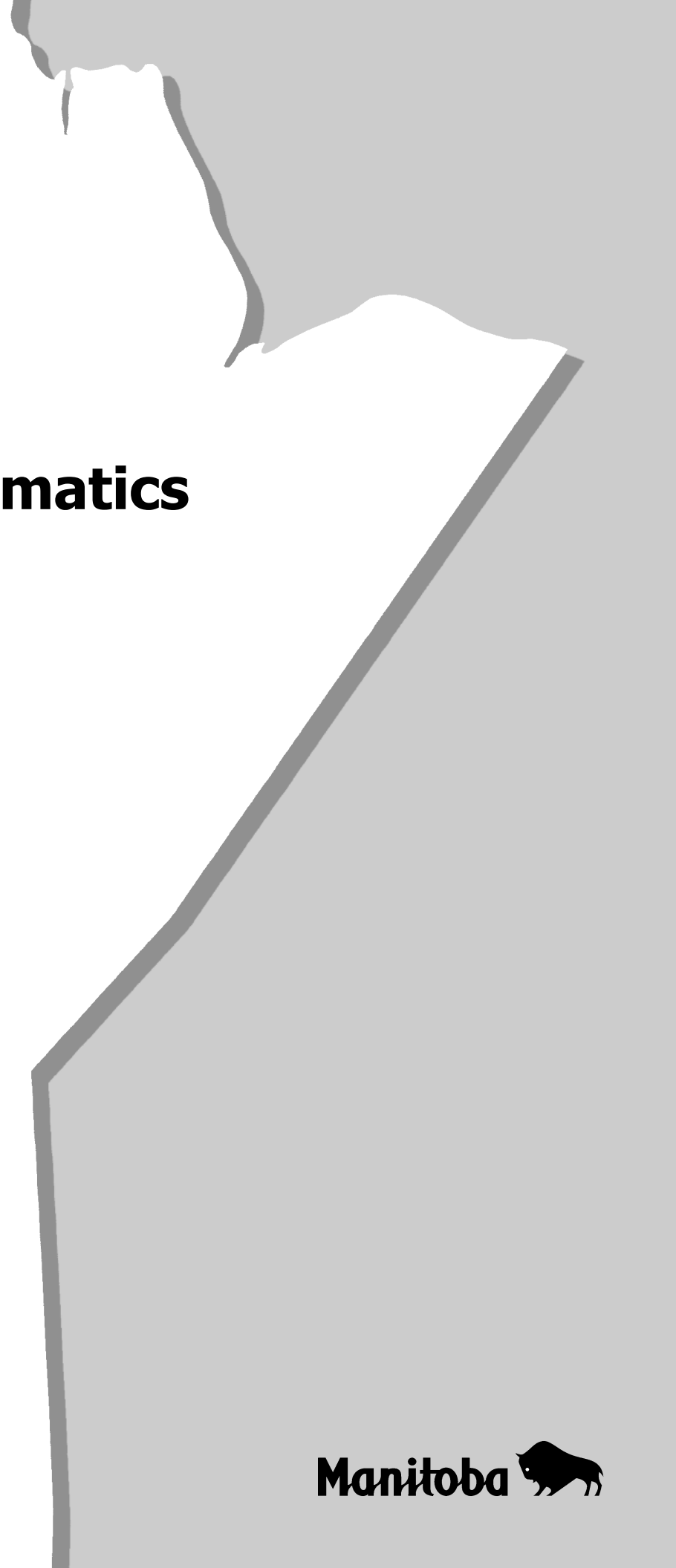
Grade 9 Mathematics (10F)

A Course for

Independent Study



Manitoba

GRADE 9 MATHEMATICS (10F)

A Course for Independent Study

2012
Manitoba Education

Manitoba Education Cataloguing in Publication Data

Grade 9 mathematics (10F) : a course for independent study

ISBN-13: 978-0-7711-5120-0

1. Mathematics—Study and teaching (Secondary).
2. Mathematics—Programmed instruction.
3. Mathematics—Study and teaching (Secondary)—Manitoba.
4. Correspondence schools and courses—Manitoba.
5. Distance education—Manitoba.
I. Manitoba. Manitoba Education
510.76

Manitoba Education
School Programs Division
Winnipeg, Manitoba, Canada

Ce document est disponible en français.

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Acknowledgements

Manitoba Education gratefully acknowledges the contributions of the following individuals in the development of *Grade 9 Mathematics (10F): A Course for Independent Study*.

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GRADE 9 MATHEMATICS (10F)

Introduction

Course Introduction

Overview

Welcome to Grade 9 Mathematics! This course is a continuation of the concepts you have been studying in previous years, as well as an introduction to new topics. This course develops foundational skills and ideas that you will need to continue studying math in the future. Problem solving, communication, reasoning, and mental math are some of the themes you will discover in each module.

As you work through the content of this course, you will discover, learn, and apply mathematical ideas to the world around you. There are four main areas you will be exploring: numbers; relations and patterns; geometry; and statistics and probability. After completing this course, you will have gained critical skills and knowledge that you will need for future math learning and confidence in your abilities as a mathematician!

There are two appendices in this course. Appendix A contains the blueprints for algebra tiles, which you can cut out and use. Appendix B is a glossary of important terms and definitions.

This course is divided into eight modules, organized as follows:

- Module 1: Statistics
- Module 2: Number Sense
- Module 3: Powers
- Module 4: Polynomials
- Module 5: Linear Relations
- Module 6: Circle Geometry
- Module 7: 2-D and 3-D Objects
- Module 8: Symmetry

What Will You Need?

Please note that you do not need a textbook to complete this course. All of the content is included with this package.

Required Resources

Here is a list of things that you must have to complete this course:

- a scientific or graphing calculator
- a metric ruler
- a protractor
- some graph paper
- A notebook or binder to collect your completed learning activities (These are activities that you will complete, but not send in for assessment.)

Optional Resources

- Access to a computer with spreadsheet and graphing capabilities is an advantage but not a requirement. Use of the Internet is suggested as a resource in some places, but if you do not have access to an online computer you can still complete the related learning activities and assignments without it.
- Access to a photocopier would be helpful because it would let you make a copy of your assignments before you send them to your tutor/marker. That way, if you and your tutor/marker want to discuss an assignment, you both have a copy to refer to.
- Newspapers or magazines may be used as sources where media information is required.

Resource Sheet

When you write your midterm and final exams, you will be allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. It is to be submitted with your exam. The Exam Resource Sheet is not worth any marks. The Midterm Resource Sheet will be returned to you.

Many students have found making a resource sheet an excellent way to review. It also provides you with a summary of the important facts of each module available when you need it. Each student is asked to complete a resource sheet for each module to help with your studying and reviewing. The Lesson Summaries are written for you to use as a guide, as are the Module Summaries at the end of each module.

As you complete each module's resource sheet, you will then be able to try to summarize the sheets from all of the modules to prepare your Exam Resource Sheet. The midterm exam is based only on Modules 1 to 4, while the final exam is based on Modules 1 to 8.

How Will You Know How You're Doing?

You will know how well you are learning by your successful completion of the following course components:

Learning Activities

One of the easiest and fastest ways to find out how much you have learned is by completing the learning activities. These have been designed to let you assess yourself by comparing your answers with the answer keys at the end of each module. They are found in most lessons. Some lessons have more than one. You will need a notebook to write your answers in.

Make sure you complete each learning activity. Besides giving you feedback, it will help you practice what you have learned and prepare you to successfully complete your assignments and exams. Many of the questions on the exams will be similar to the questions in the learning activities. **Do not mail learning activities to your tutor/marker.**

Each Learning Activity has two parts—Part A has BrainPower questions and Part B has questions related to the content in the lesson. The BrainPower questions are provided as a warm-up activity for you to do before trying the other questions. Each question should be completed quickly and without the use of a calculator or pen and paper. Some of the questions will directly relate to content you are learning in this course. Some of the questions will be a review of content from previous courses. Being able to do these questions in a few minutes will be helpful to you as you continue with your studies in mathematics.

If you are finding it is taking you longer to do the questions, you can try one of the following:

1. work with your learning partner to find more efficient strategies for completing the questions
2. ask your tutor/marker for help with the questions
3. search online for websites that help you practice the computations so you can become more efficient at completing the questions.

Your work on the BrainPower questions will not be directly assessed. None of the assignment questions or exam questions will require you to do the calculations quickly or without a calculator. However, it is for your benefit to complete the questions as they will help you in the course. They will help to build your confidence in your mathematics skills. BrainPower questions are like a warm-up you would do before competing in a sporting event.

Remember, the questions in the learning activities are similar to the ones that will be on your assignments and exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them. Don't skip ahead without learning. If you do, you will be wasting your time, and you won't be able to complete subsequent lessons.

Midterm and Final Exams

The course contains a midterm exam and a final exam. You will write them both under supervision. The midterm exam is based on Modules 1 to 4 and is worth 15% of the final mark of the course. You will write it when you have completed Module 4. In order to do well on the midterm exam, you should review all of the work that you have completed from Modules 1 to 4, including all Learning Activities and Assignments. You will need the following supplies to complete the midterm exam: pen, pencil, paper, and scientific or graphing calculator.

The final exam is cumulative and based on Modules 1 to 8. It is worth 25% of the final mark of the course. You will write it when you have completed Module 8. In order to do well on the final exam, you should review all of the work that you have completed from Modules 1 to 8, including all learning activities and assignments. You will need the following supplies to complete the final exam: pen, pencil, paper, scientific calculator, metric ruler, and protractor.

You are responsible to apply for the exams and make arrangements to have the exams sent to your proctor from the Independent Study Option office. Before you finish Module 4, you will need to make arrangements to write your midterm exam. Before you finish Module 8, you will need to make arrangements to write your final exam. When you write either of these exams, you will be supervised by a proctor. Contact the Independent Study Option at 1-800-465-9915 if you need help arranging this.

Here is how you apply for an exam:

If you are attending school, ask your school's ISO Facilitator to add your name to the ISO exam eligibility list. Do this ***at least three weeks prior*** to the next scheduled exam week.

If you are not attending school, check the **Examination Request Form** for options available to you. The Examination Request Form was mailed to you with this course. Fill in this form and mail or fax it three weeks before you are ready to write your exam. The address is:

ISO Registration
555 Main St.
Winkler, MB R6W 1C4
Fax: 204-325-1719
Phone: 1-800-465-9915

Practice Exams and Answer Keys

To help you succeed in your midterm and final exams, you may want to write the practice exams that are found at: <www.edu.gov.mb.ca/k12/dl/downloads/index.html>.

These exams are very similar to the actual exams that you will be writing. They also include an answer key, so that you can check your answers when you have finished writing them. This will give you the confidence that you need to do well on your exams. If you do not have access to the Internet, contact the Independent Study Option at 1-800-465-9915 to get a copy of the practice exams.

Assignments

Every module contains assignments that you will be sending to your tutor/marker for assessment. The assignment component of this course is worth 60% of your final mark. In order to do well on your assignments, you should complete all learning activities first and check your answers in the answer key provided.

The first assignment in each module is called a Cover Assignment. These are a little different from other assignments because they ask things that you have learned in previous grades or previous modules in this course. Remember to keep all assignments that have been marked and returned to you, as you will need to review them for the exams.

What if You Need Help?

Here are two people who can help you be successful in your course.

Your Tutor/Marker

The first person who can help you is your tutor/marker. Tutor/markers are experienced teachers who tutor ISO students and mark assignments and exams. If you are having difficulty at any time during this course, be sure to contact your tutor/marker by phone or email. They are there to help you. If you are not sure how to contact your tutor/marker, phone the Independent Study Option at 1-800-465-9915

Your Learning Partner

The next person who can help you with your course is your learning partner. A learning partner is someone you choose who will help you learn. It may be someone who knows something about math, but it doesn't have to be. A learning partner could be someone else who is taking this course, a teacher, parent, sibling, or a friend, or anybody else who can help you. Most importantly, a learning partner should be someone you feel comfortable with and who will support you as you work through this course.

Your learning partner can help you keep on schedule, check your work, help you make sense of assignments, read your course with you, or look at your learning activities and respond to them. You may even study for your exam with your learning partner.

One of the best ways that your learning partner can help you is by reviewing your midterm and final practice exams with you. These are found here: <www.edu.gov.mb.ca/k12/dl/downloads/index.html>, along with their answer keys. Your learning partner can administer your practice exam, check your answers with you, and then help you learn the things that you missed.

How Much Time Will You Need?

Learning through independent study has several advantages over learning in the classroom. You are in charge of how you learn and can choose how quickly you will complete the course. You don't have to wait for your teacher or classmates, and you can work as quickly as you want. You can also complete as many lessons at a time as you want. Read the next few pages to get an idea of how to pace yourself. You have one full year from the date of your registration to complete this course, but the pace at which you complete it is up to you.

Chart A: Semester 1

Here is a **suggested timeline** that you can follow if you start your course in September and need to complete it by the end of January.

Module	Completion Date
Module 1	mid-September
Module 2	late September
Module 3	mid-October
Module 4 and Midterm Exam	early November
Module 5	mid-November
Module 6	early December
Module 7	mid-December
Module 8 and Final Exam	mid-January

Chart B: Semester 2

Here is a **suggested timeline** that you can follow if you start your course in January and need to complete it by June.

Module	Completion Date
Module 1	mid-February
Module 2	late February
Module 3	mid-March
Module 4 and Midterm Exam	early April
Module 5	mid-April
Module 6	early May
Module 7	late May
Module 8 and Final Exam	mid-June 15

Chart C: Full School Year (Not Semestered)

Here is a **suggested timeline** that you can follow if you have registered for this course in September and would like to complete it by June.

Module	Completion Date
Module 1	late September
Module 2	late October
Module 3	late November
Module 4 and Midterm Exam	mid-January
Module 5	mid-February
Module 6	mid-March
Module 7	mid-April
Module 8 and Final Exam	late May

Do not wait until the last minute to complete your work, since your tutor/marker may not be available to mark it immediately. Make sure that you leave enough time for your work to travel through the mail, as it might take over a week. It may also take a few weeks for your tutor/marker to mark everything and send the marks to your school.

If you need this course to graduate this school year, remember to schedule and complete your final exam by June 1.

How Do You Send in Your Assignments?

In this course, you must mail your assignments to your tutor/marker. The instructions for doing so are found in the course, at the times when you will be sending them in. Here is a chart showing exactly what you will be sending in during the five mailings.

Mailing	Cover Sheet	Assignments That You Will Be Sending In
Mailing 1	Cover Sheet 1	Assignments 1.1 to 1.7
Mailing 2	Cover Sheet 2	Assignments 2.1 to 2.6
Mailing 3	Cover Sheets 3 and 4	Assignments 3.1 to 4.7
Mailing 4	Cover Sheets 5 and 6	Assignments 5.1 to 6.7
Mailing 5	Cover Sheets 7 and 8	Assignments 7.1 to 8.4

Each time you **mail** something, you must include the print version of the applicable Cover Sheet (found at the end of this introduction). Complete the information at the top of the Cover Sheet before mailing it along with your assignments.

Before you mail your completed assignments, please make a photocopy of all of the materials first so that you will have a copy in case your package goes missing. You will need to place the applicable module Cover Sheet and assignments in an envelope and address it to

ISO Tutor/Marker
555 Main St.
Winkler MB R6W 1C4

Your tutor/marker will mark your work and return it to you by mail.

What Are the Guide Graphics For?

Graphics have been placed inside the margins of the course to identify a specific task. Each graphic has a specific purpose to guide you. A description of each graphic is provided below:

Lesson Introduction
The introduction sets the stage for the lesson. It may draw upon prior knowledge or briefly describe the organization of the lesson. It also lists the outcomes for the lesson. These describe what you will learn.

Assignment
This icon indicates an assignment that you complete and send to your tutor/marker. You will be sending in your assignments in accordance with the chart found under "When and How Do You Send in Your Assignments?"

Learning Activity
This icon indicates a learning activity that will help you review or practice what you have learned and prepare for your assignment and exam. You will not send learning activities to your tutor/marker.

Mail-in
This icon indicates when it is time to mail in your assignments.

Learning Partner
This icon indicates a part of the lesson when your learning partner could really help you learn or do something.

Resource Sheet:
This icon indicates material that may be valuable to include on your Resource Sheet.

Good Luck with the Course!

And remember, if you need help at any point during this course, contact your tutor/marker.

GRADE 9 MATHEMATICS

Module 1: Statistics
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 1: Statistics

Date Received: _____ Date Returned: _____

	Marks
❑ Assignment 1.1: Cover Assignment	____ /14
❑ Assignment 1.2: Populations and Samples	____ /21
❑ Assignment 1.3: Bias in Data Collection	____ /15
❑ Assignment 1.4: Case Study	____ /14
❑ Assignment 1.5: Displaying Data	____ /12
❑ Assignment 1.6: Data Analysis	____ /14
❑ Assignment 1.7: Developing a Project	____ /35
Total:	____ /125

Remarks:

_____

GRADE 9 MATHEMATICS

Module 2: Number Sense
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 2: Number Sense

Date Received: _____ Date Returned: _____

	Marks
❑ Assignment 2.1: Cover Assignment	____ /23
❑ Assignment 2.2: Decimal Numerals	____ /18
❑ Assignment 2.3: Operations with Fractions	____ /14
❑ Assignment 2.4: Conversions	____ /14
❑ Assignment 2.5: Perfect Square Numbers	____ /15
❑ Assignment 2.6: Estimating Square Roots	____ /18
Total:	____ /102

Remarks:

GRADE 9 MATHEMATICS

Module 3: Powers
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 3: Powers

Date Received: _____ Date Returned: _____

(Remember: Module 3 and Module 4 assignments are to be submitted together at the completion of Module 4.)

	Marks
❑ Assignment 3.1: Cover Assignment	____ /21
❑ Assignment 3.2: The Meaning and Uses of Powers	____ /26
❑ Assignment 3.3: Order of Operations	____ /24
❑ Assignment 3.4: Operations with Powers	____ /20
❑ Assignment 3.5: Powers with Like Bases	____ /20
❑ Assignment 3.6: More Power Laws	____ /16
Total:	____ /127

Remarks:

GRADE 9 MATHEMATICS

Module 4: Polynomials
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 4: Polynomials

Date Received: _____ Date Returned: _____

(Remember: Module 3 and Module 4 assignments are to be submitted together at the completion of Module 4.)

	Marks
❑ Assignment 4.1: Cover Assignment	____ /23
❑ Assignment 4.2: Introduction to Polynomials	____ /23
❑ Assignment 4.3: Collecting Like Terms	____ /20
❑ Assignment 4.4: Distribute the Negatives	____ /23
❑ Assignment 4.5: Multiplication	____ /18
❑ Assignment 4.6: Division and Equivalents	____ /17
❑ Assignment 4.7: Geometry and Polynomials	____ /16
Total:	____ /140

Remarks:

GRADE 9 MATHEMATICS

Module 5: Linear Relations
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 5: Linear Relations

Date Received: _____ Date Returned: _____

(Remember: Module 5 and Module 6 assignments are to be submitted together at the completion of Module 6.)

	Marks
❑ Assignment 5.1: Cover Assignment	____ /15
❑ Assignment 5.2: Patterns and Translations	____ /16
❑ Assignment 5.3: Writing Equations	____ /20
❑ Assignment 5.4: Working with Graphs	____ /24
❑ Assignment 5.5: Operations with Equations	____ /24
❑ Assignment 5.6: Inequalities and Graphing	____ /22
❑ Assignment 5.7: Inequalities and Problems	____ /15
Total:	____ /136

Remarks:

GRADE 9 MATHEMATICS

Module 6: Circle Geometry
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 6: Circle Geometry

Date Received: _____ Date Returned: _____

(Remember: Module 5 and Module 6 assignments are to be submitted together at the completion of Module 6.)

	Marks
❑ Assignment 6.1: Cover Assignment	____ /15
❑ Assignment 6.2: Measurement and Terminology	____ /30
❑ Assignment 6.3: Central and Inscribed Angles	____ /18
❑ Assignment 6.4: More Inscribed Angles	____ /18
❑ Assignment 6.5: Tangents	____ /14
❑ Assignment 6.6: Perpendicular Bisectors	____ /19
❑ Assignment 6.7: Solving Problems	____ /26
Total:	____ /140

Remarks:

_____

GRADE 9 MATHEMATICS

Module 7: 2-D and 3-D Objects
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 7: 2-D and 3-D Objects

Date Received: _____ Date Returned: _____

(Remember: Module 7 and Module 8 assignments are to be submitted together at the completion of Module 8.)

	Marks
❑ Assignment 7.1: Cover Assignment	____ /16
❑ Assignment 7.2: Composite Objects	____ /24
❑ Assignment 7.3: Similarity	____ /24
❑ Assignment 7.4: Drawing Polygons	____ /15
❑ Assignment 7.5: Scale Drawings	____ /15
Total:	____ /94

Remarks:

GRADE 9 MATHEMATICS

Module 8: Symmetry
Cover Sheet

Please place on top of your assignments to assist in proper recording of your work.

Send to:
ISO Tutor/Marker
555 Main Street
Winkler, MB R6W 1C4

Name: _____ Phone: _____

Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes Email: _____

School Name: _____

For Office Use Only

Module 8: Symmetry

Date Received: _____ Date Returned: _____

(Remember: Module 7 and Module 8 assignments are to be submitted together at the completion of Module 8.)

	Marks
❑ Assignment 8.1: Cover Assignment	____ /16
❑ Assignment 8.2: Line Symmetry	____ /18
❑ Assignment 8.3: Rotation Symmetry	____ /18
❑ Assignment 8.4: Art and Tessellations	____ /30
Total:	____ /82

Remarks:

Released 2012

Printed in Canada
Imprimé au Canada

GRADE 9 MATHEMATICS (10F)

Module 1: Statistics

MODULE 1: STATISTICS

Introduction

You are exposed to a lot of statistical information through advertising, media, and the Internet. This module will help you to analyze that information, and explain how to determine if it is biased or slanted towards one view. It will illustrate how the method of data collecting and the wording of the survey question can seriously affect the results. You will see how making graphs to illustrate the data helps to communicate the results and draw conclusions about it. You will apply these considerations to case studies and, in a project where you will collect and organize data, display it and draw conclusions based on your interpretation of the statistics.

As you work through this module, try to become aware of the use of statistics and probability in society while reading magazines and newspapers, listening to radio and television, or surfing the net.

Assignments in Module 1

You will be assessed on how well you complete the following assignments, which are found within the lessons themselves.

Lesson	Assignment Number	Assignment Title
1	Assignment 1.1	Cover Assignment
2	Assignment 1.2	Populations and Samples
3	Assignment 1.3	Bias in Data Collection
4	Assignment 1.4	Case Study
5	Assignment 1.5	Displaying Data
6	Assignment 1.6	Data Analysis
7	Assignment 1.7	Developing a Project

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 1. You might use your Module 1 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 1 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 1 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

Lesson 1: Cover Assignment

Throughout this *Grade 9 Mathematics* course, you will be required to use reasoning skills in problem-solving situations. When solving a problem, you often make assumptions that determine the number and range of solutions to the problem. An **assumption** is simply the conditions on which you base an argument or solution. Check the following examples.

Example 1

Two players play five games of chess. Each person wins three games. How is this possible?

Solution:

Assumption: The players do not play the games against each other.

Answer: Up to 10 or more different games may have been going on.

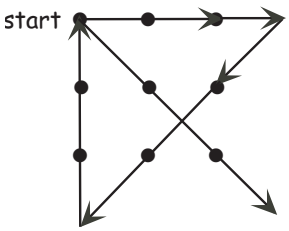
Example 2

Join all nine dots with four straight lines without lifting your pencil and without overlapping:

Solution:

Assumption: The lines may go outside the square.

Answer:



NOTES

Assignment 1.1: Cover Assignment

Total: 14 marks

Solve each problem and note any **assumptions** you make in order to arrive at your solution. Please write your **answers** in the space provided on this page.

1. The rhinoceros beetle can grow to about 150 mm in length. How many finger widths is that? *(2 marks)*

2. By the beginning of the 1990s cars were being produced at the rate of 60 cars per minute worldwide. How many cars were produced worldwide in one year? *(2 marks)*

3. The average person eats 12 L of ice cream in one year. If a person lived for 80 years, how much ice cream would he/she have eaten? *(2 marks)*

4. Find a rectangular piece of paper and complete the following activities. *(5 marks)*

   a) Fold it in half. Record the number of folds and the number of equal parts into which the paper is divided.

   b) Fold the paper in half again and record your results in the table below.

   c) Repeat as often as you can to complete the chart.

Folds	Parts
1	
2	
3	
4	
5	

   d) How many equal parts will there be after 20 folds?

5. Make up a problem of your own that requires the solver to make assumptions. For the problem, state your assumption and your solution. You may want to work with your learning partner to check your problem, assumption, and solution. *(3 marks)*

Remember, you will forward this assignment to the tutor/marker at the end of Module 1.

NOTES

LESSON 2: POPULATIONS AND SAMPLES

Lesson Focus

When you finish this lesson, you will be able to

- ❑ distinguish between a sample and a census
- ❑ describe when each should be used

Lesson Introduction

Data can be collected from an entire population or from a small sample of that population. The "population" of a study is everything in the group that is being studied. A "sample" is a representative part of the whole group. In this lesson you will decide if a population or a sample is most appropriate.

Is it a Census or Is it a Sample?

A census is used when an entire population is being surveyed. A population could be the members of a volleyball team, the apples on a given tree, the bacteria on a specific doorknob, or the residents of a city. When you collect data from or about each individual member of a population, you are taking a census. When you elect the president of your school's student council, every student in the school is allowed to vote. In this example, the population of the study would be all the students in the school. Not every single student is necessarily present to vote, but the intent is to include every student in the census.

When the Government of Canada wants to survey every adult in the country, it can be a very costly and lengthy process. According to Statistics Canada, in July 2007 there were 33 390 141 Canadian citizens, about 26 million of whom were adults. Creating and printing forms, hiring staff, and paying postage to contact that many people costs millions of dollars. The time from the start of the process until all the information has been collected can be many years. For more details, you can check out the "Stat Can" website at <www.statcan.ca>.

Therefore, statisticians will often use a sample, or a part of the population, to collect data. A good sample will represent the entire group, so statisticians can generalize about the entire population from the information collected from the sample. A sample from the populations listed above could be: the volleyball team members with birthdays on odd numbered days, every 10th apple picked off the tree, the bacteria picked up on one swab of the doorknob, or the first name on each page of the city phone book.

A produce manager in a grocery store is watching a shipment of oranges, looking for the percentage of fruit that might be rotten. He can't possibly open every box in the shipment. He might open every 10th box, or every 50th box, to check. This would be a random sample. A random sample is a representative part of the population, selected in such a way that each item has an equal chance of being selected. In this example, the population for this study would be all of the oranges. If the manager just checked the first 10 boxes at the front of the truck, he would not be taking a random sample.

Another example of a random sample would be picking a student from your class. For the sample to be truly random, every student must have an equal opportunity of being selected. To accomplish this, the teacher could put the name of each student in a hat, and draw a name.

Before you continue, include the definitions for *census* and *sample* in your own words. **Note:** Define the two ways that sample is used in this lesson (see Appendix B: Glossary).

Example 1

Determine whether the surveyed set is a census, or a sample.

a) Your town council wants to determine if people want to build a new hockey rink, so they survey the coaches of the hockey teams.

Solution:

Sample: they picked a small portion of the voting population

b) Your teacher wants the students to select a movie for Friday afternoon. She asks every student in the class to select one of the three available choices.

Solution:

Census: everyone in the class was consulted

Learning Activities

Each lesson in this course will contain learning activities for you to practice what you have just learned. Complete each learning activity question to the best of your ability. Then check your answers using the answer key at the end of the module. You will need a notebook to compile your answers.

Learning Activity 1.1

Census and Samples

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Find 10% of 30.
2. Calculate 15 + 45.
3. Calculate 4 x 25.
4. You have $10.00. You spend $3.50 on candy. How much do you have left?
5. Which is larger: $\frac{1}{4}$ or $\frac{1}{5}$?
6. Which is smaller: 0.45 or 0.405?
7. You have $20.00. You want to buy 3 boxes of doughnuts that cost $6.00 each (no tax). Do you have enough money?
8. What is half of 48?

continued

Learning Activity 1.1: Census and Samples (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify whether you would use a sample or a census in each situation, and why:
   a) Electing a prime minister for Canada
   b) Calculating your final mark in math class
   c) Finding the favourite shoes of 14-year-olds
   d) Determining the percentage of vehicles on the road with only one occupant
2. You are in charge of buying pizza for a party for all Grade 9 students. To help you select the toppings, you go into each classroom with Grade 9 students, and ask the opinion of the person sitting in the front desk of each row. Is this a census or a sample? If it is a sample, is it random or not?

There are many factors to consider when deciding whether to use a sample or a census when collecting data from a population: the size of the population, the method of data collection used, the nature of the study and its outcomes, cost, time required, and so on. If you were to survey drivers in Winnipeg to see what percentage wears a seatbelt, you couldn't possibly do a census. Carefully choosing a random sample, where every driver has a chance to be surveyed, is the only option. If you were researching an e-coli outbreak in a small community you may want to interview each family, but if you wanted to know which restaurant was the most popular, you could use a sample.

How you collect your data can affect the appropriateness of your sample. Telephone interviews, mail-in replies, web polls, observation, or face-to-face questionnaires are all methods you could use. To pick a random sample however, everyone has to have the chance of being selected. If, for the seatbelt survey, you only polled drivers on a Sunday morning, daily commuters—those who drive to work and back each day during the week—would not be a part of your sample. That would create a *biased* sample. You will work more with bias in the next lesson.

Lesson Summary

In this lesson, you learned the difference between a sample and a population. You considered when you should use a census or a sample for collecting data. You also learned what makes for a good random sample.

In the next lesson, you will consider how to collect the data in order to avoid biased results.

If you have questions or comments while you are working on your course, you are encouraged to contact your tutor/marker by phone or email at any time.

NOTES

Assignment 1.2: Populations and Samples

Total: 21 marks

1. List three reasons why the Canadian government takes a census every five years instead of every year. *(3 marks)*

2. What is the difference between a sample and a census? *(2 marks)*

3. Is a census a collection of data based on a sample or the entire population? *(1 mark)*

4. List two reasons why a statistician would use a sample instead of the entire population. *(2 marks)*

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5. Name two populations for which your English class would be a sample. *(2 marks)*

6. The student council is planning a dance. They need a decision on whether the students would prefer the dance on Thursday or Friday. How can 10 students from the school be selected for a random sample? *(2 marks)*

7. Describe the population, and explain whether they used a sample or a census in each of the following scenarios. *(6 marks)*

   a) Jason wants to know which ice cream sundae topping is the favourite among the 600 students in his high school. He asks each of the students in his Grade 9 math class. *(2 marks)*

   b) To elect a new student council president, each student casts a vote. *(2 marks)*

   c) A Brandon rock music radio station wants to determine the favourite songs of teenagers, so they phone every high school principal. *(2 marks)*

8. Describe a situation where you could survey a population to answer a question, and justify why you used a census instead of a sample.
(3 marks)

NOTES

Lesson 3: Bias in Data Collection

Lesson Focus

When you finish this lesson, you will be able to

- ❑ identify bias in sample selection and question wording and method
- ❑ identify questions that might violate someone's privacy or cultural sensitivity
- ❑ describe the effect these have on data collection
- ❑ describe ethical behaviour when collecting statistics

Lesson Introduction

A survey is only valid if the data collected accurately represents the entire population. This lesson highlights how biased sample selection, question wording, method of data collection and ethical, cultural, and privacy issues all affect the data collected.

Data Collection

When data collected do not accurately represent the whole population, or the results over-emphasize the opinion of a small group, the results are said to be biased. Bias can be caused by many things.

As a reminder to yourself, include on your Resource Sheet the following ways that bias can appear in a survey.

Bias in Sample Selection

When you select a sample for a statistical study, you must ensure that the sample accurately represents your population. You wouldn't ask only senior citizens how they feel about raising the legal minimum driving age to 18. That would be a biased sample.

Your sample should include people of all ages. Sample size must be appropriate as well. If you have a population of 1000 students, a sample of 1 may not be reliable.

Question Wording

If you were to ask 100 people on the street "do you believe there is an increase in violence among wild, unruly teenagers?" your data would be affected by the wording. The words "wild" and "unruly" suggest the teenagers are all badly behaved, and that could affect someone's response to the question. Similarly, asking "should the government fund a wonderful, fabulous, new stadium for the Blue Bombers?" might lead those answering the question towards a positive response. The wording of the question can affect how people answer.

Method, Cost, Time, and Timing

Mail-in and long-distance phone surveys can be quite costly. Personally interviewing hundreds of people could take countless hours. And different people respond in various ways to different approaches. Those who take the time to complete and send in mail-in surveys are usually those who are not happy with the survey issue. People who are content with a product or an issue tend not to respond to mail-in surveys. This creates a biased result because not everyone in the sample participates.

Ethics

Ethical behavior (i.e., doing things according to a moral code of right and wrong) is essential when collecting statistics. Many companies collect financial data about the spending habits of their customers. It would be unethical of a company to sell that data to other companies. If you are surveying workers to collect data on how effective they feel their supervisor is, the responses would have to be anonymous and confidential so the boss couldn't know who said what. If there was a chance that the boss could find out who said negative things about him, the workers may not be honest for fear that they might lose their jobs! Another example of the unethical behaviour of a statistician is to "pad" the data. For example, if you are conducting a survey to find the average wage paid to teenage workers with the intention of presenting this data to your employer, it would be unethical of you to falsely include higher wages to raise the average.

Cultural Issues

Canada is a multicultural country whose people embrace the traditions, beliefs, foods, religions, and behaviours of their homeland. A statistician has to be careful to not offend any cultural sensitivity. An example of this might be asking about eating habits. Some cultures do not allow eating of certain foods, and participants in the survey may not wish to be open about their habits for fear of ridicule. Their lack of participation, or possible wrong answers, may affect your data.

Privacy Issues

Finally, you do not want to ask questions that might invade someone's privacy or hurt their feelings. Questions about income, weight, religion, or even politics might offend some people. They may not answer truthfully, and their answers might affect the data you collect.

When designing an unbiased statistical study, you want to be sure that the wording of the question is neutral and sensitive to cultural and other issues, and that the sample resembles the entire population with the sample size large enough to generate a reliable conclusion. The method must be ethical.

Learning Activity 1.2

Bias

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate 9 x 9.
2. Calculate 900 ÷ 30.
3. What is 10% of 5790?
4. What is the area of a rectangle with width 3 cm and length 5 cm?
5. You buy coffee for $1.55. You give the cashier $2.05. How much change should you get?
6. Which is larger: $\frac{5}{6}$ or $\frac{5}{4}$?
7. If the diameter of a circle is 60 cm, what is the radius?
8. You want to survey 100 people. Each survey will take 3 minutes to complete. How many minutes will this take?

continued

Learning Activity 1.2: Bias (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the bias in each of these questions, and describe how they might affect the data collected.
   a) A shoe manufacturing company wants to find out which shoes are most popular with 15- and 16-year-olds, so he surveys all the players at a basketball tournament.
   b) You are comparing two brands of nacho chips and asking for opinions. You ask "which do you prefer, this fabulous, tasty, crunchy, to-die-for brand, or this other bland, boring chip?"
   c) The Manitoba government wants to know how Manitobans feel about banning cell phone use while driving, so they ask 10 people driving down Main Street in Winkler on a Saturday afternoon.
   d) You are doing a study on how often people cut their hair. Some people in your class who come from a culture requiring the constant wearing of headdress refuse to answer your questions, and are made to feel uncomfortable.
2. The prime minister announces that a recent survey showed 82% of Canadians were in favour of going to war. The details of his data collection reveal that 100 people living on a military base were polled. Discuss why this generalization might not be valid for all Canadians.
3. You decide to collect data from every high school in the province regarding their hat policy. Postage will cost $0.52 for each mailing, and another $0.52 for the return envelope. How might this affect how you collect your data?

Lesson Summary

This lesson discussed various concerns you should have when designing a statistical project. Bias can be the result of poor wording, inappropriate sample selection, method, cost, and timing as well as ethical issues, privacy, and cultural sensitivity. You will use these considerations in the next lesson to analyze a case study for potential problems.

NOTES

Assignment 1.3: Bias in Data Collection

Total: 15 marks

1. Identify the source of bias in each example. *(3 marks)*

   a) A morning radio show invites listeners to phone in and express their opinion on building a rapid transit system in Winnipeg. *(1 mark)*

   b) To find the average income of a working person in Manitoba, all the doctors and nurses in the province were surveyed. *(1 mark)*

   c) People were asked to express a preference for small grocery stores or big chain stores by answering the following question. *(1 mark)*

   Place an X in the box next to your preference:

   ❑ The friendly corner grocery store

   ❑ The huge chain grocery store

2. A committee of three people from your class of 30 students is to be randomly selected. Describe two ways this can be accomplished. *(2 marks)*

3. A radio station manager wants to determine which bands are the favourites of 15-year-olds, so he phones every high school principal. *(2 marks)*

   a) What bias exists in this survey? *(1 mark)*

   _____

   _____

   b) Suggest a better method to randomly collect these data. *(1 mark)*

   _____

   _____

4. Why would more teenagers buy an energy drink named "Raging Bull" than one named "Sleeping Hound?" *(2 marks)*

   _____

   _____

   _____

5. Your science teacher wants to measure the weight of each student, and graph the data. What potential problems might she encounter with her data collection? *(2 marks)*

   _____

   _____

6. A classmate is doing a survey to find the household income for each student's family. *(4 marks)*

   a) Why would some people not participate in this survey? *(2 marks)*

   _____

   _____

   b) How might a lack of participation affect your data? *(2 marks)*

   _____

   _____

Lesson 4: Analyzing a Case Study

Lesson Focus

When you finish this lesson, you will be able to

- ❑ analyze a case study and identify potential problems related to data collection: bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity

Lesson Introduction

In this lesson, you will apply each of the considerations from the previous lesson to the design of a statistical study.

Use the following definitions to help you identify bias and other potential problems in statistical studies.

Add to your Resource Sheet any influences that can create bias or alter the results of a survey that you did not include from the previous lesson.

Bias: A biased result is skewed rather than representative. Be sure that your sample truly represents the population of your study.

Use of Language: The wording of your question can strongly affect your results. The wording must remain neutral so it does not influence the response.

Ethics: Refers to what is generally acceptable to others. Some data must remain confidential. Statisticians cannot create data to achieve their goal.

Cost: Consideration must be given to the cost of how you plan to collect your data. Travel, hotels, phone charges, postage, and staffing can all be very costly.

Time and Timing: Can you collect and organize all your data within the time available to you? Mail surveys can take weeks to collect data. People might not appreciate phone surveys late in the evening or early in the morning. Also, the time when you do your survey can affect the results. Asking people to identify their favourite team while they are leaving a Blue Bomber football game after a huge win is an example of when timing can affect your data collection.

Privacy: Many people do not appreciate questions about weight, religion, money, family, etc. Also, adding names, addresses, telephone numbers, or other personal information would violate privacy.

Cultural Sensitivity: Your questions cannot insult or belittle any group. Asking a group of very poor people about expensive, luxury cars would be insensitive to their situation.

Example 1

Tony has three weeks to finish a statistical study. He has decided to go to high schools in Thompson, Dauphin, Brandon, and Morden, along with three schools in Winnipeg, to determine the favourite brand of MP3 player students use. His sample will consist of the first 20 students he meets at each school. The question will be: "Which is your favourite MP3 player, the cool iPod or something else?" Tony will publish detailed data containing information on who owns which player. Determine the potential problems.

Solution:

Potential Problems

a) **Bias:** His sample doesn't give everyone in each school the opportunity to be polled. He might run into a large group of friends who all have the same MP3 players.

b) **Use of Language:** The question uses language that may affect people's response. Mentioning only the iPod, and using the word "cool" slants the responses.

c) **Ethics:** Publishing the names of owners is ethically wrong. It could lead to thieves targeting them.

d) **Cost:** Travel to each community would be costly, along with food and hotel costs

e) **Time and Timing:** Will he have enough time to arrange each trip, get permission from the principals, collect his data, and organize it?

f) **Privacy:** Some students may not want the fact that they own an expensive MP3 player to be public knowledge

g) **Cultural Sensitivity:** If he polled some students who could not afford this item, they might be offended. Some cultures do not allow children to listen to music, and this survey would be insensitive to their situation.

Learning Activity 1.3

Analyzing a Case Study

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. There is a walking club of 10 people who use a pedometer each day to count the number of steps they take. Here are the data for one day (rounded to the nearest hundred):

7500	8500	3400	6700	7500	9200	5400	5700	7500	8500

   What is the mode of these data?

2. Calculate 15 x 30.
3. How many minutes in three hours?
4. You buy a sandwich for $3.65. You pay with two toonies. How much change do you get?
5. If the area of a rectangle is 60 cm^2 and the width is 10 cm, what is the length?
6. Arrange the following numbers in order from smallest to largest:

   $\frac{2}{3}$ 0.45 $\frac{6}{5}$ 1.8

7. What is half of 840?
8. You are bringing cookies to your mathematics class of 25 students. You want each student to have three cookies. How many cookies should you bring?

continued

Learning Activity 1.3: Analyzing a Case Study (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Identify and discuss any potential problems with this data collection project.

You have a part-time job in a restaurant. Everyone complains about the supervisor. You decide to survey your friends at work regarding their feelings toward the boss. You decide to question your friends at the end of a busy shift, and will publish their responses in a document showing who was asked and how they responded. The question you will ask reads: "Do you feel our supervisor is mean, picks on some workers, and has favourites on staff?" The results will be sent to the regional manager.

Lesson Summary

In this lesson, you applied data collection considerations to a case study and analyzed it for bias and potential problems. In the next lesson, you will take the results of a statistical study and display the data on graphs or charts.

Assignment 1.4: Case Study

Total: 14 marks

Sally had two weeks to complete a statistics project. She decided to mail a survey to each home in Tuxedo, a generally wealthy or affluent area of Winnipeg. The question she asked was "How many flat-panel high definition (HD) televisions do you have in your home?" She did not get many responses, so she made up some data to show more televisions in homes. Sally will use these data to project how many HD televisions are owned by average Manitoba families.

Discuss any potential problems Sally might have with respect to the following:

a) Bias *(2 marks)*

b) Use of language *(2 marks)*

c) Ethics *(2 marks)*

d) Cost *(2 marks)*

e) Time and timing *(2 marks)*

f) Privacy *(2 marks)*

g) Cultural sensitivity *(2 marks)*

LESSON 5: DISPLAYING DATA

Lesson Focus

When you finish this lesson, you will be able to

- ❑ display data in an appropriate manner

Lesson Introduction

Once data are collected, the results are often displayed on a chart or graph so they can be easily understood. Using programs with graphing capabilities like Microsoft Office Excel or Graphical Analysis, you can quickly make very attractive graphs.

Charts and Graphs

Once you have collected your data, you will need to display your results so that an observer can understand them. The most common methods include charts, bar graphs, or pie graphs.

This lesson will focus on graphs made with technology, but hand-drawn graphs are acceptable if you do not have access to a computer. If you make your graphs in this manner, remember that the independent variable (or category) goes along the horizontal or x-axis, and the dependent variable (or value) goes along the vertical or y-axis. Whether they are hand-drawn or made using technology, all bar graphs must have labels on each axis and a title. Pie graphs must have a legend on the side or use text boxes to add labels for each section.

On your Resource Sheet, include the characteristics that you need to have for each type of graph (bar graphs and pie graphs).

The following example was created using Windows XP and Microsoft Office Excel. You can use other operating systems and spreadsheet programs. Your school has access to the program Graphical Analysis that you can use. Check with your math or computer teacher. Consult the appropriate help menu, manuals or check for online tutorials to find what works with what you have. In the Excel spreadsheet program, you can click on the (?) icon at the top

right of the screen for help with specific questions about making and labelling your graph. Try the following websites or type "making graphs" in your search engine. There is a lot of helpful information out there!

Windows XP

www.internet4classrooms.com/excel_create_chart.htm

Windows Vista

office.microsoft.com/home/video.aspx?assetid=ES102018931033&width=884&height=540&startindex= 0&CTT=11&Origin=HA102004991033

With practice, you will soon be making graphs that look great!

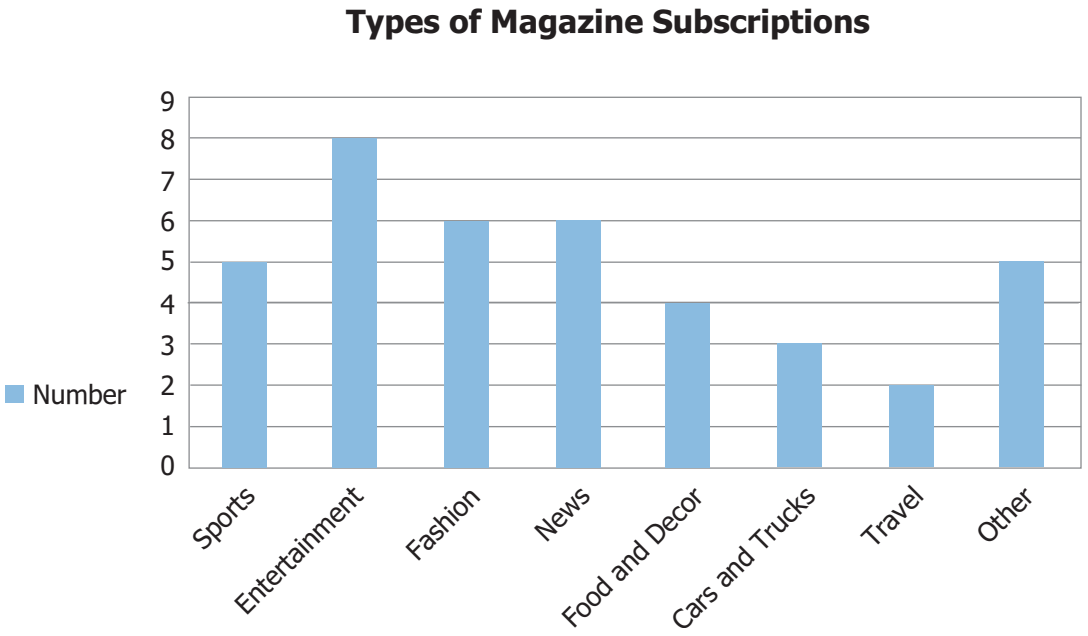
The following is a list of a library's different types of magazine subscriptions, as well as the number of subscriptions.

Magazine Type	Number
Sports	5
Entertainment	8
Fashion	6
News	6
Food and Decor	4
Cars and Trucks	3
Travel	2
Other	5

Open your spreadsheet program and enter the data from this chart into columns A and B.

Highlight the data in the cells, including the headings, and click on "Insert" from the task bar. Choose "chart" and "column" as the type of chart. Experiment with different legend placements, titles, font size, and any other options available to you in your spreadsheet program. One way to create this graph is shown below.

If you are not familiar with making graphs on the computer, you should include the steps on your Resource Sheet *while you complete this module.* You will not have access to a computer while you are writing the exam, so you will not need to include these steps on your Exam Resource Sheets.



Learning Activity 1.4

Displaying Data

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate 40 x 30.
2. How many eggs in three dozen?
3. You are making pancakes but only need to make half the recipe. The recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar will you use to make half the recipe?
4. One-quarter of the Grade 9 students are going skating at The Forks. There are 80 Grade 9 students in the school. How many are going skating at The Forks?
5. How many centimetres in 3.5 m?
6. Which of the following fractions are equivalent?

   $\frac{4}{8}$ $\frac{5}{12}$ $\frac{7}{14}$ $\frac{9}{16}$
7. What is the probability of tossing a coin and getting heads?
8. If 100 cm of snow is equivalent to 50 mm of water, 150 cm of snow is equvalent to how much water?

continued

Learning Activity 1.4: Displaying Data (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

The following chart represents the average hourly wage in dollars across Canada in 2007. It is from Statistics Canada's website at <www.statcan.ca.>

	2008
Canada	**20.16**
Newfoundland and Labrador	19.48
Prince Edward Island	16.51
Nova Scotia	17.83
New Brunswick	18.20
Quebec	18.98
Ontario	20.31
Manitoba	18.75
Saskatchewan	20.12
Alberta	23.14
British Columbia	20.21
Yukon	20.85
Northwest Territories	24.86
Nunavut	22.51

Adapted From Statistics Canada. <http://www40.statcan.gc.ca/l01/cst01/labr80-eng.htm>. Extracted on May 13, 2009.

Draw a bar graph based on the above chart. If you have access to a computer, you may be able to use the chart wizard in Excel, or Graphical Analysis, or a similar program. Or you can draw it by hand. Be sure to label the axes. Check your answer in the learning activity keys found at the end of this module.

Lesson Summary

In this lesson, you displayed data using charts or graphs. In the next lesson, you will analyze various charts and graphs.

NOTES

Assignment 1.5: Displaying Data

Total: 12 marks

While the use of technology is strongly recommended, hand-drawn graphs are acceptable.

1. Use Excel or Graphical Analysis to create a bar graph to display the following data. Be sure to include labels on each axis and a title. Print the graph and attach it to this assignment. *(6 marks)*

Monthly Expenses	Amount
rent	820
food	480
car payment	325
loan	275
utilities	410
savings	125
entertainment	315

2. Use Excel or Graphical Analysis to create a pie graph to display the following data. Be sure to include labels and a title. Print the graph and attach it to this assignment. *(6 marks)*

Favourite Ice Cream	Number
chocolate	29
vanilla	30
strawberry	12
rollo	33
bubblegum	16
marble	22

NOTES

LESSON 6: DATA ANALYSIS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ analyze charts and graphs
- ❑ draw conclusions

Lesson Introduction

Once a graph is constructed, you need to be able to read it or analyze it correctly so that the conclusions you make about the data are accurate.

Analyzing Charts

You have been analyzing charts and graphs for many years now, so this lesson should mostly be review for you!

Looking at a chart, you can see patterns. You should be able to see maximum or minimum values, peak periods, etc.

Example 1: Body Mass Index, Changes from 1994/1995 to 2006/2007, by Gender

The following chart contains information about the change in people's body mass index (BMI) from 1994/1995 to 2006/2007. BMI is useful in estimating a healthy weight based on a person's height. People with a high BMI are likely to be overweight. It is calculated by using a person's height and weight.

	Percent
Increased BMI from 1994/1995 to 2006/2007	
Both genders	30.5%
Males	31.6%
Females	29.4%
Same BMI from 1994/1995 to 2006/2007	
Both genders	63.6%
Males	63.4%
Females	63.9%
Decreased BMI from 1994/1995 to 2006/2007	
Both genders	5.8%
Males	5.0%
Females	6.7%

Adapted From Statistics Canada <http://www40.statcan.ca/l01/cst01/hlth68-eng.htm>. Extracted on May 12, 2009.

Analyze the chart above to answer the following questions related to change in BMI between 1994/1995 and 2006/2007.

1. In which gender did more people experience an increase in BMI?
2. Overall, did most people's BMI decrease, or stay the same, or increase?
3. On the average, did more people of both genders experience an increase or a decrease in their BMI?
4. An increase in BMI is often associated with increased health risks. In general terms, what does this chart indicate in regards to the health of males and females?

Solutions:

1. Males. They had the highest increase at 31.6% while the increase for females was 29.4%.
2. Same. Overall, 63.6% of people's BMI stayed the same, while only 30.5% experienced an increase and 5.8% experienced a decrease.
3. Increase. Only 5.8% of people from both genders experienced a decrease in BMI, while 30.5% experienced an increase in BMI.
4. Since more males experienced an increase in BMI and more females experienced a decrease in BMI, then, in general terms, males have increased their health risks.

Learning Activity 1.5

Data Analysis

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. If 12 people each contribute equally to a gift costing $144, how much will each person pay?
2. Convert 350 mm to cm.
3. Calculate 8 x 8.
4. Calculate 80 x 9.
5. Name a fraction that is between $\frac{3}{8}$ and $\frac{3}{5}$.
6. If you work 35 hours in five days, how many hours did you average per day?
7. What is the value of the expression $2t - 8$ if $t = 5$?
8. Find the area of a square with side length of 5 cm.

continued

Learning Activity 1.5: Data Analysis (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

The following chart shows the average hourly earnings by province or territory for 2004 and 2008. Use the data in the chart to answer the following questions. Check your answers in the learning activity keys found at the end of this module.

Average Hourly Earnings for Hourly Paid Employees By Province and Territory		
	2004	**2008**
Canada	**17.69**	**20.16**
Newfoundland and Labrador	17.41	19.48
Prince Edward Island	14.68	16.51
Nova Scotia	16.06	17.83
New Brunswick	15.71	18.20
Quebec	16.79	18.98
Ontario	18.43	20.31
Manitoba	16.34	18.75
Saskatchewan	16.50	20.12
Alberta	18.46	23.14
British Columbia	17.74	20.21
Yukon	18.03	20.85
Northwest Territories	22.44	24.86
Nunavut	19.72	22.51

Adapted from Statistics Canada. <http://www40.statcan.gc.ca/l01/cst01/labr80-eng.htm.> Extracted on May 13, 2009.

Learning Activity 1.5: Data Analysis (continued)

a) Which provinces or territories showed an increase in hourly earnings?

b) Which province or territory showed had the smallest increase in hourly earnings?

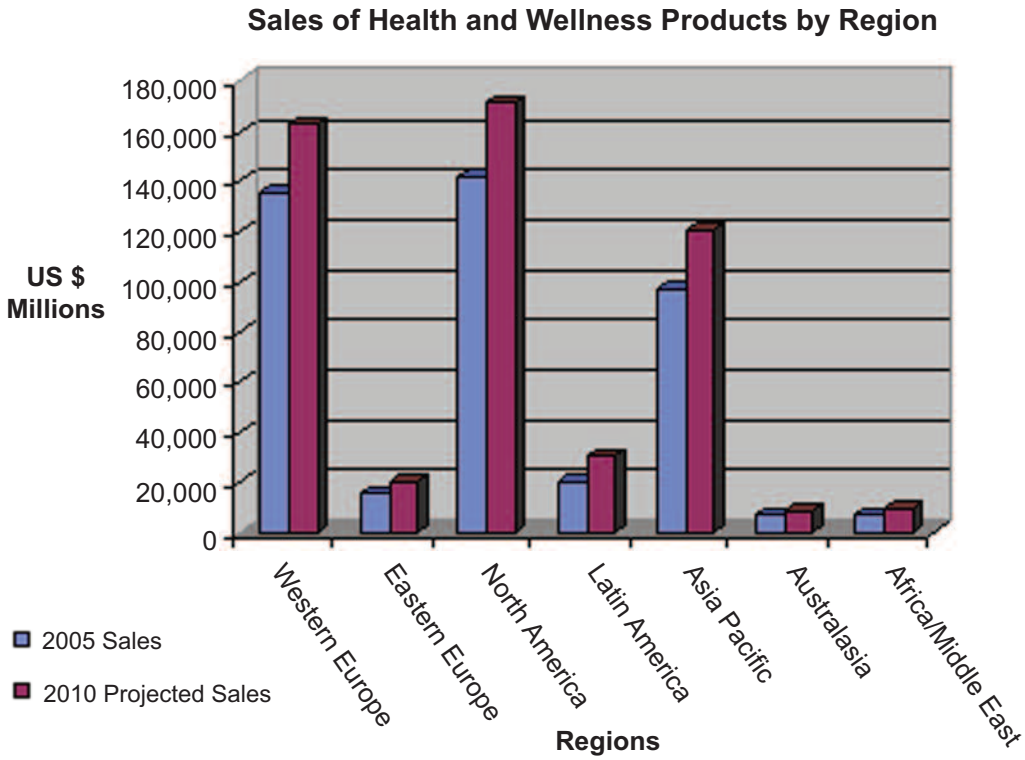
c) Which province or territory showed had the greatest increase in hourly earnings?

d) In your opinion, why might the hourly wage increase more in some provinces and territories than in others?

Analyzing Graphs

You can use bar or pie graphs to help analyze data. You can quickly see which parts are the largest or smallest, etc.

Example 2



a) Which region bought the most products in 2005?

Answer: North America

b) Which area is projected to buy the most products in 2010?

Answer: North America

c) In which region do you see the most growth in projected sales between 2005 and 2010?

Answer: Looks like a three-way tie among Western Europe, North America, and Asia Pacific

d) Why might Australia have such low sales in health and wellness products?

Answer: Healthier lifestyle and eating habits

Lesson Summary

In this lesson, you analyzed charts and graphs and made conclusions based on the data displayed. Now you are ready to set up your own statistical project using all of the previous lessons!

Assignment 1.6: Data Analysis

Total: 14 marks

1. Given the following chart based on a restaurant's menu items, answer the questions below. *(4 marks)*

Hercules' Hamburgers	Calories
Hamburger	270
Cheeseburger	310
Minotaur Burger	570
The Pantheon Burger	530
Mushroom Burger	490
Chicken Fillet	480
Grilled Chicken	350

a) Which menu item has the highest amount of calories? *(1 mark)*

_____

b) Which item has the lowest number of calories? *(1 mark)*

_____

c) How many more calories does the Chicken Fillet have compared to the Grilled Chicken? *(1 mark)*

_____

d) Why might the Minotaur Burger have the highest number of calories? *(1 mark)*

_____

_____

2. Analyze the following charts, comparing menu items from two different restaurants, and answer the questions. *(5 marks)*

Greek Submarines

Double Meat Sandwich (12 in.)
33% of calories from fat

Nutrition Facts

Serving Size:
612 (g)

Amount Per 1 Serving	
Calories 1,260	Calories from fat 420
	% Daily Value*
Total Fat 70 g	108%
Saturated Fat 28 g	140%
Trans Fat 0.0 g	
Cholesterol 200 mg	67%
Sodium 5,640 mg	235%
Total Carbs 98 g	33%
Dietary Fibre 8 g	32%
Sugars 20 g	
Protein 68 g	

*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.

	Calories	**2,000**
Total Fat	**Less than**	**65g**
Sat. Fat	**Less than**	**20 g**
Cholesterol	**Less than**	**300 mg**
Sodium	**Less than**	**2,400 mg**
Total Carbohydrates		**300 g**
Dietary Fibre		**25 g**

Nero's Hamburgers

Flame-Broiled Burger
29% of calories from fat

Nutrition Facts

Serving Size:
117 (g)

Amount Per 1 Serving	
Calories 280	Calories from fat 80
	% Daily Value*
Total Fat 9 g	14%
Saturated Fat 4 g	18%
Trans Fat 0.5 g	
Cholesterol 30 mg	10%
Sodium 590 mg	25%
Total Carbs 34 g	11%
Dietary Fibre 1 g	4%
Sugars 7 g	
Protein 15 g	

*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.

	Calories	**2,000**
Total Fat	**Less than**	**65g**
Sat. Fat	**Less than**	**20 g**
Cholesterol	**Less than**	**300 mg**
Sodium	**Less than**	**2,400 mg**
Total Carbohydrates		**300 g**
Dietary Fibre		**25 g**

a) Which item has the lowest amount of fat content? *(1 mark)*

b) Which item has the highest number of calories? *(1 mark)*

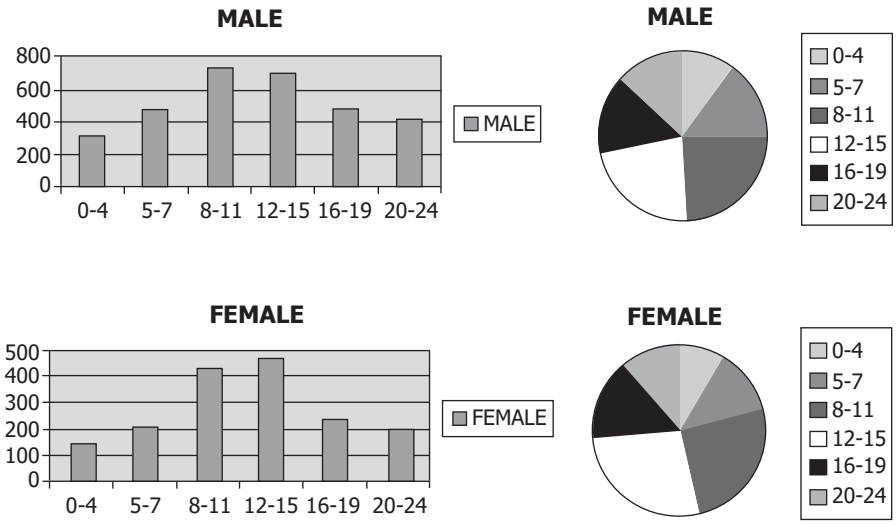
c) How many grams of sugar are in the double meat sandwich? *(1 mark)*

d) How many more calories are in the double meat sandwich than the flame-broiled burger? *(1 mark)*

e) Compare the serving sizes of these two menu items. *(1 mark)*

3. Analyze the following graphs and answer the questions. *(5 marks)*

These graphs show pedestrian-vehicular accidents in England organized by age groups and gender.



a) Which age group and gender had the most accidents? *(1 mark)*

b) Which age group and gender had the least amount of accidents? *(1 mark)*

c) Who has more accidents—boys or girls? *(1 mark)*

_____

d) Approximately how many accidents happened to girls aged 5–11? *(1 mark)*

_____

e) Why do boys have more accidents than girls? *(1 mark)*

_____

LESSON 7: DEVELOPING A PROJECT

Lesson Focus

When you finish this lesson, you will be able to

- ❑ design and implement a statistical project
- ❑ use a rubric to assess your work

Lesson Introduction

You are now ready to design your own project, collect data, organize and display the data, and offer some analysis or conclusions based on your findings.

Designing a Project

What is something you are interested in knowing about? How could you find the answer to your question? This lesson allows you to create a question or survey, collect the data based on your question, display and analyze the data, and then assess the results and the process you went through by using a rubric.

Rubric

A rubric is an outline of how your project will be marked. Ideally, you would develop your own marking guidelines and apply them to your work. A sample rubric follows. You may use it to assess your own project or you may create your own. If you choose to create your own rubric, ask your learning partner to help you create the rubric. Phone your tutor/marker to discuss the rubric or email it to get some feedback before completing your project. You must include the rubric when you send this assignment to your tutor/marker.

Topic and Marks	Criteria	Rating Scale		
Question *(5 marks)*	■ Uses neutral language ■ Asks for a clear question that can be answered with data ■ Does not invade privacy ■ Sensitive to cultural issues	All elements appear and are thoroughly developed *(4 or 5 marks)*	Some or all elements with limited development *(2 or 3 marks)*	Information is incomplete or presented in a vague manner *(0 or 1 mark)*
Population or sample *(5 marks)*	■ Population identified ■ Choice of sample or census explained ■ Description of how sample was selected (random, size) ■ Any bias identified	All elements appear and are thoroughly developed *(4 or 5 marks)*	Some or all elements with limited development *(2 or 3 marks)*	Information is incomplete or presented in a vague manner *(0 or 1 mark)*
Data collection *(10 marks)*	■ Method appropriate, well explained ■ Time and timing considerations ■ Cost considerations	All elements appear and are thoroughly developed *(8 to 10 marks)*	Some or all elements with limited development *(4 to 7 marks)*	Information is incomplete or presented in a vague manner *(0 or 2 marks)*
Chart or graph *(10 marks)*	■ Chart or graph included ■ Labels and titles appropriate ■ Easy to read ■ Accurately shows data	All elements appear and are thoroughly developed *(8 to 10 marks)*	Some or all elements with limited development *(4 to 7 marks)*	Information is incomplete or presented in a vague manner *(0 or 2 marks)*
Concluson *(5 marks)*	■ Question answered ■ Trends identified	All elements appear and are thoroughly developed *(4 or 5 marks)*	Some or all elements with limited development *(2 or 3 marks)*	Information is incomplete or presented in a vague manner *(0 or 1 mark)*

Lesson Summary

After you have completed your project, you will have applied what you learned in earlier assignments by designing, implementing, and assessing a statistical study.

Assignment 1.7: Developing a Project

Total: 35 marks

Develop your own statistical project plan and complete it. You will need to follow these guidelines for collection, displaying, and analyzing data:

- Formulate a question for investigation
- Select the whole population or a sample of the population
- Choose a data collection method
- Collect the data
- Display the collected data in a chart or graph
- Draw conclusions from your data
- Use the rubric provided or develop your own rubric for marking your project, and complete a self-assessment.

Describe each step listed above in detail. Take time to clearly address each of the guidelines. The graph, whether hand-drawn or generated through a program, must have a title, labels, and clearly show the results of your study. If you designed your own rubric, be sure to include it when you submit your project.

Self-assessment is a great tool for you to use to check your understanding of this material. Honestly mark your work. If you do not like the mark you feel you have earned, you can always go back and improve your project! Your self-assessment will be worth 25% of the final mark for this project.

Things To Remember

Include these key points for data collection on your Resource Sheet.

Question: Use neutral, clear language, and be sure the topic does not invade people's privacy or sensitivity. Be sure to ask a question that can be answered with data.

Census or Sample: Decide who or what will be polled, and explain if this represents a census or a sample.

Data Collection: You must explain how you intend to collect your data, remembering to be considerate of people's time. Will cost be a factor in your data collection? Will you be able to complete your data collection, organize it, and display it within your timeline? Then follow your plan and collect the data using the method described.

Displaying Data: Use charts or graphs, making sure you include labels and titles.

Conclusions /Analysis: Answer the question posed in your study, and make comments on what you observed.

MODULE 1 SUMMARY

Congratulations! You have finished the first module in this course!

Statistics are used everywhere. They are used for determining car insurance rates and athletic rankings, in elections, and in calculating health risks, to name a few. After completing this module, you are able to choose a sample or population and create your own statistical study. You can describe your methods of collecting and displaying data, and analyze the data in terms of bias, wording, ethics, cost, time and timing, privacy, and cultural sensitivity, as outlined in these lessons.

The next module will focus on number sense and using decimals, fractions, percentages, and square numbers.

It is now time to send the seven assignments from Module 1 to your tutor/marker for feedback on how you are doing.

Mailing Your Assignments

Before you mail your completed assignments, please photocopy them first. That way, you will have a copy of your assignments in case they go missing. Please organize your material in the following order:

- Cover Sheet for Module 1
- Assignment 1.1: Cover Assignment
- Assignment 1.2: Populations and Samples
- Assignment 1.3: Bias in Data Collection
- Assignment 1.4: Case Study
- Assignment 1.5: Displaying Data
- Assignment 1.6: Data Analysis
- Assignment 1.7: Developing a Project

Place all of these assignments in an envelope and address it to:

ISO Tutor/Marker
555 Main St.
Winkler MB R6W 1C4

Remember: You can call or email your tutor at any time if you have questions!

NOTES

GRADE 9 MATHEMATICS (10F)

Module 1: Statistics

Learning Activity Answer Keys

MODULE 1: STATISTICS

Learning Activity 1.1: Census and Samples

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Find 10% of 30.
2. Calculate 15 + 45.
3. Calculate 4 x 25.
4. You have $10.00. You spend $3.50 on candy. How much do you have left?
5. Which is larger: $\frac{1}{4}$ or $\frac{1}{5}$?
6. Which is smaller: 0.45 or 0.405?
7. You have $20.00. You want to buy three boxes of doughnuts that cost $6.00 each (no tax). Do you have enough money?
8. What is half of 48?

Answers:

1. 3
2. 60
3. 100
4. $6.50
5. $\frac{1}{4}$
6. 0.405
7. Yes (3 x 6 = 18 which is less than 20)
8. 24

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify whether you would use a sample or a census in each situation, and why:
   a) Electing a prime minister for Canada
   b) Calculating your final mark in math class
   c) Finding the favourite shoes of 14-year-olds
   d) Determining the percentage of vehicles on the road with only one occupant

   Answers:

   a) census—every voter in Canada should vote
   b) census—the teacher would want all of your math marks included
   c) sample—it would be difficult to survey every 14-year-old
   d) sample—observing every car on all roads is impossible

2. You are in charge of buying pizza for a party for all Grade 9 students. To help you select the toppings, you go into each classroom with Grade 9 students and ask the opinion of the person sitting in the front desk of each row. Is this a census or a sample? If it is a sample, is it random or not?

   Answer:

   This is a random sample of all the Grade 9 students in the school.

Learning Activity 1.2: Bias

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate 9 x 9.
2. Calculate 900 ÷ 30.
3. What is 10% of 5790?
4. What is the area of a rectangle with width 3 cm and length 5 cm?
5. You buy coffee for $1.55. You give the cashier $2.05. How much change should you get?
6. Which is larger: $\frac{5}{6}$ or $\frac{5}{4}$?
7. If the diameter of a circle is 60 cm, what is the radius?
8. You want to survey 100 people. Each survey will take three minutes to complete. How many minutes will this take?

Answers:

1. 81
2. 30
3. 579
4. 15 cm^2
5. 50 cents
6. $\frac{5}{4}\left(\frac{5}{4} \text{ is larger than } 1; \frac{5}{6} \text{ is smaller than } 1\right)$
7. 30 cm (radius is half the diameter)
8. 300 minutes

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the bias in each of these questions, and describe how they might affect the data collected.

   a) A shoe manufacturing company wants to find out which shoes are most popular with 15- and 16-year-olds so he surveys all the players at a basketball tournament.

   Answer:

   Bias: This is not a random sample as basketball players may not favour the same shoes as students in general. They might prefer the high-tops for added ankle support.

   b) You are comparing two brands of nacho chips and asking for opinions. You ask "which do you prefer, this fabulous, tasty, crunchy, to-die-for brand, or this other bland, boring chip?"

   Answer:

   Bias: The wording of the question leads people to choose the first brand.

   c) The Manitoba government wants to know how Manitobans feel about banning cell phone use while driving, so they ask 10 people driving down Main Street in Winkler on a Saturday afternoon.

   Answer:

   Bias: This sample is very small, and doesn't fairly represent the one million residents of Manitoba. Also, people living in places other than Winkler did not have an opportunity to respond to this survey.

   d) You are doing a study on how often people cut their hair. Some people in your class who come from a culture requiring the constant wearing of headdress refuse to answer your questions, and are made to feel uncomfortable.

   Answer:

   Bias: You have encountered a situation of cultural insensitivity. Their refusal to participate skews your data.

2. The prime minister announces that a recent survey showed 82% of Canadians were in favour of going to war. The details of his data collection reveal that 100 people living on a military base were polled. Discuss why this generalization might not be valid for all Canadians.

   Answer:

   People in the military are trained to go to war, but they only represent a small number of average citizens.

3. You decide to collect data from every high school in the province regarding their hat policy. Postage will cost $0.52 for each mailing, and another $0.52 for the return envelope. How might this affect how you collect your data?

   Answer:

   The cost is prohibitive. With over 800 public schools in Manitoba, the cost of paper, envelopes, and stamps needed to contact them is too high.

Learning Activity 1.3: Analyzing a Case Study

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. There is a walking club of 10 people who use a pedometer each day to count the number of steps they take. Here is the data for one day (rounded to the nearest hundred):

7500	8500	3400	6700	7500	9200	5400	5700	7500	8500

   What is the mode of these data?
2. Calculate 15 x 30.
3. How many minutes in three hours?
4. You buy a sandwich for $3.65. You pay with two toonies. How much change do you get?
5. If the area of a rectangle is 60 cm^2 and the width is 10 cm, what is the length?
6. Arrange the following numbers in order from smallest to largest:

   $\frac{2}{3}$ 0.45 $\frac{6}{5}$ 1.8
7. What is half of 840?
8. You are bringing cookies to your mathematics class of 25 students. You want each student to have three cookies. How many cookies should you bring?

Answers:

1. 7500 (the mode is the number that shows up the most often, 7500 is the data set 3 times, 8500 is the data set twice, and all other numbers only appear once)
2. 450
3. 180 (there are 60 minutes in one hour)
4. $0.35 (two toonies is $4.00)
5. 6 cm (60 ÷ 10)
6. 0.45 $\frac{2}{3}$ $\frac{6}{5}$ 1.8

   (compare the numbers to the benchmarks of 0.5, 1, and 1.5: 0.45 is less than 0.5; $\frac{2}{3}$ is more than 0.5 but less than 1; $\frac{6}{5}$ is more than 1 but less than 1.5; 1.8 is greater than 1.5)

7. 420
8. 75 cookies

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Identify and discuss any potential problems with this data collection project.

> You have a part-time job in a restaurant. Everyone complains about the supervisor. You decide to survey your friends at work regarding their feelings toward the boss. You decide to question your friends at the end of a busy shift, and will publish their responses in a document showing who was asked and how they responded. The question you will ask reads: "Do you feel our supervisor is mean, picks on some workers, and has favourites on staff?" The results will be sent to the regional manager.

Answer:

a) **Bias:** Your sample is biased because it doesn't give everyone working at the restaurant the opportunity to be polled. Your friends might share the same feelings toward the supervisor.

b) **Use of Language:** The question uses language that may affect people's responses. Using the words "mean," "picks on," and "favourites" might influence the responses given.

c) **Ethics:** Publishing the names of co-workers and their feelings is ethically wrong. It could lead to their being fired.

d) **Cost:** No problems with cost.

e) **Time and Timing:** Asking the question at the end of a busy shift, when everyone is probably tired and short-tempered, will affect the responses given.

f) **Privacy:** Some workers may be unhappy with the supervisor, but prefer to keep their feelings private. They may not answer, or may not answer truthfully, which would affect your data.

g) **Cultural Sensitivity:** Many families raise their children with the adage "if you have nothing positive to say, say nothing." Other cultures do not allow people to ever question authority figures. This survey would go against their established customs. They may not respond at all, or may not answer truthfully. Either reaction would affect your data.

Learning Activity 1.4: Displaying Data

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate 40 x 30.
2. How many eggs in three dozen?
3. You are making pancakes but only need to make half the recipe. The recipe calls for $\frac{1}{2}$ cup of sugar. How much sugar will you use to make half the recipe?
4. One-quarter of the Grade 9 students are going skating at The Forks. There are 80 Grade 9 students in the school. How many are going skating at The Forks?
5. How many centimetres in 3.5 m?
6. Which of the following fractions are equivalent?

   $\frac{4}{8}$ $\frac{5}{12}$ $\frac{7}{14}$ $\frac{9}{16}$
7. What is the probability of tossing a coin and getting heads?
8. If 100 cm of snow is equivalent to 50 mm of water, 150 cm of snow is equivalent to how much water?

Answers:

1. 1200
2. 36 (12 eggs in a dozen)
3. $\frac{1}{4}$ (half of a half is a quarter)
4. 20 students $\left(80 \times \frac{1}{4} \text{ or } 80 \div 4\right)$
5. 350 cm (100 cm in 1 m)
6. $\frac{4}{8}$ or $\frac{7}{14}$ $\left(\text{both are equivalent to } \frac{1}{2}\right)$
7. $\frac{1}{2}$
8. 75 mm (half the amount and change the units)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

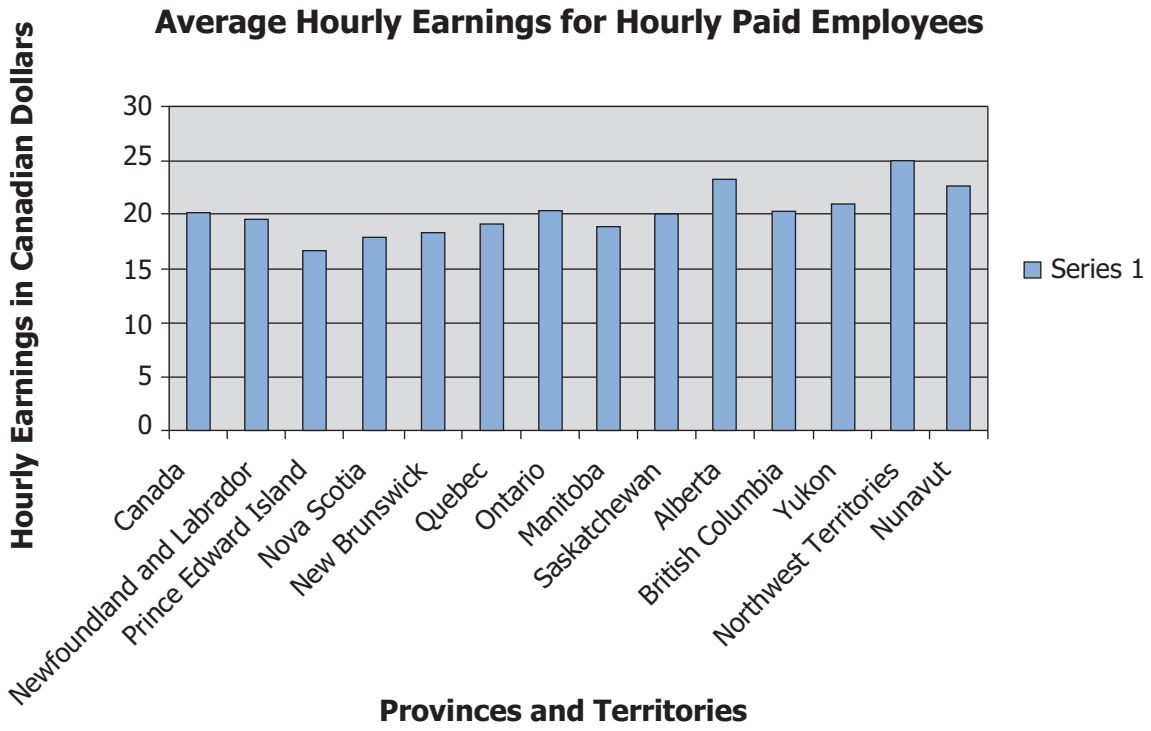
The following chart represents the average hourly wage in dollars across Canada in 2007. It is from Statistics Canada's website at <www.statcan.ca.>

	2008
Canada	**20.16**
Newfoundland and Labrador	19.48
Prince Edward Island	16.51
Nova Scotia	17.83
New Brunswick	18.20
Quebec	18.98
Ontario	20.31
Manitoba	18.75
Saskatchewan	20.12
Alberta	23.14
British Columbia	20.21
Yukon	20.85
Northwest Territories	24.86
Nunavut	22.51

Adapted From Statistics Canada. <http://www40.statcan.gc.ca/l01/cst01/labr80-eng.htm>. Extracted on May 13, 2009

Draw a bar graph based on the above chart. If you have access to a computer, you may be able to use the chart wizard in Excel, or Graphical Analysis, or a similar program. Or, you can draw it by hand. Be sure to label the axes.

Answer:



Learning Activity 1.5: Data Analysis

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. If 12 people each contribute equally to a gift costing $144, how much will each person pay?
2. Convert 350 mm to cm.
3. Calculate 8 x 8.
4. Calculate 80 x 9.
5. Name a fraction that is between $\frac{3}{8}$ and $\frac{3}{5}$.
6. If you work 35 hours in 5 days, how many hours did you average per day?
7. What is the value of the expression $2t - 8$ if $t = 5$?
8. Find the area of a square with side length of 5 cm.

Answers:

1. $12 (144 ÷ 12 = 12)
2. 35 cm (10 mm = 1 cm)
3. 64
4. 720
5. Answers will vary. Possible answers: $\frac{3}{7}, \frac{3}{6}$ or $\frac{1}{2}, \frac{17}{40}, \ldots$.

   Note: $\frac{3}{8} = 0.375$ and $\frac{3}{5} = 0.6$, so any fraction that is equivalent to a decimal between 0.375 and 0.6 is acceptable. Also, $\frac{3}{8} = \frac{15}{40}$ and $\frac{3}{5} = \frac{24}{40}$, so any fraction with 40 as a denominator and a numerator between 15 and 40 is acceptable as well.
6. 7 hours (35 ÷ 5)
7. 2 (2(5) — 8 = 10 — 8 = 2)
8. 25 cm^2 (each side of a square is the same length so the area is 5 cm x 5 cm = 25 cm^2)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

The following chart shows the average hourly earnings by province or territory for 2004 and 2008. Use the data in the chart to answer the following questions. Check your answers in the learning activity keys found at the end of this module.

Average Hourly Earnings for Hourly Paid Employees By Province and Territory

	2004	2008
Canada	**17.69**	**20.16**
Newfoundland and Labrador	17.41	19.48
Prince Edward Island	14.68	16.51
Nova Scotia	16.06	17.83
New Brunswick	15.71	18.20
Quebec	16.79	18.98
Ontario	18.43	20.31
Manitoba	16.34	18.75
Saskatchewan	16.50	20.12
Alberta	18.46	23.14
British Columbia	17.74	20.21
Yukon	18.03	20.85
Northwest Territories	22.44	24.86
Nunavut	19.72	22.51

Adapted from Statistics Canada. <http://www40.statcan.gc.ca/l01/cst01/labr80-eng.htm.>
Extracted on May 13, 2009.

a) Which provinces or territories showed an increase in the hourly earnings?

Answer:

All of them.

b) Which province or territory showed has the smallest increase in hourly earnings?

Answer:

Nova Scotia

c) Which province or territory showed has the greatest increase in hourly earnings?

Answer:

Alberta

d) In your opinion, why might the hourly wage increase more in some provinces and territories than in others?

Answer:

The economy in provinces like Alberta may be stronger than in places like Nova Scotia. Therefore, employers need to pay a higher wage, and are able to do so.

NOTES

GRADE 9 MATHEMATICS (10F)

Module 2: Number Sense

MODULE 2: NUMBER SENSE

Introduction

Number sense is tough to define but once you've got it, it makes math so much more fun! Number sense includes an understanding of the value of numbers, and their order or magnitude. It helps you to see patterns and relationships, and understand the effect of operations on numbers. The lessons in this module aim to provide you with useful strategies to improve your number sense abilities. You will work with big and small, positive and negative numbers, fractions, decimals, percentages, and square numbers and their roots. You will use these different representations of values to help you put numbers in order, perform operations on numbers, solve problems, and practice mental math and estimating with perfect square numbers and roots.

Assignments in Module 2

You will need to complete the following six assignments and send them to your tutor/marker when you have completed this module.

Lesson	Assignment Number	Assignment Title
1	Assignment 2.1	Cover Assignment
2	Assignment 2.2	Decimal Numbers
3	Assignment 2.3	Operations with Fractions
4	Assignment 2.4	Conversions
5	Assignment 2.5	Perfect Square Numbers
6	Assignment 2.6	Estimating Square Roots

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 2. You might use your Module 2 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 2 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 2 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

Lesson 1: Cover Assignment

The ability to draw logical conclusions is an important skill when learning to use mathematical reasoning. Read through the following examples and then complete the assignment.

Example 1

What conclusions can be drawn from the following argument:

> If I eat pizza after midnight, then I will have nightmares. If I have nightmares, then I will get very little sleep.

Solution:

Conclusion: If I eat pizza after midnight, then I will get very little sleep.

In mathematics, precise language is very important. Some everyday words can take on special meaning in mathematics. The small word "if" is among the most important words in the language of logic and reasoning. It is used all the time in everyday language, but in mathematics it is used very carefully. The examples below will help you understand the mathematical use of the word "if."

Example 2

Change the following statements into ***if—then*** statements.

a) A hexagon is a six-sided polygon.

b) People who live in glass houses shouldn't throw stones.

Solutions:

a) ***If*** a figure is a hexagon, ***then*** it is a six-sided polygon.

b) ***If*** you live in a glass house, ***then*** don't throw stones.

Example 3

Fill in the conclusions below.

a) ***If*** there is snow on the ground, ***then*** . . .

b) ***If*** a figure is a triangle, ***then*** . . .

Solutions:

a) . . . it is winter. (This is one possible answer.)

b) . . . it has three sides. (This is one possible answer.)

Example 4

Assume that x represents any of the numbers 1, 2, 3, 4, 5, ... Tell whether the statement is always true, sometimes true, or never true.

$x + 1 > x$

Solution:

This is always true. No matter which value from the set above is chosen, the statement will always be true. It is possible to prove this mathematically but that is not the purpose of this lesson.

Assignment 2.1: Cover Assignment

Total: 23 marks

1. State a conclusion that can be drawn from the following arguments. *(3 marks)*

   a) If a young person is an athlete, then he or she will score well on tests of spatial visualization. If a person scores high on spatial visualization tests, then he or she will do well in geometry. *(1 mark)*

   _____

   _____

   b) If you are open-minded, then you will listen to both sides of a story. If you listen to both sides of a story, then you will make a more intelligent decision. If you make an intelligent decision, then you will have a more successful career. *(1 mark)*

   _____

   _____

   c) If two base angles of a triangle are congruent, then the triangle will have two sides congruent. If two sides of a triangle are congruent, then the triangle is isosceles. *(1 mark)*

   _____

   _____

2. Translate the following statements into ***if—then*** statements. *(5 marks)*

   a) I must pass my test to graduate. *(1 mark)*

   _____

   b) World peace is in jeopardy due to increasing world tensions. *(1 mark)*

   _____

   c) A network has five nodes. *(1 mark)*

   _____

d) A square is a four-sided polygon. *(1 mark)*

_____

e) All Irish setters are dogs. *(1 mark)*

_____

3. Assume that x represents the numbers 1, 2, 3, 4, 5, ... Tell whether the following statements are always true, sometimes true, or never true. Explain your reasoning in each case. *(6 marks)*

   a) $x + 1 = 2x$ *(2 marks)*

   b) $1 > x \div x$ *(2 marks)*

   c) $x < 2x$ *(2 marks)*

4. Fill in an appropriate conclusion for the statements below. *(5 marks)*

   a) If I study math every night . . . *(1 mark)*

   _____

   b) If today is Friday . . . *(1 mark)*

   _____

   c) If the temperature is –42° . . . *(1 mark)*

   _____

   d) If a polygon is an octagon . . . *(1 mark)*

   _____

   e) If two even numbers are added . . . *(1 mark)*

   _____

5. Diophantus was a famous Greek mathematician who lived about 200 C.E. He has been called the "Father of Algebra" because of his contributions to that field. After his death, a student composed this puzzle about his life:

   His boyhood lasted for a sixth of his life.

   His beard grew after another twelfth of his life had passed.

   He married after another seventh of his life had passed.

   A son was born five years after his marriage.

   The son lived exactly half as long as his father.

   The father died four years after his son.

   How old was Diophantus when he died?

   Hint: A student started to solve the puzzle.

   Let x = Diophantus' age.

   Then:

   $$\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{7}\right)x = \text{his age when he married}$$

   $$\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{7}\right)x+5 = \text{his age when his son was born}$$

   $x - 4$ = his age when his son died

   $$\text{So, } (x-4)-\left[\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{7}\right)x+5\right] = \text{number of years the son lived}$$

   Therefore:

   $$x-4-\left[\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{7}\right)x+5\right]=\frac{x}{2}$$

   Solve for x to find Diophantus' age when he died. Show your work. *(4 marks)*

NOTES

Lesson 2: Decimal Numbers

Lesson Focus

When you finish this lesson, you will be able to

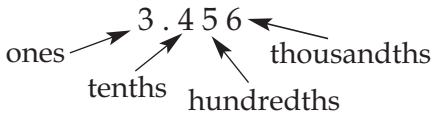
- ❑ review names of place values
- ❑ work with positive and negative decimal numerals
- ❑ convert percentages to decimal numerals
- ❑ estimate and perform basic operations to solve problems with decimal numerals

Lesson Introduction

To work with numbers in decimal form, you need a solid understanding of place value. This lesson reviews operations on positive and negative numbers with digits in the tenths, hundredths, and thousandths places. You will practise estimating to solve problems with decimals, and convert percentages to decimals.

Naming the Digits: Place Value

Recall:



So, 0.4 would be four-tenths or $\frac{4}{10}$ or $\frac{40}{100}$.

0.41 would be $\frac{4}{10}$ and $\frac{1}{100}$ or $\frac{41}{100}$

Therefore, 0.41 is larger than 0.4.

If you have a hard time remembering which decimal place goes with which name, you should include this diagram on your Resource Sheet.

Put the following numbers in order from smallest to largest:

Example 1

0.7 –7.0 7.0 1.071 –1.7 1.07 –0.71 –0.70

Solution:

–7.0 –1.7 –0.71 –0.70 0.7 1.07 1.071 7.0

Adding and Subtracting Decimals: Working with Negative Values

In earlier grades you worked with positive and negative integers (for example, 5 – 9 = –4). In this example, there are more negatives (9) than positives (5), so the answer is negative. Decimal numerals work exactly the same.

Example 1

Evaluate: 2.34 – 5.89

Solution:

Looking at the ones place, there are 2 positives but 5 negatives, so that would leave you with 3 more negatives or –3. Subtracting the hundredths you have $\frac{34}{100}$ positives and $\frac{89}{100}$ negatives. You have $\frac{55}{100}$ more negatives than positives, so combining these results you would get: (–3) and $\left(-\frac{55}{100}\right)$ or –3.55.

2.34 – 5.89 = –3.55

Example 2

Evaluate: 1.059 – 22.6

Solution:

Looking at the whole numbers, 1 positive and 22 negatives, you have many more negatives than positives, so the answer must be negative. There are 21 more negatives, so the answer begins with –21. Consider the thousandths: there are $\frac{59}{1000}$ positives and $\frac{600}{1000}$ negatives, so $\frac{541}{1000}$ more negatives. Combining these results, (–21) and $\left(-\frac{541}{1000}\right)$ you get –21.541.

1.059 – 22.6 = –21.541

Example 3

Evaluate: –6.48 + 9.23

Solution:

Looking at the ones place, there are 6 negatives and 9 positives. There are 3 more positives, so the answer begins with 3. The final answer will be positive.

Looking at the hundredths, there are $\frac{48}{100}$ negatives and $\frac{23}{100}$ positives, so 25 hundredths more negatives. The answer would then be (+3) and $\left(-\frac{25}{100}\right)$ or +2.75.

–6.48 + 9.23 = 2.75

Note: You could rewrite the question as 9.23 – 6.48.

What if the signs in example 3 were switched around?

6.48 – 9.23

Looking at the ones place, there are about 6 positives and about 9 negatives. Our final answer will be negative.

6.48 – 9.23 = –2.75

Notice the digits in the solution are the same as in example 3, but the sign is different.

Example 4

Place the decimal in the second term to reach the answer given:

23.5 – 3 6 4 1 = –12.91

Solution:

Using estimation, 23 – 36 would leave you with a negative 13, which is close to –12.91, so the decimal must be placed in the second term so its value is about –36.

23.5 – 36.41 = –12.91

Check this answer on your calculator.

Multiplying Decimals: Converting Percents to Decimals

Percent means "out of a hundred"—like 100 cents in a dollar, or 100 centimetres in a metre. A percentage can be written as a fraction with a denominator of 100, or converted to a decimal by moving the decimal spot two spaces to the left.

So, 62% is $\frac{62}{100}$ or 62 hundredths, which is written as 0.62.

This can also be calculated by dividing 62 by 100. The line in middle of the fraction is another symbol for division.

$$\frac{62}{100} = 62 \div 100 = 0.62$$

$$7\% \rightarrow \frac{7}{100} \rightarrow 0.07$$

$$13\frac{1}{2} \rightarrow \frac{13.5}{100} \rightarrow \frac{135}{1000} \rightarrow 0.135$$

When doing calculations using percentages, convert the percent to a decimal and then perform the mathematical operations.

Sample Problems

1. You have two pieces of cloth material, the first measuring 2.4 m in length, and the second piece measuring 3.7 m. If you cut off 8% of the first piece, and $7\frac{1}{2}$% of the second piece, what is the total length of the two pieces you cut off?

   Solution:

   $$\begin{array}{rr} 2.4 \text{ m} & 3.7 \text{ m} \\ \times\ 0.08 & \times\ 0.075 \\ \hline 0.192 \text{ m} & 0.2775 \text{ m} \end{array}$$

   0.192 + 0.2775 = 0.4695 m

2. Your job pays you $8.60 per hour. The boss likes your work and tells you to expect a 5% raise. Find the amount of the raise and the new hourly pay rate.

   Solution:

   $8.60 * 0.05 = $0.43

   (Note: In high school mathematics, it is more common to use the * sign to indicate multiplication rather than x. This is because the x can be easily mistaken for a variable, *x*)

   Then, $8.60 + $0.43 = $9.03.

Learning Activity 2.1

Decimal Numerals

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You have $2 a week. How much money will you have saved in one year?
2. Write two equivalent fractions for $\frac{4}{6}$.
3. Estimate the answer to 645.37 + 492.17.
4. If 15% of 40 is 6, what is 30% of 40?
5. You are buying a DVD for $9.53 including tax. You give the cashier $20.03. How much change do you get?
6. Convert 45 g to kilograms.
7. Calculate 11 x 11.
8. Calculate 63 ÷ 9.

continued

Learning Activity 2.1: Decimal Numbers (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert each of these percentages to a decimal numeral. If it helps, write each as a fraction over 100.
   a) 17%
   b) $96\frac{1}{2}\%$
   c) 3%
   d) $5\frac{1}{4}\%$
   e) 112%
2. Place the decimal in the second term to reach the answer:
   a) 674.2 – 6 2 0 5 = 612.15
   b) 13.86 – 1 5 4 7 = –140.8
3. The value of your car has dropped $3\frac{1}{2}\%$ since last year. If the car was worth $18 000 last year, find its value now.
4. You wrote three math quizzes, and earned a total of 72.8 marks. You have two of the quizzes, one scored 22.5 and the other scored 29.4. Find the value of the third quiz.
5. Gilles added 6.7 and 0.25, getting an answer of 9.2. Explain the error he made in the addition, and give the correct answer.
6. Place the decimal point in the correct position in the answer, and explain your thinking.

   2.95 ∗ 3.12 = 9 2 0 4

7. Place the decimal in the second term to reach the answer, and explain your thinking:

   35.08 – 5 5 5 5 = –520.42

Lesson Summary

In this lesson, you reviewed basic operations with positive and negative decimal numerals, converting percentages to decimals, and putting decimals in order. You used mental math and estimation to help you solve problems. In the next lesson, you will focus on operations with fractions.

Assignment 2.2: Decimal Numbers

Total: 18 marks

1. Add or subtract. *(2 marks)*
   a) 9.5 + 0.41 = _____ *(1 mark)*
   b) 16.92 – 175.8 = _____ *(1 mark)*

2. Multiply or divide. *(2 marks)*
   a) 17.8 x 0.03 = _____ *(1 mark)*
   b) 18.25 ÷ 0.5 = _____ *(1 mark)*

3. Place the decimal in the proper position, and explain your reasoning. (Hint: Use estimation.) *(4 marks)*
   a) 3.06 + 0.75 = 3 8 1 *(2 marks)*

   _____

   _____

   b) 6.7 ∗ 1 0 9 = 7.303 *(2 marks)*

   _____

   _____

4. Using estimation, explain the error, and give the correct answer. *(4 marks)*
   a) 12.5 + 1.03 = 1.353 *(2 marks)*

   _____

   _____

   b) 12.2 ∗ 2.06 = 251.32 *(2 marks)*

   _____

   _____

5. Put these decimal numerals in order from the least to the greatest. *(2 marks)*

   4.21 –2.4 4.12 –0.42 2.41

   _____

6. Put these decimal numerals in order from the largest to the smallest. *(2 marks)*

   0.101 –0.011 –0.01 0.011 –0.101

   _____

7. A business experienced a 3% drop in sales last month. If they normally sell $21 500 worth of goods, find the amount of sales for last month. *(2 marks)*

   _____

   _____

Lesson 3: Operations with Fractions

Lesson Focus

When you finish this lesson, you will be able to

- ❑ perform basic operations with fractions

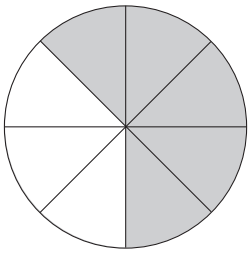
Lesson Introduction

This lesson reviews basic operations using rational numbers in fraction form, and solving problems using fractions.

What Is a Fraction?

A fraction can be used to describe several different ideas.

- It can be a part of a whole.
  You can cut a round pizza into eight equal pieces. This diagram shows five out of the eight pieces missing. $\frac{5}{8}$ of the pizza is gone.



- A fraction can be a part of a group.
  Three of the four members on the committee were male. $\frac{3}{4}$

- It could be a measurement.

  The point halfway between 0 and 1 on a ruler is $\frac{1}{2}$.

- Or a fraction could indicate division.

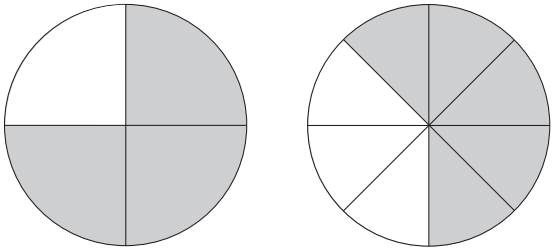
$$\frac{1}{4} = 1 \div 4 = 0.25$$

Any rational number can be written as a fraction, and its decimal equivalent will either terminate or repeat. For example:

$$\frac{1}{2} = 0.5 \text{ or } -\frac{1}{3} = -0.33333...$$

A fraction is made up of two parts—the numerator (or top) and denominator (bottom). When performing operations on fractions, having common denominators makes calculations much easier.

If you and a friend ate these pizzas, how much did you eat in total?

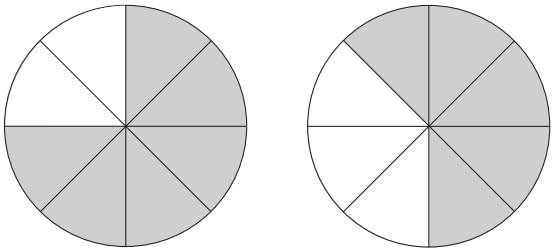


You can see that you ate $\frac{3}{4}$ of one pizza and $\frac{5}{8}$ of another.

$$\frac{3}{4} + \frac{5}{8} = ?$$

It would be much easier to find an answer if the pieces of pizza were the same size. To do that, you can write equivalent fractions with a common denominator. The lowest common multiple of 4 and 8 is 8. To write an equivalent fraction, multiply $\frac{3}{4}$ by $\frac{2}{2}$. Remember that $\frac{2}{2} = 1$, so we are not changing the value of the fraction, only the "size of the pieces."

$$\frac{3}{4} * \frac{2}{2} = \frac{6}{8}$$



Now each piece of pizza is the same size, $\frac{1}{8}$ of the pie, and you can add them easily:

$$\frac{6}{8} + \frac{5}{8} = \frac{11}{8}.$$

You have eaten 11 pieces of pizza that are each $\frac{1}{8}$ of the pie, or $\frac{11}{8}$.

Remember: $\frac{8}{8} = 1$

So, $\frac{8}{8} + \frac{3}{8} = \frac{11}{8}$

$\frac{11}{8}$ or $1\frac{3}{8}$

Each of the following questions already have common denominators, or the value on the bottom, so all you have to do is add the numerators, or numbers, on top. Final answers should be written in lowest terms (simplified) and when the fraction is larger than one, it could be written as a mixed numeral (a whole number and a fraction).

In your own words, explain on your Resource Sheet how when you add or subtract fractions, they need to have a common denominator.

Learning Activity 2.2

Adding and Subtracting Fractions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. If 12% of 400 is 48, what is 6% of 400?
2. Convert 25% to a decimal.
3. Write two equivalent fractions for $\frac{12}{15}$.
4. Is the answer to 63.459 + 4.678 closer to 70 or 700?
5. Calculate 400 ÷ 20.
6. Is the answer to 42 – 67 positive or negative?
7. Solve for n: $3n = 27$.
8. If you save $2 a week, how much will you have saved at the end of two years?

continued

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Solve each addition or subtraction, and then check your answers in the learning activity keys found at the end of this module.

1. $\frac{3}{7} + \frac{5}{7} =$

2. $\frac{4}{5} - \frac{1}{5} =$

3. $\frac{2}{3} + \frac{5}{3} =$

4. $1\frac{3}{8} - \frac{7}{8} =$

Practice Finding Common Denominators

You learned how to find common denominators in earlier grades, but as this is a really important step in adding and subtracting fractions, here is some review and practice.

Example 1

Evaluate: $\frac{1}{4} + \frac{2}{3}$

Solution:

The lowest common multiple of 3 and 4 is 12. Sometimes you can just multiply the two denominators together to find the lowest common multiple.

The next step is to write equivalent fractions using the LCM as the new denominator. To do this, multiply each fraction by a value of 1, written as a fraction with the number needed to multiply to get to the LCM.

$\frac{3}{3} = 1$

$\frac{1}{4} * \frac{3}{3}$ gives us $\frac{3}{12}$

Now, for the second fraction, to make the denominator 12, we have to multiply both top and bottom by 4

$$\frac{4}{4} = 1$$

$$\frac{2}{3} * \frac{4}{4} \text{ gives us } \frac{8}{12}$$

Now, add the numerators

$$\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

Learning Activity 2.3

Equivalent Fractions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Is the answer to 26.35 x 4.63 closer to 100 or 1000?
2. Calculate $\frac{1}{4} + \frac{3}{4}$.
3. Convert $2\frac{3}{4}$ to an improper fraction.
4. Evaluate $5t + 3$ if $t = -2$.
5. What two numbers add to 7 and multiply to 6?
6. Solve for c: $7 = 10 + c$.
7. Convert 0.15 to a percent.
8. What is the area of a rectangle with width 6 mm and length 5 mm?

continued

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write equivalent fractions with common denominators, and add or subtract as indicated. Simplify your answers.

1. $\frac{3}{5} + \frac{2}{3} =$

2. $\frac{2}{3} - \frac{4}{7} =$

3. $\frac{4}{5} + \frac{7}{8} =$

Multiplication and Division

It is not necessary to find a common denominator when multiplying or dividing fractions.

When multiplying, multiply the top numbers (numerators), multiply the bottom numbers (denominators), and simplify to lowest terms.

Include on your Resource Sheet instructions on how to multiply fractions. You may want to include an example as well.

Example 1

Evaluate: $\frac{2}{3} * \frac{3}{5}$

Solution: $\frac{2}{3} * \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

Example 2

Evaluate: $3\frac{3}{4} * 1\frac{1}{2}$

Solution:

Convert mixed numerals to improper fractions to calculate.

$$3\frac{3}{4} * 1\frac{1}{2} = \frac{15}{4} * \frac{3}{2}$$
$$= \frac{45}{8} \text{ or } 5\frac{5}{8}$$

You have likely been taught that when dividing, you invert the second fraction, and then multiply. Do you know why?

If you are given $8 \div 2$, dividing by two is the same thing as finding half, or multiplying by $\frac{1}{2}$.

$$\begin{array}{ccc} 8 & \div & 2 \\ \downarrow & & \downarrow \\ 8 & * & \frac{1}{2} \end{array}$$

You have changed the division to multiplication and inverted or found the reciprocal of 2.

2 can be written as $\frac{2}{1}$ and the reciprocal of $\frac{2}{1}$ is $\frac{1}{2}$.

So, written as fractions with a denominator of 1, you have:

$$\frac{8}{1} \div \frac{2}{1} = \frac{8}{1} * \frac{1}{2} = \frac{8}{2} = 4$$

Include on your Resource Sheet instructions on how to divide fractions. You should include an example as well.

Example 3

Evaluate: $5 \div \frac{1}{3}$

Solution:

This question is asking you to find how many thirds you can divide 5 into.

$5 * \frac{3}{1} = \frac{15}{1}$

Five can be divided into 15 "thirds"

Example 4

Evaluate: $\frac{4}{5} \div \frac{1}{4}$

Solution:

$\frac{4}{5} \div \frac{1}{4} = \frac{4}{5} * \frac{4}{1} = \frac{16}{5}$ or $3\frac{1}{5}$

Example 5

Evaluate: $\frac{3}{4} \div 1\frac{2}{3}$

Solution:

$\frac{3}{4} \div 1\frac{2}{3}$

$= \frac{3}{4} \div \frac{5}{3}$

$= \frac{3}{4} * \frac{3}{5}$

$= \frac{9}{20}$

Learning Activity 2.4

Multiplying and Dividing Fractions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write two equivalent fractions for $\frac{8}{12}$.
2. Convert $\frac{3}{10}$ to a decimal.
3. Add $\frac{5}{8}+\frac{5}{8}$.
4. What two numbers have a product of 24 and a difference of 5?
5. In January, you saved $3 each day. How much did you save by the end of January?
6. Calculate 100 x 36.5.
7. Calculate 70 x 70.
8. Convert 0.25 to a fraction.

continued

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Multiply or divide as indicated. Simplify your answers.

1. $\frac{3}{5} * \frac{4}{7} =$

2. $\frac{6}{7} \div \frac{3}{4} =$

3. $1\frac{2}{3} * \frac{4}{5} =$

4. $\frac{-3}{4} \div 1\frac{4}{5} =$

Working with Negative Values

The same principles you applied earlier to working with positive and negative integers and decimals also apply to rational numbers. Recall:

2 – 6 = –4

You have more negatives than positives, so the answer will be negative.

With fractions, once you find the common denominator, you collect the numerators. If you have more negative values than positive values, the answer will be negative.

$$\frac{1}{5} - \frac{4}{5} = \frac{-3}{5}$$

There are $\frac{1}{5}$ positives and $\frac{4}{5}$ negatives so there are $\frac{3}{5}$ more negatives than positives.

Also, you need to remember the rules when multiplying or dividing with negative values:

Negative * negative = positive

Negative * positive = negative

Word Problems

Use fractions and the correct operation to solve the following word problems.

Example 1

You have a 12-inch submarine sandwich. You offer your friend a bite, and he takes $\frac{1}{3}$ of the sub. How many inches of the sandwich did he bite off, and how many inches remain?

Solution:

$$12 \times \frac{1}{3} = \frac{12}{3}$$
$$= 4$$

Your friend bit off 4 inches.

12 – 4 = 8 inches remain.

Example 2

Your car's fuel tank holds 50 litres of gas. A trip to the lake will consume $\frac{4}{5}$ of a full tank. How much gas remains in the tank once you arrive at the lake?

Solution:

Since the trip will use $\frac{4}{5}$ of the gas, then the remaining gas would be:

$$1 - \frac{4}{5} = \frac{1}{5}$$

$$50 * \frac{1}{5} = \frac{50}{5} = 10 \text{ litres remain}$$

Another way to solve this problem would be to find the fuel used to drive to the lake, and then subtract that amount from the 50 litres.

$$50 * \frac{4}{5} = \frac{200}{5}$$
$$= 40 \text{ litres consumed}$$
$$50 - 40 = 10 \text{ litres remaining}$$

Learning Activity 2.5

Word Problems

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You changed your cell phone plan and saved $5 a month. How much do you save in one year?
2. You want to watch a movie that is 150 minutes long but you have to leave the house in two hours. Do you have enough time to watch the movie before leaving the house?
3. Add $\frac{2}{7}+\frac{3}{7}$.
4. Which is larger: $\frac{2}{3}$ or $\frac{3}{4}$?
5. If the radius of a circle is 4.5 mm, what is the diameter?
6. Write 2 equivalent fractions for $\frac{1}{5}$.
7. Is 456.3 ÷ 4.6 closer to 100 or 1000?
8. Calculate 6400 ÷ 80.

continued

Learning Activity 2.5: Word Problems (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Four friends are having lunch. The bill is $48.60, and the friends agree to split the cost evenly. Use fractions to determine how much each person must pay.
2. If $\frac{1}{6}$ of people have green eyes, and $\frac{11}{20}$ of the students in your school are girls, find how many girls with green eyes attend your school of 840 students.
3. Find the error, and correct: $4 - 8 \div \frac{2}{3} = 4 - \frac{16}{3} = \frac{12}{3} - \frac{16}{3} = -\frac{4}{3}$.
4. Why does $6 \div \frac{1}{2}$ give you an answer with a value greater than 6? Solve and explain your answer.

Lesson Summary

In this lesson, you reviewed the basic operations of addition, subtraction, multiplication, and division with fractions, and used them to solve word problems. In the next lesson, you will combine what you have done in the past two lessons and apply what you have learned in order to convert fractions to decimal numerals, and to percents.

Assignment 2.3: Operations with Fractions

Total: 14 marks

1. Add. *(2 marks)*

   a) $\frac{1}{3} + \frac{4}{5} =$ *(1 mark)*

   b) $\frac{5}{6} + \frac{3}{4} =$ *(1 mark)*

2. Subtract. *(2 marks)*

   a) $\frac{4}{5} - \frac{1}{2} =$ *(1 mark)*

   b) $\frac{3}{4} - \frac{4}{5} =$ *(1 mark)*

3. Multiply. *(2 marks)*

   a) $-\frac{5}{7} * \frac{3}{4} =$ *(1 mark)*

   b) $\frac{3}{5} * \frac{1}{6} =$ *(1 mark)*

4. Divide. *(2 marks)*

   a) $\frac{2}{3} \div \frac{1}{3} =$ *(1 mark)*

   b) $-\frac{3}{5} \div 1\frac{5}{7} =$ *(1 mark)*

5. Jimmy bought a 12-metre length of garden hose and cut off $\frac{3}{4}$ of it. How long is the piece he removed? *(1 mark)*

6. The lumber yard is selling off scrap pieces of lumber that would normally be eight feet long. One piece is $\frac{2}{3}$, and another piece is $\frac{3}{4}$ the normal length. If you bought both pieces, how much lumber would you have in total? *(2 marks)*

7. The skill-testing question for a contest was: "What is 20 divided by $\frac{1}{2}$?" Your friend answered 10 and did not win the prize! Describe the error he made and correct it. Explain why your answer is correct. *(3 marks)*

Lesson 4: Converting Fractions, Percents, and Decimal Numbers

Lesson Focus

When you finish this lesson, you will be able to

- ❑ convert fractions to decimal numerals
- ❑ convert decimal numerals to percents
- ❑ place rational numbers in order
- ❑ solve problems using operations with fractions and decimal numerals

Lesson Introduction

In this lesson, you will continue to work with rational numbers, converting fractions, percents, and decimal numerals so you can compare and order them.

Fractions to Decimal Numerals

In the last lesson, you learned that a fraction can mean different things. The horizontal line in the middle of the fraction can indicate division. It means the same thing as the ÷ symbol. So one way to convert a fraction is to divide the numerator by the denominator. Remember: "top divided by bottom."

Example 1

Convert $\frac{-4}{5}$ to a decimal.

Solution:

$\frac{-4}{5}$ means the same thing as -4 ÷ 5.

To convert this fraction to a decimal numeral, take –4 (top) divided by 5 (bottom) = –0.8.

Example 2

Convert $1\frac{3}{4}$ to a decimal.

Solution:

$1\frac{3}{4}$ converted to a decimal numeral would be 1 plus 3 divided by 4 or 1.75.

You could also change this to an improper fraction and then divide:

$$\frac{7}{4} = 1.75$$

Decimals to Percents

Refer back to Lesson 2 to review how to convert percents to decimal numerals. The reverse holds true for converting decimals back into percentages.

Example 1

Convert 0.65 to a percent.

Solution:

0.65 is read as 65 hundredths or $\frac{65}{100}$ and can be written as 65%.

Example 2

Convert 0.032 to a percent.

Solution:

0.032 means $\frac{32}{1000}$ or $\frac{3.2}{100}$ and can be written as 3.2%.

Learning Activity 2.6

Converting Fractions, Percents, and Decimal Numbers

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert $\frac{17}{3}$ to a mixed numeral.
2. What two numbers have a product of –18 and a sum of –3?
3. Calculate –15 x 3.
4. If 28% of 330 is 92.4, what is 14% of 320?
5. Calculate $\frac{4}{5}-\frac{2}{5}$.
6. Calculate 720 ÷ 9.
7. If you have 3 quarters, 1 dime, and 1 nickel, how much money do you have?
8. How many minutes in 1.5 hours?

continued

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Convert the value on each line to all three forms.

Fraction	Decimal	Percent
$\frac{1}{5}$		
	0.35	
		7
$\frac{2}{3}$		

Put Fractions, Decimals, and Percents in Order

Now that you have learned a method to convert all numbers to decimal numerals, you can place them in any given order.

Example 1

Place the following rational numbers in order from smallest to largest:

$$\frac{5}{7} \qquad -0.25 \qquad -\frac{1}{3} \qquad 0.001 \qquad -0.01 \qquad \frac{3}{4} \qquad -\frac{2}{5}$$

Solution:

First convert them all to decimal numerals:

$$0.741 \qquad -0.25 \qquad -0.3333 \qquad 0.001 \qquad -0.01 \qquad 0.75 \qquad -0.4$$

Rearrange them from smallest to largest:

$$-0.4 \qquad -0.3333 \qquad -0.25 \qquad -0.01 \qquad 0.001 \qquad 0.714 \qquad 0.75$$

Then rewrite each in the form it was given:

$-\frac{2}{5}$ $-\frac{1}{3}$ –0.25 –0.01 0.001 $\frac{5}{7}$ $\frac{3}{4}$

Example 2

Place the following rational numbers in order from largest to smallest:

$3\frac{1}{2}\%$ 0.036 $-\frac{1}{30}$ $\frac{2}{59}$ –3% 0.03666... –0.035

Solution:

Convert all to decimal numerals:

0.035 0.036 –0.0333... 0.0339 –0.03 0.0366... –0.035

Now order them as instructed:

0.03666... 0.036 0.035 0.0339 –0.03 –0.0333 –0.035

Then rewrite them as they were given:

0.03666... 0.036 $3\frac{1}{2}\%$ $\frac{2}{59}$ –3% $-\frac{1}{30}$ –0.035

Numbers Between Numbers

Using conversions or common denominators, you can find a number between any two given numbers.

You should be familiar with both methods. Add them to your Resource Sheet as you read them.

Method 1: Convert to Decimals

Find a rational number in between $\frac{1}{5}$ and $\frac{2}{7}$

Convert them to decimal numerals: $1 \div 5 = 0.20$ and $2 \div 7 = 0.2857$

0.25 is a number in between the two values.

To convert that back to a fraction, 0.25 is 25 hundredths or $\frac{25}{100}$, which simplifies to $\frac{1}{4}$.

Method 2: Common Denominators.

The common denominator for $\frac{1}{5}$ and $\frac{2}{7}$ is 35.

So $\frac{1}{5} * \frac{7}{7} = \frac{7}{35}$ and $\frac{2}{7} * \frac{5}{5} = \frac{10}{35}$

Pick a number between the two fractions; $\frac{8}{35}$ is a possible answer.

Lesson Summary

In this lesson, you converted rational numbers from one form to another. In the next lesson, you will review your understanding of perfect square numbers and square roots.

