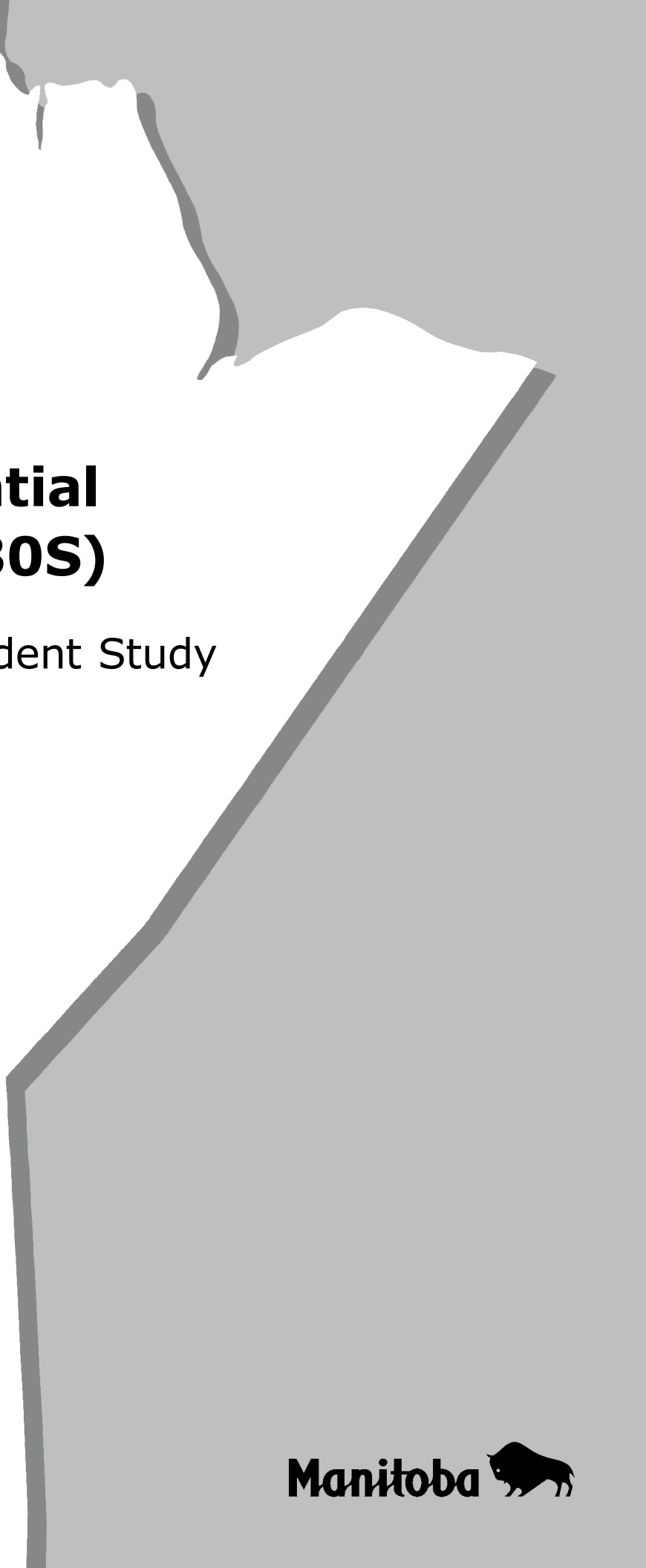
Grade 11 Essential Mathematics (30S)

A Course for Independent Study



Manitoba

Grade 11 Essential Mathematics (30S)

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CONTENTS

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Grade 11 Essential Mathematics (30S)

Introduction

Introduction to the course

Overview

Welcome to Grade 11 Essential Mathematics!

This course is a continuation of the concepts you have been studying in previous years, as well as an introduction to new topics. You will put to use many of the skills that you have already learned to solve problems and do basic arithmetic operations. This course helps you develop the skills, ideas, and confidence that you will need to continue studying math in the future.

As a student enrolled in a distance learning course, you have taken on a dual role—that of a student and a teacher. As a student, you are responsible for mastering the lessons and completing the learning activities and assignments. As a teacher, you are responsible for checking your work carefully, noting areas in which you need to improve, and motivating yourself to succeed.

What Will You Learn in This Course?

In this course, Grade 11 Essential Mathematics, you build on the knowledge and skills you gained while studying Grade 10 Essential Mathematics. In general, the Essential Mathematics curriculum emphasizes consumer applications, problem solving, decision making, and spatial sense. The specific topics studied in Grade 11 are listed below. After completing this course, you will be well-prepared to complete Grade 12 Essential Mathematics.

How Is This Course Organized?

The Grade 11 Essential Mathematics course consists of the following seven modules:

- Module 1: Interest and Credit
- Module 2: 3-D Geometry
- Module 3: Statistics
- Module 4: Managing Money
- Module 5: Relations and Patterns
- Module 6: Trigonometry
- Module 7: Design Modelling

Each module in this course consists of several lessons, which contain the following components:

- **Lesson Focus:** The Lesson Focus at the beginning of each lesson identifies one or more specific learning outcomes (SLOs) that are addressed in the lesson. The SLOs identify the knowledge and skills you should have achieved by the end of the lesson.
- **Introduction:** Each lesson begins with an explanation of what you will be learning in that lesson.
- **Lesson:** The main body of the lesson is made up of the content that you need to learn. It contains text, explanations, images, diagrams, and completed examples.
- **Learning Activities:** Many (most) lessons include one or more learning activities that will help you learn about the lesson topics and prepare you for the assignments, the midterm examination, and the final examination. Once you complete a learning activity, check your responses against those provided in the Learning Activity Answer Key found at the end of each applicable module. You will not submit the completed learning activities to the Distance Learning Unit.
- **Assignments:** Assignments are found at the end of each lesson that has an assignment. You will mail or electronically submit all your completed assignments to the Distance Learning Unit for assessment at the end of each module. In total, all assignments are worth seventy-five percent (75%) of your final course mark.
- **Summary:** Each lesson ends with a brief review of what you just learned.

This course also includes the following appendices:

- **Appendix A: Unit Conversions:** At the end of the course, you will find an appendix that contains Conversion Formula Tables. These unit conversions are for reference use only and are not required for the examinations.
- **Appendix B: Glossary:** The glossary at the end of the course provides definitions for an alphabetical list of the terms identified throughout the course. You can use the glossary to review terms used in the course.

What Resources Will You Need for This Course?

You do not need a textbook for this course. All the content is provided directly within the course. You will, however, need access to a variety of resources.

The required and optional resources for this course are identified below.

Required Resources

For this course, you will need access to the following resources. If you do not have access to one or more of these resources, contact your tutor/marker.

- **A calculator:** Use a graphing or scientific calculator as you work through this course. You will also need the calculator for the examination(s).
- **A metric and imperial ruler:** Use the rulers as you work through this course. You will also need both rulers for the examination(s).
- **A geometry set:** Use a geometry set including a set of compasses, a protractor, and a set square for this course. You will also **need** the geometry set for the examination(s).

Electronic Resources

For this course, you will need the following electronic resource. If you do not have access to the Internet, or if you need a copy of the resource, contact the Distance Learning Unit at 1-800-465-9915.

- Module 7 contains a number of diagrams that represent 2-D views of 3-D objects. Module 7: Design Modelling is available in the learning management system (LMS).

Optional Resources

It would be helpful if you had access to the following resources:

- **Access to a computer with spreadsheet and graphing capabilities:** Access to a computer with spreadsheet software and graphing capabilities will be an advantage but not a requirement.
- **A computer with Internet access:** Use of the Internet may be suggested as a resource in some places, but if you do not have access to an online computer you can still complete the related learning activities and assignments without it.

- **Access to a photocopier:** With access to a photocopier/scanner, you could make a copy of your assignments before submitting them so that if your tutor/marker wants to discuss an assignment with you over the phone, each of you will have a copy. It would also allow you to continue studying or to complete further lessons while your original work is with the tutor/marker. Photocopying or scanning your assignments will also ensure that you keep a copy in case the originals are lost.

Who Can Help You with This Course?

Taking an independent study course is different from taking a course in a classroom. Instead of relying on the teacher to tell you to complete a learning activity or an assignment, you must tell yourself to be responsible for your learning and for meeting deadlines. There are, however, two people who can help you be successful in this course: your tutor/marker and your learning partner.

Your Tutor/Marker

Tutor/markers are experienced educators who tutor Independent Study Option (ISO) students and mark assignments and examinations. When you are having difficulty with something in this course, contact your tutor/marker, who is there to help you. Your tutor/marker's name and contact information were sent to you with this course. You can also obtain this information in the learning management system (LMS).

Your Learning Partner

A learning partner is someone **you choose** who will help you learn. It may be someone who knows something about mathematics, but it doesn't have to be. A learning partner could be someone else who is taking this course, a teacher, a parent or guardian, a sibling, a friend, or anybody else who can help you. Most importantly, a learning partner should be someone with whom you feel comfortable and who will support you as you work through this course.

Your learning partner can help you keep on schedule with your coursework, read the course with you, check your work, look at and respond to your learning activities, or help you make sense of assignments. You may even study for your examination(s) with your learning partner. If you and your learning partner are taking the same course, however, your assignment work should not be identical.

How Will You Know How Well You Are Learning?

You will know how well you are learning in this course by how well you complete the learning activities, assignments, and examinations.

Learning Activities

Each learning activity has two parts—Part A has BrainPower questions and Part B has questions related to the content in the lesson

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for you before trying the other questions. Each question should be completed quickly and without using a calculator, and most should be completed without using pencil and paper to write out multiple steps. Some of the questions will relate directly to content of the course. Some of the questions will review content from previous courses—content that you need to be able to answer efficiently.

Being able to do these questions in a few minutes will be helpful to you as you continue with your studies in mathematics. If you are finding it is taking you longer to do the questions, you can try one of the following:

- work with your learning partner to find more efficient strategies for completing the questions
- ask your tutor/marker for help with the questions
- search online for websites that help you practice the computations so you can become more efficient at completing the questions.

None of the assignment questions or examination questions will require you to do the calculations quickly or without a calculator. However, it is for your benefit to complete the questions as they will help you in the course. Also, being able to successfully complete the BrainPower exercises will help build your confidence in mathematics. BrainPower questions are like a warm-up you would do before competing in a sporting event.

Part B: Course Content Questions

One of the easiest and fastest ways to find out how much you have learned is to complete Part B of the learning activities. These have been designed to let you assess yourself by comparing your answers with the answer keys at the end of each module. There is at least one learning activity in each lesson. You will need a notebook or loose-leaf pages to write your answers.

The learning activities in this course will help you to review and practise what you have learned in the lessons. You will not submit the completed learning activities to the Distance Learning Unit. Instead, you will complete the learning activities and compare your responses to those provided in the Learning Activity Answer Key found at the end of each module.

Make sure you complete the learning activities. Doing so will not only help you to practise what you have learned, but will also prepare you to complete your assignments and the examination(s) successfully. Many of the questions on the examination(s) will be similar to the questions in the learning activities. Remember that you **will not submit learning activities to the Distance Learning Unit**.

Assignments

Each module in this course contains assignments, which you will complete and submit to the Distance Learning Unit for assessment. The assignments are worth a total of 75% of your final course mark.

There are two types of assignments that you must submit to your tutor/marker. Each module has a cover assignment, which you can complete at any time during the module. Lesson assignments are located throughout the modules, and include questions similar to the questions in the learning activities of previous lessons. The cover assignments and lesson assignments have space provided for you to write your answers on the question sheets. **You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate).**

The tutor/marker will mark your assignments and return them to you. Remember to keep all marked assignments until you have finished the course so that you can use them to study for your examinations.

Resource Sheet

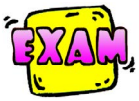
When you write your midterm and final examinations, you will be allowed to take an Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. It is to be submitted with your examination. The Examination Resource Sheet is not worth any marks.

Creating your own resource sheet is an excellent way to review. It also provides you with a convenient reference and quick summary of the important facts of each module. Each student is asked to complete a resource sheet for each module to help with studying and reviewing.

The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module. Refer to these when you create your own resource sheet. Then, go to Appendix B: Glossary (at the end of the course) to check the information on your resource sheet.

After you complete each module's resource sheet, you should summarize the sheets from all of the modules to prepare your Examination Resource Sheet. The midterm examination is based only on the first three modules of the course, while the final examination is based on Modules 4 to 7.

Midterm and Final Examinations



This course contains a midterm examination and a final examination.

- The midterm examination is based on Modules 1 to 3, and is worth 12.5% of your final mark in this course. You will write the midterm examination when you have completed Module 3.

  Formulas are not provided on the examination. As a student, you can use your Midterm Examination Resource Sheet to bring any formulas you have not memorized into the examination with you. You will be required to bring the following supplies to the midterm examination: pens and pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Midterm Examination Resource Sheet.

- The final examination is based on Modules 4 to 7, and is worth 12.5% of your final mark in this course. You will write the final examination when you have completed Module 7.

  You can use your Final Examination Resource Sheet to bring any formulas you have not memorized into the examination with you. Formulas are not provided on the examination. You will be required to bring the following supplies to the final examination: pens and pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Final Examination Resource Sheet. You may also bring coloured pencils for the 2-D and 3-D drawings.

The two examinations are worth a total of 25% of your final course mark. You will write both examinations under supervision.

To do well on each examination, you should review all the work you have completed from the modules, including all learning activities and assignments.

Practice Examinations and Answer Keys

To help you succeed in your examinations, you will have an opportunity to complete a Midterm Practice Examination and a Final Practice Examination. These examinations, along with the answer keys, are found in the learning management system (LMS). If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to obtain a copy of the practice examinations.

These practice examinations are similar to the actual examinations you will be writing. The answer keys enable you to check your answers. This will give you the confidence you need to do well on your examinations.

Requesting Your Examination(s)

You are responsible for making arrangements to have the examinations sent to your proctor from the Distance Learning Unit. Please make arrangements before you finish Module 3 to write the midterm examination. Likewise, you should begin arranging for your final examination before you finish Module 7.

To write your examinations, you need to make the following arrangements:

- **If you are attending school,** your examination will be sent to your school as soon as all the applicable assignments have been submitted. You should make arrangements with your school's ISO school facilitator to determine a date, time, and location to write the examination.

- **If you are not attending school,** check the Examination Request Form for options available to you. Examination Request Forms can be found on the Distance Learning Unit's website, or look for information in the learning management system (LMS). Two weeks before you are ready to write the examination, fill in the Examination Request Form and mail, fax, or email it to

  Distance Learning Unit
  500–555 Main Street
  PO Box 2020
  Winkler MB R6W 4B8
  Fax: 204-325-1719
  Toll-Free Telephone: 1-800-465-9915
  Email: distance.learning@gov.mb.ca

How Much Time Will You Need to Complete This Course?

Learning through independent study has several advantages over learning in the classroom. You are in charge of how you learn and you can choose how quickly you will complete the course. You can read as many lessons as you wish in a single session. You do not have to wait for your teacher or classmates.

From the date of your registration, you have a maximum of **12 months** to complete this course, but the pace at which you proceed is up to you. Read the following suggestions on how to pace yourself.

Chart A: Semester 1

If you want to start the course in September and complete it in January, you can follow the timeline suggested below.

Module	Completion Date
Module 1	Middle of September
Module 2	End of September
Module 3	Middle of October
Midterm Examination	Beginning of November
Module 4	Middle of November
Module 5	End of November
Module 6	Middle of December
Module 7	Middle of January
Final Examination	End of January

Chart B: Semester 2

If you want to start the course in February and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Module 1	Middle of February
Module 2	End of February
Module 3	Middle of March
Midterm Examination	End of March
Module 4	Beginning of April
Module 5	Middle of April
Module 6	End of April
Module 7	Beginning of May
Final Examination	Middle of May

Chart C: Full School Year (Not Semestered)

If you want to start the course in September and complete it in May, you can follow the timeline suggested below.

Module	Completion Date
Module 1	End of September
Module 2	End of October
Module 3	End of November
Midterm Examination	Beginning of January
Module 4	End of January
Module 5	End of February
Module 6	End of March
Module 7	End of April
Final Examination	Middle of May

Timelines

Do not wait until the last minute to complete your work, since your tutor/marker may not be available to mark it immediately. It may take a few weeks for your tutor/marker to assess your work and return it to you or your school.

If you need this course to graduate this school year, all coursework must be received by the Distance Learning Unit on or before the first Friday in May, and all examinations must be received by the Distance Learning Unit on or before the last Friday in May. Any coursework or examinations received after these deadlines may not be processed in time for a June graduation. Assignments or examinations submitted after these recommended deadlines will be processed and marked as they are received.

When and How Will You Submit Completed Assignments?

When to Submit Assignments

While working on this course, you will submit completed assignments to the Distance Learning Unit seven times. The following chart shows you exactly what assignments you will be submitting at the end of each module.

Submission of Assignments

Submission	Assignments You Will Submit
1	**Module 1: Interest and Credit** Module 1 Cover Sheet Module 1 Cover Assignment: Loyalty Programs Assignment 1.1: Interest Assignment 1.2: What Is Credit? Assignment 1.3: Credit Payment Calculations Assignment 1.4: Credit Card Calculations
2	**Module 2: 3-D Geometry** Module 2 Cover Sheet Module 2 Cover Assignment: Geometric Patterns Assignment 2.1: Surface Area Assignment 2.2: Volume Assignment 2.3: Applications of Volume and Capacity
3	**Module 3: Statistics** Module 3 Cover Sheet Module 3 Cover Assignment: Statistics in Your Life Assignment 3.1: Circle Graphs, Bar Graphs, and Histograms Assignment 3.2: Line Graphs Assignment 3.3: Interpreting Graphs
4	**Module 4: Managing Money** Module 4 Cover Sheet Module 4 Cover Assignment: How Time Affects the Value of Money Assignment 4.1: Budgets Assignment 4.2: Bank Accounts Assignment 4.3: Cheques, Deposit Slips, and Registers

continued

Submission of Assignments (continued)	
Submission	Assignments You Will Submit
5	**Module 5: Relations and Patterns** Module 5 Cover Sheet Module 5 Cover Assignment: Applying Patterns Assignment 5.1: Scatterplots Assignment 5.2: Patterns and Linear Relations Assignment 5.3: Slope Assignment 5.4: Scale
6	**Module 6: Trigonometry** Module 6 Cover Sheet Module 6 Cover Assignment: Math Operations Assignment 6.1: Applying Trigonometry
7	**Module 7: Design Modelling** Module 7 Cover Sheet Module 7 Cover Assignment: Four-Colour Problem Assignment 7.1: Three 2-D Views of 3-D Objects Assignment 7.2: One-Point Perspective Drawings Assignment 7.3: Exploded Views and Component Parts

How to Submit Assignments

In this course, you have the choice of submitting your assignments either by mail or electronically.

- **Mail:** Each time you **mail** something, you must include the print version of the applicable Cover Sheet (found at the end of this Introduction). Complete the information at the top of each Cover Sheet before submitting it along with your assignments.
- **Electronic submission:** You do not need to include a cover sheet when submitting assignments electronically.

Submitting Your Assignments by Mail

If you choose to mail your completed assignments, please photocopy/scan all the materials first so that you will have a copy of your work in case your package goes missing. You will need to place the applicable module Cover Sheet and assignment(s) in an envelope, and address it to

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Your tutor/marker will mark your work and return it to you by mail.

Submitting Your Assignments Electronically

Assignment submission options vary by course. Sometimes assignments can be submitted electronically and sometimes they must be submitted by mail. Specific instructions on how to submit assignments were sent to you with this course. In addition, this information is available in the learning management system (LMS).

If you are submitting assignments electronically, make sure you have saved copies of them before you send them. That way, you can refer to your assignments when you discuss them with your tutor/marker. Also, if the original hand-in assignments are lost, you are able to resubmit them.

Your tutor/marker will mark your work and return it to you electronically.

The Distance Learning Unit does not provide technical support for hardware-related issues. If troubleshooting is required, consult a professional computer technician.

What Are the Guide Graphics For?

Guide graphics are used throughout this course to identify and guide you in specific tasks. Each graphic has a specific purpose, as described below.

Lesson Introduction: The introduction sets the stage for the lesson. It may draw upon prior knowledge or briefly describe the organization of the lesson. It also lists the learning outcomes for the lesson. Learning outcomes describe what you will learn.

Learning Partner: Ask your learning partner to help you with this task.

Learning Activity: Complete a learning activity. This will help you to review or practise what you have learned and to prepare for an assignment or an examination. You will not submit learning activities to the Distance Learning Unit. Instead, you will compare your responses to those provided in the Learning Activity Answer Key found at the end of the applicable module.

Assignment: Complete an assignment. You will submit your completed assignments to the Distance Learning Unit for assessment at the end of a given module.

Mail or Electronic Submission: Mail or electronically submit your completed assignments to the Distance Learning Unit for assessment at this time.

Phone or Email: Telephone or email your tutor/marker.

Resource Sheet: Indicates material that may be valuable to include on your resource sheet.

Examination: Write your midterm or final examination at this time.

Note: Take note of and remember this important information or reminder.

Remember: If you have questions or need help at any point during this course, contact your tutor/marker or ask your learning partner for help.

Good luck with the course!

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Module 1: Interest and Credit
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 1 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 1: Loyalty Programs	_____ /15	_____ /15
❑ Assignment 1.1: Interest	_____ /25	_____ /25
❑ Assignment 1.2: What Is Credit?	_____ /8	_____ /8
❑ Assignment 1.3: Credit and Payment Calculations	_____ /32	_____ /32
❑ Assignment 1.4: Credit Card Calculations	_____ /15	_____ /15
	Total: ____ /95	**Total: ____ /95**
For Tutor/Marker Use		
Remarks:		

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Module 2: 3-D Geometry
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 2 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 2: Geometric Patterns	_____ /24	_____ /24
❑ Assignment 2.1: Surface Area	_____ /30	_____ /30
❑ Assignment 2.2: Volume	_____ /28	_____ /28
❑ Assignment 2.3: Applications of Volume and Capacity	_____ /30	_____ /30
	Total: ____ /112	**Total: ____ /112**
For Tutor/Marker Use		
Remarks:		

Grade 11 Essential Mathematics (30S)

Module 3: Statistics
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 3 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 3: Statistics in Your Life	_____ /11	_____ /11
❑ Assignment 3.1: Circle Graphs, Bar Graphs, and Histograms	_____ /27	_____ /27
❑ Assignment 3.2: Line Graphs	_____ /16	_____ /16
❑ Assignment 3.3: Interpreting Graphs	_____ /17	_____ /17
	Total: ____ /71	**Total: ____ /71**
For Tutor/Marker Use		
Remarks:		

Grade 11 Essential Mathematics (30S)

Module 4: Managing Money
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 4 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 4: How Time Affects the Value of Money	_____ /9	_____ /9
❑ Assignment 4.1: Budgets	_____ /30	_____ /30
❑ Assignment 4.2: Bank Accounts	_____ /18	_____ /18
❑ Assignment 4.3: Cheques, Deposit Slips, and Registers	_____ /19	_____ /19
	Total: ____ /76	**Total: ____ /76**
For Tutor/Marker Use		
Remarks:		

Grade 11 Essential Mathematics (30S)

Module 5: Relations and Patterns
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 5 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 5: Applying Patterns	_____ /14	_____ /14
❑ Assignment 5.1: Scatterplots	_____ /17	_____ /17
❑ Assignment 5.2: Patterns and Linear Relations	_____ /28	_____ /28
❑ Assignment 5.3: Slope	_____ /29	_____ /29
❑ Assignment 5.4: Scale	_____ /19	_____ /19
	Total: ____ /107	**Total: ____ /107**
For Tutor/Marker Use		
Remarks:		

Grade 11 Essential Mathematics (30S)

Module 6: Trigonometry
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 6 Assignments	**Attempt 1**	**Attempt 2**
Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	_____ Date Received	_____ Date Received
❑ Cover Assignment 6: Math Operations	_____ /10	_____ /10
❑ Assignment 6.1: Applying Trigonometry	_____ /27	_____ /27
	Total: ____ /37	**Total: ____ /37**

For Tutor/Marker Use
Remarks:

Grade 11 Essential Mathematics (30S)

Module 7: Design Modelling
Cover Sheet

Please complete this sheet and place it on top of your assignments to assist in proper recording of your work. Submit the package to:

Drop-off/Courier Address

Distance Learning Unit
555 Main Street
Winkler MB R6W 1C4

Mailing Address

Distance Learning Unit
500–555 Main Street
PO Box 2020
Winkler MB R6W 4B8

Contact Information

Legal Name: _____ Preferred Name: _____

Phone: _____ Email: _____

Mailing Address: _____

City/Town: _____ Postal Code: _____

Attending School: ❑ No ❑ Yes

School Name: _____

Has your contact information changed since you registered for this course? ❑ No ❑ Yes

Note: Please keep a copy of your assignments so that you can refer to them when you discuss them with your tutor/marker.

For Student Use	For Office Use Only	
Module 7 Assignments Which of the following are completed and enclosed? Please check (✓) all applicable boxes below.	**Attempt 1** _____ Date Received	**Attempt 2** _____ Date Received
❑ Cover Assignment 7: Four-Colour Problem	_____ /8	_____ /8
❑ Assignment 7.1: Three 2-D Views of 3-D Objects	_____ /21	_____ /21
❑ Assignment 7.2: One-Point Perspective Drawings	_____ /15	_____ /15
❑ Assignment 7.3: Exploded Views and Component Parts	_____ /16	_____ /16
	Total: ____ /60	**Total: ____ /60**
For Tutor/Marker Use		
Remarks:		

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Grade 11 Essential Mathematics (30S)

Module 1
Interest and Credit

Module 1:
Interest and Credit

Introduction

In Grade 10 Essential Mathematics, you studied payment for work, including gross pay and net pay. This module will focus on two topics that relate to what you do with the money you earn. The first two lessons deal with interest, both on investments and loans. The following lessons talk about credit—where you can get it and how to use it. You may wonder what this has to do with you, but interest and credit will probably be an important part of your financial world. Money deposited in a savings account earns interest. On the other hand, if you borrow money or use a credit card, you will likely pay interest—sometimes a lot. Sometimes you may need credit to purchase an expensive item, such as a car, laptop, or even a house!

Before beginning the first lesson of this course, complete Learning Activity 1.1 at the end of this introduction. It will prepare you for the course you are about to start.

Assignments in Module 1

When you have completed the assignments for Module 1, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Loyalty Programs
2	Assignment 1.1	Interest
3	Assignment 1.2	What Is Credit?
5	Assignment 1.3	Credit Payment Calculations
6	Assignment 1.4	Credit Card Calculations

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the following list of instructions to help you with preparing your resource sheet for the material in Module 1. On this sheet, you should record mathematics terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1, 2, and 3 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 3.

Resource Sheet for Module 1

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important mathematics terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how each formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Module 1 Cover Assignment: Loyalty Programs

Consumer loyalty is important for any business. To encourage loyalty, many retail stores and banking institutions offer incentive programs in the form of awarding points for purchases. These incentive programs reward the best customers for investing in their business and they encourage all shoppers to continue to invest their money at the same bank or retail store in the future. Is the amount of money spent at a store realistic when compared to the value of the reward that the store gives to their customer? Is the value of the reward the same for each company's incentive program or is the reward offered by some institutions better than others?

This activity will help you to answer these questions as you investigate the costs and rewards of the incentive programs historically offered by Zellers, National Bank, Royal Bank, and Scotia Bank.

Notes

Module 1 Cover Assignment

Loyalty Programs

Total: 15 marks

When you purchase a product and use a credit card issued by your bank, you can earn points as part of a loyalty program. Each bank has its own loyalty program. The points that you earn can be exchanged for merchandise. Loyalty programs have been around for a long time and have changed over the years, based on market research done by each participating company. It is often difficult to determine which loyalty program to join in order to save the most money. The information in the chart below is from a report entitled *Reframing the Conversation on Loyalty Programs in Canada*. The report was produced by Environics Research Group and was commissioned by COLLOQUY. At the time of this writing, the details of the Environics report were available online at http://compareloyaltyprograms.ca/pdf/Reframing_the_Conversation_on_Loyalty_Programs_in_Canada.pdf.

Non-Credit Loyalty Cards

Some loyalty programs are associated with a credit card, while others do not involve credit cards (referred to as non-credit card loyalty programs). There are 13 non-credit loyalty card programs listed in the following chart that were evaluated in the loyalty programs report. Some programs are for loyalty at one specific store, such as Canadian Tire. Others are coalition loyalty programs where members can earn points through hundreds of retail partners, as opposed to just one. The research found that the largest coalition loyalty reward programs in Canada are AIR MILES and Aeroplan. Both of these programs are synonymous with travel rewards and have the lowest "time to reward" in all reward categories, not just travel (Environics, 17).

continued

Module 1 Cover Assignment: Loyalty Programs (continued)

Loyalty Programs	
Cash Equivalent	**Time to $100 reward (months)**
AIR MILES® Reward Program Membership Card	5.87
Shoppers Optimum® Card	6.69
Aeroplan® Membership Card	7.29
Canadian Tire Money®	7.82
metro&moi™ Program	9.74
The More Rewards Program	11.46
PETRO-POINTS™	14.79
Esso Extra®	16.88
The Club Sobeys Program	24.44
Hudson's Bay Rewards™ Card	28.25
SCENE® Card	34.87
plum® rewards	142.33
Best Buy® Reward Zone™	252.13

Source: Environics, 17.

"Time to Reward" Algorithm

In order to conduct an apples-to-apples comparison of all loyalty programs in the report, an evaluation algorithm was developed that predicts "time to reward" as accurately as possible, using appropriate variables collected across every loyalty program. The "time to reward" algorithm was developed to predict how many months it takes to get to $100 CAD worth of reward (Environics, 9).

continued

Module 1 Cover Assignment: Loyalty Programs (continued)

Answer the following:

1. The determination of the best loyalty program depends partly on the particular spending habits of each individual. The table below entitled High-Level Shopping Categories shows the average monthly dollars spent on each high-level shopping category according to 2012 data from Statistics Canada.

High-Level Shopping Categories	Average monthly dollar spent (Statistics Canada, 2012)
Clothing and accessories (such as shoes, jewellery, hats, and children's clothing)	$285.42
General merchandise (such as books, music, movies, entertainment, electronics, and children's wear)	$126.92
Restaurants and dining	$178.58
Grocery, liquor, and drugstore (including pharmaceuticals and over-the-counter medication products)	$570.50
Health, beauty, and personal care (such as personal grooming and nutrition products)	$231.08
Home and office goods and services	$181.92
Automotive care and services (such as fuel, car parts/accessories, maintenance services, and cleaning supplies)	$840.58
Travel and accommodations	$213.83
Total	$2628.83

   a) Complete the table below for each category by determining the percent of the total spent per month by an average Canadian. Round percents to the nearest tenth. *(4 marks)*

High-Level Shopping Categories	Monthly Spending	Percent
Clothing and accessories	$285.42	
General merchandise	$126.92	
Restaurants and dining	$178.58	
Grocery, liquor, and drugstore	$570.50	
Health, beauty, and personal care	$231.08	
Home and office goods and services	$181.92	
Automotive care and services	$840.58	
Travel and accommodations	$213.83	

continued

b) Ava-May spends money on the shopping categories in the same proportion as the average Canadian, as calculated in the table above. However, after paying other bills (such as rent and utilities), the total amount she has to spend on the high-level shopping categories is $1200. Complete the table below for each category by determining the amount of money Ava-May will budget for each category. Round amounts to the nearest dollar. *(4 marks)*

High-Level Shopping Categories	Monthly Budget ($)
Clothing and accessories	
General merchandise	
Restaurants and dining	
Grocery, liquor, and drugstore	
Health, beauty, and personal care	
Home and office goods and services	
Automotive care and services	
Travel and accommodations	

2. The following table shows the time, in months, required to earn a reward equivalent to $100 for various loyalty programs. Use the table to answer the questions that follow.

Loyalty Programs	
Cash Equivalent	**Time to $100 reward (months)**
AIR MILES® Reward Program Membership Card	5.87
Shoppers Optimum® Card	6.69
Aeroplan® Membership Card	7.29
Canadian Tire Money®	7.82
metro&moi™ Program	9.74
The More Rewards Program	11.46
PETRO-POINTS™	14.79
Esso Extra®	16.88
The Club Sobeys Program	24.44
Hudson's Bay Rewards™ Card	28.25
SCENE® Card	34.87
plum® rewards	142.33
Best Buy® Reward Zone™	252.13

continued

Module 1 Cover Assignment: Loyalty Programs (continued)

a) How long (in years) would it take to earn a reward value of $400 using the Club Sobeys Program? *(1 mark)*

b) When taking care of their automobiles, Ava-May goes to businesses that allow her to use her AIR MILES card and Clark goes to Canadian Tire to collect Canadian Tire Money. Who will take longer (and by how many months) to earn $500 worth of rewards using their chosen loyalty program? *(2 marks)*

c) If Clark regularly used his SCENE card when going out, how long would it take him to earn a reward value of $20? *(2 marks)*

3. The table above shows that the AIR MILES card is the best loyalty program for the average Canadian. However, it will not always be the best loyalty program for every Canadian. Describe a circumstance that might have someone choose to join a loyalty program other than the AIR MILES card. *(2 marks)*

Notes

Learning Activity 1.1

This learning activity is the only one that does not include a BrainPower section, although it does have two parts. Be sure to complete this learning activity before you begin your first lesson.

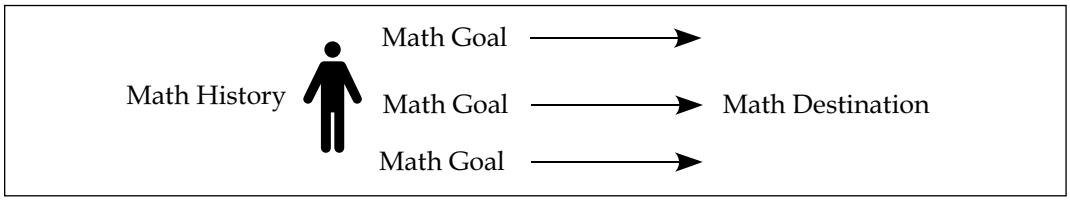
The first learning activity involves you having a conversation with your tutor/marker. Having this conversation with your tutor/marker has two important purposes. First, it introduces you to a very valuable resource—your tutor/marker. He or she is available for you to answer questions, explain concepts, and guide you through this course. You can discuss your math learning and progress. Feel free to contact your tutor by phone or email at any time during this course.

The second important purpose of this assignment is to get you thinking about your math goals. You may have a future career in mind and this course is getting you one step closer to it by completing a prerequisite for a future required course. There may be specific skills or topics you are interested in learning about and they are covered in this course.

If you are unsure of your math goals or why they are important, consider this:

- goals give you a sense of direction and purpose in taking this course
- goals help motivate you to learn and do your best, even when it's tough
- when you accomplish your goals, there is a great sense of achievement and success.

Good goals need to be realistic and specific, and they should reflect what is important to you. They should give you direction and take you further down the path from where you have been to where you want to go.



Goals can be long term or short term, but they are the pathway that takes **you** from where you were/are, closer to where you want to go.

continued

Learning Activity 1.1 (continued)

Part A: Your Tutor/Marker

Fill in the following blanks using information provided with your course:

My tutor/marker's name is _____

I can phone my tutor/marker at 1-8_____

My tutor/marker's email is _____

Be ready to discuss the following topics and the reasons for your answers with your tutor/marker during your phone conversation. If you'd like, make some notes below before you call in order to help you feel prepared. Feel free to add other questions or comments that you may have as well.

1. I am taking this course by distance education because…

_____

_____

_____

_____

2. What I like about math and can do mathematically is… (favourite topic, skill, where you use math, etc.)

_____

_____

_____

_____

continued

Learning Activity 1.1 (continued)

3. What I dislike about math or have difficulty doing is …

4. Previous math experiences that influence the way I feel about math are…

5. The next math course I would like to take is…

6. What I am hoping this course will help me accomplish and learn for the future is…

continued

7. I will organize things to help me succeed in this course by…

_____

_____

_____

_____

During your phone conversation, jot down a sentence or two in the spaces above about what you and your tutor/marker talk about. For example, if you are taking this course because it doesn't fit into your schedule at school or because you travel a lot with your basketball team and this is more convenient, state that in the space below question 1.

Part B: Your Math Pathway

Use the answers to the questions from the conversation with your tutor/marker as a starting point and fill in the following diagram. In the Math History box, jot down point-form notes about your prior experience and knowledge about math (Questions 2, 3, and 4). In the Math Destination box, jot down what completing this course will help you accomplish in the future (Questions 5 and 6).

In the Pathway box, write down what you will need to do to move down the pathway from your History to your Destination.

Math History	Pathway	Math Destination

continued

Learning Activity 1.1 (continued)

For example, if your destination includes needing a 75% in this course so that you can feel confident going into Grade 12 Essential Mathematics, or you need to learn how to make smart consumer decisions, what will help you accomplish this? It may mean figuring out how you best learn and study math. It may mean setting up a schedule so you complete the assignments on time. You may need to find your calculator manual and figure out how to use it, set up regular appointments with your learning partner, research a topic on the Internet, or read a textbook about a certain math concept or skill. Your pathway is unique to you.

As you move through this course and work on achieving your goals, self-assessment becomes important. It is the way for you to determine if you are getting closer to your destination, and if the steps along your pathway are taking you in the right direction. You will need to periodically ask yourself: Am I doing my assignments? Are my note-taking skills improving? How often have I contacted my tutor/marker or worked with my learning partner? Have I found useful homework websites? Is my schedule working? What do I need to change or adjust so I can get to my destination?

Several times during this course, you will go through this cycle of looking at where you have been, where you want to go, and where you currently are. At any time, you may want to revise your goals or set new ones, as you evaluate your own progress and learning.

- Look back/history—reflect on what you know, how far you have come
- Look around/pathway—assess if you are achieving your goals, determine if new learning or understanding has occurred, and check your progress
- Look forward/destination—determine what you want to know, set goals

Each time you go through these steps you will become better at mathematics!

It is important that you keep the chart of your pathway handy, as you will revisit it at other points in this course.

Notes

Lesson Focus

In this lesson, you will

- ❑ use the simple interest formula to calculate interest and the other variables

Lesson Introduction

When you invest money, you are able to earn interest on your investment. Conversely, when you repay a loan, you must pay interest in addition to the amount you originally borrowed. In this lesson, we will explore the most basic form of interest, called **simple interest**.

Simple Interest

Financial institutions borrow and lend money. When you deposit money into a savings account, you are lending the financial institution money for a period of time. The financial institution pays you interest for borrowing your money. In turn, the financial institution lends your money to individuals who need it. These individuals must pay interest for the money they borrow. The interest rate they pay the financial institution is higher than the interest rate you receive from the same institution. In this way, the financial institution earns a profit on these transactions.

Calculating Simple Interest

When you invest some money in a financial institution, the institution pays you interest for using your money. When you borrow money from a financial institution, you pay interest to the institution. The mathematical formula for calculating simple interest is:

$I = Prt$, where
I = interest
P = principal, which is the original amount invested or borrowed
r = annual (yearly) rate of interest expressed as a decimal
t = length of time in years

You should include this formula on your resource sheet.

Note that the time in the formula, $I = Prt$, must be in years.

- When the length of time is in months, divide by 12 to convert it into years.
- When the length of time is in days, divide by 365.

The mathematical formula $I = Prt$ is a calculation for simple interest. **Simple interest** is calculated from the *original amount* invested or borrowed, not on an amount that includes interest already charged.

Most financial institutions calculate interest based on amounts including previously charged interest (known as compound interest). This may sound complicated, but it will be explained in the next lesson.

In this course, when you are asked to calculate interest, you must calculate **simple interest** unless you are asked to calculate **compound interest**.

Example 1

Olive Branch invests \$1500 in a financial institution that offers her an interest rate of 4% per annum (per year). Calculate the interest Olive will earn at the end of

a) three years

b) seven months

c) 100 days

Solution

a) In the formula $I = Prt$, $P = \$1500$, $r = 4\%$ or 0.04, $t = 3$.

$I = Prt$
$= 1500 \times 0.04 \times 3$
$= 180$

At the end of three years, Olive will earn \$180.

b) In the formula $I = Prt$, $P = \$1500$; $r = 4\%$ or 0.04; $t = \frac{7}{12}$.

$$I = Prt$$

$$= 1500 \times 0.04 \times \frac{7}{12}$$

$$= 35$$

At the end of seven months, Olive will earn \$35.

c) In the formula $I = Prt$, $P = \$1500$, $r = 4\%$ or 0.04, and $t = \frac{100}{365}$.

$$I = Prt$$

$$= 1500 \times 0.04 \times \frac{100}{365}$$

$$= 16.44$$

At the end of 100 days, Olive will earn \$16.44.

Using the Simple Interest Formula

Not only can we use the formula $I = Prt$ to calculate interest, but we can also use it to calculate the other variables in the formula (principal, rate, and time).

To calculate the interest, use:

$I = Prt$

To calculate the rate, you must isolate the variable r.

$I = Prt$ Divide both sides by variables you already know, Pt.

$\frac{I}{Pt} = \frac{Prt}{Pt}$ Simplify the right-hand side by cancelling Pt.

$$\frac{I}{Pt} = r$$

$$r = \frac{I}{Pt}$$

To find the principal, **divide** the interest by the product of the rate and the time.

$$P = \frac{I}{rt}$$

To find the time, **divide** the interest by the product of the principal and the time.

$$t = \frac{I}{Pr}$$

You may want to include these formulas on your resource sheet, but you don't need to if you understand how to calculate them from the simple interest formula.

Example 1

Treya Pine invested a certain sum of money in a financial institution and earned $200 interest after four years. If the annual interest rate was 5 percent, what amount did Treya invest?

Solution

In the formula $I = Prt$, $I = \$200$, $r = 5\%$ or 0.05, $t = 4$, $P = ?$

$$P = \frac{I}{rt}$$

$$= \frac{200}{(0.05 \times 4)}$$

$$= \$1000$$

Treya invested $1000.

You can calculate the above using the following keys on your calculator.

200	÷	(	0.05	×	4	)	=

You can also solve this problem by substituting the values into the $I = Prt$ formula and *then* solving the equation.

$$I = Prt$$

$$200 = P(0.05)(4)$$

$$200 = P(0.2)$$

$$\frac{200}{0.2} = \frac{P(0.2)}{0.2}$$

$$\frac{200}{0.2} = P$$

$$1000 = P$$

Treya invested $1000.

Example 2

Brooke Poole has $2400 to invest in a financial institution. Calculate the annual rate of interest if she plans to earn $300 on her investment at the end of two years.

Solution

$I = \$300$, $P = \$2400$, $t = 2$, $r = ?$

$$r = \frac{I}{Pt}$$
$$= \frac{300}{(2400 \times 2)}$$
$$= 0.0625$$
$$= 6.25\%$$

Brooke requires an annual rate of 6.25% if she wants to earn $300 on her investment at the end of two years.

Example 3

Wade Lake borrows $5000 from the bank. He is charged interest at a rate of 4% per year. Calculate the number of days Wade kept the money if he owes $360 in interest.

Solution

$I = \$360$, $P = \$5000$, $r = 4\%$ or 0.04, $t = ?$

$$t = \frac{I}{Pr}$$
$$= \frac{360}{(5000 \times 0.04)}$$
$$= 1.8 \text{ year}$$
$$= 1.8 \times 365 = 657 \text{ days}$$

Wade kept the money for 657 days.

Note that the time is always in years when in the formula. To calculate the number of days, multiply the answer, 1.8 years, by 365.

The following learning activity will help you put into practice what you have just learned. When you have finished, check your answers in the Learning Activities Answer Key at the end of this module.

Learning Activity 1.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate for $y = 4$: $4y + (-6)$
2. A right triangle has sides with lengths of 12, 20, and 16. What is the length of the hypotenuse?
3. How many times do you have to reflect an image over a line so that you get the original image back?
4. There is a square kite in a picture, drawn to scale. The scale ratio is 1 mm : 20 cm. If the kite has a side length of 3 mm, how large is the life-size kite?.
5. Denise earns 15% of her sales at her family restaurant. If she sells $1400 in food, how much money will she earn?

Part B: Simple Interest Formula

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand. Don't forget that you can ask your learning partner or tutor/marker for help if you are having a hard time understanding.

1. Find the simple interest for each of the following. Round to the nearest cent.

Interest	Principal	Rate	Time
	$1000	5%	2 years
	$1000	5%	6 months
	$1000	5%	100 days
	$10,000	7¾%	1 year
	$10,000	7¾%	360 days
	$10,000	7¾%	18 months

continued

Learning Activity 1.2 (continued

2. Rayna Fontaine invests $20,000 in a financial institution at 10%. Calculate the number of days it will take her investment to earn $1200 in interest.
3. Douglas Fir borrows money from his financial institution at an interest rate of 6.25% per year. If he pays $397.50 in interest after four years, calculate the amount of his loan.
4. Luke Wharm has two years to save $2800 for a winter vacation. He has $10,000 to invest in a financial institution.
   a) Calculate the interest rate he requires to earn enough for his vacation.
   b) Is it likely that Luke will find a financial institution that will offer him this rate? Check with a financial institution or a newspaper.
5. Preston Fawcett has $1000 to invest in a financial institution. He decides to purchase a step bond that guarantees him $4\frac{3}{4}\%$ for the first year, $5\frac{1}{2}\%$ for the second year, and $6\frac{3}{4}\%$ for the third year.
   a) Calculate the total interest he will earn in three years.
   b) Calculate the average interest he will earn in three years

---

Lesson Summary

In this lesson, you learned about simple interest. The formula for simple interest is $I = Prt$. You used this formula to determine the amount of interest earned on an investment. You also changed the formula so that you could solve for the other variables. Now that you understand simple interest, we will move on to compound interest in the next lesson.

Before you move on, be sure you have the formulas and examples that you find helpful on your resource sheet.

Notes

Lesson 2: Compound Interest

Lesson Focus

In this lesson, you will

- ❑ solve problems using the compound interest formula
- ❑ solve problems using the rule of 72
- ❑ study the advantages and disadvantages of different types of investments

Lesson Introduction

Most interest calculations are for compound interest instead of simple interest. A quick way to estimate the rate of compound interest is to use the Rule of 72. In this lesson, you will learn how to calculate compound interest using the compound interest formula, which is somewhat more complex than the simple interest formula. You will also be encouraged to think about earning money by making investments.

Using Compound Interest

An investment earns compound interest when the interest from each time period is added to the principal, and then earns interest in the following time periods. As the principal grows, the rate at which you earn interest grows as well, because you are earning "interest on interest." Compounding makes a significant difference in the final value of an investment. Compounding increases the amount you earn when investing, but increases the costs when you borrow money.

Comparing Simple Interest and Compound Interest

The following examples illustrate the difference between simple interest and compound interest. The first example involves simple interest, whereas the second example involves compound interest.

Example 1

Miles West invests $5000 in a financial institution at 6% per annum (simple interest). Calculate his interest at the end of three years.

Solution

In the formula $I = Prt$, I = ?, P = $5000, r = 6% or 0.06, and t = 3.

$$I = Prt$$
$$= 5000 \times 0.06 \times 3$$
$$= 900$$

At the end of three years, Miles earns $900 in interest.

Example 2

Miles West invests $5000 in a financial institution at 6% per annum, compounded annually. Calculate his interest at the end of three years.

Solution

Interest Period	$I = Prt$	Amount
0		$5000.00
1	$I = 5000.00 \times .06 \times 1 = 300.00$	$5000.00 + $300.00 = $5300.00
2	$I = 5300.00 \times .06 \times 1 = 318.00$	$5300.00 + $318.00 = $5618.00
3	$I = 5618.00 \times .06 \times 1 = 337.08$	$5618.00 + 337.08 = $5955.08
	Total Interest 955.08	

To find the interest earned in the first year, use the formula:

$I = Prt$, where P = 5000, r = 6% = 0.06, and t = 1.

The interest is $300. We add the interest ($300) to the principal ($5000). The investment is then worth $5300 at the end of the first year. The amount $5300 becomes the "new" principal, and we use it to calculate the interest for the second year.

To find the interest earned in the second year, use the formula:

$I = Prt$, where P = 5300, r = 0.06, and t = 1.

The interest is $318. We add the interest ($318) to the last principal amount ($5300). The investment is then worth $5618 at the end of the second year. The amount of $5618 is the new principal, and we use it to calculate the interest for the third year.

To find the interest earned in the third year, use the formula:

$I = Prt$, where $P = 5618$, $r = 0.06$, and $t = 1$.

The interest is \$337.08. We add the interest (\$337.08) to the past principal amount (\$5618). The investment is then worth \$5955.08 at the end of the third year.

The total interest Miles earns in three years = \$5955.08 − \$5000 = \$955.08.

The difference in the interest Miles earns when the interest is compounded (Example 2) compared to when it is simple interest (Example 1) is equal to \$955.08 − \$900 = \$55.08.

Note that the amount of interest earned when we use compound interest is greater than when we use simple interest.

Compound Interest Formula

Computing compound interest as you did in Example 2 is a lengthy procedure, especially as the length of time is increased. You can set up a spreadsheet to do these calculations, but you may not always have access to a computer. Fortunately, there is a formula for calculating compound interest, making the calculations less time-consuming.

The formula for computing compound interest is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}, \text{ where}$$

A = final amount (principal + interest)

P = principal or the amount invested or borrowed

r = annual (yearly) rates of interest expressed as a decimal

n = number of interest periods a year

t = length of time in years

It is a good idea to include this formula on your resource sheet.

Note: There are some special words we use to describe the number of interest periods in a year. Here are some examples, along with an explanation and a value for n.

Number of Compounding Periods	Explanation	Value of *n*
annually	1 time a year	1
semi-annually	2 times a year (or at each half)	2
quarterly	4 times a year	4
bi-weekly	every 2 weeks	26
weekly	every week	52
daily	every day	365

Example 1

If Miles West invests $5000 in a financial institution at 6% per annum compounded annually, calculate his interest at the end of three years using the compound interest formula.

Solution

In the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, $P = 5000$, $r = 0.06$, $n = 1$, and $t = 3$.

Note that as the interest is compounded only one time a year, $n = 1$.

$$A = 5000\left(1 + \frac{0.06}{1}\right)^{1\times 3}$$

$$= 5000(1 + 0.06)^3$$

$$= 5000(1.06)^3$$

$$= 5000(1.191016)$$

$$= 5955.08$$

After three years, the investment is worth $5955.08.

After three years, the interest earned is equal to:

$\$5955.08 - \$5000.00 = \$955.08$.

To calculate $(1.06)^3$ on your calculator, press [1.06], then [y^x], then [3].

If your calculator does not have a [y^x] button, check the manual to see how to calculate exponents. You could also use the definition of exponents to calculate:

$$1.06^3 = 1.06 \times 1.06 \times 1.06$$
$$= 1.1191016$$

The value $5000\left(1+\frac{0.06}{1}\right)^{1\times 3}$ may be calculated directly on your scientific or graphing calculator as follows:

5000 [×] [(] 1 [+] 0.06 [÷] 1 [)] [y^x] [(] 1 [×] 3 [)] [=]

Check to see what keying sequence works with your calculator. You may want to write a note about using your calculator on your resource sheet.

Note that the compound interest formula calculates the final amount (A) an investment is worth, including Principal plus Interest. To calculate the amount of interest (I) earned, you must subtract the principal (P) from the calculated amount (A).

The Effect of Compounding Frequency

Example 1

If Miles West invests $5000 in a financial institution at 6% per annum compounded quarterly for a period of three years, calculate his interest at the end of three years using the compound interest formula.

Solution

Since the interest is compounded quarterly or four times a year, $n = 4$.

In the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$, $P = 5000$, $r = 0.06$, $n = 4$, and $t = 3$.

$$A = 5000\left(1+\frac{0.06}{4}\right)^{4\times 3}$$
$$= 5000(1+0.015)^{12}$$
$$= 5000(1.015)^{12}$$
$$= 5000(1.195618)$$

After three years, the investment is worth $5978.09.

After three years, the interest earned is equal to $5978.09 − $5000 = $978.09.

Note that the investment compounded quarterly (that is, four times a year) earns interest of $978.09, while the investment compounded annually earns interest of $955.08. When the interest is compounded quarterly, it is reinvested into the principal more often. Since the principal grows more frequently, the final investment is worth more. In general, the more often an investment is compounded, the greater the amount of interest it earns.

Many financial institutions now offer investments compounded daily. These investments grow more rapidly because they are compounded more often. When investing or borrowing money from a financial institution, it is important to consider not only the interest the institution offers, but how often the interest is compounded.

The Rule of 72

When making the decision to invest your money, it is important to consider everything that influences how much money you will earn. The rule of 72 is a method that uses the value of 72 to estimate the number of years and the annual interest rate it takes for an investment to double when interest is compounded annually.

The **rule of 72** states that an investment doubles in value when the interest rate multiplied by the number of years of the investment equals 72. For example, if you invest a sum of money at an annual interest rate of 3% for 24 years, it will approximately double in value ($3 \times 24 = 72$).

In this example, the product of the annual interest rate and the number of years of the investment is equal to 72. The formula for the rule of 72 is

$rt = 72$, where

r = percent rate compounded annually (ignore the percent, do not change into decimal form)

t = number of years

You should include this formula on your resource sheet.

You may also want to remind yourself on your resource sheet that you only use this formula when interest is compounded annually.

Example 1

Ernest Dollar invests $4000 at an interest rate of 6%, compounded annually. Using the rule of 72, estimate how many years it will take Ernest's investment to double.

Solution

To find the number of years it will take for the investment to double, we must change the formula so

$$rt = 72$$

$$\frac{rt}{r} = \frac{72}{r}$$

$$t = \frac{72}{r}$$

In the formula $t = \frac{72}{r}$, $r = 6$

$$t = \frac{72}{6}$$

$$t = 12$$

It will take approximately 12 years for the investment to double in value.

Again, you can solve this equation by substituting the values you already know into the equation, *then* isolating the variable t.

$$r = 6$$

$$(6)t = 72$$

$$\frac{(6)t}{6} = \frac{72}{6}$$

$$t = 12$$

It will take approximately 12 years for the value of an investment to double.

The rule of 72 does not give an exact value. It approximates the value of an investment after a given number of years. The rule is useful as it is a simple method of finding how much an investment is worth without having to use formulas or tables. You can determine how closely the rule of 72 approximates an actual value by using the compound interest formula. Consider the following example.

Example 2

Refer to Example 1. Using the rule of 72, the number of years it takes for an investment to approximately double in value at 6% is 12.

a) Use the compound interest formula to determine the actual value of a $4000 investment with an interest rate of 6 percent compounded annually for 12 years.

b) Find the difference between the actual value of the investment and the doubled value obtained from the rule of 72.

Solution

a) In the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, $P = 4000$, $r = 0.06$, $n = 1$, and $t = 12$.

Note that as the interest is compounded only one time a year, $n = 1$.

$$A = 4000\left(1 + \frac{0.06}{1}\right)^{1\times 12}$$

$$= 4000(1 + 0.06)^{12}$$

$$= 4000(1.06)^{12}$$

$$= 4000(2.0122)$$

$$= 8048.79 \text{ (rounded to the nearest cent)}$$

After four years, the investment is worth $8048.79.

b) The difference between the actual value of the investment and the doubled value obtained from the rule of 72 = $8048.79 − $8000 = $48.79.

The purpose of this learning activity is to give you some practice in using the compound interest formula. This is important because you will be doing this in your assignments and in your examination. When you have finished, check your answers in the Learning Activities Answer Key at the end of this module.

Learning Activity 1.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If $12 \times 15 = 180$, what is 13×15?
2. You are paid $9.50 per hour. If you work 20 hours this week, how much will you be paid?
3. The base of a triangle is 4 cm. The height of the same triangle is 5 cm. What is the area of the triangle? $\left(A = \frac{1}{2} \times bh\right)$
4. Convert the following decimal to a percent: 0.013
5. Jordan is twice as old as Carly. Carly is one-third as old as Marc. If Marc is 18, how old is Jordan?

Part B: Compound Interest

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. A principal of $10,000 is invested for four years at 8% compounded annually. Complete the following chart to calculate the interest earned.

Interest Period	*I = Prt*	Amount
0		
1		
2		
3		
4		

continued

Learning Activity 1.3 (continued)

2. a) Use the compound interest formula to solve question 1.
   b) Is your answer the same when you use the chart and the compound interest formula?

3. Copy and complete the following chart.

Principal (*P*)	**Compound Interest Rate (*r*)**	**Time in Years (*t*)**	**Number of Interest Periods (*n*)**	$\frac{r}{n}$	***nt***	$A = P\left(1 + \frac{r}{n}\right)^{nt}$
\$1000	7% compounded annually	7				
\$250	10% compounded semi-annually	5				
\$50,000	8% compounded daily	3				
\$4000	6% compounded monthly	6				
\$800	9% compounded quarterly	2				
\$25,000	$5\frac{1}{2}\%$ compounded semi-annually	4				

4. Chase South has \$10,000 to invest for a period of five years. Institution A offers him an interest rate of $7\frac{1}{2}\%$ compounded semi-annually.

   Institution B offers him an interest rate of $8\frac{1}{4}\%$ compounded semi-annually.

   a) Using the compound interest formula, calculate how much the final investment will be worth at Institution A.
   b) Using the compound interest formula, calculate how much the final investment will be worth at Institution B.
   c) Calculate how much more interest Chase would earn by investing at Institution B.

5. Wanda Waite invests \$25,000 for 18 years. At the end of this time, her investment will be worth \$50,000. Use the rule of 72 to estimate the interest rate.

Deciding How to Invest

As you may have guessed, investing your money is an easy way to increase the amount of money you have. Banks offer a number of different options for investing, so it is important to compare them and decide what is best for you based on your financial goals.

High-Risk Investments

In 2007, the world entered a recession (a period when unemployment rises and the value of goods produced by a country decreases). Governments and some people were in a financial crisis. This recession had a massive impact on high-risk investments, and many people lost a lot of money.

High-risk investments are investments that may increase or decrease in value very rapidly. These investments may produce a large return, but this is not guaranteed. There is the risk that you may lose some or most of the money you invested. Examples of high-risk investments are the stock market and real estate.

Low-Risk Investments

You may have a low-risk investment already! If you have a savings account or a chequing account that pays interest, then you have a low-risk investment.

A **low-risk investment** is one where you will not likely lose any of the money you invest. Your investment will grow reliably and slowly. For example, money deposited into a savings account will be safe, and will pay small amounts of interest into your account on a regular basis.

Comparing the two major categories of investments, low-risk investments are normally safer than high-risk ones.

Now work with your learning partner to explore decision making related to investments in Learning Activity 1.4.

Learning Activity 1.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate for $x = 5$: $3x - 8$
2. If 13×14 is 182, what is 13×15?
3. You are planting a garden in your backyard with the dimensions 1 m by 3 m. What is the area of your garden?
4. Ivana is in the food court at the mall. One store's special costs $8.50, and includes a sandwich, a drink, and a bag of chips. Another store does not have a special, so it costs $5 for the sandwich, $2.00 for the drink, and $2.00 for the chips. Which store should she go to?
5. Convert the following to a decimal: 38%

Part B: Exploring Investment Decisions

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

In this learning activity, you will investigate different types of investments.

1. Discussing finances is a personal matter for many people. Ask your learning partner, a parent, or a relative, if he or she would be comfortable discussing his or her experiences with investments. When you find someone who is comfortable, ask him or her to respond to the following questions:
   - Has this person invested money before?
   - Was it a high- or low-risk investment?
   - Why did they choose a high- or low-risk investment?
   - Did they find it beneficial to invest?

continued

Learning Activity 1.4 (continued)

2. What are some of your current financial goals, and what financial goals might you have in the future? If possible, discuss this with the person you worked with in question 1.
3. The investment world is changing constantly, so the options of how to invest are also changing. Visit your local bank and speak with a personal banker about what investment options they offer or research the answers on the Internet. Questions you may ask include the following:
   - What are some of the high-risk investments that you offer?
   - What are some of the reasons people choose high-risk investments?
   - What are some of the low-risk investments that you offer?
   - What are some of the reasons people choose low-risk investments?

   Choosing one of the financial goals discussed in question 2, you may ask the personal banker which investment option would be best.

Before you move on to the assignment at the end of this lesson, be sure to check your answers at the end of the module.

Lesson Summary

In this lesson, you explored the use of simple interest to calculate compound interest. You used the compound interest formula to calculate investment values. In addition, you used the rule of 72 to estimate time periods and rates required to double the value of an investment. Finally, you read about and discussed the pros and cons of different types of investments. In the next lesson, you will be learning about credit.

Now that you have learned about simple and compound interest and have demonstrated the relationship between the two, it is time to do Assignment 1.1. You will hand in this assignment along with the other assignments from this module after they have all been completed.

Notes

Assignment 1.1

Interest

Total: 25 marks

Note to Students: Have you made a resource sheet for this module? Do you have the definitions and formulas on your resource sheet? If so, you should use it now. If not, now would be a good time to make one.

1. Gale Storm invests $5000 in her financial institution. Calculate the amount of interest she will earn if she invests the money for $3\frac{1}{2}$ years at a simple interest rate of $5\frac{3}{4}\%$. *(1 mark)*

2. Fill in the blanks in the following chart. *(6 marks – one for each blank)*

Interest	Principal	Rate	Time
$160.00	$1000	8%	_____ years
$510.00		6.5%	7 years
$342.53	$5000		100 days
$43.50	$800	9%	_____ days
$14.00		7%	50 days
$150.00	$2000		4 months

continued

Assignment 1.1: Interest (continued)

3. A principal of $2000 is invested for two years at 10 percent compounded semi-annually. Complete the following chart to calculate the interest earned. *(5 marks)*

Interest Period	*I = Prt*	Amount
0		
1		
2		
3		
4		

4. a) Using the compound interest formula, solve question 3. *(1 mark)*

   b) Is your answer the same using the chart and the compound interest formula? *(1 mark)*

5. Using the compound interest formula, calculate both the value of the investment and the interest earned after the given time periods.
   a) $4000.00 for five years at 7% compounded semi-annually *(1 mark)*

   b) $600.00 for 10 years at 9% compounded monthly *(1 mark)*

   c) $2500.00 for two years at 8% compounded daily *(1 mark)*

continued

d) $100,000.00 for four years at 12% compounded quarterly *(1 mark)*

e) $900.00 for eight years at 10% compounded annually *(1 mark)*

6. Flo East has $25,000 to invest in a financial institution for a period of three years. Institution A offers her an interest rate of 6% compounded annually. Institution B offers her an interest rate of 6% compounded monthly. Using the compound interest formula, determine which financial institution would pay her more interest, and by how much. *(3 marks)*

7. Senta Cash invests $350 at 6% compounded annually.
   a) Use the rule of 72 to estimate how many years it will take her investment to double in value. *(1 mark)*

   b) Use the compound interest formula to determine the actual value of $350 at 6% compounded annually for the number of years indicated in part (a). *(1 mark)*

   c) Determine the difference between the actual value of the investment and the doubled value found with the rule of 72. *(1 mark)*

Notes

Lesson 3: Credit Options

Lesson Focus

In this lesson, you will

- ❑ learn the meaning of credit
- ❑ study the types of credit options that are available

Lesson Introduction

The next four lessons will focus on credit. In this lesson, you learn the meaning of credit and the options you have when you need to use credit. Unlike interest, you may not yet have firsthand experience with credit, but you most likely will in the future. Keep this in mind, and do not be afraid to ask your learning partner, tutor/marker, or parent if you have any questions about any of the topics mentioned in the next four lessons.

What is Credit?

Credit plays a large role in personal finance because almost everyone uses it. In order to buy a house, a car, furniture, or even to go on holidays, we may use credit.

Credit is borrowing money with the promise to pay it back. Credit is often used for the large purchases listed above, but even using a *credit* card is a form of credit (as you may have guessed, since it is called a credit card). In the case of credit cards, you may be under the impression that real money is not involved, but you must still pay that money back to the credit card company.

Lesson 2 of this module discussed the recession that began in late 2007. One of the contributing factors to this recession was that many people were not able to pay off their credit. This affected credit card companies and banks because they are the major sources of money lending, and they started running out of money to lend.

In the past, people used credit very little, but it has become a lot more common in recent years. This is because funds are more available and access to them is much easier. In addition, the average amount of money a person earns is higher now than it was before, and so people are better able to pay back the money they have borrowed.

Why We Use Credit

We borrow money for a variety of reasons. Each person is different and every purchase is different, but the following are common reasons for using credit.

It would be helpful to have a list on your resource sheet (in point form) of the reasons for using credit.

The most frequent reason people borrow money is for the "big ticket" items, such as houses and cars. Many people cannot afford these items without using credit because they are very expensive (tens of thousands of dollars). You may have heard your parents talk about mortgage payments—a mortgage is credit used to purchase a house.

School and training are other common reasons for people to borrow money. This is seen as "investing in the future," since one result of education is usually an increased income. People might use credit to pay for university and trade certification.

Sometimes people need to use credit because they encounter unexpected costs that do not fit in their budget (we will talk about budgets in Module 4). An unexpected cost may be that you would like to buy a car in the future but you find the car you want at an amazing price today. If you do not think that you will see a deal this good again, you may choose to use credit to buy the car now. On the other hand, you may also want to consider that the low price could be because it is defective or damaged.

Similar to unexpected costs, people may use credit if they suddenly become unemployed. The use of credit in this situation is beneficial only if they do not expect to use it for a long period of time. Hopefully, they will be able to recover quickly from their financial difficulties.

The last reason that people borrow money may surprise you. People sometimes use credit so that they can increase the principal value they invest. At this point, you may be thinking, "Why would you borrow money to invest money?" This is a valid question. The reason some people would do this could be that they expect to earn a large amount of money from the investment in a short amount of time, and then they could pay back the borrowed money and still have earned a lot of money. The calculations below demonstrate why someone would do this (using simple interest).

Your Investment	Your Investment + Borrowed Money
P = \$500, r = 6% or 0.06, t = 5 years $I = Prt$ $= 500 \times 0.06 \times 5$ = \$150	P = \$500 + \$1000, r = 0.06, t = 5 years $I = Prt$ $= 1500 \times 0.06 \times 5$ = \$450

As you can see, you would earn more money if you invested more money. You need to keep in mind that you would also be charged interest on the \$1000 borrowed, but we will explore this in Lesson 5. For now, complete the following learning activity and do not forget to check your answers! The answer key is at the back of the module.

Learning Activity 1.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate for $z = 3$: $6z - 9$
2. Heather is $\frac{4}{5}$th the height of Crystal. Ashley is 15 cm taller than Heather.

   If Crystal is 150 cm tall, how tall is Ashley?
3. You are having a day of shopping. At European Clothing, you spend \$38.00; at Discount you spend \$10.00; at Micki's you spend \$66.00; and at Alarms you spend \$23.00. How much money did you spend all together?
4. Solve for d: $2d - 7 = 29$
5. Evaluate: $\frac{2}{3} \times \frac{5}{7}$

continued

Part B: What Is Credit?

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Brainstorm the following question with your learning partner. There are no right or wrong answers, as your responses are based on

- the information that you just read
- your personal opinions
- your experiences

1. a) With your learning partner, brainstorm different expenses for which you would use or have used credit.

   b) Sort the expenses you came up with in part (a) and write them into a suitable category using a table similar to the one shown below.

Big Ticket Item	School and Training	Unexpected Costs	Costs While Unemployed	Investments

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Credit Approval

Now that you know why you would want to use credit, where do you get it?

Pre-approved credit cards are a common source of credit that is very accessible. This form of credit simply requires a signature.

Banks and other financial institutions approve credit based on a number of factors including your credit rating (a score based on how well you paid past debts, and your financial status), any current debt and credit you have, and your net worth. Net worth is a comparison of your assets (money and investments you have) and your liabilities (money you owe for other credit, such as a credit card).

You can also offer your assets as security, such as your house, car, or investments. By doing this, you are able to get more credit and maybe even a lower interest rate. The danger with this is that if you do not pay the credit and interest, you could lose whatever you offered as security. This is why many people in the US lost their homes in the 2007 recession.

Comparing Credit Options

So far, you have learned what credit is, why we use it, and how to get it. We will now look at the types of credit available.

Credit Cards

As mentioned before, credit cards are a frequently used source of credit. People feel comfortable using them because it does not feel as though they are using money. There are many companies involved in the credit card business, which means there are many credit cards from which to choose. As great as this all sounds, it is important to remember that using a credit card just means that you have to pay later. If you do not pay off the balance on your credit card by the due date (usually around the end of the month), the company will charge interest, and credit card interest rates are usually very high—most ranging from 15% to 20%.

Another way to use your credit card is to withdraw cash from a bank machine. Financial experts do not recommend this because, unlike the balance on your credit card, you will be required to pay interest from the time you withdraw the cash until the amount is repaid.

Personal Line of Credit

A personal line of credit is a credit account from which you can withdraw funds up to a predetermined limit. You pay interest on only the amount you withdraw, and you can repay the loan in part or completely at any time. As you repay the loan, you can again withdraw funds from the line of credit account. In this way, it resembles a credit card.

The borrower may or may not provide some assets as security in order to get the line of credit. When compared with credit cards, personal lines of credit have much lower interest rates, and so they are a better option if you are not able to pay off the debt immediately.

Personal and Consumer Loans

A consumer loan is an amount of money lent to an individual for personal, family, or household purposes. Usually the person borrowing the money does not offer any security for this loan. This type of credit is very flexible because the amount of money borrowed depends on the purpose of the loan. Depending on the institution granting the loan, the loans can vary in the amount borrowed, how long you can take to repay them, and the rate of interest charged. Factors that affect the interest rate will be discussed in greater detail in the next lesson. The typical length of time a person takes to repay a personal or consumer loan is between five and seven years. Again, these loans have a lower interest rate than credit cards.

Overdraft Protection

Overdraft protection is short-term credit that allows you to withdraw more money from your bank account than you have in the account. The amount that you can overdraw the account is usually quite small. This feature is for the client's convenience, and it helps the client maintain a good credit rating by avoiding bounced cheques (cheques "bounce" when there is not enough money in the account to pay for the cheque). Banks may charge an overdraft fee and/or charge interest on the money overdrawn. You will learn about avoiding this situation in Module 4.

It is important to understand how you get credit, and what credit options are available. The following learning activity will help you. In it, you will compare the different credit options. Ask your learning partner to help you if you get stuck, or refer to the answers at the back of the module for help—but be sure you understand them and don't just copy the answers.

Learning Activity 1.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate: $\frac{5}{6} + \frac{2}{3}$
2. Sarah walked to school this morning, which is 2 km away from her house. She then walked 500 m to the convenience store after school before going home (1.5 km). How far did Sarah walk in total?
3. What is 10% of 1400?
4. You have to buy a gift for your sister. You get her some makeup, which costs $25.75 including tax. If you give the cashier $40, how much change should you get back?
5. Your best friend wants to hang out this week. You work Monday, Tuesday, Thursday, and Saturday. You have soccer on Wednesday and Sunday. Is there any day that you are free?

Part B: Credit Options

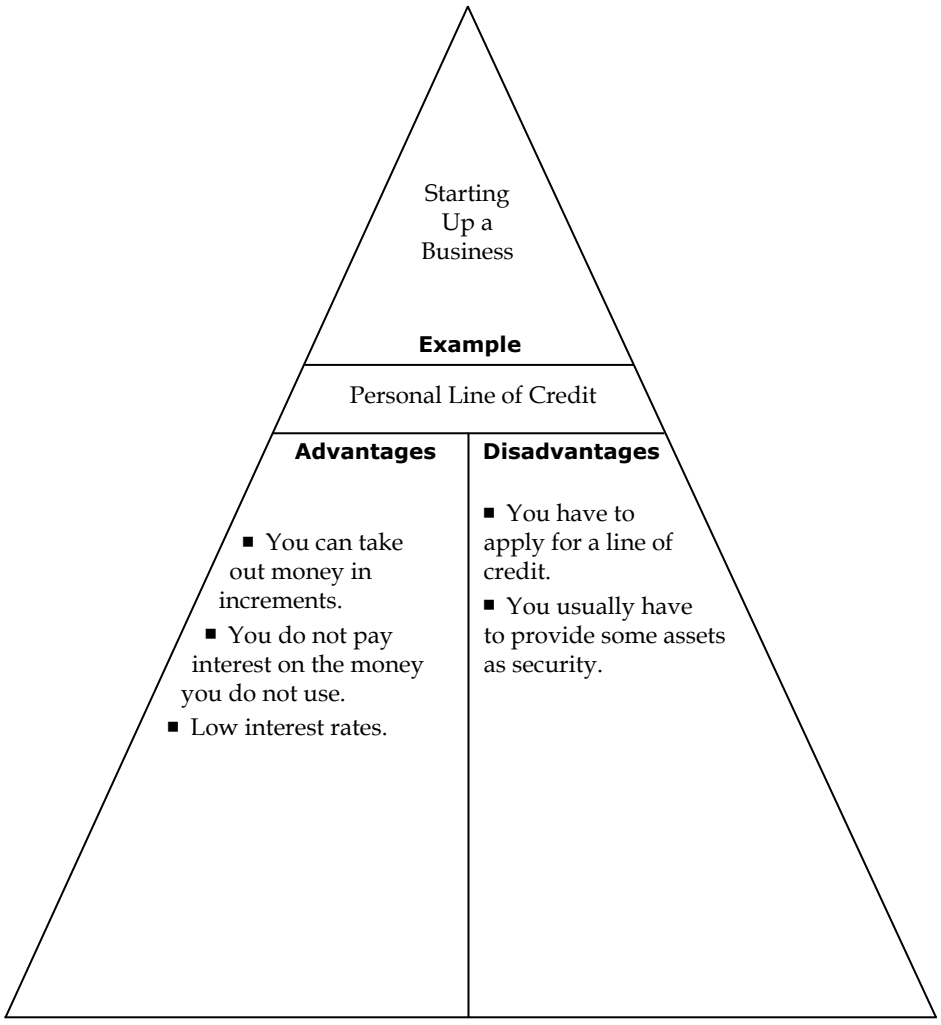
Fill in the "TRI to Remember" triangles on the next page for the three credit options you read about in the previous section to compare the advantages and disadvantages of each. (Do not use "personal line of credit," as it is in the example.)

It would be a good idea to include the advantages and disadvantages of different credit options on your resource sheet.

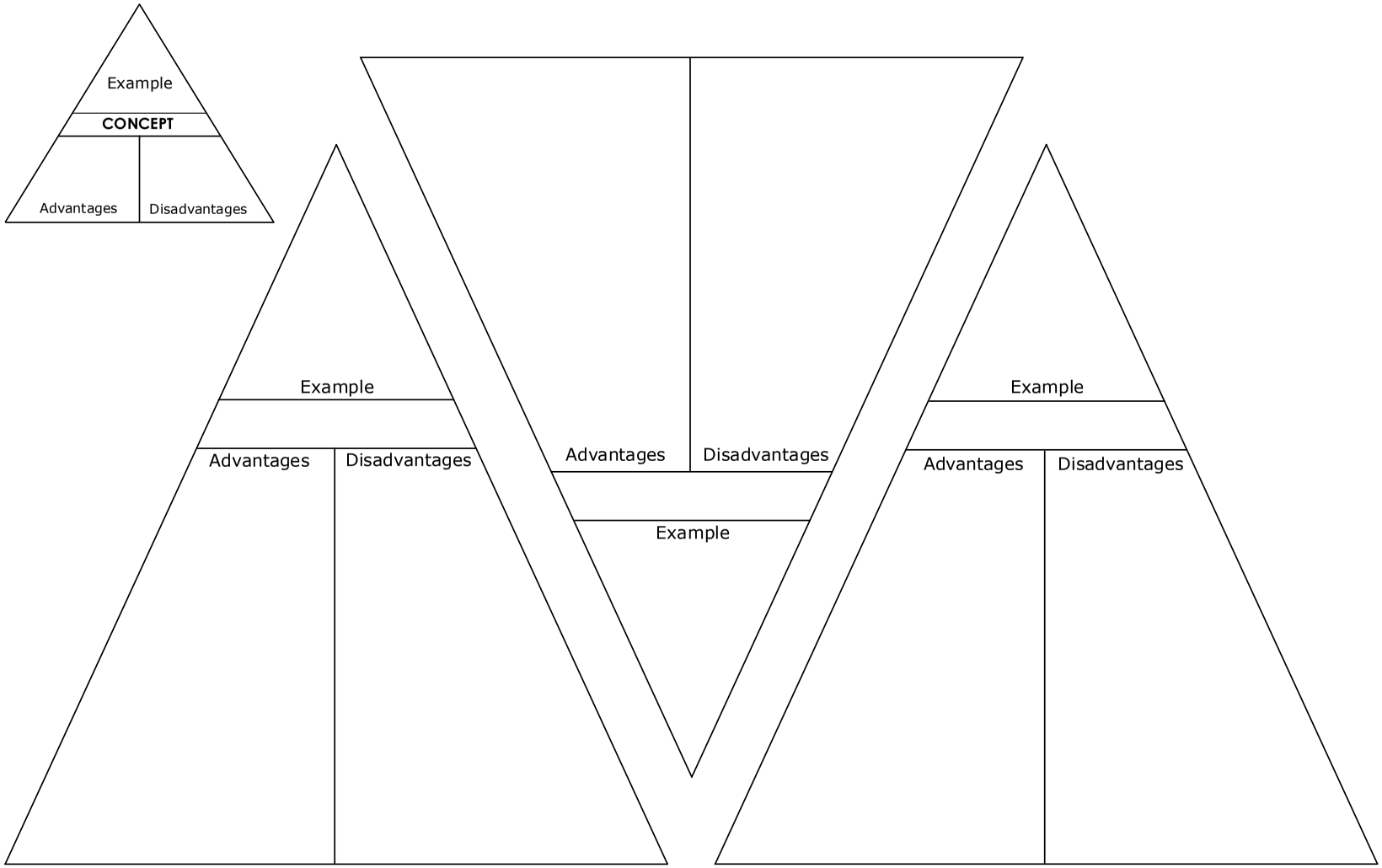
Lesson Summary

In this lesson, you learned about credit. You learned why we use credit, and explored what types of credit are available. Complete Assignment 1.2 before you move on to the next lesson. In the next lesson, you will be learning about personal loans.

TRI to Remember Example



TRI to remember



Notes

Assignment 1.2

What Is Credit?

Total: 8 marks

Note to Students: Do you have the definitions on your resource sheet? If so, you would be able to use it now. If not, now would be a good time to add the definitions to your resource sheet.

1. Write a definition for credit in your own words. *(1 mark)*

2. In two to five sentences, write a story about someone who needs to use credit. Include both why this person is using credit (e.g., to renovate the house), and the type of credit he or she should use (e.g., a personal line of credit). Explain why this type of credit is the best option. *(3 marks)*

(continued)

Assignment 1.2: What Is Credit? (continued)

3. Draw a line to connect the example where credit may be used to the type of credit that would probably be used. *(4 marks)*

Buying groceries	Personal loan
Family vacation	Personal line of credit
Using more money than is in your account	Credit card
Buying a boat	Overdraft protection

Lesson 4: Personal Loans

Lesson Focus

In this lesson, you will

- ❑ calculate a monthly payment based on the amount of the loan, as well as the term and interest rate
- ❑ determine the term and interest rate, given the monthly payment and amount of the loan

Lesson Introduction

In the previous lesson, you studied different types of credit, one of which was the personal loan. In this lesson, you will focus on calculations related to personal loans. The reason that you may not have any experience with personal loans at this time is that you must be 18 or older to get a loan. When learning about personal loans, it is important to remember that although you may not have a personal loan at this time, you probably will in the future when you are buying a car, going to post-secondary school, or renovating a house. Your parents have probably used personal loans, so if you have any questions, they may be able to answer them. Also, your learning partner and/or tutor marker have probably had personal loans and might be able to answer any questions you might have.

Personal Loans

When discussing loans, you must consider four components:

- the term
- the interest rate
- the amount of the loan
- the monthly payment

The **term** of the loan is the amount of time during which the conditions of the loan are in effect. Although you may choose to take a longer time to repay or **amortize** (amortize means to pay back a loan) a loan in full, the interest rate you negotiate with the lending institution is fixed for a maximum of five years. On the last day of a term, you must pay off the loan or renew it. Most people repay their personal loans within five years.

The **interest rate** you are charged depends on many factors. One factor is the amount of money you borrow. Generally, the greater the amount of money you borrow, the lower the interest rate. Other factors that determine interest rate are your past financial record, your present financial situation, and the amount of security you can offer. Financial institutions sometimes offer "variable interest rates," which means the interest rate fluctuates. We will be focusing on "fixed interest rates" with monthly payments.

Using an Amortization Table

Refer to the Amortization Table found at the end of this lesson. This table indicates the monthly payment required to pay a $1000 loan for a given time period and at a given interest rate. Amortization tables are available for other terms, and function in a similar manner.

Example 1

Bea Noble requires a personal loan of $10,000 for home renovations. Her financial institution offers her a three-year loan at a fixed interest rate of $10\frac{1}{4}\%$.

a) How much must Bea pay her financial institution each month?

b) How much does Bea pay in interest over the three years?

Solution

a) Refer to the Amortization Table found at the end of this lesson. This table gives the monthly payments required to repay a loan of $1000 at interest rates from 6% to 14% from one to five years. Note that $10\frac{1}{4}\% = 10.25\%$.

Move down the first column at the left, titled Annual Rate, until you find the entry 10.25%. Then move across that row until you are in the column titled 3 Years. The monthly payment required for a $1000 loan is $32.38.

Since the loan Bea Noble requires is for $10,000, the value of $32.38 must be multiplied by 10 ($10 000 ÷ $1000 = 10).

Bea's monthly payment: 32.38 × 10 = $323.80.

b) Since Bea is repaying the loan in three years, and there are 12 months in a year, she makes a total of (12 × 3) 36 payments.

The total amount Bea pays over three years: $323.80 × 36 = $11,656.80.

The interest Bea pays over three years: $11,656.80 − $10,000 = $1656.80.

The amount of interest an individual pays over the term of the loan is the cost of borrowing. In the above example, Bea pays an additional $1656.80 in interest for the loan over a three-year period.

Example 2

Bea Noble chooses to amortize her $10,000 loan in five years rather than three years. Her financial institution offers her a fixed interest rate of 9.5%.

a) How much must Bea now pay her financial institution each month?

b) How much does Bea pay in interest over the five years?

c) Does Bea pay more interest if she chooses to amortize her loan over five years rather than three years (in example 1)?

Solution

a) Refer to the Amortization Table found at the end of this lesson. Move down the first column at the left, titled Annual Rate, until you find the entry 9.5%. Then move across that row until you are in the column titled 5 Years. The monthly payment required for a $1000 loan is $21.00.

Since the loan Bea Noble requires is for $10,000, the value of $21.00 must be multiplied by 10 ($10 000 ÷ $1000 = 10).

Bea's monthly payment: $21.00 × 10 = $210.00.

b) Since Bea is repaying the loan in five years, and there are 12 months in a year, she makes (12 × 5) 60 payments.

The total amount Bea pays over five years: $210.00 × 60 = $12600.00.

The interest Bea pays over five years: $12600.00 − $10 000 = $2600.00.

c) Bea pays more interest when she chooses to amortize her loan over five years than when she amortizes over three years. Note that the interest rate changed from $10\frac{1}{4}\%$ to 9.5%. Even with a lower interest rate, Bea will still pay more interest.

In general, the longer it takes you to repay a loan, the greater the amount of interest you pay (even though the interest rate is usually lower). With your learning partner, discuss some reasons for choosing five years instead of three years to repay a loan (even though you eventually pay more interest to the lending institution).

Comparing Options

Being able to find the interest rate and term is just as important as finding the monthly payment. If you are able to do this, you can compare your options before deciding where to get your loan. You may want to do this when buying a car. Many car advertisements include a possible monthly payment. You can use the monthly payment as well as the price of the car to determine the interest rate and term the car dealership is offering, and you can compare it with the interest rate and term the bank may offer you. It is always a good idea to compare and consider your financial options so that you can save as much money as possible.

Example 1

Bea Noble is making payments of $462.60 per month to pay her $10,000 loan. What is the interest rate and term of her loan?

Solution

Paying $462.60 per month for a $10,000 loan is the same as paying $46.26 for a $1000 loan.

$$\left(1000 \div 10000 = \frac{1}{10} \text{ so } \$462.60 \times \frac{1}{10} = \$46.26\right)$$

On your amortization table, find this monthly payment. The monthly payment is $46.26 if the interest is 10.25% and the term is two years.

At the end of this lesson, there is an amortization table for you to use while completing the following learning activity. Once you have finished the activity, check your answers at the back of the module.

Learning Activity 1.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Isabella does not want to walk around a parking lot with dimensions 30 m by 40 m. If she walks from corner to corner (diagonally), how far will she walk?
2. Fill in the blanks in the pattern: 1, 2, 4, 8, _____ , _____
3. Write two equivalent fractions for $\frac{16}{28}$.
4. Solve for c: $c - 4 = 15$
5. Jack and Jill climbed up a hill. They covered a distance of 400 m in eight minutes. How fast did they climb?

Part B: Personal Loans

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Refer to the Amortization Table found at the end of this lesson, as required.

continued

Learning Activity 1.7 (continued)

1. Complete the following chart:

Amount of Loan	Annual Interest Rate	Amortization Period (years)	Monthly Payment
$5000	$10\frac{1}{2}\%$	4	
$13,500	$9\frac{3}{4}\%$	1	
$7800	11%	5	
$10,750	$10\frac{1}{4}\%$	3	
$18,250	$8\frac{3}{4}\%$	2	

2. Bea Wright requires a personal loan of $15 000 for home renovations. Her financial institution offers her a five-year loan at a fixed interest rate of $10\frac{1}{4}\%$.

   a) How much must Bea pay her financial institution each month?

   b) How much does Bea pay in interest over the five years?

3. If Bea Wright decides to amortize her loan of $15,000 over one year rather than five years, her financial institution offers her a fixed interest rate of $9\frac{3}{4}\%$.

   a) How much must Bea now pay her financial institution each month?

   b) How much must Bea pay in interest over the year?

   c) What is the difference in the interest she pays when the loan has an amortization period of one year instead of five years?

4. Bea Reddy has a personal loan of $5000 to purchase appliances for her new house. Her monthly payment is $125. What is the interest rate of her loan, and how long is the term?

Lesson Summary

In this lesson, you calculated the monthly payment of a personal loan, and used the monthly payment to find the interest rate and term of the loan. In the next lesson, you will study in-store credit.

The next assignment is at the end of the next lesson, and it includes personal loan calculations. Before you move on, be sure that you have any helpful hints or examples from this assignment written on your resource sheet.

Notes

Amortization Table					
Amortization Period Monthly Payment Per $1000 Loan Proceeds					
Annual Rate	1 Year Monthly	2 Years Monthly	3 Years Monthly	4 Years Monthly	5 Years Monthly
6.00%	$86.07	$44.33	$30.43	$23.49	$19.34
6.25%	$86.18	$44.44	$30.54	$23.61	$19.46
6.50%	$86.30	$44.56	$30.66	$23.72	$19.57
6.75%	$86.41	$44.67	$30.77	$23.84	$19.69
7.00%	$86.53	$44.78	$30.88	$23.95	$19.81
7.25%	$86.64	$44.89	$31.00	$24.07	$19.93
7.50%	$86.76	$45.01	$31.11	$24.19	$20.05
7.75%	$86.87	$45.12	$31.23	$24.30	$20.16
8.00%	$86.99	$45.24	$31.34	$24.42	$20.28
8.25%	$87.10	$45.34	$31.45	$24.53	$20.40
8.50%	$87.22	$45.46	$31.57	$24.65	$20.52
8.75%	$87.34	$45.57	$31.68	$24.71	$20.64
9.00%	$87.45	$45.68	$31.80	$24.89	$20.76
9.25%	$87.57	$45.80	$31.92	$25.00	$20.88
9.50%	$87.68	$45.91	$32.03	$25.12	$21.00
9.75%	$87.80	$46.03	$32.15	$25.24	$21.12
10.00%	$87.92	$46.14	$32.27	$25.36	$21.25
10.25%	$88.03	$46.26	$32.38	$25.48	$21.37
10.50%	$88.15	$46.38	$32.50	$25.60	$21.49
10.75%	$88.27	$46.49	$32.62	$25.72	$21.62
11.00%	$88.38	$46.61	$32.74	$25.85	$21.74
11.25%	$88.50	$46.72	$32.86	$25.97	$21.87
11.50%	$88.62	$46.84	$32.98	$26.09	$21.99
11.75%	$88.73	$46.96	$33.10	$26.21	$22.12
12.00%	$88.85	$47.07	$33.21	$26.33	$22.24
12.25%	$88.97	$47.19	$33.33	$26.46	$22.37
12.50%	$89.08	$47.31	$33.45	$26.58	$22.50
12.75%	$89.20	$47.42	$33.57	$26.70	$22.63
13.00%	$89.32	$47.54	$33.69	$26.83	$22.75
13.25%	$89.43	$47.66	$33.81	$26.95	$22.88
13.50%	$89.55	$47.78	$33.94	$27.08	$23.01
13.75%	$89.67	$47.89	$34.06	$27.20	$23.14
14.00%	$89.79	$48.01	$34.18	$27.33	$23.27

Lesson 5: Sales Promotions

Lesson Focus

In this lesson, you will

- ❑ calculate the cost, including interest, of items purchased using a sales promotion
- ❑ determine when it is cost effective to pay for an item right away, or why you may take advantage of a sales promotion

Lesson Introduction

In the previous lesson, you did calculations related to personal loans. This lesson will focus on calculations related to in-store credit. You are probably familiar with some of the in-store credit options, although you may not realize it. Have you seen commercials for furniture saying, "Buy now and don't pay for two years!"? This is an example of an in-store credit, which we will be looking at in this lesson.

In-Store Credit

Companies provide credit in a variety of ways to encourage you to buy their products. You may be more familiar with some forms of credit than others. The most commonly advertized form is the "buy-now, pay-later" promotion.

Buy-Now, Pay-Later Promotions

Furniture and appliance stores often use this promotional option, and the commercials are everywhere. But what does it mean to buy now and pay later? Customers get to postpone payment for a specified amount of time. On one hand, this option is great because the buyer does not have to pay interest during this specified time. On the other hand, this option can be risky. If you do not pay by the end of the specified time, then you will need to pay interest.

Do not be misled. Although it sounds like you do not have to pay anything when you get the furniture or appliance, most companies require the customer to pay certain costs at the time of purchase. These costs include taxes on the purchase, any delivery charges, and an administration fee. The administration fee is the amount a company charges its customers for the work involved in administering the buy-now, pay-later plan.

As a consumer, you have to consider all promotions carefully. The buy-now, pay-later promotion is no exception. With a buy-now, pay-later plan, you have to pay an extra administration fee. As well, some companies offer an alternate price if you pay at the time of purchase. This pay-now price is often significantly lower than the pay-later price. Again, it is up to you as a consumer to be sure you are getting the best possible price.

For all the examples, use the GST rate as 5% and PST rate as 7%. These are the values of these taxes in Manitoba in July 2012.

Example 1

Amelia Battison Cameron purchases a sofa from Company A. She can either pay for the sofa at the time of purchase or purchase it on the company's buy-now, pay-later plan.

If she chooses to pay for the sofa at the time of purchase, the price is $924.95 (plus taxes). The delivery charge for the sofa is $25.

If she chooses to purchase the sofa using the company's buy-now, pay-later option, the price of the sofa is $995.95 (plus taxes). At the time of purchase, she must pay the taxes, delivery charge, and a $49.99 administration fee. She has a one-year period to pay for her purchase. If she pays within the year, she is not charged interest.

a) If Amelia pays for the sofa at the time of purchase, calculate her total pay-now price.

b) If Amelia chooses to purchase the sofa using the buy-now, pay-later option, how much will she pay at the time of purchase?

c) If Amelia chooses to purchase the sofa using the buy-now, pay-later option and pays for it within one year, calculate her total pay-later price.

d) Calculate the difference between the total pay-now price and the total pay-later price.

e) Express as a rate of percent the difference in the two prices compared to the total pay-now price.

Solution

a) The total pay-now price of the sofa

= pay-now price + PST + GST + delivery charge

PST is 7%, so $924. 95 × 0.07 = $64.75

GST is 5%, so $924. 95 × 0.05 = $46.25

= $924. 95 + $64.75 + $46.25 + $25 = $1060.95

b) At the time of purchase, Amelia will pay the taxes, the delivery charge, and the administration fee.

Stores rarely indicate the total selling price of an item (which includes both GST and PST). However, as a consumer, you should consider the total price before making a purchase. To quickly determine the cash selling price of a product including taxes (PST, GST), you can multiply the selling price by 1.12. Occasionally, this method results in a difference of a cent or two due to rounding. Or, to get an estimate of the taxes, find 10% of the selling price. This will be lower than the actual taxes, which are 12%, but will give you a good estimate. For example, if the selling price is $995.95, then the total taxes is a little more than $99.59, since 10% of $995.95 is the same as 0.10×995.95. We do not demonstrate this method in this course, but you can use it if you prefer.

The amount Amelia will pay at the time of purchase

= PST + GST + delivery charge + administration fee

PST: $\$995.95 \times 0.07 = \$69.72$

GST: $\$995.95 \times 0.05 = \$49.80$

= $\$69.72 + \$49.80 + \$25 + \$49.99 = \$194.51$

c) The total pay-later price of the sofa = $\$995.95 + \$194.51 = \$1190.46$.

d) The difference between the pay-now and pay-later prices

= $\$1190.46 - \$1060.95 = \$129.51$.

e) Below are two methods for calculating the rate of percent. Use whichever method you prefer. In both methods, we must compare the difference between the pay-later and pay-now prices (the answer from part (d) to the cost of the pay-now price (the answer from part (a).

Method 1

Let x represent the unknown rate in the following proportion:

$$\frac{x}{100} = \frac{129.51}{1060.95} \quad \begin{matrix}\text{(difference in cost)} \\ \text{(cost if you pay now)}\end{matrix}$$

Multiply both sides by 100

$$(100)\frac{x}{100} = \frac{129.51}{1060.95}(100)$$

Round to the nearest tenth:

$x = 12.2$

The rate of percent = 12.2%

Method 2

Rate of percent

$$= \frac{\text{difference}}{\text{pay-now cost}} \times 100\%$$

$$= \frac{129.51}{1060.95} \times 100\% = 12.2\%$$

Example 2

Breslin has to buy a computer for school. The store that has the computer she wants has two purchasing options:

Option 1: To pay for the computer upfront, for $999.89 plus taxes and $40 for installation.

Option 2: To pay the taxes and installation fee now, but take 18 months to pay for the computer.

a) How much would Breslin pay for the computer if she pays cash?

b) Breslin cannot pay for the computer after 18 months—she still owes $400. If she pays off the remaining balance three months late, how much will she pay in interest if the store charges:

   i) 5.1% simple annual interest, charged monthly, starting after the original 18 months?

   ii) 5.1% compound annual interest, compounded monthly, starting from the original date of purchase?

Solution

a) PST: $0.07 \times 999.89 = \$69.99$

GST: $0.05 \times 999.89 = \$49.99$

Total cost: $999.89 + 69.99 + 49.99 + 40 = \$1159.87$

b) i) number of months = 3

$$I = (999.89)(0.051)\left(\frac{3}{12}\right)$$

$$= (999.89)(0.051)(0.25)$$

$$= \$12.75$$

ii) Number of months: $18 + 3 = 21$

$$A = (999.89)\left(1 + \frac{0.051}{12}\right)^{21}$$

$$= (999.89)(1 + 0.00425)^{21}$$

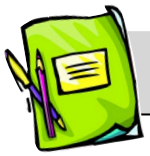
$$= (999.89)(1.00425)^{21}$$

$$= (999.89)(1.093147)$$

$$= \$1093.03 \text{ including the principal}$$

$1093.03 - 999.89 = \$93.14$ interest

Did you notice how different the two interest amounts are? If Breslin is only charged interest for three months and if it is not compounded, she would only have to pay $12.75, but if she is charged interest from the time of purchase and it is compounded monthly, she would have to pay $93.14! That is a difference of over $80! For this reason, it is important to read the fine print in credit agreements or any other contract that you sign, because small details such as when you start to pay interest, or if the interest is compounded, can greatly alter how much you pay if you miss a deadline. No matter what the contract is, the best practice is to pay within the deadline so that you do not have to pay any interest.



Learning Activity 1.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If 25% of 2720 is 680, what is 50% of 2720?
2. The sides of a right-angle triangle are 28, 45, 53. What are the two possible ratios for tangent?
3. List four types of credit.
4. It is your friend's birthday. You collect $5 from each of your other friends to buy him a hoodie that costs $40.00 including tax. If you and nine other people pay $5 for the hoodie, will you be able to afford it?
5. Evaluate: $\frac{3 \times 4}{6} + 8$

continued

Learning Activity 1.8 (continued)

Part B: Buy Now, Pay Later

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

For questions that include taxes, PST = 7% and GST = 5%.

1. Complete the following chart. The rate of percent expresses the ratio of the difference between the two prices and the pay-now price. Do not include taxes. Where necessary, round the rate of percent to the nearest tenth.

Pay-Later Price	Pay-Now Price	Difference between the Two Prices	Rate of Percent
$12,000	$10,000		
	$780	$160	
$580	$500		
$1400		$75	
	$1250		10%
	$840		7.5%
$3250	$2900		
$1200		$200	

2. Bartholomew Upton York purchases a washing machine and dryer from Company Y. He can either pay for the washing machine and dryer at the time of purchase, or purchase them on the company's buy-now, pay-later plan. He chooses to purchase them on the company's buy-now, pay-later plan.

   On the buy-now, pay-later plan, the price is $698.98 (plus taxes) for the washing machine and $509.98 (plus taxes) for the dryer. At the time of purchase, Bartholomew must pay the taxes, a delivery charge, and a $35 administration fee per item. The delivery charge for both the washing machine and dryer is $35. He has six months to pay for his purchase. If he pays within this period of time, he will not be charged interest.

   a) How much will Bartholomew pay at the time of purchase (upfront)?

   b) If Bartholomew pays for the appliances within six months, calculate his total pay-later price of the appliances.

continued

Learning Activity 1.8 (continued)

3. Terrence Andrews Xavier purchases a television set from Company X on October 1. He can either pay for the television set at the time of purchase, or purchase it on the company's buy-now, pay-later plan. He chooses to purchase it on the company's buy-now, pay-later plan.

   On the buy-now, pay-later plan, the price is $698.98 (plus taxes). At the time of purchase, Terrence must pay the taxes and a $45 administration fee. There is no delivery fee. He has a six-month period to pay for his purchase. If he pays during this time period, he will not be charged interest. After six months, Company X charges customers 4% interest per annum, compounded monthly, on any outstanding amount.

   a) Calculate Terrence's total cost if he pays for the purchase on March 1 of the following year.

   b) Calculate Terrence's total cost if he pays for the purchase on May 31 of the following year. Assume that the company charges interest only after the "pay-later" period.

Installment Buying

An alternative promotion to buy-now, pay-later is installment buying. When you pay for a product in installments, you pay a down payment at the time of purchase. You pay the rest of the purchase price in equal amounts, called installment payments, over a given number of equal time periods. Therefore, the **installment price** is the sum of the down payment plus all the installment payments. The installment price is usually higher than the cash selling price. The difference between the installment price and the cash selling price is **the finance or carrying charge**.

Companies offer installment buying because it benefits business. A company is more likely to sell a product if a consumer does not have to pay its full price at the time of purchase. Similar to buy-now, pay-later, this is particularly the case with larger purchases, such as furniture and appliances.

Installment buying is a form of credit buying. Companies often charge a substantial amount for the privilege of purchasing on credit. As a wise consumer, you have to know how much the cost of installment buying will be before deciding to pay with installments.

Example 1

At Palindrome Warehouse Company, all prices read the same forwards and backwards. The company offers customers the option of purchasing its products in installments. Mark Kahn decides to purchase a PVR from the Palindrome Warehouse Company. The PVR has a cash price of $389.83 plus taxes. The installment terms are $50 down and $75 a month for six months.

a) Calculate the cash selling price of the PVR.

b) Calculate the installment price of the PVR.

c) Calculate the finance charge.

d) In your opinion, should Mark purchase the PVR in installments?

Solution

a) The PST = 7% × $389.83 = 0.07 × $389.83 = $27.29

The GST = 5% × $389.83 = 0.05 × $389.83 = $19.49

The cash selling price of the PVR = $389.83 + $27.29 + $19.49 = $436.61

b) The total monthly installment payments = $75 × 6 = $450

The installment price = $50 + $450 = $500

c) The carrying charge = $500 − $436.61 = $63.39

d) Because there is such a big price difference, he would be best to either buy the PVR in cash right now or save $75 per month and then buy the PVR when he has enough money to pay in cash. In reality, it may not be possible to buy the PVR in cash right now. If that is the case and he needs it right now, he would have to buy it in installments.

In general, when making a decision whether to purchase in installments, there are factors to consider in addition to the carrying charge. When you are trying to decide whether you should pay cash now or buy in installments, in addition to considering the carrying charge, ask yourself the following questions:

- Do you have enough money to pay the cash selling price right now?
- Do you need the item immediately?
- If you do not have enough money to pay now and you do not need it immediately, can you save enough money to pay in cash in six months?

You may want to include these questions on your resource sheet.

Another way to determine whether you should purchase a product in installments is to compare the finance charge to the cash selling price. You can then determine what percent the finance charge is of the cash selling price before you decide to purchase in installments.

Example 2

Express the carrying charge from Example 1 as a percentage of the cash selling price of the PVR.

Solution

Method 1

Let x represent the unknown rate in the following proportion:

$$\frac{x}{100} = \frac{63.39}{436.61}$$

Multiply both sides by 100

$$(100)\frac{x}{100} = \frac{63.39}{436.61}(100)$$

$x = 14.5187\ldots$

Round to the nearest tenth:

The rate of percent = 14.5%

Method 2

$$\text{Rate of percent} = \frac{\text{carrying charge}}{\text{cash selling price}} \times 100\%$$

$$= \frac{63.39}{436.61} \times 100\% = 14.5\%$$

In the following learning activity, you will practise calculating both the cash selling price and installment price of a purchase. Don't forget to check your answers.

Learning Activity 1.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the pattern: 100 000, 10 000, 1 000, _____ , _____
2. Estimate the taxes on a shirt that costs $15.00.
3. Germany, England, and Greece are in a group for a soccer tournament. Germany wins against Greece. Greece loses to England. Germany beats England. In what order do they finish? (First, Second, Third)
4. A watch chimes every hour, on the hour. Between 4:23 am and 7:54 pm, how many times does it chime?
5. Write two equivalent fractions for $\frac{45}{81}$.

Part B: Installment Buying

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

To calculate taxes, use 7% for PST and 5% for GST.

1. Copy and complete the following chart.

Cash-Selling Price (without tax)	GST (5%)	PST (7%)	Cash-Selling Price (with tax)	Down Payment	Monthly Payment	Months	Installment Price	Carrying Charge
a) 249.98				$35	$50	6		
b) 998.98				$75	$100	12		
c) 1879.99				$100	$300	8		
d) 5998.95				$500	$400	18		

continued

Learning Activity 1.9 (continued)

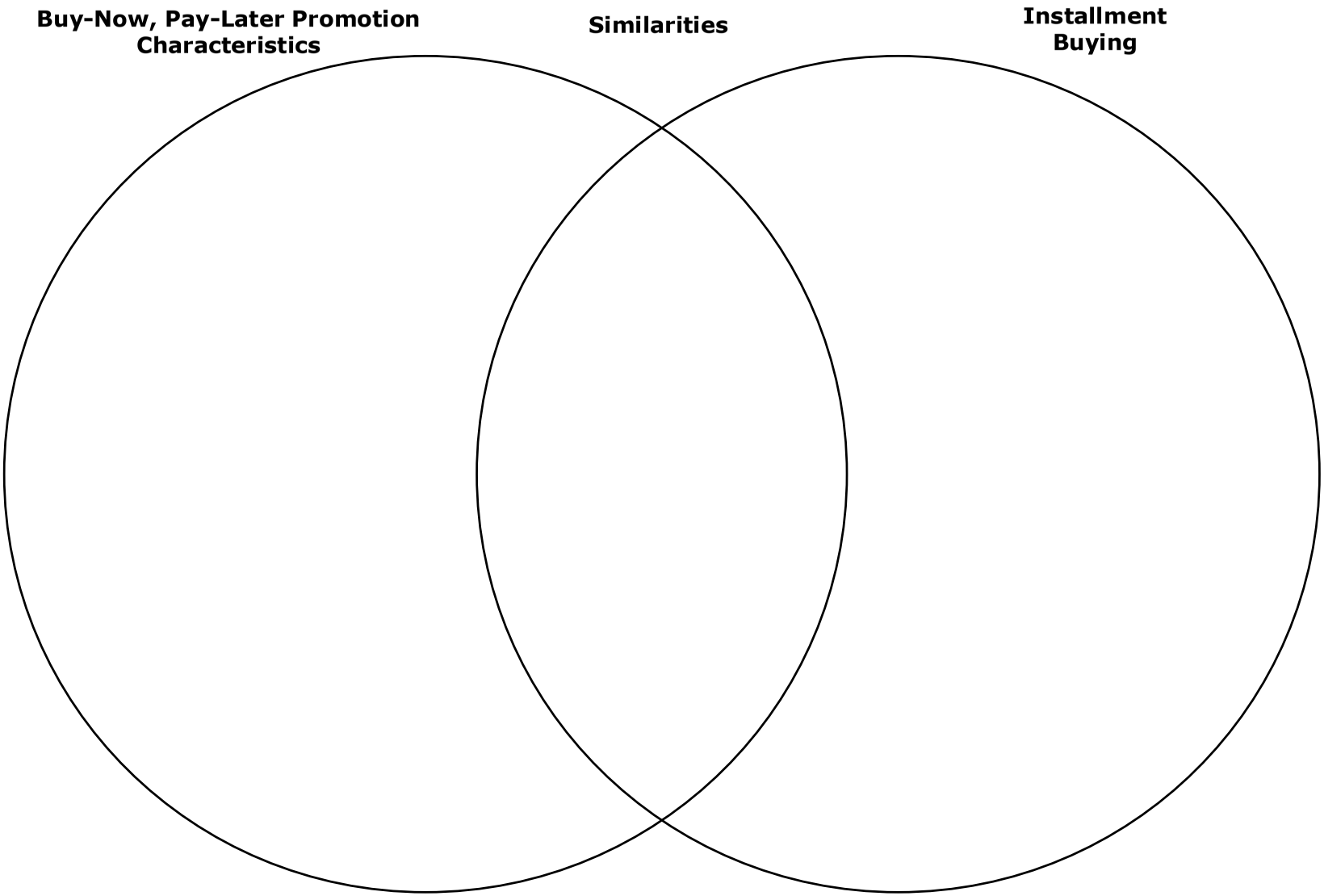
2. Robert Trebor decides to purchase a television set at Palindrome Warehouse Company. The television set has a cash selling price of $999.99 plus taxes. The installment terms are $200 down plus $45 a month for 24 months.
   a) Calculate the cash selling price of the television set.
   b) Calculate the installment price of the television set.
   c) Calculate the carrying charge.
   d) Calculate the percent rate of the carrying charge compared to the cash selling price.
   e) In your opinion, should Robert purchase the TV set in installments?
3. In the Venn Diagram on the next page, write down key characteristics of the buy-now, pay-later promotion and installment buying promotion.

Lesson Summary

In this lesson, you studied in-store credit and why companies offer it to customers. As you calculated cash prices and carrying charges, you observed that it is less expensive to pay cash upfront. This, however, is not always an option. As you start making larger purchases, remember to ask yourself the questions:

- "Can I pay for this right now?"
- "Do I need this right away?"
- "Can I save enough money to pay in cash in six months?"

Complete assignment 1.4 before moving on to the next lesson. This assignment includes questions related to loans, as well as buy-now pay-later and installment buying options.


Buy-Now, Pay-Later Promotion Characteristics
Similarities
Installment Buying

Assignment 1.3

Credit Payment Calculations

Total: 32 marks

Note to Students: Have you been adding definitions and examples to your resource sheet as you completed the last two lessons? Remember that your resource sheet should contain information that you find helpful, and it does not need to have the resource sheet icon beside it.

For questions that include taxes, PST = 7% and GST = 5%.

1. Bea Fehr requires a personal loan to purchase appliances that cost $5000. She meets with the loan officers of two financial institutions. Institution A offers her a fixed interest rate of $9\frac{3}{4}\%$ for a three-year loan; Institution B offers her a fixed interest rate of $11\frac{1}{2}\%$ for a three-year loan.

   a) How much will Bea pay Institution A each month? *(3 marks)*

   b) How much will Bea pay Institution B each month? *(2 marks)*

   c) With which financial institution will Bea pay more each month? How much more will she pay? *(1 mark)*

   d) What is the difference in interest over the three years between the two financial institutions? *(4 marks)*

continued

Assignment 1.3: Credit Payment Calculations (continued)

2. Cassandra Donaldson Steinway purchases a sound system from Company C. She can either pay for the sound system at the time of purchase or purchase it on the company's buy-now, pay-later plan.

   If she chooses to pay for the sound system at the time of purchase, the price is $795 (plus taxes). The delivery charge for the sound system is $25.

   If she chooses the buy-now, pay-later option, the price of the sound system is $869.55 (plus taxes). At the time of purchase, she must pay the taxes, delivery charge, and a $49.99 administration fee. She has 18 months to pay for her purchase. If she pays during this time period, she will not be charged interest.

   a) If Cassandra pays for the sound system at the time of purchase, calculate her total pay-now price. *(3 marks)*

   b) If Cassandra chooses to purchase the sound system using the company's buy-now, pay-later option, how much will she pay at the time of purchase? *(3 marks)*

   c) Cassandra purchases the sound system in October and pays for it the following June. How much will she pay in total for the sound system? *(1 mark)*

   d) Calculate the difference between the total pay-now price and the total pay-later price. *(1 mark)*

   e) Express the difference in the two prices, compared to the total pay-now price, as a rate of percent. *(1 mark)*

continued

Assignment 1.3: Credit Payment Calculations (continued)

3. Frederick Easton Edison purchases a television from Company F on August 15. He can either pay for the television set at the time of purchase, or purchase it on the company's buy-now, pay-later plan. He chooses to purchase it on the company's buy-now, pay-later plan.

   On the buy-now, pay-later plan, the price is $1995.95 (plus taxes). At the time of purchase, Frederick must pay the taxes and a $40 administration fee. He has one year to pay for his purchase. If he pays during this time period, he will not be charged interest. After one year, Company F charges customers 2.25% simple interest per annum, charged monthly, on any outstanding amount. Assume that he is charged interest from the date of purchase on the outstanding amount.

   How much will Frederick pay for the television if he pays the entire amount on September 15 of the following year? *(5 marks)*

continued

Assignment 1.3: Credit Payment Calculations (continued)

4. Andra Ardna decides to purchase a refrigerator at Palindrome Warehouse Company. The refrigerator has a cash selling price of $1299.21 plus taxes. The installment terms are $75.00 down plus $150 each month for one year.

   a) Calculate the cash selling price of the refrigerator. *(3 marks)*

   b) Calculate the installment price of the refrigerator. *(1 mark)*

   c) Calculate the carrying charge. *(1 mark)*

   d) Calculate the percent rate by which the installment price exceeds the cash selling price. *(1 mark)*

   e) In your opinion, should Andra purchase the refrigerator in installments? Explain your answer. *(2 marks)*

Lesson 6: Credit Cards

Lesson Focus

In this lesson, you will

- ❑ compare different credit card options
- ❑ solve problems related to credit card bills

Lesson Introduction

So far, you have solved problems involving personal loans, plus buy-now pay-later and installment buying promotions. In this lesson, you will explore credit cards. As mentioned in the previous two lessons, consumers have to be conscious of the financial decisions they make. It is best to shop around before making a large purchase or getting a loan. The same rule applies to credit cards. You need to choose the one that suits your spending habits.

Comparing Credit Cards

You may not have a credit card at this time, or you may have one in your parent's name. This is because people under the age of 18 are not eligible to have a credit card.

Before we continue with the lesson, complete the following learning activity to explore the decisions people make when using credit cards.

Learning Activity 1.10

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

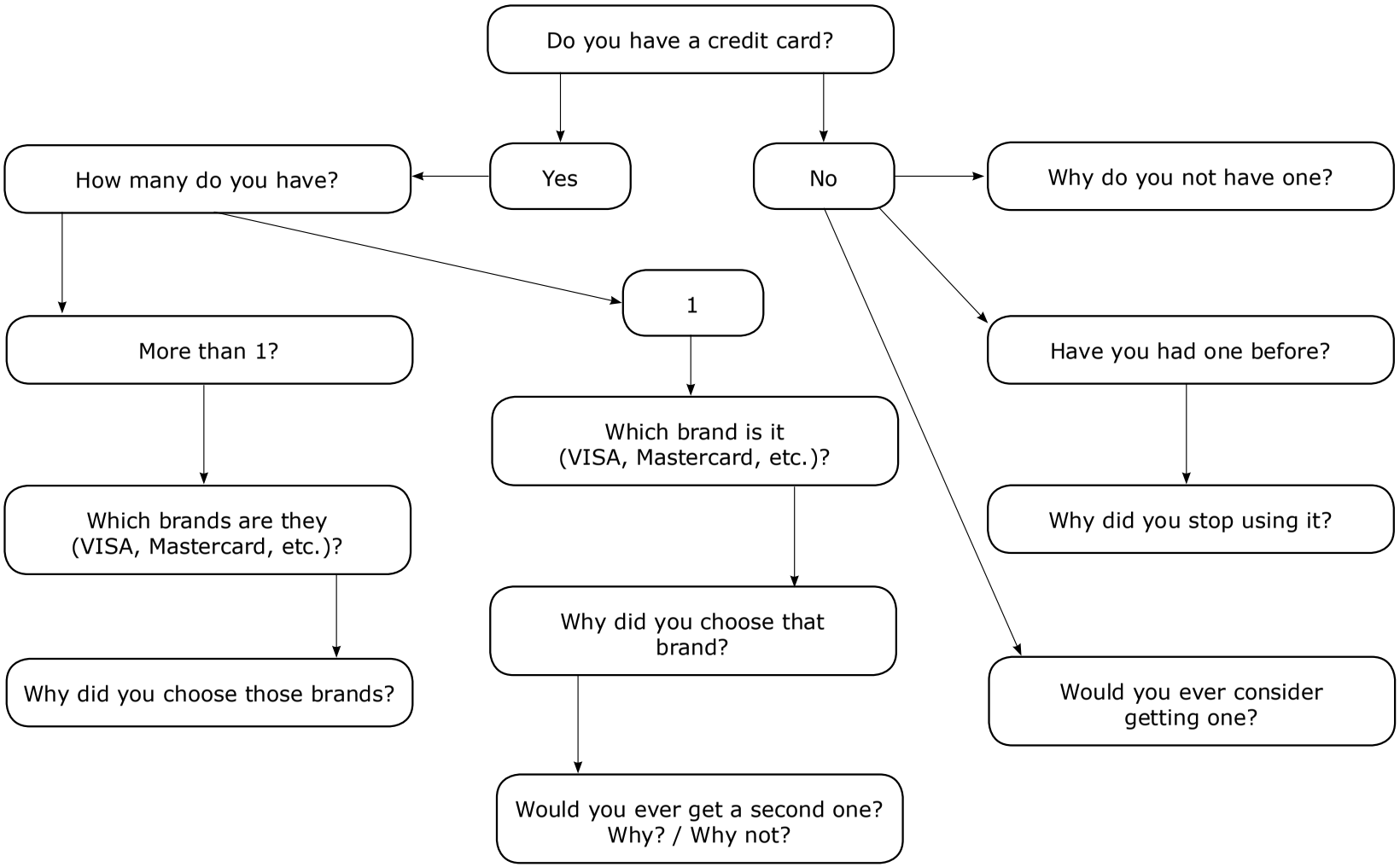
1. Evaluate: $5 - 3 + 6 \times 2 - 8 \div 4$
2. It is New Year's Eve, and you are excited for midnight. If it is 17:25 right now, how much time is there between now and midnight?
3. It has rained 354 mm in four hours. How much rain is this in cm?
4. There are 195 countries in the world, and 19 of these countries have an ecological footprint larger than 5.0 gha/person. Estimate the percentage of countries that have an ecological footprint larger than 5.0 gha/person.
5. Solve for d: $3d - 2 = 25$

Part B: Credit Cards in Your House

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Interview an adult you know. Use the flow chart on the next page to ask questions about this person's credit card use, and record the answers on a separate piece of paper.

Note: For some people, questions about their financial habits may be personal and/or private. For each question you ask, allow the interviewee to decline to answer the question if he/she is uncomfortable. Remember that learning activities are not worth marks; they are for you to review your understanding of the material provided in the lesson. If, therefore, you are unable to find someone who is willing to answer these questions, simply leave this learning activity and move on in the course.


Do you have a credit card?
Yes
No
How many do you have?
Why do you not have one?
1
More than 1?
Have you had one before?
Which brand is it
(VISA, Mastercard, etc.)?
Which brands are they
(VISA, Mastercard, etc.)?
Why did you stop using it?
Why did you choose that
brand?
Why did you choose those brands?
Would you ever consider
getting one?
Would you ever get a second one?
Why? / Why not?

Credit Card Decisions

As you went through the flow chart in the learning activity, did you notice how many decisions the person you were talking to had to make when choosing a credit card, or why the person stopped using it? There are many factors that help you decide which credit card is the best for you. We will be focusing on interest rates, annual fees, and benefits.

While you read through these factors, you may want to make a list of decisions you need to make on your resource sheet.

Interest Rates

When getting a credit card, the interest rate may be one of the most important characteristics to compare.

The first thing you want to find out about the interest rate of a credit card is if it has an **interest-free period** for purchases, such as not having to pay interest for the first 30 days. It is good to have an interest-free period because you do not have to start paying interest right away. Even better, if you pay the balance before the interest-free period is over, you do not pay any interest! A lot of credit card companies offer this, but not all of them. Remember that you are charged interest on cash withdrawn with a credit card starting the day it is taken out, whether or not you have an interest-free period.

If you have an interest-free period, the next question is, "What is the interest rate?" If you pay your bill completely at the end of every month, the interest rate will not be an important factor in your decision making about a credit card. Alternatively, if you carry a balance on your card from month to month, the interest rate will play a large role in your decision making. In this case, you will look for a credit card with a lower interest rate so that you are charged less money by the company.

Benefits

Unlike interest rates, not all credit cards offer benefits. Benefits are offered by credit card companies to encourage people to choose their credit card. Examples of benefits include points used for purchases, points used for travel, and credit cards joined with a store so that the points you collect result in discounts at that store. Having benefits attached to your credit card can help you collect points and rewards that will save you money, but you still have to be cautious when choosing a card with benefits. Some cards that have benefits also have a fee you have to pay. In this case, the question you have to ask is, "Will the cost of the benefit be worthwhile when compared with the benefit?"

Example 1

Your credit card company is offering a benefit so that you can collect points to use to buy coffee at a popular coffee shop. The benefit will cost $20 per year to have on your card. If you do not drink coffee, would it be worthwhile to have this benefit?

Solution

No, it would not be worthwhile to have this benefit. If you do not drink coffee, then you do not spend money to buy coffee. With this in mind, why would you spend $20 per year to collect points for buying coffee?

Annual Fees

Just as benefits are not a necessity, not all companies charge an annual fee for using their credit card. Companies charge an annual fee for convenience and/or because of the benefits the card offers. If a credit card has an annual fee, the price range can be quite large, ranging from approximately $12 to $350. Again, as a consumer, your decision needs to be based on what is best for you. If there is an option to have a credit card with no fee or to have a credit card with a fee, you should choose the card that fulfils your spending needs.

Example 2

Kellen is getting a credit card now that he has turned 18. Both credit cards that he is comparing offer a 21-day interest-free period. MYZA has a credit card with no annual fee but an interest rate of 26 percent. Victory Card offers a card with an annual fee of $20 and an interest rate of 16 percent. Kellen plans to carry a balance on his credit card. Which should he choose?

Solution

If the interest Kellen pays for the MYZA card is more than $20 over the interest paid on the Victory Card, then the Victory Card is the better choice (e.g., if MYZA = $145 and Victory Card = $120, he would pay $25 more for MYZA). Otherwise, the MYZA card would be the better choice.

The following page is an example of the information a company may provide you when it wants you to choose its credit card.

Annual Interest Rates

The annual percentage rate (APR) is the interest rate that the cardholder has to pay on the new balance of the card. The default percentage rate (DPR) is the interest rate applied to the balance of the card if the minimum balance has not been paid by the due date. The DPR is 5% higher than the APR.

No Fee Card	*APR*	*DPR*
On Purchases	17.99%	22.99%
On Cash Advances	19.99%	24.99%
Low Rate Card		
On Purchases	11.99%	16.99%
On Cash Advances	19.99%	24.99%
Premium Card		
On Purchases	19.50%	24.50%
On Cash Advances	19.99%	24.99%

There is an introductory rate (1.99%) available for the first eight months after acquiring the card.

Interest-Free Grace Period

Interest will not be charged on purchases for 21 days from the statement date as long as the previous statement balance has been paid in full. There is no interest-free grace period on cash advances.

Determination of Interest

Interest is charged from the transaction date until it is paid in full. Interest is calculated daily but is added on each monthly statement. The daily interest rate is the APR ÷ 365 days in a year.

Minimum Payment

The total minimum payment due on a monthly statement is 3% of the balance or $10—whichever is greater—*plus* any other money owed from past months. New balances under $10 must be paid in full.

Foreign Currency Conversion

All foreign currency transactions are converted to Canadian dollars at the foreign exchange rate set by the credit card company. The currency may be directly converted into Canadian dollars or first into US dollars then Canadian dollars, depending on the country.

A cross-border fee of 2.5% is charged for each foreign transaction, plus the exchange rate explained above.

Annual Fees

Annual fees for Primary Cardholders

- No Fee Card $0
- Low Rate Card $29
- Premium Card $99

This fee is applied once the card is used, and appears on the first or second statement. Following this, the annual fees are applied again on the anniversary of the card's activation.

Annual fee for each Authorized User

- No Fee Card $0
- Low Rate Card $29
- Premium Card $29

If the authorized user(s) are added when the primary card is activated, then the annual fees are applied at the same time as they are to the primary card. If the authorized user(s) are added after the account has been opened, the annual fee will appear on the statement following the first use of the card, and will be applied on the anniversary of the card's first transaction.

Other Fees

Cash Advance	
(ATM—Canada)	$2.50 per transaction
(ATM—International)	$5.00 per transaction
(Cash back—Canada)	$7.50 per transaction
(Cash back—International)	$7.50 per transaction
Statement Copy	
Within 12 months	$5.00 per statement
Over 12 months	$7.50 per statement
Returned/Refused Payment/Cheques	$20.00 per transaction
Over limit	$30.00 per statement
Stop payment	$10.00 per transaction

Example 3

Emmett is getting a credit card. Comparing two credit card companies, he sees that they both do not offer an interest-free period, and the interest rate of the cards is 20%. Both cards have a benefit plan to collect points for movies. MYZA (one of the credit card companies) charges $15 annually to have this benefit. Canadian Direct (the other credit card) charges $25 annually to have this benefit. If Emmett usually spends $20 going to the movies each year, which credit card would be the better choice?

Solution

If Emmett spends $20 on movies each year, then he should choose the benefit that will save him money. MYZA would be the better choice because it would cost $15 instead of his usual $20.

Credit Card Interest Rate Calculations

When you are using your credit card, it is best to pay the balance by the due date so that you do not have to pay interest on the purchase. If you do not pay the balance by the due date, the company will charge you interest. To calculate the interest charged by a credit card company, use the simple interest formula.

Example 1

On January 5, Minnie Payment makes a purchase of $400 on her credit card. The purchase appears on her monthly statement issued January 20. Minnie does not pay for the purchase by the due date indicated on the January statement. Her next monthly statement is issued February 20. Calculate the interest added to her February statement for the purchase. The bank charges her an annual interest rate of 21%.

Solution

As Minnie does not pay for the $400 purchase by the due date indicated on the statement, the company charges her daily interest from January 5 to February 20.

The number of days from January 5 to February 20 = 47 (there are 27 days left in January and 20 days in February).

$$\begin{aligned} I &= Prt \\ &= 400 \times 0.21 \times \frac{47}{365} \\ &= 10.82 \end{aligned}$$

Interest charge on the purchase appearing on the February statement = $10.82.

Notice that when you are counting the number of days, January 5 and February 20 are included. This is true for all credit card interest calculations.

Example 2

Brigitte makes a purchase for $350 on her credit card on June 3. The purchase appears on the monthly statement issued on June 25, but she does not pay for the purchase by the due date on the June statement. The next monthly statement is issued on July 25. What is the interest added to her July bill for this purchase if the annual interest rate is 18% and if she has an interest-free period of 30 days?

Solution

From June 3 to July 25 is 53 days.

$$I = Prt$$

$$= \$350 \times 0.18 \times \frac{53}{365}$$

$$= \$9.15 \text{ in the interest added to the July statement}$$

Note: Even though Brigitte has an interest-free period on her credit card, she is still charged interest from the date of purchase. The interest-free period is effective only if you pay within the time period—in this example, within the first 30 days. If you do not pay within the interest-free period, interest is still charged from the *date of purchase*.

You should include this information as a reminder on your resource sheet.

The previous calculations included only one purchase. Calculating the interest of multiple purchases becomes very complicated because you have to treat each purchase separately. Because of this, it is easier to do these calculations on a spreadsheet. This is not required in this course. If a question includes multiple purchases, it will state the amount of interest that appears on the statement.

Example 3

William Payment's monthly statement shows a previous balance of $963.45. It also indicates that during the month, Bill made a payment of $500 and purchased goods totaling $626.95. Assume his interest charges for the month are $17.50. Calculate his new balance.

Solution

The new balance = previous monthly balance − monthly payment + purchases + interest

= $963.45 − $500 + $626.95 + $17.50 = $1107.90.

Note: In order to compute the new balance, subtract the monthly payment from the previous monthly balance, and then add the new purchases and the interest charges.

Example 4

In the previous example, Bill's minimum monthly payment will be 5% of the ending balance or $10, whichever is greater.

a) Calculate Bill's minimum monthly payment.

b) What are the consequences if Bill decides to pay only the minimum monthly balance?

Solution

a) 5% of the new balance = 5% × $1107.90 = 0.05 × $1107.90 = $55.40

Since $55.40 is greater than $10, the minimum monthly payment is $55.40.

b) If Bill pays only the minimum monthly balance, he will be charged interest on his purchases.

Note: Continually paying only the minimum amount means you will take a long time to pay off your credit card and get out of debt. Some credit card companies will make a note on your statement to tell you how long it will take to pay off the debt if you only make minimum payments. This time estimate is only valid if you don't make any more purchases on your card. For example, a balance of $3000 on a credit card may take 17 years to pay if only the minimum balance is paid each month.

Protecting Your Credit Card

Once you have a credit card, you need to be aware of what you have purchased using your credit card and when your payments are due. At the end of every month, be sure to check your credit card statements to make sure that you were not charged twice for the same purchase. This can happen because of an error in the computer system. Also, you want to be sure that only purchases you made are on your bill. If other purchases appear on your bill, you need to inform the company that issued your card for two reasons:

1. You do not want to pay for anything that you didn't buy, and if you don't pay for it then you will be charged interest (if you don't tell the company that you did not authorize that transaction).
2. Your credit card number may have been stolen, so you should cancel the card and have another one issued (which will have a different number).

It is very important that you protect your credit card number so that no one can steal it. A credit card thief can spend a lot of money and ruin your credit rating. This can have a negative effect on you in the future when you apply for a loan or personal line of credit, because the bank will look at your credit rating to determine whether you are reliable at paying back money that you have borrowed.

The following is a list of ways you can protect your credit card number:

- Do not give your credit card to someone as a "security deposit."
- Keep your receipts because they have part of your card number on them. Don't leave them at the till or put them in a public garbage can.
- If you are making an online purchase, be sure that the website is secure before entering your credit card number. You can tell that a website is safe if it has https:// at the beginning of the URL, rather than just http://.
- Do not give out your credit card number over the phone. This is very important because you do not know who is on the phone with you.

Now that you have read about decision making and calculations related to credit cards, complete the learning activity to be sure that you understood the lesson. Check your answers to be sure.

Learning Activity 1.11

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Convert the following percentage to a decimal: 14.5%
2. Your dad is in England for a business trip. In England, they are six hours ahead of us in Manitoba. If it is 4:30 pm here, what time is it in England?
3. Dario went to the store to buy some new clothes for the next school year. He spends $16.71 on t-shirts, $48.13 on pants, and $24.42 on a sweater. Estimate how much Dario spends in total, to the nearest dollar (not including taxes).
4. Evaluate for $x = 8$: $4x - 15$
5. The area of your property is 150 m^2. If the dimensions of your house are 8 m by 10 m, what is the area of your yard, not including the area taken up by your house?

Part B: Credit Card Calculations

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Yembe wants to get a credit card now that he is 18. He is comparing the following two credit card options, which both offer an interest-free period of 30 days. Consonant Two (a credit card company) has an interest rate of 19 percent and an annual fee of $25. Victory Card (another credit card company) offers a 22% interest rate but no annual fee. If Yembe plans to pay the balance on his credit card by the due date each time, which company should he choose?

continued

Learning Activity 1.11 (continued)

2. On September 12, Max Payment makes a purchase of $3000 on his credit card. The purchase appears on his monthly statement issued September 22. Max does not pay for the purchase by the due date indicated on his September statement. His next monthly statement is issued on October 22. Calculate the interest he is charged for the purchase on his October statement. Assume his lending institution charges him an annual interest rate of 20.5%.

3. If the new balance on a monthly credit card statement is $689.32, and if the minimum monthly payment corresponds to at least 5% of the ending balance or $10, whichever is greater, calculate the minimum monthly payment.

4. Joy Purchase's monthly statement has a previous balance of $1638.92. The statement indicates that Joy made a payment of $650 during the month and purchased more goods totalling $879.54. Assume her interest charges for the month are $32.75. Joy's minimum monthly payment corresponds to at least 5% of her ending balance or $10, whichever is greater.
   a) Calculate Joy's new balance.
   b) Calculate Joy's minimum monthly payment.

Lesson Summary

In this lesson, you explored the decisions you have to make when choosing a credit card. You also performed basic calculations related to credit card bills. These calculations included application of the simple interest formula. As you may have noticed, the last four lessons emphasized the importance of studying your credit options and making informed decisions about credit. The next lesson will continue exploring credit card safety, as well as decision making related to all forms of credit.

Lesson 7: Strategies for Credit Use

Lesson Focus

In this lesson, you will

- ❑ learn how to use credit wisely
- ❑ learn how to reduce accumulated debt

Lesson Introduction

In this module, you have done calculations related to different forms of credit. In this lesson, you will read about different strategies for using credit. It is important to be well informed about the different types of credit so that you can decide whether to use credit, and which is the best form of credit to use in any given situation.

Be Credit Smart

Once you have credit, it is important that you use it wisely so that you will be able to use credit in the future. You also want to think about how you are using credit, because if you are not cautious, you could end up with a large amount of debt.

The following is a list of questions you should ask yourself, or at least think about, before you use credit to pay for something:

- Do you really need it? Just because you do not feel like you are spending money by "putting it on plastic," you still are. Be conscious of impulse buying.
- Is this the best value? Remember from Grade 10 that comparing prices is important when making decisions about a purchase. This is still true when you are using credit to pay for an item.
- Is this the better option? Similar to comparing price values, do not forget to shop around when you are applying for credit. As you learned in the past four lessons, not every company or financial institution offers the same payment plans for the same type of credit.
- Is the statement correct? Check your monthly statement and be sure that you are not charged extra, double charged, or charged for something you didn't buy.

- How many credit cards do you have? There is no need to have a large number of credit cards. This increases your risk of forgetting to pay the balance, or you may spend more than you can afford.

Dealing with Debt

If you think about the suggested questions (above) before making decisions related to credit, you are unlikely to get into financial trouble over debt. **Debt** refers to any money that you owe that you have committed to pay back. When someone is said to be in debt, it means that the person owes money and has not paid it all back.

Getting In Debt

There are many indicators that you are in financial trouble over debt or at risk of getting into financial trouble over debt. If you are having a hard time paying the minimum amount on your credit card(s) each month, you are at a high risk of getting into financial trouble over debt. Also, using credit to pay for your daily living costs can put you at risk of getting into financial trouble over debt. In general, you are at risk of getting into financial trouble over debt if you spend more than you earn. It is important to keep this in mind. Always be aware of how much you owe.

Getting Out of Debt

Some people lose track of how much they owe and suddenly realize that they owe a lot more than they can afford. Anyone in this situation can take several steps to recover from debt.

- **Stop** using your credit cards. If you continue to use them, you will go deeper into debt. The same thing goes for other credit—stop all borrowing.
- **Contact** the company/companies that you owe. Because they are interested in you paying back the money you owe, they will probably be willing to help you out by changing your payment plan.
- **Consolidate** (combine) your debts by using a consumer loan. If you do this, instead of owing money to a number of companies, you only have to pay back one form of credit. This is also a good idea because the interest rate of a consumer loan is much lower than the interest rate of a credit card.
- **Cash in** some of your investments (if you have any) in order to pay off some of your debt.
- **Seek advice** from a counselor, or even someone close to you that you trust, if you are having trouble decreasing your debt without help.

- **Evaluate** how you got into debt. Create a plan to maintain a debt-free lifestyle once you have recovered.

The most important thing to do when you realize that you have a lot of debt is NOT to panic. Stay calm and start making a plan to deal with your debt.

Lesson Summary

In this lesson, you learned about being credit smart. You read about debt and how to cope with it. In the 2007 recession, many people realized that they were in financial trouble over debt to the point where they could not pay back the money they owed. Many people lost their homes and had to give up the lifestyles they loved. Keep this in mind as you begin to explore the world of credit, and remember to live within your means.

Complete the following assignment and then send all five assignments for this module and the cover sheet to the Distance Learning Unit. You're almost done this module!

Notes

Assignment 1.4

Credit Card Calculations

Total: 15 marks

Note to Students: You are able to use your resource sheet to complete this assignment. Be sure that it is complete, including examples that help you understand how to solve a question.

1. On May 6, May Payment makes a purchase of $258.50 on her credit card. The purchase appears on her monthly statement issued May 18. May does not pay for the purchase by the due date indicated on her May statement. Her next monthly statement is issued on June 18. Calculate the interest she is charged for her purchase on her June statement. Assume her lending institution charges her an annual interest rate of 19.75%, and she has an interest-free period of 21 days. *(3 marks)*

2. If the new balance on a monthly credit card statement is $157.42, and if the minimum monthly payment corresponds to at least 5% of the ending balance or $10, whichever is greater, calculate the minimum monthly payment. *(2 marks)*

continued

Assignment 1.4: Credit Card Calculations (continued)

3. Mary Purchase's monthly statement has a previous balance of $384.26. The statement indicates that during the month Mary made a payment of $275 and purchased no goods. Assume her interest charges for the month are $4.50. Mary's minimum monthly payment corresponds to at least 5% of her ending balance or $10, whichever is greater.

   a) Calculate Mary's new balance. *(2 marks)*

   b) Calculate Mary's minimum monthly payment. *(1 mark)*

continued

Assignment 1.4: Credit Card Calculations (continued)

4. Read the following paragraph. Once you have read it, answer the questions below.

   DeWayne has two credit cards, which he uses to pay for *everything*. DeWayne does not check his credit card statements regularly, so sometimes, such as when a machine at a store malfunctions, he is charged *twice* for the same purchase. DeWayne makes sure that he pays the minimum payment on each credit card, but most of the time he does not have enough money to pay *more* than the minimum payment. Recently, something has happened so that DeWayne *cannot* even pay the minimum payments on his credit cards. DeWayne has *stopped* making payments for his mortgage (a loan used to buy a house) because he does not have enough money. He is worried because he doesn't have any investments or large savings accounts. On a rainy Wednesday, DeWayne receives a phone call from the bank. He is in debt.

   a) Use the list "How to get out of debt" to make five different suggestions to help DeWayne deal with his debt. *(5 marks)*

   b) Using the information from the story, state two possible reasons why DeWayne is in debt. *(2 marks)*

Notes

Module 1 Summary

Congratulations, you have finished Module 1!

In this module, you learned and explored the topics of interest and credit. Many of you already have experience with interest, and in this module you learned how to do a variety of interest calculations related to both investments and credit. Although you may not have a lot of experience with credit now, it is helpful to know what it is and how to use it responsibly before you get started. This is the best way to avoid debt and maintain a good credit rating.

There are many details to remember in this module, so it is a good idea to complete your resource sheet for Module 1. Refer to Appendix B: Glossary at the end of the course to help you create your resource sheet.

In the next module, you will study 3-D shapes and geometry.

Submitting Your Assignments

It is now time for you to submit the Module 1 Cover Assignment and Assignments 1.1 to 1.4 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 1 assignments and organize your material in the following order:

- ❑ Module 1 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 1 Cover Assignment: Loyalty Programs
- ❑ Assignment 1.1: Interest
- ❑ Assignment 1.2: What Is Credit?
- ❑ Assignment 1.3: Credit Payment Calculations
- ❑ Assignment 1.4: Credit Card Calculations

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Grade 11 Essential Mathematics (30S)

Module 1
Interest and Credit

Learning Activity Answer Keys

Module 1:
Interest and Credit

Learning Activity 1.1

There is no answer key for this learning activity.

Learning Activity 1.2

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate for $y = 4$: $4y + (-6)$
2. A right triangle has sides with lengths of 12, 20, and 16. What is the length of the hypotenuse?
3. How many times do you have to reflect an image over a line so that you get the original image back?
4. There is a square kite in a picture, drawn to scale. The scale ratio is 1 mm : 20 cm. If the kite has a side length of 3 mm, how large is the life-size kite?.
5. Denise earns 15%of her sales at her family restaurant. If she sells $1400 in food, how much money will she earn?

Answers:

1. 10 (4(4) + (−6) = 16 − 6 = 10)
2. 20 (Remember that the hypotenuse is always the longest side of a right triangle.)
3. Twice (The line of reflection doesn't move, so the first reflection moves the image onto the other side of the line of reflection and the second reflection brings it back.)
4. 60 (3 × 20 = 60 cm)
5. 210 (10% of $1400 = $140; 5% of 1400 is half of 10% = $70; so 15% = 140 + 70 = $210)

Part B: Simple Interest Formula

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand. Don't forget that you can ask your learning partner or tutor/marker for help if you are having a hard time understanding.

1. Find the simple interest for each of the following. Round to the nearest cent.

   Answer:

Interest	Principal	Rate	Time
$100	$1000	5%	2 years
$25	$1000	5%	6 months
$13.70	$1000	5%	100 days
$775	$10,000	7¾%	1 year
$764.38	$10,000	7¾%	360 days
$1162.50	$10,000	7¾%	18 months

2. Rayna Fontaine invests $20,000 in a financial institution at 10%. Calculate the number of days it will take her investment to earn $1200 in interest.

   Answer:

   $I = Prt$

   $$t = \frac{I}{Pr}$$

   $$t = \frac{1200}{(20\ 000)(0.10)}$$

   $t = 0.6$ years

   $t = 0.6 \times 365 = 219$ days

3. Douglas Fir borrows money from his financial institution at an interest rate of 6.25% per year. If he pays $397.50 in interest after four years, calculate the amount of his loan.

   Answer:

   $$P = \frac{I}{rt}$$

   $$P = \frac{397.50}{(0.0625)(4)}$$

   $P = \$1590$

4. Luke Wharm has two years to save $2800 for a winter vacation. He has $10,000 to invest in a financial institution.

   a) Calculate the interest rate he requires to earn enough for his vacation.

   Answer:

   $$r = \frac{2800}{(10\ 000)(2)}$$

   $r = 0.14$ or 14%

   b) Is it likely that Luke will find a financial institution that will offer him this rate? Check with a financial institution or a newspaper.

   Answer:

   No, it is not likely that a financial institution would offer such an interest rate.

5. Preston Fawcett has $1000 to invest in a financial institution. He decides to purchase a step bond that guarantees him $4\frac{3}{4}\%$ for the first year, $5\frac{1}{2}\%$ for the second year, and $6\frac{3}{4}\%$ for the third year.

   a) Calculate the total interest he will earn in three years.

   Answer:

   Interest earned in year 1 = $1000 \times 0.0475 \times 1 = \$47.50$

   Interest earned in year 2 = $1000 \times 0.055 \times 1 = \$55.00$

   Interest earned in year 3 = $1000 \times 0.0675 \times 1 = \$67.50$

   Total interest = $\$47.50 + \$55.00 + \$67.50 = \$170.00$

   b) Calculate the average interest he will earn in three years.

   Answer:

   Average interest = $\frac{170}{3} = \$56.67$

Learning Activity 1.3

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If $12 \times 15 = 180$, what is 13×15?
2. You are paid $9.50 per hour. If you work 20 hours this week, how much will you be paid?
3. The base of a triangle is 4 cm. The height of the same triangle is 5 cm. What is the area of the triangle? $\left(A = \frac{1}{2} \times bh\right)$
4. Convert the following decimal to a percent: 0.013
5. Jordan is twice as old as Carly. Carly is one-third as old as Marc. If Marc is 18, how old is Jordan?

Answers:

1. 195 (Since multiplying is repeated addition, 13×15 means add 15 to itself thirteen times. You already know $12 \times 15 = 180$ so add another 15 to get 195.)

   Another approach:

   $12 \times 15 = 180$

   $13 \times 15 = (12 \times 15) + (1 \times 15)$

   $= 180 + 15$

   $= 195$

2. $190 (10 hours of work = $95, so 20 hours = $2 \times \$95 = \$190$)
3. 10 cm^2 ($0.5 \times 4 \times 5 = 0.5 \times 20 = 10$)
4. 1.3% (Remember that percent is out of 100, so you can move the decimal two places to the left or divide by 100.)
5. 12 years old (Carly = $18 \div 3 = 6$. Jordan = $2 \times 6 = 12$)

Part B: Compound Interest

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. A principal of $10 000 is invested for four years at 8 percent compounded annually. Complete the following chart to calculate the interest earned.

   Answer:

Interest Period	*I = Prt*	Amount
0		$10,000.00
1	$I = 10{,}000 \times 0.08 \times 1 = \$800$	10,000 + 800 = $10,800
2	$I = 10{,}800 \times 0.08 \times 1 = \$864$	10,800 + 864 = $11,664
3	$I = 11{,}664 \times 0.08 \times 1 = \$933.12$	11,664 + 933.12 = $12,597.12
4	$I = 12{,}597.12 \times 0.08 \times 1 = \$1007.77$	12,597.12 + 1.007.77 = $13,604.89

2. a) Use the compound interest formula to solve question 1.

   Answer:

$$A = 10{,}000\left(1 + \frac{0.08}{1}\right)^{1\times 4} = \$13{,}604.89$$

   b) Is your answer the same using the chart and the compound interest formula?

   Answer:

   The answer is nearly the same as the figure calculated using the chart. (If there is a difference, it is due to rounding off too early in the chart.)

3. Copy and complete the following chart.

   Answer:

Principal (*P*)	Compound Interest Rate (*r*)	Time in Years (*t*)	Number of Interest Periods (*n*)	r *n*	*nt*	$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$1000	7% compounded annually	7	1	0.07	7	$1605.78
$250	10% compounded semi-annually	5	2	0.05	10	$407.22
$50,000	8% compounded daily	3	365	*0.0002191	1095	$63,560.79
$4000	6% compounded monthly	6	12	0.005	72	$5728.16
$800	9% compounded quarterly	2	4	0.0225	8	$955.86
$25,000	$5\frac{1}{2}$% compounded semi-annually	4	2	0.0275	8	$31,059.50

* Use calculator steps to avoid round-off error: 50 000 × (1 + 0.08 ÷ 365) y^x (365 × 3)

4. Chase South has $10,000 to invest for a period of five years. Institution A offers him an interest rate of $7\frac{1}{2}\%$ compounded semi-annually.

   Institution B offers him an interest rate of $8\frac{1}{4}\%$ compounded semi-annually.

   a) Using the compound interest formula, calculate how much the final investment will be worth at Institution A.

      Answer:

      Investment value at Institution A $= 10,000\left(1 + \frac{0.075}{2}\right)^{2\times 5} = \$14,450.44$

   b) Using the compound interest formula, calculate how much the final investment will be worth at Institution B.

      Answer:

      Investment value at Institution B $= 10,000\left(1 + \frac{0.0825}{2}\right)^{2\times 5} = \$14,981.32$

   c) Calculate how much more interest Chase would earn by investing at Institution B.

      Answer:

      Difference = $14,981.32 − $14,450.44 = $530.88

5. Wanda Waite invests $25,000 for 18 years. At the end of this time, her investment will be worth $50,000. Use the rule of 72 to estimate the interest rate.

   Answer:

   To find the interest rate required for an investment to double, use the formula:

   $r = \frac{72}{t}$ (in the formula, $t = 18$)

   $r = \frac{72}{18}$

   $r = 4$

   The approximate interest rate is 4%.

Learning Activity 1.4

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate for $x = 5$: $3x - 8$
2. If 13×14 is 182, what is 13×15?
3. You are planting a garden in your backyard with the dimensions 1 m by 3 m. What is the area of your garden?
4. Ivana is in the food court at the mall. One store's special costs $8.50, and includes a sandwich, a drink, and a bag of chips. Another store does not have a special, so it costs $5 for the sandwich, $2.00 for the drink, and $2.00 for the chips. Which store should she go to?
5. Convert the following to a decimal: 38%

Answers:

1. 7 ($3(5) - 8 = 15 - 8 = 7$)
2. 195 (Remember that multiplying is repeated addition. So if I know 13 added to itself fourteen times is 182, then 13 added to itself fifteen times is 182 + 13 or 195.)

   Another approach:

   $13 \times 14 = 182$

   $13 \times 15 = (13 \times 14) + (13 \times 1)$

   $= 182 + 13$

   $= 195$
3. A = 3 m^2 (Area = length × width)
4. The store with the special ($5 + $2 + $2 = $9. Compared to the $8.50, she would save $0.50.)
5. 0.38 (Remember to divide by 100.)

Part B: Exploring Investment Decisions

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

In this learning activity, you will investigate different types of investments.

1. Discussing finances is a personal matter for many people. Ask your learning partner, a parent, or a relative, if he or she would be comfortable discussing his or her experiences with investments. When you find someone who is comfortable, ask him or her to respond to the following questions:
   - Has this person invested money before?
   - Was it a high- or low-risk investment?
   - Why did they choose a high- or low-risk investment?
   - Did they find it beneficial to invest?

   Answer:

   Answers may vary depending on who you talked to and their experiences with investments.

2. What are some of your current financial goals, and what financial goals might you have in the future? If possible, discuss this with the person you worked with in question 1.

   Answer:

   Answers may vary depending on your personal goals. Some examples could be saving to buy a car/laptop/sound system/camera/etc., or saving for university/college.

3. The investment world is changing constantly, so the options of how to invest are also changing. Visit your local bank and speak with a personal banker about what investment options they offer or research the answers on the Internet. Questions you may ask include the following:
   - What are some of the high-risk investments that you offer?
   - What are some of the reasons people choose high-risk investments?
   - What are some of the low-risk investments that you offer?
   - What are some of the reasons people choose low-risk investments?

   Choosing one of the financial goals discussed in question 2, you may ask the personal banker which investment option would be best.

   Answer:

   Answers may vary depending on both the bank and the questions asked.

Learning Activity 1.5

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate for $z = 3$: $6z - 9$
2. Heather is $\frac{4}{5}$th the height of Crystal. Ashley is 15 cm taller than Heather.

   If Crystal is 150 cm tall, how tall is Ashley?
3. You are having a day of shopping. At European Clothing, you spend $38.00; at Discount you spend $10.00; at Micki's you spend $66.00; and at Alarms you spend $23.00. How much money did you spend all together?
4. Solve for d: $2d - 7 = 29$
5. Evaluate: $\frac{2}{3} \times \frac{5}{7}$

Answers:

1. 1.9 $(6(3) - 9 = 18 - 9 = 9)$
2. 135 cm (Heather: $150 \times \frac{4}{5} = \frac{150 \times 4}{5} = 30 \times 4 = 120$ cm $(150 \div 5 = 30)$

   Ashley: 120 + 15 = 135 cm)
3. 137 (Add the 1s: 8 + 6 + 3 = $17; add the 10s: 30 + 10 + 60 + 20 = 120; Add them together: 17 + 120 = $137.00)
4. $d = 18$ (Add 7 to both sides: $2d = 29 + 7$. Divide both sides by 2: $d = 36 \div 2 = 18$)
5. $\frac{10}{21}$ $\left(\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}\right)$

Part B: What Is Credit?

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Brainstorm the following question with your learning partner. There are no right or wrong answers, as your responses are based on

- the information that you just read
- your personal opinions
- your experiences

1. a) With your learning partner, brainstorm different expenses for which you would use or have used credit.

   Answer:

   Answers will vary. Possible answers include:

   Buying a car, buying a house, going on a big vacation, taking a year off work, going to university/college, getting your carpentry/electrical certification, getting your masters/Ph.D., transmission on your car stops working, water damage to your house, buying stocks, the laptop you want is on sale

   b) Sort the expenses you came up with in part (a) and write them into a suitable category using a table similar to the one shown below.

   Answer:

Big Ticket Item	School and Training	Unexpected Costs	Costs While Unemployed	Investments
buying a car buying a house going on a big vacation	going to university/ college getting your carpentry/ electrical certification getting your Masters/PhD	transmission on your car stops working water damage to your house the laptop you want is on sale	taking a year off work	buying stocks

Learning Activity 1.6

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate: $\frac{5}{6} + \frac{2}{3}$
2. Sarah walked to school this morning, which is 2 km away from her house. She then walked 500 m to the convenience store after school before going home (1.5 km). How far did Sarah walk in total?
3. What is 10% of 1400?
4. You have to buy a gift for your sister. You get her some makeup, which costs $25.75 including tax. If you give the cashier $40, how much change should you get back?
5. Your best friend wants to hang out this week. You work Monday, Tuesday, Thursday, and Saturday. You have soccer on Wednesday and Sunday. Is there any day that you are free?

Answers:

1. $\frac{3}{2}\left(\frac{5}{6} + \frac{2}{3} = \frac{5}{6} + \frac{4}{6} = \frac{5+4}{6} = \frac{9}{6} \text{ or } \frac{3}{2}\right)$
2. 4 km (500 m = 0.5 km, so 2 + 0.5 + 1.5 = 4 km)
3. 140
4. 14.25 (Count up to the amount paid from the amount owed:
   $25.75 + $0.25 = $26.00
   $26 + $4 = $30.00
   $30 + $10 = $40.00 so your change is $0.25 + $4 + $10 = $14.25)
5. Friday (Process of elimination: ~~S~~ ~~M~~ ~~T~~ ~~W~~ ~~T~~ F ~~S~~
   So you could see your friend on Friday.)

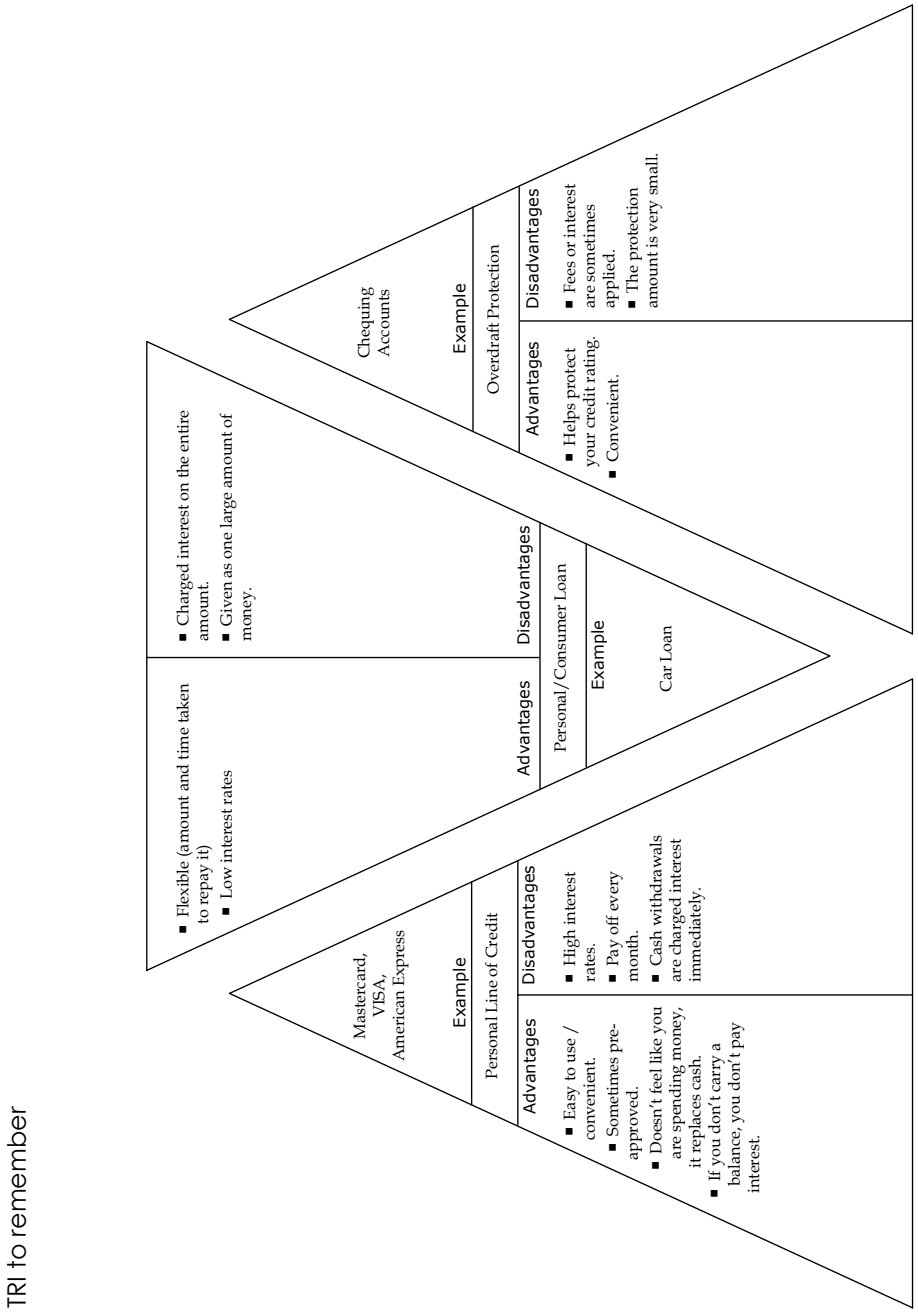
Part B: Credit Options

Fill in the "TRI to Remember" triangles on the next page for the three credit options you read about in the previous section to compare the advantages and disadvantages of each. (Do not use "personal line of credit," as it is in the example.)

It would be a good idea to include the advantages and disadvantages of different credit options on your resource sheet.

Answers will vary. Possible responses are:

TRI to remember



Learning Activity 1.7

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Isabella does not want to walk around a parking lot with dimensions 30 m by 40 m. If she walks from corner to corner (diagonally), how far will she walk?
2. Fill in the blanks in the pattern: 1, 2, 4, 8, ____ , ____
3. Write two equivalent fractions for $\frac{16}{28}$.
4. Solve for c: $c - 4 = 15$
5. Jack and Jill climbed up a hill. They covered a distance of 400 m in eight minutes. How fast did they climb?

Answers:

1. 50 m (3, 4, 5 is the simplest right angle triangle with whole number dimensions. since $3 \times 10 = 30$ and $4 \times 10 = 40$, $5 \times 10 = 50$ m diagonally)
2. 32 (The pattern is to multiply the last number by two to get the next number. 1, $1 \times 2 = 2$, $2 \times 2 = 4$, $4 \times 2 = 8$, $8 \times 2 = 16$, $16 \times 2 = 32$)
3. There are many possible answers. A few include $\frac{4}{7}, \frac{8}{14}, \frac{32}{56}, \frac{12}{21}$.
4. $c = 19$ (Add 4 to both sides.)
5. 50 m/min. $\left(\frac{400 \text{ m}}{8 \text{ min.}} = 50 \text{ m/min. since } 40 \div 8 = 5\right)$

Part B: Personal Loans

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Refer to the Amortization Table found at the end of this lesson, as required.

1. Complete the following chart:

   Answer:

Amount of Loan	Annual Interest Rate	Amortization Period (years)	Monthly Payment
\$5000	$10\frac{1}{2}\%$	4	\$128.00
\$13,500	$9\frac{3}{4}\%$	1	\$1185.30
\$7800	11%	5	\$169.57
\$10,750	$10\frac{1}{4}\%$	3	\$348.09
\$18,250	$8\frac{3}{4}\%$	2	\$831.65

2. Bea Wright requires a personal loan of \$15 000 for home renovations. Her financial institution offers her a five-year loan at a fixed interest rate of $10\frac{1}{4}\%$.

   a) How much must Bea pay her financial institution each month?

   Answer:

   Cost per \$1000 per month = \$21.37

   Monthly cost = 15×21.37 = \$320.55

   b) How much does Bea pay in interest over the five years?

   Answer:

   In five years there are $12 \times 5 = 60$ months.

   The total cost after five years is thus $60 \times \$320.55 = \$19{,}233$.

   Total interest = total cost − loan amount = \$19,233 − \$15,000 = \$4233.

3. If Bea Wright decides to amortize her loan of $15,000 over one year rather than five years, her financial institution offers her a fixed interest rate of $9\frac{3}{4}\%$.

   a) How much must Bea now pay her financial institution each month?

      Answer:

      Cost per $1000 per month = $87.80

      Monthly cost = 15 × $87.80 = $1317.00

   b) How much must Bea pay in interest over the year?

      Answer:

      Total interest = total cost − loan amount

      Total cost = 12 × $1317 = $15,804

      Total interest = $15,804 − $15,000 = $804

   c) What is the difference in the interest she pays when the loan has an amortization period of one year instead of five years?

      Answer:

      Difference in interest = $4233 − $804 = $3429

4. Bea Reddy has a personal loan of $5000 to purchase appliances for her new house. Her monthly payment is $125. What is the interest rate of her loan, and how long is the term?

   Answer:

   Since she is paying $125 per month for a $5000 loan, it would be $25 per month for a $1000 loan (125 ÷ 5 = 25). Find this in your amortization table. The interest rate for her loan is 9.25%, and the term is four years.

Learning Activity 1.8

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If 25% of 2720 is 680, what is 50% of 2720?
2. The sides of a right-angle triangle are 28, 45, 53. What are the two possible ratios for tangent?
3. List four types of credit.
4. It is your friend's birthday. You collect $5 from each of your other friends to buy him a hoodie that costs $40.00 including tax. If you and nine other people pay $5 for the hoodie, will you be able to afford it?
5. Evaluate: $\frac{3 \times 4}{6} + 8$

Answers:

1. 1360 ($2 \times 680 = 1360$)
2. $\frac{28}{45}$ and $\frac{45}{28}$ (Tangent ratio involves opposite and adjacent (not the hypotenuse, the longest side) so the two ratios are $\frac{28}{45}$ and $\frac{45}{28}$).
3. credit card, personal line of credit, personal/consumer loan, overdraft protection
4. Yes (You + 9 friends = 10 people; $10 \times 5 = \$50$ collected. Since $50 > 40$, you can afford it.)
5. 10 ($3 \times 4 = 12$, $12 \div 6 = 2$, $2 + 8 = 10$)

Part B: Buy Now, Pay Later

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

For questions that include taxes, PST = 7% and GST = 5%.

1. Complete the following chart. The rate of percent expresses the ratio of the difference between the two prices and the pay-now price. Do not include taxes. Where necessary, round the rate of percent to the nearest tenth.

   Answer:

Pay-Later Price	Pay-Now Price	Difference between the Two Prices	Rate of Percent
\$12,000	\$10,000	\$2000	$\frac{2000}{1000} \times 100 = 20\%$
\$940	\$780	\$160	$\frac{160}{780} \times 100 = 20.5\%$
\$580	\$500	\$80	$\frac{80}{500} \times 100 = 16\%$
\$1400	\$1325	\$75	$\frac{75}{1325} \times 100 = 5.7\%$
\$1375	\$1250	\$125	$\frac{125}{1250} \times 100 = 10\%$
\$903	\$840	\$63	$\frac{63}{840} \times 100 = 7.5\%$
\$3250	\$2900	\$350	$\frac{350}{2900} \times 100 = 12.1\%$
\$1200	\$1000	\$200	$\frac{200}{1000} \times 100 = 20\%$

2. Bartholomew Upton York purchases a washing machine and dryer from Company Y. He can either pay for the washing machine and dryer at the time of purchase, or purchase them on the company's buy-now, pay-later plan. He chooses to purchase them on the company's buy-now, pay-later plan.

   On the buy-now, pay-later plan, the price is $698.98 (plus taxes) for the washing machine and $509.98 (plus taxes) for the dryer. At the time of purchase, Bartholomew must pay the taxes, a delivery charge, and a $35 administration fee per item. The delivery charge for both the washing machine and dryer is $35. He has six months to pay for his purchase. If he pays within this period of time, he will not be charged interest.

   a) How much will Bartholomew pay at the time of purchase (upfront)?

      Answer:

      The total cost before taxes = washing machine + dryer

      = $698.98 + $509.98 = $1208.96

      PST = $1208.96 × 0.07 = $84.63

      GST = $1208.96 × 0.05 = $60.45

      Total cost = PST + GST + delivery charge + administration fee for dryer + administration fee for washer

      = $84.63 + $60.45 + $35 + $35 + $35

      = $250.08 upfront

   b) If Bartholomew pays for the appliances within six months, calculate his total pay-later price of the appliances.

      Answer:

      Total cost to him is $250.08 + $1208.96 = $1459.04

3. Terrence Andrews Xavier purchases a television set from Company X on October 1. He can either pay for the television set at the time of purchase, or purchase it on the company's buy-now, pay-later plan. He chooses to purchase it on the company's buy-now, pay-later plan.

   On the buy-now, pay-later plan, the price is $698.98 (plus taxes). At the time of purchase, Terrence must pay the taxes and a $45 administration fee. There is no delivery fee. He has a six-month period to pay for his purchase. If he pays during this time period, he will not be charged interest. After six months, Company X charges customers 4 percent interest per annum, compounded monthly, on any outstanding amount.

   a) Calculate Terrence's total cost if he pays for the purchase on March 1 of the following year.

      Answer:

      March 1 is within the 6-month period so he will not pay interest.

      PST = $698.98 × 0.07 = $48.93

      GST = $698.98 × 0.05 = $34.95

      The total cost = $698.98 + $48.93 + $34.95 + $45 = $827.86

   b) Calculate Terrence's total cost if he pays for the purchase on May 31 of the following year. Assume that the company charges interest only after the "pay-later" period.

      Answer:

      Because he is charged interest for only two months, the interest will be

$$A = (698.98)\left(1 + \frac{0.04}{12}\right)^2$$

$$= (698.98)(1 + 0.0033333)^2$$

$$= (698.98)(1.0033333)^2$$

$$= (698.98)(1.0066778)$$

$$= \$703.65$$

      Total cost = 703.65 + 48.93 + 34.95 + 45 = $832.53

Note: In the compound interest formula, you divide the 4% per annum interest rate by 12 because the interest is compounded monthly. The decimal form of the monthly interest rate is 0.0033333. Since he is charged interest for only two months, the exponent is 2.

Learning Activity 1.9

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the pattern: 100 000, 10 000, 1 000, _____ , _____
2. Estimate the taxes on a shirt that costs $15.00.
3. Germany, England, and Greece are in a group for a soccer tournament. Germany wins against Greece. Greece loses to England. Germany beats England. In what order do they finish? (First, Second, Third)
4. A watch chimes every hour, on the hour. Between 4:23 am and 7:54 pm, how many times does it chime?
5. Write two equivalent fractions for $\frac{45}{81}$.

Answers:

1. 100, 10 (To find the next number, divide the last number by 10. Each number has one less zero than the one before it.)
2. A good estimate would be between $1.50 and $2.00. (Taxes = 5% and 7% = 12% total. 10% = $1.50, 1% of 15 = $0.15, 2% = $0.30 so 12% = $1.50 + $0.30. The actual amount is $1.80. Estimates should be around this number.)
3. First = Germany (2 wins), second = England (1 win, 1 loss), third = Greece (2 losses)
4. 15 (The watch will chime at 5 am, 6 am, 7 am, 8 am, 9 am, 10 am, 11 am, and 12 pm plus 1 to 7 pm (7 times) for a total of 15 times.)
5. There are many possible answers. Some are $\frac{5}{9}; \frac{10}{18}; \frac{90}{162} \ldots$

Part B: Installment Buying

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

To calculate taxes, use 7% for PST and 5% for GST.

1. Copy and complete the following chart.

   Answer:

Cash-Selling Price (without tax)	GST (5%)	PST (7%)	Cash-Selling Price (with tax)	Down Payment	Monthly Payment	Months	Installment Price	Carrying Charge
a) 249.98	$12.50	$17.50	$279.98	$35	$50	6	35 + 50 × 6 = $335	335 − 279.98 = $55.02
b) 998.98	$49.95	$69.93	$1118.86	$75	$100	12	75 + 100 × 12 = $1275	1275 − 1118.86 = $156.14
c) 1879.99	$94.00	$131.60	$2105.59	$100	$300	8	100 + 300 × 8 = $2500	2500 − 2105.59 = $394.41
d) 5998.95	$299.95	$419.93	$6718.83	$500	$400	18	500 + 400 × 18 = $7700	7700 − 6718.83 = $981.17

2. Robert Trebor decides to purchase a television set at Palindrome Warehouse Company. The television set has a cash selling price of $999.99 plus taxes. The installment terms are $200 down plus $45 a month for 24 months.

   a) Calculate the cash selling price of the television set.

      Answer:

      PST = $999.99 × 0.07 = $70.00

      GST = $999.99 × 0.05 = $50.00

      Total cash selling price = $999.99 + $70.00 + $50.00 = $1119.99

   b) Calculate the installment price of the television set.

      Answer:

      Installment price = 24 × $45 + $200 = $1080 + $200 = $1280.00

   c) Calculate the carrying charge.

      Answer:

      Carrying charge = $1280.00 − $1119.99 = $160.01

d) Calculate the percent rate of the carrying charge compared to the cash selling price.

Answer:

$$\text{percent rate} = \frac{160.01}{1119.99} \times 100\% = 14.3\%$$

e) In your opinion, should Robert purchase the TV set in installments?

Answer:

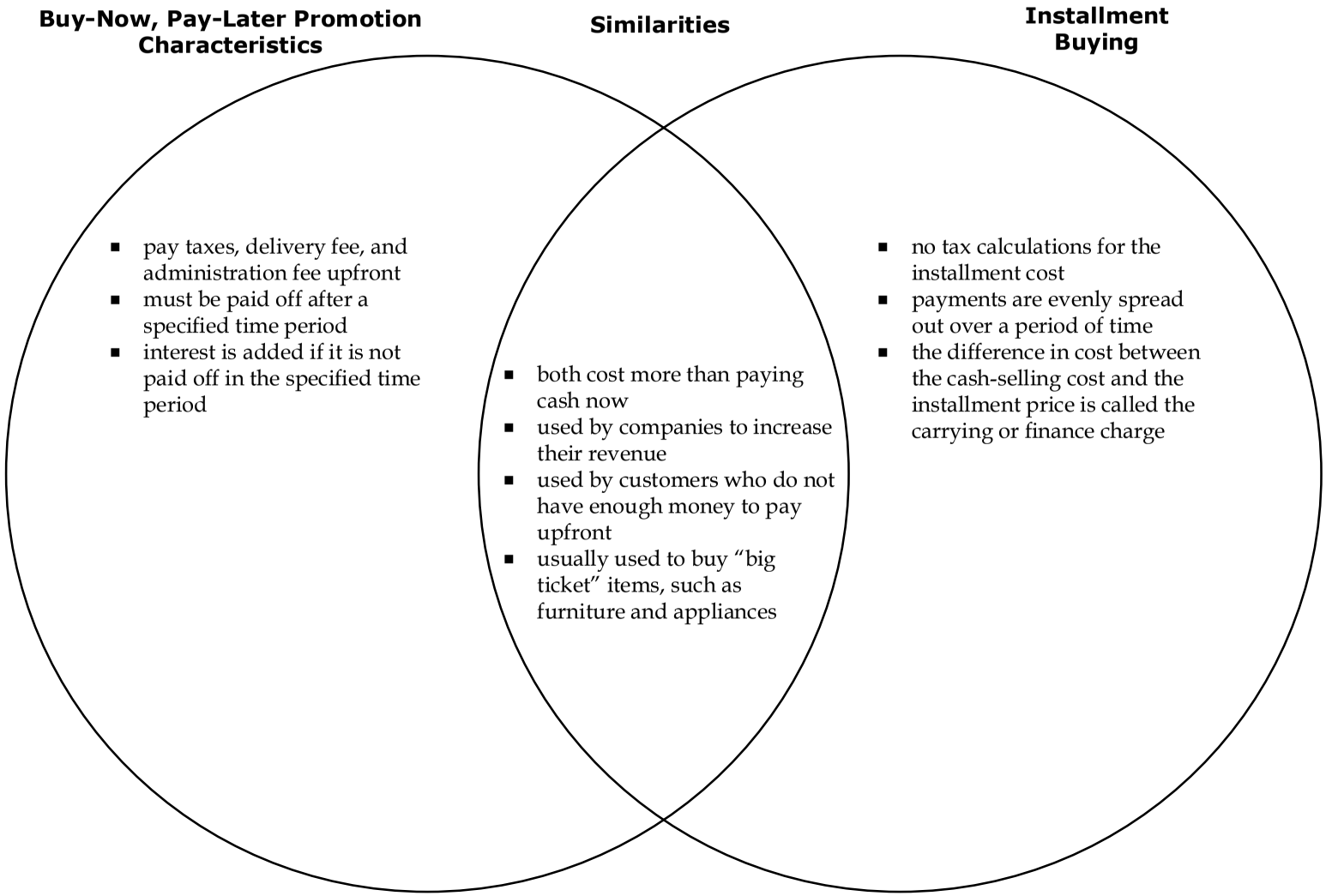
The answer should always be no if you have enough money now to pay for something. To answer this question, refer back to the three questions you should ask yourself about installment buying:

- Does Robert have enough money to pay the cash selling price right now?
  - If he does, he should pay the cash selling price.
  - If he doesn't, *move on to the next question…*
- Does Robert need the item immediately?
  - Odds are he probably doesn't need a TV right away. If this is the case, *move on to the next question…*
  - If he thinks he does, then installment buying is the only way to get it because he doesn't have enough money.
- If Robert doesn't need it immediately, can he save enough money to pay in cash in six months?
  - If Robert saves $100 from every paycheque for 12 paydays, he will have enough money to pay the cash selling price. Since most people are paid every two weeks, 12 paydays should add up to 6 months (12 × 2 = 24 weeks, and there are 52 weeks in a year so six months is approximately 52 ÷ 2 = 26 weeks).

3. In the Venn Diagram on the next page, write down key characteristics of the buy-now, pay-later promotion and installment buying promotion.

Answer:

There are many possible answers. Some are included in the diagram on the following page. Did you come up with any characteristics that are not listed? Discuss any differences with your learning partner or tutor/marker.


