + 10x + 3x - 5) \\
&= -2(2x(3x + 5) + 1\ 3x + 5)) \\
&= -2(2x + 1)(3x + 5)
\end{aligned}
$$

Notes

Lesson 5: Factoring a Difference of Squares

Lesson Focus

In this lesson, you will

- ❑ factor a polynomial that is a difference of squares
- ❑ complete a strategy map to identify problem-solving steps in factoring

Lesson Introduction

The trinomial factoring you have done in the last two lessons has resulted in solutions consisting of two binomial factors (and possibly a common factor). In this lesson, you will discover a special case where two binomial factors are multiplied to give you a binomial product. It is called a difference of squares.

Special Cases for Factoring

Example 1

Apply the distributive property to the factors to find each product. Simplify.

a) $(x + 3)(x + 3)$

b) $(x - 3)(x - 3)$

c) $(x - 3)(x + 3)$

d) $(x + 4)(x + 4)$

e) $(x - 4)(x - 4)$

f) $(x - 4)(x + 4)$

Solution:

	Factors	Product	Simplify
a)	$(x + 3)(x + 3)$	$= x^2 + 3x + 3x + 9$	$= x^2 + 6x + 9$
b)	$(x - 3)(x - 3)$	$= x^2 - 3x - 3x + 9$	$= x^2 - 6x + 9$
c)	$(x - 3)(x + 3)$	$= x^2 + 3x - 3x - 9$	$= x^2 + 0x - 9 = x^2 - 9$
d)	$(x + 4)(x + 4)$	$= x^2 + 4x + 4x + 16$	$= x^2 + 8x + 16$
e)	$(x - 4)(x - 4)$	$= x^2 - 4x - 4x + 16$	$= x^2 - 8x + 16$
f)	$(x - 4)(x + 4)$	$= x^2 + 4x - 4x - 16$	$= x^2 + 0x - 16 = x^2 - 16$

What patterns do you notice?

You should recognize that (a), (b), (d), and (e) result in perfect square trinomials. You are multiplying a factor times itself, or squaring it. You end up with a trinomial with a leading coefficient and a constant that are perfect square numbers (in this case, 1, 9, or 16). The coefficient of the middle term is two times the square root of the product of the leading coefficient and the constant.

Recall the steps:

$(1)(9) = 9 \qquad \sqrt{9} = 3 \qquad 2 \times 3 = 6$

$(1)(16) = 16 \qquad \sqrt{16} = 4 \qquad 2 \times 4 = 16$

The factors could be written as $(x + 3)^2$, $(x - 3)^2$, $(x + 4)^2$, and $(x - 4)^2$, respectively.

In (c) and (f) the factors are not exactly alike. The signs are different. When you apply the distributive property to the factors and simplify the expression, you are left with a product that can be expressed as a binomial.

$(x - 3)(x + 3) = x^2 - 9$

$(x - 4)(x + 4) = x^2 - 16$

When you multiply two binomial factors that are the same except for the sign, you end up with a binomial product. This is called a **difference of squares**.

You probably want to have this definition recorded along with an example on your Resource Sheet.

$(m + 5)(m - 5)$

$= m^2 - 5m + 5m - 25$

$= m^2 - 25$

Notice that the factors are the square root of the terms in the "difference of squares" binomial.

$$m^2 - 25 = \left(\sqrt{m^2} + \sqrt{25}\right)\left(\sqrt{m^2} - \sqrt{25}\right)$$

The "difference of squares" binomial is a special case of factoring a trinomial in the form

$ax^2 + bx + c$, when $b = 0$ and $c < 0$

Example 2

Factor the following expressions as a difference of squares.

a) $y^2 - 36$

b) $100r^2 - 49$

c) $2x^2 - 2$

d) $k^2 - 50$

Solution:

a) $y^2 - 36$
$= (y - 6)(y + 6)$

b) $100r^2 - 49$
$= (10r + 7)(10r - 7)$

c) $2x^2 - 2$ Remove any common factors.
$= 2(x^2 - 1)$
$= 2(x - 1)(x + 1)$

d) $k^2 - 50$ 50 is not a perfect square number, but this binomial may still be factored in a manner similar to a perfect square binomial.

Leave answers as exact values, and do not round.

$= \left(k + \sqrt{50}\right)\left(k - \sqrt{50}\right)$

Example 3

Identify and explain the error in the following factorization. State the correct answer.

$$w^2 + 16 = (w + 4)(w - 4)$$

Solution:

This binomial is not a difference of squares. If you apply the distributive property to the factors, you end up with

$(w + 4)(w - 4)$
$= w^2 - 4w + 4w - 16$
$= w^2 - 16$

This is not the same as the original question.

The terms in $w^2 + 16$ have no common factors, and it is not a difference of squares (it is the sum of squares), so it is not factorable. Recall the definition given above: The difference of squares binomial is a special case of factoring a trinomial in the form $ax^2 + bx + c$ when $b = 0$ and **$c < 0$. c must be negative**. The binomial must express the difference, not the sum of terms.

A polynomial that is not factorable is considered “prime.”

Learning Activity 6.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Factor: $m^2 + 3m - 54$.
2. You know that your glass holds 1 cup of milk. Would this be a good referent to use to find out how much water your water bottle can hold?
3. Andy has been to the Great Pyramid of Egypt. He bought a miniature version of the pyramid, which was to scale. The ratio comparing the miniature and the real pyramid is 1 cm: 70 royal cubits (an ancient unit of measurement). If the height of the miniature is 4 cm, how tall is the real pyramid?
4. Is 3^{-6} rational or irrational?
5. What is the GCF of 34 and 17?
6. Simplify: $\left(2^2\right)^{\frac{-1}{5}}$.
7. Solve for n: $4n - 3 = 2 + 19$.
8. You have 2 older brothers and 3 older sisters. Your parents had 11 children. How many of your siblings are younger than you?

continued

Part B: Difference of Squares and Module Review

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Why does applying the distributive property to the factors of a difference of square result in a binomial? Give an example.
2. Given the polynomial $ax^2 - c$ where a and c are perfect square numbers, write its binomial factors.
3. Factor completely.
   a) $x^2 - 36$
   b) $9y^2 - 49$
   c) $x^2 - 256$
   d) $2m^2n - 2n$
4. Use your factoring strategies to factor the following polynomials.
   a) $3mn - 6np$
   b) $a(b + 3) + c(b + 3)$
   c) $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
   d) $x^2 - 11x + 28$
   e) $x^2 - 3x - 28$
   f) $4x^4 - 20x^3 + 24x^2$
   g) $16x^2 - 24x + 9$
   h) $8x^2 - 40x + 50$
   i) $x^2 - 81$
   j) $4y^2 - 9$
   k) $20x^2y - 5y$
5. If you are feeling confident in your ability to factor trinomials with $a \in I$ and difference of squares, you do not have to do these questions. If you want more practice, work through as many as you need.
   a) $x^2 - 16$
   b) $36t^2 - 1$
   c) $4a^2 - b^2$
   d) $8c^2 - 72$
   e) $81 - (x + 7)^2$
   f) $(x - 1)^2 - (x + 1)^2$
   g) $x^8 - y^{12}$
   h) $4x^2 - 1$
   i) $4m^2 - 25y^4$
   j) $121x^2 - 196y^2$

continued

Learning Activity 6.5 (continued)

6. You are now three-quarters of the way through this course. Take a few minutes now to look back at Module 1 and the goals you set for yourself at the beginning of this course, as well as the revised version written after Module 4. Which goals have you accomplished or completed by now? Are you progressing towards achieving your other goals in a timely manner? How can you modify or adapt your steps to ensure success? How will you celebrate your achievements and continue to strive for the rest of your goals! You are almost done!

Lesson Summary

Given a trinomial in the form of $ax^2 + bx + c$, the values a, b, and c have a large impact on how you will factor the trinomial. You have considered trinomials where $a = 1$, and where $a \in I$, ($a \neq 0$). In this lesson, you saw what happened if $b = 0$ and $c < 0$. This special case is called a difference of squares binomial, and you now know how to factor it.

This is the last lesson in this module, but you will come back to multiplying and factoring polynomials in many more places as you continue learning mathematics.

Assignment 6.5

Difference of Squares and Module Review

Total Marks = 40

Note to Students: Have you made a resource sheet for this module? Do you have the definitions and formulas on your resource sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Factor completely. *(2 marks each × 10 = 20 marks)*

   a) $5ab + 10a^3$

   b) $5v(r + t) - 7q(r + t)$

   c) $2x^2 - 4x - 30$

   d) $2x^2 + 5x + 3$

   e) $4x^2 + 28x + 49$

   f) $144v^2 - 169$

g) $x^2 - 100$

h) $9x^2 - 16y^2$

i) $4 - \frac{1}{9}x^2$

j) $x^4 - 1$

2. Strategy map *(20 marks)*

   You have used a variety of strategies to factor polynomials. Which strategy is best to use with any given polynomial? What process can you go through to decide how to most efficiently factor a polynomial? You must consider common factors, binomials, trinomials, leading coefficients, perfect square trinomials, and difference of squares.

   One way to streamline your problem-solving process is to outline it in a strategy map, similar to the one in Module 4, Lesson 3. That one helps you plan your steps when solving right triangles.

   Using that idea and format, create a strategy map you can follow when presented with a polynomial (similar to the ones in this module) that must be factored. Use arrows to show possible pathways through your framework to arrive at a solution. You can indicate steps in the strategies or describe characteristics of the factors. If possible, show how multiplication and factoring are related. The map should be as concise as possible and take up no more than one page. Include it when you send in your work to be marked. A blank sheet is provided for your final copy.

   Marking Guide:

   Easy to follow *(6 marks)*

   Includes all information *(14 marks)*

Strategy Map

MODULE 6 SUMMARY

Congratulations! You have finished the sixth module in the course.

Multiplying and factoring polynomials go hand in hand. One “undoes” the other, so they can be used to check the accuracy of answers.

You are now armed with a variety of strategies and skills to factor and multiply, ones that will continue to be used in new applications. If you have any questions or hesitation about multiplying or factoring polynomials, go back now and review what you have learned. Contact your tutor/marker for help. These are really important concepts and you will use them again. Make sure you have a good understanding of them.

As we have pointed out in some of the other modules, certain methods used in mathematics are linked to the applied math courses in Grades 11 and 12, while other methods and ideas are linked to pre-calculus math. This module has links to both branches of math, but there are many more connections to pre-calculus than applied math.

In the next module, you will continue to work on some of the geometry concepts that were introduced in Modules 1 and 5. Look back on these modules to refresh your memory regarding the skills and strategies you used there, so you will be ready to work with linear equations.

Submitting Your Assignments

It is now time for you to submit Assignments 5.1 to 5.3 from Module 5 and Assignments 6.1 to 6.5 from Module 6 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 5 and Module 6 assignments and organize your material in the following order:

Cover Sheet for Modules 5 and 6 (found at the end of the Introduction)

Assignment 5.1 Relations and Functions

Assignment 5.2 Domain and Range Notation

Assignment 5.3 Functional Notation

Assignment 6.1 Describing Polynomials and Multiplying Binomials

Assignment 6.2 Multiplying Polynomials

Assignment 6.3 Factoring Binomials and Trinomials

Assignment 6.4 Factoring Trinomials When $a \in I$

Assignment 6.5 Difference of Squares and Module Review

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 6
Polynomials

Learning Activity Answer Keys

Module 6: Polynomials

Learning Activity 6.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Write the following equation as a function: $x + y = 76$.
2. If you are $\frac{3}{2}$ taller than your brother, and your brother is 4 feet tall, how tall are you?
3. The sides of a right triangle are 3, 5, 4. How long is the hypotenuse?
4. Is this relation a function: {(0,1), (3,6), (4, 8), (0, 10)}?
5. There are 120 employees at your work. Your boss says that three-quarters of the staff are coming to the meeting on Saturday morning. How many people will be attending the meeting?
6. Usain Bolt can run 100 m in 10 s. What is his average speed?
7. You need to make exact change for a customer at your work. They have given you \$60 and their bill is \$42.60. How much money will you give them?
8. Evaluate: $\left(6^2\right)^{\frac{1}{2}}$.

Answers:

1. $f(x) = 76 - x$
2. 6 feet $\left(\frac{3}{2} \times 4\right)$
3. 5 (The hypotenuse is always the longest side.)
4. No, it is just a relation. (The input value of 0 has two outputs, 1 and 10.)
5. 90 $\left(120 \times \frac{3}{4}\right)$
6. 10 m per second (100 m ÷ 10 s)
7. \$17.40 (\$42.60 + 0.40 = 43.00, 43 + 7 = 50, 50 + 10 = \$60, so 0.4 + 7 + 10 = \$17.40)
8. 6

Part B: Binomial Multiplication

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write a quadratic trinomial of degree 2 with the variable m, coefficients –3 and 1 and a constant of 5.

   Answer:

   Two possible solutions are $-3m^2 + m + 5$ or $m^2 - 3m + 5$. (Remember that you always want to start with the term that has the highest exponent.)

2. Write 3 like terms that are not exactly the same.

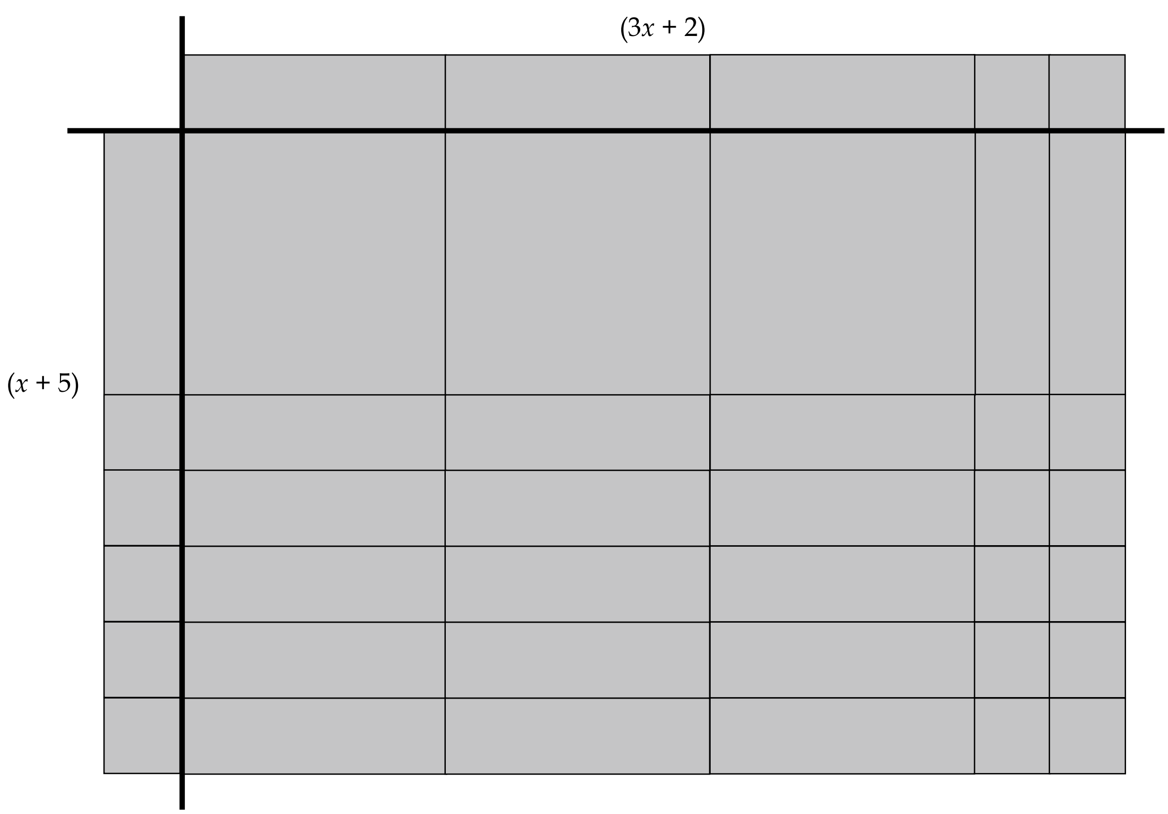
   Answer:

   Any terms that have matching variables and degree but different coefficients are like terms. For example, $5x^2y$, $-3x^2y$, and x^2y are like terms.

3. Illustrate the following products using tiles, and write your solution steps to show how it may be simplified.

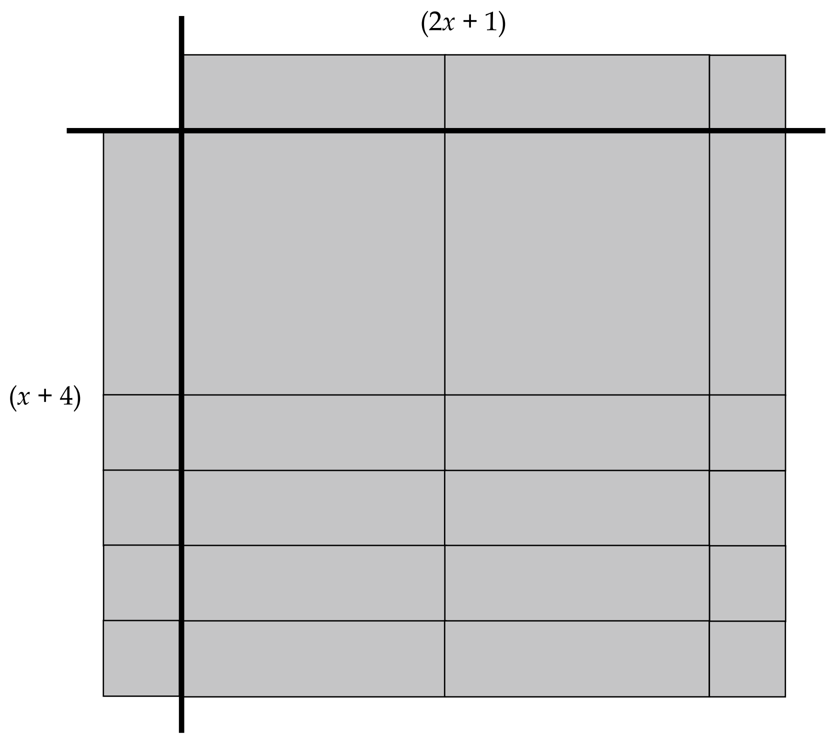
   a) $(x + 5)(3x + 2)$

   Answer:



$(x + 5)(3x + 2)$
$= 3x^2 + 2x + 15x + 10$
$= 2x^2 + 17x + 10$

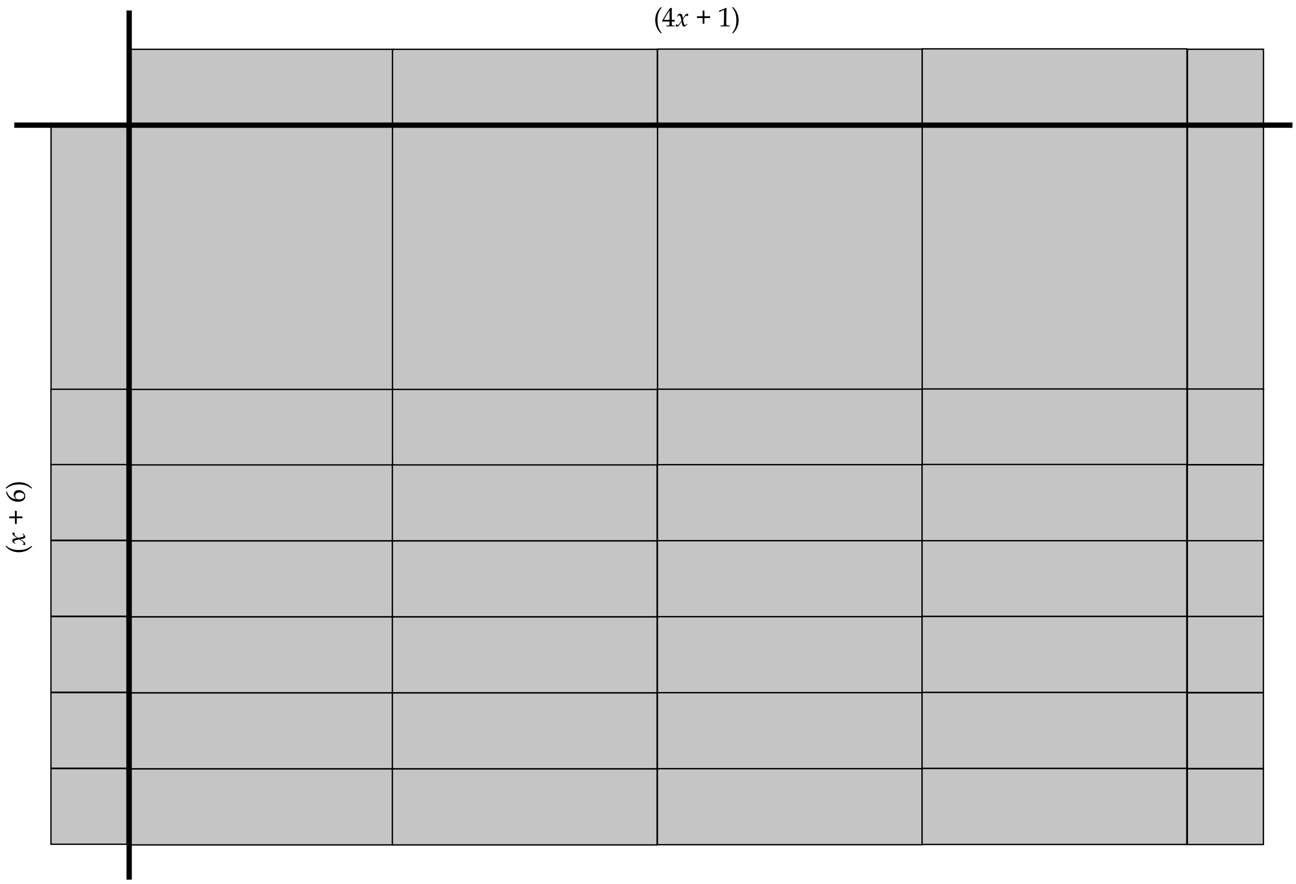
b) $(2x + 1)(x + 4)$
Answer:



$(x + 4)(2x + 1)$
$= 2x^2 + 1x + 8x + 4$
$= 2x^2 + 9x + 4$

c) $(4x + 1)(x + 6)$

Answer:



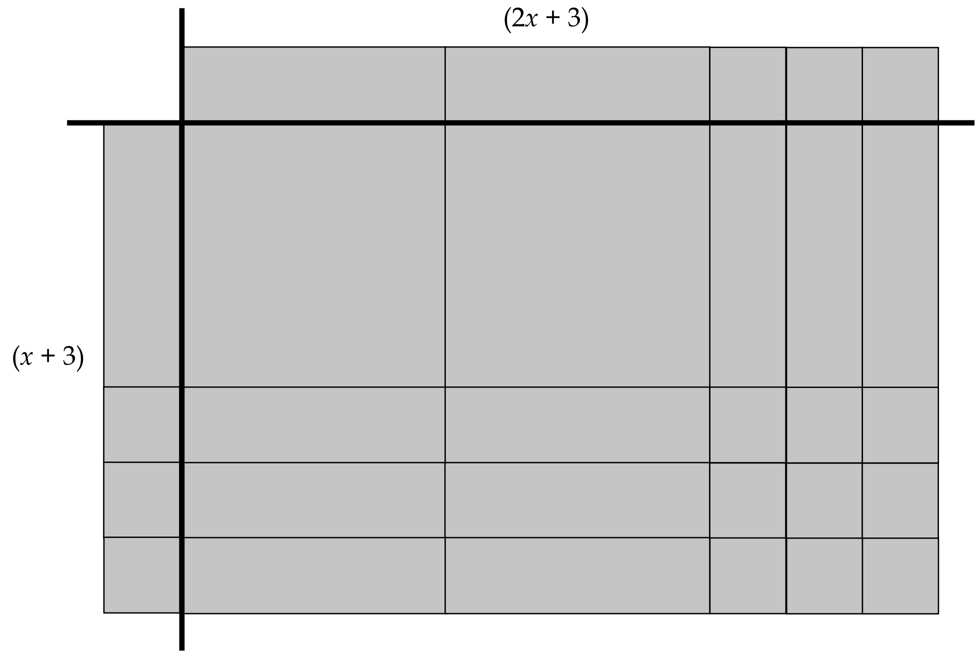
$(4x + 1)(x + 6)$

$= 4x^2 + 24x + x + 6$

$= 4x^2 + 25x + 6$

d) $(x + 3)(2x + 3)$

Answer:



$(2x + 3)(x + 3)$

$= 2x^2 + 6x + 3x + 9$

$= 2x^2 + 9x + 9$

Learning Activity 6.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. At your dad's office they have carpet made out of squares. Each square is 2 feet by 2 feet. If your dad's cubicle is 5 squares deep and 5 squares wide, what is the area of your dad's cubicle in feet?
2. Evaluate: $\sqrt{2^{-4}}$.
3. 1 yard^3 = ________ feet^3.
4. What is the domain of the function $f(x) = x + 4$?
5. It costs $4.00 for a package of 3 chocolate bars. Geri spends $20 on chocolate bars. How many chocolate bars does she buy?
6. I have 9 letters in my name. Is it possible that half of those letters are vowels?
7. What is the LCM of 3 and 5?
8. Solve: $\frac{6}{5} + \frac{2}{3}$.

Answers:

1. 100 ft.^{2} ($5 \times 2 = 10$ ft. for both the depth and width of the cubicle. $10 \times 10 = 100$)
2. $\frac{1}{4}\left(\sqrt{2^{-4}} = 2^{-\frac{4}{2}} = 2^{-2} = \frac{1}{2^2}\right)$
3. 27 feet^3 (1 yard = 3 feet; $V = 3 \times 3 \times 3$)
4. $(-\infty, \infty)$ or $\{x \in \Re\}$ (You can have any value for x.)
5. 15 chocolate bars ($20 \div 4 = 5$ packages, 5×3 chocolate bars per package = 15 chocolate bars.)
6. No (You cannot divide 9 evenly in half, and you cannot have half a vowel. We are considering 'y' to be a half vowel.)
7. 15 (Because 5 and 3 are both prime, their LCM is their product.)
8. $\frac{28}{15}\left(\frac{6}{5} + \frac{2}{3} = \frac{6 \times 3}{5 \times 3} + \frac{2 \times 5}{3 \times 5} = \frac{18+10}{15}\right)$

Part B: Multiplying Polynomials

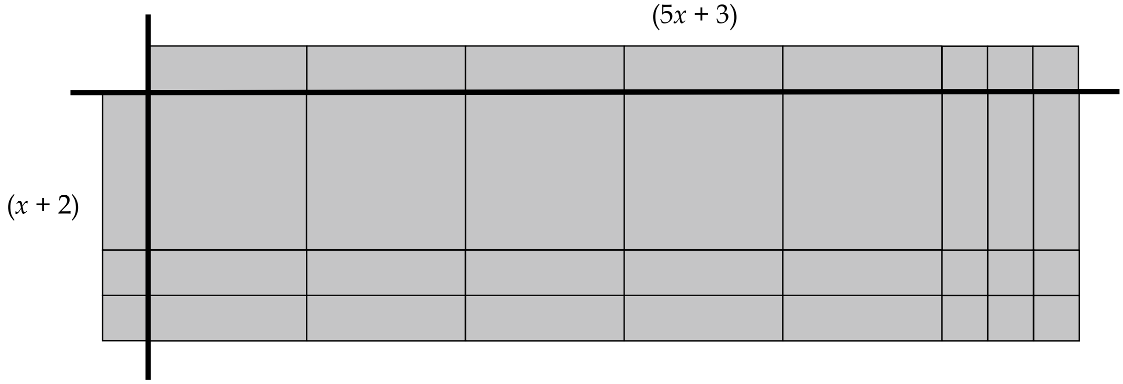
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

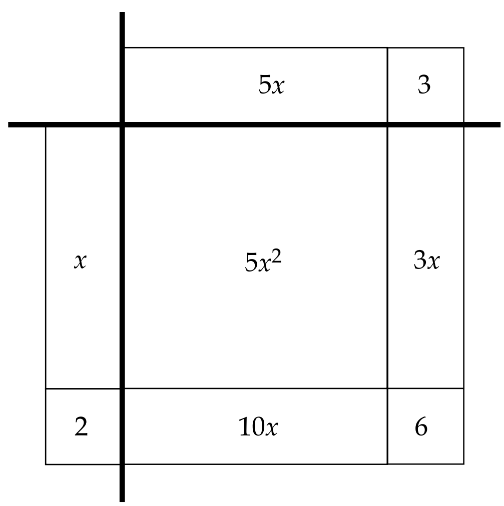
1. Simplify each product. Show the steps pictorially and symbolically.

   a) $(5x + 3)(x + 2)$

   Answer:

   All coefficients and constants are positive, so a diagram or an area model could be drawn.





$(x + 2)(5x + 3)$

$= 5x^2 + 3x + 10x + 6$

$= 5x^2 + 13x + 6$

FOIL or the distributive property can be used to show the product symbolically.

$(5x + 3)(x + 2)$

$= 5x^2 + 10x + 3x + 6$
$= 5x^2 + 13x + 6$

Alternatively,

$5x(x + 2) + 3(x + 2)$

$= 5x^2 + 10x + 3x + 6$

$= 5x^2 + 13x + 6$

b) $(4h - 5)(-3h + 7)$

Answer:

The area model would work to illustrate this product.

	$-3h$	7
$4h$	$-12h^2$	$28h$
-5	$+15h$	-35

$-12h^2 + 28h + 15h - 35$
$= -12h^2 + 43h - 35$

FOIL

$(4h - 5)(-3h + 7)$

$= -12h^2 + 28h + 15h - 35$
$= -12h^2 + 43h - 35$

or

Distributive Property

$4h(-3h + 7) - 5(-3h + 7)$

$= -12h^2 + 28h + 15h - 35$

$= -12h^2 + 43h - 35$

c) $(x^2 - x)(x + 2)$

Answer:

Use the area model.

	x^2	$-x$
x	x^3	$-x^2$
2	$2x^2$	$-2x$

$x^3 + 2x^2 - x^2 - 2x$
$= x^3 + x^2 - 2x$

FOIL

$(x^2 - x)(x + 2)$
$= x^3 + 2x^2 - x^2 - 2x$
$= x^3 + x^2 - 2x$

or

Distributive Property

$x^2(x + 2) - x(x + 2)$
$= x^3 + 2x^2 - x^2 - 2x$
$= x^3 + x^2 - 2x$

d) $(x + y)(x^2 + 2x - 1)$

Answer:

The area model can be used to illustrate this product.

	x^2	$2x$	-1
x	x^3	$2x^2$	$-x$
y	x^2y	$2xy$	$-y$

$x^3 + 2x^2 - x + x^2y + 2xy - y$

$= x^3 + x^2y + 2x^2 + 2xy - x - y$

Rearrange terms in descending order of power.

The distributive property.

$(x + y)(x^2 + 2x - 1)$

$= x^3 + 2x^2 - x + x^2y + 2xy - y$

$= x^3 + x^2y + 2x^2 + 2xy - x - y$

or

$x(x^2 + 2x - 1) + y(x^2 + 2x - 1)$

2. Simplify each product and verify your solution.

   a) $(2x - 1)^2$

   Answer:

   $(2x - 1)^2 = (2x - 1)(2x - 1)$

	$2x$	-1
$2x$	$4x^2$	$-2x$
-1	$-2x$	1

$4x^2 - 2x - 2x + 1$
$= 4x^2 - 4x + 1$

$(2x - 1)(2x - 1)$ or $2x(2x - 1) - 1(2x - 1)$

$= 4x^2 - 2x - 2x + 1$

$= 4x^2 - 4x + 1$

Verify: use $x = 1$

$(2x - 1)^2$	$4x^2 - 4x + 1$
$(2(1) - 1)^2$	$4(1)^2 - 4(1) + 1$
$(2 - 1)^2$	$4 - 4 + 1$
1^2	1

$1 = 1$

The solutions agree.

b) $(x + 3)^3$ (Hint: Write out the multiplication like you did in (a), then multiply only two polynomials together at a time)

Answer:

$(x + 3)^3 = (x + 3)(x + 3)(x + 3)$

Using the associative property $a(bc) = (ab)c$, you can group the first two binomials, calculate their product, and multiply that by the third binomial.

$= [(x + 3)(x + 3)](x + 3)$

$= [(x + 3)(x + 3)](x + 3)$

$= [x^2 + 3x + 3x + 9](x + 3)$
$= [x^2 + 6x + 9](x + 3)$

The area model works when you have only two expressions.

	x^2	$6x$	9
x	x^3	$6x^2$	$9x$
3	$3x^2$	$18x$	27

Apply the distributive property. It works in either order (commutative property), $ab = ba$.

$(x^2 + 6x + 9)(x + 3)$

or

$(x + 3)(x^2 + 6x + 9)$

$= x^3 + 6x^2 + 9x + 3x^2 + 18x + 27$

$= x^3 + 9x^2 + 27x + 27$

Verify the solution with $x = 1$.

$(x + 3)^3$	$x^3 + 9x^2 + 27x + 27$
$(1 + 3)^3$	$1^3 + 9(1)^2 + 27(1) + 27$
43	$1 + 9 + 27 + 27$
64	64

$64 = 64$

The solutions agree.

3. Simplify $(x + y + z)^2$.

   Answer:

   $(x + y + z)^2$

	x	y	z
x	x^2	xy	xz
y	xy	y^2	yz
z	xz	yz	z^2

   $= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$

   or

   $(x + y + z)(x + y + z)$ or $x(x + y + z) + y(x + y + z) + z(x + y + z)$

   $= x^2 + xy + xz + yx + y^2 + yz + xz + yz + z^2$

   $= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$

4. Rita simplified the product of $(x + 4)(x - 2)$ as $x^2 - 8$.

   Identify and explain the error she made, and show how to correct it.

   Answer:

   Rita did not apply the distributive property or use the FOIL strategy correctly when multiplying the binomials. She only multiplied the two first terms and the two last terms.

   Rita's incorrect strategy steps:

   $(x + 4)(x - 2) = x^2 - 8$

   The correct solution using FOIL or an area diagram would be:

   $(x + 4)(x - 2)$
   $= x^2 - 2x + 4x - 8$
   $= x^2 + 2x - 8$

   or

	x	-2
x	x^2	$-2x$
4	$4x$	-8

5. Multiply the following polynomials.

   a) $\left(2x^2y\right)\left(3xy^2\right)$

   b) $\left(\dfrac{-2}{3}a^3b\right)\left(-6ab^3\right)$

   c) $\left(3x^2\right)\left(4x^3\right)\left(5x^4\right)$

   d) $2x(x+1)$

   e) $\left(-2x^2\right)\left(x^3+3x^2-x\right)$

   f) $\left(-3-5p+9p^2\right)(-2p)$

   g) $(x+1)(x+2)$

   h) $(2x-4)\left(3x^2+x-2\right)$

   i) $(2x-3y)(3x+y)$

   j) $(x-2y)\left(x^2+xy-4y^2\right)$

   k) $(3x-2)^2$

   l) $(a+b-c)(a-b+c)$

   m) $(x+3)\left(x^2-3x+9\right)$

   n) $\left(1-2x+x^2\right)(1+3x)$

Answers:

a) $6x^3y^3$

b) $\dfrac{-2 \cdot -\cancel{6}^{2}}{\cancel{3}^{1}} a^3 \cdot a \cdot b \cdot b^3$
$= 4a^4b^4$

c) $60x^9$

d) $2x^2 + 2x$

e) $-2x^2\left(x^3\right) - 2x^2\left(3x^2\right) - 2x^2(-x)$
$= -2x^5 - 6x^4 + 2x^3$

f) $-3(-2p) - 5p(-2p) + 9p^2(-2p)$
$= 6p + 10p^2 - 18p^3$

g) $x^2 + 2x + 1x + 2$
$= x^2 + 3x + 2$

h) $2x\left(3x^2 + x - 2\right) - 4\left(3x^2 + x - 2\right)$
$= 6x^3 + 2x^2 - 4x - 12x^2 - 4x + 8$
$= 6x^3 - 10x^2 - 8x + 8$

i) $2x(3x + y) - 3y(3x + y)$
$= 6x^2 + 2xy - 9xy - 3y^2$
$= 6x^2 - 7xy - 3y^2$

j) $x\left(x^2 + xy - 4y^2\right) - 2y\left(x^2 + xy - 4y^2\right)$
$= x^3 + x^2y - 4xy^2 - 2x^2y - 2xy^2 + 8y^3$
$= x^3 - x^2y - 6xy^2 + 8y^3$

k) $(3x - 2)(3x - 2)$
$= 9x^2 - 6x - 6x + 4$
$= 9x^2 - 12x + 4$

l) $(a + b - c)(a - b + c)$
$= a^2 - ab + ac + ab - b^2 + bc - ac + bc - c^2$
$= a^2 - b^2 + 2bc - c^2$

m) $(x + 3)\left(x^2 - 3x + 9\right)$
$= x^3 - 3x^2 + 9x + 3x^2 - 9x + 27$
$= x^3 + 27$

n) $1 + 3x - 2x - 6x^2 + x^2 + 3x^3$
$= 1 + x - 5x^2 + 3x^3$ or
$= 3x^3 - 5x^2 + x + 1$

Answers for Practice Questions Set A

1. $x^2 + 10x + 24$
2. $x^2 + 6x - 7$
3. $x^2 - 4x - 60$
4. $x^2 + 5x + 6$
5. $x^2 - 3x - 4$
6. $x^2 - 6x + 5$
7. $x^2 - 4x + 3$
8. $x^2 - 4x$
9. $x^2 - x - 6$
10. $x^2 - 2x - 15$
11. $2x^2 + x - 1$
12. $x^2 - 10x + 21$
13. $x^2 + 14x + 48$
14. $x^2 + 15x + 54$
15. $x^2 + 12x + 27$
16. $x^2 + 6x - 27$
17. $x^2 + 8x + 15$
18. $x^2 + 2x - 15$
19. $2x^2 + 7x + 3$
20. $2x^2 - 5x - 3$

Answers for Practice Questions Set B

1. $x^2 + 5x + 6$
2. $x^2 - 5x + 6$
3. $x^2 + x - 6$
4. $x^2 - x - 6$
5. $x^2 + 7x + 6$
6. $x^2 - 7x + 6$
7. $x^2 - 5x - 6$
8. $x^2 + 5x - 6$
9. $x^2 + 7x + 12$
10. $x^2 - 7x + 12$
11. $x^2 + x - 12$
12. $x^2 - x - 12$
13. $x^2 + 13x + 12$
14. $x^2 - 13x + 12$
15. $x^2 - 11x - 12$
16. $x^2 + 11x - 12$
17. $x^2 + 8x + 12$
18. $x^2 - 8x + 12$
19. $x^2 + 4x - 12$
20. $x^2 - 4x + 12$

What patterns do you notice? We'll explore some of these patterns in the next lesson.

Learning Activity 6.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Samantha has twice as many dresses as she has shirts and three times as many shirts as she has pants. If Samantha has 12 shirts, how many pairs of pants does she have?
2. Multiply: $(x + 2)(x + 3)$.
3. Write the following as a function: $y - x = 25$.
4. Convert: 1 foot^2 = __________ inches^2.
5. The slope of a line is $\frac{4}{5}$. One point on the line is (2, 3). What are the coordinates of another point on this line?
6. Heather is packing her suitcase for Europe. If the dimensions of the suitcase are 100 cm by 80 cm by 25 cm, what is the volume of the suitcase in metres?
7. Team A has won 5 out of their last 9 games. Team B has won 4 out of their last 7 games. Which team has won a greater percentage of their games?
8. You and your friend want to split the bill for dinner. The total is $35.00 for the meal. How much will you pay for dinner?

Answers:

1. 4 pairs of pants $(12 \div 3)$
2. $x^2 + 5x + 6$ (When the two binomials do not have coefficients, the product's middle term coefficient is the sum of the constants $(2 + 3)$ and the last term of the product is the product of the constants (2×3).)
3. $f(x) = x + 25$
4. 144 inches^2 (1 foot = 12 inches, 1 foot^2 = (12 inches)^2)
5. There are many answers. (7, 7) or $(-3, -1)$ are two.
   Slope is $\frac{\text{rise}}{\text{run}}$ so $(2 + 5, 3 + 4)$ is another point on the line. So is $(2 - 5, 3 - 4)$.
6. 20 m^2 ($100 \times 80 = 8000$ cm^2, $8000 \times 25 = (8 \times 25) \times 1000 = 200 \times 1000 =$ 200 000 cm^3. 1 m^3 = 10 000 cm^3 so 200 000 cm^3 = 20 m^3)
7. Team B has the greater percentage. $\left(\frac{5}{9} = \frac{35}{63} \text{ vs } \frac{4}{7} = \frac{36}{63}\right)$
8. $17.50 ($35 is odd, so it is not divisible by 2. $35 = 34 + 1$ so $34 \div 2 = 17$ and $1 \div 2 = 0.50$. The total you pay is $17 + 0.50 =$ $17.50.)

Part B: Factoring Binomials and Trinomials

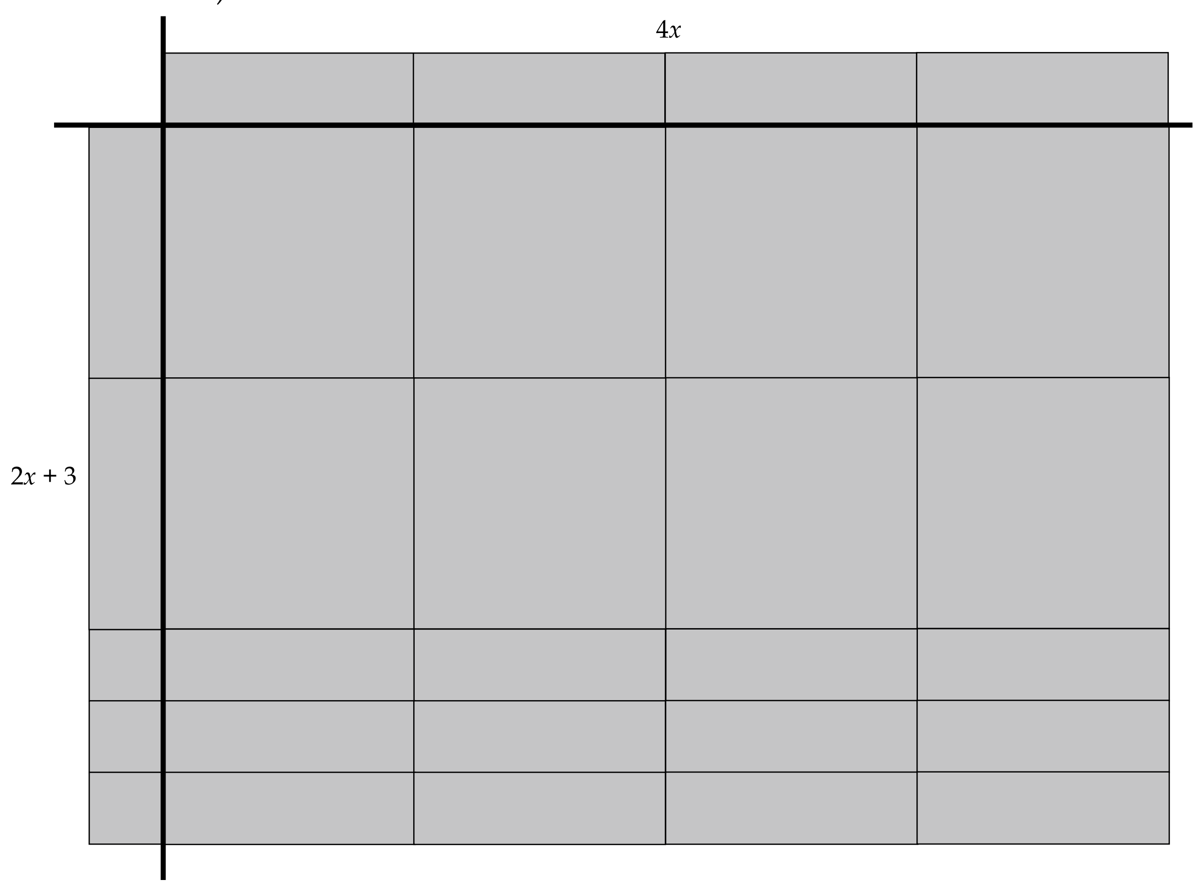
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Sketch a rectangular arrangement of tiles to represent the following expressions. Use the tiles to determine the factors of the polynomial. Verify your answer by multiplying the factors.

a) $8x^2 + 12x$	b) $2x^2 + 6x$
c) $12x^2 + 3x$	d) $x^2 + x$
e) $x^2 + 12x + 36$	f) $x^2 + 7x + 10$
g) $x^2 + 7x + 12$	h) $x^2 + 7x + 6$

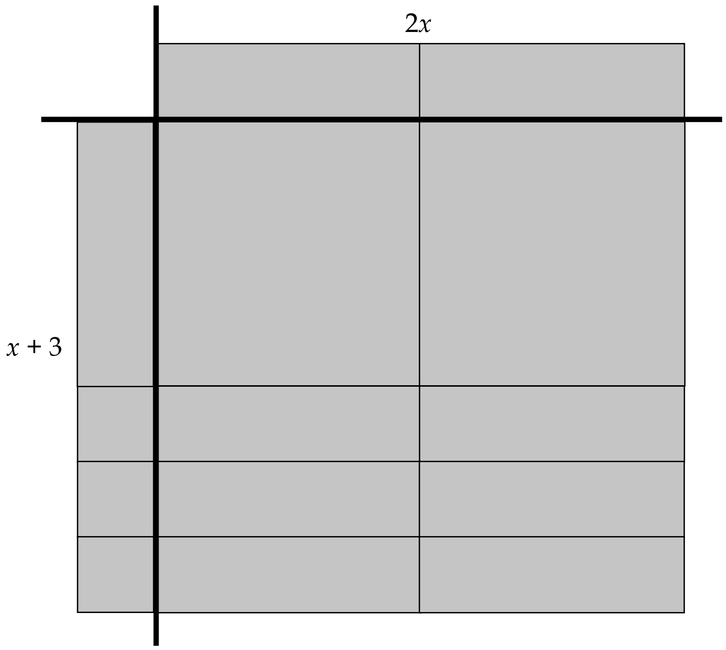
Answers:

a) $8x^2 + 12x$



Verify:

$4x(2x + 3) = 8x^2 + 12x$

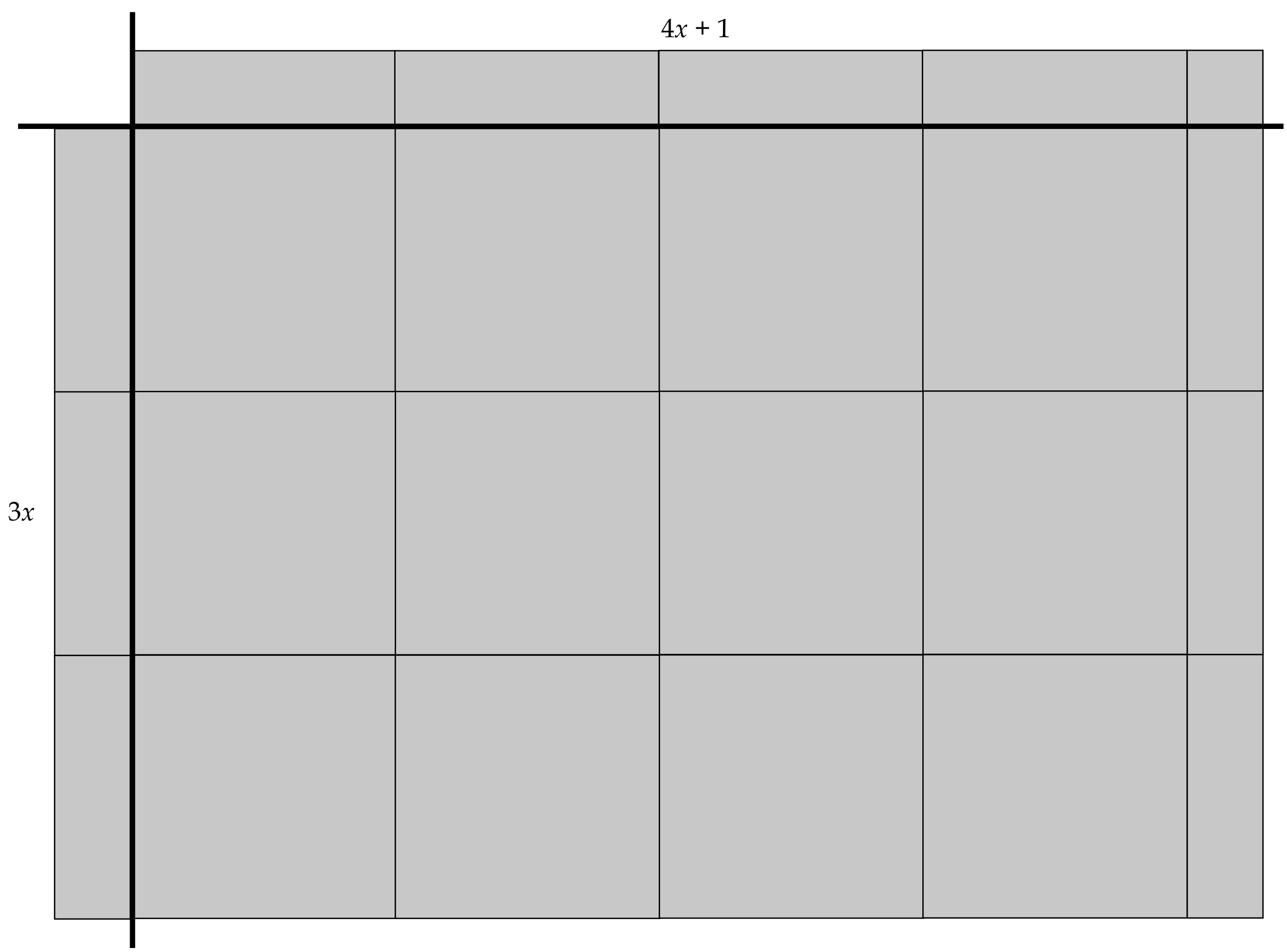
b) 

$2x^2 + 6x$

Verify:

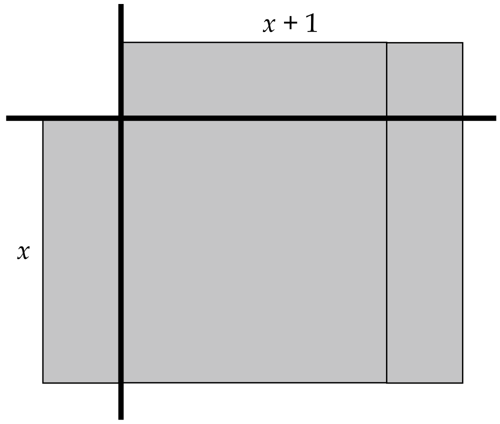
$2x(x + 3) = 2x^2 + 6x$

c) $12x^2 + 3x$



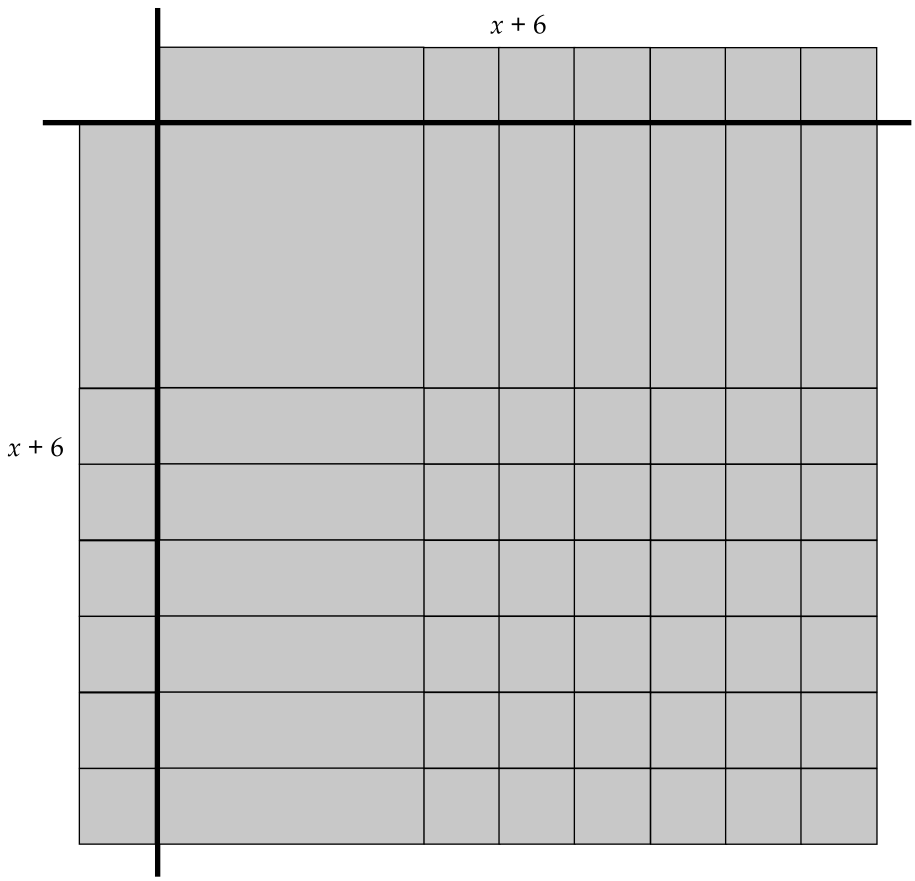
Verify: $3x(4x + 1) = 12x^2 + 3x$

d) $x^2 + x$



Verify: $x(x + 1) = x^2 + x$

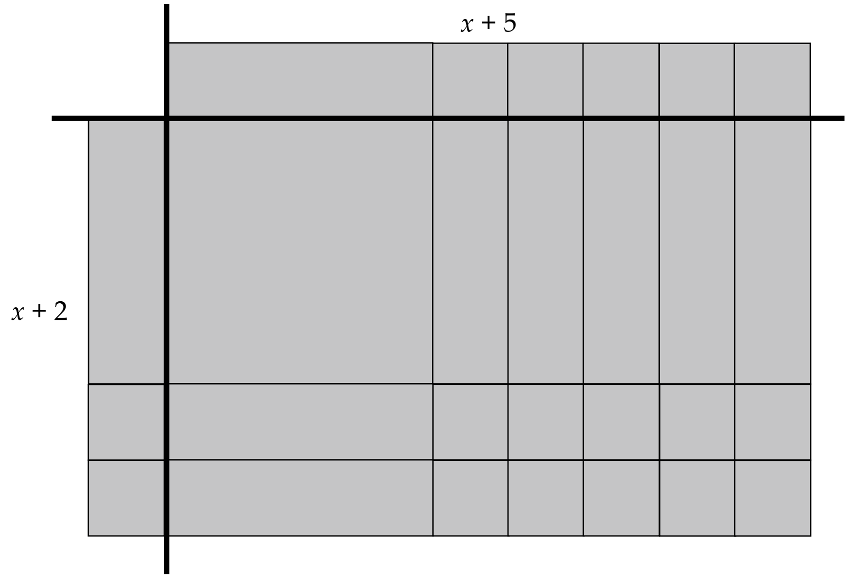
e) $x^2 + 12x + 36$



Verify: $(x + 6)(x + 6) = x^2 + 6x + 6x + 36$

$= x^2 + 12x + 36$

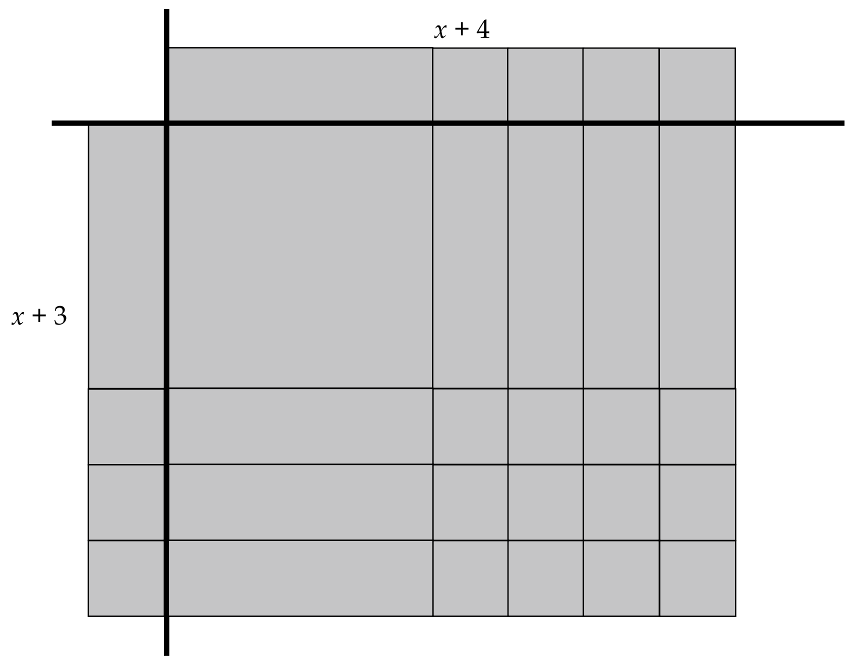
f) $x^2 + 7x + 10$



Verify: $(x + 5)(x + 2) = x^2 + 2x + 5x + 10$

$= x^2 + 7x + 10$

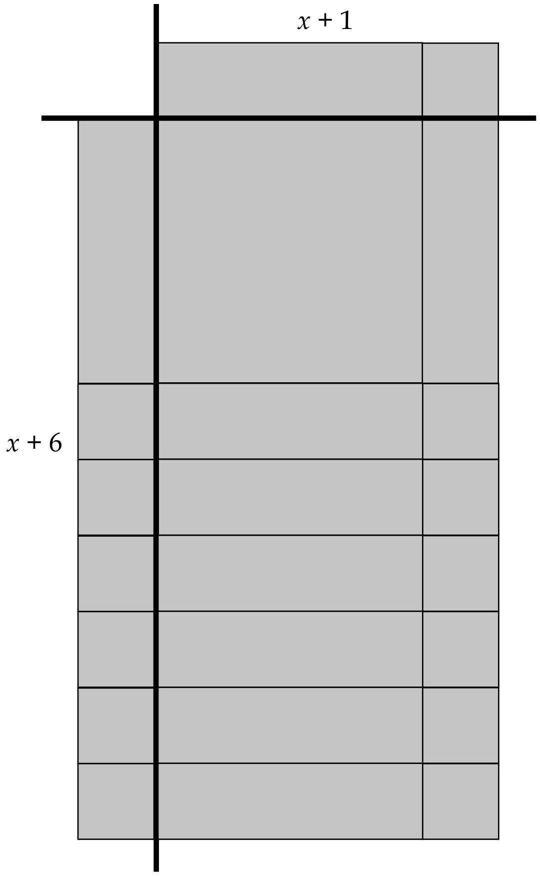
g) $x^2 + 7x + 12$



Verify: $(x + 4)(x + 3) = x^2 + 3x + 4x + 12$

$= x^2 + 7x + 12$

h) $x^2 + 7x + 6$



Verify: $(x + 6)(x + 1) = x^2 + x + 6x + 6$

$= x^2 + 7x + 6$

2. Complete the following chart to identify patterns in factoring the given trinomials.

Answers:

Trinomial	Coefficient of x^2	Coefficient of x	Constant	Binomial Factors	Sum of Constants in Binomials	Product of Constants in Binomials
$x^2 + 5x + 6$	1	5	6	$(x + 2)(x + 3)$	$2 + 3 = 5$	$(2)(3) = 6$
$x^2 + 8x + 12$	1	8	12	$(x + 2)(x + 6)$	$2 + 6 = 8$	$(2)(6) = 12$
$x^2 + 8x + 15$	1	8	15	$(x + 5)(x + 3)$	$5 + 3 = 8$	$(5)(3) = 15$
$x^2 + x - 2$	1	1	-2	$(x + 2)(x - 1)$	$(-1) + 2 = 1$	$(2)(-1) = -2$
$x^2 - 2x - 8$	1	-2	-8	$(x - 4)(x + 2)$	$(-4) + 2 = -2$	$(-4)(2) = -8$

3. a) Given a trinomial in the form $ax^2 + bx + c$, where a is the coefficient of x^2, b is the coefficient of x, and c is a constant, complete the following chart.

Answers:

Trinomial $ax^2 + bx + c$	***a***	***b***	***c***	**List all factor pairs of *c*. State what signs are needed to produce the sign of the product**	**State factor pair of *c* that gives you the sum of *b*, including the signs**	**State the binomial factors of the trinomial**
$x^2 + 4x - 21$	1	4	−21	1,21; 3,7 signs: (+) (−)	−3, 7	$(x - 3)(x + 7)$
$x^2 + 9x + 20$	1	9	20	1,20; 2,10; 4,5 signs: (+) (+)	4, 5	$(x + 4)(x + 5)$
$x^2 + 2x - 48$	1	2	−48	1,48; 2,24; 3,16; 4,12; 6,8 signs: (+) (−)	−6, 8	$(x - 6)(x + 8)$
$x^2 - 11x + 28$	1	−11	28	1,28; 2,14; 4,7 signs: (−)(−)	−7, −4	$(x - 4)(x - 7)$

b) Write a summary statement describing the relationship between the constants in the binomial factors and b and c in the trinomial (the coefficient of x and the constant in the trinomial).

Answer:

The constants in the binomials are the factor pair of c that gives you the sum of b. The two values must multiply to c and add to b.

4. a) Verify the binomial factors in the last column of the chart in 3(a) above by applying the distributive property of multiplication to each.

Answer:

$(x - 3)(x + 7)$
$= x^2 + 7x - 3x - 21$
$= x^2 + 4x - 21$

$(x + 4)(x + 5)$
$= x^2 + 5x + 4x + 20$
$= x^2 + 9x + 20$

$(x - 6)(x + 8)$
$= x^2 + 8x - 6x - 48$
$= x^2 + 2x - 48$

$(x - 4)(x - 7)$
$= x^2 - 7x - 4x + 28$
$= x^2 - 11x + 28$

b) Explain, using examples, the relationship between multiplication and factoring.

Answer:

When you multiply two polynomials together, you end up with a product. The factors of that product are the two polynomials you multiplied. To factor a polynomial, you are doing the reverse of multiplying. You start with the product and determine what it takes to multiply to get that answer. With multiplying, you start with the factors and multiply them to determine the product. Factoring is the reverse of the distributive law.

factor × factor = product	$(x + 2)(x - 7) = x^2 - 5x - 14$
product = factor × factor	$x^2 - 5x - 14 = (x + 2)(x - 7)$

5. Factor the polynomials. Verify your answer using distributive multiplication.

   a) $x^2 - 7x - 18$
   b) $a^2 - 10a - 11$
   c) $x^2 - 13x + 40$
   d) $y^2 + 13y + 36$
   e) $2x^2 + 8x - 64$

   Hint: Find a common factor for all the coefficients and constants, then try to factor the trinomial that doesn't include the common factor (e.g., $3x^2 + 6x - 9 = (3)(x^2 + 2x - 3)\ldots$).

   Answers:

   a) $x^2 - 7x - 18 = (x - 9)(x + 2)$
   Verify:
   $(x - 9)(x + 2)$
   $= x^2 + 2x - 9x - 18$
   $= x^2 - 7x - 18$

   b) $a^2 - 10a - 11 = (a - 11)(a + 1)$
   Verify:
   $(a - 11)(a + 1)$
   $= a^2 + a - 11a - 11$
   $= a^2 - 10a - 11$

   Note: Any letter may be used to represent the variable.

   c) $x^2 - 13x + 40 = (x - 8)(x - 5)$
   Verify:
   $(x - 8)(x - 5)$
   $= x^2 - 5x - 8x + 40$
   $= x^2 - 13x + 40$

   d) $y^2 + 13y + 36 = (y + 4)(y + 9)$
   Verify:
   $(y + 4)(y + 9)$
   $= y^2 + 9y + 4y + 36$
   $= y^2 - 13y + 36$

   e) $2x^2 + 8x - 64 = 2(x^2 + 4x - 32) = 2(x - 4)(x + 8)$
   Always factor out common factors.
   Verify:
   $2(x - 4)(x + 8)$
   $= 2(x^2 + 8x - 4x - 32)$
   $= 2(x^2 + 4x - 32)$
   $= 2x^2 + 8x - 64$

6. Identify and explain errors in the following factorizations. Show the correct solution.

   a) $x^2 - 5x - 6 = (x - 3)(x + 2)$

      Answer:

      The constants in the binomial, -3 and 2, give you the product of -6 but do not combine to -5. The correct factors are $(x - 6)(x + 1)$.

   b) $18y^2 - 12y = 2(9y^2 - 6y)$

      Answer:

      2 is not the greatest common factor of the terms in the binomial. The correct factorization is $6y(3y - 2)$.

   c) $3x^2 - 3x - 6 = (3x - 6)(x + 1)$

      Answer:

      If you were to multiply the factors given, you would get the trinomial given. However, the common factor of 3 would not be factored out. The correct answer is $3(x - 2)(x + 1)$.

   d) $x^2 + 20x + 9 = (x + 4)(x + 5)$

      Answer:

      The constants in the binomials give you the product of b and the sum of c, rather than the product of c (constant) and sum of b (coefficient of x) in the trinomial. The factor pairs of 9 are 3, 3 and 9, 1. Neither of these pairs will work to combine to 20, so this trinomial does not have two binomial factors. It would be impossible to arrange $x^2 + 20x + 9$ tiles into a rectangular shape.

7. Factor the given expressions.

   Note: If you are feeling confident that you are able to factor binomials and trinomials, only do (a) to (d) and (i) to (l). If you want more practice, work through (a) to (p). If you are struggling with these concepts, do not forget that you can contact your tutor/marker, and you can ask your learning partner a question or work with him or her.

   a) $12m - 24p$
   b) $a - ar^3y$
   c) $2a^2 - 12ab + 14ac$
   d) $6x^2 - 18z^6y - 6ax^3z$
   e) $3r^2 - 15rh$
   f) $4n^3 - 4n^2$
   g) $32x^2y + 4x^3y$
   h) $3mn + 6n^2m^2$
   i) $x^2 - 7x + 12$
   j) $x^2 - 10x - 24$
   k) $x^2 + 25x + 24$
   l) $x^2 - 4x - 12$
   m) $x^2 + x - 72$
   n) $c^2 - 4c - 12$
   o) $4 - 5c + c^2$
   p) $x^2 - x - 6$

   Answers:

   a) $12(m - 2p)$
   b) $a(1 - r^3y)$
   c) $2a(a - 6b + 7c)$
   d) $6(x^2 - 3z^6y - ax^3z)$
   e) $3r(r - 5h)$
   f) $4n^2(n - 1)$
   g) $4x^3y(8x + 1)$
   h) $3mn(1 + 2mn)$
   i) $(x - 4)(x - 3)$
   j) $(x + 2)(x - 12)$
   k) $(x + 1)(x + 24)$
   l) $(x - 6)(x + 2)$
   m) $(x + 9)(x - 8)$
   n) $(c - 6)(c + 2)$
   o) $(4 - c)(1 - c)$ or $(c - 4)(c - 1)$
   p) $(x - 3)(x + 2)$

Learning Activity 6.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. What two numbers have a product of -8 and a sum of -2?
2. There are 20 cheese sticks in a package. You only eat one each day, and only on weekdays (Monday to Friday). How many weeks will it take you to finish the whole package?
3. Solve: $4 - 6 + 2 \times (3 - 8)$.
4. You are playing baseball on a co-ed team. There are 16 people on your team. If you need $\frac{1}{4}$ of the team to be girls, how many have to be girls?
5. Multiply: $(x + 5)(x - 9)$.
6. The last time you counted, you had 54 DVDs. Your house was broken into last night and now you only have 32. How many DVDs were stolen?
7. Complete the pattern: -1, 0, ____, 0, ____, 0, 1.
8. You use your left hand to type more than your right when you have your hands on the keyboard properly. Typing the word *factor*, you use your left hand for 5 letters and your right hand for one. Write the fraction that represents how many times you use your right hand in total.

Answers:

1. -4, 2
2. 4 weeks (You eat one per day on the weekdays, so that is 5 per week; $20 \div 5 = 4$.)
3. -12 $(4 - 6 + 2 \times (-5) = 4 - 6 + (-10) = -2 - 10)$
4. 4 girls $\left(16 \times \frac{1}{4}\right)$
5. $x^2 - 4x - 45$ $(5 - 9 = -4, 5 \times (-9) = -45)$
6. 22 DVDs ($32 + 2 = 34$ (make the ones the same), $34 + 20 = 54$ (original total))
7. 1, -1
8. $\frac{1}{6}$ (It is out of 6 because that is the total number of letters that you had to type. If you put 5, that would be the fraction comparing right to left.)

Part B: Factoring Trinomials when *a* ∈ I

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Factor completely.

   a) $2x^2 + 11x + 15$ b) $4x^2 - 21x + 5$

   c) $5x^2 - 4x - 12$ d) $6x^2 + 5x - 6$

   e) $2x^3 + x^2 - 15x$ f) $2x^3 - 22x^2 + 36x$

   g) $-12x^2 + 26x + 10$

   Answers:

   a) $2x^2 + 11x + 15$ $(2)(15) = 30$ 30: 1, 30 (+)(+)
   2, 15
   3, 10
   5, 6
   $(5) + (6) = 11$
   $= 2x^2 + 5x + 6x + 15$
   $= x(2x + 5) + 3(2x + 5)$
   $= (2x + 5)(x + 3)$

   b) $4x^2 - 21x + 5$ $(4)(5) = 20$ 20: 1, 20 (−)(−)
   2, 10
   4, 5
   $(5) + (6) = 11$
   $= 4x^2 - 1x - 20x + 5$
   $= x(4x - 1) - 5(4x - 1)$
   $= (x - 5)(4x - 1)$

   c) $5x^2 - 4x - 12$ $(5)(-12) = -60$ 60: 1, 60 (+)(−)
   2, 30
   3, 20
   4, 15
   5, 12
   6, 10
   $(6) + (-10) = -4$
   $= 5x^2 - 10x + 6x - 12$
   $= 5x(x - 2) + 6(x - 2)$
   $= (x - 2)(5x + 6)$

d) $6x^2 + 5x - 6$ $\quad (6)(-6) = -36 \quad$ 36: 1, 36 $\quad (+)(-)$

2, 18

3, 12

4, 9

6, 6

$(9) + (-4) = 5$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

e) $2x^3 + x^2 - 15x$ (Always remove common factors first.)

$= x(2x^2 + x - 15)$ $\quad (6)(-15) = -30 \quad$ 30: 1, 30 $\quad (+)(-)$

2, 15

3, 10

5, 6

$(-5) + (6) = 1$

$= x(2x^2 - 5x + 6x - 15)$

$= x(x(2x - 5) + 3(2x - 5))$

$= x(2x - 5)(x + 3)$

f) $2x^3 - 22x^2 + 36x$ (Remove common factor.)

$= 2x(x^2 - 11x + 18)$ $\quad (1)(18) = 18 \quad$ 18: 1, 18 $\quad (-)(-)$

2, 9

3, 6

$(-2) + (-9) = -11$

$= 2x(x^2 - 9x - 2x + 18)$

$= 2x(x(x - 9) - 2(x - 9))$

$= 2x(x - 9)(x - 2)$

g) $-12x^2 + 26x + 10$ (Remove common factor, including negative sign.)

$= -2(6x^2 - 13x - 5)$ $\quad (6)(-5) = -30 \quad$ 30: 1, 30 $\quad (+)(-)$

2, 15

3. 10

5, 6

$(-15) + (2) = -13$

$= -2(6x^2 - 15x + 2x - 5)$

$= -2(3x(2x - 5) + 1(2x - 5))$

$= -2(2x - 5)(3x + 1)$

2. Factor each expression. (If you are feeling confident in your ability to factor trinomials with $a \in I$, you do not have to do these questions. If you want more practice, work through as many as you need.)

a) $2x^2 + 5x + 3$	b) $5x^2 + 6x + 1$
c) $5a^2 - 16a + 3$	d) $3y^2 + 4y + 1$
e) $24x^2 + 2x - 1$	f) $6y^2 + 20 + 23y$
g) $10 + y - 2y^2$	h) $60y^2 - 27y - 60$
i) $15x^2 + 37x + 20$	j) $15a^2 + 8a - 12$

Answers:

a) $(2x + 3)(x + 1)$	b) $(5x + 1)(x + 1)$
c) $(5a - 1)(a - 3)$	d) $(3y + 1)(y + 1)$
e) $(6x - 1)(4x + 1)$	f) $(2y + 5)(3y + 4)$
g) $(5 - 2y)(2 + y)$	h) $3(5y + 4)(4y - 5)$
i) $(5x + 4)(3x + 5)$	j) $(5a + 6)(3a - 2)$

3. For what integral values (whole numbers) of k can $4x^2 + kx + 3$ be factored? Write out all possible trinomials as a product of its factors.

Answer:

$4x^2 + kx + 3$

$(4)(3) = 12$ Signs must be (+)(+) Factors of 12: 1, 12
2, 6
3, 4

Possible combinations of the factor pairs when both are positive are 13, 8, and 7.

The possible values for k are 13, 8, and 7.

$4x^2 + 13x + 3$
$4x^2 + 8x + 3$
$4x^2 + 7x + 3$

The factor pairs of 4 are 4, 1 and 2, 2.

The factor pairs of 3 are 1, 3.

Possible factor pair combinations are

$(4x + 1)(1x + 3) = 4x^2 + 13x + 3$
$(4x + 3)(1x + 1) = 4x^2 + 7x + 3$
$(2x + 1)(2x + 3) = 4x^2 + 8x + 3$
$(2x + 3)(2x + 1) = 4x^2 + 8x + 3$

(Note that the last two result in the same trinomial.)

4. Fill in the space so that each trinomial is a perfect square trinomial.
   a) $4x^2 + \_\_\_\_ + 4$

      Answer:

      $4x^2 + 8x + 4$ $(4)(4) = 16$ $\sqrt{16} = 4$ $2 \times 4 = 8$

   b) $25x^2 + \_\_\_\_ + 9$

      Answer:

      $25x^2 + 30x + 9$ $(25)(9) = 225$ $\sqrt{225} = 15$ $2 \times 15 = 30$

   c) $x^2 + 14\,x + \_\_\_\_$

      Answer:

      $x^2 + 14x + 49$ $14 \div 2 = 7$ $7^2 = 49$ $(1)(49) = 49$

5. Factor each perfect square trinomial.
   a) $x^2 - 8x + 16$

      Answer:

      $x^2 - 8x + 16 = (x - 4)^2$

   b) $4x^2 - 4x + 1$

      Answer:

      $4x^2 - 4x + 1 = (2x - 1)^2$ or $(-2x + 1)^2$

6. Identify and explain the errors in the factorization, and state the correct solution.

   $x^2 - 8x + 16 = (x + 4)^2$

   Answer:

   The sign in the middle of the binomial factor is incorrect. It should be the same as the sign of the coefficient x. The correct answer is $(x - 4)^2$.

Learning Activity 6.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Factor: $m^2 + 3m - 54$.
2. You know that your glass holds 1 cup of milk. Would this be a good referent to use to find out how much water your water bottle can hold?
3. Andy has been to the Great Pyramid of Egypt. He bought a miniature version of the pyramid, which was to scale. The ratio comparing the miniature and the real pyramid is 1 cm: 70 royal cubits (an ancient unit of measurement). If the height of the miniature is 4 cm, how tall is the real pyramid?
4. Is 3^{-6} rational or irrational?
5. What is the GCF of 34 and 17?
6. Simplify: $\left(2^2\right)^{\frac{-1}{5}}$.
7. Solve for n: $4n - 3 = 2 + 19$.
8. You have 2 older brothers and 3 older sisters. Your parents had 11 children. How many of your siblings are younger than you?

Answers:

1. $(m - 6)(m + 9)$
2. Yes (Be aware though that it would only be accurate to the nearest cup. If there is a half cup, you cannot be precise.)
3. 280 royal cubits (4×70)
4. Rational (It can be written as a fraction because of the negative exponent.)
5. 17
6. $\dfrac{1}{\sqrt[5]{4}}$
7. $n = 6$ ($4n - 3 = 21$; $4n = 24$; $n = 6$)
8. 5 younger siblings ($11 - (5 + 2) - 1$ [you])

Part B: Difference of Squares and Module Review

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Why does applying the distributive property to the factors of a difference of square result in a binomial? Give an example.

   Answer:

   In a difference of squares, the factors are the same except for opposite signs. When you apply the distributive property, the coefficients of x are opposite and cancel to zero, leaving $ax^2 + 0x - c$, which simplifies to the binomial $ax^2 - c$.

