Grade 12 Physics (40S)

A Course for
Independent Study

Manitoba

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2013, 2019

Manitoba Education and Training

Manitoba Education and Training Cataloguing in Publication Data

Grade 12 physics (40S) : a course for independent study

ISBN: 978-0-7711-5251-1

1. Physics—Programmed instruction.
2. Physics—Study and teaching (Secondary).
3. Physics—Study and teaching (Secondary)—Manitoba.
4. Correspondence schools and courses—Manitoba.
5. Distance education—Manitoba.
I. Manitoba. Manitoba Education and Training.
530

Manitoba Education and Training
Winnipeg, Manitoba, Canada

This resource was published in 2013 and updated in 2019.

Disponible en français.

Available in alternate formats upon request.

Contents

Acknowledgements

Manitoba Education and Training gratefully acknowledges the contributions of the following individuals in the development of *Grade 12 Physics (40S): A Course for Independent Study*.

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Introduction

INTRODUCTION

Overview

Welcome to Grade 12 Physics: A Course for Independent Study.

Grade 12 Physics continues the study of basic concepts that will form the foundation for you to study physics in the future. You have already studied many concepts in physics. For example, in Grade 9 Science you studied electricity. In Grade 10 Science you studied the motion of vehicles and passengers, specifically how and why they move. In Grade 11 Physics you studied kinematics and dynamics in more detail, plus you investigated waves, sound, and light.

As a student enrolled in an independent study course, you have taken on a dual role—that of a student and a teacher. As a student, you are responsible for mastering the lessons and completing the learning activities and assignments. As a teacher, you are responsible for checking your work carefully, noting areas in which you need to improve and motivating yourself to succeed.

What Will You Learn in This Course?

In this course, you will be referring to concepts that you have studied in the past as necessary prior knowledge for the new concepts you will learn in this course. Grade 12 Physics will expand upon, and more fully complete, the investigation of some ideas already considered, as well as introduce many new topics. Many topics studied later in the course require that you understand and are able to apply knowledge from earlier in the course.

How Is This Course Organized?

The Grade 12 Physics course consists of four major topics. The four major topics are broken down into modules for a total of ten modules. The breakdown of topics and modules is shown below:

Topic 1: Mechanics

- Module 1: Kinematics
- Module 2: Dynamics
- Module 3: Projectiles and Circular Motion
- Module 4: Work and Energy
- Module 5: Momentum

Topic 2: Fields

- Module 6: Exploration of Space and Low Earth Orbit
- Module 7: Electric and Magnetic Fields

Topic 3: Electricity

- Module 8: Electric Circuits
- Module 9: Electromagnetic Induction

Topic 4: Medical Physics

- Module 10: Medical Physics

Each module in this course consists of several lessons, which contain the following components:

- **Lesson Focus:** The Lesson Focus at the beginning of each lesson identifies one or more specific learning outcomes (SLOs) that are addressed in the lesson. The SLOs identify the knowledge and skills you should have achieved by the end of the lesson. (For a complete list of the SLOs identified for Grade 12 Physics refer to Appendix G at the end of this course.)
- **Key Words:** This list identifies the important words that are used in the lesson. The key words are highlighted in bold within the text and identified by key word icons. They are defined in the Glossary at the end of the course in Appendix F.
- **Introduction:** The introduction sets the stage for the lesson. It may draw upon prior knowledge or briefly describe the organization of the lesson.
- **Lesson:** The main body of the lesson is made up of the content that you need to learn. It contains explanations, diagrams, and fully completed examples.
- **Learning Activities:** Most lessons include one or more learning activities that will help you learn about the lesson topics and prepare you for the assignments, the midterm examination, and the final examination. Once you complete a learning activity, check your responses against those provided in the Learning Activity Answer Key found at the end of the module. Do not send your learning activities to the Distance Learning Unit for assessment.
- **Assignments:** Assignments are found throughout each module within this course. At the end of each module, you will mail or electronically submit all your completed assignments from that module to the Distance Learning Unit for assessment. All assignments combined will be worth a total of 50 percent of your final mark in this course.
- **Video Files:** Some lessons refer to the *Grade 12 Physics (40S) Laboratory Activities* video files, which you are required to view for this course.
- **Summary:** Each lesson ends with a brief review of what you just learned.

This course also includes the following section:

- **Appendices:** At the end of the course, you will find seven appendices, which contain information that is required for the course or that could be useful in helping you to complete the course. Here is a list of the appendices found a the end of the course:
  - **Appendix A: Equation Sheet** contains a list of equations grouped according to the topics addressed in this course. This list of equations will be provided with the exams that you will be writing. It is imperative that you learn which concept each equation refers to, the quantity associated with each symbol, and its unit of measurement.
  - **Appendix B: Rounding Off** provides the rules for rounding off numbers correctly.
  - **Appendix C: Significant Digits** provides the rules and examples for determining where to round off your answer based on the information provided in the question and the type of calculation that is performed.
  - **Appendix D: Space Exploration** contains information about the planets and the sun, which is required to solve problems in Module 6.
  - **Appendix E: Grade 12 Physics Websites** contains a list of websites for each of the modules. These websites may contain small programs called applets, or they may contain animations, either of which will illustrate a specific concept discussed in the lessons of that module. Check this appendix when you reach the end of a lesson to see whether there was an applet or animation on the web that would help you to understand the concepts for that lesson.
  - **Appendix F: Glossary** defines terms used within this course.
  - **Appendix G: List of Specific Learning Outcomes** contains the complete list of SLOs to be achieved in Grade 12 Physics. You will not be using this appendix. It has been placed here to help classroom teachers who are using this course as a resource.

Use of Significant Digits

When submitting any assignments or when writing the exams, you are expected to round off your answers to the correct number of significant digits. In all the work you will do, the answers should be rounded off.

In all learning activities and assignments, an effort has been made to provide values for quantities using three significant digits. When in doubt, round off your answer to three significant digits.

Refer to Appendix B: Rounding Off and Appendix C: Significant Digits. Please read these over and practise using these rules. With the diligent application of the rules for significant digits, you should use significant digits as automatically as you use punctuation when you write a sentence.

What Resources Will You Need for This Course?

You do not need a textbook for this course. All the content is provided directly within the course. You will, however, need access to a variety of resources.

The required resources for this course are identified below.

Required Resources

For this course, you will need access to the following resources. If you do not have access to one or more of these resources, contact your tutor/marker.

- **A calculator:** Use a graphing or scientific calculator as you work through this course. You will also need the calculator for the examination(s).
- **Equipment to view video files:** You will need equipment to view the *Grade 12 Physics (40S) Laboratory Activities* video files.

Electronic Resources

For this course, you will need the following electronic resource(s). If you do not have access to the Internet, or if you need a copy of the resource(s), contact the Distance Learning Unit at 1-800-465-9915.

- **Video(s):** You will have an opportunity to view the *Grade 12 Physics (40S) Laboratory Activities* video files, which are available in learning management system (LMS). If you do not have access to the Internet or if you need a copy of the video, contact the Distance Learning Unit at 1-800-465-9915.

Optional Resources

It would be helpful if you had access to the following resources:

- **A photocopier/scanner:** With access to a photocopier/scanner, you could make a copy of your assignments before submitting them so that if your tutor/marker wants to discuss an assignment with you over the phone, each of you will have a copy. It would also allow you to continue studying or to complete further lessons while your original work is with the tutor/marker. Photocopying or scanning your assignments will also ensure that you keep a copy in case the originals are lost.
- **Resource people:** Access to local resource people, such as teachers, school counsellors, and librarians, would help you complete the course.

- **A computer with spreadsheet software:** Access to spreadsheet software (e.g., Microsoft Excel) would help you to present and analyze data graphically.
- **A computer with Graphical Analysis software:** This software will help you to produce and analyze graphs. If you do not have access to this software and you are attending school, ask your ISO school facilitator how you can obtain access. If you are not attending school, you will have received the software with the course.

- **A computer with Internet access:** Some lessons suggest website links as sources of information or for supplementary reference and reading. If you do not have Internet access, you will still be able to complete the course, but you will need to find different ways of accessing information.

Who Can Help You with This Course?

Taking an independent study course is different from taking a course in a classroom. Instead of relying on the teacher to tell you to complete a learning activity or an assignment, you must tell yourself to be responsible for your learning and for meeting deadlines. There are, however, two people who can help you be successful in this course: your tutor/marker and your learning partner.

Your Tutor/Marker

Tutor/markers are experienced educators who tutor Independent Study Option (ISO) students and mark assignments and examinations. When you are having difficulty with something in this course, contact your tutor/marker, who is there to help you. Your tutor/marker's name and contact information were sent to you with this course. You can also obtain this information in the learning management system (LMS).

Your Learning Partner

A learning partner is someone **you choose** who will help you learn. It may be someone who knows something about physics, but it doesn't have to be. A learning partner could be someone else who is taking this course, a teacher, a parent or guardian, a sibling, a friend, or anybody else who can help you. Most importantly, a learning partner should be someone with whom you feel comfortable and who will support you as you work through this course.

Your learning partner can help you keep on schedule with your coursework, read the course with you, check your work, look at and respond to your learning activities, or help you make sense of assignments. You may even study for your examination(s) with your learning partner. If you and your learning partner are taking the same course, however, your assignment work should not be identical.

How Will You Know How Well You Are Learning?

You will know how well you are learning in this course by how well you complete the learning activities, assignments, and examinations.

Learning Activities

The learning activities in this course will help you to review and practise what you have learned in the lessons. You will not submit the completed learning activities to the Distance Learning Unit. Instead, you will complete the learning activities and compare your responses to those provided in the Learning Activity Answer Key found at the end of each module.

Make sure you complete the learning activities. Doing so will not only help you to practise what you have learned, but will also prepare you to complete your assignments and the examinations successfully. Many of the questions on the examinations will be similar to the questions in the learning activities. **Remember that you will not submit learning activities to the Distance Learning Unit.**

Assignments

Lesson assignments are located throughout the modules and include questions similar to the questions in the learning activities of previous lessons. The assignments have space provided for you to write your answers on the question sheets. You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate).

Once you have completed all the assignments in a module, you will submit them to the Distance Learning Unit for assessment. The assignments are worth a total of 50 percent of your final course mark. You must complete each assignment in order to receive a final mark in this course. You will mail or electronically submit these assignments to the Distance Learning Unit along with the appropriate cover page once you complete each module.

For some assignments, you will need to view the video *Grade 12 Physics (40S) Laboratory Activities*, gather data from the video, and write a laboratory report.

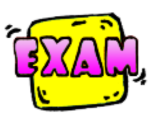
The video contains the following laboratory activities:

- *Forces in Equilibrium*
- *Circular Motion*
- *Hooke's Law*
- *Collisions in Two Dimensions*
- *Coulomb's Law*
- *Ohm's Law*
- *Half-Life Simulation*

You will submit the laboratory reports, along with the other assignments, to the Distance Learning Unit for assessment.

The tutor/marker will mark your assignments and return them to you. Remember to keep all marked assignments until you have finished the course so that you can use them to study for your examinations.

Midterm and Final Examinations



The course contains a midterm examination and a final examination.

- The **midterm examination** is based on Modules 1 to 5 and is worth 20 percent of the final mark for the course. You will write the midterm examination when you have completed Module 5.
- The **final examination** is based on Modules 1 to 10, and is worth 30 percent of your final mark in this course. You will write the final examination when you have completed Module 10.

The two examinations are worth a total of **50 percent** of your final course mark. You will write both examinations under supervision.

In order to do well on the examinations, you should review all of the work that you have completed from Modules 1 to 5 for your midterm examination and Modules 1 to 10 for your final examination, including all learning activities and assignments. Please note that most of the final examination (25% of your final mark) is based on Modules 6 to 10. A small portion (5% of your final mark) is based on Modules 1 to 5.

Practice Examinations and Answer Keys

To help you succeed in your examinations, you will have an opportunity to complete a Midterm Practice Examination and a Final Practice Examination. These examinations, along with the answer keys, are found in the learning management system (LMS). If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to obtain a copy of the practice examinations.

These practice examinations are similar to the actual examinations you will be writing. The answer keys enable you to check your answers. This will give you the confidence you need to do well on your examinations.

Requesting Your Examinations

You are responsible for making arrangements to have the examinations sent to your proctor from the Distance Learning Unit. Please make arrangements before you finish Module 5 to write the midterm examination. Likewise, you should begin arranging for your final examination before you finish Module 10.

To write your examinations, you need to make the following arrangements:

- **If you are attending school,** your examination will be sent to your school as soon as all the applicable assignments have been submitted. You should make arrangements with your school's ISO school facilitator to determine a date, time, and location to write the examination.
- **If you are not attending school,** check the Examination Request Form for options available to you. Examination Request Forms can be found on the Distance Learning Unit's website, or look for information in the learning management system (LMS). Two weeks before you are ready to write the examination, fill in the Examination Request Form and mail, fax, or email it to

  Distance Learning Unit
  500–555 Main Street
  PO Box 2020
  Winkler MB R6W 4B8
  Fax: 204-325-1719
  Toll-Free Telephone: 1-800-465-9915
  Email: distance.learning@gov.mb.ca

How Much Time Will You Need to Complete This Course?

Learning through independent study has several advantages over learning in the classroom. You are in charge of how you learn and you can choose how quickly you will complete the course. You can read as many lessons as you wish in a single session. You do not have to wait for your teacher or classmates.

From the date of your registration, you have a maximum of **12 months** to complete the course, but the pace at which you proceed is up to you. Read the following suggestions on how to pace yourself.

Chart A: Semester 1

If you want to start this course in September and complete it in January, you can follow the timeline suggested below.

Module	Completion Date
Modules 1 and 2	End of September
Modules 3 and 4	Middle of October
Module 5	Beginning of November
Midterm Examination	Middle of November
Modules 6 and 7	Beginning of December
Modules 8 and 9	End of December
Module 10	Middle of January
Final Examination	End of January

Chart B: Semester 2

If you want to start the course in February and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Modules 1 and 2	Middle of February
Modules 3 and 4	End of February
Module 5	Beginning of March
Midterm Examination	Middle of March
Modules 6 and 7	Beginning of April
Modules 8 and 9	Middle of April
Module 10	Beginning of May
Final Examination	Middle of May

Chart C: Full School Year (Not Semestered)

If you want to start the course in September and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Modules 1 and 2	Middle of October
Modules 3 and 4	Middle of November
Module 5	Middle of December
Midterm Examination	Beginning of January
Modules 6 and 7	End of February
Modules 8 and 9	Middle of April
Module 10	Beginning of May
Final Examination	Middle of May

Timelines

Do not wait until the last minute to complete your work, since your tutor/marker may not be available to mark it immediately. It may take a few weeks for your tutor/marker to assess your work and return it to you or your school.

If you need this course to graduate this school year, all coursework must be received by the Distance Learning Unit on or before the first Friday in May, and all examinations must be received by the Distance Learning Unit on or before the last Friday in May. Any coursework or examinations received after these deadlines may not be processed in time for a June graduation. Assignments or examinations submitted after these recommended deadlines will be processed and marked as they are received.

Why Is the Course so Large?

This course package includes all the things you would find in a regular face-to-face physics course taught in a classroom, such as the following:

- all the information that is normally found in a textbook
- all the notes that a teacher would hand out in class
- all the assignments that a teacher would hand out in class
- all the learning activities that a teacher would hand out in class, along with their answer keys
- all the explanations and instructions that a teacher would either say or write on a blackboard or whiteboard

This course also contains many diagrams and graphs, which tend to take up more room than straight text. This makes the course seem larger than it actually is.

When and How Will You Submit Completed Assignments?

When to Submit Assignments

While working on this course, you will submit completed assignments to the Distance Learning Unit six times. The following chart shows you exactly what assignment you will be submitting at the end of each module.

Submission of Assignments	
Submission	Assignments You Will Submit
1	**Module 1: Kinematics / Module 2: Dynamics** Module 1/Module 2 Cover Sheet Assignment 1.1: Equations of Motion Assignment 1.2: Relative Motion Assignment 2.1: Forces of Friction and Motion Assignment 2.2: Adding Vectors Using the Component Method Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium Assignment 2.4: Objects in Equilibrium Assignment 2.5: Forces Acting at an Angle
2	**Module 3: Projectiles and Circular Motion / Module 4: Work and Energy** Module 3/Module 4 Cover Sheet Assignment 3.1: Vertical Motion of a Bullet Assignment 3.2: Vector Nature of Projectile Motion Assignment 3.3: Projectile Motion of a Cannonball Assignment 3.4: Video Laboratory Activity: Circular Motion Assignment 3.5: Circular Motion of the Moon Assignment 3.6: Uniform Circular Motion of a Satellite Assignment 4.1: Calculating Work Assignment 4.2: Work and Kinetic Energy Assignment 4.3: Conservation of Mechanical Energy in a Roller Coaster Assignment 4.4: Video Laboratory Activity: Hooke's Law Assignment 4.5: Spring Potential Energy
3	**Module 5: Momentum** Module 5 Cover Sheet Assignment 5.1: Calculating Impulse, Momentum, and Force Assignment 5.2: Conservation of Linear Momentum Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions
4	**Module 6: Exploration of Space and Low Earth Orbit /** **Module 7: Electric and Magnetic Fields** Module 6/Module 7 Cover Sheet Assignment 6.1: Making a Decision Assignment 6.2: Working with Kepler's Laws Assignment 6.3: Newton's Universal Law of Gravity Assignment 6.4: The Different Energies of a Satellite of Earth Assignment 6.5: A Satellite of Jupiter Assignment 6.6: Space Travel: Problems and Some Possible Solutions Assignment 7.1: Coulomb's Law Assignment 7.2: Video Laboratory Activity: Coulomb's Law Assignment 7.3: Coulomb's Law in Two Dimensions Assignment 7.4: Electric Field between the Plates of a Parallel Plate Capacitor Assignment 7.5: The Charged Parallel Plate Capacitor Assignment 7.6: Energy Relationships in the Parallel Plate Capacitor Assignment 7.7: The Motion of Charges Moving through a Parallel Plate Capacitor Assignment 7.8: Moving Charges and Magnetic Forces Assignment 7.9: Mass Spectrometer

Submission of Assignments (continued)	
Submission	Assignments You Will Submit
5	**Module 8: Electric Circuits / Module 9: Electromagnetic Induction** Module 8/Module 9 Cover Sheet Assignment 8.1: Circuits: Current, Charge, and EMF Assignment 8.2: Electric Circuits Assignment 8.3: Developing a Good Scientific Theory Assignment 8.4: Video Laboratory Activity: Ohm's Law Assignment 8.5: Ohm's Law Assignment 8.6: Analyzing a Parallel Circuit Assignment 8.7: Analyzing a Network Circuit Assignment 9.1: Inducing EMF in a Moving Rod Assignment 9.2: Magnetic Flux Assignment 9.3: Applying Faraday's Law and Lenz's Law Assignment 9.4: A Bicycle Generator Assignment 9.5: Transformers and Electricity Transmission
6	**Module 10: Medical Physics** Module 10 Cover Sheet Assignment 10.1: Describing the Nuclear Atom Assignment 10.2: Mass Defect and Binding Energy of Boron Assignment 10.3: Alpha, Beta, and Gamma Radiation Assignment 10.4: Video Laboratory Activity: Half-Life Simulation Assignment 10.5: Rate of Radioactive Decay and Medical Treatment Assignment 10.6: Positive Effects of Ionizing and Non-ionizing Radiation Assignment 10.7: Medical Use of Ultrasound and Electromagnetic Radiation Assignment 10.8: Medical Diagnosis and Treatment Using Nuclear Medicine

How to Submit Assignments

In this course, you have the choice of submitting your assignments either by mail or electronically.

- **Mail:** Each time you **mail** something, you must include the print version of the applicable Cover Sheet (found at the end of this Introduction). Complete the information at the top of each Cover Sheet before submitting it along with your assignments.
- **Electronic submission:** You do not need to include a cover sheet when submitting assignments electronically.

Submitting Your Assignments by Mail

If you choose to mail your completed assignments, please photocopy/scan all the materials first so that you will have a copy of your work in case your package goes missing. You will need to place the applicable module Cover Sheet and assignment(s) in an envelope, and address it to

> Distance Learning Unit
> 500–555 Main Street
> PO Box 2020
> Winkler MB R6W 4B8

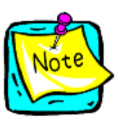
Your tutor/marker will mark your work and return it to you by mail.

Submitting Your Assignments Electronically

Assignment submission options vary by course. Sometimes assignments can be submitted electronically and sometimes they must be submitted by mail. Specific instructions on how to submit assignments were sent to you with this course. In addition, this information is available in the learning management system (LMS).

If you are submitting assignments electronically, make sure you have saved copies of them before you send them. That way, you can refer to your assignments when you discuss them with your tutor/marker. Also, if the original hand-in assignments are lost, you are able to resubmit them.

Your tutor/marker will mark your work and return it to you electronically.



The Distance Learning Unit does not provide technical support for hardware-related issues. If troubleshooting is required, consult a professional computer technician.

What Are the Guide Graphics For?

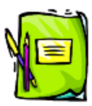
Guide graphics appear in the margins of the course to identify specific tasks. Each graphic has a specific purpose, as described below:

Lesson Focus/Specific Learning Outcomes SLOs): Note that these SLOs will be addressed within the lesson. (A complete list of the Grade 12 Physics SLOs can be found in the Appendix at the end of this course.)

Internet: Use the Internet, if you have access to it, to obtain more information. Internet access is optional for this course.

Learning Partner: Ask your learning partner to help you with this task.

Phone Your Tutor/Marker: Telephone your tutor/marker.

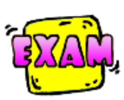


Learning Activity: Complete a learning activity. This will help you to review or practise what you have learned and to prepare for an assignment or an examination. You will not submit learning activities to the Distance Learning Unit. Instead, you will compare your responses to those provided in the Learning Activity Answer Key found at the end of the applicable module.

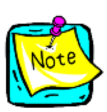
Assignment: Complete an assignment. You will submit your completed assignments to the Distance Learning Unit for assessment at the specified times.

Video: View a video.

Mail or Electronic Submission: Mail or electronically submit your completed assignments to your tutor/marker for assessment.



Examination: Write your midterm or final examination at this time.



Note: Take note of and remember this important information or reminder.

Remember: If you have questions or need help at any point during this course, contact your tutor/marker or ask your learning partner for help.

Good luck with this course!

NOTES

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Modules 1 and 2 Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________________ Preferred Name: __________________________

Phone: ________________________________ Email: ___________________________________

Mailing Address: ____

City/Town: ____ Postal Code: _______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 1 and 2 Assignments	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 1.1: Equations of Motion	_______ /5	_______ /5
❑ Assignment 1.2: Relative Motion	_______ /7	_______ /7
❑ Assignment 2.1: Forces of Friction and Motion	_______ /8	_______ /8
❑ Assignment 2.2: Adding Vectors Using the Component Method	_______ /8	_______ /8
❑ Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium	_______ /20	_______ /20

continued

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Modules 1 and 2 Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address
Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address
Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________ Preferred Name: ____________________

Phone: ____________________ Email: ____________________

Mailing Address: ____

City/Town: ____________________ Postal Code: ____________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 1 and 2 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 2.4: Objects in Equilibrium	______ /6	______ /6
❑ Assignment 2.5: Forces Acting at an Angle	______ /8	______ /8
	Total: ____ /62	**Total: ____ /62**

For Tutor/Marker Use

Remarks:

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Modules 3 and 4 Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address
Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address
Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: ____________________________ Preferred Name: __________________________

Phone: ________________________________ Email: ___________________________________

Mailing Address: ____

City/Town: ____ Postal Code: _______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 3 and 4 Assignments	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	__________ Date Received	__________ Date Received
❑ Assignment 3.1: Vertical Motion of a Bullet	_______ /9	_______ /9
❑ Assignment 3.2: Vector Nature of Projectile Motion	_______ /5	_______ /5
❑ Assignment 3.3: Projectile Motion of a Cannonball	_______ /10	_______ /10
❑ Assignment 3.4: Video Laboratory Activity: Circular Motion	_______ /20	_______ /20
❑ Assignment 3.5: Circular Motion of the Moon	_______ /5	_______ /5
❑ Assignment 3.6: Uniform Circular Motion of a Satellite	_______ /6	_______ /6
❑ Assignment 4.1: Calculating Work	_______ /5	_______ /5
❑ Assignment 4.2: Work and Kinetic Energy	_______ /5	_______ /5

continued

Grade 12 Physics (40S)

Modules 3 and 4 Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 3 and 4 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 4.3: Conservation of Mechanical Energy in a Roller Coaster	_____ /10	_____ /10
❑ Assignment 4.4: Video Laboratory Activity: Hooke's Law	_____ /20	_____ /20
❑ Assignment 4.5: Spring Potential Energy	_____ /8	_____ /8
	Total: ___ /103	**Total: ___ /103**

For Tutor/Marker Use
Remarks:

Grade 12 Physics (40S)

Module 5 Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 5 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** __________ Date Received	**Attempt 2** __________ Date Received
❑ Assignment 5.1: Calculating Impulse, Momentum, and Force	_____ /5	_____ /5
❑ Assignment 5.2: Conservation of Linear Momentum	_____ /5	_____ /5
❑ Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions	_____ /20	_____ /20
	Total: ____ **/30**	**Total:** ____ **/30**

For Tutor/Marker Use
Remarks:

Grade 12 Physics (40S)

Modules 6 and 7 Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: ____

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Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 6 and 7 Assignments	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_______________ Date Received	_______________ Date Received
❑ Assignment 6.1: Making a Decision	________ /15	________ /15
❑ Assignment 6.2: Working with Kepler's Laws	________ /7	________ /7
❑ Assignment 6.3: Newton's Universal Law of Gravity	________ /8	________ /8
❑ Assignment 6.4: The Different Energies of a Satellite of Earth	________ /5	________ /5
❑ Assignment 6.5: A Satellite of Jupiter	________ /10	________ /10
❑ Assignment 6.6: Space Travel: Problems and Some Possible Solutions	________ /6	________ /6
❑ Assignment 7.1: Coulomb's Law	________ /5	________ /5
❑ Assignment 7.2: Video Laboratory Activity: Coulomb's Law	________ /20	________ /20
❑ Assignment 7.3: Coulomb's Law in Two Dimensions	________ /6	________ /6
❑ Assignment 7.4: Electric Field between the Plates of a Parallel Plate Capacitor	________ /5	________ /5
❑ Assignment 7.5: The Charged Parallel Plate Capacitor	________ /6	________ /6

continued

Grade 12 Physics (40S)

Modules 6 and 7 Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 6 and 7 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 7.6: Energy Relationships in the Parallel Plate Capacitor	______ /7	______ /7
❑ Assignment 7.7: The Motion of Charges Moving through a Parallel Plate Capacitor	______ /7	______ /7
❑ Assignment 7.8: Moving Charges and Magnetic Forces	______ /7	______ /7
❑ Assignment 7.9: Mass Spectrometer	______ /4	______ /4
	Total: ____ /118	**Total: ____ /118**

For Tutor/Marker Use
Remarks:

Grade 12 Physics (40S)

Modules 8 and 9 Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: ____

City/Town: ___________________________________ Postal Code: _______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 8 and 9 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** __________ Date Received	**Attempt 2** __________ Date Received
❑ Assignment 8.1: Circuits: Current, Charge, and EMF	_______ /5	_______ /5
❑ Assignment 8.2: Electric Circuits	_______ /5	_______ /5
❑ Assignment 8.3: Developing a Good Scientific Theory	_______ /7	_______ /7
❑ Assignment 8.4: Video Laboratory Activity: Ohm's Law	_______ /20	_______ /20
❑ Assignment 8.5: Ohm's Law	_______ /5	_______ /5
❑ Assignment 8.6: Analyzing a Parallel Circuit	_______ /5	_______ /5
❑ Assignment 8.7: Analyzing a Network Circuit	_______ /5	_______ /5
❑ Assignment 9.1: Inducing EMF in a Moving Rod	_______ /5	_______ /5
❑ Assignment 9.2: Magnetic Flux	_______ /5	_______ /5

continued

Grade 12 Physics (40S)

Modules 8 and 9 Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: ____

City/Town: ______________________________ Postal Code: __________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Modules 8 and 9 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	______ Date Received	______ Date Received
❑ Assignment 9.3: Applying Faraday's Law and Lenz's Law	______ /11	______ /11
❑ Assignment 9.4: A Bicycle Generator	______ /5	______ /5
❑ Assignment 9.5: Transformers and Electricity Transmission	______ /6	______ /6
	Total: ____ **/84**	**Total:** ____ **/84**

For Tutor/Marker Use
Remarks:

Grade 12 Physics (40S)

Module 10 Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: ____

City/Town: ____________________________ Postal Code: ______________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 10 Assignments	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	__________ Date Received	__________ Date Received
❑ Assignment 10.1: Describing the Nuclear Atom	_______ /4	_______ /4
❑ Assignment 10.2: Mass Defect and Binding Energy of Boron	_______ /5	_______ /5
❑ Assignment 10.3: Alpha, Beta, and Gamma Radiation	_______ /7	_______ /7
❑ Assignment 10.4: Video Laboratory Activity: Half-Life Simulation	_______ /20	_______ /20
❑ Assignment 10.5: Rate of Radioactive Decay and Medical Treatment	_______ /5	_______ /5
❑ Assignment 10.6: Positive Effects of Ionizing and Non-ionizing Radiation	_______ /5	_______ /5

continued

Grade 12 Physics (40S)

Module 10 Cover Sheet *(continued)*

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

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Mailing Address: ____

City/Town: ______________________________ Postal Code: __________

Attending School: ❑ No ❑ Yes

School Name: ____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 10 Assignments *(continued)*	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	________ Date Received	________ Date Received
❑ Assignment 10.7: Medical Use of Ultrasound and Electromagnetic Radiation	_____ /5	_____ /5
❑ Assignment 10.8: Medical Diagnosis and Treatment Using Nuclear Medicine	_____ /5	_____ /5
	Total: ___ **/56**	**Total:** ___ **/56**

For Tutor/Marker Use
Remarks:

Released 2019

Printed in Canada
Imprimé au Canada

Grade 12 Physics (40S)

Topic 1: Mechanics

Topic 1: Mechanics

Introduction to Topic 1

This first topic of the course deals with mechanics, and is made up of five modules. It is a continuation of what you learned in the Grade 11 Physics course.

The first three modules are based largely on Isaac Newton's ideas of mechanics. Mechanics deals with the relationships among matter, force, and energy, especially as they affect the motion of objects. Module 1 continues the study of kinematics or the study of motion. Module 2 deals with dynamics or why objects move as they do. Module 3 looks at motion and forces in two dimensions.

In Module 4, the world of Newtonian physics is left behind. Module 4 extends the study of forces and examines mechanics in a new way through the introduction of the principles of work and energy. Module 5 deals with momentum and the conservation of momentum. These concepts were introduced in Grade 10 Science with the study of impulse and momentum and how they affect events during an automobile collision. Module 5 applies the ideas of impulse and momentum to many different situations, such as motion from the propulsion of rocket engines and falling rain.

NOTES

GRADE 12 PHYSICS (40S)

Module 1: Kinematics

This module contains the following:

- Introduction to Module 1
- Lesson 1: Laboratory Activity: Analysis of an Experiment
- Lesson 2: Kinematics: Bringing You Up To Date
- Lesson 3: Deriving Equations for Motion Involving Uniformly Accelerated Motion
- Lesson 4: Using Kinematics Equations for Constant Acceleration to Solve Problems
- Lesson 5: A Review of Working with Vectors
- Lesson 6: Relative Motion
- Module 1 Summary

Module 1: Kinematics

Introduction to Module 1

Welcome to the first module of Grade 12 Physics.

This first module introduces this Independent Study Option (ISO) course and provides some of the information you will require to complete it successfully. The study of kinematics will tap into your prior knowledge by linking concepts you have already studied to some new ideas and applications. Kinematics refers to the study of motion. You will be building on the knowledge of motion you acquired from previous science courses.

There are six lessons in this module dealing with kinematics.

Lesson 1: Laboratory Activity: Analysis of an Experiment is a “dry lab” in which you will analyze data from a given experiment. You can then use the data analysis procedure to help you predict the outcome of similar experiments. The important concept involves an explanation of how to determine relationships between variables from given data. This lesson also illustrates how to make and analyze data graphically, using a spreadsheet such as Excel.

Lesson 2: Kinematics: Bringing You Up To Date is a review of the concepts of position, displacement, velocity, and acceleration, as well as the four different ways of representing motion (physical/conceptual, numerical, graphical, and symbolic).

Lesson 3: Deriving Equations for Motion Involving Uniformly Accelerated Motion involves the derivation of the special equations for constant acceleration. Basically, you will relate the graphical representation of motion to some new symbolic relationships.

Lesson 4: Using Kinematics Equations for Constant Acceleration to Solve Problems introduces a method of solving problems called the “GUESS” method. You will employ this method to solve problems for accelerated motion along a straight line.

Lesson 5: A Review of Working with Vectors is a review of vectors. Here, you work with parallel, antiparallel, and perpendicular vectors.

Lesson 6: Relative Motion deals with relative motion, an application of vectors to the motion of objects in a frame of reference that is also moving.

Assignments in Module 1

When you complete Module 2, you will submit your Module 1 assignments, along with your Module 2 assignments, to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
4	Assignment 1.1	Equations of Motion
6	Assignment 1.2	Relative Motion


As you work through this course, remember that your learning partner and your tutor/marker are available to help you if you have questions or need assistance with any aspect of the course.

LESSON 1: LABORATORY ACTIVITY: ANALYSIS OF AN EXPERIMENT (1 HOUR)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ analyze data to determine the mathematical relationship between two variables
- ❑ graph data using a spreadsheet program such as Excel or other graphing software
- ❑ straighten the line of a relation that is a curve by manipulating the data
- ❑ write a report for an experiment

Note to Student

In this lesson, you will study a sample laboratory experiment that describes a procedure for analyzing experimental data, which you will then use in later video laboratory activities found in the learning management system (LMS). You will not write a lab report for this sample experiment.

Introduction

The presentation and analysis of experimental results is an essential part of physics laboratory exercises. In this exercise, data from a sample experiment is presented and analyzed in a form that you will be able to use to predict the outcome of similar experiments. The lesson illustrates how to make and analyze data graphically, using a spreadsheet program such as Excel.

Sample Experiment

The sample experiment is an investigation of the time it takes a specific amount of water to pour out of a can through a hole in the bottom of the can. Experience tells us that the amount of time it takes for the water to drain depends on the size of the hole in the can. In the experiment, four identical containers of water are emptied through openings of different diameters. The experiment is repeated several times and the averages of the times (in seconds) that each container takes to empty the water is shown in the Data Table.

Purpose

To find the relationship between the time it takes for a specific amount of water to drain from a container and the diameter of the opening in the container.

Apparatus

Can, water, stopwatch, spreadsheet program such as Excel

Procedure

1. Graph time (s) versus diameter (cm) for the data provided in the Data Table. You can plot the graph yourself on graph paper or enter the data into an Excel spreadsheet and use Excel to plot the graph.
2. "Adjust" the data on the *x*-axis to reflect the inverse nature of the first graph. Graph time (s) versus 1/diameter (cm^{-1}).
3. "Adjust" the data on the *x*-axis to reflect the power nature of the second graph. Graph time (s) versus 1/diameter^2 (cm^{-2}).
4. State the proportionality and find the constant of proportionality.

Data and Calculations

Data Table

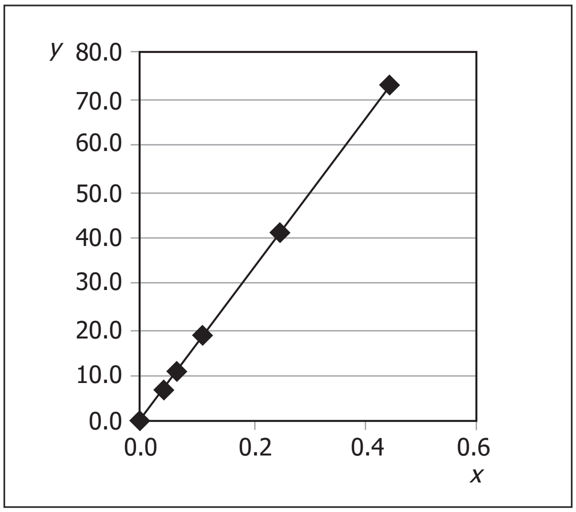
Diameter (cm)	Time (s)
1.5	73.0
2.0	41.2
3.0	18.4
5.0	6.8

Discussion

Generally, in high school physics we investigate only linear, power, and inverse relationships.

Linear Relationship

If the data you graph yields a straight line, then you have a **direct proportion**.



Whatever is on the y-axis is directly proportional to whatever is on the x-axis (that is, $y \alpha x$, where α means "is proportional to"). Whenever you have proportionality (a straight line on the graph), you can convert it to an equation by introducing a constant; that is,

$$y \alpha x$$

$$y = kx$$

where k is the constant of proportionality and can be found from the slope of the graph. In the graph above, the slope = $\dfrac{73.0-10.8}{0.44-0.06}=165.$ Thus, $y = 165x$.

You should recognize this type of relationship from your mathematics class. This resembles the classic linear relationship

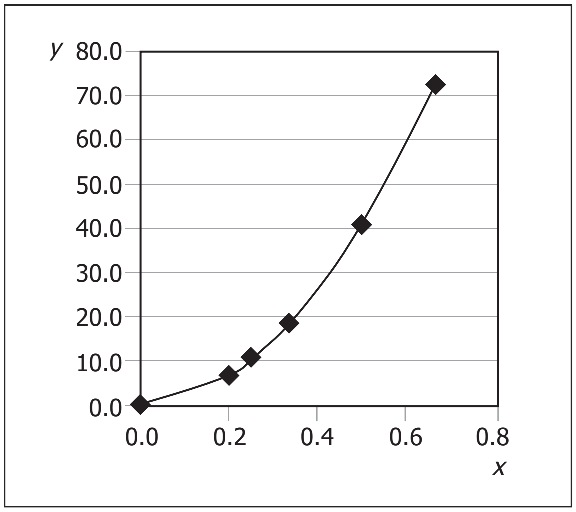
$$y = mx + b$$

where m = slope of the line and b = the y-intercept.

The goal is to make equations resemble this form for a linear relationship.

Power Relationship

If the data you graph yields a curve where the y-values are increasing at a faster rate than the x-values, then you have a **power curve**.

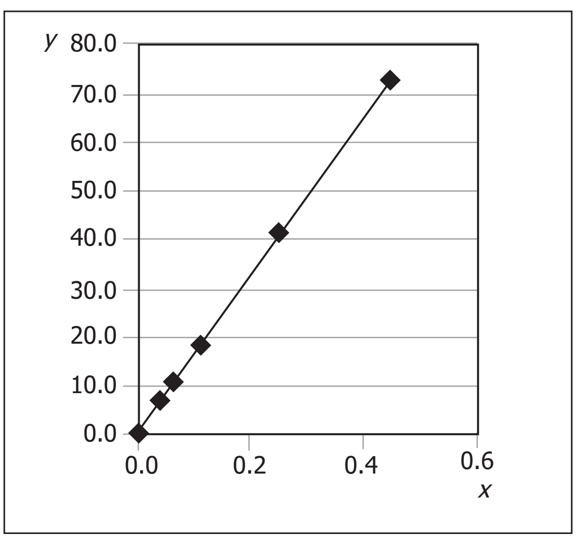


Try to "straighten" the data by graphing y versus x^2. Square the x-data and redraw your graph. If it is a straight line, then whatever is on the y-axis is directly proportional to whatever is on the x-axis. In this case,

$$y \,\alpha\, x^2$$

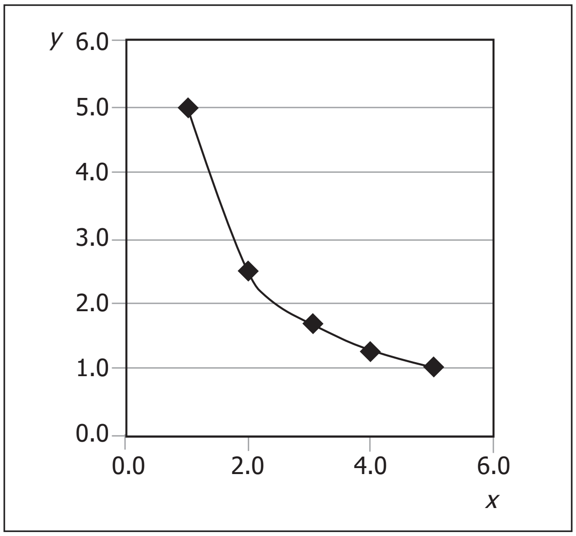
$$y = kx^2$$

where k is the constant of proportionality and can be found from the slope of the graph.



Inverse Relationship

If the data you graph yields a curve where the y-values are decreasing as the x-values are increasing, then you have an **inverse curve**.

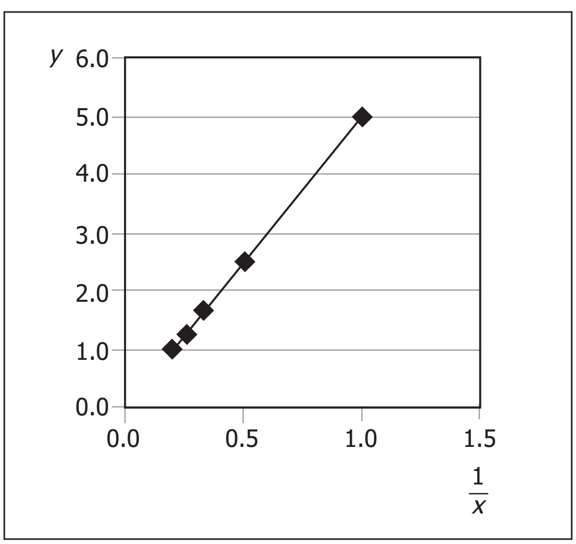


Try to "straighten" the data by graphing y versus $\frac{1}{x}$. Take the inverse $\left(\frac{1}{x}\right)$ of the x-data and redraw your graph. If it is a straight line, then whatever is on the y-axis is directly proportional to whatever is on the x-axis. In this case,

$$y \alpha \ x$$

$$y = k\frac{1}{x}$$

where k is the constant of proportionality and can be found from the slope of the graph.

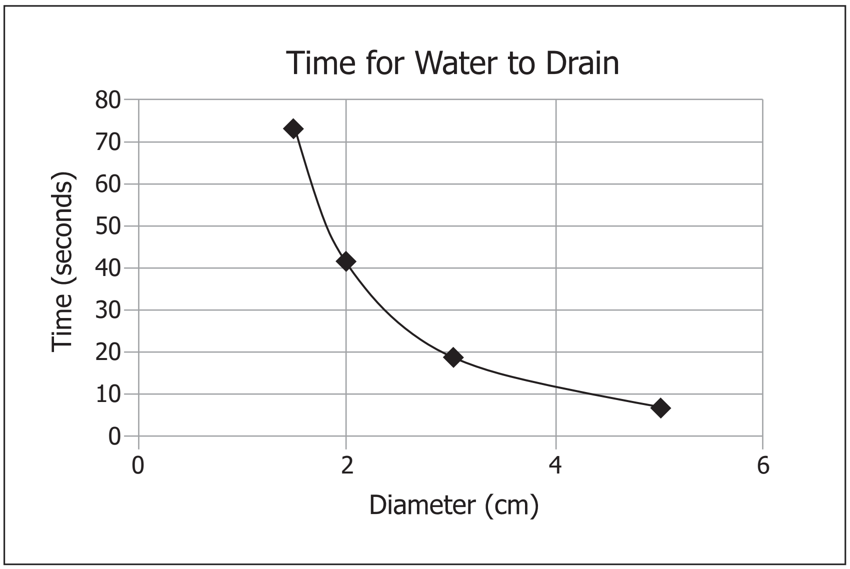


Analysis

Data Table

d (cm)	Time (s)
1.5	73.0
2.0	41.2
3.0	18.4
5.0	6.8

Begin to analyze these data by plotting a graph with diameter, *d* (cm), the independent variable, on the *x*-axis, and time (s), the dependent variable, on the *y*-axis.



The graph of the raw data yields a curve that looks like the curve of an **inverse relationship**. Either time varies inversely with diameter $\left(\mathrm{T}\ \alpha\ \frac{1}{d}\right)$ or time varies inversely with the square of the diameter $\left(\mathrm{T}\ \alpha\ \frac{1}{d^2}\right)$.

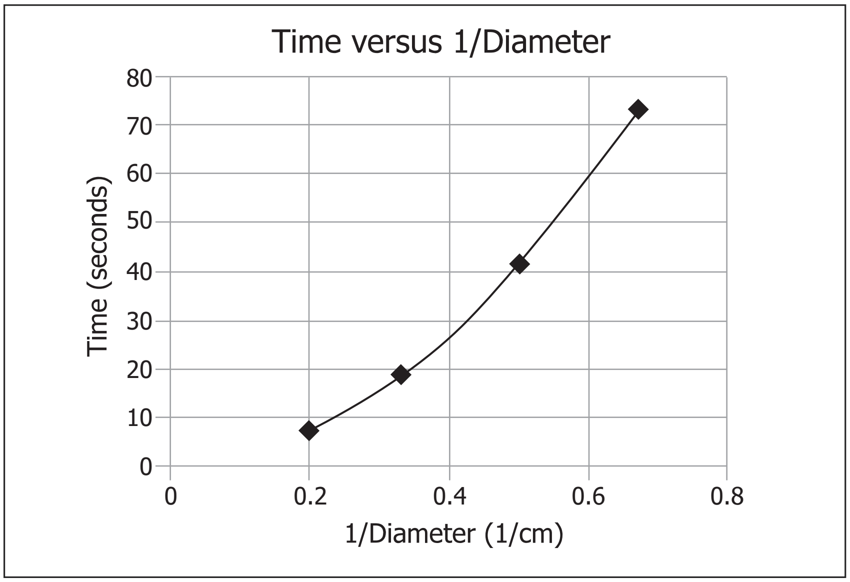
You now must "straighten the line." Since this is an inverse relationship with two options, try the simple inverse first. Then, if this does not work, try the inverse square option.

First, you need to adjust the data (independent variable or the *x*-variable) so that you can plot time versus $\frac{1}{\text{diameter}}$.

Data Table

d (cm)	$1/d$ (cm^{-1})	Time (s)
1.5	0.67	73.0
2.0	0.50	41.2
3.0	0.33	18.4
5.0	0.20	6.8

Plot the graph of time versus $\frac{1}{\text{diameter}}$ in Excel to see whether the line has been straightened.



You can see that the line is still curved.

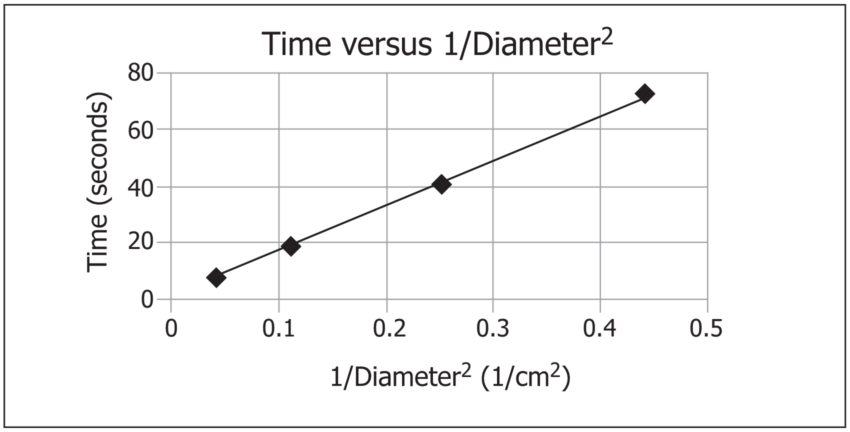
This indicates that the relationship is not "time varies inversely with diameter" $\left(T \; \alpha \; \frac{1}{d}\right)$ but "time varies inversely with the square of the diameter" $\left(T \; \alpha \; \frac{1}{d^2}\right)$.

The next step is to plot time versus $\frac{1}{\text{diameter}^2}$. Again, you need to adjust the data.

Data Table

d (cm)	$1/d^2$ (cm^{-2})	Time (s)
1.5	0.44	73.0
2.0	0.25	41.2
3.0	0.11	18.4
5.0	0.040	6.8

Now, plot the graph of time versus $\frac{1}{\text{diameter}^2}$ in Excel to see whether the line has been straightened.



Finally, you have a "straightened line." So you can change the proportionality $\left(T \; \alpha \; \frac{1}{d^2}\right)$ into an equation by replacing the proportionality sign with an equal (=) sign (= means constant).

The equation has the form $T = (k)\left(\frac{1}{d^2}\right)$.

The last task is to determine *k*, which is the slope of this straight line.

In the graph shown above, the slope = $\dfrac{73.0 \text{ s} - 10.8 \text{ s}}{0.44 \text{ cm}^{-2} - 0.06 \text{ cm}^{-2}} = 165 \text{ s/cm}^{-2}$.

The $\dfrac{1}{\text{cm}^{-2}}$ can also be written as cm^2.

Thus, time = $\left(165 \text{ s} \cdot \text{cm}^2\right)\left(\dfrac{1}{\text{diameter}^2}\right)$ or $T = \left(165 \text{ s} \cdot \text{cm}^2\right)\dfrac{1}{d^2}$.

Using Excel to find an Equation for Data

Once you have the graph of your "straightened line" drawn in Excel as a scatter plot, you can add a trendline and find the equation for this line.

1. Click on "Chart" in the menu at the top of the screen, and then click "Add Trendline."
2. Choose the type of curve (linear).
3. Click on the "Options" tab at the top of the dialogue box and check the box next to "Display Equation on Chart."
4. Click on "OK."
5. The equation is displayed on the graph in the form *y* = *mx* + *b*, where *m* is the slope and *b* is the *y*-intercept.

Conclusion

Generally, when we analyze data from an experiment we are trying to find a linear, power, or inverse relationship. Note that it is very common in physics to find an inverse square relationship, where you must take the inverse and the square of your data before you are able to "straighten the curve." In the sample experiment, this was the case.

The relationship between the time it takes to drain a fixed volume of water from a can and the diameter of the hole through which the water drains is $T = \left(165\,s \cdot \mathrm{cm}^2\right)\frac{1}{d^2}$, where time is measured in seconds and the diameter of the hole is measured in centimetres.

Summary

This lesson provides the tools you will need to analyze the data from upcoming experiments. When you come across the task of finding the mathematical relationship between variables, refer back to this lesson.

There is no assignment for this lesson.

NOTES

Video - Excel 10 (Problem 16) Experimental data analysis, curve fitting, linear regression, trendline.

This video illustrates how to use Excel to draw a graph or chart as Excel calls a graph.

https://youtu.be/peA7ubscWrU

(0-4:40) This part of the video shows how 3 sets of measurements of the diameters and circumferences of metal cylinders are analyzed to determine a mathematical relationship between these variables. In the first part the average diameters and average circumferences are calculated and a new data table is produced in Excel.

These average values are selected. Then Insert → Chart is selected from the taskbar. A menu of the types of charts appears. Choose the chart type Scatterplot.

The skeleton graph appears containing only the data points with no line.

The graph is then formatted with labels, units and a title.

(8:14 – 13:32) This part of the video illustrates how this graph of a linear relationship can be analyzed to produce an equation for these data.

This time all of the diameter-circumference ordered pairs are plotted on a scatterplot.

The skeleton graph is formatted with labels, units and a title and then completed by inserting the trendline (line of best fit).

Lesson 2: Kinematics: Bringing You Up To Date (1 hour)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define terms such as position, distance, displacement, speed, velocity, and acceleration
- ❑ describe motion in words, using a table of data, on a graph, and using equations

Key Words

position	distance	displacement
speed	velocity	acceleration
time instant	time interval	scalar
vector	rise	run
slope	reference point	uniform motion

Introduction

This first module reviews some of the terms and concepts that you studied in Grade 10 Science and in Grade 11 Physics. In Grade 10 Science, you were asked to describe motion in words, using numbers from measurements in a data table, using a graph of the measurements, and finally by using equations. You will recall that the graphs were particularly useful. The graphs that you drew—that is, position-time graphs, velocity-time graphs, and acceleration-time graphs—were all just different versions of the same story of the motion of an object. They not only described a motion, but they also allowed you to calculate some other quantities useful in describing motion.

In Grade 11 Physics, these concepts were expanded upon. You were required to convert one type of graph into another, such as a position-time graph into a velocity-time graph, or a velocity-time graph into an acceleration-time graph, by taking the slope of the lines of the first graph. Then you discovered that you could go in the opposite direction. You could start with an acceleration-time graph and reconstruct a velocity-time graph, or start with a velocity-time graph and reconstruct an acceleration-time graph. This second set of conversions required that you find the area between the line on the graph and the horizontal axis.

Also, in Grade 11 Physics, equations for kinematics were derived from the graphs and used to solve problems. In Grade 11 Physics, you were not required to learn the derivations of these equations. The derivations were provided simply to demonstrate to you the origin of these equations. Your task was to be able to use the equations to solve kinematics problems.

In Grade 12 Physics, you will be going one step further. Not only should you be able to describe motion in words, using a data table, using a graph of position-time, velocity-time, or acceleration-time, and using the appropriate equations, you must also be able to derive the equations for kinematics from the appropriate graph. Physics is not only about having a list of equations into which you can plug numbers. The equations of physics are the symbolic summary of the concepts involved in a particular topic. The concepts must be understood first before you can successfully work with problem solving.

So here's a bit of advice to ensure that you will have some success in the problem-solving process in Grade 12 Physics. Try to understand the big picture about the phenomenon that you are studying. What is the phenomenon? Into which topic does it fit? What are the variables or names of the factors that affect the situation? What symbols are used to represent these variables? Are they vectors or scalars? What units do they have? If you can't answer these questions, then even though you will be given the equations for that situation, you will be unable to solve problems.

In this lesson, then, let's put this method of studying physics into practice!

A Review of Kinematics

You have already seen that there are many terms that are associated with describing motion: time instant, time interval, position, displacement, speed, velocity, and acceleration. Before we begin to talk about kinematics, which is the study of motion, it would be wise to review all of these terms.

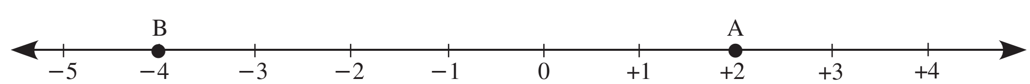
> A **time instant** is just a clock reading, such as 3:14 PM.
>
> A **time interval** is simply the time that elapses between two clock readings or two instants in time. For example, the time interval between 3:14:25 PM and 3:14:39 PM is 14 seconds.

Time is a scalar quantity, meaning we can specify time using only magnitude. In SI, the unit we use for time is the second or s.

In order to talk about motion, we must be able to indicate where an object is located. An object moves when it changes its location.

> The **reference point** is the zero location in a coordinate system or frame of reference.
>
> **Position** is the location of an object in relation to the location of a specific point called the reference point.

Since in this section we are dealing only with motion along a straight line or straight-line kinematics, you can think of positions as falling along a straight line. A classic example would be the number line. The position of each number (for example, each integer) is marked on the line relative to the position of 0, the reference point. Numbers to the right of zero are considered to be positive numbers, and numbers to left the zero are considered to be negative numbers. In the same way, we can mark positions along a line.



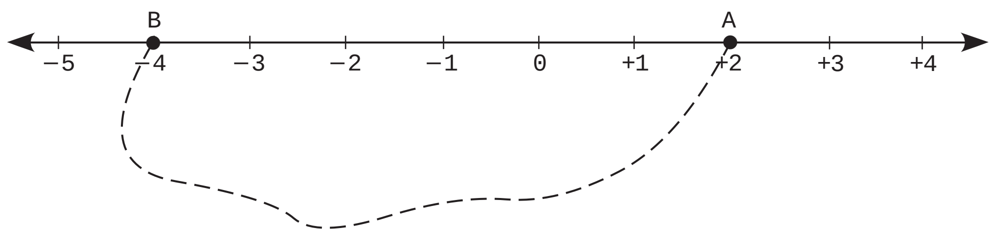
The reference point is zero, which we mark as 0 m.

The point A is located at a position of +2 m—that is, 2 m to the right of the reference point. Normally, we consider motion to the right or up as positive. In a given question, you should specify the directions for the motions.

The point B is located at a position of –4 m—that is, 4 m to the left of the reference point. Again, we consider motion to the left or down to be negative.

The points A and B represent two positions. A represents position 1 (pos_1) and B represents position 2 (pos_2). If the object moved from position A over to position B, then position A is called the initial position (pos_1) and position B is called the final position (pos_2).

Let's say a ladybug was to walk from position A to position B. Now, the ladybug is not likely to walk in a straight line, but rather to wander. A possible path is indicated below with the dashed line.



Along the path indicated by the dashed line, the ladybug travels a distance of 11 m during a time of 45 seconds.

> **Distance** (d) is the length of the path travelled by an object. It has no direction and is therefore a scalar quantity.

In this case, the distance travelled is 11 m, d = 11 m.

The time interval, Δt, is 45 s.

Speed

From these data, we can calculate how fast the ladybug was travelling. How fast an object travels is called its **speed**. Average speed is calculated by taking the total distance travelled over the total time interval.

Speed is a measure of how fast an object is travelling. Speed is calculated by the distance travelled over the time interval.

$$v = \frac{d}{\Delta t}$$

Quantity	**Symbol**	**Unit**
Speed	v	metres/second (m/s)
Distance	d	metres (m)
Time interval	Δt	seconds (s)

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time interval}}$$

$$v_{\text{avg}} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}}$$

Here, the speed of the ladybug would be:

$$v_{\text{avg}} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}}$$

$$v_{\text{avg}} = \frac{11\text{ m}}{45\text{ s}}$$

$$v_{\text{avg}} = 0.24\text{ m/s}$$

Since speed has no direction associated with it, it is a scalar.

Displacement

Now, **displacement** represents the change in position—that is, the final position minus the initial position. The symbol for displacement is $\vec{d}$. Displacement represents not only how far the object has travelled, but also in which direction it has travelled. Both of these must be specified—that is, magnitude and direction—for displacement. Therefore, displacement is a **vector** quantity.

Displacement represents the change in position of an object. Displacement is calculated by subtracting as follows: final position – initial position.

$$\vec{d} = \overrightarrow{\text{pos}}_2 - \overrightarrow{\text{pos}}_1$$

Quantity	**Symbol**	**Unit**
Initial position	$\overrightarrow{\text{pos}}_1$	metres (m)
Final position	$\overrightarrow{\text{pos}}_2$	metres (m)
Displacement	$\vec{d}$	metres (m)

Displacement is a *vector* quantity.

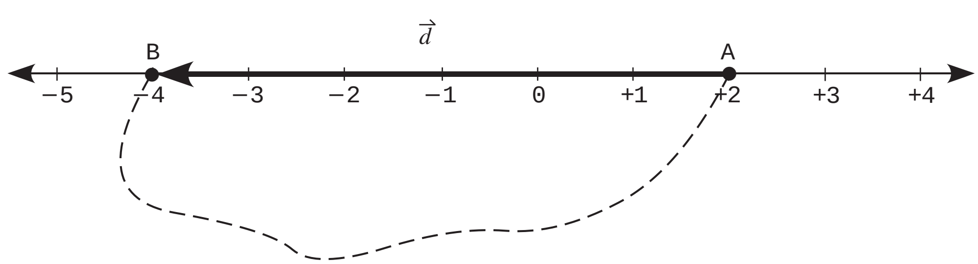
The ladybug's initial position was +2 m and its final position was –4 m.

$$\overrightarrow{\text{pos}}_1 = +2\text{ m}$$
$$\overrightarrow{\text{pos}}_2 = -4\text{ m}$$

displacement = $\overrightarrow{\text{pos}}_2 - \overrightarrow{\text{pos}}_1$

$$\vec{d}_{\text{total}} = \overrightarrow{\text{pos}}_2 - \overrightarrow{\text{pos}}_1$$
$$\vec{d}_{\text{total}} = -4\text{ m} - (+2\text{ m}) = -6\text{ m}$$

The displacement of the ladybug is –6 m or 6 m to the left.



To represent a vector, we use a directed line segment—an arrow. The **tail** of the arrow (the end of the arrow without a head) is placed at the initial position (point A). The **head** (the end of the arrow where the arrowhead is) is placed at the final position (point B).

The displacement vector $\vec{d}$ is drawn in the diagram above.

Velocity

Once you understand displacement and are able to determine displacement, you can move on to finding velocity. Velocity is defined as the rate of change of position. It is calculated by taking the displacement over the time interval.

Velocity is the rate of change of position. Velocity is calculated by the displacement over the time interval.

$$\vec{v} = \frac{\vec{d}}{\Delta t} = \frac{\overrightarrow{\text{pos}}_2 - \overrightarrow{\text{pos}}_1}{\Delta t}$$

Quantity	Symbol	Unit
Initial position	$\overrightarrow{\text{pos}}_1$	metres (m)
Final position	$\overrightarrow{\text{pos}}_2$	metres (m)
Displacement	$\vec{d}$	metres (m)
Velocity	$\vec{v}$	metres/second (m/s)
Time interval	Δt	seconds (s)

Velocity is a *vector* quantity.

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time interval}}$$

$$\vec{v}_{\text{avg}} = \frac{\vec{d}_{\text{total}}}{\Delta t_{\text{total}}}$$

Velocity is a vector quantity. Its direction will be the same as the direction of the displacement.

Velocity will have units of metres over seconds (m/s) in SI units. You may be familiar with other velocity units like kilometres per hour (km/h). Units like these should be converted to m/s before doing calculations.

You can think of velocity as speed plus direction.

Getting back to the ladybug, you can see that the displacement was –6 m during a time of 45 s.

Using, $\vec{v}_{avg} = \dfrac{\vec{d}_{total}}{\Delta t_{total}}$

$$\vec{v}_{avg} = \frac{-6\text{ m}}{45\text{ s}} = -0.13\text{ m/s}$$

The velocity of –0.13 m/s indicates that the ladybug would have travelled from A to B and during 45 seconds if it had maintained a constant speed of 0.13 m/s in the negative direction or to the left. Since the ladybug had wandered from the straight-line path, this velocity represents the average velocity for this journey.

As you work through this course, remember that your learning partner and your tutor/marker are available to help you if you have questions or need assistance with any aspect of the course.

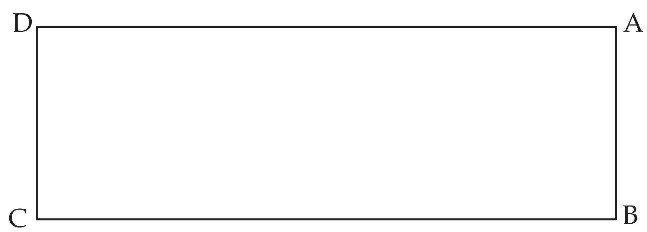
Learning Activity 1.1

Working with Distance, Displacement, Speed, and Velocity

Test your understanding of the ideas you are reviewing by answering the practice questions below. Remember, you do not submit learning activities for assessment. Instead, you complete them in order to prepare yourself for the assignments, which **are** submitted for assessment. Once you have completed this learning activity, check your work in the answer key provided at the end of Module 1.

To answer questions 1 to 3, use the information below.

A city block is laid out in a grid running in the north-south and east-west directions. The blocks measure 135 m in length in the east-west direction, and 45.0 m in width in the north-south direction. A city block is drawn below.



1. On your bicycle, you travel from A to B during 9.00 s.
   a) What is your average speed?
   b) What is your average velocity?
2. If you travel from A to B to C to D, what is your
   a) distance travelled?
   b) displacement?
3. If the journey in #2 took 55.0 s, calculate
   a) your average speed.
   b) your average velocity.

Acceleration

Some motions occur with the speed and the direction of motion always being the same. In other words, the velocity is constant.

The motion of an object with constant velocity is called **uniform motion**.

Not all motion is uniform motion. Objects may speed up, slow down, or change direction. In these cases, we say that the object is accelerating.

Acceleration is defined as the rate of change of velocity. Think of it as how the velocity changes with time. For example, an acceleration of +2 m/s/s indicates that an object is changing its velocity by 2 m/s, every second, in the positive direction.

Acceleration can be calculated as the change in velocity over the time interval.

Acceleration is defined as the rate of change of velocity. Acceleration is calculated by taking the change in velocity and dividing it by the time interval.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \text{ or } \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

Quantity	**Symbol**	**Unit**
Acceleration	$\vec{a}$	metres/second squared (m/s^2)
Change in velocity	$\Delta\vec{v}$	metres/second (m/s)
Time interval	Δt	seconds(s)
Initial velocity	$\vec{v}_1$	metres/second (m/s)
Final velocity	$\vec{v}_2$	metres/second (m/s)

Acceleration is a *vector* quantity.

The equation would look like:

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$= \frac{\text{final velocity minus initial velocity}}{\text{final time minus initial time}}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

This brings us up-to-date as far as a review of terms for kinematics is concerned.

Learning Activity 1.2

Acceleration Calculations

Test your understanding of the ideas you are reviewing by answering the practice questions below. Remember, you do not submit learning activities for assessment. Instead, you complete them in order to prepare yourself for the assignments, which are submitted for assessment. Once you have completed this learning activity, check your work in the answer key provided at the end of Module 1.

1. Alberto reaches the bottom of the hill coasting along at 9.25 m/s. He begins to coast up a second hill where the average acceleration is –1.20 m/s/s. What is the change in Alberto's velocity during 3.00 s of coasting up this hill? What is his final velocity?

2. A dragster racing on a quarter-mile track (about 400.0 m) has an average acceleration of 11.2 m/s/s [E] reaching a velocity of 72.0 m/s [E]. What was the time needed to race this distance?

Describing Motion

In studying physics, we have four different ways of representing or describing any given phenomenon. These are visual, numerical, graphical, and symbolic. The following example will illustrate these different modes of representation.

Visual Representation

This first description deals with the visual (that is, what you see happening).

In this description, we will start with our everyday terms. We will deal only with motion along a straight line.

You are walking down the hallway from English class to geography class. It is 50 metres east from the door of the English room to the door of the geography room. It takes 25 seconds to walk this distance at a steady pace.

The steady pace tells us that this is uniform motion.

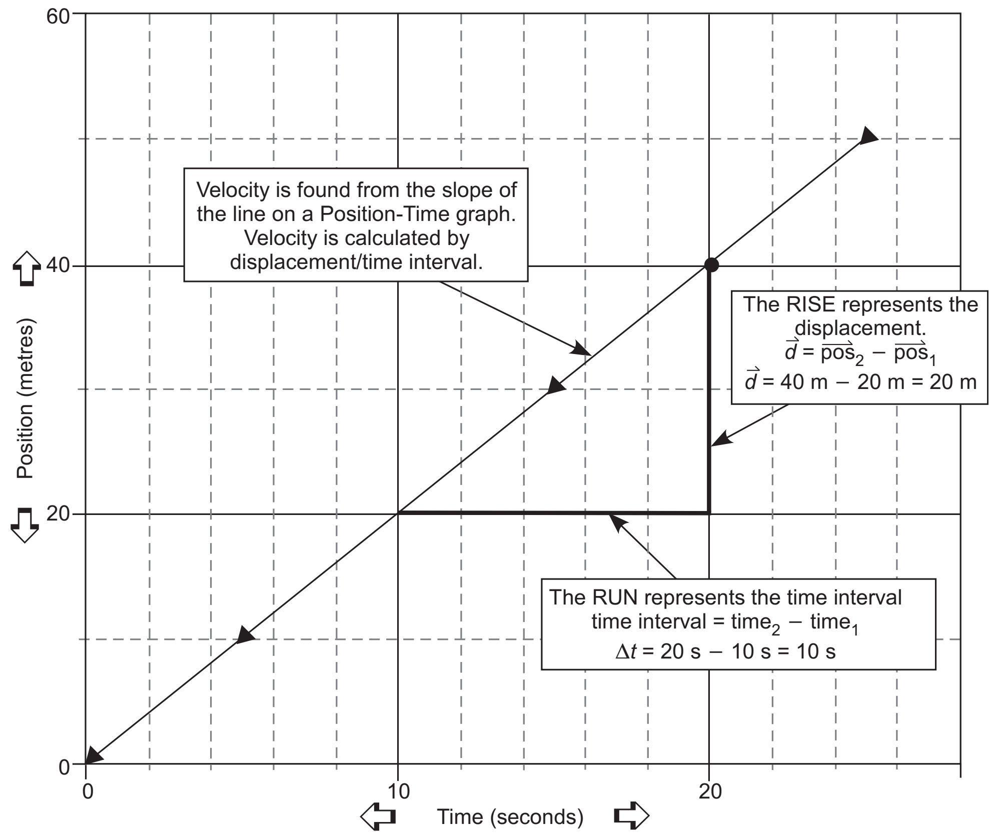
Numerical Representation

The description above can also be represented in a data table of time and position. We must indicate from where all the positions are measured—that is, the reference point. In this case, the reference point will be the door to the English room. From the description, we see that you would travel 50 metres east in 25 seconds or 10 metres east for every five seconds.

Time (seconds)	Position (metres east)
0	0
5	10
10	20
15	30
20	40
25	50

Graphical Representation

The information above can be plotted on a graph. Time would be plotted on the **horizontal axis**. Position will be plotted on the **vertical axis**.



You can see on the graph that the line is straight. A straight line on a position-time graph represents uniform motion.

If you have a position-time graph, there is some information that you can read directly from the graph. A position-time graph tells the story of where an object is at an instant in time.

For example, from the position-time graph above, the position at 12 seconds would be 24 metres east. The object is at 35 metres east at about 17.5 seconds.

Symbolic Form (Equations)

You can write a mathematical equation to describe this motion.

From a graph, you can read information directly. However, you can also calculate quantities—that is, find indirect information. There are only two things that can be found from a graph indirectly. One is the slope of the line (slope = rise/run); the second is the area beneath the curve or line, between the line and the horizontal axis.

You can see from the data table that the position in metres east is always twice as large as the time in seconds. The position measures how far you are located from the origin. Travelling from the origin to this position is a change in position or **displacement**. The time it took you to walk is a **time interval**.

Your equation would be:

Displacement = (2.0 m/s east) (time interval)

The 2.0 m/s east is a speed with direction. This is **velocity**.

But what does this mean on the graph? Is this direct or indirect information? If it is indirect, is it the slope or the area?

Displacement is calculated by the change in position, $\vec{d}_{\text{total}} = \overrightarrow{\text{pos}}_2 - \overrightarrow{\text{pos}}_1$.

On the graph this represents the **RISE**.

Time interval is $\Delta t_{\text{total}} = t_2 - t_1$.

On the graph, this represents the **RUN**.

Together, the equation relating these three quantities is written as:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\vec{v}_{\text{avg}} = \frac{\overrightarrow{\text{pos}}_2 - \overrightarrow{\text{pos}}_1}{t_2 - t_1} \text{ or } \vec{v}_{\text{avg}} = \frac{\vec{d}_{\text{total}}}{\Delta t_{\text{total}}}$$

In conclusion, **velocity** is just the **slope** of a position-time graph!

All four of these modes of representation are equal. If you are given one of these, the other three modes of representation can be obtained. Normally, when you begin the study of the new concept, it is best to begin with the visual representation. You know the old saying, "A picture is worth a thousand words." In this course, it is wise to use diagrams as a starting point for explanations for problem solving. The visual clues provided by diagrams help to clarify the situation and avoid confusion.

For problem solving, of which there will be a great deal in this course, the focus is on the symbolic mode of representation—that is, the equation. Again, while using the equation is an efficient way to deal with problems, a proper understanding of the situation is required. This is where the visual representation, the diagram, plays its very important role.

Learning Activity 1.3

Modes of Analyzing Motion

Check your understanding of the concepts in this lesson by answering the following practice questions. You may check your work against the answer key provided at the end of Module 1.

1. Consider the following situation. You are jogging at a constant pace around a track. This is the typical track that might be found around the perimeter of a football field. The distance around the track is 360.0 m.
   a) Draw a diagram of the situation.
   b) It takes you 90.0 seconds to jog completely around the track and return to your starting point. Calculate your average speed and your average velocity.
   c) On a straight section of the track, it takes you 25.0 seconds to jog 100.0 m. What is your average velocity? Hint: Refer to your diagram.
   d) Are you accelerating while you are jogging along one of the straight sections of the track? Explain.
   e) Are you accelerating while you are jogging along one of the curved sections of the track? Explain.
2. The car is travelling along a section of the road that forms one-half of a circle with a radius of 176 m. The car requires 22.0 seconds to travel around the curve.
   a) Draw a diagram of the situation.
   b) What is the average speed of the car while it travels around this half-circle?
   c) What is the average velocity of the car while it travels around this half-circle?

Lesson Summary

In this lesson, you reviewed some ideas on kinematics.

Assignment

Most lessons will have an assignment at this point in the lesson. This lesson does not have an assignment.

Assignments are to be completed and submitted for evaluation. Assignments are part of your evaluation. Together they represent 50% of your total mark in this course. After completing a particular section of the course, you will be instructed to submit all of the assignments from a particular module. Submit assignments to your tutor-marker when you are instructed to do so.

Video - Position Distance & Displacement - Average Speed & Velocity Word Problems & Graphs - Physics

This video reviews the terms position, distance and displacement.

An initial position and displacement are used to find the final position.

https://youtu.be/uTQ4_AOae1g

Video - Introduction to Displacement and the Difference between Displacement and Distance

This video introduces displacement, the change in position of an object. The difference between distance and displacement are identified. Different references systems are introduced. These reference systems are used to provide a means of assigning directions to displacement.

In the video the symbol x is used for position and Δx is used for displacement. In our course we use the symbol for displacement.

https://youtu.be/uTQ4_AOae1g

Video - Velocity and Speed are Different: Example Problem

This video calculates speed and velocity for a journey. It is demonstrated that these are not the same quantity.

The video also illustrates important steps in the problem solving process. Start by reading the problem, finding the given information and converting to required units, identifying the unknown, finding an equation to link the unknown to the given quantities and finally substituting and solving for the answer.

https://youtu.be/0kQrz4dfxDw

Video - Understanding Uniformly Accelerated Motion

This video introduces the meaning of accelerated motion without equations. The video attempts to develop an understanding the concept of uniformly accelerated motion by observing the motion of a basketball rolling down a sloping driveway and secondly rolling up a sloping driveway.

https://youtu.be/0kQrz4dfxDw

Video - Introduction to Uniformly Accelerated Motion with Examples of Objects in UAM

This video introduces the variables and equations for uniformly accelerated motion (UAM).

Note: These equations use delta x (Δx) for displacement instead of the vector $\vec{d}$ (d with a little arrow over the d).

Also the equation d = v Δt (our equation number 1) is used for the case where acceleration = 0 m/s/s which is not a case of uniformly accelerated motion.

Also note the 5-4-3-2-1 sequence.

https://youtu.be/WCR2Ki6hFf4

Lesson 3: Deriving Equations for Motion Involving Uniformly Accelerated Motion (1.5 hours)

Lesson Focus

When you have completed this lesson, you should be able to

- ❑ derive the following equation from the definition of acceleration and from finding the slope of a velocity-time graph for constant acceleration

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

- ❑ derive the following equation from a velocity-time graph for constant acceleration

$$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)\Delta t$$

- ❑ derive the following equation from a velocity-time graph for constant acceleration

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

- ❑ derive the following equation from a velocity-time graph for constant acceleration

$$v_2^2 = v_1^2 + 2ad$$

- ❑ relate the shape of the area on a velocity-time graph for constant acceleration to the equations that are derived from the graph

Key Words

uniformly accelerated motion

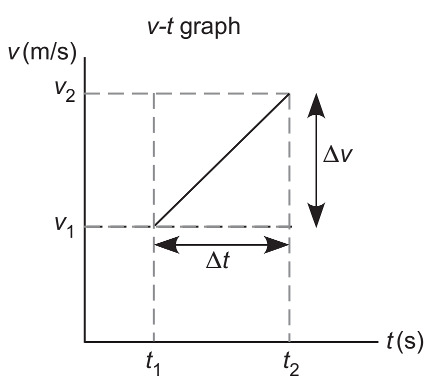
Introduction

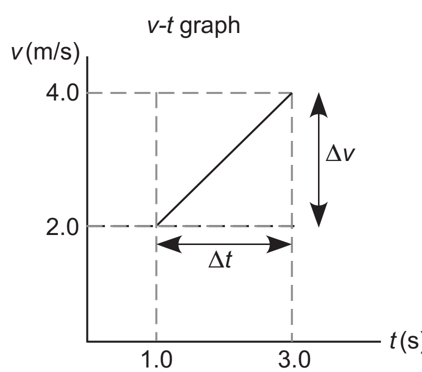
In this lesson, we will examine a velocity-time graph and derive from it four very useful equations for uniformly accelerated motion. These equations will become important in solving problems involving constant acceleration. The beauty of this lesson is that equations, which will be very useful to us later, are in fact being derived. This will help you to understand and appreciate where these equations come from, and therefore how they can be applied to solving problems.

The equations being derived here are for **uniformly accelerated motion**. The motion of the object is one where the object is speeding up or slowing down. What you should see on a velocity-time graph is a line sloping up or down indicating that the velocity is changing.

Deriving the Equation for Acceleration

One graph that represents motion at a constant acceleration is the one shown below on the left. The graph on the right has values of velocity and time added to it.





In Grade 11 Physics, we learn that uniform acceleration can be defined as the change in velocity over the change in time. Mathematically, this can be written as

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \text{ or } \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

In general, the slope of a straight line is given by the change in y-value divided by the change in the x-value or $\Delta y/\Delta x$. For the velocity-time graph above, the change in y-value is $\Delta\vec{v}$, and the change in x-value is Δt. By dividing $\Delta\vec{v}\ /\ \Delta t$, we can see that this is also the definition of acceleration. Thus, for constant acceleration, the slope of a velocity-time graph gives acceleration.

The equation above may be rewritten as

$$\vec{v}_2 - \vec{v}_1 = \vec{a}\Delta t \text{ or}$$

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t \text{ or}$$

$$\vec{v}_2 = \vec{v}_1 + \vec{a}t$$

For the graph with values of velocity and time, the acceleration can be found using

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

The value of acceleration for this graph is

$$\vec{a} = \frac{(4.0 \text{ m/s} - 2.0 \text{ m/s})}{(3.0 \text{ s} - 1.0 \text{ s})} = 1.0 \text{ m/s}^2$$

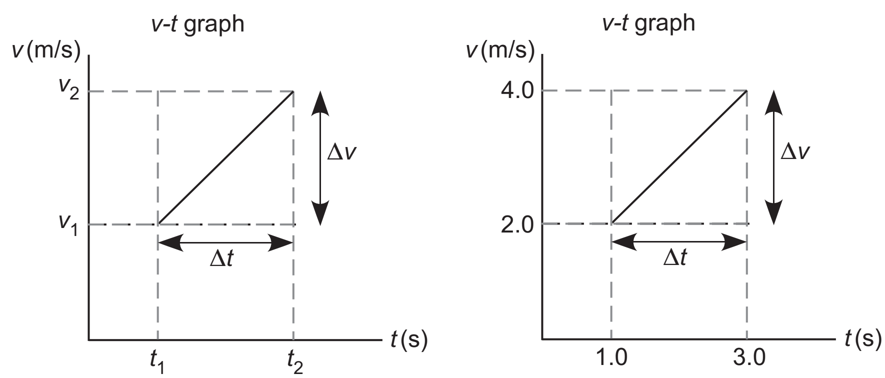
The value of the second velocity is already given as 4.0 m/s.

To confirm that the second velocity can be found using our derived equation of $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$, substitute in the appropriate values. The second velocity becomes

$$\vec{v}_2 = 2.0 \text{ m/s} + (1.0 \text{ m/s}^2)(2.0 \text{ s}) = 4.0 \text{ m/s}$$

Deriving the Equation For Displacement Given Initial Velocity, Final Velocity, and Time

Consider again the graphs used previously.



To find the displacement of an object undergoing uniformly accelerated motion, determine the area under the velocity-time graph.

One way to determine the area for the graph shown above is to find the area of a trapezoid. In general, the area of a trapezoid is given by

$$\text{area of trapezoid} = \frac{1}{2}(\text{sum of the parallel sides})(\text{height})$$

For the example above,

$$\text{area of trapezoid} = \frac{1}{2}(v_1 + v_2)(\Delta t)\text{, or in other words}$$

$$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)\Delta t$$

(The symbol Δx is often used to indicate a change of position. For the purposes of this discussion, we will use $\vec{d}$ to represent displacement.)

To determine numerically the value of the displacement for the graph above, substitute the values.

$$\vec{d} = \frac{1}{2}\left(+2.0 \text{ m/s} + \left(+4.0 \text{ m/s}\right)\right)\left(2.0 \text{ s}\right) = +6.0$$

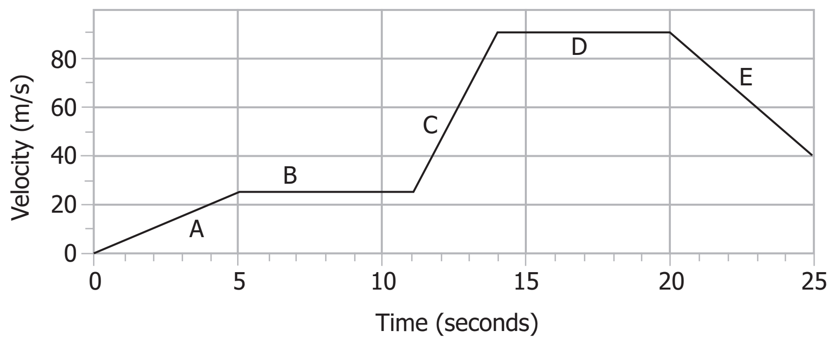
Note that in this case the area that represents the displacement lies above the horizontal time axis. Therefore, the area and hence the displacement is positive.

Learning Activity 1.4

Linking Kinematics Graphs and Equations

Test your understanding of the derivation of the equations for uniformly accelerated motion by answering the practice questions below. You may check your work against the answer key provided at the end of Module 1.

Velocity-Time Graph

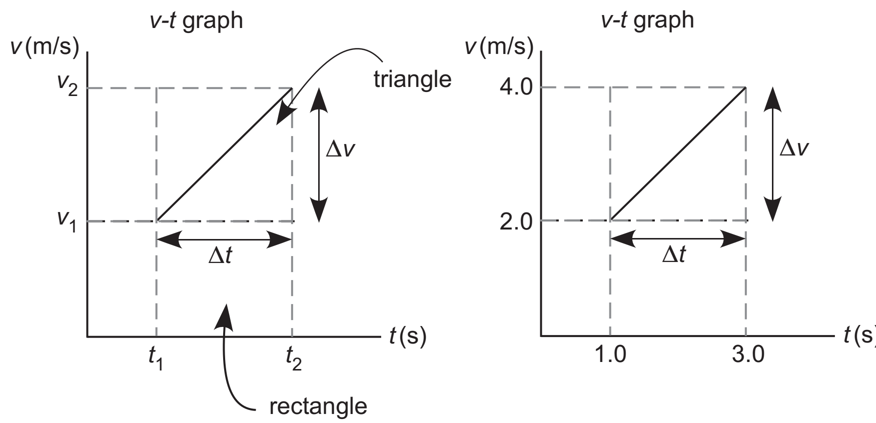


All the questions refer to the velocity-time graph given above.

1. What on the velocity-time graph represents acceleration? Displacement?
2. Which of the two equations that have been derived so far in this lesson should be used to find
   a) the acceleration for the time interval A?
   b) the displacement for the time interval C?
   c) the displacement for the time interval B?
3. Calculate the displacement during time interval C.

Deriving the Equation for Displacement Given Initial Velocity, Acceleration, and Time

Another useful equation can be derived from the graph by considering the area of the rectangle and the area of the triangle under the solid line.



To find the displacement of an object undergoing uniformly accelerated motion, determine the area under the velocity-time graph.

total area = area of rectangle + area of triangle

The area of a rectangle is given by the length times the width. For the rectangle above,

area of rectangle = $v_1\Delta t$

The area of a triangle is given by one-half the base times the height.

area of triangle = $\frac{1}{2}\Delta v\Delta t$

total area = $v_1\Delta t + \frac{1}{2}\Delta v\Delta t$

Since the area under a velocity-time graph gives displacement,

$$\vec{d} = v_1\Delta t + \frac{1}{2}\Delta v\Delta t$$

We know from the definition of acceleration that $\Delta v = a\Delta t$. Therefore,

$\vec{d} = v_1\Delta t + \frac{1}{2}(a\Delta t)\Delta t$, then adding vector notation,

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Applying this to the graph with the values stated,

$$\begin{aligned}\vec{d} &= \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2 \\ &= (+2.0\text{ m/s})(3.0\text{ s} - 1.0\text{ s}) + \frac{1}{2}(+1.0\text{ m/s}^2)(3.0\text{ s} - 1.0\text{ s})^2 \\ &= +4.0\text{ m} + (+2.0\text{ m}) = +6.0\text{ m}\end{aligned}$$

This is the same value for displacement that we obtained in the section above.

Deriving the Equation Relating Displacement, Initial Velocity, Final Velocity, and Acceleration

A final useful equation can be derived from two of the equations above. Here we will eliminate Δt from the two equations. This will yield an equation that does not contain a time interval. This relationship is very useful in some situations.

One of the equations was $\vec{v}_2 = \vec{v}_1 + \vec{a}t$. Square both sides of this equation.

$$v_2^2 = v_1^2 + 2av_1t + a^2t^2$$

Factor out the "2a" from the last two terms of the equation.

$$v_2^2 = v_1^2 + 2a\left[v_1t + \frac{1}{2}at^2\right]$$

The expression $v_1t + \frac{1}{2}at^2$ is equal to the displacement given by another derived equation above. Therefore,

$$v_2^2 = v_1^2 + 2ad$$

Note that Δt has disappeared. In this case, you can dispense with the vector notation, as the equation would require you to square a vector and multiply vectors (which are beyond the scope of this course).

To find displacement using this equation, solve for "d".

$$d = \frac{v_2^2 - v_1^2}{2a} = \frac{\left[(4.0\text{ m/s})^2 - (2.0\text{ m/s})^2\right]}{2(1.0\text{ m/s}^2)} = 6.0\text{ m}$$

Since the area is found above the time axis, the displacement is positive. So, $\vec{d} = +6.0\text{ m}$. This is the same value as obtained by the previous methods.

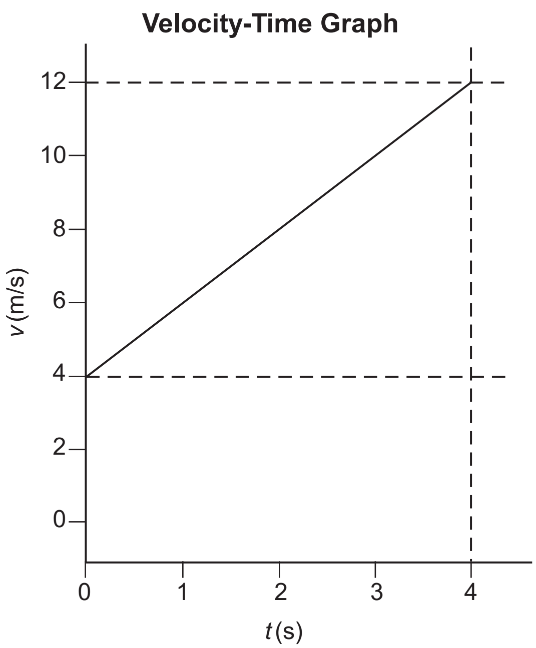
Learning Activity 1.5

Graphs and Equations for Uniform Acceleration

There are four practice questions in this learning activity. An answer key is available at the end of Module 1 for you to check your work after you have answered the questions.

The physics of uniformly accelerated motion and the graph.

1. The four parts of this assignment refer to the following graph.



   a) Using the equation $\vec{v}_2 - \vec{v}_1 = \vec{a}\Delta t$, determine the acceleration of the object.

   b) Using the equation $\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$, determine the displacement of the object above over the four-second interval.

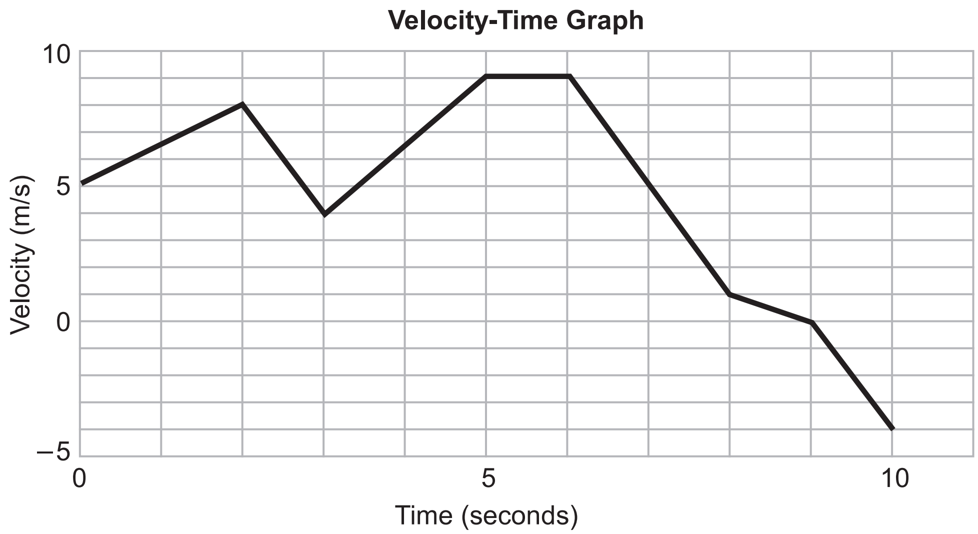
   c) Using the equation $\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$, verify that the displacement is the value you calculated above in part (b).

   d) Using the equation $v_2^2 = v_1^2 + 2ad$, verify that the displacement is the value you calculated above.

(continued)

Learning Activity 1.5: Graphs and Equations for Uniform Acceleration (continued)

2. On the following velocity-time graph, indicate during which intervals of time the acceleration is positive, negative, or zero. This graph does not represent a real-life situation. It is presented simply to help reinforce the ideas about interpreting graphs.



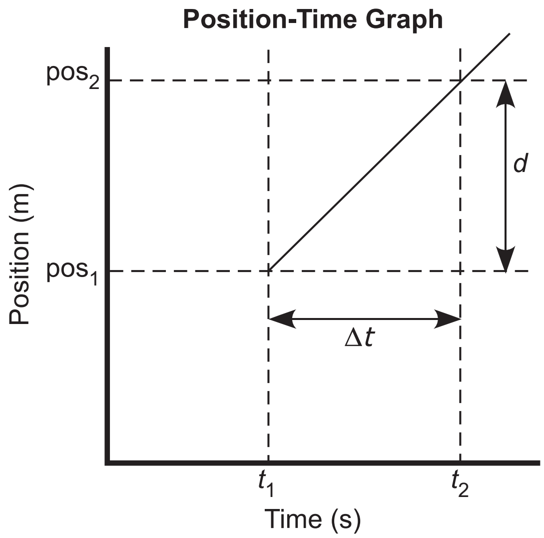
3. For the graph in question 2, determine the value of the acceleration for each time interval. Remember that the slope on a velocity-time graph gives the acceleration. The acceleration for the first interval of time has been calculated in the table below. Complete the other calculations in the table below.

Time Interval	Change in Velocity (m/s)	Acceleration (m/s/s)
0 s – 2 s	+8 – (+5) = +3	+3 m/s/2 s = +1.5

4. For each of the time intervals for the graph in question 2, determine the displacement using one of the equations for kinematics that were derived in this lesson. You should be able to use all four of the kinematics equations that involve displacement at least once each.

Lesson Summary

In Grade 11 Physics, a graph of position-time was used to derive an equation for uniform motion. In this case, the acceleration is 0 m/s/s.

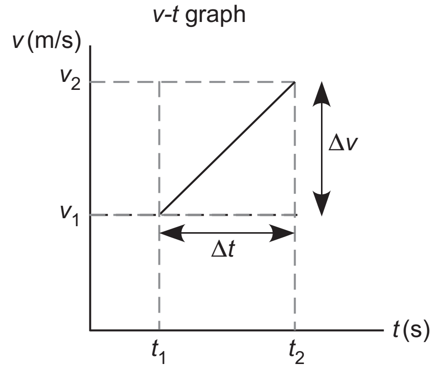


The slope of this position-time graph yielded the first equation for kinematics. From the graph, you can see that in order to calculate the slope of the line, you must find the rise that gives the change in position (displacement) $\vec{d}$. You can also see that the run is the time interval Δt. Therefore, the slope of the position-time graph represents displacement over time. This is the definition of velocity.

Kinematics Equation 1: $\vec{v} = \dfrac{\vec{d}}{\Delta t}$

In this lesson, we derived four very useful equations for uniformly accelerated motion from a velocity-time graph. We will be using these equations when we do problems involving the kinematics of uniformly accelerated motion. The next lesson will illustrate the use of these equations.

The equations can be derived from a velocity-time graph, such as the one shown below.



The equations 2 through 5 in the table below are derived from the graph illustrated above.

The equations for straight-line kinematics or for motion along a straight line are:

Kinematics Equation 1: $\vec{v} = \dfrac{\vec{d}}{\Delta t}$

Kinematics Equation 2: $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$ or $\vec{a} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

Kinematics Equation 3: $\vec{d} = \dfrac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$

Kinematics Equation 4: $\vec{d} = \vec{v}_1 \Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$

Kinematics Equation 5: $v_2^2 = v_1^2 + 2ad$

Quantity	**Symbol**	**Unit**
Displacement	$\vec{d}$	metres (m)
Velocity	$\vec{v}$	metres/seconds (m/s)
Acceleration	$\vec{a}$	metres/second^2 (m/s^2)
Time interval	Δt	seconds (s)
Initial velocity	$\vec{v}_1$	metres/seconds (m/s)
Final velocity	$\vec{v}_2$	metres/seconds (m/s)

From a velocity-time graph, you should be able to derive kinematics equations two through five. You should also be able to apply the rules of algebra to isolate one of the variables in an equation.

For example, you should be able to rewrite the equation $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ as $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ or $\vec{v}_2 - \vec{v}_1 = \vec{a}\Delta t$ or $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$. Being able to do this will make problem solving much more efficient.

This applet shows motion with constant acceleration. Graphs of position-time, velocity-time, and acceleration-time are generated along with the motion. [Motion with Constant Acceleration](https://www.walter-fendt.de/html5/phen/acceleration_en.htm)

https://www.walter-fendt.de/html5/phen/acceleration_en.htm

Video - Understanding and Walking Position as a function of Time Graphs

In this video it is shown that the slope of a position-time graph is velocity.

So starting with the position-time version of a motion, we can deduce the velocity-time version of the motion by taking the slope on the position-time graph.

Then given the position-time graph a person will walk the motion.

https://youtu.be/Mjnu5ePzXDM

Video - Walking Position, Velocity and Acceleration as a Function of Time Graphs

(0:00 to 7:10 minutes) The slope of a velocity-time graph is defined as the acceleration. Given a velocity-time graph a student must walk the graph.

(7:10 to 12:50 minutes) Here the velocity on the velocity-time graph at given instants in time is used to draw the position-time graph.

The idea of tangent is introduced.

The graphs of position-time, velocity-time and acceleration-time are arranged one beneath the other to summarize concisely that slope is used going down the diagram from P-T to V-T to A-T and areas are used to go up the diagram from A-T to V-T to P-T.

(12-50 to 18:17 minutes) The motion in a second example is analyzed using the principles identified in the first 3 sections of the video.

(18:17 to 22:55) The motion in a second example is analyzed using the principles identified in the first 3 sections of the video.

(22:55-24:53) The part of the video discusses ideal versus real data and plotting the graphs.

https://youtu.be/fhOqbAF1Uis

Video - Graphing Summary Kinematics High School & College Physics Tutorial

The video summarizes how to use slope and area to convert between the 3 types of kinematics graphs used to describe linear motion.

This short video summarizes the key concepts for relating the 3 different versions of a motion- position, velocity and acceleration.

https://youtu.be/ezn2FGBfUDE

Lesson 4: Using Kinematics Equations for Constant Acceleration to Solve Problems (2 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❏ apply the problem-solving strategy to the solution of problems involving uniformly accelerated motion

Key Words

"GUESS" method
"mks"

Introduction

There is an assignment at the end of the lesson that must be submitted. DO NOT submit the assignment until you have completed Modules 1 and 2.

In a previous lesson, it was demonstrated how the kinematics equations were derived from a velocity-time graph. You saw that each kinematics equation for uniformly accelerated motion could be related to a slope or to an area beneath the curve of a velocity-time graph.

Now we are ready to apply the equations for uniformly accelerated motion to problems. The lesson will begin with a brief introduction to a method of solving problems of this type. It is important, especially when first practicing solving problems, to follow the method. It really helps to organize your efforts and make sure the process followed is clear to anyone who looks at the solution. This is followed by a discussion of two examples where the problem-solving strategy and the use of the equations are applied.

Problem-Solving Strategy for Kinematics Problems—The “Guess” Method

When applying the equations for kinematics, or when solving physics problems in general, it is a good idea to keep in mind a problem-solving strategy. This problem-solving strategy (called the “GUESS” method) is not a substitute for understanding the concepts of physics. Rather, it is an organizational tool that will make it more likely that you will be successful in solving problems.

The word “GUESS” is an acronym that stands for the following:

Given
Unknown
Equation
Substitute
Solve

Here, in more detail, is an explanation of each of these steps.

Given

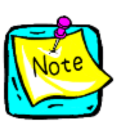
You begin any problem by first reading it carefully, and extracting the relevant information from it. The ”GIVEN” refers to the information that is given in the question.

If possible, make a drawing that represents the situation being studied. The drawing will help you to visualize the situation. This will help you to develop a reasoning strategy and will also help to explain the solution to anyone reading your answer.

Decide which direction is positive and which is negative.

It is important to decide which directions are positive (+) and which are negative (–) with respect to the reference point or origin of the coordinate system. Stick to this system while you are working through the problem.

Write down in symbolic form what you are given. Do this by writing down the values of the variables in the problem, paying careful attention to the sign of these variables.



You determine which variable you have by looking at its unit. In the part of the SI system that we use in physics, we use another acronym “**mks**” for the units of **length**, **mass**, and **time**. The unit for **length** is the metre (m); the unit for **mass** is the kilogram (kg); the unit for **time** is the second (s). Almost all of the other units that you will be using can be derived from these three basic units.

For the section on kinematics, important variables are **acceleration**, $\vec{a}$, with units of m/s/s; **velocity**, $\vec{v}$, with units of m/s; **time**, t, with units of s; and **displacement**, $\vec{d}$, or distance, d, both of which have units of m. Some of the data may be implied, such as the phrase "starts from rest." This means that the initial velocity is zero and should be written as $\vec{v} = 0 \text{ m/s}$.

For any quantities that are vectors, you must include the direction.

This first step in the problem-solving process is the most important. If you cannot extract the given information on the problem, then there's no way of solving it!

Unknown

Every problem asks you to determine an answer. This part of the problem-solving process requires you to identify the variable you're being asked to determine.

Equation

Now that you know the variables that were given in the question and the unknown that you looking for, you must use an equation that links what is given to the unknown that you are required to find.

At this point, you may rearrange the equation to isolate the variable that represents the unknown. This usually facilitates the algebra that you must perform to solve the problem.

Substitute

Now that you have the equation, substitute the values of the given variables into the equation.

At the beginning, it may be wise to include the units with the variables. One method of checking your answers is to see that the units on the left side will be the same as the units on the right side of your equation.

Solve

The last step in this process is to do the mathematics and determine the value of the unknown.

You should include a statement at the end of the problem stating the value of the unknown, its proper unit, and, if it is a vector, include a direction. Be sure to round off the answer to the correct number of significant digits.

Note: In some problems, there may be no equation that directly relates the given information to the unknown variable. In cases like these, it may be necessary to solve for some other variable. Once you have this other variable, you can then solve for the variable that is required using a different equation.

Obviously, you need to know some physics to be able to solve problems correctly. If you do not understand the concept about which the problem is inquiring, then there is little hope you'll be able to solve the problems successfully.

In the previous lesson, five equations for kinematics were developed.

The first equation was for uniform motion, where the acceleration is 0 m/s/s.

This equation was $\vec{v} = \dfrac{\vec{d}}{\Delta t}$.

The other four kinematics equations that were derived were for uniformly accelerated motion.

They were

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \text{ or } \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \text{ or } \vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

$$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)\Delta t$$

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$v_2^2 = v_1^2 + 2ad$$

There are a couple of other items to take note of: the signs of velocity and acceleration, and the units that are not given as standard SI units.

The Signs of Velocity and Acceleration

When you observe the motion of an object, it is relatively simple to determine the direction of the displacement and velocity because it is obvious to you. If an object moves to the right, it has a velocity to the right and a displacement to the right. If an object moves to left, it has a displacement to the left and a velocity to the left. In place of right and left, you could use east and west, up and down, north and south, positive and negative, or any other convenient reference system.

The direction of the acceleration is less obvious. To ascertain the direction of the acceleration, use the following rule:

> If the object is **speeding up**, the velocity and acceleration have the **same direction**.
>
> If the object is **slowing down**, the velocity and acceleration have the **opposite direction**.

Units

In solving problems, you must be sure that units are consistent. You cannot have a velocity in units of **km/h** and a time in **minutes**. In order to solve problems like this, you should convert the given units into "mks" (metres, kilograms, and seconds).

Converting from one unit to another is relatively simple. Basically, it involves taking the original given units and replacing them with an equivalent amount of the required units. A common conversion you might have to perform would be changing kilometres per hour into metres per second.

Recall that 1 kilometre equals 1000 m and that one hour equals 60 minutes x 60 seconds/minute or 3600 seconds.

Here is how you would convert 90 kph to metres per second:

$$90 \text{ kph} = \frac{90 \text{ km}}{1 \text{ h}} = \frac{90\,000 \text{ m}}{3600 \text{ s}} = 25 \text{ m/s}$$

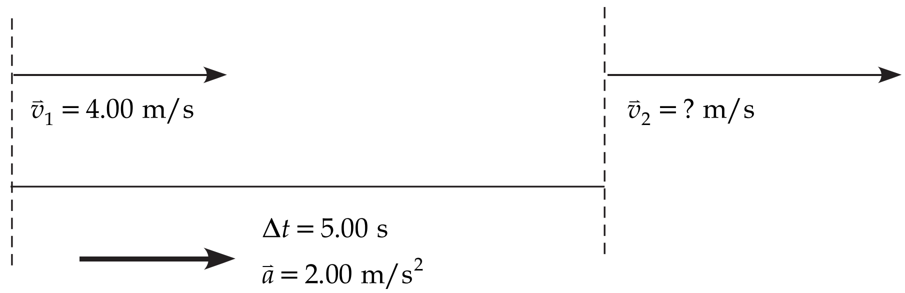
We will now look at examples of using the equations in solving problems, and at the same time applying the problem-solving strategy.

Example 1: Using Equations for Constant Acceleration, Finding Velocity

A speedboat has an acceleration of 2.00 m/s^2. What would the final velocity of the speedboat be after 5.00 seconds if the initial velocity of the speedboat is 4.00 m/s?

Let us apply the problem-solving strategy strictly to the solution of this problem.

Given: Make a drawing.



The drawing shows a vector on the left representing the initial velocity and another longer vector on the right representing the second velocity.

Decide which direction is positive and which is negative.

It is convenient to make the direction to the right positive and the left negative.

Write down in symbolic form what you are given.

The symbols for velocity, acceleration, and time are shown beside the diagrams. It is clear what is given.

Unknown: final velocity $\vec{v}_2 = ?$

Equation: Select the appropriate equation and solve the problem.

The equation must contain $\vec{v}_1$, $\vec{a}$, and Δt as the known quantities. It should also contain $\vec{v}_2$ as the unknown.

In this case, the appropriate equation is $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$. This form of kinematics equation 2 is chosen because the unknown $\vec{v}_2$ is already isolated.

Substitute: $\vec{v}_2 = 4.00 \text{ m/s} + \left(2.00 \text{ m/s}^2\right)\left(5.00 \text{ s}\right)$

Solve: $\vec{v}_2 = 4.00 \text{ m/s} + \left(2.00 \text{ m/s}^2\right)\left(5.00 \text{ s}\right) = +14.0 \text{ m/s}$

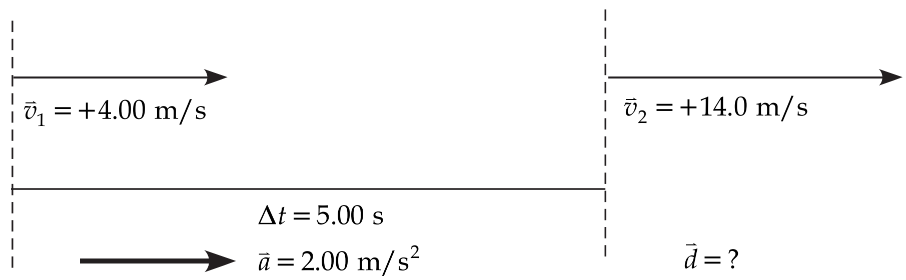
The final velocity of the speedboat is +14.0 m/s.

Example 2: Using Equations for Constant Acceleration, Finding Displacement

This example is a continuation of example 1, using the newly found final velocity to help us solve another problem.

A speedboat has an acceleration of 2.00 m/s^2. What would the final velocity of the speedboat be after 5.00 seconds if the initial velocity of the speedboat is 4.00 m/s? What is the displacement of the speedboat during the 5.00 s time interval?

Given: The diagram above would still apply with the addition of the second velocity, which we now know, and the symbol $\vec{d}$ for the displacement.



Unknown: displacement $\vec{d} = ?$

Equation: One appropriate equation would be

$$\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t.$$

Substitute:

$$\vec{d} = \frac{1}{2}(+4.00 \text{ m/s} + 14.0 \text{ m/s})(5.00 \text{ s})$$

Solve:

$$\vec{d} = \frac{1}{2}(+18.0 \text{ m/s})(5.00 \text{ s})$$
$$= (+9.00 \text{ m/s})(5.00 \text{ s}) = +45.0 \text{ m}$$

The displacement of the boat during the 5.00 s time interval is 45.0 m in a positive direction—that is, to the right.

Another appropriate equation would be $\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$.

$$\vec{d} = (4.00 \text{ m/s})(5.00 \text{ s}) + \frac{1}{2}(2.00 \text{ m/s}^2)(5.00 \text{ s})^2 = 45.0 \text{ m}$$

Note that this is the same solution as was obtained using the other equation. This example illustrates that in physics problems, there is often more than one method to obtain an answer. As long as the reasoning is sound, any method may be used.

Learning Activity 1.6

The “GUESS” Method

Try solving the following problems to practise the “GUESS” method. You may check your work against the answer key available at the end of Module 1.

1. A child on a toboggan starts from rest and accelerates down a snow-covered hill at 0.800 m/s/s. How long does it take the child to reach the bottom of the hill if it is 25.0 m away?
2. A car accelerates uniformly from a velocity of 21.8 m/s [W] to a velocity of 27.6 m/s [W]. The car travels 36.5 m [W] during this acceleration.
   a) What was the acceleration of the car?
   b) Determine the time interval over which this acceleration occurred.

Example 3: Using Equations for Constant Acceleration for Motion in Two Parts, Each with a Different Acceleration

A motorcycle starts from rest and accelerates at +2.50 m/s^2 for a distance of 150.0 m. It then slows down with an acceleration of –1.50 m/s^2 until the velocity is +10.0 m/s. Determine the total displacement of the motorcycle.

We will do this question in two parts. The first part of the problem occurs when the motorcycle is accelerating and the second part is when it is decelerating. The total displacement will be the sum of the two displacements. The displacement during the first part of the journey is already given ($\vec{d}$ = +150.0 m). Before the second part of the problem can be done, it is necessary to determine the velocity of the object at the start of this segment. This velocity will be the final velocity for the first segment.

Part A: Motorcycle Speeding Up

Given: Make a drawing.

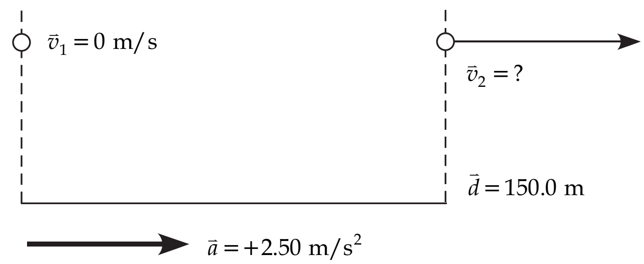
The drawing shows the second velocity vector, the displacement vector, and the acceleration vectors all pointing to the right.

Decide which direction is positive and which is negative.

It is convenient to let the direction to the right be positive and the direction to the left be negative.

Write down in symbolic form what you are given.

The symbols for velocity, displacement, and acceleration are shown.



Unknown: final velocity for the first part of the journey

$$\vec{v}_2 = ?$$

Equation: In this case, we will use kinematics equation 5,

$$v_2^{\,2} = v_1^{\,2} + 2ad$$

Substitute:

$$v_2^{\,2} = v_1^{\,2} + 2ad$$

$$v_2^{\,2} = (0\text{ m/s})^2 + 2(+2.50\text{ m/s}^2)(+150.0\text{ m})$$

Solve:

$$v_2 = \sqrt{(0\text{ m/s})^2 + 2(+2.50\text{ m/s}^2)(+150.0\text{ m})}$$

$$\vec{v}_2 = +27.4\text{ m/s}$$

The motorcycle is travelling at 27.4 m/s in the positive direction or to the right.

Part B: Motorcycle Slowing Down

Given: Make a drawing.

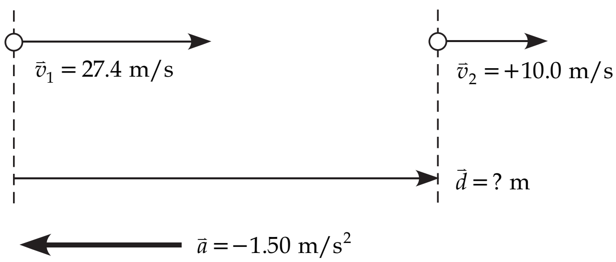
The drawing shows the second velocity vector and the displacement vector pointing to the right, and the acceleration vector pointing to the left.

Decide which direction is positive and which is negative.

It is convenient to let the direction to the right be positive and the direction to the left be negative.

Write down in symbolic form what you are given.

The symbols for velocity, displacement, and acceleration are shown.



Unknown: The unknown is the displacement while the motorcycle is slowing down.
$\vec{d} = ?$

Equation: In this case, the most appropriate equation is $v_2^2 = v_1^2 + 2ad$.

Solving for the displacement, $d = \dfrac{v_2^2 - v_1^2}{2a}$.

Substitute:

$$d = \frac{(10.0 \text{ m/s})^2 - (27.4 \text{ m/s})^2}{2(-1.50 \text{ m/s}^2)}$$

Solve:

$$= \frac{100 - 751}{-3.00}$$

$$= \frac{-651}{-3.00}$$

$$= +217 \text{ m}$$

Therefore, the total displacement is +150.0 m + (+217 m) = +367 m.

Learning Activity 1.7

Motion with Different Accelerations

Try solving the following problem to practise dividing a motion into different sections according to the acceleration. You may check your work against the answer key provided at the end of Module 1.

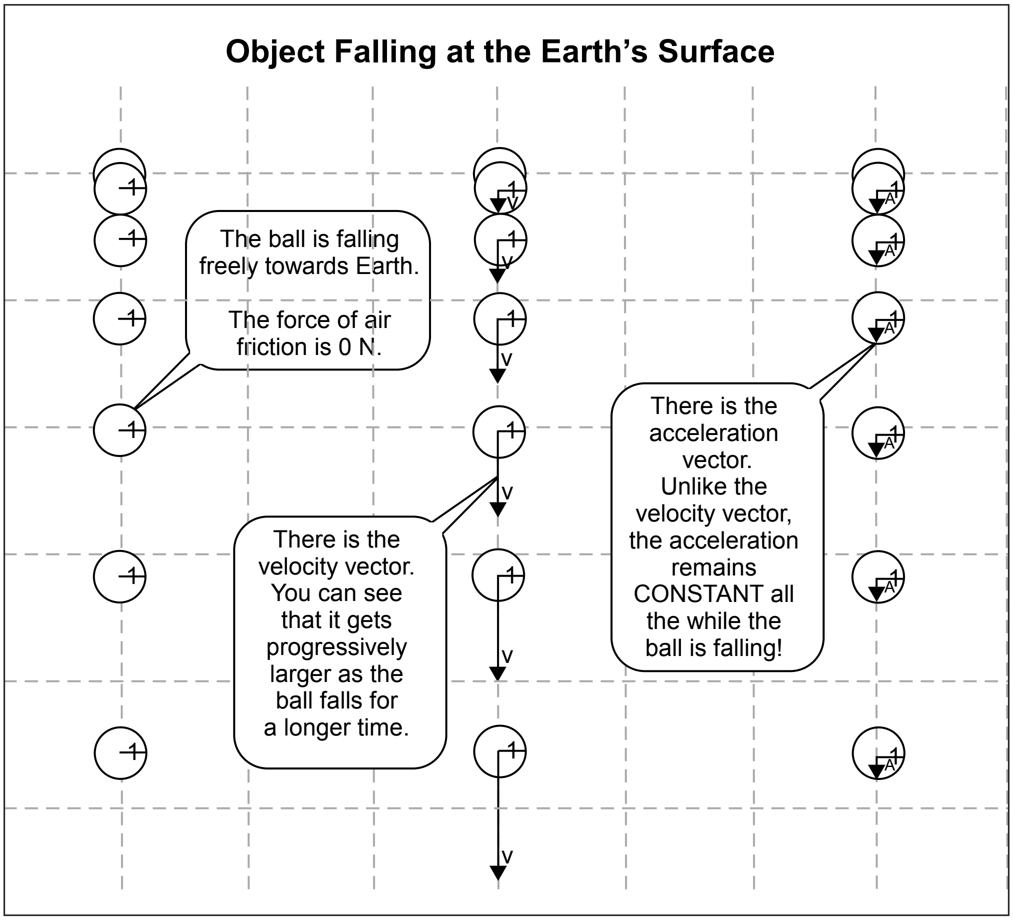
1. A ball rolls down an inclined plane, across the horizontal surface of a table, and then up a second inclined plane. Describe the type of motion that the ball undergoes on each of the surfaces. Include velocity and acceleration and their signs.

Vertical Motion at Earth's Surface

In the topic of "Fields," which is studied in Grade 11 Physics, you studied gravitational fields, and found that everywhere near Earth's surface the gravitational field is very constant at 9.80 N/kg pointing towards the centre of Earth. You also discovered that, when dropped, all objects regardless of their mass would fall towards the centre of Earth, accelerating at 9.80 m/s/s towards the centre of Earth if the force of air friction was reduced to 0 N.

The implications of these ideas are that, if the force of air friction is 0 N, any object moving freely near Earth's surface will be accelerating at 9.80 m/s/s towards the centre of Earth. If you confine the motion of an object to the vertical direction (straight up or down), then the acceleration in that situation will always be 9.80 m/s/s pointing towards the centre of Earth (down).

The following graphic illustrates the velocity vectors and acceleration vector for a free-falling object.



Example 4: Using the Equations for Constant Acceleration to Analyze the Motion of Objects Moving in the Vertical Direction

All of us have thrown an object straight upwards. Consider a baseball that is tossed straight upwards at 20.0 m/s. Air friction is negligible (equals 0 N).

a) To what height above the point of release will the ball rise?

b) At what time will the ball be at its maximum height?

c) Where will the ball be 3.00 seconds after being released?

It may seem that there is insufficient information given in the question for you to be able to solve the problem.

However, for motion in the vertical direction, you can assume that, if air friction is negligible, the value of the acceleration is 9.80 m/s/s pointing towards the centre of Earth (down). You can also assume that at the top of its flight, the object momentarily comes to rest. This is the point of the maximum height, and you can assign a velocity at that point to be 0 m/s. Thirdly, you can divide the journey of the object into two parts: part one while it is rising, and part two while it is falling. With these additional ideas, the analysis of motion in the vertical direction becomes quite simple.

a) To what height above the point of release will the ball rise?

For this part of the problem, you should consider only the motion of the ball while it is rising.

Make a drawing.

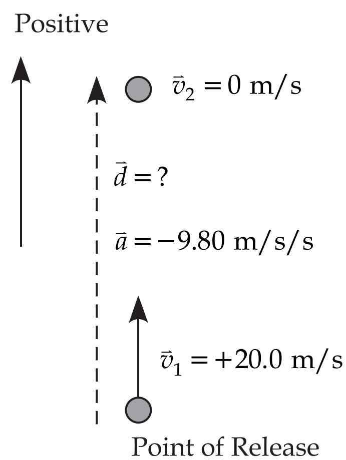
The drawing shows the initial velocity vector, the acceleration vector, and the unknown displacement vector. The initial velocity vector points upwards, the acceleration vector points downwards, and the unknown displacement vector points upwards.

Decide which direction is positive and which is negative. It is convenient to let up be positive and down be negative.

Write down in symbolic form what you are given.

The symbols for the velocities, displacement, and acceleration are shown.

Given:



Unknown: The unknown is the displacement while the ball is rising, slowing down as it rises.

$\vec{d} = ?$

Equation: In this case, the most appropriate equation would be $v_2^2 = v_1^2 + 2ad$.

Solving for the displacement $d = \dfrac{v_2^2 - v_1^2}{2a}$.

Substitute: $\vec{d} = \dfrac{(0\ \text{m/s})^2 - (+20.0\ \text{m/s})^2}{2(-9.80\ \text{m/s/s})}$

Solve: $\vec{d} = \dfrac{0-(400)}{(-19.6)} = \dfrac{-400}{-19.6} = +20.4\ \text{m}$

The ball rises to a height of 20.4 m above the point of release.

b) At what time will the ball be at its maximum height?

Make a drawing.

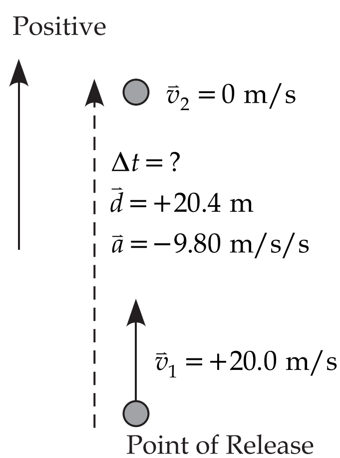
The drawing shows the initial velocity vector, the acceleration vector, and the displacement vector. The initial velocity vector points upwards, the acceleration vector points downwards, and the displacement vector points upwards.

Decide which direction is positive and which is negative. It is convenient to let up be positive and down be negative.

Write down in symbolic form what you are given.

The symbols for the velocities, displacement, acceleration, and time interval are shown.

Given:



Unknown: The unknown is the time interval while the ball is rising, slowing down as it rises.
$\Delta t = ?$

Equation: In this case, you have a choice of several equations that can be used to find the time interval. One of your choices is

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}.$$

You can rearrange the equation to isolate the time interval giving $\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$.

Substitute: $$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{(0\text{ m/s}) - (+20.0\text{ m/s})}{-9.80\text{ m/s/s}}$$

Solve: $$\Delta t = \frac{(-20.0)}{(-9.80)} = 2.04\text{ s}$$

The ball takes 2.04 seconds to rise to its maximum height.

c) Where will the ball be 3.00 seconds after being released?

Since it takes 2.04 seconds for the ball to reach its maximum height, after 3.00 seconds the ball should have risen to the top of its flight, stopped, and be falling downwards. Therefore, the ball should be somewhere below its maximum height.

This problem will require a different diagram since the final velocity is not 0 m/s, as it was when the ball was at its maximum height. This is a common error by students. For these vertical motion problems, students wrongly assume that the final velocity is always 0 m/s. This is only the case when the ball is at the top of its flight.

Make a drawing.

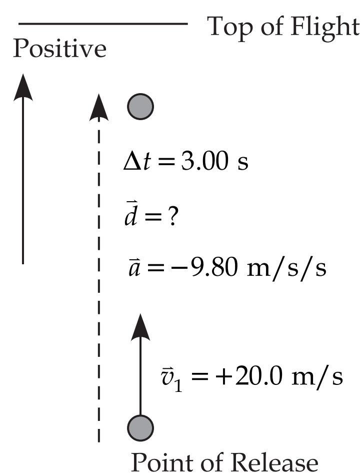
The drawing shows the initial velocity vector, the acceleration vector, and the displacement vector. The initial velocity vector points upwards, the acceleration vector points downwards, and the displacement vector points upwards.

Decide which direction is positive and which is negative. It is convenient to let up be positive and down be negative.

Write down in symbolic form what you are given.

The symbols for the initial velocity, acceleration, and time interval are shown.

Given:



Unknown: The unknown is the displacement after 3.00 seconds.

$\vec{d}_{3.00\text{s}} = ?$

Equation: The appropriate equation to use in this case is kinematics equation number four:

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Substitute:

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$= (+20.0 \text{ m/s})(3.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s/s})(3.00 \text{ s})^2$$

Solve:

$$\vec{d}_{3.00\text{s}} = (+60.0 \text{ m}) + \frac{1}{2}(-88.2 \text{ m})$$

$$= (+60.0 \text{ m}) + (-44.1 \text{ m})$$

$$= +15.9 \text{ m}$$

At 3.00 seconds after the moment the ball is released, the ball is located at a position of +15.9 m or 15.9 m above the point of release.

Learning Activity 1.8

Motion in the Vertical Direction

There are six practice questions in this learning activity. An answer key is available at the end of Module 1 for you to check your work after you have answered the questions.

The physics of acceleration and a permanent stop (conceptual)

1. It is possible for a car moving with a constant acceleration to slow down. Can the car ever come to a permanent stop if its acceleration truly remains constant? Explain.

The physics of the signs of velocity and acceleration (conceptual)

2. For each of the following situations, give the directions of the velocity and acceleration.
   a) A ball is dropped and falls to the floor, speeding up as it falls.
   b) A ball is thrown upwards and slows down as it rises.
   c) A car moving to the east coasts to a stop.
   d) You are riding a bicycle down the hill. You and your bicycle go faster and faster.

The physics of an accelerating spacecraft

3. A spacecraft is moving at a speed of +3550 m/s. The retrorockets of the spacecraft are then fired and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0 m/s^2. Assume that the displacement of the craft is +225 km. What is the velocity of the craft after this displacement?

The physics of an accelerating skier

4. a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a velocity of –8.00 m/s when going down a slope for 4.00 s?
   b) How far does the skier travel in this time?

The physics of an accelerating rocket

5. During an interval of 20.0 s, a rocket's velocity increased from 255 m/s to 555 m/s. What was the displacement of the rocket during this time interval?

The physics of an object moving in the vertical direction near Earth's surface

6. You are standing on a bridge that spans a ravine. You throw a stone downwards from the bridge towards the bottom of the ravine. The stone travels for 2.50 seconds from the moment it leaves your hand until it strikes the bottom of the ravine 58.1 m below. What was the velocity of the stone when it left your hand?

Lesson Summary

In this lesson, we concentrated on applying the equations for uniformly accelerated motion to four problems. In doing the problems, it is a good idea to keep in mind the "GUESS" problem-solving strategy.

The word "GUESS" is an acronym that stands for the following:

Given
Unknown
Equation
Substitute
Solve

Remember that in the "GIVEN" part of the process, you should include a diagram to help visualize and clarify the situation. As well, be sure to include a coordinate system to give you the directions for any variables that are vectors. The units for the variables given in the question will indicate to you which variable is represented.

In solving problems, it is important to follow a good strategy and to be organized in your approach.

As you work through this course, remember that your learning partner and your tutor/marker are available to help you if you have questions or need assistance with any aspect of the course.

Assignment 1.1

Equations of Motion (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. **Submit this assignment to the Distance Learning Unit, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.**

The physics of an accelerating electron

An electron is accelerated from rest to a velocity of 2.00×10^7 m/s.

a) If the electron travelled 0.100 m while it was being accelerated, what was its acceleration?

(continued)

Assignment 1.1: Equations of Motion (continued)

b) How long did the electron take to attain its final velocity? In your answer, be sure to include all the steps for solving kinematics problems.

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for drawing an appropriate diagram with labels showing what is given and what is required in question (a)
- 1 mark for selecting the appropriate equation and doing the algebra correctly in question (a)
- 1 mark for the correct solution with the correct units in question (a)
- 1 mark for using an appropriate equation in question (b)
- 1 mark for the correct solution with the correct units in question (b)

Video - Introduction to Acceleration with Prius Brake Slamming Example Problem

A problem involving accelerated motion is solved by the class. There are several steps in converting given information into SI units. Mistakes are made along the way and corrected.

https://youtu.be/mzpwV75Beeo

Video - A Basic Acceleration Example Problem and Understanding Acceleration Direction

A person on a bicycle demonstrates several motions – speeding up, slowing down, moving right, and moving left- and these motions are related to the direction of the acceleration.

https://youtu.be/Mg8NsHpaDrY

Relating the Motion of an Object to the Sign of the Acceleration (II)

Launch Concept Builder

https://www.physicsclassroom.com/Concept-Builders/Kinematics/Acceleration

Click on the link Launch Concept Builder. Practice identifying the direction of the acceleration given some information about the motion.

Physics Lecture: Uniform Acceleration Motion

[Physics Lecture: Uniform Acceleration Motion](https://youtu.be/00pRpgfBOjU)

https://youtu.be/00pRpgfBOjU

This video presents a summary of the 4 equations for uniformly accelerated motion.

While the symbols for the quantities of motion are the same as those used in our course materials, the order of the equations differs from the order used in the course materials.

A strategy is presented whereby a student can select the correct equation by analyzing the quantities given in the question.

Video - Uniformly Accelerated Motion Examples

This video works through 2 sample problems of uniformly accelerated motion. The instructor follows a systematic approach to problem solving similar to the GUESS method but with the U first and then G.

For displacement the instructor uses X – X0 where X represents the final position and X0 represents the initial position.

The instructor mentions several strategies that typically must be employed to interpret correctly the information given in the question.

https://youtu.be/cGvruPrbaRg

Video - Kinematic Equations

The kinematics equations are used to solve problems involving straight line motion. Some problems involve horizontal motion and some involve vertical motion.

https://youtu.be/nHmZEunBWgY

Video - 1D KINEMATIC MOTION PRACTICE - Acceleration Example Problem

Two sample problems involving uniformly accelerated motion are solved using the acceleration equation.

https://youtu.be/VVueCq2HUJQ

Assignment 1.1

Equations of Motion (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. **Submit this assignment to the Distance Learning Unit, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.**

The physics of an accelerating electron

An electron is accelerated from rest to a velocity of 2.00×10^7 m/s.

a) If the electron travelled 0.100 m while it was being accelerated, what was its acceleration?

(continued)

Assignment 1.1: Equations of Motion (continued)

b) How long did the electron take to attain its final velocity? In your answer, be sure to include all the steps for solving kinematics problems.

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for drawing an appropriate diagram with labels showing what is given and what is required in question (a)
- 1 mark for selecting the appropriate equation and doing the algebra correctly in question (a)
- 1 mark for the correct solution with the correct units in question (a)
- 1 mark for using an appropriate equation in question (b)
- 1 mark for the correct solution with the correct units in question (b)

Lesson 5: A Review of Working with Vectors (1 hour)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ add parallel and antiparallel vectors
- ❑ add perpendicular vectors
- ❑ resolve vectors into their components
- ❑ subtract both parallel and antiparallel vectors and perpendicular vectors

Key Words

vectors	vector addition	vector subtraction
vector components	resolving vectors	resultant
parallel vectors	antiparallel vectors	

Introduction

In Grade 11 Physics, you were introduced to vectors and how to work with vector quantities when solving physics problems. Since vectors are such a prevalent part of the work that we will be doing in this course, it will be useful to have a quick review of how to work with vectors.

Representing Vectors on a Diagram

To begin, recall that vector quantities are those quantities that have both a magnitude or size and a direction.

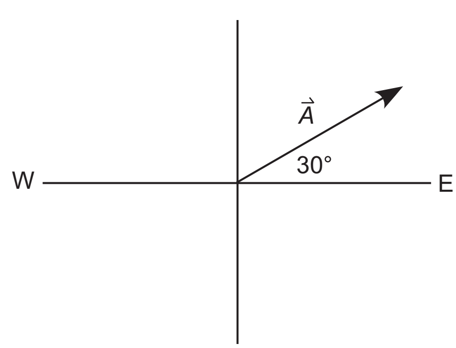
> A **vector** is a quantity that requires both magnitude and direction in order to be completely described.
>
> A **scalar** is a quantity that can be expressed completely by single numbers with appropriate units. Direction is not required.

Back in Lesson 1, you saw that **speed** represents how fast one is travelling, while **velocity** represents how fast one is travelling in a particular direction. Speed is a scalar. The fact that velocity requires a direction makes velocity a vector.

Vectors are represented using a **directed line segment (an arrow)**.

The **length** of the vector represents the **magnitude** of the quantity. The arrow points in the direction of the vector.

To represent a vector, we use a directed line segment commonly called an arrow. The length of the arrow represents the magnitude or size of the vector. The end of the arrow with the arrowhead is called the **head** of the vector, and the end of the arrow without the arrowhead is called the **tail** of the vector. The arrowhead points in the direction of the vector. The **direction** of the vector, though, is measured at the **tail of the vector**. When drawing a vector, we always mark the direction at the tail of the vector between the vector and one of the axes of our coordinate system, as indicated in the diagram below.



The vector being represented is the vector $\vec{A}$. Let's say that $\vec{A}$ represents a displacement of 10.0 m [30° north of east].

In drawing a scale diagram, we must choose a scale to represent how the sizes of lines on the diagram are related to the real-life quantity. In this case, an appropriate scale would be 1 cm = 2 m.

The length of the vector arrow should then be 5.0 cm.

To draw this vector, you would first draw in and label the coordinate system. You would want to place the tail of the vector at the origin.

The direction given for the vector [30° north of east] indicates that the vector will be drawn in the northeast quadrant. For this "Degree-Direction-Direction" notation, you start measuring the angle from the axis given by the second "Direction," in this case east. The angle is then made by moving towards the first "Direction," in this case north.

To draw the vector pointing in the correct direction, you must measure the angle at the foot of the vector and mark it on the diagram. Place the crosshairs of a protractor at the origin and, starting from east, measure an angle going 30° north. Place a mark on your paper to indicate this direction.

The next step is to draw in the vector arrow. The length represents the size or magnitude of the vector.

Finally, draw in the vector arrow 5.0 cm long, starting at the origin out towards the mark, indicating 30° north of east. Be sure to place an arrowhead at the head of the vector.

In this course, you will not be required to solve problems using a scale diagram for vectors. You will be using an algebraic method for that purpose. However, at this point, it is very important that you understand how to interpret the given angle and place the vector correctly on the diagram.

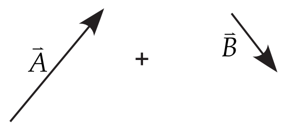
Vector Addition

When you add two numbers together, you call the answer the "sum." For vectors, we may call the answer to an addition problem the sum, but we instead often refer to it as the **resultant** $\vec{R}$. The word "resultant" simply means the answer you get when adding vectors.

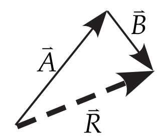
Graphical Technique of Vector Addition

One method for adding vectors is called the graphical technique.

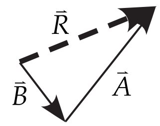
In the graphical technique, you draw the vectors carefully with a ruler, to scale, and place them tip to tail. The resultant can then be found by drawing in a new arrow that starts at the tail of the first vector, and extends straight out to the tip of the last arrow, as shown on the following page. Notice that relocating a vector does not change the vector as long as the size and orientation are not changed.



First place tip-to-tail



Or, you can add vector $\vec{A}$ to vector $\vec{B}$.



The newly drawn resultant could then be measured with a ruler, and its direction determined with a protractor. This technique is limited by the exactness of the drawing and measurements.

Tip-to-Tail Vector Addition

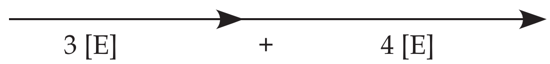
1. Draw a coordinate system and sketch the first vector with its tail at the origin.
2. Draw the second vector with its tail on the head of the first vector.
3. Draw the resultant vector with its tail beginning on the tail of the first vector and its head drawn at the head of the last vector.
4. Determine the magnitude and direction of the resultant vector using the scale and a protractor or the theorem of Pythagoras and trigonometry.

Adding Parallel Vectors

With some thought, you may realize that it is not necessary to go through the steps of carefully drawing out vectors to be added. Vectors may be added mathematically. Parallel vectors are the simplest. They are vectors that point in the exact same direction. When these kinds of vectors are to be added, the resultant will simply be the length of the two vectors combined, and will be pointing in the same direction. This can be nicely seen in the following example:

Example: Add the vectors $\vec{A} = 3\,[\mathrm{E}]$ and $\vec{B} = 4\,[\mathrm{E}]$.

Rather than carefully drawing them, consider a rough sketch of the vectors placed tip-to-tail:



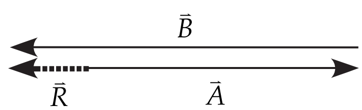
The resultant is the new vector found from the tail of the first vector, and ending at the tip of the last vector. Clearly, this will be the vector that is 7 [E]

This means that the question can now be answered as 3[E] + 4[E] = 7[E].

Adding Antiparallel Vectors

Antiparallel vectors are vectors that point in exactly opposite directions. Again, rather than carefully drawing them, it should be clear from a rough sketch that the resultant will now be the difference (subtract!) between the two, and will point in the direction of the larger vector.

Example: Add the vectors $\vec{A} = 3\,[E]$ and $\vec{B} = 4\,[W]$.



This means that the question can now be answered as 3 [E] + 4 [W] = 1 [W].

Learning Activity 1.9

Introduction to Vectors

Answer the following questions to check your understanding of working with vectors. You may check your work against the answer key provided at the end of Module 1.

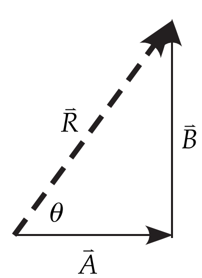
1. Sketch the following vectors. You do not have to make an accurate scale diagram, but try to draw these in an appropriate size.
   a) 80 m [20° E of N]
   b) 500 m [20° S of W]
2. Add the following vectors:
   a) $\vec{A}$ = 202 m [W] and $\vec{B}$ = 357 m [W]
   b) $\vec{A}$ = 202 m [W] and $\vec{B}$ = 357 m [E]

Adding Perpendicular Vectors

Upon adding perpendicular vectors with a rough sketch, you will find that a **right triangle** is formed. The resultant will be the **hypotenuse**, while the two vectors being added are the **legs**. Finding the size of the resultant is done with the Pythagorean theorem, and the direction it is pointing can be found by using trigonometry (inverse tangent to be exact). Note that the angle θ must be placed inside the triangle at the tail end of the resultant.

Example: Add the vectors $\vec{A}$ = 3.00 [E] and $\vec{B}$ = 4.00 [N]; call the answer the resultant $\vec{R}$.

Placing $\vec{A}$ and $\vec{B}$ tip-to-tail and drawing in the resultant gives the right triangle shown (note that the angle *theta* was also labelled at the tail end of the resultant):



Using the Pythagorean theorem, we find $\vec{R}$ = 5.00, and, using the inverse tangent, we find that θ = 53.1°. Looking at the sketch, we can then say that $\vec{R}$ = 5.00 [53.1° N of E].

Subtracting Vectors: The subtraction of vectors is accomplished by changing the subtraction question into an addition question by "adding the opposite" of the vector being subtracted.

$$\vec{A} - \vec{B} = \vec{R} \qquad \text{becomes} \qquad \vec{A} + \left(-\vec{B}\right) = \vec{R}$$

Resolving Vectors

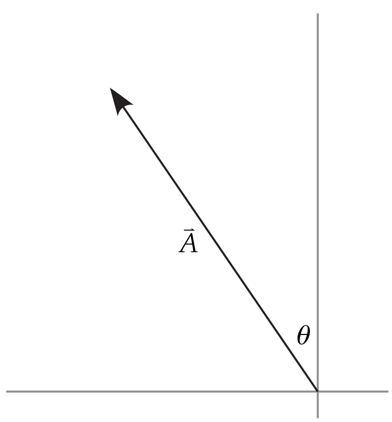
Perhaps surprisingly, the opposite of adding vectors is not subtracting vectors. Adding vectors is the process of combining two vectors into one. **Resolving** is the process of taking one vector and "breaking it" into two. Each of the "vector fragments" is called a **component** of the original vector.

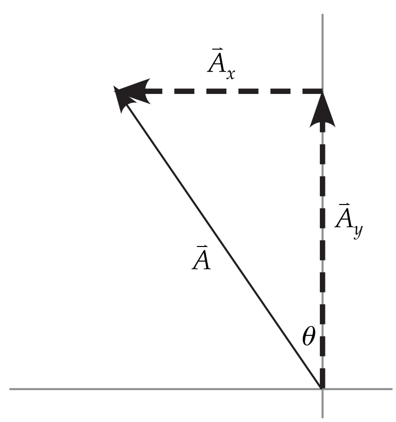
The main reason why this will need to be done is that we often prefer vectors that point in convenient directions, such as north, east, south, and west. Vectors that do not point in a convenient direction can be made into a pair of vectors—each of these vectors pointing in one of the preferred directions.

To resolve a vector, simply consider the vector to be the hypotenuse of a soon-to-be-drawn right triangle, which has one leg that is horizontal (the "x-component"), and one that is vertical (the "y-component"). By knowing the size of the original vector and the direction it is pointing, the sizes of the components can be found with trigonometry.

Example: Resolve the vector $\vec{A}$ = 10.0 m [30.0° W of N].

The vector is drawn here. Notice that the vector slants between north and west, which means that it has a component in each of these directions.





The components have been drawn in with a perpendicular from the tip of the vector to one of the axes, so that a right triangle is formed that encloses the given angle. Since the original vector was labelled $\vec{A}$, the components have been labelled $\vec{A}_x$ and $\vec{A}_y$.

The size of the components can now be found. Note that this will always involve using sine and cosine, but which component is found with cosine and which with sine can change. Remember that the length of the hypotenuse is known to be 10.0 m.

$$\sin 30.0° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\vec{A}_x}{10.0\text{ m}}, \text{ which means when we solve for } \vec{A}_x$$

$$\vec{A}_x = (10.0\text{ m})\sin(30.0°), \text{ so}$$

$$\vec{A}_x = 5.00\text{ m [W]}$$

$$\cos 30.0° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\vec{A}_y}{10.0\text{ m}}, \text{ which means when we solve for } \vec{A}_y$$

$$\vec{A}_y = (10.0\text{ m})\cos(30.0°), \text{ so}$$

$$\vec{A}_y = 8.66\text{ m [N]}$$

Components of Vectors

To sum up components of vectors, we can list three properties that are worth remembering:

1. Components of a vector are mutually perpendicular and each component will point in a direction along one of the major axes of your coordinate system. For example, a vector pointing in the northeast direction will have a component pointing in a northerly direction and a second component pointing in the easterly direction.
2. The components of a vector add up to give the original vector. We will be using this idea later on to add vectors together.
3. The components of the vector are independent of each other. This is less obvious. What this means is that for a velocity vector pointing in a northeasterly direction, the northerly component can be dealt with separately from the easterly component. You can treat these motions separately, as motion in the northerly direction or as motion in the easterly direction. This will be a great help in future lessons.

Learning Activity 1.10

Adding, Subtracting, and Resolving Vectors

Answer the following questions to help you recall and reinforce the ideas about vectors that have been discussed in this lesson. You may check your work against the answer key provided at the end of Module 1.

1. Sketch the following vectors. You do not have to make an accurate scale diagram, but try to draw these in an appropriate size.
   a) $\vec{A}$ = 35.0 m [40.0° W of S]
   b) $\vec{B}$ = 50.0 m [20.0° S of W]
   c) $\vec{C}$ = 20.0 m [30.0° S of E]
2. Resolve the vectors in #1 into their components.
3. Add the following vectors:
   a) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [W]
   b) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [E]
   c) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [S]
4. Perform the subtractions as indicated. Remember to treat subtraction as "the addition of the opposite."
   a) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [W], find $\vec{A} - \vec{B}$
   b) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [W], find $\vec{B} - \vec{A}$
   c) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [S], find $\vec{B} - \vec{A}$

Lesson Summary

A **vector** is a quantity that requires both magnitude and direction in order to be completely described.

Scalar is a quantity that can be expressed completely by single numbers with appropriate units. Direction is not required.

A vector can be **resolved** into two components that are mutually perpendicular.

Resolving a vector is completed in the following fashion:

1. Draw a coordinate system and sketch the vector with its tail at the origin. Mark the angle at the tail of the vector.
2. From the head of the vector, drop a perpendicular to one of the coordinate axes so as to include the angle at the tail of the vector. This should form a right triangle consisting of the given vector as the hypotenuse, and the two legs representing the two components of the vector.
3. Using the trigonometric functions (sine, cosine, and tangent), determine the magnitude of the two components.

The addition of vectors involves the **tip-to-tail** method. In this process, follow these steps:

1. Draw a coordinate system and sketch the first vector with its tail at the origin.
2. Draw the second vector with its tail on the head of the first vector.
3. Draw the resultant vector with its tail beginning on the tail of the first vector and its head drawn at the head of the last vector.

To add **parallel vectors**, add the magnitudes of the given vectors to obtain the magnitude of the resultant. The direction of the resultant is the same as the direction of the given vectors.

To add **antiparallel vectors**, obtain the magnitude of the resultant vector by finding the difference between the magnitudes of the two given vectors. The direction of the resultant is given by the direction of the larger of the given vectors.

To add vectors that are **perpendicular** to each other, draw a sketch following the tip-to-tail method. To determine the magnitude of the resultant vector, use the theorem of Pythagoras. To determine the direction of the resultant vector, use the tangent function.

To **subtract vectors**, change the question from subtraction to addition by **”adding the opposite”** of the vector that is being subtracted. Then add the vectors, as outlined in the steps above. The **“opposite”** of a given vector is just a vector of the same length pointing in the opposite direction.

NOTES

Vector Components

Vector Components

https://www.walter-fendt.de/html5/phen/forceresolution_en.htm

For the link above, set the two angles so that their sum is 90 degrees. This will result in the applet drawing the components in the x-y coordinate system.

Video - Introduction to Tip-to-Tail Vector Addition, Vectors and Scalars

This video is a good review of vector and scalars. There are many examples provided. The video also shows how to use the tip to tail method of arranging vectors to add the vectors together.

https://youtu.be/ZYl9-iz7nR8

Video Lecture - How to use Cardinal Directions with Vectors

The video describes how to state the direction of a vector using the "direction of direction" format for the angles direction.

https://youtu.be/UWn3u6kv1Wk

Video - Introductory Tip-to-Tail Vector Addition Problem

This video illustrates how to add 2 perpendicular vectors together. The motion along with velocity vectors are demonstrated very well.

https://youtu.be/256Yn47knH4?list=PLPyapQSxH6mY_hbPFnqgb_Ru_gKos6mab

Video - Using a Data Table to Make Vector Addition Problems Easier

The Video page layout includes an embedded YouTube video, which can scale in any browser or mobile device. Embedding videos in the page is a great way to present video content, while accompanying it with supporting context, explanations and activities.

Swap out the content and replace the video on this page to create your own page. Instructions for replacing videos are provided below.

https://youtu.be/nwqu0RIsvV4

Video - Using a Data Table to Make Vector Addition Problems Easier

A data table of the vectors and their components is constructed. This table simplifies the organization and tracking of the vectors and their components when determining the components of the resultant vector.

https://youtu.be/nwqu0RIsvV4

Video - A Visually Complicated Vector Addition Problem using Component Vectors

This video illustrates the use of the component method to determine the sum of three displacement vectors.

It is noted that the order in which the 3 vectors are added is immaterial. The sum of the 3 vectors is the same regardless of which order they are added.

https://youtu.be/e0rZI2JYGkY

Video - Adding Vectors: How to Find the Resultant of Three or More Vectors

The solution for a problem where 3 vectors are added together. All of the steps are demonstrated.Note: The chart of the component values have the vectors listed across the top of the table in the chart and the components listed along the side of the table in the chart of components.

https://youtu.be/g_TnqKX5ybY

Assignment 1.2

Relative Motion (7 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of a ferryboat and a river

A ferryboat, whose speed in still water is 4.00 m/s, must cross a river whose current is 3.00 m/s. The river runs from west to east and is 128 m wide. The boat is pointed north.

a) If the boat does not compensate for the flow of the river water and allows itself to be pushed off course, what would be the new velocity of the boat? In answering this question, draw a vector diagram with the appropriate labels for the vectors, and state both the magnitude and direction of the velocity of the boat relative to the riverbank.

(continued)

Assignment 1.2: Relative Motion (continued)

b) In part (a), what is the velocity of the boat across the river? How long would it take the boat to travel across the river?

c) The boat now compensates for the river current so that it travels straight across the river to the north without being pushed off course. Draw a labelled vector diagram for this situation, and determine the angle of the boat relative to the shore.

(continued)

Assignment 1.2: Relative Motion (continued)

Method of Assessment

The total of seven marks for this assignment will be determined as follows:

- 1 mark for drawing and labelling an appropriate vector diagram in part (a)
- 1 mark for determining the magnitude of the velocity in part (a)
- 1 mark for determining the direction relative to the river bank in part (a)
- 1 mark for determining the component of the velocity of the boat across the river in part (b)
- 1 mark for determining the time that it takes boat to cross the river in part (b)
- 1 mark for drawing and labelling an appropriate vector diagram in part (c)
- 1 mark for determining the angle of the boat relative to the shore in part (c)

Lesson 6: Relative Motion (1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ use subscripts to properly describe relative motion
- ❑ use the subscript notation to set up the proper order in which one must add vectors for relative motion
- ❑ determine the relative motion of one object relative to another in a one-dimensional situation
- ❑ determine the relative motion of one object relative to another in a two-dimensional situation when an object is being moved off course without trying to compensate for the current
- ❑ determine the relative motion of one object relative to another in a two-dimensional situation when an object being moved off course is trying to compensate for the current

Key Words

relative motion

Introduction

You have likely experienced a ride up or down an escalator. As the stairs of the escalator carried you along, you realized that you were moving because the surroundings were moving next to you or relative to you. All motion is relative. By this, we mean that the only way we know something is moving is if it changes its position relative to, or compared to, some reference point.

Now you may have had some fun on the escalator by walking up as it was carrying you upwards. What you experienced was a greater velocity than when you were just standing and letting the escalator carry you. You also may have walked down an escalator that was carrying you downwards. Again, you experienced a greater velocity than when you are just standing on the escalator. Finally, you may have walked up the down escalator or down the up escalator and found that you didn't move relative to your surroundings. In other words, your velocity was zero.

You can see in these last situations that what you perceive as your motion next to your surroundings consists of your motion next to the escalator and the escalator's motion next to the surroundings.

In a nutshell, this is what **relative motion** is all about.

Relative motion really deals with the vector nature of motion. In this lesson, you will be combining what you learned just recently about kinematics with concepts concerning vectors that you studied in Grade 11 Physics. Again, there will be a need to rely on your prior knowledge about vectors.

When describing the motion of an object, it is important to be clear about the point of view of the observer. For example, imagine that you are on a bus looking out the window at another bus very close to your window. It may appear to you at some point that your bus is backing up because the bus in the window looks like it is moving forward. But that is just your point of view. To a person on the ground, it may appear that your bus is moving ahead and the bus beside you is also moving ahead, but at a faster rate. This is a one-dimensional situation. A two-dimensional situation might involve a boat moving in a cross-current. What would the boat pilot say is the velocity of his or her boat, and what would a ground observer on the shore say about the velocity of the boat from his or her point of view?

Relative Velocity in One Dimension

In relative motion in one dimension, an object is moving relative to its surroundings. The surroundings in turn are moving relative to the ground. This motion occurs in one dimension when the objects and the surroundings are moving along the same line.