Buy-Now, Pay-Later Promotion Characteristics
Similarities
Installment Buying
pay taxes, delivery fee, and administration fee upfront
must be paid off after a specified time period
interest is added if it is not paid off in the specified time period
both cost more than paying cash now
used by companies to increase their revenue
used by customers who do not have enough money to pay upfront
usually used to buy "big ticket" items, such as furniture and appliances
no tax calculations for the installment cost
payments are evenly spread out over a period of time
the difference in cost between the cash-selling cost and the installment price is called the carrying or finance charge

Learning Activity 1.10

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate: $5 - 3 + 6 \times 2 - 8 \div 4$
2. It is New Year's Eve, and you are excited for midnight. If it is 17:25 right now, how much time is there between now and midnight?
3. It has rained 354 mm in four hours. How much rain is this in cm?
4. There are 195 countries in the world, and 19 of these countries have an ecological footprint larger than 5.0 gha/person. Estimate the percentage of countries that have an ecological footprint larger than 5.0 gha/person.
5. Solve for d: $3d - 2 = 25$

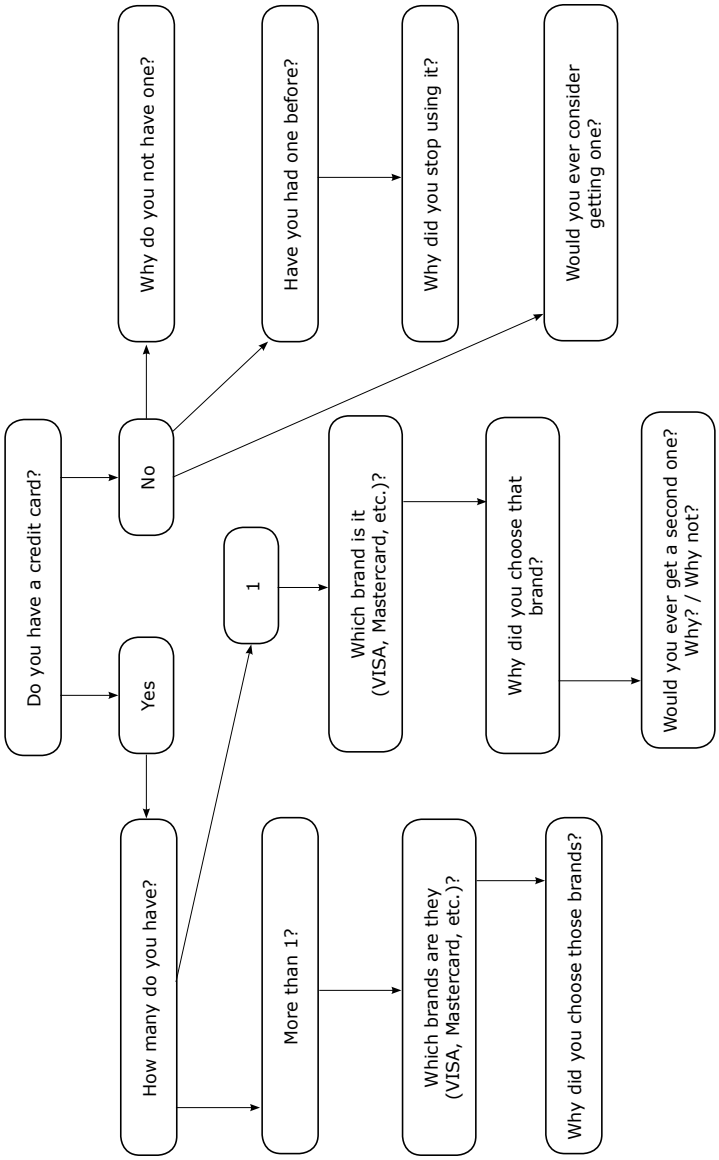
Answers:

1. 12 ($5 - 3 + 12 - 2 = 12$)
2. 6 h 35 min. (Count up. 17:25 + 00:35 = 18:00, 18:00 + 6:00 = 24:00 = 00:00. Total: 6 h 35 min.)
3. 35.4 cm (1 cm = 10 mm so 354 mm = 35.4 cm)
4. A good estimate would be between 9% and 10%. (10% of 195 = 19.5 so 19 countries would be about 9.5%. The actual percentage is 9.7%.)
5. $d = 9$ (Add 2 to both sides, $3d = 25 + 2$. Divide both sides by $3d = 27 \div 3 = 9$.)

Part B: Credit Cards in Your House

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Interview an adult you know. Use the flow chart on the next page to ask questions about this person's credit card use, and record the answers on a separate piece of paper.



Answer:

Answers will vary.

Learning Activity 1.11

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Convert the following percentage to a decimal: 14.5%
2. Your dad is in England for a business trip. In England, they are six hours ahead of us in Manitoba. If it is 4:30 pm here, what time is it in England?
3. Dario went to the store to buy some new clothes for the next school year. He spends $16.71 on t-shirts, $48.13 on pants, and $24.42 on a sweater. Estimate how much Dario spends in total, to the nearest dollar (not including taxes).
4. Evaluate for $x = 8$: $4x - 15$
5. The area of your property is 150 m^2. If the dimensions of your house are 8 m by 10 m, what is the area of your yard, not including the area taken up by your house?

Answers:

1. 0.145
2. 10:30 pm (Six hours ahead ⇒ add 6 hours to 4:30 pm)
3. A good estimate would be between $85 and $95. ($16.71 rounds to $17, $48.13 rounds to $48, and $24.42 rounds to $24. 17 + 48 + 24 = $89 or around $90.)
4. 17 (4(8) − 15 = 32 − 15 = 17)
5. 70 (Area of your house = 8 m × 10 m = 80 m^2, 150 − 80 = 70 m^2)

Part B: Credit Card Calculations

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Yembe wants to get a credit card now that he is 18. He is comparing the following two credit card options, which both offer an interest-free period of 30 days. Consonant Two (a credit card company) has an interest rate of 19 percent and an annual fee of $25. Victory Card (another credit card company) offers a 22% interest rate but no annual fee. If Yembe plans to pay the balance on his credit card by the due date each time, which company should he choose?

   Answer:

   Because Yembe plans to pay the balance on his credit card, the interest rate does not play a large role in his decision of which credit card to use. Since the Victory Card option does not have an annual fee, it is the better choice for Yembe.

2. On September 12, Max Payment makes a purchase of $3000 on his credit card. The purchase appears on his monthly statement issued September 22. Max does not pay for the purchase by the due date indicated on his September statement. His next monthly statement is issued on October 22. Calculate the interest he is charged for the purchase on his October statement. Assume his lending institution charges him an annual interest rate of 20.5%.

   Answer:

   Max will be charged interest from September 12 to October 22. The total number of days is 41.

   The daily interest rate is $\frac{20.5}{365} = 0.056\%$.

   The interest charged on the purchase made in September would be:

   $41 \times 0.00056 \times 3000 = \$68.88$

Note: This value will vary based on rounding of the percent value used. For example, $3000 \times 0.205 \times 41 \div 365 = \$69.08$.

3. If the new balance on a monthly credit card statement is $689.32, and if the minimum monthly payment corresponds to at least 5% of the ending balance or $10, whichever is greater, calculate the minimum monthly payment.

   Answer:

   5% of $689.32 is $34.47. Since that is greater than $10, the minimum monthly payment would be $34.47.

4. Joy Purchase's monthly statement has a previous balance of $1638.92. The statement indicates that Joy made a payment of $650 during the month and purchased more goods totalling $879.54. Assume her interest charges for the month are $32.75. Joy's minimum monthly payment corresponds to at least 5% of her ending balance or $10, whichever is greater.

   a) Calculate Joy's new balance.

      Answer:

      Joy's new balance would be her previous balance − payment made + interest charged on remaining balance + her new purchases.

      $1638.92 − $650 + $32.75 + $879.54 = $1901.21

   b) Calculate Joy's minimum monthly payment.

      Answer:

      5% of $1901.21 is $95.06, so that would be Joy's minimum charge.

Grade 11 Essential Mathematics (30S)

Module 2
3-D Geometry

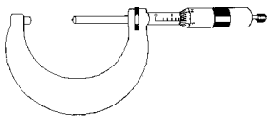
Module 2:
3-D Geometry

Introduction

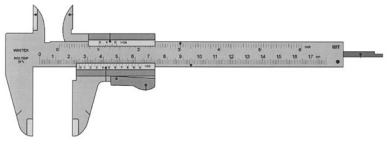
In Grade 10 Essential Math, you studied imperial and metric systems of measurement. You explored linear measurement and two-dimensional (2-D) geometry. In this module, you move on to three-dimensional (3-D) geometry, where you will study prisms, pyramids, spheres, cylinders, and cones. You will learn about surface area, volume, and capacity, and make comparisons among the three ways of measuring 3-D objects. Part of these comparisons will be finding patterns to help you improve your understanding of 3-D geometry.

For this module, you will need a ruler with both metric and imperial units (centimetres and inches). You will find a paper template for a ruler on page 5 of this module. You may want to cut this out to use it in the course. Having access to a tape measure, a caliper, and a micrometer would be beneficial to your understanding of 3-D geometry, but these items are not necessary to complete this module.

tape measure



micrometer



caliper

Geometry and measurement are a part of everyday life. For these reasons, this is an important and interesting topic. Good luck.

Assignments in Module 2

When you have completed the assignments for Module 2, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Geometric Patterns
2	Assignment 2.1	Surface Area
4	Assignment 2.2	Volume
6	Assignment 2.3	Applications of Volume and Capacity

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

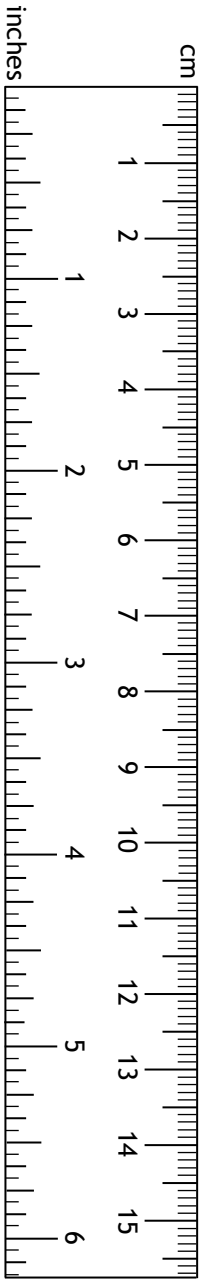
You may use the following list of instructions to help you with preparing your resource sheet for the material in Module 2. On this sheet, you should record mathematics terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1, 2, and 3 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 3.

Resource Sheet for Module 2

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

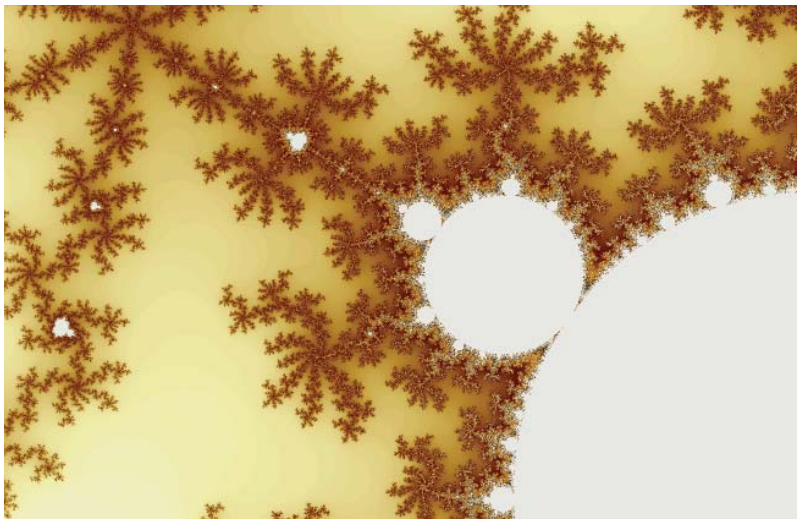
1. List all the important mathematics terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how each formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.



Note: Due to photocopying, this ruler may not be accurate. It can be used as a good estimate but it would be better if you used an actual ruler for measurement.

Module 2 Cover Assignment: Geometric Patterns

Geometric shapes are used for various purposes, not all of which are math-related. We see geometric shapes in paintings, stained glass windows, and floor tile patterns. One type of geometric shape that has emerged with the onset of computer technology is the fractal. A fractal is a shape that repeats a pattern of itself within itself, with each shape getting smaller and smaller.

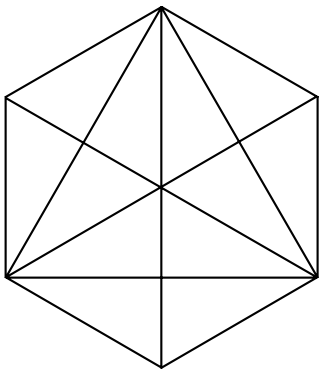


You can find some interesting fractal-generating websites by running a google search on the words "fractal generator."

You will be studying more traditional shapes such as triangles, rectangles, and circles. As is the case with fractals, there can be many traditional shapes within a shape.

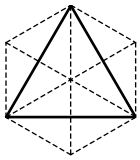
Example 1

How many triangles are in the following image?

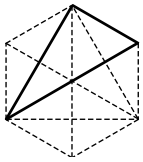


Solution

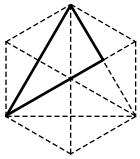
There is one triangle that is this size and shape.



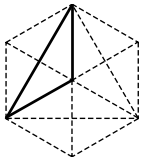
There are six triangles that are this size and shape.



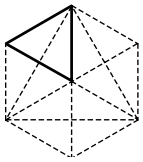
There are six triangles that are this size and shape.



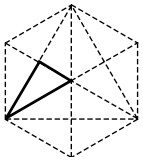
There are six triangles that are this size and shape.



There are six triangles that are this size and shape.



There are 12 triangles that are this size and shape.



The total number of triangles: 1 + 6 + 6 + 6 + 6 + 12 = 37 triangles.

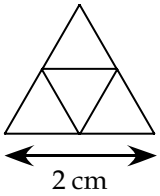
Module 2 Cover Assignment

Geometric Patterns

Total: 24 marks

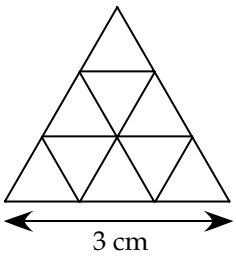
1. How many triangles are in each of the following diagrams? *(12 marks)*

a)



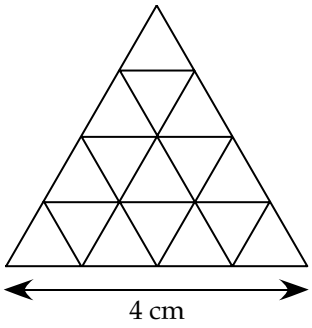
small: _____
medium: _____
Total: _____

b)



small: _____
medium: _____
large: _____
Total: _____

c)



small: _____
medium: _____
large: _____
extra large: _____
Total: _____

(continued)

Module 2 Cover Assignment: Geometric Patterns (continued)

2. a) Draw the next diagram in the pattern. *(2 marks)*

b) How many triangles are in this diagram? *(6 marks)*

small: _____

medium: _____

large: _____

extra large: _____

extra-extra large: _____

Total: _____

3. Explain two patterns that you notice. Focus on the number of triangles of each size as they compare to the other diagrams (e.g., the largest triangle in each diagram…). *(4 marks)*

Lesson 1: Surface Area

Lesson Focus

In this lesson, you will

- ❑ observe how area and surface area are related
- ❑ review the characteristics of prisms, pyramids, and spheres
- ❑ use nets, the faces approach, and formulas to calculate surface area of 3-D objects
- ❑ calculate the surface area of composite 3-D objects

Lesson Introduction

In previous math courses, you calculated perimeters (the distance around a closed 2-D figure) and areas (the surface covered up by 2-D figures) of two-dimensional (2-D) objects. This lesson will build on this knowledge by adding another dimension: you will calculate surface areas of three-dimensional (3-D) objects. Calculating surface area can be a valuable skill that you may use frequently. For example, you may need to calculate the surface area of a room before painting the walls and ceiling, so that you can buy the correct amount of paint.

Transition from Two Dimensions to Three Dimensions

Review of Grade 10 Geometry

Before you study surface area, you need to review what you have learned previously about perimeter and area. This is a quick review.

Perimeter

To find the perimeter of a 2-D geometric figure, add the lengths of all its sides. The circumference is the perimeter of a circle. The circumference of a circle is about three times its diameter, and the diameter is the distance across a circle measured through its centre. The formula for the circumference of a circle is:

$C = \pi d$

Since the diameter is twice as long as the radius, this formula can be written:

$C = 2\pi r$

The value of π (pronounced pi) is an irrational number (a never-ending and never-repeating decimal). The first 100 digits of π are:

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706 79

For this course, we will often consider it as approximately 3.14.

No matter what the size of the circle,

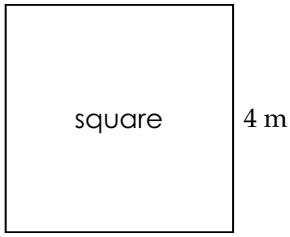
$C \div d = \pi$

You may want to include this information on your resource sheet as a reminder.

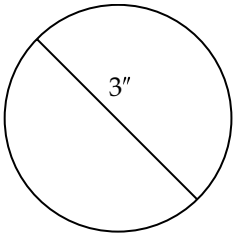
Example 1

Find the perimeter of each of the following figures:

a)



b)



Solutions

a) A square has four equal sides. You find the perimeter by adding all the sides together.

Perimeter = 4 + 4 + 4 + 4 = 16 m

b) The perimeter of a circle is the circumference.

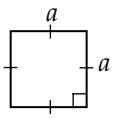
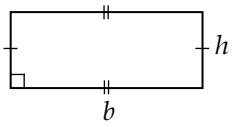
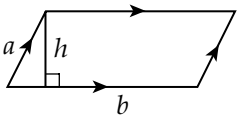
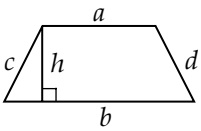
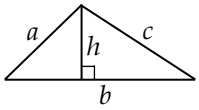
$C = (3)\pi = 9.42$ inches.

Remember that you may enter this into your calculator using the π key (you may have to press the second function button first).

Area

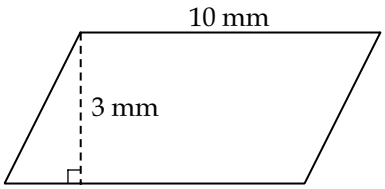
The word *area*, when used in geometry, refers to the measure of the surface covered by a region. The area of a geometric figure is the number of square units needed to cover the interior region of a 2-D geometric figure.

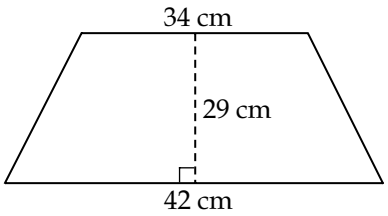
The following table includes the area formulas for different shapes. Include this on your resource sheet for this module so that you do not have to flip back and forth while completing each lesson.

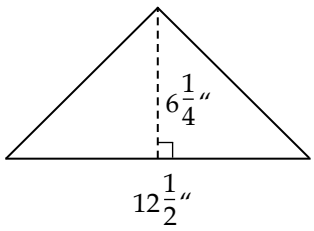
Area Formulas		
Figure	**Diagram**	**Area (in square units)**
Square		a^2
Rectangle		bh
Parallelogram		bh
Trapezoid		$\frac{1}{2}(a+b)h$
Triangle		$\frac{1}{2}bh$
Circle		πr^2

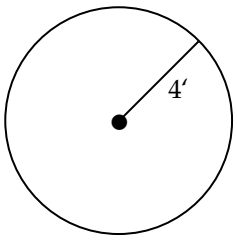
Example 2

Find the area of each of the following figures:

a) 

b) 

c) 

d) 

Solutions

a) $A = bh = (10)(3) = 30 \text{ mm}^2$

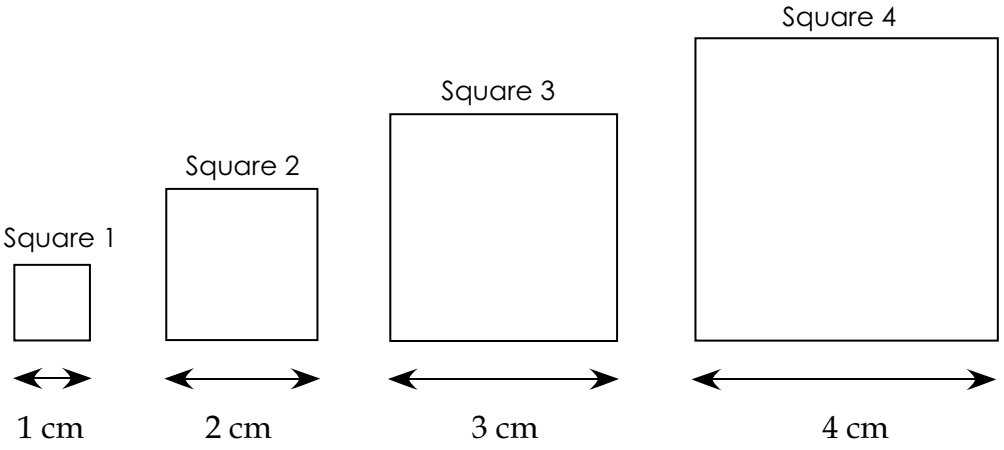
b) $A = \frac{1}{2}(a + b)h = \frac{1}{2}(34 + 42)(29) = 1102 \text{ cm}^2$

c) $A = \frac{1}{2}bh = \frac{1}{2}(12.5)(6.25) = 39.1 \text{ inches}^2$

d) $A = \pi r^2 = \pi(4)^2 = 50.3 \text{ feet}^2$
(The answer using 3.14 instead of the π button is 50.2 ft.^{2})

Comparing Areas

As you saw in the example above, calculating the area of different shapes requires different formulas. However, what is the relationship between the areas of two figures that have the *same shape* but *different sizes*?



Area of Square 1 = (1)(1) = 1 cm^2

Area of Square 2 = (2)(2) = 4 cm^2

Area of Square 3 = (3)(3) = 9 cm^2

Area of Square 4 = (4)(4) = 16 cm^2

Before you read on, jot down some patterns that you notice about the areas of these squares.

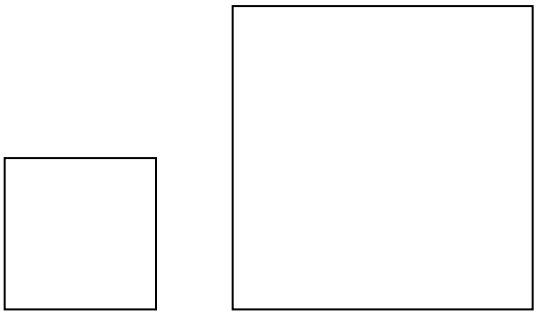
Three things that you may have noticed are:

1. each time you increase the side length by 1, you increase the area by consecutive odd numbers: 1, 1 + 3 = 4, 4 + 5 = 9, 9 + 7 = 16
2. the numbers go odd, even, odd, even
3. the areas are square numbers: $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$

Did you notice anything else?

In general, we can say that when we double the dimensions of the sides of a square, the area of the square is increased two squared or four times. When the side lengths of a square are tripled, the area is increased three squared or nine times. When the side lengths of a square are increased by a factor of four, its area is increased four squared or 16 times.

What is interesting is that this statement is true for all squares!



Square A
3 feet

Square B
6 feet

The side lengths of square B are double the side lengths of square A.

Area of square A = (3)(3) = 9 ft.^{2}

Area of square B = (6)(6) = 36 ft.^{2}

The area of square B is four times the area of square A.

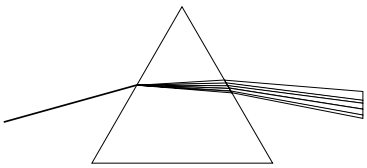
The pattern for finding the areas of similar shapes is the same as the pattern for squares shown above. For example, when the dimensions are twice as large, the area is four times as large.

3-D Shapes

Now that you have reviewed the 2-D geometry from Grade 10, it is time to look at 3-D objects, but you must first recognize and know the names of 3-D objects. You may want to include a sketch of each type of object on your resource sheet.

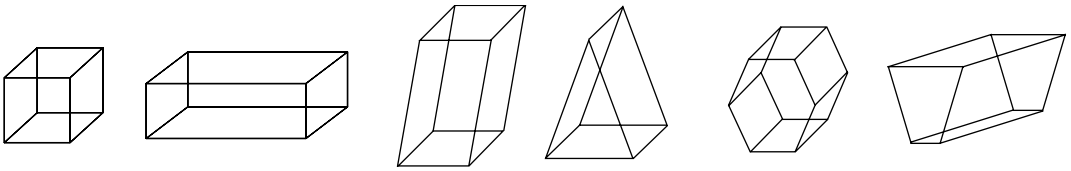
Prisms

You may already be familiar with triangular prisms from science.



A prism is a three-dimensional figure (solid) that has two sides that are the same shape and size. These two sides are opposite each other and parallel to each other, and are the bases of the prism. The other faces are rectangles (or other parallelograms) that connect the two bases.

Examples

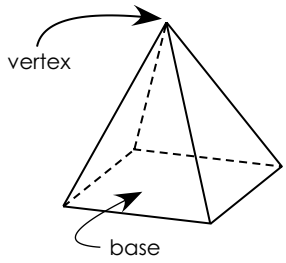


This module will focus on right prisms where the bases are perpendicular to the other sides, and the other sides (called faces) are all rectangles. It is important to understand that the base is not necessarily the bottom of the prism—it's the shape that is consistent throughout the entire object. In the rectangular prisms pictured above, any pair of parallel sides can act as the bases, but with the three objects on the right, the base is the surface at the front and back: a triangle, hexagon, and trapezoid respectively. The other faces of the three objects are all rectangles. With this in mind, we can say that the shape of the base determines the name of a prism. The faces that are not the bases are called **lateral** faces.

Pyramids

In a prism, the shape of the base remains consistent throughout the entire height of the object. A pyramid is an object in which the shape of the base reduces to a single point throughout the height of the object. The base of a pyramid is a polygon and its lateral faces are triangles that share a common vertex. Similar to prisms, the shape of the base determines the name of the pyramid.

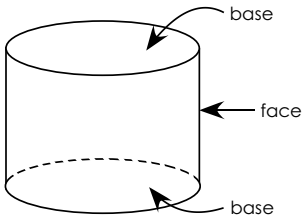
Example



Cylinders

A cylinder is a prism, but because it has a round base, it only has one rectangular face that wraps around the two circular ends to connect them.

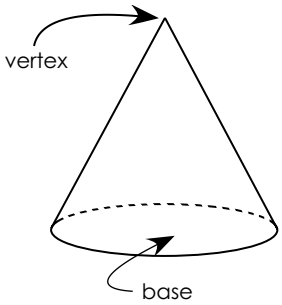
Example:



Cones

Similar to a cylinder, a cone is a form of a pyramid with a round base. As a result of the round base, instead of having *multiple* triangular lateral faces (like a pyramid), a cone has one lateral face that wraps around the circular base and comes to a point (or vertex).

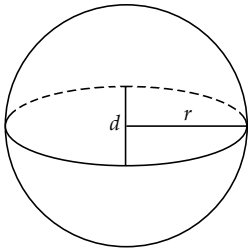
Example



Spheres

A sphere is a 3-D ball-shaped object in which all points are equidistant (the same distance) from the centre.

Example

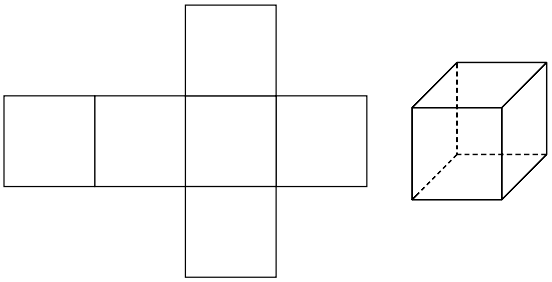


Measuring Surface Area

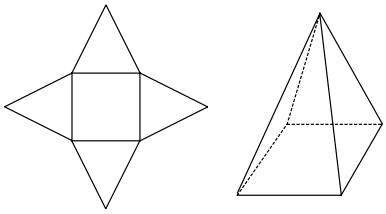
You already know what area is, but what is surface area? **Surface area** is the area of the entire surface of a 3-D object. To calculate the *surface area* of a 3-D object, you must find the *total area* of *all* the faces of the object. This means that the units for surface area are the same as the units for area (units^2, such as m^2 or in.^{2}). You will learn three different ways to calculate surface area throughout this lesson.

Method 1: Finding Surface Area Using Nets

One way to demonstrate how area and surface area are related is to use nets. A net is a flat (2-D) diagram of all the faces of a 3-D object. We will only use nets for describing objects with straight sides, which does not include cylinders, cones, or spheres.



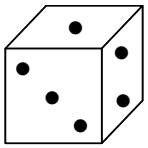
Net Object



Net Object

To draw a net, you must imagine that you are unfolding the object. Each face of the object, once it is unfolded, must still be touching *at least* one of the faces with which it shared an edge in the 3-D version. The order of faces that form a straight line in a net must be the same as their 3-D order.

For example, imagine that you are looking at a die (singular form of dice). You notice that the top is a 1, the front is a 3, the bottom is a 6, and the back is a 4.



If you make a net so that the top, front, bottom, and back are all in a straight line, they must be in one of the following orders:

(i) | 1 | 3 | 6 | 4 | (ii) | 3 | 6 | 4 | 1 | (iii) | 6 | 4 | 1 | 3 | (iv) | 4 | 1 | 3 | 6 |

or if you turned the net over

(v) | 4 | 6 | 3 | 1 | (vi) | 1 | 4 | 6 | 3 | (vii) | 3 | 1 | 4 | 6 | (viii) | 6 | 3 | 1 | 4 |

The 1 is *always* between the 4 and 3, the 3 is *always* between the 1 and 6, the 4 is *always* between the 6 and 1, and the 6 is *always* between the 3 and 4.

We will use nets to find the surface area of rectangular and square prisms. Before you learn how to do this, complete the following learning activity.

Learning Activity 2.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If 12% of 250 is 30, what is 12% of 500?
2. There are 52 cards in a deck, 13 cards of each suit. If the suits are hearts (red), spades (black), clubs (black), and diamonds (red), what percentage of the cards are black?
3. If $x = 2$, evaluate $4x - 18$.
4. You earn $9 per hour plus 5% commission. If you work 20 hours this week and your sales are $700, how much will your gross pay be?
5. There are 5 people assigned to a group. Sheniqua will not sit beside Arjun and Tapanga. Tapanga refuses to sit beside Soloman. Dejon wants to sit beside Tapanga. If they are sitting in a circle, who will be beside Sheniqua?

continued

Part B: What Is a Net?

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand. Don't forget that you can ask your learning partner or tutor/marker for help if you are having a hard time understanding.

The following learning activity will help you understand the meaning of *net*. It requires materials that you can find in your house or classroom. Ask your parent or ISO facilitator where you can find them. If you do not have access to the materials that you need to complete one of the two exercises, do not worry—but be sure to complete at least one of them.

Materials:

Activity 1:

- a 6-sided die (alternatives: sugar cube, child's alphabet block, etc.)
- a piece of paper
- a writing instrument

Activity 2:

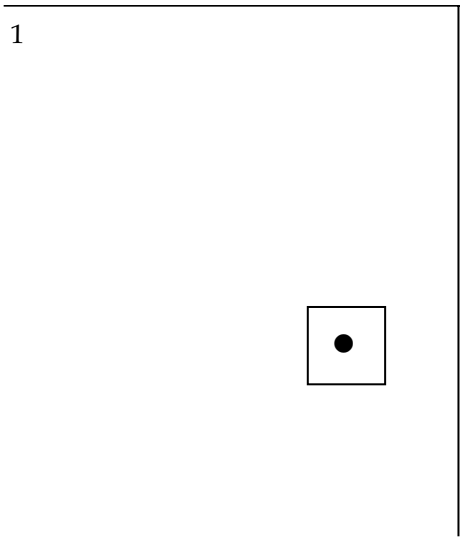
- a 6-sided box you can cut apart
- scissors
- a thick marker
- a piece of paper
- a writing instrument

1. To complete this question, you will need a 6-sided die and a piece of paper. You will draw a net of the die by following the step-by-step instructions. The images that accompany each step are examples of what you should see on your page. (**Note:** You may start with any number facing up. The example below starts with the number 1 facing up.)

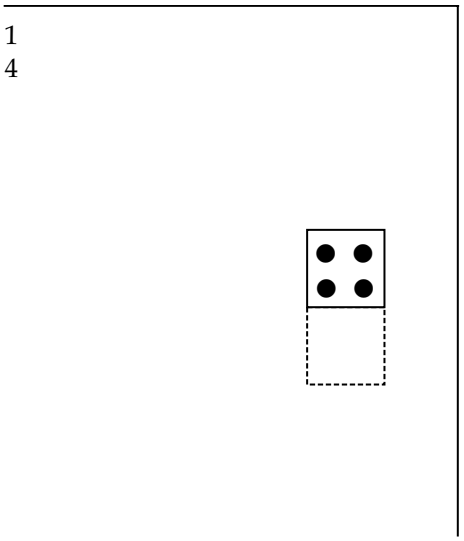
continued

Learning Activity 2.1 (continued)

A. Place the die on the piece of paper about 1 cm from the right edge and about 5 cm below the top edge. On the left side of the page, write down the number that is facing up on the die (based on the diagram below, this number would be 1). Holding the die in place, trace the shape of the die.



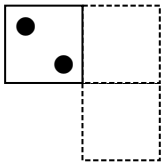
B. Turn the die over once, toward the top of the page. The bottom edge of the die in the new position should be against the top line of the die you traced in step A. Write down the new number that is facing up on the left side of the page, under the last number that was facing up (based on the diagram below, you would write 4 but you may be seeing a different number). Holding the die in place, trace the shape of the die. The dotted lines in the following diagram represent the lines that you traced in the previous step.



continued

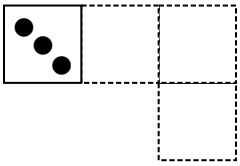
C. Turn the die over once to the left. The right edge of the die in the new position should be against the left line you traced in step B. Write down the new number that is facing up on the left side of the page, under the last number that was facing up. Holding the die in place, trace the shape of the die.

1
4
2



D. Turn the die over once to the left. The right edge of the die in the new position should be against the left line you traced in step C. Write down the new number that is facing up on the left side of the page. Hold the die in place as you trace its shape.

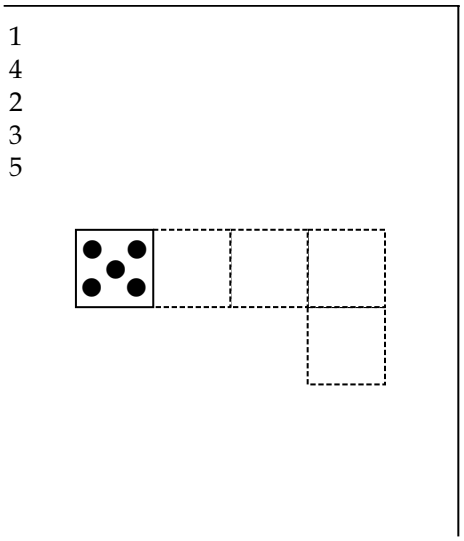
1
4
2
3



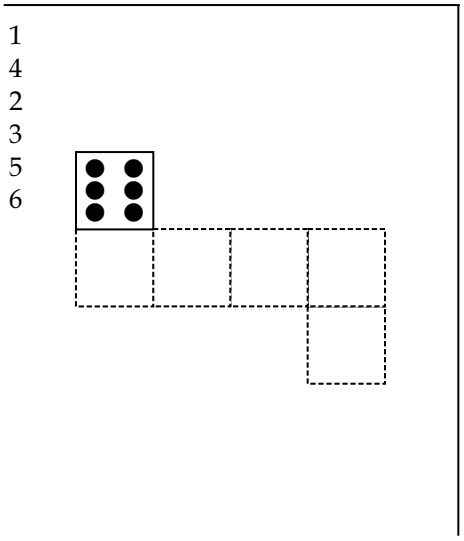
continued

Learning Activity 2.1 (continued)

E. Turn the die over once to the left. The right edge of the die in the new position should be against the left line you traced in step D. Write down the new number that is facing up on the left side of the page. Hold the die in place as you trace its shape.



F. Turn the die over once, toward the top of the page. The bottom edge of the die in the new position should be against the top line you traced in step E. Write down the new number that is facing up on the left side of the page, under the last number that was facing up. Holding the die in place, trace the shape of the die.



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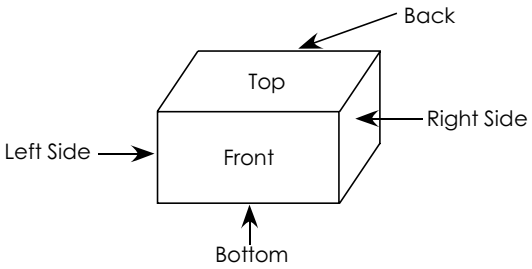
Before you check your answer, be sure that each number (1 to 6) only appears once in the list you made on the left side of the page. If you are missing a number and/or have a repeated number, this means that you traced the same face twice, so your net will have two pieces that overlap if you fold it into its shape. Restart the exercise and be sure you follow the instructions precisely. If you have done the steps correctly, the drawing on your paper is a net diagram of the die.

2. Find a six-sided box around your house that you will be able to cut apart, such as an empty cereal box.
   A. With a marker, trace the edges of each face. Label each face (front, back, top, bottom, left side, right side).



   B. Do your best to draw the net of this box on a piece of paper without cutting the box apart. The drawing does not need to be the actual size. You can reduce the heights of the sides. The drawing does not need to be to scale either. Label the sections according to the faces you think they represent.
   C. Cut the box along the edges and be sure to keep each face connected to at least one other face. The box should still be in one piece. You should be able to lay the cut-up box flat on a table or the floor.
   D. Lay the box down with the marker side facing up. Sketch what you see on a piece of paper (this can be done on the same piece of paper on which you did Step B). Include the labels.

Do your diagram and the actual cut-out look the same? If so, good work! If not, is it possible that the net you drew is also correct?

How to Draw a Net

There are many ways to draw a net. The two nets drawn in Learning Activity 2.1 are possibly different in appearance. The general steps to draw a net for a rectangular prism are as follows:

Step 1: Choose a face. Draw this face on grid paper.

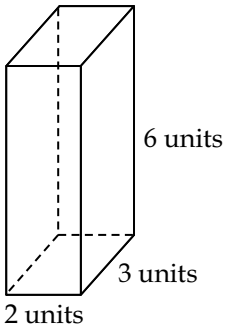
Step 2: Choose a face that shares an edge with the face you drew in Step 1. Draw this face so that it shares a side with the face from Step 1.

Step 3: Continue drawing faces that share an edge with one that you have already drawn. Be sure that you do not draw the same face twice.

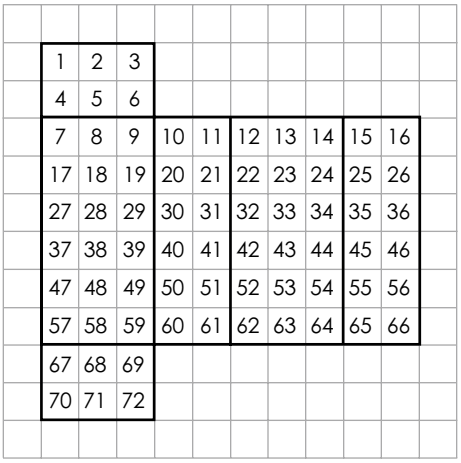
This is a very broad description of how to draw a net. As you work through the examples below, try starting with different faces or extending in different directions so that you can determine which way you prefer.

Now that you are able to draw a net, ask yourself, "How does this relate area to surface area?" If you are trying to find the surface area of a rectangular or square prism, you can draw the net of the object on grid paper. This will give you a visual representation of the area of the entire surface of the object, based on the number of grid squares inside the net.

Try to draw the net for this prism on another piece of paper before you continue.

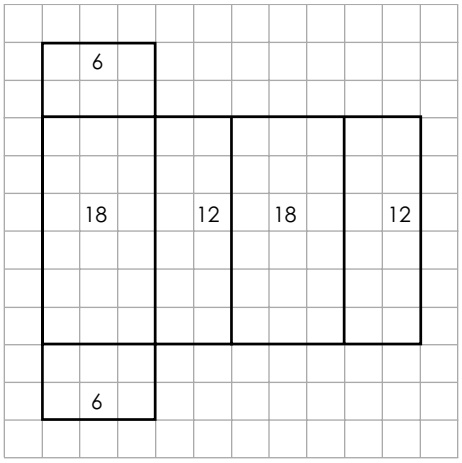


Count each square individually.



Total: 72 units^2

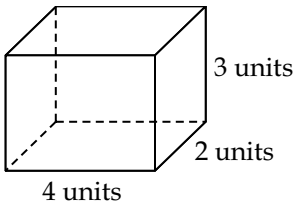
Alternatively, you can find the area of each segment of the net, and then add them together to find the total area.



6 + 18 + 6 + 12 + 18 + 12 = 72 units^2

Example 1

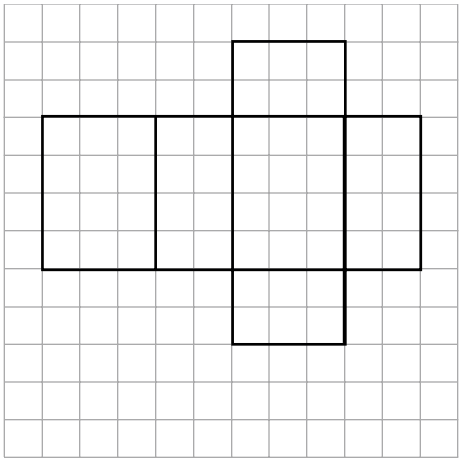
Find the surface area of the rectangular prism using a net (use graph or grid paper).



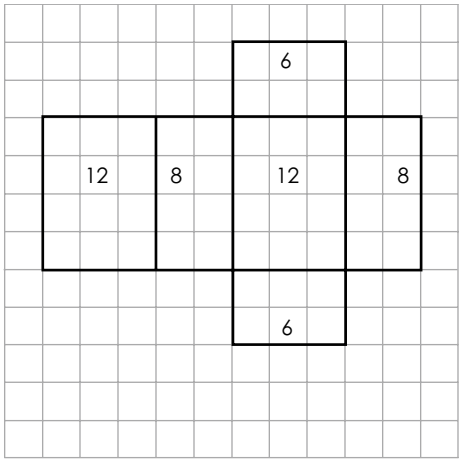
Grid paper is located at the end of this lesson. You can also use graph paper.

Solution

Your net for this prism may look like this.



If you count the squares in each segment of the net, and then add them, you get a surface area of 12 + 8 + 12 + 8 + 6 + 6 = 52 units^2.



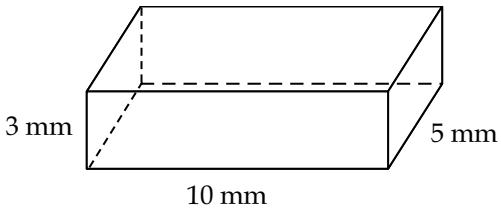
Method 2: Finding Surface Area Using the Faces of the Object

Another way to find the *surface area* of an object is to calculate the *area* of each face of the object, and then add all these areas together. We will call this method the **faces approach**.

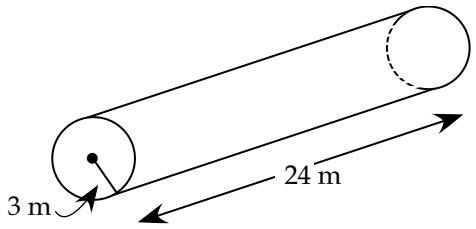
Example 2

For the following, name the type of 3-D object, and find the surface area using the faces approach. Draw the net for (a).

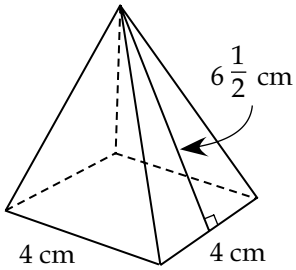
a)



b)

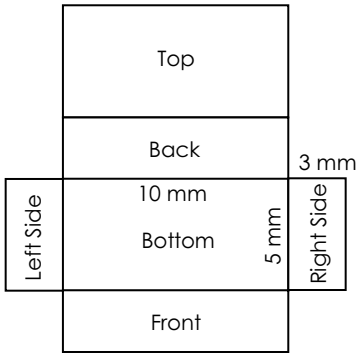


c)



Solutions

a) This shape is a rectangular prism.



Area of the top and bottom of the prism = lw

Area of the front and back of the prism = lh

Area of the two sides of the prism = wh

Surface area = $2lw + 2lh + 2wh$

$= 2(10)(5) + 2(10)(3) + 2(5)(3)$

$= 100 + 60 + 30 = 190 \text{ mm}^2$

b) This shape is a cylinder.

Area of the top and bottom (circles) = πr^2

Area of the side is a rectangle that has been curved into the shape of a tube. The length of the rectangle is the circumference of the circle.

Area of the side = $Ch = (2\pi r)h$ (Note: C is circumference.)

Surface area = 2(area of circle) + area of rectangle

Surface area = $2(\pi r^2) + (2\pi r)h$

$= 2\pi(3)^2 + 2\pi(3)(24)$

$= 18\pi + 144\pi$

$= 56.55 + 452.39$

$= 508.94 \text{ m}^2$

c) This shape is a square pyramid.

Area of the base (square) = lw

Area of the faces along the length of the square base = $\frac{1}{2}lh$

Surface area = area of square + 4 (area of triangle)

Surface area $= lw + 4\left(\frac{1}{2}lh\right)$

$= (4)(4) + 4\left[\frac{1}{2}(4)(6.5)\right]$

$= 16 + 4(13) = 68 \text{ cm}^2$

In this example, we found the area of each of the faces of the object and then added the areas. This method most clearly relates area and surface area.

Method 3: Finding Surface Area Using Formulas

The third way to find the surface area of a 3-D object is to use one of the general formulas for prisms, pyramids, and spheres. These formulas show a pattern that was found when solving for surface areas of specific objects.

When you find the total surface area of a **prism**, the general formula including both bases is:

$SA = 2B + Ph$, where

B = is the area of the base

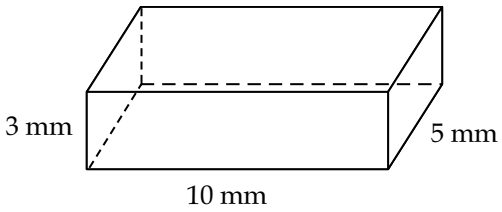
P = perimeter of the base (which is why circumference is used in Example 2(b))

h = height of the prism

You should include this formula on your resource sheet.

Example 3

Use the surface area formula for a prism to calculate the surface area of the rectangular prism from Example 2(a).



Solution

Because it is a rectangular prism, you can choose any two opposite sides to be the bases. For this solution, we will choose the top and bottom of the prism as the bases.

$B = lw = (10)(5) = 50 \text{ mm}^2$

$P = 2l + 2w = 2(10) + 2(5) = 30 \text{ mm}$

$SA = 2B + Ph$

$=2(50) + (30)(3)$

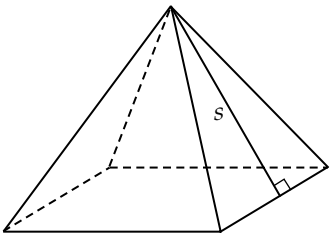
$= 100 + 90 = 190 \text{ mm}^2$

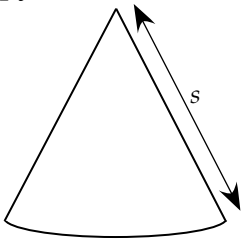
As expected, this answer is the same as the answer we found in Example 2(a).

Just as there is a general formula for the surface area of prisms, there is a general formula for the surface area of **square pyramids** and **cones**.

$$SA = B + \frac{1}{2}Ps, \text{ where}$$

B = the area of the base
P = the perimeter of the base
s = the slanted height of the pyramid

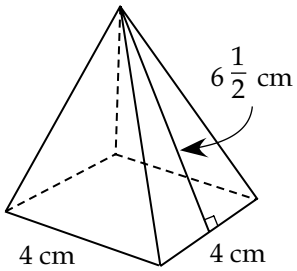




You should include this formula on your resource sheet.

Example 4

Use the general formula for the surface area of a square pyramid to find the surface area of the pyramid in Example 2(c).



Solution

$B = lw$
$= (4)(4) = 16 \text{ cm}^2$
$P = 2l + 2w$
$= 2(4) + 2(4)$
$= 8 + 8 = 16 \text{ cm}$

$$SA = B + \frac{1}{2}Ps$$
$$= 16 + \frac{1}{2}(16)(6.5)$$
$$= 16 + 52 = 68 \text{ cm}^2$$

As expected, this answer is the same as the answer we found in Example 2(c).

The third general formula that you should include on your resource sheet is the formula for calculating the surface area of a **sphere**.

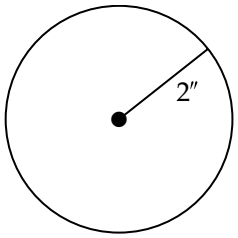
$SA = 4\pi r^2$

This formula is quite different when compared to the formulas for the surface areas of prisms and pyramids, but it resembles the formula for the area of a circle, which is the following: $A = \pi r^2$. The surface area of a sphere is actually four times the area of a circle with the same radius!

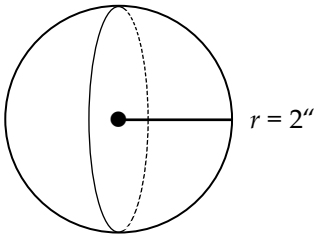
Example 5

Calculate the area/surface area of the following objects.

a)



b)



Solution

a) $A = \pi r^2$
$= \pi(2)^2$
$= 4\pi = 12.6 \text{ inches}^2$

b) $SA = 4\pi r^2$
$= 4\pi(2)^2$
$= 12.6 \times 4$
$= 4\pi(4) = 50.3 \text{ inches}^2$

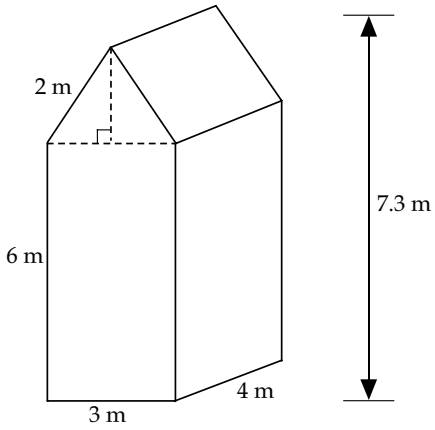
Composite Objects

Now that you can calculate the surface area of basic 3-D objects, you can apply these methods to composite objects. A **composite object** is an object made up of more than one 3-D shape. The following examples will demonstrate how to solve a problem involving a composite object.

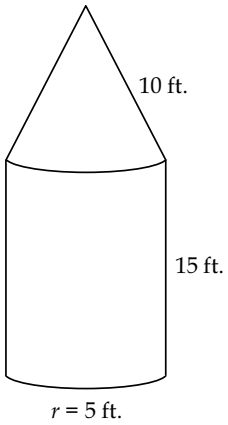
Example 1

Calculate the surface area of the following objects.

a)



b)



Solution

a) This object is made up of a rectangular prism and a triangular prism. Since they are not part of the outside surface, the top face of the rectangular prism and the bottom face of the triangular prism are not included in the calculation.

Rectangular Prism

The top and bottom of the prism are the base faces. Therefore, we only include the bottom face in the calculation.

$SA = B + Ph$

$B = 3 \times 4 = 12 \text{ m}^2$

$P = 3 + 3 + 4 + 4 = 14 \text{ m}$

$SA = 12 + (14 \times 6) = 96 \text{ m}^2$

Triangular Prism

$h_t = 7.3 - 6 = 1.3 \text{ m}$

each base: $\frac{1}{2}bh_t = \frac{1}{2}(3 \times 1.3) = 1.95 \text{ m}^2$

each face: $2 \times 4 = 8 \text{ m}^2$

$SA = 2(1.95) + 2(8)$

$SA = 3.9 + 16 = 19.9 \text{ m}^2$

The total surface area is $96 + 19.9 = 115.9 \text{ m}^2$.

b) This object is made up of a cone and cylinder. The base of the cone and one of the bases of the cylinder are not included in the surface area, as they are not part of the outside surface.

Cylinder

One base is not included, so the formula for the surface area is

$SA = B + Ph$

$B = \pi r^2 = \pi(5)^2 = \pi(25) = 78.54 \text{ ft.}^2$

$P = 2\pi r = 2\pi(5) = (10)\pi = 31.42 \text{ ft.}$

$SA = 78.54 + (31.42 \times 15)$

$= 78.54 + 471.24 = 549.8 \text{ ft.}^2$

Cone

The base is not included, so the formula for the surface area is

$$SA = \frac{1}{2}Ps$$

$$P = 2\pi r = 2\pi(5) = (10)\pi = 31.42 \text{ ft.}$$

$$SA = \frac{1}{2}(31.42)(10) = 157.1 \text{ ft.}^2$$

The total surface area of the object is $549.8 + 157.1 = 706.9 \text{ ft.}^2$.

Alternately, 3 ft. = 1 yd. so $1 \text{ yd.}^2 = (3 \text{ ft.})^2 = 9 \text{ ft.}^2$

So the total surface area is also $706.9 \div 9 = 78.5 \text{ yd.}^2$.

Now that you have learned how to find the surface area of a 3-D object using three different methods, it is time to try them out. The following learning activity has an exercise that requires an orange, some paper, and a compass for further exploration of surface area. Also, you will have a chance to practise the different methods of calculating surface area.

Learning Activity 2.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Travis invests $1500 for one year. The investment has an interest rate of 5%. How much will his investment be worth after one year?
2. Evaluate: $\frac{4}{8} \times 14$
3. Liu and Goh went out for dinner and the total cost of the meal was $28.50, including taxes. Goh pays by leaving $30 on the table to include the tip. How much is the tip?
4. Fran has an accuracy rating of 60% in volleyball. Fran's accuracy in basketball is 20% higher than volleyball. In badminton, Fran's accuracy rating is half of his basketball rating. What is Fran's accuracy in badminton?
5. Write two fractions that are equivalent to 0.20.

Part B: Surface Area Calculations

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. This question is another hands-on exercise. You will need an orange (navel oranges work best, but other kinds, as well as other fruit, may also work well—you want something that is spherical and that you can peel easily), two pieces of paper, a ruler, and a compass. If you do not have access to an orange, use some other spherical object from which you can easily remove the top layer.
   A. Use your ruler to measure the diameter of the orange in three different directions (e.g., top to bottom, side to side, and diagonally). Record each of these measurements on the corner of your paper.

continued

B. Find the average of these three measurements. This is the approximate diameter of the orange. Calculate the radius of the orange.

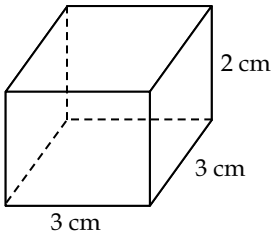
C. Adjust your compass to have the same radius as the orange in Part B. Use your ruler to measure the distance between the arms of your compass. Draw six circles with this radius, three on each page (do this on separate sheets of paper, as you will not be able to turn them over).

D. Peel your orange in small pieces, approximately 1 cm^2 or smaller. Fit the pieces into the circles that you have drawn on your paper, so you cannot see any paper inside the circle without making the pieces overlap. Try to fit your pieces of orange into the fewest number of circles possible.

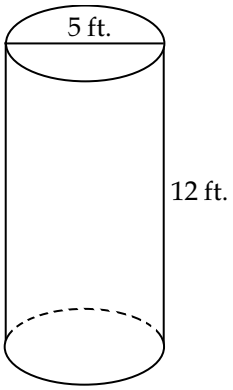
How many circles did you fill? What does this tell you about the surface area of a sphere? Enjoy your orange while you do the rest of this learning activity.

2. Name and calculate the surface area of the following objects. Use the method of calculation stated beside the object.

a) Find the surface area using a net.



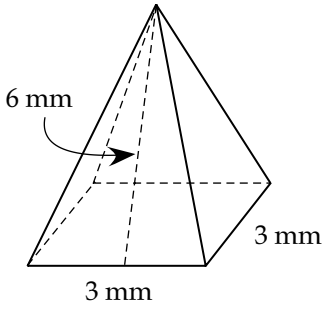
b) Find the surface area using the formula.



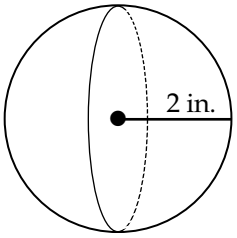
continued

Learning Activity 2.2 (continued)

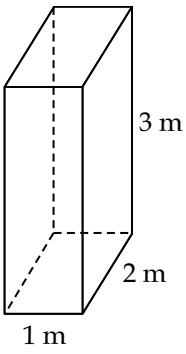
c) Find the surface area using the faces approach. The base is a square.



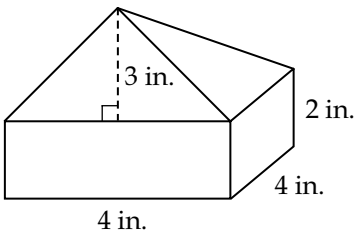
d) Find the surface area using the formula.



e) Find the surface area using a net.



f) Find the surface area using the formulas OR the faces approach.



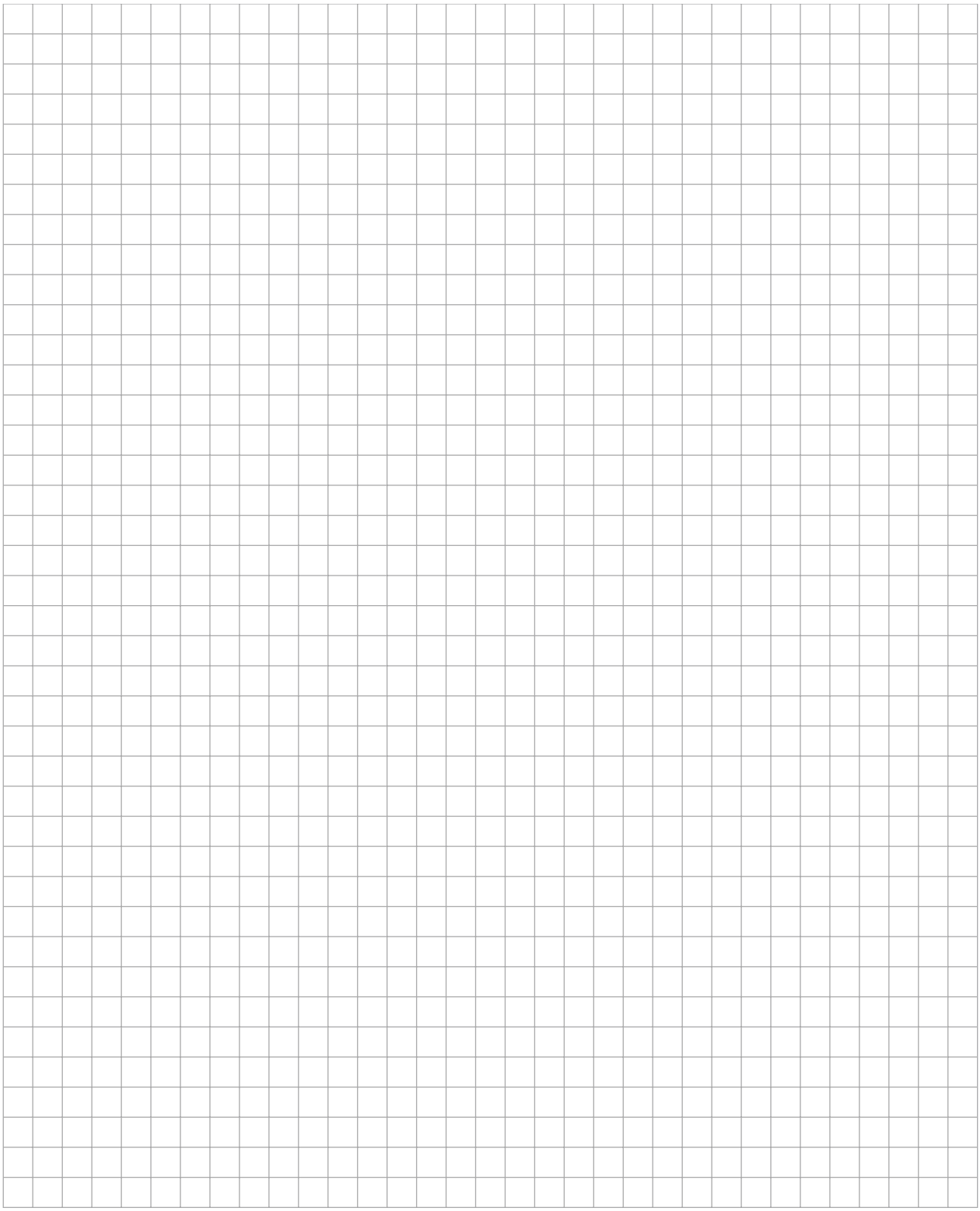
3. Briefly describe how you draw the net of a prism. Include where you start and what your steps are.

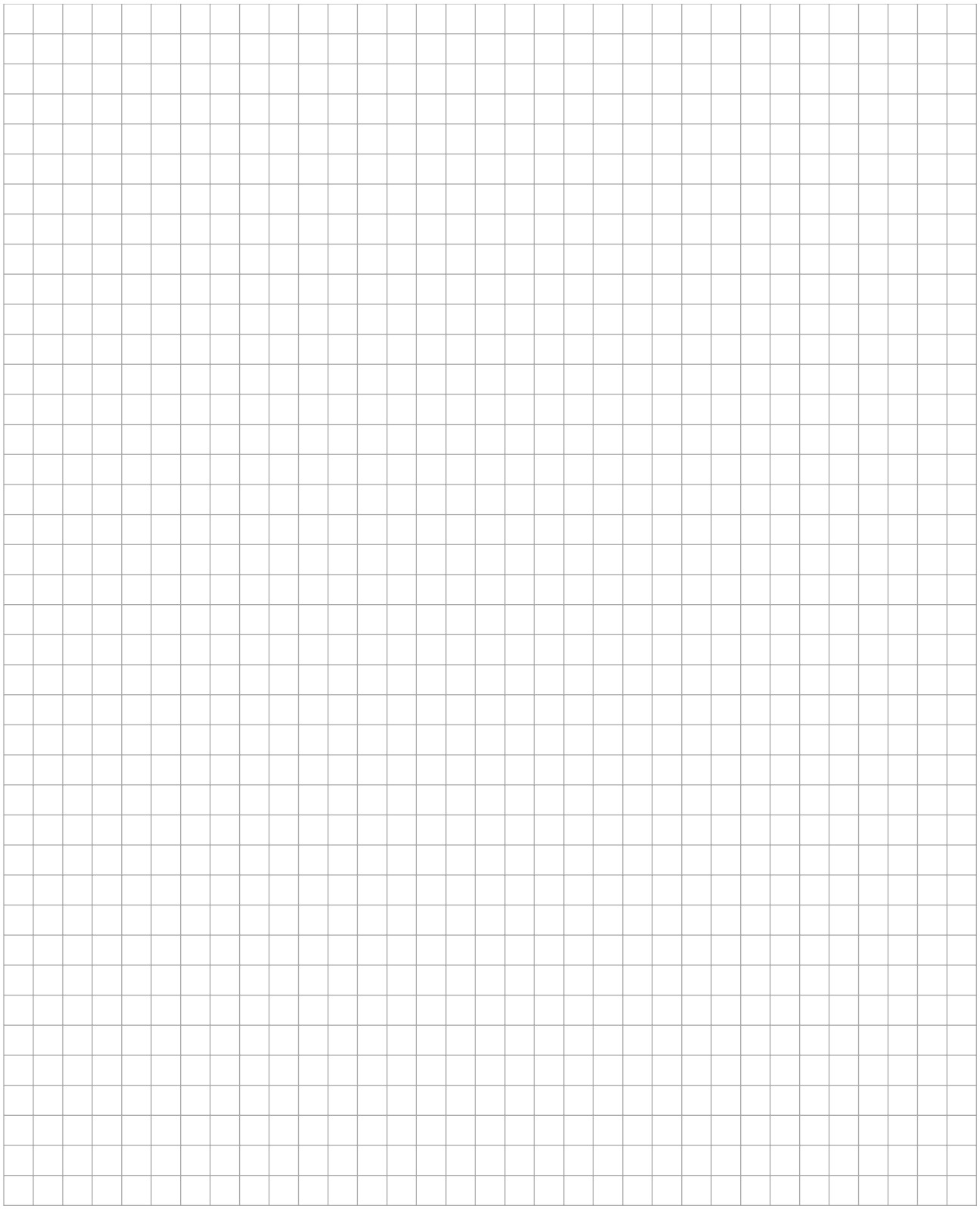
Lesson Summary

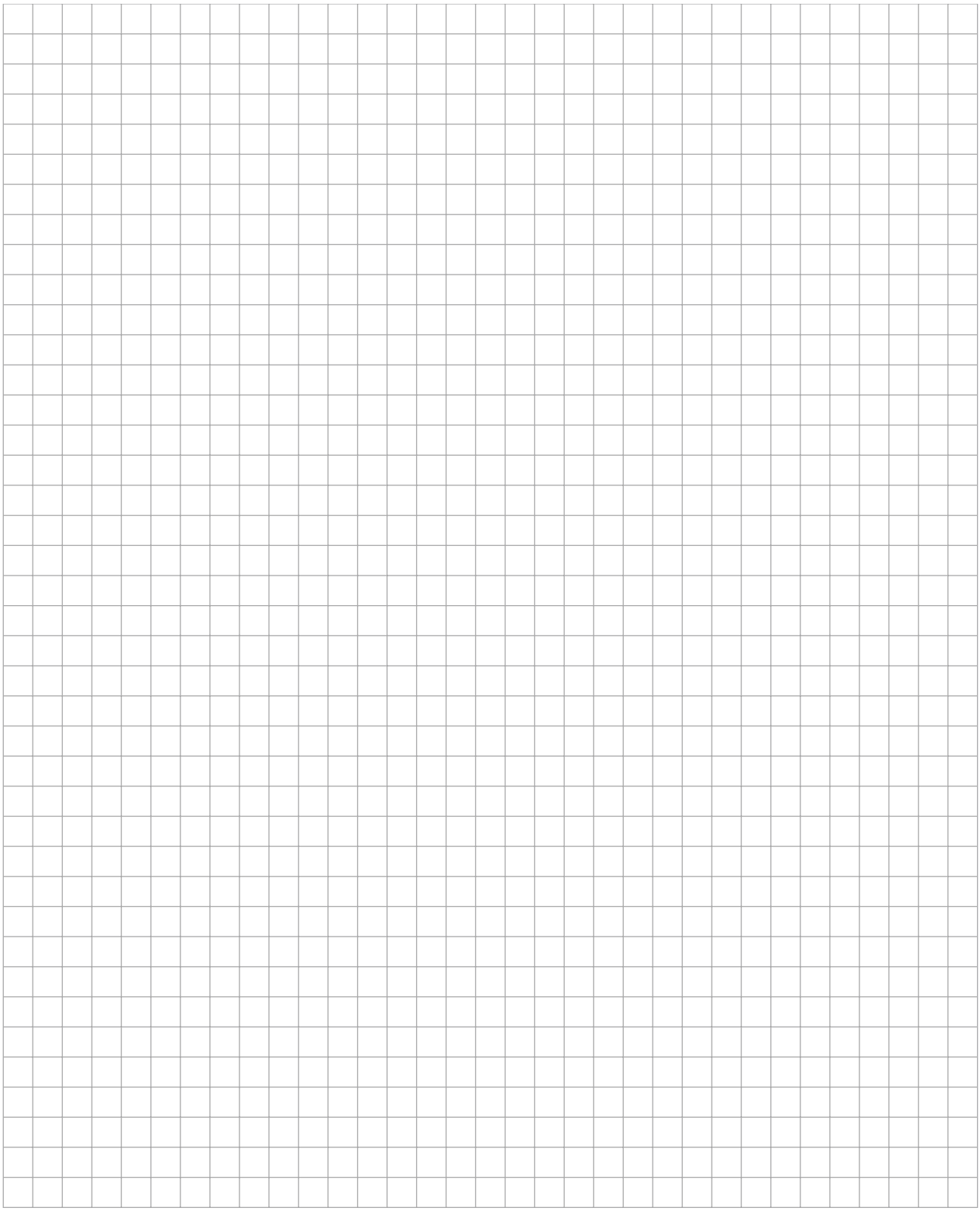
In this lesson, you calculated surface area of a variety of 3-D objects, including composite objects, using three different methods. The methods were

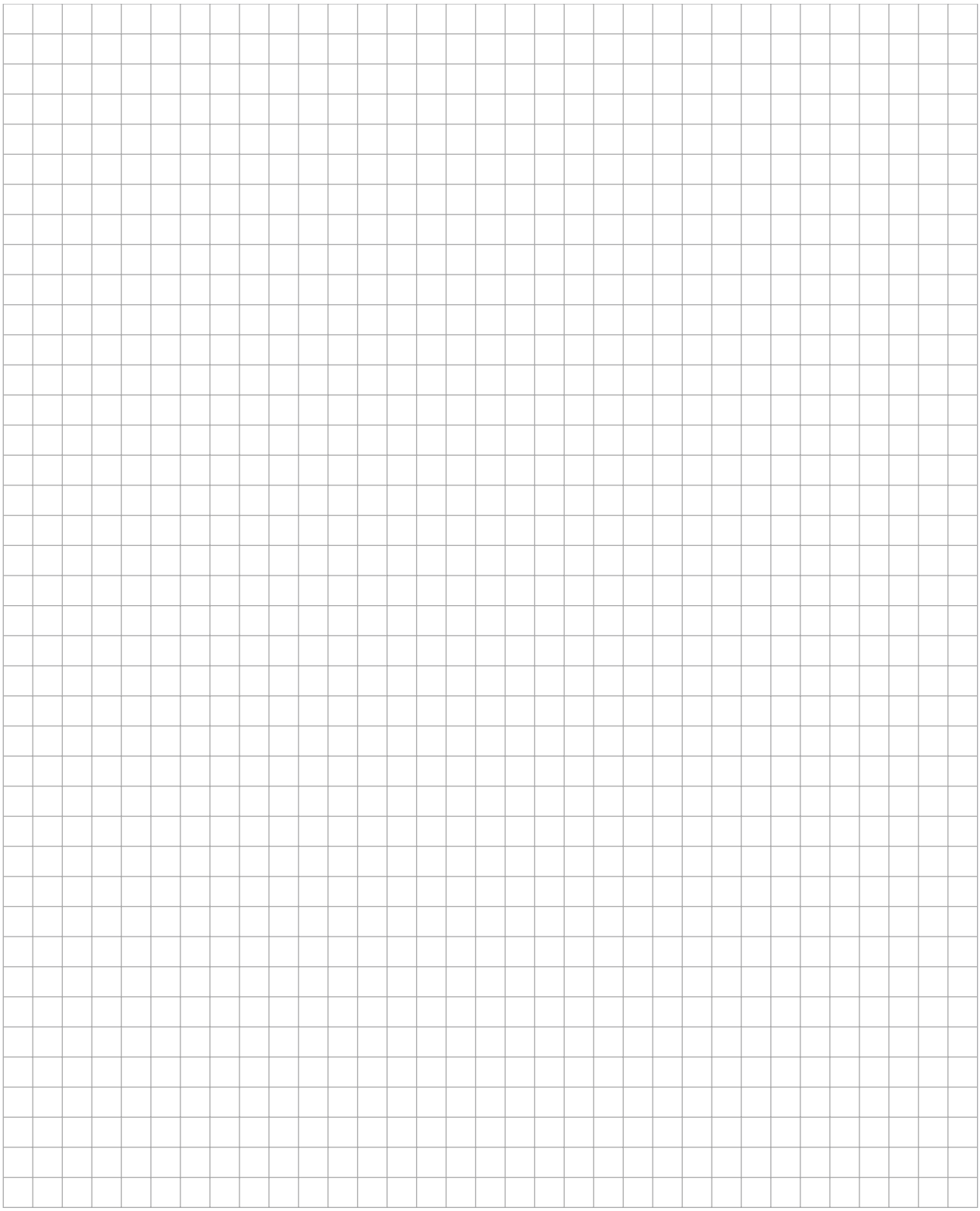
- drawing a net
- using the faces approach
- using surface area formulas

You will apply the information from this lesson in real-life situations in the next lesson.









Lesson 2: Applications of Surface Area

Lesson Focus

In this lesson, you will

- ❑ observe how changes in dimensions affect the surface area of objects
- ❑ solve real-world problems involving surface area, including the surface area of composite objects
- ❑ estimate surface area using referents

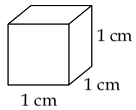
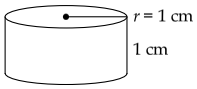
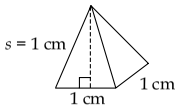
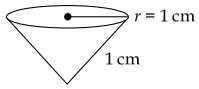
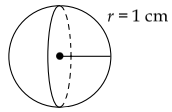
Lesson Introduction

This lesson will explore applications of surface area in day-to-day life. A very common use of surface area is in home renovations, but there are many others. You will calculate the changes in surface area when the dimensions of an object change. In some cases, it is useful to calculate approximate dimensions and surface areas without using measuring devices or calculators.

Changing Dimensions

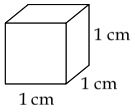
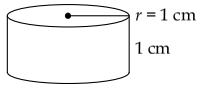
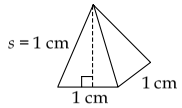
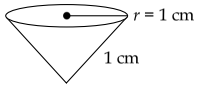
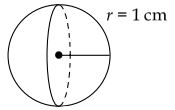
When you change one or more dimensions of an object, the surface area of the object also changes. Each object (prism, pyramid, and sphere) is affected differently by a change in dimensions, because we calculate the area of different shapes in different ways. The following table demonstrates the effect of doubling different dimensions of an object. (To keep it simple, we will write the answers in terms of π instead of using a calculator.)

Surface Area

	Rectangular Prism	Cylinder	Rectangular Pyramid	Cone	Sphere
					
Original Object	Front/back: $lh = 1 \times 1 = 1$ cm^2 2 Sides: $wh = 1 \times 1 = 1$ cm^2 Top/bottom: $lw = 1 \times 1 = 1$ cm^2 Surface area: $(2 \times 1) + (2 \times 1) + (2 \times 1)$ $= 6$ cm^2	Top/bottom: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $Ch = 2\pi r \times h$ $= 2\pi(1) \times 1$ $= 2\pi$ cm^2 Surface area: $(2 \times \pi) + 2\pi = 4\pi$ cm^2	Base: $lw = 1 \times 1 = 1$ cm^2 4 Sides: $\frac{1}{2}ls = \frac{1}{2}(1 \times 1) = \frac{1}{2}$ cm^2 Surface area: $1 + \left(4 \times \frac{1}{2}\right) = 3$ cm^2	Base: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(1) \times 1$ $= \pi$ cm^2 Surface area: $\pi + \pi = 2\pi$ cm^2	Surface area: $4\pi r^2 = 4\pi(1)^2 = 4\pi$ cm^2
Double the length (width and height = 1 cm)	Front/back: $lh = 2 \times 1 = 2$ cm^2 2 Sides: $wh = 1 \times 1 = 1$ cm^2 Top/bottom: $lw = 2 \times 1 = 2$ cm^2 Surface area: $(2 \times 2) + (2 \times 1) + (2 \times 2)$ $= 10$ cm^2		Base: $lw = 2 \times 1 = 2$ cm^2 2 Sides: $\frac{1}{2}ls = \frac{1}{2}(2 \times 1) = 1$ cm^2 2 Sides: $\frac{1}{2}ws = \frac{1}{2}(1 \times 1) = \frac{1}{2}$ cm^2 Surface area: $2 + (2 \times 1) + \left(2 \times \frac{1}{2}\right)$ $= 5$ cm^2		
Double the width (length and height = 1 cm)	Front/back: $lh = 1 \times 1 = 1$ cm^2 2 Sides: $wh = 2 \times 1 = 2$ cm^2 Top/bottom: $lw = 1 \times 2 = 2$ cm^2 Surface area: $(2 \times 1) + (2 \times 2) + (2 \times 2)$ $= 10$ cm^2		Base: $lw = 1 \times 2 = 2$ cm^2 2 Sides: $\frac{1}{2}ls = \frac{1}{2}(1 \times 1) = \frac{1}{2}$ cm^2 2 Sides: $\frac{1}{2}ws = \frac{1}{2}(2 \times 1) = 1$ cm^2 Surface area: $2 + \left(2 \times \frac{1}{2}\right) + (2 \times 1)$ $= 5$ cm^2		

(continued)

Surface Area (continued)

	Rectangular Prism	Cylinder	Rectangular Pyramid	Cone	Sphere
					
Double the height/slant (length and width OR radius = 1 cm)	Front/back: $lh = 1 \times 2 = 2$ cm^2 2 Sides: $wh = 1 \times 2 = 2$ cm^2 Top/bottom: $lw = 1 \times 1 = 1$ cm^2 Surface area: $(2 \times 2) + (2 \times 2) + (2 \times 1) = 10$ cm^2	Top/bottom: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $Ch = 2\pi r \times h$ $= 2\pi(1) \times 2$ $= 4\pi$ cm^2 Surface area: $(2 \times \pi) + 4\pi = 6\pi$ cm^2	Base: $lw = 1 \times 1 = 1$ cm^2 2 Sides: $\frac{1}{2}ls = \frac{1}{2}(1 \times 2) = 1$ cm^2 2 Sides: $\frac{1}{2}ws = \frac{1}{2}(1 \times 2) = 1$ cm^2 Surface area: $1 + (2 \times 1) + (2 \times 1) = 5$ cm^2	Base: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(1) \times 2$ $= \frac{1}{2}\pi$ cm^2 $= 2\pi$ cm^2 Surface area: $\pi + 2\pi = 3\pi$ cm^2	
Double the radius (height/slant = 1 cm)		Top/bottom: $\pi r^2 = \pi(2)^2 = 4\pi$ cm^2 Side: $Ch = 2\pi r \times h$ $= 2\pi(2) \times 1$ $= 4\pi$ cm^2 Surface area: $(2 \times 4\pi) + 4\pi = 12\pi$ cm^2		Base: $\pi r^2 = \pi(2)^2 = 4\pi$ cm^2 Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(2) \times 1$ $= 2\pi$ cm^2 Surface area: $4\pi + 2\pi = 6\pi$ cm^2	Surface area: $4\pi r^2 = 4\pi(2)^2 = 16\pi$ cm^2
Double all dimensions	Front/back: $lh = 2 \times 2 = 4$ cm^2 2 Sides: $wh = 2 \times 2 = 4$ cm^2 Top/bottom: $lw = 2 \times 2 = 4$ cm^2 Surface area: $(2 \times 4) + (2 \times 4) + (2 \times 4) = 24$ cm^2	Top/bottom: $\pi r^2 = \pi(2)^2 = 4\pi$ cm^2 Side: $Ch = 2\pi r \times h$ $= 2\pi(2) \times 2$ $= 8\pi$ cm^2 Surface area: $(2 \times 4\pi) + 8\pi = 16\pi$ cm^2	Base: $lw = 2 \times 2 = 4$ cm^2 2 Sides: $\frac{1}{2}ls = \frac{1}{2}(2 \times 2) = 2$ cm^2 2 Sides: $\frac{1}{2}ws = \frac{1}{2}(2 \times 2) = 2$ cm^2 Surface area: $4 + (2 \times 2) + (2 \times 2) = 12$ cm^2	Base: $\pi r^2 = \pi(2)^2 = 4\pi$ cm^2 Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(2) \times 2$ $= 4\pi$ cm^2 Surface area: $4\pi + 4\pi = 8\pi$ cm^2	Surface area: $4\pi r^2 = 4\pi(2)^2 = 16\pi$ cm^2

As you look down each column, you may notice that there is no obvious pattern between surface areas when you change just one dimension. Because there is no pattern, **when you do not change all of the dimensions, you must find the surface area as usual by adding all the faces together**. It may be helpful to include this on your resource sheet as a reminder.

Observe what happens when you compare the surface areas of the original objects with the surface areas of similar objects with **all** the dimensions doubled.

	Prism	Cylinder	Pyramid	Cone	Sphere
Original	6 cm^2	4π cm^2	3 cm^2	2π cm^2	4π cm^2
Doubled	24 cm^2	16π cm^2	12 cm^2	8π cm^2	16π cm^2

When you double all the dimensions, the **dimensional multiplier is 2**. Also, if you double all of the dimensions, the surface area becomes **four times larger**. This means that the **surface area multiplier is 4**. This is because the units used to measure surface area are units^2. Therefore:

Original	Doubled
dimensions × 1	dimensions × 2
dimension unit: cm	dimension unit: cm
surface area unit: cm^2	surface area unit: cm^2
Since the dimension unit is squared, the dimensional multiplier is also squared.	
surface area × $(1)^2$ = × 1	surface area × $(2)^2$ = × 4

A **multiplier** is the number by which you multiply. When you double something, you multiply by 2, so we call 2 the multiplier. When you triple something, 3 is the multiplier because you multiply by 3.

This principle works for all dimension changes as long as all the dimensions change by the same amount. You should include this rule in your own words on your resource sheet as a reminder.

This principle also agrees with what you learned about converting area units in previous mathematics courses. Remember,

$1\text{ m} = 100\text{ cm}$

$(1\text{ m})^2 = (100\text{ cm})^2$

$1\text{ m}^2 = 10\,000\text{ cm}^2$

To further explore the effect of changing dimensions, complete the following learning activity. Check your answers after you have completed the learning activity.

Learning Activity 2.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

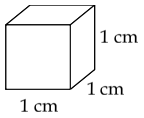
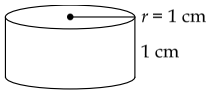
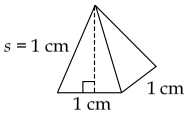
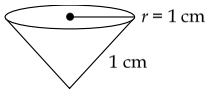
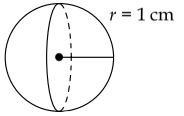
1. Sharmaine has to wake up at 5:00 am, and cannot function with less than nine hours of sleep. What is the latest time that Sharmaine should go to bed?
2. A cube has a side length of 4 cm. What is the surface area of the cube?
3. What is the mean of this data: 4 6 8 2
4. Shannon is four years younger than Cheri. If Cheri's age is double Shannon's, how old is Shannon?
5. There are five pieces of pepperoni per slice of pizza at Santa Monica's Pizza Parlour. If there are eight slices of pizza, how many pieces of pepperoni are on the whole pizza?

Part B: Dimension Changes

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. a) Complete the table on the following page.
   b) What is the pattern relating the original surface area to the surface area of the object with dimensions three times as large? How is this related to the pattern we found when we doubled all of the dimensions?

continued

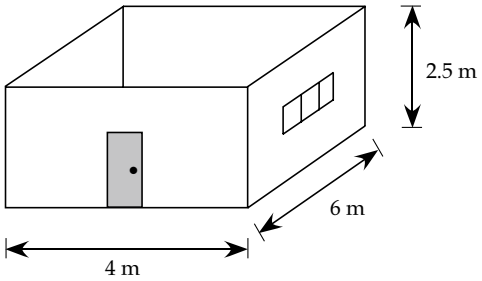
Surface Area					
	Rectangular Prism 	**Cylinder** 	**Rectangular Pyramid** 	**Cone** 	**Sphere** 
Original Object	Front/back: $lh = 1 \times 1 = 1$ cm^2 2 Sides: $wh = 1 \times 1 = 1$ cm^2 Top/bottom: $lw = 1 \times 1 = 1$ cm^2 Surface area: $(2 \times 1) + (2 \times 1) + (2 \times 1)$ $= 6$ cm^2	Top/bottom: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $Ch = 2\pi r \times h$ $= 2\pi(1) \times 1$ $= 2\pi$ cm^2 Surface area: $(2 \times \pi) + 2\pi = 4\pi$ cm^2	Base: $lw = 1 \times 1 = 1$ cm^2 4 Sides: $\frac{1}{2}ls = \frac{1}{2}(1 \times 1) = \frac{1}{2}$ cm^2 Surface area: $1 + \left(4 \times \frac{1}{2}\right) = 3$ cm^2	Base: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(1) \times 1$ $= \pi$ cm^2 Surface area: $\pi + \pi = 2\pi$ cm^2	Surface area: $4\pi r^2 = 4\pi(1)^2 = 4\pi$ cm^2
All dimensions $\times 3$					

Calculating Surface Area in Everyday Life

The following examples demonstrate applications of surface area. As you work through these examples, if there is something that you find confusing, ask your learning partner about it. Don't forget that you can include examples and useful hints on your resource sheet.

Example 1

You want to paint the inside of your bedroom, which is 4 m wide and 6 m long. The walls are 2.5 m tall. Your door and window have a combined area of 1.8 m^2, which you will not paint. A gallon of paint covers 37.2 m^2. How many gallons of paint will you need to put two coats on the walls? If a can contains one gallon of paint, how many cans do you need to buy? (Assume you are painting only the **walls** of your room.)



Solution

You can calculate the total surface area of the walls (ignoring the door and window) by eliminating the bases from the original prism surface-area formula.

$SA = Ph$

$P = 2(4) + 2(6) = 20$ m

$SA = (20)(2.5) = 50$ m^2 if there were no door or window.

The surface area of the walls, if you do not paint the door and window, is: $50 - 1.8 = 48.2$ m^2. Since you are applying two coats of paint, you must double the surface area, so it is: $48.2 \times 2 = 96.4$ m^2.

1 gallon of paint covers 37.2 m^2. You need to cover 96.4 m^2.

Step 1: Set up a ratio comparing what you have and what you need. You can calculate the number of gallons you need using proportional reasoning. Be sure to align gallons together in one column and m^2 in the other column.

1 gallon	37.2 m^2
g gallons	96.4 m^2

$$\frac{1}{g} = \frac{37.2}{96.4}$$

Step 2: Cross-multiply

$(1)(96.4) = (37.2)(g)$

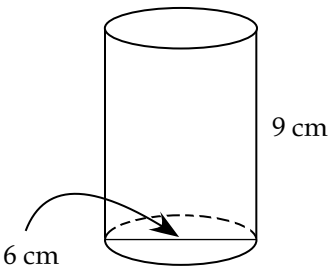
Step 3: Solve the equation

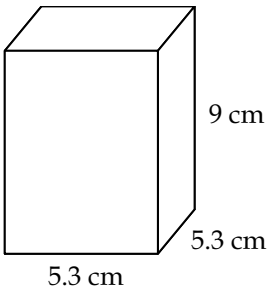
$$g = \frac{96.4}{37.2} = 2.59 \text{ gallons}$$

You would need 2.6 gallons of paint. You would have to buy three gallons of paint since you cannot buy part of a can of paint.

Example 2

Second Squeeze, a juice company, is trying to decide whether to sell its new juice in a can or a juice box. The company has "gone green" so it wants to use a minimal amount of packaging to reduce its waste. Which should it choose if both the can and the box hold the same amount of juice?





Solution

Can

$SA = 2B + Ph$

$B = \pi r^2 = \pi(3)^2 = 28.274 \text{ cm}^2$

$P = C = \pi d = \pi(6) = 18.850 \text{ cm}$

$SA = 2(28.274) + (18.850)(9)$

$= 56.548 + 169.65 = 226.198 \text{ cm}^2$

The area of material needed to make the can is 226.2 cm^2.

Box

$SA = 2B + Ph$

$B = (5.3)(5.3) = 28.09 \text{ cm}^2$

$P = 2(5.3) + 2(5.3) = 21.2 \text{ cm}$

$SA = 2(28.09) + (21.2)(9)$

$= 56.18 + 190.8 = 246.98 \text{ cm}^2$

The area of material needed to make the box is 247.0 cm^2.

Second Squeeze should choose the can because 226.2 cm^2 is less than 247.0 cm^2, so it requires less material to build the can than the box.

Example 3

Cherida has put siding on her house. Each piece of siding is 10 cm by 2 m. She used 1344 pieces of siding. Cherida did not cover the windows and doors, which have a total surface area of 7.2 m^2. What is the total surface area of Cherida's house?

Solution

The dimensions of one piece of siding are 10 cm by 2 m. Before you calculate the area, you need to have all the measurements in the same unit.

10 cm = 0.1 m.

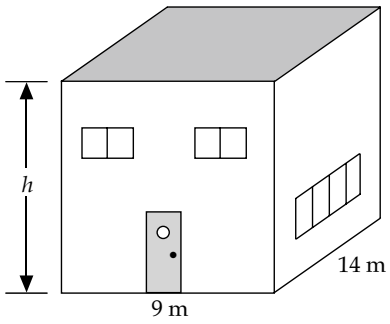
Area of one piece of siding = $0.1 \times 2 = 0.2 \text{ m}^2$

Area of all the siding used = $1344 \times 0.2 = 268.8 \text{ m}^2$. This is the surface area of the house, not including the windows and doors.

Total surface area of the house = $268.8 + 7.2 = 276 \text{ m}^2$.

Example 4

Oliver is measuring the dimensions of Cherida's house (from Example 3). He has measured the width to be 9 m and the length is 14 m. Based on the surface area of the four walls, Oliver calculates the height of the house to be 12 m. Is he correct? If he is not, what is the correct answer?



Solution

Note: A good way to find an error in calculation is to solve the question as if you do not have the answer.

$SA = Ph$ $\qquad P = 2(9) + 2(14) = 46$ m

276 m$^2 = (46)h$ $\qquad$ Divide both sides by 46.

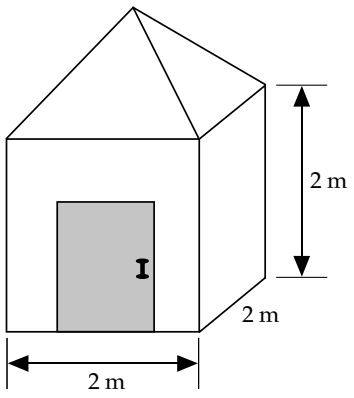
$276 \div 46 = 46h \div 46$

$h = 276 \div 46 = 6$ m

Oliver is incorrect. Cherida's house is 6 m tall.

Example 5

Jeremiah is building a shed. He has 20 m^2 of sheet metal that he will be using for the sides and the roof but not the door. The door is 1 m by 1.5 m. Does he have enough sheet metal for the shed if the slant height of the roof is 1.5 m?



Solution

The total surface area of the shed is the combined surface area of the sides of a square prism and the sides of a square pyramid, excluding the area of the door.

Square Prism (no top or bottom)

1 side = $2 \times 2 = 4$ m^2
all 4 sides = $4 \times 4 = 16$ m^2

Square Pyramid (no base)

1 side = $\frac{1}{2}bs = \frac{1}{2}(2)(1.5) = 1.5$ m^2

4 sides = $4 \times 1.5 = 6$ m^2

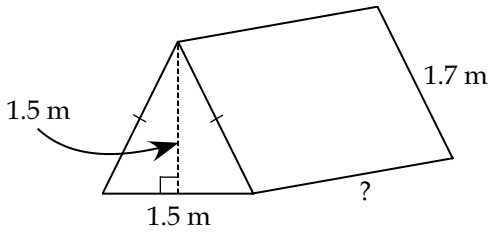
Door

$1 \times 1.5 = 1.5$ m^2

The total area of sheet metal that Jeremiah needs is $16 + 6 - 1.5 = 20.5$ m^2 of sheet metal. He only has 20 m^2, so he does not have enough.

Example 6

Rosselle knows that she has used 12.05 m^2 of fabric to make a tent, including the floor of the tent. How long is the tent from one end to the other?



Solution

The formula for the surface area of the tent is

$$SA = 2B + Ph$$

$$B = \frac{1}{2}bh = \frac{1}{2}(1.5 \times 1.5) = 1.125 \text{ m}^2$$

$$P = 1.5 + 1.7 + 1.7 = 4.9 \text{ m}$$

$$12.05 = 2(1.125) + (4.9 \times h)$$ (Subtract 2.25 from both sides.)

$$12.05 = 2.25 + (4.9 \times h)$$ (Divide both sides by 4.9.)

$$12.05 - 2.25 = (4.9 \times h)$$

$$9.8 \div 4.9 = (4.9 \times h) \div 4.9$$

$$2 \text{ m} = h$$

The length of the tent is 2 m.

Estimating Surface Area

You may be wondering why you have to learn how to estimate when you already know how to calculate the surface area of an object. If you carry a ruler or measuring tape around in your back pocket or purse all the time, then perhaps you will not have to estimate, but odds are that you may not have a ruler or other measurement tool with you at some point, so you will have to estimate lengths and sizes of objects. In Grade 10, you learned about referents (a **referent** is a known quantity that helps you to estimate or compare). If you cannot use a measuring tool, you can use a referent to estimate the measurement of an object.

The use of referents for estimating surface area is different than the use of referents to measure straight lines. This is because surface area has many components (all the faces added together), so there is not a specific pattern connecting the change in one dimension and the change in the overall surface area. For this reason, when estimating surface area, we use referents to find the dimensions of an object. We then solve for the surface area. The following examples demonstrate how you may use a referent to estimate surface area.

Example 1

Bing is given a model pyramid with a square base that has a length of 5 cm and a slant height of 6 cm. Errant's square base pyramid appears to be four times the size of Bing's (all the dimensions seem to be four times larger).

a) What is the surface area of Bing's pyramid?

b) Estimate the surface area of Errant's pyramid. Write your answer in cm^2 and in m^2.

Solution

a) $SA = B + \frac{1}{2}Ph$

$B = 5 \times 5 = 25 \text{ cm}^2$

$P = 5 + 5 + 5 + 5 = 20 \text{ cm}$

$SA = 25 + \frac{1}{2}(20 \times 6)$

$= 25 + 60 = 85 \text{ cm}^2$

Bing's pyramid has a surface area of 85 cm^2.

b) Based on what you learned at the beginning of this lesson, since the dimensions of Errant's pyramid are approximately four times larger:
The surface area should be about: $4^2 = 16$ times larger.
The approximate area of Errant's pyramid: $85 \text{ cm}^2 \times 16 = 1360 \text{ cm}^2$
Converted to m^2: $1360 \text{ cm}^2 \div (100)^2 \approx 0.14 \text{ m}^2$

Example 2

Dmitri has a piece of green paper. He doesn't know the dimensions of it, but he wants to know the surface area of this paper. He also has a piece of regular computer paper (8.5″ by 11″). Dmitri compares the two pieces of paper and finds that the width of the green piece of paper is roughly the same as the length of the computer paper. He also finds that the length of the green paper is about two times the width of the computer paper. What is the approximate surface area of the green paper?

Solution

Assume that 11″ is the width of the green paper.

The width of the green piece of paper is approximately 11″.

The length of the green paper is about $2 \times 8.5 = 17$″.

The surface area of the paper is $11 \times 17 = 187$ inches^2.

Alternately, if we assumed that 8.5″ was the width of the computer paper we would have found:

The width of the green piece of paper is approximately 8.5″.

The length of the green paper is about 2×11″ $= 22$″.

The surface area of the paper is $8.5 \times 22 = 187$ inches^2.

Both answers would be correct, but the first is more probable because 11″ by 17″ is a common paper size while 8.5″ by 22″ is not.

Example 3

Tory has two more presents in boxes to wrap for his family. The width and height of both boxes are the same, but the length of the one box is almost triple the other. How much more paper will Tory have to use for the larger box if the dimensions of the smaller box are 1′ by 1′ by 1′?

Solution

Small box	Large box
top/bottom: $lw = 1 \times 1 = 1$ ft.^{2}	top/bottom: $lw = 3 \times 1 = 3$ ft.^{2}
front/back: $lh = 1 \times 1 = 1$ ft.^{2}	front/back: $lh = 3 \times 1 = 3$ ft.^{2}
2 sides: $wh = 1 \times 1 = 1$ ft.^{2}	2 sides: $wh = 1 \times 1 = 1$ ft.^{2}
Surface Area = $2(1) + 2(1) + 2(1)$	Surface Area = $2(3) + 2(3) + 2(1)$
$= 6$ ft.^{2}	$= 14$ ft.^{2}

In this case, Tory will need about 8 ft.^{2} more wrapping paper for the larger box.

The following list includes other referents that you can use to estimate the dimensions of an object:

- The doorknob on a standard door is about 1 yard above the floor.
- A standard paper clip is about 2 inches long.
- When you spread your fingers as wide as possible, the distance from thumb to pinky is about 20 cm or 8 inches (this distance is commonly known as a "span").
- The perimeter of a sheet of paper is 99 cm (almost 1 m).
- The diameter of wire used in a standard paper clip, the thickness of a dime, and the thickness of a credit card are all approximately 1 mm.
- Two steps or one pace, and the distance from floor to average woman's waist are approximately 1 m.
- The diameter of a $1 coin (loonie), the width of a thumb, and the thickness of a hockey puck are about 1 inch.
- The distance a fit person can run in seven minutes, the distance between country roads, and about 15–20 city blocks (depending on how the streets are organized, this may be more or less) are all referents for 1 mile.

Check out some of the referents through measurement.

Now that you have seen some possible uses for surface area, and how to estimate surface area, complete the following learning activity to be sure that you understand what you have read. Be sure to check your answers, and ask your learning partner for help if needed.

Learning Activity 2.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. You are buying a top for $10.00, a pair of pants for $25.00, and a sweater for $20.00. Everything in the store is 10% off. How much will your bill be before tax?
2. A pack of three pens is $10, while a pack of five pens is $15. Which is the better deal?

(continued)

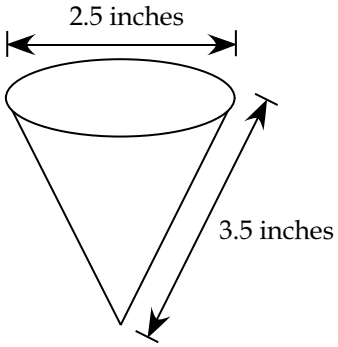
Learning Activity 2.4 (continued)

3. Jia runs or cycles every day except Friday. If Jai runs on Monday, Wednesday, and Saturday how many days does Jai cycle?
4. Complete the pattern: 36, 33, 30, _____ , _____
5. Solve for m: $3 - m = 10$

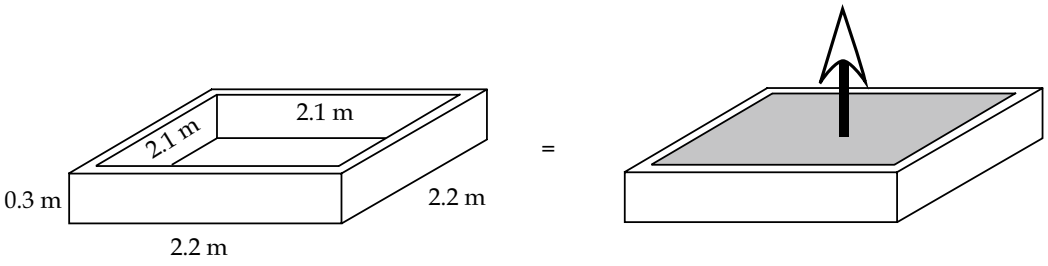
Part B: Applying Surface Area

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. The NBA is making a new basketball for next season. The ball must have a circumference of 29.5". How much material will they need to make the basketball?
2. How much paper do you need to make a cone paper cup?



3. Teela and Pedro are making a sandbox for their kids. Once they have cut the wood and put it together, they would like to paint it. The dimensions they measure outside of the wood frame are 2.2 m by 2.2 m. Inside the frame, the dimensions are 2.1 m by 2.1 m.

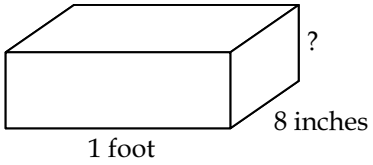


   a) What is the surface area of the sandbox?
   b) If a can of spray paint covers 10 m^2, how many cans do they need to buy to paint the sandbox?

(continued)

Learning Activity 2.4 (continued)

4. Mac has wrapped a box using 352 in.^{2} of wrapping paper, with no overlap. What is the height of the box?



5. Which is the larger amount of pizza: 2 small pizzas (11") or 1 large pizza (15")? (The measurements are the diameters.)
6. Find the surface area of a 3-D object in your house or at school. The object could be something like a box, a ball, a can, or anything else you might find.
   a) Use one of the referents from the lesson (or a different one that you have thought of) to estimate the surface area of the object.
   b) Use a ruler or other measuring tool to measure the dimensions of the object in the previous question, and then use the measurements to recalculate the surface area. Was your estimate accurate? Why might your estimate be different from the actual surface area?

Lesson Summary

In this lesson, you explored how changing the dimensions of a 3-D object affects the surface area. You also saw many examples of how we use surface area in everyday life, and learned how to estimate the dimensions of various objects using referents.

Before you start the next lesson, complete the following assignment, which you will hand in at the end of the module.

Assignment 2.1

Surface Area

Total: 30 marks

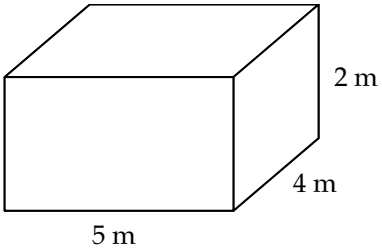
Note to Students: Before you start this assignment, be sure that your resource sheet has all of the formulas from the past two lessons, and any definitions that you want to include. You are encouraged to use your resource sheet to complete this assignment. If your resource sheet does not have the information you need to solve a problem, you should add the information now.

1. In your own words, describe the relationship between area and surface area. Include how they are different and how they are similar. You may use examples as part of your explanation. *(2 marks)*

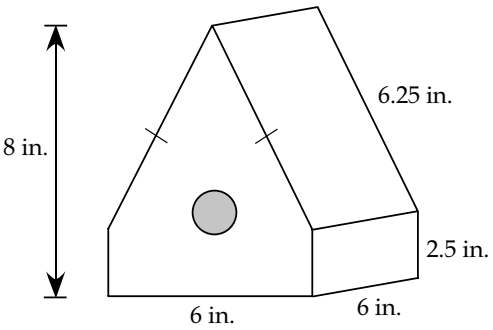
continued

Assignment 2.1: Surface Area (continued)

2. Draw the net and find the surface area of the prism. Show your work. *(2 marks)*



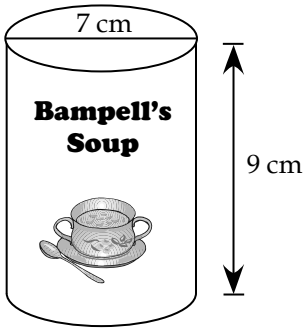
3. Maddry is painting the outside of a birdhouse she built. Each container of paint she has will cover 80 square inches. How many containers of paint will she need to buy? The diameter of the hole is two inches, and there is only the one hole. Solve this question using the faces method. Show your work. *(6 marks)*



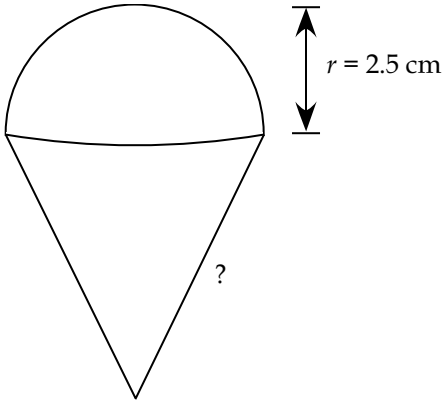
continued

Assignment 2.1: Surface Area (continued)

4. Bampell's Soup Company makes the labels for its cans of soup from 23 cm by 28 cm paper. If all of its soup cans look like the diagram below with the same dimensions, how many labels can the company cut from one piece of paper? Is there any unused paper? Show your work. *(4 marks)*



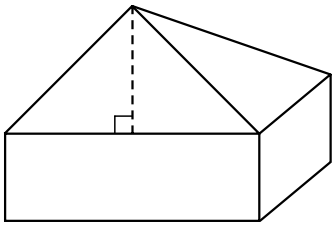
5. The surface area of an ice cream cone with half a sphere of ice cream is 117.8 cm^2. The radius of the cone and ice cream is 2.5 cm. What is the slant height of the ice cream cone? Show your work. *(6 marks)*



continued

Assignment 2.1: Surface Area (continued)

6. The diagram shows a building with a pyramidal roof. Use the diagram to write a problem dealing with surface area, and then solve the problem. Show your calculations. Also, be sure to label the dimensions on the building. Show your work. *(8 marks)*



7. What would be an appropriate referent to find the surface area of the following?
   a) baseball *(1 mark)*

   b) DVD case *(1 mark)*

Lesson 3: Units of Volume

Lesson Focus

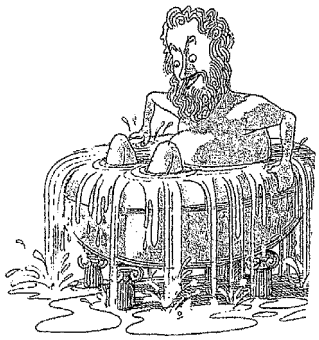
In this lesson, you will

- ❑ calculate volumes using both imperial and metric units
- ❑ convert units of volume from imperial to imperial or metric to metric
- ❑ manipulate a formula to calculate one dimension if the volume is given

Lesson Introduction

You have already studied surface area in this module. Volume is another way to describe a 3-D object. This lesson will be the introduction to volume, and Lesson 4 will explore it further. Here is a story about volume that changed history:

Archimedes was a Greek mathematician, who was well known for his brilliance. One day, the King approached Archimedes with a problem that he needed to solve. The King had given pure gold to a goldsmith so that the goldsmith could make a crown for the King. The King did not trust the goldsmith—he was afraid that the goldsmith had added silver to the crown. This would, of course, reduce the density of the gold in the crown. He wanted Archimedes to determine the density of the crown without melting it down. This last detail made everything more difficult, because he wasn't sure how to determine the volume of something that is so oddly shaped, and he needed the volume to determine the density. As Archimedes took a bath, he realized that as he got in, the water level went up, and as he got out, the water level went down. He figured out that the increased water level was equal to the volume of water being displaced when he got into the tub. Therefore, he reasoned that if he submerged the crown in water, he could determine the volume of the crown by measuring the increase in the level of water in the container.



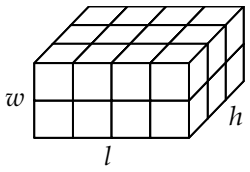
This method of finding the volume of an object is still used today. You may have done an experiment like this (perhaps not in the bathtub) in a previous grade.

Calculating Volume

Before you learn how to calculate volume, you should know what it is. **Volume** is the amount of 3-D space occupied by an object. (This is why the bathtub experiment was so effective.) Volume is measured in cubic units.

Prisms

You can measure the volume of the object below by counting the number of cubes the figure contains.



The top layer of the object is represented by 12 cubes. The object has two identical layers; therefore, the object's volume can be calculated as follows:

No. of cubes/layer × No. of layers (e.g., 12 × 2 = 24 cubes).

Counting cubes works for finding the volume of rectangular objects. However, for non-rectangular objects, you will need another method. You can find the volume of any prism by multiplying the area of the base by the height.

If you examine the base of the above diagram, you would find it to be 12 square units.

$$\begin{aligned} A &= lw \\ &= 4(3) \\ &= 12 \text{ u}^2 \end{aligned}$$

If this area is maintained throughout the height of the figure, as it is in this case, the area can be simply multiplied by the height in order to get the volume.

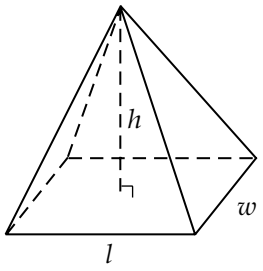
$V = Bh$ where B is area of the base and h is height of the figure.

For the above example: $V = 12(2)$
$= 24 \text{ u}^3$ (cubic units)

Similarly, you can find the volume for any "prism" (a 3-D figure where the shape of the base is maintained throughout the height) by multiplying the area of the base by the height.

Pyramids

A pyramid is a 3-D figure in which the shape of the base reduces to a single point throughout the height of the figure. To calculate the volume for these figures, you take one-third of the volume of its corresponding prism or $V = \frac{1}{3}Bh$.



The following are formulas for volume as well as for surface area. It would be helpful to include the volume formulas (either the general ones or the specific ones) on your resource sheet for this module.

Surface Area and Volume

	Volume			Surface Area
	General Formula	Base	Specific Formula	
Rectangular/ Triangular Prism	Bh	Rectangle: lw	$lw \times h$	$2B + Ph$
		Triangle: $\frac{1}{2}bh_t$	$\frac{1}{2}bh_t \times h$	$2B + Ph$
Cylinder	Bh	πr^2	$\pi r^2 \times h$	$2B + Ph$
Rectangular/ Triangular Pyramid	$\frac{1}{3}Bh$	Rectangle: lw	$\frac{1}{3}(lw) \times h$	$B + \frac{1}{2}Ph$
		Triangle: $\frac{1}{2}bh_t$	$\frac{1}{3}\left(\frac{1}{2}bh_t\right) \times h$	$B + \frac{1}{2}Ph$
Cone	$\frac{1}{3}Bh$	πr^2	$\frac{1}{3}(\pi r^2) \times h$	$B + \frac{1}{2}Ps$
Sphere	$\frac{4}{3}\pi r^3$			$4\pi r^2$

Reminder: B is the area of the base, and P is the perimeter of the base.

It is important to notice that the volume formulas for pyramids and cones do not involve the slant height but the height from the peak **perpendicular to the base.**

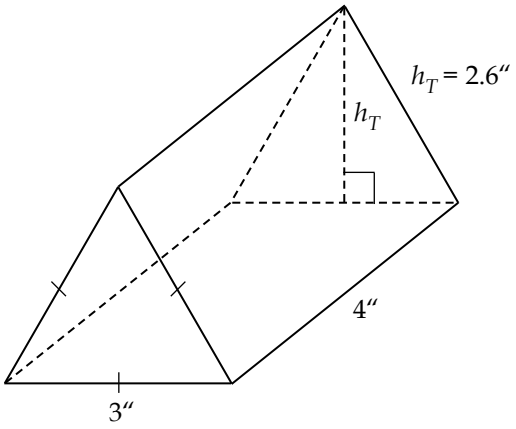
The units for volume are also different than the units for surface area. Volume is a true 3-D measurement, unlike surface area, so instead of units^2, we use units^3 or cubic units for volume. With this in mind, be careful to answer the questions with the appropriate units.

Metric Units	Imperial Units
Cubic centimetres (cm^3) = millilitres (mL) Cubic metres (m^3)	Cubic inches (in.^{3}) Cubic feet (ft.^{3})

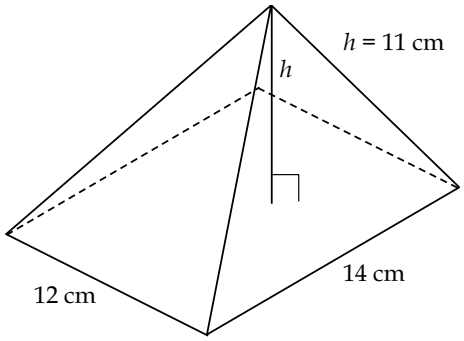
Example 1

Calculate the volumes of the following objects:

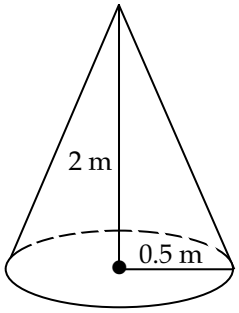
a)



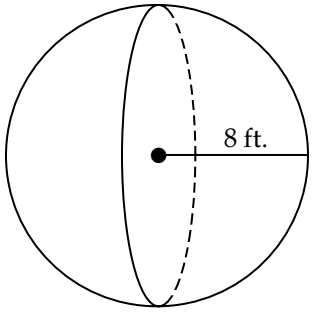
b)



c)



d)



Solutions

a) $B = \frac{1}{2}bh_t$

$= \frac{1}{2}(3)(2.6) = 3.9 \text{ in.}^2$

$V = Bh = 3.9 \times 4 = 15.6 \text{ in.}^3$

b) $B = lw$

$= 12 \times 14 = 168 \text{ in.}^2$

$V = \frac{1}{3}Bh = \frac{1}{3}(168)(11)$

$= 616 \text{ cm}^3 = 616 \text{ mL}$

c) $B = \pi r^2$

$= \pi (0.5)^2 = 0.785 \text{ m}^2$

$V = \frac{1}{3}Bh = \frac{1}{3}(0.785)(2) = 0.52 \text{ m}^3$

d) $V = \frac{4}{3}\pi (8)^3$

$= \frac{4}{3}\pi (512) = 2145 \text{ ft.}^3$

You can use the volume of an object to find one of the dimensions if you know the other dimensions.

Example 2

The volume of a sphere is 41 m^3. What is the diameter of the sphere?

Solution

The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3,$$

so you only need to know the volume in order to solve for the radius (it is the only other variable—remember that π is actually a number).

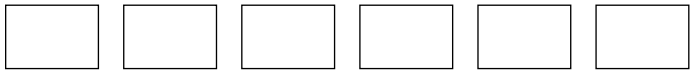
$41 = \frac{4}{3}\pi r^3$ You are solving for r, so divide both sides of the equation by $\left(\frac{4}{3}\pi\right)$.

$41 \div \left(\frac{4}{3}\pi\right) = \frac{4}{3}\pi r^3 \div \left(\frac{4}{3}\pi\right)$

$9.7880 = r^3$ To solve for r, you need to calculate the cube root of 9.7880.

The sequence of buttons you need to press depends on the brand of calculator you are using.

In the boxes below, record the steps you need to use on your calculator.



$\sqrt[3]{9.7880} = \sqrt[3]{r^3}$

$2.14 = r$

Since $d = 2r$, the diameter of the sphere is $2 \times 2.14 = 4.28$ m.

Example 3

The volume of a rectangular prism is 720 mm^3. If the width is 6 mm and the height is 10 mm, what is the length?

Solution

The formula for volume for a rectangular prism is

$V = Bh = lwh$	
$720 = l \times 6 \times 10$	Since length is the only variable that we do not know, we can solve for it.
$720 = l \times 60$	Divide by 60 on both sides of the equation.
$720 \div 60 = (l \times 60) \div 60$	
$12 = l$	

The length of the prism is 12 mm.

Converting Volume Measurements within a System

Calculating volume is somewhat similar to calculating surface area because, for the most part, you use the same variables. Sometimes when the answer is very large, you may want to convert the volume to different units so that it is easier to visualize the answer. For example, the answer in Example 1(d) is 2145 $ft.^3$, and this answer might be easier to visualize if it were converted to $yd.^3$.

You have learned how to convert between metric units and imperial units for linear measurements (units) and surface area ($units^2$) in previous math courses. Now you will learn how to convert the units of volume measurements.

Metric Units

As demonstrated in the previous lesson, since

$1\text{ m} = 100\text{ cm}$

$(1\text{ m})^2 = (100\text{ cm})^2$

$1\text{ m}^2 = 10\,000\text{ cm}^2$

You can take this calculation one step further to determine how many cm^3 are in 1 m^3.

$1\text{ m} = 100\text{ cm}$

$(1\text{ m})^3 = (100\text{ cm})^3$

$(1 \times 1 \times 1)\text{ m}^3 = (100 \times 100 \times 100)\text{ cm}^3$

$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$

You can apply this procedure to all units that are equal, so you can calculate how many cubic millimetres are in 1 cubic centimetre.

Length	Volume
$1\text{ cm} = 10\text{ mm}$ $1\text{ m} = 100\text{ cm}$ $1\text{ km} = 1000\text{ m}$	$1\text{ cm}^3 = (10\text{ mm})^3 = 1000\text{ mm}^3$ $1\text{ m}^3 = (100\text{ cm})^3 = 1\,000\,000\text{ cm}^3$ $1\text{ km}^3 = (1000\text{ m})^3 = 1\,000\,000\,000\text{ m}^3$

Imperial Units

Just like metric units, you can convert between imperial units for length. This also means that you can convert between different units of volume.

Using the same method as before, you know that

$1\text{ yd.} = 3\text{ ft.}$

$(1\text{ yd.})^3 = (3\text{ ft.})^3$

$(1 \times 1 \times 1)\text{ yd.}^3 = (3 \times 3 \times 3)\text{ ft.}^3$

$1\text{ yd.}^3 = 27\text{ ft.}^3$

Overall,

Length	Volume
$1\text{ ft.} = 12\text{ in.}$ $1\text{ yd.} = 3\text{ ft.}$ $1\text{ mile} = 1760\text{ yd.}$	$1\text{ ft.}^3 = (12\text{ in.})^3 = 1728\text{ in.}^3$ $1\text{ yd.}^3 = (3\text{ ft.})^3 = 27\text{ ft.}^3$ $1\text{ mi.}^3 = (1760\text{ yd.})^3 = 5\,451\,776\,000\text{ yd.}^3$

You may want to include the metric and imperial conversions on your resource sheet so that you do not have to flip back and forth when solving problems in the rest of the lesson and module.

Conversion Ratio

You can convert from one unit to another unit using proportional reasoning if you know the unit ratios.

Example 1

A truck hauls 216 ft.^{3} of gravel. It is easier to visualize this volume of gravel when it is measured in cubic yards. Convert the volume to cubic yards.

Solution

Use proportional thinking to do this conversion.

From the table above, you know that 1 yd.^{3} : 27 ft.^{3}.

Be sure to align cubic feet together in one column and cubic yards in another column.

27 ft.^{3}	1 yd.^{3}
216 ft.^{3}	x yd.^{3}

$$\frac{27}{216} = \frac{1}{x} \qquad \text{(Cross-multiply and solve for } x\text{.)}$$

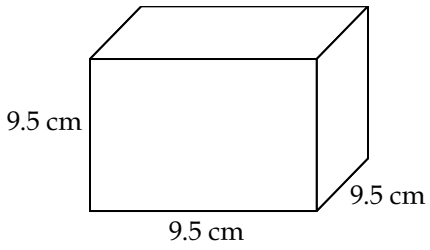
$$(27)(x) = (216)(1)$$

$$x = \frac{216}{27}$$

$$x = 8 \text{ yd.}^3 \text{ of gravel}$$

Example 2

The volume of a fish tank is 857.4 cm^3. What is the volume of the fish tank in m^3? Is this number easier to visualize?



Solution

To convert from cm^3 to m^3, use the conversion ratio:

$$\frac{1\ m^3}{1\ 000\ 000\ cm^3}$$

Be sure to align cm^3 together in one column, and m^3 in another column.

1 000 000 cm^3	1 m^3
857.4 cm^3	x m^3

$$\frac{1\ 000\ 000}{857.4} = \frac{1}{x} \qquad \text{(Cross-multiply and solve for } x\text{.)}$$

$$(1\ 000\ 000)(x) = (857.4)(1)$$

$$x = \frac{857.4}{1\ 000\ 000}$$

$$x = 0.0008574\ m^3$$

This number is quite small, so it would be best to leave the volume as 857.4 cm^3. Also, 1 cm^3 = 1 mL so 857.4 cm^3 = 857.4 mL, which is almost a litre.

Before we move on to talk about how surface area and volume are related, complete the following learning activity to check your understanding. Be sure that you are answering the questions correctly by looking in the answer key at the end of the module. If you do not understand how to use the formulas talk to your learning partner or tutor/marker.

Learning Activity 2.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Bjork wants to buy a computer that costs $700 before taxes. If Bjork has only $750, will he be able to pay for the computer (taxes = 12% total)?
2. Marco is on page 150 of the book he has to read for English class. If the book has 500 pages, has he read half the book?
3. Ina is five inches shorter than Ra. If Ra is half a foot taller than Horus, then who is the shortest?
4. The score with one minute to go in a football game is 28 to 34. What is the minimum number of points the losing team must score in order to win the game?
5. Evaluate $4x - 32$ if $x = 9$.

(continued)

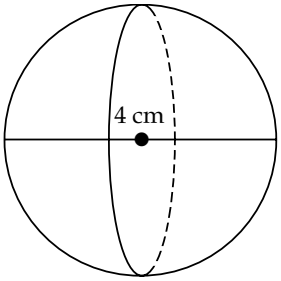
Learning Activity 2.5 (continued)

Part B: Calculating and Converting Volume

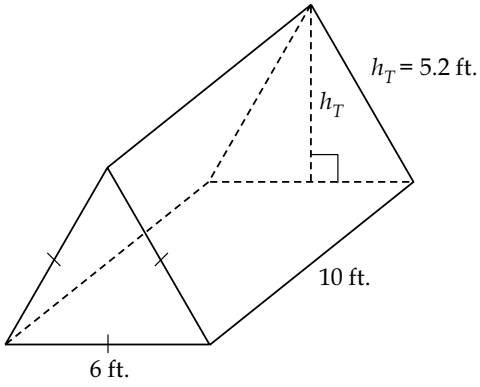
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Calculate the volume of each of the following objects. Convert any solutions that are very small or very large to appropriate units.

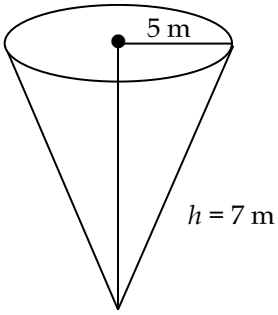
a)



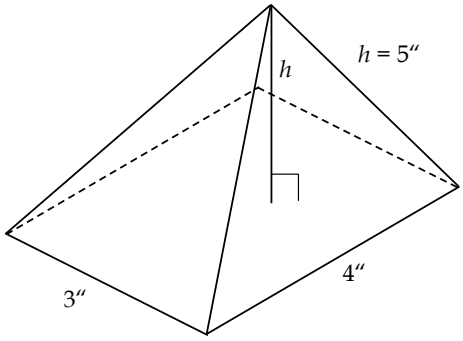
b)



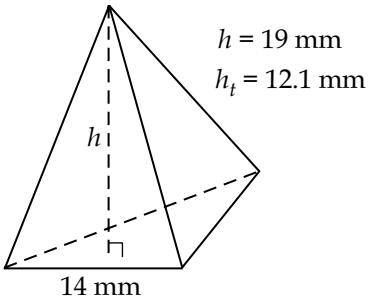
c)



d)



e)



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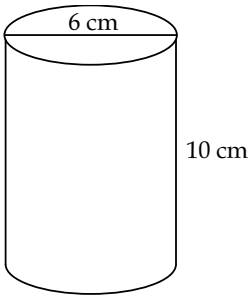


(continued)

Learning Activity 2.5 (continued)

2. Convert the following:
   a) 9280 mm^3 to cm^3
   b) 319 $in.^3$ to $ft.^3$
   c) 4 $yd.^3$ to cubic inches
   d) 0.637 m^3 to cubic centimetres
3. Solve for the missing dimension.
   a) A cone has a volume of 5236 $inches^3$. If the radius is 2 feet, how tall is the cone?
   b) A triangular prism has a volume of 1950 cm^2. The height of the triangle is 13 cm, and the height of the prism is 20 cm. What is the perimeter of the triangle (assuming all the sides are equal)?
4. Find a 3-D object in your house, such as a box or ball or (perhaps) a model pyramid. Use a ruler, tape measure, caliper, or micrometer to measure the dimensions of the object. You know which dimensions you have to measure based on the volume formula—you must know the values for everything except the volume itself. Calculate the volume.

Lesson Summary

In this lesson, you studied units used to measure volume. This included metric units (mm^3, cm^3, and m^3) and imperial units ($in.^3$, $ft.^3$, and $yd.^3$). You used the formulas to calculate volume. In some questions, you were given the volume and then had to calculate one dimension. You also converted units within the same system of measurement (both metric and imperial).

In the next lesson, you will continue to learn about volume by examining the effect of changing one or more dimensions of a variety of 3-D objects. You will also investigate the relationships between surface area and volume.

Lesson 4: Volumes of Various Objects

Lesson Focus

In this lesson, you will

- ❑ study the relationship between the volumes of prisms and pyramids
- ❑ observe how changing dimensions of an object changes the volume of an object
- ❑ explore the relationship between surface area and volume

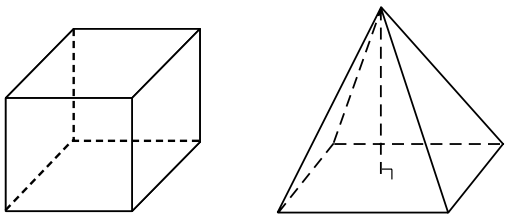
Lesson Introduction

In Lesson 3, you learned how to calculate volume and the units used to measure volume. In this lesson, you will look at the volumes of different types of objects and compare them. You will also analyze how surface area and volume are related. One of the best ways to make something stick in your brain is to discover relationships between different skills you have already learned. You have already learned about volume through calculations; now it's time to look at volume another way.

Comparing Volumes

Prisms and Pyramids

Prisms and pyramids look like two completely different 3-D objects when you put them side-by-side.



prism pyramid

Prisms have two bases, while pyramids have one base; pyramids have triangular sides, while prisms have rectangular sides. Despite these differences, their volumes are related.

Complete the learning activity to discover this relationship.

Learning Activity 2.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

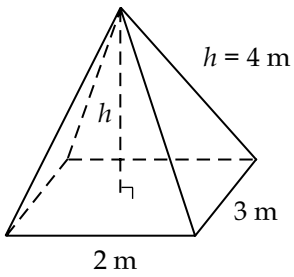
1. Aesir works 25 hours per week and earns $11 per hour. Calculate Aesir's gross pay after two weeks.
2. There are six red, four yellow, four orange, four green, and two purple candies in a package. What percentage of the candies are green?
3. A triangle has an area of 20 m^2. If the base is 10 m, what is the height of the triangle?
4. After adding 20 songs to your mp3 player, you have used up 100 MB (megabytes) of memory. What is the average number of megabytes per song?
5. Evaluate: $3 + 5 - (8 \times 3) + 1$

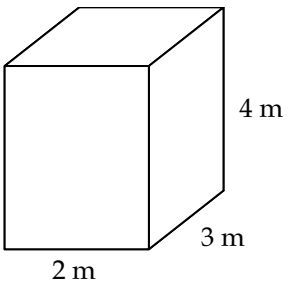
Part B: Prism and Pyramid Volumes

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Calculate the volumes of the pairs of prisms and pyramids. Record the volume values in a table (left column = prism volume, right column = pyramid volume).

a)

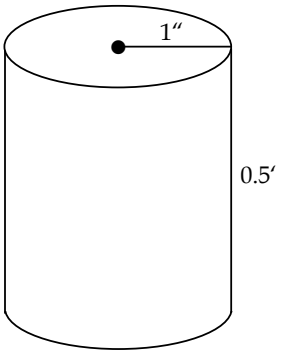


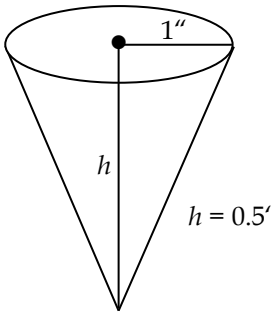


(continued)

Learning Activity 2.6 (continued)

b)





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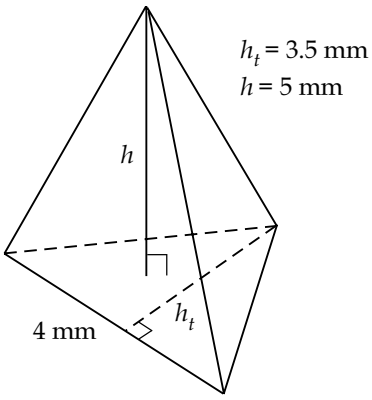
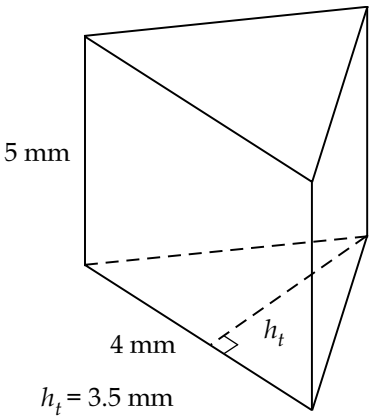




2. Compare the values in the left and right columns of Question 1. What do you notice?

Relating Prism and Pyramid Volumes

What you noticed in this learning activity is true for all prisms and pyramids that have equal dimensions. **A pyramid with the same base and height as a prism will have a volume equal to one-third of the prism's volume.**

You should include that on your resource sheet.

This statement is reflected in the general and specific formulas for prisms and pyramids:

Formulas for Prisms and Pyramids		
	Volume	
	General Formula	Specific Formula
Rectangular Prism	Bh	$lw \times h$
Triangular Prism		$\frac{1}{2}bh_t \times h$
Cylinder		$\pi r^2 \times h$
Rectangular Pyramid	$\frac{1}{3}Bh$	$\frac{1}{3}(lw) \times h$
Triangular Pyramid		$\frac{1}{3}\left(\frac{1}{2}bh_t\right) \times h$
Cone		$\frac{1}{3}\left(\pi r^2\right) \times h$

Everything after the "one-third" in the pyramid formula is the same as the prism formula.

Changing Dimensions

The table shows the effect of changing one, two, or three dimensions of prisms, pyramids, cylinders, cones, and spheres.

Changing Dimensions

	Rectangular Prism	**Cylinder**	**Rectangular Pyramid**	**Cone**	**Sphere**
Original Object	$B = lw = 1 \times 1 = 1\ \text{cm}^2$ Volume: $V = Bh = 1 \times 1 = 1\ \text{cm}^3$	$B = \pi r^2 = \pi(1)^2 = 1\pi\ \text{cm}^2$ Volume: $V = Bh = 1\pi \times 1 = \pi\ \text{cm}^3$	$B = lw = 1 \times 1 = 1\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(1 \times 1) = \frac{1}{3}\ \text{cm}^3$	$B = \pi r^2 = \pi(1)^2 = 1\pi\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(1\pi \times 1) = \frac{1}{3}\pi\ \text{cm}^3$	Volume: $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi (1)^3 = \frac{4}{3}\pi\ \text{cm}^3$
Double the length (width and height = 1 cm)	$B = lw = 2 \times 1 = 2\ \text{cm}^2$ Volume: $V = Bh = 2 \times 1 = 2\ \text{cm}^3$		$B = lw = 2 \times 1 = 2\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(2 \times 1) = \frac{2}{3}\ \text{cm}^3$		
Double the width (length and height = 1 cm)	$B = lw = 1 \times 2 = 2\ \text{cm}^2$ Volume: $V = Bh = 2 \times 1 = 2\ \text{cm}^3$		$B = lw = 1 \times 2 = 2\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(2 \times 1) = \frac{2}{3}\ \text{cm}^3$		
height/slant (length and width OR radius = 1 cm)	$B = lw = 1 \times 1 = 1\ \text{cm}^2$ Volume: $V = Bh = 1 \times 2 = 2\ \text{cm}^3$	$B = \pi r^2 = \pi(1)^2 = 1\pi\ \text{cm}^2$ Volume: $V = Bh = 1\pi \times 2 = 2\pi\ \text{cm}^3$	$B = lw = 1 \times 1 = 1\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(1 \times 2) = \frac{2}{3}\ \text{cm}^3$	$B = \pi r^2 = \pi(1)^2 = 1\pi\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(1\pi \times 2) = \frac{2}{3}\pi\ \text{cm}^3$	
Double the radius (height/slant = 1 cm)		$B = \pi r^2 = \pi(2)^2 = 4\pi\ \text{cm}^2$ Volume: $V = Bh = 4\pi \times 1 = 4\pi\ \text{cm}^3$		$B = \pi r^2 = \pi(2)^2 = 4\pi\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(4\pi \times 1) = \frac{4}{3}\pi\ \text{cm}^3$	Volume: $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi (2)^3 = \frac{32}{3}\pi\ \text{cm}^3$
Double all dimensions	$B = lw = 2 \times 2 = 4\ \text{cm}^2$ Volume: $V = Bh = 4 \times 2 = 8\ \text{cm}^3$	$B = \pi r^2 = \pi(2)^2 = 4\pi\ \text{cm}^2$ Volume: $V = Bh = 4\pi \times 2 = 8\pi\ \text{cm}^3$	$B = lw = 2 \times 2 = 4\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(4 \times 2) = \frac{8}{3}\ \text{cm}^3$	$B = \pi r^2 = \pi(2)^2 = 4\pi\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(4\pi \times 2) = \frac{8}{3}\pi\ \text{cm}^3$	Volume: $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi (2)^3 = \frac{32}{3}\pi\ \text{cm}^3$

Note that changing one dimension affects different objects in different ways.

For triangular or rectangular objects,

- if one dimension is doubled, then the volume also doubles. The dimension multiplier is 2, and the total multiplier is also 2.
- if two dimensions are doubled, then the volume increases by a factor of (2×2) or 4. The total multiplier in this case is 4.
- if all three dimensions are doubled, the volume increases by a factor of $(2 \times 2 \times 2)$ or 8. In this case, the dimension multiplier for each dimension is 2, and the total multiplier is 8.

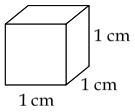
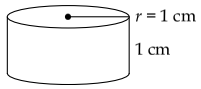
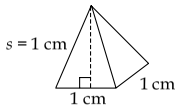
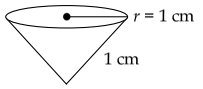
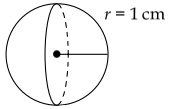
For cylinders and cones,

- if the height is doubled, the volume also doubles. The dimension multiplier for height is 2, and the total multiplier is also 2.
- if the radius doubles, the volume increases by a factor of 4 ($r^2 = 2^2 = 4$). The dimension multiplier for radius is 2, and the total multiplier is 4.

For spheres,

- a sphere has only one dimension, which is radius, and if radius doubles, the volume ($r^3 = 2^3 = 8$) increases by a factor of 8. In this case, the dimension multiplier is 2, and the total multiplier is 8.

Changing Dimensions

	Rectangular Prism	Cylinder	Rectangular Pyramid	Cone	Sphere
					
Original Object	$B = lw = 1 \times 1 = 1\ \text{cm}^2$ Volume: $V = Bh = 1 \times 1 = 1\ \text{cm}^3$	$B = \pi r^2 = \pi(1)^2 = 1\pi\ \text{cm}^2$ Volume: $V = Bh = 1\pi \times 1 = \pi\ \text{cm}^3$	$B = lw = 1 \times 1 = 1\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(1 \times 1) = \frac{1}{3}\ \text{cm}^3$	$B = \pi r^2 = \pi(1)^2 = 1\pi\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(1\pi \times 1) = \frac{1}{3}\pi\ \text{cm}^3$	Volume: $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi\,(1)^3 = \frac{4}{3}\pi\ \text{cm}^3$
Triple all dimensions	$B = lw = 3 \times 3 = 9\ \text{cm}^2$ Volume: $V = Bh = 9 \times 3 = 27\ \text{cm}^3$	$B = \pi r^2 = \pi(3)^2 = 9\pi\ \text{cm}^2$ Volume: $V = Bh = 9\pi \times 2$ $= 27\pi\ \text{cm}^3$	$B = lw = 3 \times 3 = 9\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(9 \times 3) = \frac{27}{3}\ \text{cm}^3$	$B = \pi r^2 = \pi(3)^2 = 9\pi\ \text{cm}^2$ Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(9\pi \times 3) = \frac{27}{3}\pi\ \text{cm}^3$	Volume: $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi\,(3)^3 = \frac{108}{3}\pi\ \text{cm}^3$

Therefore, for all geometric objects,

- if all the dimensions are doubled, the volume increases by a factor of (2^3) or 8. Each dimension multiplier is 2, and the total multiplier is 8.

Similarly,

- if all the dimensions are increased by a factor of 3, the volume increases by a factor of (3^3) or 27. Each dimension multiplier is 3, and the total multiplier is 27. Note the table shown above.

Also,

- if all dimensions increased by a factor of 1.5, the volume increases by a factor of 1.5^3, which is approximately 3.38. In this case, each of the dimension's multiplier is 1.5, and the total multiplier is 3.38.

These principles will be helpful to remember, especially as we discuss estimating volume in the next lesson. You may want to include a hint or two on your resource sheet.

Surface Area vs. Volume

Surface area and volume both refer to quantities of 3-dimensional objects. Volume is a measure of the amount of 3-dimensional space that an object occupies. The units for volume are always cubed (cm^3, m^3, $ft.^3$, $yd.^3$). In Lesson 5, you will learn about units of capacity, which include gallons and litres. These are similar, but not identical to units of volume.

Surface area is a combined measure of the areas of all of the 2-dimensional surfaces of an object. A rectangular prism like a box or a die has six 2-dimensional surfaces. The units for surface area are always squared (cm^2, m^2, $ft.^2$, $yd.^2$)

The following table compares a variety of 3-D objects.

Changing Dimensions

	Rectangular Prism	Cylinder	Rectangular Pyramid	Cone	Sphere
Surface Area	Front/back: $lh = 1 \times 1 = 1$ cm^2 2 Sides: $wh = 1 \times 1 = 1$ cm^2 Top/bottom: $lw = 1 \times 1 = 1$ cm^2 Surface area: $(2 \times 2) + (2 \times 1) + (2 \times 1)$ $= 6$ cm^2	Top/bottom: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $Ch = 2\pi r \times h$ $= 2\pi(1) \times 1$ $= 2\pi$ cm^2 Surface area: $(2 \times \pi) + 2\pi = 4\pi$ cm^2	Base: $lw = 1 \times 1 = 1$ cm^2 4 Sides: $\frac{1}{2}ls = \frac{1}{2}(1 \times 1.1)$ $= 0.55$ cm^2 Surface area: $1 + (4 \times 0.55) = 3.2$ cm^2	Base: $\pi r^2 = \pi(1)^2 = \pi$ cm^2 Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(1) \times 1.4$ $= 1.4\pi$ cm^2 Surface area: $\pi + 1.4\pi = 2.4\pi$ cm^2	Surface area: $4\pi r^2 = 4\pi(1)^2 = 4\pi$ cm^2
Volume	$B = lw = 1 \times 1 = 1$ cm^2 Volume: $V = Bh = 1 \times 1 = 1$ cm^3	$B = \pi r^2 = \pi(1)^2 = \pi$ cm^2 Volume: $V = Bh = \pi \times 1 = \pi$ cm^3	$B = lw = 1 \times 1 = 1$ cm^2 Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(1 \times 1) = \frac{1}{3}$ cm^3	$B = \pi r^2 = \pi(1)^2 = \pi$ cm^2 Volume: $V = \frac{1}{3}Bh$ $= \frac{1}{3}(\pi \times 1) = \frac{1}{3}\pi$ cm^3	Volume: $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$ cm^3

Note that the units for surface area (units squared) and volume (units cubed) are different and so do not provide the same information. Also, area is a 2-D measure and volume is a 3-D measure.

Lesson Summary

In this lesson, you learned about volume relationships between different types of objects, and between objects with different dimensions. You also compared surface area and volume to find out that they are very different types of measurement, although they both describe 3-D objects. In the next lesson, you will study capacity, which is related to volume.

Before you move on to the next lesson, complete the following assignment. You will hand it in with the other assignments from this module once you have completed the whole module.

Assignment 2.2

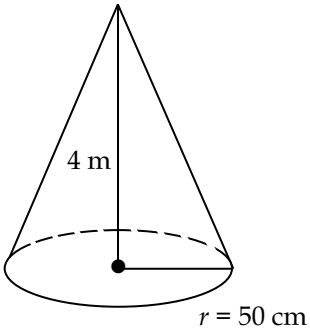
Volume

Total: 28 marks

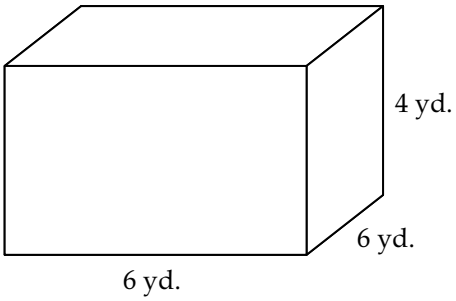
Note to Students: You are encouraged to use your resource sheet as you complete this assignment. At any time during the assignment, if you realize that something is missing from the resource sheet, add any missing information to it. It is a tool to help you.

1. Calculate the volume of each of the following objects. Show your work.

   a) *(3 marks)*



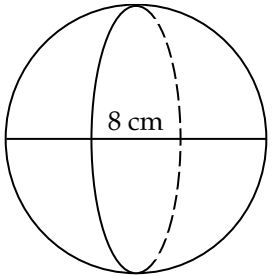
   b) *(2 marks)*



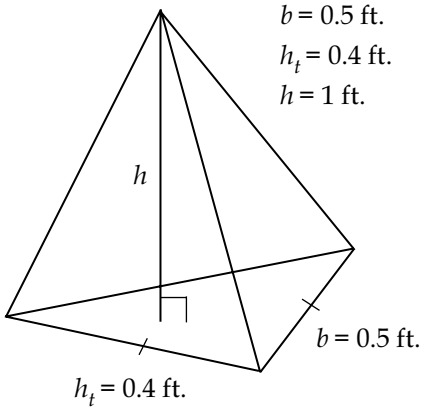
continued

Assignment 2.2: Volume (continued)

c) *(3 marks)*



d) *(2 marks)*



continued

Assignment 2.2: Volume (continued)

2. Convert the following. Show your work. *(3 marks)*

   a) 2915 $in.^3$ to cubic feet

   b) 0.267 cubic yards to $ft.^3$

   c) 0.0000264 m^3 to mm^3

continued

Assignment 2.2: Volume (continued)

3. Solve for the missing dimension. Show your work.
   a) The volume of a rectangular pyramid is 0.045 m^3. The height of the pyramid is 60 cm, and the length is 0.5 m. What is the width of the pyramid? *(4 marks)*

   b) A cylinder has a volume of 14.1 $ft.^3$. If the diameter of the cylinder is 3 ft., what is the height of the cylinder? *(4 marks)*

4. A cylinder has a volume of 30 $in.^3$. What is the volume of a cone with the same radius and height? Explain how you know. *(2 marks)*

continued

Assignment 2.2: Volume (continued)

5. In your own words, explain how changing the dimension(s) of an object affect(s) the volume. You can use example(s) in your explanation. *(2 marks)*

6. Based on what you have learned about surface area and volume, explain the relationship between the two in your own words. How are they similar? How are they different? *(3 marks)*

Notes

Lesson 5: Applications of Surface Area

Lesson Focus

In this lesson, you will

- ❑ explore the relationship between volume and capacity
- ❑ learn the units used to measure capacity
- ❑ calculate the capacity of a container
- ❑ convert between units of volume and capacity
- ❑ estimate volume and capacity

Lesson Introduction

In the previous two lessons, you studied volume. Capacity is very similar to volume—so much so that people often use them interchangeably. You may have heard the term *capacity* used to describe a pail, an airplane, or a hockey arena. In this lesson, you will explore how volume and capacity are similar and how they are different.

Comparing Volume and Capacity

The following statements compare capacity and volume. Capacity describes the amount a container can hold. Volume describes the actual space taken up by an object or substance. For example, a measuring cup may have a capacity of eight fluid ounces, and contain five fluid ounces of water.

Below are some examples:

Capacity	Volume
A carton can hold up to 1 L of milk.	There is 1 L of milk in the carton.

The capacity states *how much the carton is able to contain*, while the volume is describing the *amount of milk*.

A container can store three cups of rice.	There are two cups of rice in the container.

The capacity states *how much the container is able to hold*, while the volume is describing the *amount of rice*.

Capacity	Volume
A binder can have a maximum of 400 pages in it.	There are 350 pages in the binder.

The capacity states *how much the binder is able to hold*; the volume is describing *the number of pages*.

Units

Volume is measured in cubic units such as mm^3, cm^3, and m^3 in the metric system, and $in.^3$, $ft.^3$, and $yd.^3$ in the imperial system. The units used to measure capacity are normally not the same as the cubic units used to measure volume. The units to describe capacity vary greatly and are not restricted to just metric and imperial units. When you describe the capacity of an airplane, the unit used is people or seats. Similarly, when we stated the capacity of a binder, the unit used was pages. This section will highlight some of the common metric and imperial units used to describe capacity. It is a good idea to include these units on your resource sheet for this module.

The metric system measures capacity in millilitres (mL), litres (L), and, infrequently, kilolitres (kL).

Metric Capacity

1000 mL = 1 L

1000 L = 1 kL

On the other hand, the imperial system of measurement has a variety of units for capacity and they vary from country to country. These units were developed because it was hard to measure capacity in cubic units a few centuries ago, so they used "standard" containers as a referent. The most frequently used terms are *fluid ounces* and *gallons*. It is important to state *fluid* ounces because stating only "ounces" refers to mass.

Imperial Capacity

(American)

128 US fl. oz. = 1 US gallon

(Canadian)

160 CDN fl. oz. = 1 CDN gallon

Note that 1 US fl. oz. is slightly larger than 1 CDN fl. oz.

We use many different units to describe capacity in everyday life.

Standard Name	US Imperial	Canadian Imperial	Metric Capacity
cup*	8 fl. oz.	8 fl. oz.	250 mL
quart	32 fl. oz.		946 mL (almost 1 L)
gallon	128 fl. oz.	160 fl. oz.**	3.785 L per US gallon 4.546 L per CDN gallon
bushels, wet	8 gallons 1024 fl. oz.	8 gallons 1280 fl. oz.	30.28 L per US bushel 36.37 L per CDN bushel

* Note: One cup US imperial is more than one cup CDN imperial, and both are less than 250 mL. Be very careful about which system you are using. using 8 fl. oz. in a cup is a standard measurement in a kitchen.

** Using 160 fl. oz. in a gallon is prescribed by the Canadian government.

Notice that in Canada the imperial measurements sometimes agree with the British system and sometimes they agree with the US system. The same words are used but the measures are different. Converting measures from one imperial system to another can be very confusing. The following two websites may be useful for converting between Canadian imperial, American imperial, and metric units:

- http://convert.french-property.co.uk/
- www.onlineconversion.com/

Calculating Capacity

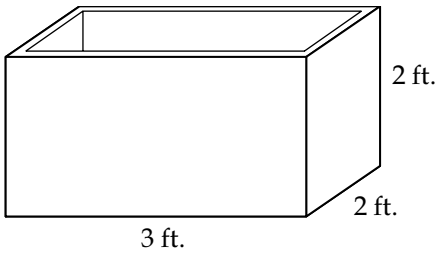
Now that you know which units describe capacity, you are ready to learn how to calculate capacity.

To calculate the capacity of an object, you must know its *inside* dimensions. To find the dimensions of the inside, you can either measure them directly OR measure the outside dimensions and the thickness of the sides and bottom.

One way to think of capacity problems is to imagine that you are finding the volume of the inside of the container.

Example 1

Calculate the capacity of the container below if the sides are all 0.25 feet thick, and the bottom is 0.5 feet thick.



Solution

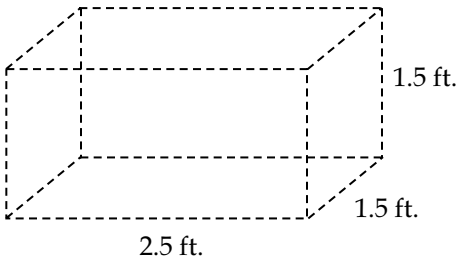
Dimensions of the inside:

length = 3 ft. − (2 × 0.25) = 2.5 ft.

width = 2 ft. − (2 × 0.25) = 2 − 0.5 = 1.5 ft.

height = = 2 ft. − (0.5) = 2 − 0.5 = 1.5 ft.

Inside



Capacity = $Bh = lwh$

$= 2.5 \times 1.5 \times 1.5 = 5.625$

Therefore, the capacity of this container is about 5.6 ft.^{3}.

Wait one second! This is not a unit to describe capacity—a cubic foot is a unit of volume.

Converting From Volume to Capacity

You will need to convert between units describing volume and units describing capacity. Given that 1 cm^3 (volume) = 1 mL (capacity), it is possible to determine, for example, the number of litres in one cubic metre.

$$1\ cm^3 = 1\ mL$$

$$1\ 000\ 000 \times 1\ cm^3 = 1\ mL \times 1\ 000\ 000$$

and we know that 1 000 000 cm^3 = 1 m^3, so . . .

$$1\ 000\ 000\ cm^3 = 1\ 000\ 000\ mL$$

$$1\ m^3 = 1\ 000\ 000\ mL$$

We also know that 1000 mL = 1 L, so . . .

$$1\ m^3 = \left(\frac{1\ 000\ 000}{1000}\right)L$$

$$1\ m^3 = 1000\ L$$

Conversions between Volume and Capacity

Metric

1 cm^3 = 1 mL	1000 cm^3 = 1 L
1 m^3 = 1 000 000 mL	
1 m^3 = 1000 L	0.001 m^3 = 1 L
1 m^3 = 1 kL	

Imperial

Because the US uses imperial measurements, we will use US fluid ounces.

1 $in.^3$ = 0.554 fl. oz.	1 fl. oz. = 1.81 $in.^3$
1 $ft.^3$ = 958 fl. oz.	1 fl. oz. = 0.0010 $ft.^3$
1 $ft.^3$ = 7.5 gallons	1 gallon = 0.13 $ft.^3$
1 $yd.^3$ = 35 853 fl. oz.	
1 $yd.^3$ = 202 gallons	1 gallon = 0.005 $yd.^3$

You may want to note these conversions on your resource sheet, so you don't have to flip back and forth throughout this lesson and the next.

To complete Example 1, you can use the conversion information above to find the capacity.

Use the conversion: 1 $ft.^3$ = 7.5 gallons

1 $ft.^3$	7.5 gal.
5.6 $ft.^3$	x gal.

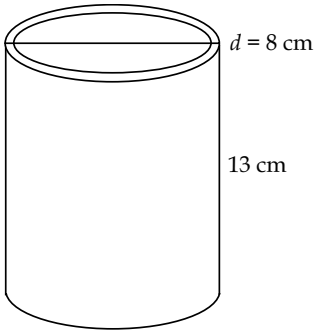
$$\frac{1}{5.6} = \frac{7.5}{x}$$

$$(1)(x) = (5.6)(7.5)$$

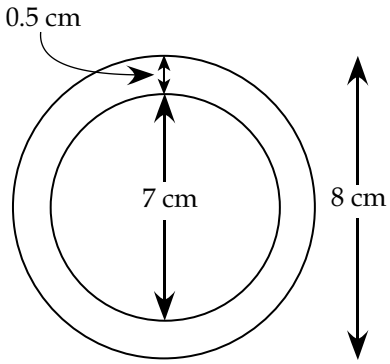
$$x = 42 \text{ gallons}$$

Example 2

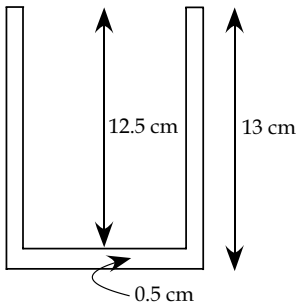
Calculate the capacity of the cylinder below if the sides and bottom are 0.5 cm thick.



Solution



The inside diameter is 7 cm, so the radius of the inside of the cylinder is 3.5 cm.



The inside height or "depth" is 12.5 cm.

Capacity = Volume = $\pi r^2 \times h$

$= \pi(3.5)^2 \times (12.5)$

$= 481.06 \text{ cm}^3$

Volume of 1 cm^3 = capacity of 1 mL

Therefore, 481.06 cm^3 = 481.06 mL

The capacity of the cylinder is 481 mL.

In the following learning activity, you are asked to calculate the capacities of various containers. Be sure to check your answers after completing the questions.

Learning Activity 2.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. What is the mean of the following data?

   5 8 2 7 3

2. Draco, Dora, and Luna have to share the $750 that their parents gave them for their trip to Europe. If it is split evenly among the three of them, how much will each get?
3. Solve for q: $4q - 10 = 14$
4. If 19% of 1053 is 200, what is 38% of 1053?
5. The three sides of a right triangle are 3.6, 6.7, and 5.6. Which side is the hypotenuse?

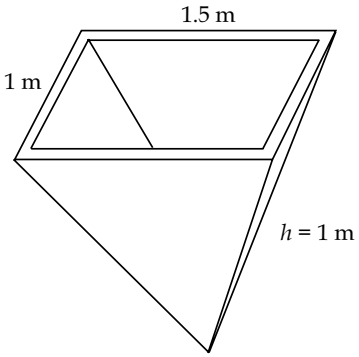
continued

Learning Activity 2.7 (continued)

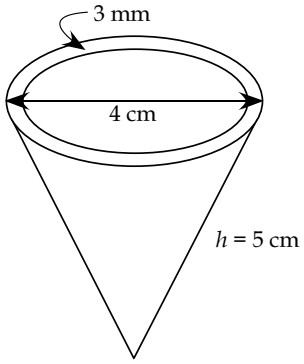
Part B: Calculating Capacity

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

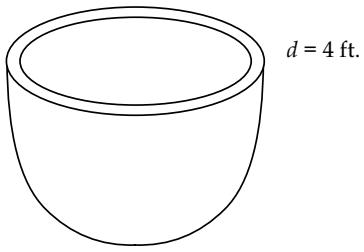
1. Calculate the capacity of the following objects.
   a) The sides of this pyramid are 15 cm thick, and that makes the inside height 96.6 cm.



   b) The sides of this cone are 3 mm thick and that makes the inside height 4.25 cm.



   c) The thickness of this semi-sphere is 1 inch.



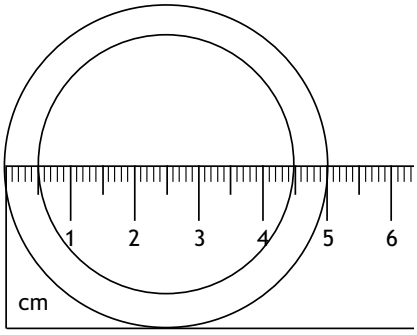
continued

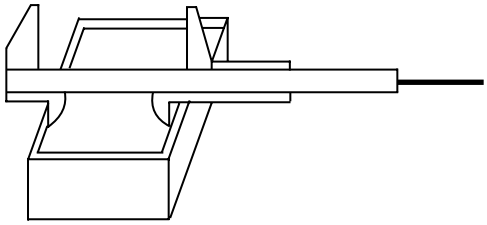
Learning Activity 2.7 (continued)

2. Write down your strategy to calculate capacity. Include step-by-step instructions.

3. Find an object at your house that has an opening so that it can store something (e.g., a box, jewelry box, bottle cap, an open can, etc.).

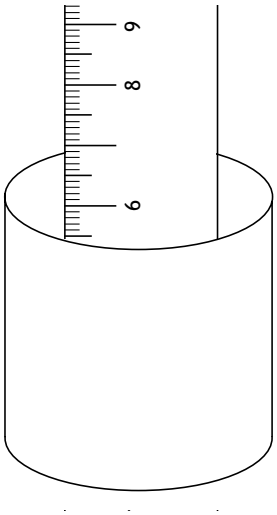
   Step 1: Use a ruler or calipers to measure the inside length and width or diameter of the object. Remember that if you are using calipers, use the inside jaws.

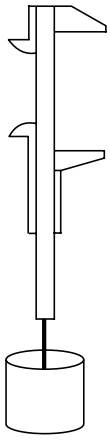




   If you have measured the diameter, calculate the radius.

   Step 2: Use the same ruler or calipers to measure the depth of the inside. If you are using calipers, you can use the depth probe for this.





   Step 3: Use the measurements from Steps 1 and 2 to calculate the volume of the space inside the object.

   Step 4: Convert the units of volume to capacity.

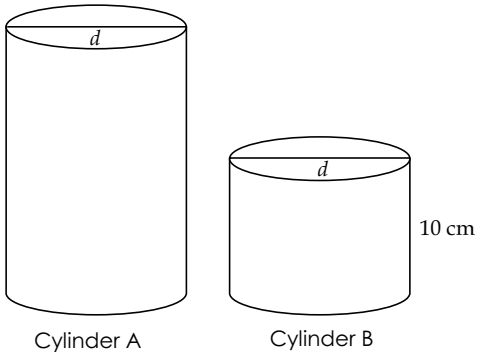
Estimating the Size of an Object

Volume

Earlier in this module, you used referents to estimate the surface area of an object. When estimating the volume of an object, it is always easiest to compare it with an object that has the same shape but different size. As you did with surface area, you would compare the dimensions in order to approximate how much the volume would change. In Lesson 4, you learned how changing one, two, or all of the dimensions of an object affected the volume of the object. The patterns you saw varied depending on the shape of the bases (rectangular and triangular objects were affected differently than circular objects). The following examples show how to apply the principles outlined in the previous lesson to estimate the volume of an object.

Example 1

You have Cylinder A and Cylinder B. The diameter of both objects is the same. Cylinder B is 10 cm tall, and the volume is 503 cm^3. If you stacked two of Cylinder B on top of each other, they would have the same height as Cylinder A. Estimate the volume of Cylinder A.



Solution

Since the diameter of Cylinders A and B are the same, their radii are also the same. This means that in order to build Cylinder A from Cylinder B, the only difference would be in the height. The height of Cylinder A is double that of Cylinder B, so the volume is also doubled.

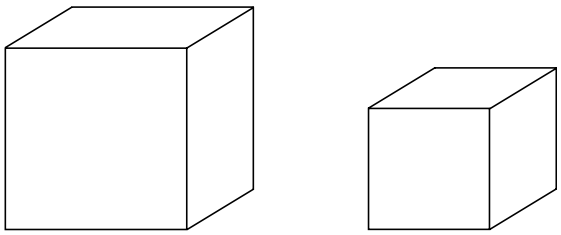
$2 \times 503 = 1006\ cm^3$

In this example, we used the smaller object to estimate the volume of the larger object.

The volume of the smaller container is multiplied by 2 because only one dimension was changed and the multiplier is 2. (Remember that a multiplier is the number by which you multiply the dimension of the original object to get the dimension of the new object.)

Example 2

There are two boxes. Box A appears to be one-and-a-half times taller and one-and-a-half times longer than Box B. The width of both boxes is about the same. The volume of Box A is 512 in.^{3}. What is the approximate volume of Box B?



Solution

If Box A has 1.5 times the height and 1.5 times the length of Box B, then the volume of Box A should be about 1.5 × 1.5 = 2.25 times the volume of Box B. In an equation, this would look like

2.25 × V (Box B) = V (Box A)	Insert the value(s) that you know.
2.25 × V (Box B) = 512	You want to know V (Box B), so divide both sides of the equation by 2.25.
[2.25 × V (Box B)] ÷ 2.25 = 512 ÷ 2.25	
V (Box B) = 227.555…	The volume of Box B is 228 in.^{3}.

This example is the reverse of Example 1. We use the larger object to estimate the volume of the smaller object. The multiplier in this example (2.25) is a combination of two multipliers (1.5 and 1.5) because two dimensions are changed. We multiply 1.5 by 1.5, because when we calculate volume, we multiply the dimensions (length, width, and height) and we treat the multipliers the same way we treat the dimensions.

You can use many referents when estimating the volume of an object, but you must use the same shape (rectangular prism with rectangular prism, sphere with sphere). This would be a good hint to include on your resource sheet.

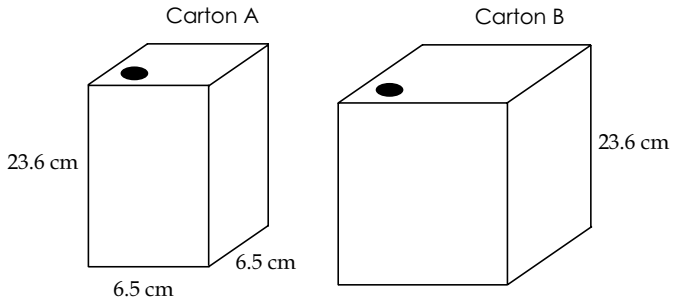
The next learning activity requires that you find a referent in your house and use it to estimate the volume of another object.

Capacity

You can use the same procedures you applied for estimating the volume of an object to estimate the capacity of a container.

Example 3

Carton A has a capacity of 1 L. The height of Carton B is the same as Carton A, but both the length and width are 1.4 times the size of Carton A's length and width. What is the approximate capacity of Carton B to the nearest litre?



Solution

To estimate the capacity of Carton B, you can find the total multiplier comparing Cartons A and B, and then apply it to the capacity of Carton A:

The changes in dimension are 1.4 times and 1.4 times, so

$1.4 \times 1.4 = 1.96 \approx 2$

Since 2 is the total multiplier, you would expect the capacity of Carton B to be two times larger than Carton A, so the estimated capacity of Carton B is

$2 \times 1 \text{ L} = 2 \text{ L}$

To check your work:

l and $w = 1.4 \times 6.5 = 9.1$ cm

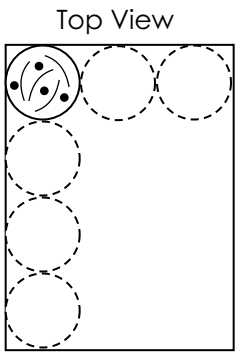
$V = lwh = (9.1)(9.1)(23.6)$

$= 1954.3 \text{ cm}^3$

Capacity = 1954.3 mL $\approx$ 2 L, which is the same as our estimation.

You can also estimate the capacity by using the intended contents of the container as a referent. The following is a situation where this is possible.

Luca has just made some muffins and needs to put them in a container. Luca uses the size of one muffin to estimate the number of muffins he can place in a rectangular container.



Luca estimates that he will be able to place three muffins across the width, four muffins along the length of the container, and only one layer of muffins, so the container's capacity is approximately (3 × 4 × 1) 12 muffins. Luca will be able to use this container to store 12 muffins.

Examples of capacity referents you can use include:

- a sugar cube is about 1 cm^3 or 1 mL
- a teaspoon is 5 mL
- a tablespoon is 15 mL
- a short water glass can hold approximately 1 cup

In general, the steps to estimate the volume or capacity are:

I. Compare each dimension of the known and unknown objects. Approximately how many times longer/wider/taller is one than the other? Instead of saying, for example, that one is 3 cm longer than the other, state that it is 1.5 times longer. In this way, you are identifying the multiplier.

II. Combine the multipliers as if you were using them in the formula. For example, if you estimate that the radius of a sphere is doubled, then the volume should be $2^3 = 8$ times larger, because the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$ (the radius is cubed).

III. Apply the combined multiplier to the volume of the known object in order to calculate the volume of the unknown object.

The following learning activity has some questions where you estimate volume and capacity. Check the answer key at the end of the module if you need help, or ask your learning partner.

Learning Activity 2.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. How many reflections do you need to return to the original position?

2. Solve for k: $8 - 2k = 2$
3. The length of a book is $\frac{3}{4}$ of a foot. What is the length in inches?
4. You finish school at 3:30 pm. It takes you 20 minutes to get to work by bus. You work for five hours. You get a ride home, and this takes only 10 minutes. You do homework for an hour before going to bed. At what time do you go to bed?
5. After having your car filled with gas, the total cost was $36.75. You handed the gas jockey $40.00, and asked for a toonie. How much did you tip the gas jockey?

Part B: Estimating Volume and Capacity

Unlike other learning activities, these questions are not like the ones that will be on your assignments and midterm examination since you will not be asked to directly measure an object. However, doing these problems correctly will deepen your understanding of using referents for estimation of volume and capacity. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. To complete this question, you will need to find two objects:
   i) an object whose volume you don't know, such as a box
   ii) an object such as a sugar cube, where you know the volume, and where the shape is the same as Object (i)

continued

Learning Activity 2.8 (continued)

You don't have to use a box. Any 3-D object will work, although using an oddly shaped object that you haven't used before will make this exercise more challenging.

Once you have these two objects, use the method of finding the total multiplier, comparing the two objects in order to estimate the volume of Object (i). Use a ruler or tape measure to find the actual measurements of Object (i), and calculate the volume. Was your estimate close? If not, why?

2. To complete this question, you will need to find two objects:
   i) An object whose capacity you don't know, such as a milk carton or jar.
   ii) An object that is the same shape as Object (i), where you know the capacity, such as a larger/smaller milk carton or a can.

   Once you have these two objects, use the method of finding the total multiplier comparing the two objects in order to estimate the capacity of Object (i). Use a ruler or tape measure to find the actual measurements of Object (i), and calculate the capacity. Was your estimate close? Why or why not?

Lesson Summary

In this lesson, you studied the relationship between capacity and volume. You also identified the units for capacity and volume, and discovered that they are not the same. In the next lesson, you will solve problems involving volume and capacity in everyday life.

Notes

Lesson 6: Applications of Volume and Capacity

Lesson Focus

In this lesson, you will

- ❑ calculate the volumes and capacities of 3-D objects, including composite objects

Lesson Introduction

The fuel tank of a car has a certain capacity, perhaps 75 litres. When you buy gasoline, you may need 50 litres to fill the tank. The volume of gasoline you buy is 50 litres. The capacity is the amount of material a container can hold, and the volume is the amount of material in the container—in this case, the number of litres of gasoline in the tank.

The following problems are applications of volume and capacity calculations in everyday situations.

Everyday Life

You estimate volume and capacity as though it is second nature. Can you think of any other examples of where you use or estimate volume or capacity frequently?

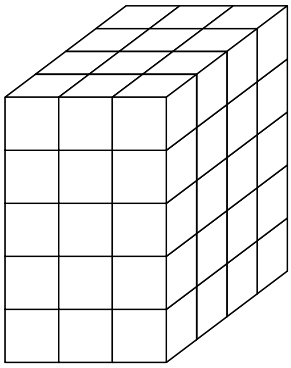
The following examples demonstrate where and how we calculate volume and capacity in the real world, after which you will have a chance to practise in the learning activity for this lesson.

Example 1

A storage company stacks boxes three across, four deep, and five layers high. What volume of boxes does this represent?

Solution

Sketch a diagram of this situation.



The area of the base of this stack is 3 by 4 boxes or 12 boxes.

$B = 3 \times 4$

$B = 12$

The height of the stack is 5, so the volume is

$V = Bh$

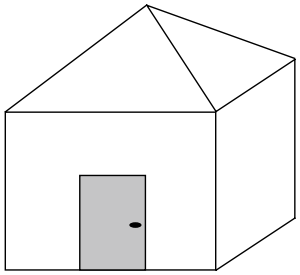
$V = 12(5)$

$V = 60$

The stack is 60 boxes.

Example 2

A tool shed has the following shape. Determine the storage space inside the shed.



Interior dimensions:

Total height: 12 ft.

Height of rectangular prism: 8 ft.

Length: 7.5 ft.

Width: 4.5 ft.

Solution

In this case, the best unit to describe the storage space (capacity) is the same as the volume unit—$ft.^3$. Note that the dimensions given are the interior dimensions, so we don't need to worry about the thickness of the walls.

This object is composed of a rectangular prism and a rectangular pyramid. Calculate the volume of the two spaces and combine them.

Prism

$V = Bh$

The base is rectangular, so its area is

$A = L \times W$

$A = 7.5 \times 4.5$

$A = 33.75 \text{ ft.}^2$

so

$V = 33.75 \times 8$

$V = 270 \text{ ft.}^3$

Pyramid

The pyramid has the same shaped base as the prism, so its area is also 33.75 ft.^{2}. The height of the roof is the difference between the total height and the height of the prism (12 − 8 or 4).

The volume of the pyramid is

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}(33.75)(4)$$

$V = 45 \text{ ft.}^3$

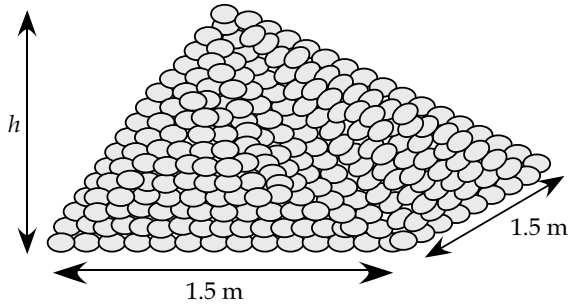
Total volume = 270 + 45

Total volume = 315

The shed has a storage space of 315 cubic feet.

Example 3

A grocery store displays 1 cubic metre of oranges (ignoring spaces between the oranges) arranged in a pyramid with a square base, 1.5 m along one side. How high is the display?



Solution

In this question, you know the volume and must solve for a missing dimension—in this case, the height of the pyramid.

Write the formula, substituting all known values, and solve for the unknown dimension.

$$V = \frac{1}{3}Bh, \text{ where } B = 1.5^2$$

$$1 = \frac{1}{3}(1.5)^2 h$$

$$1 = 0.75h$$

$$h = \frac{1}{0.75}$$

$$h = 1.333$$

The display of oranges is 1.3 m high.

Example 4

Find the volume of a volleyball if its circumference is 26″.

Solution

First, you need to calculate the length of the radius.

$$C = 2\pi r$$

$$26 = 2\pi r$$

$$r = \frac{26}{(2\pi)}$$

$$r = 4.13802852 \text{ inches}$$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (4.13802852)^3$$

$$V = 296.8035205$$

The volume of the volleyball is approximately 296.8 cubic inches.

Example 5

Find the volume of a carrot that is nine inches long and one inch across at the top.

Solution

A carrot is approximately cone shaped.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}\left(\pi r^2\right)h$$

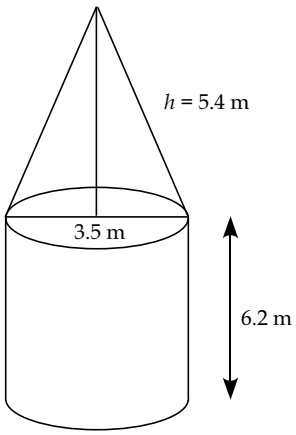
$$V = \frac{1}{3}\left(\pi 0.5^2\right)9$$

$$V \approx 2.4$$

The volume of this carrot is approximately 2.4 in.^{3}.

Example 6

The turret on a castle has a cone-shaped roof above a cylindrical structure. This turret will be a tourist area, and safety engineers must know its air capacity because they need to design a fan that replaces the air inside the turret every hour. Determine the capacity of the turret, assuming the measurements given are the interior dimensions. Round your final answer to the nearest hundredth of a cubic metre.



Solution

Cone

$$V = \frac{1}{3}\pi r^2 \times h$$
$$= \frac{1}{3}\pi (1.75)^2 \times (5.4)$$
$$= 17.3180\ldots$$

Pyramid

$$V = \pi r^2 \times h$$
$$= \pi (1.75)^2 \times (6.2)$$
$$= 59.65099\ldots$$

The total capacity of the turret is 17.32 + 59.65 = 76.97 m^3.

Learning Activity 2.9

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the pattern: 1, 4, 9, 16, _____ , _____
2. At the hockey game, Murphy bought a hot dog for $3.50, a soft drink for $2.75, and some popcorn for $3.25. How much is his total?
3. Murphy gives the cashier a $20 bill and gets $11 back. Is this the correct change?
4. At Deklin's school, they have 72-minute classes, except on Friday, when their classes are 45 minutes long. If Deklin is in math this term, how much time will he spend in the math classroom in one week?
5. Solve for t: $4t - 8 = 0$

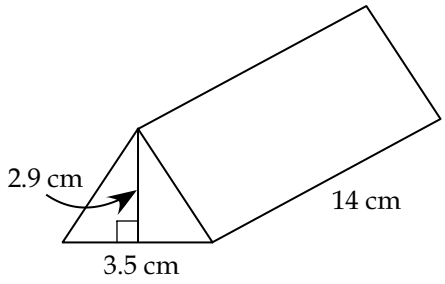
continued

Learning Activity 2.9 (continued)

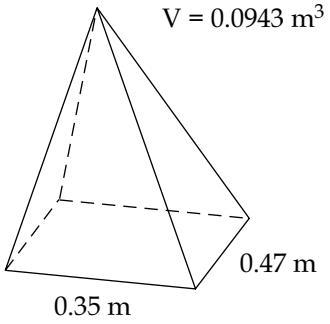
Part B: Applying Volume and Capacity

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

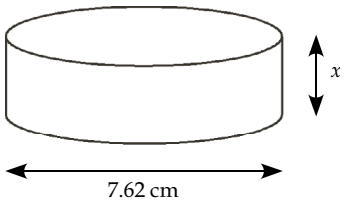
1. A chocolate bar shaped like a triangular prism is 14 cm long. Its triangular base is 3.5 cm long and 2.9 cm high. Calculate the volume of chocolate in the bar.