Assignment 2.4: Converting Fractions, Percents, and Decimal Numbers

Total: 14 marks

1. Convert the following fractions and percents to decimal numerals, rounding to a maximum of four decimal places. *(2 marks)*

   a) $\frac{4}{7}$ *(1 mark)*

   b) $-6\frac{1}{2}\%$ *(1 mark)*

2. Convert the following to fractions in most simplified form. Show your steps. *(2 marks)*

   a) –0.35 *(1 mark)*

   b) 16% *(1 mark)*

3. Convert the following to percents, rounding to one decimal place. *(2 marks)*

   a) 0.075 *(1 mark)*

   b) $-\frac{1}{8}$ *(1 mark)*

4. Your new car uses 5% less fuel than your previous car. If your previous car used 12 litres for every 100 km, find the fuel economy per 100 km for the new car. Show your calculations. *(2 marks)*

5. You were happy to receive a $3\frac{1}{2}\%$ raise at work. Your friend is bragging that he did better with a raise of $\frac{1}{30}\text{th}$ his pay. Who gets the larger increase? Show your calculations. *(2 marks)*

6. Use any method to pick a rational number between $\frac{3}{4}$ and $\frac{4}{7}$. Show your work. *(2 marks)*

7. You scored 80% on a math quiz. If the quiz had a total of 20 questions, how many questions did you get wrong? Show your work. *(2 marks)*

LESSON 5: PERFECT SQUARE NUMBERS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ list perfect square numbers
- ❑ find square roots of perfect square numbers
- ❑ find perfect square numbers given their roots

Lesson Introduction

This lesson reviews how to find perfect square numbers and square roots using mental math.

What Is a Perfect Square Number (PSN)?

A **perfect square number** is found by multiplying a number (or root) by itself.

In your own words, add this definition to your Resource Sheet. You may want to include the abbreviation in case you forget what it means (PSN).

Example 1

Is 16 a perfect square number?

Solution:

$4 * 4 = 16$

So, 16, by definition, is a perfect square number, and its square root is 4

Another way to write this is 4^2, which reads as "4 squared '' or 4^2, which can be read as "four to the power of two" or "four squared."

You should be able to recognize and state the first few perfect square numbers from memory, since they were used a lot throughout your middle school math courses. The list of perfect square numbers begins as 1, 4, 9, 16, 25...

Finding the squares of larger numbers is not difficult.

Example 2

What is the perfect square number if the root is 20?

Solution:

20^2 is written as 20 x 20

But, we can break that down to 2 x 2 * 10 x 10

= 4 * 100

= 400

Learning Activity 2.7

Perfect Square Numbers

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert $\frac{2}{5}$ to a decimal.
2. What is half of 74?
3. You saved $208 in one year by putting aside the same amount each week. How much did you save each week?
4. You worked 7.5 hours on four different days one week. How many hours did you work for the week?
5. Solve for *b*: $7b = 21$.
6. Solve for *n*: $\frac{n}{4} = 25$.
7. You know that 60 x 4 is 240. What is 60 x 8?
8. How many quarters in $3.25?

continued

Learning Activity 2.7: Perfect Square Numbers (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Complete the following chart. Use mental math to find the perfect square numbers given their roots.

Root	Multiplication	PSN
1	1 x 1	1
2	2 x 2	4
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

2. Find the squares of these numbers, without using a calculator.
   a) 30
   b) 40
   c) 50
   d) 60

Finding Square Roots

Now that you know the perfect square numbers, you can do the exact opposite operation to find square roots, given a PSN.

The square root of 25 is the number that, multiplied by itself, gives you 25.

5 x 5 = 25, so 5 is the square root of 25

The symbol used for square roots is the radical sign: $\sqrt{}$

$\sqrt{25} = 5$ because $5 * 5 = 25$

Now consider that (–5) * (-5) = 25. When you square a negative number, the result is positive, so the square root of 25 can be both positive and negative 5, or ± 5.

However, when you are asked to find the root of a number, it usually means just the positive value. When the radical sign $\sqrt{}$ is used, it refers to the principal root or the positive root.

Can you find the root of a negative number? For example, $\sqrt{-36}$. No. There is no root or number that you can multiply by itself to come up with a negative value. This is because a negative multiplied by a negative is a positive.

Given the numbers 7 and 12, can you identify a number between these two that would have a square root? You can list off the initial PSNs by memory. The number 9 is a perfect square number, and is between 7 and 12.

Learning Activity 2.8

Finding Square Roots

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate $4^2 + 3^2$.
2. 52 is between what two perfect squares?
3. You worked 10 hours last week. You get paid $7.75 an hour. How much money did you earn last week?
4. Convert 32 litres to millilitres.
5. What is the volume of a cube with side length 2 cm?
6. Write two equivalent fractions for $\frac{20}{25}$.
7. Is $\frac{2}{3}+\frac{4}{5}$ greater or less than 1?
8. Calculate $8^2 - 4^2$.

continued

Learning Activity 2.8: Finding Square Roots (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the list of PSNs, find the positive square root of each.

PSN	Root
36	
81	
100	
25	
49	
121	
4	
144	
64	
9	
169	
225	

Lesson Summary

In this lesson, you reviewed the first 15 perfect square numbers and found the square root of each. The next lesson deals with estimating square roots of numbers that are not perfect square numbers.

Assignment 2.5: Perfect Square Numbers

Total: 15 marks

1. Given the following roots, find each perfect square number: *(4 marks)*
   a) 6 *(1 mark)*
   b) (–9) *(1 mark)*
   c) 12 *(1 mark)*
   d) 7 *(1 mark)*

2. Given these perfect square numbers, find each positive square root: *(4 marks)*
   a) 100 *(1 mark)*
   b) 25 *(1 mark)*
   c) 169 *(1 mark)*
   d) 16 *(1 mark)*

3. Find a perfect square number between 25 and 49. *(1 mark)*

4. Find a perfect square number between 50 and 70. *(1 mark)*

5. Find two perfect square numbers between 75 and 120. *(2 marks)*

6. The carpet in your living room measures 4 m by 4 m. What is the area in square metres of the carpet? *(1 mark)*

7. You have a square tarp to cover your boat. The area of the tarp is $36m^2$. Find the length of one side. *(1 mark)*

8. Explain why +7 and –7 are both roots of 49. *(1 mark)*

LESSON 6: ESTIMATING SQUARE ROOTS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ find approximate square roots with and without technology
- ❑ find approximate square numbers given approximate roots with and without technology
- ❑ identify errors in square root calculations

Lesson Introduction

Not all numbers are perfect square numbers with rational roots. This lesson shows you how to estimate and find approximate square roots for numbers that are not perfect squares.

Estimating square roots

Not all numbers are perfect squares, so not all numbers have square roots that are whole numbers.

Example 1

Estimate the square root of 10 $\left(\sqrt{10}\right)$.

Solution:

The number 10 does not have a whole number square root. There is no whole number that can be multiplied by itself to get 10. But we know that 3^2 is 9 and 4^2 is 16, and since 10 is between 9 and 16, the approximate root of 10 must be between 3 and 4. Given that 10 is very close to 9, a good estimate of its root might be 3.1 or 3.2.

The actual root of 10 is an irrational number: 3.16227766… It is not an "exact" answer because the decimal never ends (your calculator screen just runs out of room and rounds the answer). So 3.16 is said to be the approximate square root of 10.

Example 2

Estimate the square root of 75 $\left(\sqrt{75}\right)$.

Solution:

75 is between the perfect square numbers of 64 and 81. Their square roots are 8 and 9, so the answer must be somewhere between those numbers.

75 is 11 units away from 64, and only 6 units away from 81, so it is more than half-way toward 9, in fact it is almost $\frac{2}{3}$ of the distance. So, a good estimate of the root of 75 would be 8.67. This is an approximate value. Using a calculator to check, you can find that $\sqrt{75}$ shows 8.660254038…

Squares and Roots Using Technology

So far, you have been finding PSNs and their roots using mental math. In this lesson, the use of a calculator will make quick work of checking your estimations of approximate square numbers and roots.

Different calculators require different keystrokes to find squares and roots. Check your calculator. What keystrokes does it require to square a number? Does it have a shortcut? Many calculators use a button with $\boxed{y^x}$ or $\boxed{x^y}$, or a key with $\boxed{x^2}$. Graphing calculators and computers use the $\boxed{\wedge}$ key. Press $\boxed{\text{SHIFT}}$ or $\boxed{\text{2nd FUNCTION}}$ and $\boxed{6}$ on a computer keyboard to indicate an exponent.

Use your calculator to show that $17^2 = 289$. Write the order of keystrokes you need to find square numbers here as a reminder:

$\square\ \square\ \square\ \square\ \square$

For example,

$\boxed{1}\ \boxed{7}\ \boxed{x^2}\ \boxed{=}$ or $\boxed{1}\ \boxed{7}\ \boxed{\wedge}\ \boxed{2}\ \boxed{\text{ENTER}}$

What about calculating square roots? Use your calculator to show that $\sqrt{484} = 22$.

Do you have to press the $\sqrt{\ }$ sign or the value first? Do you have to press $\boxed{=}$? To access the $\sqrt{\ }$ sign on a graphing calculator, you need to first press the $\boxed{\text{2nd FUNCTION}}$ key and then $\boxed{x^2}$.

Write the sequence of keystrokes needed to find a square root using your calculator here:

$\square\ \square\ \square\ \square\ \square$

You can also use a spreadsheet program to find square roots. Remember to start all "calculating" formulas with an = sign. Type the following command into a cell, and it will instantly find the square root of the value in the bracket.

=sqrt(25)

This will find the square root of 25, which is 5.

Closest Perfect Square Number

If you are given an approximate square root, you can easily estimate its approximate square number.

Example 1

If the approximate square root is 7.3, find its approximate square number.

Solution:

We know that 7.3 is between 7 and 8, so the answer will be somewhere between 49 and 64. Looking at the decimal place, 3 tenths means the value is closer to the 7 than it is to the 8. Therefore, the number must be closer to 49 than 64. Also, since 0.3 is roughly one-third of the distance, and the distance between 49 and 64 is 15, we can estimate that the number would be about 5 units (one-third of 15) away from 49. A good estimate for the approximate square number would be 49 + 5 = 54.

Remember, you are using approximate values, not exact values.

The actual answer for 7.32 is 53.29, so the estimate was very close!

Learning Activity 2.9

Estimating Square Roots

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Find $\sqrt{81}$.
2. Calculate 225 ÷ 15.
3. You started your math homework at 4:30 pm and finished at 5:15 pm. How many minutes did you spend on your math homework?
4. What perfect square is closest to 45?
5. Is the answer to –15 + 40 positive or negative?
6. Calculate 11^2.
7. If 13 x 12 is 156, what is 14 x 12?
8. Calculate $7^2 - 6^2$.

continued

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Estimate the approximate roots of the following values, then check your answers with a calculator.
   a) $\sqrt{60}$
   b) $\sqrt{90}$
   c) $\sqrt{180}$
2. Given these approximate square roots, write down the steps you take to find the approximate square number of each. Check your answers with a calculator.
   a) 9.7
   b) 11.4
   c) 5.2

Identify the Error

A friend has asked you to help him with his math. He wrote that the positive square root of 6.4 is 3.2. What was his mistake?

Lesson Summary

In this lesson, you learned how to make good estimates and find the approximate square and square roots of numbers, with and without technology.

NOTES

Assignment 2.6: Estimating Square Roots

Total: 18 marks

1. Use technology to find the approximate square root of the following numbers. Round your answers to 2 decimal places. *(4 marks)*
   a) 40 _____ *(1 mark)*
   b) 200 _____ *(1 mark)*
   c) 750 _____ *(1 mark)*
   d) 1234 _____ *(1 mark)*

2. Given these square roots, find the square numbers. *(4 marks)*
   a) 7.5 _____ *(1 mark)*
   b) 6.22 _____ *(1 mark)*
   c) 25.642 _____ *(1 mark)*
   d) 17.7 _____ *(1 mark)*

3. Find a perfect square number between 8.2 and 11.7. *(1 mark)*

   _____

   _____

4. Write the steps you would take to estimate the approximate square root of 75. *(3 marks)*

   _____

   _____

   _____

   _____

5. Between which two integers does each square root lie? *(6 marks)*

a) $\sqrt{38}$ *(2 marks)*

b) $\sqrt{130}$ *(2 marks)*

c) $-\sqrt{105}$ *(2 marks)*

MODULE 2 SUMMARY

Congratulations, you have finished two of the eight modules in this course.

This module provided opportunities for you to review and practice estimating, mental math, conversions and operations with decimals, fractions, and percentages. These skills will strengthen your number sense and are a strong foundation on which to build math concepts. In the next module, you will continue to develop your understanding of number sense and will work with the order of operations, exponents, and powers.

It is now time to send the six assignments from Module 2 to your tutor/marker for feedback on how you are doing.

Mailing Your Assignments

Before you mail your completed assignments, please photocopy them first. That way, you will have a copy of your assignments in case they go missing. Please organize your material in the following order:

- Cover Sheet for Module 2
- Assignment 2.1: Cover Assignment
- Assignment 2.2: Decimal Numerals
- Assignment 2.3: Operations with Fractions
- Assignment 2.4: Conversions
- Assignment 2.5: Perfect Square Numbers
- Assignment 2.6: Estimating Square Roots

Place all of these assignments in an envelope and address it to:

ISO Tutor/Marker
555 Main St.
Winkler MB R6W 1C4

Remember: You can call or email your tutor/marker at any time if you have questions!

NOTES

GRADE 9 MATHEMATICS (10F)

Module 2: Number Sense

Learning Activity Answer Keys

MODULE 2: NUMBER SENSE

Learning Activity 2.1: Decimal Numerals

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You have $2 a week. How much money will you have saved in one year?
2. Write two equivalent fractions for $\frac{4}{6}$.
3. Estimate the answer to 645.37 + 492.17.
4. If 15% of 40 is 6, what is 30% of 40?
5. You are buying a DVD for $9.53 including tax. You give the cashier $20.03. How much change do you get?
6. Convert 45 g to kilograms.
7. Calculate 11 x 11.
8. Calculate 63 ÷ 9.

Answers:

1. Since there are 52 weeks in a year, you saved $104.
2. Answers will vary; some possible fractions are: $\frac{2}{3}, \frac{8}{12}, \frac{12}{18}, \frac{40}{60}, \ldots$.

   To be equivalent, both numerator and denominator must be multiplied or divided by the same number.
3. 645.37 is close to 650; 492.17 is close to 500; an estimate might be 1150. Any answer between 1100 and 1200 would be a good estimate.
4. 12 (twice as much)
5. $10.50 ($0.50 gets the 9.53 to 10.03, then $10 more to make the $20)
6. Since there are 1000 g in a kilogram, 45 ÷ 1000 = 0.045 kg.
7. 121
8. 7

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Convert each of these percentages to a decimal numeral. If it helps, write each as a fraction over 100.
   a) 17% *Answer:* 0.17
   b) $96\frac{1}{2}\%$ *Answer:* 0.965
   c) 3% *Answer:* 0.03
   d) $5\frac{1}{4}\%$ *Answer:* 0.0525
   e) 112% *Answer:* 1.12

2. Place the decimal in the second term to reach the answer:
   a) 674.2 – 6 2 0 5 = 612.15 *Answer:* 62.05
   b) 13.86 – 1 5 4 7 = –140.8 *Answer:* 154.7

3. The value of your car has dropped $3\frac{1}{2}\%$ since last year. If the car was worth $18 000 last year, find its value now.

   Answer:

   $18 000 x 0.035 = $630

   Then $18 000 – $630 = $17 370.

4. You wrote three math quizzes, and earned a total of 72.8 marks. You have two of the quizzes, one scored 22.5 and the other scored 29.4. Find the value of the third quiz.

   Answer:

   22.5 + 29.4 = 51.9

   Then, 72.8 – 51.9 = 20.9.

5. Gilles added 6.7 and 0.25, getting an answer of 9.2. Explain the error he made in the addition, and give the correct answer.

   Answer:

   Gilles added 6.7 and 2.5. The decimal was wrongly placed. The answer should be 6.95.

6. Place the decimal point in the correct position in the answer, and explain your thinking.

   2.95 ∗ 3.12 = 9 2 0 4

   Answer:

   2.95 is approximately 3 and 3 ∗ 3 = 9, so the answer should be close to 9. The decimal is placed so the answer is 9.204.

7. Place the decimal in the second term to reach the answer, and explain your thinking:

   35.08 – 5 5 5 5 = –520.4

   Answer:

   To reach –520, we need to subtract 555 from 35, so the decimal is placed so the term is 555.5.

Learning Activity 2.2: Adding and Subtracting Fractions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. If 12% of 400 is 48, what is 6% of 400?
2. Convert 25% to a decimal.
3. Write two equivalent fractions for $\frac{12}{15}$.
4. Is the answer to 63.459 + 4.678 closer to 70 or 700?
5. Calculate 400 ÷ 20.
6. Is the answer to 42 – 67 positive or negative?
7. Solve for n: $3n = 27$.
8. If you save $2 a week, how much will you have saved at the end of two years?

Answers:

1. 24 (half as much)
2. $0.25 \left(25\% \text{ means } \frac{25}{100} \right)$
3. Answers will vary; some possible fractions are: $\frac{4}{5}, \frac{8}{10}, \frac{24}{30}, \frac{120}{150}, \ldots$.

   To be equivalent, both numerator and denominator must be multiplied or divided by the same number.
4. closer to 70 (estimate 60 + 4 or 64)
5. 20
6. negative since 67 is larger than 42
7. 9 (27 ÷ 3 = 9)
8. Since there are 52 weeks in a year, and 104 weeks in two years, you saved $208.

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Solve each addition or subtraction, and then check your answers in the learning activity keys found at the end of this module.

1. $\frac{3}{7} + \frac{5}{7} =$

2. $\frac{4}{5} - \frac{1}{5} =$

3. $\frac{2}{3} + \frac{5}{3} =$

4. $1\frac{3}{8} - \frac{7}{8} =$

Answers:

1. $\frac{3}{7} + \frac{5}{7} = \frac{8}{7}$ or $1\frac{1}{7}$

2. $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

3. $\frac{2}{3} + \frac{5}{3} = \frac{7}{3}$ or $2\frac{1}{3}$

4. $1\frac{3}{8} - \frac{7}{8} = \frac{11}{8} - \frac{7}{8} = \frac{4}{8}$

$= \frac{1}{2}$ (simplify)

Note: Depending on the context for adding and subtracting fractions, you may want to leave the fraction as an improper fraction (larger numerator than denominator), or you may want to write it as a mixed fraction (a whole number and a fraction).

Learning Activity 2.3: Equivalent Fractions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Is the answer to 26.35 x 4.63 closer to 100 or 1000?
2. Calculate $\frac{1}{4}+\frac{3}{4}$.
3. Convert $2\frac{3}{4}$ to an improper fraction.
4. Evaluate $5t + 3$ if $t = -2$.
5. What two numbers add to 7 and multiply to 6?
6. Solve for c: $7 = 10 + c$.
7. Convert 0.15 to a percent.
8. What is the area of a rectangle with width 6 mm and length 5 mm?

Answers:

1. 100 (since 26 is close to 25 and 25 x 4 = 100)
2. 1
3. $\frac{11}{4}$ $\left(2 \text{ can be written as } \frac{8}{4} \text{ so } \frac{8}{4}+\frac{3}{4}=\frac{11}{4}\right)$
4. –7 (5(–2) + 3 = –10 + 3 = –7)
5. 6 and 1
6. –3 = c (7 – 10 = –3)
7. 15% $\left(0.15 \text{ means } \frac{15}{100}\right)$
8. 30 mm^2 (6 mm x 5 mm = 30 mm^2)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write equivalent fractions with common denominators, and add or subtract as indicated. Simplify your answers.

1. $\frac{3}{5} + \frac{2}{3} =$

2. $\frac{2}{3} - \frac{4}{7} =$

3. $\frac{4}{5} + \frac{7}{8} =$

Answers:

1. $\frac{3}{5} + \frac{2}{3} = \frac{3}{5} * \frac{3}{3} + \frac{2}{3} * \frac{5}{5} = \frac{9}{15} + \frac{10}{15} = \frac{19}{15}$ or $1\frac{4}{15}$

2. $\frac{2}{3} - \frac{4}{7} = \frac{2}{3} * \frac{7}{7} - \frac{4}{7} * \frac{3}{3} = \frac{14}{21} - \frac{12}{21} = \frac{2}{21}$

3. $\frac{4}{5} + \frac{7}{8} = \frac{4}{5} * \frac{8}{8} + \frac{7}{8} * \frac{5}{5} = \frac{32}{40} + \frac{35}{40} = \frac{67}{40}$ or $1\frac{27}{40}$

Learning Activity 2.4: Multiplying and Dividing Fractions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write two equivalent fractions for $\frac{8}{12}$.
2. Convert $\frac{3}{10}$ to a decimal.
3. Add $\frac{5}{8}+\frac{5}{8}$.
4. What two numbers have a product of 24 and a difference of 5?
5. In January, you saved $3 each day. How much did you save by the end of January?
6. Calculate 100 x 36.5.
7. Calculate 70 x 70.
8. Convert 0.25 to a fraction.

Answers:

1. Answers will vary; some possible fractions are: $\frac{2}{3}, \frac{16}{24}, \frac{4}{6}, \frac{18}{120}, ...$

   To be equivalent, both numerator and denominator must be multiplied or divided by the same number.
2. 0.3 (both are read as three-tenths)
3. $\frac{10}{8}$ or $\frac{5}{4}$ or $1\frac{2}{8}$ or $1\frac{1}{4}$
4. 3 and 8
5. Since January has 31 days, you saved $93.
6. 3650
7. 4900
8. $\frac{25}{100}$ or $\frac{1}{4}$ $\left(\text{or any equivalent fraction}\right)$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Multiply or divide as indicated. Simplify your answers.

1. $\frac{3}{5} * \frac{4}{7} =$

2. $\frac{6}{7} \div \frac{3}{4} =$

3. $1\frac{2}{3} * \frac{4}{5} =$

4. $\frac{-3}{4} \div 1\frac{4}{5} =$

Answers:

1. $\frac{3}{5} * \frac{4}{7} = \frac{12}{35}$

2. $\frac{6}{7} \div \frac{3}{4} = \frac{6}{7} * \frac{4}{3} = \frac{24}{21}$ or $1\frac{3}{21}$ or $1\frac{1}{7}$

3. $1\frac{2}{3} * \frac{4}{5} = \frac{5}{3} * \frac{4}{5} = \frac{20}{15}$ or $1\frac{5}{15}$ or $1\frac{1}{3}$

4. $\frac{-3}{4} \div 1\frac{4}{5} = \frac{-3}{4} \div \frac{9}{5} = \frac{-3}{4} * \frac{5}{9} = \frac{-15}{36} = \frac{-5}{12}$

Learning Activity 2.5: Word Problems

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You changed your cell phone plan and saved $5 a month. How much do you save in one year?
2. You want to watch a movie that is 150 minutes long but you have to leave the house in two hours. Do you have enough time to watch the movie before leaving the house?
3. Add $\frac{2}{7}+\frac{3}{7}$.
4. Which is larger: $\frac{2}{3}$ or $\frac{3}{4}$?
5. If the radius of a circle is 4.5 mm, what is the diameter?
6. Write two equivalent fractions for $\frac{1}{5}$.
7. Is 456.3 ÷ 4.6 closer to 100 or 1000?
8. Calculate 6400 ÷ 80.

Answers:

1. Since there are 12 months in a year, you save 12 x 5 or $60 a year.
2. No, 2 hours is 120 minutes and the movie is 150 minutes.
3. $\frac{5}{7}$
4. $\frac{3}{4}$ is larger. $\left(\frac{2}{3}\right.$ is approximately 67% while $\frac{3}{4}$ is 75%; if you write these as equivalent fractions, $\frac{2}{3}=\frac{8}{12}$ and $\frac{3}{4}=\frac{9}{12}\Big)$
5. Diameter is twice the radius so 9 mm.
6. Answers will vary; some possible fractions are: $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{10}{50}, \ldots$.

   To be equivalent, both numerator and denominator must be multiplied or divided by the same number.
7. 100 (can be estimated to be 400 ÷ 4 = 100)
8. 80

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Four friends are having lunch. The bill is $48.60, and the friends agree to split the cost evenly. Use fractions to determine how much each person must pay.

   Answer:

   Since there are four friends, each person would pay $\frac{1}{4}$ of the cost.

   $$\$48.60 * \frac{1}{4} = \frac{\$48.60}{4}$$
   $$= \$12.15$$

2. If $\frac{1}{6}$ of people have green eyes, and $\frac{11}{20}$ of the students in your school are girls, find how many girls with green eyes attend your school of 840 students.

   Answer:

   $$840 * \frac{11}{20} = \frac{9240}{20} = 462 \text{ girls}$$

   $$462 \times \frac{1}{6} = 77 \text{ girls with green eyes}$$

3. Find the error, and correct: $4 - 8 \div \frac{2}{3} = 4 - \frac{16}{3} = \frac{12}{3} - \frac{16}{3} = -\frac{4}{3}$.

   Answer:

   The error was that the reciprocal of $\frac{2}{3}$ was not used. It was not inverted before being multiplied.

   $$4 - 8 \times \frac{3}{2}$$
   $$= 4 - \frac{24}{2}$$
   $$= 4 - 12$$
   $$= -8$$

4. Why does $6 \div \frac{1}{2}$ give you an answer with a value greater than 6? Solve and explain your answer.

   Answer:

   $6 \div \frac{1}{2}$ means "how many halfs" are in 6

   $6 * \frac{2}{1} = 12$

   There are 12 halves in 6. Since you are finding how many fractional parts there are in something, the answer has a bigger value.

Learning Activity 2.6: Converting Fractions, Percents, and Decimal Numbers

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert $\frac{17}{3}$ to a mixed numeral.
2. What two numbers have a product of –18 and a sum of –3?
3. Calculate –15 x 3.
4. If 28% of 330 is 92.4, what is 14% of 320?
5. Calculate $\frac{4}{5} - \frac{2}{5}$.
6. Calculate 720 ÷ 9.
7. If you have 3 quarters, 1 dime, and 1 nickel, how much money do you have?
8. How many minutes in 1.5 hours?

Answers:

1. $5\frac{2}{3}$
2. –6 and 3
3. –45
4. 46.2 (half of 92.4)
5. $\frac{2}{5}$
6. 80
7. ninety cents
8. 90 minutes (60 minutes in 1 hour, 30 minutes in 0.5 hours)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Convert the value on each line to all three forms.

Fraction	Decimal	Percent
$\frac{1}{5}$	0.2	20
$\frac{35}{100} = \frac{7}{20}$	0.35	35
$\frac{7}{100}$	0.07	7
$\frac{2}{3}$	0.666...	66.7

Learning Activity 2.7: Perfect Square Numbers

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert $\frac{2}{5}$ to a decimal.
2. What is half of 74?
3. You saved $208 in one year by putting aside the same amount each week. How much did you save each week?
4. You worked 7.5 hours on 4 different days one week. How many hours did you work for the week?
5. Solve for b: $7b = 21$.
6. Solve for n: $\frac{n}{4} = 25.$
7. You know that 60 x 4 is 240. What is 60 x 8?
8. How many quarters in $3.25?

Answers:

1. $0.4 \left(\text{since } \frac{2}{5} \text{ is equivalent to } \frac{4}{10} \right)$
2. 37
3. $4.00 (52 weeks in a year so 208 ÷ 52. There are 52 weeks in a year, so if you saved $1 a week, you'd have $52; $2 a week would be double that or $104, so you need to double again for $4 a week to get you to $208).
4. 30 hours (7.5 hours per day for 2 days is 15 hours; double that to get 30 hours)
5. $b = 3$ (21 ÷ 7 = 3)
6. $n = 100$ (25 x 4 = 100)
7. 480 (double 240)
8. 13 quarters (4 quarters in $1; 8 quarters in $2; 12 quarters in $3 so 13 quarters is $3.25)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Complete the following chart. Use mental math to find the perfect square numbers given their roots.

Root	Multiplication	PSN
1	1 x 1	1
2	2 x 2	4
3	3 x 3	9
4	4 x 4	16
5	5 x 5	25
6	6 x 6	36
7	7 x 7	49
8	8 x 8	64
9	9 x 9	81
10	10 x 10	100
11	11 x 11	121
12	12 x 12	144
13	13 x 13	169
14	14 x 14	196
15	15 x 15	225

2. Find the squares of these numbers without using a calculator.

   a) 30
   b) 40
   c) 50
   d) 60

   Answers:

   a) 30 x 30 = 3 x 3 ∗ 10 x 10 = 9 x 100 = 900
   b) 1600
   c) 2500
   d) 3600

Learning Activity 2.8: Finding Square Roots

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate $4^2 + 3^2$.
2. 52 is between what two perfect squares?
3. You worked 10 hours last week. You get paid $7.75 an hour. How much money did you earn last week?
4. Convert 32 litres to millilitres.
5. What is the volume of a cube with side length 2 cm?
6. Write two equivalent fractions for $\frac{20}{25}$.
7. Is $\frac{2}{3}+\frac{4}{5}$ greater or less than 1?
8. Calculate $8^2 - 4^2$.

Answers:

1. 25 (16 + 9)
2. 49 and 64
3. $77.50
4. 32000 mL (1 L = 1000 mL)
5. 8 cm^3
6. Answers will vary; some possible fractions are: $\frac{4}{5}, \frac{40}{50}, \frac{8}{10}, \frac{200}{250}, ...$

   To be equivalent, both numerator and denominator must be multiplied or divided by the same number.
7. Greater (both $\frac{2}{3}$ and $\frac{4}{5}$ are greater than one-half. Therefore, the sum is greater than 1).
8. 48 (64 – 16)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the list of PSNs, find the positive square root of each.

PSN	Root
36	6
81	9
100	10
25	5
49	7
121	11
4	2
144	12
64	8
9	3
169	13
225	15

Learning Activity 2.9: Estimating Square Roots

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Find $\sqrt{81}$.
2. Calculate 225 ÷ 15.
3. You started your math homework at 4:30 pm and finished at 5:15 pm. How many minutes did you spend on your math homework?
4. What perfect square is closest to 45?
5. Is the answer to –15 + 40 positive or negative?
6. Calculate 11^2.
7. If 13 x 12 is 156, what is 14 x 12?
8. Calculate $7^2 - 6^2$.

Answers:

1. 9
2. 15
3. 45 minutes (30 minutes from 4:30 pm to 5:00 pm plus 15 minutes to get to 5:15 pm)
4. 49 (36 is the closest square smaller than 45, but 49 is only 4 away instead of 9 away)
5. positive (40 is larger than 15)
6. 121
7. 168 (12 more than 156)
8. 13 (49 – 36)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Estimate the approximate roots of the following values, then check your answers with a calculator.

   a) $\sqrt{60}$

   b) $\sqrt{90}$

   c) $\sqrt{180}$

Answers:

1. a) 60 is about $\frac{2}{3}$ of the way between the PSNs 49 and 64, so the root of 60 would be between 7 and 8. A good estimate might be 7.67. The actual root is approximately 7.75.

   b) 90 is about halfway between the PSNs 81 and 100, so the root of 90 would be between 9 and 10. A good estimate might be 9.50. The actual root is approximately 9.49.

   c) 180 is about $\frac{2}{3}$ of the way between the PSNs 169 and 196, so the root of 180 would be between 13 and 14. A good estimate might be 13.40. The actual root is approximately 13.42.

2. Given these approximate square roots, write down the steps you take to find the approximate square number of each. Check your answers with a calculator.
   a) 9.7
   b) 11.4
   c) 5.2

Answers:

2. a) 9.7 is between 9 and 10, so 9.72 would be about $\frac{3}{4}$ of the way between 81 and 100. The difference between 81 and 100 is about 20, and $\frac{3}{4}$ of 20 is about 15, so a good estimate might be 80 + 15 = 95. The actual value is approximately 94.09.

   b) 11.4 is between 11 and 12, so 11.42 would be a little less than half way between 121 and 144. A good estimate might be 121 + 10 = 131. The actual value is approximately 129.96.

   c) 5.2 is between 5 and 6, so 5.22 would be about $\frac{1}{4}$ of the way between 25 and 36. The difference between 25 and 36 is about 10, and $\frac{1}{4}$ of 10 is about 2, so a good estimate might be 25 + 2 = 27. The actual value is approximately 27.04.

Identify the Error

A friend has asked you to help him with his math. He wrote that the positive square root of 6.4 is 3.2. What was his mistake?

Answer:

Your friend simply divided by 2, or found half of the number. To find square root, he must see that 6.4 is between PSNs 4 and 9, whose roots are 2 and 3. So, his answer should be between 2 and 3!

NOTES

GRADE 9 MATHEMATICS (10F)

Module 3: Powers

MODULE 3: POWERS

Introduction

In the last module, you learned about working with decimals. In this module, you will learn about working with powers. Powers are numbers or variables with exponents. You started learning about powers in Grade 8. Remember that 4 to the power of 2 is written as 4^2. It does not mean 4 multiplied by 2. Rather, it means 4 multiplied by itself. 4^2 can be written as 4 x 4, or 16.

Powers are interesting and important. For example, if you are studying astronomy in Grade 9 Science, you will use powers to describe really long distances, like from here to the sun. Here is a riddle that demonstrates how powers can help you work with large numbers.

Pretend that you have a wealthy relative who wants to give you some money every day for a month – 31 days. But, he tells you to choose one of the following two options, and gives you only a minute to make your choice.

- Option A: Your relative will give you $1000 per day for 31 days.
- Option B: Your relative will give one penny on the first day, double it the next day, and continue doubling it every day for 31 days.

You want to pick the option that will give you the most money. Option A is easy to calculate, but how do you calculate Option B? Later in this module, you will learn how to use exponents to do just that. And, the answer will surprise you.

Assignments in Module 3

You will need to complete the following six assignments and send them to your tutor/marker when you are done this module.

Lesson	Assignment Number	Assignment Title
1	Assignment 3.1	Cover Assignment
2	Assignment 3.2	The Meaning and Uses of Powers
3	Assignment 3.3	Order of Operations
4	Assignment 3.4	Operations with Powers
5	Assignment 3.5	Powers with Like Bases
6	Assignment 3.6	More Power Laws

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 3. You might use your Module 3 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 3 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 3 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

NOTES

Lesson 1: Cover Assignment

The study of geometry began long ago in Egypt and Babylonia. Long ago mathematicians took measurements and applied procedures that seemed to give correct answers to practical problems such as dividing up land and building canals. They used **inductive reasoning** to solve many of these everyday problems.

Inductive reasoning is the process of observing data, recognizing patterns, and making generalizations from your observations.

You have had some experience with inductive reasoning in the past. This kind of reasoning is very important for geometry and most scientific inquiry. When mathematicians or scientists use inductive reasoning, they start with a **generalization**, sometimes called a **conjecture**.

Example 1

Look at the pattern of numbers and make a conjecture about the next three numbers in each sequence.

a) 1, 2, 4, 8, 16, 32, . . .

b) 1, 5, 14, 30, 55, 91, . . .

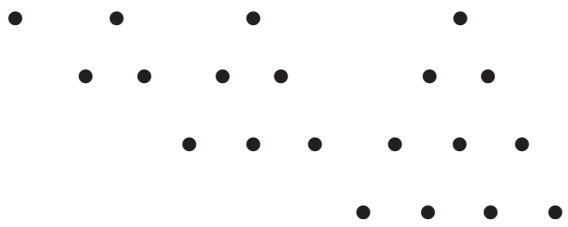
Solutions:

a) The next three numbers are 64, 128, and 256. The pattern is that each number is twice the previous number.

b) The next three numbers are 140, 204, 285. Each number is obtained by adding the next square number, $1 + \boxed{4} = 5$, $5 + \boxed{9} = 14$, $14 + \boxed{16} = 30$, etc.

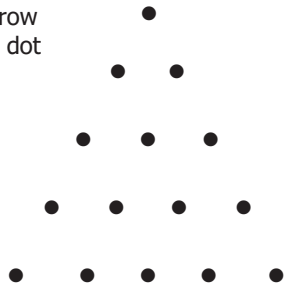
Example 2

Draw the next shape in the pattern below.



Solution:

add a bottom row with one more dot



Example 3

Find an expression for the value in the sequence in terms of its position, n.

Position	1	2	3	4	5	...	n
Value	5	6	7	8	9	...	?

Solution:

Each term is arrived at by adding 4 to the position. Therefore the value of the term n will be $n + 4$.

Assignment 3.1: Cover Assignment

Total: 21 marks

Show your calculations or indicate the strategy you used in each of the following questions.

1. Fill in the blanks with the missing sequences below. *(4 marks)*

   a) 0, 10, 21, 33, 46, 60, _____ , _____ , ... *(1 mark)*

   b) $\frac{1}{2}$, 9, $\frac{2}{3}$, 10, $\frac{3}{4}$, 11, _____ , _____ , ... *(1 mark)*

   c) 1, 1, 2, 3, 5, 8, 13, _____ , _____ , ... *(1 mark)*

   d) $1 + (9 \cdot 0) = 1$ *(1 mark)*

   $2 + (9 \cdot 1) = 11$

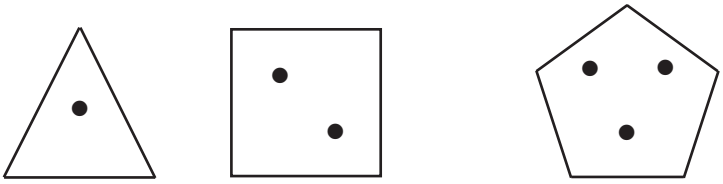
   $3 + (9 \cdot 12) = 111$

   $4 + (9 \cdot 123) = \square$

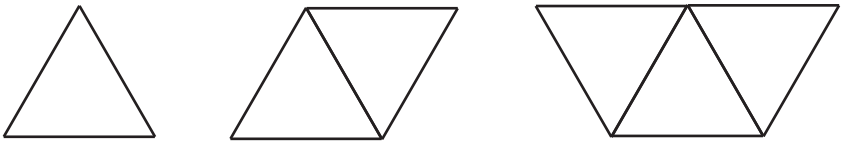
   $\square = 11111$

2. Describe the next shape in the picture patterns shown. You may use a drawing as part of your description. *(2 marks)*

a)



b)



3. Find an expression for the value of the sequence in terms of the term position, *n*. *(2 marks)*

a)

term position	1	2	3	4	5	...	*n*
value of term	1	4	9	16	25	...	

b)

term position	1	2	3	4	5	...	*n*
value of term	0	3	8	15	24	...	

4. Complete the table below. Make a conjecture about the value of the *nth* term. *(2 marks)*

1	2	3	4	5	6	7	8	*n*
2	6	12	20	30	42	56	?	

5. What is the sum of the first 4000 odd numbers? Show your strategy. *(3 marks)*

6. How much would 2.6 kg of hamburger cost if it is sold for $1.58/kg?
(1 mark)

7. Find the Lowest Common Multiple of 10, 12, and 14. Show your strategy.
(2 marks)

8. a) Find all possible digits for *a* that would make 519*a* divisible by 3.
(2 marks)

b) Find all possible digits for *a* that would make 519*a* divisible by 11.
(2 marks)

9. The speed of a train, in km/h, can be found using the formula

$$s = \frac{c}{2}$$

where c is the number of "clicks" you hear in 25 s. If you hear 124 clicks in 25 s, how fast is the train going? *(1 mark)*

Remember, you will forward this assignment to the tutor/marker at the end of Module 3.

NOTES

LESSON 2: THE MEANING AND USES OF POWERS

Lesson Focus

When you finish this lesson, you will be able to

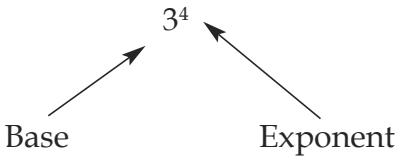
- ❑ identify powers, the base, and exponent
- ❑ express a given power as a repeated multiplication
- ❑ express a given repeated multiplication as a power
- ❑ demonstrate why a number with an exponent of 0 equals 1

Lesson Introduction

Powers can be used to express very large or very small numbers that are the result of repeated multiplications. In this lesson, you will learn how to write powers in exponential and expanded form, and apply powers to scientific and financial situations. You will explore patterns in powers, and determine the first of six "power laws."

Terminology

Study the following example, and identify each part.



3^4 together is called a power

Many people get confused by the terms base and exponent, so you should include this diagram on your Resource Sheet (at least until you are comfortable with these terms).

Powers are used as a shortcut, or as a symbolic way to indicate repeated multiplication. 3^4 means that 3 is multiplied by itself four times or $3 \times 3 \times 3 \times 3$.

Brackets Make a Difference

It's important to note where the negative sign is placed. If it is included inside the bracket, then it is part of the base and is included in each repeated multiplication.

Example 1

Evaluate $(-3)^4$.

Solution:

$(-3)^4$ means $(-3) * (-3) * (-3) * (-3)$

$= 81$

The base is –3, and the exponent is 4.

If the negative sign is not included inside a bracket, then it is not included in the multiplication.

Example 2

Evaluate -3^4.

Solution:

-3^4 means $-(3 * 3 * 3 * 3)$

$= -81$

Because the negative sign is not inside a bracket, the base is 3. After the power is calculated, take the opposite of the answer or multiply the answer by –1.

Learning Activity 3.1

Identifying Components

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. When rolling a standard six-sided die, what is the probability of rolling a 3?
2. Calculate $\sqrt{81}$.
3. You save $900 each month for five months. How much money have you saved?
4. Estimate $\sqrt{37}$.
5. Convert 127% to a decimal.
6. Calculate $\frac{2}{3}-\frac{4}{3}$.
7. Calculate 6^2.
8. Will $\frac{1}{4}-\frac{1}{5}$ be positive or negative?

continued

Learning Activity 3.1: Identifying Components (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Identify each base, exponent, and power.

Term	Base	Exponent	Power
2^5			
$(-4)^3$			
26^7			
-3^6			

Expanded Form

10^3 is in exponential form and it means 10 is multiplied by itself three times. You could also say "10 is a factor 3 times."

This can be written in expanded form as 10 x 10 x 10.

Learning Activity 3.2

Using Forms

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You need to save $1000 for a trip. You have five months before the trip. How much do you need to save on average each month?
2. Calculate $3 + 4 \times 2$.
3. Calculate $\frac{3}{7} \cdot \frac{2}{7}$.
4. Find 10% of 50.
5. Find 5% of 50.
6. Find 15% of 50.
7. Is $\sqrt{42}$ greater or less than 7?
8. Calculate $\sqrt{1600}$.

continued

Learning Activity 3.2: Using Forms (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write these powers out in the expanded form:
   a) 2^5
   b) 4^9
   c) -2^4
   d) $(-3)^5$

2. Given the expanded form, write the term in exponential form:
   a) $5 * 5 * 5 * 5 * 5 * 5$
   b) $-3 * -3 * -3$ (Be careful here. Ask yourself, "What is the base?")
   c) $6 * 6 * 6 * 6$

Using Technology

Refer back to Module 2, Lesson 5 for the keystrokes your calculator uses to square numbers. Exponents may work in a similar way. You will likely use one of the following keys to find the value of a power:

y^x x^y ^

These buttons instruct the calculator to expand the power as a repeated multiplication.

For example, 3^4 may be entered into the calculator as:

[3] [y^x] [4] [=] or [3] [^] [4] [ENTER]

which really means $3 * 3 * 3 * 3$.

You should also find the bracket keys, as you will need them too.

( )

If you are using a spreadsheet program, make sure to start each cell entry with the = sign to indicate you are entering a formula.

Calculators are all designed slightly differently from each other, so you will need to experiment with yours to learn how it works. Practice with the following powers. Calculate them using technology to ensure you get the correct answers.

a) $2^3 = 8$ b) $4^5 = 1024$

c) $-3^6 = -729$ d) $(-2)^8 = 256$

Some calculators are programmed to perform "implied multiplication" and the correct order of operations. Check if your calculator does this.

Type in 2(3) on your calculator.

Some calculators display the bracket symbol when it is pressed, and some do not.

Does the answer show 6? If not, you will have to remember to use the multiplication key: $2 * 3$.

Type in 2((24–6)/9).

Does the answer show 4?

If yes, then your calculator is programmed to use "implied" multiplication using brackets.

Type in 2+3*4.

Does the answer show 14?

If yes, then your calculator automatically performs the order of operations.

Where Are Powers Used?

Science

When scientists study bacteria they use mathematical ideas about exponents to help them accurately express the value of very large numbers. Bacteria are grown in the laboratory in "gardens '' or "cultures '' in small petri dishes. Bacteria grow through a process of cell division, where each cell divides itself into two more cells. Each of those cells divide into two more cells. In this way, bacterial growth keeps doubling, and is "exponential. '' By counting the number of cells over a period of time, the scientists can learn about bacterial growth under certain conditions.

The table below records the number of bacteria observed over 4 hours. The scientists started with 500 cells.

Time	Number of Bacteria
08:00	500
09:00	500 x 2 = 1000
10:00	(500 x 2) x 2 = 2000
11:00	(500 x 2 x 2) x 2 = 4000
12:00	(500 x 2 x 2 x 2) x 2 = 8000

After 24 hours of growth at this rate, the bacterial count would reach:

500 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 or 8 388 608 000 cells!!

This calculation is very large, so scientists write it using a power:

$$500 \times 2^{24}$$

Can you see how the exponent means that the base of 2 is used as a factor 24 times? The 500 represents the original number of cells.

Money Question

Do you remember the riddle in the module introduction? It went like this:

A wealthy relative wants to give you some money every day for 31 days, and gives you only a minute to choose one of the following options:

- Option A: Your relative will give you $1000 per day for 31 days.
- Option B: Your relative will give one penny on the first day, double it the next day, and continue doubling it every day for 31 days.

Many people chose Option A because they multiply $ 1 000 by 31 and conclude that they will receive a total of **$31 000**. That's pretty easy to calculate. But how can you calculate the total for Option B?

The easiest way to do that is to create a chart and calculate the amount for each day, then totalling the amounts for all 31 days. That's what we did in the chart on the following page.

- The column on the left lists the days, from day 1 to day 31.
- The column in the middle shows the exponent that you can use to calculate the amount for each day.
- The column on the right shows the amount of money that your wealthy relative will give you.

So, on Day 1, he gives you one cent, which is $0.01. The next day, he gives you two cents, or $0.02, and the number doubles every day for 31 days. It's easy to calculate the amount for the first few days, but it keeps getting harder with each day. So, by the time you get to Day 31, it is very difficult to keep track of the number of times that you have multiplied the number '2'.

Note: On Day 1, you didn't multiply by 2 because you started with $0.01. On Day 2, you multiplied by 2 for the first time. By Day 31, you have only multiplied by 2 thirty times. Check your answer using the chart on the next page.

Or, instead of creating the chart, you could use your calculator to find 2^{30}. Try it on your calculator: .01 * 2 ^ 30 = $10 737 418.24.

Day	Exponent	Amount
1	$.01 * 2^0$	$0.01
2	$.01 * 2^1$	$0.02
3	$.01 * 2^2$	$0.04
4	$.01 * 2^3$	$0.08
5	$.01 * 2^4$	$0.16
6	$.01 * 2^5$	$0.32
7	$.01 * 2^6$	$0.64
8	$.01 * 2^7$	$1.28
9	$.01 * 2^8$	$2.56
10	$.01 * 2^9$	$5.12
11	$.01 * 2^{10}$	$10.24
12	$.01 * 2^{11}$	$20.48
13	$.01 * 2^{12}$	$40.96
14	$.01 * 2^{13}$	$81.92
15	$.01 * 2^{14}$	$163.84
16	$.01 * 2^{15}$	$327.68
17	$.01 * 2^{16}$	$655.36
18	$.01 * 2^{17}$	$1 310.72
19	$.01 * 2^{18}$	$2 621.44
20	$.01 * 2^{19}$	$5 242.88
21	$.01 * 2^{20}$	$10 485.76
22	$.01 * 2^{21}$	$20 971.52
23	$.01 * 2^{22}$	$41 943.04
24	$.01 * 2^{23}$	$83 886.08
25	$.01 * 2^{24}$	$167 772.16
26	$.01 * 2^{25}$	$335 544.32
27	$.01 * 2^{26}$	$671 088.64
28	$.01 * 2^{27}$	$1 342 177.28
29	$.01 * 2^{28}$	$2 684 354.56
30	$.01 * 2^{29}$	$5 368 709.12
31	$.01 * 2^{30}$	$10 737 418 24
Total for 31 days		**$21 474 836.47**

Are you surprised at the answer? You may have thought that Option A was the best, but you would end up with only $ 31 000. With Option B, you end up with over $ 21 000 000! This riddle shows the power of exponents.

What if the Exponent is 0?

Study the patterns in this example:

$2^4 = 2 * 2 * 2 * 2 = 16$

$2^3 = 2 * 2 * 2 = 8$ ÷ 2

$2^2 = 2 * 2 = 4$

$2^1 = 2 = 2$

$2^0 = = ?$

In the first column the exponent is decreasing by 1 each time. In the middle column there is one less factor of two being multiplied. In the final column, the answer keeps getting divided by 2. So, if this pattern is continued 2^0 has to equal $\frac{2}{2}$ or 1.

Try a different number:

$6^4 = 6 * 6 * 6 * 6 = 1296$

$6^3 = 6 * 6 * 6 = 216$

$6^2 = 6 * 6 = 36$

$6^1 = 6 = 6$

$6^0 = = ?$

Each answer is the previous value divided by 6, since 6 is used as a factor one less time for each step.

So, the final value would be $\frac{6}{6} = 1$.

$6^0 = 1$

This is the Zero Exponent law: Any base with an exponent of zero = 1.

$4369^0 = 1$ $(-2)^0 = 1$

It is easy to forget the Zero Exponent Law when you first learn it, so include it on your Resource Sheet.

Lesson Summary

In this lesson, you identified the base and the exponent in a power. You expressed powers as repeated multiplications and in exponential form, and applied them in real-world situations. You used patterns to demonstrate the first power law that states any number with an exponent of 0 equals 1. In the next lesson, you will review the correct order of operations to use when simplifying and solving expressions that include exponents.

Assignment 3.2: The Meaning and Uses of Powers

Total: 26 marks

1. Identify the base in each term. *(4 marks)*
   a) 2^5 _____ *(1 mark)*
   b) -3^4 _____ *(1 mark)*
   c) 26^{19} _____ *(1 mark)*
   d) $(-2)^6$ _____ *(1 mark)*

2. Identify the exponent in each term. *(4 marks)*
   a) 9^8 _____ *(1 mark)*
   b) -4^5 _____ *(1 mark)*
   c) 22^{23} _____ *(1 mark)*
   d) $(-5)^6$ _____ *(1 mark)*

3. Write each power in expanded form to show its meaning. *(4 marks)*
   a) 3^6 _____ *(1 mark)*
   b) $(-2)^4$ _____ *(1 mark)*
   c) -5^3 _____ *(1 mark)*
   d) 6^5 _____ *(1 mark)*

4. Given each expanded form, write it as a power. *(4 marks)*
   a) $3 * 3 * 3 * 3$ _____ *(1 mark)*
   b) $-4 * 4 * 4 * 4 * 4$ _____ *(1 mark)*
   c) $(-2) * (-2) * (-2)$ _____ *(1 mark)*
   d) $1 * 1 * 1 * 1 * 1 * 1 * 1$ _____ *(1 mark)*

5. Use your calculator to find the value of each power. *(4 marks)*

   a) 2^9 _____ *(1 mark)*

   b) 3^8 _____ *(1 mark)*

   c) $(-5)^6$ _____ *(1 mark)*

   d) -6^4 _____ *(1 mark)*

6. Write each power in expanded form and calculate. State which has the larger value. *(2 marks)*

   a) 2^3 or 3^2 *(1 mark)*

   b) 4^5 or 5^4 *(1 mark)*

7. Use patterns to demonstrate how $3^0 = 1$. *(2 marks)*

8. A snowball with a diameter of 20 cm rolling down a hill will double its diameter every 4 seconds. If the snowball rolled for 12 seconds, find its diameter. *(2 marks)*

LESSON 3: ORDER OF OPERATIONS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ apply the order of operations to solve problems
- ❑ solve problems without using technology
- ❑ solve problems using technology

Lesson Introduction

The order in which the steps in mathematical problems are solved is constant across the world. Many calculators and spreadsheet programs are designed to do this automatically, but you still need to understand the process to ensure your mental calculations are correct. In this lesson, you will learn the correct order of operations to perform when your calculations include powers.

Order of Operations

In earlier mathematics courses, you worked with the order of operations. Remember, brackets or parentheses are done first because they are a mathematical notation to indicate priority. In earlier grades, you performed multiplication and division in order from left to right and then addition and subtraction from left to right. Now, we need to deal with exponents.

Since exponents represent repeated multiplication, exponents need to be dealt with before other operations.

You have just won a prize—but to claim it, you must first correctly answer a skill-testing question.

The question is: $(5 - 3) * 4^2 + 8 \div 2$

Which is the correct answer?

a) $64 + 4 = 68$

b) $5 - 12^2 + 4 = -135$

c) $2 * 24 \div 2 = 24$

d) $2 * 16 + 4 = 36$

There is only one correct answer and it can only be found by following the correct order of operations.

So the correct answer to the skill testing question is (d).

$(5 - 3) * 4^2 + 8 \div 2 = (2) * 16 + 4$

$= 32 + 4$

$= 36$

Example 1

Evaluate: $(2 + 3^2) - 4$

Solution:

The operations inside the brackets must be simplified first. In this case, the exponent must be done before the addition.

$(2 + 3^2) - 4$	Simplify inside the bracket first, beginning with the exponent
$= (2 + 9) - 4$	Do the addition inside the bracket.
$= 11 - 4$	Then do the subtraction.
$= 7$	

Example 2

Evaluate: $5^3 - (2 + 6 * 3) - 4^2$

Solution:

$= 5^3 - (2 + 18) - 4^2$	Begin inside the bracket with the multiplication and then the addition.
$= 5^3 - (20) - 4^2$	
$= 125 - (20) - 16$	Apply the exponents.
$= 125 - 20 - 16$	Now do the subtraction.
$= 89$	

To help you remember the order of operations, you can memorize "BEDMAS" or "Please Excuse My Dear Aunt Sally." Each upper case letter refers to the operation in the correct order:

Brackets or **P**arentheses

Exponents

Division or **M**ultiplication in the order they appear

Addition or **S**ubtraction in the order they appear

Include the order of operations on your Resource Sheet. You can include both or choose one. You may also want to include an example.

Learning Activity 3.3

Applying Order of Operations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Rewrite in exponential form: $6 \cdot 6 \cdot 6 \cdot 6$.
2. Is 5.67 x 105.32 closer to 50 or 500?
3. What two numbers have a sum of 5 and a product of 6?
4. If the radius of a circle is 4 cm, what is the diameter?
5. You have to leave for work in 3 hours. You want to watch a movie that is 150 minutes long. Do you have enough time?
6. Which is larger: 0.605 or 0.65?
7. Arrange the following numbers in order from smallest to largest:

   $-\frac{3}{4}$ $\quad -0.45$ $\quad -\frac{4}{3}$ $\quad -0.54$

8. Is $\sqrt{67}$ greater or smaller than 8?

continued

Learning Activity 3.3: Applying Order of Operations (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Simply these expressions following the order of operations, and explain each step.

   a) $\dfrac{\left(3^2 - 6 * 4\right)}{5}$

   b) $16 + (12 + 5^2) - 4^3$

2. Simplify these expressions following the order of operations, and show each step.

   a) $4 * (2^3 - 5)$

   b) $6 + (2^4 - 4^2) + 9^0$

   c) $4 + (3 - 2^3) - (5^3 - 5^2)$

3. Use the correct order of operations to simplify these expressions.

   a) $4^4 - 16 * 3 + 8$

   b) $12 + 6^3 * (-2)$

   c) $53 - \dfrac{8^2}{8^2} + 6 * (-4)$

Brackets within Brackets

Whenever you have a question with two or more sets of brackets, you must work from the "inside out."

Example 1

Evaluate: $[3 * (2 + 3 * 4)]$

Solution:

Simplify the inner bracket first, doing the multiplication before the addition:

$=[3 * (2 + 12)]$

$=[3 * 14]$

Now that the inner bracket is simplified, work with the outer bracket.

$= 42$

Example 2

Evaluate: $[(6 - 2^2) - (4^3 - 5 * 4)]$

Solution:

This problem looks difficult, but you can solve it correctly by following the established order of operations.

Step 1: Simplify exponents within each bracket.

$=[(6 - 4) - (64 - 5 * 4)]$

Step 2: Simplify any multiplication or division within each bracket.

$=[(6 - 4) - (64 - 20)]$

Step 3: Simplify each inner bracket.

$=[2 - 44]$

Step 4: Simplify the outer bracket.

$= -42$

Double Brackets and Technology

Using a calculator or a spreadsheet to perform calculations involving two or more brackets is not difficult. You just have to be very careful with how many brackets you use, and where they are placed.

Some calculators do not have the square brackets [ ], so you will use the round brackets ( ) instead.

Example

Evaluate: [(5 – 2) – 4(2 + 3)]

Solution:

If the question reads [(5 – 2) – 4(2+3)] and you only have round brackets on your calculator, this is how you would enter the data:

((5 – 2) – 4(2 + 3))

Note that the number of brackets that open must equal the number of brackets that close.

Enter the above question into your calculator, carefully using the brackets. If everything is entered properly, your display should show an answer of -17.

This method works exactly the same on a spreadsheet. Remember to start the spreadsheet formula with the = sign.

Learning Activity 3.4

Using Technology

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What two numbers have a product of 18 and a sum of 9?
2. Rewrite as a product: $(-3)^5$.
3. What is the volume of a swimming pool that measures 7 m by 3 m by 2 m?
4. What is the cost of 8 bottles of water if each costs $0.99?
5. Is $\sqrt{90}$ greater than or less than 9?
6. Evaluate $5^2 - 3^2$.
7. When rolling a standard six-sided die, what is the probability of rolling a number that is a multiple of 3?
8. Will $\frac{1}{2} - \frac{1}{3}$ be positive or negative?

continued

Learning Activity 3.4: Using Technology (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use technology to simplify each expression.

a) $\left[\frac{\left(6^2+4^3\right)}{5^2}\right]$

b) $[3^4 - (5^3 + 4^2)]$

c) $[8^3 - (6^2 - 4^3) + (5^3 + 3^5)]$

Identify the Error

A classmate had this work marked wrong, and he couldn't see his error. Identify the error, and correct it.

$[4 * (2^3 - 5)]$

$= [4 * (8 - 5)]$

$= [32 - 5]$

$= 27$

Lesson Summary

In this lesson, you learned that expressions must be simplified following an established order of operations, with and without technology. This order will be followed for all calculations in this course, including operations on powers, which is the topic of the next lesson.

Assignment 3.3: Order of Operations

Total: 24 marks

1. Simplify each expression, showing at least 2 steps of each solution. *(8 marks)*

   a) $6 * (4 + 12)$ *(2 marks)*

   b) $(3 - 5 * 3) - 1$ *(2 marks)*

   c) $3^3 - (4^2 + 3^2)$ *(2 marks)*

   d) $125 - (2^4 - 4^2)$ *(2 marks)*

2. Using technology, simplify the following expressions. *(4 marks)*

   a) $18 - (6 + 7 * 4)$ _____ *(1 mark)*

   b) $(18^2 - 4^5) \div 10^2$ _____ *(1 mark)*

   c) $(3^2 - 4^2) - 2^3$ _____ *(1 mark)*

   d) $17^3 + (3^{10} * 2^3)$ _____ *(1 mark)*

3. Simplify each expression, showing at least 2 steps of each solution. *(8 marks)*

a) $[(6 - 4^3) - (5^2 + 3^4)]$ *(2 marks)*

b) $[8 - (6^2 + 4^3) + (8^2 + 4^3)]$ *(2 marks)*

c) $[(4^2 + 5^3) - 4(3 + 2^3)]$ *(2 marks)*

d) $\left[\left(\frac{5^3}{5^2} - \left(3^2\right)^0\right)\right]$ *(2 marks)*

4. Using technology, simplify the following expressions. *(4 marks)*

a) $[(7^3 + 3^7) - (6^4 - 4^6)]$ _____ *(1 mark)*

b) $\left[\left(\frac{8^4}{4^3}\right) - \left(2^3 * 2^3\right)\right]$ _____ *(1 mark)*

c) $[(164 - 5^2) + (4^2 * 3)]$ _____ *(1 mark)*

d) $\left[\frac{\left(3^4 + 3^2\right)}{9} + \left(\frac{6^2}{2^2}\right)\right]$ _____ *(1 mark)*

LESSON 4: OPERATIONS WITH POWERS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ perform basic operations on powers with like and unlike bases
- ❑ identify errors made in basic calculations

Lesson Introduction

The next two lessons will show you the correct order of operations to use when simplifying powers. First, you will look at adding and subtracting powers with like and unlike bases. Next, you will learn how to multiply and divide powers with unlike bases. In Lesson 5, you will learn how to multiply and divide powers with like bases.

Adding or Subtracting Powers with Like and Unlike Bases

When adding or subtracting powers with like or unlike bases, the first step is to apply the exponents, or find the value of each power, then perform the operation. An exponent means repeated multiplication so it must be done before addition or subtraction.

Because there are a lot of rules that you will have to remember for the different modules in this course, you should include how to add/subtract powers on your Resource Sheet. You may want to include an example as well.

Example 1

Evaluate: $4^3 + 4^4$

Solution:

This example has like bases.

This means $4 * 4 * 4 \quad + \quad 4 * 4 * 4 * 4$

The proper order of operations is to do the multiplication before the addition:

$= 4 * 4 * 4 \quad + \quad 4 * 4 * 4 * 4$

$= 64 \quad + \quad 256$

$= 320$

Example 2

Evaluate: $3^4 - 3^2$

Solution:

This means $3 * 3 * 3 * 3 \quad - \quad 3 * 3$

The proper order of operations is to do the multiplication before the subtraction:

$= 3 * 3 * 3 * 3 \quad - \quad 3 * 3$

$= 81 \quad - \quad 9$

$= 72$

Example 3

Evaluate: $2^3 + 3^4$

Solution:

This example has unlike bases.

This means $2 * 2 * 2 \quad + \quad 3 * 3 * 3 * 3$

Apply the exponents first and find the value of each power.

$2 * 2 * 2 \quad + \quad 3 * 3 * 3 * 3$

$= 8 \quad + \quad 81$

$= 89$

Example 4

Evaluate: $4^3 - (-2)^4$

Solution:

Carefully check the base, and apply the exponent.

$= 4^3 \quad - \quad (-2)^4$

$= 4 * 4 * 4 \quad - \quad (-2) * (-2) * (-2) * (-2)$

$= 64 \quad - \quad 16$

$= 48$

Example 5

Evaluate: $2^7 - (-3)^3$

Solution:

$= 128 - (-27)$ (the opposite of negative 27 is positive 27)

$= 128 + 27$

$= 155$

Remember, when the negative sign is inside the bracket, it is included in the multiplication! Watch what happens with a negative base if the exponent is an odd or even number.

$(-3)^3 = (-3) * (-3) * (-3)$ negative base, odd exponent

$= -27$

$(-3)^4 = (-3) * (-3) * (-3) * (-3)$ negative base, even exponent

$= +81$

Multiplying or Dividing Powers with Unlike Bases

Multiplying and dividing powers with unlike bases requires the same steps as addition and subtraction of powers. First, apply the exponent and calculate each term, and then perform the required operation.

You should include how to multiply/divide powers on your Resource Sheet, especially because it is different than adding. You may choose to include an example.

Example 1

Evaluate: $5^3 * 2^6$

Solution:

$= (5) * (5) * (5)$ x $(2) * (2) * (2) * (2) * (2) * (2)$

$= 125$ x 64

$= 8000$

Example 2

Evaluate: $6^5 \div 3^4$

Solution:

$= (6) * (6) * (6) * (6) * (6) \quad \div \quad (3) * (3) * (3) * (3)$

$= 7776 \quad \div \quad 81$

$= 96$

Learning Activity 3.5

Problems with Unlike Bases

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert 33.5% to a decimal.
2. Which is larger: 2^3 or 3^2?
3. Estimate $\sqrt{56}$.
4. What two numbers have a sum of 5 and a product of 4?
5. How many centimetres in 567 mm?
6. At your part-time job, you make $7.50 an hour. You worked 4 hours today. How much money did you earn?
7. Calculate –7 + (–4) + 5.
8. Your laundry needs 57 minutes to wash and 43 minutes for drying and folding. You need to leave the house in 1.5 hours. Do you have enough time to finish a load of laundry?

continued

Learning Activity 3.5: Problems with Unlike Bases (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Do the exponent calculations first, then add, subtract, multiply, or divide as indicated.
   a) $4^5 - 3^3$
   b) $2^4 * 3^2$
   c) $\frac{4^2}{2^3}$
   d) $5^6 + 4^5$
   e) $(-2)^3 - (-3)^3$
2. Express each power as a repeated multiplication, and find the value.
   a) $3^4 + 4^3$
   b) $5^3 - 5^2$
   c) $2^4 - 3^4 * 2^3$
   d) $\frac{(3^3 * 3)}{3^2}$

Identify the Error

A classmate shows you her work, and asks for your help showing her where she made an error. Can you identify the mistake made in each question?

1. $2^3 + 4^2$
   $= 6 + 8$
   $= 14$

2. $2^3 * 5^2$
   $= 10^6$
   $= 1\,000\,000$

Lesson Summary

This lesson showed you how to apply the correct order of operations when adding or subtracting powers with like or unlike bases, as well as multiplying or dividing powers with unlike bases only. In the next lesson, you will learn some shortcuts for multiplying and dividing powers with like bases.

Assignment 3.4: Operations with Powers

Total: 20 marks

1. Add the following powers. *(4 marks)*
   a) $5^3 + 8^2$ _____ *(1 mark)*
   b) $6^4 + 7^3 + 2^5$ _____ *(1 mark)*
   c) $9^3 + 2^4 + 5^4$ _____ *(1 mark)*
   d) $25^4 + 6^5$ _____ *(1 mark)*

2. Subtract the following powers. *(4 marks)*
   a) $6^5 - 3^3$ _____ *(1 mark)*
   b) $4^7 - 6^4$ _____ *(1 mark)*
   c) $4^8 - 6^3 - 2^4$ _____ *(1 mark)*
   d) $16^4 - 12^3$ _____ *(1 mark)*

3. Multiply the following powers. *(4 marks)*
   a) $3^5 * 6^2$ _____ *(1 mark)*
   b) $5^3 * (-2)^4$ _____ *(1 mark)*
   c) $(-4)^5 * (-2)^3$ _____ *(1 mark)*
   d) $6^2 * 3^5 * (-1)^4$ _____ *(1 mark)*

4. Divide the following powers. *(4 marks)*
   a) $\frac{2^8}{4^2}$ _____ *(1 mark)*
   b) $\frac{6^4}{3^4}$ _____ *(1 mark)*
   c) $\frac{5^3}{5^2}$ _____ *(1 mark)*
   d) $\frac{8^3}{2^8}$ _____ *(1 mark)*

5. Identify the error made, and correct the answer. *(4 marks)*

a) $3^3 + 4^2$ *(2 marks)*

$= 7^5$

$= 16807$

b) $\frac{10^6}{2^3}$ *(2 marks)*

$= 5^2$

$= 25$

LESSON 5: POWERS WITH LIKE BASES

Lesson Focus

When you finish this lesson, you will be able to

- ❑ apply an exponent law when multiplying powers with like bases
- ❑ apply an exponent law when dividing powers with like bases
- ❑ identify the error in calculations, and make corrections

Lesson Introduction

The first exponent law you learned is that any power with an exponent of 0 equals 1. In this lesson, you will examine patterns in multiplying and dividing powers with like bases to come up with two more exponent laws. They will make it faster and easier to multiply and divide powers with like bases.

Product Law

As you saw in the previous lesson, there are several steps involved in adding or subtracting powers and in multiplying and dividing powers with unlike bases. It is possible to do calculations with powers using some basic rules that will reduce the amount of work required. In this lesson, you will learn to use two exponent laws that will help you when multiplying or dividing powers with like bases.

$5^6 * 5^7$ like bases: both 5

You can use an exponent law to simplify the process

$4^3 * 6^2$ unlike bases: 4 and 6

You cannot use any exponent law with unlike bases

The product law states: When you multiply powers with like bases, **add** the exponents.

$3^5 * 3^4 = (3 * 3 * 3 * 3 * 3) * (3 * 3 * 3 * 3)$

Include the product law on your Resource Sheet. You may want to include that it is true for *only* powers with like bases.

In expanded form, you can see that 3 is a factor nine times. This can be written as 3^9.

When you apply the product law, simply add the exponents when multiplying like bases.

$$3^5 * 3^4 = 3^{(5+4)}$$

$$= 3^9$$

Remember, you must be multiplying **like** bases to use this exponent law.

Learning Activity 3.6

Using the Product Law

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What two numbers have a sum of –8 and a product of –20?
2. Estimate $\sqrt{102}$.
3. Is $\frac{2}{3} - \frac{3}{4}$ positive or negative?
4. Which is larger: $\frac{2}{7}$ or $\frac{2}{5}$?
5. If the diameter of a circle is 14 cm, what is the radius?
6. Write two equivalent fractions for $\frac{9}{12}$.
7. Solve for d: $5d = 125$.
8. You buy 3 items at the drugstore that total $3.52 after taxes. You give the cashier $5.02. How much change do you get back?

continued

Learning Activity 3.6: Using the Product Law (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the product exponent law to simplify (not solve) these powers, if possible.

a) $5^8 * 5^6$

b) $7^4 * 7^3 * 7^5$

c) $5^3 * 4^2$

d) $(-2)^7 * (-2)^2$

Quotient Law

The quotient law states: When dividing powers with like bases, **subtract** the exponents.

Include the quotient law on your Resource Sheet. You may select an example to include as well.

In expanded form, you can see what can be simplified.

$$\frac{3^5}{3^3} = \frac{3 * 3 * 3 * 3 * 3}{3 * 3 * 3}$$

You know that $\frac{3}{3} = 1,$ so each time you have $\frac{3}{3}$, they can be simplified to 1.

So, $\frac{\cancel{3} * \cancel{3} * \cancel{3} * 3 * 3}{\cancel{3} * \cancel{3} * \cancel{3}}$ leaves us with

$= 1 * 1 * 1 * 3 * 3$ or simply $3 * 3$ which can be written as

$= 3^2$

When you apply the quotient law, simply subtract the exponents.

$$\frac{3^5}{3^3}$$
$$= 3^{(5-3)}$$
$$= 3^2$$

Remember, you must be dividing **like** bases to use this exponent law.

Learning Activity 3.7

Using the Quotient Law

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Arrange the following numbers from greatest to least:

   $\frac{1}{3}$ $\quad -0.4 \quad$ 0.5 $\quad -1.2$

2. If 14% of 600 is 84, what is 28% of 600?
3. Evaluate $6d - 2$ if $d = -1$.
4. Calculate 5^3.
5. Convert 15 mm to cm.
6. In April, you saved $4.00 each day. How much did you save by the end of April?
7. Solve for n: $7 = n - 3$.
8. Write as an expression with a single exponent: $3^2 \cdot 3^4$.

continued

Learning Activity 3.7: Using the Quotient Law (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the quotient exponent law to simplify (not solve) these powers, if posible.

a) $\frac{5^6}{5^2}$

b) $\frac{8^8}{8^7}$

c) $\frac{9^7}{8^2}$

d) $\frac{(-6)^5}{(-6)^5}$

Identify the Error

A classmate shows you his work, and asks for your help in showing him where he made an error.

a) $4^5 * 4^6$

$= 16^{30}$

b) $\frac{5^8}{5^4}$

$= 5^2$

Find the Value

Using either the product or the quotient exponent laws, simplify the expression, and then find its value.

a) $4^5 * 4^6$

$= 4^{(5+6)}$

$= 4^{(11)}$

$= 4\,194\,304$

b) $\dfrac{(-5)^8}{(-5)^3}$

$= (-5)^{(8-3)}$

$= (-5)^5$

$= -3125$

Mixed Operations

You can now combine the mathematical operations when using powers. Be sure you take note if the bases are like or unlike, and that you follow the proper order of operations:

- brackets or parentheses
- exponents
- multiplication and division (in the order they appear)
- addition and subtraction (in the order they appear)

Example 1

Evaluate: $8^2 + \dfrac{4^5}{4^3}$

Solution:

There are unlike bases in this question, so first you must deal with the exponents, simplifying each power.

$= 64 + 4^2$

$= 64 + 16$ (Now do the addition.)

$= 80$

Example 2

Evaluate: $3^3 * 3^5 - 4^2 * 4^3$

Solution:

Here you are multiplying like bases, but subtracting unlike bases

$=3^{(3+5)} - 4^{(2+3)}$ (Use product exponent law to simplify.)

$=3^8 - 4^5$

$=6561 - 1024$ (Simplify each power before subtracting.)

$= 5537$

Learning Activity 3.8

Solving Mixed Operations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write as an expression with a single exponent: $\frac{6^6}{6^2}$.
2. Calculate $53 \div \frac{1}{2}$.
3. If 15% of 400 is 60, what is 30% of 400?
4. Calculate 5% of 480.
5. What is the perimeter of a rectangular yard that is 50 m by 75 m?
6. Convert $3\frac{2}{3}$ to an improper fraction.
7. Ealuate $3n + 4$ when $n = -2$.
8. You want to survey 50 people. Each survey takes 5 minutes. You have 3 hours to complete the task. Do you have enough time?

continued

Learning Activity 3.8: Solving Mixed Operations (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Perform the indicated mixed operations, and solve.

a) $6^5 * 6^2 - 8^3$

b) $(-4)^3 - \frac{5^6}{5^3}$

c) $\frac{6^8}{6^6} - \frac{7^4}{7^2}$

Lesson Summary

In this lesson, you learned two short cuts—the product and quotient exponent laws, and how to use them when multiplying or dividing powers with like bases. In the next lesson, you will learn about three more exponent laws.

Assignment 3.5: Powers with Like Bases

Total: 20 marks

1. Use the product exponent law to simplify (not solve) the following powers, if possible. *(4 marks)*

   a) $6^3 * 6^4$ _____ *(1 mark)*

   b) $(-2)^7 * (-2)^7$ _____ *(1 mark)*

   c) $3^4 * 3^5 * 3^6$ _____ *(1 mark)*

   d) $2^3 * 3^2$ _____ *(1 mark)*

2. Use the quotient exponent law to simplify (not solve) the following powers, if possible. *(4 marks)*

   a) $\frac{(-7)^{10}}{(-7)^6}$ _____ *(1 mark)*

   b) $\frac{12^4}{6^2}$ _____ *(1 mark)*

   c) $\frac{6^6}{6^6}$ _____ *(1 mark)*

   d) $\frac{2^{20}}{2^{19}}$ _____ *(1 mark)*

3. Find the value of each expression. *(4 marks)*

   a) $3^4 * 3^5$ _____ *(1 mark)*

   b) $(-2)^3 * (-2)^4$ _____ *(1 mark)*

   c) $\frac{(-6)^5}{(-6)^3}$ _____ *(1 mark)*

   d) $\frac{12^4}{12^4}$ _____ *(1 mark)*

4. Perform the indicated operations, using exponent laws to simplify. Show your steps. *(8 marks)*

a) $2^3 + \frac{4^3}{4^2}$ *(2 marks)*

b) $\frac{10^4}{10^2} - 3^4$ *(2 marks)*

c) $(-3)^3 + (-2)^4 * (-2)^3$ *(2 marks)*

d) $2^3 * 2^4 - 3^2 * 3^4$ *(2 marks)*

LESSON 6: MORE POWER LAWS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ use the power of a power law to simplify expressions
- ❑ use the power of a product law to simplify expressions
- ❑ use the power of a quotient law to simplify expressions

Lesson Introduction

You have learned to add exponents when multiplying powers with like bases, and to subtract exponents when dividing powers with like bases. You know that anything to the exponent 0 = 1. In this lesson, you will learn three more exponent laws dealing with powers that have a product or a quotient as a base, or a base with two or more exponents.

You should include these power laws on your Resource Sheet. You should include an explanation of which law is which so that you don't get them confused. You may want to include an example for each law.

Power of a Power Law

So far, you have worked with powers that have integers (positive and negative whole numbers) for the base. What do you do when the base of a power is a power? Or a product? Or a quotient? There is a power law for each of these situations to make simplifying and evaluating powers quick and easy to remember.

> The power of a power law states: When a base has two or more exponents, multiply the exponents.

Example 1

Express $(2^3)^2$ as one exponent, keeping the base the same.

Solution:

Here, the base has two exponents. Write this as a series of repeated multiplications.

$(2^3)^2$	This means 2^3 is multiplied by itself two times. Rewrite this in expanded form.
$= (2^3) * (2^3)$	Now you are multiplying like bases, so add the exponents.
$= 2^6$	Remember, the base remains the same.

The power of a power law states that you can simply multiply the exponents.

$(2^3)^2$

$=(2)^{(3*2)}$

$= (2)^6$

Example 2

Express as a power with one exponent. Keep the base the same.

$((-3)^4)^5$

Solution:

Using the power of a power law, multiply the exponents.

$= (-3)^{(4*5)}$

$= (-3)^{20}$

Example 3

Express as a power with one exponent. Keep the base the same.

$(-4)^6)^3$

Solution:

Note that the base is now 4 since (-4^6) can be written as $4 * 4 * 4 * 4 * 4 * 4$. The negative sign is not part of the base.

Therefore, $(-4)^6)^3 = -4^{18}$.

Example 4

Express the following as one exponent. Keep the base the same.

$[5^{(4)(3)}]^{(2)}$

Solution:

Sometimes different brackets are used to show the various steps. When using brackets, work from the inside the expression toward the outside.

$[5^{(4)(3)}]^{(2)}$

$= [5^{(12)}]^{(2)}$

$=5^{(12*2)}$

$=5^{24}$

Power of a Product Law

> The power of a product law states: When a base is a product, apply the exponents to each factor.

Example 1

Evaluate: $(3 * 4)^2$

Solution:

Here, the base is a product of two integers. Written as repeated multiplication, this means $(3 * 4)(3 * 4)$.

This power can be solved in two ways.

a) If you follow the order of operations, simplify what is inside the brackets first, and then apply the exponent.

$(3 * 4)^2$

$= (12)^2$ Simplify

$12^2 = 144$ Apply the exponent

b) If you follow the power of a product law, apply the exponent to each factor, simplify, and find the value.

$(3 * 4)^2$

$=(3^2 * 4^2)$ Apply the exponent to each factor

$=9 * 16$ Simplify

$=144$ Find the value

You get the same answer as with the order of operations.

Example 2

Apply the power of a product law, but do not solve.

$(6 * 9)^3$

Solution:

The base of this power is the bracket with the product of 6 and 9

$= (6^3 * 9^3)$ Apply the exponent to each factor.

Example 3

Apply the power of a product law, but do not solve.

$(2 * 3 * 4)^5$

Solution:

The base of this power is the bracket with the product $(2 * 3 * 4)$.

$= (2^5 * 3^5 * 4^5)$ Apply the exponent to each factor

Power of a Quotient Law

The power of a quotient law states: When the base is a quotient, apply the exponent to each base in the numerator and denominator.

Example 1

Write as a fraction without exponents: $\left(\frac{3}{4}\right)^2$

Solution:

This power has a base that is a fraction.

Written as repeated multiplication, this becomes:

$$\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}$$

This can be simplified using the power of a quotient law by applying the exponent to both the numerator and the denominator.

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

Example 2

Apply the power of a quotient law, but do not solve.

$$\left(\frac{8}{9}\right)^4$$

Solution:

$$=\left(\frac{8^4}{9^4}\right)$$

More than one exponent law can be used to solve a power.

Example 3

Apply the power of a quotient law first, and then simplify by using the other power laws.

$$\left(\frac{5^4}{5^2}\right)^3$$

Solution:

The power of a quotient law states that you must apply the exponent to numerator and the denominator

$$=\frac{\left(5^4\right)^3}{\left(5^2\right)^3}$$ Simplify this using the power of a power law.

$$=\frac{5^{12}}{5^6}$$ Then use the quotient law.

$$=5^{(12-6)}$$

$$=5^6$$

If you apply the power laws in a different order, you will still end up with the same answer. Try it!

The power of a quotient law can also be used to show why the zero exponent law works. Remember that anything to the power of 0 equals 1.

Example 4

Apply the power of a quotient law first, and then simplify using the other power laws.

$$\left(\frac{6}{6}\right)^3$$

Solution:

$=\frac{6^3}{6^3}$	Apply the power of a quotient law.
$=6^{(3-3)}$	Now simplify using the quotient law.
$=6^0$	According to the zero law, this should equal 1.

Look back at the base you started with: $\left(\frac{6}{6}\right)$ is equal to 1, so an equivalent expression for the original power could be $(1)^3$. One, multiplied by itself any number of times, will always equal 1, so it follows that if

$$\left(\frac{6}{6}\right)^3 = 1^3 = 1, \text{ then } \left(\frac{6}{6}\right)^3 = \left(\frac{6^3}{6^3}\right) = 6^0 = 1.$$

Learning Activity 3.9

Applying Power Laws

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil-and-paper.

1. Which is larger: $\frac{4}{7}$ or $\frac{4}{9}$?
2. If the radius of a circle is 35 mm, what is the diameter?
3. The nine starting players for the Winnipeg Goldeyes were asked to do as many sit-ups as they could in one minute. The data are shown below. What is the median of the data?

39	39	41	42	49	50	50	50	56

4. Estimate $\sqrt{415}$.
5. Calculate –87 + 10.
6. Write two equivalent fractions for $\frac{5}{4}$.
7. Convert $\frac{1}{5}$ to a percent.
8. Convert $\frac{1}{5}$ to a decimal.

continued

Learning Activity 3.9: Applying Power Laws (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Express as a power with one exponent. Keep the base the same.
   a) $(4^5)^3$
   b) $(-5^2)^5$
   c) $[(4)^{(2)}]^3$
   d) $\{[9^{(2)(3)}]^{(4)}\}$
2. Apply the power of a product law or power of a quotient law but do not solve.
   a) $\left(\frac{-1}{4}\right)^9$
   b) $(13 * 49)^7$
3. Simplify the following powers using any of the power laws. Show your steps.
   a) $\left(\frac{3^3}{3}\right)^2$
   b) $(7^0 * 1^9)^8$

Identify the Error

The following work was marked wrong. Identify the error, and correct the work.

$$\left(\frac{8}{2}\right)^2$$
$$= \frac{64}{2}$$
$$= 32$$

Lesson Summary

This lesson explained what to do when working with powers that have unique bases, such as products, fractions or another power. You learned and applied the power of a power law, the power of a product law and the power of a quotient law to simplify powers.

NOTES

Assignment 3.6: Powers with Like Bases

Total: 16 marks

1. Use the power law to express as a power with one exponent. Keep the base the same. *(4 marks)*

   a) $(6^2)^3$ _____ *(1 mark)*

   b) $(-3^4)^5$ _____ *(1 mark)*

   c) $8^{(5)(8)}$ _____ *(1 mark)*

   d) $(-22^4)^{(4)(2)}$ _____ *(1 mark)*

2. Apply the power of a product law or the power of a quotient law to simplify the following. Do not solve. *(4 marks)*

   a) $\left(\frac{10}{2}\right)^3$ _____ *(1 mark)*

   b) $\left(\frac{-2}{1}\right)^4$ _____ *(1 mark)*

   c) $(7 * 8)^2$ _____ *(1 mark)*

   d) $\left[(5^2)(2^5)\right]^3$ _____ *(1 mark)*

3. Identify the error, and correct it. *(8 marks)*

a) $(3 * 4)^3$ *(2 marks)*

$= 9 * 12$

$= 108$

b) $8^0 + 5^3$ *(2 marks)*

$= 8 + 125$

$= 133$

c) $\left(\frac{8^3}{2^4}\right)^2$ *(2 marks)*

$= \left(\frac{8^3}{2^8}\right)$

$= 2$

d) $\left(3^4 - 2^5\right) * 6^3$ *(2 marks)*

$= 81 - 6912$

$= -6831$

MODULE 3 SUMMARY

Congratulations, you have finished three of the eight modules in this course.

This module provided opportunities for you to learn and apply the correct order of operations in calculations involving powers. You also learned six exponent laws to help you to compute powers quickly and accurately: the zero exponent law, the product law, the quotient law, the power of a power law, the power of a product, and the power of a quotient law. Knowing how and when to apply these laws, and in what order, will strengthen your number sense skills. This will be helpful in the next module as well. If you are having trouble remembering the laws, check your answers by writing all exponents as repeated multiplication. The laws are simply a shortcut that may be helpful to you. Module 4 introduces variables as bases and as parts of a term in a mathematical expression.

Submitting Your Assignments

You will not be sending your assignments to your tutor/marker at this time. Instead, you will submit your Modules 3 and 4 assignments together when you have completed Module 4.

NOTES

GRADE 9 MATHEMATICS (10F)

Module 3: Powers

Learning Activity Answer Keys

MODULE 3: POWERS

Learning Activity 3.1: Identifying Components

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. When rolling a standard six-sided die, what is the probability of rolling a 3?
2. Calculate $\sqrt{81}$.
3. You save $900 each month for five months. How much money have you saved?
4. Estimate $\sqrt{37}$.
5. Convert 127% to a decimal.
6. Calculate $\frac{2}{3}-\frac{4}{3}$.
7. Calculate 6^2.
8. Will $\frac{1}{4}-\frac{1}{5}$ be positive or negative?

Answers:

1. $\frac{1}{6}$ (there is one 3 and six possible numbers)
2. 9
3. $4500
4. 6.1 ($\sqrt{38}=6$ and 37 is just slightly larger than 36)
5. 1.27
6. $-\frac{2}{3}$
7. 36
8. positive since $\frac{1}{4}$ is larger than $\frac{1}{5}$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Identify each base, exponent, and power.

Term	Base	Exponent	Power
2^5	2	5	2^5
$(-4)^3$	-4	3	$(-4)^3$
26^7	26	7	26^7
-3^6	3	6	-3^6

Learning Activity 3.2: Using Forms

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You need to save $1000 for a trip. You have five months before the trip. How much do you need to save on average each month?
2. Calculate $3 + 4 \times 2$.
3. Calculate $\frac{3}{7} \cdot \frac{2}{7}$.
4. Find 10% of 50.
5. Find 5% of 50.
6. Find 15% of 50.
7. Is $\sqrt{42}$ greater or less than 7?
8. Calculate $\sqrt{1600}$.

Answers:

1. $200 each month ($1000 \div 5$)
2. 11 (remember to do multiplication first)
3. $\frac{6}{49}$
4. 5
5. 2.5 (half of 10%)
6. 7.5 (10% + 5%)
7. less than 7 (since $\sqrt{49} = 7$)
8. 40

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write these powers out in the expanded form:
   a) 2^5

      Answer:

      $2 * 2 * 2 * 2 * 2$

   b) 4^9

      Answer:

      $4 * 4 * 4 * 4 * 4 * 4 * 4 * 4 * 4$

   c) -2^4

      Answer:

      $-(2 * 2 * 2 * 2)$

   d) $(-3)^5$

      Answer:

      $(-3) * (-3) * (-3) * (-3) * (-3)$

2. Given the expanded form, write the term in exponential form:
   a) $5 * 5 * 5 * 5 * 5 * 5$

      Answer:

      5^6

   b) $-3 * -3 * -3$ (Be careful here. Ask yourself, "What is the base?")

      Answer:

      $(-3)^3$

   c) $6 * 6 * 6 * 6$

      Answer:

      6^4

Learning Activity 3.3: Applying Order of Operations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Rewrite in exponential form: $6 \cdot 6 \cdot 6 \cdot 6$.
2. Is 5.67 x 105.32 closer to 50 or 500?
3. What two numbers have a sum of 5 and a product of 6?
4. If the radius of a circle is 4 cm, what is the diameter?
5. You have to leave for work in 3 hours. You want to watch a movie that is 150 minutes long. Do you have enough time?
6. Which is larger: 0.605 or 0.65?
7. Arrange the following numbers in order from smallest to largest:

   $-\frac{3}{4}$ $\quad -0.45$ $\quad -\frac{4}{3}$ $\quad -0.54$

8. Is $\sqrt{67}$ greater or smaller than 8?

Answers:

1. 6^4
2. 500 (estimate the calculation as 5 x 100)
3. 2 and 3
4. 8 cm (radius is half the diameter)
5. Yes, because 3 hours is equivalent to 180 minutes (3 x 60).
6. 0.65 (0.65 can be written as 0.650 and read as six hundred fifty thousandths; 0.605 is six hundred five thousandths)
7. $-\frac{4}{3}$ $\quad -\frac{3}{4}$ $\quad -0.54$ $\quad -0.45$

   (Compare the numbers to the benchmarks of –1, –0.5, and 0.

   $-\frac{4}{3}$ is less than –1, $-\frac{3}{4}$ is –0.75 and smaller than both –0.54 and –0.5,

   and –0.45 is larger than –0.5.)
8. greater since $\sqrt{64} = 8$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Simply these expressions following the order of operations, and explain each step.

   a) $\dfrac{\left(3^2 - 6 * 4\right)}{5}$

   Answer:

   $= \dfrac{\left(9 - 6 * 4\right)}{5}$ Exponents inside bracket first

   $= \dfrac{(9 - 24)}{5}$ Multiplication inside brack

   $= \dfrac{-15}{5}$ Simplify within the bracket

   $= -3$ Division

   b) $16 + (12 + 5^2) - 4^3$

   Answer:

   $= 16 + (12 + 25) - 4^3$ Exponents inside brackets first

   $= 16 + 37 - 4^3$ Simplify inside the brackets

   $= 16 + 37 - 64$ Apply exponent outside of the brackets

   $= 53 - 64$ Addition and subtraction in the order they appear

   $= -11$

2. Simplify these expressions following the order of operations, and show each step.

   a) $4 * (2^3 - 5)$

   Answer:

   $= 4 * (8 - 5)$

   $= 4 * 3$

   $= 12$

b) $6 + (2^4 - 4^2) + 9^0$

Answer:

$= 6 + (16 - 16) + 9^0$

$= 6 + 0 + 1$

$= 7$

c) $4 + (3 - 2^3) - (5^3 - 5^2)$

Answer:

$= 4 + (3 - 8) - (125 - 25)$

$= 4 + (-5) - 100$

$= -1 - 100$

$= -101$

3. Use the correct order of operations to simplify these expressions.

a) $4^4 - 16 * 3 + 8$

Answer:

$256 - 48 + 8 = 216$

b) $12 + 6^3 * (-2)$

Answer:

$12 + 216 * (-2) = -420$

c) $53 - \frac{8^2}{8^2} + 6 * (-4)$

Answer:

$53 - 1 + (-24) = 28$

Learning Activity 3.4: Using Technology

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What two numbers have a product of 18 and a sum of 9?
2. Rewrite as a product: $(-3)^5$.
3. What is the volume of a swimming pool that measures 7 m by 3 m by 2 m?
4. What is the cost of 8 bottles of water if each costs $0.99?
5. Is $\sqrt{90}$ greater than or less than 9?
6. Evaluate $5^2 - 3^2$.
7. When rolling a standard six-sided die, what is the probability of rolling a number that is a multiple of 3?
8. Will $\frac{1}{2} - \frac{1}{3}$ be positive or negative?

Answers:

1. 6 and 3
2. $(-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3)$
3. 42 m^2 (7 x 3 x 2 or 21 x 2 or 7 x 6)
4. $7.92 (if each bottle cost $1.00, then 8 would be $8.00; since each bottle is 1 cent less, 8 bottles is 8 cents less or $7.92)
5. greater than 9 since $9^2 = 81$, and 90 is greater than 81
6. 16 (25 – 9)
7. $\frac{2}{6}$ or $\frac{1}{3}$

   (multiples of 3 on a standard six-sided die are 3 and 6; there are 6 possible numbers on the die)
8. positive since $\frac{1}{2}$ is larger than $\frac{1}{3}$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use technology to simplify each expression.

a) $\left[\frac{\left(6^2+4^3\right)}{5^2}\right]$

Answer:

4

b) $[3^4 - (5^3 + 4^2)]$

Answer:

–60

c) $[8^3 - (6^2 - 4^3) + (5^3 + 3^5)]$

Answer:

908

Identify the Error

A classmate had this work marked wrong, and he couldn't see his error. Identify the error, and correct it.

$[4 * (2^3 - 5)]$

$= [4 * (8 - 5)]$

$= [32 - 5]$

$= 27$

Answer:

The classmate correctly simplified the exponent first. His mistake was not simplifying the bracket before performing the multiplication.

Corrected:

$[4 * (2^3 - 5)]$

$= [4 * (8 - 5)]$

$= [4 * 3]$

$= 12$

Learning Activity 3.5: Problems with Unlike Bases

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert 33.5% to a decimal.
2. Which is larger: 2^3 or 3^2?
3. Estimate $\sqrt{56}$.
4. What two numbers have a sum of 5 and a product of 4?
5. How many centimetres in 567 mm?
6. At your part-time job, you make $7.50 an hour. You worked 4 hours today. How much money did you earn?
7. Calculate –7 + (–4) + 5.
8. Your laundry needs 57 minutes to wash and 43 minutes for drying and folding. You need to leave the house in 1.5 hours. Do you have enough time to finish a load of laundry?

Answers:

1. 0.335
2. $3^2 = 9$
3. 56 is between 49 and 64 but closer to 49. So, $\sqrt{56} \approx 7.5$, since it is almost halfway between 49 and 64.
4. 4 and 1
5. 56.7 cm
6. $30 (double 7.50 to get 15 and double again to get 30)
7. –6
8. No, 1.5 hours is 90 minutes. 57 + 43 > 90

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Do the exponent calculations first, then add, subtract, multiply, or divide as indicated.
   a) $4^5 - 3^3$

      Answer:

      1024 – 27 = 997

   b) $2^4 * 3^2$

      Answer:

      16 * 9 = 144

   c) $\frac{4^2}{2^3}$

      Answer:

      $\frac{16}{8} = 2$

   d) $5^6 + 4^5$

      Answer:

      15625 + 1024 = 16649

   e) $(-2)^3 - (-3)^3$

      Answer:

      (–8) – (–27) = –8 + 27 = 19

2. Express each power as a repeated multiplication, and find the value.
   a) $3^4 + 4^3$

      Answer:

      3 * 3 * 3 * 3 + 4 * 4 * 4

      = 81 + 64

      = 145

b) $5^3 - 5^2$

Answer:

$5 * 5 * 5 - 5 * 5$

$= 125 - 25$

$= 100$

c) $2^4 - 3^4 * 2^3$

Answer:

$2 * 2 * 2 * 2 - 3 * 3 * 3 * 3 * 3 * 2 * 2 * 2$

$= 16 - 81 * 8$

$= 16 - 648$

$= -632$

d) $\dfrac{(3^3 * 3)}{3^2}$

Answer:

$$\frac{(3 * 3 * 3 * 3)}{3 * 3}$$

$$= \frac{81}{9}$$

$$= 9$$

Identify the Error

A classmate shows you her work, and asks for your help showing her where she made an error. Can you identify the mistake made in each question?

1. $2^3 + 4^2$

   $= 6 + 8$

   $= 14$

   Answer:

   You should be able to see that she multiplied the base by the exponent. The correct procedure is to apply the exponent to each base, and then add.

   $2^3 + 4^2$

   $= 2 * 2 * 2 + 4 * 4$

   $= 8 + 16$

   $= 24$

   Note: You don't have to show the repeated multiplication each time, but if you are having trouble getting the right answer, go back to the definition of exponents (repeated multiplication).

2. $2^3 * 5^2$

   $= 10^6$

   $= 1\ 000\ 000$

   Answer:

   This person multiplied the bases and then the exponents. To correct the error, you have to apply the exponent first to each term, and then multiply!

   $2^3 * 5^2$

   $= 2 * 2 * 2 + 5 * 5$

   $= 8 * 25$

   $= 200$

Learning Activity 3.6: Using the Product Law

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What two numbers have a sum of –8 and a product of –20?
2. Estimate $\sqrt{102}$.
3. Is $\frac{2}{3} - \frac{3}{4}$ positive or negative?
4. Which is larger: $\frac{2}{7}$ or $\frac{2}{5}$?
5. If the diameter of a circle is 14 cm, what is the radius?
6. Write two equivalent fractions for $\frac{9}{12}$.
7. Solve for d: $5d = 125$.
8. You buy 3 items at the drugstore that total $3.52 after taxes. You give the cashier $5.02. How much change do you get back?

Answers:

1. –10 and 2
2. 102 is between 100 and 121 but much closer to 100. So, $\sqrt{102} \approx 10.1$.
3. Negative since $\frac{3}{4} > \frac{2}{3}$.
4. $\frac{2}{5}$

   (Since the numerators are the same, compare the denominators. The 5 represents a whole cut into 5 equal pieces, while the 7 represents the same whole cut into 7 equal pieces. The 5 pieces will be bigger than the 7 pieces.)
5. 7 cm (half the diameter)
6. Answers will vary; some possible fractions are: $\frac{3}{4}, \frac{6}{8}, \frac{18}{24}, \frac{36}{48}, \ldots$

   (To be equivalent, both numerator and denominator must be multiplied or divided by the same number.)
7. $d = 25$
8. $1.50 (0.50 + 3.52 = 4.02, so you need one more dollar to get to $5.02.)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the product exponent law to simplify (not solve) these powers:

a) $5^8 * 5^6$

Answer:

5^{14}

b) $7^4 * 7^3 * 7^5$

Answer:

7^{12}

c) $5^3 * 4^2$

Answer:

Not like bases—cannot be simplified.

d) $(-2)^7 * (-2)^2$

Answer:

$(-2)^9$

Learning Activity 3.7: Using the Quotient Law

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Arrange the following numbers from greatest to least:

   $\frac{1}{3} \qquad -0.4 \qquad 0.5 \qquad -1.2$

2. If 14% of 600 is 84, what is 28% of 600?
3. Evaluate $6d - 2$ if $d = -1$.
4. Calculate 5^3.
5. Convert 15 mm to cm.
6. In April, you saved $4.00 each day. How much did you save by the end of April?
7. Solve for n: $7 = n - 3$.
8. Write as an expression with a single exponent: $3^2 \cdot 3^4$.

Answers:

1. $0.5 \qquad \frac{1}{3} \qquad -0.4 \qquad -1.2$

   $\left(\frac{1}{3} = 0.33,\right.$ which is less than 0.5 so for the positive numbers 0.5 is greatest.

   For the negative numbers, $-1.2 < -0.4$.)
2. 168 (twice 84)
3. –8 ($6(-1) - 2 = 6 - 2 = -8$)
4. 125 (5 x 5 x 5 = 25 x 5=125
5. 1.5 cm (10 mm = 1 cm, so divide by 10)
6. $120 (30 days in April so 30 x 4 = 120)
7. $10 = n$
8. 3^6 ($3^2 \cdot 3^4 = (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = 3^6$)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use the quotient exponent law to simplify (not solve) these powers.

a) $\frac{5^6}{5^2}$

Answer:

$5^{(6-2)} = 5^4$

b) $\frac{8^8}{8^7}$

Answer:

$8^{(8-7)} = 8^1 = 8$

c) $\frac{9^7}{8^2}$

Answer:

Not like bases—cannot be simplified.

d) $\frac{(-6)^5}{(-6)^5}$

Answer:

$(-6)^{(5-5)} = (-6)^0 = 1$

Remember: Any base with the exponent 0 = 1.

Identify the Error

A classmate shows you his work, and asks for your help in showing him where he made an error.

a) $4^5 * 4^6$

$= 16^{30}$

Answer:

You can see that he multiplied the bases and the exponents. The correct procedure when multiplying like bases is to keep the base in your answer, and add the exponents.

$4^5 * 4^6$

$= 4^{(5+6)}$

$= 4^{11}$

Note: To help your friend, you can go back to the definition of exponents—repeated multiplication. So, $4^5 * 4^6 = (4 * 4 * 4 * 4 * 4) * (4 * 4 * 4 * 4 * 4 * 4)$ or 4 multiplied by itself 11 times = 4^{11}.

b) $\frac{5^8}{5^4}$

$= 5^2$

Answer:

You can see that your friend divided the exponents instead of subtracting them. The correct response should be:

$\frac{5^8}{5^4}$

$= 5^{(8-4)}$

$= 5^4$

Note: To help your friend, you can go back to the definition of exponents—repeated multiplication. So,

$$\frac{5^8}{5^4} = \frac{5 * 5 * 5 * 5 * 5 * 5 * 5 * 5}{5 * 5 * 5 * 5}$$

which can be simplified to 5^4.

Learning Activity 3.8: Solving Mixed Operations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write as an expression with a single exponent: $\frac{6^6}{6^2}$.
2. Calculate $53 \div \frac{1}{2}$.
3. If 15% of 400 is 60, what is 30% of 400?
4. Calculate 5% of 480.
5. What is the perimeter of a rectangular yard that is 50 m by 75 m?
6. Convert $3\frac{2}{3}$ to an improper fraction.
7. Ealuate $3n + 4$ when $n = -2$.
8. You want to survey 50 people. Each survey takes 5 minutes. You have 3 hours to complete the task. Do you have enough time?

Answers:

1. $6^4 \left(\frac{6^6}{6^2} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6} = 6^4 \right)$
2. $106 \left(53 \div \frac{1}{2} = 53 \cdot 2 = 106 \right)$
3. 120 (twice 60)
4. 24 (10% is 48, so 5% is half of 48 or 24)
5. 250 m (50 + 75 + 50 + 75 = 100 + 150)
6. $\frac{11}{3} \left(3 = \frac{9}{3} \text{ and } \frac{9}{3} + \frac{2}{3} = \frac{11}{3} \right)$
7. –2 (3(–2) + 4 = –6 + 4)
8. No, 3 hours is 180 minutes; you need 50 x 5 or 250 minutes (just over 4 hours).

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Perform the indicated mixed operations, and solve.

a) $6^5 * 6^2 - 8^3$

Answer:

$= 6^7 - 8^3$

$= 279\,936 - 512$

$= 279\,424$

b) $(-4)^3 - \frac{5^6}{5^3}$

Answer:

$= -64 - 125$

$= -189$

c) $\frac{6^8}{6^6} - \frac{7^4}{7^2}$

Answer:

$= 36 - 49$

$= -13$

Learning Activity 3.9: Applying Power Laws

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Which is larger: $\frac{4}{7}$ or $\frac{4}{9}$?
2. If the radius of a circle is 35 mm, what is the diameter?
3. The nine starting players for the Winnipeg Goldeyes were asked to do as many sit-ups as they could in one minute. The data are shown below. What is the median of the data?

39	39	41	42	49	50	50	50	56

4. Estimate $\sqrt{415}$.
5. Calculate: –87 + 10.
6. Write two equivalent fractions for $\frac{5}{4}$.
7. Convert $\frac{1}{5}$ to a percent.
8. Convert $\frac{1}{5}$ to a decimal.

Answers:

1. $\frac{4}{7}$

   (Since the numerators are the same, compare the denominators. The 7 represents a whole cut into 7 equal pieces, while the 9 represents the same whole cut into 9 equal pieces. The 7 pieces will be bigger than the 9 pieces.)
2. 70 mm (diameter is twice the radius)
3. 49 (median is the middle number when the data is arranged in order)
4. 415 is between 400 and 441 but closer to 400. So, $\sqrt{415} \approx 20.4$.
5. –77
6. Answers will vary; some possible fractions are: $\frac{10}{8}, \frac{15}{12}, \frac{30}{24}, \frac{60}{48}, \ldots$.

   (To be equivalent, both numerator and denominator must be multiplied or divided by the same number.)

7. 20% $\left(\text{since } \frac{1}{5} = \frac{2}{10} = \frac{20}{100}\right)$

8. 0.2 $\left(\text{since } \frac{1}{5} = \frac{2}{10}\right)$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Express as a power with one exponent. Keep the base the same.

   a) $(4^5)^3$

   Answer:

   $= 4^{15}$

   b) $(-5^2)^5$

   Answer:

   $= -5^{10}$

   c) $[(4)^{(2)}]^3$

   Answer:

   $= 4^6$

   d) $\{[9^{(2)(3)}]^{(4)}\}$

   Answer:

   $= 9^{24}$

2. Apply the power of a product law or power of a quotient law but do not solve.

a) $\left(\frac{-1}{4}\right)^9$

Answer:

$$\frac{(-1)^9}{4^9}$$

b) $(13 * 49)^7$

Answer:

$= 13^7 * 49^7$

3. Simplify the following powers using any of the power laws. Show your steps.

a) $\left(\frac{3^3}{3}\right)^2$

Answer:

$$= \frac{\left(3^3\right)^2}{3^2}$$

$$= \frac{3^6}{3^2}$$

$= 3^4$ or 81

b) $(7^0 * 1^9)^8$

Answer:

$= (7^0)^8 * (1^9)^8$

$= 7^0 * 1^{72}$

$= 1 * 1$

$= 1$

Laws may be applied in a different order, and will result in the same answer. For example:

$$\left(\frac{3^3}{3}\right)^2$$

$$=\left(3^{3-1}\right)^2$$

$$=\left(3^2\right)^2$$

$$=3^4$$

Or $(7^0 * 1^9)^8$

$= (1)^8 * (1)^8$

$= 1 * 1$

$= 1$

Identify the Error

The following work was marked wrong. Identify the error, and correct the work.

$$\left(\frac{8}{2}\right)^2$$

$$=\frac{64}{2}$$

$$=32$$

Answer:

The student applied the exponent to the numerator but not the denominator.

Corrected:

$$\left(\frac{8}{2}\right)^2$$

$$=\frac{64}{4}$$

$$=16$$

GRADE 9 MATHEMATICS (10F)

Module 4: Polynomials

MODULE 4: POLYNOMIALS

Introduction

You communicate with others using expressions composed of words. Mathematics uses expressions composed of numbers and letters. In mathematics, a polynomial is an expression that is constructed from one or more terms including parts like variables and constants, using the operations of addition, subtraction, multiplication, and division. In this module, you will learn to communicate using polynomial expressions.

Assignments in Module 4

You will need to complete the following seven assignments and send them to your tutor/marker when you are done this module.

Lesson	Assignment Number	Assignment Title
1	Assignment 4.1	Cover Assignment
2	Assignment 4.2	Introduction to Polynomials
3	Assignment 4.3	Collecting Like Terms
4	Assignment 4.4	Distribute the Negatives
5	Assignment 4.5	Multiplication
6	Assignment 4.6	Division and Equivalents
7	Assignment 4.7	Geometry and Polynomials

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 4. You might use your Module 4 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 4 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 4 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

When you get to the end of this module, you will be writing the midterm exam supervised by a proctor. You need to make the following arrangements about three weeks before writing your exam. Here's what you need to do:

- **If you are attending school,** ask your school's Independent Study Option (ISO) Facilitator to add your name to the ISO exam eligibility list.
- **If you are not attending school,** check the **Examination Request Form** for options available to you. The Examination Request Form was mailed to you with this course. Fill in this form and mail or fax it. The address is:

  ISO Registration
  555 Main St.
  Winkler, MB R6W 1C4
  Fax: 204-325-1719
  Phone: 1-800-465-9915

NOTES

LESSON 1: COVER ASSIGNMENT

When solving problems in mathematics, you may need to first make conjectures about what the answer will be. A conjecture is a conclusion based on known or observed information, but without conclusive proof. This will help you decide on the best way to solve the problem. There are many problems that can assist you in practising making conjectures. A lot of these are more like puzzles or games.

Example 1

Solve the number puzzle below.

$$\begin{array}{r} 372 \\ 384 \\ +\ 9B4 \\ \hline C7CA \end{array}$$

A, B, and C represent one of the digits 0, 1, 2, . . ., 9

Solution:

Conjecture: Before trying to solve this you might think about what some possible values for C are, since the third column has to add to "C7."

Validate

One solution to this problem is . . .

A = 0, B = 5, C = 1. Validate this solution.

NOTES

Assignment 4.1: Cover Assignment

Total: 23 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Imagine you have a supply of letter tiles, and you use them to make word triangles like this one.

A			
L	A		
A	L	E	
R	E	A	L

Rules: Each horizontal row contains the letters of the previous one, plus one more. It's OK to scramble the letters from one row to the next.

a) Extend this word triangle for two more rows. *(2 marks)*

b) Make a word triangle with your own letters. Your triangle should have no less than 5 rows and no more than 9 rows. *(5 marks)*

c) How many letter tiles are used in a five-row word triangle? *(1 mark)*

_____

d) Complete the table below. *(1 mark)*

Rows	Tiles
1	1
2	3
3	6
4	
5	

e) How many letter tiles are used in a 10-row word triangle? *(1 mark)*

_____

f) The numbers you found in the table are called triangular numbers. You will see them again in this course. Explain how you found this answer. *(1 mark)*

_____

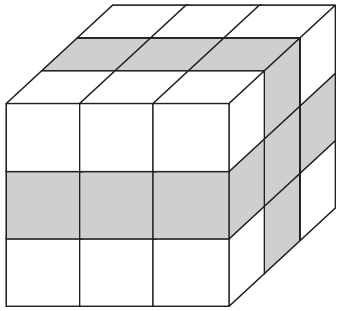
_____

_____

2. a) A digit is represented by the letter **S**. What is one value of S if S6SS6 is divisible by 4? *(1 mark)*

b) A rule that states under what condition(s) a number is divisible by another number is called a divisibility test. What is the divisibility test for 4? (What is a quick method of determining whether a number is or is not divisible by 4?) You may remember this from previous mathematics courses, or you may need to do research to find out. *(1 mark)*

3. The diagram shown below represents a cube consisting of 27 smaller cubes.



a) Two stripes are painted onto the shape as shown. Suppose the shape is taken apart to show individual cubes. What fraction of these cubes would have at least one side painted? *(1 mark)*

b) What percent of the cubes would have no sides painted? *(1 mark)*

c) If you threw all the cubes into a bag and picked one out without looking, what is the chance that the one you picked would have a painted side? *(1 mark)*

4. Make up a word or number puzzle of your own. Try it out on your learning partner. Include a copy of the puzzle and the solution. *(2 marks)*

5. There are approximately 600 000 words in the English language. A person uses about 5000 of these words. What percent of the total number of words does a person use? *(1 mark)*

6. John had the following bowling scores: 95, 103, 96, 110, 109, 110, 103, 116, and 103.

   a) What was his mean score? *(1 mark)*

   b) What was the mode of John's score? *(1 mark)*

7. Replace the "•" with digits so that the sum is correct. The "•" may represent different digits. *(2 marks)*

$$\begin{array}{r} 7\bullet 82 \\ +\ \bullet 9\bullet\bullet \\ \hline 10329 \end{array}$$

8. Simplify. *(1 mark)*

$$\frac{3}{7} \times \frac{5}{6} \times \frac{14}{15}$$

Remember, you will forward this assignment to the tutor/marker at the end of Module 4.

Lesson 2: Introduction to Polynomials

Lesson Focus

When you finish this lesson, you will be able to

- ❑ identify terms, variables, numerical coefficients, constants, and exponents
- ❑ draw pictures of expressions using algebra tiles
- ❑ write algebraic expressions from diagrams
- ❑ describe a situation using a polynomial expression

Lesson Introduction

This lesson introduces some new terminology to refer to polynomials and their components. You will also use drawings of basic algebra tiles to illustrate polynomial expressions.

Terminology

You should include the terminology for this module on your Module Resource Sheet, so that you do not have to come back to this page while you work through the lessons. As you work through this module, you may find that you do not need to refer to the terminology as often as you do in the beginning.

Variable: A letter or symbol that represents an unknown value.

x, y, n, θ

Exponent: A superscript to the right of a base that indicates repeated multiplications of the base.

3^4 (The 4 is the exponent on the base of 3.)

x^5 (the 5 is the exponent on the base of x.)

x (Here, x is the base, and it has an implied exponent of 1.)

Degree: Usually the exponent on the variable indicates the degree of the term.

x^2 would be a term with a degree of 2

y would be the same as y^1, so the degree here is 1

xy is the same as x^1y^1, so the degree for this term would be the sum of the exponents, or 2

$3n^4 - n^2$ (If there is more than one term with exponents in an expression, the degree of the polynomial is from the term with the highest degree. This expression would be degree 4.)

Constant: A term with no variables, usually a number.

Numerical coefficient: The value in front of the variable.

$-3x^2$ (Here the numerical coefficient is –3.)

r (Since there is no numerical value stated in front, it is assumed to be one r, and the numerical coefficient is 1. It could also be read as "1 times r.")

Term: Variables, numerical coefficients, or constants in a polynomial expression. Terms are separated by addition or subtraction signs.

$3x$ is a term

–5 is a term

$-4x^2y$ is a term

$x + 3$ has two terms, separated by the addition sign

$-7a + 3b - 4$ has three terms, each separated by an addition or subtraction sign

Monomial: A polynomial with one term.

Binomial: A polynomial with two terms.

Trinomial: A polynomial with three terms.

Polynomial: A mathematical expression with one or more terms.

Learning Activity 4.1

Identifying Components

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate $10 - 3 \times 6 \div 2$.
2. How many dozen eggs are there in 60 eggs?
3. What two numbers have a product of –81 and a sum of 0?
4. The Winnipeg Blue Bombers scored 3 touchdowns (7 points each) and two field goals (3 points each). The Montreal Alouettes scored 25 points altogether. Who won the game?
5. What is the radius of a circle whose diameter is 46 cm?
6. Convert 600 g to kg.
7. Calculate $3^2 + 2^3$.
8. Which is larger: $\frac{1}{6}$ or $\frac{1}{7}$?

continued

Learning Activity 4.1: Identifying Components (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the numerical coefficients, variables, exponents, degree, constants, and number of terms in each polynomial.

Polynomial	$6x$	$-3n + 1$	$n^2 - 3n + 4$	7
Numerical Coefficient				
Variable				
Exponent				
Degree				
Constant				
Number of Terms				

2. Write a polynomial with:
   a) two terms, one variable, and one constant
   b) three terms, two variables, one constant, one positive numerical coefficient
   c) one term, two variables, negative numerical coefficient

Algebra Tiles

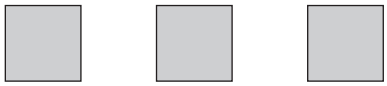
Algebra tiles will be used to illustrate how polynomials are created. You can cut out your own tiles with the template provided, and use them to help you work with polynomials. **The template is in Appendix A, at the end of this course.**

This module will use three basic shapes.

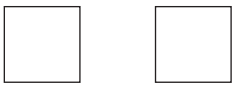
The first shape will have dimensions of 1 unit by 1 unit, and an area of 1 x 1 = 1. The value of this tile is 1.

If the tile is shaded it will represent +1, and without shading it will represent –1.

So, this arrangement of tiles, or diagram, would represent 3.

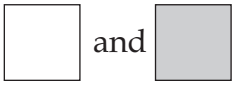


While this diagram would represent 2 negatives, or –2.



Cancelling Tiles to Zero.

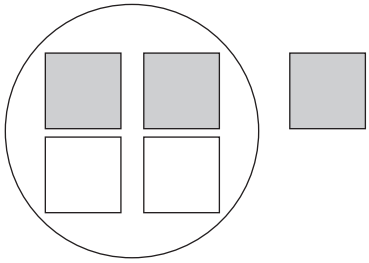
Tiles represent values, and so negative and positive tiles behave in the same way as positive and negative values. If we combine the following two tiles,



they would cancel each other out, or represent a value of zero because (–1) and (+1) = 0.