   Example:

   (Your example may be different but the pattern will be the same.)

$64x^2 - 100$	
$= (8x + 10)(8x - 10)$	Factors
$= 64x^2 - 80x + 80x - 100$	Apply the distributive property and simplify by cancelling the opposite terms.
$= 64x^2 - 100$	

2. Given the polynomial $ax^2 - c$ where a and c are perfect square numbers, write its binomial factors.

   Answer:

   The factors of $ax^2 - c$ are written as $\left(\sqrt{ax^2}+\sqrt{c}\right)\left(\sqrt{ax^2}-\sqrt{c}\right)$.

3. Factor completely.

   a) $x^2 - 36$

      Answer:

      $x^2 - 36 = (x - 6)(x + 6)$

   b) $9y^2 - 49$

      Answer:

      $9y^2 - 49 = (3y - 7)(3y + 7)$

c) $x^2 - 256$

Answer:

$x^2 - 256 = (x - 16)(x + 16)$

d) $2m^2n - 2n$

Answer:

$2m^2n - 2n = 2n(m^2 - 1) = 2n(m - 1)(m + 1)$

4. Use your factoring strategies to factor the following polynomials.

a) $3mn - 6np$

b) $a(b + 3) + c(b + 3)$

c) $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

d) $x^2 - 11x + 28$

e) $x^2 - 3x - 28$

f) $4x^4 - 20x^3 + 24x^2$

g) $16x^2 - 24x + 9$

h) $8x^2 - 40x + 50$

i) $x^2 - 81$

j) $4y^2 - 9$

k) $20x^2y - 5y$

Answers:

a) $3mn - 6np$
$= 3n(m - 2p)$

b) $a(b + 3) + c(b + 3)$
$= (b + 3)(a + c)$

c) $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi\left(R^3 - r^3\right)$

d) $x^2 - 11x + 28$
$= (x - 7)(x - 4)$

e) $x^2 - 3x - 28$
$= (x - 7)(x + 4)$

f) $4x^4 - 20x^3 + 24x^2$
$= 4x^2(x^2 - 5x + 6)$
$= 4x^2(x - 3)(x - 2)$

g) $16x^2 - 24x + 9 \quad 16 \times 9 = 144 \quad (-12)(-12) = 144 \quad (-12) + (-12) = -24$
$= 16x^2 - 12x - 12x + 9$
$= 4x(4x - 3) - 3(4x - 3)$
$= (4x - 3)(4x - 3)$
$= (4x - 3)^2$

h) $8x^2 - 40x + 50$
$= 2(4x^2 - 20x + 25)$
$= 2(2x - 5)^2$

i) $x^2 - 81$
$= (x - 9)(x + 9)$

j) $4y^2 - 9$
$= (2y - 3)(2y + 3)$

k) $20x^2y - 5y$
$= 5y(4x^2 - 1)$
$= 5y(2x - 1)(2x + 1)$

5. If you are feeling confident in your ability to factor trinomials with $a \in \mathrm{I}$ and difference of squares, you do not have to do these questions. If you want more practice, work through as many as you need.

a) $x^2 - 16$
b) $36t^2 - 1$
c) $4a^2 - b^2$
d) $8c^2 - 72$
e) $81 - (x + 7)^2$
f) $(x - 1)^2 - (x + 1)^2$
g) $x^8 - y^{12}$
h) $4x^2 - 1$
i) $4m^2 - 25y^4$
j) $121x^2 - 196y^2$

Answers:

a) $(x + 4)(x - 4)$
b) $(6t + 1)(6t - 1)$
c) $(2a + b)(2a - b)$
d) $8(c - 3)(c + 3)$
e) $(2 - x)(16 + x)$
f) $((x - 1) - (x + 1))((x - 1) + (x + 1)) = -4x$
g) $(x^4 - y^6)(x^4 + y^6)$
$= (x^2 - y^3)(x^2 + y^3)(x^4 + y^6)$
h) $(2x + 1)(2x - 1)$
i) $(2m - 5y^2)(2m + 5y^2)$
j) $(11x - 14y)(11x + 14y)$

6. You are now three-quarters of the way through this course. Take a few minutes now to look back at Module 1 and the goals you set for yourself at the beginning of this course, as well as the revised version written after Module 4. Which goals have you accomplished or completed by now? Are you progressing towards achieving your other goals in a timely manner? How can you modify or adapt your steps to ensure success? How will you celebrate your achievements and continue to strive for the rest of your goals! You are almost done!

Answer:

List your successes, modifications, and new steps (or new goals) in a place where you will be reminded and motivated to bring them to completion. Share your successes with someone. Ask them to keep you accountable as you work to accomplish your other goals.

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 6
Polynomials

Learning Activity Answer Keys

Module 6: Polynomials

Learning Activity 6.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Write the following equation as a function: $x + y = 76$.
2. If you are $\frac{3}{2}$ taller than your brother, and your brother is 4 feet tall, how tall are you?
3. The sides of a right triangle are 3, 5, 4. How long is the hypotenuse?
4. Is this relation a function: {(0,1), (3,6), (4, 8), (0, 10)}?
5. There are 120 employees at your work. Your boss says that three-quarters of the staff are coming to the meeting on Saturday morning. How many people will be attending the meeting?
6. Usain Bolt can run 100 m in 10 s. What is his average speed?
7. You need to make exact change for a customer at your work. They have given you $60 and their bill is $42.60. How much money will you give them?
8. Evaluate: $\left(6^2\right)^{\frac{1}{2}}$.

Answers:

1. $f(x) = 76 - x$
2. 6 feet $\left(\frac{3}{2} \times 4\right)$
3. 5 (The hypotenuse is always the longest side.)
4. No, it is just a relation. (The input value of 0 has two outputs, 1 and 10.)
5. 90 $\left(120 \times \frac{3}{4}\right)$
6. 10 m per second (100 m ÷ 10 s)
7. \$17.40 (\$42.60 + 0.40 = 43.00, 43 + 7 = 50, 50 + 10 = \$60, so 0.4 + 7 + 10 = \$17.40)
8. 6

Part B: Binomial Multiplication

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write a quadratic trinomial of degree 2 with the variable m, coefficients –3 and 1 and a constant of 5.

   Answer:

   Two possible solutions are $-3m^2 + m + 5$ or $m^2 - 3m + 5$. (Remember that you always want to start with the term that has the highest exponent.)

2. Write 3 like terms that are not exactly the same.

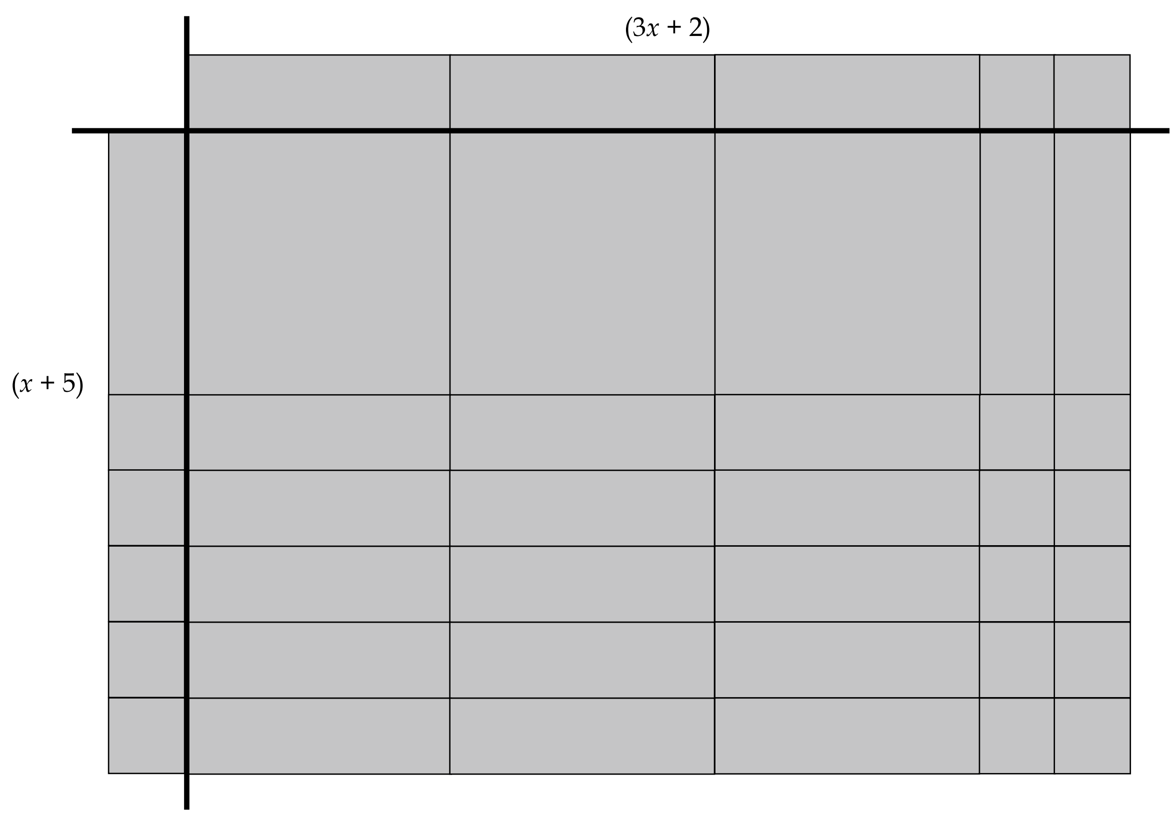
   Answer:

   Any terms that have matching variables and degree but different coefficients are like terms. For example, $5x^2y$, $-3x^2y$, and x^2y are like terms.

3. Illustrate the following products using tiles, and write your solution steps to show how it may be simplified.

   a) $(x + 5)(3x + 2)$

   Answer:



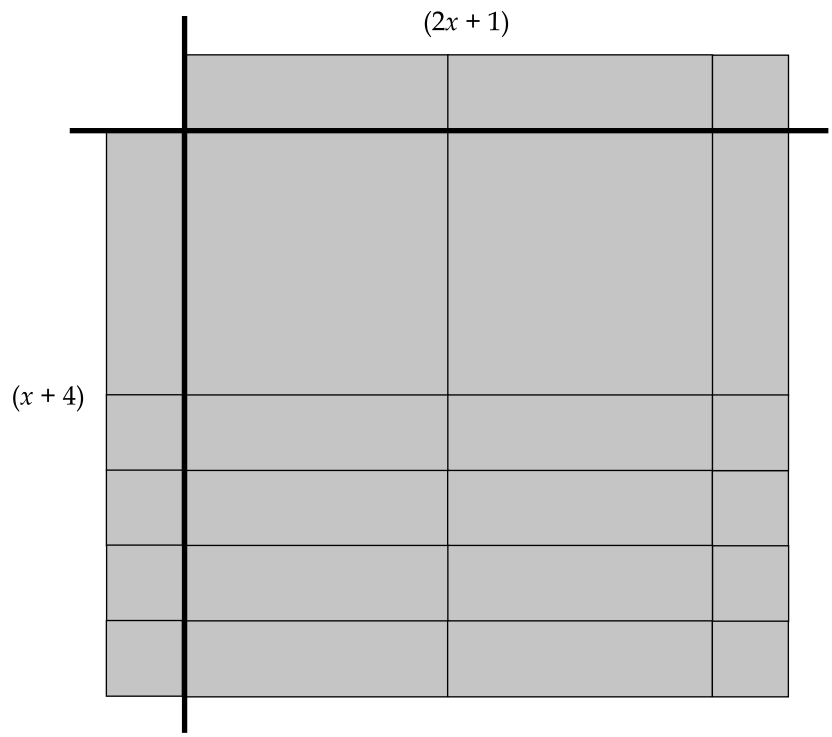
$(x + 5)(3x + 2)$

$= 3x^2 + 2x + 15x + 10$

$= 2x^2 + 17x + 10$

b) $(2x + 1)(x + 4)$

Answer:



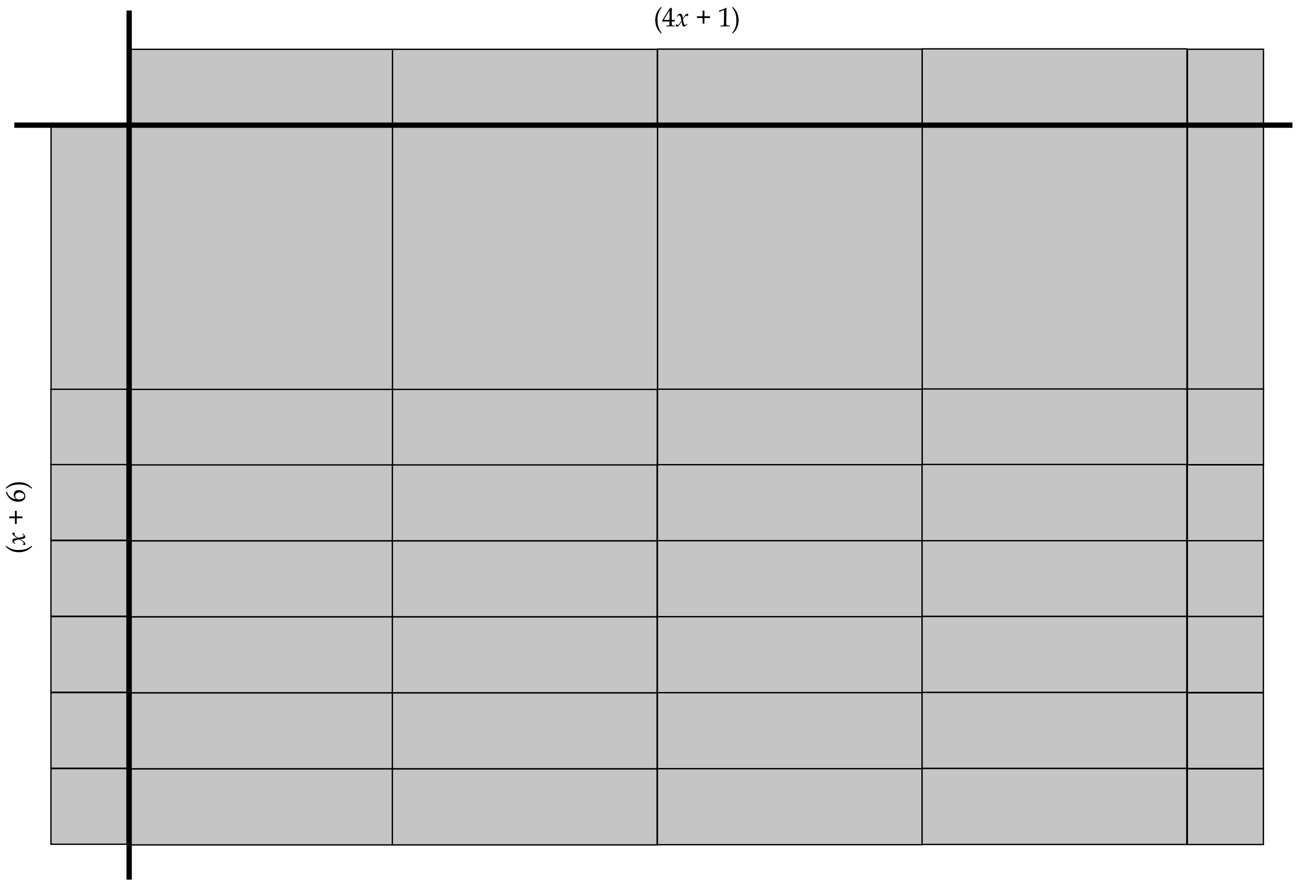
$(x + 4)(2x + 1)$

$= 2x^2 + 1x + 8x + 4$

$= 2x^2 + 9x + 4$

c) $(4x + 1)(x + 6)$

Answer:



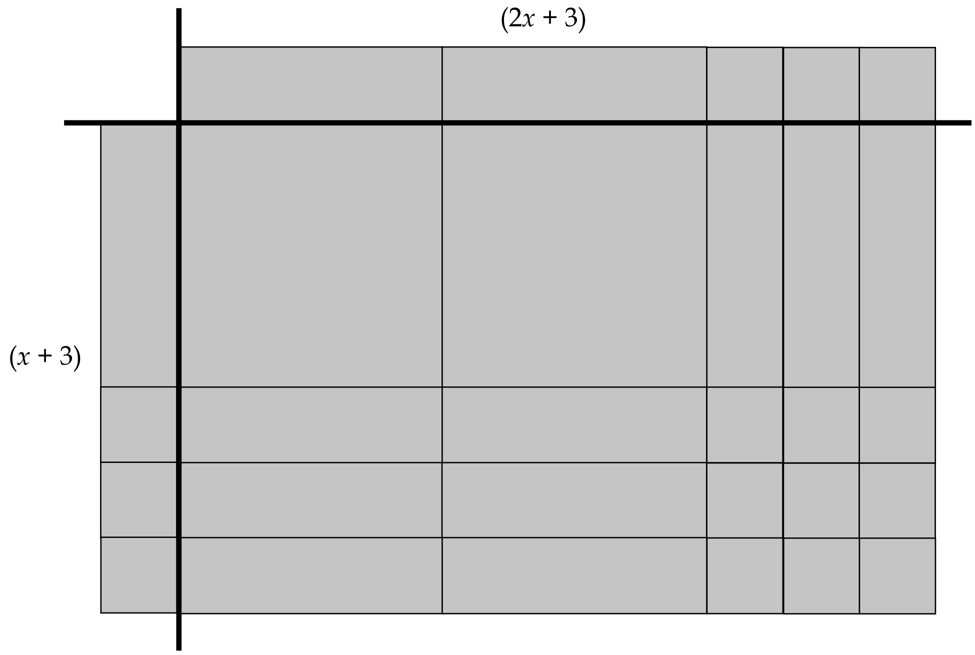
$(4x + 1)(x + 6)$

$= 4x^2 + 24x + x + 6$

$= 4x^2 + 25x + 6$

d) $(x + 3)(2x + 3)$

Answer:



$(2x + 3)(x + 3)$

$= 2x^2 + 6x + 3x + 9$

$= 2x^2 + 9x + 9$

Learning Activity 6.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. At your dad's office they have carpet made out of squares. Each square is 2 feet by 2 feet. If your dad's cubicle is 5 squares deep and 5 squares wide, what is the area of your dad's cubicle in feet?
2. Evaluate: $\sqrt{2^{-4}}$.
3. 1 yard^3 = ________ feet^3.
4. What is the domain of the function $f(x) = x + 4$?
5. It costs $4.00 for a package of 3 chocolate bars. Geri spends $20 on chocolate bars. How many chocolate bars does she buy?
6. I have 9 letters in my name. Is it possible that half of those letters are vowels?
7. What is the LCM of 3 and 5?
8. Solve: $\frac{6}{5} + \frac{2}{3}$.

Answers:

1. 100 ft.^{2} ($5 \times 2 = 10$ ft. for both the depth and width of the cubicle. $10 \times 10 = 100$)
2. $\frac{1}{4}\left(\sqrt{2^{-4}} = 2^{-\frac{4}{2}} = 2^{-2} = \frac{1}{2^2}\right)$
3. 27 feet^3 (1 yard = 3 feet; $V = 3 \times 3 \times 3$)
4. $(-\infty, \infty)$ or $\{x \in \Re\}$ (You can have any value for x.)
5. 15 chocolate bars ($20 \div 4 = 5$ packages, 5×3 chocolate bars per package = 15 chocolate bars.)
6. No (You cannot divide 9 evenly in half, and you cannot have half a vowel. We are considering 'y' to be a half vowel.)
7. 15 (Because 5 and 3 are both prime, their LCM is their product.)
8. $\frac{28}{15}\left(\frac{6}{5} + \frac{2}{3} = \frac{6 \times 3}{5 \times 3} + \frac{2 \times 5}{3 \times 5} = \frac{18+10}{15}\right)$

Part B: Multiplying Polynomials

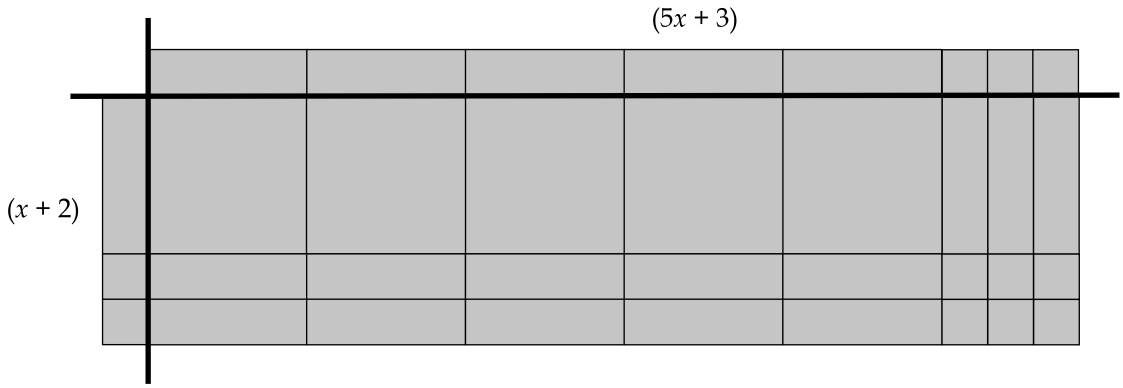
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

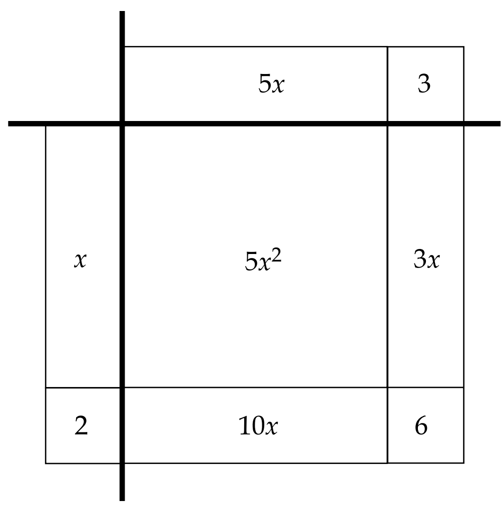
1. Simplify each product. Show the steps pictorially and symbolically.

   a) $(5x + 3)(x + 2)$

   Answer:

   All coefficients and constants are positive, so a diagram or an area model could be drawn.





$(x + 2)(5x + 3)$

$= 5x^2 + 3x + 10x + 6$

$= 5x^2 + 13x + 6$

FOIL or the distributive property can be used to show the product symbolically.

$(5x + 3)(x + 2)$

$= 5x^2 + 10x + 3x + 6$
$= 5x^2 + 13x + 6$

Alternatively,

$5x(x + 2) + 3(x + 2)$

$= 5x^2 + 10x + 3x + 6$

$= 5x^2 + 13x + 6$

b) $(4h - 5)(-3h + 7)$

Answer:

The area model would work to illustrate this product.

	$-3h$	7
$4h$	$-12h^2$	$28h$
-5	$+15h$	-35

$-12h^2 + 28h + 15h - 35$
$= -12h^2 + 43h - 35$

FOIL

$(4h - 5)(-3h + 7)$

$= -12h^2 + 28h + 15h - 35$
$= -12h^2 + 43h - 35$

or

Distributive Property

$4h(-3h + 7) - 5(-3h + 7)$

$= -12h^2 + 28h + 15h - 35$

$= -12h^2 + 43h - 35$

c) $(x^2 - x)(x + 2)$

Answer:

Use the area model.

	x^2	$-x$
x	x^3	$-x^2$
2	$2x^2$	$-2x$

$x^3 + 2x^2 - x^2 - 2x$
$= x^3 + x^2 - 2x$

FOIL or Distributive Property

FOIL

$(x^2 - x)(x + 2)$

$= x^3 + 2x^2 - x^2 - 2x$

$= x^3 + x^2 - 2x$

Distributive Property

$x^2(x + 2) - x(x + 2)$

$= x^3 + 2x^2 - x^2 - 2x$

$= x^3 + x^2 - 2x$

d) $(x + y)(x^2 + 2x - 1)$

Answer:

The area model can be used to illustrate this product.

	x^2	$2x$	-1
x	x^3	$2x^2$	$-x$
y	x^2y	$2xy$	$-y$

$x^3 + 2x^2 - x + x^2y + 2xy - y$

$= x^3 + x^2y + 2x^2 + 2xy - x - y$

Rearrange terms in descending order of power.

The distributive property.

$(x + y)(x^2 + 2x - 1)$

$= x^3 + 2x^2 - x + x^2y + 2xy - y$

$= x^3 + x^2y + 2x^2 + 2xy - x - y$

or

$x(x^2 + 2x - 1) + y(x^2 + 2x - 1)$

2. Simplify each product and verify your solution.

   a) $(2x - 1)^2$

   Answer:

   $(2x - 1)^2 = (2x - 1)(2x - 1)$

	$2x$	-1
$2x$	$4x^2$	$-2x$
-1	$-2x$	1

$4x^2 - 2x - 2x + 1$

$= 4x^2 - 4x + 1$

$(2x - 1)(2x - 1)$ or $2x(2x - 1) - 1(2x - 1)$

$= 4x^2 - 2x - 2x + 1$

$= 4x^2 - 4x + 1$

Verify: use $x = 1$

$(2x - 1)^2$	$4x^2 - 4x + 1$
$(2(1) - 1)^2$	$4(1)^2 - 4(1) + 1$
$(2 - 1)^2$	$4 - 4 + 1$
1^2	1

$1 = 1$

The solutions agree.

b) $(x + 3)^3$ (Hint: Write out the multiplication like you did in (a), then multiply only two polynomials together at a time)

Answer:

$(x + 3)^3 = (x + 3)(x + 3)(x + 3)$

Using the associative property $a(bc) = (ab)c$, you can group the first two binomials, calculate their product, and multiply that by the third binomial.

$= [(x + 3)(x + 3)](x + 3)$

$= [(x + 3)(x + 3)](x + 3)$

$= [x^2 + 3x + 3x + 9](x + 3)$
$= [x^2 + 6x + 9](x + 3)$

The area model works when you have only two expressions.

	x^2	$6x$	9
x	x^3	$6x^2$	$9x$
3	$3x^2$	$18x$	27

Apply the distributive property. It works in either order (commutative property), $ab = ba$.

$(x^2 + 6x + 9)(x + 3)$

or

$(x + 3)(x^2 + 6x + 9)$

$= x^3 + 6x^2 + 9x + 3x^2 + 18x + 27$

$= x^3 + 9x^2 + 27x + 27$

Verify the solution with $x = 1$.

$(x + 3)^3$	$x^3 + 9x^2 + 27x + 27$
$(1 + 3)^3$	$1^3 + 9(1)^2 + 27(1) + 27$
43	$1 + 9 + 27 + 27$
64	64

$64 = 64$

The solutions agree.

3. Simplify $(x + y + z)^2$.

Answer:

$(x + y + z)^2$

	x	y	z
x	x^2	xy	xz
y	xy	y^2	yz
z	xz	yz	z^2

$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$

or

$(x + y + z)(x + y + z)$ or $x(x + y + z) + y(x + y + z) + z(x + y + z)$

$= x^2 + xy + xz + yx + y^2 + yz + xz + yz + z^2$

$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$

4. Rita simplified the product of $(x + 4)(x - 2)$ as $x^2 - 8$.

   Identify and explain the error she made, and show how to correct it.

   Answer:

   Rita did not apply the distributive property or use the FOIL strategy correctly when multiplying the binomials. She only multiplied the two first terms and the two last terms.

   Rita's incorrect strategy steps:

   $$(x + 4)(x - 2) = x^2 - 8$$

   The correct solution using FOIL or an area diagram would be:

   $$(x + 4)(x - 2)$$
   $$= x^2 - 2x + 4x - 8$$
   $$= x^2 + 2x - 8$$

   or

	x	-2
x	x^2	$-2x$
4	$4x$	-8

5. Multiply the following polynomials.

   a) $\left(2x^2y\right)\left(3xy^2\right)$

   b) $\left(\dfrac{-2}{3}a^3b\right)\left(-6ab^3\right)$

   c) $\left(3x^2\right)\left(4x^3\right)\left(5x^4\right)$

   d) $2x(x+1)$

   e) $\left(-2x^2\right)\left(x^3+3x^2-x\right)$

   f) $\left(-3-5p+9p^2\right)(-2p)$

   g) $(x+1)(x+2)$

   h) $(2x-4)\left(3x^2+x-2\right)$

   i) $(2x-3y)(3x+y)$

   j) $(x-2y)\left(x^2+xy-4y^2\right)$

   k) $(3x-2)^2$

   l) $(a+b-c)(a-b+c)$

   m) $(x+3)\left(x^2-3x+9\right)$

   n) $\left(1-2x+x^2\right)(1+3x)$

Answers:

a) $6x^3y^3$

b) $\dfrac{-2 \cdot -\cancel{6}^2}{\cancel{3}^1} a^3 \cdot a \cdot b \cdot b^3$

$= 4a^4b^4$

c) $60x^9$

d) $2x^2 + 2x$

e) $-2x^2\left(x^3\right) - 2x^2\left(3x^2\right) - 2x^2(-x)$

$= -2x^5 - 6x^4 + 2x^3$

f) $-3(-2p) - 5p(-2p) + 9p^2(-2p)$

$= 6p + 10p^2 - 18p^3$

g) $x^2 + 2x + 1x + 2$

$= x^2 + 3x + 2$

h) $2x\left(3x^2 + x - 2\right) - 4\left(3x^2 + x - 2\right)$

$= 6x^3 + 2x^2 - 4x - 12x^2 - 4x + 8$

$= 6x^3 - 10x^2 - 8x + 8$

i) $2x(3x + y) - 3y(3x + y)$

$= 6x^2 + 2xy - 9xy - 3y^2$

$= 6x^2 - 7xy - 3y^2$

j) $x\left(x^2 + xy - 4y^2\right) - 2y\left(x^2 + xy - 4y^2\right)$

$= x^3 + x^2y - 4xy^2 - 2x^2y - 2xy^2 + 8y^3$

$= x^3 - x^2y - 6xy^2 + 8y^3$

k) $(3x - 2)(3x - 2)$

$= 9x^2 - 6x - 6x + 4$

$= 9x^2 - 12x + 4$

l) $(a + b - c)(a - b + c)$

$= a^2 - ab + ac + ab - b^2 + bc - ac + bc - c^2$

$= a^2 - b^2 + 2bc - c^2$

m) $(x + 3)\left(x^2 - 3x + 9\right)$

$= x^3 - 3x^2 + 9x + 3x^2 - 9x + 27$

$= x^3 + 27$

n) $1 + 3x - 2x - 6x^2 + x^2 + 3x^3$

$= 1 + x - 5x^2 + 3x^3$ or

$= 3x^3 - 5x^2 + x + 1$

Answers for Practice Questions Set A

1. $x^2 + 10x + 24$
2. $x^2 + 6x - 7$
3. $x^2 - 4x - 60$
4. $x^2 + 5x + 6$
5. $x^2 - 3x - 4$
6. $x^2 - 6x + 5$
7. $x^2 - 4x + 3$
8. $x^2 - 4x$
9. $x^2 - x - 6$
10. $x^2 - 2x - 15$
11. $2x^2 + x - 1$
12. $x^2 - 10x + 21$
13. $x^2 + 14x + 48$
14. $x^2 + 15x + 54$
15. $x^2 + 12x + 27$
16. $x^2 + 6x - 27$
17. $x^2 + 8x + 15$
18. $x^2 + 2x - 15$
19. $2x^2 + 7x + 3$
20. $2x^2 - 5x - 3$

Answers for Practice Questions Set B

1. $x^2 + 5x + 6$
2. $x^2 - 5x + 6$
3. $x^2 + x - 6$
4. $x^2 - x - 6$
5. $x^2 + 7x + 6$
6. $x^2 - 7x + 6$
7. $x^2 - 5x - 6$
8. $x^2 + 5x - 6$
9. $x^2 + 7x + 12$
10. $x^2 - 7x + 12$
11. $x^2 + x - 12$
12. $x^2 - x - 12$
13. $x^2 + 13x + 12$
14. $x^2 - 13x + 12$
15. $x^2 - 11x - 12$
16. $x^2 + 11x - 12$
17. $x^2 + 8x + 12$
18. $x^2 - 8x + 12$
19. $x^2 + 4x - 12$
20. $x^2 - 4x + 12$

What patterns do you notice? We'll explore some of these patterns in the next lesson.

Learning Activity 6.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Samantha has twice as many dresses as she has shirts and three times as many shirts as she has pants. If Samantha has 12 shirts, how many pairs of pants does she have?
2. Multiply: $(x + 2)(x + 3)$.
3. Write the following as a function: $y - x = 25$.
4. Convert: 1 foot^2 = __________ inches^2.
5. The slope of a line is $\frac{4}{5}$. One point on the line is (2, 3). What are the coordinates of another point on this line?
6. Heather is packing her suitcase for Europe. If the dimensions of the suitcase are 100 cm by 80 cm by 25 cm, what is the volume of the suitcase in metres?
7. Team A has won 5 out of their last 9 games. Team B has won 4 out of their last 7 games. Which team has won a greater percentage of their games?
8. You and your friend want to split the bill for dinner. The total is $35.00 for the meal. How much will you pay for dinner?

Answers:

1. 4 pairs of pants $(12 \div 3)$
2. $x^2 + 5x + 6$ (When the two binomials do not have coefficients, the product's middle term coefficient is the sum of the constants $(2 + 3)$ and the last term of the product is the product of the constants (2×3).)
3. $f(x) = x + 25$
4. 144 inches^2 (1 foot = 12 inches, 1 foot^2 = (12 inches)^2)
5. There are many answers. (7, 7) or (−3, −1) are two.
   Slope is $\frac{\text{rise}}{\text{run}}$ so $(2 + 5, 3 + 4)$ is another point on the line. So is $(2 - 5, 3 - 4)$.
6. 20 m^2 ($100 \times 80 = 8000$ cm^2, $8000 \times 25 = (8 \times 25) \times 1000 = 200 \times 1000 =$ 200 000 cm^3. 1 m^3 = 10 000 cm^3 so 200 000 cm^3 = 20 m^3)
7. Team B has the greater percentage. $\left(\frac{5}{9} = \frac{35}{63} \text{ vs } \frac{4}{7} = \frac{36}{63}\right)$
8. $17.50 ($35 is odd, so it is not divisible by 2. $35 = 34 + 1$ so $34 \div 2 = 17$ and $1 \div 2 = 0.50$. The total you pay is $17 + 0.50 =$ $17.50.)

Part B: Factoring Binomials and Trinomials

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Sketch a rectangular arrangement of tiles to represent the following expressions. Use the tiles to determine the factors of the polynomial. Verify your answer by multiplying the factors.

   a) $8x^2 + 12x$ b) $2x^2 + 6x$

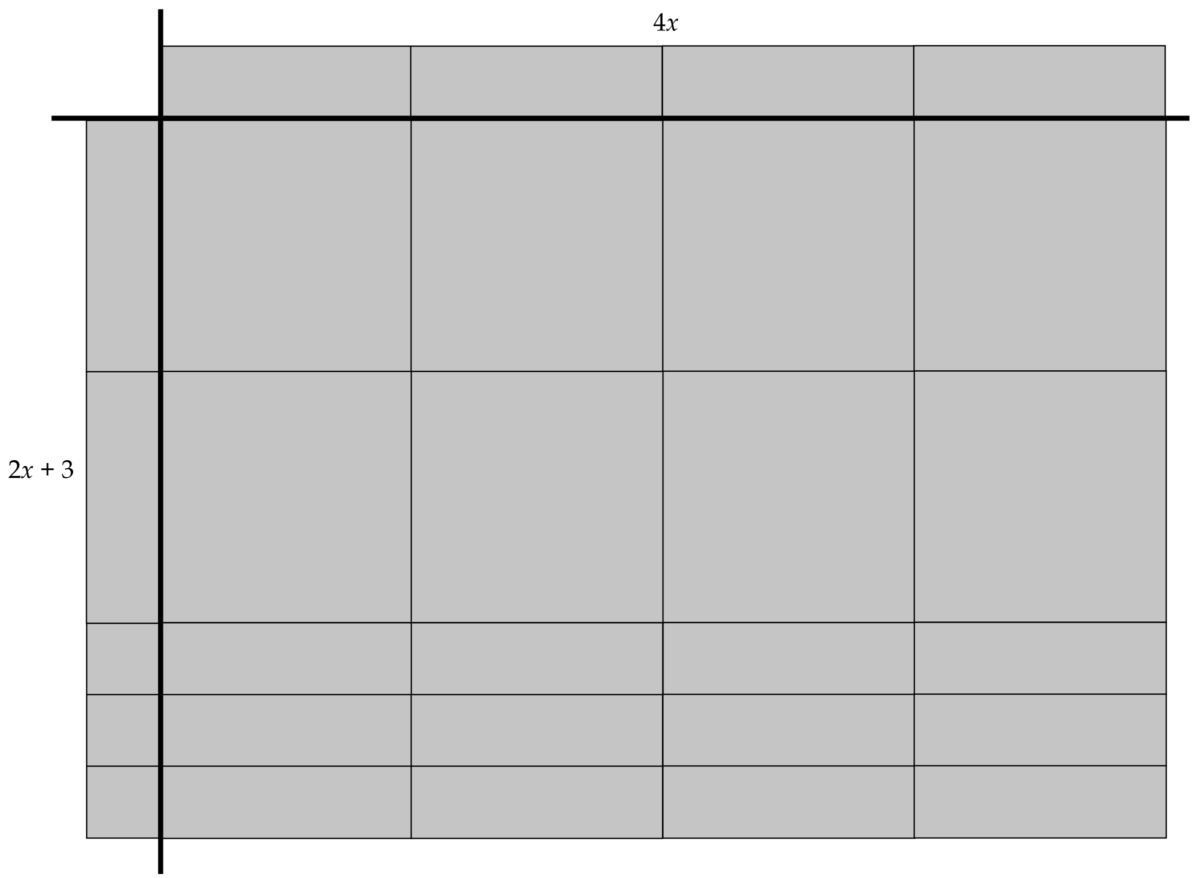
   c) $12x^2 + 3x$ d) $x^2 + x$

   e) $x^2 + 12x + 36$ f) $x^2 + 7x + 10$

   g) $x^2 + 7x + 12$ h) $x^2 + 7x + 6$

   Answers:

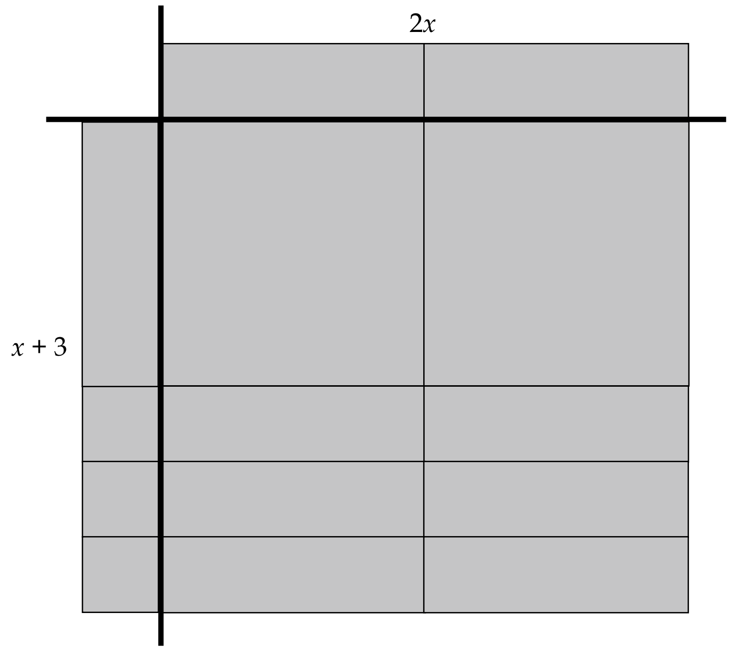
   a) $8x^2 + 12x$



Verify:

$4x(2x + 3) = 8x^2 + 12x$

b)

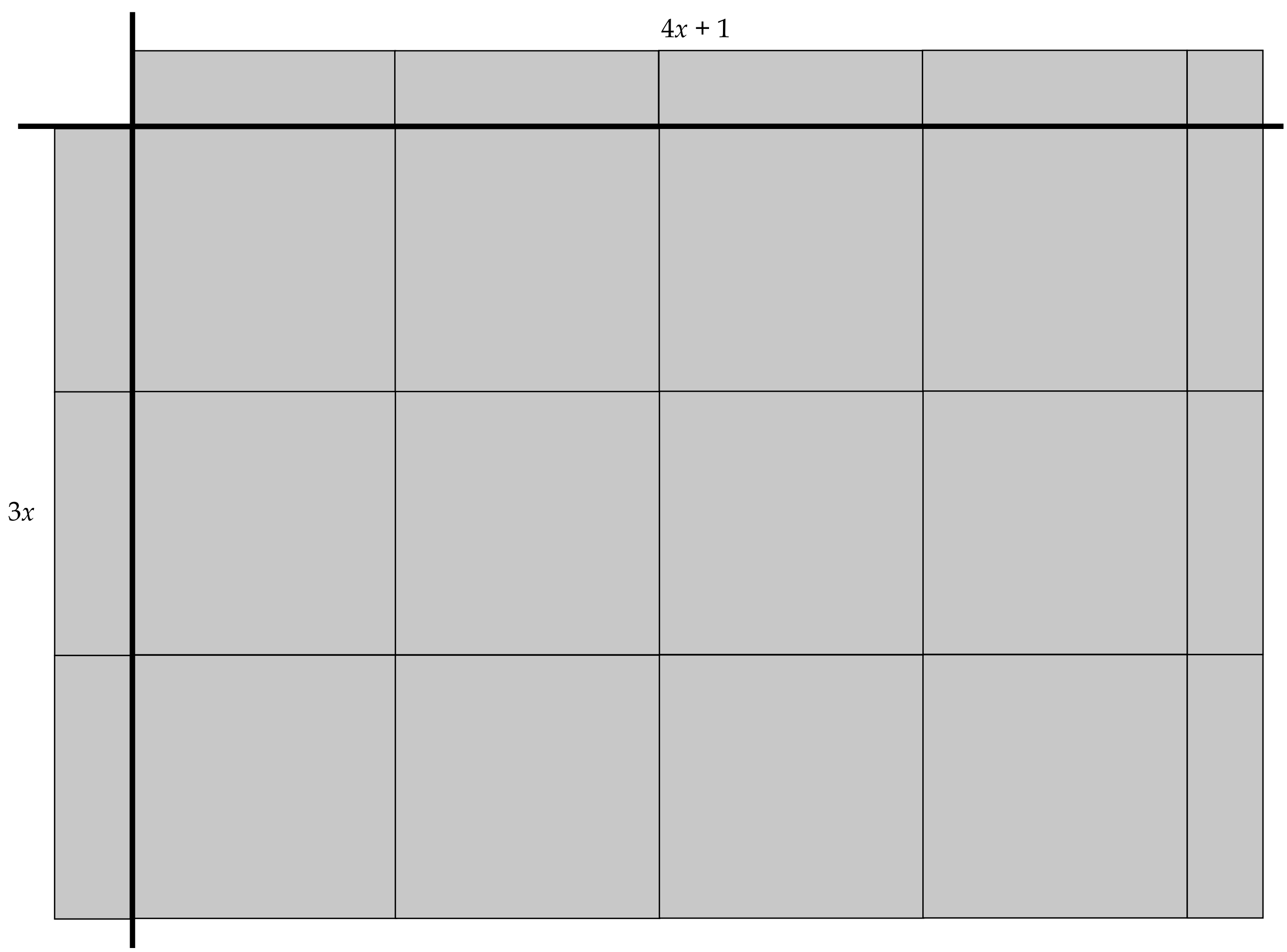


$2x^2 + 6x$

Verify:

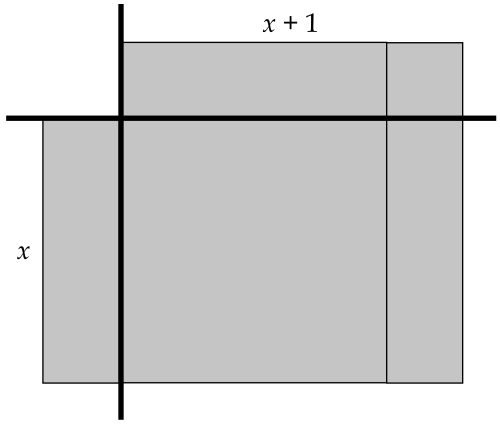
$2x(x + 3) = 2x^2 + 6x$

c) $12x^2 + 3x$



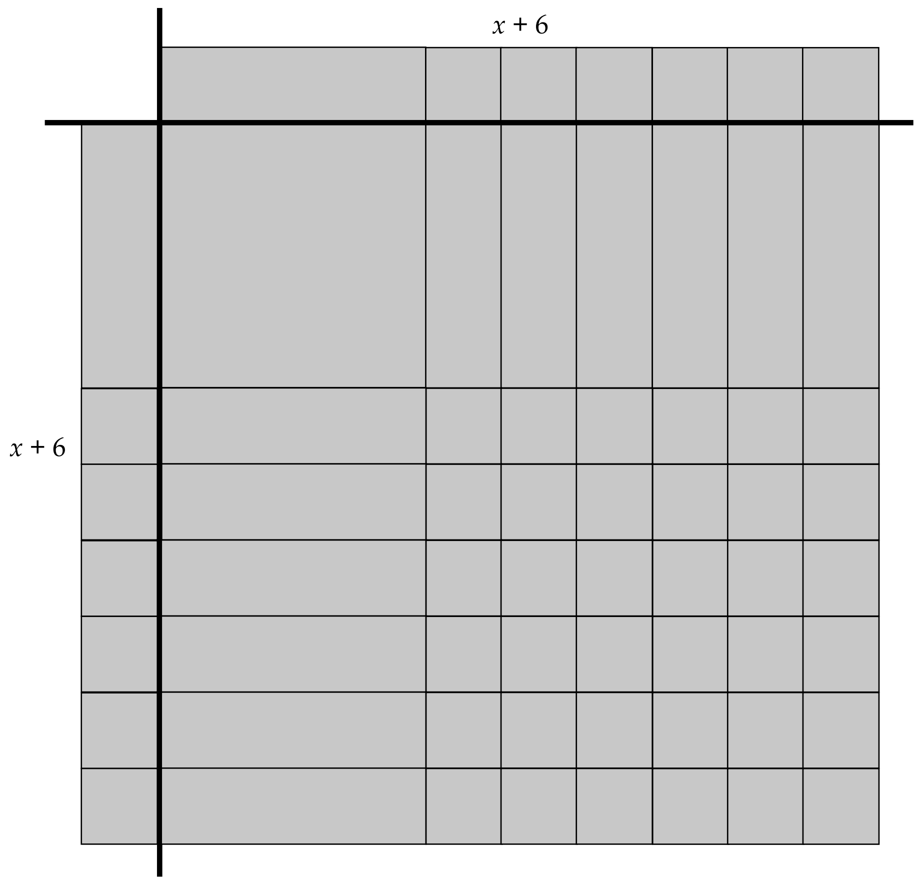
Verify: $3x(4x + 1) = 12x^2 + 3x$

d) $x^2 + x$



Verify: $x(x + 1) = x^2 + x$

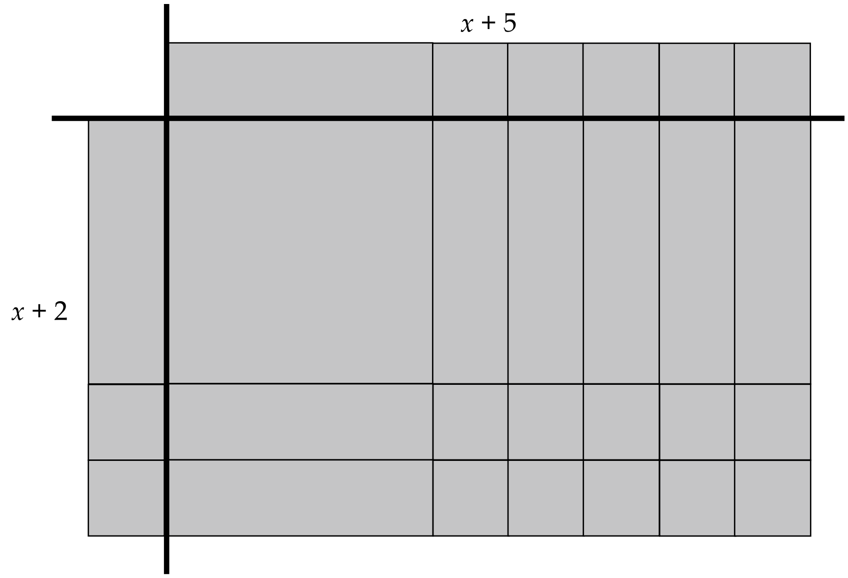
e) $x^2 + 12x + 36$



Verify: $(x + 6)(x + 6) = x^2 + 6x + 6x + 36$

$= x^2 + 12x + 36$

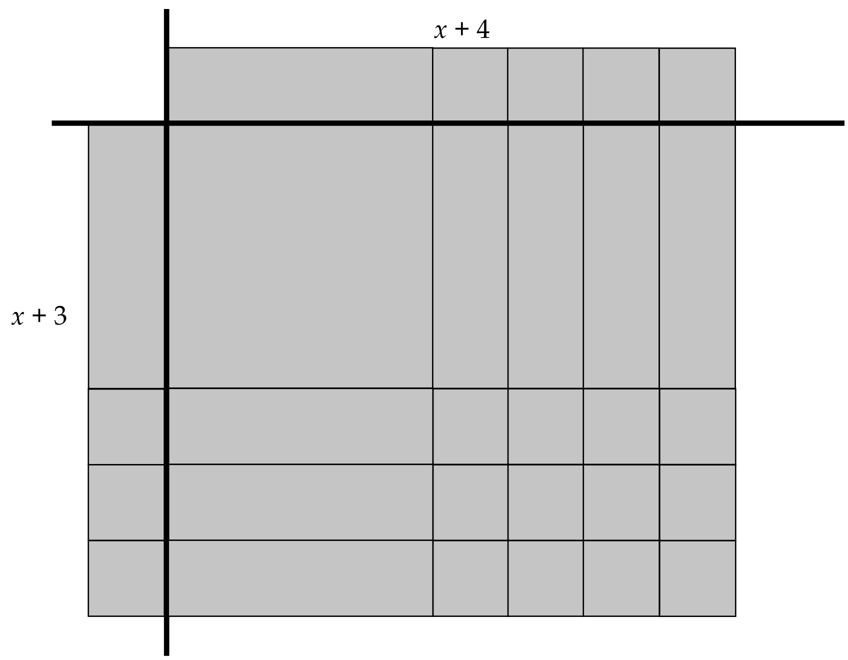
f) $x^2 + 7x + 10$



Verify: $(x + 5)(x + 2) = x^2 + 2x + 5x + 10$

$= x^2 + 7x + 10$

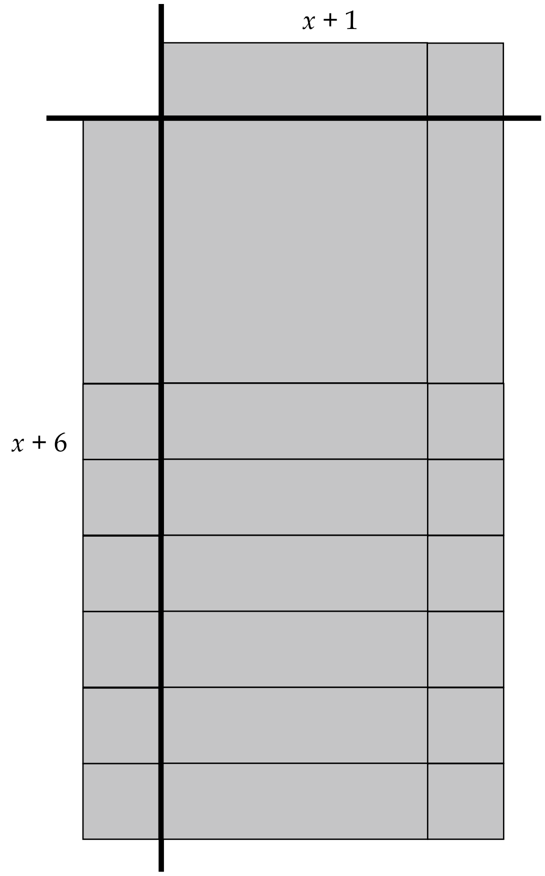
g) $x^2 + 7x + 12$



Verify: $(x + 4)(x + 3) = x^2 + 3x + 4x + 12$

$= x^2 + 7x + 12$

h) $x^2 + 7x + 6$



Verify: $(x + 6)(x + 1) = x^2 + x + 6x + 6$
$= x^2 + 7x + 6$

2. Complete the following chart to identify patterns in factoring the given trinomials.

Answers:

Trinomial	Coefficient of x^2	Coefficient of x	Constant	Binomial Factors	Sum of Constants in Binomials	Product of Constants in Binomials
$x^2 + 5x + 6$	1	5	6	$(x + 2)(x + 3)$	$2 + 3 = 5$	$(2)(3) = 6$
$x^2 + 8x + 12$	1	8	12	$(x + 2)(x + 6)$	$2 + 6 = 8$	$(2)(6) = 12$
$x^2 + 8x + 15$	1	8	15	$(x + 5)(x + 3)$	$5 + 3 = 8$	$(5)(3) = 15$
$x^2 + x - 2$	1	1	-2	$(x + 2)(x - 1)$	$(-1) + 2 = 1$	$(2)(-1) = -2$
$x^2 - 2x - 8$	1	-2	-8	$(x - 4)(x + 2)$	$(-4) + 2 = -2$	$(-4)(2) = -8$

3. a) Given a trinomial in the form $ax^2 + bx + c$, where a is the coefficient of x^2, b is the coefficient of x, and c is a constant, complete the following chart.

Answers:

Trinomial $ax^2 + bx + c$	***a***	***b***	***c***	**List all factor pairs of *c*. State what signs are needed to produce the sign of the product**	**State factor pair of *c* that gives you the sum of *b*, including the signs**	**State the binomial factors of the trinomial**
$x^2 + 4x - 21$	1	4	−21	1,21; 3,7 signs: (+) (−)	−3, 7	$(x - 3)(x + 7)$
$x^2 + 9x + 20$	1	9	20	1,20; 2,10; 4,5 signs: (+) (+)	4, 5	$(x + 4)(x + 5)$
$x^2 + 2x - 48$	1	2	−48	1,48; 2,24; 3,16; 4,12; 6,8 signs: (+) (−)	−6, 8	$(x - 6)(x + 8)$
$x^2 - 11x + 28$	1	−11	28	1,28; 2,14; 4,7 signs: (−)(−)	−7, −4	$(x - 4)(x - 7)$

b) Write a summary statement describing the relationship between the constants in the binomial factors and b and c in the trinomial (the coefficient of x and the constant in the trinomial).

Answer:

The constants in the binomials are the factor pair of c that gives you the sum of b. The two values must multiply to c and add to b.

4. a) Verify the binomial factors in the last column of the chart in 3(a) above by applying the distributive property of multiplication to each.

Answer:

$(x - 3)(x + 7)$
$= x^2 + 7x - 3x - 21$
$= x^2 + 4x - 21$

$(x + 4)(x + 5)$
$= x^2 + 5x + 4x + 20$
$= x^2 + 9x + 20$

$(x - 6)(x + 8)$
$= x^2 + 8x - 6x - 48$
$= x^2 + 2x - 48$

$(x - 4)(x - 7)$
$= x^2 - 7x - 4x + 28$
$= x^2 - 11x + 28$

b) Explain, using examples, the relationship between multiplication and factoring.

Answer:

When you multiply two polynomials together, you end up with a product. The factors of that product are the two polynomials you multiplied. To factor a polynomial, you are doing the reverse of multiplying. You start with the product and determine what it takes to multiply to get that answer. With multiplying, you start with the factors and multiply them to determine the product. Factoring is the reverse of the distributive law.

factor × factor = product $(x + 2)(x - 7) = x^2 - 5x - 14$

product = factor × factor $x^2 - 5x - 14 = (x + 2)(x - 7)$

5. Factor the polynomials. Verify your answer using distributive multiplication.

a) $x^2 - 7x - 18$

b) $a^2 - 10a - 11$

c) $x^2 - 13x + 40$

d) $y^2 + 13y + 36$

e) $2x^2 + 8x - 64$

Hint: Find a common factor for all the coefficients and constants, then try to factor the trinomial that doesn't include the common factor (e.g., $3x^2 + 6x - 9 = (3)(x^2 + 2x - 3)\ldots$).

Answers:

a) $x^2 - 7x - 18 = (x - 9)(x + 2)$
Verify:
$(x - 9)(x + 2)$
$= x^2 + 2x - 9x - 18$
$= x^2 - 7x - 18$

b) $a^2 - 10a - 11 = (a - 11)(a + 1)$
Verify:
$(a - 11)(a + 1)$
$= a^2 + a - 11a - 11$
$= a^2 - 10a - 11$

Note: Any letter may be used to represent the variable.

c) $x^2 - 13x + 40 = (x - 8)(x - 5)$
Verify:
$(x - 8)(x - 5)$
$= x^2 - 5x - 8x + 40$
$= x^2 - 13x + 40$

d) $y^2 + 13y + 36 = (y + 4)(y + 9)$
Verify:
$(y + 4)(y + 9)$
$= y^2 + 9y + 4y + 36$
$= y^2 - 13y + 36$

e) $2x^2 + 8x - 64 = 2(x^2 + 4x - 32) = 2(x - 4)(x + 8)$
Always factor out common factors.
Verify:
$2(x - 4)(x + 8)$
$= 2(x^2 + 8x - 4x - 32)$
$= 2(x^2 + 4x - 32)$
$= 2x^2 + 8x - 64$

6. Identify and explain errors in the following factorizations. Show the correct solution.

   a) $x^2 - 5x - 6 = (x - 3)(x + 2)$

      Answer:

      The constants in the binomial, -3 and 2, give you the product of -6 but do not combine to -5. The correct factors are $(x - 6)(x + 1)$.

   b) $18y^2 - 12y = 2(9y^2 - 6y)$

      Answer:

      2 is not the greatest common factor of the terms in the binomial. The correct factorization is $6y(3y - 2)$.

   c) $3x^2 - 3x - 6 = (3x - 6)(x + 1)$

      Answer:

      If you were to multiply the factors given, you would get the trinomial given. However, the common factor of 3 would not be factored out. The correct answer is $3(x - 2)(x + 1)$.

   d) $x^2 + 20x + 9 = (x + 4)(x + 5)$

      Answer:

      The constants in the binomials give you the product of b and the sum of c, rather than the product of c (constant) and sum of b (coefficient of x) in the trinomial. The factor pairs of 9 are 3, 3 and 9, 1. Neither of these pairs will work to combine to 20, so this trinomial does not have two binomial factors. It would be impossible to arrange $x^2 + 20x + 9$ tiles into a rectangular shape.

7. Factor the given expressions.

Note: If you are feeling confident that you are able to factor binomials and trinomials, only do (a) to (d) and (i) to (l). If you want more practice, work through (a) to (p). If you are struggling with these concepts, do not forget that you can contact your tutor/marker, and you can ask your learning partner a question or work with him or her.

a) $12m - 24p$ b) $a - ar^3y$

c) $2a^2 - 12ab + 14ac$ d) $6x^2 - 18z^6y - 6ax^3z$

e) $3r^2 - 15rh$ f) $4n^3 - 4n^2$

g) $32x^2y + 4x^3y$ h) $3mn + 6n^2m^2$

i) $x^2 - 7x + 12$ j) $x^2 - 10x - 24$

k) $x^2 + 25x + 24$ l) $x^2 - 4x - 12$

m) $x^2 + x - 72$ n) $c^2 - 4c - 12$

o) $4 - 5c + c^2$ p) $x^2 - x - 6$

Answers:

a) $12(m - 2p)$ b) $a(1 - r^3y)$

c) $2a(a - 6b + 7c)$ d) $6(x^2 - 3z^6y - a\mathrm{x}^3z)$

e) $3r(r - 5h)$ f) $4n^2(n - 1)$

g) $4x^3y(8x + 1)$ h) $3mn(1 + 2mn)$

i) $(x - 4)(x - 3)$ j) $(x + 2)(x - 12)$

k) $(x + 1)(x + 24)$ l) $(x - 6)(x + 2)$

m) $(x + 9)(x - 8)$ n) $(c - 6)(c + 2)$

o) $(4 - c)(1 - c)$ or $(c - 4)(c - 1)$ p) $(x - 3)(x + 2)$

Learning Activity 6.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. What two numbers have a product of -8 and a sum of -2?
2. There are 20 cheese sticks in a package. You only eat one each day, and only on weekdays (Monday to Friday). How many weeks will it take you to finish the whole package?
3. Solve: $4 - 6 + 2 \times (3 - 8)$.
4. You are playing baseball on a co-ed team. There are 16 people on your team. If you need $\frac{1}{4}$ of the team to be girls, how many have to be girls?
5. Multiply: $(x + 5)(x - 9)$.
6. The last time you counted, you had 54 DVDs. Your house was broken into last night and now you only have 32. How many DVDs were stolen?
7. Complete the pattern: -1, 0, ____, 0, ____, 0, 1.
8. You use your left hand to type more than your right when you have your hands on the keyboard properly. Typing the word *factor*, you use your left hand for 5 letters and your right hand for one. Write the fraction that represents how many times you use your right hand in total.

Answers:

1. $-4, 2$
2. 4 weeks (You eat one per day on the weekdays, so that is 5 per week; $20 \div 5 = 4$.)
3. -12 $(4 - 6 + 2 \times (-5) = 4 - 6 + (-10) = -2 - 10)$
4. 4 girls $\left(16 \times \frac{1}{4}\right)$
5. $x^2 - 4x - 45$ $(5 - 9 = -4, 5 \times (-9) = -45)$
6. 22 DVDs ($32 + 2 = 34$ (make the ones the same), $34 + 20 = 54$ (original total))
7. $1, -1$
8. $\frac{1}{6}$ (It is out of 6 because that is the total number of letters that you had to type. If you put 5, that would be the fraction comparing right to left.)

Part B: Factoring Trinomials when *a* ∈ I

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Factor completely.

   a) $2x^2 + 11x + 15$ b) $4x^2 - 21x + 5$

   c) $5x^2 - 4x - 12$ d) $6x^2 + 5x - 6$

   e) $2x^3 + x^2 - 15x$ f) $2x^3 - 22x^2 + 36x$

   g) $-12x^2 + 26x + 10$

   Answers:

   a) $2x^2 + 11x + 15$ $(2)(15) = 30$ 30: 1, 30; 2, 15; 3, 10; 5, 6 $(+)(+)$

   $(5) + (6) = 11$

   $= 2x^2 + 5x + 6x + 15$
   $= x(2x + 5) + 3(2x + 5)$
   $= (2x + 5)(x + 3)$

   b) $4x^2 - 21x + 5$ $(4)(5) = 20$ 20: 1, 20; 2, 10; 4, 5 $(-)(-)$

   $(5) + (6) = 11$

   $= 4x^2 - 1x - 20x + 5$
   $= x(4x - 1) - 5(4x - 1)$
   $= (x - 5)(4x - 1)$

   c) $5x^2 - 4x - 12$ $(5)(-12) = -60$ 60: 1, 60; 2, 30; 3, 20; 4, 15; 5, 12; 6, 10 $(+)(-)$

   $(6) + (-10) = -4$

   $= 5x^2 - 10x + 6x - 12$
   $= 5x(x - 2) + 6(x - 2)$
   $= (x - 2)(5x + 6)$

d) $6x^2 + 5x - 6$ $\quad (6)(-6) = -36 \quad 36$: 1, 36 $\quad (+)(-)$

2, 18

3, 12

4, 9

6, 6

$(9) + (-4) = 5$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

e) $2x^3 + x^2 - 15x$ (Always remove common factors first.)

$= x(2x^2 + x - 15)$ $\quad (6)(-15) = -30 \quad 30$: 1, 30 $\quad (+)(-)$

2, 15

3, 10

5, 6

$(-5) + (6) = 1$

$= x(2x^2 - 5x + 6x - 15)$

$= x(x(2x - 5) + 3(2x - 5))$

$= x(2x - 5)(x + 3)$

f) $2x^3 - 22x^2 + 36x$ (Remove common factor.)

$= 2x(x^2 - 11x + 18)$ $\quad (1)(18) = 18 \quad 18$: 1, 18 $\quad (-)(-)$

2, 9

3, 6

$(-2) + (-9) = -11$

$= 2x(x^2 - 9x - 2x + 18)$

$= 2x(x(x - 9) - 2(x - 9))$

$= 2x(x - 9)(x - 2)$

g) $-12x^2 + 26x + 10$ (Remove common factor, including negative sign.)

$= -2(6x^2 - 13x - 5)$ $\quad (6)(-5) = -30 \quad 30$: 1, 30 $\quad (+)(-)$

2, 15

3. 10

5, 6

$(-15) + (2) = -13$

$= -2(6x^2 - 15x + 2x - 5)$

$= -2(3x(2x - 5) + 1(2x - 5))$

$= -2(2x - 5)(3x + 1)$

2. Factor each expression. (If you are feeling confident in your ability to factor trinomials with $a \in I$, you do not have to do these questions. If you want more practice, work through as many as you need.)

a) $2x^2 + 5x + 3$	b) $5x^2 + 6x + 1$
c) $5a^2 - 16a + 3$	d) $3y^2 + 4y + 1$
e) $24x^2 + 2x - 1$	f) $6y^2 + 20 + 23y$
g) $10 + y - 2y^2$	h) $60y^2 - 27y - 60$
i) $15x^2 + 37x + 20$	j) $15a^2 + 8a - 12$

Answers:

a) $(2x + 3)(x + 1)$	b) $(5x + 1)(x + 1)$
c) $(5a - 1)(a - 3)$	d) $(3y + 1)(y + 1)$
e) $(6x - 1)(4x + 1)$	f) $(2y + 5)(3y + 4)$
g) $(5 - 2y)(2 + y)$	h) $3(5y + 4)(4y - 5)$
i) $(5x + 4)(3x + 5)$	j) $(5a + 6)(3a - 2)$

3. For what integral values (whole numbers) of k can $4x^2 + kx + 3$ be factored? Write out all possible trinomials as a product of its factors.

Answer:

$4x^2 + kx + 3$

$(4)(3) = 12$ Signs must be (+)(+) Factors of 12: 1, 12
2, 6
3, 4

Possible combinations of the factor pairs when both are positive are 13, 8, and 7.

The possible values for k are 13, 8, and 7.

$4x^2 + 13x + 3$
$4x^2 + 8x + 3$
$4x^2 + 7x + 3$

The factor pairs of 4 are 4, 1 and 2, 2.

The factor pairs of 3 are 1, 3.

Possible factor pair combinations are

$(4x + 1)(1x + 3) = 4x^2 + 13x + 3$
$(4x + 3)(1x + 1) = 4x^2 + 7x + 3$
$(2x + 1)(2x + 3) = 4x^2 + 8x + 3$
$(2x + 3)(2x + 1) = 4x^2 + 8x + 3$

(Note that the last two result in the same trinomial.)

4. Fill in the space so that each trinomial is a perfect square trinomial.

   a) $4x^2 + \_\_\_\_ + 4$

   Answer:

   $4x^2 + 8x + 4$ $\quad (4)(4) = 16 \quad \sqrt{16} = 4 \quad 2 \times 4 = 8$

   b) $25x^2 + \_\_\_\_ + 9$

   Answer:

   $25x^2 + 30x + 9$ $\quad (25)(9) = 225 \quad \sqrt{225} = 15 \quad 2 \times 15 = 30$

   c) $x^2 + 14\,x + \_\_\_\_$

   Answer:

   $x^2 + 14x + 49$ $\quad 14 \div 2 = 7 \quad 7^2 = 49 \quad (1)(49) = 49$

5. Factor each perfect square trinomial.

   a) $x^2 - 8x + 16$

   Answer:

   $x^2 - 8x + 16 = (x - 4)^2$

   b) $4x^2 - 4x + 1$

   Answer:

   $4x^2 - 4x + 1 = (2x - 1)^2$ or $(-2x + 1)^2$

6. Identify and explain the errors in the factorization, and state the correct solution.

   $x^2 - 8x + 16 = (x + 4)^2$

   Answer:

   The sign in the middle of the binomial factor is incorrect. It should be the same as the sign of the coefficient x. The correct answer is $(x - 4)^2$.

Learning Activity 6.5

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Factor: $m^2 + 3m - 54$.
2. You know that your glass holds 1 cup of milk. Would this be a good referent to use to find out how much water your water bottle can hold?
3. Andy has been to the Great Pyramid of Egypt. He bought a miniature version of the pyramid, which was to scale. The ratio comparing the miniature and the real pyramid is 1 cm: 70 royal cubits (an ancient unit of measurement). If the height of the miniature is 4 cm, how tall is the real pyramid?
4. Is 3^{-6} rational or irrational?
5. What is the GCF of 34 and 17?
6. Simplify: $\left(2^2\right)^{\frac{-1}{5}}$.
7. Solve for n: $4n - 3 = 2 + 19$.
8. You have 2 older brothers and 3 older sisters. Your parents had 11 children. How many of your siblings are younger than you?

Answers:

1. $(m - 6)(m + 9)$
2. Yes (Be aware though that it would only be accurate to the nearest cup. If there is a half cup, you cannot be precise.)
3. 280 royal cubits (4×70)
4. Rational (It can be written as a fraction because of the negative exponent.)
5. 17
6. $\dfrac{1}{\sqrt[5]{4}}$
7. $n = 6$ ($4n - 3 = 21$; $4n = 24$; $n = 6$)
8. 5 younger siblings ($11 - (5 + 2) - 1$ [you])

Part B: Difference of Squares and Module Review

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Why does applying the distributive property to the factors of a difference of square result in a binomial? Give an example.

   Answer:

   In a difference of squares, the factors are the same except for opposite signs. When you apply the distributive property, the coefficients of x are opposite and cancel to zero, leaving $ax^2 + 0x - c$, which simplifies to the binomial $ax^2 - c$.

   Example:

   (Your example may be different but the pattern will be the same.)

$64x^2 - 100$	
$= (8x + 10)(8x - 10)$	Factors
$= 64x^2 - 80x + 80x - 100$	Apply the distributive property and simplify by cancelling the opposite terms.
$= 64x^2 - 100$	

2. Given the polynomial $ax^2 - c$ where a and c are perfect square numbers, write its binomial factors.

   Answer:

   The factors of $ax^2 - c$ are written as $\left(\sqrt{ax^2}+\sqrt{c}\right)\left(\sqrt{ax^2}-\sqrt{c}\right)$.

3. Factor completely.

   a) $x^2 - 36$

   Answer:

   $x^2 - 36 = (x - 6)(x + 6)$

   b) $9y^2 - 49$

   Answer:

   $9y^2 - 49 = (3y - 7)(3y + 7)$

c) $x^2 - 256$

Answer:

$x^2 - 256 = (x - 16)(x + 16)$

d) $2m^2n - 2n$

Answer:

$2m^2n - 2n = 2n(m^2 - 1) = 2n(m - 1)(m + 1)$

4. Use your factoring strategies to factor the following polynomials.

a) $3mn - 6np$

b) $a(b + 3) + c(b + 3)$

c) $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

d) $x^2 - 11x + 28$

e) $x^2 - 3x - 28$

f) $4x^4 - 20x^3 + 24x^2$

g) $16x^2 - 24x + 9$

h) $8x^2 - 40x + 50$

i) $x^2 - 81$

j) $4y^2 - 9$

k) $20x^2y - 5y$

Answers:

a) $3mn - 6np$
$= 3n(m - 2p)$

b) $a(b + 3) + c(b + 3)$
$= (b + 3)(a + c)$

c) $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi\left(R^3 - r^3\right)$

d) $x^2 - 11x + 28$
$= (x - 7)(x - 4)$

e) $x^2 - 3x - 28$
$= (x - 7)(x + 4)$

f) $4x^4 - 20x^3 + 24x^2$
$= 4x^2(x^2 - 5x + 6)$
$= 4x^2(x - 3)(x - 2)$

g) $16x^2 - 24x + 9$ $\quad 16 \times 9 = 144 \quad (-12)(-12) = 144 \quad (-12) + (-12) = -24$
$= 16x^2 - 12x - 12x + 9$
$= 4x(4x - 3) - 3(4x - 3)$
$= (4x - 3)(4x - 3)$
$= (4x - 3)^2$

h) $8x^2 - 40x + 50$
$= 2(4x^2 - 20x + 25)$
$= 2(2x - 5)^2$

i) $x^2 - 81$
$= (x - 9)(x + 9)$

j) $4y^2 - 9$
$= (2y - 3)(2y + 3)$

k) $20x^2y - 5y$
$= 5y(4x^2 - 1)$
$= 5y(2x - 1)(2x + 1)$

5. If you are feeling confident in your ability to factor trinomials with $a \in \mathrm{I}$ and difference of squares, you do not have to do these questions. If you want more practice, work through as many as you need.

a) $x^2 - 16$
b) $36t^2 - 1$
c) $4a^2 - b^2$
d) $8c^2 - 72$
e) $81 - (x + 7)^2$
f) $(x - 1)^2 - (x + 1)^2$
g) $x^8 - y^{12}$
h) $4x^2 - 1$
i) $4m^2 - 25y^4$
j) $121x^2 - 196y^2$

Answers:

a) $(x + 4)(x - 4)$
b) $(6t + 1)(6t - 1)$
c) $(2a + b)(2a - b)$
d) $8(c - 3)(c + 3)$
e) $(2 - x)(16 + x)$
f) $((x - 1) - (x + 1))((x - 1) + (x + 1)) = -4x$
g) $(x^4 - y^6)(x^4 + y^6)$
$= (x^2 - y^3)(x^2 + y^3)(x^4 + y^6)$
h) $(2x + 1)(2x - 1)$
i) $(2m - 5y^2)(2m + 5y^2)$
j) $(11x - 14y)(11x + 14y)$

6. You are now three-quarters of the way through this course. Take a few minutes now to look back at Module 1 and the goals you set for yourself at the beginning of this course, as well as the revised version written after Module 4. Which goals have you accomplished or completed by now? Are you progressing towards achieving your other goals in a timely manner? How can you modify or adapt your steps to ensure success? How will you celebrate your achievements and continue to strive for the rest of your goals! You are almost done!

Answer:

List your successes, modifications, and new steps (or new goals) in a place where you will be reminded and motivated to bring them to completion. Share your successes with someone. Ask them to keep you accountable as you work to accomplish your other goals.

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 7
Coordinate Geometry

Module 7: Coordinate Geometry

Introduction

The game of Battleship® involves strategizing to sink your opponent's fleet of ships by calling out coordinates. The game is based on a modified Cartesian grid, using letters and positive integers along the x- and y-axis. This module will make extensive use of the Cartesian grid—you will plot points, graph equations, and calculate distances and midpoints using ordered pairs. You will make use of some of the skills and concepts you learned in previous modules, and write linear equations in different forms, and use technology to help you find a way to mathematically describe the relationship between the x- and y-variables.

Assignments in Module 7

When you complete Module 8, you will submit your Module 7 assignments, along with your Module 8 assignments, to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 7.1	Distance and Midpoint
2	Assignment 7.2	Linear Relations Formulas
3	Assignment 7.3	Writing Linear Equations Based on Different Information
4	Assignment 7.4	Line of Best Fit and Correlation

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions that follows to help you with preparing your resource sheet for the material in Module 7. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 1 to 8.