2. Pepper has made a rectangular pyramid sculpture with a volume of 0.0943 m^3. The pyramid with base dimensions of 0.35 m by 0.47 m is shown below. Determine its height.

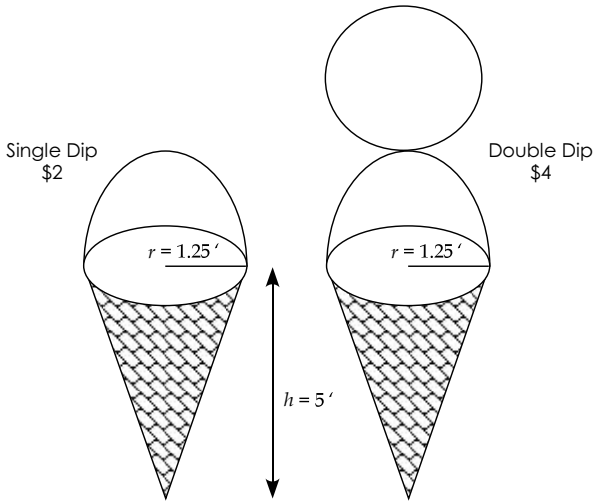


3. A hockey puck has a diameter of 7.62 cm and a volume of 115.83 cm^3. Calculate the thickness of a hockey puck.



continued

4. An ice cream cone has a radius of 1.25″ and a height of 5″. If you buy a single-dip cone for $2, the cone is completely filled with ice cream and there is a half-sphere shaped scoop above the cone. If you order the double dip for $4, they put an extra sphere-shaped scoop of ice cream on top. Which is the better deal? Explain.



5. Fred wants to estimate the weight of a large wooden beam. He has a length of 2 × 6 wood that he weighs, and the weight is 12 pounds. He uses his piece of wood as a referent to determine the approximate weight of the beam. After measuring the beam against his piece of wood, he discovers that the beam is 12 times as long as the piece of wood, 6 times as wide, and 4 times as deep.
   a) Calculate the weight multiplier.
   b) Calculate the approximate weight of the beam.

Lesson Summary

In this lesson, you have explored many different applications for volume and capacity, and have solved problems related to this. The following assignment is the last in the module and focuses on what you have learned in this lesson as well as in Lesson 5.

Assignment 2.3

Applications of Volume and Capacity

Total: 30 marks

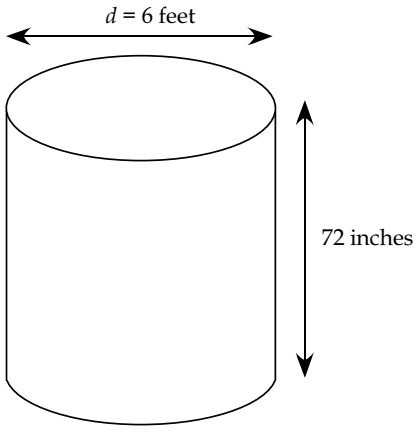
Note to Students: Be sure that your resource sheet is complete. You are encouraged to use it as an aid as you complete this assignment, so feel free to add to it as needed.

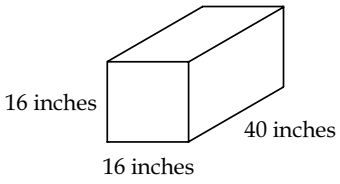
1. In your own words, write the definition of capacity. *(1 mark)*

2. A farmer is baling his hay. He has the option of making cylinder-shaped "round bales" or prism-shaped "square bales."

   The round bales have a diameter of 6 feet and are 72 inches long.

   The square bales have a base with 16 inches per side and are 40 inches long.





continued

a) Determine the volume of hay in each bale. Write each answer as cubic inches. Show your work. *(5 marks)*

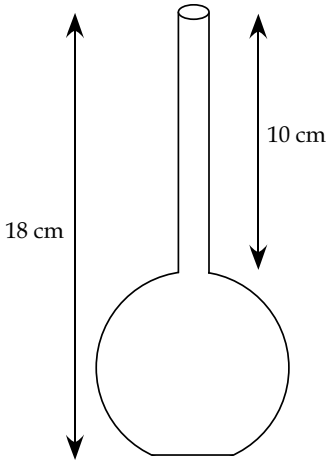
b) One round bale feeds his herd of 50 cows for one day. How many square bales would the farmer have to make to equal one round bale? *(1 mark)*

3. Calculate the circumference of a beach ball whose air capacity is 22.7 L. Round the final answer to the nearest tenth of a metre. [**Hint:** Assume the plastic skin of the ball is very thin, and convert the capacity into units related to volume.] Show your work. *(5 marks)*

continued

Assignment 2.3: Applications of Volume and Capacity (continued)

4. Angela has a flask for chemistry that is missing its capacity label. The exterior diameter of the opening is 2 cm, and the thickness of the glass for the whole flask is 3 mm.



a) What are the two shapes that make up the flask? *(2 marks)*

b) What are the inside dimensions of the two shapes? Show your work. *(3 marks)*

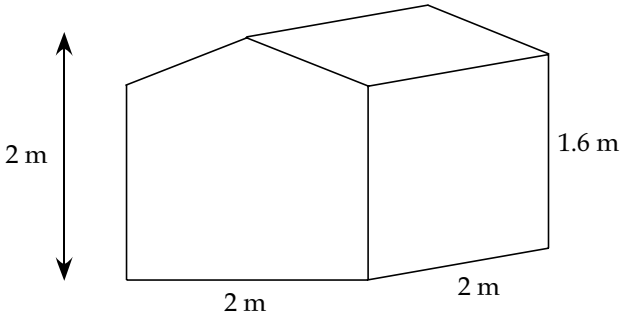
c) What is the approximate capacity of the flask in mL, to the nearest tenth? Show your work. *(5 marks)*

continued

Assignment 2.3: Applications of Volume and Capacity (continued)

5. a) Write a scenario where you would need to determine the capacity of the following building. Be sure to indicate the purpose of the building. *(1 mark)*

   b) Calculate the capacity of the building. Interior dimensions of the building are shown. Show your work. *(3 marks)*



6. Jess wants to estimate the capacity of a cylindrical water jug. The only referent she has is a 355 mL pop can. By holding the pop can next to the water jug, she discovers that the water jug is approximately 2.5 times as tall and five times as wide as the pop can. Show your work.

   a) Calculate the capacity multiplier. *(2 marks)*

   b) Calculate the approximate capacity of the water jug. Round your answer to the nearest litre. *(2 marks)*

Module 2 Summary

You have completed another module. In this module, you explored the geometry of 3-D objects, including prisms, pyramids, and spheres. You learned how to calculate and estimate the surface area, volume, and capacity of these objects, and you investigated how these three properties are related (if they are). Then you solved everyday problems using the surface area and volume formulas.

If you still have questions about the material in this lesson, then take some time to review your work, or ask your learning partner or tutor/marker for help. Also, be sure to complete your Module 2 Resource Sheet if you have not already done so. Compare the information on your resource sheet with the information in Appendix B: Glossary found at the end of this course.

The next module is very different from this one. You will study statistics, where you will focus on drawing and analyzing graphs.

Submitting Your Assignments

It is now time for you to submit the Module 2 Cover Assignment and Assignments 2.1 to 2.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 2 assignments and organize your material in the following order:

- ❑ Module 2 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 2 Cover Assignment: Geometric Patterns
- ❑ Assignment 2.1: Surface Area
- ❑ Assignment 2.2: Volume
- ❑ Assignment 2.3: Applications of Volume and Capacity

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Grade 11 Essential Mathematics (30S)

Module 2
3-D Geometry

Learning Activity Answer Keys

Module 2:
3-D Geometry

Learning Activity 2.1

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If 12% of 250 is 30, what is 12% of 500?
2. There are 52 cards in a deck, 13 cards of each suit. If the suits are hearts (red), spades (black), clubs (black), and diamonds (red), what percentage of the cards are black?
3. If $x = 2$, evaluate $4x - 18$.
4. You earn $9 per hour plus 5% commission. If you work 20 hours this week and your sales are $700, how much will your gross pay be?
5. There are 5 people assigned to a group. Sheniqua will not sit beside Arjun and Tapanga. Tapanga refuses to sit beside Soloman. Dejon wants to sit beside Tapanga. If they are sitting in a circle, who will be beside Sheniqua?

Answers:

1. 60 (500 is double 250, so 12 percent is also doubled)
2. 50% (Since each suit has the same number of cards, and two out of four suits are black, $(2 \div 4) \times 100\% = 50\%$ of the cards.)
3. -10 $(4 \times 2 - 18)$
4. $215 (10% of 700 is 70, so 5% of 700 is 35. Also, $9 \times 20 = 180$.
   Total = 180 + 35 = $215.)
5. Soloman and Dejon (She will not sit beside the other two.)

Part B: What Is a Net?

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand. Don't forget that you can ask your learning partner or tutor/marker for help if you are having a hard time understanding.

The following learning activity will help you understand the meaning of *net*. It requires materials that you can find in your house or classroom. Ask your parent or ISO facilitator where you can find them. If you do not have access to the materials that you need to complete one of the two exercises, do not worry—but be sure to complete at least one of them.

Materials:

Activity 1:

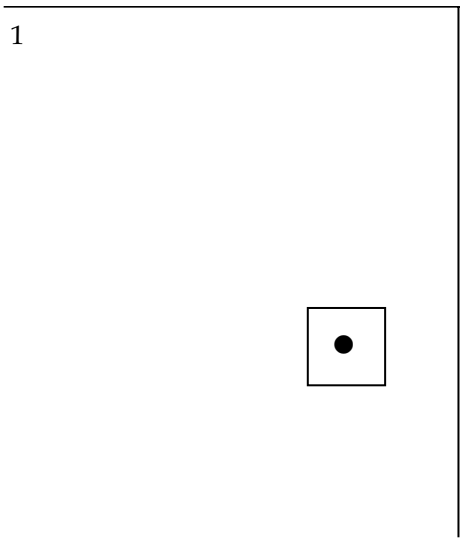
- a 6-sided die (alternatives: sugar cube, child's alphabet block, etc.)
- a piece of paper
- a writing instrument

Activity 2:

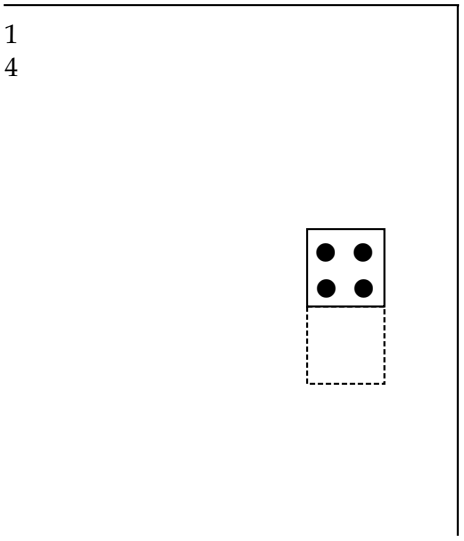
- a 6-sided box you can cut apart
- scissors
- a thick marker
- a piece of paper
- a writing instrument

1. To complete this question, you will need a 6-sided die and a piece of paper. You will draw a net of the die by following the step-by-step instructions. The images that accompany each step are examples of what you should see on your page. (**Note:** You may start with any number facing up. The example below starts with the number 1 facing up.)

A. Place the die on the piece of paper about 1 cm from the right edge and about 5 cm below the top edge. On the left side of the page, write down the number that is facing up on the die (based on the diagram below, this number would be 1). Holding the die in place, trace the shape of the die.

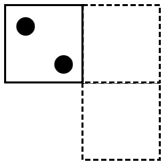


B. Turn the die over once, toward the top of the page. The bottom edge of the die in the new position should be against the top line of the die you traced in step A. Write down the new number that is facing up on the left side of the page, under the last number that was facing up (based on the diagram below, you would write 4 but you may be seeing a different number). Holding the die in place, trace the shape of the die. The dotted lines in the following diagram represent the lines that you traced in the previous step.



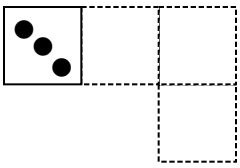
C. Turn the die over once to the left. The right edge of the die in the new position should be against the left line you traced in step B. Write down the new number that is facing up on the left side of the page, under the last number that was facing up. Holding the die in place, trace the shape of the die.

1
4
2

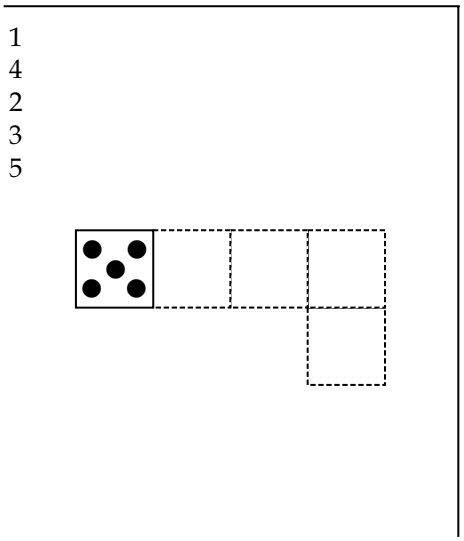


D. Turn the die over once to the left. The right edge of the die in the new position should be against the left line you traced in step C. Write down the new number that is facing up on the left side of the page. Hold the die in place as you trace its shape.

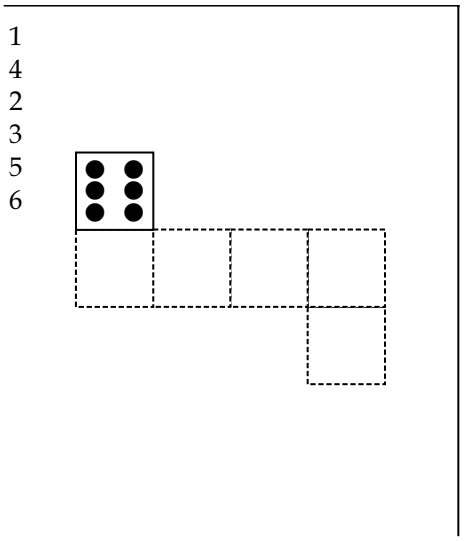
1
4
2
3



E. Turn the die over once to the left. The right edge of the die in the new position should be against the left line you traced in step D. Write down the new number that is facing up on the left side of the page. Hold the die in place as you trace its shape.



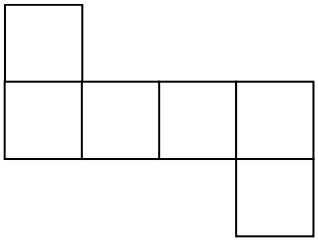
F. Turn the die over once, toward the top of the page. The bottom edge of the die in the new position should be against the top line you traced in step E. Write down the new number that is facing up on the left side of the page, under the last number that was facing up. Holding the die in place, trace the shape of the die.



Before you check your answer, be sure that each number (1 to 6) only appears once in the list you made on the left side of the page. If you are missing a number and/or have a repeated number, this means that you traced the same face twice, so your net will have two pieces that overlap if you fold it into its shape. Restart the exercise and be sure you follow the instructions precisely. If you have done the steps correctly, the drawing on your paper is a net diagram of the die.

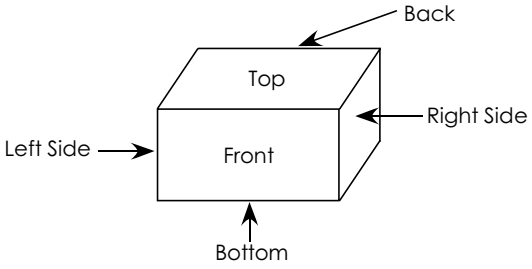
Answer:

The final diagram should look like this:



This is the net diagram of your die.

2. Find a six-sided box around your house that you will be able to cut apart, such as an empty cereal box.
   A. With a marker, trace the edges of each face. Label each face (front, back, top, bottom, left side, right side).

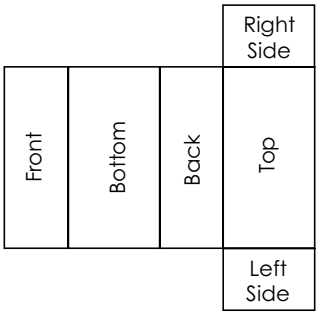


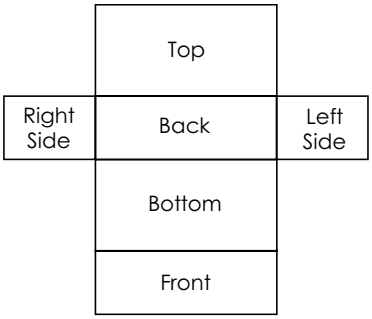
   B. Do your best to draw the net of this box on a piece of paper without cutting the box apart. The drawing does not need to be the actual size. You can reduce the heights of the sides. The drawing does not need to be to scale either. Label the sections according to the faces you think they represent.
   C. Cut the box along the edges and be sure to keep each face connected to at least one other face. The box should still be in one piece. You should be able to lay the cut-up box flat on a table or the floor.
   D. Lay the box down with the marker side facing up. Sketch what you see on a piece of paper (this can be done on the same piece of paper on which you did Step B). Include the labels.

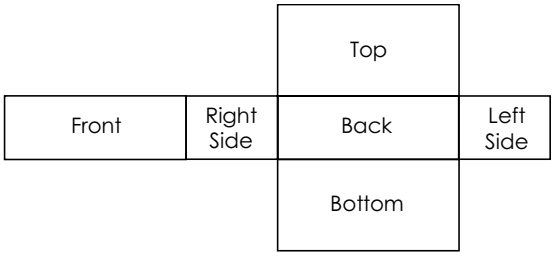
Do your diagram and the actual cut-out look the same? If so, good work! If not, is it possible that the net you drew is also correct?

Answer:

Your diagram may or may not look like the cut-out you made. You can check to see if your diagram (made in Step B) would make a box by comparing the sequence of top, front, bottom, back, and the sides. You should not have two sides touching each other because they do not touch each other when the box is constructed. The same rule applies for top/bottom and front/back. Some examples of what your net and cut-out might look like include the following:







Learning Activity 2.2

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Travis invests $1500 for one year. The investment has an interest rate of 5%. How much will his investment be worth after one year?
2. Evaluate: $\frac{4}{8} \times 14$
3. Liu and Goh went out for dinner and the total cost of the meal was $28.50 including taxes. Goh pays by leaving $30 on the table to include the tip. How much is the tip?
4. Fran has an accuracy rating of 60% in volleyball. Fran's accuracy in basketball is 20% higher than volleyball. In badminton, Fran's accuracy rating is half of his basketball rating. What is Fran's accuracy in badminton?
5. Write two fractions that are equivalent to 0.20.

Answers:

1. $1575 (5% of 1500 is half of 10%. 10% = 150 so 5% = $75. Total investment = $1575.)
2. $7 \left(\frac{4}{8} \times 14 = \frac{1}{2} \times 14 = \frac{14}{2} = 7 \right)$
3. $1.50 (30 − 28.50)
4. 40% (basketball: 60 + 20 = 80 percent

   badminton: $\frac{1}{2}$ of 80)
5. $\frac{20}{100}, \frac{2}{10}, \frac{1}{5}, \frac{3}{15}$ (List any two; other answers are possible.)

Part B: Surface Area Calculations

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. This question is another hands-on exercise. You will need an orange (navel oranges work best, but other kinds, as well as other fruit, may also work well—you want something that is spherical and that you can peel easily), two pieces of paper, a ruler, and a compass. If you do not have access to an orange, use some other spherical object from which you can easily remove the top layer.

   A. Use your ruler to measure the diameter of the orange in three different directions (e.g., top to bottom, side to side, and diagonally). Record each of these measurements on the corner of your paper.

   B. Find the average of these three measurements. This is the approximate diameter of the orange. Calculate the radius of the orange.

   C. Adjust your compass to have the same radius as the orange in Part B. Use your ruler to measure the distance between the arms of your compass. Draw six circles with this radius, three on each page (do this on separate sheets of paper, as you will not be able to turn them over).

   D. Peel your orange in small pieces, approximately 1 cm^2 or smaller. Fit the pieces into the circles that you have drawn on your paper, so you cannot see any paper inside the circle without making the pieces overlap. Try to fit your pieces of orange into the fewest number of circles possible.

   How many circles did you fill? What does this tell you about the surface area of a sphere? Enjoy your orange while you do the rest of this learning activity.

   Answer:

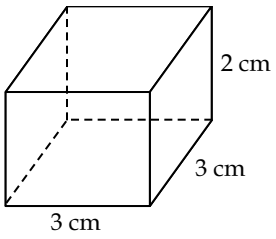
   You should have filled four circles with peel. This indicates that the surface area of your orange is four times the area of a circle with the same radius. This method of finding the surface area of a sphere is similar to the "faces approach" you learned for prisms and pyramids.

   You have just confirmed the formula for the surface area of a sphere:

   $SA = 4\pi r^2$

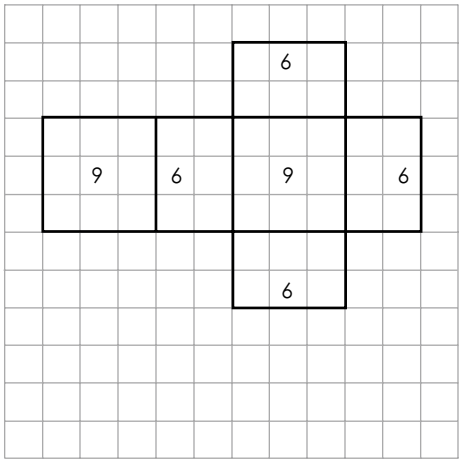
2. Name and calculate the surface area of the following objects. Use the method of calculation stated beside the object.

   a) Find the surface area using a net.



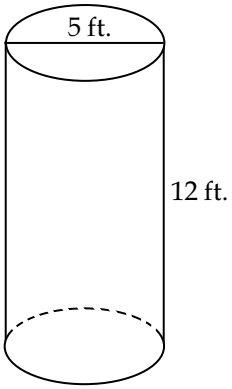
Answer:

Square or rectangular prism



$9 + 6 + 9 + 6 + 6 + 6 = 42 \text{ cm}^2$

b) Find the surface area using the formula.



Answer:

Cylinder (circular prism)

$SA = 2B + Ph$

$B = \pi r^2 \qquad \left(r = \frac{1}{2}d = 2.5 \text{ feet}\right)$

$= \pi (2.5)^2$

$= \pi (6.25) = 19.635 \text{ ft.}^2$

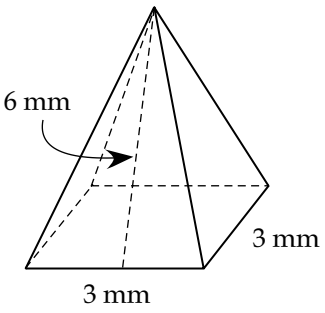
$P = C = \pi d$

$= \pi (5) = 15.708 \text{ ft.}$

$SA = 2(19.635) + (15.708)(12)$

$= 39.270 + 188.496 = 227.8 \text{ ft.}^2$

c) Find the surface area using the faces approach. The base is a square.



Answer:

Square pyramid

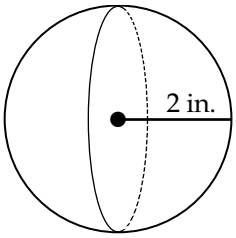
area of bottom = (3)(3) = 9 mm^2

area of front triangle = $\frac{1}{2}$(3)(6) = 9 mm^2

Total = square + 4(triangle)

Total = 9 + 4(9) = 45 mm^2

d) Find the surface area using the formula.



Answer:

Sphere

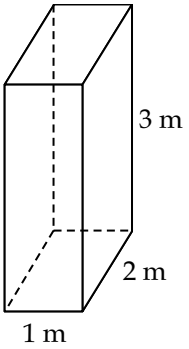
$SA = 4\pi r^2$

$= 4\pi(2)^2$

$= 4\pi^4$

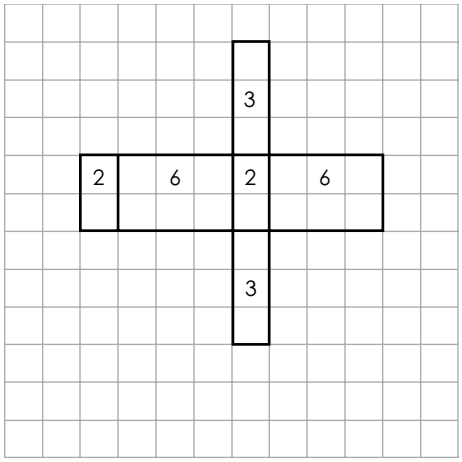
$= 16\pi = 50.3 \text{ inches}^2$

e) Find the surface area using a net.



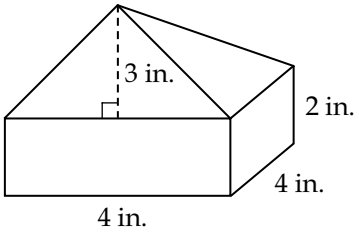
Answer:

Rectangular prism



$2 + 6 + 2 + 6 + 3 + 3 = 22 \text{ m}^2$

f) Find the surface area using the formulas OR the faces approach.



Answer:

The answer will be the same using both approaches. This solution shows how you would find the surface area using the formulas.

Square Prism

The top and bottom will be the bases. Since the top is not part of the surface area, it will not be included.

$SA = B + Ph$

$B = 4 \times 4 = 16$ in.^{2}

$P = 4 + 4 + 4 + 4 = 16$ in.

$SA = 16 + (16 \times 2)$

$= 16 + 32 = 48$ in.^{2}

Square Pyramid

The base is not part of the surface area, so it is not included in the calculation.

$$SA = \frac{1}{2}Ps$$

$P = 4 + 4 + 4 + 4 = 16$ in.

$$SA = \frac{1}{2}(16 \times 3) = 24 \text{ in.}^2$$

The total surface area is $48 + 24 = 72$ in.^{2}.

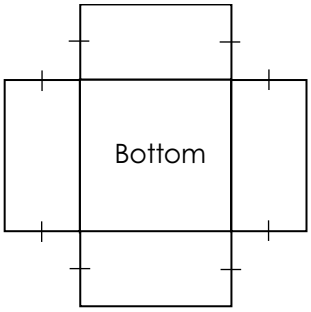
3. Briefly describe how you draw the net of a prism. Include where you start and what your steps are.

   Answer:

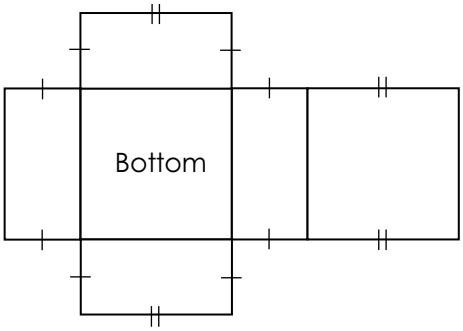
   Answers will vary. Here are some examples. If you are struggling with drawing nets, try using these to redo questions 2(a) and 2(e). If these examples do not help, talk to your learning partner or ask your tutor/marker for help. The ability to draw nets will appear again in Module 7, so it is important that you have a strategy for drawing them.

   Example 1: Start by drawing the bottom face of the prism.

   Next, draw the sides of the prism so that it looks like the sides have become detached from each other and fallen outward. Each side should extend the same distance from the bottom face.

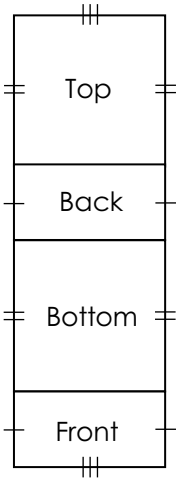


   Then draw the top of the prism attached to one of the sides.

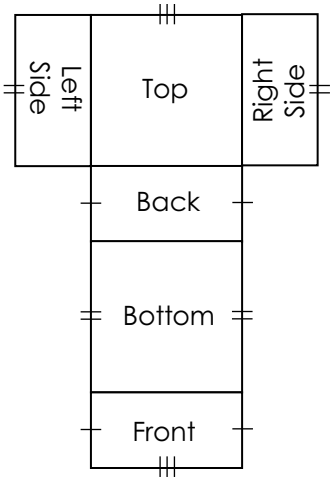


Example 2: Start by drawing the top of the prism.

Next, draw the back and then bottom and front of the prism, extending down from the top face.



Finally, draw the sides of the prism extending out from either side of the top face.



Learning Activity 2.3

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Sharmaine has to wake up at 5:00 am, and cannot function with less than nine hours of sleep. What is the latest time that Sharmaine should go to bed?
2. A cube has a side length of 4 cm. What is the surface area of the cube?
3. What is the mean of this data: 4 6 8 2
4. Shannon is four years younger than Cheri. If Cheri's age is double Shannon's, how old is Shannon?
5. There are five pieces of pepperoni per slice of pizza at Santa Monica's Pizza Parlour. If there are eight slices of pizza, how many pieces of pepperoni are on the whole pizza?

Answers:

1. 8:00 pm (Count backwards. Five hours takes you to midnight (12:00 am); account for the other four hours = $12 - 4 = 8$.)
2. 96 cm^2 (1 face = $4 \times 4 = 16$; 6 sides = $16 \times 6 = 96$)
3. 5 (mean = average so $\frac{4+6+8+2}{4} = \frac{20}{4} = 5$)
4. 4 (Shannon's age + 4 = Cheri's age; Cheri's age = 2 × Shannon's age so Shannon must be four years old because $4 + 4 = 2 \times 4$)
5. 40 (5 per slice × 8 slices = 40)

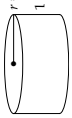
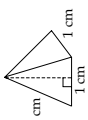
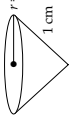
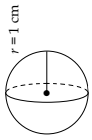
Part B: Dimension Changes

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. a) Complete the table.

Answer:

Surface Area

	Rectangular Prism 	**Cylinder** 	**Rectangular Pyramid** 	**Cone** 	**Sphere** 
Original Object	Front/back: $lh = 1 \times 1 = 1\ \text{cm}^2$ 2 Sides: $wh = 1 \times 1 = 1\ \text{cm}^2$ Top/bottom: $lw = 1 \times 1 = 1\ \text{cm}^2$ Surface area: $(2 \times 1) + (2 \times 1) + (2 \times 1)$ $= 6\ \text{cm}^2$	Top/bottom: $\pi r^2 = \pi(1)^2 = \pi\ \text{cm}^2$ Side: $Ch = 2\pi r \times h$ $= 2\pi(1) \times 1$ $= 2\pi\ \text{cm}^2$ Surface area: $(2 \times \pi) + 2\pi = 4\pi\ \text{cm}^2$	Base: $lw = 1 \times 1 = 1\ \text{cm}^2$ 4 Sides: $\frac{1}{2}ls = \frac{1}{2}(1 \times 1) = \frac{1}{2}\ \text{cm}^2$ Surface area: $1 + \left(4 \times \frac{1}{2}\right) = 3\ \text{cm}^2$	Base: $\pi r^2 = \pi(1)^2 = \pi\ \text{cm}^2$ Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(1) \times 1$ $= \pi\ \text{cm}^2$ Surface area: $\pi + \pi = 2\pi\ \text{cm}^2$	Surface area: $4\pi r^2 = 4\pi(1)^2 = 4\pi\ \text{cm}^2$
All dimensions × 3	Front/back: $lh = 3 \times 3 = 9\ \text{cm}^2$ 2 Sides: $wh = 3 \times 3 = 9\ \text{cm}^2$ Top/bottom: $lw = 3 \times 3 = 9\ \text{cm}^2$ Surface area: $(2 \times 9) + (2 \times 9) + (2 \times 9)$ $= 54\ \text{cm}^2$	Top/bottom: $\pi r^2 = \pi(3)^2 = 9\pi\ \text{cm}^2$ Side: $Ch = 2\pi r \times h$ $= 2\pi(3) \times 3$ $= 18\pi\ \text{cm}^2$ Surface area: $(2 \times 9\pi) + 18\pi = 36\pi\ \text{cm}^2$	Base: $lw = 3 \times 3 = 9\ \text{cm}^2$ 4 Sides: $\frac{1}{2}ls = \frac{1}{2}(3 \times 3) = \frac{9}{2}\ \text{cm}^2$ Surface area: $9 + \left(4 \times \frac{9}{2}\right) = 27\ \text{cm}^2$	Base: $\pi r^2 = \pi(3)^2 = 9\pi\ \text{cm}^2$ Side: $\frac{1}{2}Cs = \frac{1}{2}2\pi r \times s$ $= \frac{1}{2}2\pi(3) \times 3$ $= 9\pi\ \text{cm}^2$ Surface area: $9\pi + 9\pi = 18\pi\ \text{cm}^2$	Surface area: $4\pi r^2 = 4\pi(3)^2 = 36\pi\ \text{cm}^2$

b) What is the pattern relating the original surface area to the surface area of the object with dimensions three times as large? How is this related to the pattern we found when we doubled all of the dimensions?

Answer:

The surface area of the object that has each dimension multiplied by 3 is 9 times the surface area of the original object. Since $3^2 = 9$, this follows the same principle.

Each dimension $\times$ 2 means SA times 2^2 or times 4

Each dimension $\times$ 3 means SA times 3^2 or times 9

Learning Activity 2.4

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. You are buying a top for $10.00, a pair of pants for $25.00, and a sweater for $20.00. Everything in the store is 10% off. How much will your bill be before tax?
2. A pack of three pens is $10, while a pack of five pens is $15. Which is the better deal?
3. Jia runs or cycles every day except Friday. If Jai runs on Monday, Wednesday, and Saturday how many days does Jai cycle?
4. Complete the pattern: 36, 33, 30, _____ , _____
5. Solve for m: $3 - m = 10$

Answers:

1. $49.50 (Total without discount: $55; discount = $5.50; total = 55 − 5.50 = $49.50)
2. pack of 5 pens (10 ÷ 3 is just over $3; 15 ÷ 5 = $3, so the pack of 5 pens is the better deal.)
3. 3 days (S✓ M T✓ W T✓ F S)
4. 27, 24 (subtract 3)
5. $-7 = m$ (subtract 3, $-m = 10 - 3 = 7$; switch signs, $m = -7$ or add m to both sides, $3 = 10 + m$; subtract 10, $3 - 10 = m$)

Part B: Applying Surface Area

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. The NBA is making a new basketball for next season. The ball must have a circumference of 29.5". How much material will they need to make the basketball?

   Answer:

   The formula for the surface area of a ball is $4\pi r^2$. Before you can solve for this, you need to know the radius of the ball.

   $$C = 2\pi r$$

   $$29.5 = 2\pi r \qquad \text{(Divide by } 2\pi \text{ on both sides.)}$$

   $$29.5 \div (2\pi) = 2\pi r \div (2\pi)$$

   $$r = 29.5 \div (2\pi) = 4.7 \text{ inches}$$

   Once you know the radius of the ball, you can find the surface area using the formula:

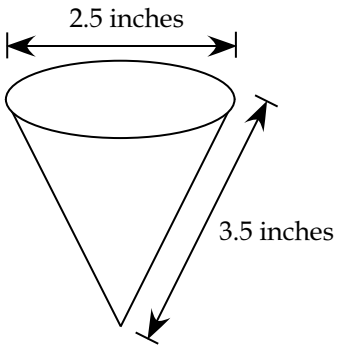
   $$SA = 4\pi r^2$$

   $$= 4\pi(4.7)^2$$

   $$= 4\pi(22.09) = 277.6 \text{ inches}^2$$

   You would need 277.6 inches^2 of material to make a basketball.

2. How much paper do you need to make a cone paper cup?



   Answer:

   Using the formula for a pyramid, you can find the surface area of the cup, if it was not open. You will have to exclude the area of the base to find the area of paper needed to make the cup.

$$SA = \frac{1}{2}Ps$$

$$P = C = 2\pi r$$

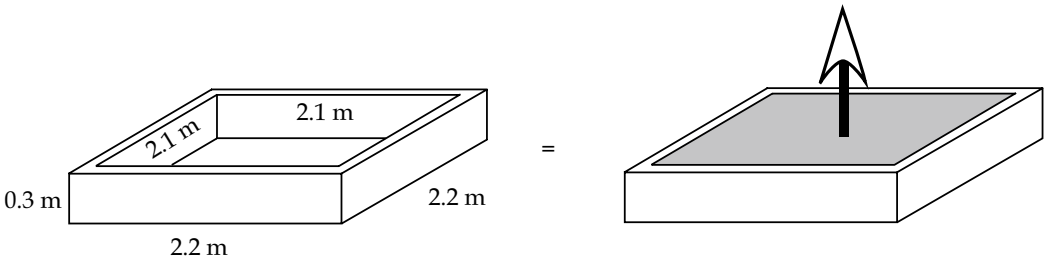
$$= 2\pi\,(1.25) = 7.854 \text{ inches}$$

$$SA = \frac{1}{2}Ps$$

$$= \frac{1}{2}(7.854)(3.5) = 13.745 \text{ inches}^2$$

You would need 13.7 inches^2 of paper to make a cone paper cup.

3. Teela and Pedro are making a sandbox for their kids. Once they have cut the wood and put it together, they would like to paint it. The dimensions they measure outside of the wood frame are 2.2 m by 2.2 m. Inside the frame, the dimensions are 2.1 m by 2.1 m.



a) What is the surface area of the sandbox?

Answer:

One way to look at this question is to think about a square prism that is 2.2 m by 2.2 m by 0.3 m. Then imagine removing a square prism from the middle that is 2.1 by 2.1 by 0.3 m.

Large Prism

Do not include the base because it is on the ground

$SA = B + Ph$

$B = 2.2 \times 2.2 = 4.84 \text{ m}^2$

$P = 4(2.2) = 8.8 \text{ m}$

$SA = (4.84) + (8.8)(0.3)$

$= 4.84 + 2.64 = 7.48 \text{ m}^2$

Removing the smaller prism alters the surface area of the larger prism. The top is now missing a 2.1 m by 2.1 m square out of it, and the interior surface area is equal to the surface area of the sides of the smaller prism.

$2.1 \times 2.1 = 4.41$ m^2 is missing from the top

One side of the small prism is $2.1 \times 0.3 = 0.63$ m^2, so all four sides add up to

$4 \times 0.63 = 2.52$ m^2 is added to the surface area of the large prism.

When you combine these areas with the area of the large prism, the surface area of the sandbox frame is $7.48 - 4.41 + 2.52 = 5.59$ m^2.

Alternate explanation:

This solution uses the faces method for finding surface area:

SA of outside sides:

$SA = 2.2 \times 0.3 \times 4 = 2.64$ m^2

SA of inside sides:

$SA = 2.1 \times 0.3 \times 4 = 2.52$ m^2

SA of top edge:

Area of top (outside dimensions) − area of top of interior (inside dimensions)

Top of sandbox: $2.2 \times 2.2 = 4.84$ m^2

Top of interior: $2.1 \times 2.1 = 4.41$ m^2

Difference $= 4.84 - 4.41 = 0.43$ m^2

$SA = 2.64 + 2.52 + 0.43 = 5.61$ m^2

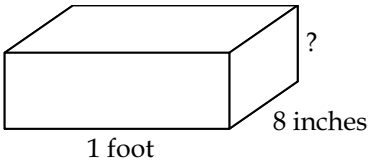
Note that the difference in the answers may be due to rounding.

b) If a can of spray paint covers 10 m^2, how many cans do they need to buy to paint the sandbox?

Answer:

They will need only one can of spray paint to cover the sandbox frame.

4. Mac has wrapped a box using 352 in.^{2} of wrapping paper, with no overlap. What is the height of the box?



Answer:

The formula used is $SA = 2B + Ph$.

1 foot = 12 inches

$B = 12 \times 8 = 96$ in.^{2}

$P = 12 + 12 + 8 + 8 = 40$ in.

$352 = 2(96) + (40 \times h)$

$352 = 192 + (40 \times h)$ (Subtract 192 from both sides.)

$352 - 192 = (40 \times h)$ (Divide both sides by 40.)

$160 \div 40 = (40 \times h) \div 40$

$4 \text{ in.} = h$

The height of the box is four inches.

5. Which is the larger amount of pizza: 2 small pizzas (11″) or 1 large pizza (15″)? (The measurements are the diameters.)

Answer:

The surface area of one small pizza is

$SA = \pi r^2$

$= \pi(5.5)^2$ The radius is $\frac{1}{2}$ of 11 = 5.5″.

$= \pi(30.25) = 95.033\ldots$

The surface area of the 11″ pizza is 95 in.^{2}.

The surface area of two small pizzas is $2 \times 95 = 190$ in.^{2}.

The surface area of one large pizza is

$SA = \pi r^2$

$= \pi(7.5)^2$ The radius is $\frac{1}{2}$ of 15 = 7.5″.

$= \pi(56.25) = 176.71\ldots$

The surface area of the 15″ pizza is 177 in.^{2}.

There is more pizza in two small pizzas than in one large. It is amazing that even though the diameter of the two pizzas are only 4″ different, the surface area of the small pizza (just one) is a lot smaller than the surface area of the large pizza—almost half the area.

6. Find the surface area of a 3-D object in your house or at school. The object could be something like a box, a ball, a can, or anything else you might find.

   a) Use one of the referents from the lesson (or a different one that you have thought of) to estimate the surface area of the object.

      Answer:

      Answers will vary. Regardless of which referent you use, you should be confident that it is close to the actual measurement. For example, you would use a sugar cube to represent (approximately) 1 cm (not 1 inch because that is not accurate).

   b) Use a ruler or other measuring tool to measure the dimensions of the object in the previous question, and then use the measurements to recalculate the surface area. Was your estimate accurate? Why might your estimate be different from the actual surface area?

      Answer:

      Answers will vary. If you have used a sphere for part a, the easiest way to find out the radius is to use a piece of string to find the circumference of the ball, and then calculate the radius.

      There are many reasons for your estimate to be different from the actual surface area, including:

      - the referent is not exactly what you use it for (your thumb might only be 0.95 inches instead of 1 inch)
      - there is space or overlap between referents so that your measurement using the referent may be shorter or longer (respectively) than it actually is.

Learning Activity 2.5

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Bjork wants to buy a computer that costs $700 before taxes. If Bjork has only $750, will he be able to pay for the computer (taxes = 12% total)?
2. Marco is on page 150 of the book he has to read for English class. If the book has 500 pages, has he read half the book?
3. Ina is five inches shorter than Ra. If Ra is half a foot taller than Horus, then who is the shortest?
4. The score with one minute to go in a football game is 28 to 34. What is the minimum number of points the losing team must score in order to win the game?
5. Evaluate $4x - 32$ if $x = 9$.

Answers:

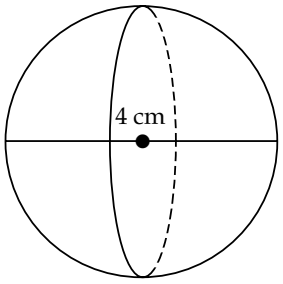
1. No (Even 10% of 700 = $70, which means that Bjork will not be able to afford the computer.)
2. No (Half of 500 = 250 pages.)
3. Horus (Half a foot = 6 inches, so Horus is shorter than Ina.)
4. 7 (34 − 28 = 6 points would tie the game, so they must score at least 7 points to win)
5. 4 (4(9) − 32 = 36 − 32 = 4)

Part B: Calculating and Converting Volume

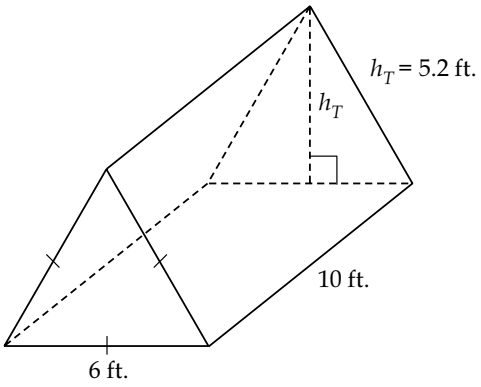
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Calculate the volume of each of the following objects. Convert any solutions that are very small or very large to appropriate units.

a)



b)



Answers:

$$r = \frac{1}{2}d = 2\ cm$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\,(2)^3$$

$$= \frac{4}{3}\pi\,(8) = 33.51\ \text{cm}^3$$

$$B = \frac{1}{2}bh_t$$

$$= \frac{1}{2}(6 \times 5.2) = 15.6\ \text{ft.}^2$$

$$V = Bh = (15.6)(10) = 156\ \text{ft.}^3$$

27 ft.^{3}	1 yd.^{3}
156 ft.^{3}	x yd.^{3}

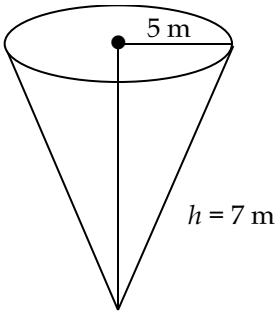
$$\frac{27}{156} = \frac{1}{x}$$

$$(27)(x) = (156)(1)$$

$$x = \frac{156}{27}$$

$$x = 5.8\ \text{yd.}^3$$

c)



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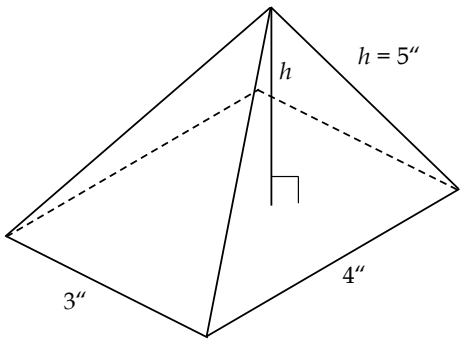


Answers:

$B = \pi r^2 = \pi (5)^2$

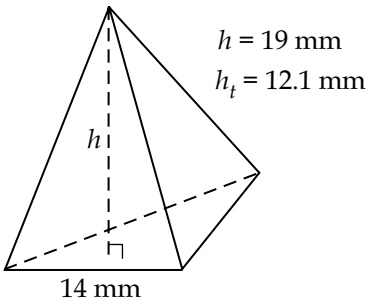
$= 25\pi = 78.5 \text{ m}^3$

$V = \frac{1}{3}Bh = \frac{1}{3}(78.5)(7) = 183.3 \text{ m}^3$

$B = lw = 3 \times 4 = 12 \text{ in.}^2$

$V = \frac{1}{3}Bh = \frac{1}{3}(12)(5) = 20 \text{ in.}^3$

e)



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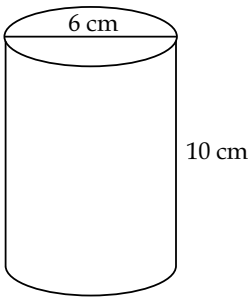


Answers:

$B = \frac{1}{2}bh_t = \frac{1}{2}(14)(12.1)$

$= 84.7 \text{ mm}^2$

$V = \frac{1}{3}Bh = \frac{1}{3}(84.7)(19)$

$= 536.4 \text{ mm}^2$

1000 mm^3	1 cm^3
536.4 mm^3	x cm^3

$B = \pi r^2 = \pi (3)^2$

$= 9\pi = 28.3 \text{ cm}^2$

$V = Bh = (28.3)(10) = 283 \text{ cm}^2$

$\frac{1000}{536.4} = \frac{1}{x}$

$(1000)(x) = (536.4)(1)$

$x = \frac{536.4}{1000}$

$x = 0.54 \text{ cm}^3$

2. Convert the following:

a) 9280 mm^3 to cm^3

Answer:

1000 mm^3	1 cm^3
9280 mm^3	x cm^3

$$\frac{1000}{9280} = \frac{1}{x}$$

$$(1000)(x) = (9280)(1)$$

$$x = \frac{9280}{1000}$$

$$x = 9.28 \text{ cm}^3$$

b) 319 $in.^3$ to $ft.^3$

Answer:

1728 $in.^3$	1 $ft.^3$
391 $in.^3$	x $ft.^3$

$$\frac{1728}{319} = \frac{1}{x}$$

$$(1728)(x) = (319)(1)$$

$$x = \frac{319}{1728}$$

$$x = 0.18 \text{ ft.}^3$$

c) 4 $yd.^3$ to cubic inches

The first step is to convert cubic yards to cubic feet.

Answer:

1 $yd.^3$	27 $ft.^3$
4 $yd.^3$	x $ft.^3$

$$\frac{1}{4} = \frac{27}{x}$$

$$(1)(x) = (4)(27)$$

$$x = 108 \text{ ft.}^3$$

The second step is to convert cubic feet to cubic inches.

1 ft.^{3}	1728 in.^{3}
108 ft.^{3}	x in.^{3}

$$\frac{1}{108} = \frac{1728}{x}$$

$$(1)(x) = (108)(1728)$$

$$x = 186\ 624 \text{ in.}^3$$

d) 0.637 m^3 to cubic centimetres

Answer:

1 m^3	1 000 000 cm^3
0.637 m^3	x cm^3

$$\frac{1\ 000\ 000}{0.657} = \frac{1}{x}$$

$$(1)(x) = (0.657)(1\ 000\ 000)$$

$$x = 637\ 000 \text{ cm}^3$$

3. Solve for the missing dimension.

a) A cone has a volume of 5236 inches^3. If the radius is 2 feet, how tall is the cone?

Answer:

The formula for volume will be V (cone) = $\frac{1}{3}(\pi r^2)h$.

Before you can solve this, you must convert either the radius into inches or the volume into feet^3. We chose to convert the volume into cubic feet so that the number is smaller and easier to work with.

The volume is

1 ft.^{3}	1728 in.^{3}
x ft.^{3}	5236 in.^{3}

$$\frac{1}{x} = \frac{1728}{5236}$$

$$(1)(5236) = (x)(1728)$$

$$\frac{5236}{1728} = x$$

$$x = 3.03 \text{ ft.}^3$$

$$3.03 = \frac{1}{3}\pi\,(2)^2 \times h$$

$$3.03 = \frac{1}{3}\pi\,(4) \times h$$

$$3.03 = \frac{4}{3}\pi \times h \qquad \left(\text{Divide both sides by } \left(\frac{4}{3}\pi\right).\right)$$

$$3.03 \div \left(\frac{4}{3}\pi\right) = \left(\frac{4}{3}\pi \times h\right) \div \left(\frac{4}{3}\pi\right)$$

$$0.7234 = h$$

The height of the cone is 0.72 feet or $0.7234 \times 12 = 8.68$ inches.

b) A triangular prism has a volume of 1950 cm^2. The height of the triangle is 13 cm, and the height of the prism is 20 cm. What is the perimeter of the triangle (assuming all the sides are equal)?

Answer:

The formula for volume is V (triangular prism) $= \frac{1}{2}bh_t \times h$.

$$1950 = \frac{1}{2}b(13) \times 20$$

$$1950 = 130b \qquad \text{(Divide both sides of the equation by 130.)}$$

$$1950 \div 130 = 130b \div 130$$

$$15 = b$$

Since the base of the triangle is 15 cm and all three sides of the triangle are equal, the perimeter is $15 \times 3 = 45$ cm.

4. Find a 3-D object in your house, such as a box or ball or (perhaps) a model pyramid. Use a ruler, tape measure, caliper, or micrometer to measure the dimensions of the object. You know which dimensions you have to measure based on the volume formula—you must know the values for everything except the volume itself. Calculate the volume.

   Answer:

   Answers may vary.

   Rectangular Prism: Measure the length, width, and height.

   Triangular prism: Measure the base of the triangle, height of the triangle, and the height of the prism.

   Cylinder: Measure either the diameter or circumference (then calculate the radius), and the height of the cylinder.

   Rectangular pyramid: Measure the length, width, and vertical height from base to peak. Hold a ruler vertically in front of the pyramid so that the base of the ruler is against the base of the pyramid (resting on a table). How high is the peak from the table?

   Triangular pyramid: Measure the base of the triangle, height of the triangle, and the height of the pyramid. Hold a ruler vertically in front of the pyramid so that the base of the ruler is against the base of the pyramid (resting on a table). How high is the peak from the table?

   Cone: Measure either the diameter or circumference (then calculate the radius), and the height of the cone. Hold a ruler vertically in front of the cone, so that the base of the ruler is against the base of the cone (resting on a table). How high is the peak from the table?

   Sphere: Measure the circumference or diameter, and then calculate the radius.

Learning Activity 2.6

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Aesir works 25 hours per week and earns $11 per hour. Calculate Aesir's gross pay after two weeks.
2. There are six red, four yellow, four orange, four green, and two purple candies in a package. What percentage of the candies are green?
3. A triangle has an area of 20 m^2. If the base is 10 m, what is the height of the triangle?
4. After adding 20 songs to your mp3 player, you have used up 100 MB (megabytes) of memory. What is the average number of megabytes per song?
5. Evaluate: $3 + 5 - (8 \times 3) + 1$

Answers:

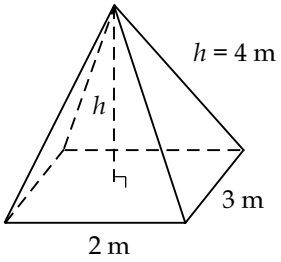
1. $550 (2 weeks = 50 hours ($50 \times 11 = 10 \times 5 \times 11 = 10 \times 55 = 550$)
2. 20% (Total candies = $6 + 4 + 4 + 4 + 2 = 20$ and $\frac{4}{20} \times 100\% = 20\%$)
3. 4 m $\left(A = \frac{1}{2}bh = 20 = \frac{1}{2}(10)(h);\ 20 = 5h \text{ so } h = 4 \text{ m}\right)$
4. 5 MB ($100 \div 20 = 5$)
5. -15 ($3 + 5 - 24 + 1 = -15$)

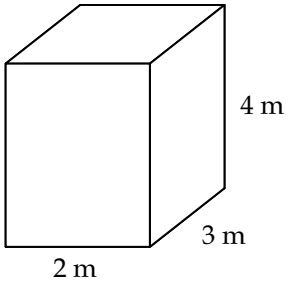
Part B: Prism and Pyramid Volumes

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Calculate the volumes of the pairs of prisms and pyramids. Record the volume values in a table (left column = prism volume, right column = pyramid volume).

a)





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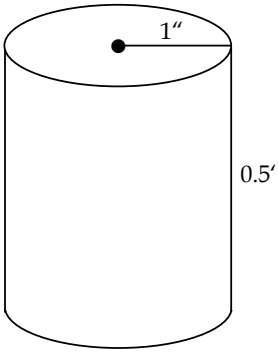
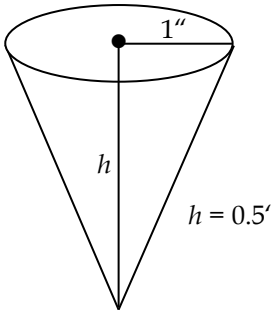
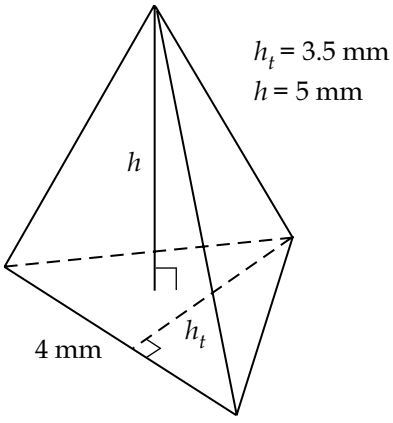
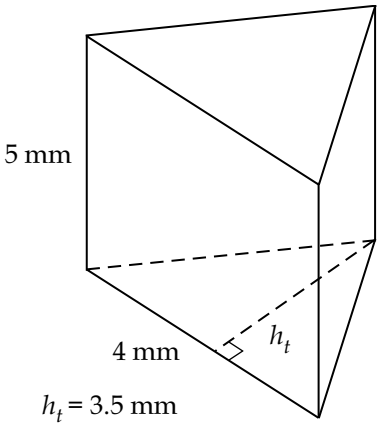




c)





	Prism	Pyramid
a)	24 m^3	8 m^3
b)	18.8 $in.^3$	6.3 $in.^3$
c)	35 mm^3	11.7 mm^3

Note: For (b), be sure to convert 0.5 feet to 6 inches.

2. Compare the values in the left and right columns of Question 1. What do you notice?

   Answer:

   The prism and pyramid in each pair have the same dimensions. The prism volume is three times the pyramid volume.

   a) $8 \times 3 = 24$

   b) $6.3 \times 3 = 18.9$ (different because of rounding)

   c) $11.7 \times 3 = 35.1$ (different because of rounding)

Learning Activity 2.7

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. What is the mean of the following data?

   5 8 2 7 3

2. Draco, Dora, and Luna have to share the $750 that their parents gave them for their trip to Europe. If it is split evenly among the three of them, how much will each get?
3. Solve for q: $4q - 10 = 14$
4. If 19% of 1053 is 200, what is 38% of 1053?
5. The three sides of a right triangle are 3.6, 6.7, and 5.6. Which side is the hypotenuse?

Answers:

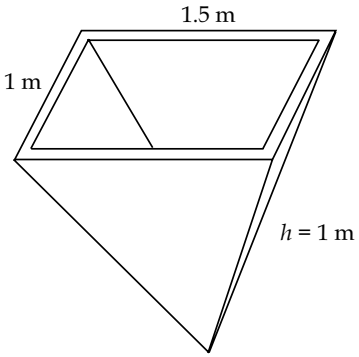
1. 5 (5 + 8 + 2 + 7 + 3 = 25; 25 ÷ 5 = 5)
2. $250 (75 ÷ 3 = 25, so each of them will get $250.)
3. 6 (add 10: $4q = 14 + 10$; divide by 4: $q = 24 \div 4 = 6$)
4. 400 (38% is double 19%, so 38% is double 200 = 400)
5. 6.7 (The longest side is the hypotenuse.)

Part B: Calculating Capacity

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Calculate the capacity of the following objects.

   a) The sides of this pyramid are 15 cm thick, and that makes the inside height 96.6 cm.



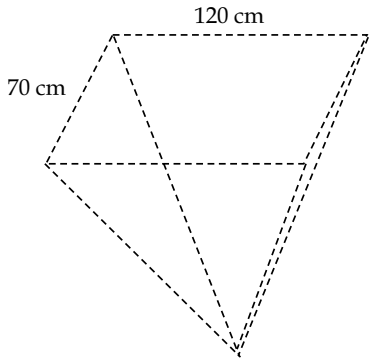
Answer:

Change the dimensions to centimetres.

inside length = 150 cm − (2 × 15) = 150 − 30 = 120 cm

inside width = 100 cm − (2 × 15) = 100 − 30 = 70 cm

inside height = 96.6 cm



$$\text{Capacity} = \text{Volume} = \frac{1}{3}lwh$$

$$= \frac{1}{3}(120 \times 70 \times 96.6)$$

$$= \frac{1}{3}(811\ 440) = 270\ 480\ \text{cm}^3$$

$1\ \text{L} = 1000\ \text{cm}^3$

$1000\ \text{cm}^3$	$1\ \text{L}$
$270\ 480\ \text{cm}^3$	$x\ \text{L}$

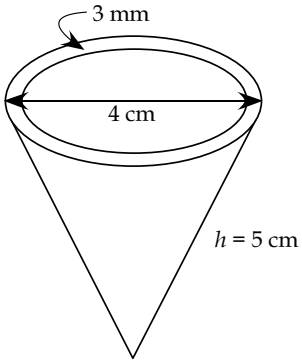
$$\frac{1000}{270\ 480} = \frac{1}{x}$$

$$(1000)(x) = (270\ 480)(1)$$

$$x = \frac{270\ 480}{1000}$$

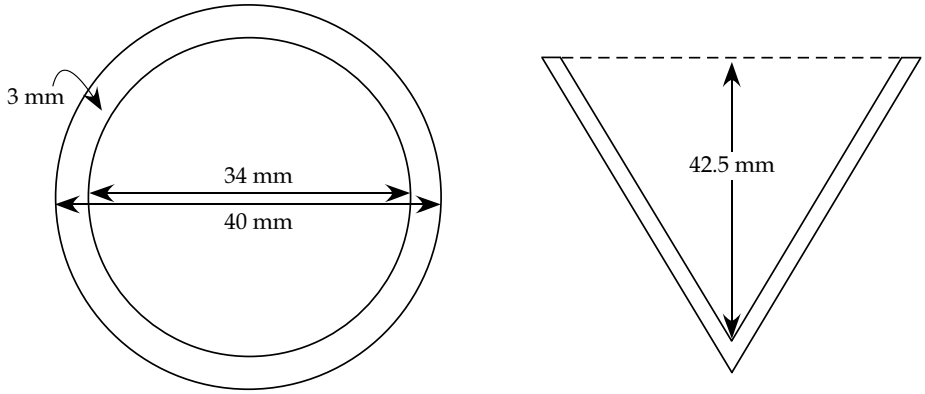
$$x = 270\ \text{L}$$

b) The sides of this cone are 3 mm thick and that makes the inside height 4.25 cm.



Answer:

Either change the height and diameter units to millimetres or the thickness measurements to centimetres.



inside diameter: 34 mm, so the inside radius is 17 mm

inside height: 42.5 mm

$$\text{Capacity} = \text{Volume} = \frac{1}{3}\left(\pi r^2\right) \times h$$

$$= \frac{1}{3}\pi\,(17)^2 \times (42.5)$$

$$= \frac{1}{3}(38586.6) = 12\ 862.2\ \text{mm}^3$$

Convert this to cm^3, which will be converted to mL.

$1000\ \text{mm}^3$	$1\ \text{cm}^3$
$12862.2\ \text{mm}^3$	$x\ \text{cm}^3$

$$\frac{1000}{12862.2} = \frac{1}{x}$$

$$(1000)(x) = (12862.2)(1)$$

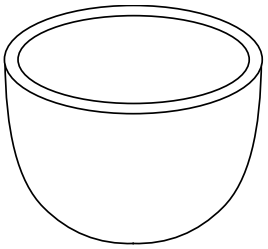
$$x = \frac{12862.2}{1000}$$

$$x = 12.9\ \text{cm}^3$$

$1\ \text{cm}^3$ (volume) = 1 mL (capacity)

capacity = 12.9 mL

c) The thickness of this semi-sphere is 1 inch.



Answer:

Convert 1 inch into feet: $1 \div 12 = 0.083$ feet

Inside diameter: $4 - (2 \times 0.083) = 4 - 0.167 = 3.83$ ft. so the radius is $3.83 \div 2 = 1.92$ feet

$$\text{Capacity} = \text{Volume} = (0.5)\frac{4}{3}\pi r^3$$

$$= (0.5)\frac{4}{3}\pi (1.92)^3$$

$$= (0.5)\frac{4}{3}\pi \times 7.078$$

$$= (0.5)29.65 \text{ ft.}^3$$

$$= 14.8 \text{ ft.}^3$$

1 ft.^{3}	7.5 gal.
14.8 ft.^{3}	x gal.

$$\frac{1}{14.8} = \frac{7.5}{x}$$

$$(1)(x) = (14.8)(7.5)$$

$$x = 111 \text{ gallons}$$

2. Write down your strategy to calculate capacity. Include step-by-step instructions.

   Answer:

   Answers may vary. This is an example of what your answer may look like if you described the method used in the Example 1 of this lesson (imagining that you are finding the volume of an object that fills the container).

   Step 1: Find the measurements of the outside of the container.

   Step 2: Find the measurements of the thickness of each side (which are usually the same) and the bottom

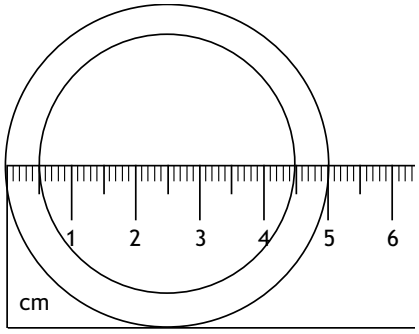
   Step 3: Calculate the inside dimensions by subtracting the thickness from the outside measurement.

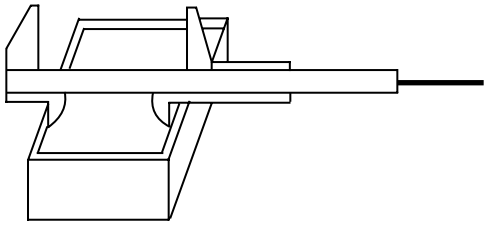
   Step 4: Use the volume formula to calculate the volume of the space.

   Step 5: Convert the units from volume to capacity.

3. Find an object at your house that has an opening so that it can store something (e.g., a box, jewelry box, bottle cap, an open can, etc.).

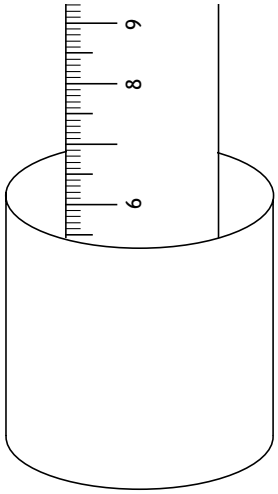
   Step 1: Use a ruler or calipers to measure the inside length and width or diameter of the object. Remember that if you are using calipers, use the inside jaws.

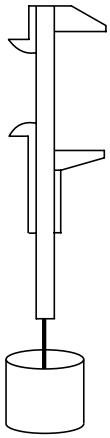




   If you have measured the diameter, calculate the radius.

   Step 2: Use the same ruler or calipers to measure the depth of the inside. If you are using calipers, you can use the depth probe for this.





Step 3: Use the measurements from Steps 1 and 2 to calculate the volume of the space inside the object.

Step 4: Convert the units of volume to capacity.

Answer:

Answers may vary. Be sure to use the correct formula based on the shape of the interior.

You can check that your calculated answer is reasonable by using a measuring cup to determine the amount of water your object holds. Compare the calculated value to the measured value; they should be very close to the same number. Be sure to use the same units.

Learning Activity 2.8

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. How many reflections do you need to return to the original position?

2. Solve for k: $8 - 2k = 2$
3. The length of a book is $\frac{3}{4}$ of a foot. What is the length in inches?
4. You finish school at 3:30 pm. It takes you 20 minutes to get to work by bus. You work for five hours. You get a ride home, and this takes only 10 minutes. You do homework for an hour before going to bed. At what time do you go to bed?
5. After having your car filled with gas, the total cost was $36.75. You handed the gas jockey $40.00, and asked for a toonie. How much did you tip the gas jockey?

Answers:

1. 2 (Once over the line, and a second to return.)
2. 3 (subtract 8: $-2k = 2 - 8$; divide by -2: $k = -6 \div (-2) = 3$)
3. 9 $\left(\frac{3}{4} \times 12 \text{ inches per foot} = 9 \text{ inches}\right)$
4. 10:00 pm (3:30 + (0:20 + 5:00 + 0:10 + 1:00) = 3:30 + 6:30 = 10:00 pm)
5. $1.25 (40 − $36.75 = $3.25 would be your change, and 3.25 − 2 = $1.25 is the tip)

Part B: Estimating Volume and Capacity

Unlike other learning activities, these questions are not like the ones that will be on your assignments and midterm examination since you will not be asked to directly measure an object. However, doing these problems correctly will deepen your understanding of using referents for estimation of volume and capacity. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. To complete this question, you will need to find two objects:
   i) an object whose volume you don't know, such as a box
   ii) an object such as a sugar cube, where you know the volume, and where the shape is the same as Object (i)

   You don't have to use a box. Any 3-D object will work, although using an oddly shaped object that you haven't used before will make this exercise more challenging.

   Once you have these two objects, use the method of finding the total multiplier, comparing the two objects in order to estimate the volume of Object (i). Use a ruler or tape measure to find the actual measurements of Object (i), and calculate the volume. Was your estimate close? If not, why?

   Answer:

   Step 1: How much larger is each dimension of the larger object (double, triple, 4 times, etc.)?

   Step 2: Based on the formula for calculating the volume of your object (prism/ pyramid/sphere), combine the dimension multipliers in the same way: (volume of a rectangular prism is *lwh*, so the combined multiplier will be (length multiplier) × (width multiplier) × (height multiplier).

   Step 3: Apply the new multiplier to the known volume to calculate the approximate volume of the unknown.

2. To complete this question, you will need to find two objects:
   i) An object whose capacity you don't know, such as a milk carton or jar.
   ii) An object that is the same shape as Object (i), where you know the capacity, such as a larger/smaller milk carton or a can.

   Once you have these two objects, use the method of finding the total multiplier comparing the two objects in order to estimate the capacity of Object (i). Use a ruler or tape measure to find the actual measurements of Object (i), and calculate the capacity. Was your estimate close? Why or why not?

   Answer:

   Step 1: How much larger/smaller is each dimension of Object (i) (half, double, triple, four times, etc.)?

   Step 2: Based on the formula for calculating the volume of your object (prism/ pyramid/sphere), combine the dimension multipliers in the same way: (volume of a rectangular prism is lwh, so the combined multiplier will be (length multiplier) × (width multiplier) × (height multiplier).

   Step 3: Apply the new multiplier to the known capacity to calculate the approximate capacity of the unknown.

   You could check your answers using a measuring cup to determine the capacities of your objects and comparing the values (using the same units) to the capacities you calculated.

Learning Activity 2.9

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the pattern: 1, 4, 9, 16, _____ , _____
2. At the hockey game, Murphy bought a hot dog for $3.50, a soft drink for $2.75, and some popcorn for $3.25. How much is his total?
3. Murphy gives the cashier a $20 bill and gets $11 back. Is this the correct change?
4. At Deklin's school, they have 72-minute classes, except on Friday, when their classes are 45 minutes long. If Deklin is in math this term, how much time will he spend in the math classroom in one week?
5. Solve for t: $4t - 8 = 0$

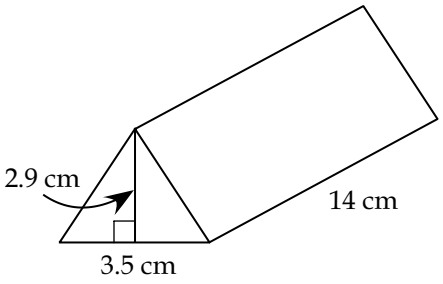
Answers:

1. 25, 36 (These are the squared numbers ($1^2 = 1$, $2^2 = 4$, $3^2 = 9$), so $5^2 = 25$ and $6^2 = 36$.)
2. $9.50 (3 + 2 + 3 + 0.50 + 0.75 + 0.25 = 8 + 1.50 = $9.50)
3. No (The cashier gave him too much. He should have gotten $10.50 back.)
4. 5 hr. 33 min. ((4 × 72) = (4 × 70) + (4 × 2) = 280 + 8 = 288; 288 + 45 = 333 min. or 5 hr. 33 min.)
5. 2 (add 8: $4t = 8$; divide by 4: $t = 8 \div 4 = 2$)

Part B: Applying Volume and Capacity

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. A chocolate bar shaped like a triangular prism is 14 cm long. Its triangular base is 3.5 cm long and 2.9 cm high. Calculate the volume of chocolate in the bar.



Answer:

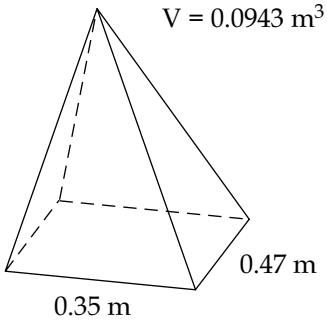
$V = Bh$

$$V = \frac{(3.5 \times 2.9)}{2} \times 14$$

$V = 71.05$

There is 71.05 cm^3 of chocolate in the bar.

2. Pepper has made a rectangular pyramid sculpture with a volume of 0.0943 m^3. The pyramid with base dimensions of 0.35 m by 0.47 m is shown below. Determine its height.



Answer:

$$V = \frac{1}{3}Bh$$

$$0.0943 = \frac{1}{3}(0.35 \times 0.47) \times h$$

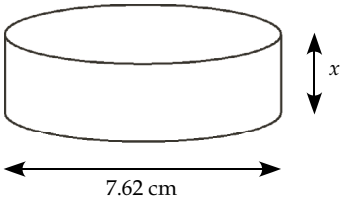
$$0.0943 = 0.05483h$$

$$h = \frac{0.0943}{0.05483}$$

$$h = 1.71986139$$

The height of the pyramid is about 1.72 m.

3. A hockey puck has a diameter of 7.62 cm and a volume of 115.83 cm^3. Calculate the thickness of a hockey puck.



Answer:

$d = 2r$

$r = \frac{d}{2}$

$r = \frac{7.62}{2}$

$r = 3.81$

$V = Bh$

$V = (\pi r^2)h$

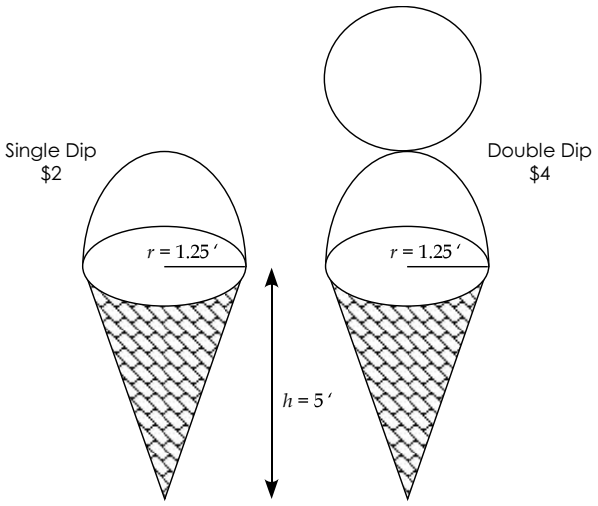
$115.83 = \pi (3.81^2)x$

$x = \frac{115.83}{\pi \times 14.5161}$

$x = 2.539926986$

The thickness of a hockey puck is 2.54 cm, which is equivalent to 1". It would be a great referent!

4. An ice cream cone has a radius of 1.25″ and a height of 5″. If you buy a single-dip cone for $2, the cone is completely filled with ice cream and there is a half-sphere shaped scoop above the cone. If you order the double dip for $4, they put an extra sphere-shaped scoop of ice cream on top. Which is the better deal? Explain.



Answer:

$V_{\text{single dip}} = V_{\text{cone}} + V_{\text{semi-sphere}}$

$V_{\text{single dip}} = \frac{1}{3}Bh + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V_{\text{single dip}} = \frac{1}{3}\pi\,(1.25)^2\,(5) + \frac{1}{2}\left(\frac{4}{3}\pi\,(1.25)^3\right)$

$V_{\text{single dip}} = 8.181230869 + 4.090615434$

$V_{\text{single dip}} = 12.2718463$

$V_{\text{double dip}} = \frac{1}{3}Bh + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \left(\frac{4}{3}\pi r^3\right)$

$V_{\text{double dip}} = 12.2718463 + 8.181230869$

$V_{\text{double dip}} = 20.45307717$

The single-dip cone has about 12 in.^{3} of ice cream, while the double-dip has about 20 in.^{3}. You pay twice as much for the double-dip, but you don't get twice as much ice cream, so the single-dip cone is a better deal.

5. Fred wants to estimate the weight of a large wooden beam. He has a length of 2 × 6 wood that he weighs, and the weight is 12 pounds. He uses his piece of wood as a referent to determine the approximate weight of the beam. After measuring the beam against his piece of wood, he discovers that the beam is 12 times as long as the piece of wood, 6 times as wide, and 4 times as deep.

   a) Calculate the weight multiplier.

      Answer:

      $V = (l)(w)(h)$

      Dimension multipliers

      $l = 12$

      $w = 6$

      $h = 4$

      Total multiplier

      $(l)(w)(h) = (12)(6)(4) = 288$

   b) Calculate the approximate weight of the beam.

      Answer:

      Weight = 12 × 288 = 3456 pounds

      This is more than the weight of most cars.

Notes

Grade 11 Essential Mathematics (30S)

Module 3
Statistics

Module 3: Statistics

Introduction

In Grade 9, you learned how to collect data, and how to display data on a graph so that it is easy to understand. In this module, you will learn more about displaying data using graphs, as well as how to interpret data. You may choose to draw the graphs by hand or with a computer.

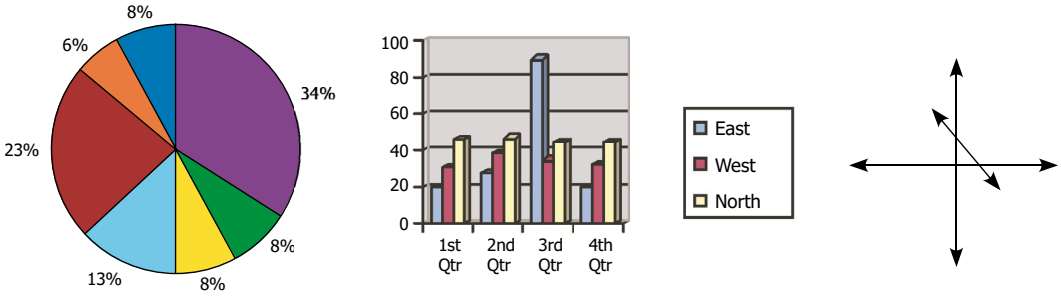
The following is an often-quoted statement said (reportedly) by Benjamin Disraeli, a distinguished British public official. The statement reads: "There are three kinds of lies: lies, damned lies, and statistics." The statement refers to the misuse of statistics.

In this module, you will learn how to select the most suitable kinds of graphs to present different types of data. You will also learn how graphs are used to emphasize a certain point of view. It is important to recognize how graphs can be used to present data in misleading ways.

The following is a list of words that you will use in this module and their definitions. These words are also included in the glossary of this course.

- **Data:** Information that is collected that is usually numerical, organized in charts, and displayed by graphs.
- **Graph:** A visual representation used to show a relationship between data.

  Examples



- **Population:** Everything or everyone in a group that is being studied.
- **Sample:** A part of the group being studied that represents the whole group.
- **Survey:** To ask either written or verbal questions for the purpose of acquiring information/data.

Assignments in Module 3

When you have completed the assignments for Module 3, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Statistics in Your Life
3	Assignment 3.1	Circle Graphs, Bar Graphs, and Histograms
4	Assignment 3.2	Line Graphs
5	Assignment 3.3	Interpreting Graphs

Writing Your Midterm Examination

You will write the midterm examination when you have completed Module 3 of this course. The midterm examination is based on Modules 1 to 3, and is worth 12.5 percent of your final mark in the course. To do well on the midterm examination, you should review all the work you complete in Modules 1 to 3, including all the learning activities and assignments. You will write the midterm examination under supervision.

Resource Sheet

When you write your midterm examination, you are encouraged to take a Midterm Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the following list of instructions to help you with preparing your resource sheet for the material in Module 3. On this sheet, you should record mathematics terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 1, 2, and 3 to prepare your Midterm Examination Resource Sheet. The midterm examination for this course is based on Modules 1 to 3.

Resource Sheet for Module 3

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important mathematics terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how each formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Notes

Module 3 Cover Assignment: Statistics in Your Life

The statistics used in this activity come from Manitoba Public Insurance, the company that provides us with the automobile insurance program known as Autopac. Read the information provided below and respond to the questions that follow.

For the period March 1, 2009, to February 28, 2010:

- Average number of Autopac claims reported per working day: 1079
- Total Autopac claims reported: 267 611
- Bodily injury claims reported: 16 671
- Property damage claims reported: 250 940
- Total theft claims reported in Winnipeg: 2291
- Total theft claims reported elsewhere in the province: 1037
- Number of Autopac policies in force (the number of cars insured)—2009 average: 955 342
- Licensed drivers in Manitoba: 776 209
- Calls taken by the Autopac line: 771 643
- Claims incurred: $614,198,000
- PIPP (Personal injury protection plan) claims incurred: $175,000,000
- Physical damage claims incurred: $330,900,000
- Population of Manitoba in 2009: 1 217 200
- Population of Winnipeg in 2009: 675 100

Notes

Module 3 Cover Assignment

Statistics in Your Life

Total: 11 marks

Each question is worth 1 mark. Show your work.

1. What percentage of the number of Autopac claims were from claims for property damage?

2. What percentage of the number of Autopac claims were from claims for bodily injury?

3. What is the average cost per claim?

4. What is the ratio of number of Autopac policies in force (the number of cars insured) to licensed drivers in Manitoba?

5. Why are the number of Autopac policies in force (the number of cars insured) not the same as the number of licensed drivers in the province?

continued

Module 3 Cover Assignment: Statistics in Your Life (continued)

6. What was the population of Manitoba in 2009?

7. What percentage of the population of Manitoba were licensed drivers in 2009?

8. What percentage of Manitoba's population lived in Winnipeg in 2009?

9. What percentage of the claims incurred by Autopac were for physical damage expenses?

10. What percentage of the claims incurred by Autopac were for PIPP (personal injury protection plan) expenses?

11. What is the total percentage of the claims incurred by Autopac for both PIPP and physical damage?

Lesson 1: Circle Graphs

Lesson Focus

In this lesson, you will

- ❑ study the meaning of discrete data
- ❑ draw circle graphs by hand or with a computer
- ❑ explore the advantages and disadvantages of using a circle graph to represent data

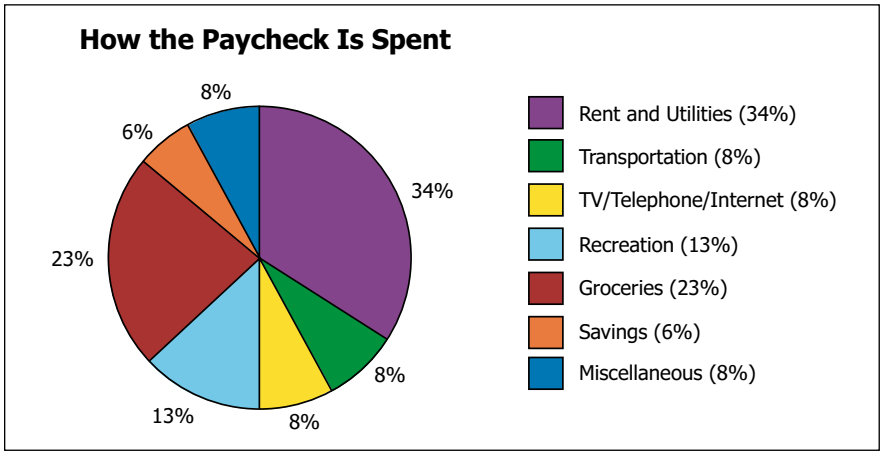
Lesson Introduction

Sometimes a picture is easier to understand than a paragraph. Similarly, sometimes a graph is easier to understand than lists of numbers. The type of graph drawn usually depends on the kind of data one wants to illustrate.

In this lesson, you will review and draw circle graphs. You will also learn what kind of data is best represented with a circle graph. Also, you will have the option of drawing the circle graph by hand or with a computer.

Using Circle Graphs

You may be thinking, "What exactly is a circle graph, again?" A **circle graph** represents different categories of data as sectors of a circle, similar to slices of a pizza. All circle graphs compare parts of a whole, where *whole* represents 100% of the data. The sections of this graph are usually labelled with a percentage, which reflects how much of the data is represented by that sector.



Circle graphs are used to display **discrete data**. This means that the data can be sorted into well defined categories that are clearly separate.

Recognizing Discrete Data

You must be able to recognize whether your data is (or is not) discrete. There cannot be any overlap between categories when drawing a circle graph. The examples below explain the definition of discrete data.

Example 1

Identify whether or not the data is discrete. Explain.

a) The number of cars in a parking lot, sorted by colour

b) The ages of people

c) The distance people travel to work

d) Favourite subject in school

e) Preferred sport

Solutions

a) This could be discrete or not, depending on how you labelled the colours. If the only categories you allowed were tan, silver, white, black, brown, blue, green, red, yellow, and orange, then the data would be discrete. If you labelled each car specifically so that you ended up having one greenish-blue car and another yellow-orange car, then there would be overlap between the categories, making the data NOT discrete.

b) Because only people on their birthday are exactly the age they say they are (16 years old, for example), age cannot be clearly defined. Although you may say that you are 16 years old, you may actually be 16 years, 3 months, 2 days old. This data would not be discrete.

c) Similar to age, distance is not discrete because you can travel part of a kilometre or other unit of measurement.

d) As you know from being in school, subjects are treated as separate. Each class is different and is allotted a certain amount of time. Therefore, a survey of favourite subjects would provide discrete data.

e) Similar to school subjects, all sports are different, and so each sport is in a different, separate category. This data is discrete.

If you can easily divide data into distinct categories, then the data is most likely discrete. You would then be able to draw a circle graph to represent the data.

Drawing Circle Graphs

The following are components of a good circle graph:

- title of the graph
- accurately drawn sectors
- percentage labels on each sector
- a legend explaining what each sector represents, and the actual values
- accurate division of the circle based on the percentage of each sector compared to the total population (larger percentages should be larger sectors)

Record the components of a good circle graph on your resource sheet.

To draw a good circle graph, you need to follow these four steps.

1. **Be sure the data you are using is appropriate for a circle graph.** When drawing a circle graph, you must use discrete data. In order to achieve this, you must be sure that the same information cannot be placed into two or more categories. An example of this type of data could be the following:

   Where are you in a 24-hour period of time, and how long are you there?

   This survey would produce appropriate data for a circle graph because you *cannot* be in two places at once.

2. **Rewrite each category of data as a percentage of the total.** As stated before, each wedge or sector of the circle graph represents a percentage of the total. Therefore, you must know the percent value of each category. Continuing with the example above, one person's response may be:

   3 hours in the car 8 hours at work 2 hours at the gym 11 hours at home

   We cannot say for sure how much of the circle will represent 11 hours until we know it as a percent.

   Total number of hours: 24

   $$\%\ \text{in the car} = \frac{3}{24} \times 100\% = 12.5\% \qquad \%\ \text{at work} = \frac{8}{24} \times 100\% = 33.3\%$$

   $$\%\ \text{at the gym} = \frac{2}{24} \times 100\% = 8.3\% \qquad \%\ \text{at home} = \frac{11}{24} \times 100\% = 45.8\%$$

   To check your answer, the sum of the percentages should be approximately 100 percent (it may not be exact because of rounding the decimals).

   $12.5 + 33.3 + 8.3 + 45.8 = 99.9\%$

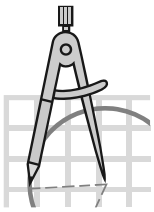
3. **You now convert the percentage of each sector into degrees.** Remember that a circle is divided into 360º.

   Car: 12.5% of a circle = 0.125 × 360º = 45º

   Work: 33.3% of a circle = 0.333 × 360º = 120º

   Gym: 8.3% of a circle = 0.083 × 360º= 30º

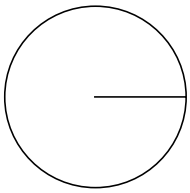
   Home: 45.8% of a circle = 0.458 × 360º= 165º



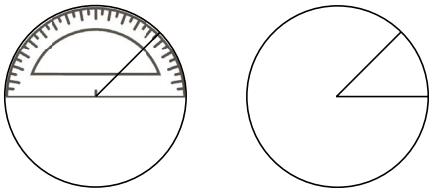
4. **Draw a circle, and then use a protractor to draw each sector.**

   a) Use a **compass** to draw the circle so that it is accurate.

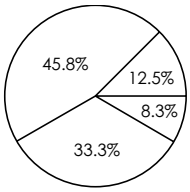
   b) Draw a radius from the centre of the circle to any point on the circumference.



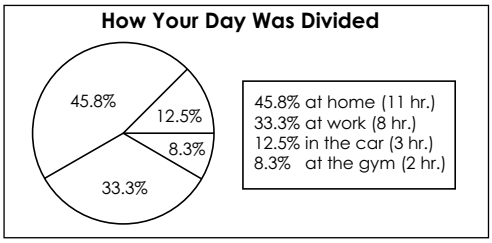
   c) Measure the angle of the segment you would like to draw from the radius, using a protractor. *car = 45º*



   d) Continue measuring the different angles until you have the correct number of sectors. As you go, label the sectors with their percent values.



   e) Your graph will need a descriptive title and a legend stating what each sector represents.



Include these four steps, in your own words, in your resource sheet for this module.

Following these steps will enable you to draw a circle graph accurately for any discrete data.

Example 1

As part of a project, a social studies class collected data about the number of countries that individual students have been to, including Canada. The class found out that 10 students have been to five countries, 26 have been to four countries, 55 have been to three countries, 100 have been to two countries, and nine have been only in Canada. None of the students surveyed have been to more than five countries. Create a circle graph with their data.

Solution

Step 1: The data is discrete; you cannot partially enter a country. Either you do or you don't.

Step 2: Total number of students = 10 + 26 + 55 + 100 + 9 = 200

5 countries; 10 students: $\frac{10}{200} \times 100\% = 5\%$ of students have been to five countries.

4 countries; 26 students: $\frac{26}{200} \times 100\% = 13\%$

3 countries; 55 students: $\frac{55}{200} \times 100\% = 27.5\%$

2 countries; 100 students: $\frac{100}{200} \times 100\% = 50\%$

1 country; 9 students: $\frac{9}{200} \times 100\% = 4.5\%$

Step 3 5 countries; 5 percent of a circle = $0.05 \times 360^o = 18^o$

4 countries; 13 percent of a circle = $0.13 \times 360^o = 46.8^o \approx 47^o$

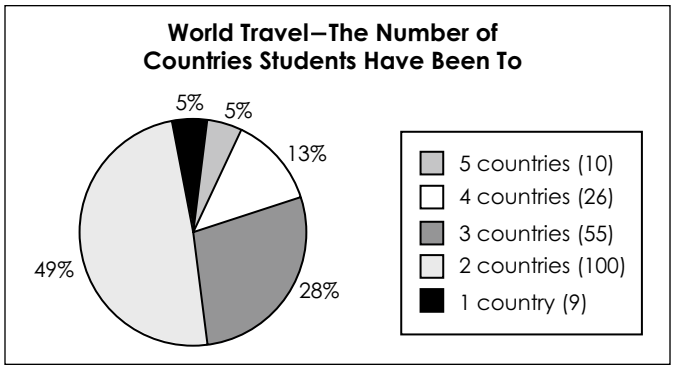
3 countries; 27.5 percent of a circle = $0.275 \times 360^o = 99^o$

2 countries; 50 percent of a circle = 180^o

1 country; 4.5 percent of a circle = $0.045 \times 360^o = 16.2^o \approx 16^o$

Because most protractors cannot measure a 10th of a degree, round the percentages to the nearest whole number, and then check your answers by adding all the angles. The sum should be 360^o.

Step 4



Notice that instead of including decimals, you rounded the percentages up for "three countries"(27.5 → 28%) and "one country" (4.5 → 5%), and decreased the percentage for "two countries" by 1%. This is done so that the percentages still add up to 100. The 1% is subtracted from the *largest section* because it will not change the shape of the larger sector very much.

Drawing a Circle Graph on the Computer

Drawing graphs on the computer can make the process much quicker. This section will demonstrate how to construct a circle graph using *Microsoft Excel* for *Windows XP,* but there are many different graphing programs available that you can use. Do not be afraid to use a different program to draw these graphs. Just use the help button in order to find out what to do. If you are in a school while you are taking this course, your school may have access to a program called *Graphical Analysis*, which can also be used to create circle graphs.

It is handy to know how to construct a circle graph on the computer because it allows you to skip a couple of steps. Note that you are not required to use a computer to draw a graph. Using a computer is an option presented in this course. **If you do not have access to a computer with graphing software, work through Example 2, keeping in mind the directions (including the four steps listed previously) to draw a good circle graph.**

Work through the following example.

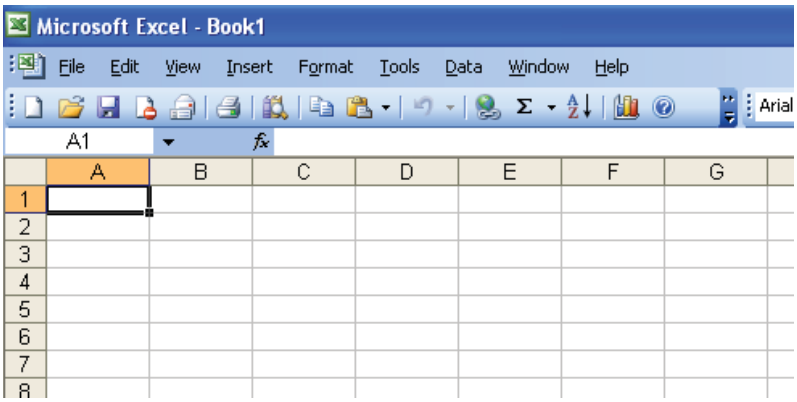
The solution shows a step-by-step outline of how to draw a circle graph with a computer.

Example 2

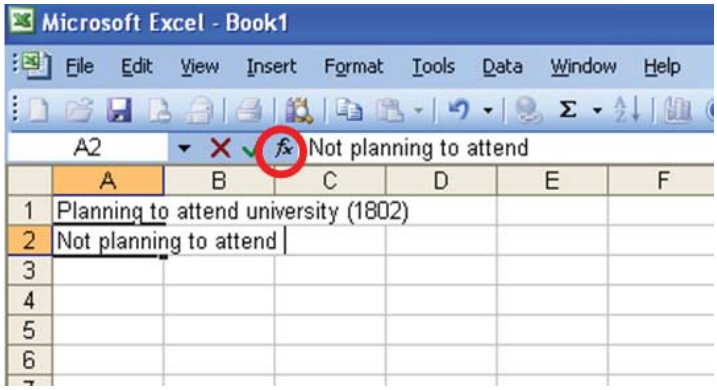
A university registrar would like to find out how many high school students in the area are planning to attend university. In the survey, 1802 students said they intended to go to university, 1163 said they were not sure whether or not they would go to university, and 826 said they were not going to attend university (although they may do some other form of post-secondary education). Create a circle graph for this data so that the surveyors can present this information to the registrar.

Solution

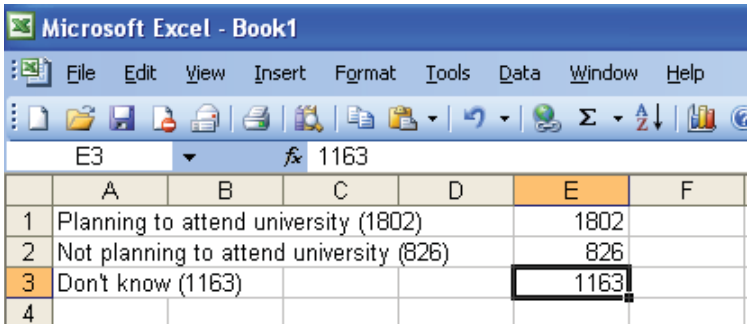
Once you have opened *Microsoft Excel*, click in the first box (A1).



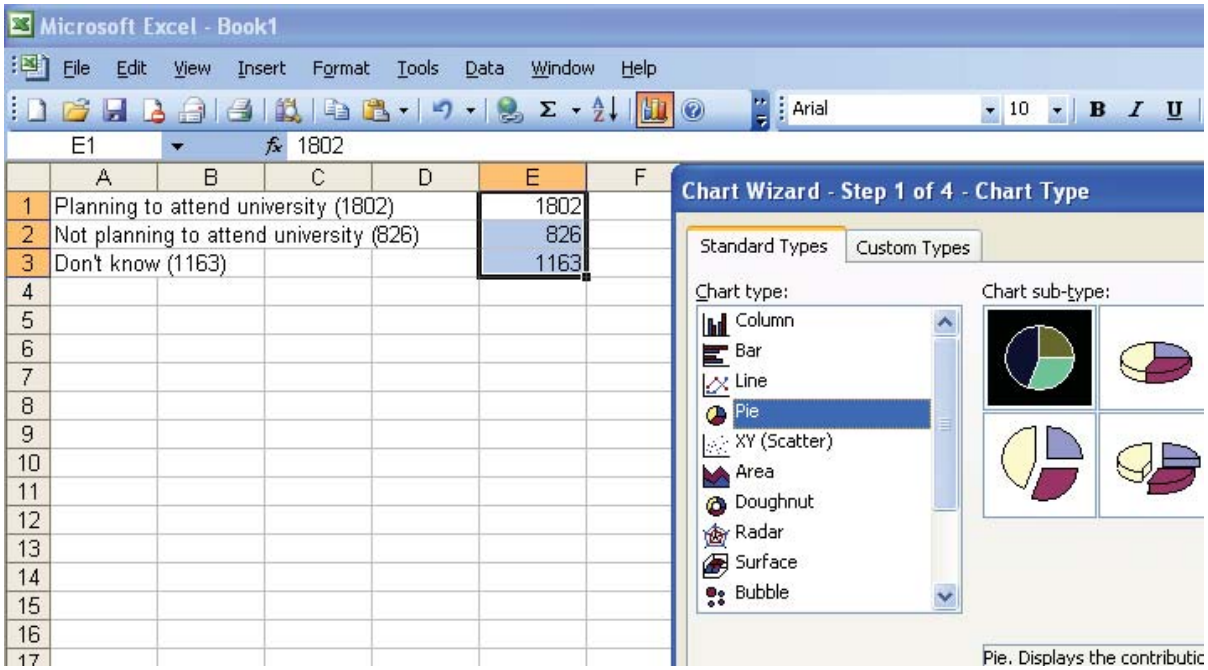
In this column (A), type in the names of each section that will appear in the circle graph. Include the number of students in brackets at the end. If you make a mistake typing and you have already clicked on a different **cell** (box), click on the cell with the mistake, and then click in the space beside '*fx*' to edit the text.



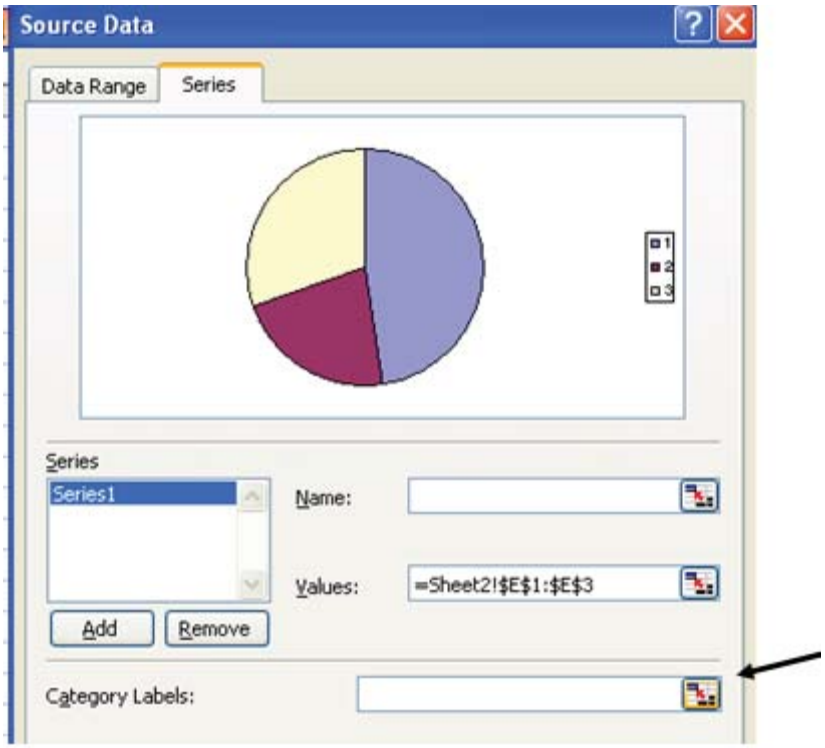
In the next open column, enter the data beside the correct section.



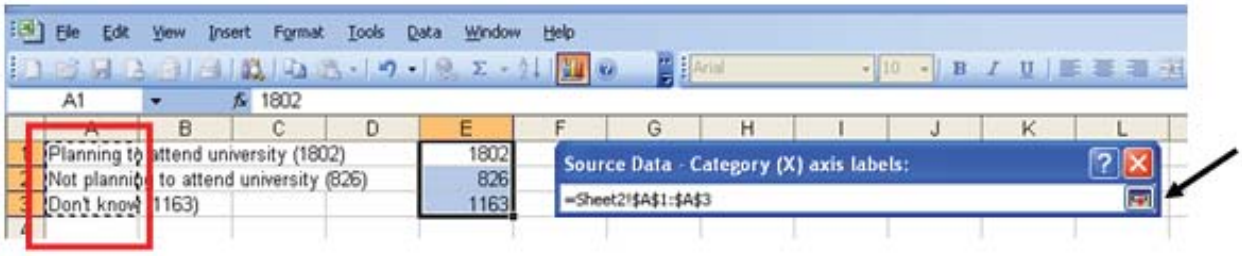
Select the column with just the numbers in it (E), and then click on the icon that looks like a graph. A window should pop up in the middle of your screen, and you can choose the type of chart you want to draw. Select "Pie," and then press the NEXT button at the bottom of the window.



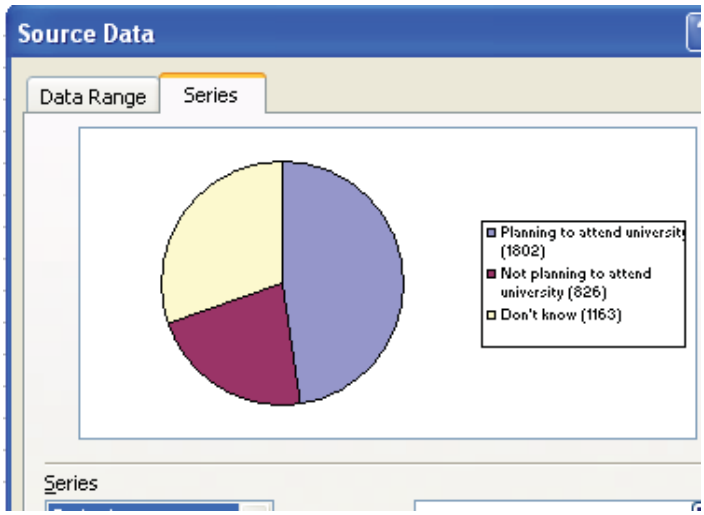
The window on your screen should change. Click on the Series tab at the top of the window. Click on the button beside the Category Labels.



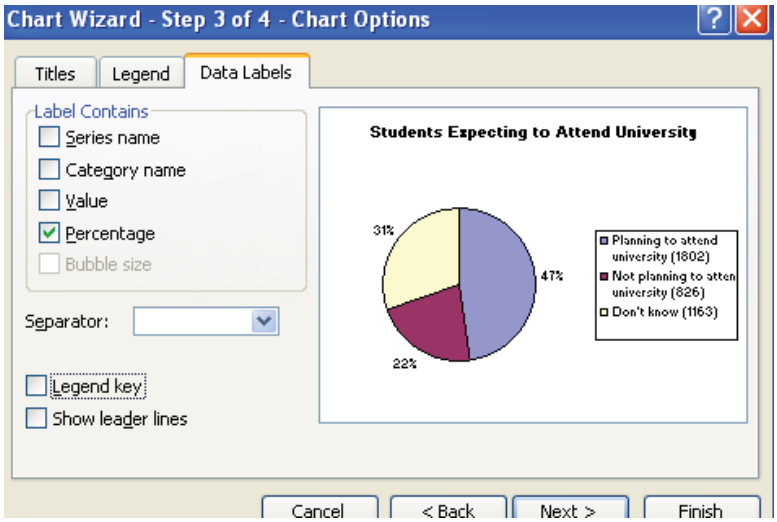
Select the names of the categories that you typed, and then press the button at the end of the bar.



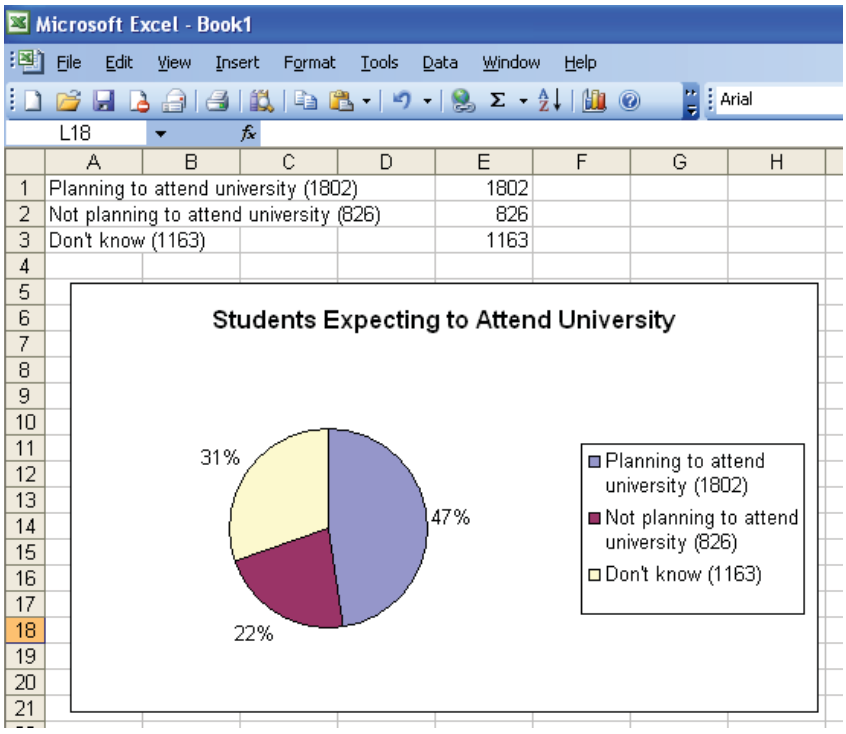
The window should be restored, but the legend should now have titles for the different sections.



Click on the NEXT button at the bottom of the window, and a new window should appear in its place. Here, you can add a title for your graph. Also, if you click on the Data Labels tab, you can add the percent values for each section.



Click on the NEXT button, and then FINISH. Your circle graph should appear in the spreadsheet.



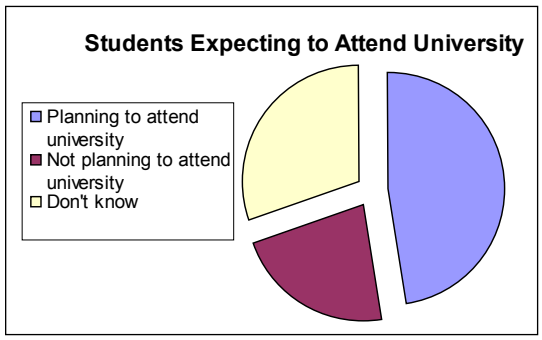
You may want to write down a few key steps on your resource sheet so that you can complete the learning activity in this module as well as the assignments for this module on your computer. It may have taken a while to draw this graph, but after doing it a few more times you will probably be able to draw a circle graph on the computer rather quickly.

Variations of Circle Graphs

In the world of business and marketing, people tend to jazz up their graphs and make them even more appealing. Various types of circle graphs communicate the same information, and you should recognize these types of graphs. You will learn more about the effect of graph choice in Lessons 5 and 6.

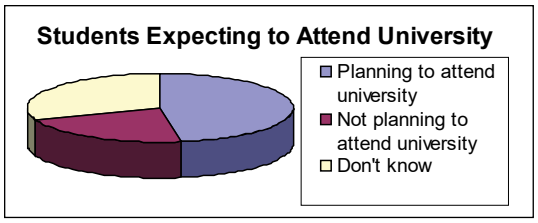
Exploded Circle Graph:

This graph highlights the individual sections of the circle graph because they are separated.



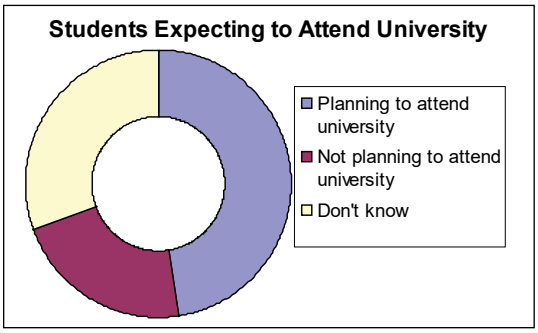
3-D Circle Graph:

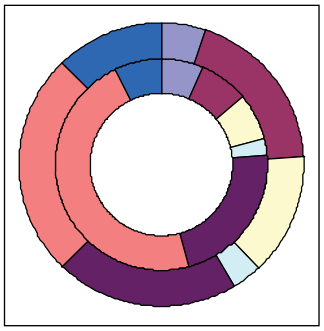
This version of a circle graph makes the graph more eye-catching.



Doughnut Circle Graph:

A doughnut graph is a circle graph with a hole in the centre. Unlike a circle graph, you can use the doughnut graph to compare sets of data, but the graph can look confusing, so it is not recommended.





(double)

Advantages and Disadvantages of Circle Graphs

Before we tell you about some of the advantages and disadvantages of circle graphs, take a moment to think about and/or talk to your learning partner about the possible pros and cons of using a circle graph. Make a list in the table below.

Advantages	Disadvantages

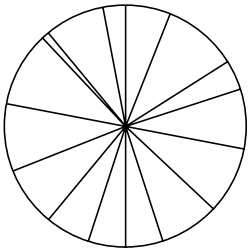
Advantages

- Circle graphs present data in a visual way that makes it easy to compare the different categories of data. In the example "How Your Day Was Divided," it is very easy to see where most of the time was spent.
- Circle graphs are a great way to summarize your results in a way that is easily understood. The largest sector is the majority; the smallest is the minority.
- Circle graphs are eye-catching and easy to understand.

Disadvantages

- Circle graphs are not a good way of showing relationships between two or more sets of data.
- Circle graphs normally show percentages and not actual data values. For example, let's imagine that students at a different school also surveyed their classmates to find out how many countries they have been to (Example 1). Instead of surveying 200 people, they only asked 50 students. The results may produce the same graph, but that does not mean that the same number of students in both surveys have been to five countries. All we can conclude from the graph is that the same percentage of students have been to five countries.
- A circle graph is not very effective if there are too many sectors. The circle graph below has 15 sectors, and it is very difficult to sort the sectors in order from largest to smallest.

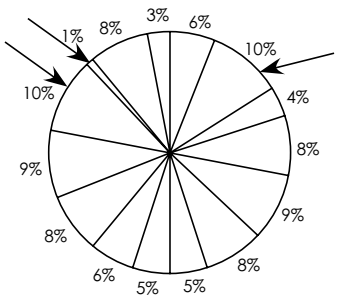
  Can you tell which section is the largest? Which is the smallest?

  Write down your "guestimates" before you look at the answer at the end of this list.
- In order to correct the issue of too many sections, you may see an *other* section in the graph. This section is made up of all the very tiny sections, which seems like a very good idea. The trouble with doing this, however, is that it is often difficult to know exactly what *other* means. Because it is not clearly defined, it is usually ignored. This is a problem because *all* the data must be represented in the graph.

  Another use of the *other* section is to group all the "misfit" data (data that fits into two or more categories) into one category. This category may be necessary because, as you learned, circle graphs represent discrete data that does not have any overlap.

  Answer: The largest sectors are 10 percent, while the smallest is 1 percent. Did you get it right?



In your own words, include the advantages and disadvantages of using a circle graph in the chart at the end of this lesson. You will be asked to complete this chart as you work through the next three lessons.

Here is your chance to practise what you have learned in this lesson. Complete the following learning activity before moving on to Lesson 2. Check your answers to be sure that you understand the content of this lesson. Circle graphs will be included in Assignment 3.1 at the end of Lesson 3.

Learning Activity 3.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Convert the following decimal to a percent: 0.005.
2. Convert the following percent to a decimal: 345%.
3. Convert the following fraction to a percent: $\frac{4}{5}$.
4. For every 10 people in the world, four are Asian. Write this as a percentage.
5. Yesterday, Malaki spent five hours in school, two hours in the gym, and nine hours sleeping. How many hours of the day are not accounted for?

continued

Learning Activity 3.1 (continued)

Part B: Circle Graphs

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand. Don't forget that you can ask your learning partner or tutor/marker for help if you are having a hard time understanding.

1. Write D for discrete or N for not discrete beside each example of data. Explain your choices.

   Note that, in some cases, you may have either D or N as your answer, but your explanation is important for backing up your decision. The way the categories are created may determine whether information is discrete or not discrete.

   a) A survey of the number of pages in various books.

   b) The genres of books at your school library.

2. Many people from countries other than the US visit Canada over the course of a year. In 2009, according to Statistics Canada, 519 000 people came by land, 3 501 000 people came by airplane, and 150 000 came by sea.

   a) Draw a circle graph for this data if possible.

   b) When looking at the graph, what do you observe about the methods of travel used by visitors to Canada?

continued

Learning Activity 3.1 (continued)

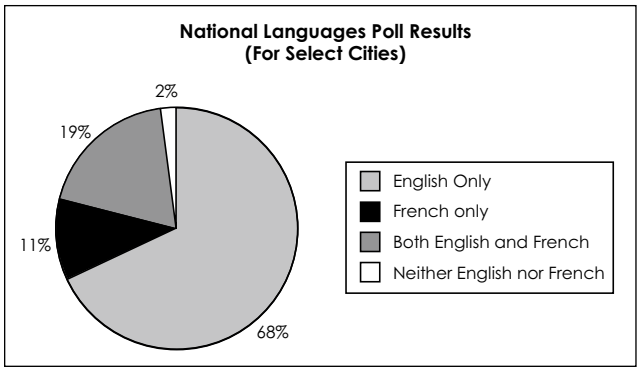
3. Suppose the Government of Canada is researching the possibility of changing our official language laws. Part of its research is data from the 2006 census by Statistics Canada. The table shows language results for people in some of our provincial capitals.

	English Only	French Only	Both	Neither
Total	6, 931,670	476,650	906,270	238,745

a) List the categories you would use for this data in a circle graph.

b) Draw the circle graph for this data.

c) The circle graph representing the results for a larger number of cities is shown below. How is this graph different from the one you drew? In what ways is it similar? Why might the results be different?



Lesson Summary

In this lesson, you have studied circle graphs—what they are, what type of data they display (discrete data), and how to create them by hand or with a computer. You can use circle graphs to compare similar things, such as the amount of money you spent on gifts for your family and friends. In the next lesson, you will learn about bar graphs, which are also used to display discrete data.

Pros and Cons of Different Graphs				
	Circle Graph	Bar Graph	Histogram	Line Graph
Pros				
Cons				
Example of Where You Would Use It				

Lesson 2: Bar Graphs

Lesson Focus

In this lesson, you will

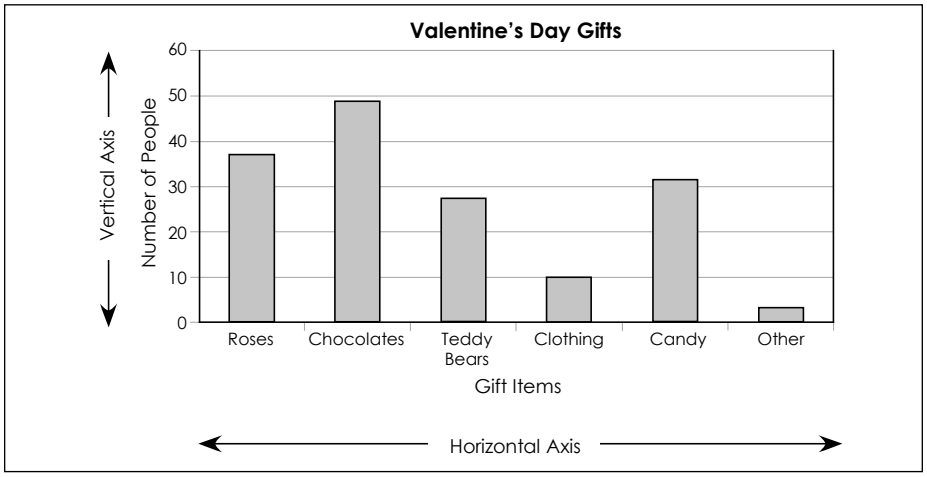
- ❑ study bar graphs and the type of data they represent
- ❑ draw bar graphs by hand or with a computer
- ❑ explore the advantages and disadvantages of using bar graphs to represent data

Lesson Introduction

Bar graphs display the same kind of information—discrete data—as circle graphs. A circle graph provides a quick visual impression when comparing the quantities of relatively few items, but a bar graph is more suitable when comparing many items. Also, a bar graph is better suited for displaying data, because each bar represents a quantity and not just a percentage of the whole amount. The media frequently uses bar graphs to display research data, such as people's incomes in various occupations, the number of drinks consumed annually per person in a variety of countries, and so on.

Bar Graphs

You may have already studied bar graphs in previous grades, but we will review them in this lesson. Bar graphs are graphs that use vertical or horizontal lines to represent discrete data.



All bar graphs have a **vertical axis** and a **horizontal axis**. The graph above is a **vertical bar graph** because the bars are vertical. When you read a vertical bar graph, the vertical axis is the **quantitative axis** (the axis describing the amount, usually labelled as the "Number of _____"). To help you remember the word *quantitative*, think of the word *quantity*, which refers to the number of items. The horizontal axis, known as the **qualitative axis**, is divided into different categories. The categories can be anything from the types of gifts to years to colours. A **horizontal bar graph** has bars that go from side to side. The horizontal axis is the quantitative axis, and the vertical axis is the qualitative axis.

Drawing Bar Graphs

Drawing a bar graph requires fewer tools than drawing a circle graph. The components of a good bar graph are

- a title for the graph
- a title and labels for the qualitative axis
- a title and scale for the quantitative axis
- accurately drawn bars
- a legend (if you are drawing a double or triple bar graph)

Include the components of a good bar graph on your resource sheet.

The following are the five steps you should follow to create a bar graph. The data is from a survey where students were asked to name their favourite class.

1. **The data must be discrete and sorted into categories.** As mentioned previously, bar graphs represent discrete data. Before you can progress to the next step, you need to sort your data so that you know which values are associated with each category.

   Phys. Ed.: 179 students *Music: 106 students*
   Art: 28 students *English: 65 students*
   Math: 14 students *Science: 47 students*
   French: 51 students

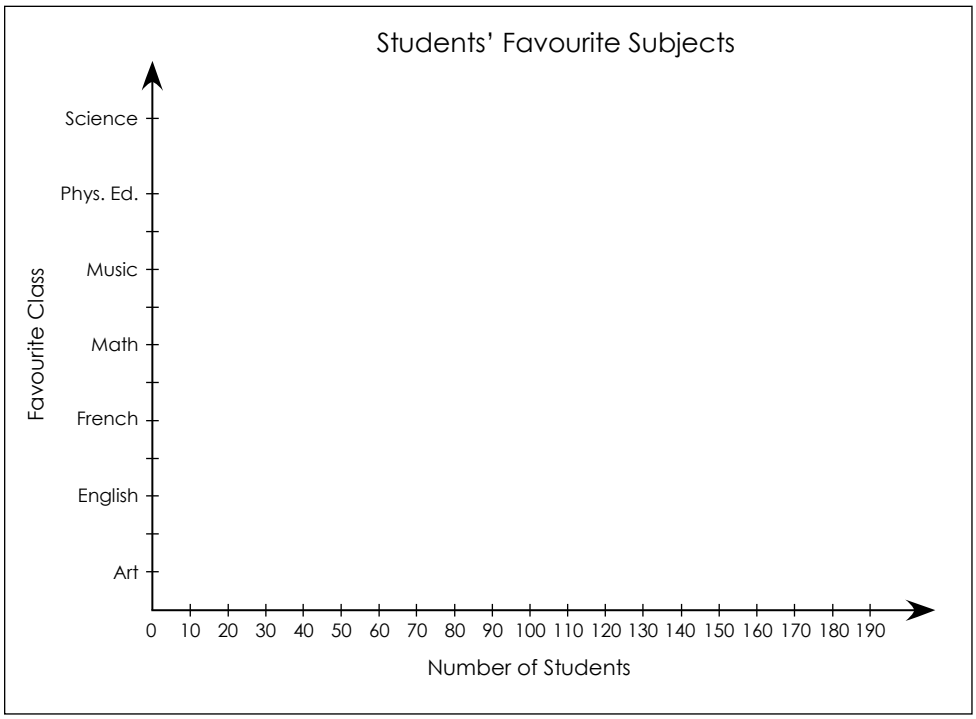
2. **Determine the maximum value of the sorted data.** In order to draw a good bar graph, you want the data spread out evenly. By knowing the highest value of any category, you know how long the longest bar will be. This will determine how you assign numbers to the quantitative axis.

   Highest value: 179 students like phys. ed. the best.

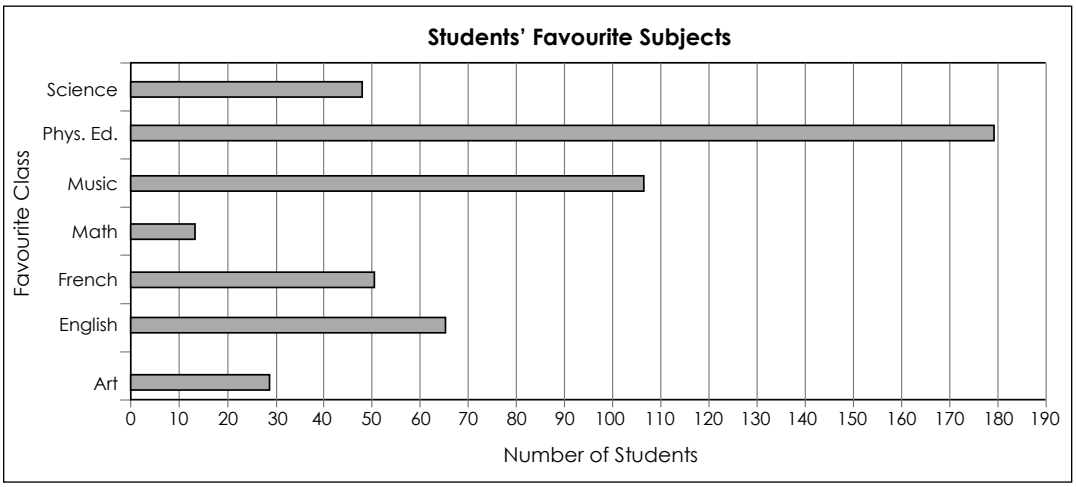
3. **Determine the scale of the quantitative axis.** For a bar graph to be drawn accurately, you need to label a reasonable number of equal increments (you don't want too few or too many). A good rule of thumb is between 5 and 15 increments, although you can have fewer or more, depending on the data.

   Continuing with this example, I refer back to the previous step and see that my maximum is 179. This is close to 180. If I make my increments 20, then I would use 10 increments to go from 0 to 200. Or I can make my increments 10 to produce the graph in Step 5.

4. **Draw and label your axes.** Using a ruler or straight edge, draw your vertical and horizontal axes. The quantitative axis should always start at zero, and should extend slightly past the maximum in Step 2.



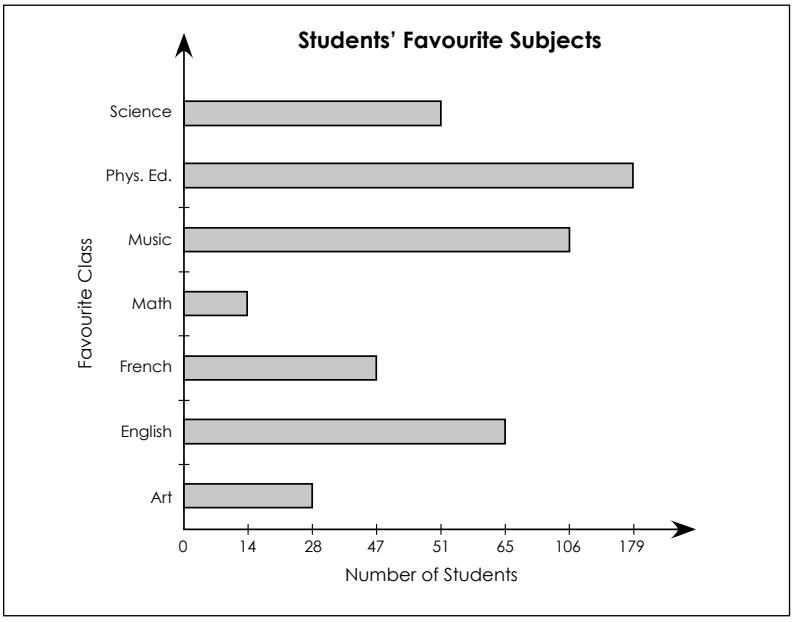
5. **Draw a bar for each category. The value of the category will determine the length of the bar.** It is important to draw the bars accurately. After the bars are drawn, the graph provides a quick way to compare categories.



As you may have noticed, vertical grid lines have been included. This makes the graph even easier to read, especially if, for example, you want to compare the number of students who like French to those who like science.

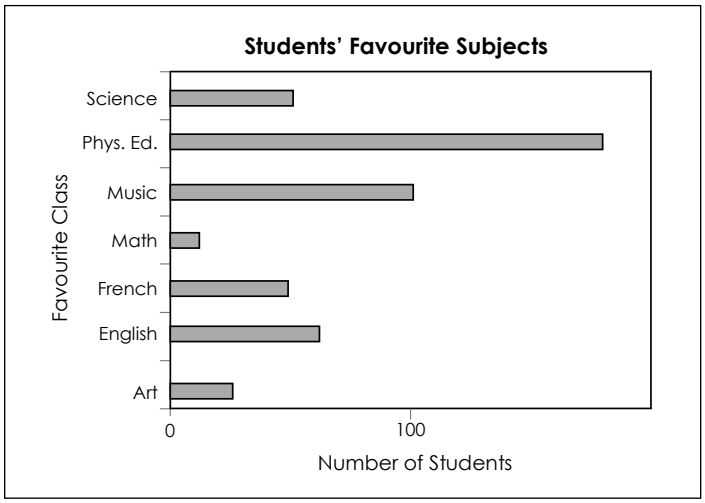
Below are some examples of badly drawn graphs, where the five steps were not followed.

Bad Graph #1:



This graph is badly drawn because the scale on the quantitative axis is not uniform. The student who drew this graph used the exact values of the data to label the axis, instead of using fixed increments.

Bad Graph #2:



This graph does not have enough divisions along the quantitative axis, so estimating the value of each bar is more difficult. Also, because it is so squashed along the quantitative axis, it becomes harder to see differences between the different subjects. Ask yourself, "Is it clear that the same number of students chose French as their favourite subject as those who chose science?" The answer should be no, because they were not chosen by the same number of students. French was chosen by 51, while science was chosen by 47 students. The effect of altering a graph will be discussed in Lesson 5.

Example 1

Fisheries and Oceans Canada monitors the number of tonnes of fish caught in the provinces. According to Statistics Canada, in 2006, Manitoba's data was as follows:

Fish	Tonnes Caught	Fish	Tonnes Caught
Carp	366	Lake Trout	40
Perch	148	Pike	1099
Sauger	172	Sucker	1808
Tullibee	63	Whitefish	2054
Yellow Pickerel	5829		

a) Using this data, construct a vertical bar graph.

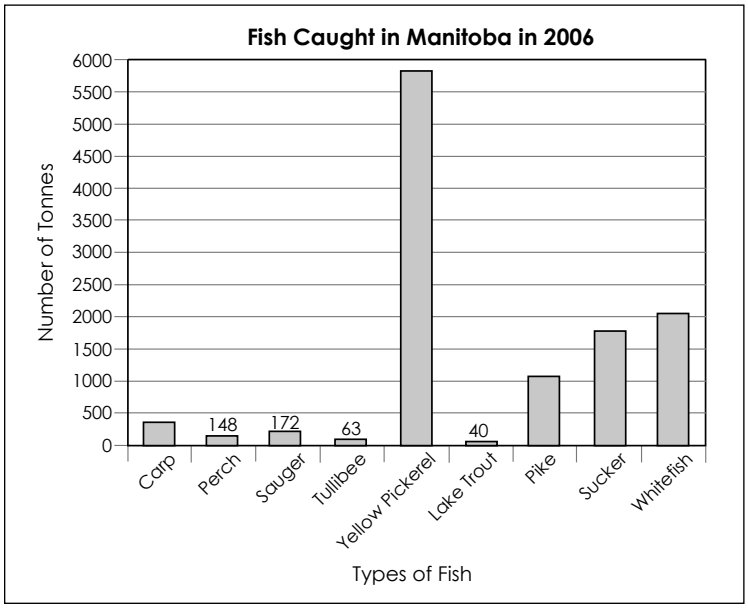
b) What conclusions could you make based on this graph?

Solution

a) I. The data is already sorted into categories by "type of fish."

II. The highest value of any category is yellow pickerel with 5829 tonnes.

III, IV, and V.



Note: Because there is such a wide range of data (40 to 5829 tonnes), it is not easy to read the values of all the bars. In this case, it is helpful to label these bars with the correct values. This way, anyone is able to read the graph.

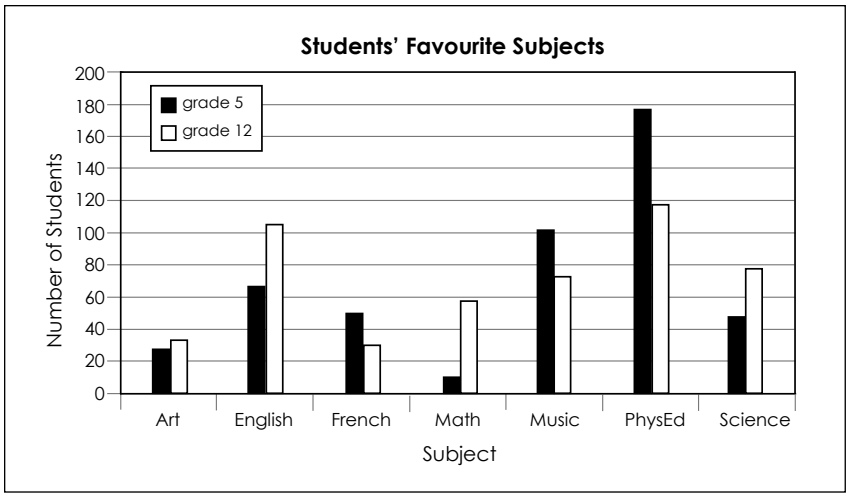
b) There are many possible answers. The following are a few examples:

- Yellow pickerel are the most commonly caught fish in Manitoba.
- We don't have a lot of lake trout and tullibee in Manitoba lakes.

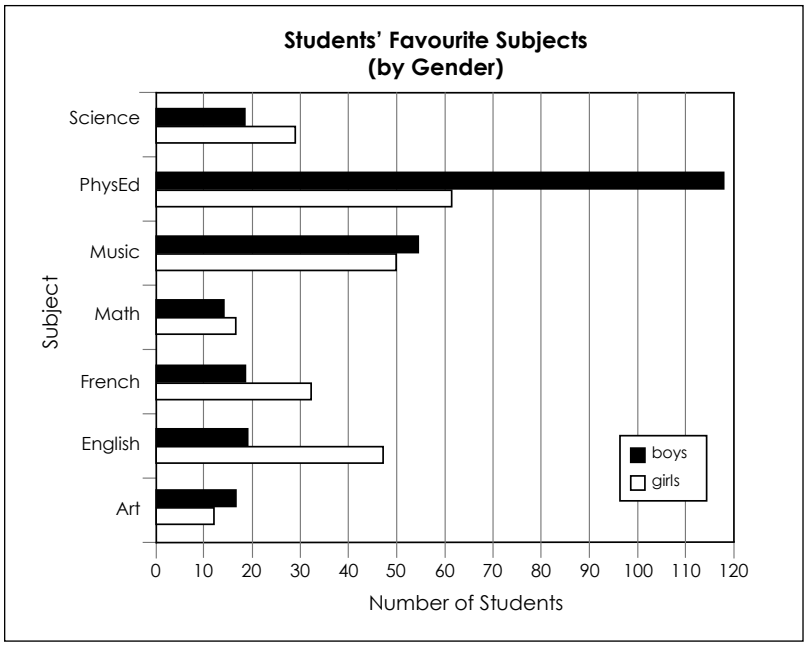
Double Bar Graphs

Double bar graphs are very similar to the bar graphs you have already studied, and you can use the same steps to draw them. With this type of graph, you can compare data from different surveys or different years, or highlight a difference that is in every category (such as boys and girls instead of students). This is an option we did not have when drawing circle graphs.

We could redo the bar graph of students' favourite subjects in many ways to draw attention to differences. For example, if the same students completed surveys in Grade 5 and Grade 12, then a graph could show how they changed.



Another possibility would be to break down the data from Grade 5 and show how many boys and how many girls like each subject.



Example 2

Statistics Canada did a survey to determine the average ticket costs for both domestic and international flights for 2008. Use the data below to draw a double bar graph comparing domestic and international flights.

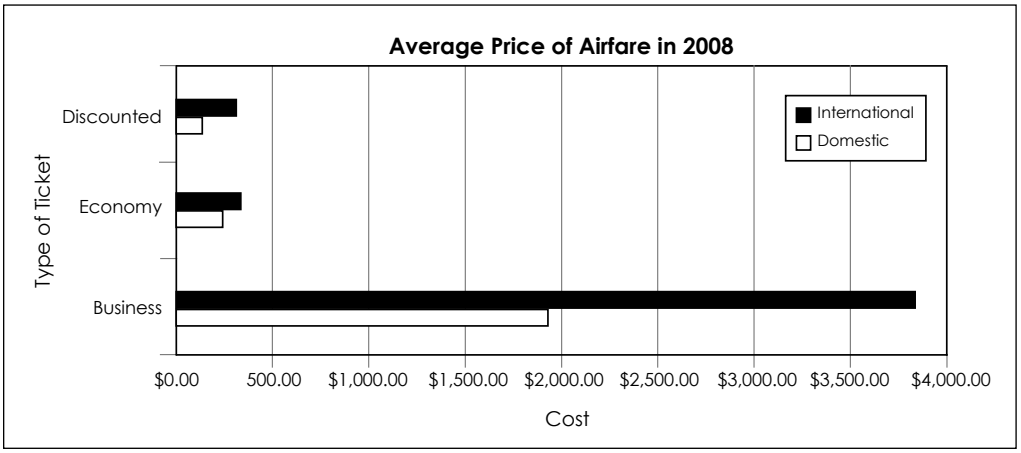
	Domestic	International
Business	$1,890.10	$3,796.90
Economy	$268.20	$345.80
Discounted	$187.60	$330.30

Solution

I. The data is already sorted by type of ticket, then by type of flight.

II. The highest value is for international business class tickets at $3 796.90

III and IV:



Creating a Bar Graph on the Computer

Colouring the bars of a bar graph can be tedious work, and so drawing them on the computer can be faster. Another benefit of creating a bar graph on the computer is that the ends of the bars will be at the exact value they should be, making the graph more accurate.

The example shows how to make a bar graph on a *Windows XP* operating system, using *Microsoft Excel*. As mentioned in the previous lesson, this is not the only graphing program available, so do not hesitate to try different software.

If you do not have a computer with graphing software, work through Example 3, keeping in mind the directions (including the five steps listed previously) to draw a good bar graph.

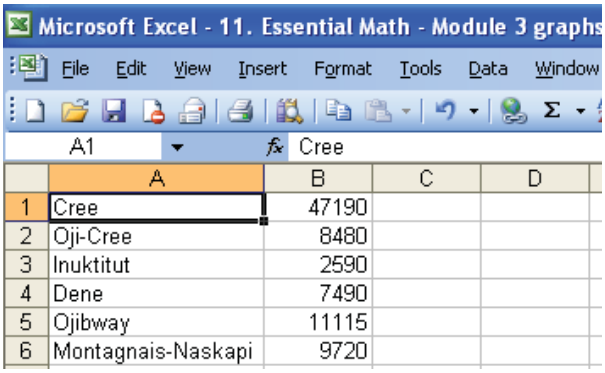
Example 3

The six most spoken Aboriginal languages in Canada are Cree, Oji-Cree, Dene, Ojibway, Montagnais-Naskapi, and Inuktitut. Use the information from Statistics Canada to draw a bar graph displaying the number of people that speak each language.

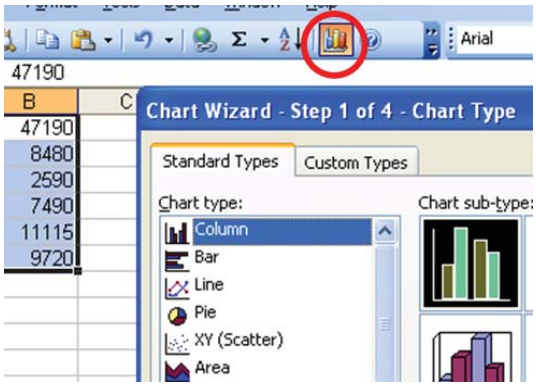
Cree	47190
Oji-Cree	8480
Inuktitut	2590
Dene	7490
Ojibway	11115
Montagnais-Naskapi	9720

Solution

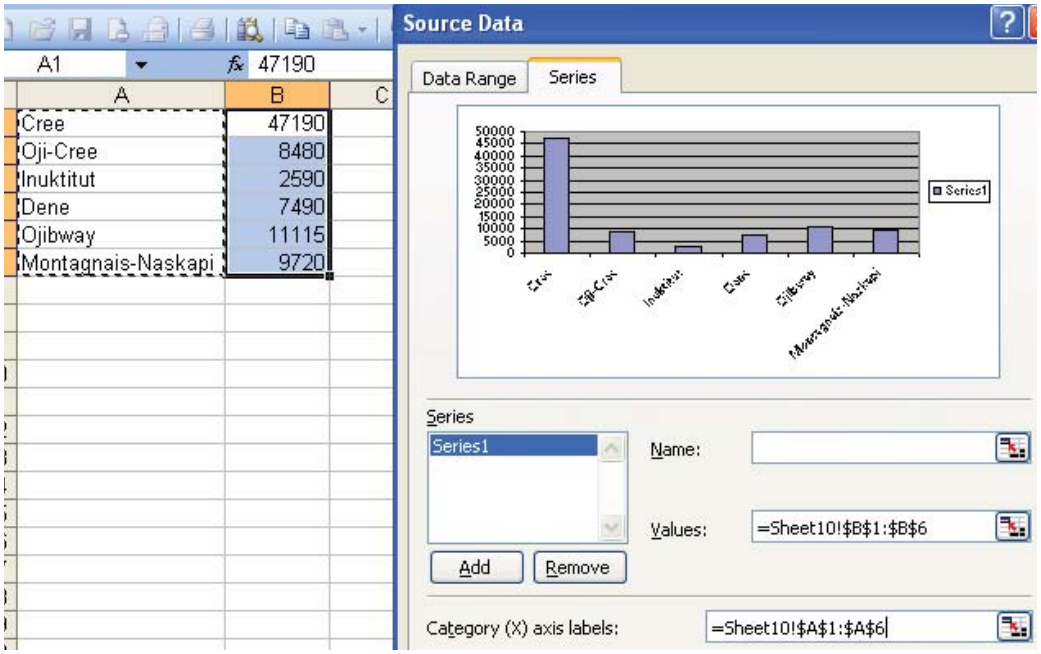
1. In one column, enter the names of the categories (languages). In the next open column, enter the number of people who speak that language.



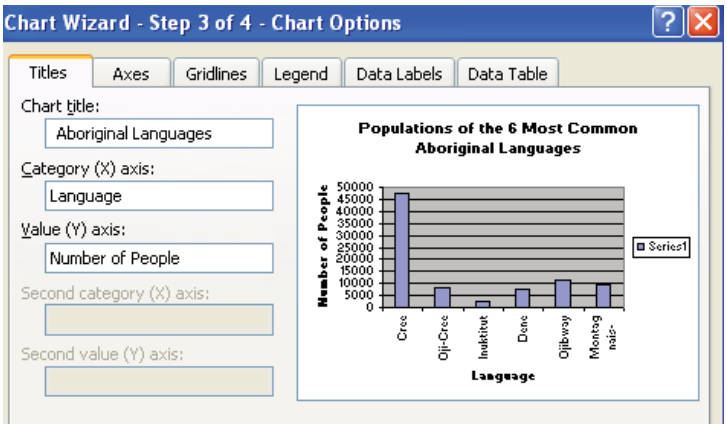
2. Highlight the data and click on the chart icon. The same window that appeared when you made your circle graph will appear now.



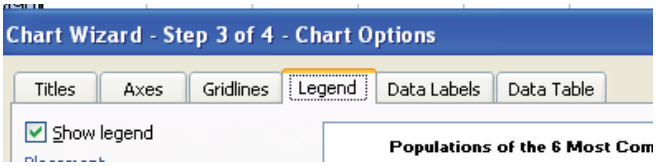
3. If you want to draw a vertical bar graph, select "Column," and then press the NEXT button. If you want to draw a horizontal bar graph, select "Bar," and then press the NEXT button. No matter which you choose, a new window will appear in place of this one. Select the SERIES tab. Press the button beside the "Category axis labels," and select the cells with the names of the languages. The labels should appear in the graph.



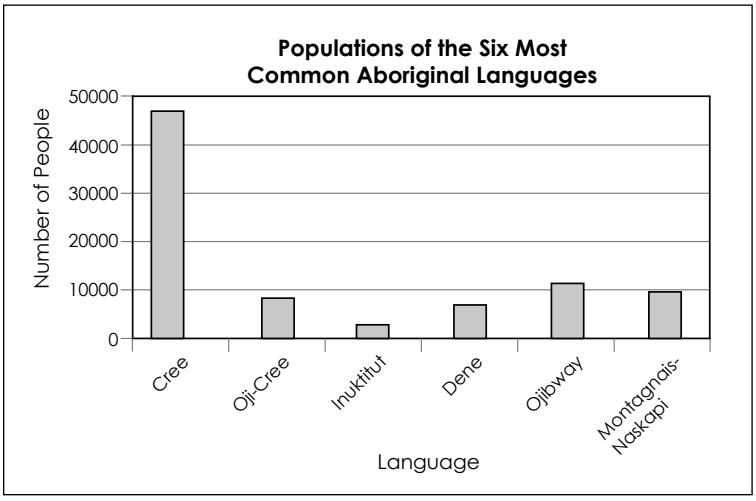
4. Click on the NEXT button to go to the next window. Here you can write the title for your graph along with the title for the "Category (X) axis," which is the *qualitative axis* and the title for the "Value (Y) axis," which is the *quantitative axis*.



Here you can also remove the legend by going to the LEGEND tab, and unclicking the "Show legend" box. You would need the legend if you were making a double bar graph.



5. Click the NEXT button, and then FINISH. A graph will appear. You can change different aspects of the graph by right-clicking on them.

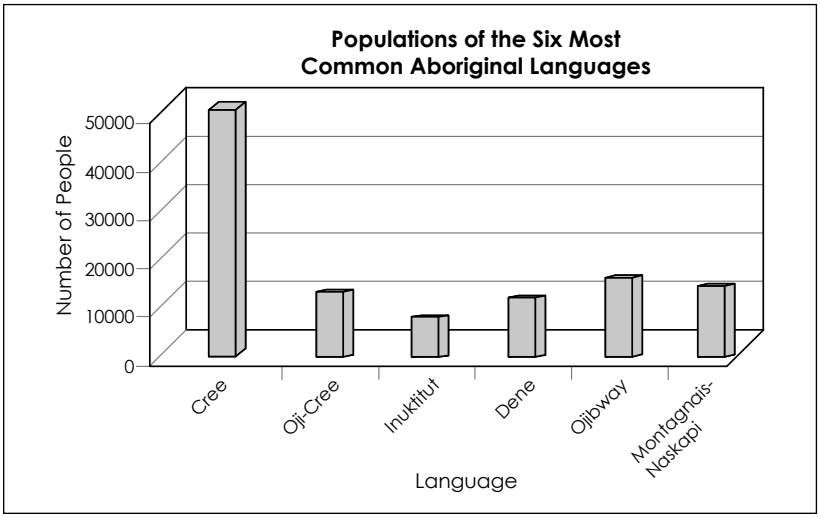


Variations of Bar Graphs

On the computer, you can alter the appearance of your graph so that it is more appealing. It can change the way the reader interprets the data. We will discuss the effects of these graphs in greater detail in Lesson 5, but you should know what they look like.

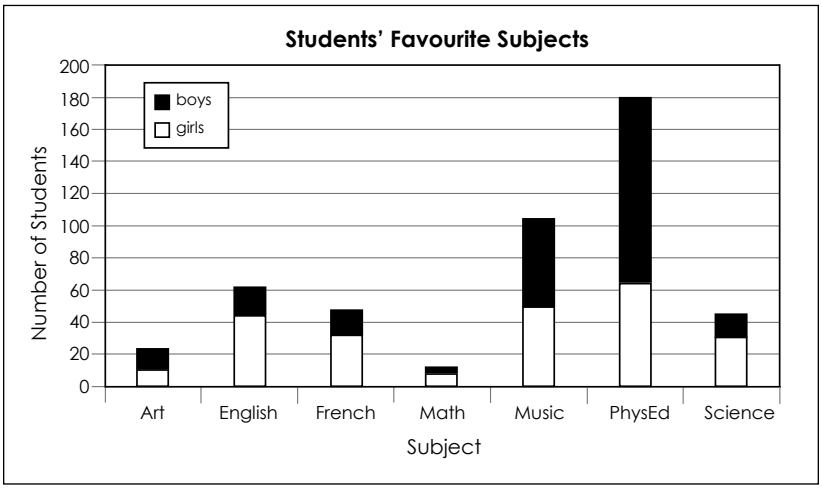
3-D Bar Graph:

This graph simply makes the columns 3-D.



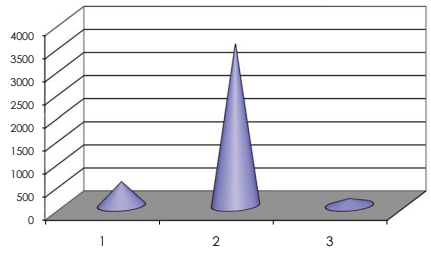
Stacked Double (or Triple) Bar Graph:

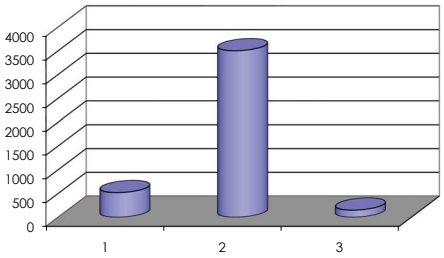
This graph stacks the values so that you can read the total for each category.

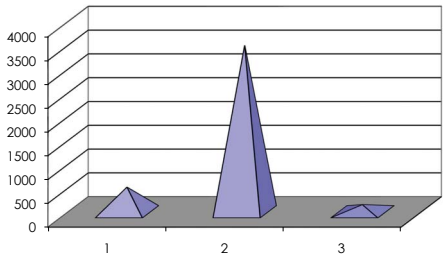


3-D Shapes:

These graphs are exactly like the 3-D bar graph, except that instead of rectangles they are cones, cylinders, and pyramids.







Advantages and Disadvantages of Bar Graphs

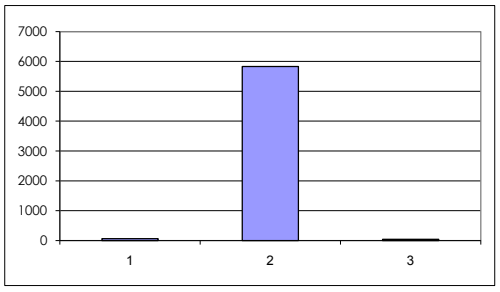
Now that you have drawn some bar graphs and looked at some different versions of bar graphs, it is time to look at some reasons for using them. Why would someone choose to use a bar graph? The advantages and disadvantages of bar graphs influence this decision.

Advantages

- Bar graphs are eye-catching and easy to understand.
- Bar graphs show the number of items in each category, not just a percentage.
- It is easy to compare sets of data (double and triple bar graphs).

Disadvantages

- You can only use discrete data when creating a bar graph.
- You have to estimate the exact length of the bar so that it represents the correct value.
- If the range of data is large, some bars may be very short and hard to estimate or even hard to see. For example, if two categories have only 40 items or fewer, while another has more than 5000 items, the graph could look like this: Can you see the other two bars?



Include these points in your own words in the pros and cons chart you started in the previous lesson. Include some of your own ideas as well if you think of something not mentioned above.

The following learning activity will help you put into practice what you have just learned. When you have finished, check your answers in the Learning Activities Answer Key at the end of this module.

Learning Activity 3.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Dania invests $5000. It takes eight years to double. Using the rule of 72, estimate the interest rate.
2. How much tax would you have to pay on an item that costs $99.99 (approximately) if the PST = 7% and the GST = 5%?
3. How long is the interest-free period if Zach buys a snowboard on November 27th and he does not have to pay until December 13th?
4. Are the following two fractions equal? $\frac{6}{8}$ $\frac{13}{16}$
5. Jeewon is at the movies with her friends. She spends $11.00 on her ticket, $4.00 on a drink, and $5.50 on ice cream. If she started with $25.00, how much does she have left?

continued

Learning Activity 3.2 (continued)

Part B: Bar Graphs

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. To kick off their Active Living course, students must survey their friends to determine their favourite ways to stay active. The teacher would like the class to record the gender of each person because they will be discussing their results in class. The teacher plans to draw a double bar graph with the data.

Gender	Activity
F	Soccer
M	Running
F	Running
F	Weightlifting
M	Soccer
M	Hockey
F	Speed Skating
M	Swimming
F	Soccer
F	Running
F	Swimming
M	Hockey
M	Hockey
M	Football
F	Baseball
F	Baseball
F	Running
M	Baseball
M	Basketball
F	Walking
M	Running
M	Cycling
M	Baseball
F	Volleyball

Gender	Activity
M	Hockey
M	Hockey
M	Cycling
F	Running
F	Basketball
M	Volleyball
F	Volleyball
F	Running
M	Weightlifting
M	Swimming
F	Cycling
F	Running
M	Weightlifting
M	Basketball
F	Volleyball
M	Football
M	Football
M	Basketball
F	Running
F	Weightlifting
F	Basketball
F	Volleyball
M	Football
M	Speed Skating

Gender	Activity
F	Hockey
F	Basketball
M	Soccer
F	Hockey
M	Running
M	Weightlifting
M	Speed Skating
F	Cycling
M	Running
F	Hockey
F	Hockey
M	Soccer
M	Volleyball
F	Basketball
M	Baseball
M	Weightlifting
M	Speed Skating
F	Cycling
M	Running
F	Hockey
F	Walking
M	Walking
M	Running
F	Cycling

continued

Learning Activity 3.2 (continued)

a) Given the data above, what are the categories that the teacher could use?
b) What will the two bars for each category represent?
c) Sort the data into categories
d) Draw a graph of the data using a double bar graph.
e) What information can you read from the graph?

Lesson Summary

In this lesson, you studied and drew bar graphs. Bar graphs are graphs where vertical or horizontal bars represent discrete data. You drew the graphs by hand or with a computer. You also explored the advantages and disadvantages of using a bar graph to represent data.

In the next lesson, you will study histograms, which are graphs that look somewhat similar to bar graphs but the data represented in them is not discrete.

Lesson 3: Histograms

Lesson Focus

In this lesson, you will

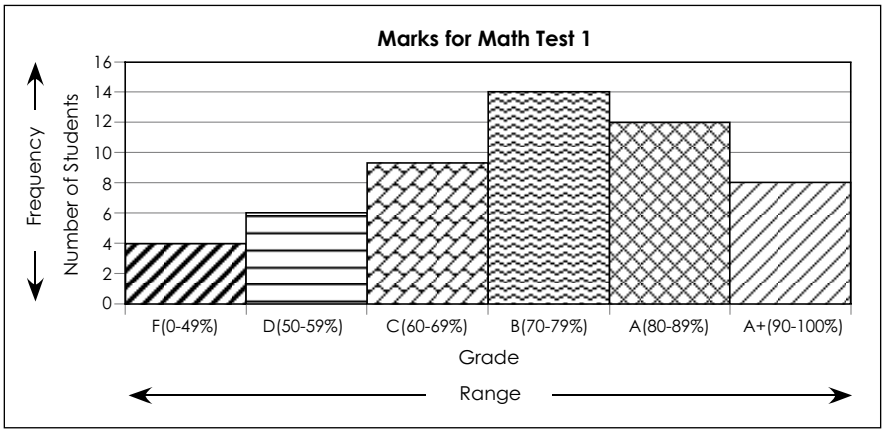
- ❑ study the meaning of continuous data and histograms
- ❑ draw histograms by hand or with a computer
- ❑ explore the advantages and disadvantages of using a histogram to represent data

Lesson Introduction

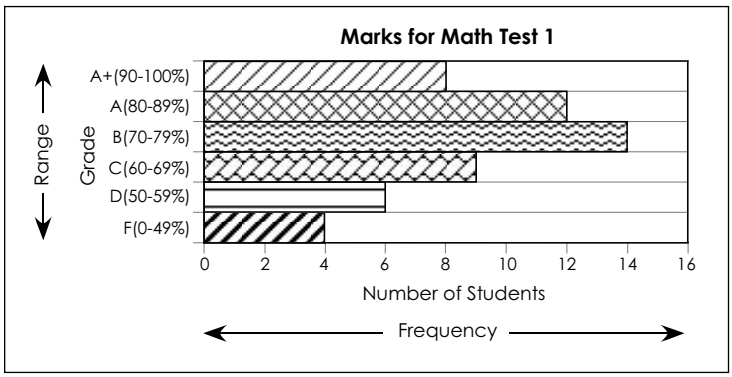
The previous two lessons have focused on circle graphs and bar graphs. These display discrete data, which is data sorted into distinct categories. Histograms look a lot like bar graphs, but the data they represent is not discrete. In this lesson, you will study histograms, and at the end of the lesson you will have the opportunity to compare bar graphs and histograms.

Histograms

Bar graphs display **discrete data**. For example, if a bar graph represents the marks of a group of students, the graph will have one bar for each student. **Histograms**, on the other hand, show continuous data. In the graph shown below, each category shows a range of marks. The **range** of each category is the difference between the minimum and maximum values.



Most histograms look like vertical bar graphs, with qualitative and quantitative axes. The horizontal (qualitative) axis displays the range, and the vertical (quantitative) axis shows frequency. If you draw the histogram horizontally, then, of course, you need to switch the axes.



Recognizing Continuous Data

The data shown in a histogram must be continuous. This means that each category represents a range of values, and it is possible to have any value within the range. For example, a student can be included in the top range of marks (90–100%), and you do not know whether the student has a mark of 92% or 98%. All you know is that eight students have marks in that category.

The opposite of continuous data is discrete data, where you can have only specific amounts and categories, and nothing in between. For example, the number of people at a birthday party is discrete because you cannot have half a person. Note that discrete data is something you can count (as in the number of people in a room), and continuous data is something you need to measure (as in measuring your height).

Example 1

State whether the data is continuous or discrete.

a) the amount of water in a water cooler as it is poured out

b) number of markers in a package at the store

c) length of a pencil

d) weight of different cats

Solutions

a) This is continuous data because you have to measure the amount of water, and the amount may be any fraction of a unit (for example, half a millilitre).

b) This is discrete, because you count the number of markers. A company would not sell half a marker.

c) This is continuous data because the result of the measurement of a pencil can be any length (within reason).

d) This, again, is continuous data. A cat's weight is measured and can be any value, including any fraction of a kilogram.

As you progress through this module, it is important that you can tell whether data is continuous or discrete, because you cannot use all graphs with any type of data.

Drawing a Histogram

The following are components of a good histogram:

- a title for the graph
- a title and labels (ranges) for the qualitative (horizontal) axis
- a title and scale for the quantitative (vertical) axis
- accurately drawn bars with no spaces between them

Include the components of a good histogram on your resource sheet.

The following four steps show the procedure for drawing a good histogram. These steps are very similar to drawing a bar graph.

1. **Sort the data into sections so that all possible data is included.** In the above graphs, the sections range from 0 to 100 percent. This does not mean that someone got 0 percent and someone got 100 percent on the test, but those are the minimum and maximum marks available.

   You do not need to divide the sections evenly across the range. As you can see, the category for an F is 0 to 49 percent while the other categories have much smaller ranges. This is done to avoid having very small bars, which, as you saw in the previous lesson, can become hard to read and do not communicate data very well.

2. **Find the maximum frequency.** By determining how high the tallest bar must be, you can determine the range of frequency you need to include on your graph.

3. **Draw and label your vertical and horizontal axes.** Before you begin to draw the bars for each section, you need to have the axes drawn and labelled. *You always draw the axis describing frequency to extend further than the maximum value.* In the above graph, the maximum frequency is 14 but the axis goes up to 16. If we were making a histogram based on data collected about customer satisfaction at a restaurant, the axes could look like this:

4. **Draw the bars for the graph.** This is the final step. Be as precise as possible when drawing the bars. Most histograms do not have spaces between the bars. This is a visual cue for: "When one category ends, the next one begins."

Make a note on your resource sheet to remind yourself that histogram bars do not have spaces between them.

If you were to complete the graph about customer satisfaction, it might look like this:

Now that you know the steps to make a good histogram, let's see how it should look when you are answering a question.

Example 1

In 2007, Statistics Canada did a survey to determine the incomes of employed people in Canada. The results, after compiling the information, are shown below.

Ranges of Income	Number of People in Each Range
under $50,000	18 395 690
$50,000–$100,000	4 759 410
$100,000–$150,000	756 930
$150,000–$200,000	197 130
$200,000–$250,000	82 020
over $250,000	160 060

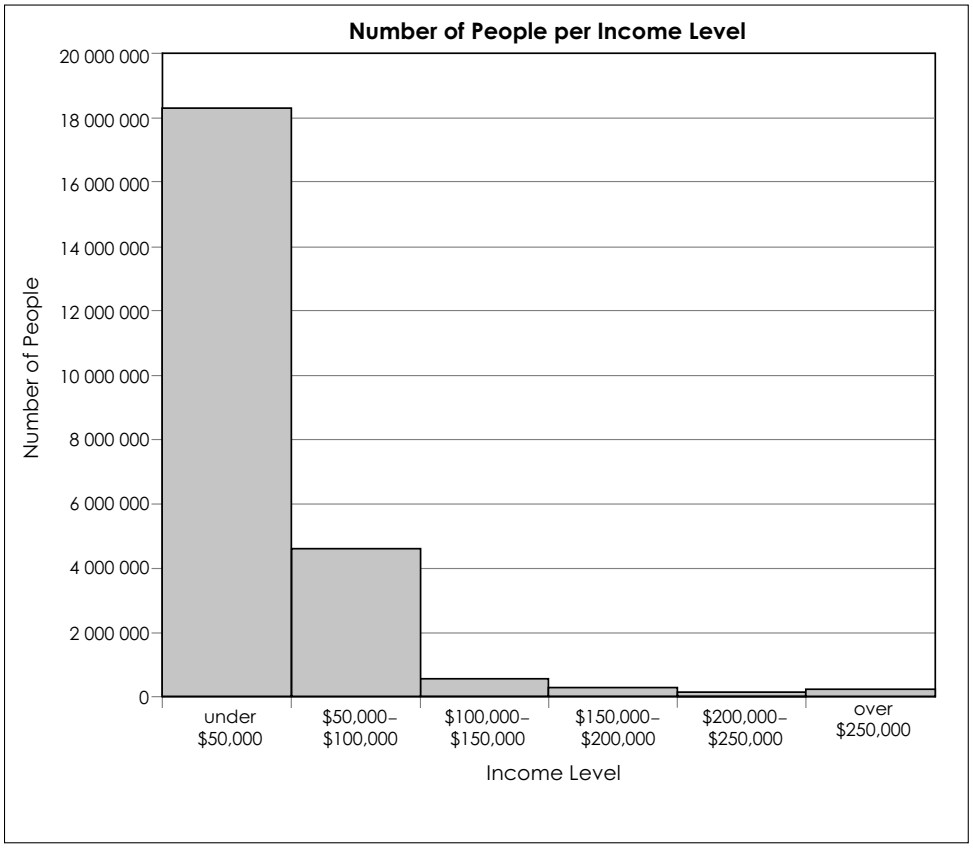
Draw a histogram to represent this information.

Solution

I. The categories have already been created.

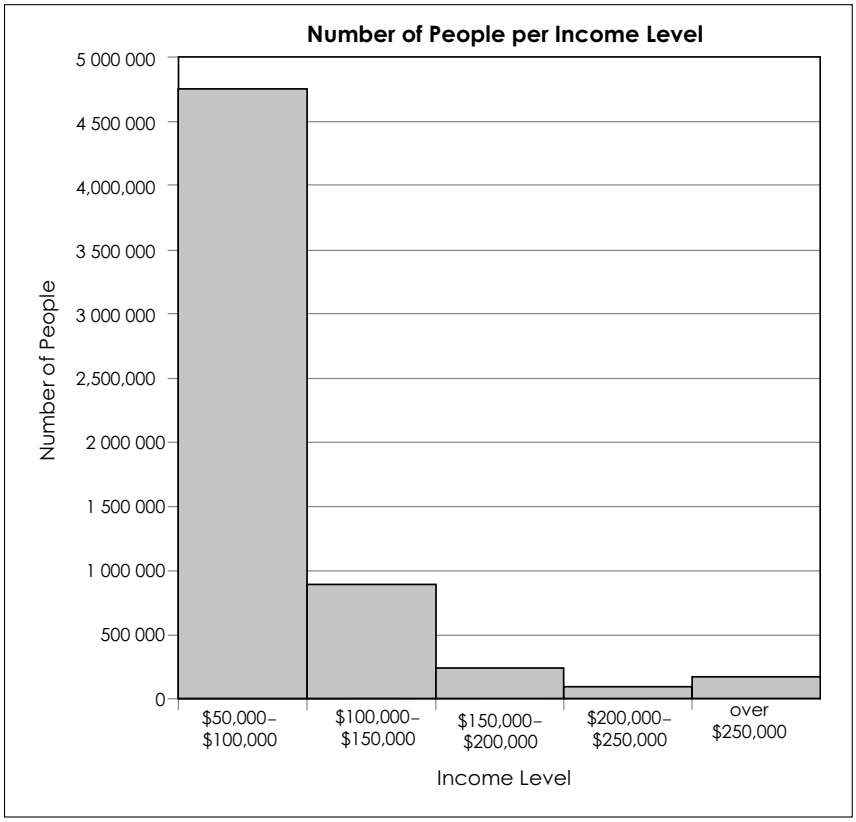
II. The maximum frequency is 18 395 690 people who make under $50,000.

III and IV:



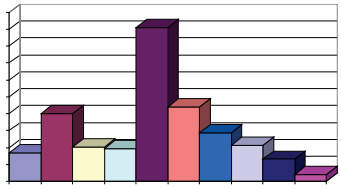
Using a histogram in this case is helpful, because we do not need to know how many people earn exactly $45,134, or if anyone even does. The histogram gives us the "big picture" by grouping the data (in this case, income amounts).

Because the data varies from 82 000 to 18 395 690 or 0.08 million to 18 million, it is hard to see a difference in the last five categories. Here is the histogram with only the last five categories showing.

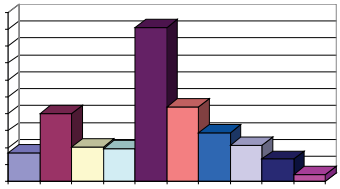


Variations of Histograms

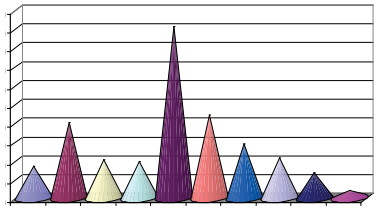
One major variation of the histogram is the 3-D histogram, which shows the same data using prisms instead of flat rectangles. It also extends to other 3-D shapes that could be used to make a bar graph, but because of their odd shapes, they do not maintain the connected appearance that the rectangles and rectangular prisms do.



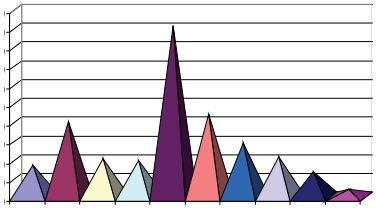
(prisms)



(cylinders)



(cones)



(pyramids)

Creating a Histogram on the Computer

The steps for drawing a histogram on the computer are very similar to drawing a bar graph. **If you do not have access to graphing software, work through Example 2, keeping in mind the directions (including the four steps listed previously) for drawing a histogram.** For those of you using a different operating system than *Windows XP* and/or different software than *Excel*, remember that the help button is very useful if you have questions.

Your learning partner may be able to help you. Also, if you have access to the Internet, searching for the answer to your problem may be just a click away.

The steps for generating a histogram are the same as for a bar graph, so we will not repeat them again. (See page 47.)

Example 2

In 2001 Statistics Canada surveyed the population to find out how many people lived in Canada. The results were as follows:

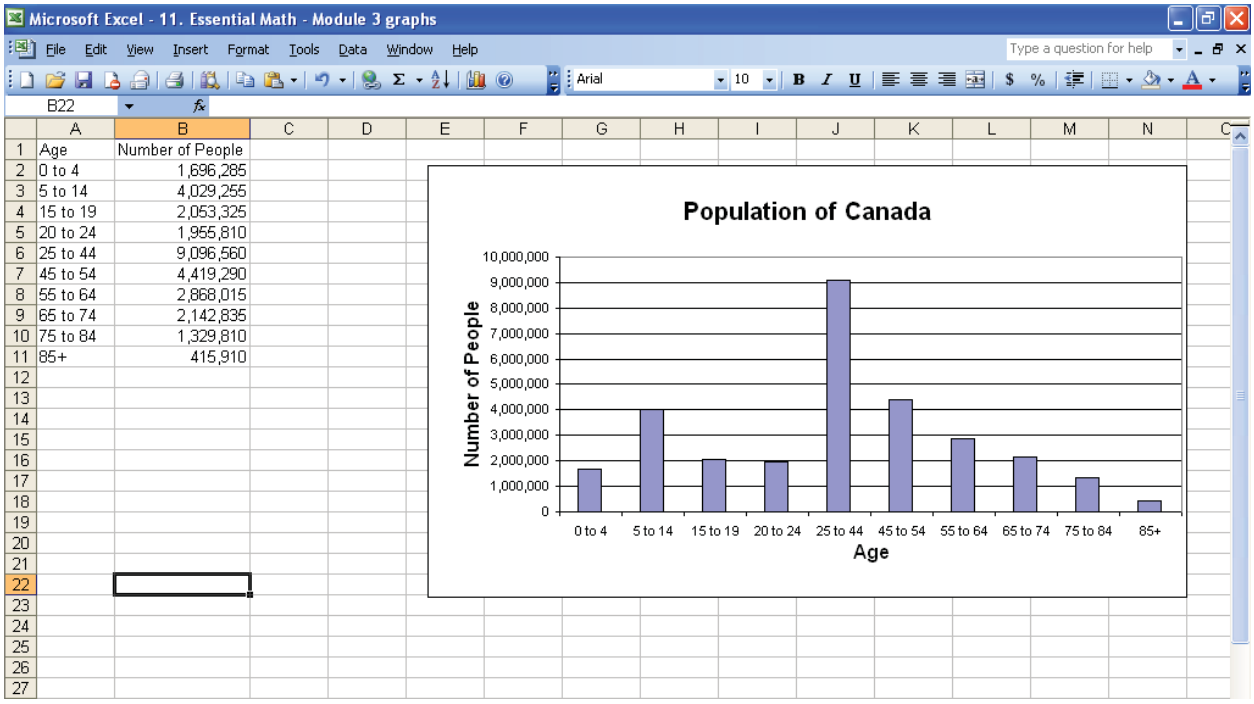
Age	Number of People
0 to 4	1,696,285
5 to 14	4,029,255
15 to 19	2,053,325
20 to 24	1,955,810
25 to 44	9,096,560
45 to 54	4,419,290
55 to 64	2,868,015
65 to 74	2,142,835
75 to 84	1,329,810
85+	415,910

a) Draw a histogram to represent this data.

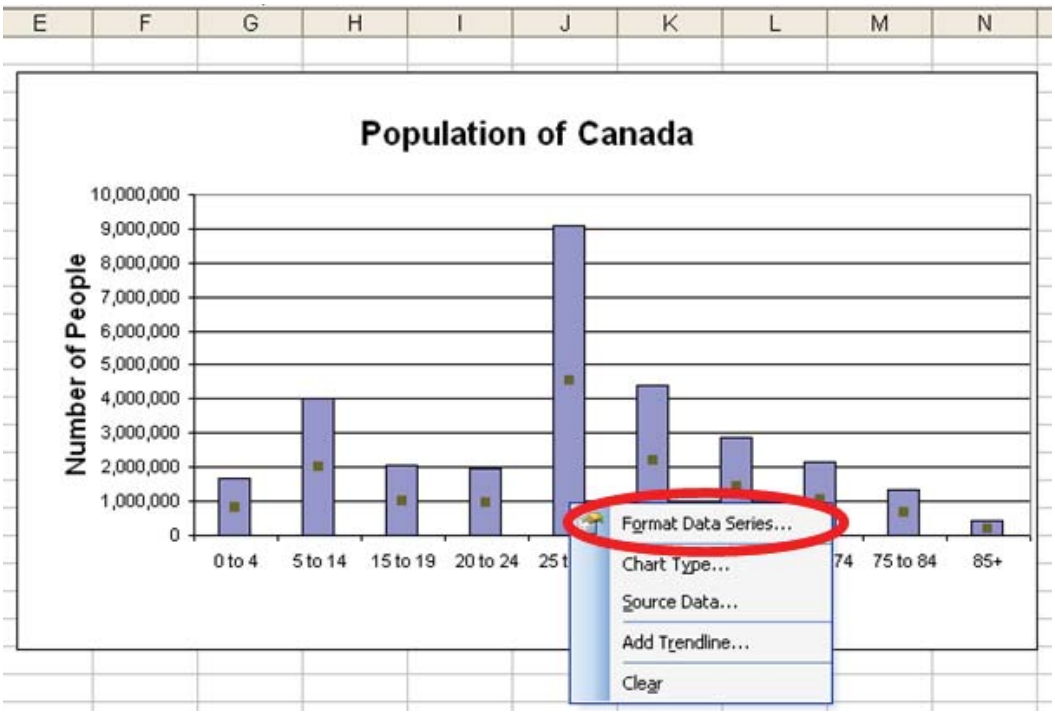
b) What conclusions can you make based on the graph?

Solutions

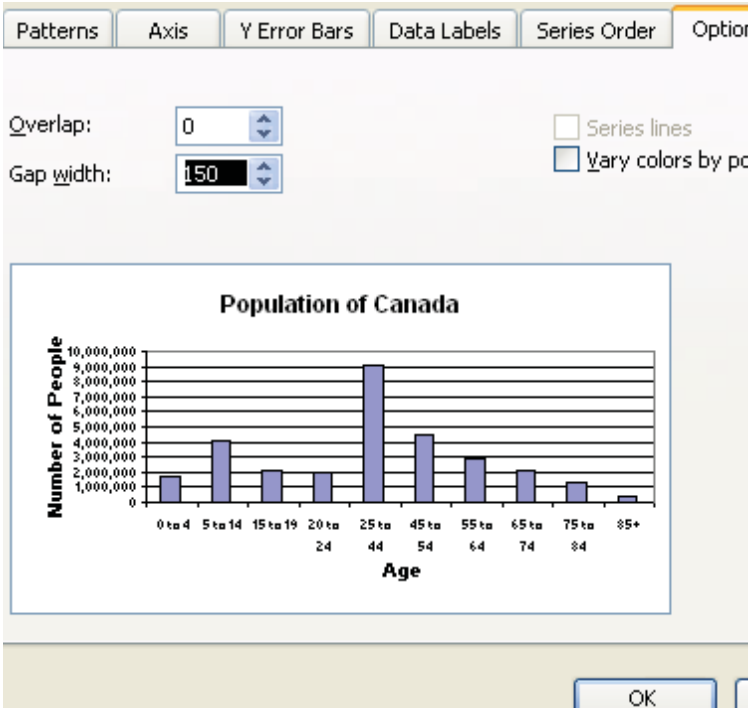
a) Once you have generated a bar graph, your screen should look like this.