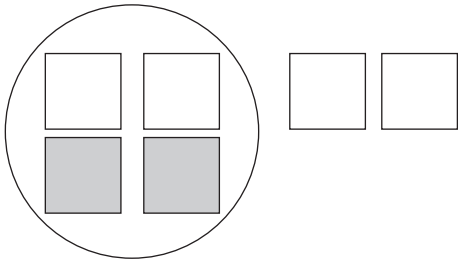
Example 1

Combine 3 positive tiles and 2 negative tiles: (+3) and (–2). Two of the positives cancel out the two negatives, so the value remaining would be one positive tile, or +1.



Example 2

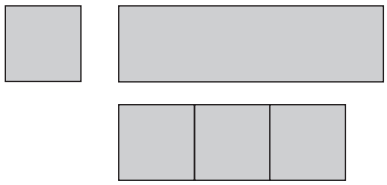
Combine 4 negative with 2 positive tiles. (-4) and (+2).



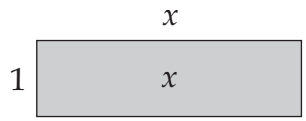
The two positive tiles are cancelled out by two of the negative tiles. What remains is the answer of –2.

The second shape looks like this.

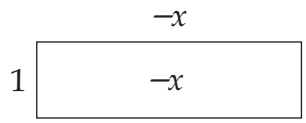
The width is the same as the previous tile (1), but the length is a little longer than three single tiles. Since the length is not an exact value, it is represented by the variable x. It can also be read as "1 times x "so it has an area of x: $(1) * (x) = 1x$. This is called the x-tile.



The measurements of the second tile are:

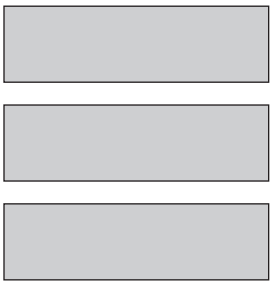


An x-tile with no shading is considered to be negative.



Example 3

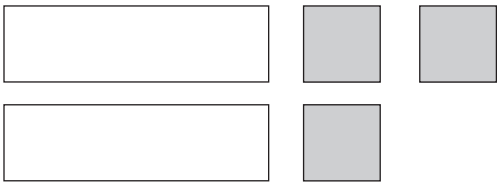
This diagram illustrates 3 x-tiles, representing $3x$.



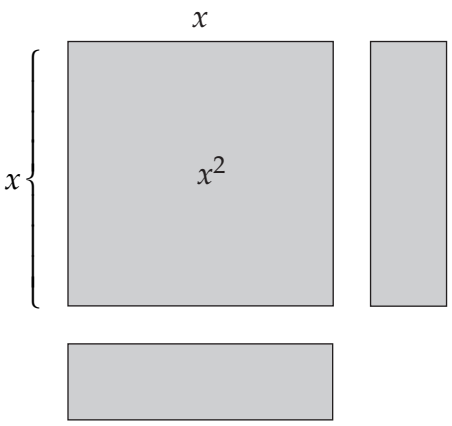
Example 4

This diagram shows two negative x-tiles ($-2x$) and three positive one tiles ($+3$).

Combine the tiles into a polynomial expression: $-2x + 3$.

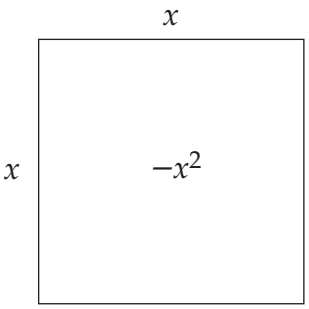


The third shape is a square tile.

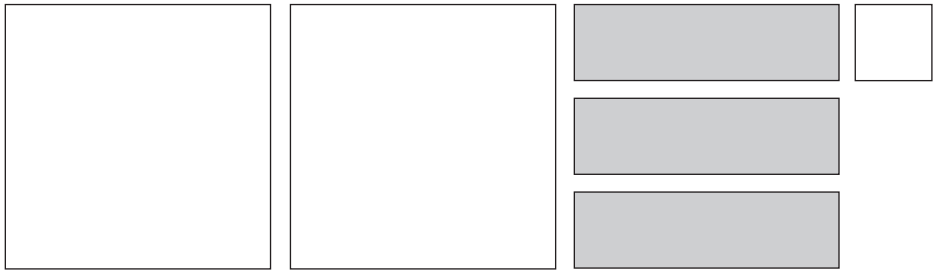


Notice it is x wide, and x long. So, its area represents $(x) * (x) = x^2$.

Again, if it has no shading, it is considered a negative value.



Example 5



This diagram shows 2 negative x-squared tiles, 3 positive x-tiles, and 1 negative tile.

Expressing this as a polynomial would look like:

$$-2x^2 + 3x - 1$$

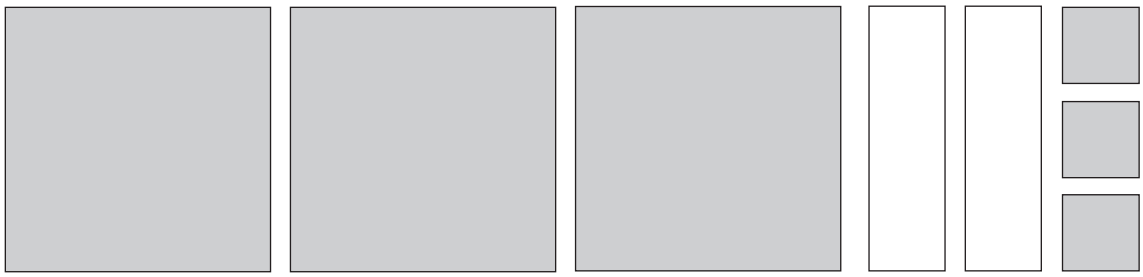
Drawing Diagrams to Represent Polynomial Expressions

Given any polynomial expression, you can illustrate it with a tile diagram.

Example 1

Given $3x^2 - 2x + 3$, represent this expression with a tile diagram.

You would use three x-squared tiles, two negative x-tiles, and three positive 1 tiles.



As you complete more of these diagrams, you may create a different system so that it takes you less time to draw the algebra tiles. For example, a square ☐ may represent the large square tile, a line | may represent the rectangular tile, and a dot • may represent the small square tile. Although you can use this when you are completing quick calculations or the learning activities, **you must draw out the full tiles when answering questions on assignments or exams** because each person could have a different code and it may not make sense to your tutor/marker.

Learning Activity 4.2

Writing Polynomial Expressions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write three equivalent fractions for $\frac{4}{6}$.
2. Calculate $\sqrt{3^2+4^2}$.
3. You buy an ice cream cone for $2.35 and give the cashier a $5 bill. How much change do you get back?
4. Sheila was born in 1995. Her mother was 25 years old when Sheila was born. What year was Sheila's mother born?
5. A third of your Grade 9 class is going on a ski trip to Duck Mountain Provincial Park. There are 90 students in your Grade 9 class. How many are not going skiing?
6. 92 is between which two perfect squares?
7. You are planning a 1.5 hour football practice. You know you need to have a 10 minute warm-up and a 10 minute warm-down session, there will be two 5-minute breaks and you want to have 20 minutes for a scrimmage. How much time does that leave for drills?
8. What is the perimeter of a rectangular field with dimensions 20 m by 30 m?

continued

Learning Activity 4.2: Writing Polynomial Expressions (continued)

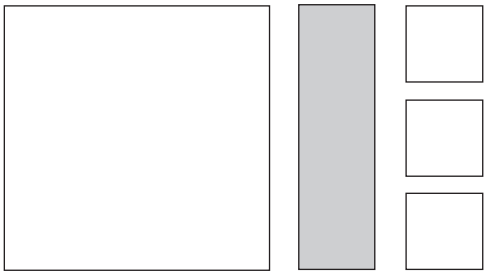
Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

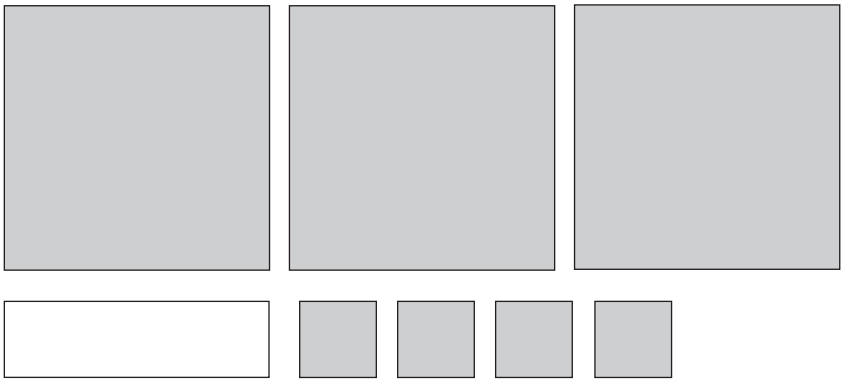
1. Write the polynomial expressions represented by the algebra tiles.

   a)

   b)



   c)



2. Draw algebra tile diagrams to represent the polynomial expressions below.
   a) $-x + 3$
   b) $2x^2 - 4x - 1$

Describing a Situation

A variable (x) can represent an unknown value in a polynomial.

Polynomial expressions can be used to describe situations where an actual value is unknown.

Example 1

You and your best friend want to host a party but you do not know how many others will come. Write a polynomial expression describing the number of people at the party.

Solution:

Guests plus you and your friend

$n + 2$

Example 2

A plumber will come to your house to work on your kitchen sink. He charges $75 for the trip plus $30 per hour that he is there. He does not know how long the repair job will take. Write a polynomial to represent how much it will cost for the plumber to fix your sink.

Solution:

If you let h represent the number of hours, then the polynomial expression for his fee would be as follows:

$30 times the number of hours (h) plus the flat fee ($75)

$30h + 75$

Example 3

Emilie is three times older than Shay. Write the polynomial expression for Emilie's age.

Solution:

Let S = Shay's age

Emilie's age would be $3S$

Example 4

Create your own situation that could be represented by the following polynomial.

$4x - 5$

Solution:

One example would be: Jimmy's grandfather is five years less than four times Jimmy's age.

Learning Activity 4.3

Situations as Expressions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Simplify $-3 + 4$.
2. A pair of shoes that regularly cost $45 are on sale for 10% off. What is the amount of discount?
3. What two numbers have a product of 18 and a sum of 9?
4. You have $65 in your chequing account. You deposit your $350 paycheque and write cheques for $10 and $35. What is your new balance?
5. If 24% of 600 is 144, what is 12% of 600?
6. Solve for t: $3t = 21$.
7. You want to join boot camp to get fit. The cost is $400 for 8 sessions. How much does each session cost?
8. From Regina to Winnipeg is approximately 570 km. At an average driving speed of 110 km/hr, approximately how long will it take to drive from Regina to Winnipeg?

continued

Learning Activity 4.3: Situations as Expressions (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write a polynomial expression to represent each situation.
   a) Sammy hits the ball twice as far as Joshua.
   b) Your sister is four years younger than you.
   c) A repairman charges $50 to come to your house, and $42.50 per hour.
   d) Your weight is 4.5 kgs over the suggested level.
2. Create a situation which could be represented by these polynomial expressions:
   a) $3t - 2$
   b) $k + 1$

Internet Fun

There are many sites on the internet where you can move tiles around to represent expressions. Type these key words into your search engine "interactive algebra tiles," and have some fun!

Lesson Summary

In this lesson you learned new terminology to use when working with polynomials. You also learned how to use algebra tiles to illustrate polynomial expressions. In the next lesson, you will use these tiles to help understand how like terms are collected.

Assignment 4.2: Introduction to Polynomials

Total: 23 marks

1. Given the following polynomial expressions, identify the component parts. *(8 marks)*

   a) $2x - 4$

   variable _____ *(1 mark)*

   constant _____ *(1 mark)*

   numerical coefficient(s) _____ *(1 mark)*

   exponent _____ *(1 mark)*

   b) $-3w^2 + 4w + 1$

   variable _____ *(1 mark)*

   constant _____ *(1 mark)*

   numerical coefficient(s) _____ *(1 mark)*

   exponent _____ *(1 mark)*

2. Identify each polynomial expression as a monomial, binomial, or trinomial. *(3 marks)*

   a) $x^2 - 3x + 2$ _____ *(1 mark)*

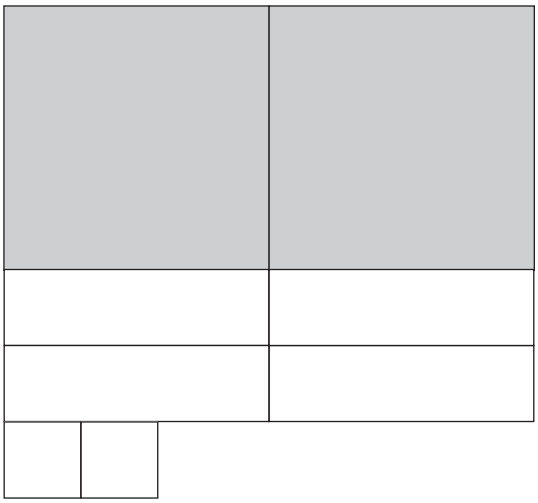
   b) $-3x^2 + 5$ _____ *(1 mark)*

   c) $73xy$ _____ *(1 mark)*

3. Given these algebra tile diagrams, write the polynomial expression for each. *(4 marks)*

   a)  *(1 mark)*

   _____

b)  *(1 mark)*

4. Draw algebra tile diagrams to represent these polynomial expressions. *(4 marks)*

   a) $-2x^2 + 4$ *(2 marks)*

   b) $3x^2 - 2x - 3$ *(2 marks)*

5. Create a situation represented by the polynomial expression $2x - 1$.
(2 marks)

6. Write a polynomial expression to represent the following situation.
(2 marks)

   Your driver's licence will cost you $55 plus $95 for each demerit you have.

NOTES

Lesson 3: Collecting Like Terms

Lesson Focus

When you finish this lesson, you will be able to

- ❑ add polynomial expressions by collecting like terms
- ❑ illustrate addition of expressions using algebra tiles
- ❑ simplify algebraic expressions from diagrams
- ❑ identify the error in a given simplification

Lesson Introduction

In this lesson, you will learn how to combine positive and negative like terms in polynomials by collecting or grouping them, and show how to simplify expanded polynomial expressions by using tiles or drawing shapes.

Like Terms

People naturally collect in groups of "like terms." Hockey players tend to hang out together at school. Your collection of friends possibly shares similar interests with you. In mathematics, like terms are collected to simplify algebraic expressions.

Like terms usually have two things in common:

- same base or variable
- same exponent

These are all like terms:

x^2 $3x^2$ $-4x^2$ $-x^2$

Notice they all have the same base (x) and they all share the same exponent (2).

Powers can have a number as a base. The same rules apply when collecting these like terms.

a) 3^2
b) 5^2
c) 3^3
d) 4^2
e) 2^3
f) 3^2
g) 5^2

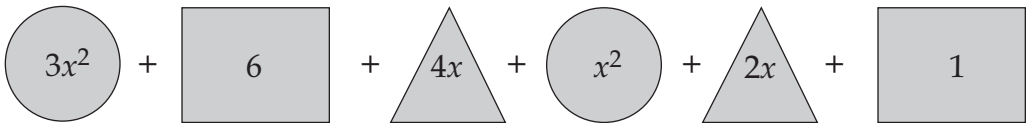
Given the above list of terms, the "like terms," or those with the same base and same exponent are (a) and (f), along with (b) and (g).

One method of collecting like terms is to use shapes. Draw circles around the x^2-terms, triangles around the x-terms, and rectangles around the constant (numbers) terms. Then bring the like terms together, or "collect" them.

Example 1

Collect the like terms: $3x^2 + 6 + 4x + x^2 + 2x + 1$

Solution:



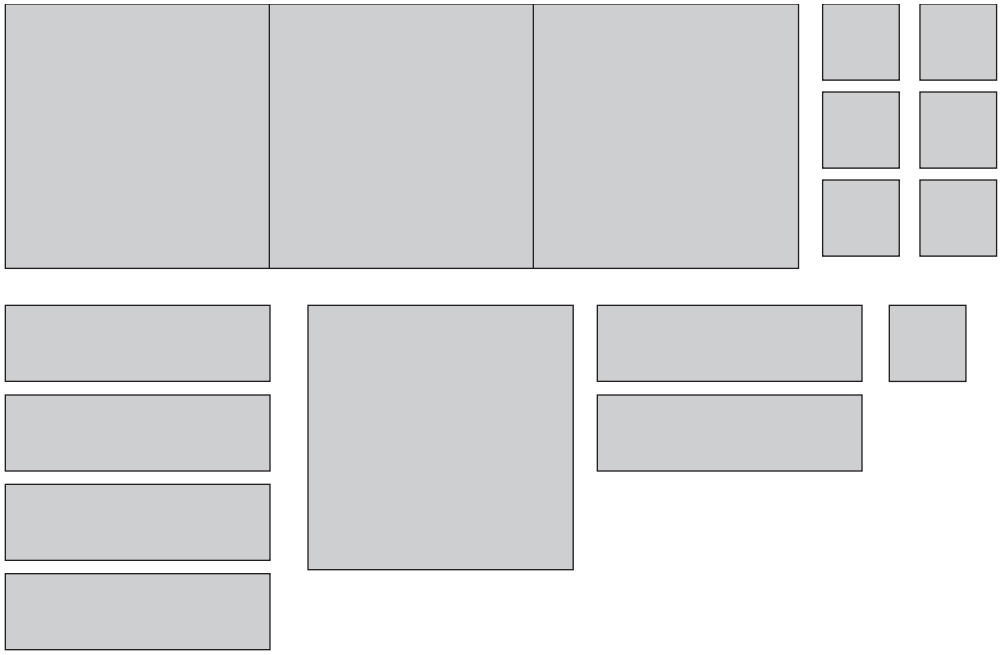
Collecting the circles, you should see a total of $4x^2$ ($3x^2 + 1x^2$).

Collecting the triangles, you should see a total of $6x$ ($4x + 2x$).

Collecting the rectangles, you should see a total of 7 (6 + 1).

So the answer would be $4x^2 + 6x + 7$.

Tiles could also be used to show how to collect the like terms:
$3x^2 + 6 + 4x + x^2 + 2x + 1$.

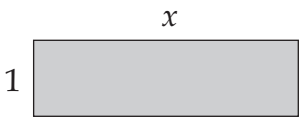


When you collect the like tiles, you get $4x^2 + 6x + 7$.

Finding Perimeter

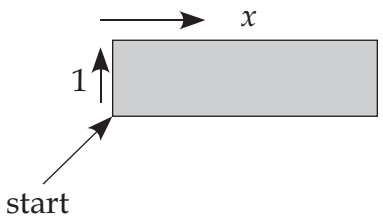
Perimeter is the distance around something. To find the perimeter, you simply add the lengths of each side.

Example 1

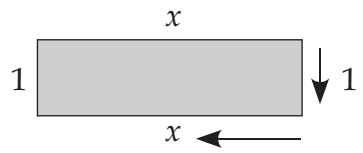


Solution:

This x-tile has four sides. To find its perimeter, add up all four side lengths.



If you start at the bottom left corner and move upward, you can see that there is a 1 and an x. But you must also add the remaining two sides.



Moving down, and to the left, you can see an additional side with a value of 1, and another side with a value of x.

So, you have $1 + x + 1 + x$.

Collecting all the x-terms gives us $2x$ (x + x).

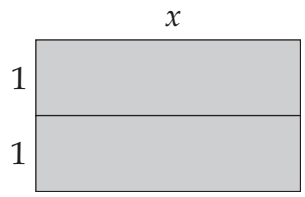
Collecting all the constant terms gives us 2 (1 + 1).

The perimeter around this tile is $2x + 2$.

$1 + x + 1 + x = 2x + 2$

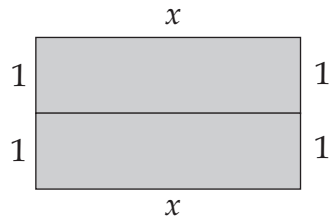
Example 2

Find the perimeter of the following object.



Solution:

Carefully write in the measurements of each side before collecting the like terms.



Now collect the like terms.

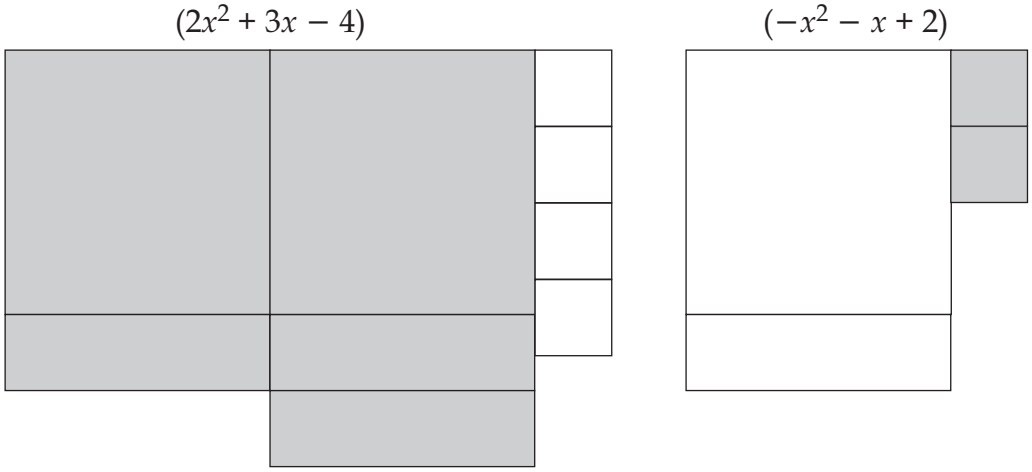
$1 + 1 + x + 1 + 1 + x = x + x + 1 + 1 + 1 + 1 = 2x + 4$

$2x + 4$ is the perimeter of this object.

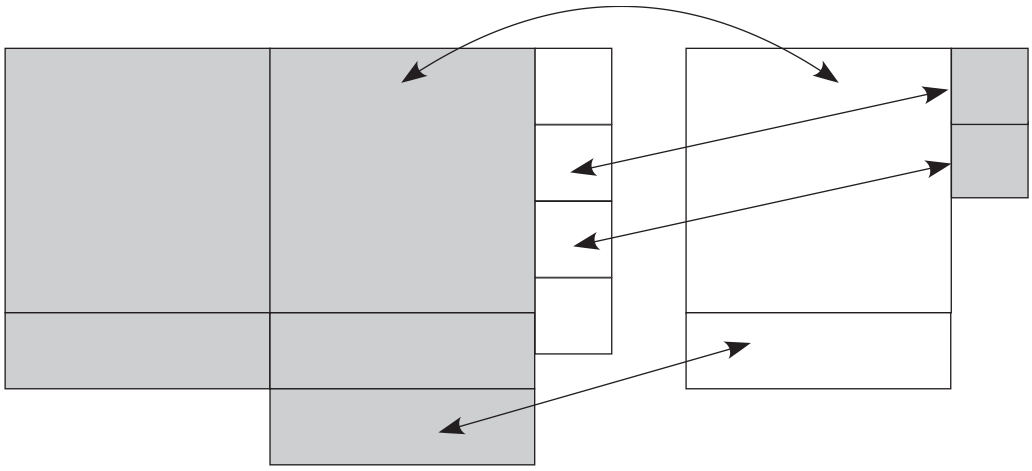
Collecting Positive and Negative Terms

Algebra tiles can be used to show how to combine positive and negative values in polynomials. When you combine opposite tiles, for example ($-x^2$) and (x^2), they cancel each other out because ($-x^2$) and (x^2) = 0. If you cancel all the zero pairs when combining polynomials, you simplify the expression.

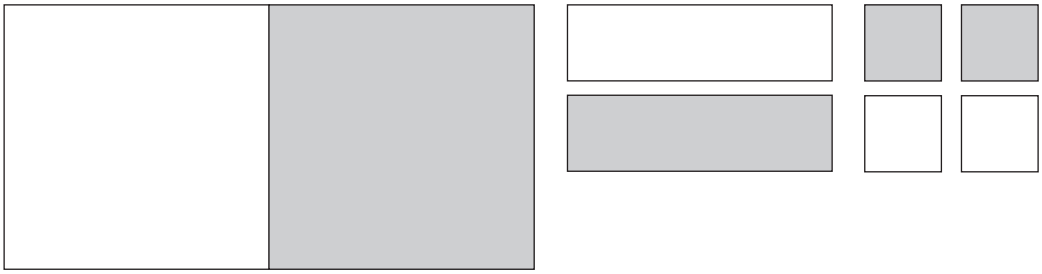
Combine ($2x^2 + 3x - 4$) and ($-x^2 - x + 2$).



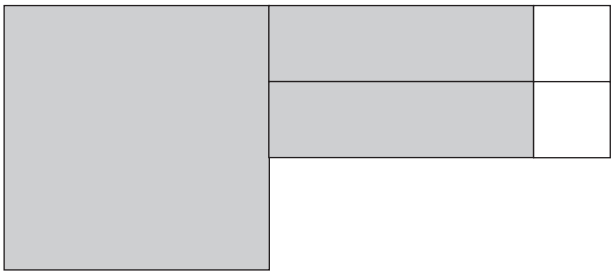
Now, pair off the opposite tiles that cancel each other out. The answer will be the tiles that remain.



Or



The tiles that remain are:

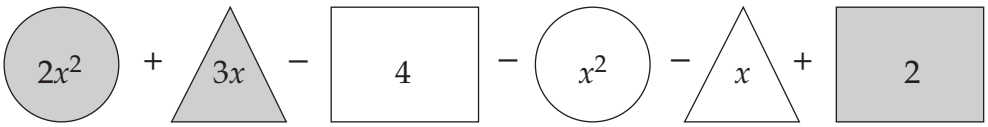


So the simplified polynomial expression is $x^2 + 2x - 2$.

This can also be represented using shapes as in an earlier example.

Combine $(2x^2 + 3x - 4)$ and $(-x^2 - x + 2)$.

Draw circles around the x^2-terms, triangles around the x-terms, and rectangles around the constant (numbers) terms. Notice the shading indicates if the terms are positive or negative. When you bring the like terms together consider if you have more positive or more negative terms of the same shape.



Collect the circles: There are 2 positive x^2 and 1 negative x^2 so there is 1 more positive x^2 than negative. ($2x^2$ and $-x^2 = 1x^2$)

Collect the triangles: There are 3 positive x's and 1 negative x so there are 2 more positive x's. ($3x$ and $-x = 2x$)

Collect the rectangles: There are 4 negatives and 2 positives, so there are 2 more negatives than positives. (-4 and $2 = -2$)

So the simplified expression for $(2x^2 + 3x - 4)$ and $(-x^2 - x + 2)$ is $x^2 + 2x - 2$.

You can see how the same rules you used to combine whole numbers, decimals, and fractions apply to polynomials.

Example 1

Evaluate: 2 – 5

Solution:

There are 2 positives, and 5 negatives. Since there are more negatives than positives, the answer will be negative. Further, since 2 positives cancel out 2 negatives, you are left with 3 negatives.

2 – 5 = –3

You could use the simple "two-questions method":

- Which do I have more of—positives or negatives? (negatives)
- How many more? (3)

Some things to keep in mind when collecting like negatives:

- –4 – 5 (You are not multiplying but "collecting" like terms. Four negatives combined with 5 more negatives gives —9.)
- $-3x - 4x$ (Three negative x's combined with four more negative x's gives $-7x$.)

When dealing with simplifying polynomial expressions, apply the same rules.

Example 2

Simplify: $-3x + x$

Solution:

- Which do I have more of? (negatives)
- How many more? (2)

The simplified answer of $-3x + x = -2x$.

Example 3

Simplify: $2y^2 - 8y^2 + 3y^2 - y^2$

Solution:

First, group all the positive terms and the negative terms together.

$2y^2 + 3y^2 - 8y^2 - y^2 = 5y^2 - 9y^2$

- Which do I have more of? (negatives)
- How many more? ($4y^2$)

The simplified answer is $-4y^2$.

Learning Activity 4.4

Writing Polynomial Expressions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Arrange the following numbers in ascending order (from smallest to largest):

   $\frac{2}{3}$ 0.53 0.76 $\frac{6}{5}$

2. What is the mode of the following data:

   1, 1, 2, 3, 3, 3, 3, 3, 5, 5, 5, 6, 6, 6, 7, 8, 9

3. What is the median of the following data:

   1, 1, 2, 3, 3, 3, 3, 3, 5, 5, 5, 6, 6, 6, 7, 8, 9

4. Solve for n: $3 = 5 + 2n$.
5. Estimate the 15% tip on a restaurant bill of $150.
6. Add $\frac{2}{3} + \frac{4}{3}$.
7. Estimate $\sqrt{56}$.
8. Calculate $3^3 - 2^3$.

continued

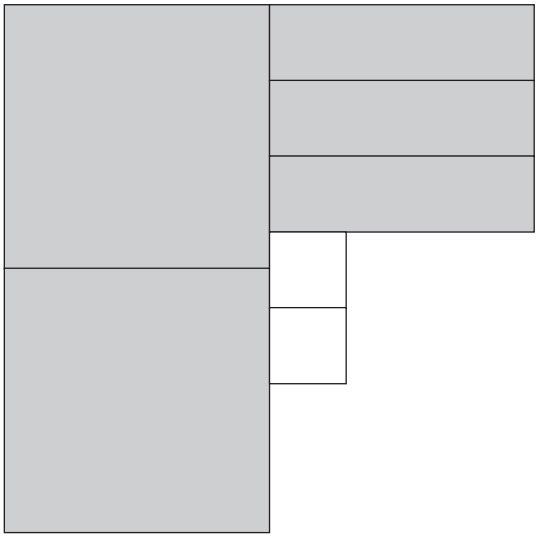
Learning Activity 4.4: Writing Polynomial Expressions (continued)

Part B

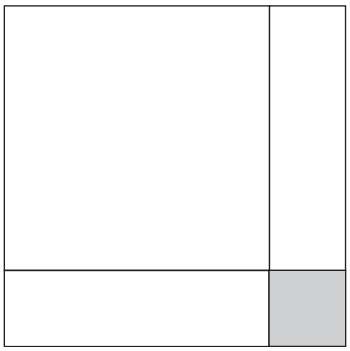
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the polynomials illustrated by these algebra tiles.

   a)



   b)



2. Draw a picture of the algebra tiles needed to represent these polynomials.
   a) $-x^2 + 2x + 1$
   b) $2x^2 - 3x - 4$

3. Use circles, triangles, and rectangles to identify like terms, and collect them.
   a) $2x^2 - 4x + 30 - 2x + 3 + x^2$
   b) $-x^2 - 3x + 6 - 2x - x^2 + 1$

Simplifying Expressions

Given an expanded expression, use your knowledge of combining like terms, and your skills working with positive and negative values, to simplify it.

Use the algebra tile template provided, or draw shapes to collect like terms, if it helps.

Example 1

Simplify: $5x^2 - 4x - 3x^2 - x + 2$

Solution:

- collect the x^2 terms: $5x^2 - 3x^2 = 2x^2$
- collect the x terms: $-4x - x = -5x$
- collect the constants: 2

Simplified: $2x^2 - 5x + 2$

An expression may have more than one variable.

Example 2

Simplify: $4f^2 + 3g^2 - 33f + 5g - 2f^2 - 2g^2 - 8g + f - 5$

Solution:

- collect f^2 terms: $4f^2 - 2f^2 = 2f^2$
- collect g^2 terms: $3g^2 - 2g^2 = g^2$
- collect f terms: $-33f + f = -32f$
- collect g terms: $5g - 8g = -3g$
- collect constants: -5
- answer: $2f^2 + g^2 - 32f - 3g - 5$

Identify the Error

Your friend made errors simplifying the following expression. Identify the errors made, and correct them.

$x^2 - 3x + 5 - 4x^2 + x - 4$

$-5x^2 + 2x + 1$

Two mistakes were made here:

a) When collecting the x^2 terms, he made them both negative. There is 1 positive x^2, and 4 negative. When collected, that gives $-3x^2$.

b) Also, $-3x + x$ has more negatives than positives. The answer should be $-2x$.

All together, the corrected answer is $-3x^2 - 2x + 1$.

Learning Activity 4.5

Simplifying Expressions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. The distance from Flin Flon to Thompson is approximately 380 km. At an average speed of 90 km/h, how long will it take to drive from Flin Flon to Thompson?
2. Evaluate $3t - 4t^2$ if $t = -1$.
3. Solve for g: $5g + 2 = -3$.
4. What two numbers have a difference of 4 and a product of 12?
5. Is 4.56 x 24.875 closer to 1000, 100 or 10?
6. You buy lunch for $5.05. You give the cashier a $10 bill and a nickel. How much change do you get?
7. Simplify: $5^2 - 4^2$.
8. Simplify: $3^2 + 3^3$.

continued

Learning Activity 4.5: Simplifying Expressions (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Simplify the following expressions by collecting like terms.
   a) $3a^2 - 6 + a^2 - 5a + 2$
   b) $-4v^2 - v + 5 - 2v^2 + 6v - 2$
2. Simplify the following expressions by collecting like terms.
   a) $3x^2 - 2y^2 - 3x - 6y + 4x^2 - 4y^2 - 6y + 4x$
   b) $-6m^2 + 3n - 5n^2 + 4 - m^2 + 2n^2 - 3m + n - 5$
3. Identify the error, and correct it.

   $-3x^2 + 4x - x^2 + 9 - x$

   $3x^4 + 3x + 9$

Lesson Summary

In this lesson, you combined and simplified polynomial expressions by collecting positive and negative like terms. In the next lesson, you will learn how to subtract polynomial expressions.

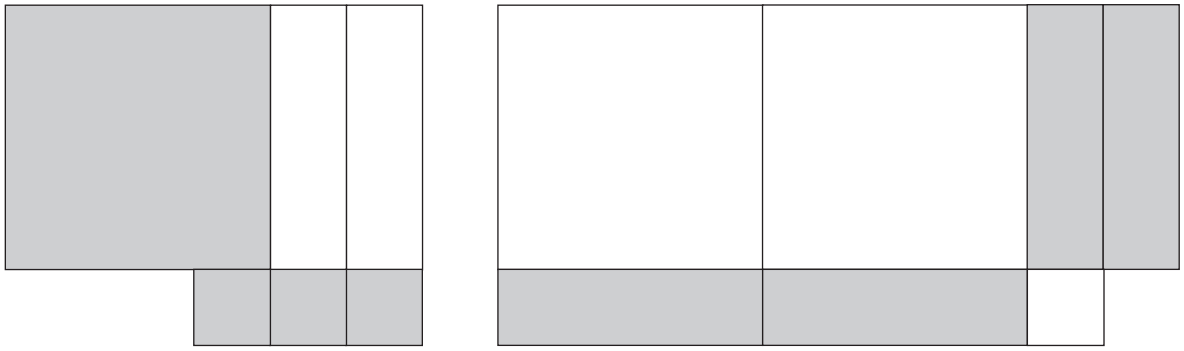
Assignment 4.3: Collecting Like Terms

Total: 20 marks

1. Draw algebra tile diagrams to illustrate each polynomial, then illustrate the answer. *(5 marks)*

   $x^2 + 3x - 4$ combined with $-2x^2 - 2x + 2$

2. Given these algebra tile diagrams, write a polynomial for each, and then combine and simplify the expression. *(2 marks)*



3. Use any method to simplify these polynomial expressions. *(4 marks)*

   a) $2x^2 - 3x + x^2 + 5x - 7$

   b) $-5n^2 + 7 - 3n - n^2 + 2n - 8$

4. Find the perimeter of the following figure. *(2 marks)*

5. Simplify the following expanded expressions. *(4 marks)*

   a) $a^2 - 3a + 5b^2 - 4a^2 - b - 3b^2 - 5b - a$

   b) $3x^2 - 4y - y^2 + 5 - 6x + 8y - 3y^2 + x^2 - 2 + x$

6. Identify the error(s) made, and simplify correctly. *(3 marks)*

$-3x^2 - 5x + 6 + x^2 - x - 8$

$= 2x^2 + 6x - 2$

LESSON 4: DISTRIBUTE THE NEGATIVES

Lesson Focus

When you finish this lesson, you will be able to

- ❑ subtract polynomial expressions
- ❑ show how negative values are distributed
- ❑ identify errors made

Lesson Introduction

In this lesson, you will see that subtracting polynomials can be thought of as combining the opposite of the given expression. To show how the negative is applied to the polynomial, you will use arrows to indicate the distribution.

Using Negative Signs

One of the most common mistakes made in mathematics involves the improper use of negative signs. This lesson focuses on using arrows to show how a negative value is distributed through an expression. Using brackets to identify the polynomial expression that needs a negative value applied, and then using arrows to show the actual distribution should eliminate any errors.

Subtraction

(x) and ($-x$) are said to be opposites. When subtracting values, you can think of it as performing the opposite operation of adding. To use similar vocabulary as in the last lesson, we could read this expression: $4 - (2)$ as "combining 4 and the opposite of 2." This is because the negative sign in front of the bracket means you need to take the opposite of whatever is inside the bracket.

When subtracting polynomials, for example $4 - (2 + x)$, you need to subtract, or combine the opposite of each term in the bracket. The subtract sign is a negative sign that needs to be applied, or distributed, to each term in the bracket. Then the expression can be written without the brackets and like terms can be collected to simplify the polynomial.

Example 1

Subtract $2x + 1$ from the polynomial expression $3x + 5$.

Solution:

First, write this operation using brackets around the expression to be subtracted.

$3x + 5 - (2x + 1)$

The bracket indicates which terms need to be subtracted.

Next, the negative sign must be applied to each term in the bracket. You can think of it as taking the opposite of each term in the bracket, or as multiplying each term in the bracket by –1. The negative sign without a value or constant means –1.

To show this, you can use arrows to apply or distribute the negative to each term.

$= 3x + 5 - (2x + 1)$

You can see how we took the negative value of 1 and distributed it to each term in the second expression. If you force yourself to draw in the arrows each time, you will force your mind to process each step needed, and you will make fewer mistakes!

Negative 1 times positive $2x$ gives us $-2x$, and negative 1 times positive 1 gives us –1.

Or, the opposite of $(2x + 1)$ is $-2x - 1$.

Remember that negative times positive gives a negative, and that negative times negative gives us a positive answer.

$= 3x + 5 - 2x - 1$

Notice that by distributing the negative value, the brackets have been eliminated.

Now collect the like terms to simplify the two expressions:

$= x + 4$

You may want to include an example of how to distribute a negative on your Resource Sheet.

Example 2

$-4x + 3$ subtract $2x - 5$

Solution:

$= -4x + 3 - (2x - 5)$

Notice how the second expression is put in a bracket. This identifies where the negative has to be distributed, or which terms you need to take the opposite of.

Using arrows, show the actual distribution of the negative through the bracket. Note that 3 is not being distributed, only the negative 1 immediately in front of the bracket.

$= -4x + 3 - (2x - 5)$

$= -4x + 3 - 2x + 5$

$-1 * -2x = -2x$

$-1 * -5 = 5$

Or, the opposite of $(2x - 5)$ is $-2x + 5$.

Collect and combine the like terms.

$= -6x + 8$

The previous examples used only binomials. The same method works with trinomials.

Simply bracket the part to be subtracted, and distribute the negative value throughout the bracket, or take the opposite of all the terms inside the bracket.

Example 3

Subtract $-2f^2 - 3f + 2$ from $6f^2 + 5f - 3$.

Solution:

$= 6f^2 + 5f - 3 - (-2f^2 - 3f + 2)$

The opposite of $(-2f^2 - 3f + 2)$ is $2f^2 + 3f - 2$

Or $-1 * -2f^2 = 2f^2$

$-1 * -3f = 3f$

$-1 * 2 = -2$

$= 6f^2 + 5f - 3 + 2f^2 + 3f - 2$

Now that the brackets are gone, collect and combine the like terms.

$= 8f^2 + 8f - 5$

Learning Activity 4.6

Applying Negative Signs

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write three equivalent fractions for $\frac{6}{10}$.
2. Convert 350 cm to m.
3. Calculate $2^4 - 2^2$.
4. Collect like terms: $3n + 2m - 5m - 7n$.
5. Write as a power with a single exponent: $\left(2^3\right)^4$.
6. What two numbers have a sum of –12 and a product of –45?
7. How much will you pay for 40 litres of gas if the cost is $0.99 per litre?
8. Two chocolate bars cost $1.50. How much will 6 chocolate bars cost?

continued

Learning Activity 4.6: Applying Negative Signs (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use arrows and brackets to show how you are distributing the negative value through the bracket, and simplify. Check your answers in the Learning Activity Keys found at the end of this module.

1. $6x - 4 - (2x + 3)$
2. $-5g + 4 - (-3g - 2)$
3. $-2y^2 - 3y + 4 - (3y^2 - 5y + 6)$
4. $3x - 7$ subtract $5x - 3$
5. Subtract $4p - 2$ from $6p - 5$

Identify the Error

Nicole showed her work in answering the following question, and still the teacher marked it wrong. Find the error she made, and correct it.

$5b - 4 - (-2b + 3)$

$= 5b - 4 - 2b - 3$

$= 3b - 7$

Lesson Summary

In this lesson, you learned how to use arrows to show the distribution of negative values through a bracket to indicate when you are taking the opposite of a term. The next lesson deals with multiplication of polynomial expressions.

NOTES

Assignment 4.4: Distribute the Negatives

Total: 23 marks

1. Simplify the following polynomial expressions. *(4 marks)*
   a) $3x - 4 - 5x - 1$ *(2 marks)*

   b) $6y + 2 - y - 3$ *(2 marks)*

2. Use arrows to show the distribution of the negative, and simplify. *(4 marks)*
   a) $-(x - 4)$ *(2 marks)*

   b) $-(3v^2 - 4v + 6)$ *(2 marks)*

3. Use arrows to show the distribution of the negative, and simplify. *(4 marks)*

   a) $-5k + 4 - (-4k - 3)$ *(2 marks)*

   b) $26d - 16 - (-8 + 10d)$ *(2 marks)*

4. Subtract. *(4 marks)*

   a) $9x^2 - 6x + 3 - (8x^2 - 6x + 4)$ *(2 marks)*

   b) $-7r^2 + 5r - 4 - (-3r^2 - 3r + 1)$ *(2 marks)*

5. $14w - 8$ subtract $-5w - 2$ *(2 marks)*

6. Subtract $5x - 6$ from $-3x + 7$. *(2 marks)*

7. Identify the error, and correct it. *(3 marks)*

   $3x - 4 - (-2x - 3)$

   $3x - 4 + 2x - 3$

   $5x - 7$

NOTES

LESSON 5: MULTIPLICATION

Lesson Focus

When you finish this lesson, you will be able to

- ❑ model multiplication of polynomials with tile diagrams
- ❑ model multiplication of polynomials with grid diagrams
- ❑ use arrows to show distribution
- ❑ identify errors made in simplifying expressions

Lesson Introduction

So far in this module, you have been combining (or adding) and subtracting polynomial expressions. This lesson deals with multiplication. It will show three methods of illustrating the actual process of multiplying polynomials.

Multiplying Monomials Review

$x * 2x = 2x^2$

$3x * -4x = -12x^2$ **Note:** Multiply the similar components:

Coefficients: $3 * -4 = -12$

Variables: $x * x = x^2$

Then combine the results as a monomial: $-12x^2$

Many people forget how to multiply monomials until they have had a lot of practice, so you may want to include these instructions on your Resource Sheet for this module.

$x * y = xy$

$(4a)(-2b) = -8ab$ **Note:** Brackets can be used to indicate multiplication.

Multiplying Polynomials

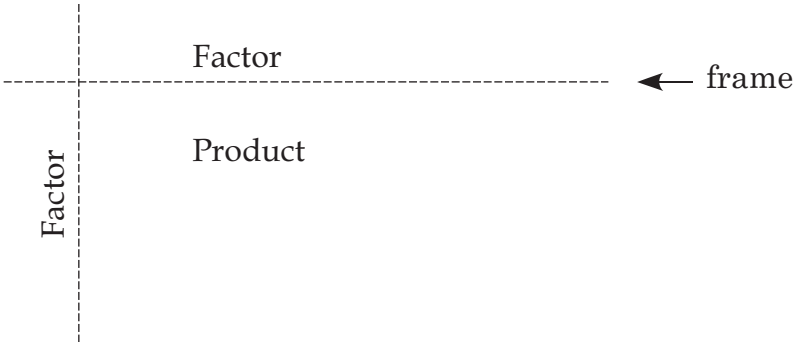
In your own words, you may include on your Resource Sheet the steps on how to multiply polynomials for each of the three methods described in this lesson.

1. Multiplication with tiles
2. Multiplication through the grid method
3. Distribute using arrows

Multiplication with Tiles

Algebra tile diagrams can be used to show the multiplication process. You can model this process using the tiles you created from the template in Appendix A.

To calculate the area of a rectangle, the dimensions of length and width are multiplied. This concept is the basis for showing multiplication using tiles. Tiles representing the factors (think length and width) are arranged across the top, and down the side of a frame, and the rectangular area between them represents the product or answer to the multiplication question.

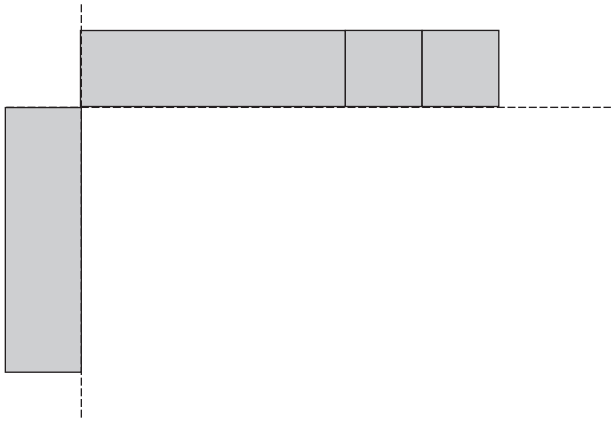


Example 1

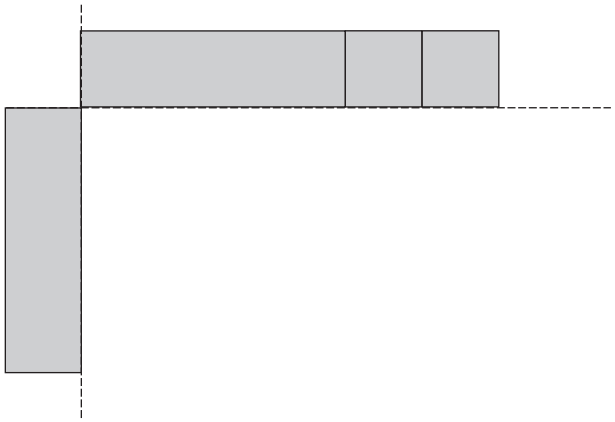
Simplify: $x(x + 2)$

Solution:

$x + 2$ multiplied by x would be arranged as shown below.

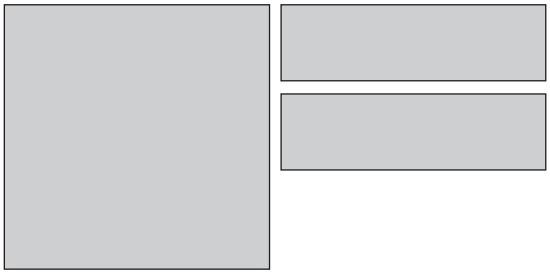


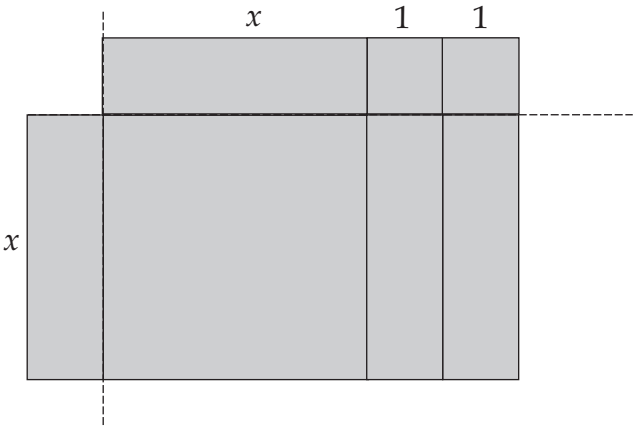
The goal is to fill the empty space to create a rectangle. The tiles within that rectangle form the answer to the multiplication.



Start by sliding in the largest tile that would fit, matching the dimensions of the factors along the frame. Then work your way down to the smallest tiles needed to make a complete rectangle.

These are the tiles you will need for this product!





The rectangle is now filled, and you can read the answer.

There is one x^2-tile, and two x-tiles. Combine the like terms, and the product, or answer, is

$$x^2 + 2x$$

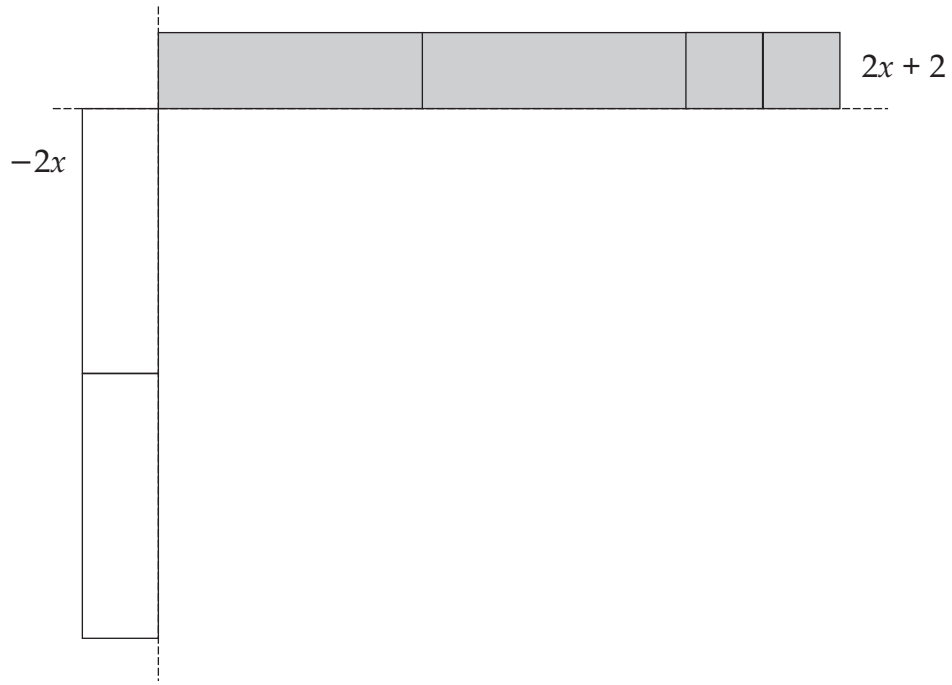
Now, it is very evident that $x * x = x^2$ because that shape fills in the area between the two x-tile factors. $1 * x = x$ because that tile completes the rectangle shape.

Example 2

Simplify: $2x + 2$ multiplied by $-2x$

Solution:

Place the necessary tiles representing the factors along the top and side of the frame.



Now, select the appropriate tiles to fill the rectangle. Watch the negative and positive values, and remember you are multiplying them.

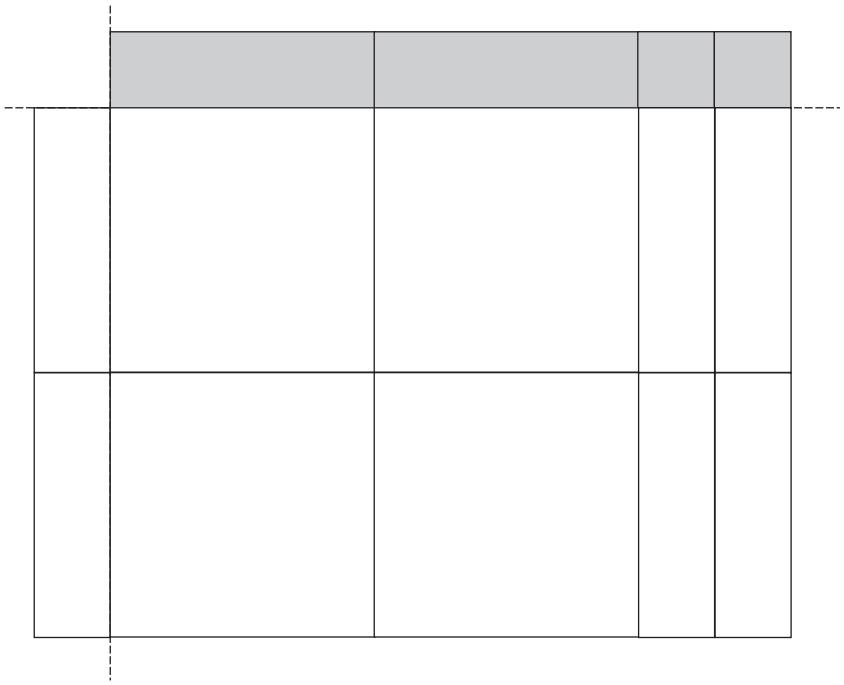
$(x)(-x) = -x^2$

This is represented by a white square tile. You will need 4 of these.

$(1)(-x) = -x$

This is represented by a white x-tile. You will also need 4 of these.

Now, fill these tiles into the space for the rectangle.



Now that the rectangle is full of tiles, the answer can be read as

$$-4x^2 - 4x$$

Showing Multiplication Through the Grid Method

Many students use the grid method to multiply polynomials. The same examples that were used for the tile method to multiply polynomials are used to illustrate the grid method.

Example 1

Simplify: $(x + 2)(x)$

Solution:

Set up a grid with the terms of the first expression across the top, and the second along the side. These terms are all positive.

	x	2
x		

Multiply the corresponding terms to fill in the grid, similar to how the factors were multiplied with the tiles.

x times x is x^2

x times 2 is $2x$

	x	2
x	x^2	$2x$

The answer for $(x)(x + 2)$ is the terms in the grid or $x^2 + 2x$.

Example 2

Simplify: $(2x + 2) * (-2x)$

Solution:

Set up a grid with the terms of the binomial across the top, and the monomial along the side. Notice the negative sign is included.

	$2x$	2
$-2x$		

Multiply the terms to fill in the grid. Watch the signs.

$-2x$ times $2x$ is $-4x^2$ (negative times positive is negative)

$-2x$ times 2 is $-4x$

	$2x$	2
$-2x$	$-4x^2$	$-4x$

The product or answer is $-4x^2 - 4x$.

Distribute Using Arrows

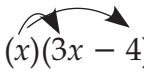
The third method used to represent multiplication with polynomials uses arrows to show the monomial being multiplied or distributed over the second polynomial expression.

Example 1

Simplify: $(x)\ (3x - 4)$

Solution:

Use arrows to show that the $3x$ is multiplied by the x, and also the -4 is multiplied by the x.



x times $3x$ gives us $3x^2$

x times -4 gives us $-4x$

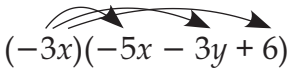
The answer then is $3x^2 - 4x$.

Example 2

Simplify: $(-3x)(-5x - 3y + 6)$

Solution:

Use arrows to show that the $-3x$ is distributed, or multiplies, each term in the second polynomial.



$-3x$ times $-5x$ is $15x^2$

$-3x$ times $-3y$ is $9xy$

$-3x$ times 6 is $-18x$

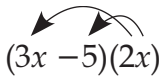
Combine each of these products to get the answer of $15x^2 + 9xy - 18x$.

Example 3

Simplify: $(3x - 5)(2x)$

Solution:

Sometimes the monomial is placed on the right side. The arrows showing distribution would then go from right to left.



$(3x - 5)(2x)$

$2x$ times $3x$ is $6x^2$

$2x$ times -5 is $-10x$

Collect these to show an answer of $6x^2 - 10x$.

Finding Area

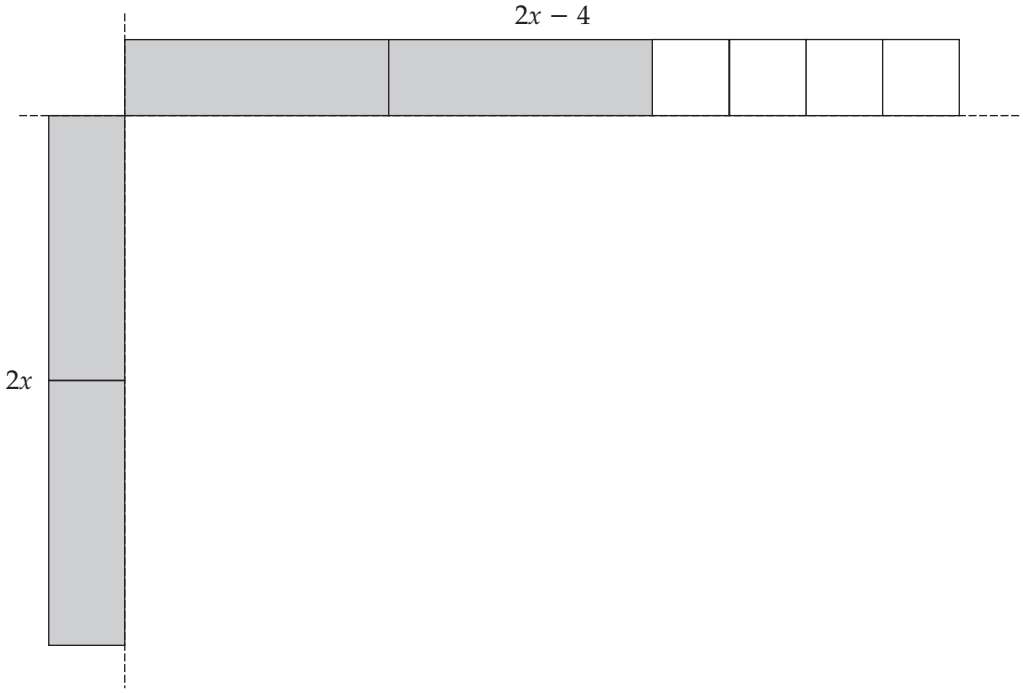
If a room measures 3 m by 5 m, the area can quickly be calculated as $15m^2$ (Area = length $*$ width).

If the dimensions of a figure are represented by polynomials, you can calculate an expression representing the area by using your choice of methods: algebra tiles, grids, or arrows.

Find the area of a figure $2x - 4$ long, and $2x$ wide.

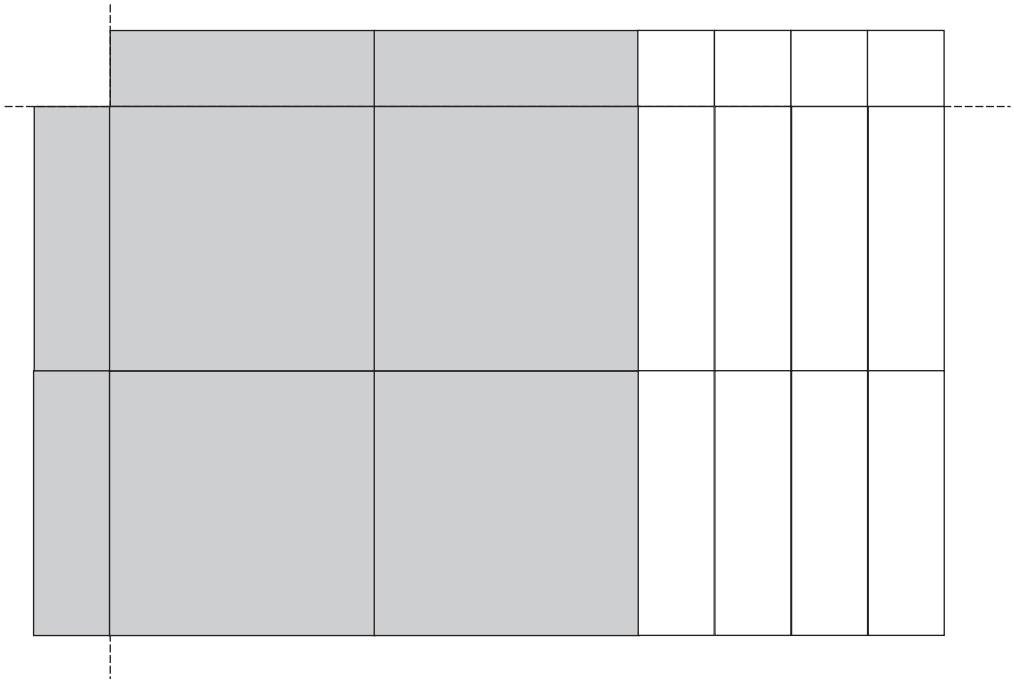
Method A: Algebra Tiles

Use tiles to represent the factors along the top and side of the frame



Fill in the rectangle area with tiles to find the answer.

Remember to watch the sign when multiplying negative and positive values.



The tiles within the rectangle show $4x^2 - 8x$.

Method B: Use a Grid

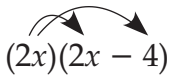
	$2x$	-4
$2x$		

Multiply the values for each part of the grid.

	$2x$	-4
$2x$	$4x^2$	$-8x$

Combining your terms, the answer is $4x^2 - 8x$.

Method C: Use Arrows



So $2x * 2x = 4x^2$

$2x * (-4) = -8x$

The answer is $4x^2 - 8x$.

Learning Activity 4.7

Multiplication with Algebra Tiles

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Simplify $-3m + 6n - 4m$.
2. You worked 8 hours last week and are paid $9.50 an hour. How much did you earn?
3. Calculate $\frac{1}{2} + \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$.
4. Write as a power with a single exponent: $\left(4^5\right)\left(4^7\right)$.
5. Is $\frac{4}{5} - \frac{3}{4}$ positive or negative?
6. The Winnipeg Goldeyes baseball stadium has 20 sections. Each section holds approximately 400 people. How many people would make a sell-out crowd?
7. What is the area of a square that has sides 4 m long?
8. Evaluate $5h - 7$ if $h = 2$.

Learning Activity 4.7: Multiplication with Algebra Tiles (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use algebra tiles to illustrate this multiplication, and state the answer.

   $(2x)(-x + 2)$

2. Use the grid method to illustrate this multiplication, and state the answer.

   $(-3x)(4x - 7)$

3. Use arrows to show the distribution of the first term, and state the answer.

   $(5x)(3x - 4y - 6)$

4. Describe the error made in this multiplication, and correct it.

   $(-3a)(-4a + 6)$

	$-4a$	6
$-3a$	$12a^2$	$18a$

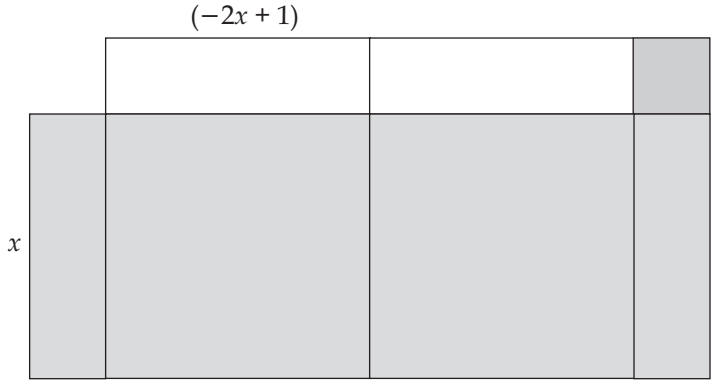
   $12a^2 + 18a$

continued

Learning Activity 4.7: Multiplication with Algebra Tiles (continued)

Identify the Error

1. Find the error in this algebra tile illustration, and correct it.



2. Find the errors in this grid illustration, and correct them.

	$-3x$	4
$-5x$	$-15x^2$	$20x$

3. Find the errors in the following arrow illustration, and correct them.

$(-x)(-5x - 3y + 1)$

$5x - 3xy - 1$

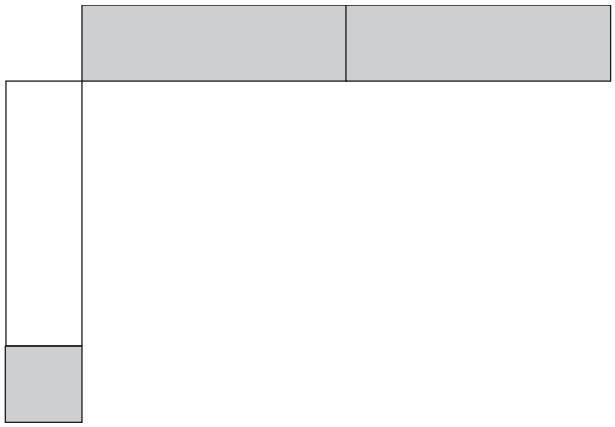
Lesson Summary

In this lesson, you used algebra tiles, the grid method, and arrows to show how polynomials are multiplied. The next lesson deals with the division of polynomial expressions, and with recognizing equivalent expressions.

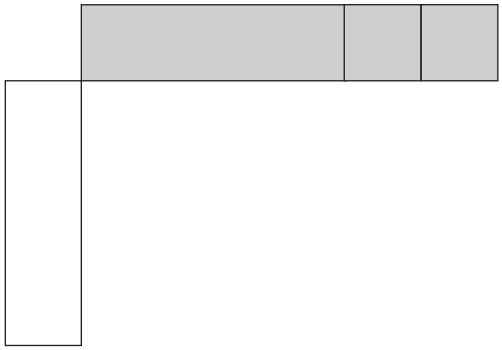
Assignment 4.5: Multiplication

Total: 18 marks

1. Find the product, given the following tile arrangement. *(2 marks)*



2. Find the product, given the following tile arrangement. *(2 marks)*



3. Find the product, given the following grid arrangement. *(2 marks)*

	$-3x$	5
$-x$		

4. Find the product using arrows to illustrate the distribution. *(4 marks)*

a) $(3x)(-2x + 2)$ *(2 marks)*

b) $(5x - 4)(-x)$ *(2 marks)*

5. Use any method to simplify these polynomial expressions. *(4 marks)*

a) $(-2x)(x - 2y + 4)$ *(2 marks)*

b) $(8x - 9)(-3y)$ *(2 marks)*

6. Identify the error(s) made, and correct it. *(4 marks)*

$(3x)(4x - 2y - 3)$

$(3x)(4x - 2y - 3)$

$12x^2 + 6xy - 6x$

NOTES

LESSON 6: DIVISION AND EQUIVALENTS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ model division of polynomials with tile diagrams
- ❑ simplify polynomial expressions using division
- ❑ identify errors made in simplifying expressions
- ❑ identify equivalent expressions

Lesson Introduction

If addition and subtraction are opposite operations, then division is the opposite of multiplication. In this lesson, you will learn how to simplify polynomial expressions using division. Also, you will explore equivalent expressions, and how to simplify them.

Multiplication versus Division

Consider 6 x 3. We know that this operation results in 18. The opposite operation could be 18 divided by 3 equals 6 or 18 divided by 6 equals 3.

$$6 \times 3 = 18 \quad \rightarrow \quad \frac{18}{3} = 6 \text{ or } \frac{18}{6} = 3$$

Another example would be –5 times 4 equals –20. An opposite operation is –20 divided by 4 equals –5.

$$-5 \times 4 = -20 \quad \rightarrow \quad \frac{-20}{4} = -5$$

In the powers unit, you learned that:

$(x^3)(x^4) = x^7$

An opposite operation of division finds:

$$\frac{x^7}{x^4} = x^3$$

If you have both variables and coefficients, $2x * 5x = 10x^2$, and an opposite operation would find $10x^2 \div 5x = 2x$.

$$(2x)(5x) = 10x^2 \quad \rightarrow \quad \frac{10x^2}{5x} = 2x$$

It is important that you recognize multiplication is the opposite, or inverse, operation to division. In the last lesson, you used the terms "factor" and "product " when talking about multiplication and the different parts of the tile diagrams.

(factor) * (factor) = (product)

	Factor
Factor	Product

When talking about division, the correct mathematical terms to use are:

(dividend) ÷ (divisor) = (quotient)

In a tile diagram, they would be placed as follows:

	Quotient
Divisor	Dividend

Dividing Polynomials with a Monomial

You may want to include on your Resource Sheet the steps for dividing polynomials by a monomial for each of the two methods described in this lesson.

1. Division using algebra tiles
2. Division using arrows

Division Using Algebra Tiles

Example 1

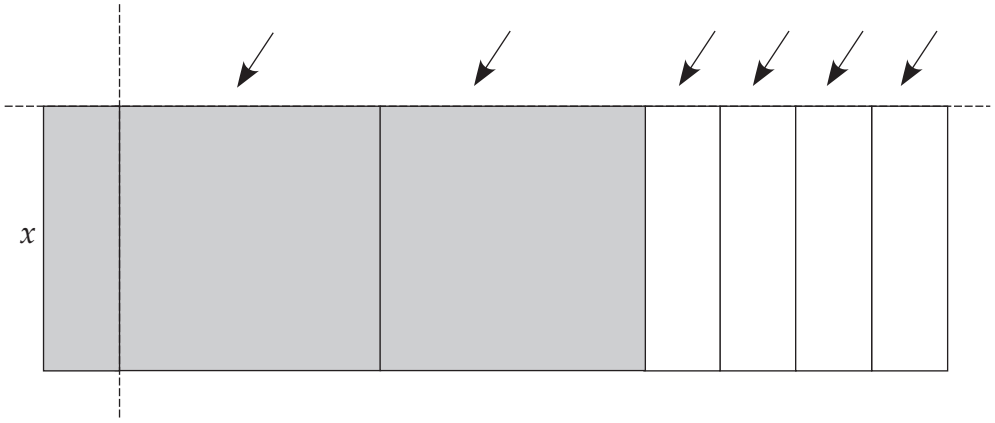
Simplify: $(2x^2 - 4x) \div x$

Solution:

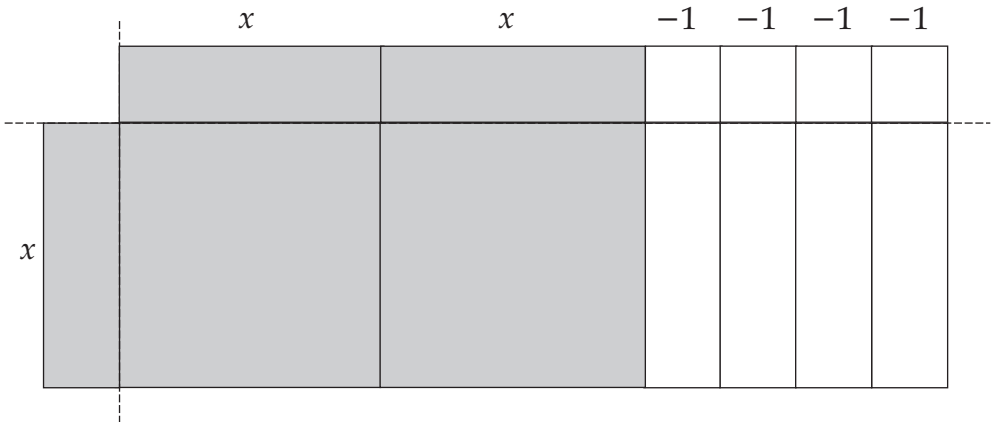
Given a polynomial expression, you can use tiles to find the answer, or the quotient.

First, place the tile representing the divisor along the vertical edge of the frame.

Next, place the tiles representing the dividend inside the frame, in a rectangle arrangement that lines up with the divisor.



You can see the dividend $2x^2 - 4x$ arranged as a rectangle inside the frame, lined up with the divisor of x along the vertical side. Your task is to find what tiles can replace the arrows above. This would be the quotient when $2x^2 - 4x$ is divided by x.



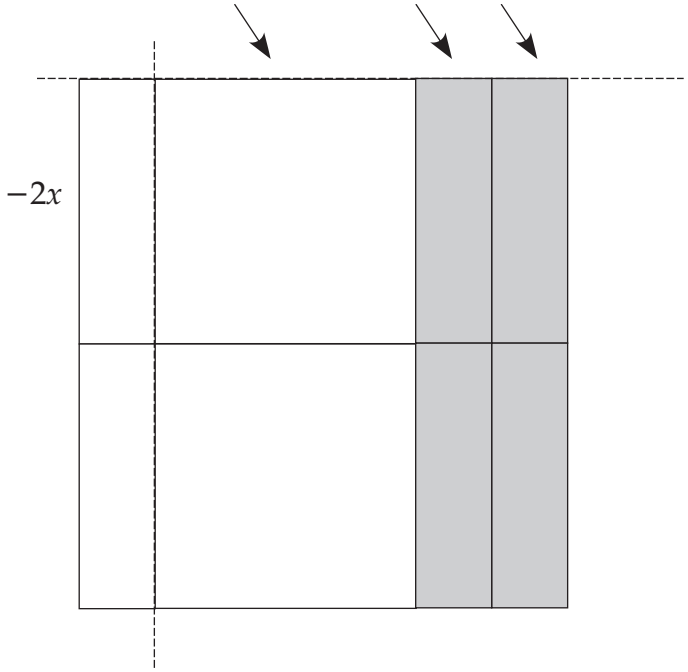
This illustration shows two x-tiles, and four –1 tiles fill in the top row.

So the answer to $(2x^2 - 4x) \div x = 2x - 4$.

Example 2

Use algebra tiles to find $(-2x^2 + 4x) \div (-2x)$.

Solution:



The dividend is arranged so the tiles make a rectangle that matches the dimensions of the divisor along the vertical edge.

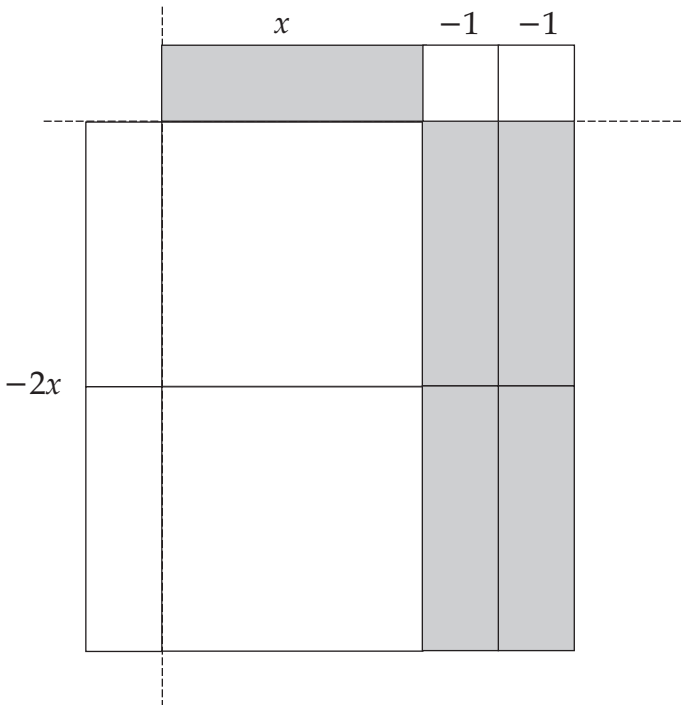
Now place tiles to fill in the top row. This will be the answer. Be careful with the negative values here!

x times $-x$ is $-x^2$

-1 times $-x$ is positive x

Answer:

$(-2x^2 + 4x) \div (-2x) = x - 2$



Division Using Arrows

Recall: $\frac{10x^2}{5x} = 2x$

It is important to remember that similar components of the terms are divided:

coefficient ÷ coefficient: 10 ÷ 5 is 2

variable ÷ variable: $x^2 \div x = x$

So, $\frac{10x^2}{5x} = 2x.$

When a polynomial dividend has two or more terms, each term must be divided by the divisor!

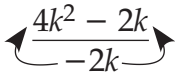
(recall: $\frac{\text{dividend}}{\text{divisor}}$ = quotient)

Write each term over the divisor to remind you that each must be divided by the value in the denominator.

$$\frac{4k^2-2k}{-2k} \text{ splits into } \frac{4k^2}{-2k}-\frac{2k}{-2k}$$

(As always, be careful with the signs in the numerator!)

Arrows can also be used to show the value in the denominator dividing or going into each term along the top.



This is really the same as splitting the terms.

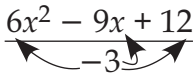
$$\frac{4k^2}{-2k}-\frac{2k}{-2k}$$

To solve, $\frac{4k^2}{-2k}$ is $-2k$, and $\frac{-2k}{-2k}$ is 1, so the answer is $-2k + 1$.

Using arrows is especially helpful when there is a longer expression to work with.

$$\frac{6x^2-9x+12}{-3}$$

In this example, the –3 has to be divided into each of the terms in the dividend. Use arrows to direct your thinking for each step:



Following each arrow:

$$\frac{6x^2}{-3} \text{ is } -2x^2$$

$$\frac{-9x}{-3} \text{ is } 3x$$

$$\frac{12}{-3} \text{ is } -4$$

The answer is $-2x^2 + 3x - 4$.

Learning Activity 4.8

Division with Algebra Tiles

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Simplify $5t - 6s + 7s - 9t$.
2. You go grocery shopping and buy 3 items for $0.99 each, 2 items for $2.25 each, and one item worth $1.59. You have a $10 bill with you. Do you have enough money?
3. Dauphin Country Fest had 300 volunteers who worked 3500 hours to clean up the site after the event was over. Approximately how many hours did each volunteer spend helping out?
4. Write as a power with a single exponent: $\frac{5^6}{5^2}$.
5. You made $540 last week for working 30 hours. What is your hourly wage?
6. Budgeting guidelines suggest you budget 30% of your gross earnings for housing expenses. If your gross monthly earnings are $2000, how much should you be spending on housing expenses?
7. What two numbers have a sum of –7 and a product of –30?
8. Calculate $3^2 - 2^3$.

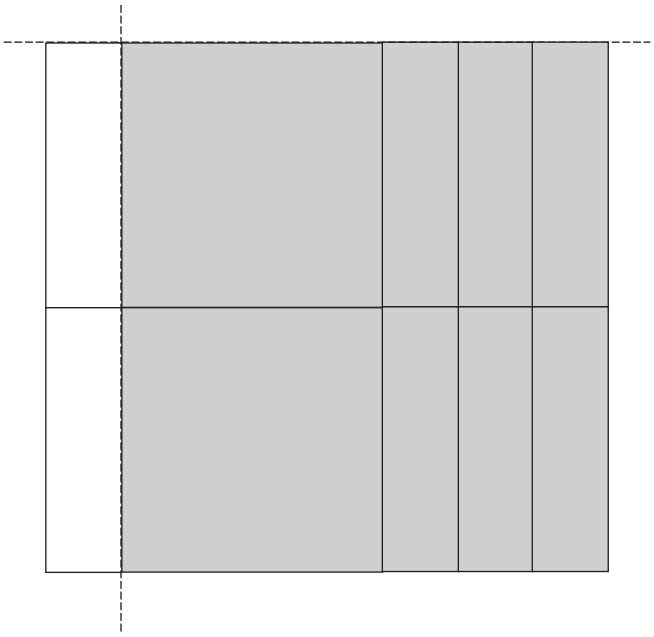
continued

Learning Activity 4.8: Division with Algebra Tiles (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given this illustration of algebra tiles, find the quotient of $(2x^2 + 6x) \div -2x$.



2. Draw an arrangement of algebra tiles used to find the quotient of $(x^2 - 4x) \div x$.
3. Use arrows to divide the bottom value into each of the terms in the numerator, and find the quotient.

$$\frac{7x^2 - 14x + 21}{-7}$$

4. Find the error in the following work.

$$\frac{6x^2 - 9x}{3x}$$
$$= 2x^2 - 3$$

Equivalents

The number 5 can be expressed in a variety of ways.

It could be 2 + 3 or 25 ÷ 5.

These are all equivalent expressions.

In algebra there are also different ways to represent the same expression.

Example 1

Write the expression $2(x + 3)$ in three different ways.

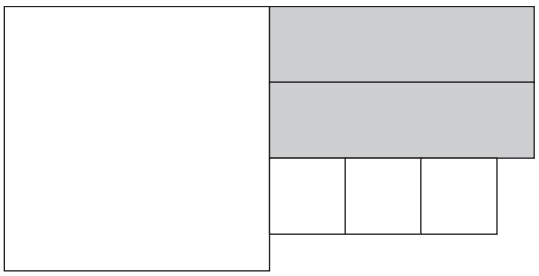
Solution:

$2x + 6$ $(x + 5) + (x + 1)$ $2(x - 1) + 8$

Some expressions are listed in a different order, and may need to be rearranged.

$-4x + 2 + 3x^2$ is equivalent to $3x^2 - 4x + 2$

Equivalent expressions can be identified from tile diagrams. Usually they just need to be rearranged.



These polynomial expressions are equivalent.

Learning Activity 4.9

Expressing Equivalents

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert 4 kg to grams.
2. What two numbers have a product of 11 and a sum of 12?
3. Simplify $m(2m + 3)$.
4. Solve for d: $12 - 2d = 4$.
5. The area of a rectangluar pool is 50 m^2. One side measures 5 m. What is the length of the other side?
6. Heather is finished 65% of her homework, and Amanda is finished $\frac{3}{4}$ of hers. Who is finished more of her homework?
7. A shirt you want to buy is on sale for 40% off. The original price is $60. What is the sale price?
8. Write as a power with a single exponent: $\frac{3^3 \cdot 3^7}{3^5}$.

continued

Learning Activity 4.9: Expressing Equivalents (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Which of the following expressions are equivalent to $3x - 6$?
   a) $3x + 6$
   b) $6x - 4$
   c) $3(x - 2)$
   d) $x - 3 + 2x - 3$
   e) $3 + x - 6$
2. Write the expression $-4x + 8$ in two different ways.
3. Draw two tile arrangements illustrating $2x^2 + 3x - 2$.

Lesson Summary

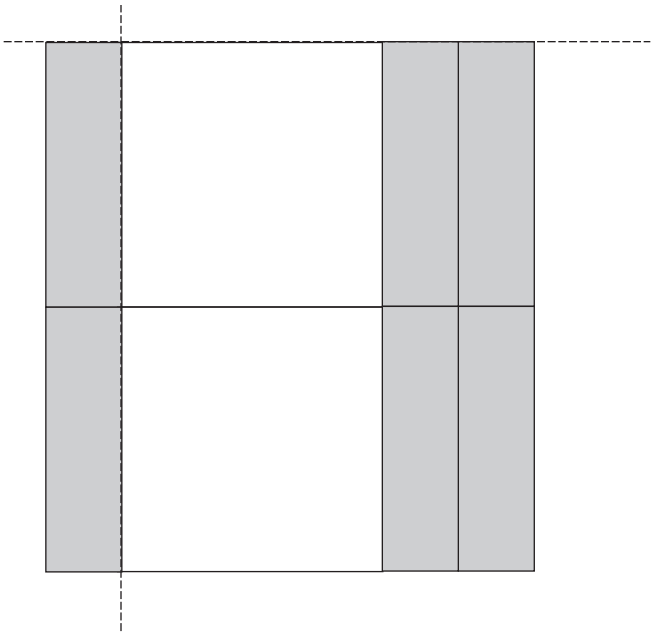
In this lesson, you learned how to divide polynomial expressions. You also learned how to recognize and write equivalent polynomial expressions. The next lesson deals with finding perimeter and area using polynomial expressions in geometric shapes.

NOTES

Assignment 4.6: Division and Equivalents

Total: 17 marks

1. Given the following algebra tile diagram, find the quotient. *(2 marks)*



2. Illustrate $(x^2 + 3x) \div (-x)$ by drawing an algebra tile diagram and give the answer. *(2 marks)*

3. Illustrate using arrows $(4g^2 - 2g - 8) \div (-2)$ and state the answer. *(3 marks)*

4. Use any method to divide the following polynomial expressions. Show the steps you use, and state the answers. *(6 marks)*

   a) $\frac{18d^2 - 21d + 12}{-3}$ *(3 marks)*

   b) $\frac{-3x^2 - 3x}{-3x}$ *(3 marks)*

5. Identify which of the following expressions are equivalent to $-3m + 2$. *(2 marks)*

   a) $m + 6 - 4m - 4$

   b) $-3 + m + 2$

   c) $-6m + 4$

   d) $8 - 3m - 6$

   e) $8 - 3m + 6$

6. Write two equivalent expressions to $-2x - 4$. *(2 marks)*

NOTES

LESSON 7: GEOMETRY AND POLYNOMIALS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ add polynomial expressions based on a geometric shape
- ❑ find the perimter of geometric shapes
- ❑ write algebraic expressions from diagrams
- ❑ describe a situation using a polynomial expression

Lesson Introduction

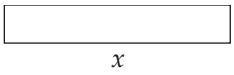
Often shapes have incomplete measurements. You can still write expressions to represent the shape's area or perimeter by assigning a variable to the unknown dimension and using it in calculations.

If you find any of the examples in this lesson particularly helpful, include them on your Resource Sheet.

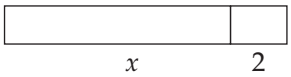
Shapes and Equivalent Expressions

Example 1

This boat dock is x metres long.



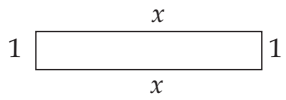
A long, dry summer is lowering the water level in the lake, and you need to add 2 m to the dock. The new dock is $x + 2$ metres long.



If the width has a value of 1 metre, find the perimeter of the both the old and the new docks.

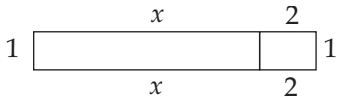
Solution:

To find the perimeter or distance around the dock, you have to measure the length of all exterior edges.



The old dock's perimeter is $x + 1 + x + 1$ or $2x + 2$ metres.

You can write $2(x) + 2(1)$ or $2(x + 1)$ as equivalent expressions.

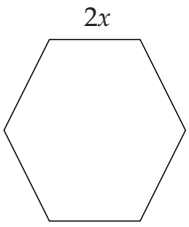


The new dock has dimensions of $x + 2 + 1 + 2 + x + 1$. Simplified, that reads as $2x + 6$ metres.

You can write $2(x) + 2(2) + 2(1)$ or $2(x + 3)$ as equivalent expressions.

Example 2

You are building a six-sided (hexagon) flower garden. The length of each side is $2x$. Find the total length of the sides.



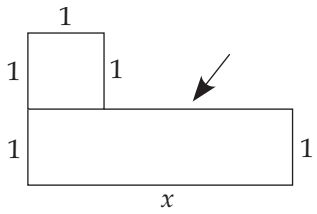
Solution:

You could add them as follows: $2x + 2x + 2x + 2x + 2x + 2x = 12x$.

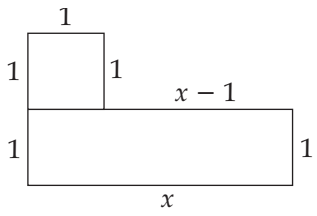
Or, you could write an equivalent expression $6(2x)$.

Finding Unknown Lengths in Perimeter

To find the perimeter of this figure, carefully label the length of each side. What value does the side with the arrow have?



The bottom side is x-units long. The side in question is 1 unit less than this, because the square reduces its length by 1. Its measurement can be expressed as $x - 1$. Once you label that on the diagram, you have the dimensions of each section and you can combine them to find the perimeter.

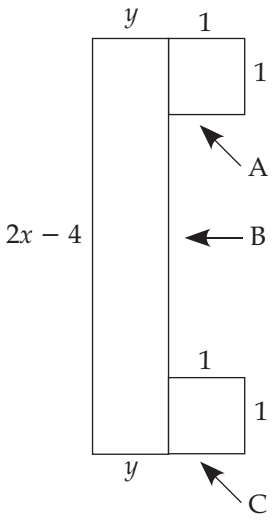


$5(1) + x + (x - 1)$ is an expression that represents all sides.

It can be simplified into an equivalent expression of $2x + 4$.

Example 1

Find the perimeter of this shape, carefully labelling each unknown side based on lengths you do know.



Solution:

The letters and arrows indicate the sides with missing measurements. Sides A and C are easily identified as having a value of 1. Side B is more of a challenge. The entire left side has a measurement of $2x - 4$. To find the length of Side B you have to subtract a value of 1 off the top of this side, and a value of 1 off the bottom.

So, the length of side B is $(2x - 4) - 1 - 1$.

Equivalent expressions for this side would be $(2x - 4) - 2$ or $2x - 6$.

Now that you have all the lengths, combine them to find the perimeter.

$$(2x - 4) + 2(y) + (2x - 6) + 6$$

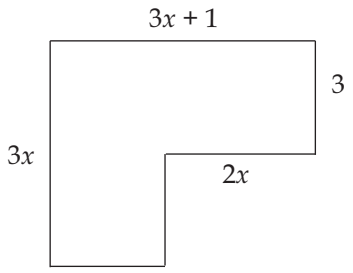
This simplifies to: $4x + 2y - 4$.

Area and Shapes

The area of a rectangular shape can be found by multiplying the length by the width. If a figure is made up of more than one rectangle, find the area of each, and add them up. The first step is to label the length of each side.

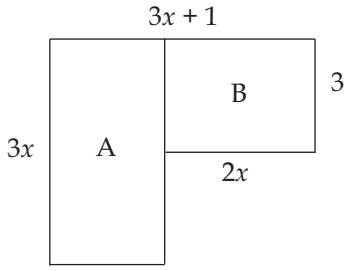
Example 1

Find the area of the following figure.

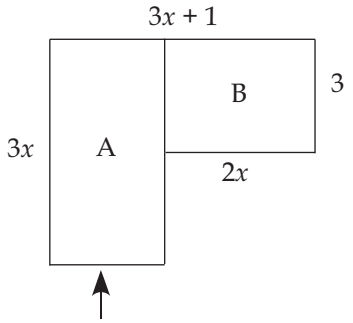


Solution:

First divide the figure into rectangular shaped parts and label them.



Find any missing dimensions using the given expressions.



This length would be the difference between $(3x + 1)$ and $(2x)$.

$$3x + 1 - (2x) = x + 1$$

Calculate the area of parts A and B.

A: $(x + 1)(3x) = 3x^2 + 3x$

B: $(3)(2x) = 6x$

Combine the two areas to find the total.

$$3x^2 + 3x + 6x = 3x^2 + 9x$$

Learning Activity 4.10

Shapes and Equivalent Expressions

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

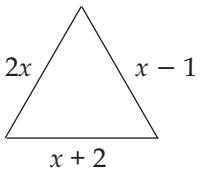
1. What two numbers have a product of –8 and a sum of 2?
2. On his last mathematics test, Gilles got 6 out of 9 questions correct. What is his approximate percentage grade on the test?
3. You start work at 8:45 and work until 16:15. How long did you work?
4. You have $460 in your chequing account. You write cheques for $65, $130, and $35. You deposit $150 in your chequing account. What is your new balance?
5. Collect like terms: $9h - (3g + 2h) + 5g$.
6. Convert 1.25% to a decimal.
7. Convert 0.2 to a fraction.
8. Order the following from smallest to largest: –0.34 –0.43 0.34 0.43

continued

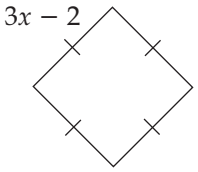
Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

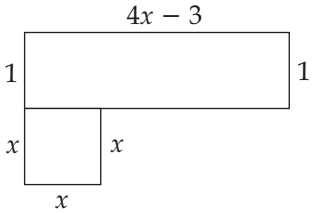
1. Find the perimeter of the following figure.



2. Find the perimeter of the following figure.

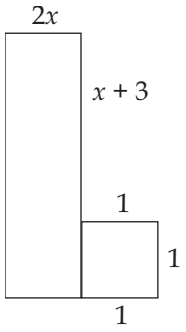


3. Find the perimeter of the following figure.



Learning Activity 4.10: Shapes and Equivalent Expressions (continued)

4. Find the area of the following figure.



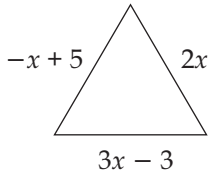
Lesson Summary

In this last lesson, you used polynomial expressions based on shapes to find perimeter and area. You will continue to use the algebra skills developed in these lessons in the next modules, which focus on linear relations and geometry.

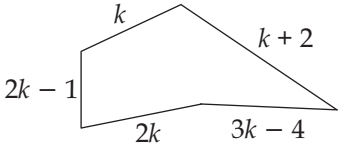
Assignment 4.7: Geometry and Polynomials

Total: 16 marks

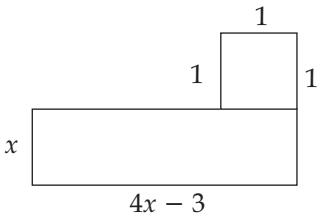
1. Find the perimeter of the following figure. *(2 marks)*



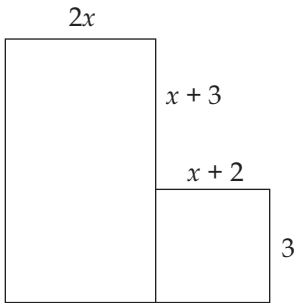
2. Find the perimeter of the following shape. *(2 marks)*



3. Find the perimeter of the following shape. *(3 marks)*



4. Find the area of the following figure. *(4 marks)*



5. Design a garden plot using the following measurements: *(5 marks)*
   - one variable
   - one monomial
   - one binomial

   Find the perimeter and the area of your garden plot. Include a diagram and show your calculations.

MODULE 4 SUMMARY

Congratulations, you have finished half of the modules in this course.

You are learning to communicate mathematically! Using and understanding polynomials is foundational for continued success in your math career. Representing unknown quantities with tiles or variables and performing operations on polynomials will be skills that you practice and perfect as you continue to learn and communicate mathematically. Your understanding of how addition and subtraction, multiplication and division are opposite operations, and how to identify and write equivalent expressions will help you in the rest of the modules in this course, and in future math courses you take.

It is now time to send all of the assignments from Modules 3 and 4 to your tutor/marker for feedback on how you are doing.

Mailing Your Assignments

Before you mail your completed assignments, please photocopy them first. That way, you will have a copy of your assignments in case they go missing. Please organize your material in the following order:

- Cover Sheet for Module 3
- Assignment 3.1: Cover Assignment
- Assignment 3.2: The Meaning and Uses of Powers
- Assignment 3.3: Order of Operations
- Assignment 3.4: Operations with Powers
- Assignment 3.5: Powers with Like Bases
- Assignment 3.6: More Power Laws
- Cover Sheet for Module 4
- Assignment 4.1: Cover Assignment
- Assignment 4.2: Introduction to Polynomials
- Assignment 4.3: Collecting Like Terms
- Assignment 4.4: Distribute the Negatives
- Assignment 4.5: Multiplication
- Assignment 4.6: Division and Equivalents
- Assignment 4.7: Geometry and Polynomials

Place all of these assignments in an envelope and address it to:

ISO Tutor/Marker
555 Main St.
Winkler MB R6W 1C4

Remember: You can contact your tutor/marker at any time if you have questions.

Applying for Your Midterm Exam

You are now ready to write the midterm exam. It is worth 100 marks (15% of your final mark). You will complete this exam while being supervised by a proctor. You should already have made arrangements to have the exam sent to the proctor from the Independent Study Option office. If you have not done so, contact the Independent Study Option at 1-800-465-9915.

In order to do well on the midterm exam, you should review all of the work that you have completed from Modules 1 to 4, review all of your notes, learning activities, and assignments.

You need to bring the following when you write the exam: pens/pencils (2 or 3 of each), blank paper, a calculator, a ruler, and a protractor.

You will have a maximum of **2 hours** to complete this exam. When you have completed the exam, you will hand it in to your proctor, who will then forward it to your tutor/marker for assessment.

Here is how you apply for an exam:

If you are attending school, ask your school's ISO Facilitator to add your name to the ISO exam eligibility list. Do this ***at least three weeks prior*** to the next scheduled exam week.

If you are not attending school, check the **Examination Request Form** for options available to you. The Examination Request Form was mailed to you with this course. Fill in this form and mail or fax it three weeks before you are ready to write your exam. The address is:

ISO Registration
555 Main St.
Winkler, MB R6W 1C4
Fax: 204-325-1719
Phone: 1-800-465-9915

Note: The Midterm Practice Exam is an excellent study aid for reviewing Modules 1 to 4. Instructions for accessing and completing the Midterm Practice Exam are found on the following page.

Completing Your *Midterm Practice Exam*

Getting the Most Out of Your Midterm Practice Exam

Like the Midterm Exam that you will be writing, your Midterm Practice Exam is based on Modules 1 to 4. It is very similar to the actual Midterm Exam. This means that if you do well on your Midterm Practice Exam, you should do well on the Midterm Exam, because you will lave learned the content. You will also feel more confident and less nervous about writing the exam.

The practice exam and answer key can be found at the following website: <www.edu.gov.mb.ca/k12/dl/downloads/index.html>. Complete the midterm practice exam and then check your answers against the key.

If you do not have access to the Internet, contact the Independent Study Option at 1-800-465-9915 to get a copy of the practice exam and its answer key.

To get the most out of your Midterm Practice Exam, follow these steps:

1. Study for the Practice Exam as if it were an actual exam.
2. Review those Learning Activities and Assignments from Modules 1 to 4 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Ask your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 4, including all of your notes, learning activities, and assignments.
5. Use your Module Resource Sheets to make a draft of your Midterm Exam Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following to the Midterm Practice Exam: pens/pencils (2 or 3 of each), blank paper, a calculator, a ruler, and a protractor.
7. Write your Midterm Practice Exam as if it were an actual exam. In other words, write the entire exam in one sitting, and don't check your answers until you have completed the entire thing.
8. Once you have completed the entire exam, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your Resource Sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Midterm Exam Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

NOTES

GRADE 9 MATHEMATICS (10F)

Module 4: Polynomials

Learning Activity Answer Keys

MODULE 4: POLYNOMIALS

Learning Activity 4.1: Identifying Components

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Calculate $10 - 3 \times 6 \div 2$.
2. How many dozen eggs are there in 60 eggs?
3. What two numbers have a product of –81 and a sum of 0?
4. The Winnipeg Blue Bombers scored 3 touchdowns (7 points each) and two field goals (3 points each). The Montreal Alouettes scored 25 points altogether. Who won the game?
5. What is the radius of a circle whose diameter is 46 cm?
6. Convert 600 g to kg.
7. Calculate $3^2 + 2^3$.
8. Which is larger: $\frac{1}{6}$ or $\frac{1}{7}$?

Answers:

1. 1 (multiplication and division are done before subtraction)
2. 5 dozen ($60 \div 12 = 5$)
3. –9 and 9 (the numbers must have the same value but opposite signs to have a sum of zero)
4. Winnipeg Blue Bombers won ($7 \times 3 + 2 \times 3 = 27$, which is larger than 25)
5. 23 cm (radius is half the diameter)
6. 0.6 kg (1000 g in 1 kg)
7. 17 (9 + 8)
8. $\frac{1}{6}$ (one whole divided into 6 pieces would have larger pieces than the same whole divided into 7 pieces)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the numerical coefficients, variables, exponents, degree, constants, and number of terms in each polynomial.

   Answer:

Polynomial	$6x$	$-3n + 1$	$n^2 - 3n + 4$	7
Numerical Coefficient	6	–3	1, –3	none
Variable	x	n	n	none
Exponent	1	1	1,2	none
Degree	1	1	2	0
Constant	none	1	4	7
Number of Terms	1	2	3	1

2. Write a polynomial with:
   a) two terms, one variable, and one constant
   b) three terms, two variables, one constant, one positive numerical coefficient
   c) one term, two variables, negative numerical coefficient

   Possible Answers:

   Answers will vary.

   a) $x + 6$, $a - 5$
   b) $-2r + 5s - 4$, $8x - 3y + 1$
   c) $-xy$, $-4ab$

Learning Activity 4.2: Writing Polynomial Expressions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write three equivalent fractions for $\frac{4}{6}$.
2. Calculate $\sqrt{3^2+4^2}$.
3. You buy an ice cream cone for $2.35 and give the cashier a $5 bill. How much change do you get back?
4. Sheila was born in 1995. Her mother was 25 years old when Sheila was born. What year was Sheila's mother born?
5. A third of your Grade 9 class is going on a ski trip to Duck Mountain Provincial Park. There are 90 students in your Grade 9 class. How many are not going skiing?
6. 92 is between which two perfect squares?
7. You are planning a 1.5 hour football practice. You know you need to have a 10 minute warm-up and a 10 minute warm-down session, there will be two 5-minute breaks and you want to have 20 minutes for a scrimmage. How much time does that leave for drills?
8. What is the perimeter of a rectangular field with dimensions 20 m by 30 m?

Answers:

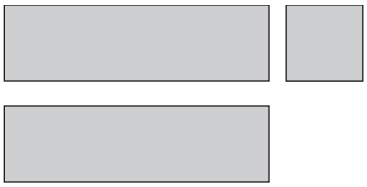
1. Answers will vary; some possible fractions are: $\frac{2}{3}, \frac{6}{9}, \frac{8}{12}, \frac{12}{18}, \frac{40}{60}...$

   To be equivalent, both numerator and denominator must be multiplied or divided by the same number.
2. $5\ \left(\sqrt{9+16}=\sqrt{25}=5\right)$
3. $2.65 (0.65 to get to $3 and another $2 to get to $5)
4. 1970 (1995 – 25)
5. $60\ \left(90\times\frac{1}{3}=30;\ 90-30=60 \text{ or } \frac{2}{3} \text{ are not going; } 90\times\frac{2}{3}=60\right)$
6. 81 and 100
7. 40 minutes (1.5 hours is 90 minutes. Warm-up, warm-down, and breaks is 10 + 10 + 5 + 5 = 30 minutes, then add 20 minutes for scrimmage for a total of 50 minutes which leaves 40 minutes for drills)
8. 100 m (20 + 30 + 20 + 30)

Part B

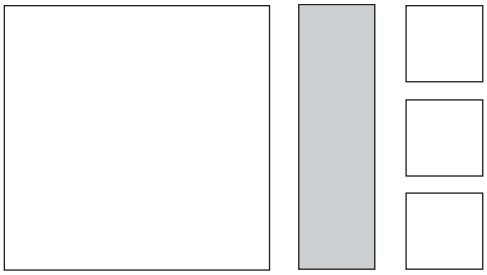
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the polynomial expressions represented by the algebra tiles.

a) 

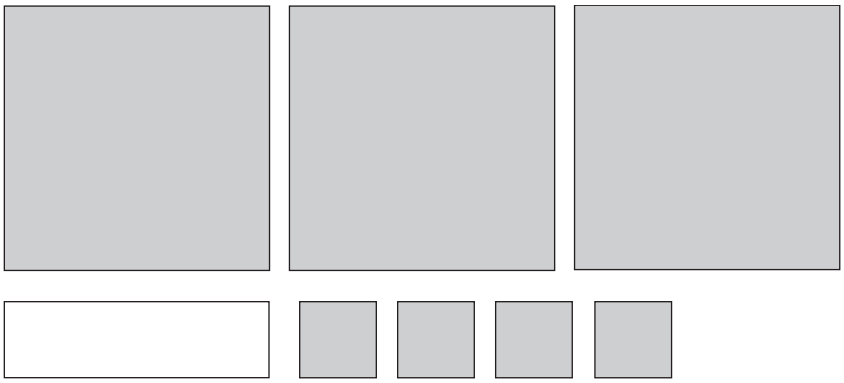
Answer:

$2x + 1$

b) 

Answer:

$-x^2 + x - 3$

c) 

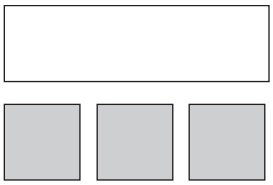
Answer:

$3x^2 - x + 4$

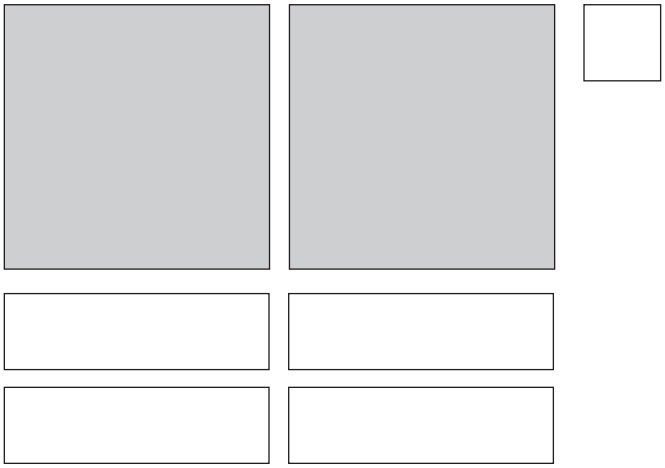
2. Draw algebra tile diagrams to represent the polynomial expressions below.

a) $-x + 3$

Answer:



b) $2x^2 - 4x - 1$



Learning Activity 4.3: Situations as Expressions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Simplify –3 + 4.
2. A pair of shoes that regularly cost $45 are on sale for 10% off. What is the amount of discount?
3. What two numbers have a product of 18 and a sum of 9?
4. You have $65 in your chequing account. You deposit your $350 paycheque and write cheques for $10 and $35. What is your new balance?
5. If 24% of 600 is 144, what is 12% of 600?
6. Solve for t: $3t = 21$.
7. You want to join boot camp to get fit. The cost is $400 for 8 sessions. How much does each session cost?
8. From Regina to Winnipeg is approximately 570 km. At an average driving speed of 110 km/hr, approximately how long will it take to drive from Regina to Winnipeg?

Answers:

1. 1
2. $4.50 (10% of $45 is $4.50)
3. 3 and 6
4. $370 (65 + 350 – 10 – 35)
5. 72 (half of 144)
6. $t = 7$ ($21 \div 3 = 7$)
7. $50 ($400 \div 8 = 50$)
8. Just over 5 hours (5 hours x 110 km/hr = 550 km. There are just 20 more km to go at the 5 hour mark.)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write a polynomial expression to represent each situation.
   a) Sammy hits the ball twice as far as Joshua.
   b) Your sister is four years younger than you.
   c) A repairman charges $50 to come to your house, and $42.50 per hour.
   d) Your weight is 4.5 kgs over the suggested level.

   Answers:

   a) $2j$, where j is the distance Joshua can hit
   b) $y - 4$, where y is your age
   c) $42.5(h) + 50$, where h is the number of hours worked
   d) $w + 4.5$, where w is your weight

2. Create a situation which could be represented by these polynomial expressions:
   a) $3t - 2$
   b) $k + 1$

   Answers:

   a) Answers will vary.
   Possible Answer: 2 less than 3 times a value, a shot put went 2 m less than 3 times another
   b) Answers will vary.
   Possible Answer: Always eats one more piece of pizza, always one degree warmer

Learning Activity 4.4: Writing Polynomial Expressions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Arrange the following numbers in ascending order (from smallest to largest):

   $\frac{2}{3}$ 0.53 0.76 $\frac{6}{5}$

2. What is the mode of the following data:

   1, 1, 2, 3, 3, 3, 3, 3, 5, 5, 5, 6, 6, 6, 7, 8, 9

3. What is the median of the following data:

   1, 1, 2, 3, 3, 3, 3, 3, 5, 5, 5, 6, 6, 6, 7, 8, 9

4. Solve for n: $3 = 5 + 2n$.
5. Estimate the 15% tip on a restaurant bill of $150.
6. Add $\frac{2}{3} + \frac{4}{3}$.
7. Estimate $\sqrt{56}$.
8. Calculate $3^3 - 2^3$.

Answers:

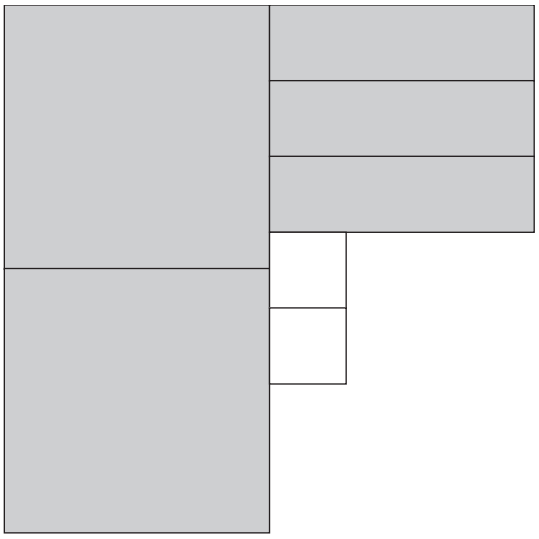
1. 0.53 $\frac{2}{3}$ 0.76 $\frac{6}{5}$
   (Compare the numbers to the benchmarks of 0.5 and 1 $\longrightarrow$ $\frac{6}{5}$ is the only value greater than 1 so it is the largest. $\frac{2}{3}$ is equivalent to 0.6666… and so it between 0.53 and 0.76.)
2. 3 (the mode is the number that appears the most in a data set)
3. 5 (the median is the middle number if the data is arranged in order)
4. $n = -1$ (subtract 5 on both sides and then divide by 2)
5. $22.50 (15% of 150 is 22.5 (since 15 x 15 is 225))
6. $2 \left(\frac{6}{3} = 2\right)$
7. Approximately 7.5 ($7^2 = 49$ and $8^2 = 64$; 56 is 7 away from 49 and 8 away from 64 so the square root of 56 is close to halfway between 7 and 8)
8. 19 (27 – 8)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Write the polynomials illustrated by these algebra tiles.

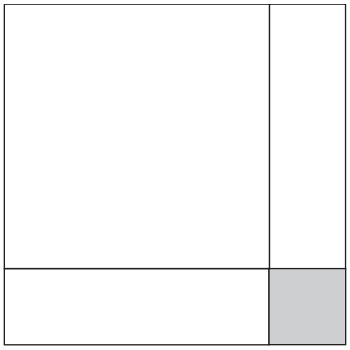
   a)



   Answer:

   $2x^2 + 3x - 2$

   b)



   Answer:

   $-x^2 - 2x + 1$

2. Draw a picture of the algebra tiles needed to represent these polynomials.

   a) $-x^2 + 2x + 1$

   Answer:

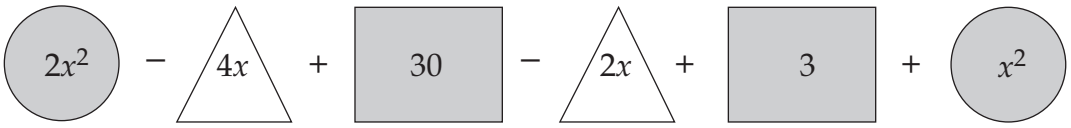
b) $2x^2 - 3x - 4$

Answer:

3. Use circles, triangles, and rectangles to identify like terms, and collect them.

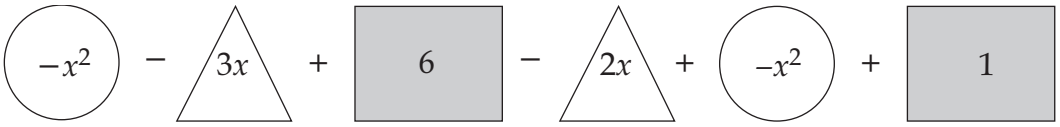
a) $2x^2 - 4x + 30 - 2x + 3 + x^2$

Answer:



$3x^2 - 6x + 33$

b) $-x^2 - 3x + 6 - 2x - x^2 + 1$



$-2x^2 - 5x + 7$

Learning Activity 4.5: Simplifying Expressions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. The distance from Flin Flon to Thompson is approximately 380 km. At an average speed of 90 km/h, how long will it take to drive from Flin Flon to Thompson?
2. Evaluate $3t - 4t^2$ if $t = -1$.
3. Solve for g: $5g + 2 = -3$.
4. What two numbers have a difference of 4 and a product of 12?
5. Is 4.56 x 24.875 closer to 1000, 100 or 10?
6. You buy lunch for $5.05. You give the cashier a $10 bill and a nickel. How much change do you get?
7. Simplify: $5^2 - 4^2$.
8. Simplify: $3^2 + 3^3$.

Answers:

1. Just over 4 hours (90 x 4 = 360, so you have 20 km left to travel past the 4 hours)
2. -7 $(3(-1) - 4(-1)^2 = -3 - 4(1) = -3 - 4 = -7)$
3. -1
4. 6 and 2
5. 100 (rounding to 4 x 25 makes the product close to 100)
6. $5 (You gave the cashier $10.05, so 10.05 – 5.05 = 5.00)
7. 9 (25 – 16)
8. 36 (9 + 27)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Simplify the following expressions by collecting like terms.
   a) $3a^2 - 6 + a^2 - 5a + 2$
   b) $-4v^2 - v + 5 - 2v^2 + 6v - 2$

   Answers:
   a) $4a^2 - 5a - 4$
   b) $-6v^2 + 5v + 3$

2. Simplify the following expressions by collecting like terms.
   a) $3x^2 - 2y^2 - 3x - 6y + 4x^2 - 4y^2 - 6y + 4x$
   b) $-6m^2 + 3n - 5n^2 + 4 - m^2 + 2n^2 - 3m + n - 5$

   Answers:
   a) $7x^2 - 6y^2 + x - 12y$
   b) $-7m^2 - 3n^2 - 3m + 4n - 1$

3. Identify the error, and correct it.

   $-3x^2 + 4x - x^2 + 9 - x$

   $3x^4 + 3x + 9$

   Answer:

   The student multiplied the $-3x^2$ by $-1x^2$ to get positive $3x^4$.

   Corrected answer is $-4x^2 + 3x + 9$.

Learning Activity 4.6: Applying Negative Signs

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write three equivalent fractions for $\frac{6}{10}$.
2. Convert 350 cm to m.
3. Calculate $2^4 - 2^2$.
4. Collect like terms: $3n + 2m - 5m - 7n$.
5. Write as a power with a single exponent: $\left(2^3\right)^4$.
6. What two numbers have a sum of –12 and a product of –45?
7. How much will you pay for 40 litres of gas if the cost is $0.99 per litre?
8. Two chocolate bars cost $1.50. How much will 6 chocolate bars cost?

Answers:

1. Answers will vary; some possible fractions are: $\frac{3}{5}, \frac{60}{100}, \frac{12}{20}, \frac{18}{30}, \frac{9}{15}\ldots$

   To be equivalent, both numerator and denominator must be multiplied or divided by the same number.
2. 3.5 m (100 cm = 1 m)
3. 12 (16 – 4)
4. $-4n - 3m$
5. 2^{12}
6. –15 and 3
7. $39.60 ($0.99 is 1 cent away from $1 so 40 litres at $1 would be $40. Subtract 40 cents for the 1 cent less to get $39.60)
8. $4.50 (3 times as much or each bar is $0.75 so 6 x 0.75 = 4.50)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Use arrows and brackets to show how you are distributing the negative value through the bracket, and simplify.

1. $6x - 4 - (2x + 3)$

   Answer:

   $6x - 4 - (2x + 3)$

   $= 6x - 4 - 2x - 3$

   $= 4x - 7$

2. $-5g + 4 - (-3g - 2)$

   Answer:

   $-5g + 4 - (-3g - 2)$

   $= -5g + 4 + 3g + 2$

   $= -2g + 6$

3. $-2y^2 - 3y + 4 - (3y^2 - 5y + 6)$

   Answer:

   $-2y^2 - 3y + 4 - (3y^2 - 5y + 6)$

   $= -2y^2 - 3y + 4 - 3y^2 + 5y - 6$

   $= -5y^2 + 2y - 2$

4. $3x - 7$ subtract $5x - 3$

   Answer:

   $3x - 7$ subtract $5x - 3$

   $= 3x - 7 - (5x - 3)$

   $= 3x - 7 - 5x + 3$

   $= -2x - 4$

5. Subtract $4p - 2$ from $6p - 5$

Answer:

$6p - 5 - (4p - 2)$

$= 6p - 5 - 4p + 2$

$= 2p - 3$

Identify the Error

Nicole showed her work in answering the following question, and still the teacher marked it wrong. Find the error she made, and correct it.