Resource Sheet for Module 7

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Lesson 1: Distance and Midpoint between Two Points

Lesson Focus

In this lesson, you will

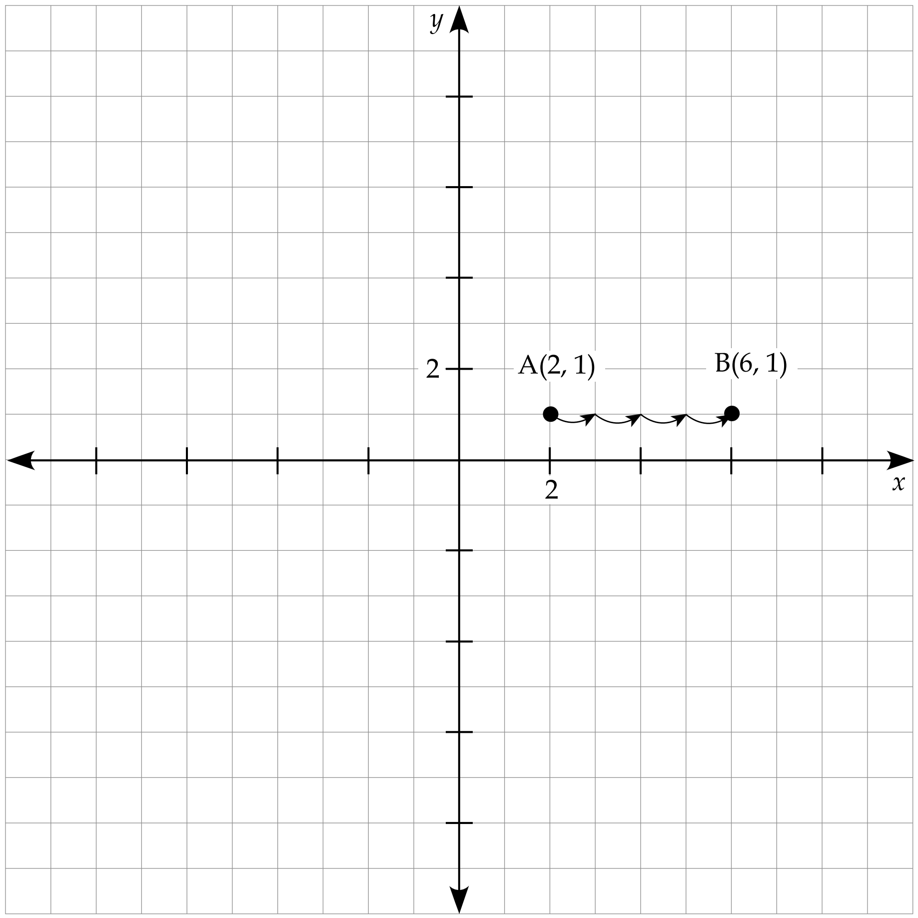
- ❑ determine the distance between two points on a Cartesian plane using a variety of strategies
- ❑ determine the midpoint of a line segment, given the endpoints of the segment, using a variety of strategies
- ❑ determine the endpoint of a line segment, given the other endpoint and the midpoint, using a variety of strategies
- ❑ solve contextual problems involving distance between points or midpoint of a line segment

Lesson Introduction

If you are walking home from school, planning a road trip vacation across the country, creating a work of art using perspective, or playing a strategy game, knowing the distance between two points and the location of the midpoint is useful information. This lesson will show you a variety of ways to do just that when working with line segments on a coordinate grid and ordered pairs.

The Distance between Two Points

Plot A(2, 1) and B(6, 1) and determine the distance between these two points.



These points can be joined in a horizontal line and the distance between them can be counted using the grid lines. It is 4 units long.

Notice the x-coordinates of points A and B.

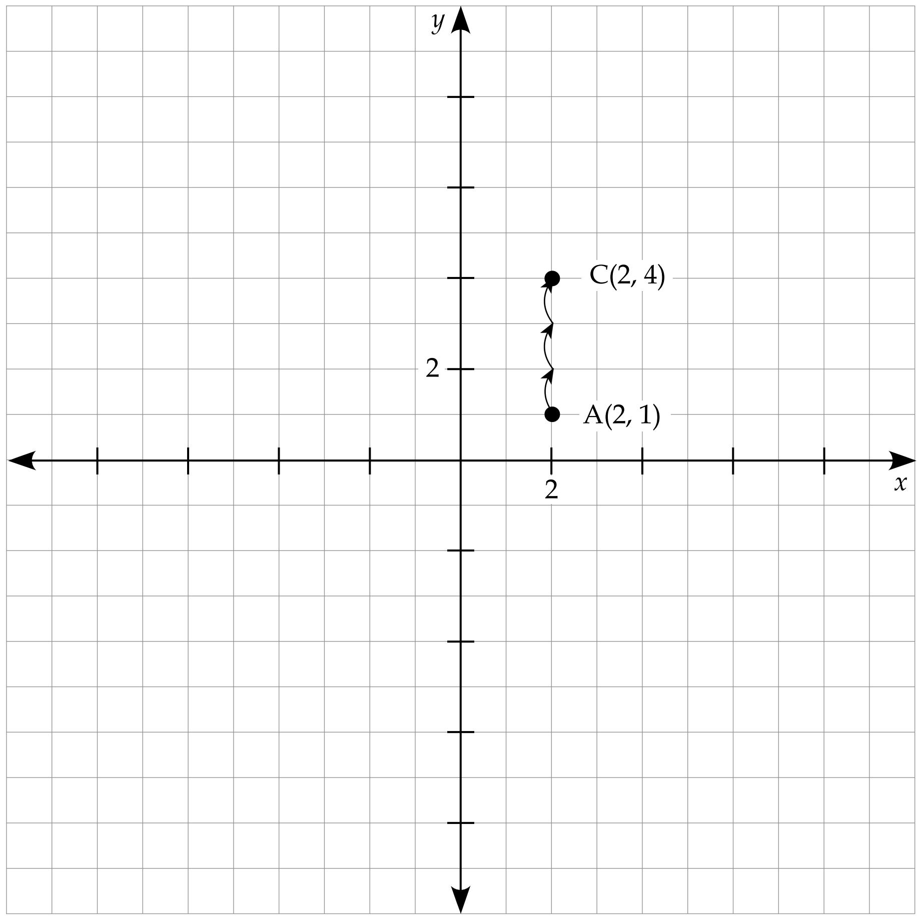
The difference between them is also equal to 4.

Label the points A(2, 1) and B(6, 1) as

$$\begin{array}{cc} \downarrow & \downarrow \\ (x_1, y_1) \text{ and } & (x_2, y_2) \end{array}$$

$x_2 - x_1 = 6 - 2 = 4$

Plot C(2, 4) and determine the distance between points A and C.



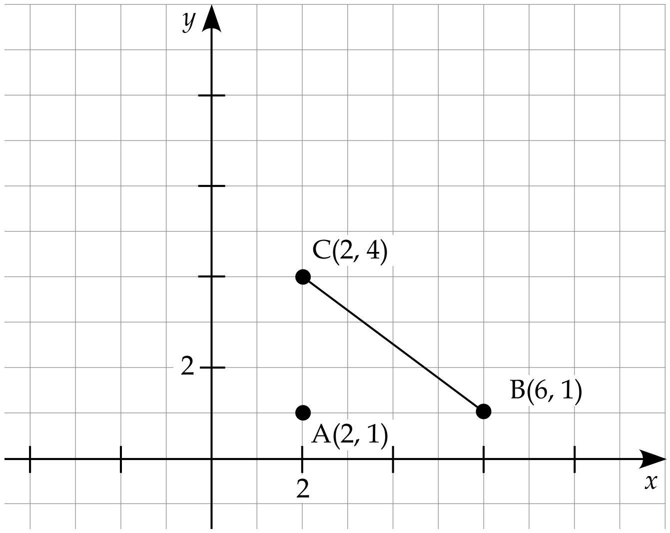
These two points can be joined with a vertical line. The distance between them can be determined by counting the grid lines. It is 3 units long. The length can also be calculated by finding the difference between the y-coordinates of these two points.

Label the points A(2, 1) and C(2, 4) as

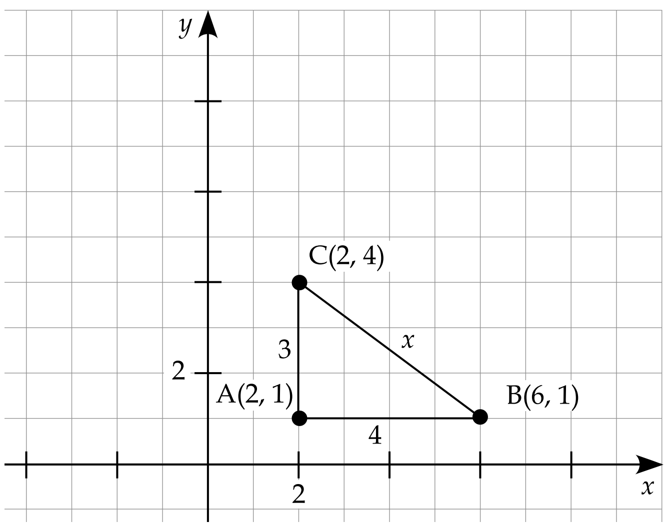
$$\begin{array}{ccc} A(2, 1) & & C(2, 4) \\ \downarrow & & \downarrow \\ (x_1, y_1) & \text{and} & (x_2, y_2) \end{array}$$

$y_2 - y_1 = 4 - 1 = 3$

Now determine the distance between the points B and C.



The line joining these two points is a straight line, but it is diagonal so its length cannot be determined by simply counting the number of spaces along the grid.



If you add line segments AB and AC to the diagram, a right triangle is formed! The length of BC can be calculated using the Pythagorean Theorem (Remember from Module 4 that $a^2 + b^2 = c^2$ where c is the hypotenuse and a and b are perpendicular side lengths in a right triangle).

$$3^2 + 4^2 = x^2$$
$$9 + 16 = x^2$$
$$25 = x^2$$
$$x = \pm 5$$

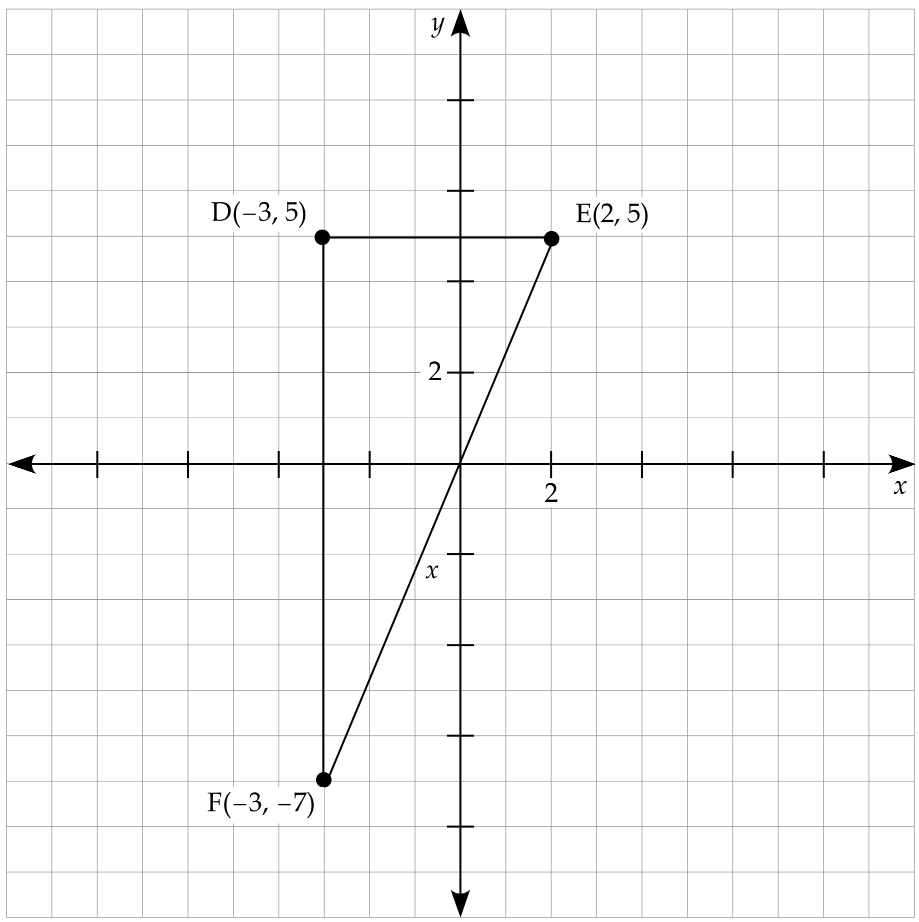
Since x is a side length it must be a positive value.

$$x = 5$$

Example 1

Plot the points D(−3, 5), E(2, 5), and F(−3, −7), and calculate the perimeter of the right triangle.

Solution:



To calculate the distance from D to E, count the spaces along the grid or find the difference in the x-coordinates.

D(−3, 5), E(2, 5)

$(x_1, y_1) \quad (x_2, y_2)$

$x_2 - x_1 = 2 - (-3) = 5$

To calculate the distance from F to D, count the spaces along the grid or find the difference in the y-coordinates.

F(−3, −7), D(−3, 5)

$(x_1, y_1) \quad (x_2, y_2)$

$y_2 - y_1 = 5 - (-7) = 12$

Calculate the distance from E to F by using the Pythagorean Theorem.

$$5^2 + 12^2 = x^2$$
$$25 + 144 = x^2$$
$$169 = x^2$$
$$x = \pm 13$$

Since x is a side length, it must be a positive value.

$$x = 13$$

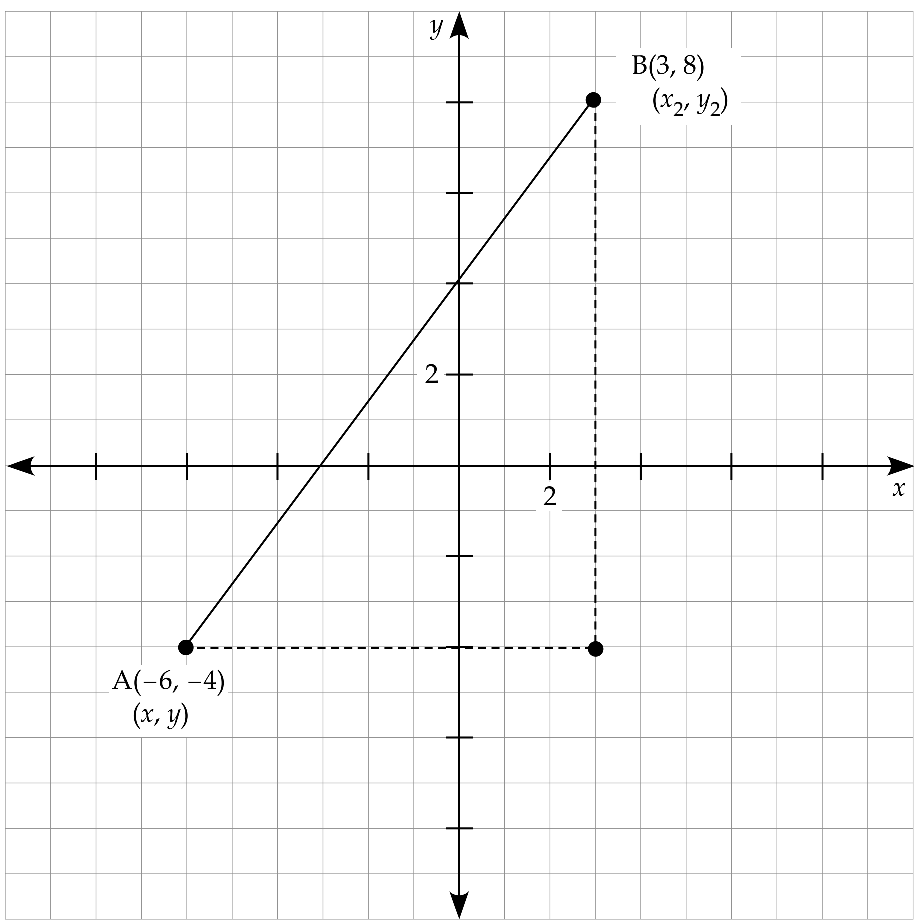
The perimeter of the triangle is the sum of all 3 side lengths.

$$5 + 12 + 13 = 30$$

The perimeter is 30 units long.

Example 2

As you may have guessed from the example above, we can calculate the length of line segment $\overline{AB}$ if A(−6, −4) and B(3, 8) without finding the third point of the triangle.



vertical distance:

$= y_2 - y_1$

$= 8 - (-4) = 12$

horizontal distance:

$= x_2 - x_1$

$= 3 - (-6) = 9$

$$\left(\overline{AB}\right)^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2$$

$$\left(\overline{AB}\right)^2 = 9^2 + 12^2$$

$$\left(\overline{AB}\right)^2 = 81 + 144$$

$$\left(\overline{AB}\right)^2 = 225$$

$$\sqrt{\left(\overline{AB}\right)} = \sqrt{225}$$

$$\overline{AB} = 15$$

You just need to find the horizontal distance (the difference between the x-coordinates) and the vertical difference (the difference between the y-coordinates) of the points A and B and plug that into the Pythagorean Theorem.

In general, given points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the length of PQ is represented by a formula that is simply a restatement of the Pythagorean Theorem:

(length of $\overline{PQ}$)^2 = (difference in x-values)^2 + (difference in y-values)^2

$$\left(\overline{PQ}\right)^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)$$

$$PQ = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)}$$

The **distance formula**, $d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$, can be used to calculate the length of a line segment between two points, (x_1, y_1) and (x_2, y_2)

It will be helpful to have this formula on your Resource Sheet.

Example 3

Use the distance formula to calculate the length of $\overline{MN}$, if M(52, 225), N(−392, −108).

Solution:

Label the points as (x_1, y_1) and (x_2, y_2).

$$\begin{array}{cc} \downarrow & \downarrow \\ M(52, 225) & N(-392, -108) \end{array}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-392 - 52)^2 + (-108 - 225)^2}$$

$$d = \sqrt{(-444)^2 + (-333)^2}$$

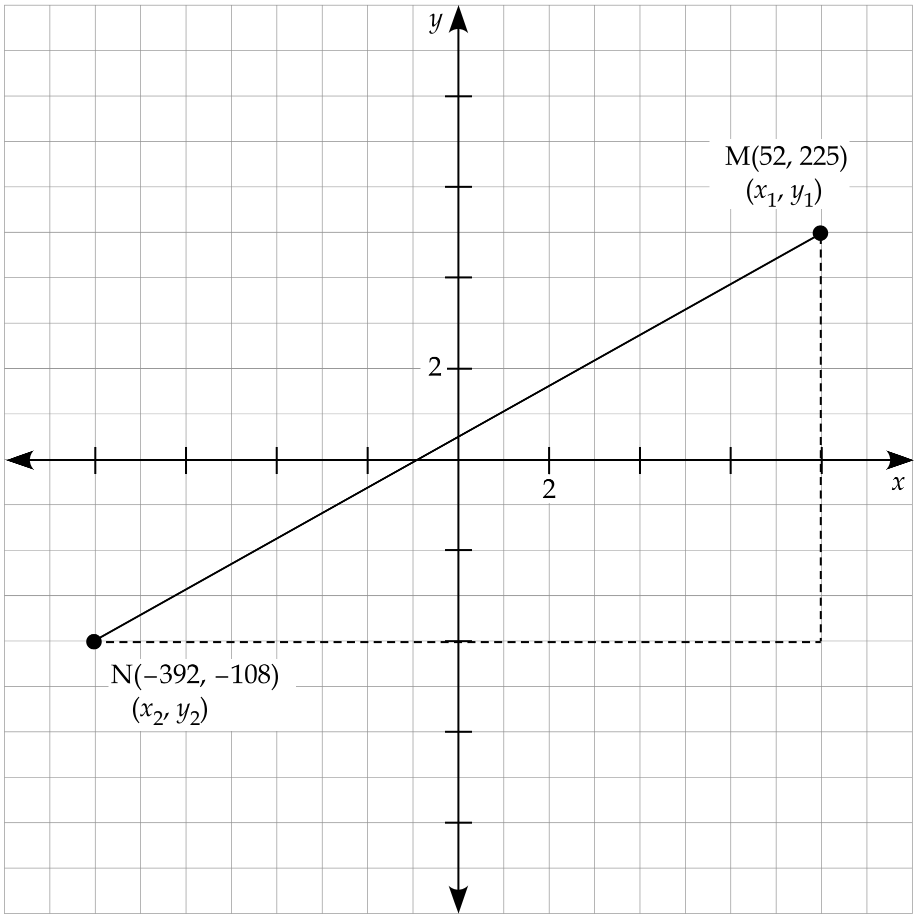
$$d = \sqrt{197\ 136 + 110\ 889}$$

$$d = \sqrt{308\ 025}$$

$$d = 555$$

The distance between points M and N (or the length of line segment $\overline{MN}$) is 555 units.

You can check your answer by sketching a diagram of a right triangle and verifying the solution with the Pythagorean Theorem.



vertical distance:

$= y_2 - y_1$

$= -108 - 225 = -333$

horizontal distance:

$= x_2 - x_1$

$= -392 - 52 = -444$

Remember, if you know the triangle is a right triangle, then $a^2 + b^2 = c^2$.

$$\left(\overline{MN}\right)^2 = (-444)^2 + (-333)^2$$

$$\left(\overline{MN}\right)^2 = 308025$$

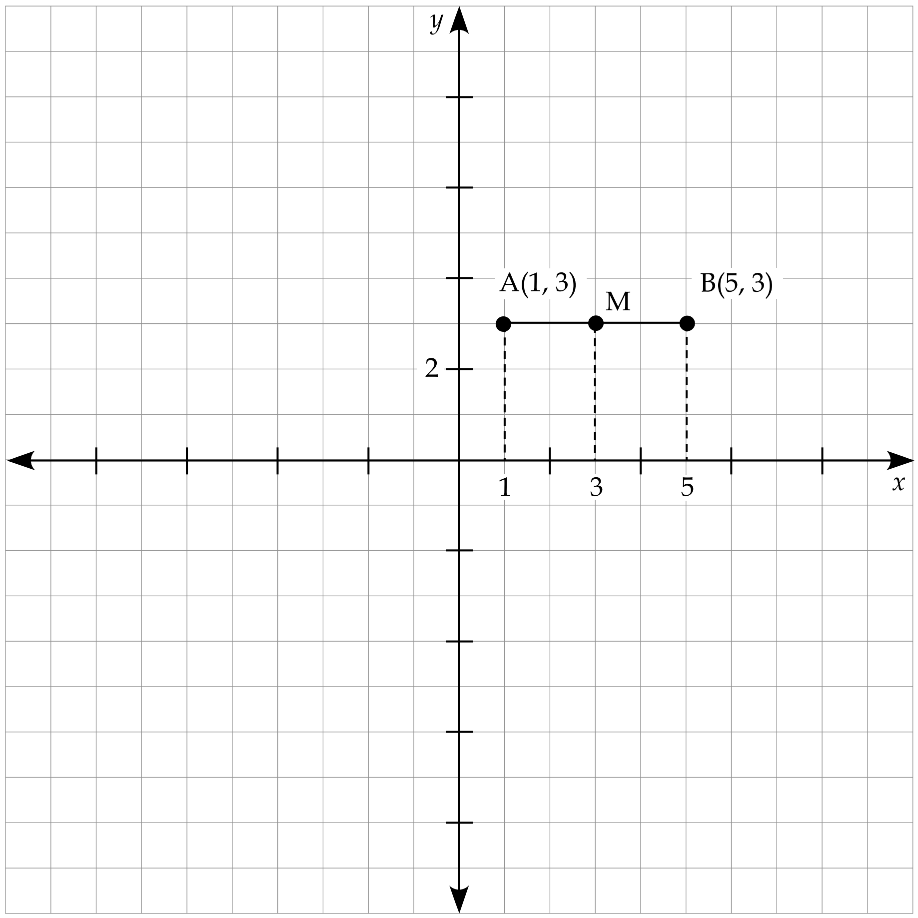
$$\overline{MN} = 555$$

$$555^2 = (-444)^2 + (-333)^2$$

The Midpoint of a Line Segment

The **midpoint of a line segment** is the point on the line that is halfway between the two end points, so it is equal distance from both.

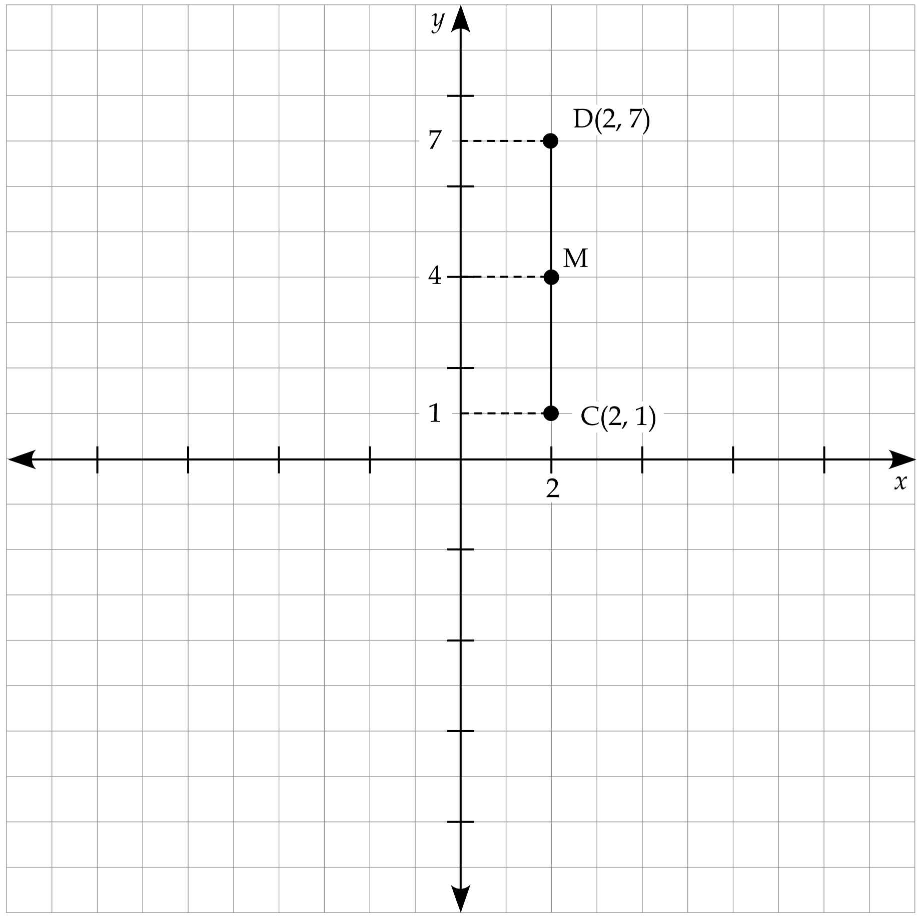
To find the midpoint, M, of a horizontal line segment $\overline{AB}$, find the average (or mean) of the *x*-coordinates of the endpoints.



$$\frac{1+5}{2}=3$$

The coordinates of M are (3, 3).

Similarly, the midpoint, M, of a vertical line segment $\overline{CD}$ will be at the average of the y-coordinates of the endpoints.



$$\frac{7+1}{2}=4$$

The coordinates of M are (2, 4).

In general, the midpoint of a line segment joining points $P(x_1, y_1)$ and $Q(x_2, y_2)$ can be represented as

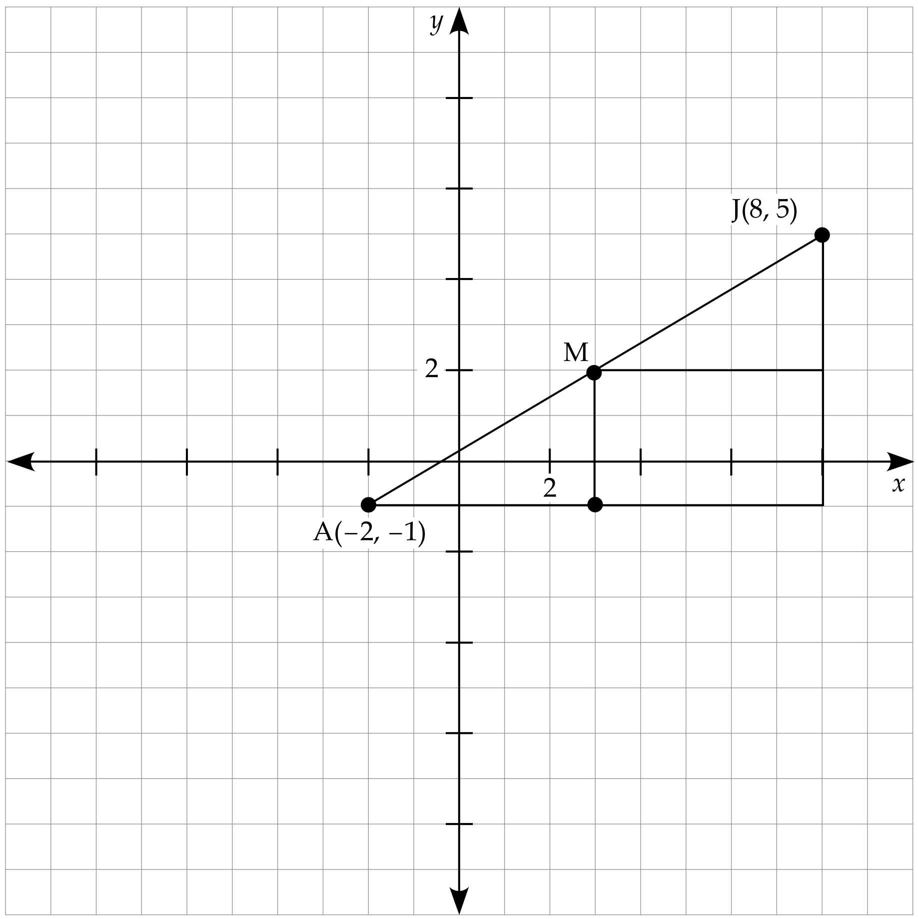
M = (average of the x-coordinates, average of the y-coordinates)

$$M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Example 4

Find the midpoint of line segment $\overline{JK}$ if J(8, 5) and K(–2, –1).

Solution:



J(8, 5) K(−2, −1)

(x_1, y_1) (x_2, y_2)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{8 + (-2)}{2}, \frac{5 + (-1)}{2}\right)$$

$$M = \left(\frac{6}{2}, \frac{4}{2}\right)$$

$$M = (3, 2)$$

The coordinates of the midpoint are (3, 2).

Example 5

Find the coordinates of A if the midpoint of $\overline{AB}$ is at M(2, −3) and the coordinates of B are (1, 2). Verify your answer.

Solution:

The midpoint formula is $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, where M is a coordinate point (x_m, y_m).

Fill in what you know.

$(2, -3) = \left(\frac{x_1 + 1}{2}, \frac{y_1 + 2}{2}\right)$ Unknown: $A(x_1, y_1)$

This formula is really made up of two parts:

$$(2, -3) = \left(\frac{x_1 + 1}{2}, \frac{y_1 + 2}{2}\right)$$

Calculates the x-coordinate of the midpoint

Calculates the y-coordinate of the midpoint

Calculate the coordinates (x_1, y_1) independently:

<table>
<tr><td colspan="2">x-coordinate midpoint: $x_m = \frac{x_1 + x_2}{2}$</td><td colspan="2">$y$-coordinate midpoint: $y_m = \frac{y_1 + y_2}{2}$</td></tr>
<tr><td>$2 = \frac{x_1 + 1}{2}$
$4 = x_1 + 1$
$3 = x_1$</td><td colspan="2">multiply both sides of equation by 2
isolate the variable

The coordinates of A are (x_1, y_1)
A(3, −8)</td><td>$-3 = \frac{y_1 + 2}{2}$
$-6 = y_1 + 2$
$-8 = y_1$</td></tr>
</table>

The answer may be verified using the distance formula. The length of the segment $\overline{\text{BM}}$ must equal the length of the segment $\overline{\text{MA}}$ if M is the midpoint of $\overline{\text{AB}}$.

$$d_{\text{BM}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{\text{BM}} = \sqrt{(2-1)^2 + (-3-2)^2}$$

$$d_{\text{BM}} = \sqrt{1^2 + (-5)^2}$$

$$d_{\text{BM}} = \sqrt{26}$$

$$d_{\text{MA}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

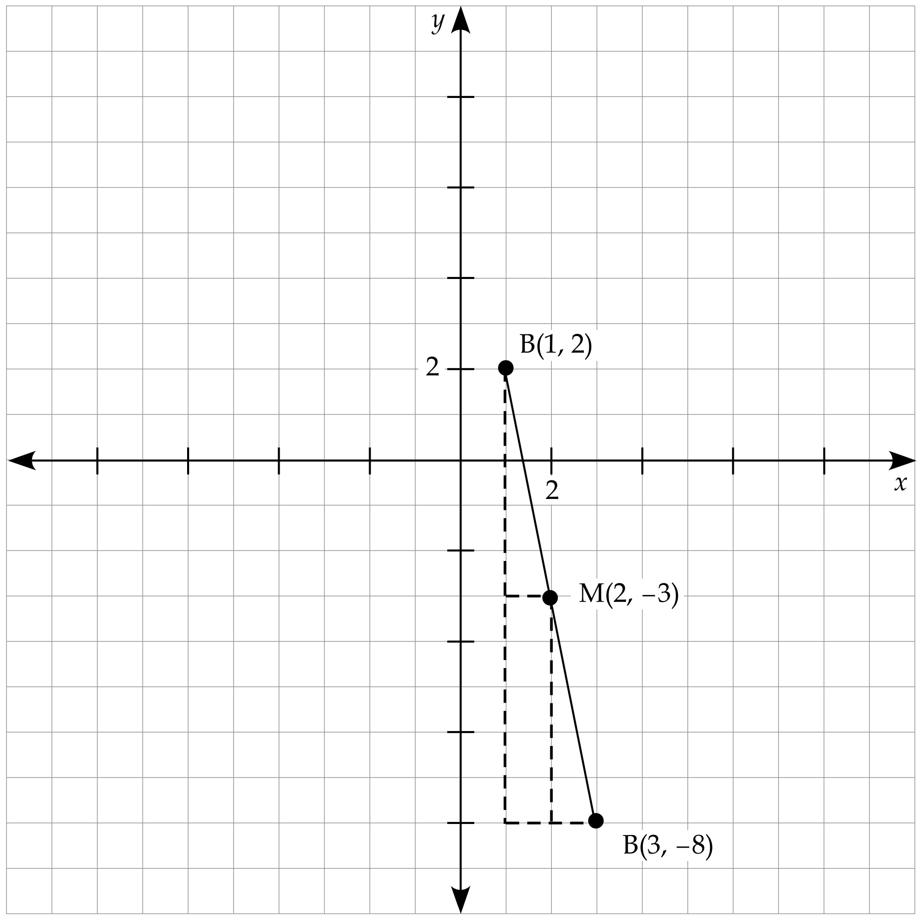
$$d_{\text{MA}} = \sqrt{(3-2)^2 + (-8-(-3))^2}$$

$$d_{\text{MA}} = \sqrt{1^2 + (-5)^2}$$

$$d_{\text{MA}} = \sqrt{26}$$

The length of $\overline{\text{BM}}$ is the same as $\overline{\text{MA}}$ so M is the midpoint of $\overline{\text{AB}}$.

Alternately, the answer may be verified visually by graphing the points.



Point M is located at the average x-value and the average y-value.

Notice, also, that the slope of the segment $\overline{\text{BM}}$ is the same as the slope of $\overline{\text{MA}}$.

$$\frac{\text{rise}}{\text{run}} = \frac{-5}{1}$$

The slope of a line is consistent along the entire length of the line and the distance travelled along the line between the points is the same.

Learning Activity 7.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. In Home Economics, the teacher asks your group to make a double recipe of lasagna for the class, while other groups are making different parts of the meal. The recipe for lasagna calls for $\frac{3}{4}$ cup of Parmesan cheese. How much Parmesan cheese will you need if you are making a double recipe?
2. Jared lost 55% of his weight when he went on his submarine sandwich diet. He originally weighed 420 pounds. How much does he weigh now?
3. The points (1, 1) and (5, 6) make a line. What is the slope of the line?
4. You work 9 am to 3 pm on Tuesday, Wednesday, Thursday, Saturday, and Sunday. You work 12 pm to 5 pm on Monday and Friday. How many hours do you work per week?
5. True or False: The area of 4 equal circles is the same as the surface area of a sphere with the same radius.
6. Factor: $4x^2 - 81$.
7. The sides of a right triangle are 5, 13, and 12. Which side is the hypotenuse?
8. Which is larger: 0.66 or $\frac{2}{3}$?

continued

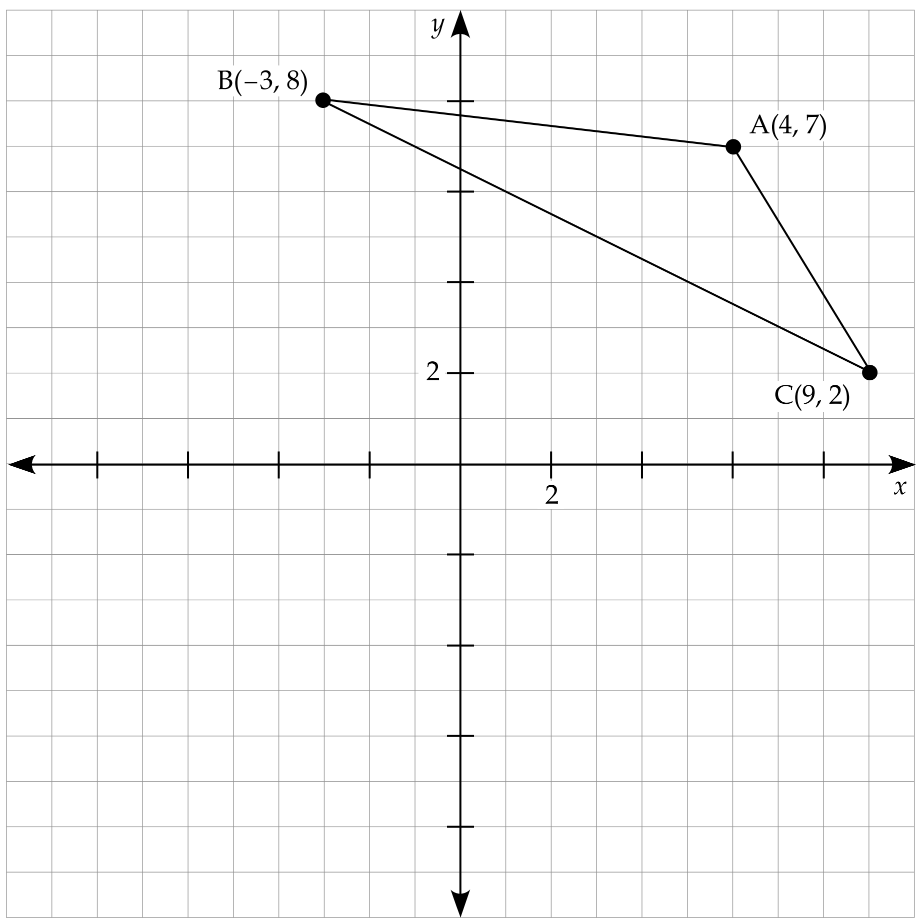
Learning Activity 7.1 (continued)

Part B: Distance and Midpoint

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. For each set of coordinates,
   a) find the length of the line segment $\overline{AB}$
   b) find the coordinates of the midpoint of $\overline{AB}$
      i) A(5, −3) and B(1, 0)
      ii) A(−1, 4) and B(14, −4)
      iii) A(2, 3) and B(0, −1)

2. Determine whether the triangle with vertices A(4, 7), B(–3, 8), and C(9, 2) is isosceles. Write the lengths in radical form. (Remember that isosceles triangles have two sides equal in length.)



continued

Learning Activity 7.1 (continued)

3. Using the diagram from Question 2, show that the line segment joining the midpoints of $\overline{AB}$ and $\overline{AC}$ is half the length of $\overline{BC}$.

4. An online map (located at www.daftlogic.com/projects-google-maps-distance-calculator.htm) plots the locations of cities in Canada using coordinates. Winnipeg is at (49. 8946, −97.0752) and Vancouver is at (49.2678, −123.1348). If one unit on the grid represents 72 km, find the distance between Winnipeg and Vancouver. Round your final answer to the nearest km.

5. A circle with centre at O(1, −2) has one endpoint of a diameter at A(−3, −1). Find the coordinates of the other endpoint of the diameter, B, using the midpoint formula. Verify your answer using another strategy.

6. The three side lengths in a triangle are 18 units, 24 units, and 30 units. Is this a right triangle?

Lesson Summary

The distance formula, as you discovered in this lesson, is really just a variation of the Pythagorean Theorem. The midpoint formula can also be used to determine the coordinates of an endpoint, and your answers can be verified using diagrams, exploring the slope, and finding the length of line segments. These two concepts, the distance and the midpoint between two points, are useful and you will continue to use them in the following lessons and in future math courses.

The next lesson will tie together topics from Modules 1 and 5. You will relate linear relations expressed in different forms and find strategies for graphing them.

Assignment 7.1

Distance and Midpoint

Total Marks = 24

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. For each set of coordinates,
   a) find the length of the line segment $\overline{AB}$
   b) find the coordinates of the midpoint of $\overline{AB}$
      a) A(3, −5) and B(9, 3) *(4 marks)*

      b) A(0, 0) and B(8, −2) *(4 marks)*

      c) A(11, 7) and B(5, −1) *(4 marks)*

2. A triangle has vertices at M(−5, 3), N(−1, −8) and P(6, −1).
   a) Determine whether triangle MNP is an isosceles triangle. Show your work. *(4 marks)*

   b) Determine whether triangle MNP is a right triangle. Show your work. *(4 marks)*

3. The midpoint of a line segment is at (9, 6). If one endpoint is at (−37, 110), find the other endpoint. *(4 marks)*

Lesson 2: Forms of Linear Relations

Lesson Focus

In this lesson, you will

- ❑ relate linear relations expressed in slope-intercept form, general form, and slope-point form to their graphs
- ❑ write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the process
- ❑ express a linear relation in different forms and graph it, with or without technology
- ❑ generalize and explain strategies for graphing each form of linear relation
- ❑ identify equivalent linear relations from a set of linear relations

Lesson Introduction

You have seen how linear relations can be expressed using words, ordered pairs, tables of values, graphs, and equations. You have analyzed the characteristics of linear relations and their graphs, including the intercepts, slope, domain, and range. You can determine whether a relation represents a function and write it in functional notation. This lesson will incorporate these ideas while concentrating on three different ways to express the equation of a linear relation. It will also outline strategies to help you create a graph from each of the three expressions.

Different Ways to Write a Linear Equation

It will be helpful to have the different formulas for a linear relation labelled and written on your Resource Sheet.

Slope-intercept form $y = mx + b$

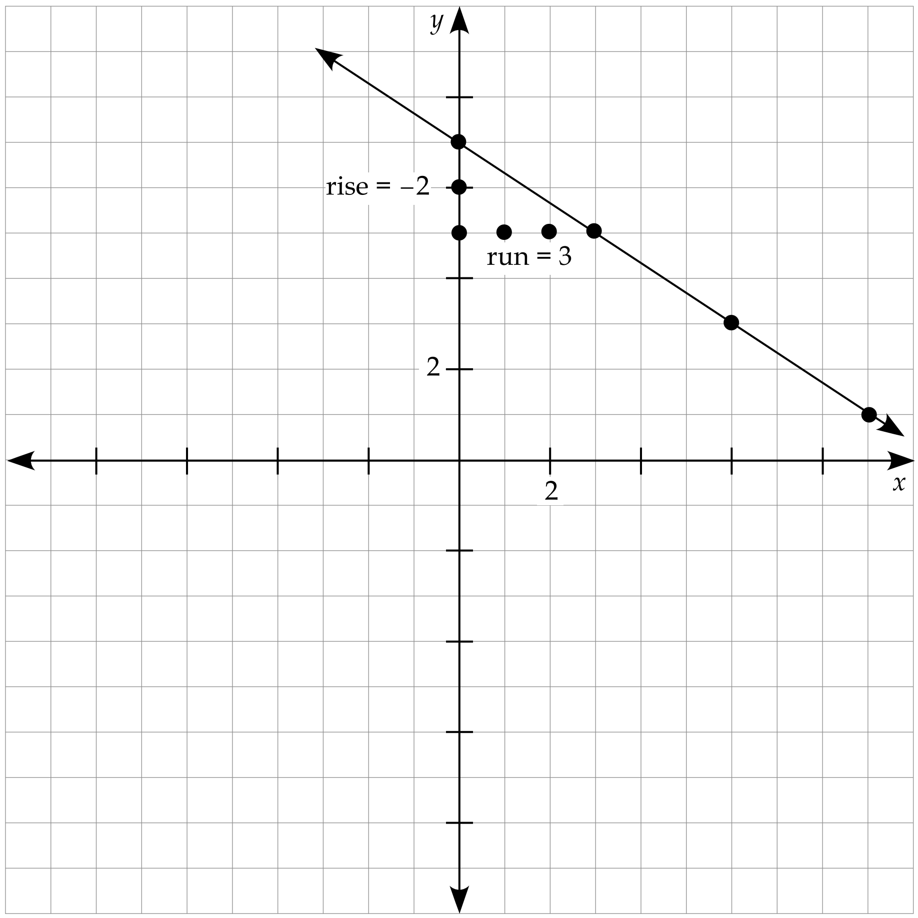
You have used the slope-intercept form of a linear equation to express a linear relation and graph it. The $y = mx + b$ form is useful because it clearly identifies the slope and y-intercept of the line. To graph an equation given in this form, you first plot the y-intercept on the vertical axis and then count the $\frac{\text{rise}}{\text{run}}$ to find subsequent points and join them with a straight line.

Example 1

Graph $y = \frac{-2}{3}x + 7.$

Solution:

Find +7 along the y-axis. From there, move two units down and three units to the right and mark the next point. Repeat. Join the points with a straight line.



General Form $Ax + By + C = 0$

When a linear equation is written in the form $Ax + By + C = 0$ where A, B, and C are integers, it is called the general form.

To graph a linear relation expressed in general form, you can solve for the x- and y-intercepts by making one of the variables equal to zero and solving for the remaining variable. Alternately, you may also make a table of values and plot the points, or convert the equation to slope-intercept form.

Example 2

Graph $5x - 4y + 20 = 0$ by solving for the intercepts.

Solution:

The x-intercept of the graph is where $y = 0$ and the y-intercept is where $x = 0$.

x-intercept

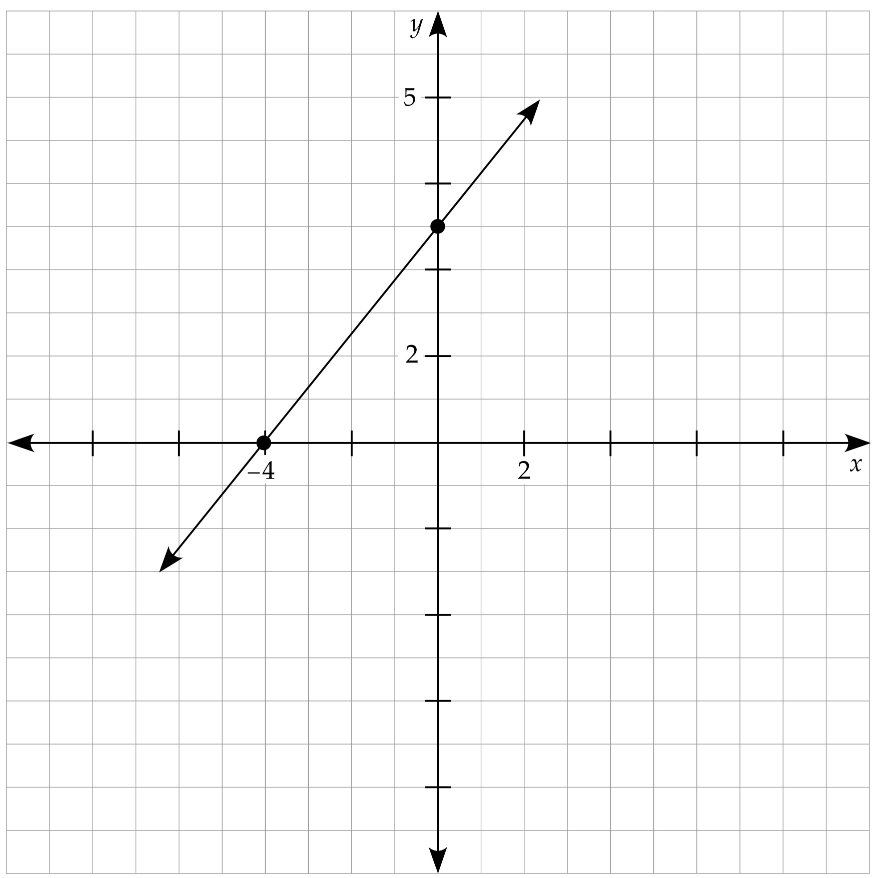
$$
\begin{aligned}
5x - 4(0) + 20 &= 0 \\
5x + 20 &= 0 \\
5x &= -20 \\
x &= -4
\end{aligned}
$$

The x-intercept is at -4.

y-intercept

$$
\begin{aligned}
5(0) - 4y + 20 &= 0 \\
-4y + 20 &= 0 \\
-4y &= -20 \\
y &= 5
\end{aligned}
$$

The y-intercept is at 5.



Point-Slope Form $y - y_1 = m(x - x_1)$

In Module 1, you learned how to calculate the slope of a line given two points on that line, using the slope formula.

Recall: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

If you know the slope and one point on the line, you can write the equation of that line using a slightly rearranged version of the slope formula.

$$m = \frac{y - y_1}{x - x_1}$$

Note: The subscript 2 is taken off the x- and y-variables so the final equation has variables in it.

$$(x - x_1)m = \frac{y - y_1}{x - x_1}(x - x_1)$$

Multiply both sides of the equation by $(x - x_1)$.

$$m(x - x_1) = y - y_1$$

or

$$y - y_1 = m(x - x_1)$$

This is the point-slope form of a linear relation.

Example 3

A linear relation has a slope of −2 and goes through the point (−6, 9). Write the equation of this line in slope-point form and sketch a graph.

Solution:

$m = -2$

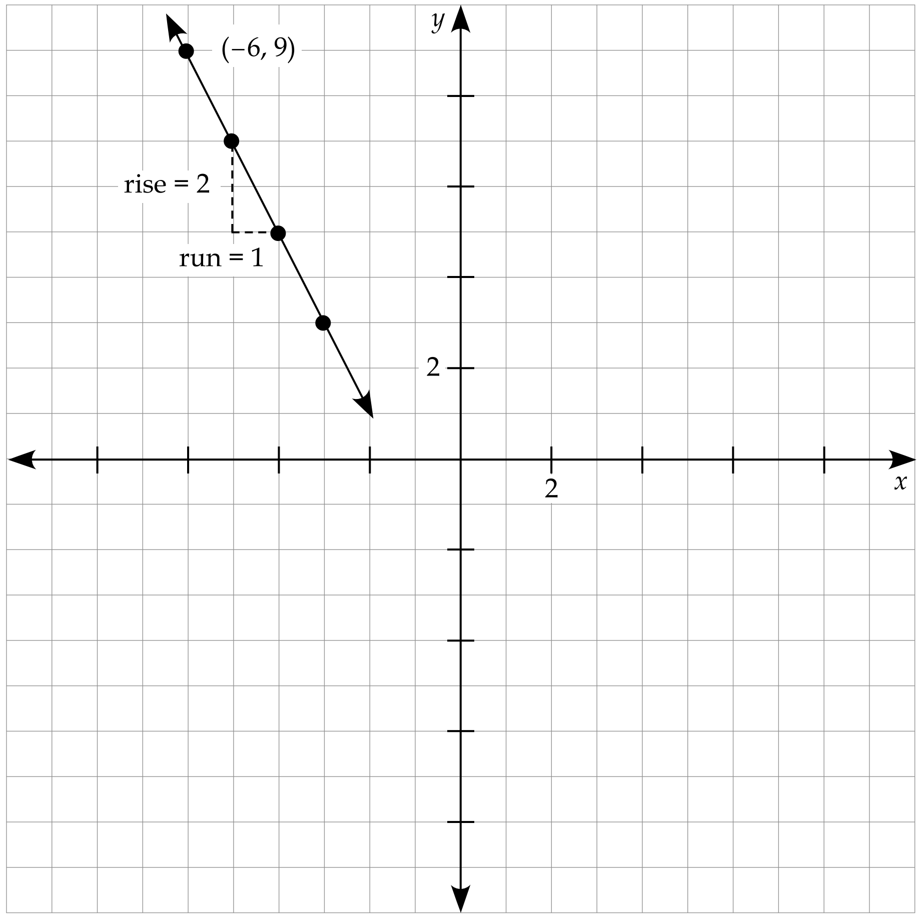
$(-6, 9)$ is (x_1, y_1)

$y - y_1 = m(x - x_1)$

$y - (9) = -2\,(x - (-6))$

Instead of first plotting the y-intercept, as you did when graphing the slope-intercept form, plot the given point (–6, 9) and then count the $\dfrac{\text{rise}}{\text{run}}, \dfrac{-2}{1}$, to find subsequent points and join them with a straight line.

$y - 9 = -2(x + 6)$



Example 4

Graph $y + 7 = 3(x + 4)$.

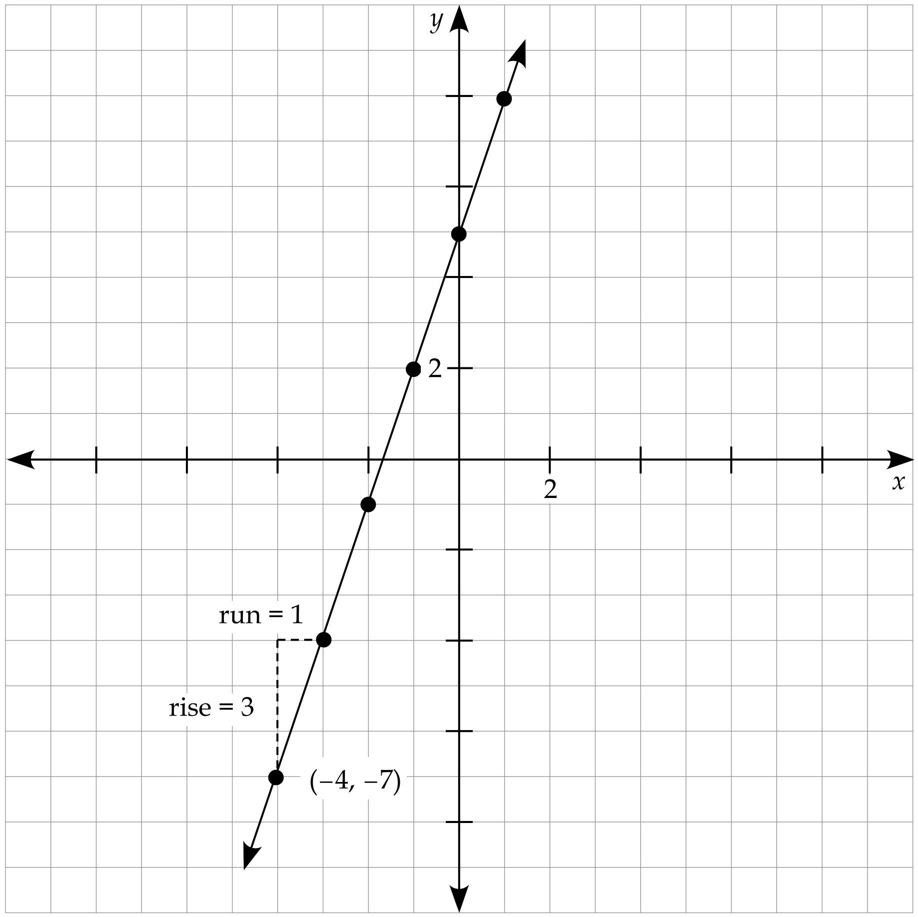
Solution:

$y - y_1 = m(x - x_1)$

point $(-4, -7)$ Watch the signs! Remember the formula has NEGATIVE signs in it, so reverse the signs when stating the point.

$m = 3$

$y + 7 = 3(x + 4)$



Graphing with Technology

Graphs of linear equations can be created by hand, by plotting points on graph paper, or by using technology. Graphing calculators, computer software like *Graphical Analysis*, spreadsheet programs like *Excel*, or online applets are all ways to make graphs. You are free to use any or all of these forms when creating your graphs. Keep in mind that for your exam, you will need to plot points and draw graphs by hand.

Graphing with technology is used in applied math quite frequently, although it is also used in pre-calculus math.

While spreadsheets can create line graphs based on a table of values, typically graphing calculators and graphing programs or software will require that equations be in the slope-intercept form. It is therefore important to know how to correctly rewrite equations in different forms.

Changing Forms

Linear equations can be written in any of the three forms as outlined in this lesson. They can be converted to and from each different expression.

Example 5

Write the equation $y = \frac{-2}{9}x + 4$ in

a) general form

b) point-slope form

Solution:

As a reminder for yourself, you may want to include these steps on your Resource Sheet.

a) general form is $Ax + By + C = 0$

$$y = \frac{-2}{9}x + 4$$

$$\frac{2}{9}x + y - 4 = 0$$

Step 1: Rearrange the terms so they are all on one side of the equation and the coefficient of x is positive.

$$(9)\frac{2}{9}x + (9)y - (9)4 = (9)0$$

$$2x + 9y - 36 = 0$$

Step 2: Multiply each term by the denominator so all coefficients are integers.

b) slope-point form is $y - y_1 = m(x - x_1)$.

The slope is given in the slope-intercept form as $m = \frac{-2}{9}$.

The y-intercept can be written as a coordinate point (0, 4).

$$y - 4 = \frac{-2}{9}(x - 0)$$

$$y - 4 = \frac{-2}{9}x$$

Example 6

The graph of a linear relation goes through the point (−3, 5) and has a slope of 4. Express this as a linear equation in three different forms and graph it.

Solution:

Slope-point form: $y - y_1 = m(x - x_1)$

$y - 5 = 4(x + 3)$ Watch the signs!

Slope-intercept form: $y = mx + b$

$y - 5 = 4(x + 3)$ Simplify the slope-point form of the equation by applying the distributive property.

$y - 5 = 4x + 12$ Isolate the y.

$y = 4x + 12 + 5$ Combine like terms.

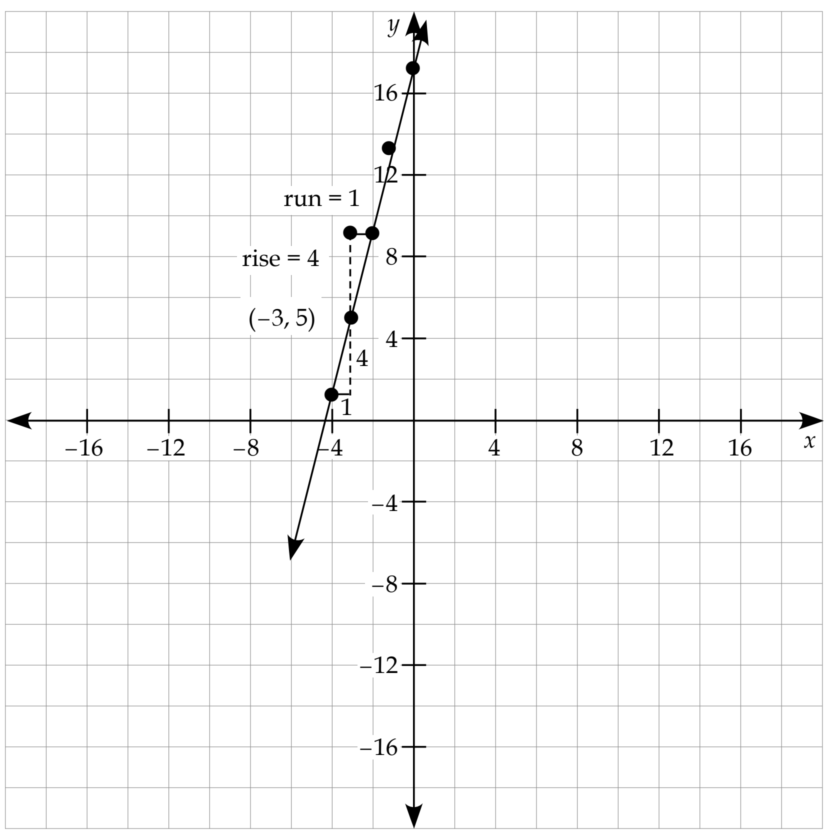
$y = 4x + 17$

General form: $Ax + By + C = 0$

$y = 4x + 17$ Start with the slope-intercept form.

$4x - y + 17 = 0$

Graph: $y - 5 = 4(x + 3)$



Slope:

$\dfrac{4}{1} = \dfrac{\text{rise}}{\text{run}}$

To plot points to the left of (−3, 5), work backwards:

down: 4

to the left: 1

Example 7

Write the general form of a linear relation $Ax + By + C = 0$ in slope-intercept form. State the slope and y-intercept of the line.

Solution:

You can manipulate literal coefficients in the same way as numerical ones, and the outcome leads to some convenient shortcuts when graphing the general form.

$Ax + By + C = 0$

$By = -Ax - C$ Rearrange the terms to isolate the term with the y–variable.

$\frac{By}{B} = \frac{Ax}{B} - \frac{C}{B}$ Divide each term by B to isolate the y.

$y = \frac{-A}{B}x - \frac{C}{B}$ The slope of the line is $\frac{-A}{B}$ and the y-intercept is at $\frac{-C}{B}$.

It may be helpful to include this formula on your Resource Sheet.

Example 8

Using the shortcuts identified in Example 7, state the slope and y-intercept of the line $6x + 5y - 2 = 0$.

Solution:

$A = 6$

$B = 5$

$C = -2$

$$\text{Slope} = \frac{-A}{B} = \frac{-6}{5}$$

$$y\text{-intercept} = \frac{-C}{B} = \frac{-(-2)}{5} = \frac{2}{5}$$

Learning Activity 7.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Factor: $2x^2 - 4x + 10$.
2. Find the y-intercept: $3y - 7x = 90$.
3. Simplify: $\dfrac{3}{4\sqrt[6]{x^9}}$.
4. Estimate the taxes, at 12%, of a pair of shoes that cost $74.89.
5. Complete the pattern: −1, 2, −3, ____, ____.
6. On average, you right-click once for every 5 left-clicks on your mouse. Because of this, the left button wears out 5 times faster. If the right button is estimated to last for 3 years, how many months will the left button last?
7. You have $4.65. If you buy a package of gum for $2.95, how much money will you have left over?
8. You ride your bike instead of taking the bus to get to work from April until October. Last summer it rained 35% of the days that you could have ridden your bike, and you don't ride in the rain. There are 240 days that you would have ridden your bike (you don't ride your bike on Hallowe'en). How many days did you ride your bike to work?

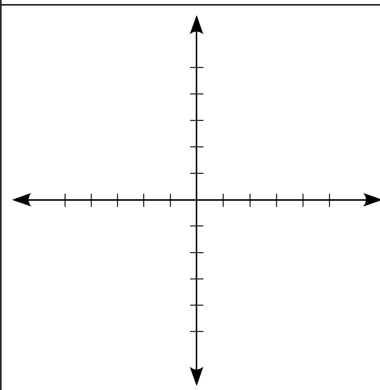
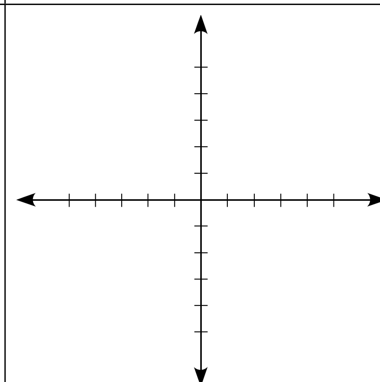
continued

Learning Activity 7.2 (continued)

Part B: Linear Relation Formulas

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Complete the following chart. Express each linear relation in all three forms and sketch a graph.

Slope-Intercept Form	$y = 2x - 4$	
General Form		$3x + y - 5 = 0$
Slope-Point Form		
Graph		

2. Write the given linear relation in slope-intercept form using two different strategies. Explain the strategies.

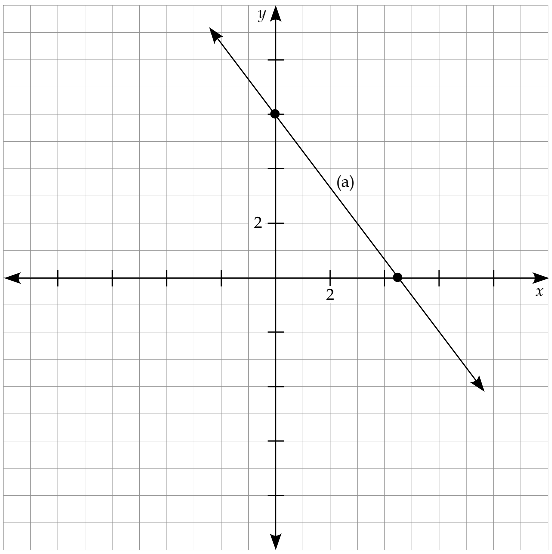
$$y + \frac{1}{20} = \frac{1}{2}\left(x - \frac{2}{5}\right)$$

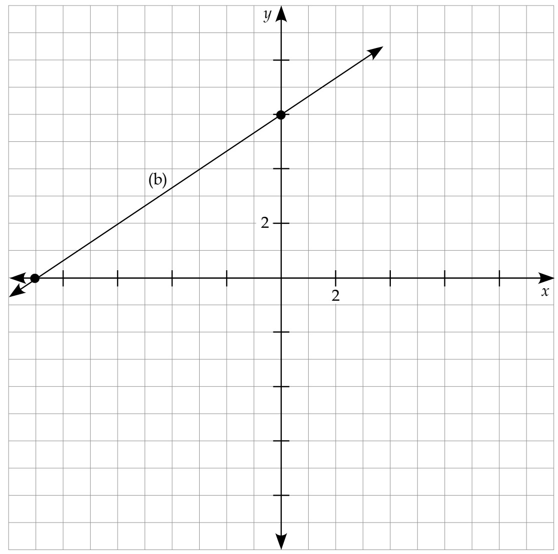
3. Explain two different strategies you could use to graph $6x - y + 3 = 0$. Sketch a graph.

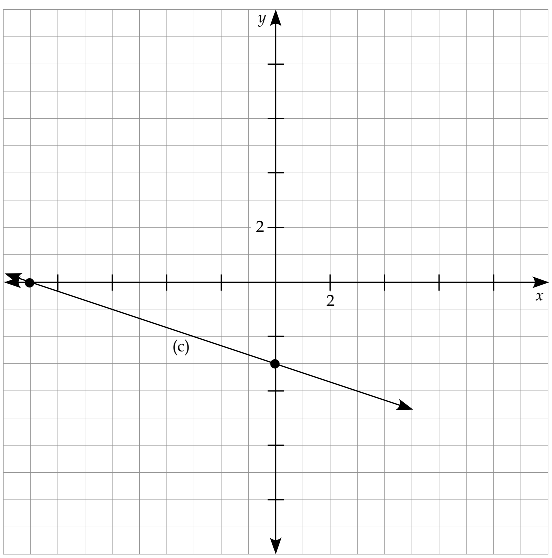
continued

Learning Activity 7.2 (continued)

4. Match each graph to its equation(s).







____ $y-8=\frac{2}{3}(x-3)$

____ $y+3=\frac{-2}{6}x$

____ $y=\frac{2}{3}x+6$

____ $4x+3y-18=0$

____ $y+2=\frac{4}{6}(x+2)$

____ $y-6=\frac{-4}{3}x$

5. The slope and y-intercept of a line are given as:

$$m=\frac{-5}{3}$$

$$b=\frac{7}{3}$$

Write the general form of the equation for this linear relation ***without*** first writing it in the slope-intercept form.

Lesson Summary

The slope-intercept form, point-slope form, and general form of a linear relation all say the same thing but in a different way. Each expresses a linear relation and can be used to graph the equation, or determine the intercepts, slope, or points along the line. This lesson highlighted different strategies to use when graphing or rewriting the different forms of linear equations. You also saw how different equations can be equivalent expressions, and you used the point and slope of a line to write an equation. In the next lesson, you will have more opportunities to write linear equations, given different characteristics of the line.

Notes

Assignment 7.2

Linear Relation Formulas

Total Marks = 30

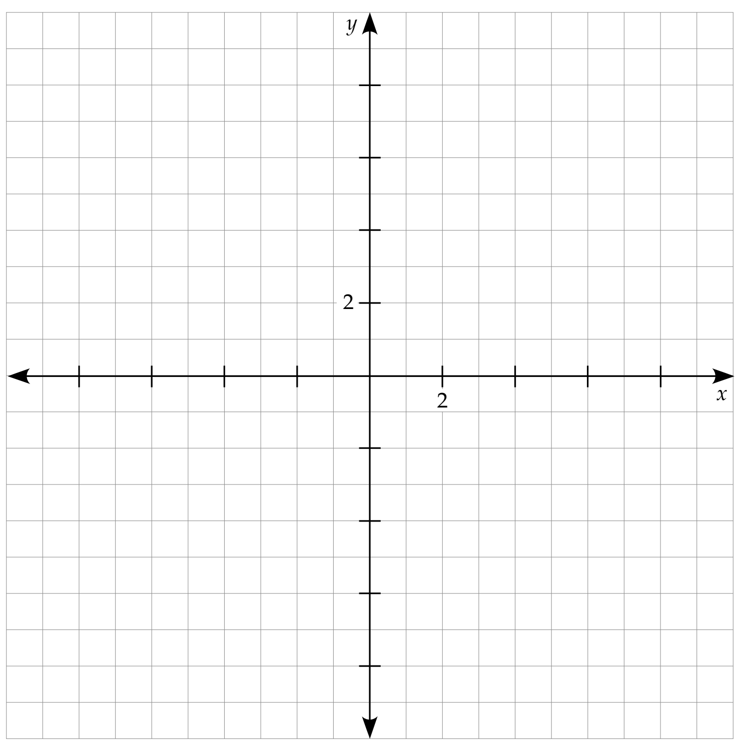
Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Rewrite the given linear relation in the form requested. Sketch the graph of each on the grid provided. Be sure to clearly label your graphs.

   a) Express $y = \frac{5}{3}x + 2$ in general form. *(3 marks)*

   b) Express $y + 4 = \frac{-2}{3}(x + 2)$ in slope-intercept form. *(3 marks)*

   c) Express $5x - y - 7 = 0$ in point-slope form. *(3 marks)*



2. State the x- and y-intercepts and the slope of the linear relation $7x - 2y + 8 = 0$ Show your work. *(4 marks)*

3. Explain the similarities and differences in your strategies for graphing a linear relation expressed in slope-intercept form and point-slope form. Give an example of each. *(5 marks)*

Assignment 7.2: Linear Relation Formulas (continued)

4. The graph of a linear relation has a slope of $\frac{1}{3}$ and the line goes through $(-1, 5)$.

   Express this as a linear relation in all three forms. Show your work and explain the steps in your process.

   a) point-slope form *(2 marks)*

   b) slope-intercept form *(3 marks)*

   c) general form *(3 marks)*

5. Circle the linear equations that are equivalent, and match the given graph. *(4 marks)*

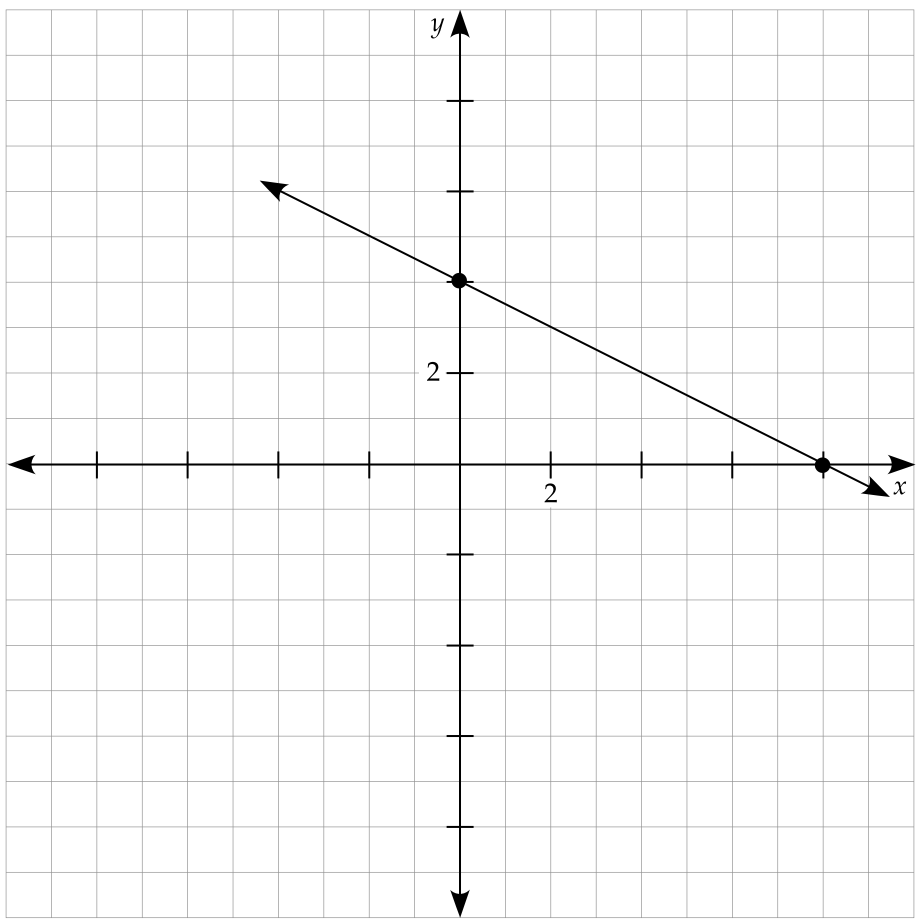
$x+2y-8=0$

$y+4=\frac{1}{2}x$

$y=-\frac{1}{2}x+4$

$y-\frac{9}{4}=\frac{2}{4}\left(x-\frac{7}{2}\right)$

$x+y+4=0$



Lesson 3: Writing Linear Equations

Lesson Focus

In this lesson, you will

- ❑ determine the equation of a linear relation given
  - the slope and coordinates of a point on a line
  - a scatterplot or graph of a line
  - the coordinates of two points on a line
  - a point and the equation of a parallel or perpendicular line

Lesson Introduction

In Module 1, you wrote linear equations in the form $y = mx + b$ to represent data in a scatterplot, and when given the slope and y-intercept of a line. In the last lesson, you wrote the equation of a linear relation when given a point and the slope of the line. In this lesson, you will investigate how to write linear equations when given a scatterplot, two points on a line, or a point and the equation of a parallel or perpendicular line.

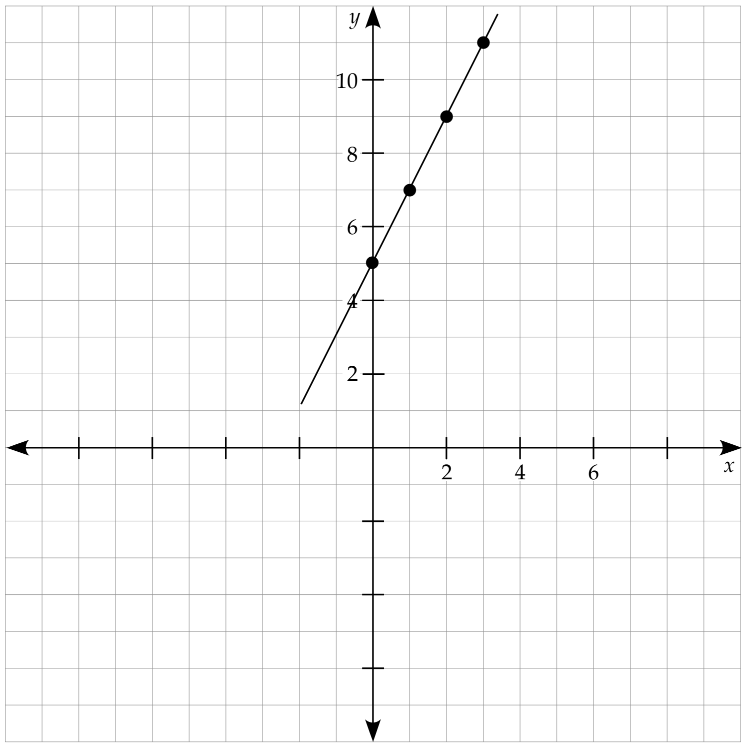
Writing a Linear Equation

There are many different strategies outlined in this lesson about how to determine the equation of a linear relation based on the information given to you. It is a good idea to write these down on your Resource Sheet in point form. We will help you out with this. Look for the resource sheet icon in the margin!

Writing an Equation from a Graph or Scatterplot

Example 1

Write the coordinates of at least two points found along the following line. Use them to determine the equation of the line in slope-intercept form.



Solution:

The points (0, 5), (1, 7), (2, 9), and (3, 11) all fall along the line.

To write the equation of this line in slope-intercept form, find the y-intercept, where $x = 0$, on the graph or use the ordered pairs above.

It is at the point (0, 5), so the y-intercept is at 5.