Solving Relative Motion Problems

Let

$\vec{v}_{OS}$ represents the velocity of the "object" relative to the "surroundings"

$\vec{v}_{SG}$ represents the velocity of the "surroundings" relative to the "ground"

$\vec{v}_{OG}$ represents the velocity of the "object" relative to the "ground"

Then

$$\vec{v}_{OG} = \vec{v}_{OS} + \vec{v}_{SG}$$

Note the order in which the vectors are added. The common frame of reference is the "surroundings." The vectors are added down so that the symbols for the "surroundings" are close together.

Example 1: Relative Motion on a Bus

Let's look at the situation of a passenger walking to the right in a moving bus. The person is moving at a velocity of 1.0 m/s to the right in the bus. But the bus is also moving to the right at 3.0 m/s relative to the ground. How might a person on the sidewalk waiting for a bus view the velocity of the person in the bus? You can probably guess that the person would see the passenger in the bus moving at a velocity of 4.0 m/s to the right. Or we can say that the velocity of the passenger relative to the ground is 4.0 m/s to the right.

In the situation described above, there are three different velocities. Using a system of subscripts, we can make clearer what the different velocities are in reference to.

$\vec{v}_{PB}$ can represent the velocity of the "passenger" relative to the "bus" = +1.0 m/s

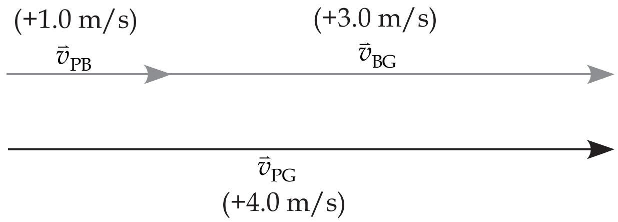
$\vec{v}_{BG}$ can represent the velocity of the "bus" relative to the "ground" = +3.0 m/s

$\vec{v}_{PG}$ can represent the velocity of the "passenger" relative to the "ground" = +4.0 m/s

Note that each velocity symbol has a two-letter subscript. The first letter refers to the body that is moving. The second letter refers to the object from which the velocity is measured.

Using the symbols above, we could summarize the situation as follows:

$$\vec{v}_{PG} = \vec{v}_{PB} + \vec{v}_{BG}$$



The vector $\vec{v}_{PG}$ is the vector sum of $\vec{v}_{PB}$ and $\vec{v}_{BG}$. We could represent this by using vectors.

Notice that when the vector sum is written as $\vec{v}_{PG} = \vec{v}_{PB} + \vec{v}_{BG}$, there is a definite order in the subscripts. The subscript (P) appears as the first subscript written on both the left-hand side and the right-hand side of the equation. The subscript (G) appears as the last subscript on both sides of the equation. The third subscript (B), the object linking the P and G, appears as the two inner subscripts.

$$\vec{v}_{PG} = \vec{v}_{PB} + \vec{v}_{BG}$$

Ordering the vectors so that the subscripts appear in this manner can help determine if the velocities are being added in the correct order. Keeping this in mind will prove to be extremely helpful in the two-dimensional situation described later.

Example 2: Relative Motion on an Escalator: Going Up the Up Escalator

Let's go back to the introduction where we talked about playing on an escalator. Suppose that the escalator is moving upwards at 1.2 m/s and that you are walking at 1.0 m/s, also upwards. What would your velocity be, as seen by someone standing nearby on the floor?

In this case, let's assume that going up the escalator is the positive direction and going down is the negative direction. Again, there will be three velocities in this situation: there will be the velocity of you relative to the escalator; the velocity of the escalator relative to the ground; and finally the velocity of you relative the ground.

$\vec{v}_{PE}$ can represent the velocity of the "person" relative to the "escalator" = +1.0 m/s

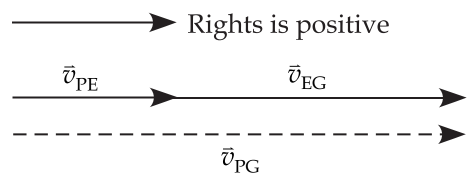
$\vec{v}_{EG}$ can represent the velocity of the "escalator" relative to the "ground" = +1.2 m/s

$\vec{v}_{PG}$ can represent the velocity of the "person" relative to the "ground" = +2.2 m/s

Your velocity relative to the ground is determined by the vector sum of your velocity relative to the escalator and the escalator's velocity relative to the ground.

This can be written as $\vec{v}_{PG} = \vec{v}_{PE} + \vec{v}_{EG}$.

The "tip-to-tail" diagram will look like this:



Since you are adding parallel vectors, you simply add the magnitudes together, giving a resultant that points in the direction of the given vectors.

$$\vec{v}_{PG} = \vec{v}_{PE} + \vec{v}_{EG} = +1.0 \text{ m/s} + (+1.2 \text{ m/s}) = +2.2 \text{ m/s}$$

Once more, notice the arrangement of the subscripts.

Example 3: Relative Motion on an Escalator: Going Down the Up Escalator

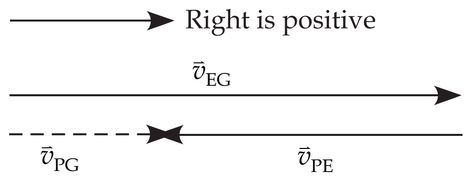
Let's consider a case where you're going down the up escalator. Suppose the escalator is still moving upwards (positive direction), but now you are moving downwards (negative direction) and the magnitudes of the velocity are the same as in the example above. What would a person standing on the floor nearby observe to be your velocity relative to the ground?

$\vec{v}_{PE}$ can represent the velocity of the "person" relative to the "escalator" = –1.0 m/s

$\vec{v}_{EG}$ can represent the velocity of the "escalator" relative to the "ground" = +1.2 m/s

$\vec{v}_{PG}$ can represent the velocity of the "person" relative to the "ground" = ?

Let's draw a sketch of the vectors involved.



Since the vectors being added lie one on top of the other, they have been drawn slightly separated to illustrate how they are being added together.

The equation for the addition is still $\vec{v}_{PG} = \vec{v}_{PE} + \vec{v}_{EG}$ and adding the vectors gives

$$\vec{v}_{PG} = \vec{v}_{PE} + \vec{v}_{EG} = -1.0 \text{ m/s} + (1.2 \text{ m/s}) = +0.2 \text{ m/s}$$

Since the vectors are antiparallel, remember that you must find the difference of the magnitudes to determine the magnitude of the resultant, then give the sign of the resultant as the sign of the larger vector.

Learning Activity 1.11

Relative Motion along a Line

Answer the following questions to check your understanding of relative motion in one dimension. You may check your work against the answer key provided at the end of Module 1.

For each of the following, identify each velocity using the two subscript notations, and determine the velocity of the object relative to the ground.

1. A person is walking at 1.2 m/s towards the back of a train travelling at 19.0 m/s [N].
2. A bird is flying through the air at 5.65 m/s [E] where the wind is blowing at 4.95 m/s [E].
3. A person is walking on a moving sidewalk at 1.80 m/s in the same direction as the sidewalk, which is moving at 3.00 m/s.

Relative Velocity in Two Dimensions: An Object Blown Off Course

The examples of relative motion we have considered so far have been for velocities in one dimension that are along a straight line. Let us now consider relative motion in two dimensions.

Two common examples of relative motion in two dimensions are boats crossing a river with a current, or airplanes moving across a wind.

Example 4: A Boat Crossing a River

Imagine that you are standing on the shore of a body of water that is not moving. A boat starts at the south shore and heads due north across this body of water. Since the water is not moving, you'd expect the boat to move due north. You would be correct.

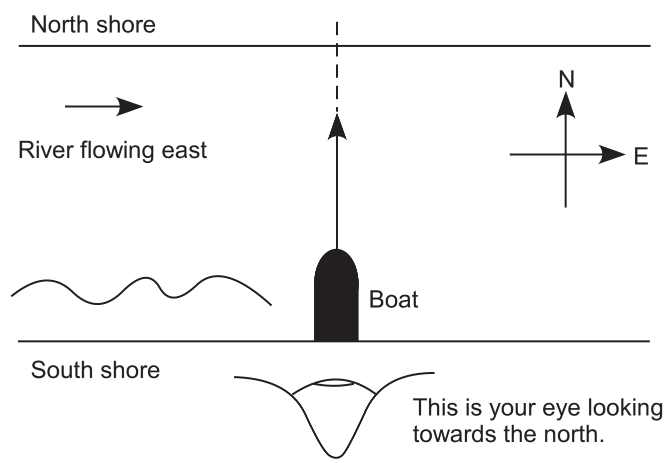
Let's look at the situation of a boat crossing a river.

Now imagine a river with a water current moving from west to east at a speed of 5.00 m/s. This is the speed of the water relative to the shore. It can be labelled as follows:

$\vec{v}_{\text{WS}}$ = velocity of water relative to the shore = 5.00 m/s [E].

Now imagine a boat pointing from south to north. It is moving at a speed of 10.00 m/s relative to the water.

$\vec{v}_{\text{BW}}$ = velocity of boat relative to the water = 10.00 m/s [N].



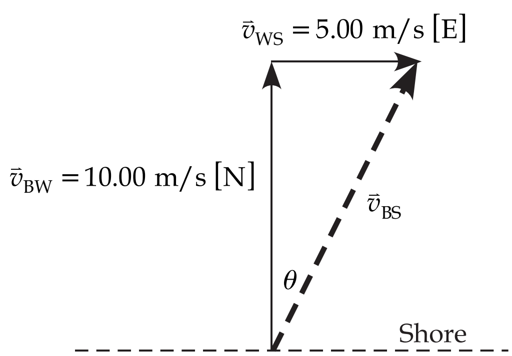
In this situation, the boat may be pointing north, but it will not actually move north relative to the shore. The water current is pushing the boat off course. For a person at rest on the shore, the boat will be pushed off to the side, towards the east side in this case, while it is crossing the river. The actual velocity of the boat relative to the shore can be labelled as follows:

$\vec{v}_{BS}$ = velocity of boat relative to the shore = ?

We can find this velocity by adding together the vectors that give the velocity of the boat relative to the water and the velocity of the water relative to the shore.

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$

Notice that the two inner subscripts on the right side of the equation are identical. The two outer subscripts on the right side are the same as the two subscripts on the left side. Using vectors, we can draw a diagram of the situation.



To determine the magnitude of the velocity of the boat relative to the shore, you can use the Pythagorean theorem.

$$\vec{v}_{BS} = \sqrt{(5.00 \text{ m/s})^2 + (+10.00 \text{ m/s})^2}$$

The direction can be determined using the inverse tangent function.

$$\theta = \tan^{-1}\frac{5.00 \text{ m/s}}{10.00 \text{ m/s}} = 26.6°$$
$$= 26.6° \text{ [E of N]}$$

or, in other words, 63.4° away from the shore.

The velocity of the boat as observed by you standing on the shore is 11.2 m/s [26.6° E of N].

Another question we can ask here is "How fast is the boat approaching the north shore of the river?"

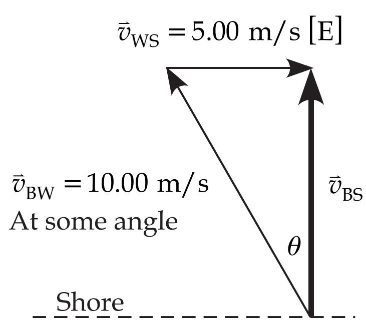
In this case, we are going to use one of the properties of vector components. This property is that vector components are independent of each other. As mentioned at the beginning of this section, if a boat is heading across still water, its velocity relative to the shore would have the same direction as the boat, since the water is not carrying it off course.

> A boat heading **directly** across a river approaches the opposite shore at whatever speed the boat is travelling. This is true whether the river is carrying the boat downstream or not.

Now, in the case with the river flowing, the velocity of the boat straight across the river is still the same as when the water is not flowing. The river is moving the boat sideways to its direction of motion across the river. The motion of the water does not cause the boat to speed up or slow down. The boat is still heading straight across the river and approaching the north shore at 10.0 m/s.

Example 5: Relative Velocity in Two Dimensions: Making a Course Correction

Now, suppose that the captain of the boat does not want the boat to be carried off to the side. Instead, the captain wants the boat to travel straight across the river. In this situation, the boat must be pointed into the current at some angle. We know that the speed of the boat in the water is 10.0 m/s and that the water is moving at 5.00 m/s. So this time the vectors must be arranged as follows:



Notice that the vectors $\bar{v}_{BW}$ and $\bar{v}_{WS}$ are still added together to give $\bar{v}_{BS}$.

$$\bar{v}_{BS} = \bar{v}_{BW} + \bar{v}_{WS}$$

The angle at which the boat must point relative to the straight-across direction from the shore is found using the sine function.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\bar{v}_{WS}}{\bar{v}_{BW}} = \frac{5.00 \text{ m/s}}{10.00 \text{ m/s}}$$

$$\theta = \sin^{-1}(0.500) = 30.0°$$

Notice that this time we use the inverse sine function, since the two known sides are the opposite side and the hypotenuse. To find the magnitude of the velocity of the boat relative to the shore, we use the Pythagorean theorem solving for the length of one of the sides, not the hypotenuse.

$$\vec{v}_{\text{BS}} = \sqrt{(10.00 \text{ m/s})^2 - (5.00 \text{ m/s})^2} = \sqrt{75.0} = 8.66 \text{ m/s}$$

Relative to the shore, the velocity of the boat is 8.66 m/s [straight across the river].

This velocity relative to the shore is slower than the speed of the boat in the water. If we wanted to know the time it took to cross the river when moving perpendicular to the shore, we would need to know the distance from one shore to the other. If, for example, the distance straight across was 40.0 m, then the time to cross the river would be found using kinematics equation number one for a constant velocity: $\vec{v}_{\text{across}} = \dfrac{\vec{d}}{\Delta t}$.

Rearranging gives you: $\Delta t = \dfrac{\vec{d}}{\vec{v}_{\text{across}}}$

Solving gives: $\Delta t = \dfrac{40.0 \text{ m}}{8.66 \text{ m/s}} = 4.62 \text{ s}$

Learning Activity 1.12

Relative Motion in One and Two Dimensions

There are five practice questions in this learning activity. An answer key is available at the end of Module 1 for you to check your work after you have answered the questions.

The physics of a hound walking on a moving boat

1. A hound walks at a speed of 2.00 m/s along the deck toward the front of a boat, which is travelling at 8.00 m/s with respect to the water.
    a) What is the velocity of the hound relative to the water?
    b) What would be the velocity of the hound if the dog were walking toward the back of the boat?

The physics of an airplane travelling in calm air

2. An Air Canada plane is travelling at 1000.0 km/h in a direction 40.0° east of north.
    a) Find the components of the velocity vector by finding the component in the northerly direction and the easterly direction.
    b) How far to the north and how far to the east would the plane travel in 4.00 h?

The physics of a vacationer and a cruise ship

3. A captain walks 4.00 km/h directly across a cruise ship whose speed relative to Earth is 12.0 km/h. What is the speed of the captain with respect to Earth?

The physics of a canoe in a current

4. A person paddling a canoe is able to make the canoe and himself travel at a speed of 1.50 m/s in still water. The paddler heads the canoe directly across a 3.00 km wide river. The current of the river flows at 0.900 m/s downstream.
    a) What is the velocity of the canoe as observed from the shore?
    b) How long does it take the canoe to cross the river?
    c) How far downstream will the canoe be upon reaching the other side of the river?

(continued)

Learning Activity 1.12: Relative Motion in One and Two Dimensions (continued)

The physics of an airplane flying in a crosswind

5. An airplane flies with an air speed of 225 km/h heading due west. At the altitude at which the plane is flying, the wind is blowing at 105 km/h heading due south.
   a) What is the velocity of the plane as observed by someone standing on the ground?
   b) How far off course would the plane, while it is heading due west, be blown by the wind during 1.50 h of flying?
   c) What heading must a plane take in order to reach its destination, which is due west of the starting point?

Lesson Summary

The use of subscripts can clarify what velocity vectors refer to. Each velocity symbol has a two-letter subscript. The first letter refers to the body that is moving; the second letter refers to the object from which the velocity is measured (the surroundings).

There is a pattern in the subscripts when the vector sum of velocity vectors is written as follows:

$$\vec{v}_{\mathrm{AC}} = \vec{v}_{\mathrm{AB}} + \vec{v}_{\mathrm{BC}}$$

The first subscript (A) on the left side of the equation is also the first subscript on the right side of the equation. Also, the last subscript (C) on the left side of the equation is also the last subscript on the right side of the equation. The third subscript (B) appears as the two inner subscripts. Ordering the vectors so that the subscripts appear in this manner can help determine if the velocities are being added in the correct order.

The relative motion of two objects in both the one-dimensional and two-dimensional situations was reviewed. The velocity vectors are added graphically and by using trigonometry, just as any two vectors would be added.

In the two-dimensional situation, an object will be carried off course by a current if no attempts are made to make a course correction. In this lesson, only two vectors at right angles are considered. The magnitude of the resultant vector can be found using the Pythagorean theorem, $h^2 = a^2 + b^2$ where "h" is the hypotenuse and "a" and "b" are the two sides. The angle the object is carried off course can be determined using the $\tan^{-1}$ function.

When an object is to move in a direction perpendicular to a current without being carried off course, it must point into the current at some angle. To determine the angle, the $\cos^{-1}$ function or the $\sin^{-1}$ function is used, depending on the reference direction required.

Notes

Assignment 1.2

Relative Motion (7 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of a ferryboat and a river

A ferryboat, whose speed in still water is 4.00 m/s, must cross a river whose current is 3.00 m/s. The river runs from west to east and is 128 m wide. The boat is pointed north.

a) If the boat does not compensate for the flow of the river water and allows itself to be pushed off course, what would be the new velocity of the boat? In answering this question, draw a vector diagram with the appropriate labels for the vectors, and state both the magnitude and direction of the velocity of the boat relative to the riverbank.

(continued)

Assignment 1.2: Relative Motion (continued)

b) In part (a), what is the velocity of the boat across the river? How long would it take the boat to travel across the river?

c) The boat now compensates for the river current so that it travels straight across the river to the north without being pushed off course. Draw a labelled vector diagram for this situation, and determine the angle of the boat relative to the shore.

(continued)

Assignment 1.2: Relative Motion (continued)

Method of Assessment

The total of seven marks for this assignment will be determined as follows:

- 1 mark for drawing and labelling an appropriate vector diagram in part (a)
- 1 mark for determining the magnitude of the velocity in part (a)
- 1 mark for determining the direction relative to the river bank in part (a)
- 1 mark for determining the component of the velocity of the boat across the river in part (b)
- 1 mark for determining the time that it takes boat to cross the river in part (b)
- 1 mark for drawing and labelling an appropriate vector diagram in part (c)
- 1 mark for determining the angle of the boat relative to the shore in part (c)

NOTES

Relative Motion

Relative Motion

https://surendranath.org/

Click on "Applets Menu" in the upper left hand corner of the page, then "Kinematics," then "Boat and River." Observe the combination of the motion of the boat and river as seen from the shore.

Change the velocities of the boat next to the water and the water next to the shore to observe the resultant velocity of the boat next to the shore.

Choose a velocity for the boat next to the water as 4 and the velocity of the water next to the shore of 2. Calculate the direction the boat must be steered to travel directly across the river. Use the applet to check your answer.

Video - An Introductory Relative Motion Problem

View the resultant velocity of a car moving next to a paper and the paper moving next to the earth.

The velocity of the car next to the earth is calculated.

In part b) the problem is extended to determine how far the car actually traveled while it moved across the paper.

https://youtu.be/CZqajGiNaiQ

Video - An Introductory Relative Motion Problem with Vector Components

The relative motion analysis is extended for a toy car moving next to the paper and the paper moving next to the ground. This time the two motions, the car next to the paper and the paper next to the ground, are not perpendicular to each other.

https://youtu.be/T8iiMPZAaRM

Boat Crossing a River Simulator

Boat Crossing a River Simulator

https://www.physicsclassroom.com/Physics-Interactives/Vectors-and-Projectiles/Riverboat-Simulator

Click on link and then "Launch Interactive"

You can adjust the speed of the boat, the direction the boat heads, the velocity of the river's current and the width of the river.

Try these:

Set the current to 0 m/s. Aim the boat straight across the river at 10.0 m/s. Where does the boat land and how long does it take to cross the river?

Set the current to 3.0 m/s. Repeat the steps above.

Set the current to 5.0 m/s. Repeat the steps above.

How does the time to cross the river compare in these 3 cases?

Does the current's velocity affect the time required for the boat to cross the river?

Try to have the boat travel straight across the river when the current is flowing at 5.0 m/s and the boat is moving at 10.0 m/s next to the water. You will have to adjust the heading of the boat until a trial shows that the boat travels straight across the river.

The simulation provides an introduction to the topic of relative velocity. The boat moves relative to the water and the water moves relative to the shore. The resulting motion is the combination of these two components of motion.

The simulation also provides an introduction into the concept of the independence of perpendicular components of motion. Changes in the river velocity have no effect upon time to cross the river. Put another way, we could say that the vertical component of motion (the river's current) has no effect upon the horizontal component (the boat velocity and the time to cross the river); this independence of perpendicular components of motion is quite obvious if you restrict their exploration to cases in which the boat *heads* straight across the river.

Module 1 Summary

Congratulations! You have completed the first module of Grade 12 Physics. Most of the concepts discussed were a review of ideas from earlier courses in science and physics. The motion of objects is often used as a starting point for the study of many of the major concepts of physics. A good grasp of kinematics is crucial to your comprehension of these concepts yet to be studied.

Submitting Your Assignments

You will not submit your Module 1 assignments to the Distance Learning Unit at this time. Instead, you will submit them, along with the Module 2 assignments, **when you have completed Module 2**.

NOTES

Grade 12 Physics (40S)

Module 1: Kinematics

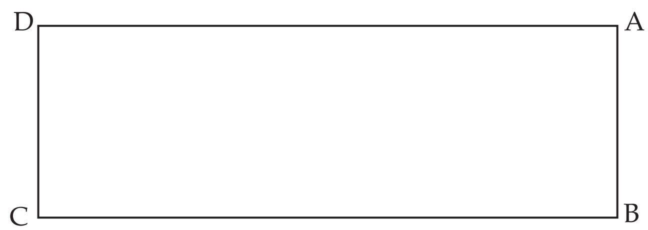
Learning Activity Answer Keys

Module 1: Kinematics

Learning Activity 1.1: Working with Distance, Displacement, Speed, and Velocity

To answer questions 1 to 3, use the information below.

A city block is laid out in a grid running in the north-south and east-west directions. The blocks measure 135 m in length in the east-west direction, and 45.0 m in width in the north-south direction. A city block is drawn below.



1. On your bicycle, you travel from A to B during 9.00 s.

   a) What is your average speed?

   Answer:

   $d = 45.0 \text{ m } [\text{S}]$

   $\Delta t = 9.00 \text{ s}$

   $v_{\text{avg}} = ?$

   $$v_{\text{avg}} = \frac{d}{\Delta t} = \frac{45.0 \text{ m}}{9.00 \text{ s}} = 5.00 \text{ m/s}$$

   b) What is your average velocity?

   Answer:

   $$\vec{v}_{\text{avg}} = \frac{\vec{d}}{\Delta t} = \frac{45.0 \text{ m } [\text{S}]}{9.00 \text{ s}} = 5.00 \text{ m/s } [\text{S}]$$

2. If you travel from A to B to C to D, what is your
   a) distance travelled?

      Answer:

      d_{AB} = 45.0 m; d_{BC} = 135 m; d_{CD} = 45.0 m

      d_{total} = 45.0 m + 135 m + 45.0 m = 225 m

   b) displacement?

      Answer:

      The displacement from A to B to C to D is the same as going directly from A to D, which is 135 m [W].

3. If the journey in #2 took 55.0 s, calculate
   a) your average speed.

      Answer:

      $$d_{total} = 225 \text{ m}$$
      $$\Delta t = 55.0 \text{ s}$$
      $$v_{avg} = ?$$
      $$v_{avg} = \frac{d}{\Delta t} = \frac{225 \text{ m}}{55.0 \text{ s}} = 4.09 \text{ m/s}$$

   b) your average velocity.

      Answer:

      $$\vec{d} = 135 \text{ m } [\text{W}]$$
      $$\Delta t = 55.0 \text{ s}$$
      $$\vec{v}_{avg} = \frac{\vec{d}}{\Delta t} = \frac{135 \text{ m } [\text{W}]}{55.0 \text{ s}}$$
      $$\vec{v}_{avg} = 2.45 \text{ m/s } [\text{W}]$$

Learning Activity 1.2: Acceleration Calculations

1. Alberto reaches the bottom of the hill, coasting along at 9.25 m/s. He begins to coast up a second hill where the average acceleration is –1.20 m/s/s. What is the change in Alberto's velocity during 3.00 s of coasting up this hill? What is his final velocity?

   Answer:

   $$\vec{v}_1 = +9.25 \text{ m/s}$$
   $$\vec{a}_{\text{avg}} = -1.20 \text{ m/s/s}$$
   $$\Delta t = 3.00 \text{ s}$$
   $$\Delta\vec{v} = ? \text{ and } \vec{v}_2 = ?$$
   $$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$
   $$-1.20 \text{ m/s/s} = \frac{\Delta\vec{v}}{3.00 \text{ s}}$$
   $$\Delta\vec{v} = (-1.20 \text{ m/s/s})(3.00 \text{ s}) = -3.60 \text{ m/s}$$

   But, $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$
   $$-3.60 \text{ m/s} = \vec{v}_2 - (+9.25 \text{ m/s})$$
   $$\vec{v}_2 = -3.60 \text{ m/s} + (+9.25 \text{ m/s})$$
   $$\vec{v}_2 = +5.65 \text{ m/s}$$

   The change in velocity is –3.60 m/s and Alberto's final velocity is +5.65 m/s.

2. A dragster racing on a quarter-mile track (about 400.0 m) has an average acceleration of 11.2 m/s/s [E] reaching a velocity of 72.0 m/s [E]. What was the time needed to race this distance?

   Answer:

   $$\vec{v}_1 = 0 \text{ m/s}$$
   $$\vec{v}_2 = 72.0 \text{ m/s [E] or } +72.0 \text{ m/s}$$
   $$\vec{a}_{\text{avg}} = 11.2 \text{ m/s/s [E] or } +11.2 \text{ m/s/s}$$
   $$\Delta t = ?$$
   $$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$
   $$+11.2 \text{ m/s/s} = \frac{+72.0 \text{ m/s} - 0 \text{ m/s}}{\Delta t}$$
   $$\Delta t = \frac{+72.0 \text{ m/s}}{+11.2 \text{ m/s/s}} = 6.43 \text{ s}$$

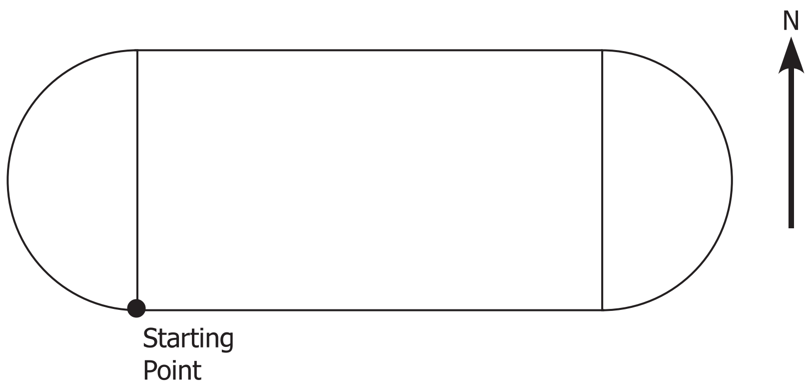
   The dragster took 6.43 seconds to race down the 400.0 m track.

Learning Activity 1.3: Modes of Analyzing Motion

1. Consider the following situation. You are jogging at a constant pace around a track. This is the typical track that might be found around the perimeter of a football field. The distance around the track is 360.0 m.

   a) Draw a diagram of the situation.

   Answer:



   b) It takes you 90.0 seconds to jog completely around the track and return to your starting point. Calculate your average speed and your average velocity.

   Answer:

   Distance travelled $d = 360.0$ m

   Time interval $\Delta t = 90.0$ s

   Average speed $v = ?$

   The equation to use is $v_{avg} = \dfrac{d_{total}}{\Delta t_{total}}$.

$$v_{avg} = \frac{360.0\ \text{m}}{90.0\ \text{s}}$$

$$v_{avg} = 4.00\ \text{m/s}$$

   Note: There are rules for significant digits that should be followed when giving the answer. Normally we will be working with three significant digits.

   To find the average velocity, you have to work with displacement, not distance travelled. In this case, since you ended up at your starting point, the change in your position is 0 m.

   Total displacement $\vec{d}_{total} = 0$ m

   Time interval $\Delta t = 90.0$ s

Average velocity $\vec{v} = ?$

$$\vec{v}_{\text{avg}} = \frac{\vec{d}_{\text{total}}}{\Delta t_{\text{total}}}$$

You can see in this case, since the total displacement is 0 m, that the average velocity must be 0 m/s.

c) On a straight section of the track, it takes you 25.0 seconds to jog 100.0 m. What is your average velocity? **Hint:** Refer to your diagram.

Answer:

In this case, you have to find average velocity, which is a vector. Therefore, you must consider the direction of the displacement. Here, you are travelling from your starting point and moving in an easterly direction.

Total displacement $\vec{d}_{\text{total}} = 100.0 \text{ m [E]}$

Time interval $\Delta t = 25.0 \text{ s}$

Average velocity $\vec{v} = ?$

$$\vec{v}_{\text{avg}} = \frac{\vec{d}_{\text{total}}}{\Delta t_{\text{total}}}$$

$$\vec{v}_{\text{avg}} = \frac{100.0 \text{ m [E]}}{25.0 \text{ s}} = 4.00 \text{ m/s [E]}$$

Your average velocity is 4.00 metres per second in an easterly direction.

d) Are you accelerating while you are jogging along one of the straight sections of the track? Explain.

Answer:

While you are jogging along one of the straight sections of the track, you are not accelerating. Since you are jogging at a constant pace, there is no change in velocity. Since there's no change in velocity, then there is no acceleration.

e) Are you accelerating while you are jogging along one of the curved sections of the track? Explain.

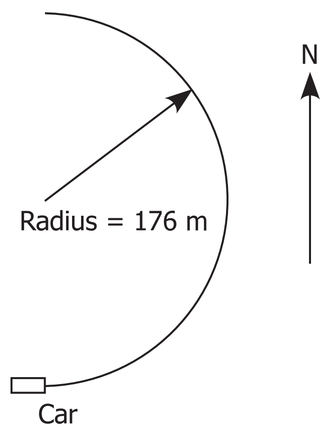
Answer:

While you are jogging along one of the curved sections of the track, you are accelerating. Acceleration shows up as an object speeding up, slowing down, or changing direction. In this case, you're not speeding up or slowing down but the direction of your velocity is changing. Therefore, if your velocity changes, there is an acceleration.

2. The car is travelling along a section of the road that forms one-half of a circle with a radius of 176 m. The car requires 22.0 seconds to travel around the curve.

   a) Draw a diagram of the situation.

   Answer:



   b) What is the average speed of the car while it travels around this half-circle?

   Answer:

   Finding the speed requires knowing the distance travelled and the time interval. Here, the distance travelled is one-half of the circumference of the circle.

   Distance travelled $d = \left(\frac{1}{2}\right)c = \left(\frac{1}{2}\right)(2\pi R) = \pi R$

   $= (3.14)(176 \text{ m}) = 552.64 \text{ m}$

   $= 553 \text{ m}$

   Time interval $\Delta t = 22.0 \text{ s}$

   Average speed $v = ?$

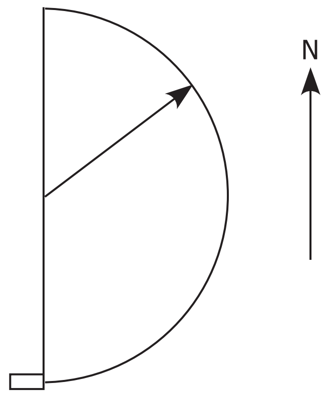
   The equation to use is $v_{avg} = \dfrac{d_{total}}{\Delta t_{total}}$.

   $$v_{avg} = \frac{553 \text{ m}}{22.0 \text{ s}}$$

   $$v_{avg} = 25.1 \text{ m/s}$$

c) What is the average velocity of the car while it travels around this half-circle?

Answer:



To calculate the average velocity as the car goes halfway around the circle, you must determine the displacement from the starting point—the initial position—to the final position. If you look at the diagram, you'll see that the car has moved from the bottom of the circle to the top of the circle, a distance equal to the diameter of the circle. In this case, a displacement vector starts out at the bottom of the circle and points to the top of the circle. The displacement points north.

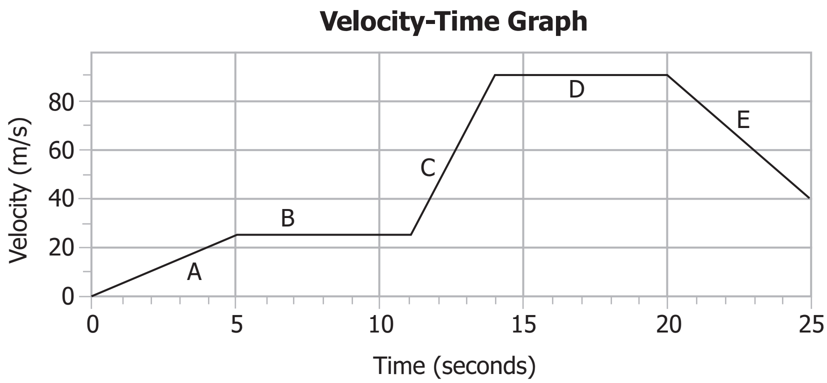
Displacement	$\vec{d}_{\text{total}} = 2R\ [\text{N}] = 2\big(176\ \text{m}\ [\text{N}]\big) = 352\ \text{m}\ [\text{N}]$
Time interval	$\Delta t = 22.0\ \text{s}$
Average velocity	$\vec{v} = ?$

$$\vec{v}_{\text{avg}} = \frac{\vec{d}_{\text{total}}}{\Delta t_{\text{total}}}$$

$$\vec{v}_{\text{avg}} = \frac{\vec{d}_{\text{total}}}{\Delta t_{\text{total}}} = \frac{352\ \text{m}\ [\text{N}]}{22.0\ \text{s}} = 16.0\ \text{m/s}\ [\text{N}$$

The average velocity is 16.0 m/s [N].

Learning Activity 1.4: Linking Kinematics Graphs and Equations



All the questions refer to the velocity-time graph given above.

1. What on the velocity-time graph represents acceleration? Displacement?

   Answer:

   On a velocity-time graph, the slope of the line gives acceleration. On the velocity-time graph, the area beneath the curve gives displacement.

2. Which of the two equations that have been derived so far in this lesson should be used to find

   a) the acceleration for the time interval A?

   Answer:

   Acceleration involves finding the slope using the formula:

   $$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

   b) the displacement for the time interval C?

   Answer:

   Displacement is found using the area formula for a trapezoid:

   $$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)\Delta t$$

   c) the displacement for the time interval B?

   Answer:

   Displacement is found using the area formula for a trapezoid:

   $$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)\Delta t$$

   In this case, the initial velocity and final velocity are the same; therefore, the average velocity is the same as the initial velocity and final velocity.

3. Calculate the displacement during time interval C.

   Answer:

   From the graph:

   $\vec{v}_1 = +25 \text{ m/s}$

   $\vec{v}_2 = +90 \text{ m/s}$

   $\Delta t = 3.0 \text{ s}$

   $$\vec{d} = \frac{(\vec{v}_1 + \vec{v}_2)}{2}\Delta t = \frac{(25 \text{ m/s} + 90 \text{ m/s})}{2}(3.0 \text{ s})$$

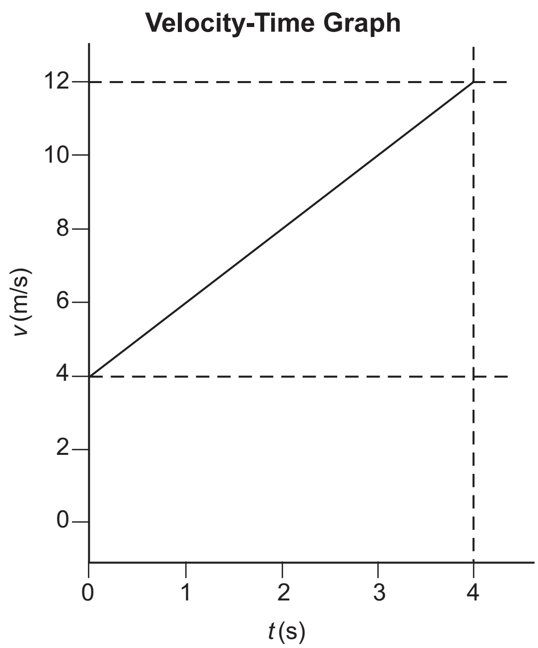
   $\vec{d} = +172.5 \text{ m} = 170 \text{ m}$

From the graph, we can estimate velocity to the nearest 5 m/s and the time to the nearest 0.1 s. This results in each measurement of velocity or time interval also having two significant digits.

Learning Activity 1.5: Graphs and Equations for Uniform Acceleration

The physics of uniformly accelerated motion and the graph

1. The four parts of this assignment refer to the following graph.



a) Using the equation $\vec{v}_2 - \vec{v}_1 = \vec{a}\Delta t$, determine the acceleration of the object.

Answer:

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t \text{ or } \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t} = \frac{12 \text{ m/s} - 4 \text{ m/s}}{4 \text{ s}} = 2 \text{ m/s}^2$$

b) Using the equation $\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$, determine the displacement of the object above the four-second interval.

Answer:

$$\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t = \frac{1}{2}(+4 \text{ m/s} + +12 \text{ m/s})4 \text{ s} = 32 \text{ m}$$

c) Using the equation $\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$, verify that the displacement is the value you calculated in part (b).

Answer:

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2 = (4 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(2 \text{ m/s}^2)(4 \text{ s})^2 = 32 \text{ m}$$

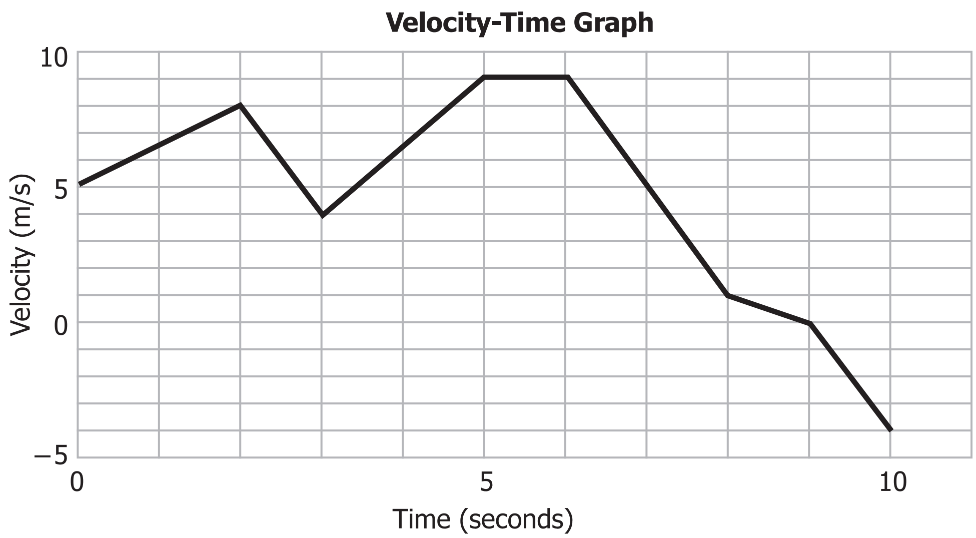
d) Using the equation $v_2^2 = v_1^2 + 2ad$, verify that the displacement is the value you calculated above.

Answer:

$$d = \frac{v_2^2 - v_1^2}{2a}$$

$$d = \frac{(12\ \text{m/s})^2 - (4\ \text{m/s})^2}{2(2\ \text{m/s}^2)} = 32\ \text{m}$$

2. On the following velocity-time graph, indicate during which intervals of time the acceleration is positive, negative, or zero. This graph does not represent a real-life situation. It is presented simply to help reinforce the ideas about interpreting graphs.



Answer:

The equation for acceleration is $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$, which represents the slope of the velocity-time graph.

The slope is positive during the time intervals 0 s – 2 s, and 3 s – 5 s.

The slope is negative during the time intervals 2 s – 3 s, 6 s – 8 s, 8 s – 9 s, and 9 s – 10 s.

The slope is zero during the interval from five seconds to six seconds.

3. For the graph in question 2, determine the value of the acceleration for each time interval. Remember that the slope on a velocity-time graph gives the acceleration. The acceleration for the first interval of time has been calculated in the table below. Complete the other calculations in the table below.

   Answer:

   The equation for acceleration is $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$, which represents the slope of the velocity-time graph.

Time Interval	Change in Velocity (m/s)	Acceleration (m/s/s)
0 s – 2 s	+8 – (+5) = +3	+3 m/s / 2 s = +1.5
2 s – 3 s	+4 – (+8) = –4	–4 / 1 = –4
3 s – 5 s	+9 – (+4) = +5	+5 / 2 = +2.5
5 s – 6 s	+9 – (+9) = 0	0 / 1 = 0
6 s – 8 s	+1 – (+9) = –8	–8 / 2 = –4
8 s – 9 s	0 – (+1) = –1	–1 / 1 = –1
9 s – 10 s	–4 – 0 = –4	–4 / 1 = –4

4. For each of the time intervals for the graph in question 2, determine the displacement using one of the equations for kinematics that were derived in this lesson. You should be able to use all four of the kinematics equations that involve displacement at least once each.

   Answer:

   The area beneath the curve of a velocity-time graph represents the displacement.

   The shape of the area for the first interval from zero to two seconds is a trapezoid.

   For this interval, we have the following information:

   Initial velocity = 5 m/s
   Final velocity = +8 m/s
   Time interval = 2 s
   Acceleration = +1.5 m/s/s

   With all of this information, you have several choices for the equations that you can use to determine the displacement.

This information allows us to use the third kinematics equation, $\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$, to find the unknown, the displacement.

$$\vec{d} = \frac{1}{2}(+5 \text{ m/s} + +8 \text{ m/s})2\text{ s} = \frac{1}{2}(+13 \text{ m/s})2 \text{ s} = +13 \text{ m}$$

For the interval from 2 s to 3 s, we have the following information:

Initial velocity	= +8 m/s
Final velocity	= +4 m/s
Time interval	= 1 s
Acceleration	= –4 m/s/s

For this one, let us use the fourth kinematics equation $\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$.

$$\vec{d} = (+8 \text{ m/s})(1 \text{ } s) + \frac{1}{2}(-4 \text{ m/s/s})(1 \text{ s})^2 = +8 \text{ m} + (-2 \text{ m}) = +6 \text{ m}$$

For the interval from 3 s to 5 s, you have the following information:

Initial velocity	= +4 m/s
Final velocity	= +9 m/s
Time interval	= 2 s
Acceleration	= +2.5 m/s/s

Again, this is accelerated motion. Here, you could use kinematics equation number 5, $v_2^2 = v_1^2 + 2ad$, to solve for displacement.

Here, we can arrange by isolating the displacement. Rearranging equations using variables tends to be easier than rearranging the same equations with numbers in them.

$$d = \frac{{v_2}^2 - {v_1}^2}{2a}$$

$$d = \frac{(+9 \text{ m/s})^2 - (+4 \text{ m/s})^2}{2(+2.5 \text{ m/s/s})} = \frac{81 - (+16)}{+5} = +13 \text{ m}$$

The next time interval from five to six seconds has an acceleration of 0 m/s/s. On a velocity-time graph, the shape of the area you must find will either be a square or a rectangle.

You could use kinematics equation 3, in which case the initial and final velocity are equal. You could use kinematics equation 4, in which case the second part of the equation disappears since the acceleration is 0 m/s/s. You cannot use kinematics equation 5, since you end up dividing by an acceleration of 0 m/s/s, which of course is impossible. Your best bet, if the acceleration is 0 m/s/s, is to use kinematics equation 1.

For the interval from 5 s to 6 s, you have the following information:

Initial velocity	= +9 m/s
Final velocity	= +9 m/s
Time interval	= 1 s
Acceleration	= 0 m/s/s

Here, you will solve for the displacement using kinematics equation 1:

$$\bar{v} = \frac{\vec{d}}{\Delta t}.$$

If we rearrange the equation to solve for the displacement, it becomes

$$\vec{d} = \bar{v}\Delta t.$$

$$\vec{d} = (+9\ \text{m/s})(1\ \text{s}) = +9\ \text{m}$$

We hope by now that you have caught on to the idea that because of the variety of equations for kinematics, you're able to use more than one equation to solve a particular problem. Another thing you should realize is that you should try to make life easy for yourself. Many times you will have options available to you. Choose the option that is the simplest to work with.

For the rest of the intervals, you are provided with the answers below. Determine the displacements and check your work.

For the time interval from six to eight seconds, the displacement is +10 m.

For the time interval from eight to nine seconds, the displacement is +0.5 m.

For the time interval from nine to ten seconds, the displacement is –2 m. Note that this displacement is negative. You'll notice on the graph that this area lies below the horizontal time axis in the negative region of area on this velocity-time graph.

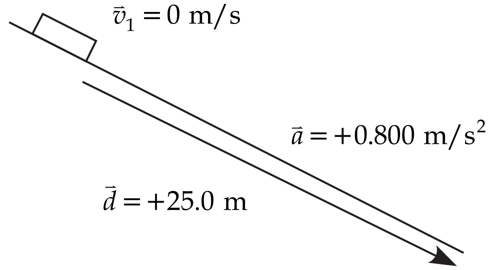
Finally, it is difficult on a graph to determine the coordinates of a point to any great degree of accuracy. So, the use of significant digit rules is difficult to apply. In these cases, try to estimate as best you can and round off your answers sensibly. For example, $\frac{+3\ \text{m}}{2\ \text{s}}$ = +1.5 m/s, not +2 m/s.

Learning Activity 1.6: The "GUESS" Method

1. A child on a toboggan starts from rest and accelerates down a snow-covered hill at 0.800 m/s/s. How long does it take the child to reach the bottom of the hill if it is 25.0 m away?

   Answer:

   Given: In the drawing, the positive direction is down the hill.



   Unknown: Time interval $\Delta t = ?$

   Equation: The appropriate equation is kinematics equation number 4:

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

   Substitute:

$$\vec{d} = \vec{v}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$+25.0\text{ m} = \left(0\text{ m/s}\right)\Delta t + \frac{1}{2} + \left(0.800\text{ m/s}^2\right)(\Delta t)^2$$

   Solve:

$$+25.0\text{ m} = \left(0\text{ m/s}\right)\Delta t + \left(+0.400\text{ m/s}^2\right)(\Delta t)^2$$

$$+25.0\text{ m} = +\left(+0.400\text{ m/s}^2\right)(\Delta t)^2$$

$$\frac{+25.0\text{ m}}{+0.400\text{ m/s}^2} = 62.5\text{ s}^2 = (\Delta t)^2$$

$$\Delta t = 7.90\text{ s}$$

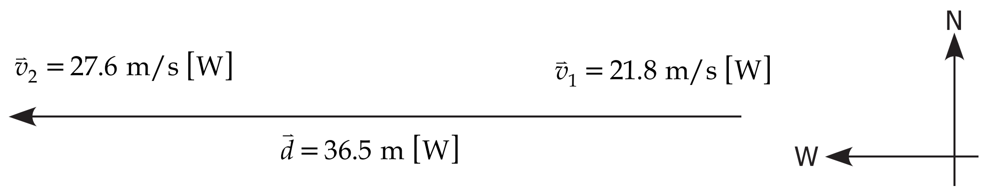
   The child and toboggan reached the bottom of the hill 7.90 seconds from the time they began to move.

2. A car accelerates uniformly from a velocity of 21.8 m/s [W] to a velocity of 27.6 m/s [W]. The car travels 36.5 m [W] during this acceleration.

   a) What was the acceleration of the car?

   Answer:

   Given:



Unknown: Acceleration $\vec{a} = ?$

Equation: This is uniformly accelerated motion. The appropriate equation from kinematics would be equation number 5:

$$v_2^2 = v_1^2 + 2ad$$

Substitute:

$$v_2^2 = v_1^2 + 2ad$$

$$(+27.6)^2 = (+21.8)^2 + 2(a)(+36.5)$$

(Use west as a positive direction and include positives and negatives to facilitate the mathematics.)

Solve:

$$761.76 = 475.24 + 73.0a$$

$$286.52 = 73.0a$$

$$\vec{a} = \frac{286.52}{73.0} = +3.92 \text{ m/s}^2$$

The car is accelerating at 3.92 m/s/s [W].

b) Determine the time interval over which this acceleration occurred.

Answer:

Given: Initial velocity $\vec{v}_1 = 21.8\text{ m/s [W]}$

Final velocity $\vec{v}_2 = 27.6\text{ m/s [W]}$

Displacement $\vec{d} = 36.5\text{ m [W]}$

Acceleration $\vec{a} = 3.92\text{ m/s}^2\text{ [W]}$

Unknown: Time interval $\Delta t = ?$

Equation: Since this is accelerated motion, an appropriate equation to use would be kinematics equation 3:

$$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)\Delta t$$

Substitute: $$+36.5\text{ m} = \frac{1}{2}\left(+21.8\text{ m/s} + \left(+27.6\text{ m/s}\right)\right)\Delta t$$

Solve: $$+36.5\text{ m} = \frac{1}{2}\left(+49.4\text{ m/s}\right)\Delta t$$

$$+36.5\text{ m} = \left(+24.7\text{ m/s}\right)\Delta t$$

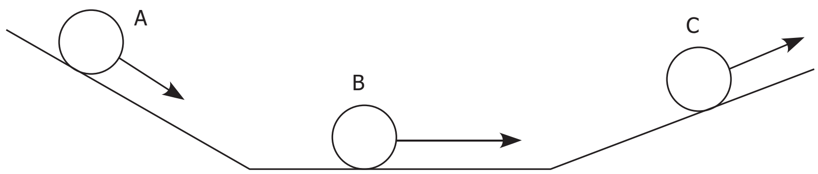
$$\Delta t = \frac{+36.5\text{ m}}{\left(+24.7\text{ m/s}\right)} = 1.48\text{ s}$$

The car accelerates during a time of 1.48 seconds.

Learning Activity 1.7: Motion with Different Accelerations

1. A ball rolls down an inclined plane, across the horizontal surface of a table, and then up a second inclined plane. Describe the type of motion that the ball undergoes on each of the surfaces. Include velocity and acceleration and their signs.

   Answer:



A: During Part A, consider “down the ramp” to be positive and “up the ramp” to be negative.

The ball rolls down the ramp with an increasing positive velocity. Since the velocity is positive and the object is speeding up, the acceleration is also positive.

B: During Part B, consider right to be positive and left to be negative. The ball rolls across this section with a constant velocity to the right. The acceleration in this section is 0 m/s/s.

C: During Part C, consider “up the ramp” to be positive and “down the ramp” to be negative. The ball rolls up the ramp with a decreasing positive velocity. Since the velocity is positive and the object is slowing down, the acceleration must be negative.

Learning Activity 1.8: Motion in the Vertical Direction

The physics of acceleration and a permanent stop (conceptual)

1. It is possible for a car moving with a constant acceleration to slow down. But can the car ever come to a permanent stop if its acceleration truly remains constant? Explain.

   Answer:

   If the acceleration vector points opposite to the direction of the velocity vector, the object will slow down with a constant acceleration. However, if the acceleration remains constant, the object will never come to a permanent stop. As time increases, the magnitude of the velocity will become smaller and smaller. Eventually, the velocity will be zero for just an instant. The magnitude of the velocity will increase in the same direction as the acceleration. In summary, if the acceleration remains constant, the object will slow down, stop for just an instant, reverse direction, and then speed up.

The physics of the signs of velocity and acceleration (conceptual)

2. For each of the following situations, give the directions of the velocity and acceleration.

   a) A ball is dropped and falls to the floor, speeding up as it falls.

      Answer:

      Let's choose up to be positive and down to be negative. Since the ball has a negative velocity (it is moving downwards) and the velocity is increasing, the velocity and acceleration must have the same direction. Therefore, the acceleration is also downwards or in the negative direction.

   b) A ball is thrown upwards and slows down as it rises.

      Answer:

      Again, let's choose up to be positive and down to be negative. This time the ball is moving upwards or in the positive direction so the velocity is positive. The ball is slowing down, which tells us that the velocity and acceleration must have opposite directions. Therefore, the acceleration is negative.

c) A car that is moving to the east coasts to a stop.

Answer:

A velocity of the car is to the east and it is slowing down. The signs of the velocity and acceleration must be opposite to each other. Therefore, the acceleration points to the west.

d) You are riding a bicycle down the hill. You and your bicycle go faster and faster.

Answer:

In this case, let's choose "down the hill" to be the positive direction. Your velocity is down the hill (positive direction) and you are speeding up. Therefore, the signs of the velocity and acceleration point in the same direction. The acceleration points down the hill or in the positive direction.

The physics of an accelerating spacecraft

3. A spacecraft is moving at a speed of +3550 m/s. The retrorockets of the spacecraft are then fired and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0 m/s^2. Assume that the displacement of the craft is +225 km. What is the velocity of the craft after this displacement?

   Answer:

   Given:

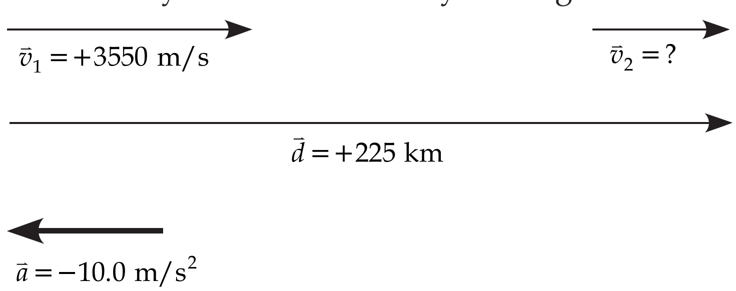
   Make a drawing.

   The drawing shows a vector pointing to the right representing the initial velocity. A second vector pointing to the right represents the second velocity. The long arrow below these two represents the displacement vector pointing to the right. The acceleration vector has been drawn and points left.

   Decide which direction is positive and which is negative.

   It is convenient to let the direction to the right be positive and the direction to the left be negative.

   Write down in symbolic form what you are given.



In this case, the displacement is given in kilometres and should be converted into metres.

$\vec{d}$ = +225 km = +225000 m = +2.25 × 10^5 m

Unknown: Final velocity $\quad \vec{v}_2 = ?$

Equation: The appropriate equation for this question would be $v_2^2 = v_1^2 + 2ad$.

Substitute: $\vec{v}_2{}^2 = \left(+3.55 \times 10^3 \text{ m/s}\right)^2 + 2\left(-10.0 \text{ m/s}^2\right)\left(+2.25 \times 10^5 \text{ m}\right)$

Solve: $\vec{v}_2 = \sqrt{\left(3.55 \times 10^3 \text{ m/s}\right)^2 + 2\left(-10.0 \text{ m/s}^2\right)\left(2.25 \times 10^5 \text{ m}\right)}$

$= +2850 \text{ m/s or } -2850 \text{ m/s}$

Both answers are possible. In the case where $\vec{v}_2$ = +2850 m/s, the spacecraft is still moving to the right after it has slowed down. This corresponds to the diagram above. In the case where $\vec{v}_2$ = –2850 m/s, the spacecraft has now reversed direction and is moving to the left. This second situation would take more time than the first.

The physics of an accelerating skier

4. a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a velocity of –8.00 m/s when going down a slope for 4.00 s?

Answer:

Calculating average acceleration

Given:

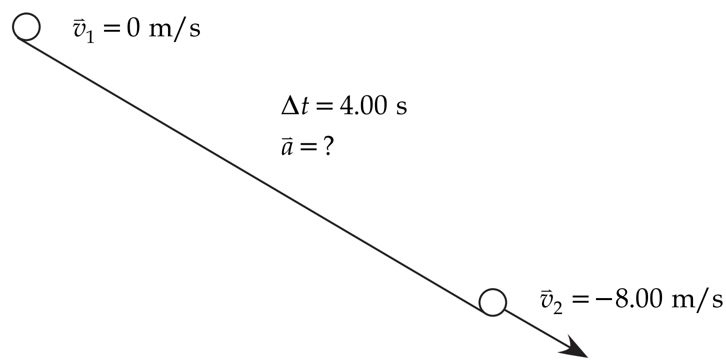
Make a drawing.

The drawing shows a point at the top from where the motion starts. At this point, the velocity is zero. Farther down the slope, the skier is shown with a velocity vector pointing down the slope. This vector represents the second velocity. The acceleration vector has not been drawn.

Decide which direction is positive and which is negative.

It is convenient to let the direction up the slope be positive and the direction down the slope be negative.

Write down in symbolic form what you are given.



Unknown: Average acceleration $\vec{a} = ?$

Equation: The appropriate equation for this question would be

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \text{ or } \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}.$$

Substitute: Solving for acceleration, the equation becomes:

$$\vec{a} = \frac{-8.00 \text{ m/s} - 0 \text{ m/s}}{4.00 \text{ s}}$$

Solve: $\vec{a} = -2.00 \text{ m/s}^2$

The acceleration of the skier is –2.00 m/s/s.

b) How far does the skier travel in this time?

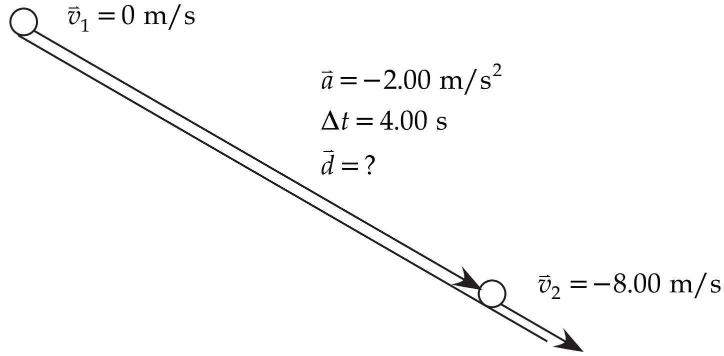
Answer:

Calculating distance travelled.

Given:

To solve for the displacement, the drawing can now be modified to what is shown below.

A second arrow has been added to show the displacement vector. The value of the acceleration is also added to the diagram.



Unknown: $\vec{d} = ?$

Equation: The appropriate equation for this question is

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2.$$

Substitute: $\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2 = (0 \text{ m/s})(4.00 \text{ s}) + \frac{1}{2}(-2.00 \text{ m/s}^2)(4.00 \text{ s})^2$

Solve: $\vec{d} = -16.0$ m

The displacement is –16.0 m. The distance travelled, which is a scalar, is 16.0 m.

There are other ways to arrive at the same answer. Other equations for displacement could be applied here. One of these equations is as follows.

$$\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)t = \frac{1}{2}(-8.00 \text{ m/s} + 0 \text{ m/s})(4.00 \text{ s}) = -16.0 \text{ m}$$

The distance travelled is 16.0 m.

The physics of an accelerating rocket

5. During an interval of 20.0 s, a rocket's velocity increased from 255 m/s to 555 m/s. What was the displacement of the rocket during this time interval?

   Answer:

   Given:

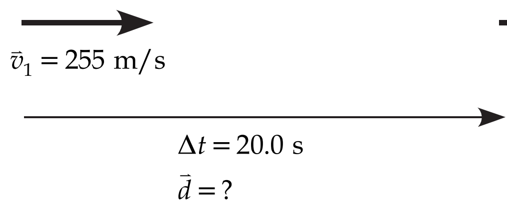
   Make a drawing.

   The drawing shows a vector indicating the initial velocity. The vector to the right shows the second velocity. The vector underneath shows the displacement.

   Decide which direction is positive and which is negative.

It is convenient to let the direction to the right be positive and the direction to the left be negative.

Write down in symbolic form what you are given.



Unknown: Displacement $\vec{d} = ?$

Equation: The appropriate equation for this problem is

$$\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t.$$

Substitute: $$\vec{d} = \frac{1}{2}(255 \text{ m/s} + 555 \text{ m/s})(20.0 \text{ s})$$

Solve: $$\vec{d} = 8100 \text{ m} = 8.10 \times 10^3 \text{ m}$$

The displacement of the rocket during this time interval is 8.10×10^3 m to the right.

The physics of an object moving in the vertical direction near Earth's surface

6. You are standing on a bridge that spans a ravine. You throw a stone downwards from the bridge towards the bottom of the ravine. The stone travels for 2.50 seconds from the moment it leaves your hand until it strikes the bottom of the ravine 58.1 m below. What was the velocity of the stone when it left your hand?

   Answer:

   Given:

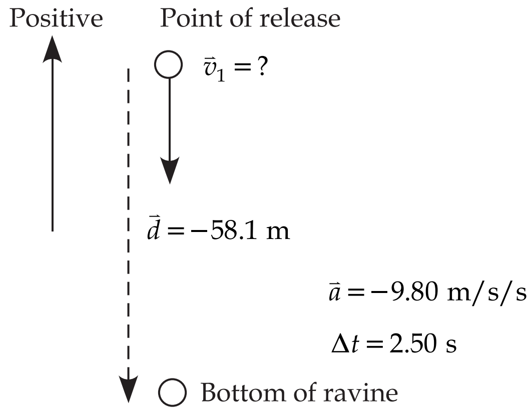
   Make a drawing.

   The drawing shows vectors indicating the initial velocity, the displacement, and the acceleration.

   Decide which direction is positive and which is negative.

   It is convenient to let up be positive and down be negative.

Write down in symbolic form what you are given.



Unknown: The unknown is the velocity of the stone at the time of release.

$\vec{v}_1 = ?$

Equation: The appropriate equation to use would be kinematics equation 4.

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Substitute:

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$-58.1 \text{ m} = \vec{v}_1 (2.50 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s/s})(2.50 \text{ s})^2$$

Solve:

$$-58.1 \text{ m} = \vec{v}_1 (2.50 \text{ s}) + (-30.6 \text{ m})$$

$$(-58.1 \text{ m}) - (-30.6 \text{ m}) = \vec{v}_1 (2.50 \text{ s})$$

$$-27.5 \text{ m} = \vec{v}_1 (2.50 \text{ s})$$

$$\vec{v}_1 = \frac{-27.5 \text{ m}}{2.50 \text{ s}} = -11.0 \text{ m/s}$$

Note: In the above calculation, you must square 2.50 seconds before you multiply by the acceleration.

The velocity of the stone as it left your hand was –11.0 m/s or 11.0 m/s downwards.

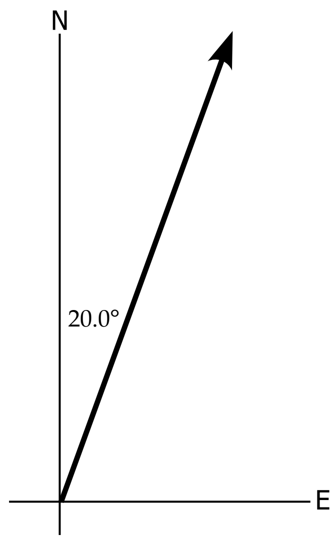
Learning Activity 1.9: Introduction to Vectors

1. Sketch the following vectors. You do not have to make an accurate scale diagram, but try to draw these in an appropriate size.

   a) 80 m [20° E of N]

   Answer:

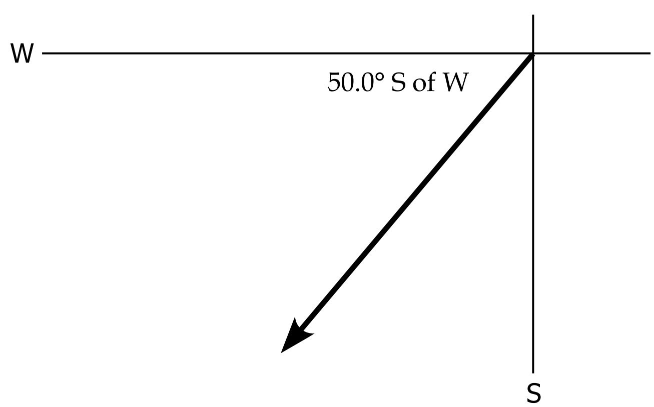
   An appropriate scale to use would be 1 cm = 10 m. Your vector should have a length of 8 cm.



   b) 500 m [50° S of W]

   Answer:

   An appropriate scale to use would be 1 cm = 100 m. So your vector should be 5 cm long.



2. Add the following vectors.

a) $\vec{A}$ = 202 m [W] and $\vec{B}$ = 357 m [W]

Answer:

These vectors are parallel, so simply add the magnitudes and keep the same direction.

$\vec{R}$ = 559 m [W]

b) $\vec{A}$ = 202 m [W] and $\vec{B}$ = 357 m [E]

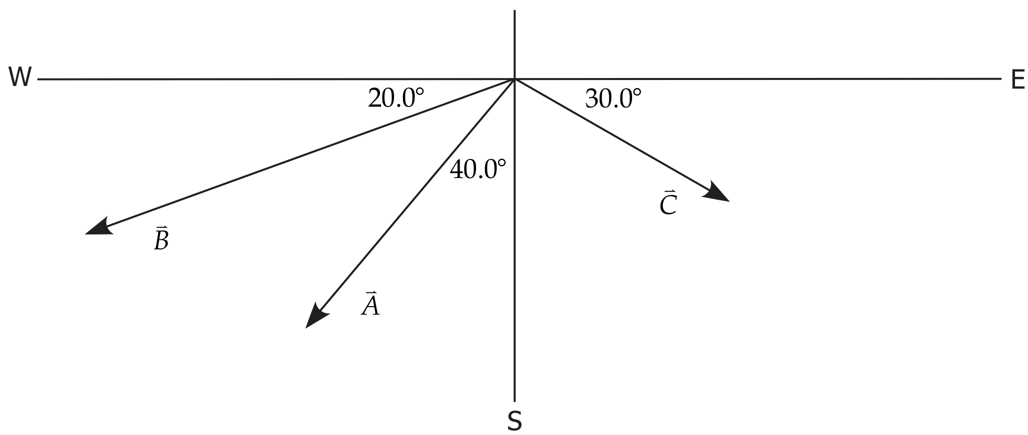
Answer:

These vectors are antiparallel, so find the difference between their magnitudes to obtain the magnitude of the resultant. The direction of the resultant is given by the direction of the larger vector.

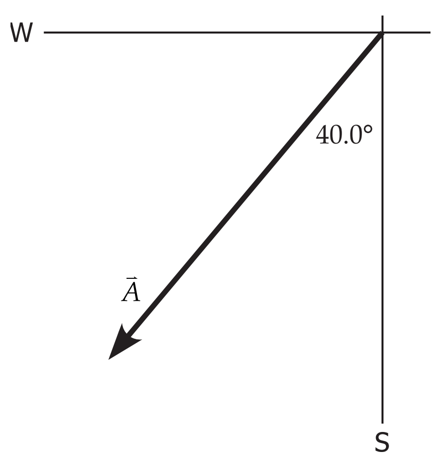
$\vec{R}$ = 155 m [E]

Learning Activity 1.10: Adding, Subtracting, and Resolving Vectors

1. Sketch the following vectors. You do not have to make an accurate scale diagram, but try to draw these in an appropriate size.
   a) $\vec{A}$ = 35.0 m [40.0° W of S]
   b) $\vec{B}$ = 50.0 m [20.0° S of W]
   c) $\vec{C}$ = 20.0 m [30.0° S of E]
   Answer:



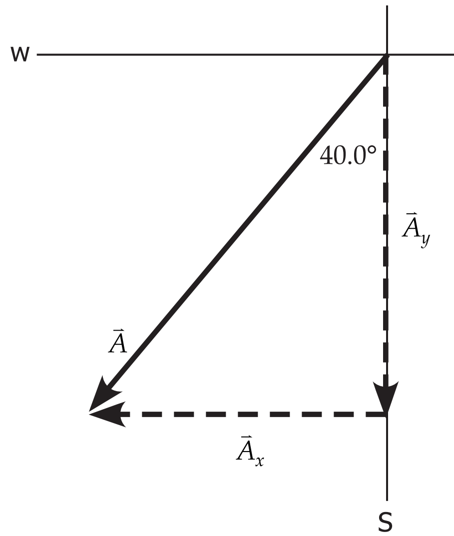
2. Resolve the vectors in #1 into their components.
   a) $\vec{A}$ = 35.0 m [40.0° W of S]
   Answer:



Now you must enclose the given angle with the two components. This is done by dropping a perpendicular from the tip of the vector to the south axis. Draw in the two components, $\vec{A}_x$ for the east-west direction and $\vec{A}_y$ for the north-south direction.

Using $\sin 40.0° = \frac{\vec{A}_x}{\vec{A}}$, we can change this to

$$\vec{A}_x = \vec{A}(\sin 40.0°) = (35.0 \text{ m})(\sin 40.0°) = 22.5 \text{ m}.$$



The x-component of the vector $\vec{A}$ is –22.5 m. The sign of this component is negative because it points in the negative x–direction.

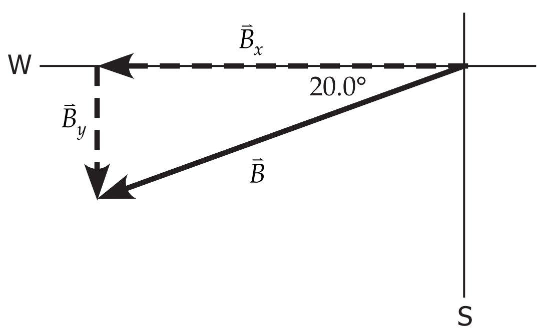
Using $\cos 40.0° = \frac{\vec{A}_y}{\vec{A}}$, we can change this to

$$\vec{A}_y = \vec{A}(\cos 40.0°) = (35.0 \text{ m})(\cos 40.0°)$$
$$= 26.8 \text{ m}.$$

The y-component of the vector $\vec{A}$ is –26.8 m. The sign of this component is negative because it points in the negative y-direction.

b) $\vec{B}$ = 50.0 m [20.0° S of W]

Answer:



Using the sin 20.0°, we can find $\vec{B}_y$, the y-component of $\vec{B}$.

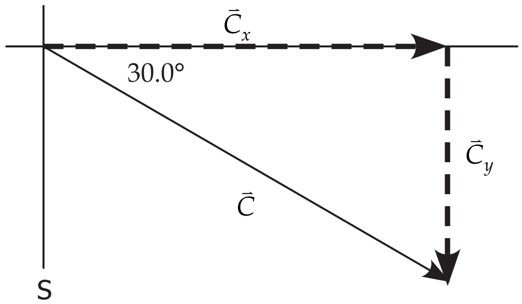
$\vec{B}_y = \vec{B}(\sin 20.0°)$
$= (50.0 \text{ m})(\sin 20.0°)$
$= –17.1 \text{ m}.$

Using the cos 20.0°, we can find $\vec{B}_x$, the x-component of $\vec{B}$.

$\vec{B}_x = \vec{B}(\cos 20.0°)$
$= (50.0 \text{ m})(\cos 20.0°)$
$= –47.0 \text{ m}.$

c) $\vec{C}$ = 20.0 m [30.0° S of E]

Answer:



Using the sin 30.0°, we can find $\vec{C}_y$, the *y*-component of vector $\vec{C}$.

$$\begin{aligned}\vec{C}_y &= \vec{C}(\sin 30.0°)\\ &= (20.0\text{ m})(\sin 30.0°)\\ &= -10.0\text{ m.}\end{aligned}$$

Using the cos 30.0°, we can find $\vec{C}_x$, the *x*-component of vector $\vec{C}$.

$$\begin{aligned}\vec{C}_x &= \vec{C}(\cos 30.0°)\\ &= (20.0\text{ m})(\cos 30.0°)\\ &= +17.3\text{ m.}\end{aligned}$$

3. Add the following vectors.

a) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [W]

Answer:

For vectors that are parallel—that is, pointed in the same direction—simply add the magnitudes of the vectors together to give the magnitude of the resultant. The resultant points in the same direction as the two component vectors.

$\vec{A}+\vec{B} = 20.0\text{ m }[\text{W}] + 25.0\text{ m }[\text{W}] = 45.0\text{ m }[\text{W}]$

b) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [E]

Answer:

For vectors that are antiparallel—that is, pointed in opposite directions—find the difference between the magnitudes to obtain the magnitude of the resultant. The resultant points in the direction of the larger vector in the question.

$\vec{A}+\vec{B} = 20.0\text{ m }[\text{W}] + 25.0\text{ m }[\text{E}] = 5.0\text{ m }[\text{E}]$

c) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [S]

Answer:

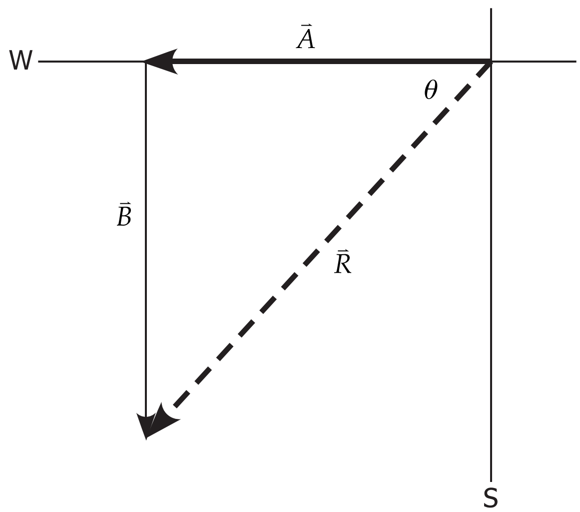
The addition of vectors requires that we start first with a coordinate system.

Draw in the first vector with its tail at the reference point. Using the tip-to-tail method, place the tail of the second vector on the head of the first vector.

The resultant is a vector, with its foot at the tail of the first vector and its head at the head of the last vector.

$\vec{A} = 20.0 \text{ m } [\text{W}]$ and $\vec{B} = 25.0 \text{ m } [\text{S}]$

$\vec{A} + \vec{B} = 20.0 \text{ m } [\text{W}] + 25.0 \text{ m } [\text{S}]$



Using the theorem of Pythagoras, you can calculate from the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ the magnitude of the resultant vector $\vec{R}$.

$\vec{R}^2 = \vec{A}^2 + \vec{B}^2 = (20.0)^2 + (25.0)^2 = 400 + 625 = 1025$

$\vec{R} = \sqrt{1025} = 32.0 \text{ m}$

You still need to know the direction of the resultant. The angle for the direction of the resultant must be drawn at the foot of the resultant. This angle is marked in the diagram by theta, θ.

You can calculate the value of theta using the tangent function:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{25.0}{20.0} = 1.25$$

Using the second function or shift button on your calculator, you can determine the angle from which the tangent value is 1.25.

$\theta = 51.3°$

Finally, state your complete answer. The sum of the vectors $\vec{A}$ and $\vec{B}$ is 32.0 m [51.3° S of W].

4. Perform the subtractions as indicated. Remember to treat subtraction as "the addition of the opposite."

   a) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [W], find $\vec{A}-\vec{B}$

   Answer:

   $$-\vec{B}=25.0 \text{ m } [\text{E}]$$
   $$\vec{A}-\vec{B}=\vec{A}+\left(-\vec{B}\right)=20.0 \text{ m } [\text{W}]+25.0 \text{ m } [\text{E}]$$
   $$=5.0 \text{ m } [\text{E}]$$

   b) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [W], find $\vec{B}-\vec{A}$

   Answer:

   $$-\vec{A}=20.0 \text{ m } [\text{E}]$$
   $$\vec{B}-\vec{A}=\vec{B}+\left(-\vec{A}\right)=25.0 \text{ m } [\text{W}]+20.0 \text{ m } [\text{E}]$$
   $$=5.0 \text{ m } [\text{W}]$$

   c) $\vec{A}$ = 20.0 m [W] and $\vec{B}$ = 25.0 m [S], find $\vec{B}-\vec{A}$

   Answer:

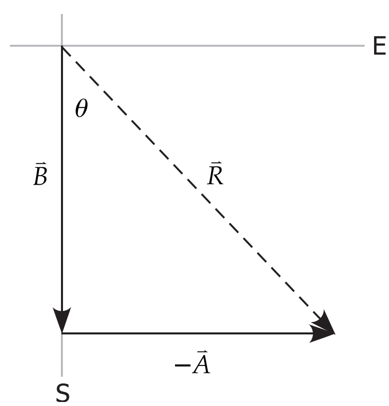
   You will be adding the opposite of the vector $\vec{A}$ on to the vector $\vec{B}$.

   The opposite of the vector $\vec{A}$ is $-\vec{A}$ = 20.0 m [E] .

   $$\vec{B}-\vec{A}=\vec{B}+\left(-\vec{A}\right)=25.0 \text{ m } [\text{S}]+20.0 \text{ m } [\text{E}]$$

   To determine the resultant in this case, you must draw a sketch of the given vectors using the tip–to-tail method, draw in the resultant vector, and mark the angle at the foot of the resultant vector inside the right triangle that appears in your diagram.

   Then you must use the theorem of Pythagoras to find the magnitude of the resultant vector and the tangent function to find the direction in which it points.



   The resultant has a magnitude of 32.0 m. The angle is 38.7°.

   The resultant for this question is 32.0 m [38.7° E of S].

Learning Activity 1.11: Relative Motion along a Line

For each of the following, identify each velocity using the two subscript notations, and determine the velocity of the object relative to the ground.

1. A person is walking at 1.2 m/s towards the back of a train travelling at 19.0 m/s [N].

   Answer:

   $\vec{v}_{PT}$ can represent the velocity of the "person" relative to the "train" = 1.2 m/s [S]

   $\vec{v}_{TG}$ can represent the velocity of the "train" relative to the "ground" = 19.0 m/s [N]

   $\vec{v}_{PG}$ can represent the velocity of the "person" relative to the "ground" = ?

   Add: $\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG}$

   Since you are adding antiparallel vectors, you simply find the difference between the magnitudes, giving a resultant that points in the direction of the larger vector.

   $\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG} = 1.2 \text{ m/s } [\text{S}] + 19.0 \text{ m/s } [\text{N}] = 17.8 \text{ m/s } [\text{N}]$

2. A bird flying through the air at 5.65 m/s [E] where the wind is blowing at 4.95 m/s from the east.

   Answer:

   The wind is really going west!

   $\vec{v}_{BA}$ can represent the velocity of the "bird" relative to the "air" = 5.65 m/s [E]

   $\vec{v}_{AG}$ can represent the velocity of the "air" relative to the "ground" = 4.95 m/s [W]

   $\vec{v}_{BG}$ can represent the velocity of the "bird" relative to the "ground" = ?

   Add: $\vec{v}_{BG} = \vec{v}_{BA} + \vec{v}_{AG}$

   Since you are adding antiparallel vectors, you simply find the difference between the magnitudes, giving a resultant that points in the direction of the larger vector.

   $\vec{v}_{BG} = \vec{v}_{BA} + \vec{v}_{AG} = 5.65 \text{ m/s } [\text{E}] + 4.95 \text{ m/s } [\text{W}] = 0.70 \text{ m/s } [\text{E}]$

3. A person is walking on a moving sidewalk at 1.80 m/s in the same direction as the sidewalk, which is moving at 3.00 m/s.

   Answer:

   Let the direction of motion be positive.

   $\vec{v}_{PS}$ can represent the velocity of the "person" relative to the "sidewalk" = +1.80 m/s

   $\vec{v}_{SG}$ can represent the velocity of the "sidewalk" relative to the "ground" = +3.00 m/s

   $\vec{v}_{PG}$ can represent the velocity of the "person" relative to the "ground" = ?

   Add: $\vec{v}_{PG} = \vec{v}_{PS} + \vec{v}_{SG}$

   Since you are adding parallel vectors, you simply add the magnitudes, giving a resultant that points in the direction of the given vector.

   $\vec{v}_{PG} = \vec{v}_{PS} + \vec{v}_{SG} = +1.80 \text{ m/s} + (+3.00 \text{ m/s}) = +4.80 \text{ m/s}$

Learning Activity 1.12: Relative Motion in One and Two Dimensions

The physics of a hound walking on a moving boat

1. A hound walks at a speed of 2.00 m/s along the deck toward the front of a boat, which is travelling at 8.00 m/s with respect to the water.

   a) What is the velocity of the hound relative to the water?

   Answer:

   Let us assume, for the sake of simplicity, that the boat is moving to the right.

   The velocity of the hound with respect to the boat is $\vec{v}_{HB}$ = 2.00 m/s [Right].

   The velocity of the boat with respect to the water is $\vec{v}_{BW}$ = 8.00 m/s [Right].

   The velocity of the hound with respect to the water is $\vec{v}_{HW}$.

   $$\begin{aligned}\vec{v}_{HW} &= \vec{v}_{HB} + \vec{v}_{BW} \\ &= 2.00 \text{ m/s } [\text{Right}] + 8.00 \text{ m/s } [\text{Right}] = 10.00 \text{ m/s } [\text{Right}]\end{aligned}$$

   b) What would be the velocity of the hound if the dog were walking toward the back of the boat?

   Answer:

   In this case, the velocity of the dog is opposite to that of the boat.

   $$\begin{aligned}\vec{v}_{HW} &= \vec{v}_{HB} + \vec{v}_{BW} \\ &= 2.00 \text{ m/s } [\text{Left}] + 8.00 \text{ m/s } [\text{Right}] \\ &= 6.00 \text{ m/s } [\text{Right}]\end{aligned}$$

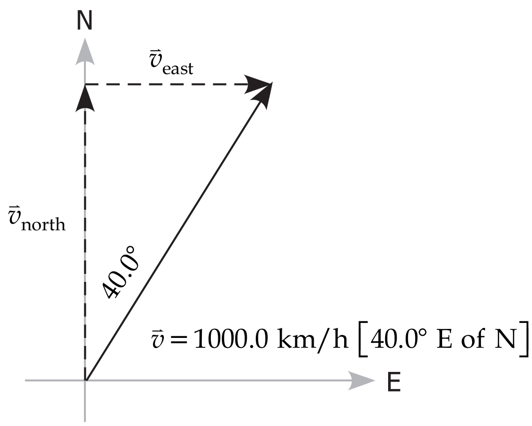
The physics of an airplane of an airplane travelling in calm air

2. An Air Canada plane is travelling at 1000.0 km/h in a direction 40.0° east of north.

   a) Find the components of the velocity vector by finding the component in the northerly direction and the easterly direction.

      Answer:

      A rough diagram of the motion will help to visualize the situation.



Since you don't have to work with any other units like time in seconds, you can keep the units for the velocity of the plane as km/h.

The component in the northerly direction is found by using the cosine function.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\vec{v}_{north}}{\vec{v}}$$

$$\cos 40.0° = \frac{\vec{v}_{north}}{1000.0 \text{ km/h}}$$

$$\vec{v}_{north} = 766 \text{ km/h}$$

The component in the easterly direction is found by using the sine function.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\vec{v}_{east}}{\vec{v}}$$

$$\sin 40.0° = \frac{\vec{v}_{east}}{1000.0 \text{ km/h}}$$

$$\vec{v}_{east} = 643 \text{ km/h}$$

The northerly component of the velocity of the plane is 766 km/h, and the easterly component of the velocity of the plane is 643 km/h.

b) How far to the north and how far to the east would the plane travel in 4.00 h?

Answer:

To find the distances travelled, use $\vec{v}_{\text{across}} = \dfrac{\vec{d}}{\Delta t}$, since the velocities are constant.

Rearranging gives you $\vec{d} = \vec{v}\Delta t$.

The distance travelled in the northerly direction is
$d = (766 \text{ km/h})(4.00 \text{ h}) = 3.06 \times 10^3 \text{ km}$.

The distance travelled in the easterly direction is
$d = (643 \text{ km/h})(4.00 \text{ h}) = 2.57 \times 10^3 \text{ km}$.

The physics of a vacationer and a cruise ship

3. A captain walks 4.00 km/h directly across a cruise ship whose speed relative to Earth is 12.0 km/h. What is the speed of the captain with respect to Earth?

Answer:

Assume that the ship is travelling towards the east and that the captain is walking north.

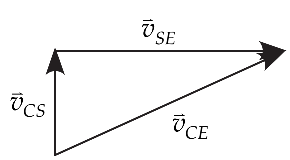
Then, $\vec{v}_{CS}$ is the velocity of the captain with respect to the ship = 4.00 km/h [N].

$\vec{v}_{SE}$ is the velocity of the ship with respect to Earth = 12.0 km/h [E].

$\vec{v}_{CE}$ is the velocity of the captain with respect to Earth.

Adding the vectors together, you get $\vec{v}_{CE} = \vec{v}_{CS} + \vec{v}_{SE}$.

Sketching the vector diagram gives:



In this case, the velocity of the captain with respect to Earth is the hypotenuse of the triangle.

$$\vec{v}_{CE} = \sqrt{(4.0 \text{ km/h})^2 (12.0 \text{ km/h})^2} = 12.6 \text{ km/h}$$

Note that the angle is not stated, since the question asked for speed and not velocity.

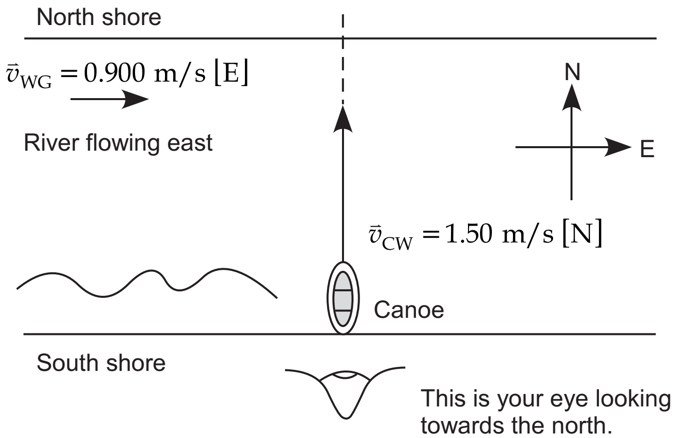
The speed of the captain with respect to Earth is 12.6 km/h.

The physics of a canoe in a current

4. A person paddling a canoe is able to make the canoe and himself travel at a speed of 1.50 m/s in still water. The paddler heads the canoe directly across a 3.00 km wide river. The current of the river flows at 0.900 m/s downstream.

   a) What is the velocity of the canoe as observed from the shore?

   Answer:

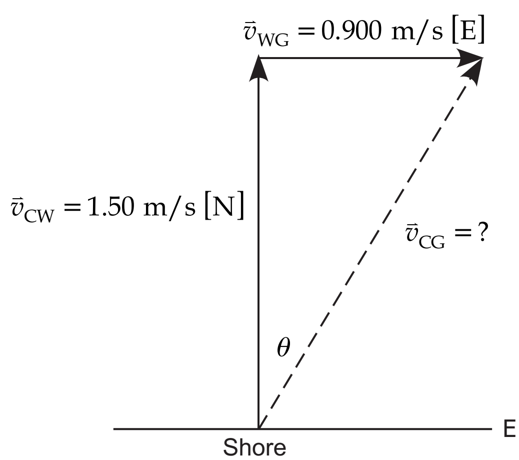


The velocity $\vec{v}_{CW}$ of the canoe relative to the water = 1.50 m/s [N].

The velocity $\vec{v}_{WG}$ of the water relative to the ground = 0.900 m/s [E].

The velocity $\vec{v}_{CG}$ of the canoe relative to the ground is the vector sum of $\vec{v}_{CW}$ and $\vec{v}_{WG}$.

$\vec{v}_{CG} = \vec{v}_{CW} + \vec{v}_{WG}$



Use the magnitudes of the two given vectors and the theorem of Pythagoras to find the magnitude of the hypotenuse.

$$\vec{v}_{CG} = \sqrt{(1.50)^2 + (0.900)^2} = \sqrt{2.25 + 0.810} = \sqrt{3.06}$$
$$\vec{v}_{CG} = 1.75 \text{ m/s}$$

Use the tangent function to relate $\vec{v}_{CW}$ and $\vec{v}_{WG}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{0.900 \text{ m/s}}{1.50 \text{ m/s}} = 0.600$$
$$\theta = \tan^{-1}(0.600) = 31.0°$$

The velocity of the canoe is 1.75 m/s [31.0° east of north].

The velocity of the canoe, as given relative to the shore, is 1.75 m/s [59.0° north of the shore].

b) How long does it take the canoe to cross the river?

Answer:

The component of $\vec{v}_{CG}$ that is perpendicular to the width of the river (points across the river) determines how fast the canoe is moving across the river. This perpendicular component points in the northerly direction and is represented by $\vec{v}_{CW}$.

$$\vec{v}_{CW} = 1.50 \text{ m/s [N]}$$
$$\vec{d} = 3.00 \text{ km [N]} = 3000 \text{ m [N]} = 3.00 \times 10^3 \text{ m [N]}$$

Here, you must convert km to m, since the velocity is given in m/s.

The unknown is the time interval.

The equation to use from kinematics is the one for constant velocity:

$$\vec{v}_{across} = \frac{\vec{d}}{\Delta t}$$

Rearranging gives you: $\Delta t = \dfrac{\vec{d}_{across}}{\vec{v}_{across}}$

Substituting and solving gives $\Delta t = \dfrac{3.00 \times 10^3 \text{ m [N]}}{1.50 \text{ m/s [N]}} = 2.00 \times 10^3$ s.

The time for the canoe to cross the river is $\Delta t = 2.00 \times 10^3$ s.

c) How far downstream will the canoe be upon reaching the other side of the river?

Answer:

The component of $\vec{v}_{CG}$ that is parallel to the direction of the current determines how far the canoe is carried downstream; this component is $\vec{v}_{WG} = 0.900 \text{ m/s [E]}$.

The canoe is being carried downstream while it is paddled across the river: $\Delta t = 2.00 \times 10^3$ s.

The unknown is the displacement downstream: $\vec{d}_{\text{downstream}}$.

Since the motion occurs with constant velocity, you can use kinematics equation 1, as you did in part (b). However, this time the equation must be rearranged to solve for the displacement downstream, $\vec{d}_{\text{downstream}}$.

$$\vec{d}_{\text{downstream}} = \vec{v}_{\text{downstream}} \Delta t$$

Substituting and solving gives $\vec{d}_{\text{downstream}} = (0.900 \text{ m/s})(2.00 \times 10^3 \text{ s}) = 1.80 \times 10^3$ m downstream.

The distance that the canoe is carried downstream while crossing the river is 1.80×10^3 m.

The physics of an airplane flying in a crosswind

5. An airplane flies with an airspeed of 225 km/h heading due west. At the altitude at which the plane is flying, the wind is blowing at 105 km/h heading due south.

   a) What is the velocity of the plane as observed by someone standing on the ground?

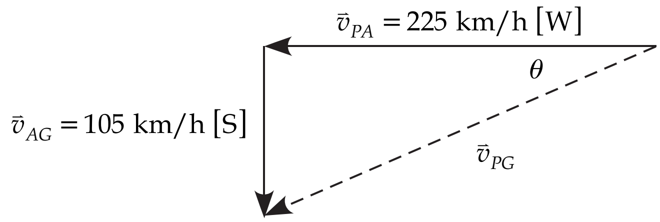
   Answer:

The velocity of the plane relative to the air	$\vec{v}_{PA}$ = 225 km/h [W]
The velocity of the air relative to the ground	$\vec{v}_{AG}$ = 105 km/h [S]
The velocity of the plane relative to the ground	$\vec{v}_{PG}$ = ?

   The velocity of the plane relative to the air plus the velocity of the air relative to the ground yields the velocity of the plane relative to the ground.

   $$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} = 225 \text{ km/h [W]} + 105 \text{ km/h [S]}$$

These vectors are perpendicular to each other. First, draw a sketch of the vectors and the resultant.



Using the theorem of Pythagoras, you can determine the magnitude of $\vec{v}_{PG}$.

Using the tangent function, you can determine theta, θ.

$v_{PG}{}^2 = (225)^2 + (105)^2 = 61650$

$\vec{v}_{PG} = 248 \text{ km/h}$

The velocity of the plane, as observed from the ground, would be 248 km/h [25.0° south of west].

$\tan\theta = \dfrac{105}{225} = 0.46667$

$\theta = \tan^{-1}(0.46667) = 25.0°$

b) How far off course would the plane, while it is heading due west, be blown by the wind during 1.50 h of flying?

Answer:

The component of the plane's velocity that is carrying it off course is the component due to the wind: $\vec{v}_{AG} = 105 \text{ km/h [S]}$

You can use kinematics equation 1 for constant velocity to determine the displacement.

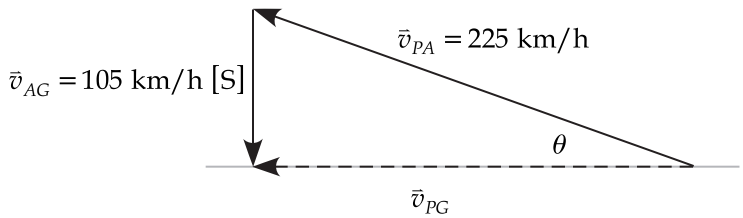
Given:	$\vec{v}_{AG} = 105 \text{ km/h [S]}$ $\Delta t = 1.50 \text{ h}$
Unknown: Displacement	$\vec{d} = ?$
Equation:	$\vec{v} = \dfrac{\vec{d}}{\Delta t}$ $\vec{d} = \vec{v}\Delta t$
Substitute:	$\vec{d} = (105 \text{ km/h [S]})(1.50 \text{ h})$
Solve:	$\vec{d} = 158 \text{ km [S]}$

The plane is blown 158 km south off course.

c) What heading must a plane take in order to reach its destination, which is due west of a starting point?

Answer:

Since the plane is blown off course to the south, the plane must head north of due west in order to prevent being blown off course. Drawing the vectors produces the following diagram.



The angle theta, θ, can be found using the sine function.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{105}{225} = 0.46667$$

$$\theta = \sin^{-1}(0.46667) = 27.8°$$

The plane must head 27.8° north of west in order to not be blown off course.

Grade 12 Physics (40S)

Module 2: Dynamics

This module contains the following:

- Introduction to Module 2
- Lesson 1: Newton's First Law of Motion
- Lesson 2: Newton's Second Law of Motion
- Lesson 3: Mass, Weight, and the Force of Gravity
- Lesson 4: The Force of Friction
- Lesson 5: The Component Method for Adding and Subtracting Vectors
- Lesson 6: Video Laboratory Activity: Forces in Equilibrium
- Lesson 7: Forces in Equilibrium
- Lesson 8: Forces at an Angle
- Lesson 9: Newton's Third Law of Motion
- Module 2 Summary

Module 2: Dynamics

Introduction to Module 2

In the previous module, we studied kinematics, which is a description of how objects move. In this module, we study dynamics, which deals with why objects move as they do. In studying dynamics, it is necessary to consider how forces affect the movement of objects. Once forces are taken into account, it is possible to describe motion in a much more realistic manner.

There are nine lessons in this module:

Lesson 1: Newton's First Law of Motion will review Newton's first law of inertia.

Lesson 2: Newton's Second Law of Motion deals with Newton's second law of inertia and shows how it can be linked to kinematics.

Lesson 3: Mass, Weight, and the Force of Gravity reviews mass and weight.

Lesson 4: The Force of Friction deals with the force of friction. These concepts were first used in Grade 10 Science and then studied further in Grade 11 Physics.

Lesson 5: The Component Method for Adding and Subtracting Vectors will introduce to you a method for adding vectors that point in directions other than perpendicular to each other.

Lesson 6: Video Laboratory Activity: Forces in Equilibrium demonstrates that forces in equilibrium have a vector sum of 0 N. A force table is employed to arrange and measure the forces.

Lesson 7: Forces in Equilibrium is the study of forces in equilibrium—that is, situations where one or more forces act on objects but the objects are at rest.

Lesson 8: Forces at an Angle considers forces acting on objects at an angle, including objects on inclined surfaces.

Lesson 9: Newton's Third Law of Motion considers action-reaction pairs of forces.

Assignments in Module 2

When you have completed the assignments for Module 2, submit your completed assignments for Module 1 and Module 2 to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
4	Assignment 2.1	Forces of Friction and Motion
5	Assignment 2.2	Adding Vectors Using the Component Method
6	Assignment 2.3	Video Laboratory Activity: Forces in Equilibrium
7	Assignment 2.4	Objects in Equilibrium
8	Assignment 2.5	Forces Acting at an Angle


As you work through this course, remember that your learning partner and your tutor/marker are available to help you if you have questions or need assistance with any aspect of the course.

LESSON 1: NEWTON'S FIRST LAW OF MOTION

(0.75 HOUR)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ describe how Newton's First Law of Motion applies in various situations
- ❑ draw free-body diagrams to aid in the analysis of problems involving forces

Key Words

inertia	force	mass
Newton's laws	free-body diagram	net force
balanced force	normal force	force of friction
dynamics		

Introduction

This lesson is a review lesson and is quite short. It should take no more than 45 minutes to complete.

In Module 1, you considered kinematics, the study of how objects move.

In this module, you will consider dynamics, the study of why objects move.

You have been subjected to Newton's laws of motion at one time or another. If you are pushing a car at rest, you know that it is difficult to get it moving. On the other hand, once the car is in motion it wants to keep moving. You know that the harder you push on the car, the greater its acceleration will be. If you push a large car with the same force as a small car, the acceleration of the large car will be less than the acceleration of the small car. As you push on the car, you can feel it pushing back on your hands with an equal but opposite force to the force your hands exert on the car.

In this one example, all three of Newton's laws of motion apply. Let's refresh your memory by considering each of these laws in more detail.

Newton's First Law of Inertia

You'll notice that each of Newton's laws of motion involve the idea of force.

> A **force** is an action, like a push or pull, that causes a change in the motion of an object.

Since Newton's First Law of Motion, also known as Newton's law of inertia, involves the concept of inertia, it is imperative that we understand the meaning of the term "inertia."

Let's study the following definitions:

> **Matter** is anything that has mass and volume. Matter also possesses inertia.
>
> **Mass** is a measure of the amount of material in an object.
>
> **Inertia** is the property of matter that opposes any change in its state of motion.

Inertia is the natural tendency of an object to remain at rest or in motion at a constant speed along a straight line (constant velocity). The mass of the object is a quantitative measure of its inertia. For example, a book (mass = 2 kg) has less inertia than a bicycle (mass = 15 kg). The larger the mass of an object, the greater the inertia. Therefore, it takes less force to start a 5 kg smooth stone moving on ice than it does to start a 10 kg stone on the same surface. Similarly, it is easier to change the speed or direction of the lighter stone because it has less inertia.

> **Newton's First Law of Motion:** If there is no net force acting on a body, the body will continue in its state of rest or will continue moving along a straight line at constant speed.

"Net force" is used in Newton's first law of motion. As you saw in Grade 11 Physics, in most cases, an object is acted upon by several forces simultaneously and the net force is the vector sum of these forces. You will recall that a free-body diagram was drawn showing all the forces. All the forces acting on the object may balance, such as the normal force cancelling the force of gravity and a force of friction cancelling the applied force. In this case, the net force on the object is 0 N and there is no net force to change the state of motion of the object.

Another way to look at this law is to ask, "What keeps an object at rest remaining at rest?" From your knowledge, you can surmise that if a book is resting on a table, there is no net force on the book since the normal force and the force of gravity cancel. You do not have to do anything to the book to keep it motionless as long as the net force is 0 N.

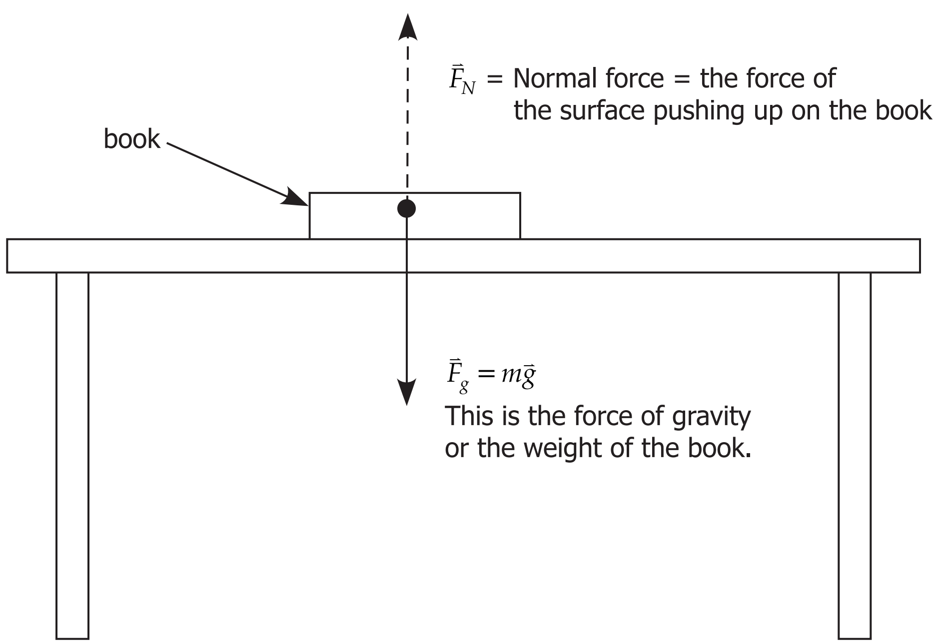
Another question to ask is, "What keeps an object in motion moving with constant velocity?" The answer, again, is a net force of 0 N acting on the object. There is no need to push or pull the object to keep it moving. It moves along all on its own.

Consider the game of curling, which uses curling rocks with a mass of 18 kg. If the curler exerts no unbalanced force on the rock, the rock will not move. Rocks sit in storage without moving as long as there is no unbalanced force acting on the rocks. This is an example of the inertia of rest. Now, consider a rock that is thrown by a curler. After the rock leaves the curler's hand, it slides down the ice. The ice is level and very slippery, so the force of friction is small. The rock slides with a fairly constant velocity, slowing down slightly due to the unbalanced force of friction.

If we could extend the ice into an infinitely long level sheet and reduce the drag of the ice and the air on the rock (reduce the net force to 0 N), then the rock would slide forever with constant velocity. There would be nothing pushing it or pulling it to keep it moving with constant velocity. This is an example of the inertia of motion.

An Example of the Inertia of Rest

Here is an example of an object that remains at rest. Carefully study the illustration below:



A book is at rest on a flat, level table. Obviously, it will remain there unless a net force is applied to it, such as your reaching out and picking it up.

Normal Force: The normal force is the force with which the surface pushes up onto an object that rests on that surface. The normal force always points perpendicularly out of the surface.

This does not mean that the book has no forces acting on it. There are at least two forces acting on it: the force of gravity pulling it downward against the table and the equal, but opposite force (**normal force**) of the table pushing upward against the book.

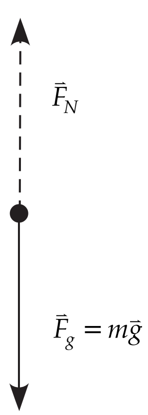
Balanced Forces: Balanced forces **do no**t cause a change in motion. They are equal in size and opposite in direction.

Since the vertically upward and the vertically downward forces have the same magnitudes, their vector sum is zero. Therefore, there is no net force acting on the book or all of the forces acting on the book are **balanced**, and the book remains at rest.

In order to illustrate the forces acting on an object, you draw a **free-body diagram**.

Free-Body Diagram: A **free-body diagram** uses a dot (•) to represent the object upon which the forces are acting. Then all of the forces acting on the object are drawn in as force vectors with the tails starting from the dot.

In the example with the book resting on the table, the free-body diagram would look like this:



The force of gravity $\left(\vec{F}_g\right)$ and the normal force $\left(\vec{F}_N\right)$ are both acting on the book (represented by the dot).

$\vec{F}_{\text{net}} = \sum \text{Forces}$

(Sum of the forces acting on the book)

Read the symbol Σ as "sum of."

$\vec{F}_{\text{net}} = \vec{F}_N + \vec{F}_g$

$\vec{F}_{\text{net}} = 0 \text{ N}$

These two forces are balanced forces.

An Example of the Inertia of Motion

Consider this example of an object that remains at constant velocity. In space, beyond the gravitational pull of the Earth or any other celestial body, an astronaut working on the outside of a spacecraft throws a hammer away from the craft. The hammer will travel away from the spacecraft in a straight line at a constant speed. It will continue to move in a straight line unless some other object bumps into it and causes it to change direction. It will continue to move at the same velocity unless some other object causes it to speed up or slow down.

As long as there are no net forces acting on the hammer, it will not change direction or speed. Undoubtedly, the hammer will travel millions of kilometres until it travels close to another object, such as a planet. Then, the force of gravity of the planet will create a net force on the hammer and it will change direction and speed.

Sometimes, it is difficult for us to comprehend Newton's First Law of Motion unless we understand the effect that friction has on an object. Consider a child pulling a wagon at a constant velocity.

Obviously, a steady force is needed to keep the wagon moving. If the child ceases exerting a force on the handle of the wagon, the wagon will stop. It appears that there is a net force acting on the wagon—the force provided by the child. If this were the case, there should be an acceleration of the wagon. However, you are told that the wagon is moving at a constant velocity. Therefore, there must be a backward force equal in magnitude to that supplied by the child so that the vector sum of the two forces is zero. This hidden force is the force of friction. Thus, we can see that Newton's first law applies to this situation.

Friction is a force that opposes the motion of one surface over another surface.

Learning Activity 2.1

Newton's First Law

There are five questions in this learning activity. You may check your answers against the answer key provided at the end of Module 2.

1. If a basketball is placed on top a low-level table, it will remain there.
   a) What forces are acting on the ball in this situation?
   b) Are there any unbalanced forces acting on the ball?
2. a) If the basketball in #1 is given a slight push, what happens to the ball?
   b) What forces act on the ball after it has been pushed?
   c) Are any of these unbalanced forces?
   d) What is the effect of the various forces on the motion of the ball?
3. Why is it particularly dangerous to drive on an icy highway? (**Hint:** Consider the inertia of the vehicle and the low frictional force between the wheels and the icy surface.)
4. Why do you "lunge forward" when your car suddenly comes to a halt? Why are you "thrown backward" when your car rapidly accelerates?
5. Why is your body pressed against the left side of the seat when the roller-coaster car in which you are riding suddenly veers to the right?

Lesson Summary

Inertia is the natural tendency of an object to remain at rest or in motion at a constant speed along a straight line (constant velocity).

Newton's laws of motion deal with the idea of **force**—an action, like a push or a pull, that causes a change in the motion of an object.

Newton's First Law of Motion (Law of Inertia) states: It there is no net force acting on a body, the body will continue in its state of rest or will continue moving along a straight line at a constant speed.

You use **free-body diagrams** to keep track of all forces that are acting on a particular object so that you can determine the **net force**, the sum of all the forces acting on the object.

The net force may consist of **balanced forces**, in which case the net force is 0 N.

If the net force has a value other than 0 N, then the forces are said to be **unbalanced forces**.

NOTES

Video - Introduction to Newton's First Law of Motion

This video introduces Newton's First Law of Motion. Examples of this law are provided. The idea of inertia is also discussed. Balanced and unbalanced forces are defined and illustrated with examples.

https://youtu.be/7kPRD0ow-hM

Video - High School Physics - Newton's 1st Law of Motion

The video introduces Newton's Law of Inertia- Newton's First Law. The video explains the law and gives examples. Balanced and unbalanced forces are defined and illustrated with examples. Inertial and gravitational mass are defined.

https://youtu.be/NxQ3hgHuvHo

Lesson 2: Newton's Second Law of Motion
(1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ describe how Newton's Second Law of Motion applies in various situations
- ❑ draw free-body diagrams to aid in the analysis of problems involving forces
- ❑ solve problems involving the forces acting on an object and the motion of the object

Key Words

Newton of force
Newton's second law

Introduction

As you have learned, Newton's first law, the law of inertia, deals with the motion of objects when the net force acting on them was 0 N. Objects that are at rest remain at rest (inertia of rest), and objects that are in motion remain in motion with the same speed and direction of motion (inertia of motion). Newton's second law deals with what happens when the net force is not zero newtons.

Newton's Second Law

Let's try to grasp the workings of Newton's second law. In a typical Physics lab there are dynamics carts—special carts with low friction bearing in the wheels. A constant force can be exerted on the carts by pulling on them with an elastic that is stretched a certain amount, say 40 cm. These carts roll along on level tables and can be loaded with weights to change their mass.

If a cart at rest is pulled along by horizontal elastic stretched to 40 cm, the cart begins to move. It accelerates. The cause of this acceleration is the force the elastic exerts on the cart. While the force is exerted by the elastic, the velocity increases and the cart accelerates.

> If the net force acting on an object is not 0 N, then an **unbalanced force** is acting on the object. **An unbalanced force** acting on an object **causes** the object **to accelerate in the direction of the force**.

Now, suppose the cart is pulled by two elastics side by side, each stretched to 40 cm. This greater force will produce a greater acceleration of the cart. Since the friction between the cart and the table is negligible, and there is little or no drag from wind resistance, **the acceleration of the cart is in direct proportion to the force applied by the stretched elastics**.

> The larger the unbalanced force acting on a given mass, the larger the acceleration of the mass. The acceleration of a given mass subjected to an unbalanced force is directly proportional to the size of the unbalanced force.
>
> $$\vec{a} \propto \vec{F}_{\text{net}}$$
>
> (acceleration is directly proportional to the net force)

Two elastics produce twice the force, which in turn produces twice the acceleration of the cart. The direction of the acceleration is linked to the direction of the unbalanced net force acting on the cart. If the cart is pulled to the right, the acceleration of the cart is also to the right. The cart will accelerate in the direction of the net force.

Another factor that comes into play in Newton's second law is the inertia or mass of the object. The mass of the cart can be doubled by adding weights to it. If one elastic stretched 40 cm is used to pull the cart by itself, it will cause an acceleration of a certain magnitude. If the same elastic stretched 40 cm is used to pull a cart doubled in mass, the acceleration decreases to one-half the magnitude of just the cart itself.

This demonstrates that the acceleration of an object is in **inverse proportion** to its mass.

A given unbalanced force acting on a given mass produces a certain acceleration. The same unbalanced force acting on a larger mass produces a smaller acceleration. In this case, for a given unbalanced force, the acceleration varies inversely with the mass of the object.

$$\vec{a} \propto \frac{1}{m}$$

(acceleration is inversely proportional to the mass)

So, if the mass of the cart and weights was triple the original mass of the cart, and the cart was pulled with one elastic stretched 40 cm, the acceleration would be one-third the magnitude of the acceleration of the cart alone.

These two proportions may be combined as

$$\vec{a} \propto \vec{F}_{\text{net}} \cdot \frac{1}{m}$$

and written as an equation

$$\vec{F}_{\text{net}} = km\vec{a}$$

Any unit of force, any unit of mass, and any unit of acceleration can be used, provided the proper value is assigned to the constant ***k***. For simplicity, it is customary to use a system of units for which ***k* has a value of one**.

A **newton of force** is the force that will give to a mass of 1 kg an acceleration of 1 m/s^2.

$$1 \text{ N} = (1 \text{ kg})(1 \text{ m/s}^2)$$

From the definition of the newton of force, we can see that this set of units, when used in the equation $\vec{F}_{\text{net}} = km\vec{a}$, will provide a value of 1 for the constant k.

$$\vec{F}_{\text{net}} = m\vec{a}$$

Newton's Second Law of Motion

The effect of a net force applied to a body is the body accelerating in the direction of the force. The acceleration is in direct proportion to the net force and in inverse proportion to the mass of the body.

$$\vec{F}_{\text{net}} = m\vec{a}$$

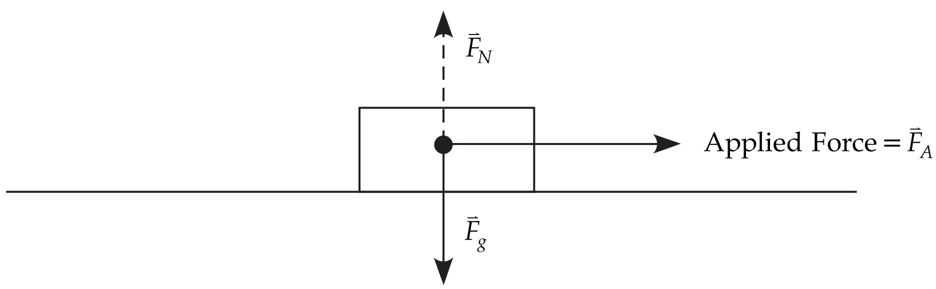
Quantity	Symbol	Unit
Net force	$\vec{F}_{\text{net}}$	newtons (N)
Mass	m	kilograms (kg)
Acceleration	$\vec{a}$	metres/second/second (m/s/s)

It is very important that you understand that the $\vec{F}_{\text{net}}$ in the formula $\vec{F}_{\text{net}} = m\vec{a}$ always means **net force**. Remember from Lesson 1 that net force is the vector sum of all of the forces acting on a body. Therefore, if more than one force is acting on a body, the vector sum of these forces must be determined.

Example 1: Newton's Second Law

How many newtons of force must a boy exert horizontally against a 10.0 kg box resting on a smooth floor (negligible friction) to give the box an acceleration of 2.00 m/s^2?

Given: Free-Body Diagram



In this case, choose right to be the positive direction.

Mass $m = 10.0$ kg

Acceleration $\vec{a} = +2.00$ m/s/s

Unknown: Net force $\vec{F}_{\text{net}} = ?$

Equation: $\vec{F}_{\text{net}} = m\vec{a}$

Substitute: $\vec{F}_{\text{net}} = (10.0\text{ kg})(+2.00\text{ m/s/s})$

Solve: $\vec{F}_{\text{net}} = +20.0\text{ N}$

Therefore, the force exerted by the boy is 20.0 N to the right.

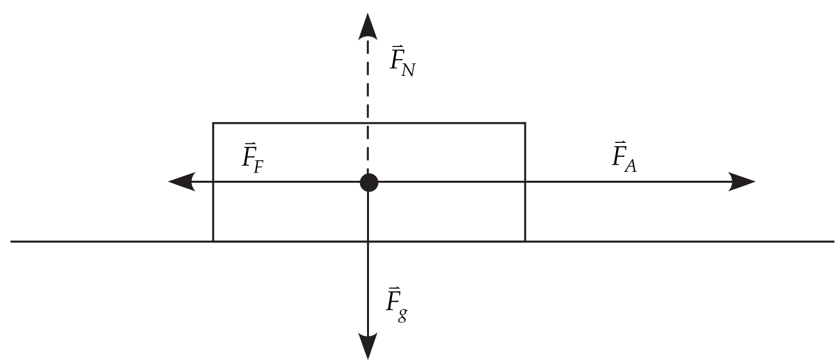
Problems relating the net force, the mass, and acceleration in a given situation are quite simple. You should be able to solve a problem where you're given several forces that must be added to give the net force. Again, remember that a free-body diagram is very useful in keeping track of the forces acting on the body.

Example 2: Newton's Second Law with Several Forces

A car of mass 1250 kg is pushed forward by its tires with the force of 2750 N. A force of friction of 1530 N also acts on the car. What is the acceleration of the car?

Given: Let right be the positive direction.

Mass of the car	$m = 1250$ kg
Applied force on the car	$\vec{F}_A = +2750$ N
Force of friction on the car	$\vec{F}_F = -1530$ N



Unknown: Acceleration of the car $\quad \vec{a} = ?$

Equation: $\vec{F}_{\text{net}} = m\vec{a}$

In this case, you do not know the net force. You must calculate it first.

$\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_F + \vec{F}_N + \vec{F}_g$

Note that the normal force and a force of gravity are equal but opposite. Since they are acting on the same body, these two forces are balanced and therefore cancel out.

So, $\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_F = (+2750\text{ N}) + (-1530\text{ N}) = +1220\text{ N}$

Use $\vec{F}_{\text{net}} = m\vec{a}$ and rearrange to get $\vec{a} = \dfrac{\vec{F}_{\text{net}}}{m}$.

Substitute and Solve: $\vec{a} = \dfrac{\vec{F}_{\text{net}}}{m} = \dfrac{+1220\text{ N}}{1250\text{ kg}} = +0.976\text{ m/s}^2$

Rounding the answer off to three significant digits, the acceleration of the car is 0.976 m/s/s [Right].

Relating Dynamics Information to Kinematics Information

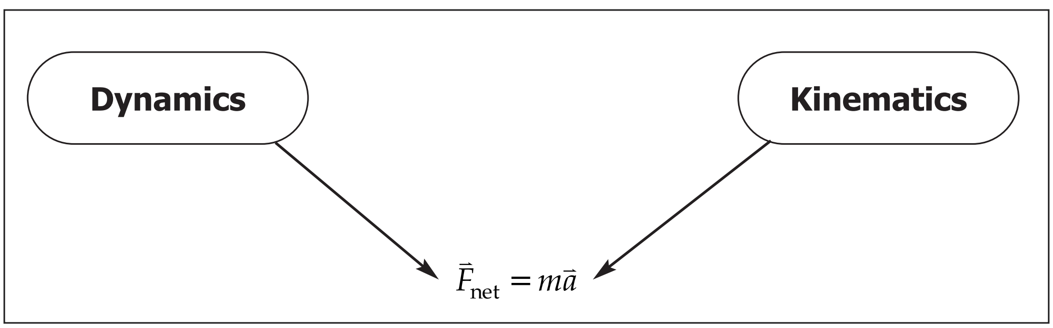
Many times in problems involving dynamics, kinematics information is also provided. Since kinematics is the study of motion and dynamics is a study of the cause of the motion, it is logical that these two should be linked together. The link between kinematics and dynamics happens to be Newton's second law.

The most efficient method for solving these kinds of problems is to take the information from the question and divide it into two categories: dynamics and kinematics.

The work that can be done using the dynamics information depends on what exactly is given. In analyzing this information, you list the forces, draw a free-body diagram, and determine the net force.

The work that can be done using the kinematics information again depends exactly on what is given. In analyzing this information, you list the displacement, the velocities, and the time intervals. The goal is to determine the acceleration.

Since the dynamics information resulted in the net force, and since the kinematics information resulted in the acceleration, the link between dynamics and kinematics is Newton's second law: $\vec{F}_{\text{NET}} = m\vec{a}$.



Here is the problem-solving strategy listing all the quantities:

Dynamics

m = mass

$\vec{F}_A$ = applied force

$\vec{F}_g$ = force of gravity

$\vec{F}_N$ = normal force

$\vec{F}_F$ = force of friction

$\vec{F}_{\text{net}}$ = net force

Forces are related using a free-body diagram.

Then, $\vec{F}_{\text{net}} = \sum \text{Forces}$

Kinematics

$\vec{d}$ = displacement

$\vec{v}_1$ = initial velocity

$\vec{v}_2$ = final velocity

Δt = time interval

$\vec{a}$ = acceleration

The kinematics information should be analyzed to determine $\vec{a}$.

$\vec{F}_{\text{net}} = m\vec{a}$

Newton's Second Law

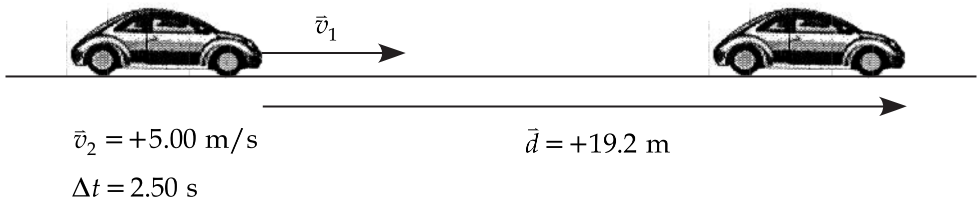
Example 3: A Problem with Dynamics and Kinematics

A car with a mass of 1175 kg is travelling at a velocity of 5.00 m/s [E]. During a time interval of 2.50 seconds the car travels 19.2 m [E]. A force of friction of 2150 N acts on the car. What is the force that is driving the car forward?

The strategy in solving this problem involves using the kinematics information to determine the acceleration, then using Newton's second law to determine the net force, then working with this net force and the other dynamics information to determine the force that is driving the car forward. The solution to this problem really has three steps to it.

Step 1: Analyzing kinematics information to determine the acceleration

Given: Set east as the positive direction.



Unknown: Acceleration $\vec{a} = ?$

Equation: Kinematics equation #4 can be used to calculate the acceleration.

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

Substitute and solve: for $\vec{a}$:

$$+19.2\ m = (+5.00 \text{ m/s})(2.50 \text{ s}) + \frac{1}{2} \vec{a} (2.50 \text{ s})^2$$

$$+19.2 = 12.5 + 3.125\vec{a}$$

$$+6.7 = 3.125\vec{a}$$

$$\vec{a} = \frac{+6.7}{3.125} = 2.144$$

$$\vec{a} = 2.1 \text{ m/s}^2 \text{ [E]}$$

Two significant digits are necessary in this case.

Step 2: Use Newton's second law to determine the net force.

$m = 1175 \text{ kg}$

$\vec{a} = 2.1 \text{ m/s}^2 \text{ [E]}$

$\vec{F}_{\text{net}} = m\vec{a}$

$\vec{F}_{\text{net}} = (1175 \text{ kg})(2.1 \text{ m/s}^2 \text{ [E]}) = 2467.5 \text{ N [E]}$

$\vec{F}_{\text{net}} = 2500 \text{ N [E]}$

Step 3: Analyze the dynamics of the situation. Since you now know the net force, you can work backwards to determine the applied force driving the car forward.

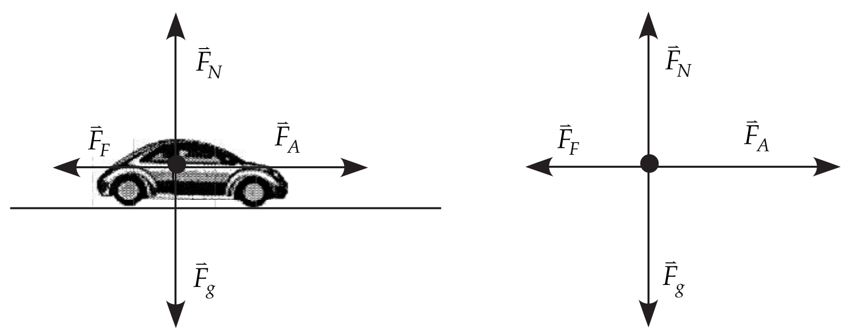
Given: Net force $\vec{F}_{\text{net}} = 2500 \text{ N [E]} = +2500 \text{ N}$

Force of friction $\vec{F}_F = 2150 \text{ N [W]} = -2150 \text{ N}$

Normal force $\vec{F}_N = ?$

Force of gravity $\vec{F}_g = ?$

In this case, the normal force and the force of gravity cancel out.



Unknown: The applied force driving the car forward $\vec{F}_A = ?$

Equation:

$$\vec{F}_{\text{net}} = \sum \text{Forces} = \vec{F}_A + \vec{F}_F + \vec{F}_N + \vec{F}_g \qquad \left(\vec{F}_N + \vec{F}_g \text{ cancel}\right)$$

$$\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_F$$

Substitute and Solve:

$$+2500 \text{ N} = \vec{F}_A + (-2150 \text{ N})$$

$$\vec{F}_A = (+2500 \text{ N}) - (-2150 \text{ N}) = +4650 \text{ N}$$

Rounding to significant digits, the applied force driving the car forward is 4600 N [E].

Learning Activity 2.2

Linking Kinematics and Dynamics

There are two problems in this learning activity. The first problem begins with dynamics information and then proceeds to kinematics information. The second problem begins with kinematics information and proceeds to dynamics information. You can check your work against the answer key found at the end of Module 2.

1. A crate of mass 30.0 kg rests on a level concrete floor. A man pushes the crate with a constant horizontal force of 75.0 N to the left. A constant force of friction of 40.0 N also acts on the crate. If the crate starts from rest, how far would it be pushed during 5.35 seconds?
2. A toboggan carrying a passenger is pulled by a person who exerts a force on a rope. The mass of a toboggan and passenger is 35.5 kg. A force of friction of 12.0 N acts on the toboggan. At one point in time, the velocity of the toboggan is 1.00 m/s [E], and after sliding 4.30 m [E] the velocity is 3.00 m/s [E]. If the person pulling the toboggan pulls horizontally towards the east, what force did he exert on the rope?

Lesson Summary

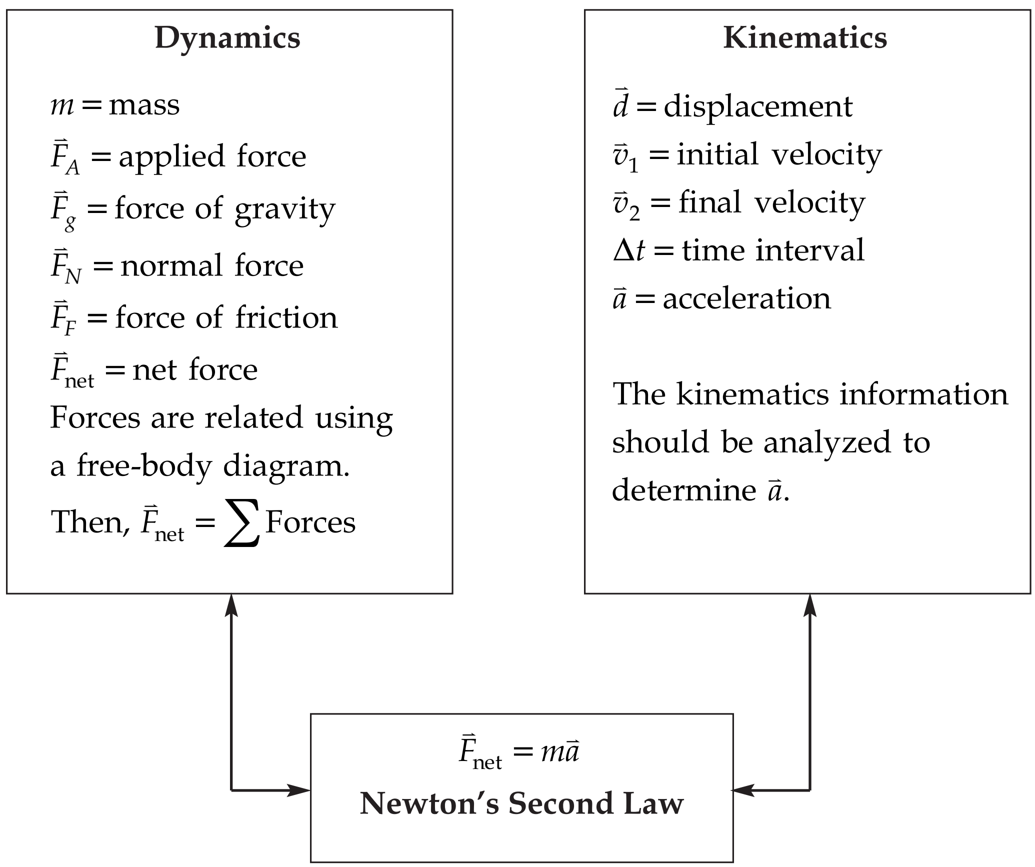
Newton's Second Law of Motion deals with how forces applied to an object affect the motion of the object.

The effect of a net force applied to a body is the body accelerating in the direction of the force. The **acceleration** is in **direct proportion to the net force** and in **inverse proportion to the mass** of the body.

The **mathematical statement** of Newton's Second Law of Motion is:

$$\vec{F}_{\text{net}} = m\vec{a}$$

The **problem-solving strategy** that allows you to link dynamics information with kinematics information is given below:



Note that you can start with dynamics information and proceed to kinematics, or you can start with kinematics information and proceed to dynamics.

NOTES

Video - Introduction to Free Body Diagrams or Force Diagrams

This video describes how to draw Free Body Diagrams which are also called Force Diagrams. In addition the definition of the force normal and the force applied are given. Free body diagrams of a book are drawn on a level surface and on an incline.

https://youtu.be/29YPIvj1zjc

Video - More on Newton's Second Law

This video from Khan Academy shows that if several forces act on an object the analysis of the forces to determine the net force can be accomplished using the property of vectors that components of vectors are independent of each other or what happens in the x direction does not affect what happens in the y direction.

So we can find the sum of the horizontal or x components of the forces by themselves. Then we can add the vertical or y components of the forces by themselves.

Then the sum of the x-components plus the sum of the y components of the forces can be added using the component method to give the net force that is acting.

If a force is acting in neither the vertical direction nor the horizontal direction, that force must be resolved into its horizontal and vertical components which are then added with the forces in the x direction or in the y direction.

https://youtu.be/24vtg9Ehr0Q

Video - Introductory Newton's 2nd Law Example Problem and Demonstration

This video show how kinematics and dynamics are related through Newton's Second Law.

A cart is placed on a track that is almost friction-less. The cart is accelerated from rest by a falling mass that is attached to the cart by a string.

There are kinematics quantities (displacement, initial velocity, time) that are measured along with the applied force of the string pulling the cart.

The question is to find the force of friction (a dynamics quantity).

The analysis requires a list of the given quantities, a free body diagram, and finding Fnet as the sum of the forces acting on the cart.

From the kinematics side the measured quantities allow acceleration to be calculated.

Then the link over to dynamics is Fnet = ma where Fnet is calculated.

In turn knowing Fnet allows the force of friction to be calculated.

https://youtu.be/sF_Ln4xWsV0

LESSON 3: MASS, WEIGHT, AND THE FORCE OF GRAVITY (1.5 HOURS)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ distinguish between gravitational mass and inertial mass
- ❑ distinguish between weight and mass
- ❑ calculate the force of gravity using $\vec{F}_g = m\vec{g}$

Key Words

gravitational field	weight	mass
gravitational mass	inertial mass $\vec{g}$	

Introduction

Weight and mass are two words that people use interchangeably every day, but in physics they have very specific meanings that are quite different from each other. What confuses people is the fact that the weight of an object is related to its mass. So let's clear up the confusion!

Mass versus Weight

> **Mass** refers to the amount of matter in an object.

Mass is **measured** on a balance by comparing the unknown mass to a known mass. Mass is conserved in physical and chemical changes, is additive (2 kg + 3 kg = 5 kg), and is the same at all locations. Your mass would be the same here on Earth, on the Moon, in space, or on some other planet. Mass has **units** of **kilograms** and is a **scalar** quantity.

On Earth, the **weight** of an object is the gravitational force that Earth exerts on the object.

The weight of an object is measured on a scale. Scales measure force. The **weight** of an object (the force of gravity on the object) always acts downward, toward the centre of Earth and, since it **is a force**, is measured in **units of newtons** (N). Weight is a **vector** quantity. Since weight is a force of gravity and the force of gravity varies from place to place, the weight of an object can change with location. While you cannot be massless, you can be truly weightless if the force of gravity acting on you is 0 N.

Let's explain the difference using an example. Suppose that a person has a mass of 100 kg. (This is a fairly large person.) How much will this person weigh on Earth?

Weight is how much the 100 kg person is attracted to Earth by Earth's gravity. This gravity exerts a force on the person that causes the person to accelerate towards the centre of the planet. We have already studied how much an object is accelerated towards the centre of Earth. The person will accelerate at 9.80 m/s^2. This acceleration is caused by the force of gravity that Earth exerts on every object that is on it.

We have all of the information that we need to calculate how much the force is. We can use the formula:

$$\vec{F} = m\vec{a}$$

The force is the mass (100 kg) multiplied by the acceleration due to gravity (which is 9.80 m/s^2). This is 100 times 9.80 or 980 N [towards the centre of Earth]. This force is what we usually call "weight."

A person who has a mass of 100 kg weighs 980 newtons here on Earth.

The equation $\vec{F} = m\vec{a}$ is usually written in a special way when we are talking about weight.

The **gravitational field intensity or strength, $\vec{g}$,** at Earth's surface measures the force of gravity acting on a unit mass (1 kg). On Earth's surface, the force of gravity acting on 1 kg of mass is 9.80 N towards the centre.

Therefore, on Earth's surface $\vec{g}$ **= 9.80 N/kg towards Earth's centre**.

The acceleration due to gravity originates in the strength of the gravitational field and is usually called "$\vec{g}$" instead of "$\vec{a}$," and the force is usually called "$\vec{F}_g$" instead of "$\vec{F}$." This makes the famous equation $\vec{F}_g = m\vec{g}$.

Weight
The weight of an object can be calculated by multiplying the mass of the object by the acceleration of gravity (gravitational field strength) at that point.

$$\vec{F}_g = m\vec{g}$$

Quantity	**Symbol**	**Unit**
Force of gravity	$\vec{F}_g$	newtons (N)
Mass	m	kilograms (kg)
Acceleration due to gravity	$\vec{g}$	metres/second/second (m/s/s)

$\vec{g}$ is also called gravitational field intensity or strength and can have units of N/kg (newton/kilogram).

Weight is a *vector* quantity.

This is still the same equation as $\vec{F} = m\vec{a}$, but we use this equation whenever we are finding weight (weight is a special kind of force and "$\vec{g}$" is a special kind of "$\vec{a}$").

On the Moon, the person's mass remains at 100 kg, but the person's weight will be different. Weight is how much a person is attracted by the planet's (Moon's) gravity. If the gravitational field is weaker ($\vec{g}$ is less), then the person will be attracted less.

The Moon, for example, is smaller than Earth. It exerts a weaker pull on objects and the acceleration due to gravity on the Moon is only one-sixth of the acceleration due to gravity on Earth. This means that, on the Moon, a person weighs only one-sixth of his or her weight on Earth. Television pictures of astronauts on the Moon show that these people can jump very high, even with heavy space suits on. This is because they weigh so little.

People can even weigh nothing! An astronaut "space walking" in outer space where there is no gravitational attraction, because there is no planet or Moon nearby, weighs zero newtons. This does not mean that the astronaut has no mass! The mass is the same as it would be on Earth or on the Moon.

If you know the gravitational field and the mass of an object, you should be able to calculate the weight of that object.

Example 1: The Weight of a Car

What does a car of mass 2250 kg weigh here on Earth's surface?

Given: In this case, let "towards the centre of Earth" (down as we observe it at Earth's surface) be negative.

Mass $m = 2250$ kg

Gravitational field $\vec{g} = 9.80$ N/kg towards Earth's centre

Unknown: Weight (force of gravity) $\vec{F}_g = ?$

Equation: $\vec{F}_g = m\vec{g}$

Substitute and solve:

$$\vec{F}_g = m\vec{g}$$
$$\vec{F}_g = (2250\ \text{kg})(-9.80\ \text{N/kg})$$
$$\vec{F}_g = -22050\ \text{N}$$
$$\vec{F}_g = -2.20 \times 10^4\ \text{N}$$

The car's weight is 2.20×10^4 N down.

Example 2: The Weight of a Football Player on the Moon

If the acceleration due to gravity on the Moon is 1.62 m/s^2, what would a 110 kg football player weigh there?

Given: In this case, let "towards the centre of the Moon" (down as we observe it at the Moon's surface) be negative.

Mass $m = 110$ kg

Gravitational field $\vec{g} = -1.62$ N/kg

Unknown: Weight (force of gravity) $\vec{F}_g = ?$

Equation: $\vec{F}_g = m\vec{g}$

Substitute and solve:

$$\vec{F}_g = m\vec{g}$$
$$\vec{F}_g = (110\ \text{kg})(-1.62\ \text{N/kg})$$
$$\vec{F}_g = -178.2\ \text{N}$$
$$\vec{F}_g = -180\ \text{N}$$

Here, you must use two significant digits in your final answer.

Gravitational Mass Versus Inertial Mass

Another property of mass with which you're familiar is gravity. Any two objects—such as you and Earth—interact and exert a gravitational force on each other. The term **gravitational mass** includes the property of masses that causes them to interact—that is, exert forces on each other. This gravitational mass is the mass with which you are familiar. This is the mass we defined earlier in the lesson as the amount of matter in an object. It is measured on a balance in units of kilograms and is a scalar quantity. Gravitational mass is additive, is conserved in chemical and physical changes, and does not vary with location.

Now there is a second property of mass called **inertia**. Inertia is the property of mass that causes it to resist a change in its motion. Objects with lots of inertial mass have lots of resistance to a change in its motion.

Inertial mass is the property of an object that causes it to resist a change in its motion. It is measured using Newton's second law. A known force is exerted on a mass and the acceleration of the mass is measured. Then the inertial mass is calculated using

$$m_{\text{inertial}} = \frac{\vec{F}_{\text{net}}}{\vec{a}}$$

Inertial mass is also additive, conserved in physical and chemical changes, and stays the same at all locations.

It turned out that inertial mass and gravitational mass are the same property of matter. They just make themselves evident in different ways.

Learning Activity 2.3

Mass and Weight

There are two questions in this learning activity. The first question deals with calculating weights on other planets, and the second deals with including the weight on a free-body diagram. You may check your work against the answer key found at the end of Module 2.

1. Acceleration due to gravity for various planets and our Moon:

Object	Acceleration at Its Surface (m/s^2)
Mercury	3.50
Venus	8.50
Earth	9.80
Mars	3.70
Jupiter	26.1
Saturn	11.1
Uranus	10.5
Neptune	13.8
Pluto	0.31
Moon	1.62

   a) If your mass were 72.0 kg on Earth, what would be your mass on the Moon?
   b) If your mass is 72.0 kg on Earth, what would you weigh on Earth? On Mars? On Saturn?
   c) Would you be able to walk on Jupiter if it had a solid surface?
   d) On which planet would you weigh the least?
   e) If your standing jump here on Earth were 0.900 m, how high would you be able to jump on Pluto?

2. A car with a mass of 1175 kg is travelling along a level road at a constant velocity. Draw a free-body diagram of the car. Label all the forces that act on the car. Calculate the force of gravity and the normal force acting on the car.

Lesson Summary

Mass refers to the amount of matter in an object. Mass has **units of kilograms** and is a **scalar** quantity.

On Earth, the **weight** of an object is the gravitational force that Earth exerts on the object. Weight has **units of newtons** and is a **vector** quantity.

The weight of an object can be calculated by multiplying the mass of the object by the acceleration of gravity (gravitational field intensity or strength) at that point.

$$\vec{F}_g = m\vec{g}$$

Since the gravitational field strength varies with location, the acceleration of gravity will be different and therefore the weight of an object will change from location to location.

Inertial mass is the property of an object that causes it to resist a change in its motion. It is measured using Newton's second law. A known force is exerted on a mass and the acceleration of the mass is measured. Then the inertial mass is calculated using $m_{\text{inertial}} = \dfrac{\vec{F}_{\text{net}}}{\vec{a}}$.

NOTES

Video - Introduction to Force

This video defines force including its dimensions (units) and the fact it is a vector. Demonstrations illustrate how force and mass affect the acceleration. The video also illustrates that a force is an interaction between two objects. Forces are also shown to be contact forces or field forces.

https://youtu.be/fiT2R88Zt58

Video - Introduction to the Force of Gravity and Gravitational Mass

This video defines force of gravity (weight) and gravitational mass.

https://youtu.be/6q4kRwRuScI

Video - Weight and Mass are Not the Same

This video compares weight and mass for objects. The weight and mass of a person are compared on the Earth and on the Moon.

https://youtu.be/nRSJ8w8FrDw

Lesson 4: The Force of Friction (1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ describe the origins of the force of friction
- ❑ distinguish between static friction and kinetic friction
- ❑ define the coefficient of static friction and the coefficient of kinetic friction
- ❑ calculate the force of static friction and the force of kinetic friction on level surfaces and on inclined planes
- ❑ draw free-body diagrams to aid in the analysis of problems involving forces

Key Words

force of friction
coefficient of static friction
static friction
static friction
coefficient of kinetic friction

Introduction

Try this little activity. On your desk or on a table (make sure it is level), stack several books (four or five). Push on the bottom book horizontally with only one finger. Start pushing gently on the middle of one side of the bottom book. Increase the force you are exerting on the book gradually until the stack of books begins to move. Then try to keep the books moving with a constant velocity.

Compare the force that was required to just start the books moving with the force that is required to keep the books moving with constant velocity. If necessary, repeat the procedure until you can make a comparison of these forces.

This activity serves to introduce you to **frictional forces**. Frictional forces are involved in practically everything you do, in some cases aiding the process, and in other cases hindering the process. In this lesson, you will study how to calculate these forces and incorporate them in the analysis of forces and the motion they cause.

Friction

Whenever an object moves while in contact with another object, frictional forces oppose the relative motion between the objects. These forces are caused by the adhesion (sticking) of one surface to the other due to an attractive electromagnetic force acting between the surface atoms of one object and those of another and by the interlocking of the irregularities of the rubbing surfaces. The force of frictional resistance depends upon the properties of the surfaces and upon the force keeping the surfaces in contact.

The effects of friction are often undesirable. Friction increases the work necessary to operate machinery; it causes wear; it generates heat, which often does additional damage. To reduce this waste of energy, friction is minimized by the use of wheels, bearings, rollers, and lubricants. Automobiles and airplanes are streamlined in order to decrease air friction, which is large at high speeds.

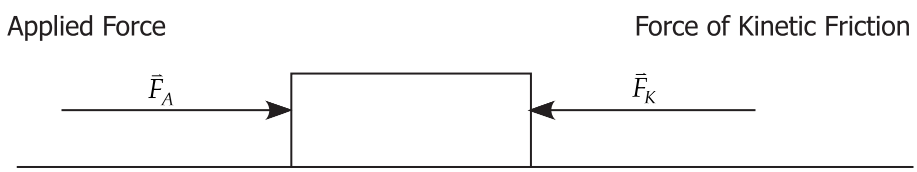
On the other hand, friction is desirable in many cases. Nails and screws hold boards together by means of friction. Power may be transmitted from a motor to a machine by means of a clutch or a friction belt. In walking, driving a car, striking a match, tying shoes, or sewing fabric together, we find friction to be a useful force. Sand is placed on rails in front of the drive wheels of locomotives, gravel is scattered on icy streets, and special materials are developed for use in brakes—all for the purpose of increasing friction where it is desirable.

> **Friction** is the force that resists the relative motion between objects that are in contact with each other.

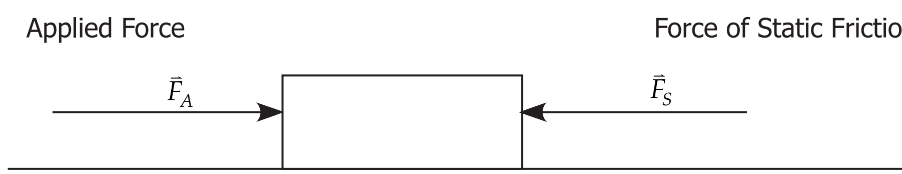
When you slide the books across a desk or table, you find that you must continue to apply a steady horizontal force to cause the books to slide uniformly over the horizontal surface. You can conclude that there is a force, parallel to the surfaces in contact, opposing the motion. This opposing force is called **kinetic friction**.

As the object begins to move, the forces between the atoms of the surfaces are overcome (slip) until the atoms of the surfaces again form an attractive force (stick). The cycle of "slip and stick" repeats itself rapidly as one object moves on the surface of another. The "slip and stick" process is responsible for the noise that is made during events, like the screeching of tires or fingernails on a blackboard.

Kinetic friction is the frictional force between objects that are sliding with respect to one another.



The object moves with uniform velocity across the surface.



When a body at rest on a horizontal surface is pushed gently sideways, it does not move because there is a frictional force just equal to the sideways push. If the push is increased, the frictional force increases until the object begins to move. If the side push exceeds the largest force of friction that exists just before the body begins to move, the body is accelerated.

Static friction is the maximum frictional force between stationary objects. It exists at the moment just before the objects begin to move relative to each other.

It is evident, then, that friction is a force that opposes the net force applied to a body, and its direction is opposite to that of the applied force. Now what you should have observed with the activity involving the books is that the applied force required just to get the books moving was larger than the force required to keep the books moving at a constant velocity as they slid across the desk or table. This tells you that the force of static friction is larger than the force of kinetic friction.

Coefficient of Friction

You can try a second activity involving pushing books horizontally across a level desk or table. This time, place only one book on the table, and then take note of the force required just to overcome the force of static friction (just to get the book moving) and the force required to keep them moving at a constant velocity across the surface of the desk or table (balancing the force of kinetic friction).

Place a second book on top of the first book and repeat this procedure.

Place a third book on top of the first two books and repeat the procedure again.

What you should notice is that as the number of books increase, a larger force is required just to get the books moving, and a larger force is required to keep the books moving with constant velocity. In other words, both the force of static friction and the force of kinetic friction have increased as more books are used.

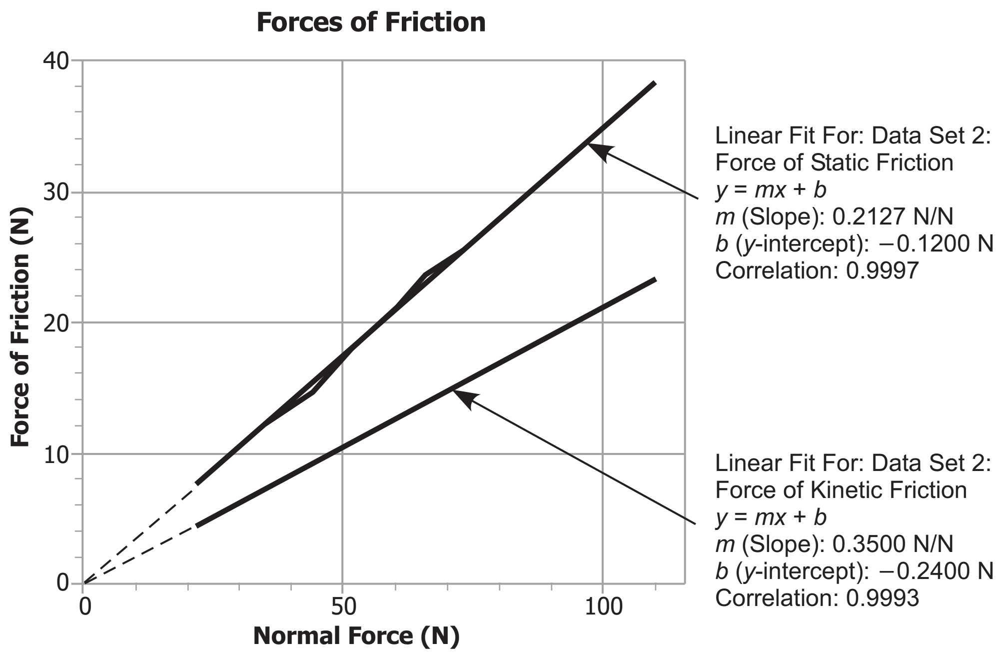
In fact, there is a mathematical relationship that exists in this situation. As more books are added, the mass of the books you are pushing increases. As a mass increases, so does the force of gravity that is pulling down on the books. This in turn increases the normal force of the surface pushing up on the bottom book. It turns out that the **force of static friction** and the **force of kinetic friction** both **depend on the normal force**.

If you were to measure the forces involved in this situation, you might end up with results such as those shown below.

If you used identical physics books, a typical weight for each book might be 22 N.

Number of Books	Normal Force (N)	Force of Static Friction (N)	Force of Kinetic Friction (N)
1	22	7.7	4.6
2	44	14.5	9.3
3	66	23.4	13.9
4	88	30.5	18.3
5	110	38.2	23.5

Graphical Analysis of Data showing the relationship between static friction and the normal force and the force of kinetic friction and the normal force.



You can see that the data linking the force of static friction to the normal force gives a direct proportion between the two variables.

$\vec{F}_S \; \alpha \; \vec{F}_N$

You can change this into an equation by introducing a constant of proportionality, μ_s.

Then the relationship becomes $\vec{F}_S = \mu_s \vec{F}_N$.

If you rearrange this equation to isolate μ_s, it becomes $\mu_s = \dfrac{\vec{F}_S}{\vec{F}_N}$.

In this case, we drop the vector notation since we have not used the operation of division with vectors.

$$\mu_s = \frac{F_S}{F_N}$$

The value of this constant for static friction for the data is 0.35. The value of this constant, called the **coefficient of friction**, depends on the object and the surface in the particular situation under consideration. “Slippery” surfaces tend to have low coefficients of friction while “sticky” surfaces tend to have higher coefficients of friction.

> The **coefficient of friction** is the ratio of the force of friction to the normal force pressing the surfaces together
>
> $$\mu = \frac{F_F}{F_N}$$
>
> Coefficients of friction have no units.

The same kind of analysis can be used with the force of kinetic friction and how it is related to the normal force.

You can see that the data linking the force of kinetic friction to the normal force gives a direct proportion between the two variables.

$\vec{F}_K \ \alpha \ \vec{F}_N$

You can change this into an equation by introducing a constant of proportionality, μ_k.