$5b - 4 - (-2b + 3)$

$= 5b - 4 - 2b - 3$

$= 3b - 7$

Answer:

Nicole didn't use arrows to show the distribution to each term. Had she used arrows, she would have seen that negative 1 times negative $2b$ would equal positive $2b$.

Corrected:

$5b - 4 - (-2b + 3)$

$= 5b - 4 + 2b - 3$

$= 7b - 7$

Learning Activity 4.7: Multiplication with Algebra Tiles

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Simplify $-3m + 6n - 4m$.
2. You worked 8 hours last week and are paid $9.50 an hour. How much did you earn?
3. Calculate $\frac{1}{2} + \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$.
4. Write as a power with a single exponent: $\left(4^5\right)\left(4^7\right)$.
5. Is $\frac{4}{5} - \frac{3}{4}$ positive or negative?
6. The Winnipeg Goldeyes baseball stadium has 20 sections. Each section holds approximately 400 people. How many people would make a sell-out crowd?
7. What is the area of a square that has sides 4 m long?
8. Evaluate $5h - 7$ if $h = 2$.

Answers:

1. $-7m + 6n$
2. $76 (8 x 9 = 72 and 8 x 0.50 = 4, so 72 + 4 = 76)
3. $1 \left(\text{multiplication first and } \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = \frac{1}{2}, \text{ then addition to get } \frac{1}{2} + \frac{1}{2} \right)$
4. 4^{12}
5. positive $\left(\frac{4}{5} = 0.8 \text{ and } \frac{3}{4} = 0.75\right)$
6. 8000 (20 x 400)
7. 16 m^2
8. 3 (10 – 7)

Part B

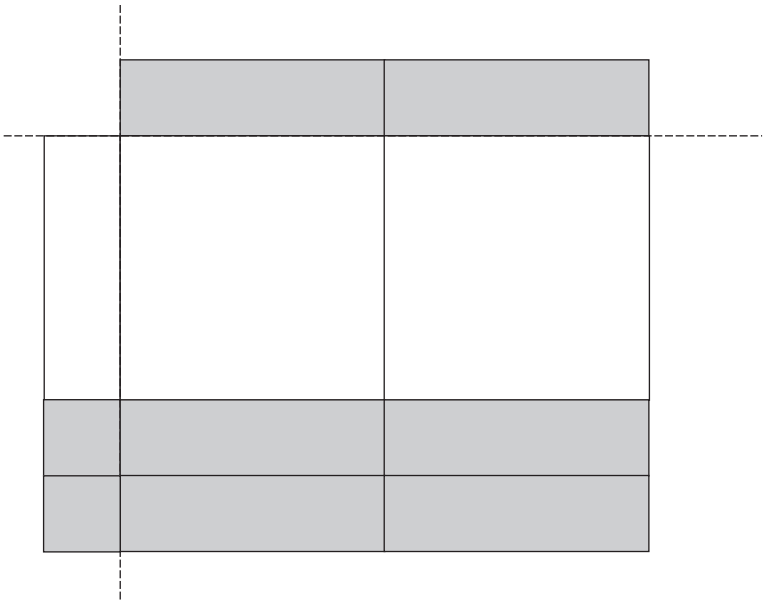
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use algebra tiles to illustrate this multiplication, and state the answer.

   $(2x)(-x + 2)$

   Answer:

   $-2x^2 + 4x$



2. Use the grid method to illustrate this multiplication, and state the answer.

   $(-3x)(4x - 7)$

   Answer:

	$4x$	-7
$-3x$	$-12x^2$	$21x$

   $-12x^2 + 21x$

3. Use arrows to show the distribution of the first term, and state the answer.

   $(5x)(3x - 4y - 6)$

   Answer:

   $(5x)(3x - 4y - 6)$

   $15x^2 - 20xy - 30x$

4. Describe the error made in this multiplication, and correct it.

   $(-3a)(-4a + 6)$

	$-4a$	6
$-3a$	$12a^2$	$18a$

   $12a^2 + 18a$

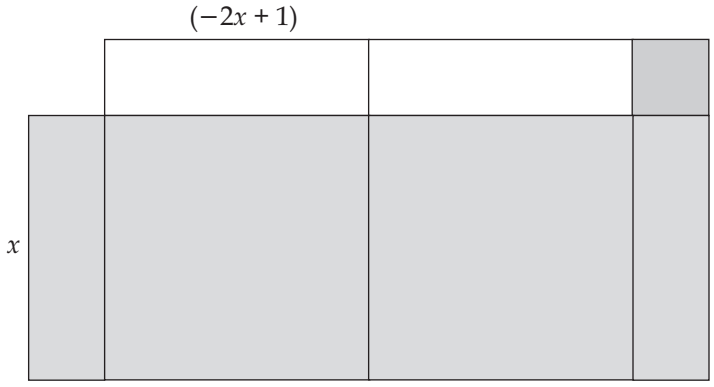
   Answer:

   $-3a$ times 6 should be negative $18a$

   $12a^2 - 18a$

Identify the Error

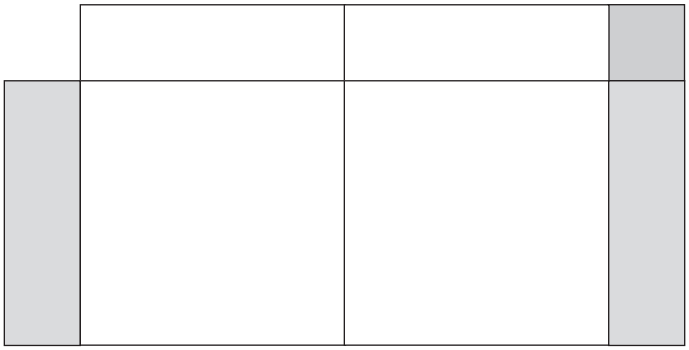
1. Find the error in this algebra tile illustration, and correct it.



Answer:

x times $-x$ should be $-x^2$, not positive x^2

$(-2x + 1)(x) = -2x^2 + x$



2. Find the errors in this grid illustration, and correct them.

	$-3x$	4
$-5x$	$-15x^2$	$20x$

Answer:

$(-5x)(-3x)$ is positive $15x^2$

$(-5x)(4)$ is $-20x$

$(-5x)(-3x + 4) = 15x^2 - 20x$

3. Find the errors in the following arrow illustration, and correct them.

$(-x)(-5x - 3y + 1)$

$5x - 3xy - 1$

Answer:

Arrow needs to go to all three terms of the trinomial.

$(-x)(-5x)$ is positive $5x^2$

$(-x)(-3y)$ is positive $3xy$

$(-x)(1)$ is negative x

$(-x)(-5x - 3y + 1) = 5x^2 + 3xy - x$

Learning Activity 4.8: Division with Algebra Tiles

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Simplify $5t - 6s + 7s - 9t$.
2. You go grocery shopping and buy 3 items for $0.99 each, 2 items for $2.25 each, and one item worth $1.59. You have a $10 bill with you. Do you have enough money?
3. Dauphin Country Fest had 300 volunteers who worked 3500 hours to clean up the site after the event was over. Approximately how many hours did each volunteer spend helping out?
4. Write as a power with a single exponent: $\frac{5^6}{5^2}$.
5. You made $540 last week for working 30 hours. What is your hourly wage?
6. Budgeting guidelines suggest you budget 30% of your gross earnings for housing expenses. If your gross monthly earnings are $2000, how much should you be spending on housing expenses?
7. What two numbers have a sum of –7 and a product of –30?
8. Calculate $3^2 - 2^3$.

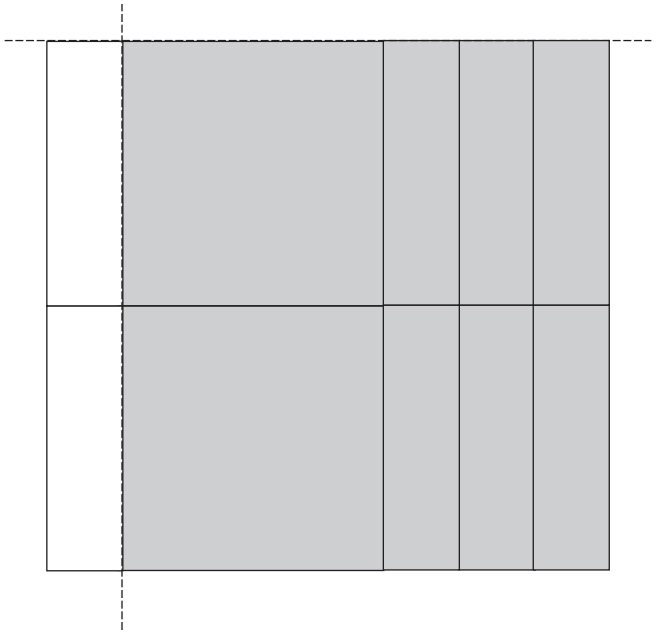
Answers:

1. $-4t + s$
2. Yes (estimating, you have $3 + $4.50 + $1.50 = $9)
3. Approximately 12 hours each (round 3500 to 3600 and 3600 ÷ 300 = 12)
4. 5^4
5. $18 (540 ÷ 30 = 54 ÷ 3 = 18)
6. $600 (2000 x 0.30)
7. –10 and 3
8. 1 (9 – 8)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given this illustration of algebra tiles, find the quotient of $(2x^2 + 6x) \div -2x$.

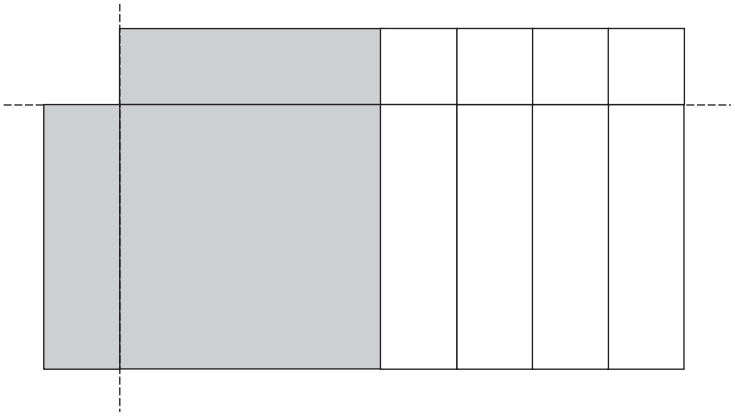


Answer:

$-x - 3$

2. Draw an arrangement of algebra tiles used to find the quotient of $(x^2 - 4x) \div x$.

Answer:



$x - 4$

3. Use arrows to divide the bottom value into each of the terms in the numerator, and find the quotient.

$$\frac{7x^2 - 14x + 21}{-7}$$

Answer:

$7x^2 - 14x + 21$

-7

$-x^2 + 2x - 3$

4. Find the error in the following work.

$$\frac{6x^2 - 9x}{3x}$$

$$= 2x^2 - 3$$

Answer:

$$\frac{6x^2 - 9x}{3x}$$

$$= 2x^2 - 3$$

$\frac{6x^2}{3x}$ should be $2x$, not $2x^2$

Answer is $2x - 3$.

Learning Activity 4.9: Expressing Equivalents

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert 4 kg to grams.
2. What two numbers have a product of 11 and a sum of 12?
3. Simplify $m(2m + 3)$.
4. Solve for d: $12 - 2d = 4$.
5. The area of a rectangluar pool is 50 m^2. One side measures 5 m. What is the length of the other side?
6. Heather is finished 65% of her homework, and Amanda is finished $\frac{3}{4}$ of hers. Who is finished more of her homework?
7. A shirt you want to buy is on sale for 40% off. The original price is $60. What is the sale price?
8. Write as a power with a single exponent: $\frac{3^3 \cdot 3^7}{3^5}$.

Answers:

1. 4000 g
2. 11 and 1
3. $2m^2 + 3m$
4. 4 ($-2d = -8$; $d = 4$)
5. 10 m (50 ÷ 5)
6. Amanda $\left(\frac{3}{4} = 0.75 \text{ or } 75\%, \text{ which is more than } 65\%\right)$
7. $36 (60 x 0.40 = 24; 60 – 24 = 36 or 40% off means 60% left so 60 x 0.60 = 36)
8. 3^5

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Which of the following expressions are equivalent to $3x - 6$?
   a) $3x + 6$
   b) $6x - 4$
   c) $3(x - 2)$
   d) $x - 3 + 2x - 3$
   e) $3 + x - 6$

   Answer:

   Only (c) and (d) are equivalent.

2. Write the expression $-4x + 8$ in two different ways.

   Possible Answers::

   Answers can vary.

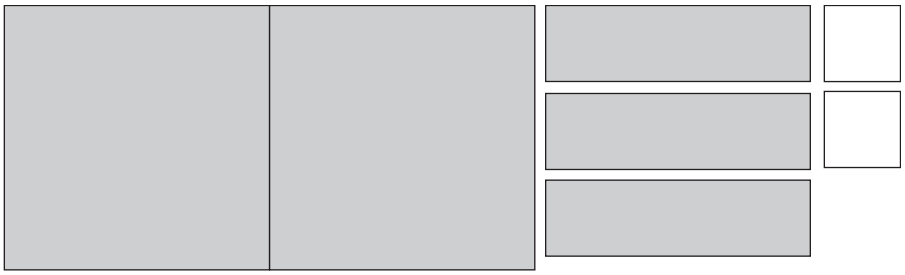
   $-4(x - 2)$

   $-3x + 2 - x + 6$

3. Draw two tile arrangements illustrating $2x^2 + 3x - 2$.

   Answer:

   Any tile arrangement that includes the following tiles.



Learning Activity 4.10: Shapes and Equivalent Expressions

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What two numbers have a product of –8 and a sum of 2?
2. On his last mathematics test, Gilles got 6 out of 9 questions correct. What is his approximate percentage grade on the test?
3. You start work at 8:45 and work until 16:15. How long did you work?
4. You have $460 in your chequing account. You write cheques for $65, $130, and $35. You deposit $150 in your chequing account. What is your new balance?
5. Collect like terms: $9h - (3g + 2h) + 5g$.
6. Convert 1.25% to a decimal.
7. Convert 0.2 to a fraction.
8. Order the following from smallest to largest: –0.34 –0.43 0.34 0.43

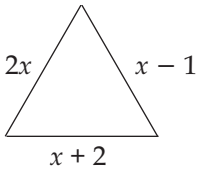
Answers:

1. 4 and –2
2. 67% $\left(\frac{6}{9} = \frac{2}{3} = 0.6666...,\text{ which can be converted to approximately } 67\%\right)$
3. 7 hours and 30 minutes (from 9:00 to 16:15 is 7 hours and 15 minutes, add 15 minutes to go back to 8:45)
4. $380 (460 – 65 – 130 – 35 + 150 = 460 – 65 – 35 – 130 + 150 = 460 – 100 + 20 = 380)
5. $7h + 2g$
6. 0.0125 (1.25 ÷ 100)
7. $\frac{1}{5}\left(0.2 = \frac{2}{10} = \frac{1}{5}\right)$
8. –0.43, –0.34, 0.34, 0.43

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

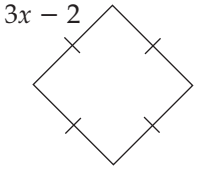
1. Find the perimeter of the following figure.



Answer:

$2x + x - 1 + x + 2 = 4x + 1$

2. Find the perimeter of the following figure.



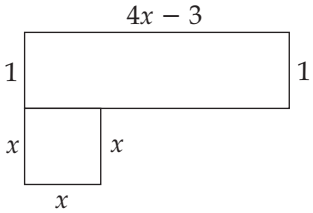
Answer:

The tick marks on the sides indicate that all sides have the same measurement.

$4(3x - 2)$

$= 12x - 8$

3. Find the perimeter of the following figure.



Answer:

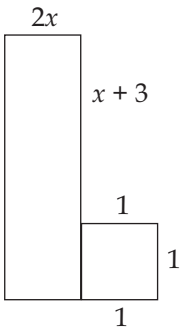
Determine the length of the unlabelled side, it's $4x - 3 - x$.

Add all sides:

$4x - 3 + 2 + 3x + (4x - 3 - x)$

$= 10x - 4$

4. Find the area of the following figure.



Answer:

$(2x)(x + 4) + (1)(1)$

$= 2x^2 + 8x + 1$

GRADE 9 MATHEMATICS (10F)

Module 5: Linear Relations

MODULE 5: LINEAR RELATIONS

Before You Get Started . . .

Before you begin working through the next part of the course, now would be a good time to look at the Resource Sheet that you used for your midterm. Answer the following questions in the space below so that when you are making your Final Exam Resource Sheet, you will make changes that you think will be helpful.

1. Was there any information that you wished you had put on your Resource Sheet?

2. Was there any information on your Resource Sheet that you did not need?

3. Was there any information on your Resource Sheet that you did not remember or didn't understand when you looked at it during the midterm?

Use your answers to these questions as you work through the second half of this course. Look back to find out what type of information you found most/least useful on the midterm. This will help you make your Resource Sheet for the final exam.

Introduction

TV shows with crime scene investigation themes capture viewers' attention because it's interesting to figure things out, to look at clues or patterns and try to find a solution. This is the basic concept behind linear relations. In this module, you will find patterns and describe them to help you solve problems using equations, tables, and graphs. You will be able to verify or prove your solutions by substituting values back into the expression or equation to make sure it is correct. You will also look at situations where there is more than one possible answer.

Assignments in Module 5

You will need to complete the following seven assignments and send them to your tutor/marker when you are done this module.

Lesson	Assignment Number	Assignment Title
1	Assignment 5.1	Cover Assignment
2	Assignment 5.2	Patterns and Translations
3	Assignment 5.3	Writing Equations
4	Assignment 5.4	Working with Graphs
5	Assignment 5.5	Operations with Equations
6	Assignment 5.6	Inequalities and Graphing
7	Assignment 5.7	Inequalities and Problems

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 5. You might use your Module 5 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 5 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 5 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

NOTES

Lesson 1: Cover Assignment

The ability to use logical reasoning to solve problems is an important mathematical skill. Logic is a way of organizing thoughts or ideas in order to explain reasons or rules.

Some problems in mathematical logic can be solved using a matrix, or a chart that organizes information in a specific way.

Example 1

A gorilla, a donkey, a cat, and an aardvark are named Gabby, Debbie, Corey, and Art, but not necessarily in that order. Use the clues provided to match the animal with its name:

- Corey is the oldest and is a good friend of the cat or Gabby.
- Debbie is more stubborn than either the cat or Gabby.
- The gorilla does not get along with Gabby.
- Only one animal and its name begin with the same letter.

Solution:

	Aardvark	Cat	Donkey	Gorilla
Art				
Corey		✗		
Debbie		✗		
Gabby		✗		✗

From the first statement we can conclude that Corey cannot be the cat. The second statement tell us that neither Debbie nor Gabby is the cat. The third statement tells us that the gorilla is not Gabby.

From the matrix you can conclude that Art is the cat and Debbie is the donkey because of the clue about the first letter. Therefore, Corey is the gorilla and Gabby is the aardvark.

	Aardvark	Cat	Donkey	Gorilla
Art	✗	❍	✗	✗
Corey	✗	✗	✗	❍
Debbie	✗	✗	❍	✗
Gabby	❍	✗	✗	✗

Logic also requires that you read carefully to look for clues.

Example 2

Boris and Natasha each own a very unique pet. One owns a monkey and the other owns an alligator. The alligator owner would like to have a golden retriever, but she is allergic to animal hair. Who owns the monkey?

Solution:

The alligator owner is female (she), therefore, Boris owns the monkey.

Assignment 5.1: Cover Assignment

Total: 15 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Read the following clues and use the matrix to help you determine what happened on the first four dates for Andy and Anita, Brady and Belinda, Chad and Connie, and Doug and Denise. Each couple went out to a movie, out for supper, to a concert or to a hockey game, but no one "double dated" (each couple went on a different date each night). *(5 marks)*

	1st date	2nd date	3rd date	4th date
Andy and Anita				
Brady and Belinda				
Chad and Connie				
Doug and Denise				

For their second dates, Andy and Anita went to a movie while Chad and Connie went for supper. On their third date, Brady and Belinda went to see a hockey game and Doug and Denise went to a concert. For the fourth date, Chad and Connie went to a concert while Doug and Denise went out for supper.

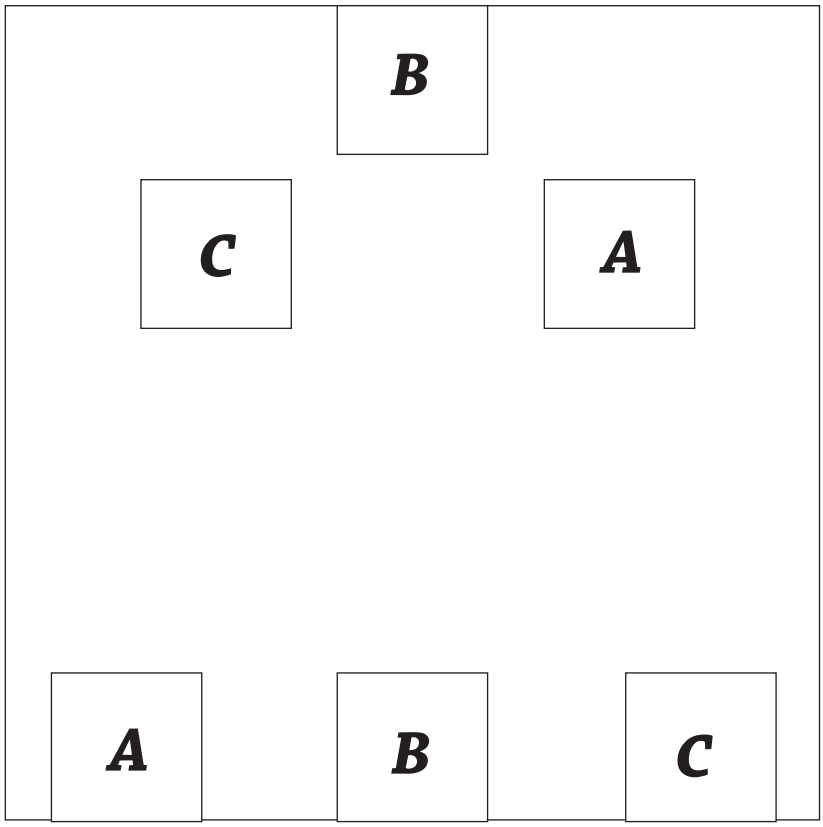
2. Mary's mother has four children.
   Her first child is named Rose.
   The second is Lilly.
   The third is Chrysanthemum.
   What is the name of the fourth child?
   (5 marks)

_____

_____

3. Sometimes, the logical solution to a problem requires some creativity! Your goal with the diagram below is to use lines to join Box A to the other Box A, Box B to the other Box B, and Box C to the other Box C. The lines cannot cross any other lines. The lines must remain inside the large rectangle.

   Use a pencil that can be erased, and find the solution. There are two possible answers to this puzzle. (You only need to find one.) *(5 marks)*



LESSON 2: PATTERNS AND TRANSLATIONS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ write an expression describing a pattern
- ❑ write patterns from graphs
- ❑ translate words into mathematical expressions

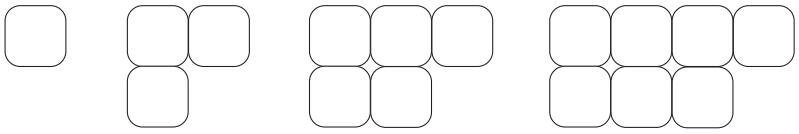
Lesson Introduction

Identifying patterns is something you do nearly every day. In this lesson, you will learn to translate words into math symbols, and write patterns using mathematical expressions.

Patterns in Words and Expressions

Example 1

Look at the pattern of shapes below. What happens to each group of shapes?



Solution:

The pattern is that two squares are added to the left side of the shape each time.

To write a mathematical expression showing this pattern, assign a variable to the shape, (n for example) and then show that each time you add two more.

$n + 2$

Note: You can use any letter to represent the unknown or original shape.

In this case, let n = the number of shapes in the previous iteration. An iteration is an instruction repeated on previous results. In this case, you add 2 in each subsequent iteration.

Given one square in the first iteration (n), this can be written as $n = 1$.

The second iteration would be $n + 2$ or $1 + 2$.

The second group is 3 squares.

To determine how many squares would be in the next iteration, and following the pattern, use the 3 squares from the previous iteration, and add 2 more.

$n + 2$ or $3 + 2 = 5$ squares

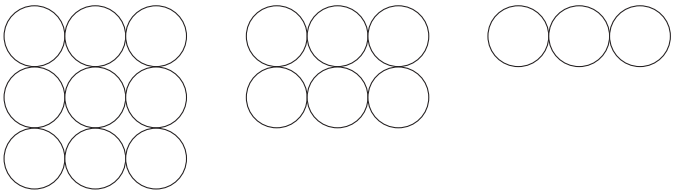
If there were 27 squares in a given iteration, and you wanted to find how many would be in the next step, substitute the number you have for the previous result and add 2.

$$\begin{aligned} & n + 2 \\ &= 27 + 2 \\ &= 29 \end{aligned}$$

The next group would have 29 squares.

Example 2

Find the pattern shown with the following shapes.



Solution:

You can see that the pattern started with 9 circles, the next step had 6 circles, then 3 circles.

The pattern is to subtract 3 circles each time.

To write a mathematical expression for this pattern, assign a variable (c) to indicate the number of circles, and then subtract 3.

$c - 3$

Had you started with 42 circles and were asked to find how many would be in the next iteration, you would use the mathematical expression. Substitute 42 in place of the variable c, and then subtract 3.

$$\begin{aligned} & c - 3 \\ &= 42 - 3 \\ &= 39 \end{aligned}$$

The next group would have 39 circles.

Example 3

Five students in your class are comparing how much money they have. Write an expression to show the pattern.

Student	Money
Albert	\$1
Bruno	\$2
Cam	\$4
Dinah	\$8
Eve	\$16

Solution:

To write a mathematical expression showing the pattern, you must analyze the chart. Each student has twice as much money as the previous student.

The mathematical expression for this situation would be found by assigning a variable to the money (m), and multiplying it by 2.

$2m$

If another student, Faye, was to join the group, and followed the same pattern, how much money would she have?

Since the last student had \$16, and the pattern is $2m$.

$$\begin{aligned} & 2m \\ = \ & 2(16) \\ = \ & \$32 \end{aligned}$$

Learning Activity 5.1

Analyzing Patterns

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. How many half-dozen eggs are there in 420 eggs?
2. Evaluate 32 ÷ 0.5.
3. At the hardware store, the price of a $300 tool box is reduced by 40%. What is the sale price for the tool box?
4. If Andre completed $\frac{3}{8}$ of his homework and Patricia completed $\frac{2}{3}$ of hers, who is closer to finishing their homework?
5. If a notebook costs $0.27, how much would 100 notebooks cost??
6. Calculate 12 x 12.
7. If you saved $208 in a year, how much money did you save each week?
8. You bought three DVDs valued at $9.99 each, taxes included. What is the cost of your total purchase?

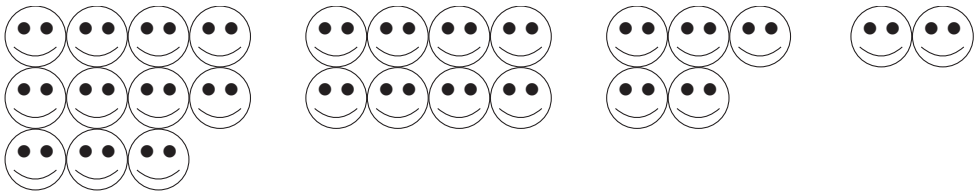
continued

Learning Activity 5.1: Analyzing Patterns (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the following pattern of shapes, write a mathematical expression showing the changes for each iteration.



2. When you fail to return a library book by the due date, the late fee is $0.25 for each day you are late. Write a mathematical expression for this situation.

3. Using online banking on your web phone, you want to monitor the amount of money in your account during a shopping spree. Initially, you had $120 in the account. This chart shows the balance each time you check. Write a mathematical expression showing the pattern.

Time	Balance
08:00	$120
10:00	$60
12:00	$30
14:00	$15

4. Your grandmother gives you money for your birthday. The following chart shows how much you received each year. Write a mathematical expression showing the pattern.

Age	Gift
10	$10
11	$15
12	$20
13	$25
14	$30

Note: The variables can be any letter you choose.

Translations

You must be able to translate words into mathematical symbols. Some basic words for each operation are listed below.

Addition	Subtraction
sum	difference
plus	take away
increased by	reduced by
more than	less than

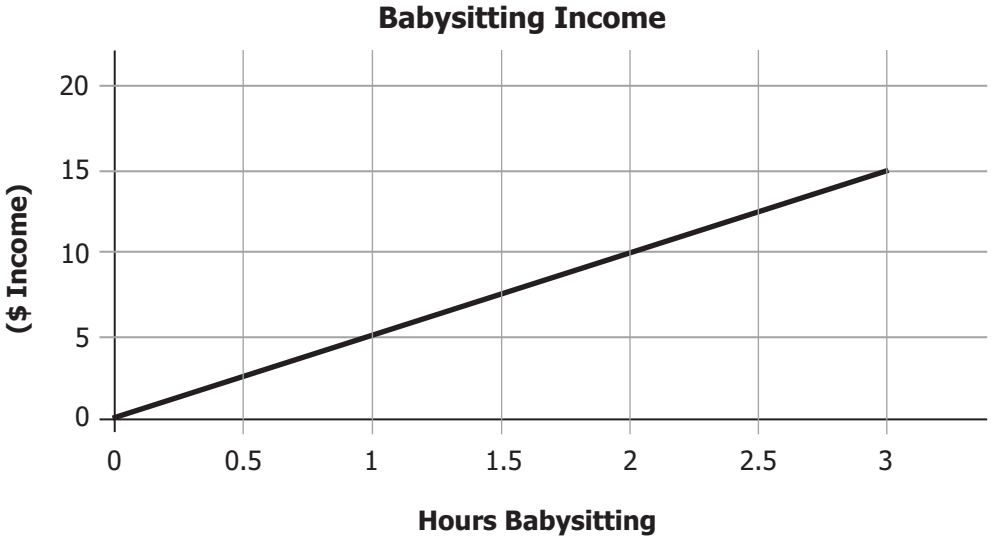
Multiplication	Division
doubled	into
tripled	quotient
product of	over
half of	

If you have difficulty remembering what something means in "math language," you should write this term on your Resource Sheet and/or include the examples below.

Written Using Words	Translated to Math
Four more than a number	$n + 4$
Twice a number	$2h$
Two-thirds of a number reduced by one	$\frac{2k}{3} - 1$

Patterns from Graphs

Analyze this graph, and write a mathematical expression that describes the pattern.



You can see the line shows your income based on how many hours you babysit.

Each hour you earn $5.

An expression showing this would be $5h$, or $5 per hour.

Calculate your earnings if you babysat for 7 hours. Use the expression:

$$\begin{aligned} & 5h \\ &= 5(7) \\ &= 35 \end{aligned}$$

You would earn $35.

This is an example of **extrapolation**, where you calculate values beyond the available data range. **Interpolation** is finding values inside the data field. Using expressions, it is quick and simple. You will do more problems using interpolation and extrapolation in a later lesson.

Include the definitions for *extrapolation* and *interpolation* on your Resource Sheet, as they are easily confused.

Sequences

Once you find the pattern, fill in the next three values in this sequence:

Example 1

2, 7, 12 , 17 ___ , ___ , ___

Solution:

The pattern is to add 5 to each number, so the next three values would be 22, 27, and 32.

Example 2

3, -6, 12, -24 ___ , ___ , ___

Solution:

Here, the pattern is to multiply each value by –2. The next three values would be 48, -96, and 192.

Example 3

1, 2, 5, 14, ___ , ___ , ___

Solution:

Some patterns involve two operations.

In this example, take the value, multiply it by 3 and subtract 1 ($3v - 1$).

So the next 3 values would be 41, 122, and 365.

Learning Activity 5.2

Translating Words into Equations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Estimate an 11% tip on a $45.00 bill.
2. Subtract $\frac{4}{5} - \frac{3}{25}$.
3. How many days are there in 7 weeks?
4. If 12% of 400 is 48, what is 4% of 400?
5. Add 78 + 33 + 22.
6. What distance do you travel if you walk at 5 m/s for 40 seconds?
7. How much change do you get if you pay a bill of $12.40 with 2 ten dollar bills?
8. Which value is larger: $\frac{3}{8}$ or 0.5?

continued

Learning Activity 5.2: Translating Words into Equations (continued)

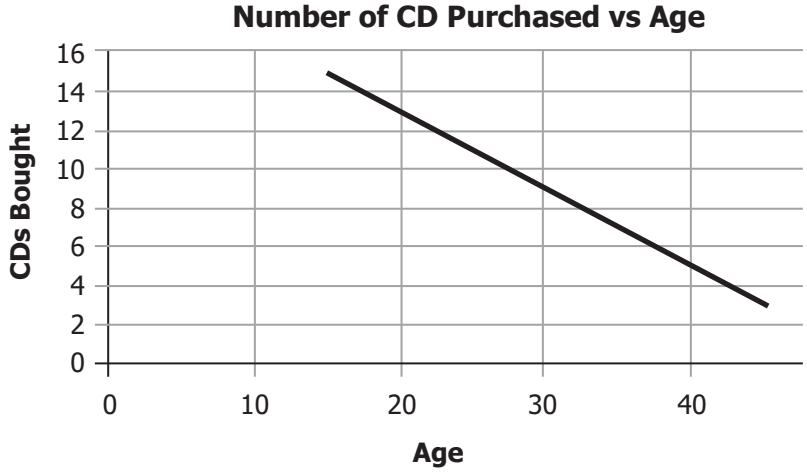
Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Match the written statement with the algebraic translation:

a) Six less than a number	i) $3x$
b) The quotient of 2 and a number	ii) $\frac{n}{6}$
c) A number tripled	iii) $2m+3$
d) Three more than twice a number	iv) $\frac{2}{n}$
	v) $n-6$
	vi) $2n$

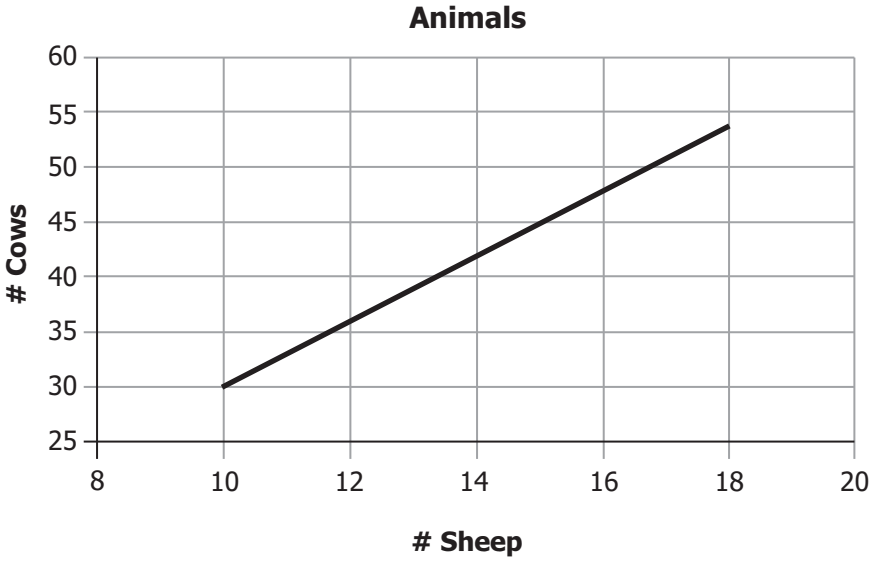
2. Use the graph below to answer the questions that follow the graph.



a) Write a statement about the general buying pattern of people indicated in this graph.
b) Approximately how many CDs will a 20-year-old purchase?
c) How many CDs would you expect a 15-year-old to buy?
d) Based on this graph, how many CDs would a 60-year-old person buy?
e) Somebody bought 6 CDs. How old would he be?

Learning Activity 5.2: Translating Words into Equations (continued)

3. The graph below shows the number of sheep and cows a farmer has over the course of one season.



a) Find an expression that compares the number of cows to the number of sheep.

b) If the farmer had 33 cows, approximately how many sheep would he have?

c) If the farmer had 25 sheep, approximately how many cows would he have?

d) Write an expression showing the total number of animals at any given time.

Lesson Summary

In this lesson, you learned to express words using mathematical expressions. You also identified patterns in groups of shapes, and from graphs. In the next lesson, you will apply these skills to working with equations.

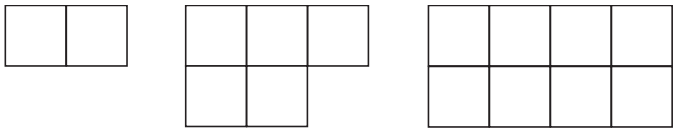
NOTES

Assignment 5.2: Patterns and Translations

Total: 16 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Write an algebraic expression showing the following pattern. *(2 marks)*



2. Translate the following expressions into words. *(2 marks)*

   a) $3x$ *(1 mark)*

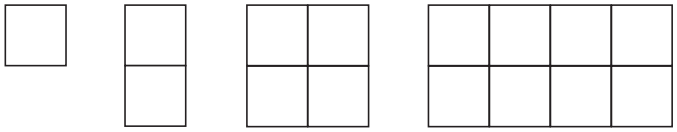
   b) $n - 3$ *(1 mark)*

3. Translate the following words into mathematical expressions. *(2 marks)*

   a) A number increased by seven *(1 mark)*

   b) A number divided by 3 *(1 mark)*

4. Identify the pattern illustrated by these shapes. *(2 marks)*

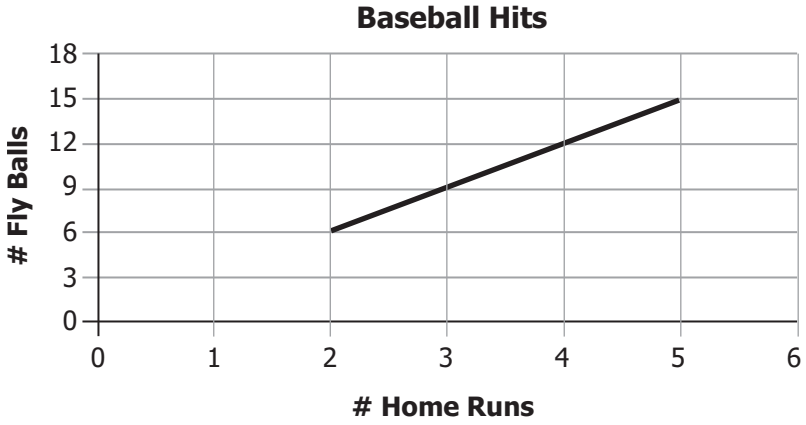


5. Identify the pattern, and find the next 2 values. *(6 marks)*

   a) 37, 33, 29, 25, _____, _____ *(3 marks)*

   b) –22, –16, –10, –4, _____, _____ *(3 marks)*

6. Identify the pattern displayed in the graph. *(2 marks)*



LESSON 3: WRITING EQUATIONS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ write equations representing given situations
- ❑ create a situation fitting an equation
- ❑ substitute given values into equations
- ❑ verify equations by checking values

Lesson Introduction

In this lesson, you will create equations to represent given situations. You will learn to use variables to represent unknown values, and then substitute what you do know in place of the variable. Finally, you will examine charts and write equations to represent the patterns shown

Developing Equations

Equations are used to simplify calculations. Once you learn how to create equations, calculations are not as difficult.

As you work through this lesson, you should include on your Resource Sheet examples of how to write equations.

Example 1

You are working at a part-time job that pays $9 per hour. If you work for 10 hours, how much will you be paid?

Solution:

You could calculate your pay with a pictorial representation

Let ☐ represent each hour worked.

$9	$9	$9	$9	$9	$9	$9	$9	$9	$9
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

The most basic way to calculate your pay would be to add each icon of $9 to find the total. This is not only cumbersome, but the chance of making a mistake is higher.

$$9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9$$

If you set up an equation using a variable (h) to indicate the number of hours, you can find your pay quickly.

If your pay (P) is calculated as $9 per hour ($h$), you could write it as $P = 9h$.

If you work 10 hours, substitute that value in place of the variable.

$$P = 9h$$
$$P = 9(10)$$
$$P = 90$$

You would be paid $90.

Example 2

Your job involves working with cash. At the end of your shift, you have 26 five dollar bills, and you have to count them! How would you do this?

Solution:

You could group them into groups of 10, but that doesn't work evenly

$5	$5	$5	$5	$5
$5	$5	$5	$5	$5

$5	$5	$5	$5	$5
$5	$5	$5	$5	$5

$5	$5	$5	$5	$5
$5				

You could start adding 5 + 5 + 5 + 5… This is also cumbersome.

An easier method of finding the total value of these $5 bills is to set up an equation! Establish the variables that you will use, and use them in an equation.

Number of bills (b) and total value (V) would work.

Since each bill is worth $5, your equation would look like

$$5(b) = V$$

or "$5 for each bill equals the value."

Now substitute in the number of bills you have to find the total value.

Let b = number of \$5 bills, and V = total value.

$5(b) = V$
$5(26) = V$
$130 = V$

You had \$130 in \$5 bills.

Example 3

Some people drive too fast, and get caught by "red light cameras." The fine is \$185.

a) If you get caught on three occasions, how would you calculate the total amount owing?

Solution:

- You could add \$185 + 185 + 185 to find the total.
- You could set up a basic equation using the variable, s, for each time you were caught speeding and the variable, T, for total owing.

  Let s = the number of speeding violations.

  $185(s) = T$

  Now substitute in the number of times you were caught, and solve for the total owing, T.

  $185(3) = T$
  $555 = T$

  You would owe \$555.

b) If you were caught 10 times, how could you calculate the total fines owed?

Solution:

$185(s) = T$
$185(10) = 1850$

You would owe \$1850.

c) If your total fines are \$1295, how could you calculate the number of times you were caught speeding?

Solution:

Use the same equation, and solve for the unknown variable.

$185(s) = T$

$185(s) = 1295$

Divide each side by 185 to isolate the unknown variable, s.

$$\frac{185(s)}{185} = \frac{1295}{185}$$

$$s = 7$$

You were caught speeding 7 times.

Learning Activity 5.3

Equations for Situations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Evaluate $4 - 6t$ if $t = -3$.
2. What two numbers have a sum of 8 and a product of 15?
3. Rodelyn earns $24 per shift. How much money does she earn after five shifts?
4. How many quarters are there in $5.75?
5. Add $\frac{9}{2} + \frac{5}{2}$.
6. If the $600 rent on your apartment is increased by 2%, what is the new rent?
7. Put in ascending order: $\frac{3}{4}, \frac{2}{5}, \frac{4}{6}$.
8. What is the total cost for four pens that cost $1.97 each?

continued

Learning Activity 5.3: Equations for Situations (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write an equation for each situation.

1. Finding the cost of B.C. apples at $3 per kg.

   Let k represents kilograms and C represents the total cost.

2. Finding the cost of gas at $1.35 per litre.

   Let L represents litres, and C represents the cost.

3. Receiving $5 for every goal scored in hockey.

   Let G represent each goal, and M represent money received.

4. You received $100 for mowing 5 lawns. Set up an equation to show how to calculate your fee per lawn.

   Let F represent the fee for each lawn mowed, and M represent the money received

Patterns in Tables

The following table shows values in two columns. The second column is always found using a pattern, or equation applied to the value in the first column.

Example 1

Write an equation to describe the data in this table.

A	*B*
2	8
5	20
–3	–12

Solution:

The first value, or value in the A column, is multiplied by 4 to find the value in the B column. This pattern repeats itself each time. This can be represented mathematically:

B represents $4A$
$B = 4A$

To check or verify this pattern, take any A value, and substitute it into the equation to see if you get the corresponding B value from the table.

Take the –3, and substitute it into the equation $B = 4A$.

$B = 4(-3)$
$B = -12$

Therefore, the equation works.

Check the other A values also:

$A = 2$

$B = 4A$
$= 4(2)$
$= 8$

The equation works.

$A = -3$

$B = 4A$
$= 4(-3)$
$= -12$

The equation works.

Sometimes it is more challenging to find the equation from a table of values. There could be 2 mathematical steps in the equation!

Example 2

Examine the values in the following table, to see what pattern is applied each time. Write an equation to describe the pattern.

C	***D***
–2	–3
3	7
15	31

Solution:

What was done to the *C* value to result in the *D* value? Many times you have to use "trial and error" to find the pattern. In this example, the pattern used is to multiply the *C* value by 2, and then add 1. The equation would be:

$$D = 2C + 1$$

Now take each value in the *C* column, and substitute it into the equation to prove that it works every time. Remember, the *C* value in the table replaces the *C* in the equation.

For $C = -2$

$$\begin{aligned} D &= 2C + 1 \\ &= 2(-2) = 1 \\ &= -4 + 1 \\ &= -3 \end{aligned}$$

It works.

For $C = 3$

$$\begin{aligned} D &= 2C + 1 \\ &= 2(3) = 1 \\ &= 6 + 1 \\ &= 7 \end{aligned}$$

It works.

For $C = 15$

$$\begin{aligned} D &= 2C + 1 \\ &= 2(15) = 1 \\ &= 30 + 1 \\ &= 31 \end{aligned}$$

It works.

An Extra Challenge

Here is a table showing numbers of candy bars purchased compared to the total price. Your goal is to write an equation representing the pattern.

***n* (number)**	***T* (total)**
4	7.20
7	12.60
11	19.80

It is understood that each bar costs the same amount. At this point, you are not looking for the actual cost of each bar. You are simply learning how to set up an equation. You will solve equations in a later lesson.

To write the equation for this pattern, let a variable represent the unknown. Since you are looking for cost, use the variable *C*.

In words, four bars times the cost of each bar equals $7.20.

Using math symbols, $4(C) = 7.20$, or using a different value from the table:

$7(C) = 12.60$
$n(C) = T$

Create a Situation

Here is an equation, and your task is to create a situation it could be describing. You can be creative with your answers, but they must make sense.

Example 1

$$C = \frac{1}{2}m + 3$$

Solution:

C could represents my cousin's age, and *m* is my age.

"I have a cousin who is 3 years older than half my age."

Another situation it could describe is, "You have $3 more than one-half of my money."

Example 2

$a = 3b - 1$

Solution:

Someone (a) spends 1 hour less than three times as long studying at home, compared to his friend (b).

The fuel economy in L /100 km of your Hybrid car (b) is 1 less than three times the value compared to my SUV (a).

Learning Activity 5.4

Representing Equations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Estimate the value of 101.55 x 44.3.
2. If 9 of 12 members of a professional volleyball team are taller than 2.0 m, what fraction of players does this represent?
3. Evaluate $3^2 + 4^2 + 5^2$.
4. What is the area of a rectangle with length 12 mm and width 4 mm?
5. Convert 0.03 to a percent.
6. The square root of 162 is found between which two consecutive whole numbers?
7. Jacques is 37 years old. His son, Pierre, is 13 years old. How old was Jacques when Pierre was born?
8. What two numbers have a sum of 9 and a product of –22?

continued

Learning Activity 5.4: Representing Equations (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write an equation for each situation.

1. Write an equation representing the pattern shown in the table of values below.

x	y
–4	–8
3	6
$\frac{1}{3}$	$\frac{2}{3}$

2. Write an equation representing the pattern shown in the table of values below.

R	Q
2	3
5	9
–3	–7

3. Create a situation that could be represented by the following equation.

   $m = 2y + 7$

Lesson Summary

In this lesson, you learned to set up equations to represent patterns and situations. In the next lesson, you will work closely with analyzing graphs.

Assignment 5.3: Writing Equations

Total: 20 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Write an equation representing each situation. *(2 marks)*
   a) Mary is twice as old as Bob, where M = Mary and B represents Bob. *(1 mark)*

   b) Tomatoes cost twice as much as potatoes, where T represents tomatoes, and P represents potatoes. *(1 mark)*

2. Write an equation showing my height in cm being 3 cm more than half the height of my cousin. Let m = my height and C represent my cousin's height. *(2 marks)*

3. Write equations to represent each situation. *(6 marks)*
   a) Find the cost of a bag of apples at \$3.25/kg , where C = the cost and K represents each kilogram. *(2 marks)*

   b) Total number of woodscrews when there are 75 screws in a box. Use T to represent the total number of screws and b is the number of boxes. *(2 marks)*

c) Cost of gas at $1.30 per litre, where C represents the cost and L represents the number of litres. *(2 marks)*

4. a) Write an equation representing the pattern shown in the table of values below. *(2 marks)*

f	g
8	4
10	5
2	1
–4	–2

b) Use your equation to prove that if f has a value of 10, g is 5. *(2 marks)*

5. a) Write an equation representing the pattern shown in the table of values below. *(2 marks)*

R	S
–2	–4
1	5
3	11

b) Substitute $R = 3$ into the equation to prove $S = 11$. (2 marks)

6. Describe a situation that could be represented by the following equations. *(2 marks)*

a) $d = 4h$ *(1 mark)*

b) $k = j + 3$ *(1 mark)*

NOTES

Lesson 4: Working with Graphs

Lesson Focus

When you finish this lesson, you will be able to

- ❑ read graphs and write corresponding statements
- ❑ create a graph given a linear relation
- ❑ match equations to graphs
- ❑ use interpolation and extrapolation
- ❑ solve problems using graphs

Lesson Introduction

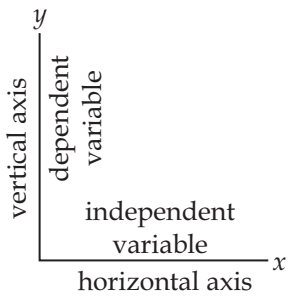
In this lesson, you will examine graphs and describe in words what the graph is telling you. You will create graphs from relations, discuss interpolation and extrapolation, and solve problems using graphs.

Creating Graphs—Independent vs Dependent Variables

Graphs can be used to display data and to predict outcomes. Creating graphs requires a basic understanding of dependent and independent variables. Also, you must create evenly spaced units or a scale along each axis.

Graphs display the relationship between two variables. Usually one variable depends on the other. The graphs are constructed with the independent variable along the horizontal axis and the dependent variable is along the vertical axis. Math teachers will often refer to the horizontal axis as the "x" axis, and the vertical axis as the "y" axis.

Include this diagram on your Resource Sheet.

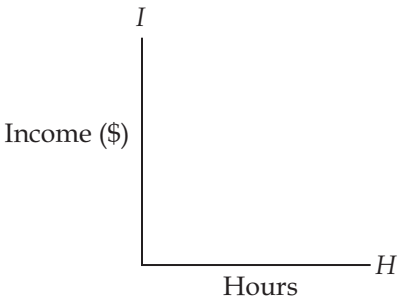


Example 1

Your income (I) is compared to the number of hours (H) you work. Draw and label the axes of a graph comparing income and hours worked.

Solution:

The more hours you work, the higher your income. So, the income depends on the hours. The income, because it is the dependent variable, goes along the vertical, y-axis. The hours, being the independent variable, would go along the horizontal x-axis.



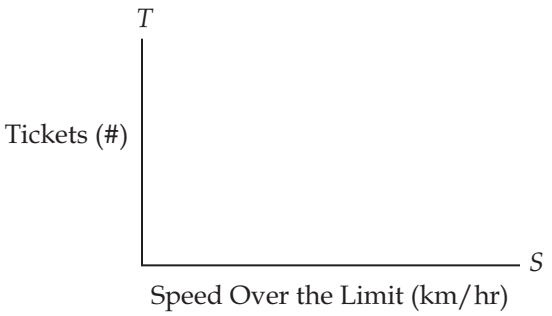
Remember: The independent variable always is placed along the horizontal axis.

Example 2

You are creating a graph to show your average speed (S) over the posted limit and the number of speeding tickets (T) you receive in a year. You must identify the variables.

Solution:

The tickets are given when you speed, and are not given when you do not speed. So the number of tickets you receive depend on how fast you drive. Tickets received would be the dependent variable, and the speed would be the independent variable. Your graph would be set up as follows:



Many times one could argue that the reverse is true! You could say that the speed at which you drive depends on how many tickets you have received. After paying lots of money in fines, people learn to slow down. Many of these examples could be argued in reverse order. For the purposes of this course, use the most obvious relationship between the variables.

Learning Activity 5.5

Variables

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write two equivelent fractions for $\frac{9}{15}$.
2. Marc has 42¢ more than Nicole. If Nicole has $1.74, how much money does Marc have?
3. If you buy a sports shirt worth $13.65 with a $20 bill, how much change do you get?
4. Reduce the ratio 35:15 to its most simplest form.
5. Evaluate 14 + 6 x 3.
6. A cake cooks for 1.75 hours. How many minutes is that?
7. The price of a basketball ticket is $15. How many tickets can you buy with $225?
8. You achieve $\frac{17}{20}$ on your mathematics test. What is your result as a percent?

continued

Learning Activity 5.5: Variables (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the dependent (D) and the independent (I) variables for each situation, and justify your choices.
   a) fast food meals per week _____ vs weight gain _____
   b) championships _____ vs games won _____
   c) profits _____ vs sales _____
   d) temperature outside _____ vs ice cream sales _____
   e) hours studying _____ vs marks _____
2. Draw a basic graph placing the independent variables along the horizontal (x) axis, and the dependent variable along the vertical (y) axis, for each situation listed above.

Setting the Scale for Each Axis

Sometimes the increments used in the scale are obvious, and sometimes you have to set your own scale. The rule is that each unit must be equal. These units can be any value: 1 or 5, 0.5 or 100, etc.

For example, if you were given the following data,

2, 5, 3, 3, 2, 1

you would likely set your scale at units or increments of 1 each. The number line may look like the following.

Notice each unit is evenly spaced, and the distance between each scale is equal. Start at zero and include the 4 and the 6, even though no data exists for those values.

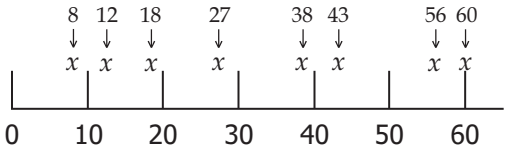
Example 1

State what scale you would use to graph the data: 8, 12, 27, 38, 43, 18, 56, 60

Solution:

If you use a scale with units of 1 to graph the above data points, the line would be far too long, and the points would be too spread out. A different scale is needed.

Look at the highest value (60) and the lowest (8). A scale of 10 would be appropriate. Start at zero.



The little *x* shows each value graphed on the number line.

The scale on the axis is evenly spaced. The value 38 should be placed between 30 and 40, and closer to the 40. When you look at this line graph, you should be able to guess the value to be about 38.

Since graphs usually involve both the horizontal and the vertical axis, you have to establish an appropriate scale for each.

Example 2

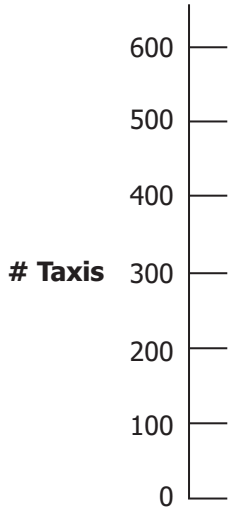
The given values show the number of taxis compared to city population.

Population	Taxis
50 000	10
125 000	26
246 000	55
385 000	110
625 000	305
1 350 000	550

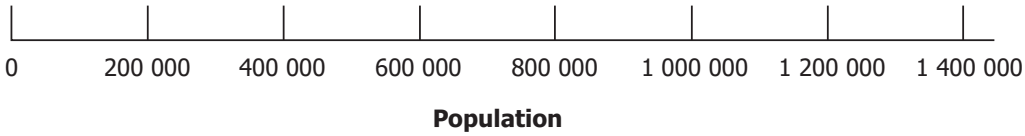
What scales would you use to represent the data (the number of taxis and the population)?

Solution:

The number of taxis depends on the population of the city. So, the taxi data would go along the vertical axis. Now, an evenly spaced but appropriate scale must be set. Since the number of taxis goes from 10 to 550, you could start at zero and use increments of 100 along the vertical axis.

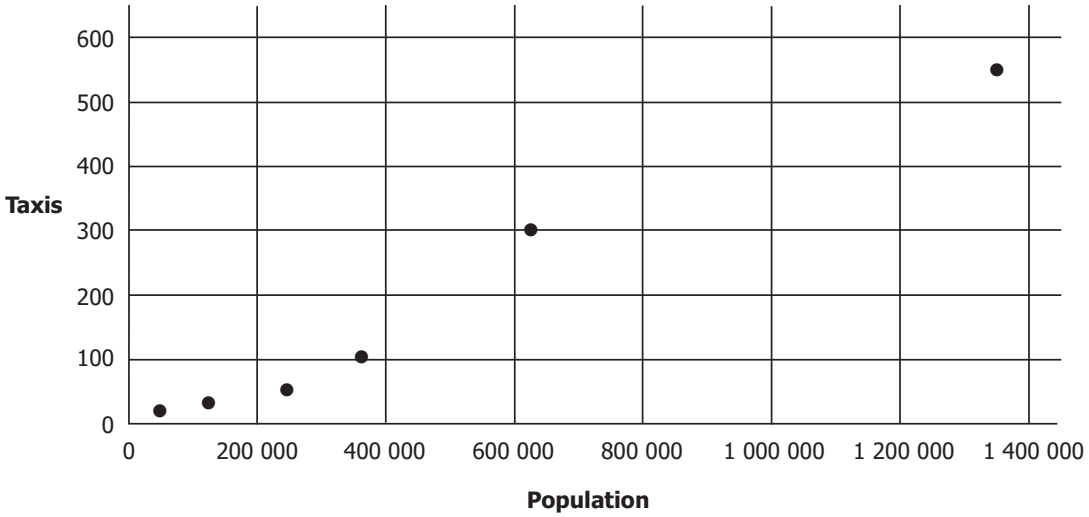


Now set the scale along the horizontal, or independent axis. The data ranges from 50 000 to 1 350 000. Choose increments that would be easy to read, and will not require a line longer than the width of the page. Increments of 200 000 would work well.



Combine the two axes and plot the data points as found in the following table of values.

Population	Taxis
50 000	10
125 000	26
246 000	55
385 000	110
625 000	305
1 350 000	550



Creating a Graph from an Equation

Note: Graphs may be generated using a computer spreadsheet program or drawn by hand on graph paper. Strive for neatness and accuracy.

The following steps walk you through how to create a graph from an equation. Include the steps, in your own words, on your Resource Sheet so that you can refer back to them and use them as a checklist.

Given an equation, create a table of values with at least 4 points for graphing purposes.

$y = 2x$

Step 1: Set up a table of values, and choose 4 different values for x. While these values can be random, you will want to keep them within a range that can be easily graphed.

x	y
0	
1	
–1	
2	

Step 2: Substitute each of these x-values into the equation to find the corresponding y-values.

if $x = 0$ $\quad y = 2x$

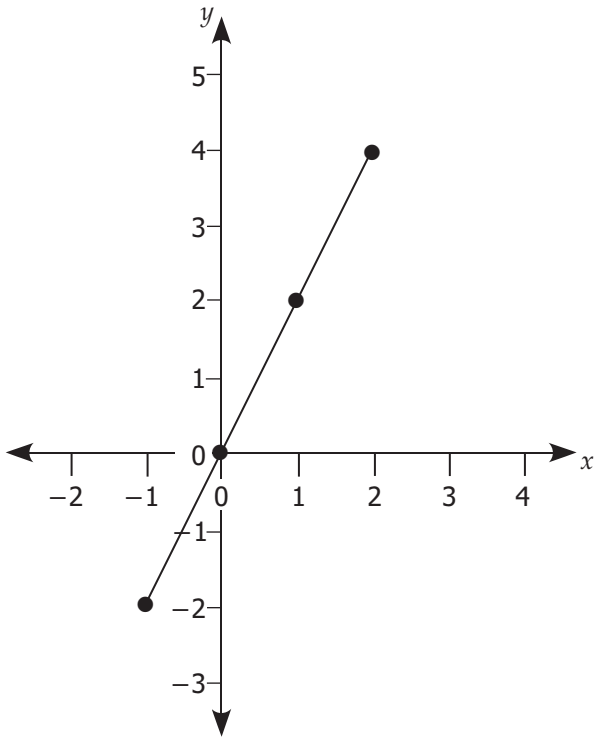
$y = 2(0)$

$y = 0 \qquad (0, 0)$

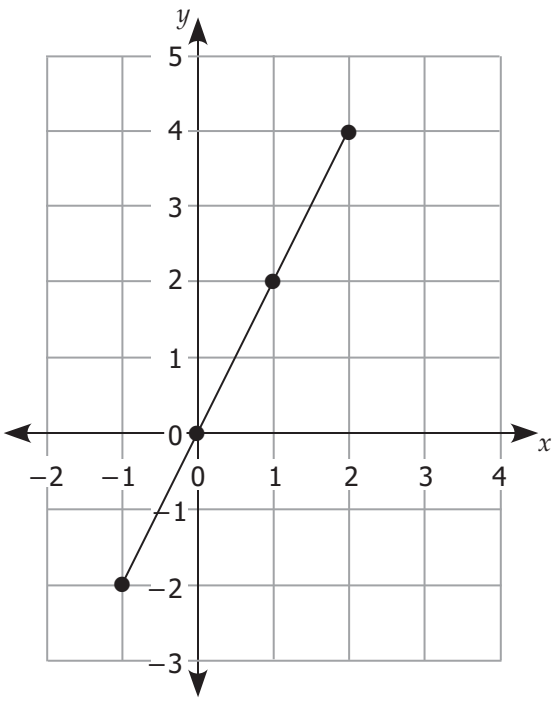
if $x = 1$	$y = 2x$	
	$y = 2(1)$	
	$y = 2$	(1, 2)
if $x = -1$	$y = 2x$	
	$y = 2(-1)$	
	$y = -2$	(–1, –2)
if $x = 2$	$y = 2x$	
	$y = 2(2)$	
	$y = 4$	(2, 4)

x	y
0	0
1	2
–1	–2
2	4

Step 3: Now set up the graph using these data points. Choose appropriate increments for the scales. Using a ruler, join the points with a line:



If you insert the gridlines, you can look up each point along the x or horizontal axis, and slide over to see where it corresponds to, or how it relates to the value along the y or vertical axis. Go to 1 along the horizontal axis, then up to the dot on the line. Now slide to your left, and notice that 1 on the horizontal axis relates to 2 on the vertical axis.



Interpolation and Extrapolation

As mentioned earlier in this module, these terms deal with finding values on a graph that were not given in the table of values. Interpolation means to find a value "inside" the data, or in between the given points. Extrapolation means to find a value outside the given points.

Example 1

Use the graph below to answer the following questions.

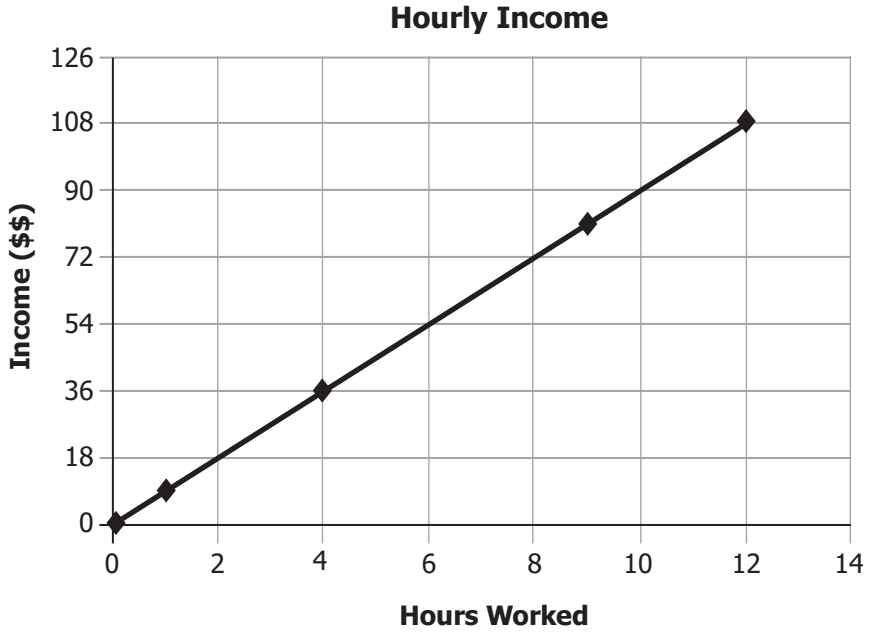
a) Find the hourly wage

b) Use interpolation to find income earned after 6 hours.

c) Use extrapolation to find income earned after 14 hours.

d) If you earn \$72, how many hours did you work?

e) Was question (d) an example of interpolation or extrapolation?



Solutions:

a) The easiest point to read shows 4 hours earned \$36, so that would be $\frac{\$36}{4} = \$9$ per hour.

b) 6 hours relates (where it meets the line) to \$54.

c) 14 hours would hit the line (if extended) at \$126.

d) \$72 meets the line directly above 8 hours along the *x*-axis.

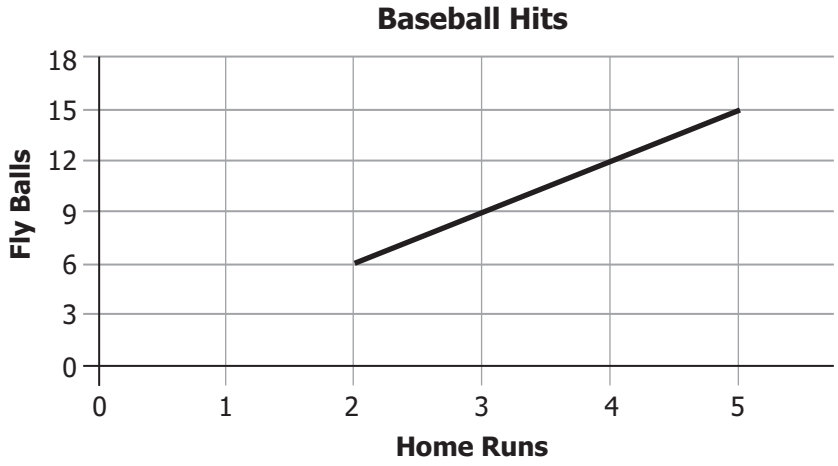
e) This would be using data between given points, so it is interpolation.

Writing Equations from a Graph

You have used given points to create graphs. You have created points from an equation, and then created a graph. In earlier lessons you identified patterns from graphs. Now you will write equations from what you observe on graphs.

Here is a graph you looked at earlier, comparing the number of fly balls to the number of home runs. The data about home runs is along the horizontal axis, and the fly balls are along the vertical axis. You can see there were three times as many fly balls as there were home runs. If you think, "three times the home runs equals the number of fly balls," you can quickly set the equation:

$F = 3H$



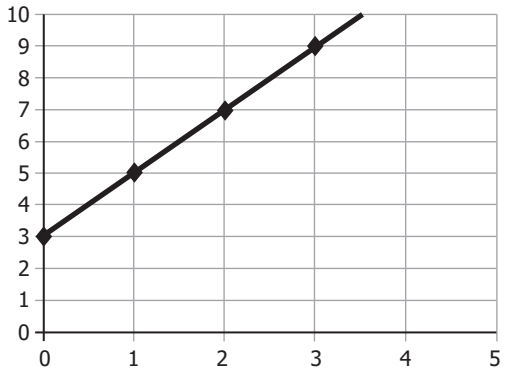
Take any point along the graph and substitute it into the equation to see if it is a true statement. Take $H = 3$. Slide up, and you can see that it hits the line where $F = 9$. Substitute $H = 3$ into the equation to see if in fact it does equal nine.

$F = 3H$
$F = 3(3)$
$F = 9$

You proved that the equation fits the data shown on the graph for that point. You might want to test other points along the graph to ensure the equation is accurate every time.

Example 1

Given the following graph, write the equation represented by the data.



Solution:

To find the equation of this line, you need to look for a pattern. Setting up a table of values using the points along the line is helpful. When $h = 0$, $v = 3$, etc. (h = horizontal axis, while v represents the vertical axis).

h	v
0	3
1	5
2	7
3	9

If you try $v = h + 3$, it will work for the first point on the line, but not for any of the others. As you keep looking for a pattern, you will find $v = 2h + 3$ works for each point on the line. That is the equation of the line.

Using extrapolation, extend the line to find v when $h = 5$. It appears that v would equal 13. To prove it, substitute $h = 5$ into the equation:

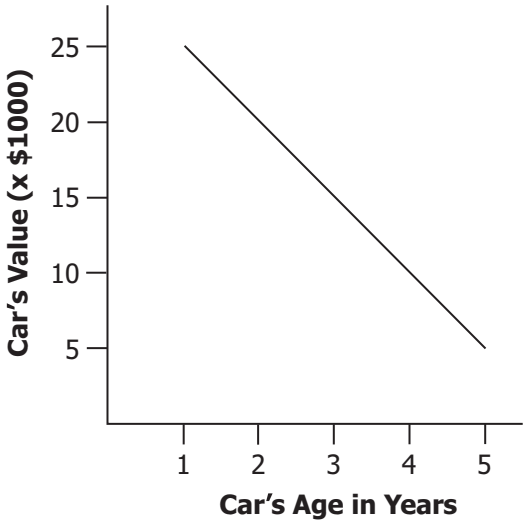
$v = 2h + 3$
$v = 2(5) + 3$
$v = 10 + 3$
$v = 13$

What Do These Graphs Tell Us?

When we look at basic graphs, you must be able to put into words what you see.

Example 1

State what conclusion we can make from looking at the graph.

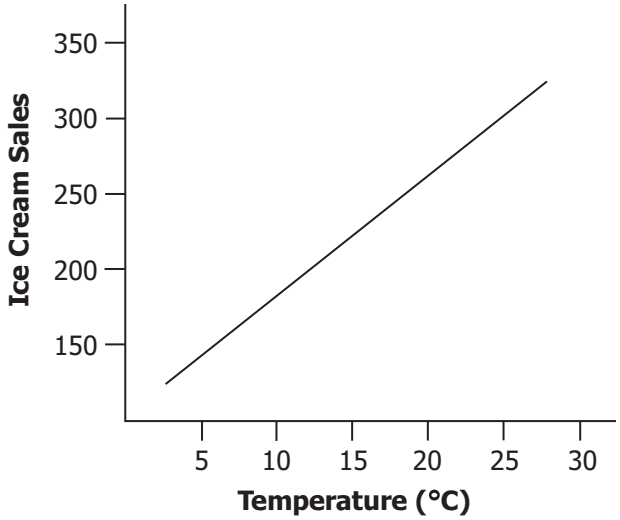


Solution:

This graph tells us that as a car ages, its value decreases.

Example 2

State what conclusion we can make from looking at the graph.



Solution:

This graph indicates that as temperature increases, so does the sale of ice cream.

Matching a Graph to an Equation

Using your skills with substitution, you can quickly match an equation to a graph. Pick any point along the line of the graph, and substitute those horizontal and vertical values into the equation to see if they satisfy the equation.

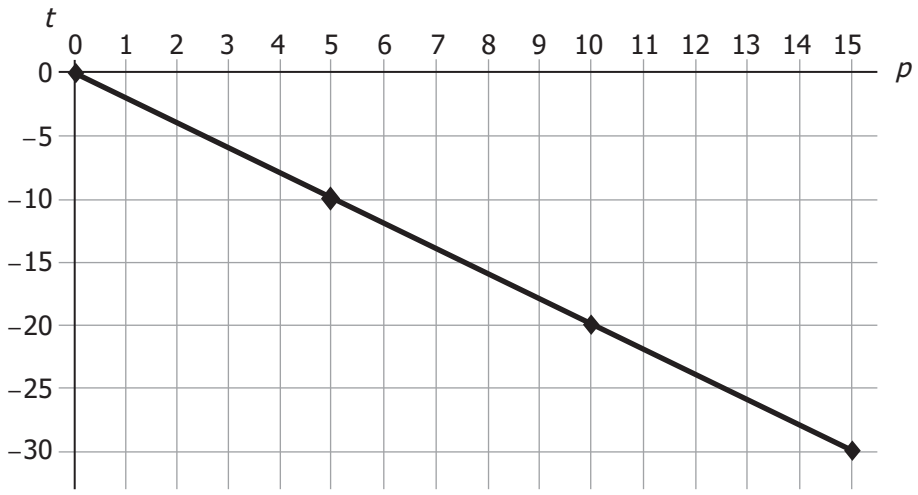
Example 1

Which equation matches this graph (t represents the vertical axis, and p represents the horizontal axis)?

a) $t = 3p$

b) $t = -2p$

c) $t = p + 2$



Solution:

Take the point $p = 5$ and substitute it into the first equation $t = 3p$.

$t = 3(5)$

$t = 15$

But, when you look at the graph, when $p = 5$, $t = -10$. The first equation does not match the graph.

Substitute $p = 5$ into the second equation.

$t = -2(5)$

$t = -10$

Look at the graph, when $p = 5$, $t = -10$, so the second equation matches the graph.

To be sure, find a second point on the line, and plug the values into the second equation to test it.

The point (10, –20) is indicated with a dot ($p = 10$, $t = -20$).

Substitute those values into the second equation, $t = -2p$, to see if they work.

$t = -2p$
$-20 = -2(10)$
$-20 = -20$

This is true, so that point also satisfies the equation.

Learning Activity 5.6

Graphs and Equations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Estimate the value of 126.35 x 4.13.
2. A $50 kite is reduced by 30%. What is the price of the kite after the discount?
3. Solve $12 + c = 51$.
4. Solve $9 = 35 + t$.
5. If it was 7°C Monday, 10°C Tuesday, 11°C Wednesday, 9°C Thursday, and 13°C Friday, what was the average temperature for these five days?
6. A $155 raincoat is reduced by 10%. What is the amount of the discount?
7. Jacob can swim 10 lengths of the pool in 5 minutes. At that pace, how many lengths can he swim in half an hour?
8. Subtract $\frac{5}{2} - \frac{1}{2}$.

continued

Learning Activity 5.6: Graphs and Equations (continued)

Part B

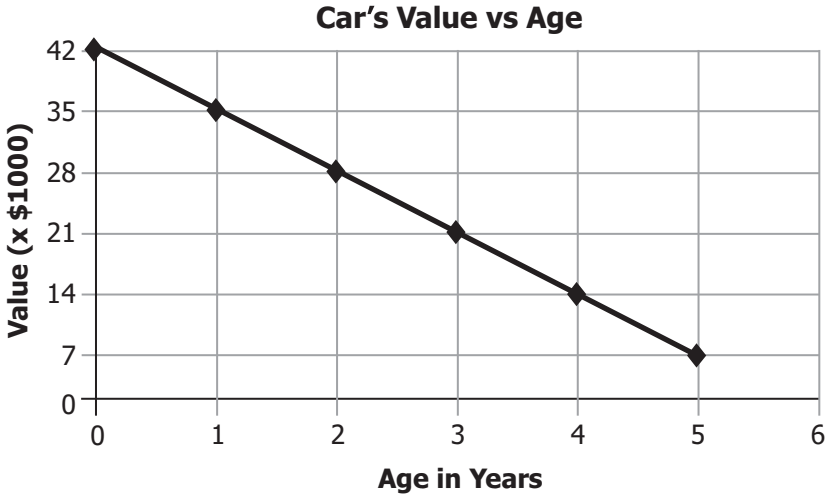
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Set up a graph using an appropriate scale, labelled axes, and accurately plotted points joined by a line for these values, and write an equation for the graph.

   The following chart shows the total number of toddlers in a nursery compared to the number of toddlers with colds in the nursery. Which is the dependent variable? **Hint:** The more toddlers present, the more toddlers with colds.

Toddlers with Colds	Total Toddlers in Nursery
2	10
8	40
15	75
26	130

2. Use the graph below to answer the questions that follow.



   a) If you had a maximum of $21 000 available to purchase a car, how old a car would you be buying?

   b) If your car is 4 years old, what is its approximate value?

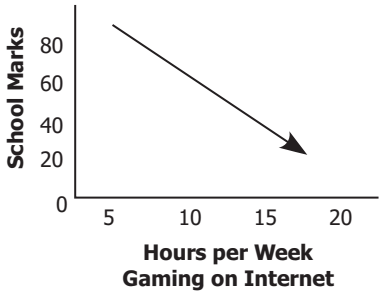
continued

c) Following the graph, what is a 7-year-old car worth?

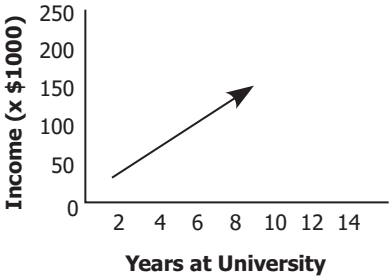
d) What is the approximate value of a car that is $2\frac{1}{2}$ years old?

3. Write a sentence describing what each of the graphs below shows.

a)



b)

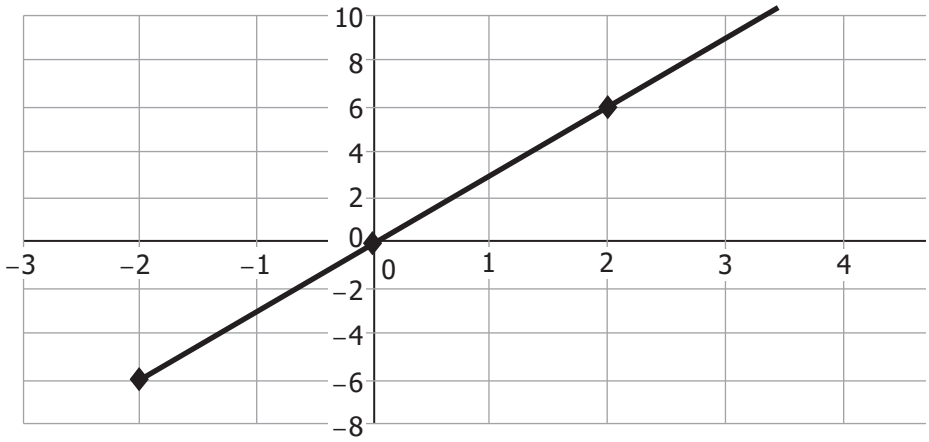


4. Which equation matches the graph below.

a) $v = -3h$

b) $v = h + 4$

c) $v = 3h$



Lesson Summary

In this lesson, you analyzed graphs; looking for patterns and writing equations for the graph. You learned to set appropriate scales when creating graphs from equations. You also developed skills in problem solving using interpolation and extrapolation.

Assignment 5.4: Working with Graphs

Total: 24 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. For each situation, identify the dependent variable (D) and the independent variable (I), and justify your answer. *(4 marks)*

   a) school marks _____ vs. hours studying _____ *(2 marks)*

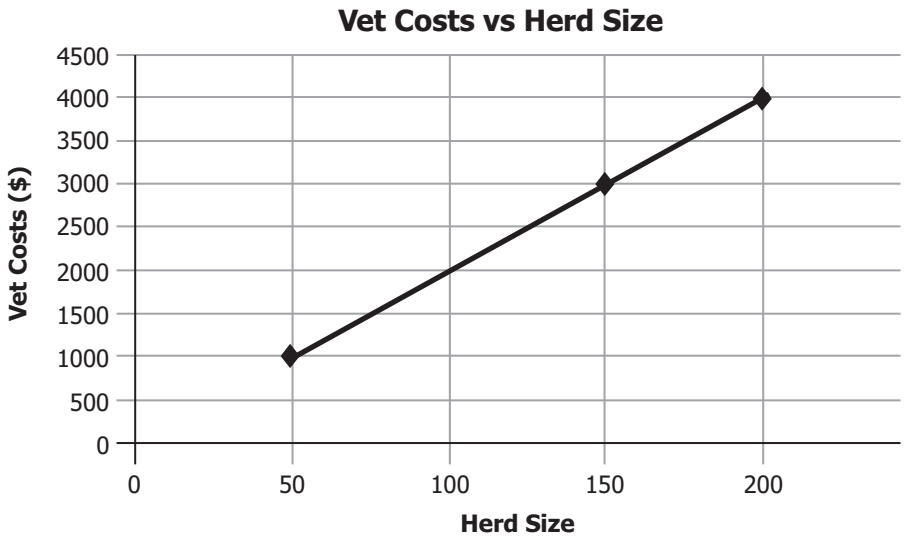
   b) hourly wage _____ vs. income _____ *(2 marks)*

2. Generate at least 4 points from this equation, and draw a graph. Be sure to label the axis, and to use an appropriate scale with consistent increments. *(5 marks)*

   $n = 2m + 2$

m	*n*

3. The graph below shows the veterinarian costs for a cattle rancher. *(5 marks)*



a) Using interpolation, find the cost of treating a herd of 100 animals.

_____

b) Using extrapolation, estimate how many cattle could be treated for $6000.

_____

c) Write an equation showing the pattern on the graph.

_____

d) The bill from the vet is $3500.

i) How many animals did he treat?

_____

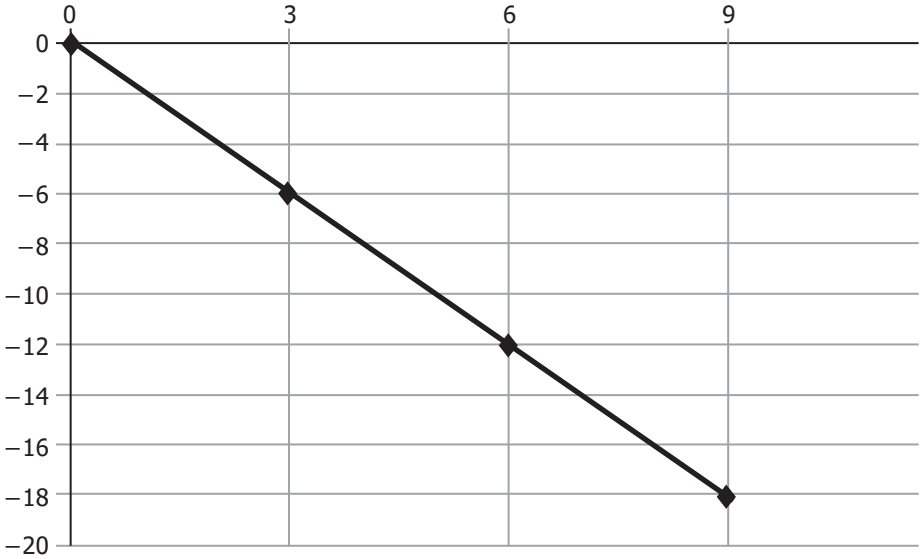
ii) Is this using interpolation or extrapolation, and why?

_____

_____

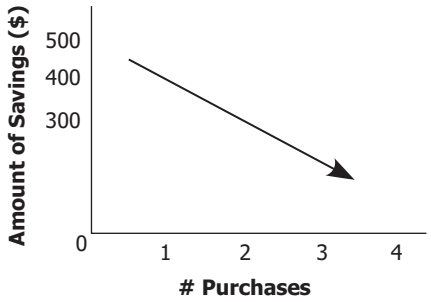
4. Identify the pattern represented with this graph, and write the equation. *(2 marks)*

Let v = vertical axis, and h represent the horizontal axis.

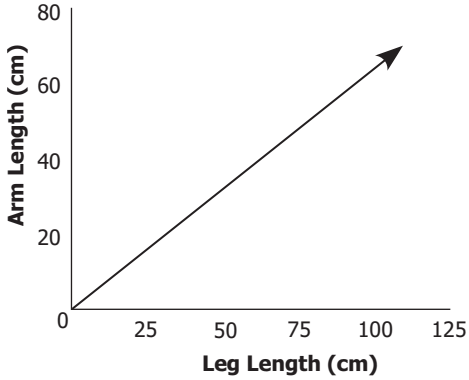


5. Write a word phrase to describe each of the graphs below. *(4 marks)*

a)

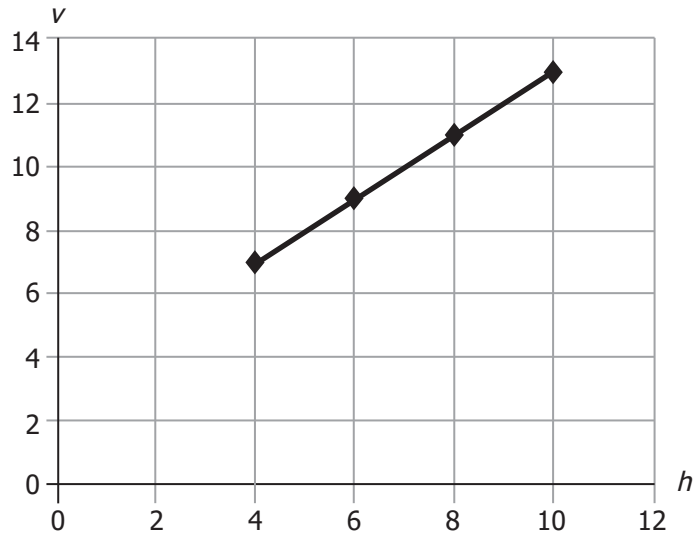


b)



6. Select the equation that matches the graph. *(4 marks)*

Graph A

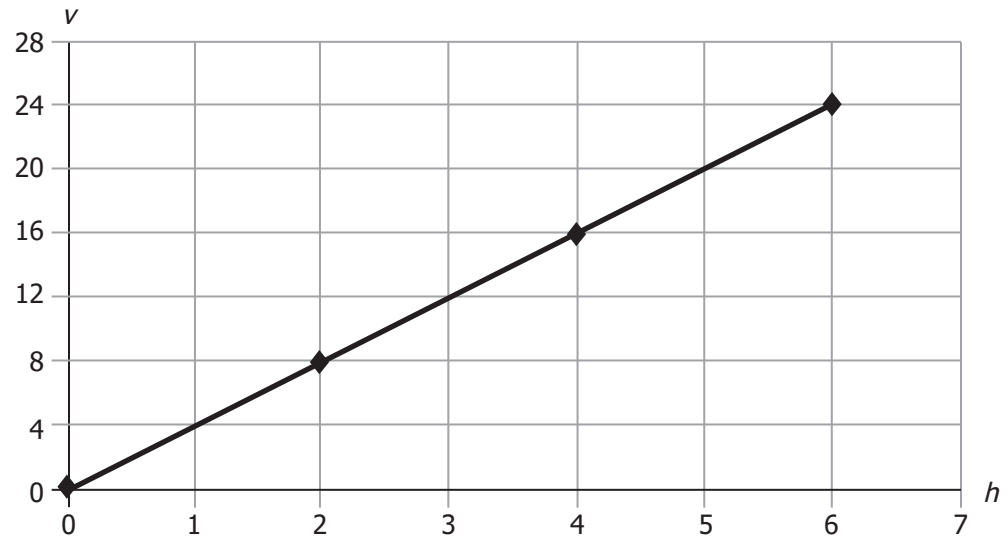


a) $v = h - 3$

b) $v = h + 2$

c) $v = h + 3$ _____

Graph B



a) $v = -4h$

b) $v = 4h$

c) $v = \frac{h}{4}$ _____

Lesson 5: Operations with Equations

Lesson Focus

When you finish this lesson, you will be able to

- ❑ perform addition and subtraction on equations to isolate the variable
- ❑ perform multiplication and division on equations to isolate the variable
- ❑ match equations to graphs
- ❑ identify the error, and make corrections
- ❑ solve problems using equations

Lesson Introduction

In this lesson, you will add, subtract, multiply, and divide to solve equations.

Solving for Variables—Equality and Isolation

Equality is everywhere. If you have brothers and sisters, you want to be sure that your parents give each sibling equal amounts of treats or gifts. If you are working at a job, you want to be sure you are receiving equal pay for equal work. Teachers work very hard to ensure all students are assessed equally, without having favourites!

In math, you can "solve" problems by finding what the variable or unknown "equals." Phrases like "solve the problem," "find the value of," or "what does x equal?" basically mean the same thing.

When little kids behave badly, sometimes parents will "isolate" them in a quiet space. Hospital patients with contagious diseases will often be placed in "isolation" to protect them and others. In algebra, you always want to "isolate" the variable, or get it by itself. Once the variable is isolated, you have the answer and can determine what the variable equals.

In previous years, you learned basic mathematical operations. This lesson will review them in the context of solving equations with variables, before moving onto more challenging equations and problems.

Addition and Subtraction Review

If your friend gives you $5, and then takes away $5, they cancel each other out. This is an example of using opposite operations to cancel a value. Addition is the opposite operation to subtraction, and multiplication is the opposite operation to division.

To cancel a –4, add 4.

To cancel a 3, subtract 3.

The rule is "if you perform an operation on one side of an equation, you must do the same on the other side of the = sign." Think of a teeter-totter. If you add a weight to one end, you need to add an equal weight to the other end to keep it balanced. This is the "balanced scale" method of solving equations. You want to keep the equation balanced while isolating the variable on one side in order to solve for the value of the unknown.

Example 1

Given the following equation, solve for x.

$x - 2 = 5$

Solution:

To isolate the variable x, you need to remove the –2. The opposite operation to subtraction is addition. If you add 2 to the left side of the equation, it will "zero out" or cancel the –2, leaving the variable x by itself.

But being an equation where both sides are equal, what you do on one side, must be done equally on the other.

$$x - 2 = 5$$

$$x \cancel{-2} + \cancel{2} = 5 + 2$$

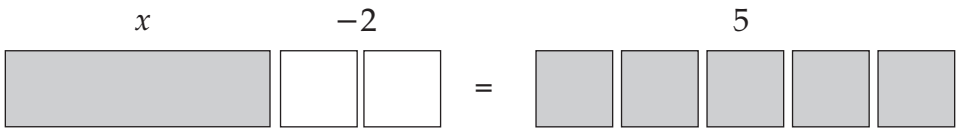
The +2 cancels out the –2 on the left side of the equation, isolating the variable. Then combine the two values on the right-hand side.

$x = 7$

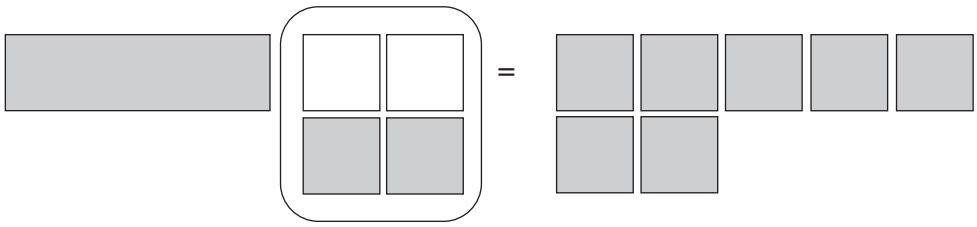
You have solved the equation by isolating the variable, using a "balanced scale" method.

You used the opposite operation of adding 2 to cancel the subtract 2. And, to keep the equation balanced, you added 2 to each side of the equation so they remained equal.

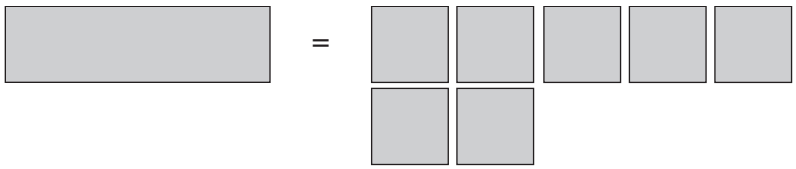
You can use algebra tiles to show how to isolate the variable.



Knowing that you need to isolate the x-tile, cancel out the two negative 1 tiles by introducing two positive 1 tiles. But if you do it on one side, you have to do it equally on the other.



The two positive tiles cancel out the two negative tiles. You are left with the following diagram.



$x = 7$

Example 2

Solve for x:

$x + 3 = 2$

Solution:

To isolate the variable, introduce a –3 to cancel out the +3. And if you do it on one side, do it equally on the other!

$$x + 3 - 3 = 2 - 3$$

$$x = -1$$

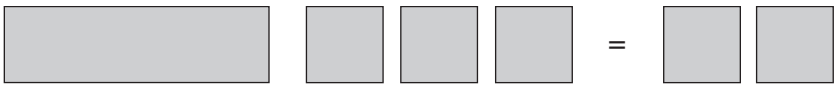
Remember:

2 – 3 "What do you have more of?" (Negatives)

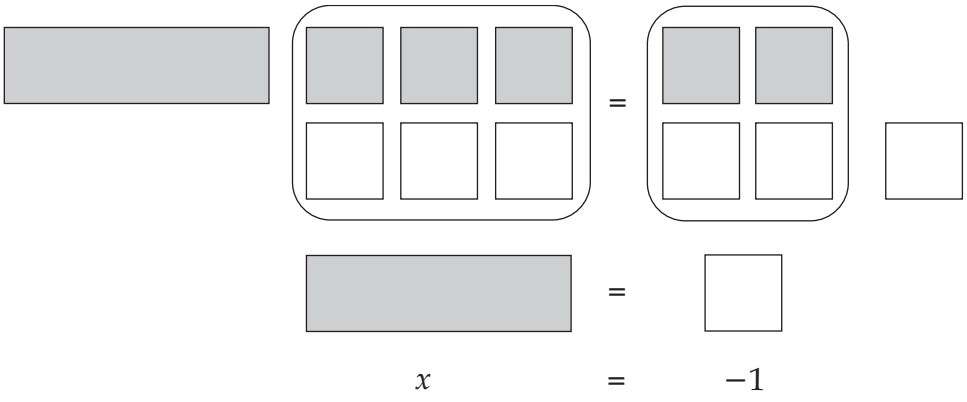
"How many more do you have?" (One)

$x = -1$

This can be shown using algebra tiles.



To isolate the variable, you need to cancel out those 3 positive 1 tiles by introducing 3 negative 1 tiles. And if you do it on one side, do it equally on the other!



$x = -1$

Multiplication and Division Review

A term such as "$2x$" means "2 times x", or "2 multiplied by x."

An equation such as "$2x = -6$," can be read as "2 times x equals negative 6."

If you are given the equation $2x = -6$, as usual, you need to isolate the variable. The opposite operation to multiplication is division. So, if you want to remove the "times 2," you need to divide by 2. And, what you do to one side must be done equally to the other.

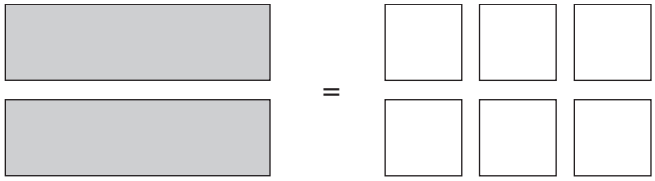
$$2x = -6$$

$$\frac{2x}{2} = \frac{-6}{2}$$

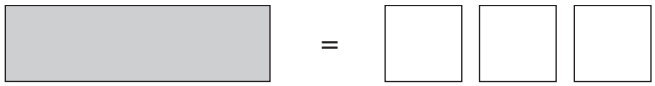
$$x = -3$$

This equation can be represented using algebra tiles.

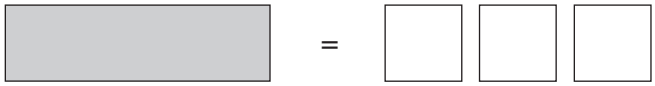
$2x = -6$



To isolate x, divide these tiles into two equal groups, each with an x-tile isolated.



and



Both groups agree that $x = -3$.

Example 1

Solve for x, given $\frac{x}{3} = -2$.

Solution:

An equation written in the form $\frac{x}{3} = -2$ means "x divided by three equals –2."

Since multiplication is the opposite operation to division, multiply both sides equally by 3 to cancel out the division, and isolate the variable. Use (3) to indicate multiplication.

$$\frac{x}{3}(3) = -2(3)$$

$$x = -6$$

Combination Review

When terms with combined operations are presented, the method to solve for the variable is to isolate the **term** with the variable, and then isolate the **variable**.

You should include these two steps *in the correct order* on your Resource Sheet.

Example 1

Solve for x: $-2x + 3 = 9$

Solution:

To isolate the **term** with the variable x, first cancel out the +3 by subtracting 3 from both sides of the equation.

$$-2x + 3 - 3 = 9 - 3$$

$$-2x = 6$$

To isolate the **variable**, divide both sides equally by –2. You want to solve for a positive variable, so divide by negative 2. Negative $2x$ divided by negative 2 leaves you with a positive x.

$$\frac{-2x}{-2} = \frac{6}{-2}$$

$$x = -3$$

Example 2

Solve for x: $\frac{x}{5} - 4 = -2$

Solution:

To isolate the term with the variable first, add 4 to each side, cancelling out the –4.

$$\frac{x}{5} - 4 + 4 = -2 + 4$$

$$\frac{x}{5} = 2$$

To isolate the variable, multiply both sides equally by 5 to cancel out the division.

$$\frac{x}{\cancel{5}}(\cancel{5}) = 2(5)$$

$$x = 10$$

Learning Activity 5.7

Solving for Variables

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert 0.75 to a fraction.
2. Convert $\frac{7}{10}$ to a decimal.
3. Convert $\frac{17}{5}$ to a mixed fraction.
4. If 24% of 500 is 120, what is 48% of 500?

continued

Learning Activity 5.7: Solving Variables (continued)

5. Calculate 6300 ÷ 70.
6. What is triple 34?
7. Which is larger: $\frac{3}{2}$ or $\frac{4}{3}$?
8. You ate 6 of 8 pieces of tourtière. What percent of the tourtière remains?

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Solve the following equations by isolating the variable using opposite operations. Show all your work.

a) $h-3=2$

b) $p+7=-5$

c) $-4x=6$

d) $\frac{x}{-3}=5$

e) $2k-4=-6$

f) $\frac{x}{2}+5=4$

Collecting Like Terms

You learned how to collect like terms in Module 4 so that you could simplify polynomials. Similarly, the ability to collect like terms and to perform basic operations (addition, subtraction, multiplication, and division) are important when you are solving equations at all levels of difficulty. Although other strategies are used at higher levels of difficulty, being able to collect like terms and execute basic operations are essential skills..

Example 1

Given an equation in the form $2x = 10 - 3x$, solve for x.

Solution:

In all the previous examples, the variable was on one side, and a constant was on the other. In this case, you will first need to collect the like terms by moving the terms with the variable onto one side, and the constants to the other side of the equation.

$$2x = 10 - 3x$$

In this situation, if you add $3x$ to each side, it will cancel out the $-3x$ on the right-hand side, and you will have all the variables on one side. Combine the like terms.

$$2x + 3x = 10 - \cancel{3x} + \cancel{3x}$$

$$5x = 10$$

Then isolate the variable by dividing both sides by 5.

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

A good habit is to check your answer. Make sure both sides of the equation balance, or are equal.

$2x$	$10 - 3x$	Start with the original equation
$2(2)$	$10 - 3(2)$	Substitute the value calculated for the variable.
4	$10 - 6$	Simplify
4	4	Equality

Both sides are the same, so the answer is correct.

Example 2

Solve for k: $-6k - 8 = 4k + 17$

Solution:

This equation has terms with variables on both sides, and terms with constants on both sides. The goal is to get the variables on one side, and the constants on the other. In this situation, you have the option of adding $6k$ to each side, or subtracting $4k$ from each side. Either approach is correct. The method that is easier is to keep the result positive, so adding $6k$ to each side leaves you with positive $10k$. If you establish this habit of keeping the result positive, it will reduce many opportunities for mistakes, especially in the next lesson.

$$-6k - 8 = 4k + 17$$

$$\cancel{-6k} - 8 + \cancel{6k} = 4k + 17 + 6k$$

Add $6k$ to each side to move all the terms with the variable on to one side

$$-8 = 10k + 17$$

Combine like terms.

$$-8 - 17 = 10k + \cancel{17} - \cancel{17}$$

Subtract 17 from each side to isolate the term with the variable.

$$\frac{-25}{10} = \frac{\cancel{10}k}{\cancel{10}}$$

$$-2.5 = k$$

Divide each side by 10 to isolate the variable.

Dividing by a Variable

Sometimes the variable is placed in the denominator, such as:

$$\frac{3}{x} = 6$$

To isolate the variable, you need to move it out of the fraction format. Remember that the opposite operation to division is multiplication. If you multiply both sides equally by x, the fraction is gone!

You should make a note of how to get rid of the fraction on your Resource Sheet.

$\frac{3}{\cancel{x}}(\cancel{x}) = 6(x)$ $\qquad$ $\frac{x}{x}$ equals 1

$3(1) = 6x$

You can see how the variable is now out of the fraction format. Now divide both sides equally by 6 to isolate the variable.

$$\frac{3}{6} = \frac{6x}{6}$$

$$\frac{1}{2} = x$$

Check the answer.

$\frac{3}{x}$	6
$\frac{3}{\frac{1}{2}}$	6

Remember that $\frac{3}{\frac{1}{2}}$ means $3 \div \frac{1}{2}$ or 3 x 2 which is 6.

6	6

Both sides are the same, so the answer $x = \frac{1}{2}$ must be correct.

Example 1

Solve for b: $\frac{-27}{b} = 3$

Solution:

With the variable in the denominator, multiply both sides by b.

$$\frac{-27}{b}(b) = 3(b)$$

The variables on the left side cancel because $\frac{b}{b} = 1$.

$$-27 = 3b$$

Now divide each side by 3 to isolate the variable.

$$\frac{-27}{3} = \frac{3b}{3}$$
$$-9 = b$$

Check your answer.

$\frac{-27}{b}$	3
$\frac{-27}{-9}$	3
3	3

Working with Brackets

Recall your previous work with brackets.

When dealing with the order of operations, any operations inside the brackets must be dealt with first.

$3 * (5+7) = 3 * (12) = 36$

But brackets can also indicate multiplication.

$6(4) = 6 * 4 = 24$

Example 1

Solve for x: $2(x - 1) = 6$

Solution:

There is nothing inside the bracket that can be simplified, and so the next step would be to consider multiplication. The 2 outside the bracket must be multiplied by each term inside the bracket. Using arrows to show each step, distribute the 2 to each term inside the bracket, and multiply.

$$2(x - 1) = 6$$

$$2x - 2 = 6$$

Now with the brackets gone, isolate the term with the variable, and then isolate the variable, following the steps used previously.

$$2x - \cancel{2} + \cancel{2} = 6 + 2$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

When presented with brackets on each side, follow the same steps.

- Distribute the value in front of the brackets.
- Place terms with the variable on one side, terms with constants on the other.
- Isolate the variable.

As a reminder for yourself, include the steps to solve for a variable inside a set of brackets on your Resource Sheet.

Example 2

Solve for f: $-3(f + 3) = 2(f + 3)$

Solution:

$$-3(f + 3) = 2(f + 3)$$

$$-3(f + 3) = 2(f + 3)$$

$$-3f-9=2f+6$$ Add $3f$ to each side to collect the variables and keep the sign positive.

$$-3f-9+3f=2f+6+3f$$ Cancel and then combine like terms.

$$-9=5f+6$$

$$-9-6=5f+6-6$$ Subtract 6 from each side to isolate the term with the variable.

$$\frac{-15}{5}=\frac{5f}{5}$$ Divide both sides by 5 to isolate the variable.

$$-3=f$$

Check your answer.

$-3(f+3)$	$2(f+3)$
$-3(-3+3)$	$2(-3+3)$
$-3(0)$	$2(0)$
0	0

Both sides are equal, so the answer $f = -3$ is correct.

Example 3

Solve for x: $5(2x - 3) = -2(-4x + 5)$

Solution:

- Distribute the value in front of the brackets.
- Place terms with variable on one side, terms with constants on the other.
- Isolate the variable.

$$5(2x-3)=-2(-4x+5)$$

$$10x-15=8x-10$$ Subtract $8x$ from each side.

$$10x-15-8x=8x-10-8x$$

$$2x-15=10$$ Add 15 to each side.

$$2x-15+15=-10+15$$

$$2x=5$$ Divide each side by 2.

$$x=\frac{5}{2}\left(\text{or } 2\frac{1}{2}\right)$$

Learning Activity 5.8

Solving Advanced Equations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Evaluate $6 + 3 \times 4 \div 2$.
2. If the diameter of a circle is 10 cm, estimate the circumference.
3. Eric has 256 hockey cards. Tristan has two times as many. How many cards do both boys have?
4. Estimate the square root of 63.
5. How many minutes is there between 13:15 and 17:00?
6. Estimate the value of $4.99 + $5.02 + $1.99 + $3.98.
7. Your savings account balance is $47 and you deposit $56. What is the new balance?
8. There is a shipping charge of 4% on the $250 bicycle you purchased. What is the amount of the shipping charge?

continued

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Solve each equation by isolating the variable. Show all your work.

1. $3x = 5 + 2x$
2. $-5g - 4 = 5g$
3. $5w + 6 = 2w - 3$
4. $2v - 7 = 6v + 13$
5. $\frac{5}{x} = 10$
6. $2^3 = \frac{16}{x}$
7. $3(k - 2) = 9$
8. $-2(r + 5) = 3(r + 2)$
9. $5(3d - 3) = 3(3d + 5)$
10. $-4(x + 6) = 3(2x - 4)$

Problem Solving

Use your skills with solving equations to solve problems.

Example 1

Your job pays $8.75 per hour. You have 8 weeks over the summer to work, and you plan on working 32 hours per week. Use an equation to determine your total income (I) for the summer.

Solution:

Income = hourly rate of pay x hours worked per week x number of weeks

$I = 8.75 * 32 * 8$

$I = 2240$

You will earn an income of $2240

Example 2

You need to earn $3237.50 over the summer to buy a used car. Your job pays $8.75 per hour. Use an equation to determine how many hours (h) you will have to work to generate this income.

Solution:

$$3237.50 = 8.75h$$

$$\frac{3237.50}{8.75} = \frac{8.75h}{8.75}$$

$$370 = h$$

You will have to work for 370 hours.

Learning Activity 5.9

Problem Solving

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Express $\frac{9}{20}$ as a decimal.
2. Find $\sqrt{169}$.
3. Divide a $640 bill equally among eight people.
4. An $800 boat now sells for $600. What is the percent rate of discount?
5. What is the median of the following: 1, 6, 8, and 10?
6. Estimate a 15% tip on a bill of $48.75.
7. How many centimetres are there in 49.4 m?
8. Write an equivalent fraction for 0.6.

continued

Learning Activity 5.9: Problem Solving (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Set up an equation and solve the given problem.

1. Textbooks for university courses cost an average of $110 each, and tuition will be $850 per course. You plan on taking 4 courses requiring one textbook each. Find the total estimated cost of books and tuition for your first year.
2. You have been asked to play hockey for a great team. The basic fee is $2600 plus $450 per tournament. You have a total amount of $4900 available for hockey expenses this year. Use an equation to determine how many tournaments you will be able to participate in.
3. Your friend was given the following problem, and his work was marked wrong. Identify the error, and correct it.

$$3(2x - 4) = 5(3x + 3)$$
$$6x - 12 = 15x + 15$$
$$9x = 27$$
$$x = 3$$

Lesson Summary

In this lesson, you learned to perform various operations with equations. The next lesson introduces you to inequalities.

Assignment 5.5: Operations with Equations

Total: 24 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. a) $3x = 10 - 2x$ *(2 marks)*

b) $25 + 3r = 4r$ *(2 marks)*

2. a) $3(x - 5) = 9$ *(3 marks)*

b) $9 = -2(x - 4)$ *(3 marks)*

3. $4g - 7 = 9g + 3$ *(3 marks)*

4. $7(w - 5) = 5(w + 3)$ *(3 marks)*

5. a) $3(2v + 3) = -3(-3v + 4)$ *(3 marks)*

b) $-2(4t - 6) = 5(3t - 2)$ *(3 marks)*

6. $\frac{14}{x} = -2$ *(2 marks)*

NOTES

Lesson 6: Inequalities and Graphing

Lesson Focus

When you finish this lesson, you will be able to

- ❑ solve linear inequalities
- ❑ apply the rule for multiplying an inequality by a negative
- ❑ graph linear inequalities

Lesson Introduction

Not everything has an exact value. This lesson deals with inequalities, using terms like "greater than" and "less than." You will also learn how to graph inequalities.

As you work through this lesson, write down on your Resource Sheet the inequality symbols and what they mean. The Resource Sheet icon has been included where the inequality symbols are found. You should also include instructions about how to plot inequalities on a number line.

Expressing Inequalities

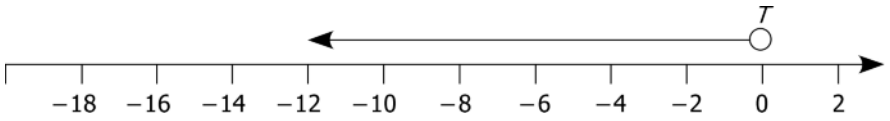
Manitoba's temperature in January is generally below 0°C. But, it doesn't stay at a constant temperature. It can be any value "less than " 0. This means the temperature could be –35, –18, or –4, but not anything equal to or above 0. This is an inequality.

In math, this symbol "<" is used to show "less than."

You would read "$T < 0$" as "T is less than 0," where T stands for temperature.

Notice how the T is at the small end of the symbol, indicating it is less than the value at the large end.

On a number line, this inequality would be shown as follows:



The arrow indicates the line continues to include all values less than 0.

The open circle indicates that 0 is not included in the answer.

If you are 14 years old and you have younger brothers and sisters, their ages would be less than yours. You would write this in math terms as:

Let A = their ages

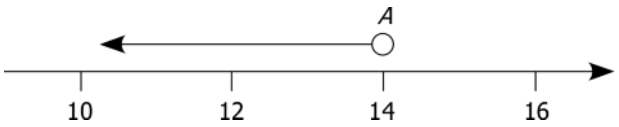
$A < 14$

This would be read as follows:

"A is less than 14."

Notice the variable is on the small end of the symbol, and the 14 is at the large end.

This could be shown on a number line as follows:



The arrow and line indicate all values that are less than 14.

Imagine a cousin drops in to visit and she is 16 years old. Would her age "satisfy" the inequality? The answer is no, because she is not less than 14.

Now, in the summer months, our temperature is usually above 0°C. If we let T = temperature, you would write an inequality representing this situation as:

$T > 0$

This is reads "temperature is greater than 0." Notice the variable is at the large end of the symbol, indicating it is greater than the value at the smaller end.

On a number line, this would be shown as follows:

All values that could touch the line are greater than 0.

Inequality Symbol Drill

Place the appropriate < or > symbol between these values to make true statements.

a) 5 9

b) 2 –5

c) 16 12

d) –4 –8

e) 21 –21

f) –6 –2

Answers:

a) Since 5 is less than 9, you would insert the < symbol.

b) 2 is greater than –5 (think temperature), so 2 > –5

c) >

d) >

e) >

f) <

Working with positive and negative values can be confusing sometimes. If you compare the numbers to values on a thermometer, it might help you get a better sense of which are less than or greater than.

Returning to the earlier example about younger siblings, imagine you have a twin sister, who is obviously the same age as you. If you are both 14 and the rest of your siblings are younger, you now must include the 14 in your inequality statement, so it would read: "age is less than or equal to 14"

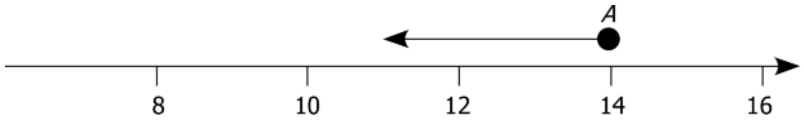
In math, when there is some equality included, draw a line under the inequality symbol. So, if your siblings are less than or equal to 14, it would be written as follows:

$$A \leq 14$$

Notice that the "less than" symbol is still there. You have just added the line underneath it to include your sister, whose age is equal to yours. This is read as:

"Age is less than or equal to 14."

When you illustrate this on a number line, the circle must now be filled in to show the "equal to" part of the expression.



Previously, when you graphed the inequality $A < 14$, it had an empty circle.

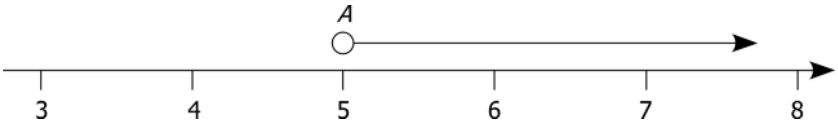
Here, you have $A \leq 14$, and the "equal to" portion requires that the circle be filled in.

Legend

< or >	→	○
≤ or ≥	→	●

Reading a Number Line

Given the number line below, write the inequality.

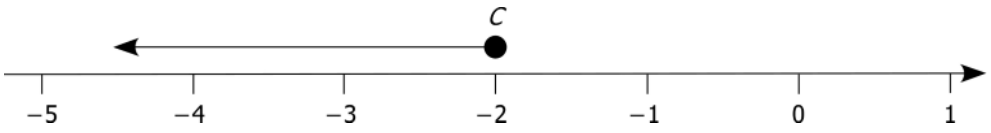


The values represented on the number line show greater than, but not including 5.

$A > 5$

Can you think of a situation that could be expressed using the inequality of A > 5? For example, the City Public Works department will clear the snow on residential streets if there is more than 5 cm of snow.

Write the inequality expressed by the following number line:



The line shows values less than or equal to –2.

$C \leq -2$

Adding, Subtracting, Multiplying, or Dividing

As with equations, when solving inequalities, follow the same steps:

a) Get all terms with variables on one side.

b) Isolate the term with the variable.

c) Then, isolate the variable.

If you have not already included these steps on your Resource Sheet, do so now.

Example 1

Solve for d: $2d + 5 < 9$

Solution:

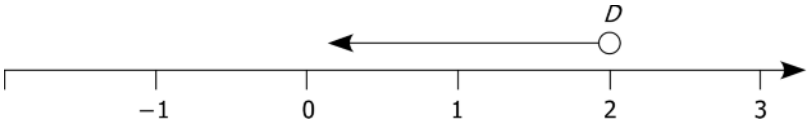
$2d + 5 - 5 < 9 - 5$ Subtract 5 from each side to isolate the term with the variable.

$2d < 4$

$\frac{2d}{2} < \frac{4}{2}$ Divide each side by 2 to isolate the variable.

$d < 2$

To graph this inequality, set up a horizontal number line, draw an open circle above the 2, and draw the arrow so that it refers to the values that are less than 2.



Example 2

Solve for c: $3c - 4 \geq 8$

Solution:

$$3c - 4 + 4 \geq 8 + 4$$

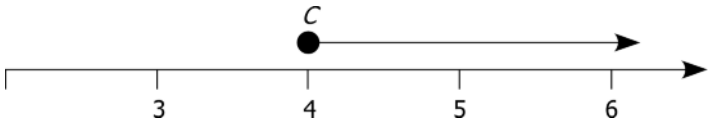
Add 4 to each side to isolate the term and the variable.

$$3c \geq 12$$

$$\frac{3c}{3} \geq \frac{12}{3}$$

Divide each side by 3 to isolate the variable.

$$c \geq 4$$



Example 3

Solve for k: $5k + 7 > 7k + 13$

Solution:

In this example, place all the terms with the variables on one side. You have two choices, both of which would be correct. But, as with equations, it is a good idea to keep the coefficient of the variable positive if possible. This helps to avoid errors. You will understand more about this a little later in the lesson.