Count the $\frac{\text{rise}}{\text{run}}$ between any two points on the graph, or use the formula and any two coordinate points stated above to calculate the slope.

$$\frac{\text{rise}}{\text{run}} = \frac{2}{1}$$

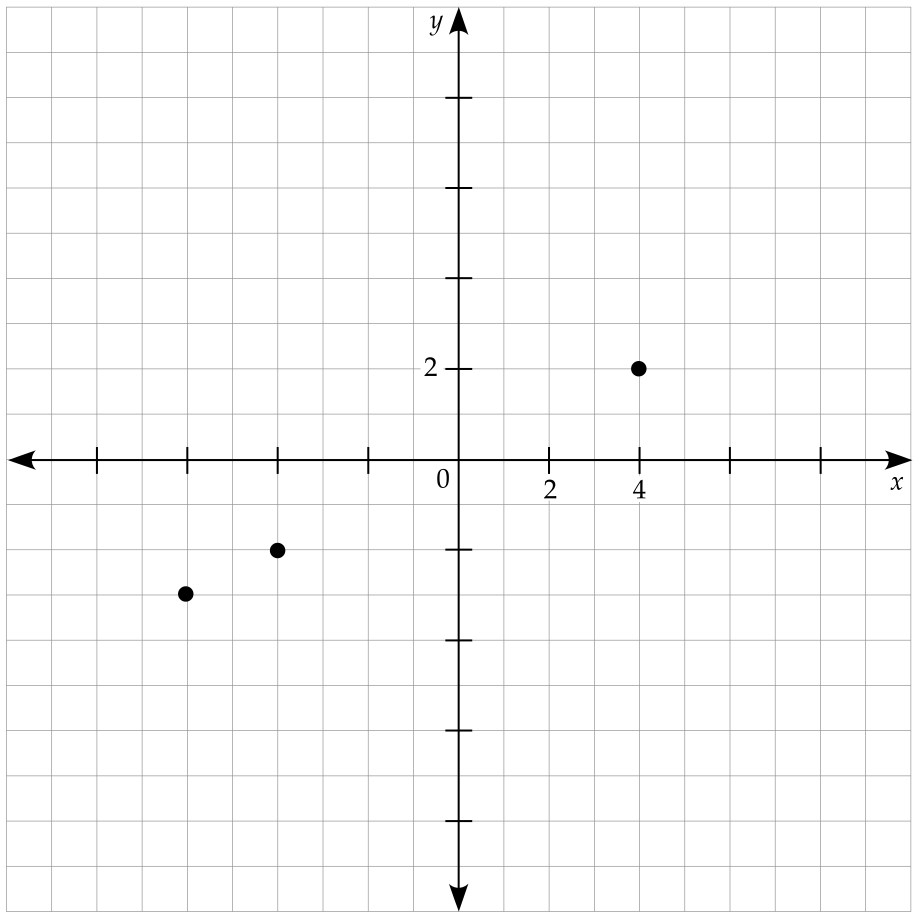
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-7}{2-1} = \frac{2}{1}$$

The equation of the line is $y = 2x + 5$.

Compare the ordered pairs to this equation. Each y-value is five more than two times the corresponding x-value, which matches this formula.

Example 2

For the graph shown below, determine the equation of the linear relation.



Solution:

The three points can be joined using a straight line. The slope of the line is $\frac{1}{2}$, found by counting the units of rise and run, and it passes through the origin so the y-intercept is 0. The equation of the line is $y = \frac{1}{2}x$.

Writing an Equation Given Two Points on the Line

You can write an equation given the slope and a point (either the y-intercept or another point) on the line. What if all you were given were the coordinates of two points on the line? Use them to calculate the slope, and then using the slope and either point, write the equation.

Example 3

Write the general form of the linear equation for the line that passes through the points (1, 2) and (3, −2).

Solution:

Calculate the slope of the line between these two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-2}{3-1} = \frac{-4}{2} = -2$$

Substitute the slope and either point into the point-slope formula.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= -2(x - 1) \\ y - 2 &= -2x + 2 \\ y &= -2x + 4 \\ 2x + y - 4 &= 0 \end{aligned}$$

Rearrange this equation into the general form.

Example 4

Write the equation of the line that passes through (−0.6, 3.9) and the origin.

Solution:

The origin is at (0, 0).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3.9}{0+0.6} = \frac{-3.9}{0.6} = -6.5$$

$$\begin{aligned} y - y_1 &= m(x + x_1) \\ y - 0 &= -6.5(x - 0) \\ y &= -6.5x \end{aligned}$$

Example 5

Write the general form of the equation of the line that goes through the midpoint of the line segment with endpoints $(-14, -21)$ and $(24, 15)$ and has the same x-intercept as the line $3x - y - 9 = 0$.

Solution:

You are given a lot of information about other lines in this question. You just need to manipulate what you are given until you have found two coordinate points on the line you want, and then ignore all the other stuff.

First point: midpoint of line segment with endpoints at $(-14, -21)$ and $(24, 15)$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{-14 + 24}{2}, \frac{-21 + 15}{2}\right)$$

$$M = \left(\frac{10}{2}, \frac{-6}{2}\right)$$

$M = (5, -3)$ This is the first coordinate point.

Second point: same x-intercept as the line $3x - y - 9 = 0$.

The x-intercept is found where $y = 0$.

$3x - (0) - 9 = 0$

$3x - 9 = 0$

$3x = 9$

$x = 3$ The coordinates of the second point are the x-intercept at $(3, 0)$.

Now, use these ordered pairs, $(5, -3)$ and $(3, 0)$, to determine the equation of the line that goes through these points.

Calculate the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-3)}{3 - 5} = \frac{3}{-2} = \frac{-3}{2}$$

Use the slope and one of the points in the point-slope formula

$$
\begin{aligned}
y - y_1 &= m\left(x - x_1\right) \\
y - (-3) &= \frac{-3}{2}(x - 5) \\
y + 3 &= \frac{-3}{2}x + \frac{15}{2} \\
y &= \frac{-3}{2}x + \frac{15}{2} - 3 \\
y &= \frac{-3}{2}x + \frac{15}{2} - \frac{6}{2} \\
y &= \frac{-3}{2}x + \frac{9}{2}
\end{aligned}
$$

$$\frac{3}{2}x + y - \frac{9}{2} = 0$$

$$3x + 2y - 9 = 0$$

Writing an Equation Given a Point and a Parallel or Perpendicular Line

No matter what configuration of information you are given, as long as you can find the slope and a coordinate point along a line, you can write the equation of that line. In the last examples, you used information about other lines and points to establish what you needed to know about the required equation.

In Module 1, you learned that parallel lines have the same slope, and that perpendicular lines have negative reciprocal slopes. This information can be incorporated into solving for equations of linear lines.

Example 6

Write the slope-intercept form of the equation of the line that is parallel to $y = 2x + 5$ and goes through (2, 8).

Solution:

Parallel lines have equivalent slopes. The slope of the parallel line you were given is $m = 2$, so the slope you want to use is $m = 2$.

You are given the point (2, 8). Write the equation.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 8 &= 2(x - 2) \\
y - 8 &= 2x - 4 \\
y &= 2x - 4 + 8 \\
y &= 2x + 4
\end{aligned}
$$

Example 7

Write the point-slope form of the linear equation that is perpendicular to the line $5x - 4y - 40 = 0$ and has a y-intercept at 11.

Solution:

To find the slope you want, rewrite the equation of the perpendicular line in slope-y-intercept form.

$$
\begin{aligned}
5x - 4y - 40 &= 0 \\
\frac{5x}{4} - \frac{40}{4} &= \frac{4y}{4} \\
\frac{5}{4}x - 10 &= y \\
y &= \frac{5}{4}x - 10
\end{aligned}
$$

The slope of the perpendicular line is $\frac{5}{4}$ so the slope you want to use is the negative reciprocal of that: $\frac{-4}{5}$.

The coordinates of the point you are given are (0, 11).

In point-slope form, the equation is

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 11 &= \frac{-4}{5}(x - 0) \\
y - 11 &= \frac{-4}{5}x
\end{aligned}
$$

Learning Activity 7.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Is this relation a function: {(2, 4), (5, 8), (6, 1), (3, 7)}?
2. Find the midpoint of the line segment with end points (2, 6) and (4, 8).
3. Convert: 300 m = _________ km.
4. Evaluate: $\sqrt[4]{81}$.
5. June is a big birthday month for you. Your brother's is on June 9th, your nephew's is on June 20th, plus Father's Day is in June! If you want to spend $30 on each present and you have $85.00 saved up, will this be possible?
6. Identify the type of angle that has a measure of 345°.
7. What is the range of the following relation?
   {(1, 2), (2, 4), (3, 6), (4, 8)}
8. Write as an improper fraction: $\frac{19}{16}$.

Part B: Writing Linear Equations Based on Different Information

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Explain the process you go through to write the equation of a linear relation when you are given the coordinates of two points on the line.
2. Write the equation of a linear relation in slope-intercept form, given $m = \frac{-9}{2}$ and $b = \frac{1}{2}$.

continued

3. A line has a slope of $\frac{8}{3}$ and goes through the point (−72, −94). Write the equation in point-slope form and general form.
4. Write the equation of the line that goes through the points (26, 9) and (43, −6). State your answer in $y = mx + b$ form.
5. A line crosses the x-axis at 14 and the y-axis at 35. Write the equation of the line in general form. Use two different methods to arrive at the answer.
6. A line is parallel to $y = -3x - 55$ and goes through the point (−8, 19). Write the equation of the line in point-slope form.
7. Write the equation of the line that is perpendicular to $5x + 6y - 72 = 0$ and has an x-intercept of −4 in general form. Compare the coefficients in both equations. What do you notice?
8. Write the equation of the line that is the perpendicular bisector (bisector means "cuts in half") of the line segment between (−3, −8) and (15, 6). State the answer in point-slope form. (Hint: The line must pass through the midpoint of the line segment.)

   You should include the definition of a bisector on your resource sheet.

9. Write the equation of a line that is perpendicular to the line $y = -12$, and explain your answer.

Lesson Summary

This lesson outlined several different methods you can use to write the equation of a linear relation. The one you choose depends on what you were given, and what form you need to write the answer in. The first step is generally to find the slope and a point and then substitute them into the correct form. You can find the slope either using the formula, counting the rise/run, or by finding the slope of a parallel or perpendicular line. The point you choose to use may be either of the two points you used to find the slope, it may be an x- or y-intercept, or it may be a point it shares with another line. Just find a point and a slope, substitute them into the formula, and then simplify and rewrite it in another form if needed.

The next lesson will look at writing equations based on data generated from a specific context. You will use technology to graph data, draw a line of best fit, and describe the relationship between the variables with a correlation coefficient, or r-value.

Notes

Assignment 7.3

Writing Linear Equations Based on Different Information

Total Marks = 25

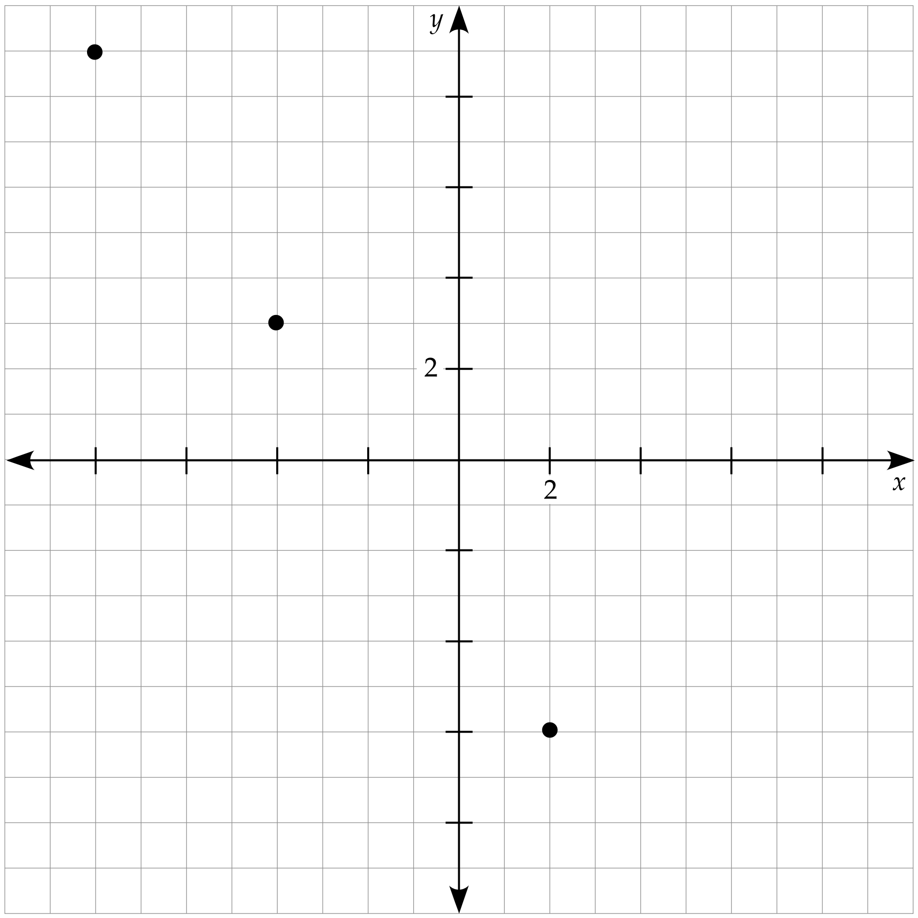
Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Explain the process you go through to write the equation of a linear relation when you are given a point and the equation of a perpendicular line. *(3 marks)*

2. The graph of a linear relation has a slope of $\frac{-3}{4}$ and a y-intercept at 15. Write the equation of this line in the three different forms, using different methods. *(4 marks)*

Assignment 7.3: Writing Linear Equations Based on Different Information (continued)

3. Write the slope-intercept form of the equation for the linear relation illustrated in the scatterplot. *(3 marks)*



Assignment 7.3: Writing Linear Equations Based on Different Information (continued)

4. A line goes through the point $(-12, 23)$ and has a slope of $\frac{-7}{3}$. Write the equation of this linear relation in point-slope form. *(2 marks)*

5. Write the general form of the linear equation for a line that goes through $(-54, -17)$ and $(9, 11)$. *(3 marks)*

Assignment 7.3: Writing Linear Equations Based on Different Information (continued)

6. A line is parallel to $y = \frac{1}{2}x + 37$ and has the same y-intercept as the line $4x - 5y + 9 = 0$. Write the equation of the line in point-slope form. *(3 marks)*

7. Write the equation of a line in general form that is perpendicular to $9x - 5y + 3 = 0$ and goes through the point $(18, -12)$. *(3 marks)*

Assignment 7.3: Writing Linear Equations Based on Different Information (continued)

8. Write the equation of the perpendicular bisector of the line segment with endpoints at $(-8, 5)$ and $(14, 11)$. State your answer in point-slope form. *(4 marks)*

Notes

Lesson 4: Correlation of Data

Lesson Focus

In this lesson, you will

- ❑ determine the equation of a linear relation given data in context
- ❑ determine the equation of the line of best fit from a scatterplot using technology, and discuss the correlation
- ❑ solve contextual problems using the equation of a linear relation

Lesson Introduction

Real-world situations do not always follow theoretical rules and mathematical equations. This lesson will examine data in context, and you will write a linear equation to describe the relationship between variables based on the information given about a specific situation. You will also use technology to draw a line of best fit, and calculate its equation when given data in context. You will discuss the correlation between data sets or variables using an r-value.

What is "Good" Data?

Graphs of Linear Data

Data that represent a linear relation can be found in many situations. Graphing the data can help you identify the resulting equation, which can then be used to solve problems.

Example 1

Allycia gets paid $195 for a five-hour shift, and $237 for an eight-hour shift.

a) Graph these points on a scatterplot.

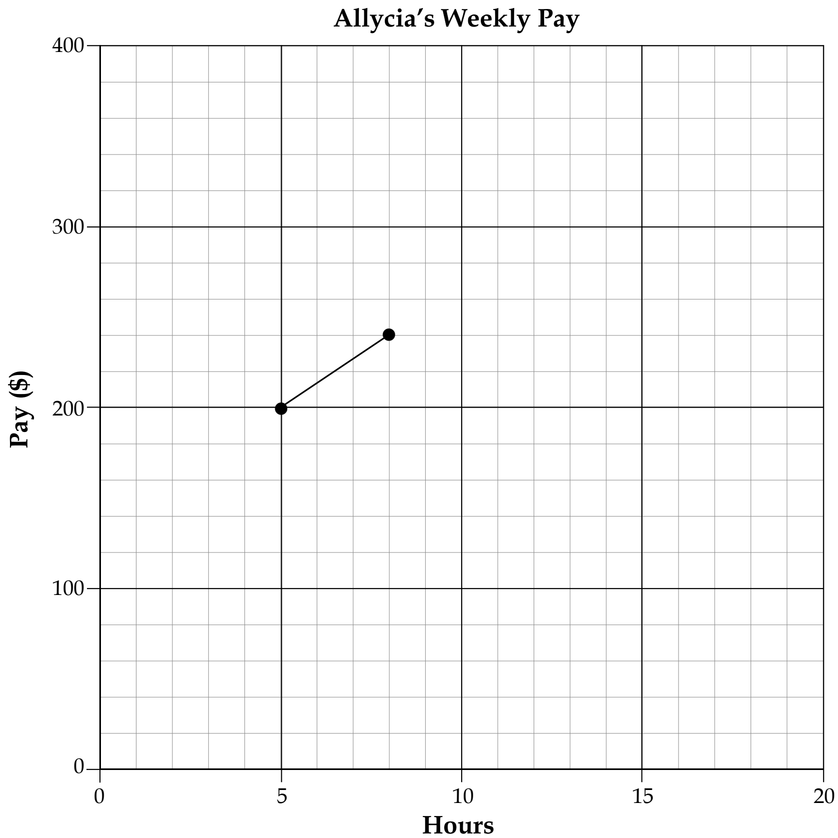
b) Determine the linear equation her boss uses to calculate her pay each week. Write the answer in slope-intercept form.

c) Explain the significance of the slope and y-intercept in this context.

d) What would her maximum pay be per week if she worked a 40-hour week?

Solution:

a) Graph



b) To write the equation of this line, you need the slope and y-intercept.

Calculate the slope using the coordinates of the two points representing (hours, pay) on the graph:

(5, 195), (8, 237)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{237 - 195}{8 - 5}$$

$$m = \frac{42}{3} = 14$$

The slope of this line is 14.

Use the slope and one point from the graph to determine the y-intercept.

$$\begin{aligned} y &= mx + b \\ 195 &= 14(5) + b \\ 195 - 70 &= b \\ 125 &= b \end{aligned}$$

The y-intercept is at 125.

The equation of this line is $y = 14x + 125$.

c) The slope of 14 represents the amount Allycia gets paid per hour. For each vertical increase of \$14, there is a horizontal increase of one hour. If you extend the line on the graph above, you notice it intercepts the y-axis at 125. The coordinate point (0, 125) represents (hours, pay), so for 0 hours worked Allycia gets paid \$125. She gets a set salary of \$125 per week plus an hourly rate of \$14.

d) If Allycia worked a full 40-hour week, she would earn \$685.

$y = 14x + 125$

$y = 14(40) + 125$

$y = \$685$

Example 2

When you travel to the United States, you must pay for your purchases in American dollars (USD). When you exchange your Canadian cash for American dollars, the bank's rate of exchange determines how much you will get. In March 2009, one Canadian dollar was worth 79 cents USD. That means \$100 CDN would be \$79 USD while \$500 CND would be \$395 USD.

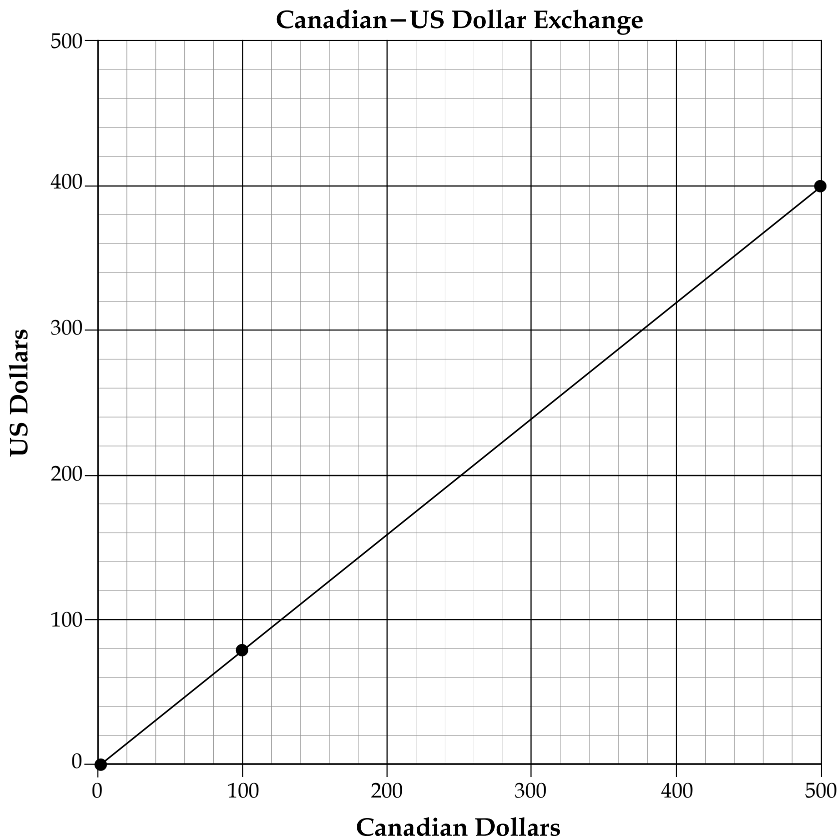
a) Use a graph to represent this linear relationship by plotting the points on a grid and joining them with a straight line.

b) Write a linear equation to represent this line.

c) Use the equation above to determine the cost in Canadian dollars to purchase a camera that is \$250 USD.

Solution:

a) Graph



b) To write an equation representing these data, consider the slope and y-intercept.

$m = \dfrac{0.79}{1} = 0.79$ (Since this is the exchange rate, you could also use the two points to calculate the slope.)

The graph goes through the origin.

The equation of this line is $y = 0.79x$.

c) $y = 0.79x$

where y is the cost in U.S. dollars and x is the cost in Canadian dollars.

$$(250) = 0.79x$$

$$\frac{(250)}{0.79} = \frac{0.79x}{0.79}$$

$$x = 316.46$$

The camera would cost the equivalent of \$316.46 in Canadian dollars.

Graphs of Data that are Approximately Linear

Not all situations generate data that fall nice and neatly into a perfect linear relationship. Often, data will follow a linear trend but, when graphed, the points cannot all be connected with a single straight line.

Example 3

Suppose you collected data to find the relationship between the height and weight of male students in your school.

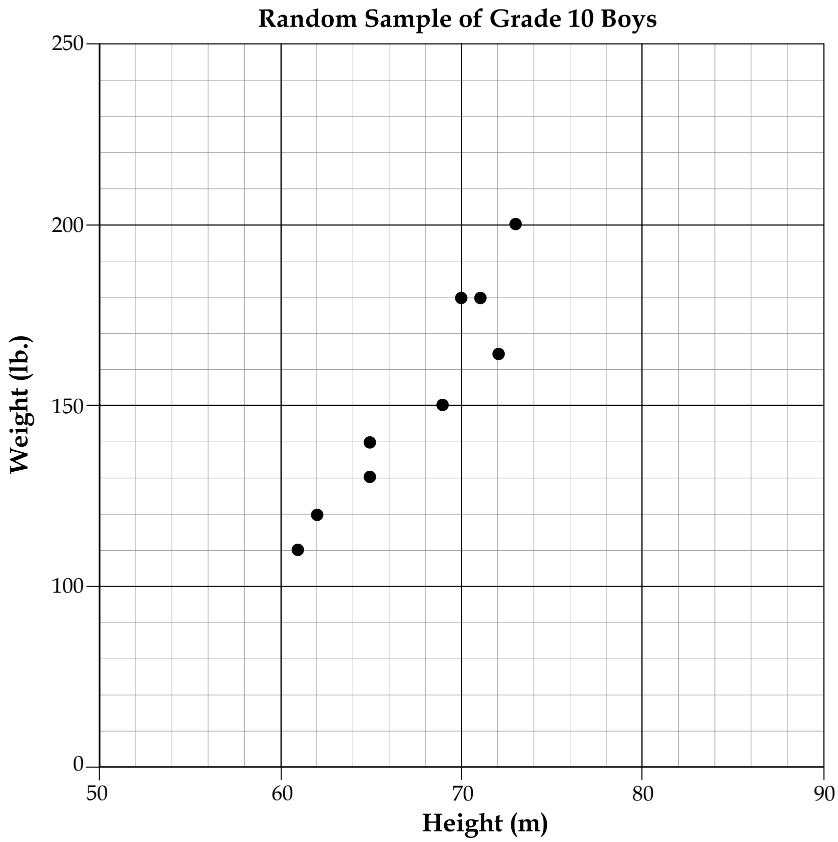
Height (inches)	Weight (pounds)
61	110
69	150
71	180
62	120
65	140
73	200
65	130
72	165
70	180

Use a graphing program like *Graphical Analysis 3*, a spreadsheet, or a graphing calculator to plot the given data.

Solution:

If you need help using technology to graph these data, call your tutor/marker or ask your learning partner for help.

If you are using a graphing calculator, you may want to record the steps you take to enter the data on your Resource Sheet.

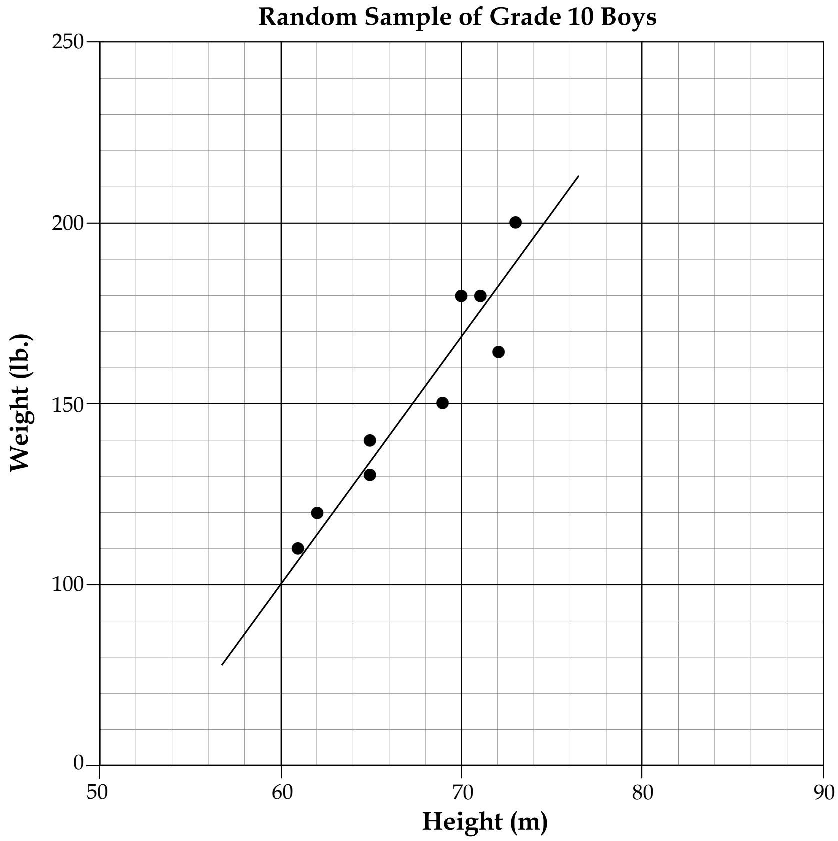


Drawing a Line of Best Fit

The data points in the graph above could be considered approximately linear. Generally speaking, the taller a male student is, the more he weighs.

It would be possible to draw a line on this graph to approximately represent the linear trend of the data, and it could be used to draw conclusions and make predictions. To best reflect the data, this ”line of best fit” should be drawn through as many points as possible. There will be some points that do not fall on the ”line of best fit.” Half of the remaining points should be above the line and the other half of the remaining points should be below the line. The line is not required to go through the origin.

It may look something like this:



It would be a good idea to include how to draw a "line of best fit" in your Resource Sheet.

To write the equation of the line, choose two points on the line and determine the slope. Use the slope and a point to write the equation.

The two points along the line that you can read the most precisely from the grid are (66, 140) and (72, 180). Locate these points on the line above.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{180 - 140}{72 - 66}$$

$$m = \frac{40}{6} = \frac{20}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 140 = \frac{20}{3}(x - 66)$$

$$y - 140 = \frac{20}{3}x - 440$$

$$y = \frac{20}{3}x - 440 + 140$$

$$y = \frac{20}{3}x - 300$$

You can use this equation to predict the height and weight of boys in your class.

Example 4

How much would you expect a boy who is 5′ 7″ tall to weigh?

Solution:

5 feet, 7 inches is 67 inches. Substitute 67 into the formula and solve for weight, y.

$$y = \frac{20}{3}x - 300$$

$$y = \frac{20}{3}(67) - 300$$

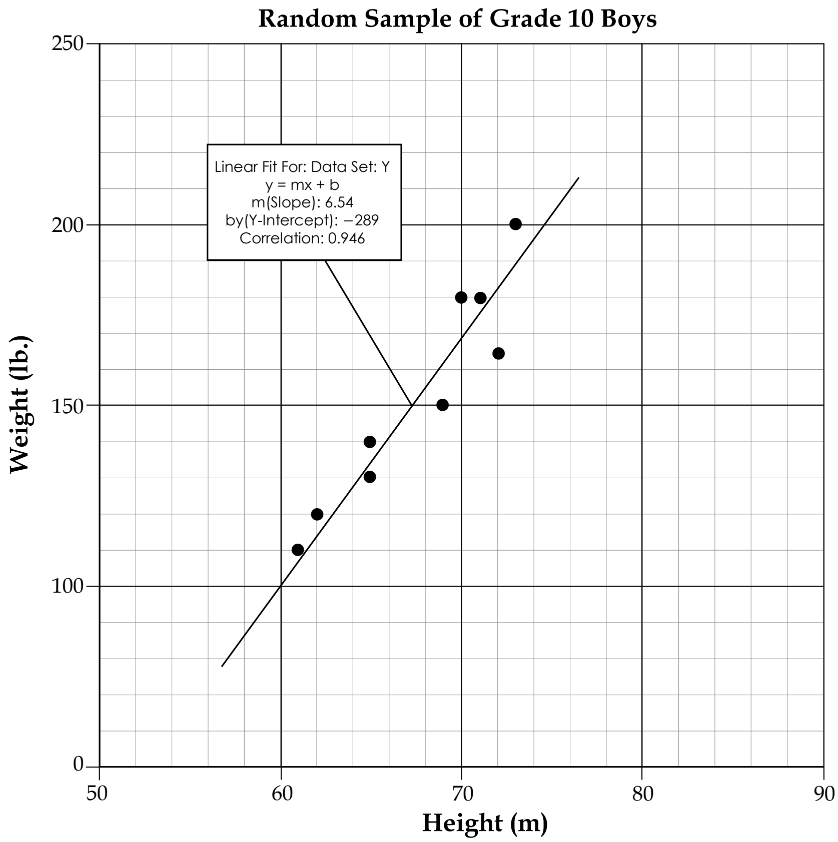
$$y = 146.6\overline{6}$$

He would weigh approximately 147 pounds.

Using Technology to Determine the Equation of the Line of Best Fit

Graphing calculators and software can be used to draw and calculate the equation of the line of best fit. Read the Help file or the manual that came with your graphing device to determine how your calculator or program performs that operation.

Graphical Analysis 3 draws the line of best fit (also called a linear regression), and calculates the equation like this:



Using the data in the graph's text box, substitute the values given for *m* and *b* into the slope-intercept form of the equation:

$$y = mx + b$$

$$y = 6.54x - 289$$

This is very close to the equation of the line of best fit determined above.

Correlation and the Correlation Coefficient (*r*-value)

The line of best fit on the graph above rises to the right. As the *x*-variable (height) increases, the *y*-variable (weight) also increases. This means the data have a positive correlation (and we see a positive slope). The points fall quite close to the line, so the relationship between the variables, or the correlation, is quite strong.

> **Correlation** refers to the relationship between the *x*-variable and *y*-variable, and can be either positive or negative. The strength or weakness of a correlation depends on whether the data points fall close to the "line of best fit." The closer the points are to the line, the stronger the correlation.

This definition would be handy to have on your Resource Sheet.

The line is a good representation of the data. Notice in the text box on the graph above that the correlation is given as 0.946. It is positive and close to 1. This number is also called the correlation coefficient (*r*).

Note: If you are using *Microsoft Excel*, you are given the R^2-value, so you have to take the square root $\left(\sqrt{}\right)$ of it to get the *R*-value.

Note: *Microsoft Excel* uses the capital "*R*" for correlation coefficient. When referring to the correlation coefficient in statistical terms, the lower case "*r*" is more commonly used. Either the upper- or lower-case letter is acceptable.

Example 5

Baseball statisticians discuss the Power-Speed number (#) of professional baseball players. It is a calculation based on the number of home runs and stolen bases a player has in a given season. The following data compare the ages of some professional baseball players and their Power-Speed numbers.

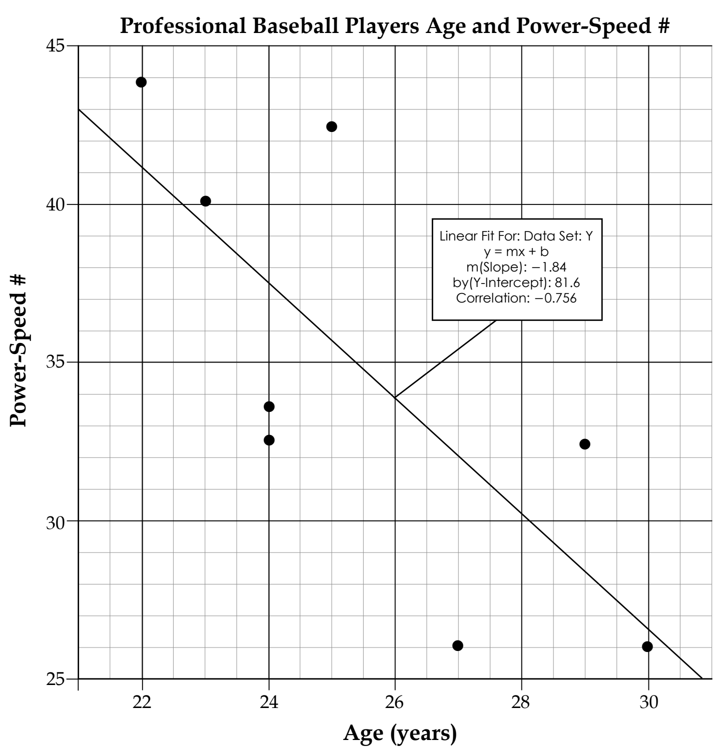
Age (years)	Power-Speed (#)
22	43.9
25	42.5
30	26.0
24	33.7
23	40.9
29	32.5
27	26.2
24	32.6

(data taken from www.baseball-reference.com/leaders/power_speed_number_season.shtml.)

Graph the data and find the equation of the line of best fit and correlation coefficient using technology.

Solution:

Using *Graphical Analysis 3*:



The line of best fit on the graph above falls as it moves to the right. As the x-variable (age) increases, the y-variable (Power-Speed #) decreases. This means the data have a negative correlation. The software calculated the equation of the line of best fit as $y = -1.84x + 81.6$. Notice that the slope is a negative value.

While the above line reflects the general trend of the data points, the points are quite spread out from the line. The correlation between age and number is not very strong. Notice in the text box on the graph above that the correlation coefficient is given as -0.756.

The following will serve as a helpful reference on your Resource Sheet.

> The **correlation coefficient** (denoted by the letter r) is a numerical indicator of the relationship between two sets of data. It measures whether the relationship is positive or negative, and whether it is strong or weak. The r-value ranges from -1 to $+1$.

The r-value will be **positive** if there is a positive relationship between the variables.

- as the x-variable increases in value, the y-variable also increases
- the graph of the line rises from left to right
- the slope of the line of best fit is positive

The r-value will be **negative** if there is a negative relationship between the variables.

- as the x-variable increases in value, the y-variable decreases in value
- the graph of the line falls from left to right
- the slope of the line of best fit is negative

The r-value will be **zero** if there is no relationship between the variables.

- the points are scattered randomly
- it is impossible to draw a line of best fit as the data do not display either a positive or negative trend

The correlation is **strong** if the r-value is close to –1 or +1.

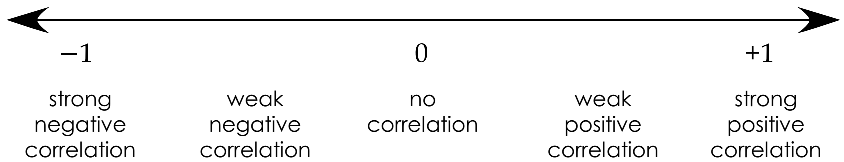
- most of the points tend to fall along the line or close to the line
- the closer the points are to the line, the stronger the correlation, either positive or negative

The correlation is **weak** if the r-value is closer to 0.

- most of the points do not fall along or near the line

- the farther the points are away from the line, the weaker the correlation, either positive or negative

The r-value will only be equal to exactly –1 or +1 if there is a perfect linear relationship between the two variables. Typically, the r-value is a decimal value between −1 and +1. The stronger the relationship between the two variables, the closer it will be to either −1 or +1. The weaker the relationship, the closer the r-value will be to 0.



The correlation coefficient is calculated using the following formula:

$$r = \frac{n\sum(xy) - \sum x \sum y}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2}\sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

where r = correlation coefficient

n = number of pairs of data

(x, y) = the points on the scatterplot

Σ is the capital Greek letter *sigma* and is used to indicate the sum. So Σx means you add all the x-values together. Σxy means you add all the products of xy together.

You will not be expected to know this for any assignments or exams in this course. You may come across the Σ notation in future mathematics courses.

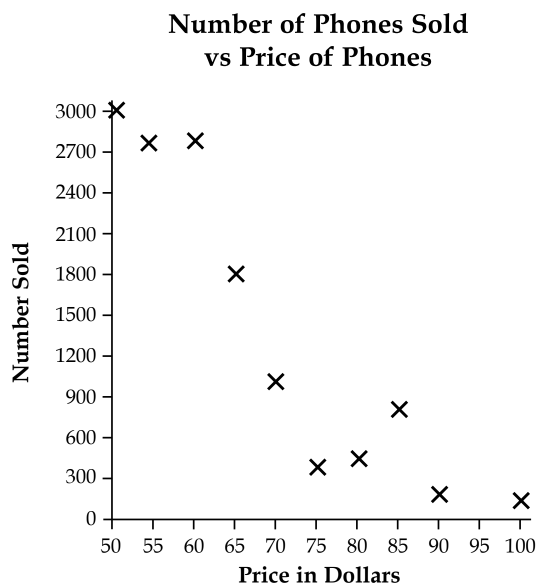
The formula is very complex, and you are not expected to use it. It is more important that you understand how to determine it using technology, that you know what it means, and that you are able to explain it.

Although there are many other types of correlations possible, all of the correlations considered in this course are linear. For that reason, all of the lines of best fit you draw will be straight lines.

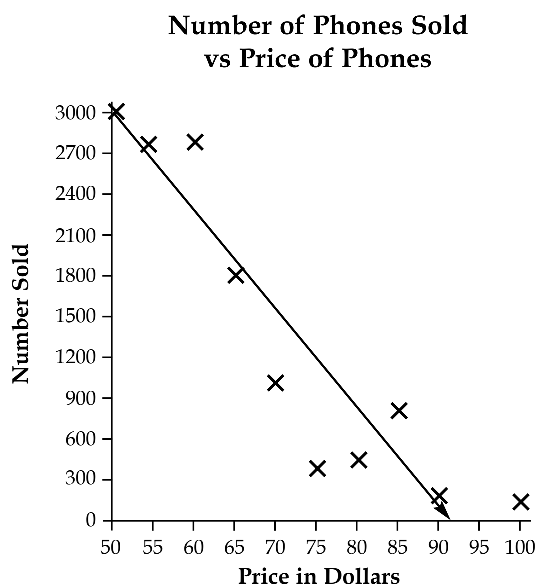
It is also important to note that you must be cautious when you give meaning to a calculated correlation coefficient. Even if a strong correlation exists, it does not necessarily mean that a change in one variable will cause a change in the other variable. There may be many other factors that affect the relationship between the two sets of data, as you will see in some of the examples that follow.

Example 6

Draw a line of best fit on the graph below. Describe the correlation between the two sets of data and estimate the *r*-value.



Solution:

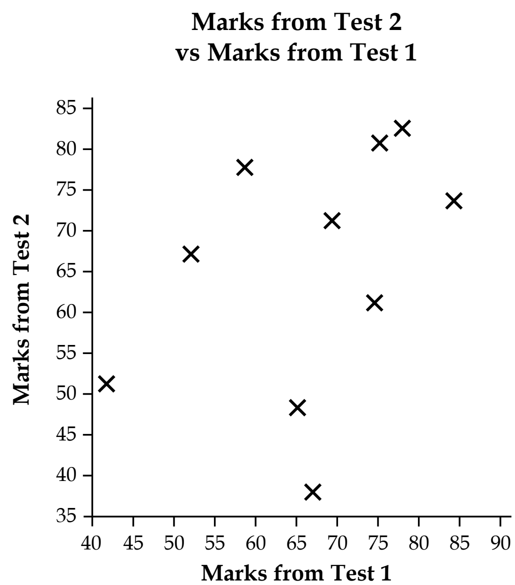


As the price in dollars increases, the number of phones sold decreases, so the correlation is negative. The points are quite close to the line, so the correlation is strong. A good estimate for the *r*-value would be between −0.85 to −0.95.

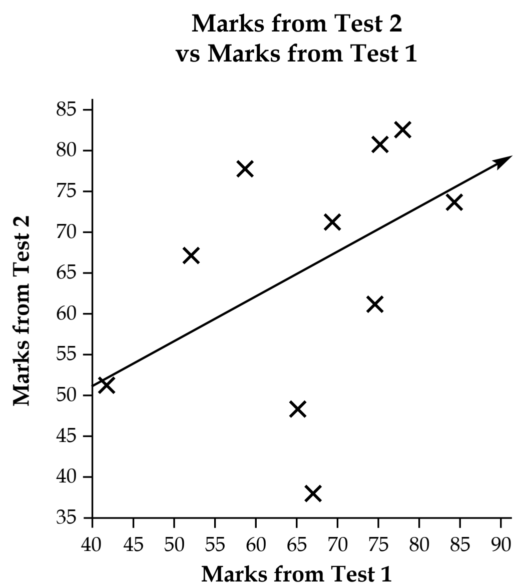
Using technology, the actual correlation coefficient is −0.922.

Example 7

Draw a line of best fit on the graph below. Describe the correlation between the two sets of data, and estimate the *r*-value.



Solution:



From the scatterplot, it appears that the students with higher marks on the first test have somewhat higher marks on the second test. It seems like a positive correlation exists between the marks on the two tests; however, the points are quite spread out from the line so the correlation is weak. The *r*-value will likely be between 0.2 and 0.5. In actuality, the *r*-value is equal to 0.370.

Example 8

There is a strong positive correlation between the number of fire engines that report to a fire and the amount of damage caused by the fire. Since the correlation is so strongly positive, can you conclude that sending more fire engines causes more damage? Explain.

Solution:

This conclusion is faulty because the number of engines sent is a response to the fire, not a cause of its severity. The more damage a fire causes, the more trucks are required, but it is the fire that causes the damage, not the trucks.

Note

Note: A common misconception with correlation coefficients is to say that because the correlation is high, the first variable causes the second variable. A correlation does not imply causality.

Learning Activity 7.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. If the intercepts of a line are $x = 3$ and $y = 9$, what is the slope of the graph?
2. Which has a larger volume with the same dimensions: rectangular prism or a rectangular pyramid?
3. Which variable is independent: comparing the amount of rain to the number of mosquitos?
4. Write the following as a function: $y + 2x = 4$.
5. What two numbers have a sum of -6 and a product of 5?
6. Evaluate: $\left(4^2\right)^{\frac{-1}{4}}$.
7. Jared has a big crush. He sits 6 desks to the left of his crush. If the desks are each 80 cm wide and there are no spaces in between them, how close does he sit to his crush (in metres)?
8. Multiply: $(x + 5)(2x + 1)$.

continued

Part B: Line of Best Fit and Correlation

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

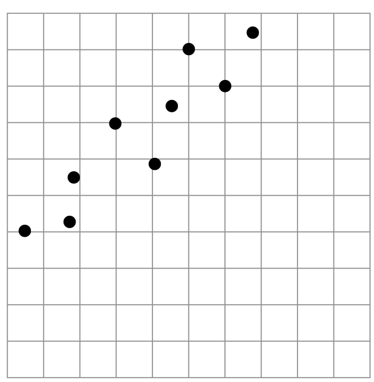
1. You record the cost of filling the fuel tank on your truck and the number of litres of gasoline purchased.

Litres (L)	34	42	86	91
Cost ($)	17.68	21.84	44.72	47.30

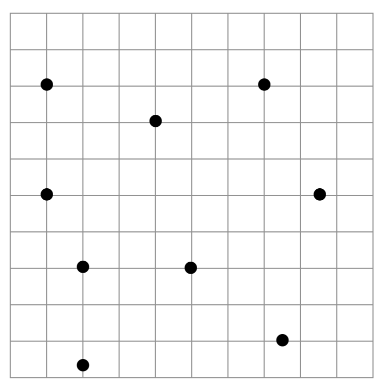
   a) Create a scatterplot of these data on graph paper, and draw the line of best fit.
   b) Use ordered pairs to write the equation of the resulting line in slope-intercept form. What does the slope of the equation represent?
   c) Use the equation to determine how much 15 L would cost you.
   d) Use the equation to determine how many litres you could purchase for $50.00.

2. Describe the correlation illustrated by the following scatterplots as strong or weak, and positive or negative, or no correlation. Estimate an *r*-value for each.

a)

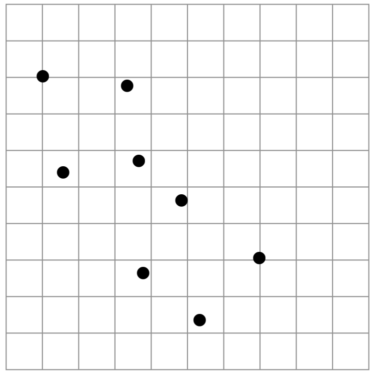


b)

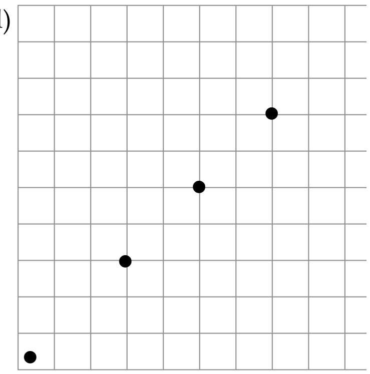


continued

c)



d)



3. a) Use technology to graph the following data.
   b) Estimate the *r*-value.
   c) Use technology to draw the line of best fit, and calculate the correlation coefficient.
   d) Explain what the correlation coefficient indicates about the data.
   e) Use the equation of the line of best fit to calculate the possible record time for the 2016 Summer Olympics. Does your answer seem reasonable to you?

Women's 400 m World Track Records	
Year	**Approximate Time (seconds)**
1920	65
1930	59
1940	57
1950	56
1960	53
1970	51
1980	48

continued

Learning Activity 7.4 (continued)

4. a) Use technology to graph the following data.
   b) Use technology to draw the line of best fit, determine the equation of the line, and calculate the correlation coefficient.
   c) Explain what the correlation coefficient indicates about the data. Can you use the equation to predict the score of the next Grey Cup game?

Grey Cup Final Scores	
Winning Score	**Losing Score**
39	20
46	10
38	16
38	9
27	17
26	21
27	10
31	19
35	31
32	14
21	17
16	6
24	7
14	7
24	3
16	13
23	7
16	6
33	14
35	10

Lesson Summary

In this lesson, you took data based on a specific context and graphed them to help you determine the linear equation of the resulting line. You used technology to plot points in a scatterplot, draw the line of best fit, and calculate the correlation coefficient. You can describe the relationship between two sets of data using an r-value between −1 and +1, and explain what that means in terms of the two variables. You also solved contextual problems based on linear relations.

Assignment 7.4

Line of Best Fit and Correlation

Total Marks = 51

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. The following chart compares temperatures in degrees Celsius and degrees Fahrenheit.

°F	°C
−40	−40
32	0
212	100

a) Draw a graph representing this linear relationship and draw a line of best fit. *(4 marks)*

Assignment 7.4: Line of Best Fit and Correlations (continued)

b) Use the points to calculate the equation of the line in point-slope form. *(3 marks)*

c) Use the equation to calculate the equivalent of 60°F. *(2 marks)*

d) Use the equation to calculate the equivalent of 28°C. *(2 marks)*

Assignment 7.4: Line of Best Fit and Correlations (continued)

2. a) Stack 8 pennies in a column. Measure the height of the column in mm. Record your measurement in the chart below. Build 6 other stacks of different heights, and measure and record the data. *(4 marks)*

Number of Pennies Stacked	Height (mm)
8	

b) Construct a scatterplot of your results. Draw a line of best fit. *(4 marks)*

c) Use points on the line to determine the equation of the line in slope-intercept form. (**Hint:** 0 pennies have 0 height.) *(3 marks)*

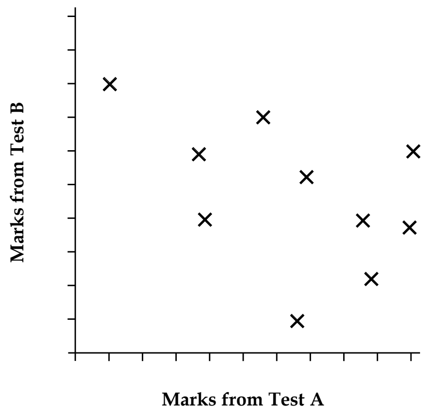
d) Use the equation to determine how tall a stack of 88 pennies would be. *(2 marks)*

e) Use the equation to determine the value (in dollars) of a stack of pennies that is 15 cm tall. *(2 marks)*

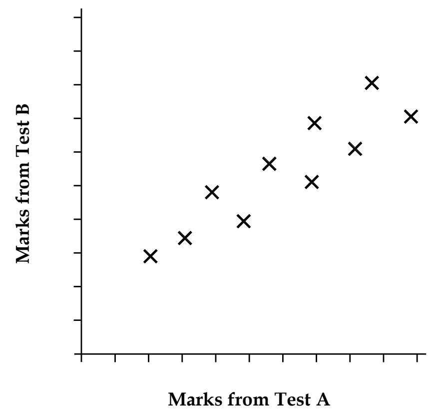
Assignment 7.4: Line of Best Fit and Correlations (continued)

3. Describe the correlation illustrated in the following scatterplots. *(5 marks)*

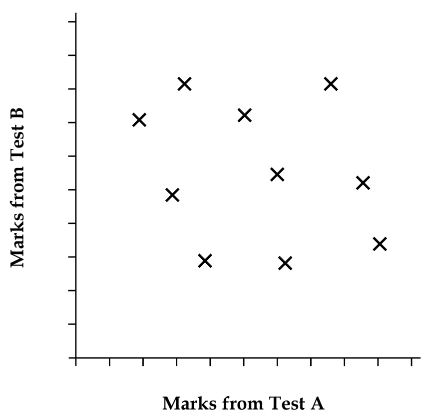
   a) ______________________________



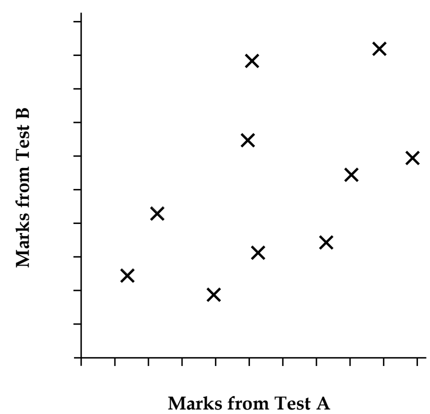
   b) ______________________________



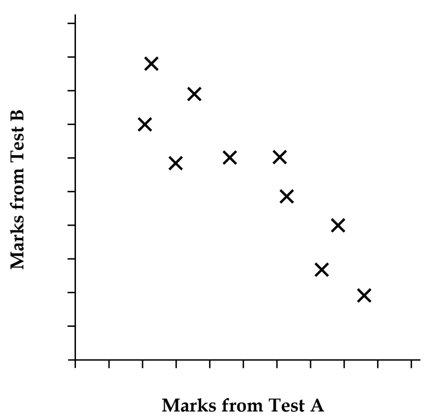
   c) ______________________________



   d) ______________________________



   e) ______________________________



4. The Hamilton High School basketball coach has kept track of a player's "shots attempted vs. shots made" statistics.

Shots Attempted	Shots Made
45	10
60	30
25	5
15	15
45	25
70	25
60	25
35	5
35	20
25	15
55	25
50	25
65	35
15	10
5	0
70	35
55	20
25	10
45	15
35	15

a) Determine the equation of the line of best fit and the correlation coefficient of the following data using technology. Print out a copy of your graph or include a hand-drawn copy. *(5 marks)*

b) Explain what the r-value means in this context. *(2 marks)*

c) Write the equation of the line in functional notation and find the following, rounded to the nearest whole number. *(4 marks)*

i) $f(18)$

ii) $f(43)$

iii) $f(100)$

Assignment 7.4: Line of Best Fit and Correlations (continued)

5. Understanding and using mathematical vocabulary correctly is important. To organize and review definitions, formulas, and examples of mathematical concepts and the related vocabulary, complete a "TRI to remember" frame. These triangular frames have three areas for you to fill in:
   - the formula pertaining to the concept with an explanation of what the variables represent, or a synonym (a word that has a similar meaning)
   - a definition of the concept, written in your own words, that explains the significance of the concept or how it is used
   - an example including the solution steps and/or a labelled diagram to illustrate the concept

   Study the examples provided and then complete three TRI-frames. Choose three concepts from the list below, write each one in a central band on a triangle, and complete the three areas of the frame for each. There is an extra sheet of frames for you to use as a draft copy if you wish.

   Concept choices (choose 3) *(3 × 3 = 9 marks)*

midpoint	slope-intercept form
parallel	correlation coefficient
distance formula	point-slope form
line of best fit	

Examples

TRI to remember

Concepts in *Coordinate Geometry*

formula or synonym

CONCEPT

definition

example and/or diagram

Write the formula and explain what the variables represent. Provide another word or idea that means the same.

Concept

Define the concept in your own words. Explain the significance of the concept or how it is used.

Create an example to illustrate the concept. Show the steps in the solution. Provide a labelled diagram.

An equation that describes how two variables, x and y, are related when the set of ordered pairs (x, y) form a straight line.

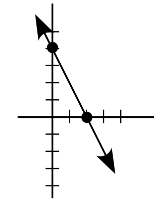
It can be found by rearranging the slope-intercept or point-slope forms so A, B, and C are integers and the equation is set equal to zero.

To graph a linear equation in general form, find the x- and y-intercepts by solving when y and x are equal to zero, or by using $m = \frac{-A}{B}$ and $b = \frac{-C}{B}$.

$2x + y - 4 = 0$

$A = 2 \quad B = 1 \quad C = -4$

$m = \frac{-2}{1} \qquad b = \frac{4}{1}$



General form of a linear equation

$Ax + By + C = 0$ where A, B, and C are integers and (x, y) is a point on the line.

$m = \frac{-A}{B}$

$b = \frac{-C}{B}$

90°

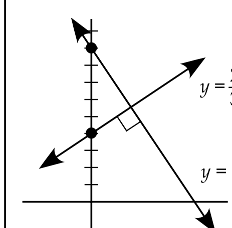
Perpendicular

Two lines are perpendicular when they intersect at 90° or right angles. The slopes of perpendicular lines are negative reciprocals.

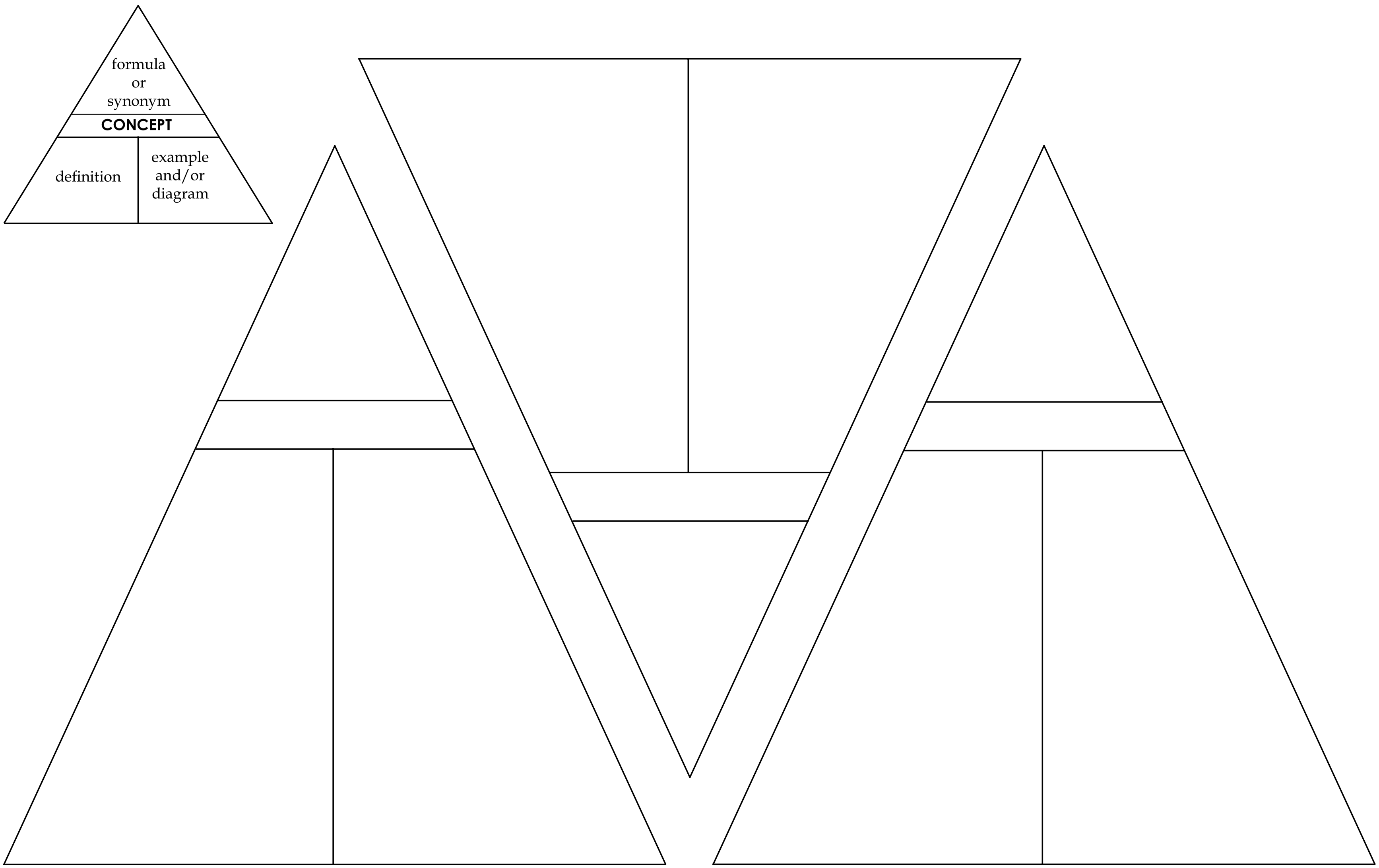
$y = \frac{2}{3}x + 4$

$y = \frac{-3}{2}x + 9$

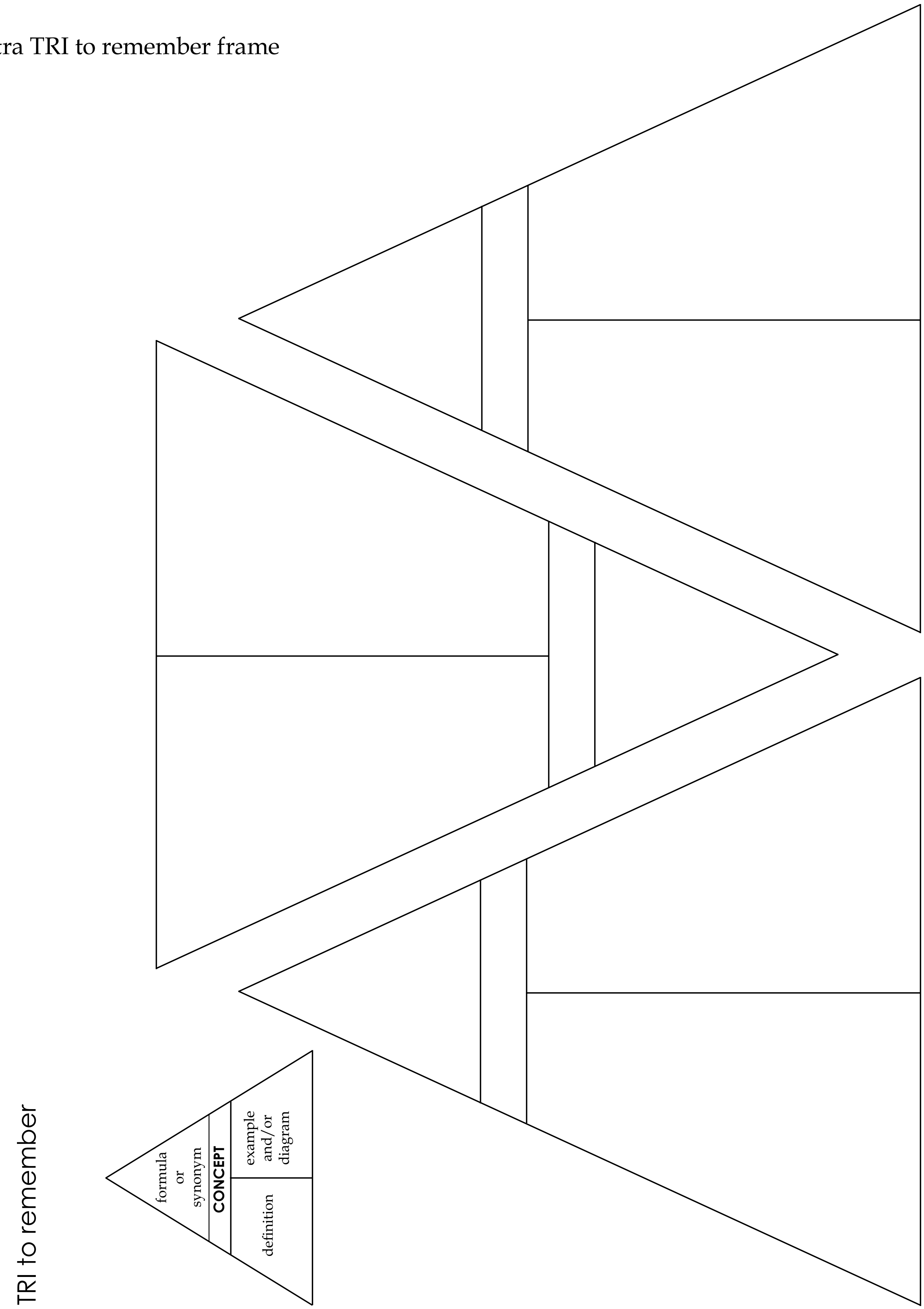
These lines are perpendicular.



TRI to remember



Extra TRI to remember frame



Module 7 Summary

Congratulations! You have finished Module 7. Just one more to go!

This module made extensive use of the coordinate grid: you plotted points, calculated distances, and found midpoints. You wrote equations in three different forms, given a variety of information about the line, and you graphed the equations and identified equivalent relations. Using technology, you found the correlation coefficient and the equation of the line of best fit, and used them to describe the data and solve contextual problems.

Many of the concepts in this module were extensions of skills and knowledge from earlier lessons, and even previous math courses. In turn, you will continue to build on these ideas in future courses. If you have any questions or concerns about what you learned in this module, please contact your tutor/marker to have things cleared up before you move on.

Submitting Your Assignments

You will not submit your Module 7 assignments to the Distance Learning Unit at this time. Instead, you will submit them, along with the Module 8 assignments, when you have completed Module 8.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 7
Coordinate Geometry

Learning Activity Answer Keys

Module 7: Coordinate Geometry

Learning Activity 7.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. In Home Economics, the teacher asks your group to make a double recipe of lasagna for the class, while other groups are making different parts of the meal. The recipe for lasagna calls for $\frac{3}{4}$ cup of Parmesan cheese. How much parmesan cheese will you need if you are making a double recipe?
2. Jared lost 55% of his weight when he went on his submarine sandwich diet. He originally weighed 420 pounds. How much does he weigh now?
3. The points (1,1) and (5,6) make a line. What is the slope of the line?
4. You work 9 am to 3 pm on Tuesday, Wednesday, Thursday, Saturday, and Sunday. You work 12 pm to 5 pm on Monday and Friday. How many hours do you work per week?
5. True or False: The area of 4 equal circles is the same as the surface area of a sphere with the same radius.
6. Factor: $4x^2 - 81$.
7. The sides of a right triangle are 5, 13, and 12. Which side is the hypotenuse?
8. Which is larger: 0.66 or $\frac{2}{3}$?

Answers:

1. $1\frac{1}{2}$ cups $\left(2 \times \frac{3}{4} = \frac{6}{4}\right)$
2. 189 pounds (50% of 420 is 210; 5% is 21; 210 − 21 = 189)
3. $\frac{5}{4}$ (rise is 6 − 1 or 5; run is 5 − 1 or 4)
4. 40 hours (9 to 3 is 6 hours, worked 5 days so 6 × 5 = 30 hours. 12 to 5 is 5 hours, worked 2 days so 5 × 2 = 10 hours.)
5. True (The formula for the area of a circle is πr^2; the formula for the surface area of a sphere is $4\pi r^2$.)

6. $(2x - 9)(2x + 9)$
7. 13 (The longest side is the hypotenuse.)
8. $\frac{2}{3}$ (Although they are very close, if you wrote this as a decimal the 6 would be repeating. If you rounded it to two decimal places, it would be 0.67.)

Part B: Distance and Midpoint

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. For each set of coordinates,
   a) find the length of the line segment $\overline{AB}$
   b) find the coordinates of the midpoint of $\overline{AB}$
      i) A(5, −3) and B(1, 0)
      ii) A(−1, 4) and B(14, −4)
      iii) A(2, 3) and B(0, −1)

   Answers:

   a) find the length of the line segment $\overline{AB}$

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$$

   i) $\mathrm{AB} = \sqrt{(1-5)^2 + (0-(-3))^2}$

   $= \sqrt{16 + 9} = \sqrt{25} = 5$

   ii) $\mathrm{AB} = \sqrt{(14-(-1))^2 + (-4-4)^2}$

   $= \sqrt{15^2 + (-8)^2} + \sqrt{225 + 64}$

   $= \sqrt{289} = 17$

   iii) $\mathrm{AB} = \sqrt{(0-2)^2 + (-1-3)^2}$

   $= \sqrt{4 + 16} = \sqrt{20}$

   $= \sqrt{4}\sqrt{5} = 2\sqrt{5}$

b) find the coordinates of the midpoint of $\overline{AB}$

Midpoint $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

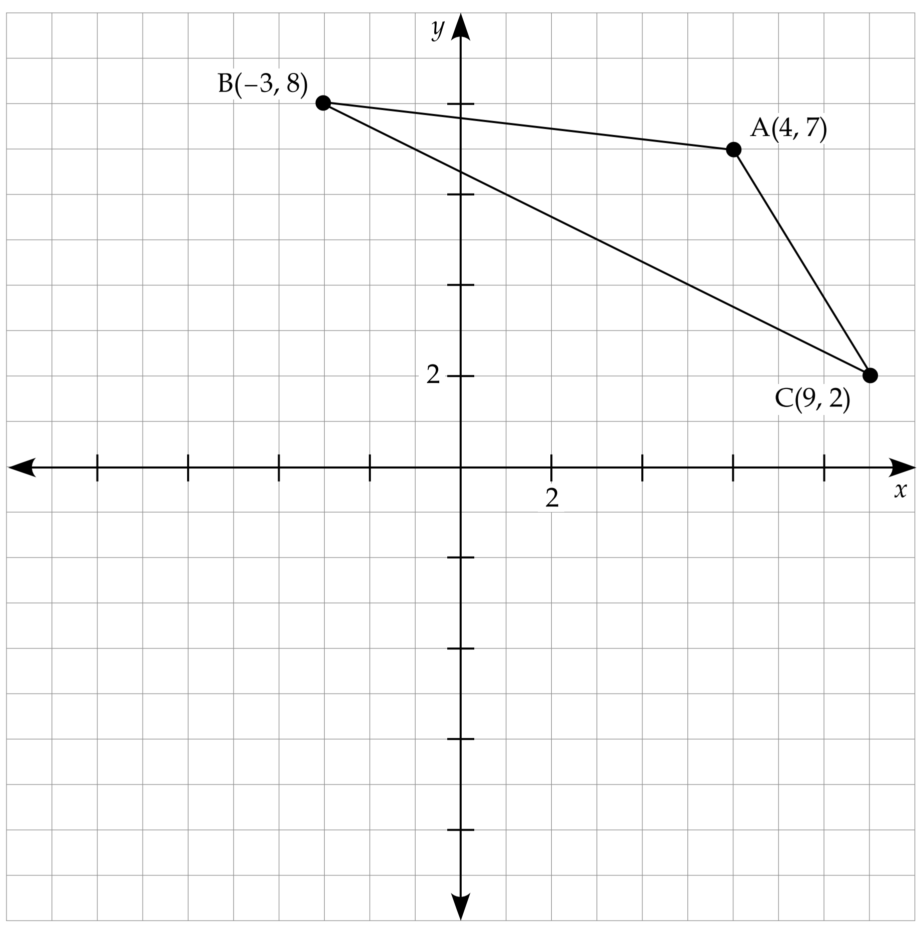
Note: The midpoint is the average of the x-values and the average of the y-values of the two endpoints.

i) $\left(\dfrac{5+1}{2}, \dfrac{-3+0}{2}\right)=\left(3, -\dfrac{3}{2}\right)$

ii) $\left(\dfrac{-1+14}{2}, \dfrac{4+(-4)}{2}\right)=\left(\dfrac{13}{2}, 0\right)$

iii) $\left(\dfrac{2+0}{2}, \dfrac{3+(-1)}{2}\right)=(1, 1)$

2. Determine whether the triangle with vertices A(4, 7), B(−3, 8), and C(9, 2) is isosceles. Write the lengths in radical form. (Remember that isosceles triangles have two sides equal in length.)



Answer:

From the diagram, *if* triangle ABC is isosceles, it looks like AB = AC. Check this assumption using the distance formula.

$$\begin{aligned}\text{AB} &= \sqrt{(4-(-3))^2+(7-8)^2}\\ &= \sqrt{49+1}\\ &= \sqrt{50}\end{aligned}$$

$$\begin{aligned}\text{AC} &= \sqrt{(4-9)^2+(7-2)^2}\\ &= \sqrt{25+25}\\ &= \sqrt{50}\end{aligned}$$

$\therefore$ AB = AC

$\therefore$ ΔABC is isosceles

3. Using the diagram from Question 2, show that the line segment joining the midpoints of $\overline{\text{AB}}$ and $\overline{\text{AC}}$ is half the length of $\overline{\text{BC}}$.

Answer:

If M is the midpoint of $\overline{\text{AB}}$, the coordinates of M are

$$\left(\frac{-3+4}{2}, \frac{8+7}{2}\right) = \left(\frac{1}{2}, \frac{15}{2}\right)$$

If N is the midpoint of $\overline{\text{AC}}$, the coordinates of N are

$$\left(\frac{4+9}{2}, \frac{7+2}{2}\right) = \left(\frac{13}{2}, \frac{9}{2}\right)$$

Compare the lengths of $\overline{\text{BC}}$ and $\overline{\text{MN}}$.

$$d = \sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$$

$$\begin{aligned}\therefore \overline{\text{BC}} &= \sqrt{(9-(-3))^2+(2-8)^2}\\ &= \sqrt{144+36}\\ &= \sqrt{180}\\ &= 6\sqrt{5}\end{aligned}$$

$$\overline{\text{MN}} = \sqrt{\left(\frac{13}{2}-\frac{1}{2}\right)^2+\left(\frac{9}{2}-\frac{15}{2}\right)^2}$$

$$= \sqrt{36+9}$$

$$= \sqrt{45}$$

$$= 3\sqrt{5}$$

$$\therefore \overline{\text{MN}} = 3\sqrt{5}$$

$$= \frac{1}{2}\left(6\sqrt{5}\right)$$

$$= \frac{1}{2}\overline{\text{BC}}$$

4. An online map (located at www.daftlogic.com/projects-google-maps-distance-calculator.htm) plots the locations of cities in Canada using coordinates. Winnipeg is at (49. 8946, −97.0752) and Vancouver is at (49.2678, −123.1348). If one unit on the grid represents 72 km, find the distance between Winnipeg and Vancouver. Round your final answer to the nearest km.

   Answer:

   $$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

   $$d = \sqrt{(49.8946-49.2678)^2 + (-97.0752-(-123.1348))^2}$$

   $$d = \sqrt{(0.6268)^2 + (26.0596)^2}$$

   $$d = \sqrt{679.4956304}$$

   $$d = 26.0671 \text{ units}$$

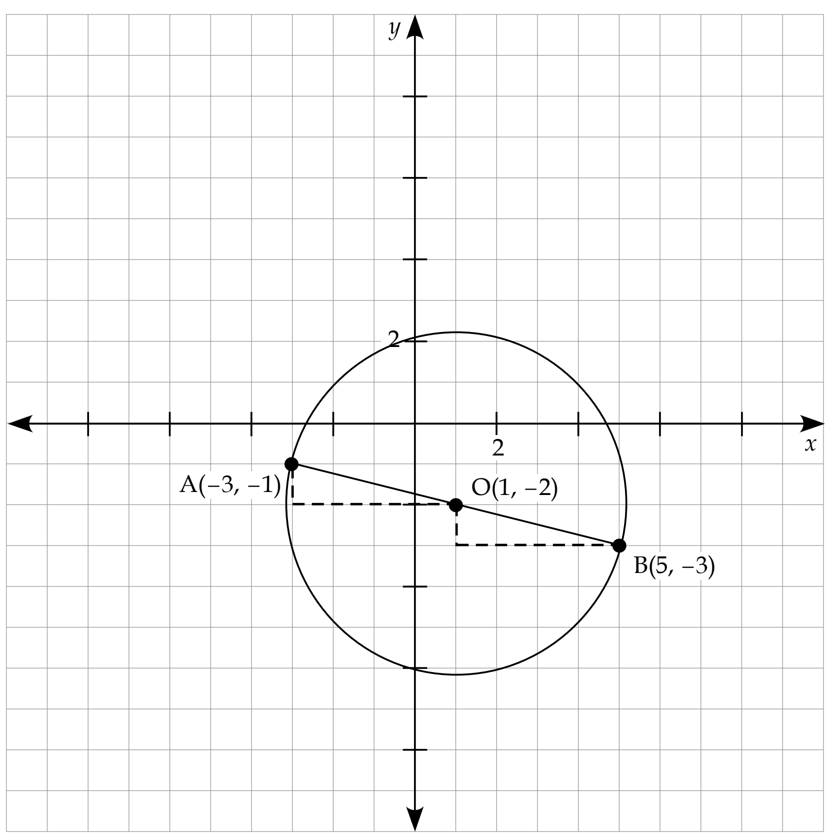
   If one unit is 72 km, then the total distance between Vancouver and Winnipeg is 72 × 26.0671 = 1877 km “as the crow flies.”

5. A circle with centre at O(1, −2) has one endpoint of a diameter at A(−3, −1). Find the coordinates of the other endpoint of the diameter, B, using the midpoint formula. Verify your answer using another strategy.

Answer:

$$(1, -2) = \left(\frac{x_1 + (-3)}{2}, \frac{y_1 + (-1)}{2}\right)$$

x-coordinate midpoint: $x_m = \dfrac{x_1 + x_2}{2}$		y-coordinate midpoint: $y_m = \dfrac{y_1 + y_2}{2}$
$1 = \dfrac{x_1 + (-3)}{2}$	multiply both sides of equation by 2	$-2 = \dfrac{y_1 + (-1)}{2}$
$2 = x_1 - 3$	isolate the variable	$-4 = y_1 - 1$
$5 = x_1$		$-3 = y_1$
	The coordinates of the endpoint are B(5, −3)	



You could verify your answer using the Pythagorean Theorem or the distance formula.