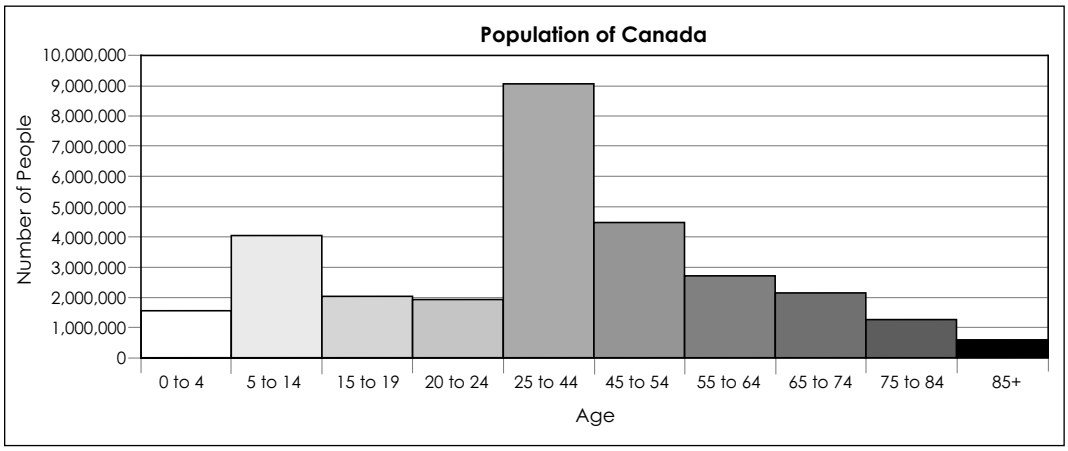
Click on one of the bars so that a square appears in the middle of every bar. Right-click on the bar and select "Format Data Series."



A window will appear. Go to the OPTIONS tab. Change the Gap Width to zero.

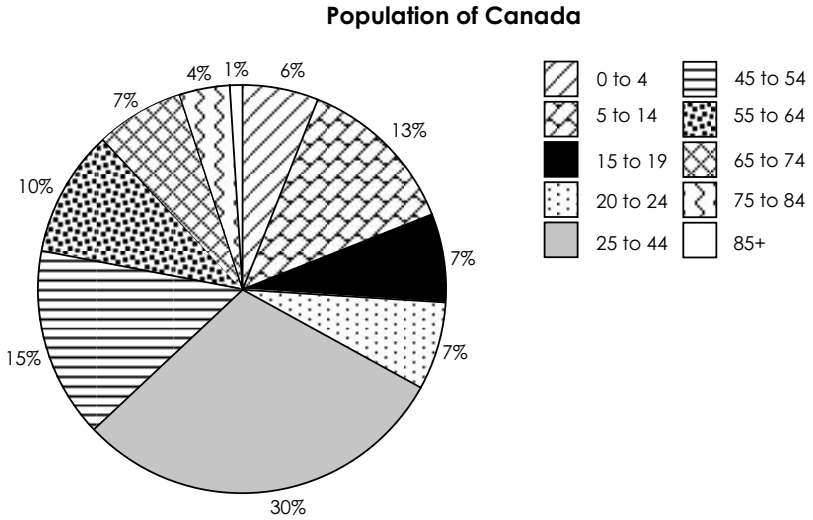


If you want each bar to be a different colour, you can also select the "Vary colours by point" box. Click OK. The window will disappear and your graph should look like this:



b) From looking at the histogram, we can tell that the *most common ages* are 25 to 44. *Be careful how you word your answers.*

If someone said the *majority* of the population is between the ages of 25 and 44, this would be **wrong**. The word *majority* implies more than 50 percent of the total. If we presented this data in a circle graph, we would see that the category "25 to 44" is less than 50 percent of the population.



Also, you should notice that the age range of the tallest bar is 25 to 44—almost 20 years! The majority of the other bars have a range of either 5 or 10 years. This means that if the age range of "25 to 44" were split up into smaller categories, each bar would be shorter than the one currently in the graph. Unfortunately, it cannot be broken down because the data provided is already sorted into these categories—there is no **raw data**.

The least common ages are above 85 years old.

Advantages and Disadvantages of Histograms

As with the other two types of graphs you have studied, there are advantages and disadvantages associated with histograms. Write some advantages and disadvantages you can think of on the chart below. Also, if possible, talk with your learning partner about some advantages and disadvantages of using a histogram, and add these to the chart as well.

Advantages	Disadvantages

Advantages

The following are some of the advantages of drawing histograms:

- Histograms summarize data.
- Histograms group data into categories so that it is easier to see patterns.
- Histograms are useful when describing a range of data all related to one topic (e.g., incomes of Canadians).

Disadvantages

The following are a few disadvantages of using histograms:

- Histograms do not include exact data values (e.g., from the graph in Example 2, you cannot tell how many people are 25 or 44).
- Histograms are hard to compare with other data *if both graphs do not have the same categories/sections*, because you do not know how many data points from one category would be in another if you changed the range. For example, in Example 2, you wouldn't be able to divide the "25 to 44" category into two parts because you wouldn't know that (perhaps) only 15 percent of those people are older than 34.
- Histograms can be misread, as demonstrated in Example 2(b). You must use the correct words when analyzing a graph.

Before you move to the learning activity for this lesson, you should complete the column of pros and cons related to histograms. The table is at the end of Lesson 1.

Learning Activity 3.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Write two equivalent fractions for $\frac{9}{6}$.

2. A cube is 1 m tall, 2 m wide, and 3 m deep. What is the volume of the cube?

3. From your house, you walk 10 m south, 8 m east, 10 m north, and 6 m west. How far are you from your house?

4. A cylinder has a volume of 60 mm^3. What is the volume of a cone with the same radius and height?

5. If 10% of people do not like the colour brown and there are five people in your family, how many people do not like the colour brown in your family?

Part B: Histograms

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Compare and contrast bar graphs and histograms using a chart similar to the one below.

	Bar Graphs	Histograms
Differences		
	Both	
Similarities		

continued

Learning Activity 3.3 (continued)

2. As part of his law class, Jeremiah is doing a project about jails. As part of his presentation, he wants to include statistics about the sentences of prisoners in a high security prison. The data he collected is in the table below.

Sentence Time	Number of Prisoners
under 1 year	845
1 year and up to 2 years	1758
2 years and up to 5 years	2486
5 years and up to 10 years	6784
10 years and up to 15 years	5917
15 years and up to 25 years	4375
25 years and over	583

a) Draw a histogram that Jeremiah could use in his presentation about Canadian jails.

b) The provincial government is responsible for prisoners who serve terms less than two years, while the federal government is responsible for prisoners serving two years or more in jail. Based on the graph above, which government is responsible for the larger number of prisoners?

Lesson Summary

In this lesson, you studied and drew histograms. You studied differences between discrete and continuous data. Histograms are graphs that use vertical bars to represent continuous data. You drew the graphs by hand or with a computer. You also explored the advantages and disadvantages of using a histogram to represent data.

In the next lesson, you will study line graphs, which also display continuous data. But before you move to the next lesson, complete the following assignment. This is part of your evaluation for Module 3.

Notes

Assignment 3.1

Circle Graphs, Bar Graphs, and Histograms

Total: 27 marks

Note to Students: Use your resource sheet to help you complete this assignment. Do not hesitate to add to your resource sheet as you complete the assignment. You can draw any graphs using a computer. Just remember to attach a print of each graph to the assignment when you submit your work at the end of the module.

1. Statistics Canada surveyed Canadians 15 years old and older to find out how involved they are in sports. The three categories of involvement were (with their results):

   1. Regularly Participate — 8309 thousand people
   2. Participate Through a Club or Organization — 4599 thousand people
   3. Participate in Competitions or Tournaments — 2992 thousand people

   a) Is this data discrete or continuous? Circle your answer below. *(1 mark)*

   discrete / continuous

   b) Choose which graph you would use to describe this data (circle graph/bar graph/histogram). Explain why you chose this graph. *(2 marks)*

continued

c) The government also recorded the gender of the people taking this survey. Based on this information, the results are:

Participation	Male	Female
Regularly Participate	5 140 000	3 169 000
Through a Club/Organization	2 338 000	2 261 000
In Competitions or Tournaments	2 076 000	916 000

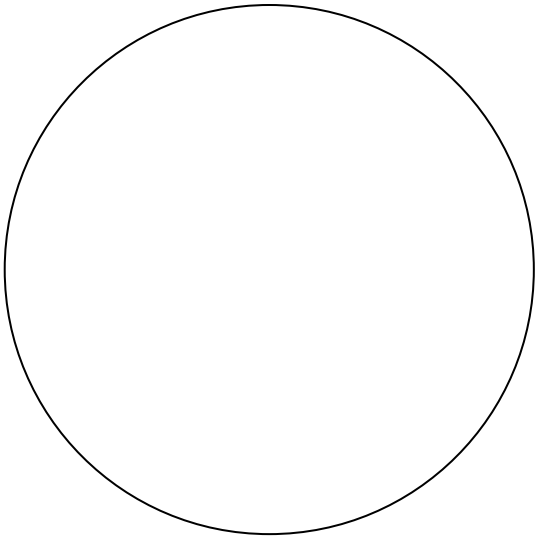
Create a double bar graph to represent this data. *(5 marks)*

continued

Assignment 3.1: Circle Graphs, Bar Graphs, and Histograms (continued)

2. The staff at Vaton, a restaurant, organizes a tip pool for the kitchen staff as well as the bussers, bartenders, and hosts who work there. The kitchen staff receives 50% of the tip pool, the bartenders receive 25% of the tip pool, the bussers receive 10% of the tip pool, and the hosts receive 15% of the tip pool.

   a) Draw and label a circle graph to show how the tip pool is divided. *(7 marks)*



   b) If the tip pool for Saturday night is $600, how much money goes to the kitchen staff? *(1 mark)*

   c) If there are 12 kitchen staff working on Saturday night, how much money will each of them get? *(1 mark)*

continued

Assignment 3.1: Circle Graphs, Bar Graphs, and Histograms (continued)

3. Winnipeggers experience a wide range of temperatures over the course of a year. The following data is the average temperature per month.

Month	Average Temperature (°C)	Month	Average Temperature (°C)
January	−18	July	20
February	−14	August	19
March	−6	September	13
April	4	October	6
May	12	November	−4
June	17	December	−14

a) What are the maximum and minimum average temperatures? *(1 mark)*

b) Sort the data into **at least three** categories that can be used to draw a histogram (e.g., from −20 to −5 °C; from −4 to 10 °C; from 11 to 22 °C). *(2 marks)*

c) Draw a histogram to represent the data. *(6 marks)*

d) Based on the graph, estimate the average temperature for Winnipeg overall. *(1 mark)*

Lesson 4: Line Graphs

Lesson Focus

In this lesson, you will

- ❑ study line graphs
- ❑ draw line graphs by hand or with a computer
- ❑ explore the advantages and disadvantages of using a line graph to represent data
- ❑ interpolate and extrapolate values from a graph

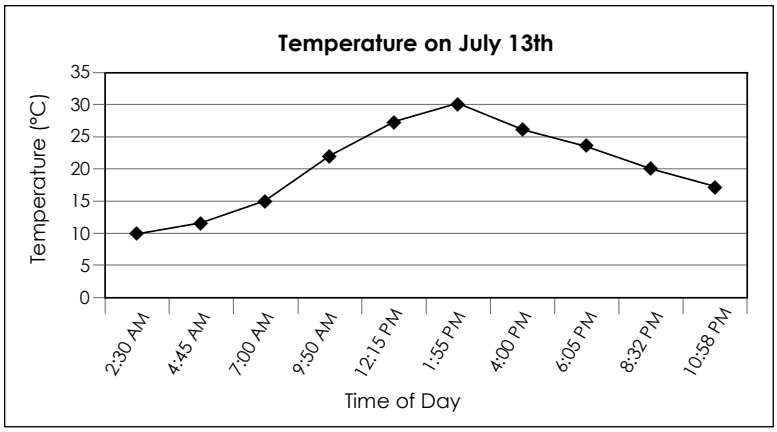
Lesson Introduction

Line graphs, like histograms, display continuous data. A line graph provides a quick visual impression of the relation between two quantities or groups of data. For example, a line graph can show the relationship between distance and time in physics, or between the number of litres of water flowing through a garden hose each minute while watering the lawn. A line graph can also be used to show the temperature each hour of a day.

Line Graphs

So far, we have looked at circle graphs where a circle is divided into sectors, and we have looked at bar graphs and histograms that use rectangles to illustrate data. A line graph uses points on a graph to illustrate data. These points are then connected by drawing line segments from one point to the next. Like a histogram, line graphs display continuous data using two axes. Line graphs can also be used to illustrate discrete data that represents *average values* (such as the average temperature for different months of the year).

The two axes describe different characteristics of each data point. Using the example mentioned in the introduction of this lesson, if we were to measure and draw a graph of the temperature over the course of the day, it could look like this:



Identifying Variables

One axis describes the time of day while the other describes the temperature. These two characteristics will be called **variables** from now on, because when one changes, the other changes as well. The variable on the *horizontal axis* is the **independent variable** because it is not affected by the other variable. In this graph, *time* is the independent variable because time does not depend on the temperature. The variable along the *vertical axis* is the **dependent variable**. The dependent variable (temperature) depends on the independent variable (time of day), because it is the time of day that determines how hot it is.

Example 1

Identify the independent variable and the dependent variable in each statement.

a) The distance you are from home compared to the amount of time you have spent driving as you go to work

b) The number of pizzas needed for a party compared to the number of people at the party

c) The amount of time spent doing homework compared to the amount of homework you have left to do

d) age compared to the height of a person

Solutions

a) **independent variable:** time

dependent variable: distance

The amount of time that has passed always increases, and it is unaffected by how far you have driven. The distance you have driven depends on the amount of time you have spent driving, and so distance is the dependent variable.

b) **independent variable:** number of people at the party

dependent variable: number of pizzas needed

The amount of food you supply for your guests depends on the number of people at the party. Therefore, the number of pizzas depends on the number of people, and so the number of pizzas is the dependent variable.

c) **independent variable:** time

dependent variable: homework left to do

Time is the independent variable because even if you do not do any homework, time will still pass. The amount of homework you have left to do depends on the amount of time spent doing it. Therefore, the amount of homework left to do is the dependent variable.

d) **independent variable:** age

dependent variable: height

Age is the independent variable because we always get older, even if you don't get any taller or shorter. Height increases from the time we are a baby until we reach our teens or adulthood, and so height is the dependent variable.

You may want to include definitions or an example of independent and dependent variables on your resource sheet. This topic will be revisited in Module 5.

Drawing a Line Graph

To draw a good line graph, you need

- a title for the graph
- titles on each axes
- scales or labels for each axes
- accurately plotted points, connected with straight line segments

You may want to include the components of a good line graph on your resource sheet.

Follow the five steps below to create a line graph.

1. **Identify which variable is independent and which is dependent.** Recall that the horizontal axis represents the independent variable, and the vertical axis represents the dependent variable.

   For example, if you are drawing a graph that shows the thickness of the ice on Lake Winnipeg each month of the year, the thickness of the ice is the dependent variable and the month is the independent variable.

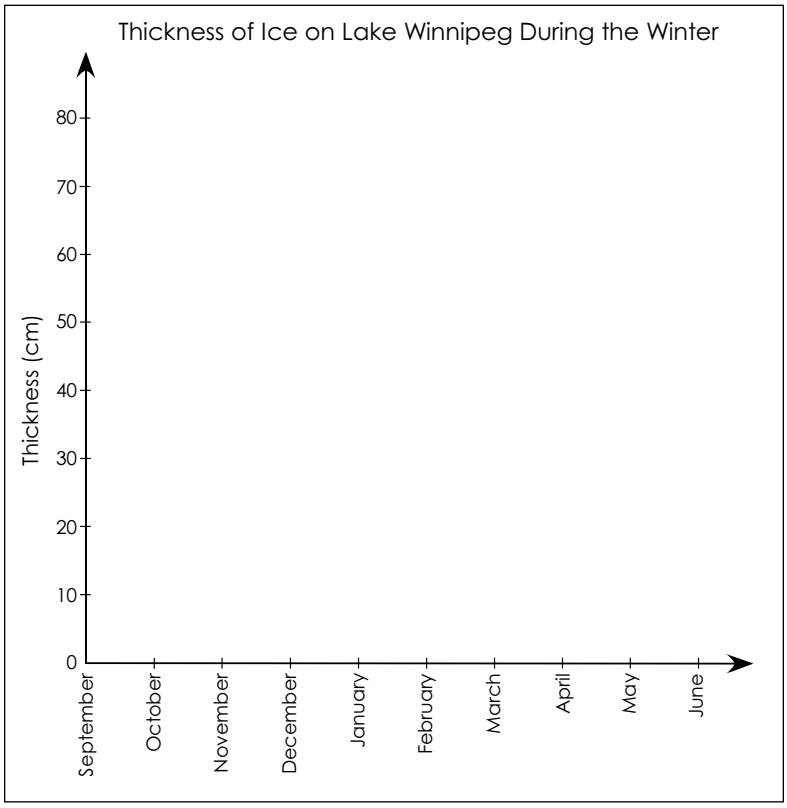
2. **Determine the range of data for both variables.** You did this previously for bar graphs and histograms. You need this information to write the scales and labels onto the axis.

   For the example above:

October	*0 cm*	*March*	*55 cm*
November	*10 cm*	*April*	*2 8 cm*
December	*30 cm*	*May*	*6 cm*
January	*75 cm*	*June*	*0 cm*
February	*75 cm*		

   The months (independent variable) range from October to June, and the thickness (dependent variable) varies from 0 to 75 cm.

3. **Draw and label the axes of the graph.** Do this before adding the points.



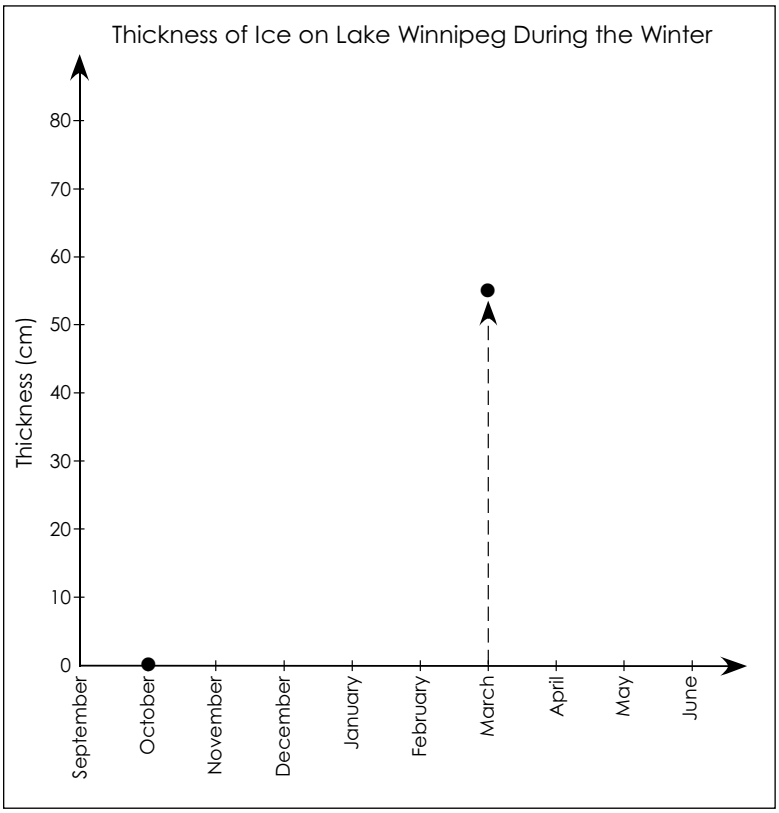
   The scale must be uniform so that the graph does not present misleading information.

4. **Plot the points onto the graph.** This is trickier than drawing a bar graph or histogram because each point must line up with the correct values on the horizontal and vertical axes. The values describing these points may be written as **ordered pairs**. An ordered pair indicates exactly where a point is located on a graph, and is written as:

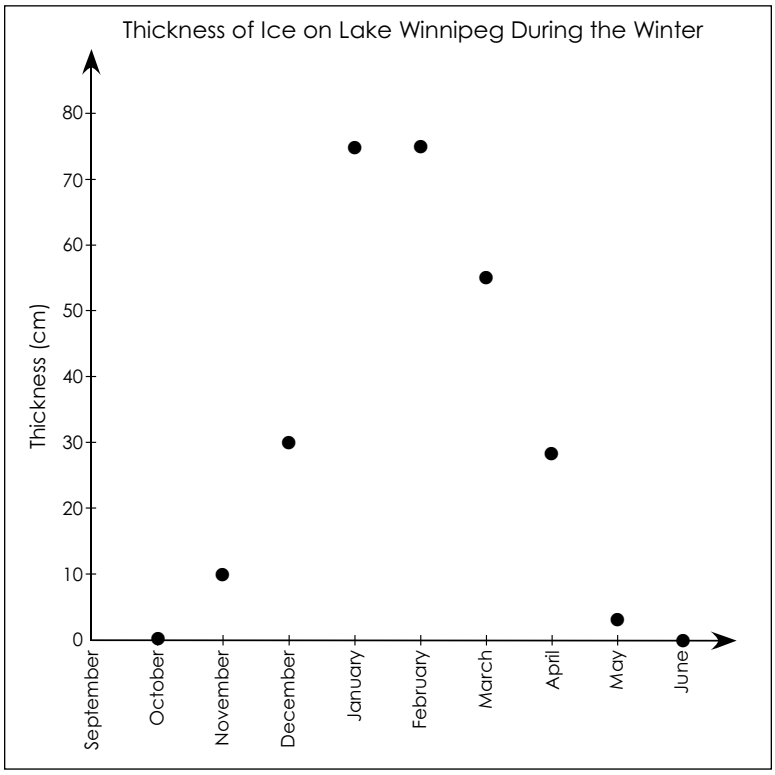
   (independent variable, dependent variable)

   (October, 0) and (March, 55) are ordered pairs for the above data.

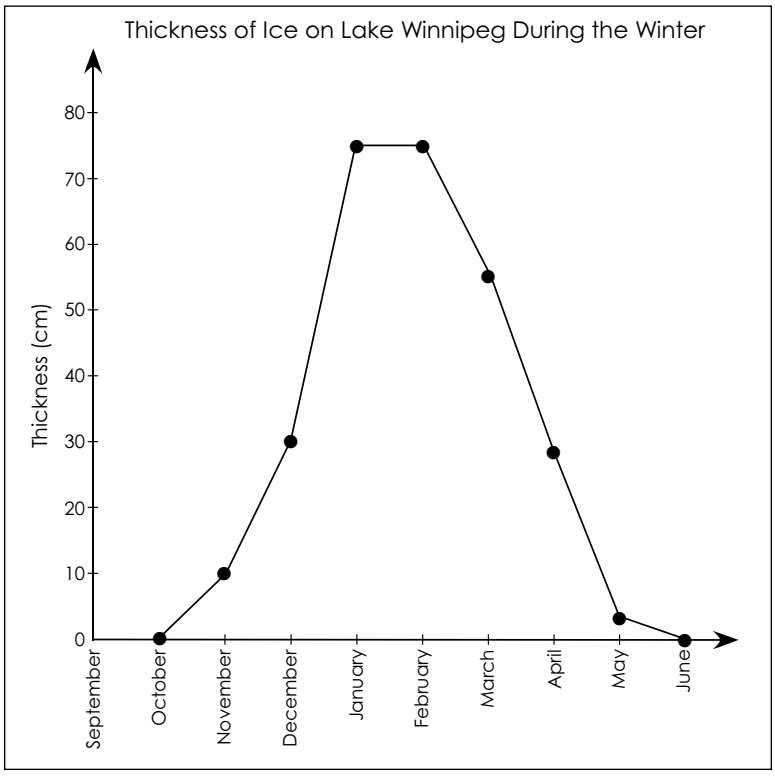
   In order to put this point on the graph, you must find the independent variable on the horizontal axis. Move directly up from the independent variable until you reach the dependent variable—this is where you plot the point for the ordered pair.



Do this for all of the points.



5. **Connect the points using line segments.** Use a ruler and pencil to connect each point with the next.



The following example has a step-by-step solution. Try to draw the graph before looking at the answer. On the other hand, if you want to see how it is done, try to complete one step at a time and then check your answer before you move on to the next step.

Example 1

Ronin has lived in the same house since he was born. From the time he could stand, his father measured his height on his birthday and marked it on the door frame in the kitchen. The measurements are shown below. Draw a line graph showing Ronin's height compared to his age.

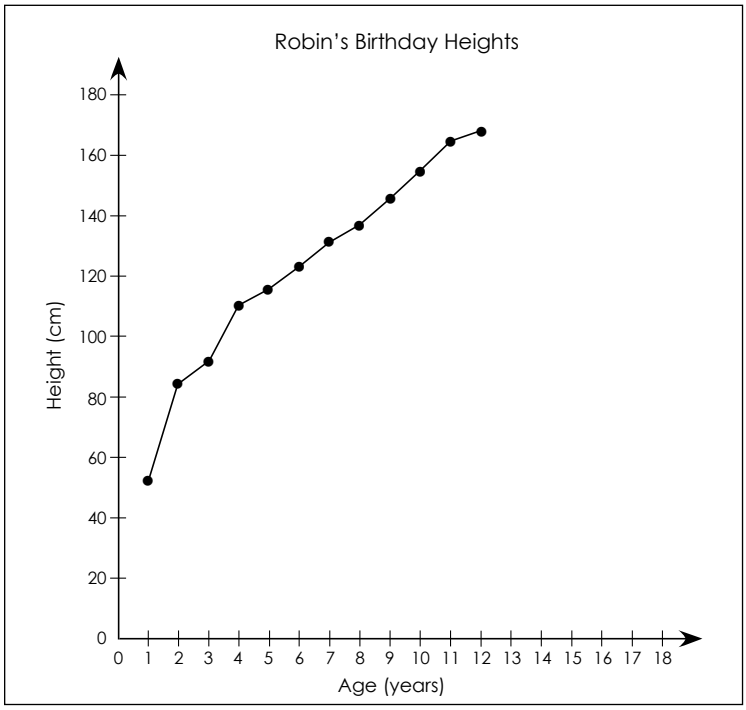
Age	Height	Age	Height
1	52 cm	7	131 cm
2	84 cm	8	137 cm
3	91 cm	9	146 cm
4	110 cm	10	155 cm
5	116 cm	11	163 cm
6	122 cm	12	168 cm

Solution

Step 1: independent variable: age; dependent variable: height

Step 2: his age is from 1 to 12 years old, his height is from 52 cm to 168 cm

Step 3/4/5



Double Line Graphs

Just as you could have single and double bar graphs, you can have single and double line graphs. Double line graphs are useful when making comparisons. The steps you follow for drawing the graph are the same as the steps for a single line graph. Be sure to determine the ranges of both sets of data before you draw the axis with the scale. You will need to include a legend to identify each line.

Example 2

The value of money today is different than it was many years ago. For example, 50 cents would have bought a lot more in 1900 than it would buy today. Statistics Canada collected the following data so that it could monitor the increase in prices. The data collected was the prices of similar items at different times.

	09-Jun	10-May	10-Jun
food	$122.20	$122.90	$123.00
shelter	$121.30	$123.00	$123.30

Draw a double line graph using the data above.

Solution

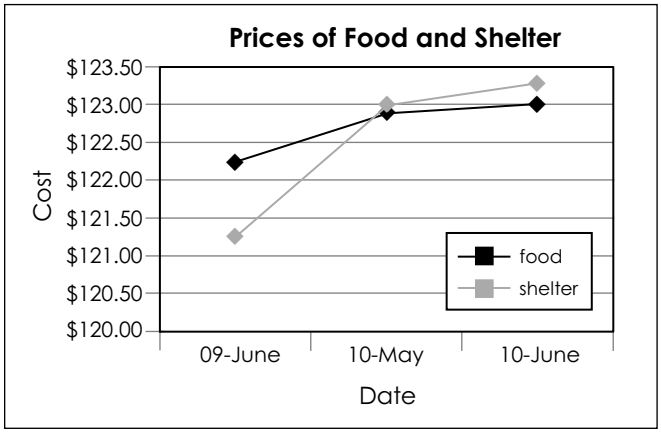
Step 1: independent: date, dependent: cost

Step 2: food: $122.20 to $123.00

shelter: $121.30 to $123.30

June/09 to June/10

Step 3/4/5



Creating a Line Graph on the Computer

Drawing a line graph by hand can be tedious work if there is a large number of points or a large range of data. Drawing the graph with a computer may save you some time. **If you do not have access to a computer with graphing software, complete Example 3 using the five steps to draw a line graph.**

This example is done in *Microsoft Excel* in *Windows XP*. If you are using different software, do not hesitate to press the help button or ask your learning partner for help.

Example 3

Seamus is a manager at a car factory. For one week, he compared the number of workers present to the number of cars produced. This is what he noticed.

Monday:	80 workers produce 12 cars
Tuesday:	55 workers produce 8 cars
Wednesday:	91 workers produce 13 cars
Thursday:	108 workers produce 17 cars
Friday:	67 workers produce 10 cars

Create a line graph for Seamus' data.

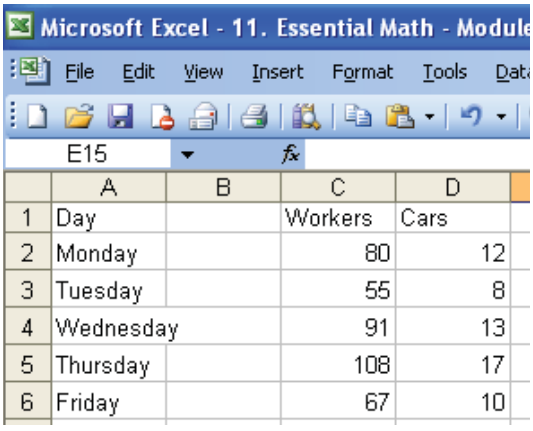
Solution

Step 1: independent variables = number of workers; dependent variables = number of cars

Step 2: number of workers ranges from 55 to 108; number of cars ranges from 8 to 17

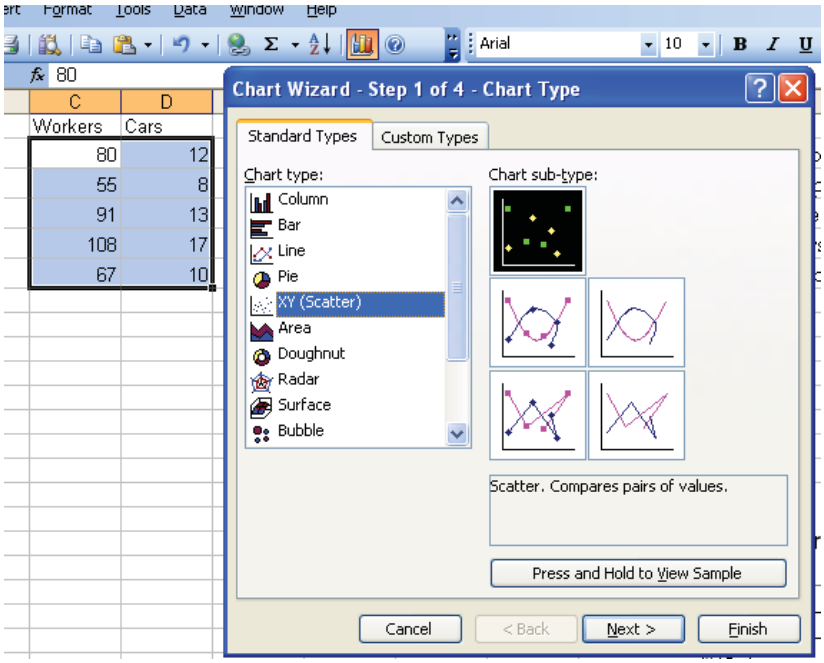
Step 3/4/5

Enter the information into the spreadsheet.

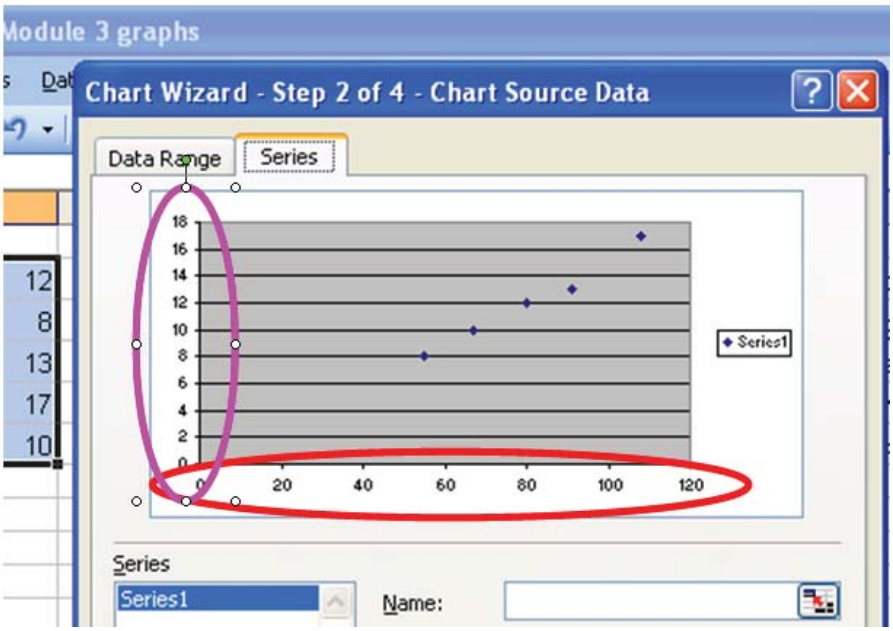


	A	B	C	D
1	Day		Workers	Cars
2	Monday		80	12
3	Tuesday		55	8
4	Wednesday		91	13
5	Thursday		108	17
6	Friday		67	10

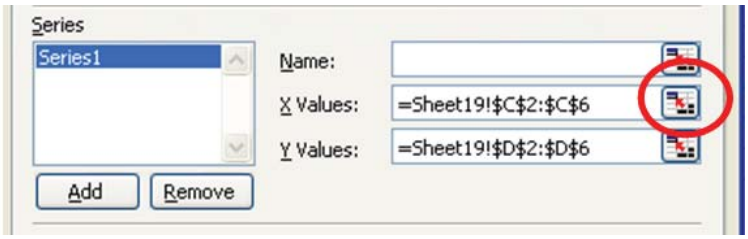
Select both columns of data, and then press the chart icon. In the window that pops up, under XY (SCATTER), select the SCATTER graph (without lines).



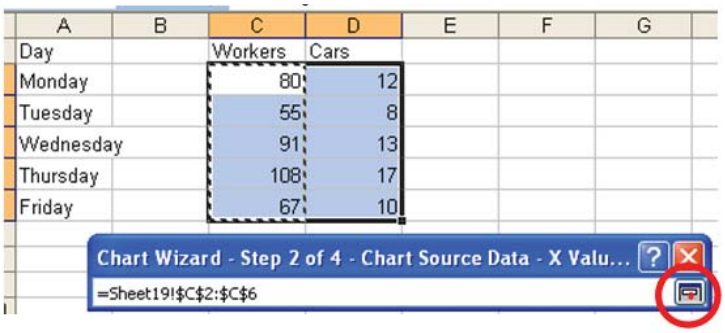
Click NEXT. Check the graph in the window to be sure that the workers data is along the horizontal axis and the cars data is along the vertical axis. You can do this by checking the scales of each axis.



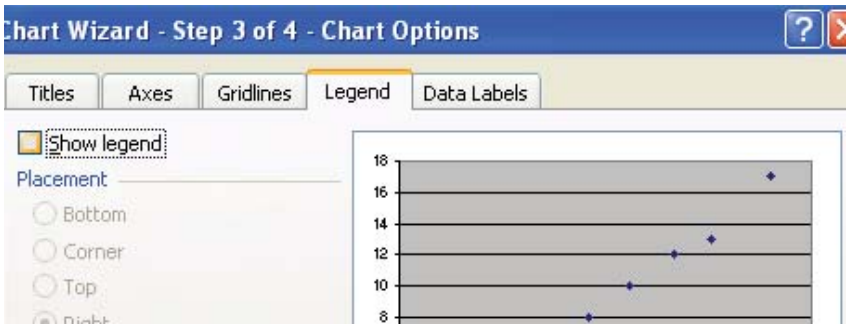
If the computer switches them around, you can correct this by pressing the button at the end of the *X Values* bar.



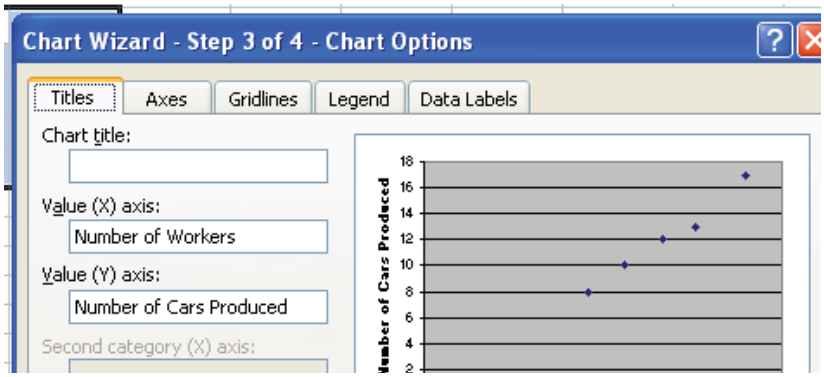
Select the column that contains the number of workers. Press the button at the end of the narrow bar to return to the larger window.



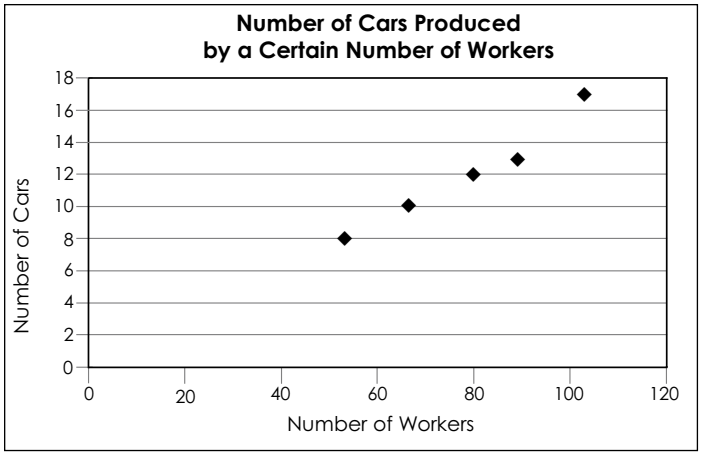
Repeat the same procedure to change the vertical axis by pressing the button at the end of the *Y Values* bar, and then selecting the column containing the number of cars produced. Press NEXT. You do not need a legend for this graph, so you can go to the LEGEND tab and click on the checkmark beside the *Show legend* text.



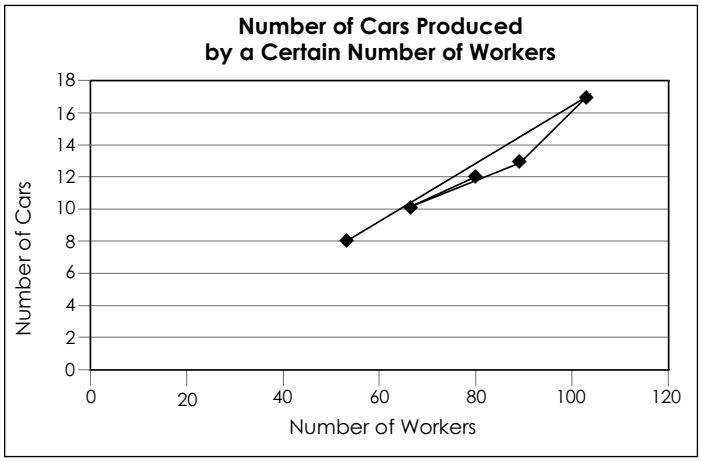
Under the TITLES tab, name your graph and the axes. The X axis is the horizontal axis; the Y axis is the vertical axis.



Press the NEXT button, and then the FINISH button. Your graph should appear on the screen.



Notice that the dots are not connected. Once you have printed the graph, you will have to connect them by hand. If you tried to use the scatter graph with lines, the computer would connect the points in the order of the data in the chart. Your graph would look like this:



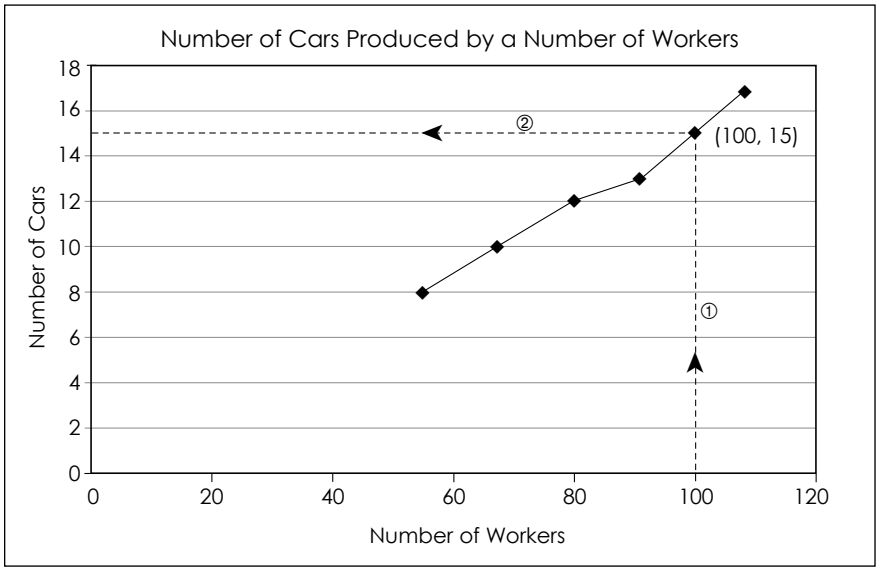
For our purposes, it is easier to have the computer create a scatterplot (i.e., a graph made up of separate points) without lines, and then add the lines later by drawing them in by hand.

Reading Line Graphs

Unlike the three other types of graphs that you have studied, line graphs can provide information *in addition to* the data collected. You can use a line graph to predict other data points both *inside and outside* the range of data.

Interpolation

The term *interpolation* means interpreting information from inside a set of data. We use interpolation to estimate values that are not included in our data. In the graph you created in the previous example, you could use interpolation to determine the approximate number of cars you could produce with 100 workers.



Steps:

Starting with the Independent Variable

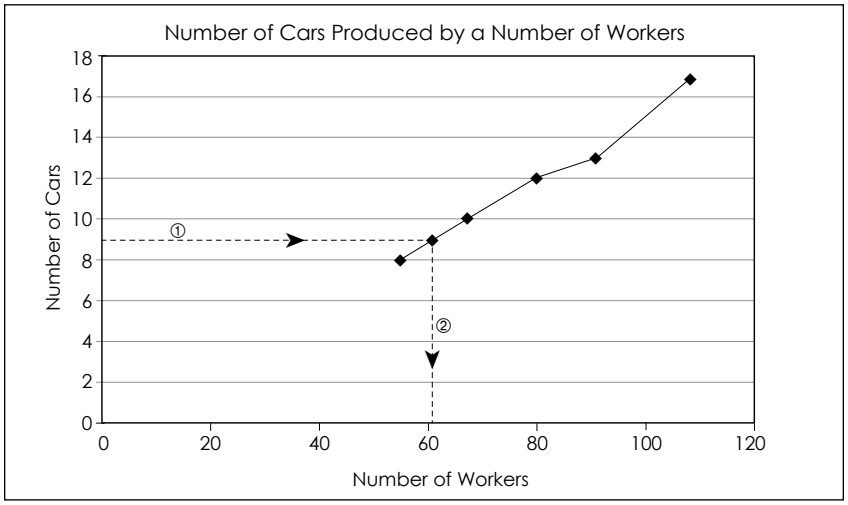
1. Use a straight edge (such as a ruler) to draw a *vertical line* up from 100 on the *horizontal axis* until it crosses the graph (the *independent* variable).
2. Then use the straight edge to draw a *horizontal line* from the point where your vertical line crossed the graph to the *vertical axis* (the *dependent* variable).

In the diagram above, the point on the graph that represents (approximately) 100 workers is in line with (approximately) 15 cars. This means that if 100 workers were at the factory, we could expect them to produce 15 cars.

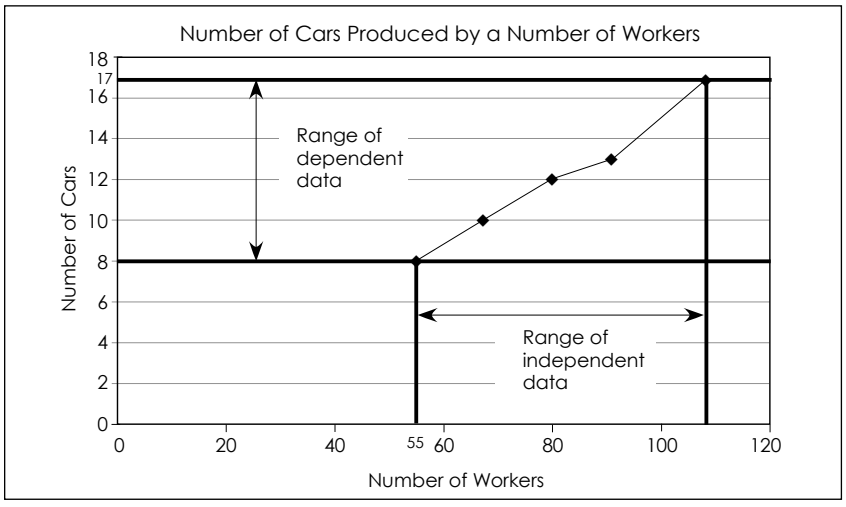
You can also determine how many workers you would need to produce a certain number of cars, using steps similar to those used in the previous example. For example, how many workers would be required to produce nine cars?

Starting with the Dependent Variable

1. Using a straight edge, draw a *horizontal line* from the *vertical axis* in line with the number of cars you want to produce (9, the *dependent* variable) until it crosses the graph.
2. Using the straight edge, draw a *vertical line* from the point where your horizontal line crossed the graph to the *horizontal axis* (the *independent* variable). As you can see, you would need approximately 60 workers to produce nine cars.



You can only interpolate within the range of data. For the graph shown above, you can only interpolate for any number of cars between 8 and 17 or for any number of workers between 55 and 108.

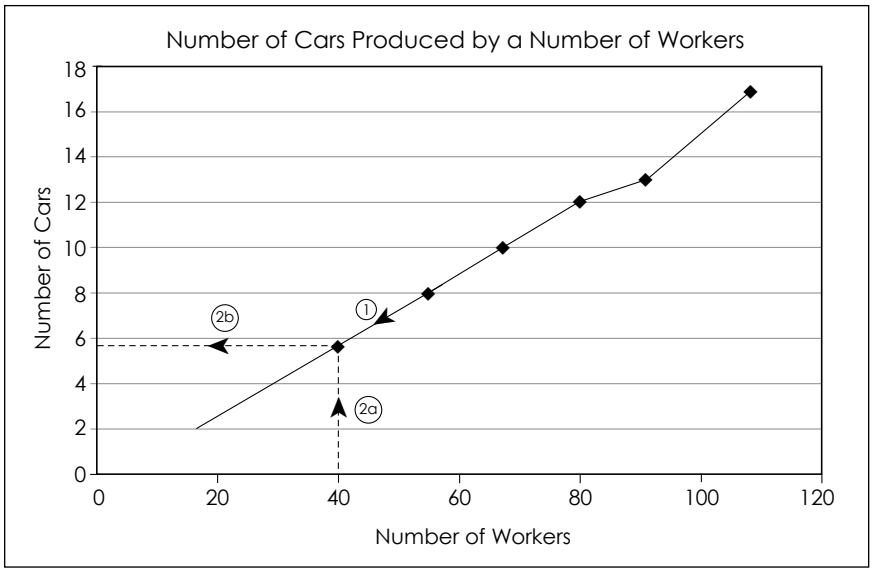


You are also able to estimate points *outside* the range of data.

Extrapolation

If you want to find a point outside of the range of data, you must extrapolate. **Extrapolation** is the estimation of information *outside* the data set. Still using the same graph, you can estimate where points may lie outside of the graph by extending the lines. For example, if you wanted to estimate how many cars 40 people could produce, you must

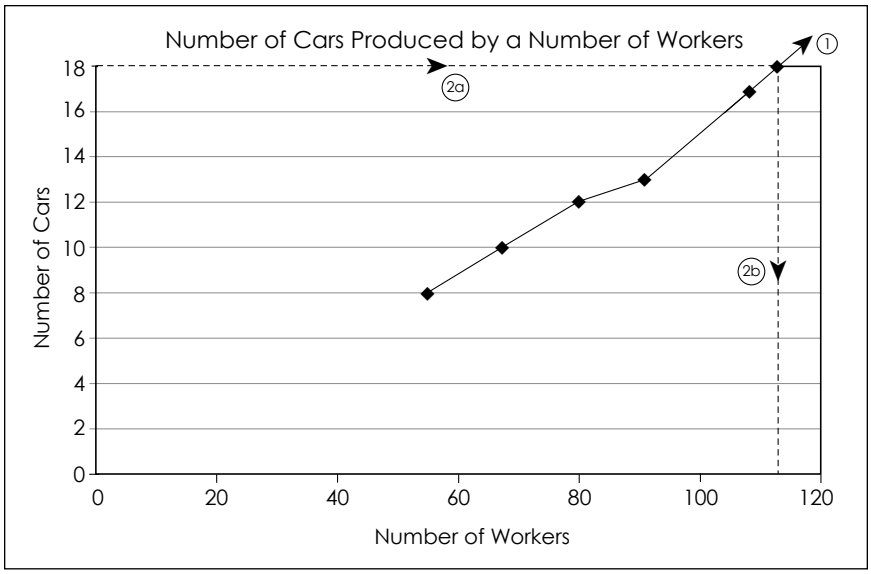
1. extend the line to the left of 55 people until it *passes* 40 people
2. follow the steps used for interpolation, **starting with the independent variable**



Based on the graph, 40 people would produce five cars (rounded down because we only want finished cars).

Also, you can extrapolate data that is greater than the data you already have. If you wanted to estimate the number of workers required to produce 18 cars, you would have to

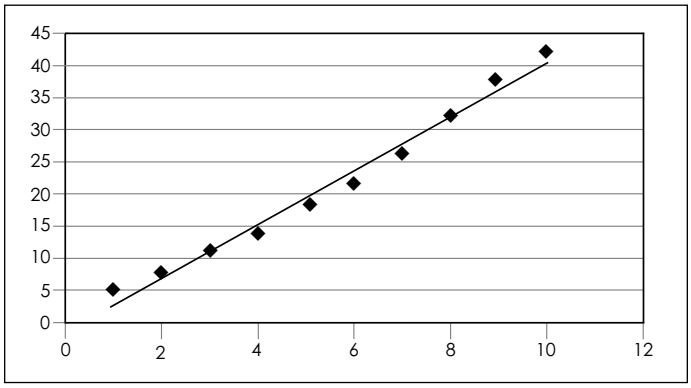
1. extend the line from 17 cars to the right until it *passes* 18 cars
2. follow the steps used for interpolation, **starting with the dependent variable**



Based on the graph, you would need approximately 113 workers to produce 18 cars.

Line of Best Fit

What happens if you need to interpret a graph that has **no lines**, only the data points? An alternative to connecting each point to the next with a line segment is to draw a line of best fit. A **line of best fit** is a line drawn onto a scatterplot (a graph made up of separate points) that shows the pattern of the points on the graph, but does not need to include all or any of the data points. It is similar to an *average* for the graph. We will not discuss this option in detail in this course. This topic is presented in other math courses.



The following learning activity will help you put into practice what you have just learned.

Learning Activity 3.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Sara and Leon are sharing the cost of a car so that they don't have to use their parents' car anymore. If the car will cost $14 000, how much will each of them have to pay?

2. Are the following fractions equal? $\frac{25}{100}$ and $\frac{7}{28}$

3. There are 18 marbles in a bag—6 green and 12 blue. Write the number of blue marbles as a fraction of the total number of marbles.

4. Insu spent $45.75 at the mall on Boxing Day. If he used a $100 gift card to pay, how much money is left on the card?

5. Your car has a hole in the dashboard for a stereo. The dimensions are 15 cm wide by 4 cm tall by 12 cm deep. The stereo you want to buy is 10 cm deep by 16 cm wide by 5 cm tall. Will this stereo fit into the dashboard?

continued

Learning Activity 3.4 (continued)

Part B: Line Graphs

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Everyone needs food to survive. Canadians spend millions of dollars every year on food. Statistics Canada collected the following data about how much money Canadians spend on food each year.

Year	Millions of Dollars Spent
2005	59464.9
2008	69701.1

   a) Draw a line graph for the data. The scale of the horizontal axis should be from 2002 until 2014. The scale of the vertical axis should be from 40,000 million to 90,000 million dollars.

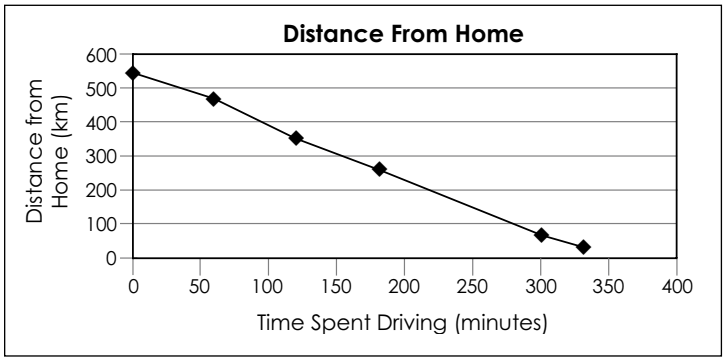
   b) How much money would you expect Canadians to spend on food in the year 2012?

   c) Estimate how much money Canadians spent on food in 2003. Use extrapolation.

   d) Estimate which year Canadians spent approximately $65,000 million on food. Use interpolation.

continued

2. Given the graph below, describing the distance you are from your home as you drive, answer the following questions.



   a) Are you leaving your house or coming home?
   b) How far are you from home after 2.5 hours of driving?
   c) How long will you have been driving before you are 0 km from your house?

Advantages and Disadvantages of Line Graphs

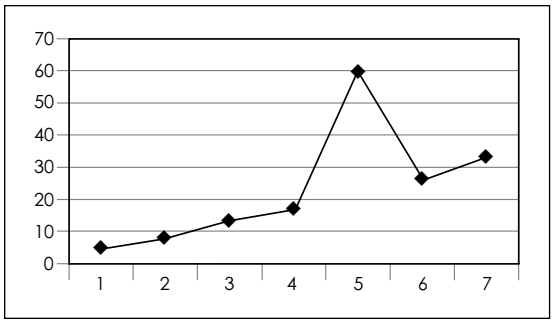
Now that you have had a chance to draw and interpret line graphs, state some advantages and disadvantages you can think of.

Advantages	Disadvantages

Here are some of our ideas:

Advantages

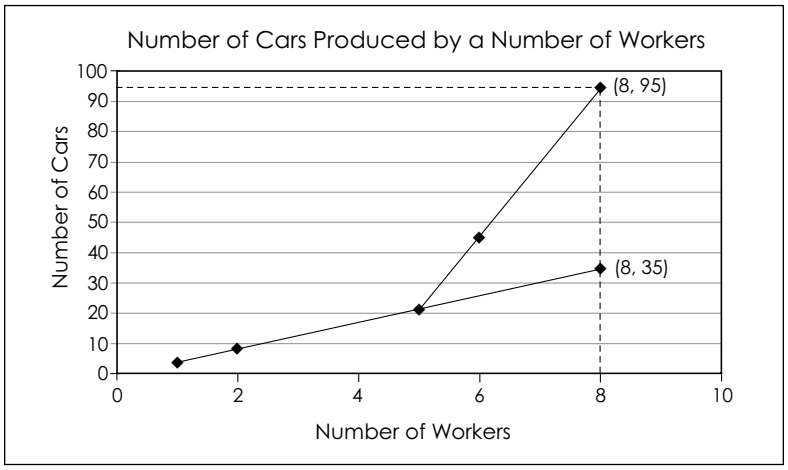
- It is easy to compare two sets of data in double line graphs
- Line graphs make relationships within the data clear (increasing, decreasing, or fluctuating)
- When you display data in a line graph, any data that does not fit the trend becomes obvious. For example, if you have a graph that looks like this:



It is immediately apparent that the point above 5 on the horizontal axis does not fit in with the rest of the data.

Disadvantages

- If you use discrete data, such as the first question in the learning activity, you are restricted by the sections. In that question, you could not have read a point as 2008.5 years, because there is no such thing. This is why we rounded the answer up to 2007 years for Part (c) of the question.



- If the trend of the data is not a straight line, then your estimation based on extrapolation could be less accurate.
- In this graph, if you extrapolate from the point at (6, 45), you would say that the point above 8 might be at (8, 95). If you extrapolate from (5, 22), you would find that the point above 8 is at (8, 35). The second answer is more logical when you look at the whole graph because it follows a relatively straight line. Line graphs, however, do not have to be straight, so you would need more data before you could be sure.

In your own words, include these points, as well as any other ones you can think of, in your pros and cons table at the end of Lesson 1.

Lesson Summary

In this lesson, you studied and drew line graphs. Line graphs show the relationship between independent and dependent variables. When drawing the graph, you plot points (ordered pairs), and then join the points with line segments to represent continuous data. You drew the graphs by hand or with a computer. After a graph is drawn, you can estimate other values by interpolation or extrapolation. You also explored the advantages and disadvantages of using a line graph to represent data.

In the next lesson, you will explore how some people use graphs to convey certain messages—in some cases misleading messages. Before you move onto the next lesson, complete the following assignment. This is part of your evaluation for Module 3.

Notes

Assignment 3.2

Line Graphs

Total: 16 marks

Note to Students: Using your resource sheet while completing this assignment is a good way to see whether it has all the necessary information. Do not hesitate to add to your resource sheet as you complete the assignment. You may draw any graphs using a computer. Just remember to attach a print of each graph to the assignment when you submit your work at the end of the module.

During gym class, Fowzia is wearing a heart rate monitor that also measures body temperature. She periodically checks the heart rate and temperature readings throughout the class and collects the following data. Although both heart rate and body temperature could be considered the independent variables, we will choose heart rate as the independent variable for this question.

Heart Rate (beats per minute)	Body Temperature (°F)
80	98.6
90	98.8
100	98.9
120	99.1
150	99.5

a) Draw a line graph to represent the data. Be sure to include all the components of a good line graph. The range of the independent axis should be from 40 to 180 beats per minute. The scale on the dependent axis should include values from 98 to 100 °F. *(7 marks)*

continued

Assignment 3.2: Line Graphs (continued)

b) Write a statement that describes the relationship between heart rate and body temperature, as displayed on the graph. *(1 mark)*

c) What is Fowzia's body temperature when the heart rate monitor reads 140 beats per minute? *(2 marks)*

continued

Assignment 3.2: Line Graphs (continued)

d) If Fowzia's normal body temperature is 98 ^{o}F, what is her approximate resting heart rate? *(3 marks)*

e) What will Fowzia's body temperature be if her heart rate is 160 beats per minute? *(3 marks)*

Notes

Lesson 5: Interpreting Graphs

Lesson Focus

In this lesson, you will

- ❑ identify different interpretations of the same graph and the reasoning for each interpretation
- ❑ modify a graph in order to emphasize a point of view

Lesson Introduction

In the first four lessons, you have studied four different types of graphs. These four types—circle graph, bar graph, histogram, and line graph—are frequently used by media, advertisers, and others to provide statistical information in a way that can be quickly and easily understood. In this lesson, you will study how some people use graphs to influence other people by modifying the way a graph is drawn. In some cases, an "honest" graph can present misleading information.

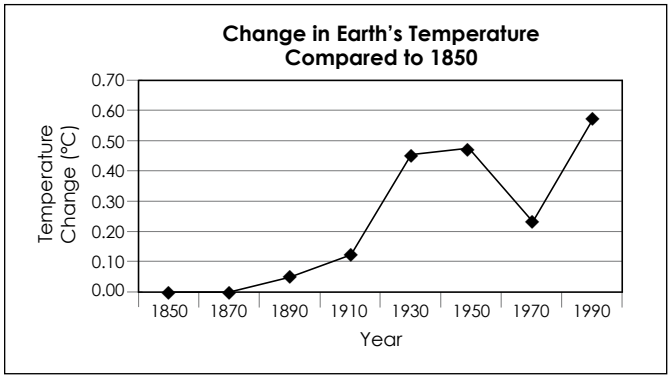
Interpretations of Graphs

Multiple Interpretations of One Graph

Your interpretation of a graph may be determined

- by your personal beliefs, or
- by the factual data (numbers on the axis) presented by the graph

The following graph describes the average temperature changes on Earth since 1850 CE.



What do you conclude about the temperature on Earth when you look at the graph?

Based on this graph, you can draw many conclusions about Earth's temperature. Here are some examples:

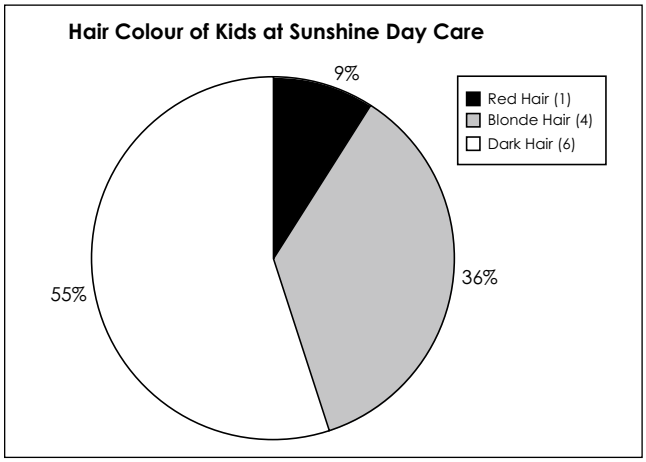
1. Whatever we did between 1950 and 1970 was effective in lowering Earth's temperature.
2. As it did between 1930 and 1950, Earth's temperature may plateau (stay approximately the same).
3. Earth's temperature is increasing rapidly.
4. We do not need to be concerned about Earth's temperature because it is not increasing by that much.

The basis for the first two statements might be **personal beliefs**. If someone is very concerned about an increase in Earth's temperature, this person may conclude that human intervention was effective in lowering Earth's temperature (first statement). On the other hand, if someone believes that human activities do not affect Earth's temperatures, this person may think that the fluctuations of temperature are just part of a natural process (second statement).

The basis for the last two statements might be the **factual data** presented by the graph. Someone might be very concerned about the sharp vertical rise in temperature from 1970 to 1990 (third statement). Someone else might not be at all concerned about a temperature increase of 0.6°C in 100 years (fourth statement).

Example 1

State two different ways someone could interpret the graph. Include one possible reason for each interpretation.



Solution

Interpretation 1: Most people have dark-coloured hair.

Reason: The largest part of the circle represents people with dark hair, and it is larger than 50 percent of the population

Interpretation 2: Blonde and dark hair colours are almost equally common.

Reason: There are four kids with blonde hair and six with dark hair. This is not a large difference.

Multiple Graphs for the Same Data

Not only is it possible for different people to read the same graph differently, it is also possible to draw a variety of graphs using the same data, and each graph will emphasize a certain point.

Example 1

The following chart lists the annual sales of the Whatsit Widget Company. The sales are for the years 2004–2008.

Year	2004	2005	2006	2007	2008
Sales of Widgets	136 000	140 000	144 000	148 000	155 000

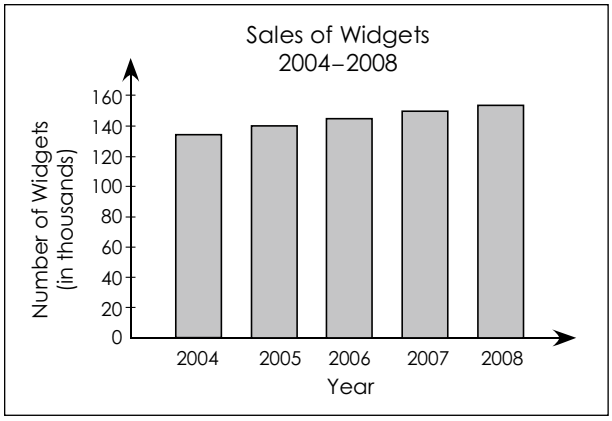
a) Draw a bar graph to represent this data accurately.

b) Mr. Hiram Sayles, the president of the Whatsit Widget Company, would like the increase in sales over the past five years to appear as large as possible. Draw a graph to represent the data in a way that makes the increase in sales appear much greater.

c) Ms. Ida Better, the president of the Better Widget Company, would like the increase in sales of the Whatsit Widget Company to look as small as possible. Draw a graph to represent the data in a way that makes the increase in sales appear much smaller.

Solution

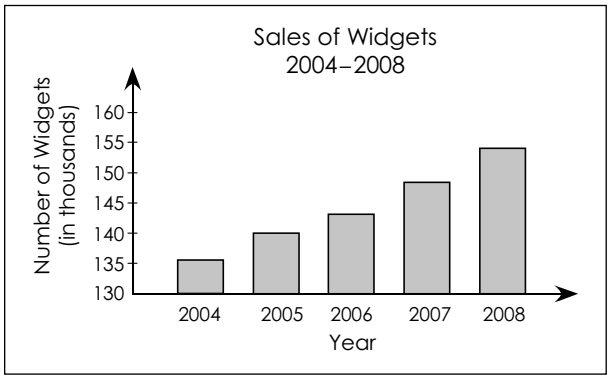
a) Choose a graph with a vertical axis beginning at 0 and ending at 160 000, with intervals of 20 000.



The graph represents the data accurately because of the following:

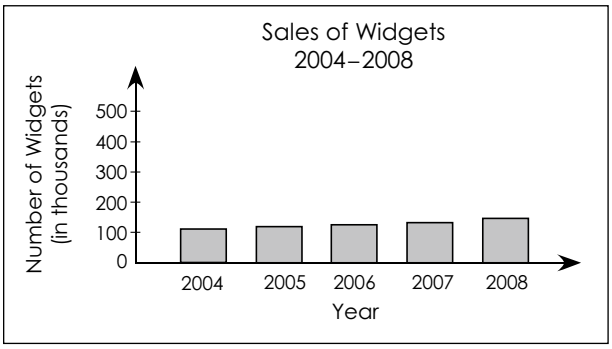
- the vertical axis begins at 0
- the maximum value on the vertical axis matches the maximum value in data
- the bars all have the same width

b) Choose a graph with a vertical axis beginning at 130 000 and ending at 160 000, with intervals of 5000.



The placement of the data on a graph can create different impressions. By making the range of the vertical scale smaller, the differences between the bars of a graph are exaggerated.

c) Choose a graph with a vertical axis beginning at 0, ending at 500 000, with intervals of 100 000.

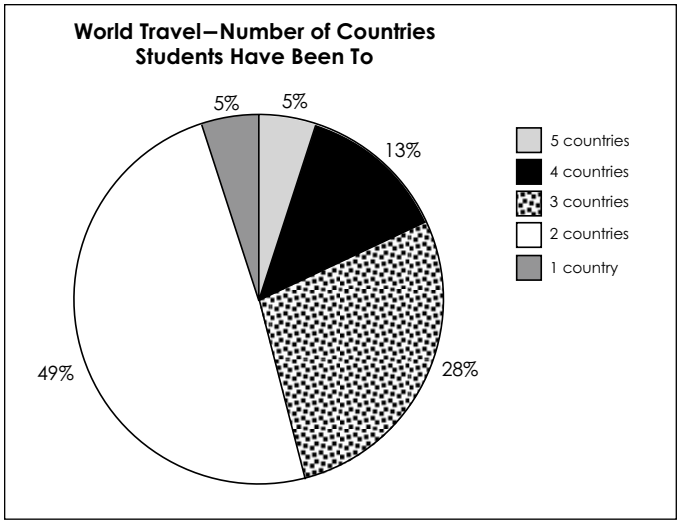


This graph shows another way to present data to create a different impression. By choosing a maximum value on the vertical axis that is much greater than the maximum value in the data, the differences between the heights of the bars are very small.

Overall, the vertical scale of a graph can have a large impact on the appearance of the graph. If the scale is large, the graph looks flat, and the differences are less obvious. If the scale is small, the differences between data points are exaggerated.

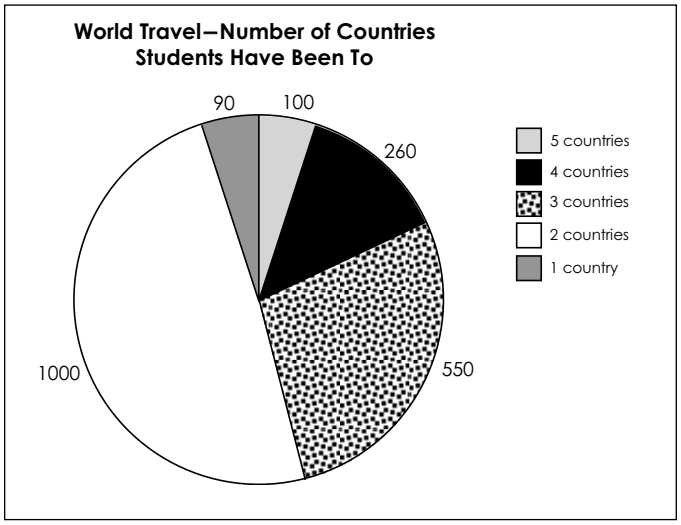
Example 2

As part of a project, a social studies class collected data about the number of countries that individual students had been to, including Canada. Here is one graph the class created to represent the data they collected.



a) State three conclusions you might have after looking at the first graph.

b) Do you still stand by your conclusions now that you can see the percentages and actual values for each wedge? The following chart shows the same data but with values instead of percentages.



Solution

a) Some possible conclusions include:

- The most common response/number of countries students have visited is two.
- Approximately the same number of students have been to one or five countries.
- The total number of students that have been to one or five countries is approximately the same as the number of students who have been to four countries.
- The total number of students that have been to one, four, or five is about the same as the number of students who have been to three countries.

b) If you compare the number of students instead of the percentages, you may not agree with the original conclusions.

- The most common response is still two countries.
- There are about the same number of people who have been to one or five countries (90 vs. 100 is only a difference of 10 out of 2000).
- The total of the number of students who have been to one or five countries is 190, while the number of students who have been to four countries is 260. The difference is 70 students. This is a large number of people.

- The total number of students who have been to one, four, or five countries is 450. This is 100 less than the number of students who have been to three countries.

Work through the questions in the learning activity for this lesson to practise interpreting graphs in multiple ways. The assignment for this lesson will have questions that are similar to the ones in the learning activity.

Learning Activity 3.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. The volume of a square pyramid is 8 m^3. What is the volume of a prism with the same dimensions (length, width, and height)?

2. Seok Yong plays on a school sports team. Based on the following clues, decide whether he plays on the basketball team, hockey team, soccer team, or volleyball team.
   - He does not like to be cold.
   - There are no extra pads required.
   - He scored 34 points in their last game.

3. Convert the fraction to a percent. $\frac{25}{50}$

4. There is a cube with each side length equal to 1 cm. What is the surface area of the cube?

5. Complete the pattern. 10, 100, 1000, _____ , _____ ...

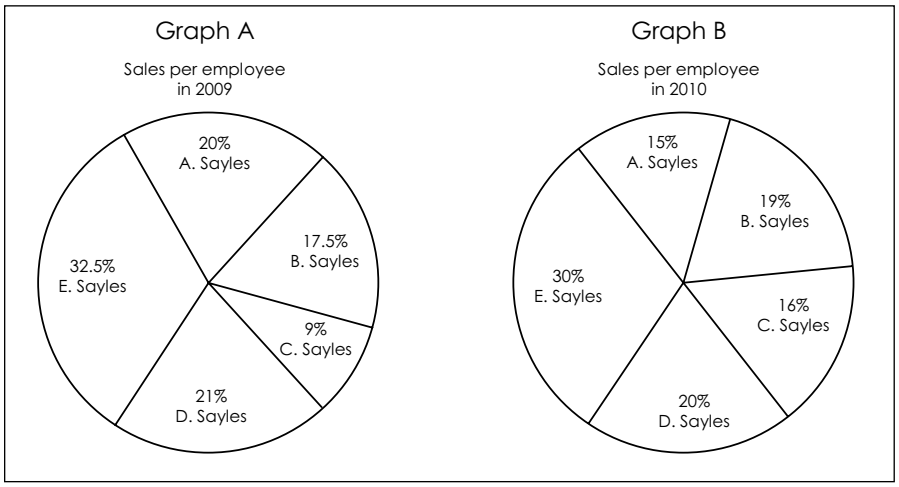
continued

Learning Activity 3.5 (continued)

Part B: Interpreting Graphs

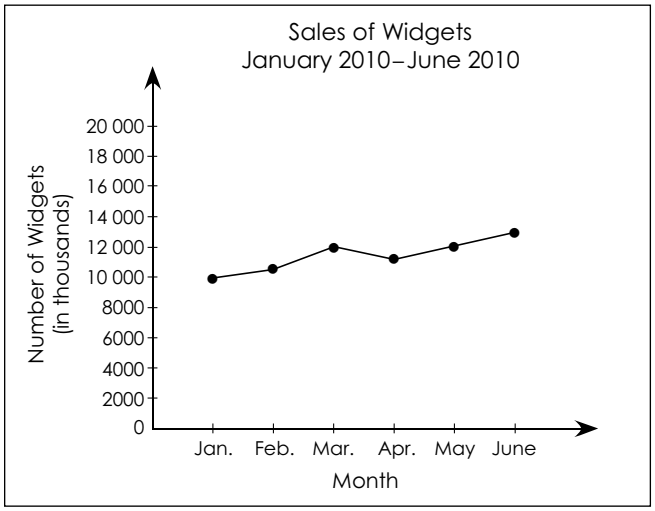
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Compare the two graphs below. State two ways that someone could interpret the data. Include a reason for each interpretation.



2. The following three graphs all represent sales of widgets at the Whatsit Widget Company from January 2010 to June 2010.

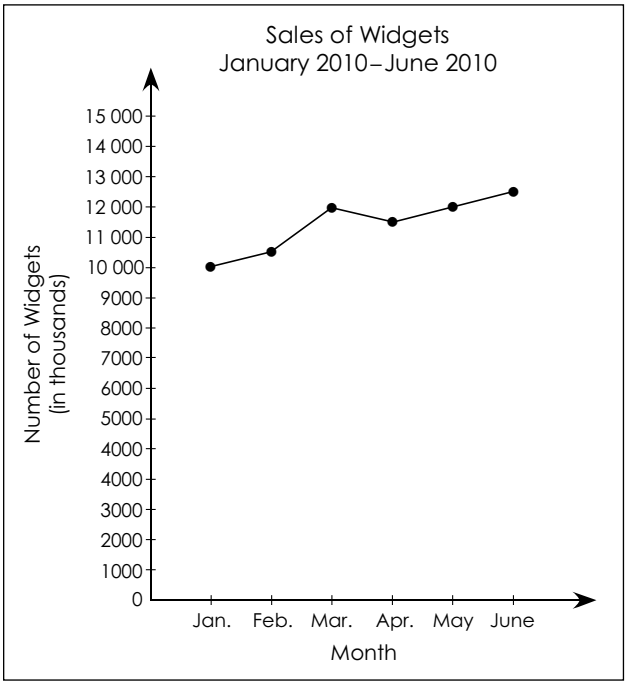
   Graph A



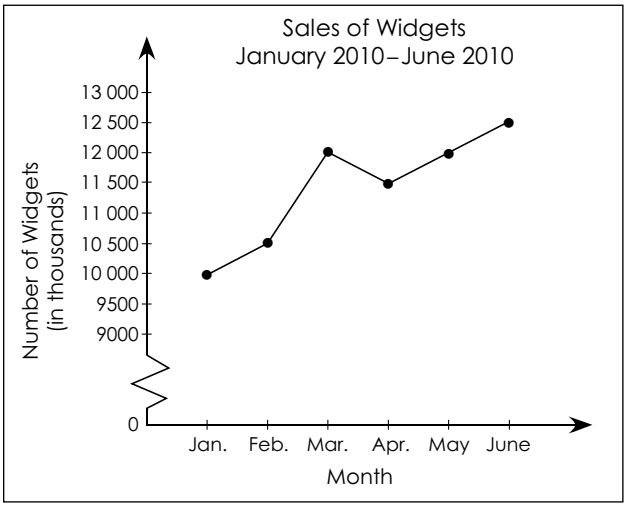
continued

Learning Activity 3.5 (continued)

Graph B



Graph C



a) Which graph seems to show the greatest increase in the sale of widgets at the Whatsit Widget Company?

b) Which graph seems to show the least increase in the sale of widgets at the Whatsit Widget Company?

c) Explain why the three graphs give different impressions.

Lesson Summary

In this lesson, you learned how changing the vertical scale on a graph can change the appearance of a graph. This technique is sometimes used to emphasize a certain point of view—sometimes a misleading point of view. You also learned that different people might interpret the same graphs differently. It is important to look at graphs critically so that you understand the information being presented.

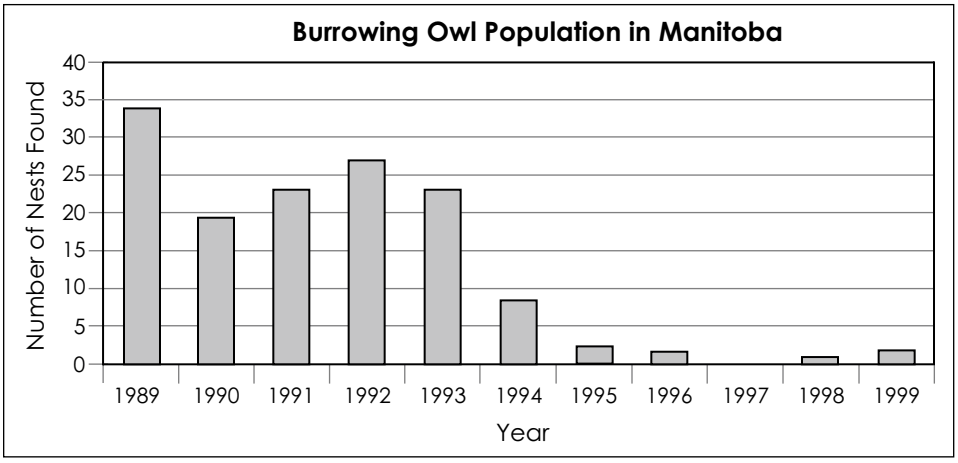
Assignment 3.3

Interpreting Graphs

Total: 17 marks

Note to Students: Use your resource sheet to help you complete this assignment. Do not hesitate to add to your resource sheet as you answer the following questions. You can draw any graphs using a computer. Just remember to attach a print of each graph to the assignment when you submit your work at the end of the module.

1. The data for the graph was provided by *Manitoba Wildlife*. The graph shows the population of the burrowing owl in Manitoba.

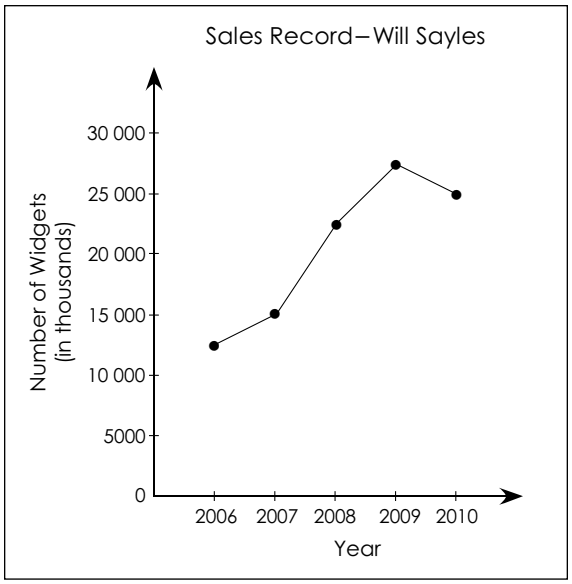


List two different interpretations of the graph. Explain your reason for each interpretation. *(4 marks)*

continued

Assignment 3.3: Interpreting Graphs (continued)

2. One of the salespeople of the Whatsit Widget Company, Will Sayles, is asked to report his sales figures for the years 2009 and 2010. He presents his sales figures using the following graph.



a) Why do you think he includes his sales figures for the years 2006 to 2010? *(1 mark)*

b) How is he using the data to his advantage? *(1 mark)*

continued

Assignment 3.3: Interpreting Graphs (continued)

3. The president of the Whatsit Widget Company, Mr. Hiram Sayles, would like to present the annual profits of the company for the years 2004–2008 with a graph. The figures representing the profit for each of the five years are as follows:

Year	2004	2005	2006	2007	2008
Profit ($)	75 000	78 500	81 000	83 500	85 000

a) Draw a graph that accurately represents the data. *(3 marks)*

b) Draw a graph in which the yearly profit does not appear to change very much. *(3 marks)*

continued

Assignment 3.3: Interpreting Graphs (continued)

c) Draw a graph that exaggerates the increase in yearly profit. *(3 marks)*

d) If you were the president of the company, which graph would you choose, and why? *(2 marks)*

Module 3 Summary

You have completed Module 3! In this module, you studied four types of graphs, two kinds of data, and a variety of interpretations of graphs. In Grade 12 Essential Mathematics, you will learn more about statistics and analyzing data, and the probability of events.

Once you have completed your midterm examination, you can begin the next module, where you will expand your understanding of personal finances. In Module 1, you studied interest and credit. In Module 4, you will study budgets and managing finances.

Submitting Your Assignments

It is now time for you to submit the Module 3 Cover Assignment and Assignments 3.1 to 3.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 1 assignments and organize your material in the following order:

- ❑ Module 3 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 3 Cover Assignment: Statistics in Your Life
- ❑ Assignment 3.1: Circle Graphs, Bar Graphs, and Histograms
- ❑ Assignment 3.2: Line Graphs
- ❑ Assignment 3.3: Interpreting Graphs

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Midterm Examination

Congratulations, you have finished Module 3 in the course. The midterm examination is out of 100 marks and worth 12.5% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 3.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens and pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Midterm Examination Resource Sheet. A maximum of 2.5 hours is available to complete your midterm examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point, you will also have to combine your Resource Sheets from the first four modules onto one 8½" × 11" paper (you may use both sides). Make sure that you have all of the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you.

This is a sample:

Front	**Back**
Module 1	Module 3
Module 2	

Examination Review

You are now ready to begin preparing for your midterm examination. Please review the content, learning activities, and assignments from Modules 1 to 3.

The midterm practice examination is also an excellent study aid for reviewing Modules 1 to 3.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the midterm examination determines 12.5% of your final mark in this course and you will have 2.5 hours to complete the examination.

Midterm Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The midterm practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your midterm practice examination, follow these steps:

1. Study for the midterm practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 1 to 3 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 3, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Midterm Examination Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following to the midterm practice examination: pens and pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Midterm Examination Resource Sheet.

7. Write your midterm practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the midterm examination is 2.5 hours.
8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Midterm Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Grade 11 Essential Mathematics (30S)

Module 3
Statistics

Learning Activity Answer Keys

Module 3: Statistics

Learning Activity 3.1

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Convert the following decimal to a percent: 0.005.
2. Convert the following percent to a decimal: 345%.
3. Convert the following fraction to a percent: $\frac{4}{5}$.
4. For every 10 people in the world, four are Asian. Write this as a percentage.
5. Yesterday, Malaki spent five hours in school, two hours in the gym, and nine hours sleeping. How many hours of the day are not accounted for?

Answers:

1. 0.5% (0.005 × 100 = 0.5%)
2. 3.45 (345 ÷ 100 = 3.45)
3. 80% $\left(\frac{4}{5} = \frac{80}{100} \text{ or } 80\%\right)$
4. 40% $\left(\frac{4}{10} = \frac{40}{100} \text{ or } 40\%\right)$
5. 8 hours (24 − (5 + 9 + 2) = 8)

Part B: Circle Graphs

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand. Don't forget that you can ask your learning partner or tutor/marker for help if you are having a hard time understanding.

1. Write D for discrete or N for not discrete beside each example of data. Explain your choices.

   Note that, in some cases, you may have either D or N as your answer, but your explanation is important for backing up your decision. The way the categories are created may determine whether information is discrete or not discrete.

   a) A survey of the number of pages in various books.

      Answer:

      N. This is not discrete because every book has a different number of pages, so you could not have categories for 100 pages, 200 pages, 300 pages, etc. You would have to have a section for every possible number of pages.

      D. If you created categories such as < 100 pages, 101–200 pages, 201–300 pages, 301–400 pages, and > 400 pages.

   b) The genres of books at your school library.

      Answer:

      D. Genres are distinct groupings—fantasy and history are clearly separate entities.

      N. There are some books that fit into multiple genres (for example, there are romance novels, history books, and there are historical romance books). Which category would the third type fit into?

2. Many people from countries other than the US visit Canada over the course of a year. In 2009, according to Statistics Canada, 519,000 people came by land, 3,501,000 people came by airplane, and 150,000 came by sea.

   a) Draw a circle graph for this data if possible.

      Answer:

      i) It is possible to draw a circle graph from this data because it is discrete.

ii) Total people: 519 000 + 3 501 000 + 150 000 = 4 170 000 people

By land: $\frac{519,000}{4,170,000} \times 100\% = 12.4\%$

By airplane: $\frac{3,501,000}{4,170,000} \times 100\% = 84\%$

By sea: $\frac{150,000}{4,170,000} \times 100\% = 3.6\%$

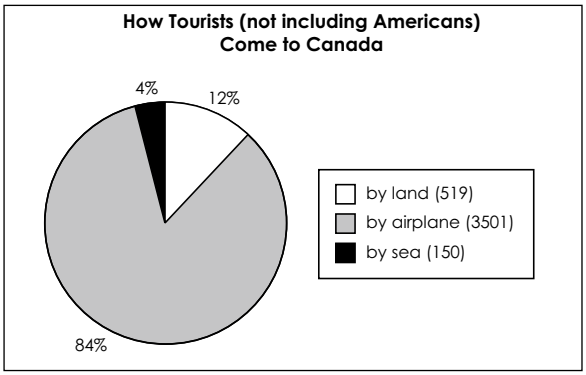
iii) By land: $0.124 \times 360^o = 44.64^o \approx 45^o$

By airplane: $0.84 \times 360^o = 302.4^o \approx 302^o$

By sea: $0.036 \times 360^o = 12.96^o \approx 13^o$

Check: $45 + 302 + 13 = 360^o$

iv)



b) When looking at the graph, what do you observe about the methods of travel used by visitors to Canada?

Answer:

Answers may vary.

- Not many people come by land and sea.
- Most people come by airplane.

3. Suppose the Government of Canada is researching the possibility of changing our official language laws. Part of its research is data from the 2006 census by Statistics Canada. The table shows language results for people in some of our provincial capitals.

	English Only	French Only	Both	Neither
Total	6, 931,670	476,650	906,270	238,745

a) List the categories you would use for this data in a circle graph.

Answer:

English only, French only, Both, and Neither

b) Draw the circle graph for this data.

Answer:

i) The data can be graphed as shown in the table.

ii) Total number of people: 6 931 670 + 476 650 + 906 270 + 238 745 = 8 553 335

$$\text{Percent English} = \frac{6,931,670}{8,553,335} \times 100\% = 81\%$$

$$\text{Percent French} = \frac{476,650}{8,553,335} \times 100\% = 5.6\%$$

$$\text{Percent Both} = \frac{906,270}{8,553,335} \times 100\% = 10.6\%$$

$$\text{Percent Neither} = \frac{238,745}{8,553,335} \times 100\% = 2.8\%$$

Check: $81 + 5.6 + 10.6 + 2.8 = 100\%$

iii) English angle: $0.81 \times 360^o = 291.6^o \approx 292^o$

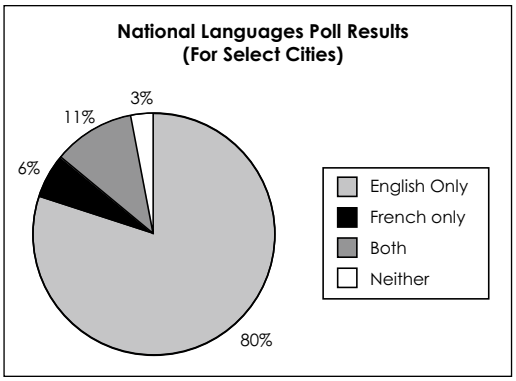
French angle: $0.056 \times 360^o = 20.16^o \approx 20^o$

Both angles: $0.106 \times 360^o = 38.16^o \approx 38^o$

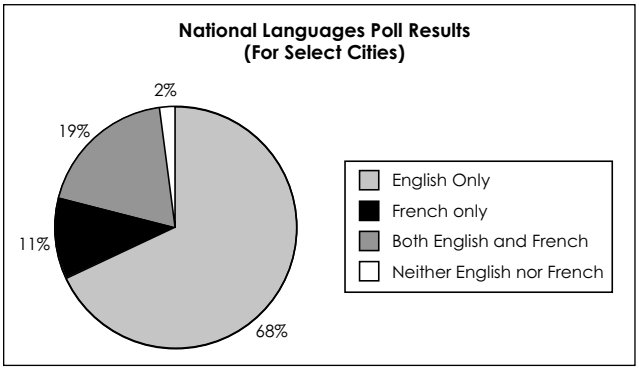
Neither angle: $0.028 \times 360^o = 10.08^o \approx 10^o$

Check: $292 + 20 + 38 + 10 = 360^o$

iv)



c) The circle graph representing the results for a larger number of cities is shown below. How is this graph different from the one you drew? In what ways is it similar? Why might the results be different?



Answer:

Differences: Larger percentage of French-only and bilingual people in the larger survey; smaller percentage of people who do not speak either national language or who speak English only.

Similarities: Largest to smallest percentage = English, bilingual, French, neither; English is the majority.

Reasons may vary:

- Quebec City was not included in the first graph
- More people who live in not-capital cities speak French or are bilingual
- People who speak neither French nor English tend to live in large cities

Learning Activity 3.2

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Dania invests $5000. It takes eight years to double. Using the rule of 72, estimate the interest rate.
2. How much tax would you have to pay on an item that costs $99.99 (approximately) if the PST = 7% and the GST = 5%?
3. How long is the interest-free period if Zach buys a snowboard on November 27th and he does not have to pay until December 13th?
4. Are the following two fractions equal? $\frac{6}{8}$ $\frac{13}{16}$
5. Jeewon is at the movies with her friends. She spends $11.00 on her ticket, $4.00 on a drink, and $5.50 on ice cream. If she started with $25.00, how much does she have left?

Answers:

1. 9% $\left(\frac{72}{8} = 9\%\right)$
2. Approximately $12 (7 + 5 = 12% of $100)
3. 15 days (3 + 12)
4. No (An equivalent fraction for $\frac{6}{8}$ would be $\frac{12}{16}$, which is less than $\frac{13}{16}$.)
5. $4.50 (25 − (11 + 4 + 5.5) = $4.50)

Part B: Bar Graphs

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. To kick off their Active Living course, students must survey their friends to determine their favourite ways to stay active. The teacher would like the class to record the gender of each person because they will be discussing their results in class. The teacher plans to draw a double bar graph with the data.

Gender	Activity
F	Soccer
M	Running
F	Running
F	Weightlifting
M	Soccer
M	Hockey
F	Speed Skating
M	Swimming
F	Soccer
F	Running
F	Swimming
M	Hockey
M	Hockey
M	Football
F	Baseball
F	Baseball
F	Running
M	Baseball
M	Basketball
F	Walking
M	Running
M	Cycling
M	Baseball
F	Volleyball

Gender	Activity
M	Hockey
M	Hockey
M	Cycling
F	Running
F	Basketball
M	Volleyball
F	Volleyball
F	Running
M	Weightlifting
M	Swimming
F	Cycling
F	Running
M	Weightlifting
M	Basketball
F	Volleyball
M	Football
M	Football
M	Basketball
F	Running
F	Weightlifting
F	Basketball
F	Volleyball
M	Football
M	Speed Skating

Gender	Activity
F	Hockey
F	Basketball
M	Soccer
F	Hockey
M	Running
M	Weightlifting
M	Speed Skating
F	Cycling
M	Running
F	Hockey
F	Hockey
M	Soccer
M	Volleyball
F	Basketball
M	Baseball
M	Weightlifting
M	Speed Skating
F	Cycling
M	Running
F	Hockey
F	Walking
M	Walking
M	Running
F	Cycling

continued

a) Given the data above, what are the categories that the teacher could use?

Answer:

The categories would be the activities: running, cycling, baseball, basketball, volleyball, soccer, hockey, speed skating, football, weightlifting, swimming, and walking

b) What will the two bars for each category represent?

Answer:

The two bars for each category will represent the two genders.

c) Sort the data into categories

Answer:

M	Soccer	M	Running	F	Basketball	F	Weightlifting
F	Soccer	F	Running	F	Basketball	F	Weightlifting
M	Soccer	F	Running	M	Basketball	M	Weightlifting
M	Soccer	M	Running	F	Basketball	M	Weightlifting
F	Soccer	F	Running	M	Basketball	M	Weightlifting
		M	Running	M	Basketball	M	Weightlifting
		F	Running	F	Basketball		
M	Hockey	F	Running	M	Cycling	M	Speedskating
M	Hockey	F	Running	F	Cycling	M	Speedskating
F	Hockey	F	Running	F	Cycling	M	Speedskating
F	Hockey	M	Running	F	Cycling	F	Speedskating
M	Hockey	M	Running	F	Cycling		
F	Hockey	M	Running	M	Cycling		
F	Hockey						
M	Hockey						
F	Hockey						
M	Hockey						
F	Volleyball	M	Football	M	Baseball	M	Swimming
F	Volleyball	M	Football	F	Baseball	M	Swimming
M	Volleyball	M	Football	F	Baseball	F	Swimming
M	Volleyball	M	Football	M	Baseball	F	Walking
F	Volleyball			M	Baseball	F	Walking
F	Volleyball					M	Walking

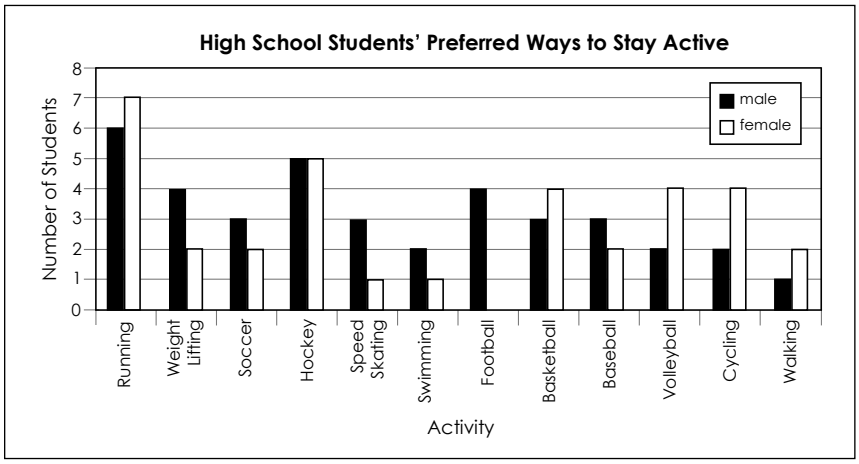
Note: You could also have used the following chart and tally marks.

Total:

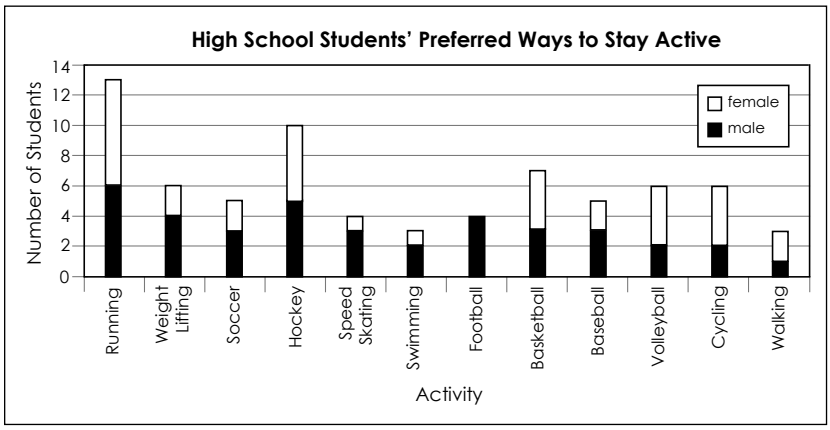
Activity	Gender				Total
	Male		Female		
Running	𝍸 I	6	𝍸 II	7	13
Weightlifting	IIII	4	II	2	6
Soccer	III	3	II	2	5
Hockey	𝍸	5	𝍸	5	10
Speedskating	III	3	I	1	4
Swimming	II	2	I	1	3
Football	IIII	4		0	4
Basketball	III	3	IIII	4	7
Baseball	III	3	II	2	5
Volleyball	II	2	IIII	4	6
Cycling	II	2	IIII	4	6
Walking	I	1	II	2	3
Total		38		34	72

d) Draw a graph of the data using a double bar graph.

Answer:



You could also use a stacked double bar graph so that the total number of people in each category is obvious and the students will be able to estimate how many of those people were from each gender.



e) What information can you read from the graph?

Answer:

Answers will vary. Possible answer:

Running is the most popular, while walking and swimming are the least commonly chosen. Only boys chose football. More girls than boys chose running, basketball, volleyball, cycling, and walking. More boys than girls chose weightlifting, soccer, speed skating, swimming, football, and baseball. There is an equal number of boys and girls who chose hockey.

Learning Activity 3.3

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Write two equivalent fractions for $\frac{9}{6}$.

2. A cube is 1 m tall, 2 m wide, and 3 m deep. What is the volume of the cube?

3. From your house, you walk 10 m south, 8 m east, 10 m north, and 6 m west. How far are you from your house?

4. A cylinder has a volume of 60 mm^3. What is the volume of a cone with the same radius and height?

5. If 10% of people do not like the colour brown and there are five people in your family, how many people do not like the colour brown in your family?

Answers:

1. Answers will vary. Possible answers include: $\frac{3}{2}, \frac{18}{12}, \frac{6}{4}, \frac{12}{8}, \frac{15}{10}, \frac{21}{14}, \ldots$

2. 6 m^3 ($1 \times 2 \times 3 = 6$)

3. 2 m west (You walked the same distance south and north, so they cancel out, but you walked 8 m east and only 6 m west.)

4. 20 mm^3 $\left(\frac{60}{3} = 20\right)$

5. Technically, by doing the math, you would find that half a person would like the colour brown. Since you cannot count people as halves, you could answer either 1 or 0, or you can say there is no answer.

Part B: Histograms

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Compare and contrast bar graphs and histograms using a chart similar to the one below.

   Answer:

	Bar Graphs	Histograms
Differences	■ sorted into qualitative categories ■ discrete data ■ qualitative and quantitative axes	■ sorted into categories that describe a range of data ■ continuous data ■ axes represent range of data and frequency
	Both	
Similarities	■ use rectangles (bars) to display the data ■ easy to read the values of each bar	

2. As part of his law class, Jeremiah is doing a project about jails. As part of his presentation, he wants to include statistics about the sentences of prisoners in a high security prison. The data he collected is in the table below.

Sentence Time	Number of Prisoners
under 1 year	845
1 year and up to 2 years	1758
2 years and up to 5 years	2486
5 years and up to 10 years	6784
10 years and up to 15 years	5917
15 years and up to 25 years	4375
25 years and over	583

a) Draw a histogram that Jeremiah could use in his presentation about Canadian jails.

Answer:

Since the data is already sorted, you can start at Step 2. The maximum value is 6784, so the frequency marked on the vertical axis should go up to approximately 7000. The graph would look like this.

b) The provincial government is responsible for prisoners who serve terms less than two years, while the federal government is responsible for prisoners serving two years or more in jail. Based on the graph above, which government is responsible for the larger number of prisoners?

Answer:

The federal government is responsible for more prisoners. You can tell by looking at the graph because the two sections that show the provincial government's responsibilities are shorter than the four sections that show the federal government's responsibilities.

Learning Activity 3.4

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Sara and Leon are sharing the cost of a car so that they don't have to use their parents' car anymore. If the car will cost $14 000, how much will each of them have to pay?

2. Are the following fractions equal? $\frac{25}{100}$ and $\frac{7}{28}$

3. There are 18 marbles in a bag—6 green and 12 blue. Write the number of blue marbles as a fraction of the total number of marbles.

4. Insu spent $45.75 at the mall on Boxing Day. If he used a $100 gift card to pay, how much money is left on the card?

5. Your car has a hole in the dashboard for a stereo. The dimensions are 15 cm wide by 4 cm tall by 12 cm deep. The stereo you want to buy is 10 cm deep by 16 cm wide by 5 cm tall. Will this stereo fit into the dashboard?

Answers:

1. $7000 (14 000 ÷ 2 = 7000)

2. Yes (Both can be simplified to $\frac{1}{4}$.)

3. $\frac{12}{18}$ or $\frac{2}{3}$

4. $54.25 (100 − 45.75 = 54.25 or count up from $45.75: $0.25 + $45.75 = $46.00; $46 + $4 = $50; $50 + $50 = $100, so 0.25 + 4 + 50 = 54.25)

5. No (The stereo is too wide, 16 cm is greater than 15 cm; and too tall, 5 cm is greater than 4 cm.)

Part B: Line Graphs

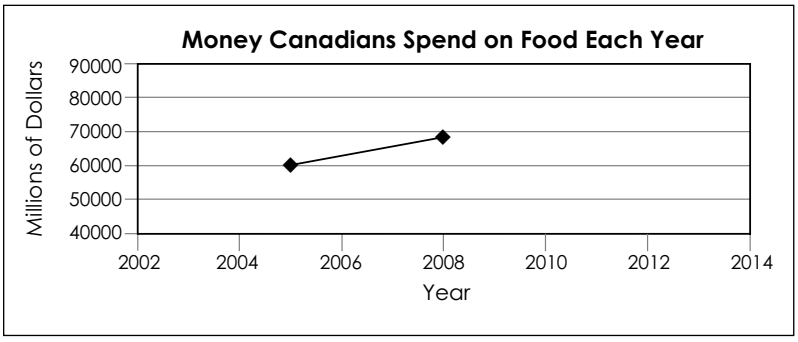
Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Everyone needs food to survive. Canadians spend millions of dollars every year on food. Statistics Canada collected the following data about how much money Canadians spend on food each year.

Year	Millions of Dollars Spent
2005	59464.9
2008	69701.1

   a) Draw a line graph for the data. The scale of the horizontal axis should be from 2002 until 2014. The scale of the vertical axis should be from 40,000 million to 90,000 million dollars.

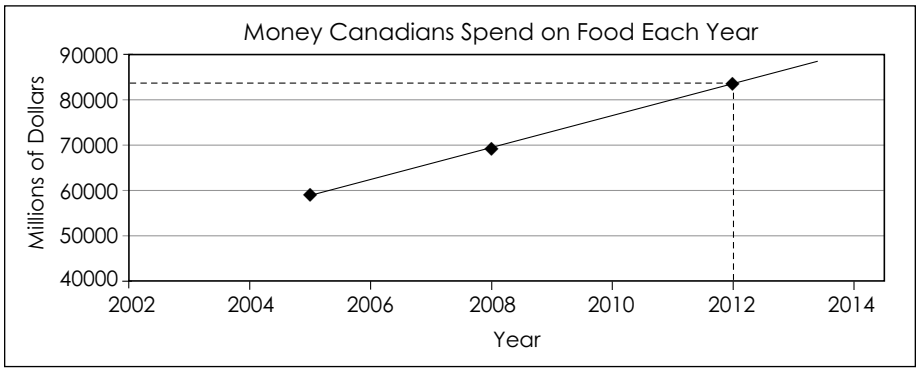
   Answer:



b) How much money would you expect Canadians to spend on food in the year 2012?

Answer:

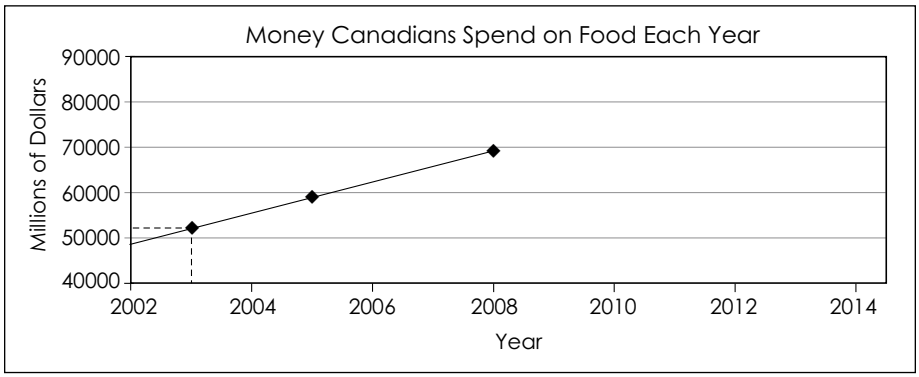
You have to extend the line from 2009 past 2012. Find the point on the graph in line with 2012. Determine where the point lines up on the vertical axis.



Based on the graph, in 2012 Canadians will spend approximately $84,000 million on food. If your answer is within the range of $83,000 million and $85,000 million, that is acceptable.

c) Estimate how much money Canadians spent on food in 2003. Use extrapolation.

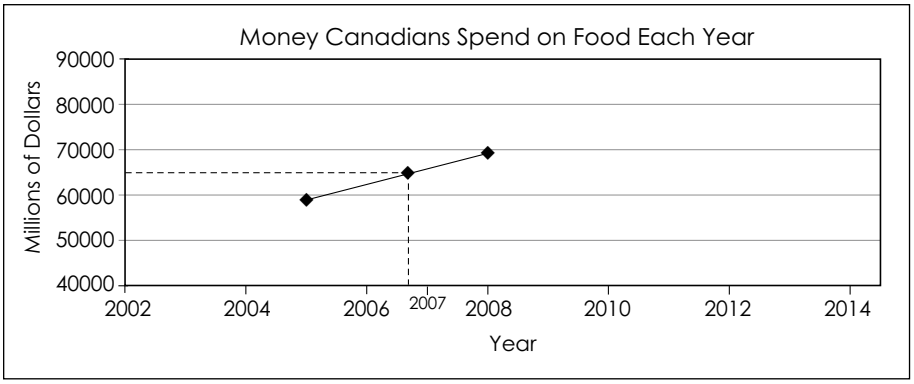
Answer:



Based on the graph, Canadians spend approximately $53,000 million on food in 2003. If your answer is within the range of $52,000 and $54,000 million, that is acceptable.

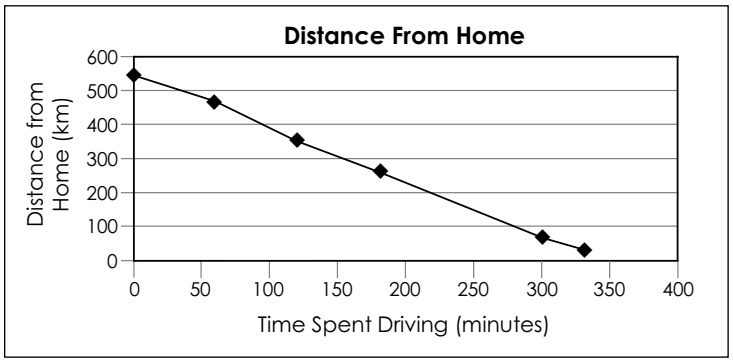
d) Estimate which year Canadians spent approximately $65,000 million on food. Use interpolation.

Answer:



Although this point does not line up with any year in particular, we can round it up to 2007 because it is closer to 2007 than 2006.

2. Given the graph below, describing the distance you are from your home as you drive, answer the following questions.



a) Are you leaving your house or coming home?

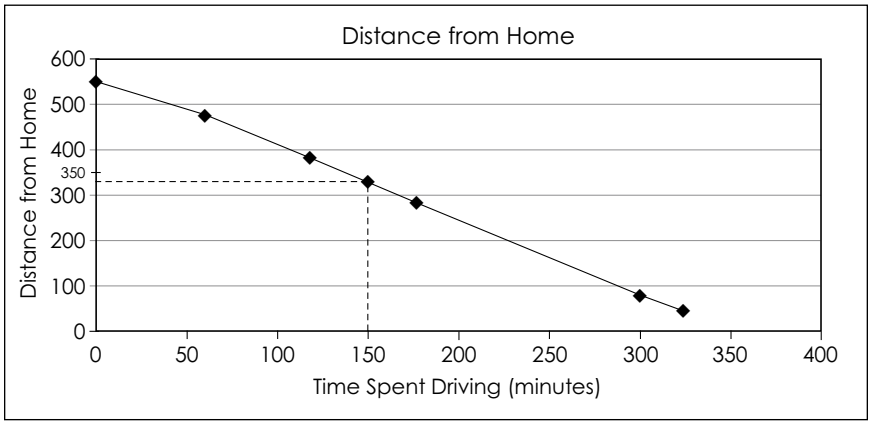
Answer:

Because your distance from home is getting smaller, you are going home.

b) How far are you from home after 2.5 hours of driving?

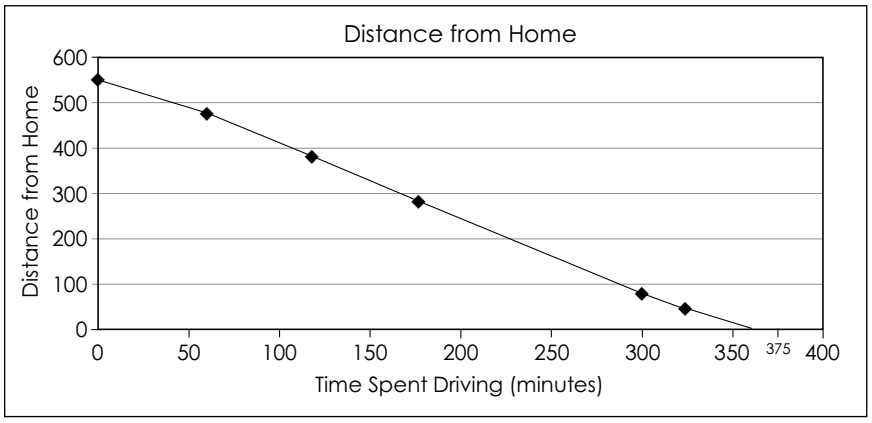
Answer:

At 2.5 hours, time = 150 minutes. You are about 340 km from home.



c) How long will you have been driving before you are 0 km from your house?

Answer:



Based on the graph above, you would drive for 360 to 370 minutes before you reached your house.

Learning Activity 3.5

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. The volume of a square pyramid is 8 m^3. What is the volume of a prism with the same dimensions (length, width, and height)?
2. Seok Yong plays on a school sports team. Based on the following clues, decide whether he plays on the basketball team, hockey team, soccer team, or volleyball team.
   - He does not like to be cold.
   - There are no extra pads required.
   - He scored 34 points in their last game.
3. Convert the fraction to a percent. $\frac{25}{50}$
4. There is a cube with each side length equal to 1 cm. What is the surface area of the cube?
5. Complete the pattern. 10, 100, 1000, _____ , _____ ...

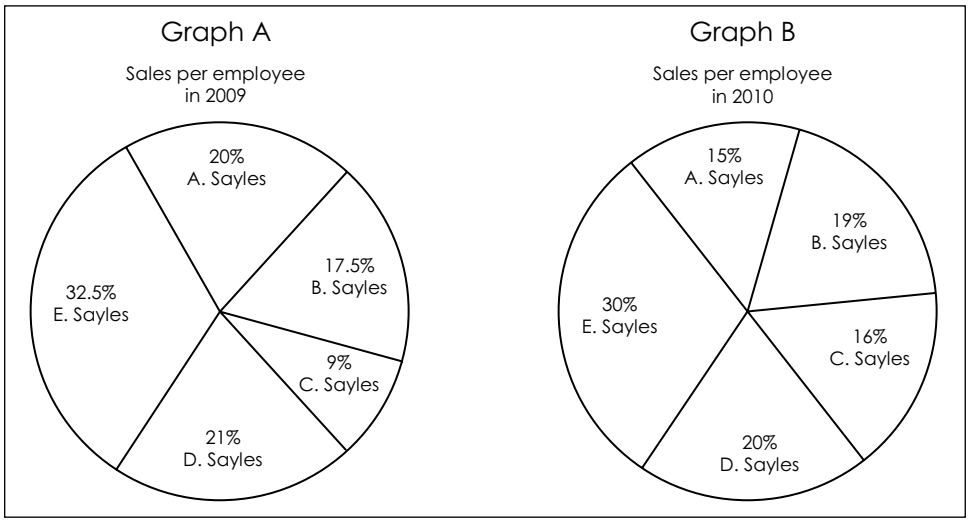
Answers:

1. 24 m^3 (8 × 3 = 24)
2. He plays basketball. He does not play hockey because it's on ice and rinks are cold. He does not play soccer because he would need shin pads. He does not play volleyball because games typically go up to only 25 points, and the points are not usually attributed to one person.
3. 50%
4. 6 cm^2 (Each face is 1 cm^2 and there are six faces to a cube, so area = 1 × 6 = 6 cm^2.)
5. 10 000, 100 000

Part B: Interpreting Graphs

Remember, these questions are similar to the ones that will be on your assignments and midterm examination. If you are able to answer them correctly, you are likely to do well on your assignments and midterm examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Compare the two graphs below. State two ways that someone could interpret the data. Include a reason for each interpretation.



Answer:

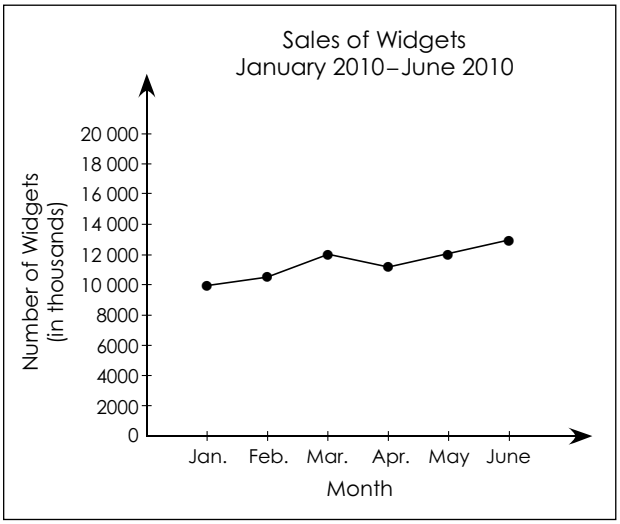
Other answers may be correct as well. Ask your learning partner to help you decide whether your answers would be a reasonable interpretation of the graphs.

I A. Sayles had fewer sales in 2010 than in 2009, while everyone else's sales increased. This interpretation is based on the percentages provided in the graphs. For this to be true, the sales would have to be the same or less in 2010 than in 2009.

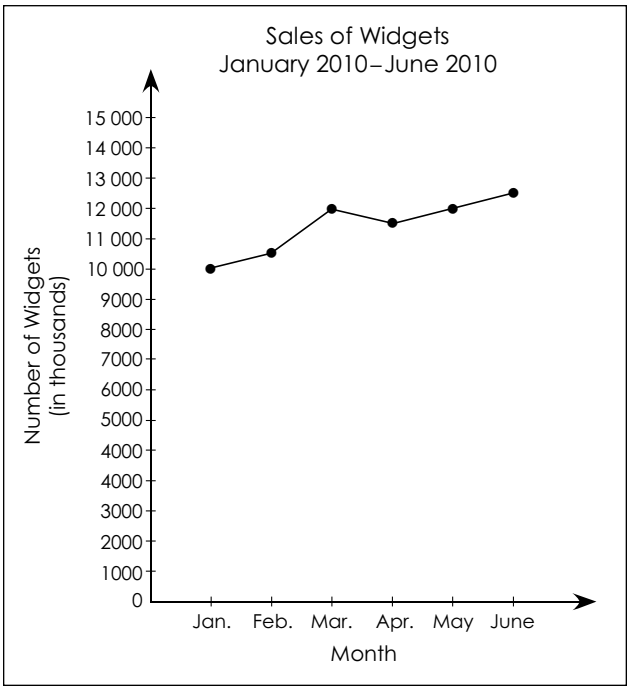
II Everyone's sales increased from 2009 to 2010. This interpretation is based on the idea that the sales increased so much from 2009 to 2010 that even A. Sayles' sales increased, despite having a lower percentage of the total sales.

2. The following three graphs all represent sales of widgets at the Whatsit Widget Company from January 2010 to June 2010.

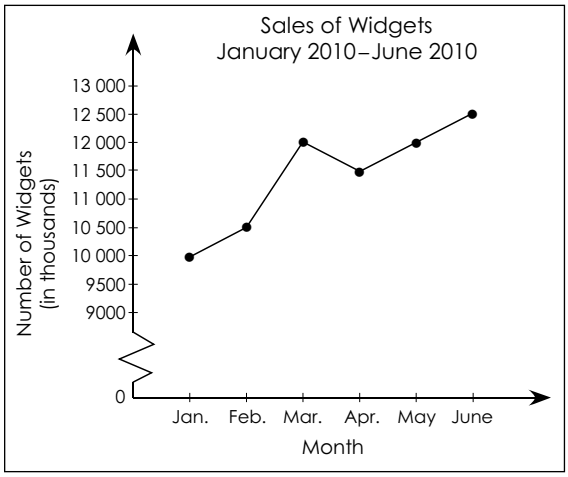
Graph A



Graph B



Graph C



a) Which graph seems to show the greatest increase in the sale of widgets at the Whatsit Widget Company?

Answer:

Graph C

b) Which graph seems to show the least increase in the sale of widgets at the Whatsit Widget Company?

Answer:

Graph A

c) Explain why the three graphs give different impressions.

Answer:

Graph A: Although the scale starts at 0, the vertical axis scale goes much higher (20 000) than the data (approximately 12 500). This "squishes" the graph because the range of data is only a very small section of the vertical scale.

Graph B: This graph is the most "honest" because the vertical scale starts at zero and extends just past the range of data.

Graph C: This graph spreads out the data more than B because of the vertical scale. Instead of starting at zero, it starts at 9000. Although this makes the data easier to read, it also provides an inaccurate visual representation because it appears that the sales increase is *huge* when it is only $2500.

Overall, the vertical scale influences the appearance of the graph so that it looks like there is a large or small increase in Widget sales. If the scale is unnecessarily large, the graph will look compressed. If the scale is small (does not include zero), the graph will look stretched.

Grade 11 Essential Mathematics (30S)

Module 4
Managing Money

Module 4:
Managing Money

Before You Get Started

Before you begin working through the next part of the course, now would be a good time to look at the resource sheet you used for your midterm examination. Answer the following questions so that when you prepare your Final Examination Resource Sheet, you may make changes that you think will be helpful.

- During the midterm examination, did you wish that you had included more information on your resource sheet?
- Was there any information on your resource sheet that you did not need?
- Had you forgotten how to use some of the information on the resource sheet?

Use your answers to these questions as you work through the second half of this course. Look back to find out what type of information you found most/least useful on the Midterm Examination Resource Sheet. This will help you to prepare a better resource sheet for your final examination.

Introduction

You began this course by studying credit and interest in Module 1. In this module, you will expand your understanding of personal finance by focusing on personal budgets and managing your finances. You will learn how to create and analyze a personal budget, and how to use a budget to set financial goals and prepare for unexpected expenses. You will also study a variety of services provided by banks and other financial institutions.

Assignments in Module 4

When you have completed the assignments for Module 4, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	How Time Affects the Value of Money
2	Assignment 4.1	Budgets
3	Assignment 4.2	Bank Accounts
5	Assignment 4.3	Cheques, Deposit Slips, and Registers

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the following list of instructions to help you with preparing your resource sheet for the material in Module 4. On this sheet, you should record mathematics terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 4, 5, 6, and 7 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 4 to 7.

Resource Sheet for Module 4

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important mathematics terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how each formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Notes

Module 4 Cover Assignment: How Time Affects the Value of Money

The following two charts indicate the year-end values of two types of investments. Abra begins investing at age 25 and invests $2000 a year for 10 years. Barbara begins investing at age 35 and invests $2000 a year until the age of 65. Both girls reinvest the principal as well as all the interest until the age of 65 (compound interest). The money is invested at an interest rate of 5.4%, compounded annually.

Abra's Investment

Age	Year	Yearly Contribution	Year End Value
25	1	$2,000	$2,108
26	2	$2,000	$4,330
27	3	$2,000	$6,672
28	4	$2,000	$9,140
29	5	$2,000	$11,741
30	6	$2,000	$14,484
31	7	$2,000	$17,374
32	8	$2,000	$10,420
33	9	$2,000	$23,630
34	10	$2,000	$27,015
35	11	0	$28,473
36	12	0	$30,011
37	13	0	$31,631
38	14	0	$33,340
39	15	0	$35,140
40	16	0	$37,037
41	17	0	$39,037
42	18	0	$41,145
43	19	0	$43,367
44	20	0	$45,709
45	21	0	$48,177
46	22	0	$50,779
47	23	0	$53,521
48	24	0	$56,411
49	25	0	$59,457
50	26	0	$62,668
51	27	0	$66,052
52	28	0	$69,619
53	29	0	$73,378
54	30	0	$77,341
55	31	0	$81,517
56	32	0	$85,919
57	33	0	$90,559
58	34	0	$95,449
59	35	0	$100,603
60	36	0	$106,036
61	37	0	$111,762
62	38	0	$117,797
63	39	0	$124,158
64	40	0	$130,863
65	41	0	$137,929

Barbara's Investment

Age	Year	Yearly Contribution	Year End Value
25	1	0	0
26	2	0	0
27	3	0	0
28	4	0	0
29	5	0	0
30	6	0	0
31	7	0	0
32	8	0	0
33	9	0	0
34	10	0	0
35	11	$2,000	$2,108
36	12	$2,000	$4,330
37	13	$2,000	$6,672
38	14	$2,000	$9,140
39	15	$2,000	$11,741
40	16	$2,000	$14,484
41	17	$2,000	$17,374
42	18	$2,000	$20,420
43	19	$2,000	$23,630
44	20	$2,000	$27,015
45	21	$2,000	$30,581
46	22	$2,000	$34,341
47	23	$2,000	$38,303
48	24	$2,000	$42,479
49	25	$2,000	$46,881
50	26	$2,000	$51,521
51	27	$2,000	$56,411
52	28	$2,000	$61,565
53	29	$2,000	$66,998
54	30	$2,000	$72,724
55	31	$2,000	$78,759
56	32	$2,000	$85,120
57	33	$2,000	$91,824
58	34	$2,000	$98,891
59	35	$2,000	$106,339
60	36	$2,000	$114,189
61	37	$2,000	$122,463
62	38	$2,000	$131,184
63	39	$2,000	$140,376
64	40	$2,000	$150,065
65	41	$2,000	$160,276

Notes

Module 4 Cover Assignment

How Time Affects the Value of Money

Total: 9 marks

Refer to the two charts and answer the following questions. Each question is worth 1 mark.

1. How much money does Abra invest in the 10 years from the ages of 25–34?

2. What is the year-end value of Abra's investment at the age of 35?

3. What is the year-end value of Abra's investment at the age of 65?

4. Explain why there is a difference between the value of Abra's investment at age 35 and age 65.

5. How much money does Barbara invest from the ages of 35 through to 65?

continued

Module 4 Cover Assignment: How Time Affects the Value of Money (continued)

6. What is the year-end value of Barbara's investment at the age of 65?

7. What are Abra's net earnings?

8. What are Barbara's net earnings?

9. Explain in your own words what is meant by the "time value of money."

Lesson 1: Building a Budget

Lesson Focus

In this lesson, you will

- ❑ learn about incomes and expenses that should be included in personal budgets
- ❑ study the decisions that must be made when developing a budget
- ❑ learn how to prepare a monthly budget

Lesson Introduction

Why is it important to learn how to budget your money?

It is important to prepare a budget because, if you do not, you can put yourself at risk of going into debt by overspending, or you may be spending your money on the wrong things. Some people believe that they do not earn enough money to create a budget. **This is false.** In fact, the less money you earn, the more important it is to budget because you have a higher risk of running out of money! A budget can help you

- live within your income
- identify financial priorities
- allocate funds to meet expenses
- meet financial emergencies and reduce credit use
- reduce uncertainty and stress
- gain a sense of financial independence and control
- save and invest to reach financial goals

What Is a Budget?

A **budget** is an estimate of future income and expenses over a given period of time, and a financial plan (setting goals that include income, spending, and savings) based on this estimate. People create budgets to ensure that they have enough money to afford the lifestyle they have or would like. Budget goals can be short-term (such as paying off a credit card bill) or long-term (such as paying for university). No matter what goals you have, your budget should be flexible (easily adjusted). This is necessary because your situation can change quickly (you may get a promotion or lose your job, which changes your income dramatically) and the demands on your finances can also change quickly (such as your air conditioner breaking when it is 35°C outside).

Before You Get Started

There are two things you need before you can start creating your budget:

1. **Know what is important to you.** You need to be aware of your spending habits before you begin formulating your budget.
   - Are you the type of person who likes to spend now? Would you be willing to use credit to achieve a financial goal sooner (such as going on a holiday)? Do you use some of your savings to purchase new items (upgrading your phone or buying a new car)?
   - Do you like to pay off IOUs and loans as soon as possible? Is it important to you to save up for the future (such as college or buying a house)?
   - What type of shopper are you? Are you a bargain hunter, or do you look for name-brand products? Do you need to have the latest and greatest, or is what you already have good enough?

   You should answer these questions honestly. If you do not, you may not be able to follow your budget plan, which would defeat the purpose of having a budget.

2. **Decide what your financial goals are.** If you cannot think of any financial goals you may have, start by making a list of life goals that you have or discuss them with someone you trust. This will aid you in setting financial goals that will let you achieve your life's goals. For example, if you would like to move to a place of your own in five years, then a financial goal could be to save enough money for a down payment.

You should include these two points on your resource sheet before moving on.

In addition to knowing these two things, it is a good idea to keep track of

- how much money you earn
- how much money you spend
- where you spend your money

Do this for at least a few months before you start developing your budget. It is beneficial to do this because you can see how much you are paid and where your money goes. This helps you to estimate how much you will spend on certain things next month and in the future.

Preparing a Budget

In order to keep all the information organized, here is a budget form (found on the following page).

As you can see on the budget form, the monthly budget is divided into five sections:

- Net income
- Monthly savings
- Monthly expenses
- Annual expenses
- Summary

You should note any key points in the descriptions of each section (below) on your resource sheet as a reminder to yourself.

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***		
1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Expenses refer to any outflow (spending) of money to another person or company to pay for an item or service. They are also known as costs. **Recurring expenses** are costs that appear on a regular basis, such as groceries, gas, or phone bills. **Unexpected expenses** are costs that do not appear on a regular basis and are not planned. This can include things like car repairs, broken glasses, or parking tickets.

1. Net Income

Your first step in preparing a monthly budget is to accurately estimate your monthly income. On the budget form in this course, the category for net income is subdivided into three parts:

- primary annual income
- secondary annual income
- other annual income

Because this budget form is general and will be used for families as well as individuals, a primary as well as secondary annual income is included. The "other annual income" category includes income such as scholarships, bonuses, tips and gratuities, income tax refunds, child tax credits, and inheritances.

	Weekly Amount	Annual Amount	***AVERAGE MONTHLY INCOME***
1) ***NET INCOME***			1) $ _____
Primary Income.....	$ _____	$ _____	
Secondary Income.....	$ _____	$ _____	
Other Income.....	$ _____	$ _____	
Total Annual Income.....		$ _____	

Note that the monthly income in the budget form is net income and not gross income. You may recall from previous math courses that this is take-home pay—that is, the income you receive after deductions such as Canada Pension Plan contributions, Employment Insurance premiums, and income tax. Also remember that you have to divide the annual income by 12 to find the average monthly amount.

2. Monthly Savings

The second section is the savings component. It is recommended you set aside 10 percent of your average monthly income. This is what you can use as an emergency fund to pay debts or to invest for the future.

		MONTHLY SAVINGS
2) ***MONTHLY SAVINGS*** (10% of Average Monthly Income)		2) $ _____

Many financial planners suggest that, to save money, you should "pay yourself first." By this, financial planners mean that you should set aside a certain percentage of your net income as soon as you receive it and live off the remaining amount. Paying yourself first will provide you with financial security. You should not use the amount you set aside to buy a DVD player or to take a vacation. You could use the amount you set aside to establish an emergency fund, pay off debt, or invest for your retirement. Most financial planners recommend that you set aside 10 percent of your net income. They also suggest you begin the habit of paying yourself first *as soon as you start earning money*.

3. Monthly Expenses

The third section consists of expenses that vary each month. Examples of these expenses are groceries, clothing, and car maintenance.

3) ***MONTHLY EXPENSES***		
Mortgage or Rent	$ _____	
Car Payments	$ _____	
Telecommunications	$ _____	
Hydro	$ _____	
Other Utilities	$ _____	
Cable	$ _____	
Groceries	$ _____	
Clothing	$ _____	
Car Maintenance	$ _____	
Gasoline	$ _____	
Health and Disability Insurance	$ _____	
Personal Care	$ _____	
Entertainment	$ _____	***TOTAL MONTHLY EXPENSES***
Other	$ _____	
Other	$ _____	
TOTAL MONTHLY EXPENSES		3) $ _____

4. Annual Expenses

The fourth section consists of annual expenses. Although you pay annual expenses only once a year, you should budget for them each month. You are then prepared for them when you have to pay them.

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

Note that you have to divide annual expenses by 12 to calculate the average monthly contribution.

5. Summary

To complete the summary,

a) rewrite the average monthly income in the space provided

b) find the sum of the savings, the total monthly expenses, and the total monthly contributions

c) subtract the sum (from step (b)) from your average monthly income

If the difference is *positive*, you can spend more or add to savings. If the difference is *negative*, you have a *shortfall* or **deficit**. If there is a deficit, it is necessary to examine the budget and either earn more money or spend less money.

Recommendations for Your Budget

Financial planners are great aids when you are preparing a budget, especially if you do not know how much you should be spending in the different categories. The recommended levels of spending in the various categories of a budget depend on the subject's age, income, and family situation. The following table of recommended levels of spending provides only a general guideline.

Category	Percent
Housing	30%
Food	25%
Transportation	12½%
Clothing	7½%
Health and Fitness	5%
Recreation	5%
Insurance	5%
Savings	10%

Study the following example.

Example 1

Jason Williams and Cammy Grimes are a young married couple who have just purchased their first home. Jason is a desktop publisher and receives a gross salary of $480 a week. After deductions, his net salary is $370 a week. Cammy is a pharmacy technician and receives a gross salary of $882 biweekly. After deductions, her net biweekly salary is $742.

Jason and Cammy prepare a budget each month. If their budget indicates a shortfall or deficit, they adjust the budget to remove the deficit. Jason and Cammy would like to purchase living room furniture during the month of March. They expect their other expenses for the month of March to include the following:

- mortgage payment is $580.70
- car payment is $385.50
- telecommunications payment is $42.50
- hydro payment is $90.00
- heating and water payments are $154.00
- cable payment is $21.20
- groceries are $370.00
- clothing is $180.00
- gasoline expense is $92.00
- health and disability insurance is $62.80 a month
- personal care expenses are $45.00
- entertainment is $146.70
- furniture is $525.40
- gifts are $48.20

Their annual expenses include the following:

- Autopac payment is $836.50
- life insurance premiums are $370.14
- property taxes on their new home are $2764.80
- home insurance is $398.20
- newspapers and periodicals are $220 a year

Solution

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ 3,211.00
Primary Income	$ 370 x 52 =	$ 19,240.00	
Secondary Income	$ 742 x 26 =	$ 19,292.00	
Other Income	$	$	
Total Annual Income		$ 28,532.00	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income) 3,211 x 10%			2) $ 321.10
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ 580.70	
Car Payments		$ 385.50	
Telecommunications		$ 42.50	
Hydro		$ 90.00	
Other Utilities		$ 154.00	
Cable		$ 21.20	
Groceries		$ 370.00	
Clothing		$ 180.00	
Car Maintenance		$	
Gasoline		$ 92.00	
Health and Disability Insurance		$ 62.80	
Personal Care		$ 45.00	
Entertainment		$ 146.70	***TOTAL MONTHLY***
Other (furniture)		$ 525.40	***EXPENSES***
Other (gifts)		$ 48.20	
TOTAL MONTHLY EXPENSES			3) $ 2,744.00

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 836.50	$ 69.71	
Life Insurance	$ 370.14	$ 30.85	
Property Taxes	$ 2,764.80	$ 230.40	
Home Insurance	$ 398.20	$ 33.18	
Vacations	$	$	***TOTAL***
Newspapers & Periodicals	$ 220.00	$ 18.33	***MONTHLY***
Other	$	$	***CONTRIBUTIONS***
TOTAL MONTHLY EXPENSES			4) $ 382.47

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 3,211.00
2. SAVINGS	2)	$ 321.10	
3. TOTAL MONTHLY EXPENSES	3)	$ 2,744.00	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 382.47	
TOTAL AMOUNTS 2 + 3 + 4			$ 3,447.57
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ (236.57)

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Deficits

The brackets around the amount in the last line mean that Jason and Cammy spend more money than they earn. This is the **deficit** that was mentioned in the Section 5 summary. If your budget shows a deficit, you must correct this problem by adjusting the amount of money you spend.

As you can see in the example, it is possible (and quite common) for people to have a deficit in a given month. It is best to avoid a deficit because it must be repaid in the following month or months. Also, a deficit can very quickly lead to a large debt. Deficits are often created when credit cards are used to purchase goods you cannot afford.

Prioritizing purchases

You can prevent a deficit by prioritizing your purchases. This means that you may postpone a purchase until you can afford it, or you may decide to not purchase it at all. When you are analyzing a budget and are trying to eliminate a deficit, there are three rules you should follow. You might want to include these rules on your resource sheet.

Rule #1: As has been stated before, *always pay yourself first*.

Many people will take the money budgeted for savings to reduce the deficit. This is a bad idea because you will end up saving less than the recommended 10 percent, which means that you will be less prepared if an unexpected expense comes up. Also, it will take longer to achieve your long-term goals, or you may abandon these long-term goals altogether.

Rule #2: Do not sacrifice living expenses to eliminate the deficit.

Other than your savings, there are certain important expenses to budget for because they maintain your standard of living. These expenses include the following:

- mortgage payments and property taxes so that you can continue living in your house
- hydro, heating, and water payments, which allow you to function within your house (especially in the winter when we need electricity and heating)
- groceries, because you need to eat to survive
- health and disability insurance, life insurance premiums, and home insurance, which all provide you with protection and peace of mind in case something happens to you or your house (such as needing glasses, breaking an arm, or dealing with a flooded basement)

Rule #3: Ask yourself, "Do I really need all the stuff I'm buying?"

It is important that you evaluate your spending habits if you are at risk of having a deficit. Do you really need to buy a whole new wardrobe each season? Is there something wrong with the snowboard you have, or do you just want to get a different one? There are some spending habits you should try to eliminate completely. Impulse buying (buying something just because you see it and/or want it, but maybe don't need it) should be the first thing you try to eliminate when you are working at reducing your deficit.

If you do decide that you need an item, then the next question you can ask yourself is, "Do I need it right away or can it wait?" This question is similar to one you asked yourself in Lesson 5 of Module 1, when you were deciding whether or not to use in-store credit. You should remember that just because you don't buy it this month, it doesn't mean that you should never buy it. You may be able to use the money you save this month to pay for it next month.

Notice that car payments, gasoline, and Autopac are not included under Rule #2. This is because a car may not be a necessity. If you make a judgment call and decide that you absolutely need a car, then these three expenses would fall under Rule #2. Otherwise, you might consider getting a bus pass because it would save you a lot of money each month.

Example 2

Refer back to Example 1. How can Jason and Cammy adjust their spending so that they do not have a deficit?

Solution

The couple's budget has a deficit of $236.57. Jason and Cammy have a number of options for adjusting their budget. One option would be to wait a month to buy the furniture. They should be able to save enough money in two months for the furniture. Another option would be to buy the furniture and spend less on some other items.

By preparing a budget, Jason and Cammy realize that they will have a shortfall or deficit in their budget if they purchase the furniture. They will have to decide whether to buy the furniture in March.

Now that you have seen how to set up a budget, it is time to practise and be sure you understand. When you have finished, check your answers in the Learning Activities Answer Key at the end of this module.

Learning Activity 4.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Jonghun has $50 to spend on groceries. He needs milk ($5.00), vegetables ($15.00), fruit ($15.00), and some sort of protein. Chicken costs $12.50, pork costs $15.50, and steak costs $20. Which type(s) of meat could he afford?

2. Rewrite the fraction in lowest terms: $\frac{750}{500}$.

3. On the Canadian flag, the maple leaf has three main segments. Each segment has three points on it. How many points are there in total?

4. When comparing team statistics, you can look at a team's record (how many games it has won, lost, or tied) as well as the goals for (the number of goals the team has scored) and the goals against (the number of goals other teams have scored against it). If a team has not won a game, is it possible for it to have more goals for than against?

5. Solve for g: $5g - 10 = -30$

continued

Learning Activity 4.1 (continued)

Part B: Creating a Budget

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. a) Prepare a budget for Kirk Donner for the month of October. Use one of the budget forms provided at the end of the lesson, or create a worksheet of your own.

   Kirk is a university student who lives at home with his parents. During the past year, he worked for a lawn service in the summer months and earned a net income of $4800. He also works part-time during the year, and earns a net income of $90 per week. Kirk was awarded a scholarship of $1000.

   Kirk's expenses for the month of October will include the following:

   - payment of $150.00 to his parents for board and room
   - car payment is $105.80
   - gasoline expense is $65.00
   - clothing is $140.00
   - personal care expense is $25.00
   - entertainment expense is $100.00
   - gifts are $35.00
   - CD expense is $20.00

   Kirk's annual expenses will include the following:

   - university tuition is $3200
   - books and supplies are $550
   - Autopac payment is $470.50
   - vacation is $750
   - magazine subscription is $28 a year
   - Christmas gifts are $240.70

   b) Is Kirk's budget in a deficit position?

   c) Kirk has to pay for his textbooks. Will he be able to?

continued

Learning Activity 4.1 (continued)

2. a) Prepare a budget for Daria Smythe for the month of May. Use one of the budget forms provided at the end of the lesson, or create a worksheet of your own.

   Daria has completed a welding course and has just started working. She has a biweekly net salary of $1090. Daria is trying to save money to buy a car. Her expenses for the month of August will include the following:

   - rent payment is $525.00, and she shares this with her two roommates
   - bus pass is $55.85
   - telecommunications payment is $30.00, which she shares with her two roommates
   - hydro payment is $56.00, which she shares with her two roommates
   - water payment is $36.00, which she shares with her two roommates
   - cable payment is $21.22, which she shares with her two roommates
   - groceries are $140.00
   - clothing is $200.00
   - health and disability insurance is $42.80
   - personal care expense is $30.00
   - entertainment expense is $80.00
   - gifts are $21.40
   - student loan payment is $180.00
   - charitable contribution is $25.00

   Daria's annual expenses will include the following:

   - tenant's insurance is $252, which she shares with her two roommates
   - vacation is $1500
   - newspaper subscription is $168.60, which she shares with her two roommates
   - gym membership is $450

   b) Is Daria's budget in a deficit position?

   c) Daria would like to purchase a car for about $8000 in the near future. How much is Daria able to set aside in May to purchase a car? How long will it take Daria until she is able to afford a car if she sets aside the same amount each month?

continued

Learning Activity 4.1 (continued)

3. This question has two parts. Complete *at least one* of the two parts.

   a) Interview a parent/guardian/relative or your learning partner. Collect all of the information you need to create a monthly budget for your family or the person you interviewed. It would be helpful to fill out a budget form (found at the end of this lesson) as you interview the person to ensure that you get all the information that you need.

   Does this person have a deficit? If so, what recommendations could you make to help him or her get rid of it?

   b) Below are listed some career choices, along with the average net Manitoba salary for each career. Select one on the careers that you find interesting.

Restaurant Manager	$33 300	Retail Sales Manager	$43 700
Accountant	$58 600	Executive Assistant	$43 000
General Office Clerk	$33 700	Air Traffic Controller	$76 400
Civil Engineer	$74 600	Computer Systems Analyst	$51 600
Web Designer	$42 000	Dentist	$124 100
Doctor	$183 000	Radiation Therapist	$68 800
Community Service Worker	$35 600	Journalist	$46 200
Translator	$46 200	Chef	$36 600
Esthetician	$26 200	Firefighter	$59 600
Police Officer	$63 800	Carpenter	$40 800
Heavy Equipment Operator	$38 800	Mechanic	$38 500
Farm/Hog Barn Manager	$35 100	Underground Miner	$58 300

   (This information was taken from *Manitoba Prospects 2010*. If you are interested in finding out more about different careers in Manitoba, visit mb.jobfutures.org.)

   Complete a budget form (found at the end of this lesson), using your family's expenses.

   Could you afford your current lifestyle with the salary for that career? If you have a deficit, what would you have to change so that you would not have a deficit?

Lesson Summary

In this lesson, you learned the importance of preparing a personal or family budget. You then prepared a few sample budgets. If the budget showed a deficit, you used the ”Three Rules” to eliminate the deficit. In the next lesson, you will continue to analyze personal budgets, and try to answer some ”What if this were to happen?“ questions.

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount	*AVERAGE MONTHLY INCOME*
1)	***NET INCOME***			1) $ _____
	Primary Income	$ _____	$ _____	
	Secondary Income	$ _____	$ _____	
	Other Income	$ _____	$ _____	
	Total Annual Income		$ _____	***MONTHLY SAVINGS***
2)	***MONTHLY SAVINGS***			
	(10% of Average Monthly Income)			2) $ _____
3)	***MONTHLY EXPENSES***			
	Mortgage or Rent		$ _____	
	Car Payments		$ _____	
	Telecommunications		$ _____	
	Hydro		$ _____	
	Other Utilities		$ _____	
	Cable		$ _____	
	Groceries		$ _____	
	Clothing		$ _____	
	Car Maintenance		$ _____	
	Gasoline		$ _____	
	Health and Disability Insurance		$ _____	
	Personal Care		$ _____	
	Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
	Other		$ _____	
	Other		$ _____	
	TOTAL MONTHLY EXPENSES			3) $ _____

		Annual Amount	Monthly Amount	
4)	***ANNUAL EXPENSES***			
	Car Insurance	$ _____	$ _____	
	Life Insurance	$ _____	$ _____	
	Property Taxes	$ _____	$ _____	
	Home Insurance	$ _____	$ _____	
	Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
	Newspapers & Periodicals	$ _____	$ _____	
	Other	$ _____	$ _____	
	TOTAL MONTHLY EXPENSES			4) $ _____

5)	***SUMMARY***			
	1. AVERAGE MONTHLY INCOME			1) $ _____
	2. SAVINGS	2)	$ _____	
	3. TOTAL MONTHLY EXPENSES	3)	$ _____	
	4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ _____	
	TOTAL AMOUNTS 2 + 3 + 4			$ _____
	5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount	*AVERAGE MONTHLY INCOME*
1)	***NET INCOME***			1) $ _____
	Primary Income	$ _____	$ _____	
	Secondary Income	$ _____	$ _____	
	Other Income	$ _____	$ _____	
	Total Annual Income		$ _____	***MONTHLY SAVINGS***
2)	***MONTHLY SAVINGS***			
	(10% of Average Monthly Income)			2) $ _____
3)	***MONTHLY EXPENSES***			
	Mortgage or Rent		$ _____	
	Car Payments		$ _____	
	Telecommunications		$ _____	
	Hydro		$ _____	
	Other Utilities		$ _____	
	Cable		$ _____	
	Groceries		$ _____	
	Clothing		$ _____	
	Car Maintenance		$ _____	
	Gasoline		$ _____	
	Health and Disability Insurance		$ _____	
	Personal Care		$ _____	
	Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
	Other		$ _____	
	Other		$ _____	
	TOTAL MONTHLY EXPENSES			3) $ _____

		Annual Amount	Monthly Amount	
4)	***ANNUAL EXPENSES***			
	Car Insurance	$ _____	$ _____	
	Life Insurance	$ _____	$ _____	
	Property Taxes	$ _____	$ _____	
	Home Insurance	$ _____	$ _____	
	Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
	Newspapers & Periodicals	$ _____	$ _____	
	Other	$ _____	$ _____	
	TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1.	AVERAGE MONTHLY INCOME		1) $ _____
2.	SAVINGS	2) $ _____	
3.	TOTAL MONTHLY EXPENSES	3) $ _____	
4.	TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
	TOTAL AMOUNTS 2 + 3 + 4		$ _____
5.	AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount	*AVERAGE MONTHLY INCOME*
1)	***NET INCOME***			1) $ _____
	Primary Income	$ _____	$ _____	
	Secondary Income	$ _____	$ _____	
	Other Income	$ _____	$ _____	
	Total Annual Income		$ _____	***MONTHLY SAVINGS***
2)	***MONTHLY SAVINGS***			
	(10% of Average Monthly Income)			2) $ _____
3)	***MONTHLY EXPENSES***			
	Mortgage or Rent		$ _____	
	Car Payments		$ _____	
	Telecommunications		$ _____	
	Hydro		$ _____	
	Other Utilities		$ _____	
	Cable		$ _____	
	Groceries		$ _____	
	Clothing		$ _____	
	Car Maintenance		$ _____	
	Gasoline		$ _____	
	Health and Disability Insurance		$ _____	
	Personal Care		$ _____	
	Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
	Other		$ _____	
	Other		$ _____	
	TOTAL MONTHLY EXPENSES			3) $ _____

		Annual Amount	Monthly Amount	
4)	***ANNUAL EXPENSES***			
	Car Insurance	$ _____	$ _____	
	Life Insurance	$ _____	$ _____	
	Property Taxes	$ _____	$ _____	
	Home Insurance	$ _____	$ _____	
	Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
	Newspapers & Periodicals	$ _____	$ _____	
	Other	$ _____	$ _____	
	TOTAL MONTHLY EXPENSES			4) $ _____

5)	***SUMMARY***		
	1. AVERAGE MONTHLY INCOME		1) $ _____
	2. SAVINGS	2) $ _____	
	3. TOTAL MONTHLY EXPENSES	3) $ _____	
	4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
	TOTAL AMOUNTS 2 + 3 + 4		$ _____
	5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount	*AVERAGE MONTHLY INCOME*
1)	***NET INCOME***			1) $ _____
	Primary Income	$ _____	$ _____	
	Secondary Income	$ _____	$ _____	
	Other Income	$ _____	$ _____	
	Total Annual Income		$ _____	***MONTHLY SAVINGS***
2)	***MONTHLY SAVINGS***			
	(10% of Average Monthly Income)			2) $ _____
3)	***MONTHLY EXPENSES***			
	Mortgage or Rent		$ _____	
	Car Payments		$ _____	
	Telecommunications		$ _____	
	Hydro		$ _____	
	Other Utilities		$ _____	
	Cable		$ _____	
	Groceries		$ _____	
	Clothing		$ _____	
	Car Maintenance		$ _____	
	Gasoline		$ _____	
	Health and Disability Insurance		$ _____	
	Personal Care		$ _____	
	Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
	Other		$ _____	
	Other		$ _____	
	TOTAL MONTHLY EXPENSES			3) $ _____

		Annual Amount	Monthly Amount	
4)	***ANNUAL EXPENSES***			
	Car Insurance	$ _____	$ _____	
	Life Insurance	$ _____	$ _____	
	Property Taxes	$ _____	$ _____	
	Home Insurance	$ _____	$ _____	
	Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
	Newspapers & Periodicals	$ _____	$ _____	
	Other	$ _____	$ _____	
	TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1.	AVERAGE MONTHLY INCOME		1) $ _____
2.	SAVINGS	2) $ _____	
3.	TOTAL MONTHLY EXPENSES	3) $ _____	
4.	TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
	TOTAL AMOUNTS 2 + 3 + 4		$ _____
5.	AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount	*AVERAGE MONTHLY INCOME*
1)	***NET INCOME***			1) $ _____
	Primary Income	$ _____	$ _____	
	Secondary Income	$ _____	$ _____	
	Other Income	$ _____	$ _____	
	Total Annual Income		$ _____	***MONTHLY SAVINGS***
2)	***MONTHLY SAVINGS***			
	(10% of Average Monthly Income)			2) $ _____
3)	***MONTHLY EXPENSES***			
	Mortgage or Rent		$ _____	
	Car Payments		$ _____	
	Telecommunications		$ _____	
	Hydro		$ _____	
	Other Utilities		$ _____	
	Cable		$ _____	
	Groceries		$ _____	
	Clothing		$ _____	
	Car Maintenance		$ _____	
	Gasoline		$ _____	
	Health and Disability Insurance		$ _____	
	Personal Care		$ _____	
	Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
	Other		$ _____	
	Other		$ _____	
	TOTAL MONTHLY EXPENSES			3) $ _____

		Annual Amount	Monthly Amount	
4)	***ANNUAL EXPENSES***			
	Car Insurance	$ _____	$ _____	
	Life Insurance	$ _____	$ _____	
	Property Taxes	$ _____	$ _____	
	Home Insurance	$ _____	$ _____	
	Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
	Newspapers & Periodicals	$ _____	$ _____	
	Other	$ _____	$ _____	
	TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1.	AVERAGE MONTHLY INCOME		1) $ _____
2.	SAVINGS	2) $ _____	
3.	TOTAL MONTHLY EXPENSES	3) $ _____	
4.	TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
	TOTAL AMOUNTS 2 + 3 + 4		$ _____
5.	AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY***
Other		$ _____	***EXPENSES***
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY***
Newspapers & Periodicals	$ _____	$ _____	***CONTRIBUTIONS***
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

AVERAGE MONTHLY INCOME

	Weekly Amount	Annual Amount	
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount	
				AVERAGE MONTHLY INCOME
1)	***NET INCOME***			1) $ _____
	Primary Income	$ _____	$ _____	
	Secondary Income	$ _____	$ _____	
	Other Income	$ _____	$ _____	
	Total Annual Income		$ _____	***MONTHLY SAVINGS***
2)	***MONTHLY SAVINGS***			
	(10% of Average Monthly Income)			2) $ _____
3)	***MONTHLY EXPENSES***			
	Mortgage or Rent		$ _____	
	Car Payments		$ _____	
	Telecommunications		$ _____	
	Hydro		$ _____	
	Other Utilities		$ _____	
	Cable		$ _____	
	Groceries		$ _____	
	Clothing		$ _____	
	Car Maintenance		$ _____	
	Gasoline		$ _____	
	Health and Disability Insurance		$ _____	
	Personal Care		$ _____	
	Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
	Other		$ _____	
	Other		$ _____	
	TOTAL MONTHLY EXPENSES			3) $ _____

		Annual Amount	Monthly Amount	
4)	***ANNUAL EXPENSES***			
	Car Insurance	$ _____	$ _____	
	Life Insurance	$ _____	$ _____	
	Property Taxes	$ _____	$ _____	
	Home Insurance	$ _____	$ _____	
	Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
	Newspapers & Periodicals	$ _____	$ _____	
	Other	$ _____	$ _____	
	TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1.	AVERAGE MONTHLY INCOME		1) $ _____
2.	SAVINGS	2) $ _____	
3.	TOTAL MONTHLY EXPENSES	3) $ _____	
4.	TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
	TOTAL AMOUNTS 2 + 3 + 4		$ _____
5.	AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Lesson 2: Modifying Budgets

Lesson Focus

In this lesson, you will

- ❑ explore how different scenarios affect a budget
- ❑ learn how to alter a budget in order to avoid overspending or to reach a financial goal

Lesson Introduction

Now that you know how to create a budget, you will be able to map your savings ahead of time. But what does this mean for you? As you know from the previous lesson, this helps you avoid having a deficit for the month. In addition, once you have a budget, you can use it to plan for expenses in advance or before you even know they exist. We are, again, referring to financial goals and unexpected expenses.

The Flexible Budget

As stated in the previous lesson, budgets must be flexible. This is because no matter how much you plan, you never know what may happen.

Unexpected Expenses

Have you ever been in a situation where something happened that you did not anticipate? Did this "something" result in you having to spend more money than you had originally thought?

These are unexpected expenses and they happen to everyone. The following examples will examine how these different unexpected expenses affect your budget, and how you can be ready for them.

Example 1

Sabet shares an apartment with Francis. They split the cost of rent, cable, hydro, water, apartment insurance, and the telecommunications bill.

- rent is $780 per month
- telecommunications payment is $40.00
- hydro payment is $47.00
- water payment is $25.00
- cable payment is $50.00
- apartment insurance is $500 annually

Sabet works as a server, earning $663.19 net biweekly plus around $400 in tips. Sabet's other expenses for June include the following:

- bus pass is $56.00
- groceries cost $210
- clothing costs $50
- health insurance costs $38.25
- personal care costs are $15.35
- entertainment expenses are $115
- long distance charges on the telecommunications bill cost $6.71

Sabet's annual expenses also include the following:

- vacation for $900
- cooking magazine subscription is $28 per year
- gym membership costs $275

a) Prepare a budget for Sabet for the month of June.

b) After they have a fight, Francis decides to move back home. Adjust Sabet's budget accordingly.

c) Can Sabet afford to continue living in the apartment without Francis? Explain your answer.

Solution

a)

MONTHLY BUDGET FORM

	Bi-Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ 2,303.57
Primary Income	$ 663.19 x 52/2	$ 17,242.94	
Secondary Income	$	$	
Other Income	$ 400.00 x 52/2	$ 10,400.00	
Total Annual Income		$ 27,642.94	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 230.36
3) ***MONTHLY EXPENSES***			
Mortgage or Rent	$780 ÷ 2	$ 390.00	
Car Payments		$	
Telecommunications	$40 ÷ 2 = $20 + $6.71	$ 26.71 (including long distance)	
Hydro	$47 ÷ 2	$ 23.50	
Other Utilities	$25 ÷ 2	$ 12.50	
Cable	$50 ÷ 2	$ 25.00	
Groceries		$ 210.00	
Clothing		$ 50.00	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$ 38.25	
Personal Care		$ 15.35	
Entertainment		$ 115.50	***TOTAL MONTHLY EXPENSES***
Other (Bus Pass)		$ 56.30	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 962.31

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$	$	
Life Insurance	$	$	
Property Taxes	$	$	
Home Insurance (Appt) $500 ÷ 2	$ 250.00	$ 20.83	
Vacations	$ 900.00	$ 75.00	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ 28.00	$ 2.33	
Other (Gym Membership)	$ 275.00	$ 22.92	
TOTAL MONTHLY EXPENSES			4) $ 121.08

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ 2,303.57
2. SAVINGS	2) $ 230.36	
3. TOTAL MONTHLY EXPENSES	3) $ 962.31	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ 121.08	
TOTAL AMOUNTS 2 + 3 + 4		$ 1,313.75
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ 989.82

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

b)

MONTHLY BUDGET FORM

	Bi-Weekly Amount	Annual Amount	*AVERAGE MONTHLY INCOME*
1) ***NET INCOME***			1) $ 2303.57
Primary Income	$ 663.19 x 52/2	$ 17,242.94	
Secondary Income	$	$	
Other Income	$ 400.00 x 52/2	$ 10,400.00	
Total Annual Income		$ 27,642.94	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 230.36
3) ***MONTHLY EXPENSES***			
Mortgage or Rent	$780 ÷ 2	$ 390.00 /780.00	
Car Payments		$	
Telecommunications	$40 ÷ 2 = $20 + $6.71	$ 26.71 (including long distance)	40 + 6.71 = $46.71
Hydro	$47 ÷ 2	$ 23.50/$47.00	
Other Utilities	$25 ÷ 2	$ 12.50/25.00	
Cable	$50 ÷ 2	$ 25.00/50.00	
Groceries		$ 210.00	
Clothing		$ 50.00	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$ 38.25	
Personal Care		$ 15.35	
Entertainment		$ 115.50	***TOTAL MONTHLY EXPENSES***
Other (Bus Pass)		$ 56.30	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 962.31/1,433.31

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$	$	
Life Insurance	$	$	
Property Taxes	$	$	
Home Insurance (Appt) $500 ÷ 2	$ 250.00/500.00	$ 20.83/41.67	
Vacations	$ 900.00	$ 75.00	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ 28.00	$ 2.33	
Other (Gym Membership)	$ 275.00	$ 22.92	
TOTAL MONTHLY EXPENSES			4) $ 121.08/141.92

5) ***SUMMARY***		
1. AVERAGE MONTHLY INCOME		1) $ 2,303.57
2. SAVINGS	2) $ 230.36	
3. TOTAL MONTHLY EXPENSES	3) $ 962.31/1,433.31	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ 121.08/141.92	
TOTAL AMOUNTS 2 + 3 + 4		$ 1,313.75/1,805.59
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ 989.82/497.98

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

c) Sabet can continue living in the apartment because the budget still has a surplus (the opposite of deficit).

Note that you do not need to complete a whole new form for part (b) if you do not want to. Just write the new values beside the original values, preferably in a different colour so that it is easy to distinguish them.

Example 2

The Jiang family, which includes three children, is on an August road trip from Portage la Prairie to the East Coast. Hun Jiang earns $56,000 net per year while Bohyun Jiang earns $500 net biweekly. Their monthly costs include the following:

- mortgage payment is $800
- payments for the van cost $189
- telecommunications payments are $50
- hydro bill is $92
- water bill is $63
- groceries cost $530.31
- clothing costs $218.59
- gas totals around $280
- health and disability insurance costs $74
- personal care costs $52.64
- entertainment costs $60
- music lessons cost $400 per child
- Internet costs $38.00

The Jiang family's annual expenses are the following:

- car insurance is $1200
- life insurance is $543.18
- property taxes cost $9968
- home insurance is $483.79
- vacation costs $5300

a) Prepare a budget for the Jiang family for the month of August.

b) Between Steinbach and Kenora, the Jiang family gets a flat tire. The nearest mechanic says that it will cost $65 to tow the van, $67 for a new tire, and $80 for the labour. Do the Jiangs have enough money to pay for this without creating a deficit?

Solution

a)

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ 5,750.00
Primary Income	$	$ 56,000.00	
Secondary Income	$ 500 x 52/2	$ 13,000.00	
Other Income	$	$	
Total Annual Income		$ 69,000.00	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 575.00
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ 800.00	
Car Payments		$ 189.00	
Telecommunications		$ 50.00	
Hydro		$ 92.00	
Other Utilities		$ 63.00	
Cable		$	
Groceries		$ 530.37	
Clothing		$ 218.59	
Car Maintenance		$	
Gasoline		$ 280.00	
Health and Disability Insurance		$ 74.00	
Personal Care		$ 52.64	
Entertainment		$ 60.00	***TOTAL MONTHLY EXPENSES***
Other (music lessons) $400 x 3 =		$ 1,200.00	
Other (Internet)		$ 38.00	
TOTAL MONTHLY EXPENSES			3) $ 3,647.54

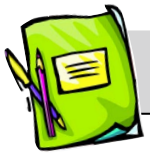
	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 12.00 ÷ 12	$ 100.00	
Life Insurance	$ 543.18 ÷ 12	$ 45.27	
Property Taxes	$ 9,968.00 ÷ 12	$ 830.67	
Home Insurance	$ 483.79 ÷ 12	$ 40.32	
Vacations	$ 5,300.00 ÷ 12	$ 441.67	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$	$	
Other	$	$	
TOTAL MONTHLY EXPENSES			4) $ 1,457.93

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 5,750.00
2. SAVINGS	2)	$ 575.00	
3. TOTAL MONTHLY EXPENSES	3)	$ 3,647.54	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 1,457.93	
TOTAL AMOUNTS 2 + 3 + 4			$ 5,680.47
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 69.53

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

b) The total cost of replacing the tire would be 65 + 67 + 80 = $212.00. The Jiangs only have a surplus of $69.53, so they would have a deficit after paying to replace the tire.

As you can see from these examples, sometimes you have a large enough surplus that you can easily pay for unexpected expenses, like Sabet. Other people may not have a large surplus and may find it difficult to pay for unexpected expenses. If you start with a deficit, you definitely cannot pay for unexpected expenses without creating a greater deficit.



Learning Activity 4.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate the following: $10 + 3 \times 6 - \frac{12}{2}$

2. There are a lot of special occasions in Brennan's family in June. His sisters' birthdays are on the 4th, so he buys the two of them $30 gift cards each for workout clothes. His brother's birthday is on the 12th, so he pays $20 for a rare CD for his brother. And it's Father's Day, so he spends $50 on a luxury car wash for his dad. How much money did Brennan spend on gifts in June?

3. Write two equivalent fractions for $\frac{45}{90}$.

4. High school students are collecting mittens for a shelter. If each student donates two pairs of mittens and there are 140 students at the school, how many pairs of mittens will the school donate?

5. A turkey must be roasted 15 minutes for every pound. If a turkey weighs 20 pounds, how long (in hours) would you have to roast the turkey?

continued

Learning Activity 4.2 (continued)

Part B: Unexpected Expenses

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. With your learning partner, brainstorm different types of unexpected expenses that you may face.

2. Emery has to save some money for university. Right now his monthly budget for September looks like this:

EMERY'S MONTHLY BUDGET FOR SEPTEMBER

	Bi-Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ 1,436.06
Primary Income	$ 462.80	$ 12,032.80	
Secondary Income	$	$	
Other Income	$ 200.00 (tips)	$ 5,200.00	
Total Annual Income		$ 17,232.80	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS*** (10% of Average Monthly Income)			2) $ 143.61
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$	
Car Payments		$ 180.00	
Telecommunications		$ 75.00	
Hydro		$	
Other Utilities		$	
Cable		$	
Groceries		$	
Clothing		$ 100.00	
Car Maintenance		$	
Gasoline		$ 80.00	
Health and Disability Insurance		$	
Personal Care		$ 22.41	
Entertainment		$ 191.46	***TOTAL MONTHLY EXPENSES***
Other		$	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 648.87

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 988.46	$ 82.37	
Life Insurance	$ 120.91	$ 10.08	
Property Taxes	$	$	
Home Insurance	$	$	
Vacations	$ 1,600.00	$ 133.34	
Newspapers & Periodicals	$ 35.00	$ 2.17	***TOTAL MONTHLY CONTRIBUTIONS***
Other (university)	$ 4,039.61	$ 336.63	
Other (books)	$ 728.54	$ 60.71	
TOTAL MONTHLY EXPENSES			4) $ 625.30

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 1,436.06
2. SAVINGS	2)	$ 143.61	
3. TOTAL MONTHLY EXPENSES	3)	$ 648.87	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 625.30	
TOTAL AMOUNTS 2 + 3 + 4			$ 1,417.78
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 18.28

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

continued

Learning Activity 4.2 (continued)

a) Can Emery afford to pay for his books?

b) Last year, he got into two car accidents, so his car insurance is now $1620 per year. Can he still afford his books?

c) Emery has $6000 in his bank account after he has paid his tuition for this school year. A couple of weeks into university, he realizes that he has to quit his job because he has too much homework. How long can he afford to keep the same budget before he runs out of money? Answer this question based on the original budget provided, not including the information from Part (b).

Assessing Financial Goals

A budget can also be used to assess your financial goals. You can then

- make a plan based on your budget
- alter the goals so that they become more realistic

Example 1

Celine would like to buy a car in May before she graduates. It is the beginning of her Grade 11 school year (September).

a) Prepare a budget for Celine. She is paid $230.55 net weekly. Her monthly costs include the following:
   - her cell phone bill is $26.78
   - clothing expenses are $95.48
   - personal care costs $35.00
   - entertainment costs $62.81

   Her annual costs are the following:
   - vacation expenses are $600
   - a magazine subscription costs $19

b) The car Celine wants to buy will cost $16,999. Will she be able to save enough money to pay cash for it before graduating? If not, is it possible to adjust the budget so that she could afford to pay cash for the car?

Solution

a)

CELINE'S MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ 999.05
Primary Income	$ 230.55	$ 11,988.60	
Secondary Income	$	$	
Other Income	$	$	
Total Annual Income		$ 11,988.60	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 99.90
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$	
Car Payments		$	
Telecommunications		$ 26.78	
Hydro		$	
Other Utilities		$	
Cable		$	
Groceries		$	
Clothing		$ 95.48	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$	
Personal Care		$ 35.00	
Entertainment		$ 62.81	***TOTAL MONTHLY EXPENSES***
Other		$	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 220.07

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$	$	
Life Insurance	$	$	
Property Taxes	$	$	
Home Insurance	$	$	
Vacations	$ 600.00	$ 50.00	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ 19.00	$ 1.58	
Other	$	$	
TOTAL MONTHLY EXPENSES			4) $ 51.58

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ 999.05
2. SAVINGS	2) $ 99.90	
3. TOTAL MONTHLY EXPENSES	3) $ 220.07	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ 51.58	
TOTAL AMOUNTS 2 + 3 + 4		$ 371.55
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ 627.50

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

b) Based on Celine's budget, she should be able to designate $627.50 each month to buying a car. From September of Grade 11 to May of Grade 12 is 21 months.

   Savings: 21 × 627.50 = $13,177.50

   Celine would not be able to pay cash for the car. She is approximately $4000 short.

   Celine would need to save an additional $4000 ÷ 21 = $190.48 each month. Because her monthly expenses are only $220.07, it is not realistic to adjust Celine's budget to afford the car. She would be left with approximately $30 spending money per month.

In the above example, it is important to note that we have determined that this financial goal is *too high* given the period of time. If Celine wanted to wait another seven months, she would have enough money. This might be a better option for her to consider.

Another alternative would be for Celine to use a loan to pay for the last $4000. As you learned in Module 1, credit is often used to pay for big-ticket items.

Example 2

This is Emil's monthly budget. He has a full-time job.

While attending university, he has lived at home where he pays $200 rent per month, but now that he has a job, he would like to move out as soon as possible.

EMIL'S MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
1) ***NET INCOME***			***AVERAGE MONTHLY INCOME***
Primary Income	$	$ 30,000.00	1) $ 3,433.53
Secondary Income	$ 430.86 bi-weekly	$	
Other Income	$	$ 11,202.36	
Total Annual Income		$ 41,202.36	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 343.35
3) ***MONTHLY EXPENSES***			
Mortgage or (Rent)		$ 200.00	
Car Payments		$ 200.00	
Telecommunications		$ 35.62	
Hydro		$	
Other Utilities		$	
Cable		$	
Groceries		$	
Clothing		$ 80.43	
Car Maintenance		$ 45.89	
Gasoline		$ 79.77	
Health and Disability Insurance		$ 36.45	
Personal Care		$ 16.21	
Entertainment		$ 84.98	***TOTAL MONTHLY EXPENSES***
Other		$	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 779.35

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 750.00	$ 62.50	
Life Insurance	$ 103.10	$ 8.59	
Property Taxes	$	$	
Home Insurance	$	$	
Vacations	$ 1,100.00	$ 91.67	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$	$	
Other	$	$	
TOTAL MONTHLY EXPENSES			4) $ 162.75

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME			1) $ 3,433.53
2. SAVINGS	2)	$ 343.35	
3. TOTAL MONTHLY EXPENSES	3)	$ 779.35	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 162.75	
TOTAL AMOUNTS 2 + 3 + 4			$ 1,285.46
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 2,148.07

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

a) When he finishes university, Emil has $8000 saved to buy his first house. He would like to save a total of $35,000 before he buys a house. How long will it take him to save $35,000?

b) The following would be his costs of owning a house and living in it:

- mortgage payments are $1200 monthly
- hydro bill costs $56 monthly
- water bill costs $31 monthly
- heating costs $60 monthly
- groceries are $175 per month
- property taxes total $5762 annually
- home insurance would cost $1600 annually

Prepare a budget for Emil, including all of this information in addition to the information in the original budget. Will he be able to afford owning a house?

Solution

a) He already has $8000 saved, so he still needs to save $35000 − 8000 = $27,000

Time: 27000 ÷ 2148.07 = 12.6

It would take Emil approximately 13 months (1 year and 1 month) to save up enough money.

b)

EMIL'S MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
1) ***NET INCOME***			1) $ 3,433.53
Primary Income	$	$ 30,000.00	
Secondary Income	$ 430.86 bi-weekly	$	
Other Income	$	$ 11,202.36	
Total Annual Income		$ 41,202.36	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS*** (10% of Average Monthly Income)			2) $ 343.35
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ 1,200.00	(He is now paying for a house; he doesn't live at home so he doesn't have to pay rent.)
Car Payments		$ 200.00	
Telecommunications		$ 35.62	
Hydro		$ 56.00	
Other Utilities (water & heating/31 + 60 =)		$ 91.00	
Cable		$	
Groceries		$ 175.00	
Clothing		$ 80.43	
Car Maintenance		$ 45.89	
Gasoline		$ 79.77	
Health and Disability Insurance		$ 36.45	
Personal Care		$ 16.21	
Entertainment		$ 84.98	***TOTAL MONTHLY EXPENSES***
Other		$	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 2,101.35

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 750.00	$ 62.50	
Life Insurance	$ 103.10	$ 8.59	
Property Taxes	$ 5,762.00	$ 480.17	
Home Insurance	$ 1,600.00	$ 133.33	
Vacations	$ 1,100.00	$ 91.67	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$	$	
Other	$	$	
TOTAL MONTHLY EXPENSES			4) $ 776.25

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 3,433.53
2. SAVINGS	2)	$ 343.35	
3. TOTAL MONTHLY EXPENSES	3)	$ 2,101.35	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 776.25	
TOTAL AMOUNTS 2 + 3 + 4			$ 3,220.95
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 212.58

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Yes, Emil does have enough money to live on his own and own a house. He can even use some of the remaining money to get cable!

In these two examples you can see how budgets can be used to calculate how long it may take to save enough money for something, or you can use a budget to calculate whether you can afford what it is you want. As you learned in Module 1, you can use credit to obtain what you cannot afford on your own. Just remember that you should include the monthly payment of any credit in your budget (under the "other" category of monthly expenses) so that you get the most accurate depiction of your finances.

Complete the following learning activity where you need to complete a budget, and then use it to determine whether a financial goal is reasonable. Check your answers to be sure you understand this lesson.

Learning Activity 4.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Convert the fraction to a decimal and a percent: $\frac{21}{84}$.

2. To bisect an angle means to cut in half. What is the name of the angle when you bisect a straight line?

3. If Jonah has $81 and he buys four DVDs for $15 each, will he have enough money?

4. Aldin's sales were $1500 over the past two weeks. If he makes 20% commission, how much will he be paid (gross)?

5. Complete the pattern: 100, 75, 50, _____ , _____ , _____ .

continued

Learning Activity 4.3 (continued)

Part B: Assessing Financial Goals

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Yagabi Umbago earns $1800 net each month. Rafi Umbago earns $488 weekly.