Subtract $5k$ from each side, which will leave you with a positive $2k$ on the right hand side. Then subtract 13 from each side

$$5k + 7 - 5k > 7k + 13 - 5k$$

Subtract $5k$ from each side to leave you with a positive $2k$ on the right hand side.

$$7 > 2k + 13$$

$$7 - 13 > 2k + 13 - 13$$

Subtract 13 from each side to isolate the term with the variable.

$$-6 > 2k$$

$$\frac{-6}{2} > \frac{2k}{2}$$

Divide both sides by 2 to isolate the variable.

$$-3 > k$$

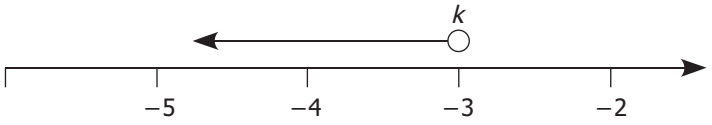
This presents you with something new! The variable is on the right-hand side. Notice the variable is at the small end of the symbol (less than) and the –3 is at the large end (greater than). Reading from left to right, read this as "–3 is greater than k."

Reading from right to left, "k is less than –3."

Some people prefer to rewrite the expression as:

$$k < -3$$

This takes some practice to perfect, but it is crucial that you understand it when drawing the number line. The values represented under the arrow must be less than –3.



Example 4

Solve for v: $2v - 7 \leq 3v - 3$

Solution:

$$2v - 7 - 2v \leq 3v - 3 - 2v$$

Subtract $2v$ from each side to get the variables all on one side.

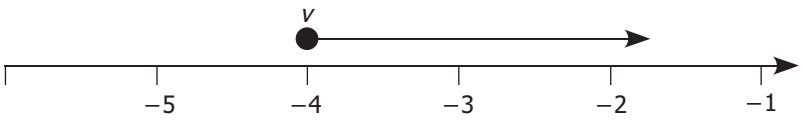
$$-7 \leq v - 3$$

$$-7 + 3 \leq v - 3 + 3$$

Add 3 to each side to isolate the variable.

$$-4 \leq v$$

The variable v is on the large end of the symbol, so it is greater than or equal to the value –4. When you draw the number line, the values under the line must be greater than –4. And, the circle must be filled in, because it is greater than or equal to (≥) –4.



Example 5

Solve for p: $\frac{p}{3} - 2 > 2p + 3$

Solution:

This can be simplified by adding 2 to each side, isolating the fraction.

$$\frac{p}{3} - \cancel{2} + \cancel{2} > 2p + 3 + 2$$

$$\frac{p}{3} > 2p + 5$$

Here you have an inequality with a fraction. The first step, just like when working with equations, is to eliminate the fraction. To accomplish this, multiply each term by 3. Remember, multiplying by 3 is the opposite operation to dividing by 3, and $\frac{3}{3}$ =1.

$$\frac{p}{\cancel{3}}(\cancel{3}) > 2p(3) + 5(3)$$

Notice that **each term** has to be multiplied by 3.

$$p > 6p + 15$$

$$p - p > 6p + 15 - p$$ Subtract p from each side to get all variables on one side.

$$0 > 5p + 15$$

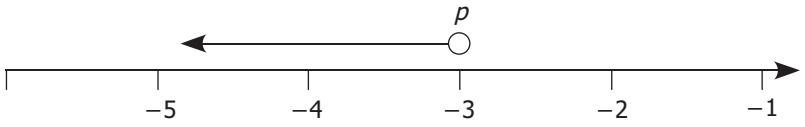
$$0 - 15 > 5p + 15 - 15$$ Subtract 15 from each side to isolate the term with the variable.

$$-15 > 5p$$

$$\frac{-15}{5} > \frac{5p}{5}$$ Divide each side by 5 to isolate the variable.

$$-3 > p$$

When you construct the number line, make sure you read the inequality correctly! The variable is on the small end of the symbol, so it is less than the value. All numbers under the line on your graph must be less than -3. And, the circle must be left unfilled, because it is only "less than", not "less than or equal to."



The Special Rule for Multiplying and Dividing with Negatives

You can add or subtract equal values on both sides of an inequality statement, and the inequality sign does not change. However, when you multiply or divide both sides of an inequality statement by a ***negative*** value, you must reverse the inequality sign. Consider the following example.

Make a note of this on your Resource Sheet as a reminder to yourself.

Example 1

Solve for R: $-2R > 4$

Solution:

There are two ways to solve this inequality. You could switch the term with the variable to the other side of the inequality sign in order to keep the sign of the coefficient positive. This longer method would require you to add $2R$ to each side, and then subtract 4 from each side to give the term with the variable a positive coefficient.

$-2R + 2R > 4 + 2R$	Add $2R$ to each side.
$0 > 4 + 2R$	
$0 - 4 > 4 + 2R - 4$	Subtract 4 from each side.
$-4 > 2R$	The term with the variable has a positive coefficient.
$\frac{-4}{2} > \frac{2R}{2}$	Divide both sides by 2 to isolate the variable.
$-2 > R$	R is ***less*** than –2.

What would happen if you didn't make the coefficient of the variable positive? Here is another method:

$-2R > 4$	
$\frac{-2R}{-2} > \frac{4}{-2}$	Divide both sides by –2.
$R > -2$	R is ***greater*** than –2.

This answer is the opposite to method 1! Which one is correct? Method 1 results in the correct answer, but there is a shortcut to all those steps!

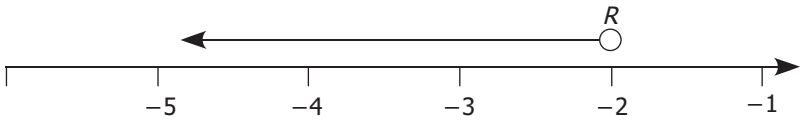
Here is the "special rule."

When multiplying or dividing an inequality by a negative value, switch the direction of the inequality symbol to make it a true statement.

$-2R > 4$ Same example as above.

$\frac{-2R}{-2} > \frac{4}{-2}$ Divide both sides by –2.

$R < -2$ Switch the direction of the inequality sign.



This method is much easier than the longer example shown above. But you must remember to switch the direction of the symbol whenever multiplying or dividing by a negative value!

Why?

When you multiply or divide a value by a negative, you make it the opposite sign. You must switch the direction of the inequality sign so the opposite statement is now true.

Consider the following:

6 < 12 **true** This will be considered the original inequality.

Add 3 to both sides to the inequality above.

9 < 15 true

Subtract 3 from both sides of the original inequality.

3 < 9 true

Multiply both sides of the original inequality by 2.

12 < 24 true

Divide both sides of the original inequality by 6.

1 < 2 true

Multiply both sides of the original inequality by -2.

–12 < –24 FALSE To make this a true statement, reverse the symbol –12 > –24.

Divide both sides of the original inequality by –2.

–3 < –6 FALSE To make this a true statement, reverse the symbol –3 > –6.

In order to keep an inequality statement true, you must **switch the direction of the inequality sign** when you ***multiply or divide by a negative number***.

Example 2

Solve for q: $\frac{q}{-3}+3\le -2$

Solution:

Isolate the term with the variable by subtracting 3 from each side.

$$\frac{q}{-3}+3-3\le -2-3$$

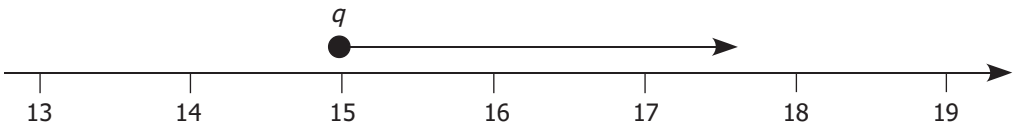
$$\frac{q}{-3}\le -5$$

$$\frac{q}{-3}(-3)\le -5(-3)$$

Multiply each side by –3 to cancel out the division by –3.

$$q\ge 15$$

Remember to switch the direction of the symbol, since you multiplied by a negative value.



Learning Activity 5.10

Operations with Inequalities

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Add $\frac{3}{6}+\frac{2}{3}$.
2. Lynne travelled for 280 days. How many weeks was she gone?
3. Multiply 5 x 73 x 2
4. What is the perimeter of a rectangular school yard that measures 900 m by 300 m?
5. What two numbers have a product of –54 and a sum of 3?
6. What two numbers have a product of –144 and a sum of zero?
7. Convert 1000 cm^2 to m^2.
8. Estimate the value of the product of 42 and 38.

continued

Learning Activity 5.10: Operations with Inequalities (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Isolate the variable, and draw a number line showing the solution to each inequality.

1. a) $a > -1$
   b) $b \leq 6$
2. a) $3c \geq -12$
   b) $d + 2 < 6$
3. a) $2e - 4 < -2$
   b) $\frac{f}{5} + 2 \leq 3$
4. a) $2g - 4 > 5g - 7$
   b) $6h + 4 \geq -2h + 16$
5. a) $-3i > -2$
   b) $4j \leq 0.6$
6. a) $\frac{k}{-3} + 2 < 2k + 9$
   b) $2m \geq \frac{m}{-2} + 1$

Lesson Summary

In this lesson, you practiced operations with inequalities, and graphed the solutions on number lines. You also learned the rule that applies when multiplying or dividing an inequality by a negative value. In the next lesson, you will compare inequalities to equations and solve problems.

NOTES

Assignment 5.6: Inequalities and Graphing

Total: 22 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Describe a situation that could be represented by the inequality $x < 20$. *(2 marks)*

2. Show all your work in solving these inequalities. Graph each one on a number line. *(6 marks)*

   a) $4f > 16$ *(3 marks)*

   b) $3v - 2 \leq 7$ *(3 marks)*

3. Show all your work in solving these inequalities. Graph each one on a number line. *(6 marks)*

   a) $3k - 4 \leq k + 2$ *(3 marks)*

   b) $15p + 3 > 10p + 8$ *(3 marks)*

4. Write, in your own words, the rule that applies when multiplying or dividing an inequality by a negative value. *(2 marks)*

5. Solve these inequalities, and graph the solutions on a number line. *(6 marks)*

a) $-3g + 10 \leq -2$ *(3 marks)*

b) $-5q - 2 > 2^3$ *(3 marks)*

NOTES

LESSON 7: INEQUALITIES AND PROBLEMS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ compare solving equations to solving inequalities
- ❑ test for values that satisfy inequalities
- ❑ set up, solve, and graph inequalities from word problems

Lesson Introduction

In this lesson, you will learn how to set up inequalities from a word problem, how to graph their solutions, and test values to see if they are part of the solution set.

Solving Inequalities

There are many similarities between solving equations and solving inequalities. The actual process of how to solve each is almost identical, except for the rule you learned in the last lesson about multiplying and dividing inequalities by negative values. But a significant difference is that equations generally will only have one answer, while inequalities can have an infinite number of possible solutions.

You may want to include this difference on your Resource Sheet.

For example,

$3x + 2 = 14$ (equation) compared to $3x + 2 < 14$ (inequality)

$$3x + 2 - 2 = 14 - 2 \qquad 3x + 2 - 2 < 14 - 2$$

$$3x = 12 \qquad 3x < 12$$

In both cases, subtract 2 from each side to isolate the term with the variable,

$$\frac{3x}{3} = \frac{12}{3} \qquad \frac{3x}{3} < \frac{12}{3}$$

and then divide each side by 3 to isolate the variable.

$$x = 4 \qquad x < 4$$

With this example, the process for solving each question was identical.

Example 1

Compare the following equation and inequality.

$$\frac{x}{2} - 0.4 = 1.2 \qquad \frac{x}{2} - 0.4 \geq 1.2$$

Solution:

$$\frac{x}{2} - 0.4 + 0.4 = 1.2 + 0.4 \qquad \frac{x}{2} - 0.4 + 0.4 \geq 1.2 + 0.4$$

In both cases, add 0.4 to each side to isolate the term with the variable.

$$\frac{x}{2} = 1.6 \qquad \frac{x}{2} \geq 1.6$$

In both cases, multiply each side by 2 to isolate the variable

$$x = 3.2 \qquad x \geq 3.2$$

With this example, the process for solving each question was also identical.

Example 2

Compare the following equation and inequality.

$-5x + 10 = -5$ $\qquad$ $-5x + 10 > -5$

Solution:

$-5x + 10 - 10 = -5 - 10$ $\qquad$ $-5x + 10 - 10 > -5 - 10$

In both cases, subtract 10 from each side to isolate the term with the variable.

$$-5x = -15 \qquad -5x > -15$$

In both cases, divide each side by –5 to isolate the variable.

$$\frac{-5x}{-5} = \frac{-15}{-5} \qquad \frac{-5x}{-5} > \frac{-15}{-5}$$

This next step is unique to the inequality—because you divided by a negative value, you must switch the direction of the inequality sign to make it a true statement.

$x = 3$ $\qquad\qquad$ $x < 3$

Verifying Answers

You need to check your answers, or verify that they satisfy the equation or inequality.

With solving equations, this process is simply to take the solution you found to the equation and "plug it back into" the original equation to determine if it actually works.

First, solve the equation to find the value of x that makes it a true statement.

Given: $2x - 1 = 5$

$2x = 6$

$x = 3$

To check it, substitute the 3 in place of the x in the original equation.

$2(3) - 1$	5
$6 - 1$	5
5	5

true

It works evenly, so you can conclude that $x = 3$ satisfies the equation and is the correct answer.

A question like: "Does $x = -4$ satisfy the equation $2x - 6 = 3x - 3$?" can be addressed two ways.

a) You can solve for x, and compare the answer to the question

$2x - 6 = 3x - 3$

$2x - 6 - 2x = 3x - 3 - 2x$

$-6 = x - 3$

$-6 + 3 = x - 3 + 3$

$-3 = x$

So, $x = -4$ would not satisfy this equation.

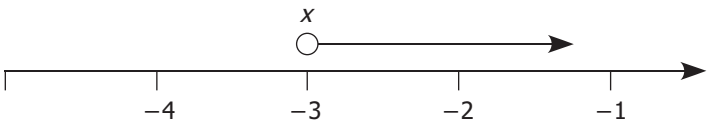
b) The second manner in which you can verify whether the value is or is not a possible answer is to simply replace the variable with the value, and see if the equation works out evenly.

$2x - 6$	$3x - 3$
$2(-4) - 6$	$3(4) - 3$
$-8 - 6$	$-12 - 3$
-14	-15

test for $x = -4$

-14 does not equal -15, so this proves that x cannot equal -4 for this equation.

With inequalities, the process is a little more involved, since there will be a multitude of possible values that satisfy the original inequality. Remember the number line.



The arrow indicates that all values greater than -3 would satisfy this inequality.

$x > -3$

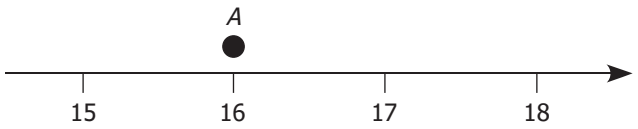
Would +2 satisfy this inequality? You can replace the x with the +2, and ask yourself "Is 2 greater than –3?" It is, so 2 is a solution to this inequality. Alternately, you can examine the graph, looking to see if +2 is included in the numbers indicated by the arrow. The arrow is pointing from –3 towards all positive values, so 2 would be included.

Check to see if –5 satisfies the inequality. First, replace the variable x with –5. Then ask yourself "Is –5 greater than –3?" It is less than, not greater than –3, so –5 would not satisfy the inequality. Then, look at the number line. The arrow does not include –5.

The solutions to inequalities are very different from solutions for equations. If you were asked the question "How old must you be to drive in Manitoba?" and you answered 16, then that would be a specific equation answer. A more correct answer would be "at least 16." This is an inequality with a solution including everyone 16 and older.

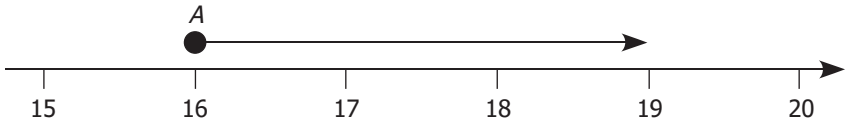
The graphs would look very different from each other.

$A = 16$



The solution to an equation has only one possible answer.

$A \geq 16$



The solution to the inequality has an infinite number of answers greater than or equal to 16.

Learning Activity 5.11

Equations and Inequalities

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Is the result of $\frac{2}{5} - \frac{2}{4}$ going to be positive or negative?
2. There is a sale of 20% on a sweater that costs $48.90. Estimate the sale price of the sweater.
3. You have a dentist appointment at 11:00. It is 10:32 now and it takes 18 minutes to get to the dentist's office. Will you be late or early?
4. Convert 4550 g to kilograms.
5. You pay for your $11.30 meal with a $10 bill and a $5 bill. How much change do you get?
6. If an HD television costs $1200 and there are taxes of 12% to pay in addition, what is the price of the TV?
7. Calculate $900 \div 30$
8. Which fraction is larger: $\frac{3}{5}$, $\frac{9}{15}$, or $\frac{18}{30}$?

continued

Learning Activity 5.11: Equations and Inequalities (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Show why or why not $x = -4$ is a solution to the equation: $3x + 2 = -14$.
2. Show why or why not $x = 0.8$ is a solution to the equation: $4x - 5 = 2x - 3.4$.
3. Show why or why not $v = 0.8$ is a solution to the inequality $v \leq 1$.
4. Show why or why not $A = \frac{-3}{4}$ is a solution to the inequality $A > \frac{-1}{2}$.
5. Describe the difference between the solution for $3C = -21$ and $3C < -21$.

Setting up an Inequality to Solve a Problem

When given a word problem, you must carefully read the question, and decide if it will be an equation or an inequality. Certain words will dictate the direction you need to go.

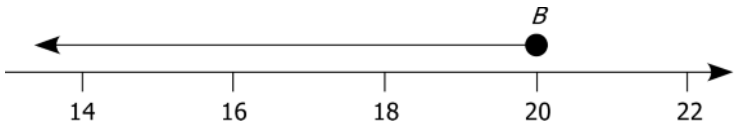
Terms like "no more than," "greater than," "at least," "maximum," "minimum," or "at most" indicate the need for an inequality.

Example 1

You decide to raise bunnies, and have room for no more than 20 animals. How many bunnies can you have at any time?

Solution:

The key words are "no more than." They tell you that you can have anywhere from 0 to 20 bunnies, but no more than 20. The inequality would look like this:



$B \leq 20$

Example 2

Your friend is raising pairs of doves. They must come in pairs, and stay together. She has room for up to 60 birds. How many pairs can she have?

Solution:

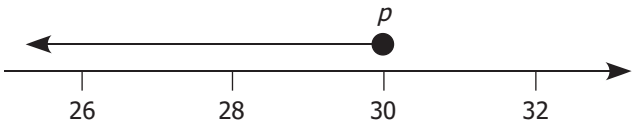
Because the term "up to" is in the problem, it indicates the need for an inequality. She could have fewer than 60 birds, but not more. But watch the wording of the question—it is a little tricky. It is not asking for how many birds, but for how many pairs. Since there are 2 birds in each pair, the inequality would be set up as follows:

Let p = number of pairs of doves

Since there are two doves in a pair

$2p \leq 60$

$p \leq 30$



She could have 30 or fewer pairs.

Lesson Summary

In this lesson, you compared solving equations to solving inequalities, and identified key words that indicate when each is to be used. You learned how to verify if certain values satisfy the given equations and inequalities. Finally, you used inequalities to solve problems. This completes your work with linear relations.

Assignment 5.7: Inequalities and Problems

Total: 15 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Solve. Compare and explain the differences in the following two solutions. *(3 marks)*

   $3x = -15$ $3x < -15$

2. Solve. Compare and explain the process used to find each of the two solutions below. *(3 marks)*

   $4r + 10 = 6r$ $4r + 10 \geq 6r$

3. Solve. Compare and explain the process used to find each of the two solutions below. *(3 marks)*

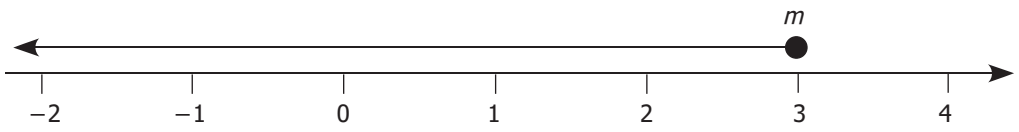
$-5y > -10$ $-5y = -10$

4. Explain how the following values all satisfy the inequality $m \leq 3$. *(2 marks)*

a) 2.5

b) –2.5

c) –22.5



5. You are working at a summer job to save enough money to buy a guitar. You will need at least $440 and your wage is $9 per hour. You want to find the minimum number of hours you will need to work to afford the guitar.

   a) Would you use an equation or an inequality to solve the problem? Explain your thinking. *(2 marks)*

   b) Using algebra, find the minimum number of hours you will need to work to afford the guitar. *(2 marks)*

NOTES

MODULE 5 SUMMARY

Congratulations, you have finished five of the modules in this course.

In the TV show *NUMB3RS*, brothers Charlie and Don work to crack FBI cases using mathematical equations to answer interesting criminal questions. While this module only uses basic operations to solve equations and inequalities, the principles are the same. Find patterns, describe them using words, symbols, or graphs, and use the equations or inequalities to solve problems. Now that you have learned how to work with linear equations, who knows where these skills will be applied!

In the next three modules, the focus will shift from algebra to geometry and you will study circles, 2D and 3D shapes, as well as transformations.

Submitting Your Assignments

Please note that you will not be submiting your assignments for Module 5 at this time. Once you have completed Module 6, you will send in your assignments for Module 5 and Module 6 to your tutor/marker.

NOTES

GRADE 9 MATHEMATICS (10F)

Module 5: Linear Relations

Learning Activity Answer Keys

MODULE 5: LINEAR RELATIONS

Learning Activity 5.1: Analyzing Patterns

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. How many half-dozen eggs are there in 420 eggs?
2. Evaluate 32 ÷ 0.5.
3. At the hardware store, the price of a $300 tool box is reduced by 40%. What is the sale price for the tool box?
4. If Andre completed $\frac{3}{8}$ of his homework and Patricia completed $\frac{2}{3}$ of hers, who is closer to finishing their homework?
5. If a notebook costs $0.27, how much would 100 notebooks cost??
6. Calculate 12 x 12.
7. If you saved $208 in a year, how much money did you save each week?
8. You bought three DVDs valued at $9.99 each, taxes included. What is the cost of your total purchase?

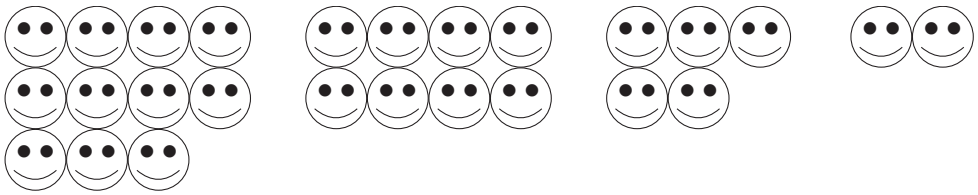
Answers:

1. 70 (There are 6 eggs in a half-dozen, so 420 ÷ 6.)
2. 64 (there are 64 halves in 32)
3. $180 (40% of $300 = $120; $300 – $120 = $180; 60% of $300 is another way to calculate)
4. Patricia $\left(\frac{3}{8}\text{ is less than half; }\frac{2}{3}\text{ is more than half}\right)$
5. $27
6. 144
7. $4 (there are 52 weeks in a year, so 208 ÷ 52)
8. $29.97 (since $9.99 is 1¢ less than $10, subtract 3¢ from $30)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Given the following pattern of shapes, write a mathematical expression showing the changes for each iteration.



Answer:

$f - 3$, where f is the number of faces in the previous iteration.

2. When you fail to return a library book by the due date, the late fee is \$0.25 for each day you are late. Write a mathematical expression for this situation.

Answer:

\$0.25(d), where d is the number of days you are late.

3. Using online banking on your web phone, you want to monitor the amount of money in your account during a shopping spree. Initially, you had \$120 in the account. This chart shows the balance each time you check. Write a mathematical expression showing the pattern.

Time	Balance
08:00	\$120
10:00	\$60
12:00	\$30
14:00	\$15

Answer:

$\frac{1}{2}(m)$, where m is the amount of money the last time you checked.

4. Your grandmother gives you money for your birthday. The following chart shows how much you received each year. Write a mathematical expression showing the pattern.

Age	Gift
10	$10
11	$15
12	$20
13	$25
14	$30

Answer:

$p + 5$, where p equals the amount of money you got last year.

Learning Activity 5.2: Translating Words into Equations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Estimate an 11% tip on a $45.00 bill.
2. Subtract $\frac{4}{5} - \frac{3}{25}$.
3. How many days are there in 7 weeks?
4. If 12% of 400 is 48, what is 4% of 400?
5. Add 78 + 33 + 22.
6. What distance do you travel if you walk at 5 m/s for 40 seconds?
7. How much change do you get if you pay a bill of $12.40 with 2 ten dollar bills?
8. Which value is larger: $\frac{3}{8}$ or 0.5?

Answers:

1. $5 is a good estimation (10% of $45 equals $4.50, and then you have to add another 1%)
2. $\frac{17}{25}$ $\left(\frac{4}{5} \text{ is equivalent to } \frac{20}{25}\text{; so } \frac{20}{25} - \frac{3}{25} = \frac{17}{25}\right)$
3. 49 days (7 x 7)
4. 16 (three times less)
5. 133 (add 78 and 22 to start)
6. 200 m (5 x 40)
7. $7.60 ($20 – $12 = $8; subtract another 40¢ from $8)
8. 0.5 $\left(\frac{3}{8} \text{ is less than half}\right)$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Match the written statement with the algebraic translation:

a) Six less than a number	i) $3x$
b) The quotient of 2 and a number	ii) $\frac{n}{6}$
c) A number tripled	iii) $2m+3$
d) Three more than twice a number	iv) $\frac{2}{n}$
	v) $n-6$
	vi) $2n$

Answers:

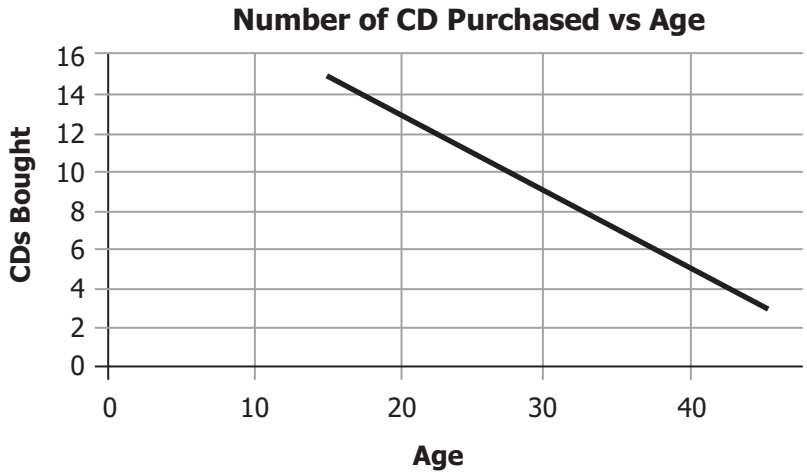
a) – v)

b) – iv)

c) – i)

d) – iii)

2. Use the graph below to answer the questions that follow the graph.

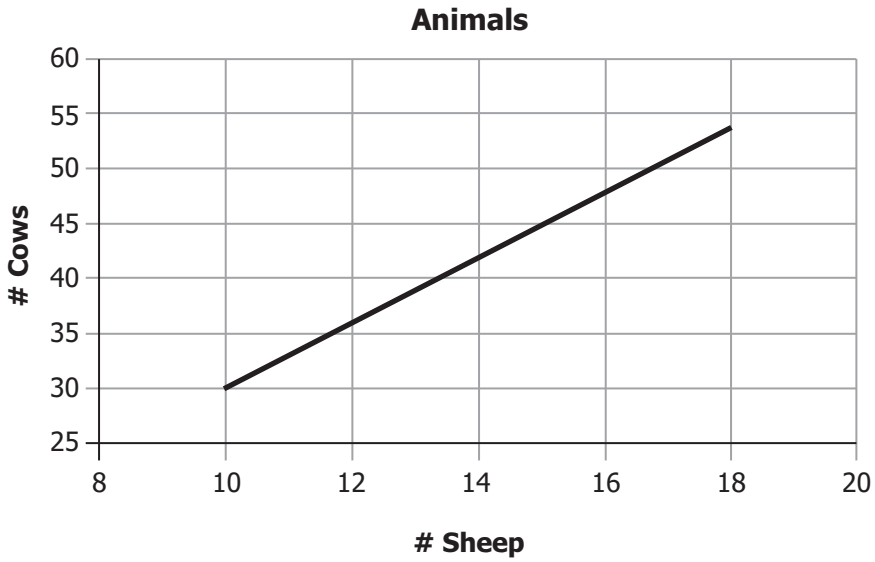


a) Write a statement about the general buying pattern of people indicated in this graph.
b) Approximately how many CDs will a 20-year-old purchase?
c) How many CDs would you expect a 15-year-old to buy?
d) Based on this graph, how many CDs would a 60-year-old person buy?
e) Somebody bought 6 CDs. How old would he be?

Answers:

a) As they get older, people buy fewer CDs.
b) 13
c) 15
d) 0
e) 36 (a number greater than 35 and closer to but less than 40)

3. The graph below shows the number of sheep and cows a farmer has over the course of one season.



a) Find an expression that compares the number of cows to the number of sheep.

b) If the farmer had 33 cows, approximately how many sheep would he have?

c) If the farmer had 25 sheep, approximately how many cows would he have?

d) Write an expression showing the total number of animals at any given time.

Answers:

a) sheep = x
cattle = $3x$

b) 11

c) 75

d) $x + 3x$

Learning Activity 5.3: Equations for Situations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Evaluate $4 - 6t$ if $t = -3$.
2. What two numbers have a sum of 8 and a product of 15?
3. Rodelyn earns $24 per shift. How much money does she earn after five shifts?
4. How many quarters are there in $5.75?
5. Add $\frac{9}{2} + \frac{5}{2}$.
6. If the $600 rent on your apartment is increased by 2%, what is the new rent?
7. Put in ascending order: $\frac{3}{4}, \frac{2}{5}, \frac{4}{6}$.
8. What is the total cost for four pens that cost $1.97 each?

Answers:

1. 22 ((4 – 6(–3)) = (4 + 18))
2. 3 and 5
3. $120 (5 x 20 + 5 x 4)
4. 23 quarters (20 quarters in $5 plus 3 quarters in $0.75)
5. $7 \left(\frac{14}{2} \text{ is equivalent to } 7 \right)$
6. 612 (2% of 600 equals 12)
7. $\frac{2}{5}, \frac{4}{6}, \frac{3}{4} \left(\frac{2}{5} \text{ is less than half and } \frac{4}{6} \text{ and } \frac{3}{4} \text{ are more than half; } \frac{4}{6} \text{ is equivalent to } \frac{2}{3} \text{ which is less than } \frac{3}{4} \right)$
8. $7.88 (since $1.97 is 3¢ less than $2.00, it is 12¢ (4 x 3¢) less than $8 (4 x $2.00))

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write an equation for each situation.

1. Finding the cost of B.C. apples at $3 per kg.

   Let k represents kilograms and C represents the total cost.

   Answer:

   $\$3(k) = C$

2. Finding the cost of gas at $1.35 per litre.

   Let L represents litres, and C represents the cost.

   Answer:

   $\$1.35(L) = C$

3. Receiving $5 for every goal scored in hockey.

   Let G represent each goal, and M represent money received.

   Answer:

   $\$5(G) = M$

4. You received $100 for mowing 5 lawns. Set up an equation to show how to calculate your fee per lawn.

   Let F represent the fee for each lawn mowed, and M represents the money received

   Answer:

   $5(F) = M$

   $$F = \frac{M}{5}$$

   $$F = \frac{100}{5}$$

   $$F = 20$$

   You charge a fee of $20 per lawn.

Learning Activity 5.4: Representing Equations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Estimate the value of 101.55 x 44.3.
2. If 9 of 12 members of a professional volleyball team are taller than 2.0 m, what fraction of players does this represent?
3. Evaluate $3^2 + 4^2 + 5^2$.
4. What is the area of a rectangle with length 12 mm and width 4 mm?
5. Convert 0.03 to a percent.
6. The square root of 162 is found between which two consecutive whole numbers?
7. Jacques is 37 years old. His son, Pierre, is 13 years old. How old was Jacques when Pierre was born?
8. What two numbers have a sum of 9 and a product of –22?

Answers:

1. 4500 (100 x 45)
2. $\frac{3}{4}$ (9 and 12 are both divisible by 3)
3. 50 (9 + 16 + 25)
4. 48 mm^2 (12 mm x 4 mm)
5. 3% (0.03 is read as 3 hundredths)
6. 12 and 13 ($12^2 = 144$ and $13^2 = 169$)
7. 24 years old (37 years – 13 years)
8. 11 and –2

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Write an equation for each situation.

1. Write an equation representing the pattern shown in the table of values below.

x	y
–4	–8
3	6
$\frac{1}{3}$	$\frac{2}{3}$

Answer:

$y = 2x$

2. Write an equation representing the pattern shown in the table of values below.

R	Q
2	3
5	9
–3	–7

Answer:

$Q = 2R - 1$

3. Create a situation that could be represnted by the following equation.

$m = 2y + 7$

Possible Answers:

Answers may vary.

"My mark is seven more than twice your mark."

"The number of mauve pencil crayons is seven more than twice the number of yellow pencil crayons."

Learning Activity 5.5: Variables

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write two equivelent fractions for $\frac{9}{15}$.
2. Marc has 42¢ more than Nicole. If Nicole has $1.74, how much money does Marc have?
3. If you buy a sports shirt worth $13.65 with a $20 bill, how much change do you get?
4. Reduce the ratio 35:15 to its most simplest form.
5. Evaluate 14 + 6 x 3.
6. A cake cooks for 1.75 hours. How many minutes is that?
7. The price of a basketball ticket is $15. How many tickets can you buy with $225?
8. You achieve $\frac{17}{20}$ on your mathematics test. What is your result as a percent?

Answers:

1. Answers will vary; some examples are: $\frac{3}{5}, \frac{6}{10}, \frac{18}{30}, \frac{90}{150}, \ldots$.

   To have equivalent fractions, the numerators and denominators must be multiplied by the same number.
2. 216¢ or $2.16 ($1.74 + $0.42)
3. $6.35 ($6 + $13.65 = $19.65; you need $0.35 to get to $20)
4. 7:3 (35 and 15 can be simplified by dividing by 5)
5. 32 (order of operations says to multiply 6 by 3 before adding 14)
6. 105 minutes (there are 60 minutes in an hour and 45 minutes in 0.75 of an hour or three-quarters of an hour)
7. 15 (225 is the square of 15 or 225 ÷ 15 = 15)
8. 85% (multiply the numerator and the denominator by 5)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify the dependent (D) and the independent (I) variables for each situation, and justify your choices.
   a) fast food meals per week __I___ vs weight gain __D___

      Because an increase in weight may depend on the number of fast food meals you eat.

   b) championships ___D__ vs games won __I___

      Because championships depend on the number of games won.

   c) profits __D___ vs sales __I___

      Because profit depends on the sales.

   d) temperature outside __I___ vs ice cream sales __D___

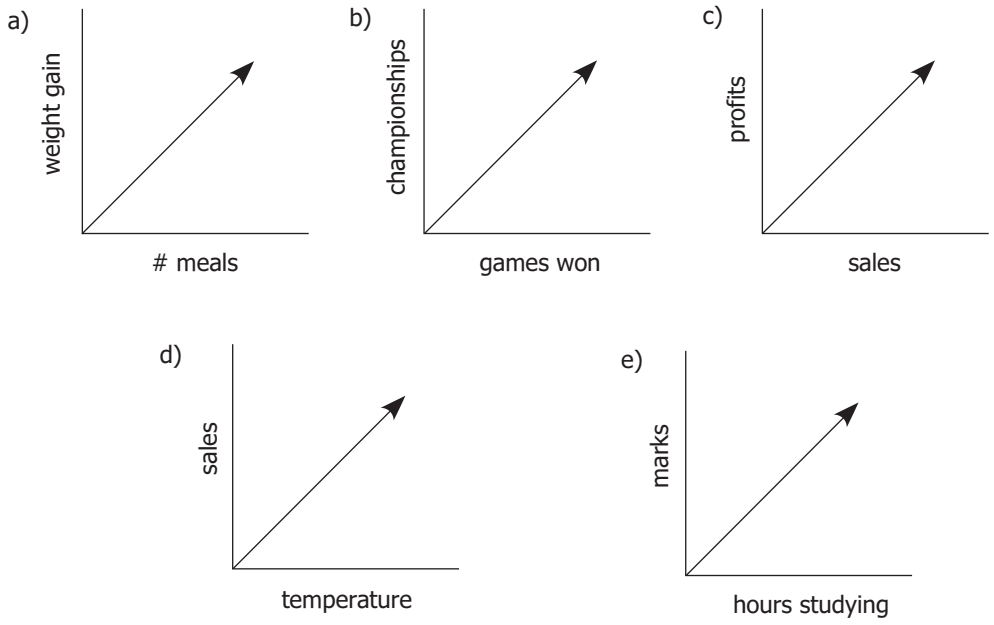
      Because ice cream sales may depend on the temperature outside.

   e) hours studying __I___ vs marks __D___

      Because your marks usually depend on the number of hours you study.

2. Draw a basic graph placing the independent variables along the horizontal (x) axis, and the dependent variable along the vertical (y) axis, for each situation listed above.

   Answer:



Learning Activity 5.6: Graphs and Equations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Estimate the value of 126.35 x 4.13.
2. A $50 kite is reduced by 30%. What is the price of the kite after the discount?
3. Solve $12 + c = 51$.
4. Solve $9 = 35 + t$.
5. If it was 7°C Monday, 10°C Tuesday, 11°C Wednesday, 9°C Thursday, and 13°C Friday, what was the average temperature for these five days?
6. A $155 raincoat is reduced by 10%. What is the amount of the discount?
7. Jacob can swim 10 lengths of the pool in 5 minutes. At that pace, how many lengths can he swim in half an hour?
8. Subtract $\frac{5}{2} - \frac{1}{2}$.

Answers:

1. 500 (100 x 4 = 400; 26.35 x 4.13 is larger than 100 because 26.35 and 4.13 are larger than 25 and 4; so 400 + 100)
2. $35 (30% of $50 equals $15 and $50 – $15 = $35 or 70% of $50 = $35)
3. 39 (51 – 12)
4. –26 (9 – 35)
5. 10°C (7°C and 13°C and then 11°C and 9°C give an average of 10°C; since it was 10°C on Tuesday, the overall average is 10°C)
6. $15.50 (10% of 155 = 0.10 x 155 = 15.5)
7. 60 lengths (there are 6 periods of 5 minutes in 30 minutes (half-hour), so 6 x 10)
8. $2\left(\frac{4}{2} \text{ is equivalent to } 2\right)$

Part B

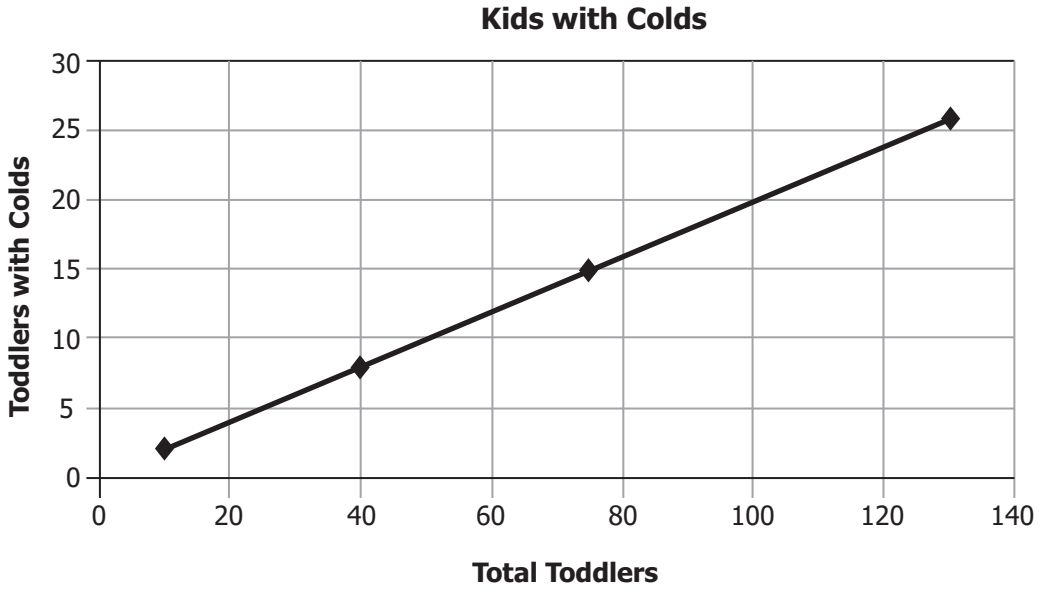
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Set up a graph using an appropriate scale, labelled axes, and accurately plotted points joined by a line for these values, and write an equation for the graph.

   The following chart shows the total number of toddlers in a nursery compared to the number of toddlers with colds in the nursery. Which is the dependent variable? **Hint:** The more toddlers present, the more toddlers with colds.

Toddlers with Colds	Total Toddlers in Nursery
2	10
8	40
15	75
26	130

Answer:

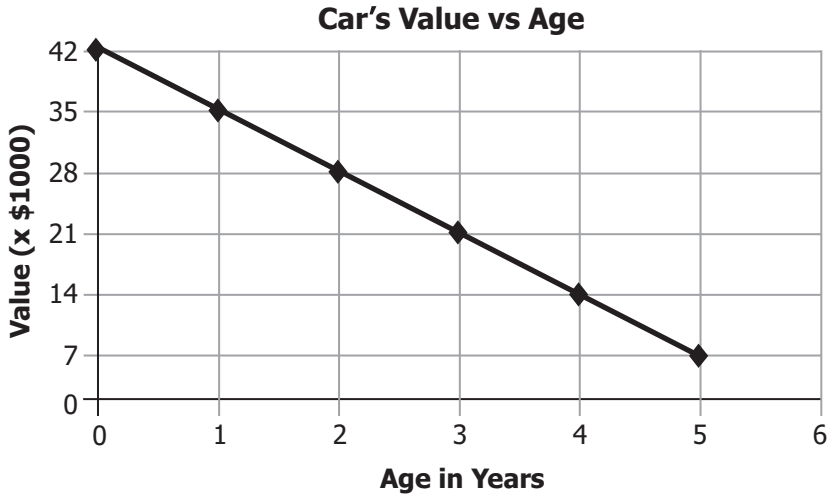


The number of kids with colds depends on the number of kids in total, so those with colds must be along the vertical axis. As the total number of toddlers increases, the number of kids with colds increases.

$$C = \frac{1}{5}T$$

where C represents those with colds, and T represents the total number of toddlers.

2. Use the graph below to answer the questions that follow.



a) If you had a maximum of $21 000 available to purchase a car, how old a car would you be buying?

b) If your car is 4 years old, what is its approximate value?

c) Following the graph, what is a 7-year-old car worth?

d) What is the approximate value of a car that is $2\frac{1}{2}$ years old?

Answers:

a) 3 years

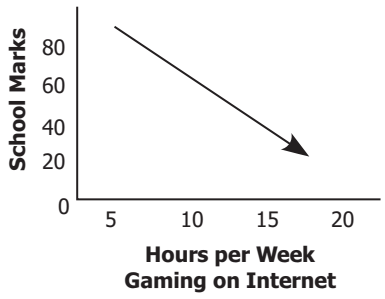
b) $14 000

c) 0

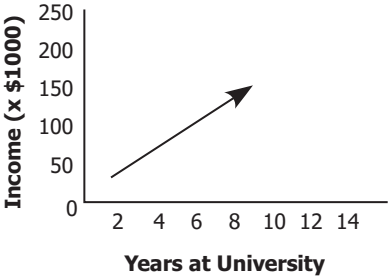
d) $22 500

3. Write a sentence describing what each of the graphs below shows.

a)



b)



Answer:

a) The more hours spent gaming on the Internet, the lower your school marks.

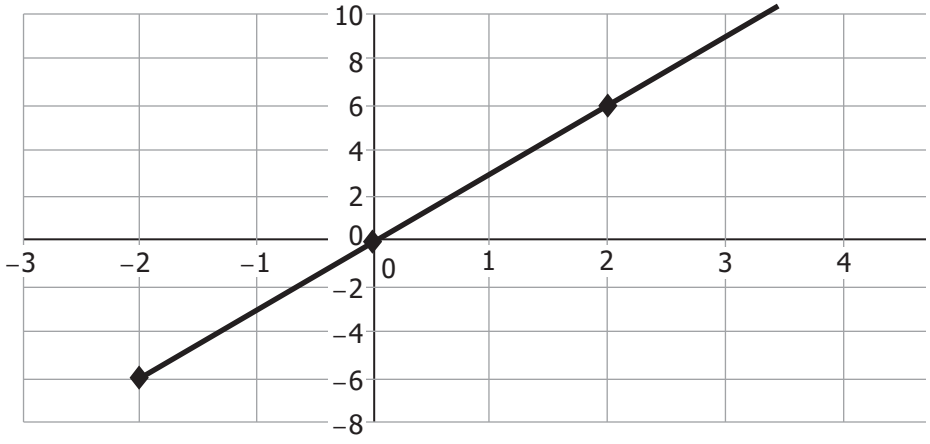
b) The more years spent in university, the higher your income.

4. Which equation matches the graph below.

a) $v = -3h$

b) $v = h + 4$

c) $v = 3h$



Answer:

c) $v = 3h$

Learning Activity 5.7: Solving for Variables

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert 0.75 to a fraction.
2. Convert $\frac{7}{10}$ to a decimal.
3. Convert $\frac{17}{5}$ to a mixed fraction.
4. If 24% of 500 is 120, what is 48% of 500?
5. Calculate 6300 ÷ 70.
6. What is triple 34?
7. Which is larger: $\frac{3}{2}$ or $\frac{4}{3}$?
8. You ate 6 of 8 pieces of tourtière. What percent of the tourtière remains?

Answers:

1. $\frac{75}{100}$ or $\frac{3}{4}$ (or any equivalent fraction)
2. 0.7 (both are read as 7 tenths)
3. $3\frac{2}{5}\left(\frac{15}{5}+\frac{2}{5}\right)$
4. 240 (double 120)
5. 90 (63 ÷ 7 = 9 and 100 ÷ 10 = 10)
6. 102 (double 34 is 68; add 30 to 68 to get 98; then add 4)
7. $\frac{3}{2}\left(\frac{3}{2}\text{ is }\frac{1}{2}\text{ larger than 1 but }\frac{4}{3}\text{ is only }\frac{1}{3}\text{ larger than 1}\right)$
8. $25\%\left(\frac{2}{8}\text{ or }\frac{1}{4}\right)$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Solve the following equations by isolating the variable using opposite operations. Show all your work.

a) $h-3=2$

$h-\cancel{3}+\cancel{3}=2+3$

$h=5$

b) $p+7=-5$

$p+\cancel{7}-\cancel{7}=-5-7$

$p=-12$

c) $-4x=6$

$\frac{\cancel{-4}x}{\cancel{-4}}=\frac{6}{-4}$

$x=\frac{-3}{2}$

d) $\frac{x}{-3}=5$

$\frac{x}{\cancel{-3}}\cancel{(-3)}=5(-3)$

$x=-15$

e) $2k-4=-6$

$2k-\cancel{4}+\cancel{4}=-6+4$

$2k=-2$

$\frac{\cancel{2}k}{\cancel{2}}=\frac{-2}{2}$

$k=-1$

f) $\frac{x}{2}+5=4$

$\frac{x}{2}+\cancel{5}-\cancel{5}=4-5$

$\frac{x}{2}=-1$

$\frac{x}{\cancel{2}}\cancel{(2)}=-1(2)$

$x=-2$

Learning Activity 5.8: Solving Advanced Equations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Evaluate $6 + 3 \times 4 \div 2$.
2. If the diameter of a circle is 10 cm, estimate the circumference.
3. Eric has 256 hockey cards. Tristan has two times as many. How many cards do both boys have?
4. Estimate the square root of 63.
5. How many minutes is there between 13:15 and 17:00?
6. Estimate the value of $4.99 + $5.02 + $1.99 + $3.98.
7. Your savings account balance is $47 and you deposit $56. What is the new balance?
8. There is a shipping charge of 4% on the $250 bicycle you purchased. What is the amount of the shipping charge?

Answers:

1. 12 (the multiplication and the division are done before the addition)
2. More than 30 cm (10π which is more than 3×10)
3. 768 cards (256 + 512)
4. Slightly less than 8 (the square of 8 is 64)
5. 225 minutes (there is 45 minutes until 14:00 and there are 3 hours or 180 minutes from 14:00 to 17:00)
6. $16 (5 + 5 + 2 + 4)
7. $103 (add 50 to 56 then subtract 3)
8. $10 (4 percent of 250 = $0.04 \times 250 = 4 \times 2.50$)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Solve each equation by isolating the variable. Show all your work.

1.
$$3x = 5 + 2x$$
$$3x - 2x = 5 + 2x - 2x$$
$$x = 5$$

2.
$$-5g - 4 = 5g$$
$$-5g - 4 + 5g = 5g + 5g$$
$$-4 = 10g$$
$$\frac{-4}{10} = g$$
$$\frac{-2}{5} = g$$

3.
$$5w + 6 = 2w - 3$$
$$5w + 6 - 2w = 2w - 3 - 2w$$
$$3w + 6 = -3$$
$$3w + 6 - 6 = -3 - 6$$
$$\frac{3w}{3} = \frac{-9}{3}$$
$$w = -3$$

4.
$$2v - 7 = 6v + 13$$
$$2v - 7 - 2v = 6v + 13 - 2v$$
$$-7 = 4v + 13$$
$$-7 - 13 = 4v + 13 - 13$$
$$\frac{-20}{4} = \frac{4v}{4}$$
$$-5 = v$$

5.
$$\frac{5}{x} = 10$$
$$\frac{5}{x}(x) = 10(x)$$
$$5 = 10x$$
$$\frac{5}{10} = \frac{10x}{10}$$
$$\frac{1}{2} = x$$

6.
$$2^3 = \frac{16}{x}$$
$$8 = \frac{16}{x}$$
$$8(x) = \frac{16}{x}(x)$$
$$\frac{8x}{8} = \frac{16}{8}$$
$$x = 2$$

7. $3(k-2)=9$

$$3k-6=9$$
$$3k-6+6=9+6$$
$$3k=15$$
$$\frac{3k}{3}=\frac{15}{3}$$
$$k=5$$

8. $-2(r+5)=3(r+2)$

$$-2r-10=3r+6$$
$$-2r-10+2r=3r+6+2r$$
$$-10=5r+6$$
$$-10-6=5r+6-6$$
$$-16=5r$$
$$\frac{-16}{5}=\frac{5r}{5}$$
$$\frac{-16}{5}=r$$

9. $5(3d-3)=3(3d+5)$

$$15d-15=9d+15$$
$$15d-15-9d=9d+15-9d$$
$$6d-15=15$$
$$6d-15+15=15+15$$
$$6d=30$$
$$\frac{6d}{6}=\frac{30}{6}$$
$$d=5$$

10. $-4(x+6)=3(2x-4)$

$$-4x-24=6x-12$$
$$-4x-24+4x=6x-12+4x$$
$$-24=10x-12$$
$$-24+12=10x-12+12$$
$$-12=10x$$
$$\frac{-12}{10}=\frac{10x}{10}$$
$$x=\frac{-6}{5} \text{ or}$$
$$-1.2=x$$

Learning Activity 5.9: Problem Solving

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Express $\frac{9}{20}$ as a decimal.
2. Find $\sqrt{169}$.
3. Divide a $640 bill equally among eight people.
4. An $800 boat now sells for $600. What is the percent rate of discount?
5. What is the median of the following: 1, 6, 8, and 10?
6. Estimate a 15% tip on a bill of $48.75.
7. How many centimetres are there in 49.4 m?
8. Write an equivalent fraction for 0.6.

Answers:

1 $0.45 \left(\frac{9}{20} = \frac{4.5}{10} \right)$

2. 13
3. $80
4. 25% (the discount is $800 – $600 = $200; $200 represents $\frac{1}{4}$ the value of $800 which is 25%)
5. 7 (the median is the number in the middle when the numbers are arranged in order; since 6 and 8 are the two values in the middle, the median is 7 since it is the mean of 6 and 8)
6. $7.50 (The bill is approximately $50. 10% of $50 corresponds to $5 and you need to add 5% more, which is $2.50)
7. 4940 cm (there are 100 cm in 1 m)
8. $\frac{6}{10}$ and $\frac{60}{100}$ (Other answers are possible. For example, $\frac{3}{5}$, $\frac{9}{15}$, or $\frac{12}{20}$; 0.6 has a numerator of 0.6 and a denominator equal to 1; to find equivalent fractions, you have to multiply the numerator and the denominator by the same number.)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

Set up an equation and solve the given problem.

1. Textbooks for university courses cost an average of $110 each, and tuition will be $850 per course. You plan on taking 4 courses requiring one textbook each. Find the total estimated cost of books and tuition for your first year.

   Answer:

   $110x + 850x$ = cost (if taking 4 courses, then $x = 4$)

   $$110(4) + 850(4) = C$$
   $$440 + 3400 = C$$
   $$3840 = C$$

   Your cost will be $3840 for the first year.

2. You have been asked to play hockey for a great team. The basic fee is $2600 plus $450 per tournament. You have a total amount of $4900 available for hockey expenses this year. Use an equation to determine how many tournaments you will be able to participate in.

   Answer:

   $$2600 + 450(x) = 4900$$
   $$2600 + 450x - 2600 = 4900 - 2600$$
   $$450x = 2300$$
   $$\frac{450x}{450} = \frac{2300}{450}$$
   $$x = 5.1$$

   Since you can only participate in a "whole" tournament, not 0.1 of a tournament, the answer would be that you could afford to participate in 5 tournaments.

3. Your friend was given the following problem, and his work was marked wrong. Identify the error, and correct it.

$$3(2x - 4) = 5(3x + 3)$$
$$6x - 12 = 15x + 15$$
$$9x = 27$$
$$x = 3$$

Answer:

In the third line, your friend made an arithmetic error. By showing each step, he may have seen his error.

$$3(2x-4)=5(3x+3)$$
$$6x-12=15x+15$$
$$6x-12-15x=15x+15-15x$$
$$-9x-12=15$$
$$-9x-12+12=15+12$$
$$-9x=27$$
$$\frac{-9x}{-9}=\frac{27}{-9}$$
$$x=-3$$

or

$$3(2x-4)=5(3x+3)$$
$$6x-12=15x+15$$
$$6x-12-6x=15x+15-6x$$
$$-12=9x+15$$
$$-12-15=9x+15-15$$
$$-27=9x$$
$$\frac{-27}{9}=\frac{9x}{9}$$
$$-3=x$$

Learning Activity 5.10: Operations with Inequalities

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Add $\frac{3}{6}+\frac{2}{3}$.
2. Lynne travelled for 280 days. How many weeks was she gone?
3. Multiply 5 x 73 x 2
4. What is the perimeter of a rectangular school yard that measures 900 m by 300 m?
5. What two numbers have a product of –54 and a sum of 3?
6. What two numbers have a product of –144 and a sum of zero?
7. Convert 1000 cm^2 to m^2.
8. Estimate the value of the product of 42 and 38.

Answers:

1. $\frac{7}{6}$ $\left(\text{with a common denominator of 6, you get } \frac{3}{6}+\frac{4}{6}\right)$
2. 40 weeks (there are 7 days in a week)
3. 730 (multiply 5 by 2 to start)
4. 2400 m (2 times the sum of 900 and 300)
5. 9 and –6
6. 12 and –12 (for the sum to be zero, the two numbers have to be equal but opposite in sign)
7. 0.1 m^2 (there are 100 cm in 1 m so 1 m^2 will be 100 cm x 100 cm = 10 000 cm^2; since 1000 cm^2 is 10 times less than 10 000 cm^2, 10 times less 1 m^2)
8. 1600 (think 40 x 40)

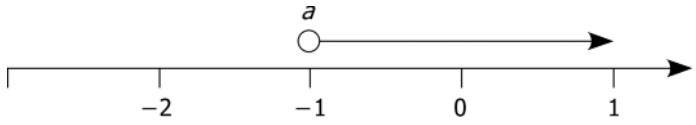
Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

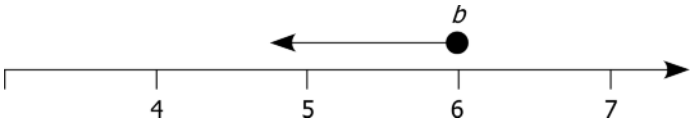
Isolate the variable, and draw a number line showing the solution to each inequality.

Answers:

1. a) $a > -1$



b) $b \leq 6$



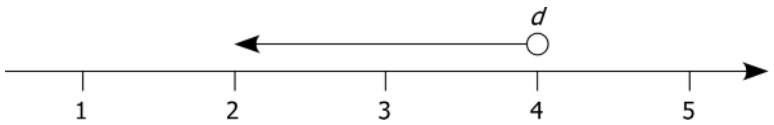
2. a) $3c \geq -12$

$$\frac{3c}{3} \geq \frac{-12}{3}$$

$$c \geq -4$$

b) $d + 2 < 6$

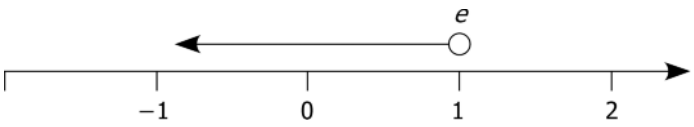
$d < 4$



3. a) $2e-4<-2$

$2e<2$

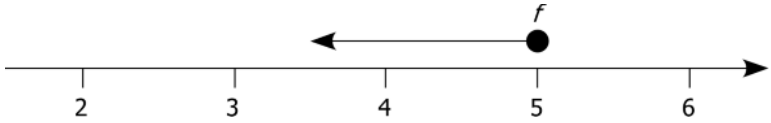
$e<1$



b) $\frac{f}{5}+2\leq 3$

$\frac{f}{5}\leq 1$

$f\leq 5$

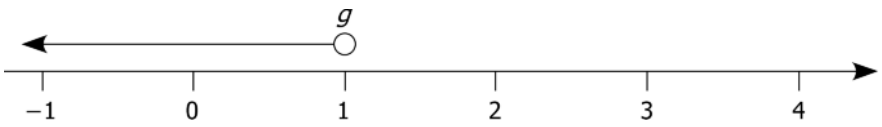


4. a) $2g-4>5g-7$

$-4>3g-7$

$3>3g$

$1>g$

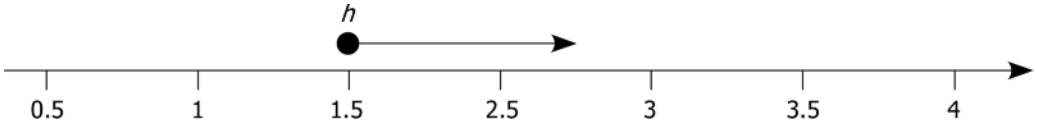


b) $6h+4\geq -2h+16$

$8h+4\geq 16$

$8h\geq 12$

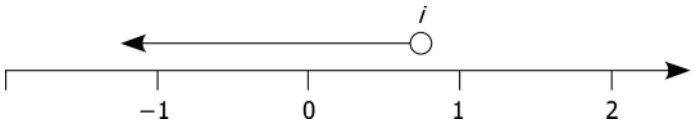
$h\geq \frac{3}{2}$



5. a) $-3i > -2$

$$\frac{-3i}{-3} > \frac{-2}{-3}$$

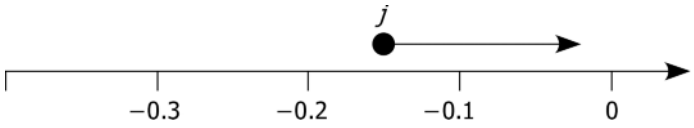
$$i < \frac{2}{3}$$



b) $-4j \leq 0.6$

$$\frac{-4j}{-4} \leq \frac{0.6}{-4}$$

$$j \geq -0.15$$



6. a) $\frac{k}{-3} + 2 < 2k + 9$

$$\frac{k}{-3} + 2 - 2 < 2k + 9 - 2$$

$$\frac{k}{-3} < 2k + 7$$

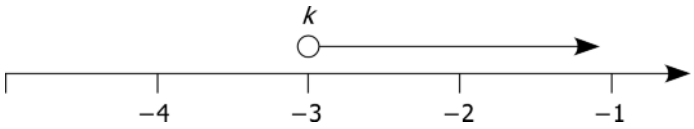
$$k > -6k - 21$$

$$7k > -21$$

$$k > -3$$

You may start by getting the fraction term by itself.

Multiply each term by –3 to remove the denominator and switch the sign.



b)

$$2m \geq \frac{m}{-2} + 1$$

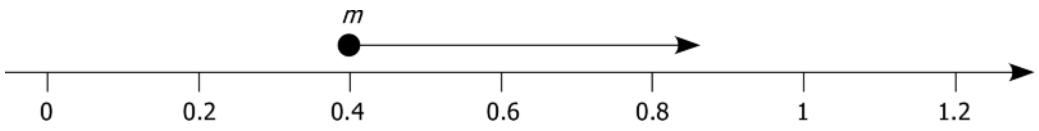
Multiply each term by –2 to eliminate the denominator and switch the direction of the sign.

$$-4m \leq m - 2$$

$$-5m \leq -2$$

$$m \geq \frac{2}{5}$$

Divide by –5 and switch the sign again.



Learning Activity 5.11: Equations and Inequalities

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Is the result of $\frac{2}{5} - \frac{2}{4}$ going to be positive or negative?
2. There is a sale of 20% on a sweater that costs $48.90. Estimate the sale price of the sweater.
3. You have a dentist appointment at 11:00. It is 10:32 now and it takes 18 minutes to get to the dentist's office. Will you be late or early?
4. Convert 4550 g to kilograms.
5. You pay for your $11.30 meal with a $10 bill and a $5 bill. How much change do you get?
6. If an HD television costs $1200 and there are taxes of 12% to pay in addition, what is the price of the TV?
7. Calculate 900 ÷ 30
8. Which fraction is larger: $\frac{3}{5}$, $\frac{9}{15}$, or $\frac{18}{30}$?

Answers:

1. Negative (since the fractions have the same numerator, the fraction with the largest denominator is the smallest, so $\frac{2}{5}$ is less than $\frac{2}{4}$)
2. Less than $40 (20% of $50 is equivalent to $10; $10 off of $50 gives $40)
3. Early (32 minutes + 18 minutes = 50; it won't be past 11:00)
4. 4.55 kg (1000 g in 1 kilogram)
5. $3.70 ($15 – $11 = $4; you have to subtract another $0.30 from $4)
6. $1344 (12% of 1200 = 0.12 x 1 200 = 12 x 12 = 144 so $1220 + $144)
7. 30
8. They are all equivalent.

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Show why or why not $x = -4$ is a solution to the equation: $3x + 2 = -14$.

   Answer:

$3x - 2$	-14
$3(-4) + 2$	-14
$-12 + 2$	-14
-10	-14

or

$$3x + 2 = -14$$
$$3x + 2 - 2 = -14 - 2$$
$$3x = -16$$
$$x = \frac{-16}{2}$$

   This is not true, so -4 is not the solution.

2. Show why or why not $x = 0.8$ is a solution to the equation: $4x - 5 = 2x - 3.4$.

   Answer:

$$4x - 5 = 2x - 3.4$$
$$2x = 1.6$$
$$x = 0.8$$

or

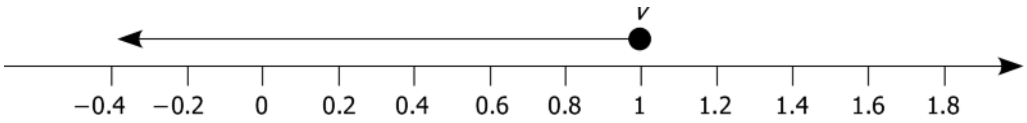
$4x - 5$	$2x - 3.4$
$4(0.8) - 5$	$2(0.8) - 3.4$
$3.2 - 5$	$1.6 - 3.4$
-1.8	-1.8

   Yes, 0.8 is the solution to this equation.

3. Show why or why not $v = 0.8$ is a solution to the inequality $v \leq 1$.

Answer:

$v \leq 1$

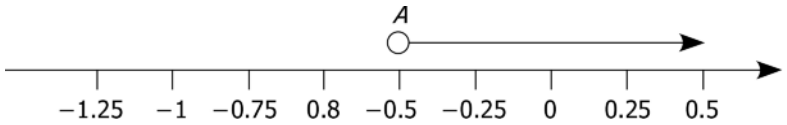


0.8 is less than 1, and falls under the line graph.

4. Show why or why not $A = \frac{-3}{4}$ is a solution to the inequality $A > \frac{-1}{2}$.

Answer:

$\frac{-3}{4}$ is less than $\frac{-1}{2}$, and the arrow goes the other direction. So, $\frac{-3}{4}$ cannot be part of the solution $A > \frac{-1}{2}$.



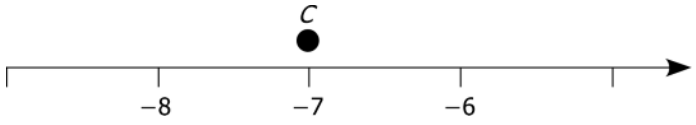
5. Describe the difference between the solution for $3C = -21$ and $3C < -21$.

Answer:

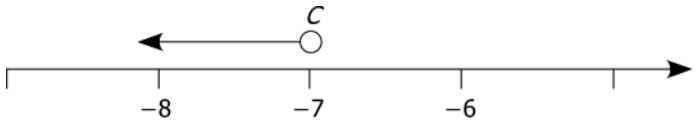
Both examples are simplified by dividing each side by 3.

$3C = -21$	$3C < -21$
$C = -7$	$C < -7$

The equation has only one possible answer, –7.



The inequality has any value less than –7 as a possible answer.



NOTES

GRADE 9 MATHEMATICS (10F)

Module 6: Circle Geometry

MODULE 6: CIRCLE GEOMETRY

Introduction

The next three modules of this mathematics course focus on geometry. In Module 6, the spotlight is on circles, and then in Modules 7 and 8, the focus shifts to 2-D and 3-D shapes. Module 6 investigates four unique properties of circles. You will learn new terminology and review your angle measurement skills. Next, you will determine how angles, lines, arcs, tangents, and chords are related in circles. You will also apply the properties of circles to problem-solving situations, and learn how to find the exact centre of a circle. To begin, however, the cover assignment for this module looks back at some of the ideas introduced in Module 1: Statistics. You will investigate examples from the media that use probability in decision making and report on them.

This unit requires that you have a protractor and a ruler.

Assignments in Module 6

You will need to complete the following seven assignments and send them to your tutor/marker when you are done this module.

Lesson	Assignment Number	Assignment Title
1	Assignment 6.1	Cover Assignment
2	Assignment 6.2	Measurement and Terminology
3	Assignment 6.3	Central and Inscribed Angles
4	Assignment 6.4	More Inscribed Angles
5	Assignment 6.5	Tangents
6	Assignment 6.6	Perpendicular Bisectors
7	Assignment 6.7	Solving Problems

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 6. You might use your Module 6 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 6 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 6 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

Lesson 1: Cover Assignment

- The weatherman reports: "There is a 40% chance of rain today." Do you take an umbrella?
- In Manitoba 2.9% of children, aged 10-14, have injuries while riding a bicycle. 27% of these injuries are head injuries. Therefore, the probability of you having a head-related injury while riding your bike is 0.78%. Many organizations claim that wearing a helmet will reduce head injuries. The decision you must make is, "Do I wear a bike helmet?"
- We are repeatedly exposed to advertising on radio, television, and in print media about the possibility of winning multi-million dollar prizes. The probability of winning the Lotto 6/49™ jackpot is nearly 1 in 14 million. Do you spend your hard-earned money buying a ticket?
- About 20-30% of adults do not wear seatbelts, possibly because they think that their behaviour only affects them. Unfortunately, studies have shown that children mimic adults and when a driver does not wear a seatbelt, 70% of the time the children in that vehicle are also unbuckled. Does an adult have a right to not wear their seatbelt?
- 90% of lung cancer is caused by smoking. Should you take that first puff?

We are constantly making decisions. Many of our decisions are based on comparing the probabilities of various outcomes, as well as our understanding of three aspects of probability—theoretical probability, experimental probability, and subjective judgment. Consider the following example.

The Saskatoon City Council recently voted against implementing a helmet law for cyclists because they felt it would reduce the number of people who went cycling. They based this decision on current environmental concerns and their desire to increase opportunities for people to get physical exercise. In this example, the council had to balance the probability of head injuries against the probability of decreased participation.

On the Environment Canada website, you can read that, during the month of May, Winnipeg typically has 15 rainy days. In theory (theoretical probability—what should happen), there is a 50% probability of rain each day of the month. Now let's say it's May 12th and it hasn't rained for the last three days (experimental probability—what has actually happened). So looking at a sunny, cloudless sky, knowing it hasn't rained in three days, you believe (subjective judgement) it will rain today and wear a raincoat to school.

NOTES

Assignment 6.1: Cover Assignment

Total: 15 marks

1. Find at least 3 examples from print and electronic media (newspaper or internet articles, magazine ads, etc.) where probability is being used to support a decision or opinion. Print or cut out the selections, and paste each to a 8-½ x 11 sheet of paper. Include the source (name and date of newspaper or magazine or the website URL). Circle or highlight the probability used in each example. *(6 marks)*

2. Choose ***one*** of the examples and, on a separate piece of paper, discuss the following:
   - **Explain** how probability is being used to support a decision or opinion, and whether or not the same data could be used to support an opposing decision. *(3 marks)*
   - **Describe** how the theoretical, experimental, and/or subjective components of probability factor into the decision in the example. *(3 marks)*
   - **Identify** any assumptions associated with the given probability and **explain** their limitations. For example, you could question the sample group or size, the data collection methods, etc. Review Module 1 for ideas. *(3 marks)*

NOTES

Lesson 2: Angle Measurement and Terminology

Lesson Focus

When you finish this lesson, you will be able to

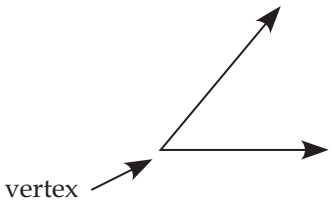
- ❑ identify parts of circles and names of angles
- ❑ measure angles using a protractor

Lesson Introduction

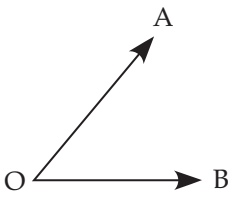
This lesson introduces terminology used when discussing circles and angles. It also reviews angle names and measurement.

Naming Angles

Angles are formed by two rays which meet at a point. The point where they meet is called the vertex.

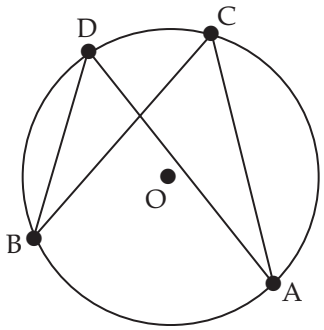


Angles are given names, usually letters of the alphabet. The vertex is always the middle letter of the name.



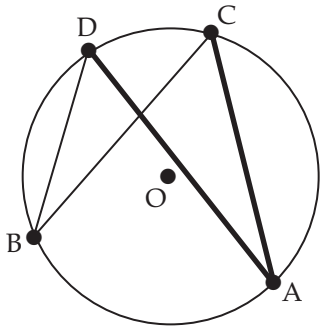
This angle is named "angle AOB" or "angle BOA." This can also be written as ∠AOB or ∠BOA. Notice that the vertex is the middle letter. The ray OB and the ray OA meet at the vertex, O.

It is important to understand exactly which angle a name refers to. There are four labelled angles in this diagram.



To find angle DAC, we need to look for point D, and then move from there along the line segment to point A and then to point C. Think "D to A to C." If you know that the middle letter is always the vertex, or the point where the angles change direction, you can quickly identify the required angle.

Here is the diagram with the requested angle highlighted.

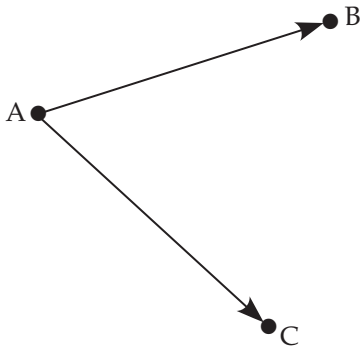


Measuring Angles

To measure an angle correctly, place the base line of the protractor along one of the rays. The centre point of the base line must be at the vertex. Then, using the appropriate scale on the edge of the protractor, carefully note the number of degrees the second ray is from the first ray.

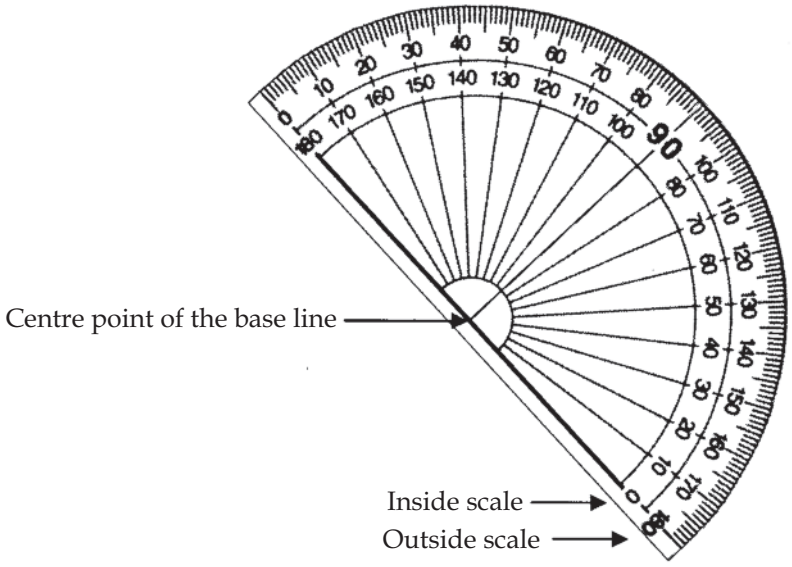
Example 1

Find the measure of ∠BAC.



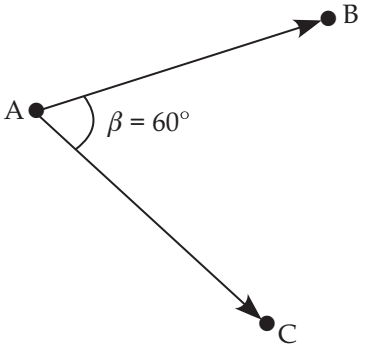
Solution:

Slide the protractor onto the angle as described.



In this case, the base line of the protractor is along ray AC, with the vertex A at the centre point. C is pointing towards the zero on the inside scale and so you need to read the degree markings from 0° to 60° on the inside scale of the protractor. Always ensure the correct scale is used for measuring.

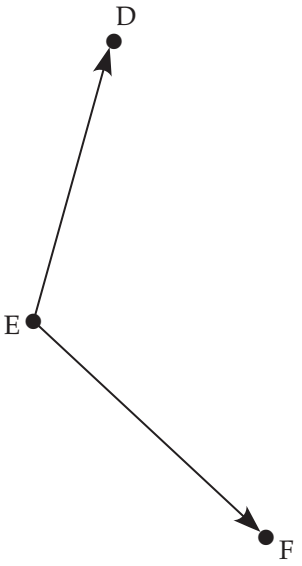
The measurement of ∠BAC is 60°.

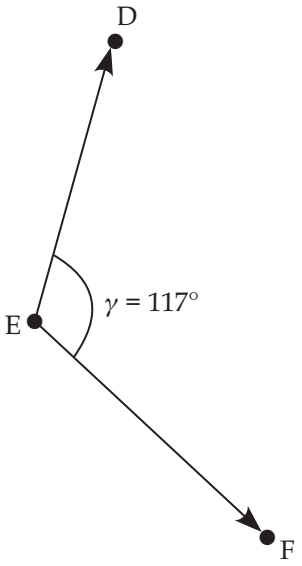


Example 2

Measure ∠DEF

Solution:

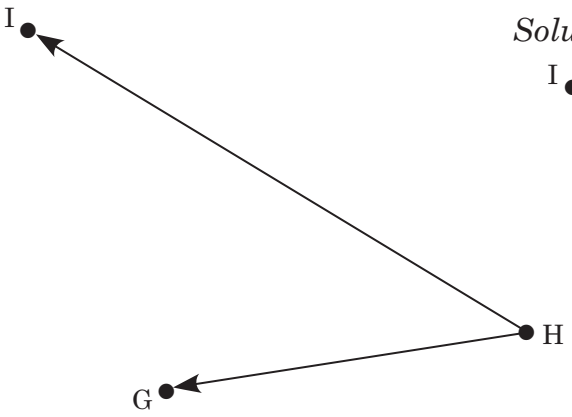




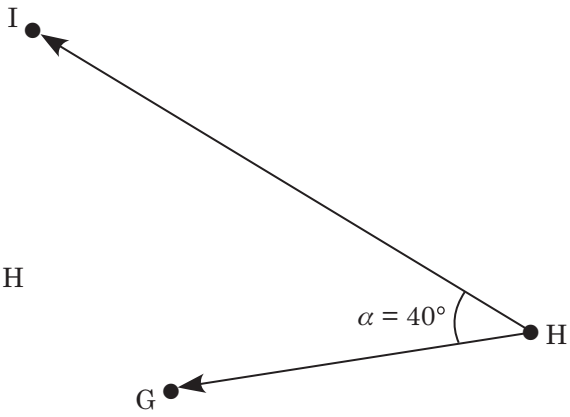
The measurement for ∠DEF is 117°.

Example 3

Measure ∠GHI



Solution:



∠GHI = 40°

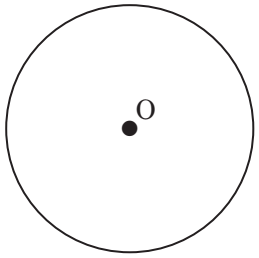
With the vertex H at the centre point of the base line, ray HG points to the zero on the outside scale of the protractor.

Circle Terminology

You will need to become familiar with the following terminology. You should include the terms and examples of them in your Resource Sheet.

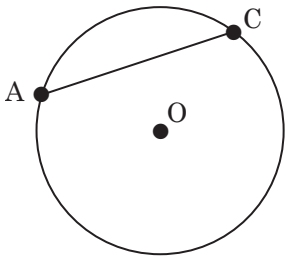
Circle with centre O

(a circle has 360°)



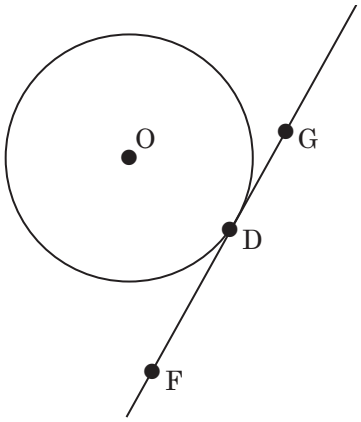
Chord AC

(a line segment with its endpoints on the outside edge or circumference of the circle)



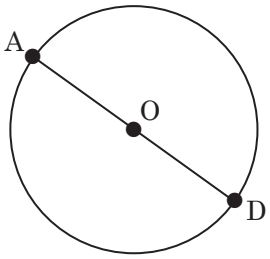
Tangent GF

(a line touching the circle at one point; D is the point of tangency)



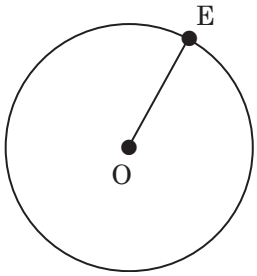
Diameter AD

(a chord through the centre of the circle)



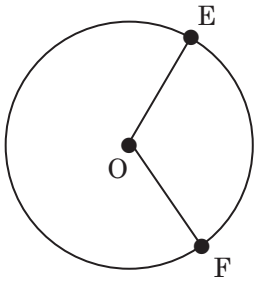
Radius OE

(a line segment from the centre of the edge of the circle; plural is radii)



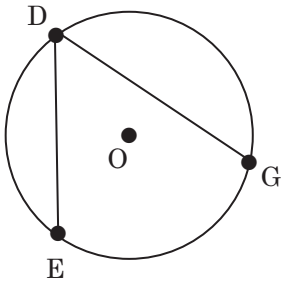
Central Angle EOF

(an angle with its vertex at the centre and its endpoints on the circle)



Inscribed Angle EDG

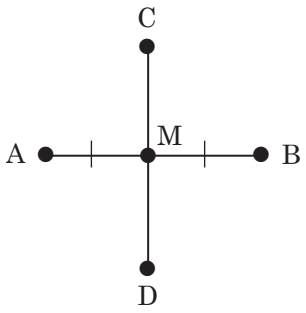
(an angle with its vertex and endpoints on the circumference of the circle)



Bisector

(divides a line or angle into two equal parts)

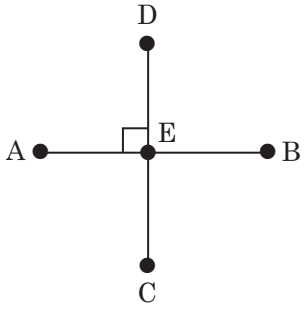
M is the midpoint of AB
CD bisects AB into equal parts
AM = BM



Perpendicular

(at right angles two lines meeting at 90°)

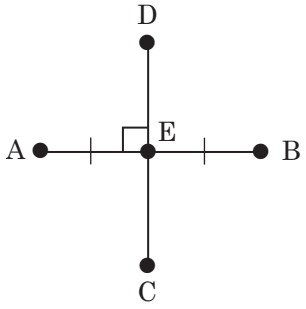
DC ⊥ AB



Perpendicular Bisector

(a line dividing a line into two equal parts, and meeting the line at a 90° angle)

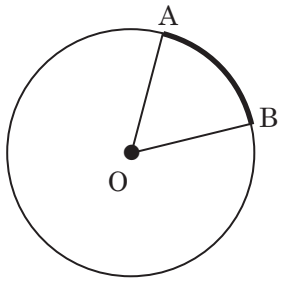
DC ⊥ bisector of AB



Arc AB $\left(\overset{\frown}{AB}\right)$

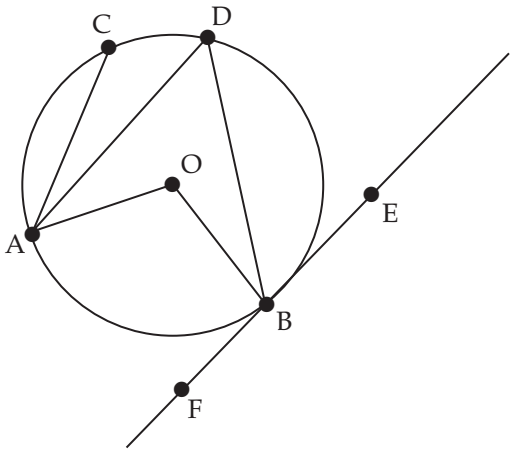
(a segment of the circumference of a circle)

$\overset{\frown}{AB}$ is between the endpoints of ∠AOB



Parts of a Circle

Given the following diagram, identify the requested components.



chords

AC, AD, and DB are chords with endpoints on the circumference of the circle

tangent

EF touches the circle at point B, the point of tangency

central angle

$\angle AOB$ is the only angle whose vertex is at the centre

inscribed angles

$\angle CAD$ and $\angle ADB$ have all three points on the outside of the circle (circumference)

arcs

$\overset{\frown}{AB}$, $\overset{\frown}{BD}$, $\overset{\frown}{CD}$, and $\overset{\frown}{AC}$ are segments of the circumference of the circle

Learning Activity 6.1

Measuring Angles

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What percent represents 3 months of a year?
2. What is the area of the top of a rectangular cake that measures 20 cm by 60 cm?
3. What is the price per m^3 of soil if 30 m^3 costs $900?
4. How much money do you have if you have 10 quarters, 12 dimes, and 3 nickels?
5. Calculate the product of 48 and 0.25.
6. A cushion costs $80. What is the sale price of the cushion if the price is reduced by 40%?
7. Two numbers have a sum of 15 and a product of 36. What are the two numbers?
8. Evaluate 10 – 3 x 4.

continued

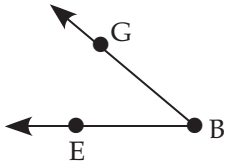
Learning Activity 6.1: Measuring Angles (continued)

Part B

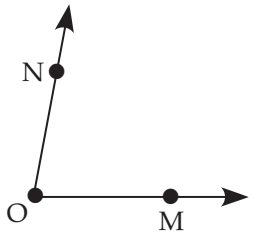
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Measure the following angles using your protractor.

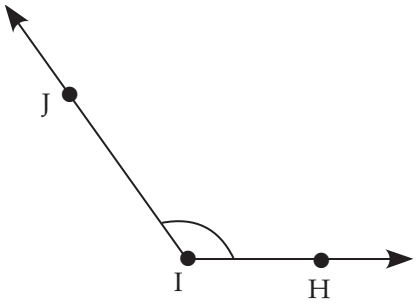
a)



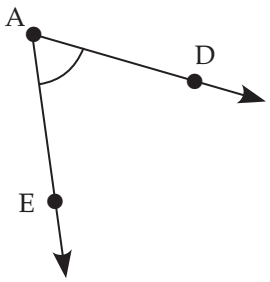
b)



c)



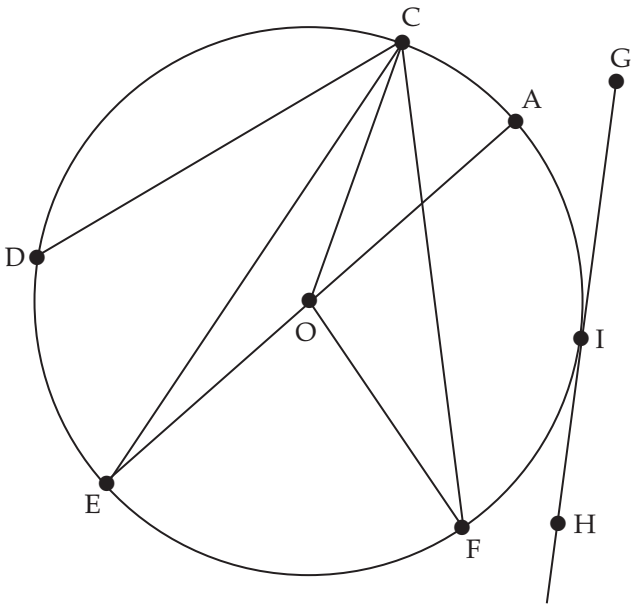
d)



Note: When measuring angles, full marks will be given for answers up to 3 degrees above or below the precise measurement. If you are measuring an angle of 50°, your answer could be between 47° and 53°. We call this "plus or minus" 3 (±3). This is to accommodate any slight variance with reprinting the diagrams, pencil lead width, or slight errors while using the protractor.

Learning Activity 6.1: Measuring Angles (continued)

2. Given the following diagram, identify the requested components.



a) Identify four different chords _____ , _____ , _____ , _____

b) Identify the diameter _____

c) Identify four radii _____ , _____ , _____ , _____

d) Identify five central angles (less than 180°) _____ , _____ , _____ , _____, _____

e) Identify four inscribed angles _____, _____ , _____ , _____

f) Identify a tangent line _____

g) Identify the point of tangency _____

h) Identify six arc segments _____ , _____ , _____ , _____ , _____ , _____

Lesson Summary

This lesson reviewed how to name angles and how to measure angles using a protractor. It also introduced you to some terminology used in studying circles. In the next lesson, you will use some of this new terminology.

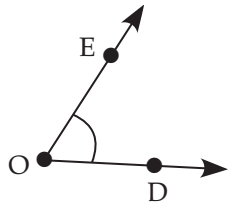
Assignment 6.2: Measurement and Terminology

Total: 30 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

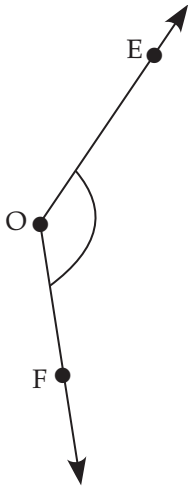
1. Use a protractor to measure each angle. *(4 marks)*

a) *(1 mark)*



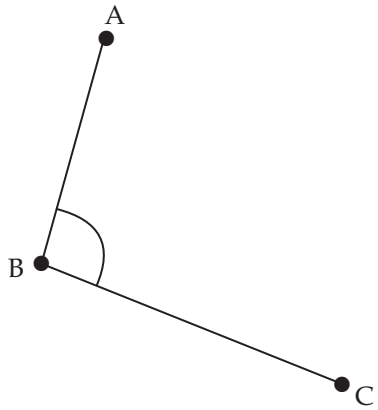
_____

b) *(1 mark)*



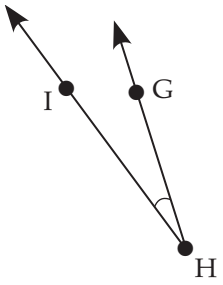
_____

c) *(1 mark)*



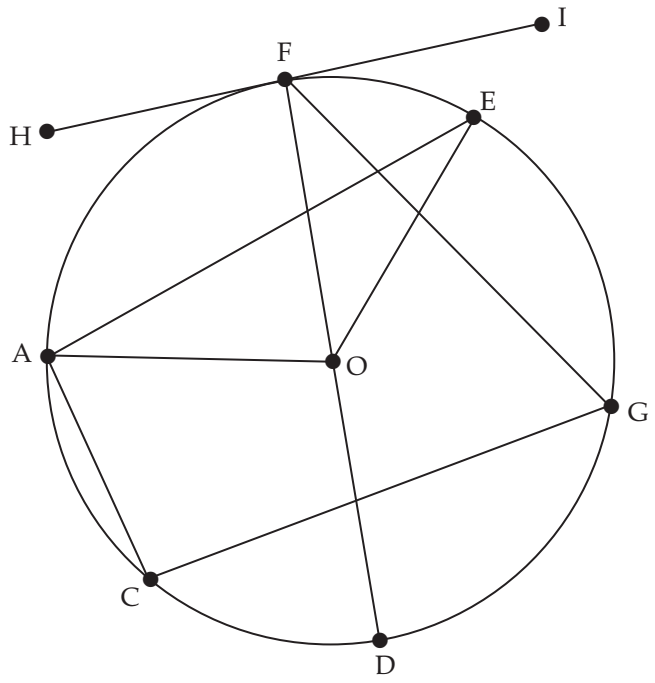
_____

d) *(1 mark)*



_____

2. Given the following circle, identify the requested components. *(26 marks)*



a) Identify five different chords _____ , _____ , _____ , _____ , _____ *(5 marks)*

b) Identify the diameter _____ *(1 mark)*

c) Identify four radii _____ , _____ , _____ , _____ *(4 marks)*

d) Identify five central angles (less than 180°) _____ , _____ , _____ , _____ , _____ *(5 marks)*

e) Identify three inscribed angles (less than 180°) _____ , _____ , _____ *(3 marks)*

f) Identify a tangent line _____ *(1 mark)*

g) Identify the point of tangency _____ *(1 mark)*

h) Identify six arcs _____ , _____ , _____ , _____ , _____ , _____ *(6 marks)*

Lesson 3: Inscribed and Central Angles

Lesson Focus

When you finish this lesson, you will be able to

- ❑ discover the connection between the measurements of inscribed and central angles
- ❑ determine the measurement of an angle inscribed by a semicircle

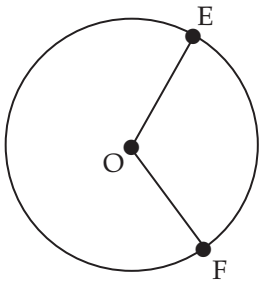
Lesson Introduction

There is a connection between the measurements of inscribed and central angles that have the same endpoints. Using a protractor, you will discover that relationship.

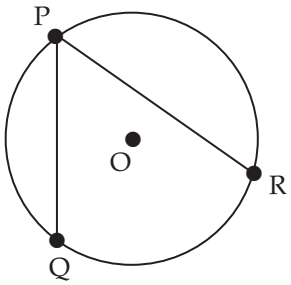
Defining Central and Inscribed Angles

Central angles have their vertex at the centre of the circle. The vertex of an inscribed angle is on the circumference of the circle.

Central Angle

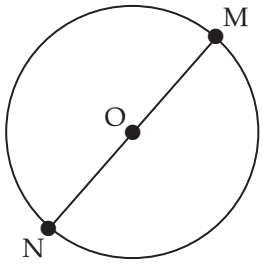


Inscribed Angle



Notice their end points. The central angle's endpoints are at E and F, with the vertex at O. The inscribed angle has endpoints Q and R, with the vertex at P.

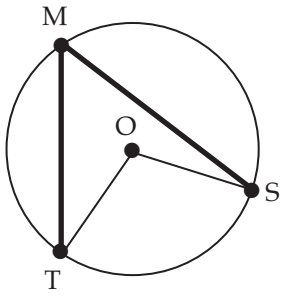
A unique central angle is the semicircle. This 180° angle is formed along the diameter with endpoints at M and N, and its vertex at the centre.



Finding Inscribed and Central Angles Sharing Endpoints

The following circle contains both a central and an inscribed angle. The lines of the inscribed angle are highlighted to help you see it compared to the central angle.

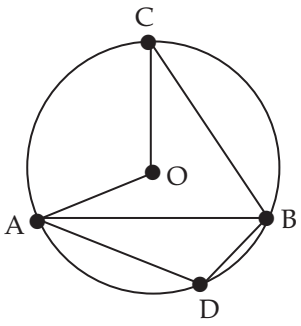
Notice that these angles share the same endpoints, S and T.



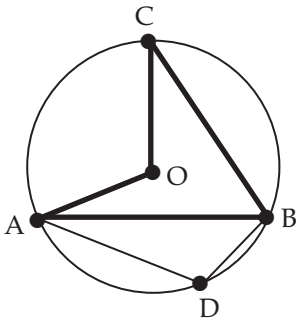
When two angles have the same endpoints, they share the arc between these endpoints. The mathematical term used to describe an angle sharing an arc or two endpoints is "subtend." ∠TMS and ∠TOS both subtend $\overset{\frown}{TS}$.

Example 1

Which inscribed and central angles share the same endpoints?



Solution:



There is only one central angle in this example. It has endpoints A and C.

Looking closer, you can see that inscribed angle ABC shares the same endpoints as the central angle. ∠AOC and ∠ABC both subtend $\overset{\frown}{AC}$.

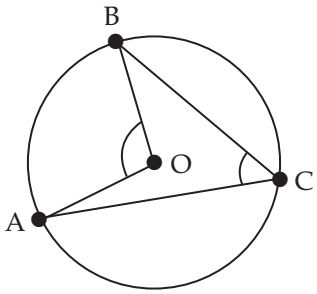
Measuring Inscribed and Central Angles that Share Endpoints

Now that you can identify central and inscribed angles that share the same endpoints, your task is to discover how these two angles are related.

Use your protractor to measure both the central and inscribed angles in each example.

Example 1

Measure ∠ACB and ∠AOB.



Solution:

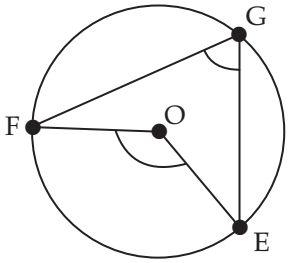
Measurements:

Central Angle: ∠AOB is 100°

Inscribed Angle: ∠ACB is 50°

Example 2

Measure ∠FGE and ∠FOE.



Solution:

Measurements:

Central Angle: ∠FOE is 132°

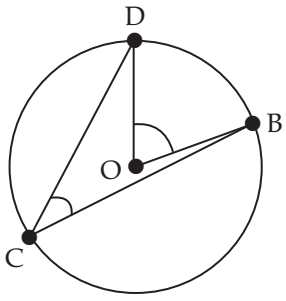
Inscribed Angle: ∠FGE is 66°

Can you see how the central angles are related to the inscribed angles?

There is a trend happening! Try two more examples with your protractor:

Example 3

Measure ∠BOD and ∠BCD.



Solution:

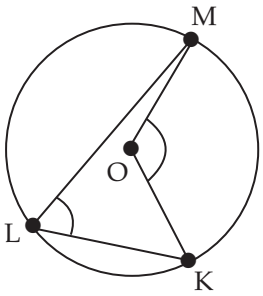
Measurements:

Central Angle: ∠BOD is 70°

Inscribed Angle: ∠BCD is 35°

Example 4

Measure ∠KOM and ∠KLM.



Solution:

Measurements:

Central Angle:	∠KOM is 122°
Inscribed Angle:	∠KLM is 61°

Once again, look at the measurements you found, and determine the relationship.

What is the pattern? The measure of the inscribed angle is equal to half the measure of the central angle. If the two angles share the same endpoints, this pattern always exists.

Another way to state this property is that the measure of a central angle is double the measure of the inscribed angle sharing the same endpoints. Most circle properties can be stated in two ways. They say the same thing, only in the reverse order.

Include this property in your Resource Sheet.

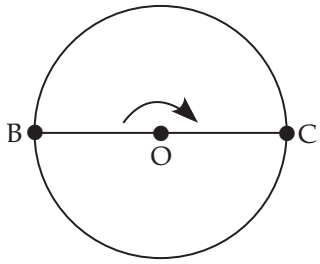
If your measurements did not show this, go back to the four examples and measure them again.

Remember: An inscribed angle sharing the same endpoints as a 100° central angle would be 50°.

A central angle sharing the same endpoints as a 50° inscribed angle would be 100°.

Measuring Inscribed Angles that Subtend a Diameter

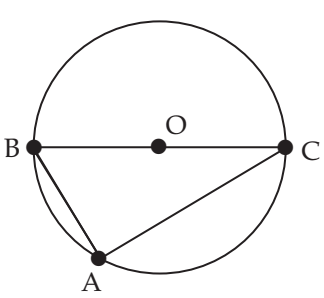
The diameter is a chord that passes through the centre of the circle, creating two semicircles.

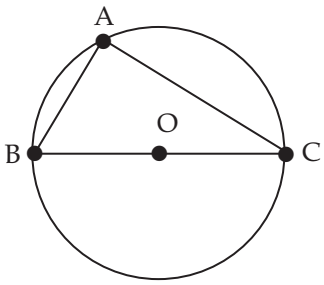


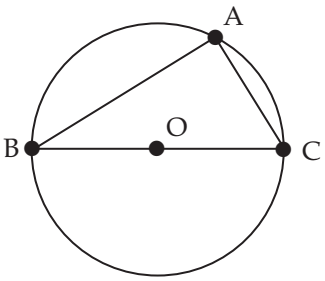
Central angle BOC is a straight angle that measures 180°.

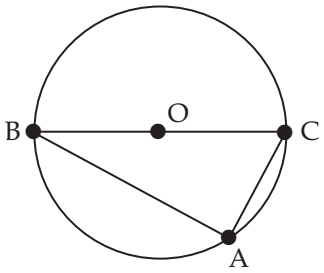
The endpoints are B and C. If we use those same endpoints to create an inscribed angle, ∠BAC, what would its measurement be?

Use your protractor, and measure ∠BAC in each of the following examples.









In each case, you should have discovered that the inscribed angle is 90°.

Can you explain why?

Remember the semicircle or straight angle BOC is 180°.

Since this angle goes through the centre, it would be a central angle.

If an inscribed angle shares the same endpoints as the diameter, it has to be half the central angle, or 90°.

To summarize, inscribed angles sharing the same endpoints as central angles are half the measure of the central angle. Or, in reverse order, central angles sharing the same endpoints as inscribed angles would be double the measure of the inscribed angle. This includes semicircles with a central angle measurement of 180°. Any inscribed angle that subtends a diameter would be 90°.

Learning Activity 6.2

Measuring Angles

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write two equivalent fractions for $\frac{8}{12}$.
2. Express $\frac{6}{8}$ as a percent.
3. Convert 8.5 cm to mm.
4. Add $\frac{3}{5} + \frac{7}{10}$.
5. Multiply 101 x 101.
6. What is the remainder after the division: 465 x 2?
7. How much does 50 L of gas cost at 99¢ per litre?
8. If 7 boxes of candy cost $3.50, how much does 4 boxes of candy cost?

continued

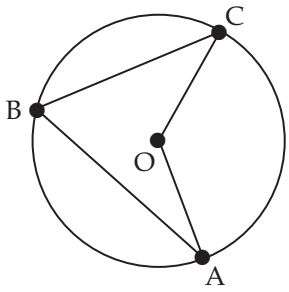
Learning Activity 6.2: Measuring Angles (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use your protractor to find the measurement of both the central and inscribed angles.

a)

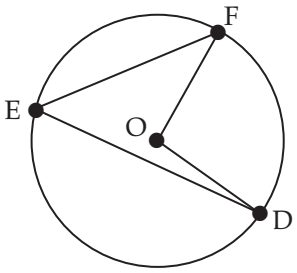


∠ABC = _____

∠AOC = _____

∠ABC and ∠AOC both subtend arc _____

b)

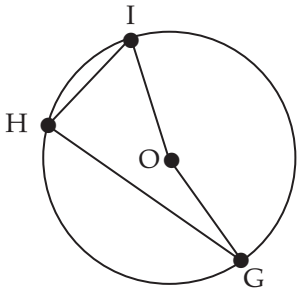


∠DOF = _____

∠DEF = _____

∠DOF and ∠DEF both subtend arc _____

c)

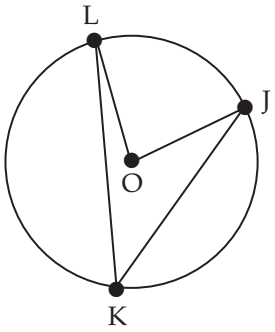


∠GHI = _____

∠GOI = _____

∠GHI and ∠GOI both subtend arc _____

d)



∠JOL = _____

∠JKL = _____

∠JOL and ∠JKL both subtend arc _____

2. An inscribed angle sharing the same endpoints as a 210° central angle would be _____°.

3. An inscribed angle that subtends a diameter would be _____°.

Lesson Summary

In this lesson, you discovered that the measure of an inscribed angle is always half the measure of the central angle that shares the same endpoints, or subtends the same arc. You also learned that circle properties can be stated in reverse order: central angles will always be double the measure of inscribed angles. You observed a special case involving an inscribed angle that shares the same endpoints as a diameter or semicircle—it will always measure 90°. In the next lesson, we will explore inscribed angles a little deeper.

NOTES

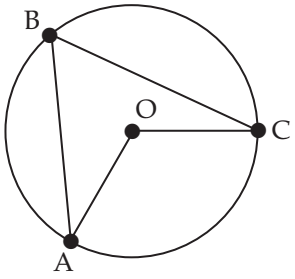
Assignment 6.3: Central and Inscribed Angles

Total: 18 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Use your protractor to find the measurement of each central and inscribed angle. *(8 marks)*

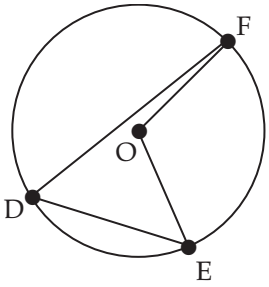
a) *(2 marks)*



∠AOC = _____

∠ABC = _____

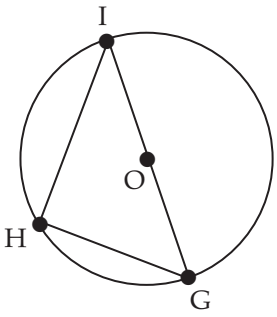
b) *(2 marks)*



∠FOE = _____

∠FDE = _____

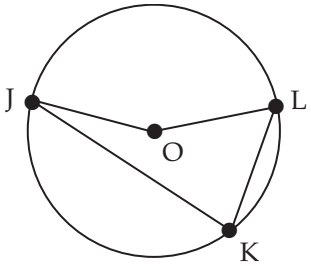
c) *(2 marks)*



∠IOG = _____

∠IHG = _____

d) *(2 marks)*

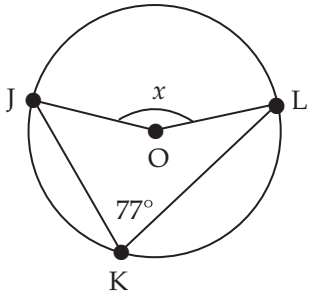


∠JKL = _____

∠JOL = _____

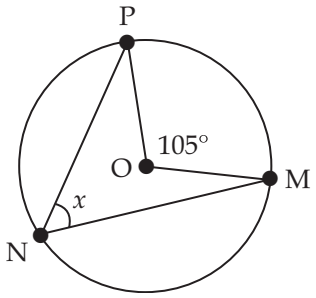
2. Find the value of x in each circle without using a protractor. *(4 marks)*

a) *(2 marks)*



x = _____

b) *(2 marks)*



x = _____

3. A circle with a central angle of 120° has an inscribed angle sharing the same endpoints. What is the measurement of the inscribed angle? *(2 marks)*

_____

4. A circle with an inscribed angle of 47° has a central angle sharing the same endpoints. What is the measurement of the central angle? *(2 marks)*

_____

5. An inscribed angle sharing the same endpoints as a semicircle will measure _____°. *(2 marks)*

Lesson 4: More Inscribed Angles

Lesson Focus

When you finish this lesson, you will be able to

- ❑ define and apply the concept of congruency
- ❑ discover that inscribed angles that subtend the same arc are congruent

Lesson Introduction

In the previous lesson, you learned that there is a relationship between the measures of central and inscribed angles when they share the same endpoints. Now you will focus on the measurements of inscribed angles that subtend the same arc or have the same endpoints.

The Meaning of Congruency

Angles that have the same measurements are said to be "congruent." This means they are alike in every respect. Identical twins are a great example of this. They are totally alike in all respects, yet they remain different people. They would be "congruent."

So, if $\angle A = 85°$ and $\angle B = 85°$, then $\angle A$ is congruent to $\angle B$.

The mathematical symbol used to denote congruency is $\cong$.

This symbol is made up of two parts:

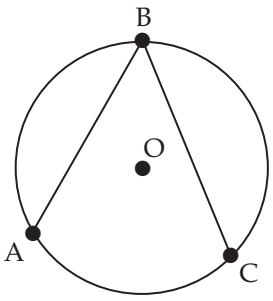
~ which means the same shape (similar), and

= which means the same size (equal)

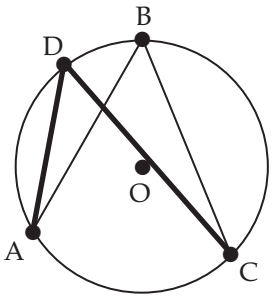
So, $\angle A \cong \angle B$.

In your own words, include the definition of *congruency* on your Resource Sheet.

Congruent Inscribed Angles



In this circle, $\angle ABC$ is an inscribed angle with endpoints at A and C. It subtends $\overset{\frown}{AC}$.



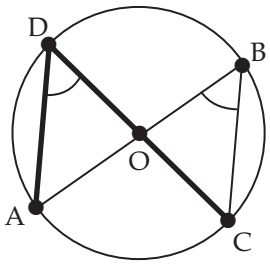
Here is an example of two inscribed angles subtending the same arc.

Notice how angles ABC and ADC have the same endpoints.

Your task is to discover the relationship between inscribed angles that subtend the same arc.

Example 1

Measure inscribed angles ADC and ABC with your protractor. What do you notice?

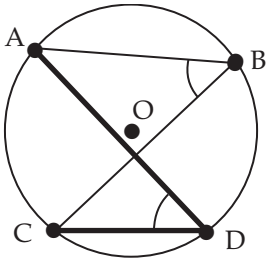


Solution:

They have equal measures. $\angle ADC \cong \angle ABC$.

Example 2

Measure ∠ABC and ∠ADC. Once again, both inscribed angles have the same endpoints, A and C. What do you notice?



Solution:

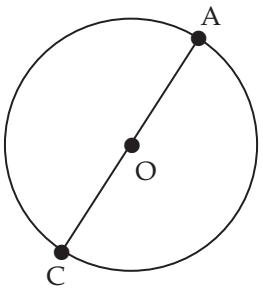
Angles ABC and ADC have the same measurements.

Angles which have the same measurements are said to be "congruent."

∠ADC ≅ ∠ABC

Example 3

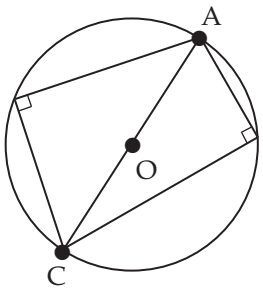
Draw and measure an inscribed angle that will subtend the arc AC.



Solution:

This next example uses a straight angle, ∠AOC. It subtends a semicircle with an arc of 180°. If you place your protractor over the diameter, you will see the 180°.

To draw an inscribed angle, place a point anywhere on the circle diagram below. Draw lines from that point to each of the endpoints, A and C, and measure the angle you created. Choose another point on the circle and again, draw lines from that point to A and C. Measure this new inscribed angle. Repeat this again. No matter how many examples you do like this, the result will always be the same!



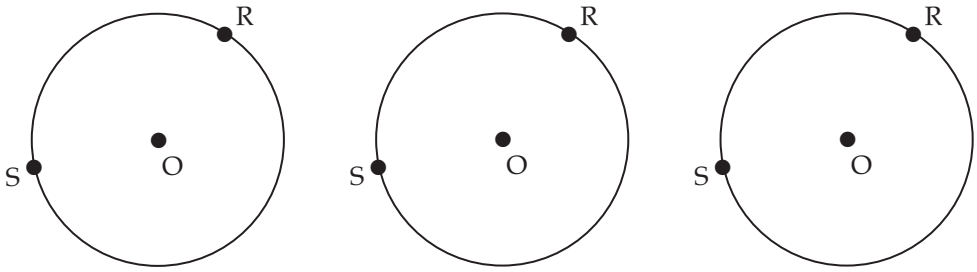
Since the inscribed angles that you created all subtend the same arc, $\overset{\frown}{AC}$, they are all congruent. And, because they subtend the same arc as a central angle, in this case a 180° angle, the inscribed angles are half the measure of the central angle, or 90°.

Example 4

Here is a circle with endpoints R and S.

Place a third point anywhere along the circumference of the circle, and join line segments from that point to each of the endpoints. Measure the angle you created.

Now place a fourth point anywhere along the edge of the circle, draw the line segments from that point to each endpoint R and S. Measure the new angle you created. Are they the same measurement?



The same circle is copied three times for you to continue this exercise until you find a pattern.

Solution:

No matter where you put your third and fourth points, all the angles should be approximately 68^{o} because all inscribed angles that subtend the same arc are equal.

You should include this rule in your Resource Sheet if you haven't already.

Learning Activity 6.3

Measuring Inscribed Angles

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. In addition to the $430 price of your plane ticket, you have to pay $34 for port fees and $60 in other charges. Can you buy this ticket if you have $514 in your chequing account?
2. The price of a $60 video game is increased by 15%. What is the new price of the video game?
3. You buy a soccer ball for $45, which was a savings of $15. What percentage does this represent?
4. What two numbers have a sum of 12 and a difference of 2?
5. What two numbers have a sum of 12 and a product of 35?
6. What two numbers have a sum of 12 and a product of 36?
7. What number is 86 more than 33?
8. What number is 33 less than 86?

continued

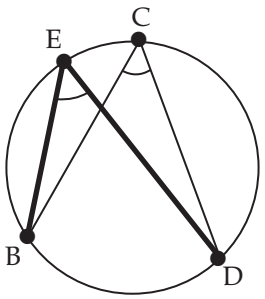
Learning Activity 6.3: Measuring Inscribed Angles (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use your protractor to find the measurements of each inscribed angle.

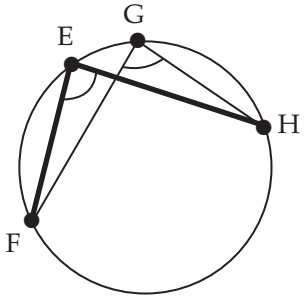
a)



∠BED = _____

∠BCD = _____

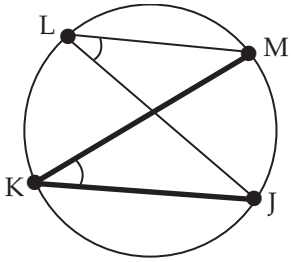
b)



∠FEH = _____

∠FGH = _____

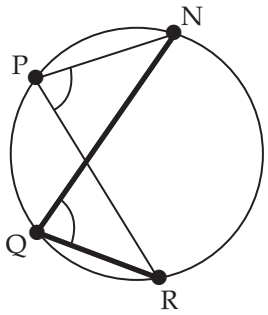
c)



∠JKM = _____

∠JLM = _____

d)



∠RQN = _____

∠RPN = _____

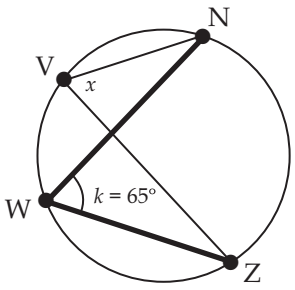
2. Inscribed angles that subtend the same arc are _____ .

continued

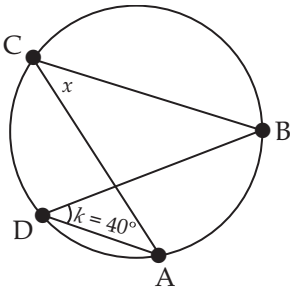
Learning Activity 6.3: Measuring Inscribed Angles (continued)

3. State the measure of the angle with the x without using a protractor.

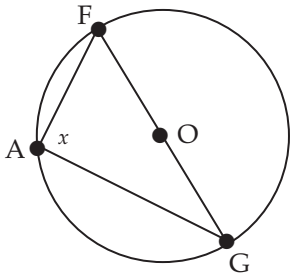
a)



b)

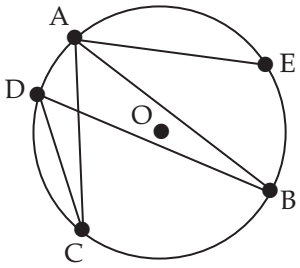


c)



Where $\overline{GOF}$ is the diameter of the circle.

4. Indentify the congruent angles in the diagram below and justify your answer:



Lesson Summary

In this lesson, you discovered that inscribed angles that subtend the same arc or share endpoints have the same measure. These angles are said to be congruent. In the next lesson, you will be introduced to tangents, and will discover their special characteristics in relation to circles.