The $\frac{\text{rise}}{\text{run}}$ of the line segment $\overline{AO}$ is $\frac{-1}{4}$ and the slope of $\overline{OB}$ is $\frac{-1}{4}$. These values also represent the vertical and horizontal distances between the points and can be used in the Pythagorean Theorem.

$$\left(\overline{AO}\right)^2 = (-1)^2 + (4)^2 \qquad \left(\overline{OB}\right)^2 = (-1)^2 + (4)^2$$

$$\left(\overline{AO}\right)^2 = 17 \qquad \left(\overline{OB}\right)^2 = 17$$

$$\overline{AO} = \sqrt{17} \qquad \overline{OB} = \sqrt{17}$$

Or using the coordinates of points A, B, and O and the distance formula.

$$d_{AO} = \sqrt{(1-(-3))^2 + (-2-(-1))^2} \qquad d_{OB} = \sqrt{(5-1)^2 + (-3-(-2))^2}$$

$$d_{AO} = \sqrt{(4)^2 + (-1)^2} \qquad d_{OB} = \sqrt{(4)^2 + (-1)^2}$$

$$d_{AO} = \sqrt{17} \qquad d_{OB} = \sqrt{17}$$

The lengths $\overline{OB}$ and $\overline{AO}$ are the same. The slopes of $\overline{OB}$ and $\overline{AO}$ are the same. A, O, and B are colinear (all 3 points are on the same line), so O is the midpoint of $\overline{AB}$. The coordinates of the endpoint B are correct.

6. The three side lengths in a triangle are 18 units, 24 units, and 30 units. Is this a right triangle?

   Answer:

   If the three sides can be substitutes into the Pythagorean Theorem and make a true statement, the triangle must be a right triangle.

$a^2 + b^2$	c^2
$18^2 + 24^2$	30^2
324 + 576	900
900	900

   This is a true statement.

   A triangle with side lengths of 14, 24, and 30 units is a right triangle.

Learning Activity 7.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Factor: $2x^2 - 4x + 10$.
2. Find the y-intercept: $3y - 7x = 90$.
3. Simplify: $\dfrac{3}{4\sqrt[6]{x^9}}$.
4. Estimate the taxes, at 12%, of a pair of shoes that cost $74.89.
5. Complete the pattern: −1, 2, −3, ____, ____.
6. On average, you right-click once for every 5 left-clicks on your mouse. Because of this, the left button wears out 5 times faster. If the right button is estimated to last for 3 years, how many months will the left button last?
7. You have $4.65. If you buy a package of gum for $2.95, how much money will you have left over?
8. You ride your bike instead of taking the bus to get to work from April until October. Last summer it rained 35% of the days that you could have ridden your bike, and you don't ride in the rain. There are 240 days that you would have ridden your bike (you don't ride your bike on Hallowe'en). How many days did you ride your bike to work?

Answers:

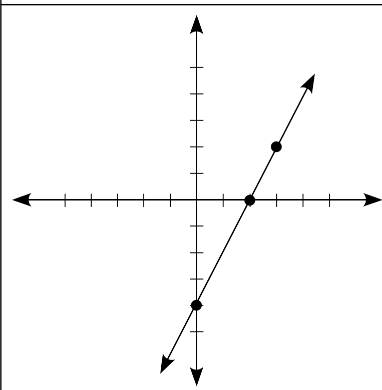
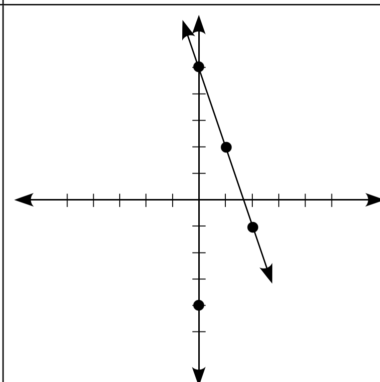
1. $2(x^2 - 2x + 5)$ ($x^2 - 2x + 5$ cannot be factored)
2. $y = 30$ ($3y - 7(0) = 90$)
3. $\dfrac{3}{4}x^{-\frac{9}{6}} = \dfrac{3}{4}x^{-\frac{3}{2}}$
4. $9 (10% of $74.89 is $7.49. 15% is approximately $11.25. Since 12% is about halfway between 10% and 15%, $9 is about halfway between the dollar values.)
5. 4, −5
6. Just over 7 months (3 years = 36 months. 36 ÷ 5 is a little bit larger than 7 because 35 ÷ 5 = 7.)
7. $1.70 ($2.95 + 1.00 = 3.95, $3.95 + 0.05 = $4.00, $4 + 0.65 = $4.65 so you still have $1.00 + $0.05 + $0.65)
8. 156 days (If you didn't ride for 35%, then you did ride for 65%= 15% + 50%. 10% = 24 days, 5% = 12 days, 50% = 120 days.)

Part B: Linear Relation Formulas

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Complete the following chart. Express each linear relation in all three forms and sketch a graph.

   Answers:

Slope-Intercept Form	$y = 2x - 4$	$y = -3x + 5$
General Form	$2x - y - 4 = 0$	$3x + y - 5 = 0$
Slope-Point Form	$m = 2$ $(0, -4)$ $y + 4 = 2(x - 0)$ $y + 4 = 2x$	$m = -3$ $(0, 5)$ $y - 5 = -3(x - 0)$ $y - 5 = -3x$
Graph		

2. Write the given linear relation in slope-intercept form using two different strategies. Explain the strategies.

$$y + \frac{1}{20} = \frac{1}{2}\left(x - \frac{2}{5}\right)$$

Answer:

The first strategy could be to determine the slope and point from the given equation, substitute that into $y = mx + b$, and then solve for b.

$$m = \frac{1}{2}$$

$$\left(\frac{2}{5}, \frac{-1}{20}\right)$$

$$y = mx + b$$

$$\frac{-1}{20} = \frac{1}{2}\left(\frac{2}{5}\right) + b$$

$$\frac{-1}{20} = \frac{1}{5} + b$$

$$\frac{-1}{20} - \frac{1}{5} = b$$

$$\frac{-1}{20} - \frac{4}{20} = b$$

$$\frac{-5}{20} = \frac{-1}{4} = b$$

$$\therefore y = \frac{1}{2}x - \frac{1}{4}$$

Another strategy could be to simplify the given equation by applying the distributive property and combining like terms.

$$y+\frac{1}{20}=\frac{1}{2}(x-\frac{2}{5})$$

$$y+\frac{1}{20}=\frac{1}{2}x-\frac{2}{10}$$

$$y=\frac{1}{2}x-\frac{2}{10}-\frac{1}{20}$$

$$y=\frac{1}{2}x-\frac{4}{20}-\frac{1}{20}$$

$$y=\frac{1}{2}x-\frac{5}{20}$$

$$\therefore y=\frac{1}{2}x-\frac{1}{4}$$

3. Explain two different strategies you could use to graph $6x - y + 3 = 0$. Sketch a graph.

   Answer:

   The equation $6x - y + 3 = 0$ could be graphed by

   a) finding the x- and y-intercepts of the line

      $6x - y + 3 = 0$

      x-intercept is where $y = 0$

      $6x - (0) + 3 = 0$

      $6x = -3$

      $$x=\frac{-3}{6}=\frac{-1}{2}$$

      y-intercept is where $x = 0$

      $6(0) - y + 3 = 0$

      $-y = -3$

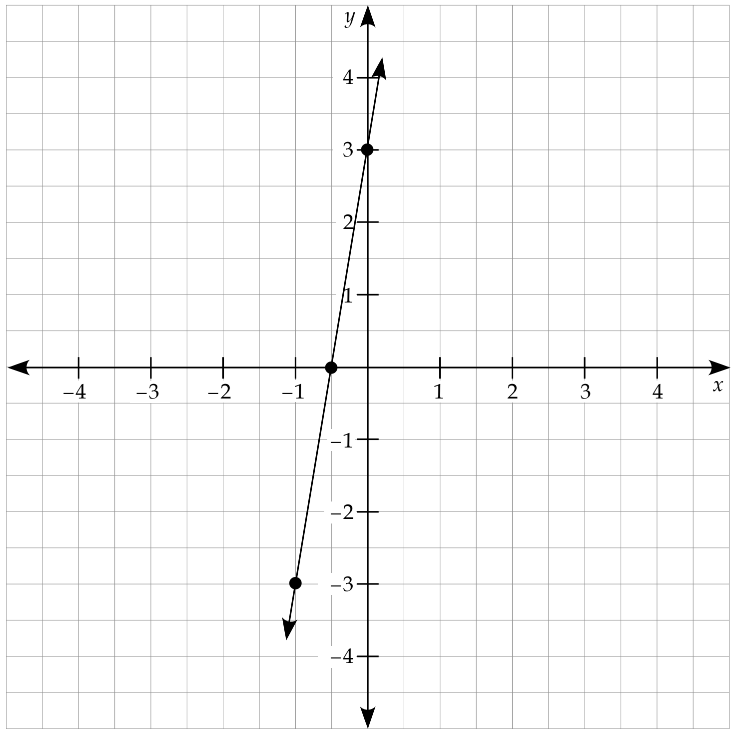
      $y = 3$

   b) using the coefficients A, B, and C to determine the slope and y-intercept of a linear equation written in general form

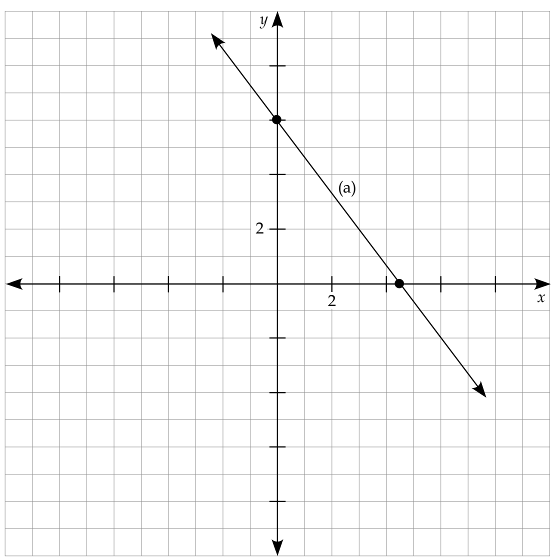
      $\mathrm{A}=6,\ \mathrm{B}=-1,\ \mathrm{C}=3$

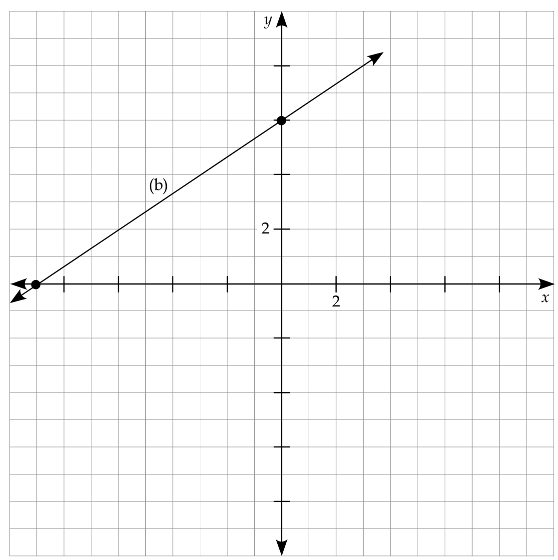
      $$m=\frac{-\mathrm{A}}{\mathrm{B}}=\frac{-6}{-1}=6$$

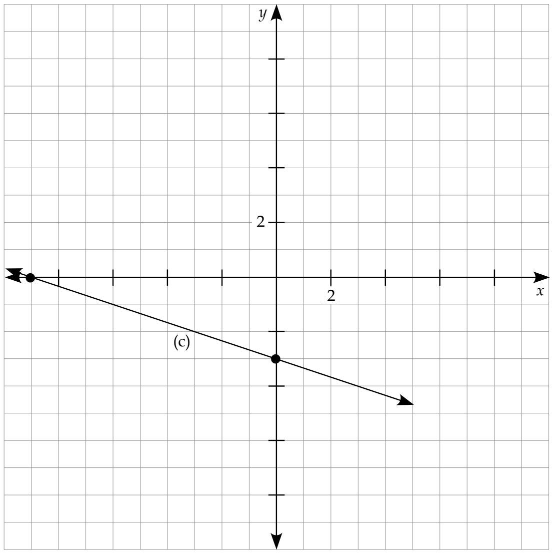
      $$b=\frac{-\mathrm{C}}{\mathrm{B}}=\frac{-3}{-1}=3$$



4. Match each graph to its equation(s).







Answers:

(b) $y - 8 = \frac{2}{3}(x - 3)$

(c) $y + 3 = \frac{-2}{6}x$

(b) $y = \frac{2}{3}x + 6$

(a) $4x + 3y - 18 = 0$

(none) $y + 2 = \frac{4}{6}(x + 2)$

(a) $y - 6 = \frac{-4}{3}x$

5. The slope and y-intercept of a line are given as:

$$m = \frac{-5}{3}$$

$$b = \frac{7}{3}$$

Write the general form of the equation for this linear relation ***without*** first writing it in the slope-intercept form.

Answer:

The general form is written as $Ax + By + C = 0$.

$$m = \frac{-A}{B} = \frac{-5}{3}$$

$$b = \frac{-C}{B} = \frac{7}{3}$$

so

$-A = -5$ or $A = 5$

$B = 3$

$-C = 7$ or $C = -7$

$5x + 3y - 7 = 0$

Learning Activity 7.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Is this relation a function: {(2, 4), (5, 8), (6, 1), (3, 7)}?
2. Find the midpoint of the line segment with end points (2, 6) and (4, 8).
3. Convert: 300 m = ________ km.
4. Evaluate: $\sqrt[4]{81}$.
5. June is a big birthday month for you. Your brother's is on June 9th, your nephew's is on June 20th, plus Father's Day is in June! If you want to spend $30 on each present and you have $85.00 saved up, will this be possible?
6. Identify the type of angle that has a measure of 345°.
7. What is the range of the following relation?
   {(1, 2), (2, 4), (3, 6), (4, 8)}
8. Write as an improper fraction: $\frac{19}{16}$.

Answers:

1. Function (No input values are repeated.)
2. (3, 7) ((2 + 4) ÷ 2 = 3, (6 + 8) ÷ 2 = 7)
3. 0.3 km (Remember, 1 km = 1000 m)
4. 3 (If you don't recognize this immediately, $\sqrt[4]{81} = \sqrt[4]{9^2} = 9^{\frac{2}{4}} = 9^{\frac{1}{2}}$. In general, the square of a square is equal to the first square root to the power of 4. For example, $2^2 = 4$, $4^2 = 16$ so $(2^2)^2 = 2^4 = 16$.)
5. No, $30 × 3 = $90. (You only have $85.00.)
6. Reflex (Reflex angles are between 180° and 360°.)
7. Range is {2, 4, 6, 8}.
8. $1\frac{3}{16}$

Part B: Writing Linear Equations Based on Different Information

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Explain the process you go through to write the equation of a linear relation when you are given the coordinates of two points on the line.

   Answer:

   If I am given the coordinates of two points on a line, I label the points as (x_1, y_1) and (x_2, y_2). I use the coordinates to determine the slope of the line using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

   Using the slope and either one of the points given, I substitute the values into the point-slope formula $y - y_1 = m(x - x_1)$. I can leave the equation in this form or simplify it, combine the like terms, and rearrange them into the slope-intercept form or the general form of a linear equation.

2. Write the equation of a linear relation in slope-intercept form, given $m = \frac{-9}{2}$ and $b = \frac{1}{2}$.

   Answer:

   Slope-intercept form is $y = mx + b$. The equation would be $y = \frac{-9}{2}x + \frac{1}{2}$.

3. A line has a slope of $\frac{8}{3}$ and goes through the point (−72, −94). Write the equation in point-slope form and general form.

   Answer:

   $m = \frac{8}{3}, (-72, -94)$

   In point-slope form: $y - y_1 = m(x - x_1)$.

   $$y + 94 = \frac{8}{3}(x + 72)$$

In general form: $Ax + By + C = 0$.

$$y + 94 = \frac{8}{3}(x + 72)$$

$$y + 94 = \frac{8}{3}x + 192$$

$$y = \frac{8}{3}x + 192 - 94$$

$$y = \frac{8}{3}x + 98$$

$$0 = \frac{8}{3}x - y + 98$$

$$0 = 8x - 3y + 294$$

4. Write the equation of the line that goes through the points (26, 9) and (43, −6). State your answer in $y = mx + b$ form.

   Answer:

   Given: (26, 9) (43, −6)

   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

   $$m = \frac{-6 - 9}{43 - 26}$$

   $$m = \frac{-15}{17}$$

   Use the slope and one of the points, and substitute the values into $y = mx + b$ to solve for b.

$y = mx + b$

$9 = \frac{-15}{17}(26) + b$

$9 = \frac{-390}{17} + b$

$\frac{153}{17} + \frac{390}{17} = b$

$\frac{543}{17} = b$

$y = \frac{-15}{17}x + \frac{543}{17}$

5. A line crosses the x-axis at 14 and the y-axis at 35. Write the equation of the line in general form. Use two different methods to arrive at the answer.

   Answer:

   The coordinates of the two intercepts are written as (14, 0) and (0, 35). Calculate the slope and then use that and one of the points to write the equation.

   This question is challenging for some students. Don't hesitate to call your tutor/marker for help. Also, it would be helpful to include this example on your Resource Sheet.

   Method 1:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{35 - 0}{0 - 14}$

$m = \frac{35}{-14} = \frac{-35}{14} = \frac{-5}{2}$

$y - 0 = \frac{-5}{2}(x - 14)$

$y = \frac{-5x}{2} + 35$

$0 = \frac{-5x}{2} - y + 35$

$0 = -5x - 2y + 35$

$5x + 2y - 35 = 0$

Method 2:

$m = \frac{-35}{14} = \frac{-5}{2}$ $\qquad$ $m = \frac{-A}{B}$

$b = 35$ $\qquad$ $b = \frac{-C}{B}$

When using $m = \frac{-A}{B}$ and $b = \frac{-C}{B}$, B must have the same value in both equations. Rewrite the y-intercept as an equivalent fraction with a denominator of 14.

$$b = \frac{35}{1} = \frac{70}{2}$$

$$m = \frac{-A}{B} = \frac{-35}{14} = \frac{-5}{2}$$

$$b = \frac{-C}{B} = \frac{70}{2}$$

$$-A = -5$$

$$A = 5$$

$$B = 2$$

$$-C = 70$$

$$C = -70$$

$$Ax + By + C = 0$$

$$5x + 2y - 70 = 0$$

6. A line is parallel to $y = -3x - 55$ and goes through the point $(-8, 19)$. Write the equation of the line in point-slope form.

   Answer:

   Parallel lines have the same slope, so use $m = -3$ and $(-8, 19)$.

   $y - y_1 = \mathrm{m}(x - x_1)$

   $y - 19 = -3(x + 8)$

7. Write the equation of the line that is perpendicular to $5x + 6y - 72 = 0$ and has an x-intercept of -4 in general form. Compare the coefficients in both equations. What do you notice?

Answer:

Perpendicular lines have negative reciprocal slopes.

The slope of the perpendicular line is found by rewriting the equation in slope-intercept form or by using $m = \frac{-A}{B} = \frac{-5}{6}$.

$$\begin{aligned} 5x + 6y - 72 &= 0 \\ 6y &= -5x + 72 \\ y &= \frac{-5}{6}x + 12 \end{aligned}$$

So the slope of the line we are looking for is the negative reciprocal.

$$m = \frac{6}{5}$$

The x-intercept is at (–4, 0).

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{6}{5}(x + 4) \qquad \text{point form}$$

$$y = \frac{6}{5}x - \frac{24}{5} \qquad \text{slope-intercept form}$$

$$6x - 5y - 24 = 0 \qquad \text{general form}$$

The line perpendicular to this was $5x + 6y - 72 = 0$.

The coefficients have switched positions, and the sign of the y-coefficient is different.

8. Write the equation of the line that is the perpendicular bisector (bisector means "cuts in half") of the line segment between (−3, −8) and (15, 6). State the answer in point-slope form. (Hint: The line must pass through the midpoint of the line segment.)

You should include the definition of a bisector on your Resource Sheet.

Answer:

The line will go through the midpoint of the segment and its slope will be the negative reciprocal of the slope of the line segment.

Given: (−3, −8) and (15, 6)

Midpoint of the segment:

$$\mathrm{M} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\mathrm{M} = \left(\frac{-3 + 15}{2}, \frac{-8 + 6}{2}\right)$$

$$\mathrm{M} = \left(\frac{12}{2}, \frac{-2}{2}\right)$$

$$\mathrm{M} = (6, -1)$$

The slope of the line segment:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 + 8}{15 + 3}$$

$$m = \frac{14}{18}$$

$$m = \frac{7}{9}$$

The slope of the perpendicular line will be $\frac{-9}{7}$.

The equation of the perpendicular bisector is:

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{-9}{7}(x - 6)$$

9. Write the equation of a line that is perpendicular to the line $y = -12$, and explain your answer.

Answer:

The line $y = -12$ is a horizontal line that crosses the y-axis at -12. The slope of a horizontal line is 0. The reciprocal of 0 is undefined, but any vertical line will be perpendicular to the horizontal line $y = -12$. The equation of a vertical line is $x = \Re$ where $\Re$ is any real number.

Learning Activity 7.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. If the intercepts of a line are $x = 3$ and $y = 9$, what is the slope of the graph?
2. Which has a larger volume with the same dimensions: rectangular prism or a rectangular pyramid?
3. Which variable is independent: comparing the amount of rain to the number of mosquitos?
4. Write the following as a function: $y + 2x = 4$.
5. What two numbers have a sum of −6 and a product of 5?
6. Evaluate: $\left(4^2\right)^{\frac{-1}{4}}$.
7. Jared has a big crush. He sits 6 desks to the left of his crush. If the desks are each 80 cm wide and there are no spaces in between them, how close does he sit to his crush (in metres)?
8. Multiply: $(x + 5)(2x + 1)$.

Answers:

1. $-3\left(m = \dfrac{9-0}{0-3} = \dfrac{9}{-3}\right)$
2. Prism (A prism’s volume is 3 times larger than a pyramid’s volume.)
3. Amount of rain (No one can control the rain, and if the rain does not fall, the likelihood of mosquitoes is minimal.)
4. $f(x) = 4 - 2x$
5. −5, −1
6. $\dfrac{1}{2}\left(\left(4^2\right)^{\frac{-1}{4}} = 4^{\frac{-2}{4}} = \dfrac{1}{4^{\frac{1}{2}}}\right)$
7. 4.8 m ($80 \times 6 = 480$ cm, 100 cm = 1 m)
8. $2x^2 + 11x + 10$

Part B: Line of Best Fit and Correlation

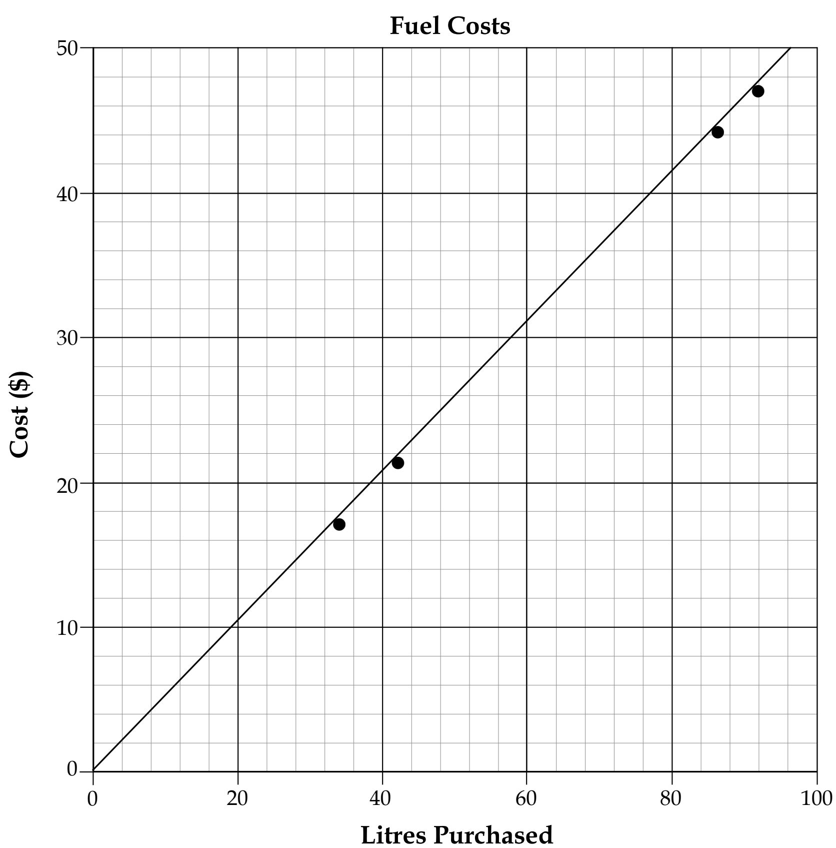
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You record the cost of filling the fuel tank on your truck and the number of litres of gasoline purchased.

Litres (L)	34	42	86	91
Cost ($)	17.68	21.84	44.72	47.30

a) Create a scatterplot of these data on graph paper, and draw the line of best fit.

Answer:



b) Use ordered pairs to write the equation of the resulting line in slope-intercept form. What does the slope of the equation represent?

Answer:

You may have chosen different points to use, but should end up with the same answer (or very close).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{21.84 - 17.68}{42 - 34}$$

$$m = \frac{4.16}{8} = 0.52$$

$$y - y_1 = m(x - x_1)$$

$$y - 17.86 = 0.52(x - 34)$$

$$y - 17.68 = 0.52x - 17.68$$

$$y = 0.52x$$

The slope represents the cost per litre. Gas is \$0.52/L.

c) Use the equation to determine how much 15 L would cost you.

Answer:

$y = 0.52x$

$y = 0.52(15)$

$y = 7.8$

15 L would cost you \$7.80.

d) Use the equation to determine how many litres you could purchase for \$50.00.

Answer:

$$y = 0.52x$$

$$50 = 0.52x$$

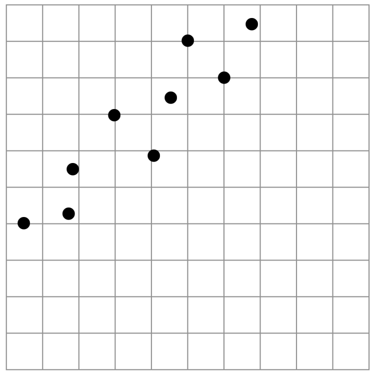
$$x = \frac{50}{0.52}$$

$$x = 96.15$$

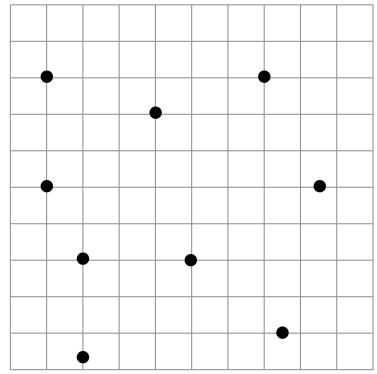
You could purchase about 96 L for \$50.00.

2. Describe the correlation illustrated by the following scatterplots as strong or weak, and positive or negative, or no correlation. Estimate an r-value for each.

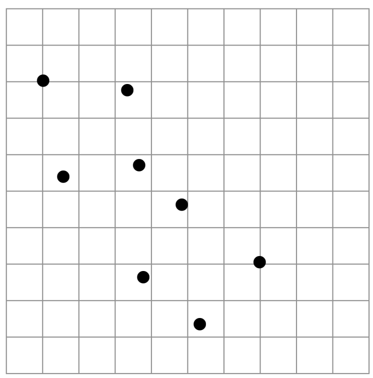
a)



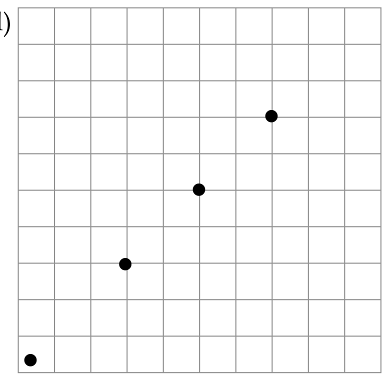
b)



c)



d)



Answers:

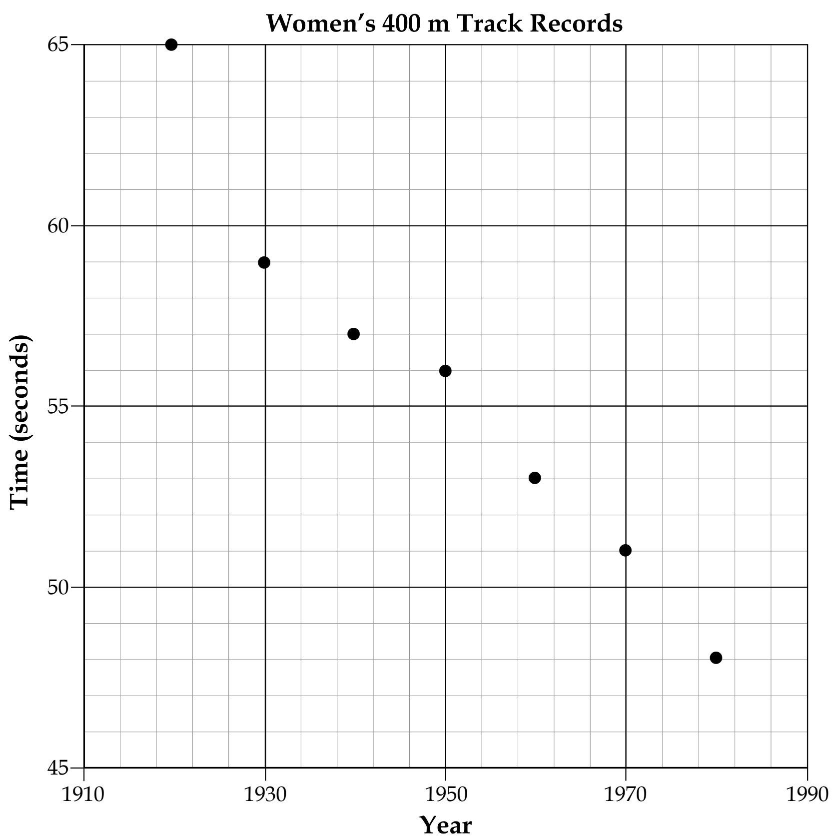
	Actual	Acceptable Range
a) strong positive correlation	$r = 0.8$	0.7 to 0.9
b) no correlation	$r = 0$	-0.1 to 0.1
c) weak negative correlation	$r = -0.6$	-0.7 to -0.5
d) strong positive correlation	$r = 1$	0.9 to 1.0

Note: The r-values provided are actual values. Your estimate should be within one-tenth of the answers given.

3. a) Use technology to graph the following data.

Women's 400 m World Track Records	
Year	**Approximate Time (seconds)**
1920	65
1930	59
1940	57
1950	56
1960	53
1970	51
1980	48

Answer:



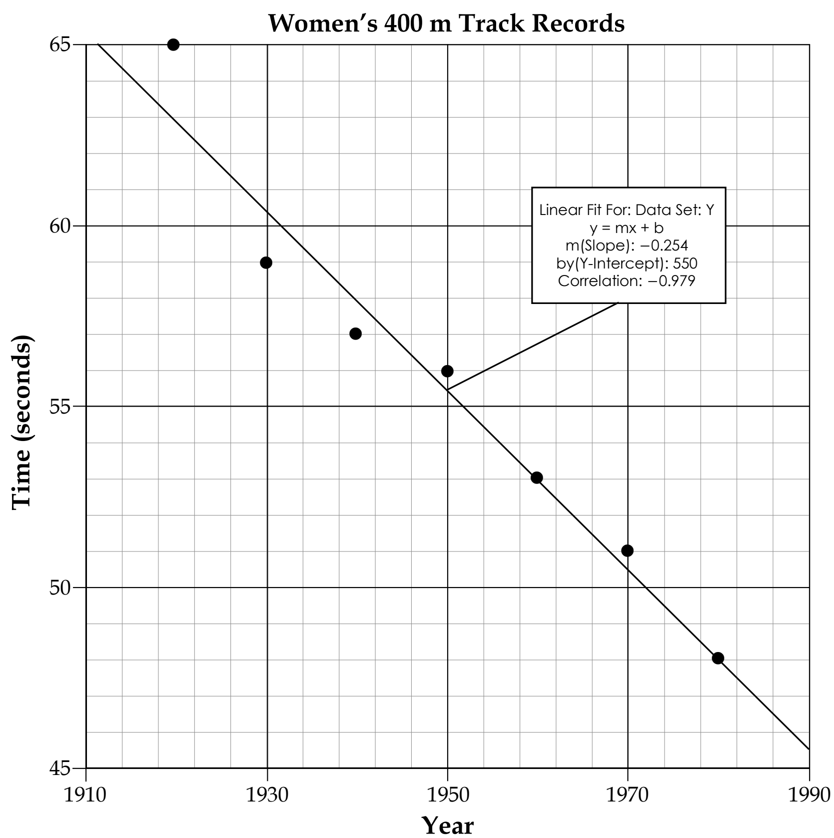
b) Estimate the *r*-value.

Answer:

The trend of the data illustrates a strong negative correlation. The points fall quite closely along a straight line so a good estimate of the *r*-value would be −0.9.

c) Use technology to draw the line of best fit, and calculate the correlation coefficient.

Answer:



The correlation coefficient is given as –0.979.

d) Explain what the correlation coefficient indicates about the data.

Answer:

The *r*-value indicates that there is a very strong negative relationship between the years and the record time of the 400 m race. As the years progress, the record time of the women's 400 m decreases. The *r*-value is very close to −1, so the relationship is nearly linear.

e) Use the equation of the line of best fit to calculate the possible record time for the 2016 Summer Olympics. Does your answer seem reasonable to you?

Answer:

The equation of the line of best fit is given as $y = -0.254x + 550$.

Substitute $x = 2016$.

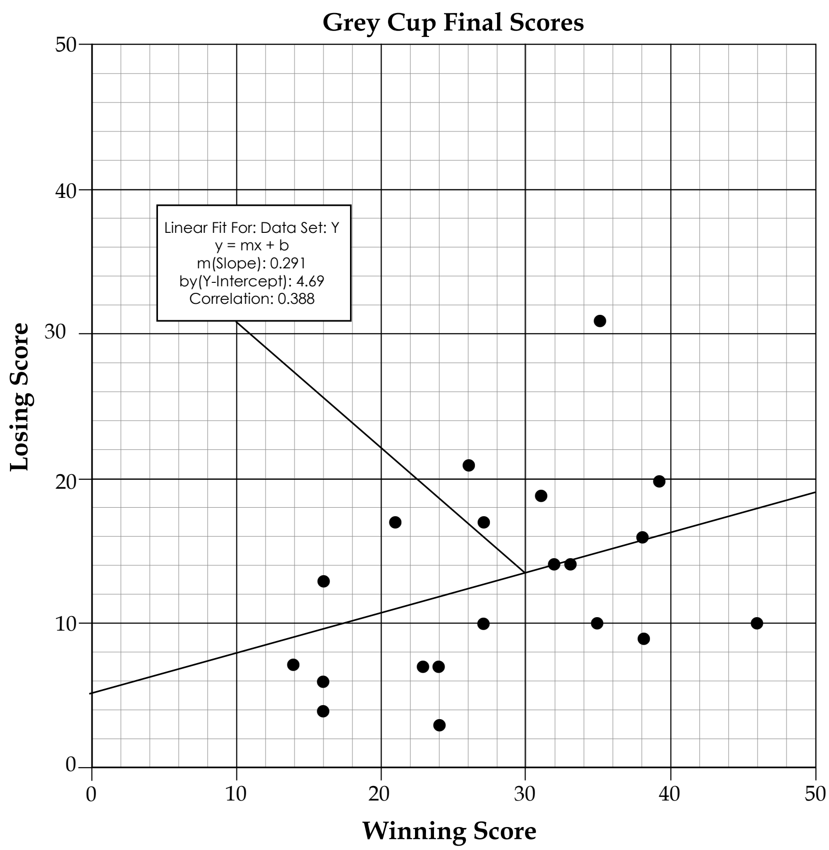
$y = -0.254(2016) + 550$

$y = 37.936$

The record could be 37.936 seconds. This is possible, but there is probably a limit to how fast a human can run, and since the current record is 47.06 seconds and was set in 1985, we may have reached it.

4. a) Use technology to graph the following data.

Answer:



b) Use technology to draw the line of best fit, determine the equation of the line, and calculate the correlation coefficient.

Answer:

$y = 0.291x + 4.69$

$r = 0.388$

c) Explain what the correlation coefficient indicates about the data. Can you use the equation to predict the score of the next Grey Cup game?

Grey Cup Final Scores	
Winning Score	**Losing Score**
39	20
46	10
38	16
38	9
27	17
26	21
27	10
31	19
35	31
32	14
21	17
16	6
24	7
14	7
24	3
16	13
23	7
16	6
33	14
35	10

Answer:

It appears that as the winning score increases, the losing team's score also increases, but the points are very spread out from the line. The correlation may be described as a very weak positive correlation. The equation of the line would be a very poor predictor of the final score in a game because factors such as which teams are playing, injuries, and even the weather affect the final outcome in a football game.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 7
Coordinate Geometry

Learning Activity Answer Keys

Module 7: Coordinate Geometry

Learning Activity 7.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. In Home Economics, the teacher asks your group to make a double recipe of lasagna for the class, while other groups are making different parts of the meal. The recipe for lasagna calls for $\frac{3}{4}$ cup of Parmesan cheese. How much parmesan cheese will you need if you are making a double recipe?
2. Jared lost 55% of his weight when he went on his submarine sandwich diet. He originally weighed 420 pounds. How much does he weigh now?
3. The points (1,1) and (5,6) make a line. What is the slope of the line?
4. You work 9 am to 3 pm on Tuesday, Wednesday, Thursday, Saturday, and Sunday. You work 12 pm to 5 pm on Monday and Friday. How many hours do you work per week?
5. True or False: The area of 4 equal circles is the same as the surface area of a sphere with the same radius.
6. Factor: $4x^2 - 81$.
7. The sides of a right triangle are 5, 13, and 12. Which side is the hypotenuse?
8. Which is larger: 0.66 or $\frac{2}{3}$?

Answers:

1. $1\frac{1}{2}$ cups $\left(2 \times \frac{3}{4} = \frac{6}{4}\right)$
2. 189 pounds (50% of 420 is 210; 5% is 21; 210 − 21 = 189)
3. $\frac{5}{4}$ (rise is 6 − 1 or 5; run is 5 − 1 or 4)
4. 40 hours (9 to 3 is 6 hours, worked 5 days so 6 × 5 = 30 hours. 12 to 5 is 5 hours, worked 2 days so 5 × 2 = 10 hours.)
5. True (The formula for the area of a circle is πr^2; the formula for the surface area of a sphere is $4\pi r^2$.)

6. $(2x - 9)(2x + 9)$
7. 13 (The longest side is the hypotenuse.)
8. $\frac{2}{3}$ (Although they are very close, if you wrote this as a decimal the 6 would be repeating. If you rounded it to two decimal places, it would be 0.67.)

Part B: Distance and Midpoint

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. For each set of coordinates,
   a) find the length of the line segment $\overline{AB}$
   b) find the coordinates of the midpoint of $\overline{AB}$
      i) A(5, −3) and B(1, 0)
      ii) A(−1, 4) and B(14, −4)
      iii) A(2, 3) and B(0, −1)

   Answers:

   a) find the length of the line segment $\overline{AB}$

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$$

   i) $$\mathrm{AB} = \sqrt{(1-5)^2 + (0-(-3))^2}$$
   $$= \sqrt{16 + 9} = \sqrt{25} = 5$$

   ii) $$\mathrm{AB} = \sqrt{(14-(-1))^2 + (-4-4)^2}$$
   $$= \sqrt{15^2 + (-8)^2} + \sqrt{225 + 64}$$
   $$= \sqrt{289} = 17$$

   iii) $$\mathrm{AB} = \sqrt{(0-2)^2 + (-1-3)^2}$$
   $$= \sqrt{4 + 16} = \sqrt{20}$$
   $$= \sqrt{4}\sqrt{5} = 2\sqrt{5}$$

b) find the coordinates of the midpoint of $\overline{AB}$

Midpoint $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

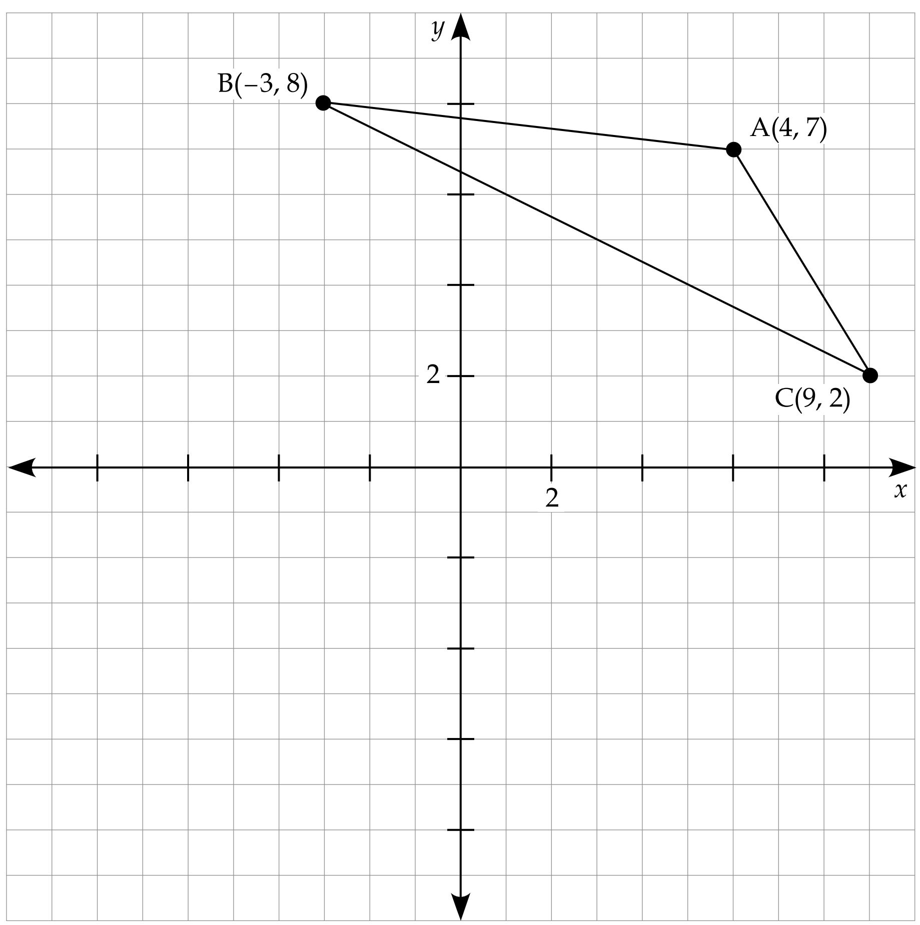
Note: The midpoint is the average of the x-values and the average of the y-values of the two endpoints.

i) $\left(\dfrac{5+1}{2}, \dfrac{-3+0}{2}\right)=\left(3, -\dfrac{3}{2}\right)$

ii) $\left(\dfrac{-1+14}{2}, \dfrac{4+(-4)}{2}\right)=\left(\dfrac{13}{2}, 0\right)$

iii) $\left(\dfrac{2+0}{2}, \dfrac{3+(-1)}{2}\right)=(1, 1)$

2. Determine whether the triangle with vertices A(4, 7), B(−3, 8), and C(9, 2) is isosceles. Write the lengths in radical form. (Remember that isosceles triangles have two sides equal in length.)



Answer:

From the diagram, *if* triangle ABC is isosceles, it looks like AB = AC. Check this assumption using the distance formula.

$$\begin{aligned}\mathrm{AB} &= \sqrt{(4-(-3))^2 + (7-8)^2}\\ &= \sqrt{49+1}\\ &= \sqrt{50}\end{aligned}$$

$$\begin{aligned}\mathrm{AC} &= \sqrt{(4-9)^2 + (7-2)^2}\\ &= \sqrt{25+25}\\ &= \sqrt{50}\end{aligned}$$

$\therefore \mathrm{AB} = \mathrm{AC}$

$\therefore \Delta\mathrm{ABC}$ is isosceles

3. Using the diagram from Question 2, show that the line segment joining the midpoints of $\overline{\mathrm{AB}}$ and $\overline{\mathrm{AC}}$ is half the length of $\overline{\mathrm{BC}}$.

Answer:

If M is the midpoint of $\overline{\mathrm{AB}}$, the coordinates of M are

$$\left(\frac{-3+4}{2}, \frac{8+7}{2}\right) = \left(\frac{1}{2}, \frac{15}{2}\right)$$

If N is the midpoint of $\overline{\mathrm{AC}}$, the coordinates of N are

$$\left(\frac{4+9}{2}, \frac{7+2}{2}\right) = \left(\frac{13}{2}, \frac{9}{2}\right)$$

Compare the lengths of $\overline{\mathrm{BC}}$ and $\overline{\mathrm{MN}}$.

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$\begin{aligned}\therefore \overline{\mathrm{BC}} &= \sqrt{(9-(-3))^2 + (2-8)^2}\\ &= \sqrt{144+36}\\ &= \sqrt{180}\\ &= 6\sqrt{5}\end{aligned}$$

$$\overline{MN} = \sqrt{\left(\frac{13}{2} - \frac{1}{2}\right)^2 + \left(\frac{9}{2} - \frac{15}{2}\right)^2}$$
$$= \sqrt{36 + 9}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5}$$

$$\therefore \overline{MN} = 3\sqrt{5}$$
$$= \frac{1}{2}\left(6\sqrt{5}\right)$$
$$= \frac{1}{2}\overline{BC}$$

4. An online map (located at www.daftlogic.com/projects-google-maps-distance-calculator.htm) plots the locations of cities in Canada using coordinates. Winnipeg is at (49. 8946, −97.0752) and Vancouver is at (49.2678, −123.1348). If one unit on the grid represents 72 km, find the distance between Winnipeg and Vancouver. Round your final answer to the nearest km.

   Answer:

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
   $$d = \sqrt{(49.8946 - 49.2678)^2 + (-97.0752 - (-123.1348))^2}$$
   $$d = \sqrt{(0.6268)^2 + (26.0596)^2}$$
   $$d = \sqrt{679.4956304}$$
   $$d = 26.0671 \text{ units}$$

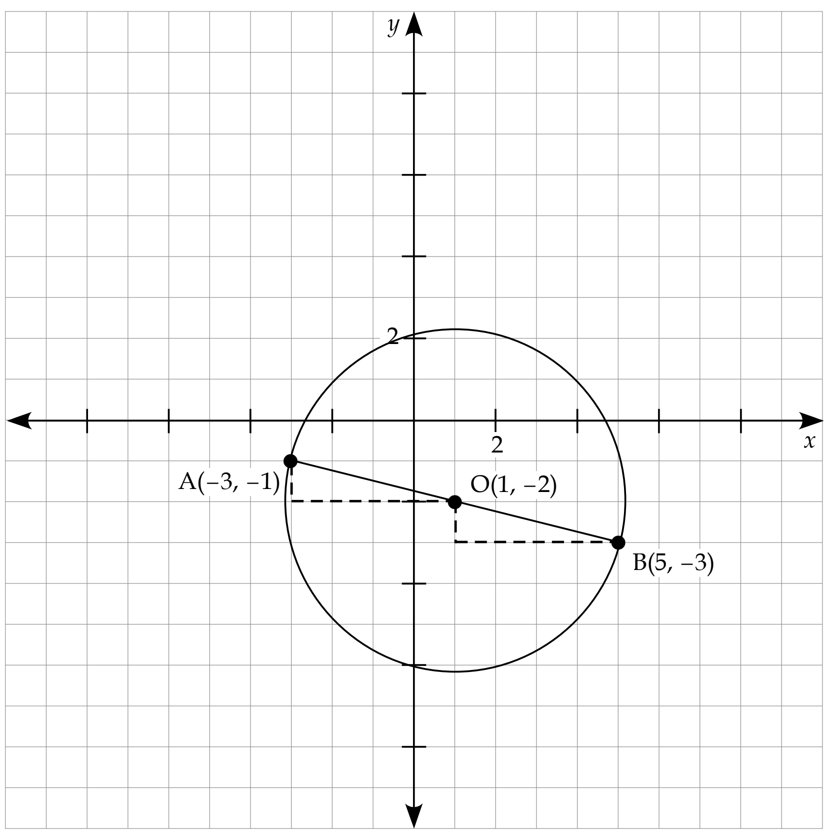
   If one unit is 72 km, then the total distance between Vancouver and Winnipeg is 72 × 26.0671 = 1877 km "as the crow flies."

5. A circle with centre at O(1, −2) has one endpoint of a diameter at A(−3, −1). Find the coordinates of the other endpoint of the diameter, B, using the midpoint formula. Verify your answer using another strategy.

Answer:

$$(1,-2)=\left(\frac{x_1+(-3)}{2},\frac{y_1+(-1)}{2}\right)$$

x-coordinate midpoint: $x_m=\frac{x_1+x_2}{2}$		y-coordinate midpoint: $y_m=\frac{y_1+y_2}{2}$
$1=\frac{x_1+(-3)}{2}$	multiply both sides of equation by 2	$-2=\frac{y_1+(-1)}{2}$
$2=x_1-3$	isolate the variable	$-4=y_1-1$
$5=x_1$		$-3=y_1$
	The coordinates of the endpoint are B(5, −3)	



You could verify your answer using the Pythagorean Theorem or the distance formula.

The $\dfrac{\text{rise}}{\text{run}}$ of the line segment $\overline{\text{AO}}$ is $\dfrac{-1}{4}$ and the slope of $\overline{\text{OB}}$ is $\dfrac{-1}{4}$. These values also represent the vertical and horizontal distances between the points and can be used in the Pythagorean Theorem.

$$\left(\overline{\text{AO}}\right)^2 = (-1)^2 + (4)^2 \qquad \left(\overline{\text{OB}}\right)^2 = (-1)^2 + (4)^2$$

$$\left(\overline{\text{AO}}\right)^2 = 17 \qquad \left(\overline{\text{OB}}\right)^2 = 17$$

$$\overline{\text{AO}} = \sqrt{17} \qquad \overline{\text{OB}} = \sqrt{17}$$

Or using the coordinates of points A, B, and O and the distance formula.

$$d_{\text{AO}} = \sqrt{(1-(-3))^2 + (-2-(-1))^2} \qquad d_{\text{OB}} = \sqrt{(5-1)^2 + (-3-(-2))^2}$$

$$d_{\text{AO}} = \sqrt{(4)^2 + (-1)^2} \qquad d_{\text{OB}} = \sqrt{(4)^2 + (-1)^2}$$

$$d_{\text{AO}} = \sqrt{17} \qquad d_{\text{OB}} = \sqrt{17}$$

The lengths $\overline{\text{OB}}$ and $\overline{\text{AO}}$ are the same. The slopes of $\overline{\text{OB}}$ and $\overline{\text{AO}}$ are the same. A, O, and B are colinear (all 3 points are on the same line), so O is the midpoint of $\overline{\text{AB}}$. The coordinates of the endpoint B are correct.

6. The three side lengths in a triangle are 18 units, 24 units, and 30 units. Is this a right triangle?

   Answer:

   If the three sides can be substitutes into the Pythagorean Theorem and make a true statement, the triangle must be a right triangle.

$a^2 + b^2$	c^2
$18^2 + 24^2$	30^2
$324 + 576$	900
900	900

   This is a true statement.

   A triangle with side lengths of 14, 24, and 30 units is a right triangle.

Learning Activity 7.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Factor: $2x^2 - 4x + 10$.
2. Find the y-intercept: $3y - 7x = 90$.
3. Simplify: $\dfrac{3}{4\sqrt[6]{x^9}}$.
4. Estimate the taxes, at 12%, of a pair of shoes that cost $74.89.
5. Complete the pattern: −1, 2, −3, ____, ____.
6. On average, you right-click once for every 5 left-clicks on your mouse. Because of this, the left button wears out 5 times faster. If the right button is estimated to last for 3 years, how many months will the left button last?
7. You have $4.65. If you buy a package of gum for $2.95, how much money will you have left over?
8. You ride your bike instead of taking the bus to get to work from April until October. Last summer it rained 35% of the days that you could have ridden your bike, and you don't ride in the rain. There are 240 days that you would have ridden your bike (you don't ride your bike on Hallowe'en). How many days did you ride your bike to work?

Answers:

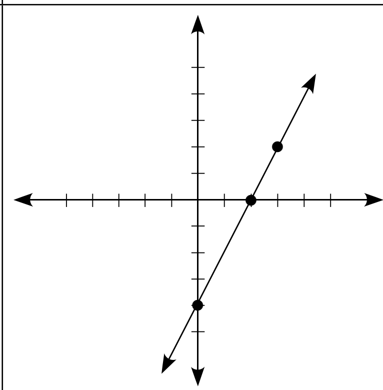
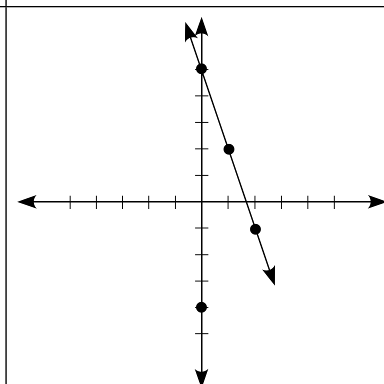
1. $2(x^2 - 2x + 5)$ ($x^2 - 2x + 5$ cannot be factored)
2. $y = 30$ ($3y - 7(0) = 90$)
3. $\dfrac{3}{4}x^{-\frac{9}{6}} = \dfrac{3}{4}x^{-\frac{3}{2}}$
4. $9 (10% of $74.89 is $7.49. 15% is approximately $11.25. Since 12% is about halfway between 10% and 15%, $9 is about halfway between the dollar values.)
5. 4, −5
6. Just over 7 months (3 years = 36 months. 36 ÷ 5 is a little bit larger than 7 because 35 ÷ 5 = 7.)
7. $1.70 ($2.95 + 1.00 = 3.95, $3.95 + 0.05 = $4.00, $4 + 0.65 = $4.65 so you still have $1.00 + $0.05 + $0.65)
8. 156 days (If you didn't ride for 35%, then you did ride for 65%= 15% + 50%. 10% = 24 days, 5% = 12 days, 50% = 120 days.)

Part B: Linear Relation Formulas

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Complete the following chart. Express each linear relation in all three forms and sketch a graph.

   Answers:

Slope-Intercept Form	$y = 2x - 4$	$y = -3x + 5$
General Form	$2x - y - 4 = 0$	$3x + y - 5 = 0$
Slope-Point Form	$m = 2$ $(0, -4)$ $y + 4 = 2(x - 0)$ $y + 4 = 2x$	$m = -3$ $(0, 5)$ $y - 5 = -3(x - 0)$ $y - 5 = -3x$
Graph		

2. Write the given linear relation in slope-intercept form using two different strategies. Explain the strategies.

$$y + \frac{1}{20} = \frac{1}{2}\left(x - \frac{2}{5}\right)$$

Answer:

The first strategy could be to determine the slope and point from the given equation, substitute that into $y = mx + b$, and then solve for b.

$$m = \frac{1}{2}$$

$$\left(\frac{2}{5}, \frac{-1}{20}\right)$$

$$y = mx + b$$

$$\frac{-1}{20} = \frac{1}{2}\left(\frac{2}{5}\right) + b$$

$$\frac{-1}{20} = \frac{1}{5} + b$$

$$\frac{-1}{20} - \frac{1}{5} = b$$

$$\frac{-1}{20} - \frac{4}{20} = b$$

$$\frac{-5}{20} = \frac{-1}{4} = b$$

$$\therefore y = \frac{1}{2}x - \frac{1}{4}$$

Another strategy could be to simplify the given equation by applying the distributive property and combining like terms.

$$y+\frac{1}{20}=\frac{1}{2}(x-\frac{2}{5})$$

$$y+\frac{1}{20}=\frac{1}{2}x-\frac{2}{10}$$

$$y=\frac{1}{2}x-\frac{2}{10}-\frac{1}{20}$$

$$y=\frac{1}{2}x-\frac{4}{20}-\frac{1}{20}$$

$$y=\frac{1}{2}x-\frac{5}{20}$$

$$\therefore y=\frac{1}{2}x-\frac{1}{4}$$

3. Explain two different strategies you could use to graph $6x - y + 3 = 0$. Sketch a graph.

   Answer:

   The equation $6x - y + 3 = 0$ could be graphed by

   a) finding the x- and y-intercepts of the line

      $6x - y + 3 = 0$

      x-intercept is where $y = 0$

      $6x - (0) + 3 = 0$

      $6x = -3$

      $$x=\frac{-3}{6}=\frac{-1}{2}$$

      y-intercept is where $x = 0$

      $6(0) - y + 3 = 0$

      $-y = -3$

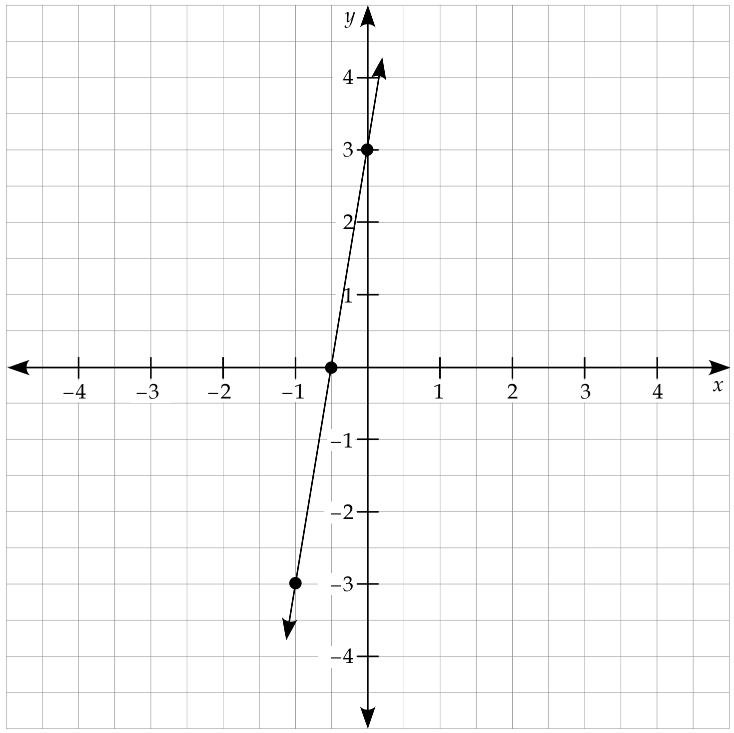
      $y = 3$

   b) using the coefficients A, B, and C to determine the slope and y-intercept of a linear equation written in general form

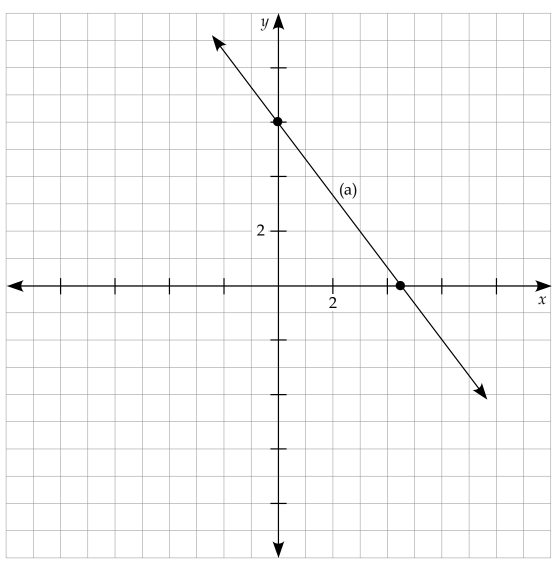
      $A = 6,\ B = -1,\ C = 3$

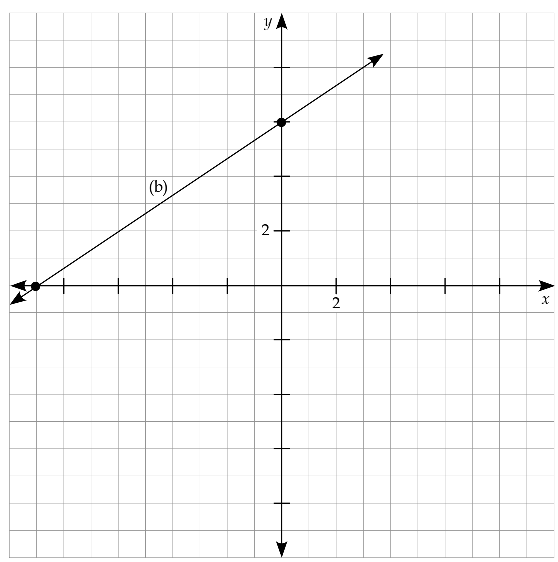
      $$m=\frac{-A}{B}=\frac{-6}{-1}=6$$

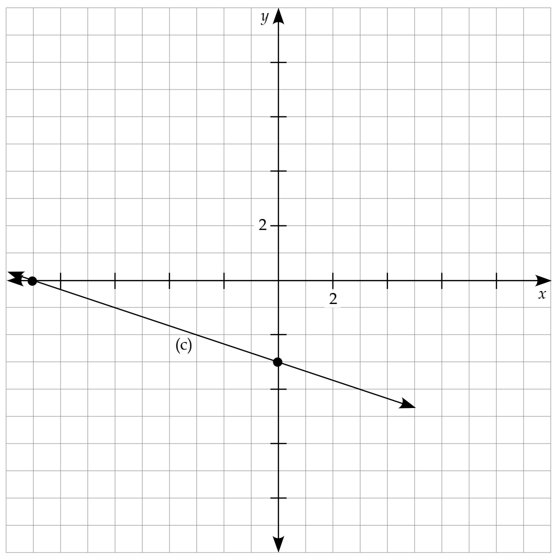
      $$b=\frac{-C}{B}=\frac{-3}{-1}=3$$



4. Match each graph to its equation(s).







Answers :

(b) $y-8=\frac{2}{3}(x-3)$

(c) $y+3=\frac{-2}{6}x$

(b) $y=\frac{2}{3}x+6$

(a) $4x+3y-18=0$

(none) $y+2=\frac{4}{6}(x+2)$

(a) $y-6=\frac{-4}{3}x$

5. The slope and y-intercept of a line are given as:

   $m=\frac{-5}{3}$

   $b=\frac{7}{3}$

   Write the general form of the equation for this linear relation ***without*** first writing it in the slope-intercept form.

   Answer:

   The general form is written as $Ax + By + C = 0$.

   $m=\frac{-A}{B}=\frac{-5}{3}$

   $b=\frac{-C}{B}=\frac{7}{3}$

   so

   $-A = -5$ or $A = 5$

   $B = 3$

   $-C = 7$ or $C = -7$

   $5x + 3y - 7 = 0$

Learning Activity 7.3

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. Is this relation a function: {(2, 4), (5, 8), (6, 1), (3, 7)}?
2. Find the midpoint of the line segment with end points (2, 6) and (4, 8).
3. Convert: 300 m = _________ km.
4. Evaluate: $\sqrt[4]{81}$.
5. June is a big birthday month for you. Your brother's is on June 9th, your nephew's is on June 20th, plus Father's Day is in June! If you want to spend \$30 on each present and you have \$85.00 saved up, will this be possible?
6. Identify the type of angle that has a measure of 345°.
7. What is the range of the following relation?
   {(1, 2), (2, 4), (3, 6), (4, 8)}
8. Write as an improper fraction: $\frac{19}{16}$.

Answers:

1. Function (No input values are repeated.)
2. (3, 7) ((2 + 4) ÷ 2 = 3, (6 + 8) ÷ 2 = 7)
3. 0.3 km (Remember, 1 km = 1000 m)
4. 3 (If you don't recognize this immediately, $\sqrt[4]{81} = \sqrt[4]{9^2} = 9^{\frac{2}{4}} = 9^{\frac{1}{2}}$. In general, the square of a square is equal to the first square root to the power of 4. For example, $2^2 = 4$, $4^2 = 16$ so $(2^2)^2 = 2^4 = 16$.)
5. No, \$30 × 3 = \$90. (You only have \$85.00.)
6. Reflex (Reflex angles are between 180° and 360°.)
7. Range is {2, 4, 6, 8}.
8. $1\frac{3}{16}$

Part B: Writing Linear Equations Based on Different Information

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Explain the process you go through to write the equation of a linear relation when you are given the coordinates of two points on the line.

   Answer:

   If I am given the coordinates of two points on a line, I label the points as (x_1, y_1) and (x_2, y_2). I use the coordinates to determine the slope of the line using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

   Using the slope and either one of the points given, I substitute the values into the point-slope formula $y - y_1 = m(x - x_1)$. I can leave the equation in this form or simplify it, combine the like terms, and rearrange them into the slope-intercept form or the general form of a linear equation.

2. Write the equation of a linear relation in slope-intercept form, given $m = \frac{-9}{2}$ and $b = \frac{1}{2}$.

   Answer:

   Slope-intercept form is $y = mx + b$. The equation would be $y = \frac{-9}{2}x + \frac{1}{2}$.

3. A line has a slope of $\frac{8}{3}$ and goes through the point (−72, −94). Write the equation in point-slope form and general form.

   Answer:

   $m = \frac{8}{3}, \left(-72, -94\right)$

   In point-slope form: $y - y_1 = m(x - x_1)$.

   $$y + 94 = \frac{8}{3}\left(x + 72\right)$$

In general form: $Ax + By + C = 0$.

$$y + 94 = \frac{8}{3}(x + 72)$$

$$y + 94 = \frac{8}{3}x + 192$$

$$y = \frac{8}{3}x + 192 - 94$$

$$y = \frac{8}{3}x + 98$$

$$0 = \frac{8}{3}x - y + 98$$

$$0 = 8x - 3y + 294$$

4. Write the equation of the line that goes through the points (26, 9) and (43, −6). State your answer in $y = mx + b$ form.

   Answer:

   Given: (26, 9) (43, −6)

   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

   $$m = \frac{-6 - 9}{43 - 26}$$

   $$m = \frac{-15}{17}$$

   Use the slope and one of the points, and substitute the values into $y = mx + b$ to solve for b.

$$y = mx + b$$

$$9 = \frac{-15}{17}(26) + b$$

$$9 = \frac{-390}{17} + b$$

$$\frac{153}{17} + \frac{390}{17} = b$$

$$\frac{543}{17} = b$$

$$y = \frac{-15}{17}x + \frac{543}{17}$$

5. A line crosses the x-axis at 14 and the y-axis at 35. Write the equation of the line in general form. Use two different methods to arrive at the answer.

   Answer:

   The coordinates of the two intercepts are written as (14, 0) and (0, 35). Calculate the slope and then use that and one of the points to write the equation.

   This question is challenging for some students. Don't hesitate to call your tutor/marker for help. Also, it would be helpful to include this example on your Resource Sheet.

   Method 1:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{35 - 0}{0 - 14}$$

$$m = \frac{35}{-14} = \frac{-35}{14} = \frac{-5}{2}$$

$$y - 0 = \frac{-5}{2}(x - 14)$$

$$y = \frac{-5x}{2} + 35$$

$$0 = \frac{-5x}{2} - y + 35$$

$$0 = -5x - 2y + 35$$

$$5x + 2y - 35 = 0$$

Method 2:

$m = \frac{-35}{14} = \frac{-5}{2}$ $\qquad m = \frac{-A}{B}$

$b = 35$ $\qquad b = \frac{-C}{B}$

When using $m = \frac{-A}{B}$ and $b = \frac{-C}{B}$, B must have the same value in both equations. Rewrite the y-intercept as an equivalent fraction with a denominator of 14.

$b = \frac{35}{1} = \frac{70}{2}$

$m = \frac{-A}{B} = \frac{-35}{14} = \frac{-5}{2}$

$b = \frac{-C}{B} = \frac{70}{2}$

$-A = -5$

$A = 5$

$B = 2$

$-C = 70$

$C = -70$

$Ax + By + C = 0$

$5x + 2y - 70 = 0$

6. A line is parallel to $y = -3x - 55$ and goes through the point $(-8, 19)$. Write the equation of the line in point-slope form.

   Answer:

   Parallel lines have the same slope, so use $m = -3$ and $(-8, 19)$.

   $y - y_1 = \mathrm{m}(x - x_1)$

   $y - 19 = -3(x + 8)$

7. Write the equation of the line that is perpendicular to $5x + 6y - 72 = 0$ and has an x-intercept of -4 in general form. Compare the coefficients in both equations. What do you notice?

Answer:

Perpendicular lines have negative reciprocal slopes.

The slope of the perpendicular line is found by rewriting the equation in slope-intercept form or by using $m = \frac{-A}{B} = \frac{-5}{6}$.

$$5x + 6y - 72 = 0$$

$$6y = -5x + 72$$

$$y = \frac{-5}{6}x + 12$$

So the slope of the line we are looking for is the negative reciprocal.

$$m = \frac{6}{5}$$

The x-intercept is at (–4, 0).

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{6}{5}(x + 4) \quad \text{point form}$$

$$y = \frac{6}{5}x - \frac{24}{5} \quad \text{slope-intercept form}$$

$$6x - 5y - 24 = 0 \quad \text{general form}$$

The line perpendicular to this was $5x + 6y - 72 = 0$.

The coefficients have switched positions, and the sign of the y-coefficient is different.

8. Write the equation of the line that is the perpendicular bisector (bisector means ″cuts in half″) of the line segment between (−3, −8) and (15, 6). State the answer in point-slope form. (Hint: The line must pass through the midpoint of the line segment.)

You should include the definition of a bisector on your Resource Sheet.

Answer:

The line will go through the midpoint of the segment and its slope will be the negative reciprocal of the slope of the line segment.

Given: (−3, −8) and (15, 6)

Midpoint of the segment:

$$\mathrm{M} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\mathrm{M} = \left(\frac{-3 + 15}{2}, \frac{-8 + 6}{2}\right)$$

$$\mathrm{M} = \left(\frac{12}{2}, \frac{-2}{2}\right)$$

$$\mathrm{M} = (6, -1)$$

The slope of the line segment:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 + 8}{15 + 3}$$

$$m = \frac{14}{18}$$

$$m = \frac{7}{9}$$

The slope of the perpendicular line will be $\frac{-9}{7}$.

The equation of the perpendicular bisector is:

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{-9}{7}(x - 6)$$

9. Write the equation of a line that is perpendicular to the line $y = -12$, and explain your answer.

Answer:

The line $y = -12$ is a horizontal line that crosses the y-axis at -12. The slope of a horizontal line is 0. The reciprocal of 0 is undefined, but any vertical line will be perpendicular to the horizontal line $y = -12$. The equation of a vertical line is $x = \Re$ where $\Re$ is any real number.

Learning Activity 7.4

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. If the intercepts of a line are $x = 3$ and $y = 9$, what is the slope of the graph?
2. Which has a larger volume with the same dimensions: rectangular prism or a rectangular pyramid?
3. Which variable is independent: comparing the amount of rain to the number of mosquitos?
4. Write the following as a function: $y + 2x = 4$.
5. What two numbers have a sum of −6 and a product of 5?
6. Evaluate: $\left(4^2\right)^{\frac{-1}{4}}$.
7. Jared has a big crush. He sits 6 desks to the left of his crush. If the desks are each 80 cm wide and there are no spaces in between them, how close does he sit to his crush (in metres)?
8. Multiply: $(x + 5)(2x + 1)$.

Answers:

1. $-3\left(m = \dfrac{9-0}{0-3} = \dfrac{9}{-3}\right)$
2. Prism (A prism's volume is 3 times larger than a pyramid's volume.)
3. Amount of rain (No one can control the rain, and if the rain does not fall, the likelihood of mosquitoes is minimal.)
4. $f(x) = 4 - 2x$
5. −5, −1
6. $\dfrac{1}{2}\left(\left(4^2\right)^{\frac{-1}{4}} = 4^{\frac{-2}{4}} = \dfrac{1}{4^{\frac{1}{2}}}\right)$
7. 4.8 m ($80 \times 6 = 480$ cm, 100 cm = 1 m)
8. $2x^2 + 11x + 10$

Part B: Line of Best Fit and Correlation

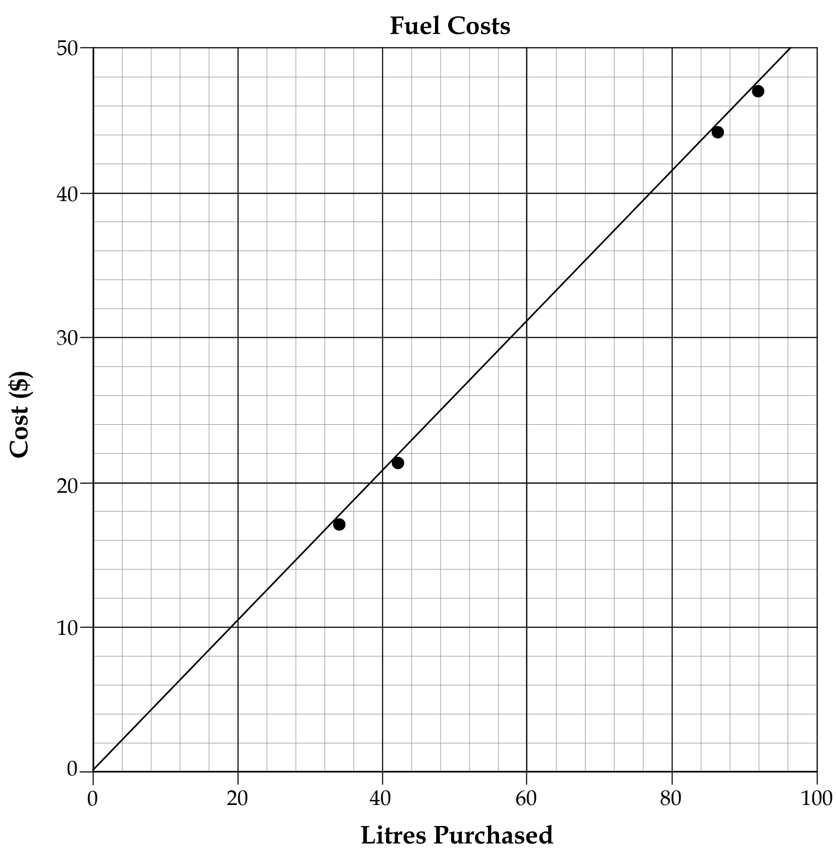
Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. You record the cost of filling the fuel tank on your truck and the number of litres of gasoline purchased.

Litres (L)	34	42	86	91
Cost ($)	17.68	21.84	44.72	47.30

a) Create a scatterplot of these data on graph paper, and draw the line of best fit.

Answer:



b) Use ordered pairs to write the equation of the resulting line in slope-intercept form. What does the slope of the equation represent?

Answer:

You may have chosen different points to use, but should end up with the same answer (or very close).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{21.84 - 17.68}{42 - 34}$$

$$m = \frac{4.16}{8} = 0.52$$

$$y - y_1 = m(x - x_1)$$

$$y - 17.86 = 0.52(x - 34)$$

$$y - 17.68 = 0.52x - 17.68$$

$$y = 0.52x$$

The slope represents the cost per litre. Gas is $0.52/L.

c) Use the equation to determine how much 15 L would cost you.

Answer:

$y = 0.52x$

$y = 0.52(15)$

$y = 7.8$

15 L would cost you $7.80.

d) Use the equation to determine how many litres you could purchase for $50.00.

Answer:

$$y = 0.52x$$

$$50 = 0.52x$$

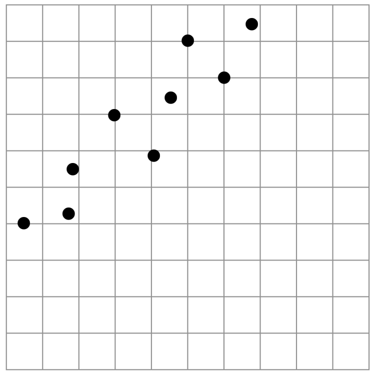
$$x = \frac{50}{0.52}$$

$$x = 96.15$$

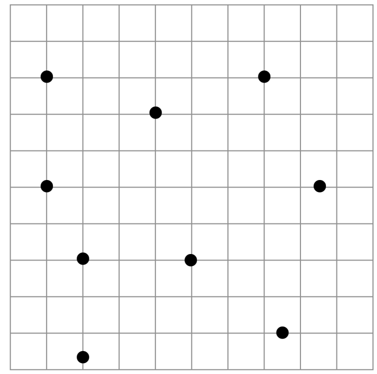
You could purchase about 96 L for $50.00.

2. Describe the correlation illustrated by the following scatterplots as strong or weak, and positive or negative, or no correlation. Estimate an r-value for each.