Then the relationship becomes $\vec{F}_K = \mu_k \vec{F}_N$.

If you rearrange this equation to isolate μ_k, it becomes $\mu_k = \frac{\vec{F}_K}{\vec{F}_N}$.

Again, we dropped the vector notation, getting: $\mu_k = \frac{F_K}{F_N}$.

Surface Friction

The magnitude of the **force of friction** acting at the surface between two objects is the product of the coefficient of friction for that surface and the magnitude of the normal force. The direction of the force of friction is always opposite to the direction of motion.

$$F_S = \mu F_N$$

Quantity	**Symbol**	**Unit**
Force of friction	F_S	newton (N)
Coefficient of friction	μ	none (coefficients of friction are unitless)
Normal force	F_N	newton (N)

Note: The vector notations for the two forces are amended since the two forces act perpendicularly to each other

Normal Forces

What does the term “normal force” mean? The word “normal,” as used here, means perpendicular. Therefore, the normal force is the perpendicular force that a surface exerts on an object that rests on that surface. More simply, you can think of it as the perpendicular force that presses the two surfaces together.

If the weight of a block resting on a horizontal surface, such as a table or floor, is 50 N, the normal force is 50 N. However, if an additional weight of 30 N is placed on the first one, the normal force will be 80 N.

Could the normal force between two surfaces ever be less than the weight of the block resting on the surface? Yes, indeed!

Let’s consider the 50 N block. Suppose that a string is attached to the top of the block. If a vertically upward force of 30 N is applied to the block through the string, then the normal force is 50 N [down] + 30 N [up] = 20 N [down].

$F_F = \mu F_N$ tells us that the force of friction is directly proportional to the normal force.

Obviously, then, when the normal force was 80 N, the force of friction was greater than it was when the normal force was 50 N. When the normal force was 20 N, the force of friction was less than when it was 50 N. This makes sense, because the force of friction will be greater when the surfaces are pressed more tightly together; it will be less when the surfaces are not tightly pressed together.

Learning Activity 2.4

Comparing Static Friction and Kinetic Friction

Answer the following questions to check your understanding of the force of friction. You may check your work against the answer key found at the end of Module 2.

1. What are the causes of the force of friction?
2. Compare and contrast the force of kinetic friction with the force of static friction. Fill the information into a template like the one below.

Compare and Contrast the Force of Static Friction and the Force of Kinetic Friction
COMPARE: How is the force of static friction the same as the force of kinetic friction?
CONTRAST: How is the force of static friction different from the force of kinetic friction?
Summary

3. In this lesson, there is a graph of force of friction and normal force. From this graph of force of friction and normal force, find:
   a) the force of kinetic friction for a normal force of 60 N. Calculate the ratio: force of kinetic friction over the normal force. What does this ratio represent?
   b) the force of static friction for a normal force of 60 N. Calculate the ratio: force of static friction over the normal force. What does this ratio represent?

Calculations with the Force of Friction

Example 1: The Force of Static Friction Compared to the Force of Kinetic Friction Acting on a Crate

A 95.0 kg crate rests on a level floor at a warehouse.

a) How large would the horizontal force have to be just to have the crate start to move? (The coefficient of static friction is 0.820.)

The horizontal pushing force required just to have the crate start to move is slightly greater than the static friction. Any force greater than static friction will cause the crate to move. To find the force of static friction, the formula $F_s = \mu_s F_N$ will be used.

Given: Mass $m = 95.0$ kg

Remember that the force of gravity or weight is given by $\vec{F}_g = m\vec{g}$

Weight (force of gravity) = (95.0 kg)(9.80 m/s/s) = 931 N [down]

Normal force $\vec{F}_N = 931 \text{ N} \left[\text{up}\right]$

Coefficient of static friction $\mu_s = 0.820$

Unknown: Force of static friction $\vec{F}_S = ?$

Equation: $F_S = \mu_s F_N$

Substitute and solve: $F_S = \mu_s F_N$

$F_S = (0.820)(931 \text{ N}) = 763.42 \text{ N}$

Therefore, a force very slightly greater than 763 N is required to start the crate in motion.

b) What horizontal force would be needed to push the crate across the floor of the warehouse at a constant speed? (The coefficient of sliding friction is 0.375.)

Since the coefficient of sliding (kinetic) friction is less than the coefficient of static friction, the force of sliding (kinetic) friction will be less than the force of static friction. As soon as the crate starts to move, the force applied to the crate must be reduced; otherwise, there will be a net force acting on the crate and it will accelerate. The crate is to be pushed at a constant speed. This can happen only if there is no net force acting on the crate. In other words, the pushing force, acting in one direction, must be exactly the same as the force of kinetic friction, acting in the opposite direction. What is the force of sliding friction? Let's find out!

$F_K = \mu_k F_N$
$F_K = (0.375)(931\text{ N}) = 349.125\text{ N}$

A force of 349 N would be required to keep the object moving with a constant velocity.

Notice that static friction is 763 N. Any force greater than 763 N will move the crate. Note, also, that the kinetic friction is 349 N. Therefore, as soon as the crate starts to move, the applied force must be reduced by 414 N, down to 349 N, in order that the crate will move at a steady speed (no acceleration). If the applied force is not reduced, there will be 414 N of net force.

Example 2: The Force of Static Friction, the Net Force, and Motion

Another crate whose weight is 85.2 N rests on the level horizontal warehouse floor. The coefficient of static friction is 0.723; the coefficient of kinetic friction is 0.312. A horizontal force of 65.0 N is applied to the crate. Will this force cause the crate to move? If so, what will be the acceleration of the crate?

The crate will move under the influence of the 65.0 N of force applied horizontally to it, if 65.0 N of force is greater than the static friction. Therefore, to answer the question, you need to know the force of static friction. Let's determine it!

Given: Mass $m = ?\text{ kg}$

Weight (force of gravity) $\vec{F}_g = 85.2\text{ N [down]}$

Remember that the force of gravity or weight is given by $\vec{F}_g = m\vec{g}$

85.2 N [down] = (m)(9.80 m/s/s)

$m = 8.69\text{ kg}$

Normal force $\vec{F}_N = 85.2\text{ N [up]}$

Coefficient of static friction $\mu_s = 0.723$

Unknown: Force of static friction $\vec{F}_S = ?$

Equation: $F_S = \mu_s F_N$

Substitute and solve:

$F_S = \mu_s F_N$
$F_S = (0.723)(85.2\text{ N}) = 61.5996\text{ N}$
$F_S = 61.6\text{ N}$

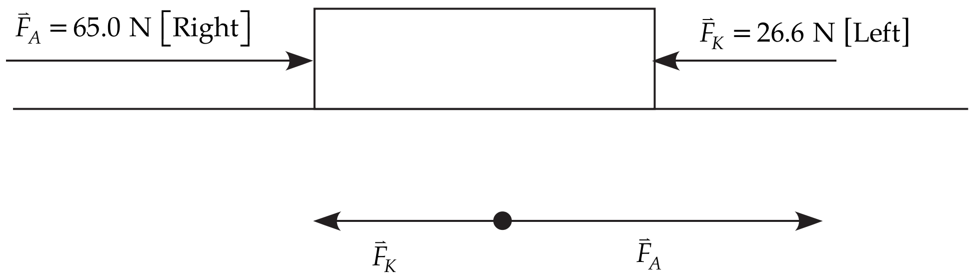
You have just determined the force of static friction to be 61.6 N. Since the horizontal force applied to the crate is 65.0 N, the crate will start to move. (The horizontal force applied is greater than the force of static friction.)

As soon as the crate commences to move, sliding (kinetic) friction begins to operate. Now, you must find the force of kinetic friction that opposes the applied force of 65.0 N.

Coefficient of sliding friction $\mu_k = 0.312$

$F_K = \mu_k F_N$
$F_K = (0.312)(85.2 \text{ N}) = 26.6 \text{ N}$

The force applied to the crate is 65.0 N; the force of kinetic friction is 26.6 N.



Since these two forces have opposite directions, the net force is 38.4 N [Right].

(65.0 N [Right] + 26.6 N [Left] = 38.4 N [Right])

What acceleration will a net force of 38.4 N [Right] give to the crate that weighs 85.2 N?

Having determined the mass of the crate, we now have all the information we need to find the acceleration of the crate using Newton's Second Law of Motion. The net force (accelerating force) is 38.4 N and the mass of the crate is 8.69 kg.

Given:	Mass	$m = 8.69 \text{ kg}$
	Net force	$\vec{F}_{\text{net}} = 38.4 \text{ N [Right]}$
Unknown:	Acceleration	$\vec{a} = ?$
Equation:		$\vec{F}_{\text{net}} = m\vec{a}$
Substitute and solve:		$\vec{F}_{\text{net}} = m\vec{a}$

$$38.4 \text{ N [Right]} = (8.69 \text{ kg})\vec{a}$$

$$\vec{a} = \frac{38.4 \text{ N [Right]}}{8.69 \text{ kg}} = 4.42 \text{ m/s}^2 \text{ [Right]}$$

The crate accelerates at 4.42 m/s/s [Right].

Learning Activity 2.5

The Effect of Friction on Motion

All the problems in this learning activity deal with the force of friction and the effect of friction on the motion of an object. You may check your work against the answer key provided at the end of Module 2.

1. The coefficient of static friction of unlubricated steel on steel is 0.740. The coefficient of kinetic friction of unlubricated steel on steel is 0.570. A block of steel of mass 65.8 kg rests on a level, smooth steel countertop.
   a) What is the minimum horizontal force that must be exerted just to put a block of steel in motion?
   b) If you can exert a horizontal force of 525 N, would you be able to move the block? If you can move the block, then what kind of motion would it show?
2. A toboggan and passenger are sliding along on level ice at 3.52 m/s. The mass of the toboggan and passenger is 40.0 kg. The toboggan hits a section of rough ice, where it slows down to 1.25 m/s, while sliding 8.20 m.
   a) Calculate the force of kinetic friction.
   b) Calculate the coefficient of kinetic friction.
3. For the situation in #2, suppose the passenger's big brother pulls on the toboggan rope with a force of 62.5 N. The rope makes an angle of 30.0° with horizontal. He pulls the toboggan across a level surface covered with snow. The toboggan moves with a constant velocity.
   a) What is the force of kinetic friction acting on the toboggan?
   b) What is the coefficient of kinetic friction for the surface?

Lesson Summary

Friction is the force that resists the relative motion between objects that are in contact with each other.

Kinetic friction is the frictional force between objects that are sliding with respect to one another.

Static friction is the maximum frictional force between stationary objects. It exists at the moment just before the objects begin to move relative to each other.

The **coefficient of friction** is the ratio of the force of friction to the normal force pressing the surfaces together.

$$\mu = \frac{F_F}{F_N}$$

Coefficients of friction have no units and vary with the surfaces of the objects involved.

Surface Friction

The magnitude of the **force of friction** acting at the surface between two objects is the product of the coefficient of friction for that surface and the magnitude of the normal force. The direction of the force of friction is always opposite to the direction of motion.

$$F_S = \mu F_N$$

Quantity	Symbol	Unit
Force of friction	F_S	newton (N)
Coefficient of friction	μ	none (coefficients of friction are unitless)
Normal force	F_N	newton (N)

Note: The vector notations for the two forces are amended since the two forces act perpendicularly to each other.

NOTES

Assignment 2.1

Forces of Friction and Motion (8 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. This assignment consists of one question with four parts worth eight marks in total. Be sure to show all your work and explain the method of arriving at your answers. Also, pay attention to directions for vector answers, and round off the answers according to the correct number of significant digits. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of pushing a car

Your car has stalled and has come to rest on a level section of a busy street. You and a passenger decide to push the car to the side of the road so it will not interfere with traffic. Together, you and your friend can exert a combined horizontal force of 1320 N. The coefficient of static friction is 0.0850 and the coefficient of kinetic friction is 0.0312. The car has a mass of 1250 kg.

a) Calculate the force of gravity on the car.

b) Calculate the normal force on the car.

(continued)

c) Will the force you and your friend exert be able to start the car moving? Explain your answer.

d) Assuming that the car will move when you push it, what will be its acceleration?

Method of Assessment

The total of eight marks for this assignment will be determined as follows:

- 1 mark for correctly determining the force of gravity on the car in part (a)
- 1 mark for correctly determining the normal force on the car in part (b)
- 1 mark for determining the force of static friction in part (c)
- 1 marks for determining whether the car will move in part (c)
- 1 mark for determining the force of kinetic friction in part (d)
- 1 mark for drawing a correct free-body diagram for part (d)
- 1 mark for determining the net force in part (d)
- 1 mark for determining the acceleration of the car in part (d)

Video - High School Physics - Friction

This video defines friction and differentiates between static friction and kinetic friction. It identifies the factors that affect the force of friction. The video discusses the coefficient of static friction and the coefficient of kinetic friction and compares these values for various surfaces. The equation $F_f = \mu F_N$ is used to calculate the force of friction and is added to the list of forces used in the

Dynamics ↔ Fnet = ma ↔ Kinematics approach for solving problems.

Finally a series of problems involving the force of friction are solved.

https://youtu.be/0iTDu45_9mc

Video - Mechanics: Friction (1 of 14) What is Friction?

This video describes the force of friction and its causes.

https://youtu.be/dsW4FSIXuTE

Video - Mechanics: Friction (2 of 14) What is Coefficient of Friction?

In this video the coefficient of friction is explained. Also static friction and kinetic friction are compared.

https://youtu.be/1EKgiYBwEi0

Video - Mechanics: Friction (3 of 14) What is the Friction Force?

This video illustrates how the force of static friction and the force of kinetic friction is calculated for an object on a level horizontal surface.

https://youtu.be/6e_MM1yjxo0

Assignment 2.1

Forces of Friction and Motion (8 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. This assignment consists of one question with four parts worth eight marks in total. Be sure to show all your work and explain the method of arriving at your answers. Also, pay attention to directions for vector answers, and round off the answers according to the correct number of significant digits. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

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a) Calculate the force of gravity on the car.

b) Calculate the normal force on the car.

(continued)

Assignment 2.1: Forces of Friction and Motion (continued)

c) Will the force you and your friend exert be able to start the car moving? Explain your answer.

d) Assuming that the car will move when you push it, what will be its acceleration?

Method of Assessment

The total of eight marks for this assignment will be determined as follows:

- 1 mark for correctly determining the force of gravity on the car in part (a)
- 1 mark for correctly determining the normal force on the car in part (b)
- 1 mark for determining the force of static friction in part (c)
- 1 marks for determining whether the car will move in part (c)
- 1 mark for determining the force of kinetic friction in part (d)
- 1 mark for drawing a correct free-body diagram for part (d)
- 1 mark for determining the net force in part (d)
- 1 mark for determining the acceleration of the car in part (d)

Lesson 5: The Component Method for Adding and Subtracting Vectors

(1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ add vectors that are parallel or antiparallel to each other
- ❑ add vectors that are perpendicular to each other
- ❑ add vectors together that are oriented at any angle to each other
- ❑ resolve a vector into its components
- ❑ draw free-body diagrams to aid in the analysis of problems involving forces

Key Words

component method
resolve vectors

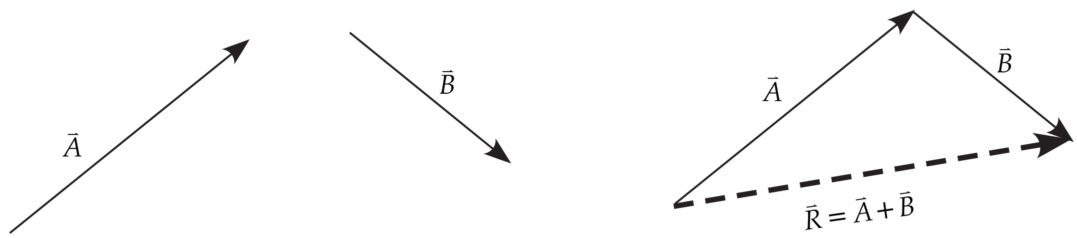
Introduction

Up until now, you have only dealt with vectors that are parallel (point in the same direction), antiparallel (point in the opposite direction), or perpendicular (point 90° to each other). Adding or subtracting parallel vectors or antiparallel vectors is much like working with integers. You may have found working with vectors that were perpendicular to each other to be a little more complicated. Here, you found the components pointing along the major axes. You combined magnitudes of the components of the resultant vector by using the theorem of Pythagoras to obtain the magnitude of the resultant vector. You then used one of the trigonometric functions to determine the direction in which the vector pointed.

In this lesson, we are going to extend this algebraic method so that you may add or subtract vectors that are oriented in any direction relative to each other.

Component Method of Adding Vectors

The next question to address is how you add vectors that look like the following two:



You know that you will arrange these using the "tip-to-tail" method for adding vectors together. You could use the law of sines or the law of cosines to solve this triangle. What we will do though is extend the algebraic method that we have used so far to add two or more vectors together.

Component Method of Adding Vectors

1. Designate a reference system using the *x*-axis and the *y*-axis.
2. Draw each of the given vectors on its own in the reference system with its tail at the origin.
3. Resolve each of the vectors into its *x*-component and *y*-component. Place these into a table as follows:

Vector	*x*-component	*y*-component
Vector 1		
Vector 2		
Resultant		

4. Add all of the *x*-components of the given vectors to find the *x*-component of the resultant. Add all of the *y*-components of the given vectors to find a *y*-component of the resultant.
5. Add the *x*-component of the resultant and the *y*-component of the resultant using the theorem of Pythagoras to determine the magnitude of the resultant, and using a trigonometric function determine the direction in which the resultant points.

Note

This process can be used to add any number of vectors together at the same time.

Example 1: Adding Vectors using the Component Method

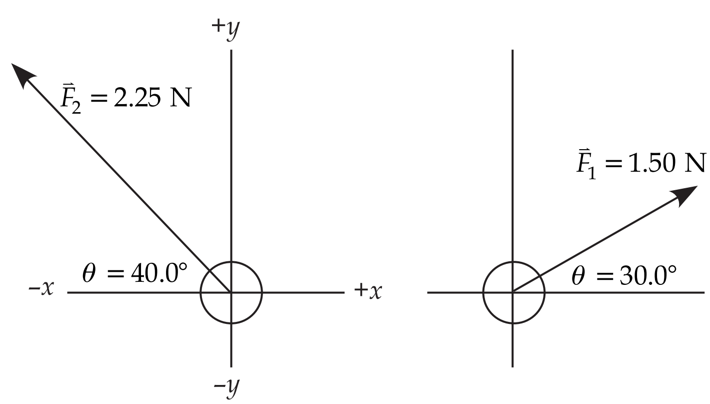
Let's examine an example involving Newton's second law.

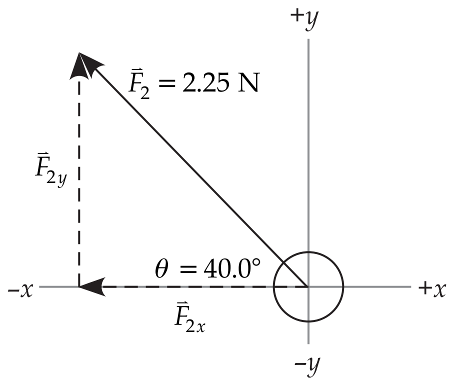
Consider something called a dry ice puck. A dry ice puck consists of a base (a heavy metal disk with a hole in the centre) upon which rests a canister that can be filled through an opening at the top. Dry ice (solid carbon dioxide) is placed in a canister and the canister is sealed. The dry ice sublimes into gaseous carbon dioxide, which is then forced out of the hole at the bottom of the disk, making the puck float on a layer of carbon dioxide. This essentially eliminates any frictional forces.

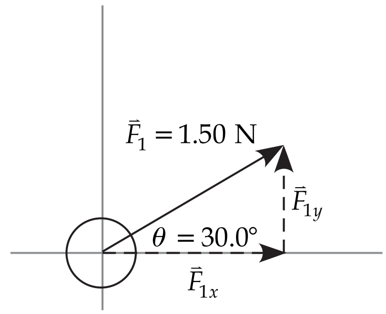
A dry ice puck has a mass of 1.78 kg. It is pulled across a level table by two forces, each of which acts in the horizontal direction. The first force is 1.50 N pulling 30.0° north of east, and the second force is 2.25 N pulling 40.0° north of west. Determine the net force acting on the dry ice puck, and describe its motion.

A sketch of the situation will help you visualize what is occurring. In this case, you should use a frame of reference where you're looking from the top, straight down onto the table. You do not have to worry about the normal force and the force of gravity in this situation, since you know they will be equal and opposite to each other, and therefore they will cancel out. Here is the sketch.

1. Designate a reference system using the *x*-axis and the *y*-axis.
2. Draw each of the given vectors on their own in the reference system with their tail at the origin.







3. Resolve each of the vectors into their x-component and y-component. Each vector has its components drawn in to enclose the given angle. You could use sin θ and cos θ to determine each of the components for the two forces.

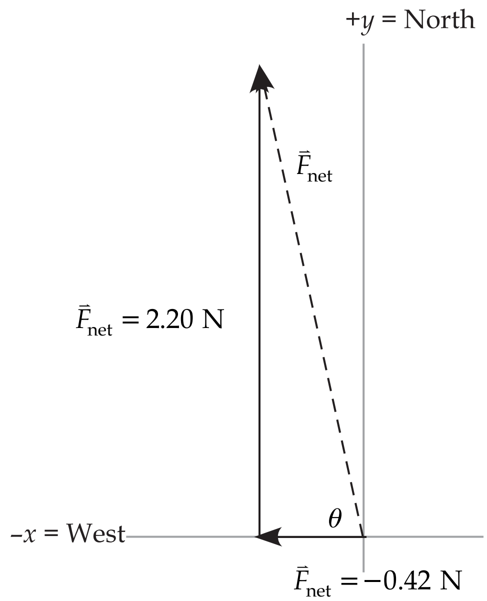
Vector	x-component	y-component
$\vec{F}_1$	$(\cos 30.0°)(\vec{F}_1) =$ $(\cos 30.0°)(1.50\text{ N}) = +1.30\text{ N}$	$(\sin 30.0°)(\vec{F}_1) =$ $(\sin 30.0°)(1.50\text{ N}) = +0.750\text{ N}$
$\vec{F}_2$	$-(\cos 40.0°)(\vec{F}_2) =$ $-(\cos 40.0°)(2.25\text{ N}) = -1.72\text{ N}$	$+(\sin 40.0°)(\vec{F}_2) =$ $+(\sin 40.0°)(2.25\text{ N}) = +1.45\text{ N}$
$\vec{F}_{\text{net}}$	-0.42 N	2.20 N

Note: The x-component of $\vec{F}_2$ points in the negative x-direction! Watch those signs!

4. Add all of the x-components of the given vectors to find the x-component of the resultant. Add all of the y-components of the given vectors to find the y-component of the resultant.

5. Add the x-component of the resultant and the y-component of the resultant using the theorem of Pythagoras to determine the magnitude of the resultant and using a trigonometric function to determine the direction in which the resultant points.

a) Sketch the components and add them, as you did earlier.



To find the magnitude of the net force, use the theorem of Pythagoras.

$$\vec{F}_{net}{}^2 = \vec{F}_{netx}{}^2 + \vec{F}_{nety}{}^2$$

$$\vec{F}_{net}{}^2 = (-0.42)^2 + (2.20)^2$$

$$\vec{F}_{net}{}^2 = 5.0164$$

$$\vec{F}_{net} = 2.24 \text{ N} = 2.2 \text{ N}$$

Using

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{2.20}{0.42} = 5.2381$$

$$\theta = \tan^{-1} 5.2381 = 79.2° = 79°$$

The net force acting on the dry ice puck is 2.2 N [79° north of west].

b) Describe the motion of the dry ice puck.

Since we are told nothing about the motion of the dry ice puck, all we can do is determine the acceleration of the puck from the net force that is acting upon it.

Using the net force and the mass of 1.78 kg, calculate the acceleration by taking

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{2.2 \text{ N}[79° \text{ N of W}]}{1.78 \text{ kg}} = 1.2 \text{ m/s}^2 \, [79° \text{ N of W}]$$

The acceleration of the puck is 1.2 m/s/s [79° N of W].

Example 2: Subtracting Vectors using the Component Method

Let's consider how you can use the component method to subtract vectors. Again, the example will involve both kinematics and dynamics.

You are travelling in your car along a highway at 27.8 m/s [34.0° south of west] when you enter a bend in the road. You maintain your speed, exiting the curve at 27.8 m/s [51.8° east of south]. The mass of your car and passengers is 1475 kg, and this change in velocity occurs during a time interval of 20.0 seconds.

a) What is the change in velocity of your car?

b) What was the average acceleration during this interval of time?

c) What was the average force of friction between the tires of your car and the road that caused this acceleration?

a) Given: Kinematics

$\vec{v}_1$ = 27.8 m/s [34.0° south of west]

$\vec{v}_2$ = 27.8 m/s [51.8° east of south]

Δt = 20.0 s

The change in velocity is found by subtracting the final velocity from the initial velocity. Even though the velocities have the same magnitude, their directions are different, which indicates that the velocity did in fact change. So, you must determine the change in velocity using the component method.

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

Remember that you must add the opposite. So, $\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1)$

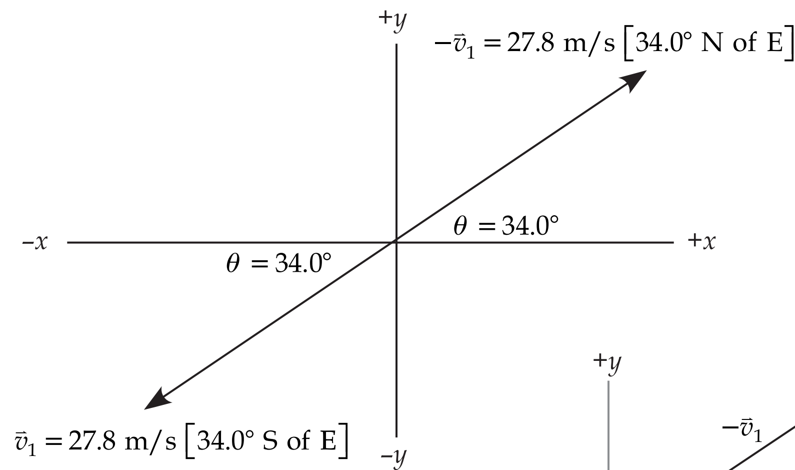
$\Delta\vec{v}$ = 27.8 m/s [51.8° east of south] – 27.8 m/s [34.0° south of west]

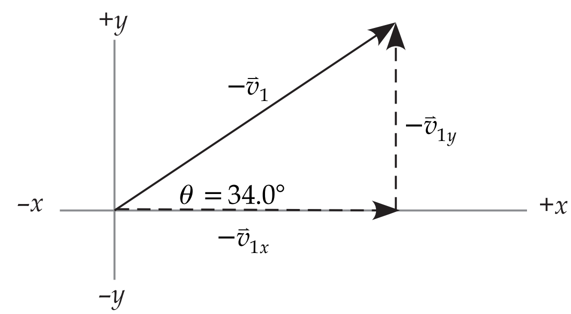
$\Delta\vec{v}$ = 27.8 m/s [51.8° east of south] + 27.8 m/s [34.0° north of east]

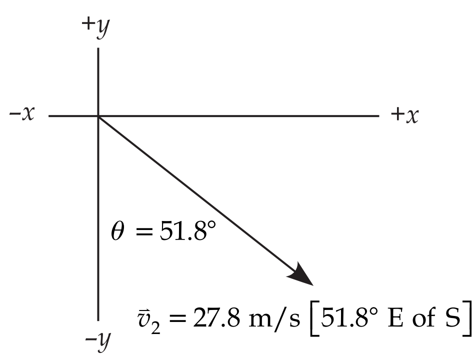
Note: $-\vec{v}_1$ points opposite the south of west, which is north of east.

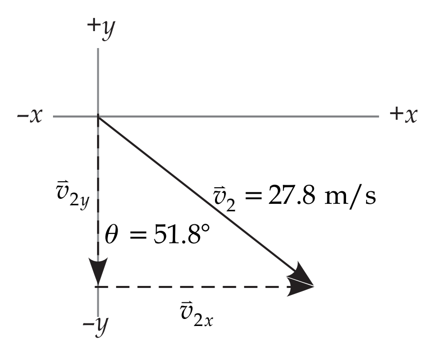
Using the component method, we can now add these vectors together.

1. Designate a reference system using the x-axis and the y-axis.
2. Draw each of the given vectors on its own in the reference system with its tail at the origin.









3. Resolve each of the vectors into its *x*-component and *y*-component. Each vector has at its components drawn in to enclose the given angle. You could use sin θ and cos θ to determine each of the components for the two forces.

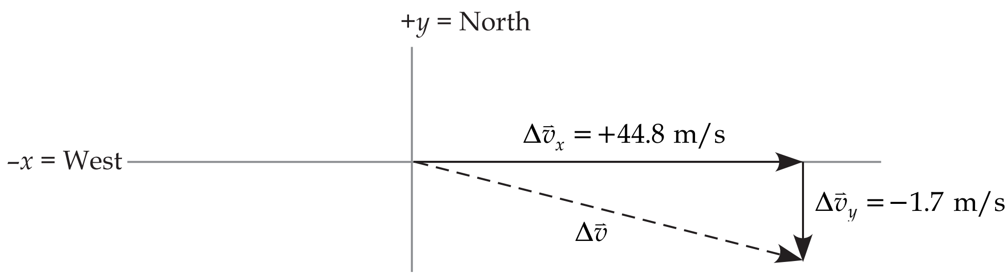
Vector	*x*-component	*y*-component
$-\vec{v}_1$	$(\cos 34.0°)(-\vec{v}_1) =$ $(\cos 34.0°)(27.8 \text{ m/s}) = +23.0 \text{ m/s}$	$(\sin 34.0°)(-\vec{v}_1) =$ $(\sin 34.0°)(27.8 \text{ m/s}) = +15.5 \text{ m/s}$
$\vec{v}_2$	$(\sin 51.8°)(\vec{v}_2) =$ $(\sin 51.8°)(27.8 \text{ m/s}) = +21.8 \text{ m/s}$	$-(\cos 51.8°)(\vec{v}_2) =$ $-(\cos 51.8°)(27.8 \text{ m/s}) = -17.2 \text{ m/s}$
$\Delta\vec{v}$	$+44.8 \text{ m/s}$	-1.7 m/s

Note: The *y*-component of $\vec{v}_2$ points in the negative *y*-direction! Watch those signs!

4. Add all of the *x*-components of the given vectors to find the *x*-component of the resultant. Add all of the *y*-components of the given vectors to find the *y*-component of the resultant.

5. Add the *x*-component of the resultant and the *y*-component of the resultant using the theorem of Pythagoras to determine the magnitude of the resultant, and using a trigonometric function to determine the direction in which the resultant points.

Sketch the components and add them as you did earlier.



Now, to find the magnitude of the change in velocity, use the theorem of Pythagoras.

$$\Delta\vec{v}^2 = \Delta\vec{v}_X{}^2 + \Delta\vec{v}_Y{}^2$$

$$\Delta\vec{v}^2 = (44.8)^2 + (-1.7)^2$$

$$\Delta\vec{v}^2 = 2007.04 + 2.89 = 2009.93$$

$$\Delta\vec{v} = \sqrt{2009.93} = 44.83224 = 45 \text{ m/s}$$

Using $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1.7}{44.8}$

$$\theta = \tan^{-1}\frac{1.7}{44.8} = 2.2°$$

The change in velocity of your car is 45 m/s [2.2° south of east].

b) To determine the acceleration, you must use kinematics equation 2: $\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ since you know the change in velocity and the time interval Δt = 20.0 s.

$$\vec{a} = \frac{45\ \text{m/s}\ 2.2°\ [\text{south of east}]}{20.0\ \text{s}} = 2.2\ \text{m/s/s}\left[2.2°\ \text{south of east}\right]$$

The average acceleration of your car is 2.2 m/s/s [2.2° south of east].

Recall that an even number trailed by a 5 is left unchanged when rounding it off.

c) To determine now the average force, you can use Newton's second law: $\vec{F}_{\text{net}} = m\vec{a}$.

The mass of the car is 1475 kg and acceleration is 2.2 m/s/s [2.2° south of east].

$\vec{F}_{\text{net}}$ = (1475 kg) (2.2 m/s/s [2.2° south of east]) = 3245 = 3200 N [2.2° south of east]

The net force required to change the direction of motion of your car is 3200 N [2.2° south of east].

Learning Activity 2.6

Vectors and the Component Method

The following questions will provide some practice in using the component method to add to vectors at any orientation. You may check your work against the answer key provided at the end of Module 2.

1. A hunter is tracking a bear. The hunter travels 675 m east, then 225 m 16.0° south of east, and finally 348 m northwest.
   a) What is the displacement of the hunter?
   b) If this displacement occurred during a time interval of 20.0 minutes, what was the average velocity of the hunter?
2. A boat is pushed forward by the propeller of its motor with a force of 2350 N [W]. The boat is pulling two water skiers. The first water skier is pulling on the boat with a force of 1280 N [26.8° N of E]. The second water skier is pulling on the boat with a force of 1520 N [35.0° S of E].
   a) What is the net force acting on the boat?
   b) If the mass of the boat and the skiers is 825 kg, describe the motion of the boat and skiers.

Lesson Summary

The component method is really an extension of the algebraic method that you have used to add together vectors that are perpendicular to each other. The process is summarized below.

Component Method of Adding Vectors

1. Designate a reference system using the *x*-axis and the *y*-axis.
2. Draw each of the given vectors on its own in the reference system with its tail at the origin.
3. Resolve each of the vectors into its *x*-component and *y*-component. Place these into a table as follows:

Vector	*x*-component	*y*-component
Vector 1		
Vector 2		
Resultant		

4. Add all of the *x*-components of the given vectors to find the *x*-component of the resultant. Add all of the *y*-components of the given vectors to find a *y*-component of the resultant.
5. Add the *x*-component of the resultant and the *y*-component of the resultant using the theorem of Pythagoras to determine the magnitude of the resultant, and using a trigonometric function determine the direction in which the resultant points.

Note that you should convert other types of reference systems, like right-left, up-down, or north-south-east-west, into an *x*-*y* reference system.

NOTES

Assignment 2.2

Adding Vectors Using the Component Method (8 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. There is one question in this assignment worth eight marks in total. Be sure to show all your work and explain the method of arriving at your answers. Also, pay attention to directions for vector answers, and round off the answers according to the correct number of significant digits. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of a flying bird

A bird is flying through the air with an air speed of 12.6 m/s heading 27.7° west of north. It is a windy day with the wind blowing at 8.95 m/s heading 12.5° south of west. What is the velocity of the bird as seen by a person on the ground?

(continued)

Assignment 2.2: Adding Vectors Using the Component Method (continued)

Method of Assessment

The total of eight marks for this assignment will be determined as follows:

- 1 mark for drawing the vectors each in its own reference system
- 1 mark for drawing the correct triangle for each vector to be used to determine the components
- 2 marks for determining the components of each vector correctly
- 1 mark for completing the chart of components
- 1 mark for determining the components of the resultant
- 1 mark for determining the magnitude of the resultant
- 1 mark for determining the correct angle for the resultant

Assignment 2.2

Adding Vectors Using the Component Method (8 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. There is one question in this assignment worth eight marks in total. Be sure to show all your work and explain the method of arriving at your answers. Also, pay attention to directions for vector answers, and round off the answers according to the correct number of significant digits. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of a flying bird

A bird is flying through the air with an air speed of 12.6 m/s heading 27.7° west of north. It is a windy day with the wind blowing at 8.95 m/s heading 12.5° south of west. What is the velocity of the bird as seen by a person on the ground?

(continued)

Assignment 2.2: Adding Vectors Using the Component Method (continued)

Method of Assessment

The total of eight marks for this assignment will be determined as follows:

- 1 mark for drawing the vectors each in its own reference system
- 1 mark for drawing the correct triangle for each vector to be used to determine the components
- 2 marks for determining the components of each vector correctly
- 1 mark for completing the chart of components
- 1 mark for determining the components of the resultant
- 1 mark for determining the magnitude of the resultant
- 1 mark for determining the correct angle for the resultant

On the next page, this applet demonstrates graphically the addition of vectors using the tip to tail method.

https://www.walter-fendt.de/html5/phen/index.html

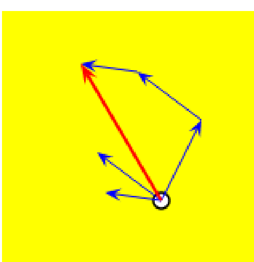
The applet begins with the addition of 2 vectors. You can change the magnitude and direction of a vector by clicking on the tip of the vector and dragging the tip with the left mouse button depressed. Try several examples to illustrate how the vectors are arranged to determine the resultant.

Change the number of vectors and repeat the steps above.

https://www.walter-fendt.de/html5/phen/resultant_en.htm

Resultant of Forces (Addition of Vectors)

This app deals with forces exerted on a body (assumed as point-sized). You can vary the number of single forces by using the choice box at the ride side. It is possible to change the sizes and directions of these forces (blue arrows) by dragging the arrowheads to the intended positions with pressed mouse button.



If you want to know the total force which is exerted on the body, you have to carry out a vector addition. As soon as you have clicked on the button "Find out resultant" the program will show you the necessary parallel translations of the force arrows and then draw the arrow of the resultant (red). The construction can be cleared by a mouse click on the lower button.

Video - Using a Data Table to Make Vector Addition Problems Easier

In this video you follow the steps to determine the resultant vector: read the problem, list the vectors and give vector names (prescribed Physics symbols) to the vectors, draw a vector diagram, resolve the vectors in to components, create a data table of the vectors and their components, add components in the columns to determine the components of the resultant, sketch the diagram for the addition of the components of the resultant, determine the magnitude of the resultant using the Pythagorean Theorem, and solve for the direction of the resultant using basic trigonometry.

https://youtu.be/nwqu0RIsvV4

Video - A Visually Complicated Vector Addition Problem using Component Vectors

This is another example of adding vectors using the component method.

https://youtu.be/e0rZI2JYGkY

Video - Adding Vectors: How to Find the Resultant of Three or More Vectors

The Video page layout includes an embedded YouTube video, which can scale in any browser or mobile device. Embedding videos in the page is a great way to present video content, while accompanying it with supporting context, explanations and activities.

Swap out the content and replace the video on this page to create your own page. Instructions for replacing videos are provided below.

https://youtu.be/g_TnqKX5ybY

Video - A Three Force Example of Newton's 2nd Law with Components

This video shows three brothers fighting over a toy turtle. the force exerted by each brother is given. The 3 force vectors are added using the component method of vector addition.

The table of the x-components and y- components is missing from this analysis. This data table is shown in the analysis of the vector addition in the next video Summing the Forces is Vector Addition.

https://youtu.be/IGtbiQt4fCQ

Video - Summing the Forces is Vector Addition

This video uses the graphical method to determine the resultant, the net force of 3 forces.

The video shows that the order in which the 3 vectors are added tip to tail results in the same net force.

It also shows that the graphical method results in the same resultant as the component method used int he previous video **A Three Force Example of Newton's 2nd La with Components.**

https://youtu.be/pr-bJTSCSUE

Lesson 6: Video Laboratory Activity: Forces in Equilibrium (1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ add two vectors with any orientation relative to each other using the component method
- ❑ determine the forces in equilibrium that are acting on an object

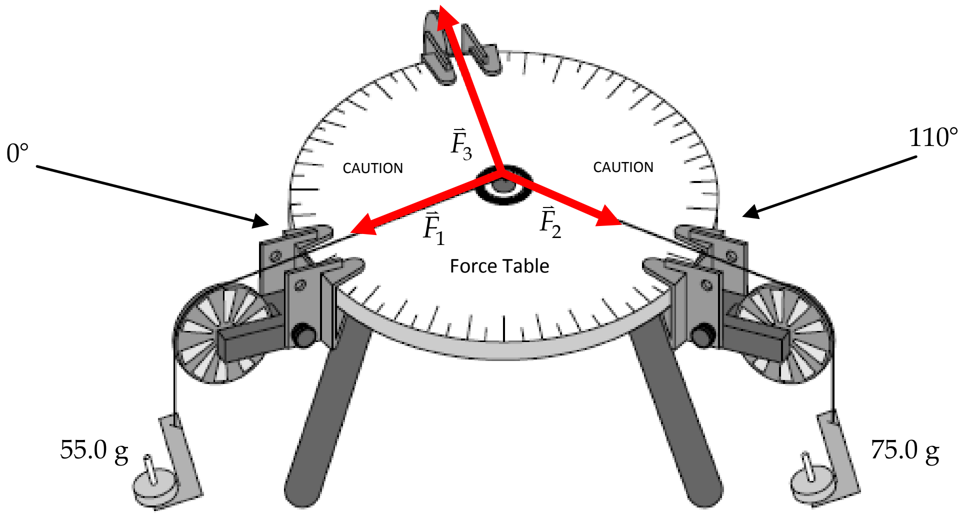
Note to Student

In this lesson, you will view *Forces in Equilibrium*, a short video laboratory activity found in the learning management system (LMS). You may need to collect some data from the video, so you should begin by reading the following introduction and reviewing Lesson 1: Laboratory Activity: Analysis of an Experiment (see Lesson 1, Module 1). Upon completion of the lab activity, you must complete Assignment 2.3, which consists of a lab report. Complete the various sections of the report in the space provided. Assignment 2.3 is to be submitted to the Distance Learning Unit for assessment at the end of Module 2.

Introduction

Two known forces are applied at different angles to a central ring on the force table by hanging masses over pulleys in different positions. The force of gravity on each of these hanging masses provides the force applied $\left(\vec{F}_1 \text{ and } \vec{F}_2\right)$. The resultant force on the ring will be the vector sum of these two forces. A balancing force ($\vec{F}_3$ in this case) can be determined experimentally by finding the angle and mass necessary to centre the ring (see Figure 1 on the next page).

Figure 1



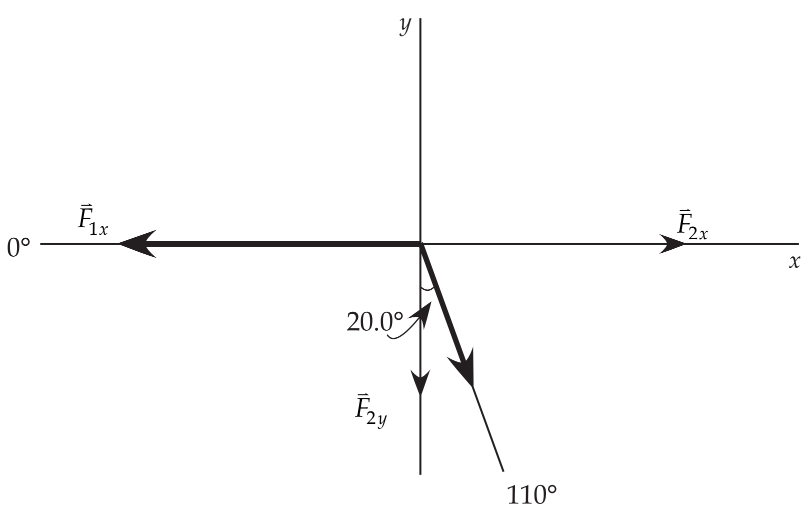
Let's suppose that we line up the 0° degree marker with $\vec{F}_1$ and we measure $\vec{F}_2$ at an angle of 110°. We know that the magnitude of the force of gravity is $\vec{F}_g = m\vec{g}$.

Therefore:

$\vec{F}_1 = 0.0550 \text{ kg} \times 9.80 \text{ N/kg} = 0.539 \text{ N [0°]}$

$\vec{F}_2 = 0.0750 \text{ kg} \times 9.80 \text{ N/kg} = 0.735 \text{ N [110°]}$

Vector Sketch



Sample Data Table

Vector	*x*-component (N)	*y*-component (N)
$\vec{F}_1$	–0.539	0
$\vec{F}_2$	0.735 × sin 20.0° = 0.251	0.735 × cos 20.0° = –0.690
$\vec{F}_3$	0.288	0.690
$\vec{F}_{net}$	0	0

We can find the *x*- and *y*-components of $\vec{F}_3$ because we know that for an object in equilibrium

$$\sum \vec{F}_x = 0 \text{ N and } \sum \vec{F}_y = 0 \text{ N}$$

$$\begin{aligned} \vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x} &= 0 \\ -0.539 + 0.251 + \vec{F}_{3x} &= 0 \\ -0.288 + \vec{F}_{3x} &= 0 \\ \vec{F}_{3x} &= 0.288 \text{ N} \end{aligned}$$

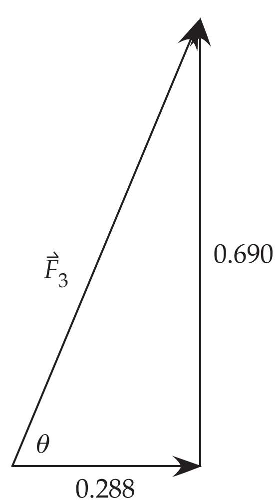
$$\begin{aligned} \vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y} &= 0 \\ 0 + -0.690 + \vec{F}_{3y} &= 0 \\ -0.690 + \vec{F}_{3y} &= 0 \\ \vec{F}_{3y} &= 0.690 \text{ N} \end{aligned}$$

Therefore:

$$\vec{F}_3 = \vec{F}_{3x} + \vec{F}_{3y}$$

$$\vec{F}_3^{\,2} = 0.288^2 + 0.690^2$$

$$\vec{F}_3 = 0.748 \text{ N } 67.3° \text{ counter-clockwise from right}$$



$$\tan\theta = \frac{0.690}{0.288}$$

$$\theta = 67.3°$$

NOTES

Assignment 2.3

Video Laboratory Activity: Forces in Equilibrium (20 MARKS)

You must complete the following lab report and submit it to the Distance Learning Unit for evaluation. Please complete your work in the space provided. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

Purpose

The purpose of this experiment is to study vector addition using a force table. The experimental value of the resultant force is compared to the calculated value of the resultant force.

Apparatus

Force table, string, 4 pulleys, 4 mass hangers, metric ruler, protractor, assorted mass set, graph paper

Procedure

1. Set up the force table as demonstrated in the video, and level the table using a carpenter's level.
2. Suspend two known masses at angles 0° and 120°. Record these masses in Data Table 1.
3. Suspend a third mass and align the pulley so that the ring is centred and the forces are balanced. Record this mass in Data Table 1.
4. Sketch the vectors at the correct angles on the Force Table template found in the Data and Calculations section that follows.
5. Calculate the forces $\left(\vec{F} = m\vec{g} \text{ where } \vec{g} = 9.80 \text{ N/kg}\right)$. Enter the data into Data Table 2.
6. Using the component method, add forces $\vec{F}_1 + \vec{F}_2$ to find the calculated value of the resultant force $\left(\vec{F}_3\right)$.
7. Compare the experimental value of the resultant force $\left(\vec{F}_3\right)$ with the calculated value.

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Video Viewing

View the video *Forces in Equilibrium*, which can be found in the learning management system (LMS).

Data and Calculations

Experimental Values (from Video)

1. Record the experimental values from the video in Data Table 1. *(2 marks)*
2. Convert all masses to kilograms. Show the sample calculation for force. *(1 mark)*

   Force 1:

   Data Table 1

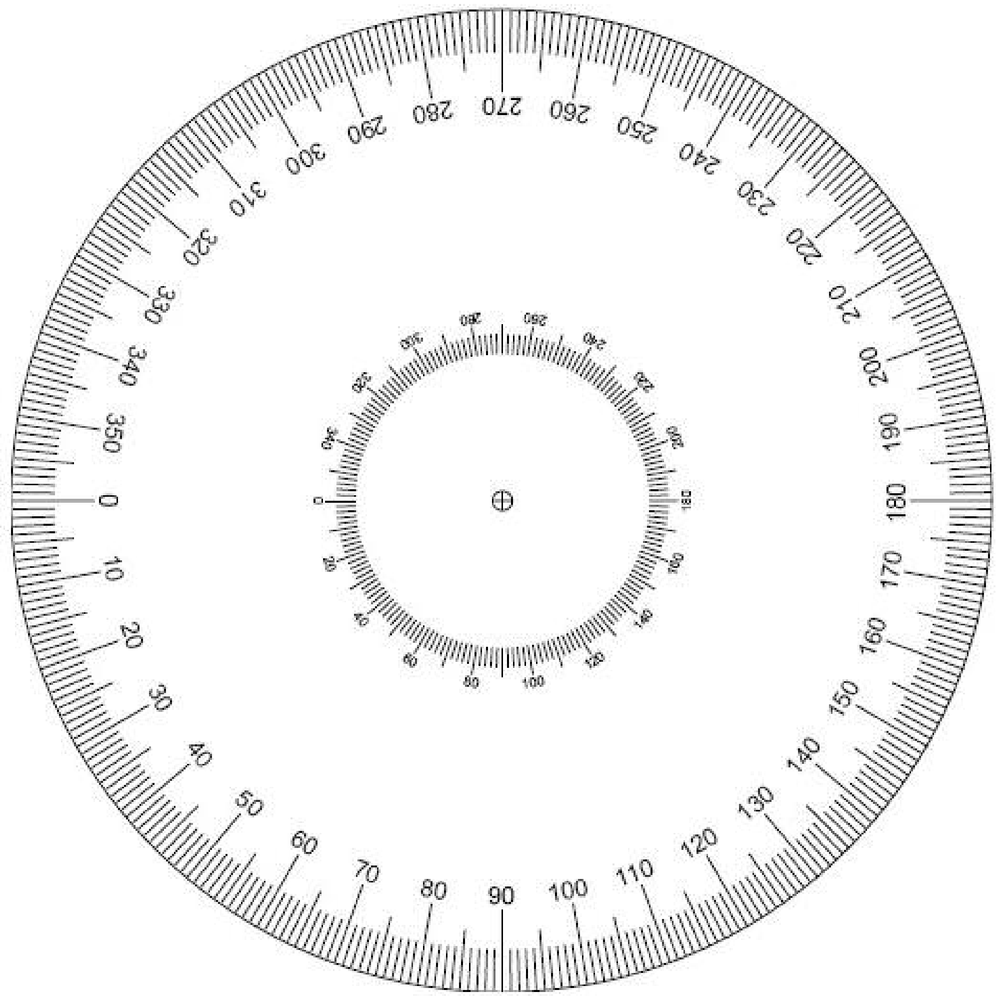
Vector	Mass (kg)	Force (N)	Angle (°)
$\vec{F}_1$			120
$\vec{F}_2$			0
$\vec{F}_3$			

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

3. On the Force Table template provided, draw in the *x*- and *y*-axes. Then draw the three force vectors on the template using an appropriate scale. *(3 marks)*

Force Table



(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Calculation of $\vec{F}_3$

4. Calculate the *x*-component and the *y*-component for each of the forces $\vec{F}_1$ and $\vec{F}_2$. Show your work in Data Table 2. *(4 marks)*

5. Calculate the *x*-component and the *y*-component for force $\vec{F}_3$. Show your work and record it in Data Table 2. *(2 marks)*

6. Calculate $\vec{F}_3$. Include a labelled sketch of the vector diagram. *(2 marks)*

Data Table 2

Vector	*x*-component (N)	*y*-component (N)
$\vec{F}_1$		
$\vec{F}_2$		
$\vec{F}_3$		

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Discussion

7. How do the calculated values for the magnitude and direction of the resultant force (Data Table 2) compare to the experimental values for the magnitude and direction (Data Table 1) found using the Force Table template? *(1 mark)*

8. Calculate the percent error for the magnitude of the resultant force. Use the following relationship. *(1 mark)*

$$\text{Percent error} = \frac{\left|\text{Calculated value} - \text{Measured value}\right|}{\text{Calculated value}} \times 100\%$$

9. What are some of the possible sources of experimental error? *(2 marks)*

10. Are there any other combinations of mass and angle that could balance the forces? Explain why or why not. *(1 mark)*

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Conclusion

11. Based on the study of vector addition using a Force Table template, how does the experimental value of the resultant force compare to the calculated value of the resultant force? *(1 mark)*

Marking Rubric for Assignment 2.3

Criteria	Possible Marks	Actual Marks
Experimental results in the Data Table	2	
Sample calculation of force from mass	1	
Drawing of the three forces on the Force Table template, including the scale	3	
Calculation of components of forces 1 and 2	4	
Calculation of x-component and y-component of force 3	2	
Calculation of force 3	2	
Discussion (percent error calculations, error analysis, questions)	5	
Conclusion	1	
Total	**20**	

Video - Force in Equilibrium

Watch the video for Assignment 2.3.

https://youtu.be/i4VHT7fOtXE?list=PLw1g3n2IMV7M72rewl81rI7b-CR0k8
Wta

Assignment 2.3

Video Laboratory Activity: Forces in Equilibrium (20 MARKS)

You must complete the following lab report and submit it to the Distance Learning Unit for evaluation. Please complete your work in the space provided. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

Purpose

The purpose of this experiment is to study vector addition using a force table. The experimental value of the resultant force is compared to the calculated value of the resultant force.

Apparatus

Force table, string, 4 pulleys, 4 mass hangers, metric ruler, protractor, assorted mass set, graph paper

Procedure

1. Set up the force table as demonstrated in the video, and level the table using a carpenter's level.
2. Suspend two known masses at angles 0° and 120°. Record these masses in Data Table 1.
3. Suspend a third mass and align the pulley so that the ring is centred and the forces are balanced. Record this mass in Data Table 1.
4. Sketch the vectors at the correct angles on the Force Table template found in the Data and Calculations section that follows.
5. Calculate the forces $\left(\vec{F} = m\vec{g} \text{ where } \vec{g} = 9.80 \text{ N/kg}\right)$. Enter the data into Data Table 2.
6. Using the component method, add forces $\vec{F}_1 + \vec{F}_2$ to find the calculated value of the resultant force $\left(\vec{F}_3\right)$.
7. Compare the experimental value of the resultant force $\left(\vec{F}_3\right)$ with the calculated value.

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Video Viewing

View the video *Forces in Equilibrium*, which can be found by visiting the Independent Study Option Audio and Video web page at www.edu.gov.mb.ca/k12/dl/iso/av.html.

Data and Calculations

Experimental Values (from Video)

1. Record the experimental values from the video in Data Table 1. *(2 marks)*
2. Convert all masses to kilograms. Show the sample calculation for force. *(1 mark)*

   Force 1:

Data Table 1

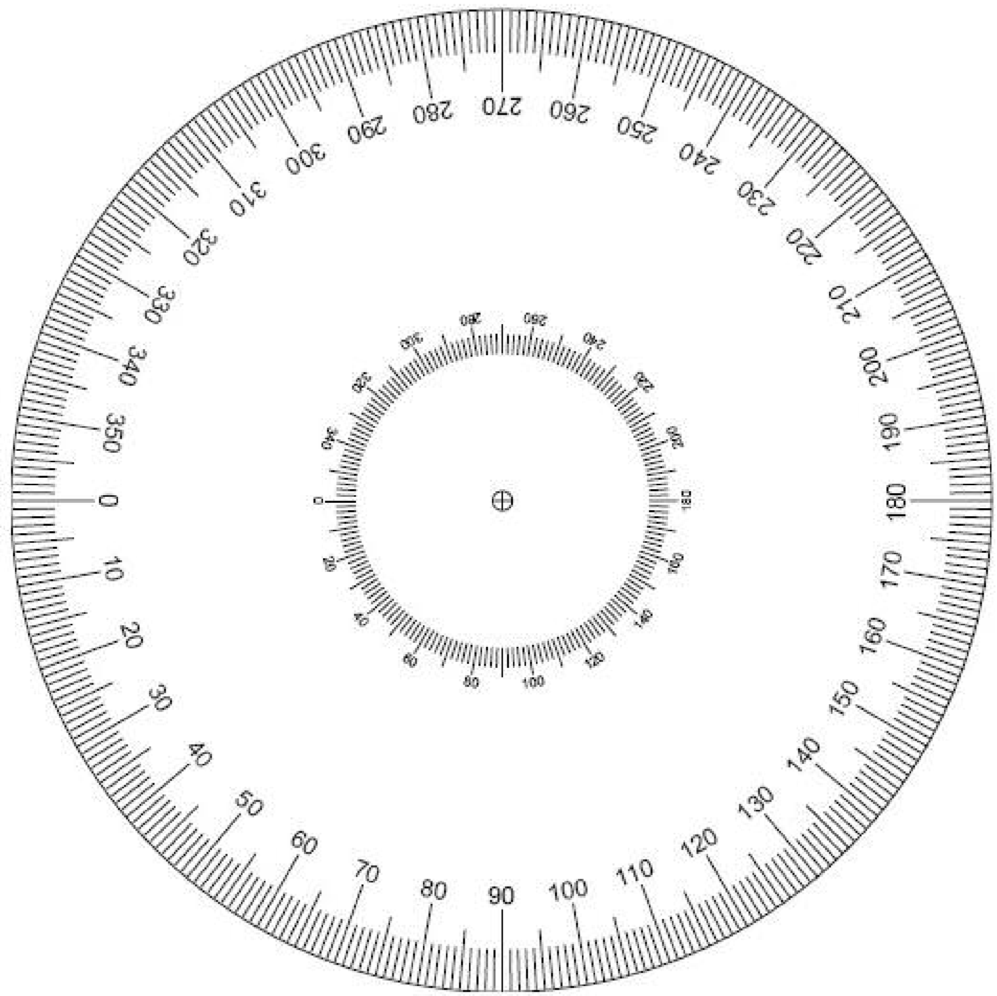
Vector	Mass (kg)	Force (N)	Angle (°)
$\vec{F}_1$			120
$\vec{F}_2$			0
$\vec{F}_3$			

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

3. On the Force Table template provided, draw in the x- and y-axes. Then draw the three force vectors on the template using an appropriate scale. *(3 marks)*

Force Table



(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Calculation of $\vec{F}_3$

4. Calculate the x-component and the y-component for each of the forces $\vec{F}_1$ and $\vec{F}_2$. Show your work in Data Table 2. *(4 marks)*

5. Calculate the x-component and the y-component for force $\vec{F}_3$. Show your work and record it in Data Table 2. *(2 marks)*

6. Calculate $\vec{F}_3$. Include a labelled sketch of the vector diagram. *(2 marks)*

Data Table 2

Vector	x-component (N)	y-component (N)
$\vec{F}_1$		
$\vec{F}_2$		
$\vec{F}_3$		

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Discussion

7. How do the calculated values for the magnitude and direction of the resultant force (Data Table 2) compare to the experimental values for the magnitude and direction (Data Table 1) found using the Force Table template? *(1 mark)*

8. Calculate the percent error for the magnitude of the resultant force. Use the following relationship. *(1 mark)*

$$\text{Percent error} = \frac{\left|\text{Calculated value} - \text{Measured value}\right|}{\text{Calculated value}} \times 100\%$$

9. What are some of the possible sources of experimental error? *(2 marks)*

10. Are there any other combinations of mass and angle that could balance the forces? Explain why or why not. *(1 mark)*

(continued)

Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium (continued)

Conclusion

11. Based on the study of vector addition using a Force Table template, how does the experimental value of the resultant force compare to the calculated value of the resultant force? *(1 mark)*

Marking Rubric for Assignment 2.3

Criteria	Possible Marks	Actual Marks
Experimental results in the Data Table	2	
Sample calculation of force from mass	1	
Drawing of the three forces on the Force Table template, including the scale	3	
Calculation of components of forces 1 and 2	4	
Calculation of *x*-component and *y*-component of force 3	2	
Calculation of force 3	2	
Discussion (percent error calculations, error analysis, questions)	5	
Conclusion	1	
Total	**20**	

Lesson 7: Forces in Equilibrium (1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the terms equilibrium and free-body diagram
- ❑ use the five-step reasoning strategy for solving equilibrium problems in one or two dimensions

Key Words

equilibrium
tension
free-body diagram

Introduction

The term “equilibrium” can be used in several ways. When you refer to a person’s state of mind, you may use the phrase “it took several days for this person to recover his or her equilibrium.” Concepts of equilibrium are important in other sciences such as chemistry and biology. In general, the term “equilibrium” can refer to a lack of change. Many of these “lack of change” situations are maintained by some processes occurring in the background. In this lesson, you will deal with the subject of equilibrium as it applies to physics.

You will study forces in equilibrium—that is, situations where one or more forces act on objects but the objects are at rest. From what you already know about Newton’s second law, if the acceleration is 0 m/s/s than the net force acting on the object is 0 N.

Defining Equilibrium

In physics, one way to look at equilibrium is the situation where an object's velocity is not changing—that is, the object is not accelerating. This could mean that the object is at rest, or that the object is moving at a constant velocity. This situation resulted when there was no net force acting on the object. The term "equilibrium" can thus be defined as follows.

> An object is in **equilibrium** when the net force acting on the object is 0 N.

For the object to have zero acceleration, the sum of the forces acting on the object must be zero. In the two-dimensional situation, the sum of all the x-components of the forces must be zero and the sum of all the y-components must be zero. This can be expressed mathematically as follows:

$$\sum \vec{F}_x = 0 \text{ N and } \sum \vec{F}_y = 0 \text{ N}$$

The forces acting on an object in equilibrium must balance. If an object is accelerating, then the forces acting on an object do not balance, and the object is not in a state of equilibrium.

Reasoning Strategy for Analyzing Equilibrium Situations

To solve equilibrium problems, the following five-step reasoning strategy is a good one to follow.

1. Select the object to be studied. This object is often called a "system." If two or more objects are connected by a rope or a cable, treat each object separately, and then follow the steps below.

2. Draw a "free-body diagram" for each object under consideration. A free-body diagram is a drawing that represents the object and the forces acting on that object. Careful attention must be paid to the direction in which the forces are drawn. Generally, a free-body diagram has the shape of a cross (+). Draw only the forces that act on that object and omit any forces that the object exerts on its surroundings.

3. Choose a set of x- and y-axes for each of the objects being analysed and resolve the forces in the free-body diagram into components that point along these axes. The final free-body diagram should contain only forces that form a cross (+). If the forces do not form a cross, the forces should be resolved into their components along the x- and y-axes.

The axes may not necessarily point vertically and horizontally or up and down the way you are accustomed to seeing them on a graph. The axes should be selected in such a way that as many forces as possible point directly along the x-axis or the y-axis. Doing so will result in fewer calculations when you apply the component method of vector addition to the situation.

4. For the equilibrium situations, set up the equations in such a way that the sum of the x-components of the forces is zero, and the sum of the y-components is also equal to zero. This can be done in a table.
5. Solve the equations for the unknown quantities you are looking for. Remember that if there are two unknowns, then, to solve this mathematically, there can be only two sets of equations.

Equilibrium in One Dimension

One common example of equilibrium in one dimension is a hanging sign suspended by vertical cords.

Example 1

Suppose you had a sign with a mass of 20.0 kg suspended by two ropes, one on each side of the sign. What would be the tension in each of the ropes?

To answer this question, follow the steps outlined above.

Step 1:

Select the object to be studied. In this case, the object is the sign.

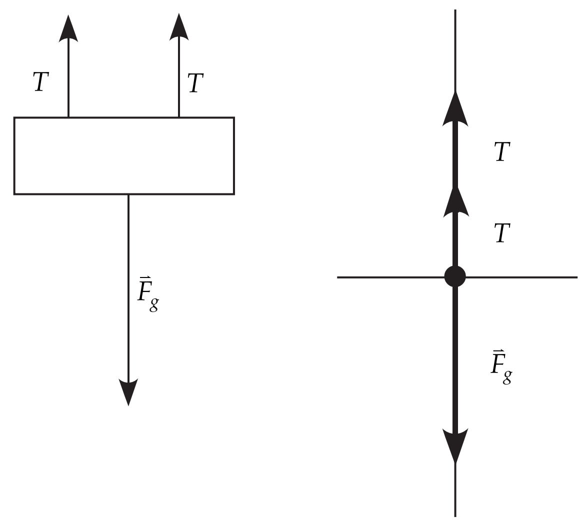
Step 2:

Draw a "free-body diagram" for each object chosen.

In the free-body diagram, there are three forces acting on this object. Gravity is acting down and this force is balanced by the two upward forces exerted by the cords. The cords are the same distance from the centre.

Strengths and wires are said to exert tension forces.

Tension is the magnitude of the force exerted on and by a cable, rope, or string.



Step 3:

Choose a set of x- and y-axes for each of the objects being analyzed, and resolve the free-body diagram into components that point along these axes.

In this case, there is only one axis: the vertical one. There are only y-components of forces. There are no x-components.

Step 4:

Set up the equations in such a way that the sum of the x-components of the forces is zero, and the sum of the y-components is also equal to zero.

There are two tension forces (T) pointing upwards and one force of gravity $\left(\vec{F}_g\right)$ pointing downwards. The sum of these forces is zero. The mathematical equation for the situation could be written as

$$2T\,[\text{up}] + \vec{F}_g\,[\text{down}] = 0\text{ N}$$

But, $\vec{F}_g = m\vec{g}$

So, $2T\,[\text{up}] + m\vec{g}\,[\text{down}] = 0\text{ N}$

Step 5:

Solve the equations for the unknown quantities you are looking for.

The tension force in each wire could therefore be found as

$$T = \frac{m\vec{g}}{2} = \frac{[(20.0)(9.80)]}{2} = 98.0\text{ N}.$$

Therefore, the force of tension on each wire was 98.0 N [up].

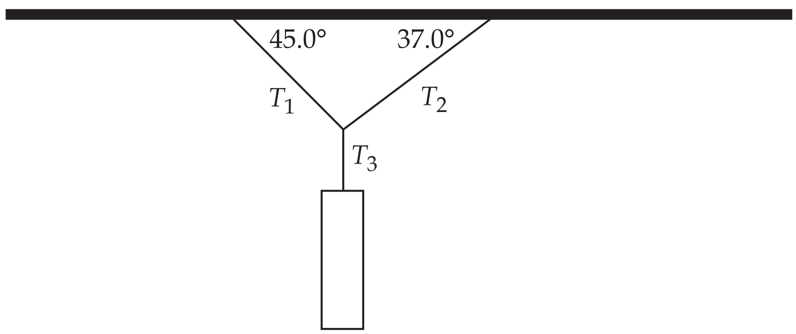
Equilibrium in Two Dimensions

The example of a hanging sign could also be applied in two dimensions.

Example 2

Given the mass of a hanging sign, determine the tension in a supporting wire.

Suppose a sign is hanging from two wires, as shown in the diagram below. The mass of the sign is 30.0 kg.



Let us apply the five steps to finding the forces acting in the two wires suspended from the ceiling.

Step 1:

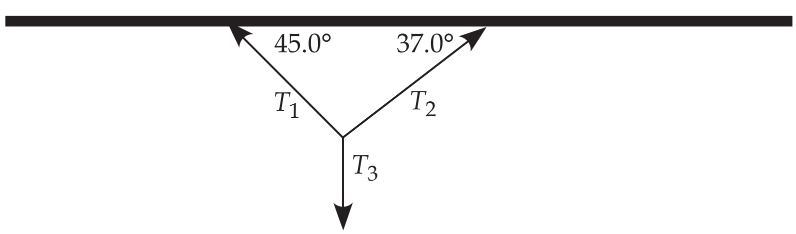
Select the object to be studied.

The object you are studying is the sign. But in this case, when you draw the free-body diagram, it would be convenient to choose the point at which the three wires join. The sign would not be a good choice for the object because there are only two forces that act on it: the force of gravity downwards and the equal and opposite force of the wires upward.

Step 2:

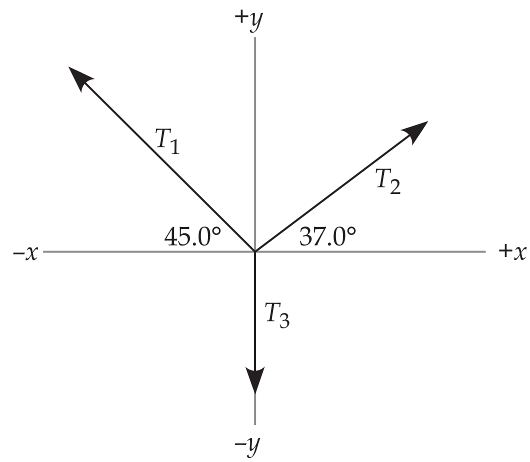
Draw a free-body diagram for each object chosen.

The free-body diagram would contain the three tensions in the wires at the angle shown in the diagram.

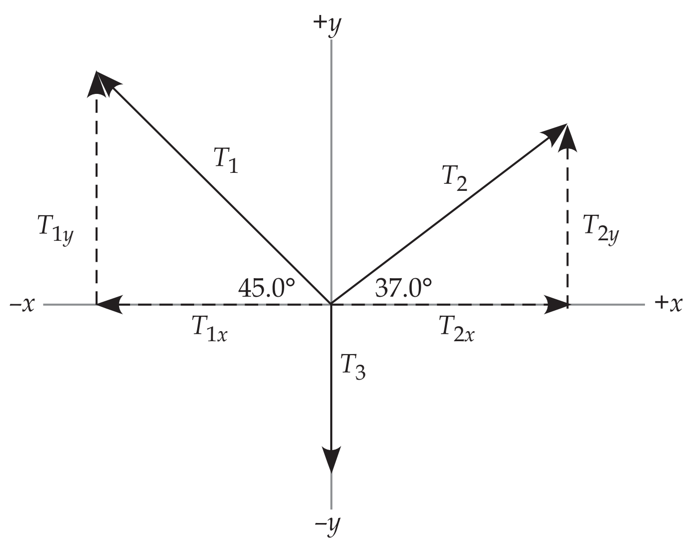


Step 3:

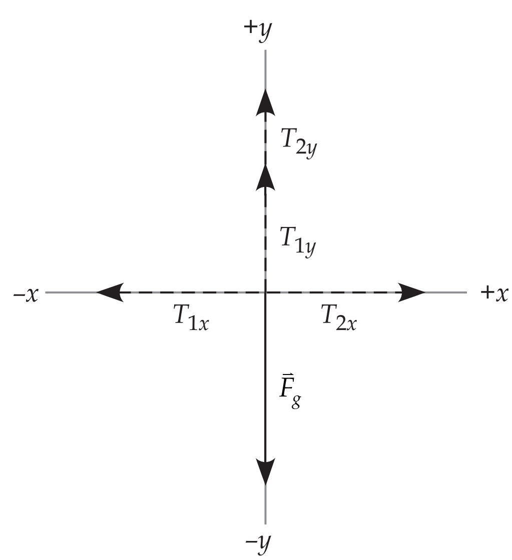
Choose a set of x- and y-axes for each of the objects being analyzed, and resolve the free-body diagram into components that point along these axes.



A convenient orientation of the axes would be in the vertical and horizontal directions.



Here are the components on the free-body diagram.



Vector	*x*-component (N)	*y*-component (N)
T_1	$-(\cos 45.0°)T_1 = -0.7071T_1$	$+(\sin 45.0°)T_1 = 0.7071T_1$
T_2	$+(\cos 37.0°)T_2 = +0.7986T_2$	$+(\sin 37.0°)T_2 = 0.6018T_2$
T_3	0	$-T_3$
$\vec{F}_{\text{net}}$	0	0

The x- and y-components of each of the vectors T_1 and T_2 are as follows:

- $T_{1x} = -T_1 \cos 45.0°$ (Note the negative direction because the force is to the left.)
- $T_{1y} = T_1 \sin 45.0°$ (Note the positive direction because the force is upwards.)
- $T_{2x} = T_2 \cos 37.0°$ (Note the positive direction because the force is to the right.)
- $T_{2y} = T_2 \sin 37.0°$ (Note the positive direction because the force is upwards.)

Step 4:

Set up the equations in such a way that the sum of the x-components of the forces is zero, and the sum of the y-components is also equal to zero.

There are two forces in the x-direction (horizontal). Their sum must be zero.

$$-T_1 \cos 45.0° + T_2 \cos 37.0° = 0$$

There are three forces in the y-direction (vertical). Their sum must be zero.

$$T_{1y} + T_{2y} - T_3 = 0$$

Note that T_3 represents the force of gravity. Therefore, $T_3 = \vec{F}_g = m\vec{g}$.

This can be rewritten as

$$T_1 \sin 45.0° + T_2 \sin 37.0° - m\vec{g} = 0$$

Step 5:

Solve the equations for the unknown quantities you are looking for. One of the ways to solve the equations for the two unknowns is as follows:

From the equations in the horizontal direction,

$$-T_1 \cos 45.0° + T_2 \cos 37.0° = 0$$

Solving for T_2:

$$T_2 = \frac{T_1 \cos 45.0°}{\cos 37.0°}$$

From the equations in the vertical direction:

$$T_1 \sin 45.0° + T_2 \sin 37.0° - m\vec{g} = 0$$

Substitute for T_2 and solve for T_1:

$$T_1 \sin 45.0° + \left(\frac{T_1 \cos 45.0°}{\cos 37.0°}\right)\sin 37.0° - m\vec{g} = 0$$

$$T_1(0.7071) + \left(\frac{T_1(0.7071)}{0.7986}\right)(0.6018) - (30.0)(9.80) = 0$$

$$0.7071T_1 + 0.5328T_1 - 294 = 0$$

$$1.2399T_1 = 294$$

$$T_1 = \frac{294}{1.2399} = 237\text{ N}$$

Placing the value for T_1 back into the expression for T_2 yields:

$$T_2 = \frac{T_1 \cos 45.0°}{\cos 37.0°} = T_2 = \frac{T_1(0.7071)}{0.7986} = \frac{(237)(0.7071)}{0.7986}$$

$$= 209.8\text{ N} = 2.10 \times 10^2\text{ N}$$

The tension in wire 1 is 237 N, and the tension in wire 2 is 2.10×10^2 N.

Example 3

Determine the mass of a hanging sign given the tensions in the supporting wires.

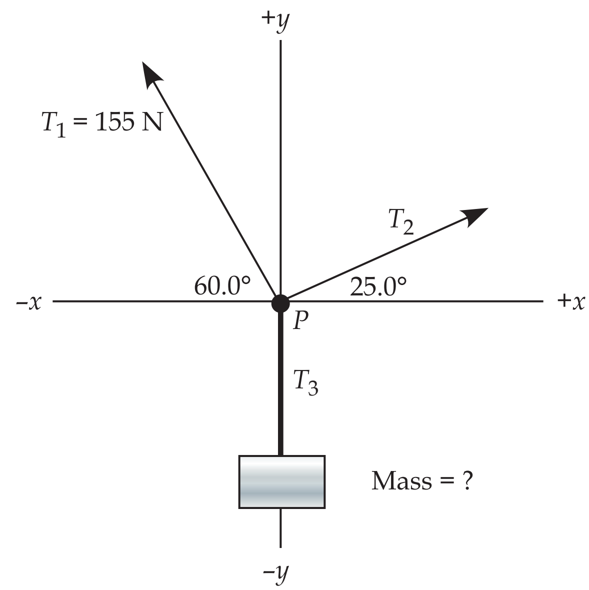
In this type of problem, you are given enough information about the tensions in the supporting wires to enable you to determine the mass of a sign being supported by those wires. The approach to be used is very similar to the previous problem. Again, there will be two unknowns: the mass of the hanging sign, and the tension in one of the wires.

For the situation below, what is the mass, m, and the tension, T_2, so that the forces acting at point P are in equilibrium?

Step 1:

Select the object to be studied.

The object you are studying is the sign. Again, in this case, when you draw the free-body diagram, it would be convenient to choose the point at which the three wires join. The sign would not be a good choice for the object because there are only two forces that act on it, the force of gravity downwards and the equal and opposite force of the wires upward.



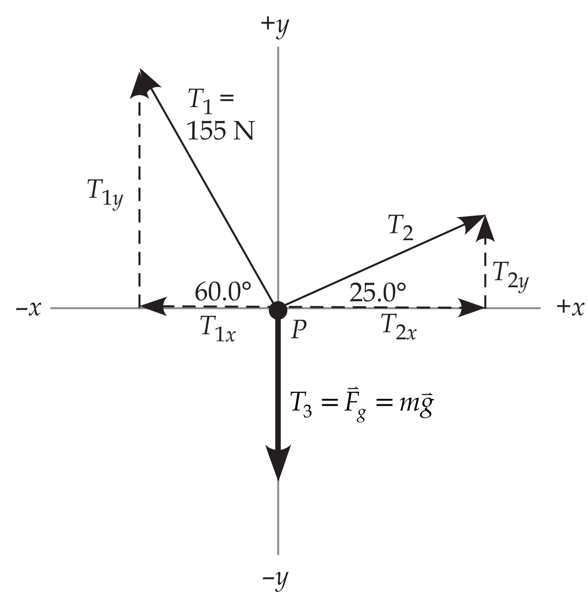
Step 2:

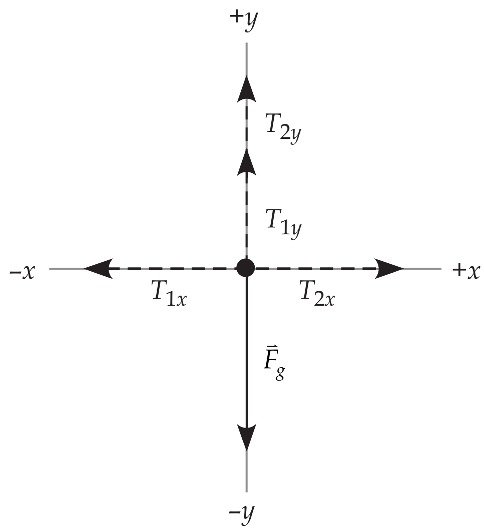
Draw a free-body diagram for each object chosen.

The free-body diagram would contain the three tensions in the wires at the angle shown in the diagram.

Step 3:

Choose a set of x- and y-axes for each of the objects being analyzed, and resolve the free-body diagram into components that point along these axes.





It is wise to arrange the vectors and their components in a table.

Vector	*x*-component (N)	*y*-component (N)
T_1	$-(\cos 60.0°)T_1 =$ $-(0.5)(155) = -77.5$	$+(\sin 60.0°)T_1 =$ $(0.8660)(155) = 134$
T_2	$+(\cos 25.0°)T_2 = 0.9063T_2$	$+(\sin 25.0°)T_2 = 0.4226T_2$
T_3	0	$-T_3$
$\vec{F}_{\text{net}}$	0	0

Step 4:

Set up the equations in such a way that the sum of the *x*-components of the forces is zero, and the sum of the *y*-components is also equal to zero.

There are two forces in the *x*-direction (horizontal). Their sum must be zero.

$$-T_1 \cos 60.0° + T_2 \cos 25.0° = 0$$

There are three forces in the *y*-direction (vertical). Their sum must be zero.

$$T_{1y} + T_{2y} - T_3 = 0$$

Step 5:

Solve the equations for the unknown quantities you are looking for.

$$-T_1 \cos 60.0° + T_2 \cos 25.0° = 0$$

$$-77.5 + (+0.9063\ T_2) = 0$$

$$+0.9063T_2 = +77.5$$

$$T_2 = \frac{77.5}{0.9063} = 85.5 \text{ N}$$

$$T_{1y} + T_{2y} - T_3 = 0$$

Note that T_3 represents the force of gravity. Therefore, $T_3 = \vec{F}_g = m\vec{g}$.

$$T_{1y} + T_{2y} - m\vec{g} = 0$$

$$+134 + (0.4226)(85.5) - m(9.80) = 0$$

$$+134 + 36.1 = 9.80m$$

$$+170.1 = 9.80m$$

$$m = 17.4 \text{ kg}$$

The mass supported is 17.4 kg and the tension in the second wire is 85.5 N.

Learning Activity 2.7

Objects in Equilibrium

There are five practice questions in this learning activity. An answer key is available at the end of Module 2 for you to check your work after you have answered the questions.

The physics of objects in equilibrium (conceptual)

1. For each of the following, explain whether or not the object can be in equilibrium:
   a) an object acted upon by a single force
   b) an object acted upon by two forces that point in mutually perpendicular directions
   c) an object acted upon by two forces that point in directions that are not mutually perpendicular

(continued)

The physics of the Queen's portrait

2. A portrait of Queen Elizabeth II is suspended from the rafters of the MTS Centre by two vertical wires. The mass of the portrait and its frame is 12.0 kg. The wires are equidistant from the left and right ends of the portrait frame. What is the tension in each wire?

The physics of a traction device

3. While a person is recuperating from a leg injury, the leg must be stretched by a traction device. Tension (T) (a force to the left) is created in the leg by the action of the weight of a mass. A cable attaches the foot of the leg to a mass hanging from a pulley. If a force of 45.0 N to the right is exerted in the knee joint of the leg, what should be the mass of the object hanging from the pulley?

The physics of a tightrope walker

4. At the circus, a tightrope walker walks along a horizontal tightrope suspended above the ground. At the moment the tightrope walker is standing at the midpoint of the rope, the tension in the rope is 2550 N. Each half of the rope makes an angle of 9.00° with respect to the horizontal. Find the force acting downwards. This is equivalent to the weight of the performer.

The physics of a hanging ornament

5. Two wires are attached to an ornament and support the weight of the ornament. One wire makes an angle of 50.5° with the horizontal. The tension in this first wire is 75.0 N. The second wire makes an angle of 72.0° with the horizontal.
   a) What is the tension in the second wire?
   b) What is the mass of the ornament?

Lesson Summary

In this lesson, we focused on the concept of equilibrium as it applies to this area of physics. You should be able to analyze both the one-dimensional and the two-dimensional situations.

An object is in equilibrium when it has zero acceleration. The sum of the forces acting on the object must be zero in both the x-direction and the y-direction.

$$\sum \vec{F}_x = 0 \text{ N and } \sum \vec{F}_y = 0 \text{ N}$$

A five-step reasoning strategy is useful in solving equilibrium problems.

1. Select the object to be studied. This object is often called a "system."
2. Draw a free-body diagram for each object chosen. A free-body diagram is a drawing that represents the object and the forces acting on that object.
3. Choose a set of x- and y-axes for each of the objects being analyzed, and resolve the free-body diagram into components that point along these axes. The free-body diagram should look like a cross ("+"). A table listing the components is useful.
4. Set up the equations in such a way that the sum of the x-components of the forces is zero, and the sum of the y-components is also equal to zero.
5. Solve the equations for the unknown quantities you are looking for.

NOTES

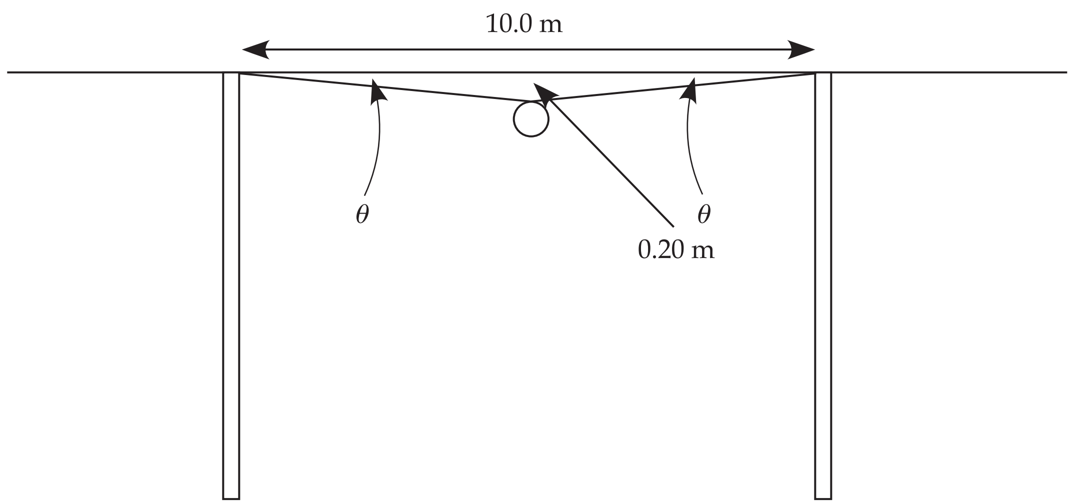
Assignment 2.4

Objects in Equilibrium (6 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of the clothesline

A clothesline is attached to two fixed ends which are 10.0 m apart. A pulley of mass 40.0 kg hangs freely in the middle of the line. The sag at the centre is 0.20 m. Find the tension in each half of the clothesline.



Step 1 and Step 2

(continued)

Assignment 2.4: Objects in Equilibrium (continued)

Step 3

Step 4

Step 5

Method of Assessment

The total of six marks for this assignment will be determined as follows:

- 1 mark for Steps 1 and 2
  Step 1: Select the object to be studied.
  Step 2: Draw a free-body diagram for each object chosen.
- 2 marks for Step 3
  Step 3: Choose a set of *x*- and *y*-axes for each of the objects being analyzed, and resolve the free-body diagram into components that point along these axes.
- 1 mark for Step 4
  Step 4: Set up the equations in such a way that the sum of the *x*-components of the forces is zero, and the sum of the *y*-components is also equal to zero.
- 2 marks for Step 5
  Step 5: Solve the equations for the unknown quantities you are looking for.

The applet on the next page allows you to control three forces that are in equilibrium.

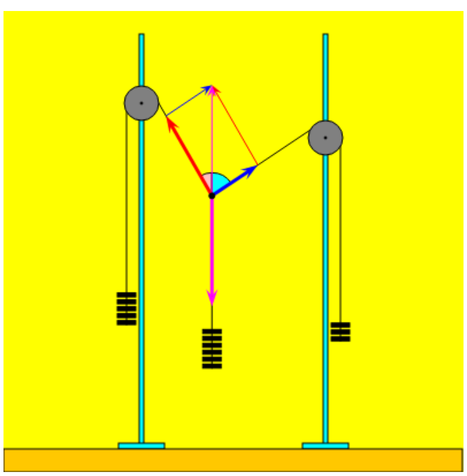
https://www.walter-fendt.de/html5/phen/

https://www.walter-fendt.de/html5/phen/equilibriumforces_en.htm

Equilibrium of Three Forces

A simple experiment concerning the equilibrium of three forces is simulated here: Weights are suspended from three tied cords. Two of the cords run over frictionless pulleys. The three forces acting on the knot (coloured arrows) are in equilibrium.

You can choose forces from 1 N to 10 N in the input fields. Notice that each force must be smaller than the sum of the other two forces! It is possible to vary the positions of the two pulleys by dragging the mouse. The parallelogram of the forces which are directed to the top left and right (red respectively blue) will be drawn if you select the corresponding option. At the bottom right you can read the angles of these two forces with respect to the vertical.



Video - Static Equilibrium

This video shows how to solve for the magnitudes of two forces supporting a known mass. The angles for the two forces are given.

This video solves 3 problems involving a body supported by two forces acting at an angle to the horizontal.

In general equilibrium means that all of the forces acting on an object are balanced. In other words the sum of all the forces acting on the object add up to 0 N.

So Fnet = F1 + F2 + F3 + ... = 0 N.

Next each force can be resolved into the x component and y component.

F1 = F1x + F1y

F2 = F2x   + F2y

F3 = F3x + F3y

These components are drawn on the free body diagram so you can clearly see their role in the situation.

Since Fnet = F1 + F2 + F3 = 0N

the components of the forces must also add to 0 N.

Then Fnet x = F1x + F2x + F3x = 0 N

and Fnety = F1y + F2y + F3y = 0 N

Depending on which forces and angles are given, the x-component and the y-component for that missing force can be calculated. From these two components the missing force can be determined.

https://youtu.be/dxM9lsbUbpw

Video - Solving Tension Problems

This video shows how to solve for the magnitudes of two forces supporting a known mass. The angles for the two forces are not given. Instead distance are provided from which angles can be calculated.

All of the steps are clearly shown and explained.

https://youtu.be/EZNSoSIurQ8

Video - Equilibrium of Force xy-component 1

This video solves 3 problems involving a body supported by two forces acting at an angle to the horizontal.

In general equilibrium means that all of the forces acting on an object are balanced. In other words the sum of all the forces acting on the object add up to 0 N.

So $F_{net} = F1 + F2 + F3 + ... = 0$ N.

Next each force can be resolved into the x component and y component.

$F1 = F1x + F1y$

$F2 = F2x + F2y$

$F3 = F3x + F3y$

These components are drawn on the free body diagram so you can clearly see their role in the situation.

Since $F_{net} = F1 + F2 + F3 = 0N$

the components of the forces must also add to 0 N.

Then $F_{net}\ x = F1x + F2x + F3x = 0$ N

and $F_{nety} = F1y + F2y + F3y = 0$ N

Depending on which forces and angles are given, the x-component and the y-component for that missing force can be calculated. From these two components the missing force can be determined.

https://youtu.be/brvKPItTEvU

Assignment 2.4

Objects in Equilibrium (6 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of the clothesline

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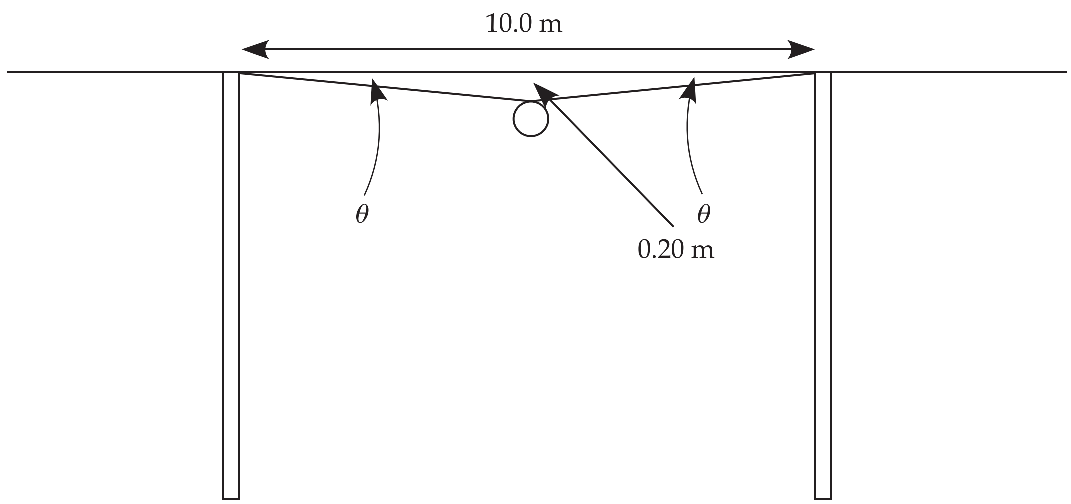


Step 1 and Step 2

(continued)

Assignment 2.4: Objects in Equilibrium (continued)

Step 3

Step 4

Step 5

Method of Assessment

The total of six marks for this assignment will be determined as follows:

- 1 mark for Steps 1 and 2
  Step 1: Select the object to be studied.
  Step 2: Draw a free-body diagram for each object chosen.
- 2 marks for Step 3
  Step 3: Choose a set of x- and y-axes for each of the objects being analyzed, and resolve the free-body diagram into components that point along these axes.
- 1 mark for Step 4
  Step 4: Set up the equations in such a way that the sum of the x-components of the forces is zero, and the sum of the y-components is also equal to zero.
- 2 marks for Step 5
  Step 5: Solve the equations for the unknown quantities you are looking for.

Lesson 8: Forces at an Angle (2.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ solve problems involving forces at an angle on a horizontal, frictionless surface
- ❑ solve problems involving forces at an angle on a horizontal surface with friction
- ❑ solve problems involving frictionless surfaces at an incline
- ❑ solve problems involving surfaces with friction at an incline

Key Words

inclined plane

Introduction

This lesson will conclude the section on forces and friction by considering forces at an angle. The object could be moving on a flat, horizontal plane with a force applied at an angle, or the object could be moving down an incline, in which case forces are also at an angle. These situations allow you to examine many realistic situations, such as the forces that might be involved in pushing a shopping cart or tobogganing down a hill. In doing so, you will be drawing free-body diagrams and analyzing forces into their components. The key will be to obtain a free-body diagram that looks like a cross, even a cross at an angle. You will also be applying some of the kinematics you learned in earlier lessons.

Friction: Basic Concepts

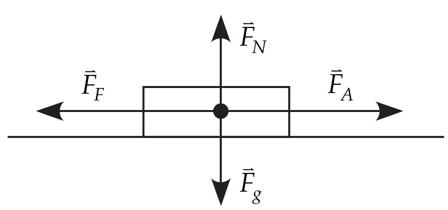
There are two basic types of frictional forces: static friction and kinetic friction.

> **Static friction** is a force that opposes the start of motion between two surfaces.

On a horizontal surface, the static frictional force is equal in magnitude to the applied force and opposite in direction. If the applied force increases again a small amount, the force of static friction increases by the same amount and the object does not move. The static frictional force is equal in magnitude to the applied force and opposite in direction. If the applied force continues to increase, there is a point where the object finally "breaks away" and starts to move.

The maximum static frictional force is dependent on two factors: the nature of surfaces in contact with each other and the normal force acting on the object. Obviously, some surfaces are more slippery than others, and the force of static friction will be less for those surfaces. The normal force measures how hard the surfaces are pressed together. The greater the force with which the surfaces are pressed together, the larger the force of static friction. The force of static friction is directly proportional to the normal force. Recall that the normal force for an object on a horizontal surface opposes the force of gravity on the object and that $\vec{F}_g = m\vec{g}$. So, in the end, the force of static friction depends on the mass of the object. However, the force of static friction does not depend on the area of contact between the object and the surface. For example, a crate resting on its smallest side experiences the same force of static friction as the same crate resting on the side with the largest area.

A free-body diagram of the forces involved in a static situation might be as follows.



The **coefficient of static friction** as the ratio of the maximum static frictional force and the normal force.

$$\mu_s = \frac{F_S}{F_N}$$

Quantity	**Symbol**	**Unit**
Coefficient of static friction	μ_s	has no units
Force of static friction	F_S	newton (N)
Normal force	F_N	newton (N)

Kinetic friction is a force that opposes the motion between two surfaces that are moving relative to each other.

The kinetic force of friction is independent of the area of contact between the surfaces of the materials (as was the case with static friction) and is independent of the speed of the sliding object if the speed is small. The magnitude of the kinetic force of friction is proportional to the magnitude of the normal force and is different for different surfaces (as was the case with static friction).

The **coefficient of kinetic friction** can be defined as the ratio of the kinetic frictional force and the normal force.

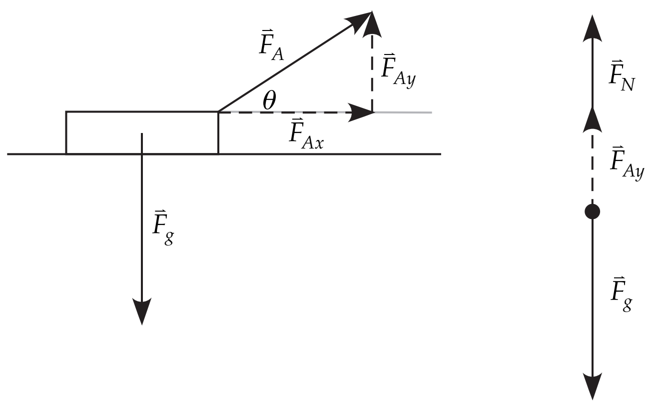
$$\mu_k = \frac{F_K}{F_N}$$

Quantity	**Symbol**	**Unit**
Coefficient of kinetic friction	μ_k	has no units
Force of kinetic friction	F_K	newton (N)
Normal force	F_N	newton (N)

The key to working with forces of friction is to determine the magnitude of the normal force that is just large enough to balance the force of gravity correctly. If objects are resting on a level horizontal surface, as in the two diagrams above, the normal force (the surface pushing up on to an object) is just large enough to balance the force of gravity pulling the object to the surface.

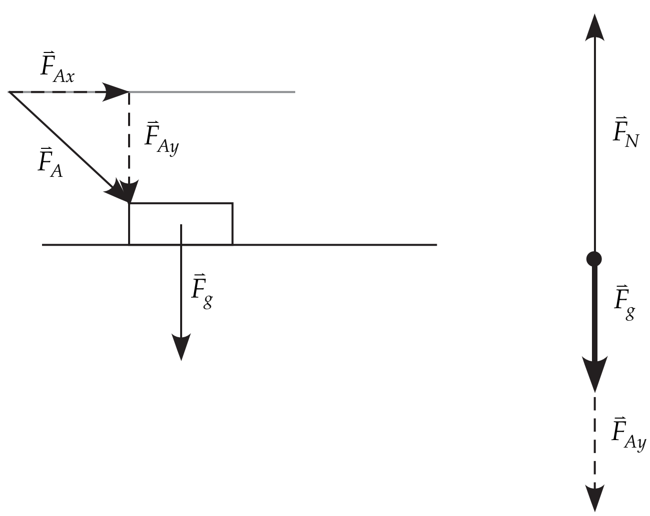
If applied forces are only acting in the horizontal direction, then the applied forces have no effect on the magnitude of the normal force. However, if applied forces act at some angle to the horizontal, then a component of the applied force will be acting in the vertical direction. This vertical component affects the magnitude of the normal force. If the applied force pulls upwards from the horizontal, then the vertical component of the applied force will decrease the normal force. If the applied force pushes or pulls the object towards the surface, then the normal force will be increased by the vertical component of the applied force.

The following examples illustrate how the normal force is affected by applied forces acting at some angle to the horizontal.



In this case, the normal force is decreased by the vertical component of the applied force since the component is pulling the object up from the surface.

If an object is being pushed or pulled towards the surface by the applied force, then the situation looks like this:

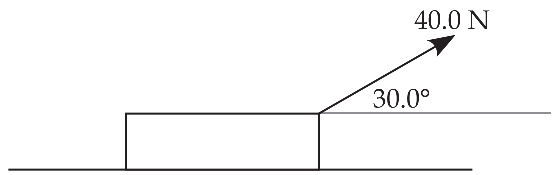


In this case, the vertical component of the applied force increases the magnitude of the normal force.

$$\vec{F}_N = -\left(\vec{F}_g + \vec{F}_{Ay}\right)$$

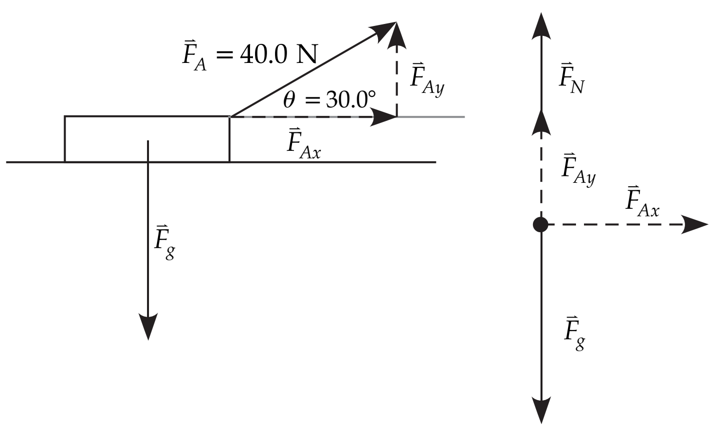
Example 1: Pulling at an Angle: No Friction

In this example, we will examine pulling a rectangular box at an angle, assuming there is no friction present. In the drawing below, a 10.0 kg box is being pulled to the right with a force of 40.0 N at an angle of 30.0°. What is the acceleration of the block?



The solution to the question involves taking the dynamics information and analyzing it to determine the net force. From the net force, you can determine the acceleration of the block.

In drawing a free-body diagram for this situation, it is useful to draw the force vectors from a dot.



In this case, the normal force is decreased by the vertical component of the applied force since the component is pulling the object up from the surface.

In the y-direction, the sum of the forces is equal to zero. The block does not rise off the table or fall into the table. The net force in the y-direction is zero.

$$\vec{F}_N + \vec{F}_A(\sin\theta) + \vec{F}_g = 0$$

$$\vec{F}_N + (40.0\text{ N})(\sin 30.0°)\,[\text{up}] + m\vec{g}\,[\text{down}] = 0$$

$$\vec{F}_N + (40.0\text{ N})(\sin 30.0°)\,[\text{up}] + (10.0\text{ kg})(9.80\text{ m/s}^2)\,[\text{down}] = 0$$

$$\vec{F}_N + (20.0\text{ N})\,[\text{up}] + (98.0\text{ N})\,[\text{down}] = 0$$

$$\vec{F}_N + (78.0\text{ N})\,[\text{down}] = 0$$

$$\vec{F}_N = 78.0\text{ N}\,[\text{up}]$$

In this case, there is only one force along the x-direction. That force is the x-component of the applied force.

$$\vec{F}_{Ax} = \vec{F}_A \cos\theta = (40.0 \text{ N})(\cos 30.0°) = 34.6 \text{ N}$$

This is the force that makes the block accelerate. It is, in fact, the net force.

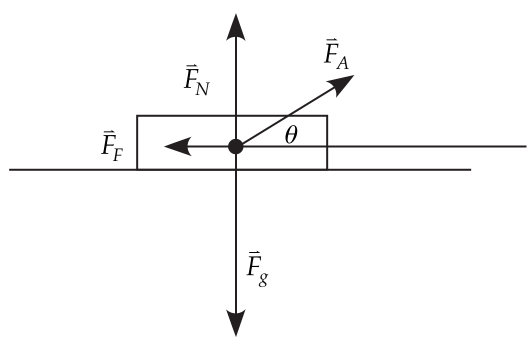
Using Newton's second law, the rate of acceleration is

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{34.6 \text{ N [right]}}{10.0 \text{ kg}} = 3.46 \text{ m/s}^2 \text{ [right]}$$

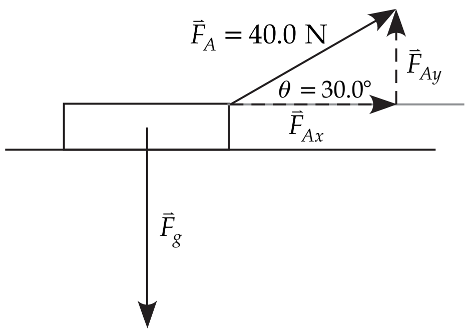
Example 2: Pulling at an Angle: With Friction

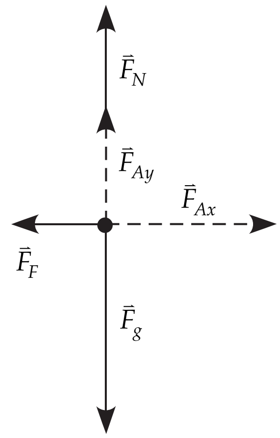
Assume that friction is now present in the previous case. The coefficient of kinetic friction is 0.300. The objective is now to determine the net force and the acceleration of the block. The new free-body diagram will show a force of friction to the left.



Again, you must consider the dynamics information in this situation, and analyze it to determine the force of friction and the net force. Again, using Newton's second law, you can determine the acceleration of the object.

The y-components of the forces do not change by adding the friction; however, the x-components of the forces do change.





In this case, the normal force is decreased by the vertical component of the applied force since the component is pulling the object up from the surface.

From the previous example, you know that:

Force of gravity = $\vec{F}_g = m\vec{g} = 98.0\ \mathrm{N}\ [\text{down}]$

Normal force = $\vec{F}_N = 78.0\ \mathrm{N}\ [\text{up}]$

Horizontal component of applied force =

$$\vec{F}_{Ax} = \vec{F}_A(\cos\theta) = (40.0\ \mathrm{N})(\cos 30.0°) = 34.6\ \mathrm{N}\ [\text{right}]$$

Now you are also given the coefficient of kinetic friction: $\mu_k = 0.300$.

The sum of the x-components of the forces is equal to the horizontal component of the applied force plus the force of friction.

A force of friction is found using $F_K = \mu_k F_N$.

$$F_K = \mu_k F_N = (0.300)(78.0\ \mathrm{N}) = 23.4\ \mathrm{N}\ [\text{left}]$$

Then, the net force is

$$\vec{F}_{\text{net}} = \vec{F}_{Ax} + \vec{F}_K$$
$$\vec{F}_{\text{net}} = 34.6\ \mathrm{N}\ [\text{right}] + 23.4\ \mathrm{N}\ [\text{left}] = 11.2\ \mathrm{N}\ [\text{right}]$$

Finally, using Newton's second law,

$$\vec{F}_{\text{net}} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{11.2\ \mathrm{N}\ [\text{right}]}{10.0\ \mathrm{kg}} = 1.12\ \mathrm{m/s^2}\ [\text{right}]$$

The acceleration of the object when friction acts is $1.12\ \mathrm{m/s^2}$ [right].

Of course, the acceleration is smaller than it was in the previous example.

Learning Activity 2.8

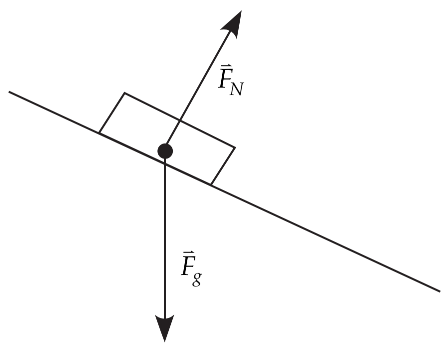
Motion of Objects Where Forces Act at an Angle

Solve the following problem to check your understanding of forces acting at an angle from the horizontal while pulling an object along a horizontal surface. An answer key is provided at the end of Module 2 for you to check your work.

1. A lawnmower of mass 18.0 kg is pushed by a horizontal force of 50.0 N acting 35.0° below the horizontal. The lawnmower moves with a constant velocity.
   a) What is the acceleration of the lawnmower?
   b) What is the net force acting on the lawnmower?
   c) What is the force of kinetic friction acting on the lawnmower?
   d) What is the normal force acting on the lawnmower?
   e) What is the coefficient of kinetic friction?

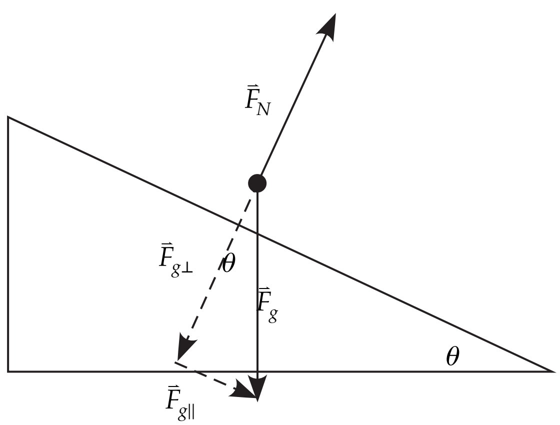
Example 3: Inclines: No Friction

In this section, we will examine an object sliding down a frictionless incline. Consider an incline as shown below, with an object sliding down the incline. Once again, the force vectors are drawn from the centre of the rectangle.



Notice that the force of gravity continues to act down to the centre of the Earth. Notice also that the normal force acts perpendicular to the surface. But since the surface is at an angle, the normal force and the force of gravity are not along the same line.

In this situation, it is convenient to set the coordinates parallel to the surface of the incline and perpendicular to the surface. It is also convenient to set the parallel direction down and to the right as the positive x-direction and the perpendicular direction, the direction of the normal vector to be the positive y-direction. In doing so, the force of gravity vector is broken into its components, as shown below.

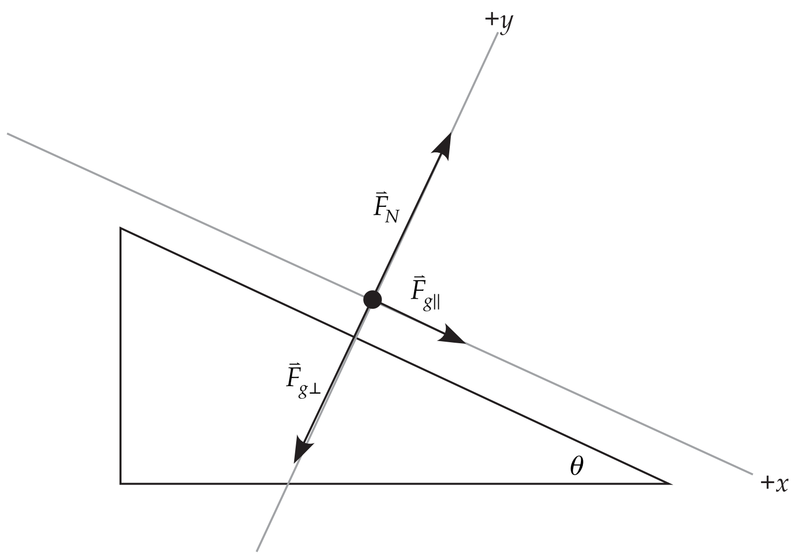


The components of $\vec{F}_g$ form a right triangle enclosing the angle, θ. This angle θ is the same as the angle θ in the ramp. The components of $\vec{F}_g$ are found by:

$$\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right)$$

$$\vec{F}_{g\parallel} = \sin\theta\left(\vec{F}_g\right)$$

Now you will have vectors only pointing along the major axes. Once again, your free-body diagram will have the shape of a cross. Remember, in a free-body diagram, your goal is always to have the forces form a cross!



If the angle of the slope is 25.0° and the mass is 50.0 kg, we can determine the magnitude of various forces.

The normal force is not equal to the gravitational force because these two forces do not point along the same line. But the perpendicular (*y*) component of the gravitational force is equal to the normal force, and the sum of these two forces is zero.

$\vec{F}_N + \vec{F}_{g\perp} = 0 \text{ N}$

$\vec{F}_N \left[\text{out of incline}\right] + \vec{F}_{g\perp}\left[\text{into incline}\right] = 0 \text{ N}$

Out of Incline = + direction

Into incline = − direction

$+\vec{F}_N - \vec{F}_{g\perp} = 0 \text{ N}$

So $\vec{F}_N = +\vec{F}_{g\perp} = m\vec{g}(\cos\theta)$

$= (50.0 \text{ kg})(9.80 \text{ m/s}^2) \cos 25.0°$

$= 444 \text{ N } \left[+\ y\text{-direction}\right]$

The force down the slope is just the *x*-component of the force of gravity. Since the incline is frictionless, it is also the net force causing the object to accelerate down the slope.

$\vec{F}_{\text{net}} = \vec{F}_{g\parallel} = m\vec{g}\sin\theta$

$= (50.0 \text{ kg})(9.80 \text{ m/s}^2)(\sin 25.0°)$

$= 207 \text{ N } [+\ x\text{-direction}]$

Using Newton's second law, you can find acceleration.

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{+207\ \text{N}}{50.0\ \text{kg}} = +4.14\ \text{m/s}^2$$

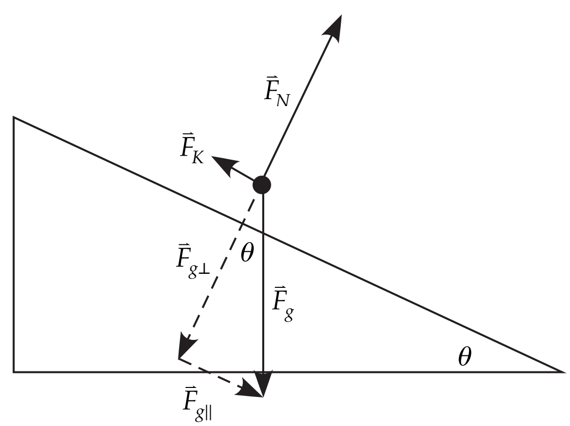
The acceleration of the object is 4.14 m/s^2 along the surface of the object.

Example 4: Inclines: With Friction

We will now examine the incline in the previous question, but with a coefficient of kinetic friction of 0.300.

Analyze the dynamics information first.

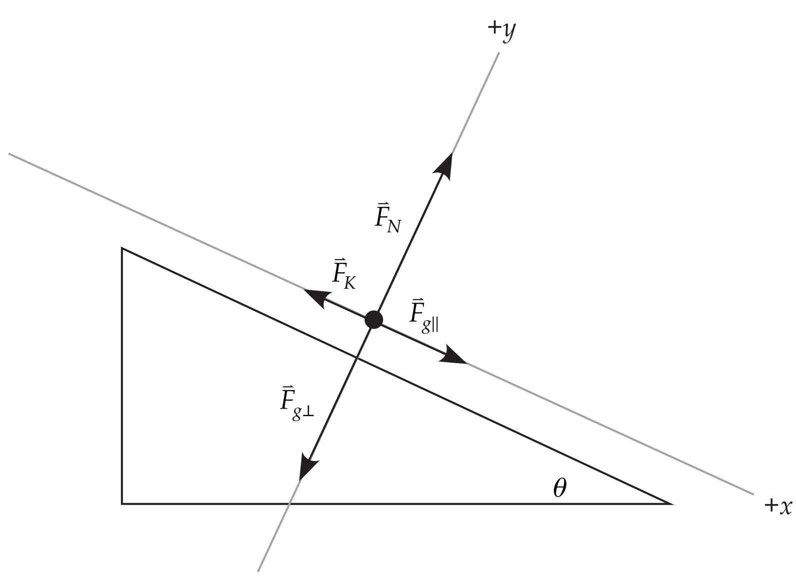
Since the object is moving down the incline, the force of friction will be up the incline.



The components of $\vec{F}_g$ form a right triangle enclosing the angle, θ. This angle θ is the same as the angle θ in the ramp. The components of $\vec{F}_g$ are found by:

$$\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right)$$

$$\vec{F}_{g\parallel} = \sin\theta\left(\vec{F}_g\right)$$



Here, the components of the force of gravity are:

$$\vec{F}_{g\perp} = \cos\theta \left(\vec{F}_g\right) = \left(\cos 25.0°\right)\left(m\vec{g}\right)$$
$$\vec{F}_{g\perp} = \left(\cos 25.0°\right)\left(50.0 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)$$
$$\vec{F}_{g\perp} = 444 \text{ N} \left[-y\text{-direction}\right]$$

$$\vec{F}_{g\|} = \sin\theta \left(\vec{F}_g\right)$$
$$\vec{F}_{g\|} = \left(\sin 25.0°\right)\left(50.0 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)$$
$$\vec{F}_{g\|} = 207 \text{ N} \left[+\, x\text{-direction}\right]$$

In adding friction, the components of the forces in the y-direction have not changed. The components of the forces in the x-direction have changed.

Along the x-direction, the parallel component of the force of gravity will pull the object down the incline. The force of kinetic friction will pull the object back up the incline.

The sum of the forces along the x-direction is equal to the net force, and it is this force that causes the object to accelerate down the incline.

$$\vec{F}_{\text{net}} = \vec{F}_{g\|} \text{ [+ } x\text{-direction] } + \vec{F}_K \text{ [} - x\text{-direction]}$$

In this example, the normal force is $\vec{F}_N = -\vec{F}_{g\perp} = 444 \text{ N}\left[\text{up}\right]$.

Using $F_K = \mu_k F_N$, you can calculate the force of kinetic friction.

In the coefficient of kinetic friction, $\mu_k = 0.300$.

$$F_K = \mu_k F_N$$
$$F_K = (0.300)(444 \text{ N}) = 133 \text{ N}$$

The force of kinetic friction is 133 N in the negative x-direction.

The net force is then

$$\vec{F}_{\text{net}} = \vec{F}_{g\|} + \vec{F}_K$$
$$\vec{F}_{\text{net}} = \left(+207 \text{ N}\right) + \left(-133 \text{ N}\right) = +74 \text{ N}$$

You can calculate the acceleration of the object using Newton's second law.

$$\vec{F}_{\text{net}} = m\vec{a}$$
$$\vec{a} = \frac{+74 \text{ N}}{50.0 \text{ kg}} = +1.48 \text{ m/s}^2$$

The object accelerates at $+1.5 \text{ m/s}^2$.

Analyzing kinematics information: You could extend the problem by applying various equations for constant acceleration as the object slides down the plane.

For example, if the object started from rest and you wanted to determine how far down the plane the object would move in 5.00 s, you could use kinematics equation #4.

Given:	Initial velocity	$\vec{v}_1 = 0 \text{ m/s}$
	Acceleration	$\vec{a} = +1.5 \text{ m/s}^2$
	Time interval	$\Delta t = 5.00 \text{ s}$
Unknown:	Displacement	$\vec{d} = ?$
Equation:		$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$
Substitute and solve:		$\vec{d} = (0 \text{ m/s})(5.00 \text{ s}) + \frac{1}{2}(+1.5 \text{ m/s}^2)(5.00)^2$
		$\vec{d} = +18.75 \text{ m}$

The displacement down the ramp is 19 m.

If you wanted to determine the new speed of the object after this time, you would do so by using

$$\vec{v}_2 = \vec{v}_1 + \vec{a}t$$

$$\vec{v}_2 = (0.00 \text{ m/s}) + (1.5 \text{ m/s}^2)(5.00 \text{ s}) = 7.5 \text{ m/s}$$

The velocity of the object after sliding for five seconds would be 7.5 m/s down the incline.

Learning Activity 2.9

Dynamics of Forces Acting at an Angle to the Motion

The practice questions in this learning activity deal with the dynamics of forces acting at an angle to the motion. An answer key is available at the end of Module 2 for you to check your work after you have answered the questions.

The physics of pushing or pulling a sled at an angle (conceptual)

1. You have the choice of either pushing or pulling a sled at a constant velocity. You can either push a sled at some angle to the horizontal, or pull the sled at the same angle to the horizontal.

   Friction is present in both situations. Will it require less force to push the sled or to pull the sled?

The physics of pushing a snow shovel across a driveway

2. A homeowner pushes a snow shovel at a uniform velocity across a driveway. The handle of the snow shovel is inclined at an angle of θ to the driveway and force $\vec{F}_A$ is applied to the handle.

   Draw a free-body diagram and write two sets of equations—one for the vertical part of the motion and one for the horizontal part. Do not solve the equations for any variables. Write your equations in terms of $\vec{F}_A$, $\vec{F}_N$, $\vec{F}_g$, $\vec{F}_F$, μ, and θ.

The physics of dragging a box at an angle

3. A 20.0 kg box is dragged along a horizontal floor with a force of 100.0 N. The force is applied at an angle of 40.0° above the horizontal and to the right. If the coefficient of kinetic friction is 0.300, what is the net force on the box, and what is the acceleration of the box?

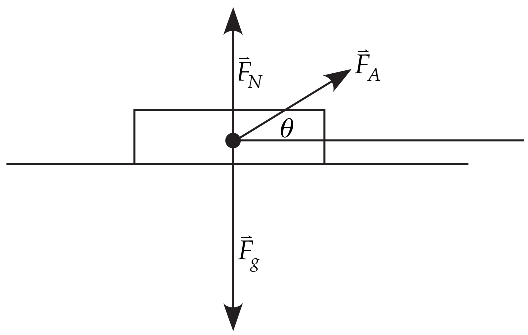
The physics of a sliding block of ice

4. A 8.00 kg block of ice slides down a ramp 15.0 m long. The ramp is inclined at 12.0° to the horizontal. If the coefficient of kinetic friction is 0.100, how long will it take the ice to reach the bottom of the ramp if it starts from rest?

Lesson Summary

In this lesson, you learned how to analyze problems involving forces at an angle.

Note: In all cases, include the directions with the force so as to determine the correct direction of the resultant force.

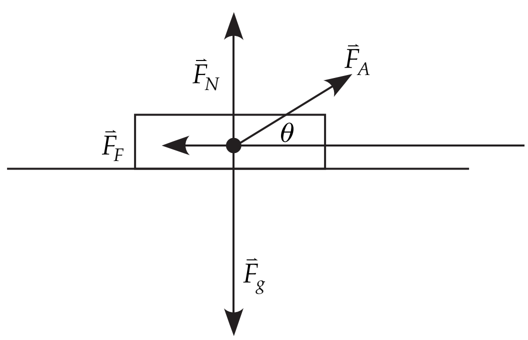


In the case of an object being pulled at an angle that is up and to the right on a frictionless surface, the net force in the y-direction is zero:

$$\vec{F}_N + \vec{F}_A \sin\theta + \vec{F}_g = 0.$$

In the x-direction, there is only one force, the x-component of the applied force:

$$\vec{F}_{Ax} = \vec{F}_A \cos\theta .$$



If friction is present, then the y-components of the force do not change.

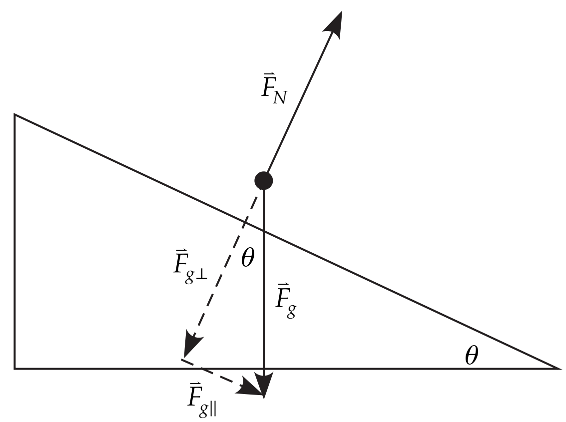
In the x-direction, the x-component of the applied force and the force of friction are acting. The net force is given by the equation becomes

$$\vec{F}_{\text{net}} = \vec{F}_{Ax} + \vec{F}_F$$

For an object on a frictionless incline, it is convenient to set the coordinates **parallel to the surface of the incline** and **perpendicular to the surface**. It is also convenient to set the direction **down and to the right** as the **positive *x*-direction** and the direction of the **normal vector** to be the **positive *y*-direction**. The equations become

$$\vec{F}_N - \vec{F}_{gy} = 0 \text{ and } \vec{F}_{gx} = m\vec{a}$$

For objects on an inclined plane, as in the diagram below, you must resolve the force of gravity into its components: one parallel to the plane and the second perpendicular to the plane. It is usually a good idea to calculate these two components of the force of gravity at the start of the question, and to use their values as required.



The components of $\vec{F}_g$ form a right triangle enclosing the angle, θ. This angle θ is the same as the angle θ in the ramp. The components of $\vec{F}_g$ are found by:

$$\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right)$$

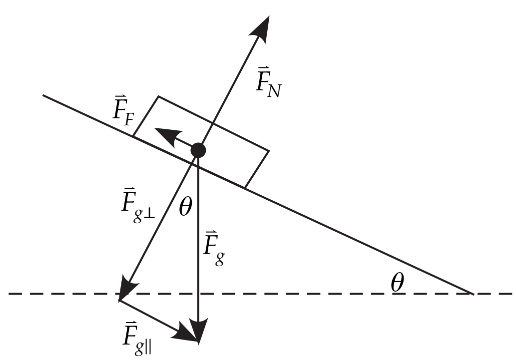
$$\vec{F}_{g\|} = \sin\theta\left(\vec{F}_g\right)$$

For this situation, the net force will just be a component of the force of gravity parallel to the inclined plane.

$$\vec{F}_{\text{net}} = \vec{F}_{g\|} = \vec{F}_g\left(\sin\theta\right)$$

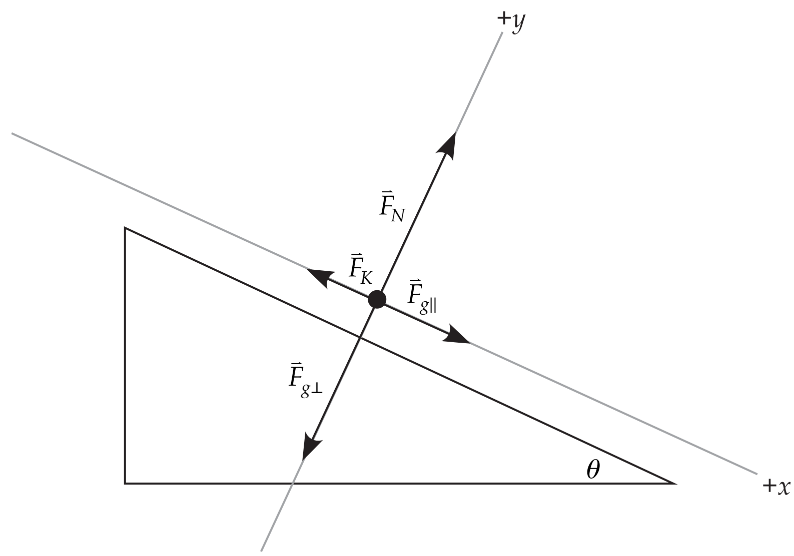
In the y-direction, the normal force balances the component of the force of gravity perpendicular to the plane.

$$\vec{F}_N = -\vec{F}_{g\perp} = -\vec{F}_g\left(\cos\theta\right)$$



For an object sliding down an incline with friction, the y-components of the forces remain the same, and in the x-direction the equation becomes $\vec{F}_{\text{net}} = \vec{F}_{g\|} + \vec{F}_F$.

The free-body diagram is drawn below.



NOTES

Assignment 2.5

Forces Acting at an Angle (8 MARKS)

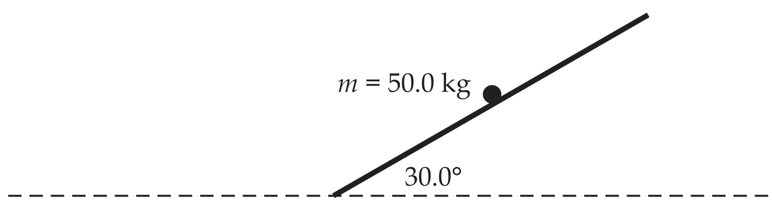
The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answer. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of tobogganing and forces at an angle

A child is tobogganing down a hillside. The child and the toboggan together have a mass of 50.0 kg. The slope is at an angle of 30.0° to the horizontal.

Assume that the positive *y*-direction is pointing in the direction of the normal force. Assume that the positive *x*-direction is down the incline.

Be sure to draw the appropriate free-body diagrams and demonstrate completely your solution to the problems.



Find the acceleration of the child

a) in the case where there is no friction.

(continued)

b) if the coefficient of friction is 0.150.

Method of Assessment

The total of eight marks for this assignment will be determined as follows:

- 1 mark for drawing the normal force and the force of gravity in the correct directions in part (a)
- 1 mark for resolving the gravity force the correct way in part (a)
- 1 mark for determining the value of the acceleration down the plane in part (a)
- 1 mark for determining the value of the normal force in part (b)
- 1 mark for determining the value of the parallel component of the force of gravity in part (b)
- 1 mark for adding the friction vector in the correct direction and for determining its value in part (b)
- 1 mark for writing the correct equation for determining the net force in part (b)
- 1 mark for determining the value of the acceleration in part (b)

On the next page, change the parameter for objects on an incline and observe the effects on components, friction, etc.

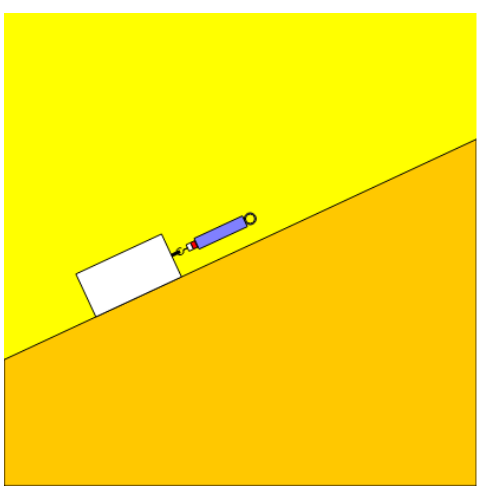
https://www.walter-fendt.de/html5/phen/index.html

https://www.walter-fendt.de/html5/phen/inclinedplane_en.htm

Inclined Plane

This HTML5 app demonstrates a motion on an inclined plane with constant velocity and the corresponding forces.

The "Reset" button brings the block to its initial position (outside of the picture). You can start or stop and continue the simulation with the other two buttons. Depending on the selected radio button the app will show a springscale from which you can read the necessary force, or the vectors of the weight force with its two components (parallel and normal to the plane), the normal force, the frictional force and the force which is necessary for the motion.



The angle of inclination, the weight of the block and the coefficient of friction can be changed within certain limits. The app will calculate the magnitudes of the mentioned forces.

Video - Using Newton's Second Law to find the Force of Friction

This video solves for the force of friction acting on a street hockey puck. A video of the sliding puck is analyzed to provided a velocity as a function of time graph with a line of best fit. the given quantities are identified, a free body diagram is drawn, the net force is found and the force of friction is determined. Some common errors crop up in solution to this problem.

https://youtu.be/IHILOnEW5Qg

Video - Breaking the Force of Gravity into its Components on an Incline

A book is resting on an inclined plane. A free body diagram is drawn showing the force of gravity, the normal force and the force of friction. Since the force of gravity is not along the direction parallel to the surface of the incline or perpendicular to the surface of the incline the force of gravity must be resolved into its components. Note that the angle between the force of gravity and the direction perpendicular to the surface of the incline is the same as the angle between the horizontal and the surface of the incline. Once the components are found the free body diagram is redrawn so that it has the shape of a cross.

https://youtu.be/B8x7-RjbtMY

Video - Introductory Static Friction on an Incline Problem

A book is resting on horizontal board. When one end of the board is raised, the book begins to slide when the board makes an angle of 15° with the horizontal direction. The analysis of this situation leads to the calculation of the coefficient of static friction.

https://youtu.be/vWzjhKifwcU

Video - High School Physics - Ramps and Inclines

The Video page layout includes an embedded YouTube video, which can scale in any browser or mobile device. Embedding videos in the page is a great way to present video content, while accompanying it with supporting context, explanations and activities.

Swap out the content and replace the video on this page to create your own page. Instructions for replacing videos are provided below.

https://youtu.be/j8R9PS_sVP8

Assignment 2.5

Forces Acting at an Angle (8 MARKS)

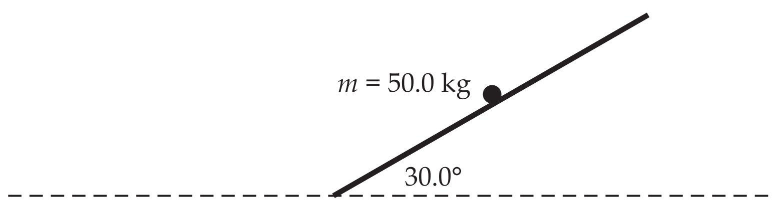
The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answer. Submit this assignment, along with all the other assignments from Modules 1 and 2, after you have completed Module 2.

The physics of tobogganing and forces at an angle

A child is tobogganing down a hillside. The child and the toboggan together have a mass of 50.0 kg. The slope is at an angle of 30.0° to the horizontal.

Assume that the positive *y*-direction is pointing in the direction of the normal force. Assume that the positive *x*-direction is down the incline.

Be sure to draw the appropriate free-body diagrams and demonstrate completely your solution to the problems.



Find the acceleration of the child

a) in the case where there is no friction.

(continued)

Assignment 2.5: Forces Acting at an Angle (continued)

b) if the coefficient of friction is 0.150.

Method of Assessment

The total of eight marks for this assignment will be determined as follows:

- 1 mark for drawing the normal force and the force of gravity in the correct directions in part (a)
- 1 mark for resolving the gravity force the correct way in part (a)
- 1 mark for determining the value of the acceleration down the plane in part (a)
- 1 mark for determining the value of the normal force in part (b)
- 1 mark for determining the value of the parallel component of the force of gravity in part (b)
- 1 mark for adding the friction vector in the correct direction and for determining its value in part (b)
- 1 mark for writing the correct equation for determining the net force in part (b)
- 1 mark for determining the value of the acceleration in part (b)

LESSON 9: NEWTON'S THIRD LAW OF MOTION

(1 HOUR)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ analyze situations that involve action-reaction pairs of forces
- ❑ explain why an action and reaction pair of forces are not balanced

Key Words

Newton's third law
action force
reaction force

Introduction

The third and final law of motion postulated by Newton deals with action forces and reaction forces. This law is often misunderstood. The key to applying this law correctly is to remember that the action force acts on one object while the reaction force acts on a different object.

Newton's Third Law

Newton's Third Law of Motion:
When one body exerts a force on another, the second body exerts on the first a force of equal magnitude in the opposite direction.

These forces are often referred to as "action" and "reaction" forces. Here, the term "action" force means the force that one body exerts on a second body, while "reaction" force means the force that the second body exerts on the first. There can be no force unless there are two bodies involved.

Remember that action and reaction forces, though equal in magnitude and opposite in direction, can never neutralize each other, for they always act on different objects. In order for two forces to neutralize each other, they must act on the same object.

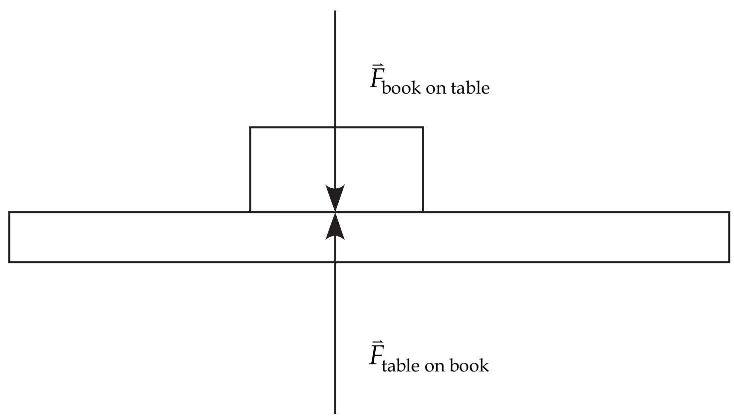
In the example of the book resting on a table from previous lessons, you will recall that the force of gravity on the book pulls the book downwards. The book exerted a force on the table that was equivalent to the force of gravity pulling down on the book.

At the same time, the table pushing up on the book is a force that is called the normal force. As hard as the book is pushing on the table (action force), the table is pushing back upwards on the book (reaction force).

In this case, the two forces are equal in magnitude but opposite in direction. In this case, the two forces also act on different objects.

$$\vec{F}_{\text{book on table}} = \vec{F}_{\text{table on book}}$$

Because the forces act on different objects, they do not cancel each other out.



Action-reaction pairs of forces are all around us!

When a baseball bat strikes a ball, it exerts a force on the ball while the two are in contact. During the same time, the ball exerts a force of the same magnitude but opposite in direction on the bat.

A freely falling body is accelerated by the net force (action force) with which Earth attracts the body. Earth, in turn, is accelerated by the opposite reaction force the body exerts on Earth. Because of the great mass of Earth, this acceleration is too small to be observed.

In throwing a light object, one has the feeling that he cannot put much effort into the throw, for he cannot exert any more force on the object than that object exerts in reaction against his hand. This reaction force is proportional to the mass of the object and to the acceleration of the object $(\vec{F} = m\vec{a})$. The thrower's arm must be accelerated along with the object thrown; hence, the larger part of the effort exerted in throwing a light object is expended in "throwing" one's arm.

When you step from a small boat to the shore, you observe that the boat is pushed away as you step. The force you exert on the boat is responsible for its motion. The force of reaction, exerted by the boat on you, is responsible for your motion toward the shore. The two forces are equal in magnitude and opposite in direction. The accelerations they produce (in boat and passenger, respectively) are inversely proportional to the masses of the objects on which they act $\left(\vec{a} = \frac{\vec{F}}{m}\right)$. Thus, a large boat will experience only a small acceleration when you step from it to shore; a small boat experiences a much greater acceleration.

A rocket ship, coasting in space, suddenly fires its rockets. The rockets shoot a blast of gas backward with a force of 1000.0 newtons (action force) and the gas exerts an equal force (–1000.0 newtons) forward on the rocket ship (reaction force). This forward force causes the rocket ship to move forward.

A gun fires a bullet forward with a force of 50.0 N (action force) and the bullet exerts an equal force (–50.0 N) backward on the gun (reaction force). This backward force is normally called the "kick" of the gun.

A skater standing on ice throws a large heavy ball with a force of 10.0 N (action force). The ball pushes back on the skater with a force of –10.0 N and the skater glides backwards.

A water skier pulls on a tow rope with a force of 1000 N (action force). The rope, in turn, pulls on the water skier with an equal but opposite force (reaction force) of –1000 N.

Example

A 92 kg astronaut, drifting just outside a 11 000 kg spacecraft, pushes on the craft with a force of 36 N.

a) What is the acceleration of the spacecraft?

The acceleration of the spacecraft is obtained easily from Newton's second law equation.

Given: Action force $\vec{F}_{\text{astronaut-spaceship}} = F_{\text{A-S}} = 36 \text{ N [right]}$

Mass of spacecraft $m_s = 11\ 000 \text{ kg}$

Unknown: Acceleration of spacecraft $\vec{a}_s = ?$

Equation: Since the only force being exerted on the spacecraft is given by the force of the astronaut on the spacecraft, the net force on the spacecraft is also 36 N [right].

Using Newton's second law, you get

$\vec{F}_{\text{net}} = m\vec{a}$.

Substitute and solve:

$$\vec{a}_s = \frac{\vec{F}_{\text{net}}}{m_s} = \frac{36 \text{ N [right]}}{11\ 000 \text{ kg}}$$
$$= 0.0033 \text{ m/s}^2 \text{ [right]}$$

As long as the astronaut pushes on the spacecraft, it will accelerate in a positive direction [right] at 0.0033 m/s^2.

b) What is the acceleration of the astronaut?

Since the spacecraft will exert a reaction force of –36 N (36 N [left]) on the astronaut, he will accelerate in a negative direction as long as the reaction force is applied to him. Again, you can use Newton's second law equation to find acceleration. In this case, you will determine the acceleration of the astronaut.

Given: Reaction force $\vec{F}_{\text{spaceship-astronaut}} = \vec{F}_{S-A} = 36 \text{ N [left]}$

Mass of astronaut $m_A = 92 \text{ kg}$

Unknown: Acceleration of astronaut $\vec{a}_A = ?$

Equation: Since the only a force being exerted on the astronaut is the force of the spacecraft, the net force on the astronaut is also 36 N [left].

Using Newton's second law, you get

$\vec{F}_{\text{net}} = m\vec{a}$.

Substitute and solve:

$$\vec{a}_A = \frac{\vec{F}_{\text{net}}}{m_S} = \frac{36 \text{ N [left]}}{92 \text{ kg}} = 0.39 \text{ m/s}^2 \text{ [left]}$$

As long as the astronaut pushes on the spacecraft, he will accelerate in a negative direction [left] at 0.39 m/s^2.

The astronaut, having a smaller mass, experiences a much larger acceleration than the spacecraft and accelerates in the opposite direction. Even though the two forces have the same magnitude, they produce different accelerations because they act on different objects that have different masses.

Learning Activity 2.10

Applying Newton's Three Laws of Motion

Answer the questions in Parts A to D of this learning activity to check your understanding of Newton's three laws of motion. An answer key is provided at the end of Module 2 for you to check your work.

Part A

In the space provided before each statement, indicate whether the statement is true or false by using T for true and F for false.

_____ 1. An electromagnetic force causes a negatively charged object to be attracted to a positively charged object.

_____ 2. The vector sum of all forces acting simultaneously on an object is known as the net force acting on the object.

_____ 3. The acceleration of an object is in inverse proportion to its mass.

_____ 4. When using Newton's second law equation, you must always be sure that mass is in newtons.

_____ 5. The terms "mass" and "weight" have the same meaning, and either can be used when you are referring to the force of gravity on an object.

_____ 6. It takes less force to start an object moving across a surface than it does to keep it moving at constant velocity.

_____ 7. Starting or sliding friction is directly proportional to the force pressing the two surfaces together.

_____ 8. The magnitudes of action and reaction forces are always the same.

_____ 9. The normal force is smaller than the force of gravity acting on an object if the applied force acts at an angle below the horizontal.

____ 10. The mass of an object is a quantitative measure of its inertia.

(continued)

Learning Activity 2.10: Applying Newton's Three Laws of Motion (continued)

Part B

Match each of the following statements in List I with a phrase from List II by placing the letter representing the phrase in the space provided.

List I

____ 11. The study of forces that cause motion or changes in motion.

____ 12. The measure of the amount of material in an object.

____ 13. The property of matter that opposes any change in its state of motion.

____ 14. If there is no net force acting on a body, it will continue in a state of rest or will continue moving along a straight line at a constant speed.

____ 15. The maximum frictional force between stationary objects.

____ 16. Represents an object and all the forces that act on it.

____ 17. When one body exerts a force on another, the second body exerts on the first a force of equal magnitude in the opposite direction.

____ 18. The ratio of the force of friction to the normal force pressing the surfaces together.

____ 19. The force that will give to a mass of 1 kg an acceleration of 1 m/s^2.

____ 20. The force that resists motion between objects that are in contact with each other.

List II

a) Definition of static friction

b) Definition of kinetic friction

c) Newton's First Law of Motion

d) Newton's Second Law of Motion

e) Newton's Third Law of Motion

f) Definition of friction

g) Definition of free-body diagram

h) Definition of matter

i) Definition of mass

j) Definition of coefficient of friction

k) Definition of dynamics of motion

l) Definition of a newton

m) Definition of inertia

n) Definition of force

o) Definition of gravity

(continued)

Learning Activity 2.10: Applying Newton's Three Laws of Motion (continued)

Part C

1. If "action forces" and "reaction forces" are equal and opposite, why can they never balance or cancel?
2. In a car crash, a passenger's head collides with the window in the door. The passenger suffers a fractured skull. The window in the door is shattered. Account for this using Newton's laws of motion.

Part D

Using the appropriate law of motion, explain the following:

1. A bullet is fired from a rifle. The rifle recoils.
2. A Judo expert attempts to break 10 boards piled on top of each other by striking them with one blow of his hand. In the process, he breaks a bone in his hand.
3. The stationary car in which a passenger sits is struck from the rear by a second car. The passenger suffers whiplash.
4. A boy throws an egg as hard as he can into a blanket held by two people. The egg is caught in the blanket without being broken.
5. A curling rock slides along the ice and slowly comes to a stop.
6. A rock is dropped from a bridge and accelerates towards Earth. The rock exerts a force on Earth. We do not notice Earth accelerating towards the rock.

Lesson Summary

Newton’s Third Law of Motion deals with action and reaction pairs of forces.

Here the term “action” force means the force that one body exerts on a second body, while “reaction” force means the force that the second body exerts on the first. There can be no force unless there are two bodies involved.

Newton’s Third Law of Motion: When one body exerts a force on another, the second body exerts on the first a force of equal magnitude in the opposite direction.

It is important to note now that a pair of action reaction forces cannot cancel out since they act on different bodies.

Video - High School Physics - Newton's 3rd Law

This is an introductory video on Newton’s Third Law. It provides many examples of Action-Reaction pairs of forces.

https://youtu.be/Q-ASVFCtvBU

Video - Introduction to Newton’s Third Law of Motion

This is another video introducing Newton’s Third Law. It includes several examples of Action-Reaction pairs of forces.

https://youtu.be/bAevUhFUhV4

Video - A Common Misconception about Newton's Third Law Force Pairs (or Action-Reaction Pairs)

This video addresses a common misconception when identifying action-reaction pairs of forces. Many students will identify pairs of forces that act on the **same** object as action-reaction force pairs.

Action-reaction pairs of forces must act on **different** objects.

Action Force	Reaction Force
1-2	2-1

The action force acts on object 2. The reaction force acts on object 1.

https://youtu.be/wmmjfbl7zG4

Module 2 Summary

Congratulations! You have finished the second module of Grade 12 Physics. Just as kinematics is important in comprehending physics, so too is dynamics. While kinematics described how objects move, dynamics reveals why objects move. As we continue our study of physics, we will continually consider the kinematics and the dynamics of a given situation so that we can arrive at a complete understanding of the phenomenon.

Submitting Your Assignments

It is now time for you to submit Assignments 1.1 and 1.2 from Module 1 and Assignments 2.1 to 2.5 from Module 2 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 1 and Module 2 assignments and organize your material in the following order:

- Modules 1 and 2 Cover Sheet (found at the end of the course Introduction)
- Assignment 1.1: Equations of Motion
- Assignment 1.2: Relative Motion
- Assignment 2.1: Forces of Friction and Motion
- Assignment 2.2: Adding Vectors Using the Component Method
- Assignment 2.3: Video Laboratory Activity: Forces in Equilibrium
- Assignment 2.4: Objects in Equilibrium
- Assignment 2.5: Forces Acting at an Angle

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

NOTES

GRADE 12 PHYSICS (40S)

Module 3: Projectiles and Circular Motion

This module contains the following:

- Introduction to Module 3
- Lesson 1: Using Equations for Constant Acceleration: Freely Falling Bodies
- Lesson 2: Projectile Motion in Two Dimensions: Vector Diagrams
- Lesson 3: Projectile Motion in Two Dimensions: Using the Equations
- Lesson 4: Uniform Circular Motion: Speed and Velocity
- Lesson 5: Video Laboratory Activity: Circular Motion
- Lesson 6: Centripetal Acceleration
- Lesson 7: Centripetal Force
- Module 3 Summary

Module 3: Projectiles and Circular Motion

Introduction to Module 3

In kinematics, you dealt with motion along the straight line—that is, motion in one dimension. In dynamics, you studied forces, which are the causes of motion. In this module, we will extend these ideas about motion and forces into two dimensions. We will consider the curved motion of an object as it flies through the air near Earth's surface, and the motion of objects that travel in circles.

In the first part of this module, we deal with a special case of motion in both one dimension and two dimensions. That motion is **projectile motion**. We begin with projectile motion in one dimension, and then discuss motion in two dimensions. It was probably Galileo who was the first to understand that two-dimensional projectile motion could be understood by considering the horizontal component and the vertical component of the motions separately.

This module consists of seven lessons. Projectile motion will be dealt with in the first three lessons.

Lesson 1: Using Equations for Constant Acceleration: Freely Falling Bodies discusses projectile motion in one dimension. That is a projectile motion only up and down and not side to side.

Lesson 2: Projectile Motion in Two Dimensions: Vector Diagrams considers two-dimensional projectile motion mainly from a non-mathematical point of view.

Lesson 3: Projectile Motion in Two Dimensions: Using the Equations adds the equations for motion at constant velocity (horizontal motion) and constant acceleration (vertical motion).

In the second part of this module, we examine a special case of motion: **circular motion**. We have already seen some motion along a curved path from our study of projectiles. There are many examples of circular motion, ranging from planets and galaxies moving around a central body to toys where an object may be twirled around at the end of a string.

Lesson 4: Uniform Circular Motion: Speed and Velocity is an introduction to some basic ideas in circular motion, such as period, frequency, speed, and velocity.

Lesson 5: Video Laboratory Activity: Circular Motion relates the kinematics of circular motion (period, frequency, speed, acceleration) to the dynamics of circular motion (centripetal force).

Lesson 6: Centripetal Acceleration examines centripetal acceleration in detail.

Lesson 7: Centripetal Force examines centripetal force, and comments on the often misunderstood, fictitious force called centrifugal force.

Assignments in Module 3

When you complete Module 4, you will submit your Module 3 assignments, along with your Module 4 assignments, to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 3.1	Vertical Motion of a Bullet
2	Assignment 3.2	Vector Nature of Projectile Motion
3	Assignment 3.3	Projectile Motion of a Cannonball
5	Assignment 3.4	Video Laboratory Activity: Circular Motion
6	Assignment 3.5	Circular Motion of the Moon
7	Assignment 3.6	Uniform Circular Motion of a Satellite

As you work through this course, remember that your learning partner and your tutor/marker are available to help you if you have questions or need assistance with any aspect of the course.

Lesson 1: Using Equations for Constant Acceleration: Freely Falling Bodies (2 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ explain what is meant by the term "free fall"
- ❑ apply the idea of symmetry in analyzing the motion of freely falling bodies
- ❑ apply the problem-solving strategy to the solution of problems involving uniformly accelerated motion

Key Words

free fall
air friction

Introduction

In the unit on kinematics, we developed equations for uniformly accelerated motion. These equations can be applied to the case of falling objects. We commonly observe this effect any time we throw an object into the air and observe it coming back down. In this lesson, we will examine only motion in an up and down sense. In Module 1, Lesson 3 we had considered one example of this. We will revisit this motion adding in the dynamics that apply in this situation. We will study the case of two-dimensional motion in the next lesson.

Free Fall

It was Galileo Galilei (1564–1642) who made important discoveries about falling bodies. Galileo discovered that all bodies fall with the same constant acceleration here at Earth's surface. It does not matter if the object is a large rock or a small rock. Both fall with the same acceleration due to Earth's gravity. This acceleration is given the special symbol $\vec{g}$ and has the value of

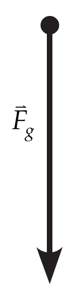
$$\vec{g} = 9.80 \text{ m/s}^2$$

An interesting consequence of this is that if air resistance could be eliminated, then a feather and a rock would fall at exactly the same rate of acceleration. You do not see this in normal circumstances because the force of air friction slows down the feather much more than the rock.

Since you probably don't have a rock and a feather but you probably do have a book and a sheet of paper, you can try this by dropping a book and a sheet of paper side by side from the same height. The book probably had a greater acceleration than the sheet of paper.