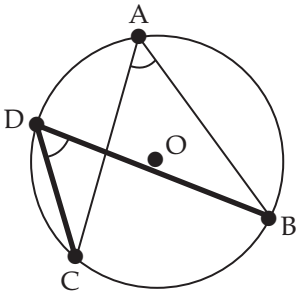
Assignment 6.4: More Inscribed Angles

Total: 18 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Use your protractor to find the measurements of the indicated angles. *(6 marks)*

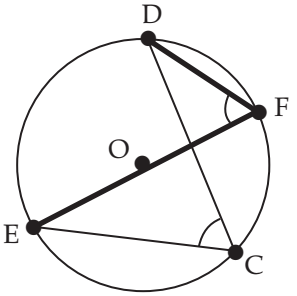
   a) *(2 marks)*



   ∠CAB = _____

   ∠CDB = _____

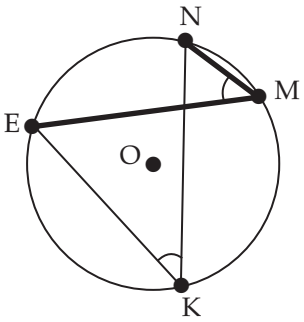
   b) *(2 marks)*



   ∠DFE = _____

   ∠DCE = _____

   c) *(2 marks)*



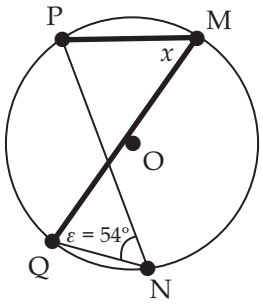
   ∠EMN = _____

   ∠EKN = _____

2. Inscribed angles that subtend the same arc are _____. *(2 marks)*

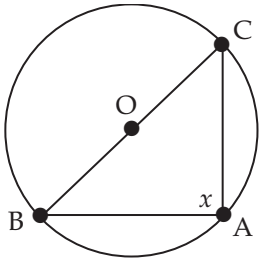
3. Without using a protractor, state the value of the angle with the x. *(8 marks)*

a) *(2 marks)*



∠QMP = _____

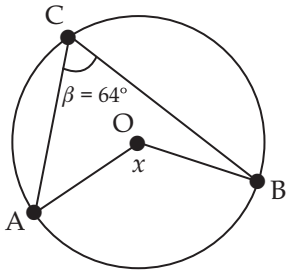
b) *(2 marks)*



Where $\overline{BOC}$ is the diameter of the circle.

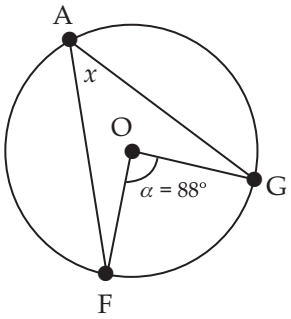
∠BAC = _____

c) *(2 marks)*



∠AOB = _____

d) *(2 marks)*



∠FHG = _____

4. Central angles are _____ the measure of inscribed angles that share the same endpoints. *(2 marks)*

LESSON 5: TANGENTS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ understand that a line tangent to a circle is perpendicular to the radius at the point of tangency
- ❑ find the point of tangency

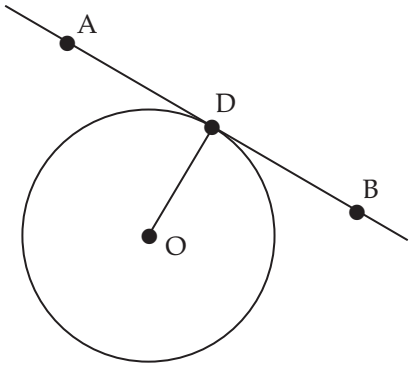
Lesson Introduction

A tangent is a line that touches the circumference of a circle at exactly one point. It has a special relationship with the circle, specifically with the radius. You will discover that property in this lesson by creating angles that are perpendicular to a tangent at the point of tangency. You will also use a right angle through the centre of a circle to find the point of tangency.

How Tangents Meet Radii

The term "perpendicular" refers to a line that meets another line at a right, or 90°, angle. The point of tangency is the exact point where the tangent line touches the circle.

Carefully examine the diagram below. AB is tangent to circle O. Point D is the "point of tangency" where the tangent line touches the circle. OD is a radius coming from the centre of the circle to the outside edge.



What happens when a tangent meets a radius?

Use your protractor to measure angles ADO and BDO. Write down the measurements you found.

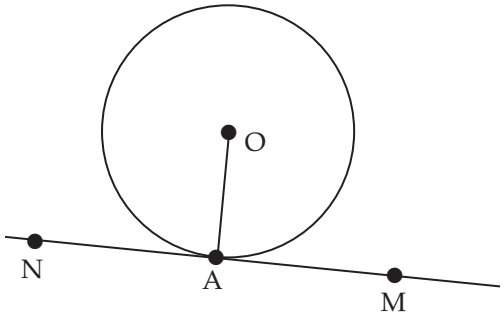
∠ADO = _____°

∠BDO = _____°

Now do the same thing with a totally different tangent on a completely different circle.

Use your protractor to measure the angles where the radius OA meets the tangent MN at point A.

Write down the measurements you found.



∠MAO = _____°

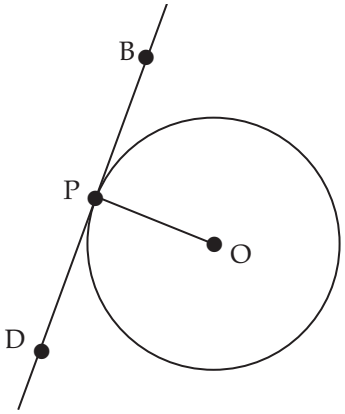
∠NAO = _____°

Do you see a pattern?

One more example.

Here, tangent BD touches the circle at the point of tangency, P. The radius is OP. Use your protractor to measure the angle at which the radius meets the tangent at the point of tangency.

Write down the measurement you found.



∠OPB = _____°

∠OPD = _____°

So what is the common occurrence?

The tangent always meets the radius at a 90° angle at the point of tangency!

In other words, the tangent is always perpendicular to the radius at the point of tangency.

If the tangent is perpendicular to the radius, then the radius must also be perpendicular to the tangent.

This property should be included on your Resource Sheet.

Finding the Point of Tangency

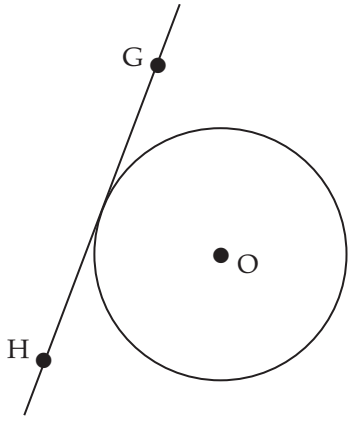
Once you understand that the radius must meet the tangent at a 90° angle, you can use that knowledge to find the exact point of tangency. (Remember that the radius always goes through the centre. You will need to keep this in mind for Lesson 6 as well.) So, if we are given the centre and the tangent, how can we find the exact point of tangency?

Look at circle O below. Somewhere along tangent GH is the point of tangency.

But where exactly?

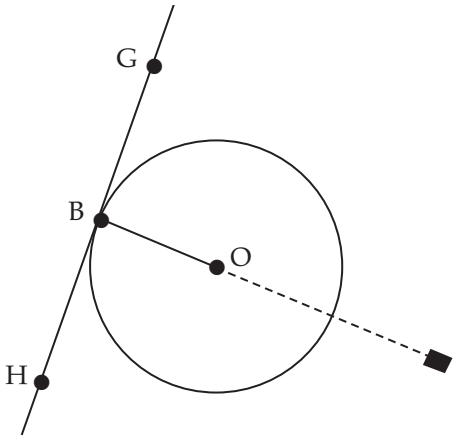
You discovered earlier in this lesson that the tangent always meets the radius at a right angle. You could guess, and draw in a radius, but would it be exactly 90°?

Place the base line of your protractor along the tangent line. Slide the protractor along this tangent line until the 90° line on the protractor sits directly over the centre of the circle. Where the centre point on the baseline touches the tangent is the exact point of tangency.



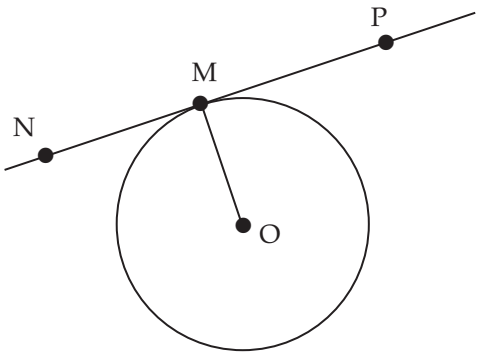
Make a mark on your paper along the top edge of your protractor at 90°. Or, make a mark under the protractor just below the centre point on the base line. Then, using a ruler, just join the centre to the tangent through your mark. The angle you created is 90°, and the point of tangency, B, is found!

The property you have illustrated is that tangents are perpendicular to the radius at the point of tangency.



Question: Does a perpendicular line from the point of tangency go through the centre?

You have seen that the radius meets the tangent at a right angle at the point of tangency. A radius, by definition, comes from the centre. So, yes, a perpendicular line from the point of tangency must go through the centre. You will need to remember this property for Lesson 6!



Learning Activity 6.4

Finding Tangents

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You buy three chocolate bars that cost $0.80 each. You give the cashier $2.50. How much money do you get back?
2. Estimate the square root of 914.
3. How many 5 kg bags of potatoes can you make with 110 kg of potatoes?
4. What is the median of the following: 64, 70, 70, 73, 82, 85, 91?
5. What is the mode of the following: 64, 70, 70, 73, 82, 85, 91?
6. If 20% of a group of 120 students play badminton, how many students don't play badminton?
7. Napoléon was born in 1769 and died in 1821. How old was he when he died?
8. What value of m satisfies the equation: $\frac{2}{5} \cdot m = 1$?

continued

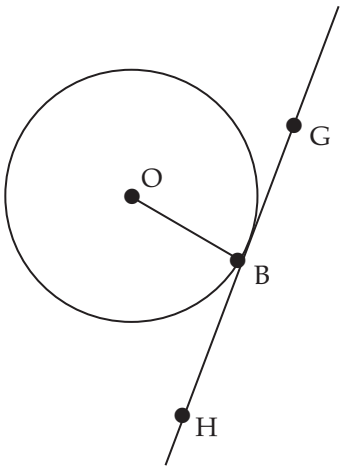
Learning Activity 6.4: Finding Tangents (continued)

Part B

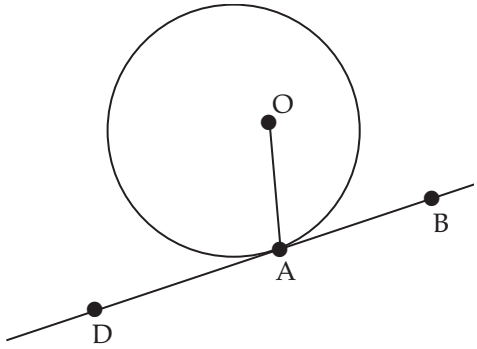
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Only one of the three circles shown has a tangent being perpendicular to the radius. Use your protractor to identify the correct drawing.

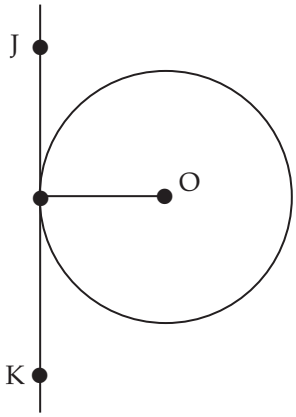
a)



b)



c)

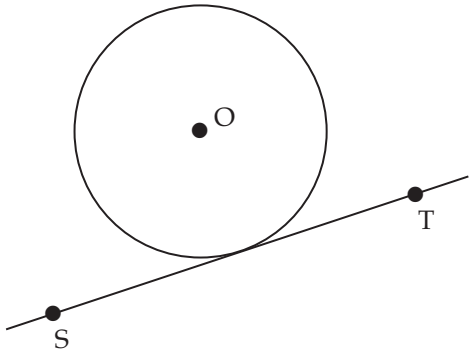


continued

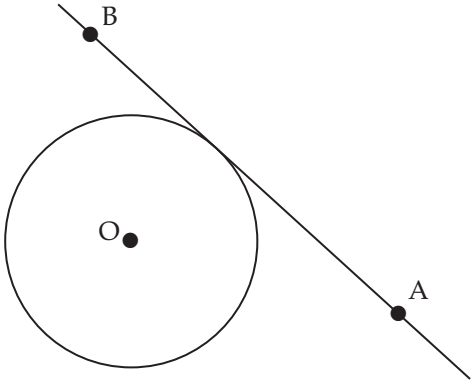
Learning Activity 6.4: Finding Tangents (continued)

2. Use your protractor to construct a radius meeting the tangent at a right angle (90°).

a)



b)



3. Tangents always meet the radius at a _____ angle at the point of tangency.

4. Perpendicular lines from the point of tangency always go through the_____ of a circle.

Lesson Summary

In this lesson, you learned that tangents touch the circle at only one point, the point of tangency. At this point, the radius meets the tangent at a right angle. This means tangents are perpendicular to the radius, at the point of tangency. You also learned that any line that is perpendicular to the tangent at the point of tangency goes through the centre. In the next lesson, you will explore perpendicular bisectors.

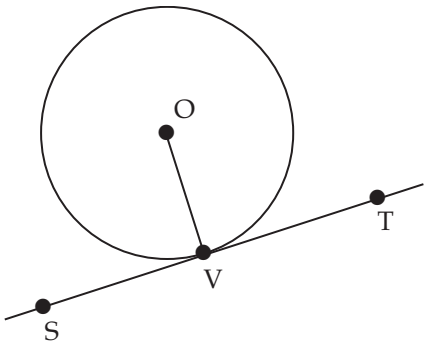
Assignment 6.5: Tangents

Total: 14 marks

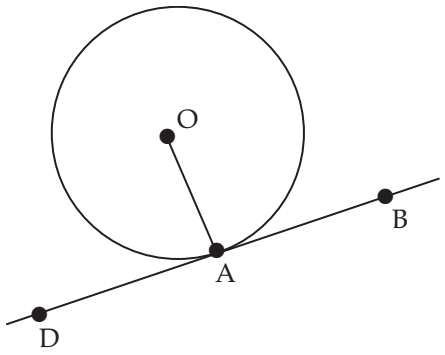
Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Only two of the three circles shown have a tangent that is perpendicular to the radius. Use your protractor to identify the correct drawings. *(4 marks)*

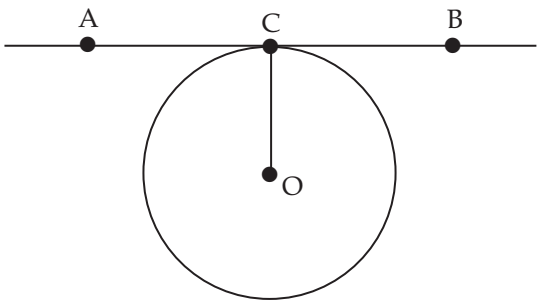
a)



b)



c)

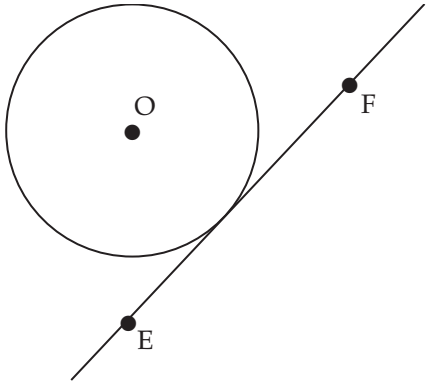


2. A line that is perpendicular to the tangent at the point of tangency goes through the _____. *(1 mark)*

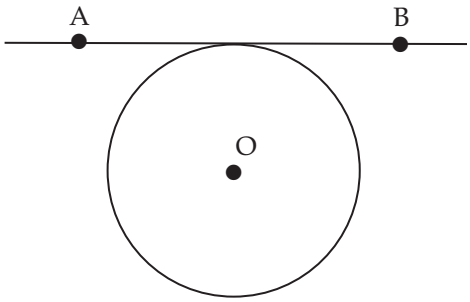
3. A radius always meets the tangent at the point of tangency at a _____ angle. *(1 mark)*

4. Use your protractor to find the exact point of tangency on each tangent, and label it. *(6 marks)*

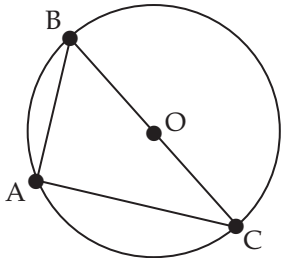
a) *(3 marks)*



b) *(3 marks)*



5. Explain in words why ∠CAB is 90°. *(2 marks)*



Lesson 6: Perpendicular Bisectors

Lesson Focus

When you finish this lesson, you will be able to

- ❑ understand how a perpendicular line from the centre of a circle bisects a chord
- ❑ understand how a line from the centre of a circle that bisects a chord is perpendicular to the chord
- ❑ understand that a perpendicular bisector to a chord always passes through the centre

Lesson Introduction

In Lesson 1, you were introduced to terminology including the definition of perpendicular bisectors. In this lesson, you will discover certain characteristics perpendicular bisectors have within circles, specifically their relationship to chords.

Defining Perpendicular Bisectors

A bisector cuts something into two equal parts.

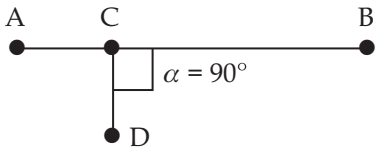
Given line AB, the point that bisects the line would be placed exactly in the middle, so that each side is the same length.

AM = BM

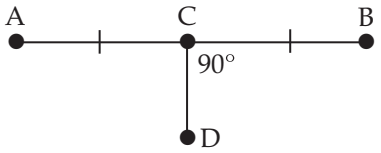
Point M has bisected the line AB into two equal parts.

Perpendicular means at right angles (90°).

In the diagram below, line segment CD is perpendicular to AB.



So, a perpendicular bisector combines these two properties. It must divide the line into two equal parts, and it must meet the line at a right angle.



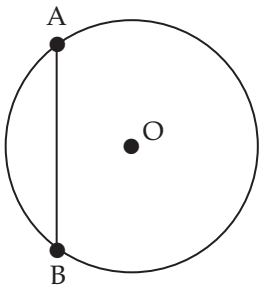
Here we have CD, a perpendicular bisector to AB. CD cuts AB into two equal parts, and is perpendicular to AB. It is called a perpendicular bisector.

Finding the Midpoint of a Chord

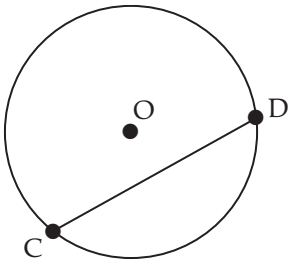
This lesson focuses on lines coming from the centre of the circle (radii) and chords.

To review some terminology from Lesson 1, a chord is a line segment with endpoints touching the circumference of the circle.

This is circle O with chord AB.



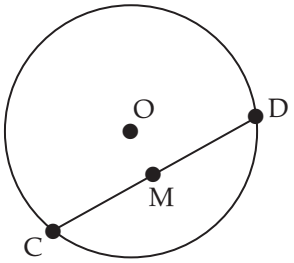
You can find the midpoint of any chord just by measuring its length, and dividing by two.



Let's assume that the length of chord CD is 6 cm.

If we were to place a point at the middle of this chord, it would be 3 cm from each side.

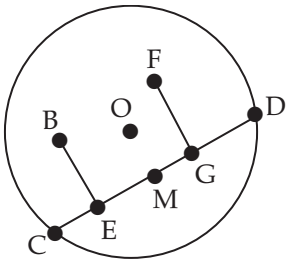
Point M is the midpoint, and it bisects the chord CD. This creates two equal parts where CM = DM



You can find the midpoint of any chord by measuring its length, and dividing by two.

Lines Perpendicular to a Chord

A perpendicular line meets the second line at a right angle (90°) to the first line. The diagram below shows two lines meeting chord, CD, at right angles.



BE is perpendicular (meets it at a right angle) to CD, and FG is perpendicular to CD.

This symbol, ⊥, shows two lines meeting at a right angle, and reads "is perpendicular to."

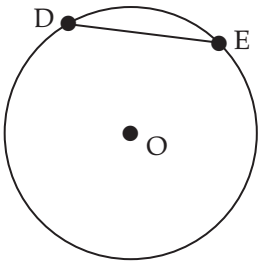
BE ⊥ CD and FG ⊥ CD

Perpendicular Bisector

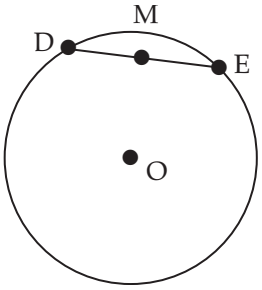
When we add the term "bisector" to the phrase, now we have to have a line which is perpendicular (⊥), but also which is at the exact centre of the chord, and bisects it into two equal parts.

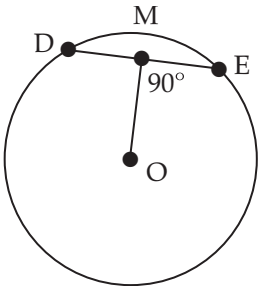
So, a perpendicular bisector has to meet the chord in the middle, and be perpendicular!

Given chord DE



Chord DE with midpoint M
(DM = EM)





This is a perpendicular bisector, where OM is ⊥ to DE, and DM = EM. The line segment OM is a perpendicular bisector to DE.

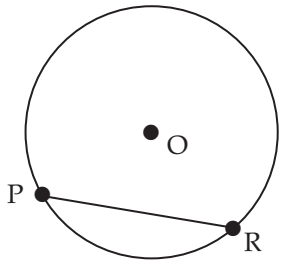
Once you understand the terminology of bisectors and perpendiculars, then they combine to read "perpendicular bisector." If these terms still confuse you, then go back and look at the examples and read the explanations again.

Method One of Creating Perpendicular Bisectors of a Chord

Take out your ruler and protractor—you'll need them to help you discover certain characteristics of perpendicular bisectors.

Example 1:

Given circle O with chord PR.



Using your ruler, measure the length of the chord, and mark the midpoint.

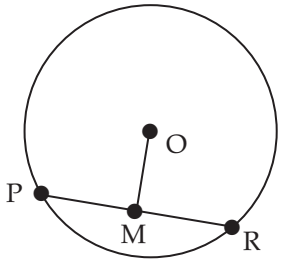
Draw a line from the centre to the midpoint. This bisects the chord.

Now place the base line of your protractor along the chord PR, with the centre being directly over the midpoint M. Measure the angle that goes through the centre.

Write down the measurement you found.

∠OMP = _____°

∠OMR = _____°



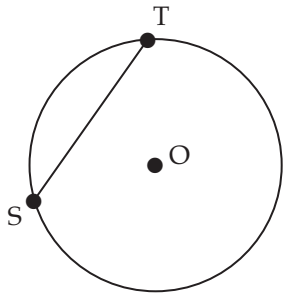
Solution:

∠OMP = 90°

∠OMR = 90°

Example 2

Given circle O with chord ST.



Using your ruler, measure the length of the chord, and mark the midpoint.

Draw a line from the centre to the midpoint. This bisects the chord.

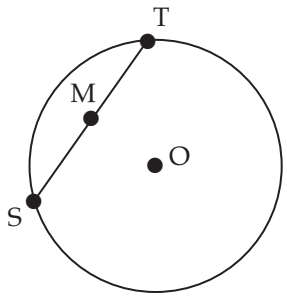
Now place the base line of your protractor along the chord ST, with the centre being directly over the midpoint M. Measure the angle that goes through the centre.

Write down the measurement you found.

$\angle OMT =$ _____°

$\angle OMS =$ _____°

Solution:



$\angle OMT = 90°$

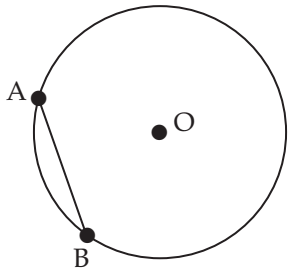
$\angle OMS = 90°$

One special characteristic you have just discovered is that a line from the centre of a circle that bisects a chord always meets the chord at a right angle. In other words, a line from the centre that bisects the chord is a perpendicular line.

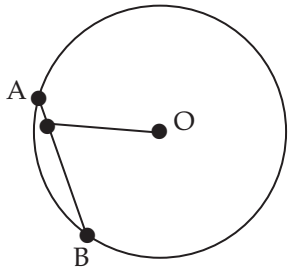
Note this property on your Resource Sheet.

Now we will look at this property from a different perspective. In the previous examples, you found the midpoint, drew the line coming from the centre to that midpoint, and then measured the angle. In these next examples, you will first find the angle of the line from the centre, and then measure to see if it bisects the chord.

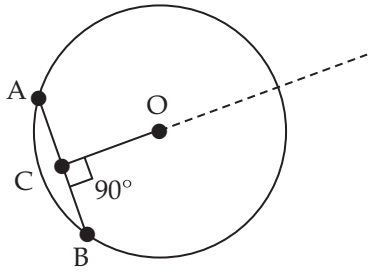
Method Two of Creating Perpendicular Bisectors of a Chord



Place the base line of your protractor along the chord AB. Line it up so the 90° line passes through the centre of the circle. Mark the right angle along the top edge of the protractor. Now use your ruler to draw a line from the centre to the chord at this spot. Label the point of intersection on the chord as C.



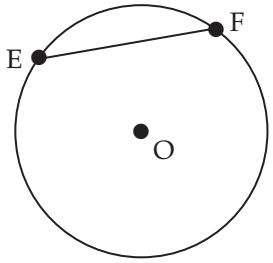
If your line looks something like the example above, where point C is placed closer to one of the endpoints on the chord, then it is wrong! The angle of the line from the centre to the chord must be 90°, as shown below.



Once you have the 90° angle set, measure the distance from A to C, and compare that to the distance from B to C. Are the distances the same?

If they are the same, then, by definition, the perpendicular line you drew from the centre has bisected the chord AB.

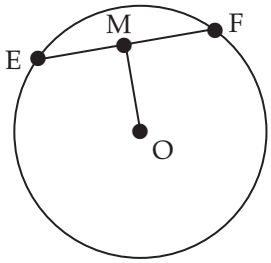
Example 1



Place the base line of your protractor along the chord EF and use it to measure a 90° angle between a line from the centre and the chord. Label the intersection point M. Draw a line joining the centre to point M.

Once you have the 90° angle drawn, measure the distance from E to M, and compare that to the distance from F to M. Are the distances the same?

Solution:



The distance from point E to point M should be equal to the distance from point F to point M. If they are not, make sure that your line from the centre of the circle is, in fact, perpendicular to the chord EF.

We can describe the characteristics you have explored in three ways: First, every time you bring a line from the centre of a circle to meet a chord at a right angle, it bisects the chord.

Or, a line from the midpoint of a chord through the centre is always perpendicular to the chord.

Thirdly, you can say that all perpendicular bisectors of a chord go through the centre of the circle.

These are the special properties of perpendicular bisectors. This will be useful information in the next lesson!

It will be helpful to have these properties on your Resource Sheet.

Learning Activity 6.5

Identifying Midpoints

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Order the following numbers in descending order (from greatest to least):

   $\frac{3}{5}$ $\quad 0.53 \quad$ $\frac{5}{3}$ $\quad 0.35$

2. Your friend lends you $200. How many weeks will it take you to pay the debt back if you give him $14 each week?
3. How many hours are there from Monday at 13:00 to Tuesday at 19:00?
4. What is the probability of an even number if you toss a regular 6-sided die?
5. A lottery prize of $4600 is to be divided equally among the 23 winners. What is the amount of each prize?
6. Calculate $31 \div \frac{1}{4}$.
7. The bank gives you 0.5% interest every 3 months. If you have $100 in your savings account, estimate the amount of interest at the end of the year.
8. Estimate the height in centimetres of a classroom if the height measures 98 inches and each inch is equivalent to approximately 2.5 cm.

continued

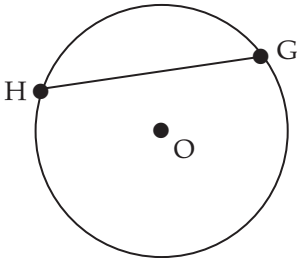
Learning Activity 6.5: Identifying Midpoints (continued)

Part B

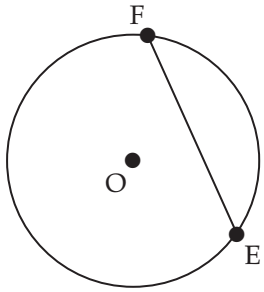
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use your ruler to find the midpoint of each chord, and then draw a line joining the midpoint to the centre. Measure the angle at which this line meets the chord.

a)



b)

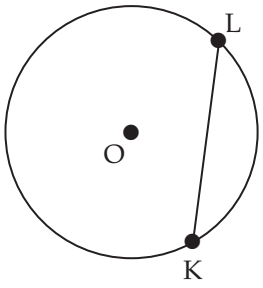


The angle measurement is _____°. The angle measurement is _____°.

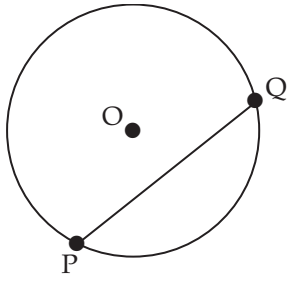
2. A line coming from the centre that bisects the chord always meets the chord at a _____ angle.

3. Use your protractor to create a 90° angle at the point where a line from the centre meets the chord. Label this point M. Then use your ruler to compare the distance from each endpoint to the midpoint.

a)



b)



KM = _____ PM = _____

LM = _____ QM = _____

Learning Activity 6.5: Identifying Midpoints (continued)

4. A line from the centre meeting the chord at a right angle always _____ the chord.
5. Perpendicular bisectors always pass through the _____ of the circle.

Lesson Summary

In this lesson, you learned that lines coming from the centre that bisect the chord always meet the chord at a right angle. You also learned that perpendicular lines coming from the centre of the circle to the chord always bisect the chord. Perpendicular bisectors of a chord always pass through the centre of the circle. In the next lesson, you will use all the properties from Lessons 1 to 5 to solve problems.

NOTES

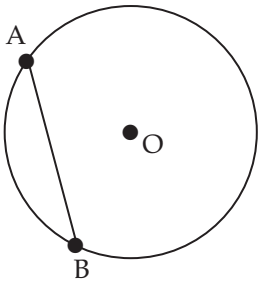
Assignment 6.6: Perpendicular Bisectors

Total: 19 marks

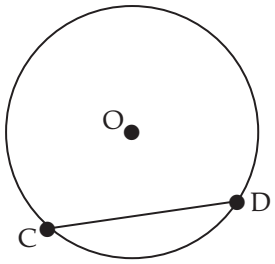
Please show all your work in the space provided below. Use a separate sheet if necessary.

1. Use your ruler to find the midpoint of each chord, and then draw a line joining the midpoint to the centre. Measure the angle at which this line meets the chord. *(6 marks)*

   a) *(3 marks)*

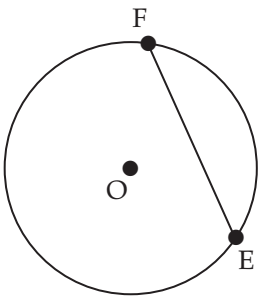
   

   b) *(3 marks)*

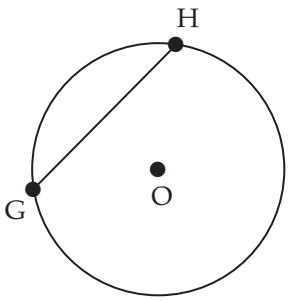
   

2. Use your protractor to create a 90° angle at the point where a line from the centre meets the chord. Label this point M. Then use your ruler to compare the distance from each endpoint to the midpoint. *(6 marks)*

   a) *(3 marks)*

   b) *(3 marks)*

3. Match the definition with the corresponding term. Write the correct letter of the definition on the line next to the term. *(3 marks)*

i) chord _____	a) touches the circle at only one point
ii) tangent _____	b) a line segment that touches the circumference of the circle at two points
iii) perpendicular bisector _____	c) a line coming from the centre and meeting the chord at a right angle

4. Use the terms below to fill in the blanks. *(4 marks)*

bisects	midpoint
right angle	centre

a) a line from the centre, bisecting the chord, meets the chord at a _____

b) a line from the centre that meets a chord at a right angle _____ the chord

c) a perpendicular line from the centre bisects the chord at the _____

d) perpendicular bisectors always pass through the _____

LESSON 7: SOLVING PROBLEMS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ find the centre of a circle using circle, angle, chord, and tangent properties
- ❑ solve problems and justify the solution strategy using circle properties

Lesson Introduction

In this lesson, you will apply the properties of circles, tangents, chords, and angles to find the centre of a circle. You will solve circle problems using the properties learned in this module.

Properties of Circles

- Inscribed angles are always half the measure of a central angle when they have the same endpoints, or subtend the same arc.
- Inscribed angles that share the same endpoints, or subtend the same arc, are always congruent.
- Inscribed angles inscribed in a semicircle are always 90°.
- Tangents are perpendicular to the radius at the point of tangency.
- Lines coming from the centre of a circle (radii) that meet a chord at a right angle always bisect the chord.
- Lines coming from the centre of a circle that bisect a chord always meet the chord at a right angle, or are perpendicular.
- Perpendicular bisectors of a chord go through the centre of the circle.

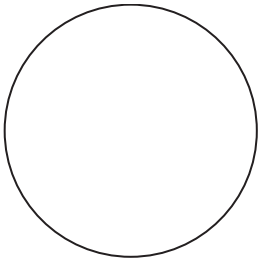
Do you have all of these properties listed on your Resource Sheet? If not, you should add any that you are missing before you move on.

Finding the Centre of a Circle Using Diameters

In Lesson 2 you discovered that angles inscribed in a semicircle are always 90°. This is because a semicircle, by definition, is a 180° angle along the diameter (which passes through the centre). The inscribed angle using the same endpoints would be half the measure of the central angle, or 90°.

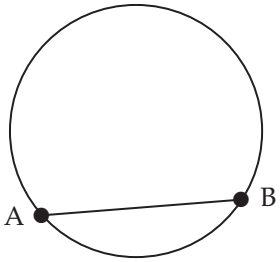
Now, you know that diameters go through the centre. If you construct two different semicircles, the point where the two diameters intersect will be the centre.

As you read through this section of the lesson, create a list of steps on your Resource Sheet that describe how to find the centre of a circle using diameters.

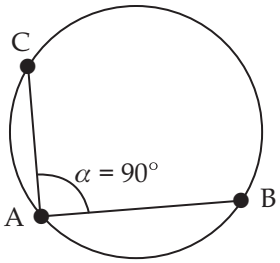


To find the centre of a circle, first construct an inscribed right angle.

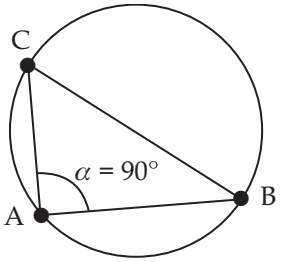
Place two points, A and B, anywhere along the outside edge (circumference) of the circle, and create a chord.



Now place the baseline of your protractor along the chord with the centre point of the protractor being over point A, and mark a 90° angle. Using a ruler and the 90° mark, draw a line joining point A to the circumference of the circle. Label this point C.

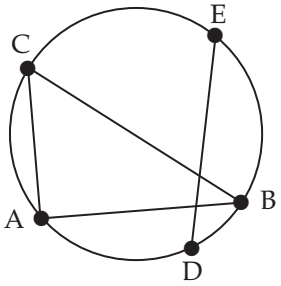


If you join C to B, you will have created a diameter, because an inscribed right angle always subtends a semicircle. But it is still uncertain as to just where the exact centre point is.

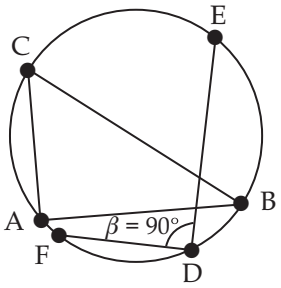


Now repeat the process using a second chord.

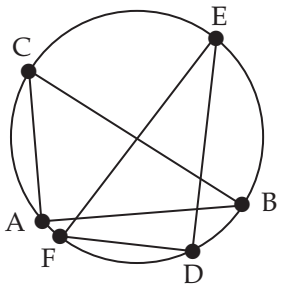
Draw another chord anywhere on the circle, and label those points D and E.



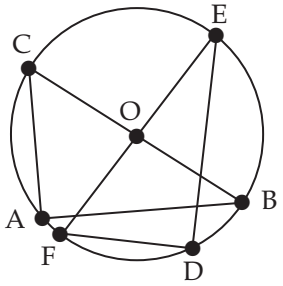
Now place the baseline of your protractor along the new chord DE with the centre point of the protractor being over point D, and mark a 90° angle. Using this mark, draw a line joining point D to the circumference of the circle. Label that intersection as point F.



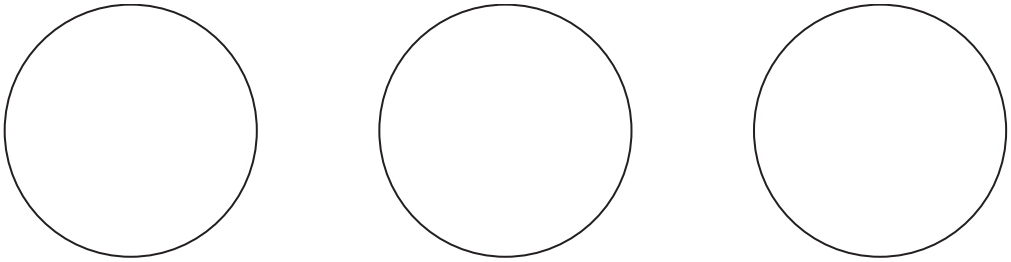
If you join F to E, you will have created another diameter.



The point where these diameters intersect is the centre of the circle!



Here are three empty circles for you to practice this method of finding the centre. Follow the steps listed above, and you will have success!



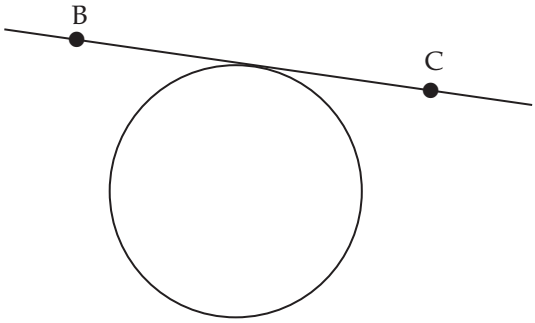
Finding the Centre Using Tangents

You learned that tangents meet the radius at a 90° angle at the point of tangency. By definition, radii come from the centre.

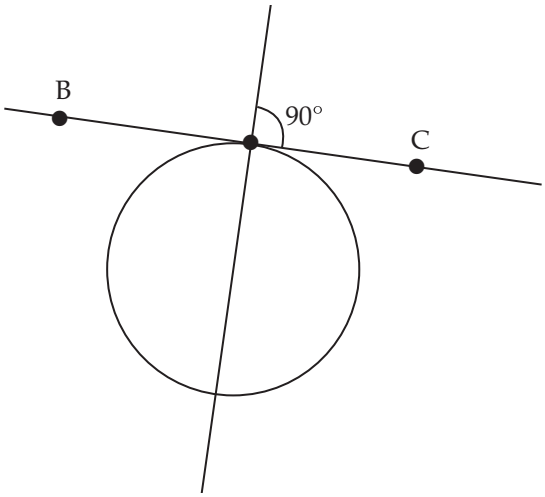
This method requires you to construct two tangents, each with a line that is perpendicular to it at the point of tangency. Where those perpendicular lines intersect, you will find the exact centre of the circle.

As you read through this section of the lesson, create a list of steps on your Resource Sheet that describe how to find the centre of a circle using tangents.

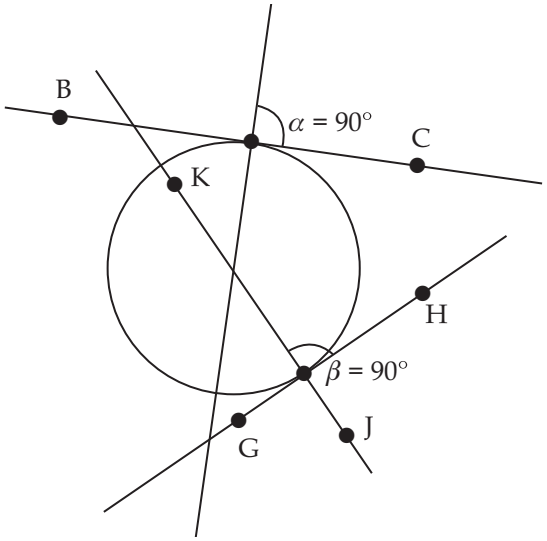
The first step involves drawing a tangent to a circle. Be careful that it only touches the circle at exactly one point.



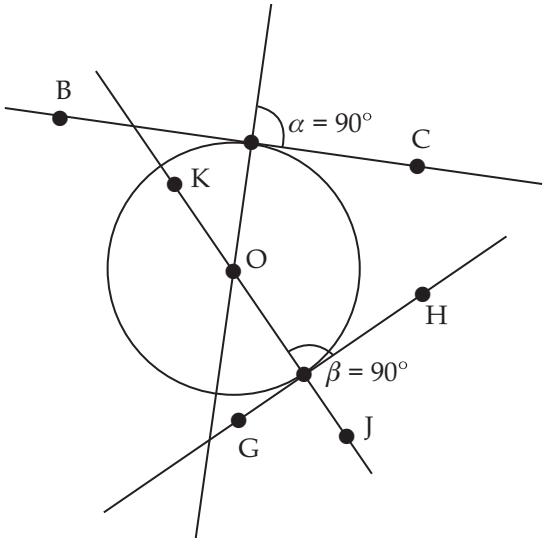
Now place the baseline of your protractor along the tangent, with the centre point at the point of tangency. Mark a 90° angle from the tangent. Draw a line joining the point of tangency with the 90° mark you measured.



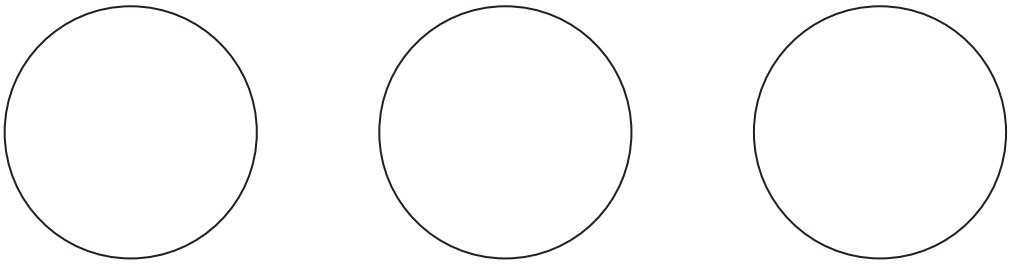
Repeat this process by drawing a new tangent and a new perpendicular line. In the diagram below, you can see the new tangent GH, and the line KJ, which is perpendicular to it.



If you measured carefully, you will have found the exact centre (O) of the circle where the two perpendicular lines intersect!



Once again, here are three empty circles for you to practice your skills with finding the centre using tangents and their perpendiculars.

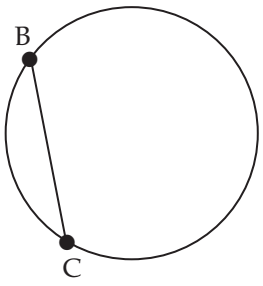


Finding the Centre Using Perpendicular Bisectors

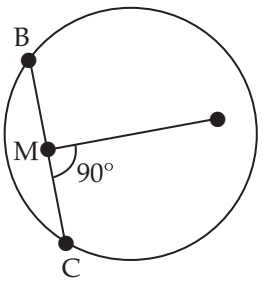
The third method you can use to find the centre of a circle applies the properties of perpendicular bisectors. Using your ruler and protractor, create two chords and their perpendicular bisectors. These bisectors go through the centre. Where they intersect will be the exact centre of the circle.

As you read through this section of the lesson, create a list of steps on your Resource Sheet that describe how to find the centre of a circle using perpendicular bisectors.

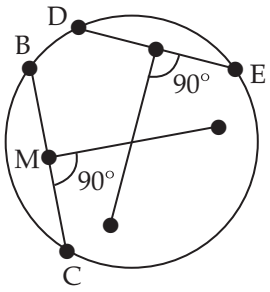
The first step is to draw a chord, BC, anywhere on the circle.

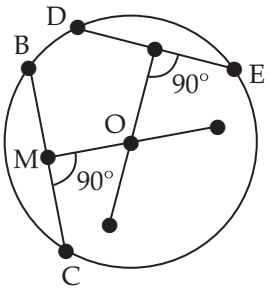


Use a ruler to find the midpoint of the chord and label it M. With a protractor, mark a 90° angle at the midpoint and draw a perpendicular line coming from the chord at M.



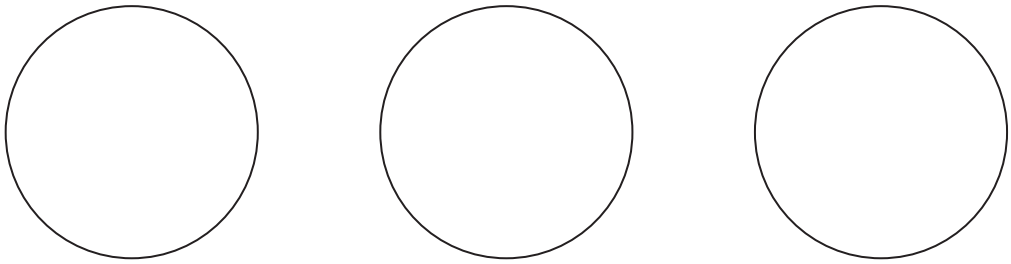
Repeat this procedure in the same circle. Draw another chord, and construct a perpendicular bisector.





The point where the two perpendicular lines intersect is the exact centre of the circle.

Here are three empty circles for you to practice your skills with finding the exact centre using perpendicular bisectors.

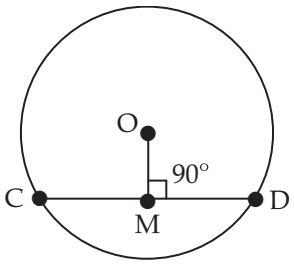


Solving Circle Problems Using Pythagoras

Remember Pythagoras? We can solve problems in circles using the Pythagorean theorem learned in Grade 8 math.

Given circle O, where CD = 8, the radius is 5, and OM ⊥ CD.

Find the length of OM.



Since CD is perpendicular to a chord, we know it bisects the chord. So, CM = 4. The radius is given as 5, so draw in OC.

To find OM, the third side of this right triangle, set up the Pythagorean formula:

$a^2 + b^2 = c^2$ where c is the hypotenuse of the right triangle

$a^2 + 4^2 = 5^2$

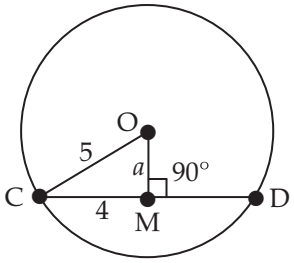
$a^2 + 16 = 25$

$a^2 = 25 - 16$

$a^2 = 9$

$a = \pm 3$

Because a is a length, it must be positive, so OM = 3.

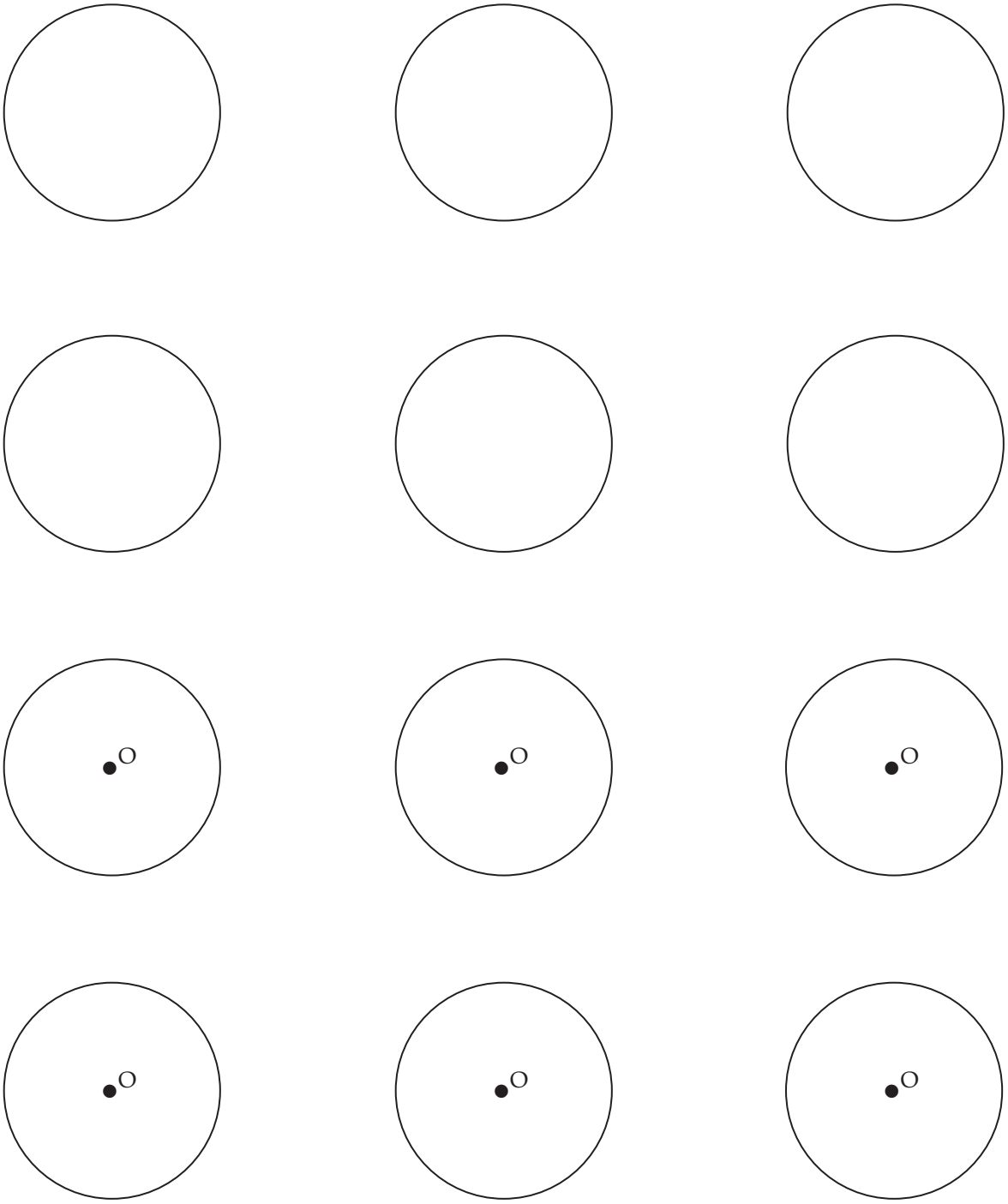


If you have a hard time remembering the Pythagorean theorem, you should include it along with this example on your Resource Sheet.

Lesson Summary

In this lesson you applied your knowledge of certain circle properties to solve problems, including how to find the centre of a circle using three different methods. This concludes our work with circle geometry.

You will find some circles on the following page for you to use to practice and study for the exam.


O
O
O
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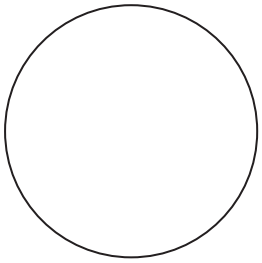
O
O

Assignment 6.7: Solving Problems

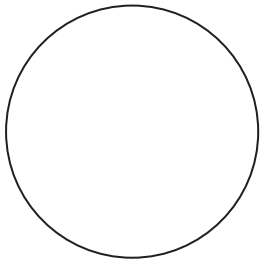
Total: 26 marks

Please show all your work in the space provided below. Use a separate sheet if necessary.

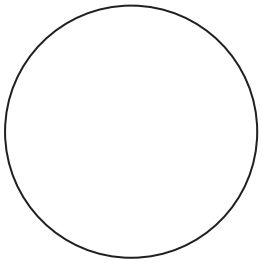
1. Demonstrate how to find the exact centre of a circle using inscribed right angles. State your strategies (what you did) and justify your solution (explain why it works) using circle properties. *(6 marks)*



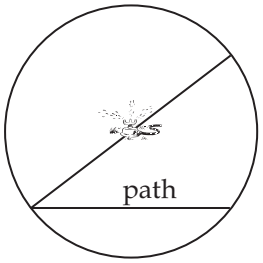
2. Demonstrate how to find the exact centre of a circle using tangents and radii. State your strategies (what you did) and justify your solution (explain why it works) using circle properties. *(6 marks)*



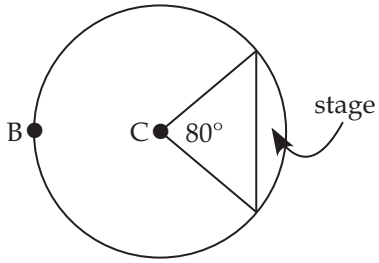
3. Demonstrate how to find the exact centre of a circle using perpendicular bisectors of a chord. State your strategies and justify your solution using circle properties. *(6 marks)*



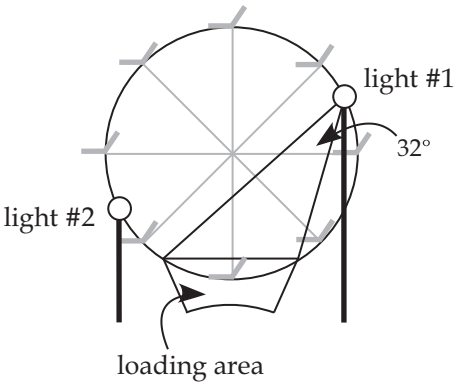
4. You have a circular flower garden with a fountain at the centre. The diameter of your garden is 10 m. There is a 6 m path (chord) through your garden. Use the Pythagorean theorem to find the perpendicular distance from the fountain to the path. Show your work. *(4 marks)*



5. You have a photographer's press pass to go to a concert at an outdoor amphitheatre. You can either sit at the centre of the ground level seating area, or along the back wall. To get a good picture of the stage from the centre, C, of the amphitheatre, you will need a 80° wide angle lens. To get the same picture from a seat along the back wall, at B, what angle lens will you need? Explain your answer. *(2 marks)*



6. A carnival company needs two spotlights next to the ferris wheel to illuminate the entire loading area. The first one lights up a 32° wide angle. Over what angle does the second light need to shine? Explain your answer. *(2 marks)*



NOTES

MODULE 6 SUMMARY

Congratulations, you have finished six of the eight modules in the course. You now can solve circle problems and justify your solutions using the following properties:

- A line from the centre of a circle that is perpendicular to a chord bisects the chord.
- A central angle will be twice the measure of an inscribed angle when they subtend the same arc, and if an inscribed angle has the same endpoints as a diameter, it will be a right angle.
- Inscribed angles that subtend the same arc are congruent.
- At the point of tangency, a tangent and radius are perpendicular.

The next module continues with geometry, but now the focus will be on 2-D and 3-D objects. You will look at surface area, similarity in polygons, scale diagrams, and symmetry.

Mailing Your Assignments

Before you mail your completed assignments, please photocopy them first. That way, you will have a copy of your assignments in case they go missing. Please organize your material in the following order:

- Cover Sheet for Module 5
- Assignment 5.1: Cover Assignment
- Assignment 5.2: Patterns and Translations
- Assignment 5.3: Writing Equations
- Assignment 5.4: Working with Graphs
- Assignment 5.5: Operations and Equations
- Assignment 5.6: Inequalities and Graphing
- Assignment 5.7: Inequalities and Problems
- Cover Sheet for Module 6
- Assignment 6.1 Cover Assignment
- Assignment 6.2 Angle Measurement and Terminology
- Assignment 6.3 Central and Inscribed Angles
- Assignment 6.4 More Inscribed Angles
- Assignment 6.5 Tangents
- Assignment 6.6 Perpendicular Bisectors
- Assignment 6.7 Solving Problems

Place all of these materials in an envelope and address it to:

ISO Tutor/Marker
555 Main St.
Winkler MB R6W 1C4

GRADE 9 MATHEMATICS (10F)

Module 6: Circle Geometry

Learning Activity Answer Keys

MODULE 6: CIRCLE GEOMETRY

Learning Activity 6.1: Measuring Angles

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What percent represents 3 months of a year?
2. What is the area of the top of a rectangular cake that measures 20 cm by 60 cm?
3. What is the price per m^3 of soil if 30 m^3 costs \$900?
4. How much money do you have if you have 10 quarters, 12 dimes, and 3 nickels?
5. Calculate the product of 48 and 0.25.
6. A cushion costs \$80. What is the sale price of the cushion if the price is reduced by 40%?
7. Two numbers have a sum of 15 and a product of 36. What are the two numbers?
8. Evaluate $10 - 3 \times 4$.

Answers:

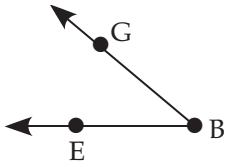
1. 25% (12 months in a year; $\frac{3}{12}$, which is equivalent to $\frac{1}{4}$)
2. 1200 cm^2 ($2 \times 6 = 12$; add two zeros)
3. \$30 ($9 \div 3$ and $100 \div 10$)
4. \$3.85 or 385¢ ($10 \times 25 + 12 \times 10 + 3 \times 5 = 250 + 120 + 15 = 385$¢)
5. 12 $\left(0.25 \text{ is equivalent to } \frac{1}{4}\right)$
6. \$48 (if the price is reduced by 40%, it is 60% of the full price and 60% of 80 is equivalent to 0.6×80)
7. 12 and 3
8. –2 (the multiplication is done first)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

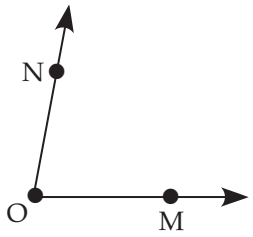
1. Measure the following angles using your protractor.

a)



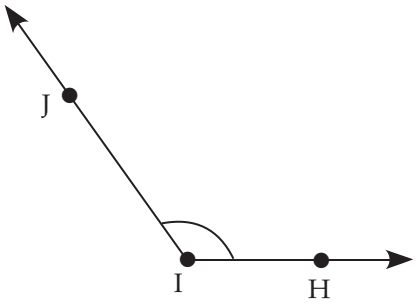
Answer:
40°

b)



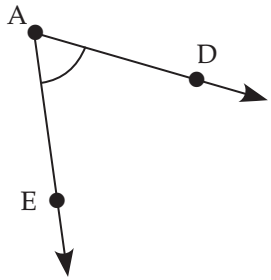
Answer:
80°

c)



Answer:
126°

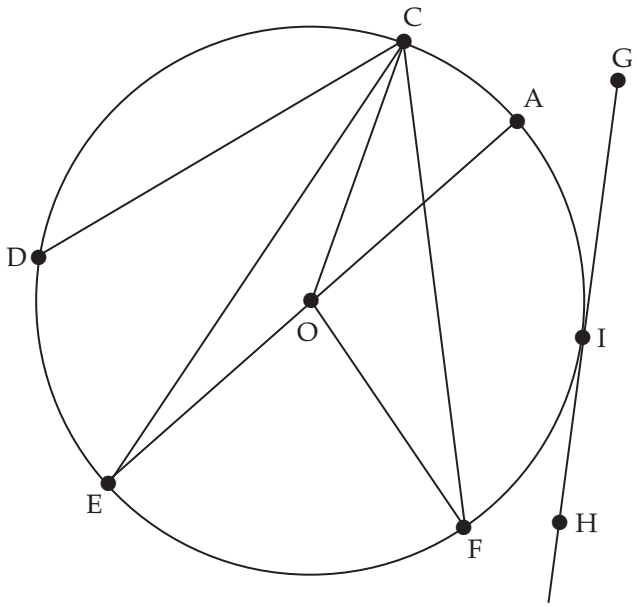
d)



Answer:
66°

Note: When measuring angles, full marks will be given for answers up to 3 degrees above or below the precise measurement. If you are measuring an angle of 50°, your answer could be between 47° and 53°. We call this "plus or minus" 3 (±3). This is to accommodate any slight variance with reprinting the diagrams, pencil lead width, or slight errors while using the protractor.

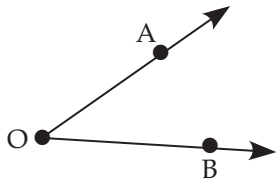
2. Given the following diagram, identify the requested components.



a) Identify four different chords. *Answer:* DC, EC, CF, EA
b) Identify the diameter. *Answer:* EA
c) Identify four radii. *Answer:* OE, OA, OF, OC
d) Identify five central angles (less than 180°).
Answer: EOF, EOC, FOC, COA, AOF
e) Identify four inscribed angles. *Answer:* DCF, ECF, CEA, DCE
f) Identify a tangent line. *Answer:* GH
g) Identify the point of tangency. *Answer:* I
h) Identify six arc segments. *Answer:* AI, IF, FE, ED, DC, CA

Note:

Angles can be labelled in two ways.
∠AOB is the same as ∠BOA



Learning Activity 6.2: Measuring Angles

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Write two equivalent fractions for $\frac{8}{12}$.
2. Express $\frac{6}{8}$ as a percent.
3. Convert 8.5 cm to mm.
4. Add $\frac{3}{5} + \frac{7}{10}$.
5. Multiply 101 x 101.
6. What is the remainder after the division: 465 x 2?
7. How much does 50 L of gas cost at 99¢ per litre?
8. If 7 boxes of candy cost $3.50, how much does 4 boxes of candy cost?

Answers:

1 Answers will vary; some examples are: $\frac{2}{3}, \frac{4}{6}, \frac{16}{24}, \frac{80}{120}, \ldots$

To have equivalent fractions, the numerators and denominators must be multiplied by the same number.

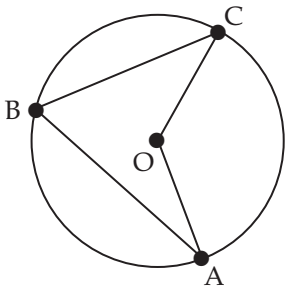
2. 75% $\left(\frac{6}{8} \text{ is equivalent to } \frac{3}{4}\right)$
3. 85 mm (10 mm in 1 cm)
4. $\frac{13}{10}\left(\frac{6}{10} + \frac{7}{10}\right)$
5. 10 201 (100 x 101 = 10 100; add another 101)
6. 1 (the units digits is 5 and is divided by 2)
7. $49.50 (50 x $1 – 50 x 1¢; so $50 – $0.50)
8. $2.00 ($3.50 ÷ 7 = $0.50 per box of candy)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

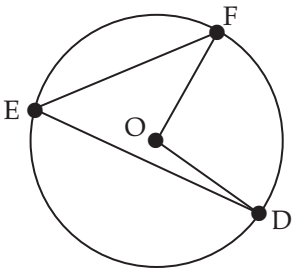
1. Use your protractor to find the measurement of both the central and inscribed angles.

a)



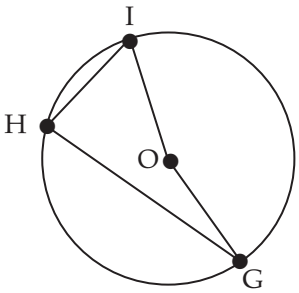
Answers:
∠ABC = 65° ±3°
∠AOC = 130° ±3°
∠ABC and ∠AOC both subtend $\overset{\frown}{AC}$

b)



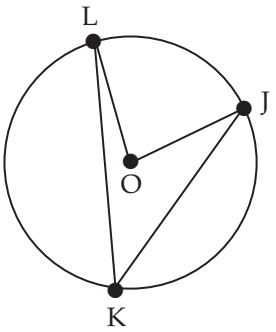
Answers:
∠DOF = 96° ±3°
∠DEF = 48° ±3°
∠DOF and ∠DEF both subtend $\overset{\frown}{DF}$

c)



∠GHI = 81° ±3°
∠GOI = 162° ±3°
∠GHI and ∠GOI both subtend $\overset{\frown}{GI}$

d)



∠JOL = 80° ±3°
∠JKL = 40° ±3°
∠JOL and ∠JKL both subtend arc $\overset{\frown}{JL}$

2. An inscribed angle sharing the same endpoints as a 210° central angle would be _____°. *Answer:* 105°

3. An inscribed angle that subtends a diameter would be _____°. *Answer:* 90°

Learning Activity 6.3: Measuring Inscribed Angles

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. In addition to the $430 price of your plane ticket, you have to pay $34 for port fees and $60 in other charges. Can you buy this ticket if you have $514 in your chequing account?
2. The price of a $60 video game is increased by 15%. What is the new price of the video game?
3. You buy a soccer ball for $45, which was a savings of $15. What percentage does this represent?
4. What two numbers have a sum of 12 and a difference of 2?
5. What two numbers have a sum of 12 and a product of 35?
6. What two numbers have a sum of 12 and a product of 36?
7. What number is 86 more than 33?
8. What number is 33 less than 86?

Answers:

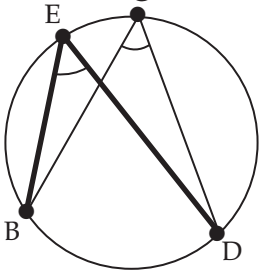
1. No (430 + 34 + 60 = 524; you are missing $10)
2. $69 (15% of $60 is equivalent to 0.15 x 60, or $9 that you add to the $60)
3. 25% (the ball was valued at $45 + $15 = $60 and $15 represents $\frac{1}{4}$ of $60)
4. 7 and 5
5. 7 and 5
6. 6 and 6
7. 119 (33 + 86)
8. 53 (86 – 33)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use your protractor to find the measurements of each inscribed angle.

a)

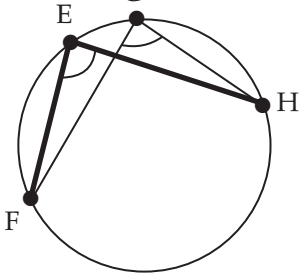


Answers:

∠BED = 50°

∠BCD = 50°

b)

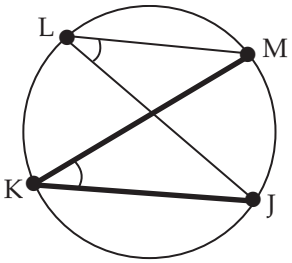


Answers:

∠FEH = 86°

∠FGH = 86°

c)

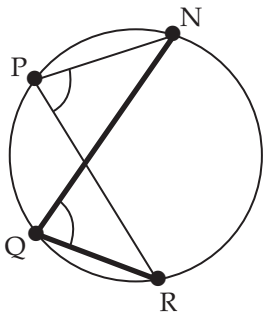


Answers:

∠JKM = 35°

∠JLM = 35°

d)



Answers:

∠RQN = 76°

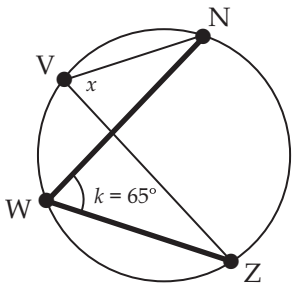
∠RPN = 76°

2. Inscribed angles that subtend the same arc are _____ .

Answer: Congruent

3. State the measure of the angle with the x without using a protractor.

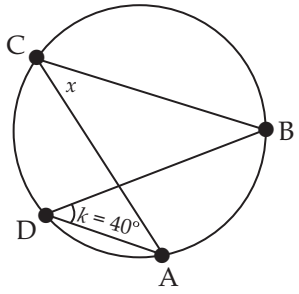
a)



Answer:

65°

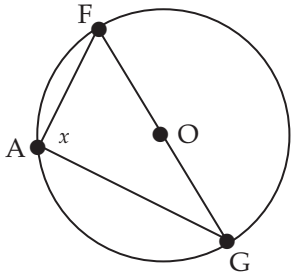
b)



Answer:

40°

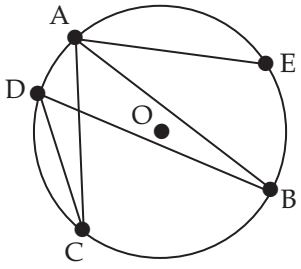
c)



Answer:

90°

4. Indentify the congruent angles in the diagram below and justify your answer.



Answer:

$\angle CDB \cong \angle CAB$ because they are both inscribed angles that subtend $\overset{\frown}{CB}$, or they are both inscribed angles with the same endpoints, C and B.

or

$\angle DCA \cong \angle DBA$ because they are both inscribed angles that subtend $\overset{\frown}{AD}$, or they are both inscribed angles with the same endpoints, A and D.

Learning Activity 6.4: Finding Tangents

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You buy three chocolate bars that cost $0.80 each. You give the cashier $2.50. How much money do you get back?
2. Estimate the square root of 914.
3. How many 5 kg bags of potatoes can you make with 110 kg of potatoes?
4. What is the median of the following: 64, 70, 70, 73, 82, 85, 91?
5. What is the mode of the following: 64, 70, 70, 73, 82, 85, 91?
6. If 20% of a group of 120 students play badminton, how many students don't play badminton?
7. Napoléon was born in 1769 and died in 1821. How old was he when he died?
8. What value of m satisfies the equation: $\frac{2}{5} \cdot m = 1$?

Answers:

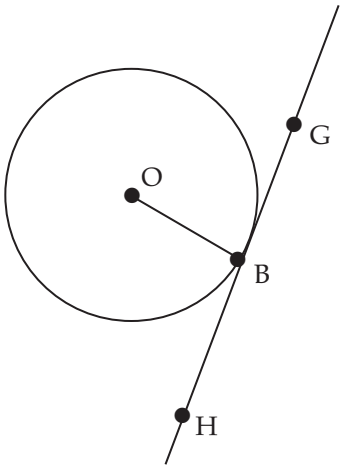
1. 10¢ (2.50 – 3 x 0.80)
2. 30 (30 x 30 = 900)
3. 22 bags (20 bags take 100 kg; two more bags take the remaining 10 kg)
4. 73 (the median is the middle value when all the values are in placed in order)
5. 70 (the mode is the value that appears the most)
6. 96 (120 x 0.20 → 12 x 2 = 24 students play badminton;
   120 – 20 = 100, 100 – 4 = 96)
7. 52 years (31 years from his birth to 1800 + 21 years after 1800)
8. $\frac{5}{2}$ (multiply by 5 and divide by 2 in order to isolate the variable m)

Part B

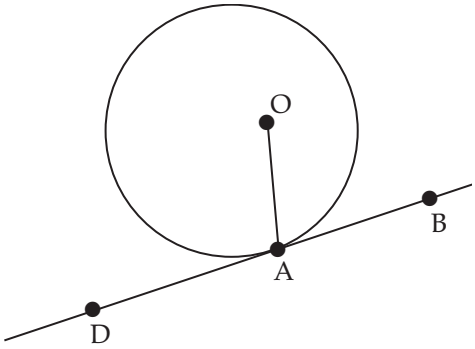
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Only one of the three circles shown has a tangent being perpendicular to the radius. Use your protractor to identify the correct drawing.

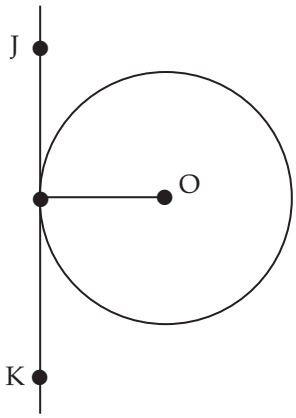
a)



b)



c)



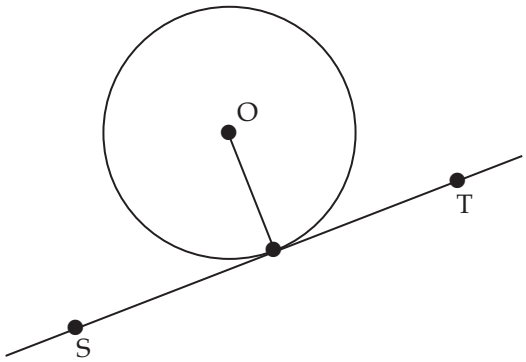
Answer:

(c)

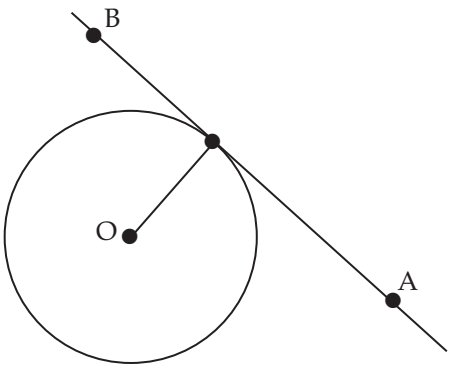
2. Use your protractor to construct a radius meeting the tangent at a right angle (90°).

Answers:

a)



b)



3. Tangents always meet the radius at a _____ angle at the point of tangency.

Answer:

right or 90°

4. Perpendicular lines from the point of tangency always go through the_____ of a circle.

Answer:

centre

Learning Activity 6.5: Identifying Midipoints

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Order the following numbers in descending order (from greatest to least):

   $\frac{3}{5}$ 0.53 $\frac{5}{3}$ 0.35

2. Your friend lends you $200. How many weeks will it take you to pay the debt back if you give him $14 each week?
3. How many hours are there from Monday at 13:00 to Tuesday at 19:00?
4. What is the probability of an even number if you toss a regular 6-sided die?
5. A lottery prize of $4600 is to be divided equally among the 23 winners. What is the amount of each prize?
6. Calculate $31 \div \frac{1}{4}$.
7. The bank gives you 0.5% interest every 3 months. If you have $100 in your savings account, estimate the amount of interest at the end of the year.
8. Estimate the height in centimetres of a classroom if the height measures 98 inches and each inch is equivalent to approximately 2.5 cm.

Answers:

1. $\frac{5}{3}$ $\frac{3}{5}$ 0.53 0.35 (Compare the numbers to the values 1, 0.5, and 0. $\frac{5}{3}$ is the only value greater than 1; $\frac{3}{5}$ is the same value as $\frac{6}{10}$ or 0.6, which is larger than 0.53 and 0.35; 0.53 is larger than 0.35)
2. 15 weeks (in 10 weeks, you have given him $140; 4 more weeks and you have given him another $56; one more week to give him the $4 that remains)
3. 30 hours (24 hours until Tuesday at 13:00; then add another 6 hours)
4. 0.5 or $\frac{1}{2}$ (3 even numbers on 6 faces)
5. $200 (46 ÷ 23 = 2; add two zeros)
6. 124 (there are 4 quarters in a unit, so 4 x 31)
7. $2 (there are 4 periods of 3 months in a year; 4 times 0.5% gives 2 %; 0.02 x 100)
8. 245 cm (100 inches is equivalent to 250 cm; 2 less inches means 5 cm less)

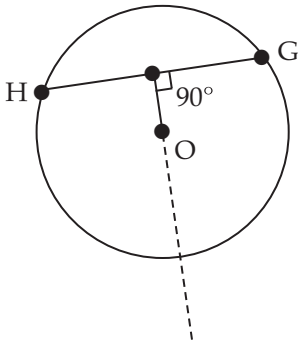
Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use your ruler to find the midpoint of each chord, and then draw a line joining the midpoint to the centre. Measure the angle at which this line meets the chord.

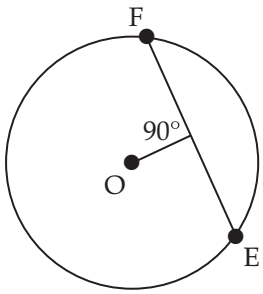
   Answers:

   a)

   The angle measurement is 90°.

   b)

   The angle measurement is 90°.

2. A line coming from the centre that bisects the chord always meets the chord at a _____ angle.

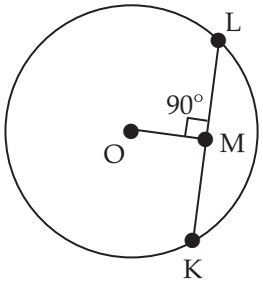
   Answer:

   right angle

3. Use your protractor to create a 90° angle at the point where a line from the centre meets the chord. Label this point M. Then use your ruler to compare the distance from each endpoint to the midpoint.

Answers:

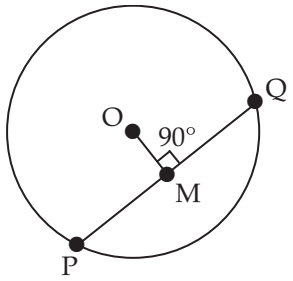
a)



KM = 1.5 cn

LM = 1.5 cm

b)



PM = 1.5 cm

QM = 1.5 cm

4. A line from the centre meeting the chord at a right angle always _____ the chord.

Answer:

bisects

5. Perpendicular bisectors always pass through the _____ of the circle.

Answer:

centre

GRADE 9 MATHEMATICS (10F)

Module 7: 2-D and 3-D Objects

MODULE 7: 2-D AND 3-D OBJECTS

Introduction

The specific outcomes of Modules 6, 7, and 8 focus on geometry. You have explored circle geometry and now in Module 7, you will concentrate on two- and three-dimensional objects of various shapes. You will determine the surface area of composite objects, work with similar polygons, and draw scale diagrams. Problem solving will help you apply your skills in these three major areas.

You will need graph paper, a ruler, and a protractor to complete the work in this module.

Assignments in Module 7

You will need to complete the following five assignments and send them to your tutor/marker when you are done this module.

Lesson	Assignment Number	Assignment Title
1	Assignment 7.1	Cover Assignment
3	Assignment 7.2	Composite Objects
4	Assignment 7.3	Similarity
5	Assignment 7.4	Drawing Polygons
6	Assignment 7.5	Scale Drawings

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 7. You might use your Module 7 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 7 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 7 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

Lesson 1: Cover Assignment

Look back at the cover assignment for Module 3 for a definition of **inductive reasoning**.

When using inductive reasoning to present an argument, it is necessary to examine a number of cases in an effort to see a pattern or a rule. The rule is then used to determine what might happen in other cases. It is important to understand that the rule is not true for sure, since it was arrived at by looking at some cases and not **all** possible cases.

Example 1

Find the last digit of 2^{1000}.

Solution

To solve, write out the first few values and look for a pattern:

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

Now look at only the last digit of each number:

2, 4, 8, 6, 2, 4, 8, ... This pattern repeats every four digits.

The last digit of 2^{1000} would be the last digit in the pattern, a 6, since in 1000 there are 250 complete groups of the four digits that repeat (1000 ÷ 4 digits in pattern = 250 with no remainder). This is not true for sure—you have only looked at some cases. In order to be sure of the value of the last digit of 2^{1000}, you would need to write out all the cases. However, in using inductive reasoning, it is not necessary to check all the possible cases. Inductive reasoning requires that you check just enough possible cases to see a pattern.

exponent	1	2	3	4	5	6	7	8	...	1000
last digit	2	4	8	6	2	4	8	6	...	

Example 2

Provide examples to show that the sum of two odd integers is always an even integer.

Solution:

Test a number of cases and look for a pattern:

$3 + 5 = 8$ $\qquad$ $11 + (-3) = 8$

$-25 - 13 = -38$ $\qquad$ $1 + 21 = 22$

In each of these cases, the sum of two odd integers is an even integer.

These are just four examples and do not prove that the sum of two odd integers is always an even integer.

When proving something using examples, you should provide at least four examples. You should try to look for important differences in the examples. Looking at the test cases above, notice that different combinations of positive and negative odd integers were used.

Assignment 7.1: Cover Assignment

Total: 16 marks

1. Find the last digit of the following. Show your work and explain your reasoning.
   a) 6^{500} *(2 marks)*

   b) 15^{1000} *(2 marks)*

2. Provide examples to show the following:
   a) The product of two positive integers is positive. *(2 marks)*

   b) The sum of two even numbers is an even number. *(2 marks)*

   c) The product of two consecutive numbers is always an even number. *(2 marks)*

3. Use inductive reasoning to tell whether the following statement is **always true**, **sometimes true**, or **never true**. Assume that n is a negative integer and p is a positive integer.

   a) $n > p$ *(2 marks)*

   b) $0 > p$ *(2 marks)*

4. Show that the GCF (greatest common factor) of an odd and an even number is always an odd number. *(2 marks)*

LESSON 2: SKILLS REVIEW

Lesson Focus

When you finish this lesson, you will be able to

- ❑ determine the area of 2-D shapes
- ❑ determine the surface area of basic 3-D shapes

Lesson Introduction

In Grade 7 Math, you practiced finding the area of 2-D (two dimensional) shapes like triangles and circles. In Grade 8 Math, you found the surface area of 3-D shapes like prisms and cylinders. This lesson will review those skills.

Calculating the Area of Shapes

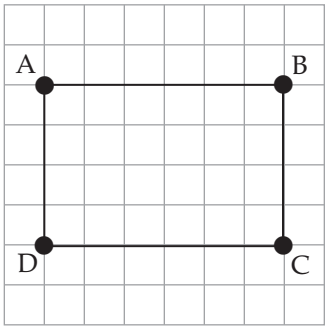
You can use a grid to determine the approximate area of a shape. In the diagrams below, assume that each square represents 1 cm^2.

Although this is a review, it will be helpful for you to know the formulas for area as you continue in this module. There will be a Resource Sheet symbol to point out where you can find a formula. When you record them on your Resource Sheet, it is good practice to label the formulas so that you know which is which.

Rectangles

Example 1

Find the area of this rectangle by counting the squares inside it.



Solution:

You should count 24 squares inside the rectangle. If each square represents 1 cm^2, then the area would be 24 cm^2.

Notice that the length of this rectangle is 6 cm, and the width is 4 cm and that 6 x 4 = 24

The area of a rectangle can be found by multiplying the length times the width. This can be written as a formula.

A = L x W

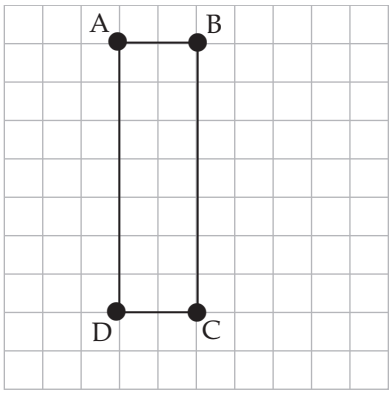
A = 6 x 4

A = 24

The area of this rectangle is 24 cm^2.

Example 2

Find the area of the rectangle by counting the squares inside it.



Solution:

This rectangle has 14 squares inside, so its area would be 14 cm^2.

The length is 7 cm and the width is 2 cm.

If you apply the formula,

L x W = A

7 x 2 = 14

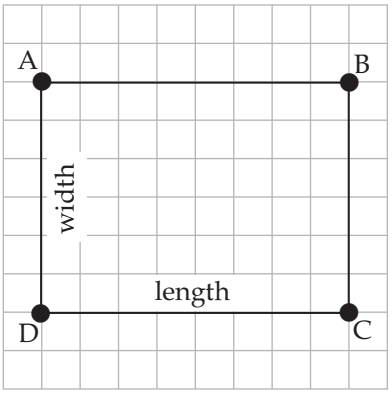
The area of this rectangle is 14 cm^2.

Note the area is always in square units.

Triangles

Right triangles are really rectangles divided in half along a diagonal.

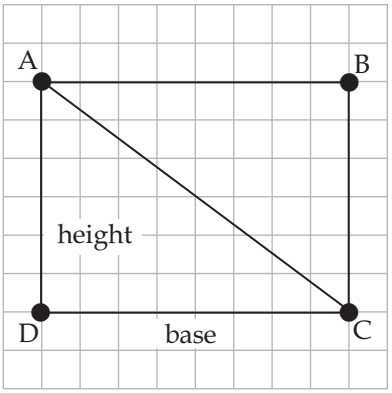
Look at this rectangle.



It measures 8 cm by 6 cm, and so has an area of 48 cm^2.

Now simply divide the rectangle into two equal parts along a diagonal, creating two equal right triangles.

If you carefully count the squares inside each triangle (include the partial squares too), you will find 24 squares in each. This is exactly half the area of the rectangle.



This can be written as a formula as well. The area of a rectangle can be found by multiplying (L)(W). The area of a triangle is half of that, or $\frac{(\mathrm{L})(\mathrm{W})}{2}$. If you rename the length of the rectangle the "base" and call the width the "height," then this formula can be written as

$$A = \frac{(BH)}{2} \text{ or } A = \frac{1}{2} B * H.$$

$$A = \frac{B * H}{2}$$

$$A = \frac{8 \times 6}{2}$$

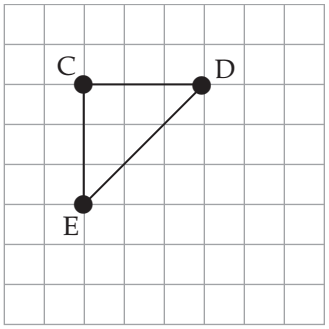
$$A = 24$$

The area of each triangle is 24 cm^2.

The one important thing to remember when finding the area of a triangle is that the base must always meet the height at a right angle. Before you can substitute values into the formula, you must identify which dimensions represent the base and height. They will always meet each other at a right angle.

Example 1

Use the formula to calculate the area of ΔCDE.



Solution:

In this triangle, the sides that meet at a right angle are CD and CE. The base could be CD, and the height CE. Each square represents 1 cm^2.

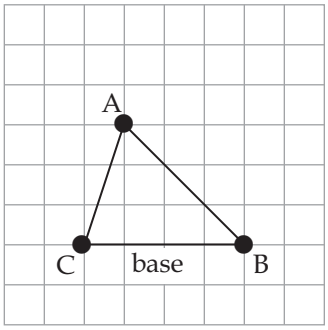
$$A = \frac{b * h}{2}$$

$$A = \frac{3 * 3}{2}$$

$$A = 4.5$$

The area of triangle CDE is 4.5 cm^2.

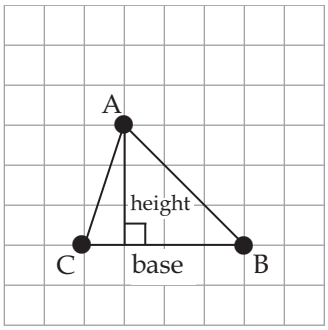
What if the triangle is not a right triangle?



The base and height must be perpendicular. You can create a line segment from a vertex that meets the base at a right angle. In this case, use vertex A and draw a line from A to side BC that is perpendicular to the base.

Example 2

Calculate the area of the triangle.



Solution:

If each square on the grid represents 1 cm^2, the base of this triangle is 4 cm and the height is 3 cm. To calculate the area of this triangle, use the formula.

$$A = \frac{b * h}{2}$$

$$A = \frac{4 * 3}{2}$$

$$A = \frac{12}{2}$$

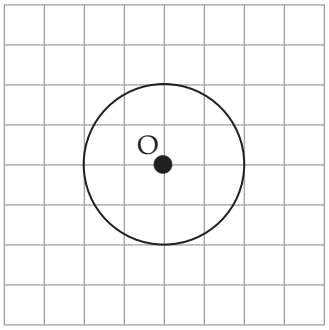
$$A = 6$$

The area of this triangle is 6 cm^2.

Circles

Count the squares inside the circle to find the area of the circle.

Assume the grid represents 1 cm^2.



It looks like about 13 squares, so the area would be about 13 cm^2.

Notice that the radius is 2 cm.

The formula for finding the area of a circle is:

Area = πr^2

The symbol π represents the mathematical constant of pi, an irrational number.

Shown to 50 decimal places, π = 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 ...

In this course, you can use 3.14 to represent the approximate value of π (pi),

$A = \pi r^2$

$= (3.14)(2^2)$

$= (3.14)(4)$

$= 12.56$

The area of this circle is approximately 12.56 cm^2.

Learning Activity 7.1

The Area of Shapes

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. How much tax (12%) do you pay on a book that costs $5.00?
2. Add –12.4 + 3.1.
3. Sophia paid $63 for a pair of jeans. Sasha paid $25 less than Sophia for an identical pair. How much did Sasha pay?
4. You got $\frac{24}{30}$ on your last mathematics test. What was your percentage grade?
5. It rained 16 mm during one April week in Vancouver. The week before, it rained 5 mm, and the week after, it rained 9 mm. What was the average weekly rainfall in Vancouver during those three weeks?
6. What is the measure of the third angle of a triangle if two of the angles measure 60° and 40°?
7. If 18% of the population is 54 people, how many are in 3% of the population?
8. Philippe completed $\frac{5}{8}$ of his homework while Nicole completed $\frac{2}{3}$ of hers. Who is closer to finishing their homework?

continued

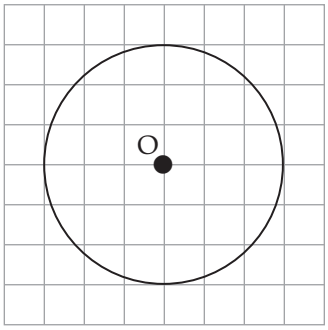
Learning Activity 7.1: The Area of Shapes (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the "counting squares" method to find area of each shape.

a)



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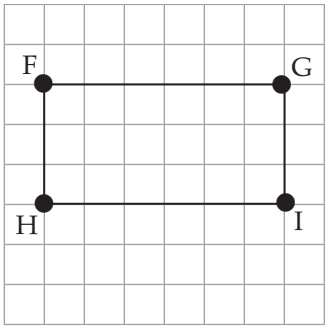

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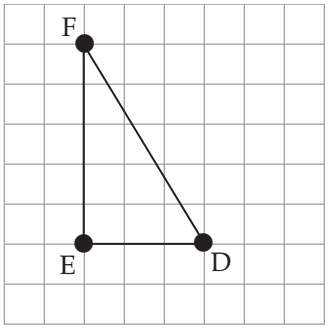


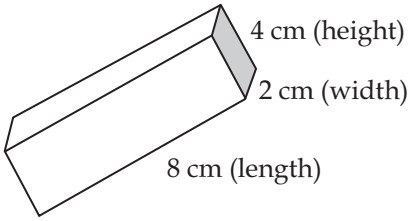
2. Now use the appropriate formula to find the area for each shape above. Let each square represent 1 cm^2.

Surface Area of 3-D Shapes

This lesson will review how to calculate the surface area of three types of 3-D shapes: rectangular prisms, triangular prisms, and cylinders.

Rectangular Prisms

Rectangular prisms have three dimensions: length, width, and height. When calculating the surface area of three dimensional (3-D) shapes, you need to carefully identify the measurements of each side, even if it cannot be seen in the diagram!



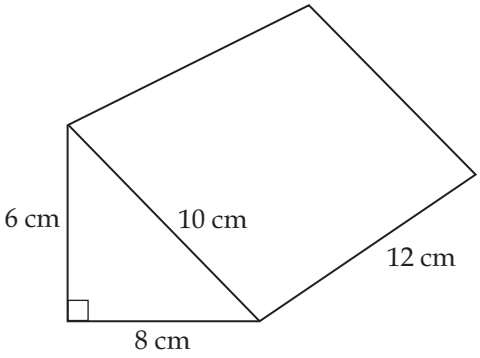
There are six sides to this prism: two end pieces, two long sides, and a top and bottom. The goal is to find the surface area of each side, and then find the total surface area for the prism.

End pieces: each is 2 cm by 4 cm	$(2 * 4) * 2 = 16$
Sides: each is 8 cm by 4 cm	$(8 * 4) * 2 = 64$
Top and bottom: each is 8 cm by 2 cm	$(8 * 2) * 2 = 32$
Total	112

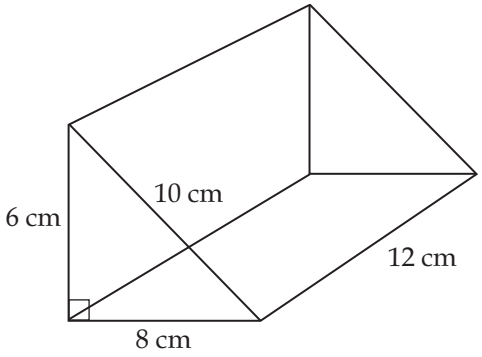
The surface area of this rectangular prism is 112 cm^2.

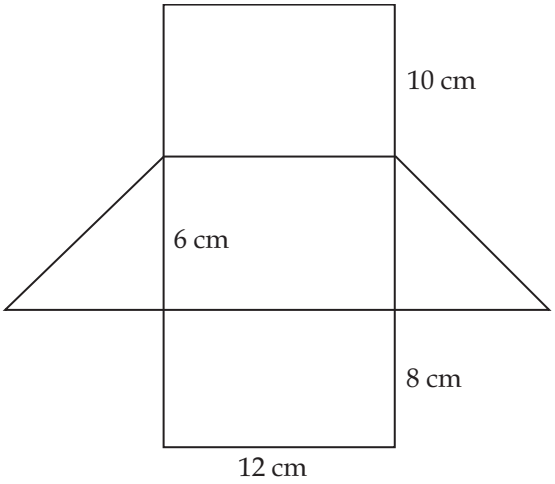
Triangular Prisms

Triangular prisms have triangular shaped ends. To calculate the total surface area of a triangular prism, you will need to carefully identify the shape and measurements of each side, count the number of sides, use the correct formulas, and find the sum of the areas of each side.



To help you visualize the sides, imagine the prism is see-through or draw a "net."





There are two triangular end pieces and three rectangles. You must find the area of each piece, and add all the surface areas together to find the total.

2 Triangular end pieces:

$$A = \frac{b * h}{2}$$

$$= \frac{6 * 8}{2} * 2$$

$$= 48$$

Front piece

$$A = L * W$$
$$= 10 * 12$$
$$= 120$$

Back piece

$$A = L * W$$
$$= 6 * 12$$
$$= 72$$

Bottom piece

$$A = L * W$$
$$= 8 * 12$$
$$= 96$$

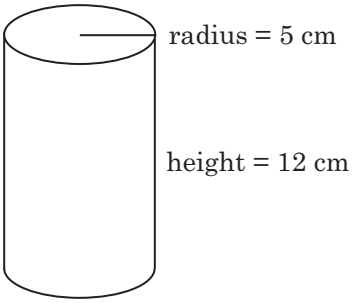
Total surface area

$$=48 + 120 + 72 + 96$$
$$= 336$$

The total surface area of this triangular prism is 336 cm^2.

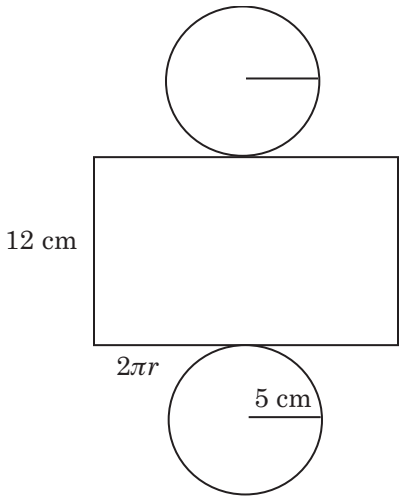
Cylinders

Take a sheet of paper and roll it into a tube shape. This cylinder has circles at each end (or base), and the sides are formed by a rectangle. The circumference of the circles at each end will depend on the width of the rectangular piece of paper.



In this case, the height of the cylinder is given, but not the width of the rectangle. It must be the same as the circumference of the circle, with a radius of 5 cm. The formula for the circumference of a circle is $C = 2\pi r$.

Imagine "unrolling" the cylinder to see each part.



To calculate the area of a rectangle, multiply the length (or height) by the width. Because the width is the same as the circumference of the circle, the formula for the surface area of this rectangular part would be $A = 2\pi r * h$.

Since this cylinder includes both a circular top and bottom, you must find the area of each circle using $A = \pi r^2$ and add that to the formula.

Surface area of a cylinder $= 2\pi r * h + 2(\pi r^2)$

$= 2(3.14)(5) * (12) + 2(3.14)(5^2)$

$= 376.8 + 157$

$= 533.8$

The total surface area of this cylinder is 533.8 cm^2.

Learning Activity 7.2

The Surface Area of 3-D Shapes

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Eight identical notebooks cost $40. What is the unit price per notebook?
2. Express $2\frac{3}{4}$ as a decimal.
3. Your chequing account balance is $62. You deposited $43 but you have to pay a bill of $35. What is the new balance on your chequing account?
4. Evaluate 50^2.
5. Marcel was born in April 1922. What age was he when he became a great-grandparent in April 2006?
6. A film lasts 105 minutes. It is now 2:00 pm and you need to leave at 3:30 pm. Do you have enough time to watch the film before you need to leave?
7. A car is travelling at 100 km/h. It has 10 km to go. How long will it take for the car to travel the remaining 10 km?
8. A CD contains 4 songs that last 3 minutes and 15 songs that last 4 minutes. What is the total length of the CD?

continued

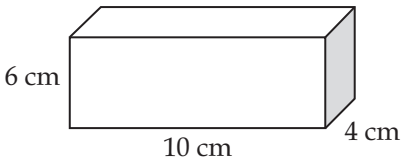
Learning Activity 7.2: The Surface Area of 3-D Shapes (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Find the surface area of each shape.

a)



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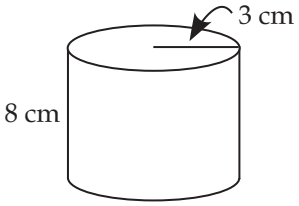

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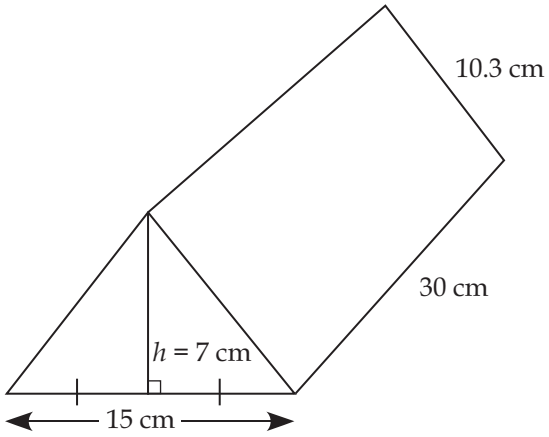


Lesson Summary

This concludes the review of skills you learned in previous grades. There will be no assignment for this lesson. In the next lesson, you will explore finding the surface area of composite objects, made up of two or more 3-D shapes.

NOTES

LESSON 3: COMPOSITE OBJECTS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ find the surface area of composite objects
- ❑ determine the area of any overlap
- ❑ solve problems using composite objects

Lesson Introduction

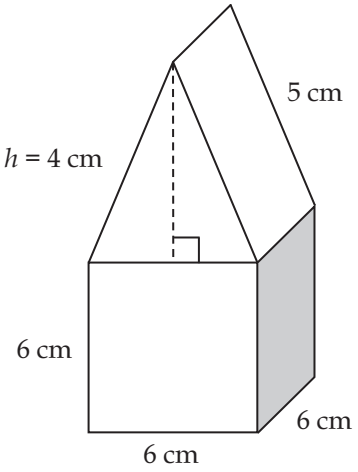
When two or more 3-D objects are combined, it is called a composite object. In this lesson, you will find the surface area of each part and add them together to find the total surface area of the composite object. Special consideration will be given to objects that may have "overlap" between the parts.

Surface Area of Composite Objects

If two shapes are arranged so that one is on top of the other, and you are to determine the total surface area, careful consideration has to be given to see exactly which surfaces overlap.

Example 1

Calculate the surface area of the object below.



Solution:

In this example, a triangular prism sits on top of a cube.

Looking at this composite object, you can see that the bottom of the triangular prism covers the top of the cube. Your calculation of the total surface area would not include these two parts, since they are not part of the outside surface.

Cube

You must include the area of the 4 square sides and the bottom.

$$A = L * W$$

$$A = (6 * 6) * 5$$

$$A = (36) * 5$$

$$A = 180$$

Triangular prism

You must include the two triangular ends and the two rectangular sides.

Triangular ends

$$A = \frac{(b * h)}{2} * 2$$

$$A = \frac{(6 * 4)}{2} * 2$$

$$A = 24$$

Rectangular sides

$A = L * W$

$A = (6 * 5) * 2$

$A = (30) * 2$

$A = 60$

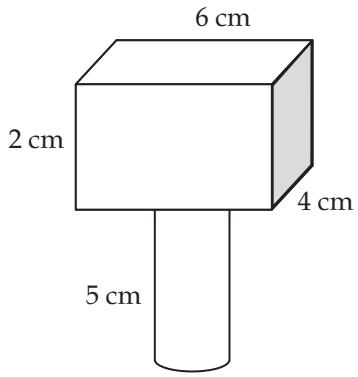
Total surface area

$180 + 24 + 60 = 264$

The total surface area of this composite object is 264 cm^2.

Example 2

Calculate the surface area of the object below.



Solution:

This composite object shows a rectangular prism on top of a cylinder. The area of overlap in this case is the top of the cylinder, and only that part of the bottom of the rectangular prism that sits atop the cylinder. Calculate the entire surface area of the rectangular prism, and then subtract the overlap.

Diameter of cylinder is 3 cm.

Rectangular prism

Front and Back

$A = L * W$

$A = (6 * 2) * 2$

$A = 24$

Top and Bottom

$A = L * W$

$A = (6 * 4) * 2$

$A = 48$

Two Ends

$A = L * W$

$A = (2 * 4) * 2$

$A = 16$

Surface area of the rectangular prism = 24 + 48 +16 = 88 cm^2.

Cylindrical prism

If the diameter is 3 cm, then the radius is half of that or $\frac{3}{2}$ cm.

Only include the bottom circle in the formula as the top of the cylinder is overlap.

$$A = 2\pi r * h + 1\left(\pi r^2\right)$$

$$A = 2(3.14)\left(\frac{3}{2}\right) * 5 + 1(3.14)\left(\frac{3}{2}\right)^2$$

$$A = 47.1 + 7.065$$

$$A = 54.165$$

Surface area of the cylinder is 54.165 cm^2.

Overlap

The top of the cylinder covers a portion of the bottom of the rectangular prism, and this area must be subtracted from the area of the rectangular prism. Calculate the area of a circle with a radius of $\frac{3}{2}$ cm.

$$A = \left(\pi r^2\right)$$

$$A = 3.14 * \left(\frac{3}{2}\right)^2$$

$$A = 7.065$$

Area of overlap is 7.065 cm^2.

Total surface area = sum of surface area of each object – area of overlap

Total surface area = 88 + 54.165 – 7.065

Total surface area = 142.165 – 7.065

Total surface area = 135.1

The total surface area of this composite object is 135.1 cm^2.

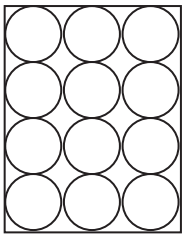
Example 3

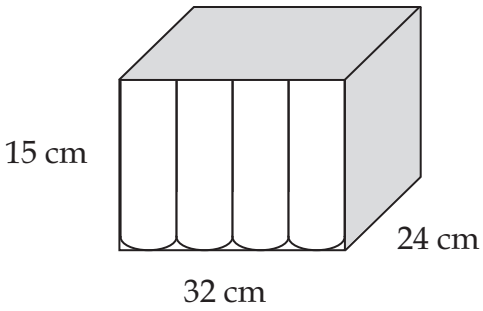
Your task is to determine how to pack 12 cans in a box so that you use the least amount of cardboard possible. The cans are 15 cm tall, with a diameter of 8 cm.

Solution:

Option A: The cans could be arranged in a 3 x 4 pattern, on one level.

Since the diameter of each can is 8 cm, the length and width of the box would have to be (3 * 8) cm by (4 * 8) cm, or 24 cm by 32 cm. The height of the box would be 15 cm.





To determine the amount of cardboard required, calculate the surface area of the box.

Each side is rectangular, so use $A = L * W$

Front and back: 32 x 15 x 2 = 960

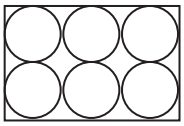
Ends: 24 x 15 x 2 = 720

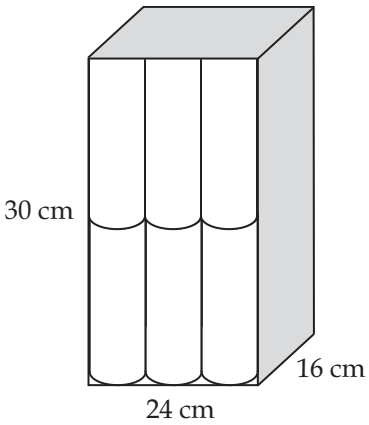
Top and bottom: 32 x 24 x 2 = 1536

Total = 960 + 720 + 1536 = 3216

This arrangement of cans would require 3216 cm^2 of cardboard.

Option B: The cans could be stacked on two levels, with 6 cans on each level





With a diameter of 8 cm, the base would measure 16 cm by 24 cm, and the height would be 30 cm.

Front and back: 24 x 30 x 2 = 1440

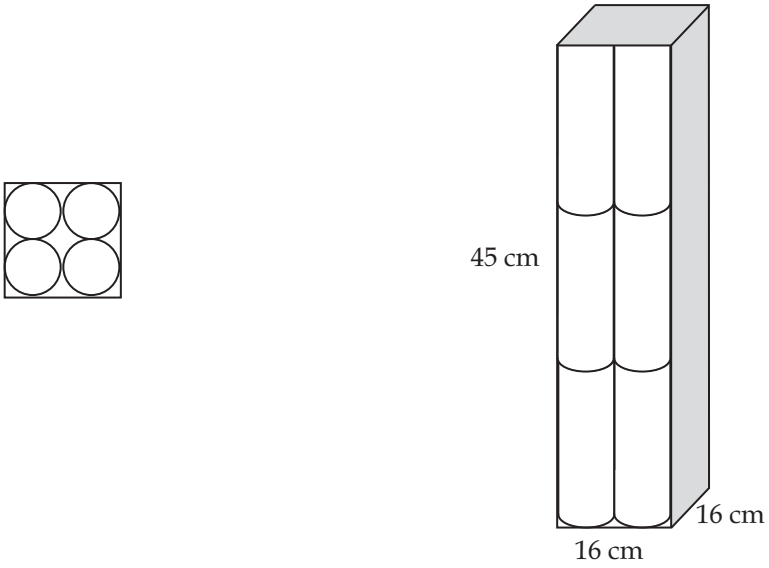
Ends: 16 x 60 x 2 = 960

Top and bottom: 24 x 16 x 2 = 768

Total = 1440 + 960 + 768= 3168

This arrangement of cans would require 3168 cm^2 of cardboard.

Option C: The cans could be stacked on three levels.



With a diameter of 8 cm per can, the base would measure 16 cm by 16 cm.

Front and back: $16 \times 45 \times 2 = 1440$

Ends: $16 \times 45 \times 2 = 1440$

Top and bottom: $16 \times 16 \times 2 = 512$

Total = 1440 + 1440 + 512 = 2792

This arrangement of cans would require 2792 cm^2 of cardboard.

Conclusion: Option C uses the least amount of cardboard to package the 12 cans.

Learning Activity 7.3

Surface Area of Composite Objects

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Solve for D: $20 = 4D + 8$
2. Which is smaller: 25% of 400, or 20% of 500?
3. You buy something for $5.82 with a $20 bill. How much change do you get?
4. What two numbers have a sum of –7 and a product of 12?
5. What two numbers have a sum of 12 and a product of –28?
6. Your telephone bill shows you spoke for 73 minutes. If each minute costs 5¢, what is the total bill in dollars?
7. How many grams in 54.5 kg?
8. How many 250 g bags of flour can be made with a single 6.25 kg bag of flour?

continued

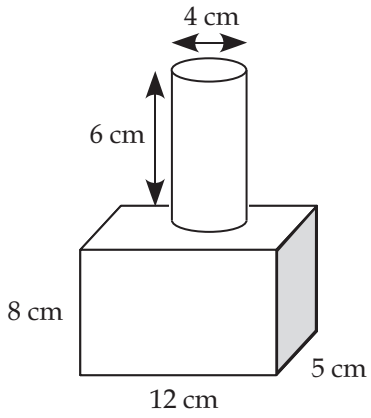
Learning Activity 7.3: Surface Area of Composite Objects (continued)

Part B

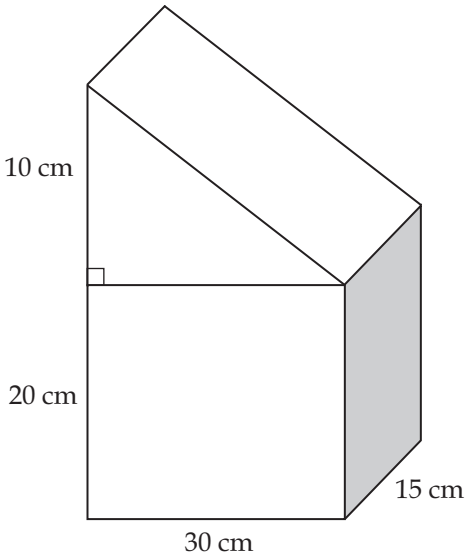
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Determine the surface area for each composite 3-D object. Assume each shape is solid and subtract any overlap.

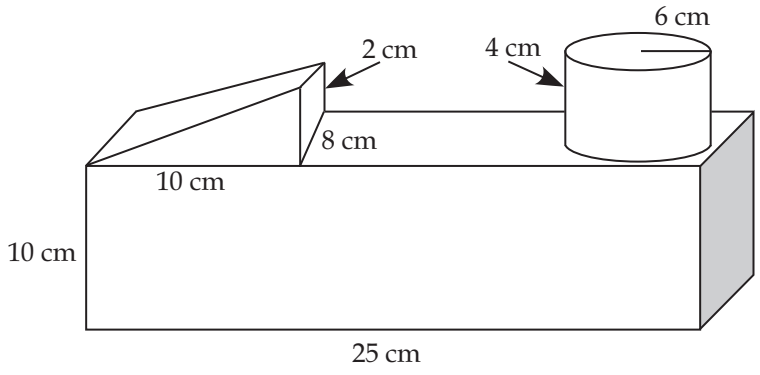
a)



b)



c)



Lesson Summary

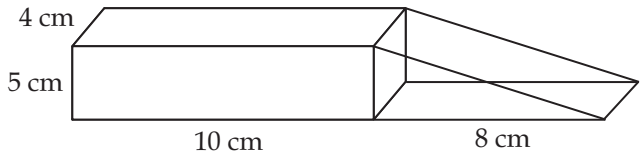
In this lesson, you learned how to find the surface area of composite 3-D objects. Consideration was given to any areas of overlap, and their effect on the calculations. In the next lesson, you will explore similarity in polygons, and solve problems using similarity.

Assignment 7.2: Composite Objects

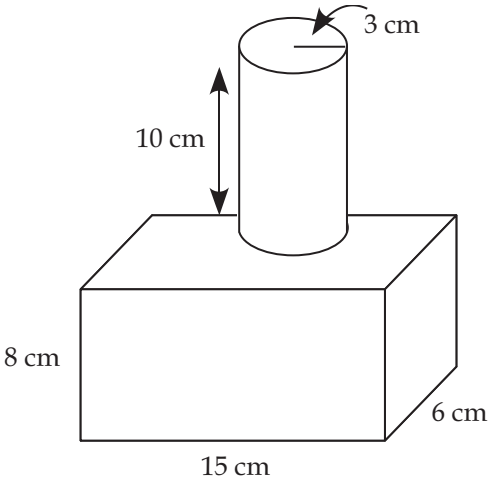
Total: 24 marks

1. Find the surface area of the following composite objects. State the area of overlap, and explain how it affects the determination of the total surface area. Show all your work.

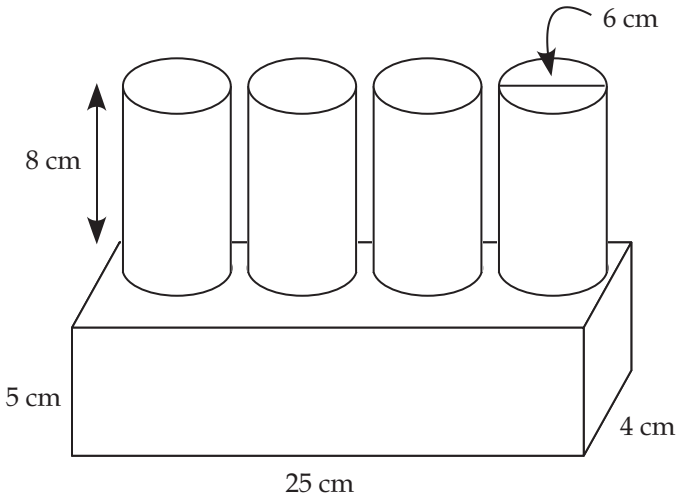
   a) *(6 marks)*



b) *(6 marks)*



c) *(6 marks)*



2. You have invented a new product that will be packaged in tin cans, and shipped in rectangular cardboard boxes. To reduce costs, you want to find the arrangement of 20 cans in a box that will require the least amount of cardboard. The cans are 16 cm tall, and have a diameter of 8 cm. Show at least 3 different arrangements, and all your calculations to determine the dimensions of the box with the smallest surface area. *(6 marks)*

Question 2 (continued)

NOTES

LESSON 4: SIMILARITY

Lesson Focus

When you finish this lesson, you will be able to

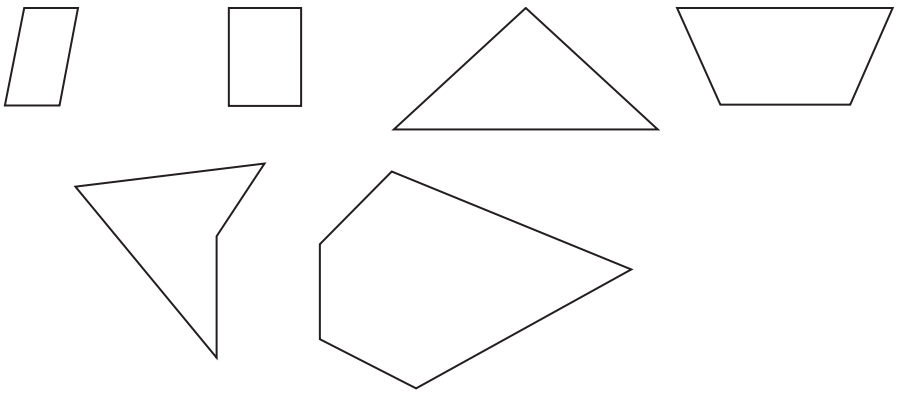
- ❑ discover the rule for similarity
- ❑ draw polygons similar to given polygons
- ❑ solve problems using similarity

Lesson Introduction

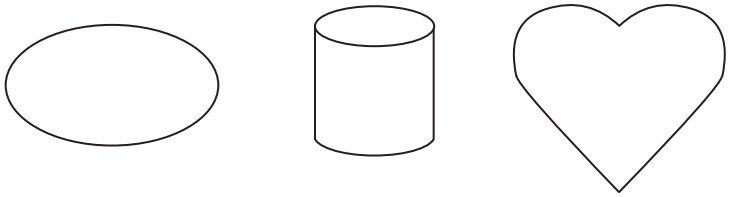
A polygon is defined as a closed figure that has straight lines as edges. It can have any number of sides. In this lesson, you will discover a general rule that defines similar polygons. It involves using ratios and proportions, so a review of methods to solve proportions is provided. Also, you will solve problems using your knowledge of similarity.