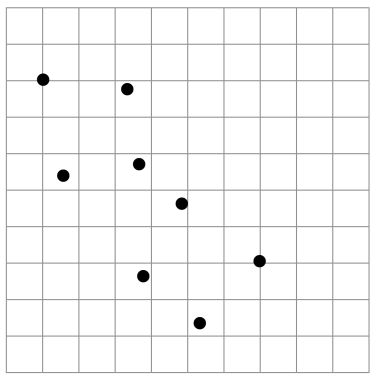
a)



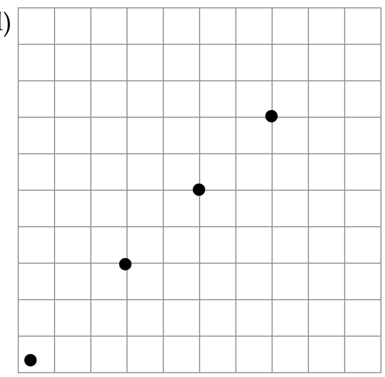
b)



c)



d)



Answers:

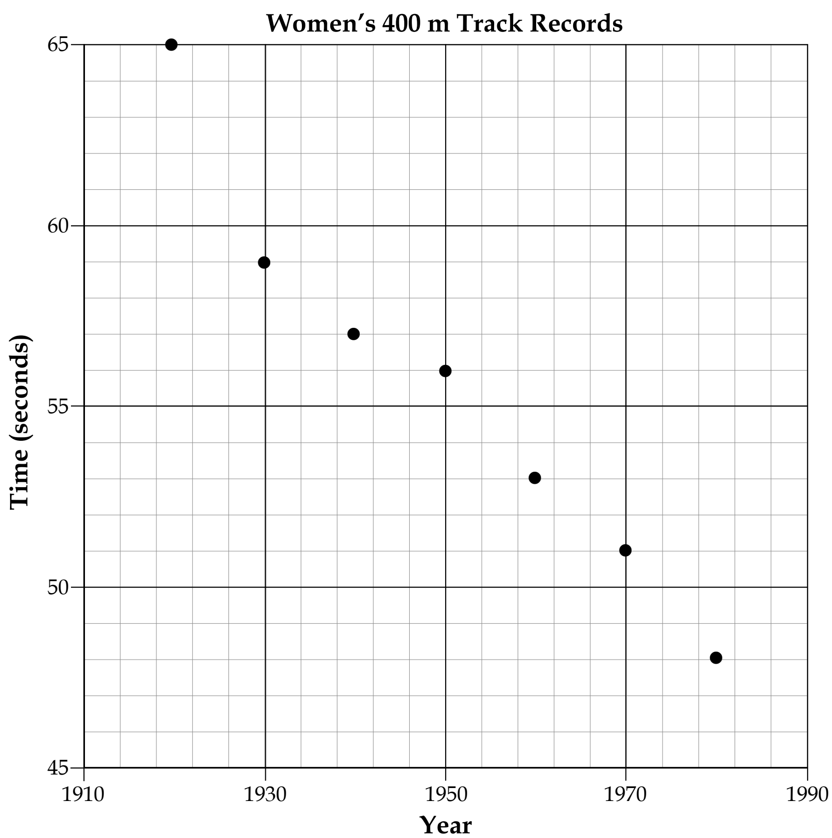
		Actual	Acceptable Range
a)	strong positive correlation	$r = 0.8$	0.7 to 0.9
b)	no correlation	$r = 0$	−0.1 to 0.1
c)	weak negative correlation	$r = -0.6$	−0.7 to −0.5
d)	strong positive correlation	$r = 1$	0.9 to 1.0

Note: The r-values provided are actual values. Your estimate should be within one-tenth of the answers given.

3. a) Use technology to graph the following data.

Women's 400 m World Track Records	
Year	**Approximate Time (seconds)**
1920	65
1930	59
1940	57
1950	56
1960	53
1970	51
1980	48

Answer:



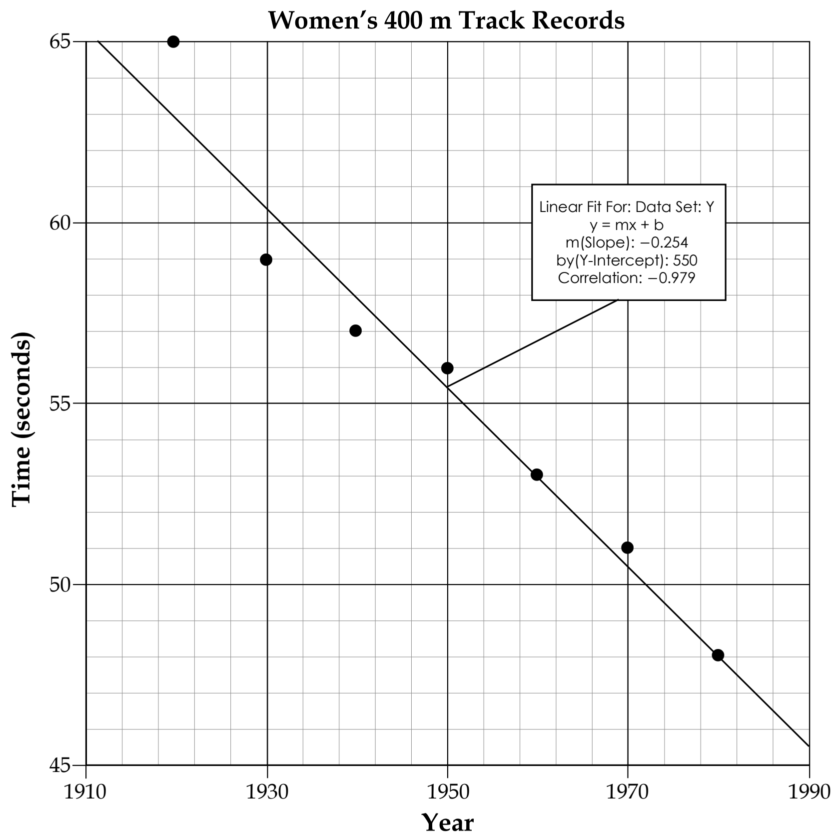
b) Estimate the r-value.

Answer:

The trend of the data illustrates a strong negative correlation. The points fall quite closely along a straight line so a good estimate of the r-value would be -0.9.

c) Use technology to draw the line of best fit, and calculate the correlation coefficient.

Answer:



The correlation coefficient is given as -0.979.

d) Explain what the correlation coefficient indicates about the data.

Answer:

The r-value indicates that there is a very strong negative relationship between the years and the record time of the 400 m race. As the years progress, the record time of the women's 400 m decreases. The r-value is very close to -1, so the relationship is nearly linear.

e) Use the equation of the line of best fit to calculate the possible record time for the 2016 Summer Olympics. Does your answer seem reasonable to you?

Answer:

The equation of the line of best fit is given as $y = -0.254x + 550$.

Substitute $x = 2016$.

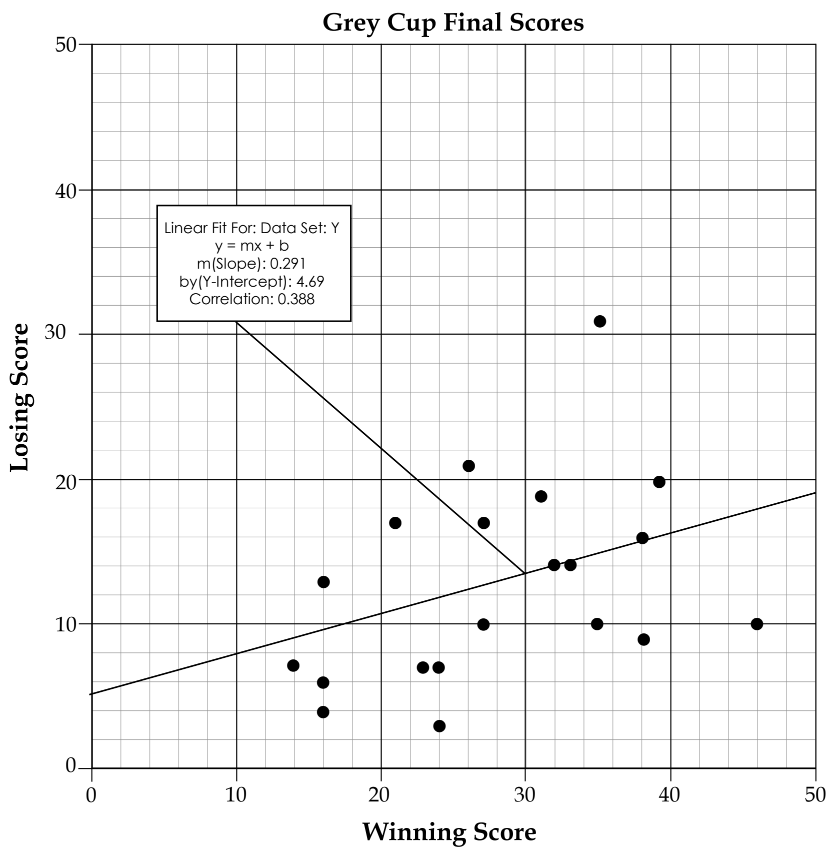
$y = -0.254(2016) + 550$

$y = 37.936$

The record could be 37.936 seconds. This is possible, but there is probably a limit to how fast a human can run, and since the current record is 47.06 seconds and was set in 1985, we may have reached it.

4. a) Use technology to graph the following data.

Answer:



b) Use technology to draw the line of best fit, determine the equation of the line, and calculate the correlation coefficient.

Answer:

$y = 0.291x + 4.69$

$r = 0.388$

c) Explain what the correlation coefficient indicates about the data. Can you use the equation to predict the score of the next Grey Cup game?

Grey Cup Final Scores	
Winning Score	**Losing Score**
39	20
46	10
38	16
38	9
27	17
26	21
27	10
31	19
35	31
32	14
21	17
16	6
24	7
14	7
24	3
16	13
23	7
16	6
33	14
35	10

Answer:

It appears that as the winning score increases, the losing team's score also increases, but the points are very spread out from the line. The correlation may be described as a very weak positive correlation. The equation of the line would be a very poor predictor of the final score in a game because factors such as which teams are playing, injuries, and even the weather affect the final outcome in a football game.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 8
Systems of Equations

Module 8: Systems of Equations

Introduction

Based on previous math lessons, you may be tempted to view a linear equation simply as a problem that needs to be solved. In this module, you will switch that around and use equations to find the solutions to practical problems. You will use pairs of equations that have the same two variables to represent a situation or context, and find solutions to problems by graphing or algebraically solving the system of equations. You will identify the three types of systems of linear equations, describe strategies to solve the systems, and verify solutions.

Having access to graphing technology is important for this lesson. You will be able to graph linear equations by hand, but for some problems, finding accurate answers will be easier with a graphing calculator, software or an online graphing tool that allows you to input equations and find the point of intersection of two lines. Familiarize yourself with your choice of technology by reading the manual or the help files that came with the product.

Assignments in Module 8

When you have completed the assignments for Module 8, submit your completed assignments for Module 7 and Module 8 to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 8.1	Solving Systems of Linear Equations Graphically
2	Assignment 8.2	Solving Systems of Equations by Elimination

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ ″ by 11″, with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions that follows to help you with preparing your resource sheet for the material in Module 8. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1 to 8 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 1 to 8.

Resource Sheet for Module 8

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Writing Your Final Examination

You will write the final examination when you have completed Module 8 of this course. The final examination is based on Modules 1 to 8, and is worth 25 percent of your final mark in the course. You will write the final examination when you have completed Module 8. To do well on the final examination, you should review all the work you complete in Modules 1 to 8, including all the learning activities and assignments. You will write the final examination under supervision.

Notes

Lesson 1: Solving Systems of Linear Equations Graphically

Lesson Focus

In this lesson, you will

- ❑ model a situation using a system of linear equations
- ❑ determine and verify the solution of a system of linear equations graphically, with or without technology
- ❑ explain the meaning of the point of intersection of a system of linear equations
- ❑ explain, using examples, why a system of equations may have no solution, one solution, or an infinite number of solutions
- ❑ solve a contextual problem that involves a system of linear equations

Lesson Introduction

In this lesson, you will see how linear equations can be written to represent situations and then be used to solve problems related to that situation. You will find the solutions to three types of systems of linear equations by graphing the equations and finding the point(s) of intersection, both with and without technology.

What is a System of Linear Equations?

Using a System of Linear Equations to Model a Situation

Example 1

Adam and Katie worked at two different electronics stores, selling TVs, computers, cameras, and other electronic gadgets. Katie was paid by straight commission. She earned 25% of all her sales. Adam was paid $300 per month plus 15% commission on his sales. Who earns more? For what amount of sales do they have the same income?

Solution:

It would be helpful to include the following steps on your resource sheet.

You can model this situation with a system of linear equations.

Step 1: Assign variables (P = amount paid, s = sales amount).

Step 2: Write equations to represent the relationships between the variables.

Katie's pay can be represented with the equation

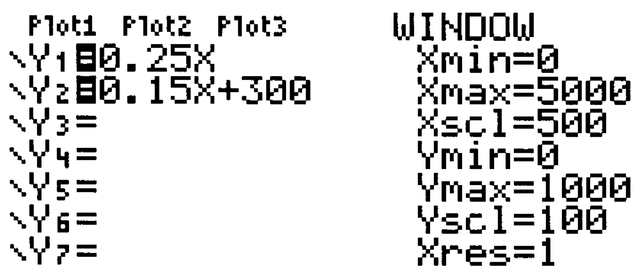
$$P = 0.25s$$

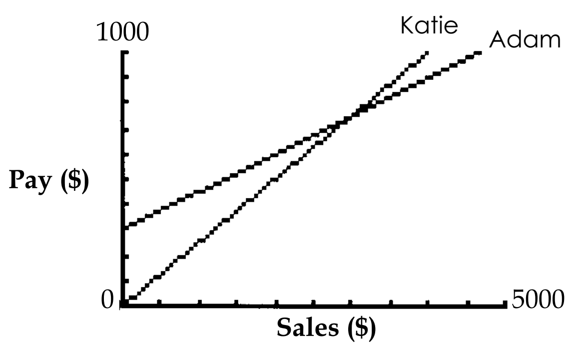
while the equation to calculate Adam's pay would look like

$$P = 0.15s + 300$$

Step 3: Graph the equations.

To see who earns more, you can graph these equations and compare them. The following screen shots were created with a TI-83 Plus graphing calculator (labels added). If you don't have access to technology, create a table of values for each equation using inputs from \$0 to \$5000 in sales and then plot the points and connect them with a ruler. Use graph paper and work as precisely as possible.

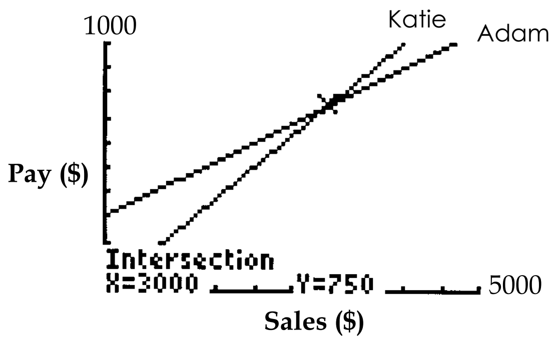




From this graph, it appears that Adam initially makes more money than Katie, but only until a certain point. For a certain amount of sales, they make the same amount, and if they sell more than that value of merchandise, Katie makes more money than Adam.

To find out the amount of sales they earn the same amount on, determine the point of intersection using technology. Check your manual for the correct steps or procedures. If the graph is hand-drawn to an acceptable degree of accuracy, you should be able to approximate the coordinates of the point of intersection. To check if it is correct, substitute the coordinates into both equations and verify that they make both equations into true statements (so that both sides are equal). Technology may provide only approximate answers as well, depending on the settings. Always verify answers received from technology.

These two lines intersect at the point (3000, 750).



If they each sell $3000 worth of electronics, they earn the same amount of money—$750.

For sales of less than $3000, Adam makes more money, and if they sell more than $3000 worth of merchandise in a month, Katie earns more money than Adam.

Given the system of equations used above,

$P = 0.25s$

$P = 0.15s + 300$

the solution of this system is (3000, 750). This is the one coordinate point (s, P) that satisfies both equations. When substituted into either equation, the ordered pair makes both equations true.

Verify:

P	$0.25s$
750	0.25(3000)
750	750

P	$0.15s + 300$
750	0.15(3000) + 300
750	750

Example 2

During a basketball game, Kali scored 18 times for 28 points. Some of the points came from free throws, worth 1 point each, and some came from field goals (a field goal in basketball is any shot other than a free throw), worth 2 points each (she did not get any 3 point baskets). How many free throws and field goals did Kali make? Use a system of linear equations and a graph to find the answer.

Solution:

Begin by writing two linear equations to represent this situation.

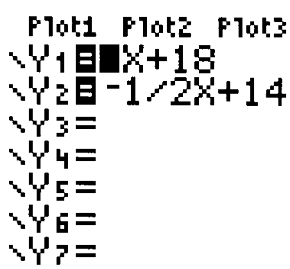
Step 1: Let x represent the number of free throws Kali made, and let y represent the number of field goals.

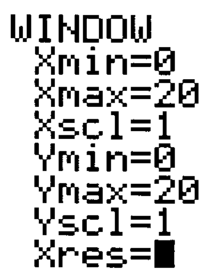
You know the number of shots she made, and the point values. Write one equation to express each statement.

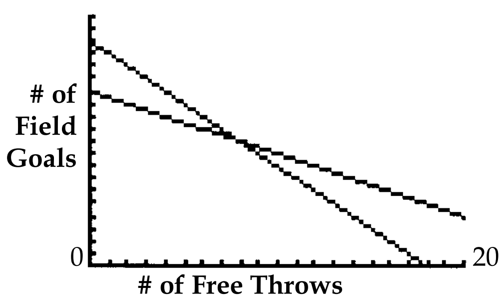
Step 2: Since she made a total of 18 shots, $x + y = 18$.

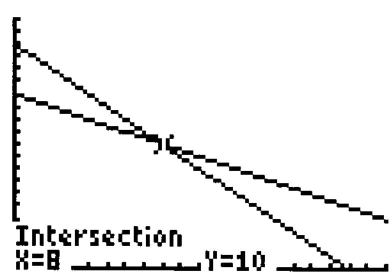
A free throw is worth 1 point and a field goal is worth 2 points. She had a total of 28 points so $1x + 2y = 28$.

Rewrite the equations in $y = mx + b$ format and graph them, with or without technology, and find the point of intersection.









The coordinates of the solution (the solution is found when you have the same x-values and y-values for both equations, seen on the graph as an intersection) are given as (8, 10). This means Kali made 8 free throws and 10 field goals.

Verify this solution.

$x + y$	18
$8 + 10$	18
18	18

$1x + 2y$	28
$1(8) + 2(10)$	28
28	28

From these examples, you can see how a system of linear equations can be used to model a situation and solve problems.

A **system of linear equations** is a set of two or more linear equations with the same variables. The solution to the system is the set of all ordered pairs that make all the equations true.

If a system of linear equations consists of two equations with two variables, it may be called a 2-by-2 system.

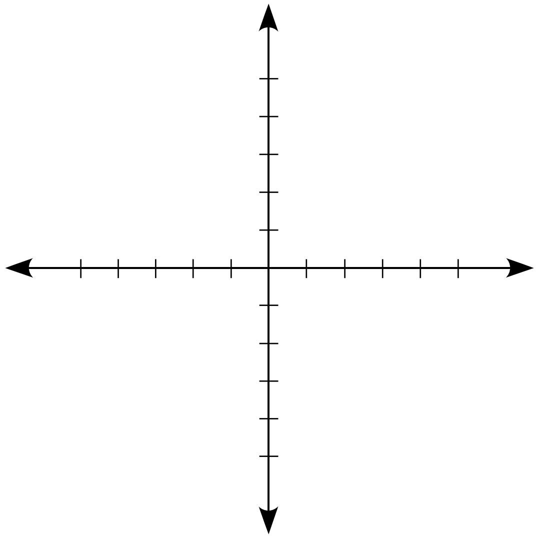
Types of Linear Systems

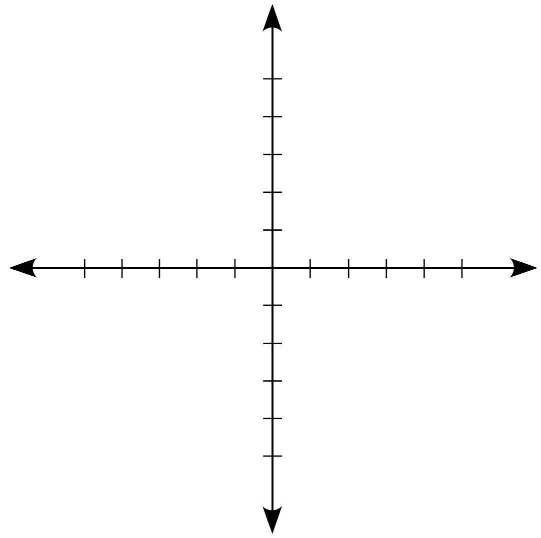
There are three different types of systems of linear equations in two categories. You can distinguish between the different types by analyzing how the lines intersect each other.

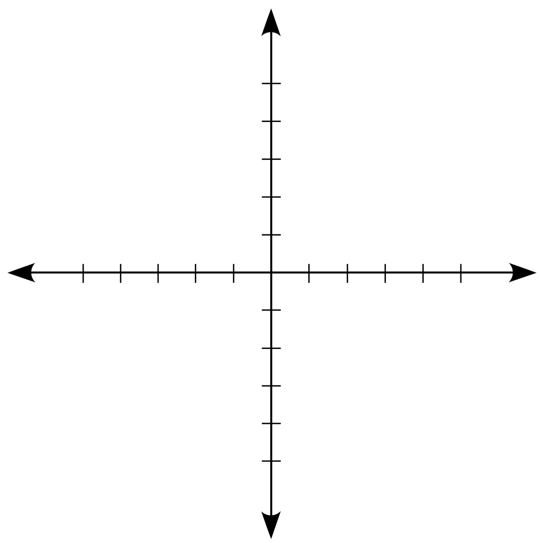
Example 3

Graph the following pairs of linear equations on the grids provided by writing the equations in slope-intercept form, or use technology and sketch the graphs below. Describe the lines and how the lines intersect.

Type 1:	Type 2:	Type 3:
l_1: $x - y - 1 = 0$	l_1: $x - y - 1 = 0$	l_1: $x - y - 1 = 0$
l_2: $2x + y = -4$	l_2: $2x - 2y - 2 = 0$	l_2: $x - y + 2 = 0$

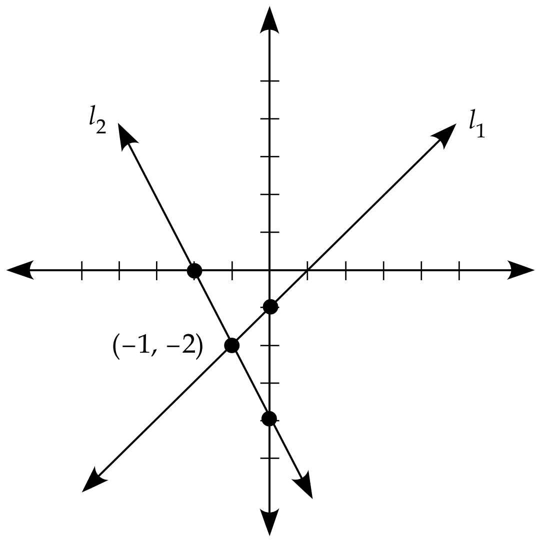




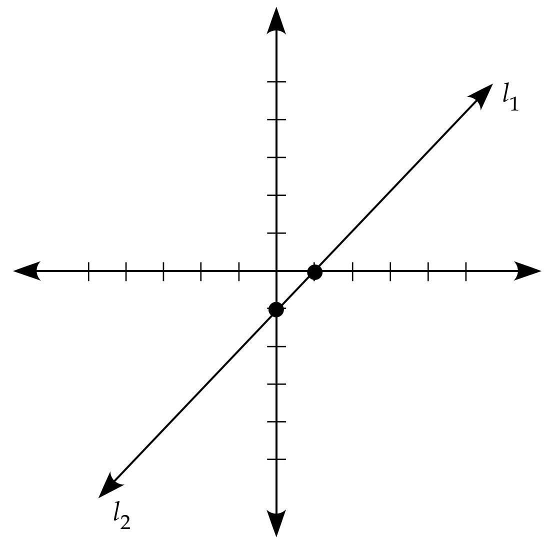


Solution:

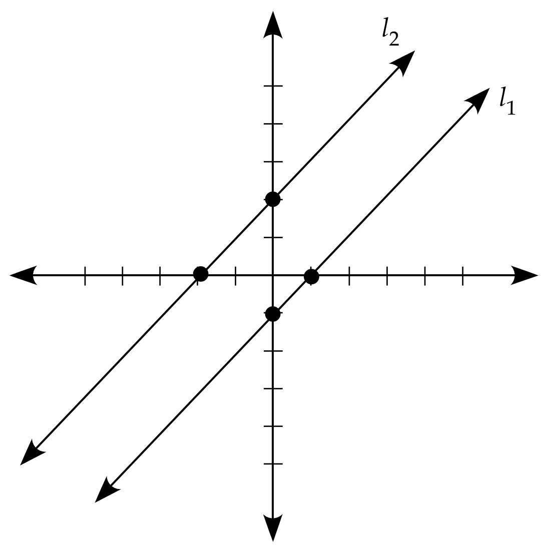
Type 1:	Type 2:	Type 3:
l_1: $y = x - 1$ l_2: $y = -2x - 4$	l_1: $y = x - 1$ l_2: $2y = 2x - 2$ $y = x - 1$	l_1: $y = x - 1$ l_2: $y = x + 2$



one unique solution



infinite solutions



no solution

> The three types of systems can be summarized as **consistent** (having at least one solution) or **inconsistent** (no solution).

Independent system: The lines in this consistent system have different slopes and different y-intercepts. Since they intersect at one point, the solution set consists of one ordered pair.

Dependent system: The equations in this consistent system represent the same line. Since the lines are the same, they have the same slope and the same y-intercept. The solution set consists of infinitely many solution points along the line.

Inconsistent system: The lines in this system are parallel and will never intersect. They have the same slope, but different y-intercepts. There is no solution to this system of equations, or the solution set is the empty set, $\varnothing$.

Learning Activity 8.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. 35% of the trees in the forest of 400 have Dutch elm disease. How many trees have this illness?
2. Evaluate: $(3 + 4x)^0$.
3. Strong or weak correlation: the points on a graph are spread out and it is hard to see the pattern they make.
4. A line has the intercepts $x = 5$ and $y = -7$. Write the equation of the line.
5. A right triangle has sides with lengths 8, 15, 17. Which is the hypotenuse?
6. Which two whole numbers is $\sqrt{150}$ between?
7. What is the sum of the first four perfect squares?
8. A dozen muffins costs $8.66. How much would you expect half a dozen muffins to cost?

continued

Learning Activity 8.1 (continued)

Part B: Solving Systems of Linear Equations Graphically

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Explain what it means if a system of linear equations has an infinite number of solutions. Give an example.
2. Write a linear system of equations that has for its solution the ordered pair (2, 5).
3. Leslie has 16 coins worth $2.20, consisting of some quarters and some nickels. Write a system of linear equations to represent this situation. Graph the system and determine how many quarters and nickels she has.
4. Three pencils and eight erasers cost $2.30. A pencil costs 18 cents more than an eraser. Use a system of equations and a graph to determine how much a pencil and eraser cost individually.
5. Determine the solution of the following systems of linear equations by graphing, with or without technology. Include a graph for each, indicate the type of system it represents, and state and verify the solution set for each.

   a) $8x - 3y = 6$

      $6x + 12y = -24$

   b) $\frac{1}{2}x - y = 8$

      $x + \frac{1}{3}y = 2$

   c) $y = \frac{-2}{3}x + 7$

      $4x + 6y = 42$

Lesson Summary

In this lesson, you graphed linear systems of equations to identify the type of system and to find the solution set for each system. This method of solving systems of linear equations is more common in applied mathematics. In the next lesson, you will use two different algebraic methods to solve systems of linear equations, which is more commonly used in pre-calculus math.

Assignment 8.1

Solving Systems of Linear Equations Graphically

Total Marks = 30

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Explain the meaning of the point of intersection of an independent system of linear equations. *(1 mark)*

2. Explain what it means if a system of linear equations has the empty set, Ø, for a solution. Give an example. *(2 marks)*

Assignment 8.1: Solving Systems of Linear Questions Graphically (continued)

3. State what type of system the following pairs of linear equations represent. State the solution to each system. Provide a graph of each system.

   a) $y - 3x = 11$ *(5 marks)*

      $2y = 5x + 19$

   b) $y = x$ *(3 marks)*

      $x = -2$

Assignment 8.1: Solving Systems of Linear Questions Graphically (continued)

c) $5y - 4x = 10$ *(5 marks)*

$\frac{-7}{3} - \frac{1}{3}y = \frac{-4}{15}x$

4. Verify if (1, 6) is the solution set for the following system of linear equations *(2 marks)*

$-3x - y = -9$

$3x - y = -3$

Assignment 8.1: Solving Systems of Linear Questions Graphically (continued)

5. Telephone company "A" charges a fixed rate of $29.99 plus $0.40 per minute for long distance phone calls (they only charge for the time used, so if you talk for part of a minute they will charge you less than $0.40). Telephone company "B" charges a fixed rate of $44.19 plus $0.20 per minute for long distance calls, with the same policy as company "A". Which company provides the cheaper service? Explain your answer. Indicate whether there is any point at which the companies have the same charge. Include your equations and a graph. *(6 marks)*

Assignment 8.1: Solving Systems of Linear Questions Graphically (continued)

6. If you drink a cola and an energy drink, you consume 111 mg of caffeine. The energy drink has 6 more than twice the number of mg of caffeine as compared to the cola. Use a system of equations and a graph to determine how many milligrams of caffeine are in an energy drink and a cola. Verify your answer. *(6 marks)*

Notes

Lesson 2: Solving Systems of Linear Equations Algebraically

Lesson Focus

In this lesson, you will

- ❑ determine and verify the solution of a system of linear equations algebraically
- ❑ describe a strategy to solve a system of linear equations

Lesson Introduction

You have found the solution to a system of linear equations by graphing the equations and finding the point(s) where the lines intersect. At times, it may be difficult to find a precise intersection point using a hand-drawn graph, and access to technology may be limited. Using one of two algebraic methods, you can solve for an exact solution to a system of linear equations. The methods are to eliminate by addition (or subtraction) or to eliminate by substitution.

Solving Systems of Equations without Graphing

Elimination by Addition or Subtraction

Trying to keep steps in order can be difficult. Include the steps written below for eliminating by addition on your Resource Sheet.

In this algebraic method of solving a system of linear equations, you will follow these steps:

Step 1: Arrange the equations with like terms in columns.

Step 2: Make the coefficients of one variable the same in both equations by multiplying each term of one or both equations by an appropriate number.

Step 3: Add or subtract the equations to eliminate the variable with matching coefficients and solve for the remaining variable.

Step 4: Substitute the value found in step 3 into either of the original equations and solve for the remaining variable.

Step 5: State the solution and verify it in each of the original equations.

These steps are demonstrated in Example 1.

Example 1

Solve the system:

$2x + 3y = 7$ Equation 1

$x + 2y - 4 = 0$ Equation 2

Solution:

Step 1

Rearrange the equations so that the x-variables are in a column, the y-variables are in a column, and the constants are both on the same side of the equal sign.

$$\begin{array}{r|r|l} 2x + & 3y = & 7 \\ x + & 2y = & 4 \end{array}$$

Step 2

You need a common coefficient for either the x-variables or the y-variables.

Multiply the second equation by 2 so there is a common x-coefficient of 2. Use the distributive property you used in Module 6.

$2x + 3y = 7 \rightarrow 2x + 3y = 7$

$2(x + 2y = 4) \rightarrow 2x + 4y = 8$

Step 3

If the signs of the common coefficients are the same, subtract the two equations. If the signs are opposite, add the equations.

$$\begin{array}{r} 2x + 3y = 7 \\ \underline{-(2x + 4y = 8)} \\ -y = 1 \\ y = 1 \end{array} \qquad 2x - (2x) = 0 \qquad 3y - (4y) = -y \qquad 7 - (8) = -1$$

Step 4

Substitute the value you found for the variable into one of the original equations, and solve for the other variable.

$$\begin{aligned} x + 2y - 4 &= 0 \\ x + 2(1) - 4 &= 0 \\ x - 2 &= 0 \\ x &= 2 \end{aligned}$$

Step 5

State and verify your solution by checking the ordered pair in both original equations.

Solution:

(2, 1)

$2x + 3y$	7
$2(2) + 3(1)$	7
$4 + 3$	7
7	7

$x + 2y - 4$	0
$(2) + 2(1) - 4$	0
$2 + 2 - 4$	0
0	0

Exactly one solution exists, so this is an independent consistent system.

Example 2

Solve the system:

$2x + 3y = 1$ Equation 1

$5x - 4y = 14$ Equation 2

Solution:

$2x + 3y = 1$ Coefficients are in columns. Multiply the first equation by 4.

$5x - 4y = 14$ Multiply the second equation by 3 to get a common y-coefficient.

$4(2x + 3y = 1) \rightarrow 8x + 12y = 4$

$3(5x - 4y = 14) \rightarrow 15x - 12y = 42$ Signs on common coefficients are opposite, so add the equations.

$$\begin{array}{l} 8x + 12y = 4 \\ \underline{+(15x - 12y = 42)} \\ 23x \qquad = 46 \end{array}$$

$8x + (15x) = 23x$ $\quad 12y + (-12y) = 0$ $\quad 4 + (42) = 46$

$\frac{23x}{23} = \frac{46}{23}$ Solve for x.

$x = 2$

Substitute this value back into an original equation.

$$\begin{aligned} 2x + 3y &= 1 \\ 2(2) + 3y &= 1 \\ 3y &= 1 - 4 \\ 3y &= -3 \\ y &= -1 \end{aligned}$$

Solution:

(2, −1)

Verify:

$2x + 3y$	1
$2(2) + 3-1)$	1
$4 - 3$	1
1	1

$5x - 4y$	14
$5(2) - 4(-1)$	14
$10 + 4$	14
14	14

This system is an independent consistent system with one solution.

Example 3

Solve:

$x - 2y = 3$ Equation 1

$-2x + 4y = 1$ Equation 2

Solution:

$x - 2y = 3$ Multiply Equation 1 by −2.

$-2x + 4y =1$

$-2(x - 2y = 3) \rightarrow -2x + 4y = -6$

$-2x + 4y = 1 \rightarrow -2x + 4y = 1$

$$\begin{array}{r} -2x + 4y = -6 \\ -2x + 4y = 1 \\ \hline 0 = -7 \end{array}$$

Subtract.

Notice that the result is a false equation. Zero is not equal to negative seven. You can conclude that there is no true solution to this system. These two equations represent parallel lines and will never intersect. This system is inconsistent.

To verify, write both equations in $y = mx + b$ form, and you will notice that they have the same slope but different y-intercepts. The lines are parallel.

Example 4

Solve:

$$9x + 6y = 48 \qquad \text{Equation 1}$$

$$\frac{3}{4}x + \frac{1}{2}y = 4 \qquad \text{Equation 2}$$

Solution:

$9x + 6y = 48$

$\frac{3}{4}x + \frac{1}{2}y = 4$ Multiply Equation 2 by 4 to change the fractions into coefficients, and then by 3 to have a common coefficient of y.

$$9x + 6y = 48 \rightarrow \qquad \rightarrow \qquad \rightarrow 9x + 6y = 48$$

$$4\left(\frac{3}{4}x + \frac{1}{2}y = 4\right) \rightarrow 3x + 2y = 16 \rightarrow 3(3x + 2y = 16) \rightarrow 9x + 6y = 48$$

$$\begin{array}{r} 9x + 6y = 48 \\ -(9x + 6y) = 48 \\ \hline 0 = 0 \end{array}$$

The signs are the same, so subtract.

0 = 0 is an equation that is ALWAYS true. You can conclude that any (x, y) pair that solves the first equation also makes the second equation true. These two equations represent the same line, so there are an infinite number of solutions. This system is a dependent consistent system.

Elimination by Substitution

The following steps should be included in your Resource Sheet as a reminder of how to execute the substitution method.

When using this second algebraic method to solve systems of linear equations, you will follow these steps:

Step 1: Solve one of the equations so that one variable is by itself on one side of the equal sign.

Step 2: Substitute this expression into the other equation, in place of the lone variable, and solve for the other variable.

Step 3: Substitute this value into either of the original equations and solve for the remaining variable.

Step 4: State the solution and verify it in both original equations.

These steps are demonstrated in Example 5.

Example 5

Solve the system using the substitution method.

$3x + 4y = -2$ Equation 1

$2x - y = 17$ Equation 2

Solution:

Step 1

It would be most convenient to solve Equation 2 for y as its coefficient is –1.

$2x - y = 17 \rightarrow y = 2x - 17$ Revised Equation 2

Step 2

Substitute the value for y from revised Equation 2 into Equation 1.

$$3x + 4y = -2$$ Insert $(2x - 17)$ in place of y.

$$3x + 4(2x - 17) = -2$$ Simplify and solve for x.

$$3x + 8x - 68 = -2$$

$$11x = 66$$

$$x = 6$$

Step 3

Substitute $x = 6$ back into the first equation and solve for y.

$$3x + 4y = -2$$

$$3(6) + 4y = -2$$

$$18 + 4y = -2$$

$$4y = -20$$

$$y = -5$$

Step 4

The solution to this independent system is $(6, -5)$.

Verify:

$3x + 4y$	-2
$3(6) + 4{-5})$	-2
$18 - 20$	-2
-2	-2

$2x - y$	17
$2(6) - (-5)$	17
$12 + 5$	17
17	17

Example 6

Solve using the substitution method:

$4x + y = 1$ Equation 1

$2x - 3y = 4$ Equation 2

Solution:

Solve Equation 1 for y because it has a coefficient of 1.

$4x + y = 1 \rightarrow y = 1 - 4x$

$2x - 3y = 4$

Substitute revised Equation 1 into Equation 2 for y.

$$2x - 3(1 - 4x) = 4$$

$$2x - 3 + 12x = 4 \quad \text{Simplify}$$

$$14x = 7 \quad \text{Solve for } x$$

$$x = \frac{1}{2}$$

Substitute the value for x into the other original equation and solve for y.

$$4x + y = 1$$

$$4\left(\frac{1}{2}\right) + y = 1$$

$$2 + y = 1$$

$$y = -1$$

Solution:

$\left(\frac{1}{2}, -1\right)$

Verify:

$4x + y$	1
$4\left(\frac{1}{2}\right) + (-1)$	1
$2 - 1$	1
1	1

$2x - 3y$	4
$2\left(\frac{1}{2}\right) - 3(-1)$	4
$1 + 3$	4
4	4

This system has one solution. It is an independent system.

Example 7

The sum of two numbers is 62 and their difference is 16. Find the two numbers using the substitution method.

Solution:

Let m and n represent the two numbers.

$m + n = 62$ Equation 1

$m - n = 16$ Equation 2

Solve Equation 1 for either variable, and substitute this value into Equation 2.

$m + n = 62 \rightarrow m = 62 - n$

$$
\begin{aligned}
m - n &= 16 \\
(62 - n) - n &= 16 \\
62 - 2n &= 16 \\
62 - 16 &= 2n \\
46 &= 2n \\
n &= 23
\end{aligned}
$$

$$
\begin{aligned}
m - n &= 16 \\
m - (23) &= 16 \\
m &= 16 + 23 \\
m &= 39
\end{aligned}
$$

The numbers are 39 and 23.

Verify:

$m + n$	62
$39 + 23$	62
62	62

$m - n$	16
$39 - 23$	16
16	16

Learning Activity 8.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. A triangular prism has a volume of 16 mm^3. The base and height of the triangle are 0.2 cm and 0.4 cm. What is the height of the prism?
2. Which is larger: 436% or $\frac{18}{5}$?
3. There are three defensemen on your soccer team. If there are 5 times as many people on the team, what percent of the team is defense?
4. You run every second day. You run 3.5 miles on Tuesday, 4 miles on Thursday, and 4.5 miles on Saturday. How far will you run on the next day that you run, and which day is that?
5. You have 6 blueberries, 4 raspberries, and 8 slices of strawberry in your bowl of cereal. If you get a piece of fruit with every mouthful, how many bites does it take to finish your breakfast?
6. You are getting ready for a barbeque you are hosting. It costs $1.50 for a package of 30 plastic cups. How much does each plastic cup cost?
7. At your barbeque, you are providing the food, but will have a collection basket for donations to cover the cost. If you buy 2 packages of hamburgers for $12.00 each and 1 package of chicken burgers for $15.00, how much must you collect to break even?
8. Simplify: $\frac{6x^5y^3z^7}{2z^9x^2y^3}$.

continued

Part B: Solving Systems of Equations by Elimination

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Solve the following system using elimination by addition or subtraction.

   $5x + 4y = 6$

   $-3y - 2x = -1$

2. Solve the following system using elimination by substitution.

   $-\frac{1}{2}x + y = 4$

   $x + 2y = 8$

3. Solve the following by elimination. You can choose either method. We suggest that you look for characteristics that make addition/subtraction the easier way, and characteristics that make substitution the easier way. For example, if the coefficients are equal for x, addition/subtraction is quicker than elimination.

   a) $2x + 3y = 12$

      $2x - 3y = 6$

   b) $3x + 4y = 15$

      $x - y = 5$

   c) $2x - 3y = 8$

      $6y - 4x = -22$

   d) $x + 3y = 7$

      $2x + 6y = 14$

   e) $5x + 10y = 50$

      $2x + 3y = 14$

4. Describe what happens when you try to solve an inconsistent system of linear equations by the following (use examples if you wish):

   a) graphing

   b) elimination

continued

5. The lengths of each of the congruent sides of an isosceles triangle are $1\frac{1}{2}$ times the length of the base. The perimeter of the triangle is 60 cm. Use a system of equations to find the length of each side of the triangle. Solve the system using elimination.
6. A hotel has 160 rooms, some singles and some doubles. The single rooms cost $45 per night and the doubles cost $60 per night. Because of a curling bonspiel (tournament), all the rooms are occupied. The sales for that night total $8700. How many of each type of room does the hotel have? Solve this problem using a system of equations to model the situation, and solve it using elimination.
7. The graphs of $ax + by = 13$ and $ax - by = -3$ intersect at (1, 4). Find a and b.

Lesson Summary

In this lesson, you used two different algebraic methods to solve 2-by-2 systems of linear equations. When it was convenient to rewrite one equation in terms of one of the variables, you used elimination by substitution to solve the system. When using elimination by addition or subtraction, you multiplied one or both of the equations by the appropriate amounts so that one variable had the same coefficient in both equations, and then you solved for the value of the variables and verified your solution.

The approaches used to solve systems of equations that you used in this lesson are used frequently, especially when you do not have access to technology and you are looking for an accurate result. With this in mind, these approaches are more commonly found in pre-calculus math.

Notes

Assignment 8.2

Solving Systems of Equations by Elimination

Total Marks = 33

Note to Students: Have you made a Resource Sheet for this module? Do you have the definitions and formulas on your Resource Sheet? If so, you would be able to use it now. If not, now would be a good time to make one.

1. Explain how you would decide to use the substitution method or the addition/subtraction method to solve a given system of linear equations. You may use examples to help you explain. *(2 marks)*

2. Solve by elimination.

   a) $y = 2x + 8$ *(3 marks)*

   $y = 10x$

Assignment 8.2: Solving Systems of Equations by Elimination (continued)

b) $3x - y + 5 = 0$ *(3 marks)*

$2y = 6x + 10$

c) $5x + 2y = -9$ *(3 marks)*

$3x - 4y = -8$

d) $2x - 3y = -12$ *(3 marks)*

$y = \frac{2}{3}x - 5$

3. Four years from now, Katelyn will be as old as Adrian is now. Seven years ago, the sum of their ages was 22. Write a system of equations and solve it using elimination to determine how old the girls are now. *(4 marks)*

Assignment 8.2: Solving Systems of Equations by Elimination (continued)

4. One number is 9 less than another number. The sum of the two numbers is 61. Use a system of equations to solve for the two numbers. *(4 marks)*

5. The perimeter of a rectangular cattle enclosure is 400 feet. It is 40 feet longer than it is wide. Using a system of equations, solve for the length and width of the enclosure. *(4 marks)*

6. The high school drama club was charged royalty fees for the privilege of using a particular script for its production. The flat rate is \$300.00, plus a fee of \$3.00 for each person attending the play. The drama club decided to charge admission fees of \$8.00 per person for the performance.

   a) One of the students wrote the following two equations to represent this situation. Explain the meaning of each. *(2 marks)*

   $y = 300 + 3x$ and $y = 8x$

   b) Use elimination to solve the system. State the solution and verify your answer. *(3 marks)*

   c) Explain the significance of the point of intersection in terms of this situation. *(2 marks)*

Assignment 8.2: Solving Systems of Equations by Elimination (continued)

7. You did it! This course (except for your final exam) is done! It's history—really! It is now part of that "Math History" you wrote about when you started this course. Now that you have reached the end of the pathway that is this course, it is a good time to do some self–assessment again.

   Look back at the History/Pathway/Destination chart you completed in Lesson 1 of Module 1, and then revised at the end of Module 4.

   When you go on a vacation. you may take photographs to help remind you of the journey and the things you did along the way. Consider this a written photo album! Write down your thoughts in regards to your trip through this course as you worked your way to achieving your goals. Include this with your Module 8 assignments when you send your work to the Distance Learning Unit.

My Photo Album

Describe your favourite part of the course or favourite distance education experience.

List the goals you have accomplished.

Describe which steps along the pathway were effective in helping you reach your destination.

What kept you from reaching your goals? What steps are left for you to take?

What would you have liked changed or liked to be different in this course?

What is your next destination? Set some new goals and describe the steps you will take to achieve them.

Notes

Module 8 Summary

Congratulations! You have finished the last module in the course!

This compact module introduced you to 2-by-2 systems of linear equations. You used graphic and algebraic methods to solve consistent and inconsistent systems, and verified the solutions. You created a system of equations to represent different contexts, and solved problems related to these situations by finding the solution to the system. You can explain the meaning of the point(s) of intersection of dependent and independent systems of equations, and explain why an inconsistent system has the empty set as its solution.

In future math courses, you may come across systems of non-linear equations, as well as 3 × 3 systems, which consist of 3 variables in 3 equations, but what you have learned in this module will be applicable to problem solving in those situations as well. You have acquired skills and knowledge that will help you model and solve practical problems using systems of equations.

Submitting Your Assignments

It is now time for you to submit Assignments 7.1 to 7.4 from Module 7 and Assignments 8.1 and 8.2 from Module 8 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 7 and Module 8 assignments and organize your material in the following order:

Cover Sheet for Modules 7 and 8 (found at the end of the Introduction)

Assignment 7.1 Distance and Midpoint

Assignment 7.2 Linear Relations and Formulas

Assignment 7.3 Writing Linear Equations Based on Different Information

Assignment 7.4 Line of Best Fit and Correlation

Assignment 8.1 Solving Systems of Linear Equations Graphically

Assignment 8.2 Solving Systems of Equations by Elimination

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction

Final Examination

Congratulations, you have finished Module 8 in the course. The final examination is out of 100 marks and worth 25% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 8.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, metric and imperial rules, and your Examination Resource Sheet. A maximum of 2 hours is available to complete your final examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point you will also have to combine your resource sheets from Modules 1 to 8 onto one 8½″ × 11″ paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you.

Sample

Front

Module 1	Module 2
Module 3	Module 4

Back

Module 5	Module 6
Module 7	Module 8

Examination Review

You are now ready to begin preparing for your final examination. Please review the content, learning activities, and assignments from Modules 1 to 8.

The final practice examination is also an excellent study aid for reviewing Modules 1 to 8.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the final examination determines 25% of your final mark in this course and you will have 2 hours to complete the examination.

Final Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The final practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your final practice examination, follow these steps:

1. Study for the final practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 1 to 8 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 8, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Final Examination Resource Sheet. You can use both sides of an 8½″ by 11″ piece of paper.
6. Bring the following to the final practice examination: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, metric and imperial rules, and your Examination Resource Sheet.
7. Write your final practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the final examination is 2 hours.
8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Final Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 8
Systems of Equations

Learning Activity Answer Keys

Module 8: Systems of Equations

Learning Activity 8.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. 35% of the trees in the forest of 400 have Dutch elm disease. How many trees have this illness?
2. Evaluate: $(3 + 4x)^0$.
3. Strong or weak correlation: the points on a graph are spread out and it is hard to see the pattern they make.
4. A line has the intercepts $x = 5$ and $y = -7$. Write the equation of the line.
5. A right triangle has sides with lengths 8, 15, 17. Which is the hypotenuse?
6. Which two whole numbers is $\sqrt{150}$ between?
7. What is the sum of the first four perfect squares?
8. A dozen muffins costs $8.66. How much would you expect half a dozen muffins to cost?

Answers:

1. 140 (10% of 400 is 40 so 30% is 120 and 5% is 20; 120 + 20 = 140)
2. 1 (Any number with an exponent of 0 = 1.)
3. Weak correlation
4. $y = \frac{7}{5}x - 7$
5. 17 (the longest side is always the hypotenuse)
6. 12 and 13 ($12^2 = 144$ and $13^2 = 169$)
7. 30 (1 + 4 + 9 + 16; 1 + 9 = 10; 4 + 16 = 20)
8. You would expect it to cost $4.33.

Part B: Solving Systems of Linear Equations Graphically

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Explain what it means if a system of linear equations has an infinite number of solutions. Give an example.

   Answer:

   If a system of linear equations has an infinite number of solutions, it means that the two equations in the system are equivalent expressions. Both equations represent the same line (same slope and y-intercept), and each coordinate point along the line is part of the solution set. The system is a dependent system.

   An example would consist of any two equivalent linear equations. One possible pair is:

   $y = 2x + 4$

   $3y = 6x + 12$

2. Write a linear system of equations that has for its solution the ordered pair (2, 5).

   Answer:

   Your answer may vary. A correct response is any two equations that include (2, 5) as an ordered pair. One possible solution is:

   $x + y = 7$

   $y = 3x - 1$

3. Leslie has 16 coins worth \$2.20, consisting of some quarters and some nickels. Write a system of linear equations to represent this situation. Graph the system and determine how many quarters and nickels she has.

Answer:

You know the number of coins and the values. Let q represent the number of quarters and n represent the number of nickels.

$$q + n = 16$$

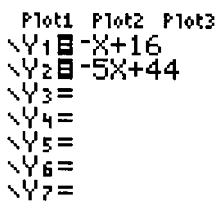
$$0.25q + 0.05n = 2.20$$

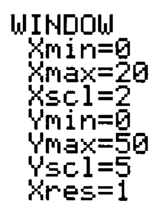
Rewrite the equations in $y = mx + b$ form, graph them, and determine the point of intersection.

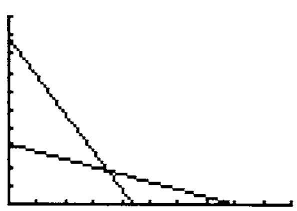
$$n = -q + 16$$

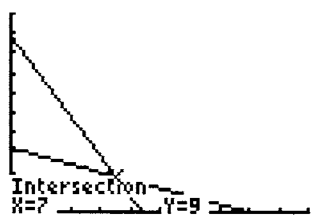
$$n = \frac{-0.25}{0.05}q + \frac{2.20}{0.05}$$

$$n = -5q + 44$$









The solution to this independent system is at (7, 9).

Leslie has 7 quarters and 9 nickels.

The solution can be verified using the original equations:

$q + n$	16
$7 + 9$	16
16	16

$0.25q + 0.05n$	2.20
$0.25(7) + 0.05(9)$	2.20
$1.75 + 0.45$	2.20
2.20	2.20

4. Three pencils and eight erasers cost \$2.30. A pencil costs 18 cents more than an eraser. Use a system of equations and a graph to determine how much a pencil and eraser cost individually.

Answer:

Let p represent the number of pencils and e represent the number of erasers.

$$3p + 8e = 2.30$$

$$p = 0.18 + e$$

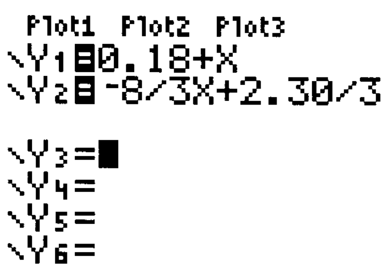
Rewrite the first equation in terms of $y = mx + b$.

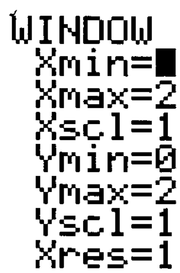
$$3p + 8e = 2.30$$

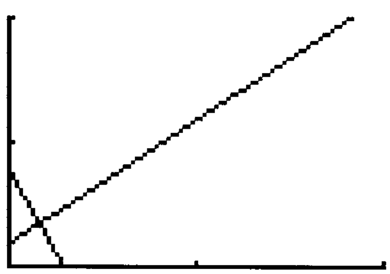
$$3p = -8e + 2.30$$

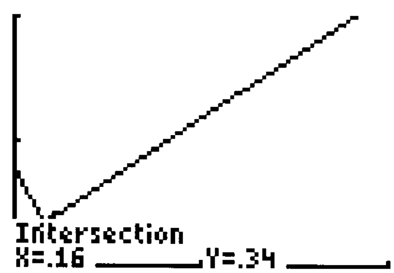
$$p = \frac{-8}{3}e + \frac{2.30}{3}$$

Use technology to graph the systems and find the point of intersection.









The solution of this independent system is at (0.16, 0.34).

Erasers are \$0.16 and pencils cost \$0.34.

Verify the solution using the original equations.

$3p + 8e$	2.30
$3(0.34) + 8(0.16)$	2.30
$1.02 + 1.28$	2.30
2.30	2.30

p	$0.18 + e$
0.34	$0.18 + (0.16)$
0.34	0.34

5. Determine the solution of the following systems of linear equations by graphing, with or without technology. Include a graph for each, indicate the type of system it represents, and state and verify the solution set for each.

   a) $8x - 3y = 6$

   $6x + 12y = -24$

   Answer:

   $8x - 3y = 6$

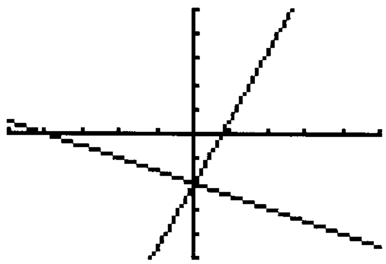
   $3y = 8x - 6$

   $y = \dfrac{8}{3x} - 2$

   $6x + 12y = -24$

   $12y = -6x - 24$

   $y = \dfrac{-1}{2}x - 2$



The solution to this independent system is at (0, −2).

Verify:

$8x - 3y$	6
$8(0) - 3(-2)$	6
6	6

$6x + 12y$	-24
$6(0) + 12(-2)$	-24
-24	-24

b) $\frac{1}{2}x - y = 8$

$x + \frac{1}{3}y = 2$

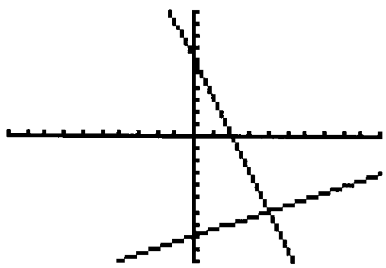
Answer:

$\frac{1}{2}x - y = 8$

$y = \frac{1}{2}x - 8$

$x + \frac{1}{3}y = 2$

$y = -3x + 6$



The solution to this independent system is at (4, −6).

Verify:

$\frac{1}{2}x - y$	8
$\frac{1}{2}(4) - (-6)$	8
$2 + 6$	8
8	8

$x + \frac{1}{3}y$	2
$(4) + \frac{1}{3}(-6)$	2
$4 - 2$	2
2	2

c) $y = \frac{-2}{3}x + 7$

$4x + 6y = 42$

Answer:

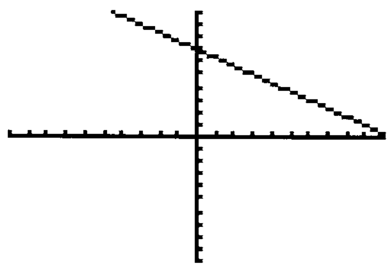
$y = \frac{-2}{3}x + 7$

$4x + 6y = 42$

$y = \frac{-4}{6}x + \frac{42}{6}$

$y = \frac{-2}{3}x + 7$

These two equations are equivalent. The system is dependent and the solution set is all points on the line.



Choose any point along the line to verify. The y-intercept is at 7, so (0, 7) is a point on the line.

y	$\frac{-2}{3}x + 7$
7	$\frac{-2}{3}(0) + 7$
7	7

$4x + 6y$	42
$4(0) + 6(7)$	42
42	42

Learning Activity 8.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. A triangular prism has a volume of 16 mm^3. The base and height of the triangle are 0.2 cm and 0.4 cm. What is the height of the prism?
2. Which is larger: 436% or $\frac{18}{5}$?
3. There are three defensemen on your soccer team. If there are 5 times as many people on the team, what percent of the team is defense?
4. You run every second day. You run 3.5 miles on Tuesday, 4 miles on Thursday, and 4.5 miles on Saturday. How far will you run on the next day that you run, and which day is that?
5. You have 6 blueberries, 4 raspberries, and 8 slices of strawberry in your bowl of cereal. If you get a piece of fruit with every mouthful, how many bites does it take to finish your breakfast?
6. You are getting ready for a barbeque you are hosting. It costs $1.50 for a package of 30 plastic cups. How much does each plastic cup cost?
7. At your barbeque, you are providing the food, but will have a collection basket for donations to cover the cost. If you buy 2 packages of hamburgers for $12.00 each and 1 package of chicken burgers for $15.00, how much must you collect to break even?
8. Simplify: $\frac{6x^5y^3z^7}{2z^9x^2y^3}$.

Answers:

1. 2 mm (0.4 cm = 4 mm, 0.2 cm = 2 mm so 16 ÷ (2 × 4))
2. 436% $\left(18 = 3\frac{3}{5} = 360\%\right)$
3. 20% $\left(3 \times 5 = 15, \frac{3}{15} = \frac{1}{5}\right)$
4. 5.0 miles on Monday (You add a half a mile each day.)
5. 18 (6 + 4 + 8)
6. $0.05 (15 ÷ 3 = 5, add the decimals (2 places—one from the original decimal in the price, the other because it is 30, not 3))
7. $39.00 ((2 × 12) + 15)
8. $3x^3z^{-2}$ (Combine like terms: $y^0 = 1$)

Part B: Solving Systems of Equations by Elimination

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Solve the following system using elimination by addition or subtraction.

$$5x + 4y = 6$$
$$-3y - 2x = -1$$

Answer:

Rearrange the equations so the same variables are lined up in columns.

$5x + 4y = 6$	Equation 1
$-2x - 3y = -1$	Equation 2

Multiply each equation by the appropriate value to get a common coefficient.

$$3(5x + 4y = 6) \rightarrow 15x + 12y = 18$$
$$4(-2x - 3y = -1) \rightarrow -8x - 12y = -4$$

The signs of the common coefficient are different, so add the equations.

$$\begin{aligned} &15x + 12y = 18 \\ +&(-8x - 12y = -4) \\ &7x = 14 \\ \hline &x = 2 \end{aligned}$$

Substitute the value for x into either equation.

$$\begin{aligned} 5x + 4y &= 6 \\ 5(2) + 4y &= 6 \\ 10 + 4y &= 6 \\ 4y &= -4 \\ y &= -1 \end{aligned}$$

Solution:

$(2, -1)$

Verify by substituting back into both original equations.

Note: You may have multiplied the two original equations by different values to get a common coefficient for x. The solution will be the same even if the process is different.

2. Solve the following system using elimination by substitution.

$$-\frac{1}{2}x + y = 4$$

$$x + 2y = 8$$

Answer:

Solve Equation 1 for y and substitute the value into Equation 2.

Alternately, solve Equation 2 for x and substitute that value into Equation 1.

Equation 1: $-\frac{1}{2}x + y = 4 \quad \rightarrow \quad y = 4 + \frac{1}{2}x$

Equation 2:

$$\begin{aligned} x + 2y &= 8 \\ x + 2(4 + \tfrac{1}{2}x) &= 8 \\ x + 8 + x &= 8 \\ 2x &= 0 \\ x &= 0 \end{aligned}$$

Now solve for y.

$$\begin{aligned} x + 2y &= 8 \\ (0) + 2y &= 8 \\ y &= 4 \end{aligned}$$

The solution is (0, 4). Verify in both original equations.

3. Solve the following by elimination. You can choose either method. We suggest that you look for characteristics that make addition/subtraction the easier way, and characteristics that make substitution the easier way. For example, if the coefficients are equal for x, addition/subtraction is quicker than elimination.

a) $2x + 3y = 12$

$2x - 3y = 6$

Answer:

$$\begin{aligned} 2x + 3y &= 12 \\ \underline{2x - 3y} &\underline{= 6} \\ 6y &= 6 \\ y &= 1 \end{aligned}$$

$$\begin{aligned} 2x + 3y &= 12 \\ 2x + 3(1) &= 12 \\ 2x &= 9 \\ x &= \frac{9}{2} \end{aligned}$$

Solution:

$\left(\frac{9}{2}, 1\right)$

b) $3x + 4y = 15$

$x - y = 5$

Answer:

$3x + 4y = 15$

$x - y = 5 \rightarrow x = 5 + y$

$$\begin{aligned} 3x + 4y &= 15 \\ 3(5 + y) + 4y &= 15 \\ 15 + 3y + 4y &= 15 \\ 7y &= 0 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} x &= 5 + y \\ x &= 5 + 0 \\ x &= 5 \end{aligned}$$

Solution:

(5, 0)

c) $2x - 3y = 8$

$6y - 4x = -22$

Answer:

$2x - 3y = 8 \rightarrow 2(2x - 3y = 8) \rightarrow 4x - 6y = 16$

$6y - 4x = -22 \rightarrow -4x + 6y = -22$

$$\begin{array}{r} 4x - 6y = 16 \\ \underline{+\left(-4x + 6y = -22\right)} \\ 0 = -6 \end{array}$$

This is a false statement. This system is an inconsistent system. They are parallel lines and there is no real solution to this system, or the solution is the empty set.

d) $x + 3y = 7$

$2x + 6y = 14$

Answer:

$x + 3y = 7$

$2x + 6y = 14$

$2(x + 3y = 7) \rightarrow 2x + 6y = 14$

$2x + 6y = 14 \rightarrow \underline{2x + 6y = 14}$ Subtract the equations.

$0 = 0$

This statement is always true. This is a dependent system. The solution consists of an infinite number of points along this line.

e) $5x + 10y = 50$

$2x + 3y = 14$

Answer:

$2(5x+10y=50) \rightarrow 10x+20y=100$

$5(2x+3y=14) \rightarrow 10x+15y=70$

$$\begin{aligned} 10x+20y&=100 \\ \underline{10x+15y}&\underline{=70} \\ 5y&=30 \\ y&=6 \\ 2x+3y&=14 \\ 2x+3(6)&=14 \\ 2x&=-4 \\ x&=-2 \end{aligned}$$

Solution:

$(-2, 6)$

These are some patterns that you may have noticed as you solved these systems of equations:

- Elimination by addition/subtraction is the easier method when the coefficients of one variable are equal.
- Elimination by addition/subtraction is the easier method if you have coefficients in front of all variables in both equations.
- Elimination by substitution is the easier method when dealing with fractional coefficients.
- Elimination by substitution is the easier method when there is no coefficient in front of one variable in one equation.

4. Describe what happens when you try to solve an inconsistent system of linear equations by the following (use examples if you wish):

   a) graphing

   Answer:

   The graphs of linear equations in an inconsistent system are parallel. They do not intersect when you graph them, so there is no solution.

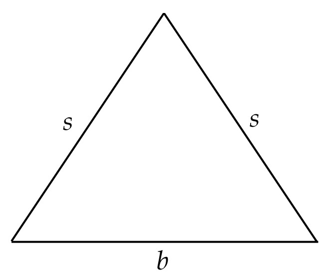
   b) elimination

   Answer:

   When you solve an inconsistent system of equations by elimination, you end up with a false statement. There is no ordered pair that will make both equations true, so there is no solution to the system (or the solution is the empty set, ∅).

5. The lengths of each of the congruent sides of an isosceles triangle are $1\frac{1}{2}$ times the length of the base. The perimeter of the triangle is 60 cm. Use a system of equations to find the length of each side of the triangle. Solve the system using elimination.

Answer:



Let s be the length of one of the congruent sides in an isosceles triangle and let b represent the length of its base.

$s = 1.5b$ Equation 1

$2s + b = 60$ Equation 2

Substitute Equation 1 into Equation 2

$$\begin{aligned} 2s + b &= 60 \\ 2(1.5b) + b &= 60 \\ 3b + b &= 60 \\ 4b &= 60 \\ b &= 15 \end{aligned}$$

Solve for s using one of the original equations.

$s = 1.5b$

$s = 1.5(15)$

$s = 22.5$

The base is 15 cm and the side lengths are 22.5 cm each.

Verify:

s	$15b$
22.5	1.5(15)
22.5	22.5

$2s + b$	60
2(22.5) + (15)	60
45 + 15	60
60	60

6. A hotel has 160 rooms, some singles and some doubles. The single rooms cost \$45 per night and the doubles cost \$60 per night. Because of a curling bonspiel (tournament), all the rooms are occupied. The sales for that night total \$8700. How many of each type of room does the hotel have? Solve this problem using a system of equations to model the situation, and solve it using elimination.

Answer:

Let s = the number of single rooms and d = the number of double rooms.

$s + d = 160$ Equation 1 Solve for either s or d and substitute into Equation 2.

$45s + 60d = 8700$ Equation 2

$$s = 160 - d$$

$$45s + 60d = 8700$$

$$45(160 - d) + 60d = 8700$$

$$7200 - 45d + 60d = 8700$$

$$15d = 1500$$

$$d = 100$$

$$s + d = 160$$

$$s + (100) = 160$$

$$s = 60$$

The hotel has 60 single rooms and 100 double rooms.

Check:

$s + d$	160
$60 + 100$	160
160	160

$45s + 60d$	8700
$45(60) + 60(100)$	8700
8700	8700

7. The graphs of $ax + by = 13$ and $ax - by = -3$ intersect at (1, 4). Find a and b.

Answer:

Substitute the ordered pair (1, 4) into both equations for (x, y).

Equation 1	Equation 2
$ax + by = 13$	$ax - by = -3$
$a(1) + b(4) = 13$	$a(1) - b(4) = -3$
$a + 4b = 13$	$a - 4b = -3$

Use elimination by subtraction to solve the system. You could also use elimination by addition (then solve for a).

$$\begin{array}{r} a + 4b = 13 \\ -\ (a - 4b = -3) \\ \hline 8b = 16 \\ b = 2 \end{array} \qquad \text{Subtract}$$

$$\begin{aligned} a + 4b &= 13 \\ a + 4(2) &= 13 \\ a &= 13 - 8 \\ a &= 5 \end{aligned}$$

$a = 5$ and $b = 2$

Verify using the solution (1, 4):

$ax + by$	13
$5(1) + (2)(4)$	13
$5 + 8$	13
13	13

$ax - by$	-3
$5(1) - (2)(4)$	-3
$5 - 8$	-3
-3	-3

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Module 8
Systems of Equations

Learning Activity Answer Keys

Module 8: Systems of Equations

Learning Activity 8.1

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

1. 35% of the trees in the forest of 400 have Dutch elm disease. How many trees have this illness?
2. Evaluate: $(3 + 4x)^0$.
3. Strong or weak correlation: the points on a graph are spread out and it is hard to see the pattern they make.
4. A line has the intercepts $x = 5$ and $y = -7$. Write the equation of the line.
5. A right triangle has sides with lengths 8, 15, 17. Which is the hypotenuse?
6. Which two whole numbers is $\sqrt{150}$ between?
7. What is the sum of the first four perfect squares?
8. A dozen muffins costs $8.66. How much would you expect half a dozen muffins to cost?

Answers:

1. 140 (10% of 400 is 40 so 30% is 120 and 5% is 20; 120 + 20 = 140)
2. 1 (Any number with an exponent of 0 = 1.)
3. Weak correlation
4. $y = \frac{7}{5}x - 7$
5. 17 (the longest side is always the hypotenuse)
6. 12 and 13 ($12^2 = 144$ and $13^2 = 169$)
7. 30 (1 + 4 + 9 + 16; 1 + 9 = 10; 4 + 16 = 20)
8. You would expect it to cost $4.33.

Part B: Solving Systems of Linear Equations Graphically

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Explain what it means if a system of linear equations has an infinite number of solutions. Give an example.

   Answer:

   If a system of linear equations has an infinite number of solutions, it means that the two equations in the system are equivalent expressions. Both equations represent the same line (same slope and y-intercept), and each coordinate point along the line is part of the solution set. The system is a dependent system.

   An example would consist of any two equivalent linear equations. One possible pair is:

   $y = 2x + 4$

   $3y = 6x + 12$

2. Write a linear system of equations that has for its solution the ordered pair (2, 5).

   Answer:

   Your answer may vary. A correct response is any two equations that include (2, 5) as an ordered pair. One possible solution is:

   $x + y = 7$

   $y = 3x - 1$

3. Leslie has 16 coins worth \$2.20, consisting of some quarters and some nickels. Write a system of linear equations to represent this situation. Graph the system and determine how many quarters and nickels she has.

Answer:

You know the number of coins and the values. Let q represent the number of quarters and n represent the number of nickels.

$$q + n = 16$$

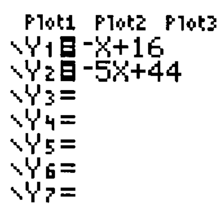
$$0.25q + 0.05n = 2.20$$

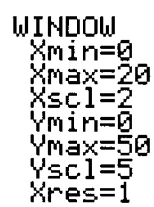
Rewrite the equations in $y = mx + b$ form, graph them, and determine the point of intersection.

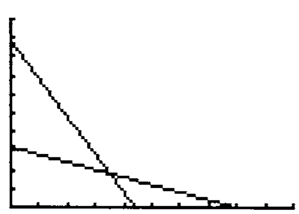
$$n = -q + 16$$

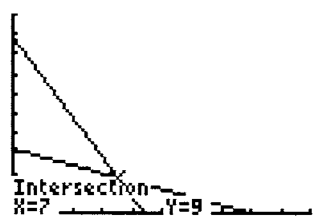
$$n = \frac{-0.25}{0.05}q + \frac{2.20}{0.05}$$

$$n = -5q + 44$$









The solution to this independent system is at (7, 9).

Leslie has 7 quarters and 9 nickels.

The solution can be verified using the original equations:

$q + n$	16
$7 + 9$	16
16	16

$0.25q + 0.05n$	2.20
$0.25(7) + 0.05(9)$	2.20
$1.75 + 0.45$	2.20
2.20	2.20

4. Three pencils and eight erasers cost \$2.30. A pencil costs 18 cents more than an eraser. Use a system of equations and a graph to determine how much a pencil and eraser cost individually.

Answer:

Let p represent the number of pencils and e represent the number of erasers.

$3p + 8e = 2.30$

$p = 0.18 + e$

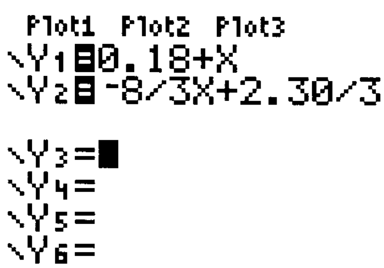
Rewrite the first equation in terms of $y = mx + b$.

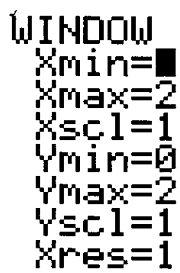
$$3p + 8e = 2.30$$

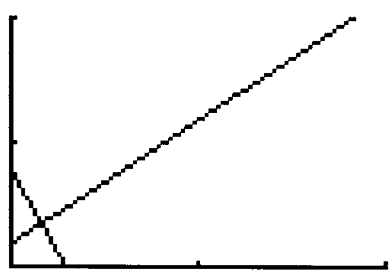
$$3p = -8e + 2.30$$

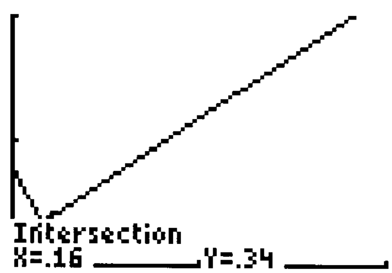
$$p = \frac{-8}{3}e + \frac{2.30}{3}$$

Use technology to graph the systems and find the point of intersection.









The solution of this independent system is at (0.16, 0.34).

Erasers are \$0.16 and pencils cost \$0.34.

Verify the solution using the original equations.

$3p + 8e$	2.30
3(0.34) + 8(0.16)	2.30
1.02 + 1.28	2.30
2.30	2.30

p	$0.18 + e$
0.34	0.18 + (0.16)
0.34	0.34

5. Determine the solution of the following systems of linear equations by graphing, with or without technology. Include a graph for each, indicate the type of system it represents, and state and verify the solution set for each.

   a) $8x - 3y = 6$

      $6x + 12y = -24$

      Answer:

      $8x - 3y = 6$

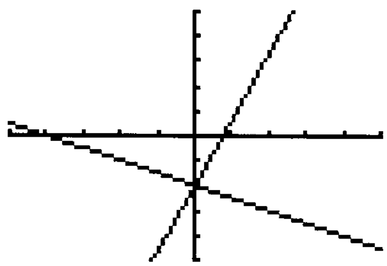
      $3y = 8x - 6$

      $y = \dfrac{8}{3x} - 2$

      $6x + 12y = -24$

      $12y = -6x - 24$

      $y = \dfrac{-1}{2}x - 2$



The solution to this independent system is at (0, −2).

Verify:

$8x - 3y$	6
$8(0) - 3(-2)$	6
6	6

$6x + 12y$	-24
$6(0) + 12(-2)$	-24
-24	-24

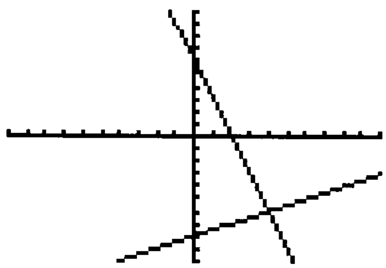
b) $\frac{1}{2}x - y = 8$

$x + \frac{1}{3}y = 2$

Answer:

$$\frac{1}{2}x - y = 8 \qquad\qquad x + \frac{1}{3}y = 2$$

$$y = \frac{1}{2}x - 8 \qquad\qquad y = -3x + 6$$



The solution to this independent system is at (4, −6).

Verify:

$\frac{1}{2}x - y$	8
$\frac{1}{2}(4) - (-6)$	8
$2 + 6$	8
8	8

$x + \frac{1}{3}y$	2
$(4) + \frac{1}{3}(-6)$	2
$4 - 2$	2
2	2

c) $y = \frac{-2}{3}x + 7$

$4x + 6y = 42$

Answer:

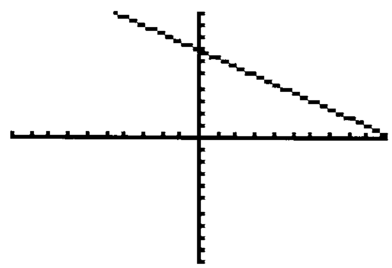
$$y = \frac{-2}{3}x + 7$$

$$4x + 6y = 42$$

$$y = \frac{-4}{6}x + \frac{42}{6}$$

$$y = \frac{-2}{3}x + 7$$

These two equations are equivalent. The system is dependent and the solution set is all points on the line.



Choose any point along the line to verify. The y-intercept is at 7, so (0, 7) is a point on the line.

y	$\frac{-2}{3}x + 7$
7	$\frac{-2}{3}(0) + 7$
7	7

$4x + 6y$	42
$4(0) + 6(7)$	42
42	42

Learning Activity 8.2

Part A: BrainPower

You should be able to complete the following eight questions in just a few minutes without using a calculator or pencil and paper.

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2. Which is larger: 436% or $\frac{18}{5}$?
3. There are three defensemen on your soccer team. If there are 5 times as many people on the team, what percent of the team is defense?
4. You run every second day. You run 3.5 miles on Tuesday, 4 miles on Thursday, and 4.5 miles on Saturday. How far will you run on the next day that you run, and which day is that?
5. You have 6 blueberries, 4 raspberries, and 8 slices of strawberry in your bowl of cereal. If you get a piece of fruit with every mouthful, how many bites does it take to finish your breakfast?
6. You are getting ready for a barbeque you are hosting. It costs $1.50 for a package of 30 plastic cups. How much does each plastic cup cost?
7. At your barbeque, you are providing the food, but will have a collection basket for donations to cover the cost. If you buy 2 packages of hamburgers for $12.00 each and 1 package of chicken burgers for $15.00, how much must you collect to break even?
8. Simplify: $\frac{6x^5y^3z^7}{2z^9x^2y^3}$.