If you could eliminate or minimize the force of air friction acting on the sheet of paper, then the book and the paper should fall together, accelerating at the same rate. To minimize the force of friction of the air acting on the paper, crumple a sheet of paper into a tight ball. If you release the book and a crumpled piece of paper at the same time from the same height, they should fall together, accelerating at the same rate.

You can approach this idea from Newton's second law of motion as well. Here is the free-body diagram of an object that is released and falling towards Earth. Let's assume that the force of air friction is zero.



Recall Newton's second law of motion:

$$\vec{F}_{\text{net}} = m\vec{a}$$

Also recall the force of gravity is given by:

$$\vec{F}_g = m\vec{g}$$

Since the force of gravity is the only force acting on the object, then the net force is equal to the force of gravity. So

$$\vec{F}_{\text{net}} = \vec{F}_g \text{ and}$$

$$m\vec{a} = m\vec{g}.$$

The mass, m, cancels, giving

$$\vec{a} = \vec{g}.$$

In the idealized case where air resistance is neglected (such as in a vacuum chamber), you can assume that all objects at the same location on Earth fall with the acceleration due to gravity. This idealized situation is called **free fall**.

> **Free fall** describes the situation in which the only force acting on the falling object is the force of gravity.
>
> For freely falling objects use the acceleration as the acceleration of gravity.
>
> $$\vec{a} = \vec{g} = 9.80 \text{ m/s}^2 \text{ [down]}$$

However, if you could get to a place with little air resistance (for instance, the surface of the Moon), we could attempt to demonstrate that two very different objects—say, a feather and a rock hammer—have similar behaviour when they fall freely under the influence of a gravity field.

In 1972, that's just what was done by Commander David Scott, one of the two Apollo 15 astronauts who were on the surface of the Moon that year. He dropped a feather and a hammer at the same time. They both hit the ground together. Here is the URL of the video clip if you are interested in viewing the event as it was broadcast on television (in mpeg format): http://www.hq.nasa.gov/office/pao/History/alsj/a15/a15v_1672206.mpg

Kinematics Equations for Uniformly Accelerated Motion

In the kinematics unit, four equations for uniformly accelerated motion were developed. They were

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \text{ or } \vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

$$\vec{d} = \frac{(\vec{v}_1 + \vec{v}_2)}{2}\Delta t$$

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$v_2{}^2 = v_1{}^2 + 2ad$$

These same equations can be applied for freely falling bodies. At times, it may be more convenient to use the letter "y" instead of "d" or "x." This is because for these kinds of problems, the motion occurs in the vertical or "y" direction. This change does not affect the use of the equations.

In the examples that follow, the same problem-solving strategy (GUESS method) for kinematics problems will be applied.

Symmetry in the Motion of Freely Falling Bodies

In doing problems involving the motion of freely falling bodies, it is useful to remember some general ideas that may apply to some problems. These ideas revolve around the idea of symmetry.

Imagine tossing a coin into the air, and then the coin falling back down to the same level it was tossed from. A number of observations related to symmetry should be apparent. Remember that you are assuming no air friction and no outside forces other than gravity acting on the falling object.

1. The time it takes the coin to reach the top of the motion is equal to the amount of time it takes the coin to fall back to the point of its release.

   $\Delta t_{\text{up}} = \Delta t_{\text{down}}$

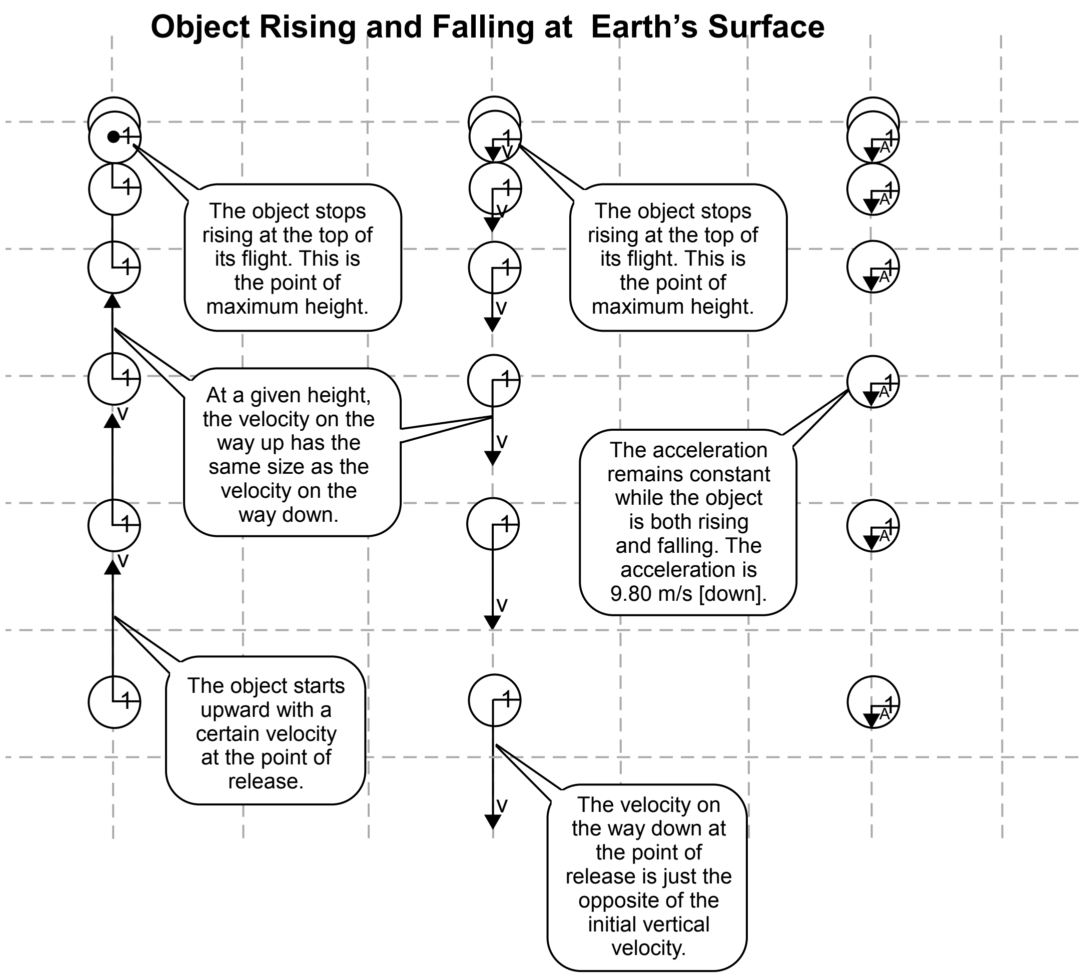
2. As the coin moves upwards, the speed of the coin at some displacement y above the point of release is the same as the speed of the coin moving downwards at the same displacement y above the point of release. The velocities will be opposite in direction, but the magnitude of the velocities (speed) will be the same.

3. The initial speed of the coin at the point of release is the same as the final speed of the coin when it reaches the point of release (remember, the directions are different).

4. The displacement of the coin up is the same as the displacement of the coin down, assuming that both are measured from the point of release.

5. The speed of the coin at the top of its motion is momentarily zero. This can be considered to be the second velocity, $\vec{v}_2$. In the second half of the motion, as the coin begins to fall, the speed at the top is still zero but this velocity is now the initial velocity, $\vec{v}_1$, for this part of the motion.

The following graphic illustrates some of these points of symmetry for objects moving vertically at Earth's surface.



Example 1: A Falling Marble

You will examine motion in the downward direction only. Imagine a marble dropped from the top of a tall building. What is the displacement of the marble after 3.00 s, and how fast is the marble moving at this point?

Given: Make a drawing.

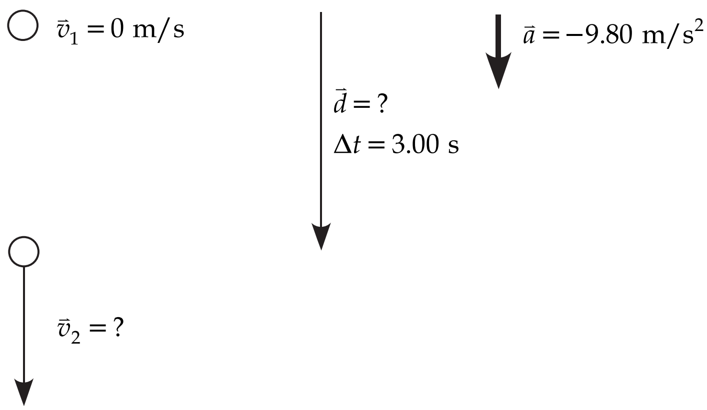
The drawing shows the vectors for the second velocity, the displacement, and the acceleration.

Decide which direction is positive and which is negative.

It is convenient to let the upward direction be positive and the downward direction be negative.

Write down in symbolic form what you are given and what you need to find.

The symbols for displacement, velocity, acceleration, and time are shown beside the diagram. Notice that the acceleration due to gravity is –9.80 m/s^2.



Unknown: Displacement $\vec{d} = ?$

Equation: Select the appropriate equation.

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Substitute:

$$\vec{d} = (0 \text{ m/s})(3.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(3.00 \text{ s})^2$$

Solve:

$$\vec{d} = -44.1 \text{ m}$$

Note that the answer for displacement is negative because the marble is falling downwards.

To determine the second velocity, you can use $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$.

$\vec{v}_2 = (0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(3.00 \text{ s}) = -29.4 \text{ m/s}$

The final velocity of the marble is –29.4 m/s or 29.4 m/s [down].

Example 2: A Rising Baseball

In this example, imagine throwing a baseball straight up at a speed of 8.00 m/s. Determine the maximum height to which the baseball rises, and the time it takes for it to reach the top of the motion.

Given: Make a drawing.

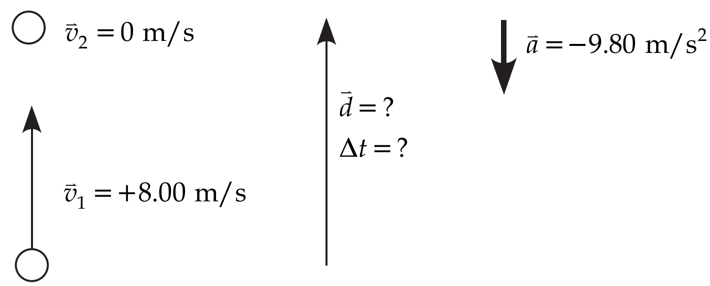
The drawing shows the vectors for the initial and final velocity, the displacement, and the acceleration for the baseball on its way upwards.

Decide which direction is positive and which is negative.

It is convenient to let the upward direction be positive and the downward direction be negative.

Write down in symbolic form what you are given.

The symbols for displacement, velocity, acceleration, and time are shown beside the diagram. Notice that the acceleration due to gravity remains as –9.80 m/s^2.



Unknown: Maximum height $\vec{d} = ?$

Equation: The appropriate equation would be $v_2^2 = v_1^2 + 2ad$.

Solving for the displacement, $d = \dfrac{v_2^2 - v_1^2}{2a}$.

Substitute and solve:

$$d = \frac{(0 \text{ m/s})^2 - (8.00 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +3.26 \text{ m}$$

The maximum height will be 3.26 m above the point of release.

The time required to reach the top of the motion can be determined using $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$.

Solving for the time, $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$.

$$\Delta t = \frac{0 \text{ m/s} - 8.00 \text{ m/s}}{-9.80 \text{ m/s}^2} = 0.816 \text{ s}$$

Note that the baseball would return to the height from which it was released at a time of 2 x Δt = 1.63 s after it was released with a velocity of 8.00 m/s [down]. (Remember the symmetry of this motion!)

Example 3: Dropping an Object from an Ascending Balloon

A balloon is ascending at a rate of 8.00 m/s. It reaches a height of 90.0 m above the ground when it releases a bag of sand. Assume that the acceleration due to gravity is 9.80 m/s^2 downwards. Determine the amount of time it takes the package to reach the top of its motion, and then the time it takes to fall after it reaches the top.

You will do this question in two parts.

First, you will determine the time it takes to reach the top of its motion.

Part I

Given: Make a drawing.

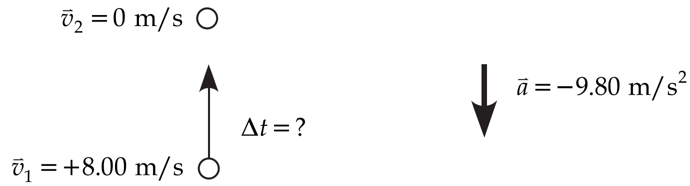
The drawing shows a vector pointing upwards representing the initial velocity.

Decide which direction is positive and which is negative.

It is convenient to let the upward direction be positive and the downward direction be negative.

Write down in symbolic form what you are given and what you need to find.

The symbols for velocity, acceleration, and time are shown beside the diagram. Notice that the acceleration due to gravity is down and is given a negative sign. Notice also that $\vec{v}_2$ has been assigned the value of zero even though this was not explicitly stated in the question. You know that the velocity of an object is zero at the top of its motion.



Unknown: Time to reach the top of its flight Δt = ?

Equation: The most appropriate equation would be $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$.

Solving for time, $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$.

Substitute and solve:

$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$$

$$\Delta t = \frac{0 \text{ m/s} - 8.00 \text{ m/s}}{-9.80 \text{ m/s}^2} = 0.816 \text{ s}$$

Now you can calculate the displacement during this time using $\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$.

From the moment the bag of sand is released, it rises an additional height of

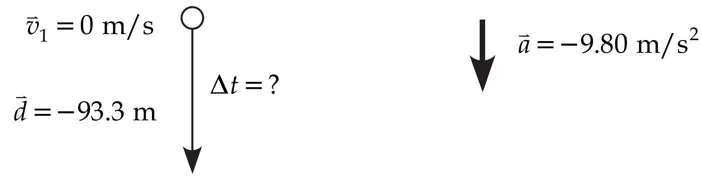
$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2 = (8.00 \text{ m/s})(0.816 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.816 \text{ s})^2.$$

$$\vec{d} = -3.26 \text{ m}$$

The sandbag stops rising at a maximum height of 90.0 m + 3.26 m = 93.26 m = 93.3 m above the ground.

Part 2

Given: For the second part of the problem, redraw the diagram and add the new set of symbols. Note that this time $\vec{v}_1$ = 0 m/s. This is because the sandbag is at the top of its motion at a height of 93.3 m, and has in effect stopped for just an instant. Notice also that the sandbag is descending so the displacement is given a negative number.



Unknown: Time it takes the sandbag to fall from its maximum height: Δt = ?

Equation: The appropriate equation in this case would be

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2.$$

Since $\vec{v}_1 = 0 \text{ m/s},$ the equation becomes

$$\vec{d} = \frac{1}{2}\vec{a}\Delta t^2.$$

Substitute and solve:

$$\Delta t = \sqrt{\frac{2\vec{d}}{\vec{a}}}$$

$$\Delta t = \sqrt{\frac{2(-93.3 \text{ m})}{-9.80 \text{ m/s}^2}} = 4.36 \text{ s}$$

Note that the total time the sandbag is in the air is 0.816 s + 4.36 s = 5.176 s or 5.18 s.

Learning Activity 3.1

Motion of Freely Falling Objects

The practice questions in this learning activity deal with freely falling objects. An answer key is available at the end of Module 3 for you to check your work after you have answered the questions.

The physics of an experimental vehicle and a ball falling (conceptual)

1. Imagine an experimental vehicle which slows down and comes to a stop with an acceleration of magnitude 9.80 m/s^2. The vehicle reverses direction and then speeds up with an acceleration of 9.80 m/s^2. Compare this situation to a ball being thrown upward, coming to a momentary stop, and then falling to Earth. Other than one situation being horizontal and the other being vertical, are the motions the same or different?

The physics of an arrow fired straight up

2. An arrow that is fired straight up can have an initial speed of 14.0 m/s. How much time will it take for the arrow to reach the top of its motion, and how long will the arrow remain in the air?

The physics of a bouncing golf ball

3. If you throw a golf ball at a hard surface and then watch it rebound, it may rebound with a speed of 4.50 m/s. In this case, how high will it rebound?

The physics of a falling sandbag

4. A balloon is descending at a velocity of –4.00 m/s. A sand bag is released from this balloon and hits the ground 8.00 s later. What was the altitude of the balloon at the time the sandbag was dropped?

Lesson Summary

In this lesson, you concentrated on applying the equations for uniformly accelerated motion to objects in free fall. **Free fall** occurs in the idealized case where air resistance is neglected, and all objects at the same location on the Earth fall with the acceleration due to gravity.

In analyzing the motion of objects in free fall under **constant acceleration**, the following equations apply.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \text{ or } \vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

$$\vec{d} = \frac{(\vec{v}_1 + \vec{v}_2)}{2}\Delta t$$

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$v_2{}^2 = v_1{}^2 + 2ad$$

If you are assuming no air friction and no outside forces acting on the falling object, the following concepts relating to the symmetry of motion apply.

1. The time it takes an object to reach the top of the motion is equal to the amount of time it takes the object to fall back to the height of the original point of release.
2. As the object moves upwards, the speed of the object at some displacement, y, above the point of release is the same as the speed of the object moving downwards at the same displacement, y, above the point of release. The velocities will be opposite in direction, but the magnitude of the velocities (speed) will be the same.
3. The initial speed of the object at the point of release is the same as the final speed of the object when it reaches the point of release (remember, the directions are different).
4. The displacement of the object up is the same as the displacement of the object down, assuming that both are measured from the point of release.
5. The speed of the object at the top of its motion is momentarily zero. This can be considered to be the second velocity, $\vec{v}_2$. In the second half of the motion, as the object begins to fall, the speed at the top is still zero but this velocity is now the initial velocity, $\vec{v}_1$, for this part of the motion.

You applied these principles to solving three situations in free fall. In each of the problems, you emphasized the “Guess” method for problem solving.

1. Given: Make a drawing.

   Decide which direction is positive and which is negative.

   Write down in symbolic form what you are given.

2. Unknown: Write down in symbolic form what you need to find.

3. Equation: Select the appropriate equation.

4. Substitute: Substitute the values that are given into the equation.

5. Solve: Solve the equation of the unknown.

NOTES

Assignment 3.1

Vertical Motion of a Bullet (9 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of a speeding bullet

A bullet is shot vertically into the air at a speed of 512 m/s.

(For this question, draw the appropriately labelled diagram for part (a) only and then follow with the correct steps for solving a problem. For parts (b) and (c), do not redraw the diagram, but do rewrite the appropriate equations with their solutions.)

a) To what maximum height does the bullet go?

(continued)

Assignment 3.1: Vertical Motion of a Bullet (continued)

b) How much time passes before the bullet stops rising?

c) What is the velocity of the bullet after 60.0 s?

Method of Assessment

The total of nine marks for this assignment will be determined as follows:

- 3 marks for drawing an appropriate diagram with labels showing what is given and what is required in part (a)
- 1 mark for selecting the appropriate equation and doing the algebra correctly in part (a)
- 1 mark for the correct solution with the correct units in part (a)
- 1 mark for using an appropriate equation in part (b)
- 1 mark for the correct solution with the correct units in part (b)
- 1 mark for using an appropriate equation in part (c)
- 1 mark for the correct solution with the correct units in part (c)

Video - Introduction to Free-Fall and the Acceleration due to Gravity

This video introduces the concept of free fall. It outlines the conditions for free fall. Objects that are in free fall show uniformly accelerated motion. The mass of the object does not affect the acceleration of an object in free fall.

https://youtu.be/vyvDzI22sOE

Video - Dropping a Ball from 2.0 Meters - An Introductory Free-Fall Acceleration Problem

This video solves a problem using the equations of motion for the motion of a medicine ball that is dropped from rest over a distance of 2.0 metres.

Note in this video that the subscript “y” is added to the symbols of velocity and acceleration and that displacement in the vertical direction is referred to as delta y (Δy).

Pay attention to some of the common errors made for vector directions.

https://youtu.be/XHsBVXbDRxk

Video - Throwing a Ball up to 2.0 Meters & Proving the Velocity at the Top is Zero

This video is an introductory problem for free fall. The analysis of the motion of a ball thrown upwards is analyzed graphically and through the use of the concepts of uniformly accelerated motion.

https://youtu.be/9VDVuD7DPF0

Video Enrichment - The Drop and Upward Throw of a Ball are Very Similar

The motions of a ball thrown upwards to a height of 2.0 m and of a ball falling from rest from a height of 2.0 metres are analyzed. If the video is reversed for the ball on its upward journey it exhibits the exact same motion as the ball falling from rest. The idea is that at the same height the speed of the object is the same on the way up and on the way down. Remember that the speed is the magnitude of the velocity. This idea is useful in projectile motion.

https://youtu.be/DTZ4ldhr76U

Assignment 3.1

Vertical Motion of a Bullet (9 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of a speeding bullet

A bullet is shot vertically into the air at a speed of 512 m/s.

(For this question, draw the appropriately labelled diagram for part (a) only and then follow with the correct steps for solving a problem. For parts (b) and (c), do not redraw the diagram, but do rewrite the appropriate equations with their solutions.)

a) To what maximum height does the bullet go?

(continued)

Assignment 3.1: Vertical Motion of a Bullet (continued)

b) How much time passes before the bullet stops rising?

c) What is the velocity of the bullet after 60.0 s?

Method of Assessment

The total of nine marks for this assignment will be determined as follows:

- 3 marks for drawing an appropriate diagram with labels showing what is given and what is required in part (a)
- 1 mark for selecting the appropriate equation and doing the algebra correctly in part (a)
- 1 mark for the correct solution with the correct units in part (a)
- 1 mark for using an appropriate equation in part (b)
- 1 mark for the correct solution with the correct units in part (b)
- 1 mark for using an appropriate equation in part (c)
- 1 mark for the correct solution with the correct units in part (c)

Lesson 2: Projectile Motion in Two Dimensions: Vector Diagrams

(1 hour)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the term "projectile motion"
- ❑ explain how the total velocity and the velocity component vectors change in both magnitude and direction for strictly vertical motion
- ❑ explain how the total velocity and the velocity component vectors change in both magnitude and direction for an object projected in the horizontal direction
- ❑ explain how the total velocity and the velocity component vectors change in both magnitude and direction for an object projected at an angle

Key Words

projectile motion
component independence trajectory

Introduction

In the previous lesson, you studied projectile motion in one dimension. This involved an object moving up or down, and you applied the equations for uniformly accelerated motion along a straight line to solving problems involving this type of projectile motion. In this lesson, we will begin to consider projectile motion in two dimensions—that is, we will examine the situation of a projectile launched into the air at an angle. To begin with, we will not do any calculations. We will examine the motion by drawing vector diagrams that describe the motion.

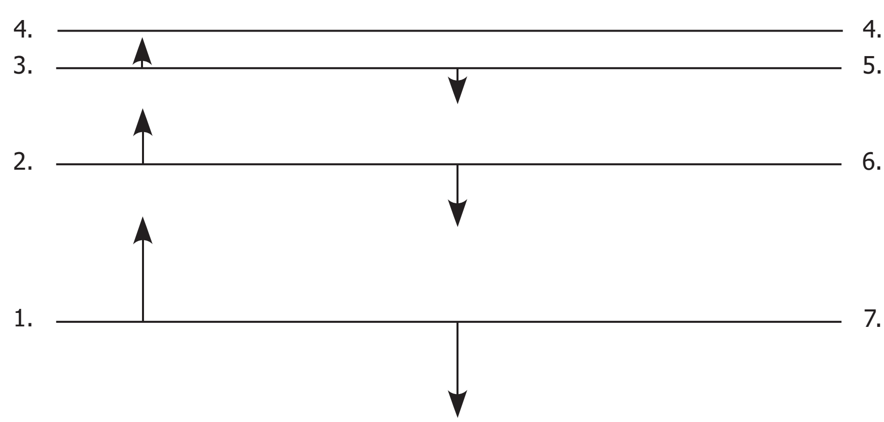
A Projectile Launched Vertically

> **Projectile motion** refers to the motion of an object projected into the air and moving only under the influence of gravity.

We will restrict ourselves to objects moving near Earth's surface so we can assume that the value of the acceleration due to gravity, $\vec{g}$, has a value of $\vec{g}$ = 9.80 m/s^2. Also, the effects of air resistance will be ignored in the following analysis. We will not be concerned with the process by which the projectile has been projected. We will study its motion after it has been projected and is moving freely through the air under the action of gravity alone. It was Galileo (1564–1642) who first accurately described the motion of projectiles. He showed that the motion could be understood by analyzing the horizontal and vertical components of the motion separately.

> **Component Independence:** Components of a vector can be treated independently of each other. The component in one direction has no effect on the component in the other direction.

Let's begin by considering only the vertical component of the motion for an object that is thrown straight upwards. The diagram below shows the velocity vectors at different parts of the projectile's path.



At position 1, the object is thrown into the air. The velocity vector is at its maximum pointing straight up. At position 2, the object has risen some distance into the air, but its velocity has decreased. At position 3, the object is near the top of its motion, and the velocity vector is very small. At position 4, the object is at the top of its motion, and it has no vertical component. At each of positions 5, 6, and 7, the object is now falling. The magnitude of the velocity at each of these positions is the same as it was when the object was rising, but the direction is completely reversed.

Notice that since the object is falling under the influence of gravity alone, this is a situation of uniformly accelerated motion. The equations we used earlier for this type of motion apply in this situation.

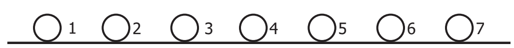
$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t \qquad v_2^2 = v_1^2 + 2ad$$

$$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)t \qquad \vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}t^2$$

A Projectile Launched Horizontally

Let’s look at the motion of a ball that is travelling at a constant velocity of 1.0 m/s along a level horizontal table. This ball is rolling from left to right. We are using a stroboscope and a camera to take seven successive pictures of the ball as it rolls.

If there is no friction, then the ball will travel 1.0 m during each second. Here is a “stroboscopic” diagram of the ball’s position every 1.0 second:



Notice that the ball travels the same distance during each second. This is because there is no friction so there is no unbalanced force acting on the ball. Without an unbalanced force acting on it, the ball doesn’t speed up or slow down: its velocity is constant. Recall that this is just Newton’s First Law of Motion!

Even if we include friction in our diagram, the picture will not change significantly. A rolling ball doesn’t slow down very much because it experiences very little friction.

The important thing to understand here is that a ball rolling horizontally will travel at a constant velocity. There is nothing to speed it up or slow it down!

Now let’s look at a ball that is dropped off the edge of a table.

You will find that the **vertical velocity** of a falling object **is independent** of the **horizontal velocity**. This is the most important idea in this unit, but it is also the most difficult to understand!

Here is a similar diagram of the same ball falling off the edge of a table.

In this case, the distance between the ball's positions is increasing during each second. This is because the ball is speeding up as it falls. It speeds up because it is being attracted by the force of gravity.

The important thing to understand is that the falling ball is speeding up because there is a force accelerating it. This force is gravity.

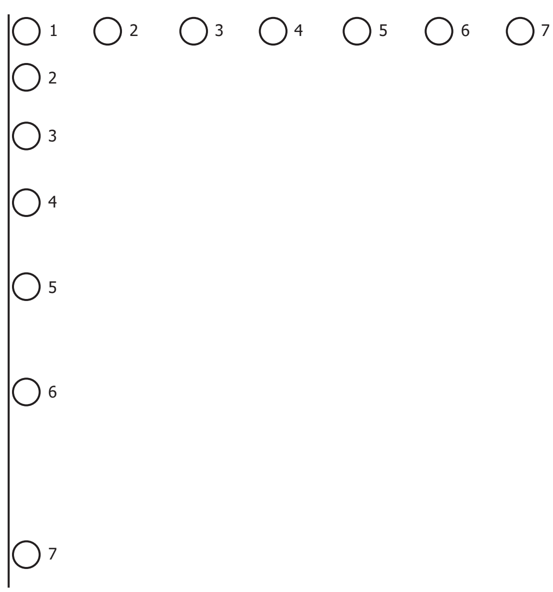
So, the same ball will travel at a constant velocity as it moves horizontally but will speed up as it falls.

What will the same ball do if it moves horizontally and falls at the same time? This can happen if a ball is rolling along a table (at constant velocity) and it suddenly rolls off the table and starts to fall downward (with increasing velocity).

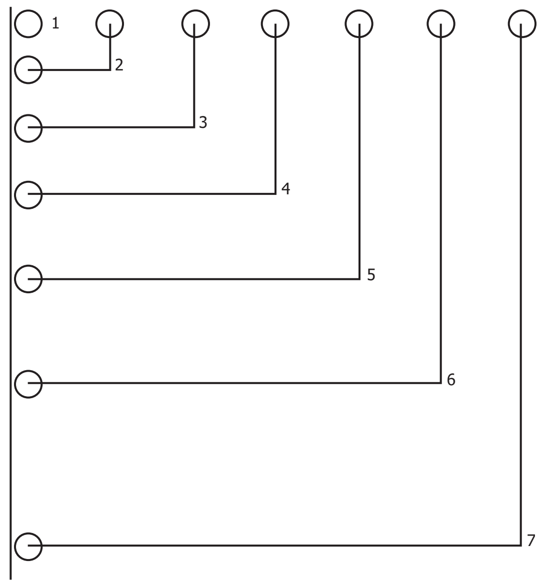
The answer is that the ball will continue to **move horizontally at a constant velocity** as it **falls with increasing vertical velocity**. The ball continues to move horizontally the same amount each second, but moves downward an increasing amount each second. This gives the ball a curved path that is called a **trajectory**.

A **trajectory** is the path followed by any projectile. In projectile motion, the trajectory has a very specific shape: a parabola.

Here are the same two paths that we saw above, but drawn on the same diagram:



Now, here is the same diagram with lines showing the actual path that the ball would take if it rolled off the table:

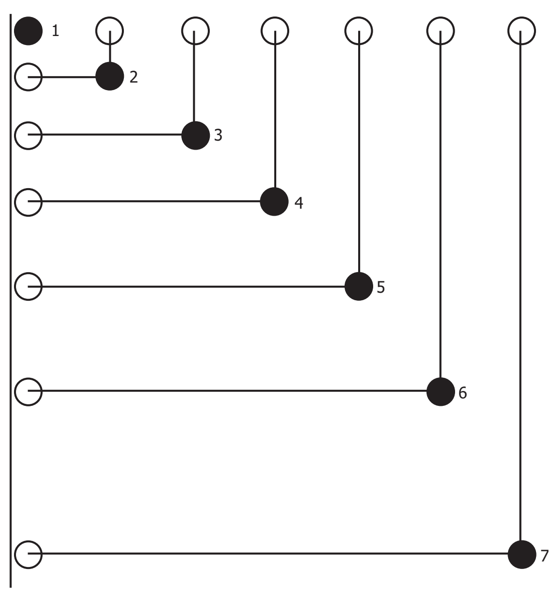


A ball rolling on the table would take the path shown by the row of balls along the top of the diagram.

A ball dropped off the edge would take the path shown by the vertical row of balls at the edge of the table.

The **intersection points** (1–7, inclusive) show the path that would be taken by a ball that is both **rolling horizontally** and **dropping vertically**. This is the path that a ball would take if it were rolling along a table and suddenly rolled off the edge. It is the trajectory of a falling ball.

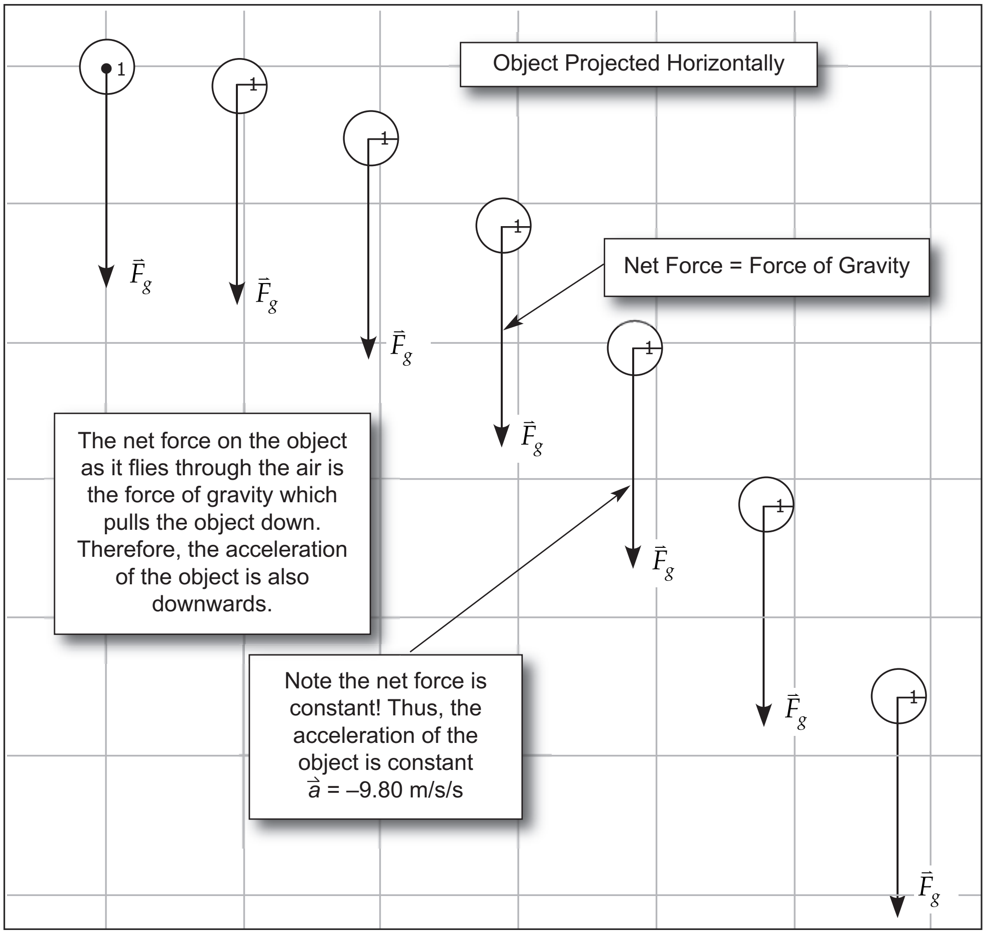
Here is the same diagram showing the path of a ball falling vertically, a ball rolling horizontally, and a ball that has rolled off the table. This last ball (which is coloured black) has the motion of both the rolling ball and the falling ball.



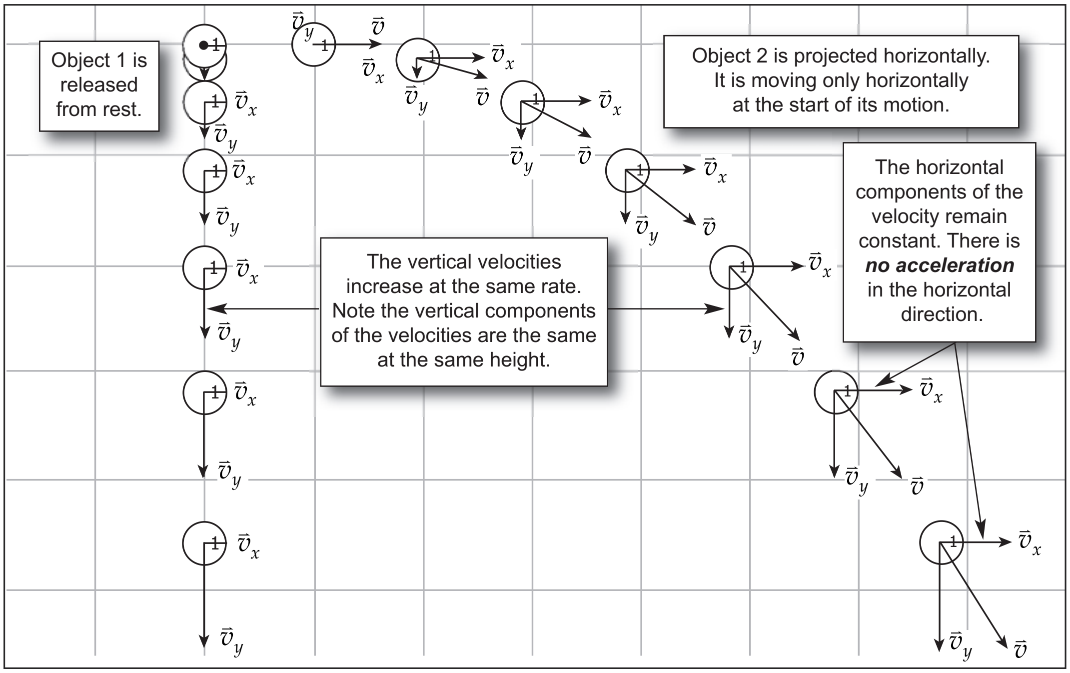
The important thing to understand is that the curved path of the falling ball is a combination of its constant horizontal velocity and its increasing vertical velocity.

This is because the ball has a force (gravity) accelerating it downward but there is no force accelerating it sideways.

Below is a graphic illustrating the forces acting on an object projected horizontally.



The graphic with the forces above links directly to the graphic with the velocities of projectiles below.



We can analyze the complex curved motion of a falling object by examining its horizontal velocity separate from its vertical velocity.

Now let's assume that a ball is launched off a table with an initial velocity, $\vec{v}_1$, in the horizontal or x-direction. The vertical component of the velocity $\left(\vec{v}_y\right)$ will gradually increase as the ball falls. The horizontal component of the velocity $\left(\vec{v}_x\right)$ will remain constant. The total velocity vector, $\vec{v}$, at each instant points in the direction of motion at that instant, and it is tangent to the path.

At any given moment, the total velocity vector, $\vec{v}$, is the vector sum of the horizontal component of the velocity $\left(\vec{v}_x\right)$ and of the vertical component of the velocity $\left(\vec{v}_y\right)$.

$$\vec{v} = \vec{v}_x + \vec{v}_y$$

In analyzing this situation, the horizontal component of velocity $\left(\vec{v}_x\right)$ has been added to the diagram on the right for projectile motion in the vertical direction.

Equations for the Horizontal Component of Projectile Motion

Notice in each of the diagrams above that the horizontal and the vertical components of velocity can be treated independently of each other. The equations for uniformly accelerated motion in the vertical direction apply here. But since there is no horizontal acceleration, the horizontal component of velocity is constant. With a constant velocity, we can use

$\vec{v}_x = \vec{v}_{1x}$ (The horizontal component of velocity at any moment is equal to the initial horizontal component.)

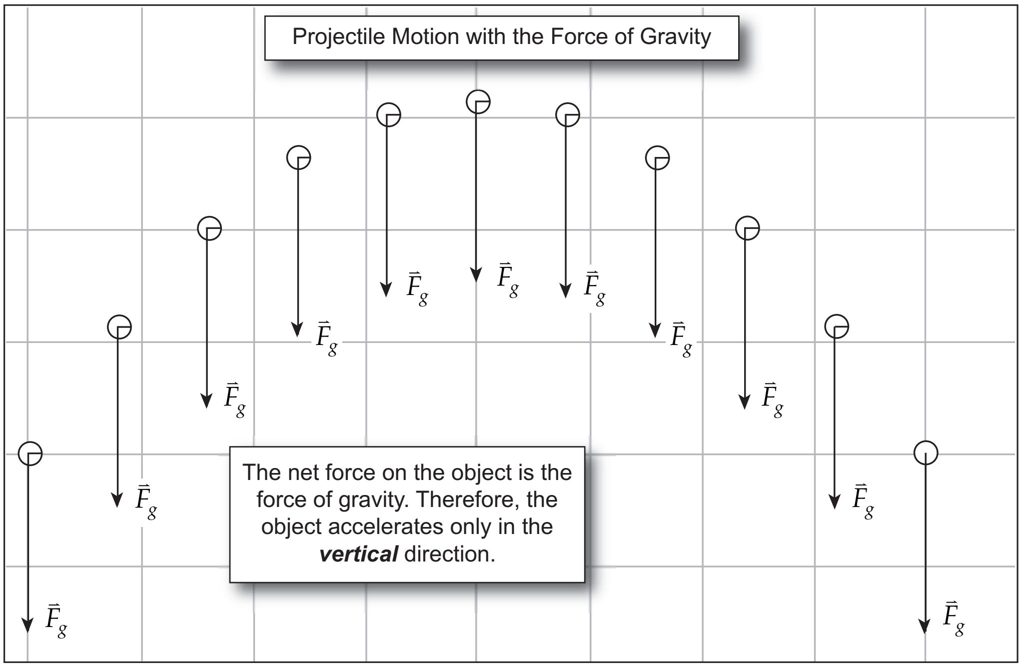
$\vec{d}_x = \vec{v}_{1x}\Delta t$ (The position of the object in the horizontal direction can be determined by multiplying the horizontal component of the velocity by the time.)

A Projectile Launched at an Angle

Now let us assume that the projectile has been launched at some angle to the horizontal.

If you've ever used a water fountain, you'll have noticed that the path followed by the water, its trajectory, looks like an upside down **U**. In fact, the trajectory follows the shape of a **parabola**. Any projectile at the surface of the Earth will follow this path. This applies to a stream of water, a baseball that has been thrown, a bullet that has been fired from a rifle, an athlete doing the broad jump, or a show horse leaping over an obstacle.

Again, if we approach this from the point of view of dynamics, once the object is really moving through the air, the only force acting on the object will be the force of gravity pulling the object downward.

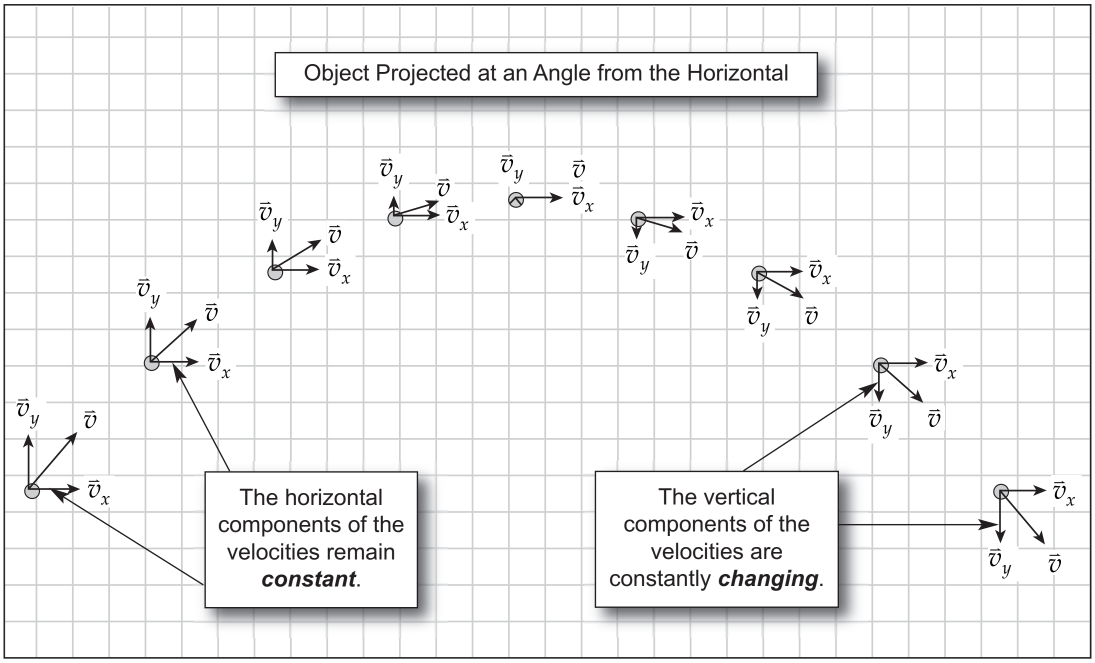


The acceleration in the vertical direction will be the acceleration due to gravity. Therefore, at Earth's surface, you can use acceleration in the vertical direction to be

$$\vec{a}_y = \vec{g}$$
$$\vec{a}_y = 9.80 \text{ m/s}^2 \text{ [down]}$$

The diagram that follows shows that the horizontal component of the motion remains constant for the whole motion, both in the up direction and the down direction.

The vertical component of the velocity changes as the object moves up and down because it is accelerating. The horizontal and vertical components of velocity can be treated separately.



The equations for horizontal motion are

$$\vec{v}_x = \vec{v}_{1x}$$

$$\vec{d}_x = \vec{v}_{1x}\Delta t$$

The equations for vertical motion are still the same as the ones for constant acceleration. With these equations, you just have to realize that the motion is in the vertical direction.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\vec{d} = \frac{(\vec{v}_1 + \vec{v}_2)}{2}\Delta t$$

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$v_2{}^2 = v_1{}^2 + 2ad$$

The total velocity vector is drawn tangent to the motion at any point. It is also the vector sum of the horizontal and vertical components of the velocities.

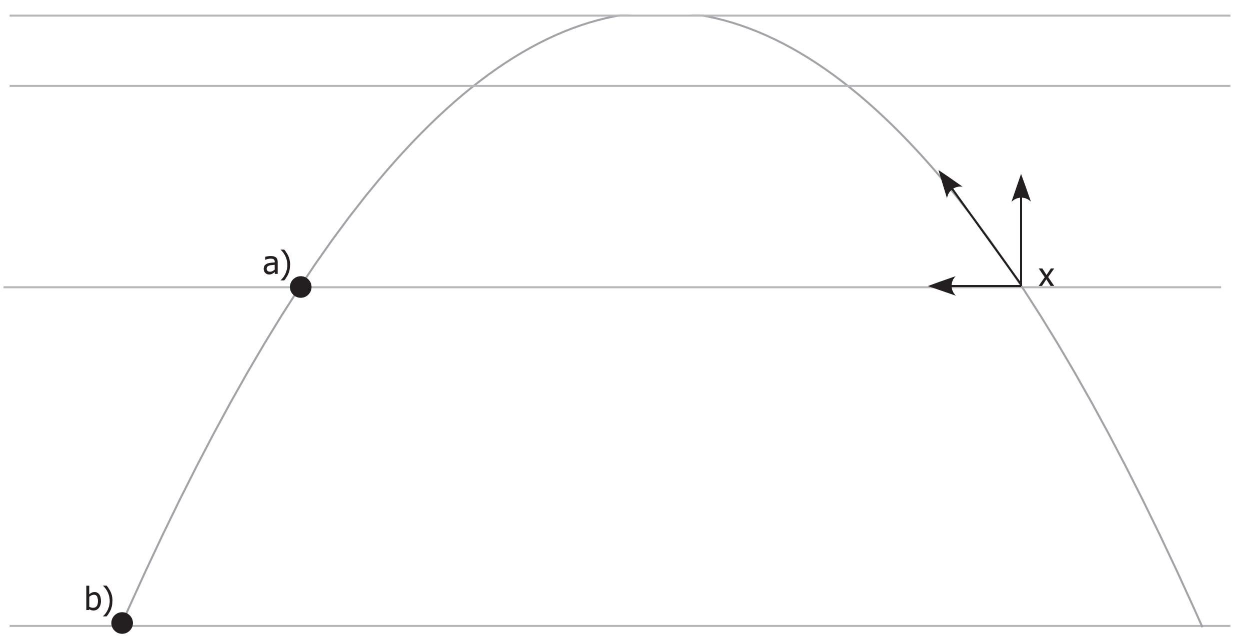
$$\vec{v} = \vec{v}_x + \vec{v}_y$$

Learning Activity 3.2

Projectile Motion: Vector Diagrams

There is one practice question with six parts in this learning activity. An answer key is available at the end of Module 3 for you to check your work after you have answered the question.

The physics of projectile motion: vector diagrams



a) Notice that the dot at point “a” is at the same vertical height as point “x.” At the dot, draw and label the horizontal component of the velocity vector, the vertical component of the velocity vector, and the total velocity vector.

b) At the dot at point “b,” estimate the length of the horizontal component, the vertical component, and the total velocity vectors, and draw them into the diagram and label them.

c) Draw in the vector representing the net force acting on the object at the position x.

(continued)

Learning Activity 3.2: Projectile Motion: Vector Diagrams (continued)

d) If you wanted to determine the distance travelled in the horizontal direction, what equation could you use?

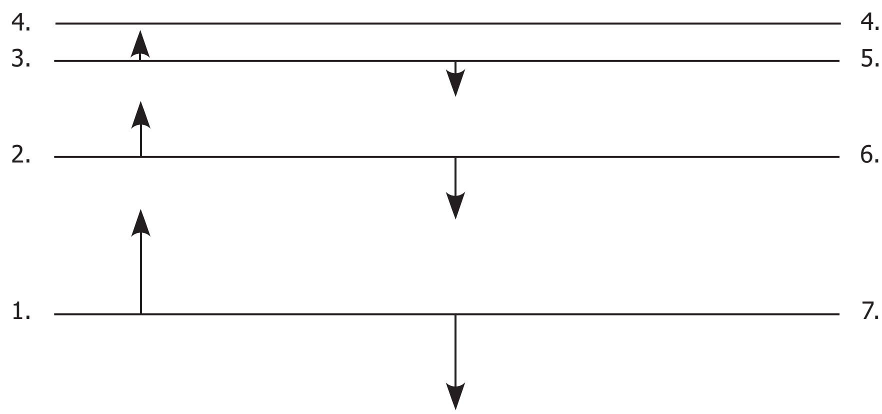
e) If you wanted to determine the distance or displacement travelled in the vertical direction, what equation could you use?

f) At what point in the motion of the projectile is the vertical component of the velocity equal to zero?

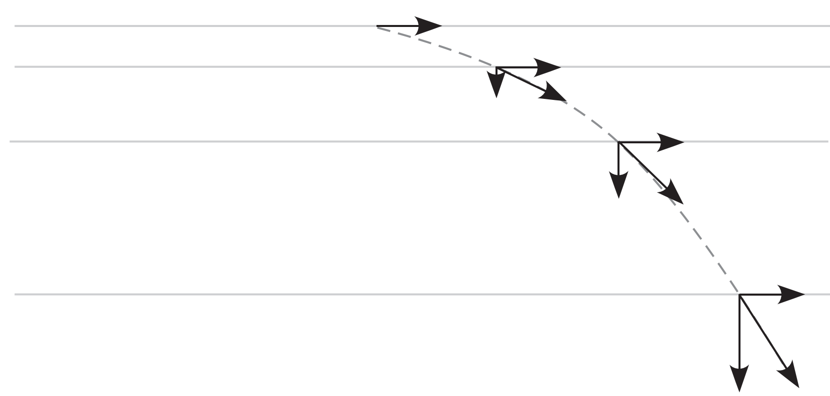
Lesson Summary

Projectile motion refers to the motion of an object projected into the air and moving only under the influence of gravity. The following diagrams show the relative magnitude and direction of the velocity vectors for objects projected vertically, horizontally, and at an angle.

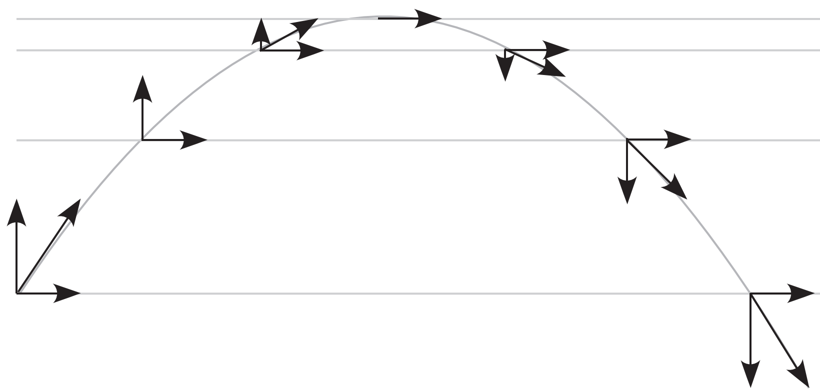
Vertical Projection: For the vertical projection of an object, only the *y*-components of the velocity vectors are shown. **The vertical velocity is constantly changing.**



Horizontal Projection: For objects projected horizontally, both the *x*- and *y*-components have been shown, as well as the total velocity vector. **The horizontal velocity is constant. The vertical velocity is changing.**



Angled Projection: For objects projected at an angle, both the x- and y-components have been shown, as well as the total velocity vector. This diagram is a combination of the two previous diagrams on horizontal and vertical projection.



The equations for horizontal motion are

$$\vec{v}_x = \vec{v}_{1x}$$

$$\vec{d}_x = \vec{v}_{1x}\Delta t$$

The equations for vertical motion are

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\vec{d} = \frac{(\vec{v}_1 + \vec{v}_2)}{2}\Delta t$$

$$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$v_2{}^2 = v_1{}^2 + 2ad$$

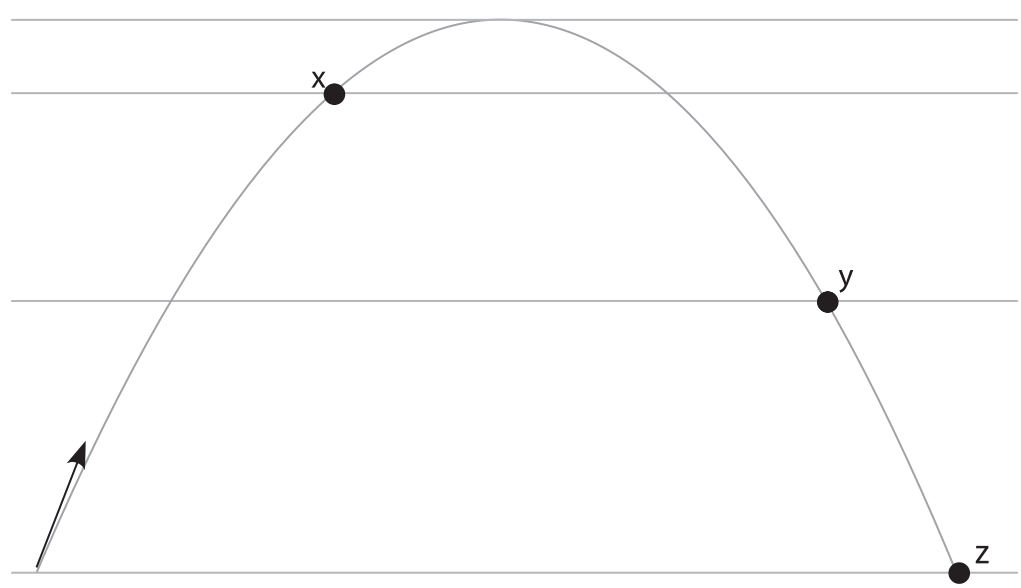
Assignment 3.2

Vector Nature of Projectile Motion (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of a projectile launched at an angle

a) The diagram below shows a projectile launched at some angle up and to the right.



Draw possible horizontal and vertical components of the velocity at point "X."

Draw a possible total velocity vector at point "Y." Be sure to demonstrate how you arrived at the answer.

(continued)

Assignment 3.2: Vector Nature of Projectile Motion (continued)

b) If an object had been projected horizontally with the same magnitude as in the depicted situation, how would the motion compare with that of the object in the diagram?

c) If an object had been projected vertically with the same initial vertical velocity as the object in the diagram, how would its vertical velocity components compare?

d) If the object had been projected from position "Z" with the horizontal components of velocity reversed (at 180°) compared to the depicted situation, how would the path of motion of that object compare with the one in the diagram?

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 2 marks for horizontal, vertical, and total velocity in part (a)
- 1 mark for comparison in part (b)
- 1 mark for comparison in part (c)
- 1 mark for comparison in part (d)

Video - Demonstrating the Components of Projectile Motion

The motion of a projectile is composed of a vertical component and a horizontal component.

An example of projectile motion is broken apart into the vertical motion and horizontal motion.   Velocity vectors representing the horizontal motion and the upward vertical and downward vertical motion are superimposed on the image. From these components the velocity for each image is constructed by adding the vertical and horizontal components.

Next the acceleration vectors are added to the images. Each image has a constant downward acceleration due to gravity.

https://youtu.be/2xCQ-MKRzlw

Video - Shoot-n-Drop

An apparatus that at once shoots a billiard ball horizontally and drops another one vertically from an equal height. Even though the two have different initial velocities, they both accelerate in the same direction and at the same rate due to Earth's gravity--this is confirmed by seeing and hearing both balls land simultaneously.

This video also illustrates the independence of the vertical and horizontal components of projectile motion.

https://youtu.be/zMF4CD7i3hg

Video - Mechanics: Motion In Two-Dimensions (1 of 21) Independent Motion in x and y

This video explains the independence of vector components and how this idea is applied to the vertical component and horizontal component of projectile motion.

https://youtu.be/Id1FLVqJKoU

In this simulation (next page) you can observe projectile motion under ideal conditions, that is, no force of air friction.

https://www.walter-fendt.de/html5/phen/index.html

https://www.walter-fendt.de/html5/phen/projectile_en.htm

To run the simulation, press Reset, make the changes, then press Reset to accept the changes.

Press Start to Run the simulation.

Observe the projectile motion with the parameters set as follow.

1. Select slow motion.

Set Initial Height at 0 m.

Set Initial Speed to 5 m/s.

Set Angle at 90 degrees. This fires the projectile straight upwards.

Select Acceleration to display the acceleration vector attached to the object. How does the acceleration vary as the object moves up then down?

Select Force to display the force vector. How does the force vary as the object moves up then down?

2. To observe the motion of an object projected horizontally:

Select slow motion.

Set Initial Height at 3 m.

Set Angle at 0 degrees. This fires the projectile horizontally to the right.

Vector Display: Set to Acceleration.

Repeat: Set to Velocity. Note the size of the horizontal component of the velocity.

3. To observe the effect of the launch angle on range (horizontal distance traveled) of the projectile.

Clear the slow motion selection.

Set Initial Height at 0 m.

Set Initial Speed to 10 m/s.

Set Angle at 80 degrees. Note the range.

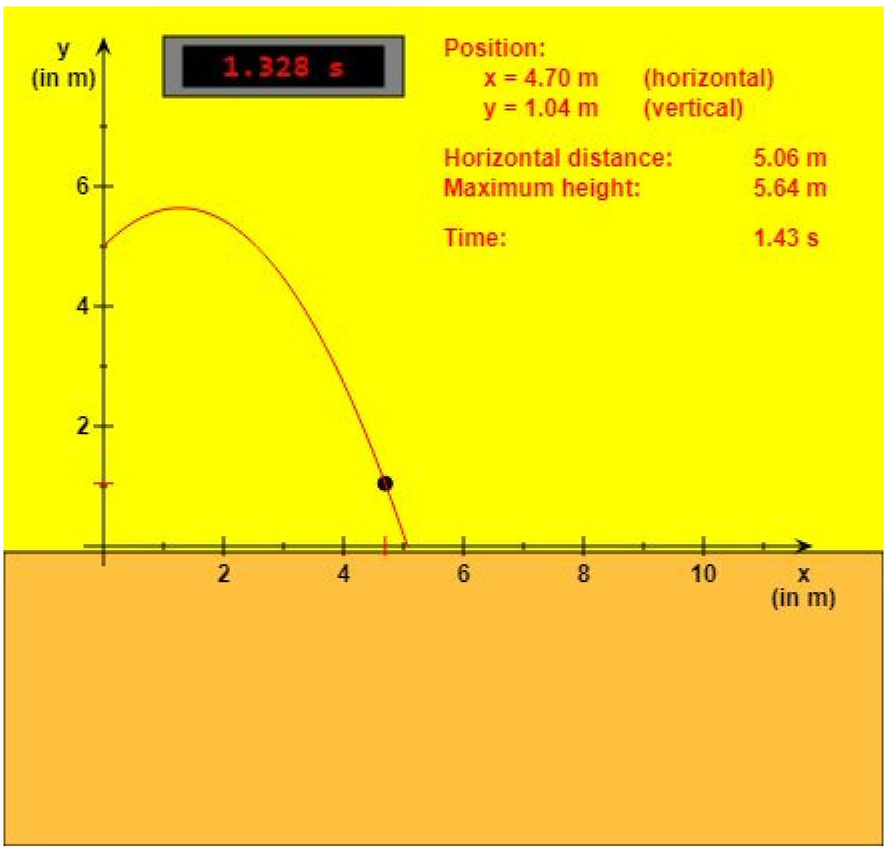
Reset the angle to 70, 60, 50, 45, 40, 30, 20, and 10 degrees. Note the range for each launch angle. Which launch angle results in the greatest range.

Projectile Motion

This HTML5 app shows the motion of a projectile.

The "Reset" button brings the projectile to its initial position. You can start or stop and continue the simulation with the other button. If you choose the option "Slow motion", the movement will be ten times slower. You can vary (within certain limits) the values of initial height, initial speed, angle of inclination, mass and gravitational acceleration. The radio buttons give the possibility to select one of five physical sizes.

The effect of air resistance is neglected.



The next page shows a great simulation where you can fire various objects out of a cannon. You can choose the initial velocity and the angle at which the object is launched from the cannon. You can observe the vectors plus the component vectors of velocity and acceleration.

Try the following:

Adjust the magnitude of the velocity so that the object will hit the target.

Keeping the angle and magnitude of the velocity the same, fire objects of different masses to see how mass affects the path of the object in projectile motion.

https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html

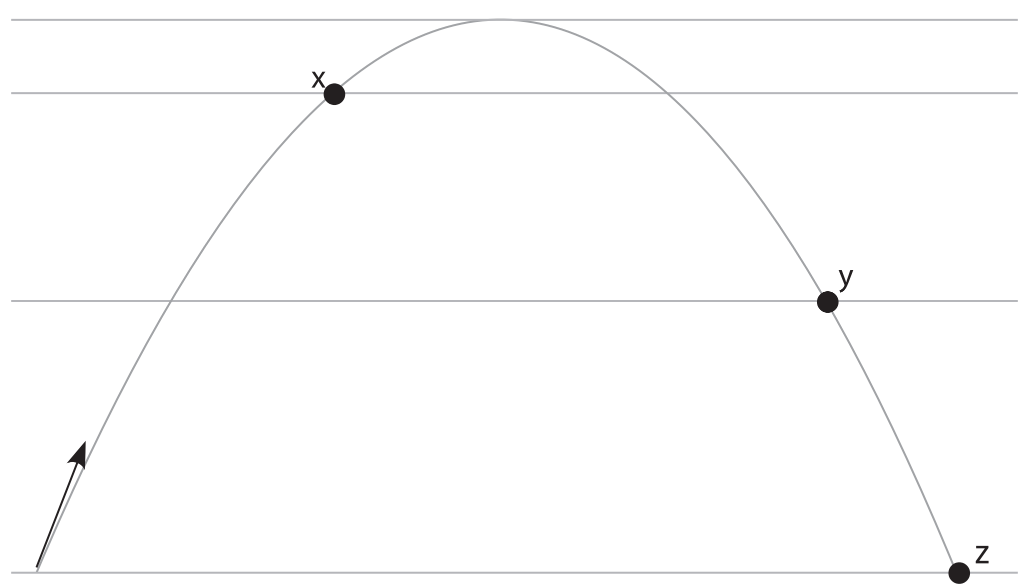
Assignment 3.2

Vector Nature of Projectile Motion (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of a projectile launched at an angle

a) The diagram below shows a projectile launched at some angle up and to the right.



Draw possible horizontal and vertical components of the velocity at point "X."

Draw a possible total velocity vector at point "Y." Be sure to demonstrate how you arrived at the answer.

(continued)

Assignment 3.2: Vector Nature of Projectile Motion (continued)

b) If an object had been projected horizontally with the same magnitude as in the depicted situation, how would the motion compare with that of the object in the diagram?

c) If an object had been projected vertically with the same initial vertical velocity as the object in the diagram, how would its vertical velocity components compare?

d) If the object had been projected from position "Z" with the horizontal components of velocity reversed (at 180°) compared to the depicted situation, how would the path of motion of that object compare with the one in the diagram?

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 2 marks for horizontal, vertical, and total velocity in part (a)
- 1 mark for comparison in part (b)
- 1 mark for comparison in part (c)
- 1 mark for comparison in part (d)

Lesson 3: Projectile Motion in Two Dimensions: Using the Equations

(3 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ analyze two-dimensional motion for projectiles that are initially projected horizontally
- ❑ analyze two-dimensional motion for projectiles that are projected at some angle to the horizontal

In analyzing these motions, do the following:

- ❑ draw a curved path of a projectile that is due to the combination of constant accelerated vertical motion and constant horizontal motion
- ❑ draw diagrams showing the components of the velocity at various points along the path of motion of a projectile
- ❑ calculate the horizontal and vertical components of velocity of a projectile at various points along its path
- ❑ use the equations for constant velocity and constant acceleration in solving problems
- ❑ solve problems to calculate how high the projectile rises, how long it stays in the air (hang time), and how far it travels (range)

Key Words

range
hang time

Introduction

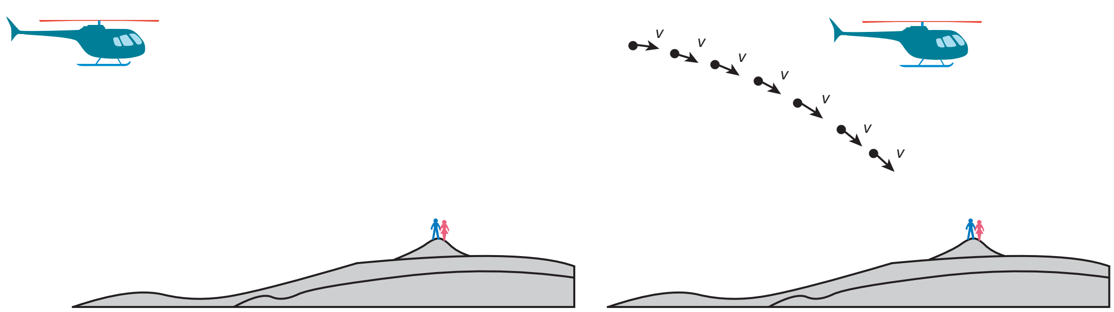
In the previous lesson, we discussed projectile motion in two dimensions by understanding the vector diagrams that represent the motion of the projectile. We learned that the horizontal component of velocity is constant, but the vertical component of velocity is changing. We learned that one set of equations for constant velocity can be applied to the horizontal part of the motion and a different set can be applied to the vertical part of the motion. In this lesson, we will use these equations to solve problems involving projectile motion in two dimensions.

Analyzing Projectile Motion using Equations

In lesson 1, we discussed the vertical part of projectile motion. In this section, we will consider the case of an object projected in a horizontal direction.

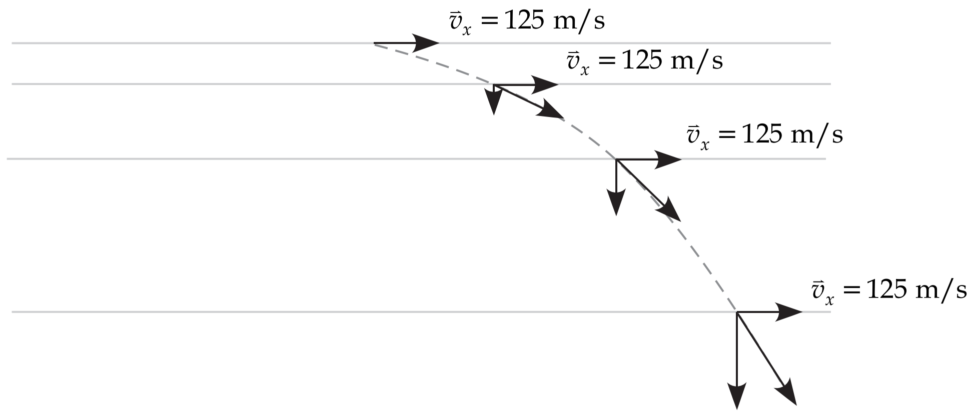
Example 1: Projectile Motion for a Horizontal Projection: A Problem

One situation like this might be an airplane flying in a horizontal direction with a constant velocity of +125 m/s and an altitude of 1250 m. The airplane then drops a food package to assist adventurers in the wilderness. The package falls in a curved path, as was discussed in the previous lesson. With this information, we can now determine various things about the motion of this package.



The initial velocity points only in the horizontal or x-direction: $\vec{v}_x = 125 \text{ m/s}$

The motion of the package would be as follows.



Recall that for the vertical component of the velocity, we can use the following equations.

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

$$\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)t$$

$$\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}t^2$$

$$v_2^2 = v_1^2 + 2ad$$

In this problem, we are given the height above the ground, d = 1250 m. Another way to state this is that the object has fallen a distance of 1250 m, so the displacement $\vec{d}$ = –1250 m. We also know that the acceleration due to gravity is $\vec{a}$ = –9.80 m/s^2. The initial component of the velocity in the vertical direction is $\vec{v}_{1y} = 0$ m/s.

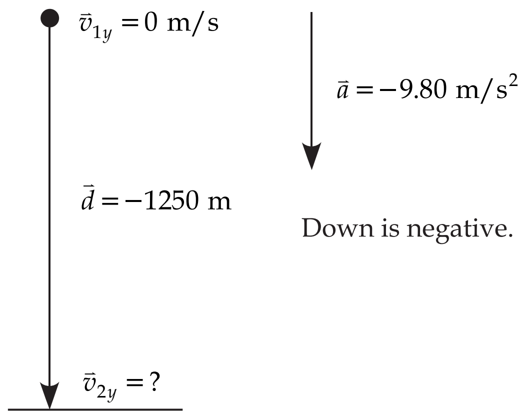
Projectile Motion for a Horizontal Projection: Solving the Problem

Here are some typical questions.

a) How long will it take the package to fall to the ground?

Time interval appears in the equations for both vertical and horizontal motion. Since you are talking about falling to the ground, you can assume that this is vertical motion.

Given:



Unknown: Time interval $\Delta t = ?$

Equation: To determine the time it takes for the food package to fall to the ground, we can use the equation $\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}t^2$.

Solving for the time and realizing that $\vec{v}_{1y} = 0$ m/s, $\Delta t = \sqrt{\frac{2\vec{d}}{\vec{a}}}$.

Substitute and solve: $\Delta t = \sqrt{\frac{2(-1250 \text{ m})}{-9.80 \text{ m/s}^2}} = 16.0 \text{ s}$

b) How fast is the package falling vertically when it strikes the ground?

You have all the information from part (a) plus the time interval.

Unknown: Vertical velocity at the moment of impact $\bar{v}_{2y} = ?$

Equation: To determine the vertical component of the velocity just as the object hits the ground, we can use the equation $v_2^2 = v_1^2 + 2ad$.

Substitute and solve: To determine the second velocity, use

$$
\begin{aligned}
v_2 &= \sqrt{v_1^2 + 2ad} \\
&= \sqrt{(0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-1250 \text{ m})} \\
&= \sqrt{+24500} = -156 \text{ m/s}
\end{aligned}
$$

Keep in mind that this is the magnitude of the velocity. Since the package is travelling in the down direction, the actual vertical component can be written as –156 m/s.

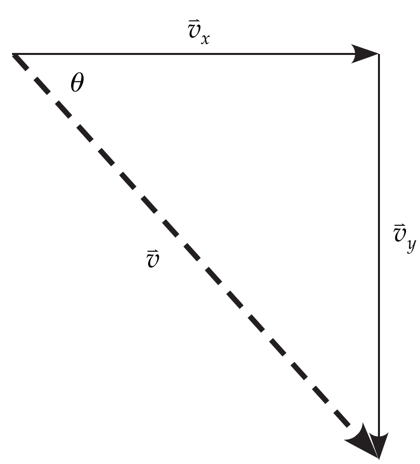
c) What is the actual velocity of the package when it strikes the ground?

To determine the actual velocity of the package when it strikes the ground, it will be necessary to add together the horizontal component and the vertical components of the velocities using trigonometry.

$$\bar{v} = \bar{v}_x + \bar{v}_y$$

The final answer will contain the magnitude of the velocity and the angle with which the package strikes the ground.

You know that the horizontal component of the velocity is 125 m/s [right] and the vertical component is 156 m/s [down].



The magnitude of the total velocity vector is therefore

$$
\begin{aligned}
\bar{v} &= \sqrt{\bar{v}_x^2 + \bar{v}_y^2} \\
&= \sqrt{(125 \text{ m/s})^2 + (156 \text{ m/s})^2} = 200 \text{ m/s} \\
&= 2.00 \times 10^2 \text{ m/s}
\end{aligned}
$$

The package makes an angle with the horizontal given by

$$\theta = \tan^{-1}\left(\frac{\bar{v}_y}{\bar{v}_x}\right) = \tan^{-1}\left(\frac{156 \text{ m/s}}{125 \text{ m/s}}\right) = 51.3°$$

Note that this angle is below the horizontal.

The velocity of the package as it hits the ground is 2.00×10^2 m/s [51.3° below the horizontal].

It is interesting to note that since the horizontal component of the velocity of the package does not change, it is always directly under the airplane as the airplane flies overhead at a constant velocity. This is assuming that there is no air friction. In reality, air resistance would slow down the package and it would fall behind the airplane. Also, if a second package were dropped from a stationary balloon at the same height as the airplane, both packages would strike Earth at the same time. Not only would they strike the Earth at the same time, but the y-components of their velocities would be equal at all points on the way down.

Note, however, that the speeds with which the packages hit the ground are not the same. The speed of the package dropped from the airplane would have a greater speed because it also has an x-component of velocity, which the package from the balloon does not.

Learning Activity 3.3

Objects Projected Horizontally

Solve the following problems to check your understanding of projectile motion. An answer key is available at the end of Module 3 for you to check your work after you have answered the questions.

1. For projectile motion, what provides the net force that controls the motion of the projectile? What implication does this have for the motion of the projectile in the horizontal direction and in the vertical direction?
2. A marble is allowed to roll off the edge of a horizontal table. The marble falls vertically 0.950 m before it strikes the floor. The marble lands a horizontal distance of 18.8 cm from the edge of the table.
   a) Calculate the time it took the marble to fall from the level of the table to the level of the floor.
   b) What was the initial velocity of the marble as it rolled off the edge of the table?
   c) What was the final vertical velocity of the marble as it struck the floor?
   d) What was the final velocity of the marble as it struck the floor?

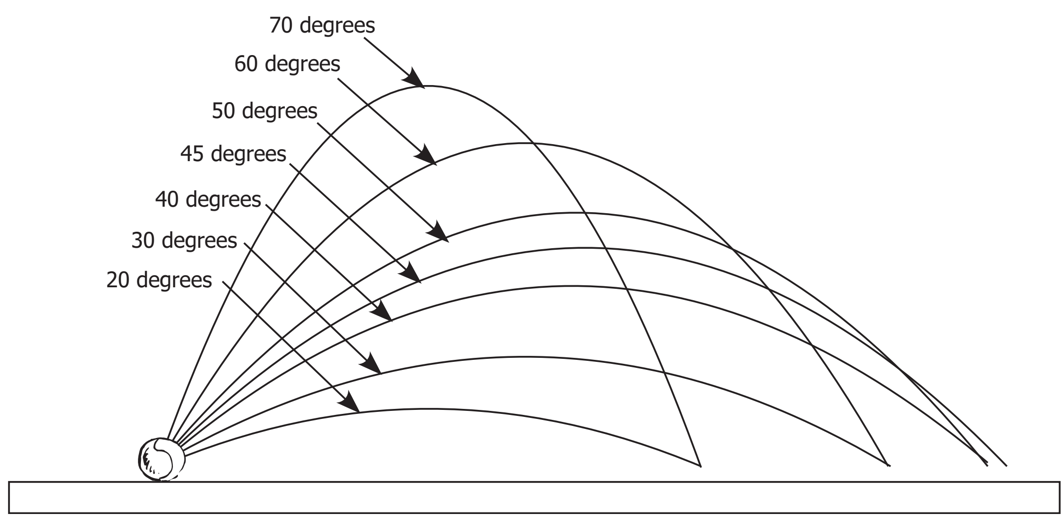
Projectile Motion for a Projection at an Angle: The Effect of the Launch Angle on Range

If you're throwing a baseball a question you might ask is "At what angle should I throw the ball so that it will travel a maximum horizontal distance in the air?"

> The horizontal distance a projectile travels while it is in the air is called the **range**.

From the graphic below, which shows the trajectories of a baseball thrown at different angles but all at the same speed of 40 m per second, you can see that the baseball will travel the largest horizontal distance when it is thrown at an angle of 45° with the horizontal.

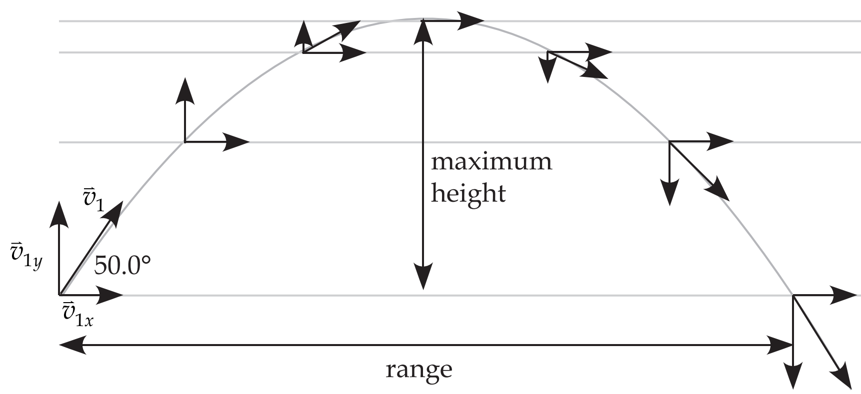
The implications of this are obvious in throwing a baseball but are also useful in games like golf and football where athletes must project the ball a certain distance.



Notice that the range of the baseball is the same for 20° and 70°, 30° and 60°, and 40° and 50°. The maximum range for the ball is at an angle of 45°.

Example 2: Projectile Motion for a Projection at an Angle: Solving the Problem

An interesting application of projectile motion for an object projected at an angle is the case of a football kicked at some angle to the horizontal.



In the diagram above, a football is kicked at an angle of 50.0° above the horizontal. It is kicked with an initial speed of 24.0 m/s. Ignore air resistance.

Calculate

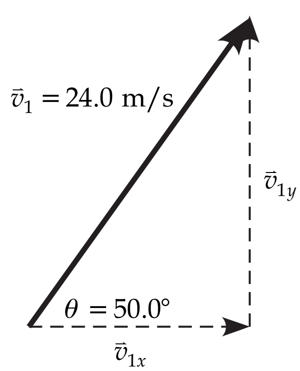
a) the maximum height reached by the football

b) its "hang time"

c) its "range"

Determining the Components of the Initial Velocity

When solving problems involving the objects launched at some angle to the horizontal, the first thing you should do is calculate the components of the initial velocity $(\vec{v}_1)$—that is, the initial velocity in the horizontal direction $(\vec{v}_{1x})$ and the initial velocity in the vertical direction $(\vec{v}_{1y})$. Any calculations that you do from that point forward must use these components of the initial velocity!



The horizontal component of the initial velocity is given by $\vec{v}_{1x} = \vec{v}_1(\cos\theta)$.

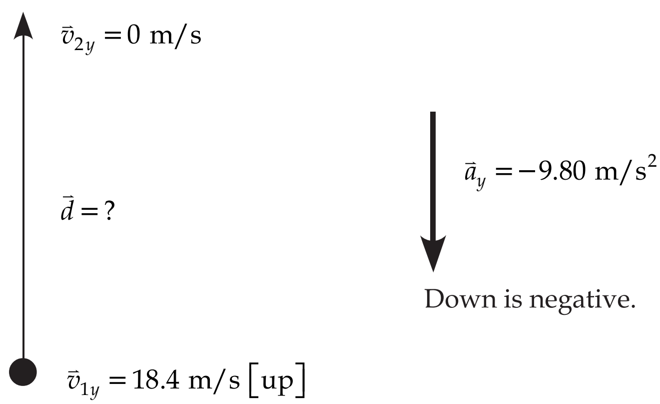
The vertical component of the initial velocity is given by $\vec{v}_{1y} = \vec{v}_1(\sin\theta)$.

In this case the horizontal component of the initial velocity is
$\vec{v}_{1x} = \vec{v}_1(\cos\theta) = (24.0 \text{ m/s})(\cos 50.0°) = 15.4 \text{ m/s [right]}$.

The vertical component of the initial velocity is
$\vec{v}_{1y} = \vec{v}_1(\sin\theta) = (24.0 \text{ m/s})(\sin 50.0°) = 18.4 \text{ m/s [up]}$.

Determining Maximum Height

To determine the maximum height that the ball reaches, remember that height is a vertical measurement. Therefore, you must use the equations for uniformly accelerated motion.



In order to find the maximum height, we can use the first half of the vertical motion. In this first half, the vertical component of the velocity at the top of the motion is 0 m/s. Also, the vertical component of the initial velocity is given by $\vec{v}(\sin\theta)$.

Since you do not yet know the time, to determine the maximum height, you must use $v_2^{\,2} = v_1^{\,2} + 2ad$.

Substituting and solving for "d"

$$d = \frac{v_2^{\,2} - v_1^{\,2}}{2a} = \frac{\left[(0 \text{ m/s})^2 - (18.4 \text{ m/s})^2\right]}{2(-9.80 \text{ m/s}^2)} = 17.3 \text{ m}$$

The ball's maximum height above the ground is 17.3 m.

This same height would have been reached if the football had been thrown straight up with an initial velocity of $\vec{v}(\sin\theta) = 18.4 \text{ m/s}$.

Determining "Hang Time"

> **"Hang time"** refers to the length of time that a projectile is in the air.

One of the ways to determine this time is to determine the time for one-half of the motion, and then multiply this by two. If you consider the first half of motion, then you can use $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$.

Solving for the time, $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \dfrac{0 \text{ m/s} - 18.4 \text{ m/s}}{-9.80 \text{ m/s}^2} = 1.88 \text{ s}.$

The total time, in other words the hang time, is 1.88 s x 2 = 3.76 s.

Determining the "Range"

> **"Range"** refers to the horizontal distance a projectile travels during its flight.

The horizontal part of the motion occurs at constant velocity. You must also use the total time of flight. From our previous study of the horizontal part of projectile motion, you use $\vec{d}_x = \vec{v}_{1x}\Delta t$.

$$\begin{aligned}\vec{d}_x = \vec{v}_{1x}\Delta t &= \left[24.0 \text{ m/s} \left(\cos 50.0°\right)\right]\left(3.76 \text{ s}\right)\\ &= \left(15.4 \text{ m/s}\right)\left(3.76 \text{ s}\right)\\ &= 58.0 \text{ m}\end{aligned}$$

The ball flies 58.0 m horizontally through the air.

The range of a projectile depends on the angle θ at which it is projected. As you saw earlier, the maximum range occurs when θ = 45°.

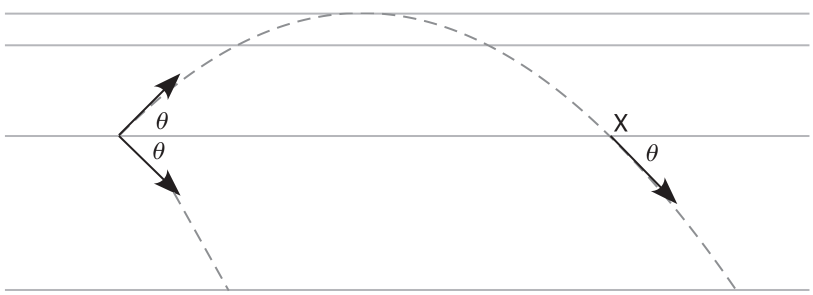
Learning Activity 3.4

Analyzing Projectile Motion

There are three practice questions in this learning activity. An answer key is available at the end of Module 3 for you to check your work after you have answered the questions.

The physics of throwing a stone (conceptual)

1. From the top of a hillside, a person throws two identical stones. The stones have identical initial speeds v_1. Stone 1 is thrown downward at some angle θ below the horizontal while stone 2 is thrown at the same angle θ above the horizontal. If we neglect air resistance, which stone, if either, strikes the surface of the ground with greater velocity?



The physics of a golf shot

2. A golf ball is struck by a club, leaving the face of the club travelling at 35.8 m/s [47.5° above the horizontal]. The ball sails over a level fairway.
   a) Determine the vertical and horizontal components of this velocity.
   b) For what length of time will the ball be in the air?
   c) Assuming the ball does not roll when it lands, what is the range of the ball?
   d) If the ball does roll after it lands, is it possible for this shot to finish in the hole? The hole is 135 m from the point where the ball is struck. Assume the ball is travelling in the correct direction.
   e) What is the velocity of the ball 4.50 seconds after it was struck?

(continued)

Learning Activity 3.4: Analyzing Projectile Motion (continued)

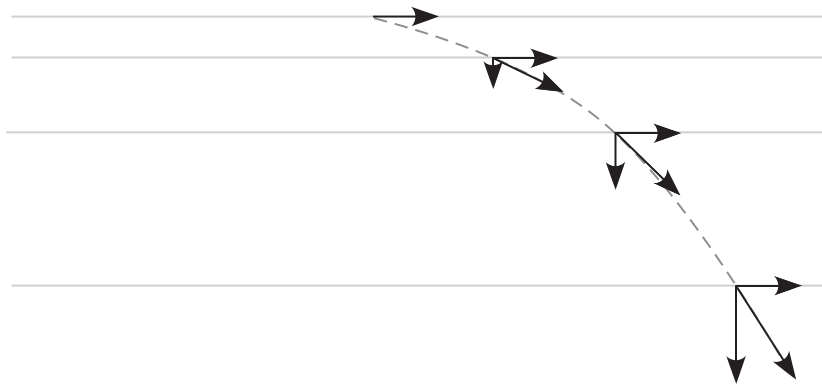
The physics of a ball launched from a rising helicopter

3. A helicopter is rising vertically with a uniform velocity of 15.0 m/s. When it is 225 m from the ground, a ball is thrown horizontally from the helicopter with a velocity of 10.0 m/s with respect to the helicopter. Determine each of the following:
   a) At what time will the ball reach the ground?
   b) Where will the ball land when it strikes the ground?
   c) With what velocity will the ball strike the ground?

Lesson Summary

In this lesson, we studied in detail the motion of objects projected horizontally and at some angle.

For objects projected horizontally, a typical diagram showing the x- and y-components of velocity, and the total velocity at different points along the motion, would be as follows:

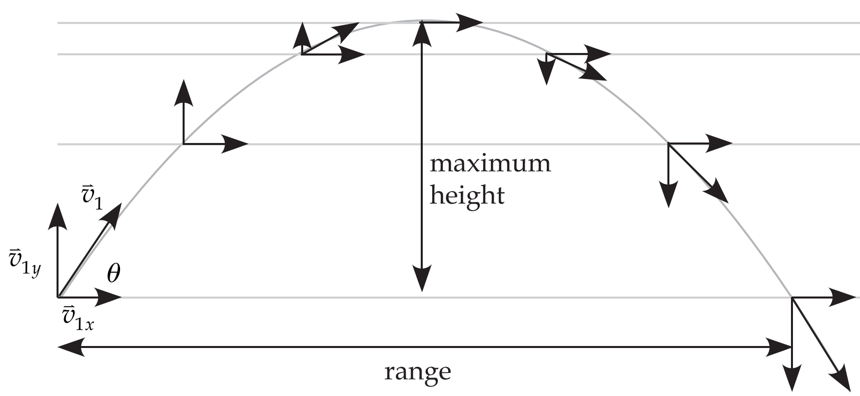


In this case, the horizontal component of the initial velocity is just equal to the initial velocity and is constant throughout the event.

$$\vec{v}_{1x} = \vec{v}_1$$

The vertical component of the initial velocity is 0 m/s. Remember the object accelerates downwards, so the vertical component of the velocity increases downwards.

For objects projected at some angle, a typical velocity diagram might look as follows:



Before you start doing calculations for these problems, determine the horizontal component of the initial velocity using $\vec{v}_{1x} = \vec{v}_1(\cos\theta)$ and the vertical component of the initial velocity using $\vec{v}_{1y} = \vec{v}_1(\sin\theta)$.

Again, the horizontal component of velocity is constant and the equation $\vec{d}_x = \vec{v}_{1x}\Delta t$ applies.

Also, in both cases, the vertical component of velocity changes while the acceleration due to gravity is constant. In this case, the equations for constant acceleration apply. They are

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

$$\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)t$$

$$\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}t^2$$

$$v_2^{\,2} = v_1^{\,2} + 2ad$$

These equations for horizontal and vertical motion can be used to solve for various aspects of projectile motion, including the time taken to rise or fall, the hang time, the *x*- and *y*-components of velocity, the total velocity, the maximum height, and the range.

Assignment 3.3

Projectile Motion of a Cannonball (10 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of a fired cannonball

A cannonball is fired with a velocity of 125 m/s at 25.0° above the horizontal.

a) Determine the horizontal and vertical components of the initial velocity.

b) Determine the maximum height the cannonball reaches in its path.

c) Determine the time it takes to reach maximum height.

(continued)

Assignment 3.3: Projectile Motion of a Cannonball (continued)

d) Determine the hang time of the cannonball.

e) Determine the range of the cannonball.

f) Draw a sketch of the motion of the projectile including the horizontal and vertical components of velocity when the projectile strikes the ground. Draw the appropriate relative lengths of the vectors for these components.

Method of Assessment

The total of 10 marks for this assignment will be determined as follows:

- 2 marks for determining the correct answer for parts (a) to (c)
- 1 mark for calculating the correct answer with the correct unit for parts (d) and (e)
- 2 marks for the component vectors showing the approximate relative lengths and the correct directions on the path of motion for part (f)

Video - Introduction to Projectile Motion

This video outlines a strategy for analyzing projectile motion problems.

Projectile motion is analyzed by considering the horizontal (x) direction and the vertical (y) direction as independent of each other.

The horizontal direction has a constant velocity with an acceleration of 0 m/s/s.

The vertical motion is controlled by the force of gravity which gives the projectile and acceleration of 9.81 m/s/s downwards. The equations of Uniformly Accelerated Motion (UAM) are applied to motion in the vertical direction.

https://youtu.be/GiiWsXtt5GE

Video - Nerd-A-Pult - An Introductory Projectile Motion Problem

This video uses a systematic approach to solving a projectile motion problem where the unknown quantity is a height (vertical displacement).

https://youtu.be/6PHwKrTGYxw

Video - (Part 1 of 2) An Introductory Projectile Motion Problem with an Initial Horizontal Velocity

This video analyzes the motion of a projectile that is initially moving in the horizontal direction.

https://youtu.be/wB_Uv8P-VAo

Video - Projectile Practice Problem 2

This problem is a variation of projectile motion where you must determine the initial velocity of the projectile, that is, the magnitude and direction of the initial velocity.

https://youtu.be/CJYHrHQgQUQ

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The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

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LESSON 4: UNIFORM CIRCULAR MOTION: SPEED AND VELOCITY (1 HOUR)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the following terms: uniform circular motion, cycle, period, frequency, hertz
- ❑ calculate the length of a cycle, the period of motion, and the frequency of motion for an object moving in a circle
- ❑ calculate the speed of motion of an object in uniform circular motion
- ❑ describe the direction of the velocity vector for an object in uniform circular motion

Key Words

cycle	uniform circular motion
hertz	frequency
period	

Introduction

In this lesson, we begin our study of circular motion. You may recall from earlier work that an object will move in a straight path if the net force acting on it is in the direction of motion. If the net force acts at some angle to the direction of motion, then the object will follow a curved path. We saw this earlier when we discussed projectile motion. Another important case of an object following a curved path is that of circular motion. There are many fascinating examples of objects moving in circular motion, ranging from simple situations such as a ball at the end of a string to a Ferris wheel at the Red River Exhibition to a jet fighter coming out of a dive. In this lesson, the focus will be to understand basic terms, and to concentrate on the ideas of speed and velocity.

Uniform Circular Motion: Where Does It Come From?

Uniform circular motion is the motion of an object travelling at a constant (uniform) speed on a circular path.

A model airplane attached to a guide wire, a car turning on a circular curve, and a person riding a merry-go-round are examples of uniform circular motion.

In uniform circular motion, the **magnitude** of the velocity (speed) **stays the same**, but the **direction** that the object is travelling is **always changing**. Velocity is a vector, which means that it has both a magnitude and a direction. If either one of these changes, there is a change in velocity and this means that there is an acceleration.

Even if a ball swinging at the end of a string doesn't speed up or slow down, its **direction is constantly changing**, so the ball is **accelerating**.

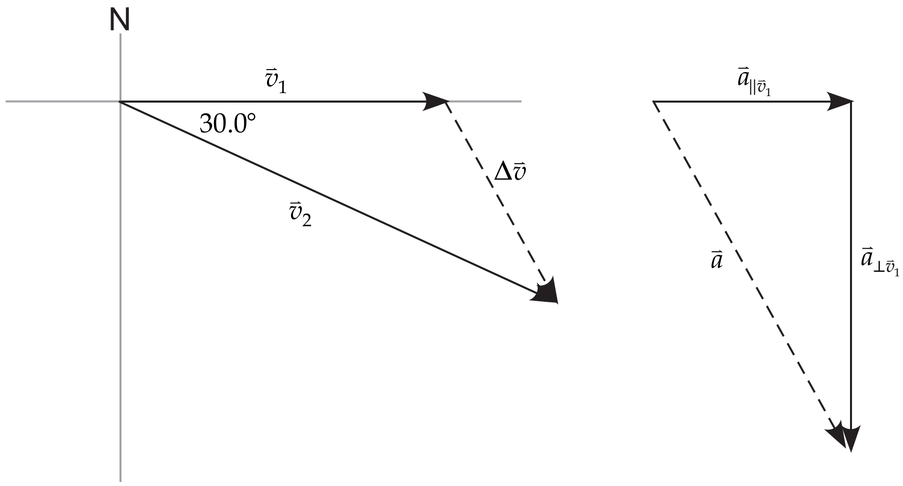
It is sometimes hard for us to understand this because if the ball never speeds up or slows down, we think that it is not accelerating. If the direction of the motion is always changing, however, it is accelerating because changing the direction of a velocity creates a new velocity and a change in velocity. Since acceleration involves how velocity changes with time, all the ingredients are present for acceleration.

Let's consider a car that is speeding up and travelling around a curve. At one moment, the car is travelling at 10.0 m/s [east] and 10.0 seconds later is travelling at 15.0 m/s 30.0° [south of east].

From this information, you can determine the acceleration. Let's begin with a vector diagram to determine the change in velocity. You know that the change in velocity is found by taking the final velocity $(\vec{v}_2)$ and subtracting from it the initial velocity $(\vec{v}_1)$.

Normally you determine the change in velocity by using the equation $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. Here you add the opposite of the initial velocity by placing the tail of the vector $-\vec{v}_1$ onto the head of the vector $\vec{v}_2$. The resultant $\Delta\vec{v}$ is drawn from the tail of $\vec{v}_2$ to the head of $-\vec{v}_1$.

Let's change the question around a little bit. You can also find the change in velocity $\Delta\vec{v}$ by rearranging the equation to $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$. Now $\vec{v}_2$ is the resultant of the addition of $\Delta\vec{v}$ to $\vec{v}_1$. The diagram, in this case, begins with the tail of the initial velocity vector and the tail of the final velocity vector drawn at the same point. The two velocity vectors are then drawn in. The change in velocity $\Delta\vec{v}$ is then drawn from the head of the initial velocity vector to the head of the final velocity vector. This will look a lot simpler in a sketch.



Recall that acceleration is found using $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$.

You can see that the change in velocity $\Delta\vec{v}$ points east of south. If you divide the change in velocity by the time interval, you'll get the average acceleration $\vec{a}$. The average acceleration vector is drawn just to the right of the velocity vectors.

You'll notice that the acceleration vector has been resolved into an easterly component and a southerly component.

The easterly component is designated by $\vec{a}_{\|\vec{v}_1}$, the component of the acceleration parallel to the initial velocity. If you ignore the other component for a moment, you'll see that the parallel component of the acceleration points in the same direction as the initial velocity. This situation where the velocity and the acceleration point in the same direction results in straight-line motion. The parallel component of the acceleration serves to change the **magnitude** of the initial velocity.

A southerly component of the acceleration is designated by $\vec{a}_{\perp\vec{v}_1}$, the component of the acceleration perpendicular to the initial velocity. If this perpendicular component of the acceleration is acting alone, the object would not speed up or slow down. Remember, speeding up or slowing down the object was the job of the parallel component of the acceleration. So if the object does not speed up or slow down, then what does the perpendicular component do? It turns out the perpendicular component's job is to change the **direction** of the initial velocity.

In this case, the acceleration had a component parallel to the initial velocity $\left(\vec{a}_{\|\vec{v}_1}\right)$ and also a component perpendicular to the initial velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$, so the acceleration changed both the magnitude and direction of the initial velocity $\vec{v}_1$. You see that the velocity changed from 10.0 m/s [east] to 15.0 m/s [30.0° S of E]. Both the **magnitude** and **direction** of the initial velocity were changed.

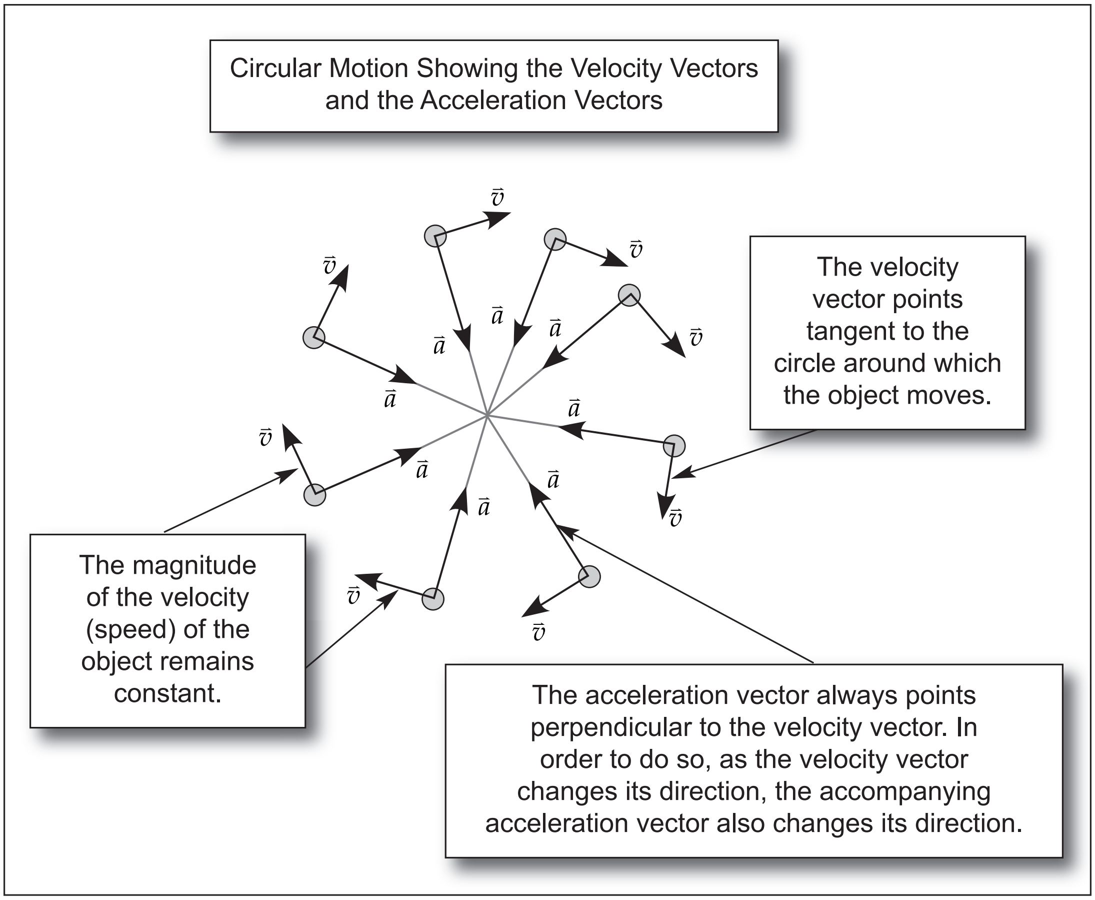
If only the component of the acceleration **parallel** to the initial velocity $\left(\vec{a}_{\|\vec{v}_1}\right)$ acts, only the **magnitude** of the initial velocity is changed. The result is **straight-line motion**.

If both the **parallel** component and the **perpendicular** component of the acceleration act, both the **magnitude** and **direction** of the initial velocity are changed. The result is a motion where the object **speeds up** and **turns**.

If only the components of the acceleration **perpendicular** to the initial velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$ acts, only the **direction** of the initial velocity is changed. This acceleration must change in direction to remain always perpendicular to the velocity. The result must be a motion where the object **does not speed up or slow down** but only **turns**. The motion that results is **uniform circular motion**.

Now, in order for there to be no change in speed (magnitude of velocity), the component of the acceleration parallel to the velocity $\left(\vec{a}_{\|\vec{v}_1}\right)$ must always be zero.

Therefore, in uniform circular motion, only the component of the acceleration perpendicular to the velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$ must be present. But the velocity in uniform circular motion is always changing its direction. So the acceleration must always change its direction to remain perpendicular to the velocity.



We'll come back to these ideas in another lesson.

Uniform Circular Motion: The Cycle, Period, and Frequency

Uniform circular motion is the motion of an object travelling at a constant (uniform) speed on a circular path.

Let's begin by examining the case of an object, such as a model airplane, moving counter-clockwise at the end of a string in a horizontal circle. To describe this motion, it is important to understand important terms.

A **cycle** can be defined as one complete motion around the circle.

So, for example, if a model airplane moves around the circle three times, then we would say that three cycles of the motion have been completed.

The **period** (T) of the motion can be defined as the time required to complete one complete cycle of the motion. Since this is a unit of time, the standard unit of time would be the second.

The **frequency** (f) of motion can be defined as the number of complete cycles per second. The unit used to measure frequency is the **hertz** (Hz), named after the scientist Heinrich Hertz (1857–1894). One hertz is one cycle per second.

The period of motion and the frequency of motion are inverses of each other.

$$\text{frequency} = \frac{\text{cycles}}{\text{time}} \qquad f = \frac{1}{T} \qquad \text{standard unit: hertz (Hz)}$$

$$\text{period} = \frac{\text{time}}{\text{cycles}} \qquad T = \frac{1}{f} \qquad \text{standard unit: second (s)}$$

For the model airplane moving in a circle, assume that the airplane completes 10.0 revolutions around the circle in a time of 5.00 s. In this case, the airplane has completed 10.0 cycles. The period of motion is the total time of 5.00 s divided by 10.0 cycles. This makes the period of motion 0.500 s. In other words, it takes 0.500 s to complete one revolution or cycle.

$$\text{period} = \frac{\text{time}}{\text{cycles}} = \frac{5.00\text{ s}}{10.0\text{ cycles}} = 0.500\text{ s}$$

The frequency of motion is the number of cycles, 10.0 cycles, divided by the time required to complete the motion, 5.00 s. Thus, the frequency of motion is 2.00 Hz. This means that 2.00 cycles are completed every second.

$$\text{frequency} = \frac{\text{cycles}}{\text{time}} = \frac{10.0\text{ cycles}}{5.00\text{ s}} = 2.00\text{ Hz}$$

Note also in this example that the period of motion and the frequency of motion are inverses of each other. For example,

$$T = \frac{1}{f}$$

$$T = \frac{1}{2.00\text{ Hz}} = 0.500\text{ s}$$

Learning Activity 3.5

Circular Motion Terms

Answer the following questions to check your understanding of circular motion. Once you have answered the questions, you may check your work against the answer key provided at the end of Module 3.

Fill in the blanks.

1. In uniform circular motion, the velocity of the object has a constant ____________ but a changing ______________.
2. In uniform circular motion, the direction of the velocity is ____.
3. The acceleration in uniform circular motion points ___________________________.
4. The time to complete one cycle is known as the _________________. It has units of _____________________.
5. The number of cycles completed per second in uniform circular motion is known as the ______________________. It has units of _________________________.
6. Describe the function of each of the following:
   a) the component of acceleration parallel to the velocity
   b) the component of the acceleration perpendicular to the velocity
7. What kind of motion will an object exhibit if
   a) only the component of acceleration parallel to the velocity acts?
   b) only the component of acceleration perpendicular to the velocity acts?
   c) both the component of acceleration parallel to velocity and the component of acceleration perpendicular to velocity act?

Uniform Circular Motion: Speed and Velocity

Let's examine again the model airplane moving in a circle. To determine the speed of the airplane, you would need to know the distance travelled and the time taken. The airplane is moving at a constant speed. In this case, you know that the speed can be determined using

$$v = \frac{d}{\Delta t} \text{ or speed} = \frac{\text{distance travelled}}{\text{time interval}}.$$

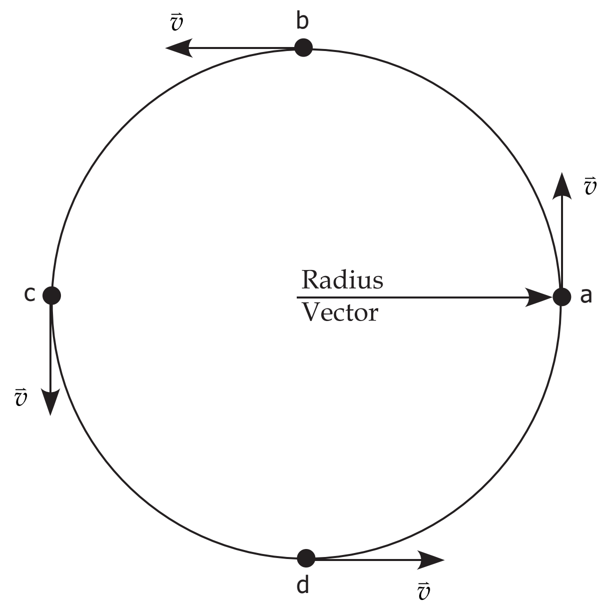
The distance the plane moves is the circumference of the circle given by $C = 2\pi R$, and the time taken is just the period of motion. Therefore, to determine the speed of any object moving in uniform circular motion, you can use $v = \frac{2\pi R}{T}$. Since the period of motion and the frequency are inverses of each other, another method to determine speed could be $v = 2\pi Rf$.

The **speed** of an object moving with uniform circular motion in a circle of radius, R, is calculated by taking the circumference of the circle, $2\pi R$, and dividing by the period.

$$v = \frac{2\pi R}{T}$$

Quantity	**Symbol**	**Unit**
Speed	v	metres/second (m/s)
Radius	R	metres (m)
Period	T	seconds (s)

Now let's look at the **direction** of the velocity vector. At any time in uniform circular motion, the direction of the velocity is **tangent** to the circle. Another way of saying this is that the velocity vector is perpendicular to the radius. The diagram below shows four different points on the circle. Note that for the object to be moving counter-clockwise, the direction of the velocity is as shown in the following diagram.



Note that in uniform circular motion, the magnitude of the velocity vector is constant. Although the direction of the velocity vector is always tangent to the circle, the direction of this vector is not constant. For example, in the diagram above, at point "a" the velocity vector is pointing straight up but at point "b" it is pointing to the left.

Uniform Circular Motion: An Example

Suppose that the model airplane referred to earlier was moving in a circle of radius 10.0 m at 30.0 revolutions per minute (rpm). Determine the speed of the airplane in m/s.

To solve this problem, you could begin by determining the distance travelled by the plane (one circumference of the circle) during the time interval of one period of the motion.

Given: Radius $R = 10.0$ m

Frequency $f = 30.0$ revolutions/minutes or

$f = 30.0$ revolutions/60 seconds $= 0.500$ Hz

Period $T = \dfrac{1}{f} = \dfrac{1}{0.500\ \text{Hz}} = 2.00$ s

Unknown: Speed $v = ?$

Equation: $v = \dfrac{2\pi R}{T}$

Substitute and solve:

$$v = \frac{2\pi R}{T}$$

$$v = \frac{2\pi\left(10.0\ \text{m}\right)}{2.00\ \text{s}} = 31.4\ \text{m/s}$$

Another way to determine the speed would be to use $v = 2\pi Rf$. The frequency must be expressed in units of Hz or revolutions per second. In this case, the frequency is

$$f = \frac{30.0 \text{ revolutions}}{60.0 \text{ seconds}} = 0.500 \text{ Hz}$$

Therefore, the speed of the airplane is $v = 2\pi(10.0 \text{ m})(0.500 \text{ Hz}) = 31.4 \text{ m/s}$.

Since speed is a scalar, no direction is required.

Learning Activity 3.6

Circular Motion: Basic Quantities

Answer the following practice questions to check your understanding of the basic quantities of uniform circular motion. An answer key is available at the end of Module 3 for you to check your work after you have answered the questions.

The physics of a car wheel moving in a circle

1. The wheel of a car has a radius of 0.280 m and is rotating at a rate of 820 revolutions per minute (rpm) (3 significant digits) as the car travels down a highway.
   a) What is the period of motion?
   b) What is the frequency of motion?
   c) What is the length of one cycle of the motion?
   d) Determine the speed of motion (in m/s) using the period of motion.
   e) Determine the speed of motion (in m/s) using the frequency of motion.
   f) Draw a diagram showing a circle. Assume that the wheel is moving clockwise. Draw a velocity vector at the top of the circle and another velocity vector at the bottom of the circle. Be sure to pay attention to both the relative lengths and directions of the vectors.

(continued)

Learning Activity 3.6: Circular Motion: Basic Quantities (continued)

The physics of blades in a food blender

2. The tips of the blades in a food blender can move at a speed of 22.0 m/s. The circle in which they move can have a radius of 0.0500 m.
   a) What is the length of one cycle of the motion?
   b) What is the period of motion?
   c) What is the frequency of motion?
   d) Draw a diagram showing a circle. Assume that the blade is moving counter-clockwise. Draw a velocity vector at the top of the circle and another velocity vector at the bottom of the circle. Be sure to pay attention to both the relative lengths and directions of the vectors.

Lesson Summary

In this lesson, we began our study of circular motion. **Uniform circular motion** is defined as the motion of an object travelling at a constant (uniform) speed on a circular path.

A cycle can be defined as one complete motion around the circle. The **period** (*T*) of the motion can be defined as the time required to complete one cycle of the motion. The unit of period is the second (s). The frequency (*f*) of motion can be defined as the number of complete cycles per second. The unit of frequency is the hertz (Hz). One hertz is one cycle per second. The period of motion and the frequency of motion are inverses of each other.

$$\text{frequency} = \frac{\text{cycles}}{\text{time}} \qquad f = \frac{1}{T} \qquad \text{standard unit: hertz (Hz)}$$

$$\text{period} = \frac{\text{time}}{\text{cycles}} \qquad T = \frac{1}{f} \qquad \text{standard unit: second (s)}$$

To determine the speed of any object moving in uniform circular motion, we can use $v = \frac{2\pi R}{T}$. Since the period of motion and the frequency are inverses of each other, another method to determine speed could be $v = 2\pi Rf$.

At any time in uniform circular motion, the direction of the velocity is tangent to the circle. Another way of saying this is that the velocity vector is perpendicular to the radius. In uniform circular motion, the magnitude of the velocity vector is constant.

Notes

Video - Circular Motion Demonstration with Sparkler

This is a simple demonstration using a sparkler attached to a drill. As the drill drives the sparkler in a circle the sparks are seen flying off the sparkler tangent to the circular motion. This is Newton's First Law.

The second demonstration uses a hula hoop and a tennis ball. The ball is projected along the inside edge of the hoop and travels in a circle around the inside of the hoop. The hoop exerts a force to push the ball into its circular path. When the hoop is removed the ball travels in a straight line again demonstrating Newton's First Law.

https://youtu.be/ID0R43My4Co

Video - Race cars with constant speed around curve

This video introduces the concept that objects moving with constant speed can be accelerating. This is due to the fact that the car has a velocity, that is, it is going in a certain direction with a certain speed. Since the direction of the velocity is changing there must be a change in velocity. A change in velocity over a time interval yields acceleration.

https://youtu.be/ITA1rW5UraU

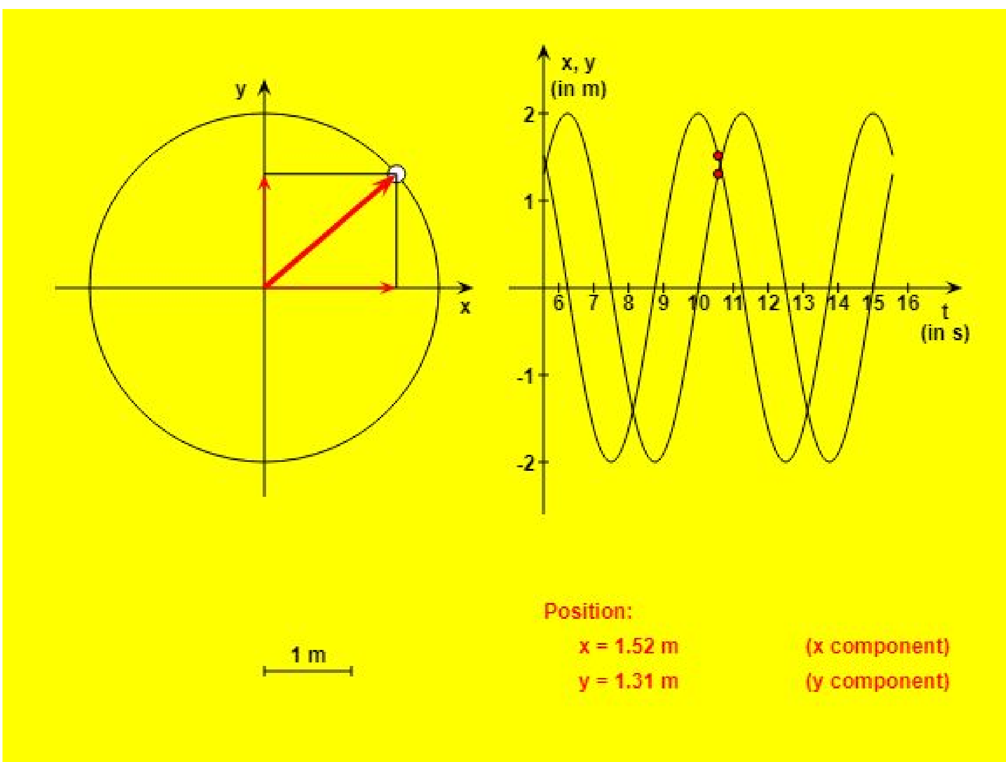
https://www.walter-fendt.de/html5/phen/

https://www.walter-fendt.de/html5/phen/circularmotion_en.htm

Uniform Circular Motion

Circular motion plays an important role in nature and technology. So, the planets move on (approximately) circular orbits around the sun. Other examples are the rotating armature of an electric motor or the crankshaft of a gasoline engine.

This HTML5 app simulates such a circular motion and demonstrates how position, velocity, acceleration and acting force vary in time. The "Reset" button brings the rotating body in its initial position. You can start or stop and continue the simulation with the other button. If you choose the option "Slow motion", the movement will be ten times slower. You can adjust radius, period and mass by using the corresponding input fields. The radio buttons give the possibility to select one of four physical sizes.



The radius vector (red) joins the center of rotation (the origin of the coordinate system) to the revolving body. The velocity vector (violet) is tangential to the circle and perpendicular to the radius vector. The vector of acceleration (blue), surprisingly, is directed inside (towards the center). Here acceleration doesn't mean an increase or decrease of speed (magnitude of velocity), but a varying direction of motion. The same is true for the force (green) on the moving body. The terms centripetal acceleration and centripetal force express that these vectors are directed towards the center of the circular motion.

Lesson 4: Uniform Circular Motion: Speed and Velocity (1 hour)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the following terms: uniform circular motion, cycle, period, frequency, hertz
- ❑ calculate the length of a cycle, the period of motion, and the frequency of motion for an object moving in a circle
- ❑ calculate the speed of motion of an object in uniform circular motion
- ❑ describe the direction of the velocity vector for an object in uniform circular motion

Key Words

cycle	uniform circular motion
hertz	frequency
period	

Introduction

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Even if a ball swinging at the end of a string doesn't speed up or slow down, its **direction is constantly changing**, so the ball is **accelerating**.

It is sometimes hard for us to understand this because if the ball never speeds up or slows down, we think that it is not accelerating. If the direction of the motion is always changing, however, it is accelerating because changing the direction of a velocity creates a new velocity and a change in velocity. Since acceleration involves how velocity changes with time, all the ingredients are present for acceleration.

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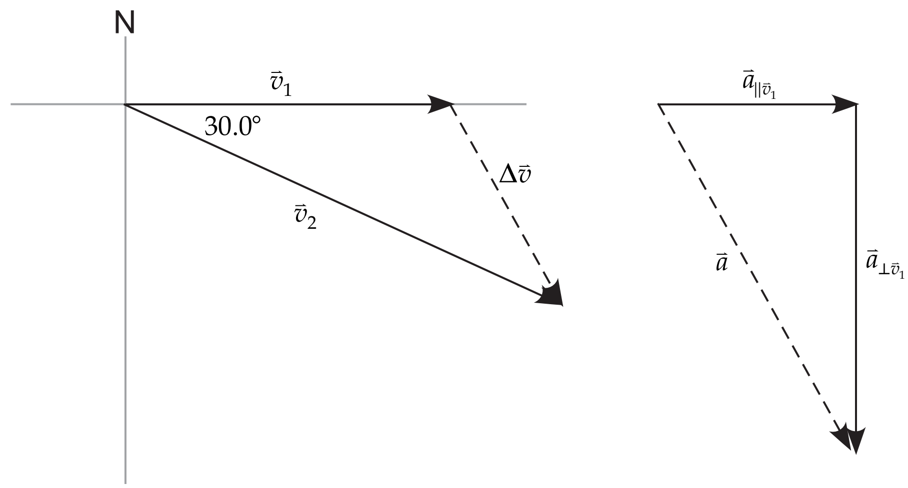
Let's change the question around a little bit. You can also find the change in velocity $\Delta\bar{v}$ by rearranging the equation to $\bar{v}_2 = \bar{v}_1 + \Delta\bar{v}$. Now $\bar{v}_2$ is the resultant of the addition of $\Delta\bar{v}$ to $\bar{v}_1$. The diagram, in this case, begins with the tail of the initial velocity vector and the tail of the final velocity vector drawn at the same point. The two velocity vectors are then drawn in. The change in velocity $\Delta\bar{v}$ is then drawn from the head of the initial velocity vector to the head of the final velocity vector. This will look a lot simpler in a sketch.



Recall that acceleration is found using $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$.

You can see that the change in velocity $\Delta\vec{v}$ points east of south. If you divide the change in velocity by the time interval, you'll get the average acceleration $\vec{a}$. The average acceleration vector is drawn just to the right of the velocity vectors.

You'll notice that the acceleration vector has been resolved into an easterly component and a southerly component.

The easterly component is designated by $\vec{a}_{\|\vec{v}_1}$, the component of the acceleration parallel to the initial velocity. If you ignore the other component for a moment, you'll see that the parallel component of the acceleration points in the same direction as the initial velocity. This situation where the velocity and the acceleration point in the same direction results in straight-line motion. The parallel component of the acceleration serves to change the **magnitude** of the initial velocity.

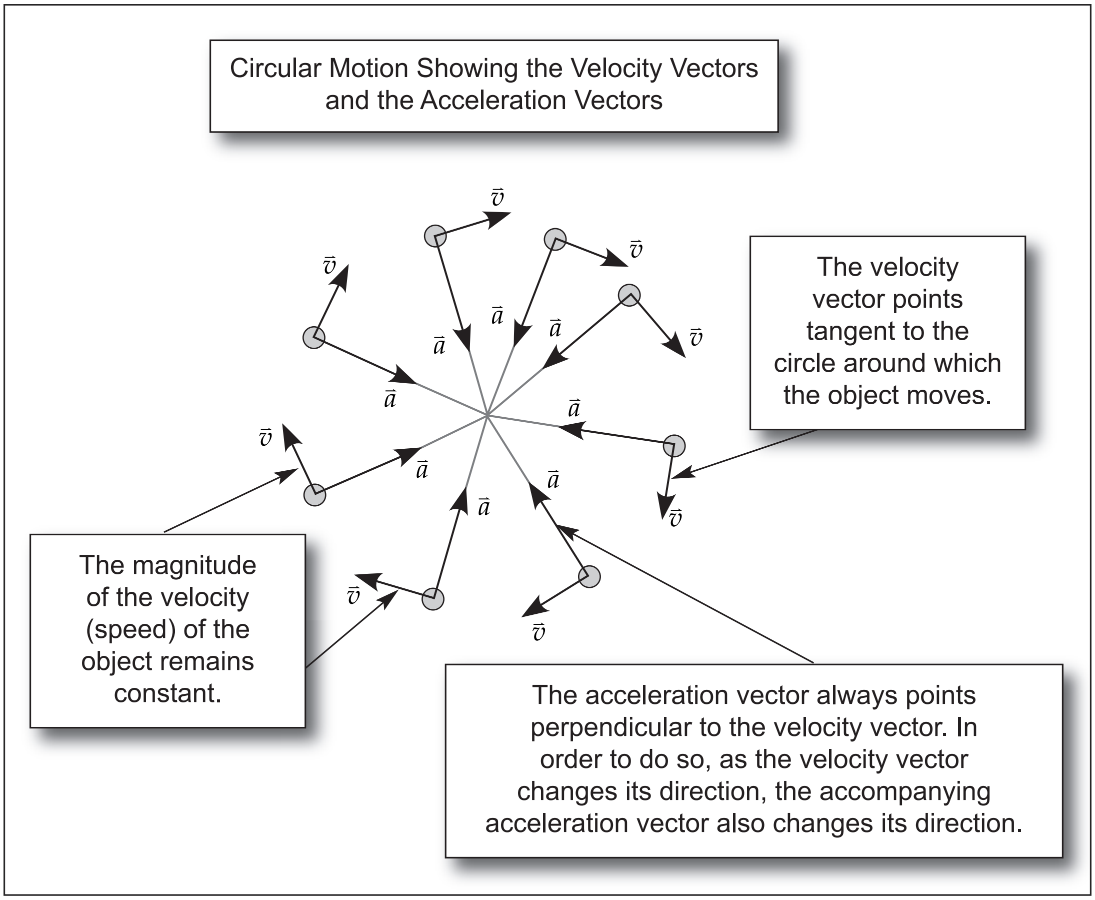
A southerly component of the acceleration is designated by $\vec{a}_{\perp\vec{v}_1}$, the component of the acceleration perpendicular to the initial velocity. If this perpendicular component of the acceleration is acting alone, the object would not speed up or slow down. Remember, speeding up or slowing down the object was the job of the parallel component of the acceleration. So if the object does not speed up or slow down, then what does the perpendicular component do? It turns out the perpendicular component's job is to change the **direction** of the initial velocity.

In this case, the acceleration had a component parallel to the initial velocity $\left(\vec{a}_{\parallel\vec{v}_1}\right)$ and also a component perpendicular to the initial velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$, so the acceleration changed both the magnitude and direction of the initial velocity $\vec{v}_1$. You see that the velocity changed from 10.0 m/s [east] to 15.0 m/s [30.0° S of E]. Both the **magnitude** and **direction** of the initial velocity were changed.

> If only the component of the acceleration **parallel** to the initial velocity $\left(\vec{a}_{\parallel\vec{v}_1}\right)$ acts, only the **magnitude** of the initial velocity is changed. The result is **straight-line motion**.
>
> If both the **parallel** component and the **perpendicular** component of the acceleration act, both the **magnitude** and **direction** of the initial velocity are changed. The result is a motion where the object **speeds up** and **turns**.
>
> If only the components of the acceleration **perpendicular** to the initial velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$ acts, only the **direction** of the initial velocity is changed. This acceleration must change in direction to remain always perpendicular to the velocity. The result must be a motion where the object **does not speed up or slow down** but only **turns**. The motion that results is **uniform circular motion**.

Now, in order for there to be no change in speed (magnitude of velocity), the component of the acceleration parallel to the velocity $\left(\vec{a}_{\parallel\vec{v}_1}\right)$ must always be zero.

Therefore, in uniform circular motion, only the component of the acceleration perpendicular to the velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$ must be present. But the velocity in uniform circular motion is always changing its direction. So the acceleration must always change its direction to remain perpendicular to the velocity.



We'll come back to these ideas in another lesson.

Uniform Circular Motion: The Cycle, Period, and Frequency

> **Uniform circular motion** is the motion of an object travelling at a constant (uniform) speed on a circular path.

Let's begin by examining the case of an object, such as a model airplane, moving counter-clockwise at the end of a string in a horizontal circle. To describe this motion, it is important to understand important terms.

> A **cycle** can be defined as one complete motion around the circle.

So, for example, if a model airplane moves around the circle three times, then we would say that three cycles of the motion have been completed.

The **period** (T) of the motion can be defined as the time required to complete one complete cycle of the motion. Since this is a unit of time, the standard unit of time would be the second.

The **frequency** (f) of motion can be defined as the number of complete cycles per second. The unit used to measure frequency is the **hertz** (Hz), named after the scientist Heinrich Hertz (1857–1894). One hertz is one cycle per second.

The period of motion and the frequency of motion are inverses of each other.

$$\text{frequency} = \frac{\text{cycles}}{\text{time}} \qquad f = \frac{1}{T} \qquad \text{standard unit: hertz (Hz)}$$

$$\text{period} = \frac{\text{time}}{\text{cycles}} \qquad T = \frac{1}{f} \qquad \text{standard unit: second (s)}$$

For the model airplane moving in a circle, assume that the airplane completes 10.0 revolutions around the circle in a time of 5.00 s. In this case, the airplane has completed 10.0 cycles. The period of motion is the total time of 5.00 s divided by 10.0 cycles. This makes the period of motion 0.500 s. In other words, it takes 0.500 s to complete one revolution or cycle.

$$\text{period} = \frac{\text{time}}{\text{cycles}} = \frac{5.00 \text{ s}}{10.0 \text{ cycles}} = 0.500 \text{ s}$$

The frequency of motion is the number of cycles, 10.0 cycles, divided by the time required to complete the motion, 5.00 s. Thus, the frequency of motion is 2.00 Hz. This means that 2.00 cycles are completed every second.

$$\text{frequency} = \frac{\text{cycles}}{\text{time}} = \frac{10.0 \text{ cycles}}{5.00 \text{ s}} = 2.00 \text{ Hz}$$

Note also in this example that the period of motion and the frequency of motion are inverses of each other. For example,

$$T = \frac{1}{f}$$

$$T = \frac{1}{2.00 \text{ Hz}} = 0.500 \text{ s}$$

Learning Activity 3.5

Circular Motion Terms

Answer the following questions to check your understanding of circular motion. Once you have answered the questions, you may check your work against the answer key provided at the end of Module 3.

Fill in the blanks.

1. In uniform circular motion, the velocity of the object has a constant ____________ but a changing ______________.
2. In uniform circular motion, the direction of the velocity is ____.
3. The acceleration in uniform circular motion points ___________________________.
4. The time to complete one cycle is known as the _________________. It has units of _____________________.
5. The number of cycles completed per second in uniform circular motion is known as the _____________________. It has units of ________________________.
6. Describe the function of each of the following:
   a) the component of acceleration parallel to the velocity
   b) the component of the acceleration perpendicular to the velocity
7. What kind of motion will an object exhibit if
   a) only the component of acceleration parallel to the velocity acts?
   b) only the component of acceleration perpendicular to the velocity acts?
   c) both the component of acceleration parallel to velocity and the component of acceleration perpendicular to velocity act?

Uniform Circular Motion: Speed and Velocity

Let's examine again the model airplane moving in a circle. To determine the speed of the airplane, you would need to know the distance travelled and the time taken. The airplane is moving at a constant speed. In this case, you know that the speed can be determined using

$$v = \frac{d}{\Delta t} \text{ or speed} = \frac{\text{distance travelled}}{\text{time interval}}.$$

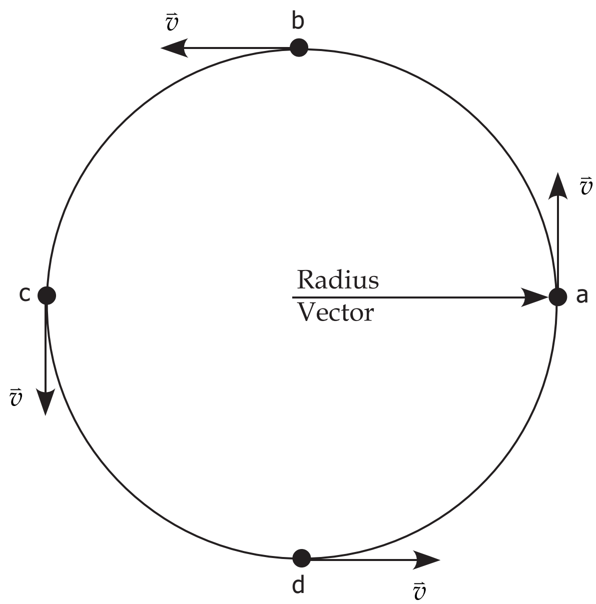
The distance the plane moves is the circumference of the circle given by $C = 2\pi R$, and the time taken is just the period of motion. Therefore, to determine the speed of any object moving in uniform circular motion, you can use $v = \frac{2\pi R}{T}$. Since the period of motion and the frequency are inverses of each other, another method to determine speed could be $v = 2\pi Rf$.

The **speed** of an object moving with uniform circular motion in a circle of radius, *R*, is calculated by taking the circumference of the circle, $2\pi R$, and dividing by the period.

$$v = \frac{2\pi R}{T}$$

Quantity	**Symbol**	**Unit**
Speed	v	metres/second (m/s)
Radius	R	metres (m)
Period	T	seconds (s)

Now let's look at the **direction** of the velocity vector. At any time in uniform circular motion, the direction of the velocity is **tangent** to the circle. Another way of saying this is that the velocity vector is perpendicular to the radius. The diagram below shows four different points on the circle. Note that for the object to be moving counter-clockwise, the direction of the velocity is as shown in the following diagram.



Note that in uniform circular motion, the magnitude of the velocity vector is constant. Although the direction of the velocity vector is always tangent to the circle, the direction of this vector is not constant. For example, in the diagram above, at point "a" the velocity vector is pointing straight up but at point "b" it is pointing to the left.

Uniform Circular Motion: An Example

Suppose that the model airplane referred to earlier was moving in a circle of radius 10.0 m at 30.0 revolutions per minute (rpm). Determine the speed of the airplane in m/s.

To solve this problem, you could begin by determining the distance travelled by the plane (one circumference of the circle) during the time interval of one period of the motion.

Given: Radius $R = 10.0$ m

Frequency $f = 30.0$ revolutions/minutes or

$f = 30.0$ revolutions/60 seconds = 0.500 Hz

Period $$T = \frac{1}{f} = \frac{1}{0.500\ \text{Hz}} = 2.00\ \text{s}$$

Unknown: Speed $v = ?$

Equation: $$v = \frac{2\pi R}{T}$$

Substitute and solve: $$v = \frac{2\pi R}{T}$$

$$v = \frac{2\pi\left(10.0\ \text{m}\right)}{2.00\ \text{s}} = 31.4\ \text{m/s}$$

Another way to determine the speed would be to use $v = 2\pi Rf$. The frequency must be expressed in units of Hz or revolutions per second. In this case, the frequency is

$$f = \frac{30.0 \text{ revolutions}}{60.0 \text{ seconds}} = 0.500 \text{ Hz}$$

Therefore, the speed of the airplane is $v = 2\pi(10.0 \text{ m})(0.500 \text{ Hz}) = 31.4 \text{ m/s}$.

Since speed is a scalar, no direction is required.

Learning Activity 3.6

Circular Motion: Basic Quantities

Answer the following practice questions to check your understanding of the basic quantities of uniform circular motion. An answer key is available at the end of Module 3 for you to check your work after you have answered the questions.

The physics of a car wheel moving in a circle

1. The wheel of a car has a radius of 0.280 m and is rotating at a rate of 820 revolutions per minute (rpm) (3 significant digits) as the car travels down a highway.
   a) What is the period of motion?
   b) What is the frequency of motion?
   c) What is the length of one cycle of the motion?
   d) Determine the speed of motion (in m/s) using the period of motion.
   e) Determine the speed of motion (in m/s) using the frequency of motion.
   f) Draw a diagram showing a circle. Assume that the wheel is moving clockwise. Draw a velocity vector at the top of the circle and another velocity vector at the bottom of the circle. Be sure to pay attention to both the relative lengths and directions of the vectors.

(continued)

The physics of blades in a food blender

2. The tips of the blades in a food blender can move at a speed of 22.0 m/s. The circle in which they move can have a radius of 0.0500 m.
   a) What is the length of one cycle of the motion?
   b) What is the period of motion?
   c) What is the frequency of motion?
   d) Draw a diagram showing a circle. Assume that the blade is moving counter-clockwise. Draw a velocity vector at the top of the circle and another velocity vector at the bottom of the circle. Be sure to pay attention to both the relative lengths and directions of the vectors.

Lesson Summary

In this lesson, we began our study of circular motion. **Uniform circular motion** is defined as the motion of an object travelling at a constant (uniform) speed on a circular path.

A cycle can be defined as one complete motion around the circle. The **period** (T) of the motion can be defined as the time required to complete one cycle of the motion. The unit of period is the second (s). The frequency (f) of motion can be defined as the number of complete cycles per second. The unit of frequency is the hertz (Hz). One hertz is one cycle per second. The period of motion and the frequency of motion are inverses of each other.

$$\text{frequency} = \frac{\text{cycles}}{\text{time}} \qquad f = \frac{1}{T} \qquad \text{standard unit: hertz (Hz)}$$

$$\text{period} = \frac{\text{time}}{\text{cycles}} \qquad T = \frac{1}{f} \qquad \text{standard unit: second (s)}$$

To determine the speed of any object moving in uniform circular motion, we can use $v = \frac{2\pi R}{T}$. Since the period of motion and the frequency are inverses of each other, another method to determine speed could be $v = 2\pi Rf$.

At any time in uniform circular motion, the direction of the velocity is tangent to the circle. Another way of saying this is that the velocity vector is perpendicular to the radius. In uniform circular motion, the magnitude of the velocity vector is constant.

NOTES

Video Assignment 3.4 - Forces in Equilibrium

Watch video or Assignent 3.4- Go to Track 2.

https://youtu.be/i4VHT7fOtXE?list=PLw1g3n2IMV7M72rewl81rI7b-CR0k8Wta

Assignment 3.4

Video Laboratory Activity: Circular Motion (20 MARKS)

You must complete the following lab report and submit it to the Distance Learning Unit for evaluation. Please complete your work in the space provided. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

Purpose

To find the relationship between the centripetal force $\left(\vec{F}_c\right)$ on an object in constant circular motion and the period of revolution (T).

Apparatus

Hollow tube, stopwatch, nylon string, rubber stopper, masses, tape, metre stick, graph paper (or Excel spreadsheet)

Procedure

To measure the time it takes for one revolution to complete, it is better to count the time for 20 revolutions and divide the time by 20 (to minimize potential error) than to count the time for only one revolution.

1. Adjust the length of the string so that the distance from the centre of the rubber stopper is about 1 m.
2. Place a piece of tape on the string a few centimetres from the bottom of the tube.
3. Determine the mass of the hanger. Record the mass in the first column of Data Table 1, and calculate the force $\left(\vec{F}_g = m\vec{g}\text{, where } \vec{g} \text{ is } 9.80 \text{ N/kg}\right)$. The weight of the hanger serves as the centripetal force acting on the object.
4. Whirl the stopper at a constant speed and radius so that the tape remains in the same place all the time.
5. Begin timing and count for 20 revolutions. Record the time in Data Table 1. Note that the small numbers on the timer are the decimal values. Round your time to one decimal place.

(continued)

6. Repeat steps 3 to 5 for different masses. Add each mass to the current mass and record the total mass in Data Table 1.
7. Graph the centripetal force $\left(\vec{F}_c\right)$ and the period (T) and determine the relationship.

Video Viewing

View the video *Circular Motion*, which can be found by visiting the Independent Study Option Audio and Video web page at www.edu.gov.mb.ca/k12/dl/iso/av.html.

Data and Calculations

1. Record the data from the video in Data Table 1. *(2 marks)*

Data Table 1

Mass (g)	**Mass (kg)**	**Time for 20 Revolutions (s)**

2. Do a sample calculation of the magnitude of the centripetal force. *(1 mark)*

$\vec{F}_c = \vec{F}_g = m\vec{g}$

(continued)

Assignment 3.4: Video Laboratory Activity: Circular Motion (continued)

3. Do a sample calculation of the period. *(1 mark)*

$$T = \frac{\text{Time for 20 revolutions}}{\text{20 revolutions}}$$

4. Calculate all the centripetal forces and periods and record their values in Data Table 2. *(4 marks)*

Data Table 2

Period (s)	**Centripetal Force (N)**

5. Plot a graph of the raw data with the centripetal force on the y-axis and the period on the x-axis. Plot the graph on graph paper or enter the data into Excel and print a copy of the graph. *(2 marks)*

6. Based on the shape of the graph, what is the relationship between the centripetal force and the period? *(1 mark)*

(continued)

7. Change the raw data (period) so that the new values will yield a straight line. Choose the correct relationship and change the values of the period. Enter the values in Data Table 3. *(2 marks)*

   Data Table 3

Period (s)	**$1/T^2$ (s^{-2})**	**Centripetal Force (N)**

8. Plot a graph of these new values to illustrate that the line has been straightened. Plot the graph on graph paper or enter the data into Excel and print a copy of the graph. *(2 marks)*
9. State the equation that represents the relationship between the centripetal force and the period, including the value of the constant of variation (*k*). *(1 mark)*

Discussion

10. Discuss possible sources of error in this experiment and how you could reduce the error. *(1 mark)*

(continued)

Assignment 3.4: Video Laboratory Activity: Circular Motion (continued)

11. Based on your data and what you observed in the video, how would you describe the relationship between the centripetal force $\left(\vec{F}_c\right)$ and the speed? *(1 mark)*

12. What would happen to the radius of revolution if the speed remained constant and the force increased? (**Hint:** Use the equation relating centripetal force, mass, speed, and radius.) *(1 mark)*

Conclusion

13. What is the relationship between the centripetal force $\left(\vec{F}_c\right)$ on an object in constant circular motion and its period of revolution (T)? *(1 mark)*

(continued)

Assignment 3.4: Video Laboratory Activity: Circular Motion (continued)

Marking Rubric for Assignment 3.4

Criteria	Possible Marks	Actual Marks
Experimental results in Data Table 1	2	
Sample calculation of centripetal force from mass	1	
Sample calculation of period	1	
Calculated forces and periods in Data Table 2	4	
Graph of centripetal force versus period	2	
Correct identification of the relationship	1	
Correct choice of the relationship and change of the values of period in Data Table 3	2	
Graph of "straightened line"	2	
Statement of equation for the relationship	1	
Discussion (error analysis, questions)	3	
Conclusion	1	
Total	**20**	

Lesson 6: Centripetal Acceleration

(1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the term “centripetal acceleration” and state the direction of this acceleration
- ❑ state the cause of centripetal acceleration
- ❑ determine the magnitude of the centripetal acceleration in circular motion

Key Words

centripetal acceleration

Introduction

In this lesson, we examine acceleration in circular motion. In doing so, you will learn that circular motion is unique in how the velocity and acceleration vectors are related to each other. The direction of the acceleration in circular motion may not be immediately obvious. The applications of this work can range from the motion of jet planes and race cars to planets and moons moving in circular orbits to a centrifuge in a medical laboratory.

Acceleration During Circular Motion: Direction

Back in Lesson 4, you saw that a car speeding up as it rounded a curve experienced a change in velocity and in acceleration. The acceleration had two components: one parallel to the initial velocity $\left(\vec{a}_{\parallel\vec{v}_1}\right)$ and one perpendicular to the initial velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$.

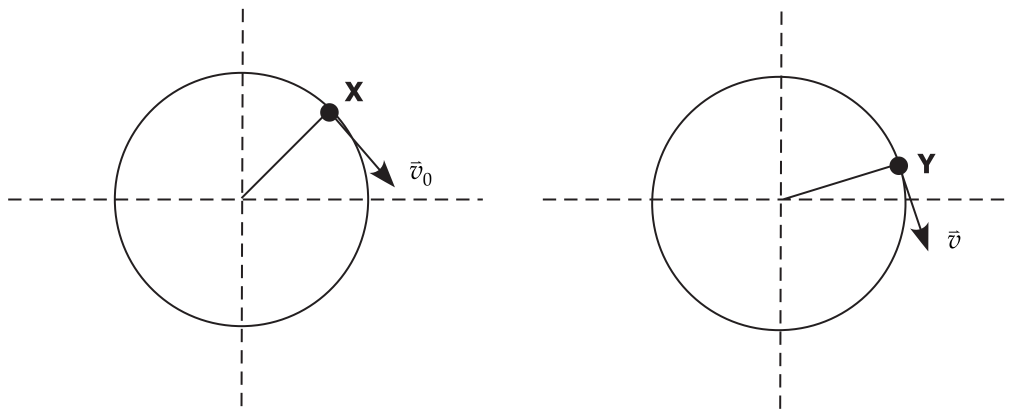
The summary of the discussion is listed below:

> If only the component of the acceleration **parallel** to the initial velocity $\left(\vec{a}_{\parallel\vec{v}_1}\right)$ acts, only the **magnitude** of the initial velocity is changed. The result is **straight-line motion**.
>
> If both the **parallel** component and the **perpendicular** component of the acceleration act, both the **magnitude** and **direction** of the initial velocity are changed. The result is a motion where the object **speeds up** and **turns**.
>
> If only the components of the acceleration **perpendicular** to the initial velocity $\left(\vec{a}_{\perp\vec{v}_1}\right)$ acts, only the **direction** of the initial velocity is changed. This acceleration must change in direction to remain always perpendicular to the velocity. The result must be a motion where the object **does not speed up or slow down** but only **turns**. The motion that results is **uniform circular motion**.

The conclusion arrived at was that, for uniform circular motion, the **acceleration must always act perpendicularly to the velocity vector**.

You saw in a previous lesson that the velocity of an object moving in a circle is always tangent to the circle (or perpendicular to the radius).

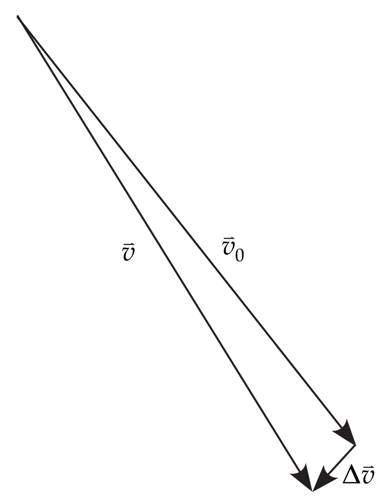
The following diagram shows an object at position "**X**" at a time to moving in a clockwise fashion. The velocity vector at this point is drawn. At a later time, *t*, the object has moved to a new position "**Y**." The velocity vector at this new position is also drawn.



The definition of acceleration is the change in velocity divided by the time interval.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t - t_0}$$

The diagram below shows the difference in the velocity vectors. The direction of the velocity vectors is the same as in the diagrams above, but the length of the vectors has been exaggerated to show the difference in velocity more clearly.



The diagram shows that the difference in the velocity vectors is pointing somewhere towards the interior of the circle. If the time interval between $\vec{v}$ and $\vec{v}_0$ is very small, then it would become more obvious that the direction of the difference in velocity, $\Delta\vec{v}$, and, therefore, the acceleration, is perpendicular to the velocity vector, $\vec{v}_0$, and, hence, along the radius towards the centre of the circle.

In fact, the object accelerates towards the centre of the circle at every moment. This acceleration is called centripetal acceleration because the word "centripetal"means "centre-seeking."

The Dynamics of Uniform Circular Motion

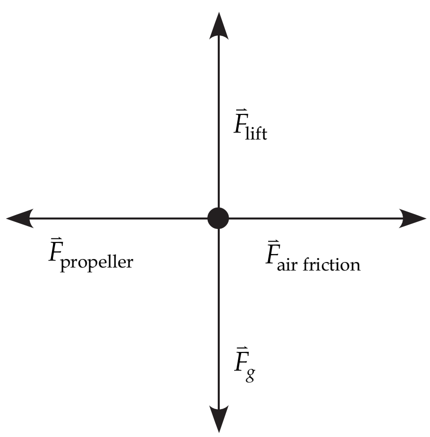
According to Newton's Second Law of Motion, $\vec{F}_{\text{net}} = m\vec{a}$, the direction of the acceleration is in the same direction as the net or unbalanced force acting on the object.

Let's consider a toy airplane, which is controlled by the operator, flying in the circle at the end of the guide line or guide wire. If the airplane is flying in a relatively flat, horizontal circle with uniform circular motion, you can assume by the lifting force of the air flowing around the wings of the plane that the force of gravity pulling the plane down is balanced. There is also the thrust of the propeller pulling the plane forward through the air and the drag of air friction pulling back on the plane. Again, these two forces are balanced. If these two pairs of balanced forces were the only forces acting on the plane, then the net force would be zero and the plane would show straight-line motion at constant velocity.

However, the plane is flying in a circle with the direction of its velocity constantly changing. In this situation, you should see that there is another force—the force of the guide wire pulling on the plane—that is causing the plane to turn.

Let's examine a couple of free-body diagrams.

Side view of motion:



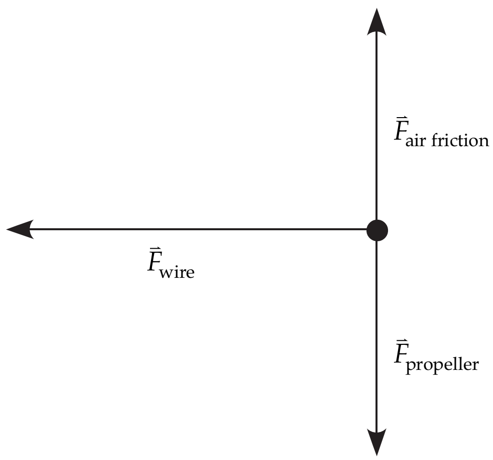
You can see that the force of gravity and the lifting force of the air will cancel out. The force of the propeller and the force of air friction also cancel out.

The net force acting on the plane appears to be 0 N, as all the forces appear to be balanced.

The problem with the diagram above is that there is a fifth force—the force of the guide wire—that is not drawn in the diagram. The force of the guide wire must point straight into the page or straight out of the page.

What you must do is take another perspective, another point of view, for this situation. If you look at this situation from the top, things will become clearer.

Top view of motion:



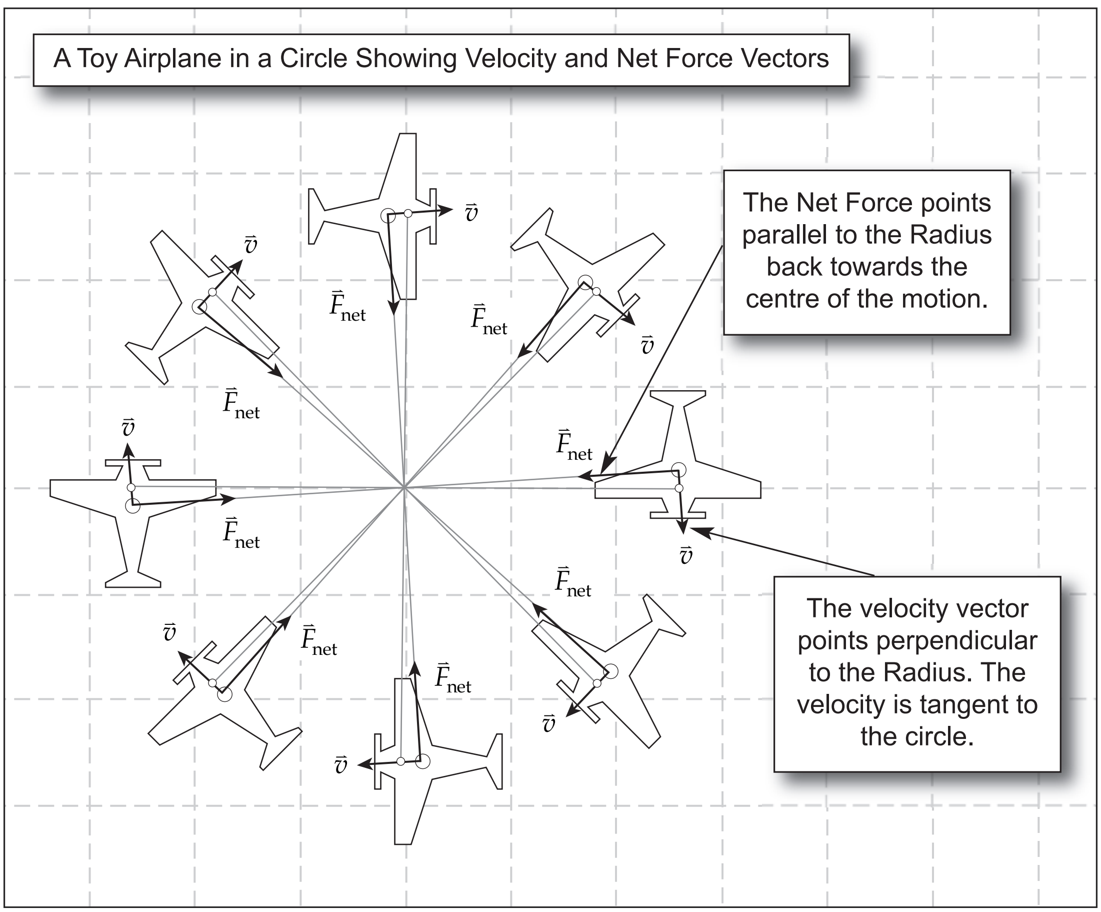
The plane is travelling south. In this view, the force of gravity and the force lifting the plane are not visible. However, you know that they balance and cancel out.