   The following are the Umbago family's monthly expenses:

   - $890 for rent
   - telecommunications payment is $34
   - utilities cost $150
   - cable costs $45
   - groceries cost $270
   - $73 for clothing
   - $200 for daycare for each of their two children
   - health and disability insurance costs $147
   - personal care costs $60
   - entertainment costs $100

   The following are their annual costs:

   - life insurance costs $176
   - apartment insurance costs $423
   - the newspaper subscription costs $50
   - one child plays hockey, which costs $1000
   - the other child plays soccer, which costs $500

   a) Prepare a budget for the Umbago family. Use one of the monthly budget forms at the end of Lesson 1 or create your own.

   b) If the rent for their apartment increases to $1100, will they still be able to afford it?

   c) The Umbago family calculates that it would cost them $8000 to go on a family vacation to Disneyland. How long would they have to save before they could go on the vacation? Use the original information provided.

continued

Learning Activity 4.3 (continued)

2. Horace Malabar would like to purchase a boat, which costs $19,450, in April next year. His monthly budget for August this year is:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
1) ***NET INCOME***			1) $ 3,501.67
Primary Income	$ 385.00	$ 20,020.00	
Secondary Income	$	$ 22,000.00	
Other Income	$	$	
Total Annual Income		$ 42,020.00	
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 350.17
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$	
Car Payments		$	
Telecommunications		$ 25.97	
Hydro		$ 52.89	
Other Utilities		$ 33.64	
Cable		$ 40.00	
Groceries		$ 198.06	
Clothing		$ 30.00	
Car Maintenance		$ 40.00	
Gasoline		$ 35.00	
Health and Disability Insurance		$ 59.81	
Personal Care		$ 25.00	
Entertainment		$ 60.00	
Other (bike maintenance)		$ 25.00	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 625.37

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 700.00	$ 58.33	
Life Insurance	$ 190.00	$ 15.83	
Property Taxes	$ 8,210.00	$ 684.17	
Home Insurance	$ 1,450.00	$ 120.83	
Vacations	$ 2,000.00	$ 166.67	
Newspapers & Periodicals	$ 70.00	$ 5.83	
Other	$	$	
TOTAL MONTHLY EXPENSES			4) $ 1,051.66

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 3,501.67
2. SAVINGS	2)	$ 350.17	
3. TOTAL MONTHLY EXPENSES	3)	$ 625.37	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 1,051.66	
TOTAL AMOUNTS 2 + 3 + 4			$ 2,027.20
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 1,474.47

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

AVERAGE MONTHLY INCOME

MONTHLY SAVINGS

TOTAL MONTHLY EXPENSES

TOTAL MONTHLY CONTRIBUTIONS

continued

Learning Activity 4.3 (continued)

a) Will he be able to buy the boat upfront if he puts his surplus each month toward the boat? If not, what type of credit should he use to pay the difference (recall from Module 1)?

b) Once he buys the boat, he also gains the following expenses:

- gas costs $60 per month
- monthly payments for the boat are $216
- maintenance per month costs $30
- boat insurance costs $650 annually

Will Horace be able to afford all of the additional expenses of having a boat?

Lesson Summary

Budgets are financial plans that need to be flexible. In this lesson, you modified budgets for individuals and families encountering new situations, including new long-range plans or unexpected expenses.

Do not forget to complete the following assignment before you move on to the next lesson.

Assignment 4.1

Budgets

Total: 30 marks

Note to Students: You are encouraged to use your resource sheet as you complete this assignment. The resource sheet should have information from the previous two lessons. Be sure to add any missing information to it as you answer the questions.

1. Explain what is meant by the statement "pay yourself first." *(1 mark)*

2. a) Prepare a budget for the Monroe family for the month of September. Use one of the budget forms provided at the end of the assignment, or create a worksheet of your own. *(10 marks)*

   Gary Monroe is an administrative assistant and earns $405.75 net per week. His wife, Janis, is a realtor and has a take-home pay of $28,000 annually. The Monroes have three school-age children. The Monroe family's expenses for the month of February will include the following:

   - mortgage payment is $715.70
   - car payment is $216.50
   - telecommunications payment is $45.50
   - hydro payment is $96.00
   - heating payment is $70.00
   - cable payment is $28.20
   - groceries are $575.00
   - clothing is $120.00
   - gasoline expenses are $85.00
   - car maintenance is $225.00
   - child care expenses are $380.20
   - music lessons cost $90

continued

- health and disability insurance is $52.90
- personal care expenses are $60.00
- children's education fund is $125.00
- house repairs are $140.00
- entertainment expense is $60.00
- gifts are $48.00

The family's annual expenses will include the following:

- Autopac payment is $620.40
- life insurance premiums are $419.30
- property taxes on their home are $2688.00
- home insurance is $382.60
- vacations cost $1200
- gifts cost $580
- newspapers and magazines are $278.40 a year

b) Is the Monroe family budget in a deficit position? Is there an additional amount the family can spend or add to its savings? *(1 mark)*

c) Gas prices suddenly spike in February. The Monroe family has to change this in their budget, so their gasoline costs will be $115 per month. Is their budget flexible enough to accommodate this change? *(1 mark)*

continued

Assignment 4.1: Budgets (continued)

2. Gary gets a new job in March, so the Monroe family has to move to Calgary. His new job will pay him $580.70 net per week. Janis continues to be a realtor, but she now takes home $34 000 per year.

   a) Prepare a new budget for the Monroe family for the month of March. *(10 marks)*

   The following are their costs for the month of March:

   - mortgage payment is $846.20
   - car payment is $216.50
   - telecommunications payment is $60.45
   - hydro payment is $176.00
   - heating payment is $68.00
   - cable payment is $48.00
   - groceries are $764.20
   - clothing is $120.00
   - gasoline expenses are $60.00
   - car maintenance costs $260.00
   - child care expenses are $500.00
   - music lessons are $110
   - health and disability insurance is $78.43
   - personal care expense is $75.00
   - children's education fund is $125.00
   - house repairs are $276.00
   - entertainment expenses are $100.00

   The family's annual expenses will include the following:

   - car insurance payment is $946.00
   - life insurance premiums are $419.30
   - property taxes on their home are $5761.00
   - home insurance is $471.94
   - vacations are $1200
   - gifts cost $580
   - newspapers and magazines cost $172.40

continued

b) Will the Monroe family be able to afford to live in Calgary? *(1 mark)*

c) After having medical issues, the life insurance premium for the Monroe family increases to $576 per year. Also, the cost of their health and disability insurance increases to $146.75 per month. Will the Monroe family still have a surplus? *(5 marks)*

d) The Monroe family would like to go camping in Banff for the weekend. Doing this will cost $945. How many months would they have to wait before they can afford to go camping (using only surplus money)? Use the original budget from part (a) of this question. *(1 mark)*

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***

1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Lesson 3: Bank Accounts

Lesson Focus

In this lesson, you will

- ❑ explore a variety of different types of bank accounts, what each type has to offer, and how to decide which is the best for your needs
- ❑ study the advantages and disadvantages of online banking and debit transactions
- ❑ learn how to be sure that your personal and financial information is secure

Lesson Introduction

In the previous two lessons, you learned how to create a budget, and then how to modify it when circumstances changed. A key part of budgeting is to "pay yourself first" by depositing at least 10 percent of your net income into a savings account. The money is usually deposited into some account in a bank or credit union because this is the most convenient way to save money. In this lesson, you will be introduced to various types of bank accounts, and the advantages of each type of account.

Types of Accounts

Banks offer several types of accounts for their customers. The following are the three most popular accounts used for everyday transactions:

- the savings account
- the chequing account
- the combination account

Another account that has become more popular recently is the electronic savings account. Before you decide what type of account to open, you need to know how you plan to use the account.

As you read through the descriptions of the different types of accounts, you should include important characteristics of each on your resource sheet.

Savings Accounts

As the name suggests, this type of account is good if you are interested in saving money because the bank pays interest. Different banks pay different interest rates for savings accounts. The interest rates may even vary for different types of savings accounts within a given bank. This type of account is recommended if you do not intend to have a lot of transactions (withdrawing money), or if you don't need your money right away. For this reason, this account would be suitable for your budgeted monthly savings or for the money you set aside each month for your annual expenses. You can withdraw or transfer your money out of a savings account at any time in several ways:

- in person
- at a bank machine (ATM)
- direct payment (typically used online)

Chequing Accounts

Typically, chequing accounts have lower interest rates (if they pay interest at all) than savings accounts, but they also have lower service charges. (Service charges are fees you pay for certain transactions). This is why chequing accounts are used for day-to-day spending activities. You could use this type of account for money you budget for monthly expenses and bills. Like the savings accounts, you can take out money at any time using the same means as a savings account, but you can also write cheques.

Combination Accounts

If you do not want to have multiple bank accounts, you can always open a combination account. This account is part savings and part chequing, so it allows you to save as well as access your money on a day-to-day basis. Like a chequing account, you can take money out in person, at an ATM, through direct payment, or by writing a cheque. One difference between a combination account and either a savings or a chequing account is the amount of interest you earn. The majority of these accounts earn interest only if the amount of money in the account is over a certain amount. This means that you would have to save up to $500, for example, before you would earn any interest.

Electronic Savings Accounts

This account is a form of savings account that has become more popular in recent years, with the rise of online banking. Electronic savings (or eSavings) accounts typically earn a higher rate of interest than other types of savings accounts. You can only access them online, at an ATM, or in person. This means that you can withdraw or transfer money only in person, at an ATM, or by direct payment online. This type of account is ideal for your monthly 10 percent savings, as well as saving for some future purchase or activity.

Other accounts that are available include tax-free savings accounts, registered education savings plans, registered retirement savings plans, guaranteed income certificates, and many others. To learn more about these accounts, you can go online to a bank's website. Some popular banks' websites are listed in Learning Activity 4.4.

Service Charges

Service charges include any fees you pay for services provided by a bank or financial institution.

As you decide which type of account is most suitable, you should be well informed about any fees (service charges) that you will be charged. A list of service charges used by different banks is shown below:

- Monthly fee: This is paid so that you can have the account.
- Overdraft protection: You learned about this in Module 1. This fee is optional.
- Interac email money transfer fee: Online money transfer to someone via email.
- Online money transfer fee: Money transferred to someone else's account directly.
- Fee for debits exceeding the monthly limit: Some accounts only allow for a certain number of free debit transactions. Once you have surpassed this number, you are charged for every additional debit transaction you make that month.
- ABM transaction fee: If you withdraw money from an automated banking machine (ABM) that does not belong to your bank, you are charged a fee by the bank. These machines are in malls, movie theatres, airports, etc. You are also charged this fee if you withdraw money from a machine associated with a bank *other than your own bank*.
- Monthly statement/bank book fee: If you request monthly statements or update your bank book using the feature on the ABM, your bank may charge you a fee.

As you can see, even if an account does not have a monthly fee, you might still have to pay service charges for your account. In addition to the fees charged by your account, there are service charges that you must pay if you use different features on an ABM. These include the following:

- Full statements and mini-statements
- Withdrawals in the United States
- Additional statement requests

Because you never want to pay more than you have to, especially when it comes to banking, here are some tips to reduce the number of service charges that you have to pay:

- Instead of going to the bank and getting the teller to pay your bills, some banks offer a lower fee if you pay your bills at an ABM or online.
- Many financial institutions offer youth and student accounts that have lower/fewer service charges in order to entice students to join their institution.
- Before you open an account, consider how often you would use your debit card to make purchases. If you expect to use your debit card frequently, it would be wise to choose an account that offers a large number of free transactions or to choose an account with an unlimited number of transactions.
- Seniors sometimes also receive special accounts because they have been loyal to the company or because they live on their pension instead of a salary.
- Minimize the number of transactions you use (if you have a maximum number free). When you go to the grocery store, make sure you don't forget anything and plan ahead for the week. If you are having company over on Saturday, be sure you buy the ingredients for dinner with the rest of your groceries so that it counts as only one transaction instead of several.
- Use your bank's ABM whenever possible.
- Use online banking and receive online statements, because these are usually less expensive or free.

As you can see, there are many details to consider when opening a bank account. Some accounts will be more convenient than others. Also, selecting the account that is best for you may save you a lot of money. Most minors (under the age of 18) will choose to open savings accounts that are specifically for youths or students. Use the following learning activity to learn more about your personal banking.

Learning Activity 4.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. In the year 2020, Tom will be 16 years old. Tara will be half Tom's age, and Tammy will be half Tara's age. What year was Tammy born?

2. Evaluate the following: $10 + \frac{4}{(2 \times 3)} \times 12 - 50$

3. From your house, you walk 10 paces straight before turning left, and then you walk 100 paces. You then turn left and walk 25 paces. Finally, you turn left again and walk 100 paces. How far are you from where you started?

4. This past week (Monday to Friday) was crazy. Monday, you did not go to school because it was a snow day; Thursday was Remembrance Day, so you didn't go to school; and then on Friday the teachers had an in-service day. How many days did you go to school this week?

5. Write your answer from Question 4 as a fraction of the number of days in a week.

Part B: Your Bank Accounts

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Option A: Talk to Someone

If you have an account with a bank, credit union, or financial institution, discuss the type and characteristics of the account with your parent/guardian, learning partner, or make an appointment with someone at the financial institution where you have your account. The following checklist includes questions you should ask about your account.

continued

Questions:

1. What type of bank account do I have?
2. What is the interest rate for my account?
3. Do I have to pay any service charges to use this account?
4. Is this the right type of account for my financial needs? (Note: To answer this properly, you need to know some of your financial goals.)

Option B: Research Banks

If you do not have a bank account or cannot talk to anyone who is aware of your finances, answer the following questions for two different banks. Use the Internet or call a bank. If you are calling, be sure to explain that you are seeking information to complete a school assignment.

Possible websites to use (although you could use other financial institutions if you prefer):

- Royal Bank: www.rbcroyalbank.com/personal.html
- CIBC: www.cibc.com/ca/chequing-savings/index.html
- TD: www.tdcanadatrust.com/accounts/index.jsp

Click on "Compare Accounts" to get more information about the different accounts each bank offers.

Questions:

1. What standard bank accounts does the bank offer?
2. Do all accounts have service charges such as monthly fees or fees for withdrawing money at the ATM, for overdraft protection, or for online money transfers?
3. Does this bank offer any student accounts? If so, what benefit(s) are there to having a student account instead of a regular account?

Advantages and Disadvantages of Banking Services

You should include lists of advantages and disadvantages of online banking and debit transactions on your resource sheet.

Online Banking

Online banking has brought money management to your fingertips. You can check balances, pay bills, and transfer funds with the click of a mouse. Automatic transfers are much easier to set up as well, so paying bills is not always something you have to think about—as long as you have enough money to pay for them.

Overall, online banking has made banking more convenient. There are, however, some disadvantages as well.

Security has become a major concern with the rise of online banking. Previously, only you and your account manager knew your account information, but now, if you're not careful, others can gain access to your personal information.

Online banking has also led to online statements. Some people prefer to check their statements online because it is more convenient and it also helps to reduce paper use. Unfortunately, some people forget to check their online statements and then forget to pay their bills on time. This may create extra service charges or other costs they should not have to pay.

Finally, if you have an online bank account (such as the eSavings account), you cannot spend the money from this account at a store. You must transfer the money into a separate savings or chequing account. This being said, it is just one more reason to use the appropriate bank account. Most online bank accounts are savings accounts and are not intended to be used for daily transactions.

Debit Transactions

Using your debit card to pay for things is quick and convenient. Debit cards look like credit cards and allow you to pay without carrying cash. Another advantage of using a debit card instead of a credit card is that if your wallet or purse is stolen, it is a lot more difficult for the thief to use your debit card because it requires a PIN (personal identification number).

A major disadvantage of using debit transactions is that in some cases

- you are allowed a limited number of transactions before the bank charges a fee, or
- you have to pay a service charge for every transaction

Another disadvantage of using a debit card in public, even at the ABM, is that there are devices that thieves can install to steal your debit card number and your PIN. With this information, the thieves can gain access to all your accounts and withdraw any money you have in them. This is true for debit cards, but not for credit cards, because using a credit card is taking a loan, and using a debit card is withdrawing money from your account. Credit card companies usually forgive debt that is the result of theft, but banks do not return money lost to debit card theft.

Protecting Your Personal Information

Now that you have heard about all the insecurities of online banking and debit transactions, here are some steps you can take to prevent your identity and your money from being stolen.

- If you are using online banking, or are purchasing something online that requires your credit card number, be sure that the protocol of the URL (the first part of the website address) is **https://**. The s stands for secure, and indicates that the page is encrypted. If the website you are on does **not** start with https://, **do not** give out any personal information such as your credit card number or social insurance number. You can also look on your computer screen for other symbols that indicate the website's security for extra assurance that it is safe, such as a closed lock or a whole key.

- Change your password! Although it is easy to continue using the same password for a long period of time, it also makes your online account more vulnerable. Change your password every three months.
- Use a combination of letters (lower case and capitals) and numbers so that it is harder to guess the password.
- Don't tell anyone your password! You may be head-over-heels in love with your partner now, but you may not be a month from now. It's best to always keep your password private.
- Have a filter on your email. Be cautious of any junk mail you receive. Some junk mail may be part of a phishing scheme (an online attempt to get your user names, passwords, credit card numbers, or other personal information).

Lesson Summary

In this lesson, you studied various types of bank accounts. You also looked at some advantages of online banking, as well as some ways of avoiding the security problems associated with online banking. You should now be well informed about the decisions you need to make when opening a bank account.

The next two lessons will explore different components of having a bank account.

Assignment 4.2

Bank Accounts

Total: 18 marks

Note to Students: Although you did not add any formulas to your resource sheet in this lesson, there may still be information you need to include. As you work through this assignment, check back in the lesson and add information to your resource sheet as needed.

1. Match the bank account that would be best for each of the following people. *(3 marks)*

Shawna would like to buy a car in two years.	Combination account
Ivan uses his debit card every day to pay for lunch at school.	Savings account
Aleena is saving to go on a trip to India, but she has to pay for her shots, medicine, and appropriate clothing before she leaves.	Chequing account

2. State three avoidable service charges that you may face when you have a bank account. Explain how you could minimize or avoid them. *(6 marks)*

continued

Assignment 4.2: Bank Accounts (continued)

3. Write an argument against using online banking. Your answer should include at least three main points. *(3 marks)*

4. List three steps you can take to ensure that your personal and financial information is secure. Explain why each of these steps is necessary. *(6 marks)*

Lesson 4: Cheques and Deposit Slips

Lesson Focus

In this lesson, you will

- ❑ write a cheque
- ❑ complete a deposit slip

Lesson Introduction

In the previous lesson, you learned about different types of bank accounts, including online banking. In this lesson, you will look at transferring money to someone else by writing a cheque. A cheque is a convenient way to pay some bills or donate money in person or through the mail. You will also learn how to prepare a deposit slip, which may be required when you deposit money into your account. Most business deposits require a deposit slip.

Writing a Cheque

A cheque is a written request to your financial institution, usually on a form supplied by the bank, directing them to give a certain amount of your money to the person or organization named on the form.

The following is an example of a cheque.

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

Pay to the
order of _____ $ _____

_____ DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

_____

:26167...001: 1234...567...

When you write a cheque, you must

- fill in the date
- write the name of the person or organization receiving the money
- write in the amount of the cheque in numbers
- write the amount of the cheque in words (Mark the cents over the number 100 [e.g., 95/100]. If there are no cents, put two zeroes above the 100 [00/100].)
- draw a line after writing the amount in words so that no additional information can be added
- sign the cheque

You may want to include this list on your resource sheet.

Example 1

Write a cheque to Dominion Automotive for $274.95. Use the date March 8, 20xx.

Solution

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

March 8 20 xx

Pay to the order of Dominion Automotive $ 274.95

two-hundred seventy-four 95/100 DOLLARS

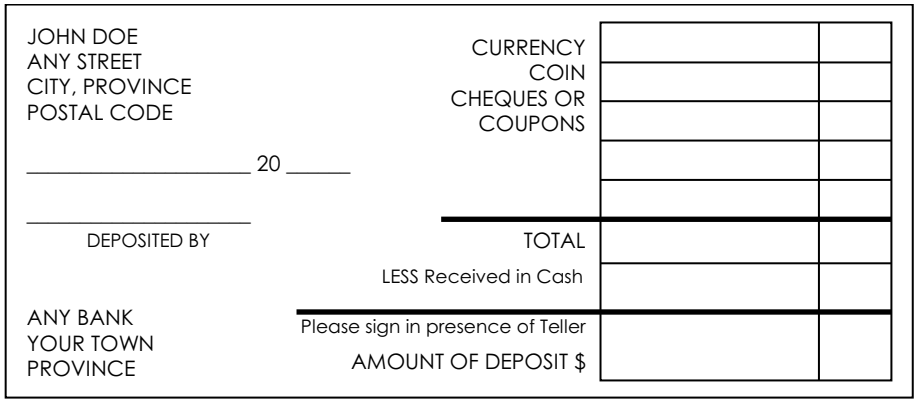
ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

Completing a Deposit Slip

Deposit slips are not commonly used for personal banking, but they are still used by companies. An example of a deposit slip is shown below.



JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

_____
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

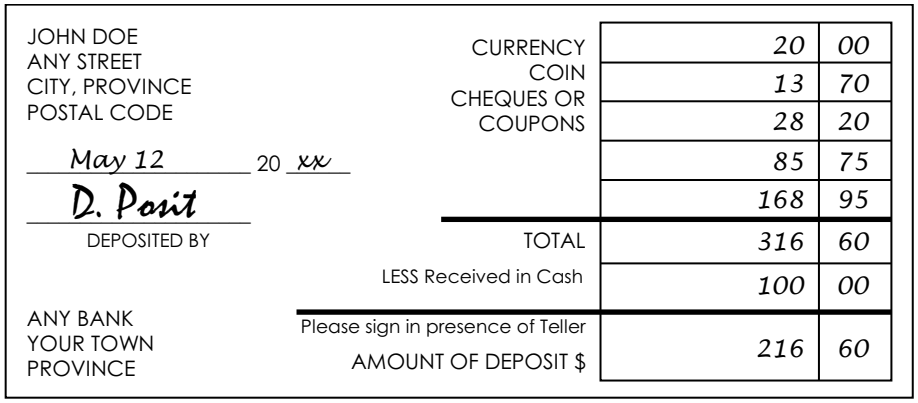
CURRENCY		
COIN		
CHEQUES OR COUPONS		
TOTAL		
LESS Received in Cash		
Please sign in presence of Teller AMOUNT OF DEPOSIT $		

Example 1

Complete a deposit slip using the following information. The date is May 12, 20xx. The amount deposited is 4 nickels, 6 quarters, 12 loonies, 2 ten-dollar bills, and 3 cheques for $28.20, $85.75, and $168.95. The amount of $100.00 is received back in cash.

Solution

- Find the total of the 2 ten-dollar bills and enter the total of $20.00 to the right of CURRENCY.
- Find the total value of 4 nickels, 6 quarters, and 12 loonies, and enter the total of $13.70 to the right of COIN.
- List each cheque separately to the right of CHEQUES OR COUPONS.
- Find the total: $316.60.
- Enter $100.00 in LESS Received in Cash, and subtract it from the total.
- Enter the difference of $216.60 to the right of AMOUNT OF DEPOSIT.



JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

May 12 20 xx

D. Posit
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY	20	00
COIN	13	70
CHEQUES OR COUPONS	28	20
	85	75
	168	95
TOTAL	316	60
LESS Received in Cash	100	00
Please sign in presence of Teller AMOUNT OF DEPOSIT $	216	60

Note that currency and cheques are listed separately, and each cheque is listed separately. Also, you write the amount of cash taken out as LESS Received in Cash. You should include these details on your resource sheet.

The amount of money you have in your account increases when you make a deposit and decreases when you write a cheque. You can calculate your new balance from your previous balance by adding the amount of your deposits and subtracting the amount of your cheques or cash you take out.

Example 2

Bianca Teller has a balance of $368.61 in her account. Last week, she made one deposit of $86.55 and wrote two cheques for $50 and $64.20. What is her new balance?

Solution

The new balance is $368.61 + $86.55 − ($50 + $64.20) = $340.96.

To be sure that you understand this lesson, complete the following learning activity, and then check your answers at the back of the module.

Learning Activity 4.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Divide 8 by 2.
2. Multiply your answer from question 1 by 3.
3. Divide your answer from question 2 by 4.
4. What is the pattern?
5. Write the next step in the pattern and solve it.

continued

Learning Activity 4.5 (continued)

Part B: Cheques and Deposit Slips

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Complete a deposit slip using the following information. The date is June 26, 20xx. The amount deposited is 5 quarters, 3 loonies, 1 five-dollar bill, 1 twenty-dollar bill, and 2 cheques for $46.83 and $125.95.

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

_____
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY		
COIN		
CHEQUES OR COUPONS		
TOTAL		
LESS Received in Cash		
Please sign in presence of Teller AMOUNT OF DEPOSIT $		

2. Complete a deposit slip using the following information. The date is Oct. 31, 20xx. The deposit is three cheques: $64.20, $136.50, and $29.28. There is $100.00 in cash taken out.

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

_____
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY		
COIN		
CHEQUES OR COUPONS		
TOTAL		
LESS Received in Cash		
Please sign in presence of Teller AMOUNT OF DEPOSIT $		

continued

Learning Activity 4.5 (continued)

3. Using the cheque blanks below, write cheques for the following:
   a) February 4, 20xx to the J & M Department Store for $48.72

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

Pay to the
order of _____ $ _____

_____ DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

_____

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   b) August 27, 20xx to State-of-the-Art Electronics for $158.95

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

Pay to the
order of _____ $ _____

_____ DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

_____

:26167...001: 1234...567...

4. There are errors in the way the following cheques are written. They are written over a two-week period. What are the errors? There may be more than one error on a cheque.
   a)

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

December 26 20 10

Pay to the
order of A. Baron $ 12

twelve DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

continued

Learning Activity 4.5 (continued)

b)

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

January 2 20 *11*

Pay to the order of *C. Baron* $ *34.10*

thirty-five *10/100* DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

:26167...001: 1234...567...

5. Ima Lender has $5396.52 in her chequing account. She makes three deposits of $382.00, $545.00, and $250.00. She pays the following bills by cheque: Hydro, $35.86; Telephone, $25.52; and Visa $86.52. Find her new balance.

Lesson Summary

In this lesson, you learned how to write cheques and complete deposit slips. You also calculated the current balance when you knew the original balance, how much was spent, and how much was deposited. In the next lesson, you will learn more about financial record keeping.

Notes

Lesson 5: Cheque Registers

Lesson Focus

In this lesson, you will

- ❑ complete a cheque register
- ❑ correct errors in a cheque register
- ❑ complete a statement of reconciliation

Lesson Introduction

Cheque registers are used to keep track of chequing account transactions. In this lesson, you will complete cheque registers, and then reconcile the register with the bank statement. You will also learn how to find and correct errors in a cheque register.

If you have a chequing account, maintaining a proper record of your transactions is important because you should always know your account balance. Overdrawing your account can be costly and can also reduce your credit rating.

Completing a Cheque Register

The following is an example of a cheque register:

Cheque Register										
DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

When you write a cheque, you should fill in the columns for

- date
- cheque number
- cheque issued to (and why)
- cheque amount
- the new balance

If you deposit money, you should fill in the columns for

- date
- description of deposit
- deposit amount
- the new balance

You may want to include a diagram that labels which columns are required for cheques and deposits on your resource sheet.

Example 1

Complete the following cheque register.

Balance: April 17, $172.68

Cheque No. 136: April 19 to Kathy's Clothing for $42.06

137: April 21 to Wholesome Foods for $63.50

138: April 25 to A & R Rental Agency for $450.00

139: April 30 to Manitoba Electric Co. for $38.12

Deposits: April 23, $650.00; May 04, $45.52

Cash withdrawn: April 27, $200

Solution

Transactions are entered in the order in which they occur. The balance in the account is calculated after every transaction.

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									172	68
Apr 19	136	TO Kathy's Clothing	42	06				CHEQUE – DEPOSIT +	42	06
		FOR						BALANCE ➜	130	62
21	137	TO Wholesale Foods	63	50				CHEQUE – DEPOSIT +	63	50
		FOR						BALANCE ➜	67	12
23		TO Deposit				650	00	CHEQUE – DEPOSIT +	650	00
		FOR						BALANCE ➜	717	12
25	138	TO A & R Rental Agency	450	00				CHEQUE – DEPOSIT +	450	00
		FOR						BALANCE ➜	267	12
27		TO Cash	200	00				CHEQUE – DEPOSIT +	200	00
		FOR						BALANCE ➜	67	12
30	139	TO Manitoba Electric Co.	38	12				CHEQUE – DEPOSIT +	38	12
		FOR						BALANCE ➜	29	00
May 4		TO Deposit				45	52	CHEQUE – DEPOSIT +	45	52
		FOR						BALANCE ➜	74	52
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

It is important to keep the cheque register up to date so that you can monitor your account(s). It is important to avoid overdrawing your account. This means that you should have enough money in your account to pay for any cheque you write. If the amount on the cheque is greater than the amount in the account, the cheque may bounce. If this happens, the cheque is returned to you, and the person receiving the cheque does not receive any money. You must then find a way to pay the amount owing. The bank usually charges a sizable fee when you overdraw the account.

Correcting a Cheque Register

But why should you fill out your cheque register? Each month your financial institution should send you an account statement (either by mail or through your online banking account). When you receive this statement, you can check your register to find out whether it has any errors and/or omissions. It is important that all your entries in your cheque register are correct.

Example 1

There are three errors and/or omissions in the cheque register. Compare the cheque register to the given monthly account statement. Make the necessary changes to correct the cheque register.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
May 1	Balance Forward			825.43
May 4	Deposit		85.00	910.40
	Cheque 237	100.00		810.40
May 10	Cheque 238	139.09		671.31
May 9	Deposit		450.00	1121.31
May 12	Cheque 240	217.87		903.44
May 15	Cheque 242	550.00		453.47

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									825	43
5/4		TO Deposit				85	00	CHEQUE – DEPOSIT +	85	00
		FOR						BALANCE ➜	910	43
	237	TO Cash	100	00				CHEQUE – DEPOSIT +	100	00
		FOR						BALANCE ➜	810	43
5/8	238	TO J & M Department Store	139	90				CHEQUE – DEPOSIT +	139	90
		FOR						BALANCE ➜	670	53
	239	TO Manitoba Charitable Association	75	00				CHEQUE – DEPOSIT +	75	00
		FOR						BALANCE ➜	595	53
5/15		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	995	53
5/21	241	TO L. Biaks	25	00				CHEQUE – DEPOSIT +	25	00
		FOR						BALANCE ➜	970	53
5/23	242	TO Rent	550	00				CHEQUE – DEPOSIT +	550	00
		FOR						BALANCE ➜	420	53
5/30	243	TO Groceries	48	93				CHEQUE – DEPOSIT +	48	93
		FOR						BALANCE ➜	371	60
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

Solution

Put a checkmark beside each entry in the cheque register that appears on the monthly statement. As you do this, check the entry in the cheque register against the monthly account statement for errors and/or omissions. Did you find the following?

- The cheque to J & M Department Store on May 8 was incorrectly entered as $139.90 rather than $139.09.
- The deposit on May 15 was incorrectly entered as $400 rather than $450.
- Cheque 240 was omitted in the cheque register.

The following three entries will correct the cheque register:

- Since the cheque to J & M Department Store on May 8 was incorrectly entered as $139.90 rather than $139.09, the balance is $0.81 less than it should be. Add the amount of $0.81 in the deposit column.
- The deposit on May 15 was incorrectly entered as $400 rather than $450. Therefore, the balance is $50 less than it should be. Add the amount of $50 in the deposit column.
- Since cheque 240 was omitted on the cheque register, the balance is $217.87 greater than it should be. Add the amount $217.87 to the cheque column.

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									825	43
5/4		TO Deposit				85	00	CHEQUE – DEPOSIT +	85	00
		FOR						BALANCE ➜	910	43
	237	TO Cash	100	00				CHEQUE – DEPOSIT +	100	00
		FOR						BALANCE ➜	810	43
5/8	238	TO J & M Department Store	139	90				CHEQUE – DEPOSIT +	139	90
		FOR						BALANCE ➜	670	53
	239	TO Manitoba Charitable Association	75	00				CHEQUE – DEPOSIT +	75	00
		FOR						BALANCE ➜	595	53
5/15		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	995	53
5/21	241	TO L. Biaks	25	00				CHEQUE – DEPOSIT +	25	00
		FOR						BALANCE ➜	970	53
5/23	242	TO Rent	550	00				CHEQUE – DEPOSIT +	550	00
		FOR						BALANCE ➜	420	53
5/30	243	TO Groceries	48	93				CHEQUE – DEPOSIT +	48	93
		FOR						BALANCE ➜	371	60
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

correction: date 5/8 cheque should be 139.09					81		372	81 41
correction: date 5/9 deposit should be 450				50	00		50 422	00 41
correction: cheque #240 omitted	217	87					217 204	87 54

You may want to add a note about cheque register corrections on your resource sheet.

Did you notice that some cheques in the cheque register were not included in the bank statement? This will happen if the cheque has not been deposited before the statement is issued. We do not adjust the register for these missing cheques because they will most likely appear on next month's statement.

Statements of Reconciliation

The statement you receive from the bank includes more than just withdrawals and deposits. It also includes service charges. Also, some of the entries in your cheque register may not appear in your bank statement. Entries that have not been processed by the bank are called **outstanding**. For these reasons, the final balances of the cheque register and the bank statement will normally not be the same.

To determine that the entries on your monthly account statement and your cheque register correspond, you need to complete a **Statement of Reconciliation**. This form enables you to determine that the entries of the two forms agree. There are many types of reconciliation forms, and one example is shown below.

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
TOTAL:	TOTAL:

Follow the steps to complete a reconciliation form:

The **left side** of the reconciliation form involves the **monthly account statement**.

1. Enter the final balance shown on the monthly account statement.
2. Add any deposits made since the statement date.
3. Find the total of the above.

The **right side** of the reconciliation form involves the **cheque register**.

1. Subtract the service charge to find the final balance shown in the cheque register.
2. Add all the cheques that do not yet appear on the statement.
3. Find the total of the above.

The two totals should balance.

You should include these steps on your resource sheet.

One of the purposes of completing a statement of reconciliation is to find any errors and/or omissions you might have made in your cheque register. You should correct these errors and/or omissions, and then complete your statement of reconciliation.

All of the questions in this lesson will refer to cheque registers that are correct or have been corrected.

Example 1

Reconcile the following monthly account statement and cheque register:

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Mar. 1	Balance Forward			238.22
	Deposit		250.00	488.22
Mar. 7	Cheque 87	29.00		
	Cheque 86	47.63		411.59
Mar. 9	Cheque 88	122.48		289.11
Mar. 12	Cheque 89	83.17		205.94
Mar. 15	Deposit		500.00	705.94
Mar. 26	Cheque 91	85.50		620.44
Mar. 31	Service Charge	5.00		615.44

Cheque Register										
DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									238	22
Mar 1		TO Deposit				250	00	CHEQUE – DEPOSIT +	250	00
		FOR						BALANCE ➜	488	22
5	86	TO Manitoba Electric Co.	47	63				CHEQUE – DEPOSIT +	47	63
		FOR						BALANCE ➜	440	59
5	87	TO Manitoba Telephone Co.	29	00				CHEQUE – DEPOSIT +	29	00
		FOR						BALANCE ➜	411	59
7	88	TO Kathy's Clothing	122	48				CHEQUE – DEPOSIT +	122	48
		FOR						BALANCE ➜	289	11
11	89	TO Zack's Electronics	83	17				CHEQUE – DEPOSIT +	83	17
		FOR						BALANCE ➜	205	94
15		TO Deposit				500	00	CHEQUE – DEPOSIT +	500	00
		FOR						BALANCE ➜	705	94
18	90	TO Dentist	94	25				CHEQUE – DEPOSIT +	94	25
		FOR						BALANCE ➜	611	69
20	91	TO O + K Plumbing	85	50				CHEQUE – DEPOSIT +	85	50
		FOR						BALANCE ➜	526	19
23	92	TO Cash	150	00				CHEQUE – DEPOSIT +	150	00
		FOR						BALANCE ➜	376	19
31		TO Deposit				375	00	CHEQUE – DEPOSIT +	375	00
		FOR						BALANCE ➜	751	19

Solution

Put a checkmark beside each entry in the cheque register that appears on the monthly account statement.

Follow the directions on the reconciliation form to complete it.

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
615.44	751.19 - 5.00 = 746.19
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
375.00	94.25
	150.00
TOTAL: 990.44	TOTAL: 990.44

Add the entry for the service charge to the cheque register.

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									238	22
Mar 1		TO Deposit			✓	250	00	CHEQUE – DEPOSIT +	250	00
		FOR						BALANCE ➜	488	22
5	86	TO Manitoba Electric Co.	47	63	✓			CHEQUE – DEPOSIT +	47	63
		FOR						BALANCE ➜	440	59
5	87	TO Manitoba Telephone Co.	29	00	✓			CHEQUE – DEPOSIT +	29	00
		FOR						BALANCE ➜	411	59
7	88	TO Kathy's Clothing	122	48	✓			CHEQUE – DEPOSIT +	122	48
		FOR						BALANCE ➜	289	11
11	89	TO Zack's Electronics	83	17	✓			CHEQUE – DEPOSIT +	83	17
		FOR						BALANCE ➜	205	94
15		TO Deposit			✓	500	00	CHEQUE – DEPOSIT +	500	00
		FOR						BALANCE ➜	705	94
18	90	TO Dentist	94	25				CHEQUE – DEPOSIT +	94	25
		FOR						BALANCE ➜	611	69
20	91	TO O + K Plumbing	85	50	✓			CHEQUE – DEPOSIT +	85	50
		FOR						BALANCE ➜	526	19
Apr 2	92	TO Cash	150	00				CHEQUE – DEPOSIT +	150	00
		FOR						BALANCE ➜	376	19
4		TO Deposit				375	00	CHEQUE – DEPOSIT +	375	00
		FOR						BALANCE ➜	751	19
service charge			5	00					746	19

The two totals are equal.

To be sure that you understand this lesson, complete the following learning activity. Then check your answers at the back of the module.

Learning Activity 4.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the following pattern: 5, −5, −15, _____ , _____

2. Write two equivalent fractions for $\frac{24}{56}$.

3. Dinoo spends $45 on winter boots, $65 on a parka, $15 on a scarf, $10 on mittens, and $8 on a toque. If Dinoo had budgeted $120 for winter clothing, does she have a deficit?

4. Muriel picks the petals off of three flowers one by one, and says the phrase "he loves me" as she picks the first petal, and then "he loves me not" as she picks the second petal, and so on. If each flower has seven petals and she says "he loves me" first, which phrase will she finish on?

5. You need 2 yd.^{2} of fabric to make a teddy bear. If you were to make five teddy bears, how many square yards of fabric would you need?

continued

Learning Activity 4.6 (continued)

Part B: Cheque Registers

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Complete the following cheque register:

   Balance: Feb. 5, $837.92

   Cheque No. 162: Feb. 10 to Manitoba Electric Co. for $58.74

   163: Feb. 10 to Manitoba Telephone Co. for $38.52

   164: Feb. 14 to Visa for $194.71

   165: Feb. 18 to Kathy's Clothing for $89.66

   Deposits: Feb. 17, $185.92; Feb. 20, $300

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT	✓	DEPOSIT AMOUNT	DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➔	

continued

2. Complete the following cheque register:

   Balance: Oct. 25, $78.72

   Cheque No. 72: Oct. 26 to Wayne's Gas Bar for $67.25

   73: Oct. 31 to C & B Rental Agencies for $398.88

   74: Nov. 04 to Baron's Department Store for $12.75

   75: Nov. 07 to the Medical Ethics Association for $75.00

   Deposits: Oct. 28, $402.96; Nov. 06, $90.82; Nov. 10, $153.72

Cheque Register										
DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

continued

Learning Activity 4.6 (continued)

3. Compare the following monthly account statement with its cheque register. There are two errors and/or omissions in the cheque register. Locate the errors and/or omissions, and make the necessary changes in the cheque register.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Jul. 1	Balance Forward			1522.71
Jul. 4	Cheque 90	231.80		1290.91
Jul. 6	Cheque 92	25.00		1265.91
Jul. 9	Deposit		484.24	1750.15
Jul. 26	Cheque 94	36.12		
	Cheque 97	104.86		1609.17
Jul. 30	Cheque 96	42.73		1566.44
Jul. 31	Cheque 95	15.00		1551.44

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1522	71
Jul 2	90	TO Econo-Airlines	231	80				CHEQUE – DEPOSIT +	231	80
		FOR						BALANCE ➜	1290	91
3	91	TO E. Cadlof	25	00				CHEQUE – DEPOSIT +	25	00
		FOR						BALANCE ➜	1265	91
5	92	TO P. Anders	25	00				CHEQUE – DEPOSIT +	25	00
		FOR						BALANCE ➜	1240	91
9		TO Deposit				484	24	CHEQUE – DEPOSIT +	484	24
		FOR						BALANCE ➜	1725	15
12	93	TO Music Lesson	28	00				CHEQUE – DEPOSIT +	28	00
		FOR						BALANCE ➜	1697	15
24	94	TO J + M Department Store	36	12				CHEQUE – DEPOSIT +	36	12
		FOR						BALANCE ➜	1661	03
24	96	TO Outdoor Supplies	42	73				CHEQUE – DEPOSIT +	42	73
		FOR						BALANCE ➜	1618	30
25	97	TO Value Foods	104	68				CHEQUE – DEPOSIT +	104	68
		FOR						BALANCE ➜	1513	62
Aug 3		TO Deposit				482	53	CHEQUE – DEPOSIT +	482	53
		FOR						BALANCE ➜	1996	15
4	98	TO Bob's 24-Hour Shopping	88	23				CHEQUE – DEPOSIT +	88	23
		FOR						BALANCE ➜	1907	92

continued

Learning Activity 4.6 (continued)

4. Compare the following monthly account statement with its cheque register. There are three errors and/or omissions in the cheque register. Locate the errors and/or omissions, and make the necessary changes in the cheque register.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Apr. 1	Balance Forward			1127.18
Apr. 3	Cheque 201	14.25		1112.93
Apr. 10	Deposit		200.00	1312.93
Apr. 16	Deposit		350.00	1662.93
Apr. 19	Cheque 204	428.00		1234.93
	Cheque 202	67.29		1167.64
Apr. 24	Deposit		250.00	1417.64
	Cheque 203	25.46		1392.18
Apr. 29	Deposit		75.00	1467.18

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1127	18
4/1	201	TO	14	25				CHEQUE – DEPOSIT +	14	25
		FOR						BALANCE ➜	1112	93
4/10		TO Deposit				200	00	CHEQUE – DEPOSIT +	200	00
		FOR						BALANCE ➜	1212	93
4/13	202	TO Computer Software	67	32				CHEQUE – DEPOSIT +	67	32
		FOR						BALANCE ➜	1245	71
4/16	203	TO Cleaning Supplies	25	46				CHEQUE – DEPOSIT +	25	46
		FOR						BALANCE ➜	1220	25
4/18	204	TO Rent	428	00				CHEQUE – DEPOSIT +	428	00
		FOR						BALANCE ➜	792	25
4/18	205	TO Groceries	113	82				CHEQUE – DEPOSIT +	113	82
		FOR						BALANCE ➜	678	43
4/18	207	TO Gift	50	00				CHEQUE – DEPOSIT +	50	00
		FOR						BALANCE ➜	628	43
4/24		TO Deposit				250	00	CHEQUE – DEPOSIT +	250	00
		FOR						BALANCE ➜	878	43
4/30	208	TO M. Holt	75	00				CHEQUE – DEPOSIT +	75	00
		FOR						BALANCE ➜	803	43
5/1		TO Deposit				200	00	CHEQUE – DEPOSIT +	200	00
		FOR						BALANCE ➜	1003	43

continued

Learning Activity 4.6 (continued)

5. Reconcile the monthly account statement to the cheque register. There are reconciliation tables at the end of the learning activity that you can use for this question and the next.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Sept. 1	Balance Forward			212.85
Sept. 3	Deposit		645.15	858.00
Sept. 8	Cheque 256	435.00		423.00
Sept. 11	Cheque 257	79.63		343.37
Sept. 16	Deposit		648.23	991.60
	Cheque 260	29.04		
	Cheque 259	36.94		925.62
Sept. 17	Cheque 258	23.16		902.46
Sept. 20	Cheque 261	94.20		808.26
Sept. 26	Cheque 263	127.89		680.37
Sept. 29	Cheque 264	61.16		619.21
Sept. 30	Deposit		647.21	1266.42
	Service Charge	5.00		1261.42

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									212	85
9/3		TO Deposit				645	15	CHEQUE – DEPOSIT +	645	15
		FOR						BALANCE →	858	00
9/6	256	TO Rent	435	00				CHEQUE – DEPOSIT +	435	00
		FOR						BALANCE →	423	00
9/10	257	TO Groceries	79	63				CHEQUE – DEPOSIT +	79	63
		FOR						BALANCE →	343	37
9/14	258	TO Manitoba Gas Co.	23	16				CHEQUE – DEPOSIT +	23	16
		FOR						BALANCE →	320	21
9/14	259	TO Manitoba Electric Co.	36	94				CHEQUE – DEPOSIT +	36	94
		FOR						BALANCE →	283	27
9/14	260	TO Manitoba Telephone Co.	29	04				CHEQUE – DEPOSIT +	29	04
		FOR						BALANCE →	254	23
9/16		TO Deposit				648	23	CHEQUE – DEPOSIT +	648	23
		FOR						BALANCE →	902	40
9/18	261	TO Baron's Dept. Store	94	20				CHEQUE – DEPOSIT +	94	20
		FOR						BALANCE →	808	26
9/22	262	TO Music Lessons	56	00				CHEQUE – DEPOSIT +	56	00
		FOR						BALANCE →	752	26
9/24	263	TO Roller Blades	127	89				CHEQUE – DEPOSIT +	127	89
		FOR						BALANCE →	624	37
9/27	264	TO Groceries	61	16				CHEQUE – DEPOSIT +	61	16
		FOR						BALANCE →	563	21
9/30		TO Deposit				647	21	CHEQUE – DEPOSIT +	647	21
		FOR						BALANCE →	1210	42
10/1	265	TO Magazine Subscription	45	98				CHEQUE – DEPOSIT +	45	98
		FOR						BALANCE →	1164	44
10/3		TO Deposit				50	00	CHEQUE – DEPOSIT +	50	00
		FOR						BALANCE →	1214	44

continued

Learning Activity 4.6 (continued)

6. Reconcile the monthly account statement to the cheque register.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
June 1	Balance Forward			1027.86
June 2	Cheque 125	100.00		927.86
June 11	Cheque 126	217.36		710.50
June 15	Deposit		500.00	1210.50
June 19	Cheque 129	194.18		
	Cheque 127	625.96		390.36
June 20	Cheque 130	26.32		364.04
	Cheque 128	73.26		290.78
June 26	Cheque 132	44.85		245.93
June 30	Cheque 133	128.43		117.50
	Service Charge	3.00		114.50

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1027	86
June 02	125	TO Cash	100	00				CHEQUE – DEPOSIT +	100	00
		FOR						BALANCE ➜	927	86
09	126	TO Mastercard	217	36				CHEQUE – DEPOSIT +	217	36
		FOR						BALANCE ➜	710	50
15		TO Deposit				500	00	CHEQUE – DEPOSIT +	500	00
		FOR						BALANCE ➜	1210	50
17	127	TO W + A Travel	625	96				CHEQUE – DEPOSIT +	625	96
		FOR						BALANCE ➜	584	54
17	128	TO Dentist	73	26				CHEQUE – DEPOSIT +	73	26
		FOR						BALANCE ➜	511	28
17	129	TO J + M Department Store	194	18				CHEQUE – DEPOSIT +	194	18
		FOR						BALANCE ➜	317	10
19	130	TO Generic Pharmacy	26	32				CHEQUE – DEPOSIT +	26	32
		FOR						BALANCE ➜	290	78
24	131	TO C. Baron	19	21				CHEQUE – DEPOSIT +	19	21
		FOR						BALANCE ➜	271	57
24	132	TO Winnipeg News	44	85				CHEQUE – DEPOSIT +	44	85
		FOR						BALANCE ➜	226	72
27	133	TO Wholesome Foods	128	43				CHEQUE – DEPOSIT +	128	43
		FOR						BALANCE ➜	98	29
30		TO Deposit				500	00	CHEQUE – DEPOSIT +	500	00
		FOR						BALANCE ➜	598	29
30	134	TO Manitoba Electric Co.	92	84				CHEQUE – DEPOSIT +	92	84
		FOR						BALANCE ➜	505	45
30	135	TO Rent	475	00				CHEQUE – DEPOSIT +	475	00
		FOR						BALANCE ➜	30	45
July 02		TO Deposit				136	00	CHEQUE – DEPOSIT +	136	00
		FOR						BALANCE ➜	166	45

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
TOTAL:	TOTAL:

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
TOTAL:	TOTAL:

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
TOTAL:	TOTAL:

Lesson Summary

In this lesson, you recorded chequing account transactions in a cheque register. You also reconciled the cheque register with the bank statement to ensure that there were no errors in the cheque register.

Be sure to complete the assignment at the end of this lesson before going on to Module 5.

Assignment 4.3

Cheques, Deposit Slips, and Registers

Total: 19 marks

Note to Students: Although you did not add any formulas to your resource sheet in this lesson, there may still be information you need to include. As you work through this assignment, check back in the lesson and add information to your resource sheet as needed.

1. Complete a deposit slip using the following information. The date is Jan. 03, 20xx. The amount deposited is 2 dimes, 18 loonies, 4 twenty-dollar bills, and a cheque for $64.20. There is $100.00 in cash taken out. *(3 marks)*

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

_____
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY		
COIN		
CHEQUES OR COUPONS		
TOTAL		
LESS Received in Cash		
Please sign in presence of Teller AMOUNT OF DEPOSIT $		

2. Using the blank cheque below, write a cheque for April 30, 20xx, to the Minister of Finance for $1573.60. *(2 marks)*

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

Pay to the
order of _____ $ _____

_____ DOLLARS

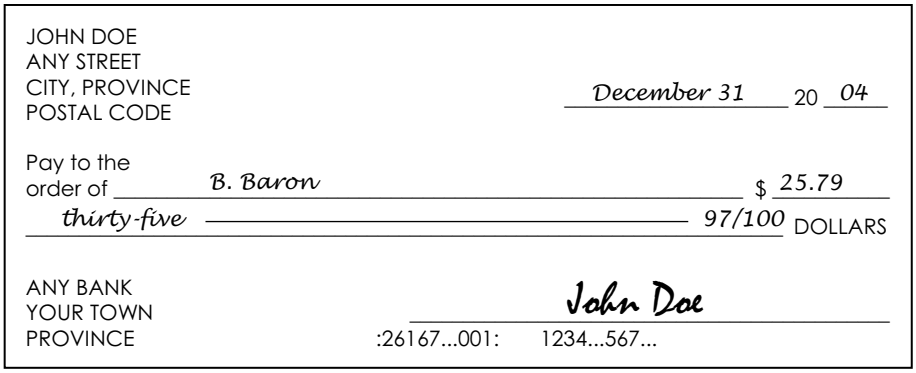
ANY BANK
YOUR TOWN
PROVINCE

_____
:26167...001: 1234...567...

continued

Assignment 4.3: Cheques, Deposit Slips, and Registers (continued)

3. There is at least one error in the way the following cheque is written. Identify the error(s). *(1 mark)*


JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

December 31 20 04

Pay to the order of B. Baron $ 25.79

thirty-five 97/100 DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

4. Ima Spender's bank balance is $62.96. She makes one deposit of $100.00, and writes cheques of $10.95, $101.80, and $50.00. Find her new bank balance. *(2 marks)*

continued

Assignment 4.3: Cheques, Deposit Slips, and Registers (continued)

5. Complete the following cheque register: *(5 marks)*

   Balance: Dec. 01, $426.97

   Cheque No. 208: Dec. 04 to the J & M Department Store for $67.25

   209: Dec. 07 to Jane's Computer Store for $398.88

   210: Dec. 10 to Wayne's Gas Bar for $32.75

   211: Dec. 14 to Value Foods for $86.12

   Deposits: Dec. 02, $49.63; Dec. 06, $250.00; Dec. 11, $48.50

   Cash withdrawn: Dec. 15, $100.

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

continued

Assignment 4.3: Cheques, Deposit Slips, and Registers (continued)

6. Compare the following bank statement with its cheque register. Locate the errors and/or omissions, and make the necessary changes in the cheque register. *(2 marks)*

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Nov. 1	Balance Forward			1156.12
Nov. 3	Deposit		396.24	1552.36
Nov. 7	Cheque 272	200.00		1332.36
Nov. 9	Cheque 273	194.27		
	Cheque 275	36.80		1121.29
Nov. 13	Deposit		125.00	1246.29
Nov. 23	Cheque 278	95.50		1150.79
Nov. 24	Cheque 276	24.12		
Nov. 28	Cheque 279	132.68		993.99

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1156	12
11/3		TO Deposit				396	24	CHEQUE – DEPOSIT +	396	24
		FOR						BALANCE ➜	1552	36
11/5	272	TO Visa	200	00				CHEQUE – DEPOSIT +	200	00
		FOR						BALANCE ➜	1352	36
11/7	273	TO Jeans and Clothing Store	194	72				CHEQUE – DEPOSIT +	194	72
		FOR						BALANCE ➜	1157	64
11/7	274	TO B. White	75	00				CHEQUE – DEPOSIT +	75	00
		FOR						BALANCE ➜	1082	64
11/8	275	TO Canadian Lumber	36	80				CHEQUE – DEPOSIT +	36	80
		FOR						BALANCE ➜	1045	84
11/13		TO Deposit				125	00	CHEQUE – DEPOSIT +	125	00
		FOR						BALANCE ➜	1170	84
11/22	276	TO Fresh Foods	24	12				CHEQUE – DEPOSIT +	24	12
		FOR						BALANCE ➜	1146	72
11/22	277	TO College Bookstore	87	98				CHEQUE – DEPOSIT +	87	98
		FOR						BALANCE ➜	1058	74
11/22	278	TO Baron's Depart. Store	95	00				CHEQUE – DEPOSIT +	95	00
		FOR						BALANCE ➜	963	74
11/26	279	TO Theatre	132	68				CHEQUE – DEPOSIT +	132	68
		FOR						BALANCE ➜	831	06

continued

Assignment 4.3: Cheques, Deposit Slips, and Registers (continued)

7. Reconcile the monthly account statement to the cheque register. Use the reconciliation forms found at the end of this assignment. *(4 marks)*

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Nov. 1	Balance Forward			1567.24
Nov. 3	Deposit		48.26	1615.50
Nov. 10	Cheque 147	348.78		1266.72
Nov. 16	Deposit		728.35	1995.07
Nov. 20	Cheque 148	43.60		1951.47
Nov. 23	Deposit		149.90	
	Cheque 149	1252.95		848.42
Nov. 30	Cheque 151	54.55		
	Cheque 150	19.21		
	Cheque 152	42.78		731.88
Nov. 30	Service Charge	5.50		726.38

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1567	24
Nov. 2		TO Deposit				48	26	CHEQUE – DEPOSIT +	48	26
		FOR						BALANCE ➜	1615	50
9	147	TO Econo-Airlines	348	78				CHEQUE – DEPOSIT +	348	78
		FOR						BALANCE ➜	1266	72
16		TO Deposit				728	35	CHEQUE – DEPOSIT +	728	35
		FOR						BALANCE ➜	1945	07
18	148	TO For-Life Health Insurance	43	80				CHEQUE – DEPOSIT +	43	80
		FOR						BALANCE ➜	1951	47
20	149	TO Computer Upgrade	1252	95				CHEQUE – DEPOSIT +	1252	95
		FOR						BALANCE ➜	698	52
23		TO Deposit				149	90	CHEQUE – DEPOSIT +	149	90
		FOR						BALANCE ➜	848	42
28	150	TO World Wide Cable	19	21				CHEQUE – DEPOSIT +	19	21
		FOR						BALANCE ➜	829	21
28	151	TO Manitoba Gas Co.	54	55				CHEQUE – DEPOSIT +	54	55
		FOR						BALANCE ➜	774	66
28	152	TO Manitoba Electric Co.	42	78				CHEQUE – DEPOSIT +	42	78
		FOR						BALANCE ➜	731	88
28	153	TO Manitoba Telephone Co.	37	32				CHEQUE – DEPOSIT +	37	32
		FOR						BALANCE ➜	694	56
Dec. 1		TO Deposit				724	32	CHEQUE – DEPOSIT +	724	32
		FOR						BALANCE ➜	1418	88
3	154	TO Car Repairs	385	46				CHEQUE – DEPOSIT +	385	46
		FOR						BALANCE ➜	1033	42

Assignment 4.3: Cheques, Deposit Slips, and Registers (continued)

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
TOTAL:	TOTAL:

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
TOTAL:	TOTAL:

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
TOTAL:	TOTAL:

Module 4 Summary

Congratulations! You have completed more than half of this course.

In this module, you extended your knowledge of personal finance by learning how to budget and use budgets to assess your financial goals and plan for the future. You learned about the various options you have when opening a bank account, and which types of accounts are best, depending on spending habits and financial goals. Finally, you learned about some of the paperwork that you must do once you have a bank account.

Before you move on to the next module, if there were any topics that you did not understand, call your tutor/marker or ask your learning partner to help you. Don't forget to check Appendix B: Glossary. If you do not deal with it now, you may forget to do so before your final examination!

Submitting Your Assignments

It is now time for you to submit the Module 4 Cover Assignment and Assignments 4.1 to 4.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 4 assignments and organize your material in the following order:

- ❑ Module 4 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 4 Cover Assignment: How Time Affects the Value of Money
- ❑ Assignment 4.1: Budgets
- ❑ Assignment 4.2: Bank Accounts
- ❑ Assignment 4.3: Cheques, Deposit Slips, and Registers

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Grade 11 Essential Mathematics (30S)

Module 4
Managing Money

Learning Activity Answer Keys

Module 4:
Managing Money

Learning Activity 4.1

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Jonghun has $50 to spend on groceries. He needs milk ($5.00), vegetables ($15.00), fruit ($15.00), and some sort of protein. Chicken costs $12.50, pork costs $15.50, and steak costs $20. Which type(s) of meat could he afford?

2. Rewrite the fraction in lowest terms: $\frac{750}{500}$.

3. On the Canadian flag, the maple leaf has three main segments. Each segment has three points on it. How many points are there in total?

4. When comparing team statistics, you can look at a team's record (how many games it has won, lost, or tied) as well as the goals for (the number of goals the team has scored) and the goals against (the number of goals other teams have scored against it). If a team has not won a game, is it possible for it to have more goals for than against?

5. Solve for g: $5g - 10 = -30$

Answers:

1. Chicken (He's already spent 5 + 15 + 15 = $35, so he has only $15 more to buy protein.)

2. $\frac{3}{2}$ (750 ÷ 250 = 3 and 500 ÷ 250 = 2)

3. 9 points (3 × 3 = 9)

4. This is impossible because the only time you get more goals for than against is when you win. If you don't win, you have never scored more goals than the opposing team.

5. −4 (−30 + 10 = −20; −20 ÷ 5 = −4)

Part B: Creating a Budget

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. a) Prepare a budget for Kirk Donner for the month of October. Use one of the budget forms provided at the end of the lesson, or create a worksheet of your own.

   Kirk is a university student who lives at home with his parents. During the past year, he worked for a lawn service in the summer months and earned a net income of $4800. He also works part-time during the year, and earns a net income of $90 per week. Kirk was awarded a scholarship of $1000.

   Kirk's expenses for the month of October will include the following:

   - payment of $150.00 to his parents for board and room
   - car payment is $105.80
   - gasoline expense is $65.00
   - clothing is $140.00
   - personal care expense is $25.00
   - entertainment expense is $100.00
   - gifts are $35.00
   - CD expense is $20.00

   Kirk's annual expenses will include the following:

   - university tuition is $3200
   - books and supplies are $550
   - Autopac payment is $470.50
   - vacation is $750
   - magazine subscription is $28 a year
   - Christmas gifts are $240.70

Answer:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	AVERAGE MONTHLY INCOME
1) **NET INCOME**			1) $ 873.33
Primary Income	$	$ 4,800.00	
Secondary Income	$ 90.00	$ 4,680.00	
Other Income	$	$ 1,000.00	
Total Annual Income		$ 10,480.00	**MONTHLY SAVINGS**
2) **MONTHLY SAVINGS** (10% of Average Monthly Income)			2) $ 87.33
3) **MONTHLY EXPENSES**			
Mortgage or Rent		$ 150.00	
Car Payments		$ 105.80	
Telecommunications		$	
Hydro		$	
Other Utilities		$	
Cable		$	
Groceries		$	
Clothing		$ 140.00	
Car Maintenance		$	
Gasoline		$ 65.00	
Health and Disability Insurance		$	
Personal Care		$ 25.00	
Entertainment		$ 100.00	**TOTAL MONTHLY EXPENSES**
Other (gifts)		$ 35.00	
Other (CD)		$ 20.00	
TOTAL MONTHLY EXPENSES			3) $ 640.80

	Annual Amount	Monthly Amount	
4) **ANNUAL EXPENSES**			
Car Insurance	$ 470.50	$ 39.21	
Life Insurance	$	$	
Property Taxes	$	$	
Home Insurance	$	$	
Vacations	$ 750.00	$ 62.50	
Newspapers & Periodicals	$ 28.00	$ 2.33	**TOTAL MONTHLY CONTRIBUTIONS**
Other (tuition/books/supplies)	$ 3,750.00	$ 312.50	
Other (Christmas gifts)	$ 240.70	$ 20.06	
TOTAL MONTHLY EXPENSES			4) $ 436.60

5) **SUMMARY**		
1. AVERAGE MONTHLY INCOME		1) $ 873.33
2. SAVINGS	2) $ 236.17	
3. TOTAL MONTHLY EXPENSES	3) $ 997.79	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ 174.18	
TOTAL AMOUNTS 2 + 3 + 4		$ 1,164.73
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ (291.40)

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

b) Is Kirk's budget in a deficit position?

Answer:

Yes, Kirk has a $291.40 deficit.

c) Kirk has to pay for his textbooks. Will he be able to?

Answer:

He will not be able to pay for his textbooks because he is short $291.40/ month.

2. a) Prepare a budget for Daria Smythe for the month of May. Use one of the budget forms provided at the end of the lesson, or create a worksheet of your own.

Daria has completed a welding course and has just started working. She has a biweekly net salary of $1090. Daria is trying to save money to buy a car. Her expenses for the month of August will include the following:

- rent payment is $525.00, and she shares this with her two roommates
- bus pass is $55.85
- telecommunications payment is $30.00, which she shares with her two roommates
- hydro payment is $56.00, which she shares with her two roommates
- water payment is $36.00, which she shares with her two roommates
- cable payment is $21.22, which she shares with her two roommates
- groceries are $140.00
- clothing is $200.00
- health and disability insurance is $42.80
- personal care expense is $30.00
- entertainment expense is $80.00
- gifts are $21.40
- student loan payment is $180.00
- charitable contribution is $25.00

Daria's annual expenses will include the following:

- tenants insurance is $252, which she shares with her two roommates
- vacation is $1500
- newspaper subscription is $168.60, which she shares with her two roommates
- gym membership is $450

Answer:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	AVERAGE MONTHLY INCOME
1) **NET INCOME**			1) $ 2,361.67
Primary Income	$ 545.00	$ 28,340.00	
Secondary Income	$	$	
Other Income	$	$	
Total Annual Income		$ 28,340.00	**MONTHLY SAVINGS**
2) **MONTHLY SAVINGS**			
(10% of Average Monthly Income)			2) $ 236.17
3) **MONTHLY EXPENSES**			
Mortgage or Rent		$ 175.00	
Car Payments		$ 55.85	
Telecommunications		$ 10.00	
Hydro		$ 18.67	
Other Utilities		$ 12.00	
Cable		$ 7.07	
Groceries		$ 140.00	
Clothing		$ 200.00	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$ 42.80	
Personal Care		$ 30.00	
Entertainment		$ 80.00	**TOTAL MONTHLY EXPENSES**
Other (gifts)		$ 21.40	
Other (student loan and charitable contribution)		$ 205.00	
TOTAL MONTHLY EXPENSES			3) $ 997.79

	Annual Amount	Monthly Amount	
4) **ANNUAL EXPENSES**			
Car Insurance	$	$	
Life Insurance	$	$	
Property Taxes	$	$	
Home Insurance	$ 84.00	$ 7.00	
Vacations	$ 1,500.00	$ 125.00	**TOTAL MONTHLY CONTRIBUTIONS**
Newspapers & Periodicals	$ 56.20	$ 4.68	
Other (gym membership)	$ 450.00	$ 37.50	
TOTAL MONTHLY EXPENSES			4) $ 174.18

5) **SUMMARY**		
1. AVERAGE MONTHLY INCOME		1) $ 2,361.67
2. SAVINGS	2) $ 236.17	
3. TOTAL MONTHLY EXPENSES	3) $ 997.79	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ 174.18	
TOTAL AMOUNTS 2 + 3 + 4		$ 1,408.14
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ 953.53

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Note: The costs for rent, telephone, hydro, water, and cable are split between her and two roommates so she only pays one-third of the cost for each.

b) Is Daria's budget in a deficit position?

Answer:

No, Daria does not have a deficit.

c) Daria would like to purchase a car for about $8000 in the near future. How much is Daria able to set aside in May to purchase a car? How long will it take Daria until she is able to afford a car if she sets aside the same amount each month?

Answer:

She can set aside $953.53 per month for the car.

$8000 ÷ $953.53 = 8.39 (Round up because eight months is not quite enough.)

Daria will have enough money to afford a car in nine months.

3. This question has two parts. Complete *at least one* of the two parts.

a) Interview a parent/guardian/relative or your learning partner. Collect all of the information you need to create a monthly budget for your family or the person you interviewed. It would be helpful to fill out a budget form (found at the end of this lesson) as you interview the person to ensure that you get all the information that you need.

Does this person have a deficit? If so, what recommendations could you make to help him or her get rid of it?

Answer:

Answers will vary. Recommendations to reduce a deficit should be based on the three rules stated in the lesson.

b) Below are listed some career choices, along with the average net Manitoba salary for each career. Select one on the careers that you find interesting.

Restaurant Manager	$33 300	Retail Sales Manager	$43 700
Accountant	$58 600	Executive Assistant	$43 000
General Office Clerk	$33 700	Air Traffic Controller	$76 400
Civil Engineer	$74 600	Computer Systems Analyst	$51 600
Web Designer	$42 000	Dentist	$124 100
Doctor	$183 000	Radiation Therapist	$68 800
Community Service Worker	$35 600	Journalist	$46 200
Translator	$46 200	Chef	$36 600
Esthetician	$26 200	Firefighter	$59 600
Police Officer	$63 800	Carpenter	$40 800
Heavy Equipment Operator	$38 800	Mechanic	$38 500
Farm/Hog Barn Manager	$35 100	Underground Miner	$58 300

(This information was taken from *Manitoba Prospects 2010*. If you are interested in finding out more about different careers in Manitoba, visit mb.jobfutures.org.)

Complete a budget form (found at the end of this lesson), using your family's expenses.

Could you afford your current lifestyle with the salary for that career? If you have a deficit, what would you have to change so that you would not have a deficit?

Answer:

Answers may vary. Be sure that you divide the yearly salary by 12 to calculate the monthly income. Also, don't forget to divide the annual expenses by 12 to calculate the monthly cost. The following are the savings you should have for each job:

Career	Salary	Monthly Income (salary ÷ 12)	Savings (10% of monthly income
Restaurant Manager	$33,300.00	$2,775.00	$277.50
Accountant	$58,600.00	$4,883.33	$488.33
General Office Clerk	$33,700.00	$2,808.33	$280.83
Civil Engineer	$74,600.00	$6,216.67	$621.67
Web Designer	$42,000.00	$3,500.00	$350.00
Doctor	$183,000.00	$15,250.00	$1,525.00
Community Service Worker	$35,600.00	$2,966.67	$296.67
Translator	$46,200.00	$3,850.00	$385.00
Esthetician	$26,200.00	$2,183.33	$218.33
Police Officer	$63,800.00	$5,316.67	$531.67
Heavy Equipment Operator	$38,800.00	$3,233.33	$323.33
Farm/Hog Barn Manager	$35,100.00	$2,925.00	$292.50
Retail Sales Manager	$43,700.00	$3,641.67	$364.17
Executive Assistant	$43,000.00	$3,583.33	$358.33
Air Traffic Controller	$76,400.00	$6,366.67	$636.67
Computer Systems Analyst	$51,600.00	$4,300.00	$430.00
Dentist	$124,100.00	$10,341.67	$1,034.17
Radiation Therapist	$68,800.00	$5,733.33	$573.33
Journalist	$46,200.00	$3,850.00	$385.00
Chef	$36,600.00	$3,050.00	$305.00
Firefighter	$59,600.00	$4,966.67	$496.67
Carpenter	$40,800.00	$3,400.00	$340.00
Mechanic	$38,500.00	$3,208.33	$320.83
Underground Miner	$58,300.00	$4,858.33	$485.83

Learning Activity 4.2

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate the following: $10 + 3 \times 6 - \frac{12}{2}$

2. There are a lot of special occasions in Brennan's family in June. His sisters' birthdays are on the 4th, so he buys the two of them $30 gift cards each for workout clothes. His brother's birthday is on the 12th, so he pays $20 for a rare CD for his brother. And it's Father's Day, so he spends $50 on a luxury car wash for his dad. How much money did Brennan spend on gifts in June?

3. Write two equivalent fractions for $\frac{45}{90}$.

4. High school students are collecting mittens for a shelter. If each student donates two pairs of mittens and there are 140 students at the school, how many pairs of mittens will the school donate?

5. A turkey must be roasted 15 minutes for every pound. If a turkey weighs 20 pounds, how long (in hours) would you have to roast the turkey?

Answers:

1. 22 ($10 + 18 - 6 = 22$)
2. \$130 ($30 \times 2 + 20 + 50 = \$130$)
3. There are many possible answers. For example, $\frac{1}{2}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{15}{30}, \ldots$
4. 280 pairs of mittens ($140 \times 2 = 280$)
5. 5 hours (Time in minutes: $20 \times 15 = 300$ minutes;
   time in hours: $\frac{300}{60} = 5$ hr. Or, if it takes 15 minutes for l pound, it takes 60 minutes or 1 hour for 4 pounds; $20 \div 4 = 5$. Therefore, it takes 5 hours.)

Part B: Unexpected Expenses

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. With your learning partner, brainstorm different types of unexpected expenses that you may face.

   Answer:

   Answers may vary. As listed in this lesson and the last, the list ranges from car maintenance to hospital bills while on vacation. These are things that you do not anticipate when making your budget.

2. Emery has to save some money for university. Right now his monthly budget for September looks like this:

EMERY'S MONTHLY BUDGET FOR SEPTEMBER

	Bi-Weekly Amount	Annual Amount	
1) ***NET INCOME***			***AVERAGE MONTHLY INCOME***
Primary Income	$ 462.80	$ 12,032.80	
Secondary Income	$	$	
Other Income	$ 200.00 (tips)	$ 5,200.00	
Total Annual Income		$ 17,232.80	1) $ 1,436.06
2) ***MONTHLY SAVINGS***			***MONTHLY SAVINGS***
(10% of Average Monthly Income)			2) $ 143.61
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$	
Car Payments		$ 180.00	
Telecommunications		$ 75.00	
Hydro		$	
Other Utilities		$	
Cable		$	
Groceries		$	
Clothing		$ 100.00	
Car Maintenance		$	
Gasoline		$ 80.00	
Health and Disability Insurance		$	
Personal Care		$ 22.41	
Entertainment		$ 191.46	***TOTAL MONTHLY EXPENSES***
Other		$	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 648.87

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 988.46	$ 82.37	
Life Insurance	$ 120.91	$ 10.08	
Property Taxes	$	$	
Home Insurance	$	$	
Vacations	$ 1,600.00	$ 133.34	
Newspapers & Periodicals	$ 35.00	$ 2.17	***TOTAL MONTHLY CONTRIBUTIONS***
Other (university)	$ 4,039.61	$ 336.63	
Other (books)	$ 728.54	$ 60.71	
TOTAL MONTHLY EXPENSES			4) $ 625.30

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 1,436.06
2. SAVINGS	2)	$ 143.61	
3. TOTAL MONTHLY EXPENSES	3)	$ 648.87	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 625.30	
TOTAL AMOUNTS 2 + 3 + 4			$ 1,417.78
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 18.28

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

a) Can Emery afford to pay for his books?

Answer:

Emery can afford to pay for his books. We know because he does not have a deficit.

b) Last year, he got into two car accidents, so his car insurance is now $1620 per year. Can he still afford his books?

Answer:

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 1620.00	$ 135.00	
Life Insurance	$ 120.91	$ 10.08	
Vacations	$ 1600.00	$ 133.34	
Periodicals	$ 26.00	$ 2.17	
University	$ 4039.61	$ 336.63	***TOTAL MONTHLY CONTRIBUTIONS***
Books	$ 728.54	$ 60.71	
Other	$	$	
TOTAL MONTHLY EXPENSES			4) $ 677.93
5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 1436.06
2. SAVINGS	2)	$ 143.61	
3. TOTAL MONTHLY EXPENSES	3)	$ 648.87	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 677.93	
TOTAL AMOUNTS 2 + 3 + 4			$ 1470.41
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ (34.35)

No, Emery cannot afford his books because he has a deficit.

c) Emery has $6000 in his bank account after he has paid his tuition for this school year. A couple of weeks into university, he realizes that he has to quit his job because he has too much homework. How long can he afford to keep the same budget before he runs out of money? Answer this question based on the original budget provided, not including the information from Part (b).

Answer:

Step 1: Determine his budget without an income (so no savings either).

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 0
2. SAVINGS	2)	$ 0	
3. TOTAL MONTHLY EXPENSES	3)	$ 648.87	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 625.30	
TOTAL AMOUNTS 2 + 3 + 4			$ 1274.17
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ (1274.17)

Step 2: How many months would it take to use up $6000?

Emery spends 1274.17 each month, so $6000 ÷ 1274.17 = 4.7 months (Emery would run out of money partway through January)

Learning Activity 4.3

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Convert the fraction to a decimal and a percent: $\frac{21}{84}$.
2. To bisect an angle means to cut in half. What is the name of the angle when you bisect a straight line?
3. If Jonah has $81 and he buys four DVDs for $15 each, will he have enough money?
4. Aldin's sales were $1500 over the past two weeks. If he makes 20% commission, how much will he be paid (gross)?
5. Complete the pattern: 100, 75, 50, _____ , _____ , _____ .

Answers:

1. 0.25, 25% (84 ÷ 21 = 4, so $\frac{21}{84} = \frac{1}{4}$. This is equivalent to 0.25 or 25%.)
2. right angle (A straight line is 180˚, so bisecting it would produce a 90˚ angle—also known as a right angle.)
3. Yes (Cost: 15 × 4 = $60, so he will have enough money.)
4. $300 (Gross: 10% of 1500 = 150 so 20% = $300)
5. 25, 0, −25

Part B: Assessing Financial Goals

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Yagabi Umbago earns $1800 net each month. Rafi Umbago earns $488 weekly.

   The following are the Umbago family's monthly expenses:

   - $890 for rent
   - telecommunications payment is $34
   - utilities cost $150
   - cable costs $45
   - groceries cost $270
   - $73 for clothing
   - $200 for daycare for each of their two children
   - health and disability insurance costs $147
   - personal care costs $60
   - entertainment costs $100

   The following are their annual costs:

   - life insurance costs $176
   - apartment insurance costs $423
   - the newspaper subscription costs $50
   - one child plays hockey, which costs $1000
   - the other child plays soccer, which costs $500

a) Prepare a budget for the Umbago family. Use one of the monthly budget forms at the end of Lesson 1 or create your own.

Answer:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ 3,914.67
Primary Income (Yagabi)	$ 1,800.00 (monthly)	$ 21,600.00	
Secondary Income (Rafi)	$ 488.00	$ 25,376.00	
Other Income	$	$	
Total Annual Income		$ 46,976.00	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 391.47
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ 890.00	
Car Payments		$	
Telecommunications		$ 34.00	
Hydro		$	
Other Utilities		$ 150.00	
Cable		$ 45.00	
Groceries		$ 270.00	
Clothing		$ 73.00	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$ 147.00	
Personal Care		$ 60.00	
Entertainment		$ 100.00	***TOTAL MONTHLY EXPENSES***
Other (daycare) 200 x 2 =		$ 400.00	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 2,169.00

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$	$	
Life Insurance	$ 176.00	$ 14.67	
Property Taxes	$	$	
Home Insurance (appt.)	$ 423.00	$ 35.28	
Vacations	$	$	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ 50.00	$ 4.17	
Other (sports) 1000 + 500 =	$ 1,500.00	$ 125.00	
TOTAL MONTHLY EXPENSES			4) $ 179.12

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 3,914.67
2. SAVINGS	2)	$ 391.47	
3. TOTAL MONTHLY EXPENSES	3)	$ 2,169.00	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 179.12	
TOTAL AMOUNTS 2 + 3 + 4			$ 2,739.59
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 1,175.08

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

b) If the rent for their apartment increases to $1100, will they still be able to afford it?

Answer:

By estimation, you can tell that the Umbago family would still be able to afford their apartment, because the rent only increases by $210 and they have a surplus of $1175.11 in their original monthly budget (from part (a)). If you were to alter their budget to check, it would look like this:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	AVERAGE MONTHLY INCOME
1) **NET INCOME**			1) $ 3,914.67
Primary Income (Yagabi)	$ 1,800.00 (monthly)	$ 21,600.00	
Secondary Income (Rafi)	$ 488.00	$ 25,376.00	
Other Income	$	$	
Total Annual Income		$ 46,976.00	**MONTHLY SAVINGS**
2) **MONTHLY SAVINGS** (10% of Average Monthly Income)			2) $ 391.47
3) **MONTHLY EXPENSES**			
Mortgage or Rent		$ 1,100.00	
Car Payments		$	
Telecommunications		$ 34.00	
Hydro		$	
Other Utilities		$ 150.00	
Cable		$ 45.00	
Groceries		$ 270.00	
Clothing		$ 73.00	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$ 147.00	
Personal Care		$ 60.00	
Entertainment		$ 100.00	**TOTAL MONTHLY EXPENSES**
Other (daycare) 200 x 2 =		$ 400.00	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 2,379.00

	Annual Amount	Monthly Amount	
4) **ANNUAL EXPENSES**			
Car Insurance	$	$	
Life Insurance	$ 176.00	$ 14.67	
Property Taxes	$	$	
Home Insurance (appt.)	$ 423.00	$ 35.28	
Vacations	$	$	**TOTAL MONTHLY CONTRIBUTIONS**
Newspapers & Periodicals	$ 50.00	$ 4.17	
Other (sports) 1000 + 500 =	$ 1,500.00	$ 125.00	
TOTAL MONTHLY EXPENSES			4) $ 179.12

5) **SUMMARY**		
1. AVERAGE MONTHLY INCOME		1) $ 3,914.67
2. SAVINGS	2) $ 391.47	
3. TOTAL MONTHLY EXPENSES	3) $ 2,379.00	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ 179.12	
TOTAL AMOUNTS 2 + 3 + 4		$ 2,949.59
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ 965.08

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

c) The Umbago family calculates that it would cost them $8000 to go on a family vacation to Disneyland. How long would they have to save before they could go on the vacation? Use the original information provided.

Answer:

The Umbagos' surplus is $1175.11.

Time: 8000 ÷ 1175.08 = 6.8

It would take the Umbago family approximately seven months to save enough money for the vacation.

2. Horace Malabar would like to purchase a boat, which costs $19,450, in April next year. His monthly budget for August this year is:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	AVERAGE MONTHLY INCOME
1) **NET INCOME**			
Primary Income	$ 385.00	$ 20,020.00	1) $ 3,501.67
Secondary Income	$	$ 22,000.00	
Other Income	$	$	
Total Annual Income		$ 42,020.00	**MONTHLY SAVINGS**
2) **MONTHLY SAVINGS**			
(10% of Average Monthly Income)			2) $ 350.17
3) **MONTHLY EXPENSES**			
Mortgage or Rent		$	
Car Payments		$	
Telecommunications		$ 25.97	
Hydro		$ 52.89	
Other Utilities		$ 33.64	
Cable		$ 40.00	
Groceries		$ 198.06	
Clothing		$ 30.00	
Car Maintenance		$ 40.00	
Gasoline		$ 35.00	
Health and Disability Insurance		$ 59.81	
Personal Care		$ 25.00	
Entertainment		$ 60.00	**TOTAL MONTHLY EXPENSES**
Other (bike maintenance)		$ 25.00	
Other		$	
TOTAL MONTHLY EXPENSES			3) $ 625.37

	Annual Amount	Monthly Amount	
4) **ANNUAL EXPENSES**			
Car Insurance	$ 700.00	$ 58.33	
Life Insurance	$ 190.00	$ 15.83	
Property Taxes	$ 8,210.00	$ 684.17	
Home Insurance	$ 1,450.00	$ 120.83	
Vacations	$ 2,000.00	$ 166.67	**TOTAL MONTHLY CONTRIBUTIONS**
Newspapers & Periodicals	$ 70.00	$ 5.83	
Other	$	$	
TOTAL MONTHLY EXPENSES			4) $ 1,051.66

5) **SUMMARY**			
1. AVERAGE MONTHLY INCOME			1) $ 3,501.67
2. SAVINGS	2)	$ 350.17	
3. TOTAL MONTHLY EXPENSES	3)	$ 625.37	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 1,051.66	
TOTAL AMOUNTS 2 + 3 + 4			$ 2,027.20
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 1,474.47

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

a) Will he be able to buy the boat upfront if he puts his surplus each month toward the boat? If not, what type of credit should he use to pay the difference (recall from Module 1)?

Answer:

August–April = 8 months

Amount saved: 8 × 1474.47 = $11,795.76 saved

Horace would not be able to afford to pay cash for the boat. Horace would most likely use a loan.

b) Once he buys the boat, he also gains the following expenses:

- gas costs $60 per month
- monthly payments for the boat are $216
- maintenance per month costs $30
- boat insurance costs $650 annually

Will Horace be able to afford all of the additional expenses of having a boat?

Answer:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ 3,501.67
Primary Income	$ 385.00	$ 20,020.00	
Secondary Income	$	$ 22,000.00	
Other Income	$	$	
Total Annual Income		$ 42,020.00	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS*** (10% of Average Monthly Income)			2) $ 350.17
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$	
Car Payments		$	
Telecommunications		$ 25.97	
Hydro		$ 52.89	
Other Utilities		$ 33.64	
Cable		$ 40.00	
Groceries		$ 198.06	
Clothing		$ 30.00	
Car Maintenance		$ 40.00	
Gasoline		$ 35.00	
Health and Disability Insurance		$ 59.81	
Personal Care		$ 25.00	
Entertainment		$ 60.00	***TOTAL MONTHLY EXPENSES***
Other (bike maintenance)		$ 25.00	
Other (boat costs) 60 + 216 + 30 =		$ 306.00	
TOTAL MONTHLY EXPENSES			3) $ 931.37

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ 700.00	$ 58.33	
Life Insurance	$ 190.00	$ 15.83	
Property Taxes	$ 8,210.00	$ 684.17	
Home Insurance	$ 1,450.00	$ 120.83	
Vacations	$ 2,000.00	$ 166.67	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ 70.00	$ 5.83	
Other (boat insurance)	$ 650.00	$ 54.17	
TOTAL MONTHLY EXPENSES			4) $ 1,105.83

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 3,501.67
2. SAVINGS	2)	$ 350.17	
3. TOTAL MONTHLY EXPENSES	3)	$ 931.37	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 1,105.83	
TOTAL AMOUNTS 2 + 3 + 4			$ 2,387.37
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 1,114.30

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

After all added expenses, Horace has a budget surplus of $1114.30. Therefore, he will be able to afford the boat.

Learning Activity 4.4

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. In the year 2020, Tom will be 16 years old. Tara will be half Tom's age, and Tammy will be half Tara's age. What year was Tammy born?

2. Evaluate the following: $10 + \frac{4}{(2 \times 3)} \times 12 - 50$

3. From your house, you walk 10 paces straight before turning left, and then you walk 100 paces. You then turn left and walk 25 paces. Finally, you turn left again and walk 100 paces. How far are you from where you started?

4. This past week (Monday to Friday) was crazy. Monday, you did not go to school because it was a snow day; Thursday was Remembrance Day, so you didn't go to school; and then on Friday the teachers had an in-service day. How many days did you go to school this week?

5. Write your answer from Question 4 as a fraction of the number of days in a week.

Answers:

1. $2016 \left(\frac{16}{2} = 8, \frac{8}{2} = 4, 2020 - 4 = 2016 \right)$

2. $-32 \left(10 + \frac{4}{6} \times 12 - 50 = 10 + 4 \times 2 - 50 = 10 + 8 - 50 = -32 \right)$

3. 15 paces (Try sketching it out.)

4. 2 days (Tuesday and Wednesday)

5. $\frac{2}{7}$ (If you got question 4 wrong, then this answer should be that number over 7.)

Part B: Your Bank Accounts

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Option A: Talk to Someone

If you have an account with a bank, credit union, or financial institution, discuss the type and characteristics of the account with your parent/guardian, learning partner, or make an appointment with someone at the financial institution where you have your account. The following checklist includes questions you should ask about your account.

Questions:

1. What type of bank account do I have?
2. What is the interest rate for my account?
3. Do I have to pay any service charges to use this account?
4. Is this the right type of account for my financial needs? (**Note:** To answer this properly, you need to know some of your financial goals.)

Answer:

The answers will vary for this option.

1. As stated in the lesson, the majority of students have savings accounts—typically student savings accounts.
2. Again, interest rates vary between and within banks.
3. These accounts have minimal service charges, if any at all.
4. Depending on whether you have a job, if you are saving up for something in particular, or if you barely make enough money to cover your expenses, you may or may not benefit from having a savings account instead of a chequing account, but remember that if you stop using a student account, you will probably have to pay higher service charges.

Option B: Research Banks

If you do not have a bank account or cannot talk to anyone who is aware of your finances, answer the following questions for two different banks. Use the Internet or call a bank. If you are calling, be sure to explain that you are seeking information to complete a school assignment.

Possible websites to use (although you could use other financial institutions if you prefer):

- Royal Bank: www.rbcroyalbank.com/personal.html
- CIBC: www.cibc.com/ca/chequing-savings/index.html
- TD: www.tdcanadatrust.com/accounts/index.jsp

Click on "Compare Accounts" to get more information about the different accounts each bank offers.

Questions:

1. What standard bank accounts does the bank offer?
2. Do all accounts have service charges such as monthly fees or fees for withdrawing money at the ATM, for overdraft protection, or for online money transfers?
3. Does this bank offer any student accounts? If so, what benefit(s) are there to having a student account instead of a regular account?

Answers:

Royal Bank	
Question 1	**Question 2**
Day-to-Day Banking	Yes
No Limit Banking	Yes
Signature No Limit Banking	Yes
VIP Banking	Yes
High Interest eSavings	Yes
Enhanced Savings	Yes
Day-to-Day Savings	Yes
Question 3: Yes, they have student accounts. Although the student accounts still have service charges, they are smaller than the service charges on regular accounts.	

CIBC	
Question 1	**Question 2**
Unlimited Chequing	Yes
Everyday Chequing	Yes
Premier Service Account	Yes
eAdvantage Savings Account	Yes
Bonus Savings Account	Yes
Premium Grown Account	Yes
Question 3: Yes, they have student accounts. The student account only has a service charge for online money transfers, and it is only $1.50.	

TD	
Question 1	**Question 2**
Value	Yes
Value Plus	Yes
Infinity	Yes
Select Service	Yes
Plan 60	Yes
Everyday Savings	Yes
High Interest Savings	Yes
Question 3: Yes, they have a youth account. The only service charge is $1.50 if you withdraw money from an ABM.	

Learning Activity 4.5

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Divide 8 by 2.
2. Multiply your answer from question 1 by 3.
3. Divide your answer from question 2 by 4.
4. What is the pattern?
5. Write the next step in the pattern and solve it.

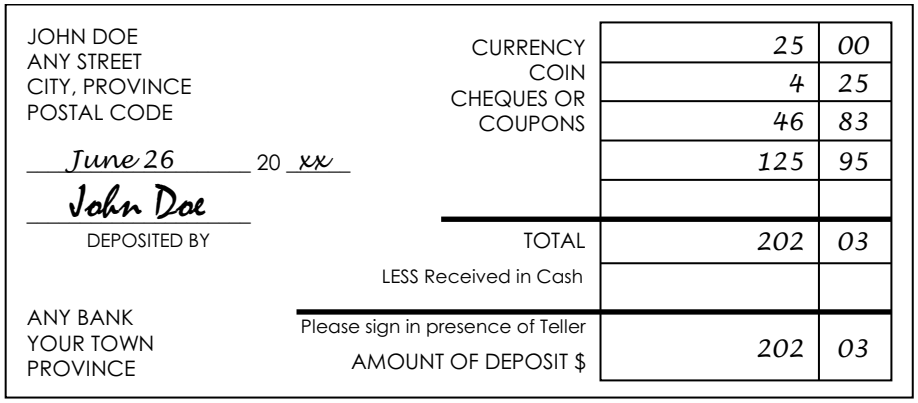
Answers:

1. 4
2. 12
3. 3
4. Alternate multiplying and dividing; the number increases by 1 each time.
5. Multiply the answer from question 3 by 5 = 15.

Part B: Cheques and Deposit Slips

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Complete a deposit slip using the following information. The date is June 26, 20xx. The amount deposited is 5 quarters, 3 loonies, 1 five-dollar bill, 1 twenty-dollar bill, and 2 cheques for $46.83 and $125.95.



JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

June 26 20 xx

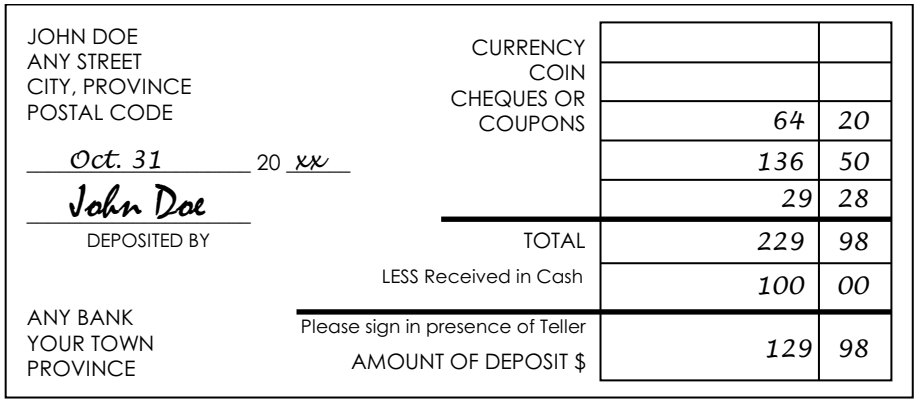
John Doe
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY	25	00
COIN	4	25
CHEQUES OR COUPONS	46	83
	125	95
TOTAL	202	03
LESS Received in Cash		
Please sign in presence of Teller AMOUNT OF DEPOSIT $	202	03

2. Complete a deposit slip using the following information. The date is Oct. 31, 20xx. The deposit is three cheques: $64.20, $136.50, and $29.28. There is $100.00 in cash taken out.

Answer:



JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

Oct. 31 20 xx

John Doe
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY		
COIN		
CHEQUES OR COUPONS	64	20
	136	50
	29	28
TOTAL	229	98
LESS Received in Cash	100	00
Please sign in presence of Teller AMOUNT OF DEPOSIT $	129	98

3. Using the cheque blanks below, write cheques for the following:
   a) February 4, 20xx to the J & M Department Store for $48.72

   Answer:

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

February 4 20 xx

Pay to the order of J + M Department Store $ 48.72

fourty-eight 72/100 DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

   b) August 27, 20xx to State-of-the-Art Electronics for $158.95

   Answer:

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

August 27 20 xx

Pay to the order of State-of-the-Art Electronics $ 158.95

one hundred fifty-eight 95/100 DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

4. There are errors in the way the following cheques are written. They are written over a two-week period. What are the errors? There may be more than one error on a cheque.

   a)

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

December 26 20 10

Pay to the order of A. Baron $ 12

twelve DOLLARS

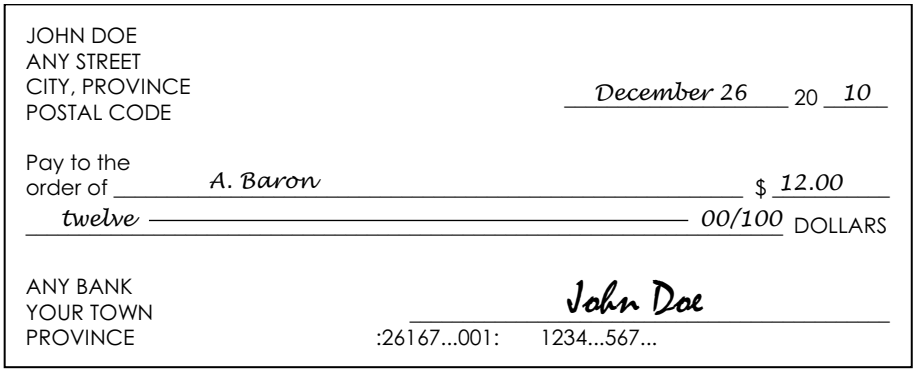
ANY BANK
YOUR TOWN
PROVINCE

John Doe

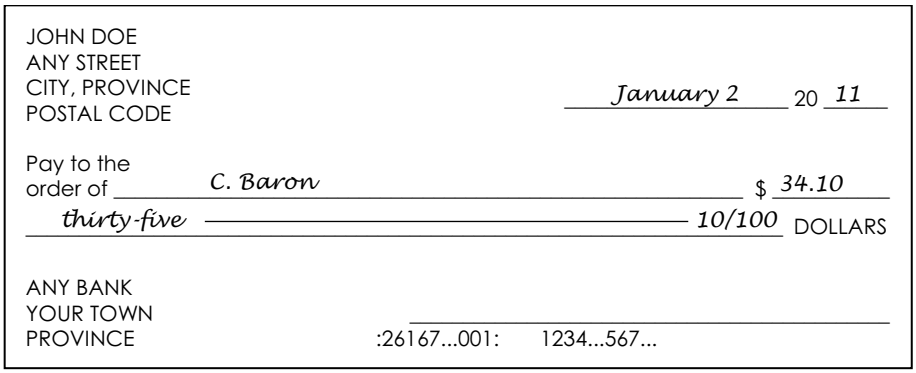
:26167...001: 1234...567...

Answer:

The number of cents is not indicated, and there is no line after the twelve. The cheque should look like this:

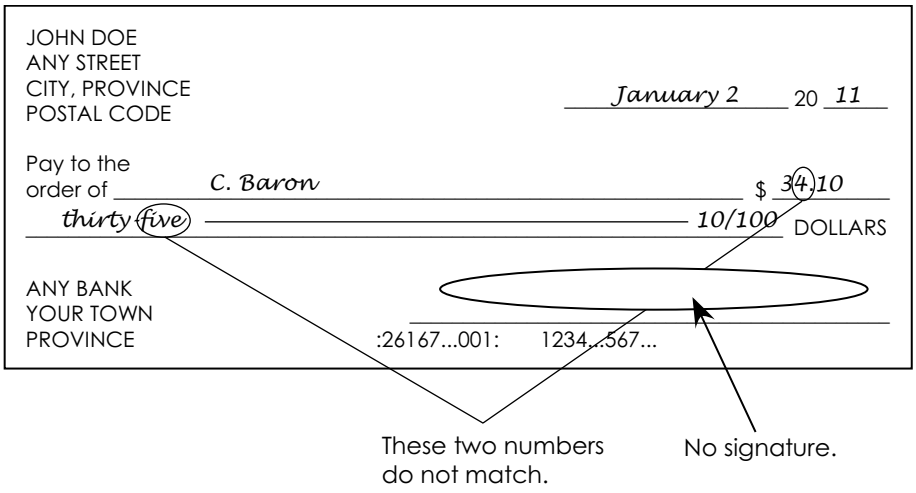


b)



Answer:

There is no signature on the cheque, and the written amount says thirty-five but the numerical amount is 34.



5. Ima Lender has $5396.52 in her chequing account. She makes three deposits of $382.00, $545.00, and $250.00. She pays the following bills by cheque: Hydro, $35.86; Telephone, $25.52; and Visa $86.52. Find her new balance.

   Answer:

   Deposits: 382 + 545 + 250 = $1177.00

   Withdrawals: 35.86 + 25.52 + 86.52 = $147.90

   Net: 5396.52 + 1177 − 147.90 = $6 425.62 is her new balance

Learning Activity 4.6

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the following pattern: 5, −5, −15, _____ , _____
2. Write two equivalent fractions for $\frac{24}{56}$.
3. Dinoo spends $45 on winter boots, $65 on a parka, $15 on a scarf, $10 on mittens, and $8 on a toque. If Dinoo had budgeted $120 for winter clothing, does she have a deficit?
4. Muriel picks the petals off of three flowers one by one, and says the phrase "he loves me" as she picks the first petal, and then "he loves me not" as she picks the second petal, and so on. If each flower has seven petals and she says "he loves me" first, which phrase will she finish on?
5. You need 2 yd.^{2} of fabric to make a teddy bear. If you were to make five teddy bears, how many square yards of fabric would you need?

Answers:

1. −25, −35 (Subtract 10 each time.)
2. $\frac{3}{7}$ or $\frac{12}{26}$ or $\frac{6}{14}$ (There are many other answers. All answers should simplify to. $\frac{3}{7}$.)
3. Yes (Spending: 45 + 65 + 15 = $125, so she is over budget before she buys the mitts and toque.)
4. "He loves me." (7 + 7 + 7 = 21 (odd number) so she will finish on "he loves me.")
5. 10 yd.^{2} ($5 \times 2 = 10$)

Part B: Cheque Registers

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Complete the following cheque register:

   Balance: Feb. 5, $837.92

   Cheque No. 162: Feb. 10 to Manitoba Electric Co. for $58.74

   163: Feb. 10 to Manitoba Telephone Co. for $38.52

   164: Feb. 14 to Visa for $194.71

   165: Feb. 18 to Kathy's Clothing for $89.66

   Deposits: Feb. 17, $185.92; Feb. 20, $300

   Answer:

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									837	92
Feb 10	162	TO Manitoba Electric Co.	58	74				CHEQUE – DEPOSIT +	58	74
		FOR						BALANCE ➜	779	18
10	163	TO Manitoba Telephone Co.	38	52				CHEQUE – DEPOSIT +	38	52
		FOR						BALANCE ➜	740	66
14	164	TO VISA	194	71				CHEQUE – DEPOSIT +	194	71
		FOR						BALANCE ➜	545	95
17		TO Deposit				185	92	CHEQUE – DEPOSIT +	185	92
		FOR						BALANCE ➜	731	87
18	165	TO Kathy's Clothing	89	66				CHEQUE – DEPOSIT +	89	66
		FOR						BALANCE ➜	642	21
20		TO Deposit				300	00	CHEQUE – DEPOSIT +	300	00
		FOR						BALANCE ➜	942	21
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

2. Complete the following cheque register:

   Balance: Oct. 25, $78.72

   Cheque No. 72: Oct. 26 to Wayne's Gas Bar for $67.25

   73: Oct. 31 to C & B Rental Agencies for $398.88

   74: Nov. 04 to Baron's Department Store for $12.75

   75: Nov. 07 to the Medical Ethics Association for $75.00

   Deposits: Oct. 28, $402.96; Nov. 06, $90.82; Nov. 10, $153.72

Answer:

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									78	72
Oct 26	72	TO Wayne's Gas Bar	67	25				CHEQUE – DEPOSIT +	67	25
		FOR						BALANCE ➜	11	47
28		TO Deposit				402	96	CHEQUE – DEPOSIT +	402	96
		FOR						BALANCE ➜	414	43
31	73	TO C & B Rental Agencies	398	88				CHEQUE – DEPOSIT +	398	88
		FOR						BALANCE ➜	15	55
Nov 4	74	TO Baron's Department Store	12	75				CHEQUE – DEPOSIT +	12	75
		FOR						BALANCE ➜	2	80
6		TO Deposit				90	82	CHEQUE – DEPOSIT +	90	82
		FOR						BALANCE ➜	93	62
7	75	TO Medical Ethics Association	75	00				CHEQUE – DEPOSIT +	75	00
		FOR						BALANCE ➜	18	62
10		TO Deposit				153	72	CHEQUE – DEPOSIT +	153	72
		FOR						BALANCE ➜	172	34
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

3. Compare the following monthly account statement with its cheque register. There are two errors and/or omissions in the cheque register. Locate the errors and/or omissions, and make the necessary changes in the cheque register.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Jul. 1	Balance Forward			1522.71
Jul. 4	Cheque 90	231.80		1290.91
Jul. 6	Cheque 92	25.00		1265.91
Jul. 9	Deposit		484.24	1750.15
Jul. 26	Cheque 94	36.12		
	Cheque 97	104.86		1609.17
Jul. 30	Cheque 96	42.73		1566.44
Jul. 31	Cheque 95	15.00		1551.44

Answer:

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1522	71
Jul 2	90	TO Econo-Airlines	231	80	✓			CHEQUE – DEPOSIT +	231	80
		FOR						BALANCE ➜	1290	91
3	91	TO E. Cadlof	25	00				CHEQUE – DEPOSIT +	25	00
		FOR						BALANCE ➜	1265	91
5	92	TO P. Anders	25	00	✓			CHEQUE – DEPOSIT +	25	00
		FOR						BALANCE ➜	1240	91
9		TO Deposit			✓	484	24	CHEQUE – DEPOSIT +	484	24
		FOR						BALANCE ➜	1725	15
12	93	TO Music Lesson	28	00				CHEQUE – DEPOSIT +	28	00
		FOR						BALANCE ➜	1697	15
24	94	TO J + M Department Store	36	12	✓			CHEQUE – DEPOSIT +	36	12
		FOR						BALANCE ➜	1661	03
24	96	TO Outdoor Supplies	42	73	✓			CHEQUE – DEPOSIT +	42	73
		FOR						BALANCE ➜	1618	30
25	97	TO Value Foods	104	68	✓			CHEQUE – DEPOSIT +	104	68
		FOR						BALANCE ➜	1513	62
Aug 3		TO Deposit				482	53	CHEQUE – DEPOSIT +	482	53
		FOR						BALANCE ➜	1996	15
4	98	TO Bob's 24-Hour Shopping	88	23				CHEQUE – DEPOSIT +	88	23
		FOR						BALANCE ➜	1907	92
		correction: cheque 95 omitted $15.00	15	00					15 1892	00 92
		correction: date 07/25 cheque 97 should be $104.86		18					1892	18 74

4. Compare the following monthly account statement with its cheque register. There are three errors and/or omissions in the cheque register. Locate the errors and/or omissions, and make the necessary changes in the cheque register.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Apr. 1	Balance Forward			1127.18
Apr. 3	Cheque 201	14.25		1112.93
Apr. 10	Deposit		200.00	1312.93
Apr. 16	Deposit		350.00	1662.93
Apr. 19	Cheque 204	428.00		1234.93
	Cheque 202	67.29		1167.64
Apr. 24	Deposit		250.00	1417.64
	Cheque 203	25.46		1392.18
Apr. 29	Deposit		75.00	1467.18

Answer:

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1127	18
4/1	201	TO	14	25	✓			CHEQUE – DEPOSIT +	14	25
		FOR						BALANCE ➜	1112	93
4/10		TO Deposit				200	00	CHEQUE – DEPOSIT +	200	00
		FOR						BALANCE ➜	1312	93
4/13	202	TO Computer Software	67	32	✓			CHEQUE – DEPOSIT +	67	32
		FOR						BALANCE ➜	1245	61
4/16	203	TO Cleaning Supplies	25	46	✓			CHEQUE – DEPOSIT +	25	46
		FOR						BALANCE ➜	1220	15
4/18	204	TO Rent	428	00	✓			CHEQUE – DEPOSIT +	428	00
		FOR						BALANCE ➜	792	15
4/18	205	TO Groceries	113	82				CHEQUE – DEPOSIT +	113	82
		FOR						BALANCE ➜	678	33
4/18	207	TO Gift	50	00				CHEQUE – DEPOSIT +	50	00
		FOR						BALANCE ➜	628	33
4/24		TO Deposit			✓	250	00	CHEQUE – DEPOSIT +	250	00
		FOR						BALANCE ➜	878	33
4/30	208	TO M. Holt	75	00				CHEQUE – DEPOSIT +	75	00
		FOR						BALANCE ➜	803	33
5/1		TO Deposit				200	00	CHEQUE – DEPOSIT +	200	00
		FOR						BALANCE ➜	1003	33
		correction: deposit omitted 4/16				350	00		350 1353	00 33
		correction: 4/13 cheque 202 should be $67.29					03		1353	03 36
		correction: deposit omitted 4/29				75	00		75 1428	00 36

5. Reconcile the monthly account statement to the cheque register. There are reconciliation tables at the end of the learning activity that you can use for this question and the next.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Sept. 1	Balance Forward			212.85
Sept. 3	Deposit		645.15	858.00
Sept. 8	Cheque 256	435.00		423.00
Sept. 11	Cheque 257	79.63		343.37
Sept. 16	Deposit		648.23	991.60
	Cheque 260	29.04		
	Cheque 259	36.94		925.62
Sept. 17	Cheque 258	23.16		902.46
Sept. 20	Cheque 261	94.20		808.26
Sept. 26	Cheque 263	127.89		680.37
Sept. 29	Cheque 264	61.16		619.21
Sept. 30	Deposit		647.21	1266.42
	Service Charge	5.00		1261.42

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									212	85
9/3		TO Deposit				645	15	CHEQUE – DEPOSIT +	645	15
		FOR						BALANCE ➜	858	00
9/6	256	TO Rent	435	00				CHEQUE – DEPOSIT +	435	00
		FOR						BALANCE ➜	423	00
9/10	257	TO Groceries	79	63				CHEQUE – DEPOSIT +	79	63
		FOR						BALANCE ➜	343	37
9/14	258	TO Manitoba Gas Co.	23	16				CHEQUE – DEPOSIT +	23	16
		FOR						BALANCE ➜	320	21
9/14	259	TO Manitoba Electric Co.	36	94				CHEQUE – DEPOSIT +	36	94
		FOR						BALANCE ➜	283	27
9/14	260	TO Manitoba Telephone Co.	29	04				CHEQUE – DEPOSIT +	29	04
		FOR						BALANCE ➜	254	23
9/16		TO Deposit				648	23	CHEQUE – DEPOSIT +	648	23
		FOR						BALANCE ➜	902	40
9/18	261	TO Baron's Dept. Store	94	20				CHEQUE – DEPOSIT +	94	20
		FOR						BALANCE ➜	808	26
9/22	262	TO Music Lessons	56	00				CHEQUE – DEPOSIT +	56	00
		FOR						BALANCE ➜	752	26
9/24	263	TO Roller Blades	127	89				CHEQUE – DEPOSIT +	127	89
		FOR						BALANCE ➜	624	37
9/27	264	TO Groceries	61	16				CHEQUE – DEPOSIT +	61	16
		FOR						BALANCE ➜	563	21
9/30		TO Deposit				647	21	CHEQUE – DEPOSIT +	647	21
		FOR						BALANCE ➜	1210	42
10/1	265	TO Magazine Subscription	45	98				CHEQUE – DEPOSIT +	45	98
		FOR						BALANCE ➜	1164	44
10/3		TO Deposit				50	00	CHEQUE – DEPOSIT +	50	00
		FOR						BALANCE ➜	1214	44

Answer:

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
1261.42	*1214.44 - 5.00 = 1209.44*
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
50.00	*45.98*
	56.00
TOTAL: *1311.42*	TOTAL: *1311.42*

6. Reconcile the monthly account statement to the cheque register.

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
June 1	Balance Forward			1027.86
June 2	Cheque 125	100.00		927.86
June 11	Cheque 126	217.36		710.50
June 15	Deposit		500.00	1210.50
June 19	Cheque 129	194.18		
	Cheque 127	625.96		390.36
June 20	Cheque 130	26.32		364.04
	Cheque 128	73.26		290.78
June 26	Cheque 132	44.85		245.93
June 30	Cheque 133	128.43		117.50
	Service Charge	3.00		114.50

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									1027	86
June 02	125	TO Cash	100	00				CHEQUE – DEPOSIT +	100	00
		FOR						BALANCE ➜	927	86
09	126	TO Mastercard	217	36				CHEQUE – DEPOSIT +	217	36
		FOR						BALANCE ➜	710	50
15		TO Deposit				500	00	CHEQUE – DEPOSIT +	500	00
		FOR						BALANCE ➜	1210	50
17	127	TO W + A Travel	625	96				CHEQUE – DEPOSIT +	625	96
		FOR						BALANCE ➜	584	54
17	128	TO Dentist	73	26				CHEQUE – DEPOSIT +	73	26
		FOR						BALANCE ➜	511	28
17	129	TO J + M Department Store	194	18				CHEQUE – DEPOSIT +	194	18
		FOR						BALANCE ➜	317	10
19	130	TO Generic Pharmacy	26	32				CHEQUE – DEPOSIT +	26	32
		FOR						BALANCE ➜	290	78
24	131	TO C. Baron	19	21				CHEQUE – DEPOSIT +	19	21
		FOR						BALANCE ➜	271	57
24	132	TO Winnipeg News	44	85				CHEQUE – DEPOSIT +	44	85
		FOR						BALANCE ➜	226	72
27	133	TO Wholesome Foods	128	43				CHEQUE – DEPOSIT +	128	43
		FOR						BALANCE ➜	98	29
30		TO Deposit				500	00	CHEQUE – DEPOSIT +	500	00
		FOR						BALANCE ➜	598	29
30	134	TO Manitoba Electric Co.	92	84				CHEQUE – DEPOSIT +	92	84
		FOR						BALANCE ➜	505	45
30	135	TO Rent	475	00				CHEQUE – DEPOSIT +	475	00
		FOR						BALANCE ➜	30	45
July 02		TO Deposit				136	00	CHEQUE – DEPOSIT +	136	00
		FOR						BALANCE ➜	166	45

Answer:

STATEMENT OF RECONCILIATION

FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
114.50	*166.45 - 3.00 = 163.45*
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES made which were not on the statement.
500.00	*19.21*
136.00	*92.84*
	475.00
TOTAL: *750.50*	TOTAL: *750.50*

Grade 11 Essential Mathematics (30S)

Module 5
Relations and Patterns

Module 5:
Relations and Patterns

Introduction

In this module, you will study mathematical relationships that are linear. An everyday example of a linear relationship is the relationship between the number of hours you work and the number of dollars you earn. You will express mathematical relationships in four ways:

- word statements
- table of values
- equations
- graphs

You will also show how a change in one quantity (e.g., hours worked) affects the change in another quantity (the number of dollars earned).

This module is challenging for many students, so it is very important to ask questions if there is something you do not understand. Also, you may call your tutor/marker or ask your learning partner for help.

Assignments in Module 5

When you have completed the assignments for Module 5, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Applying Patterns
1	Assignment 5.1	Scatterplots
3	Assignment 5.2	Patterns and Linear Relations
6	Assignment 5.3	Slope
7	Assignment 5.4	Scale

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the following list of instructions to help you with preparing your resource sheet for the material in Module 5. On this sheet, you should record mathematics terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 4, 5, 6, and 7 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 4 to 7.

Resource Sheet for Module 5

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important mathematics terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how each formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Module 5 Cover Assignment: Applying Patterns

Mathematicians have tried to make sense of the world around them by seeing and extending patterns. You studied patterns in school long before you knew that you were doing mathematics. You may realize now that mathematics is basically a study of patterns. In your past you have looked at patterns of days in a calendar, patterns of numbers on a number line, patterns in shapes, patterns in 2-dimensional graphs, and patterns in games and puzzles. Mathematicians are still doing work with patterns today.

This activity requires that you look at the patterns of shapes and the number of each shape element. Analyzing the shapes and describing the relationships between elements in the shapes is doing mathematics.

Notes

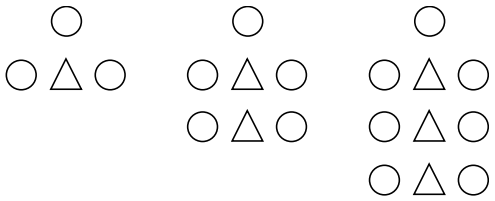
Module 5 Cover Assignment

Applying Patterns

Total: 14 marks

For each of the following problems, show your work. If you do your work on notebook paper, be sure to include it when you send your work to the Distance Learning Unit.

1. Consider the following pattern:



a) If there are 12 triangles in the pattern, how many circles are there? *(2 marks)*

b) If there are 115 circles in the pattern, how many triangles are there? *(2 marks)*

c) If there are 33 triangles in the pattern, how many circles are there? *(2 marks)*

d) Describe the pattern relating the number of circles and the number of triangles. *(2 marks)*

continued

Module 5 Cover Assignment: Applying Patterns (continued)

2. Consider the following pattern made out of toothpicks. The toothpicks create square shapes.

a) How many toothpicks are required to create seven square shapes? *(2 marks)*

b) How many square shapes are created with 52 toothpicks? *(2 marks)*

c) Describe the relation between the number of square shapes and the number of toothpicks. *(2 marks)*

Lesson 1: Scatterplots

Lesson Focus

In this lesson, you will

- ❑ identify discrete and continuous data
- ❑ draw scatterplots from data
- ❑ anticipate the shape of a graph that represents a relationship

Lesson Introduction

As Ted was growing up, his parents recorded his height on the kitchen doorpost on each of his birthdays. When he was 16, they plotted this information on a graph as a scatterplot where each point represented his height at a certain age. The pattern of points on the scatterplot showed how his height changed over time.

In Module 3, you drew circle graphs, bar graphs, histograms, and line graphs. In this lesson, you will draw and interpret **scatterplots**. A scatterplot is a graph with points that represent specific data. The points *may or may not* be connected with line segments, depending on whether the data is discrete or continuous. Scatterplots show the relationship between two sets of data.

Graphing Data

One of the goals of graphing data is to visualize a relationship between two variables. A graph is a visual representation of a numerical relationship. It is possible to predict the value of one variable when the value of the other variable is known.

In order to make such predictions, we begin by drawing the graph of a scatterplot from a data set relating two variables.

Creating a Good Scatterplot

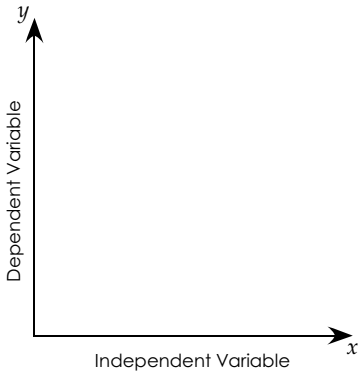
Creating a scatterplot for a set of data is very similar to drawing a line graph. Like the line graph, the **independent variable** (the variable being manipulated) is labelled along the *horizontal axis*, which we will now call the x-axis. The **dependent variable** (the variable whose value is affected by the independent variable) is labelled along the *vertical axis*, which we will now call the y-axis.

A good scatterplot needs

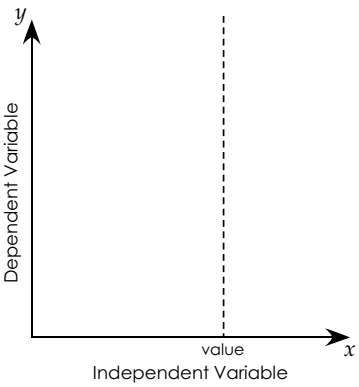
- a title
- labels for the axes
- scales for the axes
- accurately plotted points that represent the ordered pairs (in this example [age, matching height])

Steps for drawing a scatterplot: (Include these on your resource sheet.)

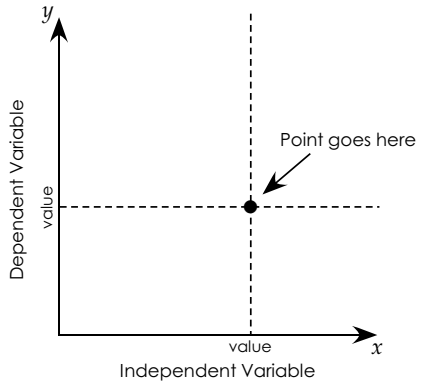
Step 1: Draw your axes and label them. Write a suitable scale on each axis.



Step 2: To plot each point accurately, you must find the location of the independent variable value on the horizontal axis.



Then you find the location of the dependent variable value on the vertical axis.



Draw the point where the two values intersect.

You must repeat Step 2 for *every* data point.

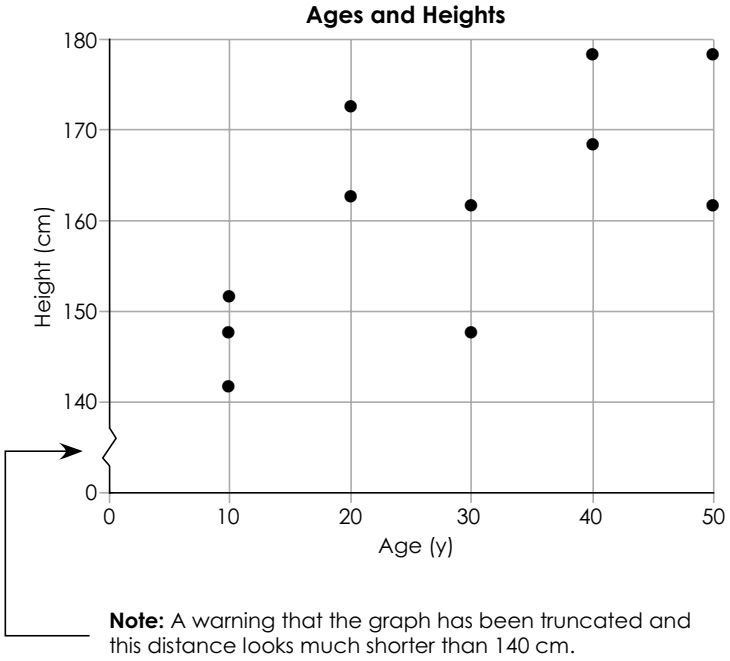
Step 3: Write a title for the graph. It should describe the relationship shown in the graph.

Example 1

Given the following data set, draw a scatterplot.

Age (years)	50	10	20	30	20	40	50	10	30	40	10
Height (cm)	162	152	173	147	163	168	178	142	162	178	147

Solution



Graphing with Technology

The steps needed to draw a scatterplot with a computer are very similar to the steps required to draw a line graph in Module 3. If you are drawing your graph using *Microsoft Excel*, you select "XY (scatter) chart" instead of a line graph.

Types of Data

As you learned in Module 3, some graphs represent **continuous data** while other graphs display **discrete data.** Scatterplots can be used to represent either type of data.

Discrete data

When you use *discrete data*, there are no decimal or fractions between the whole numbers, so the graph will *only* have *points* representing the data. Some examples of discrete data include the number of students registered in a course, the number of times a volleyball is touched before it goes to the other end of the court, the number of stitches in a scarf, or the number of modules in this course. Can you think of any other examples?

Continuous Data

When you have *continuous data*, you can have fractions and decimals between each whole number, so you can use a *line* to represent the relationship between the variables. Some examples of continuous data include the amount of milk in a glass, the amount of gasoline in your tank, or the distance travelled by an ant to get food. Can you add to this list?

When you draw a graph of continuous data, you can represent all the points that fall between the data points by drawing line segments connecting the dots.

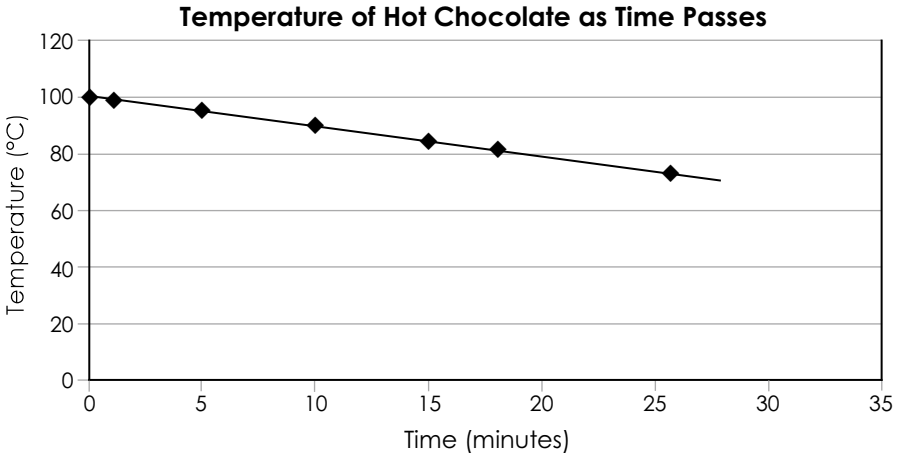
Note that for discrete data, you **count** the whole **number** of items, and for continuous data, you measure the amount of material with a certain precision (nearest tenth or nearest hundredth, etc.).

Example 1

Goldie Locks hates it when she burns her mouth on hot chocolate in winter, but she also doesn't like to drink it cold. Goldie decides to conduct an experiment to find out how long she should wait before she drinks her hot chocolate, so that it is not too hot and not too cold. Draw a graph of the data.

Time Elapsed (minutes)	0	1	5	10	15	17	26
Temperature of the Hot Chocolate (°C)	100	99	95	80	85	83	74

Solution



In this case, the points are all on the line. This will not always be the case. It is okay if some of the points are slightly beside the line. Since temperature and time can have decimals and fractions between the measured values, the data is continuous. As such, the points are joined by a line.

Anticipating Graphs

Now that you have become acquainted with scatterplots, try to predict the shape of the graph that best describes the data.

Example 1

Match each example with the most suitable graph. Explain your choices.

A. The height of a ball at the top of each bounce after it has been dropped.

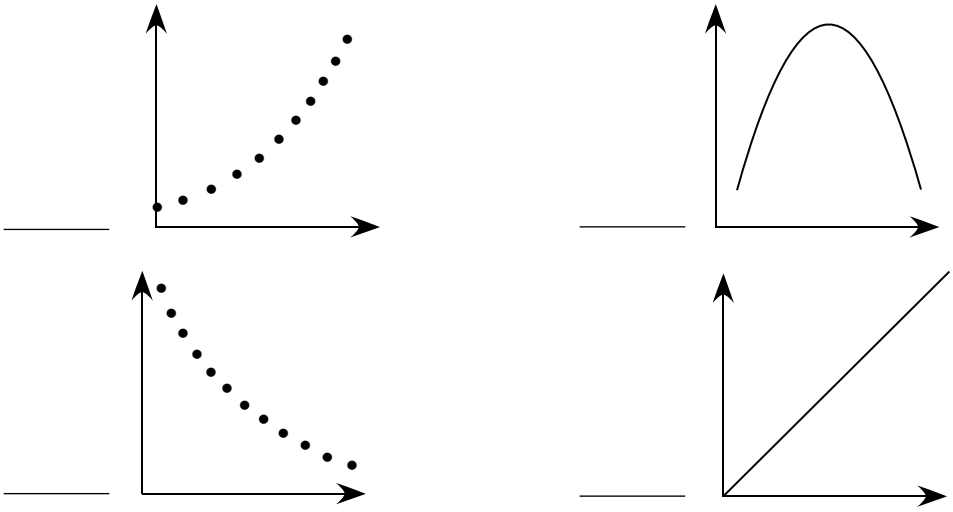
B. Your speed when you drop into a half-pipe and come up on the other side.

A half-pipe looks like this:

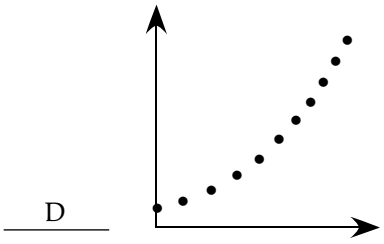
Source: Photo by Jeff Sandquist, reprinted from www.flickr.com/photos/jeffsand/166204003 under the terms of the Creative Commons Licence Agreement.

C. The amount of money you earn compared to the number of hours you work.

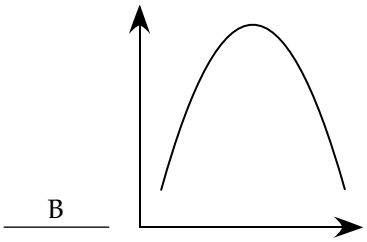
D. The value of an investment (with interest compounded annually) compared with the number of years the money is invested.



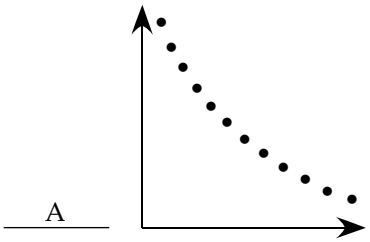
Solution



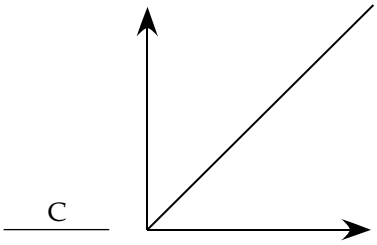
The amount of interest increases each time the interest is compounded. It is discrete because you receive specific amounts of interest at certain times of the year.



Your speed increases rapidly as you go down, and then decreases rapidly as you go up the other wall. It is continuous because your speed changes constantly.



The height of the ball will decrease with every bounce once it has been dropped. It is not continuous because there is only one maximum height per bounce.



As you spend more time working, you earn more money. The graph is continuous because you can be paid partial dollars for partial hours worked.

As you can see, we can

- predict the shapes of different graphs
- distinguish between continuous and discrete data

Here are some questions you should ask yourself when predicting the shape of a graph.

- Is the graph going to increase or decrease from left to right?
- Is the data continuous?
- Is the data going to form a straight line or will it be something else? Try to estimate what it will look like if it is not a line.

You may want to include these questions on your Module 5 Resource Sheet.

Now that you have studied scatterplots, it is time to do some practice questions.

Learning Activity 5.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Solve for h: $2h - 5 = 17$
2. Solve for m: $\frac{m}{6} = \frac{9}{54}$
3. Evaluate and reduce to lowest terms: $\frac{7}{4} \times \frac{2}{7}$
4. Complete the pattern: 1, 4, 9, 16, ____ , ____
5. Samia has four brothers. Her parents had four sons and one daughter. How many sisters does Samia have?

Part B: Scatterplots

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. For each relationship, state the independent and the dependent variables.
   a) The cost of an mp3 player compared to its memory.
      independent _____
      dependent _____
   b) The processing speed of your computer compared to the memory available on the hard drive.
      independent _____
      dependent _____

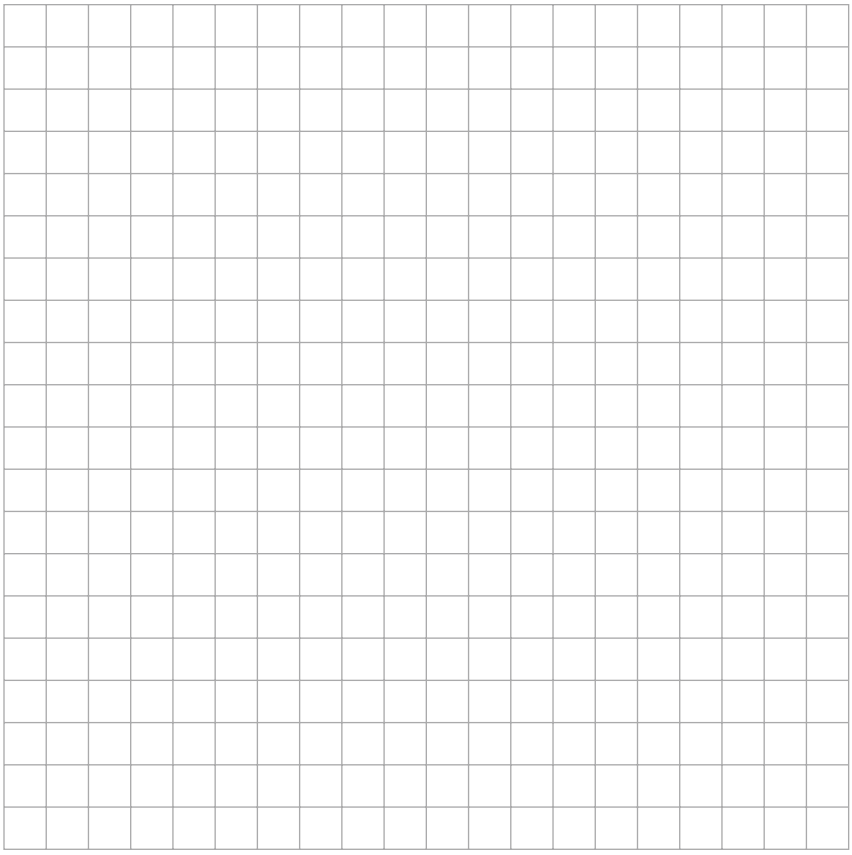
continued

Learning Activity 5.1 (continued)

2. State whether the following situations are continuous.
   a) The final examination mark and average quiz marks for the students in a Grade 11 math class.
   b) The price of a vehicle compared to its age.
   c) Productivity of a factory and the number of workers working.

3. a) Draw a scatterplot for the following data.

Depth of the Ocean (m)	0	20	100	150	200
Temperature of the Water (°C)	20	18	10	5	1

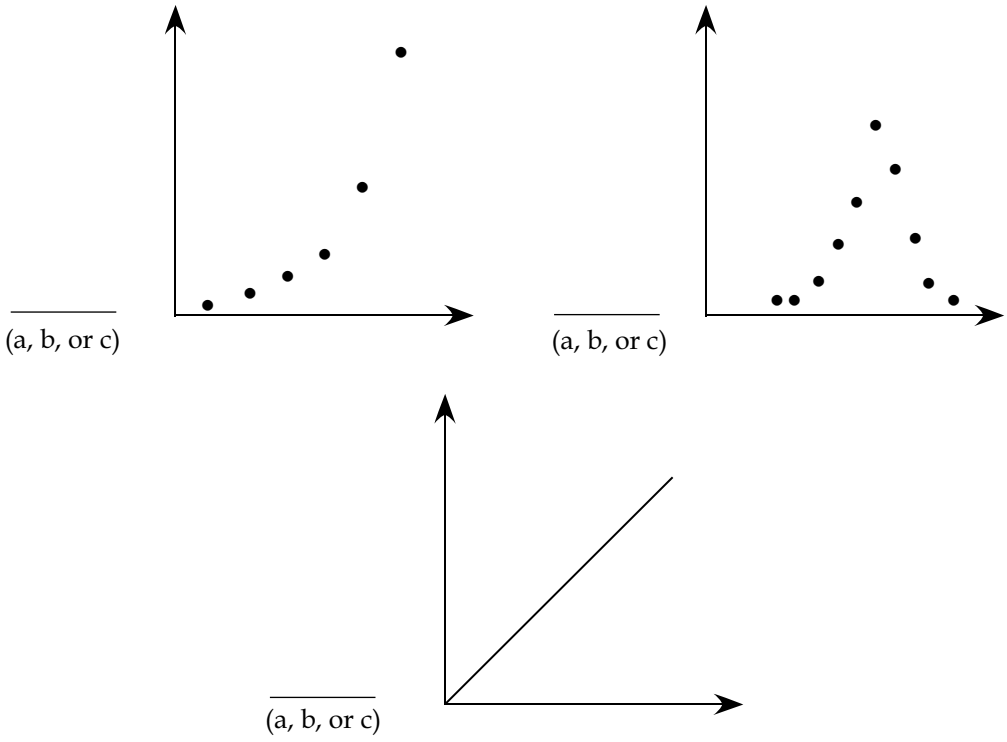


   b) Explain why you did/did not connect the dots for this graph.

continued

Learning Activity 5.1 (continued)

4. Match each situation with the most appropriate graph. Place (a), (b), or (c) in the space provided beside each graph below.
   a) For every $100 CDN you exchange at the bank, you receive $94 USD in return. Compare dollars CDN versus USD.
   b) Jerome has two children, and then his children have two children each, and so on. Compare number of children versus time.
   c) The class average was 85%. Only one person got 60 percent, and only four people got 92%. Compare mark percent versus frequency.



_____
(a, b, or c)

_____
(a, b, or c)

_____
(a, b, or c)

Lesson Summary

In this lesson, you identified data as being discrete or continuous, and drew appropriate scatterplots to represent this data. You also selected graphs that described mathematical relationships when there was no data. In the next lesson, you will examine patterns that can be seen when looking at a graph.

Assignment 5.1

Scatterplots

Total: 17 marks

Note to Students: Use your resource sheet to help you complete this assignment. Do not hesitate to add to your resource sheet as you answer the following questions. This is one of the assignments that must be sent to the Distance Learning Unit as part of your Module 5 evaluation.

1. State whether the following data is continuous or not. Explain why. *(3 marks)*

   a) Your speed related to the distance as you approach a red light.

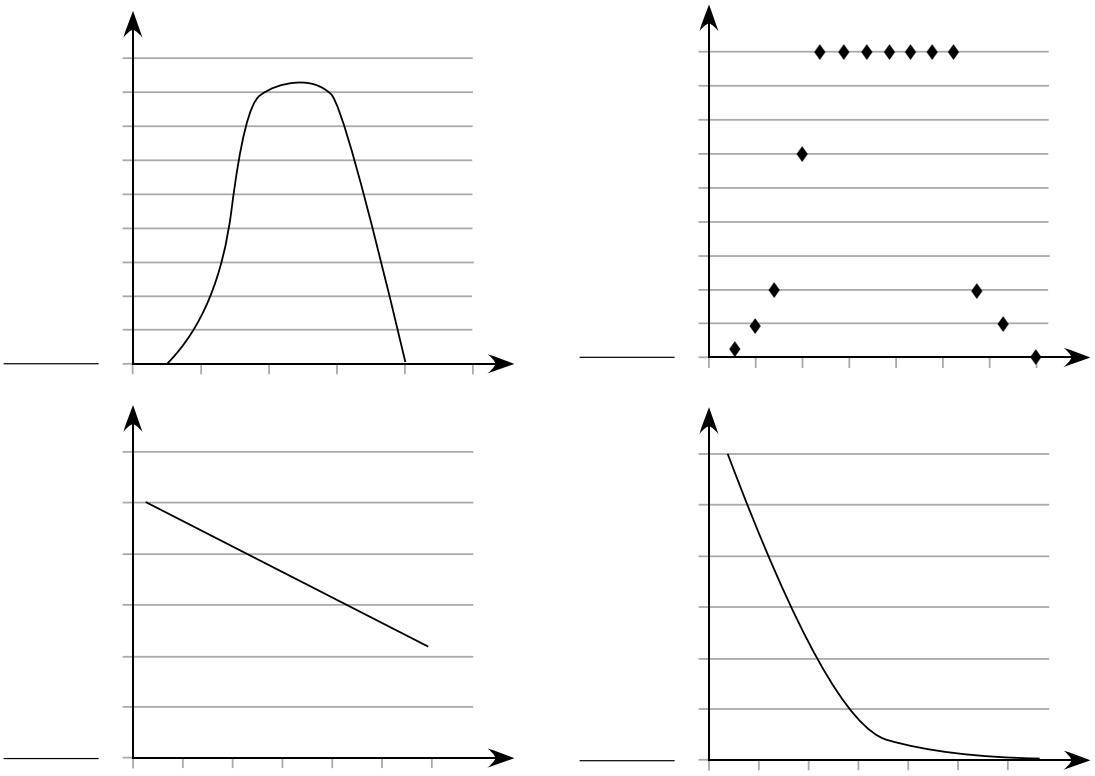
   b) The number of people in a movie theatre related to time before, during, and after a movie.

   c) The height of an airplane related to time as it lands.

continued

Assignment 5.1: Scatterplots (continued)

2. Match each of the following graphs with one of the situations from question #1. Write the letter (a, b, or c) in the blank beside the graph. One of the graphs does not relate to any of the situations. *(3 marks)*



3. Skeeter Plot has done an experiment in chemistry. He did the experiment five times so that he could check his results. The data he collected from the experiment is as follows:

Amount of salt used (g)	1.1	2.5	3.2	4.1	5.0
Time it took to dissolve (s)	10	25	30	40	50

a) State the independent variable and the dependent variable. *(2 marks)*

independent _____

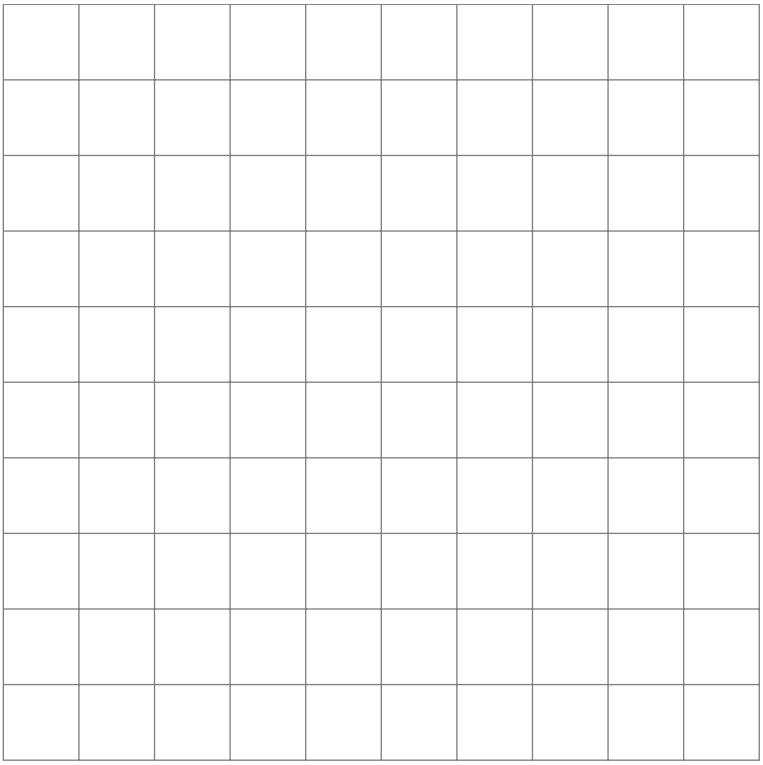
dependent _____

continued

Assignment 5.1: Scatterplots (continued)

b) Will the graph be continuous or not? Why? *(2 marks)*

c) Draw a scatterplot to represent the data. *(7 marks)*



Notes

Lesson 2: Patterns in Data

Lesson Focus

In this lesson, you will

- ❑ identify linear patterns in data, including data presented as a graph or in a table of values
- ❑ study some common characteristics of linear relationships

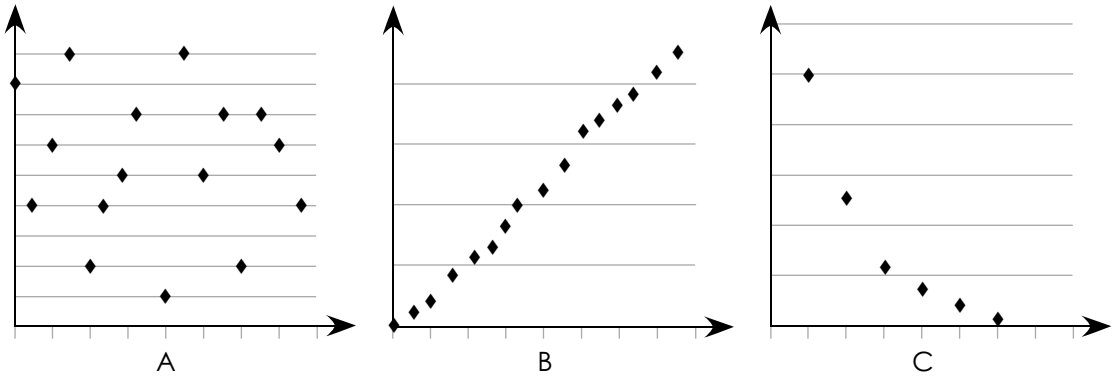
Lesson Introduction

Tess is shopping for a computer. She wants to determine whether the price increases at a constant rate as the amount of memory in the computer increases. If it does, then the relationship between the cost of the computer and the amount of memory is linear.

In the previous lesson, you drew scatterplots to display mathematical relationships. In this lesson, you will determine whether a relationship is linear, non-linear, or whether there is no identifiable pattern.

Recognizing Patterns

The graphs that represent data do not always show any distinct patterns like the ones in Lesson 1. How would you describe the following graphs?



The graphs could be described as follows:

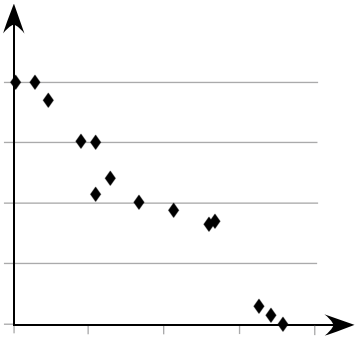
- Graph A has **no pattern**. The points seem to be randomly plotted on the grid.
- Graph B is **linear**. Although the points are not exactly in a straight line, you can see that it is very close to being a straight line. A trick to remember the meaning of linear is to think the word as "LINEar."
- Graph C is **non-linear** because it is not a straight line. It does, however, show a distinct pattern, and you could still predict (with some accuracy) where the next point should go.

Maybe you should include these three patterns on your resource sheet.

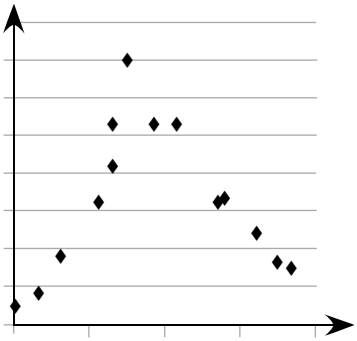
Example 1

State whether each graph is linear, non-linear, or does not show any pattern:

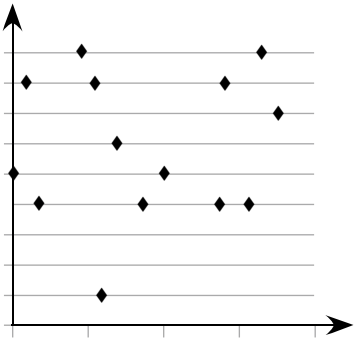
a)



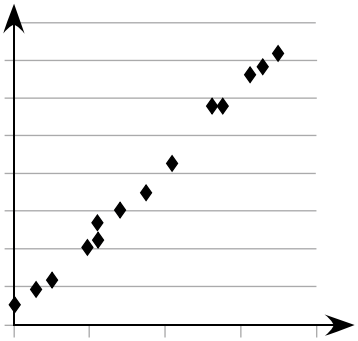
b)



c)



d)



Solution

a) Linear: Even though it is decreasing, it still forms a line.

b) Non-linear: The graph is clearly not linear, but it seems to have a pattern.

c) No pattern: The points appear random.

d) Linear: The points form a line.

Characteristics of Linear Relations

It is easy enough to see from a scatterplot whether or not a graph is linear, but there are ways to determine whether a table of values represents a linear relationship. First, read the following definition of *linear relation*.

> A linear relation is a set of data that, when plotted on a graph, looks as though a line could be drawn through it to represent the data.

Include this definition on your resource sheet.

Graphs

As you can see from the definition, linear relations are seen as lines on graphs. *The data does not necessarily need to create a perfect line.* There may be slight variations, but overall the points should form a straight line. You can estimate what the line should look like by drawing a **line of best fit** (a line of best fit is a line drawn on a scatterplot that describes the approximate relationship of the data; not all the points have to be on the line, but they should be close).

Example 1

Suppose you collected data to find the relationship between the height and weight of male students in your school.

Height (inches)	Weight (pounds)
61	110
69	150
71	180
62	120
65	140
73	200
65	130
72	165
70	180

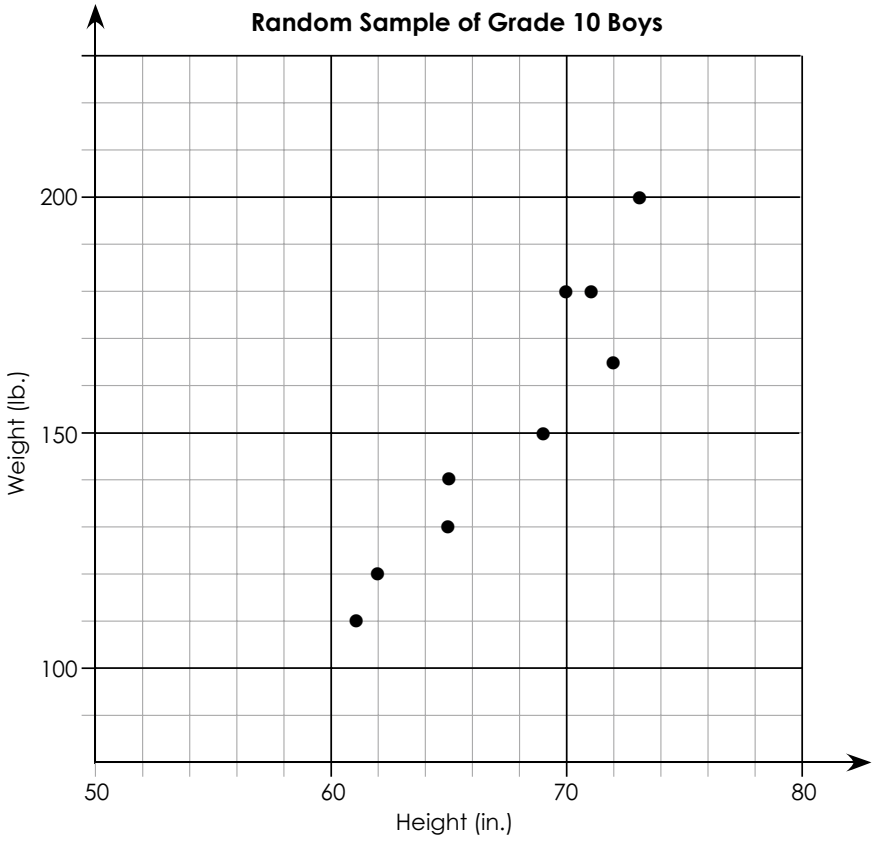
a) Draw a scatterplot of the data by hand or by using a graphing program.

b) Draw a ”line of best fit” to represent the data.

Solution

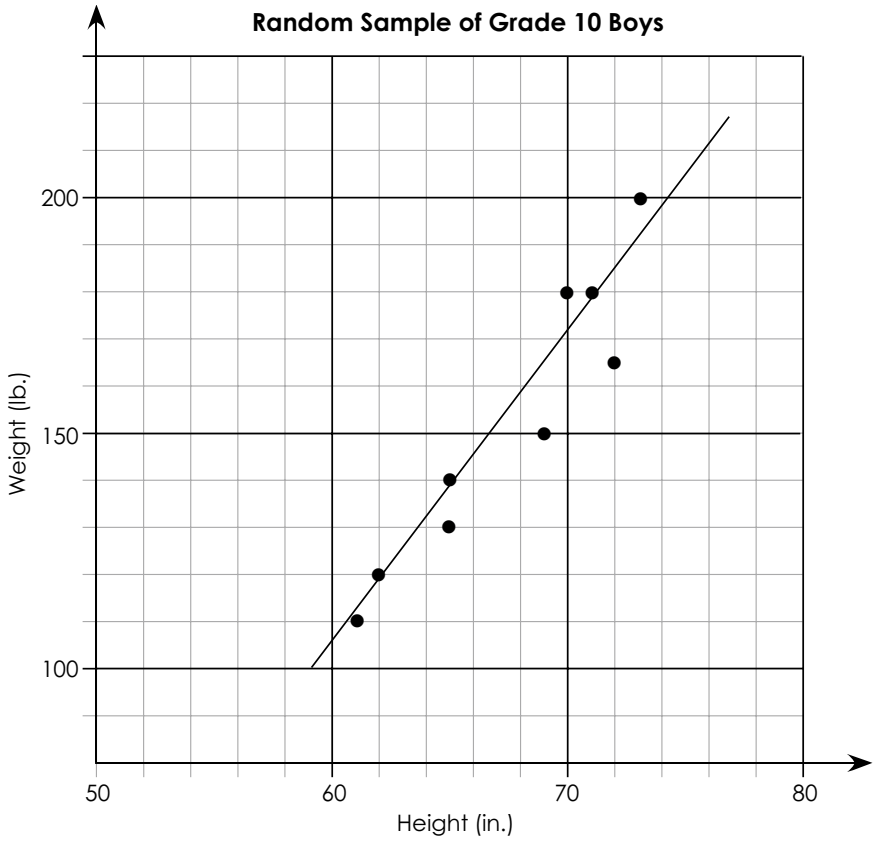
a) If you need help using technology to draw the graph, call your tutor/marker or ask your learning partner for help. You may also review Module 3, Lesson 4. Note that, if you use *Microsoft Excel*, you must select the "XY (scatter) chart" instead of the "line graph icon" to draw a scatterplot instead of a line graph.

If you are using a scientific or graphing calculator to draw a scatterplot, you may want to record the keying sequence on your resource sheet.



b) We can see that, for the most part, the taller a male student is, the more he weighs.

The line of best fit allows us to draw conclusions about the graph and make predictions. To best represent the data, the line of best fit should be drawn through as many points as possible. There will usually be some points that do not fall on the line of best fit. About half of the remaining points should be above the line, and the other half of the remaining points should be below the line. The line does *not* have to go through the origin, and it does not even have to pass through any of the points. It may look something like this:



Include the characteristics of a line of best fit on your resource sheet.

Number Patterns

If you are given the set of numbers **2, 4, 8, 16…** and you are told to write the next three numbers in the pattern, how might you find these numbers?

Perhaps there is a certain number that you can always add to the previous number to get to the next number?

2 + 2 = 4 **4** + 2 = 6 So this doesn't work.

So, if you multiply the previous number by itself each time, will you get the next number?

2 × 2 = 4 **4** × 4 = 16 No, this doesn't work.

Or if you multiply the previous number by a certain number, will you get the next number?

2 × 2 = 4 **4** × 2 = 8 **8** × 2 = 16 This does work!

The pattern 2, 4, 8, 16 … is known as a *geometric sequence*. You do not study these patterns in this course. They are taught in Grade 11 Pre-Calculus Mathematics.

Instead, we will focus on **arithmetic sequences** or **linear patterns**. These are sequences such as

1, 5, 9, 13, 17 … 6, 8, 10, 12, 14 … or 21, 20, 19, 18, 17 …

Linear patterns are seen when a set of numbers is made by adding or subtracting the same number each time.

add 1:	1 + 4 = 5	5 + 4 = 9	9 + 4 = 13	13 + 4 = 17	…
add 2:	6 + 2 = 8	8 + 2 = 10	10 + 2 = 12	12 + 2 = 14	…
subtract 1:	21 − 1 = 20	20 − 1 = 19	19 − 1 = 18	18 − 1 = 17	…

Example 2

Identify the pattern as linear or not linear. If it is a linear pattern, which number are you adding each time?

a) 11, 19, 27, 35, 43 …

b) 32, 16, 8, 4, 2 …

c) 1, 0, –1, 0, 1 …

d) 114, 101, 88, 75, 62 …

Solution

a) Linear: add 8 each time

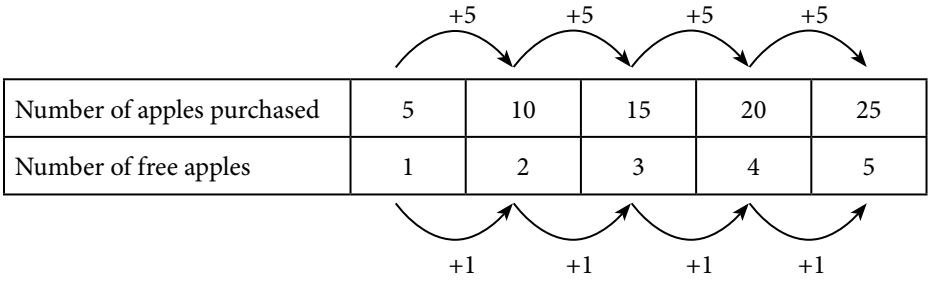
b) Not linear

c) Not linear

d) Linear: add −13 each time ***or*** subtract 13 each time (both are correct)

Table of Values

You have already used a table of values in this module. Data is frequently presented in this way. When you look for patterns in a table of values, you may notice patterns similar to the ones listed as number patterns. Note also that the independent values are in the top row, and the dependent values are in the bottom row.



The number of apples purchased goes up by 5 each time, and the number of free apples goes up by 1 each time. So, if you look at the categories separately, they are both linear patterns. Therefore, you can call the whole set of data a linear relationship. For every increase by 5 purchased apples, the number of free apples increases by 1.

There is also a pattern connecting the number of apples purchased to the number of free apples. Can you see what it is?

You could describe it by saying, "For every 5 apples you buy, you get 1 apple free." Or you could describe it using mathematical language, and say "The number of apples purchased *divided by 5* equals the number of free apples."

Number of apples purchased		Number of free apples
5	÷ 5	1
10	÷ 5	2
15	÷ 5	3
20	÷ 5	4
25	÷ 5	5

Because we know this, we can predict how many free apples we would get if we bought 100 apples!

100 ÷ 5 = 20 free apples

When you set up the proportion, you must have the top units the same (apples purchased) and the bottom units the same (free apples).

You can then solve for x:

$$\frac{5}{1} = \frac{100}{x}$$ Cross-multiply.

$$(5)(x) = (100)(1)$$

$$\frac{5x}{5} = \frac{100}{5}$$ Divide by 5 (to isolate the x on the left-hand side).

$$x = 20$$ Write the answer.

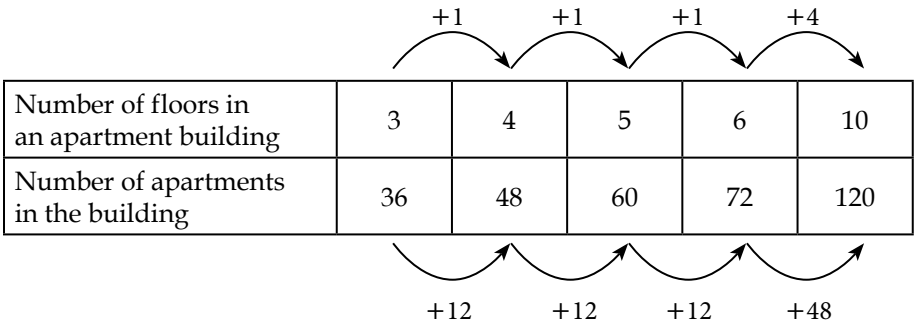
Many students find proportional reasoning challenging, so maybe you should include this example on your resource sheet as a reminder.

Example 3

Does this data represent a linear relationship?

Number of floors in an apartment building	3	4	5	6	10
Number of apartments in the building	36	48	60	72	120

Solution



This relationship seems to be linear from three to six floors, but we need to check whether 10 floors with 120 apartments are part of the linear pattern.

The pattern appears to be: When you increase the number of floors by 1, the number of apartments increases by 12. If you check the last set of differences, they increase by the same proportion:

$$\frac{1}{12} \text{ is the same as } \frac{4}{48}$$

So, the ordered pair (10, 120) is consistent with the linear pattern.

You can determine if a set of data is linear when you calculate the **rate of change** between any two data points. If the rate of change is the same for all possible sets of any two data points, then the relation is linear.

$$\text{Rate of change formula: Rate} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

You should include this on your resource sheet.

In this example, the *number of floors* is the independent variable. We know this because a building can have any number of floors. The *number of apartments* is the dependent variable because it is dictated by the number of floors in the building.

Number of floors in an apartment building	3	4	5	6	10
Number of apartments in the building	36	48	60	72	120

One possible pairing of data is (3, 36) and (4, 48). Using the rate of change formula:

$$\text{Rate} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}} = \frac{48 - 36}{4 - 3}$$

$$= \frac{12}{1}$$

$$= 12 \text{ apartments per floor}$$

Another possible pairing of data is (3, 36) and (10, 120). The rate of change is:

$$\frac{120 - 36}{10 - 3}$$

$$= \frac{84}{7}$$

$$= 12 \text{ apartments per floor}$$

The rate of change is the same for this pair as it was for the other pair. Take a moment and verify that any pair of points produces a rate of change equal to 12. (Do this in your notes.)

Linear relations have the same rate of change for any set of two points in the relation.

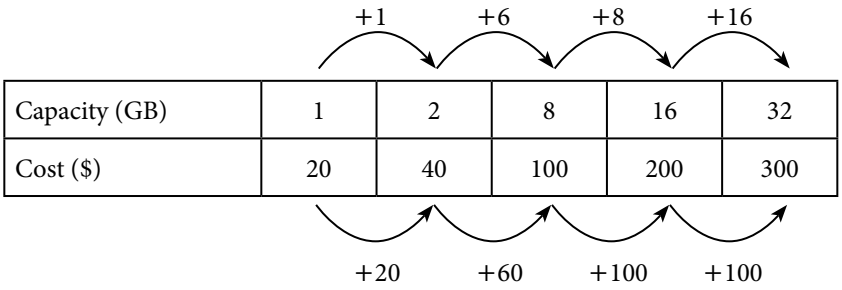
You should include this on your resource sheet.

Example 4

The table of values shows the relationship between the number of gigabytes of memory and cost. Does this data represent a linear relationship?

Capacity (GB)	1	2	8	16	32
Cost ($)	20	40	100	200	300

Solution



Because the capacity does not increase by the same amount each time, we must check whether the ratio is always the same.

The rate of change between (1, 20) and (2, 40) is

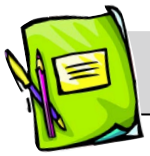
$$\frac{40-20}{2-1}$$
$$= \frac{20}{1}$$
$$= 20$$

But the rate of change between (2, 40) and (8, 100) is

$$\frac{100-40}{8-2}$$
$$= \frac{60}{6}$$
$$= \frac{10}{1}$$

Since the rate of change was 20 initially and changed to 10 later, we know that this is not a linear relation.

Recognizing linear relations in different forms becomes easier with practice. Complete the following learning activity to become more familiar with linear relations. If you have any questions, don't hesitate to ask your learning partner or tutor/marker, or refer to the answers at the end of the module for clarification.



Learning Activity 5.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the pattern: 1, 1, 2, 3, 5, 8, ____ , ____
2. A computer costs $500. Find the cost including taxes (PST = 7%, GST = 5%).
3. If 10% of 360 is 36, then 25% of 360 is _____.
4. Evaluate and reduce your answer if possible: $\frac{2}{5} + \frac{8}{15}$
5. There are three available cubicles in a row at an office for Sam, Xan, and Laura. Sam won't sit next to Xan. Can Sam sit in the middle cubicle?

continued

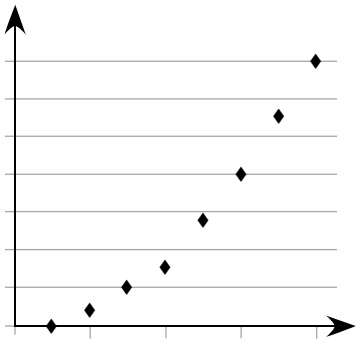
Learning Activity 5.2 (continued)

Part B: Linear Relations

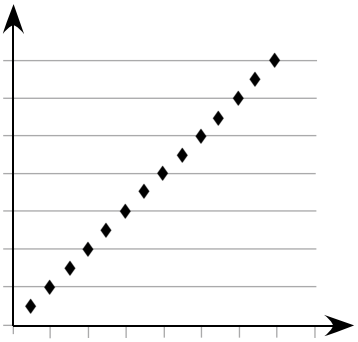
Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. State whether the graph is linear, non-linear, or does not show a pattern.

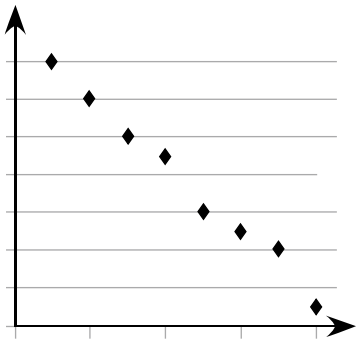
a)



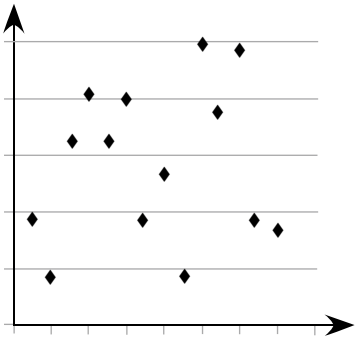
b)



c)



d)



2. State whether each pattern is linear or non-linear. If it is linear, what number do you add each time?
   a) −4, −1, 2, 5, 8 …
   b) 1, 1, 2, 3, 5 …
   c) 1, 4, 16, 64, 256 …
   d) 251, 272, 293, 314, 335 …

continued

Learning Activity 5.2 (continued)

3. Sort the following relations and patterns into linear and non-linear. (Hint: For the graphs, try to draw a line of best fit.)

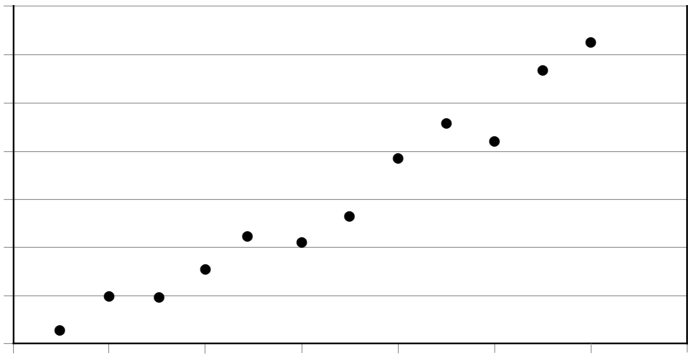
a)

x	1	2	3	4
y	7	14	21	28

b) −30, −19, −8, 3, 14, …

c) −2, −4, −6, −12, −14, …

d)

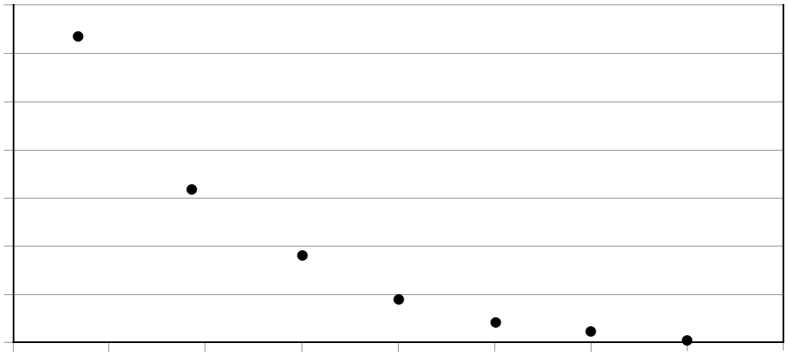


e)

Distance (km)	1	2	5	6
Time (h)	6	12	30	36

f) 6, 0, 2, 1, 0 …

g)



h)

Time (hours)	0	2	4	6
Temperature (°C)	25	23	21	16

Lesson Summary

In this lesson, data that described a relation was presented in two ways: in a table of values and as a graph. You determined whether the data was linear, non-linear, or whether the data did not seem to represent a relationship. To do this, you used a line of best fit for graphs, and proportions and rates of change for data in a table of values.

In the next lesson, you will continue studying linear relationships. You will

- draw a graph of data in a table of values
- write a table of values from data presented as a graph
- write an equation that represents the data

Lesson 3: Equations of Linear Relations

Lesson Focus

In this lesson, you will

- ❑ write equations to describe linear relations
- ❑ study linear relations that are direct or partial variations
- ❑ interpolate and extrapolate linear data

Lesson Introduction

Now that you know how to recognize a linear relation, you need to express this relation in a variety of ways. For example, if you see a graph that shows how incomes are related to education, you may need to express this relationship in words, or you may need to calculate the level of education required to earn a certain income.

Writing Equations

An equation is a mathematical sentence that describes a relation. The equation must include an equal sign (=). The letters in an equation are called **variables** (as we change one variable, another variable also changes), and usually the letters represent words or phrases. For example, in the equation for simple interest:

$I = Prt$

This whole equation is made up of letters, where:

I represents interest

P represents principal

r represents interest rate

t represents time (in years)

If we were to translate this sentence into English, it would say, "Interest is equal to the product of the principal, the interest rate, and the time." Or we could word it differently: "The interest earned on an investment is found by multiplying the principal by the interest rate and the time the investment earns interest." Either way, these words say the same thing as the equation.

The following example shows a relation expressed

- in words
- as a equation
- as a table of values
- as a graph

Example 1

Emma Ploiye is paid an hourly rate of $15. During the first week of November, she works the following hours: Monday, 6; Tuesday, 8; Wednesday, 0; Thursday, 4; Friday, 5.

a) Express the relation between daily gross pay and hours worked in words.
b) Express the relation between daily gross pay and hours worked as an equation.
c) Express the relation between daily gross pay and hours worked with a table of values.
d) Express the relation between daily gross pay and hours worked as a graph.

Solution

a) The relation can be expressed in words as follows: Emma's daily gross pay is equal to $15 multiplied by the number hours she works.

b) The relation can be expressed by the following equation:

gross *pay* = hourly rate of $15 per hour × number of *hours* worked

A shortened version of the equation can be written using letters (variables) to represent the underlined words.

$p = 15 \times h$ or $p = 15h$

Note: In the above example, p and h represent pay and hours. You do not need to use x and y.

c) Using this equation, you can complete the table of values for the hours Emma works in the first week of November. Note that the independent data is in the top row, and the dependent data is in the bottom row.

Day	Mon	Tues	Wed	Thurs	Fri
Hours Worked [h]	6	8	0	4	5
Gross Pay ($) [$p$]					

You can calculate the values for daily gross pay by substituting the corresponding values for hours worked into the equation $p = 15h$. To find Emma's gross pay for Monday, substitute the value of 6 to replace h.

$p = 15h$
$p = 15(6)$
$p = 90$

This value for daily gross pay can be entered into the table of values as follows:

Day	Mon	Tues	Wed	Thurs	Fri
Hours Worked [h]	6	8	0	4	5
Gross Pay ($) [$p$]	90				

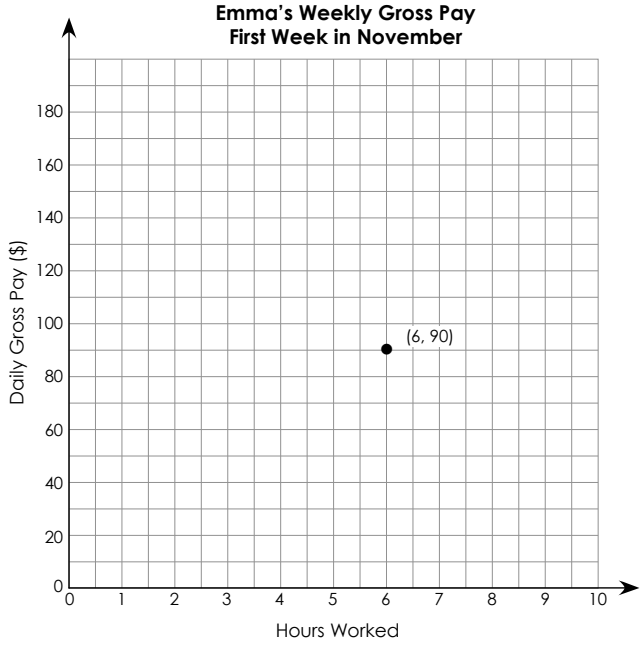
The rest of the values are calculated as follows:

Tuesday	Wednesday	Thursday	Friday
$p = 15h$	$p = 15h$	$p = 15h$	$p = 15h$
$p = 15(8)$	$p = 15(0)$	$p = 15(4)$	$p = 15(5)$
$p = 120$	$p = 0$	$p = 60$	$p = 75$

These values for gross pay can be entered into the table of values as follows:

Day	Mon	Tues	Wed	Thurs	Fri
Hours Worked [h]	6	8	0	4	5
Gross Pay ($) [$p$]	90	120	0	60	75

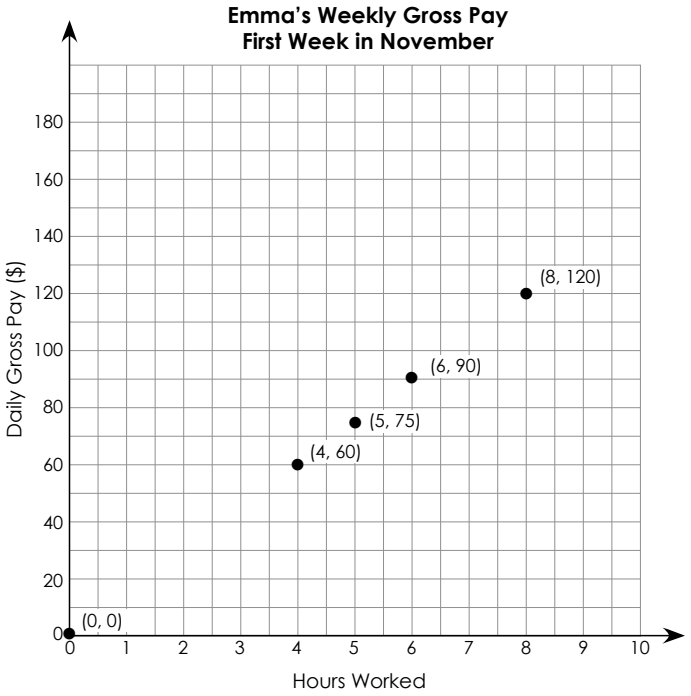
d) The value of a number of hours worked and its corresponding value of gross pay are plotted as one point on the graph. Refer to the table of values. The value of six hours has a corresponding value of $90. These two values are plotted as one point on the graph as follows:



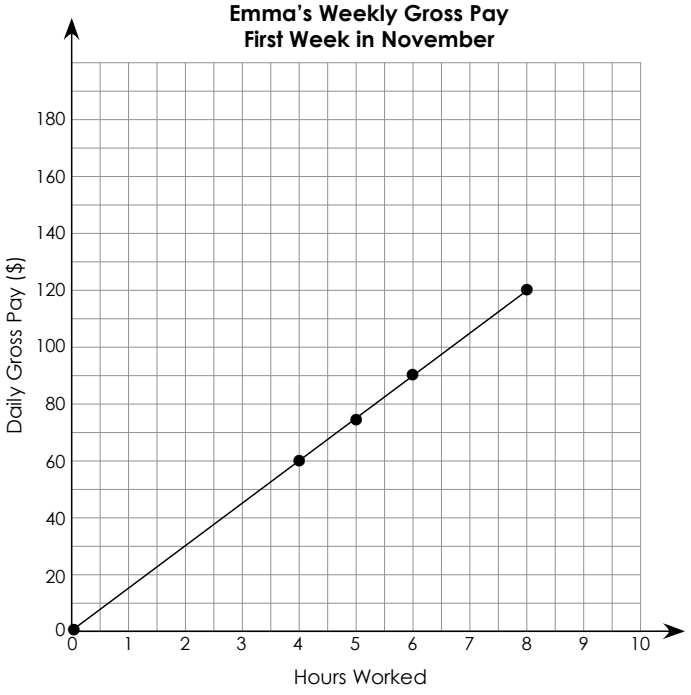
The point is identified by the ordered pair (6, 90).

Note that the independent variable (number of hours worked) is labelled on the horizontal axis, and the dependent variable (gross pay) is labelled on the vertical axis.

Plotting the data from the table of values gives the following points on the graph.



The five plotted points appear to follow a straight line. Using a ruler, draw the line of best fit.



Direct Variation

Consider the graph of the equation $p = 15h$. Did you notice that as the hours increase, so does the gross pay? Also, note that when $h = 0$, then $p = 0$. The variable p is said to vary directly as h, or to be directly proportional to h. A linear relation of the form $p = 15h$ is called a direct variation. Graphs of direct variations of the form $p = 15h$ are straight lines that pass through the point (0, 0), called the origin.