Answers:

1. 2 mm (0.4 cm = 4 mm, 0.2 cm = 2 mm so 16 ÷ (2 × 4))
2. 436% $\left(18 = 3\frac{3}{5} = 360\%\right)$
3. 20% $\left(3 \times 5 = 15, \frac{3}{15} = \frac{1}{5}\right)$
4. 5.0 miles on Monday (You add a half a mile each day.)
5. 18 (6 + 4 + 8)
6. $0.05 (15 ÷ 3 = 5, add the decimals (2 places—one from the original decimal in the price, the other because it is 30, not 3))
7. $39.00 ((2 × 12) + 15)
8. $3x^3z^{-2}$ (Combine like terms: $y^0 = 1$)

Part B: Solving Systems of Equations by Elimination

Remember, these questions are similar to the ones that will be on your assignments and final examination. So, if you were able to answer them correctly, you are likely to do well on your assignments and examination. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Solve the following system using elimination by addition or subtraction.

   $5x + 4y = 6$

   $-3y - 2x = -1$

   Answer:

   Rearrange the equations so the same variables are lined up in columns.

   $5x + 4y = 6$ Equation 1

   $-2x - 3y = -1$ Equation 2

   Multiply each equation by the appropriate value to get a common coefficient.

   $3(5x + 4y = 6) \rightarrow 15x + 12y = 18$

   $4(-2x - 3y = -1) \rightarrow -8x - 12y = -4$

   The signs of the common coefficient are different, so add the equations.

   $$\begin{array}{l} 15x + 12y = 18 \\ +(-8x - 12y = -4) \\ 7x = 14 \\ \hline x = 2 \end{array}$$

Substitute the value for x into either equation.

$$\begin{aligned} 5x + 4y &= 6 \\ 5(2) + 4y &= 6 \\ 10 + 4y &= 6 \\ 4y &= -4 \\ y &= -1 \end{aligned}$$

Solution:

(2, −1)

Verify by substituting back into both original equations.

Note: You may have multiplied the two original equations by different values to get a common coefficient for x. The solution will be the same even if the process is different.

2. Solve the following system using elimination by substitution.

$$-\frac{1}{2}x + y = 4$$

$$x + 2y = 8$$

Answer:

Solve Equation 1 for y and substitute the value into Equation 2.

Alternately, solve Equation 2 for x and substitute that value into Equation 1.

Equation 1: $-\frac{1}{2}x + y = 4 \quad \rightarrow \quad y = 4 + \frac{1}{2}x$

Equation 2: $x + 2y = 8$

$$x + 2\left(4 + \frac{1}{2}x\right) = 8$$

$$x + 8 + x = 8$$

$$2x = 0$$

$$x = 0$$

Now solve for y.

$$x + 2y = 8$$

$$(0) + 2y = 8$$

$$y = 4$$

The solution is (0, 4). Verify in both original equations.

3. Solve the following by elimination. You can choose either method. We suggest that you look for characteristics that make addition/subtraction the easier way, and characteristics that make substitution the easier way. For example, if the coefficients are equal for x, addition/subtraction is quicker than elimination.

   a) $2x + 3y = 12$

      $2x - 3y = 6$

      Answer:

      $$\begin{array}{r} 2x + 3y = 12 \\ \underline{2x - 3y = 6} \\ 6y = 6 \\ y = 1 \end{array} \qquad \begin{array}{r} 2x + 3y = 12 \\ 2x + 3(1) = 12 \\ 2x = 9 \\ x = \dfrac{9}{2} \end{array}$$

      Solution:

      $\left(\dfrac{9}{2}, 1\right)$

   b) $3x + 4y = 15$

      $x - y = 5$

      Answer:

      $$3x + 4y = 15$$
      $$x - y = 5 \rightarrow x = 5 + y$$

      $$\begin{array}{r} 3x + 4y = 15 \\ 3(5 + y) + 4y = 15 \\ 15 + 3y + 4y = 15 \\ 7y = 0 \\ y = 0 \end{array} \qquad \begin{array}{l} x = 5 + y \\ x = 5 + 0 \\ x = 5 \end{array}$$

      Solution:

      (5, 0)

c) $2x - 3y = 8$

$6y - 4x = -22$

Answer:

$2x - 3y = 8 \rightarrow 2(2x - 3y = 8) \rightarrow 4x - 6y = 16$

$6y - 4x = -22 \rightarrow -4x + 6y = -22$

$$\begin{array}{r} 4x - 6y = 16 \\ \underline{+\left(-4x + 6y = -22\right)} \\ 0 = -6 \end{array}$$

This is a false statement. This system is an inconsistent system. They are parallel lines and there is no real solution to this system, or the solution is the empty set.

d) $x + 3y = 7$

$2x + 6y = 14$

Answer:

$x + 3y = 7$

$2x + 6y = 14$

$2(x + 3y = 7) \rightarrow 2x + 6y = 14$

$2x + 6y = 14 \rightarrow \underline{2x + 6y = 14}$ Subtract the equations.

$0 = 0$

This statement is always true. This is a dependent system. The solution consists of an infinite number of points along this line.

e) $5x + 10y = 50$

$2x + 3y = 14$

Answer:

$2(5x + 10y = 50) \rightarrow 10x + 20y = 100$

$5(2x + 3y = 14) \rightarrow 10x + 15y = 70$

$$\begin{aligned} 10x + 20y &= 100 \\ \underline{10x + 15y} &\underline{= 70} \\ 5y &= 30 \\ y &= 6 \\ 2x + 3y &= 14 \\ 2x + 3(6) &= 14 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

Solution:

$(-2, 6)$

These are some patterns that you may have noticed as you solved these systems of equations:

- Elimination by addition/subtraction is the easier method when the coefficients of one variable are equal.
- Elimination by addition/subtraction is the easier method if you have coefficients in front of all variables in both equations.
- Elimination by substitution is the easier method when dealing with fractional coefficients.
- Elimination by substitution is the easier method when there is no coefficient in front of one variable in one equation.

4. Describe what happens when you try to solve an inconsistent system of linear equations by the following (use examples if you wish):

   a) graphing

   Answer:

   The graphs of linear equations in an inconsistent system are parallel. They do not intersect when you graph them, so there is no solution.

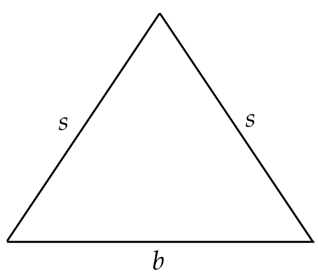
   b) elimination

   Answer:

   When you solve an inconsistent system of equations by elimination, you end up with a false statement. There is no ordered pair that will make both equations true, so there is no solution to the system (or the solution is the empty set, ∅).

5. The lengths of each of the congruent sides of an isosceles triangle are $1\frac{1}{2}$ times the length of the base. The perimeter of the triangle is 60 cm. Use a system of equations to find the length of each side of the triangle. Solve the system using elimination.

   Answer:



Let s be the length of one of the congruent sides in an isosceles triangle and let b represent the length of its base.

$s = 1.5b$ Equation 1

$2s + b = 60$ Equation 2

Substitute Equation 1 into Equation 2

$$\begin{aligned} 2s + b &= 60 \\ 2(1.5b) + b &= 60 \\ 3b + b &= 60 \\ 4b &= 60 \\ b &= 15 \end{aligned}$$

Solve for s using one of the original equations.

$s = 1.5b$

$s = 1.5(15)$

$s = 22.5$

The base is 15 cm and the side lengths are 22.5 cm each.

Verify:

s	$15b$
22.5	1.5(15)
22.5	22.5

$2s + b$	60
2(22.5) + (15)	60
45 + 15	60
60	60

6. A hotel has 160 rooms, some singles and some doubles. The single rooms cost \$45 per night and the doubles cost \$60 per night. Because of a curling bonspiel (tournament), all the rooms are occupied. The sales for that night total \$8700. How many of each type of room does the hotel have? Solve this problem using a system of equations to model the situation, and solve it using elimination.

Answer:

Let s = the number of single rooms and d = the number of double rooms.

$s + d = 160$ Equation 1 Solve for either s or d and substitute into Equation 2.

$45s + 60d = 8700$ Equation 2

$$s = 160 - d$$

$$45s + 60d = 8700$$

$$45(160 - d) + 60d = 8700$$

$$7200 - 45d + 60d = 8700$$

$$15d = 1500$$

$$d = 100$$

$$s + d = 160$$

$$s + (100) = 160$$

$$s = 60$$

The hotel has 60 single rooms and 100 double rooms.

Check:

$s + d$	160
$60 + 100$	160
160	160

$45s + 60d$	8700
$45(60) + 60(100)$	8700
8700	8700

7. The graphs of $ax + by = 13$ and $ax - by = -3$ intersect at (1, 4). Find a and b.

Answer:

Substitute the ordered pair (1, 4) into both equations for (x, y).

Equation 1

$$ax + by = 13$$
$$a(1) + b(4) = 13$$
$$a + 4b = 13$$

Equation 2

$$ax - by = -3$$
$$a(1) - b(4) = -3$$
$$a - 4b = -3$$

Use elimination by subtraction to solve the system. You could also use elimination by addition (then solve for a).

$$\begin{array}{r} a + 4b = 13 \\ -\ (a - 4b = -3) \\ \hline 8b = 16 \\ b = 2 \end{array} \quad \text{Subtract}$$

$$a + 4b = 13$$
$$a + 4(2) = 13$$
$$a = 13 - 8$$
$$a = 5$$

$a = 5$ and $b = 2$

Verify using the solution (1, 4):

$ax + by$	13
$5(1) + (2)(4)$	13
$5 + 8$	13
13	13

$ax - by$	-3
$5(1) - (2)(4)$	-3
$5 - 8$	-3
-3	-3

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Appendix A: Glossary

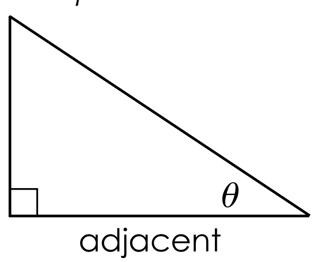
Appendix A: Glossary

accurate
How close a measurement or calculation is to the actual value.

acute
An angle less than 90°.

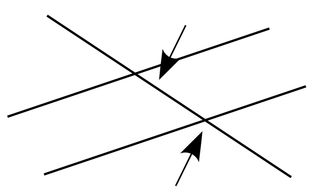
adjacent side
The side of a right triangle beside the angle (θ°)—not the hypotenuse.

Example



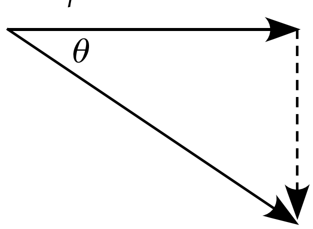
alternate angles
Angles that are opposite each other where a transversal intersects a line are equal.

Example



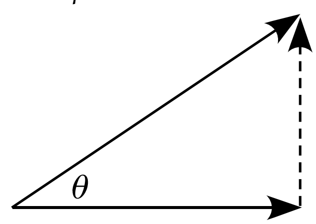
angle of depression
The angle formed between your natural line of sight (a horizontal line) and your downward line of sight.

Example



angle of elevation
The angle created between your natural line of sight (a horizontal line) and your elevated line of sight.

Example



area
The space taken up by a 2-D object.

base
The number being multiplied together with itself in a power (4 is the base in 4^3).

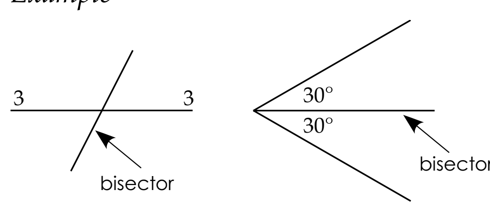
BEDMAS (Brackets, Exponents, Division, Multiplication, Addition, Subtraction)
Division and Multiplication (and Addition and Subtraction) are to be completed in the order in which they appear from left to right in the expression or equation (see **order of operations**).

binomial
A polynomial with two terms.

bisect
To cut into two equal parts.

bisector
A line that divides an angle or another line into two equal parts.

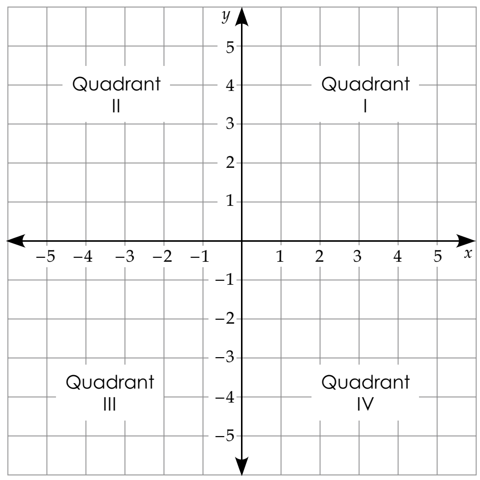
Example



Cartesian plane

The grid used to plot data and graphs.

Example



centi (c)

A metric prefix; multiplication factor = $10^{-2} = 0.01$.

circle

A shape with 1 edge (circumference) that curves around a centre point.

circumference

The distance around the edge of a circle (also known as the perimeter).

coefficient

The number multiplying the variable(s) in a term (e.g., 7 is the coefficient of $7x^3$).

common denominator

Two or more fractions that have the same number in the denominator.

complementary angles

Two angles that add up to 90°.

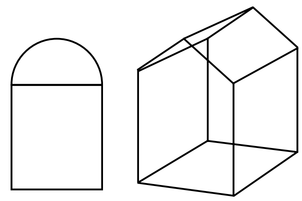
composite number

An integer greater than 1 that has *more than* two factors (not just one and itself).

composite object

An object made up of more than one 2-D or 3-D shape.

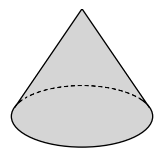
Example



cone

A geometric figure with a flat (plane) base and a curved surface (curved face).

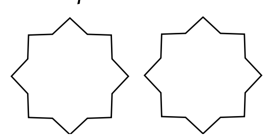
Example



congruent

Alike in every respect, but are separate. Think of identical twins, who are separate people but look exactly the same.

Example



consistent system of equations

A system that has at least one solution (independent and dependent systems).

constant

A term in an expression that has no variables (it is either a number or a symbol that represents a number, such as π).

continuous data

Data that is connected by a line on a line graph (e.g., a graph of height vs age is considered continuous data).

conversion ratio
The ratio comparing different units that are equivalent.

$$\frac{\text{new units}}{\text{old units}}$$

Example

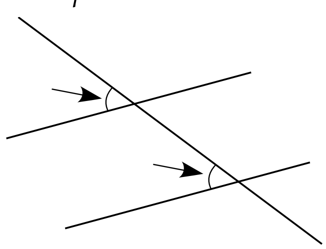
To change from centimetres to metres, the ratio would be $\frac{1\text{ m}}{100\text{ cm}}$.

convert
To change the form but not the amount of a measurement or value.

correlation (*r*)
The relationship between the x-variable and y-variable; it can be either positive or negative, either strong or weak; values range from –1 to 1.

corresponding angles
Angles formed by a transversal that are equal; these angles line up when you 'slide' one along the transversal to the other.

Example



cosine ratio
The ratio relating the adjacent side and the hypotenuse to the angle (θ°).

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

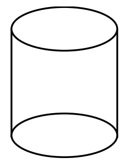
cube root
The number (factor) that, when multiplied with itself 3 times, produces the given cube.

Example

$\sqrt[3]{64} = 4$

cylinder
A prism with parallel faces that are circles.

Example



data
Information that is collected; usually numerical, organized in charts and displayed by graphs.

deci (d)
A metric prefix; multiplication factor = $10^{-1} = 0.1$.

decimal
A fractional number written in base ten form; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number).

degree
The sum of the exponents in a term or the largest exponent in an expression.

Examples

- x^2 would be a term with a degree of 2
- y would be the same as y^1, so the degree is 1
- xy is the same as x^1y^1, so the degree for this term would be the sum of the exponents, or 2
- $3n^4 - n^2 - 5$: the degree of the polynomial is from the term with the highest degree (this expression would be degree 4)

deca (da)
A metric prefix; multiplication factor = $10^1 = 10$.

denominator
The number below the line in a fraction that can state the total number of items, or the number of equal pieces that something is divided into.

Example

$\frac{2}{3} \longleftarrow$ denominator

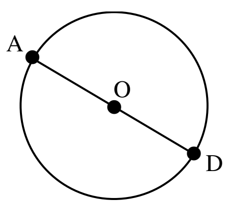
dependent system of equations
A system that has infinite solutions; these equations all represent the same line, they lie on top of each other.

dependent variable
An item being compared to another item, that is affected by the other item; graphed on the vertical or y-axis.

diameter
A chord that passes through the centre of the circle.

Example

AD is the diameter.



difference of squares principle
When you multiply two binomials that are exactly the same except for their signs, the product is also a binomial; the first term is the product for the first terms, and the second term is the product of the second terms (e.g., $(m + 2)(m - 2) = m^2 - 4$).

dimensions
Measurements of a figure (length, width, height, radius, etc.).

distance formula
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

distributive property of multiplication
$a \times (b + c) = ab + ac$

Example

$a(b + c) = ab + ac$

domain
All the x-coordinate values that are possible or reasonable for the independent variable, graphed along the x-axis.

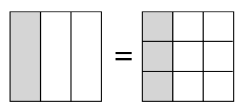
element of
The symbol, $\in$, is used to denote element of.

equation
A math sentence that states that two things are equal.

equivalent fractions
Fractions that represent the same amount.

Example

$\frac{1}{3} = \frac{3}{9}$



estimate
To find the approximate value.

evaluate
To find the value of an expression.

exponent
The number of times a number is multiplied together in a power (3 is the exponent in 4^3).

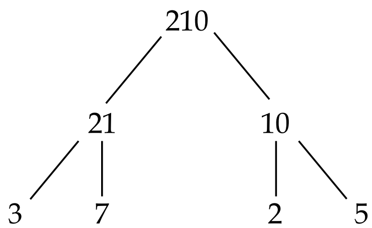
expression
One side of an equation; does not contain an equal sign or greater than/less than symbol.

extrapolation
To calculate values that are outside of the range of data.

factor
A number that, when multiplied by another number, gives a specific product (8 is a factor of 24 because $8 \times 3 = 24$).

factor tree
A tool used to find the prime factors of a number.

Example



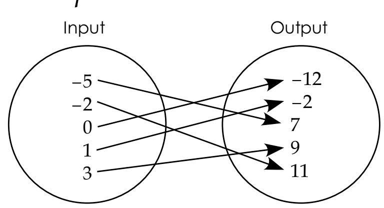
formula of a line

(slope-intercept form)	$y = mx + b$
(general form)	$Ax + By + C = 0$
(point-slope form)	$y - y_1 = m(x - x_1)$

fraction
A number that represents part of a whole that looks like $\frac{a}{b}$.

function ($f(x) = y$)
A relation that has a unique y-value for each x-value.

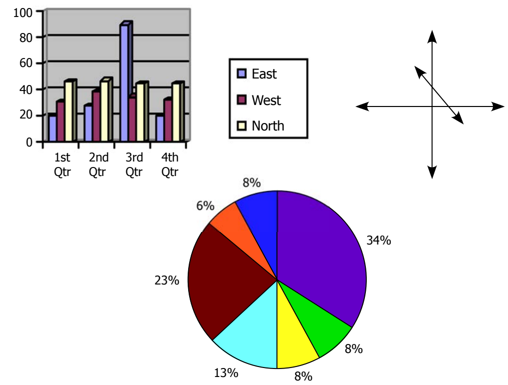
Example



geometry
The study of the position, size, and shape of figures.

graph
A visual representation used to show a relationship between data.

Example



greater than
The symbol, >, is used to show greater than.

greater than or equal to
The symbol, ≥, is used to show greater than or equal to.

greatest common factor (GCF)
The largest number that is a factor of two or more numbers (e.g., 6 is the GCF of 18 and 24).

hecto (h)
A metric prefix; multiplication factor = $10^2 = 100$.

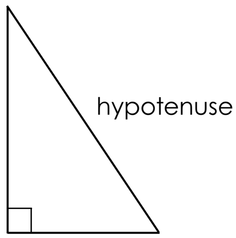
horizontal line slope
The slope is always zero ($m = 0$).

hundreds place
The place value located three places to the left of the decimal point in a number; a digit in the hundreds place has a value of 100 times the value of the digit.

hundredths place
The place value located two places to the right of the decimal point in a number; a digit in the hundredths place has a value of $\frac{1}{100}$ the digit.

hypotenuse
The side of a right triangle across from the right angle; the longest side of a right triangle.

Example



imperial system
The system of measurement used in the US, and sometimes still in Canada and Britain; includes feet, yards, pounds, gallons, and quarts.

improper fraction
A fraction that is larger than 1; the numerator is larger than the denominator $\left(\text{e.g., } \frac{9}{4}\right)$.

inconsistent system of equations
A system that has no solutions; these lines are parallel, they do not cross.

independent system of equations
A system that has one unique solution; these lines are neither parallel nor the same, they intersect at one point.

independent variable
An item being compared to another item, but is unaffected by the other item; graphed on the horizontal or x-axis.

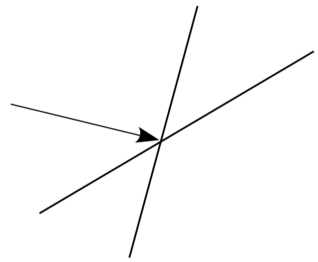
infinity (∞)
Continues forever.

integer number (I or Z)
All whole numbers as well as their negative opposites (zero is not positive or negative so it appears once) {I = … −3, −2, –1, 0, 1, 2, 3 …}.

interpolation
To calculate values between data points.

intersection point
The point where two lines cross.

Example



irrational numbers (Q′)
All numbers that you *cannot* write as a ratio (fraction); if written as a decimal they do not end or repeat (no matter how many decimal places you include) $\left(\text{e.g., } \sqrt{2} \text{ or } \pi\right)$.

kilo (k)
A metric prefix; multiplication factor = 10^3 = 1 000.

less than
The symbol, <, is used to show less than.

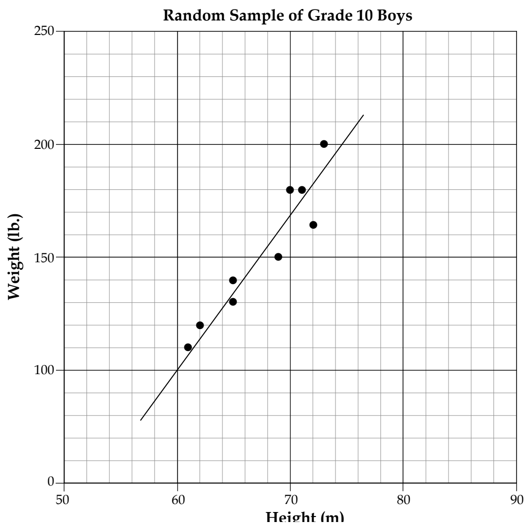
less than or equal to
The symbol, ≤, is used to show less than or equal to.

like terms
Two or more terms which have the same variable(s) with the same exponent(s).

line of best fit

A line drawn on a scatterplot that describes the approximate relationship of the data; not all the points must be on the line, but they should be close.

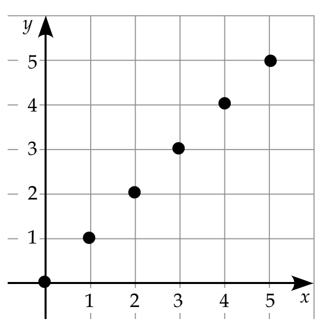
Example



linear relation

A set of data that, when plotted on a graph, looks as though a line could be drawn through it to represent the data.

Example



lowest common multiple (LCM)

The smallest number, greater than 0, that is a multiple of two or more numbers (e.g., the LCM of 20 and 25 is 100).

mega (M)

A metric prefix; multiplication factor = $10^6 = 1\ 000\ 000$.

metric system

See *Systeme Internationale*.

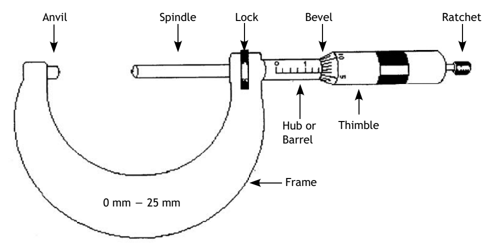
micro (μ)

A metric prefix; multiplication factor = $10^{-6} = 0.000\ 001$.

micrometer

An instrument used to make precise measurements to the nearest thousandth of a centimetre.

Example



midpoint

The point on a line segment halfway between the two endpoints.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

milli (m)

A metric prefix; multiplication factor = $10^{-3} = 0.001$.

mixed number

A number larger than 1, written as a whole number and a proper fraction $\left(\text{e.g., } 4\frac{5}{6}\right)$.

monomial

A polynomial with one term.

multiple

The product of a given number and any other integer.

nano (n)

A metric prefix; multiplication factor = $10^{-9} = 0.000\ 000\ 001$.

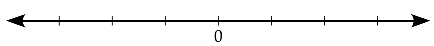
natural numbers (N)
All numbers used to count; they can be represented using objects such as rocks or fingers {N = 1, 2, 3 …}.

negative exponent rule
A number with a negative exponent is equal to the reciprocal of the base $\left(\text{e.g., } \left(\frac{4}{3}\right)^{-5} = \left(\frac{3}{4}\right)^{5} = \frac{3^5}{4^5}\right)$.

negative number
A number that is less than 0, located to the left of 0 on a horizontal number line, or located below 0 on a vertical number line.

number line
A line marked with points that represent numbers; resembles one axis of a graph.

Example



numerator
The number above the line in a fraction that states the number of parts being considered.

Example

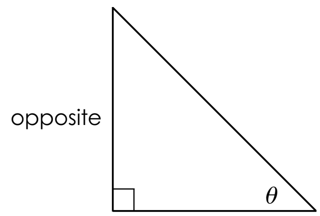
$\frac{2}{3} \longleftarrow$ numerator

numerical coefficient
The value in front of the variable ($-3x^2$, here the numerical coefficient is -3).

obtuse
An angle that is between 90º and 180º.

opposite side
The side of a triangle opposite the angle (θº)—not the hypotenuse.

Example



order of operations
A specified sequence in which mathematical operations are expected to be performed. An arithmetic expression is evaluated by following these ordered steps:

1. Simplify within grouping symbols such as parentheses or brackets, starting with the innermost grouping.
2. Apply exponents—powers and roots.
3. Perform all multiplications and divisions in order from left to right.
4. Perform all additions and subtractions in order from left to right.

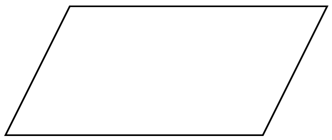
ordered pair
Set of two numbers named in a specific order, represented by (x, y); x represents the independent variable, graphed along the x-axis; y represents the dependent variable, graphed along the y-axis; also known as a coordinate pair.

parallel
Lines, faces, or edges that never cross; equidistant (always the same distance between them—they don't get closer together or farther apart).

parallel lines rule
The slopes of parallel lines are congruent.

parallelogram
A 4-sided shape with parallel opposite sides, and 4-angles that do not have to be 90°.

Example



pattern
Something that is predictable because it repeats itself or an operation is repeated over and over.

percent (%)
A number expressed in relation to 100; represented by the symbol % (e.g., 40 parts out of 100 is 40%).

perfect cube
The product of a whole number multiplied with itself 3 times (8 is a perfect cube because $2 \times 2 \times 2 = 8$).

perfect square
The product of a whole number multiplied with itself twice (9 is a perfect square because $3 \times 3 = 9$).

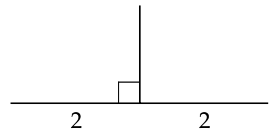
perfect square trinomials
A trinomial that is produced by squaring a binomial (e.g., $(x + 4)^2 = x^2 + 8x + 16$, so $x^2 + 8x + 16$ is a perfect square trinomial).

perimeter
The distance around the outside of a shape.

perpendicular
Two lines that meet at 90°.

perpendicular bisector
A line that divides another line in two equal parts, and forms a 90° angle with the divided line.

Example



perpendicular lines rule
The slopes of perpendicular lines are the negative reciprocals of each other.

Example

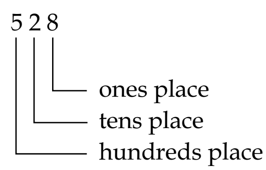
If line 1 and line 2 are perpendicular and the slope of line 1 is $m = 8$;

line 2 = $-\frac{1}{8}$.

place value
The value of a digit in a number based on its position.

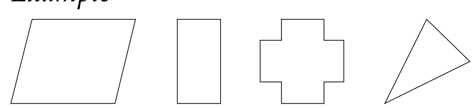
Example

In the number 528, the 5 has a value of 5 hundreds (or 500), the 2 has a value of 2 tens (or 20), and the 8 has a value of 8 ones (or 8).



polygon
A 2-D shape made up of 3 or more straight lines.

Example



polynomial
A mathematical expression with one or more terms.

positive number
Any number greater than 0; located to the right of 0 on a horizontal number line or above 0 on a vertical number line.

power
The product of a number multiplied with itself several times ($3 \times 3 \times 3 = 27$ is described as '3 to the power of 3' or 'the third power of 3').

power of a power rule
When there is an exponent inside and outside the brackets, the base stays the same and the exponents are multiplied together (e.g., $(4^3)^2 = 4^{3\times 2}$).

power of a product rule
When two or more numbers are multiplied together in the brackets and there is an exponent outside the brackets, the exponent can be applied to each number or variable being multiplied (e.g., $(3 \times 2)^3 = 3^3 \times 2^3$).

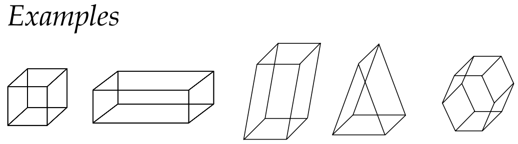
power of a quotient rule
When the base is a quotient, the exponent is applied to the numerator and denominator $\left(\text{e.g., } \left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}\right)$.

prime factorization
To write a number as the product of its prime factors (e.g., 24: $2 \times 2 \times 2 \times 3$).

prime number
An integer *greater than* 1 that has exactly 2 different factors: 1 and itself (one is not a prime number).

prism
A 3-D object that has two congruent (equal) and parallel faces, connected by parallelograms.

Examples

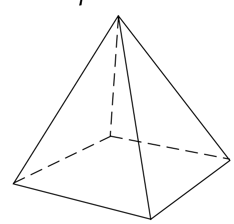


proper fraction
A fraction that is less than 1; the numerator is less than the denominator $\left(\text{e.g., } \frac{5}{8}\right)$.

proportion
Two equal ratios.

pyramid
A polyhedron whose base is a polygon and whose lateral faces are triangles that share a common vertex.

Example



Pythagorean Theorem
In a right triangle, the sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).

$a^2 + b^2 = c^2$

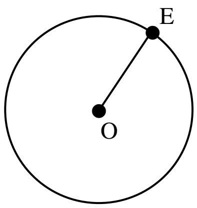
quotient of powers rule
When dividing powers with the same base, the coefficients divide and the exponents are subtracted (e.g., $46 \div 44 = 46 - 4 = 42$).

radical numbers
Any number written using the root $\left(\sqrt{\ }\right)$ symbol; they are used instead of their decimal approximation because they are more precise.

radius
A line from the centre of a circle to the edge (circumference) of the circle; half the diameter.

Example

EO is the radius



range
All the y-coordinate values that are possible or reasonable for the dependent variable, graphed along the y-axis.

rate of change
A ratio that compares different units (e.g., kilometres per hour).

ratio
A comparison of two like numbers or quantities.

rational exponent rule
If a number has a rational exponent, it can be rewritten as a radical $\left(6^{\frac{2}{3}} = \sqrt[3]{6^2}\right)$.

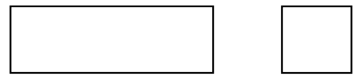
rational numbers (Q)
All numbers that can be written as a ratio (fraction); this includes decimals that repeat or terminate (end).

ray
Part of a line that has 1 end point and 1 end that goes on forever (marked by an arrow).

real numbers
All numbers that represent a quantity; all rational and irrational numbers.

rectangle
A 4-sided shape that has 4 right angles (90°), and opposite sides are congruent (equal).

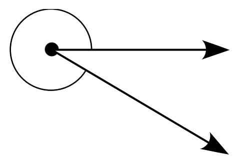
Examples



referent
An object that can be used to estimate a measurement.

reflex
An angle between 180° and 360°.

Example



relation
Any set of ordered pairs (x, y) describing the relationship between two variables.

revolution
One whole rotation around the circle (360°).

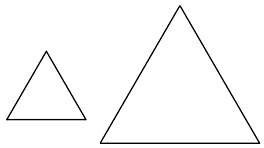
right angle
An angle that is 90°.

scale
The minimum and maximum numbers on an axis, and the divisions in between (from 0 to 10 marking each number = 0, 1, 2, 3 …; from 0 to 100 marking each multiple of 10 = 0, 10, 20, 30 …).

scientific notation
Away to write very large and very small numbers; only one digit is in front of the decimal, and the number is multiplied by a power of 10 (e.g., $4.25 \times 10^4 = 42\,500$).

similar
The same shape, proportional size.

Example



simplify
Combine like terms so that you are left with the simplest form of an equation or expression.

sine ratio
The ratio relating the opposite side and the hypotenuse to the angle (θ°).

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

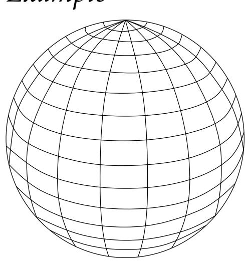
slope (*m*)
Compares how far the line moves vertically (up or down) as it moves horizontally (as it moves to the right).

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}$$

solve
Find the answer to an equation or problem, or to find the value of a variable.

sphere
A 3-D figure with a set of points in space that are the same distance from a fixed point called the centre.

Example



square root $\left(\sqrt{\ }\right)$
A number (factor) that, when multiplied by itself, produces the given square $\left(\sqrt{16} = 4\right)$.

starts at and includes ([)
The left square open bracket is used to indicate "starts at and includes."

starts at but does not include (()
The left curved open bracket is used to indicate "starts at but does not include."

straight angle
An angle of 180°; a straight line.

strong correlation
All or the majority of data points fall on or close to the line of best fit.

such that (|)
The symbol | is used to denote such that.

sum of angles
The sum of the angles in a triangle is 180° (angle 1 + angle 2 + angle 3 = 180°).

supplementary angles
Two angles that add up to 180°.

system of linear equations
A set of two or more linear equations with the same variables; the solution to the system is the set of all ordered pairs that make all the equations true.

Systeme Internationale (SI)
Measurement system based on the multiples of 10; commonly used throughout the world; also known as the metric system.

table of values
An organized list of values that shows the relationship between two variables.

tangent ratio
The ratio relating the opposite and adjacent sides to the angle (θ°).

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

tens place
The place value located two places to the left of the decimal point; a digit in the tens place has a value of 10 times the value of the digit.

tenths place
The place value located one place to the right of the decimal point; a digit in the tenths place has a value of $\frac{1}{10}$ the value of the digit.

term
Variables, numerical coefficients, or constants in a polynomial expression; separated by addition or subtraction signs.

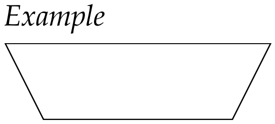
thousands place
The place value located three places to the right of the decimal point; a digit in the thousands place has a value of 1000 the value of the digit.

thousandths place
The place value located four places to the left of the decimal point in a number; a digit in the thousandths place has a value of $\frac{1}{1000}$ times the value of the digit.

title (graph)
A title indicates what the graph is about.

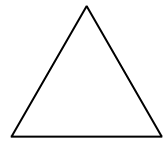
trapezoid
A 4-sided shape with at least 2 parallel sides.

Example



triangle
A 3-sided object with 3 angles; sides and angles can but don't have to be equal.

Example



trinomial
A polynomial with three terms.

trigonometry
The study of triangles.

two-dimensional (2-D)
A figure that only has two measures (a rectangle is 2-D because it is described using only length and width).

up to and including (])
The closed square bracket (]) is used to denote "up to and including."

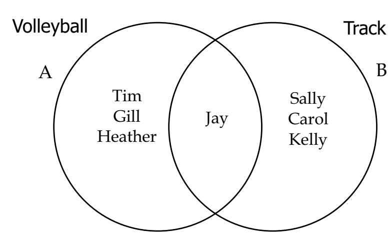
up to but not including ())
The closed round bracket ()) is used to denote "up to but not including."

variable
A letter or symbol that represents an unknown value (x, y, n, θ).

Venn diagaram
Diagram that demonstrates relationships among information.

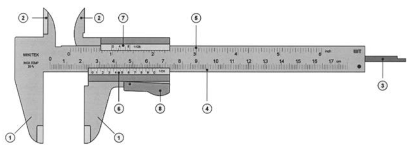
Example

The Venn diagram below shows the students who play volleyball, the students who run track, and the student who plays volleyball and runs track.



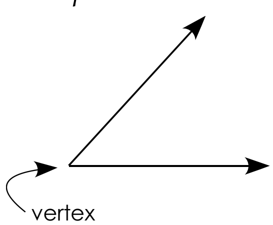
Vernier caliper
An instrument used to make precise measurements to the nearest hundredth of a centimetre.

Example



vertex
The point where two rays meet to make an angle.

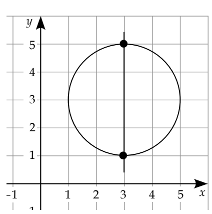
Example



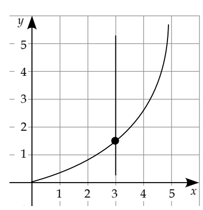
vertical line slope
Vertical lines have an undefined slope.

vertical line test
If a vertical line can be drawn anywhere on a graph and intersect the relation in more than one point, then the relation is not a function.

Examples



not a function



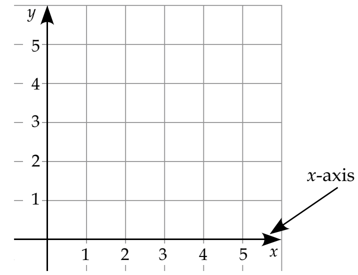
a function

weak correlation
When the data points do not fall near the line of best fit.

whole numbers (W)
All counting numbers including zero {W = 0, 1, 2, 3 …}.

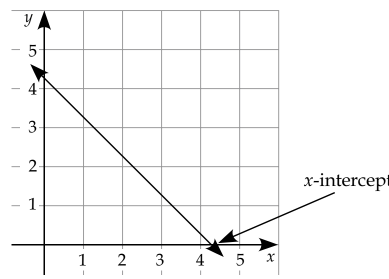
***x*-axis**
Horizontal line of the Cartesian plane.

Example



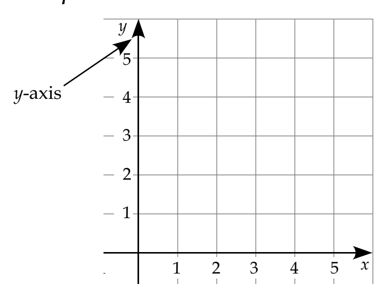
***x*-intercept**
Where the graph crosses the x-axis; $y = 0$.

Example



***y*-axis**
Vertical line of the Cartesian plane.

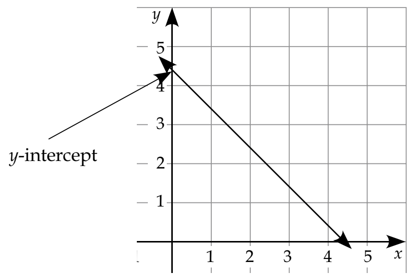
Example



y-intercept

Where the graph crosses the y-axis; $x = 0$.

Example



zero exponent rule

When a number is to the power of 0, it is equal to 1 no matter what the base is.

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Midterm Practice Exam

Grade 10 Introduction to Applied and Pre-Calculus Mathematics

Midterm Examination A-M-000

Name: ____________________

Student Number: ____________________

Attending ❑ Non-Attending ❑

Phone Number: ____________________

Address: ____________________

____

____

For Marker's Use Only

Date: ____________________

Final Mark: ________ /100 = ________ %

Comments:

Instructions

The midterm examination will be weighted as follows:

Modules 1–4	100%

The format of the examination will be as follows:

Part A: Multiple Choice	20 marks
Part B: Definitions	10 marks
Part C: Graphs and Relations	27 marks
Part D: Number Sense	7 marks
Part E: Measurement	26 marks
Part F: Trigonometry	10 marks

Time allowed: 2.5 hours

Note: You are allowed to bring a scientific calculator and your Midterm Exam Resource Sheet to the exam. Your Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module work that is submitted for marking.

You will need a metric ruler and an imperial ruler. If required, a metric/imperial ruler is provided at the end of this exam for your use.

Part A: Multiple Choice (*20 x 1 = 20 marks*)

Circle the letter of the correct answer for each question.

1. On a graph, the independent variable
   a) is graphed along the y-axis
   b) is graphed along the vertical axis
   c) is graphed along the horizontal axis
   d) is affected by changes in the other variable

2. An example of continuous data is
   a) the number of pairs of shoes you own
   b) the time it takes to run a race
   c) how many pages in a textbook
   d) the number of pizzas you order for a party

3. Calculate the slope of the line that passes through the points (2, 5) and (4, 8).
   a) $\frac{-2}{3}$
   b) $\frac{3}{2}$
   c) $\frac{2}{3}$
   d) $\frac{-3}{2}$

4. The slope of a vertical line is
   a) $m = -1$
   b) $m = 0$
   c) $m = 1$
   d) undefined

5. Given the equation of a line is $y = \frac{2}{3}x - 5$, what is the y-intercept?

a) $2x$

b) 5

c) $\frac{2}{3}$

d) -5

6. An example of a composite number is

a) 11

b) 23

c) 37

d) 51

7. The greatest common factor of 12 and 16 is

a) 2

b) 4

c) 48

d) 192

8. A possible solution for $\sqrt{16}$ is

a) 2

b) 8

c) –4

d) none of the above

9. $-\frac{5}{7}$ is best described as a(n)

a) whole number

b) integer

c) rational number

d) irrational number

10. Find the product of $(2m^2n^3)(3mn^4)$.

a) $6m^3n^7$

b) $5m^2n^{12}$

c) $6m^2n^{12}$

d) $5m^3n^7$

11. Simplify $(59x^2y)^0$.

a) -1

b) 0

c) 1

d) $59x^2y$

12. $9^{\frac{1}{2}}$ is equivalent to

a) $\sqrt{9}$

b) $\dfrac{1}{9^2}$

c) 4.5

d) -3

13. $\left(\dfrac{x}{y}\right)^{-3}$ is equivalent to

a) $\dfrac{y^3}{x^3}$

b) $-\left(\dfrac{x}{y}\right)^3$

c) $-\left(\dfrac{y}{x}\right)^3$

d) $\left(\dfrac{1}{xy^3}\right)$

14. The best choice of units to measure the distance from your town to Toronto would be
   a) metres
   b) yards
   c) miles
   d) decalitres

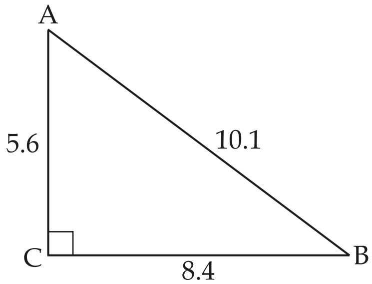
15. The surface area of a sphere with a radius of 5 inches is about
   a) 63 $in.^2$
   b) 314 $in.^2$
   c) 524 $in.^2$
   d) 3948 $in.^2$

16. If a cone has a volume of 100 cubic units, a cylinder with the same height and radius will have a volume of how many cubic units?
   a) 10
   b) 33
   c) 300
   d) 1000

17. If the two legs in a right triangle are 5 cm and 12 cm, the length of the hypotenuse is
   a) 11 cm
   b) 13 cm
   c) 17 cm
   d) 169 cm

18. The sine ratio involves the lengths of which two sides of a right triangle?
   a) $\frac{\text{opposite}}{\text{adjacent}}$
   b) $\frac{\text{adjacent}}{\text{hypotenuse}}$
   c) $\frac{\text{opposite}}{\text{hypotenuse}}$
   d) $\frac{\text{adjacent}}{\text{opposite}}$

19. Given triangle ABC, sin A = ?



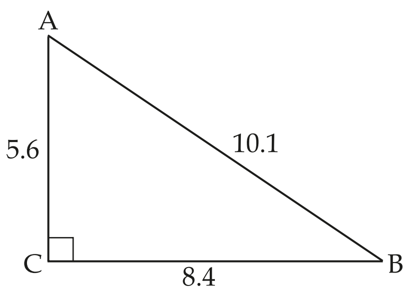
a) $\frac{5.6}{10.1}$

b) $\frac{8.4}{5.6}$

c) $\frac{5.6}{8.4}$

d) $\frac{8.4}{10.1}$

20. Given triangle ABC, the measure of angle A is



a) 37°
b) 34°
c) 90°
d) 56°

Part B: Definitions (*10 x 1 = 10 marks*)

Match each definition with the correct term from the list below. Write the correct term on the blank line with each definition. Terms are used only once. Not all terms have a definition provided

Terms

adjacent side	greatest common factor	ordered pair	similar triangles
alternate interior angles	hypotenuse	perfect cube number	sine
angle of depression	imperial system	perfect square number	slope
angle of elevation	integer	prism	sphere
cone	inverse trigonometric ratio	pyramid	square root
cosine	irrational	range	tangent
cube root	lateral surface area	rational number	total surface area
cylinder	least common multiple	referent	volume
domain	natural number	SI	whole number
graph	opposite side	similar	

1. Visual representation used to show a numerical relationship. _________________
2. A comparison of how far the line moves vertically as it moves horizontally. _________________
3. Found when an integer is multiplied by itself three times. _________________
4. The counting numbers and zero. _________________
5. A system of measurement that uses prefixes and has a decimal structure. _________________
6. 3-D object with two congruent, parallel bases and parallelogram faces. _________________
7. 3-D object in which all points are equidistant from the centre. _________________
8. Side directly across from the specified angle. _________________
9. Ratio of the opposite side and hypotenuse in a right triangle. _________________
10. Congruent angles formed on opposite sides of a line that cuts diagonally through parallel lines. _________________

Part C: Graphs and Relations (*27 marks*)

Show all calculation and formulas used for short and long answer questions. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

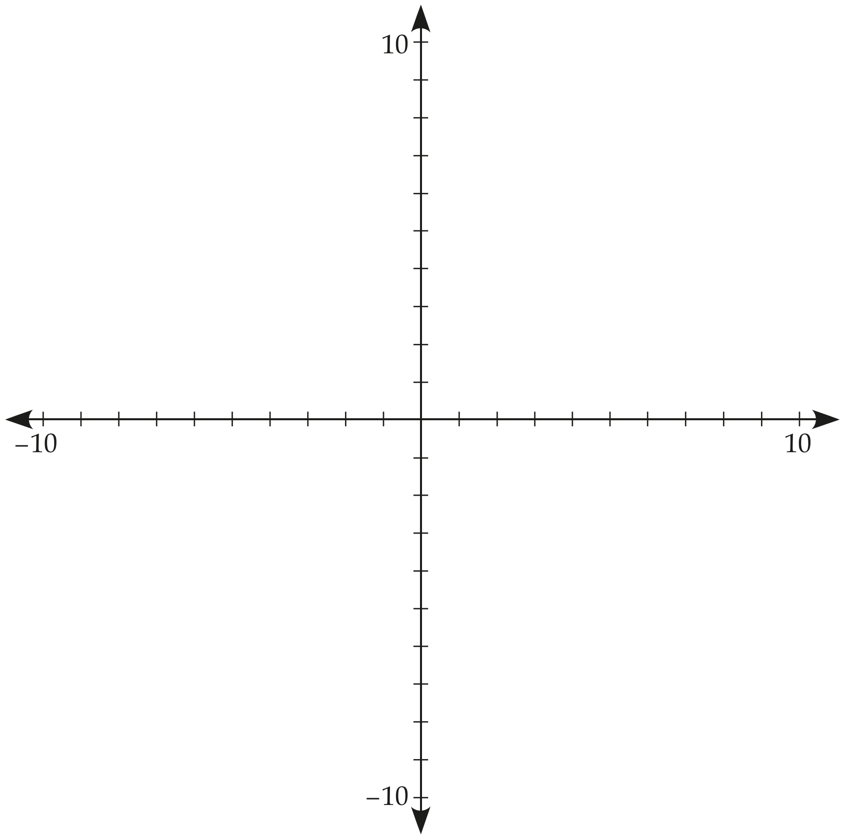
1. Given the linear equation $y = \frac{4}{3}x - 9$

   a) State the y-intercept as a value. *(1 mark)*

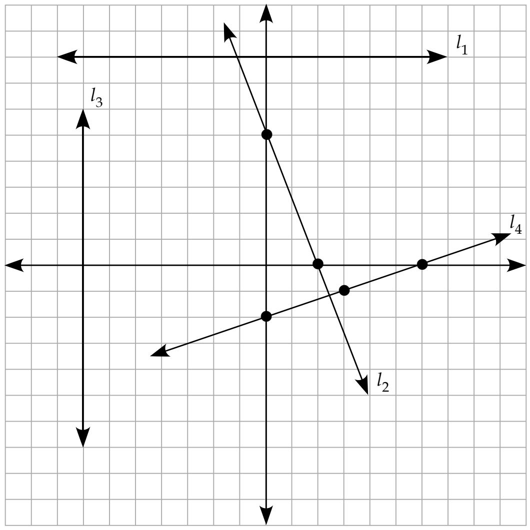
   b) State the slope of the line. *(1 mark)*

   c) Explain how you would graph the line. *(2 marks)*

d) Sketch a graph of the line. *(1 mark)*



2. State the slope of each of the following lines. *(4 marks)*



l_1 = ___________________________

l_2 = ___________________________

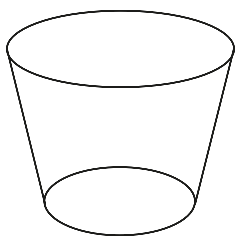
l_3 = ___________________________

l_4 = ___________________________

3. Given the equation $y = \frac{1}{3}x - 5$, state the equation of a different line that is parallel to the given line, and explain how it is parallel. *(2 marks)*

4. In line AB, A (–5, 21) and B (x, –6). Use the slope formula to find the value of x if the slope of line AB is m = –3. *(4 marks)*

5. The container below is being filled with water at a constant rate. The time (t) and height (h) of the water is graphed.

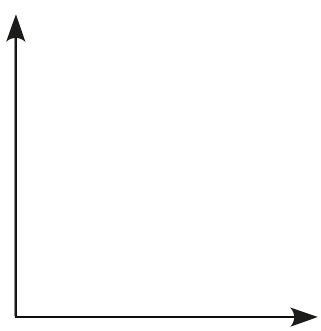


a) State the independent and dependent variables in this situation. *(1 mark)*

independent ____________________

dependent ____________________

b) Sketch a possible graph for this situation. *(1 mark)*



6. A study was undertaken to compare the age of cash registers to the cost of maintenance required on them. Nine cash registers in a department store were examined. The results were as follows.

Register #	**Age (in years)**	**Maintenance costs in dollars**
1	6	99
2	7	161
3	1	23
4	3	40
5	6	126
6	2	35
7	5	86
8	4	72
9	3	51

a) Which is the independent variable? *(0.5 mark)*

____

b) Which is the dependent variable? *(0.5 mark)*

____

c) Plot the points on the grid provided, include the components of a good graph as described in Module 1. *(4 marks)*

d) What is a reasonable domain and range for this situation? Explain your answer. *(2 marks)*

e) Is this data linear? Explain why or why not. *(2 marks)*

f) Is this data continuous? Explain why or why not. *(1 mark)*

Part D: Number Sense (*7 marks*)

Show all calculation and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Determine the prime factors of 450 using a factor tree diagram. Include your diagram. *(2 marks)*

2. Determine the prime factors of 225 and 400. Write them using exponents to indicate repeated multiplications, and use that to find the greatest common factor and least common multiple. *(4 marks)*

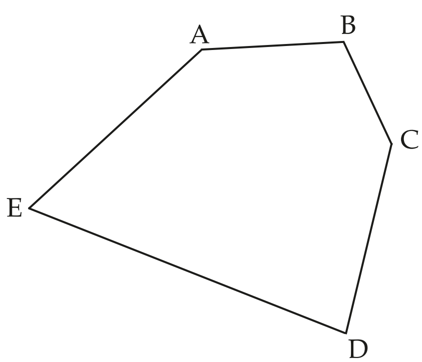
3. Write $\sqrt{252}$ as a mixed radical in simplest form. *(1 mark)*

Part E: Measurement (*26 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Explain how you could use a referent to estimate the circumference of a circular dining table. Describe your referent and measurement strategy. *(3 marks)*

2. Measure the length of each side in the following polygon to the nearest tenth of a centimetre and calculate its perimeter. *(3 marks)*



AB = _________

BC = _________

CD = _________

DE = _________

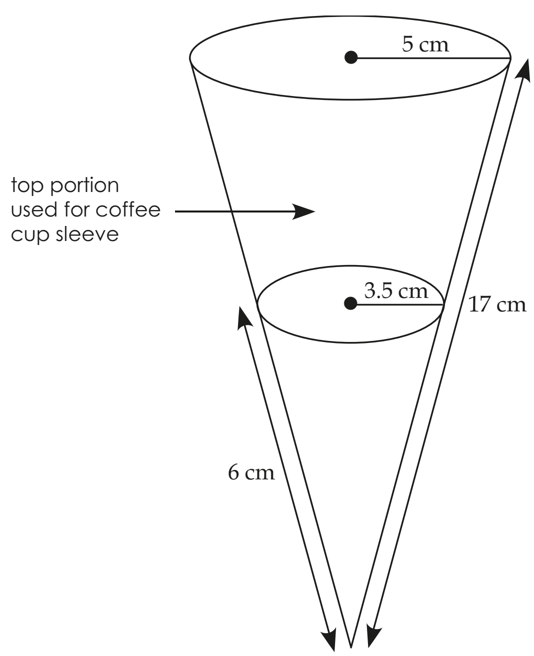
EA = _________

Perimeter = _________

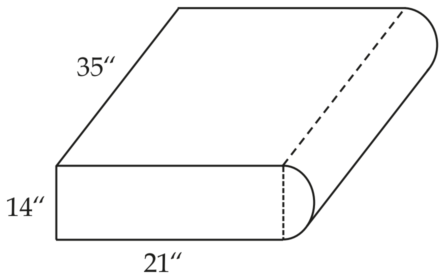
3. Convert the following measurements into the units indicated. (Round to 3 decimal places) *(8 marks)*

92 inches =	____________________	feet
19 cm =	____________________	mm
4.5 yards =	____________________	inches
11 miles =	____________________	km
5 gallons	____________________	litres
33 kg =	____________________	lb.
82 ft.^{2} =	____________________	yd.^{2}
25 000 000 cm^3	____________________	m^3

4. A take-out coffee cup has a protective paper sleeve to make it easier to hold when filled with a hot beverage. The sleeve is made by cutting the bottom off of a paper cone and using the top portion of it. Determine the lateral surface area of the part of the cone used to make the sleeve. *(4 marks)*



5. Determine the volume of this 3-D object. Its base is composed of a rectangle and a semi-circle. State your final answer in cubic feet, rounded to the nearest tenth. *(4 marks)*



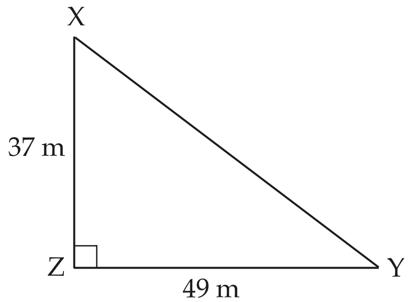
6. A beach ball has a surface area of 572.6 sq. in. Determine its diameter to the nearest $\frac{1}{2}$ inch. *(4 marks)*

Part G: Trigonometry (*10 marks*)

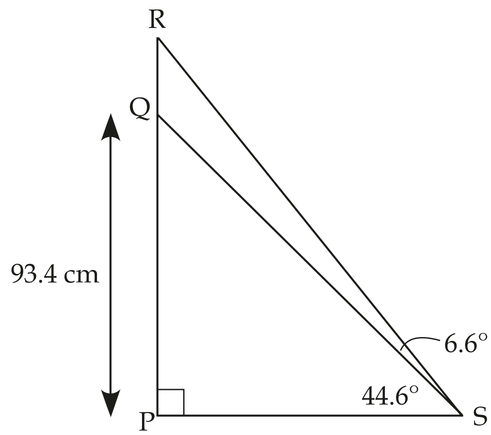
Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

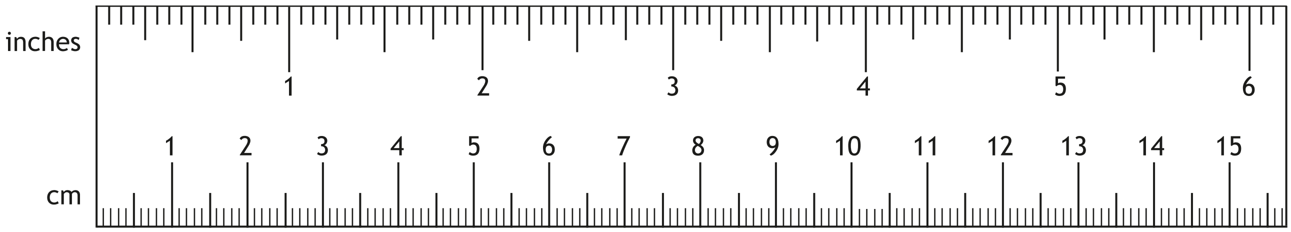
1. Calculate tan 65° to 4 decimal places and explain what this means using a diagram of a right triangle. *(3 marks)*

2. Solve the triangle. Find all angles and side lengths. Round your answers to 1 decimal place. *(3 marks)*



3. Find the length of QR to the nearest tenth. *(4 marks)*




inches
1
2
3
4
5
6
cm
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Midterm Practice Exam
Answer Key

Grade 10 Introduction to Applied and Pre-Calculus Mathematics

Midterm Practice Examination
Answer Key

Name: ___________________________________

Student Number: ___________________________

Attending ❑ Non-Attending ❑

Phone Number: ____________________________

Address: _________________________________

____

____

For Marker's Use Only

Date: _______________________________

Final Mark: _______ /100 = _______ %

Comments:

Instructions

The midterm examination will be weighted as follows:

Modules 1–4	100%

The format of the examination will be as follows:

Part A: Multiple Choice	20 marks
Part B: Definitions	10 marks
Part C: Graphs and Relations	27 marks
Part D: Number Sense	7 marks
Part E: Measurement	26 marks
Part F: Trigonometry	10 marks

Time allowed: 2.5 hours

Note: You are allowed to bring a scientific calculator and your Midterm Exam Resource Sheet to the exam. Your Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module work that is submitted for marking.

You will need a metric ruler and an imperial ruler. If required, a metric/imperial ruler is provided at the end of this exam for your use.

Part A: Multiple Choice (*20 x 1 = 20 marks*)

Circle the letter of the correct answer for each question.

1. On a graph, the independent variable
   a) is graphed along the y-axis
   b) is graphed along the vertical axis
   c) is graphed along the horizontal axis
   d) is affected by changes in the other variable (Module 1, Lesson 1)

   The independent variable is always graphed along the x-axis or the horizontal axis.

2. An example of continuous data is
   a) the number of pairs of shoes you own
   b) the time it takes to run a race
   c) how many pages in a textbook
   d) the number of pizzas you order for a party (Module 1, Lesson 1)

   Time is continuous because you calculate in fractions of a minute or a second. The other choices are all examples of things that can't be divided into fractional parts.

3. Calculate the slope of the line that passes through the points (2, 5) and (4, 8).
   a) $\frac{-2}{3}$
   b) $\frac{3}{2}$
   c) $\frac{2}{3}$
   d) $\frac{-3}{2}$

   Slope is $\frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$

   $$m = \frac{8-5}{4-2}$$

   $$m = \frac{3}{2}$$

   (Module 1, Lesson 3)

4. The slope of a vertical line is
   a) $m = -1$
   b) $m = 0$
   c) $m = 1$
   d) undefined (Module 1, Lesson 3)

   A vertical line has rise but no run so the slope would be $\frac{\text{rise}}{0}$ and since we can't divide by zero, the slope is undefined.

5. Given the equation of a line is $y = \frac{2}{3}x - 5$, what is the y-intercept?

a) $2x$

b) 5

c) $\frac{2}{3}$

d) -5

(Module 1, Lesson 3)

The equation of a line is $y = mx + b$ where b is the y-intercept. So, in $y = \frac{2}{3}x - 5$, $b = -5$.

6. An example of a composite number is

a) 11

b) 23

c) 37

d) 51

(Module 2, Lesson 1)

A composite number is a number with more than two factors. 11, 23, and 37 are all prime numbers because their only factors are 1 and themselves. 51 has factors of 1, 3, 17, and 51.

7. The greatest common factor of 12 and 16 is

a) 2

b) 4

c) 48

d) 192

(Module 2, Lesson 1)

The factors of 12 are {1, 2, 3, 4, 6, 12} and the factors of 16 are {1, 2, 4, 8, 16}. The largest number that is in both sets is 4.

8. A possible solution for $\sqrt{16}$ is

a) 2

b) 8

c) –4

d) none of the above

(Module 2, Lesson 2)

$\sqrt{16} = -4$ because $(-4)(-4)$ or $(-4)^2 = 16$.

9. $-\frac{5}{7}$ is best described as a(n)

a) whole number
b) integer
c) rational number
d) irrational number (Module 2, Lesson 3)

$-\frac{5}{7}$ is a rational number because it is represented as a fraction and would give a repeating decimal if changed to decimal form.

10. Find the product of $(2m^2n^3)(3mn^4)$.

a) $6m^3n^7$
b) $5m^2n^{12}$
c) $6m^2n^{12}$
d) $5m^3n^7$ (Module 2, Lesson 4)

$(2m^2n^3)(3mn^4) = (2 \cdot 3)(m^2m)(n^3)(n^4) = 6m^3n^7$ using the produce law of exponents to add the exponents. The coefficients are multiplied.

11. Simplify $(59x^2y)^0$.

a) –1
b) 0
c) 1
d) $59x^2y$ (Module 2, Lesson 5)

Anything raised to the power of zero equals 1.

12. $9^{\frac{1}{2}}$ is equivalent to

a) $\sqrt{9}$

b) $\dfrac{1}{9^2}$

c) 4.5

d) -3

(Module 2, Lesson 5)

$9^{\frac{1}{2}} = \sqrt{9}$. When writing in this form, the positive root is assumed so the answer would be 3. To get an answer of –3, the question would need to be $-9^{\frac{1}{2}}$, which would be written as $-\sqrt{9}$ or -3. Fractional exponents are represented by radicals.

13. $\left(\dfrac{x}{y}\right)^{-3}$ is equivalent to

a) $\dfrac{y^3}{x^3}$

b) $-\left(\dfrac{x}{y}\right)^3$

c) $-\left(\dfrac{y}{x}\right)^3$

d) $\left(\dfrac{1}{xy^3}\right)$

(Module 2, Lesson 5)

$\left(\dfrac{x}{y}\right)^{-3} = \dfrac{1}{\left(\dfrac{x}{y}\right)^3} = \dfrac{1}{\dfrac{x^3}{y^3}}$ which can be simplified to $\dfrac{y^3}{x^3}$.

14. The best choice of units to measure the distance from your town to Toronto would be

  a) metres
  b) yards
  c) miles
  d) decalitres (Module 3, Lesson 1)

  Metres and yards are inappropriate as the distance is a long one. Decalitres is not a measure of distance but a measure of capacity.

15. The surface area of a sphere with a radius of 5 inches is about

  a) 63 in.^{2}
  b) 314 in.^{2}
  c) 524 in.^{2}
  d) 3948 in.^{2} (Module 3, Lesson 6)

  The formula for surface area of a sphere is $A = 4\pi r^2$. Substituting $r = 5$, we get $A = 4\pi(25)$ or 100π or approximately 314 square inches.

16. If a cone has a volume of 100 cubic units, a cylinder with the same height and radius will have a volume of how many cubic units?

  a) 10
  b) 33
  c) 300
  d) 1000 (Module 3, Lesson 6)

  A cylinder will have three times the volume of a cone with the same height and radius. The volume of a cylinder is found by using the formula $V = \pi r^2 h$ and the volume of a cone is found by using the formula $V = \frac{1}{3}\pi r^2 h$. So, multiply 100 by 3 to get 300.

17. If the two legs in a right triangle are 5 cm and 12 cm, the length of the hypotenuse is

  a) 11 cm
  b) 13 cm
  c) 17 cm
  d) 169 cm (Module 4, Lesson 1)

  Using Pythagorus, $5^2 + 12^2 = h^2$ or $25 + 144 = h^2$ or $169 = h^2$. $h = \sqrt{169}$ or 13.
  You might recognize this as a Pythagorean triple (5, 12, 13).

18. The sine ratio involves the lengths of which two sides of a right triangle?

a) $\frac{\text{opposite}}{\text{adjacent}}$

b) $\frac{\text{adjacent}}{\text{hypotenuse}}$

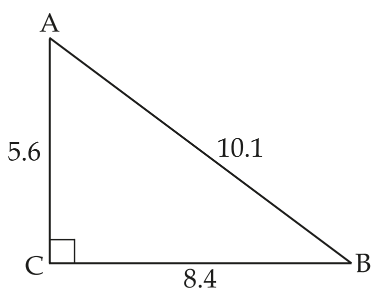
c) $\frac{\text{opposite}}{\text{hypotenuse}}$

d) $\frac{\text{adjacent}}{\text{opposite}}$

(Module 4, Lesson 2)

The definition of sine is the ratio of the opposite side to the hypotenuse.

19. Given triangle ABC, sin A = ?



a) $\frac{5.6}{10.1}$

b) $\frac{8.4}{5.6}$

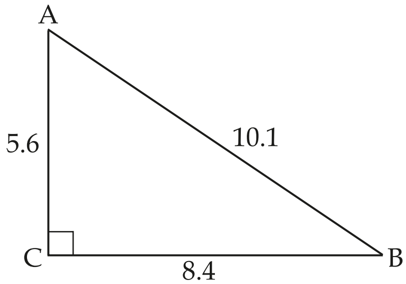
c) $\frac{5.6}{8.4}$

d) $\frac{8.4}{10.1}$

(Module 4, Lesson 2)

Since $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ and 8.4 is the length of the side opposite angle A and the hypotenuse is 10.1, $\sin A = \frac{8.4}{10.1}$.

20. Given triangle ABC, the measure of angle A is



a) 37°
b) 34°
c) 90°
d) 56° (Module 4, Lesson 3)

Using $\sin A = \frac{8.4}{10.1}$ and the inverse sine ratio on your calculator (make sure it is in degree mode), we find angle A to be 56°.

Part B: Definitions (*10 x 1 = 10 marks*)

Match each definition with the correct term from the list below. Write the correct term on the blank line with each definition. Terms are used only once. Not all terms have a definition provided

Terms

adjacent side	greatest common factor	ordered pair	similar triangles
alternate interior angles	hypotenuse	perfect cube number	sine
angle of depression	imperial system	perfect square number	slope
angle of elevation	integer	prism	sphere
cone	inverse trigonometric ratio	pyramid	square root
cosine	irrational	range	tangent
cube root	lateral surface area	rational number	total surface area
cylinder	least common multiple	referent	volume
domain	natural number	SI	whole number
graph	opposite side	similar	

1. Visual representation used to show a numerical relationship. ***graph*** (Module 1, Lesson 1)
2. A comparison of how far the line moves vertically as it moves horizontally. ***slope*** (Module 1, Lesson 3)
3. Found when an integer is multiplied by itself three times. ***perfect cube number*** (Module 2, Lesson 2)
4. The counting numbers and zero. ***whole numbers*** (Module 2, Lesson 3)
5. A system of measurement that uses prefixes and has a decimal structure. ***SI*** (Module 3, Lesson 1)
6. 3-D object with two congruent, parallel bases and parallelogram faces. ***prism*** (Module 3, Lesson 4)
7. 3-D object in which all points are equidistant from the centre. ***sphere*** (Module 3, Lesson 6)
8. Side directly across from the specified angle. ***opposite side*** (Module 4, Lesson 1)
9. Ratio of the opposite side and hypotenuse in a right triangle. ***sine*** (Module 4, Lesson 2)
10. Congruent angles formed on opposite sides of a line that cuts diagonally through parallel lines. ***alternate interior angles*** (Module 4, Lesson 4)

Part C: Graphs and Relations (*27 marks*)

Show all calculation and formulas used for short and long answer questions. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Given the linear equation $y = \frac{4}{3}x - 9$

   a) State the y-intercept as a value. *(1 mark)*

   Answer:

   $y = -9$

   b) State the slope of the line. *(1 mark)*

   Answer:

   $m = \frac{4}{3}$

   c) Explain how you would graph the line. *(2 marks)*

   Answer:

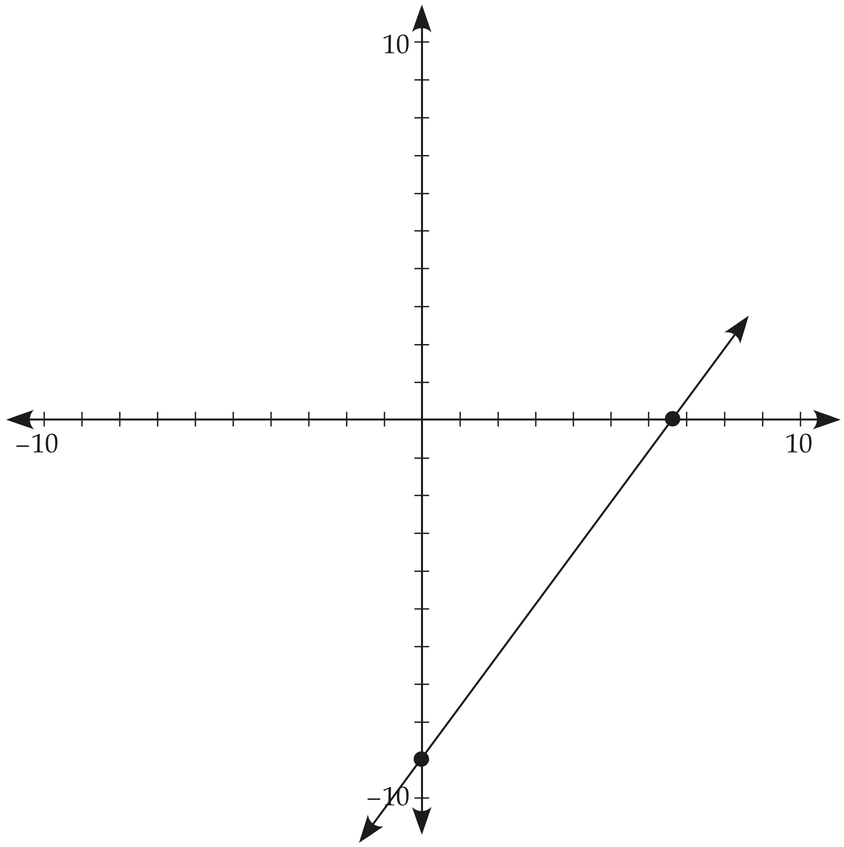
   Answers may vary. For two marks, you need to explain where you would put the y-intercept and how you would use the slope to move from the y-intercept to a second point.

   Possible answer:

   I would put a point on the y-axis at –9 since the y-intercept is –9. From that point, I would "rise" up 4 units and "run" right 3 units to get to the point (3, –5). I could also "rise" down 4 units and "run" left 3 units to get to the point (–3, –13). Using at least two points, I can draw the line.

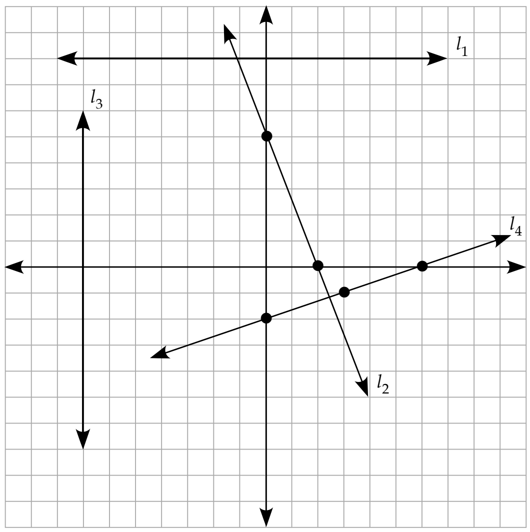
d) Sketch a graph of the line. *(1 mark)*

Answer:



(Module 1, Lesson 5)

2. State the slope of each of the following lines. *(4 marks)*



Answers:

$l_1 = 0$

$l_2 = \dfrac{-5}{2}$

$l_3 = \text{undefined}$

$l_4 = \dfrac{1}{3}$

(Module 1, Lesson 4)

3. Given the equation $y = \dfrac{1}{3}x - 5,$ state the equation of a different line that is parallel to the given line, and explain how it is parallel. *(2 marks)*

Answer:

$y = \dfrac{1}{3}x - 4$

Answers will vary. Any equation with the same slope and different y-intercept is acceptable. Students must state that their line is parallel because it has the same slope as the given line.

(Module 1, Lesson 4)

4. In line AB, A (–5, 21) and B (x, –6). Use the slope formula to find the value of x if the slope of line AB is m = –3. *(4 marks)*

   Answer:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-3 = \frac{-6 - 21}{x - (-5)}$$

$$(x + 5)(-3) = -27$$

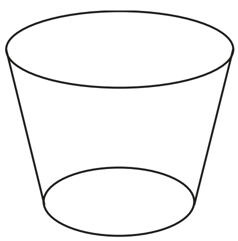
$$-3x - 15 = -27$$

$$-3x = -12$$

$$x = 4$$

(Module 1, Lesson 5)

5. The container below is being filled with water at a constant rate. The time (t) and height (h) of the water is graphed.



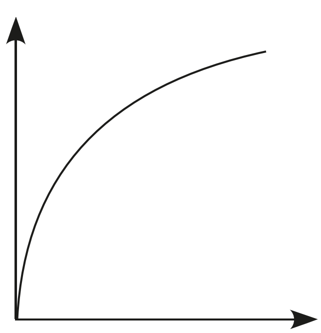
   a) State the independent and dependent variables in this situation. *(1 mark)*

   independent *Answer:* time

   dependent *Answer:* height

   b) Sketch a possible graph for this situation. *(1 mark)*

   Answer:



(Module 1, Lesson 1)

6. A study was undertaken to compare the age of cash registers to the cost of maintenance required on them. Nine cash registers in a department store were examined. The results were as follows.

Register #	**Age (in years)**	**Maintenance costs in dollars**
1	6	99
2	7	161
3	1	23
4	3	40
5	6	126
6	2	35
7	5	86
8	4	72
9	3	51

a) Which is the independent variable? *(0.5 mark)*

Answer: age

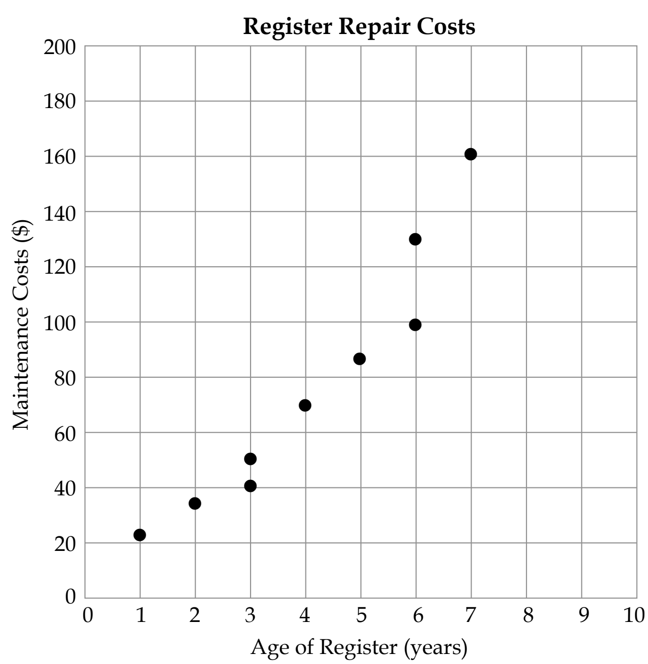
b) Which is the dependent variable? *(0.5 mark)*

Answer: maintenance costs

c) Plot the points on the grid provided, include the components of a good graph as described in Module 1. *(4 marks)*

Answer:

(Student must include labels, units, and title, and use appropriate scales for a good shape and size.)



d) What is a reasonable domain and range for this situation? Explain your answer. *(2 marks)*

Answer:

Answers may vary but should be reasonable.