Here, you see that the net force is just the force supplied by the guide wire pulling on the plane.

$$\vec{F}_{\text{net}} = \vec{F}_{\text{wire}}$$

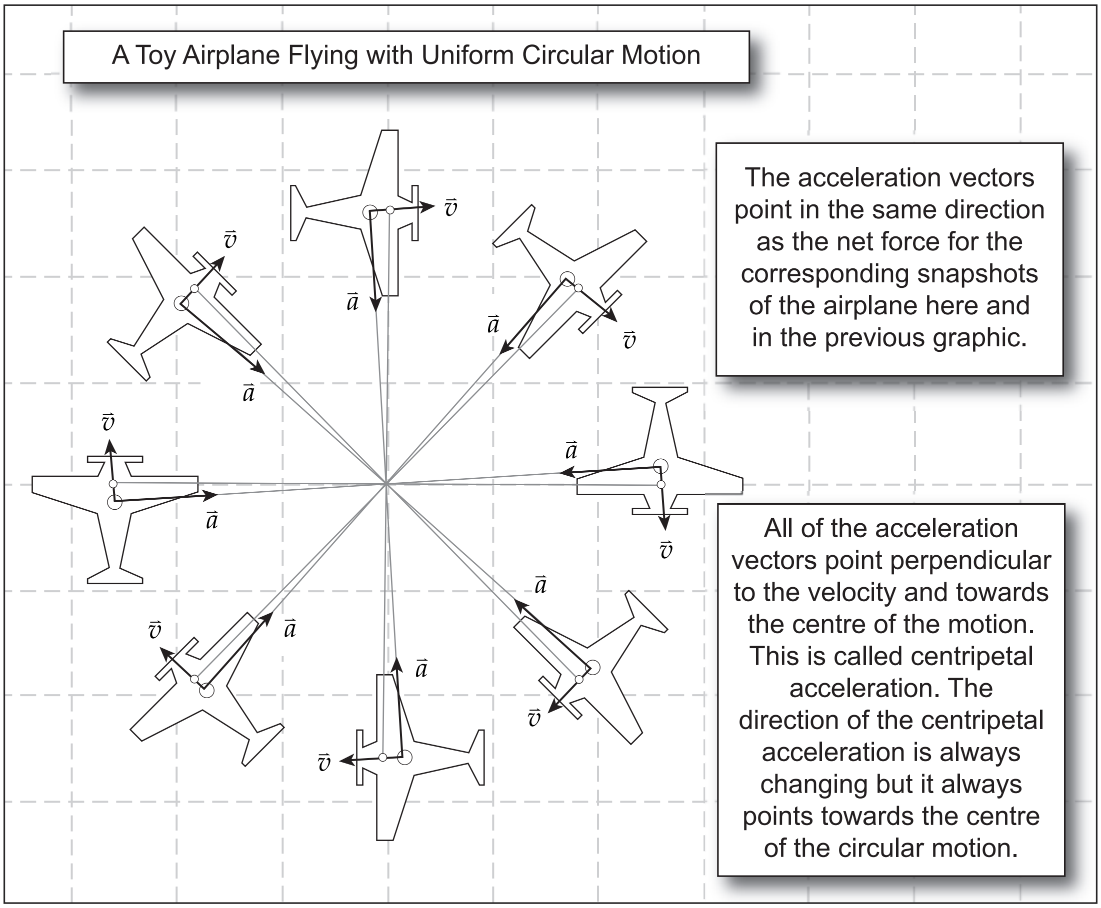
It is this force that causes the plane to change its direction of motion

Now let's look at the big picture including the plane, the guide wire, and the operator who is standing at the centre of the motion.



You can see that the net force always points towards the centre of the motion. Since that force points towards the centre of the motion, then the **acceleration** of the plane must also point towards the **centre** of the motion.

Here's another graphic of the plane, showing the velocity and acceleration vectors.

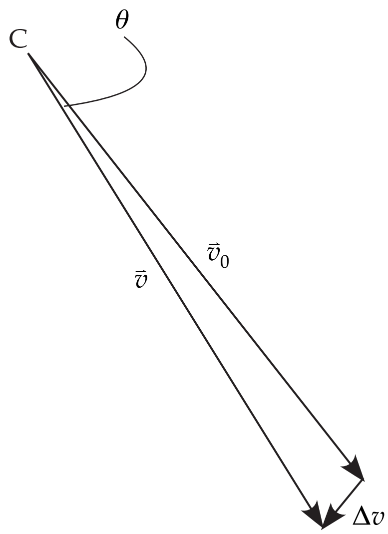


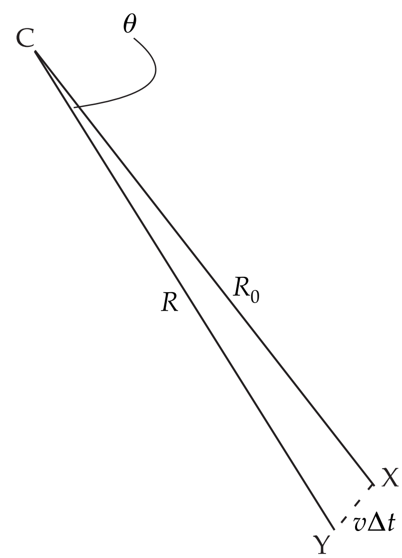
Centripetal acceleration is the acceleration of a body moving continuously in a circular path with constant speed. The centripetal acceleration is always directed towards the centre of the motion.

Since the centripetal acceleration is constantly changing direction, it is an **instantaneous acceleration**. Any given instant, the centripetal acceleration has a specific magnitude and direction. An instant later, the centripetal acceleration still has the same magnitude but a different direction. However, the directions always point towards the centre of the circular motion, so you can specify a direction for centripetal acceleration as "towards the centre."

Acceleration During Circular Motion: Magnitude

To determine how to calculate the magnitude of the centripetal acceleration, we look again at the diagram showing the difference in velocities. We also compare this diagram with another diagram showing the change in position of the object at it moves from time t_0 to t.





In moving from point “X” to point “Y,” the object travelled a distance of $v\Delta t$. For a very small elapsed time Δt, the arc length XY can be approximated as a straight line whose distance is the distance $v\Delta t$ travelled by the object. In this limit, CXY is an isosceles triangle. Because this triangle and the triangle formed by the vectors $\bar{v}$, $\bar{v}_0$, and $\Delta\bar{v}$ have the same angle θ, the triangles are geometrically similar. Therefore,

$$\frac{\Delta v}{v} = \frac{v\Delta t}{R}$$

This equation can be solved for $\Delta v/\Delta t$ to show that the magnitude a_c of the centripetal acceleration is given by

$$\frac{\Delta v}{\Delta t} = \frac{(v)(v)}{R} \text{ and}$$

$$a_c = \frac{v^2}{R}$$

We can therefore summarize our work on centripetal acceleration as follows:

Centripetal acceleration
The centripetal acceleration of an object moving with a speed, v, on a circular path of radius, R, has a magnitude, a_c, given by

$$a_c = \frac{v^2}{R}$$

Quantity	Symbol	Unit
Centripetal acceleration	a_c	metres/second/second (m/s^2)
Speed	v	metres per second (m/s)
Radius	R	metres (m)

You can combine $v = \frac{2\pi R}{T}$, the equation for the speed of an object in uniform circular motion, and $a_c = \frac{v^2}{R}$ by eliminating the speed. The centripetal acceleration is then written as $a_c = \frac{4\pi^2 R}{T^2}$.

You can also combine $v = \frac{2\pi R}{T}$, the equation for the speed of an object in uniform circular motion, and $a_c = \frac{v^2}{R}$ by eliminating the radius R. The third equation for centripetal acceleration becomes $a_c = \frac{2\pi v}{T}$.

Example: Using Equations for Uniform Circular Motion

An airplane flies in a circular path with a radius of 3450 m at a speed of 125 m/s.

a) What is the period of this motion?

b) What is the magnitude of centripetal acceleration of the plane?

c) What is the direction of the centripetal acceleration of the plane?

d) What is the magnitude of the centripetal acceleration expressed in *g*s?

The first thing you must do is differentiate whether the problem involves straight-line motion or circular motion. Each motion has its own unique set of equations.

a) What is the period of this motion?

Given:	Speed	$v = 125$ m/s
	Radius	$R = 3450$ m
Unknown:	Period	$T = ?$
Equation:		The appropriate equation to use is the one that defines the speed for uniform circular motion, $v = \frac{2\pi R}{T}$. Rearranging gives $T = \frac{2\pi R}{v}$

Substitute and solve:

$$T = \frac{2\pi R}{v}$$

$$T = \frac{2(3.14)(3450 \text{ m})}{125 \text{ m/s}}$$

$$T = 173 \text{ s}$$

It takes the plane 173 seconds to travel around the circle.

b) What is the magnitude of centripetal acceleration of the plane?

For this uniform circular motion, you know the speed, the radius, and the period. Therefore, you can use any of the three equations for centripetal acceleration to determine the magnitude of the centripetal acceleration.

Since you're given the speed and the radius, it is appropriate to use the equation involving these quantities. If you use quantities that you have previously calculated incorrectly, using them again just perpetuates the errors.

$$a_c = \frac{v^2}{R}$$

Substitute and solve:

$$a_c = \frac{(125\ \text{m/s})^2}{3450\ \text{m}}$$

$$a_c = 4.53\ \text{m/s}^2$$

The magnitude of the centripetal acceleration is $4.53\ \text{m/s}^2$.

c) What is the direction of the centripetal acceleration of the plane?

The direction of centripetal acceleration is always towards the centre of the uniform circular motion.

d) What is the magnitude of the centripetal acceleration expressed in gs?

The value of $\vec{g}$ is simply the acceleration of gravity at the surface of Earth: $\vec{g} = 9.80\ \text{m/s}^2$.

So, in this case, the centripetal acceleration is

$$a_c = \frac{4.53\ \text{m/s}^2}{\frac{9.80\ \text{m/s}^2}{\vec{g}}} = \left(4.53\ \text{m/s}^2\right)\left(\frac{\vec{g}}{9.80\ \text{m/s}^2}\right) = 0.462\ \text{g}$$

Learning Activity 3.7

Applying Equations for Uniform Circular Motion

There are six practice questions in this learning activity on uniform circular motion. An answer key is available at the end of Module 3 for you to check your work after you have answered the questions.

The physics of a model airplane and circular motion

1. A model airplane is attached to a guideline so that when it is in motion, it is moving in a circle. Suppose that it is suddenly released from its circular path. For example, the guide wire may be cut. Describe in a general way the direction of the velocity of the airplane at the moment it is released.

(continued)

Learning Activity 3.7: Applying Equations for Uniform Circular Motion (continued)

The physics of a car rounding a curve

2. A car is driven around one curve having a radius of 320 m and then around another having a radius of 960 m. In both cases, the speed is 28.0 m/s. Compare the centripetal acceleration for both turns. (Assume three significant digits.)

The physics of an orbiting satellite

3. A satellite orbiting Earth at an altitude of 500 km roughly follows a circular path. It takes a satellite 94.5 minutes to circle Earth once. The radius of this motion is 6.88×10^6 m.
   a) What is the speed of the satellite?
   b) What is the centripetal acceleration of the satellite?

The physics of a race car and circular motion

4. Computer-controlled display screens provide drivers in the Indianapolis 500 with a variety of information about how their cars are performing. For instance, as a car is going through a turn, a speed of 221 m/h (98.8 m/s) and a centripetal acceleration of 3.00 g (three times the acceleration due to gravity) are displayed. Determine the radius of the turn in metres.

The physics of centripetal acceleration and a jet plane

5. A jet plane travelling at 1800 km/h (500.0 m/s) pulls out of a dive by moving in an arc of radius 4.00 km. What is the plane's acceleration in m/s^2 and in *g*s? One g = 9.80 m/s/s.

The physics of a centrifuge and circular motion

6. A centrifuge is a device used to separate solids from a suspension of a solid in a liquid. It functions by whirling a small vessel containing the mixture at very high speed in a circular path. The acceleration experienced by a sample moving in a circle of radius 6.25 cm from the axis of rotation is 5250 g or 5250 times the acceleration of gravity. Calculate the frequency of this motion in RPMs (revolutions per minute).

Lesson Summary

In this lesson, we focused our work on the study of acceleration in circular motion.

In uniform circular motion, the direction of the acceleration vector is perpendicular to the velocity vector and along the radius towards the centre of the circle. This acceleration is called **centripetal acceleration**, meaning "centre-seeking." As an object moves around the circle, the centripetal acceleration vector continually changes direction and always points towards the centre of the circle. Therefore, centripetal acceleration can be thought of as **an instantaneous acceleration**.

The **magnitude** of the **centripetal acceleration** a_c of an object moving with a **speed** v on a circular path of **radius** R with a **period** T has a magnitude a_c given by $a_c = \frac{v^2}{R}$.

Since $v = \frac{2\pi R}{T}$, the centripetal acceleration can also be written as $a_c = \frac{4\pi^2 R}{T^2}$ and $a_c = \frac{v^2}{R}$.

Since acceleration is a vector quantity state, it is a centripetal acceleration using "towards the centre" as its direction.

Assignment 3.5

Circular Motion of the Moon (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answer. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of the Moon's circular motion

The Moon's nearly circular orbit about the Earth has a radius of about 385 000 km and a period T of 27.3 days. Determine the speed of the Moon. Determine the magnitude and direction of the Moon's acceleration.

(continued)

Assignment 3.5: Circular Motion of the Moon (continued)

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for determining the correct period of motion in seconds
- 1 mark for determining the correct radius of motion in metres
- 1 mark for determining the correct speed in m/s
- 1 mark for determining the correct magnitude of acceleration
- 1 mark for determining the correct direction of acceleration

Video - Visual understanding of centripetal acceleration formula

This video nicely illustrates the magnitude of the centripetal acceleration is given by Ac =V2/R.

By diagraming position-time for circular motion next to velocity-time it is demonstrated that the acceleration vector has a direction pointing towards the centre of the circle.

https://youtu.be/NH1_sO8QY3o

Video - Physics - Mechanics: Motion In Two-Dimensions: (16 of 21) Circular Motion and Acceleration

This video explains how motion at constant speed following a circular or curved path is undergoing an acceleration called centripetal acceleration or centre seeking acceleration.

https://youtu.be/aJ6P9beSgDc

Video - Kinematics of Uniform Circular Motion

This animation illustrates the directions of the radius vector, the velocity vector and the centripetal acceleration vector in uniform circular motion. Note that the centripetal acceleration always points towards the centre of the circular motion.

https://youtu.be/h-85rpR-mRM

Assignment 3.5

Circular Motion of the Moon (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answer. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of the Moon's circular motion

The Moon's nearly circular orbit about the Earth has a radius of about 385 000 km and a period T of 27.3 days. Determine the speed of the Moon. Determine the magnitude and direction of the Moon's acceleration.

(continued)

Assignment 3.5: Circular Motion of the Moon (continued)

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for determining the correct period of motion in seconds
- 1 mark for determining the correct radius of motion in metres
- 1 mark for determining the correct speed in m/s
- 1 mark for determining the correct magnitude of acceleration
- 1 mark for determining the correct direction of acceleration

LESSON 7: CENTRIPETAL FORCE (2 HOURS)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ derive the equation for centripetal acceleration from Newton's second law
- ❑ define the term "centripetal force"
- ❑ determine the centripetal force (magnitude and direction) acting on an object as it moves in a circle
- ❑ apply the equation for centripetal force to situations involving tension in a line, objects moving in a circle due to friction, and satellite motion
- ❑ explain why "centrifugal force" cannot be considered a real force

Key Words

centripetal force
centrifugal force

Introduction

In the previous two lessons, you learned how to apply your knowledge of velocity and acceleration to circular motion. We conclude this module with the study of forces in circular motion. You will see how Newton's second law can be applied to help in your understanding of these forces. This knowledge can be applied to many interesting situations from trapeze acts in a circus to cars negotiating turns to satellite motion.

Centripetal Force: A Definition

From Newton's second law, you know that whenever an object accelerates, there must be a net force causing the acceleration. It follows that in uniform circular motion, there must be a net force producing the centripetal acceleration. According to the second law, the net force F_c is equal to the product of the object's mass and acceleration. This net force is called the **centripetal force** and points in the same direction as the centripetal acceleration—that is, towards the centre of the circle.

> **Centripetal force** is the label given to the net force pointing toward the centre of the circular path, and this net force is the vector sum of all the force components that point along the radial direction.

According to Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$.

In the previous lesson, you learned that $a_c = \dfrac{v^2}{R}$.

Therefore, the centripetal force can be written as $\vec{F}_c = \dfrac{mv^2}{R}$.

Centripetal force equation
The **centripetal force**—the net force required to keep an object of mass m, moving with a speed v, on a circular path of radius R—has a magnitude of

$$\vec{F}_c = m\vec{a}_c = \frac{mv^2}{R}$$

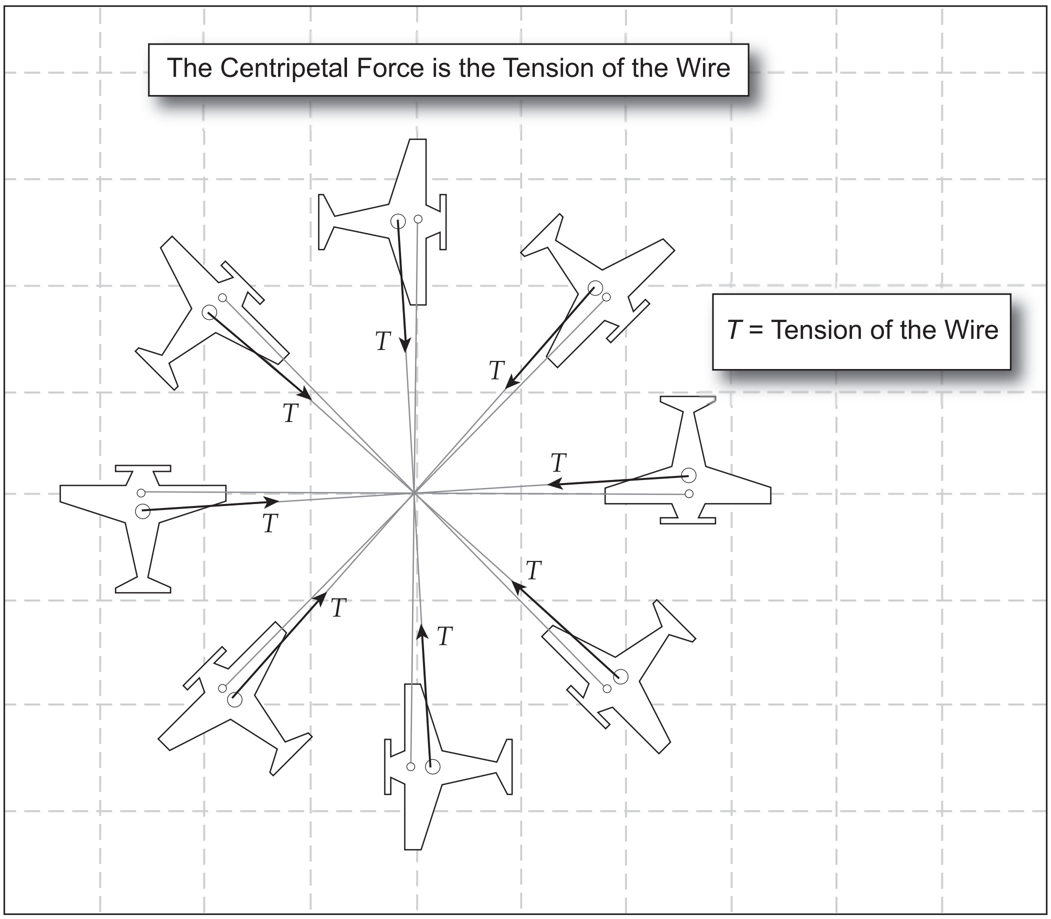
Quantity	Symbol	Unit
Centripetal force	$\vec{F}_c$	newtons (N)
Mass	m	kilograms (kg)
Speed	v	metres/second (m/s)
Radius	R	metres (m)

The centripetal force always points towards the centre of the circle and continually changes direction as the object moves.

Sources of Centripetal Force

As we saw earlier, when a model airplane on a guide wire flies in a horizontal circle, the only unbalanced force acting on the plane is the force of the guide wire (tension) pulling the plane inward. Therefore, this tension force is the centripetal force.

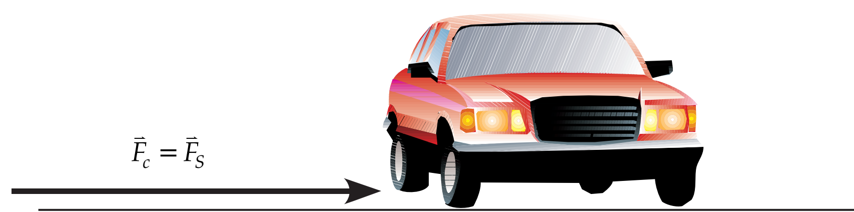
$$\bar{F}_c = T = \frac{mv^2}{R}$$



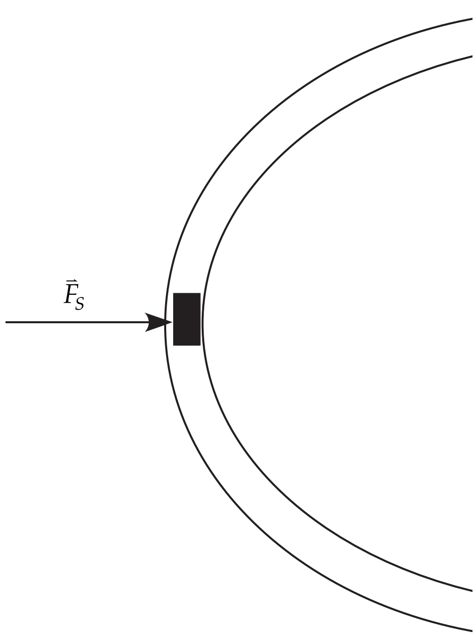
Some centripetal forces are less obvious than the tension force of the guide wire in the previous example.

Consider a car travelling at constant speed around a curve in the road. The surface of the road is horizontal or unbanked. What keeps the car moving in this circular path? The only force acting on the car is the force of friction between the tires and the road. This would be a force of static friction since the car tires are not sliding relative to the road surface, as they would if the car was skidding. If the maximum force of static friction between the tires and the road is less than the centripetal force needed to move the car in this circular path, then the car will indeed begin to skid. During our winter season when ice and snow are covering the roads, this skidding is a common occurrence. For the car to make the turn, the centripetal force is equal to the force of static friction.

$$\vec{F}_c = \vec{F}_F = \mu_s \vec{F}_N = \mu_s m\vec{g} = \frac{mv^2}{R}$$



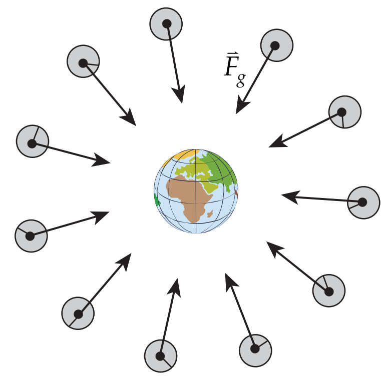
From the top, the situation would appear like this:



Here we are drawing only the force of static friction. Remember that the force of gravity is pulling the car into the road and the road is pushing back up on the car's tires (normal force). However, these two forces balance, leaving the **static force** as the **net force** or **centripetal force** acting on the car.

Today, there are many satellites in orbit around Earth. The ones in circular orbits are examples of uniform circular motion. Like a model airplane on a guide wire, each satellite is kept on its circular path by a centripetal force. The gravitational pull of Earth provides the centripetal force and acts like a guideline for the satellite.

$$\vec{F}_c = \vec{F}_g = \frac{GmM_E}{R^2} = \frac{mv^2}{R}$$



The mass of the satellite is m and its radius of orbit is R.

The force of gravity supplies the centripetal force.

Example 1: Calculating the Centripetal Force

While you can learn a whole new set of equations for centripetal force using all the different expressions for the centripetal acceleration, it is easier just to calculate the centripetal acceleration and then use Newton's second law to determine the centripetal force by substituting the value of the centripetal acceleration into $\vec{F}_c = ma_c$.

Here's a sample problem.

A child with a mass of 25.0 kg moves with a speed of 1.93 m/s when sitting 12.5 m from the centre of the merry-go-round. Calculate the centripetal force experienced by the child.

Given:	Mass	$m = 25.0$ kg
	Speed	$v = 1.93$ m/s
	Radius	$R = 12.5$ m
Unknown:	Centripetal force	$\vec{F}_c = ?$

Equation: Use the equation $\vec{F}_c = \dfrac{mv^2}{R}$.

Substitute and solve:

$$\vec{F}_c = \frac{mv^2}{R}$$

$$\vec{F}_c = \frac{(25.0\text{ kg})(1.93\text{ m/s})^2}{12.5\text{ m}}$$

$$\vec{F}_c = 7.45\text{ N}$$

The centripetal force on the child is 7.45 N towards the centre of the motion.

Alternate solution:
You could find the centripetal acceleration using $a_c = \dfrac{v^2}{R}$, then substitute the mass and this value of the centripetal acceleration into $\vec{F}_c = ma_c$.

Example 2: Calculating the Centripetal Force

A piece of string can sustain a force of 20.0 N before breaking. Can it be used to safely swing a 1.00 kg rock around a circle of radius 1.50 m if the rock is travelling at 5.00 m/s?

The question is really asking if the centripetal force necessary to keep the rock moving around the circle is greater or less than 20.0 N. If the centripetal force is larger than 20.0 N, the string will break.

Given:	Mass	$m = 1.00$ kg
	Speed	$v = 5.00$ m/s
	Radius	$R = 1.50$ m
Unknown:	Centripetal force	$\vec{F}_c = ?$

Equation: Use the equation $\vec{F}_c = \frac{mv^2}{R}$.

Substitute and solve:

$$\vec{F}_c = \frac{mv^2}{R}$$

$$\vec{F}_c = \frac{(1.00\text{ kg})(5.00\text{ m/s})^2}{1.50\text{ m}}$$

$$\vec{F}_c = 16.7\text{ N}$$

Since the centripetal force necessary in this case is 16.7 N, the string will not break.

Example 3: Calculating the Centripetal Force

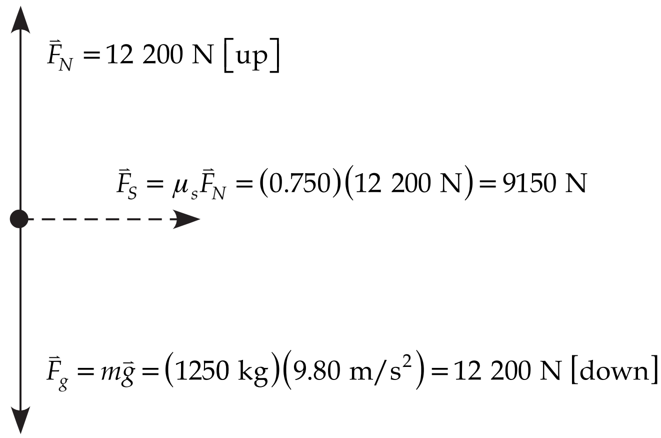
A car of mass 1250 kg is travelling on a level concrete surface. The coefficient of static friction between the tires in the road is 0.750. What is the maximum speed with which the car can travel around a curve of radius 125 m on such a surface?

Here, the centripetal force will be supplied by the force of static friction between the tires and the road. First, you'll have to calculate the force of static friction.

Given: Mass $m = 1250$ kg

Coefficient of static friction $\mu_s = 0.750$

Radius $R = 125$ m



Unknown: Speed $v = ?$

Equation: You can use the equation $\vec{F}_c = \dfrac{mv^2}{R}$ rearranged to $v = \sqrt{\dfrac{\vec{F}_c R}{m}}$.

Substitute and solve:

$$v = \sqrt{\frac{\vec{F}_c R}{m}}$$

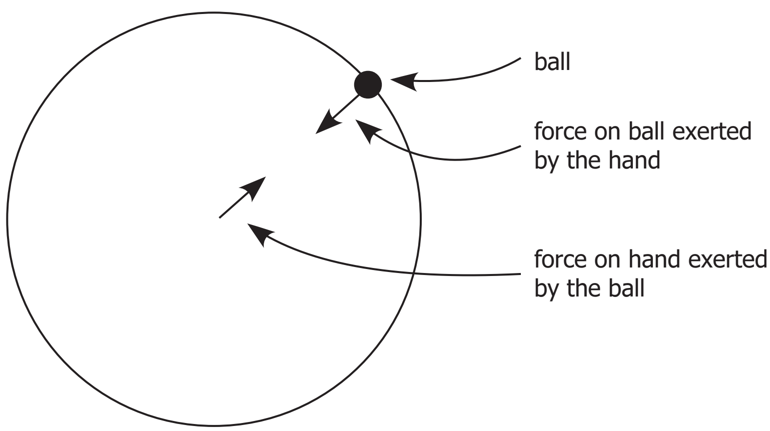
$$v = \sqrt{\frac{(9150\ \text{N})(125\ \text{m})}{1250\ \text{kg}}} = 30.2\ \text{m/s}$$

The maximum speed with which the car can travel around the curve is 30.2 m/s.

Centrifugal Force

There is a common misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal ("centre-fleeing") force.

Consider, for example, a person swinging a ball on the end of a string. If you have ever done this yourself, you know that you feel a force pulling outward on your hand. This misconception arises when this pull is interpreted as an outward "centrifugal" force pulling on the ball that is transmitted along the string to the hand. But this is not what is happening at all. To keep the ball moving in a circle, the person pulls inwardly on the ball. The ball then exerts an equal and opposite force on the hand (Newton's third law) and this is what your hand feels. The force on the ball is the one exerted inwardly on it by the person.



For even more convincing evidence that a centrifugal force does not act on the ball, consider what happens when you let go of the string. If a centrifugal force were acting, the ball would fly straight out along the radius of motion. In fact, this does not happen. The ball flies off tangentially in the direction of the velocity it had at the moment it was released, because the inward force no longer acts.

Learning Activity 3.8

Projectile Motion and Circular Motion Review

This learning activity consists of three parts. After you have answered the questions in each part of this learning activity, compare your responses to those provided in the answer key at the end of Module 3.

Part A

There are five practice questions in this part of the learning activity.

The physics of centripetal force and a trapeze act (conceptual)

1. In a circus, a man hangs upside down from a trapeze, legs bent over the bar and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight downward and is stationary or when the partner is swinging through the straight-down position?

The physics of a coin on a rotating turntable

2. A quarter rests on a record that is spinning on a record turntable. Draw a side view and a top view of a free-body diagram showing all the forces acting in this situation.

The physics of road conditions and making a turn

3. What is the speed at which a car can safely negotiate a turn of radius 50.0 m in icy weather where the coefficient of static friction is 0.100?

The physics of the orbiting Hubble space telescope

4. Determine the speed of the Hubble space telescope orbiting at a height of 596 km above Earth's surface. The radius of Earth is 6.38×10^6 m and the mass of Earth is 5.98×10^{24} kg. At this point above Earth's surface the gravitational field is 8.20 N/kg towards the centre of Earth.

The physics of a carnival ride

5. A ride at the carnival is shaped like a cylinder. A person of mass 75.0 kg stands inside the cylinder with his back to the wall and is strapped to the wall of the cylinder a distance of 7.25 m from the centre. The cylinder rotates about an axis through its centre, making one revolution every 8.40 seconds. Determine the centripetal force acting on the person.

(continued)

Learning Activity 3.8: Projectile Motion and Circular Motion Review (continued)

Part B

There are 10 multiple-choice questions in this part of the learning activity. For each of the following statements, circle the letter that represents the correct response.

1. A ball rolls across a tabletop and, upon reaching the edge, falls off. The curved path of the falling ball is a combination of its
   a) constant horizontal velocity and its constant vertical velocity
   b) increasing horizontal velocity and its constant vertical velocity
   c) constant horizontal velocity and its increasing vertical velocity
2. The curved path that a ball takes after rolling off the edge of a table is called its:
   a) orbit
   b) trajectory
   c) projectile
3. The range of a shell shot at an angle of 45° above the horizontal is the distance the shell travels:
   a) horizontally between launching and landing
   b) vertically between launching and landing
   c) along the curved path it takes between launching and landing
4. A cannonball is fired at a 30° angle above the horizontal.
   a) Its vertical velocity is dependent on its horizontal velocity.
   b) Its vertical velocity is independent of its horizontal velocity.
   c) Its vertical and horizontal velocities are equal.
5. For a shell to reach maximum range, a gun should fire it at an angle of:
   a) 30°
   b) 45°
   c) 60°
6. The centripetal acceleration of an object moving in a circular path is directed
   a) along the circumference of the circle
   b) away from the centre of the circle
   c) toward the centre of the circle

(continued)

Learning Activity 3.8: Projectile Motion and Circular Motion Review (continued)

7. If the velocity of an object moving in a circular path is doubled, its centripetal acceleration
   a) is quadrupled (4x)
   b) is doubled (2x)
   c) is not changed

8. If an object travelling in a circle completes four revolutions in two seconds, its period of revolution is
   a) 0.5 s
   b) 2 s
   c) 8 s

9. A boy, facing north, swings a ball attached to a one-metre string in a horizontal circle. When is the tension in the string the greatest?
   a) When the ball is to the right of the boy
   b) When the ball is to the left of the boy
   c) When the ball is south of the boy
   d) The tension is the same at all locations of the ball

10. If the mass of an object moving in uniform circular motion is doubled, how will the centripetal force be affected?
    a) It will not change.
    b) It will be one-half as great (1/2 x).
    c) It will double (2 x).
    d) It will be one-quarter as great (1/4 x).

(continued)

Learning Activity 3.8: Projectile Motion and Circular Motion Review (continued)

Part C

There are five true or false questions in this part of the learning activity. In the space provided before each of the following statements, indicate whether the statement is true or false by using T for true and F for false. Correct the statement if it is false.

____ 1. A freely falling object falls the same distance during each second that it is falling.

____ 2. The time required for a projectile to reach its maximum height is equal to the time spent returning to the point of release.

____ 3. A shell launched at 20° above the horizontal and one launched at 60° above the horizontal will have the same range.

____ 4. The number of revolutions per second an object travels in a circular path is known as its frequency.

____ 5. The centripetal force always points away from the centre of the circle.

Lesson Summary

In this lesson, you studied forces involved in circular motion.

Centripetal force is the label given to the net force pointing toward the centre of the circular path, and this net force is the vector sum of all the force components that point along the radial direction.

The **centripetal force** is calculated as the net force required to keep an object of mass *m*, moving at a speed *v*, on a circular path of radius *r*, using the following equation.

$$\vec{F}_c = \frac{mv^2}{R}.$$

The centripetal force always points toward the centre of the circle and continually changes direction as the object moves.

The equation for centripetal force can be derived from Newton's second law, $\vec{F}_c = ma_c$, and from the fact that the centripetal acceleration is $a_c = \frac{v^2}{R}$.

There are various sources of centripetal force.

If, for example, an object is moving in a circle due to the tension, *T*, in a cord, then

$$\vec{F}_c = T = \frac{mv^2}{R}.$$

If friction is causing an object to move in a circle, then

$$\vec{F}_c = \vec{F}_F = \mu_s m\vec{g} = \frac{mv^2}{R}.$$

If the gravitational force is the cause of circular motion, then

$$\vec{F}_c = \vec{F}_g = \frac{GmM_g}{R^2} = \frac{mv^2}{R}.$$

The so-called "centrifugal" force, in fact, does not exist. It appears to exist because the object moving in a circle exerts an equal and opposite force to the device (hand, for example) that exerts the centripetal force necessary to keep the object moving in a circle. This is a pair of action-reaction forces.

When an object is moving in a circle and the centripetal force suddenly stops acting, the object flies off tangent to the circle.

NOTES

Assignment 3.6

Uniform Circular Motion of a Satellite (6 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of satellite motion around Jupiter

A satellite is placed in a circular orbit around Jupiter. The altitude of the satellite above the surface of Jupiter is 775 km. Jupiter has a mass of 1.90×10^{27} kg and a radius of 7.14×10^{7} m.

a) Determine the radius of motion of the satellite. *(1 mark)*

b) What force is providing the centripetal force necessary for the satellite to stay in orbit? *(1 mark)*

c) In what direction is the centripetal force always acting? *(1 mark)*

(continued)

d) Using the equation for centripetal force, $\vec{F}_c = m_s \frac{v^2}{R}$, and the gravitational force, $\vec{F}_G = \frac{Gm_J m_s}{R^2}$, derive the mathematical equation that allows you to calculate the orbital speed of the satellite. *(2 marks)*

e) Calculate the orbital speed of the satellite circling Jupiter using the equation derived in part (d). The value of $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$. *(1 mark)*

Method of Assessment

The total of six marks for this assignment will be determined as follows:

- 1 mark for determining the correct radius of motion
- 1 mark for stating the force necessary for the satellite to stay in orbit
- 1 mark for stating the direction of the centripetal force
- 2 marks for the correct derivation of the formula for the orbital speed of the satellite
- 1 mark for the correct calculation of the orbital speed

Video - High School Physics - Centripetal Force

This video uses Newton’s Second Law to link centripetal acceleration and centripetal force. Several examples are used to illustrate the relationships for uniform circular motion.

https://youtu.be/ldQWTNDBSSE

Video - High School Physics - Frequency and Period

This video solves circular motion problems using all of the quantities for circular motion: period, frequency, speed, radius, centripetal acceleration and centripetal force.

https://youtu.be/845P69d3O6U

Assignment 3.6

Uniform Circular Motion of a Satellite (6 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of satellite motion around Jupiter

A satellite is placed in a circular orbit around Jupiter. The altitude of the satellite above the surface of Jupiter is 775 km. Jupiter has a mass of 1.90×10^{27} kg and a radius of 7.14×10^{7} m.

a) Determine the radius of motion of the satellite. *(1 mark)*

b) What force is providing the centripetal force necessary for the satellite to stay in orbit? *(1 mark)*

c) In what direction is the centripetal force always acting? *(1 mark)*

(continued)

Assignment 3.6: Uniform Circular Motion of a Satellite (continued)

d) Using the equation for centripetal force, $\vec{F}_c = m_s \dfrac{v^2}{R}$, and the gravitational force, $\vec{F}_G = \dfrac{Gm_J m_s}{R^2}$, derive the mathematical equation that allows you to calculate the orbital speed of the satellite. *(2 marks)*

e) Calculate the orbital speed of the satellite circling Jupiter using the equation derived in part (d). The value of $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$. *(1 mark)*

Method of Assessment

The total of six marks for this assignment will be determined as follows:

- 1 mark for determining the correct radius of motion
- 1 mark for stating the force necessary for the satellite to stay in orbit
- 1 mark for stating the direction of the centripetal force
- 2 marks for the correct derivation of the formula for the orbital speed of the satellite
- 1 mark for the correct calculation of the orbital speed

MODULE 3 SUMMARY

Congratulations, you have finished the third module in the Grade 12 Physics course.

In this third module, you have seen that the study of physics requires a basic understanding of mechanics upon which are built the explanations of more complicated phenomena. Here, you used the ideas of kinematics and dynamics (mechanics) to aid you in the description of projectile motion and circular motion. Understanding why something happens facilitates your understanding of a given phenomena. The force of gravity controls the motion of projectiles, resulting in the typical curved parabolic path. The centripetal force controls the motion of objects moving in uniform circular motion.

As you proceed through Grade 12 Physics, you will experience, over and over, this process of constructing theories and models based on what you already know and understand. It is important that you attempt to fit these new ideas into the understanding of physics that you already possess.

Submitting Your Assignments

You will not submit your Module 3 assignments to the Distance Learning Unit at this time. Instead, you will submit them, along with the Module 4 assignments, **when you have completed Module 4**.

NOTES

Grade 12 Physics (40S)

Module 3: Projectiles and Circular Motion

Learning Activity Answer Keys

Learning Activity 3.1: Motion of Freely Falling Objects

The physics of an experimental vehicle and a ball falling (conceptual)

1. Imagine an experimental vehicle that slows down and comes to a stop with an acceleration of magnitude 9.80 m/s^2. The vehicle reverses direction and then speeds up with an acceleration of 9.80 m/s^2. Compare this situation to a ball being thrown upward, coming to a momentary stop, and then falling to Earth. Other than one situation being horizontal and the other being vertical, are the motions the same or different?

   Answer:

   The two situations are identical. An experimental vehicle that comes to a momentary stop, reverses direction, and then speeds up with an acceleration of 9.80 m/s^2 is the same as a ball thrown straight upward near the surface of Earth, coming to a stop, and then falling back down. In both cases, the acceleration is constant in both magnitude and direction. In both cases, the objects begin their motion with the velocity and acceleration vectors pointing in opposite directions, and then end up with the velocity and acceleration vectors pointing in the same direction. The equations for constant acceleration could be applied the same way for both situations.

The physics of an arrow fired straight up

2. An arrow that is fired straight up can have an initial speed of 14.0 m/s. How much time will it take for the arrow to reach the top of its motion, and how long will the arrow remain in the air?

   Answer:

   Given: Make a drawing.

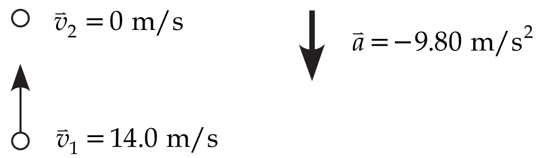
   The drawing shows a vector pointing upwards representing the initial velocity. A vector pointing downwards is the acceleration due to gravity vector.

   Decide which direction is positive and which is negative.

   It is convenient to let the upward direction be positive and the downward direction be negative.

   Write down in symbolic form what you are given.

The symbols for velocity, acceleration, and time are shown beside the diagram. Notice that the acceleration due to gravity is down and is given a negative sign. Notice also that $\vec{v}_2$ has been assigned the value of zero even though this was not explicitly stated in the question. You know that the velocity of an object is zero at the top of its motion.



Unknown: Time interval Δt

Equation: Select the appropriate equation.
In this case, the most appropriate equation would be $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$.

Solving for time, $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$.

Substitute and solve: $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \dfrac{0 \text{ m/s} - 14.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 1.43 \text{ s}$

This is for one half of the motion. For the full motion (up and down), the time would be doubled. The total time would be 2.86 s.

The physics of a bouncing golf ball

3. If you throw a golf ball at a hard surface and then watch it rebound, it may rebound with a speed of 4.50 m/s. In this case, how high will it rebound?

   Answer:

   Given: Make a drawing.

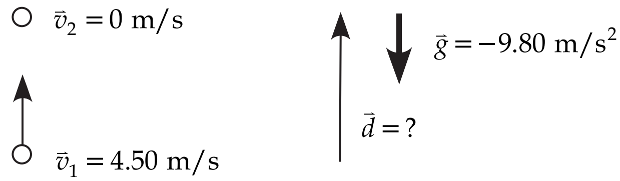
   The drawing shows a vector pointing upwards representing the initial velocity. A vector pointing downwards is the acceleration due to gravity vector. Another vector pointing upwards represents the displacement vector.

   Decide which direction is positive and which is negative.

   It is convenient to let the upward direction be positive and the downward direction be negative.

   Write down in symbolic form what you are given.

   The symbols for velocity, acceleration, and displacement are shown beside the diagram. Notice that the acceleration due to gravity is down and is given a negative sign. The velocity at the top of the motion, $\vec{v}_2$ has been assigned the value of zero.



Unknown: Displacement $\vec{d} = ?$

Equation: Select the appropriate equation.

In this case, the most appropriate equation would be $v_2^2 = v_1^2 + 2ad$.

Solving for the displacement, $d = \dfrac{v_2^2 - v_1^2}{2a}$.

Substitute and solve:

$$\frac{(0 \text{ m/s})^2 - (4.50 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 1.03 \text{ m}$$

The golf ball rebounds to a height of 1.03 m.

The physics of a falling sandbag

4. A balloon is descending at a velocity of –4.00 m/s. A sandbag is released from this balloon and hits the ground 8.00 s later. What was the altitude of the balloon at the time the sandbag was dropped?

   Answer:

   Given: Make a drawing.

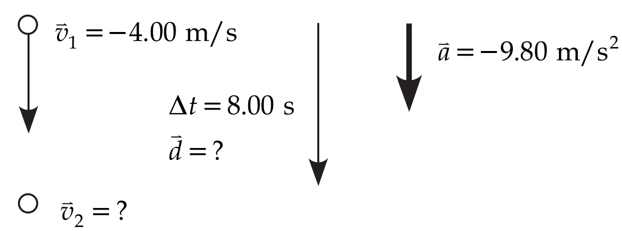
   The drawing shows a vector pointing downwards representing the initial velocity. A vector pointing downwards is the acceleration due to gravity vector. Another vector pointing downwards represents the displacement vector.

   Decide which direction is positive and which is negative.

   It is convenient to let the upward direction be positive and the downward direction be negative.

   Write down in symbolic form what you are given.

   The symbols for velocity, acceleration, displacement, and time are shown beside the diagram. Notice that the acceleration due to gravity is down and is given a negative sign.



Unknown: Altitude refers to a height above the ground $\vec{d} = ?$

Equation: Select the appropriate equation.
In this case, the most appropriate equation would be
$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$.

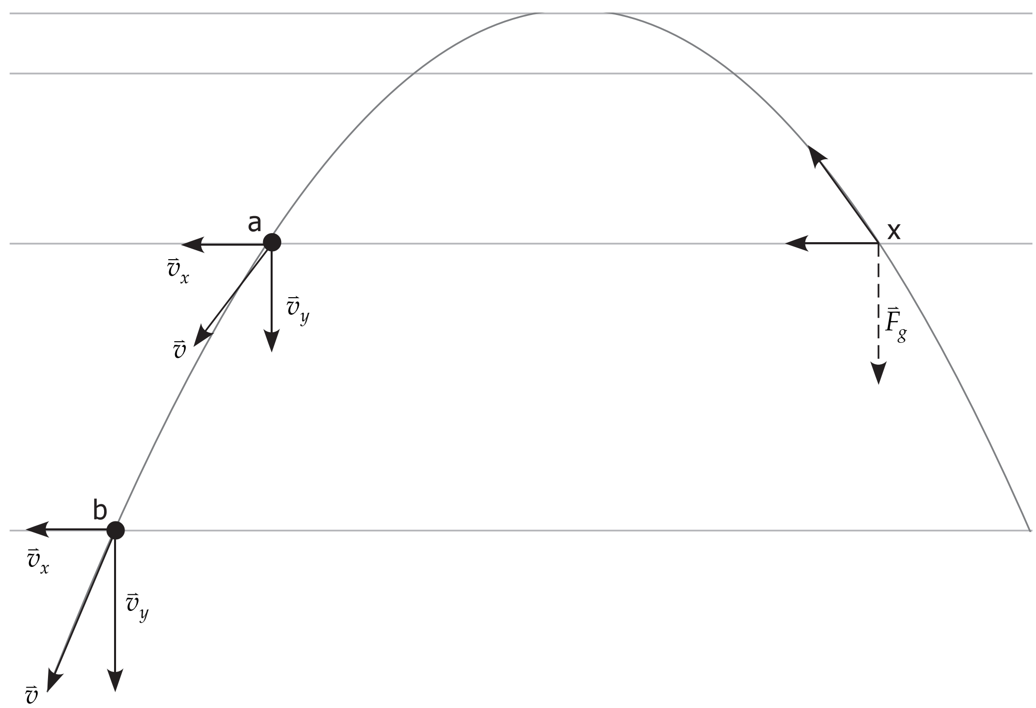
Substitute and solve: $\vec{d} = (-4.00 \text{ m/s})(8.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(8.00 \text{ s})^2$

$$= -345.6 \text{ m or } -346 \text{ m}$$

This is equivalent to the distance fallen. This is also the height above the ground that the sandbag was dropped from.

Learning Activity 3.2: Projectile Motion: Vector Diagrams

The physics of projectile motion: vector diagrams



a) Notice that the dot at point "a" is at the same vertical height as point "x." At the dot, draw and label the horizontal component of the velocity vector, the vertical component of the velocity vector, and the total velocity vector.

Answer:

The horizontal component of the vector is of the same length and direction as the vector at point "x."

The vertical component of the vector is of the same length but opposite in direction to the vertical component at "x."

The length of the total velocity vector is the same length as the total velocity vector at point "x" but pointing down and to the left instead of up and to the left.

b) At the dot at point "b," estimate the length of the horizontal component, the vertical component, and the total velocity vectors, and draw them into the diagram and label them.

Answer:

The length of the horizontal vector is the same as the horizontal component at "x" and in the same direction.

The length of the vertical component of the vector is greater than the length at "x" and opposite in direction.

The length of the total velocity vector will be greater than the one at "x." It will point tangent to the curve, and will be the sum of the horizontal and vertical components.

c) Draw in the vector representing the net force acting on the object at the position "x."

Answer:

At each position the net force is just a force of gravity pulling down on the object. The net force is constant.

d) If you wanted to determine the distance travelled in the horizontal direction, what equation could you use?

Answer:

It would be most appropriate to use the equation $\vec{d}_x = \vec{v}_{1x}\Delta t$.

e) If you wanted to determine the distance or displacement travelled in the vertical direction, what equation could you use?

Answer:

In the vertical direction you could use any of the following equations depending on what other information you were given in the question:

$$v_2^{\,2} = v_1^{\,2} + 2ad$$

$$\vec{d} = \frac{1}{2}\left(\vec{v}_1 + \vec{v}_2\right)t$$

$$\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = \vec{v}_2 t - \frac{1}{2}\vec{a}t^2$$

f) At what point in the motion of the projectile is the vertical component of the velocity equal to zero?

Answer:

The vertical component of velocity is equal to zero at the top of the motion.

Learning Activity 3.3: Objects Projected Horizontally

1. For projectile motion, what provides the net force that controls the motion of the projectile? What implication does this have for the motion of the projectile in the horizontal direction and in the vertical direction?

   Answer:

   For projectile motion, the net force that controls the motion of the projectile is the force of gravity. You assume that the force of air friction is negligible.

   Since the force of gravity, the net force, acts in the downwards direction, the only acceleration experienced by the projectile is in the downwards direction—that is, the vertical direction. The object accelerates at 9.80 m/s/s [down] at the surface of the ground.

   Since there is no force in the horizontal direction, the object does not accelerate in the horizontal direction. The velocity of the object in the horizontal direction is constant.

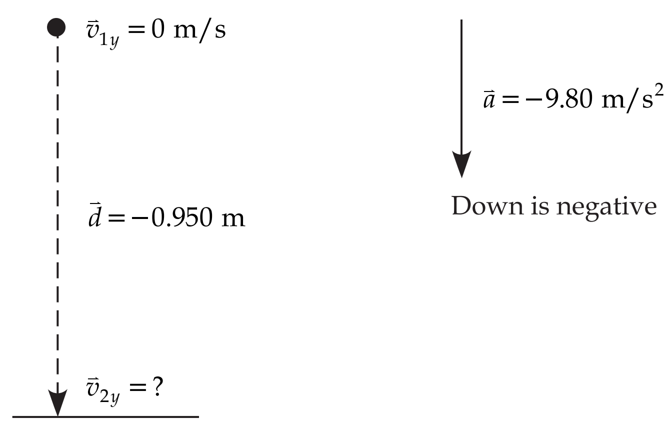
2. A marble is allowed to roll off the edge of a horizontal table. The marble falls vertically 0.950 m before it strikes the floor. The marble lands a horizontal distance of 18.8 cm from the edge of the table.

   a) Calculate the time it took a marble to fall from the level of the table to the level of the floor.

      Answer:

      Time interval appears in the equations for both vertical and horizontal motion. Since you are talking about falling to the ground, you can assume that this is vertical motion.

      Given:



Unknown: Time interval $\Delta t = ?$

Equation: To determine the time it takes for the marble to fall to the ground, we can use the equation $\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}t^2$.

Solving for the time and realizing that $\vec{v}_{1y} = 0 \text{ m/s}, \; t = \sqrt{\frac{2d}{a}}$.

Substitute and solve: $\Delta t = \sqrt{\frac{2(-0.950 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.440 \text{ s}$

It took a marble 0.440 seconds to fall from the table to the floor.

b) What was the initial velocity of the marble as it rolled off the edge of the table?

Answer:

In the horizontal direction, the acceleration was 0 m/s/s. Therefore, the marble travelled with the constant velocity $\vec{v}_{1x}$ in the horizontal direction. Since you know the time it took the marble to fall and how far it moved in the horizontal direction, you can determine the initial velocity in the horizontal direction.

Given: Time interval $\Delta t = 0.440$ s

Displacement in horizontal direction $\vec{d}$ = 18.8 cm = 0.188 m

Unknown: Initial horizontal velocity $\vec{v}_{1x} = ?$

Equation: $\vec{d}_x = \vec{v}_{1x}\Delta t$ rearranged to $\vec{v}_{1x} = \frac{\vec{d}_x}{\Delta t}$

Substitute and solve: $\vec{v}_{1x} = \frac{\vec{d}_x}{\Delta t} = \frac{0.188 \text{ m}}{0.440 \text{ s}} = 0.427 \text{ m/s}$

The marble is travelling initially at 0.427 m/s [right].

c) What was the final vertical velocity of the marble as it struck the floor?

Answer:

To find the final vertical velocity, we must deal with motion in the vertical direction.

Remember that down is the negative direction.

Given: Initial vertical velocity $\vec{v}_{1y} = 0 \text{ m/s}$

Time interval $\Delta t = 0.440 \text{ s}$

Vertical displacement $\vec{d}_y = -0.950 \text{ m}$

Acceleration $\vec{a} = -9.80 \text{ m/s}^2$

Unknown: Final vertical direction $\vec{v}_{2y} = ?$

Equation: $\vec{v}_{2y} = \vec{v}_{1y} + \vec{a}\Delta t$

Substitute and solve: $\vec{v}_{2y} = 0 \text{ m/s} + \left(-9.80 \text{ m/s}^2\right)\left(0.440 \text{ s}\right)$

$= -4.31 \text{ m/s}$

The marble is falling at 4.31 m/s when it strikes the floor.

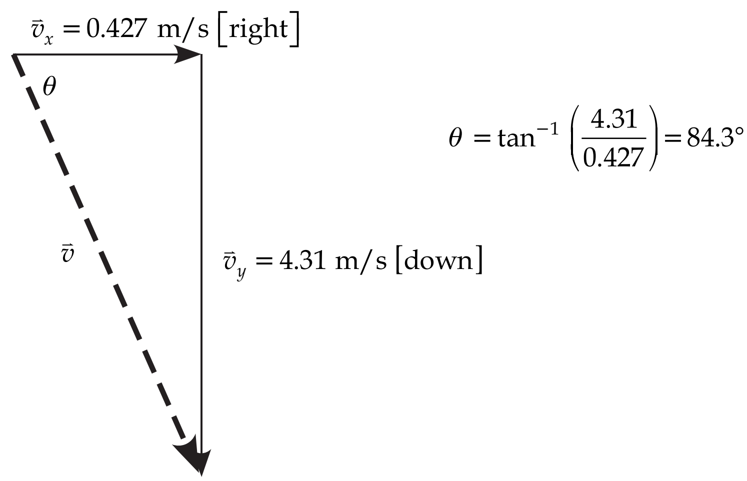
d) What was the final velocity of the marble as it struck the floor?

Answer:

The final velocity of the marble is the vector sum of the final velocity in the horizontal direction plus the final velocity in the vertical direction.

$$\vec{v} = \vec{v}_x + \vec{v}_y$$

$$\vec{v} = 0.427 \text{ m/s } [\text{right}] + 4.31 \text{ m/s } [\text{down}]$$



Find the magnitude of $\vec{v}$ using the theorem of Pythagoras.

$$\vec{v}^2 = \vec{v}_x{}^2 + \vec{v}_y{}^2 = 0.427^2 + 4.31^2 = 18.758$$

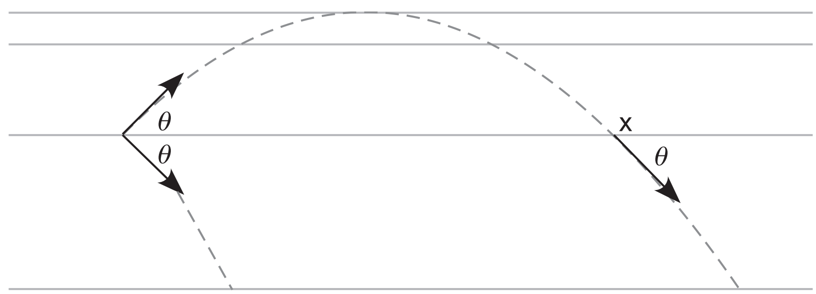
$$\vec{v} = 4.33 \text{ m/s}$$

The final velocity in the marble is 4.33 m/s [84.3° below the horizontal].

Learning Activity 3.4: Analyzing Projectile Motion

The physics of throwing a stone (conceptual)

1. From the top of a hillside, a person throws two identical stones. The stones have identical initial speeds v_1. Stone 1 is thrown downward at some angle θ below the horizontal while stone 2 is thrown at the same angle θ above the horizontal. If we neglect air resistance, which stone, if either, strikes the surface of the ground with greater velocity?



Answer:

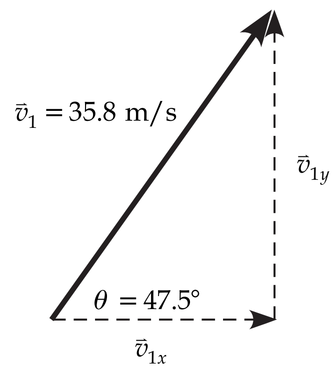
You might think that the stone thrown at a down angle would strike the ground with greater velocity. But, in fact, it does not. Look at the path of the stone thrown at an upward angle. It reaches a maximum height and then falls back to the ground. At point "x" in the drawing, when the stone returns to its original height, its speed is the same as the original speed, v_1, but at an angle θ below the horizontal, the same angle at which the other stone was thrown. Thus, the two stones have the same velocity at the same height above the ground. From this point on, the stones follow the same changes in velocity, and they strike the ground at the same velocity.

The physics of a golf shot

2. A golf ball is struck by a club, leaving the face of the club travelling at 35.8 m/s [47.5° above the horizontal]. The ball sails over a level fairway.

   a) Determine the vertical and horizontal components of this velocity.

   Answer:



The horizontal component of the initial velocity is given by $\vec{v}_{1x} = \vec{v}_1(\cos\theta)$.

The vertical component of the initial velocity is given by $\vec{v}_{1y} = \vec{v}_1(\sin\theta)$.

The horizontal component of the initial velocity is:

$$\vec{v}_{1x} = \vec{v}_1(\cos\theta)$$
$$\vec{v}_{1x} = 35.8\ \text{m/s}\ (\cos 47.5°) = 24.2\ \text{m/s}\ [\text{right}]$$

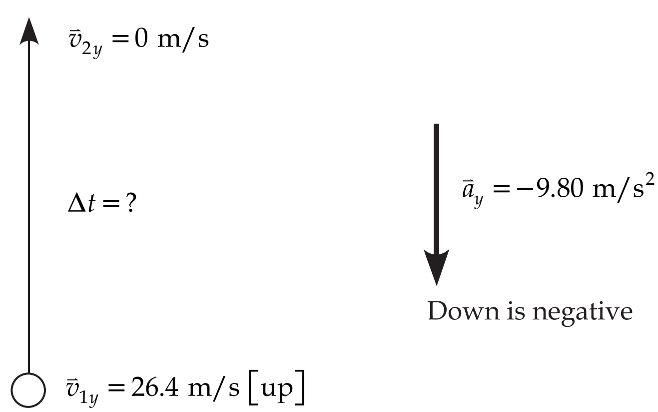
The vertical component of the initial velocity is:

$$\vec{v}_{1y} = \vec{v}_1(\sin\theta)$$
$$\vec{v}_{1y} = 35.8\ \text{m/s}\ (\sin 47.5°) = 26.4\ \text{m/s}\ [\text{up}]$$

   b) For what length of time will the ball be in the air?

   Answer:

   Here, you would consider only the first half of the ball's journey through the air.



So you could use the following equation: $\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$, which can be rearranged to solve for time interval: $\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$.

Substitute and solve: $\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{0 \text{ m/s} - (+26.4 \text{ m/s})}{-9.80 \text{ m/s}^2}$

$$\Delta t = 2.69 \text{ s}$$

The ball takes 2.69 seconds to go up and 2.69 seconds to fall back down, giving a total time in the air of 5.38 seconds.

c) Assuming the ball does not roll when it lands, what is the range of the ball?

Answer:

The range of the ball was given by the horizontal distance $(\vec{d}_x)$ the ball travels while it is in the air. The ball travels horizontally both while it is rising and falling during the time of 5.38 seconds. The velocity during this time was a constant amount of 24.2 m/s [right].

The range is calculated using the equation $\vec{d}_x = \vec{v}_{1x}\Delta t$.

$$\vec{d}_x = \vec{v}_{1x}\Delta t = 24.2 \text{ m/s [right]} (5.38 \text{ s}) = 130.196 \text{ m}$$

To three significant digits, the range of the golf ball is 1.30×10^2 m [right].

d) If the ball does roll after it lands, is it possible for this shot to finish in the hole? The hole is 135 m from the point where the ball is struck. Assume the ball is travelling in the correct direction.

Answer:

Since the hole is 135 m from the point where the ball is struck, it is possible that the ball will land 130 m from the point where it was struck, which is 5 m before the hole, then roll into the hole. Again, the direction must be correct.

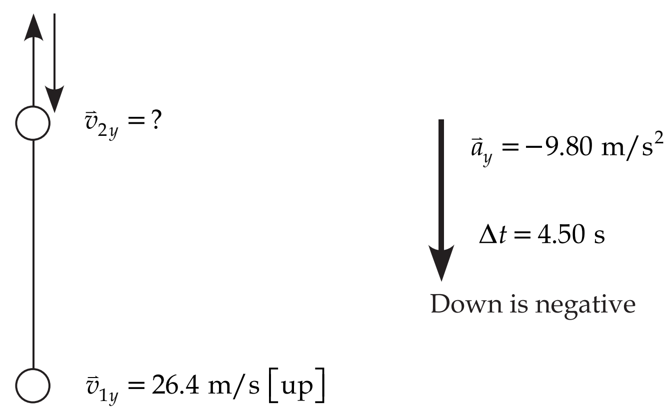
e) What is the velocity of the ball 4.50 seconds after it was struck?

Answer:

To determine the velocity of a projectile at a given instant, you must determine the horizontal component of the velocity at that instant $(\vec{v}_x)$, the vertical component of the velocity at that instant $(\vec{v}_y)$, and then add them together to give the velocity at that instant $(\vec{v})$.

Since there is no acceleration in the horizontal direction, the velocity of a golf ball at any instant while it is in the air should be the same as the horizontal component of the initial velocity. This was calculated earlier to be $\vec{v}_x$ = 24.2 m/s [right].

Calculating the component of the velocity in the vertical direction will require a bit of work. What you must do is determine the vertical velocity of the golf ball 4.50 seconds after it was struck.



You can see that at a time of 4.50 seconds, the ball is already on the way down, so the velocity will be negative.

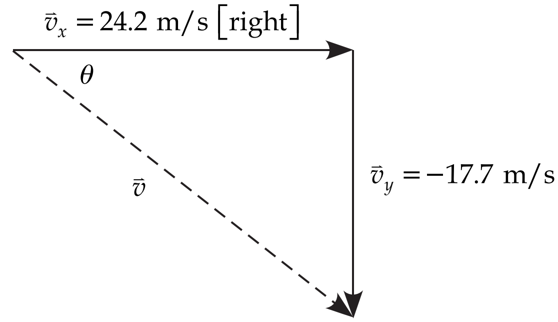
You can use the equation $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$.

Substitute and solve:

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t = (26.4 \text{ m/s}) + (-9.80 \text{ m/s}^2)(4.50 \text{ s})$$

$$\vec{v}_2 = -17.7 \text{ m/s}$$

Now you add the two components of the velocity using $\vec{v} = \vec{v}_x + \vec{v}_y$.



$v^2 = v_x{}^2 + v_y{}^2 = (24.2 \text{ m/s})^2 + (17.7 \text{ m/s})^2 = 898.93$

$\vec{v} = \sqrt{898.93} = 30.0 \text{ m/s}$

Using $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{17.7}{24.2}$ and $\theta = \tan^{-1}\left(\dfrac{17.7}{24.2}\right)$

$\theta = 36.2°$

The velocity of the ball is 30.0 m/s [36.2° below the horizontal].

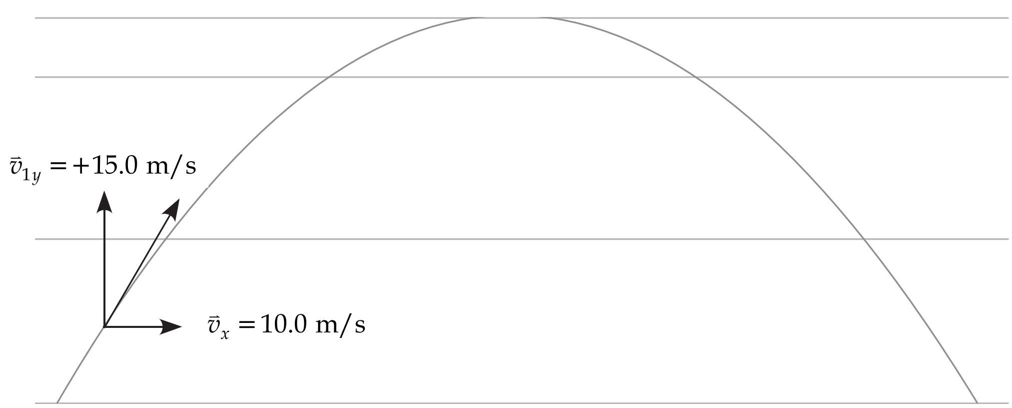
The physics of a ball launched from a rising helicopter

3. A helicopter is rising vertically with a uniform velocity of 15.0 m/s. When it is 225 m from the ground, a ball is thrown horizontally from the helicopter with a velocity of 10.0 m/s with respect to the helicopter. Determine each of the following.

   a) At what time will the ball reach the ground?

   Answer:

   First of all, it is important to understand how the ball is moving. It is projected with both a vertical and a horizontal component of velocity. Therefore, its motion will resemble what is depicted in the diagram below.



   The drawing above shows the initial vertical and horizontal components of velocity.

   To determine the time, one approach is to divide the total motion of the ball into three parts. Part 1 occurs when the ball rises to its maximum height from its present position. Part 2 is when the ball falls back down to the same height off the ground as it was when it was thrown. Part 3 occurs when the ball falls the rest of the way down to ground level.

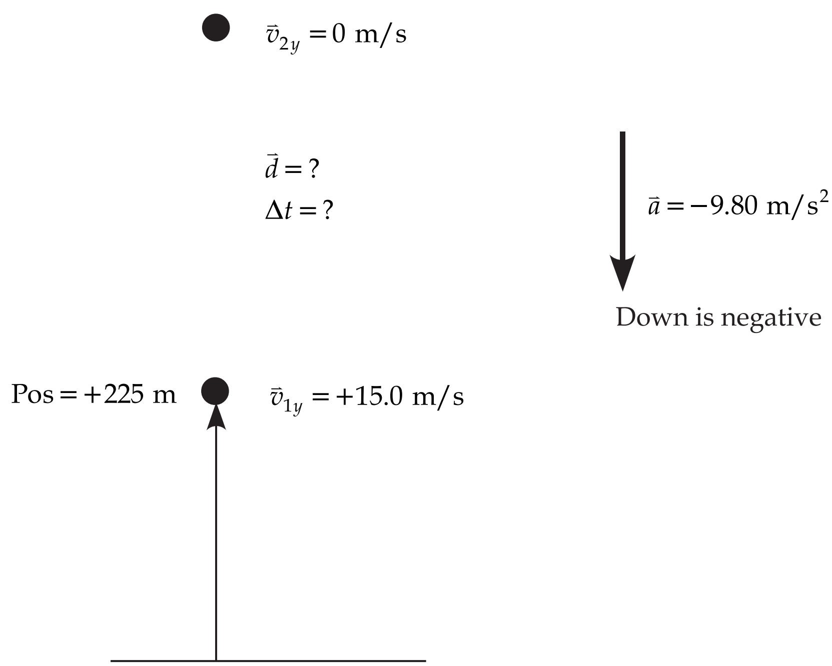
At the point of the motion shown above, the following is known:

Initial velocity vertical direction	$\vec{v}_{1y} = 15.0 \text{ m/s}$
Initial velocity horizontal direction	$\vec{v}_x = 10.0 \text{ m/s}$
Acceleration in the vertical direction	$\vec{a}_y = -9.80 \text{ m/s}^2$
Position in a vertical direction	$\text{pos} = +225 \text{ m}$

Part 1:

The ball rises to the to the top of its motion.

To determine the time for this part of the motion, we must use one of the equations for uniformly accelerated motion.



We know that at the top of the motion, the y-component of the velocity will be zero. Therefore, an appropriate equation would be $\vec{v}_2 = \vec{v}_1 + a\Delta t$.

To find the time use $t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \dfrac{0 \text{ m/s} - 15.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 1.53 \text{ s}$.

Part 2:

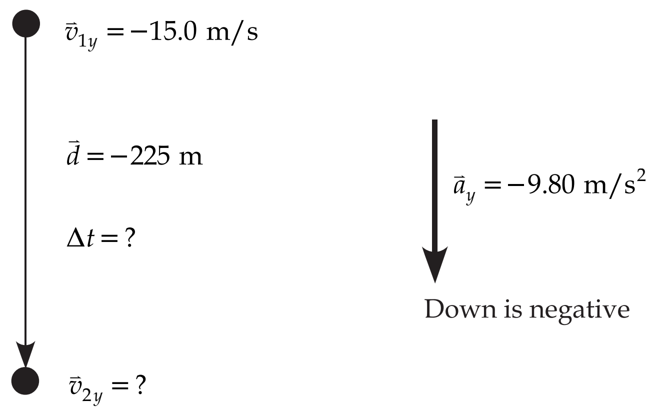
The ball falls back down to the original height above the ground.

The time it takes the ball to fall back to the same height it started from is the same time it took to rise. Therefore, the time is Δt = 1.53 s.

Part 3:

The ball falls the rest of the way to the ground.

In this part of the motion, the ball has an initial vertical velocity of $\vec{v}_{1y}$ = –15.0 m/s, which is the same vertical component it was released at but in the opposite direction.



Since we know the initial velocity, the acceleration of the object, and the distance fallen, we can determine the final velocity.

You can use the equation ${v_2}^2 = {v_1}^2 + 2ad$.

Substitute and solve:

$$\begin{aligned} v_2 = \sqrt{v_1^2 + 2ad} &= \sqrt{(-15.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-225 \text{ m})} \\ &= 68.1 \text{ m/s} \end{aligned}$$

The velocity is actually –68.1 m/s because it is falling.

The time can now be found using $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ and rearranging to

$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{-68.1 \text{ m/s} - (-15.0 \text{ m/s})}{-9.80 \text{ m/s}^2} = 5.42 \text{ s.}$$

The total time for the projectile is the sum of the three times.

Total time = 1.53 s + 1.53 s + 5.42 s = 8.48 s.

Alternative solution #1

There is another way to find the solution to the question in one step.

We can use the equation $\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$.

In using this equation, it is especially important to understand the meaning of "$\vec{d}$". If we consider the whole motion of the ball from the time it is released from the helicopter till the time it strikes the ground, then "$\vec{d}$" represents the total displacement. The total displacement is –225 m. The time in this equation then represents the total time the ball is in the air (in a time to go to its maximum height plus the time to fall back down to the ground). If we now insert the appropriate numbers into the equation, it becomes $-225\text{m} = (15.0\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$.

If we now eliminate the units and work with just the numbers, this equation becomes $4.90t^2 - 15.0t - 225 = 0$.

We can use the mathematical technique for finding the roots of this quadratic equation.

The roots of a quadratic equation are given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In this case, $a = 4.90$, $b = -15.0$, and $c = -225$.

By inserting these numbers into the quadratic equation, we arrive at the two possible roots. They are $t = 8.48$ s and –5.42 s

We can reject the negative time as impossible.

Therefore, the total time is 8.48 s.

Alternative solution #2

In the first solution, we determined the velocity of the ball when it strikes the ground in part 3 of the solution. We determined that the velocity of the ball when it strikes the ground is

$$v_2 = \sqrt{v_1^2 + 2ad} = \sqrt{(-15.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-225\ \text{m})} = 68.1\ \text{m/s}$$

or –68.1 m/s because it is falling.

The ball was initially thrown at 15.0 m/s in the positive direction. Therefore, to find the total time of flight, we can use

$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{-68.1\ \text{m/s} - (15.0\ \text{m/s})}{-9.80\ \text{m/s}^2} = 8.48\ \text{s}.$$

The point here is that there is usually more than one way to solve problems. Any logical method is acceptable. Please use the GUESS method as a means of helping you keep track of things during the problem-solving process.

b) Where will the ball land when it strikes the ground?

Answer:

To determine the horizontal component of the distance the ball travels, remember that the horizontal component of the velocity is constant $\left(\vec{v}_x = 10.0 \text{ m/s [right]}\right)$.

We can therefore use $\vec{d}_x = \vec{v}_{1x}\Delta t = \left(10.0 \text{ m/s}\right)\left(8.48 \text{ s}\right) = 84.8 \text{ m}\left[\text{right}\right]$.

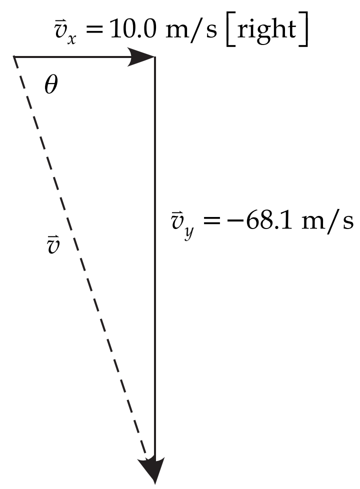
c) With what velocity will the ball strike the ground?

Answer:

The velocity of the ball when it strikes the ground will be the sum of the horizontal component and the vertical components of the velocity at that instant.

The vertical component of the velocity was determined to be –68.1 m/s.

The horizontal component of the velocity remains 10.0 m/s [right].



Therefore, the total velocity is

$$\vec{v} = \sqrt{\vec{v}_x^{\,2} + \vec{v}_y^{\,2}} = \sqrt{\left(10.0 \text{ m/s}\right)^2 + \left(68.1 \text{ m/s}\right)^2} = 68.8 \text{ m/s}.$$

The angle with which the ball strikes the ground is

$$\theta = \tan^{-1}\left(\frac{\vec{v}_y}{\vec{v}_x}\right) = \tan^{-1}\left(\frac{68.1 \text{ m/s}}{10.0 \text{ m/s}}\right) = 81.6° \text{ below the horizontal.}$$

The ball strikes the ground with the velocity of 68.6 m/s [81.6° below the horizontal].

Learning Activity 3.5: Circular Motion Terms

Fill in the blanks.

1. In uniform circular motion, the velocity of the object has a constant ***magnitude*** but a changing ***direction***.
2. In uniform circular motion, the direction of the velocity is ***tangent to the circumference of the circle***.
3. The acceleration in uniform circular motion points ***towards the centre of the circular motion. The acceleration also points in a direction perpendicular to the velocity at that moment.***
4. The time to complete one cycle is known as the ***period***. It has units of ***seconds***.
5. The number of cycles completed per second in uniform circular motion is known as the ***frequency***. It has units of ***hertz***.

6. Describe the function of each of the following:
   a) the component of acceleration parallel to the velocity

      Answer:

      The component of acceleration parallel to the velocity serves to change the magnitude of the velocity.

   b) the component of the acceleration perpendicular to the velocity

      Answer:

      The component of acceleration perpendicular to the velocity serves to change the direction of the velocity.

7. What kind of motion will an object exhibit if
   a) only the component of acceleration parallel to the velocity acts?

      Answer:

      If only the component of acceleration parallel to the velocity acts, the object will speed up or slow down (changing the magnitude of the velocity).

b) only the component of acceleration perpendicular to the velocity acts?

Answer:

If only the component of acceleration perpendicular to the velocity acts, the object will be turning (changing the direction of the velocity).

c) both the component of acceleration parallel to velocity and the component of acceleration perpendicular to velocity act?

Answer:

If both the component of acceleration parallel to the velocity and the component perpendicular to the velocity act, then the object will change both its speed and direction (changing both magnitude and direction).

Learning Activity 3.6: Circular Motion: Basic Quantities

The physics of a car wheel moving in a circle

1. The wheel of a car has a radius of 0.280 m and is rotating at a rate of 820 revolutions per minute (rpm) (3 significant digits) as the car travels down a highway.

   a) What is the period of motion?

   Answer:

   Given: Radius $R = 0.280$ m

   Frequency $f = 820$ revolutions/minute

   The period of motion is found using

   $$\frac{\text{time}}{\text{number of events}} = \frac{60.0 \text{ seconds}}{820 \text{ revolutions}} = 0.0732 \text{ s.}$$

   b) What is the frequency of motion?

   Answer:

   The frequency of motion can be found using

   $$\frac{\text{number of events}}{\text{time}} = \frac{820 \text{ revolutions}}{60.0 \text{ seconds}} = 13.7 \text{ Hz.}$$

   The frequency is the inverse of the period of motion.

   c) What is the length of one cycle of the motion?

   Answer:

   The length of one cycle of motion is the circumference of the circle.

   $C = 2\pi R = 2\pi(0.280 \text{ m}) = 1.76$ m

   d) Determine the speed of motion (in m/s) using the period of motion.

   Answer:

   To determine the speed using the period of motion, use

   $$v = \frac{2\pi R}{T} = \frac{2\pi\left(0.280 \text{ m}\right)}{0.0732 \text{ s}} = 24.0 \text{ m/s.}$$

e) Determine the speed of motion (in m/s) using the frequency of motion.

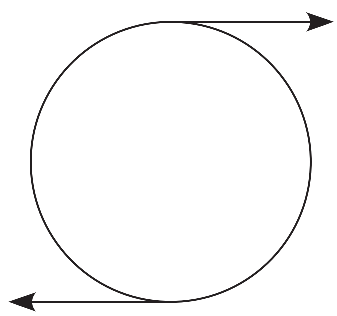
Answer:

To determine the speed using the frequency of motion, use
$v = 2\pi Rf = 2\pi(0.280 \text{ m})(13.7 \text{ Hz}) = 24.0 \text{ m/s}$

f) Draw a diagram showing a circle. Assume that the wheel is moving clockwise. Draw a velocity vector at the top of the circle and another velocity vector at the bottom of the circle. Be sure to pay attention to both the relative lengths and directions of the vectors.

Answer:

In drawing the velocity vectors, be certain that they are of the same length since the speed is constant. The velocity vectors are tangent to the circle. At the top of the circle, the velocity vector is pointing to the right. At the bottom of the circle, the velocity vector is pointing to the left.



The physics of blades in a food blender

2. The tips of the blades in a food blender can move at a speed of 22.0 m/s. The circle in which they move can have a radius of 0.0500 m.

a) What is the length of one cycle of the motion?

Answer:

Given:	Speed	$v = 22.0 \text{ m/s}$
	Radius	$R = 0.0500 \text{ m}$

The length of one cycle of the motion is given by
$C = 2\pi R = 2\pi(0.0500 \text{ m}) = 0.314 \text{ m}.$

b) What is the period of motion?

Answer:

The time for one complete revolution would involve solving for the period of motion from the basic equation for speed in circular motion.

$$v = \frac{2\pi R}{T}$$

Solving for the period of motion, $T = \frac{2\pi R}{v} = \frac{2\pi\left(0.0500\text{ m}\right)}{22.0\text{ m/s}} = 0.0143\text{ s}.$

c) What is the frequency of motion?

Answer:

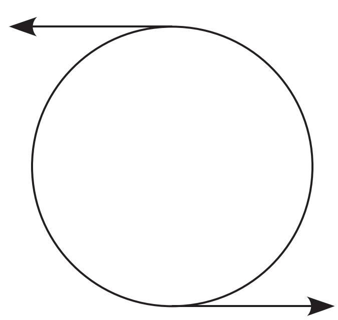
The frequency of motion can be found by finding the inverse of the period of motion.

$$f = \frac{1}{T} = \frac{1}{0.0143\text{ s}} = 69.9\,\text{Hz}$$

d) Draw a diagram showing a circle. Assume that the blade is moving counter-clockwise. Draw a velocity vector at the top of the circle and another velocity vector at the bottom of the circle. Be sure to pay attention to both the relative lengths and directions of the vectors.

Answer:

In drawing the velocity vectors, be certain that they are of the same length, since the speed is constant. The velocity vectors are tangent to the circle. At the top of the circle, the velocity vector is pointing to the left. At the bottom of the circle, the velocity vector is pointing to the right.



Learning Activity 3.7: Applying Equations for Uniform Circular Motion

The physics of a model airplane and circular motion

1. A model airplane is attached to a guideline so that when it is in motion, it is moving in a circle. Suppose that it is suddenly released from its circular path. For example, the guide wire may be cut. Describe in a general way the direction of the velocity of the airplane at the moment it is released.

   Answer:

   Newton's First Law of Motion can guide our reasoning in this question. The law states that an object continues in a state of rest or in a state of motion at a constant speed along a straight line unless compelled to change that state by a net force. When the object is suddenly released from its circular path, there is no longer a net force being applied to the object. In the case of the model airplane, the guide wire cannot apply a force since it is cut. Gravity certainly acts on the plane, but the wings provide a lift force that balances the weight of the plane. In the absence of a net force, the plane or any object would continue to move at a constant speed along a straight line in the direction it had at the time of release. As a result, the object would move along the straight line and not a circular path.

The physics of a car rounding a curve

2. A car is driven around one curve having a radius of 320 m and then around another having a radius of 960 m. In both cases, the speed is 28.0 m/s. Compare the centripetal acceleration for both turns. (Assume three significant digits.)

   Answer:

   For curve 1

Radius	R = 320 m
Speed	v = 28.0 m/s

   Case 1

   $$a_c = \frac{v^2}{R}$$

   $$a_c = \frac{(28.0\,\text{m/s})^2}{320\text{ m}}$$

   $$a_c = \frac{(28.0)^2}{320} = \frac{784}{320} = 2.45$$

   $$a_c = 2.45\text{ m/s}^2$$

   For curve 2

Radius	R = 960 m
Speed	v = 28.0 m/s

   Case 2

   $$a_c = \frac{v^2}{R}$$

   $$a_c = \frac{(28.0\text{ m/s})^2}{960\text{ m}}$$

   $$a_c = \frac{(28.0)^2}{320} = \frac{784}{960} = 0.817$$

   $$a_c = 0.817\text{ m/s}^2$$

   The acceleration for curve one and is 2.45 m/s^2, and acceleration for curve 2 is 0.817 m/s^2. Both accelerations point towards the centres of the circles of which the curves are part.

The physics of an orbiting satellite

3. A satellite orbiting Earth at an altitude of 500 km follows roughly a circular path. It takes a satellite 94.5 minutes to circle Earth once. The radius of this motion is 6.88×10^6 m.

   a) What is the speed of the satellite?

   Answer:

   Given: Period $T = 94.5 \text{ minutes} = (94.5 \text{ minutes})(60 \text{ s/minute}) = 5670 \text{ s}$

   Radius $R = 6.88 \times 10^6 \text{ m}$

   Unknown: Speed $v = ?$

   Equation: $v = \dfrac{2\pi R}{T}$

   Substitute and solve: $v = \dfrac{2\pi R}{T}$

   $$v = \frac{2(3.14)(6.88 \times 10^6 \text{ m})}{5670 \text{ s}}$$

   $$v = 7620 \text{ m/s} = 7.62 \times 10^3 \text{ m/s}$$

   b) What is the centripetal acceleration of the satellite?

   Answer:

   Since you know the speed, the period, and the radius of the motion of the satellite, you can use any of the three equations for centripetal acceleration.

   Since you're given the period and the radius of the motion, it is a good idea to use the equation that uses these values.

   $$a_c = \frac{4\pi^2 R}{T^2}$$

   $$a_c = \frac{4(3.14)^2(6.88 \times 10^6 \text{m})}{(5670 \text{ s})^2}$$

   $$a_c = 8.44 \text{ m/s}^2$$

   The centripetal acceleration of the satellite is 8.44 m/s^2 towards the centre of the motion—in this case, towards the centre of Earth.

The physics of a race car and circular motion

4. Computer-controlled display screens provide drivers in the Indianapolis 500 with a variety of information about how their cars are performing. For instance, as a car is going through a turn, a speed of 221 m/h (98.8 m/s) and a centripetal acceleration of 3.00 g (three times the acceleration due to gravity) are displayed. Determine the radius of the turn in metres.

   Answer:

Given:	Centripetal acceleration	a_c = 3.00 g = (3.00)(9.80 m/s/s)
	Speed	v = 98.8 m/s

   You need to determine the radius.

   The magnitude of the centripetal acceleration is given by $a_c = \dfrac{v^2}{R}$.

   Solving for R gives:

   $$R = \frac{v^2}{a_c} = \frac{(98.8 \text{ m/s})^2}{3.00(9.80 \text{ m/s}^2)} = 332 \text{ m}$$

   The radius of the turn is 332 m.

The physics of centripetal acceleration and a jet plane

5. A jet plane travelling at 1800 km/h (500.0 m/s) pulls out of a dive by moving in an arc of radius 4.00 km. What is the plane's acceleration in *g*s? One *g* = 9.80 m/s/s.

   Answer:

   Remember to work in the base SI units of metres, kilograms, and seconds.

Speed	v = 500.0 m/s
Radius	R = 4.00 km = 4000 m = 4.00×10^3 m

   The acceleration of the plane is:

   $$a_c = \frac{v^2}{R} = \frac{(500.0 \text{ m/s})^2}{4.00 \times 10^3 \text{ m}} = 62.5 \text{ m/s}^2$$

   This is equivalent to $\dfrac{62.5 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 6.38$ g.

The physics of a centrifuge and circular motion

6. A centrifuge is a device used to separate solids from a suspension of a solid in a liquid. It functions by whirling a small vessel containing the mixture at very high speed in a circular path. The acceleration experienced by a sample moving in a circle of radius 6.25 cm from the axis of rotation is 5250 g or 5250 times the acceleration of gravity. Calculate the frequency of this motion in RPMs (revolutions per minute).

   Answer:

   Revolutions per minute are the units of frequency. To find frequency, you must know the period of the motion, T.

   Your task, then, is to find the period of the motion.

   Given: Centripetal acceleration $a_c = 5.25 \times 10^3$ g = $(5.25 \times 10^3)(9.80$ m/s/s)

   Radius $R = 6.25$ cm = 0.0625 m

   We know that the centripetal acceleration is given by $a_c = \dfrac{v^2}{R}$.

   Therefore, the speed is:

   $$v = \sqrt{Ra_c} = \sqrt{(0.0625 \text{ m})(5.25 \times 10^3)(9.80 \text{ m/s}^2)} = 56.7 \text{ m/s}$$

   Since you know the speed and the radius, you can find the period using the equation $v = \dfrac{2\pi R}{T}$.

   The period of motion is determined using:

   $$T = \frac{2\pi R}{v} = \frac{2\pi\,(0.0625 \text{ m})}{(56.7 \text{ m/s})} = 6.92 \times 10^{-3} \text{ s } \times (1 \text{ min} / 60 \text{ s})$$

   $$= 1.15 \times 10^{-4} \text{ min}$$

   The number of revolutions per minute (f) is:

   $$f = \frac{1}{T} = \frac{1 \text{ rev}}{1.15 \times 10^{-4} \text{ min}} = 8670 \text{ rev/min}$$

   Another method is to use $a_c = \dfrac{4\pi^2 R}{T^2}$.

   Solving for the period of motion:

   $$T = \sqrt{\frac{4\pi^2 R}{a_c}} = 2\pi\sqrt{\frac{R}{a_c}} = 6.92 \times 10^{-3} \text{ s } \times (1 \text{ min} / 60 \text{ s})$$

   $$= 1.15 \times 10^{-4} \text{ min}$$

   $$f = \frac{1}{T} = \frac{1 \text{ rev}}{1.15 \times 10^{-4} \text{ min}} = 8670 \text{ rev/min}$$

Learning Activity 3.8: Projectile Motion and Circular Motion Review

Part A

The physics of centripetal force and a trapeze act (conceptual)

1. In a circus, a man hangs upside down from a trapeze, legs bent over the bar and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight downward and is stationary or when the partner is swinging through the straight-down position?

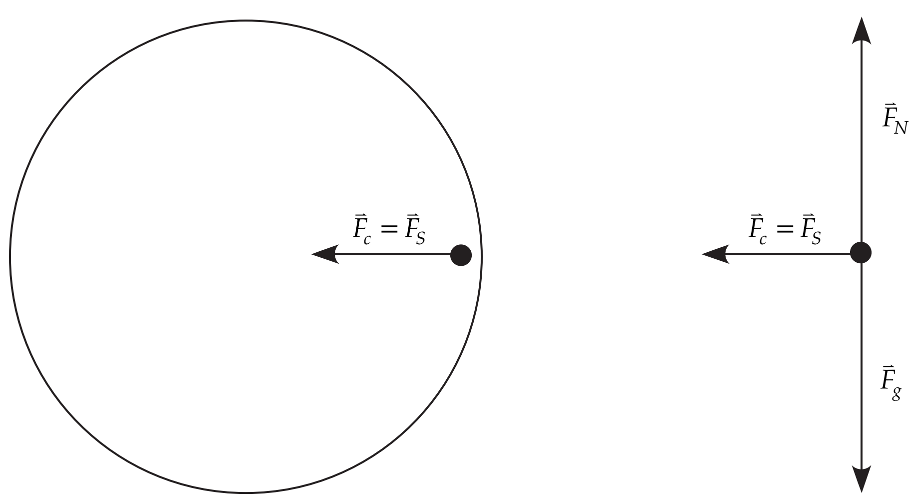
   Answer:

   When the man and his partner are stationary, the man's arms must support his partner's weight. When the two are swinging, however, the man's arms must do an additional job. Then the partner is moving in a circular arc, and has centripetal acceleration. The man's arms must exert an additional pull so that there will be sufficient centripetal force to produce this acceleration. Because of this additional pull, it is harder for the man to hold his partner while swinging rather than while stationary.

The physics of a coin on a rotating turntable

2. A quarter rests on a record that is spinning on a record turntable. Draw a side view and a top view of a free-body diagram, showing all the forces acting in this situation.

   Answer:



The physics of road conditions and making a turn

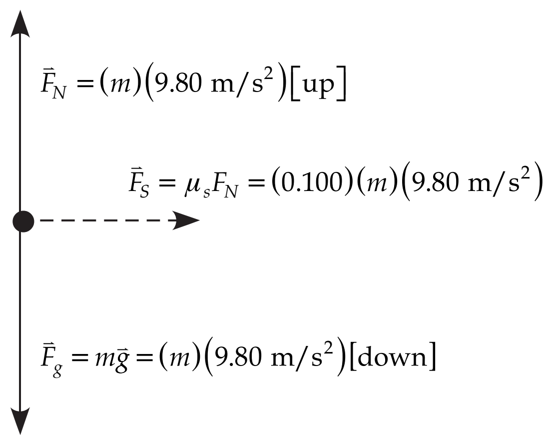
3. What is the speed at which a car can safely negotiate a turn of radius 50.0 m in icy weather where the coefficient of static friction is 0.100?

   Answer:

   The question here is whether you have enough information, since the mass of the car is not given. In situations like this, it turns out that an equation can be used as mass appears on both sides, allowing you to cancel out the mass.

   Here the centripetal force will be supplied by the force of static friction between the tires and the road. First, you'll have to calculate the force of static friction.

Given:	Mass	m = ? kg
	Coefficient of static friction	$\mu_s = 0.100$
	Radius	$R = 50.0$ m



Unknown: Speed		v = ?
Equation:		You know the force of static friction supplies the centripetal force. So $\vec{F}_c = \vec{F}_S$.

You can use the equation $\vec{F}_c = \dfrac{mv^2}{R}$ and $\vec{F}_S = \mu_s F_N = (0.100)(m)(9.80 \text{ m/s}^2)$.

Substitute and solve: $\dfrac{mv^2}{R} = \mu_s \vec{F}_N$

$$\frac{mv^2}{50.0\ \text{m}} = (0.100)(m)\left(9.80\ \text{m/s}^2\right)$$

m cancels out

$$\frac{v^2}{50.0\ \text{m}} = (0.100)(9.80\ \text{m/s}^2)$$

$$v^2 = (0.100)\left(9.80\ \text{m/s}^2\right)\left(50.0\ \text{m}\right)$$

$$v = 7.00\ \text{m/s}$$

The maximum speed with which the car can travel around the curve is 7.00 m/s.

The physics of the orbiting Hubble space telescope

4. Determine the speed of the Hubble space telescope orbiting at a height of 596 km above Earth's surface. The radius of Earth is 6.38×10^6 m and the mass of Earth is 5.98×10^{24} kg. At this point above Earth's surface the gravitational field is 8.20 N/kg towards the centre of Earth.

   Answer:

   The Hubble space telescope stays in orbit around Earth because of the gravitational attraction between Earth and the telescope. It is this gravitational attraction that is providing the centripetal force.

   Given: The gravitational field strength $\vec{g}$ = 8.20 N/kg [towards Earth's centre].

   It is important to understand what the orbital radius is in this case. We must add the radius of Earth to the height of the telescope above Earth.

   $R = 6.38 \times 10^6\ \text{m} + 0.596 \times 10^6\ \text{m} = 6.98 \times 10^6\ \text{m}$

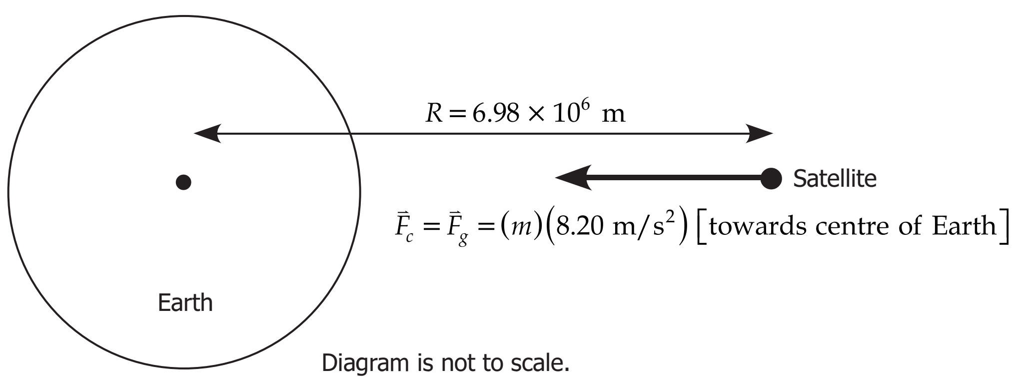


Diagram is not to scale.

Unknown: Speed $v = ?$

Equation: You use the idea that the centripetal force is supplied by the force of gravity.

$$\vec{F}_c = \vec{F}_g$$

$$\frac{m_{\text{satellite}} v^2}{R} = m_{\text{satellite}} \vec{g}$$

The mass satellite cancels out and $\frac{v^2}{R} = \vec{g}$.

Substitute and solve:
$$\frac{v^2}{6.98 \times 10^6 \text{ m}} = 8.20 \text{ m/s}^2$$

$$v = 7565 \text{ m/s}$$

The speed of the satellite must be 7.56×10^3 m/s.

Alternate solution:

The Hubble space telescope stays in orbit around Earth because of the gravitational attraction between Earth and the telescope. It is this gravitational attraction that is providing the centripetal force.

$$\vec{F}_c = \vec{F}_g$$

$$\frac{m_T v^2}{R} = \frac{G m_T M_E}{R^2}, \text{ where } m_T = \text{mass of telescope}$$

$R = 6.38 \times 10^6 \text{ m} + 0.596 \times 10^6 \text{ m} = 6.98 \times 10^6 \text{ m}$

The speed of the satellite must be:

$$v = \sqrt{\frac{(6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{kg})}{6.98 \times 10^6 \text{m}}} = 7.56 \times 10^3 \text{ m/s}$$

The physics of a carnival ride

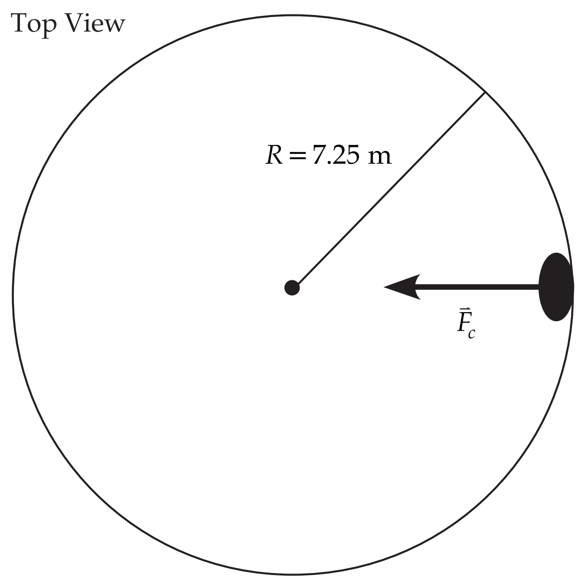
5. A ride at the carnival is shaped like a cylinder. A person of mass 75.0 kg stands inside the cylinder with his back to the wall and is strapped to the wall of the cylinder a distance of 7.25 m from the centre. The cylinder rotates about an axis through its centre, making one revolution every 8.40 seconds. Determine the centripetal force acting on the person.

   Answer:

   Given: Mass $m = 75.0$ kg

   Radius $R = 7.25$ m

   Period $T = 8.40$ s



Unknown: Centripetal force $\vec{F}_c = ?$

Equation: Since you know R and T use

$$\vec{F}_c = ma_c = \frac{m4\pi^2 R}{T^2}.$$

Substitute and solve:

$$\vec{F}_c = \frac{m4\pi^2 R}{T^2}$$

$$\vec{F}_c = \frac{(75.0\text{ kg})4(3.14)^2(7.25\text{ m})}{(8.40\text{ s})^2}$$

$$\vec{F}_c = 304\text{ N}$$

The centripetal force acting on the person is 304 N towards the centre of the motion.

Part B

For each of the following statements, circle the letter that represents the correct response.

1. A ball rolls across a tabletop and, upon reaching the edge, falls off. The curved path of the falling ball is a combination of its
   a) constant horizontal velocity and its constant vertical velocity
   b) increasing horizontal velocity and its constant vertical velocity
   c) constant horizontal velocity and its increasing vertical velocity
2. The curved path that a ball takes after rolling off the edge of a table is called its:
   a) orbit
   b) trajectory
   c) projectile
3. The range of a shell shot at an angle of 45° above the horizontal is the distance the shell travels:
   a) horizontally between launching and landing
   b) vertically between launching and landing
   c) along the curved path it takes between launching and landing
4. A cannonball is fired at a 30° angle above the horizontal.
   a) Its vertical velocity is dependent on its horizontal velocity.
   b) Its vertical velocity is independent of its horizontal velocity.
   c) Its vertical and horizontal velocities are equal.
5. For a shell to reach maximum range, a gun should fire it at an angle of:
   a) 30°
   b) 45°
   c) 60°
6. The centripetal acceleration of an object moving in a circular path is directed
   a) along the circumference of the circle
   b) away from the centre of the circle
   c) toward the centre of the circle

7. If the velocity of an object moving in a circular path is doubled, its centripetal acceleration
   a) is quadrupled (4x)
   b) is doubled (2x)
   c) is not changed
8. If an object travelling in a circle completes four revolutions in two seconds, its period of revolution is
   a) 0.5 s
   b) 2 s
   c) 8 s
9. A boy, facing north, swings a ball attached to a one-metre string in a horizontal circle. When is the tension in the string the greatest?
   a) When the ball is to the right of the boy
   b) When the ball is to the left of the boy
   c) When the ball is south of the boy
   d) The tension is the same at all locations of the ball
10. If the mass of an object moving in uniform circular motion is doubled, how will the centripetal force be affected?
    a) It will not change.
    b) It will be one-half as great (1/2 x).
    c) It will double (2x).
    d) It will be one-quarter as great (1/4 x).

Part C

In the space provided before each of the following statements, indicate whether the statement is true or false by using T for true and F for false. Correct the statement if it is false.

____ 1. A freely falling object falls the same distance during each second that it is falling.

False. A freely falling object falls increasingly larger distances during each second that it is falling.

____ 2. The time required for a projectile to reach its maximum height is equal to the time spent in returning to the point of release.

True

____ 3. A shell launched at 20° above the horizontal and one launched at 60° above the horizontal will have the same range.

False. A shell launched at 20° above the horizontal and one launched at 70° above the horizontal will have the same range.

____ 4. The number of revolutions per second an object travels in a circular path is known as its frequency.

True

____ 5. The centripetal force always points away from the centre of the circle.

False. The centripetal force always points towards the centre of the circle.

Grade 12 Physics (40S)

Module 4: Work and Energy

This module contains the following:

- Introduction to Module 4
- Lesson 1: Work
- Lesson 2: Work-Energy Theorem and Kinetic Energy
- Lesson 3: Gravity and Gravitational Potential Energy
- Lesson 4: Video Laboratory Activity: Hooke’s Law
- Lesson 5: The Spring and Spring Potential Energy
- Module 4 Summary

Module 4: Work and Energy

Introduction to Module 4

Up until now, our study of mechanics has been based largely on Newtonian ideas in mechanics. These ideas by Isaac Newton (1642–1727) were unrivalled for over a hundred years. By the beginning of the 1800s, a powerful alternative, based on the ideas of energy, was taking place. The concept of energy is a very wide one, influencing our thinking about every branch of physics, but in this module it will be applied specifically to our study of mechanics.

Over the centuries, the word energy has had different meanings. The word is derived from the Greek en (which means *in*) and ergon (which means *work*). **Energy** in a very general sense can be defined as the **capacity to do work**. The word has been used in this way since the late 1500s. Galileo (1638) employed the term *l'energia*, though he never defined it. Only in the last 200 years has the idea taken on a scientific meaning.

Energy is a property of matter and is observed indirectly through changes in speed, mass, position, and so on. There is no universal energy meter that measures energy directly. The change in energy of a system is a measure of a physical change in the system. It is through this physical change that we can measure the change in energy of the system. A force may be the cause of the change, and energy is a measure of the change.

This module will, therefore, allow us to extend our knowledge of forces and examine mechanics in a new way through the study of the principles of work and energy. As we will see later, in a given system energy is conserved—that is, it remains constant. That such quantities are conserved not only gives us a deeper insight into the nature of the world, but also gives us another way to attack practical problems.

There are five lessons in this module dealing with work and energy.

Lesson 1: Work will review what "work" means in the physical scientific sense.

Lesson 2: Work–Energy Theorem and Kinetic Energy discusses how kinetic energy and potential energy are related to work.

Lesson 3: Gravity and Gravitational Potential Energy applies the ideas of work and energy to gravity and gravitational potential energy.

Lesson 4: Video Laboratory Activity: Hooke's Law investigates the mathematical relationship between the distance a spring is stretched or compressed and the force the spring exerts.

Lesson 5: The Spring and Spring Potential Energy involves the study of energy transformations as they relate to springs.

The common thread through this module is the action of forces and the analysis of the changes that result. If your focus is the action of a force through a distance, then this leads to work and energy

Assignments in Module 4

When you have completed the assignments for Module 4, submit your completed assignments for Module 3 and Module 4 to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 4.1	Calculating Work
2	Assignment 4.2	Work and Kinetic Energy
3	Assignment 4.3	Conservation of Mechanical Energy in a Roller Coaster
4	Assignment 4.4	Video Laboratory Activity: Hooke's Law
5	Assignment 4.5	Spring Potential Energy

As you work through this course, remember that your learning partner and your tutor/marker are available to help you if you have questions or need assistance with any aspect of the course.

LESSON 1: WORK (2 HOURS)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the term "work"
- ❑ calculate the work done from the area under a force-position graph
- ❑ describe work as a transfer of energy
- ❑ give examples of various forms of energy and describe qualitatively the means by which they can perform work
- ❑ calculate the work done in moving an object at any angle between the force and the displacement
- ❑ calculate work when it is both a positive value and a negative value

Key Words

work
positive work
negative work

Introduction

In this first lesson, you will examine the idea of "work." You will see that "work" in the physics sense is different from the "work" we may use in common conversation. We will define "work" and learn how to calculate "work" in different kinds of situations. If in your experience you have ever pushed or pulled an object over some distance, then you are already familiar with the idea that it takes some energy or "work" to do so. In this lesson, you will understand at a deeper level what "work" in these kinds of situations means.

Work: A Definition

Work is a familiar concept. In your experience, you may have had to push a box along the floor. It takes work to push the box along the floor. You do more work if you push a box filled with books than if you push a box filled with polystyrene foam. More work is done while you push along the box filled with books since you have to exert a larger force to keep the box moving. You also do more work if you push the box of books for a larger distance. Force and displacement both play an important role in determining the amount of work done.

Work

Work is done when a force is exerted on an object, causing the object to move in the direction of a component of the applied force. The amount of work (W) equals the product of the force $(\vec{F})$ and the displacement $(\vec{d})$ of an object both having the same direction.

$$W = Fd$$

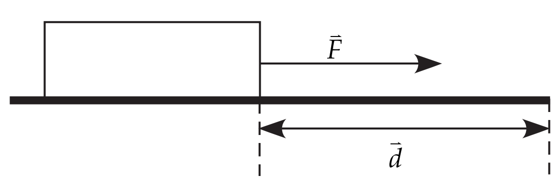
Quantity	**Symbol**	**Unit**
Work	W	joule (J)
Force	F	newton (N)
Displacement	d	metres (m)

Work is the product of two vectors: force $(\vec{F})$ and displacement $(\vec{d})$. In this case, the product of two vectors results in a scalar quantity.

Work is a scalar quantity.

It is important to note that the vector notation for force and displacement is dropped in the work equation.

The diagram below shows a constant force $\vec{F}$ of magnitude 10.0 N acting in the same direction as the displacement $\vec{d}$ of the object. The object moves 2.00 m.



For the situation above, work is defined as the magnitude of the force, $\vec{F}$, times the magnitude of the displacement, $\vec{d}$, or $W = Fd$.

The work done to push a car is the same whether the car is moved north to south or east to west, provided that the amount of force and the distance moved are the same. Work does not have a direction, and is therefore a scalar quantity.

The equation $W = Fd$ indicates that the unit of work is the unit of force times the unit of distance, or the newton · metre in SI units. One newton · metre is referred to as a *joule* (J) in honour of James Joule (1818–1889) and his work into the nature of work, energy, and heat. For the example above, the work done is $W = (10.0 \text{ N})(2.00 \text{ m}) = 20.0 \text{ J}$.

> One **joule** is the amount of work done by a force of 1 N acting through a distance of 1 m.
>
> $$1 \text{ J} = (1 \text{ N})(1 \text{ m})$$

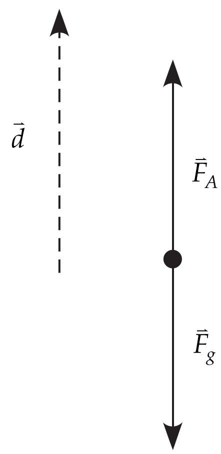
This definition of work has a feature that may be surprising. Pushing on an immovable object, such as a concrete wall, will not cause the object to move. You may break into a sweat and grow very tired, but, according to this definition of work, since the displacement is zero, the work done is zero. In physics, work requires force to be exerted, and the force must act through a distance.

Example 1: Work Done in Lifting an Object

How much work would be done in lifting a 50.0 kg bag of cement 1.25 m off the ground?

To calculate work done, you need the force and the displacement. The force required here has to be enough to overcome the force of gravity pulling down on the bag of cement.

Given:	Mass	$m = 50.0$ kg
	Displacement	$\vec{d} = 1.25$ m (Note: You can drop the vector notation.)
	Acceleration of gravity	$\vec{g} = 9.80 \text{ m/s}^2$ [down]
	Force	$\vec{F} = 490$ N (Force required to lift the bag of cement.)



Unknown: Work	$W = ?$
Equation:	$W = Fd$
Substitute and solve:	$W = (490\ \text{N})(1.25\ \text{m}) = 612.5\ \text{J}$ $W = 612\ \text{J}$

The work required is 612 J.

Positive and Negative Work

Work can be either **positive** or **negative**, depending on whether a component of the force points in the same direction as the displacement or in the opposite direction. If the **force** has a component in the **same direction** as the **displacement** of the object, the work done by the force is **positive**. On the other hand, if a component of the **force** points in the **direction opposite** to the **displacement**, the work is **negative**. If the force is **perpendicular** to the **displacement**, the force has no component in the direction of the displacement and the work is zero.

Example 2: Lifting a Box

An example could be a delivery person lifting a box of paper that has a weight of 216 N. The person lifts the box from the floor vertically upwards at a constant speed of 0.200 m/s. The force acts upwards and the displacement is upwards. While the person is lifting, the work done is

$$W = Fd = (216\ \text{N})(0.200\ \text{m}) = 332\ \text{J}$$

When the box is set back down on the floor, the force the person is exerting is up but the box is moving down. Since the force and the displacement are in opposite directions, the work done in the downward motion is $W = –332$ J.

Example 3: Work of Friction

Another example would be a person pulling a mass to the right at a constant speed with a force of 10.0 N for a distance of 2.00 m. Because the mass is moving at a constant speed, the net force is zero and there must be an equal and opposite force directed to the left. This other force could be the force of friction. The work done by the person is

$$W = Fd = (10.0 \text{ N})(2.00 \text{ m}) = 20.0 \text{ J}$$

The work done by the force of friction is –20.0 J because the direction of the force of friction is directly opposite the direction of the displacement.

Learning Activity 4.1

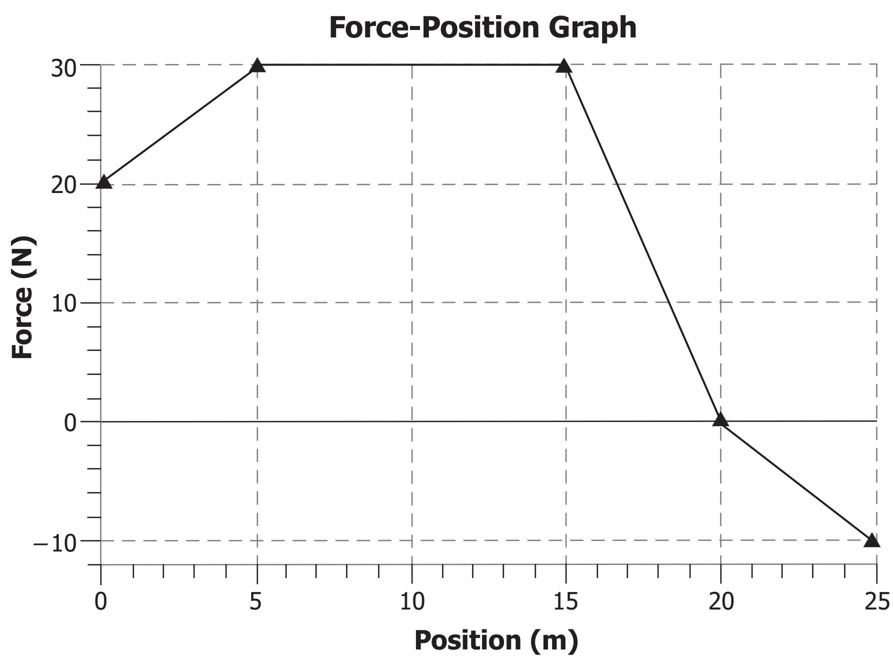
Definition of Work

Solve the following problems to check your understanding of work. You can check your answers using the answer key provided at the end of Module 4.

1. The truck exerts a force of 2150 N to pull a car 16.2 m out of the ditch. How much work is done?
2. A person pulls horizontally with a force of 375 N against a crate of mass 125 kg and moves it across the floor in a steady rate of 1.00 m/s for 3.00 seconds. How much work is done?
3. The person exerts a force of 150 newtons on a large crate for 2.50 seconds, but cannot move it. How much work has been done?

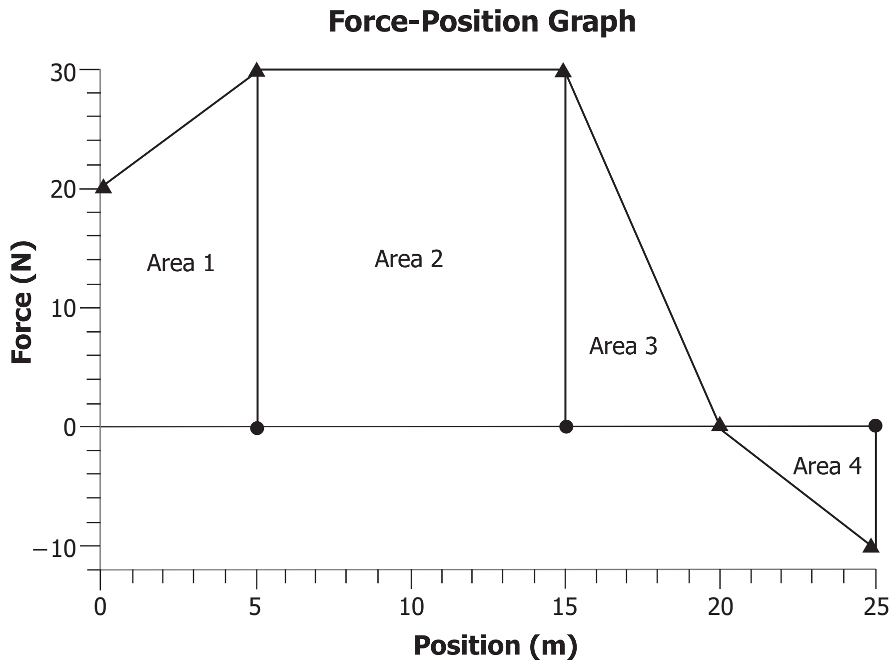
Work on a Force-Position Graph

You can plot a graph of the force applied to an object and the position of the object along a straight line.



To calculate the work done you must multiply the force by the displacement. On the graph above, you can see that the force extends upwards and the displacement (change in position) extends sideways. You can think of force as being a length and displacement as being a width, so **force times displacement** is **length times width**, which gives **area**.

To find the **work done** you must find the area between the curve and the horizontal axis. The force position graph can be nicely divided into four areas, as shown in the following graph.



To find the work done, calculate each area separately and then add them together.

Area 1 is a trapezoid, so the formula for the area is Area trapezoid = $\left[\frac{a+b}{2}\right]h$.

Area 1 = W_1 = Area trapezoid = $\left[\frac{a+b}{2}\right]h = \left[\frac{20\text{N}+30\text{N}}{2}\right]5\text{ m} = 125\text{ J}$

Area 2 = W_2 = Area rectangle = lw = (30N)(10m) = 300 J

Area 3 = W_3= Area triangle = $\frac{bh}{2} = \frac{(30\text{ N})(5\text{ m})}{2} = 75\text{ J}$

Area 4 lies below the horizontal axis. Therefore, the force has a negative value and the work done is also negative.

Area 4 = W_4 = Area triangle = $\frac{bh}{2} = \frac{(-10\text{ N})(5\text{ m})}{2} = -25\text{ J}$

So the total work done is just the sum of these four works:

$W_{TOTAL} = W_1 + W_2 + W_3 + W_4$ = 125 J + 300 J + 75 J + (–25 J) = 475 J.

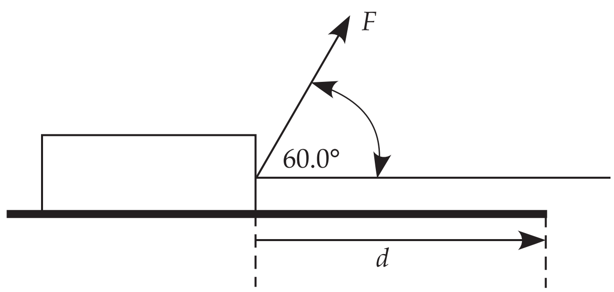
We simply add the values of the four works together.

Work and Force at an Angle

Often, the force and the displacement do not point in the same direction. **In our definition of work, it is important that the force and the magnitude of the displacement point in the same direction.** If they do not, then you must determine the component of the force along the displacement.

Example 4: Work Done Where the Force and Displacement Point in Different Directions

For example, suppose that a force of 10.0 N is applied at an angle of 60.0° above the horizontal and the object moves a distance of 2.00 m.



The component of the force that is in the direction of the displacement is $F \cos \theta$.

Therefore, the work done is $W = F(\cos \theta)d$ = (10.0 N)(cos 60.0°)(2.00 m) = 10.0 J.

When the force points in the same direction as the displacement, the $\theta = 0°$ and the equation for work remains $W = Fd$.

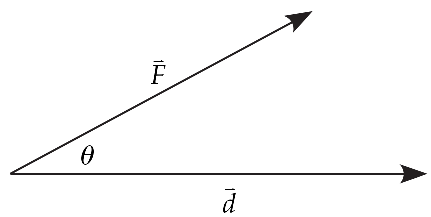
This brings us to a new equation that can be used to calculate the work done.

Work Equation

Work is calculated as the product of the force, the displacement, and the cosine of the angle between the force and displacement vectors.

$$W = Fd \cos \theta$$

Quantity	Symbol	Unit
Work	W	joule (J)
Force	F	newton (N)
Displacement	d	metres (m)
Angle between force and displacement vectors	θ	no units



Learning Activity 4.2

Calculating Work

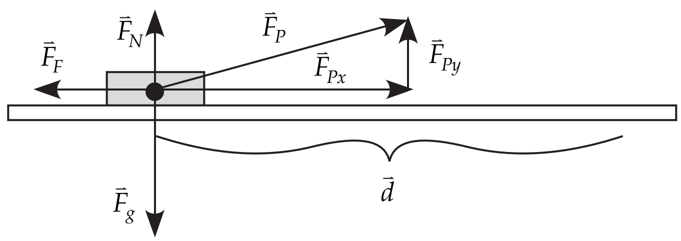
Solve the following problems to check your understanding of calculating work in situations where the force and displacement are in different directions. An answer key is provided at the end of Module 4 so that you may check your responses.

1. What work is performed in dragging a sled 15.0 m horizontally when the force of 1320 N is transmitted by a rope making an angle of 30.0° with the ground?
2. A girl pulls a wagon with a constant velocity along a level path for a distance of 45.0 m. The handle of the wagon makes an angle of 20.0° with the horizontal, and she exerts a force of 85.0 N on the handle. Find the amount of work the girl does in pulling the wagon.

Example 5: Using the General Work Equation

A 20.0 kg crate is pulled 50.0 m along a horizontal floor by a constant force exerted by a person, $\vec{F} = 1.00 \times 10^2$ N, which acts at a 20.0° angle up from the horizontal. The floor is rough and exerts a friction force $\vec{F}_F = 15.0$ N. Determine the work done by each force acting on the crate, and the net work done on the crate.

Given:	Force applied by a person	$\vec{F}_P = 1.00 \times 10^2$ N [20.0° up from horizontal]
	Force of gravity	$\vec{F}_g$
	Normal force	$\vec{F}_N$
	Force of friction	$\vec{F}_F = 15.0 \text{ N } [\text{left}]$
	Angle between force and displacement	$\theta = 20.0°$



In this case, the force of gravity and the normal force do no work, since the crate is not displaced in their direction.

The angle between the normal force and the displacement is 90.0°. The angle between the force of gravity and the displacement is 90.0°.

Using the work formula to calculate the work done by the normal force or the force of gravity, you get $W = Fd \cos \theta = Fd \cos 90.0° = 0$ J.

It turns out that the cos 90.0° is zero.

Solution A:

Add up the work done by each force.

Work done by the person (W_P).

$$\text{Work} = F_P d \cos \theta$$

$$W_P = (1.00 \times 10^2 \text{ N})(50.0 \text{ m})(\cos 20.0°) = +4.70 \times 10^3 \text{ J}$$

Work done by friction (W_F).

$$\text{Work} = F_F \Delta d \cos \theta$$

$$W_F = (15.0 \text{ N})(50.0 \text{ m})(\cos 180.0°) = -7.50 \times 10^2 \text{ J}$$

(The negative sign means work is done which opposes the motion and energy is lost by the crate.)

Net Work Done

Since work is a scalar, we can take the algebraic sum of the work done by each force.

$$W_{\text{total}} = W_P + W_F = (+4.70 \times 10^3 \text{ J}) + (-7.50 \times 10^2 \text{ J}) = +3950 \text{ J}$$

Solution B:

Find the net force on the object in the direction of motion and multiply by the displacement.

$$W_{\text{total}} = F_{\text{net}} d \cos \theta$$

where $F_{\text{net}} = F_{\text{P}} \cos \theta + F_{\text{F}} = 100 \cos 20.0° + (-15.0) = 79.0 \text{ N}$

$$W_{\text{total}} = (79.0 \text{ N})(50.0 \text{ m})$$

$$W_{\text{total}} = +3950 \text{ J}$$

Learning Activity 4.3

Calculating Work: The General Case

The practice questions in this learning activity deal with calculating work. An answer key is available at the end of Module 4 for you to check your responses after you have answered the questions.

The physics of work and pulling a crate along a surface with friction (conceptual)

1. A crate is being moved with a constant velocity $\vec{v}$ by a force *P* (parallel to $\vec{v}$) along a horizontal floor. The normal force is $\vec{F}_{N}$, the kinetic frictional force is $\vec{F}_K$, and the weight of the crate is $m\vec{g}$. Which forces do positive, zero, or negative work?

The physics of pulling a toboggan at an angle

2. A person pulls a toboggan for a distance of 35.0 m along the snow with a rope directed 25.0° above the snow. The tension in the rope is 94.0 N. How much work is done by the tension force?

The physics of work and moving up and down in an elevator

3. You are moving into an apartment. Your weight is 685 N and that of your belongings is 925 N.
   a) How much work does the elevator do in lifting you and your belongings up five stories (15.0 m) at a constant velocity?
   b) How much work does the elevator do on you (without belongings) on the downward trip, which is also made at a constant velocity?

The physics of pulling a crate at an angle

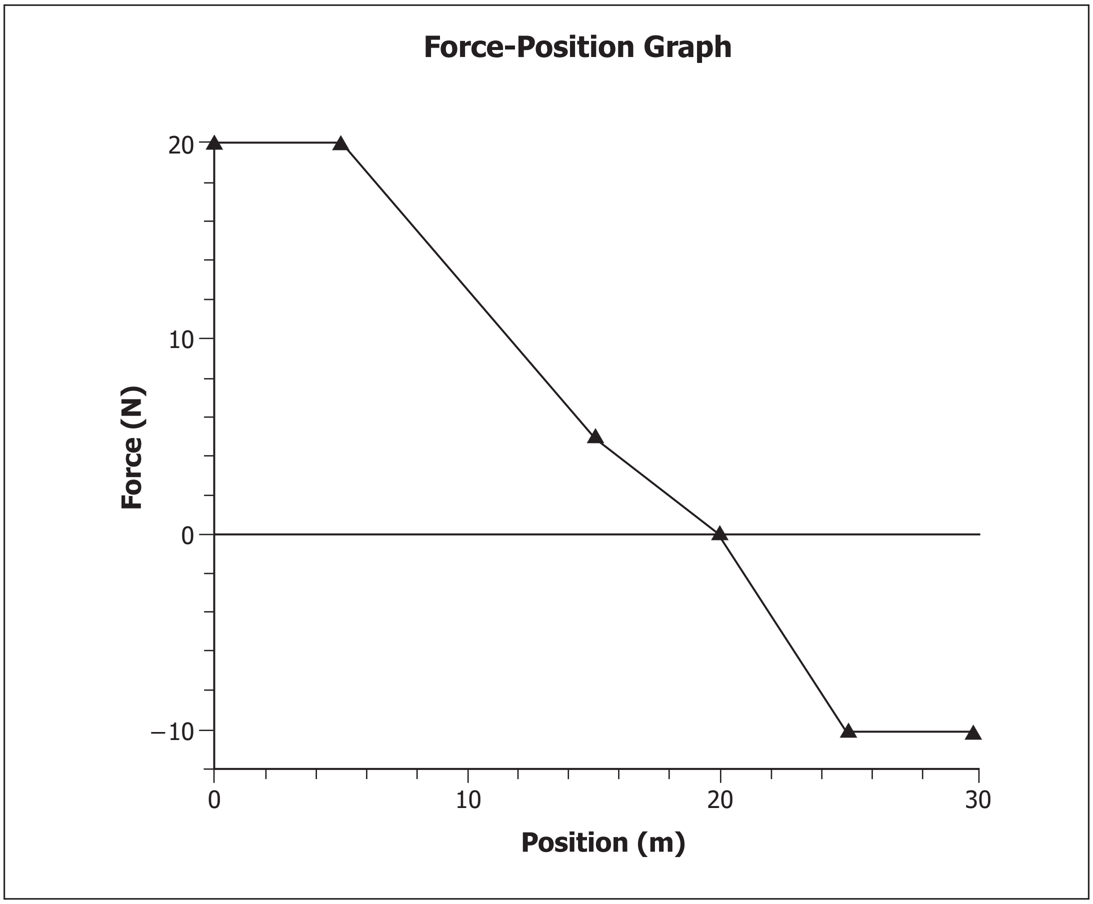
4. A 1.00 x 10^2 kg crate is being pulled across a horizontal floor by a force $\vec{F}_A$ = 203 N that makes an angle of 30.0° above the horizontal. The crate moves at a constant velocity over a distance of 2.50 m.
   a) Calculate the work done by the applied force.
   b) Calculate the normal force acting on the crate.
   c) Determine the coefficient of kinetic friction.

(continued)

Learning Activity 4.3: Calculating Work: The General Case (continued)

Calculating work from a force-position graph

5. a) Calculate the total work done according to the force-position graph below.



b) What is the meaning of a negative work in terms of force and displacement?

c) In which sections of the graph is the force uniformly changing? How is the work calculated in these regions?

Lesson Summary

In this lesson, you learned the meaning of the term "work" and how to calculate it.

The **work** done on an object by a constant force is $W = Fd$, where F is the magnitude of the force, and d is the magnitude of the displacement.

The SI unit of work is the newton·metre, also referred to as the joule (J).

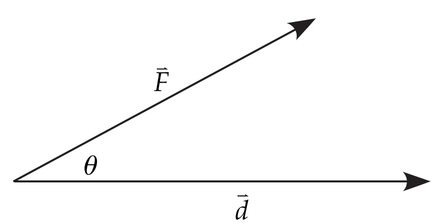
Work does not have a direction, and is therefore a scalar quantity.

In calculating work done, it is important that the force and the magnitude of the displacement point in the same direction. If they do not, then we must determine the component of the force along the displacement.

Work Equation
Work is calculated as the product of the force, the displacement, and the cosine of the angle between the force and displacement vectors.

$$W = Fd\cos\theta$$

Quantity	Symbol	Unit
Work	W	joule (J)
Force	F	newton (N)
Displacement	d	metres (m)
Angle between force and displacement vectors	θ	no units



If the force has a component in the same direction as the displacement of the object, the work done by the force is positive.

If a component of the force points in the direction opposite to the displacement, the work is negative.

If the force is perpendicular to the displacement, the force has no component in the direction of the displacement and the work is zero.

These ideas lead us to the next lesson where we will study kinetic energy and the work-energy theorem.

Assignment 4.1

Calculating Work (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of work and pulling a freezer at an angle

A homeowner exerts a force of 2.40×10^2 N while pulling an 85.0 kg freezer across the floor of a basement. The homeowner pulls at an angle of 20.0° above the surface of the floor. The coefficient of kinetic friction for this surface is 0.200, and the freezer is pulled a distance of 8.00 m. Determine the following.

a) The work done by the pulling force.

b) A free-body diagram with appropriate labels for the forces. Resolve the applied pulling force into its components and use these components in the free-body diagram, which should look like a cross (+).

c) The normal force.

(continued)

d) The force of friction.

e) The work done by the force of friction.

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each of parts (a) to (e)

Video - Work, Energy, and Power Demos

This video contains demonstrations of work, kinetic and potential energy and power.

https://youtu.be/hdWrnOgSIuw

Video - Work, Energy, and Power: Crash Course Physics #9

This video introduces work, energy, kinetic energy, gravitational potential near the Earth's surface, spring potential energy and power.

https://youtu.be/w4QFJb9a8vo

Video - Work by Constant Force

This video introduces the concept of work as defined in Physics. The equation for work is given as Work = (Force)(Displacement)(cos Θ) or W = F d cos Θ. Work is a scalar with a unit of Joule (J). The angle Θ plays an important role in calculating work. An example is provided illustrating how work is calculated.

Note: This website (AKlectures) has several other pertinent videos dealing with work, kinetic energy, potential energy, the Work-Energy Theorem and the Law of Conservation of Energy.

https://youtu.be/T92h8VtdfnA

Video - Work Done By a Variable Force Physics Problems, Force Displacement Graphs - Calculus

View this video from 0:00 to 7:00 minutes.

This video uses the concept that the area under the curve of a Force-distance graph represents work done. If the force is constant, then the area under the curve of the force-distance graph is a rectangle. In this case A= lw or on this graph W=Fd.

If the force-distance graph has varying forces, then the area under the curve of the force-distance graph is divided into regular shapes like rectangles, triangles and trapezoids.

Areas above the distance axis are positive so the work done is positive.

Areas below the distance axis are negative so the work done is negative.

Examples of these calculations are provided.

https://youtu.be/LbAxcMQ7J6c

Assignment 4.1

Calculating Work (5 MARKS)

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c) The normal force.

(continued)

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Assignment 4.1: Calculating Work (continued)

d) The force of friction.

e) The work done by the force of friction.

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each of parts (a) to (e)

Assignment 4.1

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c) The normal force.

(continued)

Assignment 4.1: Calculating Work (continued)

d) The force of friction.

e) The work done by the force of friction.

Method of Assessment

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- 1 mark for the correct answer in each of parts (a) to (e)

Lesson 2: Work-Energy Theorem and Kinetic Energy (1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the term "kinetic energy" and calculate the kinetic energy of a moving object
- ❑ state the work-energy theorem and use the theorem in solving problems
- ❑ explain how work and kinetic energy apply to an object moving in a circle

Key Words

kinetic energy	mechanical energy
potential energy	energy
work-energy theorem	

Introduction

Recall the old saying, "You can't get something for nothing." This applies to the idea of work. Work and energy are connected. In order for you to do work, you must supply your body with energy. The energy from the food you eat is used by your muscles to do work. In the process of doing work, you also expect to see a result, either a job that is done, such as pushing a box of books from one place to another, or some other change in your environment.

You can think of work as the process of transferring energy. The change in the environment can then be a change in the amount of energy possessed by the object on which the work was done. The result could be that there is a change in the "kinetic energy" or the "potential energy" of the object. The "work-energy theorem" relates the work done on an object to the change in the kinetic energy of the object. In this lesson, we will obtain this theorem by combining Newton's Second Law of Motion with the now familiar concept of work and the new idea of "kinetic energy."

Types of Energy

Energy is the capacity of a physical system to do work.

If you do work, energy is transferred.

In all cases, an object must possess some form of energy to do work. Commonly, an agent doing the work (a student, a car, a battery, a motor) will have **chemical potential** energy stored in food or fuel, which is transformed into work. In the process of doing work, the objects doing the work exchange energy—one gains and one loses. A force doing positive work on an object will transfer its energy to the object and increase the energy of that object. A force doing negative work will decrease the energy of that object and will have energy transferred from the object to the agent exerting the force.

Energy is conserved by simply being transferred from the agent to the object, or vice versa, depending on the direction of the force relative to the displacement.

We call energy that is possessed by an object, due to its motion or its stored energy of position, **mechanical** energy.

For example, when you lift an object, potential chemical energy in the glucose molecules in your muscles allow you to do work on the object. Initially, the work done by you is converted into gravitational potential energy as the object rises, as well as kinetic energy as the object is put in motion. In the end, the gain in energy equals the loss in potential chemical energy in your muscles. A curling rock, once released, has a certain speed and, therefore, kinetic energy, which will gradually be converted into thermal energy as friction acts in the opposite direction of the curling rock's movement.

Different types of energy have one thing in common: they imply a system capable of doing work. Additionally, all types of energy may be subdivided into two different categories: potential energy (PE or U) or kinetic energy (KE or E_K).

Potential energy is energy stored in a system, which could be used to do work.

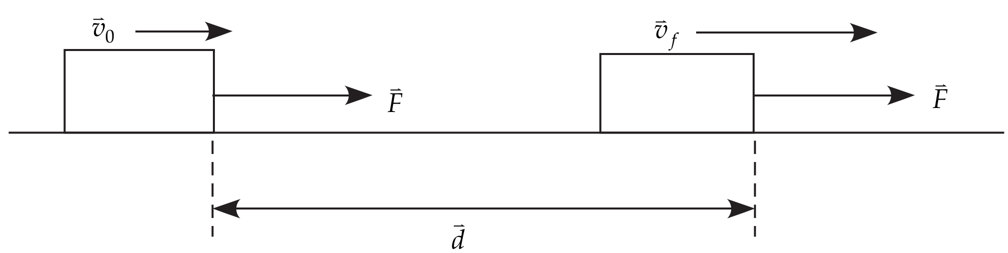
For example, water behind a dam has potential gravitational energy (PE_g or U_g), which can be used to do work to turn the turbines. A stretched spring or elastic or a compressed spring has potential elastic energy, which can be used to do work to propel an object. A gas has heat of vaporization, which can be released to do work when it condenses, as in a steam engine. A combustible material such as propane or octane has potential chemical energy, which can be released to do work once it is burned in an engine.

Kinetic energy is energy by virtue of being in motion.

Kinetic energy can be used to do work when one thing interacts with something else. For example, at the macroscopic level (visible), a swinging bat has kinetic energy and does work hitting a ball transferring the kinetic energy to the ball. At the microscopic or invisible level, heat contained in matter is due to the sum of the kinetic energies of all the particles that compose that matter. In gaseous matter, the particles are continually in motion; even in a solid, particles have a vibrational motion. The greater the motion of these particles, the greater is the heat content of that matter. The electric current in a wire is simply the motion of electrons in that wire. The kinetic energy of the electrons is called **electrical energy**, which can be used to do work as in an electric motor. Electromagnetic radiation (radio waves, infrared, visible light, ultraviolet, X-rays, gamma-rays) are photons in motion at a speed of 3.0×10^8 m/s in a vacuum

Kinetic Energy

To begin with, consider the case of a net external force, $\vec{F}$, acting on a mass, m. Recall that the net force is the vector sum of all the external forces acting on the object. The direction of the net force is in the same direction as the displacement, $\vec{d}$. The net force acting on the mass, m, produces an acceleration, $\vec{a}$, given by $\vec{a} = \frac{\vec{F}}{m}$. Due to this acceleration, the velocity of the mass increases from an initial value of $\vec{v}_0$ to a final value of $\vec{v}_f$.



In multiplying both sides of the equation, $\vec{F} = m\vec{a}$ by the distance d, we obtain $\vec{F}d = m\vec{a}d$.

We know from our study of kinematics that $v_f^2 = v_0^2 + 2\vec{a}d$.

Solving this equation for $\vec{a}d$, $\vec{a}d = \frac{1}{2}\left(\vec{v}_f^2 - \vec{v}_0^2\right)$.

Substituting this into $\vec{F}d = m\vec{a}d$ gives $\vec{F}d = \frac{1}{2}m\vec{v}_f^2 - \frac{1}{2}m\vec{v}_0^2$.

In this equation, $\vec{F}d$ represents the work done by a net external force.

$\frac{1}{2}m\vec{v}_f^2$ represents the final kinetic energy.

$\frac{1}{2}m\vec{v}_0^2$ represents the initial kinetic energy.

Thus, the quantity $\frac{1}{2}$ (mass)(speed)^2 is called the **kinetic energy**.

A formal definition of kinetic energy would be as follows:

Kinetic Energy

The kinetic energy (*KE*) of an object with mass, *m*, and speed, *v*, is given by the product of the object's mass and the square of the velocity.

$$KE = \frac{1}{2}mv^2$$

Quantity	**Symbol**	**Unit**
Kinetic energy	*KE*	joule (J)
Mass	*m*	kilogram (kg)
Speed	*v*	metres/second (m/s)

Note: When velocity is squared the product is no longer a vector. Therefore, vector notation is not used in this expression for kinetic energy.

Example 1: Calculating the Kinetic Energy of a Baseball

Calculate the kinetic energy of a 142 g baseball travelling at 40.0 m/s [south].

Given: Mass $m = 142 \text{ g} = 0.142 \text{ kg}$ (SI mass unit)

Speed $v = 40.0 \text{ m/s}$ (You do not need velocity here!)

Unknown: Kinetic energy $KE = ?$

Equation: $KE = \frac{1}{2}mv^2$

Substitute and solve: $KE = \frac{1}{2}(0.142 \text{ kg})(40.0 \text{ m/s})^2 = 114 \text{ J}$

The kinetic energy of the baseball is 114 J.

Learning Activity 4.4

Kinetic Energy

Answer the following questions to check your understanding of kinetic energy. An answer key is available at the end of Module 4 for you to check your work after you have answered the questions.

1. The baseball in example 1 above is struck by a baseball bat, leaving the bat with a velocity of 40.0 m/s [north]. Calculate
   a) the final kinetic energy of the baseball
   b) the change in the kinetic energy of the baseball
2. An object is moving at 9.00 m/s and has 215 J of kinetic energy. What is the mass of the object?

The Work-Energy Theorem

In the previous section, you saw that the SI unit for kinetic energy is the joule. The unit for kinetic energy is the same as the unit for work. **Kinetic energy**, like work, is a **scalar quantity**. The quantities of kinetic energy and work are closely related. You have seen that when work is done on an object, the kinetic energy changes. In fact, the following statement, called the **work-energy theorem**, summarizes this idea.

Work-Energy Theorem

When a net external force does work, W, on an object, the result is a change in the kinetic energy of the object. The change in the kinetic energy of the object ($KE_f - KE_0$) is equal to the work done on the object.

$$W = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

Quantity	Symbol	Unit
Work	W	joule (J)
Final kinetic energy	KE_f	joule (J)
Initial kinetic energy	KE_0	joule (J)
Mass	m	kilogram (kg)
Final velocity	v_f	metres/second (m/s)
Initial velocity	v_0	metres/second (m/s)

According to the work-energy theorem, a moving object has kinetic energy because the work done on the object transferred energy into the object. The force doing the work accelerated the object from rest to a speed, v. On the other hand, an object that possesses kinetic energy is able to do work. The kinetic energy of the object will decrease as that energy is transferred out of the object to do the work (as the object pushes or pulls a different object).

Example 2: The Work-Energy Theorem

An application of the work-energy theorem is a spaceship travelling in space.

A spaceship is travelling in space where the gravitational field intensity is 0 N/kg. The spaceship has a mass of 2.50×10^4 kg and is coasting at 5.00×10^3 m/s. The engine of the spaceship is fired and pushes the spaceship with a net force of 3.25×10^4 N along the original direction of motion of the spaceship for a distance of 8270 km.

a) Determine the work done on the spaceship by its engine.

Answer:

Given:	Mass	$m = 2.50 \times 10^4$ kg
	Initial speed	$v_0 = 5.00 \times 10^3$ m/s
	Force	$F = 3.25 \times 10^4$ N
	Displacement	$d = 8270$ km $= 8.27 \times 10^6$ m
	Angle	$\theta = 0°$

You can first find the work done on the spaceship.

$$W = Fd \cos \theta = (3.25 \times 10^4 \text{ N})(8.27 \times 10^6 \text{ m})(\cos 0°) = 2.69 \times 10^{11} \text{ J}$$

b) Determine the final speed of the spaceship.

Answer:

Since the force of the engine is the only force acting on the spaceship, it is the net external force, and the work it does causes the kinetic energy of the spaceship to change. The work is positive because net force points in the same direction as the displacement, d. According to the work-energy theorem, a positive value for W means that the kinetic energy increases. Thus, you can use the work-energy theorem to find the final kinetic energy and, hence, the final speed of the spaceship. Since the kinetic energy increases, you expect the final speed to be greater than the initial speed.

According to the work-energy theorem,

$$W = KE_f - KE_0$$

The final kinetic energy is

$$\begin{aligned} KE_f &= W + KE_0 = W + \frac{1}{2}mv_0^2 \\ &= 2.69 \times 10^{11} \text{ J} + \frac{1}{2}\left(2.50 \times 10^4 \text{ kg}\right)\left(5.00 \times 10^3 \text{ m/s}\right)^2 \\ &= 2.69 \times 10^{11} \text{ J} + 3.12 \times 10^{11} \text{ J} \\ &= 5.82 \times 10^{11} \text{ J} \end{aligned}$$

To determine the final speed, you use $KE_f = \frac{1}{2}mv_f^2$.

$$v_f = \sqrt{\frac{2\left(KE_f\right)}{m}} = \sqrt{\frac{2\left(5.82 \times 10^{11}\ \text{J}\right)}{2.50 \times 10^{4}\ \text{kg}}} = 6.82 \times 10^{3}\ \text{m/s}$$

The final velocity of the spaceship is 6.82×10^3 m/s.

In this example, the only force acting on the spaceship was the force of the engines since the gravitational field intensity is 0 N/kg. If several forces act on an object, determine the net force by adding all the forces acting on the object using vector addition, as you did in Module 2: Dynamics. The work-energy theorem allows you to link the work done by the net external force to the change in the kinetic energy of the object.

Learning Activity 4.5

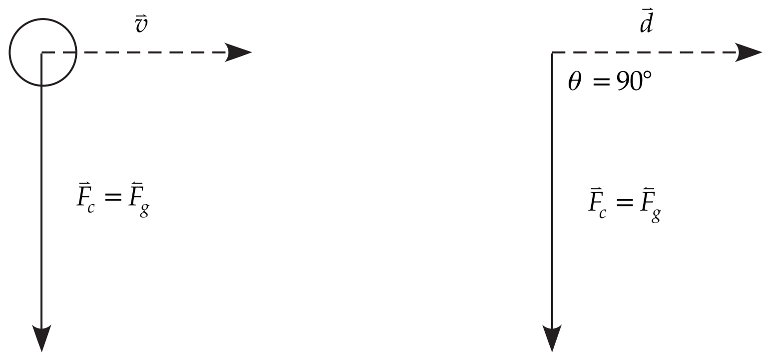
Work-Energy Theorem

Answer the following questions to test your understanding of the work-energy theorem. An answer key is available at the end of Module 4 for you to check your responses after you have answered the questions.

1. An automobile of mass 1250 kg starts from rest. It accelerates under a net force of 3250 N [west] over 25.0 m. Calculate the final kinetic energy and the final velocity of this automobile.
2. A curling rock of mass 20.0 kg is sliding along the ice with a velocity of 2.75 m/s [east]. During the next 3.00 seconds, the curling rock slows to a velocity of 1.50 m/s [east] while sliding 6.38 m [east]. Calculate
   a) the change in the kinetic energy of the curling rock.
   b) the net force that was acting on a curling rock.
   c) the coefficient of kinetic friction for the ice.

Work and Circular Motion

To understand better the special case of circular motion and work, let us study the motion of a satellite around the Earth. The only external force that acts on the satellite is the gravitational force. As with all situations involving uniform circular motion, the force is directed towards the centre of the circle and is perpendicular to the instantaneous displacement at all times. Because there is no component of force along the direction of motion, there is no work done by the force.



$W = Fd \cos \theta = Fd \cos 90° = Fd(0) = 0$ J

Since there is no work done, according to the work-energy theorem, there is **no change in kinetic energy**. The **kinetic energy** of the satellite, and therefore the **speed** of the satellite, continues to **remain** at a **constant value** everywhere on its orbit.

This analysis of the motion of objects moving with uniform circular motion should confirm what we indicated earlier. Even though there is a force—the centripetal force—acting on objects moving in a circle, the force does not change the magnitude of the velocity of the object, only its direction.

Since the magnitude of velocity is speed and the magnitude did not change, the speed of the object moving with uniform circular motion must remain constant.

Learning Activity 4.6

Work, Kinetic Energy, and the Work-Energy Theorem

There are four practice questions in this learning activity. An answer key is available at the end of Module 4 for you to check your responses after you have answered the questions.

The physics of energy and the hammer throw event

1. The hammer throw is a track and field event in which a 7.30 kg ball (the hammer), starting from rest, is whirled around in a circle several times and released. It then moves upward on the familiar curving path of projectile motion. In one throw, the hammer is given a speed of 29.0 m/s. Determine the work done to launch the motion of the hammer.

The physics of energy and an arrow

2. An archer uses a bow for which the bowstring exerts an average force of 85.2 N on the arrow over a distance of 0.825 m. If the mass of the arrow fired by the archer is 75.0 g, how fast is the arrow travelling when it leaves the bow?

The physics of energy and jogging

3. A 65.0 kg jogger is running at a speed of 5.30 m/s.
   a) What is the kinetic energy of the jogger?
   b) How much work is done by the net external force that accelerates the jogger to 5.30 m/s from rest?

The physics of energy, friction, and a curling rock

4. The speed of a curling rock decreases from 2.65 to 1.25 m/s in coasting 29.2 m along the surface of the ice. This type of shot is used to remove an opponent's rock from play. Find the coefficient of kinetic friction between the curling rock and the ice.

Lesson Summary

Energy is the capacity of a physical system to do work.

Energy comes in different forms that can be placed in two broad categories: **kinetic energy** (energy of motion) and **potential energy** (stored energy or energy of position).

When the work is done within a physical system, energy is transferred.

In this lesson, we derived the mathematical expression for kinetic energy. The **kinetic energy** (KE) of an object with mass m and speed v is given by $KE = \frac{1}{2}mv^2$.

The SI unit for kinetic energy is the joule (J).

The **work-energy theorem** states that when a net external force does work W on an object, the kinetic energy of the object changes from its initial value of KE_0 to a final value of KE_f, the difference between the two values being equal to the work:

$$W = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

For objects moving in a circle, the centripetal force is directed towards the centre of the circle and is perpendicular to the instantaneous displacement at all times. Because there is no component of force along the direction of motion, there is no work done by the force. Since there is no work done, according to the work-energy theorem, there is no change in kinetic energy.

We will extend the work of this lesson to the ideas of potential energy and the conservation of energy.

NOTES

Assignment 4.2

Work and Kinetic Energy (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of a baseball and kinetic energy

After being hit, a 0.145 kg baseball has a speed of 45.6 m/s.

a) What is the kinetic energy of the baseball after it has been hit?

b) How much work is done on the baseball by the baseball bat?

c) If the force of the baseball bat acts in the direction of the motion of the ball and the bat and ball are in contact for 0.0120 m, determine the average force applied to the baseball by the bat. In this case, the force of gravity can be ignored.

(continued)

Assignment 4.2: Work and Kinetic Energy (continued)

d) In moving through the air, the baseball slows down to a speed of 30.0 m/s. What work has the air done on the ball?

e) If the baseball travelled through the air a distance of 84.5 m, what must be the average force of resistance caused by the air?