Therefore, the following is true for the graphs of **direct variations**:

- the graph is a straight line
- the graph passes through the point (0, 0), also know as the **origin**
- the graph increases in value as you move to the right along the horizontal axis

In the equation $p = 15h$, the value 15 is known as the **constant of variation**. Note that the constant of variation is just the hourly rate of pay. As an alternative, instead of writing $p = 15h$, you can write the equations as $p = kh$ where k represents the constant of variation.

It would be helpful to include this information about direct variation in your resource sheet.

Example 1

The cost (C) of bottled water in dollars varies directly with its volume (v) in litres. The constant of variation is 2.

a) Express this variation as an equation.

b) Express this variation as a graph.

c) Explain what the constant of variation represents.

Solution

a) Since the cost (C) varies directly with the volume (v), the equation can be written as $C = kv$.

Since the constant of variation, k, is 2, the equation can be written as $C = 2v$.

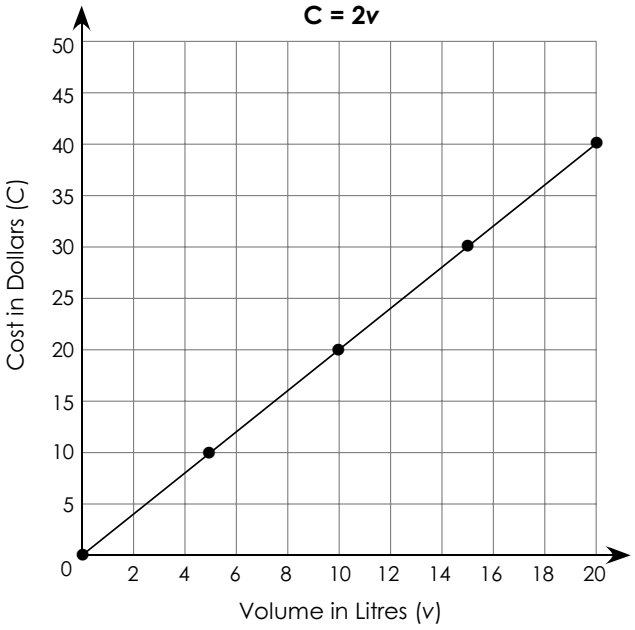
b) To help you create a graph, first construct a table of values as follows. The letters in the square brackets [ ] indicate the variables in the equation.

Volume (L) [v]	0	5	10	15	20
Cost ($) [$C$]					

$C = 2v$	$C = 2v$	$C = 2v$	$C = 2v$	$C = 2v$
$C = 2(0)$	$C = 2(5)$	$C = 2(10)$	$C = 2(15)$	$C = 2(20)$
$C = 0$	$C = 10$	$C = 20$	$C = 30$	$C = 40$

Volume (L) [v]	0	5	10	15	20
Cost ($) [C]	0	10	20	30	40

Use the points you have generated in the table of values to construct the graph. The top row is the independent variable and goes on the horizontal axis.



c) In the example, the constant of variation k is the cost of one litre of bottled water. The cost of one litre of bottled water is $2.

Based on how the constant of variation is described, you can generalize what it represents. *Unit pricing* (cost per unit) is an example of constant of variation. Also, we can say that it *compares the dependent and independent variables*. In the next lesson, you will learn more about the constant of variation.

Partial Variation

Direct variation is not the only type of variation. What if the graph *doesn't* intersect the axes at (0,0)? Consider the following example as an introduction to the second type of variation.

Example 1

Emily is paid $15 an hour. In addition to her hourly rate, she receives a bonus of $50 each day for travel expenses. Express the relation between her daily gross pay (p) and her hours worked (h)

a) as an equation

b) with a table of values

c) as a graph

Solution

a) Since daily gross pay (p) is equal to the hourly rate of $15 times number of hours worked (h), plus $50, the relation can be expressed as the following equation:

$$p = 15h + 50$$

b) Use the equation $p = 15h + 50$ to complete a table of values. Choose the following five values of h. Note that the independent data is in the top row, and the dependent data is in the bottom row.

Time (hr.) [h]	0	2	4	6	8
Gross pay ($) [$p$]					

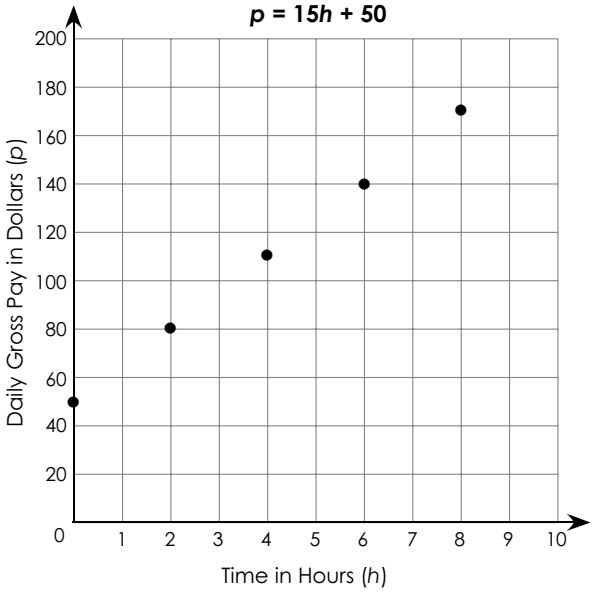
To calculate the values of the daily gross pay, substitute the values of the hours into the equation $p = 15h + 50$.

$p = 15h + 50$	$p = 15h + 50$	$p = 15h + 50$	$p = 15h + 50$	$p = 15h + 50$
$p = 15(0) + 50$	$p = 15(2) + 50$	$p = 15(4) + 50$	$p = 15(6) + 50$	$p = 15(8) + 50$
$p = 0 + 50$	$p = 30 + 50$	$p = 60 + 50$	$p = 90 + 50$	$p = 120 + 50$
$p = 50$	$p = 80$	$p = 110$	$p = 140$	$p = 170$

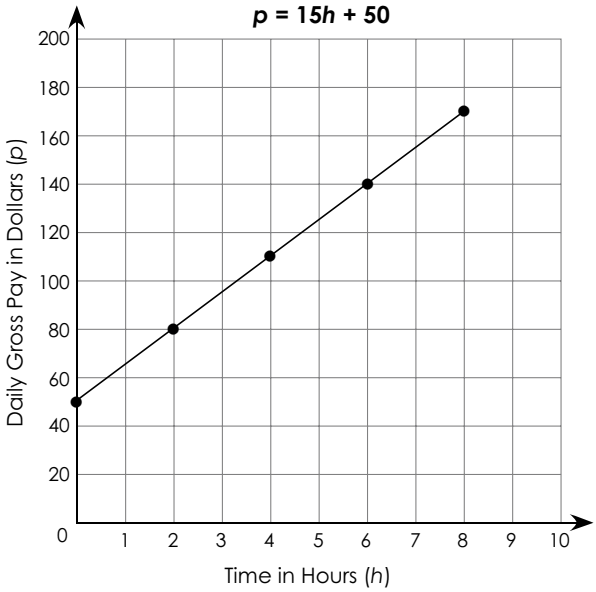
Enter these values for gross pay into the table of values as follows:

Time (hr.) [h]	0	2	4	6	8
Gross pay ($) [$p$]	50	80	110	140	170

c) Since daily gross pay depends on the time (number of hours worked), gross pay is the dependent variable, and time is the independent variable. Plot "time" on the horizontal axis, and "gross pay" on the vertical axis. Choose a scale of 0–10 for the horizontal axis and a scale 0–200 for the vertical axis. The graph should now look like this:



Draw the line of best fit. This is continuous data since you can work parts of hours and earn parts of dollars.



Compare this graph to the graph of Emma's pay in the first week of November (the first example in this lesson). Both graphs are straight lines. However, the graph of $p = 15h$ goes through the point (0, 0), while the graph of $p = 15h + 50$ goes through the point (0, 50).

Linear relations of the form $p = 15h + 50$ are known as **partial variations**. The value 15 represents the constant of variation, k. The value 50 represents the **fixed value**, F. The fixed value is the value of the dependent variable when the independent variable has the value 0. In Example 1, the fixed value tells us that when Emily works 0 hours, she still gets paid $50.

In general, a partial variation is represented by the following equation:

dependent variable = ***k*** x **independent variable + *F***

where k is the constant of variation and F is the fixed value.

Include this equation in your resource sheet.

Consider the following example.

Example 2

To rent a mid-size car, a car rental company charges $30 a day plus five cents per kilometre.

a) Identify the type of variation.

b) Express this variation as an equation.

c) Draw the graph of this relation.

Solution

a) Choose the letters C for the rental cost and d for the distance driven. Since the rental cost (C) depends both on the distance driven (d) and a fixed cost of $30, the relation is a partial variation.

b) The variation can be expressed as the following equation.

$C = 0.05d + 30$ (the 5 cents has been converted to dollar notation)

c) Use the equation $C = 0.05d + 30$ to complete the following table of values.

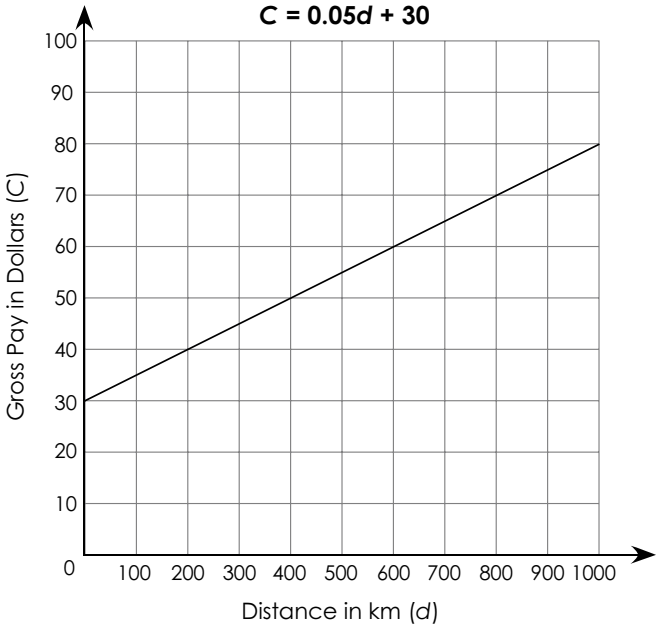
Distance (km) [d]	0	200	400	600	800
Cost ($) [$C$]					

To calculate the values of the gross pay, substitute the values of the hours into the equation $C = 0.05d + 30$.

$C = 0.05d + 30$	$C = 0.05d + 30$	$C = 0.05d + 30$	$C = 0.05d + 30$	$C = 0.05d + 30$
$p = 0.05(0) + 30$	$p = 0.05(200) + 30$	$p = 0.05(400) + 30$	$p = 0.05(600) + 30$	$p = 0.05(800) + 30$
$p = 0 + 30$	$p = 10 + 30$	$p = 20 + 30$	$p = 30 + 30$	$p = 40 + 30$
$p = 30$	$p = 40$	$p = 50$	$p = 60$	$p = 70$

Distance (km) [d]	0	200	400	600	800
Cost ($) [$C$]	30	40	50	60	70

Plot the independent variable, "distance," on the horizontal axis, and the dependent variable, "cost," on the vertical axis. Choose a scale of 0–1000 for the horizontal axis, and a scale of 0–100 for the vertical axis. The graph should now look like this:



Note that this graph of a partial variation is a straight line that passes through the point (0, 30), where $30 is the fixed cost.

Interpolation and Extrapolation

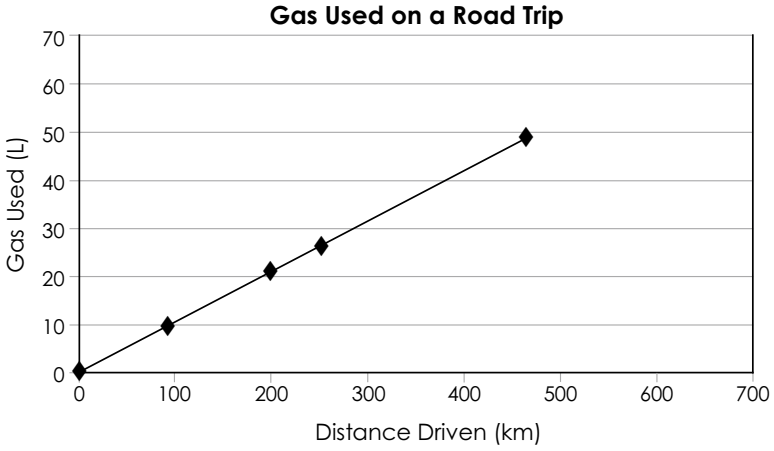
Any linear relation can be expressed in the following four ways:

- in words
- as an equation
- with a table of values
- as a graph

When you are given a linear relation represented in one of the above four ways, you can express it in any of the other ways. The next example will demonstrate how to represent a linear relation, given as a line graph, with a table of values.

Example 1

The following graph displays the amount of gas used on a road trip. The car has a 60 L gas tank.



Use the graph to

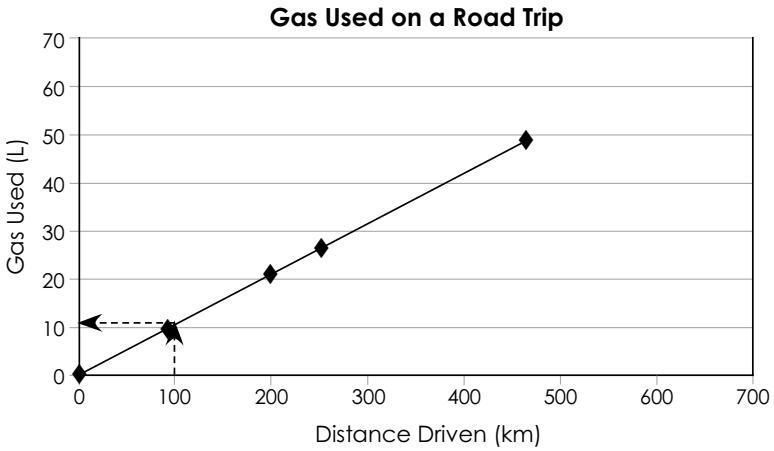
a) complete the following table of values

Distance Driven (km)	100	300
Gas Used (L)		

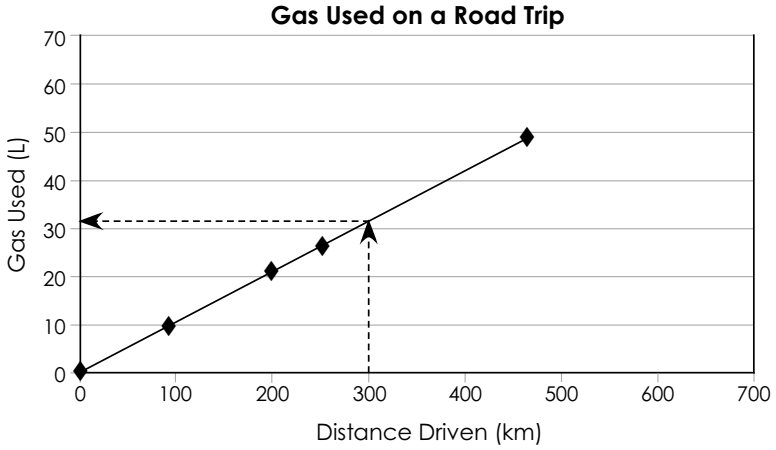
b) determine how far they could drive before running out of gas

Solution

a) To find the gas used to drive 100 km, draw a vertical line up from the *x*-axis at 100 km to the graph. From where the graph and the vertical line intersect, draw a horizontal line to the *y*-axis. It should intersect at approximately 11 L.

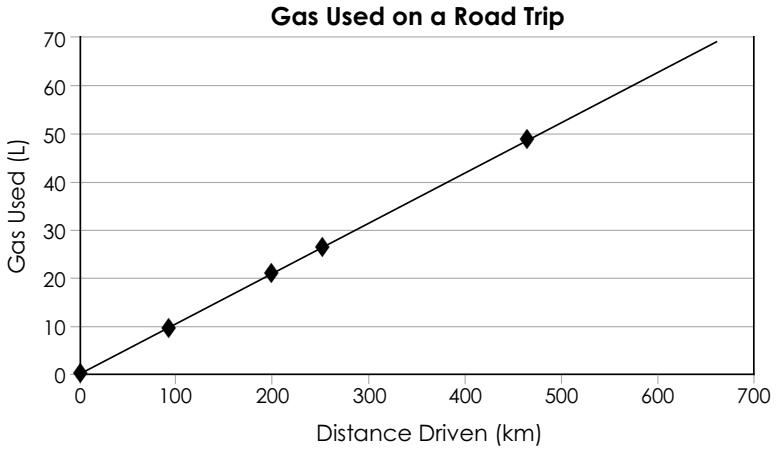


Follow similar steps to find the amount of gas used to drive 300 km. It should be at approximately 31 L.

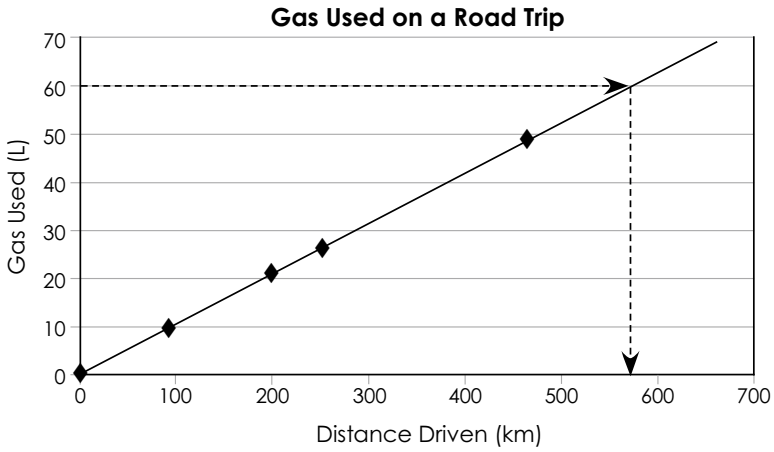


Distance Driven (km)	100	300
Gas Used (L)	11	31

b) Because the car has a gas tank with a capacity of 60 L, the most gas they can use with one tank is 60 L. First, we must extend the graph to the right.

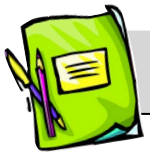


Since we know the amount of gas used, we can draw a horizontal line from 60 L (on the vertical axis) to the graph. From where the horizontal line crosses the graph, draw a vertical line down to the horizontal axis. It should be at approximately 575 km.



As you learned in Lesson 4 of Module 3, the process of determining values between known points on a graph is called **interpolation**. The process of determining values beyond known points on the graph is called **extrapolation**.

The following learning activity will help you put into practice what you have just learned. When you have finished, check your answers in the Learning Activity Answer Key at the end of this module. Talk to your tutor/marker or your learning partner if you cannot do these questions.



Learning Activity 5.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Solve the following: $5 - \frac{4 \times 5}{2} + 8$

2. Write two equivalent fractions: $\frac{20}{8}$

3. At a street vendor, hot dogs cost $4, pop costs $1.50, and chips cost $2.00. You have $6 in your pocket. Will you be able to buy one of each item?

continued

Learning Activity 5.3 (continued)

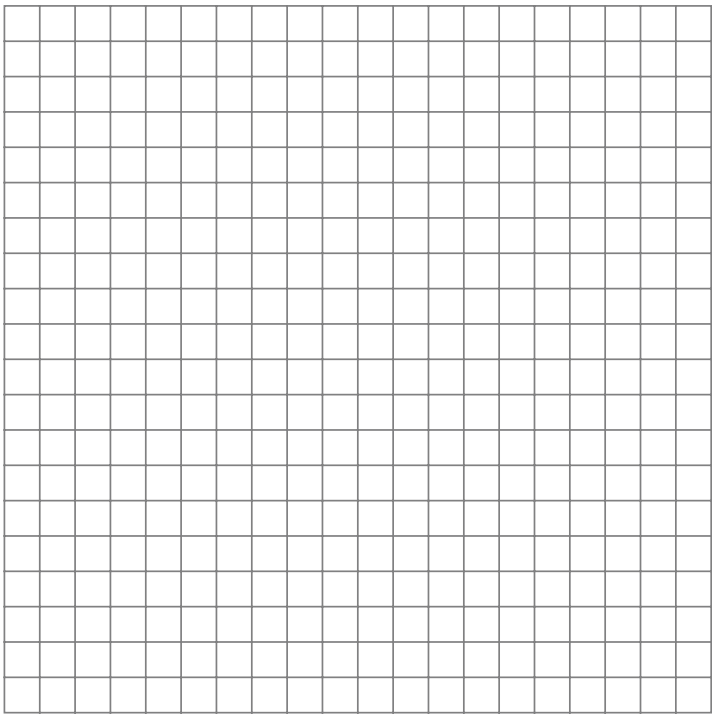
4. Which month can have a whole number of weeks (no decimal or fraction)?

5. Solve for r: $\frac{8}{r} = 2$

Part B: Direct and Partial Variation

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

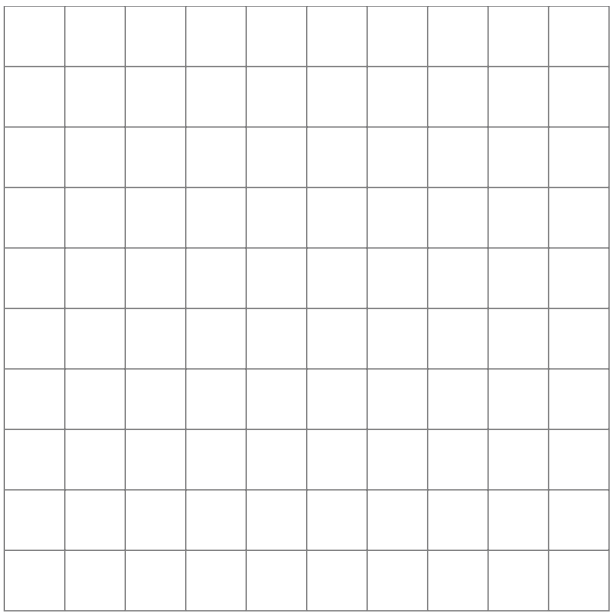
1. Under average road conditions, a vehicle can travel eight kilometres on one litre of gasoline. During the month of August, a vehicle uses the following amounts of gasoline: 30 litres, 45 litres, 10 litres, 40 litres, and 25 litres. Express the relation between the distance the vehicle travels and the amount of gasoline required to travel that distance
   a) in words
   b) as an equation
   c) with a table of values
   d) as a graph



continued

Learning Activity 5.3 (continued)

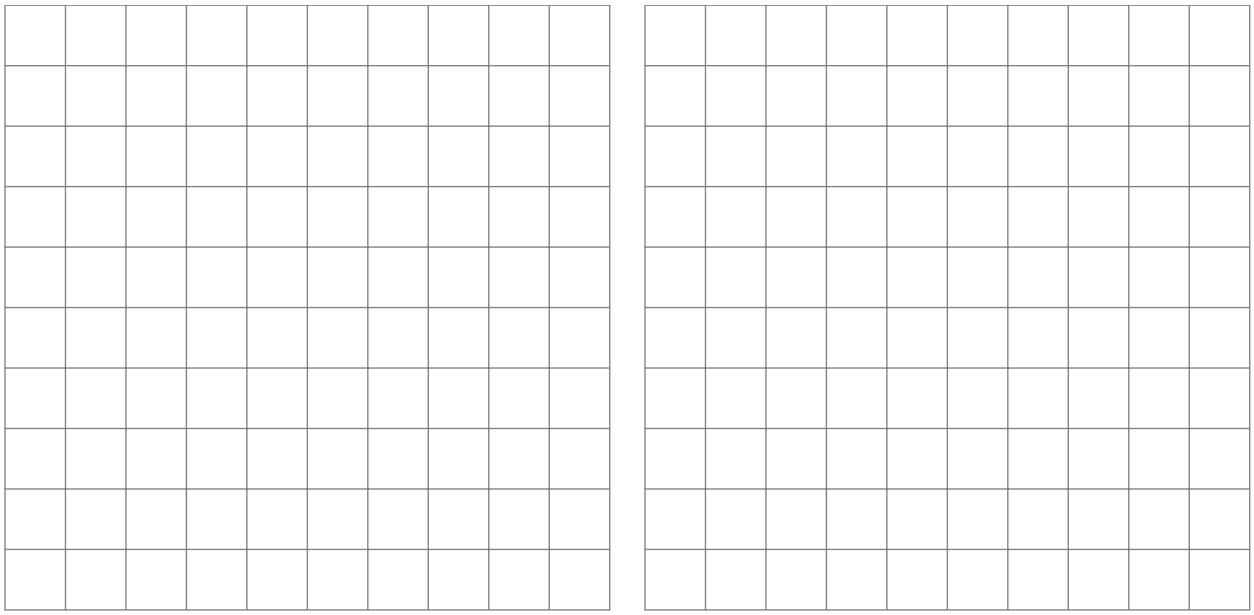
2. The cost of building a highway is $500,000 for each kilometre. Use C to represent the cost, and n the number of kilometres.
   a) State whether this relation is a direct or partial variation.
   b) State the constant of variation and the fixed value (if there is one).
   c) Express the relation as an equation.
   d) Express the relation with a table of values.
   e) Express the relation as a graph (choose appropriate scales for the coordinate axes).



continued

Learning Activity 5.3 (continued)

3. Consider the variations $d = 10t$ and $d = 10t + 40$
   a) Classify each variation as direct or partial.
   b) Create a table of values, and then draw each of the variations as a graph on the given grid lines. Choose appropriate scales for the coordinate axes.



   c) Find the value of d on both graphs when $t = 1$.
   d) Find the value of d on both graphs when $t = 4$.
   e) Indicate one way that the graphs are the same and one way in which they differ.
   f) Explain what the fixed value indicates.

Lesson Summary

In this lesson, you identified linear relations as direct or partial variations. You learned to express a given linear relation in four different ways (words, equation, table of values, and graph). Writing a linear relation in the form of an equation is an important form of communication in mathematics. Also, if you are given an equation describing a linear relation, you can create the appropriate table of values or graph.

In the next lesson, we will examine the constant of variation, exploring what it means and how it affects the graph.

Notes

Assignment 5.2

Patterns and Linear Relations

Total: 28 marks

Note to Students: Use your resource sheet to help you complete this assignment. Do not hesitate to add to your resource sheet as you answer the following questions.

1. State whether the following are linear or non-linear patterns. If they are linear, state what number you add to create the pattern. *(3 marks)*

   a) 101, 92, 83, 74, 65 …

   b) −49, −45, −41, −37, −33 …

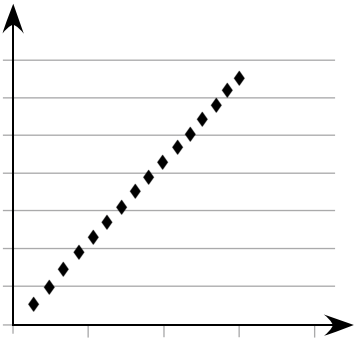
   c) 59, 61, 67, 71, 73 …

continued

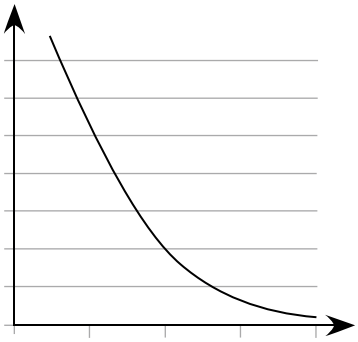
Assignment 5.2: Patterns and Linear Relations (continued)

2. Identify the following as linear, non-linear, or no pattern. *(4 marks)*

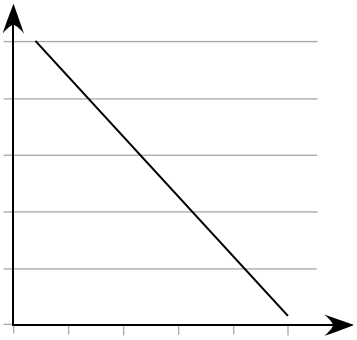
a)



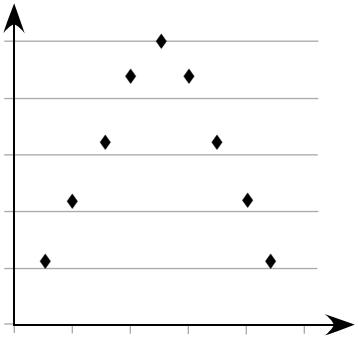
b)



c)



d)



Linear _____

Non-linear _____

No pattern _____

continued

Assignment 5.2: Patterns and Linear Relations (continued)

3. Identify the following as linear or non-linear. If linear, describe the pattern. *(4 marks)*

a)

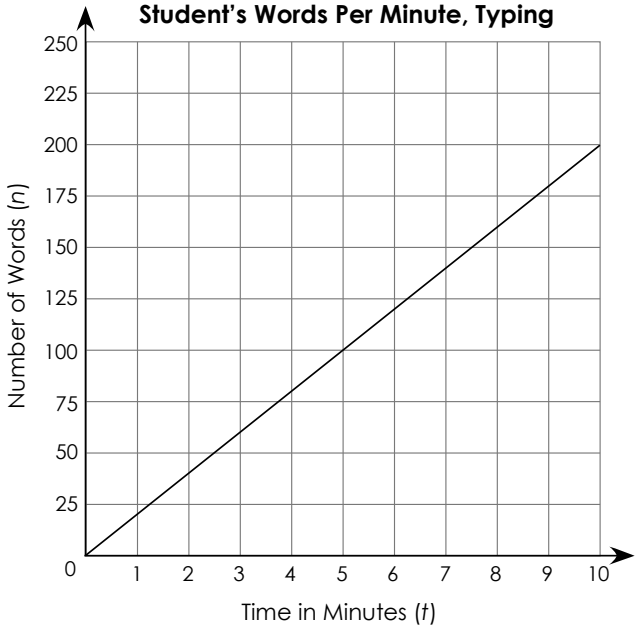
Number of questions answered	20	25	30	40
Mark (%)	40	50	60	80

b)

Canadian Geese in Manitoba	50 000	100 000	100 500	105 000	90 000
Month of the Year	May	June	July	August	September

continued

4. Consider the following graph. The graph expresses the relation between the time a student spends keyboarding and the number of words she keyboards.



a) Identify the type of variation that exists between the variables. Explain how you were able to identify the type of variation. *(2 marks)*

b) Explain what the constant of variation represents in this example. *(1 mark)*

c) Calculate the number of words the student keyboards in eight minutes. *(1 mark)*

continued

Assignment 5.2: Patterns and Linear Relations (continued)

5. The cost of renting a community centre for an event is $100 plus $5 for each person attending the event.

   a) Is the relation a direct or partial variation? *(1 mark)*

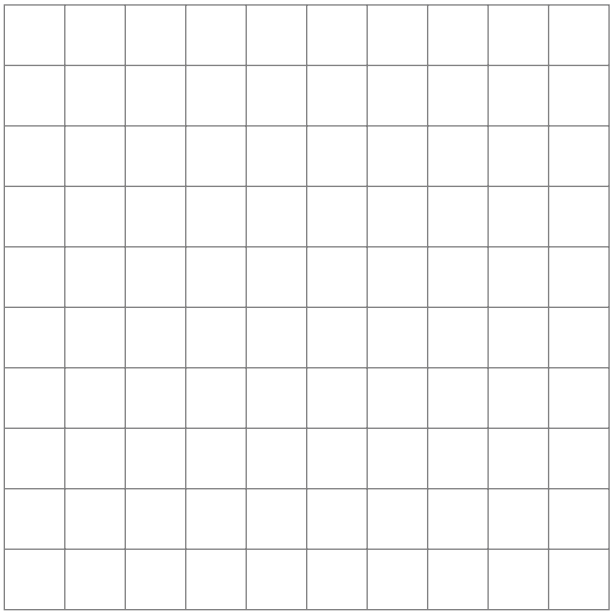
   b) Express this variation as an equation using *C* for cost and *n* for number of people. *(2 marks)*

   c) Create a table of values for 0 to 100 people using an increment of 20. *(3 marks)*

continued

Assignment 5.2: Patterns and Linear Relations (continued)

d) Express this variation as a graph. Choose appropriate scales and labels for the coordinate axes. Also include a title for the graph. *(6 marks)*



e) Explain why the graph passes through the point (0, 100). *(1 mark)*

Lesson 4: Introduction to Slope

Lesson Focus

In this lesson, you will

- ❑ use slope as part of a linear relationship
- ❑ determine the slope of a line
- ❑ identify what it means when lines have different slopes
- ❑ learn the two special cases for slope

Lesson Introduction

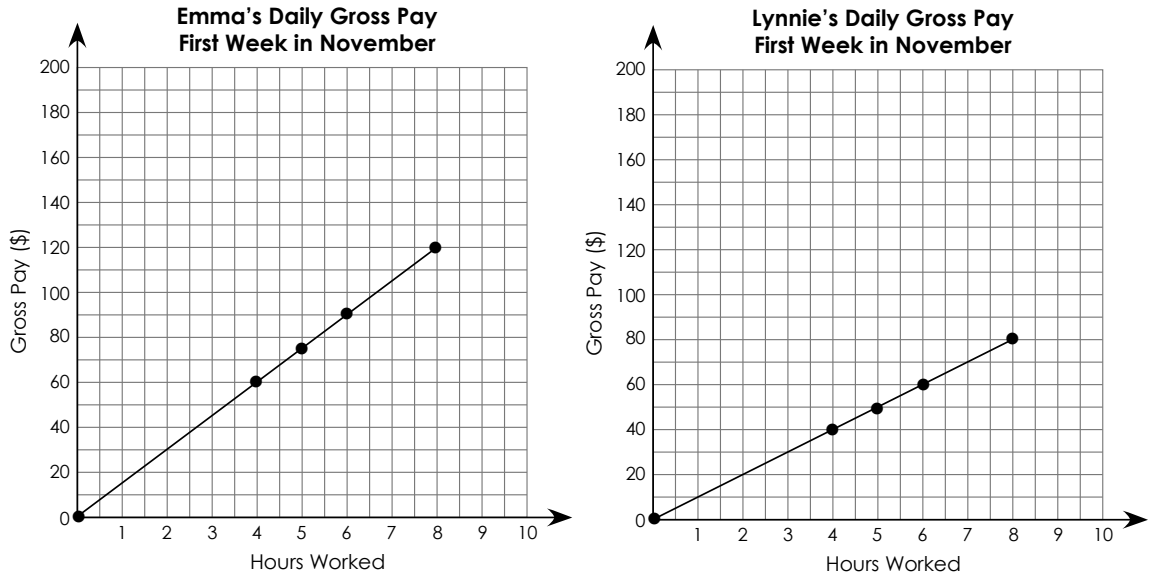
In the previous lesson, you learned how to write an equation to describe a linear relation. One component of each equation was the constant of variation. In this lesson, we will refer to the constant of variation by another name, which is slope.

What do you think of when you hear the word *slope*?

Perhaps you think of mountains (skiing down the slopes) or a skateboard ramp (the steepness of it). Maybe you think of the incline on a treadmill or the slant of your roof. In the following two lessons, you will learn about slope in everyday life. This lesson will be an introduction.

The Slope of a Line

What do you notice about the two graphs below?



There are both similarities and differences between the two graphs, but in this lesson we will focus on slope.

The graph displaying Emma's daily gross pay is much steeper than Lynnie's weekly gross pay graph. This means that Emma's graph has a steeper slope. We now need to describe slope in numerical terms.

Calculating Slope

In the previous lesson, we were told that the constant of variation was part of the equation. To determine the slope of a graph, we must work backwards.

As you work through the following example, you should make notes on your resource sheet about (a) how to calculate the slope of a line based on the graph, and (b) the meaning of slope in a linear relation.

Example 1

Find the slope of Lynnie's graph.

Solution

Before we go any further, we have to first decide whether this graph represents partial or direct variation.

Since the line passes through (0, 0), it is direct variation, so we can use the general form of the direct variation equation to find out the constant of variation: the slope.

The general formula for a direct variation is:

dependent variable = independent variable $\times$ k

To solve for the slope, k, you must choose a point from the graph.

We will use the point (4 hr., \$40) from the graph of Lynnie's Gross Pay.

Enter the coordinates into the general formula. Remember that when you write an ordered pair, it lists the independent then the dependent variable. (independent, dependent).

$\$40 = (4 \text{ hrs.}) \times k$ Divide 4 hours on both sides to isolate k.

$\frac{40}{4} = \frac{4 \times k}{4}$ The 4s cancel on the right side of the equation.

$\frac{40}{4} = k$ Solve for k.

$10k = k$

$k = 10$ dollars per hour

Although you do not include the units in the equation, the slope of Lynnie's line is 10, which means that her wage is \$10 per hour. So for every hour she works, Lynnie earns \$10.

To be sure that this is correct, start at the point (0, 0) on the graph. On the x–axis, move right one hour, then move up \$10. If you are on the line, then this slope is correct. (In this case it is.)

Slope is a number that describes how far a line moves *vertically*, compared to how far it moves *horizontally*. It describes the rate (how fast) at which a line falls ↓ or climbs ↑.

You should include this description of what slope is in your resource sheet.

To complement this description of what slope is, there is another way to calculate it, which works for any linear relation (not just direct variation). Include the steps listed in the following example on your resource sheet.

Example 2

Calculate the slope of Emma's and Lynnie's Gross Pay graphs.

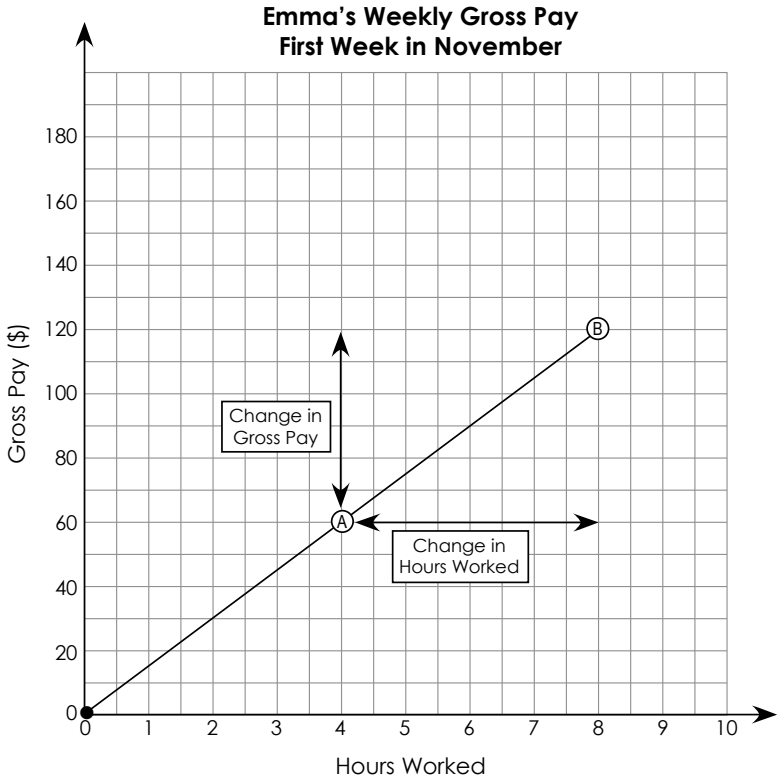
Solution

Emma's graph:

Step 1: Choose two points on the line graph. It is most convenient to choose points that are at the intersection of the two grid lines. Refer to the points chosen as Point A and Point B.

Point A represents a gross pay of \$60 and 4 hours worked.

Point B represents a gross pay of \$120 and 8 hours worked.



Step 2: Calculate the change in gross pay from Point A to Point B. This is the vertical change.

Change in pay = \$120 − \$60

= \$60

Step 3: Calculate the change in hours worked from Point A to Point B. This is the horizontal change.

Change in hours worked = 8 hours − 4 hours

= 4 hours

Note: In steps 2 and 3, the points are in the same order, from Point A to Point B. You *could* calculate the slope going from Point B to Point A, but you *must* do it that way for both steps. If you calculated from Point A to Point B in step 2, and from Point B to Point A in step 3, you would get the positive or negative sign wrong.

In Lesson 2, you already learned the Rate of Change formula:

$$\text{Rate} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

The formula for slope is the same.

Step 4: Use the following formula to calculate the slope of the line graph.

General formula for slope (for any line):

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

For this question, the formula will be:

$$\text{slope} = \frac{\text{change in gross pay}}{\text{change in hours worked}}$$

$$= \frac{60}{4} \qquad \text{Reduce the fraction.}$$

$$= \frac{15}{1}$$

$$= 15$$

Emma's wage is 15 dollars/hour.

Lynnie's graph:

We use the same steps as above, but the explanations will be brief:

Step 1: Choose two points on the line graph.

Two points are: C(4 hours, \$40) and D(8 hours, \$80)

Step 2: Calculate the *vertical* change.

Change in pay = \$80 − \$40 = \$40

Step 3: Calculate the *horizontal* change.

Change in hours worked = 8 hrs. − 4 hrs. = 4 hrs.

Step 4: Use the following formula to calculate the slope of the line graph.

General formula for slope (for any line):

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

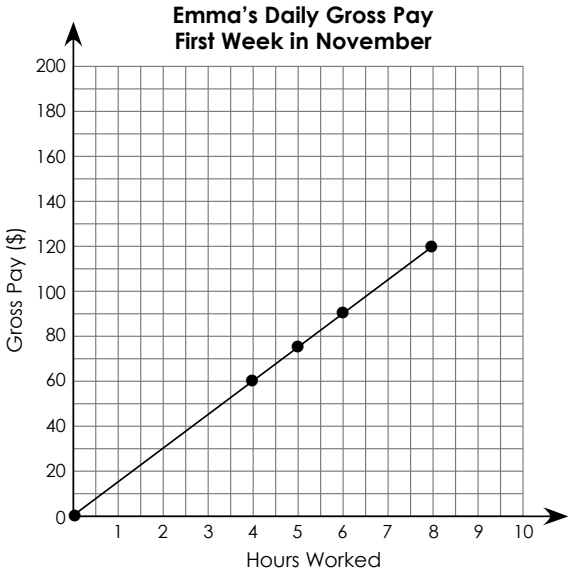
$$= \frac{40}{4}$$

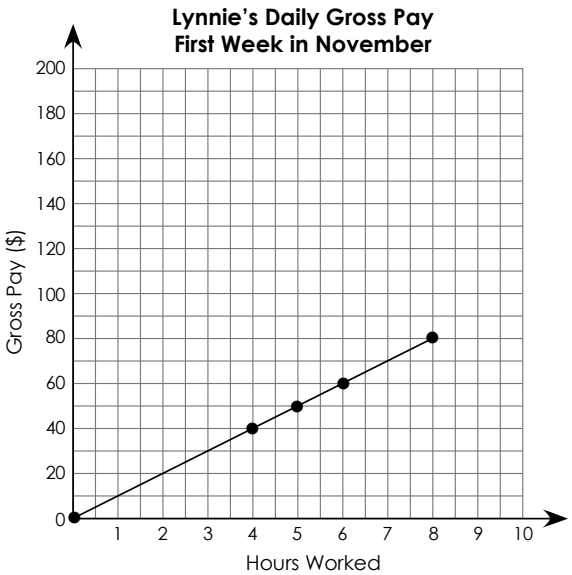
$$= 10$$

Lynnie's wage is 10 dollars/hour

Slope is a ratio, or comparison. It is a comparison of the "change in the dependent variable" to the "change in the independent variable" (as stated in a previous lesson). If you did not already write it down, include the general formula for slope on your resource sheet.

Different Slopes





Slope (Emma's) = \$15 per hour

Slope (Lynnie's) = \$10 per hour

Emma has the larger hourly rate, which means that the value of the slope is greater. Therefore, the line for Emma's graph will be steeper.

In conclusion, the greater the value of the slope, the steeper the graph of the relation. This is true of any linear relation.

In the examples you have seen so far, the slope has been a whole number. Overall, slope can be expressed in three forms:

1. As an integer (e.g., 15)
2. As a reduced fraction in lowest form (e.g., $\frac{5}{2}$)
3. As a decimal (e.g., 2.5)

A Special Case of Slope

There are two special cases where the slope has unexpected values.

Example 1

A theatre owner was interested in the relationship between time and the number of people in a movie. Throughout the 2.5-hour movie, not a single person enters or leaves the theatre. There were 80 people watching a movie.

a) State the independent and dependent variables.

b) Express, in words, the relationship between time and number of people in the theatre during the movie.

c) Express, as an equation, the relationship between time and number of people in the theatre during the movie. Use t for time and p for the number of people.

d) Draw a graph of the relation.

e) Calculate the slope of the graph.

Solution

a) independent variable: time
dependent variable: number of people

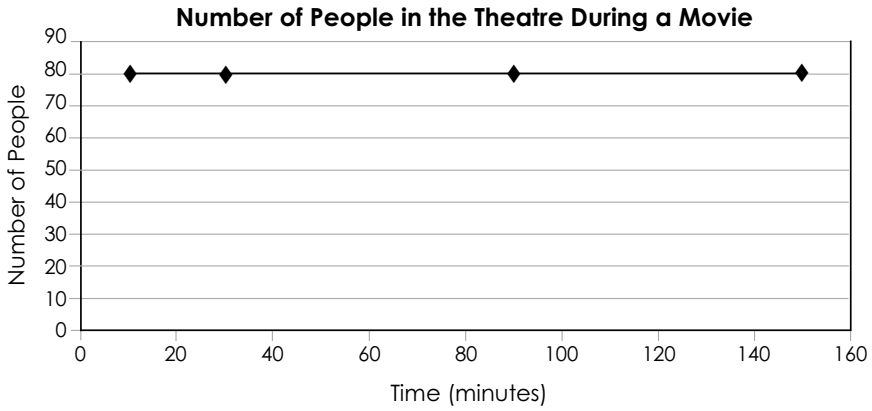
b) There are always 80 people in the theatre.

c) $p = 80$

d) The number of people *is* the dependent variable, but it is *not* affected by the amount of time that has passed. Based on the formula, no matter how much time has passed, there will still be 80 people in the theatre. This means that for *any* value of time, the number of people in the theatre will be 80.

Time passed (minutes)	10	30	90	150
Number of People in the Theatre	80	80	80	80

So the graph will be:



e) **Step 1:** Point A = (10 min., 80 people)
Point B = (150 min., 80 people)

Step 2: Change in the dependent variable = 80 people − 80 people
= 0 people

Step 3: Change in the independent variable = 150 min − 10 min
= 140 min

Step 4: $\text{Slope} = \dfrac{\text{vertical change}}{\text{horizontal change}}$

$= \dfrac{0}{140}$

= 0 people per minute left or entered the theatre

Note that the slope is zero, because the vertical change is zero, and zero divided by another number is always zero.

Special Case #1: The slope of a horizontal line is always zero.

You should include this information on your resource sheet.

The second "Special Case of Slope" will be discussed in the next lesson.

Slope in the Equation of a Line

Now that you know how to calculate slope, you can write the equation of a line based on the graph or the table of values.

Equation of a Graph

Example 1

Miguel is saving his money and is interested in tracking the value of his investment over time. His principal amount is $200. Each year, his investment increases in value by $15.

a) Is this relation a direct or partial variation?

b) Express, in words, the relation between the value of his investment and time.

c) Express, as an equation, the relation between the value (v) of his investment and time (t).

d) Draw a graph of the equation.

e) Calculate the slope of the graph.

Solution

a) Partial variation: He starts with more than $0.

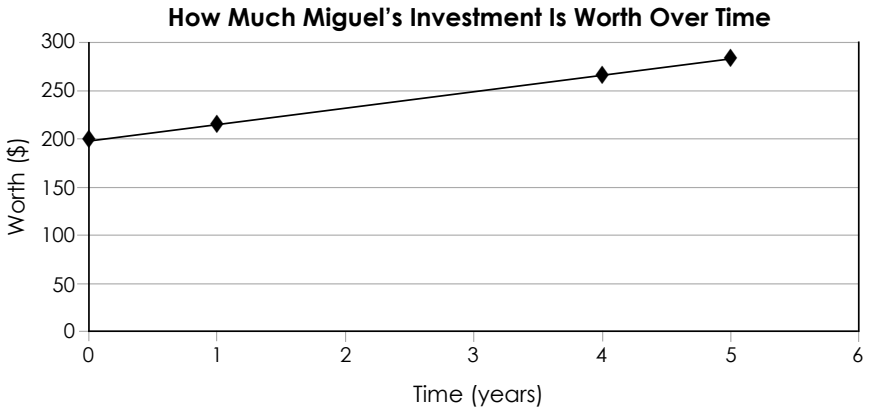
b) The value of Miguel's investment increases by $15 per year, in addition to the $200 he started with.

c) $v = 15t + 200$

d) Table of values (values were calculated by entering the number of years into the equation, as was done last lesson)

Time (years)	0	1	4	5
Value of the Investment ($)	200	215	260	275

So the graph is:



e) **Step 1:** Point A = (0, 200)

Point B = (5, 275)

Step 2: Change in dependent variable = 275 − 200 = $75

Step 3: Change in independent variable = 5 − 0 = 5 years

Step 4: $$\text{Slope} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

$$= \frac{75}{5}$$

$$= 15 \text{ dollars per year}$$

Notice that the slope (15) is the same as the number we multiplied with the independent variable: $v = 15t + 200$. This is true for all linear equations. **If the line is not horizontal or vertical, the equation for the line is**

dependent variable = (slope) independent variable + fixed value

If the graph displays direct variation, then the fixed value is zero so it is not included.

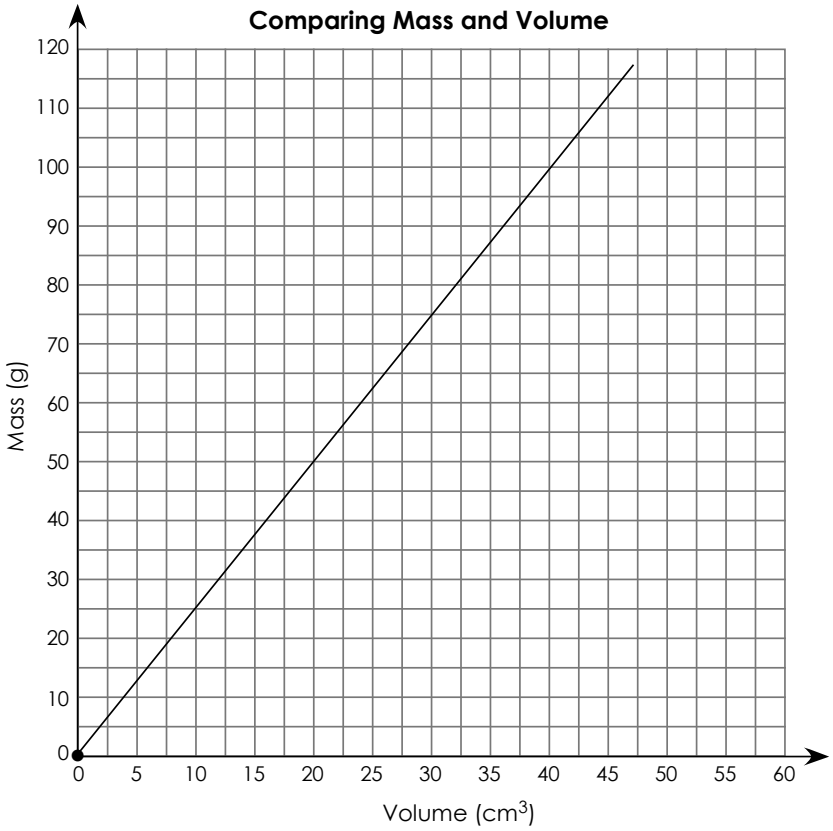
You should include this general formula in your resource sheet.

It may also be helpful to note that this formula is exactly the same as the formula for partial variation, because the *slope of the line* is the same as the *constant of variation*.

The following example uses the general form of a linear equation to translate the graph into an equation.

Example 2

A student finds the volume and mass of several samples of a substance, and expresses the relation as the following line graph.



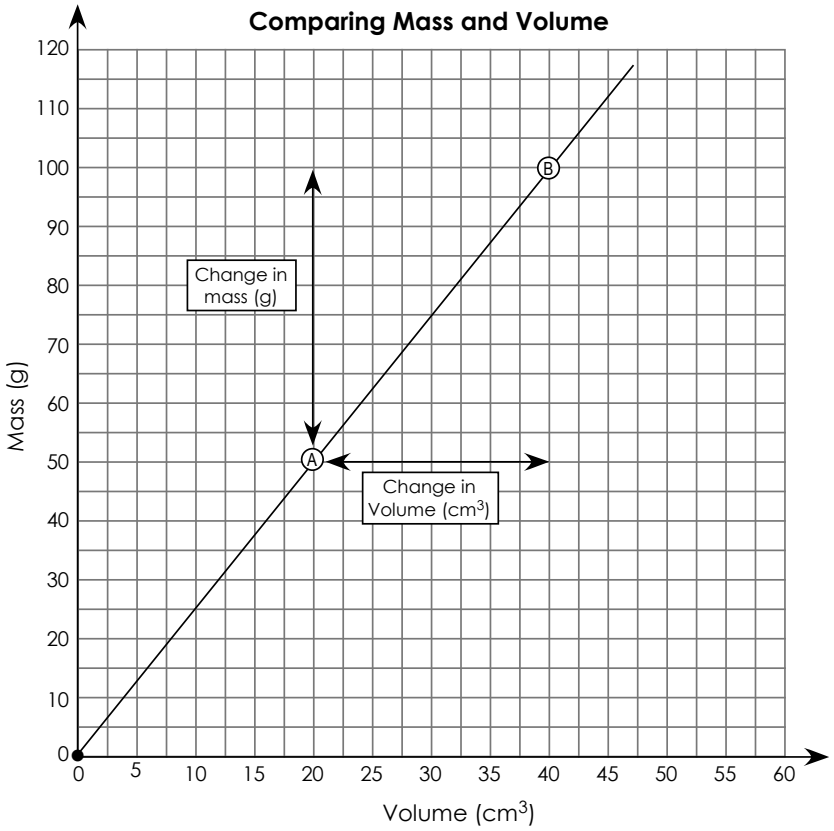
a) Calculate the slope of the line.

b) What does the slope of the line graph represent?

c) Express the relation represented by the line graph as an equation.

Solution

a) **Step 1:** Locate two points on the graph line. Remember, it is easier for you if you use points on the line that meet where the graph grid lines cross. Refer to them as Point A and Point B.



Step 2: Change in the dependent variable = 100 − 50

= 50 g

Step 3: Change in the independent variable = 40 − 20

= 20 cm^3

Step 4: Slope of the line $= \dfrac{\text{change in dependent variable}}{\text{change in independent variable}}$

$$= \frac{50}{20}$$

$$= \frac{5}{2}$$

$$= 2.5 \text{ g/cm}^3$$

This is a new way to write the formula for slope. Include it on your resource sheet.

b) The slope represents the rate of change of mass per unit of volume. It can be most easily expressed using the units g/cm^3.

You may also be interested in knowing the scientific term for the mass of 1 cm^3 of a substance is the **density**. From the student's measurements, it is likely the substance is aluminum. Aluminum has a density of 2.699 g/cm^3. As a point of interest, the densities of some other well known substances are nickel 8.92 g/cm^3, silver 10.49 g/cm^3, and gold 19.32 g/cm^3.

c) Choose the letter m to represent the mass of the substance and the letter V to represent the volume of the object. Using the general formula, the equation for this relation is $m = 2.5V$.

Equation for a Table of Values

Similar to the two examples you have just seen, you can use a table of values to calculate the slope of a linear relation, and then write the equation. The steps are outlined in the following example. They are comparable to the steps for writing an equation from a graph. Maybe you should include the steps on your resource sheet.

Example 3

A fitness club charges $50 to join, plus a monthly membership fee of $35. The table of values for some of the standard times is:

Number of Months of Membership	3	6	12	18
Total Cost ($)	155	260	470	680

a) Calculate the slope of the data.

b) Express the relation as an equation.

Solution

a) **Slope is constant in all linear relations.**

You must be sure the relation is linear. To do this, you can find the slope with different pairs of data. If the relation is linear, the slope will be the same each time.

Step 1: Choose two data pairs to use.

Pair A = (3 months, $155)

Pair B = (18 months, $680)

Step 2: Change in the dependent variable = 680 − 155 = $525

Step 3: Change in the independent variable = 18 − 3 = 15 months

Step 4: Slope of the relation $= \dfrac{\text{change in the dependent variable}}{\text{change in the independent variable}}$

$= \dfrac{525}{15}$

$= 35$ dollars per month

b) First, we must determine whether the graph is a partial or direct variation. Based on the information, the fee to join the fitness club ($50) is the fixed value so it is a partial variation. Using the general formula, we get:

$C = 35\,n + 50$

where C is the total cost and n is the number of months.

Now that you have seen many examples of how to calculate and interpret slope, and how to use slope in an equation, it is time to practise. If you have any questions, be sure to ask your tutor/marker or learning partner for help.

Learning Activity 5.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Solve for d: $\dfrac{10}{d} = 20$
2. Sara has five nickels in her pocket; Damian has one quarter. Who has more money?
3. Kudus got 75% on his final examination in science. If the exam is worth 40% of his total grade, how much is his examination mark accounted for in his total grade?
4. Elyce went to the grocery store to buy food for dinner. The total cost was $45.30 and she gave the cashier three $20 bills. How much change should she receive?
5. A checkerboard alternates black and white squares. If a checkerboard is 8 squares wide by 8 squares long, how many squares are black?

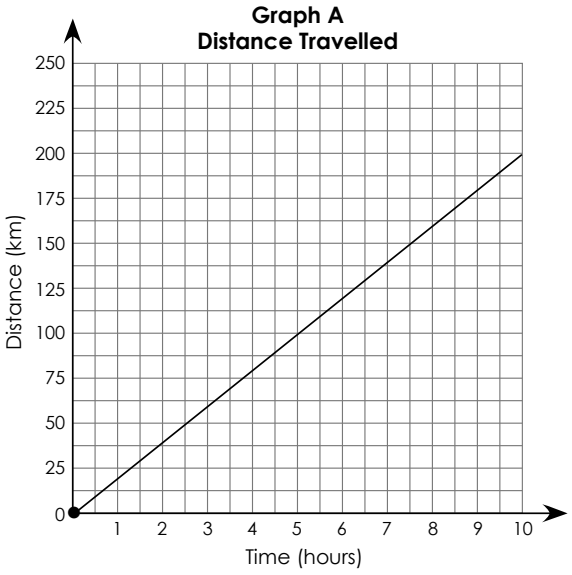
continued

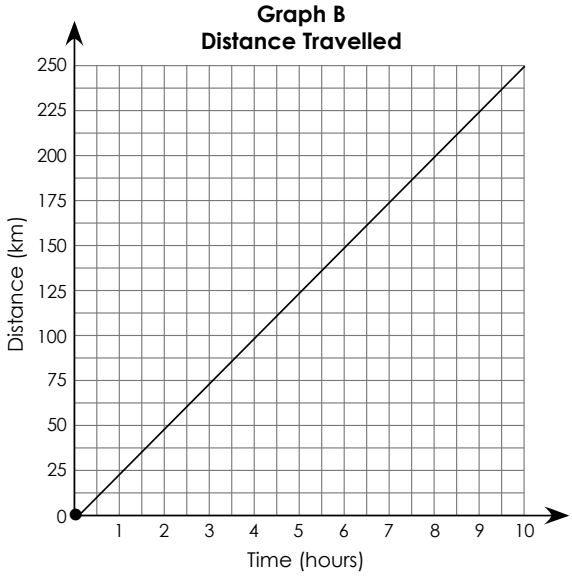
Learning Activity 5.4 (continued)

Part B: Slope

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Consider the following two line graphs modelling the distance two vehicles have travelled.





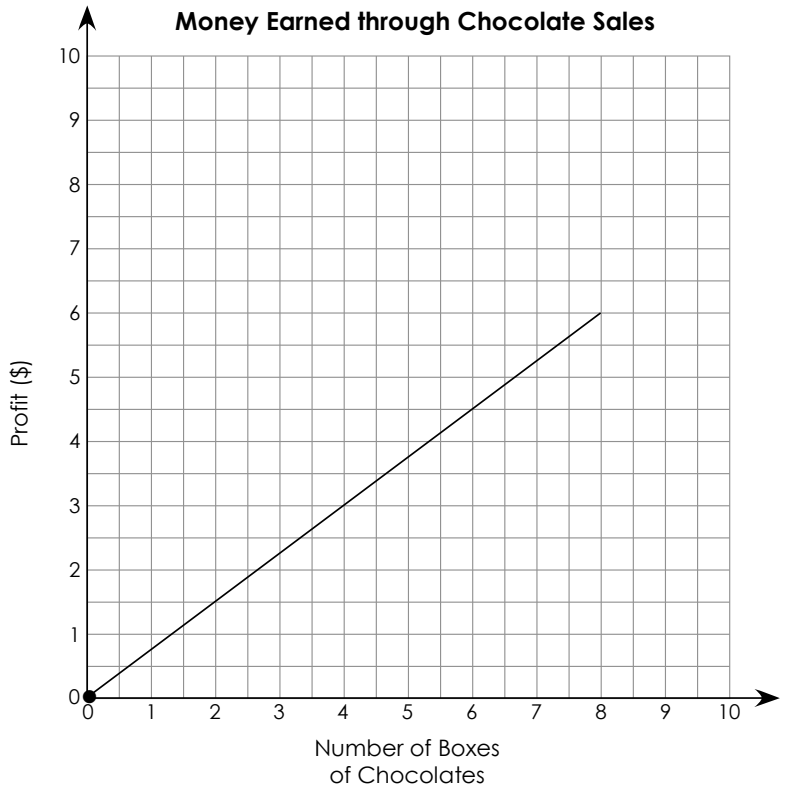
   a) What does the slope of each line graph represent?
   b) By observation, which line graph is steeper? Explain what that means in terms of the linear relations the two line graphs model.
   c) Using the formula for slope, determine the slope of each of the line graphs.
   d) How do the slopes of the line graphs compare to their steepness?

2. All horizontal lines have a slope of zero. In your own words, explain why.

continued

Learning Activity 5.4 (continued)

3. The following line graph represents the relation between the number of boxes of chocolates a student sells for a school fundraiser and the profit he earns for the school.



a) State the independent and dependent variables.

b) Determine the slope of the line graph.

c) Express the relation represented by the line graph as a formula.

d) What does the slope of the line graph represent?

Lesson Summary

In this lesson, you were introduced to slope. You calculated slope from data presented in a table of values or with a graph, using the formula:

$$\text{slope} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

You also learned that slope is a constant value in a linear equation, and represents the rate of change. If you have any questions about the learning activity or any of the material in this lesson, ask your tutor/marker or your learning partner for help.

In the following two lessons, you will continue to use slope to solve everyday problems.

Notes

Lesson 5: Slopes of Objects

Lesson Focus

In this lesson, you will

- ❑ describe slope as "rise over run"
- ❑ determine whether an object has a constant slope
- ❑ look at the safety implications of slope
- ❑ study the relationship between slope and angle of elevation

Lesson Introduction

Lynni, a safety inspector, must determine whether wheelchair ramps are safe. One safety factor is the steepness of the slope of the ramp. In Winnipeg, a city bylaw states that a ramp must have a slope not greater than 1 in 12. In this lesson, you will learn how to determine the greatest angle the ramp can make with the ground.

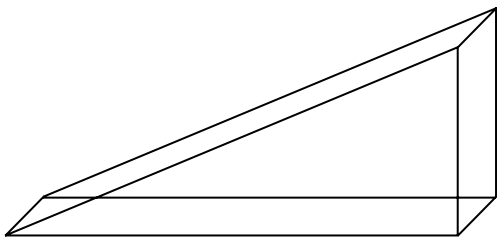
Rise over Run

In the previous lesson, we wrote the formula for slope in the following ways:

$$\text{slope} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

Both of these formulas were used when we discussed graphs. Our focus is now on tangible objects, such as a ramp or a roof, so these descriptions are not as helpful.



For example:

- Can you identify the dependent and independent variables of the ramp?

  No, you cannot, because independent and dependent variables refer to linear relationships between two variables, and this is not a linear relationship – it is an object.

- Can you state the vertical and horizontal changes of the ramp?

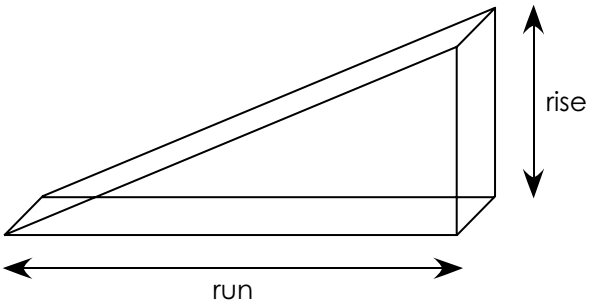
  You may answer yes. Which two values would you choose?

When we describe the vertical change as the *height* of the ramp and the horizontal change as the *length* of the ramp, we use the second formula for slope. The formula could also be written as:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$ or written in words: *rise over run*

Include this formula in your resource sheet.

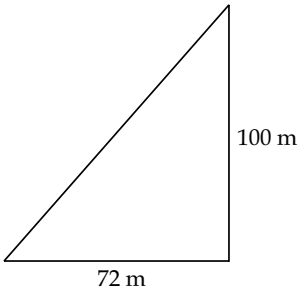
Note that the **rise** is the **vertical height**, and the **run** is the **horizontal length** (and not the slanted length of the ramp).



You have just been given three formulas for calculating slope. It is important for you to realize that they are all really the same formula. The last one given is often used because some students find it easier to remember. You need to use the version of the formula that is easiest for you.

Example 1

Ski jump hills are quite steep. A photo and a side-view diagram are shown below. Find the slope of the ski jump hill.



Solution

$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{rise} = 100 \text{ m}, \text{ run} = 72 \text{ m}$$

$$= \frac{100}{72}$$

$$= 1.388...$$

The slope of the hill is 1.4.

This means that for every one metre you move horizontally, you move up 1.4 m.

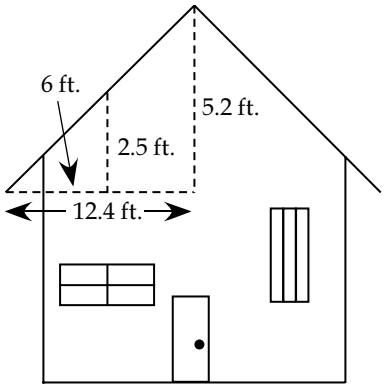
As you can see, the calculation of slope is the same here as it was for a graph. The difference is that now we are looking at measurements instead of data.

Constant Slope

Using rise-over-run to determine the slope of an object is not too difficult. But how can you be sure that the whole object has the *same* slope? This is important when looking at a roof, because you do not want a roof to be uneven.

Example 1

Pippin and Sam are looking for a house, and find an old home they like. They are, however, concerned about the amount of work they need to do to repair it. Determine whether the roof has a constant slope, because if it does not they need to rebuild it.



Solution

A "constant slope" only occurs with a straight line. To check whether a slope is constant, you need to find the slope for more than one interval. This is why two sets of measurements are given in the diagram.

Interval 1:

6 ft. from the edge of the roof (horizontally), the roof has risen 2.5 ft. (vertically)

$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{rise} = 2.5 \text{ ft., run} = 6 \text{ ft.}$$

$$= \frac{2.5}{6}$$

$$= 0.4166...$$

The slope of this section of the roof is approximately 0.42 (this means that for every one foot you move horizontally, the roof will rise 0.42 ft.).

Interval 2:

From the top of the roof (the peak), the horizontal distance is 12.4 feet from the edge of the roof, and the roof has risen 5.2 feet.

$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{rise} = 5.2 \text{ ft., run} = 12.4 \text{ ft.}$$

$$= \frac{5.2}{12.4}$$

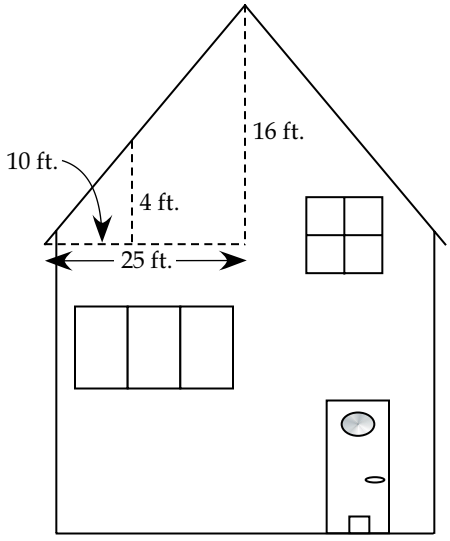
$$= 0.4193...$$

The slope over this interval is approximately 0.42 ft. Although the decimals when not rounded are not exactly the same, we will say that the roof has an approximately constant slope, because the slopes are so close. They don't need to rebuild it.

You may want to include one of these examples in your resource sheet.

Example 2

Andriko is remodelling a house. He needs to determine whether the roof needs to be repaired. Determine whether or not the roof is straight.



Solution

Interval 1:

$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{rise} = 4 \text{ feet, run} = 10 \text{ feet}$$
$$= \frac{4}{10}$$
$$= 0.4$$

Interval 2:

$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{rise} = 16 \text{ feet, run} = 25 \text{ feet}$$
$$= \frac{16}{5}$$
$$= 0.64$$

Because the slopes are so different (0.4 compared to 0.64), we know that the roof is not straight. Andriko will have to repair the roof.

Note: We used multiple points on an object the same way we used multiple points on the graph of a line to determine whether the slope is constant.

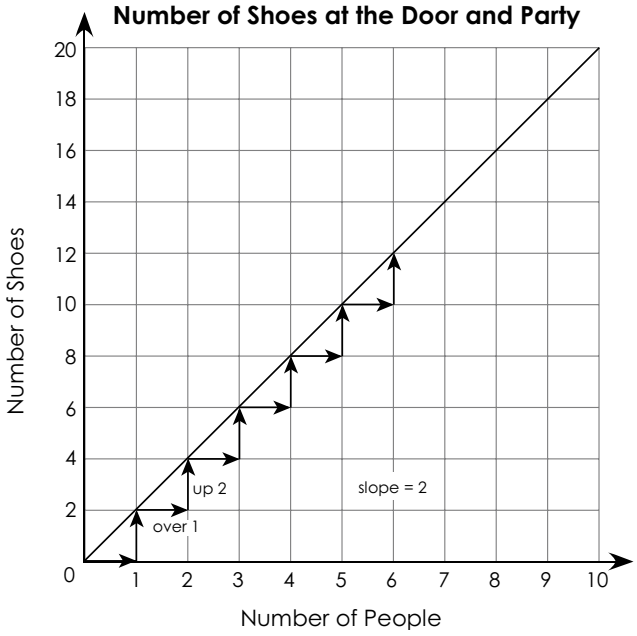
Slope and Angle of Elevation

In this lesson, you used the formula:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

What, exactly, does this number mean?

We know that as the number for slope gets larger, the slope gets steeper. A hill with a slope of 2.5 is steeper than a hill with a slope of 2. The number for slope is the vertical change if we move one unit horizontally.

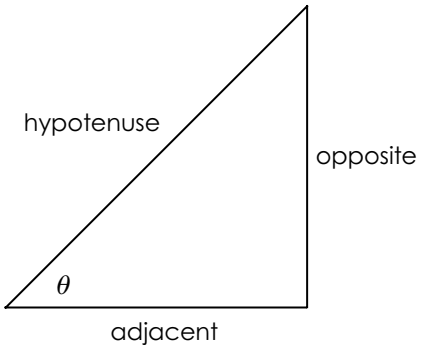


Another meaning of slope is related to trigonometry. In Grade 10 Essential Mathematics, you learned the trigonometric ratios for right angle triangles. These were:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

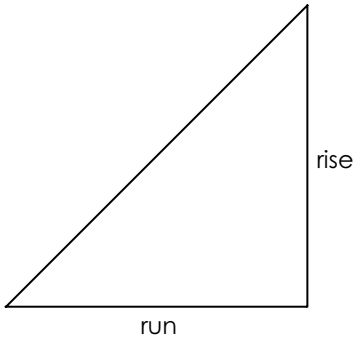
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

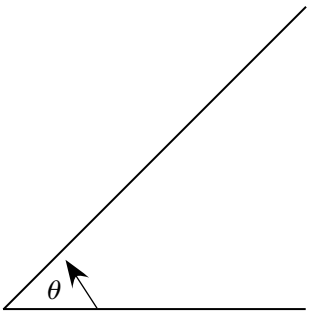


Remember that when we find the slope of an object, it is rise over run.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$



When we compare these two triangles, you see that the ratio for tangent is the same as the ratio for slope. This means that **tan θ = slope**. The angle, θ, is called the **angle of elevation**, because it is the angle between the slanted line (hypotenuse) and a horizontal line. Stated another way, it is the angle (θ) created between your natural line of sight (a horizontal line) and your elevated line of sight

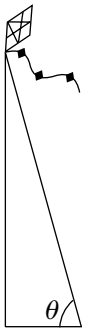


In your own words, include the relationship between slope and tangent on your resource sheet.

With the formula relating tangent and slope, we can solve for the angle of elevation.

Example 1

A kite is in the air. The slope of the string is 5. Find the angle of elevation.



A slope of 5 means the string rises up 5 units for every unit it moves horizontally. You could write the slope as $\frac{5}{1}$.

Solution

tan θ = slope

tan θ = 5

Remember that your calculator needs to be in degree mode and you must press the second function button before pressing tan on your scientific or graphing calculator when you are solving for the angle, θ.

$\theta = \tan^{-1}(5)$

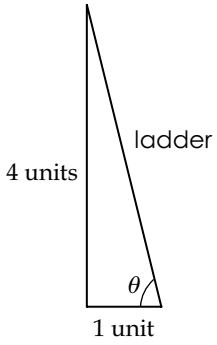
θ = 78.69…

The angle of elevation is approximately 79°. That is quite steep.

Steepness and Safety

In certain situations, there is a relation between steepness and safety. It is obviously dangerous to have things like ladders, wheelchair ramps, or roads too steep.

For example, the recommended ratio for ladder use is to have the base one foot away from the wall for every four feet of height.



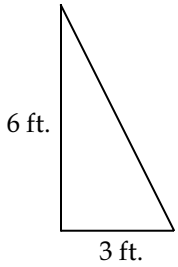
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4}{1} = 4$$

which means that the angle of elevation = $\tan^{-1}(4) = 76°$

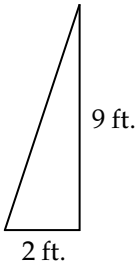
Example 1

Determine the slope of each ladder. Would it be considered safe?

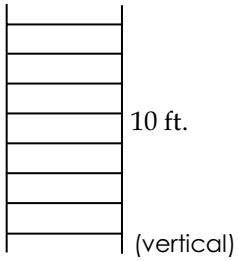
a)



b)



c)



Solution

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

a) rise = 6 ft., run = 3 ft.

$$\text{slope} = \frac{6}{3}$$

$$= 2$$

Because the slope is less than the recommended slope, this ladder would be safe.

b) rise = 9 ft., run = 2 ft.

$$\text{slope} = \frac{9}{2}$$
$$= 4.5$$

Because the slope is more than the recommended slope, the ladder is too steep—it is not safe.

c) In this case, the rise is 10 ft., but the run is zero because the ladder is vertical. You see these ladders going down the sides of tall buildings, or as part of the fire escape on apartments. Unless the ladder is firmly attached to the building, it is very unsafe.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{10}{0}$$
$$= \text{ERROR}$$

(Your calculator may give you this or a similar message because it is invalid to divide by zero.)

Do not be alarmed by this message, because you have not made a mistake. A vertical line has an infinitely large steepness so the slope cannot be described by a number. This is the second special case for slope, which was mentioned in Lesson 4.

Special Case #2: The slope of any vertical line is *undefined*. This is confirmed using the slope formula, the "run" is zero, and you cannot divide by zero.

You should include this case in your resource sheet.

Another safety concern is ramps.

- At a skate park there are ramps of all shapes and sizes. The steeper the ramp, the faster you will go, but there is a greater risk of getting hurt. Also, if a ramp is quite steep, the board might scrape the ground as you come off of it.
- Wheelchair ramps have safety concerns as well as functionality. Not only would people in a wheelchair have a problem going *down* a really steep ramp because they could lose control, but they would also have a hard time going *up* the ramp because it would take a lot of strength to roll up.

The following learning activity is designed to check your understanding of the material in this lesson. Complete all of the questions before checking your answers in the answer key at the end of the module. Also, if you find that you are missing information in your resource sheet, don't hesitate to add more.

Learning Activity 5.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

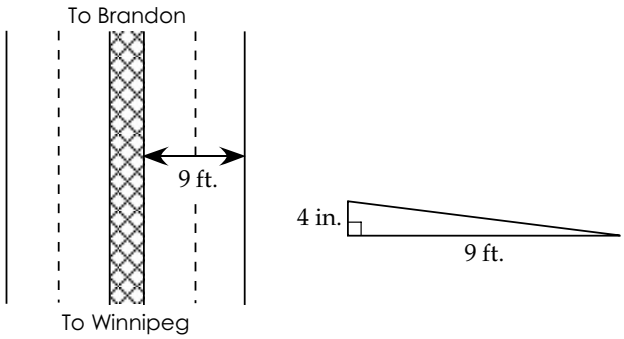
Convert the following:

1. 10 cm = _____ m
2. 12 ft. = _____ yd.
3. 15 s = _____ min.
4. 0.53 km = _____ m
5. 1.5 ft. = _____ in.

Part B: Slopes of Objects

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

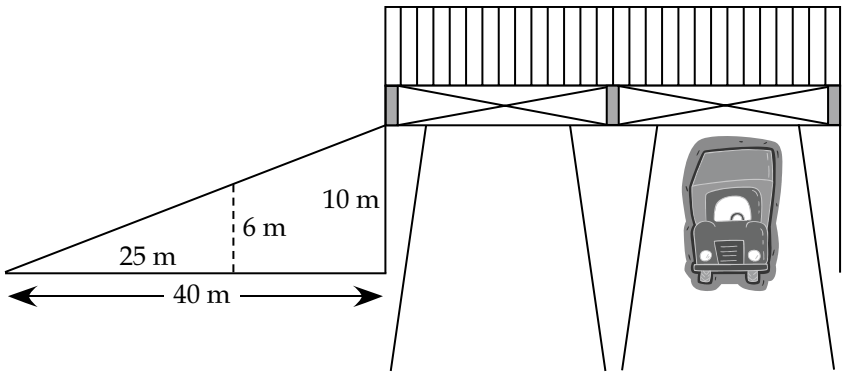
1. The Trans Canada Highway is slanted (higher in the centre than on the side of the road), so that when it rains, the water runs into the ditches. Heading from Winnipeg to Brandon, the west-bound lanes are 9 feet wide. The outside of the road is 4 inches lower than the inside. What is the road grade (slope) of the highway? Write your answer rounded to two decimal places.



continued

Learning Activity 5.5 (continued)

2. A new overpass has been built. Determine whether the incline to the top of it is constant. Write your answer rounded to two decimal places.



3. Armin is on the treadmill for her daily workout. She runs for 30 minutes with the treadmill incline set at 4. If the actual slope of the treadmill is 0.2, what is the angle of elevation of the treadmill? Write your answer rounded to the nearest degree.

4. A wheelchair ramp can have a maximum slope of 1 in 12, which means a rise of 1 inch for every 12 inches of run. Lynni inspects a ramp that has a horizontal length of 20 feet, and rises 15 inches.
   a) What is the steepest legal angle of inclination allowed for a wheelchair ramp? Write your answer rounded to one decimal place.
   b) Does the ramp being inspected by Lynni meet safety standards? Show how you arrive at your answer.

5. What are some other examples where slope is important for safety? You may discuss this with your learning partner.

Lesson Summary

In this lesson, you learned how the slope of an object is related to the slope of a line. The formulas for the slope of an object and the slope of a line are very similar:

$$\text{slope (object)} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope (line)} = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

Both formulas are ratios that compare vertical change to horizontal change. You also looked at how the safety of an object may be related to slope.

In the next lesson, you will learn about "rate of change," which is another application of slope.

Notes

Lesson 6: Rates of Change

Lesson Focus

In this lesson, you will

- ❑ describe slope as a rate of change
- ❑ use proportional reasoning to calculate slope
- ❑ use unit analysis to interpret data

Lesson Introduction

In the previous two lessons, you determined the slope of a relation from a graph or a table of values, and you calculated the slope of an object. In this lesson, you will study slope as a rate of change. This is related to the graphs in Module 4. Some examples of "rate of change" are speed (kilometres per hour) or wages (dollars per hour).

Relation to Slope

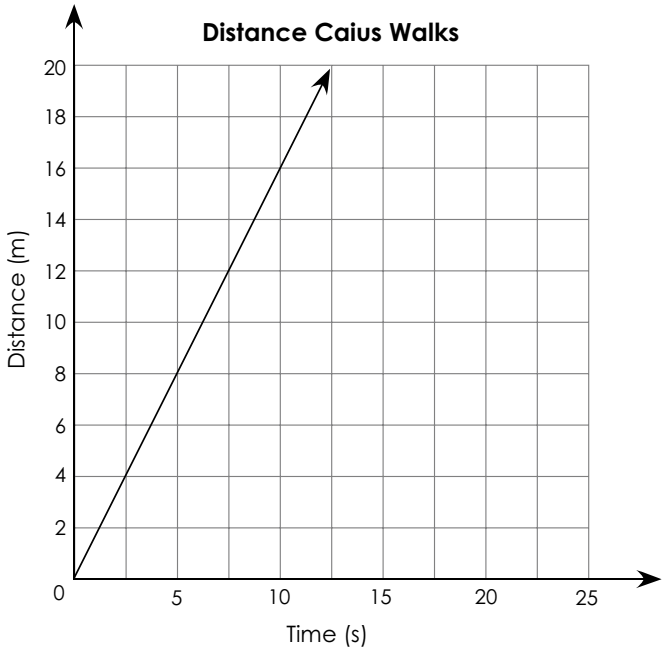
The very first example from Lesson 4 showed you how to calculate the slope of a line. The units for that slope were dollars per hour. This is the rate of change in pay, also known as the hourly wage. We know these units because when we calculated the slope, the equation was

$$\text{slope} = \frac{\$40}{4 \text{ hr.}}$$

Even before the fraction is reduced, we can see that the rate will be dollars per hour, because the amount of money is in the numerator and the time is in the denominator. In this particular case, the rate is reduced to \$10/hour, but it does not have to be in all situations.

Example 1

Use the graph to answer the questions.



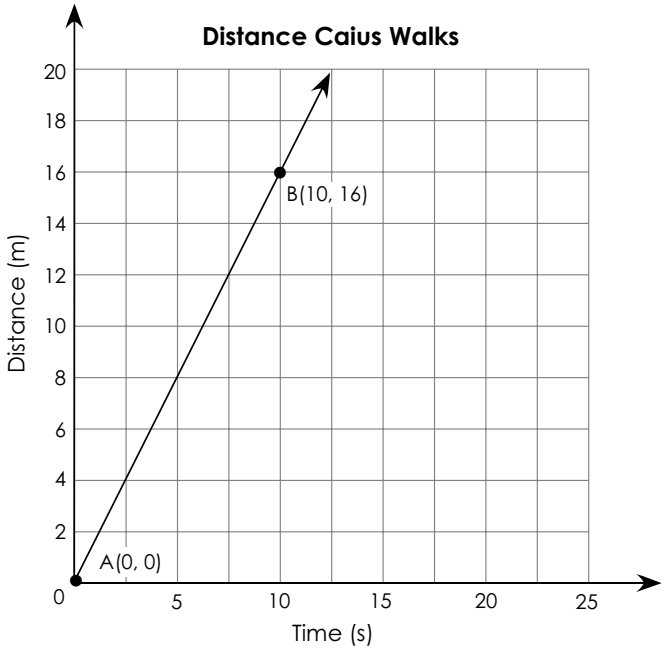
a) Calculate the slope of the graph.

b) What does the slope tell us about Caius?

c) Without using the graph, calculate how far Caius will walk in one minute.

Solution

a) Step 1: Point A = (0, 0) Point B = (10, 16)



Step 2: Difference in distance = 16 − 0 = 16 m

Step 3: Difference in time = 10 − 0 = 10 s

Step 4: $\text{Slope} = \frac{16 \text{ m}}{10 \text{ s}}$

$= 1.6 \text{ m/s}$

b) The slope is the *speed* at which Caius is walking. Caius is walking 1.6m/s. You can identify speed because its units are always some form of *distance over time*. The most common forms are kilometres per hour (km/h) and miles per hour (mph) but—like in this example—you can also describe speed as metres per second (m/s) or feet per second (ft./s).

c) Without using the graph, we can use **proportional reasoning** to calculate how far Caius will walk in 1 minute. When using proportional reasoning, it is easiest if you use the rate as part of the ratio, but it is not necessary.

It might be a good idea to include notes about proportional reasoning in your resource sheet.

Before doing the calculation, you must be sure that you are using the same units in each ratio.

1 minute = 60 seconds

Using the rate:

Let d represent the distance travelled in 60 seconds.

$$\frac{1.6 \text{ m}}{1 \text{ s}} = \frac{d}{60 \text{ s}}$$

$$(1.6)(60) = 1d$$

$$96 = d$$

Caius will walk 96 m in 1 minute.

Using another point:

Instead of using the rate, you can use another point from the graph. This second method only works if you have a direct variation. For example, if we use Point B from part (b) (10, 16),

$$\frac{16\text{ m}}{10\text{ s}} = \frac{d}{60\text{ s}}$$ where d represents the distance travelled in 60 seconds.

$$(16)(60) = 10d$$ Cross multiply.

$$960 = 10d$$ Simplify.

$$\frac{960}{10} = \frac{10d}{10}$$ Divide both sides by 10.

$$96 = d$$

Caius will walk 96 m in 1 minute.

Based on this example, as well as other questions that you have completed in Lesson 4, we can state that **rate of change** is the slope of a relation.

Complete the following learning activity and check your answers. Be sure that you understand the relationship between rate of change and slope before you start the assignment at the end of this lesson.

Learning Activity 5.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If the volume of a 2 by 2 by 2 cube is 8 units, how many times larger would the volume be of a cube with dimensions 4 by 4 by 4?
2. Complete the pattern: 4, ____ , 16, 32, ____
3. There are 30 days in September, 31 days in October, 30 days in November, and 31 days in December. How many days are there between September 1st and New Year's Eve?

continued

Learning Activity 5.6 (continued)

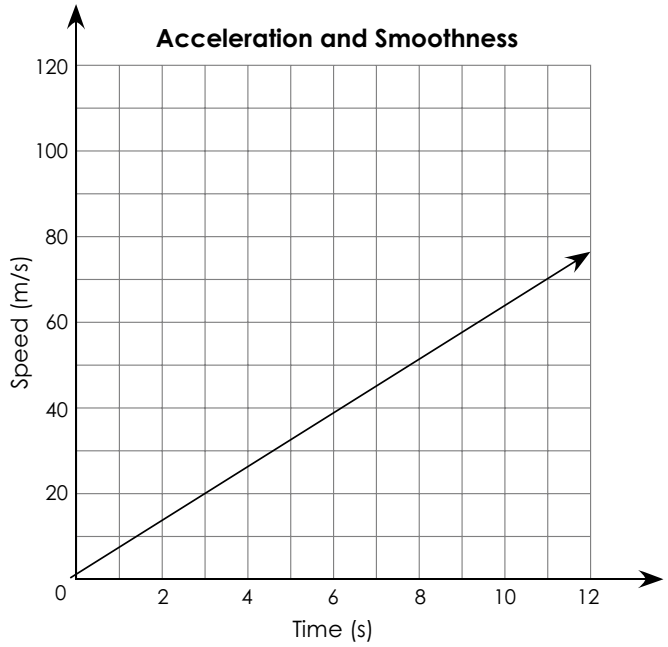
4. How many cm^2 are in 310 m^2?

5. Evaluate: $\frac{4}{9} \times \frac{1}{2}$

Part B: Rate of Change

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. As part of the testing of a new car, a company monitors the acceleration of the car to be sure that it is smooth. The graph below includes their data. Use it to answer the following questions.



   a) Calculate the slope of the graph.
   b) What does the slope mean for this situation?
   c) How long would it take the car to reach 100 km/hr.?

Converting Rates of Change

In some situations, the units for slope may not be easily understood because they are not commonly used to describe the rate of change. Alternately, you may be asked for an answer with specific units, and you must know how to convert the rate of change so that you can perform your calculations in those units.

Method 1: One unit at a time

Example 1

Miruna is driving from Brandon to Dauphin. The speed limit on a stretch of highway is 90 km/h. According to her GPS, her driving speed is 30 m/s. Is she speeding?

Solution

In this example, the speed 30 m/s is difficult to relate to because we usually describe speed as kilometres per hour or miles per hour. In addition, the speed limit is posted in kilometres per hour. In order to convert 30 m/s into more familiar units, we must apply proportional reasoning through unit analysis (unit analysis requires us to use the equivalent relationship between two units in order to convert one to the other). This example will demonstrate one method, while the following example demonstrates another method.

Step 1: Rewrite the speed so that it is a fraction.

$$\frac{30 \text{ m}}{1 \text{ s}}$$

Step 2: Decide which unit you would like to convert first. It does not make a difference which you choose, because you find the same answer either way.

Convert metres

We know that 1 km = 1000 m

Using the conversion ratio that you learned about in Module 2, Lesson 3, we can convert metres to kilometres.

$$30 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}} = \frac{30}{1000} = 0.030 \text{ km}$$

Remember that when you use the conversion ratio, the *new* unit is in the numerator (*new*merator) and the current unit is in the denominator.

Step 3: Convert the other unit.

Convert seconds

We know that 60 s = 1 min and that 60 min = 1 hr.

We can use the conversion ratio again.

$$1 \text{ s} \times \frac{1 \text{ min.}}{60 \text{ s}} \times \frac{1 \text{ hr.}}{60 \text{ min.}} = \frac{1}{3600} \text{ hr.}$$

This can be done in two steps, first converting to minutes then to hours. It is not necessary, however, because when you multiply one number by another, and then by another, you do not have to do each step separately. For example, if you had to solve 11 × 12 × 13, would you type in 11 × 12 on your calculator, press the equal button then multiply that number by 13? Perhaps, but it would be quicker to just type it all in at once.

Step 4: Create a new ratio using the converted units, and then solve.

$$\frac{30 \text{ m}}{1 \text{ s}} = \frac{0.030 \text{ km}}{\left(\frac{1}{3600} \text{ hr.}\right)} = 0.030 \times \frac{3600}{1} = 108 \text{ km/hr.}$$

Use your calculator to check this.

Miruna is speeding, because her speed is 108 km/hr. and the speed limit is only 90 km/hr.

Method 2: Single Multiplication

Example 2

Sergio is negotiating his allowance with his parents for doing work around the house. He would like $10 per hour. His dad thinks 15¢ per minute is more reasonable, since he doesn't spend very much time doing his chores. Which is the better allowance for Sergio?

Solution

We will convert 15¢ per minute to dollars per hour, because that is the more familiar way to describe wages.

Step 1: Rewrite the rate of change as a fraction

$$\frac{15 \text{ cents}}{1 \text{ min.}}$$

Step 2: Write out all of the conversion ratios that you need to use.

Converting cents to dollars: $\frac{1 \text{ dollar}}{100 \text{ cents}}$

Converting minutes to hours: $\frac{1 \text{ hour}}{60 \text{ min.}}$

Step 3: Write out the conversion calculation.

$$\frac{15 \text{ cents}}{1 \text{ min.}} \times \frac{1 \text{ dollar}}{100 \text{ cents}} \times \frac{60 \text{ min.}}{1 \text{ hour}}$$

Notice that the conversion ratio for minutes to hours is upside-down. This is because, if you look back at Example 1, we multiplied the number in the denominator by its conversion ratio, then we used this number as the denominator in the *new* ratio.

$$1 \text{ s} \times \frac{1 \text{ min.}}{60 \text{ s}} \times \frac{1 \text{ hr.}}{60 \text{ min.}} = \frac{1}{3600 \text{ hr}}$$

then

$$\frac{0.030 \text{ km}}{\left(\frac{1}{3600} \text{ hr.}\right)} = 108 \text{ km/hr.}$$

Remember from the Grade 8 and 9 Math courses that when you divide by a fraction, it is the same as when you multiply by the fraction's reciprocal (the upside-down version). Because of this, when we are writing the conversion calculation, we skip the first step and automatically flip the conversion ratio(s) for the denominator.

This holds true for *any* conversion relating to the denominator, for any set of units. You should note this in your resource sheet as a reminder.

Step 4: Check that your final answer has the desired units.

To check that all of your ratios have been set up properly, you can determine which ones cancel. In this example, the cents cancel because they appear once in the numerator and once in the denominator. Also, the minutes cancel for the same reason. This leaves us with dollars in the numerator and hours in the denominator, which is what we want.

$$\frac{15 \ \cancel{\text{cents}}}{1 \ \cancel{\text{min.}}} \times \frac{1 \text{ dollar}}{100 \ \cancel{\text{cents}}} \times \frac{60 \ \cancel{\text{min.}}}{1 \text{ hour}}$$

Step 5: Solve for the new rate of change.

$$\frac{15 \text{ cents}}{1 \text{ min.}} \times \frac{1 \text{ dollar}}{100 \text{ cents}} \times \frac{60 \text{ min.}}{1 \text{ hour}} = \frac{(15 \times 1 \times 60) \text{ dollars}}{(1 \times 100 \times 1) \text{ hours}}$$

$$= \frac{900 \text{ dollars}}{100 \text{ hours}}$$

$$= \$9 \text{ per hour}$$

Sergio would be paid less if he agreed with his dad because 15¢ per minute is equal to $9 per hour.

Both of the methods that you have seen in the examples can be used for any conversion of rates and slopes. As you work through the following learning activity, try to use both methods to figure out which you like better. Once you have done this, include the method that you prefer in your resource sheet. Check your answers to be sure that you understand converting rates of change.

If you discover a different method for converting rates or slopes, explain it to your learning partner. It is important to do this because your method may only work for certain situations, but not for all rate of change conversions.

Learning Activity 5.7

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil

1. How many hours are in 3 days?
2. Estimate 13% of $39.12.
3. If 8 students in a geometry class of 30 students can find Laos on a map, what is the fraction of the class that can find Laos on a map?
4. Evaluate: 25×16
5. What is the complementary angle to 45°?

continued

Learning Activity 5.7 (continued)

Part B: Converting Rates of Change

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Telmar is riding his bike at 8 m/s. The speed limit is 30 km/hr. Is he speeding?
2 Canadian highway speed limits are usually (around) 100 km/hr. In the US, the highway speed limit is approximately 60 mph. Which is faster?
3. Idra is filling a 4 L bucket with water. The water pours at 15 mL per second.
   a) Calculate the flow rate in L/min.
   b) How long will it take to fill the bucket?

Lesson Summary

In this lesson, you learned that slope represents ”rate of change.” You also learned how to use proportional reasoning to create conversion ratios. The conversion ratios were used to change the units of slope, when slope represents rate of change.

In the next lesson, you will continue to explore proportional reasoning. Before you move onto the next lesson, complete the following assignment, which is based on Lessons 4, 5, and 6. This is part of your evaluation for Module 5.

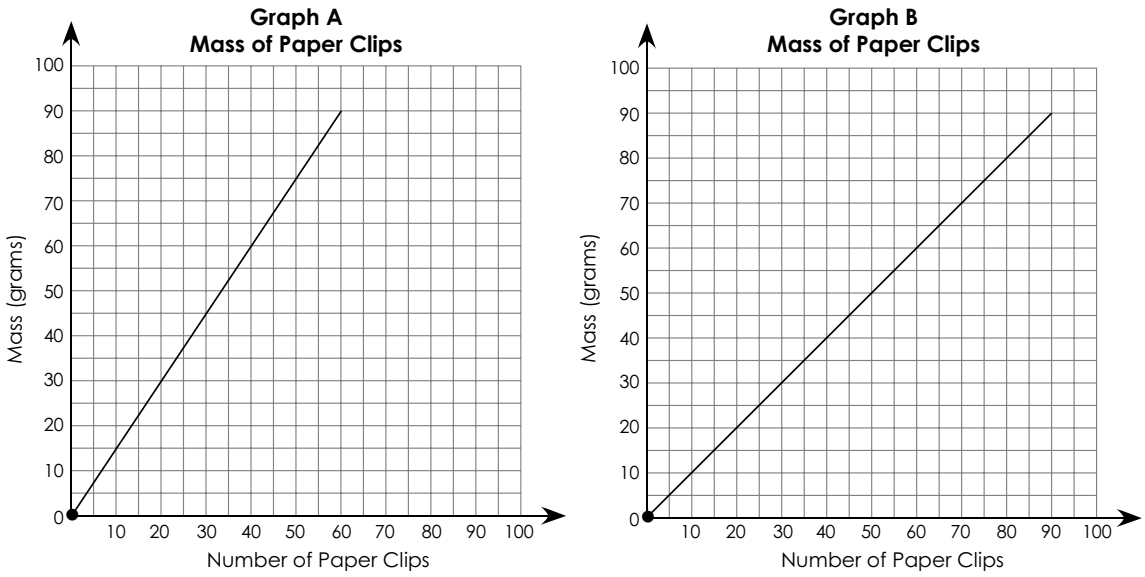
Assignment 5.3

Slope

Total: 29 marks

Note to Students: Remember to use your resource sheet as you complete this assignment. Be sure to add any missing information to your resource sheet as you do your work.

1. Consider the following two line graphs modelling the mass of two different kinds of paper clips.



a) Describe what the slope of each line graph represents. *(1 mark)*

b) By observation, which line graph is steeper? *(1 mark)*

continued

Assignment 5.3: Slope (continued)

c) Using the formula for slope, determine the slope of each of the line graphs. *(2 marks each = 4 marks)*

d) How do the slopes of the line graphs compare to their steepness? *(1 mark)*

_____

_____

e) Write the equation for each graph. Use p for paper clip and m for mass. *(2 marks)*

_____

_____

_____

continued

Assignment 5.3: Slope (continued)

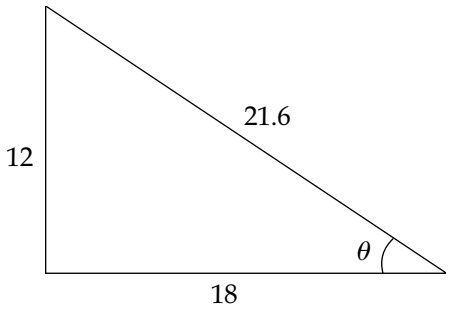
2. Explain, in your own words, why the slope of a vertical line is always undefined. *(2 marks)*

3. a) Using a diagram, explain how the trigonometric ratio *tangent* is related to slope. *(3 marks)*

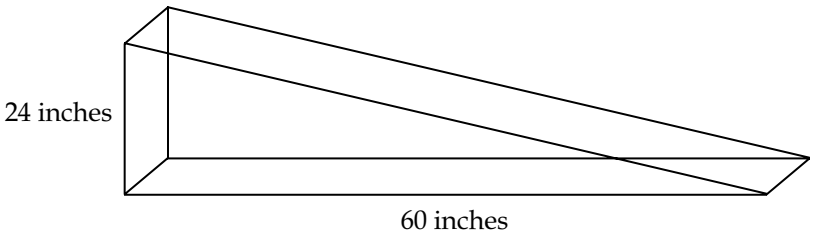
continued

Assignment 5.3: Slope (continued)

b) Calculate the number of degrees of the angle θ. Round your answer to one decimal place. *(2 marks)*



4. Calculate the slope of the ramp at the skate park. *(2 marks)*

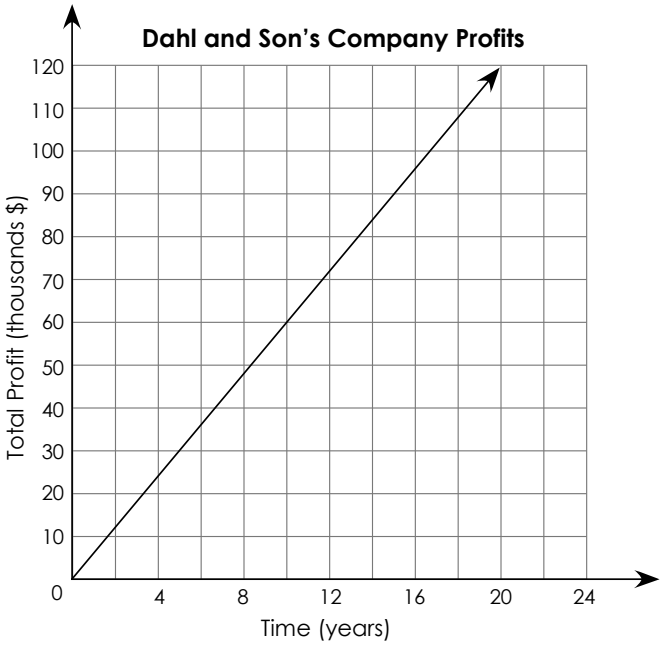


continued

Assignment 5.3: Slope (continued)

5. State and explain an example where there is a relationship between slope and the usefulness or safety of an object. *(2 marks)*

6. Dahl started his own company when he was 28 years old. Since then, his sons have joined the company. The following graph shows the total profit of the company since Dahl started it.



a) Find the slope of the line. *(2 marks)*

continued

Assignment 5.3: Slope (continued)

b) What does the slope mean in terms of the rate of change? *(1 mark)*

c) What was the profit in their fifth year? (*1 mark)*

d) At this rate, what is the expected profit after 25 years? *(2 marks)*

7. Orion is paid $12 per hour at work, and Evy is paid 12¢ per minute for her job. Who is paid more per hour? *(3 marks)*

Lesson 7: Scale

Lesson Focus

In this lesson, you will

- ❑ explore the need for scale drawings
- ❑ determine the actual size of an object based on a scale drawing
- ❑ draw a scale diagram of an object

Lesson Introduction

Sometimes it is necessary to draw something like a floor plan of a house, where the house is much larger than your piece of paper. On the other hand, if you draw a diagram of a ring, the diagram may be much larger than the actual ring. In each case, the image and the object have the same shape but not the same size. Stated another way, the dimensions of the diagram and the object are proportional. This means that all the diagram and object measures have the same ratio. The object may, for example, be 20 times as large as the drawing.

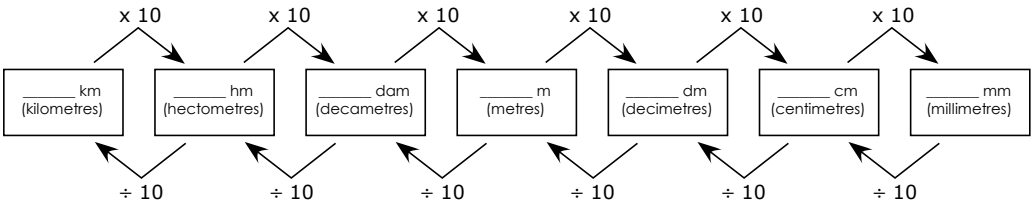
Scale Drawings

Every scale drawing has a **scale factor**, which is the ratio of the length of the drawing to the actual length of the object. Since the scale factor is a ratio, you may apply your knowledge of proportional reasoning learned previously.

You should make note of the definition of scale factor in your resource sheet.

Scale Measurement

Scale drawings involve measurements. Recall the following relationship between units of the metric system. Starting with millimetres (mm), the metric system builds by multiples of 10 (e.g., 1 cm equals 10 mm).



Consider the following example.

Example 1

Find the missing measures.

a) 2 km = _____ m

b) 4 cm = _____ m

c) 3600 km = _____ cm

d) $18\frac{1}{2}$ mm = _____ m

Solution

a) $2 \text{ km} = 2 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 2000 \text{ m}$

To change a quantity from kilometres to metres, you multiply by 1000.

b) $4 \text{ cm} = 4 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0.04 \text{ m}$

To change a quantity from centimetres to metres, you divide by 100.

c) $3600 \text{ km} = 3600 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{ m}} = 360\,000\,000 \text{ cm}$

To change a quantity from kilometres to centimetres, you multiply by 100 000.

d) $18\frac{1}{2} \text{ mm} = 18.5 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 0.0185 \text{ m}$

To change a quantity from millimetres to metres, you divide by 1000.

Scale Factors

A scale factor can be represented in various ways. The scale factor **1 cm represents 50 cm** can also be expressed as the ratio $\frac{\mathbf{1}}{\mathbf{50}}$. (Another way of expressing this ratio is with the notation **1:50**, or **1 cm:50 cm**.)

Note that the first number always represents the scale model measurement and the second number represents the actual measurement of the object.

In your resource sheet, along with your definition of a scale factor, you should include how you can write it.

Consider the following example.

Example 2

Write the following scale in two other ways.

a) 1:25

b) 1 cm represents 6 km

Solution

a) This scale factor can also be represented by $\frac{1}{25}$ or 1 unit represents 25 units.

Note: With this form, the units can be anything.

b) $6 \text{ km} = 6 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{ m}} = 600\,000 \text{ cm}$

This scale can also be represented by $\frac{1}{600\,000}$ or by 1:600 000.

Similar Figures

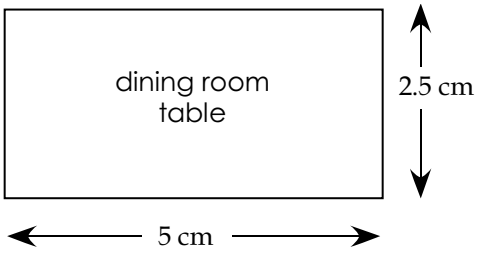
You have learned about similar figures in previous math courses (Grade 9 Math and Grade 10 Essential Math). **Similar figures** are figures that have the same shape but are not the same size. A scale drawing of an object has the same shape as the actual object and is similar to it. This means that all the measures of the diagram and the object are enlarged or reduced by the same ratio. Therefore, given a scale drawing, you can use proportional reasoning to determine the dimensions of the actual objects.

Example 3

The following scale drawing represents a dining room table. If the scale is 1:50, determine the actual dimensions of the table.

Solution

First, use your ruler to measure the dimensions of the dining room table in the scale drawing.



In the scale drawing, the length of the table is 5 cm and the width of the table is 2.5 cm.

To find the actual length of the table, set up the following proportion. Let x represent the actual length of the table. You could use a box like you used in previous courses to set up the proportion. Note that the top row is always the drawing measures, and the bottom row is the actual measures.

	ratio	length
drawing	1	5
actual	50	x

Solve for x using cross-multiplication.

$$\frac{1}{50} = \frac{5}{x}$$
$$(1)(x) = (5)(50)$$
$$x = 250$$

The actual length of the table is 250 cm or 250 ÷ 100 = 2.5 m.

To find the width of the table, follow the same procedure as in finding the length.

Set up the following proportion, letting x represent the actual width of the table.

	ratio	length
drawing	1	2.5
actual	50	x

Cross-multiply:

$$\frac{1}{50} = \frac{2.5}{x}$$
$$(1)(x) = (2.5)(50)$$
$$x = 125$$

The actual width of the table is 125 cm or 125 ÷ 100 = 1.25 m.

Notice that with a scale of 1 : 50, all dimensions increased by 50 times.

Creating a Scale Drawing

Given the actual dimensions of an object and the scale factor, you can represent the object with a scale drawing. Consider the following example.

Example 4

The length of an auditorium is 22 m, and the width is 14 m. Use a pencil and ruler to draw a scale diagram of the auditorium using the scale factor: 1 cm represents 4 m.

Solution

To find the length of the scale drawing, set up the following proportion.

	ratio	length
drawing	1 cm	x cm
actual	4 m	22 m

Let x be the scale length in cm.

$$\frac{1}{4} = \frac{x}{22}$$ Cross multiply.
$$(1)(22) = (x)(4)$$
$$22 = 4x$$
$$\frac{22}{4} = \frac{4x}{4}$$ Divide by 4 on both sides.
$$5.5 = x$$

The scale length of the auditorium is 5.5 cm.

To find the scale width of the auditorium, follow the same procedure used to find the length.

Set up the following proportion, letting x be the scale width in cm.

	ratio	length
drawing	1 cm	x cm
actual	4 m	14 m

Let x be the scale length in cm.

$$\frac{1}{4} = \frac{x}{14}$$ Cross-multiply.

$$(1)(14) = (x)(4)$$

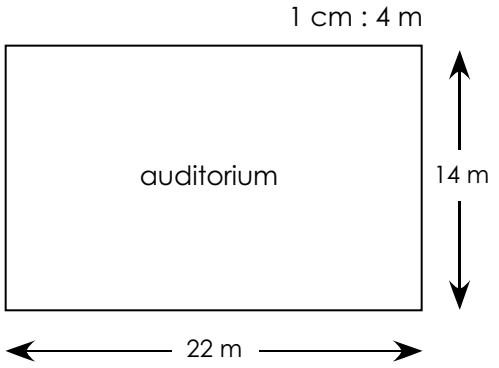
$$14 = 4x$$

$$\frac{14}{4} = \frac{4x}{4}$$ Divide by 4 on both sides.

$$3.5 = x$$

The scale length of the auditorium is 3.5 cm.

The scale drawing of the auditorium is as follows:



You need to include the actual dimensions in the drawing, and you need to include the scale factor so that anyone else looking at the drawing is able to calculate the actual dimensions of the object. Your rectangle must be drawn as 3.5 cm by 5.5 cm.

Complete the following learning activity. When you have finished, check your answers in the Learning Activity Answer Key at the end of this module.

Learning Activity 5.8

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Simon says that in five years, his dad will be double Simon's age. If Simon's dad is 43, how old is Simon?
2. You spent a total of $300 on 25 pizzas. How much did each pizza cost?
3. How many minutes are in a day?
4. What is 10% of 43?
5. It is January 18th. In 14 days, what will the date be?

Part B: Scale

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

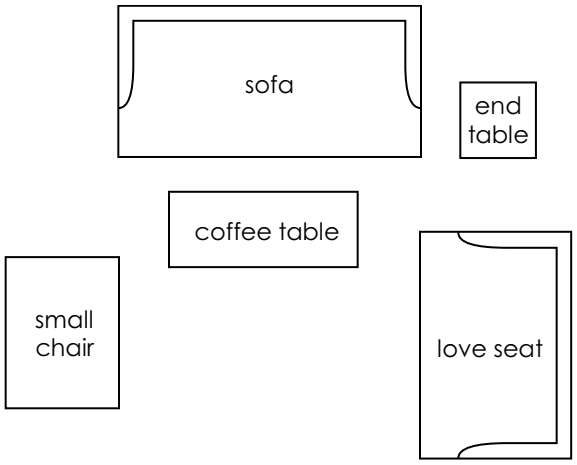
1. Complete the following chart.

Length in Drawing (cm)	Actual Length (cm)	Scale
6.8	680	
5.2	10 400	
4.5		1:10
	180	1:20

continued

Learning Activity 5.8 (continued)

2. Represent the following scale factors in two other ways.
   a) 1 cm represents 8 m
   b) $\frac{1}{10}$
   c) 1:3
3. A building is 130 m tall. If the height of the building in the scale drawing is 3.25 cm, what scale factor is used in the drawing?
4. A volleyball court measures 5 m by 8 m. If it is drawn with a scale factor of 1:300, what are the dimensions of the volleyball court in the scale drawing?
5. The following scale diagram represents an arrangement of furniture in a living room. The scale factor is 1 cm represents 60 cm. Find the actual dimensions of each piece of furniture in the room in metres.



Lesson Summary

In this lesson, you studied scale drawings and scale factors. Given a scale drawing, you calculated the actual measures of the object. Also, given an object and a scale factor, you drew a scale diagram of the object.

Complete the following assignment before you move on to Module 6.

Assignment 5.4

Scale

Total: 19 marks

Note to Students: This assignment is part of your evaluation for Module 5. Use your resource sheet to help you complete this assignment. Do not hesitate to add to your resource sheet as you answer the following questions.

1. Complete the following chart. *(3 marks)*

Length in Drawing (cm)	Actual Length (cm)	Scale
0.4	800	
	9500	1 : 1000
3.0		1 : 250

2. Represent the following scale factors in two other ways. *(3 marks)*

   a) 1 cm represents 25 km

   b) 1 : 100

   c) 1 mm represents 30 cm

continued

Assignment 5.4: Scale (continued)

3. A scale drawing of a family room has dimensions of 3.25 cm by 2.75 cm. In the scale drawing, 1 cm represents 4 m. What are the dimensions of the actual family room? Show your work. *(4 marks)*

4. Represent a rectangular window, measuring 1.6 m by 1.2 m, with a scale drawing. Use a scale factor of $\frac{1}{20}$. Show your work. *(5 marks)*

continued

Assignment 5.4: Scale (continued)

5. Create a question that requires the conversion of dimensions of a scale diagram into the actual measurements. Show your calculations in your answer. *(4 marks)*

Notes

Module 5 Summary

You have now completed Module 5!

In this module, you studied linear patterns, slope, and scale factors. Proportional reasoning was a theme throughout this module, and will be studied more extensively in Module 7 as well. If you have any questions about the material studied in Module 5, be sure to ask your tutor/marker or learning partner for help. Remember to check Appendix B: Glossary found at the end of this course.

In the next module, you will continue your study of geometry and measurement. You will be reviewing and extending your knowledge of trigonometry from Grade 10 Essential Math.

Submitting Your Assignments

It is now time for you to submit the Module 5 Cover Assignment and Assignments 5.1 to 5.4 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 5 assignments and organize your material in the following order:

- ❑ Module 5 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 5 Cover Assignment: Applying Patterns
- ❑ Assignment 5.1: Scatterplots
- ❑ Assignment 5.2: Patterns and Linear Relations
- ❑ Assignment 5.3: Slope
- ❑ Assignment 5.4: Scale

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Grade 11 Essential Mathematics (30S)

Module 5
Relations and Patterns

Learning Activity Answer Keys

Module 5:
Relations and Patterns

Learning Activity 5.1

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Solve for h: $2h - 5 = 17$
2. Solve for m: $\frac{m}{6} = \frac{9}{54}$
3. Evaluate and reduce to lowest terms: $\frac{7}{4} \times \frac{2}{7}$
4. Complete the pattern: 1, 4, 9, 16, ____ , ____
5. Samia has four brothers. Her parents had four sons and one daughter. How many sisters does Samia have?

Answers:

1. $h = 11$ (Add 5 so $2h = 22$; divide by 2 so $h = 11$.)
2. $1 \left(\frac{9}{54} \text{ reduces to } \frac{1}{6} \text{ so } m = 1 \right)$
3. $\frac{1}{2} \left(\frac{7 \times 2}{4 \times 7} = \frac{14}{28} = \frac{1}{2} \right)$
4. 25, 36 (The next numbers are 5^2 or 25, 6^2 or 36.)
5. None (She is the one daughter.)

Part B: Scatterplots

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. For each relationship, state the independent and the dependent variables.
   a) The cost of an mp3 player compared to its memory.

      Answers:

independent	memory
dependent	cost (depends on the size of memory)

   b) The processing speed of your computer compared to the memory available on the hard drive.

      Answers:

independent	memory
dependent	speed (depends on amount of memory available)

2. State whether the following situations are continuous.
   a) The final examination mark and average quiz marks for the students in a Grade 11 math class.

      Answer:

      Not continuous: Each mark is for an individual person.

   b) The price of a vehicle compared to its age.

      Answer:

      Continuous: The vehicle can be valued at any price, and the car ages continuously.

   c) Productivity of a factory and the number of workers working.

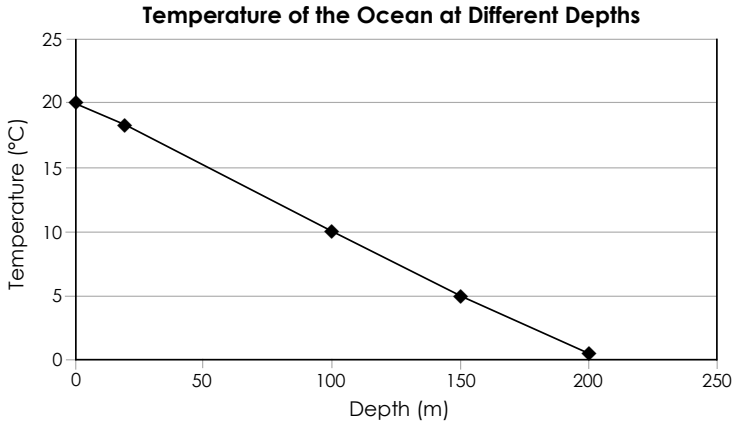
      Answer:

      Not continuous: Although productivity can be any value, you can only have a whole number of workers.

3. a) Draw a scatterplot for the following data.

Depth of the Ocean (m)	0	20	100	150	200
Temperature of the Water (°C)	20	18	10	5	1

Answer:



b) Explain why you did/did not connect the dots for this graph.

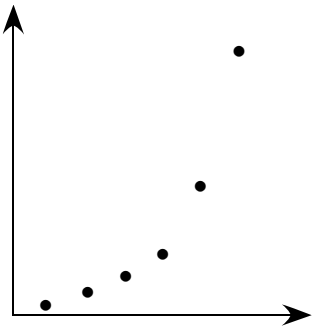
Answer:

The dots should be connected for this graph because the data is continuous. As you increase the depth even a fraction of a metre, the temperature will decrease gradually. Also, you measure (do not count) both depth and temperature.

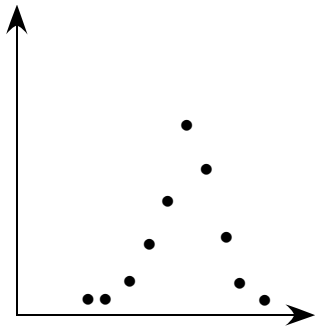
4. Match each situation with the most appropriate graph.
   a) For every $100 CDN you exchange at the bank, you receive $94 USD in return. Compare dollars CDN versus USD.
   b) Jerome has two children, and then his children have two children each, and so on. Compare number of children versus time.
   c) The class average was 85%. Only one person got 40 percent, and only four people got 92%. Compare percent versus frequency.

Answers:

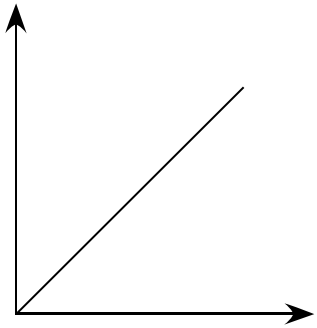
(b)



(c)



(a)



Learning Activity 5.2

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Complete the pattern: 1, 1, 2, 3, 5, 8, ____ , ____
2. A computer costs $500. Find the cost including taxes (PST = 7%, GST = 5%).
3. If 10% of 360 is 36, then 25% of 360 is ____.
4. Evaluate and reduce your answer if possible: $\frac{2}{5} + \frac{8}{15}$
5. There are three available cubicles in a row at an office for Sam, Xan, and Laura. Sam won't sit next to Xan. Can Sam sit in the middle cubicle?

Answers:

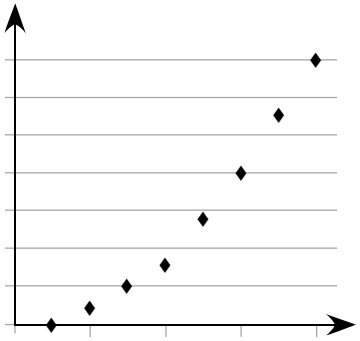
1. 13, 21 (Add the previous two items to get the next term.)
2. $560 (12% of 100 = 12 so 12 × 5 = $60 in taxes, $560 in total)
3. 90 (10% = 36, so 20% = 72, and 5% = 18, and 18 + 72 = 90)
4. $\frac{14}{15}$ $\left(\frac{2 \times 3}{5 \times 3} + \frac{8}{15} = \frac{6}{15} + \frac{8}{15} = \frac{14}{15}\right)$
5. No (If Sam won't sit next to Xan, then Sam can't be in the middle because the middle is beside both other cubicles.)

Part B: Linear Relations

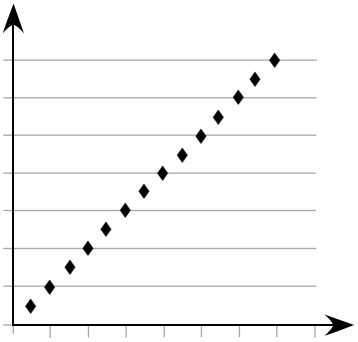
Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. State whether the graph is linear, non-linear, or does not show a pattern.

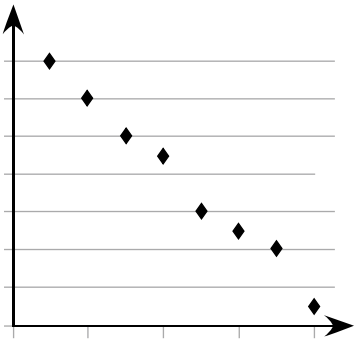
a)



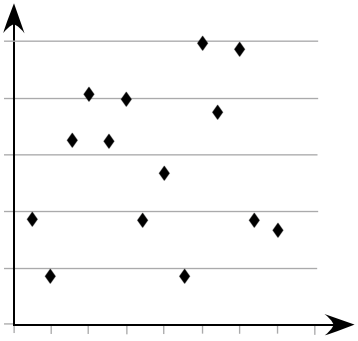
b)



c)



d)



Answers:

a) non-linear

b) linear

c) linear

d) no pattern

2. State whether each pattern is linear or non-linear. If it is linear, what number do you add each time?

   a) −4, −1, 2, 5, 8 …

   b) 1, 1, 2, 3, 5 …

   c) 1, 4, 16, 64, 256 …

   d) 251, 272, 293, 314, 335 …

   Answers:

   a) Linear: Add 3 each time.

   b) Non-linear: Although you are adding the two previous numbers to get the next number, in order for it to be linear you must add the *same number* each time.

   c) Non-linear: Each number is multiplied by 4 to get the next number.

   d) Linear: Add 21 each time.

3. Sort the following relations and patterns into linear and non-linear. (Hint: For the graphs, try to draw a line of best fit.)

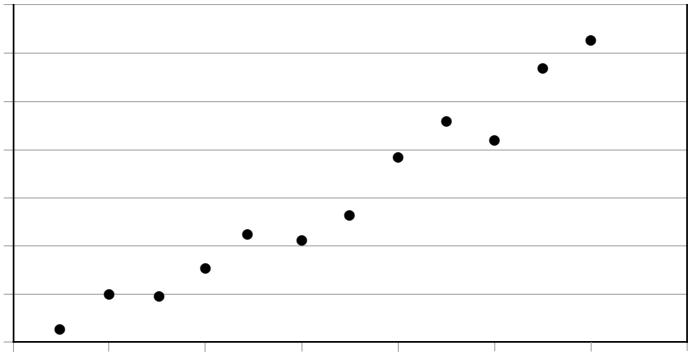
   a)

x	1	2	3	4
y	7	14	21	28

   b) −30, −19, −8, 3, 14, …

   c) −2, −4, −6, −12, −14, …

   d)

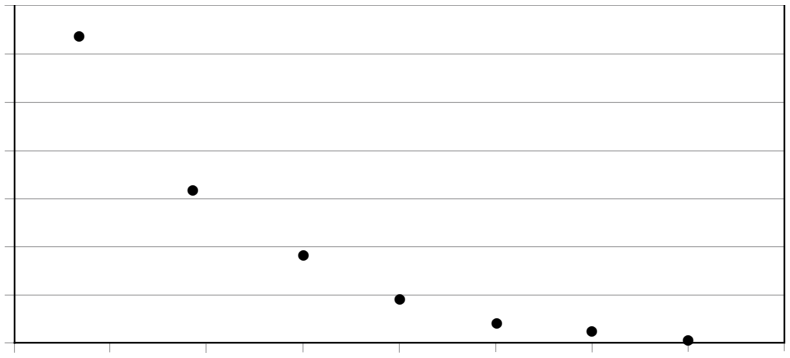


   e)

Distance (km)	1	2	5	6
Time (h)	6	12	30	36

   f) 6, 0, 2, 1, 0 …

g)



h)

Time (hours)	0	2	4	6
Temperature (°C)	25	23	21	16

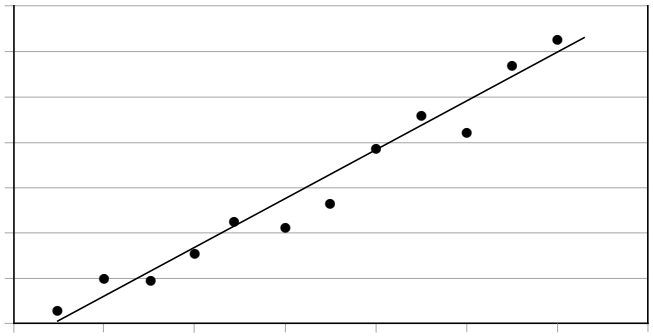
Answers:

a) This is linear because the rate of change is 7, or because all the ratios are $\frac{1}{7}$.

b) This is linear because you add 11 each time.

c) This is not linear because the difference between -6 and -12 is not 2, and the differences between other consecutive numbers is always 2.

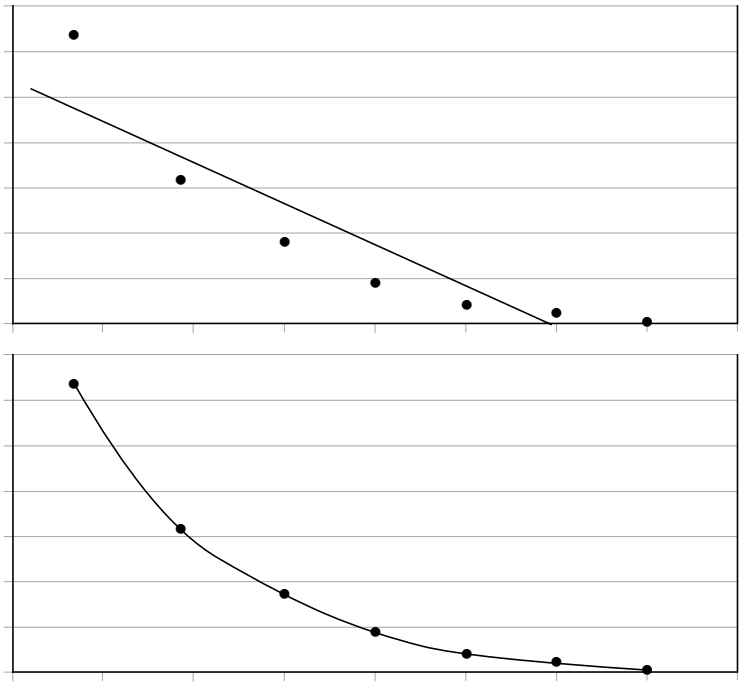
d) This is linear because you can easily draw a line of best fit.



e) This is linear because the rate of change is the same for all sets of two points: $\frac{6}{1}$, either 6 hours for 1 km or 18 hours for 3 km.

f) This is not linear because it has no obvious pattern.

g) This is not linear because it would be a very poor line of best fit. The graph is a curve.



h) This is not linear because the temperature drops 5 degrees between 4 and 6 hours, when other temperature changes are a drop of 2 degrees every 2 hours. Also, the rate of change is $\frac{-2}{2}$ or -1, except when using the last pair of numbers.

Learning Activity 5.3

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Solve the following: $5 - \frac{4 \times 5}{2} + 8$

2. Write two equivalent fractions: $\frac{20}{8}$

3. At a street vendor, hot dogs cost $4, pop costs $1.50, and chips cost $2.00. You have $6 in your pocket. Will you be able to buy one of each item?

4. Which month can have a whole number of weeks (no decimal or fraction)?

5. Solve for r: $\frac{8}{r} = 2$

Answers:

1. $3\left(5 - \frac{20}{2} + 8 = 5 - 10 + 8 = 3\right)$

2. $\frac{10}{4}$ and $\frac{5}{2}$

3. No (You could only have either a hot dog and pop ($5.50) or a hot dog and chips ($6) or chips and pop ($3.50).)

4. February (It is the only month that has a number of days (28) divisible by 7, except when it is a leap year.)

5. 4 (since 8 ÷ 4 is 2)

Part B: Direct and Partial Variation

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Under average road conditions, a vehicle can travel eight kilometres on one litre of gasoline. During the month of August, a vehicle uses the following amounts of gasoline: 30 litres, 45 litres, 10 litres, 40 litres, and 25 litres. Express the relation between the distance the vehicle travels and the amount of gasoline required to travel that distance

   a) in words

   b) as an equation

   c) with a table of values

   d) as a graph

   Answers:

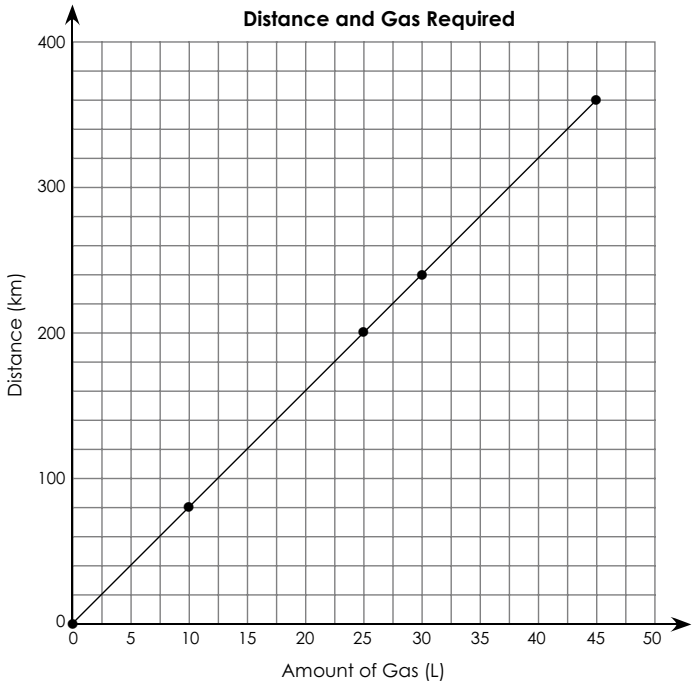
   a) The distance that can be travelled is equal to 8 multiplied by the number of litres of gasoline.

   b) $d = 8 \times g$

   c)

Amount of gas	30	45	10	25
Distance	240	360	80	200

   d)



Note: Be sure to put a title on your graph and label the axes with words, units, and numbers.

2. The cost of building a highway is $500,000 for each kilometre. Use C to represent the cost, and n the number of kilometres.
   a) State whether this relation is a direct or partial variation.
   b) State the constant of variation and the fixed value (if there is one).
   c) Express the relation as an equation.
   d) Express the relation with a table of values.
   e) Express the relation as a graph (choose appropriate scales for the coordinate axes).

Answers:

a) Direct variation, since it passes through (0, 0)

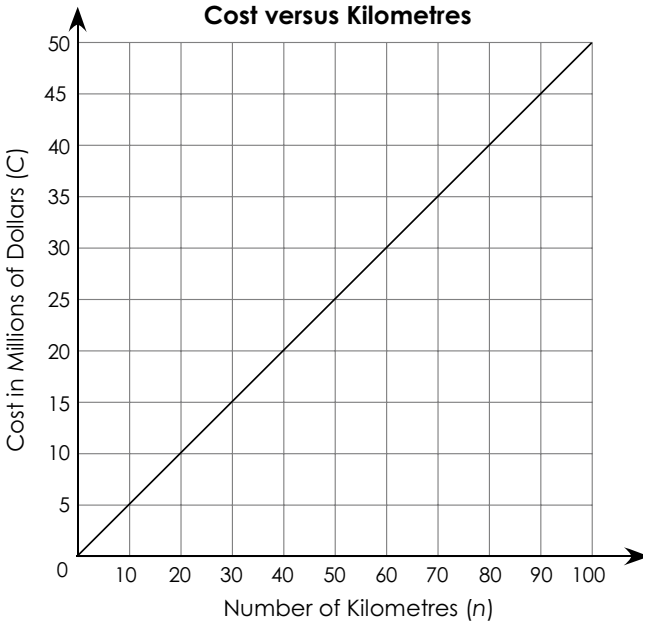
b) k = 500 000, and F = 0 (you could say there is no fixed value)

c) $C = 500\,000n$

d)

Number of kilometres (n)	0	10	20	50
Cost (C) in dollars	0	5,000,000	10,000,000	25,000,000

e)



Note: Be sure to put a title on the graph and label the axes with words, units, and numbers.

3. Consider the variations $d = 10t$ and $d = 10t + 40$

   a) Classify each variation as direct or partial.

   b) Create a table of values, and then draw each of the variations as a graph on the given grid lines. Choose appropriate scales for the coordinate axes.

   c) Find the value of d on both graphs when $t = 1$.

   d) Find the value of d on both graphs when $t = 4$.

   e) Indicate one way that the graphs are the same and one way in which they differ.

   f) Explain what the fixed value indicates.

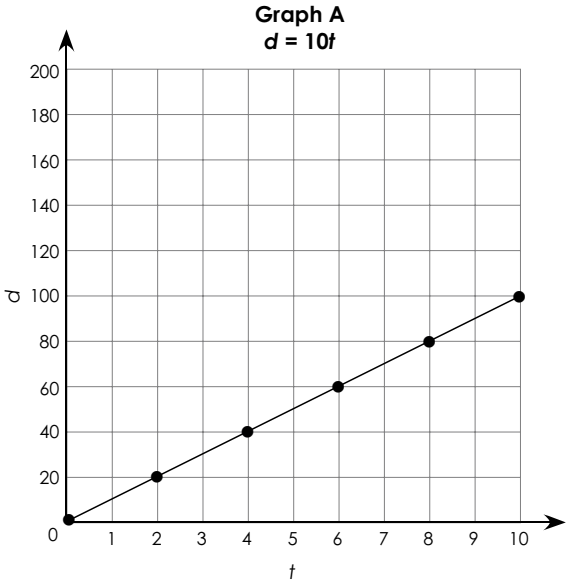
   Answers:

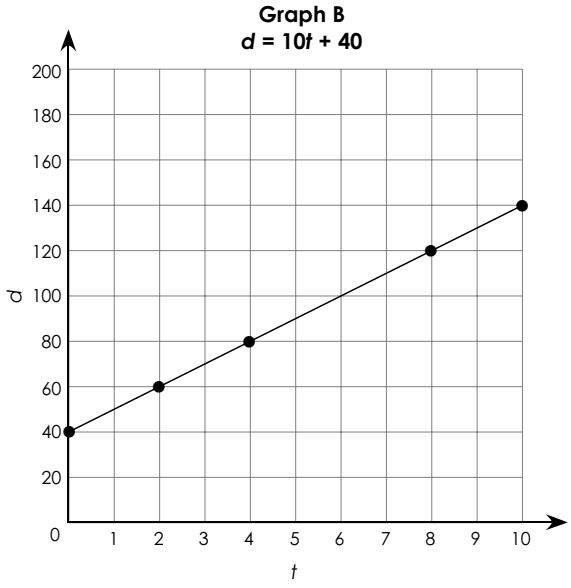
   a) The relation $d = 10t$ is a direct variation and the relation $d = 10t + 40$ is a partial variation.

   b)

t	0	2	4	8	10
$d = 10t$	0	20	40	80	100

t	0	2	4	8	10
$d = 10t + 40$	40	60	80	120	140





   c) On Graph A, where $t = 1$, $d = 10$
   On Graph B, where $t = 1$, $d = 50$

   d) On Graph A, where $t = 4$, $d = 40$
   On Graph B, where $t = 4$, $d = 80$

e) They both have the same constant of variation (the same slope) but only the graph for $d = 10t$ intersects the axes at (0, 0).

f) The fixed value indicates where the graph crosses the y-axis. For example, if the d stands for distance a used car has travelled, and the t stands for time, then the F value indicates the distance the car has travelled before you purchased it, when $t = 0$

Learning Activity 5.4

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Solve for d: $\frac{10}{d} = 20$
2. Sara has five nickels in her pocket; Damian has one quarter. Who has more money?
3. Kudus got 75% on his final examination in science. If the examination is worth 40% of his total grade, how much is his examination mark accounted for in his total grade?
4. Elyce went to the grocery store to buy food for dinner. The total cost was $45.30 and she gave the cashier three $20 bills. How much change should she receive?
5. A checkerboard alternates black and white squares. If a checkerboard is 8 squares wide by 8 squares long, how many squares are black?

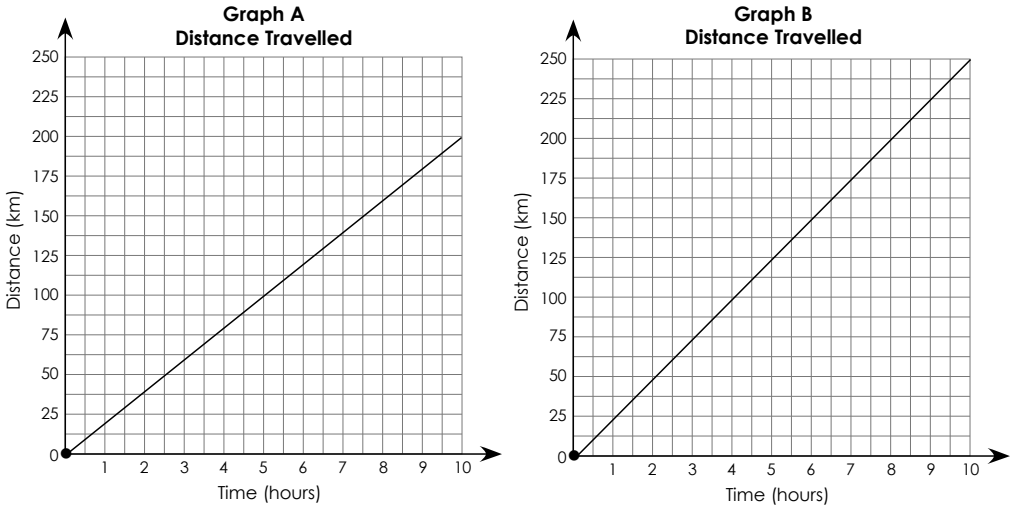
Answers:

1. $\frac{1}{2}$ $\left(10 = 20d\text{; divide by 20, so } \frac{10}{20} = d\right)$
2. They have the same amount. (A nickel is worth 5 cents so $5 \times 5 = 25$ cents. A quarter is also worth 25 cents.)
3. 30 marks (25% of 40 is 10 so 75% of 40 is 30)
4. $14.70 $(3(20) - 45.30)$
5. 32 $\left(8 \times 8 = 64, \frac{64}{2} = 32 \text{ black squares}\right)$

Part B: Slope

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Consider the following two line graphs modelling the distance two vehicles have travelled.



   a) What does the slope of each line graph represent?

   b) By observation, which line graph is steeper? Explain what that means in terms of the linear relations the two line graphs model.

   c) Using the formula for slope, determine the slope of each of the line graphs.

   d) How do the slopes of the line graphs compare to their steepness?

   Answers:

   a) The slope of each line represents the speed (in km/h) at which each vehicle was travelling.

   b) Graph B is steeper. This means that the vehicle in Graph B is faster.

   c) Using points (5, 100) and (10, 200):

   $$\text{Slope of Graph A} = \frac{200 - 100}{10 - 5} = \frac{100}{5} = 20 \text{ km/h}$$

   Using points (6, 150) and (10, 250):

   $$\text{Slope of Graph B} = \frac{250 - 150}{10 - 6} = \frac{100}{4} = 25 \text{ km/h}$$

   d) The larger slope number represents the steeper line.

2. All horizontal lines have a slope of zero. In your own words, explain why.

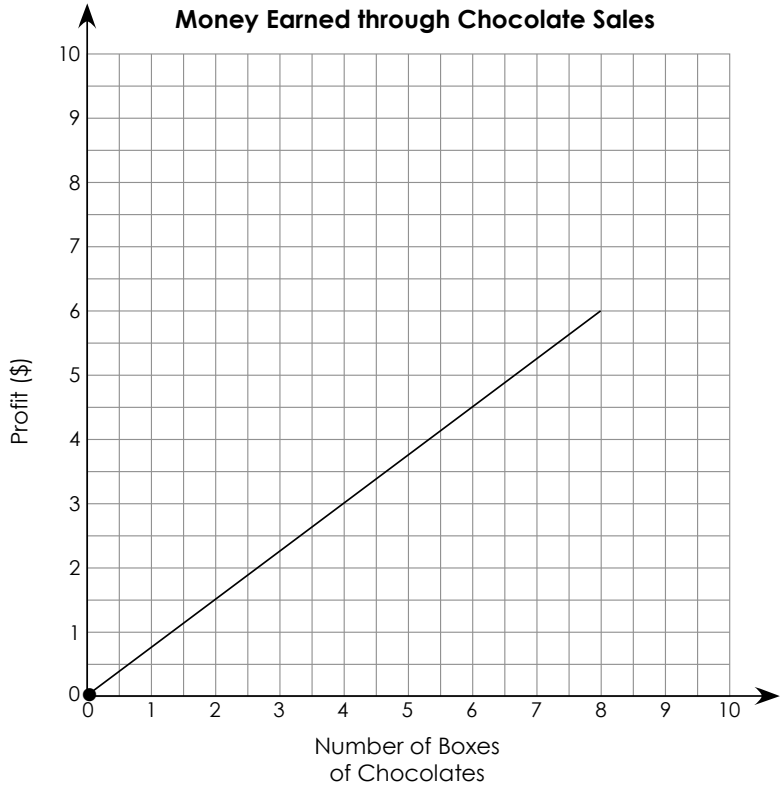
   Answer:

   Horizontal lines have a slope of zero because the change in the dependent variable (the vertical change of the graph) is zero.

   The formula for slope = change in the dependent variable/change in the independent variable.

   Since zero divided by any number is still zero, the slope is zero.

3. The following line graph represents the relation between the number of boxes of chocolates a student sells for a school fundraiser and the profit he earns for the school.



   a) State the independent and dependent variables.

   b) Determine the slope of the line graph.

   c) Express the relation represented by the line graph as a formula.

   d) What does the slope of the line graph represent?

Answers:

a) The independent variable is the number of boxes of chocolates; the dependent variable is the profit.

b) Using the points (4, 3) and (8, 6): Slope $= \frac{6-3}{8-4} = \frac{3}{4} = 0.75$ dollars per box

c) $p = 0.75 \times b$

d) The slope represents the profit earned per box of chocolates sold in dollars per box.

Learning Activity 5.5

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

Convert the following:

1. 10 cm = _____ m
2. 12 ft. = _____ yd.
3. 15 s = _____ min.
4. 0.53 km = _____ m
5. 1.5 ft. = _____ in.

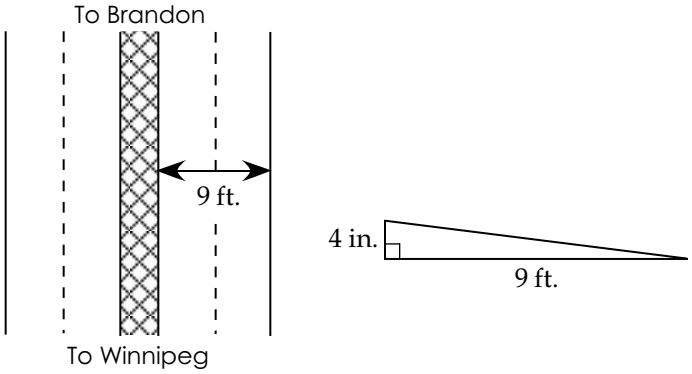
Answers:

1. 0.1 m $\left(10 \text{ cm} = \frac{10}{100} = 0.10 \text{ m}\right)$
2. 4 yd. $\left(12 \text{ ft.} = \frac{12}{3} = 4 \text{ yd.}\right)$
3. 0.25 min. $\left(15 \text{ s} = \frac{15}{60} = 0.25 \text{ min.}\right)$
4. 530 m (0.53 km = 0.53 × 1000 = 530 m)
5. 18 in. (1.5 ft. = 1.5 × 12 = 18 in.)

Part B: Slopes of Objects

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. The Trans Canada Highway is slanted (higher in the centre than on the side of the road), so that when it rains, the water runs into the ditches. Heading from Winnipeg to Brandon, the west-bound lanes are 9 feet wide. The outside of the road is 4 inches lower than the inside. What is the road grade (slope) of the highway? Write your answer rounded to two decimal places.



Answer:

(Convert both the rise and run to the same units when possible.)

rise = 4 inches = 0.333… feet

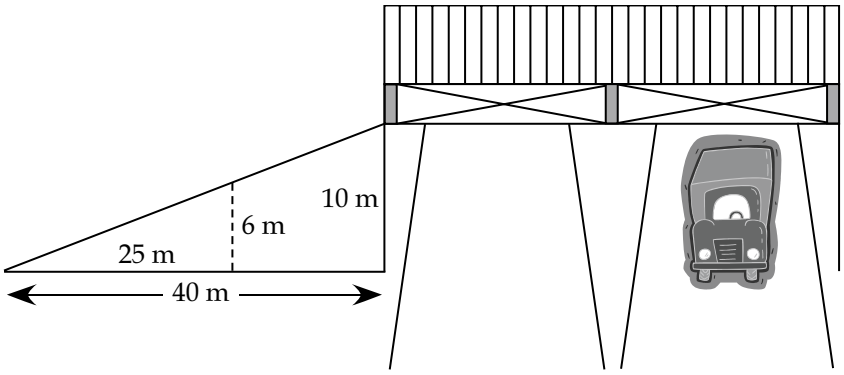
run = 9 ft.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{0.33333}{9}$$

$$= 0.04$$

The road rises 0.04 feet for every foot across.

2. A new overpass has been built. Determine whether the incline to the top of it is constant. Write your answer rounded to two decimal places.



Answer:

Interval 1:

rise = 6 m

run = 25 m

$$\text{slope} = \frac{6}{25}$$

$$= 0.24$$

Interval 2:

rise = 10 m

run = 40 m

$$\text{slope} = \frac{10}{40}$$

$$= 0.25$$

Although they are quite close, the slope isn't the same to two decimal places, so the incline is not constant. This is to be expected, because vehicles need a gradual approach to the incline. This means that the ramp is not a straight line since the slope is not constant.

3. Armin is on the treadmill for her daily workout. She runs for 30 minutes with the treadmill incline set at 4. If the actual slope of the treadmill is 0.2, what is the angle of elevation of the treadmill? Write your answer rounded to the nearest degree.

Answer:

$\tan \theta$ = slope

$\theta = \tan^{-1}(0.2)$

$\theta = 11.3^{o}$

$\theta = 11^{o}$ is the angle of elevation

4. A wheelchair ramp can have a maximum slope of 1 in 12, which means a rise of 1 inch for every 12 inches of run. Lynni inspects a ramp that has a horizontal length of 20 feet, and rises 15 inches.

   a) What is the steepest legal angle of inclination allowed for a wheelchair ramp? Write your answer rounded to one decimal place.

   Answer:

$$\tan \theta = \frac{1}{12}$$

$$\theta = \tan^{-1}\left(\frac{1}{12}\right)$$

$$\theta = 4.8°$$

   b) Does the ramp being inspected by Lynni meet safety standards? Show how you arrive at your answer.

   Answer:

   First, convert 20 feet to $20 \times 12 = 240$ inches.

$$\tan \theta = \frac{15}{240}$$

$$\theta = \tan^{-1}(0.0625)$$

$$\theta = 3.6°$$

   Therefore, the ramp meets code because its angle is less than 4.8°.

5. What are some other examples where slope is important for safety? You may discuss this with your learning partner.

   Answer:

   Other objects that have safety concerns related to slope include:

   - Skiing: Only once you prove that you are good enough are you allowed to go down the steeper hills.
   - Mudslides: Steeper mountains are more at risk to have a mudslide if the trees are removed, and the steeper a mountain is, the faster the mudslide will travel.
   - Treadmill: If you have it inclined too high for the exercise you are doing, you could fall off.
   - Arenas and stadiums: The way the seats are positioned at arenas and stadiums creates a slope. Although it is great to have a steep seating arrangement so that everyone is able to see the event easily, steep stairs could be dangerous for people as they make their way to and from their seats.

Learning Activity 5.6

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If the volume of a 2 by 2 by 2 cube is 8 units, how many times larger would the volume be of a cube with dimensions 4 by 4 by 4?
2. Complete the pattern: 4, ____, 16, 32, ____
3. There are 30 days in September, 31 days in October, 30 days in November, and 31 days in December. How many days are there between September 1st and New Year's Eve?
4. How many cm^2 are in 310 m^2?
5. Evaluate: $\frac{4}{9} \times \frac{1}{2}$

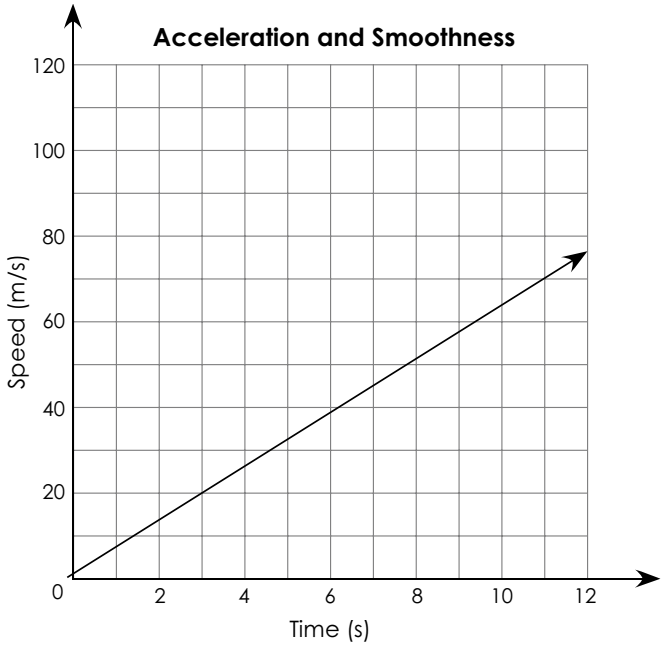
Answers:

1. 8 (Each dimension is doubled so $2^3 = 8$ times larger.)
2. 8 and 64 (Numbers are doubled each time.)
3. 120 (Since "between" means you do not count the first of September and New Year's Eve, 29 + 31 + 30 + 30 = 120 days.)
4. 3 100 000 ($1\ m^2 = (100\ cm)^2 = 10\ 000\ cm^2$ so $310\ m^2 = 3\ 100\ 000\ cm^2$)
5. $\frac{2}{9}\left(\frac{4 \times 1}{9 \times 2} = \frac{4}{18} = \frac{2}{9}\right)$

Part B: Rate of Change

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. As part of the testing of a new car, a company monitors the acceleration of the car to be sure that it is smooth. The graph below includes their data. Use it to answer the following questions.



a) Calculate the slope of the graph.

Answer:

Step 1: Point A = (0,0)

Point B = (11, 70)

Step 2: Difference in speed = 70 km/hr

Step 3: Difference in time = 11 s

Step 4: $\text{Slope} = \frac{70}{11}$

$= 6.4$ km/hr. per second

The units in this situation seem very odd because we are comparing speed (km/hr.) with time in seconds. This is because the dependent variable is a rate—speed.

b) What does the slope mean for this situation?

Answer:

The slope in this situation describes the acceleration of the car. In words, we can state that the acceleration is 6.4 km/hour per second; or that the speed increases by 6.4 km/hr. each second.

c) How long would it take the car to reach 100 km/hr.?

Answer:

You could extrapolate and extend the line to include 16 seconds or you could calculate the value as follows. We know this is a direct variation, as it goes through (0, 0), and that the rate is 6.4 km/hour per second, so we can use this as a ratio.

$$\frac{6.4 \text{ km/hr.}}{1 \text{ s}} = \frac{100 \text{ km/hr.}}{t}$$ (where t is the amount of time it will take to reach 100 km/hr.)

$$(6.4)(t) = (1)(100)$$ (Cross-multiply.)

$$6.4t = 100$$ (Simplify.)

$$\frac{6.4t}{6.4} = \frac{100}{6.4}$$ (Divide both sides by 6. to isolate t.)

$$t = 15.6$$ (Simplify.)

It would take just under 16 seconds for the car to reach 100 km/hr.

Learning Activity 5.7

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. How many hours are in 3 days?
2. Estimate 13% of $39.12.
3. If 8 students in a geometry class of 30 students can find Laos on a map, what is the fraction of the class that can find Laos on a map?
4. Evaluate: 25×16
5. What is the complementary angle to 45˚?

Answers:

1. 72 (3 days × 24 hours/day)
2. $5.10 (10% = 3.91 and 1% = 0.39, so 13% = $3.9 + 0.4 \times 3 = 5.1$)
3. $\frac{8}{30}$ (You could reduce this to $\frac{4}{15}$.)
4. 400 (25×16 is $25 \times 4 \times 4$ or 100×4)
5. 45° (Complementary means that the angles add up to 90°, so complementary to 45° is 45°, since 45 + 45 = 90.)

Part B: Converting Rates of Change

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Telmar is riding his bike at 8 m/s. The speed limit is 30 km/hr. Is he speeding?

   Answer:

   Telmar's speed: $\frac{8 \text{ m}}{\text{s}}$

   Convert metres to km: $\frac{1 \text{ km}}{1000 \text{ m}}$

   Convert seconds to hours: $\frac{1 \text{ min.}}{60 \text{ s}} \times \frac{1 \text{hr.}}{60 \text{ min.}} = \frac{1 \text{ hr.}}{3600 \text{ s}}$

   Conversion: $\frac{8 \cancel{\text{m}}}{1 \cancel{\text{s}}} \times \frac{1 \text{ km}}{1000 \cancel{\text{m}}} \times \frac{3600 \cancel{\text{s}}}{1 \text{ hr.}}$

   $= \frac{8 \times 1 \times 3600 \text{ km}}{1 \times 1000 \times 1 \text{ hr.}}$

   $= \frac{28800 \text{ km}}{1000 \text{ hr.}}$

   $= 28.8 \text{ km/hr.}$

   Telmar is not speeding.

2. Canadian highway speed limits are usually (around) 100 km/hr. In the US, the highway speed limit is approximately 60 mph. Which is faster?

   Answer:

   You could start with either one.

   Method 1:

   Canadian: $\dfrac{100 \text{ km}}{1 \text{ hr.}}$

   Convert to miles: $\dfrac{1 \text{ mi.}}{1.61 \text{ km}}$

   $$\frac{100 \cancel{\text{km}}}{1 \text{ hr.}} \times \frac{1 \text{ mi.}}{1.61 \cancel{\text{km}}} = \frac{100 \text{ mi.}}{1.61 \text{ hr.}}$$

   $$= 62 \text{ mi./hr.}$$

   Method 2:

   United States: $\dfrac{60 \text{ mi.}}{1 \text{ hr.}}$

   Convert to km: $\dfrac{1.61 \text{ km}}{1 \text{ mi.}}$

   $$\frac{60 \cancel{\text{mi.}}}{1 \text{ hr.}} \times \frac{1.61 \text{ km}}{1 \cancel{\text{mi.}}} = \frac{60 \times 1.61 \text{ km}}{1 \text{ hr.}}$$

   $$= 97 \text{ km/hr.}$$

   Using either method, you see that the Canadian speed limit is higher.

3. Idra is filling a 4 L bucket with water. The water pours at 15 mL per second.

   a) Calculate the flow rate in L/min.

   Answer:

   Flow rate: $\dfrac{15 \text{ mL}}{1 \text{ s}}$

   Convert mL to L: $\dfrac{1 \text{ L}}{1000 \text{ mL}}$

   Convert seconds to minutes:

   $\dfrac{1 \text{ min.}}{60 \text{ s}}$

   $$\frac{15 \cancel{\text{mL}}}{1 \cancel{\text{s}}} \times \frac{1 \text{ L}}{1000 \cancel{\text{mL}}} \times \frac{60 \cancel{\text{s}}}{1 \text{ min.}} = \frac{15 \times 1 \times 60 \text{ L}}{1 \times 1000 \times 1 \text{ min.}}$$

   $$= 0.9 \text{ L/min.}$$

   b) How long will it take to fill the bucket?

   Answer:

   To do this question, you need to think about what you want as your final answer. You want to be left with the time. This means that you would like to cancel out the volume you need with the volume in the rate. In order to do this, we must flip the rate upside-down.

   $$4 \cancel{\text{L}} \times \frac{1 \text{ min.}}{0.9 \cancel{\text{L}}} = \frac{4 \times 1}{0.9} \text{ min.}$$

   $$= 4.4 \text{ minutes to fill up the bucket}$$

Learning Activity 5.8

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Simon says that in five years, his dad will be double Simon's age. If Simon's dad is 43, how old is Simon?
2. You spent a total of $300 on 25 pizzas. How much did each pizza cost?
3. How many minutes are in a day?
4. What is 10% of 43?
5. It is January 18th. In 14 days, what will the date be?

Answers:

1. 19 (Dad in 5 years: $43 + 5 = 48$; Simon in 5 years: $\frac{48}{2} = 24$;
   Simon now: $24 - 5 = 19$)
2. \$12 $\left(\frac{100}{25} = 4, \text{ so } \frac{300}{25} = 3 \times \frac{100}{25} = 3 \times 4 = \$12 \text{ per pizza}\right)$
3. 1440 ($60 \times 24 = 60 \times (25 - 1) = 60 \times 25 - 60 \times 1 = 15 \times 4 \times 25 - 60 = 1500 - 60$)
4. 4.3 ($43 \div 10$)
5. February 1 (January 18 to January 31 = 13 days, so in 14 days it would be February 1.)

Part B: Scale

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Complete the following chart.

   Answers:

Length in Drawing (cm)	Actual Length (cm)	Scale
6.8	680	**1:100**
5.2	10 400	**1:2000**
4.5	**45**	1:10
9	180	1:20

   Note that scale is written as "drawing:actual."

   Also, when you are writing the scale of a drawing, you always want the number representing the drawing to be "1."

2. Represent the following scale factors in two other ways.

   a) 1 cm represents 8 m

   b) $\frac{1}{10}$

   c) 1:3

   Answers:

   a) 1 cm:8 m $\frac{1}{800}$ or 1:800

   b) 1:10 1 units represents 10 units

   c) $\frac{1}{3}$ 1 units represents 3 units

3. A building is 130 m tall. If the height of the building in the scale drawing is 3.25 cm, what scale factor is used in the drawing?

   Answer:

   First, determine the height of the building in centimetres:

   $130 \text{ m} = 130 \times 10 \times 10 = 13\,000 \text{ cm}$

   We know that 3.25 cm represents 13 000 cm, so now we must determine how much 1 cm represents.

   $$\frac{3.25}{13\,000} = \frac{1}{x}$$

   $3.25x = 13\,000$ (Divide both sides by 3.25.)

   $x = 4000$

   The scale is therefore 1:4000.

   Alternately the scale can be represented by 1 cm : 40 m.

4. A volleyball court measures 5 m by 8 m. If it is drawn with a scale factor of 1 : 300, what are the dimensions of the volleyball court in the scale drawing?

   Answer:

   First, find the width of the volleyball court in centimetres:

   $5 \text{ m} = 5 \times 10 \times 10 = 500 \text{ cm}$

   Now set up the proportions to determine the width of the court in the scale drawing:

   $$\frac{1}{300} = \frac{x}{500}$$

   $300x = 500$ (Divide both sides by 300.)

   $x = 1.7 \text{ cm}$ (to the nearest tenth)

   Follow the same steps for the length:

   $8 \text{ m} = 8 \times 10 \times 10 = 800 \text{ cm}$

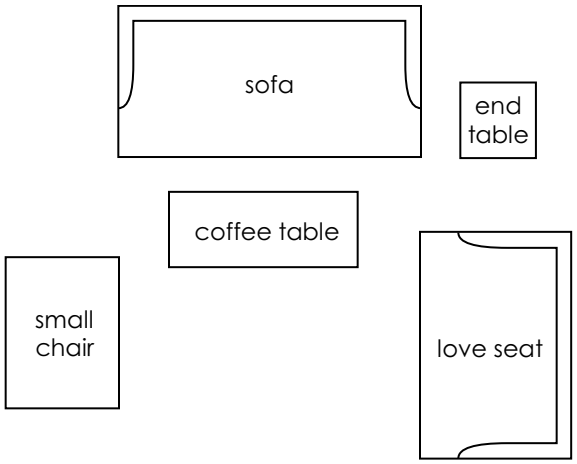
   $$\frac{1}{300} = \frac{x}{800}$$

   $300x = 800$ (Divide both sides by 300.)

   $x = 2.7 \text{ cm}$ (to the nearest tenth)

   Therefore, the dimensions of the scale drawing of the court are 1.7 cm by 2.7 cm.

5. The following scale diagram represents an arrangement of furniture in a living room. The scale factor is 1 cm represents 60 cm. Find the actual dimensions of each piece of furniture in the room in metres.



Answer:

Small Chair

The dimensions in the scale drawing are 1.5 cm by 2 cm.

First, find the actual width:

$$\frac{1}{60} = \frac{1.5}{x}$$

$x = 90$ cm (Divide by 100 to convert to metres.)

$= 0.9$ m

Find the actual length:

$$\frac{1}{60} = \frac{2}{x}$$

$x = 120$ cm (Divide by 100 to convert to metres.)

$= 1.2$ m

The chair is 0.9 m by 1.2 m.

Sofa

The dimensions in the scale drawing are 4 cm by 2 cm.

First, find the actual length:

$$\frac{1}{60} = \frac{4}{x}$$

$x = 240$ cm (Divide by 100 to convert to metres.)

$= 2.4$ m

Find the actual width:

$$\frac{1}{60} = \frac{2}{x}$$

$x = 120$ cm (Divide by 100 to convert to metres.)

$= 1.2$ m

The sofa is 2.4 m by 1.2 m.

Coffee Table

The dimensions in the scale drawing are 2.5 cm by 1 cm.

First, find the actual length:

$$\frac{1}{60} = \frac{2.5}{x}$$

$x = 150$ cm (Divide by 100 to convert to metres.)

$= 1.5$ m

Find the actual width:

$$\frac{1}{60} = \frac{1}{x}$$

$x = 60$ cm (Divide by 100 to convert to metres.)

$= 0.6$ m

The coffee table is 1.5 m by 0.6 m.

End Table

The dimensions in the scale drawing are 1 cm by 1 cm.

Since the length and width are the same, we need only do the calculations once.

$$\frac{1}{60} = \frac{1}{x}$$

$x = 60$ cm (Divide by 100 to convert to metres.)

$= 0.6$ m

The end table is 0.6 m by 0.6 m.

Love Seat

The dimensions in the scale drawing are 3 cm by 2 cm.

First, find the actual length:

$$\frac{1}{60} = \frac{3}{x}$$

$x = 180$ cm (Divide by 100 to convert to metres.)

$= 1.8$ m

Find the actual width:

$$\frac{1}{60} = \frac{2}{x}$$

$x = 120$ cm (Divide by 100 to convert to metres.)

$x = 1.2$ m

The love seat is 1.8 m by 1.2 m.

Grade 11 Essential Mathematics (30S)

Module 6
Trigonometry

Module 6: Trigonometry

Introduction

Trigonometry is a branch of mathematics involving the measurement of triangles. The word *trigonometry* is made up of two Greek words:

trigonon—triangle

metria—to measure

In Grade 9 Mathematics, you used trigonometric ratios and other formulas to calculate the side lengths and angle measures of right triangles. You will review these calculations before going on to more complex problems that involve multiple right triangles.

Assignments in Module 6

When you have completed the assignments for Module 6, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Math Operations
2	Assignment 6.1	Applying Trigonometry

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the following list of instructions to help you with preparing your resource sheet for the material in Module 6. On this sheet, you should record mathematics terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 4, 5, 6, and 7 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 4 to 7.

Resource Sheet for Module 6

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important mathematics terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how each formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Module 6 Cover Assignment: Math Operations

There are many ways to create a number using smaller numbers. For example, to create the number 30 using the same number three times, we could:

a) Add three 10s: $10 + 10 + 10 = 30$

b) Multiply two 5s, then add the third 5: $5 \times 5 + 5 = 30$

c) Multiply two 6s, then subtract the third 6: $6 \times 6 - 6 = 30$

d) Cube the number 3, then add another 3: $3^3 + 3 = (27 + 3) = 30$

For this exercise, it is very important that you understand the order of operations when evaluating a mathematical statement. You may recall the term *BEDMAS*, which is frequently used to remind us of the order in which we need to do math operations.

Brackets

Exponents

Division
Multiplication
} Divide and multiply as you read the equation (left to right)—it does not matter if you divide or multiply first.

Addition
Subtraction
} Add and subtract as you read the equation (left to right)—it does not matter if you add or subtract first.

Example 1

Write all the combinations that create 20, using the same number three times. (There are two possibilities.)

Solution

$4 \times 4 + 4 = 20$

$5 \times 5 - 5 = 20$

Example 2

Use any mathematical operations (addition, subtraction, multiplication, division, exponents, brackets, and square roots) to make the following true. Find all four solutions.

5 5 5 5 5 = 100

Solution

$(5 \times 5 \times 5) - (5 \times 5) = 100$

$(5 + 5 + 5 + 5) \times 5 = 100$

$5 \times 5 \times \left(5 - \frac{5}{5}\right) = 100$

$(5 \times 5 - 5) \times \sqrt{5 \times 5} = (20 \times 5) = 100$

Example 3

Use any mathematical operations (addition, subtraction, multiplication, division, exponents, brackets, and square roots) to make the following true. Find both solutions.

5 5 5 5 5 = 55

Solution

$5 \times 5 + 5 \times 5 + 5 = 55$

You could have brackets around the 5s being multiplied together

$(5 \times 5) + (5 \times 5) + 5 = 55$

$(5 + 5 + (5 \div 5)) \times 5 = 55$

Example 4

Use any mathematical operations (addition, subtraction, multiplication, division, exponents, brackets, and square roots) to make the following true. Find all four solutions.

9 9 9 9 9 9 = 100

Solution

$9 \times 9 + 9 + 9 + 9 \div 9 = (81 + 9 + 9 + 1) = 100$

$(9 + 9 \div 9) \times (9 + 9 \div 9) = (10 \times 10) = 100$

$99 + 99 \div 99 = 100$

$(999 - 99) \div 9 = (900 \div 9) = 100$

In the following cover assignment, you are asked to answer questions similar to the examples shown above. If you do not understand the above examples, ask your learning partner or tutor/marker for help.

Module 6 Cover Assignment

Math Operations

Total: 10 marks

Use any mathematical operations (addition, subtraction, multiplication, division, exponents, brackets, and square roots) to create true statements. If you do not need to use all the 4s provided, cross out the extra ones. An example is provided. Show your work. Each question is worth 1 mark.

Example:

4 4 4 4 = 0 *Answer:* $4 - 4 + (4 - 4) = 0$ **or** $4 - 4 - 4 + 4 = 0$

1. 4 4 4 4 = 1

2. 4 4 4 4 = 2

3. 4 4 4 4 = 3

4. 4 4 4 4 = 4

5. 4 4 4 4 = 5

continued

Module 6 Cover Assignment: Math Operations (continued)

6. 4 4 4 4 = 6

7. 4 4 4 4 = 7

8. 4 4 4 4 = 8

9. 4 4 4 4 = 9

10. 4 4 4 4 = 10

Lesson 1: Review of Right-Angled Triangles

Lesson Focus

In this lesson, you will

- ❑ review the Pythagorean theorem
- ❑ find the third angle of a triangle if two angles are known
- ❑ use trigonometric ratios to calculate side lengths and angle measures of right triangles

Lesson Introduction

This lesson will review what you learned about right triangles in previous grades. It is important that you understand these concepts because all trigonometry is based on the general properties of triangles. If you have any questions during the lesson, be sure to ask your learning partner or tutor/marker.

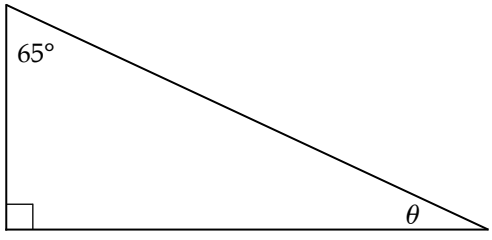
Measuring Sides and Angles

In the module introduction, you were told that the word *trigonometry* means *to measure triangles*. As the name implies, trigonometry with right-angled triangles involves finding the length of all three sides and *two* angles (because you already know that the right angle equals 90 degrees). You can use one of the following methods to find the missing angle or side:

1. Given two angles, you can find the third angle of any triangle, because the sum of the angles of any triangle is 180°.

   $\angle A + \angle B + \angle C = 180°$

   Example:

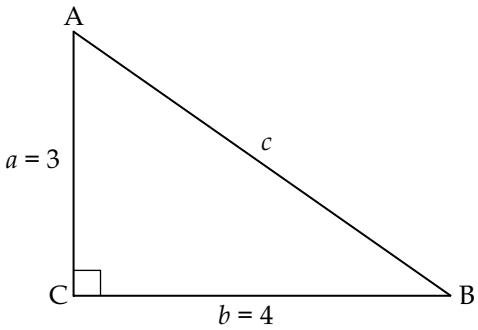
   $\theta = 180° - 90° - 65° = 25°$

2. Given two sides, you can find the third side of the right-angled triangle using the Pythagorean theorem:

   If c is the hypotenuse of a right triangle and a, b are the legs of the right triangle, then $a^2 + b^2 = c^2$.

   Example:

   Note: c is the hypotenuse.



$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 25$$
$$\sqrt{c^2} = \sqrt{25}$$
$$c = 5$$

3. If you do not have enough information to use the above formulas, use trigonometric (trig) ratios to find the measures of angles and sides of right triangles. Trig ratios will be reviewed on the next few pages.

In conclusion, to calculate the sides and angles of any right-angled triangle, apply these rules:

1. If you know two sides of a right-angled triangle, calculate the third side using the Pythagorean theorem.
2. If you know two angles of a right-angled triangle, find the third angle by applying the property that the sum of three angles in a triangle is 180°.
3. Use trig ratios for all other situations.

Include these rules in your resource sheet. They may be helpful in the following lessons in this module.

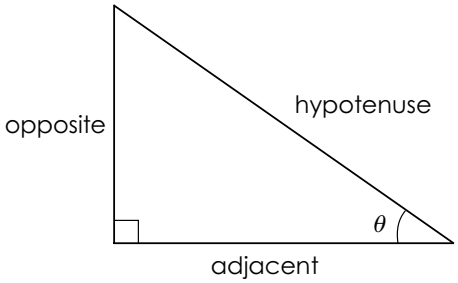
Right-Angled Trigonometry

In Grade 9, you studied three trig ratios you can use to solve right triangles. The ratios are sine, cosine, and tangent. These names are abbreviated to sin, cos, and tan.