Domain: from zero to 10 years is a reasonable age for a cash register. After about 10 years they are likely replaced as new technology develops. No negative values.

Range: from $0 to $200 seems reasonable. If repairs and maintenance were more than that, the store would probably buy a new cash register. No negative values.

e) Is this data linear? Explain why or why not. *(2 marks)*

Answer:

Yes, this data is approximately linear because a straight line drawn on the graph would go through or close to most of the data points.

f) Is this data continuous? Explain why or why not. *(1 mark)*

Answer:

Yes, the age of a cash register can be parts of years, and the costs can be fractions of dollar amounts.

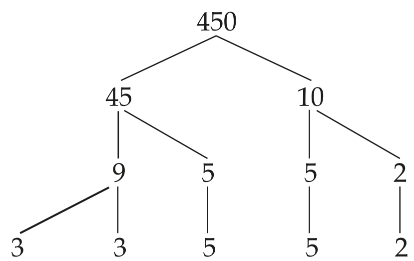
(Module 1, Lesson 2)

Part D: Number Sense (*7 marks*)

Show all calculation and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Determine the prime factors of 450 using a factor tree diagram. Include your diagram. *(2 marks)*

   Answer:



   The prime factors are $2 * 3 * 3 * 5 * 5$

   Diagrams may vary but the prime factors must be correct

(Module 2, Lesson 1)

2. Determine the prime factors of 225 and 400. Write them using exponents to indicate repeated multiplications, and use that to find the greatest common factor and least common multiple. *(4 marks)*

   Answer:

   $400 = 2^4 * 5^2$

   $225 = 3^2 * 5^2$

   GCF is $5^2 = 25$

   LCM is $2^4 * 3^2 * 5^2 = 3600$

(Module 2, Lesson 1)

3. Write $\sqrt{252}$ as a mixed radical in simplest form. *(1 mark)*

   Answer:

   $6\sqrt{7}$

(Module 2, Lesson 3)

Part E: Measurement (*26 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Explain how you could use a referent to estimate the circumference of a circular dining table. Describe your referent and measurement strategy. *(3 marks)*

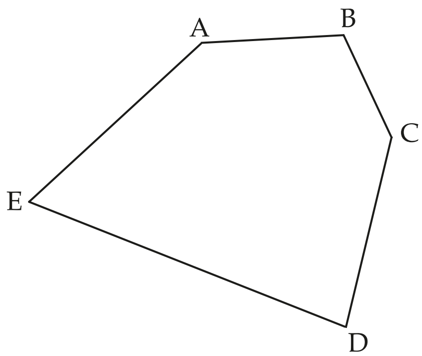
   Answer:

   Answers will vary. Students must describe a reasonable referent and strategy. A possible solution may reflect the following:

   I know that my hand span is about 7 inches long. I will mark my starting point on the table and will stretch my fingers out and place them alternately along the circumference of the table, counting the number of times it takes to return to my starting point. I will multiply the number of handspans by 7 to determine the approximate number of inches in the circumference of the table.

   (Module 3, Lesson 1)

2. Measure the length of each side in the following polygon to the nearest tenth of a centimetre and calculate its perimeter. *(3 marks)*



Answers:	Acceptable measuring range
AB = 1.9 cm	AB = 1.6 cm to 2.2 cm
BC = 1.5 cm	BC = 1.2 cm to 1.8 cm
CD = 2.6 cm	CD = 2.3 cm to 2.9 cm
DE = 4.5 cm	DE = 4.2 cm to 4.8 cm
EA = 3.1 cm	EA = 2.8 cm to 3.4 cm
Perimeter = 13.6 cm	Perimeter = 12.1 cm to 15.1 cm

Note: Each of your measurements may be off by ±3 mm due to photocopying of the figure or measurement error.

(Module 3, Lesson 1)

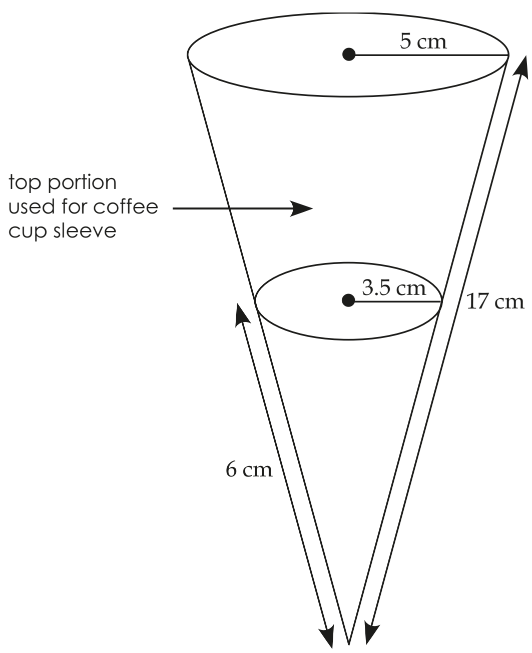
3. Convert the following measurements into the units indicated. (Round to 3 decimal places) *(8 marks)*

92 inches =	*Answer:* 7.667 feet
19 cm =	*Answer:* 190 mm
4.5 yards =	*Answer:* 162 inches
11 miles =	*Answer:* 17.699 km
5 gallons	*Answer:* 22.73 litres
33 kg =	*Answer:* 72.6 lb.
82 ft.^{2} =	*Answer:* 9.11 yd.^{2}
25 000 000 cm^3	*Answer:* 0.25 m^3

(Module 3, Lesson 2)

4. A take-out coffee cup has a protective paper sleeve to make it easier to hold when filled with a hot beverage. The sleeve is made by cutting the bottom off of a paper cone and using the top portion of it. Determine the lateral surface area of the part of the cone used to make the sleeve. *(4 marks)*

Answer:



$$\text{LSA of large cone} = \frac{1}{2}C\ell$$
$$= \frac{1}{2}2\pi(5)(17)$$
$$= 267.0353756 \text{ cm}^2$$

$$\text{LSA of small cone} = \frac{1}{2}C\ell$$
$$= \frac{1}{2}2\pi(3.5)(6)$$
$$= 65.97344573 \text{ cm}^2$$

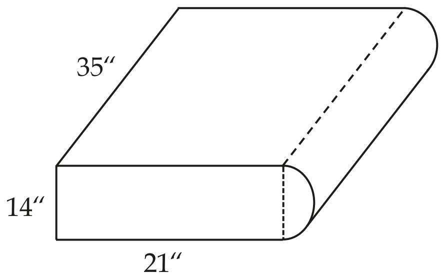
LSA of sleeve = 267.0353756 – 65.97344573

LSA of sleeve = 201.0619298

The coffee sleeve has a lateral surface area of 201.1 cm^2.

(Module 3, Lesson 6)

5. Determine the volume of this 3-D object. Its base is composed of a rectangle and a semi-circle. State your final answer in cubic feet, rounded to the nearest tenth. *(4 marks)*



Answer:

$V = Bh$

$$V = \left[(l * w) + \left(\frac{1}{2} \pi \left(r^2 \right) \right) \right] (h)$$

$$V = \left[(21 * 4) + \left(\frac{1}{2} \pi \left(7^2 \right) \right) \right] (35)$$

$$V = (294 + 76.96902001)(35)$$

$$V = 12983.9157 \text{ in.}^3$$

$$12983.9157 \text{ in.}^3 * \frac{1 \text{ ft.}^3}{1728 \text{ in.}^3} = 7.513840104 \text{ ft.}^3$$

The 3-D object has a volume of about 7.5 cubic feet.

(Module 3, Lesson 6)

6. A beach ball has a surface area of 572.6 sq. in. Determine its diameter to the nearest $\frac{1}{2}$ inch. *(4 marks)*

Answer:

$$\text{SA} = 4\pi r^2$$

$$572.6 = 4\pi r^2$$

$$\frac{572.6}{4\pi} = r^2$$

$$r^2 = 45.56606021$$

$$r = 6.750263714$$

$$d = 2r$$

$$d = (2)(6.750263714)$$

$$d = 13.5$$

The diameter of the beach ball is $13\frac{1}{2}$ inches.

(Module 3, Lesson 6)

Part G: Trigonometry (*10 marks*)

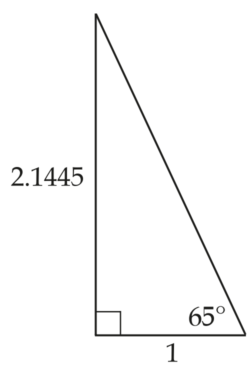
Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Calculate tan 65° to 4 decimal places and explain what this means using a diagram of a right triangle. *(3 marks)*

   Answer:

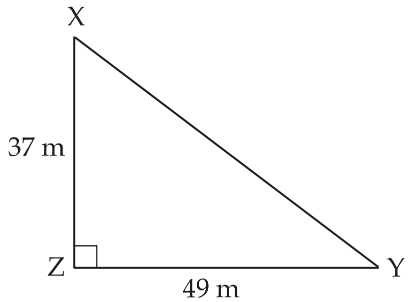
   $\tan 65° = 2.1445$

   In a right triangle with an angle of 65°, the ratio of the side lengths opposite and adjacent to the specified angle will be approximately $\dfrac{2.1445}{1}$.

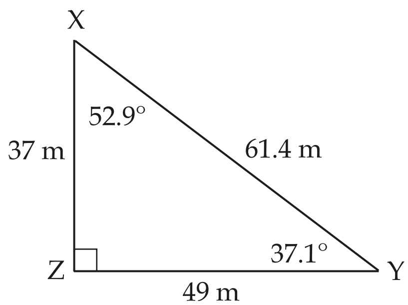


(Module 4, Lesson 1)

2. Solve the triangle. Find all angles and side lengths. Round your answers to 1 decimal place. *(3 marks)*



Answer:



$$37^2 + 49^2 = z^2$$

$$z^2 = 3770$$

$$z = 61.4 \text{ m}$$

$$\angle X = \tan^{-1}\left(\frac{49}{37}\right)$$

$$\angle X = 52.9°$$

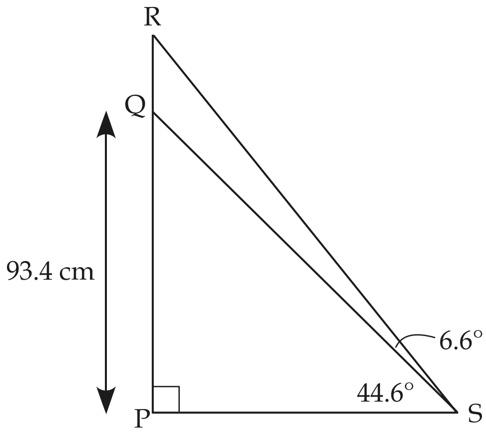
$$\angle Y = 90 - 52.9$$

$$\angle Y = 37.1°$$

Note: You may have used different trigonometric ratios to find your answers.

(Module 4, Lesson 3)

3. Find the length of QR to the nearest tenth. *(4 marks)*



Answer:

$$\tan 44.6° = \left(\frac{93.4}{PS}\right)$$

$$PS = \left(\frac{93.4}{\tan 44.6°}\right)$$

$$PS = 94.71329992$$

$$44.6 + 6.6 = 51.2$$

$$\angle RSP = 51.2°$$

$$\tan 51.2° = \frac{PR}{94.71329992}$$

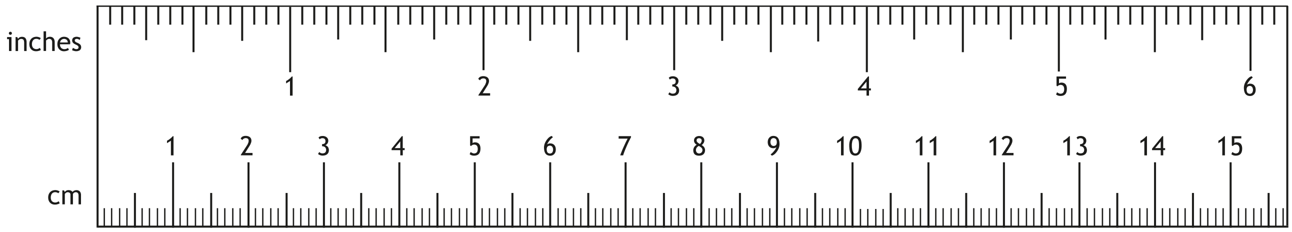
$$PR = 117.7995874$$

$$PR - PQ = QR$$

$$117.7995874 - 93.4 = 24.39958735$$

$$QR = 24.4 \text{ cm}$$

(Module 4, Lesson 4)


inches
1 2 3 4 5 6
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
cm

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Final Practice Exam

Grade 10 Introduction to Applied and Pre-Calculus Mathematics

Final Practice Exam

Name: ___________________________________

Student Number: ___________________________

Attending ❑ Non-Attending ❑

Phone Number: ____________________________

Address: _________________________________

____

____

For Marker's Use Only

Date: ________________________________

Final Mark: ________ /100 = ________ %

Comments:

Instructions

The final examination will be weighted as follows:

Modules 1–8	100%

The format of the examination will be as follows:

Part A: Multiple Choice	30 marks
Part B: Definitions	10 marks
Part C: Graphs and Relations	5 marks
Part D: Measurement	5 marks
Part E: Trigonometry	3 marks
Part F: Relations and Functions	9 marks
Part G: Polynomials	14 marks
Part H: Coordinate Geometry	20 marks
Part I: Systems	4 marks

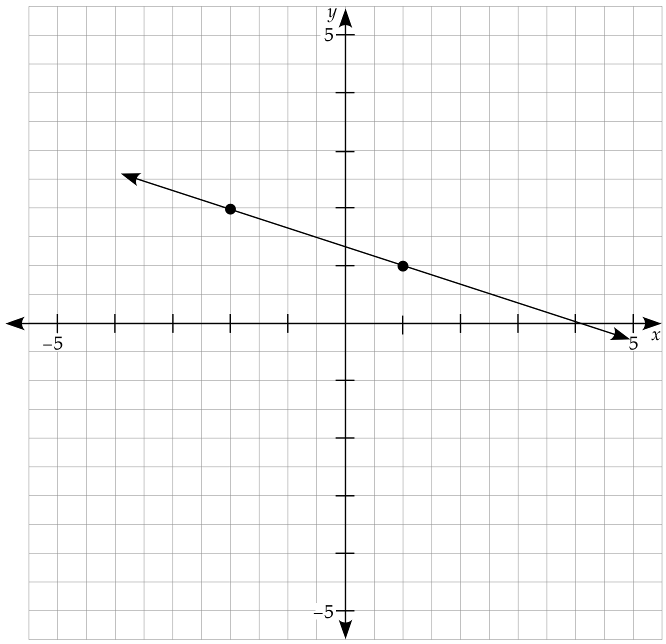
Time allowed: 2.5 hours

Note: You are allowed to bring a scientific calculator and your Resource Sheet to the exam. Your Resource Sheet must be handed in with the exam.

Part A: Multiple Choice (*30 x 1 = 30 marks*)

Circle the letter of the correct answer for each question.

1. Calculate the $\dfrac{\text{rise}}{\text{run}}$ of this line.



a) $\dfrac{1}{3}$

b) $\dfrac{3}{1}$

c) $\dfrac{-1}{3}$

d) $\dfrac{-3}{1}$

2. The x- and y-intercepts of the linear relation $3x + 5y - 15 = 0$ are at

a) (3, 0), (0, 5)

b) (5, 0), (0, 3)

c) (-3, 0), (0, –5)

d) (0, 0), 0, 0)

3. The graph of a linear relation has a slope of 2 and goes through the point (3, –5). Another point on the line is at
   a) (3, –8)
   b) (–11, 0)
   c) (5, –3)
   d) (4, –3)

4. The slope of a horizontal line is
   a) 0
   b) 1
   c) –1
   d) undefined

5. The equation of a line that is parallel to $y = 3x + 5$ is
   a) $y = -3x + 15$
   b) $y = \frac{-1}{3}x + 5$
   c) $y = -3x + 5$
   d) $y = 3x + 15$

6. Write $\sqrt[5]{x}$ with a rational exponent.
   a) $x^{\frac{5}{1}}$
   b) x^{-5}
   c) $x^{\frac{1}{5}}$
   d) $5^{\frac{1}{x}}$

7. Write $\sqrt{12}$ as a mixed radical.

a) $4\sqrt{3}$

b) $2\sqrt{3}$

c) $3\sqrt{2}$

d) $3\sqrt{4}$

8. Simplify $(3m^4n)(2m^5n)$.

a) $5m^9n$

b) $6m^9n$

c) $6m^{20}n^2$

d) $6m^9n^2$

9. The least common multiple of 32 and 20 is

a) 160

b) 640

c) 320

d) 4

10. Which of the following is a perfect cube number?

a) 324

b) 343

c) 333

d) 361

11. Convert 147 m to inches.

a) 186.37″

b) 3.73″

c) 14700″

d) 5787″

12. A pool is 6 m long and 4 m wide. If it is filled to a depth of 80 cm, how many cubic metres of water are required?

a) 1920 m^3

b) 19.2 m^3

c) 7077.888 m^3

d) 192 m^3

13. The volume of a sphere is 87 cm^3. Calculate the radius of the sphere.

a) 20.8 cm

b) 2.7 cm

c) 4.6 cm

d) 5.9 cm

14. The volume of a cone is 30 m^3. What is the volume of a cylinder with the same base and height?

a) 10 m^3

b) 30 m^3

c) 90 m^3

d) 900 m^3

15. The width of a child's pinky finger could be used as a referent for

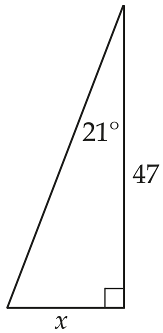
a) 1 mm

b) 1 m

c) 1 inch

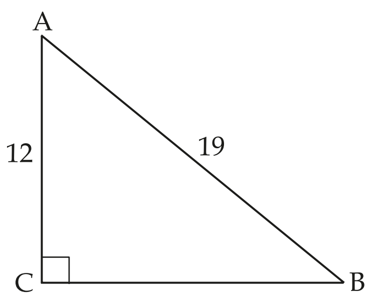
d) 1 cm

16. Solve for x.



a) 122.4
b) 18.0
c) 19.1
d) 43.9

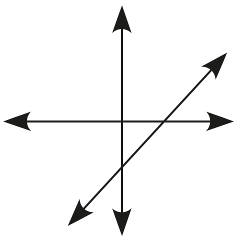
17. Solve for the measure of ∠B.



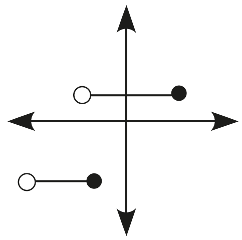
a) 32.3°
b) 39.2°
c) 50.8°
d) 57.7°

18. Which of the following does not represent a function?

a)

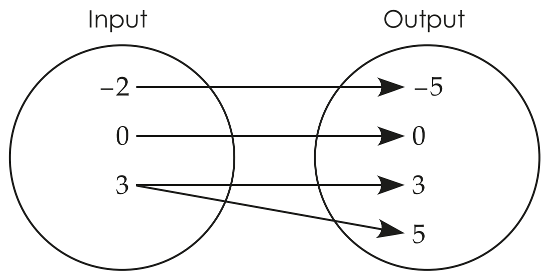


b)



c) $\{(2, 2), (3, 2), (4, 2) (5, 2)\}$

d)



a) A
b) B
c) C
d) D

19. Given the function $f(x) = \frac{3}{2}x + 9$, find $f(4)$.

a) $\frac{-10}{3}$

b) 10.5

c) 15.0

d) 19.5

20. Multiply $4(2x + 3)$.

a) $8x + 12$
b) $8x + 3$
c) $2x + 12$
d) $24x$

21. Multiply $(x + 4)(x + 9)$.
   a) $x^2 + 13$
   b) $2x + 13 + 36$
   c) $x^2 + 36$
   d) $x^2 + 13x + 36$

22. Factor $8k + 14$.
   a) $8(k + 14)$
   b) $2(4k + 7)$
   c) $4k(2 * 7)$
   d) $8(k + 6)$

23. Factor $x^2 - 4x - 12$.
   a) $(x - 6)(x + 2)$
   b) $(x + 6)(x - 2)$
   c) $(x - 6)(x - 2)$
   d) $(x + 6)(x + 2)$

24. Factor $x^2 - 25$.
   a) $(x - 5)(x - 5)$
   b) $(x + 5)(x + 5)$
   c) $(x - 5)^2$
   d) $(x + 5)(x - 5)$

25. Calculate the distance between the coordinate points (13, 5) and (–17, –9).
   a) 33.1
   b) 5.7
   c) 26.5
   d) 11.3

26. Calculate the coordinates of the midpoint of the line segment with endpoints at (–15, 9) and (7, –11).

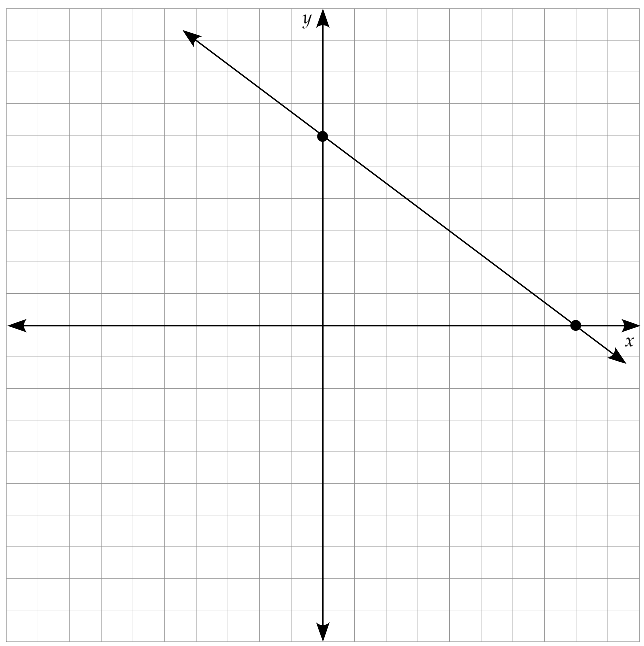
a) (–11, 10)

b) (11, –1)

c) (4, 1)

d) (–4, –1)

27. Write the equation of this line in slope-intercept form.



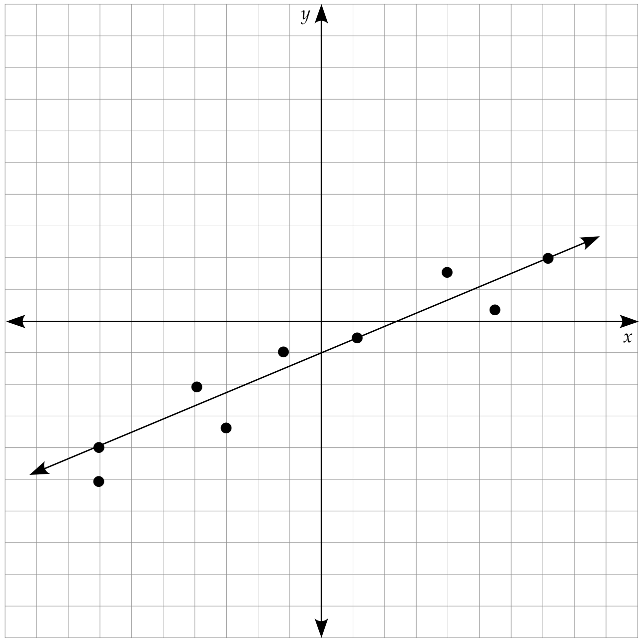
a) $y = \frac{3}{4}x + 6$

b) $y = \frac{4}{3}x + 8$

c) $y = \frac{-3}{4}x + 6$

d) $y = \frac{-3}{4}x + 8$

28. The correlation of this data is best described as



a) strong negative
b) weak negative
c) weak positive
d) strong positive

29. Three of the following linear relations are equivalent. Circle the one relation that is not equivalent to the others.

a) $2x - y + 5 = 0$
b) $y - 11 = 2(x - 3)$
c) $y = 5x + 2$
d) $3y - 6x = 15$

30. Which ordered pair is the solution to the given system of linear equations?

$x - 5y = -15$	Equation 1
$4x + 10y = -30$	Equation 2

a) $(-5, -1)$
b) $(-5, 2)$
c) $(5, -5)$
d) $(-10, 1)$

Part B: Definitions (*10 x 1 = 10 marks*)

Match each definition with the correct term or symbol from the list below. Write the correct term or symbol on the blank line with each definition. Terms are used only once. Not all terms have a definition provided.

Terms

$^\circ$ $>$ $<$ ( )
$\cdot$ $\geq$ $\leq$ [ ]
$\varnothing$
binomial
Cartesian plane
coefficient
consistent system
constant
correlation coefficient
degree
dependent system
domain
equation
function
general form
inconsistent system
independent system
like terms
linear relation
mapping
monomial
negative correlation
ordered pair
parallel lines
perpendicular lines
polynomial
positive correlation
range
relation
rule
scatterplot
simplify
slope-intercept form
slope-point form
solution
strong correlation
system of linear equations
table of values
trinomial
weak correlation
zero correlation

1. A graphic similar to a table of values that has arrows showing which input results in a given output. _________________
2. If, as the x-variable increases in value, the y-variable also increases the data displays a _________________ .
3. The equations in this linear system represent the same line. _________________
4. The coordinate system formed by a horizontal axis and a vertical axis in which a pair of numbers represents each point in the plane. _________________
5. A mathematical expression with one or more terms. _________________
6. $r = 0$ _________________
7. The highest exponent in the leading term of the polynomial, when terms are written in descending order. _________________
8. A set of two numbers named in a specific order so that the first number represents the domain value and the second number represents the range value. _________________
9. Any set of ordered pairs. _________________
10. Goes to and includes. _________________

Part C: Graphs and Relations (*5 marks*)

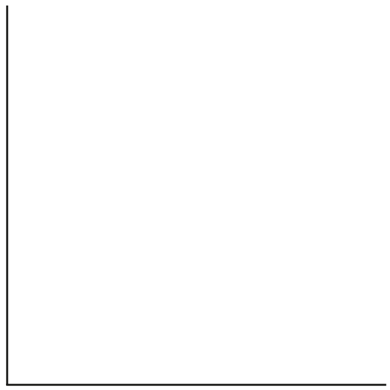
Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. The following table shows the number of fatal accidents per 10 000 000 aircraft departures for U. S. airlines for the 10 years from 1977 to 1986.

Year	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Fatal Accicents per 10 000 000	6.1	10.0	7.4	0.0	7.7	6.0	7.9	1.8	6.9	1.6

a) Create a scatterplot of this data. Include labels, units, and a title. *(3 marks)*

2. a) Sketch and label a graph that could represent the length of time spent waiting in line to get into the hockey arena and the number of people in line ahead of you. *(1 mark)*

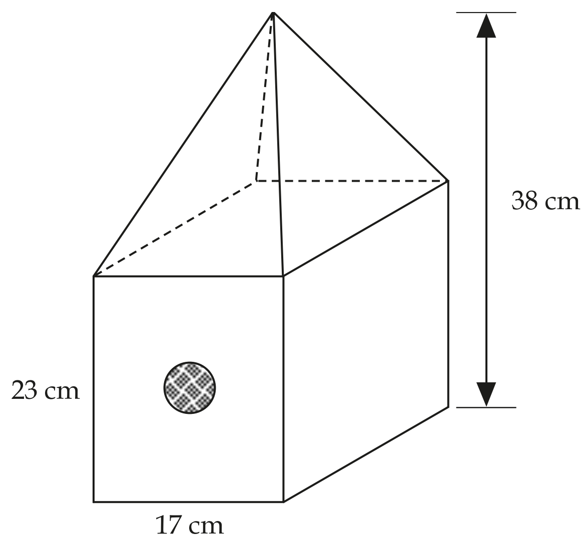


b) State a reasonable domain and range for this situation. Explain. *(1 mark)*

Part D: Measurement (*5 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

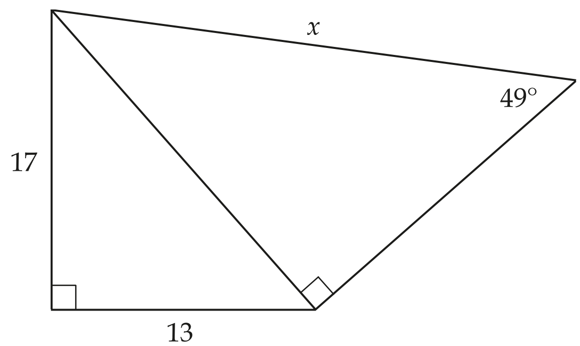
1. A birdhouse with a square base has a peaked roof as illustrated below. The total height of the birdhouse is 38 cm. Calculate the amount of space inside the birdhouse to the nearest cm^3. *(5 marks)*



Part E: Trigonometry (*3 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Solve for the length of side x. *(3 marks)*

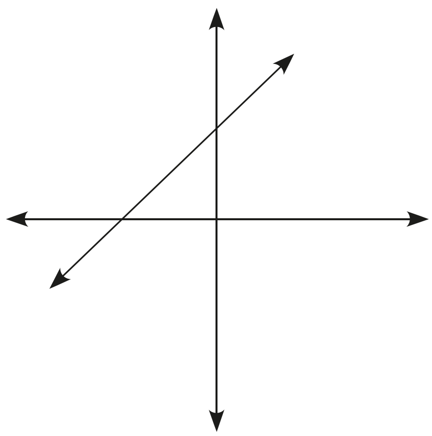


Part F: Relations and Functions (*9 marks*)

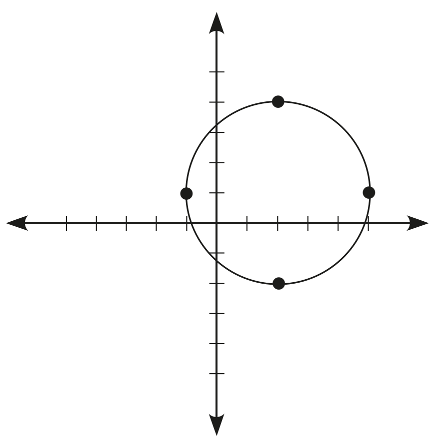
Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. State the domain and range of the following relations in both set and interval notation. *(4 marks)*

   a)



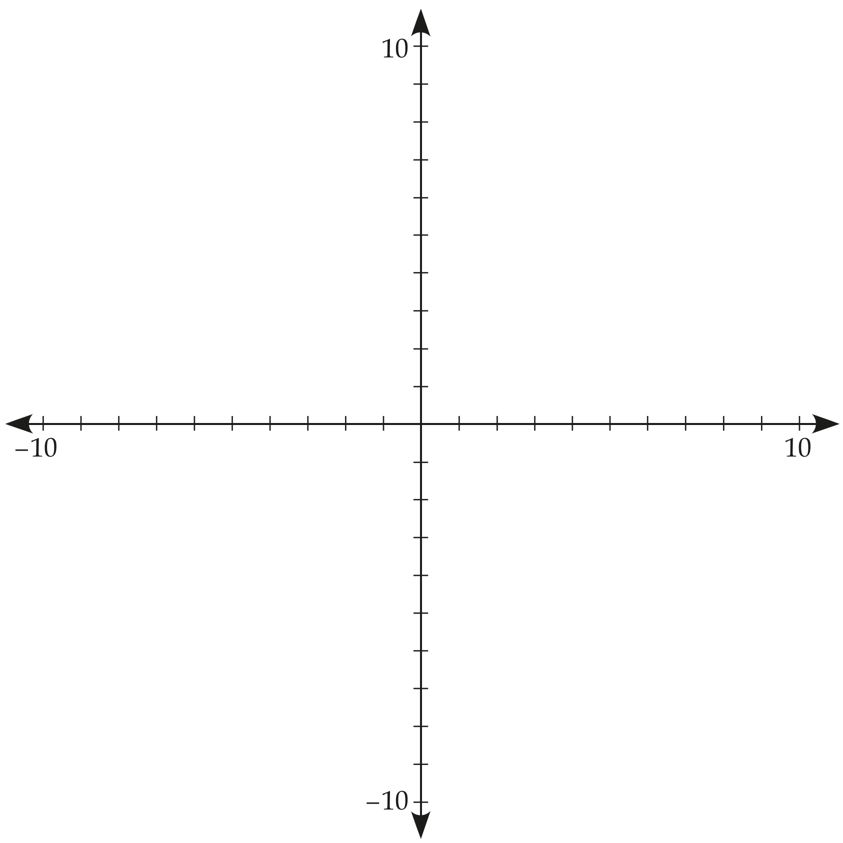
   b)



2. Given the linear equation $2x - 3y - 15 = 0$

a) Express the linear equation in functional notation. *(2 marks)*

b) Sketch the linear function. *(1 mark)*



3. Explain how you can determine whether or not a given set of ordered pairs represents a function. *(2 marks)*

Part G: Polynomials (*14 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Represent the product of $(2x + 3)(x + 4)$ pictorially. State the simplified solution. *(4 marks)*

2. Multiply and simplify the solution.

   a) $(x - 3)(3x + 5)$ *(3 marks)*

   b) $(5x + 4)(2x - 3)$ *(3 marks)*

3. Factor completely. Verify by multiplying the factors. *(4 marks)*

$$2x^2 + 7x + 6$$

Part H: Coordinate Geometry (*20 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. The centre of a circle is at (52, 34). If an endpoint of its diameter is at (61, 46), find the coordinates of the other endpoint. *(3 marks)*

2. Express the linear equation $y - 5 = \frac{2}{7}(x - 21)$ in slope-intercept form. *(2 marks)*

3. Explain a strategy for graphing a linear equation given in point-slope form. *(3 marks)*

4. The graph of a linear relation goes through the points (9, –11) and (13, –2). Write the equation of the linear relation in point-slope form. *(3 marks)*

5. The graph of a linear relation goes through the point (6, 4) and is parallel to the line $y = 5x + 10$. Write the equation of the linear relation in slope-intercept form. *(3 marks)*

6. Determine if the triangle with vertices at A(–5, 3), B(–1, –8), and C(6, –1) is an isosceles triangle. *(6 marks)*

Part I: Systems (*4 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Suzy has a German Shepherd and a Toy Poodle. The difference in height between them is 15″. Twice the height of a poodle is still 6″ shorter than a German Shepherd. Write a system of linear equations to represent this situation. Do **not** solve the system. *(1 mark)*

2. Solve the system of linear equations using elimination by addition or subtraction. *(3 marks)*

   $3x + 2y = 4$

   $x - y = 3$

Notes

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)

Final Practice Exam
Answer Key

Grade 10 Introduction to Applied and Pre-Calculus Mathematics

Final Practice Exam
Answer Key

Name: ___________________________________

Student Number: ___________________________

Attending ❑ Non-Attending ❑

Phone Number: ____________________________

Address: _________________________________

____

____

For Marker's Use Only

Date: ______________________________

Final Mark ________ /100 = ________ %

Comments:

Instructions

The final examination will be weighted as follows:

Modules 1–8	100%

The format of the examination will be as follows:

Part A: Multiple Choice	30 marks
Part B: Definitions	10 marks
Part C: Graphs and Relations	5 marks
Part D: Measurement	5 marks
Part E: Trigonometry	3 marks
Part F: Relations and Functions	9 marks
Part G: Polynomials	14 marks
Part H: Coordinate Geometry	20 marks
Part I: Systems	4 marks

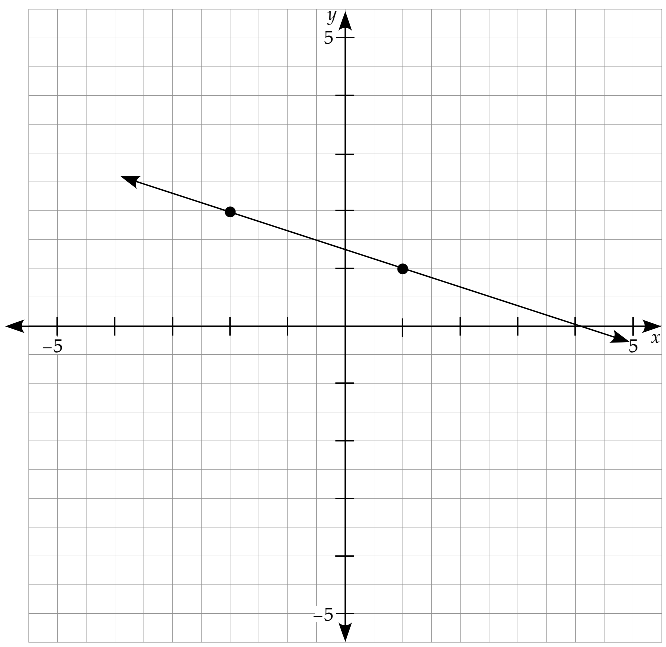
Time allowed: 2.5 hours

Note: You are allowed to bring a scientific calculator and your Resource Sheet to the exam. Your Resource Sheet must be handed in with the exam.

Part A: Multiple Choice (*30 x 1 = 30 marks*)

Circle the letter of the correct answer for each question.

1. Calculate the $\frac{\text{rise}}{\text{run}}$ of this line.



a) $\frac{1}{3}$

b) $\frac{3}{1}$

c) $\frac{-1}{3}$

d) $\frac{-3}{1}$

(Module 1, Lesson 3)

The line falls to the right, so the slope must be negative. The vertical change is 1 and the horizontal change is 3, so the correct answer is (c).

2. The x- and y-intercepts of the linear relation $3x + 5y - 15 = 0$ are at

   a) (3, 0), (0, 5)

   b) (5, 0), (0, 3)

   c) (-3, 0), (0, –5)

   d) (0, 0), 0, 0) (Module 7, Lesson 2)

   Substitute $x = 0$ into the equation and solve for the y-intercept, and then substitute $y = 0$ and solve for the x-intercept. Watch for positive and negative signs.

3. The graph of a linear relation has a slope of 2 and goes through the point (3, –5). Another point on the line is at

   a) (3, –8)

   b) (–11, 0)

   c) (5, –3)

   d) (4, –3)

   $y + 5 = 2(x - 3)$

   $y + 5 = 2x - 6$

   $y = 2x - 6 - 5$

   $y = 2x - 11$ (Module 1, Lesson 4)

   Substitute the given coordinates into the equation and see which makes a true statement.

   Or, consider that the vertical change is +2 and the horizontal change is +1.

   (3 + 1, –5 + 2) gives you (4, –3).

   Or, make a sketch of the point and count up 2 and 1 space to the right and mark the next point; alternately, move down 2 and 1 space to the left to find points to the left of the given point.

4. The slope of a horizontal line is

   a) 0

   b) 1

   c) –1

   d) undefined (Module 1, Lesson 4)

   A horizontal line has a rise of 0 and an infinite run, so the slope is 0. A slope of 1 rises to the right, and a slope of –1 falls to the right. A vertical line has an undefined slope because the rise is infinite and the run is zero. If the denominator is 0, it is undefined because you cannot divide by zero.

5. The equation of a line that is parallel to $y = 3x + 5$ is

a) $y = -3x + 15$

b) $y = \frac{-1}{3}x + 5$

c) $y = -3x + 5$

d) $y = 3x + 15$

(Module 1, Lesson 4)

Parallel lines have the same slope.

6. Write $\sqrt[5]{x}$ with a rational exponent.

a) $x^{\frac{5}{1}}$

b) x^{-5}

c) $x^{\frac{1}{5}}$

d) $5^{\frac{1}{x}}$

(Module 2, Lesson 5)

The fifth root of x would be the value that could be multiplied by itself 5 times and result in x. Using the power of a power law, $\left(x^{\frac{1}{5}}\right)^5 = x^{\frac{5}{5}} = x$.

7. Write $\sqrt{12}$ as a mixed radical.

a) $4\sqrt{3}$

b) $2\sqrt{3}$

c) $3\sqrt{2}$

d) $3\sqrt{4}$

(Module 2, Lesson 3)

12 has the perfect square factor of 4 ($4 * 3 = 12$). The square root of 4 is 2, so when moving the 4 out from under the square root sign you are left with $2 * \sqrt{3}$.

8. Simplify $(3m^4n)(2m^5n)$.

a) $5m^9n$

b) $6m^9n$

c) $6m^{20}n^2$

d) $6m^9n^2$ (Module 2, Lesson 4)

The product law states that exponents must be added when multiplying like bases. An exponent of 1 is assumed for each variable if no exponent is stated. You must also multiply the coefficients.

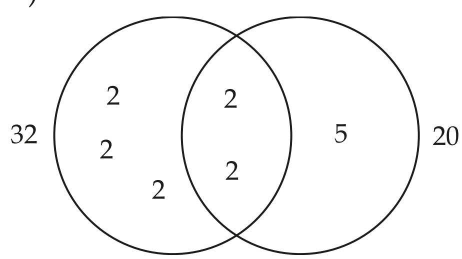
9. The least common multiple of 32 and 20 is

a) 160

b) 640

c) 320

d) 4 (Module 2, Lesson 1)



10. Which of the following is a perfect cube number?

a) 324

b) 343

c) 333

d) 361 (Module 2, Lesson 2)

324, 343, and 361 are all perfect square numbers. 333 is divisible by 3 but only 343 has a cube root ($7^3 = 343$).

11. Convert 147 m to inches.

a) 186.37″

b) 3.73″

c) 14700″

d) 5787″ (Module 3, Lesson 3)

There are 39.37 inches in a metre, so multiply $147 * 39.37 = 5787$. (a) has the conversion amount added, (b) divided 147 by the conversion factor, (c) used the conversion factor for m to cm.

12. A pool is 6 m long and 4 m wide. If it is filled to a depth of 80 cm, how many cubic metres of water are required?

a) 1920 m^3

b) 19.2 m^3

c) 7077.888 m^3

d) 192 m^3 (Module 3, Lesson 1)

($6 * 4 * 0.8 = 19.2$) If you chose (a). you didn't convert cm to m. If you chose (c), the final answer was incorrectly cubed. If you chose (d), 80 cm was converted to 8 m instead of 0.8.

13. The volume of a sphere is 87 cm^3. Calculate the radius of the sphere.

a) 20.8 cm

b) 2.7 cm

c) 4.6 cm

d) 5.9 cm (Module 3, Lesson 6)

($r^3 = 20.8$) (c) Found the square root rather than the cube root. (d) Did not use brackets when dividing by a product.

14. The volume of a cone is 30 m^3. What is the volume of a cylinder with the same base and height?

a) 10 m^3

b) 30 m^3

c) 90 m^3

d) 900 m^3 (Module 3, Lesson 6)

The volume of a cone is one-third the volume of a cylinder with the same base and height, or the volume of a cylinder is three times the volume of a cone with the same base and height. (a) is one-third the volume of the cone, not 3 times the volume. (b) is the same volume. (d) is the volume cubed.

15. The width of a child's pinky finger could be used as a referent for

a) 1 mm

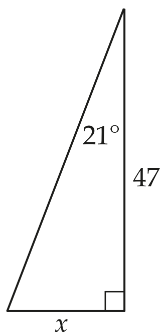
b) 1 m

c) 1 inch

d) 1 cm (Module 3, Lesson 1)

1 mm is about the width of a credit card or dime, 1 m is the approximate width of a door or a twin bed, 1 inch is the height of a hockey puck or the diameter of a loonie.

16. Solve for x.



a) 122.4

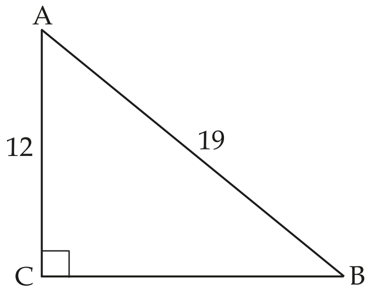
b) 18.0

c) 19.1

d) 43.9 (Module 4, Lesson 1)

In terms of the given angle, you know the adjacent side and want to find the opposite side length. The correct trig ratio is $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. (a) You exchanged the opposite and adjacent sides (c) You omitted the bracket and found tan(21) ∗ 47 instead of 47 ∗ tan(21). (d) used the cosine ratio.

17. Solve for the measure of ∠B.



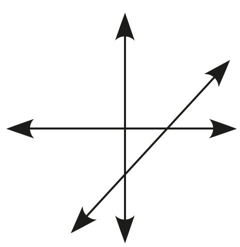
a) 32.3°
b) 39.2°
c) 50.8°
d) 57.7°

(Module 4, Lesson 3)

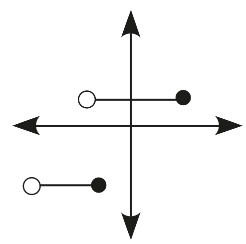
(a) used $\tan^{-1}\left(\frac{12}{19}\right)$. (c) used $\cos^{-1}\left(\frac{12}{19}\right)$. (d) used $\tan^{-1}\left(\frac{19}{12}\right)$.

18. Which of the following does not represent a function?

a)

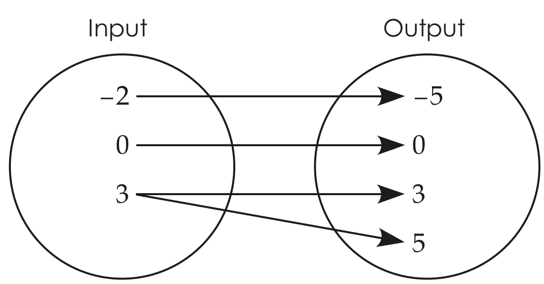


b)



c) {(2, 2), (3, 2), (4, 2) (5, 2)}

d)



a) A
b) B
c) C
d) D

(A) passes the vertical line test. (B) the hollow dots indicate that the graph goes up to that point but does not include it, the solid dot indicates the point is included, so the graph passes the vertical line test. (C) Each input has one possible output. The points would lie in a horizontal line. (D) The input of 3 has two possible outputs, so this one is not a function.

(Module 5, Lesson 1)

19. Given the function $f(x)=\frac{3}{2}x+9$, find $f(4)$.

a) $\frac{-10}{3}$

b) 10.5

c) 15.0

d) 19.5

(Module 5, Lesson 3)

(a) substituted the 4 for $f(x)$ rather than for x. (b) incorrect order of operations—multiply 4 and 3 and divide by 2 before you add 9. (d) incorrect order of operations—added the 4 and 9 and then multiplied by $\frac{3}{2}$.

20. Multiply $4(2x + 3)$.

a) $8x + 12$

b) $8x + 3$

c) $2x + 12$

d) $24x$

(Module 6, Lesson 1)

(b) didn't apply distributive property and only multiplied 4 by the first term in the bracket. (c) didn't apply distributive property and only multiplied 4 by the second term in the bracket. (d) incorrectly multiplied the terms inside the bracket and found the product of $4(6x)$.

21. Multiply $(x + 4)(x + 9)$.

a) $x^2 + 13$

b) $2x + 13 + 36$

c) $x^2 + 36$

d) $x^2 + 13x + 36$

(Module 6, Lesson 2)

(a) only multiplied the two first terms and added the two last terms. (b) added the two x terms, added the two last terms, and then multiplied the two last terms. (c) multiplied two first terms and two last terms. Missed multiplying the outside terms and the inside terms.

22. Factor $8k + 14$.

a) $8(k + 14)$

b) $2(4k + 7)$

c) $4k(2 * 7)$

d) $8(k + 6)$ (Module 6, Lesson 3)

(a) removed the coefficient of the first term but didn't divide the second term by it (8 is not a factor of 14). (c) k is not a common term and 4 is not a factor of 14. (d) $8 + 6 = 14$ but 8 is not a factor of 14.

23. Factor $x^2 - 4x - 12$.

a) $(x - 6)(x + 2)$

b) $(x + 6)(x - 2)$

c) $(x - 6)(x - 2)$

d) $(x + 6)(x + 2)$ (Module 6, Lesson 3)

(b) incorrect signs would give you $+4x$ for the middle term of the trinomial. (c) incorrect signs would give you $x^2 - 8x + 12$. (d) incorrect signs would give you $x^2 + 8x + 12$.

24. Factor $x^2 - 25$.

a) $(x - 5)(x - 5)$

b) $(x + 5)(x + 5)$

c) $(x - 5)^2$

d) $(x + 5)(x - 5)$ (Module 6, Lesson 5)

(a), (b), and (c) are all perfect square factors, not the difference of squares factors.

25. Calculate the distance between the coordinate points (13, 5) and (–17, –9).

a) 33.1

b) 5.7

c) 26.5

d) 11.3 (Module 7, Lesson 1)

a) $\sqrt{(-17-13)^2+(-9-5)^2}=33.1$

b) $\sqrt{(-17+13)^2+(-9+5)^2}=5.7$ incorrect sign inside brackets

c) $\sqrt{(-17-13)^2-(-9-5)^2}=26.5$ incorrect sign between brackets

d) $\sqrt{(13-5)^2+(-17+9)^2}=11.3$ incorrect substitution of ordered pairs

26. Calculate the coordinates of the midpoint of the line segment with endpoints at (–15, 9) and (7, –11).

a) (–11, 10)

b) (11, –1)

c) (4, 1)

d) (–4, –1)

(Module 7, Lesson 1)

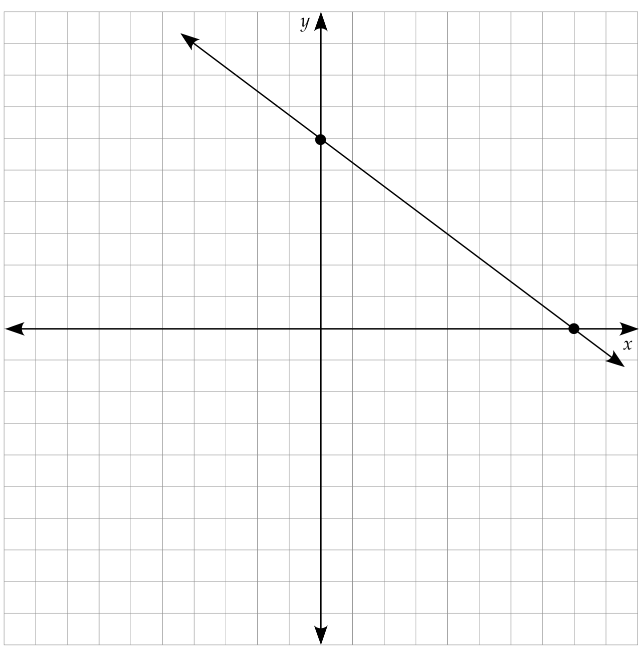
a) $\left(\frac{-15-7}{2}, \frac{9+11}{2}\right) \rightarrow (-11, 10)$ Subtracting coordinates instead of adding them

b) $\left(\frac{15+7}{2}, \frac{9-11}{2}\right) \rightarrow (11, -1)$ Incorrect signs

c) $\left(\frac{15-7}{2}, \frac{11-9}{2}\right) \rightarrow (4, 1)$ Incorrect signs

d) $\left(\frac{-15+7}{2}, \frac{9-11}{2}\right) \rightarrow (-4, -1)$

27. Write the equation of this line in slope-intercept form.



a) $y = \frac{3}{4}x + 6$

b) $y = \frac{4}{3}x + 8$

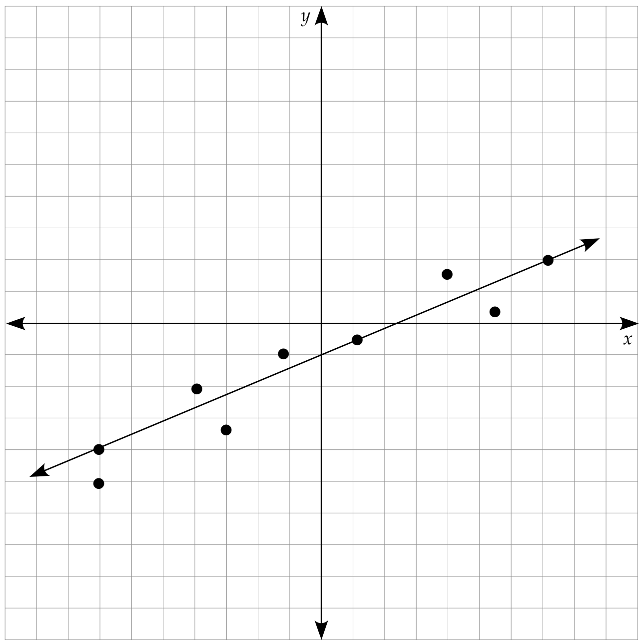
c) $y = \frac{-3}{4}x + 6$

d) $y = \frac{-3}{4}x + 8$

(Module 7, Lesson 2)

(a) slope should have a negative rise. (b) mixed up rise and run, used the x-intercept. (d) used x-intercept.

28. The correlation of this data is best described as



a) strong negative
b) weak negative
c) weak positive
d) strong positive (Module 7, Lesson 4)

The line of best fit rises to the right or has a positive slope, so the correlation is positive. The points are close to the line of best fit so the correlation is strong.

29. Three of the following linear relations are equivalent. Circle the one relation that is not equivalent to the others.

a) $2x - y + 5 = 0$
b) $y - 11 = 2(x - 3)$
c) $y = 5x + 2$
d) $3y - 6x = 15$ (Module 7, Lesson 2)

You can rewrite them all in $y = mx + b$ form and see which one is different. You may also notice that the coefficient of x simplifies to 2 in all cases except (c), so that line has a different slope.

30. Which ordered pair is the solution to the given system of linear equations?

$x - 5y = -15$ Equation 1

$4x + 10y = -30$ Equation 2

a) (–5, –1)

b) (–5, 2)

c) (5, –5)

d) (–10, 1) (Module 8, Lesson 1)

Substitute the ordered pair into both equations and see which pair makes both equations true.

(a) Only makes Equation 2 true.

(b) Only makes Equation 1 true.

(c) Only makes Equation 2 true.

Part B: Definitions (*10 x 1 = 10 marks*)

Match each definition with the correct term or symbol from the list below. Write the correct term or symbol on the blank line with each definition. Terms are used only once. Not all terms have a definition provided.

Terms

° > < ( )	domain	negative correlation	simplify
· ≥ ≤ [ ]	equation	ordered pair	slope-intercept form
∅	function	parallel lines	slope-point form
binomial	general form	perpendicular lines	solution
Cartesian plane	inconsistent system	polynomial	strong correlation
coefficient	independent system	positive correlation	system of linear equations
consistent system	like terms	range	table of values
constant	linear relation	relation	trinomial
correlation coefficient	mapping	rule	weak correlation
degree	monomial	scatterplot	zero correlation
dependent system			

1. A graphic similar to a table of values that has arrows showing which input results in a given output. ***mapping***
2. If, as the x-variable increases in value, the y-variable also increases the data displays a ***positive correlation***.
3. The equations in this linear system represent the same line. ***dependent system***
4. The coordinate system formed by a horizontal axis and a vertical axis in which a pair of numbers represents each point in the plane. ***Cartesian plane***
5. A mathematical expression with one or more terms. ***polynomial***
6. $r = 0$ ***zero correlation***
7. The highest exponent in the leading term of the polynomial, when terms are written in descending order. ***degree***
8. A set of two numbers named in a specific order so that the first number represents the domain value and the second number represents the range value. ***ordered pair***
9. Any set of ordered pairs. ***relation***
10. Goes to and includes. ≥ ≤ [ ]

Part C: Graphs and Relations (*5 marks*)

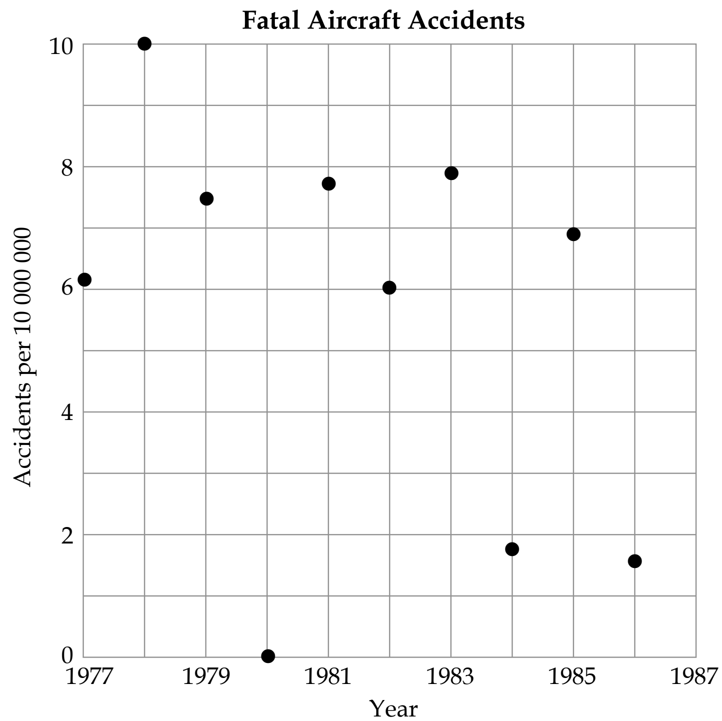
Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. The following table shows the number of fatal accidents per 10 000 000 aircraft departures for U. S. airlines for the 10 years from 1977 to 1986.

Year	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Fatal Accicents per 10 000 000	6.1	10.0	7.4	0.0	7.7	6.0	7.9	1.8	6.9	1.6

a) Create a scatterplot of this data. Include labels, units and a title. *(3 marks)*

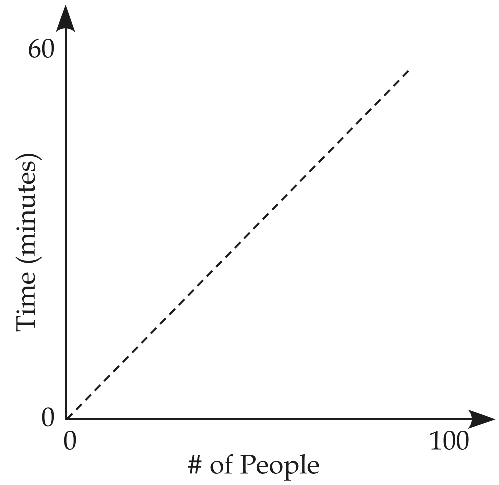
Answer:



(Module 1, Lesson 3)

2. a) Sketch and label a graph that could represent the length of time spent waiting in line to get into the hockey arena and the number of people in line ahead of you. *(1 mark)*

Answer:



b) State a reasonable domain and range for this situation. Explain. *(1 mark)*

Answer:

Answers may vary.

The number of people ahead of you could be any positive integer because you can’t have a negative number of people, but realistically, because there are multiple doors, a maximum of 100 people in a line ahead of you seems reasonable. The domain could be from zero to 100 people.

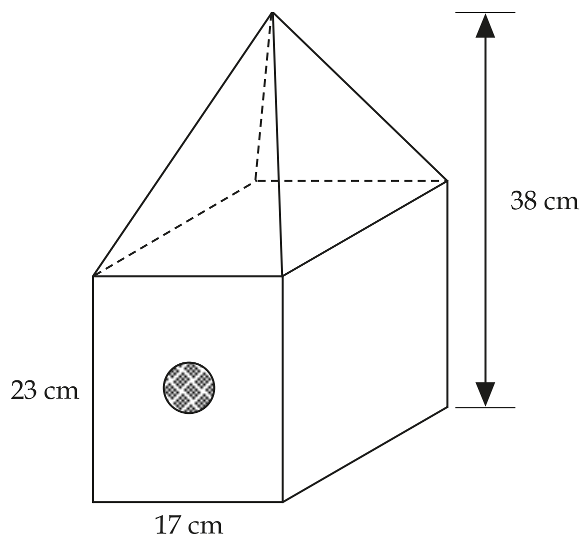
The range will also be positive values because you can’t have negative time. I don’t think you would have to wait longer than an hour to get into the arena so the range could be from zero to 60 minutes.

(Module 1, Lesson 2)

Part D: Measurement (*5 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. A birdhouse with a square base has a peaked roof as illustrated below. The total height of the birdhouse is 38 cm. Calculate the amount of space inside the birdhouse to the nearest cm^3. *(5 marks)*



Answer:

The space inside the birdhouse is the volume of the composite object.

$$V = Bh_1 + \frac{1}{3}Bh_2$$

$$B = 17^2$$

$$h_1 = 23$$

$$h_2 = 38 - 23 = 15$$

$$V = Bh_1 + \frac{1}{3}Bh_2$$

$$V = \left(17^2\right)(23) + \frac{1}{3}\left(17^2\right)(15)$$

$$V = 6647 + 1445$$

$$V = 8092$$

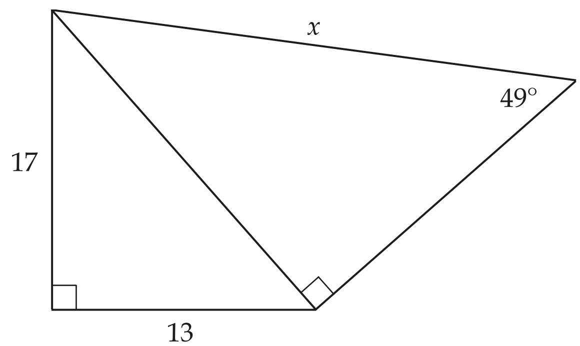
The volume of the birdhouse is 8092 cm^2.

(Module 3, Lesson 5)

Part E: Trigonometry (*3 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Solve for the length of side x. *(3 marks)*



Answer:

$$a^2 + b^2 = c^2$$

$$17^2 + 13^2 = c^2$$

$$c^2 = 21.40093456$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 49^\circ = \frac{21.40093456}{x}$$

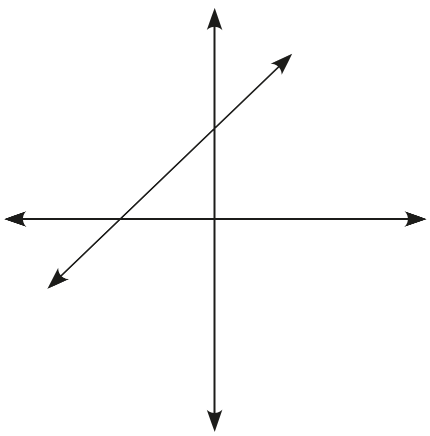
$$x = 28.36$$

(Module 4, Lesson 4)

Part F: Relations and Functions (*9 marks*)

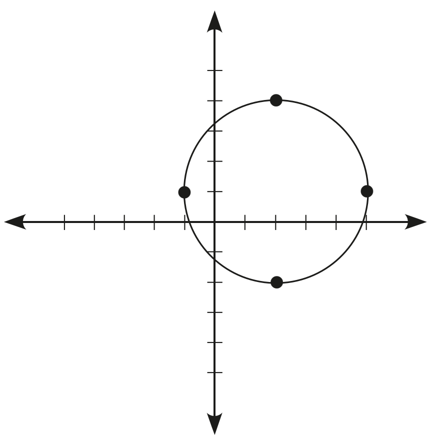
Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. State the domain and range of the following relations in both set and interval notation. *(4 marks)*

a) 

Answer:

D: $\{x \in \Re\}$
R: $\{y \in \Re\}$
D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

b) 

Answer:

D: $\{-1 \leq x \leq 5, x \in \Re\}$
R: $\{-2 \leq y \leq 4, y \in \Re\}$
D: $[-1, 5]$
R: $[-2, 4]$

(Module 5, Lesson 2)

2. Given the linear equation $2x - 3y - 15 = 0$

   a) Express the linear equation in functional notation. *(2 marks)*

   Answer:

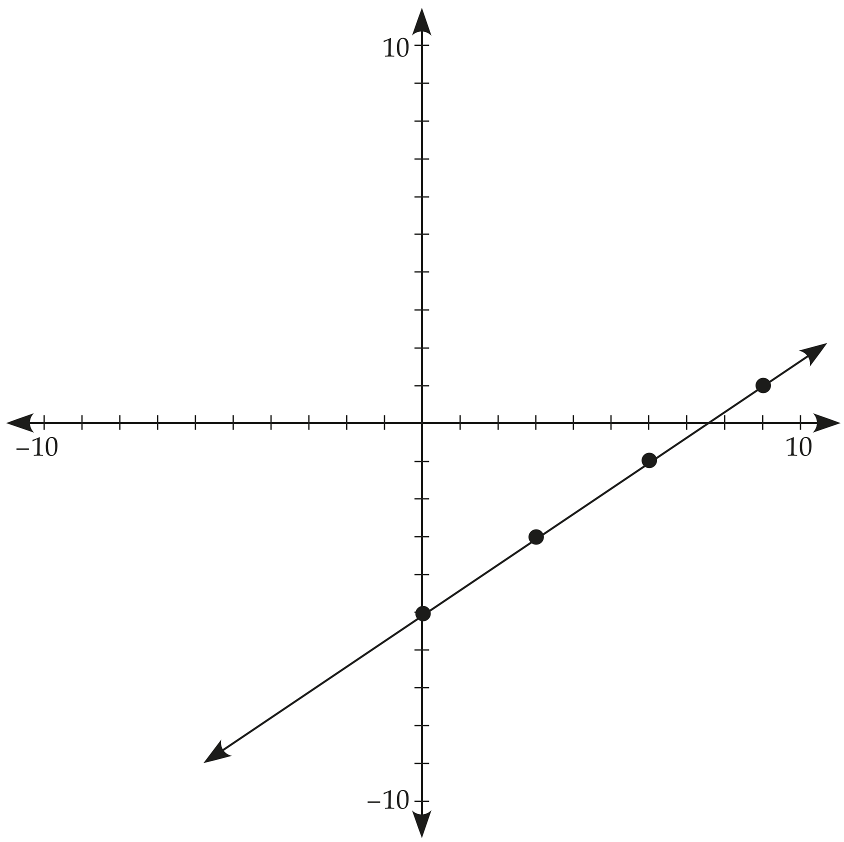
$$2x - 15 = 3y$$

$$y = \frac{2}{3}x - 5$$

$$f(x) = \frac{2}{3}x - 5$$

   b) Sketch the linear function. *(1 mark)*

   Answer:



(Module 5, Lesson 3)

3. Explain how you can determine whether or not a given set of ordered pairs represents a function. *(2 marks)*

Answer:

Look at the domain, or x-values, in the ordered pairs. If an x-value is duplicated and has two or more possible outputs, then the ordered pairs represent a relation but not a function. If each input or x-value has only one possible y-value or output, the ordered pairs represent a function.

(Module 5, Lesson 1)

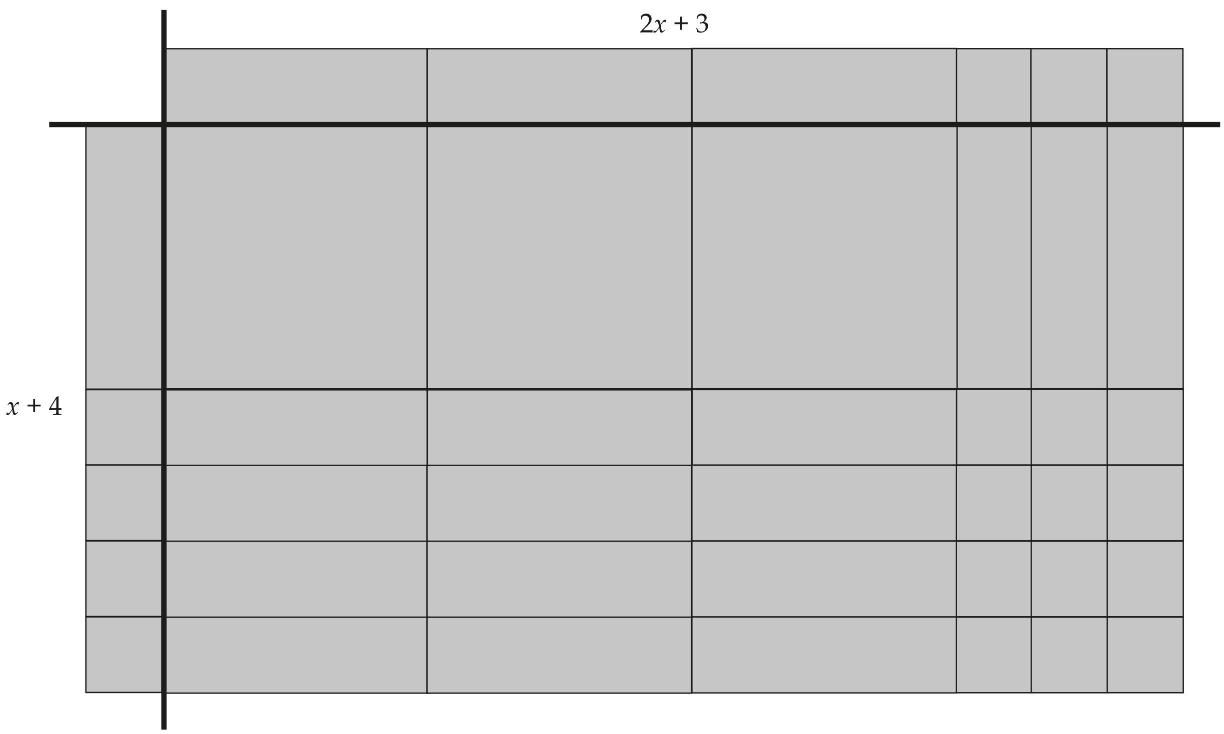
Part G: Polynomials (*14 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

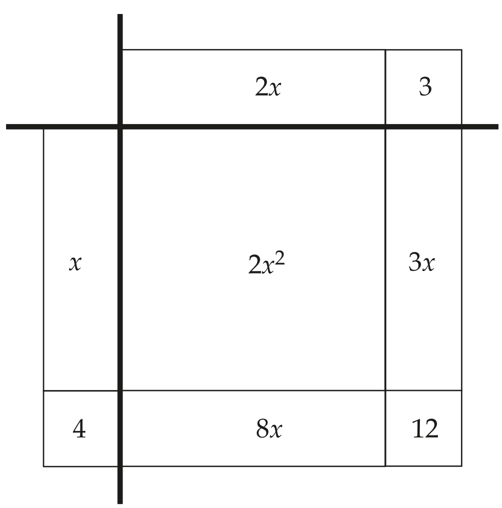
1. Represent the product of $(2x + 3)(x + 4)$ pictorially. State the simplified solution. *(4 marks)*

   Answer:

   Students may use the area model or tiles. No marks deducted if tiles not shaded.



or



$$(2x + 3)(x + 4)$$
$$= 2x^2 + 3x + 8x + 12$$
$$= 2x^2 + 11x + 12$$

(Module 6, Lesson 2)

2. Multiply and simplify the solution.

a) $(x - 3)(3x + 5)$ *(3 marks)*

Answer:

$(x - 2)(3x + 5)$

$= 3x^2 + 5x - 9x - 15$

$= 3x^2 - 4x - 15$

b) $(5x + 4)(2x - 3)$ *(3 marks)*

Answer:

$(5x + 4)(2x - 3)$

$= 10x^2 - 15x + 8x - 12$

$= 10x^2 - 7x - 12$

Students may use arrow showing the distributive property, FOIL, tiles, or the area model to show their work.

(Module 6, Lesson 2)

3. Factor completely. Verify by multiplying the factors. *(4 marks)*

$$2x^2 + 7x + 6$$

Answer:

$2 * 6 = 12$ Factor pairs of 12 are (1, 12), (2, 6), and (3, 4). The pair (3, 4) gives a sum of 7

$2x^2 + 7x + 6$

$= 2x^2 + 3x + 4x + 6$

$= x(2x + 3) + 2(2x + 3)$

$= (x + 2)(2x + 3)$

Verify:

$(x + 2)(2x + 3)$

$= 2x^2 + 3x + 4x + 6$

$= 2x^2 + 7x + 6$

(Module 6, Lesson 3)

Part H: Coordinate Geometry (*20 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. The centre of a circle is at (52, 34). If an endpoint of its diameter is at (61, 46), find the coordinates of the other endpoint. *(3 marks)*

   Answer:

   $$52 = \frac{61 + x_2}{2} \qquad 34 = \frac{46 + y_2}{2}$$

   $$104 - 61 = x_2 \qquad 68 - 46 = y_2$$

   $$x_2 = 43 \qquad y_2 = 22$$

   The other endpoint is at (43, 22).

   (Module 7, Lesson 1)

2. Express the linear equation $y - 5 = \frac{2}{7}(x - 21)$ in slope-intercept form. *(2 marks)*

   Answer:

   $$y - 5 = \frac{2}{7}(x - 21)$$

   $$y - 5 = \frac{2}{7}x - 6$$

   $$y = \frac{2}{7}x - 6 + 5$$

   $$y = \frac{2}{7}x - 1$$

   (Module 7, Lesson 2)

3. Explain a strategy for graphing a linear equation given in point-slope form. *(3 marks)*

Answer:

Answers may vary.

Point-slope form is $y - y_1 = m(x - x_1)$ where m is the slope or $\frac{\text{rise}}{\text{run}}$ and (x_1, y_1) is a point on the graph of the line. Locate the point (x_1, y_1). From that point, count up or down the number of units in the numerator of the slope or m, and to the right the number of units in the denominator (move 1 to the right if m is a whole number). Mark a point there, repeat the $\frac{\text{rise}}{\text{run}}$ to find the next point, and then connect the points with a straight line.

Note: To go from one point to the next using slope = $\frac{\text{rise}}{\text{run}}$, you have choices:

- if slope is positive, you can $\frac{\text{rise up}}{\text{run right}}$ or $\frac{\text{rise down}}{\text{run left}}$.
- if slope is negative, you can $\frac{\text{rise up}}{\text{run left}}$ or $\frac{\text{rise down}}{\text{run right}}$.

(Module 7, Lesson 3)

4. The graph of a linear relation goes through the points (9, –11) and (13, –2). Write the equation of the linear relation in point-slope form. *(3 marks)*

Answer:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2 + 11}{13 - 9}$$

$$m = \frac{9}{4}$$

$$y + 11 = \frac{9}{4}(x - 9)$$

(Module 7, Lesson 3)

5. The graph of a linear relation goes through the point (6, 4) and is parallel to the line $y = 5x + 10$. Write the equation of the linear relation in slope-intercept form. *(3 marks)*

Answer:

Steps may vary.

$y - 4 = 5(x - 6)$

$y = 5x - 26$

(Module 7, Lesson 3)

6. Determine if the triangle with vertices at A(–5, 3), B(–1, –8), and C(6, –1) is an isosceles triangle. *(6 marks)*

Answer:

Use the distance formula to show two sides are the same length.

$$d_{\mathrm{AB}} = \sqrt{(-8-3)^2 + (-1+5)^2}$$

$$d_{\mathrm{AB}} = \sqrt{121+16} = \sqrt{137}$$

$$d_{\mathrm{BC}} = \sqrt{(-1+8)^2 + (6+1)^2}$$

$$d_{\mathrm{BC}} = \sqrt{49+49} = \sqrt{98}$$

$$d_{\mathrm{AC}} = \sqrt{(-1-3)^2 + (6+5)^2}$$

$$d_{\mathrm{AC}} = \sqrt{16+121} = \sqrt{137}$$

$$\mathrm{AC} = \mathrm{AB} = \sqrt{137}$$

Triangle ABC is an isosceles triangle because two side lengths are congruent.

(Module 7, Lesson 1)

Part I: Systems (*4 marks*)

Show all calculations and formulas used for short and long answer questions. Use all decimal places in your calculations, and round the final answers to the correct number of decimal places. Include units when appropriate. Clearly state your final answer.

1. Suzy has a German Shepherd and a Toy Poodle. The difference in height between them is 15″. Twice the height of a poodle is still 6″ shorter than a German Shepherd. Write a system of linear equations to represent this situation. Do **not** solve the system. *(1 mark)*

   Answer:

   $G - P = 15$

   $2P = G - 6$

   (Module 8, Lesson 2)

2. Solve the system of linear equations using elimination by addition or subtraction. *(3 marks)*

   $3x + 2y = 4$

   $x - y = 3$

   Answer:

   $$\begin{array}{lcl} 3x+2y=4 & & 3x+2y=4 \\ 3(x-y=3) & \rightarrow & \underline{3x-3y=9} \\ & & \quad 5y=-5 \qquad \text{Subtract} \\ & & \quad\;\; y=-1 \end{array}$$

   $x - y = 3$

   $x - (-1) = 3$

   $x + 1 = 3$

   $x = 2$

   $(2, -1)$

   (Module 8, Lesson 2)

   Verify:

$3x + 2y$	4
3(2) + 2(–1)	4
6 – 2	4
4	4

$x - y$	3
2 – (–1)	3
2 + 1	3
3	3