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each of parts (a) to (e)

Video - Physics, Work Energy Theorem, An Explanation

This video introduces the Work-Energy Theorem.

When a force does work energy is transferred. If work is done on an object the transfer of energy occurs as a change in the kinetic energy of an object.

The video applies this theorem using several examples.

https://youtu.be/94VcVNIPPNk

Video - Introductory Work due to Friction equals Change in Mechanical Energy Problem

This video uses the conservation of energy to determine how far a moving puck slides given its initial velocity and the coefficient of kinetic friction. While this problem can be done using

Kinematics ↔ F = ma ↔ Dynamics

An analysis using energy is employed.

https://youtu.be/oPP83wQ8ZmU

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(continued)

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The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each of parts (a) to (e)

LESSON 3: GRAVITY AND GRAVITATIONAL POTENTIAL ENERGY (2 HOURS)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ determine the work done by gravity as an object falls from one height to another
- ❑ determine the speed of a mass as it rises or falls using the work-energy theorem
- ❑ determine the gravitational potential energy of an object
- ❑ recognize a force as being conservative or non-conservative
- ❑ define a conservative force in two ways
- ❑ explain the difference between a conservative force and a non-conservative force in terms of the work done when different paths are used
- ❑ calculate the total mechanical energy in a given situation

Key Words

gravitational potential energy
total mechanical energy
conservative force
non-conservative force

Introduction

In the previous lesson, we concentrated on the work-energy theorem and kinetic energy. In this lesson, we broaden the work-energy theorem to include the effects of gravity and gravitational potential energy. These ideas will allow us to analyze motion in a way other than using the principles of kinematics for motion with a constant acceleration. You will learn what conservative and non-conservative forces are, and you will see how important they are in actually calculating the work done when objects move from one place to another along different paths. This will lead to a very important principle called the principle of conservation of mechanical energy. These ideas will allow us to study several interesting situations involving motion, such as roller coasters.

Work Done by the Gravitational Force

The **force of gravity** is a force that can do positive or **negative** work.

The diagram below shows a ball of mass m moving vertically downward.

The only force acting on the ball is the force of gravity $m\vec{g}$.

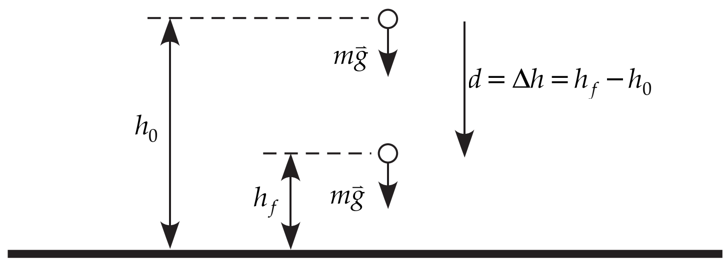
The initial height of the ball is h_0, and the final height is h_f. Both of these distances are measured from the surface of Earth.

The displacement d downward has a magnitude of $d = \Delta h = h_f - h_0$.

To calculate the work done by gravity, we use $W_{\text{gravity}} = F_g d$.

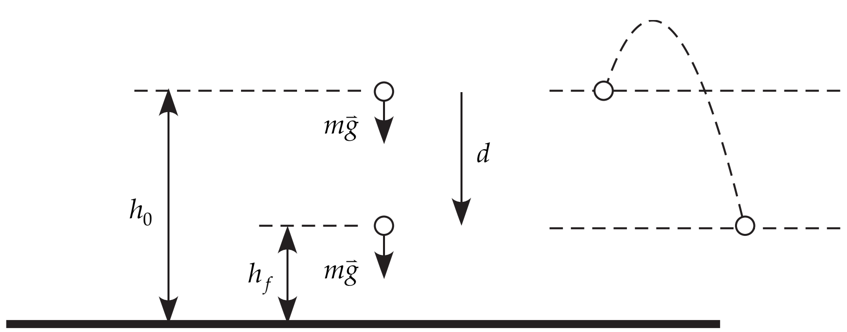
The magnitude of the force of gravity is $\vec{F}_g = m\vec{g}$. (Assume $\vec{g} = +9.80 \text{ m/s}^2$.) The force of gravity and the displacement are in the same direction, so the work done by gravity is a positive number.

$$W_{\text{gravity}} = m\vec{g}\left(h_f - h_0\right)$$



In this situation, we considered only the straight downward path of the ball. However, it turns out that this equation will work for any path by which the ball travels from the initial height to the final height, as indicated in the diagram above.

For example, suppose the ball is thrown up at an angle, and then comes back down again, as illustrated in the diagram below.



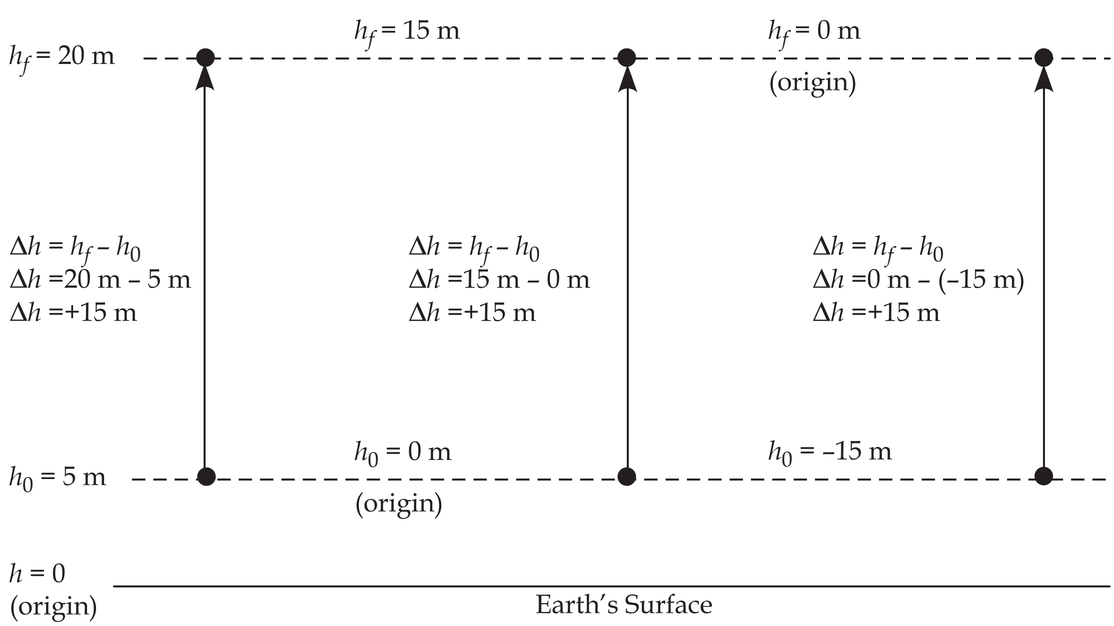
As you can see in the diagrams above, the difference in the vertical distances is the same for each path in the drawings. To calculate the work done by gravity, we use $W_{gravity} = F_g d$. In each case, the force is the same and the distance through which the force acts is the same: $d = \Delta h = h_f - h_0$. Thus, the work done by gravity is the same in each case.

We can conclude then that to calculate the work done by gravity on an object as the object rises or falls, we need only use the difference in the heights of the object ($d = \Delta h = h_f - h_0$) as the distance through which the force of gravity acts. The actual path over which the object travels is irrelevant.

In these cases, we are assuming that the difference in height is small compared to the radius of Earth, so the magnitude of $\vec{g}$, the acceleration due to gravity, is the same at every height. For positions close to Earth, we can use the value of $\vec{g} = 9.80 \text{ m/s}^2$.

Choice of Origin for Calculating Vertical Distances

The choice of the origin from which you measure vertical distances (heights) is **arbitrary**. **The choice is yours.** Usually, you would like to have positive numbers in vertical distances so you may choose the origin as the lowest point given in the question. This may not be so. You could start with the origin at the highest point mentioned in the question. It turns out that the vertical distance measured between two points using either origin will be identical.



Since only the difference in height appears in the equation for measuring the work done due to gravity, the **distances** themselves **do not need to be measured from Earth**.

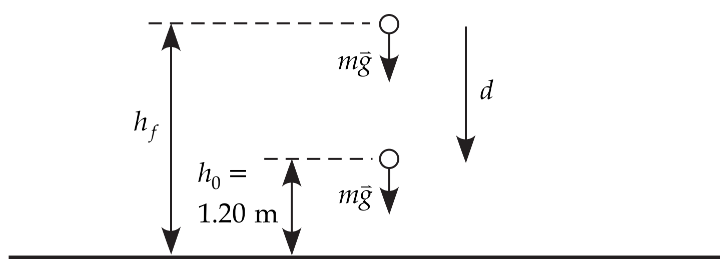
In the example above, the zero levels (origins) are all different but the vertical distance $\Delta h = h_f - h_0$ would still have the same value.

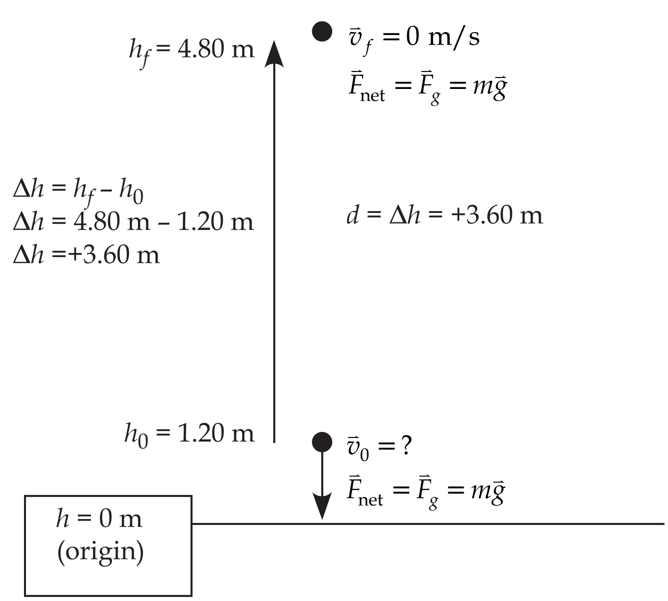
Example 1: Gravity and the Work-Energy Theorem

Part 1:

As an example of how the work due to gravity and the work-energy theorem fit together, we will study the example of throwing a ball up into the air from a height of 1.20 m. The ball reaches a maximum height of 4.80 m before falling back down. To determine the speed with which the ball leaves the hand, we could use the principles of linear kinematics and the equations for motion with a constant acceleration. But, instead, we will use the principles of work and energy.

The only force acting on the ball is the force of gravity. We assume that the force due to air friction is negligible. Since the ball is moving up but gravity is acting down, the work done due to gravity is negative.





The work done by gravity is

$$W_{\text{gravity}} = -F_g d$$

$$\vec{F}_g = -m\vec{g}$$

$$W_{\text{gravity}} = -m\vec{g}\left(h_f - h_0\right)$$

Remember to assume that $\vec{g}$ = +9.80 m/s^2.

We know from the **work-energy theorem** that the work due to gravity is

$$W_{\text{gravity}} = \Delta KE = KE_f - KE_0 = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_0^{\,2}.$$

At the highest point, the final speed $\vec{v}_f = 0\ \text{m/s}$ and the kinetic energy KE_f = 0 J, so the theorem reduces to

$$W_{\text{gravity}} = -KE_0 = -\frac{1}{2}mv_0^{\,2}.$$

Bringing together the two equations for W_{gravity}

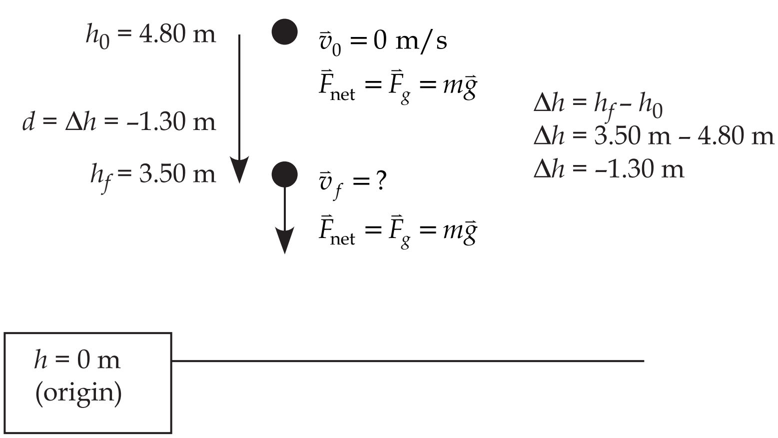
$$-m\vec{g}\left(h_f - h_0\right) = -\frac{1}{2}mv_0^{\,2}.$$

Solving for v_0,

$$v_0 = \sqrt{2\vec{g}\left(h_{\text{f}} - h_{\text{o}}\right)} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(4.80\ \text{m} - 1.20\ \text{m}\right)} = 8.40\ \text{m/s}.$$

Part 2:

If we want to determine the speed of the ball after falling back to a height of 3.50 m, we use the same principles. It is important to recognize that the initial height is h_0 = 4.80 m and h_f = 3.50 m. At the top of the path, the initial velocity $\vec{v}_0 = 0\ \text{m/s}$ and initial kinetic energy $KE_0 = \frac{1}{2}mv_0^{\,2} = 0\,\text{J}$.



The work-energy theorem indicates that

$$W_{\text{gravity}} = \Delta KE = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2.$$

Since $KE_0 = 0$ J, then

$$W_{\text{gravity}} = KE_f - KE_0 = KE_f = \frac{1}{2}mv_f^2.$$

The force due to gravity is in the down direction, so it is negative.

$$-m\vec{g}\left(h_f - h_0\right) = \frac{1}{2}mv_f^2$$

Thus, the final speed at a height of 3.50 m is

$$v_{\text{f}} = \sqrt{-2\vec{g}\left(h_{\text{f}} - h_{\text{o}}\right)} = \sqrt{-2\left(9.80 \text{ m/s}^2\right)\left(3.50 \text{ m} - 4.80 \text{ m}\right)} = 5.05 \text{ m/s}.$$

Learning Activity 4.7

Work Done by Gravity and the Work-Energy Theorem

Solve the following problem to check your understanding of the work done by gravity and the work-energy theorem. An answer key is available at the end of Module 4 for you to check your responses after you have answered the questions.

1. A stone of mass 50.0 g is thrown upwards at 22.0 m/s at a point 5.00 m above the ground.
   a) What is the work done by gravity during the time the stone rises to a new height of 19.0 m above the ground?
   b) Calculate the velocity of the stone when it is 19.0 m above the ground, using the work-energy theorem.
   c) What is the speed of the stone as it crashes into the ground?

Gravitational Potential Energy

In your work so far, you have seen that an object in motion has kinetic energy. But energy also occurs in other forms. One example of this is an object having a stored energy due to its position relative to the Earth. This kind of energy is called **gravitational potential energy**.

> The **gravitational potential energy** of an object is the potential energy an object has because of its location in a gravitational field. Objects at higher altitudes have greater gravitational potential energy than objects at lower altitudes.

You have observed gravitational potential energy in action. When you throw an object straight up into the air, as it rises it slows down, its speed decreases, and its kinetic energy decreases. Where did the kinetic energy go? The kinetic energy is simply transformed into another type of energy: **gravitational potential energy**. So as the kinetic energy of the object decreased, the gravitational potential energy increased. This increase in gravitational potential energy has to do with the change in the position of the object in the gravitational field next to the Earth.

A good example of an object with gravitational potential energy would be the piledriver. It is used by construction workers to pound "piles" or structural support beams into the ground. The piledriver contains a massive hammer that is raised to a height h above the ground and then dropped. As a result of being raised, the hammer has the potential to do the work of driving the pile into the ground. The greater the height of the hammer, the greater is the potential for doing work, and the greater is the potential energy.

To obtain an expression for the gravitational potential energy, recall that the work done by the gravitational force in moving an object from an initial height h_0 to a final height h_f is

$$W_{\text{gravity}} = m\bar{g}h_f - m\bar{g}h_0$$

This equation indicates that the work done by the gravitational force is equal to the difference between the initial and the final values of the quantity mgh. The value of mgh is larger when the height is larger and smaller when the height is smaller. The quantity mgh is referred to as the **gravitational potential energy**.

Gravitational Potential Energy Equation

The **gravitational potential energy** (PE_g) is the energy that an object of mass, m, possesses due to its position relative to the surface of Earth. The position is measured by the height, h, of the object relative to an arbitrary zero level.

The **gravitational potential energy** is calculated now as the product of the mass, m, of the object, the gravitational field strength, $\vec{g}$, and the height, h, of the object relative to an arbitrary zero level.

$$PE_g = m\vec{g}h$$

Quantity	Symbol	Unit
Gravitational potential energy	PE_g	joule (J)
Mass	m	kilogram (kg)
Acceleration due to gravity	$\vec{g}$	metres/second/second (m/s^2)
Height relative to zero level	h	metres (m)

Gravitational potential energy is a scalar quantity.

Gravitational potential energy, like work and kinetic energy, is a scalar quantity. The SI unit for gravitational potential energy is the joule, J.

The **change in gravitational potential energy** is the difference between two potential energies that is related to the work done by the force of gravity.

$$\begin{aligned} W_{\text{gravity}} &= \Delta PE \\ &= PE_{\text{final}} - PE_{\text{initial}} \\ &= m\vec{g}h_{\text{final}} - m\vec{g}h_{\text{initial}} = m\vec{g}h_f - m\vec{g}h_0 \end{aligned}$$

If you factor out the $m\vec{g}$, you obtain:

$$W_{\text{gravity}} = \Delta PE = m\vec{g}\left(h_f - h_0\right) = m\vec{g}\Delta h$$

So the change in the gravitational potential energy can then be calculated as

$$\Delta PE = m\vec{g}\Delta h.$$

As mentioned earlier, the zero level for the heights **can be taken anywhere**, as long as both h_0 and h_f are measured relative to the same zero level.

The gravitational potential energy depends on both the object and the gravitational pull (m and $\vec{g}$, respectively), as well as the height, h. Therefore, the **gravitational potential energy** belongs to the **object** and the **Earth as a system**. Normally, though, we talk about the gravitational potential energy of **the object only**.

Example 2: Gravitational Potential Energy and Changes in Gravitational Potential Energy

A person of mass 65.0 kg is rollerblading, coasting down a hill. At one point, the person is a vertical distance of 25.0 m above the bottom of the hill. Sometime later on, the person is a vertical distance of 12.0 m above the bottom of the hill. Determine

a) the initial gravitational potential energy of the person

b) the final gravitational potential energy of the person

c) the change in the gravitational potential energy of the person

Answers:

a) The key here is to determine where to set your zero level or origin from which the measurements of height will be measured. In this case, set h = 0 m at the bottom of the hill.

Given: Mass $m = 65.0$ kg

Initial height $h_i = +25.0$ m

Final height $h_f = +12.0$ m

Unknown: Initial gravitational potential energy $PE_i = ?$

Equation: $PE_i = m\bar{g}h_i$

Substitute and solve: $PE_i = (65.0 \text{ kg})(9.80 \text{ m/s}^2)(+25.0 \text{ m}) = 15900 \text{ J}$

The initial gravitational potential energy is 15900 J.

b) The final gravitational potential energy can be found in a similar fashion.

$$PE_f = m\bar{g}h_f$$
$$= (65.0 \text{ kg})(9.80 \text{ m/s}^2)(+12.0 \text{ m}) = 7640 \text{ J}$$

The final gravitational potential energy is 7640 J.

c) The change in the gravitational potential energy is just the difference between the initial and the final gravitational potential energies.

$$\Delta PE = PE_f - PE_i$$
$$= 7640 \text{ J} - 15900 \text{ J} = -8260 \text{ J} = -8300 \text{ J}$$

Alternatively, you could use

$$\Delta PE = m\bar{g}\Delta h = m\bar{g}(h_f - h_i) = (65.0 \text{ kg})(9.80 \text{ m/s/s})(12.0 \text{ m} - 25.0 \text{ m})$$
$$\Delta PE = -8281 \text{ J} = -8300 \text{ J}$$

In both cases, the person lost 8300 J (to two significant digits) of gravitational potential energy.

Learning Activity 4.8

Changes in Gravitational Potential Energy

Solve the following problems dealing with gravitational potential energy and changes in gravitational potential energy. An answer key is available at the end of Module 4 for you to check your work after you have answered the questions.

1. A piledriver of mass 475 kg falls a distance of 6.25 m before it strikes a pile.
   a) Draw a sketch of this situation, showing the origin at the initial position of the piledriver.
   b) Determine the change in gravitational potential energy of the piledriver.
2. A person of mass 75.0 kg walks up the stairs from the first floor to the fourth floor. The vertical distance between floors is 3.00 m.
   a) What is the gravitational potential energy of the person when he is standing on the landing for the third floor if you choose the first floor to be the zero level?
   b) What is the gravitational potential energy of the person when he is standing on the landing for the third floor if you choose the fourth floor to be the zero level?
   c) Calculate the change in the gravitational potential energy for the person using the fourth floor as the zero level.
3. At the Vancouver Olympics, a four-person bobsled team and the sled had a mass of 629 kg. The track was 1450 m long and fell a vertical distance of 152 m. Determine the change in gravitational potential energy of the team and sled as it travelled from its starting position to the finish line.

Conservative Forces and Non-Conservative Forces

In order to better understand the work-energy theorem, it is important to understand **conservative** and **non-conservative forces**.

Gravity is an example of a **conservative force**. The **work done** by gravity **does not depend on the choice of the path**.

For instance, when an object is moved from an initial height h_0 to a final height h_f, the object could be raised straight up or it could be moved along any curved path from the initial height to the final height. Any side-to-side motion does not affect the final work done by gravity. It is only the difference in height that is important. This example leads to one possible version of the definition of a conservative force.

> **Conservative Force:**
> **Version 1:** A force is conservative when the work it does on a moving object is independent of the path of the motion between the object's initial and final positions.

Other examples of conservative force are the elastic force of a spring and the electrical force.

There is another way to view the idea of a conservative force. For example, if you throw a book upwards, the book will rise, come to a stop, and then fall back to its original position. In situations such as these, when an object begins to travel at a given position, moves, and then ends at the same position, the object is said to travel a **closed path**.

Once the book leaves your hand, the only force acting on the book is the force of gravity. We will assume the force of air friction is negligible. On the book's **way up**, the force of gravity does **negative work**, transferring kinetic energy out of the book. All the kinetic energy is gone when the book is at the top of its flight. On the book's **way down**, the force of gravity does **positive work**, transferring kinetic energy into the book. You will recall from Module 3: Projectiles and Circular Motion that at the point of release the velocity of the book should have the same magnitude (speed) as the original velocity but in the opposite direction.

In the end, over this closed path, the force of gravity does as much positive work as negative work. Therefore, the **net work done by gravity** is 0 N.

> **Conservative Force:**
> **Version 2:** A force is conservative when it does no net work on an object moving around a closed path, starting and finishing at the same point.

Think about it like this: If a force does work and this work can be gotten back later, that force is a conservative force.

Not all forces are conservative forces.

> **Non-Conservative Force:**
> A force is non-conservative if the work it does on an object moving between two points depends on the path of motion between the points.

The kinetic frictional force is a non-conservative force. When an object slides over a surface, the kinetic frictional force points opposite to the sliding motion and does negative work. The longer the path is, the more work is done by the kinetic frictional force. If you push a book along the tabletop, you do work against friction. This work cannot be gotten back later.

Air resistance is another example of a non-conservative force. Consider the example of tossing a book upwards. On the book's way up, friction opposes the motion and transfers energy out of the book. On the book's way down, the force of air friction transfers energy out of the book as well. The book returns to its original position with less kinetic energy than it initially possessed, with the missing energy being the work done by air friction.

For a closed path, the total work done by a non-conservative force is **not** zero as it was for a conservative force.

Conservation of Mechanical Energy

The work-energy theorem has led us to consider two types of energy: kinetic energy and potential energy. The sum of these two types of energy is called the **total mechanical energy**, E, so that $E = KE + PE$. The idea of total mechanical energy will prove to be very useful in describing the motion of objects.

In the previous lesson, we stated the work-energy theorem as

$$W = KE_f - KE_0 = \Delta KE$$

In this lesson, we related the work-energy theorem to the gravitational potential energy at the surface of the Earth as

$$W = PE_f - PE_0 = \Delta PE$$

In a situation involving the kinetic energy and gravitational potential energy of a given object, we can add these works together to give

$$W = \Delta KE + \Delta PE$$

The work-energy theorem can be expressed in terms of the total mechanical energy:

$$\begin{aligned} W &= (KE_f - KE_0) + (PE_f - PE_0) \\ &= (KE_f + PE_f) - (KE_0 + PE_0) \end{aligned}$$

The expression $(KE_f + PE_f)$ is the final mechanical energy E_f.

The expression $(KE_0 + PE_0)$ is the initial mechanical energy E_0.

The work-energy theorem can, therefore, be written as $W = E_f - E_0$.

When the work-energy theorem is written in this concise way, it allows an important principle of physics to stand out. This principle is known as the **conservation of mechanical energy**.

To understand what this principle says, it is important to know if the net work done by the external forces is equal to zero. If the net work is not zero, then the conservation of mechanical energy does not apply. If the net work is zero, then the conservation of mechanical energy does apply. The equation $W = E_f - E_0$ then reduces to $E_f = E_0$.

This result indicates that the final mechanical energy is equal to the initial mechanical energy. Consequently, the total mechanical energy remains constant between the initial and final points, never varying from the initial value of E_0. A quantity that stays constant throughout the motion is said to be **conserved**. We express this principle as the Law of Conservation of Mechanical Energy.

We can state this principle in a formal way as follows:

Principle of Conservation of Mechanical Energy
The total mechanical energy ($E = KE + PE$) of an object remains constant as the object moves, provided that the net work done by external forces is zero.

For two given situations:

$$\begin{aligned} E_{\text{initial}} &= E_{\text{final}} \\ E_i &= E_f \end{aligned}$$

Therefore,

$$KE_i + PE_i = KE_f + PE_f$$

Quantity	**Symbol**	**Unit**
Total mechanical energy	E	joule (J)
Initial kinetic energy	KE_i	joule (J)
Initial potential energy	PE_i	joule (J)
Final kinetic energy	KE_f	joule (J)
Final potential energy	PE_f	joule (J)

If the work is done by **conservative forces**, then the net work done during the process is 0 N. So, mechanical energy is conserved only in those situations where the work is done by a conservative force.

Gravity is a conservative force.

Applying the principle of conservation of mechanical energy to an object moving near the Earth's surface will require an interchange of kinetic energy $\left(KE = \frac{1}{2}mv^2\right)$ and gravitational potential energy $\left(PE = m\vec{g}h\right)$.

The equation then becomes:

$$
\begin{aligned}
&E_{\text{initial}} = E_{\text{final}} \\
&KE_0 + PE_0 = KE_f + PE_f \\
&\frac{1}{2}m{v_0}^2 + m\vec{g}h_0 = \frac{1}{2}m{v_f}^2 + m\vec{g}h_f
\end{aligned}
$$

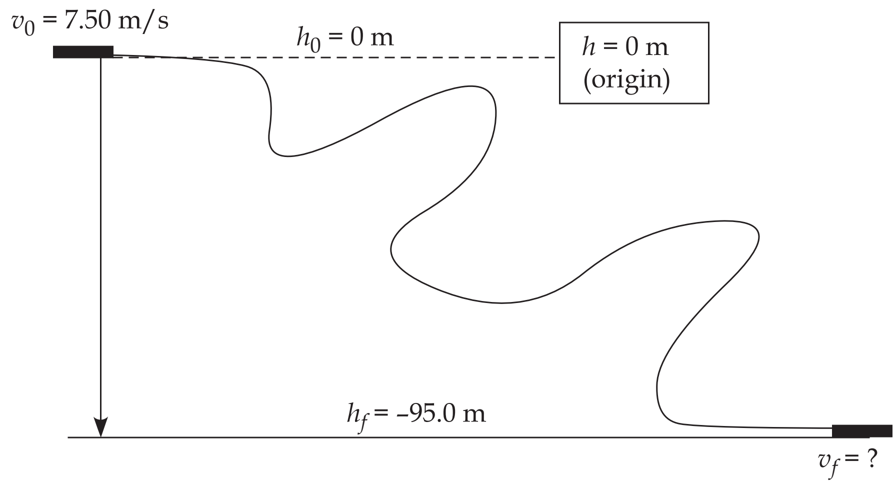
Example 3: Conservation of Mechanical Energy: An Example

The principle of conservation of mechanical energy provides a useful method of comprehending ways in which the physical universe operates. Kinetic energy of motion is converted into potential energy of position when a book rises to a greater height after being thrown upwards. As the book falls back to its original position, gravitational potential energy disappears as the height decreases, but, at the same time, the speed of the falling book increases, indicating that kinetic energy is increasing. While the sum of the kinetic and potential energies at any point is conserved, the two types of energy are converted back and forth within a given system.

A four-person bobsled and its passengers have a mass of 525 kg. The bobsled starts its run by being pushed up to a speed of 7.50 m/s and to a point 95.0 m above the finish line. What would be the final velocity of the bobsled if you ignore the energy lost to friction?

Assume that non-conservative forces such as wind resistance and friction can be ignored. The normal force is perpendicular to the path and does no work. Only the force of gravity does work, so the total mechanical energy, E, remains constant at all points on the run.

Given: Mass $m = 525$ kg



Unknown: Final speed $v_f = ?$

Equation: Here, the total mechanical energy is conserved, so you can use

$$E_{\text{initial}} = E_{\text{final}}$$

$$KE_0 + PE_0 = KE_f + PE_f$$

$$\frac{1}{2}mv_0^{\,2} + m\vec{g}h_0 = \frac{1}{2}mv_f^{\,2} + m\vec{g}h_f$$

Substitute and solve: Since the initial height is to 0 m, the initial potential energy $m\vec{g}h_0 = 0$ J.

$$\frac{1}{2}(525\text{ kg})(7.50\text{ m/s})^2$$

$$= \frac{1}{2}(525\text{ kg})v_f^{\,2} + (525\text{ kg})(9.80\text{ m/s}^2)(-95.0\text{ m})$$

$$14766 = 262.5v_f^{\,2} - 488775$$

$$v_f = \sqrt{\frac{14766 + 488775}{262.5}} = 43.8\text{ m/s}$$

The final speed of the bobsled is 43.8 m/s.

At the top of the motion, if the bobsled is moving slowly, the kinetic energy is small (14800 J) and the potential energy is at a maximum (0 J, for example). As the bobsled descends, some of the potential energy is converted to kinetic energy.

At the bottom of the run, there is a potential energy of –489000 J, and the kinetic energy becomes 504000 J.

The total mechanical energy at the bottom of the run is 15000 J, which is equal (to three significant digits) to the total mechanical energy at the top of the run (14800 J).

Learning Activity 4.9

Conservation of Mechanical Energy (Kinetic Energy and Gravitational Potential Energy)

There are four practice questions in this learning activity. An answer key is available at the end of Module 4 for you to check your work after you have answered the questions.

The physics of gravitational potential energy and an astronaut

1. In a simulation on Earth, an astronaut in his spacesuit climbs up a vertical ladder. On the Moon, the same astronaut makes the same climb. In which case does the gravitational potential energy of the astronaut change by a greater amount? Account for your answer.

The physics of gravitational potential energy and the CN Tower

2. The CN Tower in Toronto is advertised as being the world's tallest free-standing structure. At 181 stories, it has a height of 553 m. What is the gravitational potential energy of a 55.0 kg person who is at the top of the tower?

The physics of a projectile

3. A person throws a stone from the edge of a cliff at a speed of 10.0 m/s. The cliff has a height of 20.0 m. Consider the force of air friction to be negligible. Determine the speed with which the stone strikes the ground when the stone is thrown
   a) horizontally
   b) vertically straight up
   c) vertically straight down

The physics of the Magnum XL-200 roller coaster

4. One of the fastest roller coasters in the world is the Magnum XL-200 at Cedar Point Park in Sandusky, Ohio. The ride includes a vertical drop of 59.4 m. Assume that the coaster has a speed of nearly zero as it crests the top of the hill. Neglect friction and find the speed of the riders at the bottom of the hill. Determine the speed both in units of m/s and km/h.

Lesson Summary

In this lesson, we began by learning how to calculate the work done by a gravitational force as an object moves from one height to another.

The work done by gravity, as a mass m moves from an initial height of h_0 to a height h_f, is

$$W_{\text{gravity}} = mg\left(h_f - h_0\right).$$

In doing problems, only the difference in heights is important, not the actual path taken between those heights.

From the work-energy theorem, the work due to gravity is

$$W_{\text{gravity}} = KE_f - KE_0.$$

By combining equations, the change in kinetic energy can be found.

$$m\bar{g}\left(h_0 - h_f\right) = KE_f - KE_0$$

Furthermore, since $KE = \frac{1}{2}mv^2$, the initial or final speed of an object can be found as it moves from one height to another under the influence of gravity.

The **gravitational potential energy**, *PE*, is the energy that an object of mass, *m*, has due to its position relative to the surface of Earth. The position is measured by the height, *h*, of the object relative to an arbitrary zero level.

$$PE = m\bar{g}h$$

The SI unit of gravitational potential energy is the joule (J).

A **conservative force** can be defined in two ways.

Version 1: A force is conservative when the work it does on a moving object is independent of the path of the motion between the object's initial and final positions.

Version 2: A force is conservative when it does no net work on an object moving around a closed path, starting and finishing at the same point.

A force is **non-conservative** if the work it does on an object moving between two points depends on the path of motion between the points. For a closed path, the total work done by a non-conservative force is not zero as it was for a conservative force. The work that is done for a non-conservative force cannot be gotten back at some later time.

The sum of the kinetic energy and the potential energy is called the **total mechanical energy**, *E*, so that $E = KE + PE$.

The **law of conservation of mechanical energy** states that the total mechanical energy ($E = KE + PE$) of an object remains constant as the object moves, provided that the net work done by external forces is zero.

For the situation where mechanical energy for a system consists of kinetic energy and gravitational potential energy, use the following relationship:

$$E_{\text{initial}} = E_{\text{final}}$$

$$KE_0 + PE_0 = KE_f + PE_f$$

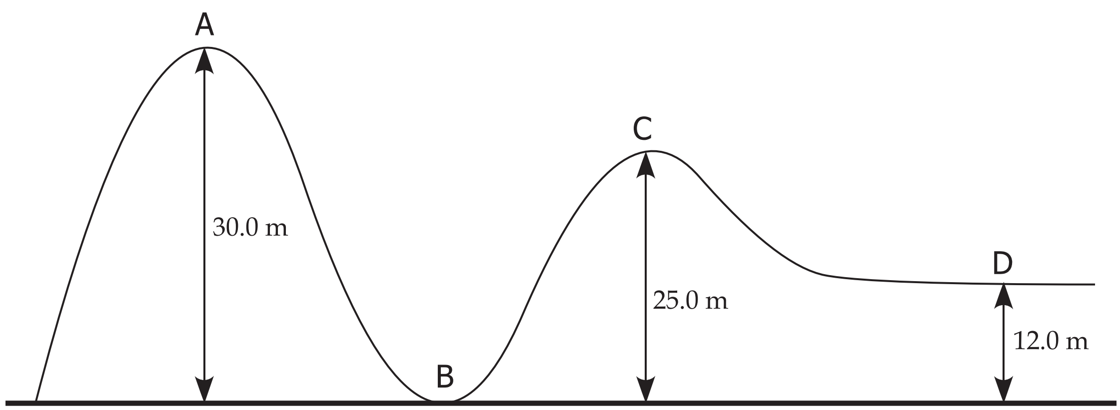
$$\frac{1}{2}mv_0^2 + m\vec{g}h_0 = \frac{1}{2}mv_f^2 + m\vec{g}h_f$$

Assignment 4.3

Conservation of Mechanical Energy in a Roller Coaster (10 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

A roller coaster of mass 1000.0 kg passes point A with a speed of 1.80 m/s. Answer the following questions about this roller coaster.



a) What is the total mechanical energy of the roller coaster at point A?

(continued)

b) What is the speed of the roller coaster at point B?

c) What is the potential energy and the kinetic energy of the roller coaster at point C?

(continued)

Assignment 4.3: Conservation of Mechanical Energy in a Roller Coaster (continued)

d) Derive an expression for the speed of the cart using the variables E, m, $\vec{g}$, and h.

e) Determine the speed of the roller coaster at point D.

Method of Assessment

The total of 10 marks for this assignment will be determined as follows:

- 2 marks for the correct answer in each of parts (a) to (e)

Notes

Video - GCSE Science Physics (9-1) Gravitational Potential Energy

This video gives a quick introduction to the idea of gravitational potential energy.

The equation $PE_g = mgh$ is introduced.

In the examples the potential energy in an object is calculated relative to the ground, that is , with the gravitational potential energy at ground level equal to 0 Joules.

https://youtu.be/63OTIdNb-TE

Video - Physics, Gravitational Potential Energy, An Explanation

This video explains what gravitational potential energy is. It also illustrated how to calculate changes in potential energy. The video shows that you must arbitrarily set a 0 J level for the gravitational potential energy. The video demonstrates mathematically that the work done lifting an object against the force of gravity equals the change in the gravitational potential energy.

The video uses U_g instead of PE_g for the symbol for gravitational potential energy. Instead of h for vertical distance the video uses y. So in the end $h_2 - h_1 = \Delta h$ becomes $y_2 - y_1 = \Delta y$.

Thus $\Delta PE_g = mg(h_2 - h_1)$ becomes $\Delta U_g = mg(y_2 - y_1)$.

https://youtu.be/z3ujg_CkslI

Video - Introduction to Conservation of Mechanical Energy with Demonstrations

This video shows the law of conservation of kinetic energy and gravitational potential energy for two situations: a dumb bell is dropped and a pendulum bob swings back and forth.

For the falling dumb bell a graph of mechanical energy as a function of time plots the kinetic energy, gravitational potential energy and the total mechanical energy of the system.

This graph reveals that the total mechanical energy of the system remains constant.

https://youtu.be/AnuLW0ZX7-Q

Video - Spacewarp - A Review of Mechanical Energies

This video uses metal spheres moving along a complicated track. The sphere starts at the top of the track, rolls along the track to a position at the bottom where it enters an elevator and is carried back to the top of the track.

The types of energy, the energy conversions and the work done are identified.

https://youtu.be/xq9gsqz9tLQ

Video - Conservation of Energy - Moving Rollercoaster

This video uses the law of conservation of energy to analyze the motion of a roller coaster. There are no frictional forces or other outside forces acting in this question.

The analysis reasons through the situation starting with the law of conservation of energy.

$$E_{initial} = E_{final}$$
$$KE_O + PE_{GO} = KE_F + PE_{GF}$$
$$\frac{1}{2}mv_O^2 + mgh_O = \frac{1}{2}mv_F^2 + mgh_F$$

The video simplifies the equations for speed or height. You may be more comfortable working with numerical values. If so substitute the given values into the final equation above and solve for the missing quantity.

Remember that the gravitational potential energy is set to 0 Joules at the lowest level in the given question.

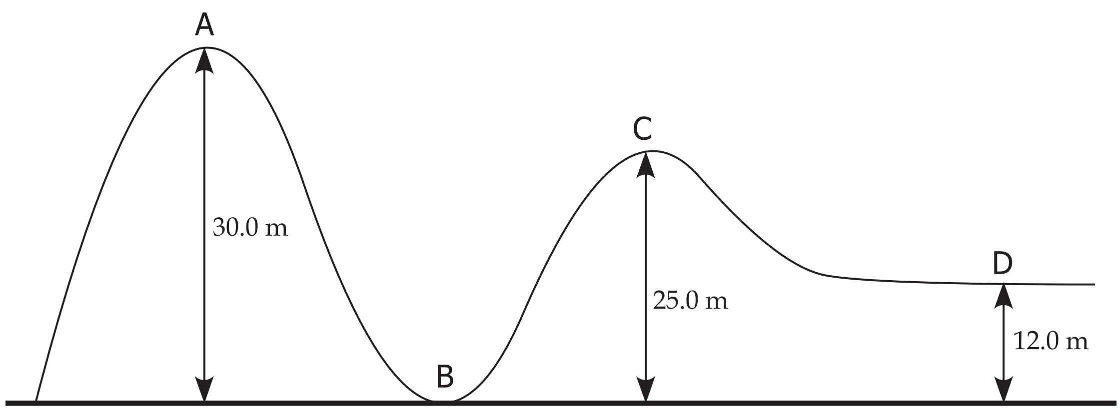
https://youtu.be/PAv9eARzP9M

Assignment 4.3

Conservation of Mechanical Energy in a Roller Coaster (10 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

A roller coaster of mass 1000.0 kg passes point A with a speed of 1.80 m/s. Answer the following questions about this roller coaster.



a) What is the total mechanical energy of the roller coaster at point A?

(continued)

Assignment 4.3: Conservation of Mechanical Energy in a Roller Coaster (continued)

b) What is the speed of the roller coaster at point B?

c) What is the potential energy and the kinetic energy of the roller coaster at point C?

(continued)

Assignment 4.3: Conservation of Mechanical Energy in a Roller Coaster (continued)

d) Derive an expression for the speed of the cart using the variables E, m, $\vec{g}$, and h.

e) Determine the speed of the roller coaster at point D.

Method of Assessment

The total of 10 marks for this assignment will be determined as follows:

- 2 marks for the correct answer in each of parts (a) to (e)

LESSON 4: VIDEO LABORATORY ACTIVITY: HOOKE'S LAW (1 HOUR)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ determine the mathematical relationship between the restoring force exerted by a coil spring and the displacement (distance that the spring is stretched)

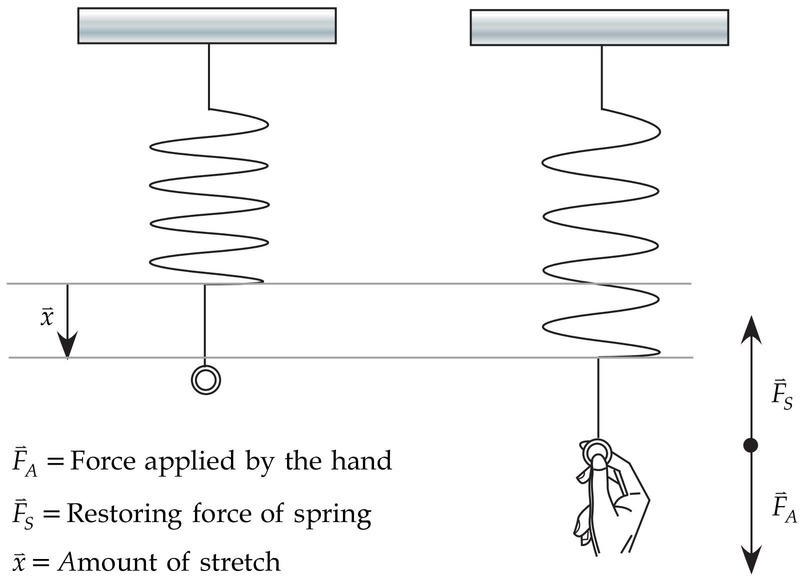
Note to Student

In this lesson, you will view *Hooke's Law*, a short video laboratory activity found in the learning management system (LMS). You may need to collect some data from the video, so you should begin by reading the following introduction and reviewing Lesson 1: Laboratory Activity: Analysis of an Experiment (see Lesson 1, Module 1). Upon completion of the lab activity, you must complete Assignment 4.4, which consists of a lab report. Complete the various sections of the report in the space provided. Assignment 4.4 is to be submitted to the Distance Learning Unit for assessment at the end of Module 4.

Introduction

Many materials possess elastic properties. When you exert an applied force to stretch a rubber band or a spring, you can feel the rubber band or spring pulling back against this applied force. The force that you feel pulling back is called the **restoring force**. The elastic materials when stretched or compressed provide this restoring force, which returns the materials to their original state.

The following diagram illustrates the applied force of a hand pulling downwards on a spring, the restoring force of the spring pulling upwards on the hand, and the amount of stretch of the spring.



The relationship, known as Hooke's law, relates the restoring force of the spring to the displacement (stretch) of the spring. Note that the amount of stretch is a vector pointing downwards, while the restoring force is a vector pointing upwards.

Sample Experiment

To determine graphical relationships, see Lesson 1: Laboratory Activity: Analysis of an Experiment (Lesson 1, Module 1).

Assignment 4.4

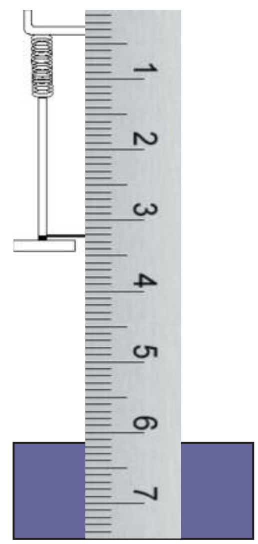
Video Laboratory Activity: Hooke's Law (20 MARKS)

You must complete the following lab report and submit it to the Distance Learning Unit for evaluation. Please complete your work in the space provided. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

Purpose

To find the relationship between the force exerted by a spring (the restoring force) and the displacement (stretch) of the spring.

Apparatus



Spring, ruler, masses, graph paper (or Excel program)

Procedure

1. Suspend a spring from a stand and place a ruler behind the spring.
2. Record the zero point of the bottom of the spring (the last complete coil on the right-hand side).
3. Add a 100 g mass to the spring and record the mass and the position of the bottom of the spring in Data Table 1.
4. Repeat steps 3 and 4 for several more 100 g masses.
5. Convert the masses from grams to kilograms and calculate the force $\left(\vec{F}_g = m\vec{g}\text{, where } \vec{g} \text{ is } 9.80\ \text{N/kg}\right)$ and the stretch or displacement of the spring in metres (final position − initial position of the spring). Record these calculations in Data Table 2.
6. Graph the restoring force $\left(\vec{F}_R\right)$ versus the stretch (x) to determine the equation relating the restoring force and the amount of stretch. Include in this equation the value of the constant of variation, k.

(continued)

Video Viewing

View the video *Hooke's Law*, which can be found in the learning management system (LMS).

Data and Calculations

1. Record the experimental results in Data Table 1. *(2 marks)*

   Initial position of the bottom of the spring = __________

   Data Table 1

Mass (g)	**Position of the Bottom of the Spring (cm)**
100	
200	
300	
400	

2. Do a sample calculation of the amount of stretch or displacement of the spring in metres. *(1 mark)*

3. Do a sample calculation of the restoring force. The restoring force, $\vec{F}_S$, is equal but opposite to the force of gravity stretching the spring. *(1 mark)*

(continued)

Assignment 4.4: Video Laboratory Activity: Hooke's Law (continued)

4. Fill in Data Table 2 with the data generated from the observations. *(5 marks)*

Data Table 2

Mass (g)	Mass (kg)	Force (N)	Position of the Bottom of the Spring (cm)	Displacement or Stretch (m)
100				
200				
300				
400				

5. Plot a graph on graph paper or print a graph generated in Excel with the restoring force on the y-axis and the amount of stretch on the x-axis. *(3 marks)*

6. Calculate the slope of the line of best fit. *(1 mark)*

Discussion

7. What are the limitations of Hooke's law? *(1 mark)*

(continued)

8. What are the units of the force constant *k*? What is the physical meaning of the force constant *k*? (**Hint:** Compare a large value of *k* with a small value.) *(1 mark)*

9. What are some examples of springs of the type discussed in this lab activity that you might find in everyday life? *(1 mark)*

10. Discuss possible sources of error in this experiment and how you could reduce the error. *(2 marks)*

(continued)

Assignment 4.4: Video Laboratory Activity: Hooke's Law (continued)

Conclusion

11. What is the relationship between the restoring force of a spring and its displacement? State the relationship as an equation using vectors. *(2 marks)*

Marking Rubric for Assignment 4.4

Criteria	Possible Marks	Actual Marks
Experimental results in Data Table 1	2	
Sample calculation of displacement	1	
Sample calculation of restoring force	1	
Completion of Data Table 2	5	
Graph of force and displacement	3	
Calculation of slope	1	
Discussion (error analysis, questions)	5	
Conclusion	2	
Total	**20**	

NOTES

Video - Watch for Assignment 4.4

Go to track 3.

https://youtu.be/i4VHT7fOtXE?list=PLw1g3n2IMV7M72rewl81rI7b-CR0k8Wta

Assignment 4.4

Video Laboratory Activity: Hooke's Law (20 MARKS)

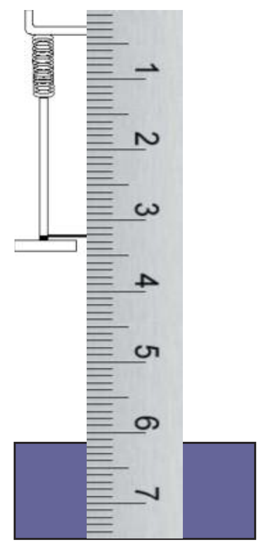
You must complete the following lab report and submit it to the Distance Learning Unit for evaluation. Please complete your work in the space provided. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

Purpose

To find the relationship between the force exerted by a spring (the restoring force) and the displacement (stretch) of the spring.

Apparatus

Spring, ruler, masses, graph paper (or Excel program)



Procedure

1. Suspend a spring from a stand and place a ruler behind the spring.
2. Record the zero point of the bottom of the spring (the last complete coil on the right-hand side).
3. Add a 100 g mass to the spring and record the mass and the position of the bottom of the spring in Data Table 1.
4. Repeat steps 3 and 4 for several more 100 g masses.
5. Convert the masses from grams to kilograms and calculate the force $\left(\vec{F}_g = m\vec{g}, \text{ where } \vec{g} \text{ is } 9.80 \text{ N/kg}\right)$ and the stretch or displacement of the spring in metres (final position − initial position of the spring). Record these calculations in Data Table 2.
6. Graph the restoring force $\left(\vec{F}_R\right)$ versus the stretch (x) to determine the equation relating the restoring force and the amount of stretch. Include in this equation the value of the constant of variation, k.

(continued)

Assignment 4.4: Video Laboratory Activity: Hooke's Law (continued)

Video Viewing

View the video *Hooke's Law*, which can be found by visiting the Independent Study Option Audio and Video web page at www.edu.gov.mb.ca/k12/dl/iso/av.html.

Data and Calculations

1. Record the experimental results in Data Table 1. *(2 marks)*

   Initial position of the bottom of the spring = __________

   Data Table 1

Mass (g)	**Position of the Bottom of the Spring (cm)**
100	
200	
300	
400	

2. Do a sample calculation of the amount of stretch or displacement of the spring in metres. *(1 mark)*

3. Do a sample calculation of the restoring force. The restoring force, $\vec{F}_S$, is equal but opposite to the force of gravity stretching the spring. *(1 mark)*

(continued)

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Assignment 4.4: Video Laboratory Activity: Hooke's Law (continued)

4. Fill in Data Table 2 with the data generated from the observations. *(5 marks)*

   Data Table 2

Mass (g)	Mass (kg)	Force (N)	Position of the Bottom of the Spring (cm)	Displacement or Stretch (m)
100				
200				
300				
400				

5. Plot a graph on graph paper or print a graph generated in Excel with the restoring force on the y-axis and the amount of stretch on the x-axis. *(3 marks)*

6. Calculate the slope of the line of best fit. *(1 mark)*

Discussion

7. What are the limitations of Hooke's law? *(1 mark)*

(continued)

8. What are the units of the force constant *k*? What is the physical meaning of the force constant *k*? (**Hint:** Compare a large value of *k* with a small value.) *(1 mark)*

9. What are some examples of springs of the type discussed in this lab activity that you might find in everyday life? *(1 mark)*

10. Discuss possible sources of error in this experiment and how you could reduce the error. *(2 marks)*

(continued)

Assignment 4.4: Video Laboratory Activity: Hooke's Law (continued)

Conclusion

11. What is the relationship between the restoring force of a spring and its displacement? State the relationship as an equation using vectors. *(2 marks)*

Marking Rubric for Assignment 4.4

Criteria	Possible Marks	Actual Marks
Experimental results in Data Table 1	2	
Sample calculation of displacement	1	
Sample calculation of restoring force	1	
Completion of Data Table 2	5	
Graph of force and displacement	3	
Calculation of slope	1	
Discussion (error analysis, questions)	5	
Conclusion	2	
Total	**20**	

Lesson 5: The Spring and Spring Potential Energy (2.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ calculate the work done by a varying force given a force-distance graph
- ❑ calculate the spring constant for an ideal spring given a force-stretch graph
- ❑ use Hooke's law in problem solving
- ❑ calculate the spring potential energy
- ❑ use the principle of the conservation of mechanical energy when problem solving with springs

Key Words

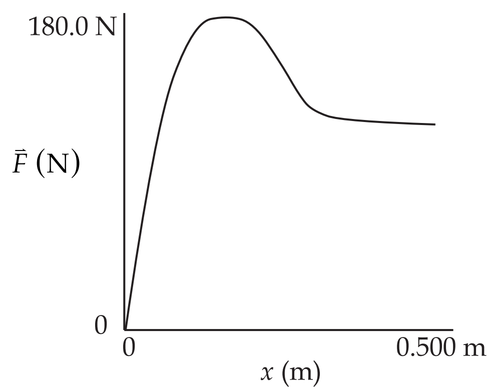
spring constant	restoring force
Hooke's law	spring potential energy

Introduction

This section of the module on work and energy is concluded by studying the spring and the energy stored in the spring. We begin with a discussion of how to determine the work done by a force that varies with distance. We extend this to the case of a force applied to a spring, and then determine the energy stored in a spring. The spring potential energy can be converted into other types of energy already studied, including kinetic energy and gravitational potential energy. There are many applications of this knowledge, especially in the use of springs used to start moving an object or to stop moving objects.

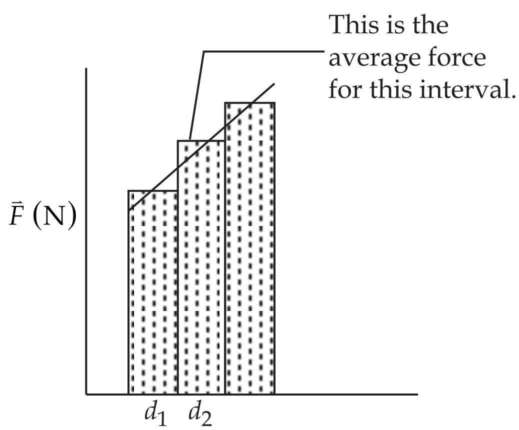
Work Done by a Variable Force

Up to now, we have considered the work done by a constant force. Quite often, situations arise where the magnitude of the force is not constant, but changes with the displacement of the object. For instance, in using a compound bow a variable force is used to draw the bow and load the arrow. A graph of such a situation is shown below.



For this kind of bow, the force rises to a maximum when the string is drawn back, and then falls to 60% of this maximum value when the string is fully drawn. The reduced force makes it easier for the archer to fully draw the bow while aiming the arrow.

When the force varies with the displacement, you **cannot** use $W = Fd$ to determine the work because this equation is **valid** only for a **constant force**. However, you can use a **graphical method** to determine the **work done** by a **variable force**.



In this method, you divide the total displacement into very small segments, d_1, d_2, and so on. For each segment, the average value of the force component in that segment is indicated by a short horizontal line.

You can use this average in $W = Fd$ and determine the approximate value for the work during the first segment. But this work is just the **area of the rectangle under the average force line**.

The word "**area**" here refers to the area of a rectangle that has a width of d_1 and a height of $\vec{F}$; it does not mean the area in square metres. In a like manner, you can calculate an approximate value for the work for each segment. Then you add the results for the segments to get, approximately, the work done by the variable force.

$W \cong F_1 d_1 + F_2 d_2 + \ldots$ (The symbol $\cong$ means "approximately equal to.")

The right side of this equation is the sum of all the rectangular areas and is an approximate value for the shaded area in the graph shown at the right. If the rectangles are made narrower and narrower by decreasing each d_1, the right side of the equation eventually becomes equal to the area under the graph. Thus, you define the **work done by a variable force** as follows:

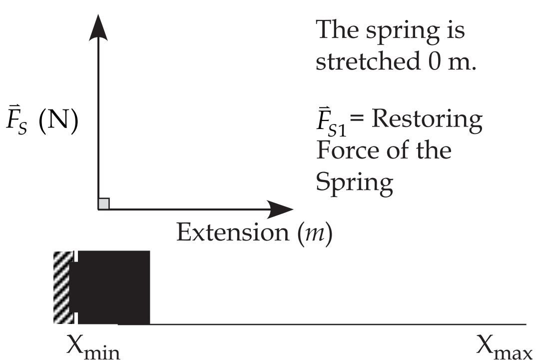
> The **work done by a variable force** in moving an object is equal to the area under the graph of force, $\vec{F}$, versus distance, d.

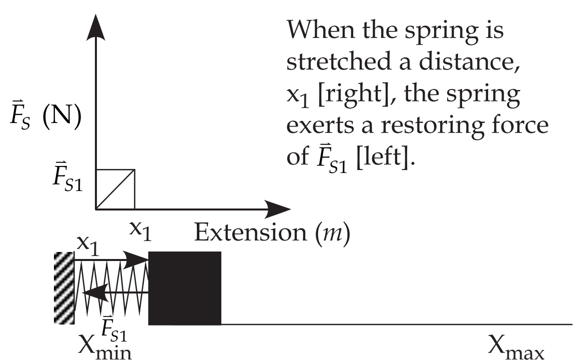
This fits in nicely with the work you had done earlier in this module in Lesson 1. You will recall that you were given a graph of force versus distance and you calculated the work done by taking the area between the curve and the horizontal axis. The only difference here is that the force varies with distance instead of being constant.

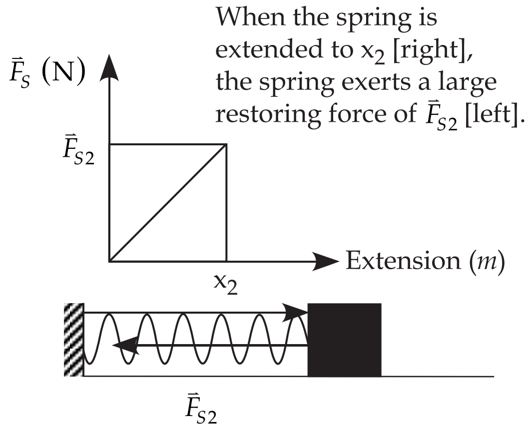
The Restoring Force of a Spring and Its Extension or Compression

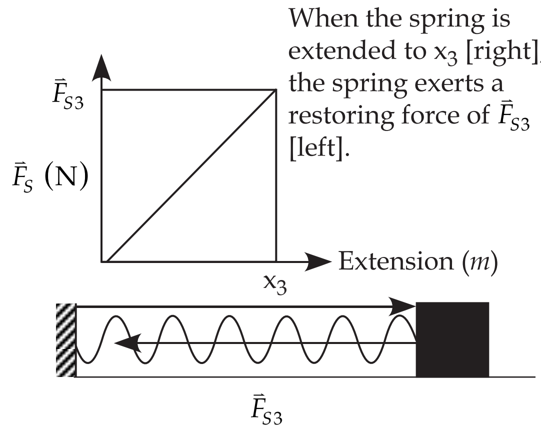
In the previous lesson, you studied Hooke's law, which related the restoring force of the spring to the amount that the spring is stretched or compressed. Let's review what was learned.

In the sequence of graphics below, a mass is attached to a horizontal spring. As the mass is moved to the right, the spring is extended and the spring tries to pull the mass back to its original position. In other words, the spring is exerting a **restoring force**, $\vec{F}_S$. The size of the restoring force varies with the extension of the spring.

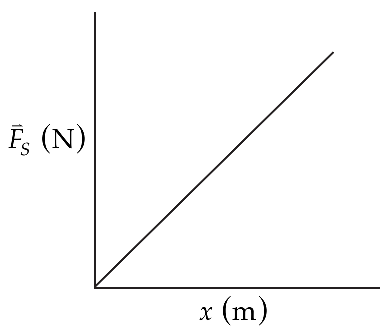








If you call the deformation of the spring x (i.e., x is the amount by which it is stretched or compressed from its normal length), a graph of the force the spring exerts $\vec{F}_S$ versus x has the form shown below.



Notice that the $\vec{F}_S - x$ graph is a straight line if the spring is not stretched too far. If the spring is stretched to a point called the **elastic limit** of the spring, then the graph is no longer linear, and the spring may not return to its original shape after the force is removed.

For the linear portion of the graph, its slope k is given by $k = \frac{\text{rise}}{\text{run}} = \frac{\Delta\vec{F}}{\Delta x}$ and the equation relating the restoring force of a spring and deformation (extension or compression) is $\vec{F}_S = kx$.

The slope of the line on the $\vec{F}_S$ versus x graph, k, is called the **spring constant** and has units of newtons/metre. The value of the spring constant, k, is related to the "stiffness" of the spring. Springs that are hard to stretch or compress are said to be "stiff" and have a larger value for k.

If you consider the vector nature of this situation, you will see that the **restoring force** of the spring is proportional to the amount of extension or compression of the spring but **OPPOSITE** in direction.

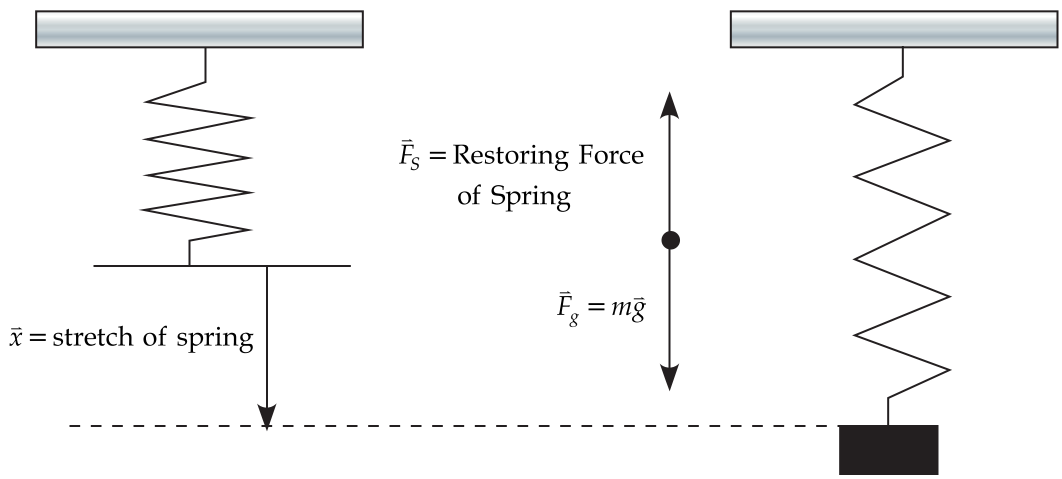
$$\vec{F}_S \ \alpha \ -\vec{x}$$

Changing this proportionality into an equation yields $\vec{F}_S = -k\vec{x}$.

Determining the Value for the Spring Constant

Here is a quick method of determining the value of the spring constant for a spring.

First, suspend a spring from a hook so that it hangs vertically. This spring has a natural unstretched length.



If a mass m is attached to the spring, the spring is extended. It stretches until the force of gravity on the mass balances the restoring force of the spring.

Now, the stretch of the spring $(\vec{x})$ is down while the restoring force of the spring $(\vec{F}_S)$ is up. These two are opposite in direction.

This restoring force of the spring is proportional to the amount of extension or compression of the spring but **OPPOSITE** in direction. A negative sign is introduced into the equation to signify that these two vectors $(\vec{F}_S \text{ and } \vec{x})$ are opposite in direction.

The equation is $\vec{F}_S = -k\vec{x}$.

In the diagram above, you can see that the restoring force of the spring and the force of gravity are equal but opposite.

$$\vec{F}_S = -\vec{F}_g$$

Substituting, you obtain $-k\vec{x} = -m\vec{g}$.

Eliminating the minus signs yields $k\vec{x} = m\vec{g}$.

Division by $\vec{x}$ gives division by a vector (which we cannot handle) so drop the vector notation, giving $k = \frac{mg}{x}$.

Notice the units for k are $\left(\frac{\text{newtons}}{\text{metres}}\right)$.

Also note that the negative sign in $\vec{F}_S = -k\vec{x}$ is not part of k. **The spring constant k is ALWAYS A POSITIVE VALUE.**

You can formally state this relationship as Hooke's law for a spring.

> **Hooke's Law**
> The restoring force $\vec{F}_S$ of an ideal spring is given by $\vec{F}_S = -k\vec{x}$, where k is the spring constant and $\vec{x}$ is the displacement of the spring from its unstretched length.
>
> $$\vec{F}_S = -k\vec{x}$$
>
> | **Quantity** | **Symbol** | **Unit** |
> |---|---|---|
> | Restoring force of spring | $\vec{F}_S$ | newton (N) |
> | Spring constant | k | newtons/metre (N/m) |
> | Amount of compression or extension of the spring | $\vec{x}$ | metres (m) |
>
> **Note:** The spring constant is a ***positive*** number.

Example 1: Hooke's Law

A force of 25.0 N stretches a spring 10.0 cm. What is the spring constant?

The applied force of 25.0 N is counteracted by the restoring force of the spring.

Given: Restoring force of spring $\vec{F}_S$ = +25.0 N

Extension of spring $\vec{x}$ = –10.0 cm = –0.100 m (opposite to force)

Unknown: Spring constant k = ?

Equation: $\vec{F}_S = -k\vec{x}$

Substitute and solve:

$$\vec{F}_S = -k\vec{x}$$

$$25.0\ \text{N} = -k(-0.100\ \text{m})$$

$$k = 250\ \text{N/m} = 2.50 \times 10^2\ \text{N/m}$$

The spring constant is 2.50×10^2 N/m.

Note: The negative sign in Hooke's law is there to indicate that the restoring force of the spring and the displacement for the extension of the spring are opposite in direction.

Example 2: Hooke's Law

A scale in a supermarket contains a spring with a spring constant of 475 N/m. What would be the extension of the spring if a 5.25 kg watermelon is placed in the basket of the scale?

Here, the spring in the scale will stretch until the restoring force of the spring balances the force of gravity pulling on the watermelon.

Given: Spring constant $k = 475$ N/m

Mass of watermelon $m = 5.25$ kg

Weight of watermelon $\vec{F}_g = (5.25 \text{ kg})(9.80 \text{ m/s}^2) = 51.4 \text{ N [down]}$

Restoring force a spring $\vec{F}_S = 51.4 \text{ N [up]}$

Unknown: Extension of spring $\vec{x} = ?$

Equation: $\vec{F}_S = -k\vec{x}$

Substitute and solve:

$$\vec{F}_S = -k\vec{x}$$

$$51.4 \text{ N } [\text{up}] = (475 \text{ N/m})(\vec{x})$$

$$\vec{x} = -0.108 \text{ m } [\text{up}] = 0.108 \text{ m } [\text{down}]$$

The spring is extended the 0.108 m.

Learning Activity 4.10

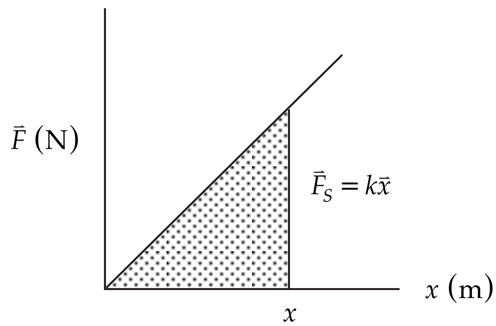
Hooke's Law

Answer the following questions to test your understanding of Hooke's law. You can check your responses against the answer key provided at the end of Module 4.

1. Springs in automobiles are used to provide a smooth ride. When a tire goes over a bump, the spring is compressed 3.00 cm. If the weight of the car supported by that tire is 3060 N, calculate the spring constant.
2. A spring scale containing a spring of constant 525 N/m stretches 2.00 cm when it is used to weigh a fish that you just caught. What is the mass of the fish?
3. When a mass of 2.16 kg is attached to a vertical spring, the spring is stretched 5.62 cm. What is the spring constant?

Spring Potential Energy (PE_S)

Earlier in this lesson, you learned that the amount of work done by a changing force over a displacement is given by the area under the corresponding force-displacement graph. In the case of the stretched spring, the displacement is just the extension, x, and the work done in extending the spring by an amount x is equal to the spring potential energy stored in the extended spring.



At the given extension, x m, the restoring force of the spring, $\vec{F}_S$, is given by $\vec{F}_S = -k\vec{x}$. Again, remember the negative sign simply indicates that the directions of the extension and the restoring force of the spring are opposite to each other.

So you can say that $\vec{F}_S = kx$.

Since the area of the force displacement graph for an ideal spring is in the shape of a triangle, the area is $\frac{1}{2}$(base)(height), which in this case is $\frac{1}{2}(x)(kx)$.

Thus, the energy stored in a spring is given by $PE_S = \frac{1}{2}kx^2$.

Potential Energy Stored in a Spring

The spring potential energy (PE_S) in a deformed spring is the product of one-half of the spring constant (k) multiplied by the square of the deformation (x) (extension or compression) of the spring.

$$PE_S = \frac{1}{2}kx^2$$

Quantity	Symbol	Unit
Spring potential energy	PE_S	joules (J)
Spring constant	k	newtons/metre (N/m)
Compression or extension	x	metres (m)

This expression makes it possible to solve a great variety of problems involving the collision between objects cushioned by springs, and the launching of objects propelled by a compressed spring.

Example 3: Potential Energy in a Compressed Spring

A spring has a spring constant of 1650 N/m. When you pull on the spring, you stretch it 5.00 cm.

a) What is the potential energy stored in the spring?

Given: Spring constant $k = 1650$ N/m

Extension $x = 5.00$ cm $= 0.0500$ m

Unknown: Spring potential energy $PE_S = ?$

Equation: $PE_S = \frac{1}{2}kx^2$

Substitute and solve:

$$PE_S = \frac{1}{2}kx^2$$

$$PE_S = \frac{1}{2}(1650 \text{ N/m})(0.0500 \text{ m})^2$$

$$PE_S = 2.06 \text{ J}$$

The potential energy stored in the spring is 2.06 J.

b) How much work is done to stretch the spring?

Here you apply the work-energy theorem to the situation involving spring potential energy. The work that you did in stretching the spring was transferred into potential energy stored in the spring. Therefore, you did 2.06 J of work.

Example 4: Compressing an Already Compressed Spring

A spring is already compressed 5.00 cm. How much work would you do to compress the spring an additional 6.00 cm? The spring constant is 125 N/m.

This question is a little tricky. You might be tempted just to say $\frac{1}{2}(125 \text{ N/m})(0.0600 \text{ m})^2$ is the work done, since you compressed the spring 6.00 cm. However, this represents the work required to compress the spring from 0 cm to 6.00 cm. What you did in this situation is compress the spring from 5.00 cm to 11.00 cm.

So we will use the work-energy theorem to solve the problem.

Work = change in spring potential energy

$$W = \Delta PE_S = PE_{Sf} - PE_{Si}$$

$$W = \Delta PE_S = PE_{Sf} - PE_{Si} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Given:	Spring constant	$k = 125$ N/m
	Initial compression	$x_i = 5.00$ cm $= 0.0500$ m
	Final compression	$x_f = 11.00$ cm $= 0.1100$ m
Unknown:	Work	$W = ?$
Equation:		Use the work energy theorem for spring potential energy. $W = \Delta PE_S = PE_{Sf} - PE_{Si} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$

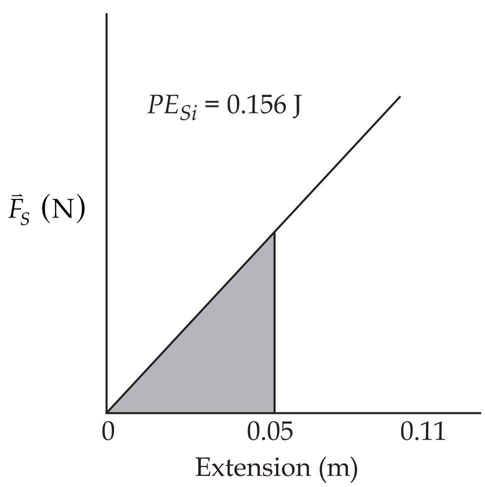
Substitute and solve:

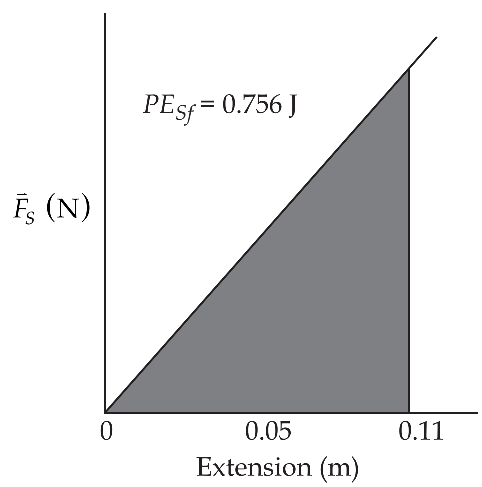
$$W = \frac{1}{2}(125 \text{ N/m})(0.1100 \text{ m})^2 - \frac{1}{2}(125 \text{ N/m})(0.0500 \text{ m})^2$$
$$W = 0.756 \text{ J} - 0.156 \text{ J} = 0.600 \text{ J}$$

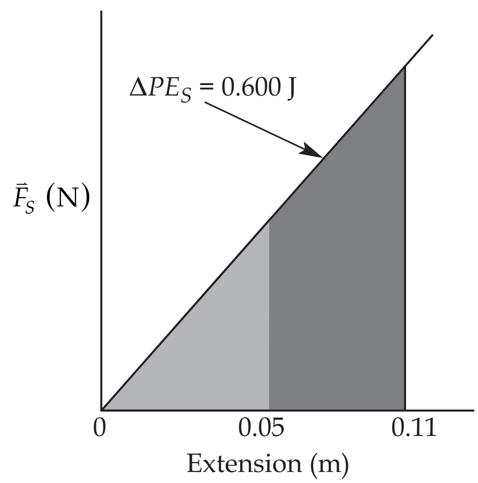
You did 0.600 J of work in compressing the spring from 5.00 cm to 11.00 cm.

An alternate way of analyzing the problem is to consider the area beneath the curve of the restoring force of the spring versus extension graph.

The graph on the left represents the amount of work done when the spring is compressed 5.00 cm. The graph on the right represents the amount of work done when the spring is compressed 11.00 cm.







You can see in the graph to the left that if you subtract the area representing the potential energy at a compression of 5.00 cm (light area) from the area representing a potential energy at a compression of 11 .00 cm (dark area), the difference is represented by the area of the trapezoid (black area to the left).

The area of the trapezoid represents the work done in compressing the spring from 5.00 cm to 11.00 cm.

Conservation of Mechanical Energy

In the last lesson, you learned that the total mechanical energy in a system is constant—that is, the sum of kinetic energy and the gravitational potential energy is constant.

$$E_{\text{initial}} = E_{\text{final}}$$
$$KE_0 + PE_0 = KE_f + PE_f$$
$$\frac{1}{2}mv_0^2 + m\vec{g}h_0 = \frac{1}{2}mv_f^2 + m\vec{g}h_f$$

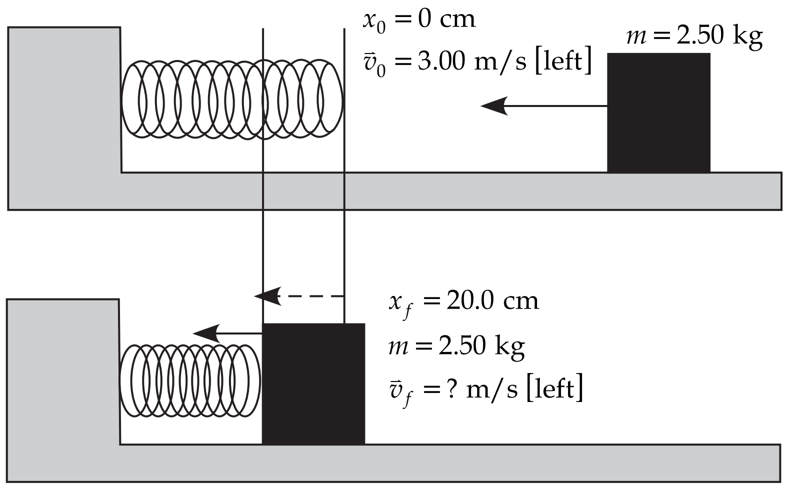
The concept of conservation of mechanical energy can be applied to a system involving a spring. The relationship above must be modified to include the potential energy of a spring rather than the gravitational potential energy. So, if you substitute the appropriate relationships into the equation above, the result is the following:

$$E_{\text{initial}} = E_{\text{final}}$$
$$KE_0 + PE_{S0} = KE_f + PE_{Sf}$$
$$\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

Don't be overwhelmed by the size of these equations. If you start with the basic idea and then replace the energies with the appropriate expressions, you can easily generate these equations.

In the following example, you will study the conversions between spring potential energy and kinetic energy.

Example 5: The Conservation of Mechanical Energy (Kinetic Energy and Spring Potential Energy)



In the situation above, the 2.50 kg mass is sliding to the left at 3.00 m/s. The mass is sliding along a level frictionless surface. The mass slides into a spring bumper, which has a spring constant of 400.0 N/m. Calculate the velocity of the mass while it is still sliding to the left but has compressed the spring by 20.0 cm.

Given:	Mass of block	$m = 2.50$ kg
	Spring constant	$k = 400.0$ N/m
	Initial velocity	$\vec{v}_0 = 3.00$ m/s [left]
	Initial compression of spring	$x_0 = 0$ m
	Final compression of spring	$x_f = 20.0$ cm $= 0.200$ m
Unknown:	Final velocity	$\vec{v}_f = ?$ m/s [left]
Equation:		You must apply conservation of mechanical energy.

$$E_{\text{initial}} = E_{\text{final}}$$

$$KE_0 + PE_{S0} = KE_f + PE_{Sf}$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

Substitute and solve: $\frac{1}{2}(2.50\ \text{kg})(3.00\ \text{m/s})^2+\frac{1}{2}(400.0\ \text{N/m})(0\ \text{m})^2$

$$=\frac{1}{2}(2.50\ \text{kg})v_f^2+\frac{1}{2}(400.0\ \text{N/m})(0.200\ \text{m})^2$$

$$11.25+0=1.25v_f^2+8.00$$

$$1.25v_f^2=3.25$$

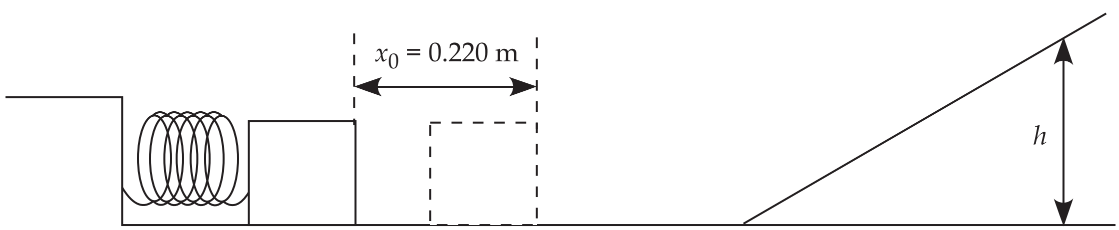
$$v_f=\sqrt{\frac{3.25}{1.25}}=1.61\ \text{m/s}$$

The final velocity of the block is 1.61 m/s [left].

Example 6: Putting it All Together: Conservation of Mechanical Energy: Kinetic Energy, Spring Potential Energy, and Gravitational Potential Energy

Let's consider an example where mechanical energy is conserved within a system involving spring potential energy, kinetic energy, and gravitational potential energy.

A 2.00 kg mass is placed against a spring of force constant 800.0 N/m. The mass has compressed the spring 0.220 m. The spring is then released, and the object moves along the horizontal, frictionless surface and up the slope.



Calculate the following:

a) the total energy of the system

b) the kinetic energy of the block at the point where the spring is still compressed 0.110 m

c) the velocity of the block as it slides on the horizontal surface

d) the vertical height of the block as it reaches above the horizontal surface and slides up the slope.

Answers:

a) the total energy of the system

To begin with, set $h = 0$ m at the level of the horizontal surface.

At the start, all of the energy of the system is the potential energy stored in a spring.

Given:	Spring constant	$k = 800.0$ N/m
	Initial compression of spring	$x_0 = 0.220$ m
Unknown:	Initial potential energy	$PE_{S0} = ?$
Equation:		The potential energy of the spring can be calculated by $PE_{S0} = \frac{1}{2}kx_0^{\,2}$
Substitute and solve:		$PE_{S0} = \frac{1}{2}(800.0\ \text{N/m})(0.220\ \text{m}^2)$ $= 19.36\ \text{J or } 19.4\ \text{J}$

This spring potential energy represents the total energy of the system.

b) the kinetic energy of the block at the point where the spring is still compressed 0.110 m

If the spring now extends partly outwards a distance of 0.110 m, then the mass has begun to move. But the sum of its kinetic energy and spring potential energy remains equal to the total mechanical energy.

$E = KE + PE$

Since the spring is still compressed 0.110 m, the stored spring potential energy is

$$PE_S = \frac{1}{2}kx^2 = \frac{1}{2}(800.0\ \text{N/m})(0.110\ \text{m})^2 = 4.84\ \text{J}.$$

So $E = KE + PE$

$$19.4\ \text{J} = KE + 4.84\ \text{J}$$

Therefore, the kinetic energy at this point is 19.4 J – 4.84 J = 14.56 J or 14.6 J.

c) the velocity of the block as it slides on the horizontal surface

Once the spring has completely extended to 0.220 m, there is no spring potential energy left. All of the mechanical energy is now kinetic energy. To find the speed of the mass after the spring has extended fully, you use $KE = \frac{1}{2}mv^2$.

$$19.4\ \text{J} = \frac{1}{2}(2.00\ \text{kg})v^2$$

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(19.4\ \text{J})}{2.00\ \text{kg}}} = 4.40\ \text{m/s}$$

The velocity of the block as it slides across the horizontal surface is 4.40 m/s [right].

d) the vertical height the block reaches above the horizontal surface as it slides up the slope

When the mass travels up the incline, all of the mechanical energy becomes gravitational potential energy.

So the gravitational potential energy at the height h is 19.4 J.

$$E = PE = m\vec{g}h$$

The vertical distance (height) to which the mass travels is

$$h = \frac{E}{m\vec{g}} = \frac{19.4\ \text{J}}{(2.00\ \text{kg})(9.80\ \text{m/s}^2)} = 0.990\ \text{m}.$$

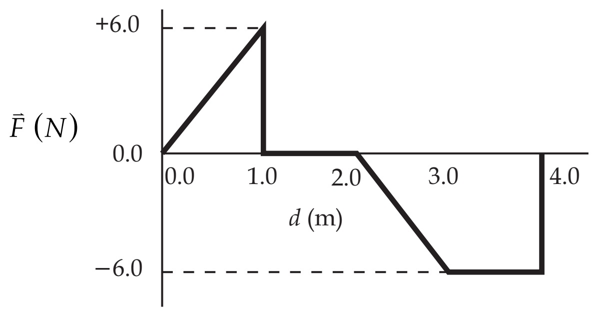
Learning Activity 4.11

Potential Energy of a Spring

There are six practice questions in this learning activity involving potential energy stored in a spring. An answer key is available at the end of Module 4 for you to check your work after you have answered the questions.

The physics of work done by a variable force

1. The force component along the displacement varies with the magnitude of the displacement as shown on the graph.



   Find the work done by the force for each of the following intervals:

   a) 0 to 1.0 m

   b) 1.0 to 2.0 m

   c) 2.0 to 4.0 m

The physics of hanging roast beef on a spring

2. A 2.50 kg roast of beef is suspended from a vertical spring on a butcher's scale whose spring constant is 200.0 N/m.

   a) What is the extension of the spring?

   b) How much energy is stored in the spring?

The physics of a moving crate of apples and a spring

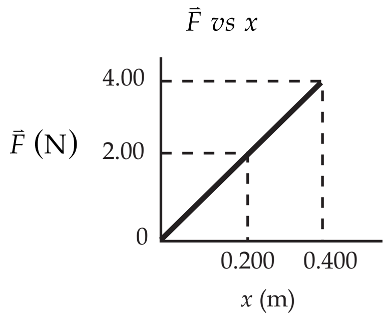
3. A 4.50 kg crate of apples is moving at a horizontal speed of 2.00 m/s towards a spring also mounted horizontally. The linear elastic spring can be compressed 1.00 cm by an applied force of 5.00 N. What is the maximum compression of the spring?

(continued)

Learning Activity 4.11: Potential Energy of a Spring (continued)

The physics of a force-compression graph and spring potential energy

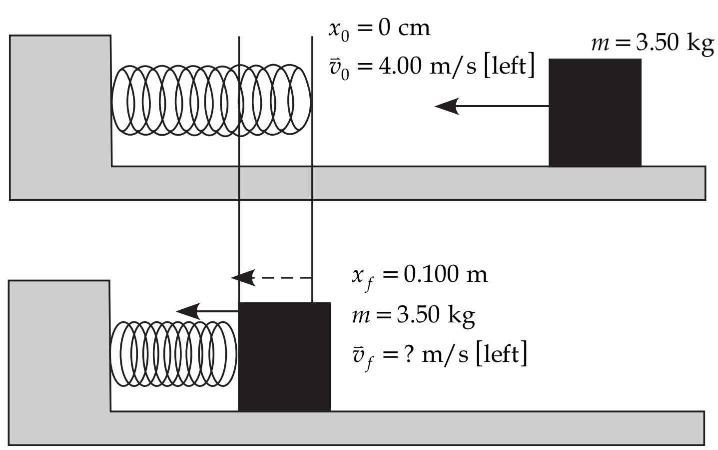
4. The force-compression graph of a hypothetical spring is shown below.



   a) How much work is done in compressing the spring 0.400 m?
   b) How much potential energy is stored in the spring when it is compressed 0.400 m?
   c) How much work is done in compressing the spring from 0.200 m to 0.400 m?

The physics of a collision between a sliding block and a spring bumper

5. A block of 3.50 kg, sliding along a frictionless horizontal surface, is moving to the left at a speed of 4.00 m/s. The block is incident on to a spring bumper, which has a spring constant of 500.0 N/m.
   a) Calculate the maximum compression of the spring.
   b) Calculate the speed of the block when the spring is compressed 0.100 m.



   c) Calculate the compression of the spring when the block is moving at 2.00 m/s.

The physics of weighing a melon

6. A scale used to measure the weight of a 1.25 kg melon stretches 2.75 cm under the weight of the melon. What is the spring constant for the spring inside the scale?

Lesson Summary

You began this lesson by discussing how to calculate the work done by a varying force. You learned that the area under a force-stretch graph could be determined by dividing the area under a curve into small rectangular segments, and finding the area under each segment.

Thus, you define the work done by a variable force as follows:

The work done by a variable force in moving an object is equal to the area under the graph of $\vec{F}$ versus d.

For an ideal spring, the restoring force of the spring, $\vec{F}_S$, versus stretch, x, graph is a straight line. For the linear portion of the graph, the slope is designated by the letter k. The equation relating force and deformation (extension or compression) is $\vec{F}_S = kx$.

The term k is a proportionality constant called the **spring constant** and has units of newtons/metre (N/m).

We can formally state this relationship as **Hooke's law** for a spring.

Hooke's Law
The restoring force $\vec{F}_S$ of an ideal spring is given by $\vec{F}_S = -k\vec{x}$, where k is the spring constant and $\vec{x}$ is the displacement of the spring from its unstretched length.

$$\vec{F}_S = -k\vec{x}$$

Quantity	**Symbol**	**Unit**
Restoring force of spring	$\vec{F}_S$	newton (N)
Spring constant	k	newtons/metre (N/m)
Amount of compression or extension of the spring	$\vec{x}$	metres (m)

Note: The spring constant is a ***positive*** number.

Potential Energy Stored in a Spring

The spring potential energy (PE_S) in a deformed spring is the product of one-half of the spring constant (k) multiplied by the square of the deformation (x) (extension or compression) of the spring.

$$PE_S = \frac{1}{2}kx^2$$

Quantity	**Symbol**	**Unit**
Spring potential energy	PE_S	joules (J)
Spring constant	k	newtons/metre (N/m)
Compression or extension	x	metres (m)

The area under the curve of the restoring force of the spring, $\vec{F}_S$, versus stretch, x, graph is the work done in compressing or stretching the spring. This work becomes potential energy stored in the spring.

In the case of a spring, the total mechanical energy in a system is constant.

$$E_{\text{initial}} = E_{\text{final}}$$
$$KE_0 + PE_{S0} = KE_f + PE_{Sf}$$
$$\frac{1}{2}mv_0^{\,2} + \tfrac{1}{2}kx_0^{\,2} = \frac{1}{2}mv_f^{\,2} + \frac{1}{2}kx_f^{\,2}$$

The energy can be interchanged between kinetic energy in an object and potential energy in the spring.

Finally, spring potential energy, kinetic energy, and gravitational potential energy can be converted from one form to another so that the total mechanical energy is conserved.

NOTES

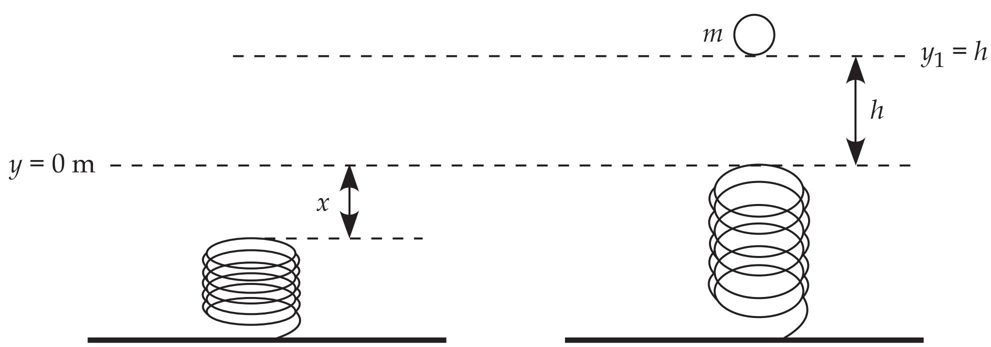
Assignment 4.5

Spring Potential Energy (8 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

The physics of a ball bearing and a spring

A ball bearing of mass m = 50.0 g, is sitting on a vertical spring whose force constant is 120.0 N/m. The initial position of the spring is at y = 0 m, as shown in the diagram below.



a) The spring is compressed downward a distance x = 0.200 m. From the compressed position, how high will the ball bearing rise? How high does the ball bearing rise above the equilibrium position at y = 0 m?

(continued)

b) What is the kinetic energy of the ball at the moment it is released from the spring? (Note: When the ball bearing leaves the spring, a small amount of the spring potential energy is converted into gravitational potential energy until the spring is at equilibrium. Ignore the conversion into gravitational potential energy and assume that all the energy is converted into kinetic energy.)

c) What is the maximum speed of the ball?

d) If the spring is compressed twice as much, how will this affect the maximum speed of the ball as it leaves the spring?

(continued)

Assignment 4.5: Spring Potential Energy (continued)

e) If the spring is compressed twice as much, how will this affect the maximum height to which the ball will rise?

Method of Assessment

The total of eight marks for this assignment will be determined as follows:

- 1 mark for the distance the ball bearing rises from the point of release in part (a)
- 1 mark for the distance the ball bearing rises above the equilibrium position in part (a)
- 1 mark for the correct answer in each of parts (b) and (c)
- 1 mark for the correct answer and 1 mark for the correct reasoning in each of parts (d) and (e)

NOTES

Video - Springs

This video discusses Hooke's Law which relates the spring constant, the stretch or compression of the spring and the restoring force of the spring.

Examples are done to calculate the restoring force, the spring constant and the amount of stretch or compression of the spring.

Then the video turns to taking the work done in distorting a spring to the amount of potential energy stored in the spring. This example of calculating work uses the area under the curve of a restoring force versus stretch graph. The area under this curve is a triangle. In the end the potential energy of a spring is calculated using $PE_S = \frac{1}{2}kx^2$.

Several problems are solved using work, spring potential energy, gravitational potential energy and kinetic energy.

https://youtu.be/6MhaPzGxfV8

Video - Conservation of Energy

This video applies the concept of the Law of Conservation of Energy to situations where the energy in a system is transferred between kinetic energy, gravitational potential energy and spring potential energy.

Several examples are done to illustrate the application of the Law of Conservation of Energy.

https://youtu.be/LKrieVc6O7A?list=PL53F202AAAA901D35

Video - Mechanics: Conservation of Energy (5 of 11) Energy Stored In A Spring

A block is placed next to a spring that has been compressed a certain distance. The block is released and it slides over a horizontal surface with a given coefficient of friction.

Using the Law of Conservation of Energy plus the work done by friction you are to calculate the distance the block will travel before it comes to rest.

https://youtu.be/dckE46X5mIQ

Video - Conservation of Energy Problem with Friction, an Incline and a Spring by Billy

View the video up to 4:52.

The law of conservation of energy using spring potential energy and kinetic energy is used up to the point where the lego piece starts moving up the incline.

Use

$$E_{Initial} = E_{Final}$$

$$KE_O + PE_{SO} = KE_F + PE_{SF}$$

$$\frac{1}{2}mv_O^2 + \frac{1}{2}kx_O^2 = \frac{1}{2}mv_F^2 + \frac{1}{2}kx_F^2$$

Cancel out the expressions that have an energy of 0 Joules to simplify the overall expression.

Then solve for the unknown.

https://youtu.be/-K5d97U7FwQ

Assignment 4.5

Spring Potential Energy (8 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Modules 3 and 4, after you have completed Module 4.

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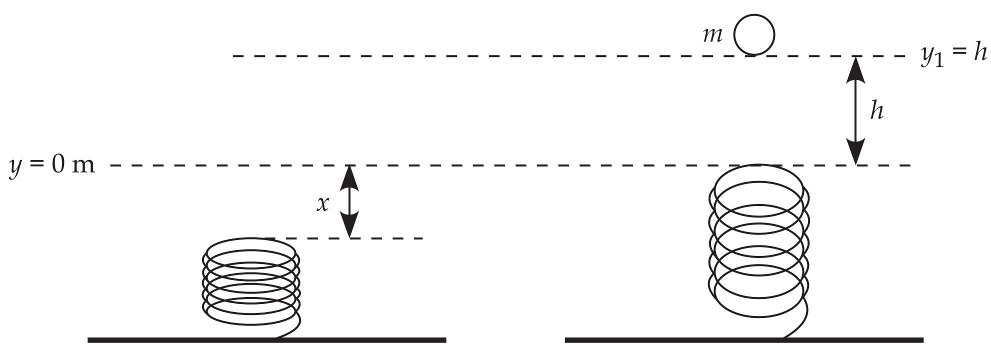

a) The spring is compressed downward a distance x = 0.200 m. From the compressed position, how high will the ball bearing rise? How high does the ball bearing rise above the equilibrium position at y = 0 m?

(continued)

b) What is the kinetic energy of the ball at the moment it is released from the spring? (Note: When the ball bearing leaves the spring, a small amount of the spring potential energy is converted into gravitational potential energy until the spring is at equilibrium. Ignore the conversion into gravitational potential energy and assume that all the energy is converted into kinetic energy.)

c) What is the maximum speed of the ball?

d) If the spring is compressed twice as much, how will this affect the maximum speed of the ball as it leaves the spring?

(continued)

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- 1 mark for the distance the ball bearing rises above the equilibrium position in part (a)
- 1 mark for the correct answer in each of parts (b) and (c)
- 1 mark for the correct answer and 1 mark for the correct reasoning in each of parts (d) and (e)

MODULE 4 SUMMARY

Congratulations! You have finished the fourth module about work and energy in this course.

As mentioned in the introduction to this module, we extended the study of mechanics by considering the relationships involving forces acting through a distance. The action of force through a distance allowed a connection to be made between work and energy. Energy is transferred when work is done (work-energy theorem). Energy, in turn, consists of two major types: kinetic energy and potential energy. The law of conservation of energy, which followed, illustrated that within a given system energy is not lost or gained but is simply converted among its various forms: kinetic energy, gravitational potential energy, and spring potential energy. Once again, we are extending our understanding of mechanics based on the concepts we have already studied.

Submitting Your Assignments

It is now time for you to submit Assignments 3.1 to 3.6 from Module 3 and Assignments 4.1 to 4.5 from Module 4 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 3 and Module 4 assignments and organize your material in the following order:

- Modules 3 and 4 Cover Sheet (found at the end of the course Introduction)
- Assignment 3.1: Vertical Motion of a Bullet
- Assignment 3.2: Vector Nature of Projectile Motion
- Assignment 3.3: Projectile Motion of a Cannonball
- Assignment 3.4: Video Laboratory Activity: Circular Motion
- Assignment 3.5: Circular Motion of the Moon
- Assignment 3.6: Uniform Circular Motion of a Satellite
- Assignment 4.1: Calculating Work
- Assignment 4.2: Work and Kinetic Energy
- Assignment 4.3: Conservation of Mechanical Energy in a Roller Coaster
- Assignment 4.4: Video Laboratory Activity: Hooke's Law
- Assignment 4.5: Spring Potential Energy

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Grade 12 Physics (40S)

Module 4: Work and Energy

Learning Activity Answer Keys

Module 4: Work and Energy

Learning Activity 4.1: Definition of Work

1. The truck exerts a force of 2150 N to pull a car 16.2 m out of the ditch. How much work is done?

   Answer:

Given:	Force	$\vec{F}_{net} = 2150\text{ N}$
	Displacement	$\vec{d} = 16.2\text{ m}$
Unknown:	Work	$W = ?$
Equation:		$W = Fd$
Substitute and solve:		$W = (2150\text{ N})(16.2\text{ m}) = 34800\text{ J}$

   The amount of work done is 34800 J.

2. A person pulls horizontally with a force of 375 N against a crate of mass 125 kg and moves it across the floor at a steady rate of 1.00 m/s for 3.00 seconds. How much work is done?

   Answer:

   To find the work, you must know the force and the displacement. Moving at 1.00 m/s for 3.00 seconds, he has a displacement of 3.00 m.

Given:	Force	$\vec{F}_{net} = 375\text{ N}$
	Displacement	$\vec{d} = 3.00\text{ m}$
Unknown:	Work	$W = ?$
Equation:		$W = Fd$
Substitute and solve:		$W = (375\text{ N})(3.00\text{ m}) = 1120\text{ J}$

   The amount of work done is 1120 J.

3. The person exerts a force of 150 N on a large crate for 2.50 seconds, but cannot move it. How much work has been done?

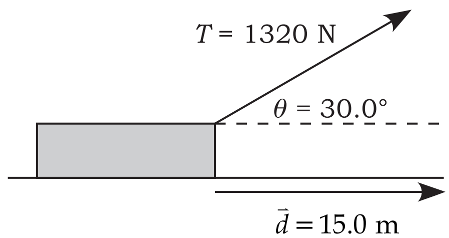
   Answer:

   Since the displacement is 0 m, $W = Fd = 0$ J. No work has been done.

Learning Activity 4.2: Calculating Work

1. What work is performed in dragging a sled 15.0 m horizontally when the force of 1320 N is transmitted by a rope making an angle of 30.0° with the ground?

   Answer:

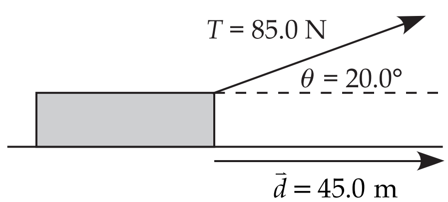
   It is important to determine the magnitude of the force along the direction of the displacement in calculating the work done.

   $W = F(\cos\theta)d = (1320\text{ N})(\cos 30.0°)(15.0\text{ m}) = 17147\text{ J} = 1.71 \times 10^4\text{ J}$

   The amount of work done is 1.71×10^4 J.

2. A girl pulls a wagon with a constant velocity along a level path for a distance of 45.0 m. The handle of the wagon makes an angle of 20.0° with the horizontal, and she exerts a force of 85.0 N on the handle. Find the amount of work the girl does in pulling the wagon.

   Answer:

   Again, it is important to determine the magnitude of the force along the direction of the displacement in calculating the work done.

   $W = F(\cos\theta)d = (85.0\text{ N})(\cos 20.0°)(45.0\text{ m}) = 3594\text{ J} = 3590\text{ J}$

   The amount of work done is 3590 J.

Learning Activity 4.3: Calculating Work: The General Case

The physics of work and pulling a crate along a surface with friction (conceptual)

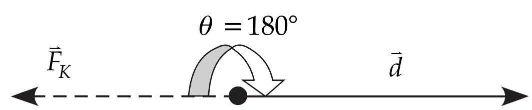
1. A crate is being moved with a constant velocity $\vec{v}$ by a force P (parallel to $\vec{v}$) along a horizontal floor. The normal force is $\vec{F}_N$, the kinetic frictional force is $\vec{F}_K$, and the weight of the crate is $m\vec{g}$. Which forces do positive, zero, or negative work?

   Answer:

   The force P acts along the displacement and therefore does positive work. Here $\theta = 0°$ and $\cos 0° = 1$.

   Both the normal force $\vec{F}_N$ and the weight $m\vec{g}$ are perpendicular to the displacement so they do zero work. Here $\theta = 90°$ and $\cos 90° = 0$. So $W = Fd \cos \theta = 0$ J.

   The kinetic frictional force $\vec{F}_K$ acts opposite to the direction of the displacement and therefore does negative work. Here $\theta = 180°$ and $\cos 180° = -1$. So $W = Fd \cos \theta < 0$ J.



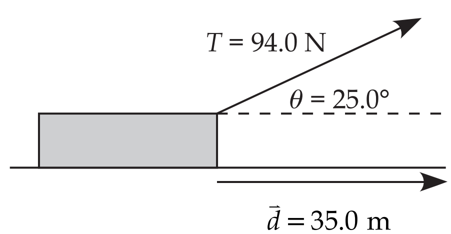
The physics of pulling a toboggan at an angle

2. A person pulls a toboggan for a distance of 35.0 m along the snow with a rope directed 25.0° above the snow. The tension in the rope is 94.0 N. How much work is done by the tension force?

   Answer:

   It is important to determine the magnitude of the force along the direction of the displacement in calculating the work done.

   $W = F(\cos \theta)d = (94.0 \text{ N})(\cos 25.0°)(35.0 \text{ m}) = 2.98 \times 10^3$ J

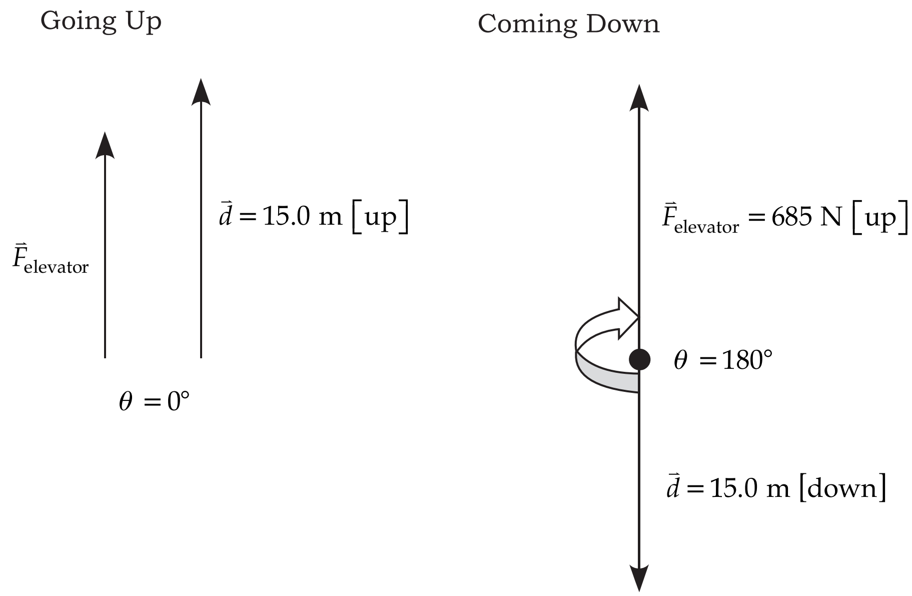


The physics of work and moving up and down in an elevator

3. You are moving into an apartment. Your weight is 685 N and that of your belongings is 925 N.

   a) How much work does the elevator do in lifting you and your belongings up five stories (15.0 m) at a constant velocity?

   Answer:



   In the upward direction, both the direction of the force and the direction of motion are the same. The force that the elevator exerts is your weight and the weight of the belongings. The total work done is therefore

   $$W_{\text{elevator}} = (F_{\text{elevator}})d(\cos 0°) = (685\ \text{N} + 925\ \text{N})(15.0\ \text{m})(1)$$
   $$= (1610\ \text{N})(15.0\ \text{m})(1) = 2.42 \times 10^4\ \text{J}$$

   b) How much work does the elevator do on you (without belongings) on the downward trip, which is also made at a constant velocity?

   Answer:

   On the downward trip, the direction of the force and the direction of motion are opposite to each other ($\theta = 180°$).

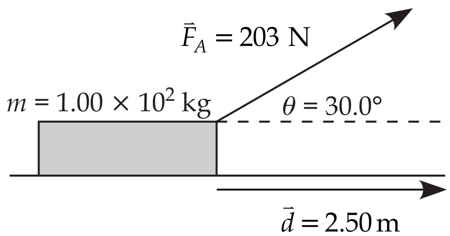
   $$W_{\text{elevator}} = (F_{\text{elevator}})d(\cos 180°) = (685\ \text{N})(15.0\ \text{m})(-1)$$
   $$= -1.03 \times 10^4\ \text{J}.$$

The physics of pulling a crate at an angle

4. A 1.00×10^2 kg crate is being pulled across a horizontal floor by a force $\vec{F}_A = 203$ N that makes an angle of 30.0° above the horizontal. The crate moves at a constant velocity over a distance of 2.50 m.

   a) Calculate the work done by the applied force.

   Answer:



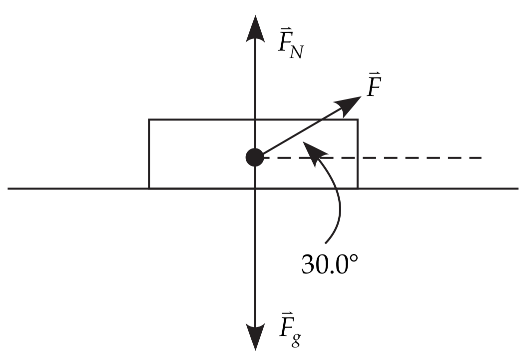
The work done by the force $\vec{F}_A$ is

$$W = F_A d \cos\theta$$
$$= (203\text{ N})(2.50\text{ m})\cos 30.0°$$
$$= 439.5\text{ J}$$
$$= 4.40 \times 10^2\text{ J}$$

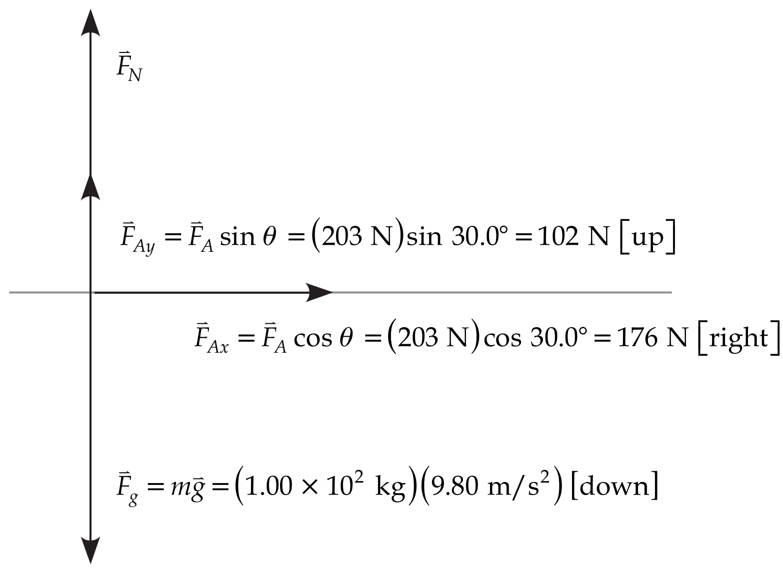
   b) Calculate the normal force acting on the crate.

   Answer:

   In order to calculate the normal force, the free-body diagram might look as follows.



Remember to resolve the forces so that the free-body diagram looks like an "x".



$$\vec{F}_{Ay} + \vec{F}_N = -\vec{F}_g$$
$$\vec{F}_N = -\vec{F}_{Ay} - \vec{F}_g$$
$$\vec{F}_N = (-102 \text{ N [up]}) - (980 \text{ N [down]})$$
$$\vec{F}_N = 878 \text{ N [up]}$$

The normal force is 878 N [up].

c) Determine the coefficient of kinetic friction.

Answer:

Since the velocity is constant the net force on the crate is 0 N. Therefore, the force of friction is balancing the horizontal component of the applied force.

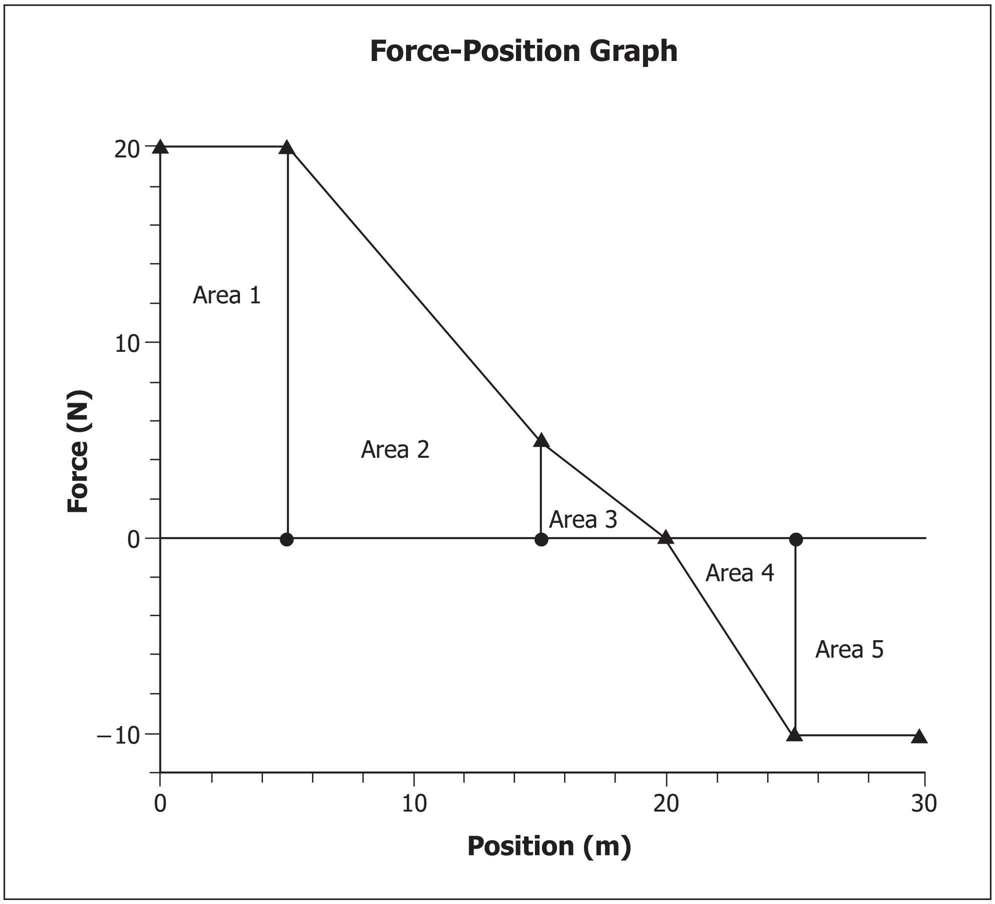
$$\vec{F}_K = 176 \text{ N}$$
$$\vec{F}_N = 878 \text{ N [up]}$$
$$F_K = \mu_k F_N$$
$$\mu_k = \frac{F_K}{F_N} = \frac{176 \text{ N}}{878 \text{ N}} = 0.200$$

The coefficient of kinetic friction is 0.200.

Calculating work from a force-position graph

5. a) Calculate the total work done according to the force-position graph below.

Answer:



To find the work done, calculate each area separately and then add them together.

Area 1 is a trapezoid so the formula for the area is Area trapezoid = $\left[\dfrac{a+b}{2}\right]h$.

Area 1 = W_1 = Area rectangle = lw = (20 N)(5 m) = 100 J

Area 2 = W_2 = Area trapezoid = $\left[\dfrac{a+b}{2}\right]h = \left[\dfrac{20\text{ N}+5\text{ N}}{2}\right](10\text{ m}) = 125\text{ J}.$

Area 3 = W_3 = Area triangle = $\dfrac{bh}{2} = \dfrac{(5\text{ N})(5\text{ m})}{2} = 12.5\text{ J}.$

Area 4 lies below the horizontal axis. Therefore, the force has a negative value and the work done is also negative.

Area 4 = W_4 = Area triangle = $\frac{bh}{2} = \frac{(-10\text{ N})(5\text{ m})}{2} = -25$ J.

Area 5 = W_5 = Area rectangle = lw = (10 N)(5 m) = –50 J.

So the total work done is just the sum of these five works:

$W_{total} = W_1 + W_2 + W_3 + W_4 + W_5$

= 100 J + 125 J + 12.5 J + (–25 J) + (–50 J)= 162.5 J = 162 J

b) What is the meaning of a negative work in terms of force and displacement?

Answer:

The work will be negative when the force and the displacement are acting in opposite directions or $\theta = 180°$.

c) In which sections of the graph is the force uniformly changing? How is the work calculated in these regions?

Answer:

The force is uniformly changing while position changes from 5 m to 15 m, 15 m to 20 m, and 20 m to 25 m.

The work is calculated now by finding the area on the force position graph. Each of these areas is shaped like a trapezoid. Therefore, you use the formula for the area of a trapezoid.

Learning Activity 4.4: Kinetic Energy

1. The baseball in the example 1 above is struck by a baseball bat, leaving the bat with a velocity of 40.0 m/s [north]. Calculate

   a) the final kinetic energy of the baseball

   Answer:

   Given: Mass m = 142 g = 0.142 kg (SI mass unit)

   Speed v = 40.0 m/s
   (You do not need velocity here!)

   Unknown: Kinetic energy KE = ?

   Equation: $KE = \frac{1}{2}mv^2$

   Substitute and solve: $KE = \frac{1}{2}(0.142 \text{ kg})(40.0 \text{ m/s})^2 = 114 \text{ J}$

   The kinetic energy of the baseball is 114 J.

   The point of this question is to show that kinetic energy is a scalar quantity. In solving for kinetic energy you need to use only speed (magnitude of velocity).

   b) the change in the kinetic energy of the baseball

   Answer:

   The change in any quantity is always the final value minus the initial value.

   The change in kinetic energy of the baseball is the final kinetic energy minus the initial kinetic energy.

   $\Delta KE = KE_F - KE_0$ = 114 J – 114 J = 0 J.

   The kinetic energy of the baseball did not change even though its direction of motion is reversed. Again, kinetic energy is a scalar quantity.

2. An object is moving at 9.00 m/s and has 215 J of kinetic energy. What is the mass of the object?

Answer:

Given: Speed $v = 9.00\ \text{m/s}$

Kinetic energy $KE = 215\ \text{J}$

Unknown: Mass $m = ?$

Equation: $KE = \frac{1}{2}mv^2$ rearranged to $m = \frac{2KE}{v^2}$

Substitute and solve: $m = \frac{2KE}{v^2} = \frac{2(215\ \text{J})}{(9.00\ \text{m/s})^2} = 5.31\ \text{kg}$

The mass of the object is 5.31 kg.

Learning Activity 4.5: Work-Energy Theorem

1. An automobile of mass 1250 kg starts from rest. It accelerates under a net force of 3250 N [west] over 25.0 m. Calculate the final kinetic energy and the final velocity of this automobile.

   Answer:

   Given: Mass $m = 1250$ kg

   Initial speed $v_0 = 0$ m/s

   Final speed $v_f = ?$ m/s

   Displacement $\vec{d} = 25.0$ m [west]

   Net force $\vec{F}_{\text{net}} = 3250$ N [west]

   Net force and displacement act in the same direction.

   Unknown: Final kinetic energy $KE_f = ?$

   Equation: Since the automobile starts from rest, all of the work done shows up in the final kinetic energy.

   $W = \Delta KE = KE_f - KE_0$

   $W = Fd = KE_f$

   Substitute and solve: $KE_f = (3250 \text{ N})(25.0 \text{ m}) = 81200 \text{ J}$

   The final kinetic energy of the automobile is 81200 J.

   Since $KE_f = \frac{1}{2}mv_f^2$, you can solve for the final velocity using $v_f = \sqrt{\frac{2KE_f}{m}}$.

   $$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(81200 \text{ J})}{1250 \text{ kg}}} = 11.4 \text{ m/s}$$

   A final velocity of the car is 11.4 m/s [west].

2. A curling rock of mass 20.0 kg is sliding along the ice with a velocity of 2.75 m/s [east]. During the next 3.00 seconds, the curling rock slows to a velocity of 1.50 m/s [east] while sliding 6.38 m [east]. Calculate

a) the change in the kinetic energy of the curling rock

Answer:

Given: Mass $m = 20.0$ kg

Initial speed $v_0 = 2.75$ m/s

Final speed $v_f = 1.50$ m/s

Time interval $\Delta t = 3.00$ s

Displacement $\vec{d} = 6.38$ m [east]

Unknown: Change in kinetic energy $\Delta KE = ?$

Equation: $$\Delta KE = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

Substitute and solve:

$$\Delta KE = \frac{1}{2}(20.0\text{ kg})(1.50\text{ m/s})^2 - \frac{1}{2}(20.0\text{ kg})(2.75\text{ m/s})^2$$

ΔKE = 22.5 J – 75.6 J = –53.1 J

The curling rock lost 53.1 J of kinetic energy. This indicates that the net force acting on the curling rock and the displacement were in opposite directions.

b) the net force that was acting on a curling rock

Answer:

Remember the net force acting on the rock in the displacement of the curling rock are opposite in direction.

Since you know the change in the kinetic energy of the curling rock, you can use the work-energy theorem to calculate the net force.

$W = \Delta KE$
$W = Fd = \Delta KE$
$F(6.38\text{ m}) = -53.1$ J
$F = -8.32$ N

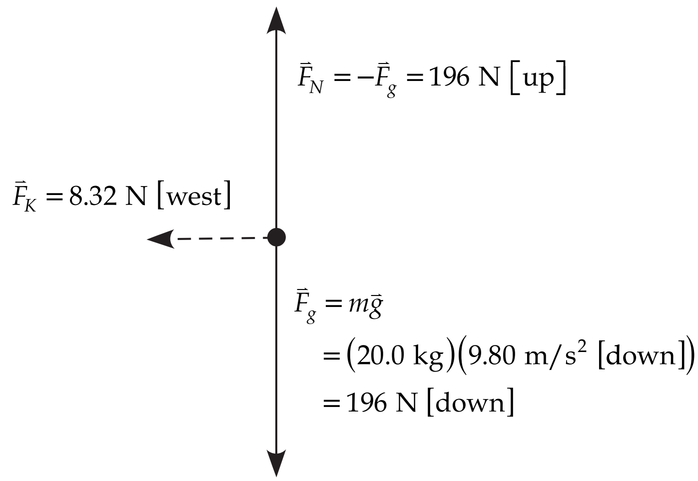
The net force is 8.32 N [west].

c) the coefficient of kinetic friction for the ice

Answer:

The net force is provided by the force of friction. You can see that the normal force is 196 N.

Use: $F_K = \mu_k F_N$ and $\mu_k = \dfrac{F_K}{F_N}$



$$\mu_k = \frac{8.32 \text{ N}}{196 \text{ N}} = 0.0424$$

The coefficient of kinetic friction for the ice is 0.0424.

Learning Activity 4.6: Work, Kinetic Energy, and the Work-Energy Theorem

The physics of energy and the hammer throw event

1. The hammer throw is a track and field event in which a 7.30 kg ball (the hammer), starting from rest, is whirled around in a circle several times and released. It then moves upward on the familiar curving path of projectile motion. In one throw, the hammer is given a speed of 29.0 m/s. Determine the work done to launch the motion of the hammer.

   Answer:

   Using the work-energy theorem, we can determine the work involved by the difference of the kinetic energies.

   $$\begin{aligned} W &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v_f^2 - v_0^2) \\ &= \frac{1}{2}(7.30 \text{ kg})\left[(29.0 \text{ m/s})^2 - (0 \text{ m/s})^2\right] \\ &= 3.07 \times 10^3 \text{ J} \end{aligned}$$

The physics of energy and an arrow

2. An archer uses a bow for which the bowstring exerts an average force of 85.2 N on the arrow over a distance of 0.825 m. If the mass of the arrow fired by the archer is 75.0 g, how fast is the arrow travelling when it leaves the bow?

   Answer:

   The work done on the arrow by the bow is given by $W = Fd$.

   The work-energy theorem gives $W = \Delta KE = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$.

   Since the initial velocity is zero, the final velocity can be determined from $W = \frac{1}{2}mv_f^2$.

   $$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fd}{m}} = \sqrt{\frac{2(85.2 \text{ N})(0.825 \text{ m})}{0.0750 \text{ kg}}} = 43.3 \text{ m/s}$$

The physics of energy and jogging

3. A 65.0 kg jogger is running at a speed of 5.30 m/s.

   a) What is the kinetic energy of the jogger?

   Answer:

   The kinetic energy of the jogger is

   $$KE = \frac{1}{2}mv^2 = \frac{1}{2}(65.0 \text{ kg})(5.30 \text{ m/s})^2 = 913 \text{ J.}$$

   b) How much work is done by the net external force that accelerates the jogger to 5.30 m/s from rest?

   Answer:

   The jogger started from rest so the jogger did not have any initial kinetic energy. Therefore, the work done is equal to the change in kinetic energy. The final kinetic energy was determined in part a above.

   $W = \Delta KE = 913$ J

The physics of energy, friction, and a curling rock

4. The speed of a curling rock decreases from 2.65 to 1.25 m/s in coasting 29.2 m along the surface of the ice. This type of shot is used to remove an opponent's rock from play. Find the coefficient of kinetic friction between the curling rock and the ice.

   Answer:

   The work done in slowing down the rock is $W = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_0^{\,2}$.

   It is friction that does the work. Also, the force of friction is opposite to the direction of motion and, therefore, negative.

   $$W = -F_f d = -\mu_k mgd$$

   These two expressions for work are equivalent to each other.

   $$\frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_0^{\,2} = -\mu_k mgd$$

   Solve for the coefficient of friction:

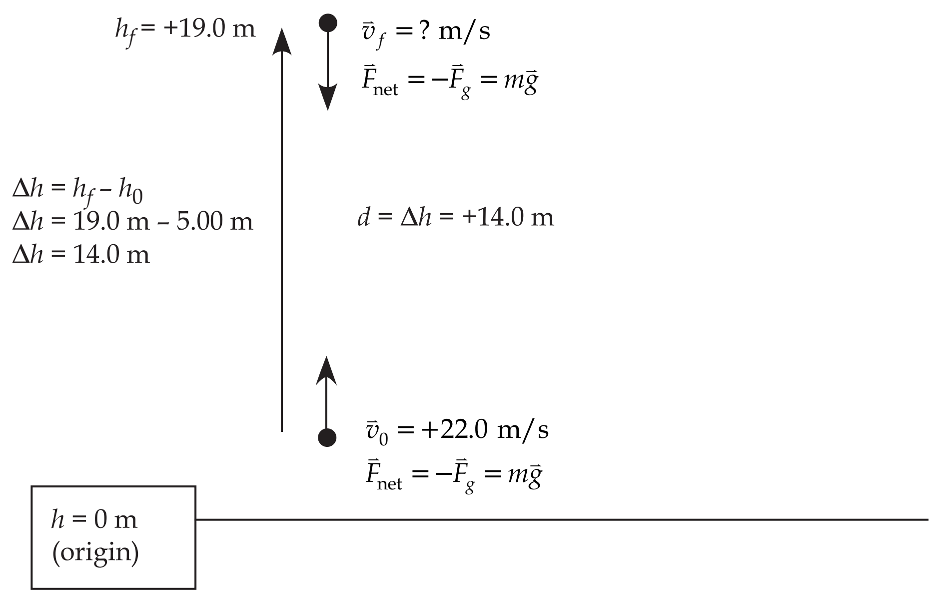
   $$\mu = \frac{-\left(v_f^{\,2} - v_0^{\,2}\right)}{2gd} = \frac{-\left[(1.25 \text{ m/s})^2 - (2.65 \text{ m/s})^2\right]}{2(9.80 \text{ m/s}^2)(29.2 \text{ m})} = 0.00954$$

Learning Activity 4.7: Work Done by Gravity and the Work-Energy Theorem

1. A stone of mass 50.0 g is thrown upwards at 22.0 m/s at a point 5.00 m above the ground.
   a) What is the work done by gravity during the time the stone rises to a new height of 19.0 m above the ground?

      Answer:

      Given: Mass m = 50.0 g = 0.0500 kg



      Unknown: W_{gravity} = ?

      Equation: $W_{\text{gravity}} = F_g d$

      The force of gravity acts downwards and the displacement is upwards. Therefore, the work will be negative.

      Substitute and solve:
      $$W_{\text{gravity}} = F_g d = -mgd$$
      $$= -(0.0500\text{ kg})(9.80\text{ m/s/s})(14.0\text{ m})$$
      $$W_{\text{gravity}} = -6.86\text{ J}$$

      Gravity does –6.86 J of work on the stone. In other words, it transfers 6.86 J of energy out of the stone.

b) Calculate the velocity of the stone when it is 19.0 m above the ground, using the work-energy theorem.

Answer:

The work done by gravity results in a change in the kinetic energy of the stone (ΔKE). You can use the work-energy theorem to relate these two quantities.

$$W = \Delta KE = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

Substitute and solve for v_f.

$$-6.86\text{ J} = \frac{1}{2}(0.0500\text{ kg})v_f^2 - \frac{1}{2}(0.0500\text{ kg})(22.0\text{ m/s})^2$$

$$-6.86\text{ J} = \frac{1}{2}(0.0500\text{ kg})v_f^2 - 12.1\text{ J}$$

$$v_f^2 = 209.6$$

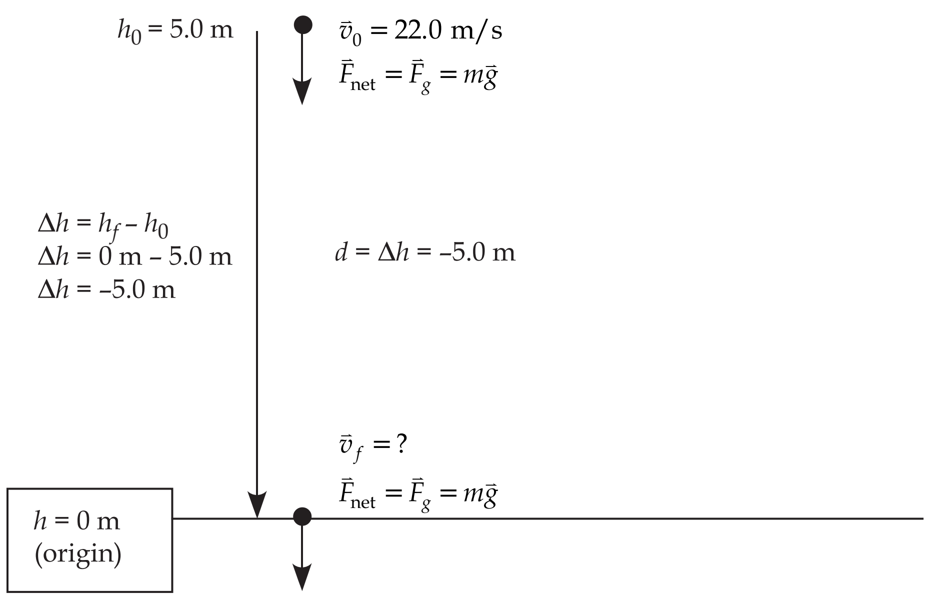
$$v_f = 14.5\text{ m/s}$$

The speed of the stone at a height of 19.0 m above the ground is 14.5 m/s.

c) What is the speed of the stone as it crashes into the ground?

Answer:

When the stone crashes into the ground, the final height is 0 m. Again, you can use the work-energy theorem to determine the final velocity.



Here, the force of gravity and the displacement both point downwards. Therefore, the work done by gravity will show up as an increase in the kinetic energy of the stone. The work done by gravity is positive.

$$W_{\text{gravity}} = \Delta KE = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

$$W_{\text{gravity}} = mgd$$

So $mgd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

$$(0.0500 \text{ kg})(9.80 \text{ m/s}^2)(5.00 \text{ m}) = \frac{1}{2}(0.0500 \text{ kg})v_f^2 - \frac{1}{2}(0.0500 \text{ kg})(22.0 \text{ m/s})^2$$

$$2.45 \text{ J} = 0.0250 v_f^2 - 12.1 \text{ J}$$

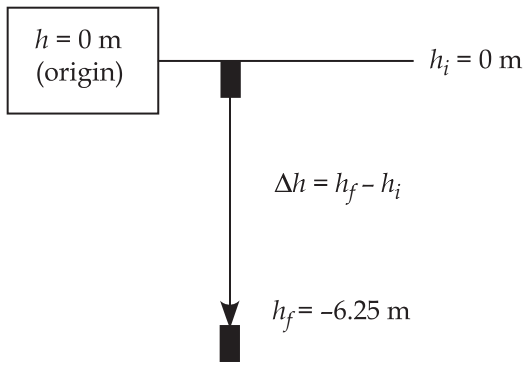
$$v_f = 24.1 \text{ m/s}$$

The stone hits the ground travelling at 24.1 m/s.

Learning Activity 4.8: Changes in Gravitational Potential Energy

1. A piledriver of mass 475 kg falls a distance of 6.25 m before it strikes a pile.
   a) Draw a sketch of this situation, showing the origin at the initial position of the piledriver.

   Answer:



   b) Determine the change in gravitational potential energy of the piledriver.

   Answer:

   $\Delta PE = mg\Delta h = mg(h_f - h_i)$

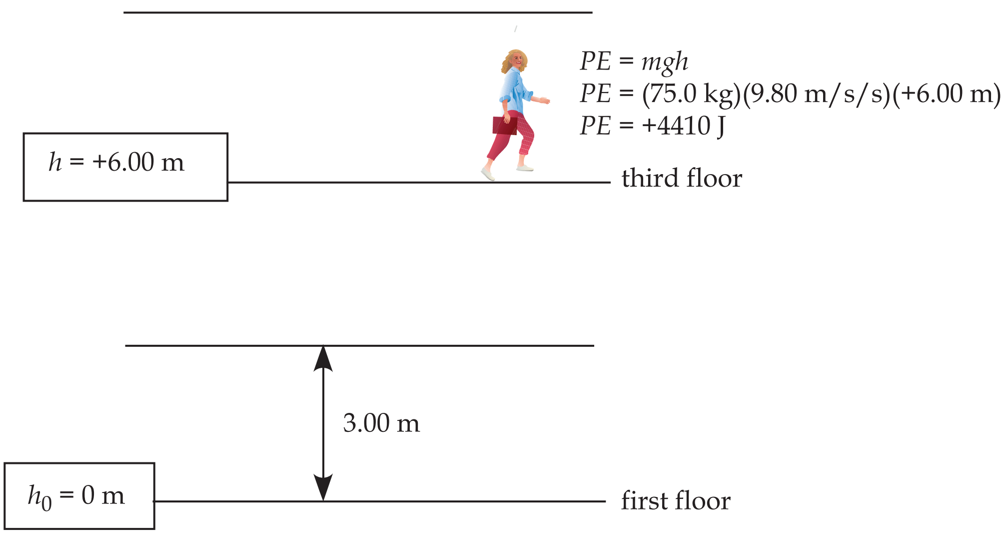
   $= (475 \text{ kg})(9.80 \text{ m/s/s})(-6.25 \text{ m} - 0 \text{ m})$

   $\Delta PE = -29093.75 \text{ J} = -29100 \text{ J}$

   The piledriver loses 29100 J of gravitational potential energy.

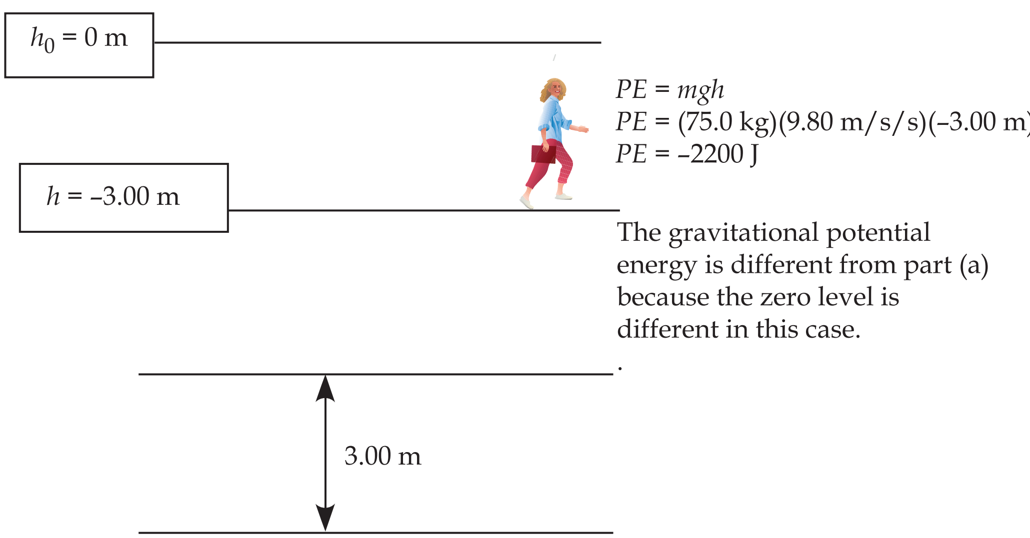
2. A person of mass 75.0 kg walks up the stairs from the first floor to the fourth floor. The vertical distance between floors is 3.00 m.
   a) What is the gravitational potential energy of the person when he is standing on the landing for the third floor if you choose the first floor to be the zero level?

      Answer:



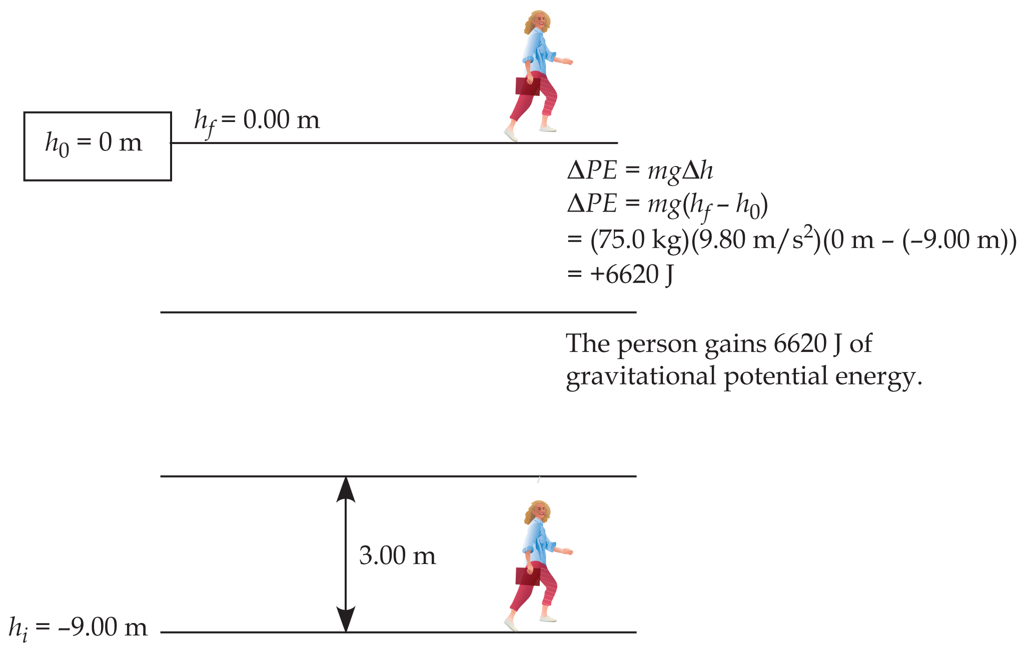
   b) What is the gravitational potential energy of the person when he is standing on the landing for the third floor if you choose the fourth floor to be the zero level?

      Answer:



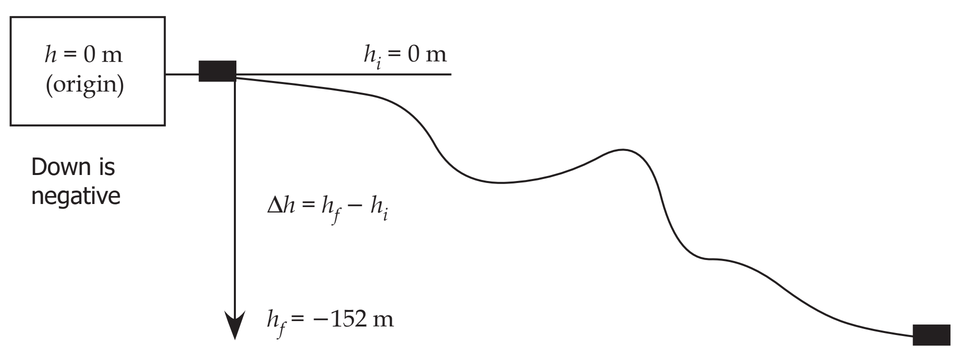
c) Calculate the change in the gravitational potential energy for the person walking up from the first floor to the fourth floor, using the fourth floor as the zero level.

Answer:



3. At the Vancouver Olympics, a four-person bobsled team and the sled had a mass of 629 kg. The track was 1450 m long and fell a vertical distance of 152 m. Determine the change in gravitational potential energy of the team and sled as it travelled from its starting position to the finish line.

Answer:



$$\Delta PE_g = m\bar{g}\Delta h = m\bar{g}\left(h_f - h_i\right)$$

$$= \left(629\ \text{kg}\right)\left(9.80\ \text{m/s/s}\right)\left(-152\ \text{m} - 0\ \text{m}\right)$$

$$\Delta PE_g = -936958.4\ \text{J} = -9.37 \times 10^5\ \text{J}$$

Learning Activity 4.9: Conservation of Mechanical Energy (Kinetic Energy and Gravitational Potential Energy)

The physics of gravitational potential energy and an astronaut

1. In a simulation on Earth, an astronaut in his spacesuit climbs up a vertical ladder. On the Moon, the same astronaut makes the same climb. In which case does the gravitational potential energy of the astronaut change by a greater amount? Account for your answer.

   Answer:

   The change in the gravitational potential energy of the astronaut is given by $\Delta PE = mg\Delta h$ where Δh is the height of the ladder. The value of g is greater on Earth than it is on the Moon. Therefore, the gravitational potential energy of the astronaut changes by a greater amount on Earth.

The physics of gravitational potential energy and the CN Tower

2. The CN Tower in Toronto is advertised as being the world's tallest free-standing structure. At 181 stories, it has a height of 553 m. What is the gravitational potential energy of a 55.0 kg person who is at the top of the tower?

   Answer:

   Choose the origin, $h = 0$ m, at the bottom of the tower.

   The gravitational potential energy is

   $$PE = mgh = (55.0\text{ kg})(9.80\text{ m/s}^2)(553\text{ m}) = 2.98 \times 10^5\text{ J}$$

The physics of a projectile

3. A person throws a stone from the edge of a cliff at a speed of 10.0 m/s. The cliff has a height of 20.0 m. Consider the force of air friction to be negligible. Determine the speed with which the stone strikes the ground when the stone is thrown

   a) horizontally

      Answer:

      Set the origin at ground level. Then, h_0 = +20.0 m and h_f = 0 m.

      If air resistance and friction can be ignored, then mechanical energy is conserved.

      Use $E_{\text{initial}} = E_{\text{final}}$

      $$KE_0 + PE_0 = KE_f + PE_f$$

      $$\frac{1}{2}mv_0^{\,2} + mgh_0 = \frac{1}{2}mv_f^{\,2} + mgh_f$$

      Notice that m can be cancelled out of this expression.

      $$\frac{1}{2}v_0^{\,2} + gh_0 = \frac{1}{2}v_f^{\,2} + gh_f$$

      Solve for v_f.

      $$v_{\text{f}} = \sqrt{v_0^{\,2} + 2g(h_0 - h_f)} = \sqrt{(10.0\ \text{m/s})^2 + 2\left(9.80\ \text{m/s}^2\right)(20.0\ \text{m})}$$
      $$= 22.2\ \text{m/s}$$

   b) vertically straight up

      Answer:

      The conservation of energy expression depends only upon the initial and final values of the speeds and heights, so the results will be the same as for part (a).

   c) vertically straight down

      Answer:

      Again, the answer will be the same as for part (a).

The physics of the Magnum XL-200 roller coaster

4. One of the fastest roller coasters in the world is the Magnum XL-200 at Cedar Point Park in Sandusky, Ohio. The ride includes a vertical drop of 59.4 m. Assume that the coaster has a speed of nearly zero as it crests the top of the hill. Neglect friction and find the speed of the riders at the bottom of the hill. Determine the speed both in units of m/s and km/h.

   Answer:

   Set the origin, h = 0 m, at the bottom of the hill. Then h_0 = +59.4 m and h_f = 0 m.

   Since we are neglecting friction, we may set the work done by the frictional force equal to zero. A normal force acts on each rider, but this force is perpendicular to the motion, so it does not do any work. Thus, the work done by the external non-conservative forces is zero, and we may use the conservation of mechanical energy to find the speed of the riders at the bottom of the hill. The principle of conservation of mechanical energy states that

$$E_{\text{initial}} = E_{\text{final}}$$
$$KE_0 + PE_0 = KE_f + PE_f$$
$$\frac{1}{2}mv_0^{\ 2} + mgh_0 = \frac{1}{2}mv_f^{\ 2} + mgh_f$$

   Notice that m can be cancelled out of this expression.

$$\frac{1}{2}v_0^{\ 2} + gh_0 = \frac{1}{2}v_f^{\ 2} + gh_f$$

   Solve for the final speed.

$$v_f = \sqrt{v_0^{\ 2} + 2g(h_0 - h_f)} = \sqrt{(0\ \text{m/s})^2 + 2\left(9.80\ \text{m/s}^2\right)(59.4\ \text{m} - 0\ \text{m})}$$
$$= 34.1\ \text{m/s} = 123\ \text{km/h}$$

   To change m/s to km/h, replace m with an equivalent number of kilometres and replace seconds with an equivalent number of hours.

$$\frac{34.1\ \text{m}}{1\ \text{s}} = \frac{0.0341\ \text{km}}{\left(\frac{1}{3600}\right)\text{h}} = (0.0341\ \text{km})\left(\frac{3600}{1\ \text{h}}\right) = 122.76\ \text{km/h} = 123\ \text{km/h}$$

Learning Activity 4.10: Hooke's Law

1. Springs in automobiles are used to provide a smooth ride. When a tire goes over a bump, the spring is compressed 3.00 cm. If the weight of the car supported by that tire is 3060 N, calculate the spring constant.

   Answer:

   Given: Restoring force of spring $\vec{F}_S = +3060 \text{ N}$

   Compression of spring $\vec{x} = -3.00 \text{ cm} = -0.0300 \text{ m}$ (opposite to force)

   Unknown: Spring constant $k = ?$

   Equation: $\vec{F}_S = k\vec{x}$

   Substitute and solve:

   $$\vec{F}_S = k\vec{x}$$

   $$k = -\frac{+3060 \text{ N}}{-0.0300 \text{ m}} = 102000 \text{ N/m}$$

   The spring constant is 102000 N/m.

2. A spring scale containing a spring of constant 525 N/m stretches 2.00 cm when it is used to weigh a fish that you just caught. What is the mass of the fish?

   Answer:

   Here, the restoring force of the spring balances the force of gravity acting on the fish.

   Given: Spring constant $k = 525 \text{ N/m}$

   Extension of spring $\vec{x} = 2.00 \text{ cm} = 0.0200 \text{ m}$

   Unknown: mass of fish $m = ?$

   Equation: $\vec{F}_S = -\vec{F}_g$

   Substitute and solve:

   $$\vec{F}_S = -\vec{F}_g$$

   $$-k\vec{x} = -m\vec{g}$$

   $$m = \frac{kx}{g} = \left(\frac{(525 \text{ N/m})(0.0200 \text{ m})}{9.80 \text{ m/s}^2}\right) = 1.07 \text{ kg}$$

   Your fish has a mass of 1.07 kg.

3. When a mass of 2.16 kg is attached to a vertical spring, the spring is stretched 5.62 cm. What is the spring constant?

Answer:

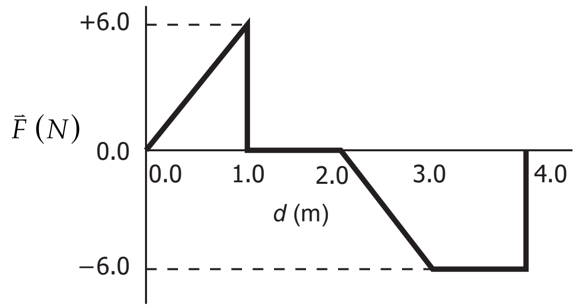
Given:	Extension of spring	$\vec{x} = 5.62 \text{ cm} = 0.0562 \text{ m}$
	Mass	$m = 2.16 \text{ kg}$
	Weight of mass	$\vec{F}_g = m\vec{g} = (2.16 \text{ kg})(9.80 \text{ m/s}^2)$
		$= 21.2 \text{ N}[\text{down}]$
	Restoring force of spring	$\vec{F}_S = 21.2 \text{ N}[\text{up}]$
Unknown:	Spring constant	$k = ? \text{ N/m}$
Equation:		$\vec{F}_S = -k\vec{x}$
Substitute and solve:		$\vec{F}_S = -k\vec{x}$
		$21.2 \text{ N}[\text{up}] = -(k)(0.0562 \text{ m}[\text{down}])$
		$k = 377 \text{ N/m}$

The spring constant is 377 N/m.

Learning Activity 4.11: Potential Energy of a Spring

The physics of work done by a variable force

1. The force component along the displacement varies with the magnitude of the displacement, as shown on the graph.



Find the work done by the force for each of the following intervals:

a) 0 to 1.0 m

Answer:

The work done during each interval is equal to the area under the force vs. the displacement curve over that interval.

The area under this part of the curve is in the shape of a triangle.

$$W = \frac{1}{2}bh = \frac{1}{2}(1.0\text{ m})(6.0\text{ N}) = 3.0\text{ J}$$

b) 1.0 to 2.0 m

Answer:

Since there is no area under the curve, the work done is zero. This is reasonable since there is no net force acting on the object during this interval.

c) 2.0 to 4.0 m

Answer:

In the interval 2.0 m to 4.0 m, the area under the curve has a triangular region for 2.0 to 3.0 m and a rectangular region from 3.0 m to 4.0 m. The total work is

$$W = \frac{1}{2}bh + bh = \frac{1}{2}(3.0\text{ m} - 2.0\text{ m})(-6.0\text{ N}) + (4.0\text{ m} - 3.0\text{ m})(-6.0\text{ N})$$
$$= 9.0\text{ J}$$

Another method to find the area is to use the area of a trapezoid.

$$W = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}\left[(4.0\text{ m} - 2.0\text{ m}) + (4.0\text{ m} - 3.0\text{ m})\right](-6.0\text{ N})$$
$$= 9.0\text{ J}$$

Notice that in the interval 2.0 m to 4.0 m, the area under the curve (and hence the work done) is negative, since the force is negative in that interval.

The physics of hanging roast beef on a spring

2. A 2.50 kg roast of beef is suspended from a vertical spring in a butcher's scale whose spring constant is 200.0 N/m.

a) What is the extension of the spring?

Answer:

The amount the spring extends can be determined from Hooke's law.

At this point, the restoring force of the spring and the force of gravity on the roast balance.

$$\vec{F}_S = -\vec{F}_g$$
$$-k\vec{x} = -m\vec{g}$$
$$x = \frac{mg}{k} = \frac{(2.50\text{ kg})(9.80\text{ N/kg})}{200.0\text{ N/m}} = 0.122\text{ m}$$

The spring is stretched 0.122 m by the weight of the roast.

b) How much energy is stored in the spring?