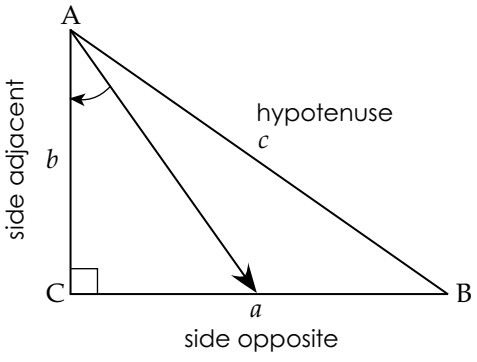
$$\sin\theta = \frac{\text{side opposite the angle } \theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{side adjacent to the angle } \theta}{\text{hypotenuse}}$$

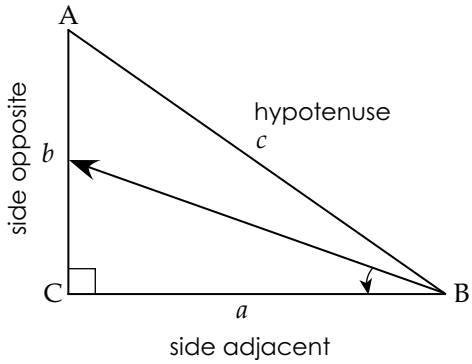
$$\tan\theta = \frac{\text{side opposite the angle } \theta}{\text{side adjacent to the angle } \theta}$$



For angle A



For angle B



Therefore,

$$\sin\angle A = \frac{a}{c} \qquad \sin\angle B = \frac{b}{c}$$

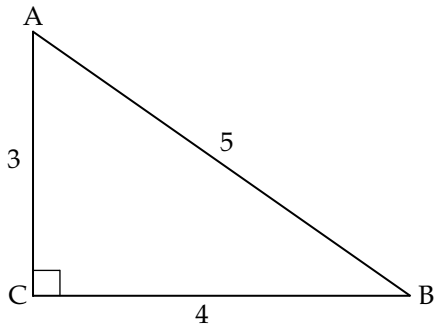
$$\cos\angle A = \frac{b}{c} \qquad \cos\angle B = \frac{a}{c}$$

$$\tan\angle A = \frac{a}{b} \qquad \tan\angle B = \frac{b}{a}$$

Example 1

a) Find sin, cos, and tan of angle A. Write your answers as fractions.

b) Find sin, cos, and tan of angle B. Write your answers as fractions.



Solution

a) $\sin \angle A = \frac{4}{5}$ b) $\sin \angle B = \frac{3}{5}$

$\cos \angle A = \frac{3}{5}$ $\cos \angle B = \frac{4}{5}$

$\tan \angle A = \frac{4}{3}$ $\tan \angle B = \frac{3}{4}$

Did you ever notice that sin ∠A = cos ∠B and cos ∠A = sin ∠B? This is because the side opposite ∠A is adjacent to ∠B, and the side opposite to ∠B is adjacent to ∠A.

When solving a right-angled triangle, draw a sketch and fill in the given information. Label the unknown angles and sides. Then, using the three trig ratios, find the missing sides and angles. This is a useful tip that you should include in your resource sheet.

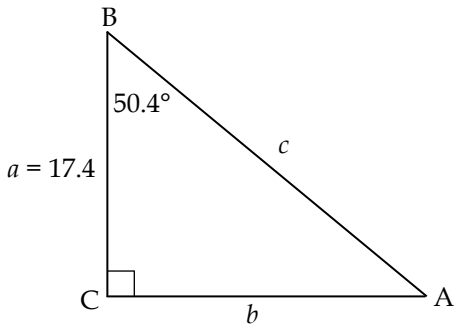
Example 2

Solve the following right-angled triangle. Note: solving a triangle means calculating the measures of any unknown parts of the triangle. In this case, it means: calculate the values for ∠A and sides *b* and *c*.

Given: $\angle C = 90^o$
$a = 17.4$
$\angle B = 50.6^o$

Solution

Sketch a right triangle, write A, B, and C to identify the angles, and *a*, *b*, and *c* to identify the sides (*a* is opposite ∠A, etc.). Then add the given information, as shown in the diagram below.



You can solve this problem in several ways, but it is a good idea to use the *given* measurements as much as possible, so that if you miscalculate a measurement it should not influence other measurement calculations. For this reason, use tan ∠B to solve for *b*, and cos B to solve for *c*.

Step 1: Solve for ∠A.

$$\angle A = 180^o - 90^o - 50.4^o = 39.6^o$$

Step 2: Solve for side *b*.

$$\tan \angle B = \frac{b}{a}$$

$$\tan 50.4° = \frac{b}{17.4} \quad \text{(Multiply both sides by 17.4.)}$$

$$(17.4)\tan 50.4° = b$$

$$21 \text{ units} = b$$

Step 3: Solve for side c.

$$\cos \angle B = \frac{a}{c}$$

$$\cos 50.4° = \frac{17.4}{c} \qquad \text{(Multiply both sides by } c\text{.)}$$

$$c \times \cos 50.4° = 17.4 \qquad \text{(Divide both sides by (cos 50.4°).)}$$

$$c = \frac{17.4}{\cos 50.4°}$$

$$c = 27.3 \text{ units}$$

$\therefore \angle A = 39.4^{o}$, $b = 21$ units, $c = 27.3$ units

Note: Be sure your calculator is set to *degrees*.

Note: To check to see if your answers are reasonable, compare the sides of the triangle with their opposite angles. If the smallest angle is opposite the shortest side and the largest angle is opposite the longest side, then your answers are reasonable.

From Example 2: $\angle A = 39.4^{o}$ $a = 17.4$

$\angle B = 50.4^{o}$ $b = 21$

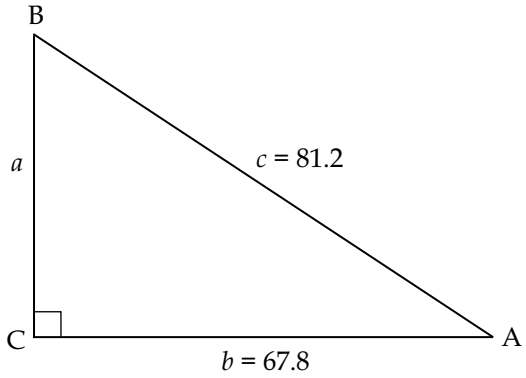
$\angle C = 90^{o}$ $c = 27.3$

The smallest angle and shortest side are $\angle A$ and side a. The largest angle and longest side are $\angle C$ and side c. The answers are reasonable.

Example 3

Solve the following right-angled triangle.

Given: $\angle C = 90°$
$b = 67.8$
$b = 81.2$



Solution

In this case, to use the given information as much as possible, we will use the Pythagorean theorem to find a, sine to find $\angle B$, and cosine to find $\angle A$.

Step 1: Solve for a.

Note: c is the hypotenuse.

$$c^2 = a^2 + b^2$$

$$81.2^2 = a^2 + 67.8^2$$

$$81.2^2 - 67.8^2 = a^2$$ (Subtract 67.8^2 from both sides.)

$$1996.6 = a^2$$ (Square root of both sides.)

$$\sqrt{1996.6} = \sqrt{a^2}$$

$$44.7 = a$$

Step 2: Solve for $\angle B$.

$$\sin \angle B = \frac{b}{c}$$

$$\sin \angle B = \frac{67.8}{81.2}$$

$$\angle B = \sin^{-1}\left(\frac{67.8}{81.2}\right)$$ (Use the second function [sin] button on your calculator to isolate $\angle B$.)

$$\angle B = 56.6°$$

Step 3: Solve for ∠A.

$$\cos \angle A = \frac{b}{c}$$

$$\cos \angle A = \frac{67.8}{81.2}$$

$$\angle A = \cos^{-1}\left(\frac{67.8}{81.2}\right)$$ (Use the second function [cos] button on your calculator to isolate ∠A.)

$$\angle A = 33.4°$$

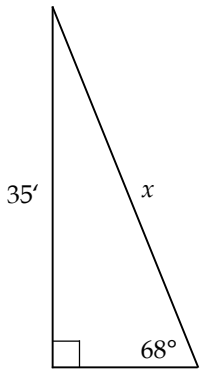
$\therefore a = 44.7$ units, $\angle B = 56.6^{o}$, $\angle A = 33.4^{o}$

Example 4

A brace cable is needed to support a satellite antenna that stands vertically on a horizontal roof. How long must the cable be if the satellite antenna is 35 feet high and the cable makes a 68° angle with the roof?

Solution

Step 1: Sketch a diagram.



Step 2: Set up a ratio where you know two values and need the third. Since you know an angle, its opposite side, and need to find the hypotenuse, use the sine ratio $\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)$.

$$\sin 68° = \frac{35}{x}$$ (Multiply both sides by x.)

$$x \times \sin 68° = 35$$ (Divide both sides by sin 68°.)

$$x = \frac{35}{\sin 68°}$$

$$x = 37.7 \text{ feet}$$

Complete the following learning activity and check your answers. Be sure that you know how to solve a right triangle. You will be doing more questions like this in the next lesson.

Learning Activity 6.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. It is 11:00 am on Monday. You have a chess tournament at 6:00 pm on Tuesday. How many hours do you have to wait?
2. How much would you have to pay for an 8 ft. long piece of wood if the hardwood store charges $1.95 per foot?
3. How many millimetres are there in 32 centimetres?
4. Simplify the fraction: $\frac{6}{20}$.
5. Solve for x: $\frac{10}{x} = \frac{5}{4}$.

Part B: Review of Right-Angle Triangle Trigonometry

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Find the measures of the missing sides and angles in the following right-angled triangles.
   a) $\angle C = 90°$

      $a = 52.7$

      $b = 39.8$

continued

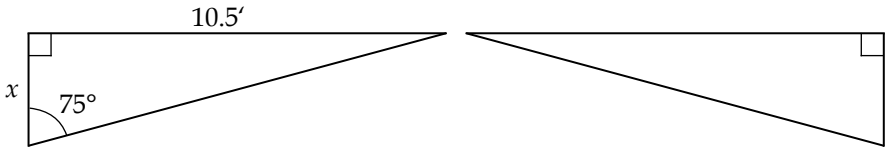
b) $\angle C = 90°$
$\angle B = 21.4°$
$a = 215.7$

c) $\angle C = 90°$
$b = 236.9$
$c = 762.6$

d) $\angle C = 90°$
$\angle A = 40.7°$
$c = 68.7$

2. To be safe, professional painters lean their ladders at an angle between 60° and 70° with the ground. If a painter uses a ladder 20 feet long to reach a window 15.7 feet off the ground, will she be safe? If not, what length of ladder, to the nearest foot, should she use?

3. An iron rod gate is made in the form of two identical right-angled triangles, as depicted below.



What total length of iron rod (to the nearest tenth of a foot) is needed to build both halves of the gate?

Lesson Summary

In this lesson, you reviewed the topics of trigonometry from Grade 10 mathematics. To complete the next lesson, you need to know how to solve a right triangle. The problems in the next lesson will include multiple right triangles in each problem.

Lesson 2: Applying Trigonometry

Lesson Focus

In this lesson, you will

- ❑ solve problems with angle of elevation and angle of depression
- ❑ solve problems that include two or more right triangles

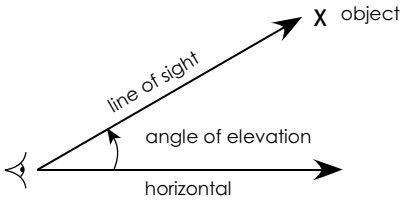
Lesson Introduction

Todd must determine the height of a large statue on top of a tall building in a city, but he cannot get onto or into the building to measure it. Therefore, he uses trigonometry to calculate the height of the statue. In this lesson, you will solve problems similar to the one Todd must solve.

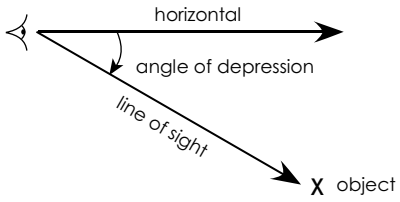
With trigonometry, we are able to calculate lengths and directions from a distance. Astronomers use trigonometry to calculate distances between celestial bodies, and physicists use trigonometry to analyze forces on an object. Tinsmiths, plumbers, and home designers also use trigonometry in their work. This lesson will focus on the application of trigonometry.

Angles of Elevation and Depression

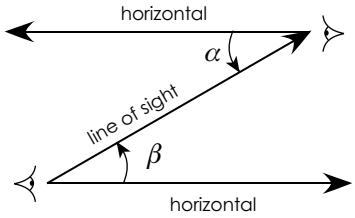
Typically, people look straight ahead horizontally. When you look up to see an object above you, your line of sight is elevated from the horizontal direction. As it was discussed in the previous module, this is called the **angle of elevation**.



Similarly, if you look down at an object, the **angle of depression** is the number of degrees below the horizon.



Using your knowledge of transversal lines from Grade 10 Essential Math, we can also state that if you look up at another person, your angle of elevation (β) is the same measure as the other person's angle of depression (α). This is true because *horizontal* lines are parallel, and α and β are alternate interior angles for parallel lines.



You may want to include examples of these three principles on your resource sheet.

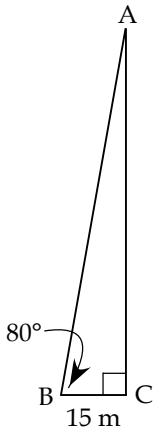
The following problems include examples of angle of elevation and angle of depression.

Example 1

From a point on the ground 15 m from the base of the Richardson Building, the angle of elevation of the top of the building is 80°. Find the height of the building to the nearest metre.

Solution

Draw a sketch that includes a right triangle, the angle of elevation, and the 15-metre distance from the building. The triangle does not need to be drawn to scale, but it must be labelled correctly. You should label the right angle with a small square.



$$\tan 80° = \frac{b}{15}$$

$$15 \times \tan 80° = b$$

$$85.1 = b$$

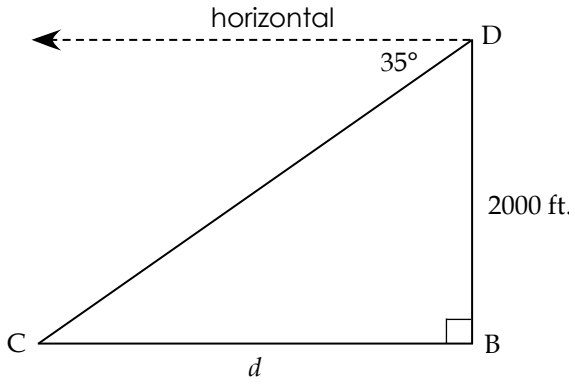
The Richardson Building is 85.1 m tall.

Example 2

An airplane has to land on an aircraft carrier in the middle of the ocean. In order for the pilot to see the aircraft carrier, he must look down with an angle of depression of 35°. If the airplane is 2000 feet above the water, how far is the airplane from the carrier? Round your answer to the nearest foot.

Solution

Draw a picture.



We know that if the angle of depression to the carrier is 35°, then the angle of elevation (∠C) from the carrier to the airplane is also 35°. Note that distance is the horizontal distance (d), and not the distance along his flight path.

$$\tan 35° = \frac{2000}{d} \qquad \text{(Multiply by } d \text{ on both sides.)}$$

$$d \times \tan 35° = 2000 \qquad \text{(Divide by } \tan 35°\text{.)}$$

$$d = \frac{2000}{\tan 35°}$$

$$d = 2856$$

The airplane is 2856 feet from the aircraft carrier.

Multiple Triangles

In addition to solving questions that require only one triangle, there are many problems that require you to use more than one triangle. It is important to sketch the correct diagram when you read a word problem. We will focus on doing this as well as solving the problem.

Example 1

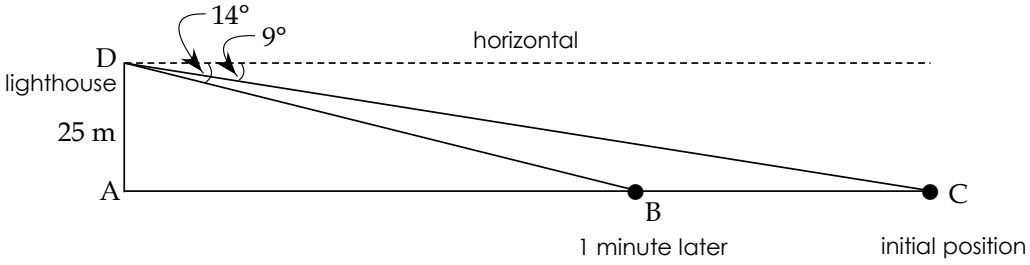
From the top of a 25 m lighthouse, the angle of depression of a boat is 9°. One minute later, the angle of depression of this boat is 14°. How fast is the boat travelling? (Assume the boat is heading toward the lighthouse and the boat is level with the base of the lighthouse.) Write your answer as metres/minute.

Solution

In order to sketch a diagram for this problem, we must identify all the triangles.

- angle of depression = 9^o
- angle of depression = 14^o

We know that both of these angles are seen from the same point and go out in the same direction, so the diagram will look like this.



We know that the angle of elevation = angle of depression from a horizontal line, so $\angle C = 9^o$ and $\angle DBA = 14^o$.

Note: There are multiple angles at point B. We want the angle in ΔADB.

In order to find out how fast the boat is going, we must find out how far the boat has travelled in one minute (remember that speed is distance divided by time). We cannot calculate this directly because we do not have a right triangle with BC as one of its sides. Instead, we must find the distance AC, and then subtract the distance of AB.

$$\tan \angle DBA = \frac{25}{AB} \qquad \tan \angle C = \frac{25}{AC}$$

$$\tan 14° = \frac{25}{AB} \qquad \tan 9° = \frac{25}{AC}$$

$$AB \times \tan 14° = 25 \qquad AC \times \tan 9° = 25$$

$$AB = \frac{25}{\tan 14°} \qquad AC = \frac{25}{\tan 9°}$$

$$AB = 100.3 \text{ m} \qquad AC = 157.8 \text{ m}$$

So the distance the boat travelled in one minute is 157.8 − 100.3 = 57.5 m. The boat's speed is 57.5 m/min. If you were to convert this, it would be 3.45 km/hr.

Example 2

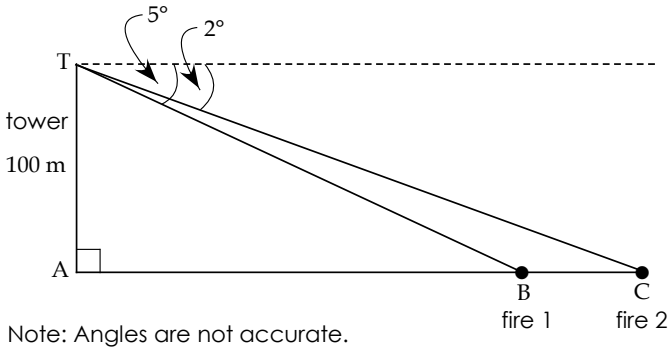
From the top of a 100 m-tall fire tower, a fire ranger observes two fires, one at an angle of depression of 5° and the other at an angle of depression of 2°. Assuming that the fires and the tower are in a straight line, how far apart are the fires if the fires are

a) on the same side of the tower?

b) on opposite sides of the tower?

Solution

a) This diagram will include two right-angle triangles, one to each fire. They will both be on the same side of the tower.



Note: Angles are not accurate. Due to space limitations, angle measurements have been increased.

Distance to fire 1:

$$\tan 5° = \frac{100}{AB}$$

$$AB \times \tan 5° = 100$$

$$AB = \frac{100}{\tan 5°}$$

$$AB = 1143 \text{ m}$$

Distance to fire 2:

$$\tan 2° = \frac{100}{AC}$$

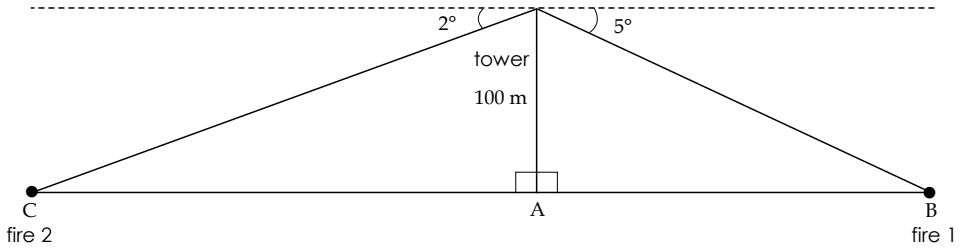
$$AC \times \tan 2° = 100$$

$$AC = \frac{100}{\tan 2°}$$

$$AC = 2864 \text{ m}$$

The fires are $2864 - 1143 = 1721$ m apart.

b) This diagram will also include two right triangles. In this case, they will be on opposite sides of the tower.



Note: Angles are not accurate.
Due to space limitations, angle measurements have been increased.

Distance to fire 1 = 1143 m (the same as part (a))

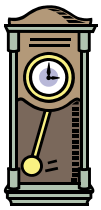
Distance to fire 2 = 2864 m (the same as part (a))

Because the fires are on opposite sides, we will add instead of subtract.

The distance between the fires is $2864 + 1143 = 4007$ m

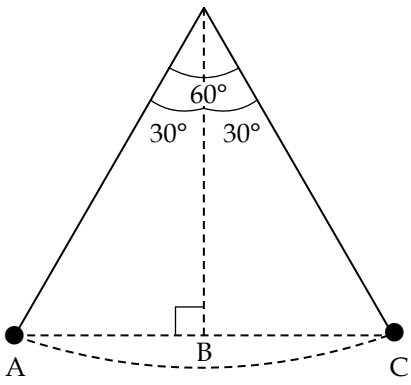
Example 3

A pendulum, one foot long, swings through an angle of 60° from one side to the other. How wide does the clock have to be so that the pendulum does not hit the sides as it swings?



Solution

The entire path of the pendulum covers 60°. Pendulums are weighted so that they swing the same amount left as they do right, which means that we can divide the swing in half to create two congruent right triangles. (Congruent means the same shape and size.) The diagram for this question should look like the following:



To answer the question, we must find out how wide the pendulum swings (AC). Because there is no right triangle with the side AC, we must use the right triangles to calculate AB and BC. Because they are equal (the triangles are congruent), we only need to calculate one of them.

$$\sin 30° = \frac{AB}{1 \text{ ft.}}$$

$$(1)\quad \sin 30° = AB$$

$$0.5 \text{ ft.} = AB$$

The width that the pendulum travels is 0.5 + 0.5 = 1 ft., so the clock must be wider than one foot.

Now that you have seen a few examples, it is time for you to work through the following exercise. Remember to carefully draw and label a diagram before doing any calculations. This is often the most difficult part of a problem. If you cannot understand a problem, contact your learning partner or tutor/marker for help.

Learning Activity 6.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. What is $\frac{2}{3}$ of 180?
2. You cut off 17 inches of a 2 ft., 5 in. board of wood. How long is it now?
3. Dagnus is 30 years old. How old is he in months?
4. Complete the pattern: 1, 2, 6, 24, 120, ____ , ____
5. How far would you travel if you drove 45 km/h for 20 minutes?

Part B: Applying Trigonometry

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. A hot-air balloon advertising for a real estate company is 250 m above ground level. The angle of depression from the balloon to the landing area is 30°. What is the distance along the ground from a point beneath the balloon to the landing area?

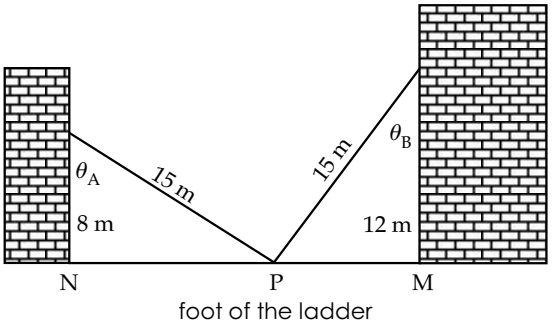
2 A surveyor uses a transit (a device used to measure angles) to measure the angle of elevation from the transit to the top of a building. The horizontal distance from the top of the transit to the building is 34 m, the angle measures 34°, and the height of the transit is 1.9 m. How high is the building? Write your answer rounded to one decimal place.

continued

Learning Activity 6.2 (continued)

3. The angle of elevation to the top of an apartment block from a window in a nearby building is 28°. The angle of depression from the window to the bottom of an apartment block is 38°, and the window is 24 feet above the ground. How tall is the apartment block (to the nearest foot)?

4. A ladder 15 m long is placed on a driveway between two buildings so that it reaches 12 m up on one building. If it is turned over, its foot being held in position, it will reach 8 m up the other building.

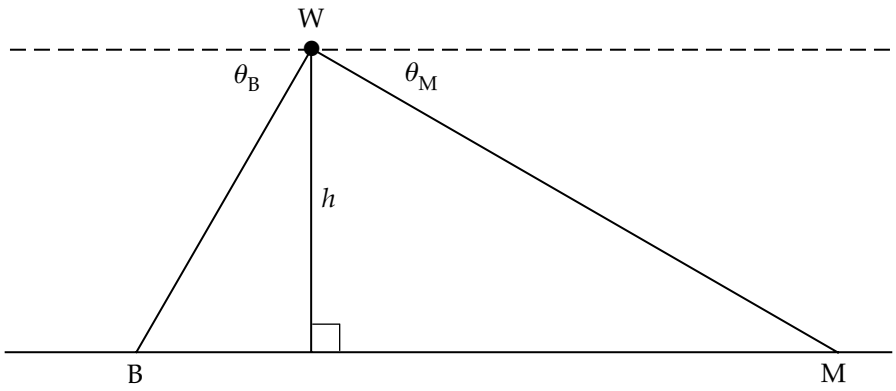


   a) How wide is the driveway from building to building?

   b) What angles (θ_A and θ_B) does the ladder make with each wall? Write your answers rounded to the nearest degree.

5. A park warden (W) is observing wild animals from an observation tower. He sees a bear (B) and a moose (M). The angle of depression of the bear (θ_B) is twice as great as the angle of depression of the moose (θ_M). The warden concludes that the moose is twice as far from the base of the tower as the bear.

   Select real values for θ_B, θ_M, and h (the height of the tower) to show that his conclusion is correct or incorrect. Be sure to show your calculations.



Lesson Summary

In this lesson, you solved problems involving angles of elevation and depression. You also solved problems that included two right triangles. Sometimes you needed to sketch and label diagrams before you could solve the problem. Be sure to complete the assignment at the end of this lesson before going on to Module 7, which is the last module in the course.

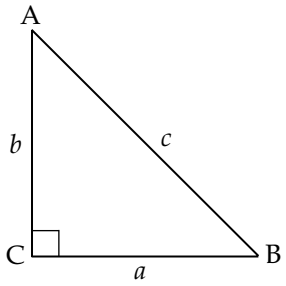
Assignment 6.1

Applying Trigonometry

Total: 27 marks

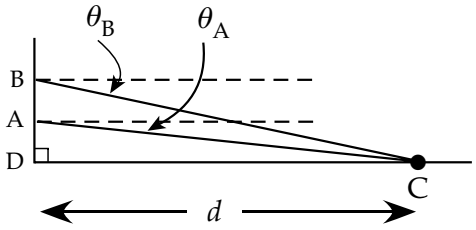
Note to Students: Use your resource sheet to help you complete this assignment. Do not hesitate to add to your resource sheet as you answer the following questions. For example, your resource sheet should include the three trig ratios.

1. Use the following diagram to explain why sin ∠A = cos ∠B. *(2 marks)*



2. Abe (A) and Beth (B) are in a tall building looking at a car (C) parked on the street at a certain distance (d) from the building. The angle of depression for Beth (θ_B) is twice the angle of depression for Abe (θ_A). Therefore, Abe and Beth conclude that Beth must be twice as far above street level as Abe because her angle of depression is twice as great.

   Select real values for θ_A, θ_B, and d to show that their conclusion is correct or incorrect. Be sure to show your calculations. *(3 marks)*



continued

Assignment 6.1: Applying Trigonometry (continued)

3. How high is a kite when the length of the string is 180 m and it makes an angle of 26° with the horizontal? Write your answer rounded to the nearest metre. Show your work. *(3 marks)*

4. From a point 133 m away from the centre of the base of the Manitoba Legislative Building, the angle of elevation to the top of the torch of the Golden Boy is 30°. From the same point, if the angle of elevation to the feet of the Golden Boy is 28°, find the height of the Golden Boy. Show your work. *(7 marks)*

continued

Assignment 6.1: Applying Trigonometry (continued)

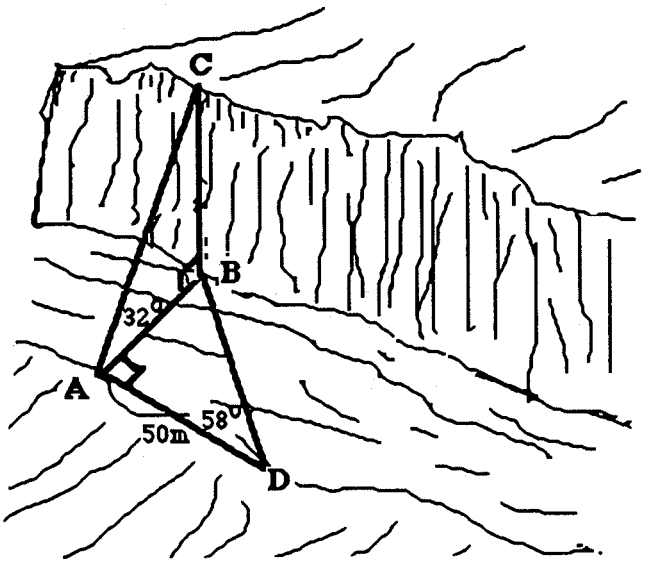
5. A building casts a shadow 57 m long when the angle of elevation of the sun is 30°.
   a) Find the height of the building. Show your work. *(3 marks)*

   b) Find the length of the shadow when the angle of elevation of the sun is 60°. *(4 marks)*

continued

Assignment 6.1: Applying Trigonometry (continued)

6. A surveyor wishes to use a cable to get from the shore of the river to the top of the cliff on the other side. To do this, she sets up her transit at A, and measures ∠CAB = 32°. She then lays off a baseline AD so that ∠BAD = 90° and AD = 50 m. She measures ∠ADB = 58°. Calculate how much rope she would need to reach the top of the cliff (point C) from point A. Show your work. *(5 marks)*



Module 6 Summary

Excellent work! You have only one more module to complete!

In this module, you solved trigonometric problems using the trig ratios sin, cos, and tan, as well as the Pythagorean theorem. Each problem included one or more right triangles. Definitions for these terms can be found in Appendix B: Glossary found at the end of the course. You learned how to interpret a word problem and draw a suitable diagram before doing the calculations. If there are problems you still cannot do, please contact your learning partner or tutor/marker.

You will continue to study trigonometry in the Grade 12 Essential Math course, where you will solve non-right-angle triangles.

The next module is related to work you have already completed in Modules 2 and 5. You may want to have your resource sheets from those modules handy as you do the Module 7 assignments.

Submitting Your Assignments

It is now time for you to submit the Module 6 Cover Assignment and Assignment 6.1 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 6 assignments and organize your material in the following order:

- ❑ Module 6 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 6 Cover Assignment: Math Operations
- ❑ Assignment 6.1: Applying Trigonometry

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Grade 11 Essential Mathematics (30S)

Module 6
Trigonometry

Learning Activity Answer Keys

MODULE 6:
TRIGONOMETRY

Learning Activity 6.1

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. It is 11:00 am on Monday. You have a chess tournament at 6:00 pm on Tuesday. How many hours do you have to wait?
2. How much would you have to pay for an 8 ft. long piece of wood if the hardwood store charges $1.95 per foot?
3. How many millimetres are there in 32 centimetres?
4. Simplify the fraction: $\frac{6}{20}$.
5. Solve for x: $\frac{10}{x} = \frac{5}{4}$.

Answers:

1. 31 hours (11 am to 11 am = 24 hours, 11 am to 12 pm = 1 hour, 12 pm to 6 pm = 6 hours, so 24 + 1 + 6 = 31 hours)
2. \$15.60 ($8 \times 1.95 = 8 \times (2 - 0.05) = 8 \times 2 - 8 \times 0.05 = 16 - 0.4 = \$15.60$)
3. 320 mm
4. $\frac{3}{10}$
5. $x = 8$ ($(10)(4) = 5x$, $5x = 40$, $x = 8$)

Part B: Review of Right-Angle Triangle Trigonometry

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Find the measures of the missing sides and angles in the following right-angled triangles.

Note: You may have used different formulas to find the values. That is okay. Just remember it is more accurate to use the given values as often as possible.

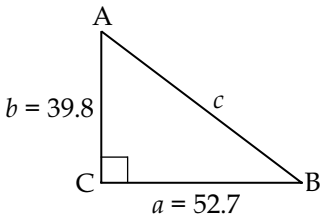
a) $\angle C = 90°$

$a = 52.7$

$b = 39.8$

Answer:

Step 1: Sketch



Step 2: $c^2 = a^2 + b^2$

$c^2 = 52.7^2 + 39.8^2$

$c = 66.0$ units

Step 3: $\tan \angle B = \dfrac{b}{a}$

$\tan \angle B = \dfrac{39.8}{52.7} = 0.7552$

$\angle B = \tan^{-1}(0.7552)$

$\angle B = 37.1°$

Step 4: $\angle A = 180° - 90° - 37.1°$

$\angle A = 52.9°$

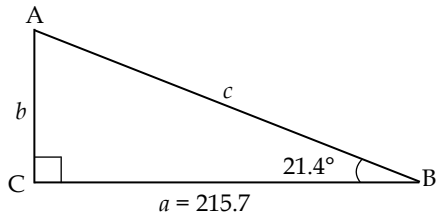
b) $\angle C = 90°$

$\angle B = 21.4°$

$a = 215.7$

Answer:

Step 1: Sketch



Step 2: $\angle A = 180° - 90° - 21.4°$

$\angle A = 68.6°$

Step 3: $\sin \angle A = \frac{a}{c}$

$$\sin 68.6° = \frac{215.7}{c}$$

$$c \sin 68.6° = 215.7$$

$$c = \frac{215.7}{\sin 68.6°}$$

$$c = 231.7 \text{ units}$$

Step 4: $c^2 = a^2 + b^2$

$$231.7^2 = 215.7^2 + b^2$$

$$b^2 = 231.7^2 - 215.7^2$$

$$b = 84.6 \text{ units}$$

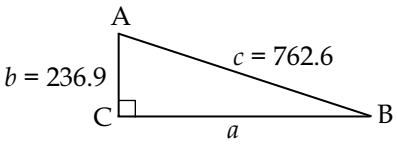
c) $\angle C = 90°$

$b = 236.9$

$c = 762.6$

Answer:

Step 1: Sketch



Step 2: $762.6^2 = a^2 + 236.9^2$

$a^2 = 762.6^2 - 236.9^2$

$a = 724.9$ units

Step 3: $\sin \angle B = \frac{b}{c}$

$\sin \angle B = \frac{236.9}{762.6}$

$\angle B = 18.1°$

Step 4: $\angle A = 180° - 90° - 18.1°$

$\angle A = 71.9°$

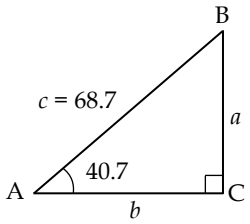
d) $\angle C = 90°$

$\angle A = 40.7°$

$c = 68.7$

Answer:

Step 1: Sketch



Step 2: $\angle B = 180° - 90° - 40.7°$

$\angle B = 49.3°$

Step 3: $\sin \angle A = \frac{a}{c}$

$$\sin 40.7° = \frac{a}{68.7}$$

$$a = 68.7 \times \sin 40.7°$$

$$a = 44.8$$

Step 4: $c^2 = a^2 + b^2$

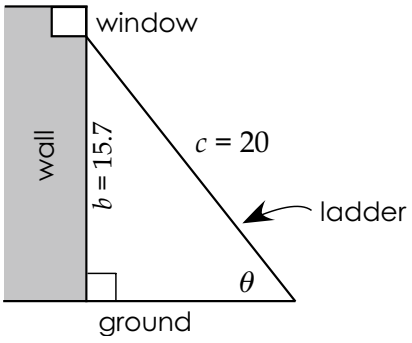
$$68.7^2 = 44.8^2 + b^2$$

$$b^2 = 68.7^2 - 44.8^2$$

$$b = 52.1 \text{ units}$$

2. To be safe, professional painters lean their ladders at an angle between 60° and 70° with the ground. If a painter uses a ladder 20 feet long to reach a window 15.7 feet off the ground, will she be safe? If not, what length of ladder, to the nearest foot, should she use?

Answer:



Solution 1:

$$\sin \angle B = \frac{b}{c}$$

$$\sin \angle B = \frac{15.7}{20}$$

$$\angle B = 51.7°$$

No, it is not safe (51.7° is less than 60°). To figure out the length of the ladder, use sin 60° and try to find the value of C.

$$\sin 60° = \frac{15.7}{c}$$

$$c \approx 18'$$

So the ladder should be close to 18′.

Solution 2:

$$\sin 60° = \frac{15.7}{c}$$

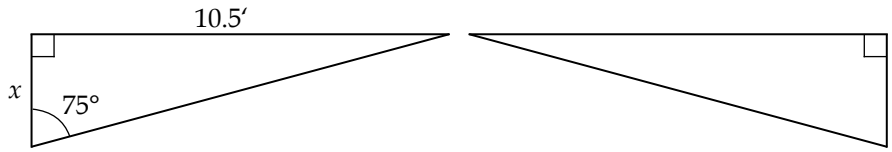
$$c \approx 18'$$

$$\sin 70° = \frac{15.7}{c}$$

$$c \approx 17'$$

You need a ladder 17′–18′ long. Since the ladder is 20′ long, it is not safe (the larger the angle, the shorter the ladder needed).

3. An iron rod gate is made in the form of two identical right-angled triangles, as depicted below.

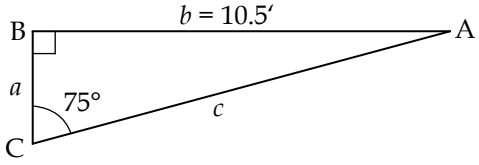


What total length of iron rod (to the nearest tenth of a foot) is needed to build both halves of the gate?

Answer:

To find the answer, you need to find the perimeter of both triangles. To do this, you need to know all the side measurements.

Step 1: Sketch the diagram and label it with the given information:



Step 2: $\tan \angle B = \frac{b}{a}$

$$\tan \angle 75° = \frac{10.5}{a}$$

$$a = \frac{10.5}{\tan \angle 75°}$$

$$a = 2.8 \text{ units}$$

Step 3: $\sin \angle B = \frac{b}{c}$

$$\sin \angle 75° = \frac{10.5}{c}$$

$$c = \frac{10.5}{\sin \angle 75°}$$

$$c = 10.9 \text{ units}$$

$P = 2(2.8 + 10.5 + 10.9)$

$P = 48.4'$ of material needed

Learning Activity 6.2

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. What is $\frac{2}{3}$ of 180?
2. You cut off 17 inches of a 2 ft., 5 in. board of wood. How long is it now?
3. Dagnus is 30 years old. How old is he in months?
4. Complete the pattern: 1, 2, 6, 24, 120, ____ , ____
5. How far would you travel if you drove 45 km/h for 20 minutes?

Answers:

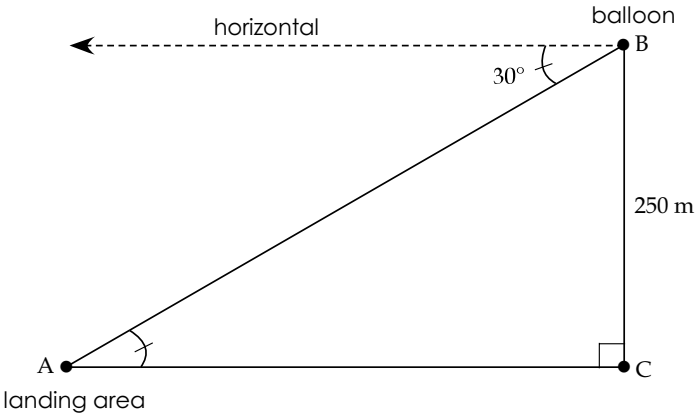
1. 120 $\left(\frac{180}{3} = 60,\ 60 \times 2 = 120\right)$
2. 1 foot or 12 inches (2'5" = 29", 29 − 17 = 12" = 1 foot)
3. 360 months (30 × 12 = 360 months)
4. 720, 5040 (Each time you multiply by the next number (× 2, × 3, × 4 …) so 120 × 6 = 720 and 720 × 7 = 5040)
5. 15 km $\left(20\ \text{min.} = \frac{1}{3}\ \text{hour, so } 45 \times \frac{1}{3} = \frac{45}{3} = 15\ \text{km}\right)$

Part B: Applying Trigonometry

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. A hot-air balloon advertising for a real estate company is 250 m above ground level. The angle of depression from the balloon to the landing area is 30°. What is the distance along the ground from a point beneath the balloon to the landing area?

   Answer:



The angle from the ground to the line of sight is equal to the angle of depression. We can then use the tangent ratio to solve for the horizontal distance between the balloon and the landing area.

$$\tan 30° = \frac{250}{AC}$$

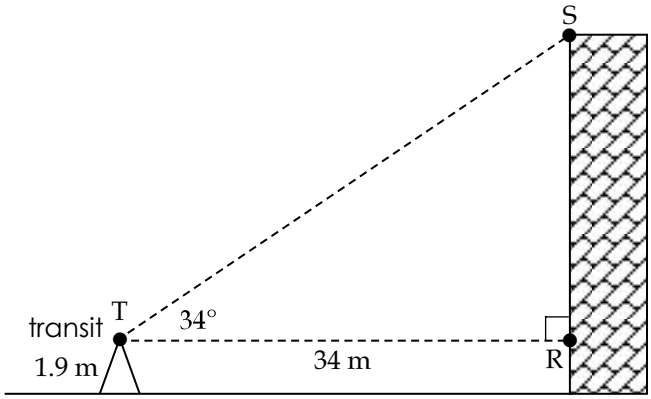
$$AC \times \tan 30° = 250$$

$$AC = \frac{250}{\tan 30°}$$

$$AC = 433 \text{ m}$$

2. A surveyor uses a transit (a device used to measure angles) to measure the angle of elevation from the transit to the top of a building. The horizontal distance from the top of the transit to the building is 34 m, the angle measures 34°, and the height of the transit is 1.9 m. How high is the building? Write your answer rounded to one decimal place.

   Answer:



Height from transit:

$$\tan 34° = \frac{RS}{34}$$

$$34 \times \tan 34° = RS$$

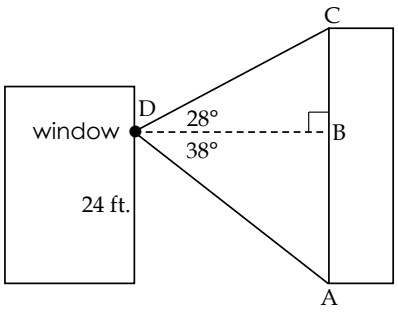
$$22.9 = RS$$

Total height of the building is 22.9 + 1.9 = 24.8 m.

3. The angle of elevation to the top of an apartment block from a window in a nearby building is 28°. The angle of depression from the window to the bottom of an apartment block is 38°, and the window is 24 feet above the ground. How tall is the apartment block (to the nearest foot)?

   Answer:

   Triangles are from the horizontal to the bottom of the building and from the horizontal to the top of the building.



   We already know that from the ground to the height of the window (to the horizontal) is 24 ft. (so AB = 24 ft.), but we need to know what the height is from the horizontal to the top of the building (or we need to find BC). Before we can solve this, we need to find out how far apart the buildings are (DB). This can be done using the lower triangle.

$$\tan 38° = \frac{24}{DB}$$

$$DB \times \tan 38° = 24$$

$$DB = \frac{24}{\tan 38°}$$

$$DB = 30.7 \text{ ft.}$$

   Now we can find the remaining height.

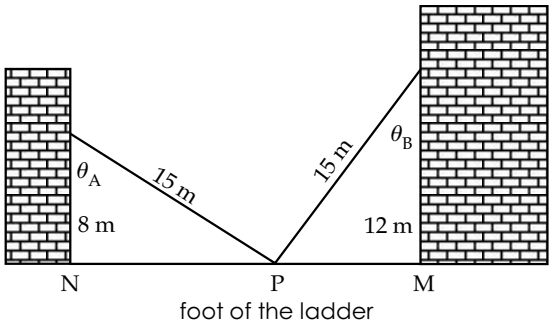
$$\tan 28° = \frac{BC}{30.7}$$

$$30.7 \times \tan 28° = BC$$

$$16.3 = BC$$

   Total height of the building = 16 + 24 = 40 ft.

4. A ladder 15 m long is placed on a driveway between two buildings so that it reaches 12 m up on one building. If it is turned over, its foot being held in position, it will reach 8 m up the other building.



a) How wide is the driveway from building to building?

Answer:

distance NP:

$$c^2 = a^2 + b^2$$
$$15^2 = 8^2 + b^2$$
$$225 - 64 = b^2$$
$$161 = b^2$$
$$\sqrt{161} = \sqrt{b^2}$$
$$12.7 \text{ m} = b$$

distance PM:

$$c^2 = b^2 + a^2$$
$$15^2 = 12^2 + a^2$$
$$225 - 144 = a^2$$
$$81 = a^2$$
$$\sqrt{81} = \sqrt{a^2}$$
$$9 \text{ m} = a$$

Total distance between buildings: 12.7 + 9 = 21.7 m.

b) What angles (θ_A and θ_B) does the ladder make with each wall? Write your answers rounded to the nearest degree.

Answer:

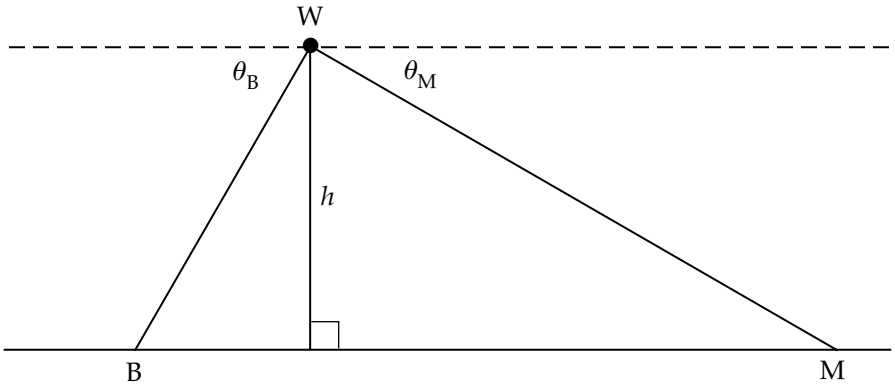
For both angles, we are given the adjacent and the hypotenuse in the question, so we will use cosine to determine the angles.

$$\cos\theta_A = \frac{8}{15}$$
$$\theta_A = \cos^{-1}\left(\frac{8}{15}\right)$$
$$\theta_A = 58°$$

$$\cos\theta_B = \frac{12}{15}$$
$$\theta_B = \cos^{-1}\left(\frac{12}{15}\right)$$
$$\theta_B = 37°$$

5. A park warden (W) is observing wild animals from an observation tower. He sees a bear (B) and a moose (M). The angle of depression of the bear (θ_B) is twice as great as the angle of depression of the moose (θ_M). The warden concludes that the moose is twice as far from the base of the tower as the bear.

   Select real values for θ_B, θ_M, and h (the height of the tower) to show that his conclusion is correct or incorrect. Be sure to show your calculations.



Answer:

Possible values could be (θ_B) = 60°, (θ_M) = 30°, and h = 50 m

Bear distance:

$$\tan 60° = \frac{50}{B}$$

$$B \times \tan 60° = 50$$

$$B = \frac{50}{\tan 60°} = 28.9 \text{ m}$$

Moose distance:

$$\tan 30° = \frac{50}{M}$$

$$M \times \tan 30° = 50$$

$$M = \frac{50}{\tan 30°} = 86.6 \text{ m}$$

Therefore, the warden's conclusion is incorrect, because 86.6 m is more than twice 28.9 m.

Notes

Grade 11 Essential Mathematics (30S)

Module 7
Design Modelling

Note to Students

Module 7 contains a number of diagrams that represent 2-D views of 3-D objects. A colour version in pdf format can be found in the learning management system (LMS). If you do not have access to the Internet or need a copy of this resource, contact the Distance Learning Unit at 1-800-465-9915.

Module 7:
Design Modelling

Introduction

This is the last module! This module will focus on representing 3-D objects in different ways. The ability to translate between 2-D drawings and 3-D objects is necessary in many professions, from construction to interior design to art to clock repairs to putting together your own furniture at home.

Assignments in Module 7

When you have completed the assignments for Module 7, submit your completed assignments to the Distance Learning Unit either by mail or electronically through the learning management system (LMS). The staff will forward your work to your tutor/marker.

Lesson	Assignment Number	Assignment Title
	Cover Assignment	Four-Colour Problem
2	Assignment 7.1	Three 2-D Views of 3-D Objects
3	Assignment 7.2	One-Point Perspective Drawings
4	Assignment 7.3	Exploded Views and Component Parts

Writing Your Final Examination

You will write the final examination when you have completed Module 7 of this course. The final examination is based on Modules 4 to 7, and is worth 12.5 percent of your final mark in the course. To do well on the final examination, you should review all the work you complete in Modules 4 to 7, including all the learning activities and assignments. You will write the final examination under supervision.

Resource Sheet

When you write your final examination, you are encouraged to take a Final Examination Resource Sheet with you into the examination. This sheet will be one letter-sized page, 8½ " by 11", with both sides in your handwriting or typewritten. You will submit it with your examination, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the following list of instructions to help you with preparing your resource sheet for the material in Module 7. On this sheet, you should record mathematics terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module's resource sheet, you may summarize the sheets from Modules 4, 5, 6, and 7 to prepare your Final Examination Resource Sheet. The final examination for this course is based on Modules 4 to 7.

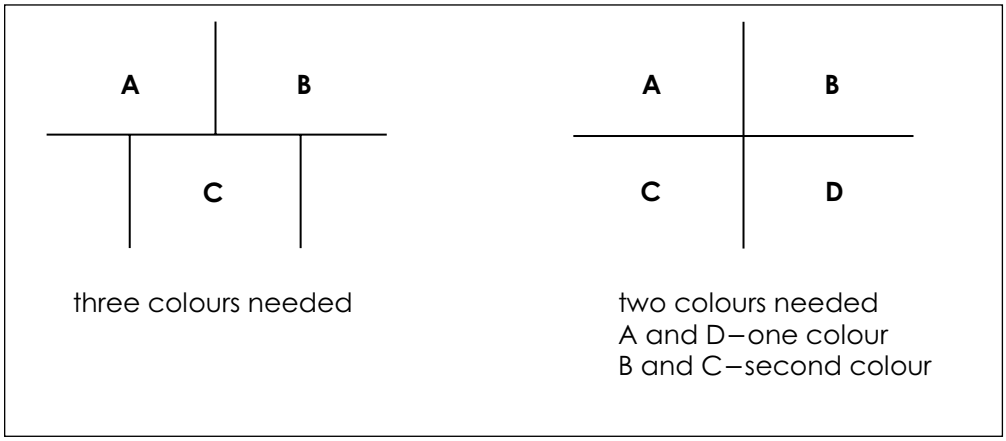
Resource Sheet for Module 7

As you go through the lessons of this module, you may want to consider the following suggestions regarding the creation of a resource sheet.

1. List all the important mathematics terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how each formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the examination. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Final Examination Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an examination.

Module 7 Cover Assignment: Four-Colour Problem

The colouring of maps is a longstanding problem in mathematics. When colouring a map, regions with adjacent sides (regions which share a side) must have different colours. The maps below show several different regions. If two regions share a common border, then the two regions must be coloured differently.



It was recognized long ago, and proven mathematically, that any map can be coloured with as few as five colours, where no two countries that share a border are the same colour. In the 1980s, it was proved that it is possible to colour a world map with only four colours.

Martin Gardner published the "map" on the following pages with a claim that it required five colours. In reality, however, it only requires four colours.

Notes

Module 7 Cover Assignment

Four-Colour Problem

Total: 8 marks (4 marks for the draft map and 4 marks for the coloured map)

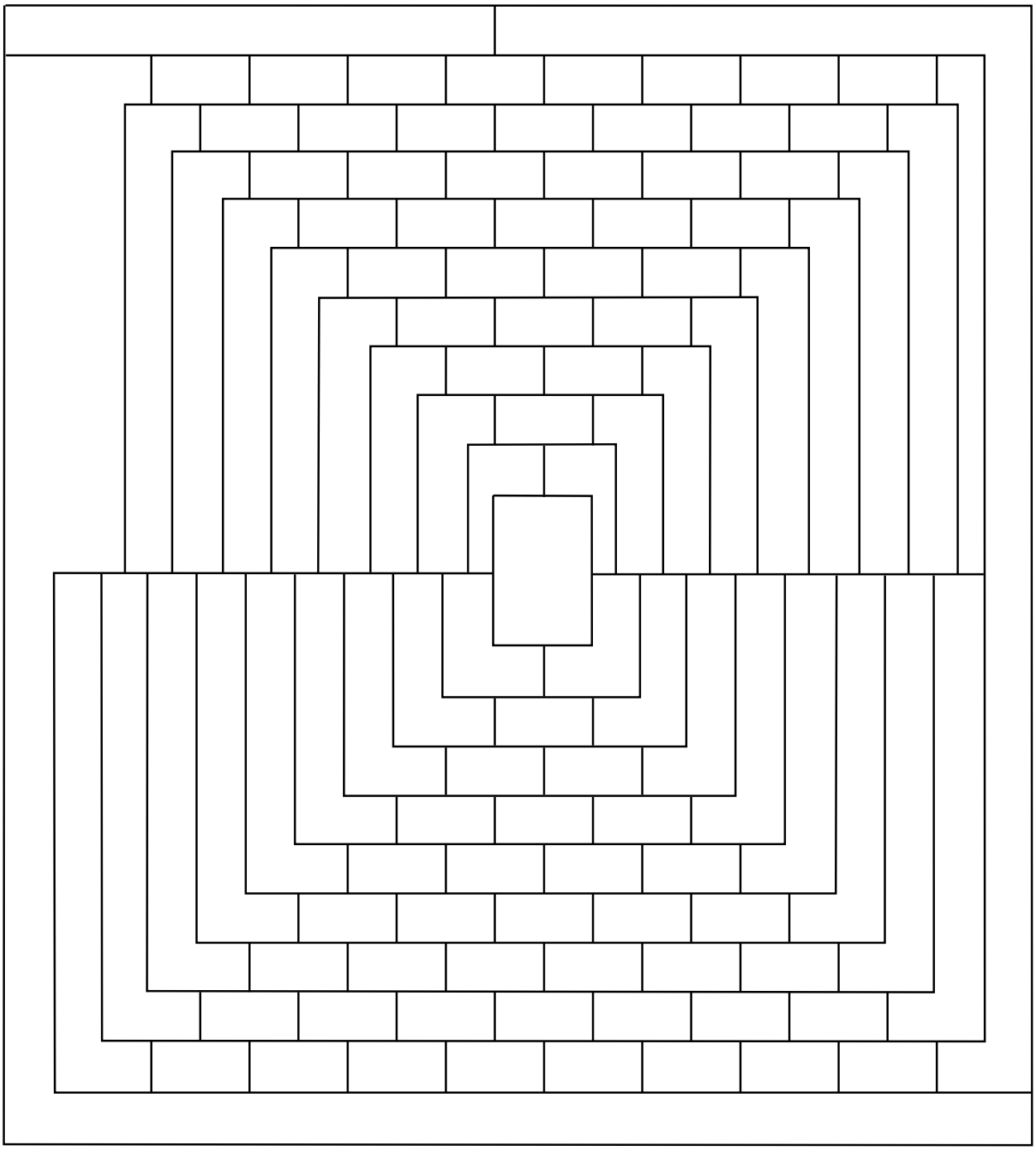
See if you can colour the map on the following page using only four colours. Note that two copies of the map are provided. Because of the difficulty of this task, it is suggested that you start with the numbers 1 to 4 rather than with colours. It is also recommended that you use a pencil. Once you have a solution using numbers, you can then choose colours and colour the map. Remember, countries that share a border must be coloured differently.

Note: Submit both the draft map and the final map to the Distance Learning Unit.

continued

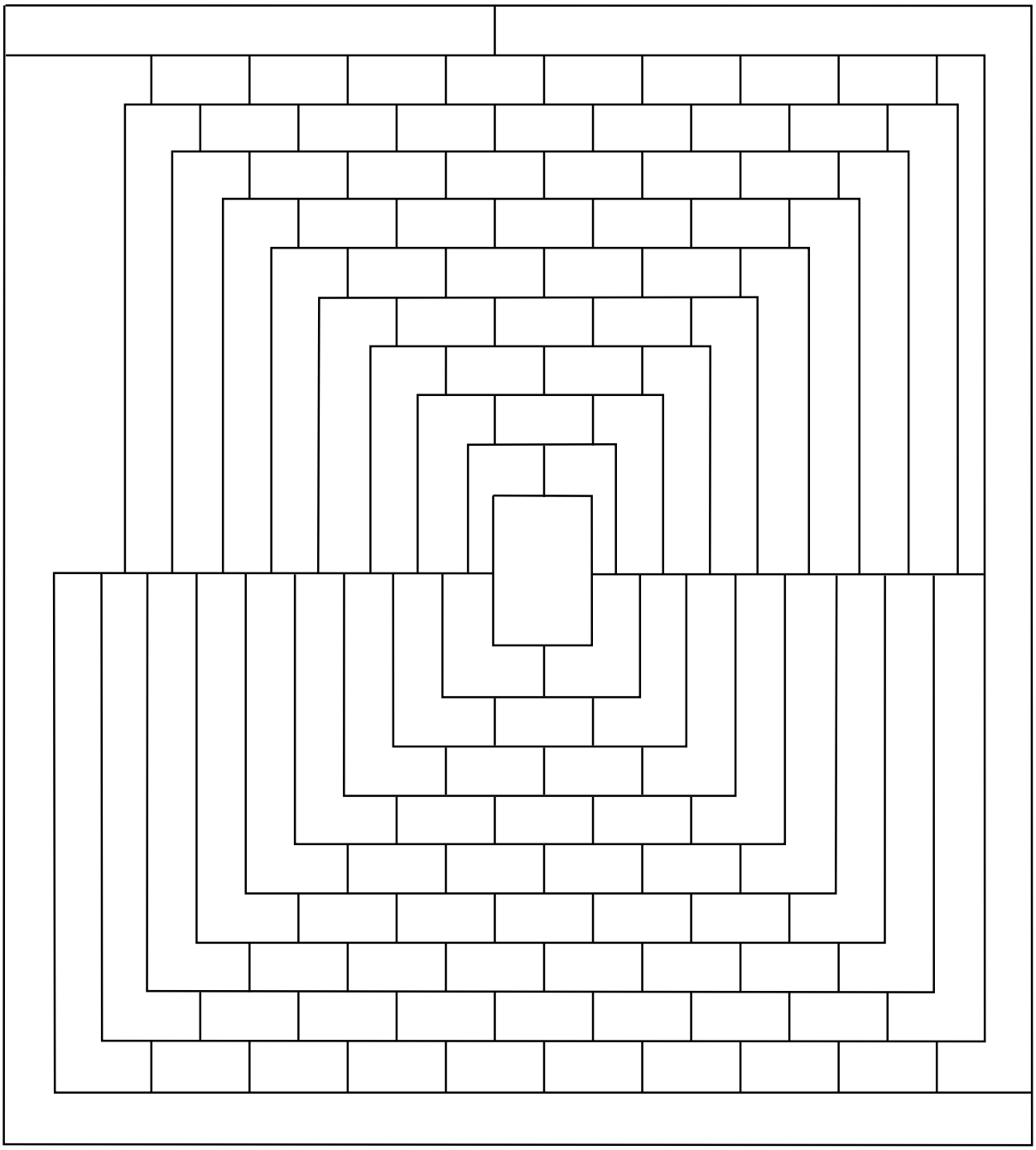
Module 7 Cover Assignment: Four-Colour Problem (continued)

Draft Copy



Module 7 Cover Assignment: Four-Colour Problem (continued)

Final Copy



Lesson 1: Isometric Dot Paper

Lesson Focus

In this lesson, you will

- ❑ be introduced to isometric dot paper
- ❑ draw a 3-D object on isometric dot paper

Lesson Introduction

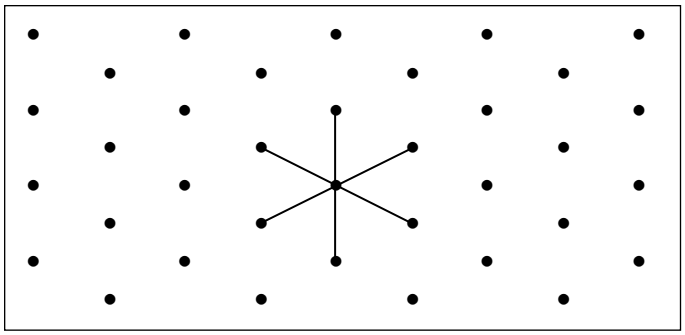
Have you ever tried to draw a 3-D (3-dimensional) object? This drawing would be a 2-D (2-dimensional) representation of a 3-D object. If you draw a sketch, it is not essential that all the angles or side lengths are drawn to scale. However, in certain professions, such as architecture and construction, it is important to make the drawing as accurate as possible. Isometric dot paper is used to draw 3-D images of rectangular or square prisms.

Drawing with Isometric Dot Paper

Isometric dot paper can be used to draw 3-D figures. Check the isometric dot paper included at the end of this lesson. Note how the dots are arranged. This pattern of arranging dots allows you to draw 3-D figures more easily.

Choose any dot on a piece of isometric dot paper. Draw a line segment from it to the next nearest dot. Note that you can draw the line segment either

- up
- down
- slanted up and to the right
- slanted up and to the left
- slanted down and to the right
- slanted down and to the left



The line segments you have drawn are all the same length. The word *isometric* means equal measure. The prefix *iso* means equal and *metric* means measure.

You should not draw horizontal lines on isometric dot paper. You should draw only vertical lines and lines that are at an angle of 30° to the horizontal.

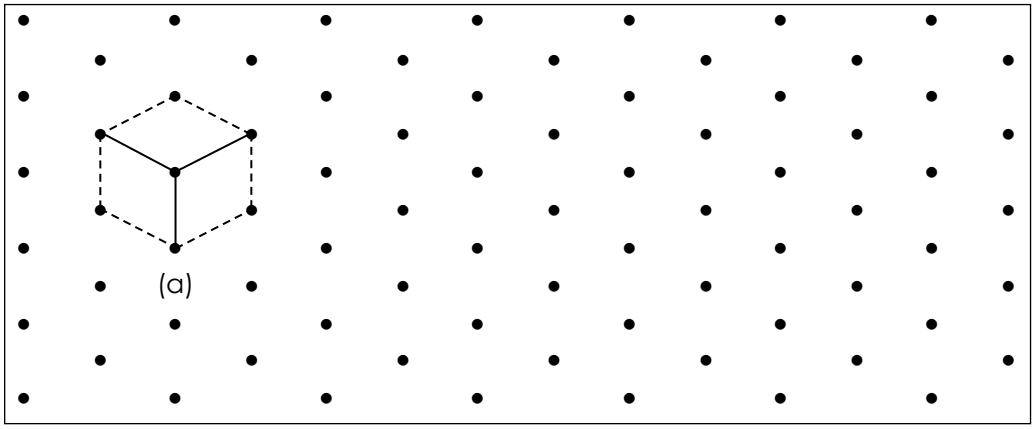
Example 1

Using isometric dot paper, draw a 1-by-1-by-1 cube.

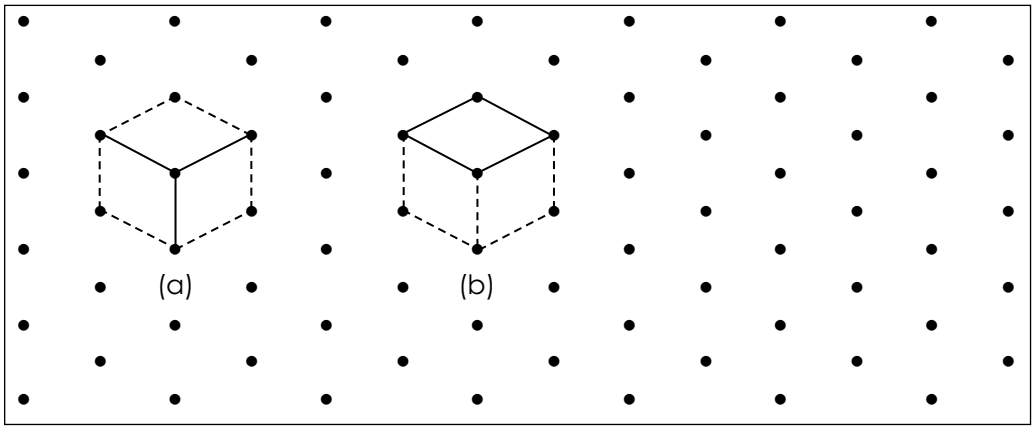
Solution

Some students find it helpful to place the object so they are looking at one of its vertical edges from a position slightly above the cube. They can then draw the object on paper as they see it.

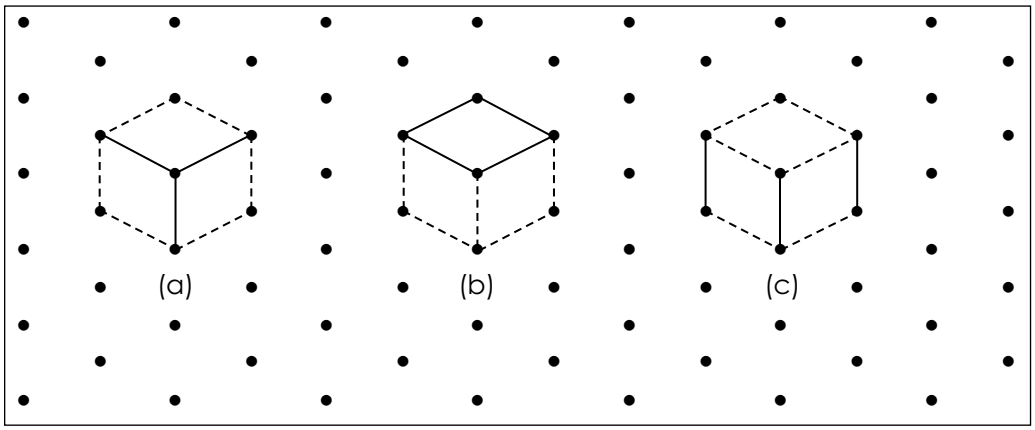
Some students find it helpful to draw a Y, and then complete the figure (a).



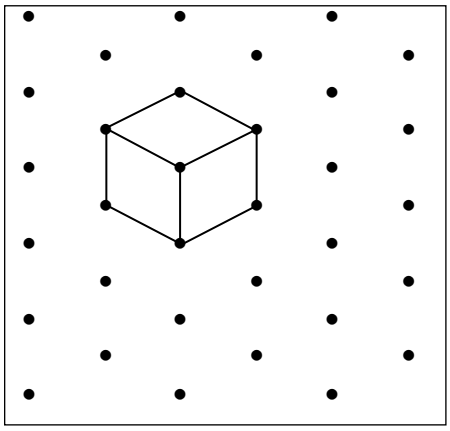
Some students draw a diamond, and then draw the lines down (b).

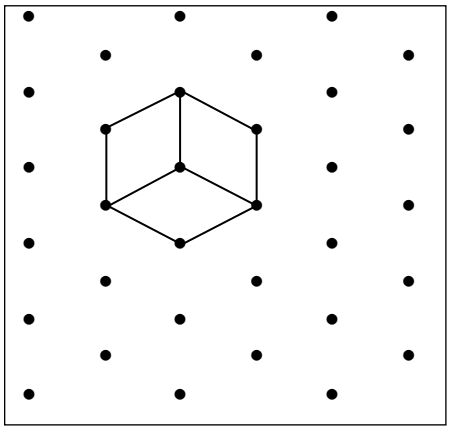


Others find it easier to draw the three vertical lines first, and then complete the figure (c).



In each case, the final diagram looks the same. An alternate drawing is shown on the right.

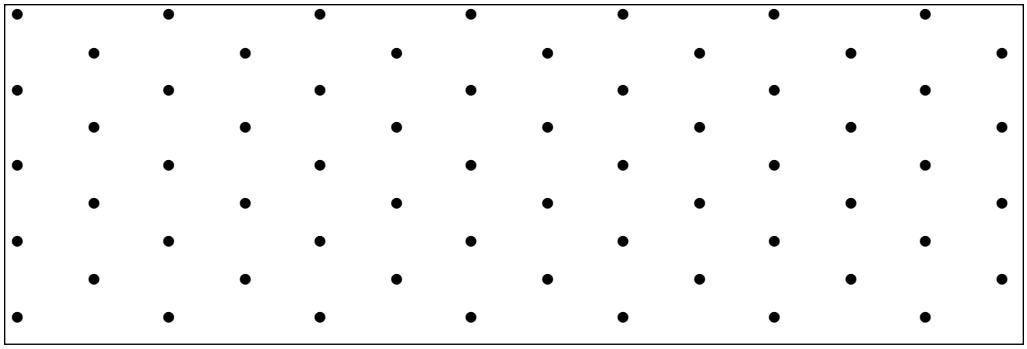




You may want to experiment with the three methods. Choose the method that works best for you.

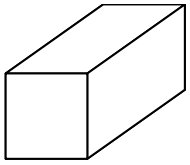
Example 2

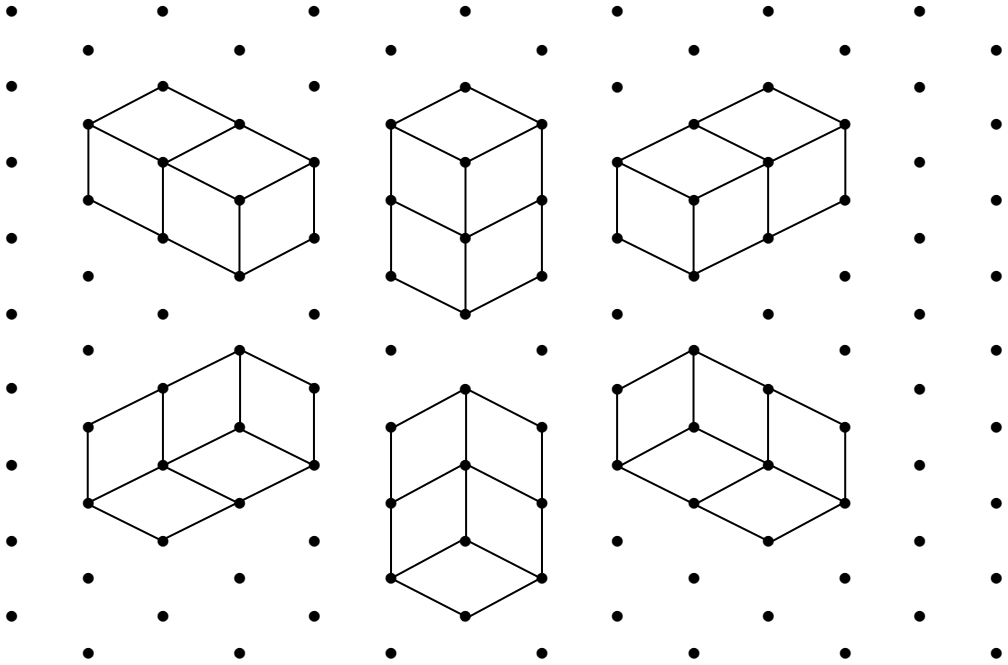
Having seen how to draw using isometric dot paper, draw two cubes side-by-side.



Solution

Although, as a solid figure, the image below would only have one shape, you can represent it on isometric dot paper in six ways.





It may be helpful to have a sketch of the six ways to represent a shape on your resource sheet. This may or may not include isometric dot paper, that is your choice. The purpose of having these sketches would be to have a reminder of the different views.

Use the following learning activity to practise drawing 3-D shapes, using isometric dot paper. Check your answers with the answer key at the end of the module.

Learning Activity 7.1

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. What is the lowest common multiple of 6 and 9?
2. How many metres are in 15.1 km?
3. Which letter comes next? J F M A M J J A S O N _____
4. Which two terms have the same value? 8, 4, 10, $\frac{12}{4}$, $\frac{20}{5}$, $\frac{8}{3}$
5. Your credit card has a balance of $450. The minimum payment will be $10 or 10% of your balance—whichever is more. How much is your minimum payment?

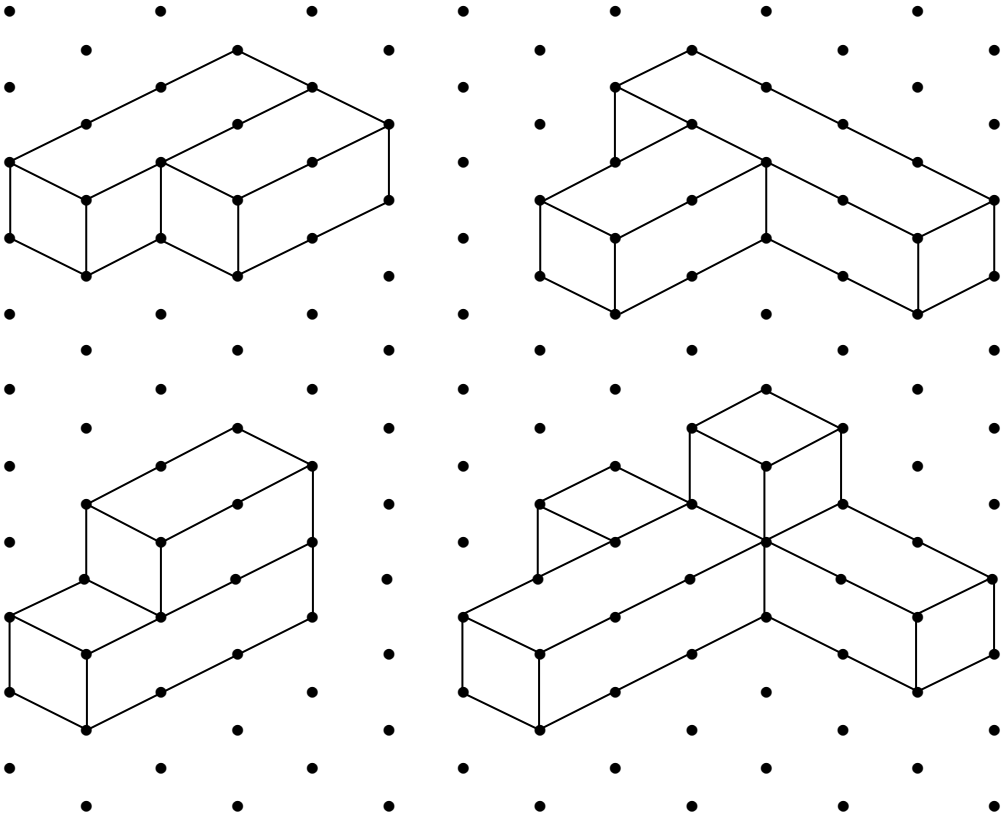
continued

Part B: Drawing Cubes and Prisms on Isometric Dot Paper

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Isometric dot paper is provided at the end of this lesson.

1. Copy the following four figures. Draw the diagrams onto the isometric dot paper provided at the end of this lesson.



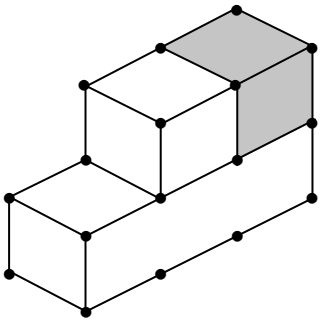
2. There are six ways to represent three cubes in a row on isometric dot paper. Draw the six different representations using the isometric dot paper provided at the end of this lesson.

continued

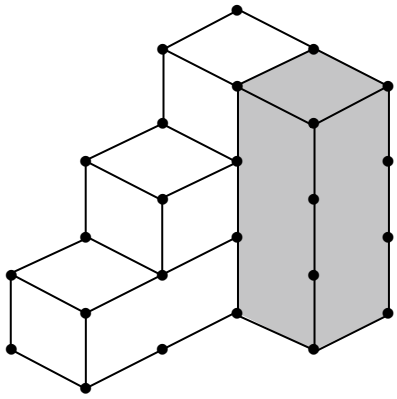
Learning Activity 7.1 (continued)

3. Remove the shaded cubes from each of the following three figures, and draw the new figures on isometric dot paper provided at the end of this lesson.

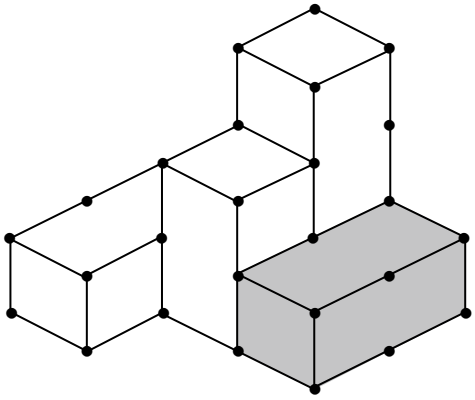
a)



b)

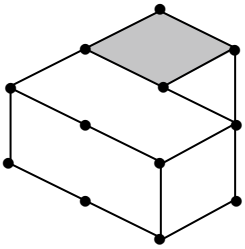


c)

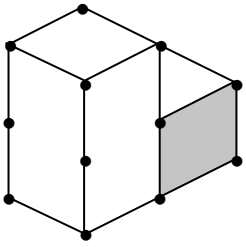


4. Add a three-unit-long prism to the shaded faces, and draw the new figures on the isometric dot paper provided at the end of this lesson.

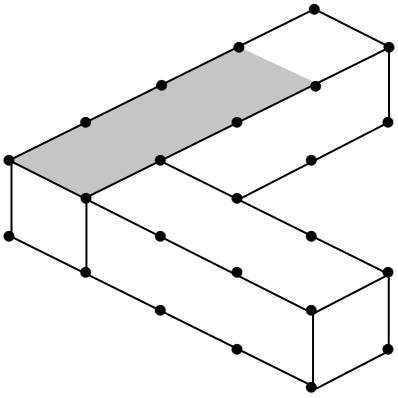
a)



b)

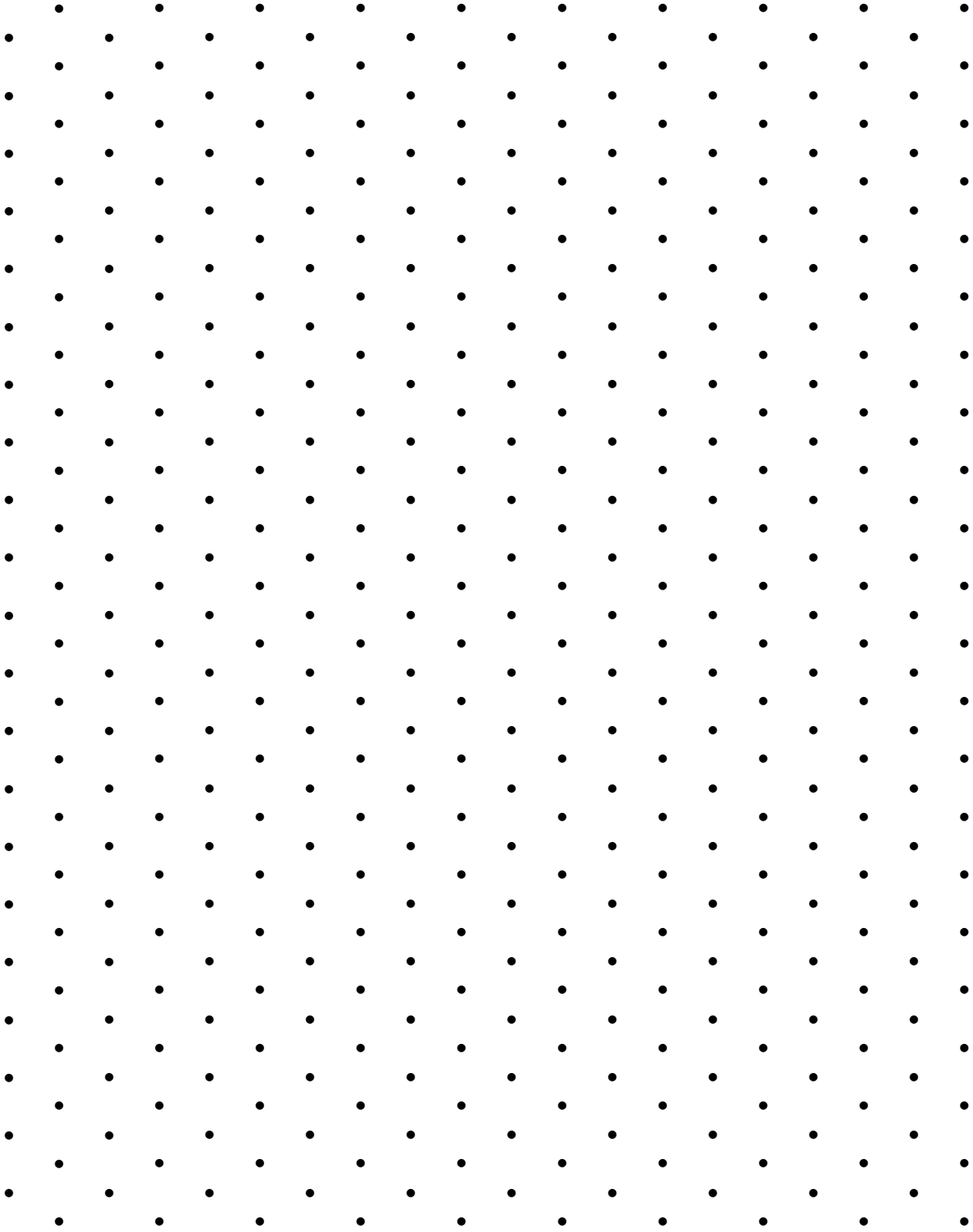


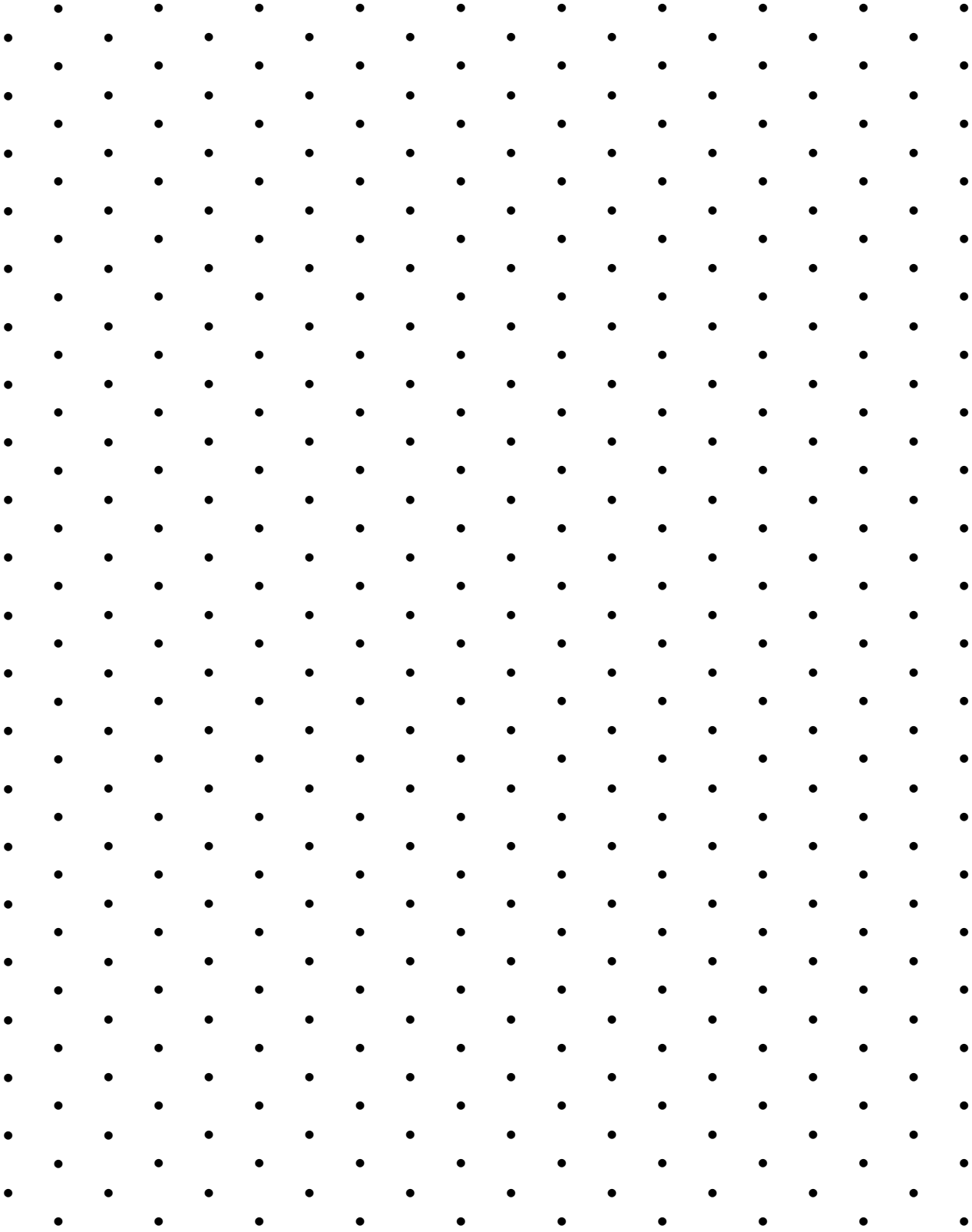
c)

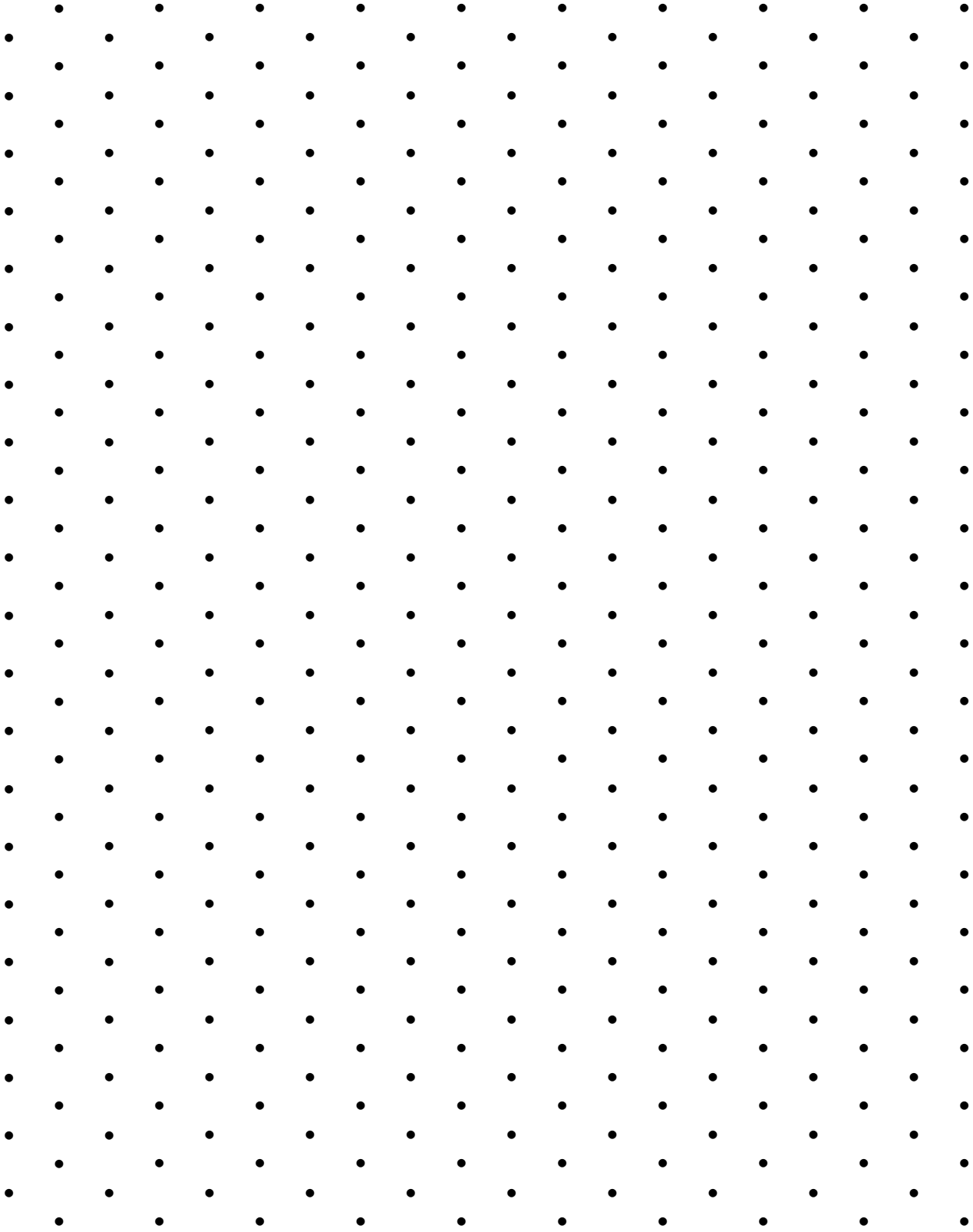


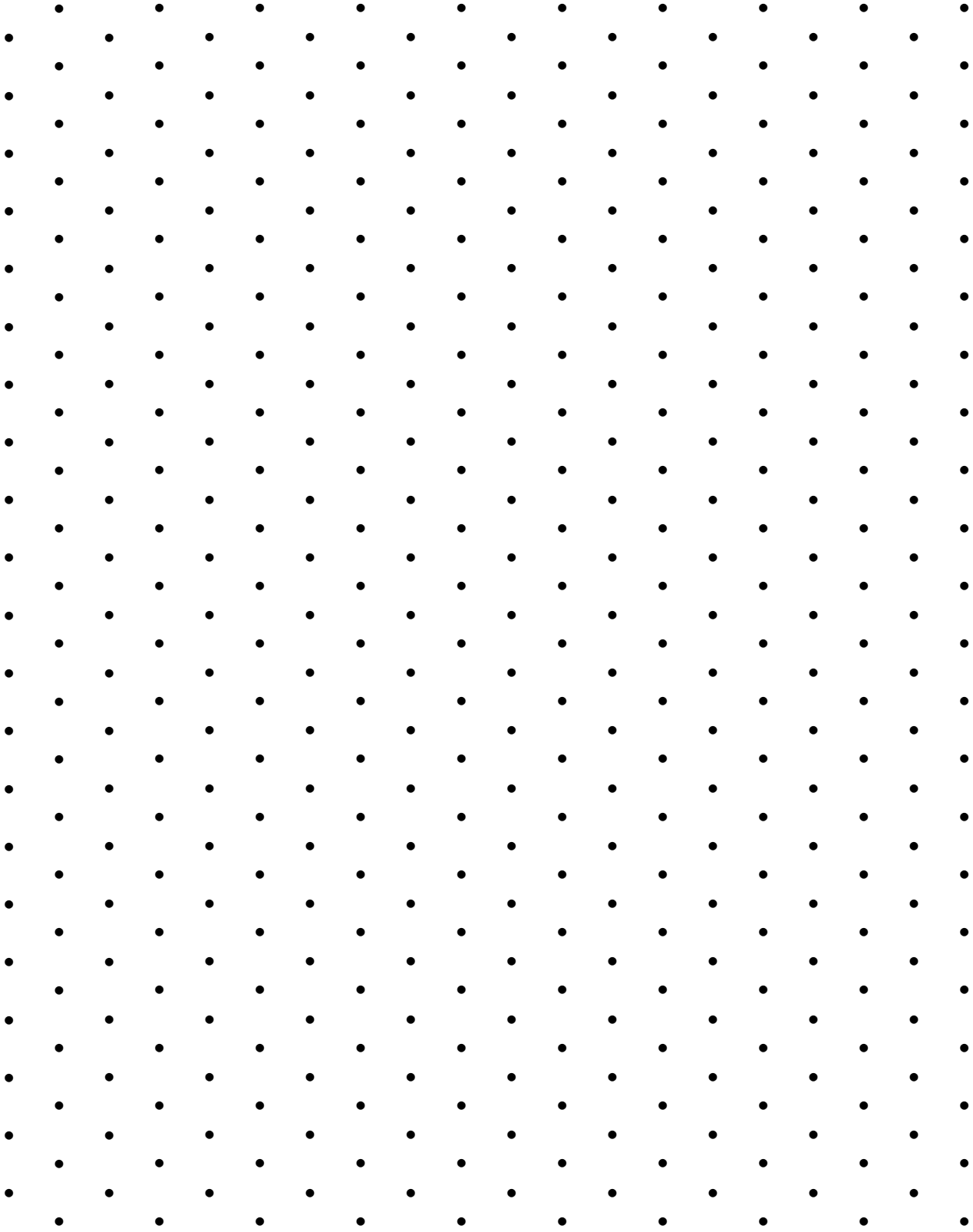
Lesson Summary

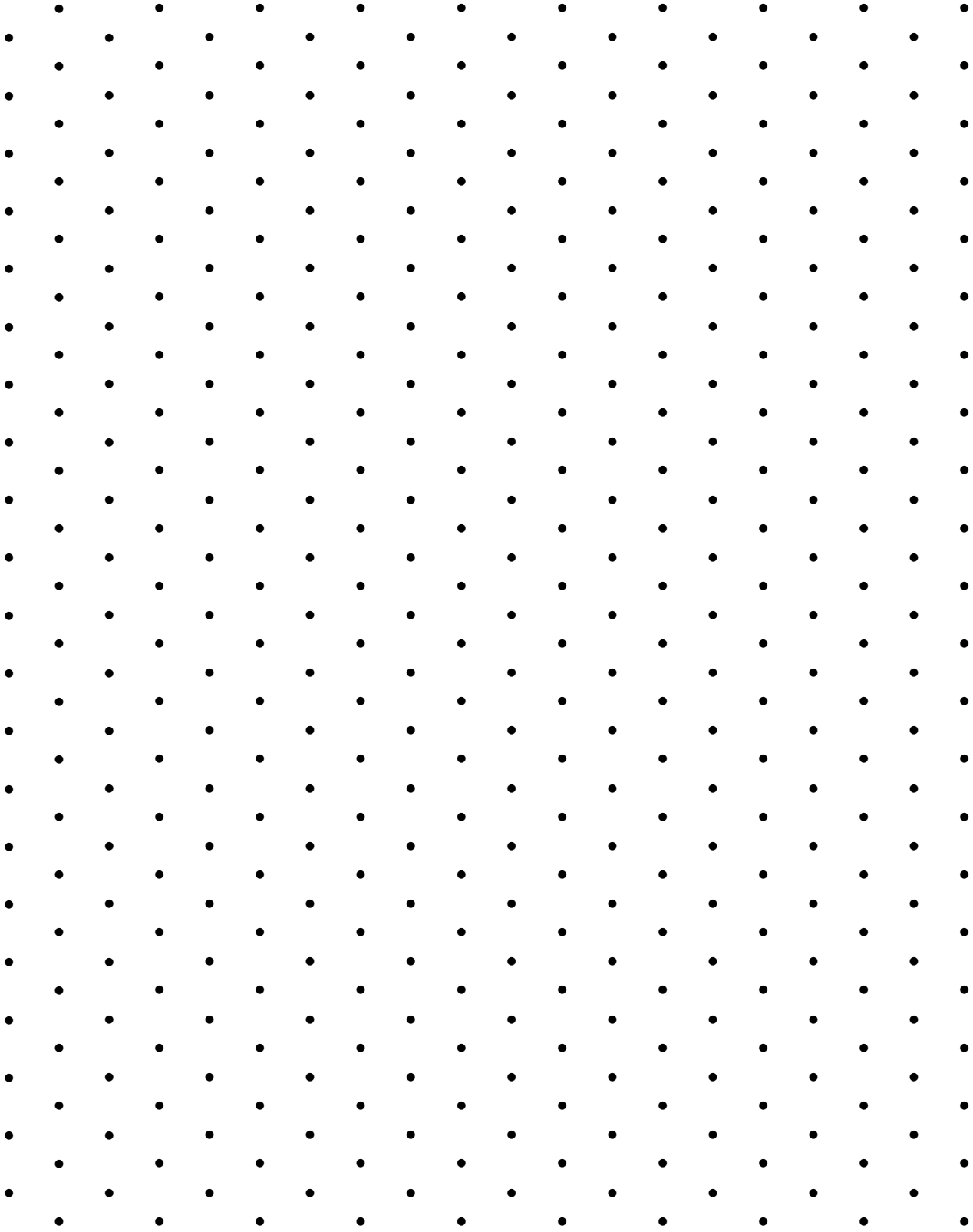
In this lesson, you learned how to draw 3-D objects on isometric dot paper. This skill is used in the next lesson, where you will draw three 2-D views of a 3-D object. If you have any questions about drawing figures on isometric dot paper, talk to your learning partner or tutor/marker before moving on to the next lesson.

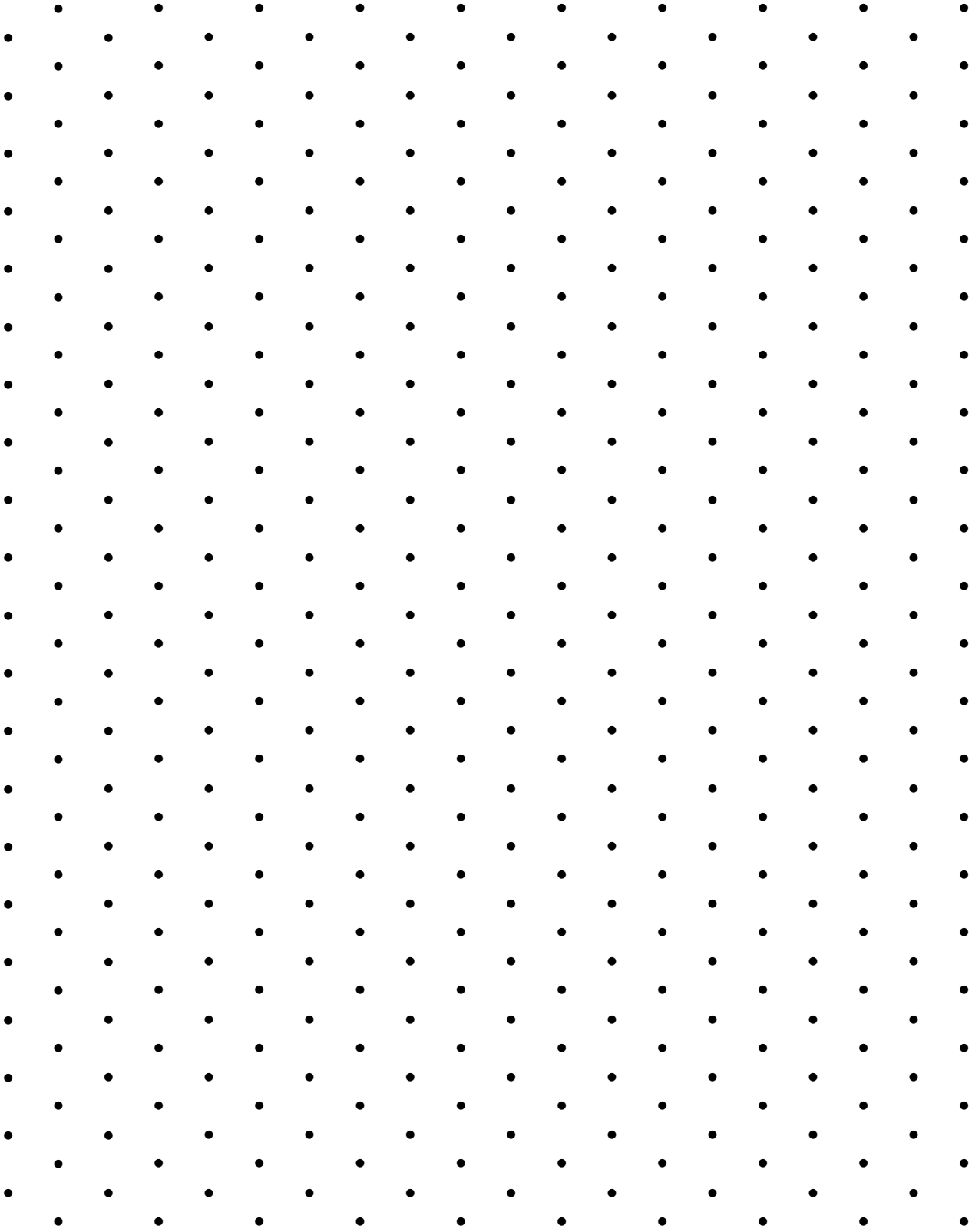












Lesson 2: Three 2-D Views of 3-D Objects

Lesson Focus

In this lesson, you will

- ❑ draw the top, front, and side views of a 3-D object
- ❑ draw and construct a 3-D object, given the top, front, and side views
- ❑ determine whether the three views of a 3-D object correctly represent the object

Lesson Introduction

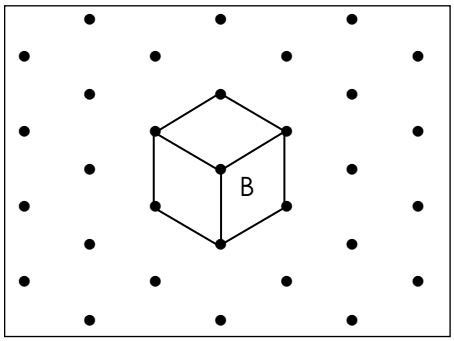
In the previous lesson, you drew 3-D figures on isometric dot paper. In this lesson, you will represent the figures with three 2-D views, and learn to translate between the three 2-D views and the 3-D drawings on isometric dot paper.

The three views are the *top view,* the *front view*, and the *side view*. These 2-D views are best drawn on square dot paper. The side view refers to the right side if you are looking at the object from the front.

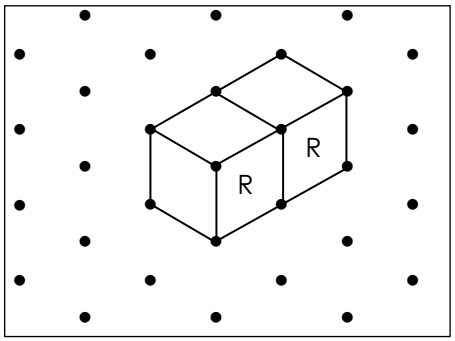
Architects frequently use these three 2-D views in their work.

In this lesson, the bars used to construct the objects have specific colours based on their length.

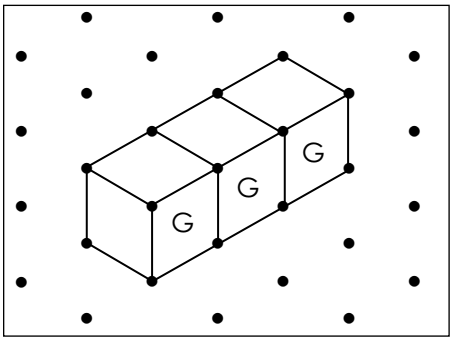
B = blue cube = 1 unit long



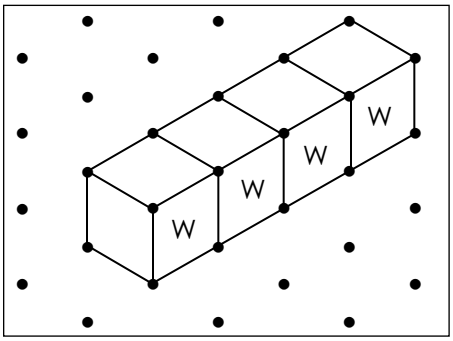
R = red cube = 2 units long



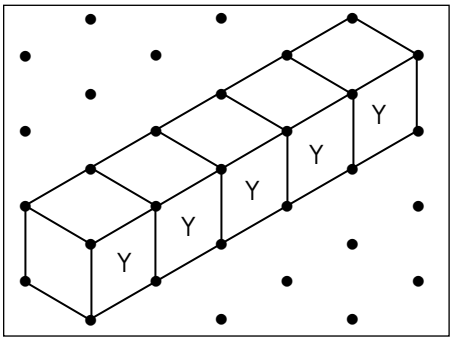
G = green bar = 3 units long



W = white bar = 4 units long



Y = yellow bar = 5 units long



Top, Front, and Side Views

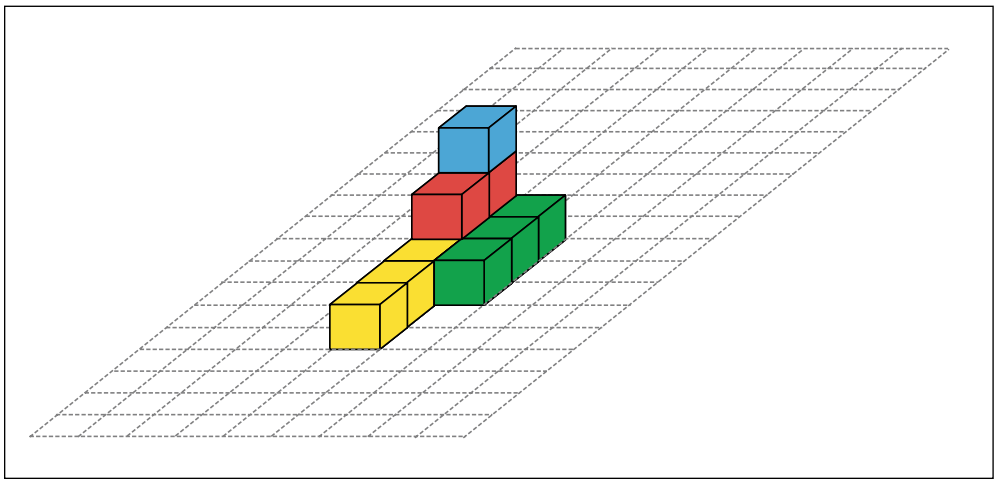
These views are similar to pictures taken with a camera showing the top, front, and right side of a 3-D object. The first part of this lesson is about drawing views similar to these pictures.

Drawing the Three 2-D Views of a 3-D Object

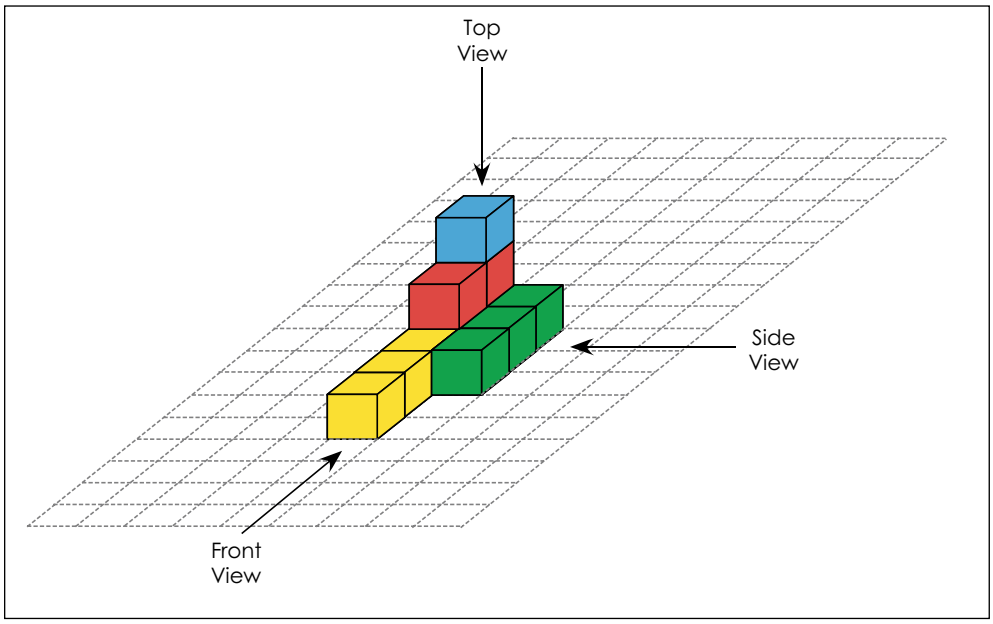
If you are given a 3-D drawing of an object, you must be able to draw the front, top, and side views.

Example 1

Draw the front, top, and side views of the following object.

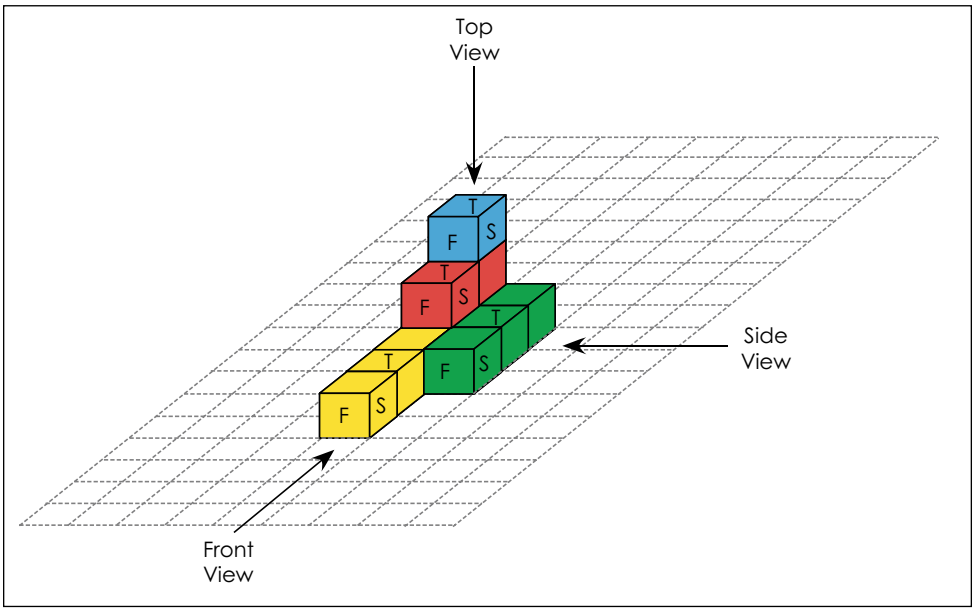


Solution

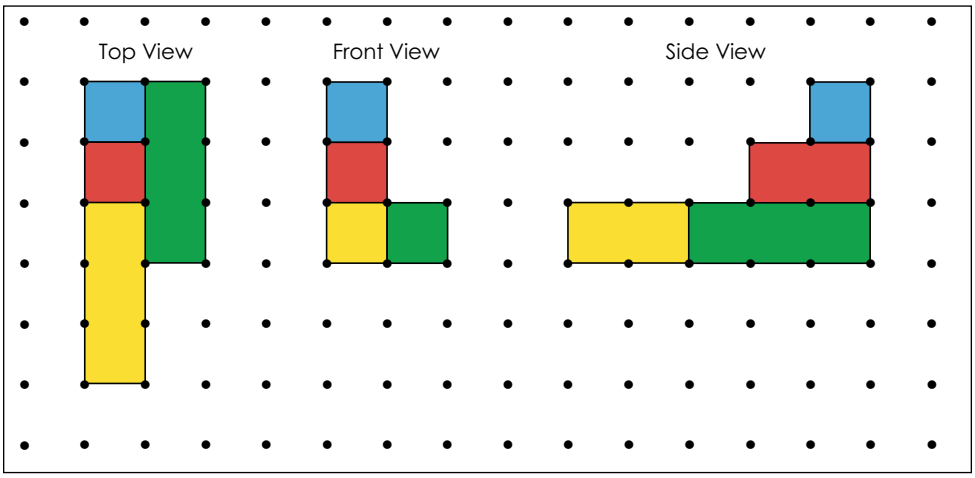


continued

It might be helpful to label the faces F (front), T (top), and S (side) so that you know which faces will be visible in a 2-D drawing.

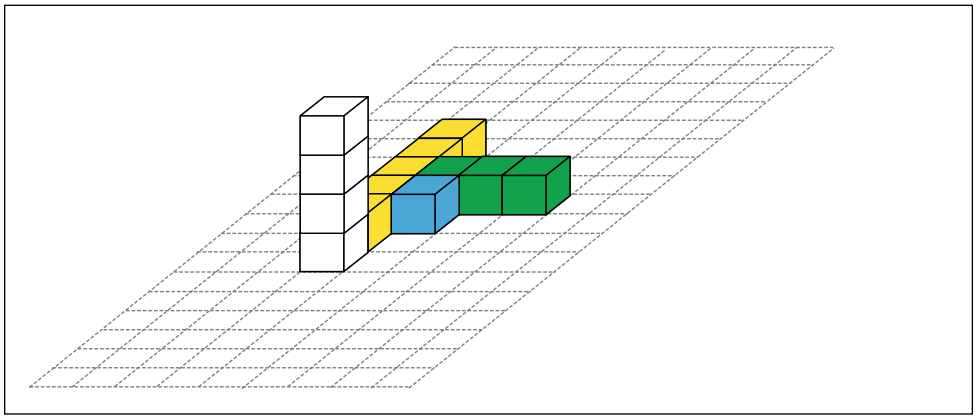


The following diagram using square dot paper shows what you should draw to answer the question.

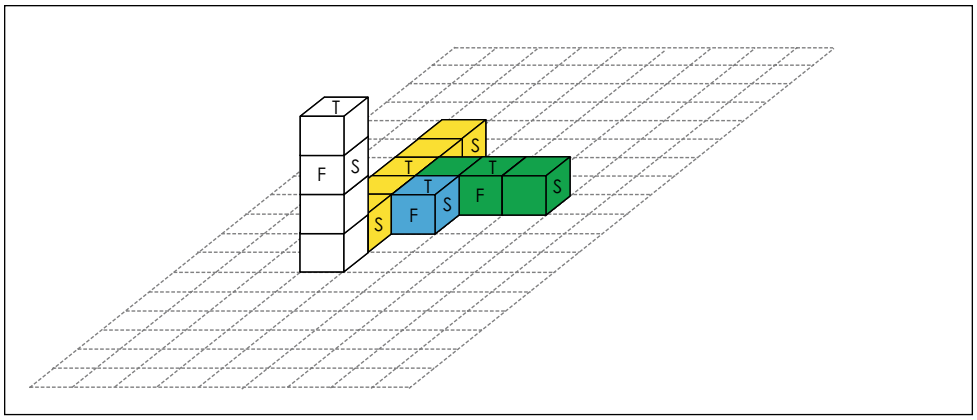


Example 2

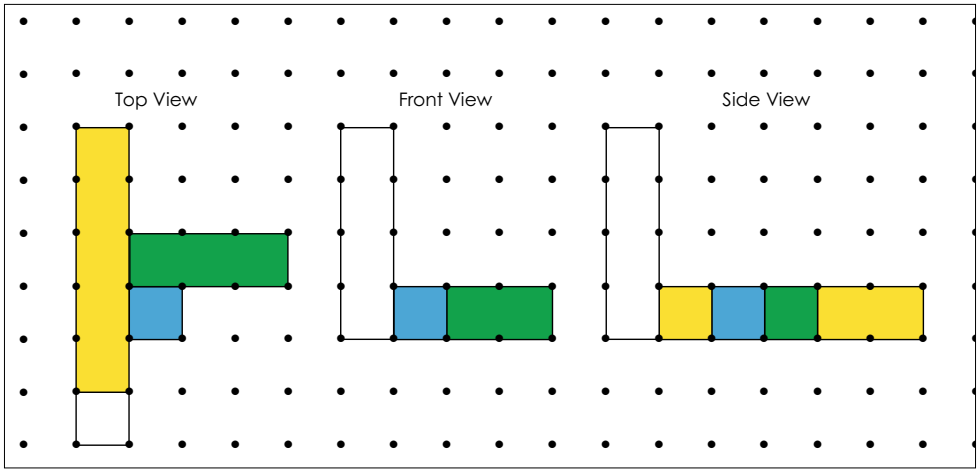
Draw the three 2-D views of this object, given the 3-D drawing below.



Solution



Note that the yellow bar does not have an F on it, so it will not appear in the front view of the object. This diagram is what you will draw to answer the question.



Interpreting the Three 2-D Drawings

It is also a useful skill to be able to translate the three views of an object to create the object, either through drawing a 3-D image of the object or constructing the object with blocks.

Drawing the 3-D object

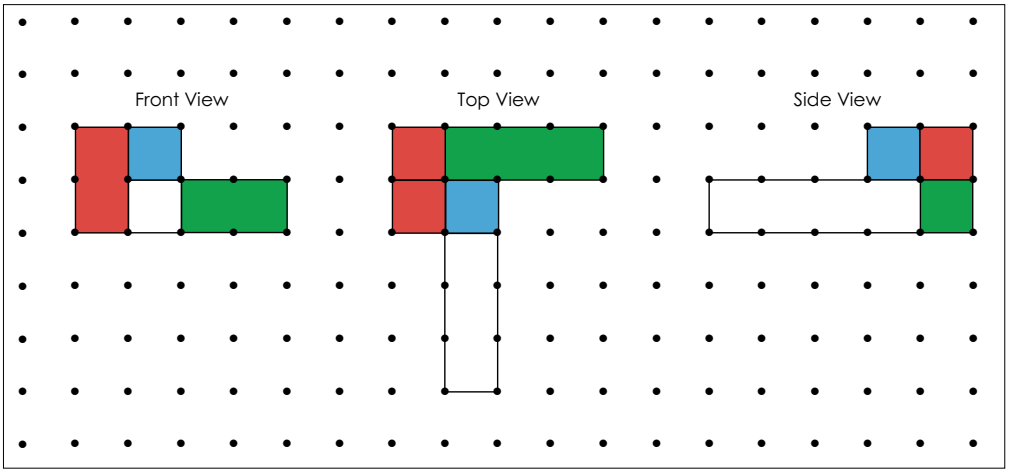
It is a good idea to follow along with the example, using isometric dot paper from the end of this lesson. This will help you better understand how to draw the 3-D objects.

Recall that the bar lengths are Blue = 1, Red = 2, Green = 3, White = 4, Yellow = 5.

Also, it will be easier for you to understand the solutions to these problems if you view the colour version. Access to the colour versions is described on the title page of this module.

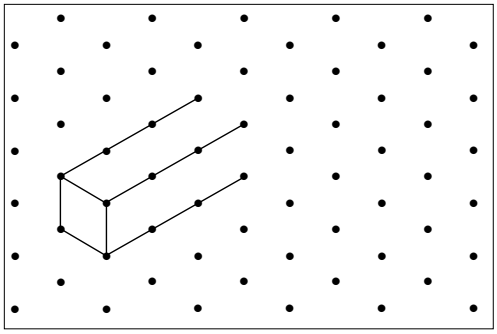
Example 1

Given the three 2-D views of the object, use isometric dot paper to sketch the object.

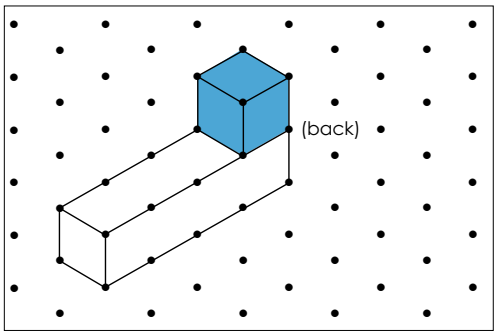


Solution

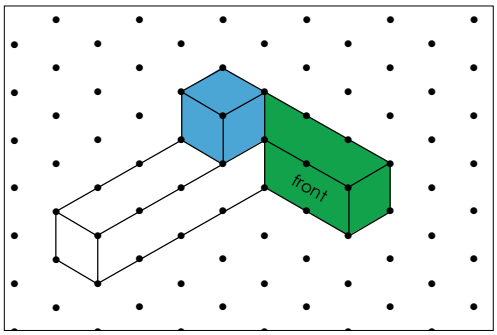
We can see in the top view that the white bar is at the front of the object and sticks out 3 units farther than the rest.



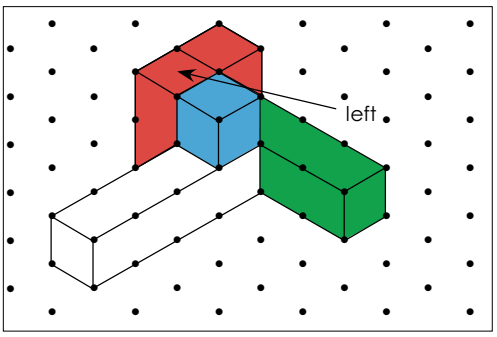
From the side view, we can see that the white bar's 4 units are all visible. Looking at the position of the blue cube, we can see that it is above the white prism (from the front view), and over top of the 4th unit (from the top and side views). We can include this in the drawing. Make sure that the cube lines up with the *back* end of the white bar.



Next, let's add the green bar. By looking at the top view, we can see that all 3 of its units are visible. With this in mind, the front view—where the green bar appears to be two units long—indicates that the green bar is behind the white bar for 1 unit. In the drawing, the *front* of the green bar should be drawn, starting at the *back* of the white bar (again). It should only extend two units from the white piece, as it does in the front view of the object.



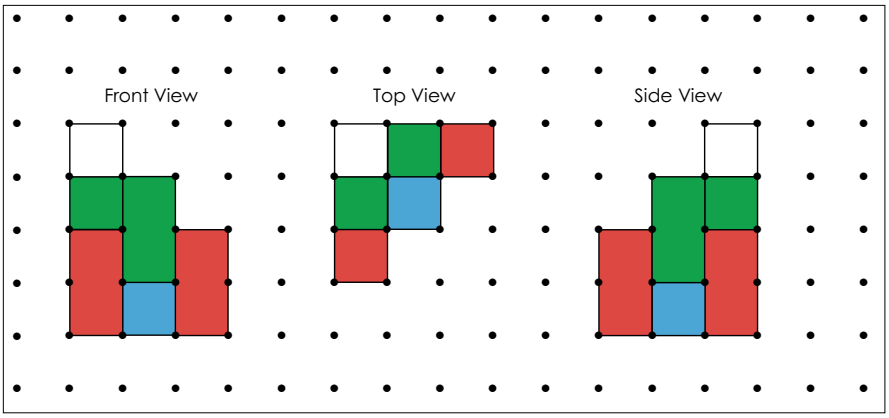
Finally, we can see in the front view that the red bar is two units tall. Based on the top view, we can see two red bars—side-by-side, on the left side of the white bar. To draw this, we start at the *left* edge of the *blue* cube, where we will draw the *tops* of the two red bars. Then draw the lines down from the tops of the red rods.



It is a good idea to double-check your 3-D sketch of the object by comparing it to the original front, top, and side views. Try it.

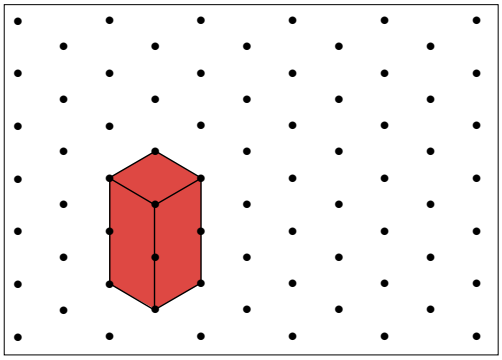
Example 2

Given the three 2-D views of the object, use isometric dot paper to sketch the object.

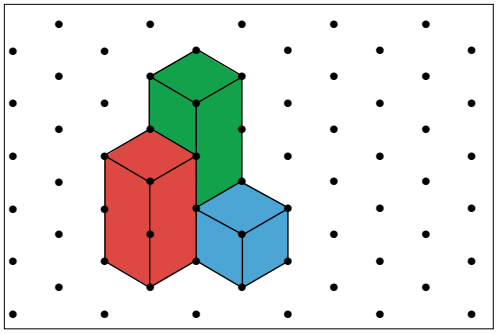


Solution

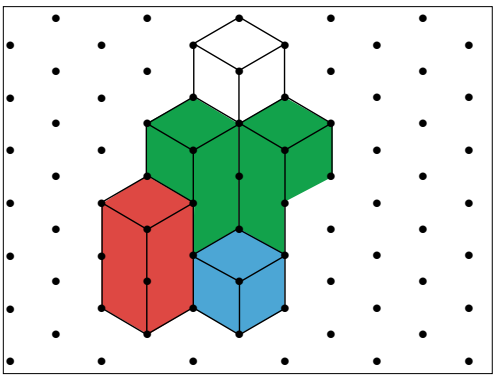
Recall: Red is two units tall, green is three units tall, white is four units tall, and blue is one unit tall. From the top view, we can see that red is at the front of the object, so we will start by drawing that.



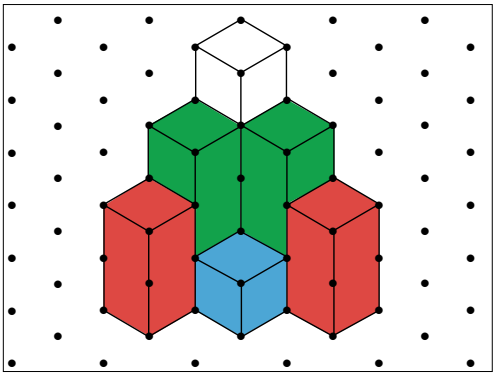
The green bar is directly behind the red bar, based on the front and top views. Based on the front view, the green bar is one unit taller than the red bar, so we can start by drawing up one unit from the *top* of the red bar. Here we can also include the blue cube, because it is in line with the green bar. Because we are drawing the object as if we are slightly *above* and to the *right* of the front, the blue cube will be in front of the green bar.



Still looking at the front and side views, we can see that the white bar is directly behind the green bar and it is taller than the green bar by one unit. Based on the right-side view, we can also see that the *other* green bar is *beside* the white bar. This means that we only see the top unit of the white bar from this angle as well.



The last piece missing is the final red bar, which is in line with the green and white bars (from the side view).



Complete the following learning activity to practise (a) drawing the three 2-D views of a 3-D object and (b) drawing the 3-D object based on the three 2-D views of it. Be sure to check your answers.

Learning Activity 7.2

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate: $\frac{3}{5} + \frac{4}{10}$

2. Solve for k: $4000 = \frac{k}{0.75}$

3. How many 125 g portions of corn are there in a 2 kg bag?

4. The perimeter of a square is 24. What is the area?

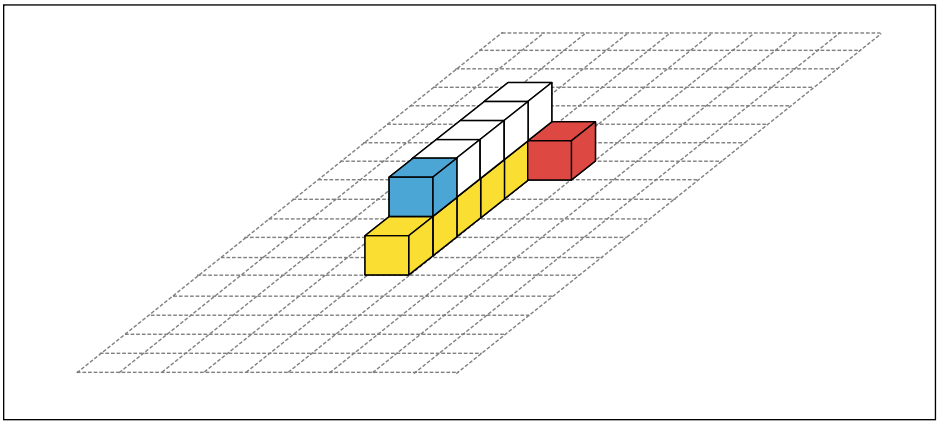
5. What is the complimentary angle to 30°?

Part B: Drawing a 3-D Object

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

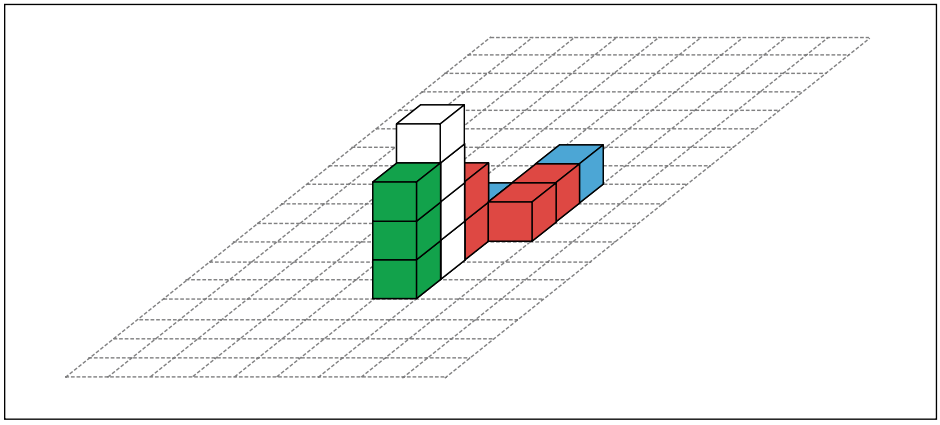
1. Draw the three 2-D views (front, top, and side) of the following objects. Use the square dot paper provided at the end of this lesson.

   a)

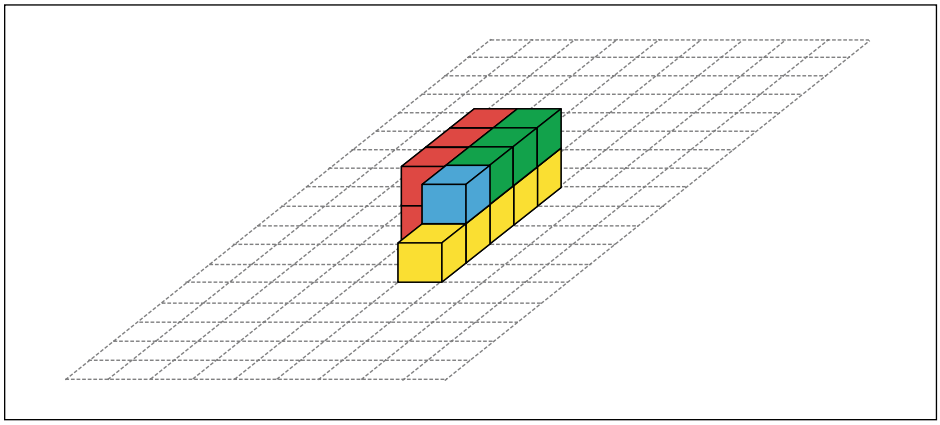


continued

b)

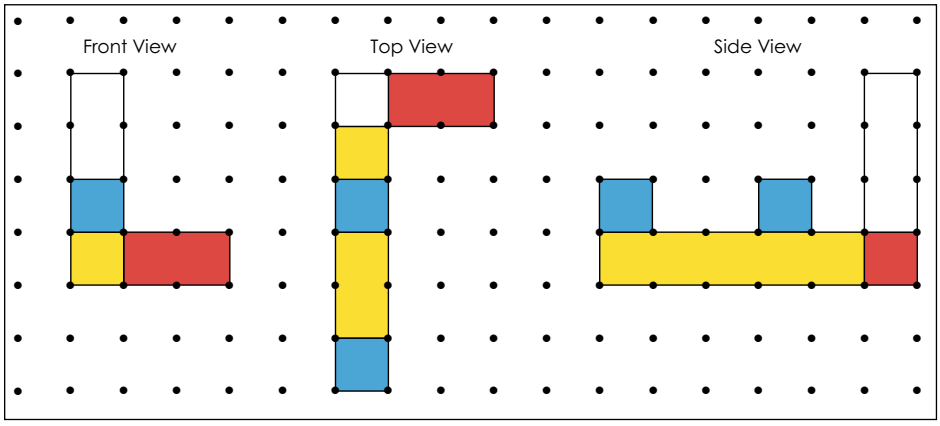


c)



2. Draw the 3-D representation of the object with the given three views. Use the isometric dot paper provided at the end of this lesson.

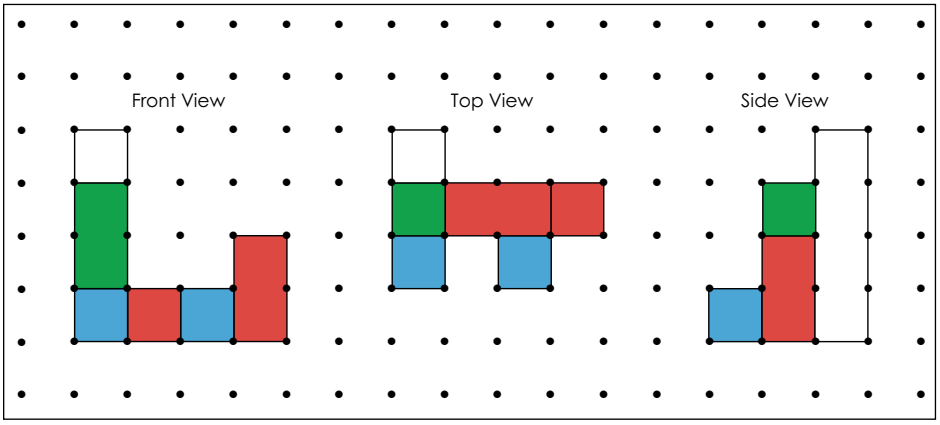
a)



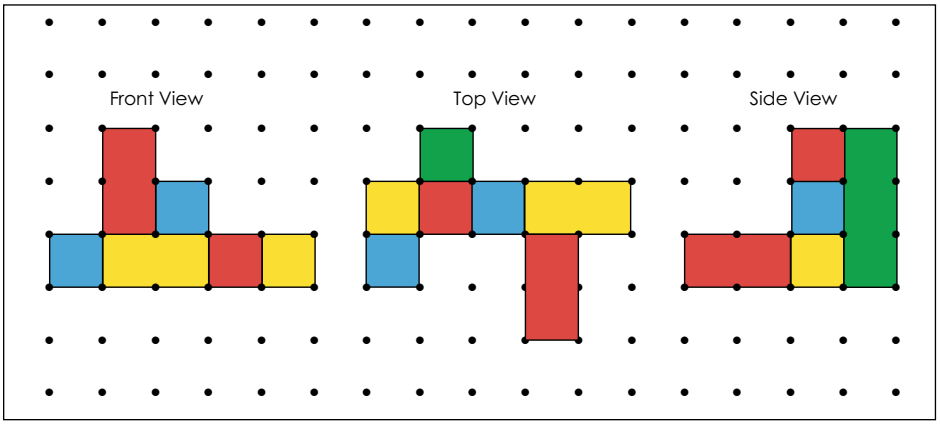
continued

Learning Activity 7.2 (continued)

b)



c)



Constructing the 3-D Object

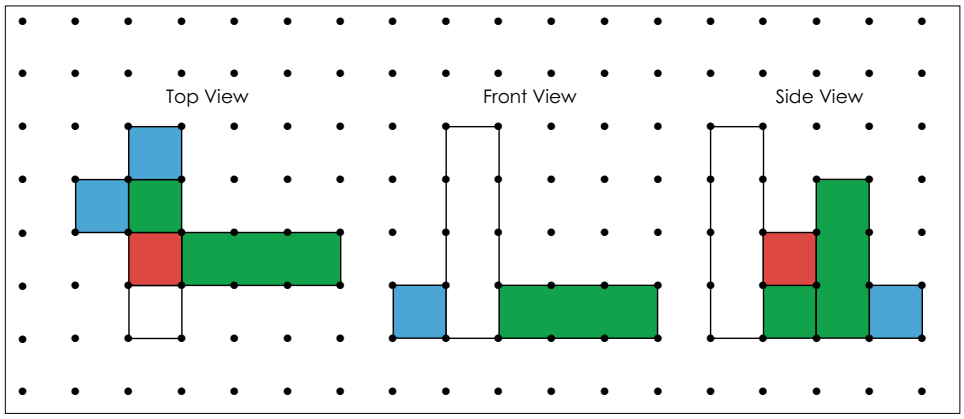
Being able to draw the 3-D object using the three views helps you to interpret the 2-D views of an object. Now that you have mastered this, you will apply what you know to constructing a 3-D object. It would be helpful to work through the example, because most people learn by doing, not just by reading the steps.

Materials you can use to construct these objects include:

- centimetre or inch cubes that attach together
- alphabet blocks
- Cuisenaire rods
- sugar cubes (you may want to use markers to distinguish between them)
- other cube objects

Example 1

Construct the object that is represented by the following three views.



Solution

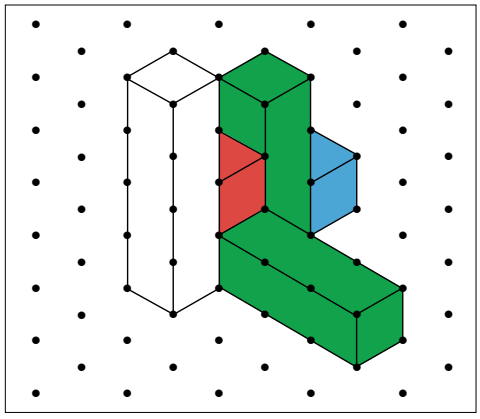
Before you begin to *construct* the object, first find out what you need in order to construct it.

Based on the top view, to build this object you need two blue cubes, two green bars (three units long each), one red bar (two units long), and one white bar (four units long).

Now that you know what you need, you can start building. From the front view, we can conclude that the red, green, and blue bars all line up behind the white bar.

The other blue cube is on the left side of this row, beside the green bar.

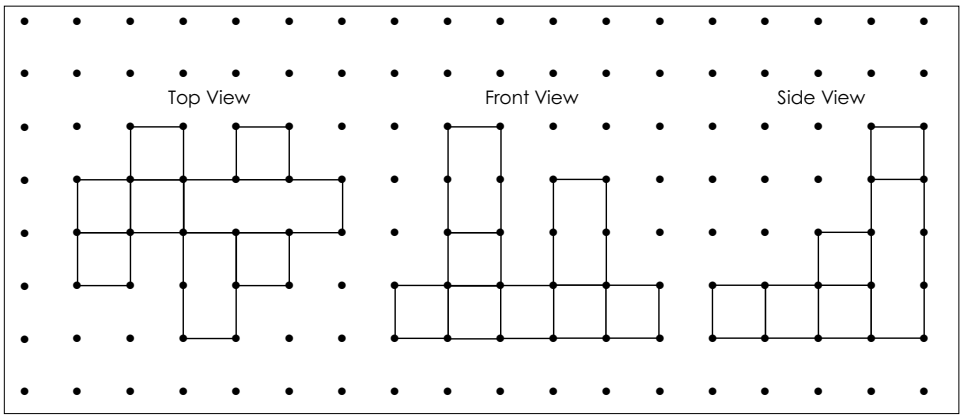
The other green bar is on the right side of the row—on its side—beside the red bar.



Not every drawing that you receive will have each piece labelled. Some will just have the drawing(s) and a list of all the parts needed. The following example demonstrates *one way* that you can construct an object if you are given the drawing(s) of an object and a list of parts.

Example 2

Construct the object that is represented by the following three views.

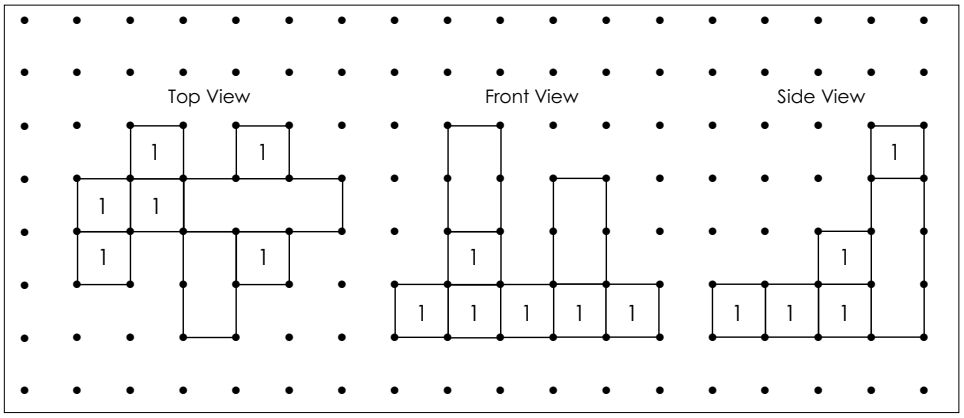


Use one yellow, one white, one green, one red, and three blue bars.

Solution

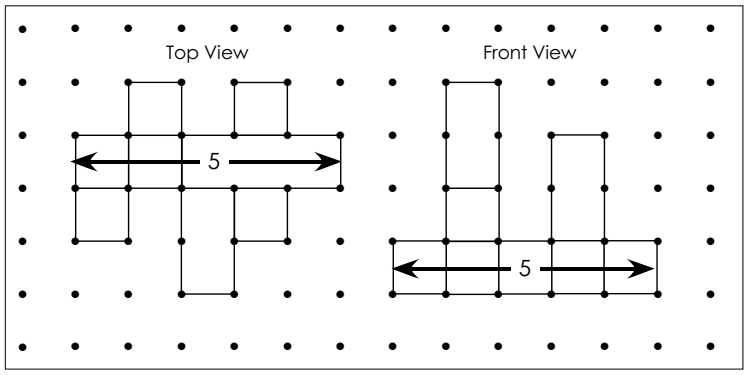
Although we do not know the colour of any piece of the drawing, we do know the lengths of the different bars (yellow = 5 units, white = 4 units, green = 3 units, red = 2 units, blue = 1 unit).

Because there are a large number of one-unit squares in each view, it would be very difficult to start by determining where the blue cubes would appear.

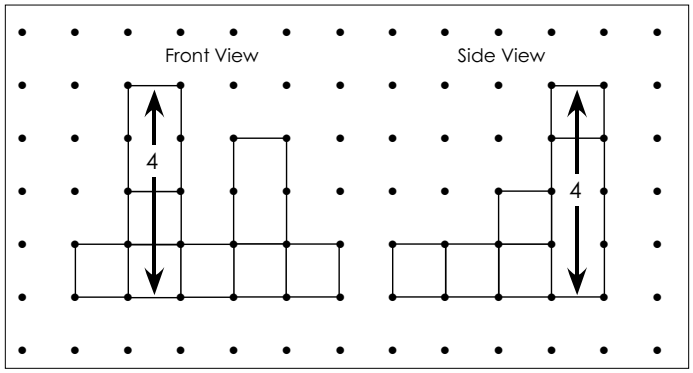


Instead, it is easier (usually) to start with the longest bar, and "work your way down." This means determining where the yellow bar is, then white, then green, etc.

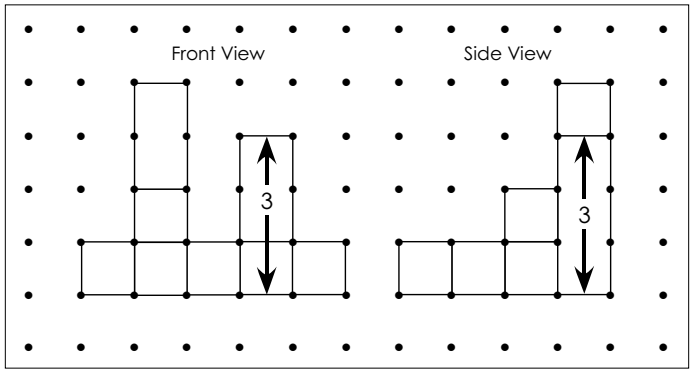
We know that the yellow bar is five units long. When we look at the three diagrams, there is only one bar that is five units long, although there are other bars on top and in front of it. This must be the yellow bar.



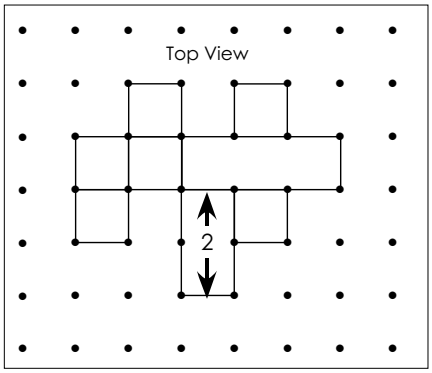
The white bar is four units long. Again, there is only one piece in the diagrams that is four units long, so it has to be the white bar.



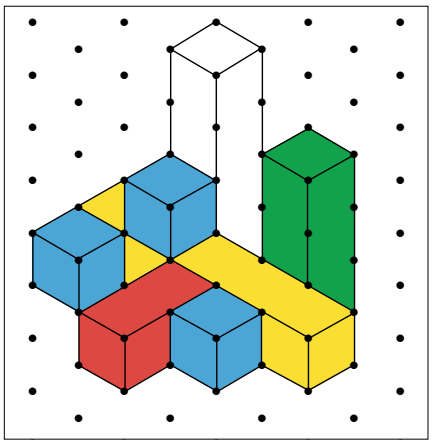
The three-unit green bar must match up with the bar that is three units long.



There is only one bar that is two units long, so it has to be the red bar.



The remaining bars are all blue. There should only be three. The final product should look something like this:



If there are certain steps that you find helpful in either example, you should add them to your resource sheet before you start the following learning activity.

Learning Activity 7.3

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If seven oranges cost $1.82, how much does each orange cost?
2. What is the average of the following numbers: 18, 19, 21, 22
3. Carmen spent 50% of her allowance for the month on a pair of shoes. Of her remaining money, she puts 50% in her bank account. If her allowance is $300, how much money does she have left?
4. A movie theatre can accommodate 350 people. If there are three showings of Parry Halfoy at the same time and all three theatres are full, how many people are seeing the movie?
5. Daris completed an average of 50 math questions per day for seven days before his examination. How many questions did he complete in total over the seven days?

Part B: Constructing Objects from the Three 2-D Drawings

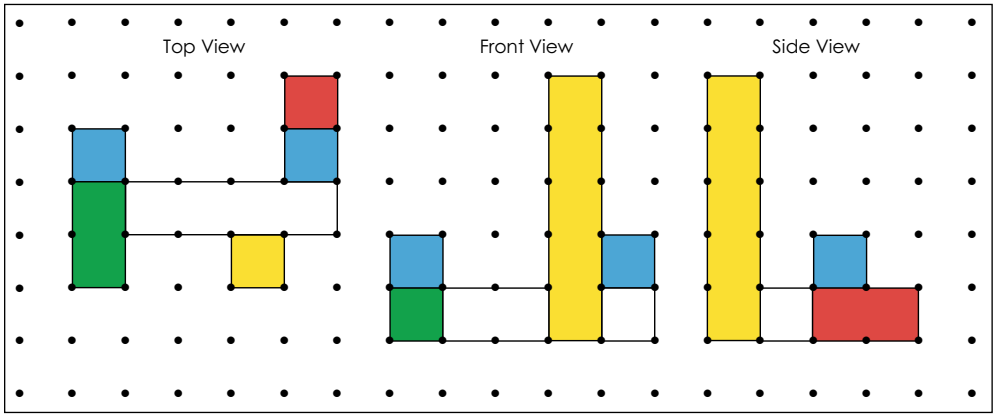
Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Construct the objects based on the three 2-D drawings provided. Each object includes one yellow bar, one white bar, one green bar, one red bar, and two blue cubes. Use the isometric dot paper provided at the end of this lesson.

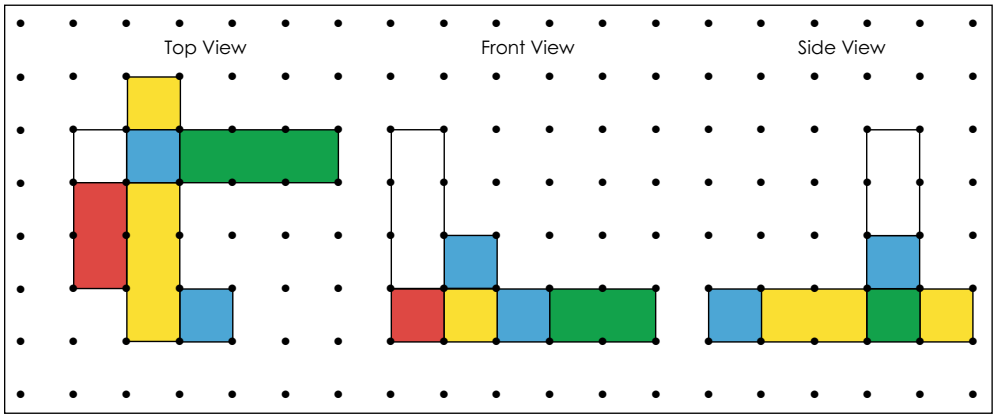
continued

Learning Activity 7.3 (continued)

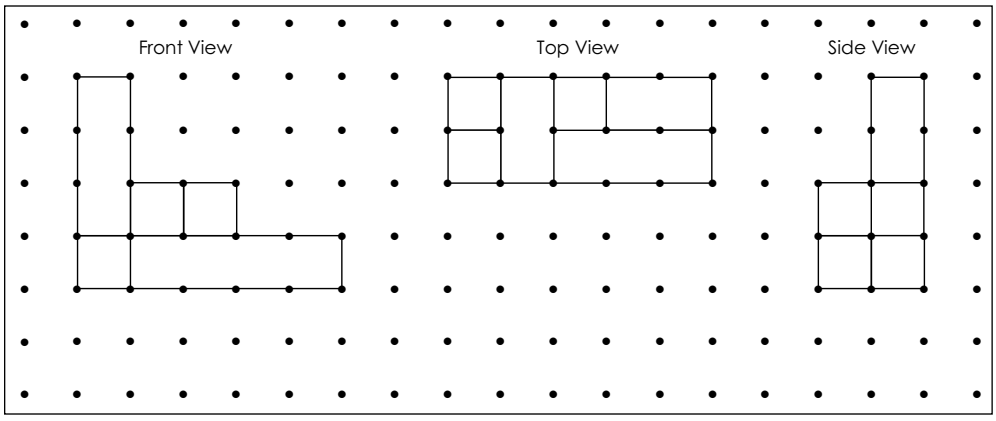
1.



2.

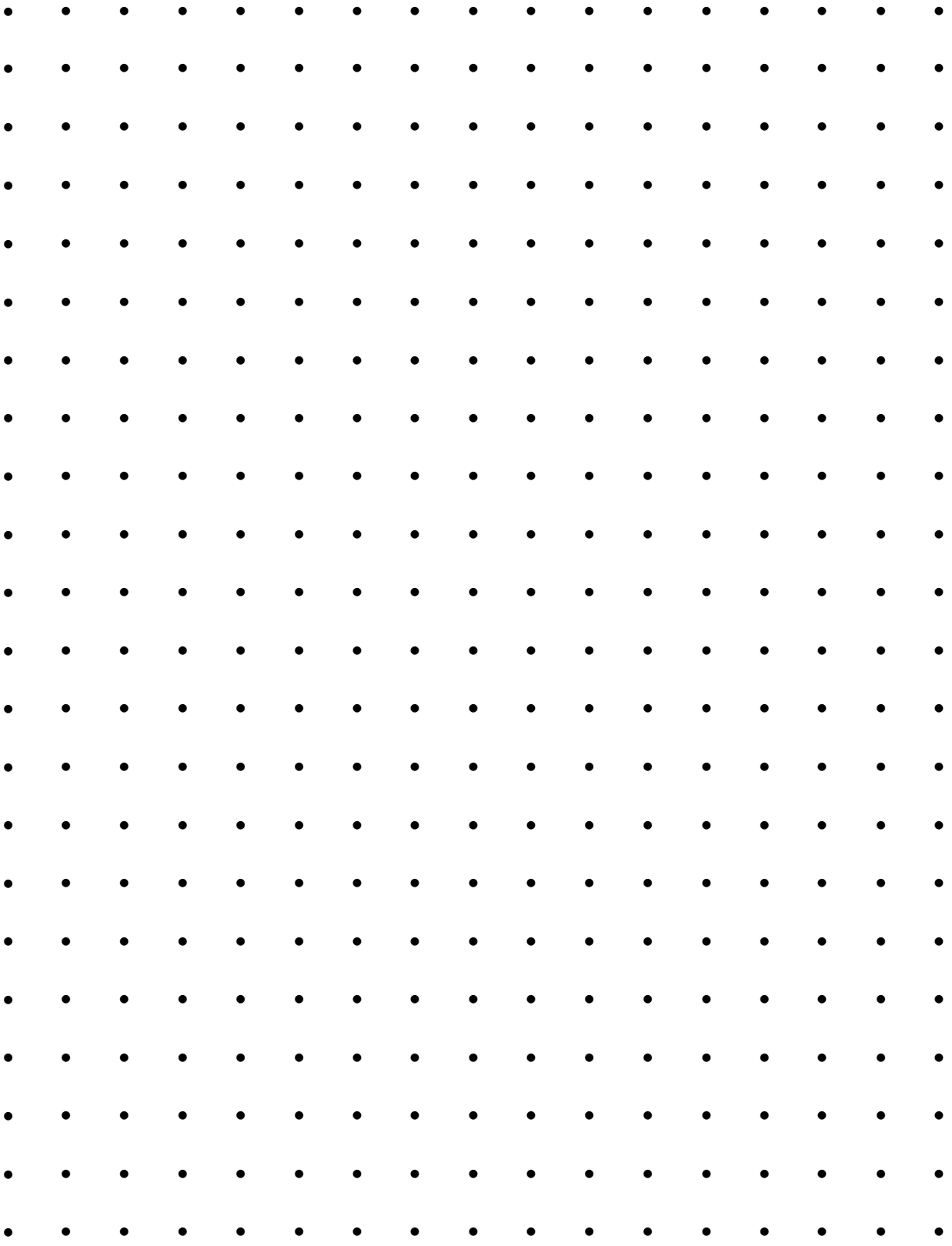


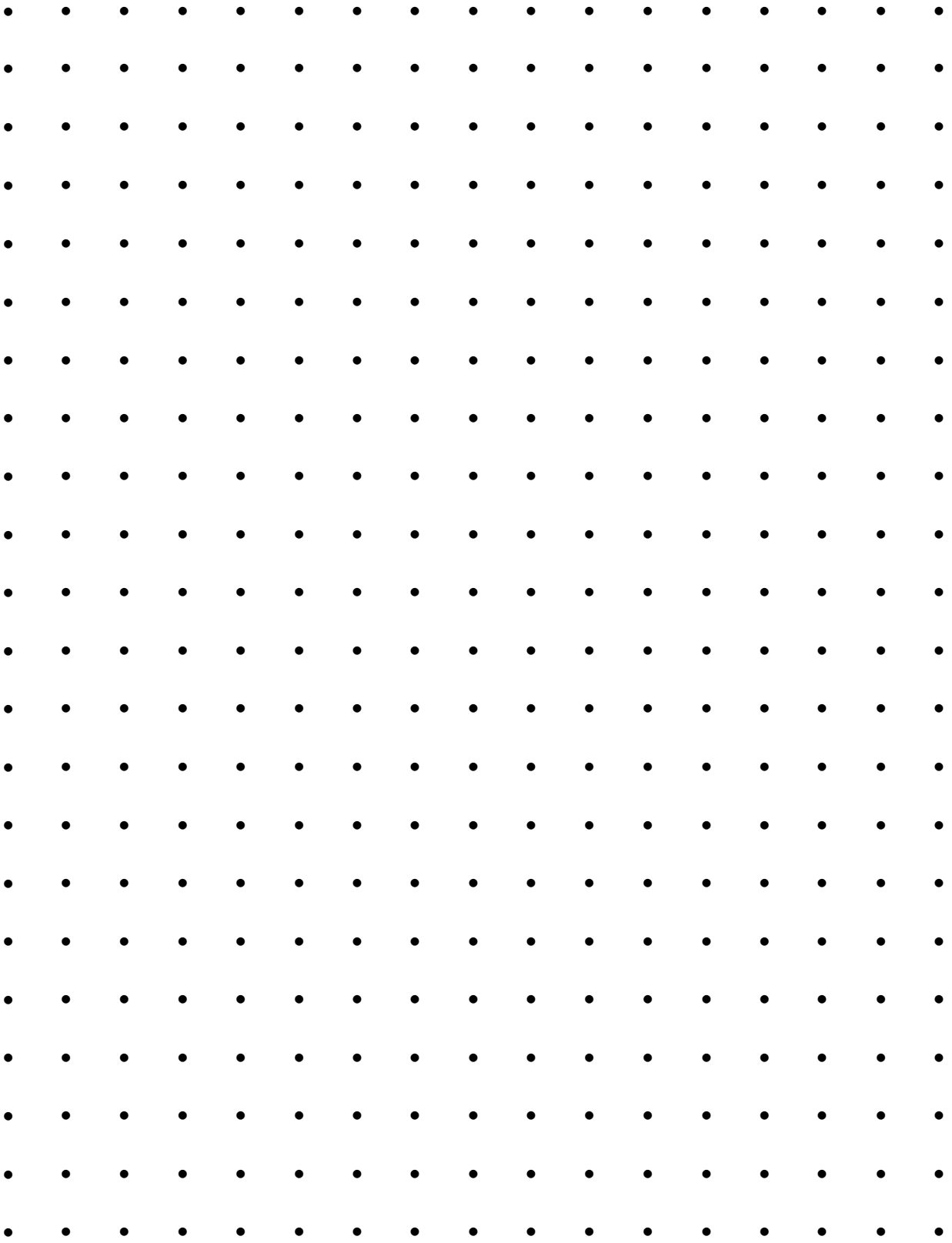
3.

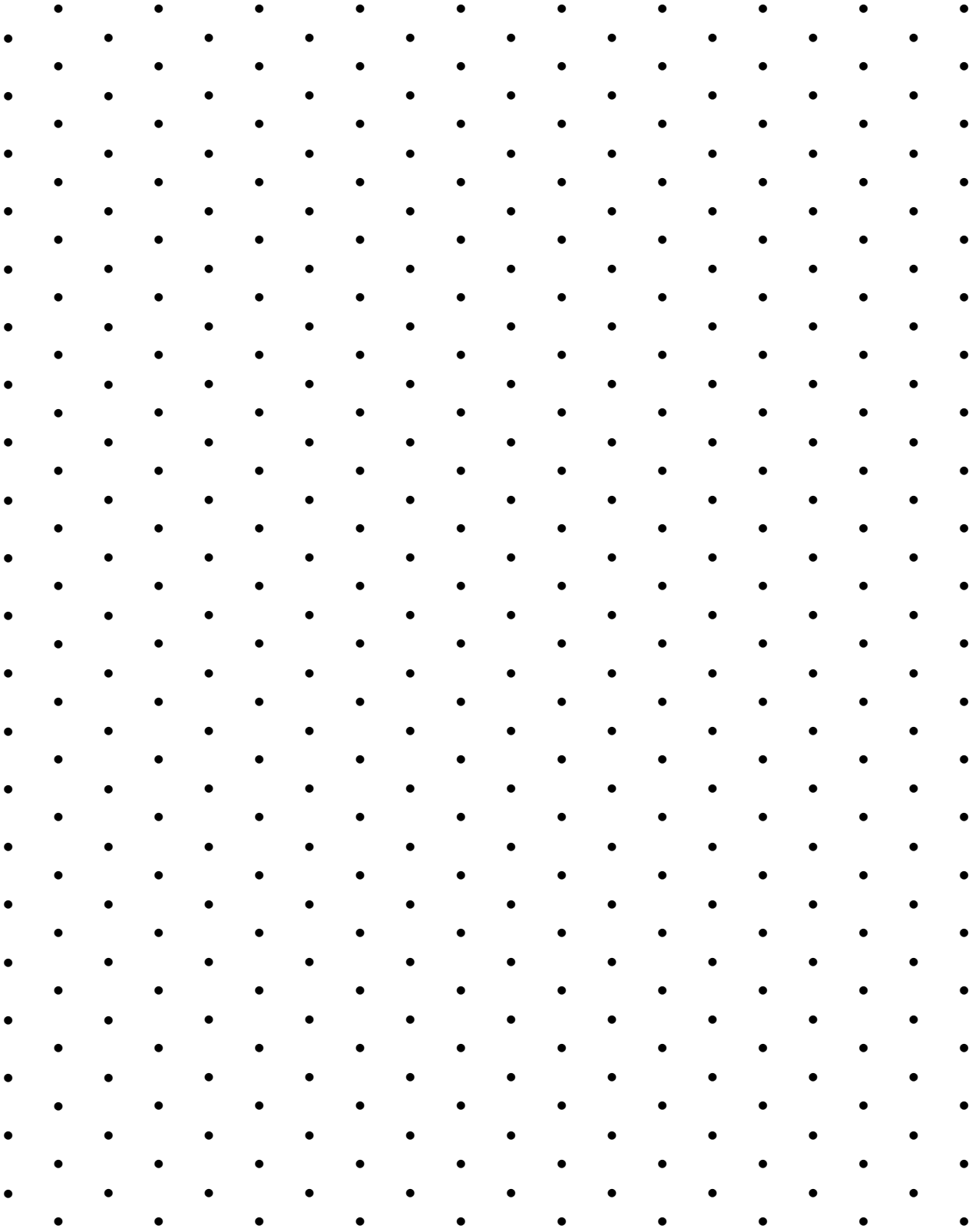


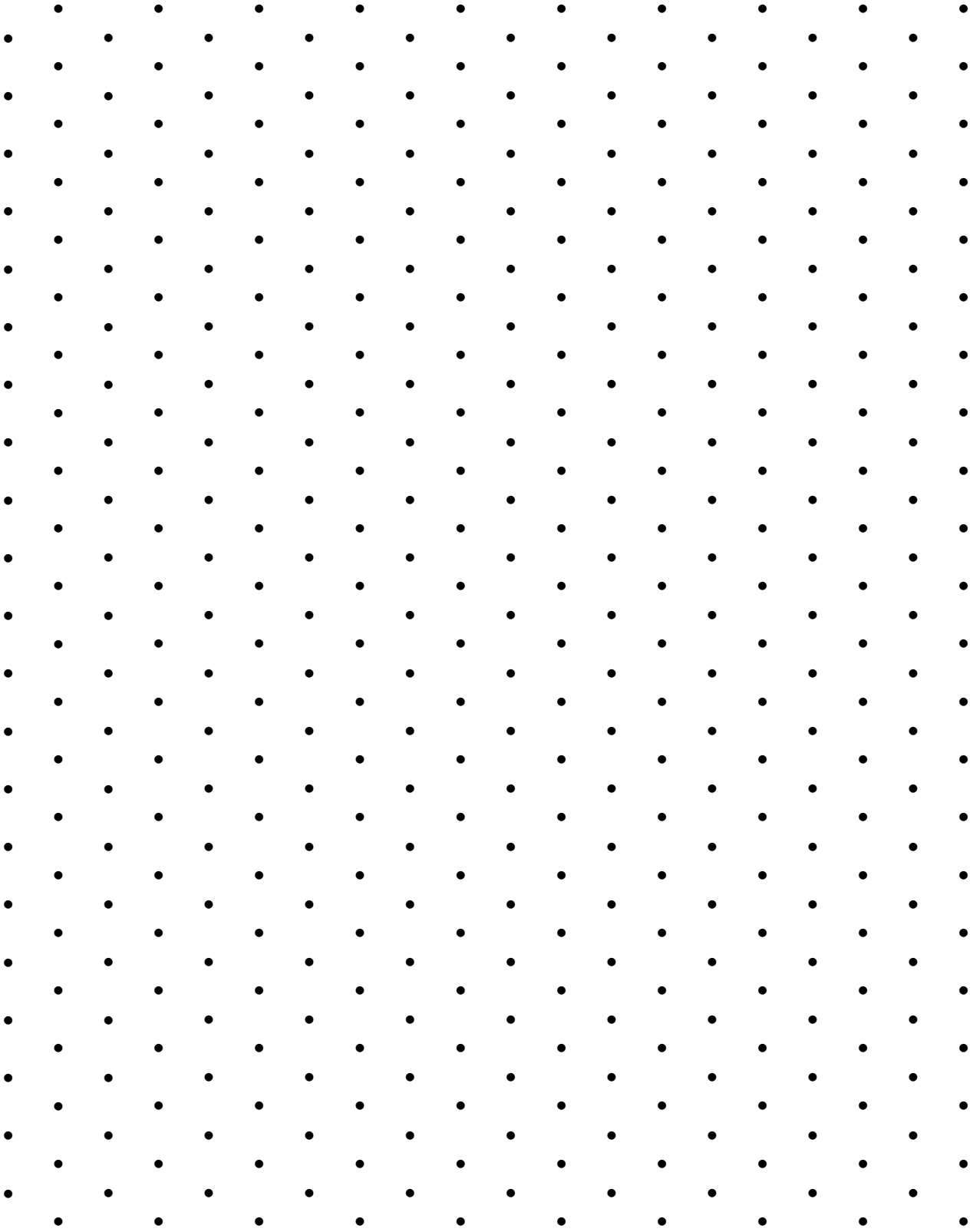
Lesson Summary

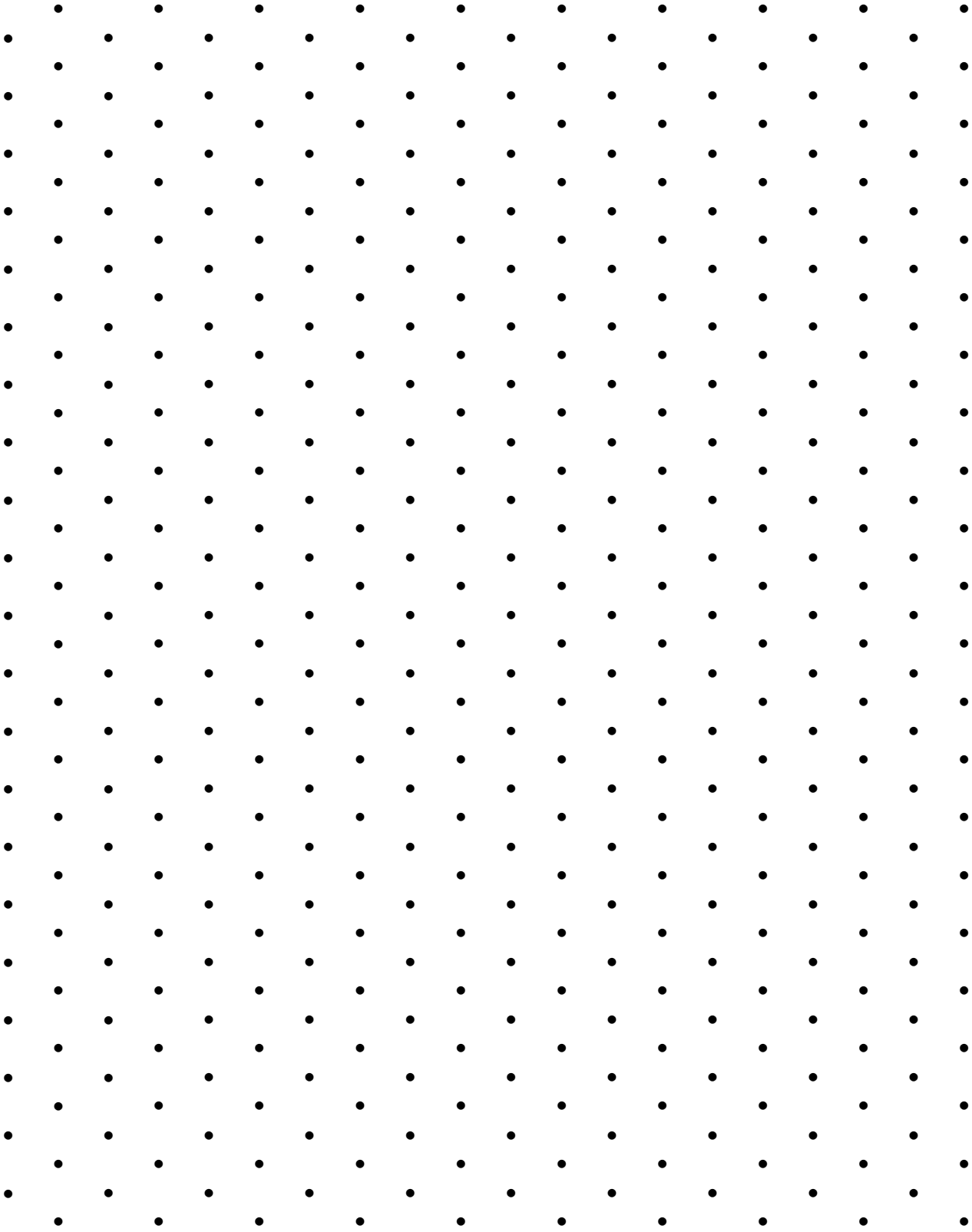
In this lesson, you learned about the three views (front, top, and side) of a 3-D object. You learned how to draw these three 2-D views. You also learned how to interpret the three views in order to draw and construct 3-D versions of the object.

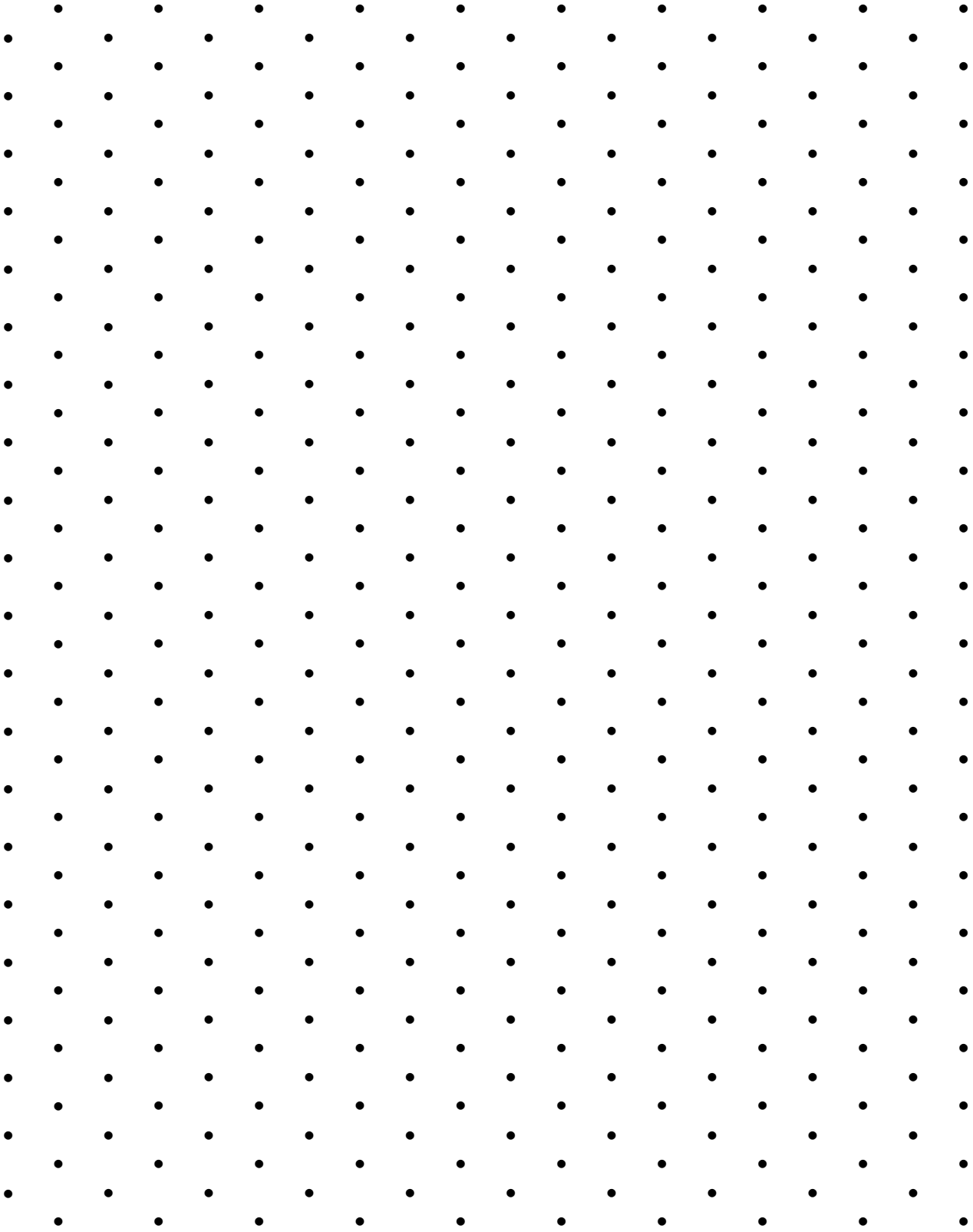












Assignment 7.1

Three 2-D Views of 3-D Objects

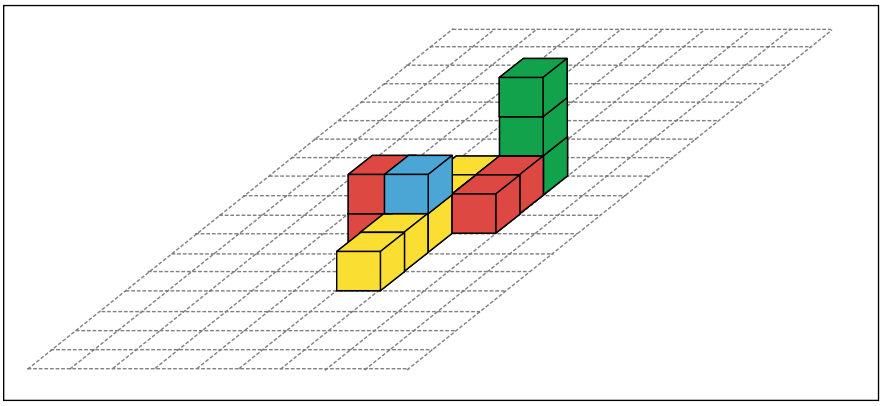
Total: 21 marks

Note to Students: Use your resource sheet to help you complete this assignment. Do not hesitate to add to your resource sheet as you answer the following questions.