Properties of Polygons

These shapes are all polygons. Notice they all have straight edges, and they are closed, meaning there are no openings along the sides.



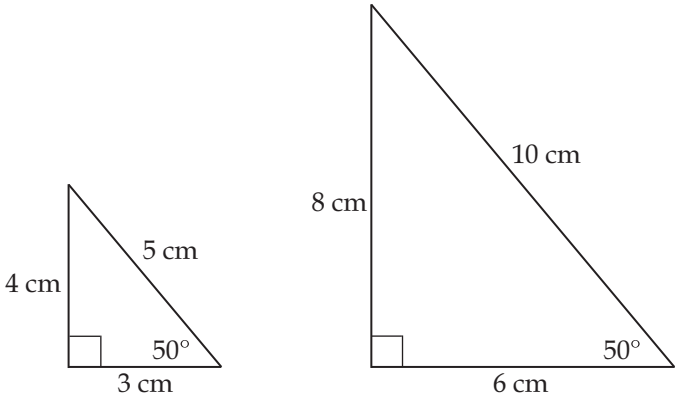
None of these shapes are polygons. Why not?



They are not polygons because the sides are not all straight edges.

Are any of the polygons shown above similar? You could say that three of them have four sides each, but that is not "similarity."

By definition, the two polygons below are similar. Compare and contrast them and list any similarities you notice in the space to the left of the drawings.



Most students will quickly see that these two triangles are the same shape, even though the one on the right is larger. Both triangles are right-angle triangles. Did you also notice that the other two corresponding angles in the triangles are congruent (or the same)? Since there are 180° in a triangle, the third angle in both triangles will be 40°. But there is another important connection. The sides in the second triangle are exactly twice as long as the corresponding sides in the first triangle. The sides are said to be proportional.

> By definition, polygons are similar if all angle measurements are the same and the side lengths are proportional. Similar figures have the same shape, but proportional size.

It will be useful to have this definition as a reminder on your Resource Sheet.

Solving proportions can be done in several different ways. Here is a review of four methods of solving proportions.

As you read through the four different methods, take note of any similarities between them. If there is one method that you feel makes the most sense, don't hesitate to include an example of it on your Resource Sheet.

Method 1: Logic

Try a simple logic puzzle: If 2 goes to 12, the 3 goes to _____.

Write your answer in the blank.

If you wrote 18, you are correct! Whether you were right or not, what logic did you follow to get your answer?

What is done mathematically to 2 to make it 12?

It is multiplied by 6. So, proportionally, you must also multiply the 3 by 6 to get the answer of 18.

The ratios are 2:12 and 3:x. You could set up a proportion to visualize it this way:

$$\frac{2}{12} = \frac{3}{x}$$
$$x = 18$$

Method 2: Multiply to Find an Equivalent Ratio

$$\frac{2}{7} = \frac{x}{21}$$

In this proportion, $7 * 3$ would give you the 21. To keep the ratio equivalent, multiply $\frac{2}{7}$ by a value of 1, written as $\frac{3}{3}$. This way the denominator is multiplied by 3, and the numerator is also multiplied by 3 to keep the proportionality. The answer is $x = 6$.

$$\frac{2}{7} * \left(\frac{3}{3}\right) = \frac{x}{21}$$
$$x = 6$$

Method 3: Divide to Find an Equivalent Fraction

$$\frac{12}{9} = \frac{x}{3}$$

Since division is the opposite operation to multiplication, sometimes it is used to find the unknown value. 9 divided by 3 would give you the 3 in the denominator, so divide by $\frac{3}{3}$.

$$\frac{12}{9} \div \left(\frac{3}{3}\right) = \frac{x}{3}$$

$$x = 4$$

Method 4: Lowest Common Multiple

Sometimes the factor by which you need to multiply is not as obvious as in the above examples. In this case, multiply each side of the proportion by the lowest common multiple of the denominator in order to eliminate the denominators.

Example 1

Solve for x: $\frac{3}{4} = \frac{x}{10}$

Solution:

Multiply both ratios by the same value, with the intent of eliminating the denominator. In this example, the lowest common multiple would be 20, since both 4 and 10 divide evenly into 20.

$(20)\frac{3}{4} = \frac{x}{10}(20)$	Multiply both ratios by 20
$\frac{60}{4} = \frac{20x}{10}$	
$15 = 2x$	Simplify
$\frac{15}{2} = x$	Isolate the variable

You can leave the answer as an improper fraction, or express it as $7\frac{1}{2}$ or 7.5.

Example 2

Solve for r: $\frac{7}{10} = \frac{12}{r}$

Solution:

Multiply both ratios by the lowest common multiple to eliminate the denominators. In this situation, with the variable in the denominator, multiply each side by $10r$.

$$(10r)\frac{7}{10} = \frac{12}{r}(10r)$$ Multiply both sides by the same value

$$\frac{70r}{10} = \frac{120r}{r}$$ Simplify

$$7r = 120$$ Isolate the variable

$$r = \frac{120}{7} \text{ or } 17\frac{1}{7}$$

Setting up Proportions

When you prepare rice for dinner, you mix 2 cups of water for each 1 cup of rice.

Now, if you need to use 4 cups of rice, how much water would you require?

The ratio is 2:1, or 2 cups of water to 1 cup of rice.

As a proportion, you can set it up in one of two ways:

$$\frac{\text{water}}{\text{rice}} = \frac{\text{water}}{\text{rice}} \text{ or } \frac{\text{water}}{\text{water}} = \frac{\text{rice}}{\text{rice}}$$

Either proportion will result in the same answer. You must just be careful to always have corresponding parts in the same place in both ratios.

Choose a method to solve for the amount of water required.

$$\frac{2}{1} = \frac{x}{4}$$

$$\frac{2}{1} * \left(\frac{4}{4}\right) = \frac{x}{4}$$

$$x = 8$$

or

$$\frac{2}{x} = \frac{1}{4}$$

$$\left(4\cancel{x}\right)\frac{2}{\cancel{x}} = \frac{1}{\cancel{4}}\left(\cancel{4}x\right)$$

$$8 = x$$

or Since you multiply the number of cups of rice by two to find the number of cups of water, 4 cups of rice, multiplied by 2 would mean you would need 8 cups of water.

Learning Activity 7.4

Working with Ratios

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What number is 7 times bigger than 21?
2. What number is 52 times bigger than 5?
3. How many rows of 20 seats are there in a room with 1200 seats?
4. Express $\frac{49}{8}$ as a mixed number.
5. Calculate 4 x 14 x 25.
6. Add $\frac{3}{7}+\frac{4}{7}$.
7. Add $\frac{3}{4}+\frac{5}{8}$.
8. You buy a tape for $20 and you have to pay 12% in taxes. What is the total cost of the tape?

continued

Learning Activity 7.4: Working with Ratios (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Solve for the unknown. Multiply or divide to find an equivalent ratio. Show your work.

   a) $\frac{2}{5} = \frac{x}{10}$

   b) $\frac{3}{4} = \frac{9}{x}$

   c) $\frac{20}{4} = \frac{x}{1}$

   d) $\frac{x}{5} = \frac{12}{15}$

2. Solve for the unknown using lowest common multiples. Show your work.

   a) $\frac{3}{5} = \frac{n}{8}$

   b) $\frac{9}{x} = \frac{5}{6}$

   c) $\frac{7}{10} = \frac{10}{p}$

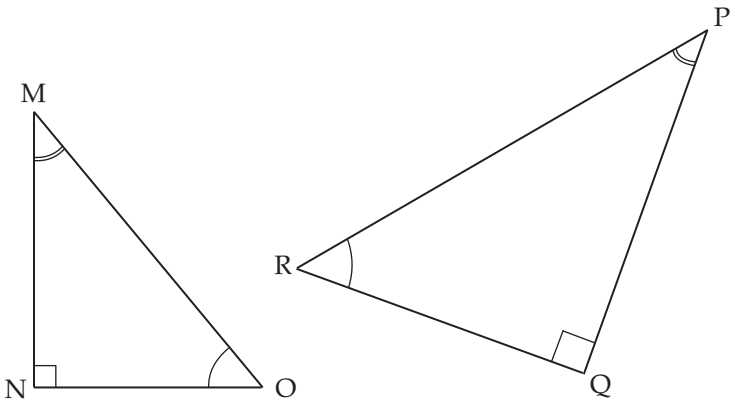
   d) $\frac{x}{7} = \frac{13}{15}$

Corresponding Sides

The definition of similar polygons states that corresponding sides in similar polygons are proportional. You now know how to solve proportions, but it is really important that you understand corresponding sides, so you set up the ratios correctly.

Example 1

Which sides correspond in these similar polygons?



Solution:

In similar polygons, the angle measures will always be the same. Even though the second triangle is flipped and turned, you can see that side NO corresponds with side QR because these two sides are both located between the same two equivalent angles: the right angle and the angle indicated with the ")" marking.

Side MN corresponds with side PQ because they are located between the right angle and the angle indicated with the "))" marking. Side MO corresponds with side PR, as they are both the hypotenuse in these similar right triangles.

To set up a proportion comparing all the corresponding sides in these similar triangles, keep the sides from one triangle on top and the sides from the other triangle on the bottom. In this case, the ratio is set up as $\frac{\text{small}}{\text{large}}$:

$$\frac{MN}{PQ} : \frac{NO}{QR} : \frac{MO}{PR}$$

To set up a proportion comparing ratios of only two sides in each triangle,

you can arrange it in two ways:

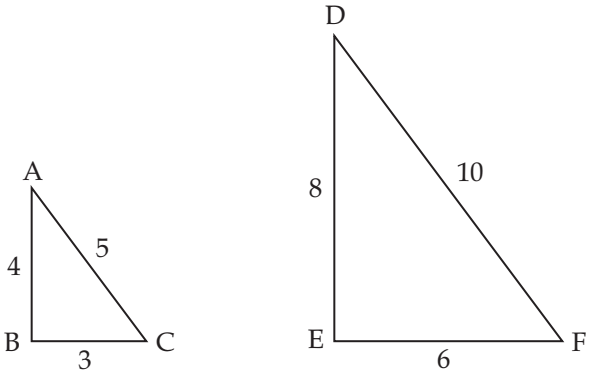
$$\frac{\text{small}}{\text{large}} : \frac{\text{small}}{\text{large}} \text{ or } \frac{\text{small}}{\text{small}} : \frac{\text{large}}{\text{large}}$$

Make sure that corresponding sides are in the same place in both ratios.

$$\frac{MN}{PQ} : \frac{NO}{QR} \text{ or } \frac{MN}{NO} : \frac{PQ}{QR}$$

Example 2

Which sides correspond in these similar polygons?



Solution:

Side AB corresponds with side DE.

Side BC corresponds with side EF.

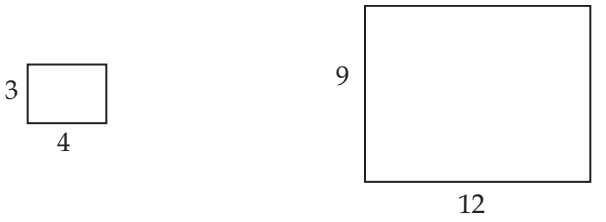
Side AC corresponds with side DF.

Set up a proportion of ratios using the corresponding sides:

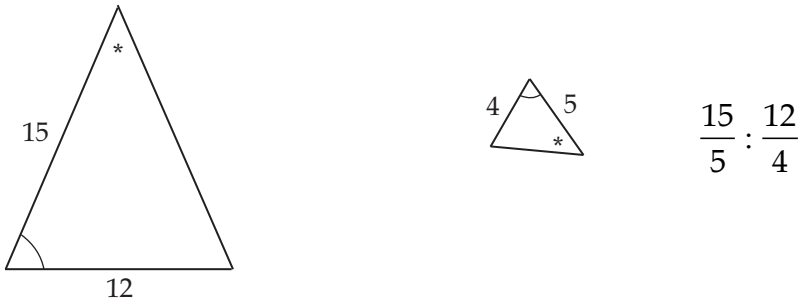
$$\frac{4}{8} : \frac{3}{6} : \frac{5}{10}$$

What do you notice about these ratios? They are all equivalent to $\frac{1}{2}$.

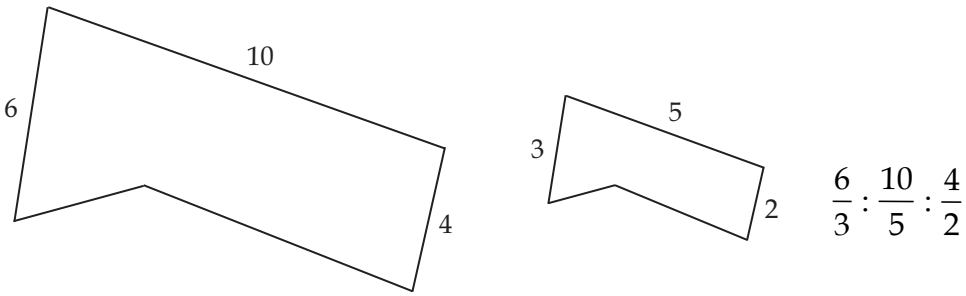
Here are more sets of similar shapes. Take careful note of how the proportions are set up. How else could they be written?



$$\frac{3}{4} : \frac{9}{12}$$



$$\frac{15}{5} : \frac{12}{4}$$

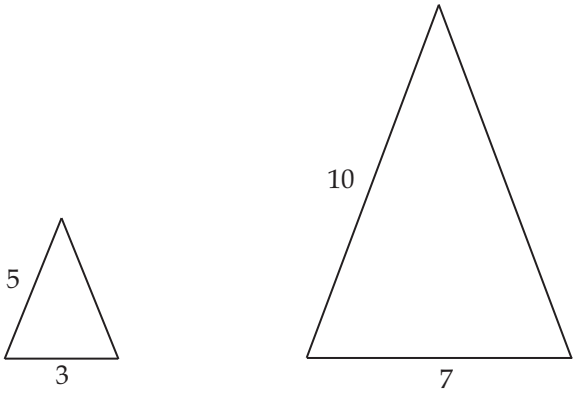


$$\frac{6}{3} : \frac{10}{5} : \frac{4}{2}$$

What do you notice about the proportions for each pair of shapes?

You should see that each of the sets of ratios is equal.

The following shapes are not similar. Why not?



Set up a proportion using corresponding sides:

$$\frac{5}{10} : \frac{3}{7}$$

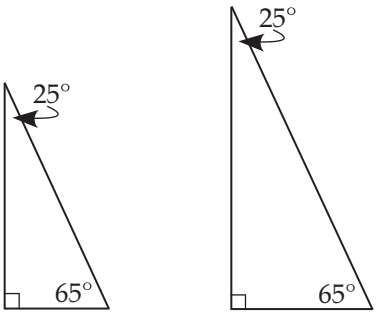
$\frac{5}{10}$ reduces to $\frac{1}{2}$ but $\frac{3}{7}$ does not.

The fractions are not equivalent so these ratios are not equal. Polygons are only similar if the corresponding sides are proportional.

This is an important property of similarity. Include it in your Resource Sheet for this module.

Angles

One other aspect of similar figures is the angles. Compare the angles in these similar shapes. What do you notice?



The angles are all the same measurement.

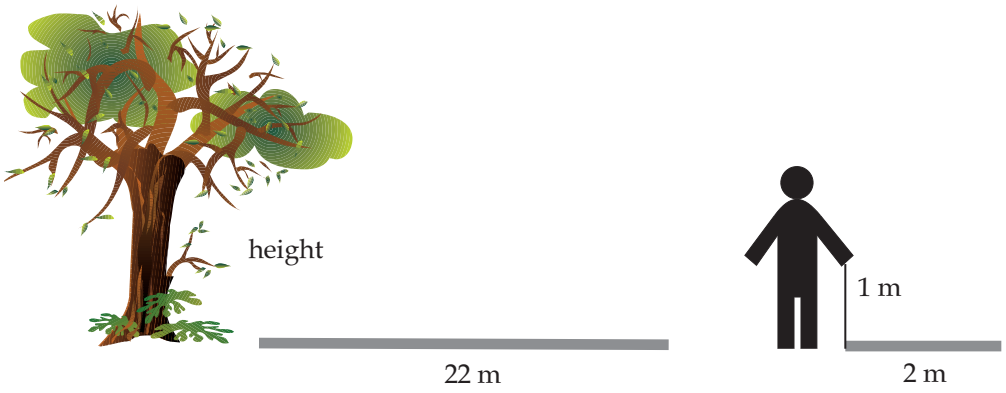
Polygons are similar if the sides are proportional, and/or the angles are all the same size.

Problem Solving

We can use the properties of similarity to solve problems.

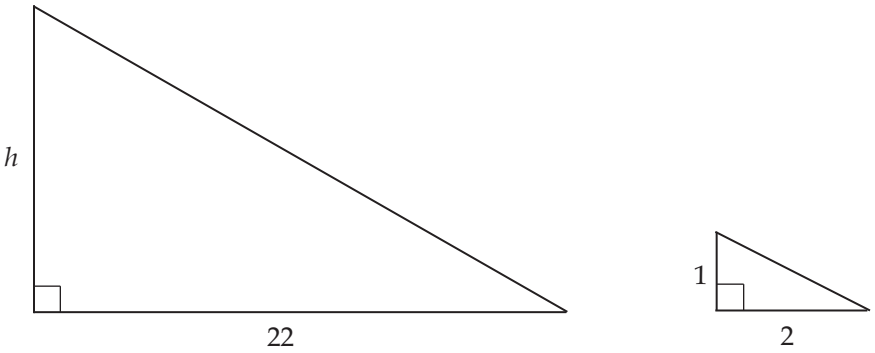
Example 1

The sun is casting a shadow along the ground next to a tall tree. You measure the length of the shadow to be 22 metres. You are holding a metre stick in an upright position, and the length of its shadow cast by the sun at the same time is 2 m. Use similarity to find the height of the tree.



Solution:

Draw similar triangles and ask yourself: "Which sides correspond in these similar triangles?"



The tree and the stick are both upright, and correspond. The shadow of the tree corresponds with the shadow of the stick.

The proportion could be set up as follows:

$$\frac{\text{height}}{\text{shadow}} = \frac{h}{22} = \frac{1}{2}$$

You can use whichever method you'd like to solve for the height (h).

One possibility would be to multiply the ratio by $\frac{11}{11}$.

$$\frac{h}{22} = \frac{1}{2} * \left(\frac{11}{11}\right)$$

$$h = 11$$

The tree is 11 m high.

Learning Activity 7.5

Corresponding Shapes

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert $3\frac{2}{5}$ into an improper fraction.
2. Evaluate $6 - 3X$, if $X = -5$.
3. Solve $10 = 22 - B$.
4. Convert 0.22 into a percent.
5. You want to save $20 per month for the next 5 years. How much money will you have at the end of 5 years?
6. Write three equivalent fractions for $\frac{18}{24}$ that all have denominators less than 24.
7. Calculate 20 x 20 x 20.
8. Since you were tired, you went to bed early one evening, at 8:30 pm. The next morning, you woke up at 6:00 am. How many minutes did you sleep?

continued

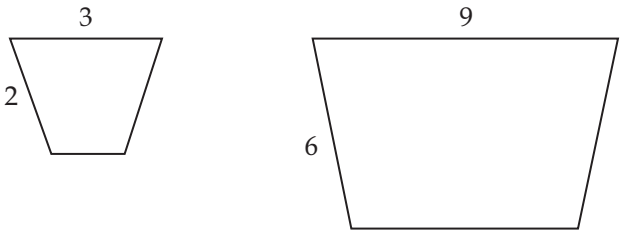
Learning Activity 7.5: Corresponding Shapes (continued)

Part B

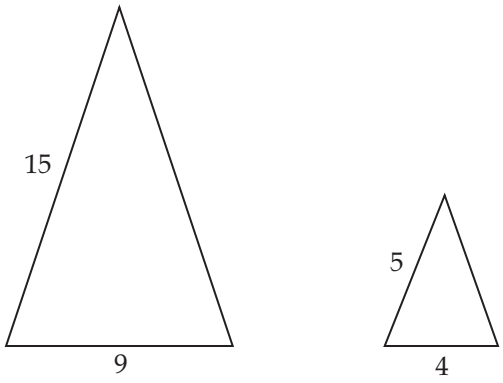
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify which set(s) of shapes are similar, and state why.

a)



b)



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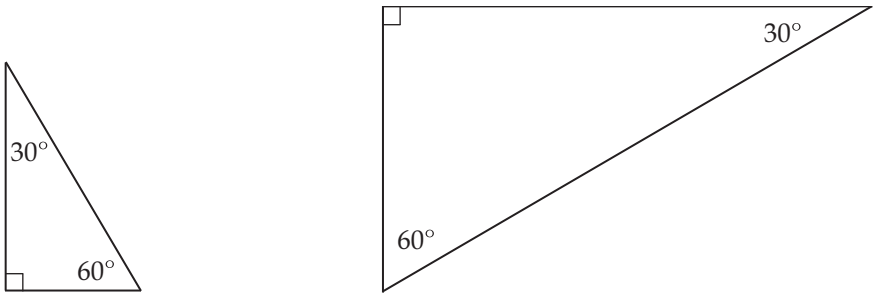
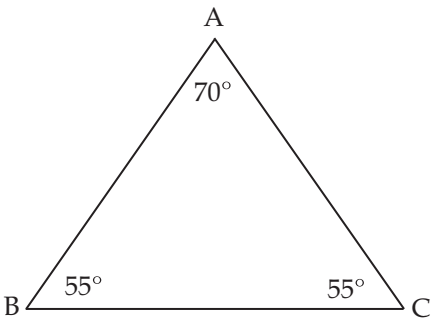
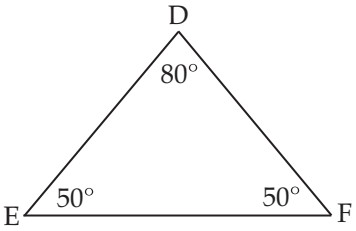


continued

Learning Activity 7.5: Corresponding Shapes (continued)

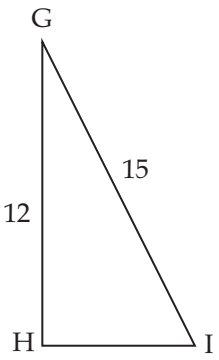
d)

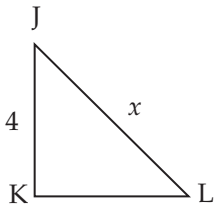




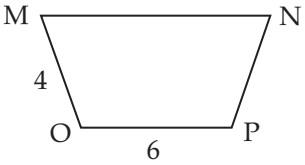
2. Set up proportions to find the missing value, x, in the similar figures below.

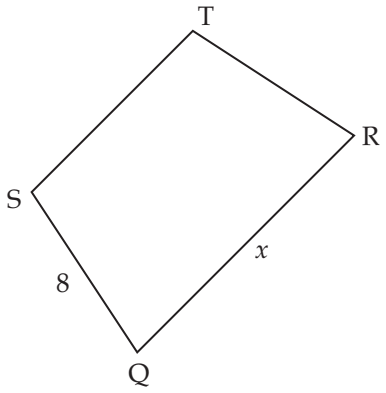
a)





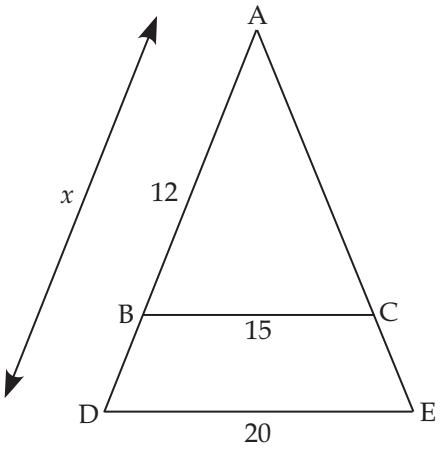
b)





continued

c) Look for the little triangle inside the larger triangle.



Lesson Summary

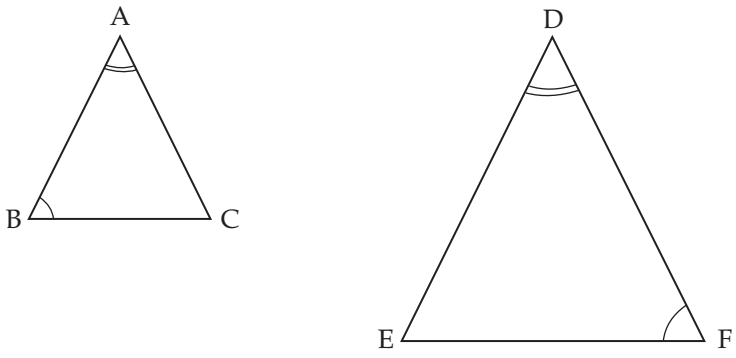
In this lesson, you discovered that similar polygons have proportional sides and equal angles. In the next lesson, you will apply the definition and actually draw similar polygons.

Assignment 7.3: Similarity

Total: 24 marks

1. Identify the corresponding sides, and express them in a proportion. *(3 marks)*

a)

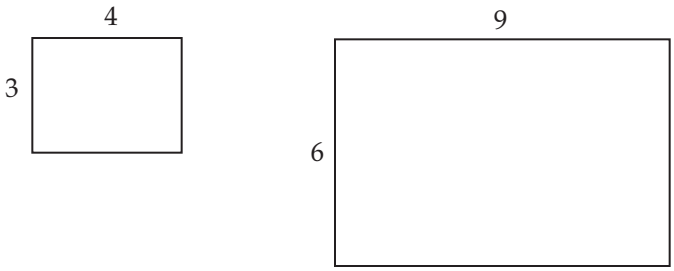


2. Fill in the blanks. *(2 marks)*

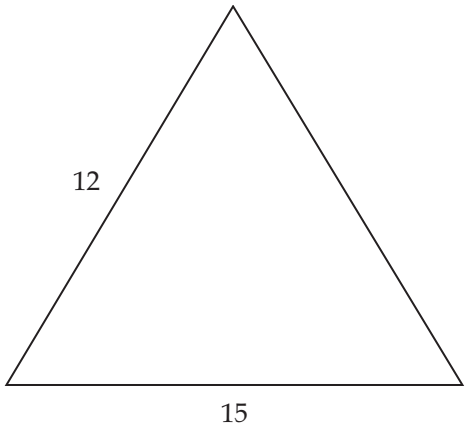
   Polygons are similar if the sides are _____ and the angle measurements are the _____.

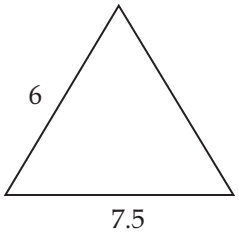
3. Identify which sets of polygons are similar, and indicate why. *(8 marks)*

a) *(2 marks)*

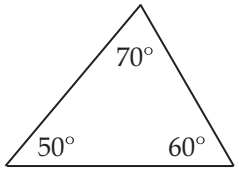


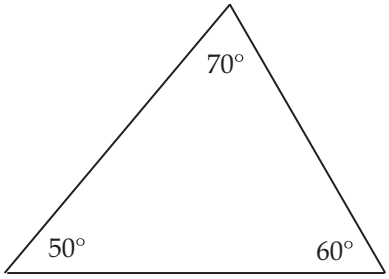
b) *(2 marks)*



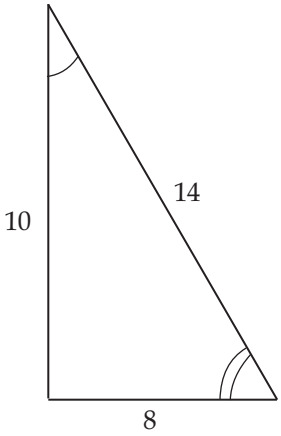


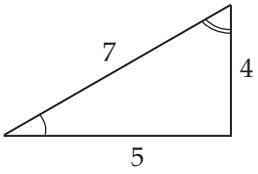
c) *(2 marks)*





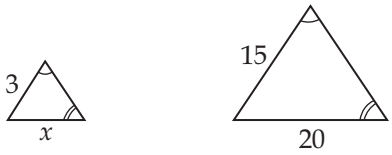
d) *(2 marks)*





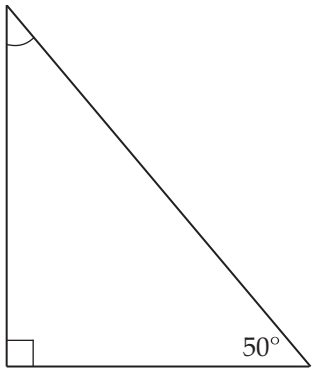
4. Find the missing value. Choose any method but show your work. *(8 marks)*

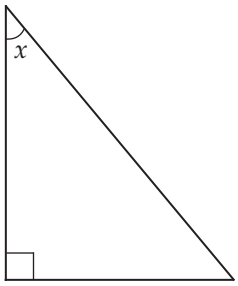
a) *(2 marks)*



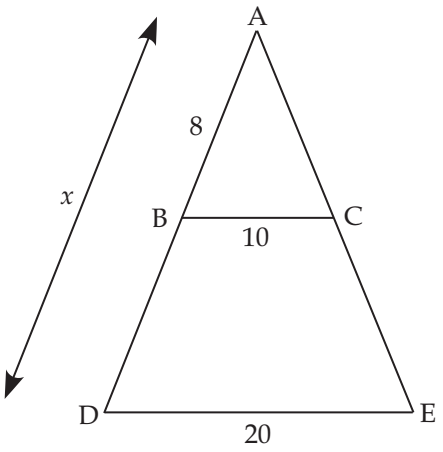
b) $\frac{5}{12} : \frac{8}{x}$ *(2 marks)*

c) *(2 marks)*



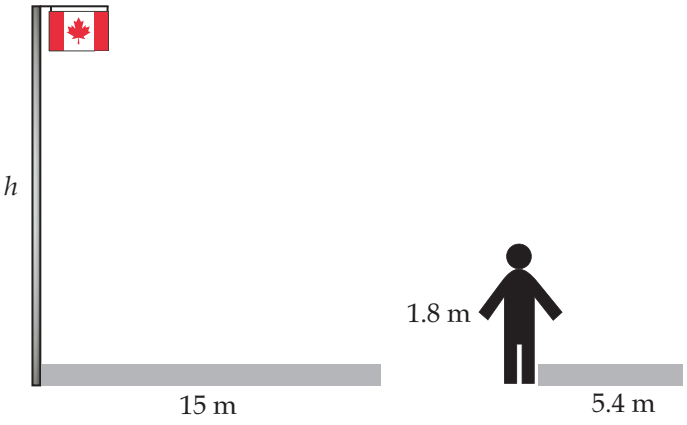


d) *(2 marks)*



5. Use similarity to solve the problem below. *(3 marks)*

   A 1.8 m tall person casts a shadow of 5.4 m. A flagpole near him casts a shadow of 15 m. How tall is the flagpole?



LESSON 5: DRAWING POLYGONS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ draw a polygon similar to a given polygon
- ❑ explain why they are similar
- ❑ solve problems using similarity

Lesson Introduction

A terrific way to ensure that you understand the properties of similar polygons is to draw some! You will need a ruler and a protractor to complete this lesson.

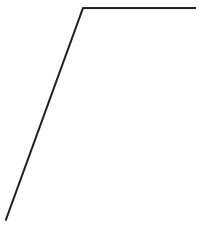
Constructing Polygons

You have learned that similar shapes have sides that are proportional. If you had a shape with side lengths of 2, 3, and 4 units, and you were to draw a polygon similar to that, you would have to decide on the amount or rate by which you would equally multiply each side. If you multiply all the dimensions by 2, then you will have sides of 4, 6, and 8.

Use your ruler, and measure this line. Then draw a line exactly twice as long as this.

A ——— B

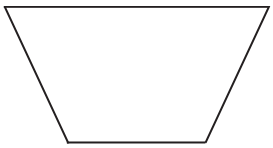
Copy this angle using your ruler and your protractor, but make each line twice as long as the original.



You should have an angle of exactly the same measurement, but each line should be exactly twice as long. To be proportional, each line in a polygon is multiplied by the same factor, but the angles are the same.

Example 1

Draw a polygon that is similar to the given shape. Explain in detail how you know it to be similar.



Solution:

You should have each side length multiplied or divided by the same value. Also, each angle must remain the same.

If the sides are proportional and the angles are the same, then your drawing is similar to the given polygon.

Learning Activity 7.6

Polygon Problems

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Which is largest, $\frac{2}{3}$, $\frac{3}{5}$, or $\frac{9}{15}$?
2. What is the area of a rectangular classroom that measures 10 m by 12 m?
3. If 25% of the area of the lawn you are mowing is 18 m^2, what is the total area of the lawn?
4. The measure of angle B is double the measure of angle A. If angle A measures 30°, what is the sum of angles A and B?

continued

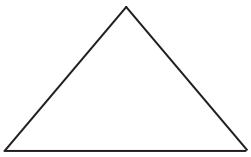
Learning Activity 7.6: Polygon Problems (continued)

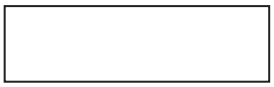
5. 16 of 20 students in your class watched the hockey game last night. What percentage does this represent?
6. Nadia earns $85 a week at work. How much does she earn after 6 weeks of work?
7. How much money do you have if your wallet contains twelve $5 bills, eight $10 bills, and fifteen $20 bills?
8. How much will 5 chocolate bars cost if each one is $0.98?

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use your ruler and protractor to draw a polygon similar to the given shape, and explain how it is similar.

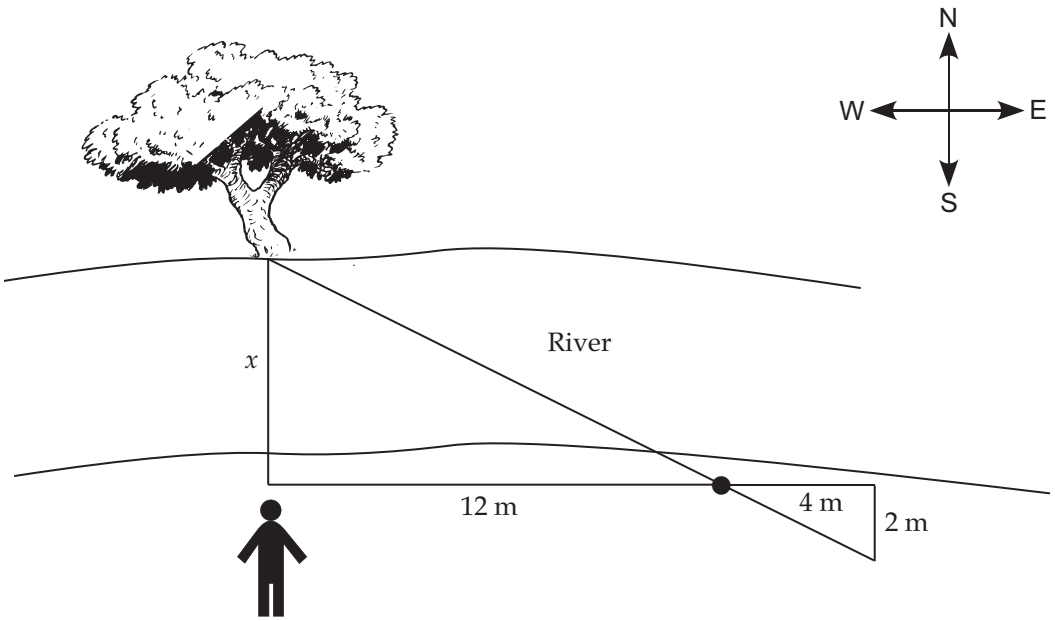
   a) 

   b) 

2. You are flying a kite and have let 60 feet of string off the spool. The wind has carried the kite so it is hovering a distance away from you, 45 feet above the ground. If you were to reel in some string so only 36 feet of string is out, how high would the kite be off the ground, if the wind remains constant? Draw similar right triangles to illustrate this situation, label them, and solve the problem.

continued

3. Refer to the diagram below. You are standing beside a river, looking north at a tree growing on the other side. You want to determine the width of the river using the properties of similarity. You walk 12 m east along the riverbank, and make a mark on the ground. Then you move another 4 m toward the east. From this point, you turn directly south for 2 m and then return to your original mark so you are in line with the tree across the river. With these measurements you can determine the width of the river.



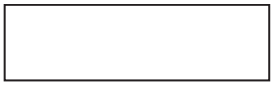
Lesson Summary

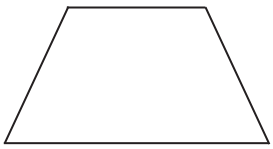
In this lesson, you used a ruler and protractor to draw similar polygons. You used diagrams of similar polygons to solve problems. In the next lesson, you will use proportions to calculate lengths in scale diagrams.

Assignment 7.4: Drawing Polygons

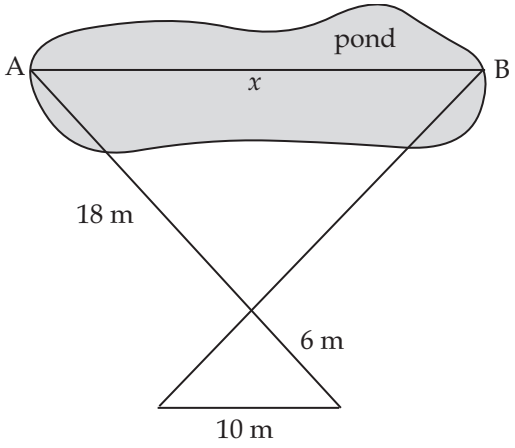
Total: 15 marks

1. Draw a polygon similar to the given shape, and explain how it is similar. *(6 marks)*

   a)  *(3 marks)*

   b)  *(3 marks)*

2. Use the similar triangles to find the distance across the pond from A to B. Show your work. *(3 marks)*



3. Draw a diagram to illustrate the problem, and then solve. *(6 marks)*

   A building casts a 10 m shadow. You are 1.8 m tall, and you cast a 1.2 m shadow. How tall is the building?

LESSON 6: SCALE DRAWINGS

Lesson Focus

When you finish this lesson, you will be able to

- ❑ identify an example of a scale drawing in the media and interpret it
- ❑ draw diagrams to scale representing enlargements or reductions
- ❑ determine scale factors of proportional drawings
- ❑ solve problems involving scale diagrams and similarity

Lesson Introduction

You are surrounded by scale drawings and you may not even notice! Every time you look at a photograph, buy an object that needs assembly and follow an instructional diagram, or look at a map, you are reading a scale drawing. This lesson will focus on understanding scale diagrams: drawing them, reading them, and finding them in the media.

What is Scale?

A scale drawing is an image that represents a real object. The rate at which the picture is enlarged or reduced, compared to the actual object, is called the scale. It is the ratio that compares the size of the image to the size of the actual object.

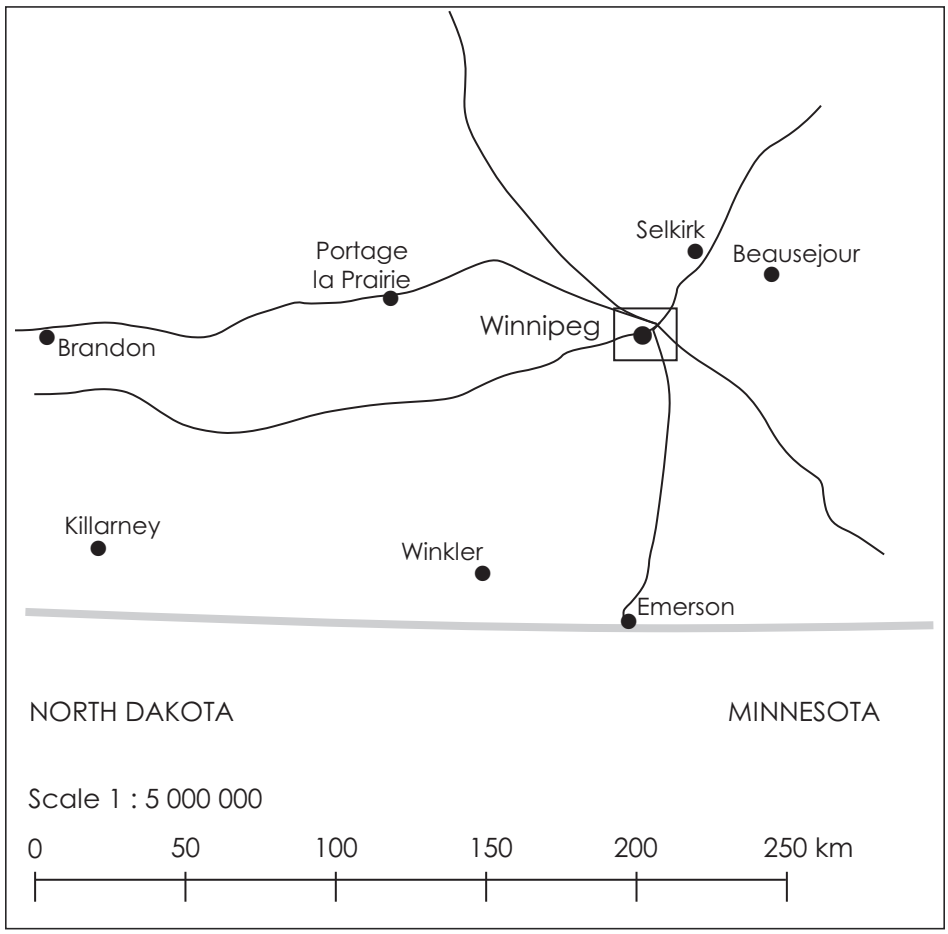
image : object or drawing : actual

For example, if you are drawing plans for building a dog house with a scale factor of 1:10, then 1 cm on your drawing would represent 10 cm on the actual doghouse.

It would be helpful to include how to write a scale ratio on your Resource Sheet so that you don't get confused when reading or writing scale ratios.

The following is a map showing Winnipeg and the area around it. Look closely at the "legend" in the bottom left corner. It shows a given length, 2 cm, on the map represents 50 km in real distance. This is a scale drawing. If you measure the distance with your ruler, and apply the scale, the distance from Portage la Prairie to the western edge of Winnipeg should be about 75 km.

Please note that this straight-line distance may be different from the highway distance, since the highway does not follow a straight line.



Other scale diagrams might show a different type of legend.

For example, 1: 5000 means 1 unit on the diagram represents 5000 similar units in real distance.

The following chart indicates the scale, size, and cost of different kinds of maps.

Map	Scale	Size	Price
Street Map Streets and street names	1:20,000	152 cm x 152 cm	$50.00
Planning Map Major zoning categories	1:37,000	91 cm x 91 cm	$30.00
Large Map Choice of features	varies	30 cm x 43 cm	$25.00

The second column is the scale used on each map. Notice that no specific units are given. On the Street Map, 1 unit of any size on the map represents 20 000 of those units in real distance. The units could be cm, inches, feet, hand widths, or whatever, as long as you only use one type of unit.

Here is an offer for a fictitious map. Notice the scale stated in the description:

> "Order our free 2015 Travel Guide and Fishing Map. The large folded map included with our guide is at approximately 1:1 000 000 scale detailing geography locations and the best fishing spots in the municipality."

This means that 1 unit on the map represents 1 million units in real distance. 1 000 000 units may be awkward to work with and hard to visualize, but using the metric system, you can restate this scale in other terms.

There are 100 centimetres in a metre, so 1 000 000 ÷100 = 10 000. If using centimetres and metres, this scale could be restated as 1 cm : 10 000 m. Now the units must be included when you state the scale.

You could also state this as 1 cm : 10 km

These are all the same scale. They are simply stated using different units.

More commonly, a scale on a map might read something like 1 cm : 200 km.

Given that Calgary is roughly 1400 km from Winnipeg, a scale drawing of that distance using the scale 1 cm : 200 km would show a line of 7 cm in length to represent this distance.

1 cm : 200 km

_____ cm : 1400 km

Since 1400 km is 7 times 200 km, multiply the 1 cm by 7 to get 7 cm.

Sometimes the most challenging part of working with scale drawings is learning to determine an appropriate scale factor.

Choosing a Scale

The distance from Winnipeg to Toronto is roughly 2200 km. If you were to draw a line that represents this distance, you would need to find an appropriate scale. This means finding a scale factor that makes your drawing readable—not too big, and not too small.

For example, if you set your scale at 1 cm : 100 km, then the drawing would have to be:

1 cm : 100 km

x : 2200 km

Use a proportion to solve for x.

Refer to previous lessons for help in setting up the proportion and for methods of solving it.

$$\frac{\text{cm}}{\text{km}} = \frac{1}{100} = \frac{x}{2200}$$

$$\frac{1}{100} * \left(\frac{22}{22}\right) = \frac{x}{2200}$$

$$x = 22$$

You would need to draw a line 22 cm long. Most notebook paper is about 20 cm wide, so this scale would not be appropriate. A scale factor of 1 cm : 250 km would certainly fit the page much better.

1 cm : 250 km

x : 2200 km

$$\frac{1}{250} = \frac{x}{2200}$$

$$2200 \div 250 = 8.8$$

$$\frac{1}{250} * \left(\frac{8.8}{8.8}\right) = \frac{x}{2200}$$

$$x = 8.8$$

The line in our scale drawing representing the distance from Winnipeg to Toronto would be 8.8 cm long.

Example 1

The average distance to the moon is 382 500 km. What scale factor could you use to draw such a diagram?

Solution:

Remember, it has to fit on your 20 cm wide paper, and cannot be so small that it is difficult to interpret. Sometimes you can use a "trial and error" approach where you just try various scales until the best one appears.

1 cm : 100 000 km

_____ cm : 382 500 km

This results in a line on your scale drawing of only 3.8 cm long. It is difficult to read such a small diagram! Now, if you double the length of your line, it is much easier to read. That means our scale factor must change to 1 cm : 50 000 km. This results in a scale drawing with a line 7.6 cm long.

Scientists use scale drawings to show enlargements of small objects. Very small insects can be enlarged on a diagram using scale factors such as 10 : 1. This indicates that measurements of the drawing are 10 times larger than the original object. 10 units on the drawing represent 1 unit on the actual object.

Example 2

Here is an enlargement of a ladybug. The scale factor is 10 : 1.

The actual insect is $\frac{1}{10}$ the size of the drawing.

Use your ruler to determine how long this ladybug actually is.

Solution:

The body of this bug is about 2.5 cm in this diagram.

$$\frac{\text{drawing}}{\text{actual}} = \frac{10}{1} = \frac{2.5}{x}$$

$$(1x)\frac{10}{1} = \frac{2.5}{x}(1x)$$

$$10x = 2.5$$

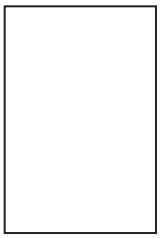
$$x = 0.25$$

This bug is actually about 0.25 cm long.

Finding the Scale Factor

Here is a scale drawing of a rectangular plot that actually measures 2 m by 3 m.

Use your ruler to determine the scale factor used.



Your measurement of the side should be 2 cm, and the length should measure 3 cm.

If 2 cm represents 2 m of the actual object, then the scale factor must be 1 cm : 1 m or 1 cm : 100 cm (1 m = 100 cm).

If the drawing is 2 cm by 3 cm, then it would be proportional to the actual object.

Learning Activity 7.7

Using Scales

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What is the square of 80?
2. What is half of 5.20?
3. What decimal is equivalent to $5\frac{2}{5}$?
4. Two consecutive numbers have a sum of 15. Which is the smaller of the two numbers?
5. In a population, 35% of people are vegetarian. What percentage of people are not vegetarians?
6. You buy a hat for $7.40 with a $20 bill. How much change do you get?
7. Evaluate $2^2 + 4^2 + 6^2$.
8. Estimate the angle made by the hands of a clock at 12:30.

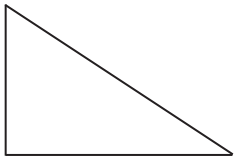
continued

Learning Activity 7.7: Using Scales (continued)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The length of the highway from Winnipeg to Thompson is about 750 km. If you use a scale factor of 1 cm : 75 km, how long would the line be on your drawing?
2. The dimensions of your rectangular-shaped room are 4 m by 2.5 m. Determine an appropriate scale factor to use in creating a scale diagram, and draw the scale diagram.
3. You have captured a mosquito and want to enlarge it on a scale drawing. The mosquito measures 3 mm by 2 mm. Determine an appropriate scale factor to use in creating a scale diagram, and draw the scale diagram.
4. A map using a scale of 1:500 000 shows the location of a lake. The length of the lake on the drawing is 2.5 cm. How long is the actual lake?
5. You have a right triangular–shaped flower garden that measures 4 m by 6 m along the sides (not the hypotenuse). Is this diagram proportional to the flower garden? If so, what scale factor was used to make this diagram?



Lesson Summary

In this lesson, you learned how to read and use scales and used scale drawings to represent really small or really large objects. Using proportions, you solved for unknown values in either the scale diagram or the real object.

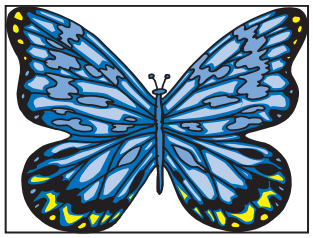
Assignment 7.5: Scale Drawings

Total: 15 marks

1. The length of the highway from Calgary to Vancouver is almost 1000 km. If you use a scale factor of 1 cm : 75 km, how long would the line representing this distance be on your drawing? Show your work. *(3 marks)*

2. The dimensions of your room are 4.5 m by 3 m. Determine an appropriate scale factor to use in creating a scale diagram, and draw the scale diagram. *(2 marks)*

3. This scale drawing is an enlargement of a small insect. The scale factor is 20 : 1. What are the actual measurements of the insect? Use your ruler, and show your calculations. *(4 marks)*



4. A map using a scale of 1:500 000 shows the distance between two towns. The distance shown on the scale drawing is 7.5 cm. How far apart are the towns? Show your calculations. *(3 marks)*

5. A cabin at the lake measures 12 m by 8 m. Is this diagram proportional to the actual measurement? If so, what scale factor was used to make this diagram? *(3 marks)*

MODULE 7 SUMMARY

Congratulations, you have finished seven of the modules in this course.

In this module, you reviewed solving for the surface area of 2-D and 3-D objects using diagrams and formulas to help organize your work. You learned how to take overlap in composite objects into consideration when calculating surface area, and solved problem situations involving 3-D shapes.

Similar polygons and scale drawings make use of ratios and proportions in solving for unknown values, and several methods to do so were presented. You learned how to identify and create similar polygons as well as scale drawings.

The next and final module in this course completes the topic of geometry by looking at symmetry.

Submitting Your Assignments

You will not be sending your assignments to your tutor/marker at this time. Instead, you will submit your Modules 7 and 8 assignments together when you have completed Module 8.

NOTES

GRADE 9 MATHEMATICS (10F)

Module 7: 2-D and 3-D Objects

Learning Activity Answer Keys

MODULE 7: 2-D AND 3-D OBJECTS

Learning Activity 7.1: The Area of Shapes

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. How much tax (12%) do you pay on a book that costs $5.00?
2. Add –12.4 + 3.1.
3. Sophia paid $63 for a pair of jeans. Sasha paid $25 less than Sophia for an identical pair. How much did Sasha pay?
4. You got $\frac{24}{30}$ on your last mathematics test. What was your percentage grade?
5. It rained 16 mm during one April week in Vancouver. The week before, it rained 5 mm, and the week after, it rained 9 mm. What was the average weekly rainfall in Vancouver during those three weeks?
6. What is the measure of the third angle of a triangle if two of the angles measure 60° and 40°?
7. If 18% of the population is 54 people, how many are in 3% of the population?
8. Philippe completed $\frac{5}{8}$ of his homework while Nicole completed $\frac{2}{3}$ of hers. Who is closer to finishing their homework?

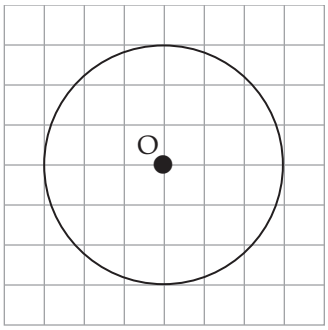
Answers:

1. $0.60 (0.12 x 5 = 0.60)
2. –9.3
3. $38 (break down 25 to 23 and 2; $63 – $23 = $40; subtract the extra $2)
4. 80% $\left(\frac{24}{30} \text{ is equivalent to } \frac{8}{10} \text{ or } 0.8\right)$
5. 10 mm per week $\left(16 + 5 + 9 = 21 + 9 = 30 \text{ and } \frac{30}{3} = 10\right)$
6. 80° (60° + 40° = 100°; 180° – 100° = 80°)
7. 9 people (54 is 6 times 9)
8. Nicole (the lowest common denominator of the two fractions is 24; Philippe completed $\frac{5}{8}$ or $\frac{15}{24}$ while Nicole completed $\frac{2}{3}$ or $\frac{16}{24}$)

Part B

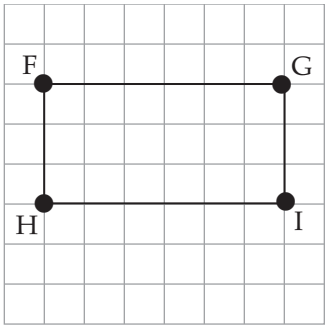
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use the "counting squares" method to find area of each shape.

a) 

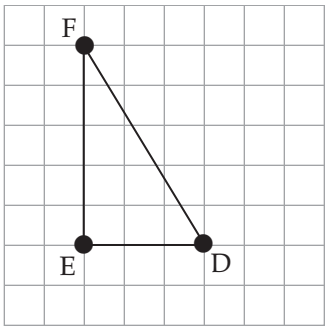
Answer:

about 28 squares

b) 

Answer:

18 squares

c) 

Answer:

about 7 or 8 squares

2. Now use the appropriate formula to find the area for each shape above. Let each square represent 1 cm^2.

   a) Area (circle) = πr^2

   $= (3.14)(3)^2$

   $= (3.14)(9)$

   $= 28.26$

   The area of the circle is about 28.26 cm^2.

   b) Area (rectangle) = $l * w$

   $= 6 * 3$

   $= 18$

   The area of the rectangle is 18 cm^2.

   c) Area (triangle) $= \frac{b * h}{2}$

   $= \frac{3 * 5}{2}$

   $= 7.5$

   The area of the triangle is 7.5 cm^2.

Learning Activity 7.2: The Surface Area of 3-D Shapes

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Eight identical notebooks cost $40. What is the unit price per notebook?
2. Express $2\frac{3}{4}$ as a decimal.
3. Your chequing account balance is $62. You deposited $43 but you have to pay a bill of $35. What is the new balance on your chequing account?
4. Evaluate 50^2.
5. Marcel was born in April 1922. What age was he when he became a great-grandparent in April 2006?
6. A film lasts 105 minutes. It is now 2:00 pm and you need to leave at 3:30 pm. Do you have enough time to watch the film before you need to leave?
7. A car is travelling at 100 km/h. It has 10 km to go. How long will it take for the car to travel the remaining 10 km?
8. A CD contains 4 songs that last 3 minutes and 15 songs that last 4 minutes. What is the total length of the CD?

Answers:

1. $\$5 \left(\frac{40}{8}\right)$
2. $2.75 \left(\frac{3}{4} \text{ is equivalent to } 0.75; \text{ so } 2+0.75\right)$
3. $70 (break down 35 to 33 and 2; 43 – 33 = 10; 10 – 2 = 8; 62 + 8 = 70)
4. 2500 ($50 = 5 \times 10$; $5^2 = 25$; $10^2 = 100$; $25 \times 100 = 2500$)
5. 84 years old (78 years until the year 2000; add 6 years)
6. No (there is only 90 minutes from 2:00 pm to 3:30 pm)
7. 6 minutes (10 is one-tenth of 100, therefore it will take one-tenth of an hour or $\frac{60 \text{ minutes}}{10}$ to travel the rest of the distance)
8. 72 minutes ($4 \times 3 = 12$ and $15 \times 4 = 60$, so 12 + 60)

Part B

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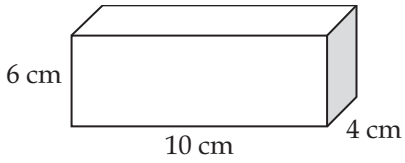
1. Find the surface area of each shape.

   a)



   Answer:

   There are 6 rectangular sides. There are two of each of the 3 sizes.

   Front and Back

   $A = L * W$

   $A = (6 * 10) * 2$

   $A = 120$

   Top and Bottom

   $A = L * W$

   $A = (4 * 10) * 2$

   $A = 80$

   Two Ends

   $A = L * W$

   $A = (4 * 6) * 2$

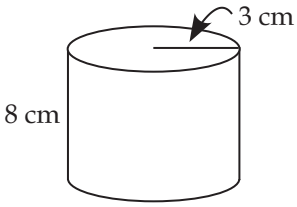
   $A = 48$

   Total = 120 + 80 + 48

   = 248

   The total surface area of this rectangular prism is 248 cm^2.

b)



Answer:

Surface area of cylinder = $2\pi rh + 2\pi r^2$

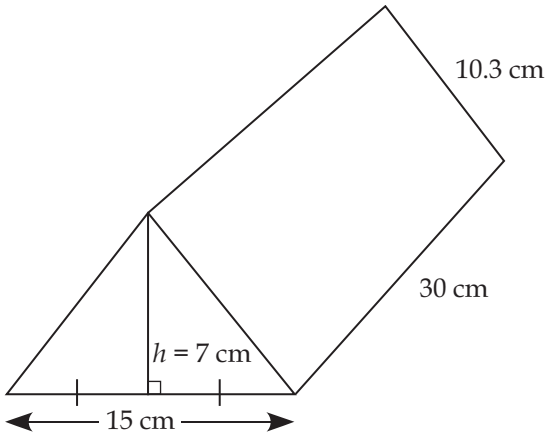
$= 2(3.14)(3)(8) + 2(3.14)(3^2)$

$= 150.72 + 56.52$

$= 207.24$

The total surface area of this cylinder is 207.24 cm^2.

c)



Answer:

This triangular prism has two congruent right triangles for its base. The height of the prism is given as 7 cm, and the base is 15 cm. The surface area is made up of the two triangular ends, two congruent sides, and the bottom of the prism.

Two Triangular Ends

$$A = \frac{b * h}{2}$$

$$A = \frac{(15 * 7)}{2} * 2$$

$$A = 52.5 * 2$$

$$A = 105$$

Two Side Pieces

$$A = L * W$$

$$A = (30 * 10.3) * 2$$

$$A = 309 * 2$$

$$A = 618$$

Bottom

$$A = L * W$$

$$A = 30 * 15$$

$$A = 450$$

Total surface area = 105 + 618 + 450 = 1173

The total surface area of this triangular prism is 1173 cm^2.

Learning Activity 7.3: Surface Area of Composite Objects

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Solve for D: $20 = 4D + 8$
2. Which is smaller: 25% of 400, or 20% of 500?
3. You buy something for $5.82 with a $20 bill. How much change do you get?
4. What two numbers have a sum of –7 and a product of 12?
5. What two numbers have a sum of 12 and a product of –28?
6. Your telephone bill shows you spoke for 73 minutes. If each minute costs 5¢, what is the total bill in dollars?
7. How many grams in 54.5 kg?
8. How many 250 g bags of flour can be made with a single 6.25 kg bag of flour?

Answer:

1. 3 ($20 - 8 = 4D$; $12 = 4D$; $3 = D$)
2. Neither; they both have a value of 100 (25% is equivalent to $\frac{1}{4}$ and 20% is equivalent to $\frac{1}{5}$; so $\frac{1}{4} \times 400 = 100$ and $\frac{1}{5} \times 500 = 100$).
3. $14.18 (18¢ to get to $6 and then $14 to get to $20)
4. –3 and –4
5. 14 and –2
6. $3.65 (5 x 70 + 5 x 3 = 365¢; you need to divide the result by 100 to get dollars)
7. 54 500 g (54.5 x 1 000)
8. 25 bags of 250 g (there are 4 bags to make 1 kg, so you need 6 x 4 = 24 bags for 6 kg; and then, there is another bag of 250 g or 0.25 kg)

Part B

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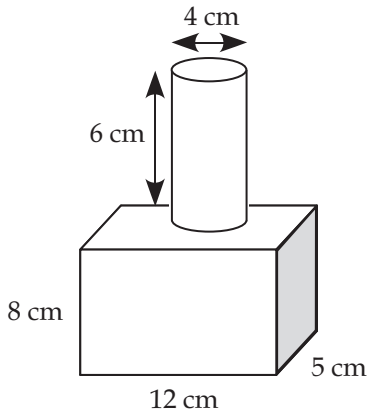
1. Determine the surface area for each composite 3-D object. Assume each shape is solid and subtract any overlap.

   a)



Answer:

Rectangular prism

$A = L * W$

Front/back 8 x 12 x 2 = 192

Ends 5 x 8 x 2 = 80

Top/bottom 12 x 5 x 2 = 120

Total = 192 + 80 + 120 = 392

Surface area of rectangular prism is 392 cm^2.

Cylindrical prism

$A = 2\pi r \times h + \pi r^2$

$A = 2 * (3.14) * (2) * (6) + (3.14) * (2)^2$

$A = 75.36 + 12.56$

$A = 87.92$

Surface area of cylinder is 87.92 cm^2.

Overlap

The base of the cylinder must be subtracted from the top surface area of the rectangular prism.

$A = \pi r^2$

$A = (3.14) * (2)^2$

$A = 12.56$

Total surface area

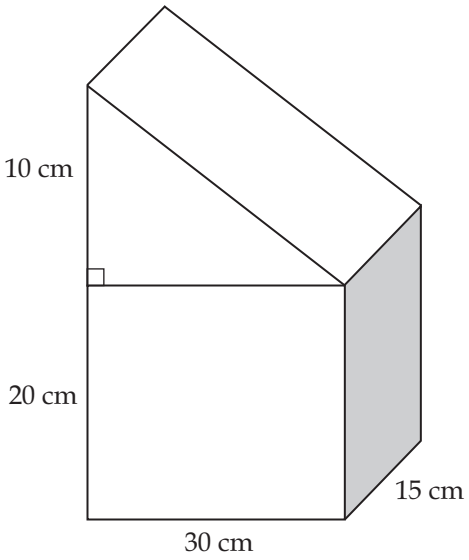
SA = 392 + 87.92 – 12.56

SA = 479.92 – 12.56

SA = 467.36

The total surface area of the composite object is 467.36 cm^2.

b)



Answer:

Triangular prism

Two triangular faces

$$A = \frac{b * h}{2}$$

$$A = \frac{(10 * 30)}{2} * 2$$

$$A = 300 \text{ cm}^2$$

Rectangular side

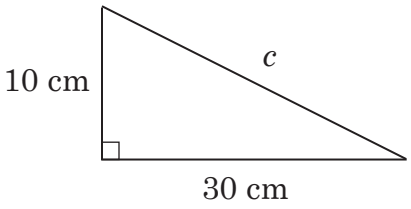
$A = L * W$

$A = 10 * 15$

$A = 150 \text{ cm}^2$

Front side

To find the area of the slanted side, you must use the Pythagorean theorem to find the slant height.



$a^2 + b^2 = c^2$

$10^2 + 30^2 = c^2$

$100 + 900 = c^2$

$1000 = c^2$

$31.62 = c$

Area of the slanted rectangular side

$A = L * W$

$A = 15 * 31.62$

$A = 474.3 \text{ cm}^2$

Bottom

Do not include the bottom of the triangular prism, since it overlaps the top of the rectangular prism.

Surface area of the triangular prism = 300 + 150 + 474.3 = 924.3 cm^2.

Rectangular prism

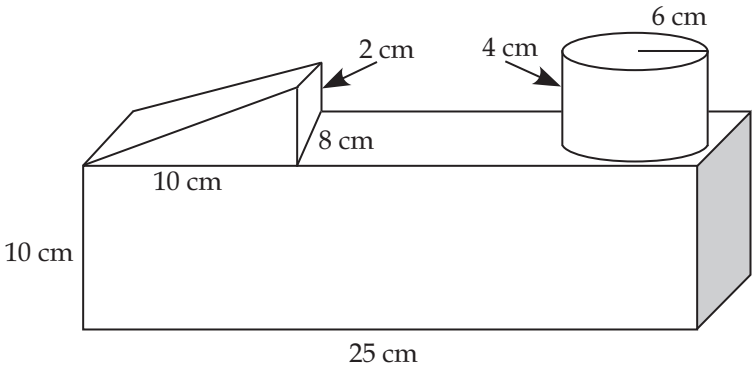
$A = L * W$

Front and back	20 x 30 x 2= 1200
Two ends	15 x 20 x 2 = 600
Top—covered by the triangular prism, so not part of the calculations	
Bottom	30 x 15 = 450
Total:	1200 + 600 + 450 = 2250 cm^2

Total surface area of the composite object = 2250 + 924.3 = 3174.3

The total surface area of this object is 3174.3 cm^2.

c)



Answer:

Triangular prism

Rectangular end piece

$A = L * W$

$A = 2 * 8$

$A = 16 \text{ cm}^2$

Two triangular sides

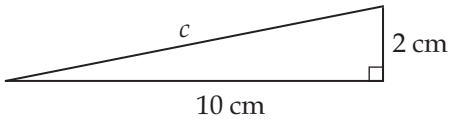
$$A = \frac{b * h}{2}$$

$$A = \frac{10 * 2}{2} * 2$$

$$A = 20 \text{ cm}^2$$

Top slanted piece

Use the Pythagorean theorem to find slant height.



$a^2 + b^2 = c^2$

$10^2 + 2^2 = c^2$

$100 + 4 = c^2$

$104 = c^2$

$10.2 = c$

Area of slanted rectangular top

$A = L * W$

$A = 10.2 * 8$

$A = 81.6 \text{ cm}^2$

Surface area of triangular prism = 16 + 20 + 81.6 = 117.6 cm^2.

Rectangular prism

$A = L * W$

Front and back	$10 * 25 * 2 = 500$
Two ends	$10 * 8 * 2 = 160$
Top and bottom	$25 * 8 * 2 = 400$

Surface area of rectangular prism = 500 + 160 + 400 = 1060 cm^2.

Cylindrical prism

$A = 2\pi r \times h + 1\pi r^2$
(only one circle as bottom overlaps the rectangular prism)

$A = 2 * (3.14) * (6) * 4 + (3.14) * (6)^2$

$A = 150.72 + 113.04$

$A = 263.76\ cm^2$

Overlap

Parts of the top of the rectangular prism are covered by the triangular and cylindrical prisms, so the area of their bases must be subtracted from the area of the rectangular prism.

Base of triangular prism

$A = L * W$

$A = 10 * 8$

$A = 80\ cm^2$

Base of cylindrical prism

$A = (3.14) * (6)^2$

$A = 113.04\ cm^2$

Total overlap area = 80 + 113.04 = 193.04 cm^2

Total surface area

Subtract overlap area from total surface area:

SA = 117.6 + 1060 + 263.76 – 193.04

SA = 1441.36 – 193.04

SA = 1248.32

The total surface area of this composite object is 1248.32 cm^2.

Learning Activity 7.4: Working with Ratios

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What number is 7 times bigger than 21?
2. What number is 52 times bigger than 5?
3. How many rows of 20 seats are there in a room with 1200 seats?
4. Express $\frac{49}{8}$ as a mixed number.
5. Calculate 4 x 14 x 25.
6. Add $\frac{3}{7}+\frac{4}{7}$.
7. Add $\frac{3}{4}+\frac{5}{8}$.
8. You buy a tape for $20 and you have to pay 12% in taxes. What is the total cost of the tape?

Answers:

1. 147 (7 x 20 + 7 x 1 = 140 + 7)
2. 260 (52 times 5 or 5 x 50 + 5 x 2 = 250 + 10)
3. 60 rows (break down 1 200 to 12 x 100; 12 ÷ 2 = 6 and 100 ÷ 10 = 10; 6 x 10 = 60)
4. $6\frac{1}{8}$ (49 divided by 8 is 6 with remainder 1)
5. 1 400 (to start, multiply 25 x 4)
6. $1\left(\frac{3}{7}+\frac{4}{7}=\frac{7}{7}=1\right)$
7. $\frac{11}{8}$ or $1\frac{3}{8}$ (make the two fractions have a common denominator; $\frac{6}{8}+\frac{5}{8}=\frac{11}{8}\Big)$
8. $22.40 ($20.00 x 0.12 = $2.40; $20.00 + $2.40)

Part B

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1. Solve for the unknown. Multiply or divide to find an equivalent ratio. Show your work.

   a) $\frac{2}{5} = \frac{x}{10}$

   b) $\frac{3}{4} = \frac{9}{x}$

   c) $\frac{20}{4} = \frac{x}{1}$

   d) $\frac{x}{5} = \frac{12}{15}$

   Answers:

   a) $\frac{2}{5} * \left(\frac{2}{2}\right) = \frac{x}{10}$

   $x = 4$

   b) $\frac{3}{4} * \left(\frac{3}{3}\right) = \frac{9}{x}$

   $x = 12$

   c) $\frac{20}{4} \div \left(\frac{4}{4}\right) = \frac{x}{1}$

   $5 = x$

   d) $\frac{x}{5} = \frac{12}{15} \div \left(\frac{3}{3}\right)$

   $x = 4$

2. Solve for the unknown using lowest common multiples. Show your work.

a) $\frac{3}{5}=\frac{n}{8}$

b) $\frac{9}{x}=\frac{5}{6}$

c) $\frac{7}{10}=\frac{10}{p}$

d) $\frac{x}{7}=\frac{13}{15}$

Answers:

a)
$$\frac{3}{5}=\frac{n}{8}$$
$$(40)\frac{3}{5}=\frac{n}{8}(40)$$
$$\frac{120}{5}=\frac{40n}{8}$$
$$24=5n$$
$$\frac{24}{5} \text{ or } 4\frac{4}{5}=n$$

b)
$$\frac{9}{x}=\frac{5}{6}$$
$$(6x)\frac{9}{x}=\frac{5}{6}(6x)$$
$$\frac{54x}{x}=\frac{30x}{6}$$
$$54=5x$$
$$\frac{54}{5} \text{ or } 10\frac{4}{5}=x$$

c)
$$\frac{7}{10}=\frac{10}{p}$$
$$(10p)\frac{7}{10}=\frac{10}{p}(10p)$$
$$7p=100$$
$$p=\frac{100}{7} \text{ or } 14\frac{2}{7}$$

d)
$$\frac{x}{7}=\frac{13}{15}$$
$$(7\cdot 15)\frac{x}{7}=\frac{13}{15}(7\cdot 15)$$
$$15x=91$$
$$x=\frac{91}{15} \text{ or } 6\frac{1}{15}$$

Learning Activity 7.5: Corresponding Shapes

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Convert $3\frac{2}{5}$ into an improper fraction.
2. Evaluate $6 - 3X$, if $X = -5$.
3. Solve $10 = 22 - B$.
4. Convert 0.22 into a percent.
5. You want to save $20 per month for the next 5 years. How much money will you have at the end of 5 years?
6. Write three equivalent fractions for $\frac{18}{24}$ that all have denominators less than 24.
7. Calculate 20 x 20 x 20.
8. Since you were tired, you went to bed early one evening, at 8:30 pm. The next morning, you woke up at 6:00 am. How many minutes did you sleep?

Answers:

1. $\frac{17}{5}\left(3 \text{ is equivalent to } \frac{15}{5}\text{; add } \frac{2}{5}\right)$
2. 21 (6 – 3(–5) or 6 + 15)
3. 12 (subtract 10 from 22)
4. $22\%\left(0.22 \text{ is equivalent to } \frac{22}{100}\right)$
5. $1 200 (5 x 20 x 12; multiply 20 by 5 to start; 20 x 5 x 12 = 100 x 12)
6. Three of the following five fractions: $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}$

   (Simplify as much as possible to find the most simple fraction, which is $\frac{3}{4}$. Then multiply the numerator and denominator of that fraction by the same number until the denominator is greater than 24.)
7. 8 000 (2 x 2 x 2 x 10 x 10 x 10)
8. 570 minutes (from 9:00 pm to 6:00 am the next mornng is 3 + 6 = 9 hours, or 540 minutes; you need to add 30 minutes to go from 8:30 pm to 9:00 pm)

Part B

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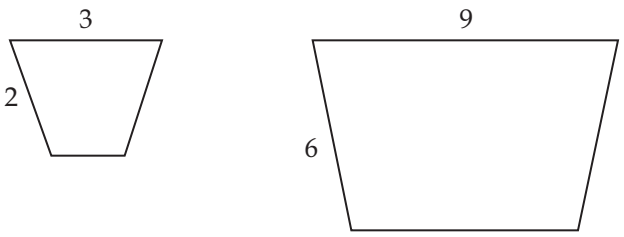
1. Identify which set(s) of shapes are similar, and state why.

a)

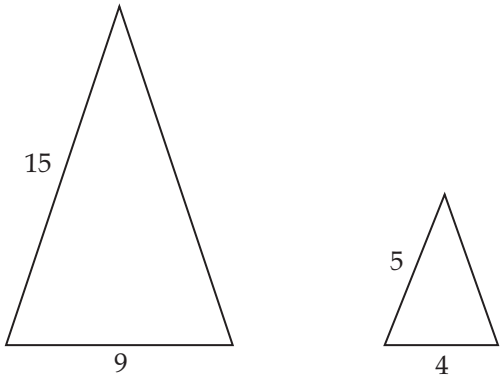


Answer:

$\frac{3}{9} : \frac{2}{6}$

Both ratios are equal to $\frac{1}{3}$, so the sides are proportional. Therefore, the shapes are similar.

b)



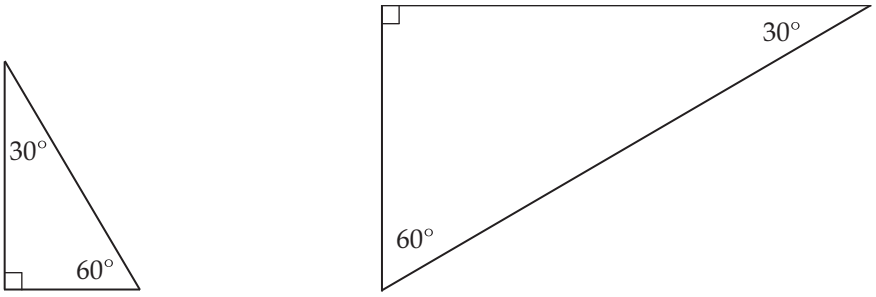
Answer:

$\frac{15}{5} : \frac{9}{4}$

$\frac{15}{3} = 3$, but $\frac{9}{4} = 2.25$

These ratios are not equal, so the shapes are not similar.

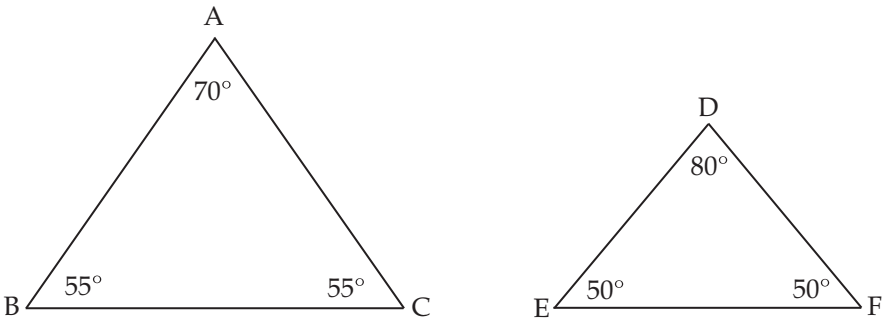
c)



Answer:

Same angles, therefore the shapes are similar.

d)

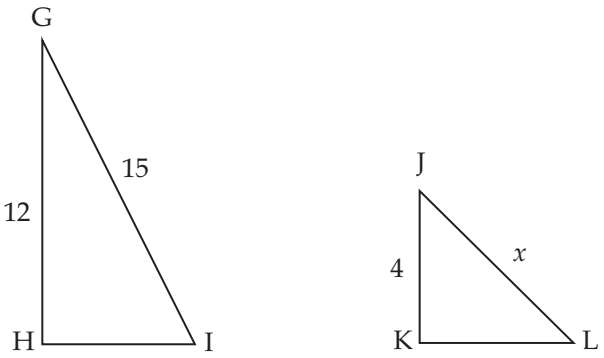


Answer:

The angles do not have the same measurements, so the shapes are not similar.

2. Set up proportions to find the missing value, x, in the similar figures below.

a)



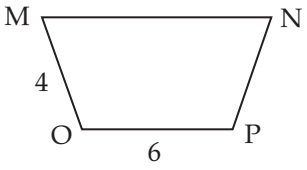
Answer:

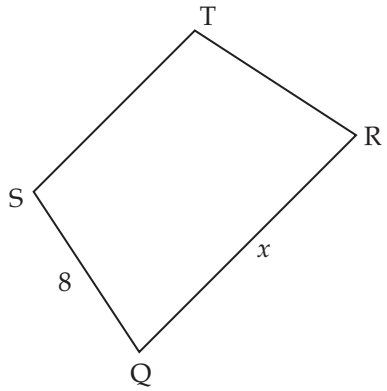
$$\frac{12}{15} : \frac{4}{x}$$

$$\frac{12}{15} \div \left(\frac{3}{3}\right) : \frac{4}{x}$$

$$x = 5$$

b)





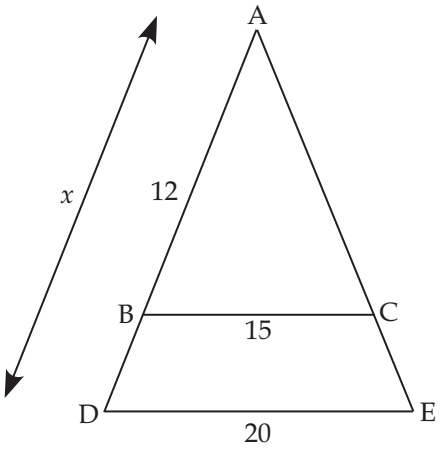
Answer:

$$\frac{6}{4} : \frac{x}{8}$$

$$\frac{6}{4} * \left(\frac{2}{2}\right) : \frac{x}{8}$$

$$x = 12$$

c) Look for the little triangle inside the larger triangle.



Answer:

$$\frac{15}{20} : \frac{12}{x}$$

$$(20x)\frac{15}{20} : \frac{12}{x}(20x) \quad \text{Multiply by LCM}$$

$$15x : 240$$

$$\frac{15x}{15} : \frac{240}{15}$$

$$x = 16$$

Learning Activity 7.6: Polygon Problems

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Which is largest, $\frac{2}{3}$, $\frac{3}{5}$, or $\frac{9}{15}$?
2. What is the area of a rectangular classroom that measures 10 m by 12 m?
3. If 25% of the area of the lawn you are mowing is 18 m^2, what is the total area of the lawn?
4. The measure of angle B is double the measure of angle A. If angle A measures 30°, what is the sum of angles A and B?
5. 16 of 20 students in your class watched the hockey game last night. What percentage does this represent?
6. Nadia earns $85 a week at work. How much does she earn after 6 weeks of work?
7. How much money do you have if your wallet contains twelve $5 bills, eight $10 bills, and fifteen $20 bills?
8. How much will 5 chocolate bars cost if each one is $0.98?

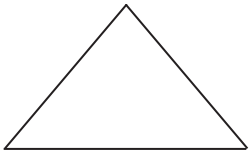
Answers:

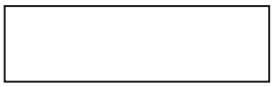
1. $\frac{2}{3}$ ($\frac{3}{5}$ and $\frac{9}{15}$ are two equivalent fractions; $\frac{2}{3}$ is equivalent to $\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$ which is larger than $\frac{9}{15}$)
2. 120 m^2
3. 72 m^2 (25% is equivalent to $\frac{1}{4}$, so the total area of the lawn is 4 x 18 or 4 x 10 + 4 x 8)
4. 90° (30° + 2 x 30° = 30° + 60°)
5. 80% ($\frac{16}{20} = \frac{8}{10} = 0.8$)
6. $510 (break down 85 into 80 and 5; 6 x 80 = 480 and 6 x 5 = 30; 480 + 30 = 510)
7. $440 (12 x 5 = 60; 8 x 10 = 80; 15 x 20 = 300; 60 + 80 + 300 = 440)
8. $4.90 ($0.98 is 2¢ short of $1; so 5 x $1 – 5 x $0.02 = $5.00 – $0.10)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Use your ruler and protractor to draw a polygon similar to the given shape, and explain how it is similar.

   a) 

   b) 

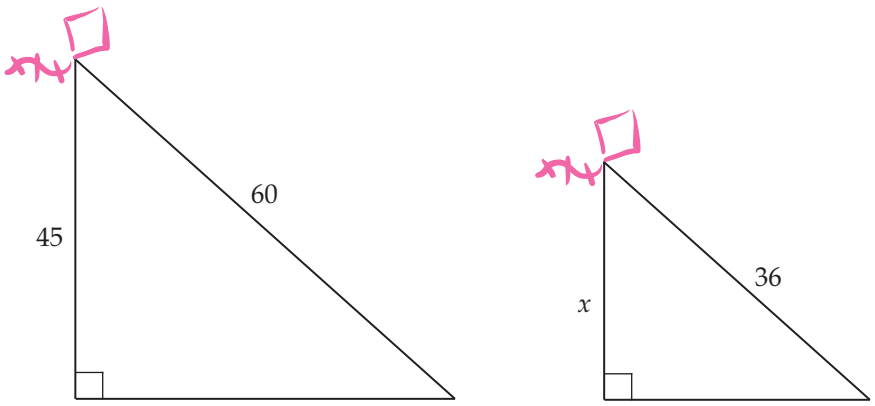
   Answer:

   For (a) and (b).

   Diagrams may vary. You can expand or shrink the given polygon, but the sides must have all been multiplied or divided by the same value and the angles must remain the same.

2. You are flying a kite and have let 60 feet of string off the spool. The wind has carried the kite so it is hovering a distance away from you, 45 feet above the ground. If you were to reel in some string so only 36 feet of string is out, how high would the kite be off the ground, if the wind remains constant? Draw similar right triangles to illustrate this situation, label them, and solve the problem.

Answer:



Using the LCM method:

$$\frac{45}{60} = \frac{x}{36}$$

$$\frac{(180)45}{60} = \frac{x(180)}{36}$$

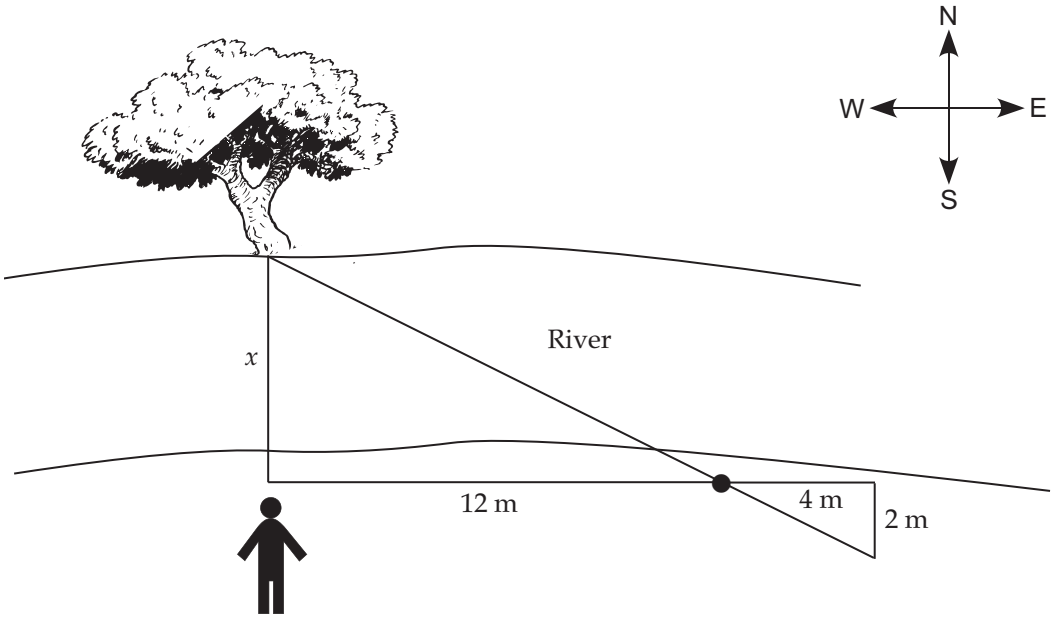
$$(3)\frac{\cancel{(180)}45}{\cancel{60}} = \frac{x\cancel{(180)}}{\cancel{36}}(5)$$

$$135 = 5x$$

$$x = 27$$

The kite would be 27 feet above the ground.

3. Refer to the diagram below. You are standing beside a river, looking north at a tree growing on the other side. You want to determine the width of the river using the properties of similarity. You walk 12 m east along the riverbank, and make a mark on the ground. Then you move another 4 m toward the east. From this point, you turn directly south for 2 m and then return to your original mark so you are in line with the tree across the river. With these measurements you can determine the width of the river.



Answer:

Be sure that you have set up the correct corresponding sides in the proportion.

$$\frac{12}{x} = \frac{4}{2}$$

$$(2x)\frac{12}{x} = \frac{4}{2}(2x)$$

$$24 = 4x$$

$$x = 6$$

The river is 6 m wide.

Learning Activity 7.7: Using Scales

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What is the square of 80?
2. What is half of 5.20?
3. What decimal is equivalent to $5\frac{2}{5}$?
4. Two consecutive numbers have a sum of 15. Which is the smaller of the two numbers?
5. In a population, 35% of people are vegetarian. What percentage of people are not vegetarians?
6. You buy a hat for $7.40 with a $20 bill. How much change do you get?
7. Evaluate $2^2 + 4^2 + 6^2$.
8. Estimate the angle made by the hands of a clock at 12:30.

Answers:

1. 6400
2. 2.60
3. $5.4 \left(\frac{2}{5} \text{ is equivalent to } \frac{4}{10} \text{ or } 0.4, \text{ so } 5+0.4 \right)$
4. 7 (7 + 8 = 15)
5. 65% (100 – 30 – 5 = 65)
6. $ 12.60 ($0.60 to get to $8 and $12 to get to $20)
7. 56 (4 + 16 + 36 = 20 + 36)
8. 180° (the two hands are diametrically opposed to make a straight angle.)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. The length of the highway from Winnipeg to Thompson is about 750 km. If you use a scale factor of 1 cm : 75 km, how long would the line be on your drawing?

   Answer:

   1 cm : 75 km

   $$\frac{\text{cm}}{\text{km}} \quad \frac{1}{75} = \frac{x}{750}$$

   $$\frac{1}{75} * \left(\frac{10}{10}\right) = \frac{x}{750}$$

   $$x = 10$$

   The line would be 10 cm long.

2. The dimensions of your rectangular-shaped room are 4 m by 2.5 m. Determine an appropriate scale factor to use in creating a scale diagram, and draw the scale diagram.

   Answer:

   1 : 100 or 1 cm : 1 m

   Draw a rectangle 4 cm x 2.5 cm.

3. You have captured a mosquito and want to enlarge it on a scale drawing. The mosquito measures 3 mm by 2 mm. Determine an appropriate scale factor to use in creating a scale diagram, and draw the scale diagram.

   Answer:

   A possible scale could be 10 : 1 or 1 cm : 1 mm. The diagram should then be 3 cm x 2 cm.

4. A map using a scale of 1:500 000 shows the location of a lake. The length of the lake on the drawing is 2.5 cm. How long is the actual lake?

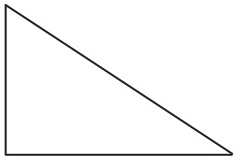
   Answer:

   1 : 500 000

   2.5 cm : 1 250 000

   There is 100 cm in a metre, or 1000 m in an kilometre, so the lake is 12 500 m long, or 12.5 km.

5. You have a right triangular–shaped flower garden that measures 4 m by 6 m along the sides (not the hypotenuse). Is this diagram proportional to the flower garden? If so, what scale factor was used to make this diagram?



   Answer:

   1 cm : 2 m or 1 : 200

   The diagram is proportional.

NOTES

GRADE 9 MATHEMATICS (10F)

Module 8: Symmetry

MODULE 8: SYMMETRY

Introduction

The word symmetry is derived from "syn," which means together, and "metron" indicating measurement. If a shape is symmetrical, it can be divided into equal parts.

This module introduces you to line symmetry and rotational symmetry. You will look at shapes, designs, art, and tessellations to see what kind of symmetry they have. Who said math couldn't be beautiful?

Assignments in Module 8

You will need to complete the following four assignments and send them to your tutor/marker when you are done this module.

Lesson	Assignment Number	Assignment Title
1	Assignment 8.1	Cover Assignment
2	Assignment 8.2	Line Symmetry
3	Assignment 8.3	Rotation Symmetry
4	Assignment 8.4	Art and Tessellations

Resource Sheet

When you write your exams, you are allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½ '' by 11", with both sides in your handwriting or typewritten. You will submit it with your exam. The Exam Resource Sheet is not worth any marks.

Many students have found making a Resource Sheet an excellent way to review. It also provides you with a summary of the important facts of each module that is available when you need it. You should complete a Resource Sheet for each module to help with your studying and reviewing. You can use the lesson summaries and module summaries, at the end of each lesson and module respectively, to check your Resource Sheet for completeness.

In an attempt to prepare you for making your Resource Sheet, a list of instructions is provided below for you to follow as you complete Module 8. You might use your Module 8 Resource Sheet for math terms, or formulas, or sample questions, or a list of places where you often make mistakes. You might write out what you need, or you might refer to page numbers in the lessons that you would like to focus on when studying for the exams. Throughout the module, helpful terms and formulas will be emphasized, but do not feel like you are restricted to *just* these things. You can include any information from the course that you think will be helpful to have in your exam.

Module 8 Resource Sheet

1. List the math terms introduced in each lesson.
2. List any the formulas stated in each lesson.
3. Which strategies for making calculations were discussed in each lesson?
4. Which questions need to be copied onto your Resource Sheet as being representative of the questions in each lesson?
5. Which questions were the most difficult? List page numbers on your Module 8 Resource Sheet so that you can redo these questions before the exam. If any of these problems are "sticklers," you could then write the problems and solutions on your Exam Resource Sheet so that you have them with you during the exam.
6. What other reminders do you need to make to yourself to help you prepare for the exam?

When you get to the end of this module, you will be writing the final exam supervised by a proctor. You need to make the following arrangements about three weeks before writing your exam. Here's what you need to do:

- **If you are attending school,** ask your school's Independent Study Option (ISO) Facilitator to add your name to the ISO exam eligibility list.
- **If you are not attending school,** check the **Examination Request Form** for options available to you. The Examination Request Form was mailed to you with this course. Fill in this form and mail or fax it. The address is:

  ISO Registration
  555 Main St.
  Winkler, MB R6W 1C4
  Fax: 204-325-1719
  Phone: 1-800-465-9915

NOTES

Lesson 1: Cover Assignment

To solve problems in mathematics, you often have to look for patterns. You have practiced this skill in previous modules. For this assignment, you are going to explore a famous problem in mathematics, Euler's formula. Identifying a pattern will help to solve the problem.

NOTES

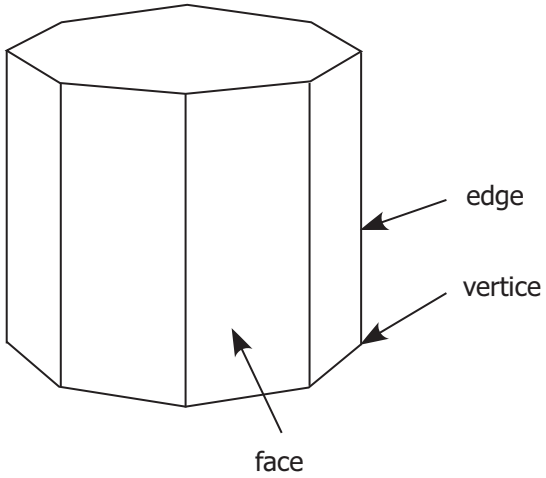
Assignment 8.1: Cover Assignment

Total: 16 marks

The mathematician Leonhard Euler discovered a relationship among the number of vertices, edges, and faces of polyhedrons. This relationship is called Euler's formula.

1. Complete the table on the following page by finding the number of vertices, v, edges, e, and faces, f, for each figure.
   (14 marks, 2 marks per line)

   For example:



2. For each figure, compare the number of edges with the sum of the number of vertices and number of faces. What pattern do you see? *(1 mark)*

   _____

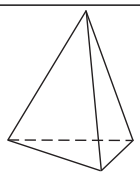
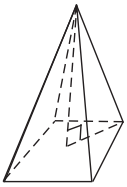
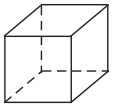
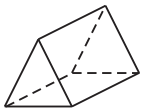
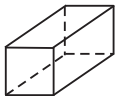
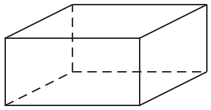
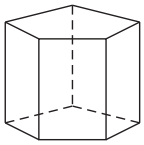
   _____

   _____

   _____

3. Write an equation to express the relationship between v, f, and e. *(1 mark)*

   _____

Polyhedron	Figure	v	f	e	$v + f$
Triangular Pyramid					
Square Pyramid					
Cube					
Triangular Prism					
Square Prism					
Rectangular Prism					
Pentagonal Prism					

Remember, you will forward this assignment to your tutor/marker at the end of Module 8.

Lesson 2: Line Symmetry

Lesson Focus

When you finish this lesson, you will be able to

- ❑ identify lines of symmetry within shapes
- ❑ complete a drawing of a shape given one half and a line of symmetry
- ❑ create or provide a piece of artwork demonstrating line symmetry

Lesson Introduction

Line symmetry involves drawing a straight line through a shape or object, dividing the shape or object into two or more equal parts. The line of symmetry can be in any direction.

Mirror Images

A mirror image of an object is a reflection of the object. The reflection in the mirror is an exact copy of the image when folded over the line of reflection. The actual placement of the mirror is the line of reflection. Since both sides of the image are equal, they are symmetrical. This is line symmetry.

Consider the letter A. Is there somewhere on this object where you could draw a line so that it divides the figure into two equal parts?

A

A vertical line drawn through the centre would create two symmetrical sides.

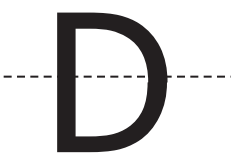
If you fold the first half across the line of symmetry, it sits nicely on top of the other side. Is there any other place on this object where you could draw a line of symmetry?

Example 1

Consider the letter D. Is there a possible line of symmetry in this image?

Solution:

A vertical line would not divide this into mirror images. A horizontal line through the centre would.

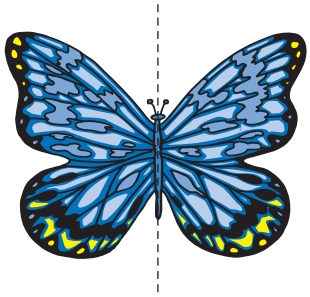


There is no other place you could draw a line of symmetry on this image.

Example 2

Here is a picture of a butterfly. Where can you draw a line of symmetry so that both sides would be perfectly matched?

Solution:

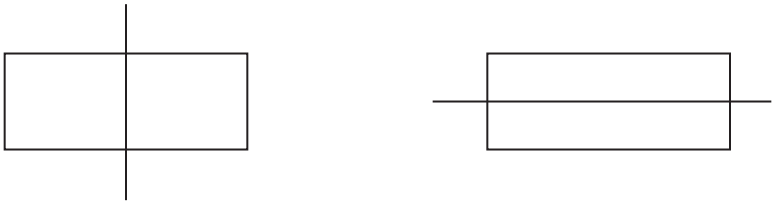


Notice how you could fold the butterfly over the line, and it would perfectly match the other side.

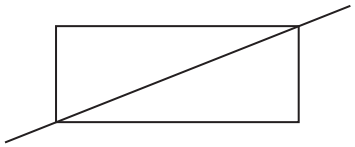
More than One Line of Symmetry

Some objects may have more than one line of symmetry.

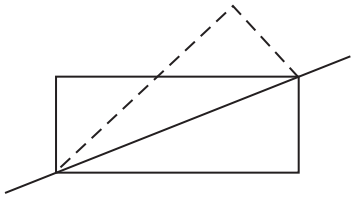
This rectangle can be reflected over a vertical line of symmetry and a horizontal line of symmetry.



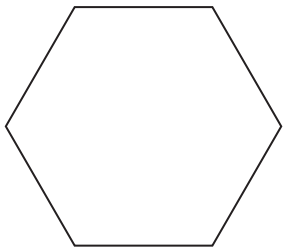
If you try to place a line of reflection along the diagonal, does it create symmetry?



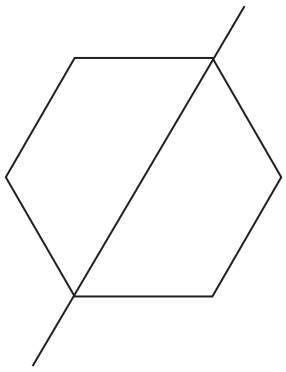
While the line does divide the rectangle into two congruent triangles, they are not symmetrical. The mirror image does not sit perfectly on top of the other half.



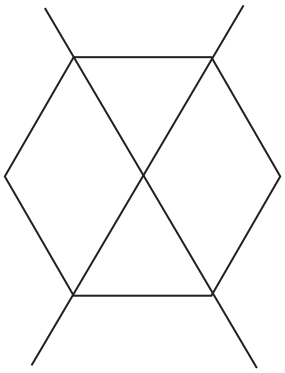
This is a regular (all sides equal) hexagon. How many lines of symmetry are possible?



Using your ruler, you could draw a line from one angle to the opposite vertex.

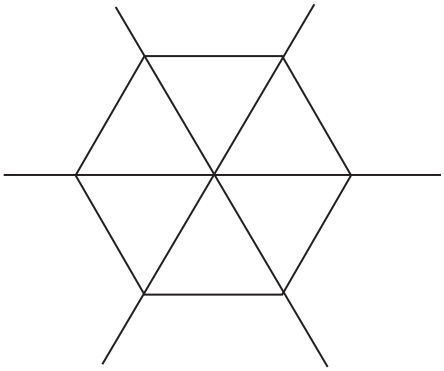


Can you see the exact copy when it is reflected over the line? Are there more possibilities?

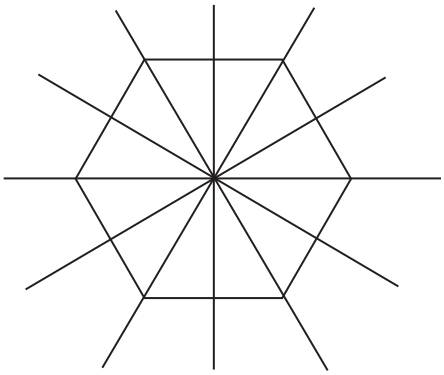


The second line also divides the object so that the one half reflects over the second half.

There are a total of 3 lines of symmetry for this object using the vertices.



But you are not limited to just the corners. More lines of symmetry can be drawn on the figure, at the midpoint of each side.



Here you can see a total of 6 lines of symmetry for this object.

Learning Activity 8.1

Determining Lines of Symmetry

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Evaluate 8 x 5 + 6 x 7.
2. Estimate 8.2 x 4.9 + 5.8 x 7.1.
3. You have walked for 3 hours at an average speed of 4.5 km/h. How far have you traveled?
4. Evaluate 5 x 6 + 18 ÷ 3.
5. In your class, $\frac{2}{9}$ of students prefer pizza to hot dogs. What fraction represents the number of students who prefer hot dogs?
6. How many bags of 5 apples can you fill with 325 apples?
7. Evaluate 3 x 3 x 3 x 3 + 3.
8. What is the amount of the discount if the price of a $32 flask is reduced by 20%?

continued

Learning Activity 8.1: Determining Lines of Symmetry (continued)

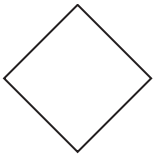
Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

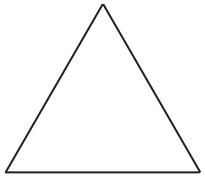
1. Given the following shapes and images, determine which have the following attributes.

   - no lines of symmetry _____
   - only one line of symmetry _____
   - only two lines of symmetry _____
   - three or more lines of symmetry _____

a)



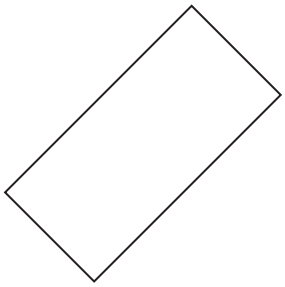
b)



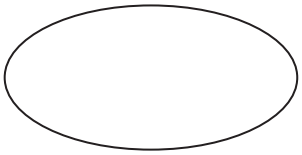
c)

d)

e)

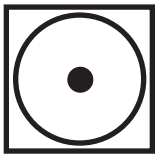


f)



g)

h)

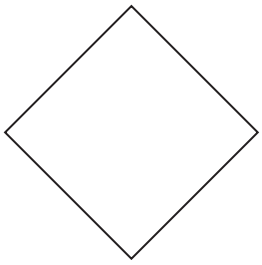


continued

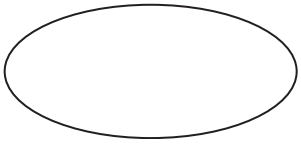
Learning Activity 8.1: Determining Lines of Symmetry (continued)

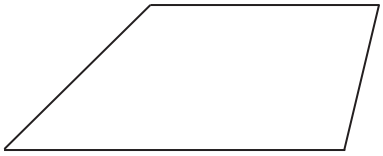
2. Draw a line of symmetry through each object or image, if possible.

a) 

b) 

c) Z

d) 

e) 

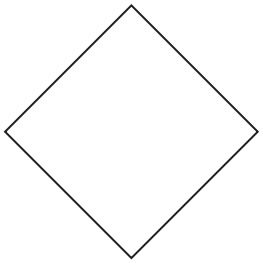
f) P

continued

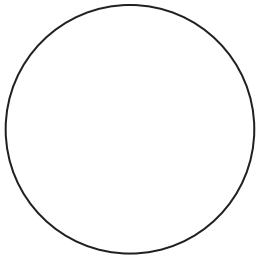
Learning Activity 8.1: Determining Lines of Symmetry (continued)

3. How many lines of symmetry can be drawn through each object? Draw them.

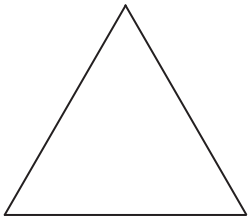
a)



b)



c)



d)

S

e)

Completing the Second Half

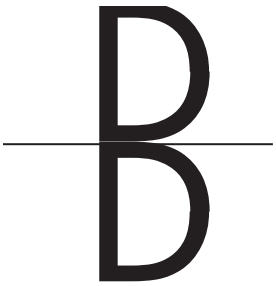
A line of symmetry occurs when you can fold an object over a line, and it sits perfectly on top of the other side.

Example 1

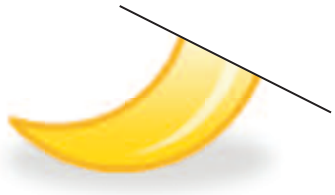
If you reflect this image over the line above it, what will the result look like? Draw it.

Solution:

It should look like the letter B.



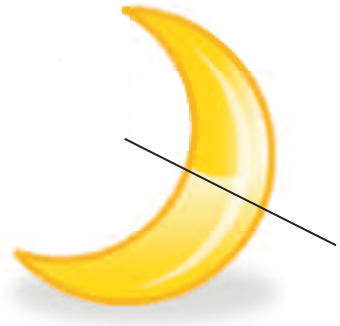
Example 2



Complete the design above by drawing the reflected image above the line of symmetry.

Solution:

Your drawing should look like this.



Lesson Summary

In this lesson, you learned about lines of symmetry. In the next lesson, you will explore rotation symmetry.

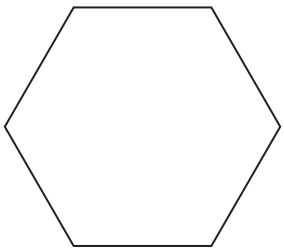
NOTES

Assignment 8.2: Line Symmetry

Total: 18 marks

1. How many lines of symmetry could be drawn on each figure? *(4 marks)*

a)

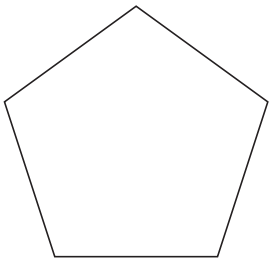


_____

b)

_____

c)



_____

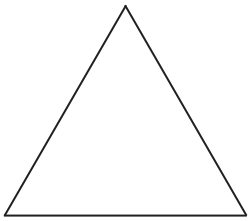
d)

_____

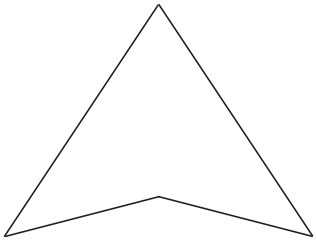
2. Draw in all possible lines of symmetry on the figures below. *(8 marks)*

a)

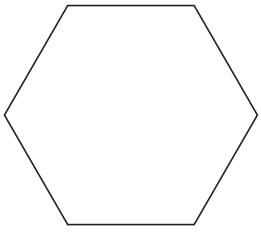
b)



c)

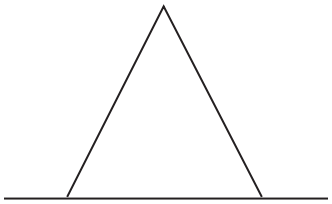


d)

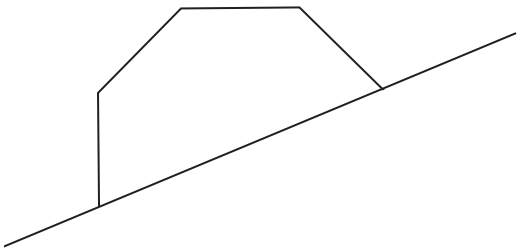


3. Draw the reflected half under the line of symmetry. *(4 marks)*

a)



b)



4. Draw an object or picture showing at least one line of symmetry. Be creative! *(2 marks)*

Lesson 3: Rotation Symmetry

Lesson Focus

When you finish this lesson, you will be able to

- ❑ describe and define rotation symmetry
- ❑ identify rotation symmetry in an object or image
- ❑ determine the order and angle of rotation in an object or image
- ❑ create or provide a piece of artwork demonstrating rotation symmetry

Lesson Introduction

Another type of symmetry is rotation symmetry. This involves the object or image turning. If it turns to a point where it looks exactly like the original, or fits onto itself, then it has rotation symmetry.

Order and Rotation

Some shapes or objects look the same, even when they are rotated.

The number of times an image repeats itself over one complete revolution (360°) is called "order." The "angle of rotation" is the smallest angle the figure needs to rotate to appear unchanged.

This would be helpful information to include on your Resource Sheet, especially since it is new math vocabulary.

If, for example, the letter S is rotated, or turned, about its centre point, it will look exactly like the original after a 180° or half turn.

S

If you then continue the rotation another 180°, it looks exactly like the original again.

In a full revolution of 360° the letter S appears unchanged two times. It has an order of 2. The angle of rotation is 180°.

You need to remember that all of the rotations are in a clockwise direction.

The angle of rotation can be calculated based on the order.

One complete revolution around a circle is 360°. If a shape repeats twice (order of 2), then $\frac{360°}{2} = 180°$.

If the shape repeats 3 times, then°, $\frac{360°}{3} = 120°$, etc.

Order of 1: 360°

Order of 2: 180°

Order of 3: 120°

Order of 4: 90°

Order of 5: 72°

Order of 6: 60°

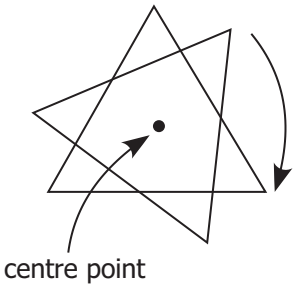
All shapes, when rotated 360°, will return to their original position. This is referred to as an order of 1. Some shapes only have an order of 1, such as this T-shape:

Any shape that has an order of 1 is not considered to have rotation symmetry, even though it repeats itself after rotating 360°. In other words, a shape that has rotation symmetry must have an order that is greater than or equal to 2.

This is an important property of rotation symmetry. Include it in your Resource Sheet.

Rotating Around the Centre

Example 1



Here is an equilateral triangle. Do you think it has rotation symmetry? If so, what order, and at what angle(s) of rotation?

Solution:

This shape will fit onto itself three times as it rotates around the centre point, so it has order 3 rotation symmetry. Because this occurs three times over 360°, or in one complete revolution, the angle of rotation is calculated to be $\frac{360°}{3} = 120°$.

Example 2

Does the crescent moon have rotation symmetry? You are mentally turning the shape, and looking for when it fits onto itself, and how many times.

Solution:

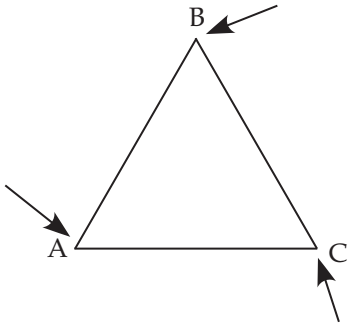
It never fits onto itself as it rotates, so it does not have rotation symmetry.

Rotating Around a Vertex

A vertex is defined as the point of an angle, or the common endpoint of two line segments (plural: vertices).

Example 1

This shape has three vertices, labelled A, B, and C:

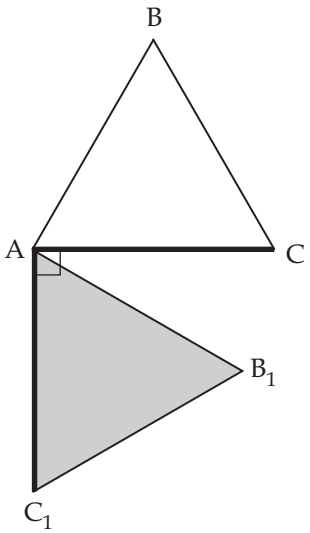


If you pick one of these vertices, and rotate the shape 90° around that vertex, what would it look like?

Solution:

Take point A, and rotate the shape 90° about point A in a clockwise direction. Point A stays where it is, and the shape rotates 90° about point A.

If this is confusing, try tracing and cutting out the shape, and manually turning it to see what is happening.

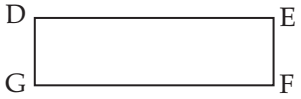


The shaded shape is the rotation 90° about vertex A.

Notice that the vertices on the rotated figure that have moved have been relabelled using the same letters as the original, but with a subscript of 1.

Rotation around a vertex can be confusing, so include an example of it on your Resource Sheet.

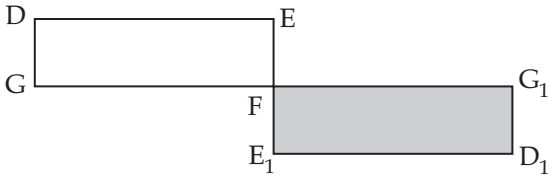
Example 2



Given rectangle DEFG, rotate the shape 180° about vertex F.

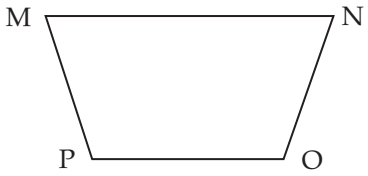
What would the resulting image look like?

Solution:



Example 3

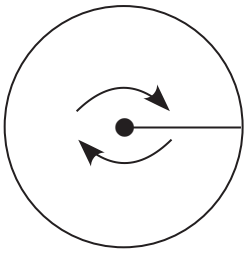
This question should challenge you! Take the shape below, and apply an angle of rotation of 60° about vertex M.

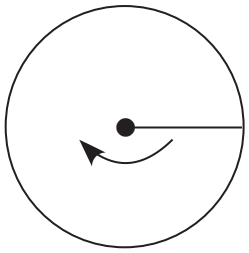


Solution:

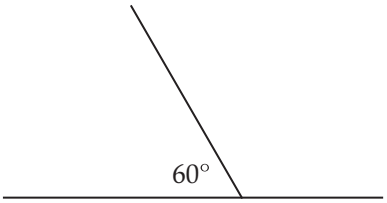
Now, an angle of 60° is fairly easy to visualize, if you break things down slowly.

One full rotation of the circle is 360°. So, a half rotation would be 180°.

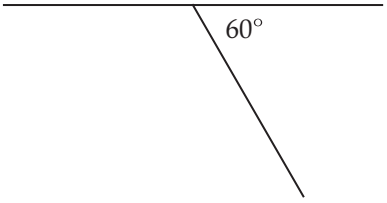




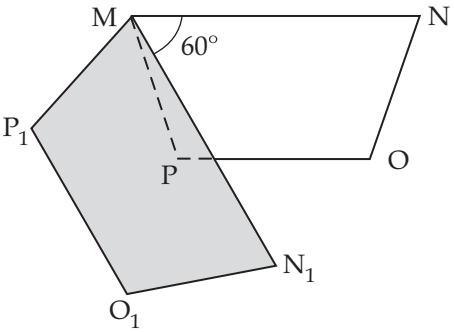
Basic mental math tells you that 60° is one-third of 180°, and that looks like this:



Because you are rotating clockwise from point M, the shape will move downward:



The image will rotate in a clockwise direction 60° about vertex M. So, vertex M stays where it is, and everything else turns 60°.



The shaded area is the trapezoid rotated 60° about vertex M.

Learning Activity 8.2

Order and Rotation Problems

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You buy a gift valued at $1.75, including taxes, for each of your three friends. Do you have enough money to buy the gifts if you pay with a $5 bill?
2. You want to travel 47 kilometres by bike. You think you can pedal at an average speed of 15 km/h. Estimate the time it will take you to complete your trip.
3. A soccer game consists of two 45-minutes halves. Between halves, there is a 15-minute break and there is approximately 5 minutes of extra time due to stoppages in play. Will a soccer game last more than 2 hours?
4. Estimate the 5% GST on a purchase of $89.
5. You buy a badminton racquet for $41 that regularly sells for $82. What is the percent price reduction?
6. Solve for m: $8 + m = -3$.
7. If a single rabbit can eat 2 flowers in 3 minutes, how many flowers can 4 rabbits eat in 9 minutes?
8. Jack painted $\frac{1}{3}$ of his room one day. At this rate, how many days will it take him to finish painting the entire room?