Answer:

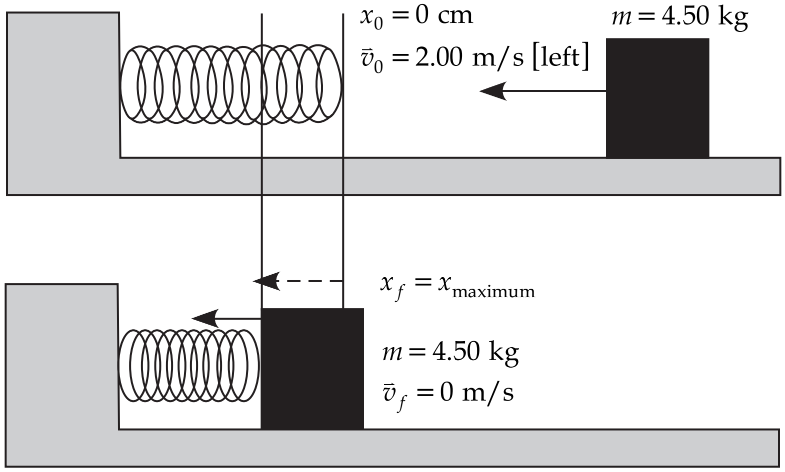
The energy stored in the spring is

$$PE_S = \frac{1}{2}kx^2 = \frac{1}{2}(200.0\text{ N/m})(0.122\text{ m})^2 = 1.49\text{ J.}$$

The physics of a moving crate of apples and a spring

3. A 4.50 kg crate of apples is moving at a horizontal speed of 2.00 m/s towards a spring also mounted horizontally. The linear elastic spring can be compressed 1.00 cm by an applied force of 5.00 N. What is the maximum compression of the spring?

   Answer:



The situation will involve the conservation of mechanical energy between the kinetic energy of the block and the potential energy of the spring. First, however, you must find the spring constant using Hooke's law.

$\vec{F}_S = -k\vec{x}$

The spring constant is $k = \dfrac{F}{x} = \dfrac{5.00 \text{ N}}{0.0100 \text{ m}} = 500 \text{ N/m} = 5.00 \times 10^2 \text{ N/m}.$

When the spring is compressed, the kinetic energy of the apple box is converted into spring potential energy.

Initial kinetic energy of the box = final potential energy of spring at maximum compression

$$\frac{1}{2}m_{\text{box}}{v_0}^2 = \frac{1}{2}k{x_{\text{maximum}}}^2$$

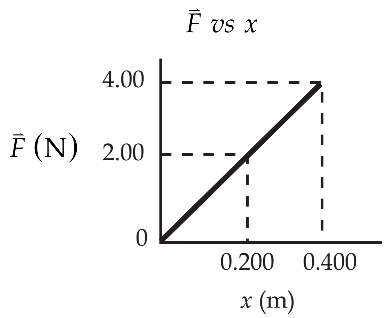
Solve for x_{maximum}

$$x_{\text{maximum}} = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{(4.50 \text{ kg})(2.00 \text{ m/s})^2}{5.00 \times 10^2 \text{ N/m}}} = 0.190 \text{ m}$$

The maximum compression of the spring is a 0.190 m.

The physics of a force-compression graph and spring potential energy

4. The force-compression graph of a hypothetical spring is shown below.



a) How much work is done in compressing the spring 0.400 m?

Answer:

The work done is found by determining the area under the curve from $x = 0$ m to $x = 0.400$ m.

$W = \frac{1}{2}(0.400 \text{ m})(4.00 \text{ N}) = 0.800 \text{ J}$

b) How much potential energy is stored in the spring when it is compressed 0.400 m?

Answer:

The energy stored in the spring is exactly equal to the work done in compressing the spring.

$PE_S = 0.800$ J.

c) How much work is done in compressing the spring from 0.200 m to 0.400 m?

Answer:

The work done in compressing the spring from 0.200 m to 0.400 m is the area of the trapezoid between 0.200 m and 0.400 m.

This is found by taking the area beneath the curve up to 0.400 m (0.800 J) and subtracting the area beneath the curve up to 0.200 m.

This second area is $\frac{1}{2}bh = \frac{1}{2}(0.200 \text{ m})(2.00 \text{ N}) = 0.200$ J.

So the work done in compressing the spring from 0.200 m to 0.400 m is 0.800 J – 0.200 J = 0.600 J.

The physics of a collision between a sliding block and a spring bumper

5. A block of 3.50 kg, sliding along a frictionless horizontal surface, is moving to the left at a speed of 4.00 m/s. The block is incident on to a spring bumper, which has a spring constant of 500.0 N/m.

   a) Calculate the maximum compression of the spring.

      Answer:

      When the spring is compressed, the kinetic energy of the block is converted into spring potential energy.

      Initial kinetic energy of block = final potential energy of spring at maximum compression

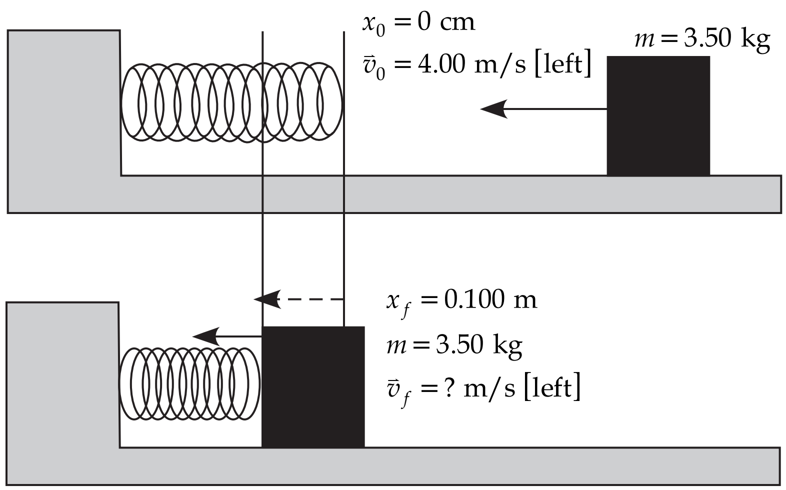
      $$\frac{1}{2}m_{\text{block}}v_0^{\;2} = \frac{1}{2}kx_{\text{maximum}}^{\;\;2}$$

      Solving for x_{maximum}

      $$x_{\text{maximum}} = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{(3.50\text{ kg})(4.00\text{ m/s})^2}{500\text{ N/m}}} = 0.335\text{ m}$$

      The maximum compression of the spring is 0.335 m.

b) Calculate the speed of the block when the spring is compressed 0.100 m.



Answer:

Given:	Spring constant	k = 500.0 N/m
	Initial velocity of block	$\vec{v}_0$ = 4.00 m/s [left]
	Initial compression of spring	x_0= 0 m
	Final compression of spring	x_f= 0.100 m
Unknown:	Final speed of the block	v_f= ? m/s [left]

Equation: Here, you use the conservation of mechanical energy involving kinetic energy and spring potential energy.

$$E_{\text{initial}} = E_{\text{final}}$$

$$KE_0 + PE_{S0} = KE_f + PE_{Sf}$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

Substitute and solve:

$$\frac{1}{2}(3.50 \text{ kg})(4.00 \text{ m/s})^2 + \frac{1}{2}(500.0 \text{ N/m})(0 \text{ m})^2$$

$$= \frac{1}{2}(3.50 \text{ kg})v_f^2 + \frac{1}{2}(500.0 \text{ N/m})(0.100 \text{ m})^2$$

$$28.0 \text{ J} + 0 \text{ J} = 1.75v_f^2 + 2.50 \text{ J}$$

$$1.75v_f^2 = 25.5 \text{ J}$$

$$v_f = 3.82 \text{ m/s}$$

The speed of the block will be 3.82 m/s when the spring is compressed 0.100 m.

c) Calculate the compression of the spring when the block is moving at 2.00 m/s.

Answer:

The analysis for this question is basically the same as for part (b), except that you are asked to find the final compression of the spring.

Given: Spring constant $k = 500.0$ N/m

Initial velocity of block $\vec{v}_0 = 4.00$ m/s [left]

Initial compression of spring $x_0 = 0$ m

Final speed of block $v_f = 2.00$ m/s

Unknown: Final compression of spring $x_f = ?$ m

Equation: Here you use the conservation of mechanical energy involving kinetic energy and spring potential energy.

$$E_{\text{initial}} = E_{\text{final}}$$

$$KE_0 + PE_{S0} = KE_f + PE_{Sf}$$

$$\frac{1}{2}mv_0^{\,2} + \frac{1}{2}kx_0^{\,2} = \frac{1}{2}mv_f^{\,2} + \frac{1}{2}kx_f^{\,2}$$

Substitute and solve:

$$\frac{1}{2}(3.50\text{ kg})(4.00\text{ m/s})^2 + \frac{1}{2}(500.0\text{ N/m})(0\text{ m})^2$$

$$= \frac{1}{2}(3.50\text{ kg})(2.00\text{ m/s})^2 + \frac{1}{2}(500.0\text{ N/m})\left(x_f\right)^2$$

$$28.0\text{ J} + 0\text{ J} = 7.00\text{ J} + 250\left(x_f\right)^2$$

$$x_f = 0.290\text{ m}$$

When the speed of the block is 2.00 m/s (this can be motion to the right or left), the spring is compressed 0.290 m.

The physics of weighing a melon

6. A scale used to measure the weight of a 1.25 kg melon stretches 2.75 cm under the weight of the melon. What is the spring constant for the spring inside the scale?

   Answer:

   Given: Extension of spring $\vec{x}$ = 2.75cm = 0.0275 m

   Mass m = 1.25 kg

   Weight of mass $\vec{F}_g = m\vec{g}$ = (1.25 kg)(9.80 m/s^2)
   = 12.2 N [down]

   Restoring force of spring $\vec{F}_S$ = 12.2 N [up]

   Unknown: Spring constant k = ? N/m

   Equation: $\vec{F}_S = -k\vec{x}$

   Substitute and solve:

   $$\vec{F}_S = -k\vec{x}$$

   $$12.2 \text{ N } [\text{up}] = -(0.0275 \text{ m } [\text{down}])k$$

   $$k = 444 \text{ N/m}$$

   The spring constant is 444 N/m.

GRADE 12 PHYSICS (40S)

Module 5: Momentum

This module contains the following:

- Introduction to Module 5
- Lesson 1: Impulse, Momentum, and the Impulse-Momentum Theorem
- Lesson 2: Conservation of Momentum in Explosions and Collisions
- Lesson 3: Video Laboratory Activity: A Collision in Two Dimensions
- Lesson 4: Conservation of Momentum in Two Dimensions
- Module 5 Summary

Module 5: Momentum

Introduction to Module 5

In the 20th century, scientists learned that momentum is one of the premier notions of physics. When there are no external forces acting on a system, its momentum remains unchanged and we have one of the great conservation laws of physics, the law of conservation of linear momentum. It can even be argued that Newton's laws are the result of the conservation of momentum.

There are four lessons in this module dealing with momentum.

Lesson 1: Impulse, Momentum, and the Impulse-Momentum Theorem will define what impulse and momentum are and how they are related in the impulse-momentum theorem.

Lesson 2: Conservation of Momentum in Explosions and Collisions discusses momentum as applied to two types of collisions: explosions and head-on collisions.

Lesson 3: Video Laboratory Activity: A Collision in Two Dimensions investigates the momentum of a system of objects before and after a collision, which results in objects moving in two dimensions.

Lesson 4: Conservation of Momentum in Two Dimensions discusses the conservation of momentum in two dimensions, which requires a review of the component method of adding vectors.

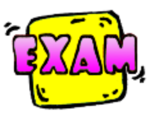
The common thread through the last two modules is the action of forces and the analysis of the changes that result. If your focus is the action of a force through a distance, then this leads to work and energy. If your focus is the action of a force over time, the results lead to impulse and momentum.

Assignments in Module 5

When you have completed the assignments for Module 5, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
1	Assignment 5.1	Calculating Impulse, Momentum, and Force
2	Assignment 5.2	Conservation of Linear Momentum
3	Assignment 5.3	Video Laboratory Activity: A Collision in Two Dimensions

Writing Your Midterm Examination



You will write the midterm examination when you have completed Module 5 of this course. The midterm examination is based on Modules 1 to 5, and is worth 20 percent of your final mark in the course. To do well on the midterm examination, you should review all the work you complete in Modules 1 to 5, including all the learning activities and assignments. You will write the midterm examination under supervision.

As you work through this course, remember that your learning partner and your tutor/marker are available to help you if you have questions or need assistance with any aspect of the course.

Lesson 1: Impulse, Momentum, and the Impulse-Momentum Theorem (1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the term "impulse" and calculate impulse
- ❑ determine the impulse imparted to an object given the force-time graph
- ❑ define the term "momentum" and calculate momentum
- ❑ derive the impulse-momentum theorem from Newton's second law
- ❑ solve practical problems using the impulse-momentum theorem

Key Words

impulse
momentum

Introduction

During the study of Grade 10 Science, you were introduced to the concepts of impulse and momentum and how they affected events during an automobile collision. Passengers were in motion and possessed momentum. In car crashes, this momentum disappeared by the application of forces over time, which you called "impulse." The size of the forces determined whether a passenger was able to stop safely or if the passenger was injured.

In the lessons of this module, you will apply the ideas of impulse and momentum to many different situations, such as motion from rocket engines being propelled by the exhaust, to many different sports situations where a ball is struck, to the common situation of rain falling on the roof of an automobile. The analyses of these situations will be performed quantitatively.

Impulse

There are many situations in which the force acting on an object varies with time.

For example, when a baseball is struck by a bat, the force the bat exerts on the baseball varies with the time of contact. The magnitude of the force is zero before the bat touches the ball. During the time the bat and ball are colliding, the force rises to a maximum and then returns to zero at the time when the ball leaves the bat. The time interval during which the bat and ball are in contact is quite short, being only a few thousandths of a second, and the maximum force is often quite large, exceeding thousands of newtons. Similar effects occur in other sports, such as a golf ball being struck by a golf club and a soccer ball being kicked.

If a baseball is to be hit well, both the size of the force and the time of contact are important. When a large average force acts on a ball and the time that the bat and ball are in contact is also large, the ball is hit solidly. The baseball leaves the bat with a large velocity.

These two factors together, the size of the average force and the time of contact, result in the quantity called **impulse**.

Impulse

The impulse, $\vec{J}$, of a force is the product of the average force, $\vec{F}_{\text{avg}}$, exerted during an interaction and the time interval, Δt, during which the force is exerted.

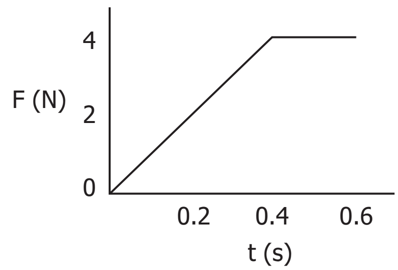
$$\vec{J} = \vec{F}_{\text{avg}} \Delta t$$

Quantity	Symbol	Unit
Impulse	$\vec{J}$	newton·second (N·s)
Average force	$\vec{F}_{\text{avg}}$	newton (N)
Time interval	Δt	seconds (s)

Impulse is a vector quantity. Its direction will be the same as the direction in which the average force is exerted. The SI unit of impulse is the newton·second (N·s).

Example 1: Impulse from a Force-Time Graph

When given a force-time graph, it is possible to determine the impulse acting on an object by finding the area under the curve. For example, the graph below shows a possible situation where force varies with time. Between 0 and 0.4 s, the force rises from 0 N to 4 N. Then between 0.4 s and 0.6 s, the force remains at a constant value of 4 N.



To determine the impulse for the graph above, find the area under the curve.

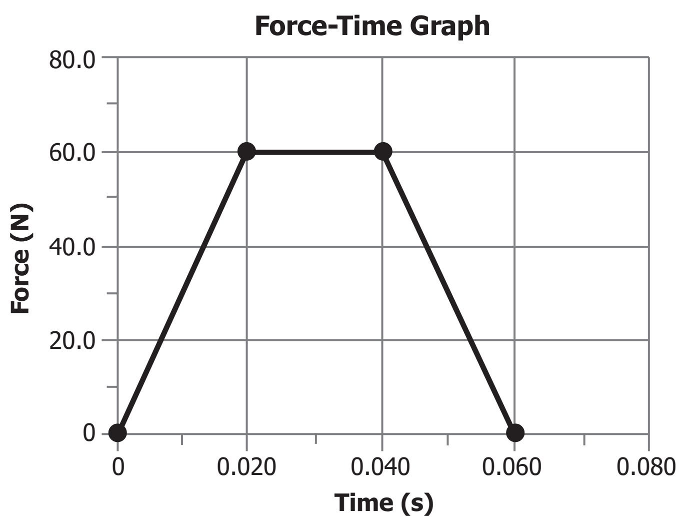
$$\vec{J} = \frac{1}{2}(4\text{ N})(0.4\text{ s}) + (4\text{ N})(0.6\text{ s} - 0.4\text{ s}) = 1.6\text{ N}\cdot\text{s}$$

Learning Activity 5.1

Calculating Impulse

Answer the following questions to check your understanding of impulse. An answer key is provided at the end of Module 5 so that you may check your work.

1. A car swerves out of control and hits a guardrail. It exerts a force of 4.00×10^3 N on the guardrail. If the guardrail exerts an impulse of -5.00×10^2 N·s, how long does it take for the car to come to a stop?
2. A water skier lets go of the tow rope and coasts to a stop. If the water exerts an average force of 2.80×10^2 N on him and he stops in 5.00 s, what is the impulse that the water exerts on him?
3. Calculate the impulse applied according to the force-time graph given below.



Momentum

When a force is exerted on an object, the object responds to the size of the impulse applied. A small impulse applied to an object, such as putting a golf ball, results in a small response in the ball. The ball rolls slowly across the green. On the other hand, when a large impulse is applied to the golf ball, such as using the driver to propel the ball off the tee, the response is much larger. The ball leaves the club travelling with a large velocity.

If, instead of putting a golf ball, you tried to putt a soccer ball, applying the same impulse as was applied to the golf ball, the response of the soccer ball would be considerably smaller. The soccer ball would exit the collision moving much more slowly than the golf ball. So, you can see, both mass and velocity play a role in how an object responds to a given impulse.

Newton combined a moving object's mass and its velocity in an expression he called the "quantity of motion." We now call this quantity **momentum**, and it is given the symbol $\vec{p}$.

Momentum

Momentum has been called "the quantity of motion." The linear momentum of an object, $\vec{p}$, is the product of the object's mass, m, and the velocity, $\vec{v}$.

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	Unit
Momentum	$\vec{p}$	kilogram·metre/second (kg·m/s)
Mass	m	kilogram (kg)
Velocity	$\vec{v}$	metres/second (m/s)

Linear momentum is a vector quantity. Its direction will be the same as the direction of the velocity. The SI unit of momentum is the kilogram·metre/second (kg·m/s).

For example, if an object of mass 10.0 kg is moving at a velocity of 2.00 m/s [up], then the momentum of this mass is $\vec{p} = m\vec{v}$ = (10.0 kg)(2.00 m/s [up]) = 20.0 kg·m/s [up].

Learning Activity 5.2

Momentum

Answer the following questions to check your understanding of momentum. An answer key is provided at the end of Module 5 so that you may check your work.

1. Which has more momentum: a golf ball of mass 45.5 g travelling at 60.0 m/s, or a baseball of mass 145 g travelling at 40.0 m/s?
2. In its orbit around the Sun, Earth travels at about 29.9 km/s. The mass of Earth is 5.98×10^{24} kg.
   a) Calculate the size of the linear momentum of Earth.
   b) Is the linear momentum of Earth constant in size? Is the linear momentum constant in direction? Describe the force that applies the impulse to cause the change in momentum of Earth.
3. Two identical golf balls have the same speed, one travelling north and one travelling south. Do these golf balls have the same momentum? Explain.

Derivation of Impulse-Momentum Theorem

Newton's Second Law of Motion can now be used to reveal a relationship between impulse and momentum.

From the definition of acceleration, you know that when a mass changes velocity from an initial value of $\vec{v}_0$ to a final value of $\vec{v}_f$ during a time interval of Δt, the acceleration of the mass can be determined as

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

According to Newton's second law, $\vec{F} = m\vec{a}$. This can be rewritten as

$$\vec{F} = m\frac{\vec{v}_f - \vec{v}_0}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_0}{\Delta t}$$

In the preceding expression, $m\vec{v}_f - m\vec{v}_0$ represents the final momentum minus the initial momentum, which is the change in momentum. Thus, the average force is given by the change in momentum per unit time.

Finally, multiplying each side of the equation by Δt results in what is known as the **impulse-momentum theorem**.

Impulse-Momentum Theorem
The **impulse-momentum theorem** states that when a net force, $\vec{F}$, acts on an object, the **impulse** is equal to the **change in momentum** of the object.

$$\vec{F}\Delta t = m\Delta\vec{v} = m\vec{v}_f - m\vec{v}_0$$

Quantity	Symbol	Unit
Force	$\vec{F}$	newton (N)
Time interval	Δt	second (s)
Mass	m	kilogram (kg)
Initial velocity	$\vec{v}_0$	metres/second (m/s)
Final velocity	$\vec{v}_f$	metres/second (m/s)

Note: Impulse is a vector quality. The direction of the impulse is the same as the direction of the **change in** the momentum.

During a collision, it is often difficult to measure the average force, $\vec{F}$, so it is not easy to determine the impulse, $\vec{F}\Delta t$, directly. On the other hand, it is usually straightforward to measure the mass and velocity of an object, so that its momentum just after and before the collision, $m\vec{v}_f$ and $m\vec{v}_i$, can be found.

Thus, the **impulse-momentum theorem** allows us to gain information about the impulse directly by simply **measuring the change in momentum** that the **impulse** causes. Then, with a knowledge of the contact time, Δt, you can evaluate the average force.

Example 2: Impulse and Momentum

As an example of applying our study of impulse and momentum, we will examine the situation of a well-hit baseball.

The baseball, of mass 0.140 kg, approaches a bat with an initial velocity of $\vec{v}_0$ = –42.0 m/s. During the collision, the bat applies a force to the ball, and the ball departs from the bat with a final velocity of $\vec{v}_f$ = +62.0 m/s. Determine the impulse applied to the ball by the bat if the time of contact was 1.60×10^{-3} s. Consider the force of gravity on the ball to be negligible compared to the force of interaction between the bat and the ball.

In hitting the ball, the bat imparts an impulse to it. You cannot use the equation $\vec{J} = \vec{F}\Delta t$ to determine the impulse since the average force is not known. You can find the impulse, however, by turning to the impulse-momentum theorem, which states that the impulse is equal to the ball's final momentum minus its initial momentum.

The initial momentum of the ball is

$$\vec{p}_0 = m\vec{v}_0 = (0.140 \text{ kg})(-42.0 \text{ m/s}) = -5.88 \text{ kg}\cdot\text{m/s}$$

The final momentum of the ball is

$$\vec{p}_f = m\vec{v}_f = (0.140 \text{ kg})(+62.0 \text{ m/s}) = +8.68 \text{ kg}\cdot\text{m/s}$$

According to the impulse-momentum theorem, the impulse applied to the ball is

$$\begin{aligned}\vec{F}\Delta t = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_0 &= +8.68 \text{ kg}\cdot\text{m/s} - (-5.88 \text{ kg}\cdot\text{m/s}) \\ &= +14.56 \text{ kg}\cdot\text{m/s or } +14.56 \text{ N}\cdot\text{s}\end{aligned}$$

Since the contact time between the ball and the bat is $\Delta t = 1.60 \times 10^{-3}$ s, you can determine the average force acting on the ball.

$$\vec{J} = \vec{F}\Delta t \quad \text{or}$$

$$\vec{F} = \frac{\vec{J}}{\Delta t} = \frac{14.56 \text{ N}\cdot\text{s}}{1.60 \times 10^{-3} \text{ s}} = +9100 \text{ N}$$

The force is $+9.10 \times 10^3$ N.

The force is positive, indicating that it points opposite to the velocity of the approaching ball. A force of 9100 N corresponds to the weight of ten 91 kg people. Such a large force is necessary to change the ball's momentum during the brief contact time.

Learning Activity 5.3

Impulse and Momentum

There are eight practice questions in this learning activity. An answer key is available at the end of Module 5 for you to check your work after you have answered the questions.

1. A 10.0 kg mass is at rest when it is acted upon by an unknown force for 1.50 seconds. If the velocity of the mass is 5.00 m/s after 1.50 seconds, what is the size of the force?

The physics of catching a freely falling parachutist

2. A girl of mass 80.0 kg jumps out of an airplane and, after free-falling for several minutes, pulls the cord on a parachute.
   a) What impulse is applied to the girl if the parachute slows her from 100.0 km/h to 10.2 km/h?
   b) If this change in velocity occurred during 11.0 seconds, what average force was applied to the girl?

The physics of catching a hockey puck

3. a) A hockey goalie catches the puck (mass = 0.170 kg) in his glove and slows it to a stop from 20.0 m/s during 0.100 seconds. What average force was exerted on his glove by the puck?
   b) By moving his hand back when catching the puck, the goalie can increase the time required to stop the puck to 0.500 seconds. What average force does the puck now exert on his hand?

The physics of catching a baseball

4. Why does a fielder draw her hand back as she catches a baseball?

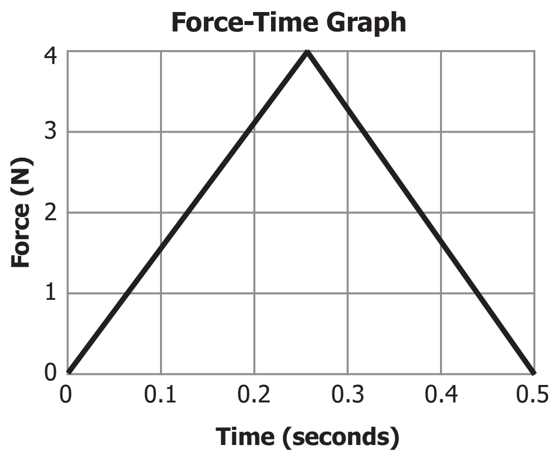
The physics of catching an egg without breaking it

5. You are participating in a Physics Olympics event called the egg toss. How could you improve your chances of catching a tossed egg without breaking it?

(continued)

The physics of impulse in sports

6. What is the impulse exerted in each of the following cases?
   a) A hockey stick exerting an average force of 120.0 N on a puck during the 0.0500 s they are in contact
   b) A force of 25.2 N east on a bowling ball for 1.05 s
   c) A billiard ball bouncing off a cushion if the force-time graph of the collision appears as below



The physics of momentum and a freight train and an automobile

7. A freight train moves due north with a speed of 1.40 m/s. The mass of the train is 4.50×10^5 kg. How fast would a 1.80×10^3 kg automobile have to be moving north to have the same momentum as the train?

The physics of rain and the force on the roof of a car

8. During a rainstorm, rain comes straight down with a velocity of $\vec{v} = -15.0$ m/s and hits the roof of the car perpendicularly. The mass of the rain that strikes the car roof is 0.0600 kg/s. Assuming that the rain comes to rest upon striking the car roof ($\vec{v}_f = 0$), find the average force exerted by the car roof on the rain.

Lesson Summary

In this lesson, the terms "impulse" and "momentum" were defined. From Newton's second law, you derived the impulse-momentum theorem. This powerful theorem allows us to understand many examples of motion that involve an applied force over a time to change the momentum and velocity of an object. In the following lesson, you will build on these important ideas and develop an important conservation law in physics.

The impulse, $\vec{J}$, of a force is the product of the average force, $\vec{F}_{\text{avg}}$, and the time interval, Δt, during which the force acts.

$$\vec{J} = \vec{F}_{\text{avg}} \Delta t$$

Impulse is a vector quantity and has the same direction as the average force.

The SI unit of impulse is the newton $\cdot$ second ($\text{N} \cdot \text{s}$).

When given a force-time graph, it is possible to determine the impulse acting on an object by finding the area under the curve.

Momentum is called quantity of motion.

The **linear momentum** of an object, $\vec{p}$, is the product of the object's mass, m, and the velocity, $\vec{v}$.

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity that points in the same direction as velocity.

The SI unit of momentum is the kilogram $\cdot$ metre/second ($\text{kg} \cdot \text{m/s}$).

The **impulse-momentum theorem** states that when a net force, $\vec{F}$, acts on an object, the **impulse** is equal to the **change in momentum** of the object.

$$\vec{F}\Delta t = m\Delta\vec{v} = m\vec{v}_f - m\vec{v}_0$$

You should be able to derive for the impulse momentum theorem from Newton's second law: $\vec{F} = m\vec{a}$.

The impulse-momentum theorem can be applied to many situations, especially situations involving sports where an object is struck by another over a very short time.

NOTES

Assignment 5.1

Calculating Impulse, Momentum, and Force (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Module 5, after you have completed Module 5.

The physics of kicking a soccer ball

A soccer player, kicking a soccer ball, gives the ball a velocity of +26.8 m/s. The mass of the ball is 0.425 kg, and the duration of the impact is 1.05×10^{-3} s.

a) What is the momentum of the soccer ball after it has been hit?

b) What is the change in momentum of the ball?

c) What is the impulse imparted to the ball?

(continued)

d) What is the average force applied to the ball by the foot of the soccer player?

e) If the ball is to have a greater velocity after being struck, should the contact time with the foot be shorter or longer? Explain your reasoning.

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each of parts (a) to (e)

Video - You Can't Run From Momentum! (a momentum introduction)

This video introduces the concept of momentum, its equation, its units and its vector nature.

https://youtu.be/K-lH-DoD6Tk

Video - Impulse, momentum and Newton's Third Law

This video gives a short introduction to impulse and change in momentum. Starting with the equation for momentum and Newton’s Second Law the Impulse Momentum Theorem is derived.

This theorem is used to determine the average force used to kick a soccer ball.

https://youtu.be/3g4v8x7xggU

Video - Physics Lab- Momentum and Impulse- Egg Baseball

This demonstration shows the effect of stopping a moving egg in a short time interval and over a long time interval.

https://youtu.be/lPzGSjIoW7c

Video - Momentum & Impulse

This video introduces the concepts of impulse and momentum and the Impulse-Momentum Theorem. A number of sample calculations are completed using these concepts.

https://youtu.be/XSR7khMBW64

Video - Calculating the Force of Impact when Stepping off a Wall

This is a slightly more complicated problem. This problem requires 2 steps – finding the velocity when the person contacts the ground (kinematics) and then find the average force using the Impulse-Momentum equation.

https://youtu.be/ILIFo2X7EUY

Video - Impulse Introduction or If You Don't Bend Your Knees When Stepping off a Wall

The average force is calculated when the instructor steps off the wall and lands without bending his knees.

https://youtu.be/6myC6S2TNHw

Assignment 5.1

Calculating Impulse, Momentum, and Force (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Module 5, after you have completed Module 5.

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a) What is the momentum of the soccer ball after it has been hit?

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c) What is the impulse imparted to the ball?

(continued)

d) What is the average force applied to the ball by the foot of the soccer player?

e) If the ball is to have a greater velocity after being struck, should the contact time with the foot be shorter or longer? Explain your reasoning.

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each of parts (a) to (e)

Lesson 2: Conservation of Momentum in Explosions and Collisions (2 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ define the terms "system" and "isolated system"
- ❑ state the law of conservation of momentum, and use the law in problem solving
- ❑ calculate the momentum and velocity of masses that result from an explosion
- ❑ calculate the momentum and velocity of masses that result from a head-on collision where objects travel along the same straight line

Key Words

head-on collision	external force
internal force	isolated system
explosion	

Introduction

In this lesson, you learn about a very important law in physics: the law of conservation of momentum. This law was experimentally observed to be true in the 17th century, and allowed Newton to formulate his third law. The law is applicable under special circumstances. You will apply this law specifically to the situation where a mass or combination of masses are at rest, and then separate from each other. This is commonly called an "explosion" situation. The second application of this law will be for head-on collisions where objects move along the same straight line.

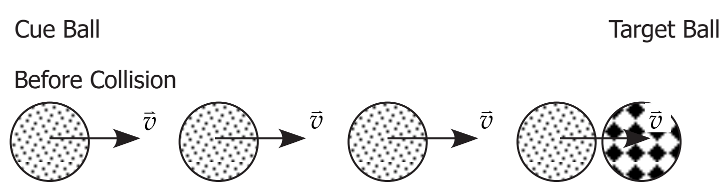
Setting the Stage

Let's consider some examples of explosions and head-on collisions and the motions that result from them.

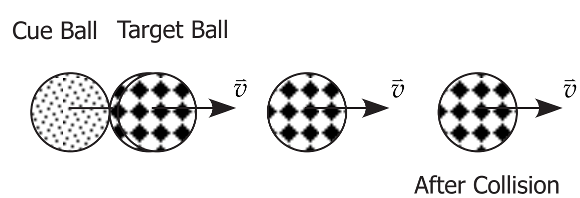
The first example is a cannon firing a cannonball. Initially, the cannon and the cannonball are both at rest. During the explosion, the cannonball is pushed in one direction by the gases expanding in the barrel of the cannon. The cannon is pushed in the other direction. The cannon is said to recoil. After the explosion, both the cannonball and the cannon are moving and therefore have momentum. These momenta are opposite in direction. How does the total momentum of the cannon and cannonball before the explosion compare to the total momentum after the explosion?

A second example involves two skaters standing facing each other on the ice. If one skater gives the other skater a push, both skaters will slide backwards. During the push (explosion), the first skater exerts a force on the second skater and, according to Newton's third law, the second skater exerts an equal but opposite force on the first skater. The forces acting on each skater provide an impulse that gives each skater a momentum. These momenta are opposite in direction. How does the total momentum of the two skaters before the explosion compare to the total momentum after the explosion?

If you've played pool or billiards, you have seen the cue ball collide with other target balls. In some of the collisions, the cue ball struck the target ball head-on.



Cue ball is moving with velocity $\vec{v}$ [right]. Target ball is stationary.



Cue ball is stationary. Target ball moves with velocity $\vec{v}$ [right].

In this case, the cue ball stopped, and the target ball proceeded along the initial line of motion of the cue ball. Before the collision, the momentum of the system was in the cue ball, since only it was moving. After the collision, the momentum of the system was in the target ball, since only it was moving. How does the total momentum of the cue ball and target ball before the collision compare to the total momentum after the collision?

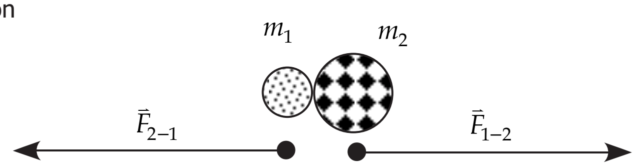
Law of Conservation of Momentum

During the 17th century, Newton and others before him had measured the momentum of colliding objects before and after a collision and discovered a strange phenomenon. They discovered that the total momentum of the colliding objects was the same after the collision as it was before.

For example, suppose two masses approach each other, as shown below.

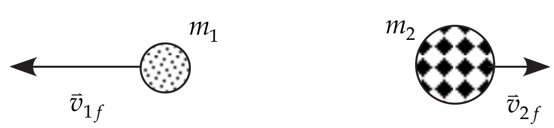
Before Collision

During Collision



During the collision, the objects exert forces on each other. According to Newton's third law, these forces are equal but opposite.

After Collision



After the collision, the masses have different velocities than they initially possessed.

How can we link the initial situation to the final situation? It turns out you can use Newton's third law and the impulse-momentum theorem to perform the task.

1. Write the impulse-momentum theorem
   for object 1 $\vec{F}_{2-1}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$
   and object 2 $\vec{F}_{1-2}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$
2. Apply Newton's third law to the forces objects 1 and 2 exert on each other. $\vec{F}_{2-1} = -\vec{F}_{1-2}$
3. But the objects exert the forces for the same time interval Δt. Multiply both force by Δt. $\vec{F}_{2-1}\Delta t = -\vec{F}_{1-2}\Delta t$
4. Substitute the expressions for the momentum changes from step 1 into the equation in step 3. $m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -\left(m_2\vec{v}_{2f} - m_2\vec{v}_{2i}\right)$
5. Rearrange the terms, placing the initial momenta on the left side and the final momenta on the right side of the equation. $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

You can see that the left side of the equation represents the **total initial momentum** and the right side represents the **total final momentum**.

In other words, total initial momentum equals total final momentum for an isolated system.

$$\vec{p}_{\text{total1}} = \vec{p}_{\text{total2}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{\mathbf{1}f} + \vec{p}_{2f}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

The two masses in this example can be referred to as a "system." A **system** can simply mean a set of objects interacting with each other.

> An **isolated system** is one in which the only forces present are those between the objects of the system.

For example, for the two masses above, the only forces acting on them are the equal and opposite forces that they exert on each other when they collide, $\vec{F}_{2-1}$ and $\vec{F}_{1-2}$. There are no external net forces. If external forces—that is, forces outside the system—do act, then momentum may not be conserved. For example, if the system above consisted of two balls rolling across a pool table, an external force of friction is acting on it and its momentum changes. Eventually, the two balls will come to rest.

Law of Conservation of Momentum

The **law of conservation of momentum** states that if the net external force acting on an object, or system of objects, is zero, then the total momentum of the object or system of objects before a collision is equal to the total momentum of the object or system of objects after the collision.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Quantity	**Symbol**	**Unit**
Mass of object 1	m_1	kilogram (kg)
Mass of object 2	m_2	kilogram (kg)
Initial velocity of object 1	$\vec{v}_{1i}$	metres/second (m/s)
Initial velocity of object 2	$\vec{v}_{2i}$	metres/second (m/s)
Final velocity of object 1	$\vec{v}_{1f}$	metres/second (m/s)
Final velocity of object 2	$\vec{v}_{2f}$	metres/second (m/s)

It is sometimes easier to remember the law of conservation of momentum as just:

total momentum before collision = total momentum after collision.

Then you can fill in the expressions for momentum, as given below.

$$\vec{p}_{\text{total}1} = \vec{p}_{\text{total}2}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{\mathbf{1}f} + \vec{p}_{2f}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Isolating a System

You know that if you're playing billiards and there is a collision between the cue ball and target ball, eventually the cue ball and target ball will both come to rest. There obviously was a loss in the momentum of the system. So how does the law of conservation of momentum apply in this situation?

As mentioned earlier, you have to isolate the system. Isolating the system can be done in two ways:

1. Isolating the system from external forces.

   Friction is the culprit that removes momentum from a system. The force of friction applies an impulse to the objects in the system $\left(\vec{J} = \vec{F}_{\text{FRICTION}}\Delta t\right)$. One way of reducing the impulse of friction is to reduce the size of the force of friction. So systems in which friction has been reduced or eliminated will exhibit the law of conservation of momentum very closely.

$\vec{J}_{\text{external}} = (0\text{ N})\Delta t = 0\text{ N}\cdot\text{s}$, so momentum is conserved.

Examples include devices like air tracks where gliders float on a cushion of air or in dry-ice pucks. The drag of friction between surfaces is basically eliminated. You may have seen something similar in air hockey games, where the puck floats on a layer of air.

2. Isolating the system in time

   Another way to reduce the impulse applied by external forces like friction is to reduce the time interval over which the external force is applied. So, in a case of collisions between billiard balls, if we consider it a very short time interval before and after the collision, during that interval a very small impulse (so small it would be negligible) would have been applied by the external forces, resulting in the momentum of the system remaining constant.

   $\vec{J}_{\text{external}} = (\vec{F}_{\text{external}})(0\text{ s}) = 0\text{ N}\cdot\text{s}$

   Again, the momentum is conserved.

Example 1: Conservation of Momentum and Explosions

One of the special cases where the law of conservation of momentum applies is one where an object explodes. In such a case, the initial momentum is the momentum of the object before the explosion. If the object is initially at rest, then the momentum of the object is zero. If the object is moving, then the momentum of the object is its mass times its velocity.

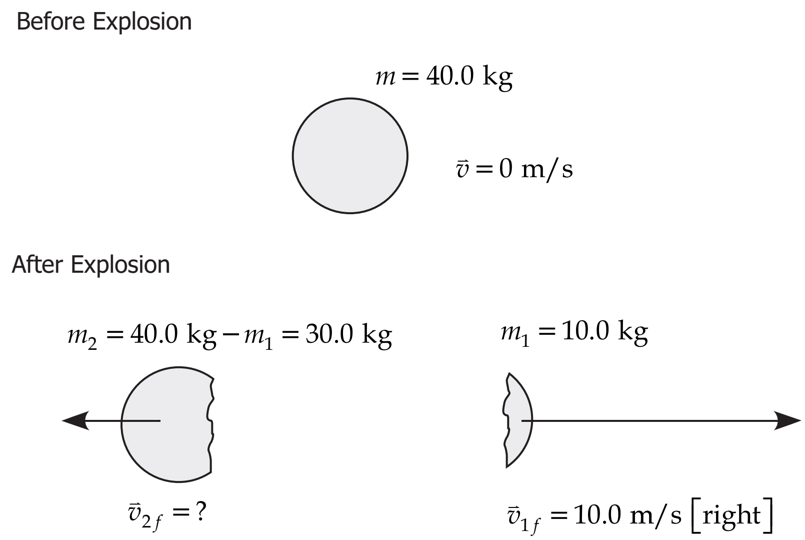
After the object explodes, the sum of the momenta of the individual pieces is equal to the momentum of the original piece.

Part 1:

A mass of 40.0 kg is initially at rest. This mass now explodes into two pieces so that one piece of mass 10.0 kg moves to the right at 10.0 m/s. What is the final velocity of the second piece?

Given: These problems involving conservation of momentum require that you are very careful in keeping track of the information. The given information should consist of a sketch representing the conditions just before the interaction plus a second sketch representing the conditions just after the interaction.

Let right be the positive direction.



Unknown: Final velocity of mass 2 $\quad \vec{v}_{2f} = ?$

Equation: This is a situation where you can use the law of conservation of momentum. You can say that the initial momentum of the system is $0 \text{ kg} \cdot \text{m/s}$.

$$\vec{p}_{\text{total1}} = \vec{p}_{\text{total2}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{\mathbf{1}f} + \vec{p}_{2f}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Substitute and solve:

$$0 \text{ kg} \cdot \text{m/s} = (10.0 \text{ kg})(+10.0 \text{ m/s}) + (30.0 \text{ kg})\vec{v}_{2f}$$
$$\vec{v}_{2f} = \frac{-100 \text{ kg} \cdot \text{m/s}}{30.0 \text{ kg}} = -3.33 \text{ m/s}$$

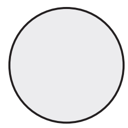
The second piece (30.0 kg) moves to the left at 3.33 m/s.

Part 2:

Suppose the original 40.0 kg mass was moving to the right at 2.00 m/s instead of being at rest. This mass explodes into two pieces so that the 10.0 kg mass moves to the right at 4.00 m/s.

Given: Before Explosion

$m = 40.0 \text{ kg}$



$\vec{v} = +2.00 \text{ m/s}$

After Explosion

$m_2 = 40.0 \text{ kg} - m_1 = 30.0 \text{ kg}$ $\quad$ $m_1 = 10.0 \text{ kg}$

$\vec{v}_{2f} = ?$ $\quad$ $\vec{v}_{1f} = 4.00 \text{ m/s [right]}$

The initial momentum is

$$\vec{p}_{\text{total initial}} = m\vec{v} = (40.0 \text{ kg})(+2.00 \text{ m/s}) = +80.0 \text{ kg}\cdot\text{m/s}$$

Unknown: Final velocity of mass 2 $\quad \vec{v}_{2f} = ?$

Equation: You use the conservation of momentum to determine the velocity of the second piece.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Substitute and solve:

$$+80.0 \text{ kg}\cdot\text{m/s} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$+80.0 \text{ kg}\cdot\text{m/s} = (10.0 \text{ kg})(+4.00 \text{ m/s}) + (30.0 \text{ kg})\vec{v}_{2f}$$

$$\vec{v}_{2f} = \frac{+80.0 \text{ kg}\cdot\text{m/s} - (10.0 \text{ kg})(+4.00 \text{ m/s})}{30.0 \text{ kg}}$$

$$= +1.33 \text{ m/s}$$

The 30.0 kg mass moves to the right at 1.33 m/s.

Applications of the principles of conservation of momentum could be recoil velocities of cannons while firing projectiles, or throwing objects from masses such as boats, which can recoil. These same principles can be applied to more than two pieces after an explosion. They can also be applied to situations where an object explodes so that pieces move in two or three dimensions.

Learning Activity 5.4

Conservation of Momentum during Explosions

Solve the following problems to check your understanding of the conservation of momentum during explosions. You may check your work by using the answer key provided at the end of Module 5.

1. In Times Square in New York City, people celebrate New Year's Eve. Some just stand around, but many more move about randomly. Consider a system comprised of all of these people. Approximately, what is the total linear momentum of this system at any given instant? Justify your answer.
2. With the engines off, a spaceship is coasting at a velocity of $+2.30 \times 10^2$ m/s through outer space. It fires a rocket straight ahead at an enemy vessel. The mass of the rocket is 1.30×10^3 kg, and the mass of the spaceship (not including the rocket) is 4.00×10^6 kg. The firing of the rocket brings the spaceship to a halt. What is the velocity of the rocket?

Example 2: Conservation of Momentum in Head-on Collisions

Let's consider a head-on collision between two curling rocks.

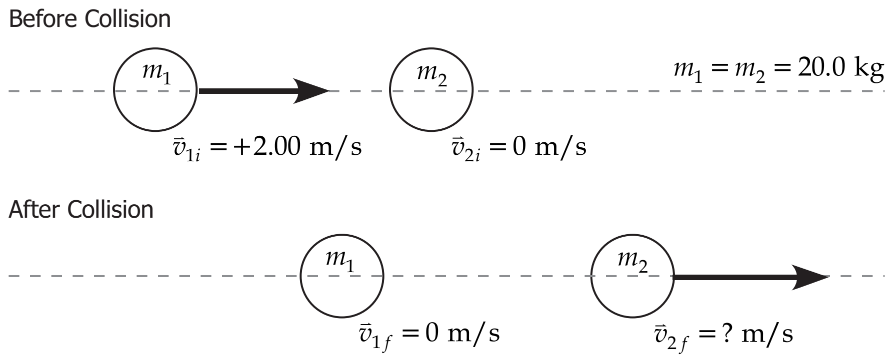
During a curling match, a curler throws a curling rock, mass 20.0 kg, with a speed of 2.00 m/s. This rock collides head-on with the stationary target stone, which also has a mass of 20.0 kg. After the collision, which lasted 0.0250 s, the first rock is stationary.

a) What is the final velocity of the target stone?
b) What is the change in momentum of the target stone?
c) What impulse was applied to the target stone?
d) What was the force of interaction between these two rocks?

Given: Let right be the positive direction.

Let the rock thrown by the curler = mass 1.

Let the target stone = mass 2.



a) Unknown: Final velocity of the target stone: $\vec{v}_{2f} = ?$ m/s

Equation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Substitute and solve: $(20.0 \text{ kg})(+2.00 \text{ m/s}) + 0 \text{ kg}\cdot\text{m/s}$

$= 0 \text{ kg}\cdot\text{m/s} + (20.0 \text{ kg})\vec{v}_{2f}$

$\vec{v}_{2f} = +2.00 \text{ m/s}$

The final velocity of the target stone is 2.00 m/s to the right. This should make sense since when the first rock comes to rest, all of the momentum should be in the second stone. Since the masses of the two stones are equal, the target stone should acquire the initial velocity of the first stone.

b) The target stone was initially at rest and therefore had no momentum. The change in momentum of the target stone should just be the final momentum of the target stone.

$$\Delta\vec{p}_2 = \vec{p}_{2f} - 0 \text{ kg}\cdot\text{m/s} = m_2\vec{v}_{2f} = (20.0 \text{ kg})(+2.00 \text{ m/s}) = +40.0 \text{ kg}\cdot\text{m/s}$$

c) The impulse applied to the target stone should just be its change in momentum.

$$\vec{J} = \Delta\vec{p}_2 = +40.0 \text{ N}\cdot\text{s}$$

d) Since you know the impulse applied to the target stone and the time interval Δt = 0.0250 s, you can find the force that acted on the target stone.

$$+40.0 \text{ N}\cdot\text{s} = \vec{F}(0.0250 \text{ s})$$

$$\vec{F} = +1600 \text{ N}$$

The force acting on the target stone was 1.60×10^3 N to the right. Using Newton's third law, the force the target stone exerted on the first rock was 1.60×10^3 N to the left.

Example 3: Conservation of Momentum in Head-on Collisions Where Colliding Objects Stick Together After the Collision

Let's consider a collision between a car and truck on an icy highway.

A car of mass 1250 kg is travelling at 14.0 m/s [west]. It collides head-on with a truck of mass 5680 kg travelling at 10.0 m/s [east]. After the collision, the two vehicles proceed, stuck together, sliding along the highway.

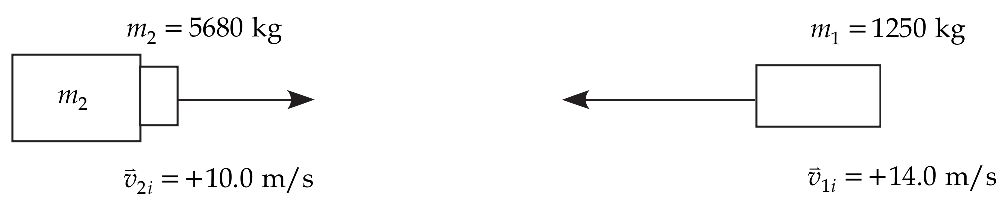
a) What is the final velocity of these two vehicles locked together?
b) What is the change in momentum of the truck?
c) What is the change in momentum of the car?
d) What impulse is applied to the truck during the collision?
e) What impulse is applied to the car during the collision?

Let east be the positive direction.

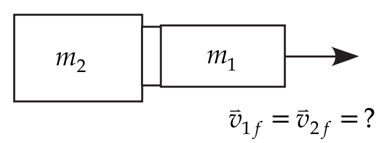
Let the car be mass 1.

Let the truck be mass 2.

Before Collision



After Collision



a) Since the two vehicles are locked together, the final velocity of the car equals the final velocity of the truck.

Unknown: Final velocity of the car and truck $\vec{v}_{1f} = \vec{v}_{2f} = ?$

Equation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Since the final velocities are equal, they can be factored out of the terms on the right side.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_{2f}$$

Substitute and solve:

$$(5680 \text{ kg})(+10.0 \text{ m/s}) + (1250 \text{ kg})(-14.0 \text{ m/s}) = (5680 \text{ kg} + 1250 \text{ kg})\vec{v}_{2f}$$

$$39300 \text{ kg}\cdot\text{m/s} = (6930 \text{ kg})\vec{v}_{2f}$$

$$\vec{v}_{2f} = +5.67 \text{ m/s}$$

The car and the truck slide along together at 5.67 m/s [east].

b) What is the change in momentum of the truck?

$$\Delta\vec{p}_2 = m_2\Delta\vec{v}_2 = m_2(\vec{v}_{2f} - \vec{v}_{2i})$$

$$\Delta\vec{p}_2 = (5680 \text{ kg})(+5.67 \text{ m/s} - (+10.0 \text{ m/s})) = -24594.4$$

$$\Delta\vec{p}_2 = -24600 \text{ kg}\cdot\text{m/s}$$

The change in momentum of the truck is 24600 kg·m/s [west].

c) What is the change in momentum of the car?

Since momentum is conserved during this collision, the change in momentum of the car should be equal but opposite to the change in momentum of the truck.

$$\Delta\vec{p}_1 = +24600 \text{ kg}\cdot\text{m/s}$$

The change in momentum of the car is 24600 kg·m/s [east].

d) What impulse is applied to the truck during the collision?

The impulse applied to the truck is equal to the change in momentum of the truck, which is 24600 kg·m/s [west].

e) What impulse is applied to the car during the collision?

The impulse applied to the car is equal to the change in momentum of the car, which is 24600 kg·m/s [east].

Learning Activity 5.5

Analyzing Head-on Collisions

Solve the following problems to check your understanding of the conservation of momentum during explosions and head-on collisions. You may check your work by using the answer key provided at the end of Module 5.

The physics of comparing kinetic energy and momentum (conceptual)

1. a) Consider a single object. Can this object have no momentum but still possess kinetic energy? Account for your answer.
   b) Consider a system of two objects. Can this system have a total momentum that is zero and a total kinetic energy that is not zero? Account for your answer.

The physics of two ice skaters pushing each other

2. a) Starting from rest, two ice skaters "push off" against each other on smooth, level ice where friction is negligible. The woman (m_1 = 54.0 kg) moves away with a velocity of $\vec{v}_{1f}$ = +2.50 m/s. What is the recoil velocity $\vec{v}_{2f}$ of the man (m_2 = 88.0 kg)?
   b) What is the change in momentum of the woman?
   c) If the push-off lasted 0.0200 seconds, what force was exerted during this push-off?

The physics of a shell and a cannon

3. A shell of mass 8.000 kg leaves the muzzle of a cannon with a horizontal velocity of 600.0 m/s. Find the recoil velocity of the cannon, if its mass is 500.0 kg.

The physics of a child and a sled

4. A 45.0 kg child runs with a horizontal velocity of +5.12 m/s and jumps onto a stationary 7.50 kg sled. Find the velocity of the child and the sled immediately after the child lands on the sled, if there was no friction.

(continued)

Learning Activity 5.5: Analyzing Head-on Collisions (continued)

The physics of two balls colliding elastically

5. A 4.00 kg ball moving to the right at 5.00 m/s collides head-on with a 2.00 kg ball moving to the left at 4.00 m/s. The 4.00 kg ball rebounds to the left at 1.00 m/s.
   a) Calculate the final velocity of the 2.00 kg ball.
   b) Calculate the change in momentum of the 4.00 kg ball.
   c) Calculate the change in momentum of the 2.00 kg ball.
   d) Calculate the force of the collision if the collision lasted 0.0750 seconds.

The physics of two colliding rail cars

6. A 28500 kg boxcar is moving at 3.25 m/s [east]. It collides with a stationary tanker car of mass 19200 kg. After the collision, the two railcars are coupled together and continue to move along the track.
   a) Calculate the final velocities of the railcars after the collision.
   b) What is the change in momentum of the boxcar?
   c) What was the impulse applied to the stationary tanker car?
   d) If the collision lasted 0.0960 seconds, calculate the force of interaction.

Lesson Summary

In this lesson, you studied the law of conservation of momentum. This important law in physics has many applications. You applied your knowledge of the conservation principle to the special situations of an explosion in one dimension and head-on collisions.

A **system** can simply mean a set of objects interacting with each other. An **isolated system** is one in which the only forces present are those between the objects of the system. The system can be isolated by making external forces very small (eliminating friction) or by considering a very small time interval before and after the interaction. Either of these conditions limits the impulse applied to the system by the external force.

Law of Conservation of Momentum

The **law of conservation of momentum** states that if the net external force acting on an object, or system of objects, is zero, then the total momentum of the object or system of objects before a collision is equal to the total momentum of the object or system of objects after the collision.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Quantity	**Symbol**	**Unit**
Mass of object 1	m_1	kilogram (kg)
Mass of object 2	m_2	kilogram (kg)
Initial velocity of object 1	$\vec{v}_{1i}$	metres/second (m/s)
Initial velocity of object 2	$\vec{v}_{2i}$	metres/second (m/s)
Final velocity of object 1	$\vec{v}_{1f}$	metres/second (m/s)
Final velocity of object 2	$\vec{v}_{2f}$	metres/second (m/s)

In two-body collisions, the impulse applied to the first body is equal but opposite to the impulse applied to the second body.

$$\vec{J}_1 = -\vec{J}_2$$

The impulse applied to one of the bodies is also equal to the change in the momentum of that body.

$$\vec{J}_1 = \vec{F}_{2-1}\Delta t = \Delta\vec{p}_1$$

If you know the change in momentum or impulse plus the time interval during which a collision occurred, you can calculate the force of the interaction.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

NOTES

Assignment 5.2

Conservation of Linear Momentum (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Module 5, after you have completed Module 5.

The physics of a person jumping onto a moving sled

1. A 75.0 kg person runs at 3.00 m/s and jumps onto a sled of mass 10.0 kg already moving in the same direction as the person at 2.00 m/s.
   a) What is the initial momentum of the sled-person system?

   b) What will the final momentum of the sled and the person be?

   c) What is the velocity of the sled and the person after the person jumps on the sled?

(continued)

Assignment 5.2: Conservation of Linear Momentum (continued)

The physics of a rifle and a bullet

2. An unconstrained rifle of mass 5.00 kg fires a 50.0 g bullet at a speed of 300.0 m/s with respect to the ground.

   a) What is the initial momentum of the rifle-bullet system?

   b) What is the recoil velocity of the rifle?

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each part of the assignment

Video - Introduction to Conservation of Momentum with Demonstrations

This video introduces the concept of conservation of momentum. The relationship for the conservation of momentum is derived.

A system of a person and a medicine ball is used to demonstrate the law of conservation of momentum.

https://youtu.be/Kf0bBxmNeec

Video - Introductory Conservation of Momentum Explosion Problem Demonstration

This video uses a catapult mounted on a low friction track projecting a sphere as a demonstration of the law of conservation of momentum in an explosion.

Measurements are made and calculations performed to show that the initial momentum of the system equals the final momentum of the system.

https://youtu.be/wP9bDRiHepc

Video - Introductory Perfectly Inelastic Collision Problem Demonstration

This video analyzes the conservation of momentum in a collision where the objects stick together after the interaction. This is an example of a perfectly inelastic collision.

The initial momentum of the system is determined by the momentum in the brass sphere. This provides the final momentum of the system also. After the interaction the sphere and the cart are moving along together so the final velocity of the cart equals the final velocity of the sphere. The final velocity of the cart and the sphere is calculated.

The velocity of the cart and the sphere is calculated from displacement and time. These velocities are then compared.

https://youtu.be/W-BCPUQXAJU

Video - Introductory Elastic Collision Problem Demonstration

This video investigates a collision between two carts on a low friction track. One cart is motionless and the second cart is moving before the collision.

After the collision both carts are moving. Position as a function of time is measured for each cart and displayed on a graph. The slope of the line on the position-time graph yields the velocity for each cart before the collision.

The law of conservation of momentum allows the final velocity of cart 1 to be calculated.

The graph of position as a function of time is extended to include the time after the collision. The final velocity of cart 1 is measured and compared to the final velocity of cart 1 that had been calculated.

The analysis continues to determine if kinetic energy was also conserved in this collision.

https://youtu.be/6Ks8VYxiugo

Assignment 5.2

Conservation of Linear Momentum (5 MARKS)

The following assignment must be submitted to the Distance Learning Unit for evaluation. Be sure to show all your work and explain the method of arriving at your answers. Submit this assignment, along with all the other assignments from Module 5, after you have completed Module 5.

The physics of a person jumping onto a moving sled

1. A 75.0 kg person runs at 3.00 m/s and jumps onto a sled of mass 10.0 kg already moving in the same direction as the person at 2.00 m/s.
   a) What is the initial momentum of the sled-person system?

   b) What will the final momentum of the sled and the person be?

   c) What is the velocity of the sled and the person after the person jumps on the sled?

(continued)

Assignment 5.2: Conservation of Linear Momentum (continued)

The physics of a rifle and a bullet

2. An unconstrained rifle of mass 5.00 kg fires a 50.0 g bullet at a speed of 300.0 m/s with respect to the ground.
   a) What is the initial momentum of the rifle-bullet system?

   b) What is the recoil velocity of the rifle?

Method of Assessment

The total of five marks for this assignment will be determined as follows:

- 1 mark for the correct answer in each part of the assignment

Lesson 3: Video Laboratory Activity: A Collision in Two Dimensions

(1.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ determine the initial and final momenta for a system of two balls involved in an elastic collision in two dimensions
- ❑ add momentum vectors using the component method
- ❑ compare and relate the initial and final momenta for a system of two balls involved in an elastic collision in two dimensions

Note to Student

In this lesson, you will view *A Collision in Two Dimensions*, a short video laboratory activity found in the learning management system (LMS). You may need to collect some data from the video, so you should begin by reading the following introduction and reviewing Lesson 1: Laboratory Activity: Analysis of an Experiment (see Lesson 1, Module 1). Upon completion of the lab activity, you must complete Assignment 5.3, which consists of a lab report. Complete the various sections of the report in the space provided. Assignment 5.3 is to be submitted to the Distance Learning Unit for assessment at the end of Module 5.

Introduction

The principle of conservation of momentum states that the total vector momentum of a closed system will be the same after a collision, in magnitude and direction, as it was before the collision. That is, the initial momentum of the system equals the final momentum when the net external force is zero. For example, consider a collision between two bodies of the same mass, *m*:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

For a collision between objects with one object initially at rest $(\vec{v}_2 = 0)$ and equal masses, we have

$$\vec{v}_1 = \vec{v}_1' + \vec{v}_2'$$

Sample Problem

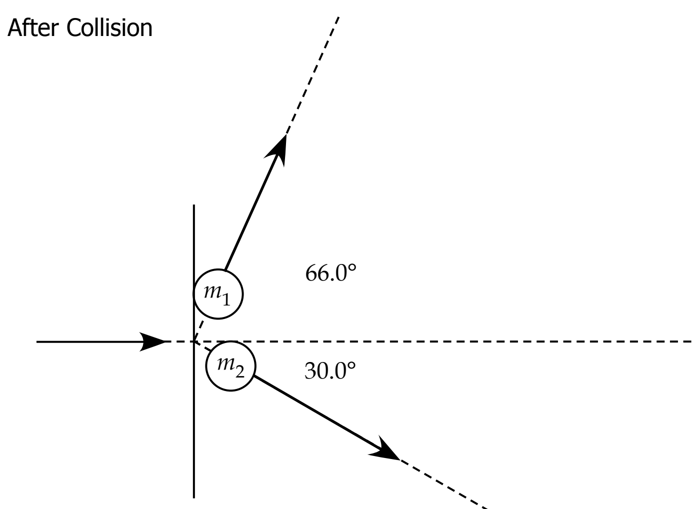
A curling rock of 20.0 kg is sliding at 2.00 m/s [E]. It collides with a stationary target rock in a glancing fashion. The final velocity of the first rock is found to be 1.20 m/s [66.0° N of E] and the final velocity of the target rock is 1.00 m/s [30.0° S of E].

Find and compare the total initial momentum and the total final momentum.

Let the shooter be mass 1. Let the target stone be mass 2.

$$\vec{p}_{\text{total initial}} = \vec{p}_{1i} + \vec{p}_{2i}$$

$$\begin{aligned}\vec{p}_{\text{total initial}} &= m_1\vec{v}_{1i} + m_2\vec{v}_{2i}\\ &= (20.0\text{ kg})(2.00\text{ m/s [E]}) + (20.0\text{ kg})(0\text{ m/s})\\ &= 40.0\text{ kg}\cdot\text{m/s [E]}\end{aligned}$$



$$\vec{p}_{\text{total final}} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$\vec{p}_{\text{total final}} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Add the vector components of the vector diagram shown on the previous page to obtain the following data.

Data Table

Vector	*x*-component (E–W)	*y*-component (N–S)
$m_1\vec{v}_{1f} = 20.0(1.20)\left[66.0° \text{ N of E}\right]$ $= 24.0 \text{ kg}\cdot\text{m/s}\left[66.0° \text{ N of E}\right]$	= 24.0 (cos 66.0°) kg·m/s = 9.76 kg·m/s	= 24.0 (sin 66.0°) kg·m/s = 21.9 kg·m/s
$m_2\vec{v}_{2f} = 20.0(1.00)\left[30.0° \text{ S of E}\right]$ $= 20.0 \text{ kg}\cdot\text{m/s}\left[30.0° \text{ S of E}\right]$	= 20.0 (cos 30.0°) kg·m/s = 17.3 kg·m/s	= –20.0 (sin 30.0°) kg·m/s = –10.0 kg·m/s
$\vec{P}_{\text{total final}}$	27.1 kg·m/s	11.9 kg·m/s

To solve for $\vec{p}_{\text{total final}}$, you combine the *x*- and *y*-components using the theorem of Pythagoras.

$$\vec{p}_{\text{total final}} = \sqrt{27.1^2 + 11.9^2} = 29.6 \text{ kg}\cdot\text{m/s}$$

$$\tan\theta = \frac{11.9}{27.1}$$
$$\theta = 23.7°$$

The final momentum is 29.6 kg·m/s [23.7° N of E].

NOTES

Assignment 5.3

Video Laboratory Activity: A Collision in Two Dimensions (20 MARKS)

You must complete the following lab report and submit it to the Distance Learning Unit for evaluation. Please complete your work in the space provided. Submit this assignment, along with all the other assignments from Module 5, after you have completed Module 5.

Purpose

To compare the initial and final momenta for a collision in two dimensions.

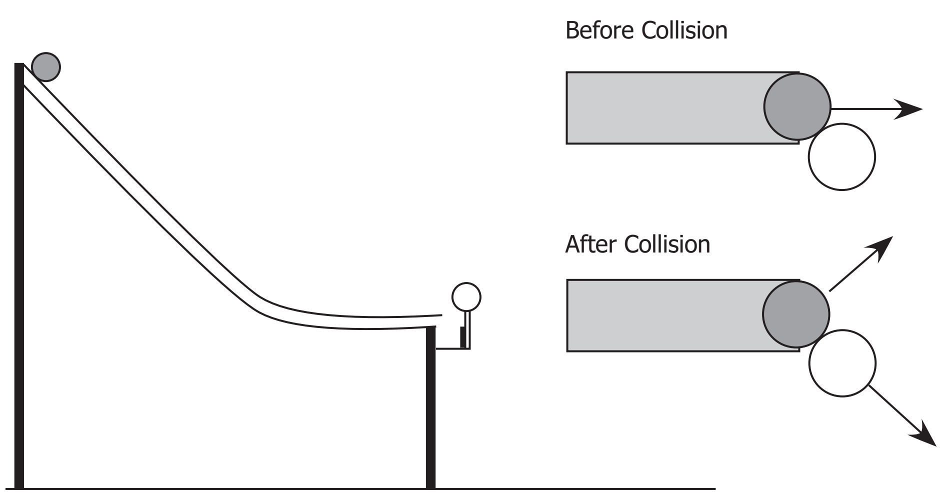
Apparatus

Ramp, metal spheres, plumb bob, paper, carbon paper

Procedure Note

In this experiment, a metal sphere rolls down a grooved ramp. At the bottom of the ramp, the motion of the metal sphere becomes horizontal and the metal sphere strikes a target sphere at rest (see Figure 1).

Figure 1



(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

We know that momentum is conserved $\left(\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}\right)$.

Therefore:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2' \qquad \text{Equation (1)}$$

For the case of the two objects with one object initially at rest (v_2 = 0) and equal masses, we have

$$\vec{v}_1 = \vec{v}_1' + \vec{v}_2' \qquad \text{Equation (2)}$$

In this case, both spheres move horizontally at a constant speed, so that $v = \frac{x}{t}$. Therefore, Equation (2) becomes

$$\frac{\vec{x}_{1\text{initial}}}{t_{1\text{initial}}} = \frac{\vec{x}_{1\text{final}}}{t_{1\text{final}}} + \frac{\vec{x}_{2\text{final}}}{t_{2\text{final}}} \qquad \text{Equation (3)}$$

Since the time it takes for the spheres to fall to the ground will be equal, Equation (3) becomes

$$\vec{x}_{1\text{initial}} = \vec{x}_{1\text{final}} + \vec{x}_{2\text{final}} \qquad \text{Equation (4)}$$

Therefore, if we record the positions of the vectors on a piece of paper on the ground, they will look exactly like the collision of the curling rocks (discussed in the sample problem at the beginning of Lesson 3).

Procedure

1. Secure the ramp on a table about 60 centimetres above the floor.
2. Place four sheets of plain white paper taped together on the floor beneath the ramp.
3. Place carbon paper over the white sheets of paper to record the positions of the spheres when they land on the floor.
4. Hang a plumb bob from the target support to record the initial position of the target sphere on the paper.
5. Using the incident sphere alone, release it from the top of the ramp and record the position where it lands on the paper. Repeat the process 10 times to get an average measurement. Draw a small circle around the edge of the sphere marks.

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

6. Place the target sphere on the support and adjust it so that the incident sphere and the target sphere are perfectly aligned for an off-centre collision.
7. Release the incident sphere from the top of the ramp and, following a clean collision, draw the incident and target vectors on the paper (a photocopy of the actual collision is included in the Data and Calculations section). Be sure to draw the vector tails from the correct position.
8. Set up a coordinate system so that the incident vector lies along the *y*-axis.
9. Record your components in the Data Table and add $\vec{x}_{1\text{final}} + \vec{x}_{2\text{final}}$.
10. Compare the initial and final displacement vectors.

Video Viewing

View the video *A Collision in Two Dimensions*, which can be found in the learning management system (LMS).

Data and Calculations

The following two pages show a photograph and a template to which you will need to refer.

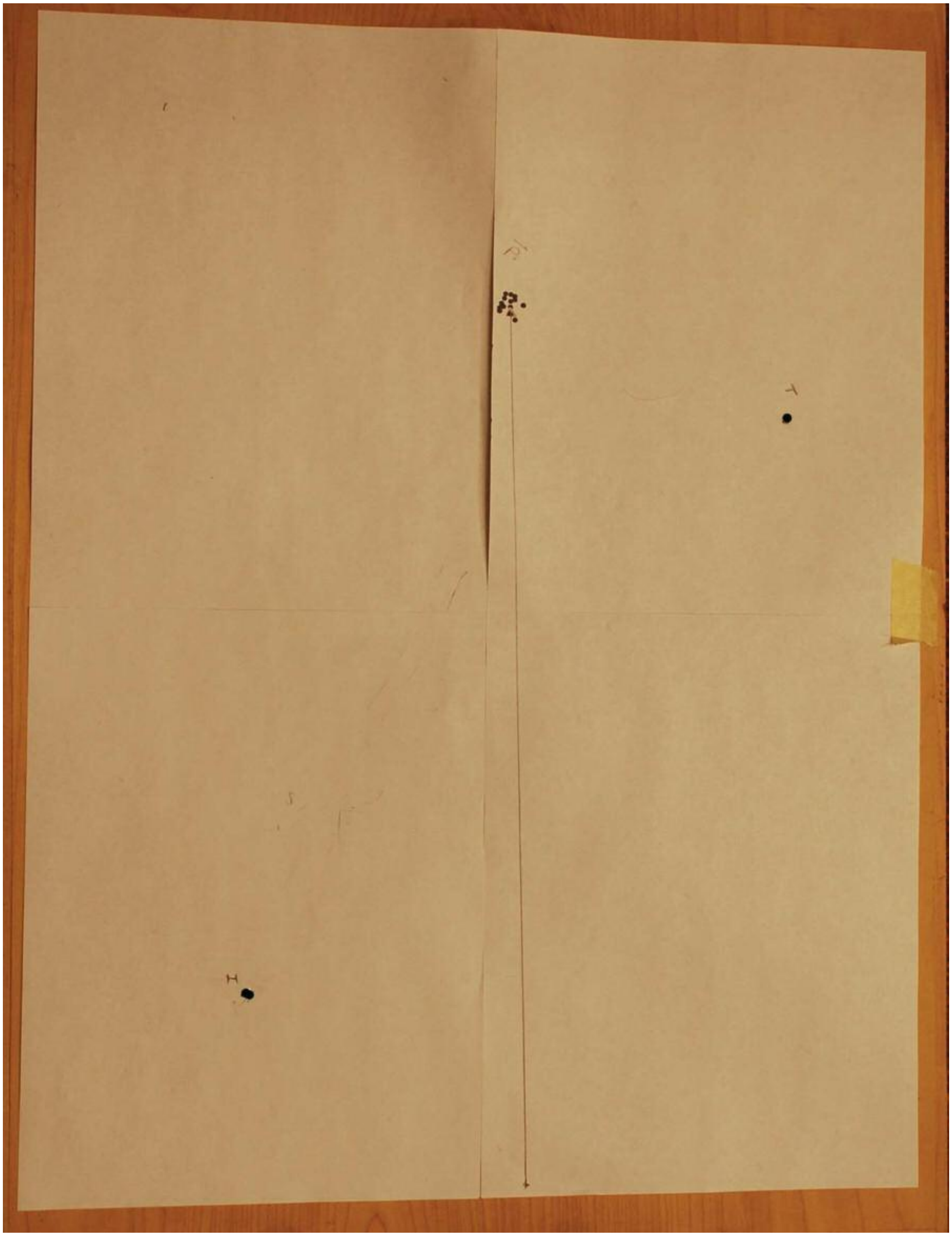
- The photo shows a group of dots indicating the positions at which the spheres landed for the 10 trials before the collision.
  - The dot on the right shows where the target sphere landed after the collision.
  - The dot on the left shows where the incident sphere landed after the collision.
- The template shows these positions on a drawing of the photo.

1. On the template provided, draw in the vectors representing $\vec{x}_{1\text{final}}$ for the incident sphere and $\vec{x}_{2\text{final}}$ for the target sphere. Label these vectors. *(2 marks)*

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

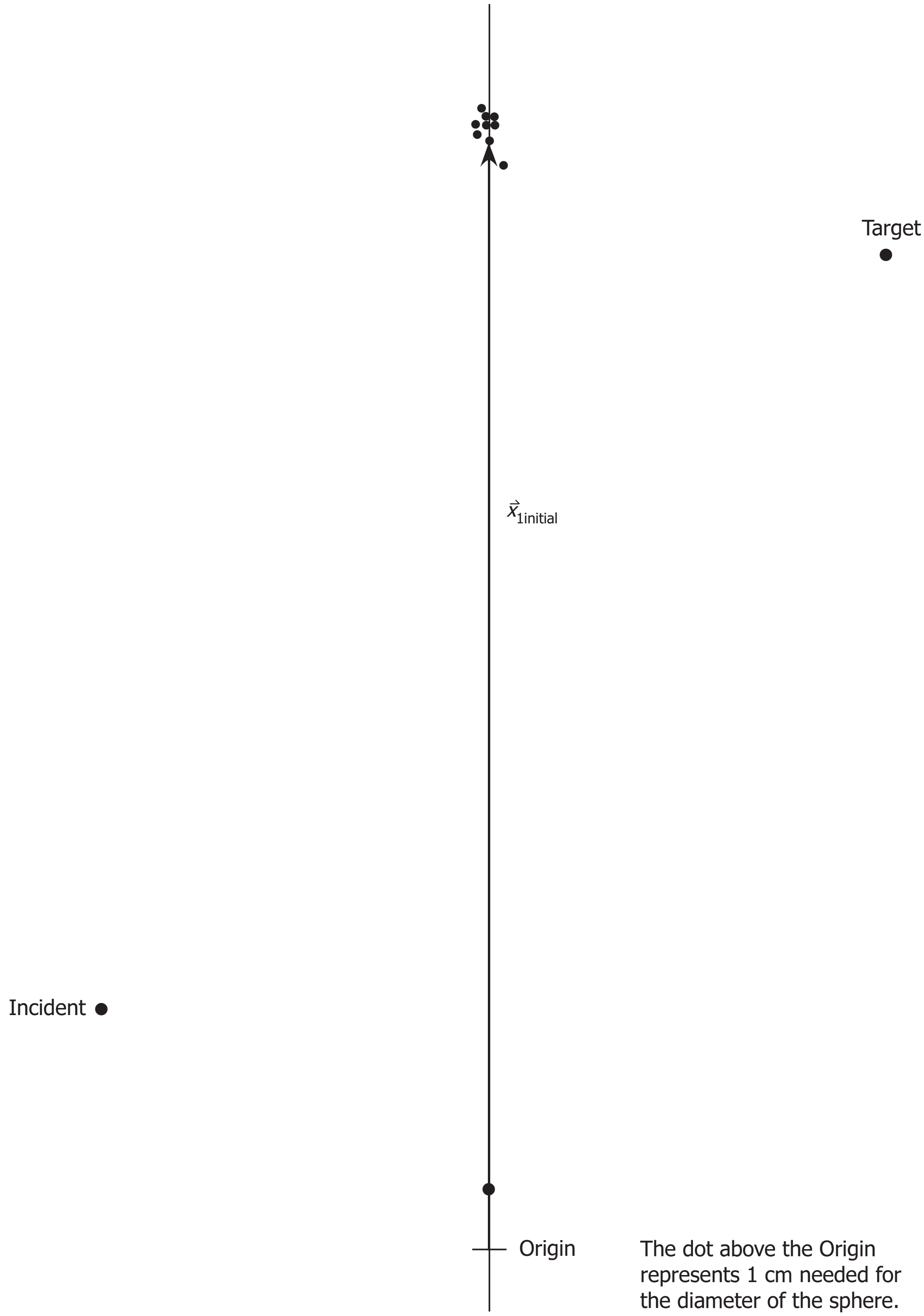
Photograph



(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

Template



The dot above the Origin represents 1 cm needed for the diameter of the sphere.

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

2. Measure the length and direction of each of the vectors $\vec{x}_{1\text{initial}}$, $\vec{x}_{1\text{final}}$, and $\vec{x}_{2\text{final}}$, and record these measurements in the Data Table. *(3 marks)*

   Use experimental values measured from the template on the previous page.

   $\vec{x}_{1\text{initial}} =$ ____________________

   Data Table

Vector	x-component	*y*-component
$\vec{x}_{1\text{final}} =$		
$\vec{x}_{2\text{final}} =$		
$\vec{x}_{1\text{initial}} = \left(\vec{x}_{1\text{final}} + \vec{x}_{2\text{final}}\right)$		

3. Calculate the *x*-component and the *y*-component of each vector in the space provided in the Data Table. *(6 marks)*
4. Calculate the magnitude and direction of $\vec{x}_{1\text{initial}}$. *(2 marks)*

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

Discussion

5. How do the calculated values for magnitude and direction of the initial momentum compare to the experimental magnitude and direction found from the experiment? What are the percent errors for each? *(3 marks)*

6. What are some of the possible sources of experimental error? *(2 mark)*

Conclusion

7. What can you say about initial momentum and final momentum in an elastic collision? *(2 marks)*

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

Marking Rubric for Assignment 5.3

Criteria	Possible Marks	Actual Marks
Correct drawing and labelling of the vectors on the template	2	
Correct determination of the magnitude and directions of the three vectors	3	
Calculation of the components of the three vectors	6	
Calculation of the magnitude and direction of $\vec{x}_{1\text{initial}}$	2	
Discussion (comparison, percent errors, error analysis)	5	
Conclusion	2	
Total	**20**	

https://youtu.be/i4VHT7fOtXE?list=PLw1g3n2IMV7M72rewl81rI7b
-CR0k8Wta

Assignment 5.3

Video Laboratory Activity: A Collision in Two Dimensions (20 MARKS)

You must complete the following lab report and submit it to the Distance Learning Unit for evaluation. Please complete your work in the space provided. Submit this assignment, along with all the other assignments from Module 5, after you have completed Module 5.

Purpose

To compare the initial and final momenta for a collision in two dimensions.

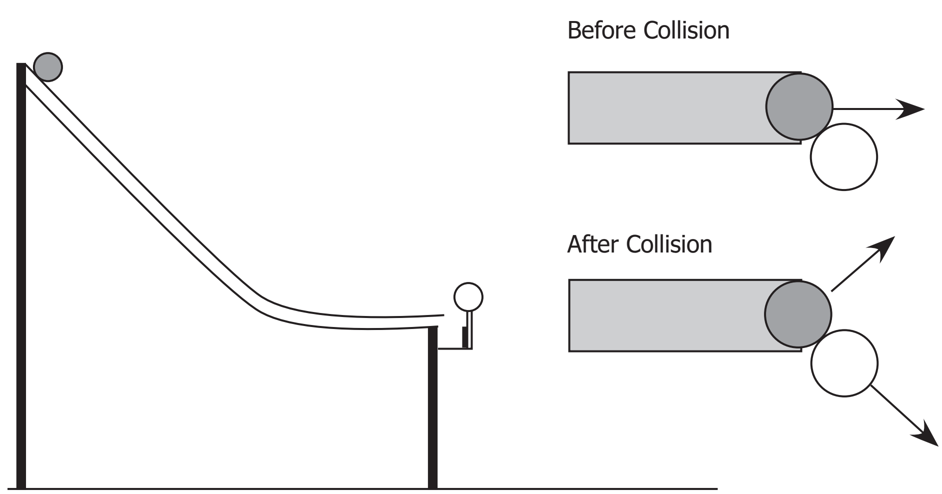
Apparatus

Ramp, metal spheres, plumb bob, paper, carbon paper

Procedure Note

In this experiment, a metal sphere rolls down a grooved ramp. At the bottom of the ramp, the motion of the metal sphere becomes horizontal and the metal sphere strikes a target sphere at rest (see Figure 1).

Figure 1



(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

We know that momentum is conserved $\left(\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}\right)$.

Therefore:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2' \qquad \text{Equation (1)}$$

For the case of the two objects with one object initially at rest ($v_2 = 0$) and equal masses, we have

$$\vec{v}_1 = \vec{v}_1' + \vec{v}_2' \qquad \text{Equation (2)}$$

In this case, both spheres move horizontally at a constant speed, so that $v = \frac{x}{t}$. Therefore, Equation (2) becomes

$$\frac{\vec{x}_{1\text{initial}}}{t_{1\text{initial}}} = \frac{\vec{x}_{1\text{final}}}{t_{1\text{final}}} + \frac{\vec{x}_{2\text{final}}}{t_{2\text{final}}} \qquad \text{Equation (3)}$$

Since the time it takes for the spheres to fall to the ground will be equal, Equation (3) becomes

$$\vec{x}_{1\text{initial}} = \vec{x}_{1\text{final}} + \vec{x}_{2\text{final}} \qquad \text{Equation (4)}$$

Therefore, if we record the positions of the vectors on a piece of paper on the ground, they will look exactly like the collision of the curling rocks (discussed in the sample problem at the beginning of Lesson 3).

Procedure

1. Secure the ramp on a table about 60 centimetres above the floor.
2. Place four sheets of plain white paper taped together on the floor beneath the ramp.
3. Place carbon paper over the white sheets of paper to record the positions of the spheres when they land on the floor.
4. Hang a plumb bob from the target support to record the initial position of the target sphere on the paper.
5. Using the incident sphere alone, release it from the top of the ramp and record the position where it lands on the paper. Repeat the process 10 times to get an average measurement. Draw a small circle around the edge of the sphere marks.

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

6. Place the target sphere on the support and adjust it so that the incident sphere and the target sphere are perfectly aligned for an off-centre collision.
7. Release the incident sphere from the top of the ramp and, following a clean collision, draw the incident and target vectors on the paper (a photocopy of the actual collision is included in the Data and Calculations section). Be sure to draw the vector tails from the correct position.
8. Set up a coordinate system so that the incident vector lies along the y-axis.
9. Record your components in the Data Table and add $\vec{x}_{1\text{final}} + \vec{x}_{2\text{final}}$.
10. Compare the initial and final displacement vectors.

Video Viewing

View the video *A Collision in Two Dimensions*, which can be found by visiting the Independent Study Option Audio and Video web page at www.edu.gov.mb.ca/k12/dl/iso/av.html.

Data and Calculations

The following two pages show a photograph and a template to which you will need to refer.

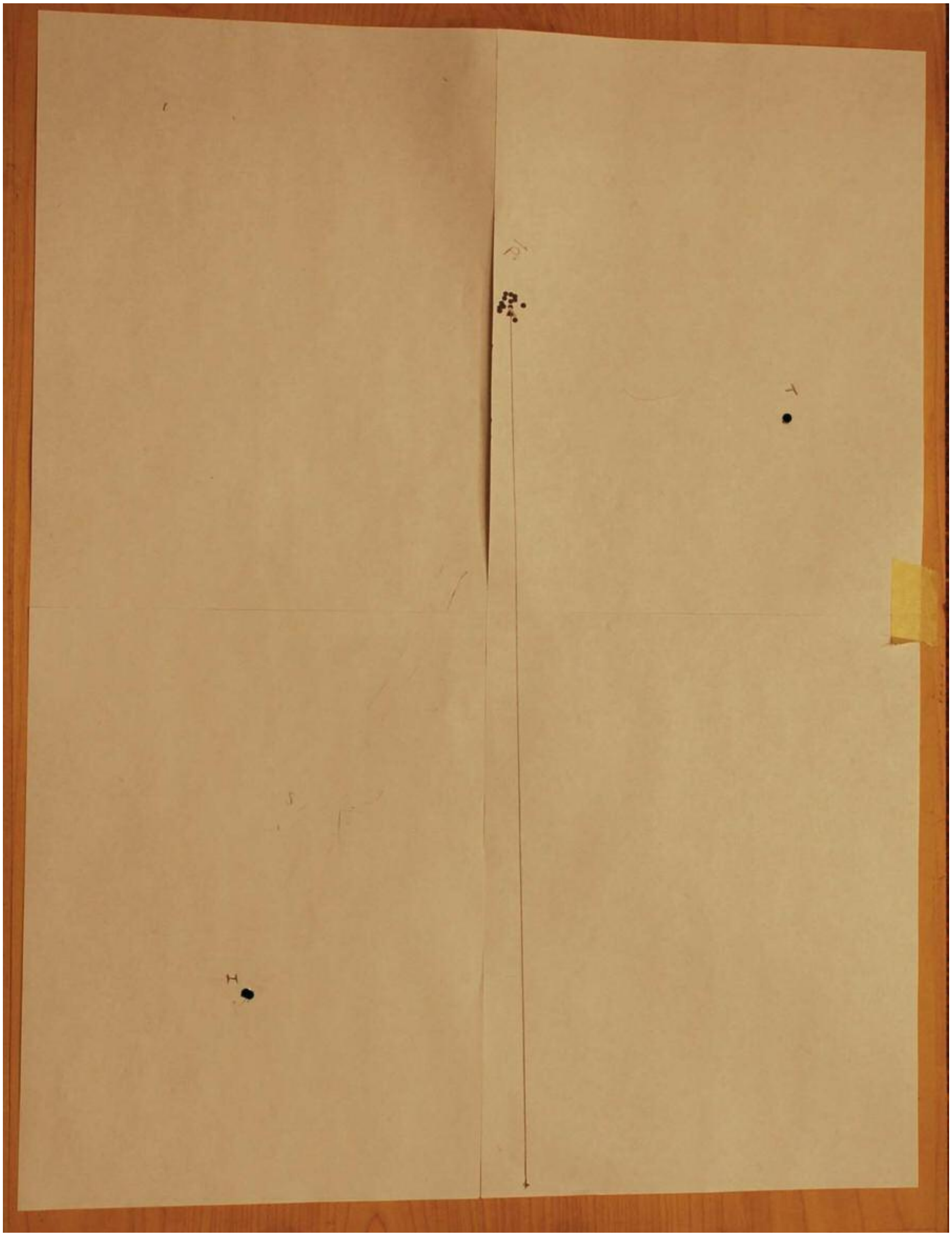
- The photo shows a group of dots indicating the positions at which the spheres landed for the 10 trials before the collision.
  - The dot on the right shows where the target sphere landed after the collision.
  - The dot on the left shows where the incident sphere landed after the collision.
- The template shows these positions on a drawing of the photo.

1. On the template provided, draw in the vectors representing $\vec{x}_{1\text{final}}$ for the incident sphere and $\vec{x}_{2\text{final}}$ for the target sphere. Label these vectors. *(2 marks)*

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

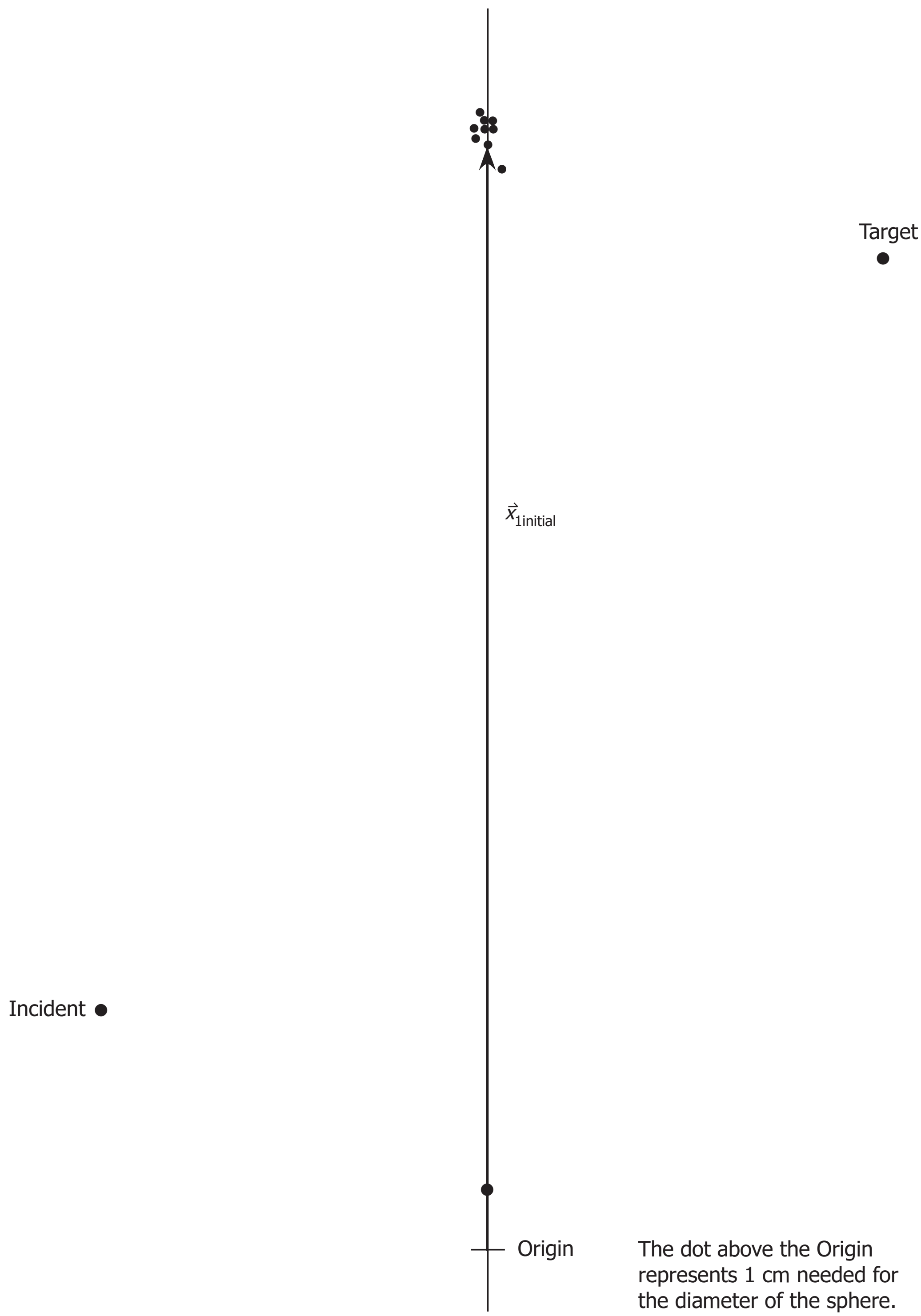
Photograph



(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

Template



(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

2. Measure the length and direction of each of the vectors $\vec{x}_{1\text{initial}}$, $\vec{x}_{1\text{final}}$, and $\vec{x}_{2\text{final}}$, and record these measurements in the Data Table. *(3 marks)*

   Use experimental values measured from the template on the previous page.

   $\vec{x}_{1\text{initial}} =$ ____________________

   Data Table

Vector	x-component	*y*-component
$\vec{x}_{1\text{final}} =$		
$\vec{x}_{2\text{final}} =$		
$\vec{x}_{1\text{initial}} = \left(\vec{x}_{1\text{final}} + \vec{x}_{2\text{final}}\right)$		

3. Calculate the *x*-component and the *y*-component of each vector in the space provided in the Data Table. *(6 marks)*

4. Calculate the magnitude and direction of $\vec{x}_{1\text{initial}}$. *(2 marks)*

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

Discussion

5. How do the calculated values for magnitude and direction of the initial momentum compare to the experimental magnitude and direction found from the experiment? What are the percent errors for each? *(3 marks)*

6. What are some of the possible sources of experimental error? *(2 mark)*

Conclusion

7. What can you say about initial momentum and final momentum in an elastic collision? *(2 marks)*

(continued)

Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions (continued)

Marking Rubric for Assignment 5.3

Criteria	Possible Marks	Actual Marks
Correct drawing and labelling of the vectors on the template	2	
Correct determination of the magnitude and directions of the three vectors	3	
Calculation of the components of the three vectors	6	
Calculation of the magnitude and direction of $\vec{x}_{1\text{initial}}$	2	
Discussion (comparison, percent errors, error analysis)	5	
Conclusion	2	
Total	**20**	

Lesson 4: Conservation of Momentum in Two Dimensions (2.5 hours)

Learning Outcomes

When you have completed this lesson, you should be able to

- ❑ determine the magnitude and direction of the momentum and the velocity of a given mass after an elastic collision in two dimensions
- ❑ determine the magnitude and direction of the momentum and the velocity of a given mass after an explosion in two dimensions

Key Words

conservation of momentum
component method

Introduction

In previous lessons, you studied collisions in one dimension and explosions in one dimension. You have seen that momentum is conserved in both situations. In this lesson, you will study collisions and explosions in two dimensions. In these kinds of situations, momentum is also conserved. You may see collisions in two dimensions in sports, such as billiards or curling, where two objects collide and then move off in directions that are different than the initial directions of motion. You may see explosions in two dimensions when an object falls and breaks apart into two or more pieces, or in radioactive decay where a nucleus may break up into two or more pieces that fly off and move away from each other.

The Approach to Momentum in Two Dimensions

In a two-dimensional collision or explosion, the total initial momentum is equal to the total final momentum, just as was the case in the one-dimensional collision. Also, in a two-dimensional collision, the momentum that is conserved is both the x-direction and the y-direction separately.

The work with momentum vectors is similar to the work you did for forces in equilibrium. There, you used the addition of vectors using the component method. You will use the component method of vector addition and subtraction to deal with the law of conservation of momentum in two dimensions.

Remember that the law of conservation of momentum states:

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

In these problems, you will be given enough information to find three of the four momenta. Your job will then be to find the missing momentum. The fact the vectors do not point along the same straight line makes the vector addition/subtraction more complicated, but the basic concept is still the law of conservation of momentum.

Example 1: A Collision in Two Dimensions Where Only One Object Is Initially Moving

A curling rock of 18.8 kg is sliding at 1.45 m/s [E]. It collides with a stationary target rock in a glancing fashion. The final velocity of the target rock is 1.00 m/s [30.0° S of E].

Calculate

a) the total initial momentum and the total final momentum
b) the final momentum of the first rock
c) the final velocity of the first rock (the shooter)
d) the change in momentum of the first rock
e) describe the motion of the centre of mass
f) calculate the impulse applied to the first rock (the shooter)

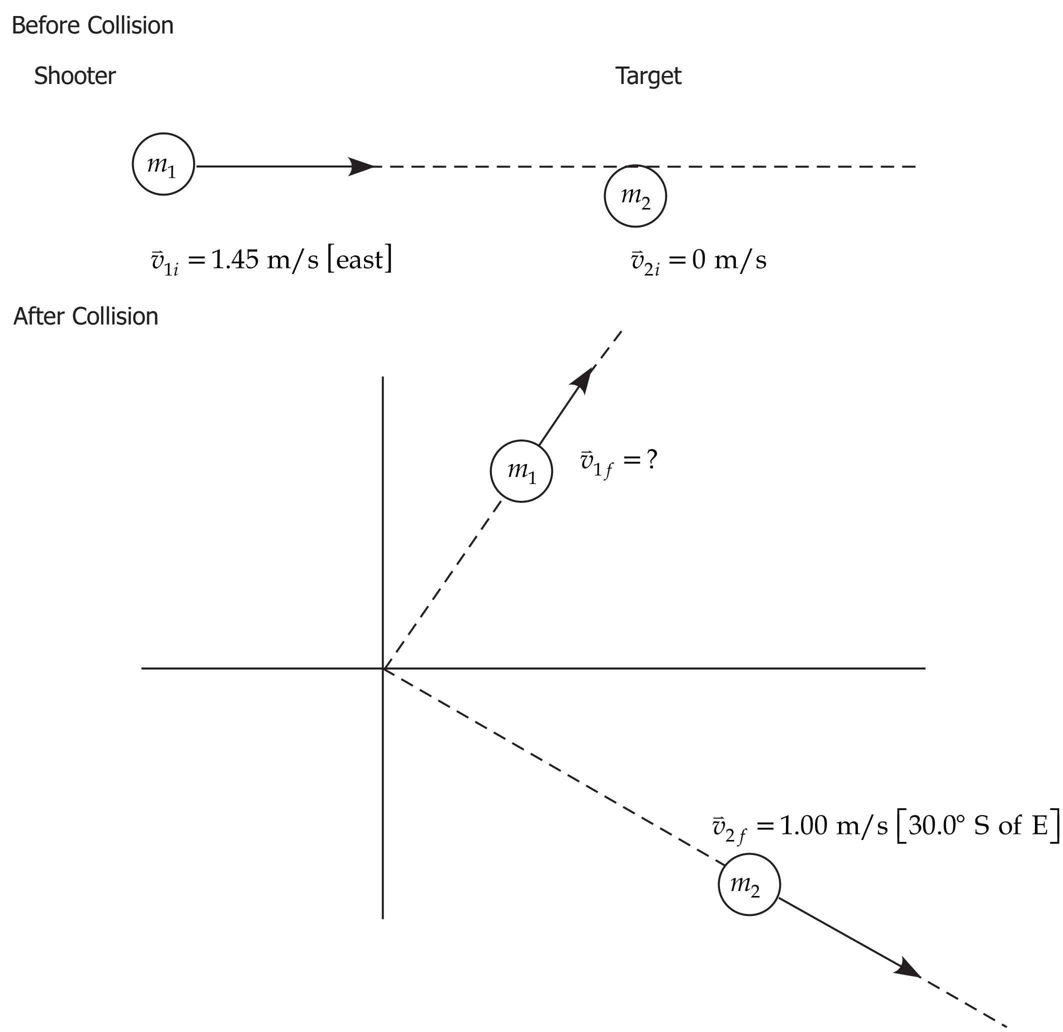
Here we go. It looks overwhelming, but do it one step at a time.

Solution:

a) Draw a set of coordinate axes and draw a diagram of the situation.

Let the shooter be mass 1.

Let the target stone be mass 2.



$$\vec{p}_{\text{total initial}} = \vec{p}_{1i} + \vec{p}_{2i}$$
$$\vec{p}_{\text{total initial}} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$$
$$= (18.8 \text{ kg})(1.45 \text{ m/s [E]}) + (18.8 \text{ kg})(0 \text{ m/s})$$
$$= 27.3 \text{ kg}\cdot\text{m/s [E]}$$

Since momentum is conserved, the total final momentum is the same.

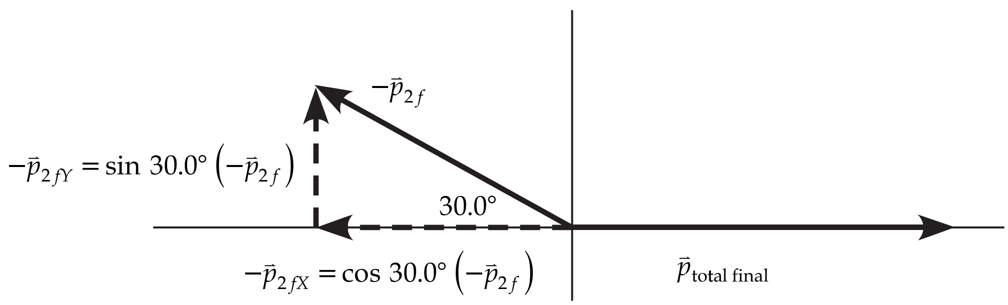
b) To calculate the final momentum of the first rock, you substitute the values obtained in part (a) into the following equation:

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$
$$\vec{p}_{\text{total final}} = \vec{p}_{1f} + \vec{p}_{2f}$$
$$\vec{p}_{\text{total final}} - \vec{p}_{2f} = \vec{p}_{1f}$$

$$27.3 \text{ kg}\cdot\text{m/s [E]} - (18.8 \text{ m/s})(1.00 \text{ m/s})[30.0° \text{ S of E}] = \vec{p}_{1f}$$
$$27.3 \text{ kg}\cdot\text{m/s [E]} - 18.8 \text{ kg}\cdot\text{m/s } [30.0° \text{ S of E}] = \vec{p}_{1f}$$

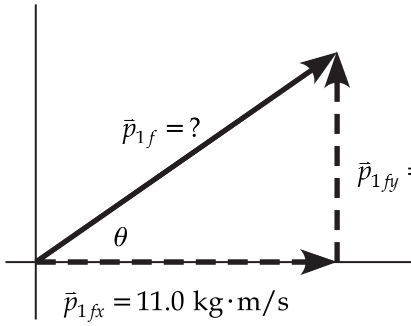
Subtract by adding the opposite.

$$27.3 \text{ kg}\cdot\text{m/s [E]} + 18.8 \text{ kg}\cdot\text{m/s } [30.0° \text{ N of W}] = \vec{p}_{1f}$$



Adding the vector components of the vector diagram shown above, you obtain the following data:

Vector	*x*-component (E–W)	*y*-component (N–S)
$\vec{p}_{\text{total final}}$	+27.3 kg·m/s	0 kg·m/s
$-\vec{p}_{2f}$	–cos 30.0° (18.8 kg·m/s) = –16.3 kg·m/s	+sin 30.0° (18.8 kg·m/s) = +9.40 kg·m/s
$-\vec{p}_{1f}$	+11.0 kg·m/s	+9.40 kg·m/s



To solve for $\vec{p}_{1f}$, you combine the components from the diagram using the theorem of Pythagoras.

$$\vec{p}_{1f}^{\,2} = (11.0)^2 + (9.40)^2 = 209.36$$

$$\vec{p}_{1f} = 14.5 \text{ kg}\cdot\text{m/s}$$

The angle at the tail of the resultant is determined using tangent:

$$\theta = \tan^{-1}\frac{9.40}{11.0} = 40.5°.$$

The resultant vector is 14.5 kg·m/s [40.5° N of E]

c) To solve for the velocity of the shooter rock, substitute values into the following equation:

$$\vec{p}_{1f} = m_1\vec{v}_{1f}$$

$$14.5 \text{ kg}\cdot\text{m/s}\left[40.5° \text{ N of E}\right] = 18.8 \text{ kg } \vec{v}_{1f}$$

$$\vec{v}_{1f} = 0.771 \text{ m/s}\left[40.5° \text{ N of E}\right]$$

d) The change in momentum of the shooter can be found by taking the difference between the final momentum and the initial momentum for the shooter. In this case, it would require a subtraction of vector quantities. However, the change in momentum of the shooter is equal but opposite to the change in momentum of the target. This can be easily calculated since the initial momentum of the target stone is 0 kg·m/s.

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2 = -\left(\vec{p}_{2f} - \vec{p}_{2i}\right)$$

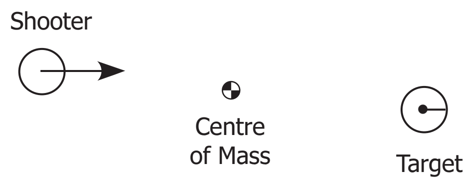
$$= -\left[18.8 \text{ kg}\cdot\text{m/s}\left[30.0° \text{ S of E}\right] - 0\right]$$

$$= 18.8 \text{ kg}\cdot\text{m/s}\left[30.0° \text{ N of W}\right]$$

e) The system of the two rocks has a centre of mass midway between them. As the rocks move, so does this centre of mass. The total momentum of the system can be thought of as the momentum of the centre of mass. The relationship for momentum can be expressed as

$$\vec{p}_{\text{total}} = m_{\text{total}}\vec{v}_{\text{centre of mass}}$$
$$27.3 \text{ kg}\cdot\text{m/s [E]} = (18.8 \text{ kg} + 18.8 \text{ kg})(\vec{v}_{\text{centre of mass}})$$
$$(\vec{v}_{\text{centre of mass}}) = 0.726 \text{ m/s [E]}$$

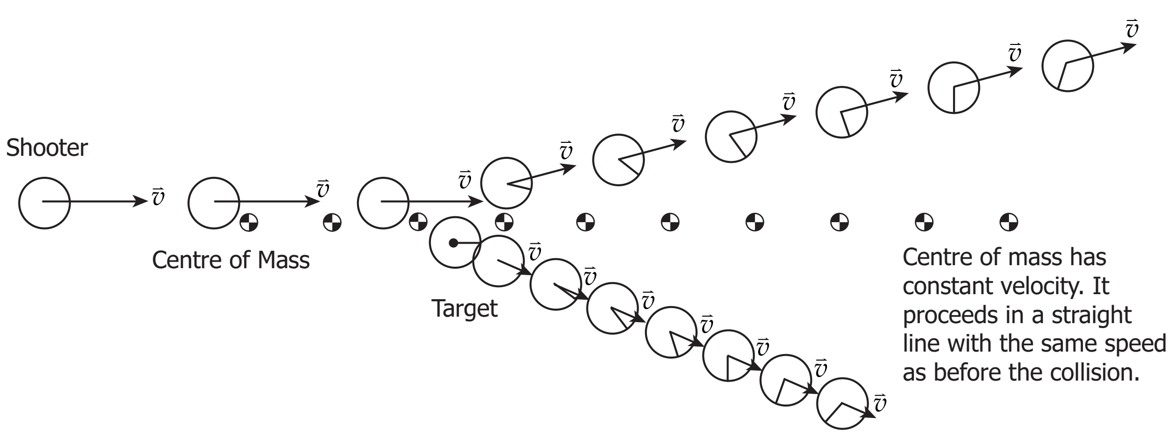
In the graphic below, the centre of mass is located midway between the two rocks since they have the same masses.



In the graphic below, the sequence shows that as the shooter slides east, the centre of mass moves to the east.

The spacing of the centre of mass positions indicates that it is moving at the same speed (equal distances in equal times). If you lay a ruler along the positions of the centre of mass, they all fall along a **straight line**.

> Since speed and direction are constant, the velocity of the centre of mass of an isolated system is constant. The velocity of the centre of mass of an isolated system of objects is the same before and after an interaction between the objects within the system.



f) Again, the impulse applied to the shooter rock causes a change in its momentum. You have already calculated the change in momentum for the shooter, so the impulse applied to the shooter equals 18.8 N·s [30.0° N of E].

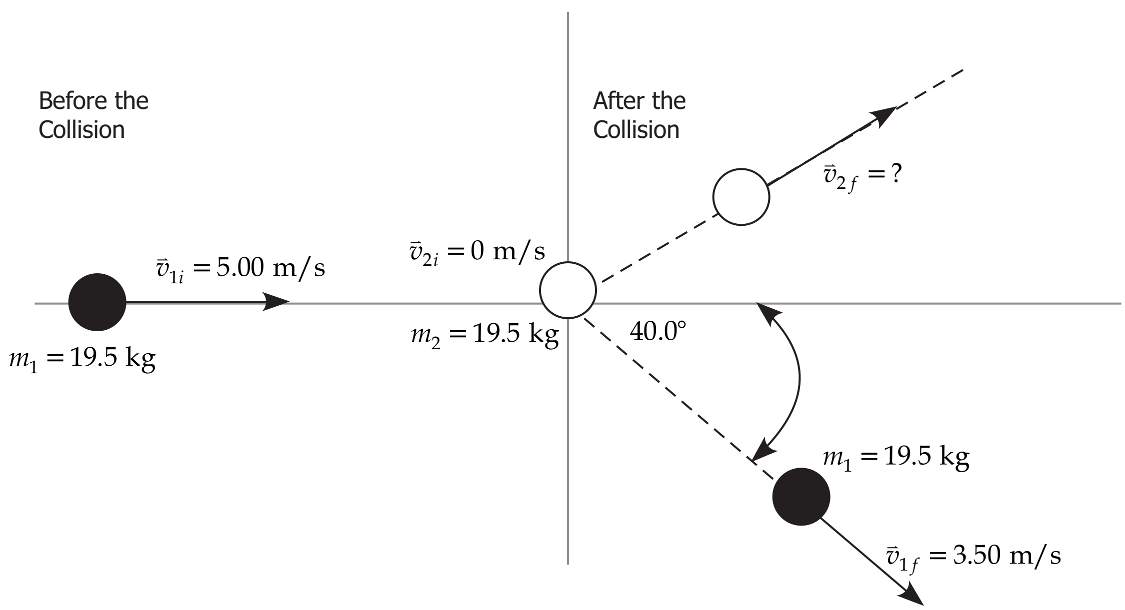
Learning Activity 5.6

Conservation of Momentum in Two Dimensions

Solve the following problem to check your understanding of the law of conservation of momentum in two dimensions. You may check your work against the answer key found at the end of Module 5.

The physics of curling stone collisions and the conservation of momentum

1. Two identical curling stones of mass 19.5 kg collide. The stone m_1 is moving at 5.00 m/s to the right towards a stationary stone of mass m_2. The collision is a glancing collision. After the collision, one of the stones m_1 moves at 3.50 m/s at an angle of 40.0° below the horizontal.
   a) What is the total initial momentum of the system?
   b) What is the total final momentum after the collision?
   c) What is the momentum of m_2 after the collision?
   d) What is the change in momentum of m_1 during the collision?
   e) What impulse is applied to rock 2 during the collision?



Example 2: Explosions in Two Dimensions

Explosions in one dimension were studied in an earlier lesson. As was the case in the one-dimensional explosions, momentum is conserved in explosions in two dimensions. It is conserved in both the x-direction and in the y-direction.

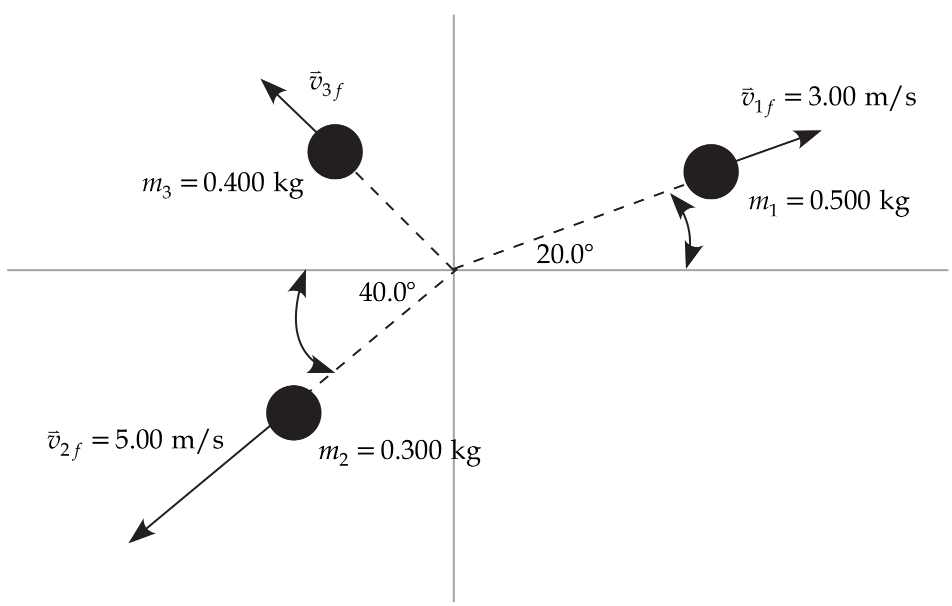
For example, consider the case of a 1.200 kg mass at rest on a smooth, frictionless surface. The mass suddenly explodes into three pieces. A mass m_1, 0.500 kg, flies off at 20.0° north of east at 3.00 m/s. A second mass m_2, 0.300 kg, flies off at 40.0° south of west with a speed of 5.00 m/s.

What are the final momentum and the final velocity of the third piece?

Given: The mass of the third piece must be
m_3 = 1.200 kg – (0.500 kg + 0.300 kg) = 0.400 kg.

Before the explosion, the mass was at rest so the initial momentum is 0 kg·m/s. Therefore, by the law of conservation of momentum, the final momentum of the system is 0 kg·m/s.

After the Explosion



Equation: $\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$

$\vec{p}_{1i} + \vec{p}_{2i} + \vec{p}_{3i} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f}$

Three pieces are formed, so three momenta are used.

Substitute and solve: $\vec{p}_{\text{total initial}} = 0 \text{ kg}\cdot\text{m/s}$

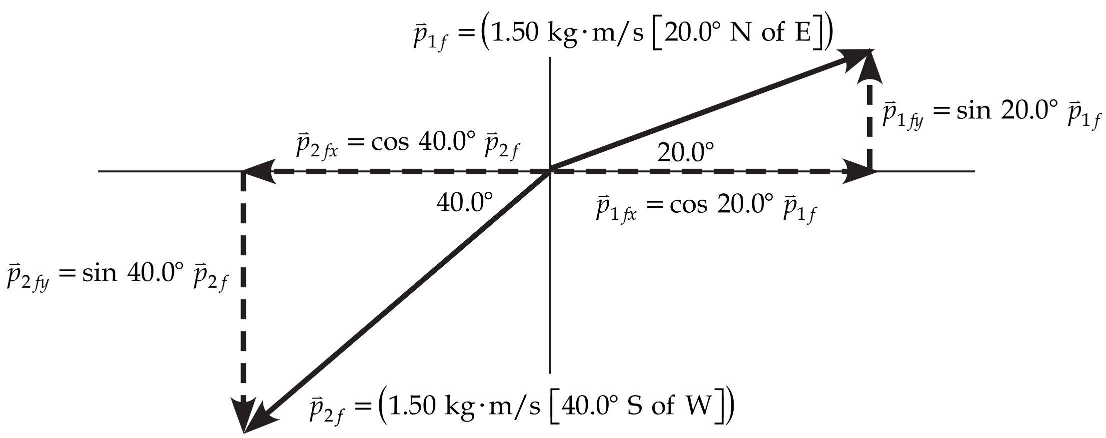
$$0 \text{ kg}\cdot\text{m/s} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f}$$

$$\vec{p}_{3f} = -\left(\vec{p}_{1f} + \vec{p}_{2f}\right)$$

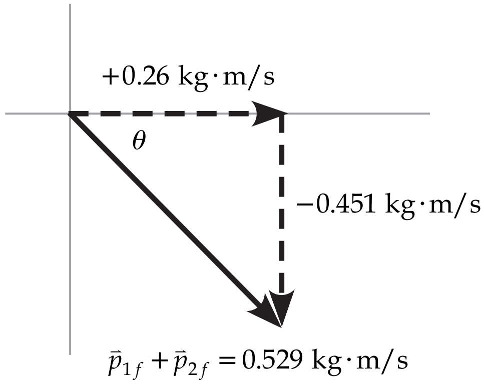
$$\vec{p}_{3f} = -\begin{pmatrix}(0.500 \text{ kg})(3.00 \text{ m/s } [20.0^\circ \text{ N of E}]) + \\ (0.300 \text{ kg})(5.00 \text{ m/s } [40.0^\circ \text{ S of W}])\end{pmatrix}$$

$$\vec{p}_{3f} = -\begin{pmatrix}(1.50 \text{ kg}\cdot\text{m/s } [20.0^\circ \text{ N of E}]) + \\ (1.50 \text{ kg}\cdot\text{m/s } [40.0^\circ \text{ S of W}])\end{pmatrix}$$

Add these two vectors using the component method.



Vector	*x*-component (E–W)	*y*-component (N–S)
$\vec{p}_{1f}$	cos 20.0° (1.50 kg·m/s) = +1.41 kg·m/s	sin 20.0° (1.50 kg·m/s) = +0.513 kg·m/s
$\vec{p}_{2f}$	–cos 40.0° (1.50 kg·m/s) = –1.15 kg·m/s	–sin 40.0° (1.50 kg·m/s) = –0.964 kg·m/s
$\vec{p}_{1f} + \vec{p}_{2f}$	+0.26 kg·m/s	–0.451 kg·m/s



Using the theorem of Pythagoras,

$$\left(\vec{p}_{1f} + \vec{p}_{2f}\right)^2 = 0.26^2 + 0.451^2 = 0.271$$

$$\vec{p}_{1f} + \vec{p}_{2f} = 0.520 \text{ kg}\cdot\text{m/s}$$

To find the angle at the tail of the resultant, use inverse tangent.

$$\theta = \tan^{-1}\frac{0.451}{0.26} = 60.0° \text{ S of E}$$

$$\vec{p}_{1f} + \vec{p}_{2f} = 0.520 \text{ kg}\cdot\text{m/s}\left[\,60.0° \text{ S of E}\right]$$

But,

$$\vec{p}_{3f} = -(\vec{p}_{1f} + \vec{p}_{2f})$$

So $\vec{p}_{3f} = 0.520 \text{ kg}\cdot\text{m/s}\left[60.0° \text{ N of W}\right]$

The final velocity of the third piece is

$$\vec{v}_{3f} = \frac{\vec{p}_{3f}}{m_3} = \frac{0.520 \text{ kg}\cdot\text{m/s}\left[60.0° \text{ N of W}\right]}{0.400 \text{ kg}}$$

$$\vec{v}_{3f} = 1.30 \text{ m/s}\ \left[60.0° \text{ N of W}\right]$$

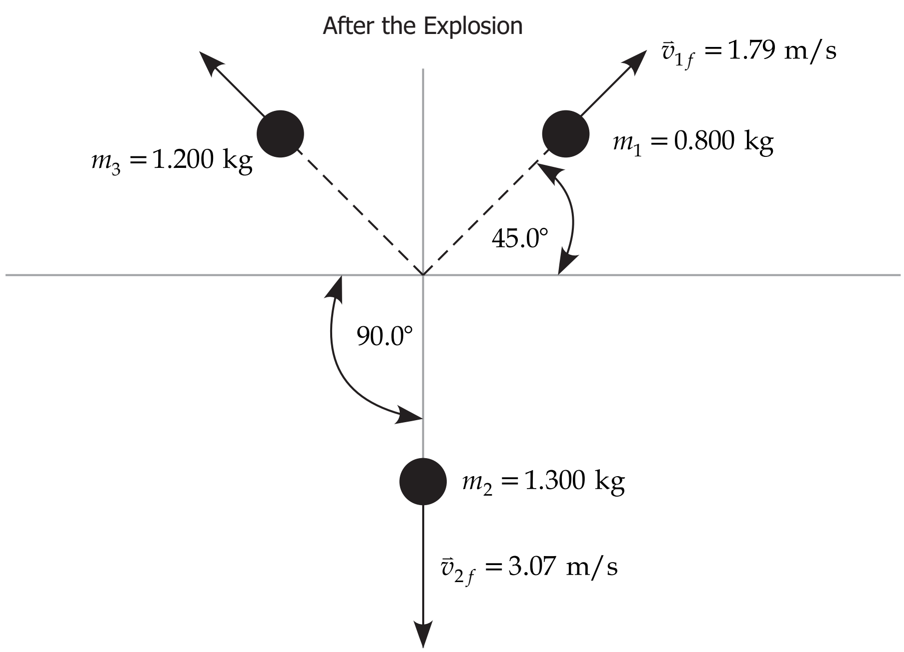
Learning Activity 5.7

Momentum and Explosions in Two Dimensions

Solve the following problem to check your understanding of explosions in two dimensions. You may check your work against the answer key found at the end of Module 5.

The physics of a shattered concrete paving stone and conservation of momentum in an explosion in 2D

1. By accident, a round concrete paving stone of mass 3.300 kg is dropped. When it strikes the floor, it breaks apart and three pieces fly off parallel to the floor. The mass m_1, 0.800 kg, moves at 1.79 m/s in a direction 45.0° north of east. The mass m_2, 1.300 kg, moves at 3.07 m/s directly south.

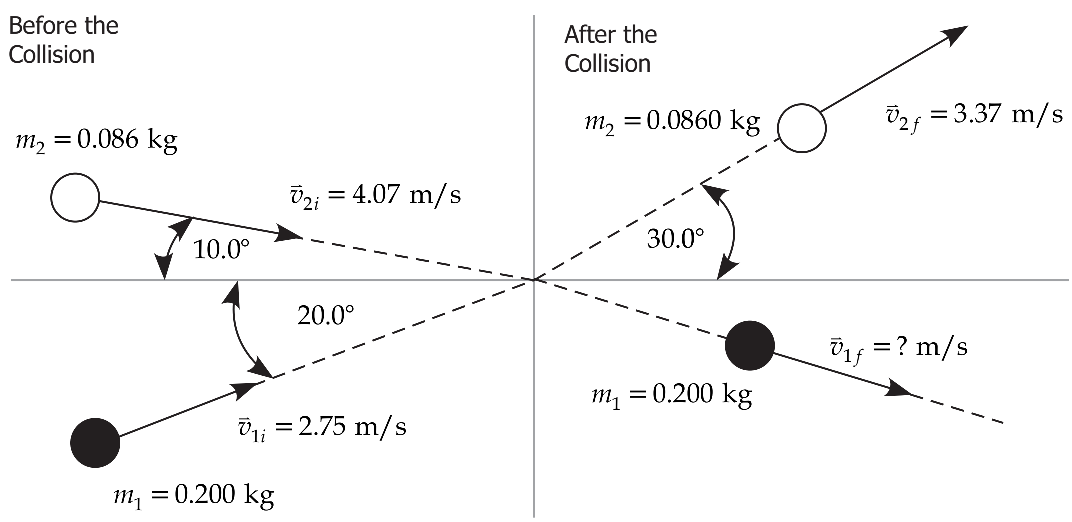


What is the magnitude and direction of the momentum, and the velocity of m_3 after the explosion?

Example 3: A Collision in Two Dimensions: Both Objects Initially Moving

The diagram below shows two balls moving towards each other and about to collide elastically. The mass m_1 = 0.200 kg is moving upwards at 2.75 m/s, and 20.0° as shown in the diagram below. The mass m_2 = 0.0860 kg is moving downwards at 4.07 m/s and 10.0°, as shown below. After the collision, the mass m_2 is moving at 3.37 m/s and 30.0° above the horizontal.

Determine the magnitude and direction of the momentum of m_1 after the collision.



Now, this is as complicated as it gets!

In the example, you can find $\vec{p}_{1i}$, $\vec{p}_{2i}$, and $\vec{p}_{2f}$ but you are missing $\vec{p}_{1f}$.

Solving $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$ for $\vec{p}_{1f}$ gives $\vec{p}_{1f} = \vec{p}_{1i} + \vec{p}_{2i} - \vec{p}_{2f}$.

Remember, you subtract a vector by adding the opposite.

$$\vec{p}_{1f} = \vec{p}_{1i} + \vec{p}_{2i} + \left(-\vec{p}_{2f}\right)$$

Given: m_1 = 0.200 kg

m_2= 0.0860 kg

$\vec{v}_{1i}$ = 2.75 m/s [20.0° North of East]

$\vec{v}_{2i}$ = 4.07 m/s [10.0° South of East]

$\vec{v}_{1f}$ = ?

$\vec{v}_{2f}$ = 3.37 m/s [30.0° North of East]

Unknown: The final momentum of m_1: $\vec{p}_{1f} = ?$

Equation:

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Solving for p_{1f} you obtain

$$\vec{p}_{1f} = \vec{p}_{1i} + \vec{p}_{2i} - \vec{p}_{2f}$$

Substitute and solve:

$$\vec{p}_{1f} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} - m_2\vec{v}_{2f}$$

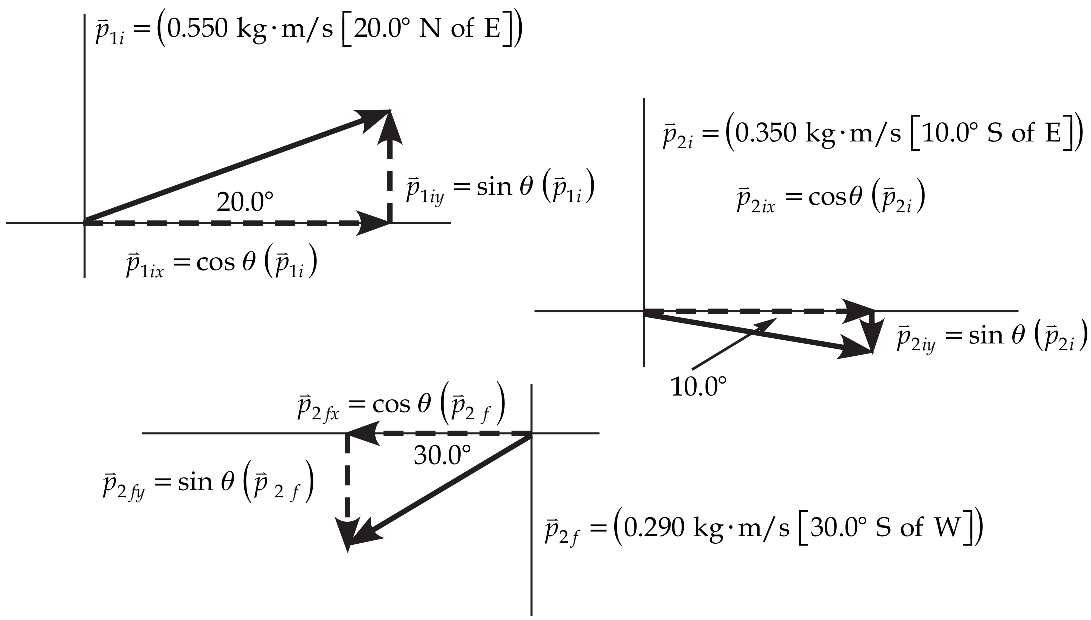
$$\vec{p}_{1f} = (0.200\text{ kg})(2.75\text{ m/s }[20.0°\text{ N of E}]) + (0.0860\text{ kg})(4.07\text{ m/s }[10.0°\text{ S of E}]) - (0.0860\text{ kg})(3.37\text{ m/s }[30.0°\text{ N of E}])$$

$$\vec{p}_{1f} = (0.550\text{ kg}\cdot\text{m/s }[20.0°\text{ N of E}]) + (0.350\text{ kg}\cdot\text{m/s }[10.0°\text{ S of E}]) - (0.290\text{ kg}\cdot\text{m/s }[30.0°\text{ N of E}])$$

$$\vec{p}_{1f} = (0.550\text{ kg}\cdot\text{m/s }[20.0°\text{ N of E}]) + (0.350\text{ kg}\cdot\text{m/s }[10.0°\text{ S of E}]) + (0.290\text{ kg}\cdot\text{m/s }[30.0°\text{ S of W}])$$

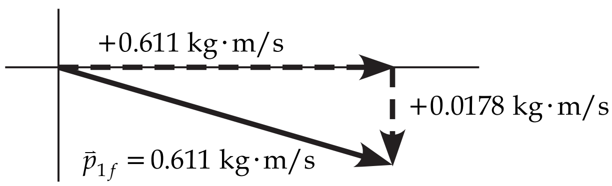
Now, this is vector addition.

Sketch the vectors and find the components in the x-direction and in the y-direction.



Set up a table with the components.

Vector	*x*-component	*y*-component
$\vec{p}_{1i}$	cos 20.0° (0.550 kg·m/s) = +0.517 kg·m/s	sin 20.0° (0.550 kg·m/s) = +0.188 kg·m/s
$\vec{p}_{2i}$	cos 10.0° (0.350 kg·m/s) = +0.345 kg·m/s	–sin 10.0° (0.350 kg·m/s) = –0.0608 kg·m/s
$-\vec{p}_{2f}$	–cos 30.0° (0.290 kg·m/s) = –0.251 kg·m/s	–sin 30.0° (0.290 kg·m/s) = –0.145 kg·m/s
$\vec{p}_{1f}$	+0.611 kg·m/s	–0.0178 kg·m/s



The components using the theorem of Pythagoras and trigonometry are used to determine $\vec{p}_{1f}$.

$$\vec{p}_{1f}{}^2 = \vec{p}_{1fX}{}^2 + \vec{p}_{1fY}{}^2 = 0.611^2 + 0.0178^2$$
$$\vec{p}_{1f} = 0.611 \text{ kg} \cdot \text{m/s}$$

The angle at the tail of the resultant can be found using inverse tangent.

$$\theta = \tan^{-1}\frac{0.0178}{0.611} = 1.69° \text{ S of E}$$
$$\theta = 1.69° \text{ below the horizontal}$$

The final momentum of the first object is 0.611 kg·m/s [1.69° south of east].

The magnitude of the final velocity is

$$\vec{v}_{1f} = \frac{\vec{p}_{1\text{f}}}{m_1} = \frac{0.611 \text{ kg} \cdot \text{m/s} \left[1.69° \text{ S of E}\right]}{0.200 \text{ kg}}$$
$$= 3.06 \text{ m/s} \left[1.69° \text{ S of E}\right]$$

The angle of the velocity vector is the same as the angle of the momentum vector.

Learning Activity 5.8

Momentum—Cumulative Exercise

There are three sets of practice questions in this learning activity. An answer key is available at the end of Module 5 for you to check your work after you have answered the questions.

Part 1

1. You're standing at rest in the middle of a pond on perfectly frictionless ice. Since the ice is frictionless, you cannot just walk across the ice as the soles of your boots cannot grip the ice. Using the ideas discussed in this module, describe a method by which you could start to move and reach the shore. Explain.
2. You and your friend notice that cars involved in car crashes are severely mangled. Your friend suggests that cars should be built more strongly so that damage is minimized during car crashes. Do you agree with your friend's suggestion? Why or why not?
3. During a fireworks display, a rocket is fired upwards. The rocket explodes just as it reaches its maximum height where it is momentarily at rest. Consider the moment just before the explosion and the moment just after the explosion when the pieces began to move, creating the display. Is kinetic energy conserved? Is momentum conserved? Explain your answers.

Part 2

Indicate whether each of the following statements is true or false by placing in the space provided a "T" for true and an "F" for false.

____ 1. The linear momentum of an object is the product of its average force and the time interval during which the force acts.

____ 2. A fielder draws his hand back as he catches a baseball to reduce the force needed to stop the ball.

____ 3. The total vector momentum of a system of objects remains constant, even though a net external force acts on the system of objects.

____ 4. If two stationary skaters "push-off" against each other, they will have exactly the same momentum.

____ 5. A force that increases linearly is illustrated on a graph by a straight line.

(continued)

____ 6. The area under the line of a force-versus-time graph is the change in velocity of an object.

____ 7. The unit for impulse is N·s.

____ 8. If a curling rock strikes a stationary rock of equal mass head on, all of the momentum is transferred to the stationary rock and the rock that was moving becomes stationary.

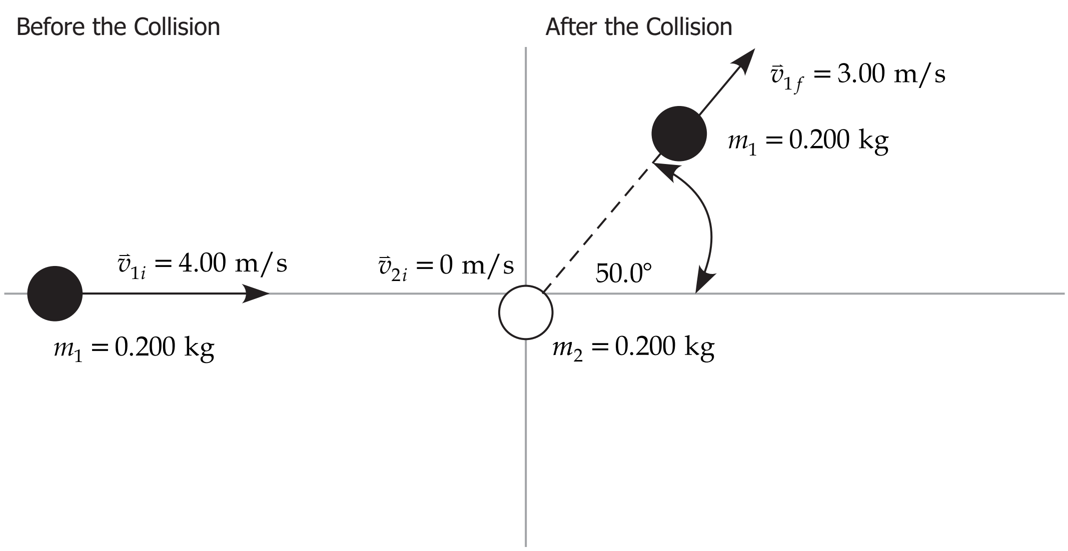
____ 9. The velocity of the centre of mass of the system of objects is larger after the collision than before the collision.

____ 10. If a moving object explodes into two pieces of equal mass, the total momentum of the system increases.

Part 3

The physics of billiard ball collisions and the conservation of momentum in 2D

1. Two identical billiard balls of mass 0.200 kg collide. The ball m_1 is moving at 4.00 m/s to the right towards a stationary billiard ball of mass m_2. The collision is not head on, but a glancing collision. One of the balls, m_1 moves at 3.00 m/s at an angle of 50.0° above the horizontal. What is the magnitude and direction of the momentum, and the velocity of m_2 after the collision?



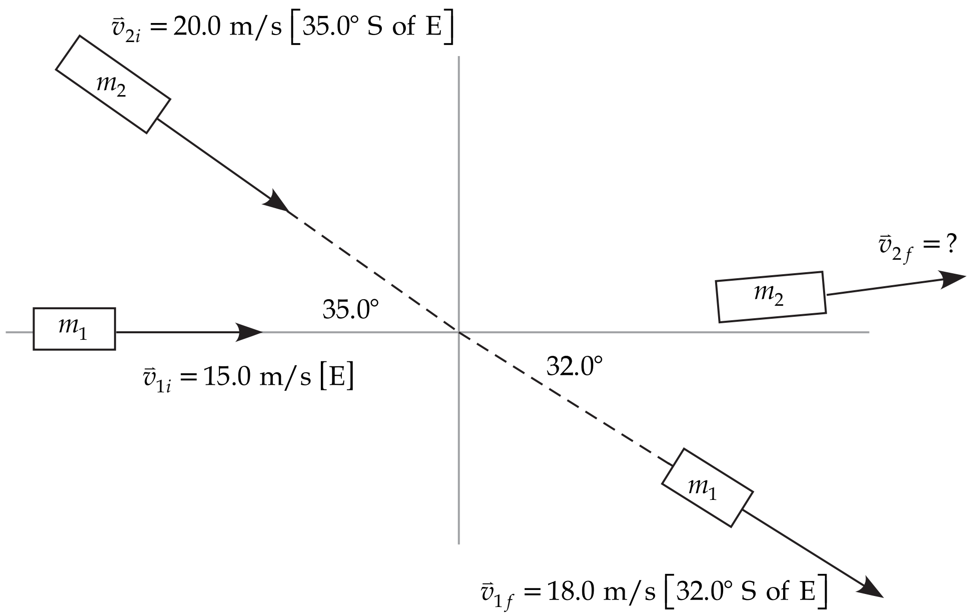
(continued)

Learning Activity 5.8: Momentum—Cumulative Exercise (continued)

The physics of vehicle collisions and the conservation of momentum in 2D

2. A car, mass m_1 = 1250 kg, is travelling east at 15.0 m/s. This car is sideswiped by a second car, mass m_2 = 1550 kg, travelling at 20.0 m/s [35.0° S of E]. After the collision the first car is travelling at 18.0 m/s [32.0° S of E].

   Determine the final momentum and final velocity of the second car.



Lesson Summary

In this lesson, you studied collisions and explosions in two dimensions.

In a two-dimensional collision or explosion, the total initial momentum is equal to the total final momentum.

You use the law of conservation of momentum:

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

In the case of an object at rest that explodes, the total momentum of the system is 0 kg·m/s. If three objects are formed from the initial object, the momentum of the third object is equal but opposite to the sum of the momenta of the first two objects.

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}} = 0\ \text{kg}\cdot\text{m/s}$$
$$0\ \text{kg}\cdot\text{m/s} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f}$$
$$\vec{p}_{3f} = -\left(\vec{p}_{1f} + \vec{p}_{2f}\right)$$

Since these vectors do not point along the same straight line, you must use the component method to add the vectors together.

Recall that the steps of the component method were:

1. Sketch each vector with its foot at the origin.
2. On the sketch, draw in the components of the vector to enclose the angle given at the foot of the vector. Assign the correct trigonometric function (sin or cos) to determine the magnitude of each component.
3. Record each component in a table such as the one below.

Vector	*x*-component	*y*-component
$\vec{p}_{1f}$		
$\vec{p}_{2f}$		
$\vec{p}_{3f} = \left(\vec{p}_{1f} + \vec{p}_{2f}\right)$		

4. Determine the x-component of the resultant by adding the values in the x-component column. Determine the y-component of the resultant by adding the values in the y-component column. Draw a sketch of these components along with the resultant vector with the angle θ at the foot of the resultant vector.

5. Using the theorem of Pythagoras, combine the magnitude of the x-component and the y-component to obtain the magnitude of the resultant.

6. The tangent function will allow you to calculate the angle θ.

$$\theta = \tan^{-1} \frac{\text{opposite}}{\text{adjacent}}$$

For collisions involving two objects with either one or both objects initially moving, you may once again apply the law of conservation of momentum to the situation. Unlike the explosion of a motionless object, the total momentum of the system is not 0 kg·m/s.

Solving these problems requires the same approach as was used earlier with collisions along a straight line. The only difference with these problems involving collisions in two dimensions is that you must use of the component method to add or subtract vectors; the concepts are identical but the analysis of collisions in two dimensions is a more lengthy process.

NOTES

Click on “Applets Menu,” then “Dynamics,” then “Collisions.” Observe collisions in two dimensions. Momentum is conserved. Change the value of “d” and see what happens.

Momentum

https://surendranath.org/

Video - 2D Momentum

This video previews the strategy for the analysis of the law of conservation of momentum in 2-dimensions.

Please focus on the method using components.

https://youtu.be/ZRq3idFDbyY

Video - Collisions in Multiple Dimensions

This video uses the component method to solve for the magnitudes of 2 balls after a collision occurs between the balls. The angles of the velocities are given for the two balls after the collision.

https://youtu.be/0Yo7Izga1q8

Video - 2D Conservation of Momentum Example using Air Hockey Discs

This video analyzes a collision between two air hockey pucks using the law of conservation of momentum. The object is to find the final velocity of one of the pucks after the collision.

The analysis uses the idea that ∑ ix = ∑fx and ∑ iy = ∑fy

Substituting mv for each of the momenta you can solve for the x-component of the missing velocity and the y-component of the missing velocity

These components can then be added to determine the missing final velocity.

Finally a check is done to determine if kinetic energy was conserved during this collision.

There are a lot of calculations in this video. Please focus on the strategy that was employed to solve the problem.

https://youtu.be/nuBE7I6-yfk

Video - Linear Momentum and Impulse Review

This review covers linear momentum, the law of conservation of momentum, types of collisions, impulse and impulse during collisions.

https://youtu.be/V54MeDvJNXo

Video - Mechanical Energy and Momentum Equations and When To Use Them!

This video compares the application of the law of conservation of energy and the law of conservation of momentum.

https://youtu.be/3KqwsjY1E60

MODULE 5 SUMMARY

Congratulations! You have finished the module on impulse and momentum.

As you have witnessed, impulse and momentum require us to consider forces and the time over which they act. You will recall the analysis of forces and distances resulted in work and energy.

The major idea in this module was momentum and the law of conservation of momentum. This law was applied for many situations, such as explosions in one and two dimensions, collisions between objects moving along a straight line, and glancing collisions between objects that resulted in motion in two dimensions. The law of conservation of momentum is always applied in the same manner. It is the details for each particular problem that supply the differences. The best approach is to start with the large, general concept and fill in the details for the given problem.

Submitting Your Assignments

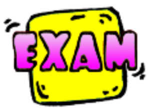
It is now time for you to submit Assignments 5.1 to 5.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 5 assignments and organize your material in the following order:

- Module 5 Cover Sheet (found at the end of the course Introduction)
- Assignment 5.1: Calculating Impulse, Momentum, and Force
- Assignment 5.2: Conservation of Linear Momentum
- Assignment 5.3: Video Laboratory Activity: A Collision in Two Dimensions

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Midterm Examination



Congratulations, you have finished Module 5 in the course. The midterm examination is out of 100 marks and worth 20% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 5.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will be allowed to use a calculator to write your examination. The calculator that you bring with you should be a graphing or scientific calculator. In addition, you will be supplied with an equation sheet and any constants that may be required.

A maximum of 3 hours is available to complete your midterm examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

Midterm Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The midterm practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

Grade 12 Physics (40S)

Module 5: Momentum

Learning Activity Answer Keys

Module 5: Momentum

Learning Activity 5.1: Calculating Impulse

1. A car swerves out of control and hits a guardrail. It exerts a force of 4.00×10^3 N on the guardrail. If the guardrail exerts an impulse of -5.00×10^2 N·s, how long does it take for the car to come to a stop?

   Answer:

   The car exerts a force of 4.00×10^3 N against the guardrail. According to Newton's Third Law of Motion, the guardrail will exert a force of 4.00×10^3 N against the car, bringing it to rest in time Δt.

   Given: Average force $\vec{F}_{avg} = -4.00 \times 10^3\ \text{N}$

   Impulse $\vec{J} = -5.00 \times 10^2\ \text{N}\cdot\text{s}$

   Unknown: Time interval $\Delta t = ?$

   Equation: $\vec{J} = \vec{F}_{avg}\Delta t$

   $$\Delta t = \frac{\vec{J}}{\vec{F}_{avg}}$$

   Substitute and solve:
   $$\Delta t = \frac{-5.00 \times 10^2\ \text{N}\cdot\text{s}}{-4.00 \times 10^3\ \text{N}} = 0.125\ \text{s}$$

   It takes the car is 0.125 seconds to come to a stop.

2. A water skier lets go of the tow rope and coasts to a stop. If the water exerts an average force of 2.80×10^2 N on him and he stops in 5.00 s, what is the impulse that the water exerts on him?

   Answer:

   Given: Average force $\vec{F}_{avg} = -2.80 \times 10^2$ N (force is opposite to the motion)

   Time interval $\Delta t = 5.00$ s

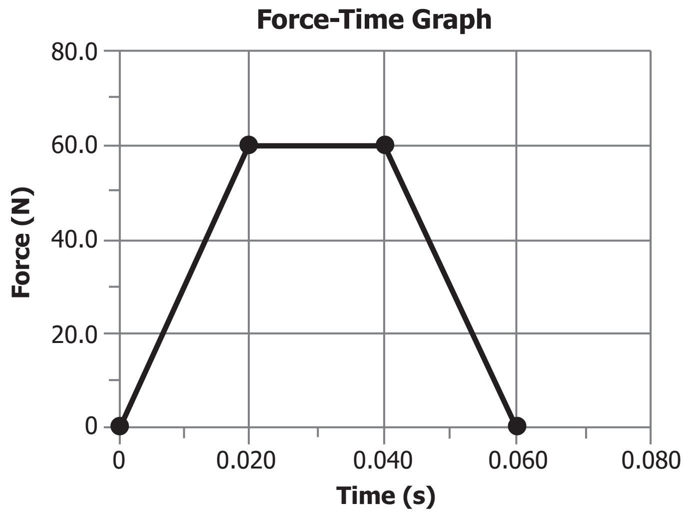
   Unknown: Impulse $\vec{J} = ?$

   Equation: $\vec{J} = \vec{F}_{avg}\Delta t$

   Substitute and solve:
   $$\vec{J} = \left(-2.80 \times 10^2\ \text{N}\right)\left(5.00\ \text{s}\right) = -1400\ \text{N}\cdot\text{s}$$
   $$= -1.40 \times 10^3\ \text{N}\cdot\text{s}$$

   The impulse applied is 1.40×10^3 N·s in a direction opposite to the original motion.

3. Calculate the impulse applied according to the force-time graph given below.



Answer:

The impulse applied is just given by the area beneath the curve. Since the forces vary, you must calculate four areas:

Area 1: 0 s to 0.020 s

Area 1 is a triangle, so use the area formula.

$$\text{Area triangle} = \frac{1}{2}bh$$

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \frac{1}{2}(60.0\ \text{N})(0.020\ \text{s}) = 0.60\ \text{N}\cdot\text{s}$$

Area 2: 0.020 s to 0.040 s

Area 2 is a rectangle.

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = (60.0\ \text{N})(0.020\ \text{s}) = 1.2\ \text{N}\cdot\text{s}$$

Area 3: 0.040 s to 0.060 s

Area 3 is another triangle.

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \frac{1}{2}(60.0\ \text{N})(0.020\ \text{s}) = 0.60\ \text{N}\cdot\text{s}$$

Area 4: 0.060 s – 0.080 s

Since the force is 0 N, the impulse is a 0 N·s.

The total impulse is just the sum of all the areas:

0.60 N·s + 1.2 N·s + 0.60 N·s = 2.4 N·s.

Learning Activity 5.2: Momentum

1. Which has more momentum: a golf ball of mass 45.5 g travelling at 60.0 m/s, or a baseball of mass 145 g travelling at 40.0 m/s?

   Answer:

   Assume both the golf ball and the baseball are travelling in the positive direction.

   For the golf ball, mass = 0.0455 kg and velocity = +60.0 m/s.

   $\vec{p} = m\vec{v}$ = (0.0455 kg)(+60.0 m/s) = +2.73 kg·m/s

   For the baseball, mass = 0.145 kg and velocity = +40.0 m/s.

   $\vec{p} = m\vec{v}$ = (0.145 kg)(+40.0 m/s) = +5.80 kg·m/s.

   Therefore, the baseball has more momentum.

2. In its orbit around the Sun, Earth travels at about 29.9 km/s. The mass of Earth is 5.98×10^{24} kg.

   a) Calculate the size of the linear momentum of Earth.

   Answer:

   The speed of 29.9 km/s should be converted to metres per second.

   $$\frac{29.9\text{ km}}{1\text{ s}} = \frac{29900\text{ m}}{1\text{ s}}$$

   $\vec{p} = m\vec{v}$ = $(5.98 \times 10^{24}$ kg)(29900 m/s) = 1.79×10^{29} kg·m/s

   The magnitude of Earth's linear momentum is 1.79×10^{29} kg·m/s.

   b) Is the linear momentum of Earth constant in size? Is the linear momentum constant in direction? Describe the force that applies the impulse to cause the change in momentum of Earth.

   Answer:

   Since the speed of Earth is fairly constant on its journey around the Sun, the size of the momentum of Earth is fairly constant.

   Since Earth travels with nearly uniform circular motion, the direction of motion and, therefore, the direction of the linear momentum are constantly changing. So, the linear momentum is not constant because the direction of the linear momentum is changing.

   The force that causes a change in momentum is the force that accelerates Earth. This is a centripetal force due to the pull of gravity of the Sun on Earth.

3. Two identical golf balls have the same speed, one travelling north and one travelling south. Do these golf balls have the same momentum? Explain.

   Answer:

   No, the golf balls do not have the same momentum. The magnitudes of their momenta will be identical, but, since momentum is a vector quantity, direction must be taken into consideration. The golf balls have opposite directions of motion; therefore, one ball has momentum pointing north and the other ball has momentum pointing south.

Learning Activity 5.3: Impulse and Momentum

1. A 10.0 kg mass is at rest when it is acted upon by an unknown force for 1.50 seconds. If the velocity of the mass is 5.00 m/s after 1.50 seconds, what is the size of the force?

   Answer:

   Assume the object is moving in the positive direction.

   Given: Mass $m = 10.0$ kg

   Time interval $\Delta t = 1.50$ s

   Initial velocity $\vec{v}_0 = 0 \text{ m/s}$

   Final velocity $\vec{v}_f = +5.00 \text{ m/s}$

   Unknown: Force $\vec{F} = ?$

   Equation: $\vec{F}\Delta t = m\vec{v}_f - m\vec{v}_0$

   Substitute and solve:

   $$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_0}{\Delta t}$$

   $$\vec{F} = \frac{(10.0 \text{ kg})(+5.00 \text{ m/s}) - 0}{1.50 \text{ s}}$$

   $$\vec{F} = +33.3 \text{ N}$$

   The force is 33.3 N in the positive direction.

The physics of catching a freely falling parachutist

2. A girl of mass 80.0 kg jumps out of an airplane and, after free-falling for several minutes, pulls the cord on a parachute.

   a) What impulse is applied to the girl if the parachute slows her from 100.0 km/h to 10.2 km/h?

      Answer:

      Assume down is the negative direction.

      Given: Mass $m = 180.0$ kg

      Initial velocity $\vec{v}_0 = -100.0 \text{ km/h} = -100000 \text{ m}/3600 \text{ s} = -27.8 \text{ m/s}$

      Final velocity $\vec{v}_f = -10.2 \text{ km/h} = -10200 \text{ m}/3600 \text{ s} = -2.83 \text{ m/s}$

      Unknown: Impulse $\vec{F} = ?$

Equation: Since you don't know the time and the force, you must find the impulse from the change in the momentum of the girl.

$$\vec{F}\Delta t = m\vec{v}_f - m\vec{v}_0$$

Substitute and solve:

$$\vec{F}\Delta t = (80.0 \text{ kg})(-2.83 \text{ m/s}) - (80.0 \text{ kg})(-27.8 \text{ m/s})$$

$$\vec{F}\Delta t = -226 \text{ kg}\cdot\text{m/s} - (-2220 \text{ kg}\cdot\text{m/s}) = +1994 \text{ N}\cdot\text{s}$$

The impulse applied to the girl was 1990 N·s in the positive or upwards direction.

b) If this change in velocity occurred during 11.0 seconds, what average force was applied to the girl?

Answer:

Since you know the impulse and the time, you can just use $\vec{J} = \vec{F}\Delta t$ to calculate the force.

$$\vec{F} = \frac{\vec{J}}{\Delta t} = \frac{+1990 \text{ N}\cdot\text{s}}{11.0 \text{ s}} = +181 \text{ N}$$

The average force applied to the girl was 181 N upwards.

The physics of catching a hockey puck

3. a) A hockey goalie catches the puck (mass = 0.170 kg) in his glove and slows it to a stop from 20.0 m/s during 0.100 seconds. What average force was exerted on his glove by the puck?

Answer:

Assume the puck is moving in the positive direction.

Given:	Mass	m = 0.170 kg
	Time interval	Δt = 0.100 s
	Initial velocity	$\vec{v}_0$ = +20.0 m/s
	Final velocity	$\vec{v}_f$ = 0 m/s
Unknown:	Force	$\vec{F} = ?$
Equation:		$\vec{F}\Delta t = m\vec{v}_f - m\vec{v}_0$

Substitute and solve:

$$\vec{F} = \frac{(0.170\text{ kg})(0\text{ m/s}) - (0.170\text{ kg})(+20.0\text{ m/s})}{0.100\text{ s}}$$

$$= \frac{-3.40\text{ kg}\cdot\text{m/s}}{0.100\text{ s}}$$

$$\vec{F} = -34.0\text{ N}$$

This represents the force that acted on the puck. By Newton's third law, the force of the puck on the glove should be +34.0 N.

b) By moving his hand back when catching the puck, the goalie can increase the time required to stop the puck to 0.500 seconds. What average force does the puck now exert on his hand?

Answer:

The change in momentum of the puck would still be the same, but the time is now lengthened to 0.500 s. The new force will be

$$\vec{F} = \frac{-3.40\text{ kg}\cdot\text{m/s}}{0.500\text{ s}} = -6.80\text{ N.}$$

The force of the puck on the glove should be +6.80 N.

This is a very important concept that is used to lessen forces during collisions involving passengers in automobiles. If the length of time over which a passenger is stopped increases, then the stopping force acting on the passenger decreases. This is the principle behind cushioning devices like airbags and padding.

The physics of catching a baseball

4. Why does a fielder draw her hand back as she catches a baseball?

Answer:

The fielder must apply an impulse to the baseball as she catches it. Since impulse is a product of the force and the time during which the force is applied, by lengthening the time during which her hand stops the ball, the fielder can lessen the force required.

The physics of catching an egg without breaking it

5. You are participating in a Physics Olympics event called the egg toss. How could you improve your chances of catching a tossed egg without breaking it?

   Answer:

   Again, an impulse must be applied to the egg in order to bring it to rest. The egg will break if the force applied to stop the egg is large. To reduce the force applied in stopping the egg, the trick is to lengthen the time over which the force acts. This could be done by bringing your hands back towards your body as you catch the egg. If you're allowed other articles, you could catch the egg with a blanket held loosely by two people. Again, the stopping time is lengthened and the stopping force is reduced.

The physics of impulse in sports

6. What is the impulse exerted in each of the following cases?

   a) A hockey stick exerting an average force of 120.0 N on a puck during the 0.0500 s they are in contact

      Answer:

      The impulse is the product of the average force and the difference in time.

      $$\vec{J} = \vec{F}\Delta t = (120.0\ \text{N})(0.0500\ \text{s}) = 6.00\ \text{N} \cdot \text{s}$$

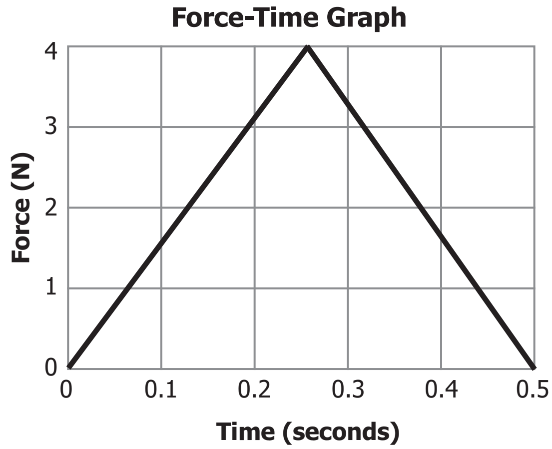
   b) A force of 25.2 N east on a bowling ball for 1.05 s

      Answer:

      In stating the impulse, we must add the direction, since it is specifically stated in the question and since impulse is a vector.

      $$\vec{J} = \vec{F}\Delta t = (25.2\ \text{N})(1.05\ \text{s}) = 26.5\ \text{N} \cdot \text{s}\ [\text{east}]$$

c) A billiard ball bouncing off a cushion if the force-time graph of the collision appears as below



Answer:

To determine the impulse, find the area under the curve.

$$\vec{J} = \frac{1}{2}(4\text{ N})(0.5\text{ s}) = 1\text{ N}\cdot\text{s}$$

For both triangles then, $\vec{J}_{\text{total}} = 2\text{ N}\cdot\text{s}$.

The physics of momentum and a freight train and an automobile

7. A freight train moves due north with a speed of 1.40 m/s. The mass of the train is 4.50×10^5 kg. How fast would a 1.80×10^3 kg automobile have to be moving north to have the same momentum as the train?

   Answer:

   In order for the automobile to have the same momentum as the train,

   $$(m_{\text{auto}})(v_{\text{auto}}) = (m_{\text{train}})(v_{\text{train}})$$

   $$v_{\text{auto}} = \frac{(m_{\text{train}})(v_{\text{train}})}{m_{\text{auto}}} = \frac{(4.50 \times 10^5\text{ kg})(1.40\text{ m/s})}{1.80 \times 10^3\text{ kg}} = 3.50 \times 10^2\text{ m/s}$$

The physics of rain and the force on the roof of a car

8. During a rainstorm, rain comes straight down with a velocity of $\vec{v}$ = –15.0 m/s and hits the roof of the car perpendicularly. The mass of the rain that strikes the car roof is 0.0600 kg/s. Assuming that the rain comes to rest upon striking the car roof ($\vec{v}_f$ = 0), find the average force exerted by the car roof on the rain.

   Answer:

   The average force needed to reduce the rain's velocity from $\vec{v}_0$ = –15.0 m/s to $\vec{v}_f$ = 0 m/s can be found using the impulse-momentum theorem.

   $$\vec{J} = \vec{F}\Delta t \text{ or } \vec{F}\Delta t = m\vec{v}_f - m\vec{v}_0$$

   $$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_0}{\Delta t}$$

   Since $\vec{v}_f$ = 0 m/s, the equation becomes

   $$\vec{F} = \frac{-m\vec{v}_0}{\Delta t}$$

   The term $m/\Delta t$ is the mass of rain per second that strikes the car roof so that

   $m/\Delta t$ = 0.0600 kg/s

   The average force acting on the rain is $\vec{F}$ = –(0.0600 kg/s)(–15.0 m/s) = +0.900 N

   This force is positive or in the upward direction. This is a reasonable result since the roof must exert an upward force on each downward-moving raindrop in order to bring it to rest. As a matter of interest, according to Newton's action-reaction law, the force exerted on the roof by the rain also has a magnitude of 0.900 N but points downward.

Learning Activity 5.4: Conservation of Momentum during Explosions

1. In Times Square in New York City, people celebrate New Year's Eve. Some just stand around, but many more move about randomly. Consider a system comprised of all of these people. Approximately, what is the total linear momentum of this system at any given instant? Justify your answer.

   Answer:

   Since linear momentum is a vector quantity, the total linear momentum of any system is the resultant of the linear momenta of the constituents. The people who are standing around have zero momentum. Those who move randomly carry momentum randomly in all directions. Since there is such a large number of people, there is, on average, just as much linear momentum in any one direction as in any other. On average, the resultant of this random distribution is zero. Therefore, the approximate linear momentum of the Times Square system is zero.

2. With the engines off, a spaceship is coasting at a velocity of $+2.30 \times 10^2$ m/s through outer space. It fires a rocket straight ahead at an enemy vessel. The mass of the rocket is 1.30×10^3 kg, and the mass of the spaceship (not including the rocket) is 4.00×10^6 kg. The firing of the rocket brings the spaceship to a halt. What is the velocity of the rocket?

   Answer:

Before Explosion

$m_{\text{spacecraft}} = 4.00 \times 10^6 \text{ kg}$ $m_{\text{rocket}} = 1.30 \times 10^3 \text{ kg}$

$\vec{v}_{\mathrm{S}i} = \vec{v}_{\mathrm{R}i} = +2.30 \times 10^2$

After Explosion

$m_{\text{spacecraft}} = 4.00 \times 10^6 \text{ kg}$ $m_{\text{rocket}} = 1.30 \times 10^3 \text{ kg}$

$\vec{v}_{\mathrm{S}f} = 0 \text{ m/s}$ $\vec{v}_{\mathrm{R}f} = ? \text{ m/s}$

Unknown: Final velocity of the rocket: $\vec{v}_{Rf} = ?\ \text{m/s}$

Equation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Adjusting the equation for the spaceship and the rocket yields:

$m_S\vec{v}_{Si} + m_R\vec{v}_{Ri} = m_S\vec{v}_{Sf} + m_R\vec{v}_{Rf}$

Substitute and solve:

$$\left(4.00 \times 10^6\ \text{kg}\right)\left(+2.30 \times 10^2\ \text{m/s}\right) + \left(1.30 \times 10^3\ \text{kg}\right)\left(+2.30 \times 10^2\ \text{m/s}\right) = 0\ \text{kg}\cdot\text{m/s} + \left(1.30 \times 10^3\ \text{kg}\right)\vec{v}_{Rf}$$

$$9.20 \times 10^8\ \text{kg}\cdot\text{m/s} = \left(1.30 \times 10^3\ \text{kg}\right)\vec{v}_{Rf}$$

$$\vec{v}_{Rf} = 7.08 \times 10^5\ \text{m/s}$$

The final velocity of the rocket is 7.08×10^5 m/s.

Learning Activity 5.5: Analyzing Head-on Collisions

The physics of comparing kinetic energy and momentum (conceptual)

1 a) Consider a single object. Can this object have no momentum but still possess kinetic energy? Account for your answer.

Answer:

Consider a massive throng in Times Square on New Year's Eve. A person in the throng (a single object) is either at rest (kinetic energy and momentum are both zero) or in motion (kinetic energy and momentum are both non-zero). So a single object cannot have kinetic energy without momentum.

b) Consider a system of two objects. Can this system have a total momentum that is zero and a total kinetic energy that is not zero? Account for your answer.

Answer:

Yes, it is possible. Consider all the people in the throng. Many are still (no kinetic energy) but many are also moving and have kinetic energy. The total energy of the system is the scalar sum of all of the individual kinetic energies.

Since linear momentum is a vector quantity, the total linear momentum of any system is the resultant of the linear momenta of the constituents. The people who are standing around have zero momentum. Those who move randomly carry momentum randomly in all directions. Since there is such a large number of people, there is, on average, just as much linear momentum in any one direction as in any other. On average, the resultant of this random distribution is zero. Therefore, the approximate linear momentum of the Times Square system is zero.

The physics of two ice skaters pushing each other

2. a) Starting from rest, two ice skaters "push off" against each other on smooth, level ice where friction is negligible. The woman (m_1 = 54.0 kg) moves away with a velocity of $\vec{v}_{1f}$ = +2.50 m/s. What is the recoil velocity $\vec{v}_{2f}$ of the man (m_2 = 88.0 kg)?

Answer:

Since the skaters start from rest, the initial momentum of the system is 0 kg·m/s. Since there are no external forces, you can assume that momentum is conserved and the final momentum of the system is also 0 kg·m/s.

Given: Let right be the positive direction.

Mass of the woman	m_1 = 54.0 kg
Final velocity of the woman	$\vec{v}_{1f}$ = +2.50 m/s
Mass of the man	m_2 = 88.0 kg

Unknown: Final velocity of the man $\vec{v}_{2f} = ?$

Equation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Substitute and solve: $0 \text{ kg}\cdot\text{m/s} = (54.0 \text{ kg})(+2.50 \text{ m/s}) + (88.0 \text{ kg})\vec{v}_{2f}$

$$(88.0 \text{ kg})\vec{v}_{2f} = -135 \text{ kg}\cdot\text{m/s}$$

$$\vec{v}_{2f} = -1.53 \text{ m/s}$$

The final velocity of the man is 1.53 m/s to the left.

b) What is the change in momentum of the woman?

Answer:

$$\Delta\vec{p}_1 = (54.0 \text{ kg})(+2.50 \text{ m/s} - 0 \text{ m/s}) = +135 \text{ kg}\cdot\text{m/s}$$

Since the woman started from rest, her change in momentum is just her final momentum: +135 kg·m/s.

c) If the push-off lasted 0.0200 seconds, what force was exerted during this push-off?

Answer:

The woman's change in momentum equals the impulse applied to her. Knowing the impulse and the time, you can calculate the force using

$$\vec{J} = \vec{F}\Delta t \text{ or } \vec{F} = \frac{\vec{J}}{\Delta t} = \frac{-135 \text{ kg}\cdot\text{m/s}}{0.200 \text{ s}} = -675 \text{ N}.$$

The push-off was a force of 675 N.

The physics of a shell and a cannon

3. A shell of mass 8.000 kg leaves the muzzle of a cannon with a horizontal velocity of 600.0 m/s. Find the recoil velocity of the cannon, if its mass is 500.0 kg.

   Answer:

   Since the cannon and the cannonball are both at rest initially, the total momentum of the system will be 0 kg·m/s both before and after the cannon is fired.

   Given: Let right be the positive direction.

Mass of the cannonball	$m_1 = 8.000$ kg
Final velocity of the cannonball	$\vec{v}_{1f} = +600.0$ m/s
Mass of the cannon	$m_2 = 500.0$ kg

   Unknown: Final velocity of the cannon: $\vec{v}_{2f} = ?$

   Equation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

   Substitute and solve:

   $$0\ \text{kg}\cdot\text{m/s} = (8.000\ \text{kg})(+600.0\ \text{m/s}) + (500.0\ \text{kg})\vec{v}_{2f}$$

   $$(500.0\ \text{kg})\vec{v}_{2f} = -4800\ \text{kg}\cdot\text{m/s}$$

   $$\vec{v}_{2f} = -9.600\ \text{m/s}$$

   The cannon recoils at 9.600 m/s to the left.

The physics of a child and a sled

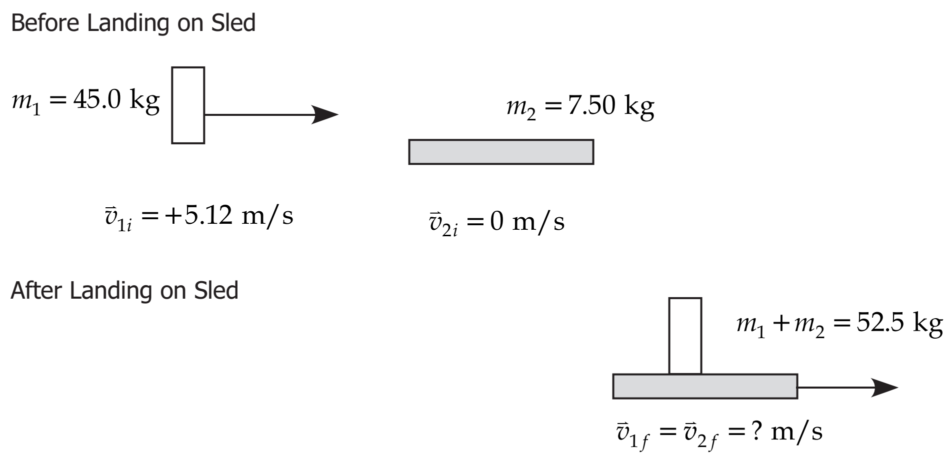
4. A 45.0 kg child runs with a horizontal velocity of +5.12 m/s and jumps onto a stationary 7.50 kg sled. Find the velocity of the child and the sled immediately after the child lands on the sled, if there was no friction.

   Answer:

   Given: Let right be the positive direction.

   Let the child be mass 1.

   Let the sled be mass 2.



   Unknown: The final velocity of the child on the sled: $\vec{v}_{1f} = \vec{v}_{2f} = ?$ m/s

   Equation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

   Since the two final velocities are the same, factor out the masses from the right side of the equation, giving

   $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_{2f}$

   Substitute and solve: $(45.0\text{ kg})(+5.12\text{ m/s}) + 0\text{ kg}\cdot\text{m/s} = (52.5\text{ kg})\vec{v}_{2f}$

   $+230.4\text{ kg}\cdot\text{m/s} = (52.5\text{ kg})\vec{v}_{2f}$

   $\vec{v}_{2f} = +4.39\text{ m/s}$

   The child and the sled move off to the right at 4.39 m/s.

The physics of two balls colliding elastically

5. A 4.00 kg ball moving to the right at 5.00 m/s collides head-on with a 2.00 kg ball moving to the left at 4.00 m/s. The 4.00 kg ball rebounds to the left at 1.00 m/s.

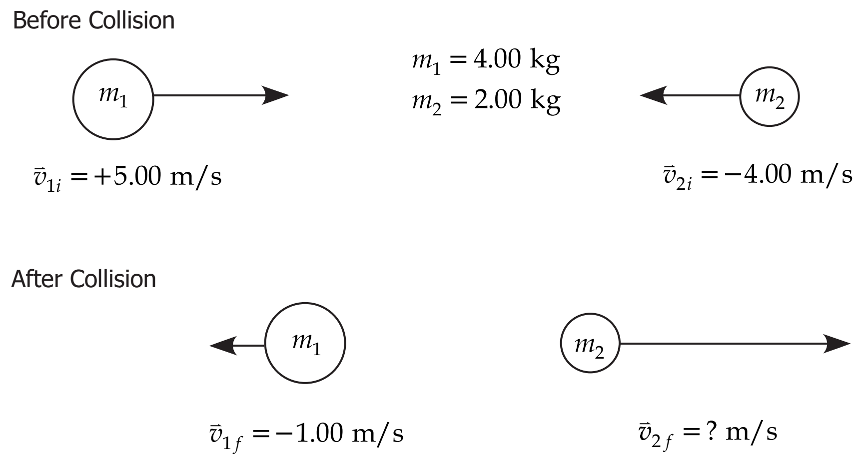
   a) Calculate the final velocity of the 2.00 kg ball.

   Answer:

   Given: Let "to the right" be the positive direction.

   Let the 4.00 kg ball be mass 1.

   Let the 2.00 kg ball be mass 2.



Unknown: Final velocity of the 2.00 kg ball: $\vec{v}_{2f} = ?$ m/s

Equation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Substitute and solve:

$$(4.00\text{ kg})(+5.00\text{ m/s}) + (2.00\text{ kg})(-4.00\text{ m/s}) = (4.00\text{ kg})(-1.00\text{ m/s}) + (2.00\text{ kg})\vec{v}_{2f}$$

$$(+20.0\text{ kg}\cdot\text{m/s}) + (-8.00\text{ kg}\cdot\text{m/s}) = (-4.00\text{ kg}\cdot\text{m/s}) + (2.00\text{ kg})\vec{v}_{2f}$$

$$\vec{v}_{2f} = \frac{+16.0\text{ kg}\cdot\text{m/s}}{(2.00\text{ kg})} = +8.00\text{ m/s}$$

The final velocity of the 2.00 kg object is 8.00 m/s [right].

b) Calculate the change in momentum of the 4.00 kg ball.

Answer:

$$\Delta\vec{p}_1 = m_1\Delta\vec{v}_1 = m_1\left(\vec{v}_{1f} - v_{1i}\right)$$
$$\Delta\vec{p}_1 = m_1\Delta\vec{v}_1 = (4.00\ \text{kg})\left(-1.00\ \text{m/s} - (+5.00\ \text{m/s})\right)$$
$$\Delta\vec{p}_1 = -24.0\ \text{kg}\cdot\text{m/s}$$

The change in momentum of the 4.00 kg ball is –24.0 kg·m/s.

c) Calculate the change in momentum of the 2.00 kg ball.

Answer:

The change in momentum of the 2.00 kg ball will be equal but opposite to the change in momentum of the 4.00 kg ball; that is, +24.0 kg·m/s.

d) Calculate the force of the collision if the collision lasted 0.0750 seconds.

Answer:

The change in momentum can be used to find the impulse applied to one of the balls.

Then, using the impulse equation, you can calculate the force.

$$\vec{J} = 24.0\ \text{kg}\cdot\text{m/s}$$
$$\vec{J} = \vec{F}\Delta t$$
$$\vec{F} = \frac{\vec{J}}{\Delta t} = \frac{24.0\ \text{N}\cdot\text{s}}{0.0750\ \text{s}} = 320 = 3.20 \times 10^2\ \text{N}$$

The average force of the interaction was 3.20×10^2 N.

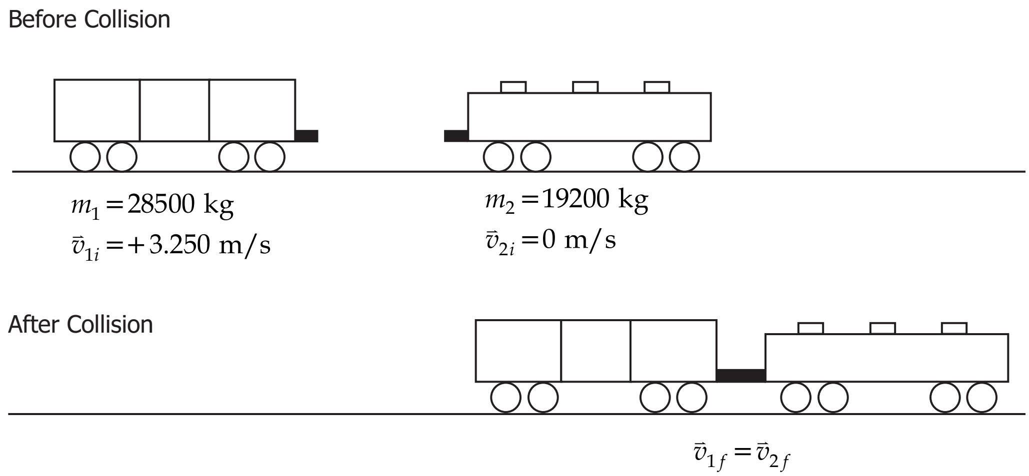
The physics of two colliding rail cars

6. A 28500 kg boxcar is moving at 3.25 m/s [east]. It collides with a stationary tanker car of mass 19200 kg. After the collision, the two railcars are coupled together and continue to move along the track.

   a) Calculate the final velocities of the railcars after the collision.

   Answer:

   Let east be the positive direction.



   Momentum is conserved.

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$(28500\text{ kg})(+3.25\text{ m/s}) + (19200\text{ kg})(0\text{ m/s}) = (28500\text{ kg})\vec{v}_{1f} + (19200\text{ kg})\vec{v}_{2f}$$

   But $\vec{v}_{1f} = \vec{v}_{2f}$, so

$$+92600\text{ kg}\cdot\text{m/s} + 0\text{ kg}\cdot\text{m/s} = (47700\text{ kg})\vec{v}_{1f}$$
$$\vec{v}_{1f} = +1.94\text{ m/s}$$

   The final velocities of both cars are +1.94 m/s or 1.94 m/s [east].

b) What is the change in momentum of the boxcar?

Answer:

$$\Delta\vec{p}_1 = m\Delta\vec{v}_1 = m\left(\vec{v}_{1f} - \vec{v}_{i0}\right)$$

S and S:

$$\Delta\vec{p}_1 = \left(28500\ \text{kg}\right)\left(\left(+1.94\ \text{m/s}\right) - \left(+3.25\ \text{m/s}\right)\right)$$

$$\Delta\vec{p}_1 = \left(28500\ \text{kg}\right)\left(-1.31\ \text{m/s}\right) = -37300\ \text{kg}\cdot\text{m/s}$$

The boxcar lost 37300 $\text{kg}\cdot\text{m/s}$ of momentum.

c) What was the impulse applied to the stationary tanker car?

Answer:

The boxcar and the tanker car will have equal but opposite momentum changes.

$$\Delta\vec{p}_2 = -\Delta\vec{p}_1 = -\left(-37300\ \text{kg}\cdot\text{m/s}\right) = +37300\ \text{kg}\cdot\text{m/s}$$

Since the impulse applied to the tanker car is its change in momentum, the impulse applied to the tanker car is $\vec{J} = \Delta\vec{p}_2 = +37300\ \text{N}\cdot\text{s}$.

The impulse applied to the tanker car is +37300 N·s or 37300 N·s [East].

d) If the collision lasted 0.0960 seconds, calculate the force of interaction.

Answer:

$$\Delta t = 0.0960\ \text{s}$$

$$\vec{F}\Delta t = \Delta\vec{p}\ \text{ rearranged to } \vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

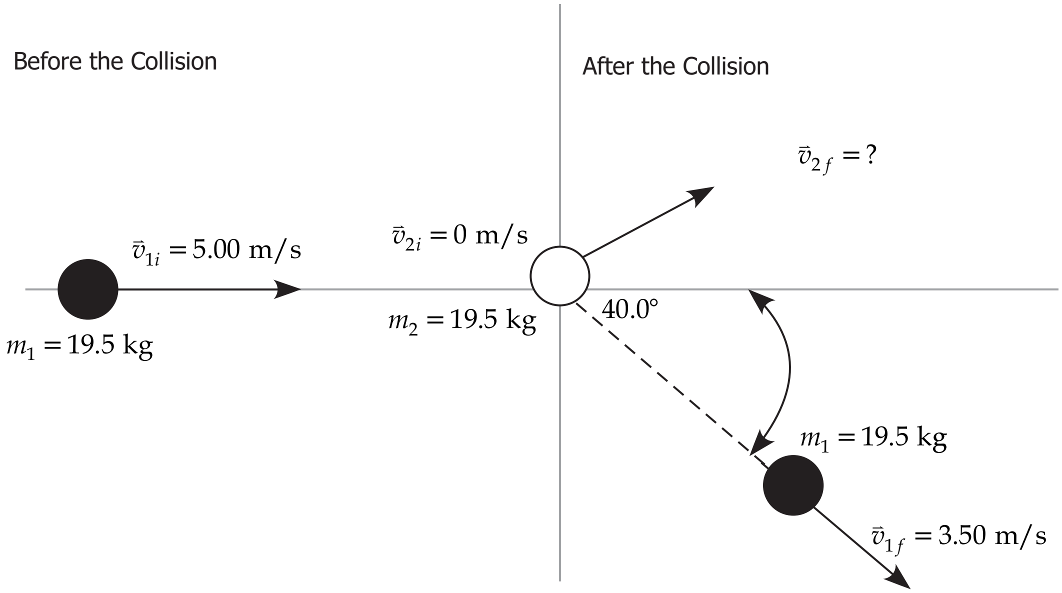
$$\vec{F} = \frac{37300\ \text{kg}\cdot\text{m/s}}{0.0960\ \text{s}} = 388000\ \text{N}$$

The force of the collision is 388000 N.

Learning Activity 5.6: Conservation of Momentum in Two Dimensions

The physics of curling stone collisions and the conservation of momentum

1. Two identical curling stones of mass 19.5 kg collide. The stone m_1 is moving at 5.00 m/s to the right towards a stationary stone of mass m_2. The collision is a glancing collision. After the collision, one of the stones m_1 moves at 3.50 m/s at an angle of 40.0° below the horizontal.



a) What is the total initial momentum of the system?

Answer:

Since only the first rock is initially moving, a total initial momentum of the system is just the initial momentum of the first rock.

$$\vec{p}_{\text{total initial}} = \vec{p}_{1i} + \vec{p}_{2i}$$
$$\vec{p}_{\text{total initial}} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$$
$$= (19.5 \text{ kg})(5.00 \text{ m/s [E]}) + (19.5 \text{ kg})(0 \text{ m/s})$$
$$= 97.5 \text{ kg}\cdot\text{m/s [E]}$$

The total initial momentum of the system is 97.5 kg·m/s [E].

b) What is the total final momentum after the collision?

Answer:

Since momentum is conserved during this collision, the total final momentum of the system is still 97.5 kg·m/s [E].

c) What is the momentum of m_2 after the collision?

Answer:

Since you know the total final momentum of the system and the final momentum of the first rock, you can calculate the final momentum of the second rock.

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$

$$\vec{p}_{\text{total final}} = \vec{p}_{1f} + \vec{p}_{2f}$$

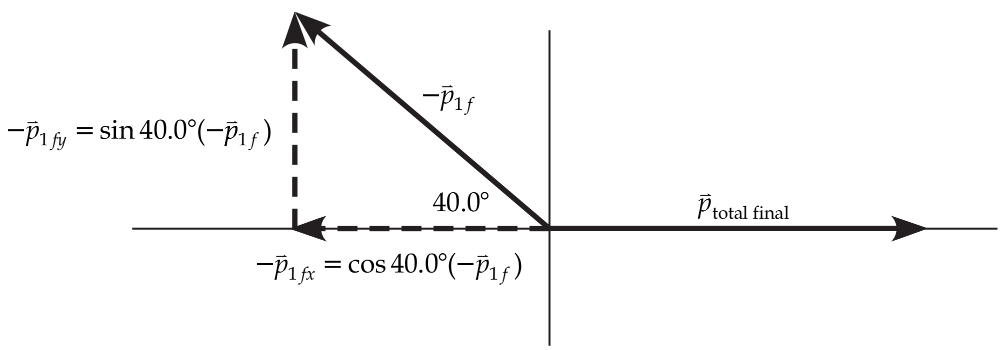
$$\vec{p}_{\text{total final}} - \vec{p}_{1f} = \vec{p}_{2f}$$

$$97.5 \text{ kg}\cdot\text{m/s [E]} - (19.5 \text{ m/s})(3.50 \text{ m/s})[40.0° \text{ S of E}] = \vec{p}_{2f}$$

$$97.5 \text{ kg}\cdot\text{m/s [E]} - 68.2 \text{ kg}\cdot\text{m/s } [40.0° \text{ S of E}] = \vec{p}_{2f}$$

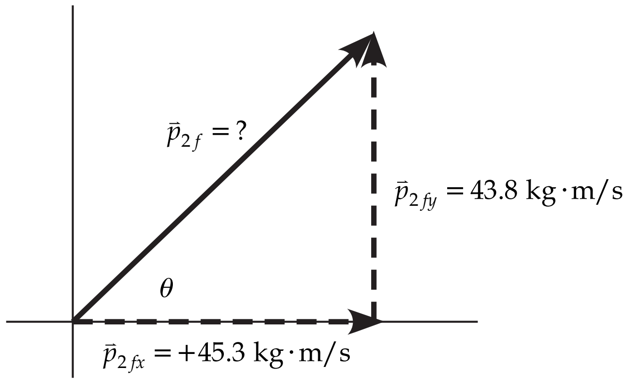
Subtract by adding the opposite.

$$97.5 \text{ kg}\cdot\text{m/s [E]} + 68.2 \text{ kg}\cdot\text{m/s } [40.0° \text{ N of W}] = \vec{p}_{2f}$$



Adding the vector components of the vector diagram shown, you obtain the following data:

Vector	*x*-component (E–W)	*y*-component (N–S)
$\vec{p}_{\text{total final}}$	+97.5 kg·m/s	0 kg·m/s
$-\vec{p}_{1f}$	–cos 40.0° (68.2 kg·m/s) = –52.2 kg·m/s	+sin 40.0° (68.2 kg·m/s) = +43.8 kg·m/s
$\vec{p}_{2f}$	+45.3 kg·m/s	+43.8 kg·m/s



To solve for $\vec{p}_{1f}$, you combine the components from the diagram using the theorem of Pythagoras.

$$\vec{p}_{2f}{}^2 = (45.3)^2 + (43.8)^2 = 3961$$

$$\vec{p}_{2f} = 63.0\ \text{kg}\cdot\text{m/s}$$

$$\theta = \tan^{-1}\frac{43.8}{45.3} = 44.0°$$

The resultant vector (final momentum of rock 2) is
63.0 kg·m/s [44.0° N of E]

d) What is the change in momentum of m_1 during the collision?

Answer:

Since rock 2 was initially not moving, you can easily find a change in momentum for rock 2.

$$\Delta\vec{p}_2 = \vec{p}_{2f} - \vec{p}_{2i} = 63.0\ \text{kg}\cdot\text{m/s}\ [44.0°\ \text{N of E}] - 0\ \text{kg}\cdot\text{m/s}$$

$$\Delta\vec{p}_2 = 63.0\ \text{kg}\cdot\text{m/s}\ [44.0°\ \text{N of E}]$$

The change in momentum for rock 1 is just equal but opposite to the change in momentum for rock 2.

$$\Delta\vec{p}_1 = 63.0\ \text{kg}\cdot\text{m/s}\ [44.0°\ \text{S of W}]$$

e) What impulse is applied to rock 2 during the collision?

Answer:

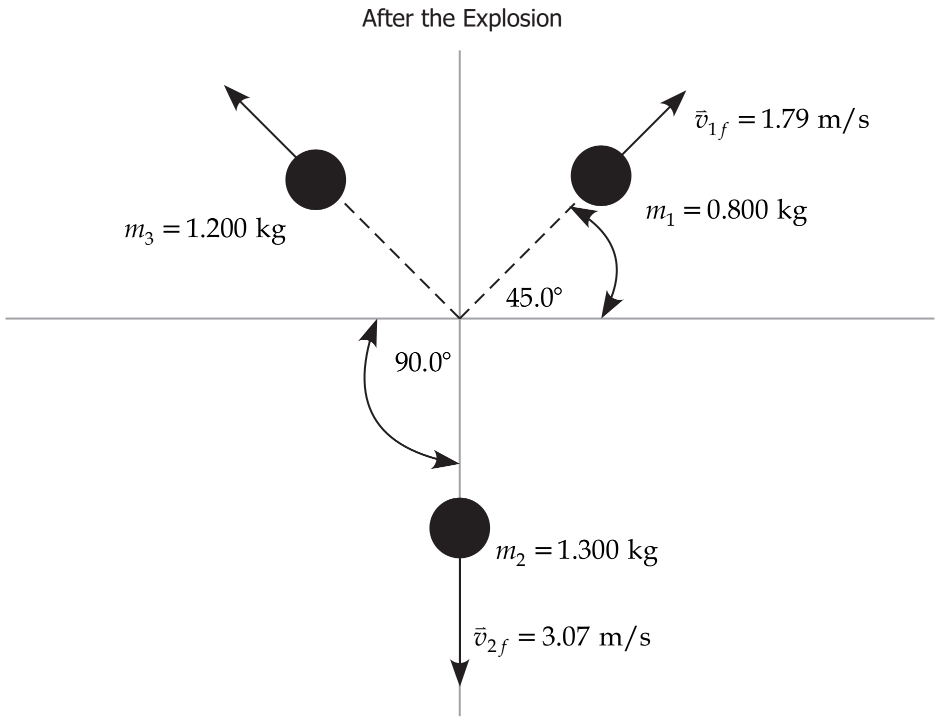
The impulse applied to rock 2 doing the collision is the same as its change in momentum.

The impulse applied to rock 2 is $\vec{J} = 63.0\ \text{N}\cdot\text{s}\ [44.0°\ \text{N of E}]$.

Learning Activity 5.7: Momentum and Explosions in Two Dimensions

The physics of a shattered concrete paving stone and conservation of momentum in an explosion in 2D

1. By accident, a round concrete paving stone of mass 3.300 kg is dropped. When it strikes the floor, it breaks apart and three pieces fly off parallel to the floor. The mass m_1, 0.800 kg, moves at 1.79 m/s in a direction 45.0° north of east. The mass m_2, 1.300 kg, moves at 3.07 m/s directly south.



What is the magnitude and direction of the momentum, and the velocity of m_3 after the explosion?

Answer:

The mass of the third piece of a plate is 3.300 kg – 1.300 kg – 0.800 kg = 1.200 kg.

You can ignore the fact that plate was falling since the vertical momentum was converted into horizontal momentum. You should only concern yourself with events that will occur in the horizontal plane.

In the horizontal direction, the plate was initially at rest so the initial momentum of the system is 0 kg·m/s.

Equation:

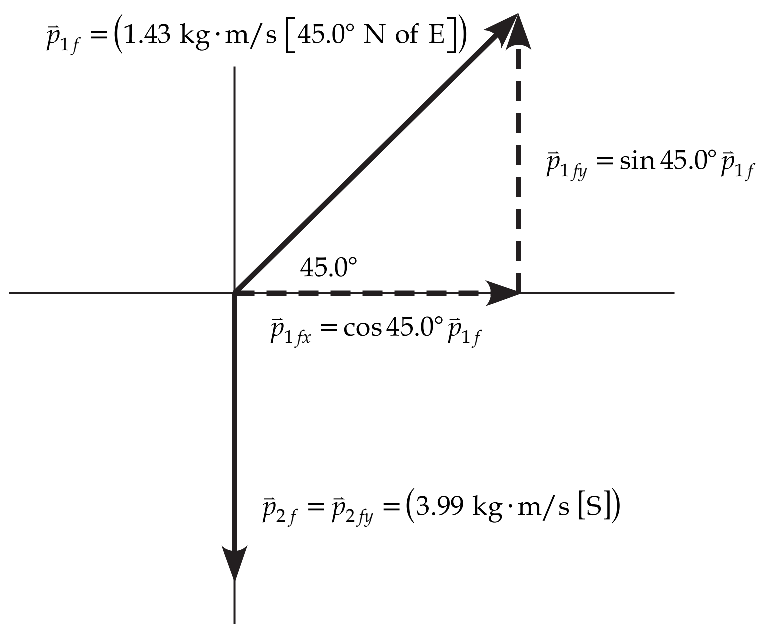
$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} + \vec{p}_{3i} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f}$$

Three pieces are formed so three momenta are used.

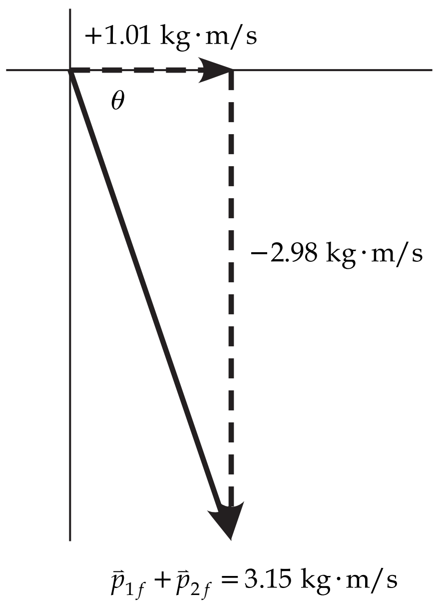
Substitute and solve:

$$\vec{p}_{\text{total initial}} = 0\ \text{kg}\cdot\text{m/s}$$
$$0\ \text{kg}\cdot\text{m/s} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f}$$
$$\vec{p}_{3f} = -\left(\vec{p}_{1f} + \vec{p}_{2f}\right)$$
$$\vec{p}_{3f} = -\Big(\big(0.800\ \text{kg}\big)\big(1.79\ \text{m/s}\ \big[45.0°\ \text{N of E}\big]\big) + \big(1.300\ \text{kg}\big)\big(3.07\ \text{m/s}\ [\text{S}]\big)\Big)$$
$$\vec{p}_{3f} = -\Big(\big(1.43\ \text{kg}\cdot\text{m/s}\ \big[45.0°\ \text{N of E}\big]\big) + \big(3.99\ \text{kg}\cdot\text{m/s}\ [\text{S}]\big)\Big)$$

Add these two vectors using the component method.



Vector	*x*-component (E–W)	*y*-component (N–S)
$\vec{p}_{1f}$	cos 45.0° (1.43 kg·m/s) = +1.01 kg·m/s	sin 45.0° (1.43 kg·m/s) = +1.01 kg·m/s
$\vec{p}_{2f}$	0 kg·m/s	–3.99 kg·m/s
$\vec{p}_{1f}+\vec{p}_{2f}$	+1.01 kg·m/s	–2.98 kg·m/s



Using the theorem of Pythagoras,

$$\left(\vec{p}_{1f}+\vec{p}_{2f}\right)^2=1.01^2+2.98^2=9.900$$

$$\vec{p}_{1f}+\vec{p}_{2f}=3.15\ \text{kg}\cdot\text{m/s}$$

Use inverse tangent to find the angle.

$$\theta=\tan^{-1}\frac{2.98}{1.01}=71.3°\left[\text{S of E}\right]$$

$$\vec{p}_{1f}+\vec{p}_{2f}=3.15\ \text{kg}\cdot\text{m/s}\left[71.3°\ \text{S of E}\right]$$

But,

$$\vec{p}_{3f}=-\left(\vec{p}_{1f}+\vec{p}_{2f}\right)$$

So $\vec{p}_{3f}=3.15\ \text{kg}\cdot\text{m/s}\left[71.3°\ \text{N of W}\right]$

The final velocity of the third piece is

$$\vec{v}_{3f}=\frac{\vec{p}_{3f}}{m_3}=\frac{3.15\ \text{kg}\cdot\text{m/s}\left[71.3°\ \text{N of W}\right]}{1.200\ \text{kg}}$$

$$\vec{v}_{3f}=2.62\ \text{m/s}\ \left[71.3°\ \text{N of W}\right]$$

Learning Activity 5.8: Momentum—Cumulative Exercise

Part 1

1. You're standing at rest in the middle of a pond on perfectly frictionless ice. Since the ice is frictionless, you cannot just walk across the ice as the soles of your boots cannot grip the ice. Using the ideas discussed in this module, describe a method by which you could start to move and reach the shore. Explain.

   Answer:

   Since there is no friction between your boots and the ice, you cannot walk to shore. In order to get yourself moving, you must be able to apply a force to some object so that the reaction force gets you going across the ice. You can do this by taking an article of clothing and throwing it horizontally, say to the east. The reaction force of the clothing on you will create an impulse that changes your momentum to the west. Once you're moving, you should continue to slide across the ice (Newton's first law) until you reach the shore.

2. You and your friend notice that cars involved in car crashes are severely mangled. Your friend suggests that cars should be built more strongly so that damage is minimized during car crashes. Do you agree with your friend's suggestion? Why or why not?

   Answer:

   The purpose of "crumple" zones at the front and rear of a car is to protect the people inside the car. Passengers inside the car experience large impulses as they are brought to rest during a car crash. Remember that impulse is a product of force and time interval. A vehicle that is built very strongly so that it does not crumple will stop very quickly during a car crash, reducing the time interval. To provide the same impulse, since the time interval is small, the force applied to the passenger must become larger. Larger forces cause more damage to passengers.

3. During a fireworks display, a rocket is fired upwards. The rocket explodes just as it reaches its maximum height where it is momentarily at rest. Consider the moment just before the explosion and the moment just after the explosion when the pieces began to move, creating the display. Is kinetic energy conserved? Is momentum conserved? Explain your answers.

   Answer:

   Kinetic energy is not conserved. The explosion represents the conversion of chemical potential energy into the kinetic energy of the particles produced by the explosion. The kinetic energy after the explosion is larger than the kinetic energy of the particles before the explosion.

   Momentum is conserved. Just before the explosion, the rocket was at rest, indicating that the total momentum was zero. After the explosion, the particles produced fly off in all directions with all sorts of velocities. However, if you were to add all of the momenta of these particles after the explosion, their total would still be zero.

Part 2

Indicate whether each of the following statements is true or false by placing in the space provided a "T" for true and an "F" for false.

F 1. The linear momentum of an object is the product of its average force and the time interval during which the force acts.

The linear momentum of an object is the product of its mass and velocity.

T 2. A fielder draws his hand back as he catches a baseball to reduce the force needed to stop the ball.

F 3. The total vector momentum of a system of objects remains constant, even though a net external force acts on the system of objects.

If a net external force acts on the system of objects, momentum is not conserved.

F 4. If two stationary skaters "push off" against each other, they will have exactly the same momentum.

The skaters will have momenta of equal magnitude but opposite direction. Since momentum is a vector, the different directions create different momenta.

T 5. A force that increases linearly is illustrated on a graph by a straight line.

F 6. The area under the line of a force-versus-time graph is the change in velocity of an object.

The area under a force-versus-time graph yields an impulse or change in momentum.

T 7. The unit for impulse is $N \cdot s$.

T 8. If a curling rock strikes a stationary rock of equal mass head-on, all of the momentum is transferred to the stationary rock and the rock that was moving becomes stationary.

F 9. The velocity of the centre of mass of the system of objects is larger after the collision than before the collision.

The velocity of the centre of mass of an isolated system of objects is the same before and after an interaction, such as a collision between objects within the system.

F 10. If a moving object explodes into two pieces of equal mass, the total momentum of the system increases.

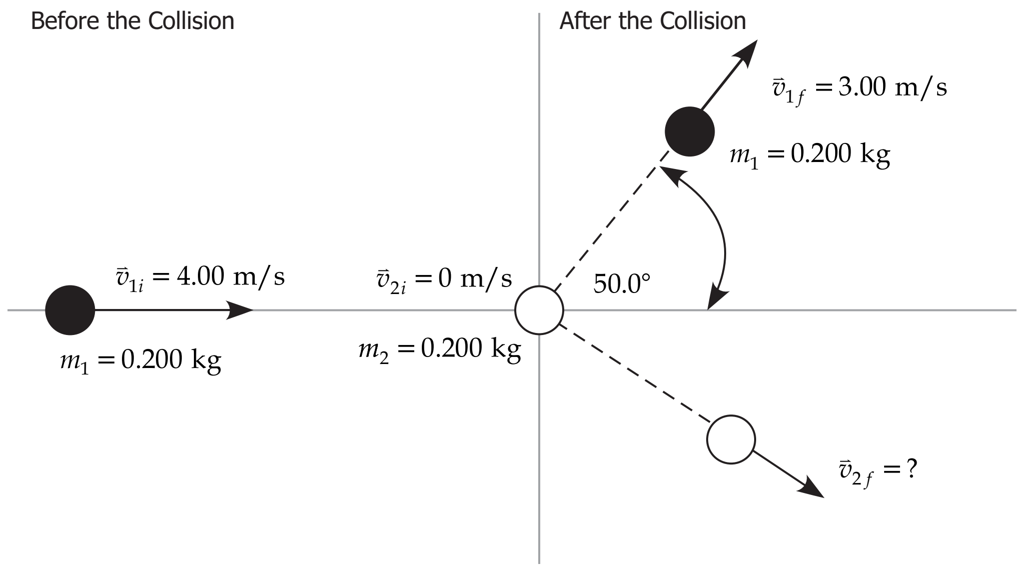
Momentum is conserved during an explosion if no external forces act on the system.

Part 3

The physics of billiard ball collisions and the conservation of momentum in 2D

1. Two identical billiard balls of mass 0.200 kg collide. The ball m_1 is moving at 4.00 m/s to the right towards a stationary billiard ball of mass m_2. The collision is not head on, but a glancing collision. One of the balls, m_1 moves at 3.00 m/s at an angle of 50.0° above the horizontal reference line. What is the magnitude and direction of the momentum, and the velocity of m_2 after the collision?

   Answer:



The first task is to determine the final momentum of mass 2.

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Since the second mass is initially not moving, the initial momentum of the second mass is $\vec{p}_{2i} = 0 \text{ kg}\cdot\text{m/s}$.

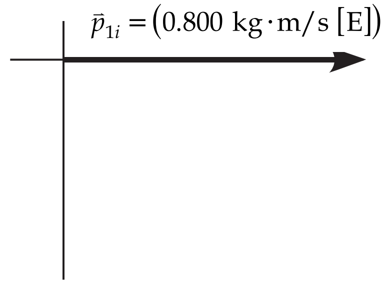
Then,

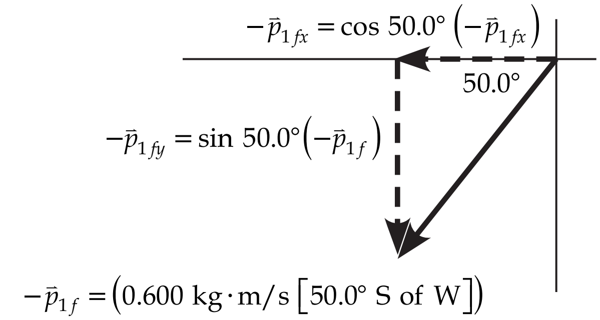
$$\vec{p}_{1i} - \vec{p}_{1f} = \vec{p}_{2f}$$
$$(0.200 \text{ kg})(4.00 \text{ m/s [E]}) - (0.200 \text{ kg})(3.00 \text{ m/s } [50.0° \text{ N of E}])$$
$$(0.800 \text{ kg}\cdot\text{m/s [E]}) - (0.600 \text{ kg}\cdot\text{m/s } [50.0° \text{ N of E}]) = \vec{p}_{2f}$$

Add the opposite.

$$(0.800 \text{ kg}\cdot\text{m/s [E]}) + (0.600 \text{ kg}\cdot\text{m/s } \lfloor 50.0° \text{ S of W} \rfloor) = \vec{p}_{2f}$$

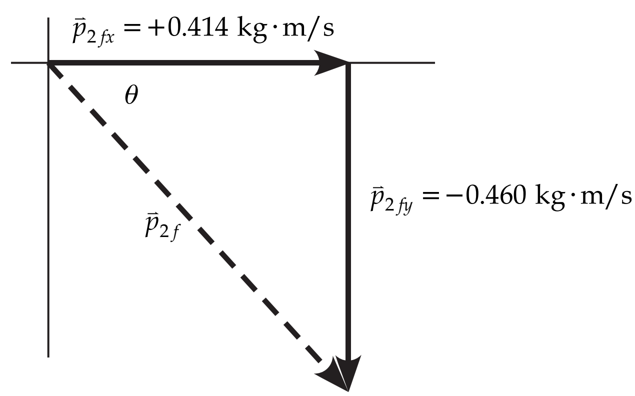
Sketch the vectors.





Adding the vector components of the vector diagram shown, you obtain the following data:

Vector	*x*-component (E–W)	*y*-component (N–S)
$\vec{p}_{1i}$	+0.800 kg·m/s	0 kg·m/s
$-\vec{p}_{1f}$	–cos 50.0° (0.600 kg·m/s) = –0.386 kg·m/s	–sin 50.0° (0.600 kg·m/s) = –0.460 kg·m/s
$\vec{p}_{2f}$	+0.414 kg·m/s	–0.460 kg·m/s



$$\theta = \tan^{-1}\frac{0.460}{0.414} = 48.0°$$

Add to the magnitudes of the components to give the magnitude of the resultant vector: $\vec{p}_{2f}$.

$$\vec{p}_{2f}^{\,2} = 0.414^2 + 0.460^2 = 0.3830$$

$$\vec{p}_{2f} = 0.619 \text{ kg}\cdot\text{m/s}$$

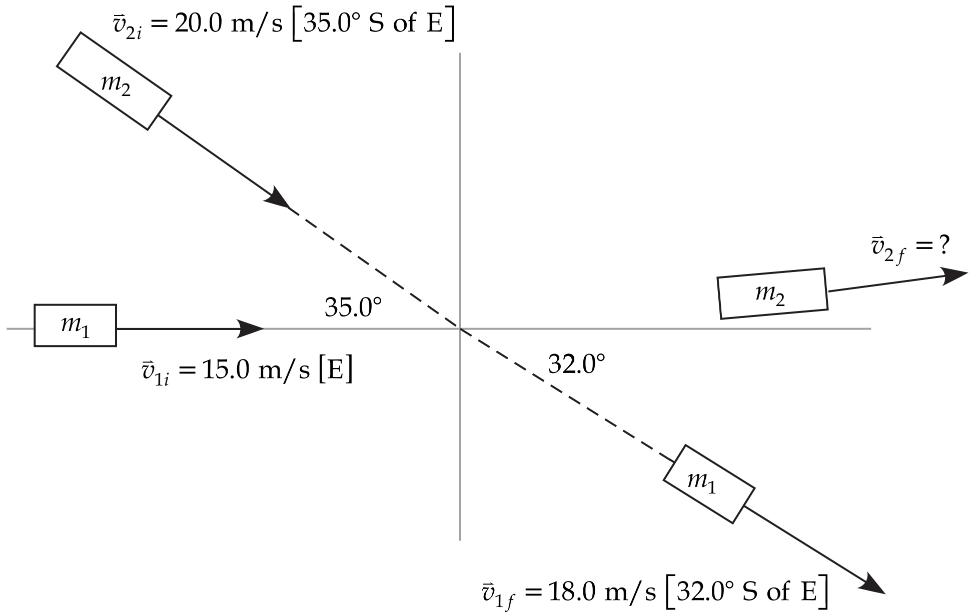
The final momentum of mass 2 is 0.619 kg·m/s [48.0° S of E].

The final velocity of mass 2 is

$$\vec{v}_{2f} = \frac{\vec{p}_{2f}}{m_2} = \frac{0.619 \text{ kg}\cdot\text{m/s}\left[48.0° \text{ S of E}\right]}{0.200 \text{ kg}} = 3.10 \text{ m/s}\left[48.0° \text{ S of E}\right]$$

2. A car, mass m_1 = 1250 kg, is travelling east at 15.0 m/s. This car is sideswiped by a second car, mass m_2 = 1550 kg, travelling at 20.0 m/s [35.0° S of E]. After the collision the first car is travelling at 18.0 m/s [32.0° S of E].

   Determine the final momentum and final velocity of the second car.



Answer:

You are given enough information to find three of the four momenta. You can find the missing momentum of the second car by using the law of conservation of momentum.

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + \vec{p}_{2f}$$

Substitute and solve:

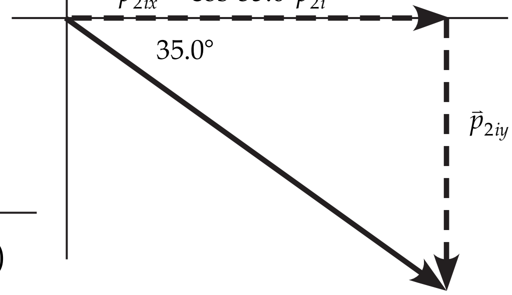
$$(1250 \text{ kg})(15.0 \text{ m/s } [\text{E}]) + (1550 \text{ kg})(20.0 \text{ m/s } [35.0° \text{ S of E}])$$
$$= (1250 \text{ kg})(18.0 \text{ m/s } [32.0° \text{ S of E}]) + \vec{p}_{2f}$$
$$(18750 \text{ kg}\cdot\text{m/s } [\text{E}]) + (31000 \text{ kg}\cdot\text{m/s } [35.0° \text{ S of E}])$$
$$= (22500 \text{ kg}\cdot\text{m/s } [32.0° \text{ S of E}]) + \vec{p}_{2f}$$
$$(18750 \text{ kg}\cdot\text{m/s } [\text{E}]) + (31000 \text{ kg}\cdot\text{m/s } [35.0° \text{ S of E}]) -$$
$$(22500 \text{ kg}\cdot\text{m/s } [32.0° \text{ S of E}]) = \vec{p}_{2f}$$

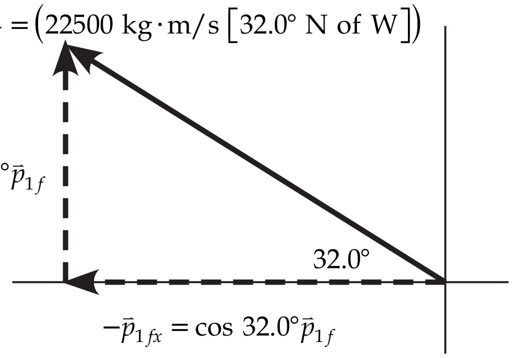
Add the opposite.

$$\left(18750 \text{ kg}\cdot\text{m/s [E]}\right)+\left(31000 \text{ kg}\cdot\text{m/s}\left[35.0^\circ \text{ S of E}\right]\right)+$$
$$\left(22500 \text{ kg}\cdot\text{m/s}\left[32.0^\circ \text{ N of W}\right]\right)=\vec{p}_{2f}$$

Add these vectors using the component method.

Sketch the vectors and find their components.



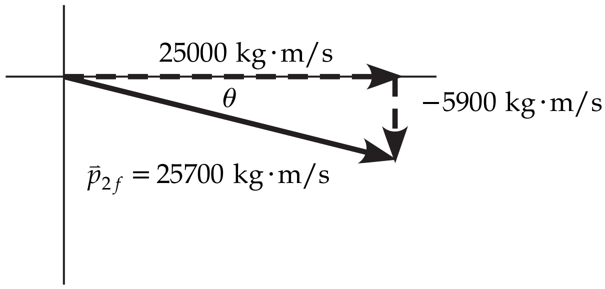


Set up a table with the components.

Vector	*x*-component	*y*-component
$\vec{p}_{1i}$	+18750 kg·m/s	0 kg·m/s
$\vec{p}_{2i}$	cos 35.0° (31000 kg·m/s) = +25400 kg·m/s	–sin 35.0° (31000 kg·m/s) = –17800 kg·m/s
$-\vec{p}_{1f}$	–cos 32.0° (22500 kg·m/s) = –19100 kg·m/s	+sin 32.0° (22500 kg·m/s) = +11900 kg·m/s
$\vec{p}_{2f}$	+25000 kg·m/s	–5900 kg·m/s

The components with the theorem of Pythagoras and trigonometry are used to determine $\vec{p}_{1f}$.

$$\vec{p}_{2f}{}^{2} = \vec{p}_{2fX}{}^{2} + \vec{p}_{2fY}{}^{2} = 25000^{2} + 5900^{2} = 659\,810\,000$$
$$\vec{p}_{2f} = 25700 \text{ kg}\cdot\text{m/s}$$



$$\theta = \tan^{-1}\frac{5900}{25000} = 13.3°$$

The final momentum of the second car is 25700 kg·m/s [13.3° south of east].

The final velocity of the second car is

$$\vec{v}_{2f} = \frac{\vec{p}_{2f}}{m_2} = \frac{25700\ \ [13.3° \text{ South of East}]}{1550 \text{ kg}} = 16.6 \text{ m/s}\ \ [13.3° \text{ South of East}].$$

Grade 12 Physics (40S)

Midterm Practice Examination

Grade 12 Physics (40S)

Midterm Practice Examination

Instructions

The midterm examination will be weighted as follows:

Modules 1–5	100%

The format of the examination will be as follows:

Part A: Multiple Choice	21 x 1 = 21 marks
Part B: Fill-in-the-Blanks	14 x 0.5 = 7 marks
Part C: Short Explanation Questions	4 x 4 = 16 marks
Part D: Problems	8 x 7 = 56 marks

Grade 12 Physics
Midterm Practice Examination

Part A: Multiple Choice

Write the letter of the choice that best completes each statement.

1.			11.			21.				
2.			12.							
3.			13.							
4.			14.							
5.			15.							
6.			16.							
7.			17.							
8.			18.							
9.			19.							
10.			20.							

Part B: Fill-in-the-Blanks

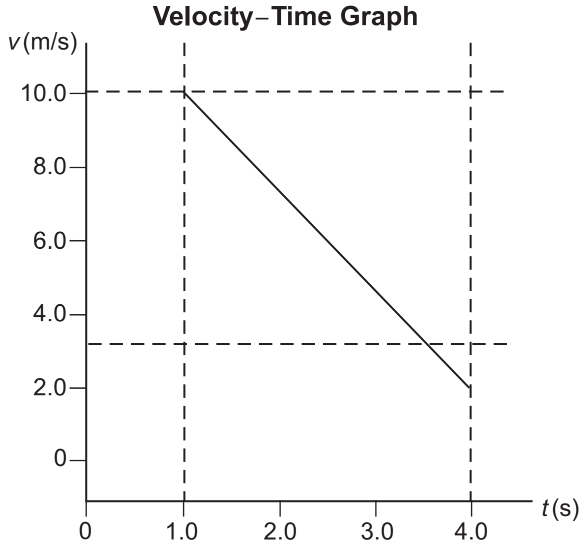
Write the word that best completes each statement in the space provided below.

1.			8.	
2.			9.	
3.			10.	
4.			11.	
5.			12.	
6.			13.	
7.			14.	

Part A: Multiple Choice (21 x 1 = 21 Marks)

Enter the letter of the choice that best answers the question on the answer sheet provided. Please print the letters of your answers clearly.

1. Consider the following velocity-time graph



The acceleration of the object calculated to two significant digits is

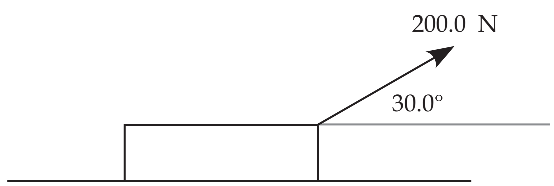
a) –2.7 ms/s^2
b) –2.5 ms/s^2
c) 2.5 ms/s^2
d) 2.7 ms/s^2

2. A river is flowing from west to east at a speed of 10.0 km/h. A boat's speed in the water is 20.0 km/h. If the boat is pointing straight north, and is blown off course, the new speed of the boat relative to the shore would be

a) 10.0 km/h
b) 17.3 km/h
c) 22.4 km/h
d) 30.0 km/h

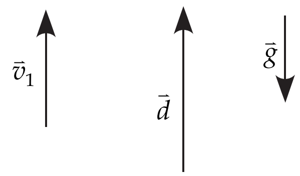
3. If a mass on a horizontal surface is pushed down at an angle (similar to pushing down on a shopping cart), then the normal force will
   a) increase and the force of gravity will not change
   b) not change and the force of gravity will increase
   c) not change and the force of gravity will also not change
   d) increase and the force of gravity will also increase

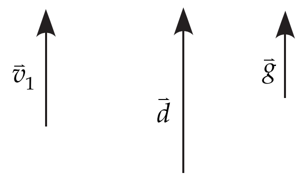
4. An object is pulled along a horizontal surface by a force that is directed up and to the right. The mass of the object is 50.0 kg. The applied force is 200.0 N directed at an angle of 30.0° above the horizontal. The magnitude force of friction is 60.0 N.

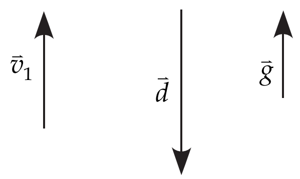


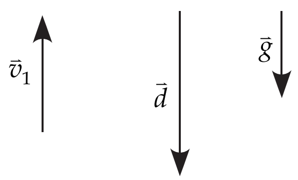
   The magnitude of the normal force is
   a) 1.00×10^2 N
   b) 3.90×10^2 N
   c) 4.90×10^2 N
   d) 5.90×10^2 N

5. A basketball is thrown straight up with a speed of 10.0 m/s. The vectors representing the initial velocity, the acceleration due to gravity, and the vertical distance travelled (height) are

a) 

b) 

c) 

d) 

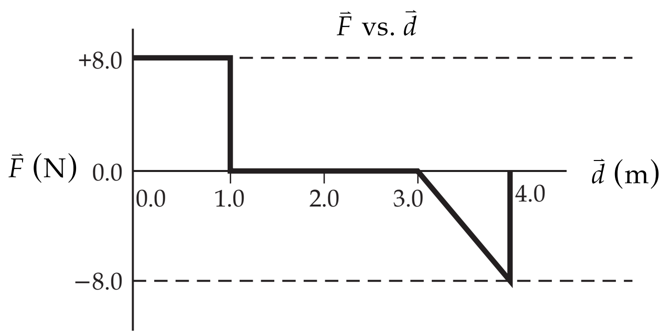
6. A ball is launched at a speed of 20.0 m/s at an angle of 40.0° above the horizontal. The maximum height reached by the ball is

a) 0.660 m
b) 0.780 m
c) 8.43 m
d) 12.0 m

7. A mass makes 20.0 revolutions in a time of 4.00 s in a circle of radius 10.0 m. The velocity of the mass is
   a) 12.6 m/s towards the centre of the circle
   b) 12.6 m/s tangent to the circle
   c) 314 m/s tangent to the circle
   d) 314 m/s towards the centre of the circle

8. When a ball is swinging in a circle at the end of a string, the hand feels a force directed away from the hand. The reason for this is that
   a) centrifugal force caused by the ball is pulling on the hand
   b) ball wants to fly out parallel to the radius of the circle
   c) centripetal force exactly balances the centrifugal force
   d) ball exerts an equal and opposite force on the hand

9. The force component along the displacement varies with the magnitude of the displacement, as shown on the graph.



   The work done by the force over the whole time interval is
   a) −4.0 J
   b) 0.0 J
   c) 4.0 J
   d) 12.0 J

10. A stopper is swung in a circle of radius 2.00 m with a period of 1.50 seconds. A centripetal force of 2.00 N acts on the stopper. What is the work done by the centripetal force during the time that the stopper travels once around the circle?

    a) 0 J
    b) 4.00 J
    c) 8.37 J
    d) 25.1 J

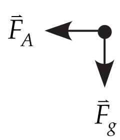
11. We can most directly derive the impulse-momentum equation from the law that states

    a) when a net external force $\vec{F}$ acts on a mass m the acceleration $\vec{a}$ that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass
    b) whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body
    c) the force of gravitation between two masses is directly proportional to the product of the two masses and inversely proportional to the separation between them squared
    d) an object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force

12. A mass of 5.00 kg is moving at a constant speed of 10.0 m/s. A force of 200.0 N then acts on the mass for 2.00 s. The new speed of the mass is

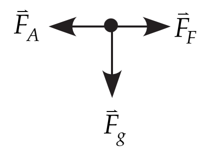
    a) 70.0 m/s
    b) 80.0 m/s
    c) 90.0 m/s
    d) 4.10×10^2 m/s

13. Which of the following free-body diagrams best represents the forces acting on an astronaut in orbit around Earth?

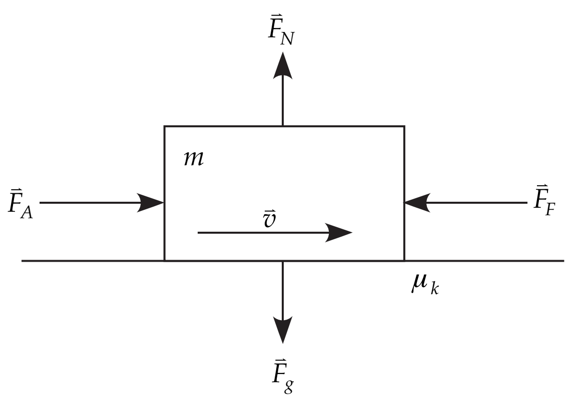
a) 

b) 

c) 

d) 

14. Study the force system diagram pictured below and select the factor that would ***not*** influence the amount of kinetic friction.



a) object's mass, m

b) coefficient of kinetic friction, μ_k

c) normal force, $\vec{F}_N$

d) applied force, $\vec{F}_A$

15. A person lifts a pail of water of mass 1.50 kg from the ground to a deck 1.00 m above the ground. How much work was done by gravitational force on the pail of water?

a) –14.7 J

b) +1.50 J

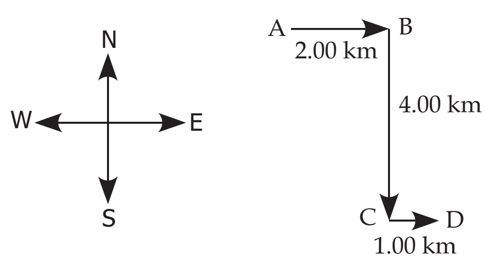
c) –1.50 J

d) +0.153 J

16. In which case is positive work done?
    a) An eastward force is applied to an eastward moving soccer ball that is already moving at a constant velocity to increase its speed in that direction.
    b) A cart is moving at a constant velocity of 10 m/s [W] when a 0.5 N [E] force is applied.
    c) Earth applies a force on the Moon as the Moon travels one complete rotation in orbit around Earth.
    d) The work done by air resistance as a baseball is thrown horizontally towards the catcher.

17. A 15.0 kg load of groceries is lifted up from the first floor to the fifth floor of an apartment building. Each floor is 5.00 m high. The potential energy of the groceries with respect to the second floor is
    a) 3.68×10^3 J
    b) 2.94×10^3 J
    c) 7.50×10^1 J
    d) 2.20×10^3 J

18. The diagram below shows the first three legs of a trip: A to B, B to C, and C to D. If a person returns from point D to point A, what is the displacement for this fourth and final leg?



    a) 7.00 km [36.8° W of N]
    b) 5.00 km [36.8° W of N]
    c) 5.00 km [36.8° E of S]
    d) 7.00 km [36.8° E of S]

19. The speed of an object moving with uniform circular motion of radius 15.0 m with a frequency of 4.00 Hz is which of the following?
    a) 3.75 m/s
    b) 23.6 m/s
    c) 60.0 m/s
    d) 377 m/s

20. A pilot flies to a destination due north from the departure point. During the flight there is a wind blowing from the west. What direction must the pilot point the plane during the flight?
    a) due east
    b) east of north
    c) due north
    d) west of north

21. An object is moving at 2.50 m/s [E]. At a time 3.00 seconds later the object is travelling at 1.50 m/s [E]. What was the displacement during this 3.00 second time interval?
    a) 6.00 m [E]
    b) 7.50 m [E]
    c) 4.50 m [E]
    d) 0.500 m [E]

Part B: Fill-in-the-Blanks (14 x 0.5 = 7 Marks)

Fill in the blanks with one of the choices in the word bank. The terms in the word bank may be used once, more than once, or not at all.

Write your answers in the space provided on the answer sheet.

acceleration	into	normal force	static
centripetal	joule	out of	two times
four times	kinetic	potential	uniform
impulse	larger	range	velocity
inertia	normal	smaller	watt

1. The force required to keep an object moving with uniform circular motion is called the ________________________ force. (Outcome S4P-1-19)
2. In uniform circular motion, if the velocity doubles, the acceleration of the object must change to be __________________________ as great as the original acceleration. (Outcome S4P-1-24)
3. The ____________________ force is always perpendicular to the surface supporting an object. (Outcome S4P-1-5)
4. The force of friction exerted on an object just before it begins to slide across a surface is called ________________________ friction. (Outcome S4P-1-7)
5. In projectile motion, the ___________________ refers to the horizontal distance the object travels. (Outcome S4P-1-18)
6. The tendency of an object to resist changes in its motion is called __________________. (Outcome S4P-1-9)
7. The area beneath a force-time graph represents ___________________. (Outcome S4P-1-11)
8. The word that best describes the motion of an object with a net force of 0 N acting on it is _______________________. (Outcome S4P-1-8)
9. The amount of friction acting on an object that is sliding across the surface of a level table depends on the coefficient of kinetic friction and the ____________________. (Outcome S4P-1-7)
10. The unit newton · metre is equivalent to the ____________________. (Outcome S4P-1-25)

11. The area beneath the force-extension graph of a spring represents _______________ energy. (Outcome S4P-1-32)
12. If negative work is done on an object, kinetic energy is transferred __________ the object. (Outcome S4P-1-27)
13. The work-energy theorem relates work done to changes in ________________ energy. (Outcome S4P-1-29)
14. If an object is pulled across a horizontal surface with a force that acts at 20° up from the horizontal, the magnitude of the normal force will be ___________________ than the force of gravity. (Outcome S4P-1-5)

Part C: Short Explanation Questions (4 x 4 = 16 Marks)

Answer any four (4) of the following questions. Be sure to indicate clearly which four questions are to be marked. Use proper English in your explanations.

Outcome S4P-1-2

1. Using sketches of the appropriate graph, derive the kinematics formula $\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}\Delta t^2$.

Outcome S4P-1-27, S4P-1-28

2. The speed of a gymnast revolving around a horizontal bar is greatest at the bottom and least at the top. Explain using the law of conservation of energy.

Outcome S4P-1-19

3. An object is travelling in a straight line with velocity $\vec{v}$. Describe the motion of the object that would result if only

   a) an acceleration parallel to the original velocity acts on the object

   b) an acceleration that constantly changes to remain perpendicular to the velocity acts on the object

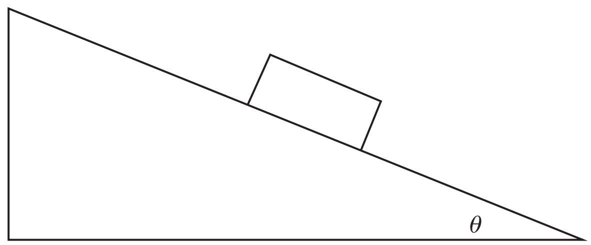
   c) an acceleration with components both parallel to and perpendicular to the original velocity acts on the object

Outcome S4P-1-32

4. Derive the equation for the potential energy of a spring $\left(PE_S = \frac{1}{2}kx^2\right)$ using Hooke's law and a force-displacement graph.

Outcome S4P-1-5

5. Draw a free-body diagram for an object of mass m resting on an inclined plane, as given in the diagram below. Label clearly the force of gravity and its components, the normal force, and the force of friction. Write an expression for the magnitude of the components of the force of gravity parallel to the surface and perpendicular to the surface.



Outcome S4P-1-14

6. Relate the impulse-momentum equation to the following real-life situations:
   a) hitting a baseball as far as possible

   b) catching a baseball with your bare hands without hurting yourself

Part D: Problems (8 x 7 = 56 Marks)

Answer any eight (8) problems. Please show your work. Number your answers clearly.

Outcomes S4P-0-2a, S4P-0-2f, S4P-0-2h, S4P-1-2, S4P-1-3

1. An airplane flies with an airspeed of 225 km/h heading due west. At the altitude at which the plane is flying, the wind is blowing at 105 km/h heading due south.
   a) What is the velocity of the plane as observed by someone standing on the ground? *(4 marks)*

b) How far off course would the plane, while it is heading due west, be blown by the wind during 1.50 h of flying? *(1 mark)*

c) What heading must a plane take in order to reach its destination, which is due west of a starting point? *(2 marks)*

2. A motorcycle starts from rest and accelerates at +3.50 m/s^2 for a distance of 175 m. It then slows down with an acceleration of –1.50 m/s^2 until the velocity is +10.0 m/s.

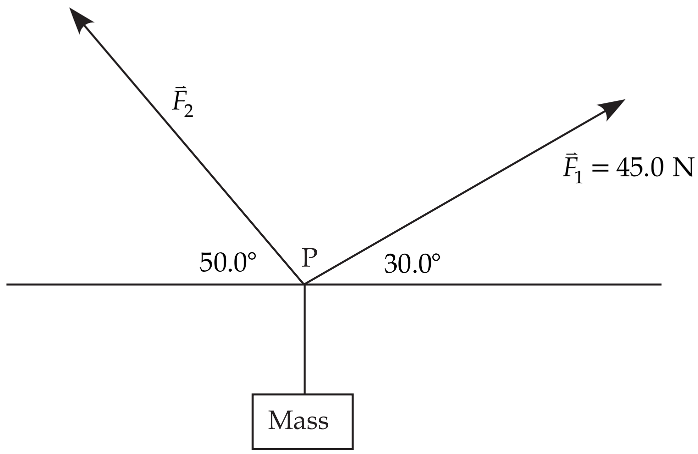
a) What is the length of time the motorcyclist takes to travel +175m? *(2 marks)*

b) What is the velocity at the end of the time interval determined in part (a)? *(2 marks)*

c) Determine the displacement of the motorcycle while it is slowing down during the second part of its journey. *(3 marks)*

Outcomes S4P-1-4, S4P-0-2h

3. What mass, M, can be supported at P so that the forces are in equilibrium at P? *(7 marks)*



Outcome S4P-1-24

4. A bicyclist and his bicycle have a mass of 85.4 kg. The cyclist is travelling around a circular track of radius 75.0 m at a constant speed of 7.96 m/s.

   a) Calculate the period of this motion. *(2 marks)*

   b) Calculate the acceleration of the cyclist. *(2 marks)*

c) Calculate the force necessary to keep the cyclist moving around the track. *(2 marks)*

d) Calculate the frequency of this motion. *(1 mark)*

Outcomes S4P-1-13, S4P-0-2h

5. A car of mass 1250 kg is travelling at 20.0 m/s [W]. At an icy intersection, the car collides with a truck of mass 2450 kg travelling at 15.0 m/s [S]. The collision lasts 0.250 seconds. After the collision, the two vehicles slide along together.

   a) What is the total momentum of the system of the car and the truck before the collision? *(5 marks)*

b) What is the velocity of the car after the collision? *(2 marks)*

Outcomes S4P-1-8, S4P-0-2h

6. A crate of mass 80.0 kg is pulled across a level concrete floor at a constant acceleration of 0.895 m/s^2. A force of 305 N acting 35.0° above the horizontal is used to move the crate.

   a) Calculate the normal force acting on the crate. *(3 marks)*

b) Calculate the force of kinetic friction acting on the crate. *(3 marks)*

c) Calculate the coefficient of kinetic friction. *(1 mark)*

Outcome S4P-1-15

7. A stone of mass 75.0 g is thrown upwards at 23.2 m/s from the height of a railing of a bridge that is 63.8 m above the surface of the water.

   a) Calculate the velocity of the stone as it strikes the water's surface. *(2.5 marks)*

b) How long after the stone is thrown is the stone 10.0 m above the surface of the water? *(3 marks)*

c) Where is the stone 3.50 seconds after being thrown?*(1.5 marks)*

Outcomes S4P-1-18, S4P-0-2h

8. A golfball is struck leaving the tee at a velocity of 45.0 m/s 47.9° from the horizontal. The ball travels over a level fairway towards a green where the hole is located 204 m from the tee.

   a) Calculate the vertical and horizontal components of the ball's velocity. *(2 marks)*

   b) Determine the time the ball is in the air. *(3 marks)*

c) If the ball is heading in the right direction, will it be possible for the golfer to score a hole in one? *(2 marks)*

Outcomes S4P-1-24, S4P-1-8

9. A satellite orbits Earth in a nearly circular orbit of radius 7.88×10^6 m with a period of 115 minutes. The satellite has a mass 238 kg.

   a) Calculate the speed of the satellite in m/s. *(2.5 marks)*

   b) Calculate the centripetal force acting on the satellite. *(3 marks)*

   c) If the weight of the satellite supplies the centripetal force, calculate the gravitational field strength at this distance from Earth. *(1.5 marks)*

Outcome S4P-1-33

10. A skier of mass 82.4 kg starts his run from rest. The skier drops 155 m vertically while skiing 795 m down the slope. The skier arrives at the bottom of the slope moving at 10.0 m/s.

    a) Determine the change in the gravitational potential energy of the skier. *(2 marks)*

b) Determine the work done by friction. *(3 marks)*

c) Determine the average force of friction that acted on the skier. *(2 marks)*

Outcome S4P-1-33

11. A dry ice puck, which slides along with no friction, has a mass of 2.30 kg and is sliding along a level horizontal surface at 2.25 m/s. The puck hits a spring bumper, which is compressed 20.0 cm, before the puck comes to rest.

    a) Determine the force constant of this spring. *(2 marks)*

b) How much is the spring compressed when the puck is sliding at 1.75 m/s towards the bumper? *(2.5 marks)*

c) If the dry ice puck was initially moving at 2.25 m/s towards the spring and the spring is compressed only 10.0 cm, calculate the speed of the puck at that moment. *(2.5 marks)*

Outcomes S4P-1-5, S4P-1-7, S4P-1-8

12. A child is sitting in a wagon on a hill, which has an incline of 14.0° from the horizontal. The mass of the child and wagon is 42.0 kg. The coefficient of kinetic friction is 0.125. The wagon begins to move.

    a) Calculate the normal force acting on the wagon. *(2.5 marks)*

b) Calculate the force of kinetic friction. *(1.5 marks)*

c) Calculate the distance the child and his wagon will move during the first 15.0 s of his trip. *(3 marks)*

Grade 12 Physics (40S)

Midterm Practice Examination

Answer Key

Grade 12 Physics (40S)

Midterm Practice Examination Answer Key

Instructions

The midterm examination will be weighted as follows:

Modules 1–5	100%

The format of the examination will be as follows:

Part A: Multiple Choice	21 x 1 = 21 marks
Part B: Fill-in-the-Blanks	14 x 0.5 = 7 marks
Part C: Short Explanation Questions	4 x 4 = 16 marks
Part D: Problems	8 x 7 = 56 marks

Grade 12 Physics
Midterm Practice Examination Key

Part A: Multiple Choice

Write the letter of the choice that best completes each statement.

1.	a		11.	a		21.	a		
2.	c		12.	c					
3.	a		13.	c					
4.	b		14.	d					
5.	a		15.	a					
6.	c		16.	a					
7.	c		17.	d					
8.	d		18.	b					
9.	c		19.	d					
10.	a		20.	d					

Part B: Fill-in-the-Blanks

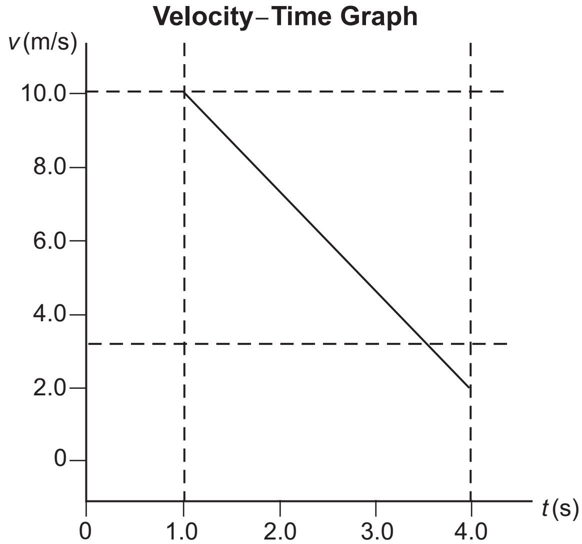
Write the word that best completes each statement in the space provided below.

1.	centripetal		8.	uniform
2.	four times		9.	normal force
3.	normal		10.	joule
4.	static		11.	potential
5.	range		12.	out of
6.	inertia		13.	kinetic
7.	impulse		14.	smaller

Part A: Multiple Choice (21 x 1 = 21 Marks)

Enter the letter of the choice that best answers the question on the answer sheet provided. Please print the letters of your answers clearly.

1. Consider the following velocity-time graph



The acceleration of the object calculated to two significant digits is

a) –2.7 ms/s^2

b) –2.5 ms/s^2

c) 2.5 ms/s^2 **Answer (a)**

d) 2.7 ms/s^2 Outcome S4P-1-1

Answer:

$$\vec{v}_2 = \vec{v}_1 + \vec{a}t$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{2\text{ m/s} - 10\text{ m/s}}{4\text{ s} - 1\text{ s}} = -2.7\text{ m/s}^2$$

2. A river is flowing from west to east at a speed of 10.0 km/h. A boat's speed in the water is 20.0 km/h. If the boat is pointing straight north, and is blown off course, the new speed of the boat relative to the shore would be

a) 10.0 km/h

b) 17.3 km/h

c) 22.4 km/h **Answer (c)**

d) 30.0 km/h Outcome S4P-1-3

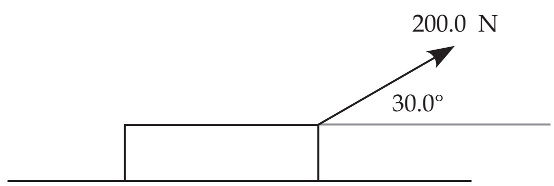
Answer:

$$v = \sqrt{(20.0\text{ km/h})^2 + (10.0\text{ km/h})^2} = 22.4\text{ km/h}$$

3. If a mass on a horizontal surface is pushed down at an angle (similar to pushing down on a shopping cart), then the normal force will
   a) increase and the force of gravity will not change
   b) not change and the force of gravity will increase
   c) not change and the force of gravity will also not change
   d) increase and the force of gravity will also increase

Answer (a)
Outcome S4P-1-8

4. An object is pulled along a horizontal surface by a force that is directed up and to the right. The mass of the object is 50.0 kg. The applied force is 200.0 N directed at an angle of 30.0° above the horizontal. The magnitude force of friction is 60.0 N.



The magnitude of the normal force is
   a) 1.00×10^2 N
   b) 3.90×10^2 N
   c) 4.90×10^2 N
   d) 5.90×10^2 N

Answer (b)
Outcome S4P-1-8

Answer:

The magnitude of the normal force is found using

$$\vec{F}_N + \vec{F}_A(\sin\theta) - \vec{F}_g = 0$$

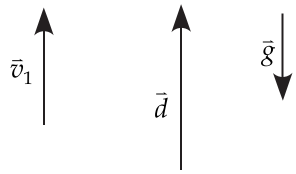
$$\vec{F}_N = \vec{F}_g - \vec{F}_A(\sin\theta) = (50.0\ \text{kg})(9.80\ \text{N/kg}) - (200.0\ \text{N})(\sin 30.0°) = 390\ \text{N}$$

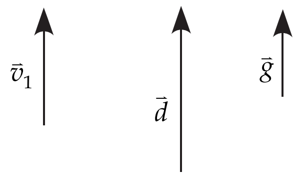
The normal force ***is not*** (50.0 kg)(9.80 N/kg) = 490 N.

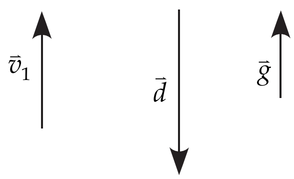
The normal force ***is not*** (200.0 N)(sin 30.0°) = 100 N.

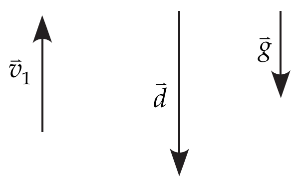
The normal force ***is not*** (50.0 kg)(9.80 N/kg) + (200.0 N)(sin 30.0°) = 590 N

5. A basketball is thrown straight up with a speed of 10.0 m/s. The vectors representing the initial velocity, the acceleration due to gravity, and the vertical distance travelled (height) are

a)


b)


c)


d)


Answer (a)

Outcome S4P-1-16

6. A ball is launched at a speed of 20.0 m/s at an angle of 40.0° above the horizontal. The maximum height reached by the ball is

a) 0.660 m
b) 0.780 m
c) 8.43 m
d) 12.0 m

Answer (c)

Outcome S4P-1-18

Answer:

To determine the maximum height, consider the first half of the motion. The vertical component of velocity at the top of motion is zero. An appropriate equation is $v_2{}^2 = v_1{}^2 + 2ad$. Solve for "d":

$$d = \frac{(0 \text{ m/s})^2 - [(20.0 \text{ m/s})(\sin 40.0°)]^2}{2(-9.80 \text{ m/s}^2)} = 8.43 \text{ m}$$

7. A mass makes 20.0 revolutions in a time of 4.00 s in a circle of radius 10.0 m. The velocity of the mass is

   a) 12.6 m/s towards the centre of the circle
   b) 12.6 m/s tangent to the circle
   c) 314 m/s tangent to the circle
   d) 314 m/s towards the centre of the circle

   Answer (c)
   Outcome S4P-1-24

   Answer:

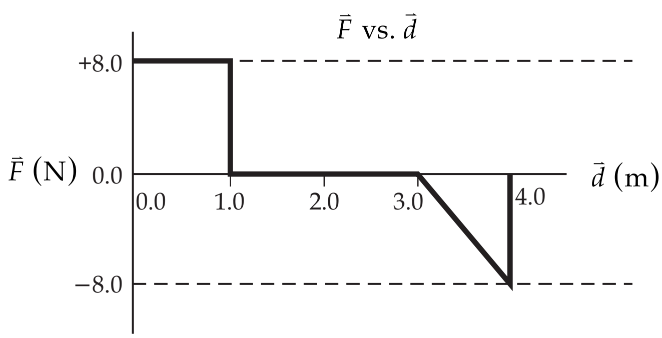
$$\vec{v} = \frac{2\pi r}{T} = \frac{2\pi\left(10.0\text{ m}\right)}{\left(4.00\text{ s}/20.0\text{ rev}\right)} = 314\text{ m/s}$$

8. When a ball is swinging in a circle at the end of a string, the hand feels a force directed away from the hand. The reason for this is that

   a) centrifugal force caused by the ball is pulling on the hand
   b) ball wants to fly out parallel to the radius of the circle
   c) centripetal force exactly balances the centrifugal force
   d) ball exerts an equal and opposite force on the hand

   Answer (d)
   Outcome S4P-1-20

9. The force component along the displacement varies with the magnitude of the displacement, as shown on the graph.



   The work done by the force over the whole time interval is

   a) −4.0 J
   b) 0.0 J
   c) 4.0 J
   d) 12.0 J

   Answer (c)
   Outcome S4P-1-26

Answer:

The work done during each interval is equal to the area under the force vs. displacement curve over that interval.

$$W = b_1 h_1 + \frac{1}{2} b_2 h_2$$

$$W = (1.0\ \text{m})(8.0\ \text{N}) + \frac{1}{2}(4.0\ \text{m} - 3.0\ \text{m})(-8.0\ \text{N}) = 4.0\ \text{J}$$

10. A stopper is swung in a circle of radius 2.00 m with a period of 1.50 seconds. A centripetal force of 2.00 N acts on the stopper. What is the work done by the centripetal force during the time that the stopper travels once around the circle?
    a) 0 J
    b) 4.00 J
    c) 8.37 J
    d) 25.1 J

Answer (a)

Outcome S4P-1-25, S4P-1-24

11. We can most directly derive the impulse-momentum equation from the law that states
    a) when a net external force $\vec{F}$ acts on a mass m the acceleration $\vec{a}$ that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass
    b) whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body
    c) the force of gravitation between two masses is directly proportional to the product of the two masses and inversely proportional to the separation between them squared
    d) an object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force

Answer (a)

Objective S4P-1-10

12. A mass of 5.00 kg is moving at a constant speed of 10.0 m/s. A force of 200.0 N then acts on the mass for 2.00 s. The new speed of the mass is

    a) 70.0 m/s
    b) 80.0 m/s
    c) 90.0 m/s
    d) 4.10×10^2 m/s

Answer (c)

Objective S4P-1-13

Answer:

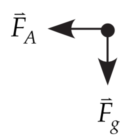
$$\vec{F}\Delta t = m(v_2 - v_1)$$

$$v_2 = \frac{\vec{F}\Delta t}{m} + v_1$$

$$v_2 = \frac{(200.0\ \text{N})(2.0\ \text{s})}{5.0\ \text{kg}} + 10.0\ \text{m/s} = 90\ \text{m/s}$$

13. Which of the following free-body diagrams best represents the forces acting on an astronaut in orbit around Earth?

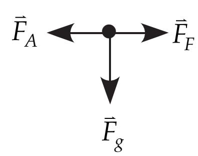
a)



b)

c)

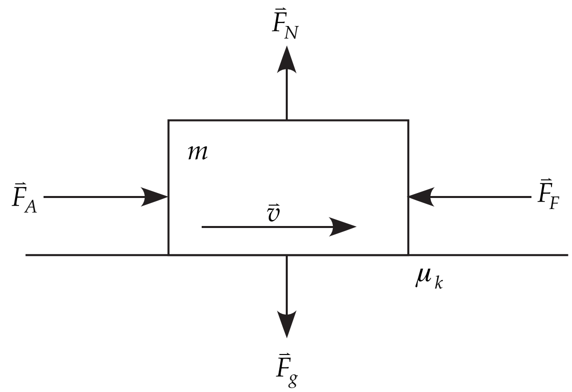
d)



Answer (c)

Outcome S4P-1-21

14. Study the force system diagram pictured below and select the factor that would ***not*** influence the amount of kinetic friction.



a) object's mass, m
b) coefficient of kinetic friction, μ_k
c) normal force, $\vec{F}_N$
d) applied force, $\vec{F}_A$

Answer (d)
Outcome S4P-1-7

15. A person lifts a pail of water of mass 1.50 kg from the ground to a deck 1.00 m above the ground. How much work was done by gravitational force on the pail of water?
a) –14.7 J
b) +1.50 J
c) –1.50 J
d) +0.153 J

Answer (a)
Outcome S4P-1-25, S4P1-27

16. In which case is positive work done?
a) An eastward force is applied to an eastward moving soccer ball that is already moving at a constant velocity to increase its speed in that direction.
b) A cart is moving at a constant velocity of 10 m/s [W] when a 0.5 N [E] force is applied.
c) Earth applies a force on the Moon as the Moon travels one complete rotation in orbit around Earth.
d) The work done by air resistance as a baseball is thrown horizontally towards the catcher.

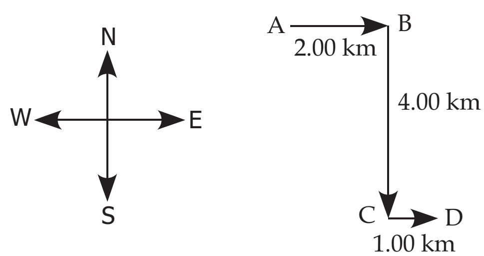
Answer (a)
Outcome S4P-1-25, S4P-1-27

17. A 15.0 kg load of groceries is lifted up from the first floor to the fifth floor of an apartment building. Each floor is 5.00 m high. The potential energy of the groceries with respect to the second floor is

a) 3.68×10^3 J
b) 2.94×10^3 J
c) 7.50×10^1 J
d) 2.20×10^3 J

Answer (d)
Outcome S4P-1-30, S4P-1-33

18. The diagram below shows the first three legs of a trip: A to B, B to C, and C to D. If a person returns from point D to point A, what is the displacement for this fourth and final leg?



a) 7.00 km [36.8° W of N]
b) 5.00 km [36.8° W of N]
c) 5.00 km [36.8° E of S]
d) 7.00 km [36.8° E of S]

Answer (b)
Outcome S4P-0-2h

19. The speed of an object moving with uniform circular motion of radius 15.0 m with a frequency of 4.00 Hz is which of the following?

a) 3.75 m/s
b) 23.6 m/s
c) 60.0 m/s
d) 377 m/s

Answer (d)
Outcome S4P-1-24

20. A pilot flies to a destination due north from the departure point. During the flight there is a wind blowing from the west. What direction must the pilot point the plane during the flight?

    a) due east
    b) east of north
    c) due north
    d) west of north

Answer (d)
Outcome S4P-1-3

21. An object is moving at 2.50 m/s [E]. At a time 3.00 seconds later the object is travelling at 1.50 m/s [E]. What was the displacement during this 3.00 second time interval?

    a) 6.00 m [E]
    b) 7.50 m [E]
    c) 4.50 m [E]
    d) 0.500 m [E]

Answer (a)
Outcome S4P-1-2

Part B: Fill-in-the-Blanks (14 x 0.5 = 7 Marks)

Fill in the blanks with one of the choices in the word bank. The terms in the word bank may be used once, more than once, or not at all.

Write your answers in the space provided on the answer sheet.

acceleration	into	normal force	static
centripetal	joule	out of	two times
four times	kinetic	potential	uniform
impulse	larger	range	velocity
inertia	normal	smaller	watt

1. The force required to keep an object moving with uniform circular motion is called the ________***centripetal***__________ force. (Outcome S4P-1-19)
2. In uniform circular motion, if the velocity doubles, the acceleration of the object must change to be _________***four times***_____________ as great as the original acceleration. (Outcome S4P-1-24)
3. The ______***normal***_________ force is always perpendicular to the surface supporting an object. (Outcome S4P-1-5)
4. The force of friction exerted on an object just before it begins to slide across a surface is called _______***static***_____________ friction. (Outcome S4P-1-7)
5. In projectile motion, the ____***range***_________ refers to the horizontal distance the object travels. (Outcome S4P-1-18)
6. The tendency of an object to resist changes in its motion is called ____***inertia***________. (Outcome S4P-1-9)
7. The area beneath a force-time graph represents ____***impulse***______. (Outcome S4P-1-11)
8. The word that best describes the motion of an object with a net force of 0 N acting on it is _______***uniform***___________. (Outcome S4P-1-8)
9. The amount of friction acting on an object that is sliding across the surface of a level table depends on the coefficient of kinetic friction and the ____***normal force***___________. (Outcome S4P-1-7)
10. The unit newton · metre is equivalent to the ______***joule***_________. (Outcome S4P-1-25)

11. The area beneath the force-extension graph of a spring represents _____***potential***_______ energy. (Outcome S4P-1-32)

12. If negative work is done on an object, kinetic energy is transferred ___***out of***______ the object. (Outcome S4P-1-27)

13. The work-energy theorem relates work done to changes in ____***kinetic***_______ energy. (Outcome S4P-1-29)

14. If an object is pulled across a horizontal surface with a force that acts at 20° up from the horizontal, the magnitude of the normal force will be _______***smaller***_______ than the force of gravity. (Outcome S4P-1-5)

Part C: Short Explanation Questions (4 x 4 = 16 Marks)

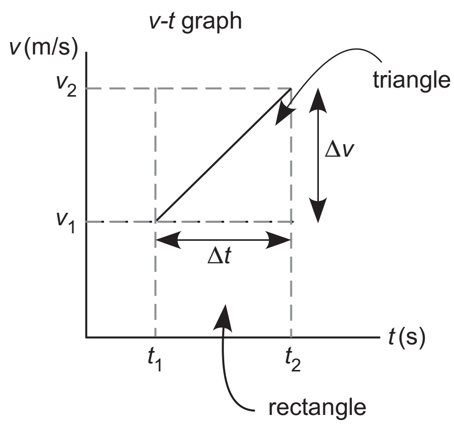
Answer any four (4) of the following questions. Be sure to indicate clearly which four questions are to be marked. Use proper English in your explanations.

Outcome S4P-1-2

1. Using sketches of the appropriate graph, derive the kinematics formula $\vec{d} = \vec{v}_1 t + \frac{1}{2}\vec{a}\Delta t^2$.

   Answer: (4 marks)

   Another useful equation can be derived from the graph by considering the area of the rectangle and the area of the triangle under the solid line.



To find the displacement of an object undergoing uniformly accelerated motion, determine the area under the velocity-time graph.

total area = area of rectangle + area of triangle

The area of a rectangle is given by the length times the width. For the rectangle above,

area of rectangle = $v_1\Delta t$.

The area of a triangle is given by one-half the base times the height.

area of triangle = $\frac{1}{2}\Delta v\Delta t$

total area = $v_1\Delta t + \frac{1}{2}\Delta v\Delta t$

Since the area under a velocity-time graph gives displacement,

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\Delta\vec{v}\Delta t.$$

We know from the definition of acceleration that $\Delta\vec{v} = \vec{a}\Delta t$. Therefore,

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}(\vec{a}\Delta t)\Delta t$$

$$\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Outcome S4P-1-27, S4P-1-28

2. The speed of a gymnast revolving around a horizontal bar is greatest at the bottom and least at the top. Explain using the law of conservation of energy.

   Answer: (4 marks)

   The law of conservation of energy states that energy is not created or destroyed but can be converted from one form to another. The total energy (kinetic energy plus potential energy) of the gymnast in this system will remain constant.

   While the gymnast is above the horizontal bar, he will possess the maximum amount of gravitational potential energy. As his body swings down around the bar, it loses gravitational potential energy but gains kinetic energy. When the gymnast is at the bottom he possesses the least amount of gravitational potential energy but the greatest amount of kinetic energy. Since kinetic energy depends on the speed, the gymnast has the greatest speed where his kinetic energy is the greatest.

Outcome S4P-1-19

3. An object is travelling in a straight line with velocity $\vec{v}$. Describe the motion of the object that would result if only

   a) an acceleration parallel to the original velocity acts on the object

      Answer: (1 mark)

      If the acceleration acts parallel to the original velocity, then only the magnitude of the original velocity will change. The result will be accelerated motion along the straight line.

   b) an acceleration that constantly changes to remain perpendicular to the velocity acts on the object

      Answer: (1 mark)

      If the acceleration acts perpendicular to the velocity and changes to remain perpendicular to the velocity, then only the direction of the velocity will change. The result will be uniform circular motion.

   c) an acceleration with components both parallel to and perpendicular to the original velocity acts on the object

      Answer: (2 marks)

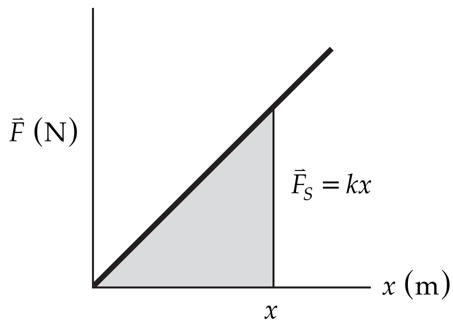
      In this case, the parallel component of the acceleration will serve to increase the magnitude of the original velocity, while the perpendicular component of the acceleration will serve to change the direction of the original velocity. The resulting motion will be the object speeding up and turning.

Outcome S4P-1-32

4. Derive the equation for the potential energy of a spring $\left(\mathrm{PE_S} = \frac{1}{2}kx^2\right)$ using Hooke's law and a force-displacement graph.

Answer: (4 marks)

The amount of work done by a changing force over a displacement is given by the area under the corresponding force-displacement graph. In the case of the stretched spring, the displacement is just the extension, x, and the work done in extending the spring by an amount x is equal to the spring potential energy stored in the extended spring.



According the Hooke's law at the given extension, x m, the restoring force of the spring, $\vec{F}_S$, is given by $\vec{F}_S = -k\vec{x}$. Again, remember of the the negative sign simply indicates that the directions of the extension and the restoring force of the spring are opposite to each other.

So, you can say that $\vec{F}_S = -kx$.

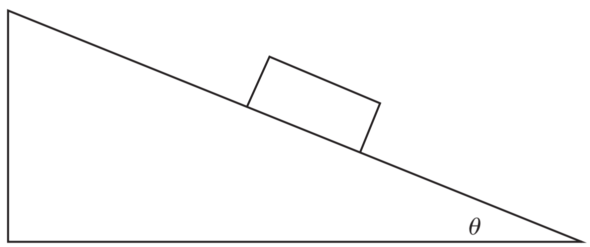
Since the area of the force displacement graph for an ideal spring is in the shape of a triangle, the area is $\frac{1}{2}$(base)(height), which in this case is $\frac{1}{2}(x)(kx)$.

Thus, the energy stored in a spring is given by

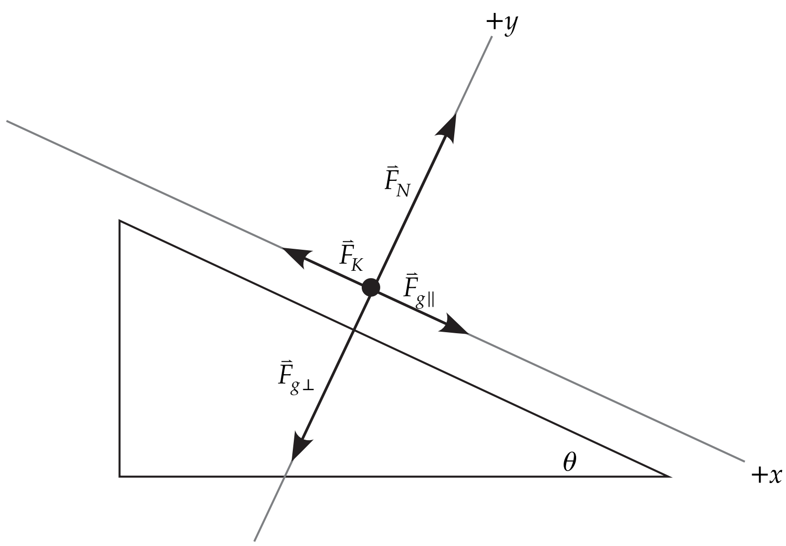
$$PE_S = \frac{1}{2}kx^2.$$

Outcome S4P-1-5

5. Draw a free-body diagram for an object of mass m resting on an inclined plane, as given in the diagram below. Label clearly the force of gravity and its components, the normal force, and the force of friction. Write an expression for the magnitude of the components of the force of gravity parallel to the surface and perpendicular to the surface.



Answer: (4 marks)



The components of the force of gravity are:

$$\vec{F}_{g\perp} = \cos\theta \left(\vec{F}_g\right)$$

$$\vec{F}_{g||} = \sin\theta \left(\vec{F}_g\right)$$

Outcome S4P-1-14

6. Relate the impulse-momentum equation to the following real-life situations:

   a) hitting a baseball as far as possible

   Answer: (2 marks)

   The impulse-momentum equation $\left(\vec{F}\Delta t = m\Delta\vec{v}\right)$ relates the impulse applied during an interaction to the change in momentum of an object. The larger the momentum of a baseball after it is hit into the air, the farther it should travel. To increase the momentum of the baseball, the hitter can do two things. First, he can exert a larger force on the ball by swinging the bat harder. Secondly, he can increase the time of the interaction by following through.

   b) catching a baseball with your bare hands without hurting yourself

   Answer: (2 marks)

   In this case, hurting your hands while catching a baseball stems from the force that the baseball exerts on your hands as you exert a force to stop the ball. The baseball must undergo a certain change in momentum. To decrease the force required to stop the ball, the time over which the ball is stopped must be lengthened. This is done by cushioning the ball by bringing your hands in towards your body as you're catching the ball.

Part D: Problems (8 x 7 = 56 Marks)

Answer any eight (8) problems. Please show your work. Number your answers clearly.

Outcomes S4P-0-2a, S4P-0-2f, S4P-0-2h, S4P-1-2, S4P-1-3

1. An airplane flies with an airspeed of 225 km/h heading due west. At the altitude at which the plane is flying, the wind is blowing at 105 km/h heading due south.

   a) What is the velocity of the plane as observed by someone standing on the ground?

   Answer: (4 marks)

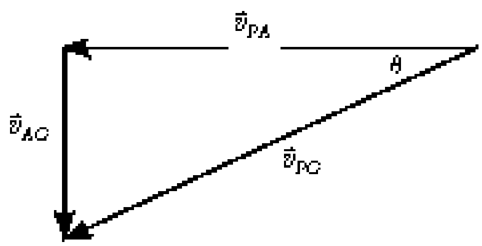
   Given: Velocity of the plane next to the air $\vec{v}_{PA} = 225 \text{ km/h } [\text{W}]$

   Velocity of the air next to the ground $\vec{v}_{AG} = 105 \text{ km/h } [\text{S}]$

   Unknown: Velocity of the plane next to the ground $\vec{v}_{PG} = ?$

   Equation: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$

   Substitute and solve: $\vec{v}_{PG} = 225 \text{ km/h } [\text{W}] + 105 \text{ km/h } [\text{S}]$



   Using the theorem of Pythagoras,

$$\vec{v}_{PG}{}^2 = \vec{v}_{PA}{}^2 + \vec{v}_{AG}{}^2$$

$$\vec{v}_{PG}{}^2 = (225 \text{ km/h})^2 + (105 \text{ km/h})^2$$

$$\vec{v}_{PG} = \sqrt{61650} = 248 \text{ km/h}$$

   Using trigonometry, you can determine the angle.

$$\theta = \tan^{-1}\frac{105}{225} = 25.0°$$

   The velocity of the plane next to the ground is 248 km/h [25.0° south of west].

b) How far off course would the plane, while it is heading due west, be blown by the wind during 1.50 h of flying?

Answer: (1 mark)

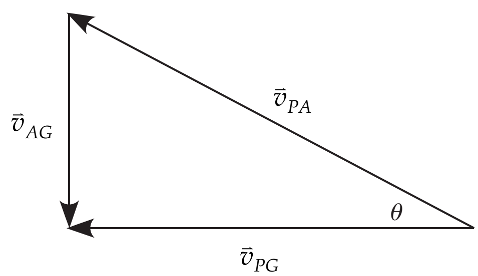
The wind pushes the plane south. The velocity of the wind is constant at 105 km/h [S] for 1.50 hours.

Using the equation $\vec{v} = \dfrac{\vec{d}}{\Delta t}$ rearranged to give $\vec{d} = \vec{v}\Delta t$, the plane is pushed off course by $\vec{d} = (105 \text{ km/s [S]})(1.50 \text{ h}) = 158 \text{ km [S]}$.

c) What heading must a plane take in order to reach its destination, which is due west of a starting point?

Answer: (2 marks)

The plane must head north of west so that the wind will push it south enough to have a final heading of due west.



In this triangle, we relate the given sides to the angle using the sine function.

$$\theta = \sin^{-1}\frac{105}{225} = 27.8°$$

The plane must head 27.8° [north of east] in order to end up flying due west.

2. A motorcycle starts from rest and accelerates at +3.50 m/s^2 for a distance of 175 m. It then slows down with an acceleration of –1.50 m/s^2 until the velocity is +10.0 m/s.

a) What is the length of time the motorcyclist takes to travel +175m?

Answer: (2 marks)

In our reference system, right will be the positive direction.

Given:	Initial velocity	$\vec{v}_1 = 0 \text{ m/s}$
	Acceleration one	$\vec{a} = +3.50 \text{ m/s}^2$
	Displacement	$\vec{d} = +175 \text{ m}$
Unknown:	Time interval	$\Delta t = ?$

Equation: $$\vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Substitute and solve:
$$+175 \text{ m} = \left(0 \text{ m/s}\right)\Delta t + \frac{1}{2}\left(+3.50 \text{ m/s/s}\right)\Delta t^2$$
$$+350 \text{ m} = \left(+3.50 \text{ m/s/s}\right)\Delta t^2$$
$$100 \text{ m/m/s}^2 = \Delta t^2$$
$$10.0 \text{ s} = \Delta t$$

It takes 10.0 seconds for the motorcycle to travel 175 m.

b) What is the velocity at the end of the time interval determined in part (a)?

Answer: (2 marks)

Unknown: Final velocity $\vec{v}_2 = ?$

Equation: $$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

Substitute and solve: $$\vec{v}_2 = 0 \text{ m/s} + \left(+3.50 \text{ m/s/s}\right)\left(10.0 \text{ s}\right) = +35.0 \text{ m/s}$$

The final velocity is plus 35.0 m /s.

c) Determine the displacement of the motorcycle while it is slowing down during the second part of its journey.

Answer: (3 marks)

Given:	Velocity at the beginning of the interval	$\vec{v}_1 = +35.0\ \text{m/s}$
	Velocity at the end of the interval	$\vec{v}_2 = +10.0\ \text{m/s}$
	Acceleration	$\vec{a} = -1.50\ \text{m/s}^2$
Unknown:	Displacement	$\vec{d} = ?$

Equation: $v_2^{\,2} = v_1^{\,2} + 2ad$

Substitute and solve:

$$(+10.0\ \text{m/s})^2 = (+35.0\ \text{m/s})^2 + 2(-1.50\ \text{m/s}^2)d$$

$$100 = 1225 - 3d$$

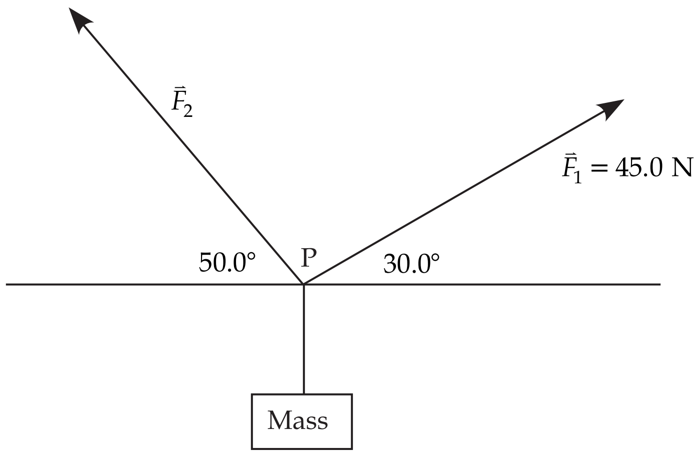
$$3d = 1125$$

$$d = +375\ \text{m}$$

The motorcycle travels +375 m while it is decelerating.

Outcomes S4P-1-4, S4P-0-2h

3. What mass, M, can be supported at P so that the forces are in equilibrium at P?



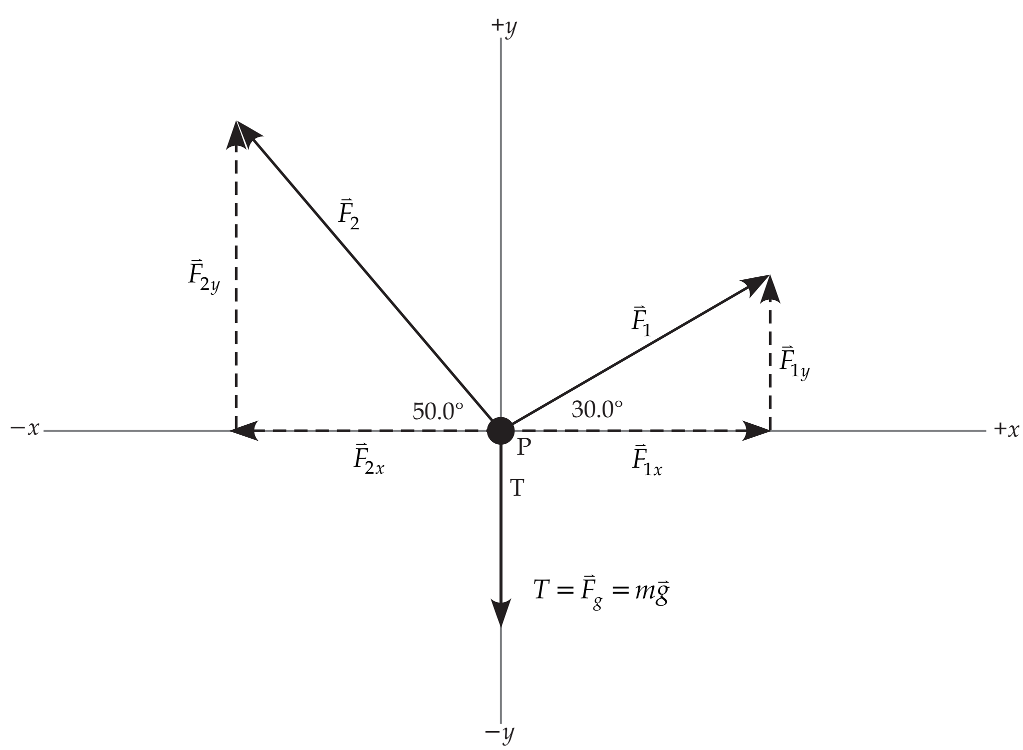
Answer: (7 marks)

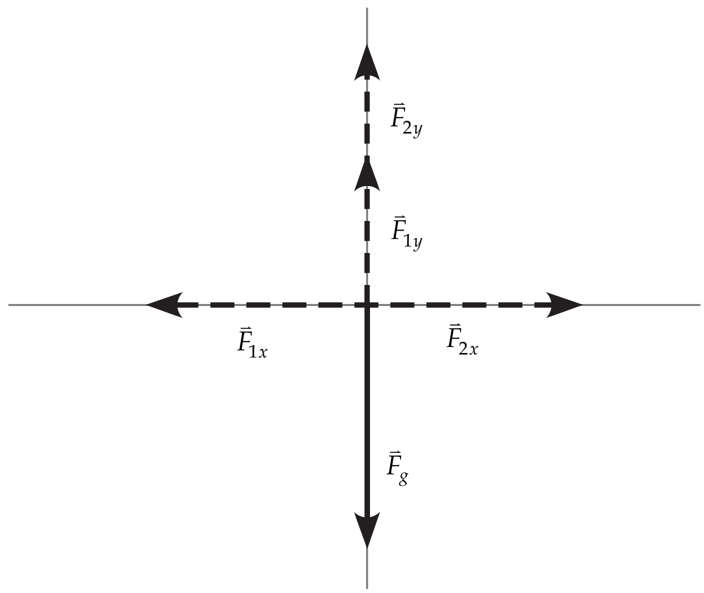
Here is the strategy.

Since you know $\vec{F}_1$ and the angle, you can determine the horizontal component and the vertical component for this force.

Next, since the forces are in equilibrium, the horizontal components of the two forces must balance. Therefore, you know the horizontal component of $\vec{F}_2$. You can then calculate the vertical component of $\vec{F}_2$.

The sum of the vertical components of the two forces will give the weight of the mass.





Vector	x-component (N)	y-component (N)
$\vec{F}_1$	$+(\cos 30.0°)\vec{F}_1 = -(0.866)(45.0\text{ N})$ $= +39.0\text{ N}$	$+(\sin 30.0°)\vec{F}_1 = -(0.5)(45.0\text{ N})$ $= +22.5\text{ N}$
$\vec{F}_2$	$-(\cos 50.0°)\vec{F}_2 = -0.6428\vec{F}_2$	$-(\sin 50.0°)\vec{F}_2 = -0.7660\vec{F}_2$
$\vec{F}_g$	0	$-\vec{F}_g$
$\vec{F}_{\text{net}}$	0	0

From the horizontal component column, you can see that the sum of the component is 0 N.

$$+39.0\text{ N} + \left(-0.6428\vec{F}_2\right) = 0\text{ N}$$
$$\vec{F}_2 = 60.7\text{ N}$$

In the vertical component, $\vec{F}_2$ is $+(\sin 50.0°)\vec{F}_2 = 0.7660(60.7\text{ N}) = 46.5\text{ N}$.

The total force pulling the mass upwards is +22.5 N + (+46.5 N) = 69.0 N.
This force just balances the force of gravity.

Using $\vec{F}_g = m\vec{g}$ rearranged to $m = \dfrac{\vec{F}_g}{\vec{g}}$ yields $m = \dfrac{69.0\text{ N}}{9.80\text{ N/kg}} = 7.04\text{ kg}$.

A mass of 7.04 kilograms is suspended at the point P.

Outcome S4P-1-24

4. A bicyclist and his bicycle have a mass of 85.4 kg. The cyclist is travelling around a circular track of radius 75.0 m at a constant speed of 7.96 m/s.

   a) Calculate the period of this motion.

   Answer: (2 marks)

   Given: Mass $m = 85.4$ kg

   Radius $R = 75.0$ m

   Speed $v = 7.96$ m/s

   Unknown: Period $T = ?$

   Equation: $v = \frac{2\pi R}{T}$ rearranged to $T = \frac{2\pi R}{v}$

   Substitute and solve: $T = \frac{2\pi (75.0 \text{ m})}{7.96 \text{ m/s}} = 59.2 \text{ s}$

   The period is 59.2 seconds.

   b) Calculate the acceleration of the cyclist.

   Answer: (2 marks)

   Equation: $a_c = \frac{v^2}{R}$

   Substitute and solve: $a_c = \frac{v^2}{R} = \frac{(7.96 \text{ m/s})^2}{75.0 \text{ m}} = 0.845 \text{ m/s/s}$

   The acceleration is 0.845 m/s^2 towards the centre of the motion.

   c) Calculate the force necessary to keep the cyclist moving around the track.

   Answer: (2 marks)

   Using $\vec{F}_c = m\vec{a}_c$

   $\vec{F}_c = (85.4 \text{ kg})(0.845 \text{ m/s/s}) = 72.2 \text{ N}$

   The centripetal force is 72.2 N towards the centre of the circle.

   d) Calculate the frequency of this motion.

   Answer: (1 mark)

   The frequency is the reciprocal of the period.

   $f = \frac{1}{T} = \frac{1}{59.2 \text{ s}} = 0.0169 \text{ Hz}$

Outcomes S4P-1-13, S4P-0-2h

5. A car of mass 1250 kg is travelling at 20.0 m/s [W]. At an icy intersection, the car collides with a truck of mass 2450 kg travelling at 15.0 m/s [S]. The collision lasts 0.250 seconds. After the collision, the two vehicles slide along together.

   a) What is the total momentum of the system of the car and the truck before the collision?

   Answer: (5 marks)

   The total momentum on the system is the momentum of the car plus the momentum of the truck added together as vectors.

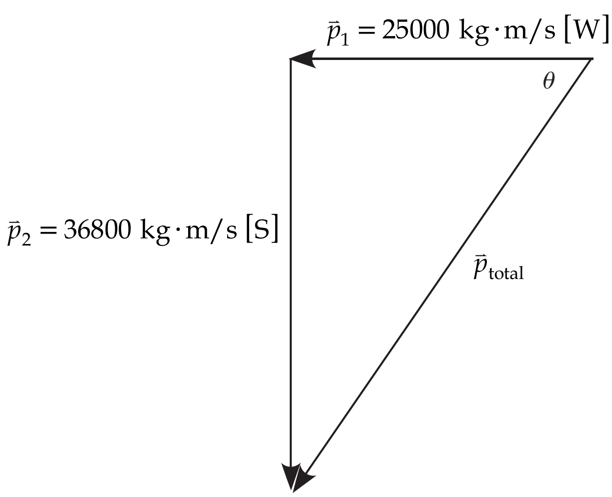
Given:	Mass of the car	$m_1 = 1250$ kg
	Initial velocity of the car	$\vec{v}_1 = 20.0$ m/s [W]
	Mass of the truck	$m_2 = 2450$ kg
	Initial velocity of the truck	$\vec{v}_2 = 15.0$ m/s [S]
Unknown: Total momentum on the system		$\vec{p}_{\text{total}} = ?$

Equation: $\vec{p}_{\text{total}} = \vec{p}_1 + \vec{p}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

Substitute and solve: $\vec{p}_{\text{total}} = (1250\text{ kg})(20.0\text{ m/s [W]}) + (2450\text{ kg})(15.0\text{ m/s [S]})$

$\vec{p}_{\text{total}} = 25000\text{ kg}\cdot\text{m/s [W]} + 36800\text{ kg}\cdot\text{m/s [S]}$

These must be added together as vectors.



$$\vec{p}_{\text{total}}{}^2 = (25000)^2 + (36800)^2 = 1.979 \times 10^9$$

$$\vec{p}_{\text{total}} = 44500\text{ kg}\cdot\text{m/s}$$

Using trigonometry, the tangent function relates the given sides.

$$\theta = \tan^{-1}\frac{36800}{25000} = 55.8°$$

The total momentum of the car and truck together is 44500 kg·m/s [55.8° south of west].

b) What is the velocity of the car after the collision?

Answer: (2 marks)

Since the car and the truck are travelling along together, they have the same velocities after the crash. You can think of the car and the truck together as a single mass moving along with a single velocity with the momentum equal to the total momentum of the system.

$$\vec{p}_{\text{total}} = m_{\text{total}}\vec{v}_{\text{total}}$$

$$44500\ \text{kg}\cdot\text{m/s}\left[55.8^\circ\ \text{S of W}\right] = \left(1250\ \text{kg} + 2450\ \text{kg}\right)\vec{v}_{\text{total}}$$

$$\vec{v}_{\text{total}} = \frac{44500\ \text{kg}\cdot\text{m/s}\left[55.8^\circ\ \text{S of W}\right]}{3700\ \text{kg}} = 12.0\ \text{m/s}\left[55.8^\circ\ \text{S of W}\right]$$

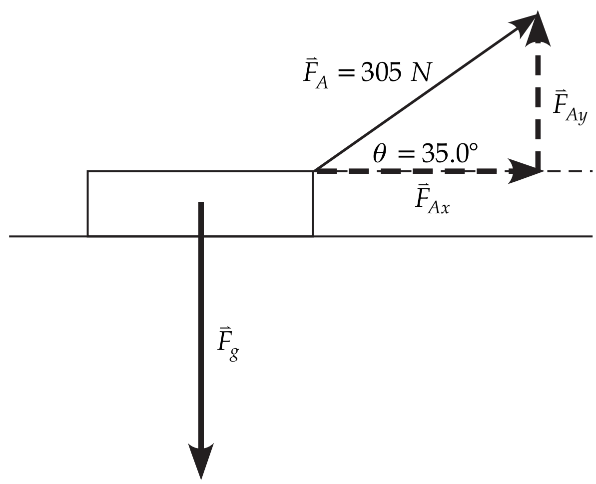
The car and the truck are moving along together at to 12.0 m/s [55.8° south of west].

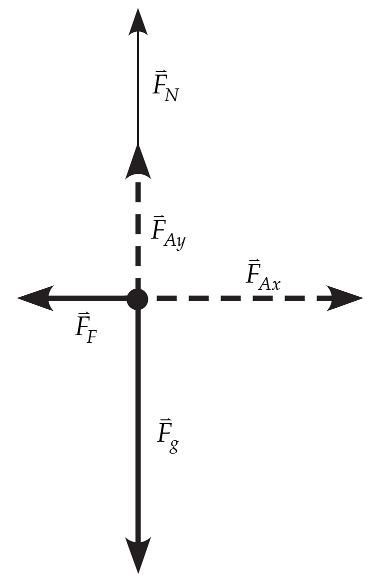
Outcomes S4P-1-8, S4P-0-2h

6. A crate of mass 80.0 kg is pulled across a level concrete floor at a constant acceleration of 0.895 m/s^2. A force of 305 N acting 35.0° above the horizontal is used to move the crate.

   a) Calculate the normal force acting on the crate.

   Answer: (3 marks)





In this case, the normal force is decreased by the vertical component of the applied force, since the component is pulling the object up from the surface.

Given: Mass $m = 80.0$ kg

Acceleration $\vec{a} = +0.895$ m/s/s

Unknown: Normal force $\vec{F}_N = ?$

Equation: From the free-body diagram, you see that

$\vec{F}_g + \vec{F}_N + \vec{F}_{Ay} = 0$ N.

You can find the force of gravity using

$\vec{F}_g = m\vec{g} = (80.0\text{ kg})(9.80\text{ m/s/s [down]}) = 784\text{ N [down]}$.

The vertical component of the applied force is found using sin 35.0° (305 N) = 175 N [up].

Substitute and solve: $784\text{ N [down]} + \vec{F}_N + 175\text{ N }\lfloor\text{up}\rfloor = 0\text{ N}$

$\vec{F}_N = 609\text{ N [up]}$

b) Calculate the force of kinetic friction acting on the crate.

Answer: (3 marks)

Since the crate is sliding in the horizontal direction, we must consider the forces acting in the horizontal direction.

$$\vec{F}_{\text{net}} = \vec{F}_{Ax} + \vec{F}_{K}$$

The net force is found using $\vec{F}_{\text{net}} = m\vec{a} = (80.0 \text{ kg})(0.895 \text{ m/s/s}) = +71.6 \text{ N}$.

The horizontal component of the applied forces is found using cos 35.0° (305 N) = +2.50 × 10^2 N.

Substitute and solve:

$$+71.6 \text{ N} = +2.50 \times 10^2 \text{ N} + \vec{F}_K$$
$$\vec{F}_K = -178 \text{ N}$$

A force of friction is 178 N [left].

c) Calculate the coefficient of kinetic friction.

Answer: (1 mark)

$$\mu_k = \frac{\vec{F}_K}{\vec{F}_N} = \frac{178 \text{ N}}{609 \text{ N}} = 0.292$$

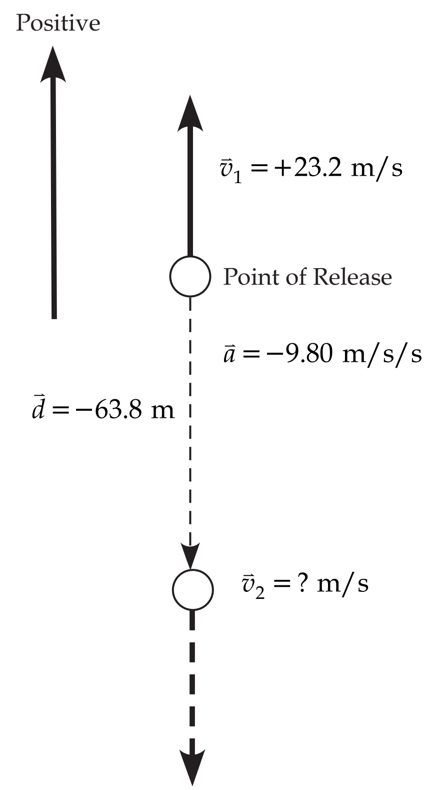
The coefficient of kinetic friction is a 0.292.

Outcome S4P-1-15

7. A stone of mass 75.0 g is thrown upwards at 23.2 m/s from the height of a railing of a bridge that is 63.8 m above the surface of the water.

   a) Calculate the velocity of the stone as it strikes the water's surface.

   Answer: (2.5 marks)



Equation: $v_2^{\,2} = v_1^{\,2} + 2ad$

Substitute and solve: $v_2^{\,2} = (23.2 \text{ m/s})^2 + 2(-9.80 \text{ m/s/s})(-63.8 \text{ m}) = 1788$

$\vec{v}_2 = -42.3$ m/s

The stone strikes the water moving at 42.3 m/s [down].

b) How long after the stone is thrown is the stone 10.0 m above the surface of the water?

Answer: (3 marks)

If the stone is 10.0 m above the surface of the water, it would have fallen 53.8 m from the point to release.

The change in this question is that the displacement will be $\vec{d} = -53.8$ m.

Unknown: Time interval $\Delta t = ?$

Equation: No single kinematics equation will yield the answer unless you use the quadratic formula.

It is simpler to do this in two steps. First, find the final velocity; then find the time interval.

$$v_2^{\,2} = v_1^{\,2} + 2ad = (23.2 \text{ m/s})^2 + 2(-9.80 \text{ m/s/s})(-53.8 \text{ m}) = 1593$$

$$\vec{v}_2 = -39.9 \text{ m/s}$$

Then use $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \dfrac{(-39.9 \text{ m/s} - 23.3 \text{ m/s})}{-9.80 \text{ m/s/s}} = 6.45 \text{ s}$

The stone is 10.0 m above the surface of the water 6.45 seconds after it was released.

c) Where is the stone 3.50 seconds after being thrown?

Answer: (1.5 marks)

Given: Time interval $\Delta t = 3.50$ s

Initial velocity $\vec{v}_1 = +23.2$ m/s

Acceleration $\vec{a} = -9.80$ m/s/s

Unknown: Displacement $\vec{d} = ?$

Equation: $\vec{d} = \vec{v}_1 \Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$

Substitute and solve: $\vec{d} = (23.2 \text{ m/s})(3.50 \text{ s}) + \dfrac{1}{2}(-9.80 \text{ m/s/s})(3.50 \text{ s})^2 = 21.2 \text{ m}$

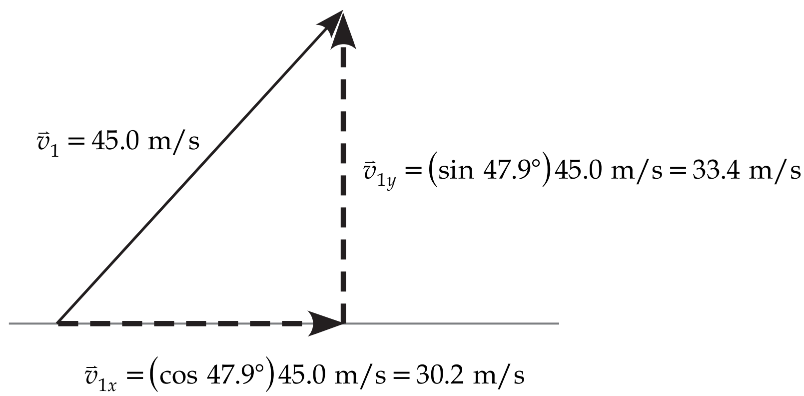
The stone is 21.2 m above the point of release.

Outcomes S4P-1-18, S4P-0-2h

8. A golfball is struck leaving the tee at a velocity of 45.0 m/s 47.9° from the horizontal. The ball travels over a level fairway towards a green where the hole is located 204 m from the tee.

   a) Calculate the vertical and horizontal components of the ball's velocity.

   Answer: (2 marks)



   The horizontal component is 30.2 m/s [right]. The vertical component is 33.4 m/s [up].

   b) Determine the time the ball is in the air.

   Answer: (3 marks)

   Consider only the vertical motion of the ball while it is rising to the top of its flight. Let up be positive.

   Given: Initial velocity $\vec{v}_1 = +33.4\text{ m/s}$

   Final velocity $\vec{v}_2 = 0\text{ m/s}$

   Acceleration $\vec{a}_y = -9.80\text{ m/s}^2$

   Unknown: Time interval $\Delta t = ?$

   Equation: $\Delta t = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$

   Substitute and solve: $\Delta t = \dfrac{0\text{ m/s} - (+33.4\text{ m/s})}{-9.80\text{ m/s}^2} = 3.41\text{ s}$

   The ball takes 3.41 seconds on the way up and another 3.41 seconds on the way down for a total time of 6.82 seconds.

c) If the ball is heading in the right direction, will it be possible for the golfer to score a hole in one?

Answer: (2 marks)

The ball must land before the hole and fly in the correct direction to land in the hole.

Given: Initial horizontal velocity $\vec{v}_{1x} = 30.2 \text{ m/s}$

Time of travel $\Delta t = 6.82 \text{ s}$

Unknown: Horizontal distance $\vec{d}_x = ?$

Equation: The horizontal distance is found using
$\vec{d}_x = \vec{v}_{1x} \Delta t$.

Substitute and solve: $\vec{d}_x = (30.2 \text{ m/s})(6.82 \text{ s}) = 206 \text{ m}$

The ball lands 2 m beyond the hole. There is no hole in one.

Outcomes S4P-1-24, S4P-1-8

9. A satellite orbits Earth in a nearly circular orbit of radius 7.88×10^6 m with a period of 115 minutes. The satellite has a mass 238 kg.

   a) Calculate the speed of the satellite in m/s.

   Answer: (2.5 marks)

   Given: Radius $R = 7.88 \times 10^6$ m

   Period $T = 115$ minutes $\times$ 60 s/min = 6900 s

   Mass $m = 238$ kg

   Unknown: Speed $v = ?$

   Equation: $$v = \frac{2\pi R}{T}$$

   Substitute and solve: $$v = \frac{2\pi R}{T} = \frac{2(3.14)(7.88 \times 10^6 \text{ m})}{6900 \text{ s}} = 7170 \text{ m/s}$$

   The satellite is travelling at 7170 m/s.

   b) Calculate the centripetal force acting on the satellite.

   Answer: (3 marks)

   Equation: $$\vec{F}_c = m\vec{a} = m\frac{v^2}{R}$$

   Substitute and solve: $$\vec{F}_c = (238 \text{ kg})\frac{(7170 \text{ m/s})^2}{7.88 \times 10^6 \text{ m}} = 1550 \text{ N}$$

   The centripetal force is 1550 N towards the centre of Earth.

   c) If the weight of the satellite supplies the centripetal force, calculate the gravitational field strength at this distance from Earth.

   Answer: (1.5 marks)

   Since $\vec{F}_g = m\vec{g}$ and $\vec{g} = \frac{\vec{F}_g}{m}$, you can determine the gravitational field strength.

   $$\vec{g} = \frac{\vec{F}_g}{m} = \frac{1550 \text{ N}}{238 \text{ kg}} = 6.51 \text{ N/kg [towards Earth's centre]}$$

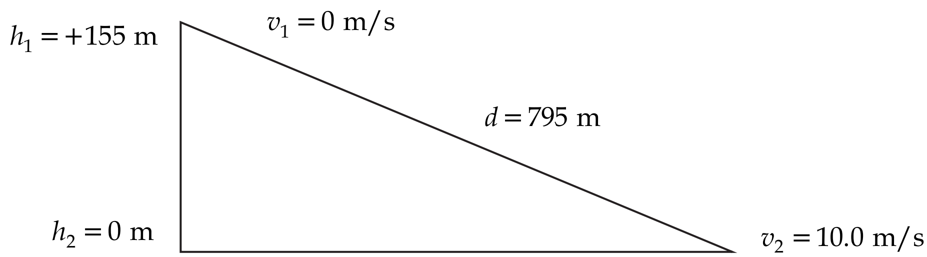
Outcome S4P-1-33

10. A skier of mass 82.4 kg starts his run from rest. The skier drops 155 m vertically while skiing 795 m down the slope. The skier arrives at the bottom of the slope moving at 10.0 m/s.

a) Determine the change in the gravitational potential energy of the skier.

Answer: (2 marks)

Given:



Mass $\quad m = 82.4$ kg

Unknown: Change in gravitational potential energy $\quad \Delta PE_g = ?$

Equation: $\quad \Delta PE_g = m\vec{g}\Delta h$

Substitute and solve: $\quad \Delta PE_g = (82.4 \text{ kg})(9.80 \text{ N/kg})(0 \text{ m} + (-155 \text{ m})) = -125000 \text{ J}$

The skier lost 125000 J of gravitational potential energy.

b) Determine the work done by friction.

Answer: (3 marks)

Friction does negative work on the skier—that is, it removes energy from the system. Therefore, the total energy of the system after the run is less than before the run. The missing energy is the work done by friction.

$$\Delta E_{\text{total}} = \Delta E_{\text{total final}} - \Delta E_{\text{total initial}}$$

$$\Delta E_{\text{total}} = \left(KE_f + PE_{gf}\right) - \left(KE_0 + PE_{g0}\right)$$

$$\Delta E_{\text{total}} = \left(\frac{1}{2}mv_f^{\,2} + mgh_f\right) - \left(\frac{1}{2}mv_0^{\,2} + mgh_0\right)$$

$$\Delta E_{\text{total}} = \left(\frac{1}{2}(82.4 \text{ kg})(10.0 \text{ m/s})\right)^2 + \left((82.4 \text{ kg})(9.80 \text{ N/kg})(0 \text{ m})\right)$$

$$- \frac{1}{2}\left((82.4 \text{ kg})(0 \text{ m/s})\right)^2 + \left((82.4 \text{ kg})(9.80 \text{ N/kg})(+155 \text{ m})\right)$$

$$\Delta E_{\text{total}} = (4120 \text{ J}) - (125000 \text{ J}) = -121000 \text{ J}$$

Work of friction = –121000 J

Since the total energy decreases by 121000 J, this is the work done by friction.

c) Determine the average force of friction that acted on the skier.

Answer: (2 marks)

Given: Work of friction $W_F = -121000$ J

Distance over which friction acts $\vec{d} = 795$ m

Unknown: Force of friction $\vec{F}_K = ?$

Equation: $W_F = \vec{F}_K \vec{d}$

Substitute and solve: $-121000 = \vec{F}_K (795)$

$\vec{F}_K = -152 \text{ N}$

The average force of friction is 152 N opposite to the motion.

Outcome S4P-1-33

11. A dry ice puck, which slides along with no friction, has a mass of 2.30 kg and is sliding along a level horizontal surface at 2.25 m/s. The puck hits a spring bumper, which is compressed 20.0 cm, before the puck comes to rest.

a) Determine the force constant of this spring.

Answer: (2 marks)

The puck will compress the spring until the puck stops moving. At that point, the initial kinetic energy of the puck will have been converted into spring potential energy. The total mechanical energy of the system is conserved.

Given:	Mass	$m = 2.30$ kg
	Initial velocity	$\vec{v}_1 = 2.25$ m/s [towards the spring]
	Final velocity	$\vec{v}_2 = 0$ m/s
	Compression of spring	$x = 20.0$ cm $= 0.200$ m
Unknown:	Spring constant	$k = ?$

Equation:

$$KE_1 + PE_{S1} = KE_2 + PE_{S2}$$

$$\frac{1}{2}mv_1^{\,2} + \frac{1}{2}kx_1^{\,2} = \frac{1}{2}mv_2^{\,2} + \frac{1}{2}kx_2^{\,2}$$

Substitute and solve:

$$\frac{1}{2}(2.30\text{ kg})(2.25\text{ m/s})^2 + \frac{1}{2}k(0\ m)^2$$

$$= \frac{1}{2}(2.30\text{ kg})(0\text{ m/s})^2 + \frac{1}{2}k(0.200\text{ m})^2$$

$$5.82\text{ J} + 0\text{ J} = 0\text{ J} + 0.0200k$$

$$k = 291\text{ N/kg}$$

The spring constant is 291 N/kg.

b) How much is the spring compressed when the puck is sliding at 1.75 m/s towards the bumper?

Answer: (2.5 marks)

Use the law of conservation of kinetic energy and spring potential energy again.

Given:	Mass	$m = 2.30$ kg
	Initial velocity	$\vec{v}_1 = 2.25$ m/s [towards the spring]
	Final velocity	$\vec{v}_2 = 1.75$ m/s
	Spring constant	$k = 291$ N/kg?
Unknown:	Compression of spring	$x_2 = ?$

Equation:

$$KE_1 + PE_{S1} = KE_2 + PE_{S2}$$

$$\frac{1}{2}mv_1^{\,2} + \frac{1}{2}kx_1^{\,2} = \frac{1}{2}mv_2^{\,2} + \frac{1}{2}kx_2^{\,2}$$

Substitute and solve:

$$\frac{1}{2}(2.30\text{ kg})(2.25\text{ m/s})^2 + \frac{1}{2}(291\text{ N/kg})(0\text{ m})^2$$

$$= \frac{1}{2}(2.30\text{ kg})(1.75\text{ m/s})^2 + \frac{1}{2}(291\text{ N/kg})(x_2\text{ m})^2$$

$$5.82\text{ J} + 0\text{ J} = 3.52\text{ J} + 145.5x_2^{\,2}$$

$$2.30\text{ J} = 145.5x_2^{\,2}$$

$$x_2^{\,2} = 0.01581$$

$$x_2 = 0.126\text{ m}$$

The spring is compressed 0.126 m.

c) If the dry ice puck was initially moving at 2.25 m/s towards the spring and the spring is compressed only 10.0 cm, calculate the speed of the puck at that moment.

Answer: (2.5 marks)

Given: Mass $m = 2.30$ kg

Initial velocity $\vec{v}_1 = 2.25$ m/s [towards the spring]

Spring constant $k = 291$ N/kg?

Compression of spring $x_2 = 10.0$ cm $= 0.100$ m

Unknown: Final velocity $\vec{v}_2 = 1.75$ m/s

Equation:

$$KE_1 + PE_{S1} = KE_2 + PE_{S2}$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Substitute and solve:

$$\frac{1}{2}(2.30 \text{ kg})(2.25 \text{ m/s})^2 + \frac{1}{2}(291 \text{ N/kg})(0 \text{ m})^2$$

$$= \frac{1}{2}(2.30 \text{ kg})\vec{v}_2^2 + \frac{1}{2}(291 \text{ N/kg})(0.100 \text{ m})^2$$

$$5.82 \text{ J} + 0 \text{ J} = 1.15\vec{v}_2^2 + 1.46 \text{ J}$$

$$\vec{v}_2^2 = \frac{4.36}{1.15} = 3.79$$

$$\vec{v}_2 = 1.95 \text{ m/s}$$

The dry ice puck is sliding at 1.95 m/s.

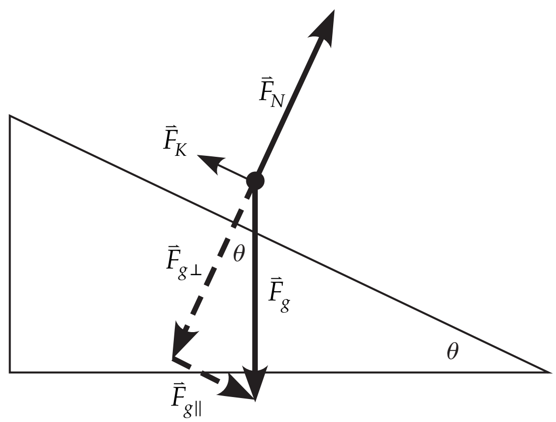
Outcomes S4P-1-5, S4P-1-7, S4P-1-8

12. A child is sitting in a wagon on a hill, which has an incline of 14.0° from the horizontal. The mass of the child and wagon is 42.0 kg. The coefficient of kinetic friction is 0.125. The wagon begins to move.

 a) Calculate the normal force acting on the wagon.

 Answer: (2.5 marks)

Given:	Mass	$m = 42.0$ kg
	Angle from the horizontal	$\theta = 14.0°$
	Coefficient of kinetic friction	$\mu_k = 0.125$
Unknown: Normal force		$\vec{F}_N = ?$



 The components of $\vec{F}_g$ form a right triangle enclosing the angle θ. This angle θ is the same as the angle θ in the ramp.

 The components of $\vec{F}_g$ are found by:

$$\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right)$$
$$\vec{F}_{g\parallel} = \sin\theta\left(\vec{F}_g\right)$$

 The force of gravity is $\vec{F}_g = m\vec{g} = (42.0\text{ kg})(9.80\text{ N/kg}) = 412\text{ N [down]}$.

 The component of the force of friction perpendicular to the plane is
 $\vec{F}_{g\perp} = \cos\theta\left(\vec{F}_g\right) = \cos 14.0°(412\text{ N}) = 400\text{ N} = 4.00 \times 10^2\text{ N}$ [into the surface].

 The normal force is then 4.00×10^2 N [out of the surface].

 b) Calculate the force of kinetic friction.

 Answer: (1.5 marks)

 $F_K = \mu_k F_N = (0.125)(400\text{ N}) = 50.0\text{ N}$

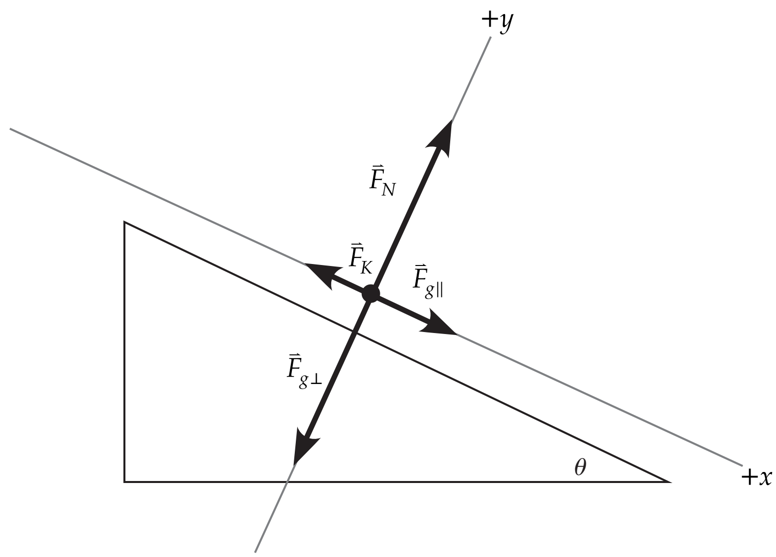
 The force of friction is 50.0 N [up the plane].

c) Calculate the distance the child and his wagon will move during the first 15.0 s of his trip.

Answer: (3 marks)

You can see that the only forces acting along the direction of motion (+*x*-direction) are the force of friction and the parallel component of gravity.

Find the net force, then the acceleration, and finally displacement.



$$\vec{F}_{g\parallel} = \sin\theta \ \vec{F}_g = \sin 14.0°(412 \text{ N}) = 99.7 \text{ N}$$

$$\vec{F}_{\text{net}} = \vec{F}_{g\parallel} + \vec{F}_K = 99.7 \text{ N [down the plane]} + 50.0 \text{ N [up the plane]}$$

$$\vec{F}_{\text{net}} = 49.7 \text{ N [down the plane]}$$

Using $\vec{F}_{\text{net}} = m\vec{a}$ rearranged to $\vec{a} = \dfrac{\vec{F}_{\text{net}}}{m}$, $\vec{a} = \dfrac{49.7 \text{ N}}{42.0 \text{ kg}} = 1.18 \text{ m/s}^2 \text{ [down the plane]}$.

Finally, use kinematics to find the displacement.

Given:	Initial velocity	$\vec{v}_1 = 0 \text{ m/s}$
	Acceleration	$\vec{a} = 1.18 \text{ m/s}^2 \text{ [down the plane]}$
	Time interval	$\Delta t = 15.0 \text{ s}$
Unknown:	Distance travelled	$\vec{d} = ?$

Equation: $\vec{d} = \vec{v}_1 \Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$

Substitute and solve: $\vec{d} = (0 \text{ m/s})(15.0 \text{ s}) + \dfrac{1}{2}(1.18 \text{ m/s}^2)(15.0 \text{ s})^2 = 133 \text{ m}$

The child and his wagon move 133 m during their 15.0 s trip.