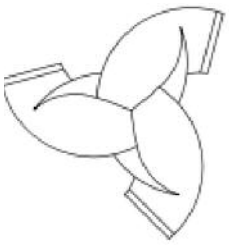
continued

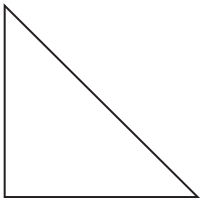
Learning Activity 8.2: Order and Rotation Problems (continued)

Part B

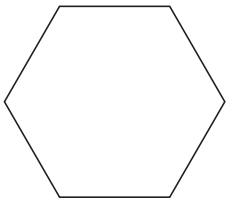
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Describe in your own words the meaning of the term "order."
2. Identify which shapes have rotation symmetry, and, if so, determine the order and angle of rotation.

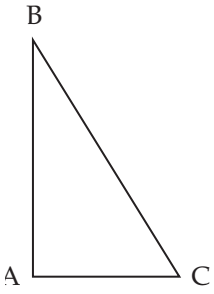
a) 

b) 

c) 

d) 

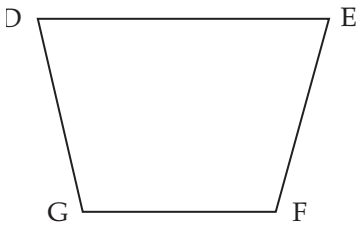
3. Rotate this shape 90° about vertex A, and draw the resulting image. Label the points on the new image.



continued

Learning Activity 8.2: Order and Rotation Problems (continued)

4. Rotate this shape 270° about vertex F, and draw the resulting image. Label the points on the new image.



Lesson Summary

In this lesson, you examined rotation symmetry and learned about order and the angle of rotation.

NOTES

Assignment 8.3: Rotation Symmetry

Total: 18 marks

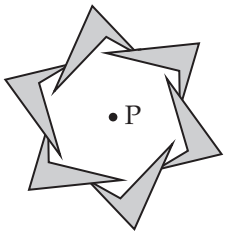
1. Describe in your own words the meaning of the term "angle of rotation." *(2 marks)*

2. Identify which shapes have rotation symmetry, and, if so, determine the order and angle of rotation. *(8 marks)*

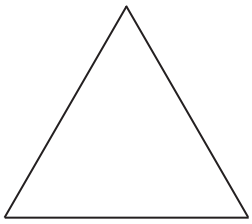
a)

b)

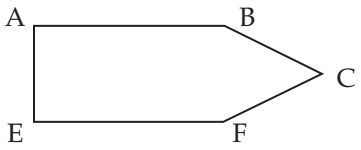
c)



d)



3. Rotate this shape 180° about vertex A, and draw the resulting image. Label the points on the new image. *(4 marks)*



4. Draw an object or picture with rotational symmetry of an order of 2 or higher. State the order and angle of rotation. Be creative! *(4 marks)*

Lesson 4: Art and Tessellations

Lesson Focus

When you finish this lesson, you will be able to

- ❑ identify and describe the types of symmetry in artwork and logos
- ❑ identify and describe the types of symmetry in tessellations

Lesson Introduction

Artwork and company logos are displayed almost everywhere you look—on the walls, in the media, on the clothes you wear, on billboards, and in bus shelters. This lesson will guide you in identifying any line or rotation symmetry in art work, logos, or tessellations you may see around you.

Identifying Symmetry

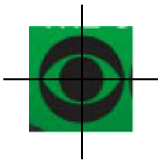
Our society regularly bombards us with pictures, company logos, and artistic patterns. Many of these shapes and diagrams have line and/or rotational symmetry to make the logo visually appealing as well as functional.

Example 1

Identify and describe the type(s) of symmetry in this logo.

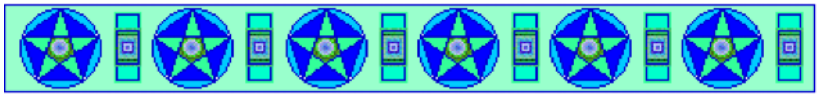
Solution:

There is both a horizontal and a vertical line of symmetry. It also has rotational symmetry of order 2. The angle of rotation is 180°.



Example 2

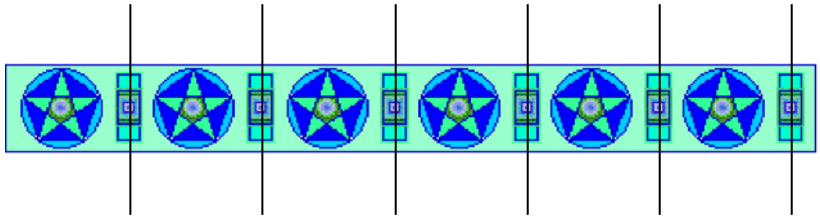
This artistic work consists of a repeated pattern. Identify and describe the symmetry.



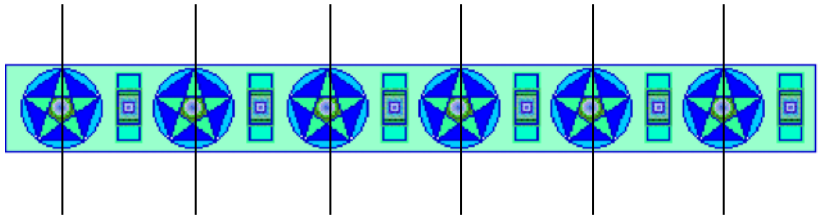
Solution:

First find any line of symmetry.

Where could you draw a line of reflection so that if the piece were folded over that line, it would fit onto the next piece? Since it is a repeated pattern, there are many possibilities.

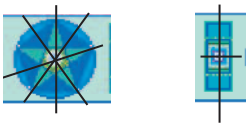


or



Can you identify any rotation symmetry in this example?

The star motif, taken individually, would have order 5 rotational symmetry and the column would have order 2.



Tessellations

A tessellation is a repeated pattern that has no gaps between the shapes.

Include this definition in your Resource Sheet.

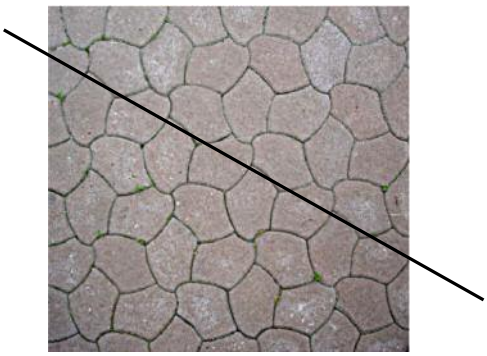
Example 1

This is a picture of paving stones in a walkway. Identify any line or rotation symmetry, and describe what you see.

Solution:

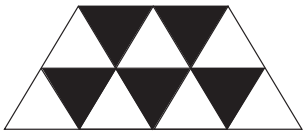
Look at one stone, and follow it as it turns. There is a pattern of three stones to a circle.

That is a rotation order of 3. The angle of rotation would be 360°/3 = 120°. This three stone pattern is repeated along the diagonal, but it does not have line symmetry.



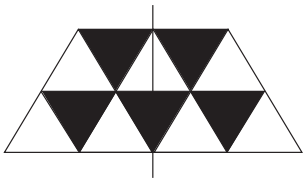
Example 2

Identify any line or rotation symmetry in this tessellation and describe it.



Solution:

Line symmetry: This line indicates where you could fold the shape and it would fit onto the reflected shape.



Rotation symmetry: Each of the individual triangles can be rotated 120° around a vertex and sit perfectly on top of another same-coloured segment.

Learning Activity 8.3

Working with Tessellations

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What two numbers have a sum of 26 and a product of 165?
2. What angle is complementary to an angle that measures 42°?
3. A football team scored as follows: 3 touchdowns for 6 points each, 2 one-point conversions, and 4 three-point field goals. How many points did they score?
4. Calculate the product of 46 and 0.5.
5. Calculate the quotient when 34 is divided by 0.5.
6. Calculate $0.5 \times 51 \div 0.5$.
7. Express $\frac{19}{5}$ as a mixed fraction.
8. The value of $\sqrt{211}$ is found between which two consecutive whole numbers?

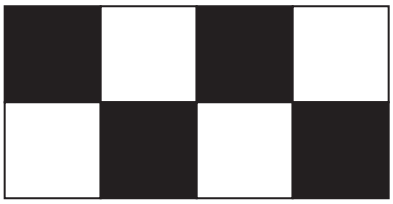
continued

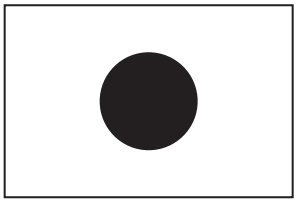
Learning Activity 8.3: Working with Tessellations (continued)

Part B

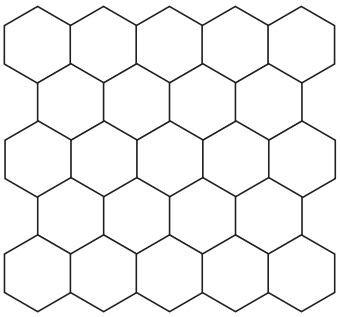
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify and describe any line symmetry and/or rotation symmetry in the following shapes and designs. Draw the line(s) of reflection and state the order and the angle of rotation.

a) 

b) 

c) 

d) 

Lesson Summary

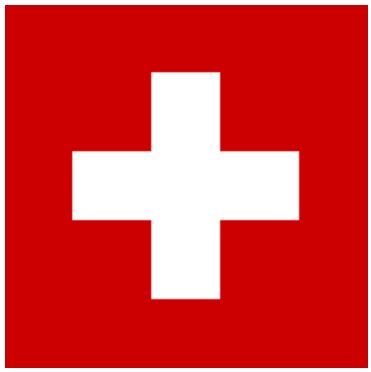
In this lesson, you looked for line and rotation symmetry in tessellations and art work.

Assignment 8.4: Art and Tessellations

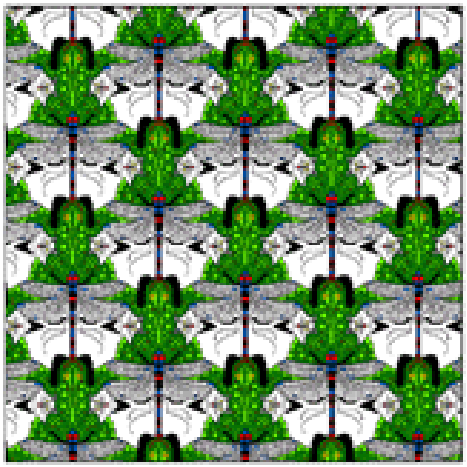
Total: 30 marks

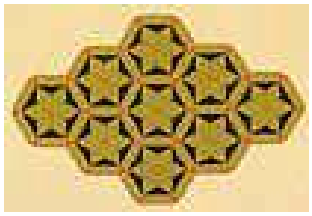
1. Identify, draw, and describe any line symmetry or rotation symmetry with the order and the angle of rotation in the following shapes and designs:

a) *(4 marks)*



b) *(4 marks)*

c)  *(4 marks)*

d)  *(4 marks)*

e)  *(4 marks)*

2. Now it is your turn to either create, or find in electronic or print media, a piece of artwork that shows line and rotation symmetry. Be creative! You may choose to draw or create your own example electronically. You must identify and describe the symmetry in your work of art in detail. If it is not your own creation, you must credit the artist and provide the source. *(10 marks)*

NOTES

MODULE 8 SUMMARY

Congratulations, you have finished this course!

This module completed the geometry component of this course and you investigated line and rotation symmetry in shapes, objects, and patterns. You saw how mathematics plays a part in creating beautiful and functional logos, and learned how to identify and describe the symmetry in art.

Mailing Your Assignments

Before you mail your completed assignments, please photocopy them first. That way, you will have a copy of your assignments in case they go missing. Please organize your material in the following order:

- Cover Sheet for Module 7
- Assignment 7.1: Cover Assignment
- Assignment 7.2: Composite Objects
- Assignment 7.3: Similarity
- Assignment 7.4: Drawing Polygons
- Assignment 7.5: Scale Drawings
- Cover Sheet for Module 8
- Assignment 8.1: Cover Assignment
- Assignment 8.2: Line Symmetry
- Assignment 8.3: Rotation Symmetry
- Assignment 8.4: Art and Tessellations

Place all of these assignments in an envelope and address it to:

ISO Tutor/Marker
555 Main St.
Winkler MB R6W 1C4

Remember: You can contact your tutor/marker at any time if you have questions.

On the following pages, you will find instructions for applying for the final exam, and information about the online practice exam.

Applying for Your Final Exam

You are now ready to write the final exam. It is worth 100 marks (25% of your final mark). You will complete this exam while being supervised by a proctor. You should already have made arrangements to have the exam sent to the proctor from the Independent Study Option office. If you have not done so, contact the Independent Study Option at 1-800-465-9915.

In order to do well on the midterm exam, you should review all of the work that you have completed from Modules 1 to 8, review all of your notes, learning activities, and assignments.

You need to bring the following when you write the exam: pens/pencils (2 or 3 of each), blank paper, a calculator, a ruler, and a protractor.

You will have a maximum of **2 hours** to complete this exam. When you have completed the exam, you will hand it in to your proctor, who will then forward it to your tutor/marker for assessment.

Here is how you apply for an exam:

If you are attending school, ask your school's ISO Facilitator to add your name to the ISO exam eligibility list. Do this ***at least three weeks prior*** to the next scheduled exam week.

If you are not attending school, check the **Examination Request Form** for options available to you. The Examination Request Form was mailed to you with this course. Fill in this form and mail or fax it three weeks before you are ready to write your exam. The address is:

ISO Registration
555 Main St.
Winkler, MB R6W 1C4
Fax: 204-325-1719
Phone: 1-800-465-9915

Note: The Final Practice Exam is an excellent study aid for reviewing Modules 1 to 8. Instructions for accessing and completing the Final Practice Exam are found on the following page.

Completing Your *Final Practice Exam*

Getting the Most Out of Your Final Practice Exam

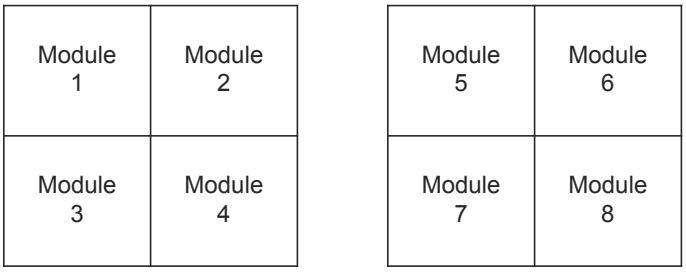
Like the Final Exam that you will be writing, your Final Practice Exam is based on Modules 1 to 8. It is very similar to the actual Final Exam. This means that if you do well on your Final Practice Exam, you should do well on the Final Exam, because you will lave learned the content. You will also feel more confident and less nervous about writing the exam.

The practice exam and answer key can be found at the following website: <www.edu.gov.mb.ca/k12/dl/downloads/index.html>. Complete the final practice exam and then check your answers against the key.

If you do not have access to the Internet, contact the Independent Study Option at 1-800-465-9915 to get a copy of the practice exam and its answer key.

To get the most out of your Final Practice Exam, follow these steps:

1. Study for the Practice Exam as if it were an actual exam.
2. Review those Learning Activities and Assignments from Modules 1 to 8 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Ask your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 4, including all of your notes, learning activities, and assignments.
5. Use your Module Resource Sheets to make a draft of your Final Exam Resource Sheet. You can use both sides of an 8½" by 11" piece of paper. One way of organizing it could be:



(front) (back)

6. Bring the following to the Final Practice Exam: pens/pencils (2 or 3 of each), blank paper, a calculator, a ruler, and a protractor.

7. Write your Final Practice Exam as if it were an actual exam. In other words, write the entire exam in one sitting, and don't check your answers until you have completed the entire thing.
8. Once you have completed the entire exam, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your Resource Sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Final Exam Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

GRADE 9 MATHEMATICS (10F)

Module 8: Symmetry

Learning Activity Answer Keys

MODULE 8: SYMMETRY

Learning Activity 8.1: Determining Lines of Symmetry

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. Evaluate 8 x 5 + 6 x 7.
2. Estimate 8.2 x 4.9 + 5.8 x 7.1.
3. You have walked for 3 hours at an average speed of 4.5 km/h. How far have you traveled?
4. Evaluate 5 x 6 + 18 ÷ 3.
5. In your class, $\frac{2}{9}$ of students prefer pizza to hot dogs. What fraction represents the number of students who prefer hot dogs?
6. How many bags of 5 apples can you fill with 325 apples?
7. Evaluate 3 x 3 x 3 x 3 + 3.
8. What is the amount of the discount if the price of a $32 flask is reduced by 20%?

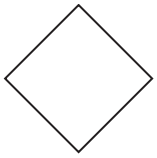
Answers:

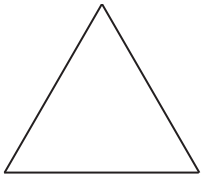
1. 82 (40 + 42; pay attention to the order of operations)
2. Approximately 82 (You can round the figures to estimate: 8 x 5 + 6 x 7.)
3. 13.5 km (3 x 4.5 is 3 x 4 + 3 x 0.5 which is 12 + 1.5)
4. 36 (30 + 6; pay attention to the order of operations)
5. $\frac{7}{9}\left(1-\frac{2}{9}=\frac{9}{9}-\frac{2}{9}\right)$
6. 65 bags (break down 325 to 300 and 25, then 300 ÷ 5 = 60 and 25 ÷ 5 = 5; 60 + 5 = 65)
7. 84 (9 x 9 + 3 = 81 + 3)
8. $\$6.40\left(\frac{20}{100}\times 32=\frac{640}{100}\right)$

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

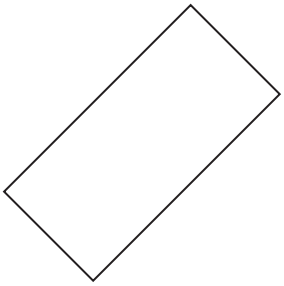
1. Given the following shapes and images, determine which have the following attributes.
   - no lines of symmetry *Answer:* (c)
   - only one line of symmetry *Answer:* (d), (g)
   - only two lines of symmetry *Answer:* (e), (f)
   - three or more lines of symmetry *Answer:* (a), (b), (h)

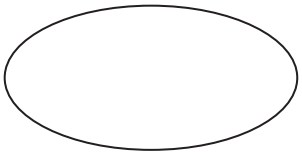
a) 

b) 

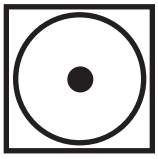
c) 

d) 

e) 

f) 

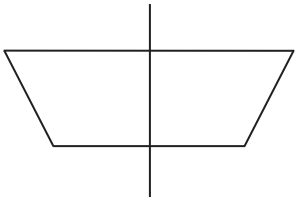
g) 

h) 

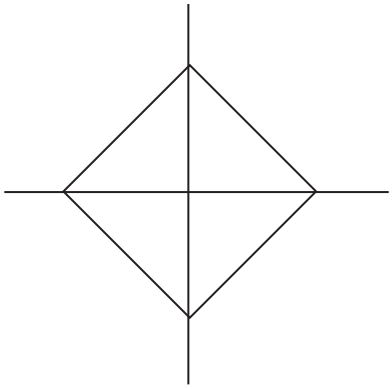
2. Draw a line of symmetry through each object or image, if possible.

Answers:

a)



b)



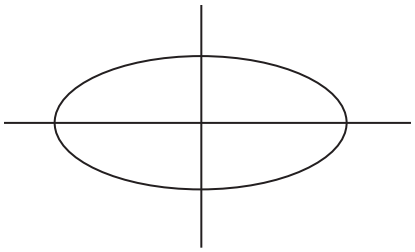
Either one is correct.

c)

Z

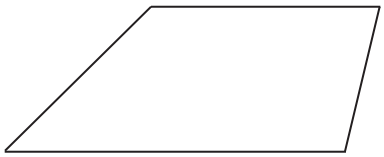
There is no possible line of symmetry.

d)



Either one is correct.

e)



There is no possible line of symmetry.

f)

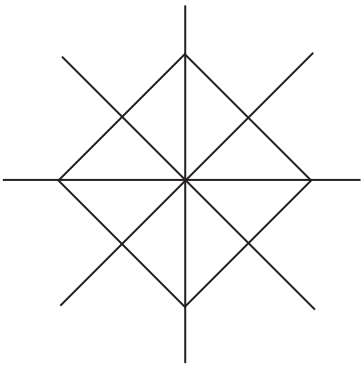
P

There is no possible line of symmetry.

3. How many lines of symmetry can be drawn through each object? Draw them.

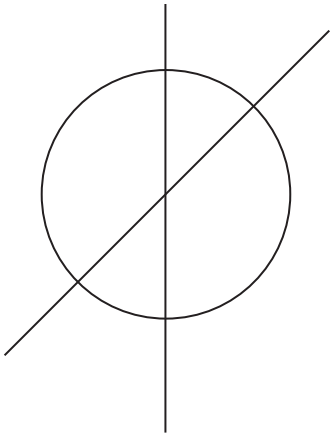
Answers:

a)



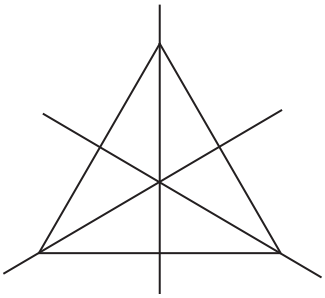
4 lines of symmetry

b)



Infinite number going through centre of circle.

c)



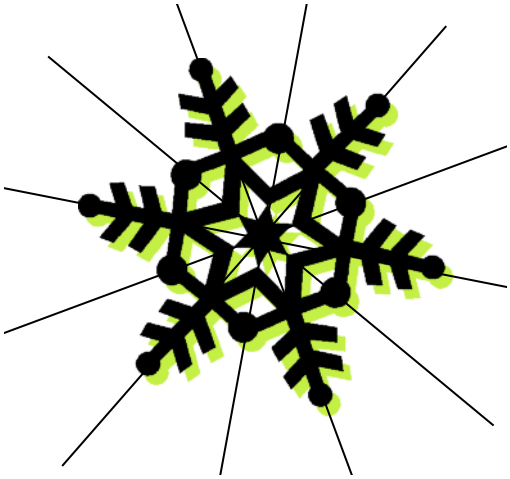
3 lines of symmetry

d)

S

No possible line of symmetry.

e)



6 lines of symmetry

Learning Activity 8.2: Order and Rotation Problems

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. You buy a gift valued at $1.75, including taxes, for each of your three friends. Do you have enough money to buy the gifts if you pay with a $5 bill?
2. You want to travel 47 kilometres by bike. You think you can pedal at an average speed of 15 km/h. Estimate the time it will take you to complete your trip.
3. A soccer game consists of two 45-minutes halves. Between halves, there is a 15-minute break and there is approximately 5 minutes of extra time due to stoppages in play. Will a soccer game last more than 2 hours?
4. Estimate the 5% GST on a purchase of $89.
5. You buy a badminton racquet for $41 that regularly sells for $82. What is the percent price reduction?
6. Solve for m: $8 + m = -3$.
7. If a single rabbit can eat 2 flowers in 3 minutes, how many flowers can 4 rabbits eat in 9 minutes?
8. Jack painted $\frac{1}{3}$ of his room one day. At this rate, how many days will it take him to finish painting the entire room?

Answers:

1. No (round $1.75 to $2, then 3 x $2 = $6; subtracting $3 x 0.25 or $0.75 gives $5.25 which is more than $5.)
2. More than 3 hours (break down 47 to 45 + 2; 45 is 15 x 3; need to add in some extra time for the 2 kilometres that remain.)
3. No (45 + 45 + 15 + 5 = 110 minutes; since there are 60 minutes in an hour, 2 hours is 2 x 60 = 120 minutes; 110 is less than 120, so less than 2 hours)
4. $4.50 $\left(\text{round 89 to 90 and } 0.05 \times 90 = \frac{5 \times 90}{100} = 4.5\right)$
5. 50% (41 is exactly half of 82)
6. –11 (need to subract 8 from –3)
7. 24 flowers (there are four times as many rabbits and 3 times more time to eat, so 4 x 3 x 2)
8. 2 days (it would take 3 days to paint the whole room and he has already spent one day painting)

Part B

Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

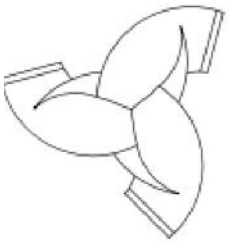
1. Describe in your own words the meaning of the term "order."

   Answer:

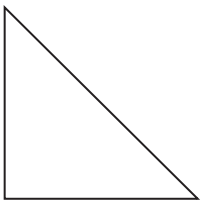
   Order is the number of times an image fits onto itself or appears unchanged, during one complete 360° revolution of the shape or object.

2. Identify which shapes have rotation symmetry, and, if so, determine the order and angle of rotation.

   Answers:

   a) 

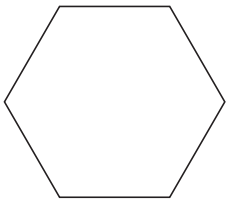
   Order 3, angle of rotation 120°

   b) 

   Does not have rotation symmetry.

   c) 

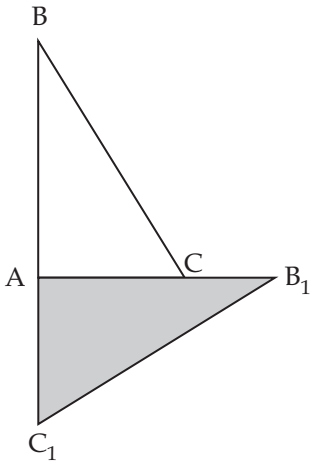
   Order 6, angle of rotation 60°

   d) 

   Order 6, angle of rotation 60°

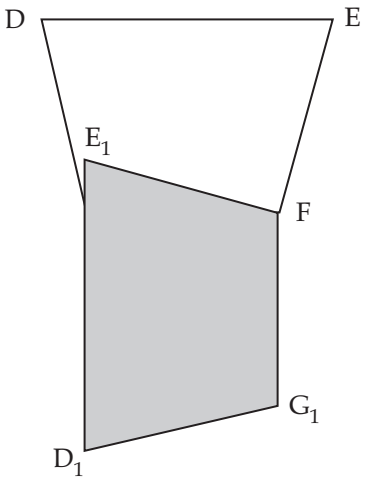
3. Rotate this shape 90° about vertex A, and draw the resulting image. Label the points on the new image.

   Answer:



4. Rotate this shape 270° about vertex F, and draw the resulting image. Label the points on the new image.

   Answer:



Learning Activity 8.3: Working with Tessellations

Part A

You should be able to complete the following eight questions in a few minutes without the use of a calculator or pencil and paper.

1. What two numbers have a sum of 26 and a product of 165?
2. What angle is complementary to an angle that measures 42°?
3. A football team scored as follows: 3 touchdowns for 6 points each, 2 one-point conversions, and 4 three-point field goals. How many points did they score?
4. Calculate the product of 46 and 0.5.
5. Calculate the quotient when 34 is divided by 0.5.
6. Calculate $0.5 \times 51 \div 0.5$.
7. Express $\frac{19}{5}$ as a mixed fraction.
8. The value of $\sqrt{211}$ is found between which two consecutive whole numbers?

Answers:

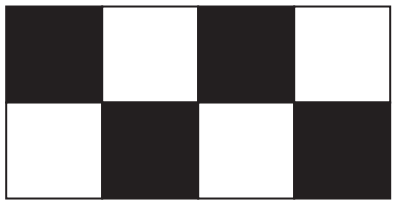
1. 15 and 11 (one of the numbers has to end in 5 and the other number has to end with 1 for the sum to end in 6 and the product to end with 5; the possible sums are 1 + 25, 5 + 21, and 11 + 15; the only numbers that give a product of 165 are 11 and 15)
2. 48° (90° – 42°)
3. 32 points ($3 \times 6 + 2 \times 1 + 4 \times 3 = 18 + 2 + 12 = 32$)
4. 23 (half of 46)
5. 68 (there are 68 halves in 34)
6. 51 (multiplication by 0.5 is cancelled by division by 0.5)
7. $3\frac{4}{5}$ (5 goes into 19 three times and the remainder is 4)
8. 14 and 15 ($15^2 = 225$ is the first perfect square after 211; so without knowing the perfect square, 14 is before 15)

Part B

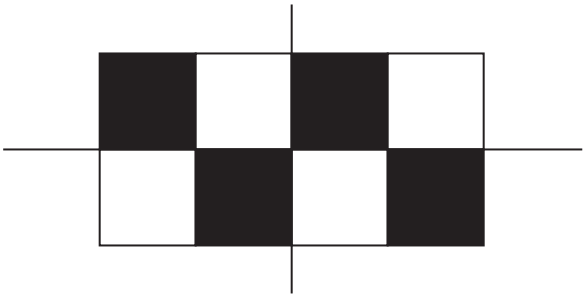
Remember, these questions are similar to the ones that will be on your assignments, your midterm, and final exams. So, if you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Identify and describe any line symmetry and/or rotation symmetry in the following shapes and designs. Draw the line(s) of reflection and state the order and the angle of rotation.

   a)



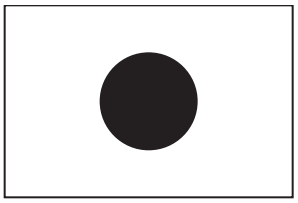
   Answer:



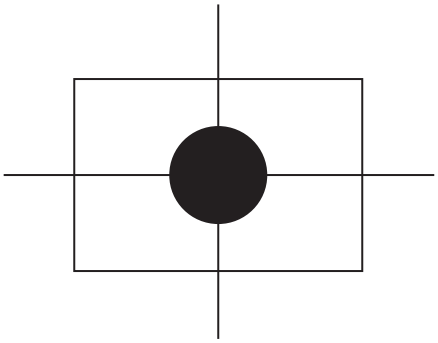
   The lines drawn onto the design demonstrate line symmetry. You could fold the shape over these lines and it would fit onto the image.

   There is rotational symmetry in the individual squares as well as in a group of 4 squares. From the centre of a group of four squares, you could rotate 180° and it would appear unchanged. From the vertex or corner of an individual square, you could rotate one square 180° and it would fit onto an identical square in the alternate level. This shows rotation symmetry of an order of 2, and with an angle of rotation of 180°.

b)



Answer:

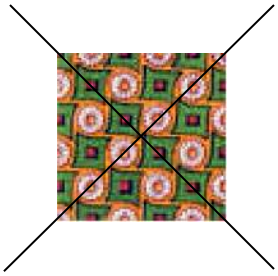


This flag has two lines of symmetry.

Using the centre point, the flag could be rotated around the centre 180° and would appear unchanged. This shows rotation symmetry with an order of 2 and an angle of rotation of 180°.

c)

Answer:

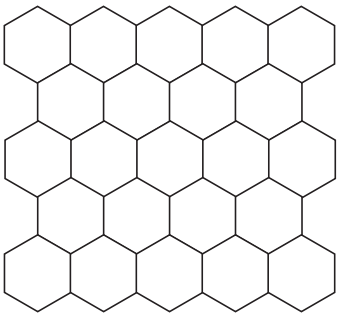


These diagonal lines drawn onto the design demonstrate line symmetry. You could fold the shape over these lines and it would fit onto the image.

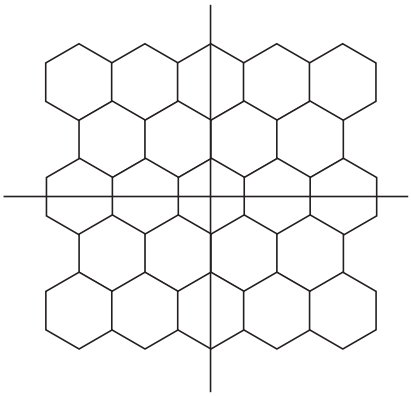
Also, any small square could be rotated 180° from one of its vertices and would fit onto an identical square in the alternate level. This shows rotation symmetry of an order of 2, and with an angle of rotation of 180°.

If you isolate either of the portions of the pattern above, you could rotate each a quarter turn and they would appear unchanged. This is rotation symmetry of an order of 4, with a 90° angle of rotation.

d)



Answer:



There are horizontal and vertical lines of symmetry possible in this tessellation. Rotation symmetry is also evident. Select the centre of one hexagon or the centre of a flower-shaped group of seven hexagons, and rotate it 60°. It will fit onto itself. If you continue this rotation six times, it returns to its original position. This is rotation symmetry with an order of 6 and an angle of rotation of 60°.

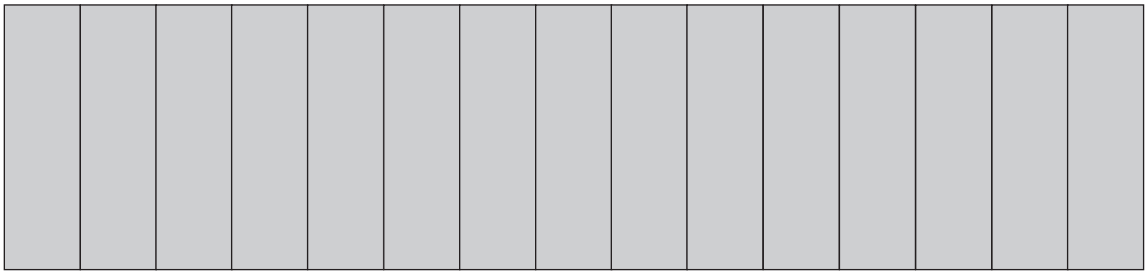
GRADE 9 MATHEMATICS (10F)

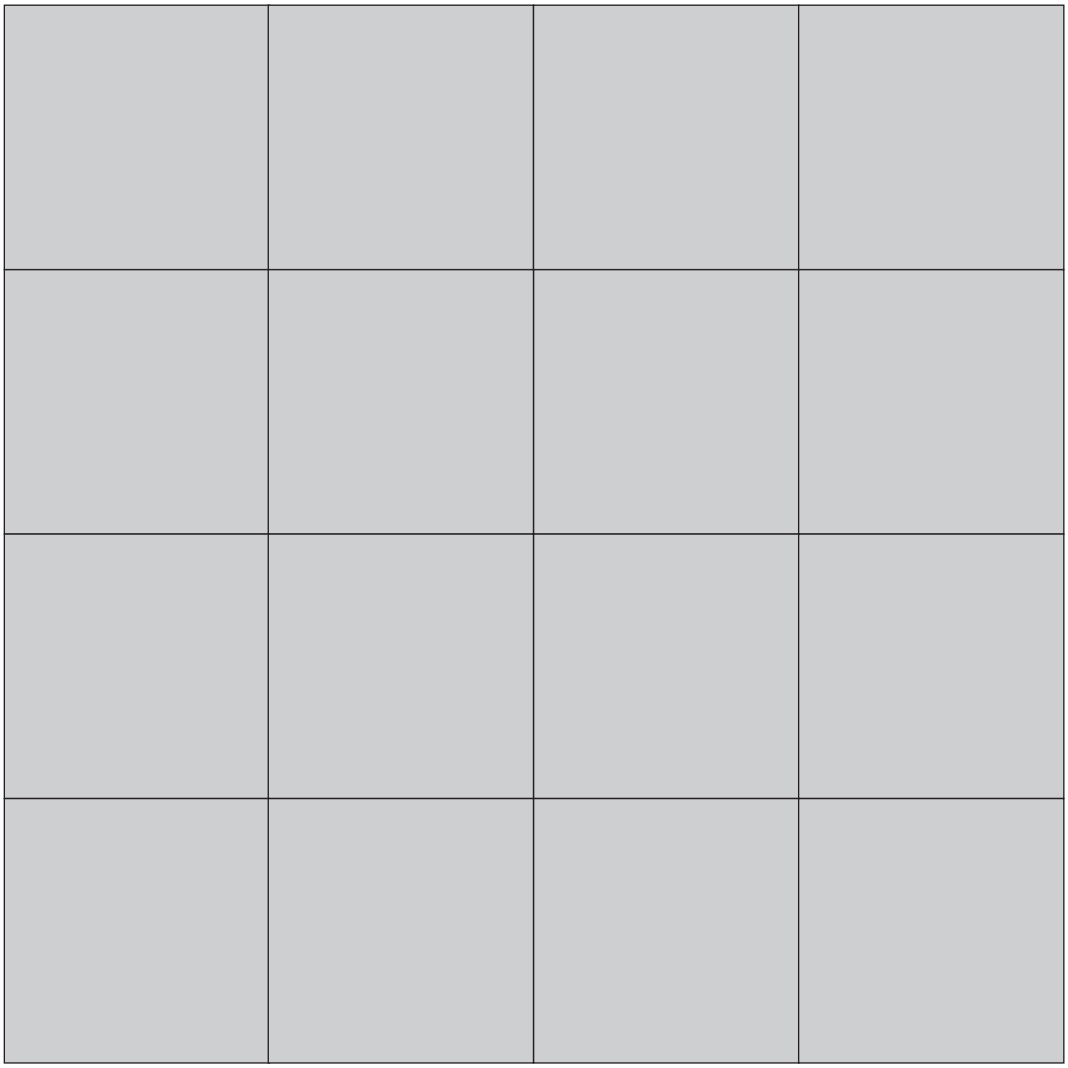
Appendices

- Appendix A: Algebra Tile Template for Module 4
- Appendix B: Glossary

Appendix A: Algebra Tile Template for Module 4

The next page includes a template that can be used to make algebra tiles. You will use these algebra tiles to learn about polynomials in Module 4. Carefully cut out the tiles with a pair of scissors.





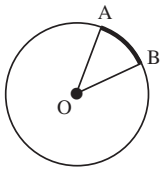
APPENDIX B: GLOSSARY

acute angle
An angle less than 90°.

addition
Combining two or more numbers to find their total (sum, plus, increased by, and more than are used to describe addition).

arc
A segment of the edge (circumference) of a circle.

Example



$\overset{\frown}{AB}$ is the arc

area
The space taken up by a 2-D object.

bar graph
A graph that uses horizontal or vertical bars to display data.

Example

base
The number being multiplied together with itself in a power (4 is the base in 4^3 and x is the base in x^5).

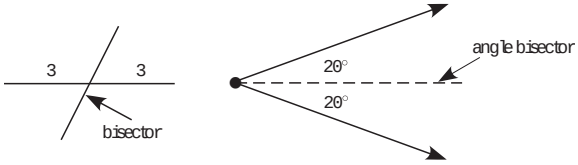
BEDMAS
Brackets, Exponents, Division, Multiplication, Addition, Subtraction. Division and multiplication (and addition and subtraction) are to be completed in the order in which they appear from left to right in the expression or equation. (*See* **order of operations**.)

bias
A sample that emphasizes one part of the population above others.

binomial
A polynomial with two terms.

bisector
A line that divides an angle or another line into two equal parts.

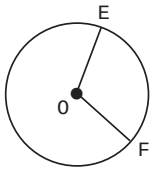
Example



census
A survey of the entire population.

central angle
An angle with its vertex on the centre and its endpoints on the edge (circumference) of the circle.

Example

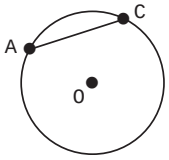


∠EOF is a central angle

chart
A diagram that illustrates information in the form of a table, graph, or picture.

chord
A line segment with its endpoints on the edge (circumference) of the circle.

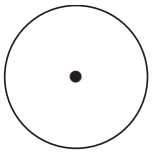
Example



circle

A shape with one edge (circumference) that curves around a centre point.

Example



$\overline{AC}$ is a chord

circumference

The distance around the edge of a circle (also known as the perimeter).

coefficient

The number multiplying the variable(s) in a term (e.g., 7 is the coefficient of $7x^3$).

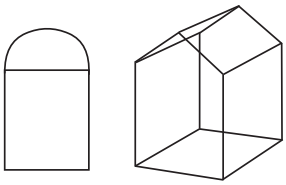
common denominator

Two or more fractions that have the same number in the denominator.

composite object

An object made up of more than one 2-D or 3-D shape.

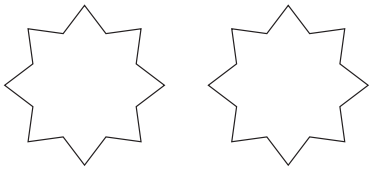
Example



congruent

Alike in every respect, but separate. Think of identical twins, who are separate people but look exactly the same.

Example



constant

A term in an expression that has no variables (it is either a number or a symbol that represents a number, such as π).

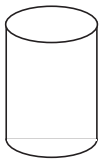
culturally sensitive

To be empathetic toward other people's beliefs or misfortunes.

cylinder

A prism with circle parallel faces.

Example



data

Information that is collected; usually numerical, organized in charts and displayed by graphs.

decimal

A fractional number written in base ten form; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number).

degree

the sum of the exponents in a term **or** largest exponent in an expression

Example

- x^2 would be a term with a degree of 2
- y would be the same as y^1, so the degree here is 1
- xy is the same as x^1y^1, so the degree for this term would be the sum of the exponents, or 2
- $3n^4 - n^2$—the degree of the polynomial is from the term with the highest degree. This expression would be degree 4.

denominator

The number below the line in a fraction that can state the total number of items, or the number of equal pieces that something is divided into.

Example

$\frac{2}{3}$ ← denominator

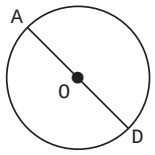
dependent variable

An item that is being compared to another item, and that is affected by the other item; graphed on the vertical or y-axis.

diameter

A chord that passes through the centre of the circle.

Example



$\overline{AD}$ is a diameter

distributive property of multiplication

$a * (b + c) = ab + ac$

$a(b + c) = ab + ac$

division

To split up a large group into a number of smaller, equal groups (into, quotient, over, and half of are used to describe division).

dividing inequalities rule

When the variable has a negative coefficient, if you divide both sides by the coefficient you have to switch the direction of the sign.

equation

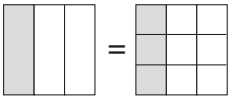
A math sentence that states that two things are equal.

equivalent fractions

Fractions that represent the same amount.

Examples

$\frac{1}{3} = \frac{3}{9}$



ethics

A code of conduct, dictating 'good' and 'bad' behaviour.

evaluate

Find the value of an expression.

exponent

The number of times a number is multiplied together in a power (3 is the exponent in 4^3 and 5 is the exponent in x^5).

expression

One side of an equation; does not contain an equal sign or greater than/less than symbol.

extrapolation

Calculate values that are outside of the range of data.

factor

A number that, when multiplied by another number, gives a specific product (8 is a factor of 24 because 8 x 3 = 24).

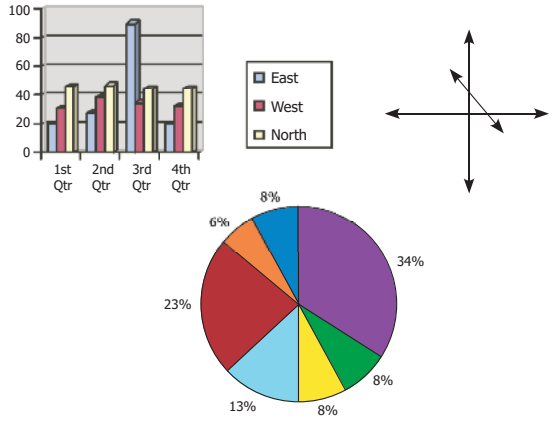
fraction

A number that represents part of a whole that looks like $\frac{a}{b}$.

graph

A visual representation used to show a relationship between data.

Example



hundreds place

The place value located three places to the left of the decimal point in a number; a digit in the hundreds place has a value of 100 times the value of the digit.

hundredths place

The place value located two places to the right of the decimal point in a number; a digit in the hundredths place has a value of $\frac{1}{100}$ the value of the digit.

independent variable

An item being compared to another item, but is unaffected by the other item; graphed on the horizontal or x-axis.

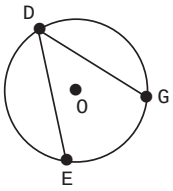
inequality

A math sentence that states that two values or expressions are not the same; use the symbols $<$, $\leq$, $>$, $\geq$, and $\neq$.

inscribed angle

An angle that has its vertex and end points on the edge (circumference) of the circle.

Example



$\angle EDG$ is an inscribed angle

interpolation

To calculate values that are between data points.

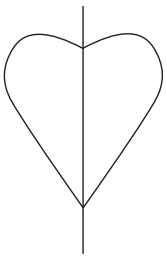
like terms

Two or more terms that have the same variable(s) with the same exponent(s).

line of symmetry

A line that divides a shape into two congruent parts so that if the shape were folded along the line, the edges would match up.

Example



lowest common multiple (LCM)

The smallest number, greater than 0, that is a multiple of two or more numbers (e.g., the LCM of 20 and 25 is 100).

midpoint

The point on a line segment halfway between the two endpoints.

monomial

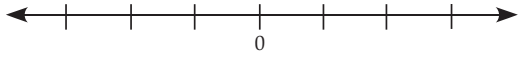
A polynomial with one term.

multiplication
Combination of equal groups into one; repeated adding (double, triple, and product of are used to describe multiplication).

negative number
A number that is less than 0, located to the left of 0 on a horizontal number line, or located below 0 on a vertical number line.

number line
A line marked with points that represent numbers; resembles one axis of a graph.

Example



numerator
The number above the line in a fraction that states the number of parts being considered.

Example

$\frac{2}{3}$ ← numerator

numerical coefficient
The value in front of the variable ($-3x^2$—here the numerical coefficient is –3).

obtuse
An angle that is between 90° and 180°.

order of operations
A specified sequence in which mathematical operations are expected to be performed. An arithmetic expression is evaluated by following these ordered steps:

1. Simplify within grouping symbols such as parentheses or brackets, starting with the innermost grouping.
2. Apply exponents—powers and roots.
3. Perform all multiplications and divisions in order from left to right.
4. Perform all additions and subtractions in order from left to right.

parallelogram
A 4-sided shape with parallel opposite sides, and 4-angles that do not have to be 90°.

Example

pattern
Something that is predictable because it repeats itself, or something is done to it over and over.

percent(%)
A number expressed in relation to 100; represented by the symbol % (e.g., 40 parts out of 100 is 40%).

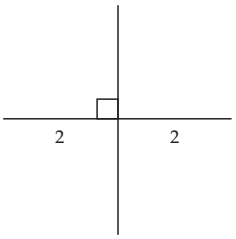
perfect square
A number resulting from multiplying a number by itself (3 x 3 = 9 so 9 is a perfect square).

perimeter
The distance around the outside of a shape.

perpendicular
Two lines that meet at 90°.

perpendicular bisector
A line that divides another line in two equal parts, and forms a 90° angle with the divided line.

Example



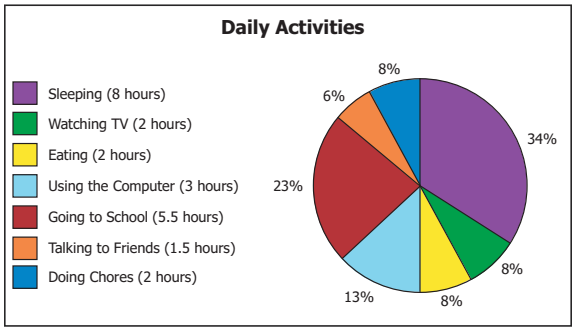
pie graph

A graph that represents data as parts of a circle (whole). The total of all the sectors should be 100% of the data.

Note that pie graphs

- show the ratio of each part to the whole, not quantities
- are almost always made from data converted to a percentage of the total
- show ratios so that comparisons can be made between different-sized quantities

Example

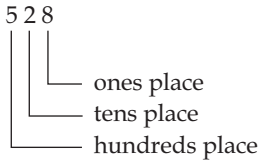


place value

The value of a digit in a number based on its position.

Example

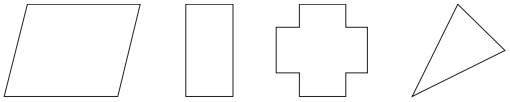
In the number 528, the 5 has a value of 5 hundreds (or 500), the 2 has a value of 2 tens (or 20), and the 8 has a value of 8 ones (or 8).



polygon

A 2-D shape made up of three or more straight lines.

Example



polynomial

A mathematical expression with one or more terms.

population

Everything or everyone in a group that is being studied.

positive number

Any number greater than 0; located to the right of 0 on a horizontal number line or above 0 on a vertical number line.

power

The product of a number multiplied with itself several times ($3 \times 3 \times 3 = 27$ is described as '3 to the power of 3' or 'the third power of 3').

power of a power rule

When there is an exponent inside and outside the brackets, the base stays the same and the *exponents are multiplied together* (e.g., $(4^3)^2 = 4^{3x2}$).

power of a product rule

When two or more numbers are multiplied together in the brackets and there is an exponent outside the brackets, the exponent can be applied to each number or variable being multiplied (e.g., $(3 * 2)^3 = 3^3 * 2^3$).

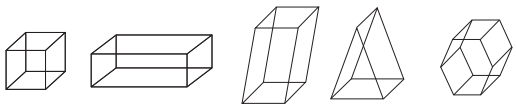
power of a quotient rule

When the base is a quotient, the exponent is applied to the numerator and denominator (e.g., $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$).

prism

A 3-D object that has two congruent (equal) and parallel faces, connected by parallelograms.

Example



product of powers rule

If the terms have the *same base*, the coefficients are multiplied and the *exponents are added* (e.g., $3^2 \times 3^4 = 3^{2+4} = 3^6$).

proportional

The ratio describing one object in relation to another remains constant.

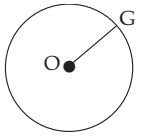
quotient of powers rule

When dividing powers with the *same base*, the coefficients divide and the *exponents are subtracted* (e.g., $4^6 \div 4^4 = 4^{6-4} = 4^2$).

radius

A line from the centre of a circle to the edge (circumference) of the circle; half the diameter.

Example



$\overline{OG}$ is a radius

random sample

A representative part of the population, selected so that every item has an equal chance of being selected.

ray

Part of a line that has one end point and one end that goes on forever (marked by an arrow).

rectangle

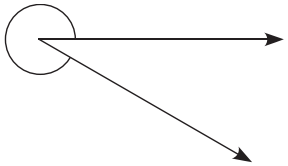
A four-sided shape that has four right angles (90°), and opposite sides are congruent (equal).

Example

reflex angle

An angle between 180° and 360°.

Example



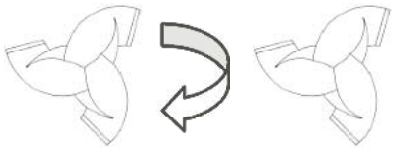
right angle

An angle that is 90°.

rotational symmetry

When you can rotate an object by a certain amount around a point and it will look the same as the original object.

Example



sample

A part of the group being studied that represents the whole group

or

A survey given to part of the population.

scale

(graphs)

The minimum and maximum numbers on an axis, and the divisions in between (from 0 to 10 marking each number = 0, 1, 2, 3…; from 0 to 100 marking each multiple of 10 = 0, 10, 20, 30…).

(drawings)

A drawing of an object that is larger or smaller than the actual object, but the two are similar (proportional).

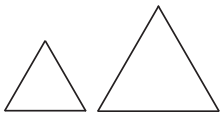
scale factor

The ratio describing the relationship between the actual object and the drawing of the object.

sequence
Numbers arranged in an order, according to some rule (2, 4, 6, 8 … are increasing even numbers).

similar
The same shape, proportional size.

Example



simplify
Combine like terms so that you are left with the simplest form of an equation or expression.

solve
Find the answer to an equation or problem, or to find the value of a variable.

square root $(\sqrt{\ })$
A number (factor) that, when multiplied by itself, produces the given square $(\sqrt{16} = 4)$.

straight angle
An angle of 180^{o}.

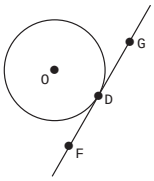
subtraction
the removal of a number from another so that you are left with the space between the two (difference, reduced by, take away, and less than are used to describe subtraction).

survey
to ask either written or verbal questions for the purpose of acquiring information/ data.

table
A way of organizing data, usually presented in rows and columns.

tangent
a line that touches the circle at only one point.

Example



$\overline{FDG}$ is the tangent

tens place
The place value located two places to the left of the decimal point; a digit in the tens place has a value of 10 times the value of the digit.

tenths place
The place value located one place to the right of the decimal point; a digit in the tenths place has a value of $\frac{1}{10}$ the value of the digit.

term
Variables, numerical coefficients, or constants in a polynomial expression; separated by addition or subtraction signs.

tessellation
a repeated pattern that has no gaps between the shapes.

Example

thousands place
The place value located four places to the left of the decimal point in a number; a digit in the thousands place has a value of 1000 times the value of the digit.

thousandths place
The place value located three places to the right of the decimal point; a digit in the thousandths place has a value of $\frac{1}{1000}$ the value of the digit.

triangle
Three-sided object with three angles; sides and angles can but don't have to be equal.

Example

trinomial
A polynomial with three terms.

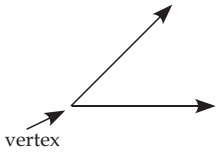
two-dimensional (2-D)
A figure that only has two measures (a rectangle is 2-D because it is described using only length and width).

variable
A letter or symbol that represents an unknown value (x, y, n, θ).

verify
Check your answer.

vertex
The point where two rays meet to make an angle.

Example



zero rule
When a number is to the power of 0, it is equal to 1 no matter what the base is.

GRADE 9 MATHEMATICS (10F)

Midterm Practice Examination

GRADE 9 MATHEMATICS (10F)

Midterm Practice Examination

Instructions

The midterm examination will be weighted as follows

Modules 1–4	100%

The format of the examination will be as follows:

Part A: Multiple Choice	15 x 1 = 15 marks
Part B: Short Answer	9 x 3 = 27 marks
Part C: Problem Solving	58 marks

Time allowed: 2 hours

Notes: You are allowed to bring the following to the exam: pen, pencil, paper, scientific calculator, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module work that is submitted for marking.

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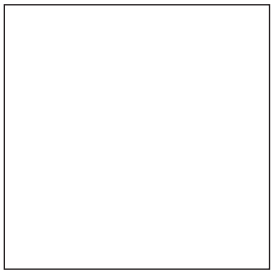
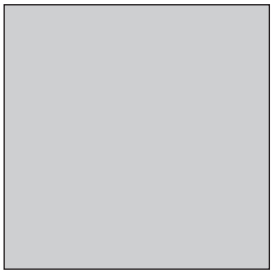
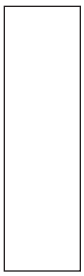
Use the following representations for algebra tiles.

 $= -x^2$

 $= x^2$

 $= -x$

 $= x$

 $= -1$

 $= 1$

Part A: Multiple Choice (15 x 1 mark each = 15 Marks)

Circle the letter of the response that represents the correct answer.

1. A method of collecting data is:
   a) telephone survey
   b) mail-in replies
   c) observation
   d) all of the above

2. Statisticians use charts and graphs to illustrate data because:
   a) they are less expensive
   b) many people can't read
   c) they like the colours
   d) they quickly show comparisons and trends

3. Surveying teachers about what teenagers like is biased because:
   a) teachers are smart
   b) schools have too many rules
   c) teachers may not know what teenagers like
   d) only parents know what teenagers like

4. When subtracting fractions:
   a) invert the second fraction and multiply
   b) multiply by 10
   c) find a common denominator, and subtract the numerators
   d) bottom number divided by the top number

5. Which of the following decimal numerals is equivalent to $\frac{5}{8}$?
   a) 0.625
   b) 0.58
   c) 58
   d) 62.5

6. 20% is equivalent to:

   a) $\frac{20}{100}$ and 0.8

   b) $\frac{10}{50}$ and 0.4

   c) $\frac{1}{5}$ and 0.20

   d) $\frac{1}{5}$ and 20

7. The quotient of $\frac{7^{10}}{7^2}$ is:

   a) 7^{12}
   b) 7^8
   c) 7^{20}
   d) 7^5

8. The simplified answer to $(-2 \cdot 3^5)^3$ is:
   a) $-6 \cdot 3^8$
   b) $6 \cdot 3^8$
   c) $8 \cdot 3^{15}$
   d) $-8 \cdot 3^{15}$

9. Simplify: $-3x^2 + 3x - 2x - 2x^2$
   a) $5x^2 + x$
   b) $5x^2 - x$
   c) $-5x^2 + x$
   d) $-5x^2 - 2x$

10. Which expression is indicated by the following tiles?

a) $-2x^2 + 2x + 4$
b) $2x^2 - 2x - 4$
c) $-2x^2 + 2x - 4$
d) $2x^2 - 2x + 4$

11. When you divide $(-9w + 6)$ by (-3), the answer is:
a) $3w - 2$
b) $-6w + 3$
c) $6w - 18$
d) $-3w - 2$

12. When simplified, $(5p^2 - 2p - 3) - (-3p^2 + p - 4)$ is:
a) $2p^2 - p + 1$
b) $2p^2 - 3p - 1$
c) $8p^2 - p + 1$
d) $8p^2 - 3p + 1$

13. When adding $\frac{5}{9}+\frac{4}{5}$, the proper first step would be:

a) $\frac{5+4}{9+5}$

b) $\frac{5}{9} * \frac{5}{4} + \frac{4}{5} * \frac{9}{5}$

c) $\frac{5}{9} * \frac{5}{5} + \frac{4}{5} * \frac{9}{9}$

d) $\frac{5}{9} * \frac{5}{5} + \frac{4}{5} * \frac{5}{5}$

14. Which of the following illustrates the correct use of the distributive property for $-5v(4v - 6) + 2v(-5v - 6)$?

a) $-5v(4v) - 5v(-6) + 2v(-5v) + 2v(-6)$

b) $-5v(4v) + 5v(-6) + 2v(-5v) + 2v(6)$

c) $5v(4v) - 5v(6) + 2v(5v) + 2v(-6)$

d) $-5v(4v) - 5v(-6) - 5v(-5v) - 5v(-6)$

15. Which of the following indicates the correct first step when dividing: $\frac{5}{9} \div \frac{3}{5}$?

a) $\frac{4}{5} * \frac{3}{5}$

b) $\frac{9}{5} \div \frac{3}{5}$

c) $\frac{5}{9} * \frac{5}{3}$

d) $\frac{5}{9} \div \frac{5}{3}$

Part B: Short Answer (9 x 3 marks each = 27 Marks)

Answer the following questions. Show all work.

1. Write $\frac{(3)^5}{(3)^3}$ in expanded notation, and find the value.

2. Illustrate using algebra tiles this expression: $-2x^2 + 3x - 1$.

3. A new car is worth $24 000, but it decreases in value by 30% after one year. Find the value of the car after one year.

4. List at least three potential problems with the following statistical statement.
   "70% of Winnipeg taxpayers would like an NHL team"

5. Place these rational and decimal numerals in order from least to greatest.

   $\frac{2}{5}$ $\quad -0.39 \quad$ $\frac{3}{7}$ $\quad -\frac{3}{8} \quad -\frac{2}{5}$

6. Divide the powers, and show your work.

   $$\frac{(-3)^{10}}{(-3)^{6}}$$

7. Divide. Show all your steps.

$$\frac{3}{4} \div \frac{4}{5}$$

8. List 2 perfect square numbers between the following values.

   82 _____ _____ 125

9. Show how you would estimate the approximate square root of 69 without using a calculator:

Part C: Problem Solving (58 Marks)

Solve each of the following problems. Show all your work and include written explanations where necessary.

1. Divide these fractions (show your work). *(5 marks)*

$$\frac{3}{5} \div 1\frac{5}{6}$$

2. Use algebra tiles to illustrate the following expressions, then combine the like terms. State the answer. *(5 marks)*

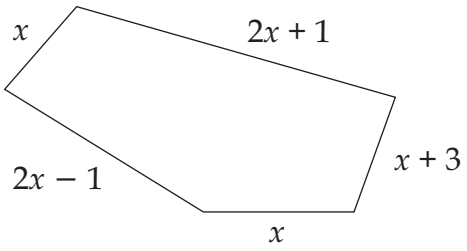
   $2x^2 - 3x - 3$ $\qquad\qquad$ $-x^2 + 4x - 1$

3. Your friend handed in the following work, and it was marked wrong. What mistakes did she make? Find the correct answer for the question. *(5 marks)*

$6 \div \frac{1}{2}$

$6 \div 2 = 3$

4. Find the perimeter of the figure below. *(6 marks)*



5. Identify the error(s) made, and do the question correctly. *(6 marks)*

$(-2 \cdot 5^3)^4$

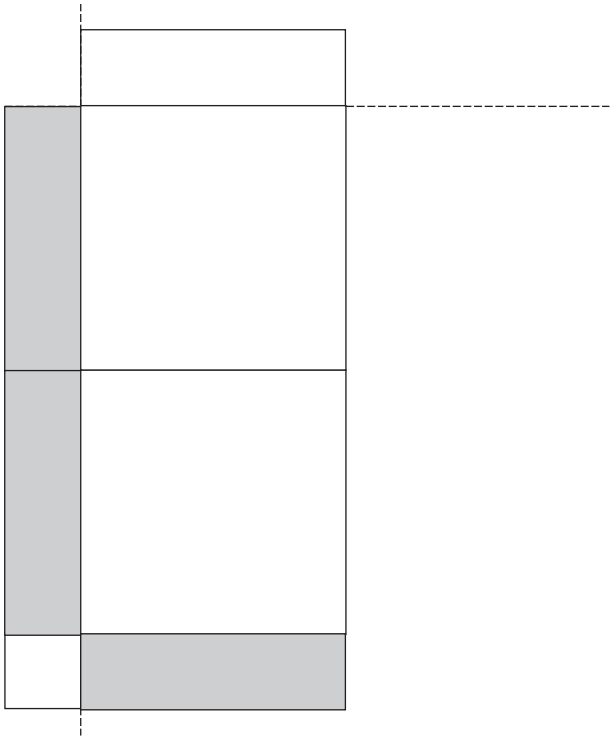
$= 8 \cdot 5^7$

6. A family budget shows a monthly income of $3200. Find the amount of money available for each category listed below. *(8 marks)*

Rent:	25%
Clothes:	20%
Utilities:	15.5%
Food:	30%

7. The number of millionaires in Canada increased by 4 ½% last year. If there were 120 000 millionaires last year, then how many live in Canada now? *(6 marks)*

8. Label the algebra tiles, and state the answer to this multiplication. *(6 marks)*



9. Use arrows to show each step to find the quotient. *(6 marks)*

$$\frac{-20d^2 - 10d + 15}{-5}$$

10. Draw a tile diagram to illustrate how you would divide $x^2 - 2x$ by x. *(6 marks)*

NOTES

GRADE 9 MATHEMATICS (10F)

Midterm Practice Examination

Answer Key

GRADE 9 MATHEMATICS (10F)

Midterm Practice Examination
Answer Key

Instructions

The midterm examination will be weighted as follows

Modules 1–4	100%

The format of the examination will be as follows:

Part A: Multiple Choice	15 x 1 = 15 marks
Part B: Short Answer	9 x 3 = 27 marks
Part C: Problem Solving	58 marks

Time allowed: 2 hours

Notes: You are allowed to bring the following to the exam: pen, pencil, paper, scientific calculator, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module work that is submitted for marking.

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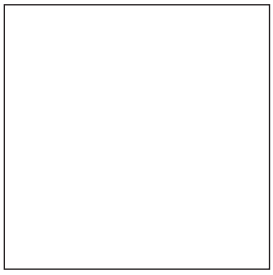
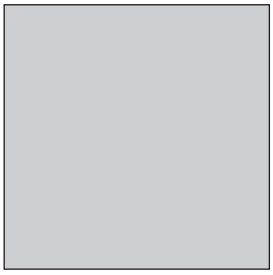
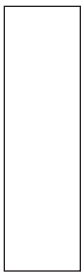
 $= -x^2$

 $= x^2$

 $= -x$

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Part A: Multiple Choice (15 x 1 mark each = 15 Marks)

Circle the letter of the response that represents the correct answer.

1. A method of collecting data is:
   a) telephone survey
   b) mail-in replies
   c) observation
   d) all of the above (Module 1, Lesson 2)

2. Statisticians use charts and graphs to illustrate data because:
   a) they are less expensive
   b) many people can't read
   c) they like the colours
   d) they quickly show comparisons and trends (Module 1, Lesson 5)

3. Surveying teachers about what teenagers like is biased because:
   a) teachers are smart
   b) schools have too many rules
   c) teachers may not know what teenagers like
   d) only parents know what teenagers like (Module 1, Lesson 3)

4. When subtracting fractions:
   a) invert the second fraction and multiply
   b) multiply by 10
   c) find a common denominator, and subtract the numerators
   d) bottom number divided by the top number (Module 2, Lesson 3)

5. Which of the following decimal numerals is equivalent to $\frac{5}{8}$?
   a) 0.625
   b) 0.58
   c) 58
   d) 62.5 (Module 2, Lesson 4)

6. 20% is equivalent to:

   a) $\frac{20}{100}$ and 0.8

   b) $\frac{10}{50}$ and 0.4

   c) $\frac{1}{5}$ and 0.20

   d) $\frac{1}{5}$ and 20

(Module 2, Lesson 4)

7. The quotient of $\frac{7^{10}}{7^2}$ is:

   a) 7^{12}

   b) 7^8

   c) 7^{20}

   d) 7^5

(Module 3, Lesson 5)

8. The simplified answer to $(-2 \cdot 3^5)^3$ is:

   a) $-6 \cdot 3^8$

   b) $6 \cdot 3^8$

   c) $8 \cdot 3^{15}$

   d) $-8 \cdot 3^{15}$

(Module 3, Lesson 4)

9. Simplify: $-3x^2 + 3x - 2x - 2x^2$

   a) $5x^2 + x$

   b) $5x^2 - x$

   c) $-5x^2 + x$

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(Module 4, Lesson 3)

10. Which expression is indicated by the following tiles?

a) $-2x^2 + 2x + 4$
b) $2x^2 - 2x - 4$
c) $-2x^2 + 2x - 4$
d) $2x^2 - 2x + 4$ (Module 4, Lesson 2)

11. When you divide $(-9w + 6)$ by (-3), the answer is:
a) $3w - 2$
b) $-6w + 3$
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12. When simplified, $(5p^2 - 2p - 3) - (-3p^2 + p - 4)$ is:
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13. When adding $\frac{5}{9}+\frac{4}{5}$, the proper first step would be:

a) $\frac{5+4}{9+5}$

b) $\frac{5}{9} * \frac{5}{4} + \frac{4}{5} * \frac{9}{5}$

c) $\mathbf{\frac{5}{9} * \frac{5}{5} + \frac{4}{5} * \frac{9}{9}}$

d) $\frac{5}{9} * \frac{5}{5} + \frac{4}{5} * \frac{5}{5}$

(Module 2, Lesson 3)

14. Which of the following illustrates the correct use of the distributive property for $-5v(4v - 6) + 2v(-5v - 6)$?

a) $-5v(4v) - 5v(-6) + 2v(-5v) + 2v(-6)$

b) $-5v(4v) + 5v(-6) + 2v(-5v) + 2v(6)$

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d) $-5v(4v) - 5v(-6) - 5v(-5v) - 5v(-6)$

(Module 4, Lesson 4)

15. Which of the following indicates the correct first step when dividing: $\frac{5}{9} \div \frac{3}{5}$?

a) $\frac{4}{5} * \frac{3}{5}$

b) $\frac{9}{5} \div \frac{3}{5}$

c) $\mathbf{\frac{5}{9} * \frac{5}{3}}$

d) $\frac{5}{9} \div \frac{5}{3}$

(Module 2, Lesson 3)

Part B: Short Answer (9 x 3 marks each = 27 Marks)

Answer the following questions. Show all work.

1. Write $\frac{(3)^5}{(3)^3}$ in expanded notation, and find the value.

   Answer:

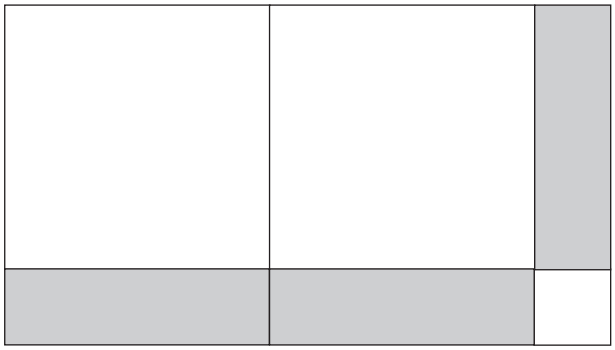
   $$\frac{\cancel{(3)}\,\cancel{(3)}\,\cancel{(3)}\,(3)(3)}{\cancel{(3)}\,\cancel{(3)}\,\cancel{(3)}} = 3^{5-3} = (3)^2 = 9$$

   (Module 3, Lesson 2)

2. Illustrate using algebra tiles this expression: $-2x^2 + 3x - 1$.

   Answer:

   Arrangement may vary.



   (Module 4, Lesson 2)

3. A new car is worth $24 000, but it decreases in value by 30% after one year. Find the value of the car after one year.

   Answer:

   $24 000 * 0.3 = 7200

   $24 000 – 7200 = $16 800

   (Module 2, Lesson 4)

4. List at least three potential problems with the following statistical statement.

   "70% of Winnipeg taxpayers would like an NHL team"

   Answer:

   Answers may vary.

   We don't know what question was posed or how the data were collected.

   Was there any incentive offered?

   Who did they ask?

   It doesn't say if they would be willing to pay for it. (Module 1, Lesson 3)

5. Place these rational and decimal numerals in order from least to greatest.

   $\frac{2}{5} \quad -0.39 \quad \frac{3}{7} \quad -\frac{3}{8} \quad -\frac{2}{5}$

   Answer:

   $-\frac{2}{5} \quad -0.39 \quad -\frac{3}{8} \quad \frac{2}{5} \quad \frac{3}{7}$ (Module 2, Lesson 4)

6. Divide the powers, and show your work.

   $\frac{(-3)^{10}}{(-3)^{6}}$

   Answer:

   $\frac{(-3)^{10}}{(-3)^{6}}$

   $= (-3)^{10-6}$

   $= (-3)^{4}$

   $= 81$ (Module 3, Lesson 5)

7. Divide. Show all your steps.

$$\frac{3}{4} \div \frac{4}{5}$$

Answer:

$$\frac{3}{4} * \frac{5}{4}$$

$$= \frac{15}{16}$$

(Module 2, Lesson 3)

8. List 2 perfect square numbers between the following values.

82 _____ _____ 125

Answer:

100 121 (Module 2, Lesson 5)

9. Show how you would estimate the approximate square root of 69 without using a calculator:

Answer:

$\sqrt{64} = 8$ $\sqrt{81} = 9$ Identify closest perfect square numbers and their roots.

69 is close to 64 State which one it is closer to.

$\sqrt{69} \approx 8.3$ Approximate the value of the square root.

Accept answers in the range 8.2 to 8.5. (Module 2, Lesson 6)

Part C: Problem Solving (58 Marks)

Solve each of the following problems. Show all your work and include written explanations where necessary.

1. Divide these fractions (show your work). *(5 marks)*

   $\frac{3}{5} \div 1\frac{5}{6}$

   Answer:

   $\frac{3}{5} \div \frac{11}{6}$

   $= \frac{3}{5} \times \frac{6}{11}$

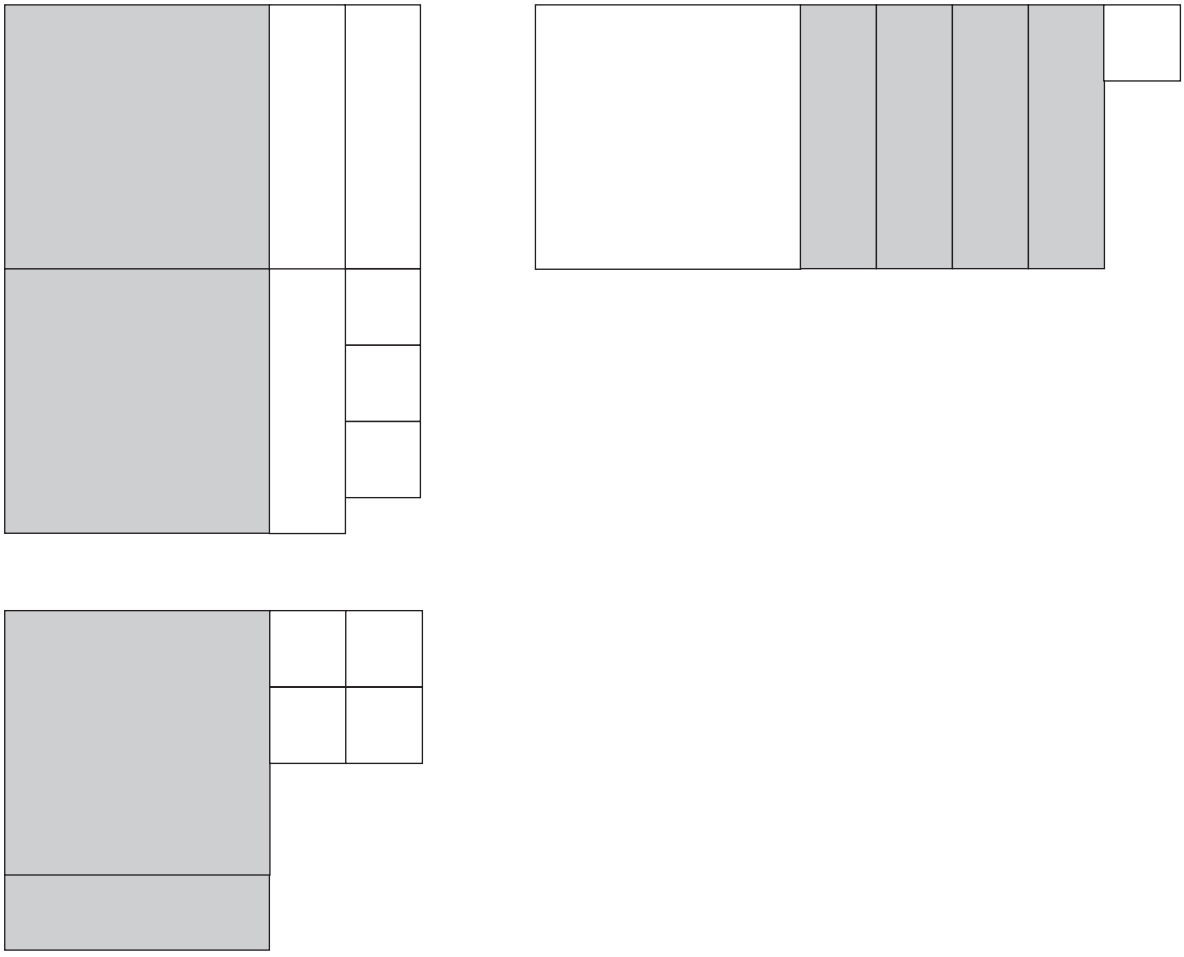
   $= \frac{18}{55}$

   (Module 2, Lesson 3)

2. Use algebra tiles to illustrate the following expressions, then combine the like terms. State the answer. *(5 marks)*

$2x^2 - 3x - 3$ $\qquad\qquad$ $-x^2 + 4x - 1$

Answer:



$x^2 + x - 4$ (Module 4, Lesson 3)

3. Your friend handed in the following work, and it was marked wrong. What mistakes did she make? Find the correct answer for the question. *(5 marks)*

$6 \div \frac{1}{2}$

$6 \div 2 = 3$

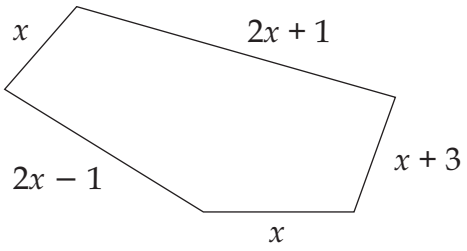
Answer:

She changed the fraction, but she still divided (instead of multiplying).

$\frac{6}{1} \times \frac{2}{1}$

$= 12$ (Module 2, Lesson 3)

4. Find the perimeter of the figure below. *(6 marks)*



Answer:

$x + (2x + 1) + (x + 3) + (2x - 1) + x$

$= x + 2x + x + 2x + x + 1 + 3 - 1$

$= 7x + 3$ (Module 4, Lesson 7)

5. Identify the error(s) made, and do the question correctly. *(6 marks)*

   $(-2 \cdot 5^3)^4$

   $= 8 \cdot 5^7$

   Answer:

   Two mistakes:

   $(-2)^4$ is not 8, but 16

   The exponents should be multiplied, not added: $3 * 4 = 12$.

   $= 16 \cdot 5^{12}$ (Module 3, Lesson 3)

6. A family budget shows a monthly income of $3200. Find the amount of money available for each category listed below. *(8 marks)*

Rent:	25%
Clothes:	20%
Utilities:	15.5%
Food:	30%

Answer:

Rent:	$3200 * 0.25 = 800$
Clothes:	$3200 * 0.2 = 640$
Utilities:	$3200 * 0.155 = 496$
Food:	$3200 * 0.3 = 960$

(Module 2, Lesson 2)

7. The number of millionaires in Canada increased by 4 ½% last year. If there were 120 000 millionaires last year, then how many live in Canada now? *(6 marks)*

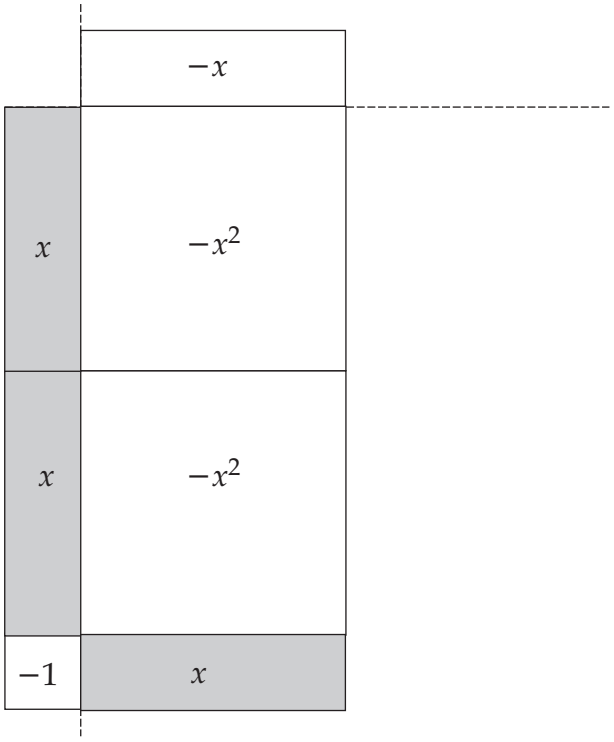
   Answer:

   120 000 x 0.045 = 5400

   120 000 + 5400 = 125 400 (Module 2, Lesson 2)

8. Label the algebra tiles, and state the answer to this multiplication. *(6 marks)*

   Answer:



   $-2x^2 + x$ (Module 4, Lesson 5)

9. Use arrows to show each step to find the quotient. *(6 marks)*

$$\frac{-20d^2 - 10d + 15}{-5}$$

Answer:

$$\frac{-20d^2 - 10d + 15}{-5}$$

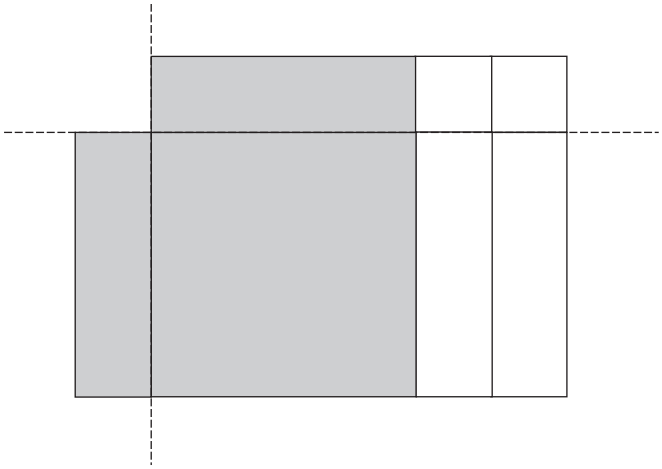
$4d^2 + 2d - 3$

(Module 4, Lesson 6)

10. Draw a tile diagram to illustrate how you would divide $x^2 - 2x$ by x. *(6 marks)*

Answer:

$x - 2$



(Module 4, Lesson 6)

NOTES

GRADE 9 MATHEMATICS (10F)

Final Practice Examination

GRADE 9 MATHEMATICS (10F)

Final Practice Examination

Instructions

The final examination will be weighted as follows

Modules 1–4	25%
Modules 5–8	75%

The format of the examination will be as follows:

Part A: Multiple Choice	22 x 2 = 44 marks
Part B: Short Answer	28 marks
Part C: Long Answer	28 marks

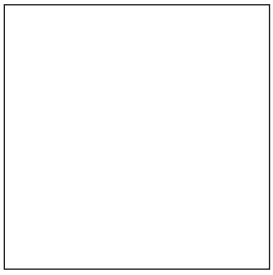
Time allowed: 2 hours

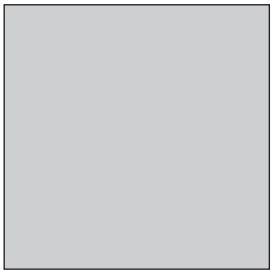
Supplies: pen, pencil, paper, scientific calculator, protractor, ruler

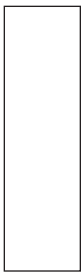
Note: See following page for instructions regarding use of algebra tiles.

The questions on this practice exam are similar to the questions you will see on your final exam. If there are any questions that you do not understand, look in the lesson where the material is taught, and ask your tutor/marker or learning partner to help you.

Use the following representations for algebra tiles.

 $= -x^2$

 $= x^2$

 $= -x$

 $= x$

 $= -1$

 $= 1$

Part A: Multiple Choice (22 x 2 = 44 Marks)

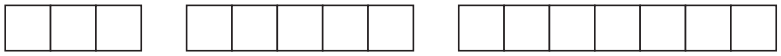
Circle the letter of the response that represents the correct answer.

1. When you roll a 6-sided cube, numbered 1 to 6, the probability of rolling a 2 is $\frac{1}{6}$. This is an example of:
   a) an experimental probability
   b) theoretical probability
   c) subjective reasoning
   d) assumption

2. An image rotated around its centre point appears unchanged after 180° and 360° turns. This is an example of:
   a) line symmetry
   b) rotation symmetry
   c) tessellation
   d) vertex

3. You survey people as they leave a Blue Bomber game and ask them to identify their favourite sport. The potential problems or bias in data is caused by:
   a) cultural insensitivity
   b) cost concerns
   c) ethical behaviour
   d) time and timing

4. Given the following pattern of shapes, choose the mathematical expression showing the changes for each iteration if b is the number of boxes in the previous iteration.



   a) $b + 1$
   b) $b + 2$
   c) $b - 1$
   d) $b - 2$

5. Add: $\frac{3}{4} + \frac{1}{2} =$

   a) $1\frac{1}{4}$

   b) $\frac{5}{4}$

   c) neither (a) nor (b) above

   d) both (a) and (b) above

6. Your income (I) is compared to the number of hours (H) you work. The variable, I, would

   a) be the dependent variable

   b) be the independent variable

   c) be constant

   d) go on the *y*-axis

7. Give the base of the following power: -3^4.

   a) 3

   b) –3

   c) –12

   d) –81

8. An inscribed angle has a measurement of 75°. A central angle shares the same endpoints. What is the measurement of the central angle?

   a) 375°

   b) 75°

   c) 150°

   d) 225°

9. When simplified, what is the exponential form of the following expression: $3^4 * 3^5 * 3^6$?

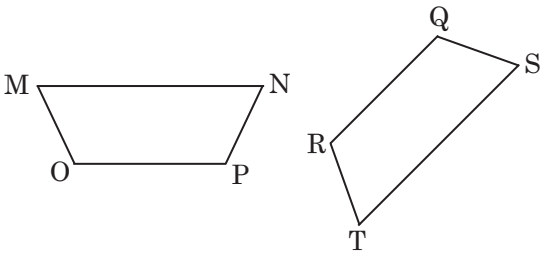
   a) 3^{15}

   b) 3^{120}

   c) 9^{15}

   d) 27^{15}

10. If shapes MOPN and SQRT are similar, which proportion is true?



a) $\frac{MO}{RT} = \frac{MN}{RQ}$

b) $\frac{MO}{RT} = \frac{OP}{TS}$

c) $\frac{NP}{QS} = \frac{OP}{TS}$

d) $\frac{MN}{ST} = \frac{MO}{SQ}$

11. A box of oranges costs $2.25. Which equation would represent the total cost, C, of any number, n, of boxes of oranges?

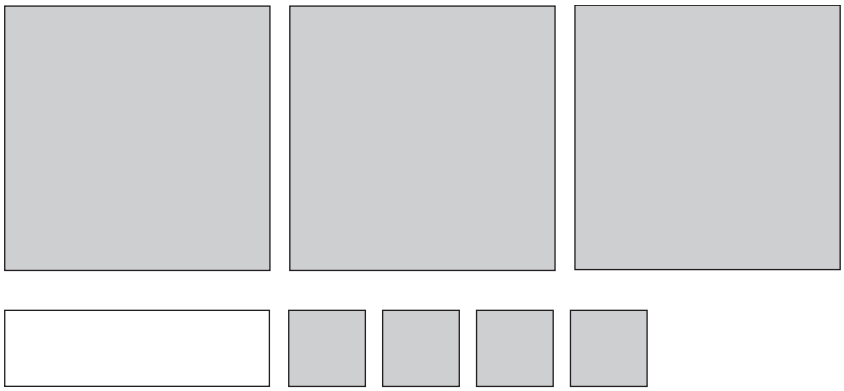
a) $C = 2.25 + n$

b) $C = 2.25 - n$

c) $C = 2.25n$

d) $C = \frac{2.25}{n}$

12. An expression representing this arrangement of tiles is:



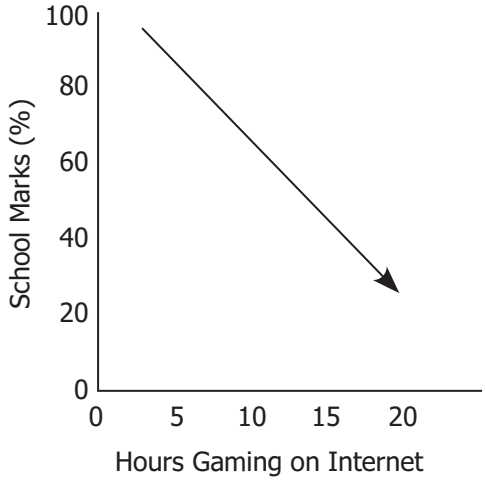
a) $-3x^2 + x - 4$

b) $-3x^2 - x + 4$

c) $3x^2 - x - 4$

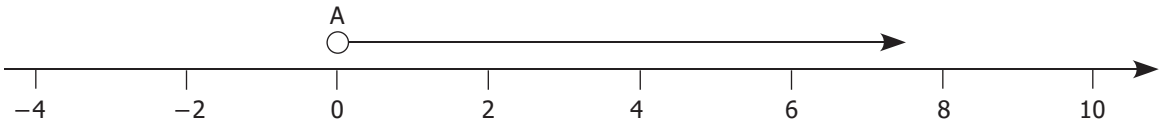
d) $3x^2 - x + 4$

13. Choose a sentence that describes in words what this graph is showing.



a) increased hours gaming on the Internet increases school marks
b) school marks decline based on increased hours gaming on the Internet
c) hours gaming on the Internet do not affect school marks
d) no hours gaming on the Internet guarantees you 100%

14. This inequality is read as:



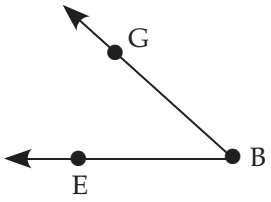
a) values less than 0
b) values less than or equal to 0
c) values greater than 0
d) values greater than or equal to 0

15. Give the next value in the following pattern: 120 90 60 ____

a) 50
b) 30
c) 80
d) 0.3

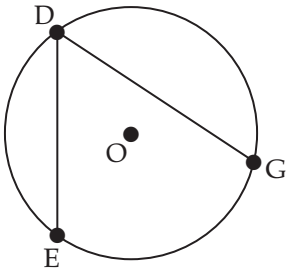
16. An inscribed angle that subtends, or has the same endpoints as a semicircle, will measure:
    a) half of 90°
    b) 90°
    c) double the measure of the semicircle
    d) 180°

17. Find the measure of the angle below, using your protractor.



    a) 42° ±3
    b) 52° ±3
    c) 32° ±3
    d) 48° ±3

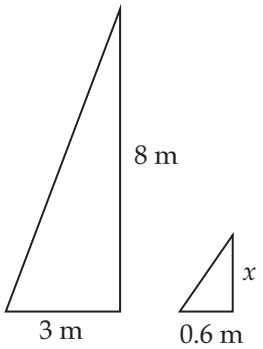
18. ∠EDG in this diagram is an example of a(n):



    a) right angle
    b) central angle
    c) perpendicular bisector
    d) inscribed angle

19. The formula for finding the circumference of a circle is:
    a) $2\pi r$
    b) πr^2
    c) $\frac{b_1 + b_2}{2}$
    d) $\frac{bh}{2}$

20. If these shapes are similar, what is the value of x?



    a) 4 m
    b) 1.6 m
    c) 1.8 m
    d) 18 m

21. Similar figures have:
    a) proportional sides and identical angles
    b) proportional sides and angles
    c) equal sides and proportional angles
    d) sides that look the same

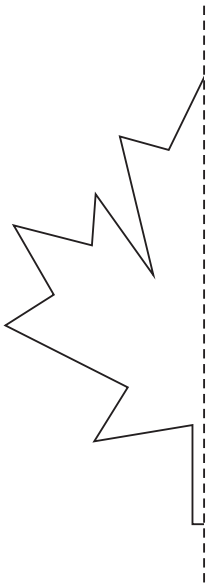
22. An object that repeats itself every 120° of rotation has rotation symmetry of order:
    a) 0
    b) 1
    c) 3
    d) 4

Part B: Short Answer (28 Marks)

Answer the following questions. Show all work.

1. Solve $4m - 2 = 5m + 7$ by isolating the variable. *(2 marks)*

2. Draw the reflected half across the line of symmetry. *(2 marks)*



3. You have a square tarp to cover your boat. The area of the tarp is 49 m^2. Find the length of one side. *(2 marks)*

4. Fill in the empty spots. *(6 marks)*

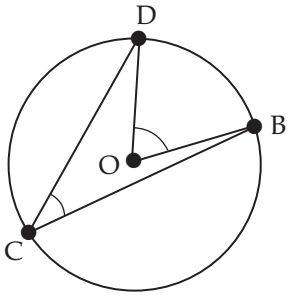
Term	Base	Exponent	Power
2^5	2		
$(-4)^3$		3	$(-4)^3$
26^7			26^7
-3^6		6	-3^6

5. Solve for g and graph your solution on a number line. *(2 marks)*

   $4g - 6 > 3g$

6. When comparing profits to sales, identify which is the dependent variable, and explain why it is dependent. *(4 marks)*

7. Use your protractor to find the measurement of both ∠DCB and ∠DOB in the following diagram. *(4 marks)*



8. The length of the highway from Calgary to Vancouver is almost 1000 km. If you use a scale factor of 1 cm : 75 km, how long would the line representing this distance be on your drawing? *(4 marks)*

9. Describe the difference between the solution for $4m = -28$ and $4m \leq -28$. *(2 marks)*

Part C: Long Answer (28 Marks)

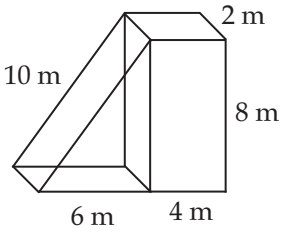
Solve each of the following problems. Show all your work, and include written explanations where necessary.

1. You want to determine the most common paint colour of cars and trucks in your town. Explain in detail how you would collect these data, and discuss why you chose that method. *(4 marks)*

2. Place the following rational numbers in order from smallest to largest. *(3 marks)*

$$\frac{5}{7} \qquad -0.25 \qquad \frac{-1}{3} \qquad 0.001 \qquad -0.01 \qquad \frac{3}{4} \qquad \frac{-2}{5}$$

3. Find the surface area of the following composite object. *(4 marks)*



4. You are working at a summer job to save money for the family winter trip to Florida. You want to have at least $500 saved. If your wage is $9.50 an hour, use a mathematical expression to find the minimum number of hours you will need to work to have enough money saved. *(3 marks)*

5. Isolate the variable, and draw a number line showing the solution to the following inequality. *(4 marks)*

   $6h + 4 \geq -2h + 16$

6. Given the following diagram, identify the listed components. *(6 marks)*

   a) 2 radii _____

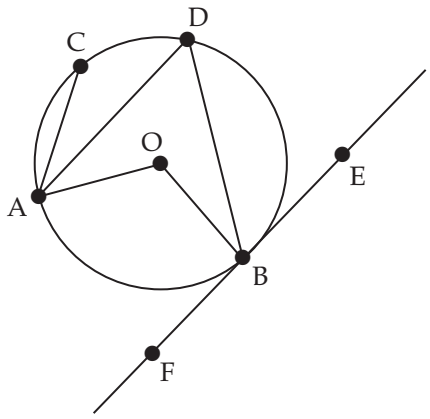
      _____

   b) tangent _____

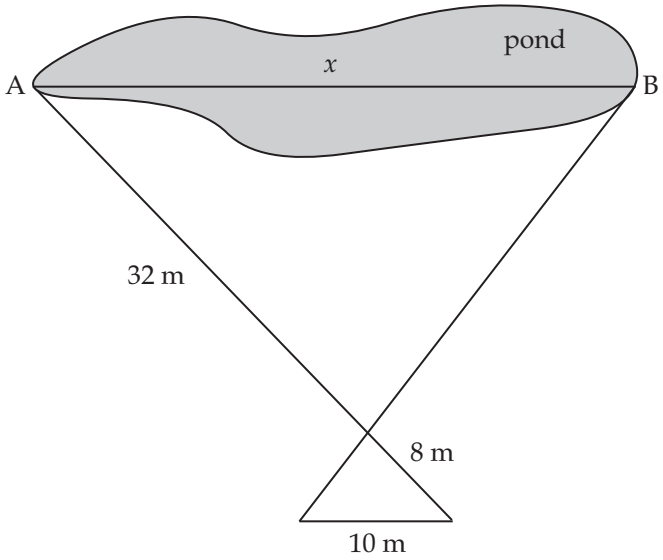
   c) 3 chords _____

      _____

      _____



7. Use proportions based on the similar triangles below to find the distance across the pond from A to B. *(4 marks)*



NOTES

GRADE 9 MATHEMATICS (10F)

Final Practice Examination

Answer Key

GRADE 9 MATHEMATICS (10F)

Final Practice Examination
Answer Key

Instructions

The final examination will be weighted as follows

Modules 1–4	25%
Modules 5–8	75%

The format of the examination will be as follows:

Part A: Multiple Choice	22 x 2 = 44 marks
Part B: Short Answer	28 marks
Part C: Long Answer	28 marks

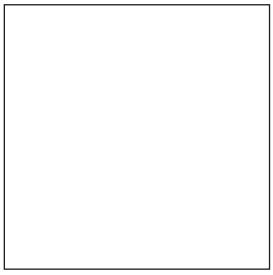
Time allowed: 2 hours

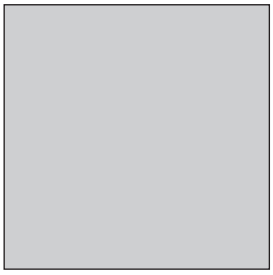
Supplies: pen, pencil, paper, scientific calculator, protractor, ruler

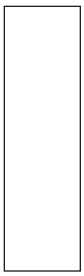
Note: See following page for instructions regarding use of algebra tiles.

The questions on this practice exam are similar to the questions you will see on your final exam. If there are any questions that you do not understand, look in the lesson where the material is taught, and ask your tutor/marker or learning partner to help you.

Use the following representations for algebra tiles.

 $= -x^2$

 $= x^2$

 $= -x$

 $= x$

 $= -1$

 $= 1$

Part A: Multiple Choice (22 x 2 = 44 Marks)

Circle the letter of the response that represents the correct answer.

1. When you roll a 6-sided cube, numbered 1 to 6, the probability of rolling a 2 is $\frac{1}{6}$. This is an example of:

   a) an experimental probability
   b) theoretical probability
   c) subjective reasoning
   d) assumption

   You have 6 options $\Rightarrow \frac{1}{6}$.

   It is theoretical because this probability is based on logic, not data.

   (Module 6, Lesson 1)

2. An image rotated around its centre point appears unchanged after 180° and 360° turns. This is an example of:

   a) line symmetry
   b) rotation symmetry
   c) tessellation
   d) vertex

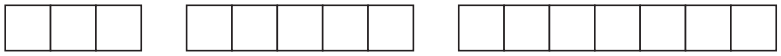
   (Module 8, Lesson 3)

3. You survey people as they leave a Blue Bomber game and ask them to identify their favourite sport. The potential problems or bias in data is caused by:

   a) cultural insensitivity
   b) cost concerns
   c) ethical behaviour
   d) time and timing

   (Module 1, Lesson 3)

4. Given the following pattern of shapes, choose the mathematical expression showing the changes for each iteration if b is the number of boxes in the previous iteration.

   a) $b + 1$
   b) $b + 2$
   c) $b - 1$
   d) $b - 2$

   (Module 5, Lesson 2)

5. Add: $\frac{3}{4} + \frac{1}{2} =$

   a) $1\frac{1}{4}$

   b) $\frac{5}{4}$

   c) neither (a) nor (b) above

   d) both (a) and (b) above

$1\frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$

(Module 2, Lesson 3)

6. Your income (I) is compared to the number of hours (H) you work. The variable, I, would

   a) be the dependent variable

   b) be the independent variable

   c) be constant

   d) go on the *y*-axis

(Module 5, Lesson 3)

7. Give the base of the following power: -3^4.

   a) 3

   b) –3

   c) –12

   d) –81

The negative is not **inside** the brackets, so it is the same as writing -1×3^4.

(Module 3, Lesson 2)

8. An inscribed angle has a measurement of 75°. A central angle shares the same endpoints. What is the measurement of the central angle?

   a) 375°

   b) 75°

   c) 150°

   d) 225°

(Module 6, Lesson 3)

9. When simplified, what is the exponential form of the following expression: $3^4 * 3^5 * 3^6$?

   a) 3^{15}

   b) 3^{120}

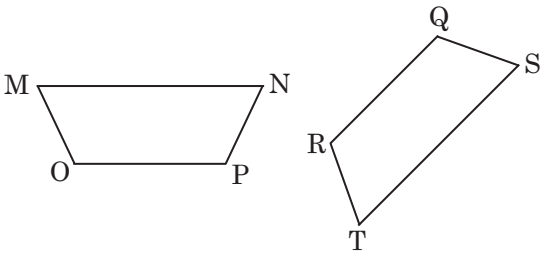
   c) 9^{15}

   d) 27^{15}

With like bases, you **add** the exponents when multiplying powers.

(Module 3, Lesson 5)

10. If shapes MOPN and SQRT are similar, which proportion is true?



a) $\frac{MO}{RT} = \frac{MN}{RQ}$

b) $\frac{MO}{RT} = \frac{OP}{TS}$

c) $\frac{NP}{QS} = \frac{OP}{TS}$

d) $\mathbf{\frac{MN}{ST} = \frac{MO}{SQ}}$ (Module 7, Lesson 3)

11. A box of oranges costs $2.25. Which equation would represent the total cost, *C*, of any number, *n*, of boxes of oranges?

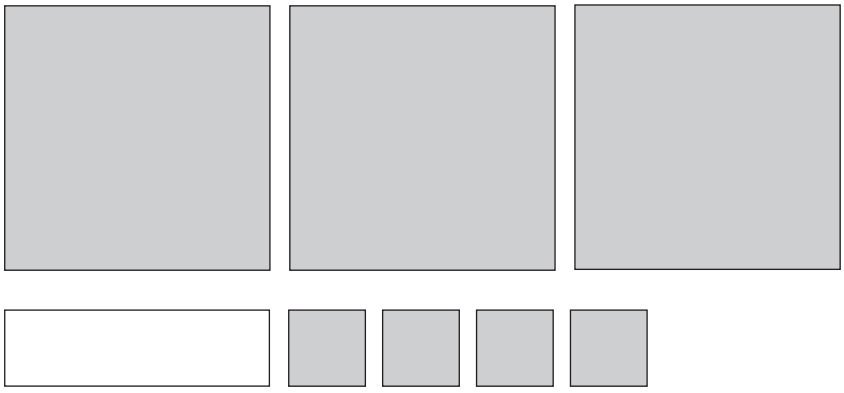
a) $C = 2.25 + n$

b) $C = 2.25 - n$

c) $\boldsymbol{C = 2.25n}$

d) $C = \frac{2.25}{n}$ (Module 5, Lesson 3)

12. An expression representing this arrangement of tiles is:



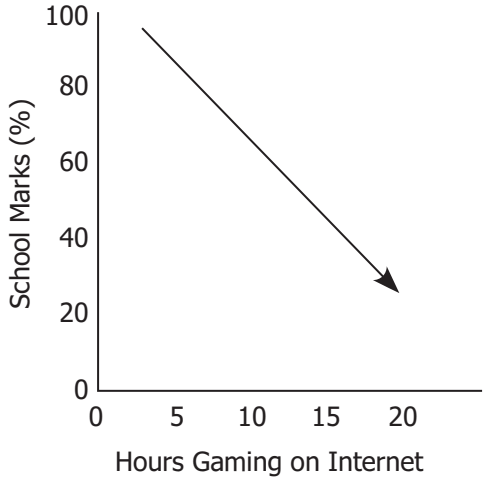
a) $-3x^2 + x - 4$

b) $-3x^2 - x + 4$

c) $3x^2 - x - 4$

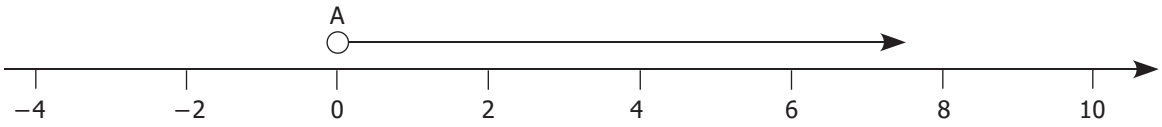
d) $\mathbf{3x^2 - x + 4}$ (Module 4, Lesson 2)

13. Choose a sentence that describes in words what this graph is showing.



a) increased hours gaming on the Internet increases school marks
b) school marks decline based on increased hours gaming on the Internet
c) hours gaming on the Internet do not affect school marks
d) no hours gaming on the Internet guarantees you 100% (Module 5, Lesson 2)

14. This inequality is read as:



a) values less than 0
b) values less than or equal to 0
c) values greater than 0
d) values greater than or equal to 0

Because the arrow is pointing to the larger numbers, it is greater than. Because the point is **not filled**, it is not equal to 0.

(Module 5, Lesson 6)

15. Give the next value in the following pattern: 120 90 60 ____

a) 50
b) 30
c) 80
d) 0.3

The pattern is to subtract 30 each time.

(Module 5, Lesson 2)

16. An inscribed angle that subtends, or has the same endpoints as a semicircle, will measure:

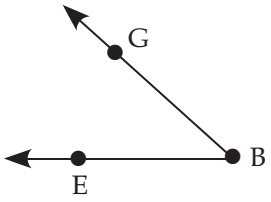
a) half of 90°

b) 90°

c) double the measure of the semicircle

d) 180° (Module 6, Lesson 3)

17. Find the measure of the angle below, using your protractor.



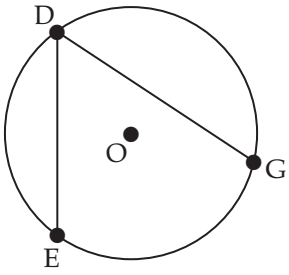
a) 42° ±3

b) 52° ±3

c) 32° ±3

d) 48° ±3 (Module 6, Lesson 2)

18. ∠EDG in this diagram is an example of a(n):



a) right angle

b) central angle

c) perpendicular bisector

d) inscribed angle (Module 6, Lesson 2)

19. The formula for finding the circumference of a circle is:

a) $2\pi r$

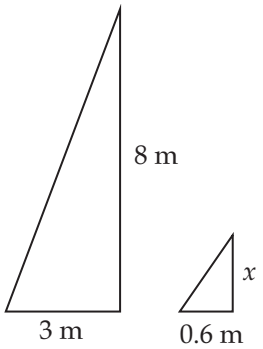
b) πr^2

c) $\frac{b_1 + b_2}{2}$

d) $\frac{bh}{2}$

(Module 7, Lesson 2)

20. If these shapes are similar, what is the value of x?



a) 4 m

b) 1.6 m

c) 1.8 m

d) 18 m (Module 7, Lesson 3)

21. Similar figures have:

a) proportional sides and identical angles

b) proportional sides and angles

c) equal sides and proportional angles

d) sides that look the same (Module 7, Lesson 3)

22. An object that repeats itself every 120° of rotation has rotation symmetry of order:

a) 0

b) 1

c) 3

d) 4 (Module 8, Lesson 3)

Part B: Short Answer (28 Marks)

Answer the following questions. Show all work.

1. Solve $4m - 2 = 5m + 7$ by isolating the variable. *(2 marks)*

   Answer:

   $4m - 2 - 4m = 5m + 7 - 4m$

   $-2 = m + 7$

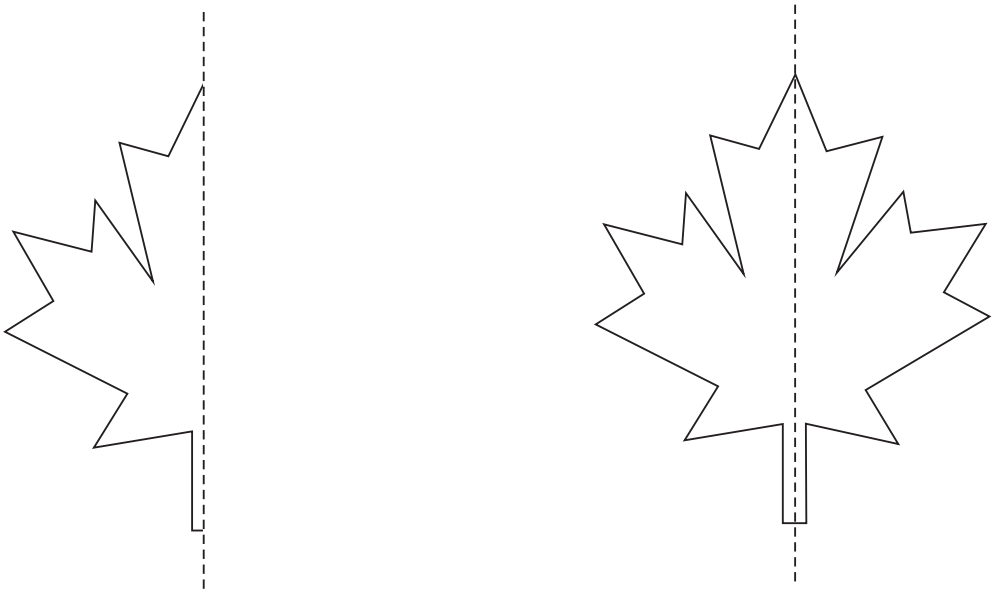
   $-2 - 7 = m + 7 - 7$

   $m = -9$

   (Module 5, Lesson 5)

2. Draw the reflected half across the line of symmetry. *(2 marks)*

   Answer:



   (Module 8, Lesson 2)

3. You have a square tarp to cover your boat. The area of the tarp is 49 m^2. Find the length of one side. *(2 marks)*

   Answer:

   7 m

   (Module 7, Lesson 2 and Module 2, Lesson 5)

4. Fill in the empty spots. *(6 marks)*

Term	Base	Exponent	Power
2^5	2		
$(-4)^3$		3	$(-4)^3$
26^7			26^7
-3^6		6	-3^6

Answer:

Term	Base	Exponent	Power
2^5	2	5	2^5
$(-4)^3$	(-4)	3	$(-4)^3$
26^7	26	7	26^7
-3^6	3	6	-3^6

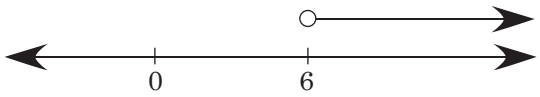
(Module 3, Lesson 2)

5. Solve for g and graph your solution on a number line. *(2 marks)*

$4g - 6 > 3g$

Answer:

$g > 6$



(Module 5, Lesson 6)

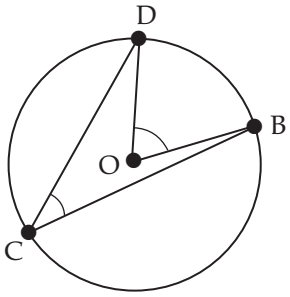
6. When comparing profits to sales, identify which is the dependent variable, and explain why it is dependent. *(4 marks)*

Answer:

The profits are the dependent variable because how much you sell determines your profit. Lower sales lessen the profit, higher sales increase the profit.

(Module 5, Lesson 4)

7. Use your protractor to find the measurement of both ∠DCB and ∠DOB in the following diagram. *(4 marks)*



Answer:

∠DOB = 70° ±3

∠DCB = 35° ±3 (Module 6, Lesson 2)

8. The length of the highway from Calgary to Vancouver is almost 1000 km. If you use a scale factor of 1 cm : 75 km, how long would the line representing this distance be on your drawing? *(4 marks)*

Answer:

1 cm : 75 km

x : 1000

x = 13.3 cm (Module 7, Lesson 5)

9. Describe the difference between the solution for $4m = -28$ and $4m \leq -28$. *(2 marks)*

Answer:

The solution to $4m = -28$ is $m = -7$ while the solution to $4m \leq -28$ is all real numbers less than or equal to –7. $4m \leq -28$ has an infinite number of solutions while $4m = -28$ has only one solution.

(Module 5, Lesson 7)

Part C: Long Answer (28 Marks)

Solve each of the following problems. Show all your work, and include written explanations where necessary.

1. You want to determine the most common paint colour of cars and trucks in your town. Explain in detail how you would collect these data, and discuss why you chose that method. *(4 marks)*

   Answer:

   Make observations on various roads at various times.

   Various roads to cover the most number of drivers, and different times to get shift workers.

   I could also cruise around parking lots to collect data (quicker).

   (Module 1, Lesson 7)

2. Place the following rational numbers in order from smallest to largest. *(3 marks)*

   $\frac{5}{7}$ $\quad -0.25$ $\quad \frac{-1}{3}$ $\quad 0.001$ $\quad -0.01$ $\quad \frac{3}{4}$ $\quad \frac{-2}{5}$

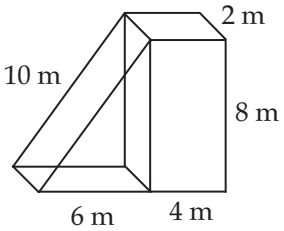
   Answer:

   0.714 $\quad -0.25$ $\quad -0.333$ $\quad 0.001$ $\quad -0.01$ $\quad 0.75$ $\quad -0.4$

   $\frac{-2}{5}$ $\quad \frac{-1}{3}$ $\quad -0.25$ $\quad -0.01$ $\quad 0.001$ $\quad \frac{5}{7}$ $\quad \frac{3}{4}$

   (Module 2, Lesson 4)

3. Find the surface area of the following composite object. *(4 marks)*



Answer:

Rectangular Prism:

$2 \times 4 \times 8 + 2 \times 4 \times 2 + 2 \times 8 \times 2 = 112 \text{ m}^2$

Triangular Prism:

$$2 \times \frac{1}{2} \times 6 \times 8 + 10 \times 2 + 6 \times 2 + 2 \times 8 = 96 \text{ m}^2$$

Overlap:

$8 \times 2 \times 2 = 32$

Total:

$112 + 96 - 32 = 176 \text{ m}^2$ (Module 7, Lesson 3)

4. You are working at a summer job to save money for the family winter trip to Florida. You want to have at least $500 saved. If your wage is $9.50 an hour, use a mathematical expression to find the minimum number of hours you will need to work to have enough money saved. *(3 marks)*

Answer:

Let h = money earned per hour

$$9.50h \geq 500$$

$$\frac{9.50h}{9.50} \geq \frac{500}{9.50}$$

$$h \geq 52.6$$

Since you work only full hours, you need to work 53 hours to earn at least $500.

(Module 5, Lesson 7)

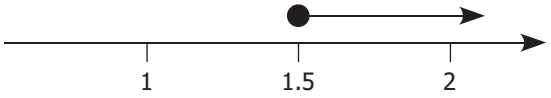
5. Isolate the variable, and draw a number line showing the solution to the following inequality. *(4 marks)*

$6h + 4 \geq -2h + 16$

Answer:

$8h \geq 12$

$h \geq \frac{3}{2}$



(Module 5, Lesson 5)

6. Given the following diagram, identify the listed components. *(6 marks)*

a) 2 radii _____

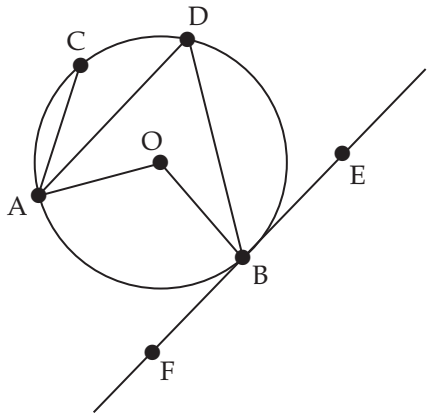
_____

b) tangent _____

c) 3 chords _____

_____

_____

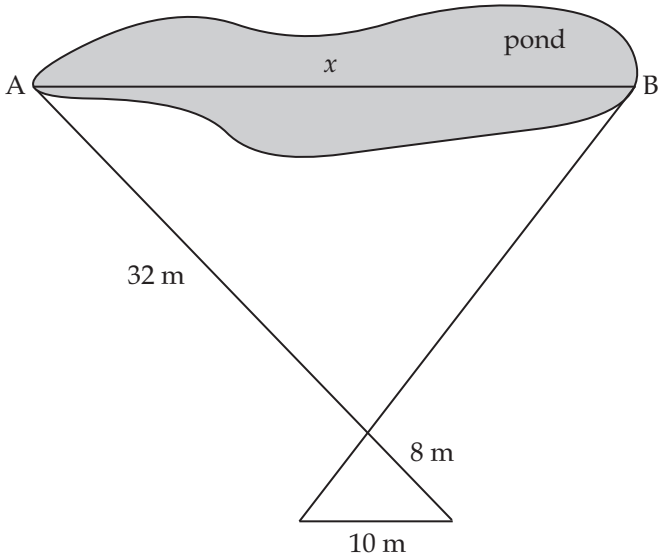


Answer:

2 radii	OA
	OB
tangent	FE
3 chords	AC
	AD
	BD

(Module 6, Lesson 2)

7. Use proportions based on the similar triangles below to find the distance across the pond from A to B. *(4 marks)*



Answer:

$$\frac{32}{x} : \frac{8}{10}$$

$$(10x)\frac{32}{x} = \frac{8}{10}(10x)$$

$$320 = 8x$$

$$40 = x$$

The pond is 40 m long. (Module 7, Lesson 4)

NOTES