Remember, the lengths are the following:

- yellow bar = 5 units,
- white bar = 4 units,
- green bar = 3 units,
- red bar = 2 units, and
- blue cube = 1 unit.

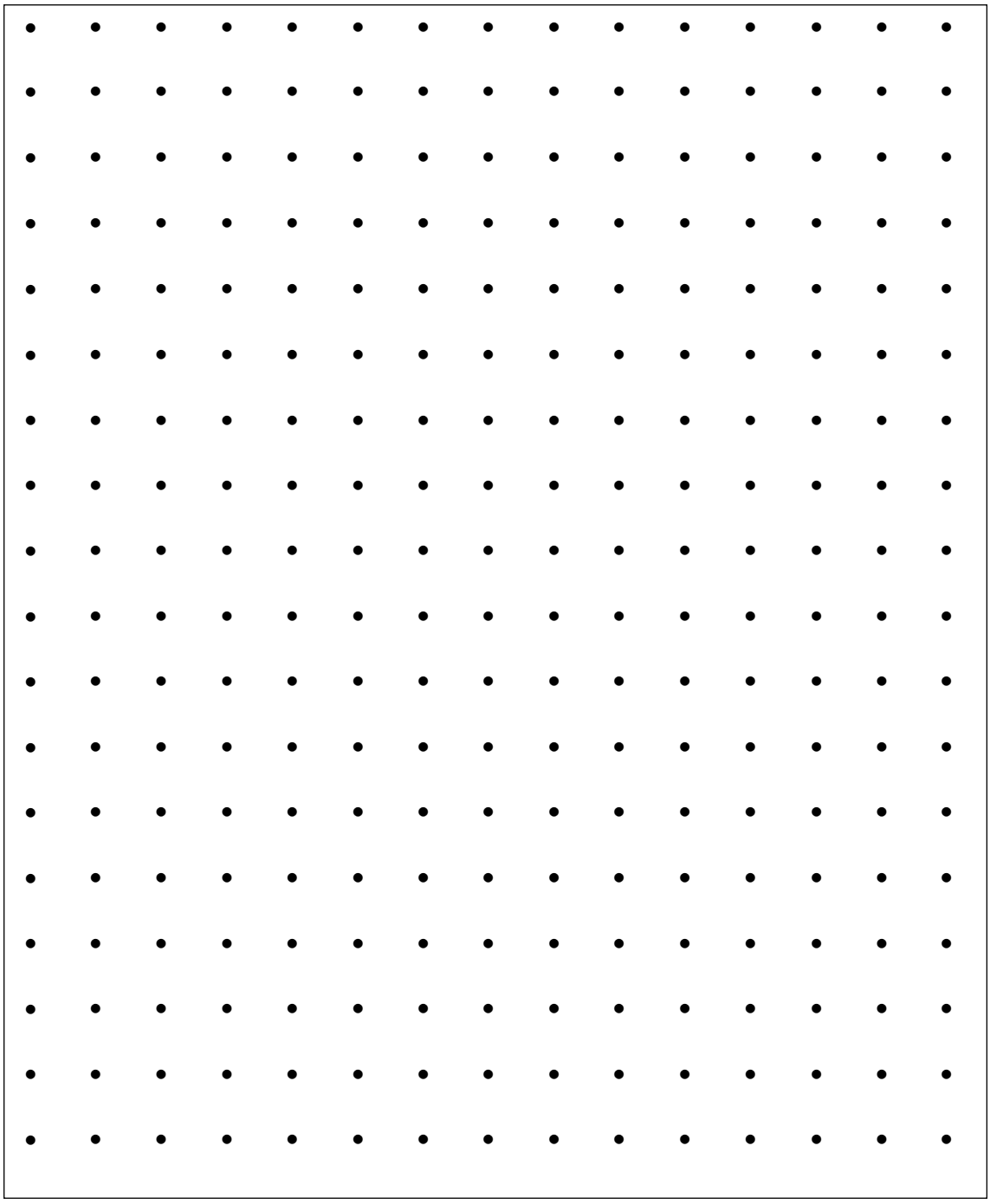
1. a) Label the faces of the object. (F = front, T = top, S = side) *(3 marks)*



continued

Assignment 7.1: Three 2-D Views of 3-D Objects (continued)

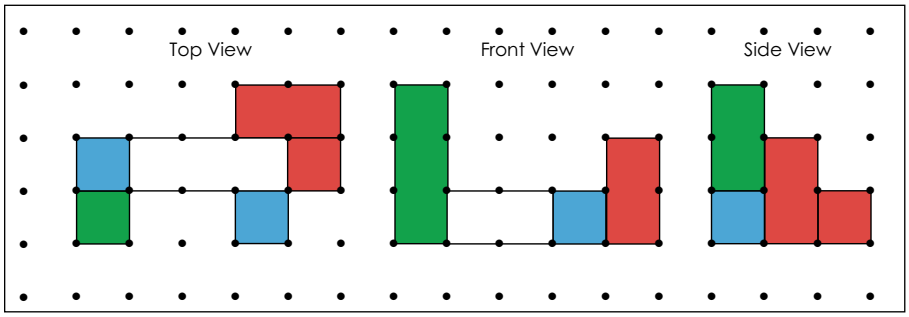
b) Draw the three 2-D views of the object, using the square dot paper below. Label them as top view, front view, and side view. If you do not colour the bars, label them with the appropriate colour name. *(6 marks)*

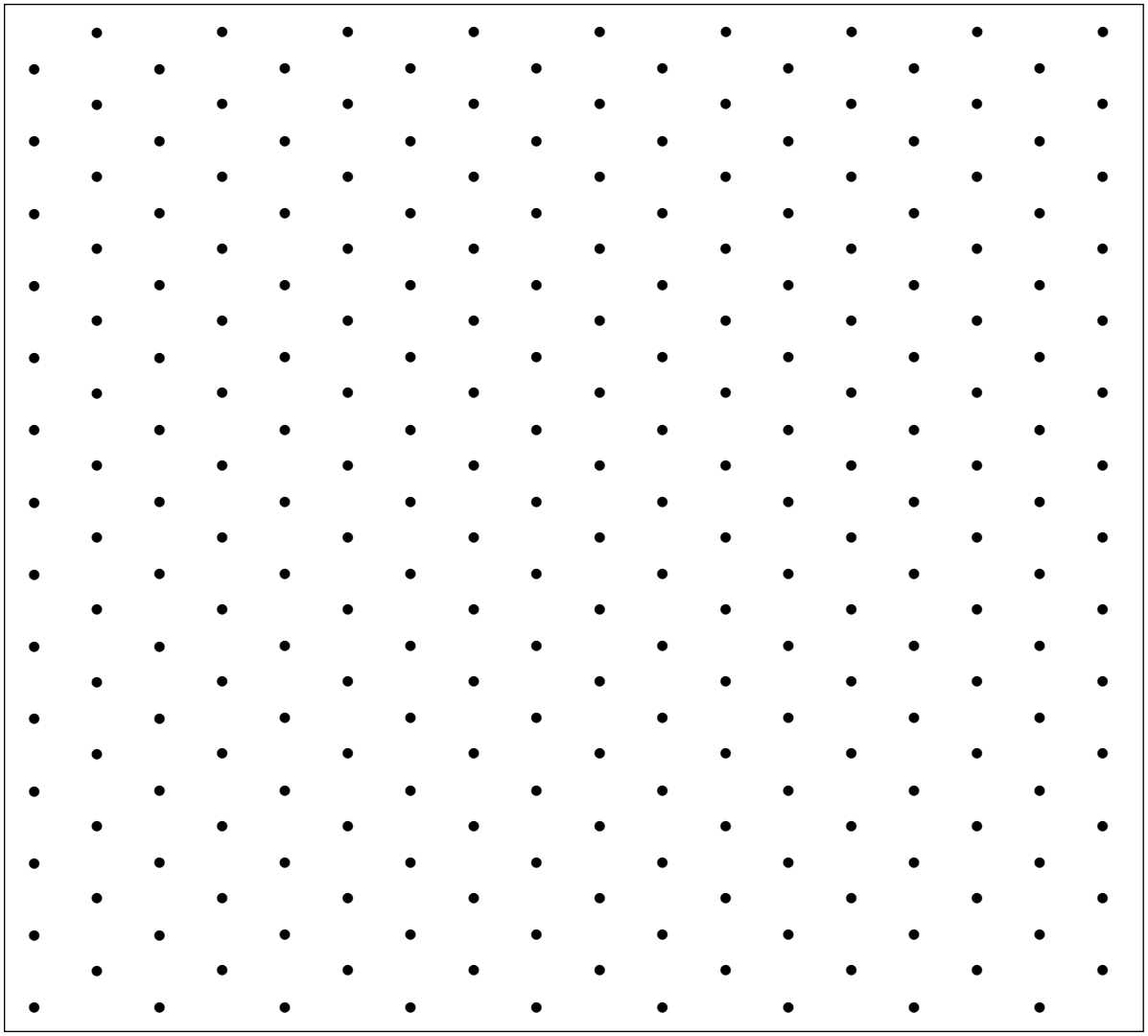


continued

Assignment 7.1: Three 2-D Views of 3-D Objects (continued)

2. Draw the object below (in 3-D) using the isometric dot paper provided. If you do not colour the bars, label them with the appropriate colour name. *(6 marks)*



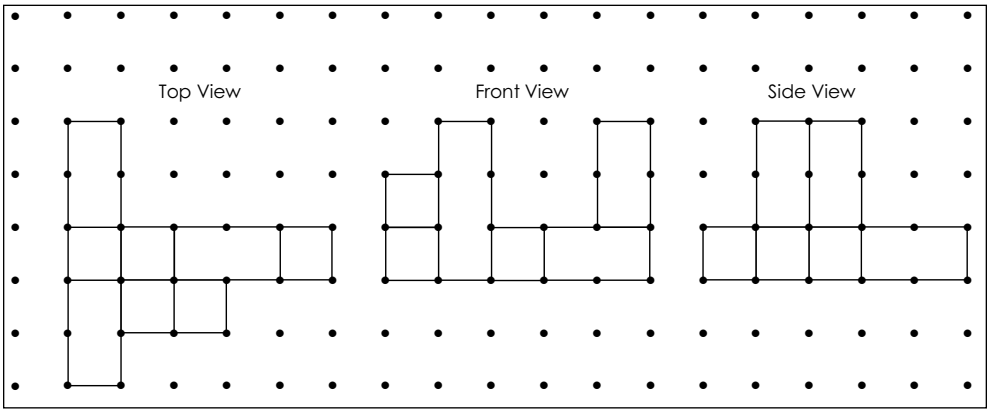


continued

Assignment 7.1: Three 2-D Views of 3-D Objects (continued)

3. Construct an object that is described by the three 2-D drawings below. Include a photograph of your construction with this assignment. If the photo is not printed in colour, label it appropriately. Use one yellow, one white, one green, two red, and two blue bars. If you do not colour the bars, label them with the appropriate colour name. *(6 marks)*

   Remember: yellow bar = 5 units long, white bar = 4 units long, green bar = 3 units long, red bar = 2 units long, blue cube = 1 unit.



Lesson 3: One-Point Perspective Drawings

Lesson Focus

In this lesson, you will

- ❑ draw a one-point perspective view of a 3-D object
- ❑ identify the point of perspective in a one-point perspective drawing

Lesson Introduction

So far in this module, you have learned to draw 3-D objects on isometric dot paper, as well as the three 2-D views of the object. In this lesson, you will learn another way to represent 3-D objects—through one-point perspective drawings. This technique is used by anyone who wants to create the illusion of depth in a 2-D image.

One-Point Perspective

You have probably noticed that if an object is closer to you, it will appear larger than an object of the same size that is farther away. Objects that are farther away from you appear to be smaller and closer together. When you look along railway tracks towards the horizon, the parallel tracks appear to meet. In perspective drawing, parallel lines running directly away from you are drawn so that they meet at *one point*, called the **vanishing point**. The vanishing point is located on the **horizon line**, which represents where the horizon would be if the object were seen in real life instead of on paper.

Drawing a One-Point Perspective

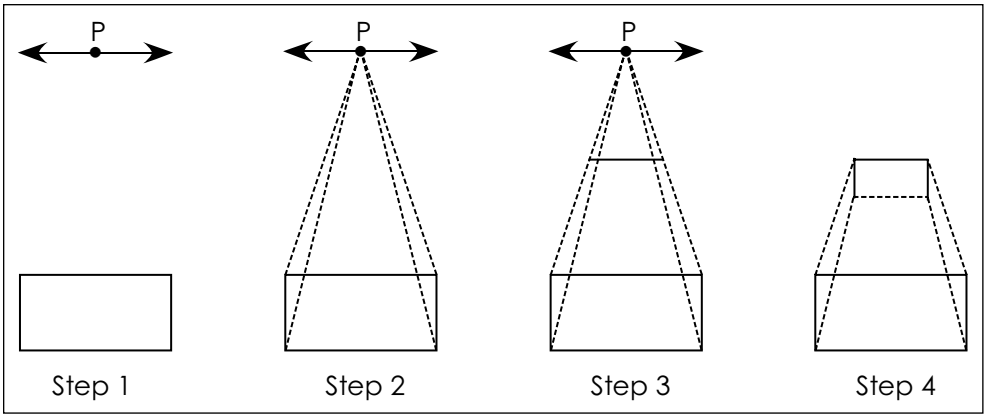
A rectangular solid is one of the simplest objects to draw in one-point perspective. Consider the following example.

Example 1

Using one-point perspective, draw a rectangular solid where you are looking directly at one face.

Solution

Follow these four steps. You should probably include these steps on your resource sheet.



Step 1: Using a pencil, draw a rectangle. This represents the front face. Then draw a horizon line parallel to the horizontal edges of your rectangle and select a vanishing point on it. Call the vanishing point **P**.

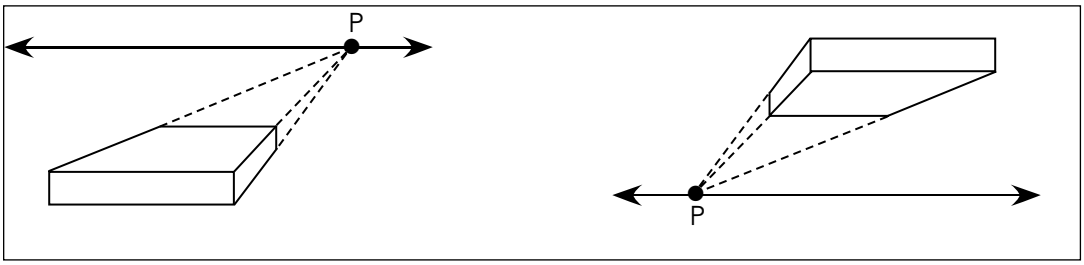
Step 2: Draw dashed lines from each of the corners of the rectangle to the vanishing point **P**.

Step 3: Draw a line parallel to the horizon line. This line forms the visible back horizontal edge.

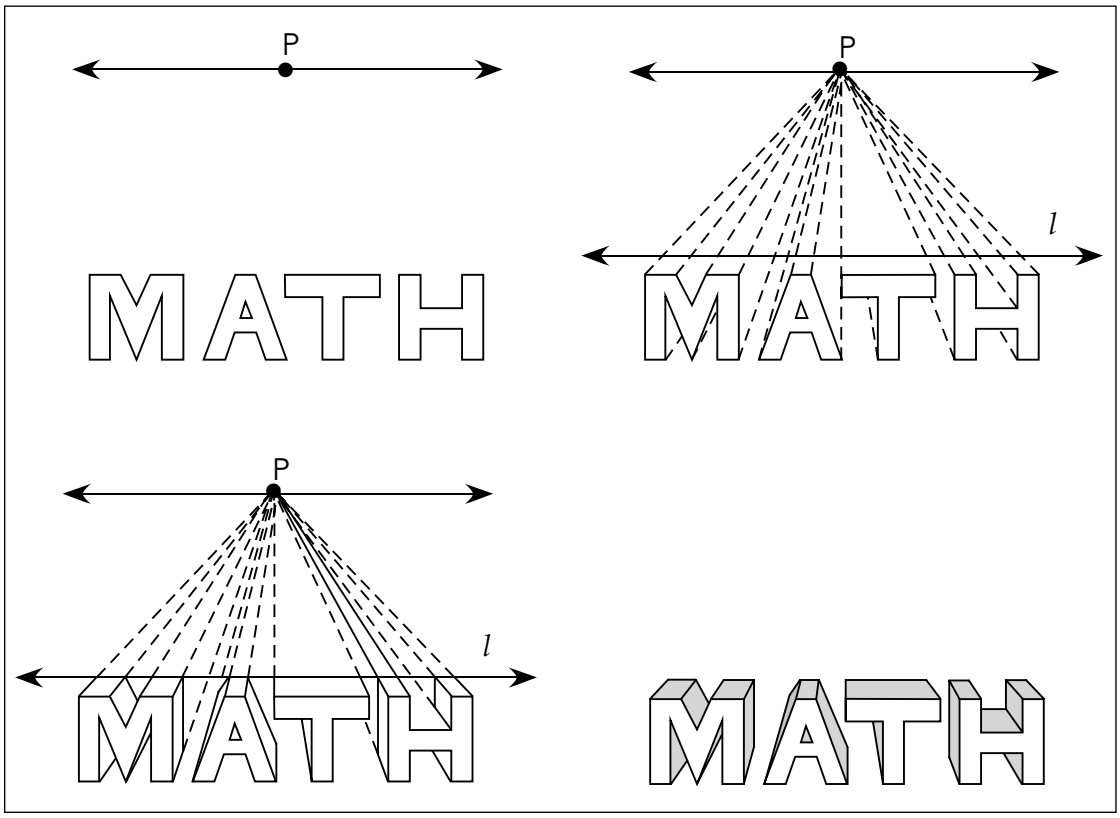
Step 4: Draw the hidden back vertical and horizontal edges with dotted lines. Erase the horizon line and the unnecessary portions of the vanishing lines.

In the above example, you were looking directly at one face. If you were to view the object from the right, then the vanishing point would be to the right. If you were to view the object from the left, then the vanishing point would be to the left.

Consider the following two drawings. In the first drawing, you are viewing the object from above and to the right. In the second drawing, you are viewing the object from below and to the left.

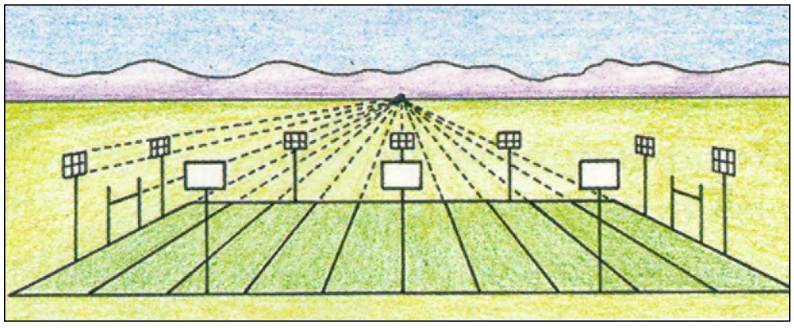


Note that you can draw words in block letters using one-point perspective. The following drawings show the word MATH being drawn in one-point perspective.



Identifying the Point of Perspective

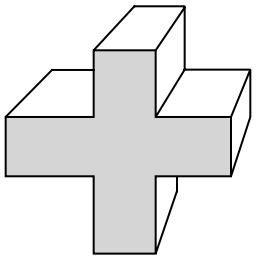
In addition to drawing one-point perspectives, we can also identify where the point of perspective for a drawing is located. If the vanishing point is visible in the drawing, the point of perspective is easily identified because the point of perspective is located at the vanishing point.



If the vanishing point is not visible, then we must use the lines that would have been drawn using the vanishing point to predict where the point is.

Example 1

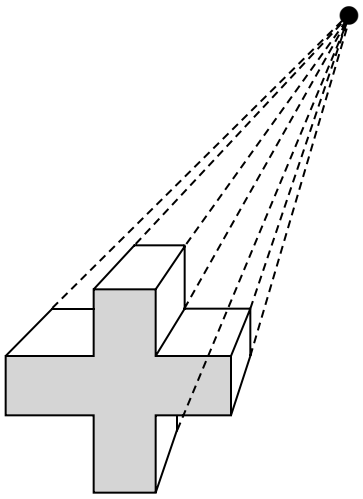
Identify the perspective of the following diagram.



Solution

Step 1: We can see if the point of perspective was above or below the original shape based on where the extension is visible. In this diagram, the point of perspective was *above* and to the right of the original shape because we can see the *top* and the right side of the object. If the point of perspective had been *below* the shape, we would see the *bottom* of the object.

Step 2: Extend the lines that are drawn off of the original shape, or the face of the object. The point where these lines cross is the vanishing point, and indicates the point of perspective.



In the following learning activity, you have a chance to practice drawing one-point perspective views. Check your answers once you have completed all of the questions.

Learning Activity 7.4

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If you travel 1800 m in 5 min., how fast are you going (in m/s)?
2. Which side is the hypotenuse of a right triangle if the side lengths are 3, 4, 5?
3. What is the measure of the third angle of a triangle if the other two angles are 80° and 60°?
4. 8′5″ = _____ in.
5. What is the total cost of lunch if a sandwich costs $2.25, a salad costs $4.05, and an orange juice costs $1.55?

Part B: One-Point Perspective

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

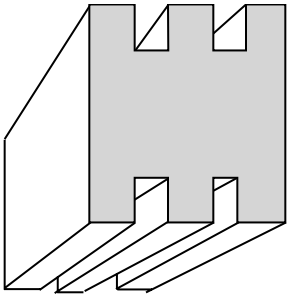
1. Draw a one-point perspective of a rectangular solid that you view from above and to the left.
2. Draw a one-point perspective of the word TEXT in block letters that you view from below.

continued

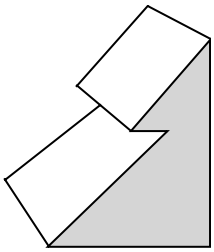
Learning Activity 7.4 (continued)

3. Identify where the point of perspective (vanishing point) is for the following drawings.

a)



b)



4. Explain how one-point perspective drawings are different from drawings made using isometric dot paper. You may use drawings to help you explain.

Lesson Summary

This lesson focused on one-point perspective drawings. You learned how to draw them, and also to identify the point of perspective. Perspective in a drawing is very important if you want to create a realistic picture that shows depth.

Assignment 7.2

One-Point Perspective Drawings

Total: 15 marks

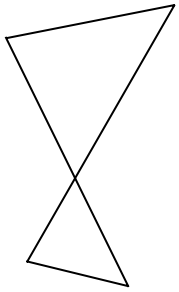
Note to Students: If you have not already added the steps for drawing a one-point perspective on your resource sheet, you should do so now. This will allow you to look at the steps while completing the assignment without flipping back and forth between the assignment and the lesson.

1. Draw a one-point perspective of a rectangular solid that you view from below and to the right. *(4 marks)*

continued

Assignment 7.2: One-Point Perspective Drawings (continued)

2. Draw a prism with two triangular bases that you view from above and to the right. *(3 marks)*



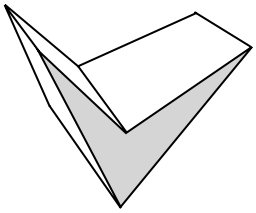
3. Draw a one-point perspective of the word EMAIL in block letters that you view from above. *(4 marks)*

continued

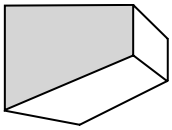
Assignment 7.2: One-Point Perspective Drawings (continued)

4. Use a pencil and ruler to locate the point of perspective (vanishing point) for each drawing. *(2 marks each × 2 = 4 marks)*

a)



b)



Notes

Lesson 4: Exploded Views and Component Parts

Lesson Focus

In this lesson, you will

- ❑ create a scale drawing of a 3-D object from an exploded view
- ❑ sketch a 2-D exploded view of a 3-D object
- ❑ draw the component parts of a 3-D object

Lesson Introduction

In the previous lessons, you learned how to create 3-D drawings of objects and how to draw the three views of a 3-D object. This lesson will focus, again, on drawing 3-D objects, but in a different way. Exploded views are drawings that show all the parts of an object and how they are oriented when the object is put together. Each piece is called a component part.

Exploded Views

In order to construct an object, you have to know how many pieces make up the object and how these pieces fit together. An exploded view provides you with this information. In an exploded view of an object, you can accomplish this by drawing each individual piece of the object separately and placing it in its relative position to the other pieces of the object. This is known as the exploded view of an object.

Note that all drawings in this lesson are on $\frac{1}{4}$ in. graph paper. You will use this type of graph paper for each of your drawings.

Note also that the dimensions in this lesson are in imperial units. Wood products are sold in imperial units, and carpenters for the most part use the imperial system of measurement.

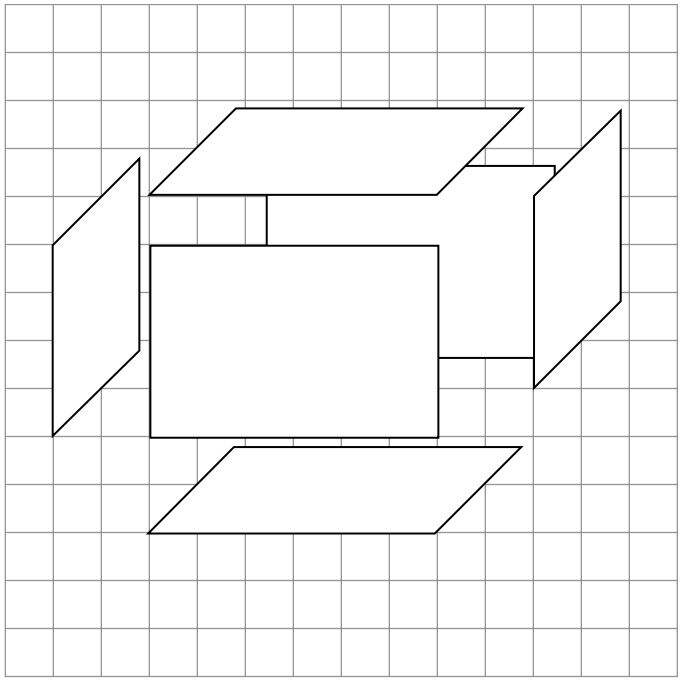
Finally, note that all exploded views in this module are drawn to scale. It is important to draw to scale because, in everyday life, people rely on these types of diagrams to be accurate depictions of the object. Include this in your resource sheet.

Creating 3-D Drawings from Exploded Views

If you are given an exploded view of an object, it is important that you know how to interpret the drawing, and that you can visualize what it would look like if it were put together. The following examples will demonstrate this.

Example 1

The following diagram is an object in exploded format.



a) What type of object is this?

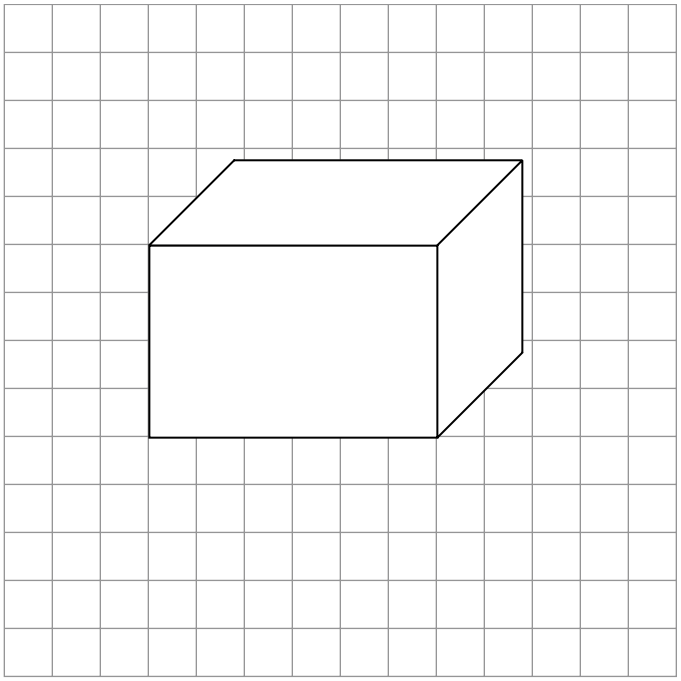
b) Sketch a 3-D version of this object.

c) If the exploded view is drawn to a scale of 1 in.: 1 ft., find the actual dimensions of the object.

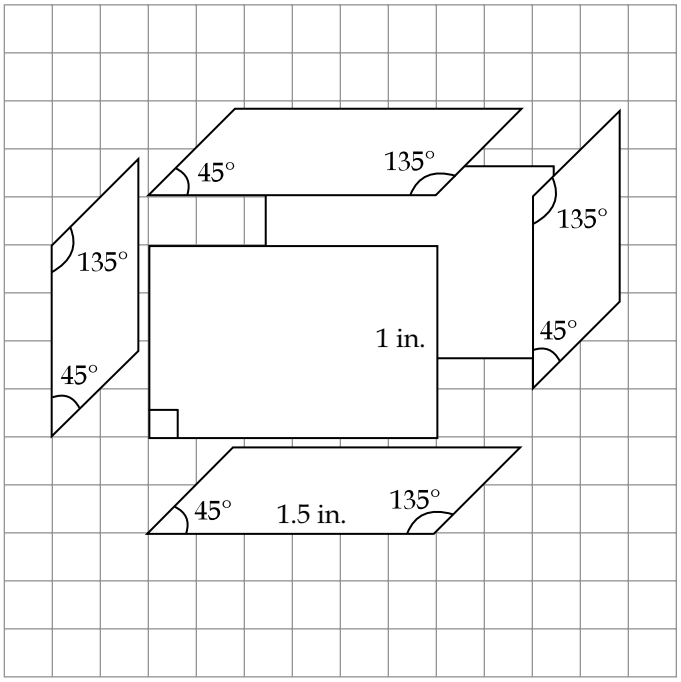
Solution

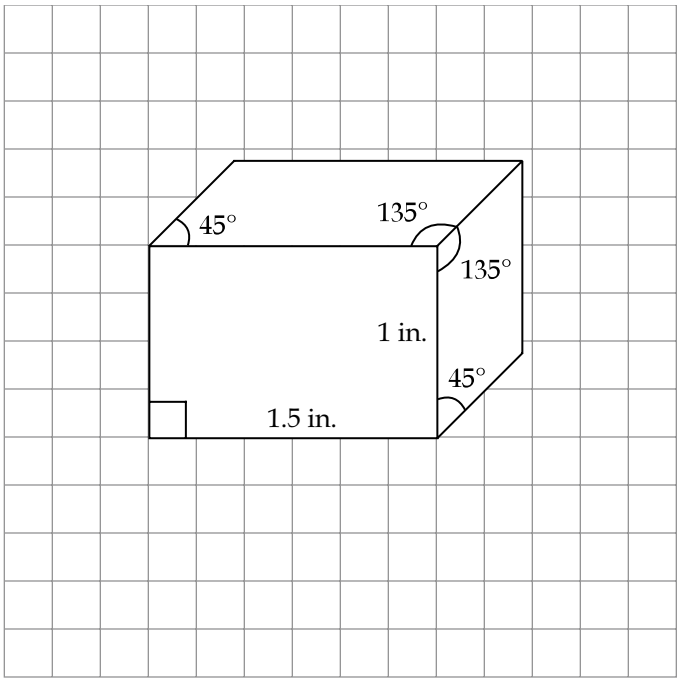
a) This object is a rectangular solid.

b) In order to sketch this object, you put the sides of the rectangular solid together as follows.



Be sure that the all the dimensions and the angle of the sides are the same as in the exploded diagram.





You should note this likeness in your resource sheet as a reminder to yourself.

c) Using the imperial scale of your ruler, measure the front surface of the rectangular solid. The front surface is $1\frac{1}{2}$ in. $\times$ 1 in.

Using the ruler, measure the third dimension (depth). The third dimension measures $\frac{5}{8}$ in. This type of sketch is called an **oblique projection**. The depth measurement in the drawing is one-half the actual measure, so the depth of the actual solid is $\frac{5}{8}$ in. $\times 2 = 1\frac{1}{4}$ in.

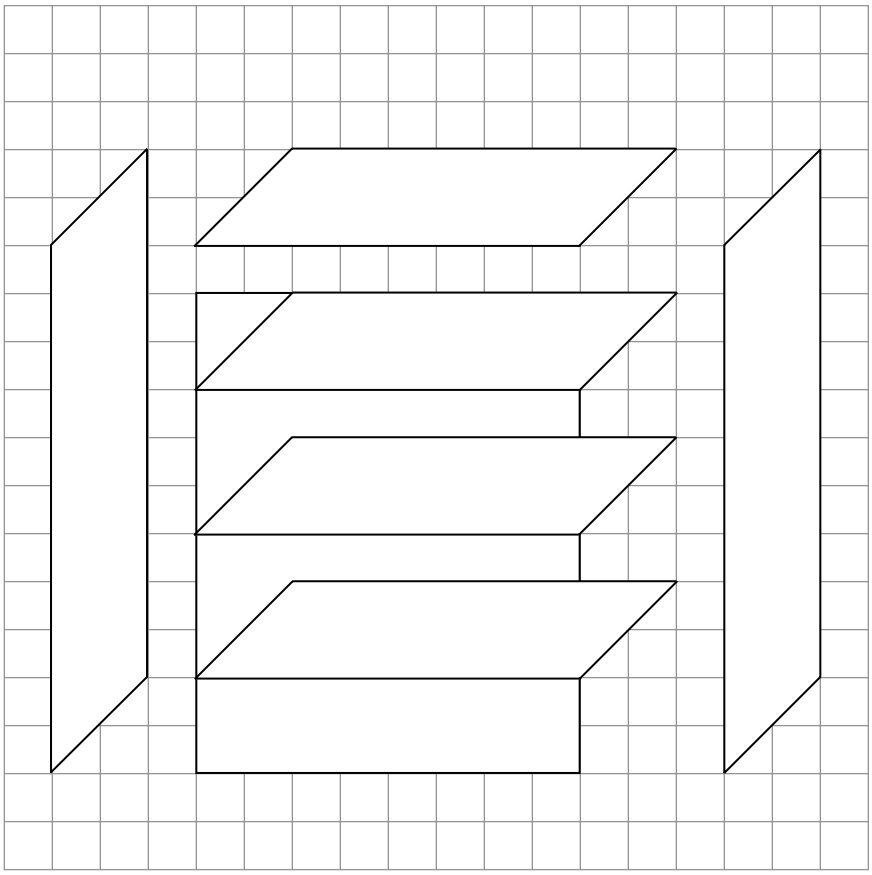
The dimensions in the drawing are $1\frac{1}{2}$ in. $\times$ 1 in. $\times 1\frac{1}{4}$ in.

Since the scale is 1 in. : 1 ft., the actual dimensions of the rectangular solid are $1\frac{1}{2}$ ft. $\times$ 1 ft. $\times 1\frac{1}{4}$ ft.

You should include this trait about depth of oblique projections in your resource sheet.

Example 2

The following is a diagram of an object in exploded format.



a) What type of object is this?

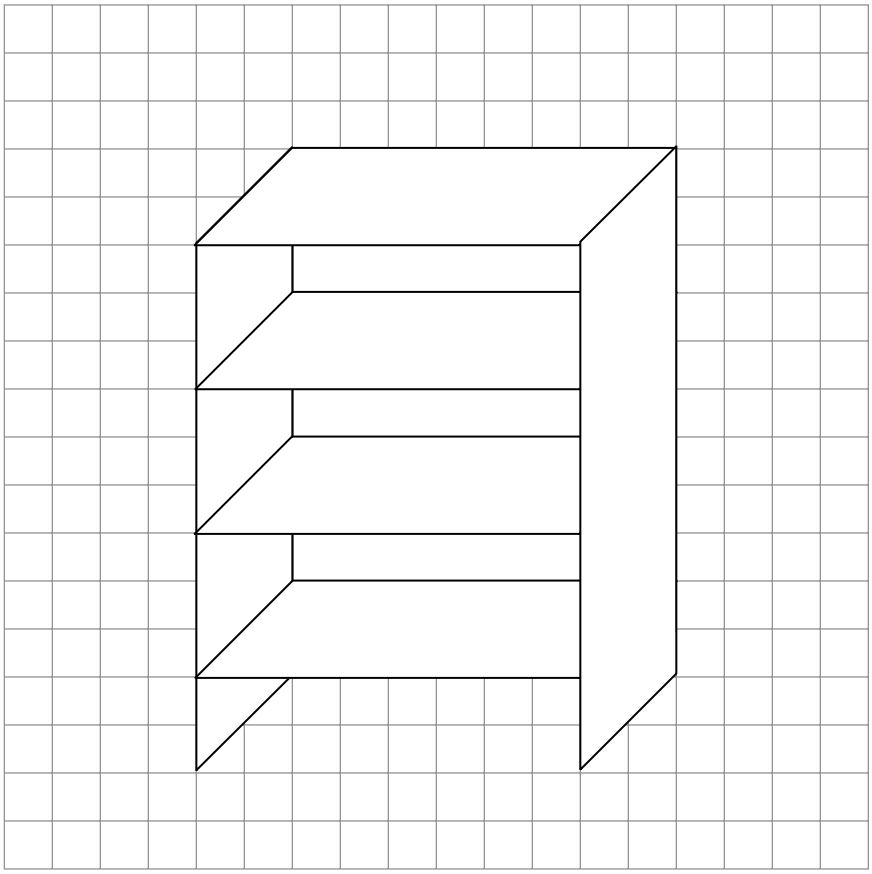
b) Sketch a 3-D version of this object.

c) If the exploded view is drawn to a scale of 1 in.:1.5 ft., find the actual dimensions of the object.

Solution

a) The object is a shelving unit.

b) In order to make a 3-D sketch of this object, you put the pieces of the shelving unit together as follows.



c) Using the imperial scale of your ruler, measure the backboard. The backboard is $2 \text{ in.} \times 2\frac{1}{2} \text{ in.}$

Measure the side panels. The height of each side panel measures $2\frac{3}{4}$ in. The depth of each side panel is $\frac{3}{4}$ in., which is the same as the depth of the shelves. Since in an oblique projection this represents one-half the actual measure, the depth of each shelf is $\frac{3}{4} \text{ in.} \times 2 = 1\frac{1}{2} \text{ in.}$ Measure the shelves. The length of each shelf measures 2 in. The scale dimensions of the bookshelf are as follows:

Backboard: $2 \text{ in.} \times 2\frac{1}{2} \text{ in.}$

Side panels: $1\frac{1}{2} \text{ in.} \times 2\frac{3}{4} \text{ in.}$

Shelves: $2 \text{ in.} \times 1\frac{1}{2} \text{ in.}$

To find the actual dimensions of the solid, use the alternate method and multiply each measure by a scale factor of 1.5 ft.

The actual dimensions of the bookshelf are as follows:

Backboard: 3 ft. × 3.75 ft.

Side panels: 2.25 ft. × 4.13 ft.

Shelves: 3 ft. × 2.25 ft.

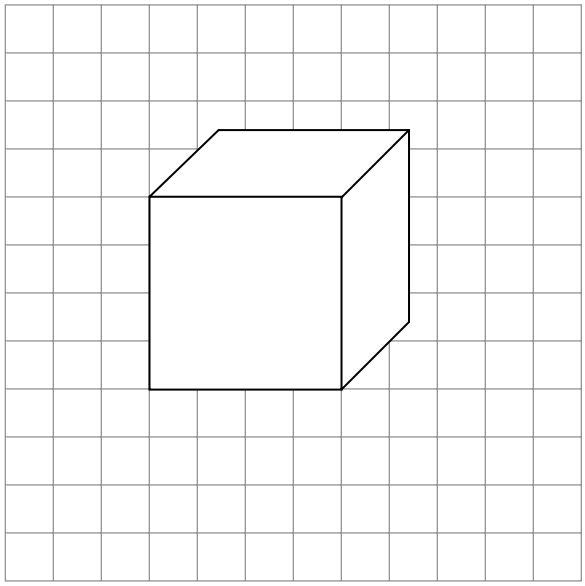
In reality, because of the thickness of the material and how the shelving unit is constructed, some of the pieces will have to be either larger or smaller. However, for the purposes of this course, you do not have to consider the thickness of the construction material. As well, you do not have to consider how the pieces are held together when constructing the object.

Drawing Exploded Views of 3-D Objects

You must also know how to create an exploded view. Each example provides the sketch of an object, and demonstrates how to create an exploded view. In order to draw an object in exploded format, you have to separate it into its components.

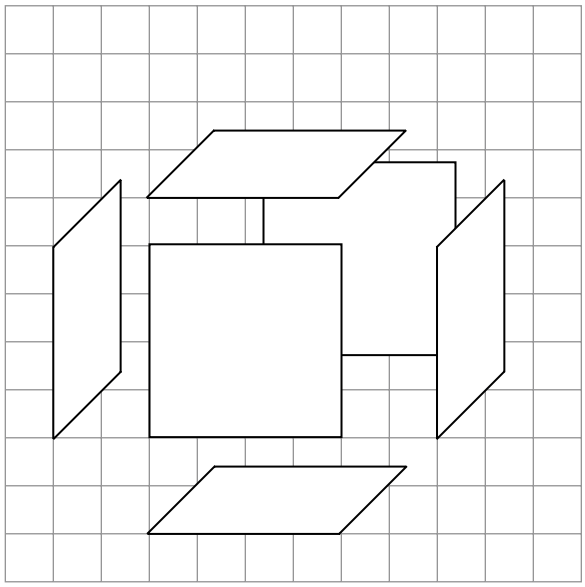
Example 3

Draw an exploded diagram of the following cube.

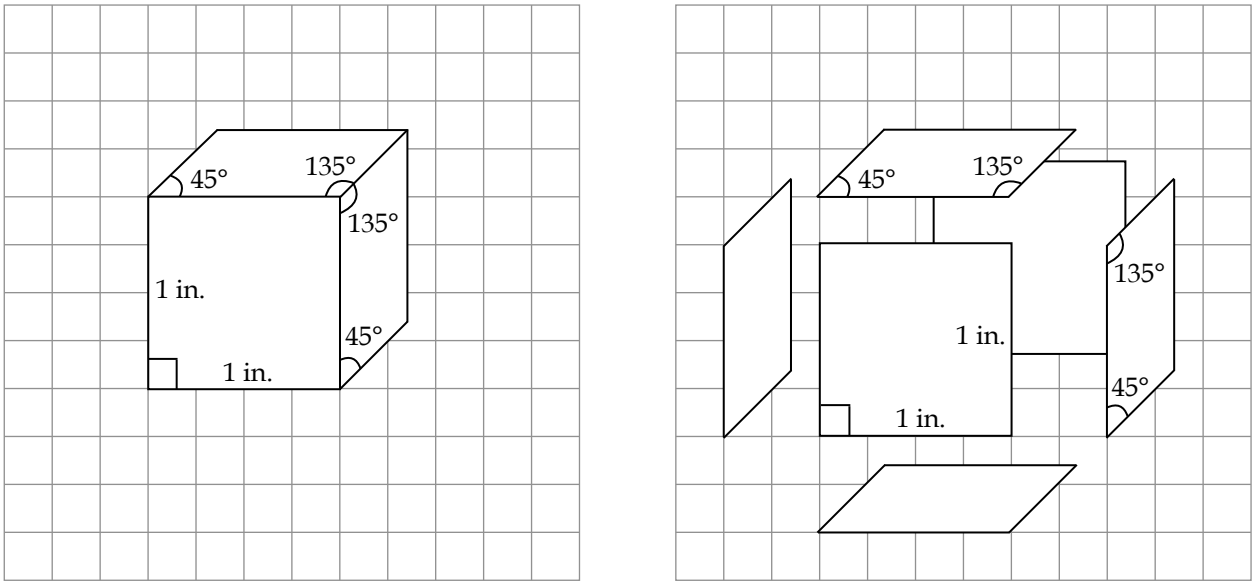


Solution

Separate the sides of the sketch as follows. Note that in an exploded view, all sides are shown.

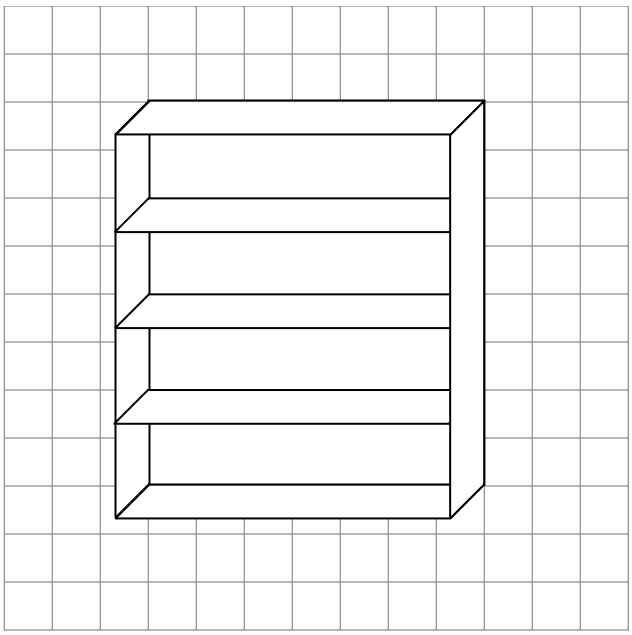


Note also that the dimensions and angles are the same in the exploded view as they were in the original drawing.



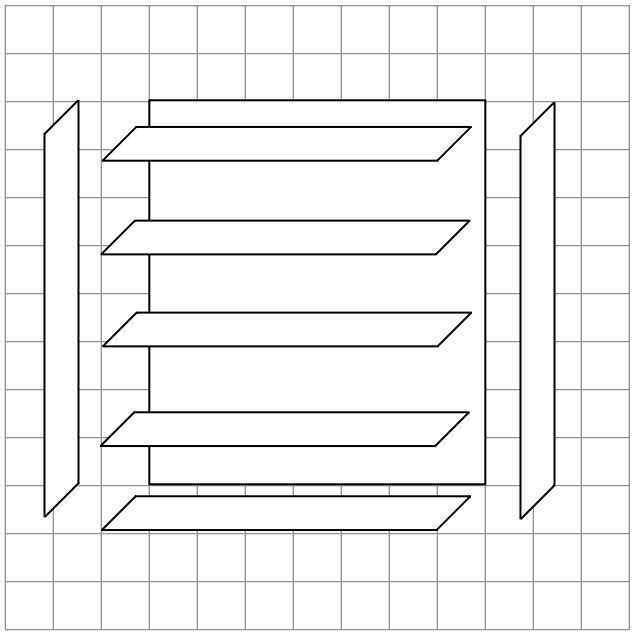
Example 4

Draw the exploded view of the following bookshelf.



Solution

Separate the sides of the sketch as follows.



Be sure to include all parts of the object, including the back of the shelving unit in this case. You may want to make a note of this in your resource sheet to remind yourself to include all parts.

Before you continue, complete the following learning activity. Be sure that you are confident in your ability to draw exploded views of objects. You can do this by checking your answers at the end of the module. Ask your learning partner or tutor/marker if you have any questions.

Learning Activity 7.5

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Gazielle was born in 1998. How old will she be in 2040?
2. What is a third of $\frac{1}{9}$?
3. How many eggs do you have if you have 11 dozen?
4. Evaluate: $75 - 50 - 25 + 25$
5. Gabi paid $72 for a pair of jeans. Chelsea paid $34 less than Gabi. How much did Chelsea pay?

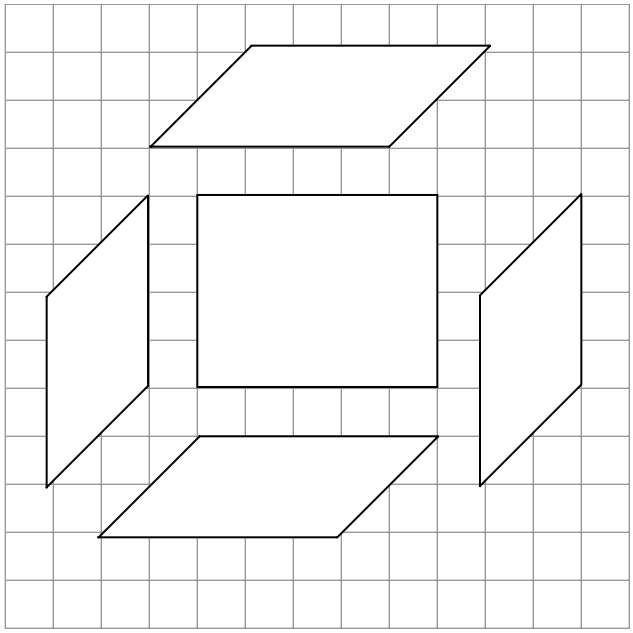
continued

Learning Activity 7.5 (continued)

Part B: Exploded Views

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. The following is the diagram of an object in exploded format.



a) What type of object is this?

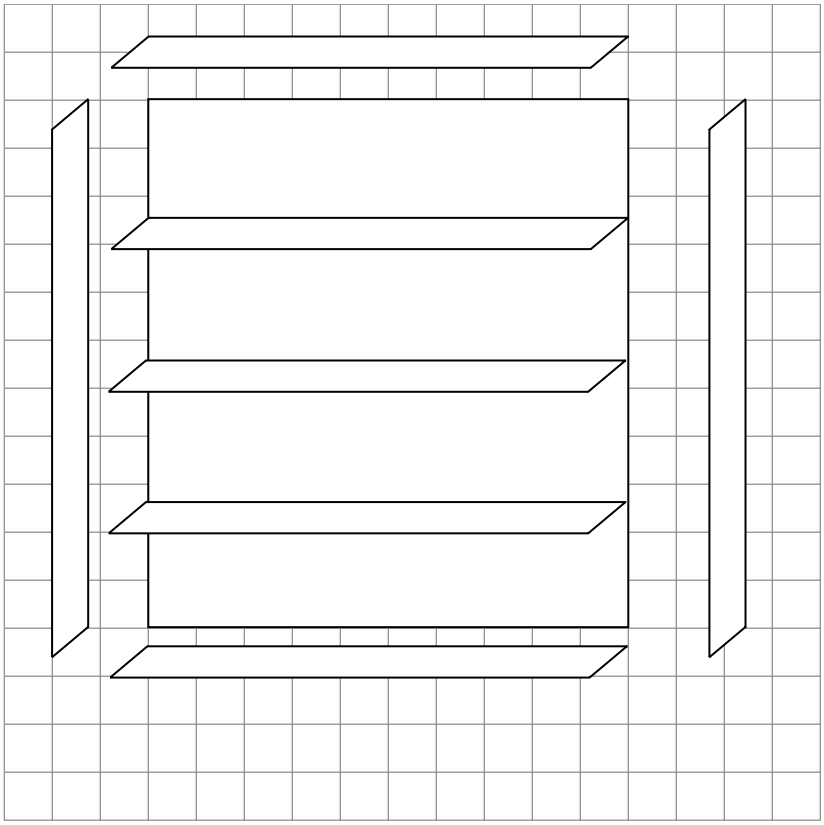
b) Sketch this object in 3-D.

c) Find the dimensions of the object if the scale of the drawing is $1 \text{ in.} : 1\frac{1}{4} \text{ ft.}$

continued

Learning Activity 7.5 (continued)

2. The following is the diagram of a bookshelf in exploded format.



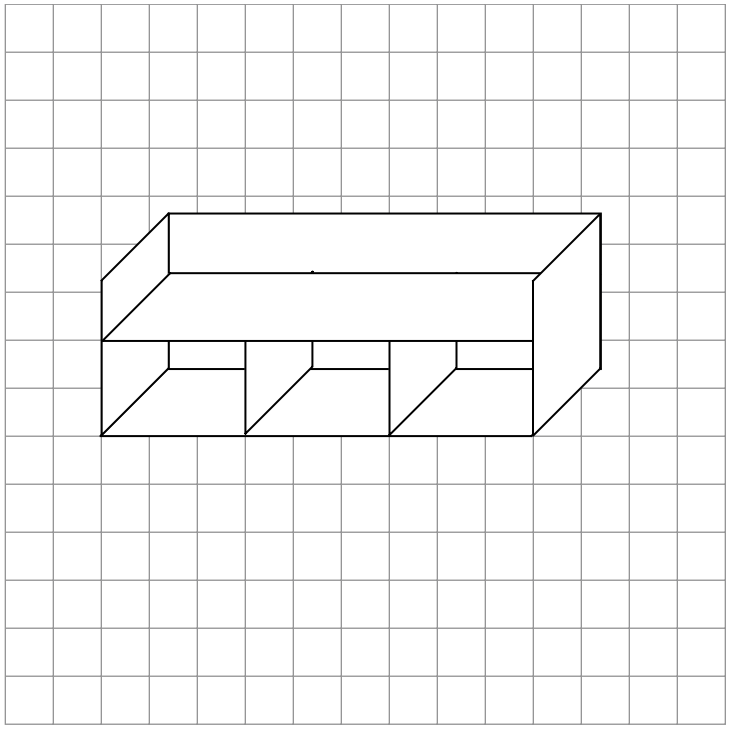
a) Draw a 3-D version of this object.

b) If the exploded view is drawn to a scale of 1 in.:2 ft., find the actual dimensions of the object.

continued

Learning Activity 7.5 (continued)

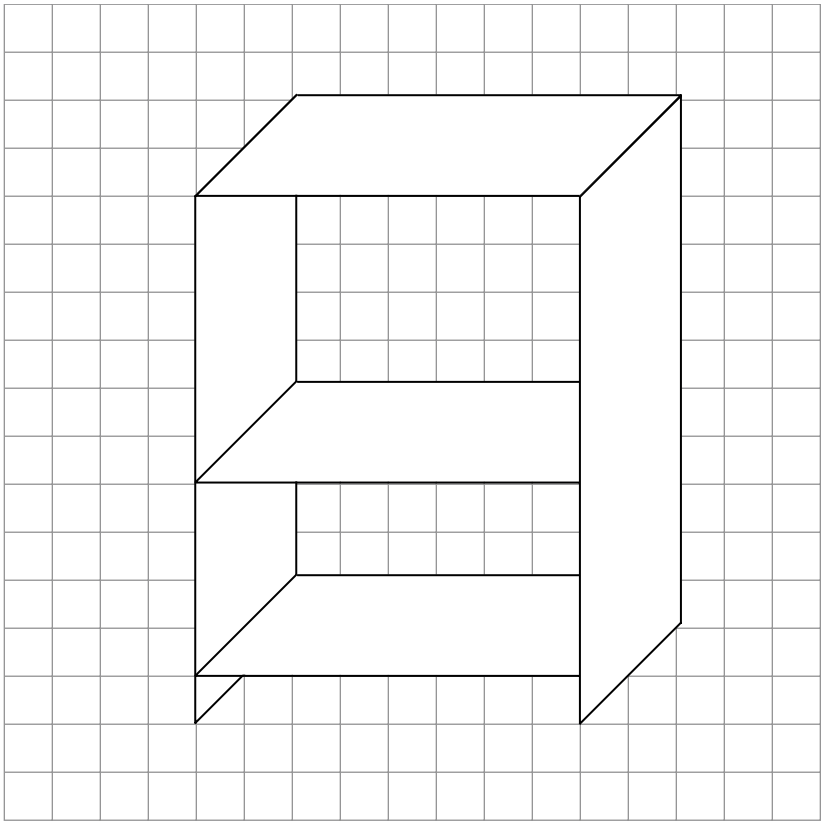
3. Draw an exploded view of the following desk organizer.



continued

Learning Activity 7.5 (continued)

4. Draw an exploded view of the following TV stand.



Component Parts

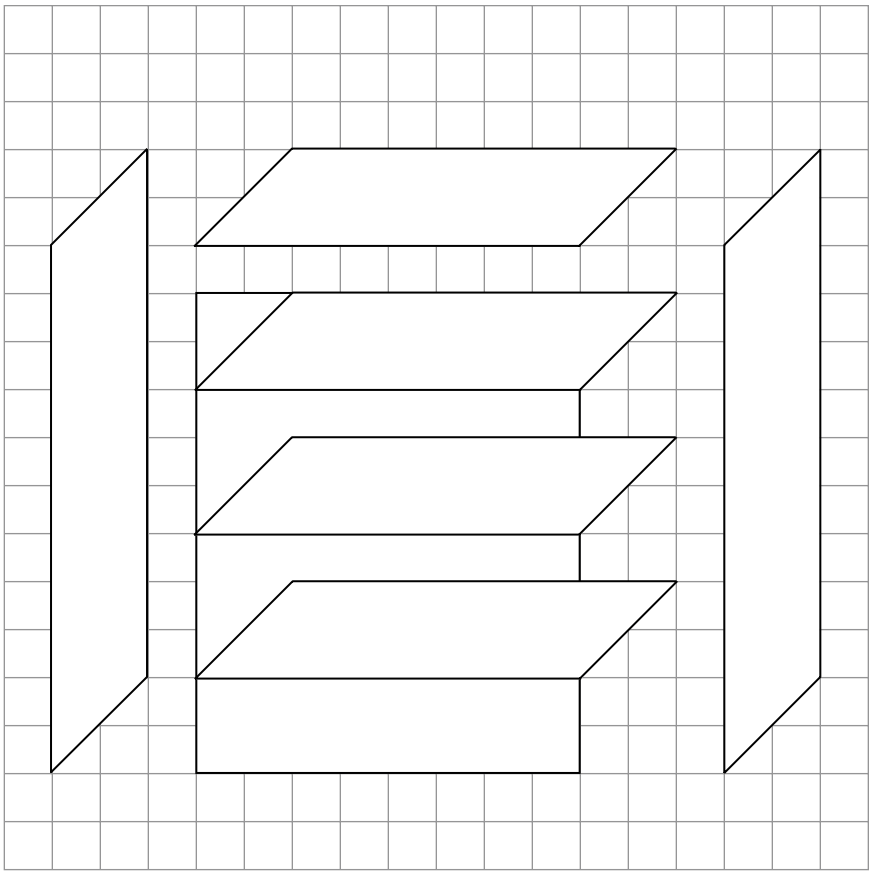
When you draw an object with its component parts, you identify each piece of an object individually (similar to an exploded view). Unlike an exploded view of an object, component parts represent the individual pieces in two dimensions, and do not show the orientation of the pieces with respect to each other. Also, in a component parts diagram, pieces with the same dimensions are only drawn once.

You should note the differences between exploded views and component part diagrams on your resource sheet.

In order to represent an object in its component parts, it is helpful to have it in exploded format.

Consider the following example.

Example 1



Draw the component parts of the shelving unit to scale.

Solution

In order to draw the component parts diagram, you have to draw these pieces to scale. In this case, the exploded view is to scale already, with a scale of 1 in. : 1 ft.

Draw the backboard $2 \text{ in.} \times 2\frac{1}{2} \text{ in.}$

Draw the side panels $1\frac{1}{2} \text{ in.} \times 2\frac{3}{4} \text{ in.}$

Draw the shelves $2 \text{ in.} \times 1\frac{1}{2} \text{ in.}$

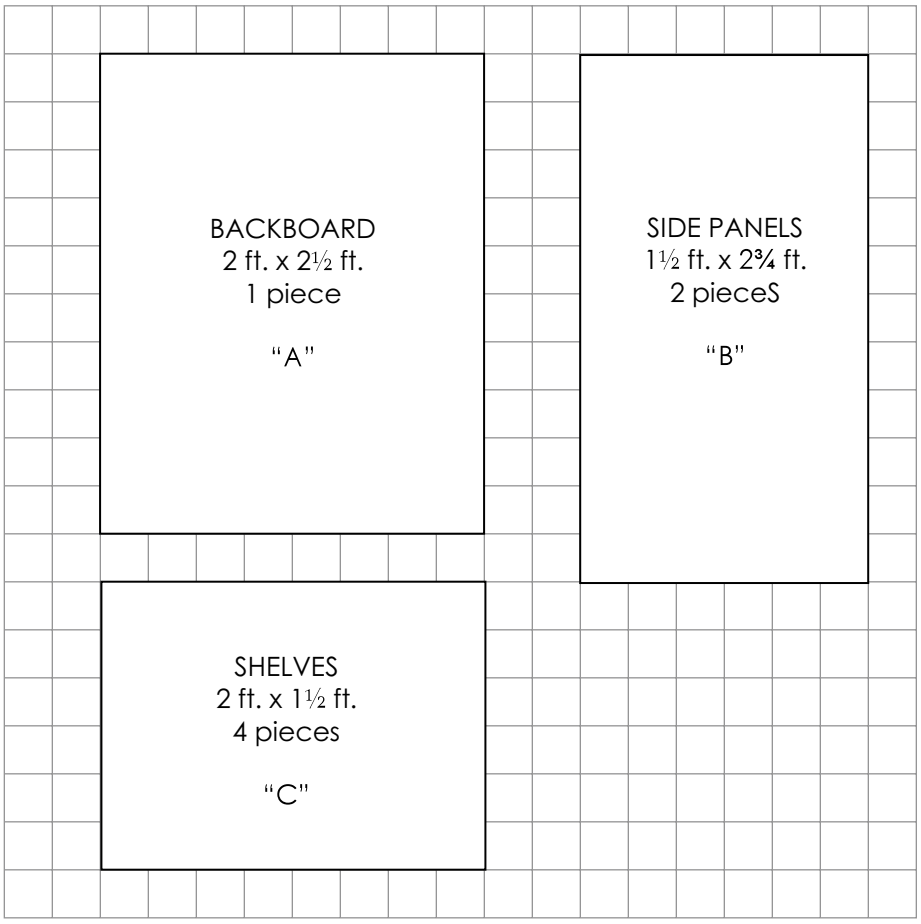
The actual dimensions are:

Backboard: $2 \text{ ft.} \times 2\frac{1}{2} \text{ ft.}$

Side panels: $1\frac{1}{2} \text{ ft.} \times 2\frac{3}{4} \text{ ft.}$

Shelves: $2 \text{ ft.} \times 1\frac{1}{2} \text{ ft.}$

From the exploded view of the shelving unit, you have one backboard, two side panels, and four shelves. Indicate this on the component parts diagram.



Scale 1:12

Note that in a component parts diagram, you have to indicate the following:

- the scale
- the actual dimensions of each component part
- the number of each component part
- what each component is (shelf, sides, back, etc.)

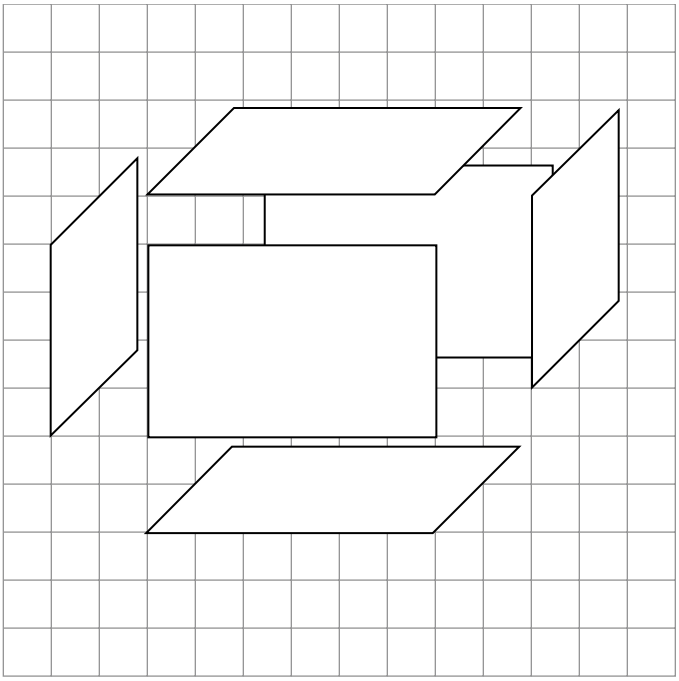
Include these four requirements of a component parts diagram in your resource sheet.

Note again that exploded views are drawn on ¼-inch graph paper. All the questions will use imperial units, and it is quite easy to create a scale drawing using imperial units on ¼-inch graph paper.

The component parts diagram indicates how much material you require to construct an object.

Example 2

Draw the component parts of the rectangular solid to scale.



Solution

In order to draw the component parts diagram, you have to draw these pieces to scale.

Draw the backboard $1\frac{1}{2}$ in. $\times$ 1 in.

Draw the side panels $1\frac{1}{4}$ in. $\times$ 1 in.

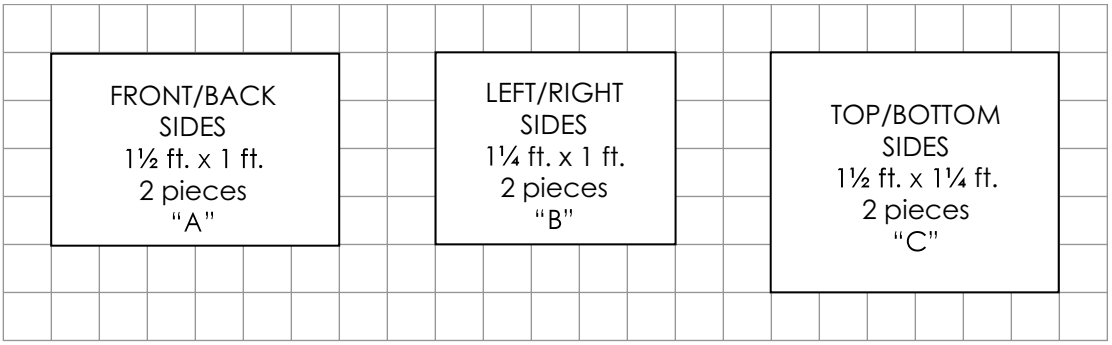
Draw the shelves $1\frac{1}{2}$ in. $\times 1\frac{1}{4}$ in.

If the scale is 1 in.:1 ft., or 1 in.:12 in., the actual dimensions of each piece are:

Front and back panels: $1\frac{1}{2}$ ft. × 1 ft.

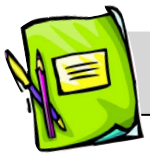
Side panels: $1\frac{1}{4}$ ft. × 1 ft.

Top and bottom panels: $1\frac{1}{2}$ ft. × $1\frac{1}{4}$ ft.



Scale 1:12

Before you attempt the assignment following this lesson, complete the following learning activity. Be sure that you understand what a component part diagram is and how to draw it before you finish this lesson.



Learning Activity 7.6

Complete the following, and check your answers in the learning activity keys found at the end of this module.

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate: $\frac{7}{8} - \frac{3}{16}$

2. Estimate the product: 41 × 49

3. There are 15 students missing from a class of 45. In lowest form, write the fraction representing the missing students in the class.

continued

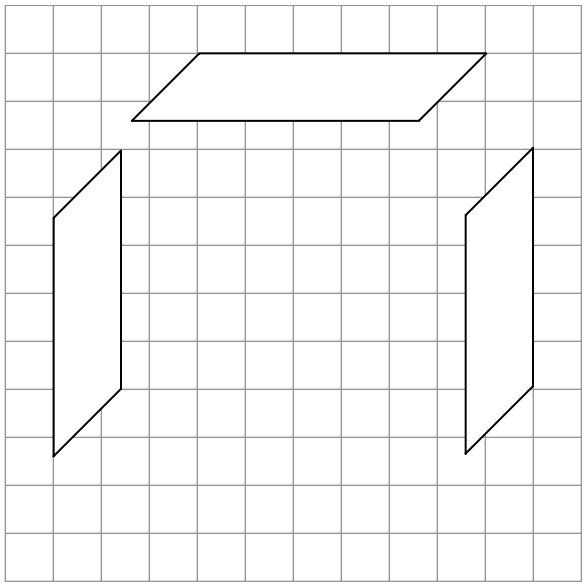
4. How many mm^3 are there in 1 cm^3?

5. The scale of the map is 1 cm : 22 km. If it is 7 cm on the map from your house to the next town, how far do you have to drive?

Part B: Component Parts

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Consider the following exploded view of a desk drawn to a scale of 1 in. : 2 ft.



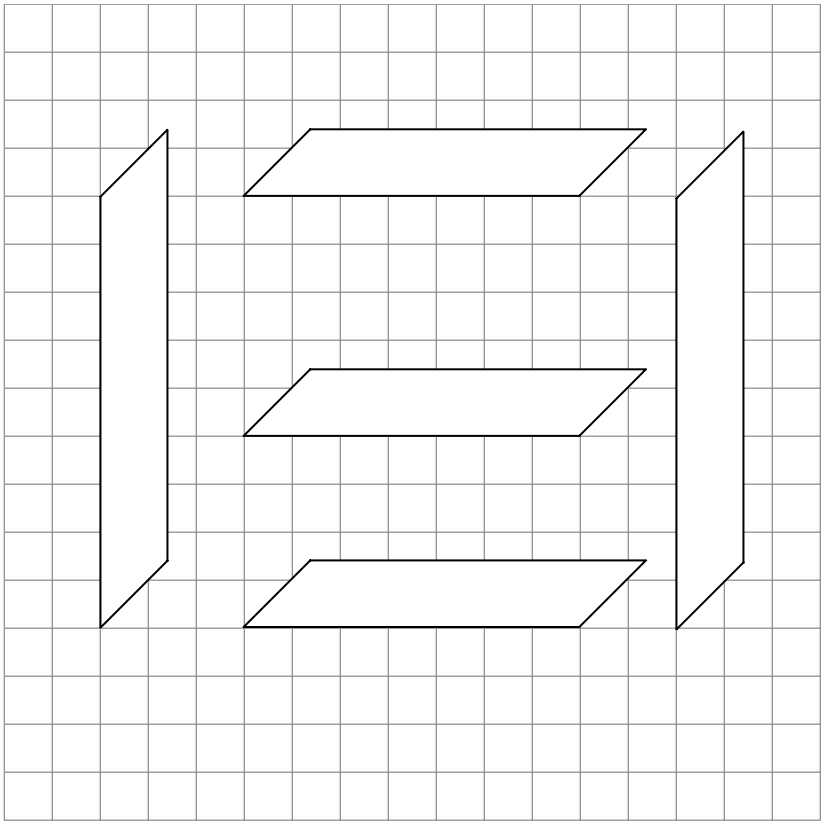
a) Find the actual dimensions of the desk.

b) Draw the component parts of the desk to a scale of 1 in. : 2 ft.

continued

Learning Activity 7.6 (continued)

2. The exploded view of the shelf is reproduced as follows:



a) Calculate the actual dimensions of the desk. The scale is 1 in. : 2 ft.

b) Draw the component parts of the shelf to scale.

Lesson Summary

In this lesson, you learned how to draw exploded views, and also how to put them together in a drawing. You also learned about component parts and how they relate to exploded views of objects. You have almost finished the course! Complete the following assignment.

Notes

Assignment 7.3

Exploded Views and Component Parts

Total: 16 marks

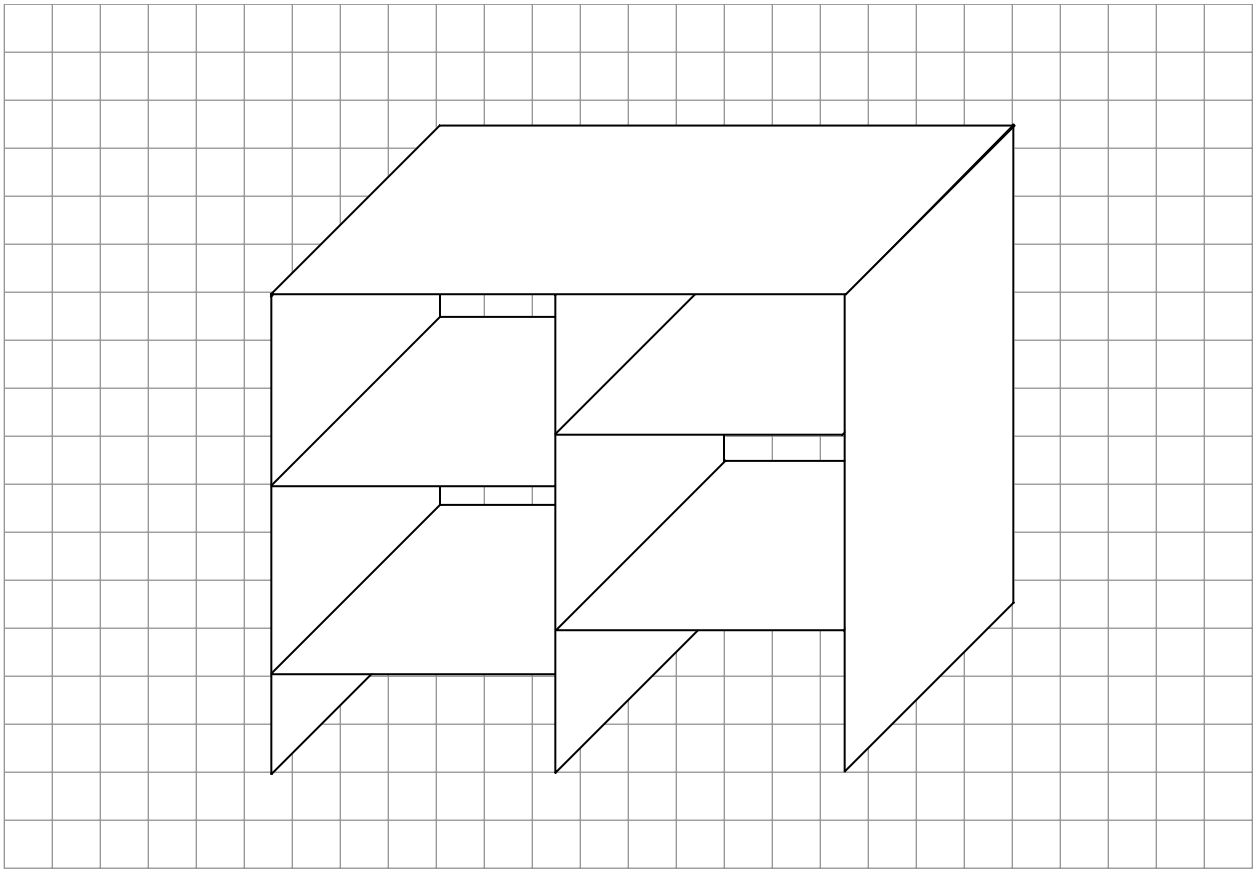
Note to Students: Remember that you can use your resource sheet as you complete this assignment. If any information is missing, go back to the lesson and add it to your resource sheet. You want to be sure that your resource sheet is complete so that when you make your resource sheet for the final examination, you have all the information you need.

1. Explain how exploded views and component parts are alike. *(2 marks)*

continued

Assignment 7.3: Exploded Views and Component Parts (continued)

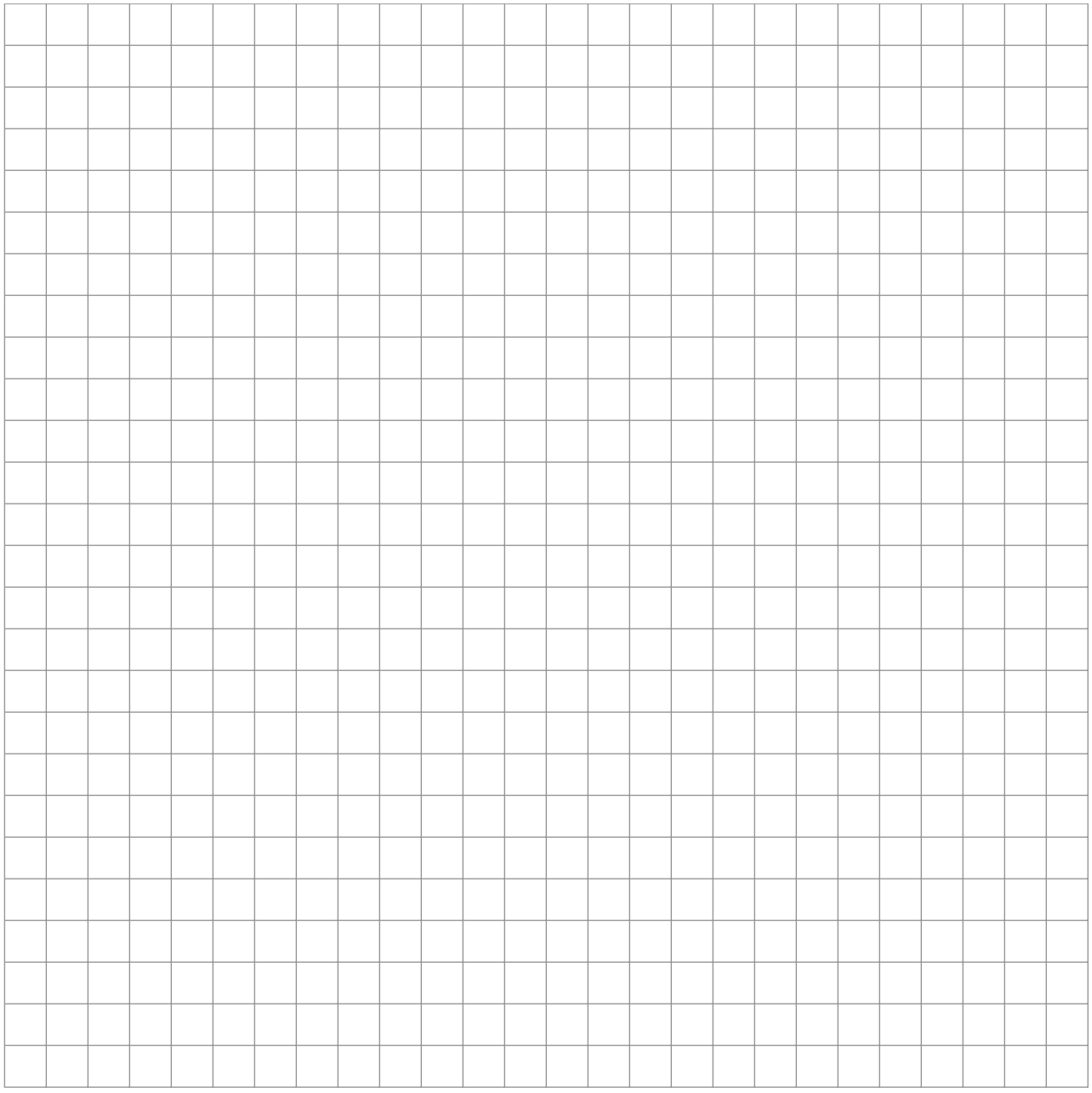
2. Use the diagram below to answer the questions that follow.



continued

Assignment 7.3: Exploded Views and Component Parts (continued)

a) Draw an exploded diagram of the TV stand. *(5 marks)*



continued

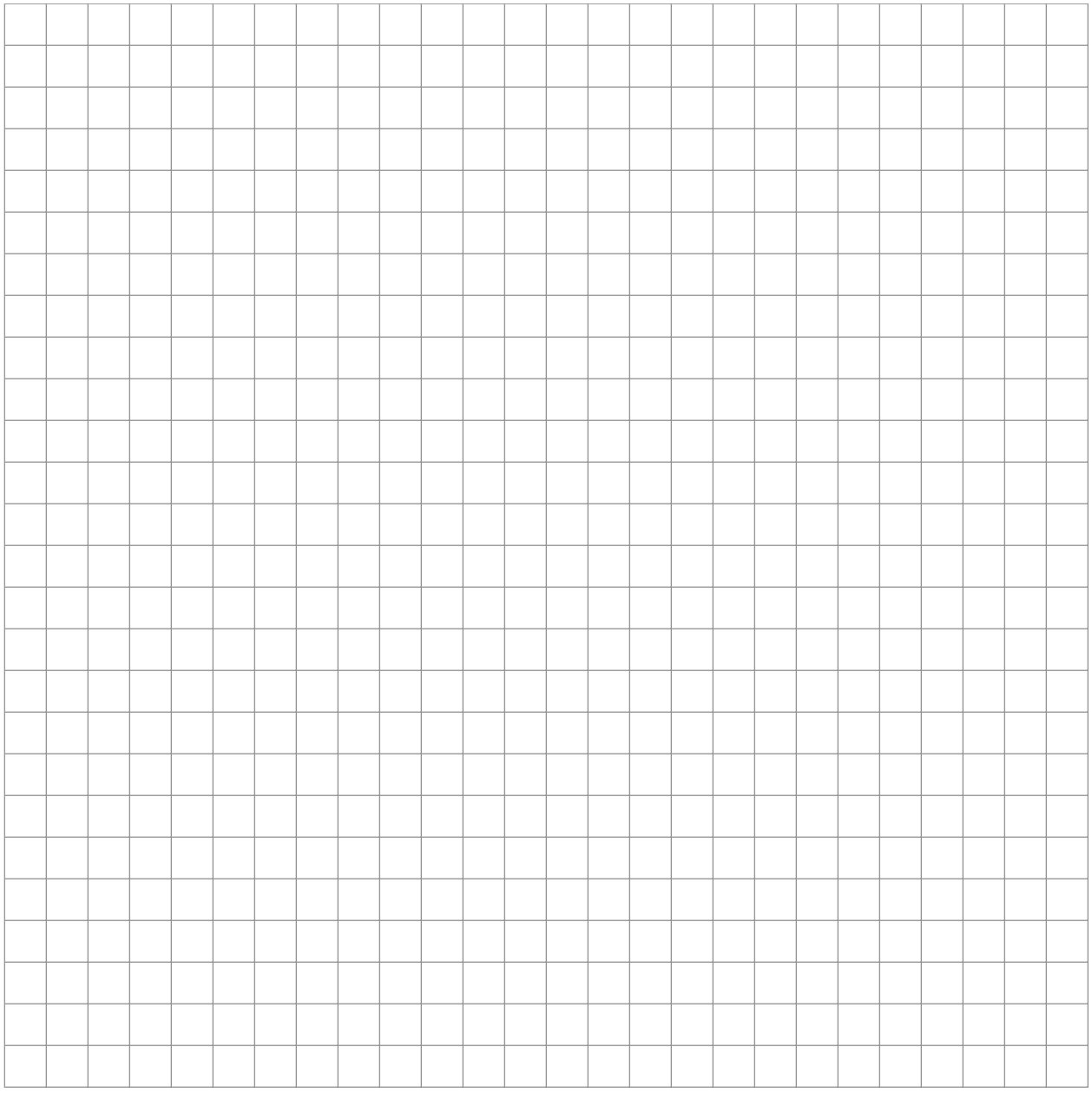
Assignment 7.3: Exploded Views and Component Parts (continued)

b) The scale of the drawing is 1 in. : $\frac{1}{2}$ ft. What are the actual measurements of the pieces of the TV stand? Show your work. *(3 marks)*

continued

Assignment 7.3: Exploded Views and Component Parts (continued)

c) Draw the component parts diagram of the TV stand. *(6 marks)*



Notes

Module 7 Summary

Congratulations! You have now completed all the modules in this course! All you have left to do is to write your final examination.

This module has focused on 3-D drawings. You learned how to use isometric dot paper, and how to draw with a one-point perspective. You also converted between the three 2-D views of a 3-D object and the object itself (drawn and constructed). Exploded views and component parts were the last two ways you learned to represent a 3-D object. Refer to Appendix B: Glossary found at the end of the course if you need help with definitions.

All of the methods you have learned in this module can be applied in a number of fields outside of mathematics. Art, architecture, construction, and mechanical work are just a few examples.

Submitting Your Assignments

It is now time for you to submit the Module 7 Cover Assignment and Assignments 7.1 to 7.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 7 assignments and organize your material in the following order:

- ❑ Module 7 Cover Sheet (found at the end of the course Introduction)
- ❑ Module 7 Cover Assignment: Four-Colour Problem
- ❑ Assignment 7.1: Three 2-D Views of 3-D Objects
- ❑ Assignment 7.2: One-Point Perspective Drawings
- ❑ Assignment 7.3: Exploded Views and Component Parts

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Final Examination

Congratulations, you have finished Module 7 in the course. The final examination is out of 100 marks and worth 12.5% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 4 to 7.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Final Examination Resource Sheet. You may also bring coloured pencils to draw 2-D and 3-D diagrams. A maximum of 2.5 hours is available to complete your final examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point you will also have to combine your resource sheets from Modules 4 to 7 onto one 8½" × 11" paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you. We suggest that you divide your paper into two quadrants on each side so that each quadrant contains information from one module.

Front

Module 4
Module 5

Back

Module 6
Module 7

Examination Review

You are now ready to begin preparing for your final examination. Please review the content, learning activities, and assignments from Modules 4 to 7.

The final practice examination is also an excellent study aid for reviewing Modules 4 to 7.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the final examination determines 12.5% of your final mark in this course and you will have 2.5 hours to complete the examination.

Final Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The final practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your final practice examination, follow these steps:

1. Study for the final practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 4 to 7 that you found most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 4 to 7, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Final Examination Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following things to the final practice examination: pens/pencils (2 or 3 of each), blank paper, a scientific or graphing calculator, a geometry set (which includes a ruler, a protractor, and a compass), and your Final Examination Resource Sheet. You may also bring coloured pencils to draw 2-D and 3-D diagrams.

7. Write your final practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the examination is 2.5 hours.

8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.

9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Final Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Grade 11 Essential Mathematics (30S)

Module 7
Design Modelling

Learning Activity Answer Keys

Module 7:
Design Modelling

Learning Activity 7.1

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. What is the lowest common multiple of 6 and 9?
2. How many metres are in 15.1 km?
3. Which letter comes next? J F M A M J J A S O N _____
4. Which two terms have the same value? 8, 4, 10, $\frac{12}{4}$, $\frac{20}{5}$, $\frac{8}{3}$
5. Your credit card has a balance of $450. The minimum payment will be $10 or 10% of your balance—whichever is more. How much is your minimum payment?

Answers:

1. 18 (6×3 and 9×2)
2. 15 100 m (15.1×1000)
3. D (for December—tricky, eh?)
4. 4, $\frac{20}{5}$ (They both equal 4.)
5. $45 (10% of 450 is 45, which is more than 10)

Part B: Drawing Cubes and Prisms on Isometric Dot Paper

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

Isometric dot paper is provided at the end of this lesson.

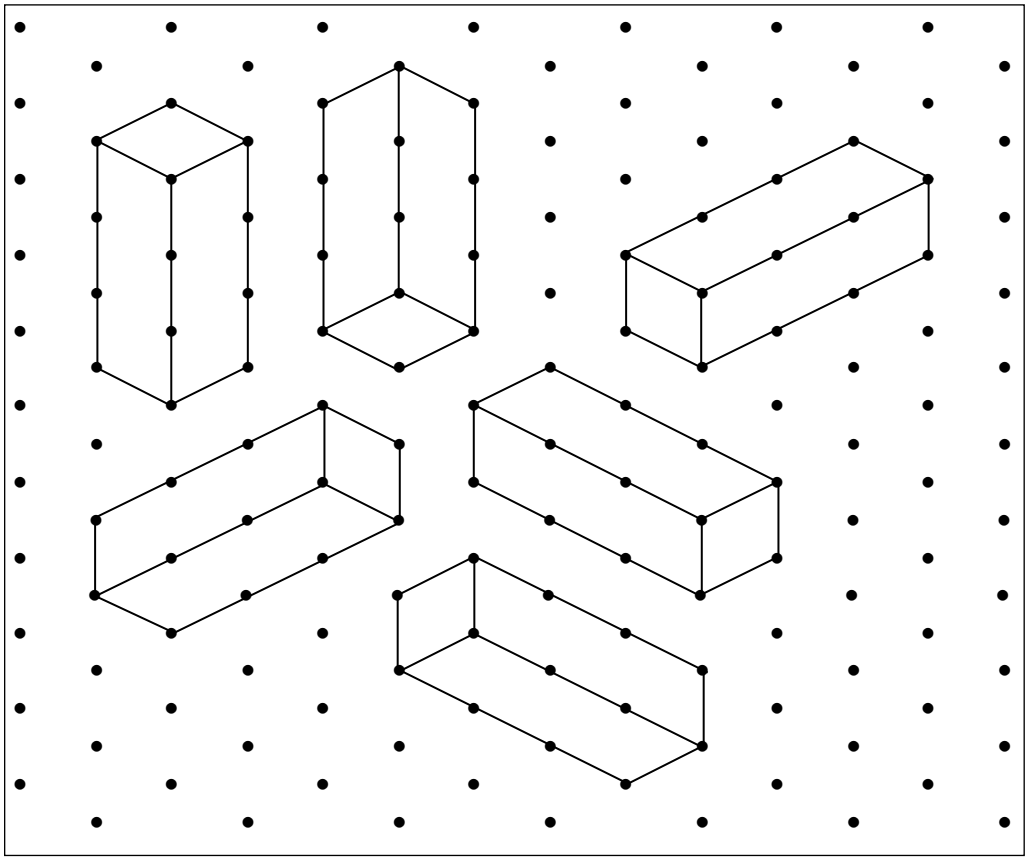
1. Copy the following four figures. Draw the diagrams onto the isometric dot paper provided at the end of this lesson.

   Answer:

   Your drawings should be identical to the drawings given in the question.

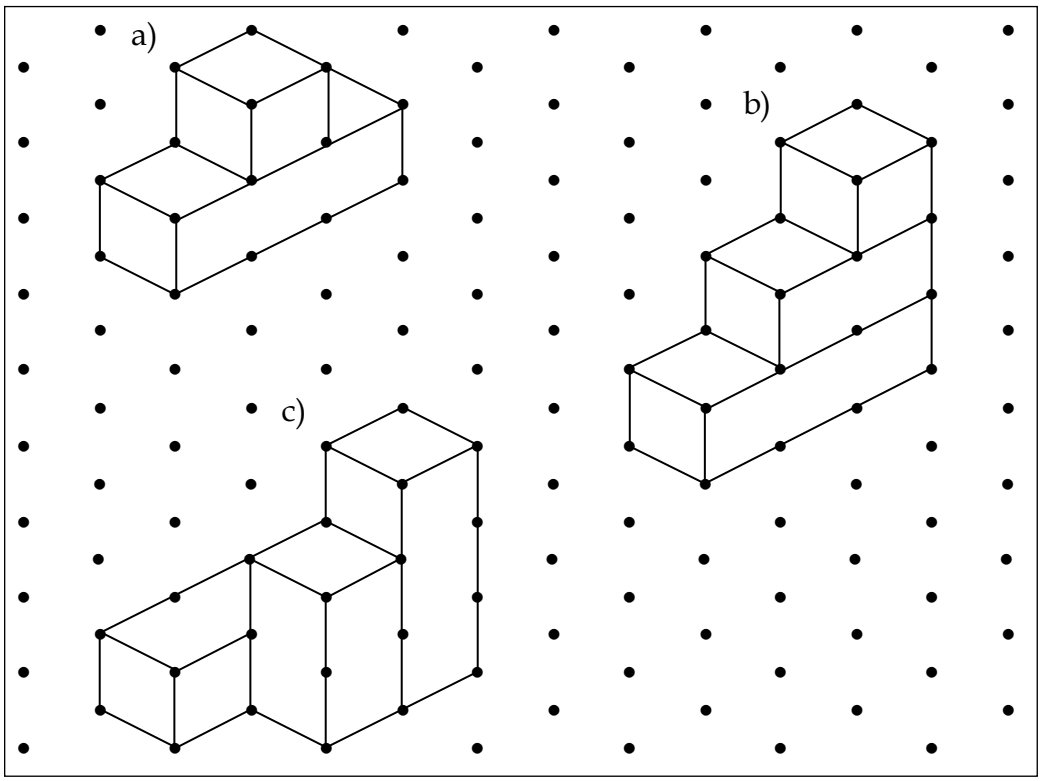
2. There are six ways to represent three cubes in a row on isometric dot paper. Draw the six different representations using the isometric dot paper provided at the end of this lesson.

   Answer:



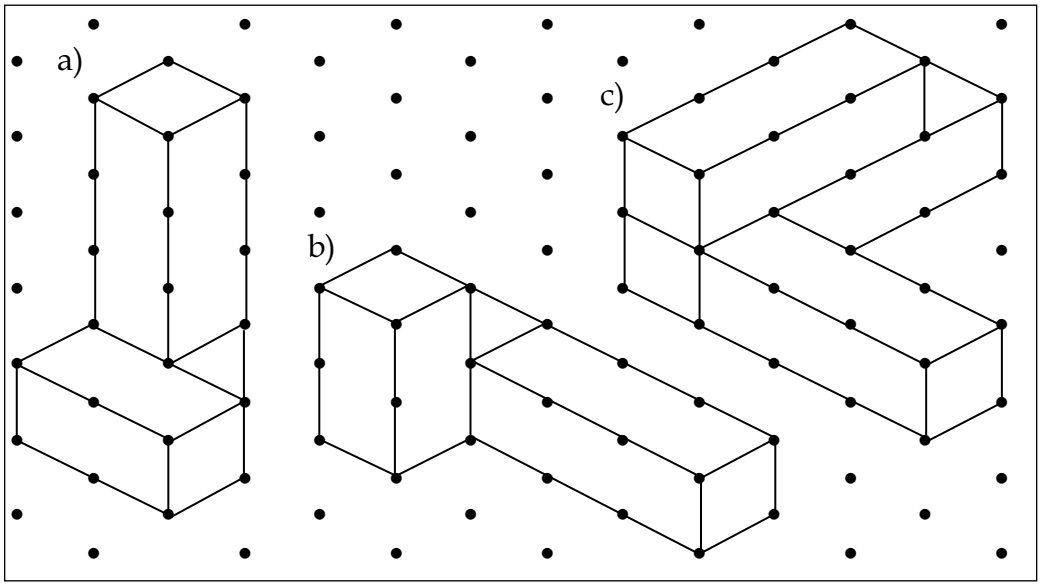
3. Remove the shaded cubes from each of the following three figures, and draw the new figures on isometric dot paper provided at the end of this lesson.

Answer:



4. Add a three-unit-long prism to the shaded faces, and draw the new figures on the isometric dot paper provided at the end of this lesson.

Answer:



Learning Activity 7.2

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate: $\frac{3}{5} + \frac{4}{10}$

2. Solve for k: $4000 = \frac{k}{0.75}$

3. How many 125 g portions of corn are there in a 2 kg bag?

4. The perimeter of a square is 24. What is the area?

5. What is the complimentary angle to 30°?

Answers:

1. $1 \left(\frac{6}{10} + \frac{4}{10} = \frac{10}{10} \right)$

2. $3000 \left(4000 \times 0.75 = 4000 \times \frac{3}{4} = 4000 \div 4 \times 3 = 1000 \times 3 \right)$

3. 16 (2 kg = 2000 g, 125 × 8 = 1000 g, so 16 portions)

4. $36 \left(\frac{24}{4} = 6 \text{ (side length)}, 6 \times 6 = 36 \text{ units}^2 \right)$

5. 60° (90 − 30 = 60°)

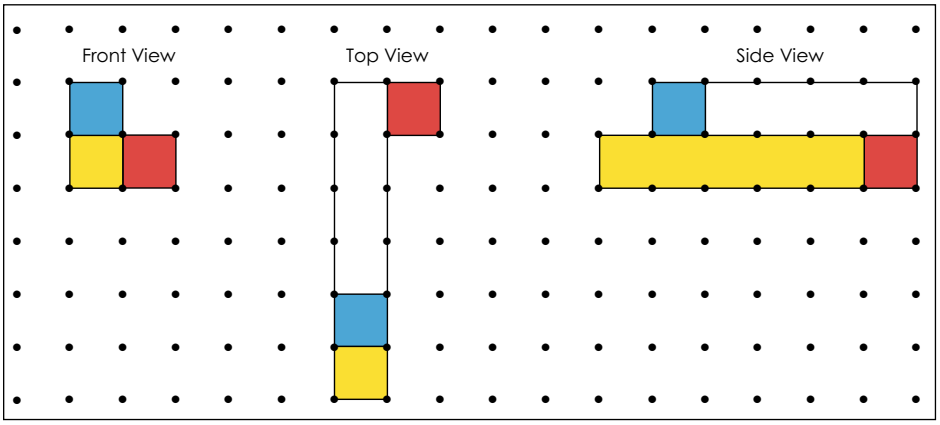
Part B: Drawing a 3-D Object

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

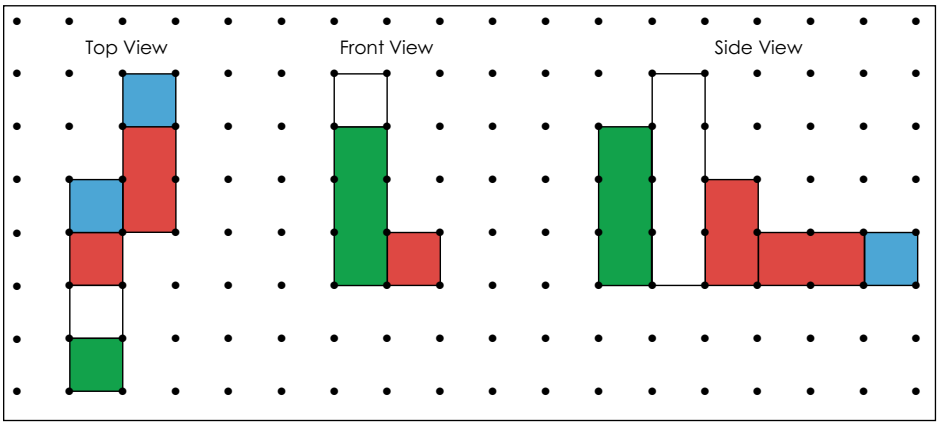
1. Draw the three 2-D views (front, top, and side) of the following objects. Use the square dot paper provided at the end of this lesson.

   Answers:

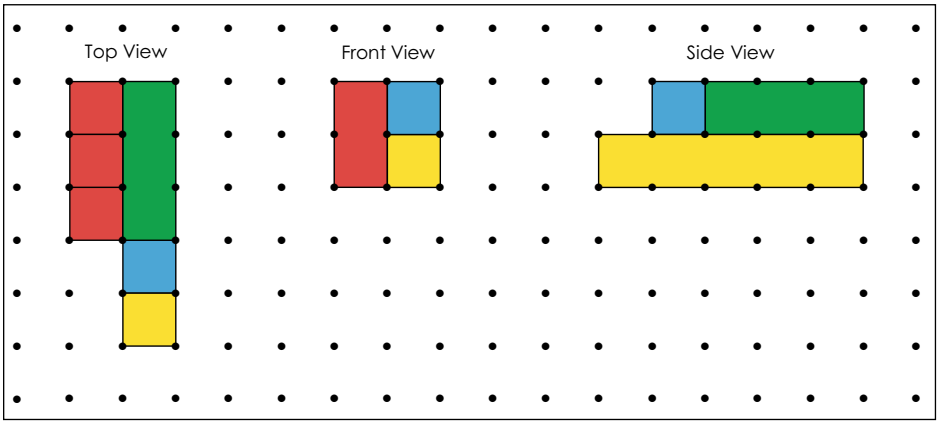
a)



b)



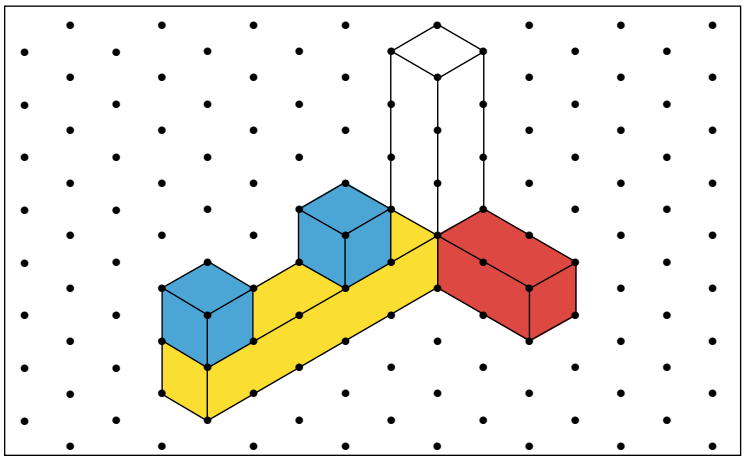
c)



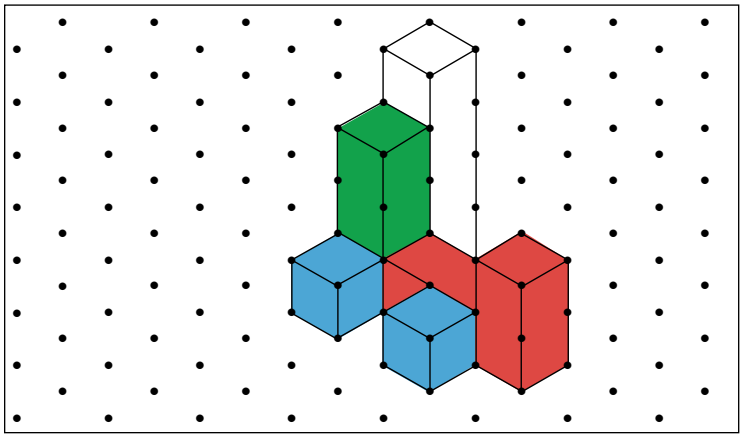
2. Draw the 3-D representation of the object with the given three views. Use the isometric dot paper provided at the end of this lesson.

Answers:

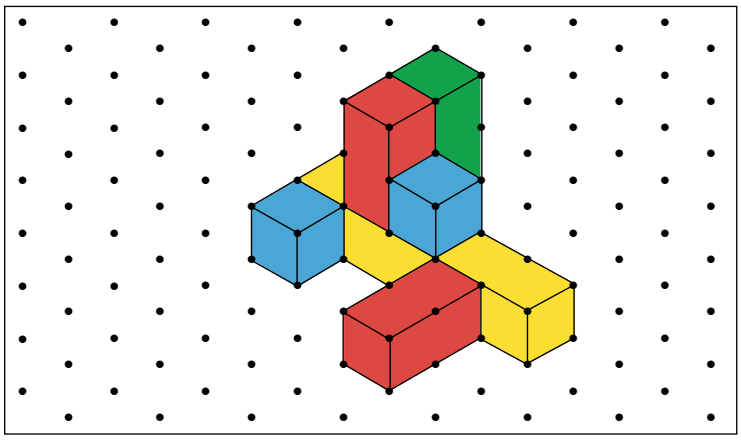
a)



b)



c)



Learning Activity 7.3

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If seven oranges cost $1.82, how much does each orange cost?
2. What is the average of the following numbers: 18, 19, 21, 22
3. Carmen spent 50% of her allowance for the month on a pair of shoes. Of her remaining money, she puts 50% in her bank account. If her allowance is $300, how much money does she have left?
4. A movie theatre can accommodate 350 people. If there are three showings of Parry Halfoy at the same time and all three theatres are full, how many people are seeing the movie?
5. Daris completed an average of 50 math questions per day for seven days before his examination. How many questions did he complete in total over the seven days?

Answers:

1. $\$0.26 \left(\frac{1.82}{7} = \frac{(1.4 + 0.42)}{7} = \frac{1.4}{7} + \frac{0.42}{7} = 0.2 + 0.06 = \$0.26 \text{ per orange} \right)$
2. 20 ((18 + 22 + 21 + 19) ÷ 4 or 80 ÷ 4)
3. $75 (30 × 0.5 = 150, 150 × 0.5 = $75 left)
4. 1050 (3 × 350 = 3 × 300 + 3 × 50 = 1050 people)
5. 350 (50 × 7 = 350 questions)

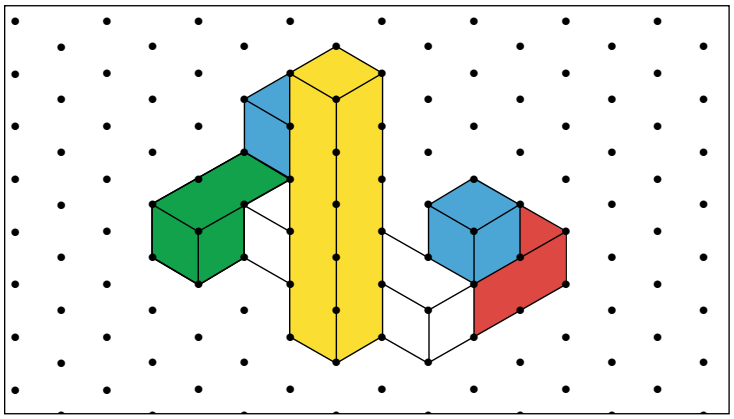
Part B: Constructing Objects from the Three 2-D Drawings

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

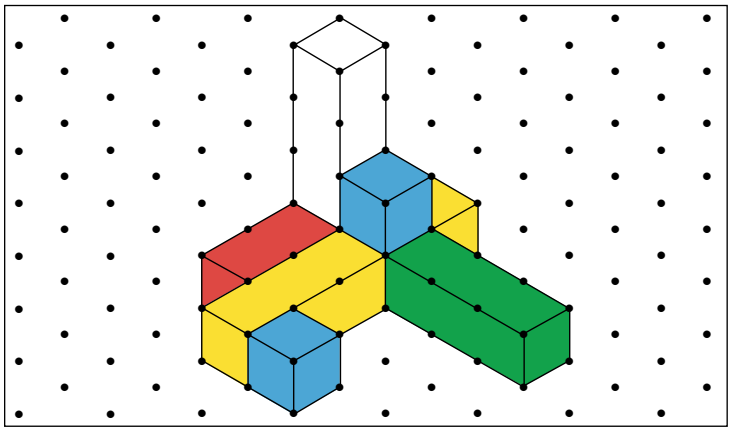
Construct the objects based on the three 2-D drawings provided. Each object includes one yellow bar, one white bar, one green bar, one red bar, and two blue cubes. Use the isometric dot paper provided at the end of this lesson.

Answers:

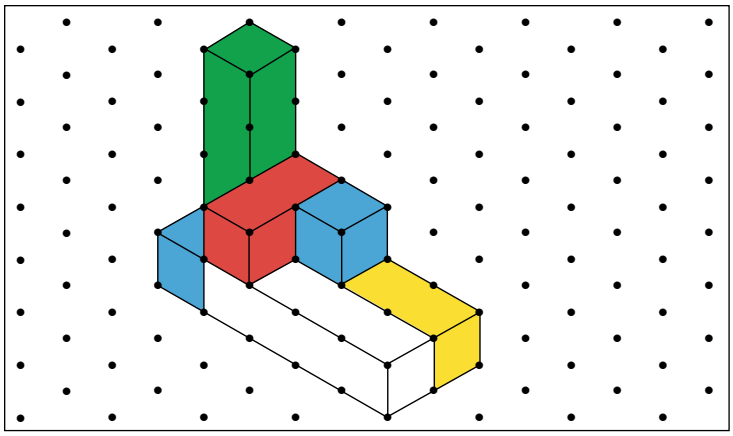
1.



2.



3.



Learning Activity 7.4

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. If you travel 1800 m in 5 min., how fast are you going (in m/s)?
2. Which side is the hypotenuse of a right triangle if the side lengths are 3, 4, 5?
3. What is the measure of the third angle of a triangle if the other two angles are 80° and 60°?
4. 8′5″ = _____ in.
5. What is the total cost of lunch if a sandwich costs $2.25, a salad costs $4.05, and an orange juice costs $1.55?

Answers:

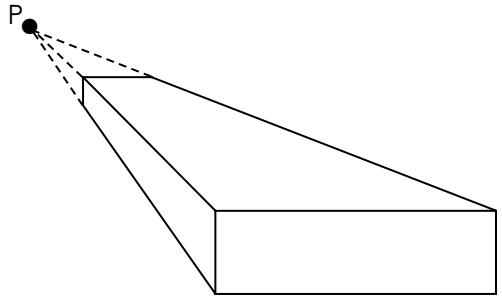
1. $6 \left(5 \text{ min.} = 300 \text{ s}, \frac{1800}{300} = 6 \text{ m/s}\right)$
2. 5 (The hypotenuse is always the longest side.)
3. 40 ($180 - (80 + 60) = 40°$; the sum of the angles in a triangle is 180°)
4. 101 ($8 \times 12 = 96$, $96 + 5 = 101$ in.)
5. $7.85 ($2 + 0.25 + 4 + 0.05 + 1 + 0.55$)

Part B: One-Point Perspective

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

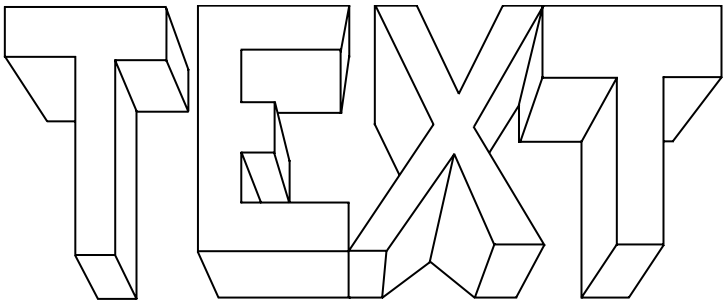
1. Draw a one-point perspective of a rectangular solid that you view from above and to the left.

   Answer:



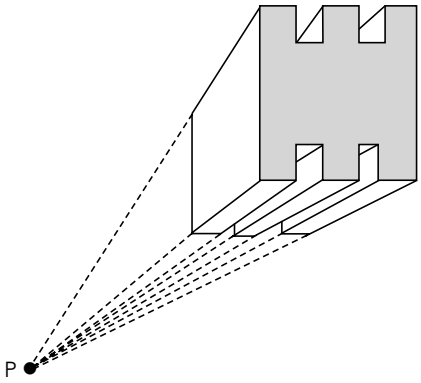
2. Draw a one-point perspective of the word TEXT in block letters that you view from below.

   Answer:

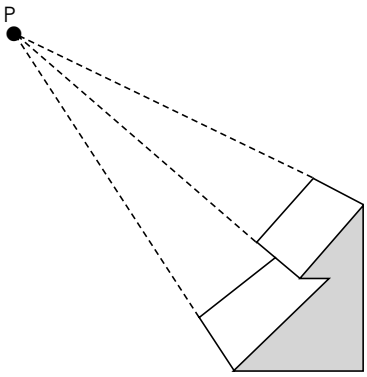


3. Identify where the point of perspective (vanishing point) is for the following drawings.

a)



b)



Answers:

a) The perspective is below and to the left.

b) The perspective is above and to the left.

4. Explain how one-point perspective drawings are different from drawings made using isometric dot paper. You may use drawings to help you explain.

Answer:

When you use isometric dot paper, the height remains constant because the dots are all equally spaced. When you draw a one-point perspective, the height at the front of the object is always larger than the height farther back, because it is closer to the vanishing point. In other words, the one-point perspective drawing is *tapered*.

Learning Activity 7.5

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Gazielle was born in 1998. How old will she be in 2040?
2. What is a third of $\frac{1}{9}$?
3. How many eggs do you have if you have 11 dozen?
4. Evaluate: $75 - 50 - 25 + 25$
5. Gabi paid $72 for a pair of jeans. Chelsea paid $34 less than Gabi. How much did Chelsea pay?

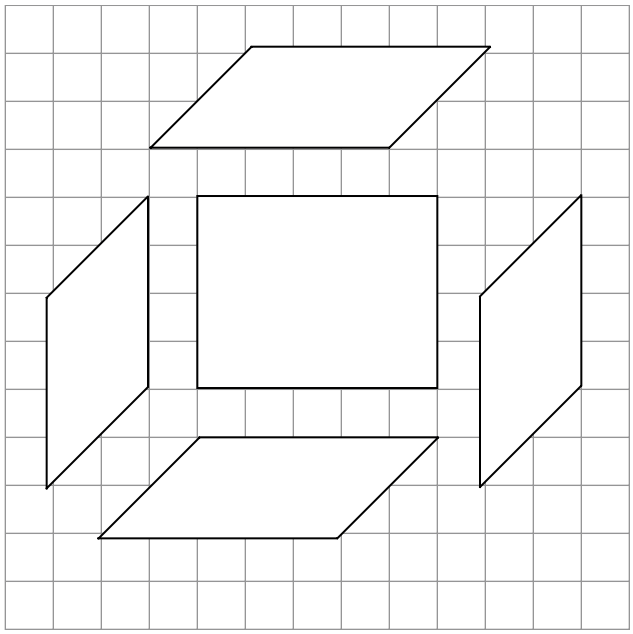
Answers:

1. 42 (1998 is 2 years to the year 2000; add 40 more)
2. $\frac{1}{27}\left(\frac{1}{9} \times \frac{1}{3} = \frac{1}{27}\right)$
3. 132 ($12 \times 11 = 132$ eggs)
4. 25
5. 38 ($72 - 34 = \$38$)

Part B: Exploded Views

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. The following is the diagram of an object in exploded format.



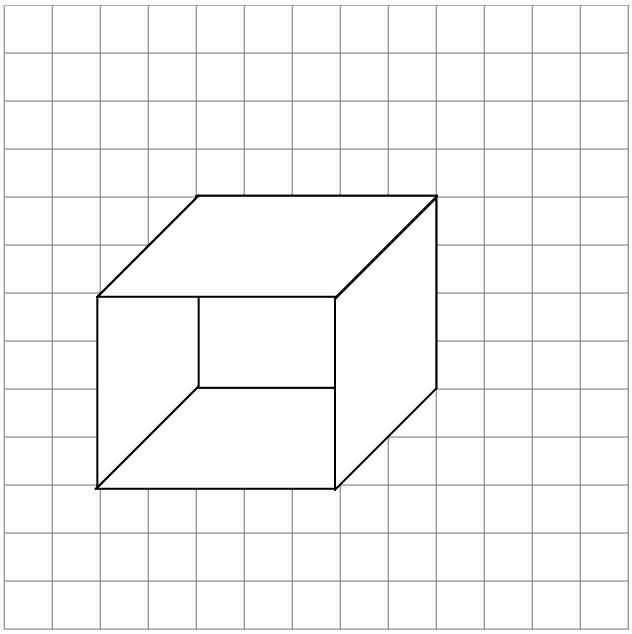
a) What type of object is this?

Answer:

The object is a rectangular solid with the front surface missing.

b) Sketch this object in 3-D.

Answer:



c) Find the dimensions of the object if the scale of the drawing is $1 \text{ in.} : 1\frac{1}{4} \text{ ft.}$

Answer:

Using the imperial scale of your ruler, measure the back surface of the rectangular solid. The back surface is $1\frac{1}{4} \text{ in.} \times 1 \text{ in.}$

Using the ruler, measure the third dimension. The third dimension measures $\frac{3}{4}$ in. Since, in an oblique projection, this represents one-half the measure, the depth of the solid is $\frac{3}{4} \text{ in.} \times 2 = 1\frac{1}{2} \text{ in.}$

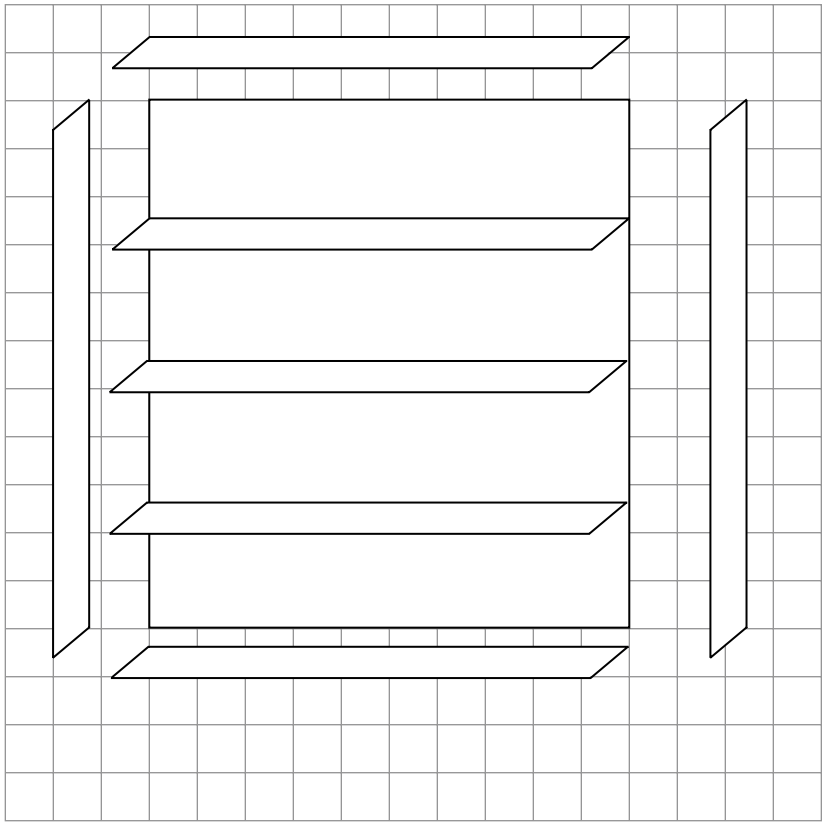
The dimensions of the solid are $1\frac{1}{4} \text{ in.} \times 1 \text{ in.} \times 1\frac{1}{2} \text{ in.}$

The scale is $1 \text{ in.} : 1\frac{1}{4} \text{ ft.}$ which is the same as 1 in. : 1.25 ft. or 15 in.

Multiply each of the dimensions by 1.25 to find the actual dimensions in feet or by 15 to find the actual dimensions of the object in inches.

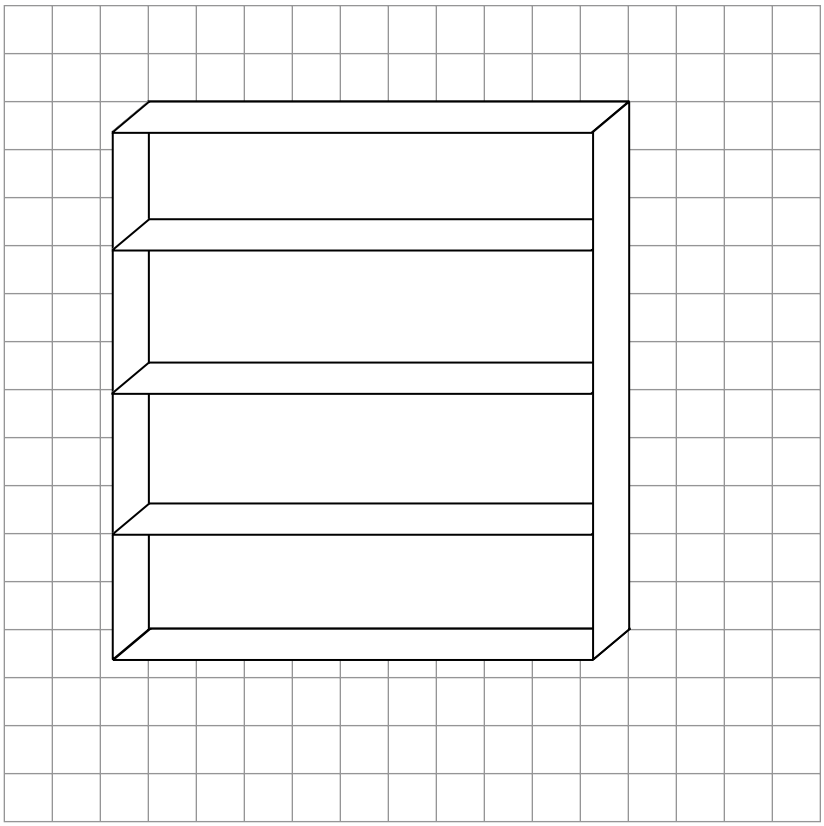
1.56 ft. × 1.25 ft. × 1.88 ft. or 18.75 in. × 15 in. × 22.5 in.

2. The following is the diagram of a bookshelf in exploded format.



a) Draw a 3-D version of this object.

Answer:



b) If the exploded view is drawn to a scale of 1 in.:2 ft., find the actual dimensions of the object.

Answer:

Using the imperial scale of your ruler, measure the backboard. The backboard is $2\frac{1}{2}$ in. $\times 2\frac{3}{4}$ in.

Measure the side panels. The height of each side panel measures $2\frac{3}{4}$ in. The depth of each side panel is $\frac{1}{4}$ in. Since, in an oblique projection, this represents one-half the measure, the depth of each shelf is $\frac{1}{4}$ in. $\times 2 = \frac{1}{2}$ in. Measure the shelves. The length of each shelf measures $2\frac{1}{2}$ in. The depth of each shelf is $\frac{1}{4}$ in. Since, in an oblique projection, this represents one-half the measure, the depth of each shelf is $\frac{1}{4}$ in. $\times 2 = \frac{1}{2}$ in.

The scale dimensions of the bookshelf are as follows:

Backboard: $2\frac{1}{2}$ in. $\times$ $2\frac{3}{4}$ in.

Side panels: $2\frac{3}{4}$ in. $\times$ $\frac{1}{2}$ in.

Shelves: $2\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in.

To find the actual dimensions of the solid, use the alternate method and multiply each measure by a scale factor of 2 ft.

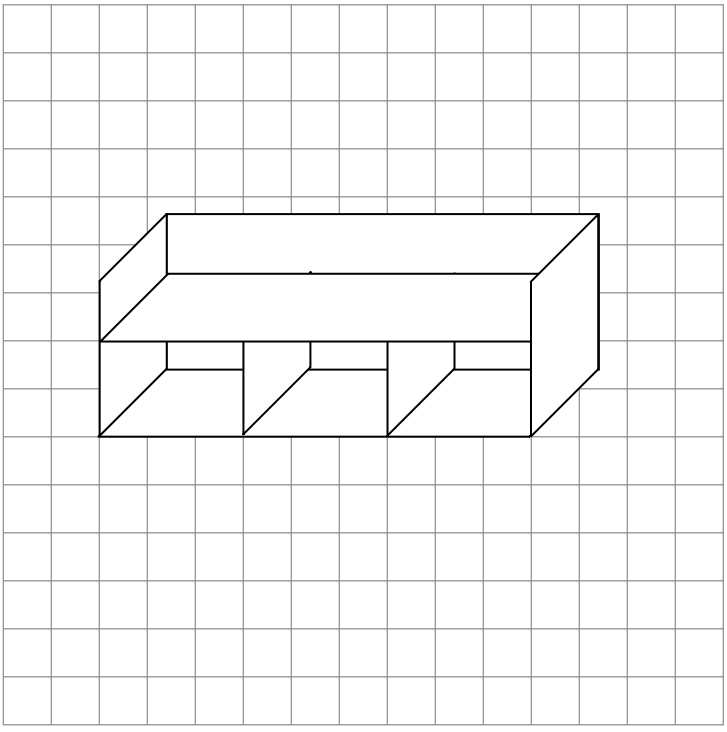
The actual dimensions of the bookshelf are as follows:

Backboard: 5 ft. $\times$ $5\frac{1}{2}$ ft.

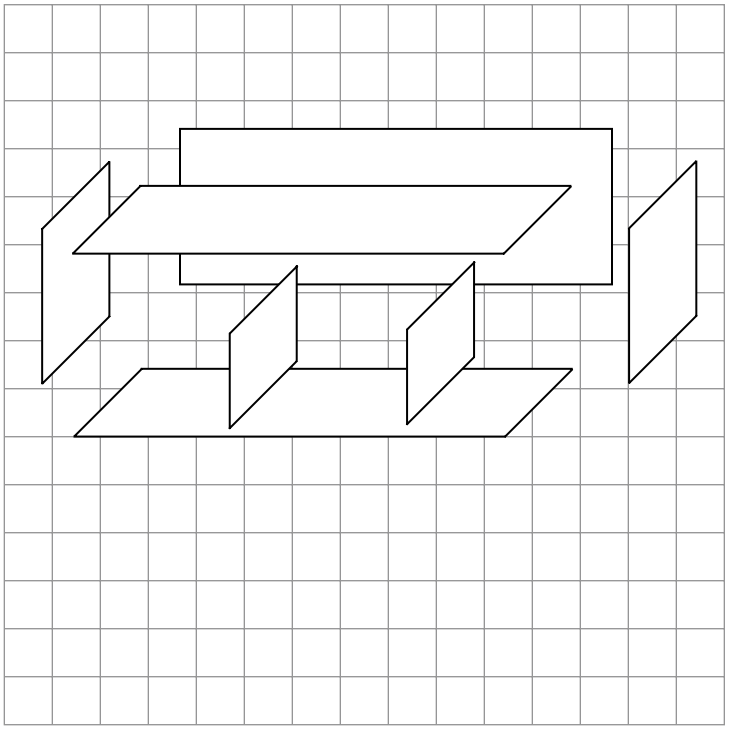
Side panels: $5\frac{1}{2}$ ft. $\times$ 1 ft.

Shelves: 5 ft. $\times$ 1 ft.

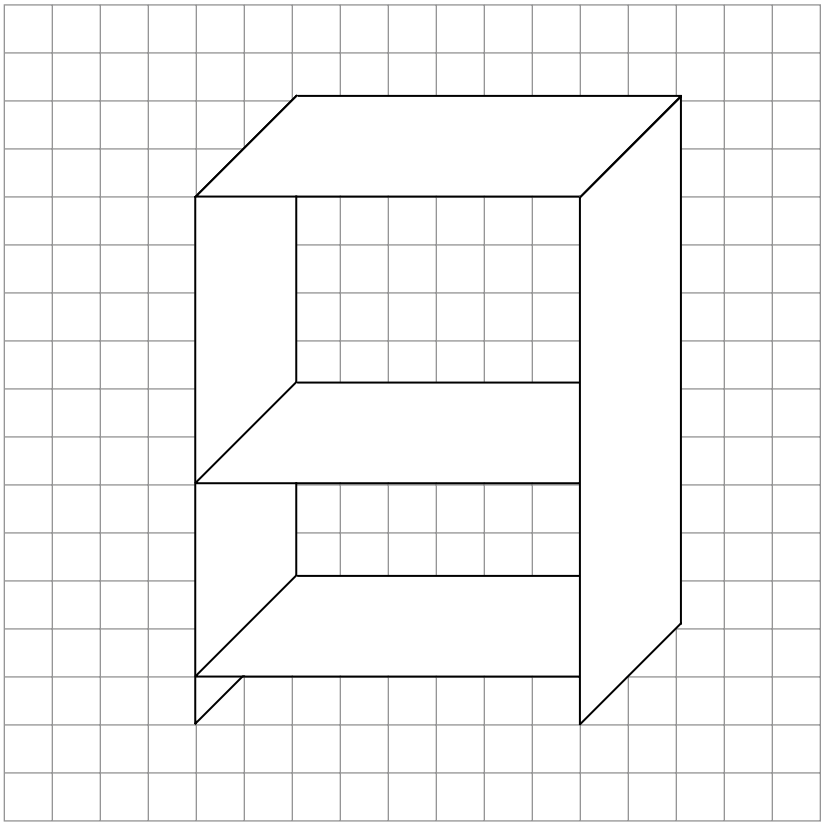
3. Draw an exploded view of the following desk organizer.



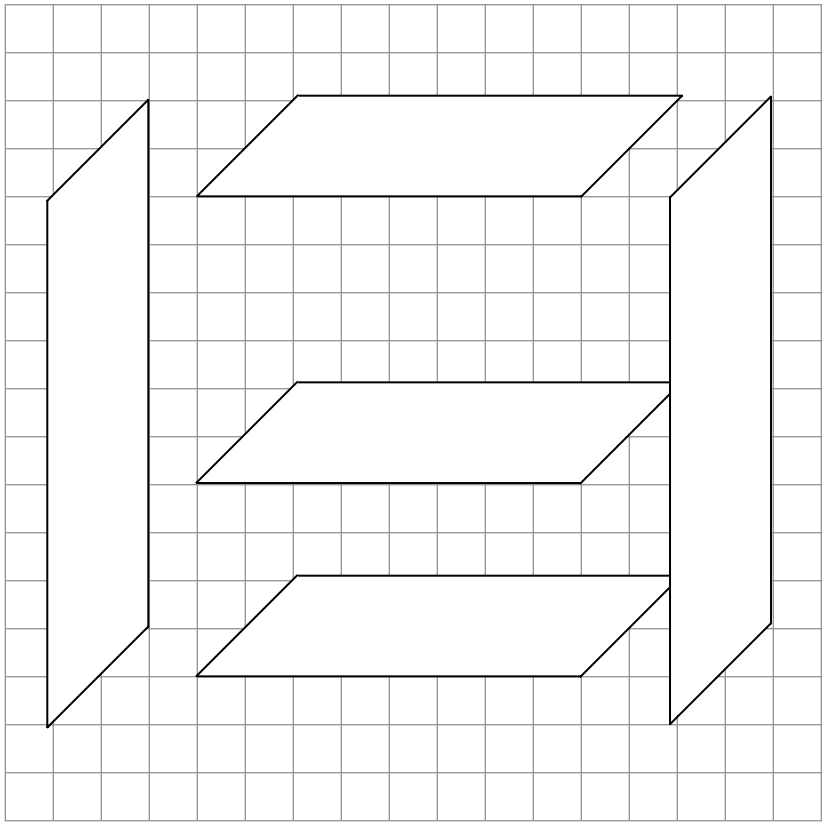
Answer:



4. Draw an exploded view of the following TV stand.



Answer:



Learning Activity 7.6

Part A: BrainPower

You should be able to complete the following five questions in just a few minutes without using a calculator or paper and pencil.

1. Evaluate: $\frac{7}{8} - \frac{3}{16}$
2. Estimate the product: 41×49
3. There are 15 students missing from a class of 45. In lowest form, write the fraction representing the missing students in the class.
4. How many mm^3 are there in 1 cm^3?
5. The scale of the map is 1 cm : 22 km. If it is 7 cm on the map from your house to the next town, how far do you have to drive?

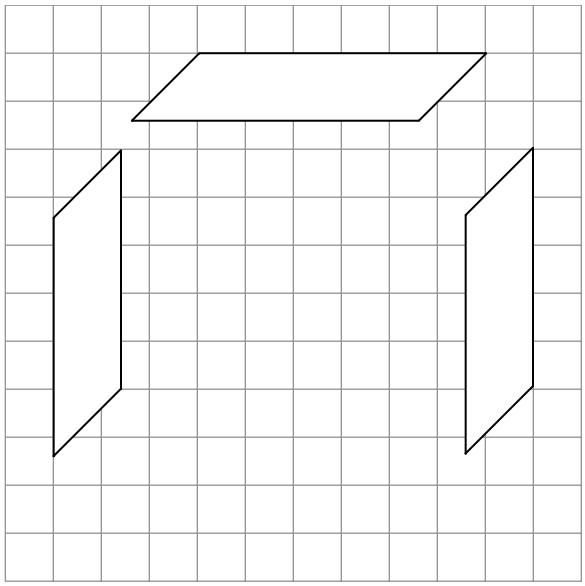
Answers:

1. $\frac{11}{16}\left(\frac{14}{16} - \frac{3}{16} = \frac{11}{16}\right)$
2. 2000 (approximately $40 \times 50 = 2000$)
3. $\frac{1}{3}\left(\frac{15}{45} = \frac{1}{3}\right)$
4. 1000 (10 mm = 1 cm so 10^3 or $10 \times 10 \times 10 = 1000$ $mm^3 = 1$ cm^3)
5. 154 km ($7 \times 22 = 154$ km)

Part B: Component Parts

Remember, these questions are similar to the ones that will be on your assignments and final examination. If you are able to answer them correctly, you are likely to do well on your assignments and final examination. If you are not able to answer them correctly, you need to go back to the lesson to review the information you do not understand.

1. Consider the following exploded view of a desk drawn to a scale of 1 in. : 2 ft.



a) Find the actual dimensions of the desk.

Answer:

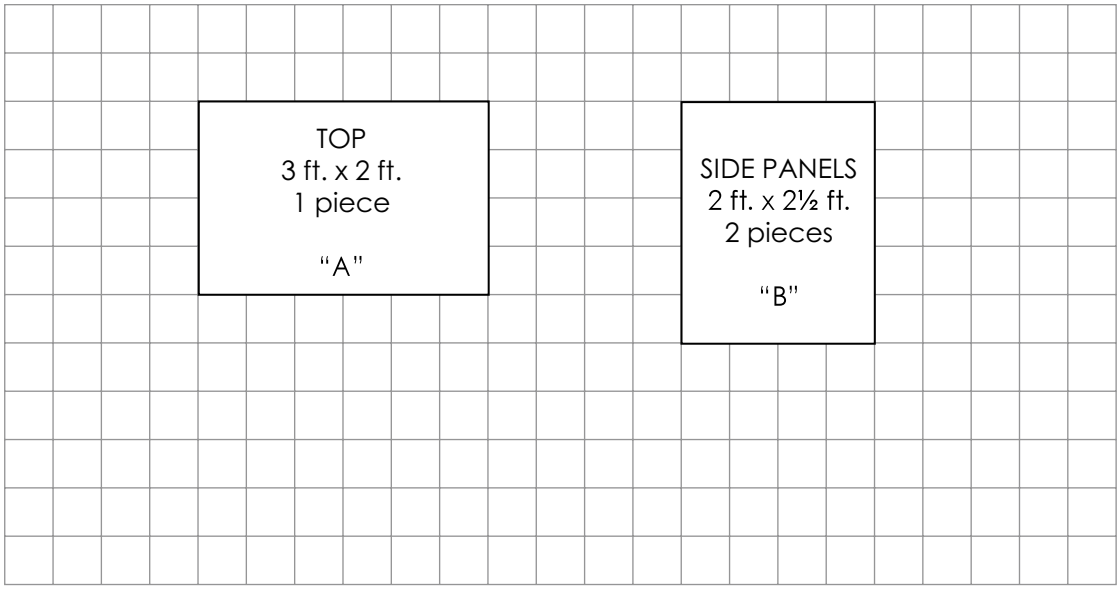
Using the imperial scale on your ruler, measure the top panel of the desk. The dimensions of the top are $1\frac{1}{2}$ in. by $\frac{1}{2}$ in. Since, in an oblique projection, the depth of $\frac{1}{2}$ in. represents one-half the measure, the depth of the top panel is $\frac{1}{2}$ in. $\times 2 = 1$ in. The dimensions of the top panel are $1\frac{1}{2}$ in. by 1 in. Using the ruler, measure the side panels. The dimensions are $\frac{1}{2}$ in. by $1\frac{1}{4}$ in. Since the depth represents one-half the measure, the depth of each side panel is $\frac{1}{2}$ in. $\times 2 = 1$ in. The dimensions of the side panels are 1 in. $\times 1\frac{1}{4}$ in.

Since the scale is 1 in.:2 ft., the actual dimensions are as follows:
The dimensions of the top panel are 3 ft. by 2 ft.

The dimensions of the side panels are $2 \text{ ft.} \times 2\frac{1}{2}$ ft.

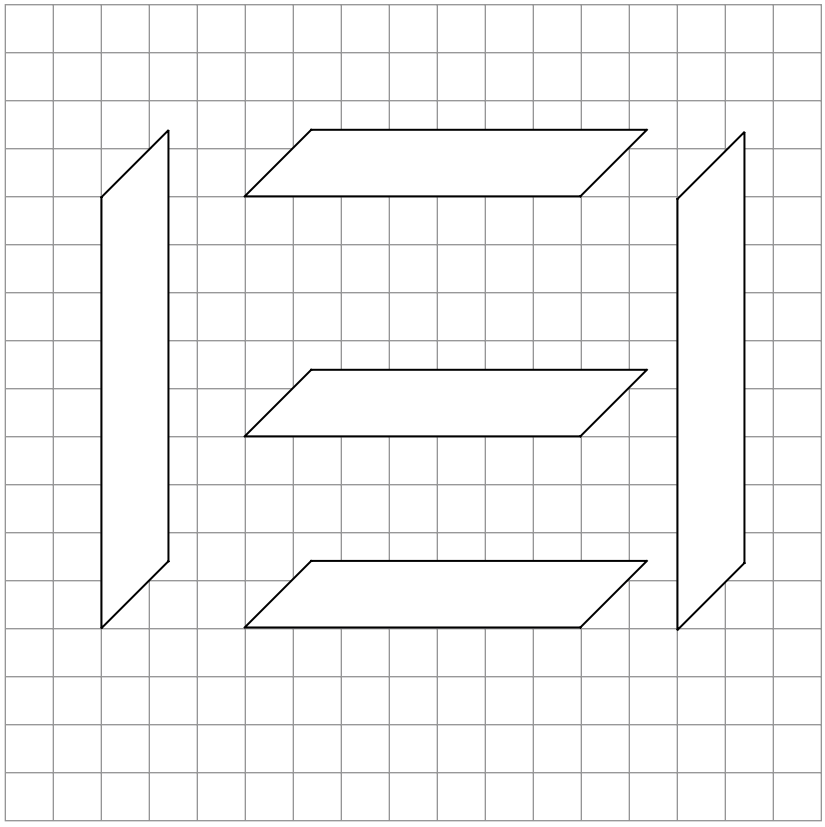
b) Draw the component parts of the desk to a scale of 1 in.:2 ft.

Answer:



Scale: (1:24)

2. The exploded view of the shelf is reproduced as follows:



a) Calculate the actual dimensions of the desk. The scale is 1 in. : 2 ft.

Answer:

Shelves:

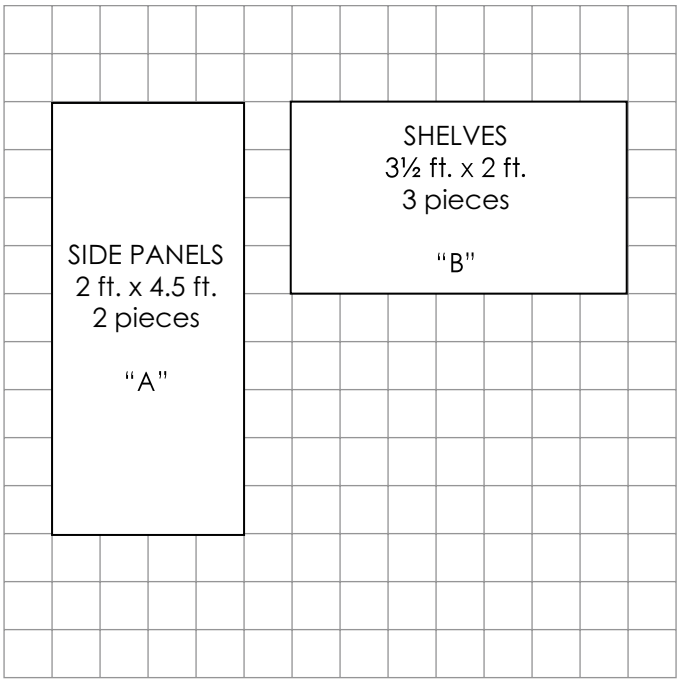
Diagram dimensions:	Actual dimensions
1.75 in. long	1.75 × 2 = 3.5 ft. tall
0.5 in. measured × 2 = 1 in. deep	1 × 2 = 2 ft. deep
1.75 in. × 1 in.	3.5 ft. × 2 ft.

Side Panels

Diagram dimension:	Actual dimensions:
2.25 in. tall	2.25 × 2 = 4.5 ft. tall
0.5 in. measured × 2 = 1 in. deep	1 × 2 = 2 ft. deep
2.25 in. × 1 in. deep	4.5 ft. × 2 ft.

b) Draw the component parts of the shelf to scale.

Answer:



Scale: 1 in.:2 ft.

Notes

GRADE 11 ESSENTIAL MATHEMATICS (30S)

Appendices

Appendix A: Unit Conversions
Appendix B: Glossary

Appendix A: Unit Conversions

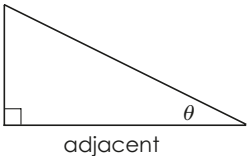
Length Conversions		
Imperial	**Metric**	**Between Systems**
1 ft. = 12 in.	1 cm = 10 mm	1 in. = 2.54 cm
1 yd. = 3 ft.	1 m = 100 cm	1 yd. = 0.9144 m
1 mi. = 5280 ft.	1 km = 1000 m	1 mi. = 1.60 km
Area Conversions		
Imperial	**Metric**	**Between Systems**
1 $ft.^2$ = 144 $in.^2$	1 cm^2 = 100 mm^2	1 $in.^2$ = 6.45 cm^2
1 $yd.^2$ = 9 $ft.^2$	1 m^2 = 10 000 cm^2	1 $yd.^2$ = 0.8361 m^2
		1 $mi.^2$ = 2.59 km^2
Volume Conversions		
Imperial	**Metric**	**Between Systems**
1 $ft.^3$ = 1728 $in.^3$	1 cm^3 = 1000 mm^3	1 $in.^3$ = 16.4 cm^3
1 $yd.^3$ = 27 $ft.^3$	1 m^3 = 1000 000 cm^3	1 $yd.^3$ = 0.765 m^3
Capacity Conversions		
Imperial	**Metric**	**Between Systems**
1 $in.^3$ = 0.554 fl. oz. (US)	1 cm^3 = 1 mL	1 gal. (CDN) = 4.54 L
1 $ft.^3$ = 7.5 gal. (US)	1 L = 1000 cm^3	1 gal. (US) = 3.785 L
		1 L = 61 $in.^3$
		1 L = 0.264 gal. (US)

Notes

APPENDIX B: GLOSSARY

adjacent side
The side of a right triangle beside the angle (θ^o)—not the hypotenuse.

Example

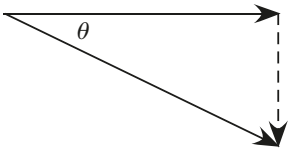


administration fee
The amount a company charges its customers for the work involved in administering a credit plan, such as the buy-now, pay-later plan.

amortize
To pay back a loan.

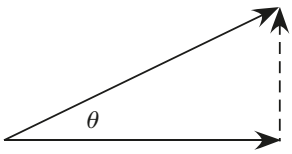
angle of depression
The angle (θ) formed between your natural line of sight (a horizontal line) and your downward line of sight.

Example



angle of elevation
The angle (θ) created between your natural line of sight (a horizontal line) and your elevated line of sight.

Example



area
The space taken up by a 2-D object; measured in square units such as in.^{2}, ft.^{2}, cm^2, m^2, etc.

arithmetic sequence
A sequence such as –5, –1, 3, 7, 11, 15 …, which has a constant difference between terms. In this case, the difference is "4." This means that you can always find the next term by adding "4" to any term. In the arithmetic sequence 15, 12, 9, 6…, you would need to add "–3" to any term to get the next term.

bar graph
A graph that uses horizontal or vertical bars to display discrete data.

Example

BEDMAS
(Brackets, Exponents, Division, Multiplication, Addition, Subtraction) Division and multiplication (and addition and subtraction) are to be completed in the order in which they appear from left to right in the expression or equation (see order of operations).

benefits
Points and incentive programs offered by credit card companies to encourage you to choose them as your credit card company.

bias
A sample that emphasizes one part of the population above others.

budget
An estimate of future income and expenses over a given period of time, and a financial plan (income, spending, and savings) based on this estimate.

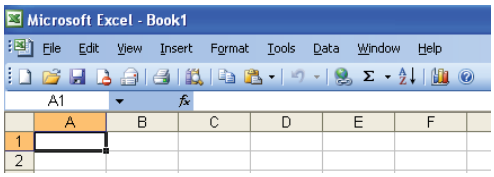
buy-now, pay-later promotion
A sales promotion that offers the option to pay the taxes and an administration fee upfront and take the item home with you, but not pay for the item until a later date (e.g., buy now but don't pay until 2020).

capacity
The amount a container can hold—similar to volume.

carrying charge
The difference between the installment price and the cash selling price. This is the same as the finance charge.

cell
A single box in a spreadsheet with specific coordinates (e.g., A1).

Example



census
A survey of the entire population.

centi (c)
A metric prefix; multiplication factor = $10^{-2} = 0.01$.

chart
A diagram that illustrates information in the form of a table, graph, or picture.

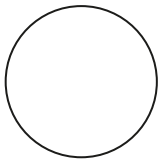
cheque
A written request to your financial institution, usually on a form supplied by the bank, directing them to give a certain amount of your money to the person or organization named on the form.

cheque register
Used to keep track of all transactions in a bank account. A small booklet is usually supplied with your chequebook for this purpose.

chequing account
Used for day-to-day spending activities. This type of account typically does not pay interest, and usually the bank charges a small monthly service fee.

circle
A two-dimensional shape made by drawing a curve that is always the same distance from a fixed point called the centre.

Example



circle graph
See *pie graph*.

circumference
The distance around the edge of a circle (also known as the perimeter).

combination account
May be used for day-to-day transactions (like a chequing account), but it pays interest (like a savings account) if the money in the account is above a certain limit.

common denominator
Two or more fractions that have the same number in the denominator.

compass
An instrument used to locate points at a given distance from a fixed point and to describe circles and arcs.

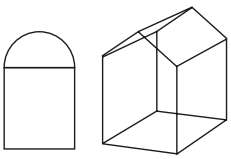
Example

component parts diagram
A 2-D scale drawing that shows one example of each piece of an object without showing how the pieces are oriented with respect to each other.

component view
This is the same as "component parts diagram."

composite object
An object made up of more than one 2-D or 3-D shape or object.

Example



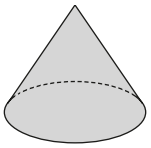
compound interest
Interest is calculated based on the current balance, including previous interest added to the principal.

$$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

where A is the amount of the investment including interest. P is the initial amount called the principal; r is the annual interest rate written in decimal form; n is the number of times interest is calculated per year; and t is the time in years.

cone
A 3-D object that rises from a circular base to a single point at the top.

Example



constant of variation
In a direct variation (e.g., if "$C = kN$" represents "Cost = 12 × Number if items bought"), the constant is: k = \$12 per item.

consumer loan
A certain amount of money is borrowed from a financial institution to pay for some specified event or item; interest is applied to the entire amount of money, even if some isn't used.

continuous data
Data where any value within the range is possible (e.g., a graph of height versus age is considered continuous data).

conversion ratio
The ratio comparing different units that are equivalent.

$$\frac{\text{new units}}{\text{old units}}$$

Example

To change from centimetres to metres, the ratio would be $\frac{1\text{ m}}{100\text{ cm}}$.

convert
To change the form but not the amount of a measurement or value.

cosine ratio
The ratio relating the adjacent side and the hypotenuse to the angle (θ^{o}).

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

costs

See *expenses*.

credit

Borrowing money with a promise to pay it back, usually with interest added, in a specified amount of time.

credit card

A card, issued by a bank or business, that entitles a person to have purchases billed to a charge account; the cardholder must repay the amount owed in monthly installments; the interest rate is usually very high.

cross-multiply

A proportion with one unknown value can be solved in the following way: (The first step is to cross-multiply.)

Given the proportion $\frac{x}{7} = \frac{5}{8}$, solve for x.

First, cross multiply.

$(8)(x) = (7)(5)$ now multiply 7×5

$8x = 35$ divide each side by 8 (to isolate the x on the left-hand side)

$\frac{8x}{8} = \frac{35}{8}$ write the answer

$x = \frac{35}{8}$

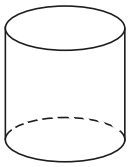
cube root $\left(\sqrt[3]{\ }\right)$

The number (factor) that, when multiplied by itself three times, produces the given cube $\left(\text{e.g., } \sqrt[3]{64} = 4\right)$.

cylinder

A prism with circular parallel faces.

Example



data

Information that is collected; usually numerical, organized in charts, and displayed by graphs.

debt

Any amount of money that you owe and have committed to pay back.

deci (d)

A metric prefix; multiplication factor = $10^{-1} = 0.1$.

deficit

In your budget, when you spend more money than you earn, the amount available for other savings is negative. It is signified by writing the number in brackets ( ).

deka (da)

A metric prefix; multiplication factor = $10^{1} = 10$.

denominator

The number below the line in a fraction that can state the total number of items, or the number of equal pieces that something is divided into.

Example

$\frac{2}{3}$ ← denominator

dependent variable

Quantities that are being created by a process; the observed result of the independent variable being manipulated. They form the range of a relation.

dimensions

Measurements of a figure (length, width, height, radius, etc.).

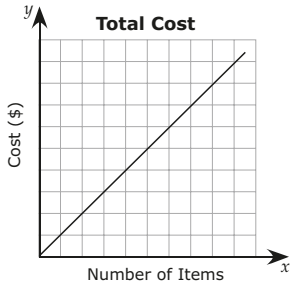
diameter

The distance across a circle measured through its centre.

direct variation

A linear relation that can be written in the form $C = kN$ (e.g., the **C**ost depends on the **N**umber of items bought), where "k" is the constant of variation, which is the price per item. The graph of a direct variation is a line graph that passes through the origin, increasing from left to right.

Example



discrete data

Data that can be sorted into well defined categories, so that each piece of information can be put into only one category; also, data that is based on counting and not measurement is discrete. For example, the number of people in a room is counted (discrete), and distance is measured (not discrete).

electronic savings account

An online savings account that you can only access (check balances of do transactions) online, at an ATM, or in person.

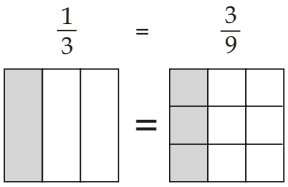
equation

A math sentence that describes a relation. An equation must include an equal sign (e.g., $I = PRT$ and $y = 2x + 3$ are equations).

equivalent fractions

Fractions that represent the same amount.

Examples



estimate

To find the approximate value.

evaluate

To use arithmetic calculations to find the value of an expression.

expenses

Any outflow (spending) of money to another person or company to pay for an item or service; also known as "costs."

exploded view

A scale, 3-D drawing of an object that allows you to see all of the component parts and how they are oriented.

exponent

The number of times a number is multiplied by itself in a power (3 is the exponent in 4^3).

expression

One side of an equation; does not contain an equal sign or "greater than/ less than" symbol.

extrapolate

Estimating information from outside the set of data.

faces approach

(for finding the surface area of a 3-D object)

Calculate the area of each face, and then add all the areas.

finance charge

The difference between the price when you do not pay cash (i.e., when you use credit) and the cash selling price. This is the same as the carrying charge.

fixed value

In a partial variation such as $C = \$3n + \$20$, which describes the cost of going to an event that includes rides, with a $20 entry fee and $3 rides, the $20 fee is the fixed value.

frequency

How often something occurs.

geometric sequence

A group of numbers where each term after the first is found by multiplying the previous one by a fixed number called "common ratio" (e.g., in the sequence 2, 6, 18, 54,…, each number is multiplied by 3 to get the next number).

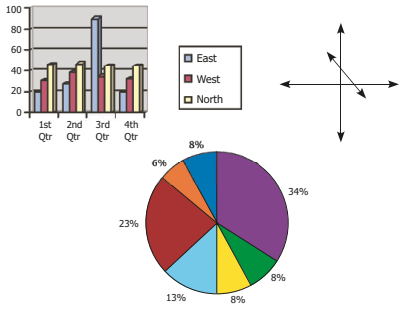
geometry

Math that involves the position, size, and shape of figures.

graph

A visual representation used to show a relationship between data.

Examples



hecto (h)

A metric prefix; multiplication factor = $10^2 = 100$.

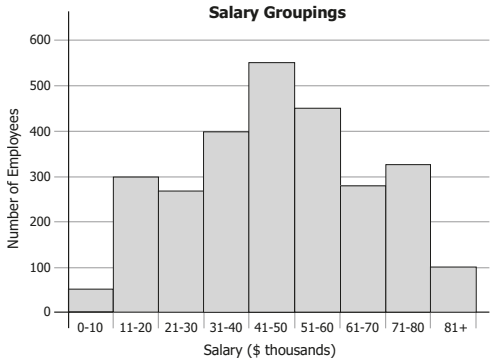
high-risk investments

Investments that have very flexible rates of return; you can make a lot of money in a short amount of time, but there is also a chance that you can lose money.

histogram

A bar graph that displays the frequency of data that has been organized into intervals; the intervals cover all possible values of data (i.e., is continuous data); therefore, there are no spaces between the bars of the graph.

Example



horizontal axis

The axis of a graph that runs from side to side/left to right.

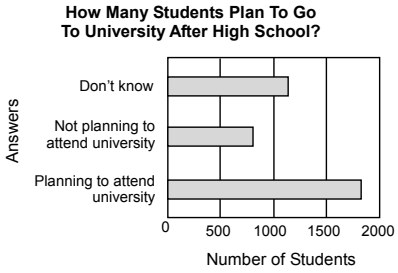
Example

The axis labelled "Gift Items" is the horizontal axis.

horizontal bar graph

A bar graph whose bars go from left to right; horizontal axis is the value axis.

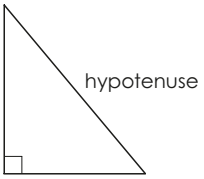
Example



hypotenuse

The side of a right triangle across from the right angle; the longest side of a right triangle.

Example



imperial system

The system of measurement used in the US, and sometimes still in Canada and Britain; includes inches , feet, yards, miles, ounces, pounds, quarts, gallons, and acres.

improper fraction

A fraction that is larger than 1; the numerator is larger than the denominator $\left(\text{e.g., } \frac{9}{4}\right)$.

independent variable

Quantities that are available at the start of a process; the variable representing the value being manipulated. They form the domain of a relation.

installment buying

A sales promotion that offers the option to pay a down payment on an item that you can take home, and then pay a certain amount every month/ two weeks/etc. for a specified time period.

installment price

The total cost of an item bought on the installment plan; the installment price includes the cash price, interest, and a service fee.

interest (I)

Money earned on an investment or charged on a loan; determined by the amount of money invested/borrowed, the interest rate, and the term.

interest-free period

A period of time after you make a purchase on a credit card, during which you do not have to pay interest.

interest rate (r)

The percent that indicates how much interest you will earn or be charged.

interpolation

To determine approximate values between data points.

isometric dot paper

A grid of dots used to draw 3-D diagrams of rectangular prisms.

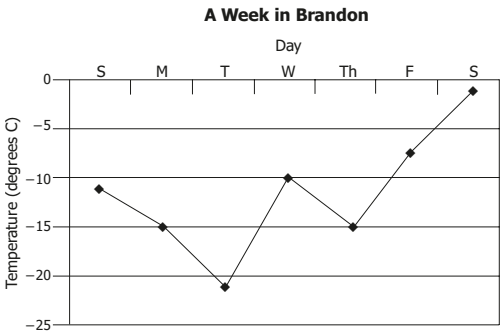
kilo (k)

A metric prefix; multiplication factor = $10^3 = 1000$.

line graph (broken line graph)

A graph that uses line segments to show changes in data; the data represents relationships such as trends or quantities changing over time.

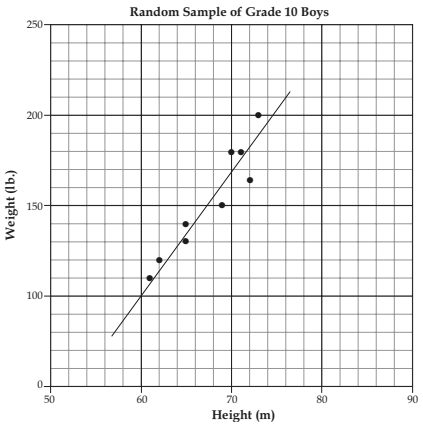
Example



line of best fit

A line drawn on a scatterplot that describes the approximate relationship of the data; not all the points must be on the line, but they should be close.

Example



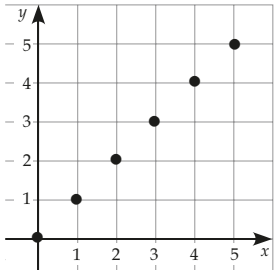
line of credit

See *personal line of credit*.

linear relation

A set of data that, when plotted on a graph, looks as though a line could be drawn through it to represent the data; it must have a constant rate of change.

Example



low-risk investments

Investments that have a small, consistent interest rate; it usually takes a long period of time to see substantial growth but you minimize your risk of losing money.

mega (M)

Aa metric prefix; multiplication factor = 10^6 = 1 000 000.

metric system

See *Système Internationale.*

micro (μ)

A metric prefix; multiplication factor = 10^{-6} = 0.000 001.

milli (m)

A metric prefix; multiplication factor = 10^{-3} = 0.001.

mixed number

A number larger than 1, written as a whole number and a proper fraction (e.g., 4).

multiplier

The number you multiply by (e.g., when you double the number 5, the multiplier is 2: double 5 = 5 + 5 = 5 × 2).

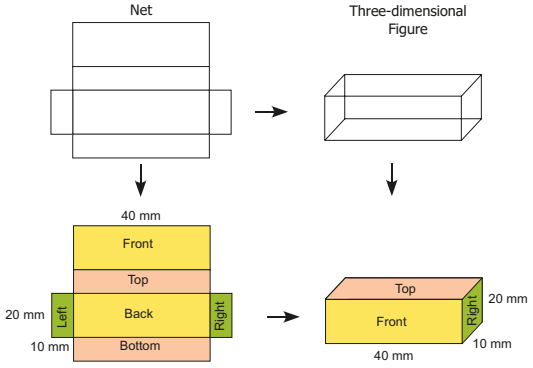
nano (n)

A metric prefix; multiplication factor = $10^{-9} = 0.000\ 000\ 001$.

net

The 2-D set of polygons that show all the faces of a 3-D object.

Example



numerator

The number above the line in a fraction that states the number of parts being considered.

Example

$\frac{2}{3}$ ← numerator

oblique projection

An exploded view, drawn to scale, that is viewed from above and to the left, and the depth measurement in the drawing is one-half of the actual measure.

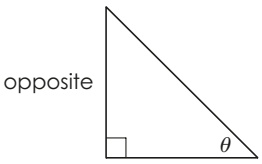
one-point perspective

A 2-D representation of a 3-D image where the size of the image decreases in proportion to the distance from the viewpoint.

opposite side

In a triangle, the side opposite angle θ is the side that does not include the two sides that form the angle θ.

Example



ordered pair

A pair of numbers that identifies the location of a point on a graph (e.g., the ordered pair [2, 5] indicates the point two spaces to the right and five spaces up from the origin).

order of operations

A specified sequence in which mathematical operations must be performed. An arithmetic expression is evaluated by following these ordered steps, sometimes referred to as BEDMAS:

1. Simplify within grouping symbols such as parentheses or brackets, starting with the innermost grouping.
2. Apply exponents—powers and roots.
3. Perform all multiplications and divisions in order from left to right.
4. Perform all additions and subtractions in order from left to right.

outstanding entry

Entries in the cheque register that do not yet appear in the bank statement.

overdraft protection

Credit that helps maintain your credit rating by allowing you to write cheques for slightly more money than you actually have in the account.

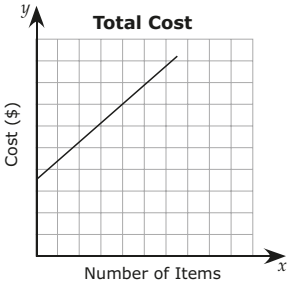
parallelogram

A four-sided polygon with opposite sides congruent (equal length) and parallel.

partial variation

A line graph that does not pass through the origin of a graph, and increases from left to right.

Example



perfect square

The product of a whole number multiplied with itself (9 is a perfect square because $3 \times 3 = 9$).

perimeter

The distance around the outside of a 2-D shape or space.

perpendicular

Meet at 90 degrees, right angles.

personal line of credit

Credit that can be used whenever you want; you do not need to use it all at once; no interest is charged on the amount of money that you do not use.

personal loan

See *consumer loan*.

pi

Normally represented by the Greek symbol π. π is the ratio of the circumference of a circle to its diameter. The approximate value of π is 3.14.

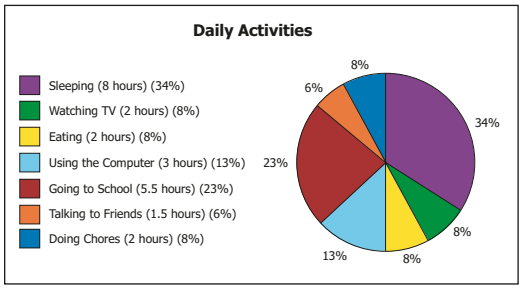
pie chart

A graph that represents data as parts of a circle (whole). The total of all the sectors should be 100% of the data.

Note that pie charts

- show the ratio of each part to the whole, not quantities
- are almost always made from data converted to a percentage of the total
- show ratios so that comparisons can be made between different-sized quantities

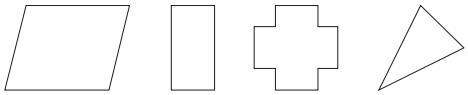
Example



polygon

A 2-D shape made up of three or more line segments.

Example



population

Everything or everyone in a group that is being studied.

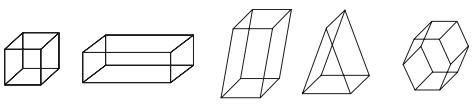
principal (P)

The original amount invested or borrowed.

prism

A 3-D object that has two congruent (equal) and parallel faces, connected by parallelograms.

Examples



proper fraction

A fraction that is less than 1; the numerator is less than the denominator $\left(\text{e.g., } \frac{5}{8}\right)$.

proportion

Two ratios that are equal.

Example

$\frac{4}{3} = \frac{400}{x}$ is a proportion

proportional reasoning

Using a proportion (two ratios) to analyze and solve a math problem.

Example

In your school, the ratio of boys to girls is 4 to 3. If your school has 400 boys, how many girls does it have?

Solution: Solve for x using cross-multiplication

$$\frac{4 \text{ boys}}{3 \text{ girls}} = \frac{400 \text{ boys}}{x \text{ girls}}$$

$$\frac{4}{3} = \frac{400}{x}$$

$$(4)(x) = (3)(400)$$

$$\frac{4x}{4} = \frac{1200}{4}$$

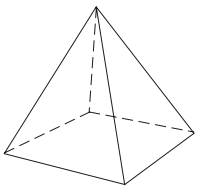
$$x = 300$$

Therefore, the school has 300 girls.

pyramid

A polyhedron whose base is a polygon and whose lateral faces are triangles that share a common vertex.

Example

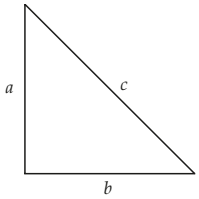


Pythagorean theorem

In a right-angled triangle, the square of the longest side (the "hypotenuse", c) is equal to the sum of the squares of the other two sides (a and b).

Example

$a^2 + b^2 = c^2$



qualitative axis

The axis on a bar graph that is divided into categories for the data to be sorted into; in vertical bar graphs it is the horizontal axis, in horizontal bar graphs it is the vertical axis.

quantitative axis

The axis on a bar graph that indicates the number of items or the amount; in vertical bar graphs it is the vertical axis, in horizontal bar graphs it is the horizontal axis; it typically has a title "Number of _____".

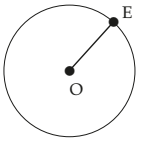
Example

The axis labelled the "Number of People" is the quantitative axis.

radius

A line from the centre of a circle to the edge (circumference) of the circle; half the diameter.

Example



random sample
A representative part of the population, selected so that every item has an equal chance of being selected.

range
The difference between the minimum and maximum values of data.

ratio
A comparison of two like numbers or quantities.

raw data
The data collected from a survey before it has been put into another form (such as a graph).

rectangle
A four-sided polygon with four right angles (90^o), and opposite sides that are congruent (equal) and parallel.

Examples

recurring expenses
Costs that appear on a regular basis (e.g., phone bill, gas, bus fare).

referent
An object that can be used to estimate a measurement (e.g., the width of a thumb is approximately 1 inch).

right angle
An angle that is 90^o.

right triangle
A triangle with a 90^o angle.

rule of 72
An investment doubles in value (approximately) when the interest rate (compounded annually) multiplied by the number of years an investment is held equals 72.

$rt = 72$

sample
A part of the group being studied that represents the whole group.

savings account
A bank account that pays interest on money deposited into the account, but does not permit you to write cheques.

scale (drawings)
A drawing of an object that is larger or smaller than the actual object, but the two are similar (proportional).

scale factor
The ratio describing the relationship between the drawing of the object and the actual object.

Example

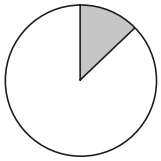
A scale factor for the diagram of a house may be written as

- 1 cm represents 2 m, or
- 1 cm : 2 m, or
- 1 : 200, or
- $\frac{1}{200}$

scatterplot
A graph with points that represents data written as ordered pairs—the points are not connected by line segments.

sector
Part of the interior of a circle, similar to a wedge of pizza or a piece of pie.

Examples



service charges
Any fees you pay for services provided by the bank or financial institution.

similar figures

Figures that have the same shape but are not necessarily the same size.

simple interest

The amount of interest is calculated as a percent of the original amount invested or borrowed (known as the principal).

$I = Prt$

where I is the amount of interest without the principal, P is the principal (the original amount of money invested or borrowed), r is the annual interest rate written in decimal form, and t is the time in years.

simplify

Combine like terms so that you are left with the simplest form of an equation or expression.

sine ratio

The ratio relating the opposite side and the hypotenuse to the angle (θ^o).

Examples

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

slope

- in a linear relation, slope = $\frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$
- a value that describes rate of change: wage rate = $\frac{\text{dollars}}{\text{hour}}$
- a value that describes the incline of a line or object: slope = $\frac{\text{rise}}{\text{run}}$

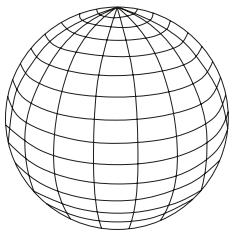
solve

To find the answer to an equation or problem, or to find the value of a variable.

sphere

A 3-D figure with a set of points in space that are the same distance from a fixed point called the centre.

Example



square root $\left(\sqrt{\ }\right)$

A number (factor) that, when multiplied by itself, produces the given square $\left(\sqrt{16} = 4\right)$.

statement of reconciliation

A form used to determine that the entries on your monthly account statement and your cheque register agree.

sum of angles

The sum of the angles in a triangle is 180^o (angle 1 + angle 2 + angle 3 = 180^o).

surface area

The sum of the areas of the faces or curved surface of a 3-D object.

survey

To ask either written or verbal questions for the purpose of acquiring information/data.

Système Internationale (SI)

Measurement system based on the multiples of 10; commonly used throughout the world; also known as the metric system.

table of values

An organized list of values that shows the relationship between two variables.

tangent ratio

The ratio relating the opposite and adjacent sides—not the hypotenuse—to the angle (θ^o).

Examples

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

term

The amount of time that the conditions of a loan are in effect.

title (graph)

A title indicates what the graph is about.

trapezoid

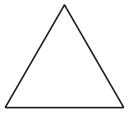
A four-sided shape with two parallel sides.

Example

triangle

A three-sided polygon.

Example



trigonometry

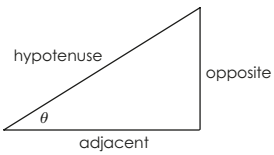
The study of triangles, which includes the calculation of side lengths and angle measures.

trigonometric ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



2-dimensional (2-D)

A figure that only has two measures (a rectangle is 2-D because it is described using only length and width).

unexpected expenses

Costs that do not appear on a regular basis and are not planned for (e.g., car repairs, home repairs, medical expenses, etc.).

vanishing point

The point at which parallel lines (e.g., railroad tracks) appear to meet as the distance increases.

variable

A letter or symbol that represents an unknown value (x, y, n, θ).

verify

Check your answer.

vertical axis

The axis of a graph that goes up and down/top to bottom.

Example

The axis labelled the "Number of People" is the vertical axis.

volume

The amount of space taken up by an object.

GRADE 11 ESSENTIAL MATHEMATICS (30S)

Midterm Practice Exam

GRADE 11 ESSENTIAL MATHEMATICS

Midterm Practice Test

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____ /100 = _____ %

Comments:

Instructions

The midterm examination is based on Modules 1 to 3 of the Grade 11 Essential Mathematics course. It is worth 12.5% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the midterm examination.

Format

The format of the examination will be as follows:

Part A: Multiple Choice	10 marks
Part B: Definitions	10 marks
Part C: Long Answer	80 marks
Total	100 marks

(see over)

Instructions (continued)

Resources Provided

- an amortization table is included at the end of this examination

Resources Required (Not Provided)

To complete this examination, you will need:

- pens/pencils (2 or 3 or each)
- blank paper
- scientific or graphing calculator
- geometry set (including a ruler, a protractor, and a compass)
- Midterm Exam Resource Sheet (The Midterm Exam Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module that is submitted for marking.)

Notes

For Part C

- show all calculations and formulas
- include units where appropriate
- clearly state your final answer
- diagrams may not be drawn to scale

Name: _____

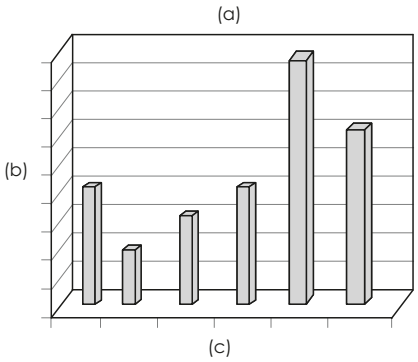
Part A: Multiple Choice *(10 x 1 = 10 marks)*

Circle the letter of the best answer for each question.

1. Draco invests $5000. Approximately how long will it take for the investment to double if the rate is 10% compounded annually?
   a) 8 years
   b) 7 years
   c) 720 years
   d) 7 months

2. If the volume of a prism is 30 m^3, what is the volume of a pyramid with the same dimensions?
   a) 10 m^3
   b) 15 m^3
   c) 60 m^3
   d) 90 m^3

3. Which axis is the qualitative axis?
   a)
   b)
   c)



4. Which of the following is not a step in getting out of debt?
   a) Stop using your credit cards
   b) Consolidate your debts
   c) Do not tell anyone you are in debt
   d) Cash in some of your investments

5. How much is the volume of a rectangular prism changed when you double one dimension?
   a) The volume stays the same.
   b) The volume is multiplied by 4.
   c) The volume is doubled.
   d) The volume is multiplied by 8.

6. Which is the most cost-efficient way to pay for a big ticket item?
   a) buy now, pay later
   b) installment buying
   c) personal loan
   d) cash

7. The surface area of a sphere is 25 cm^2. Convert this to mm^2.
   a) 2.5 mm^2
   b) 2.5 mm^3
   c) 0.25 mm^2
   d) 2500 mm^2

8. Which type of credit should Danuwa use to pay for his university costs when he moves to Toronto to go to university?
   a) credit card
   b) overdraft protection
   c) personal line of credit
   d) consumer loan

9. Devdas invests $4600 for eight years. The investment earns 5% in simple interest. How much interest will Devdas earn on the investment?
   a) $1840
   b) $184 000
   c) $11 500
   d) $115

Name: _____

10. The volume of a prism is 5184 in.^{3}. What is the volume in cubic feet?
    a) 432 ft.^{3}
    b) 36 ft.^{3}
    c) 62 208 ft.^{3}
    d) 3 ft.^{3}

Part B: Definitions *(10 x 1 = 10 marks)*

Match each definition with the correct term from the list below. Write the correct term on the line below each definition. Terms are used only once. Not all terms have a definition provided.

bar graph	continuous	histogram	personal line of credit
buy now, pay later	credit	independent	prism
capacity	credit card	installment buying	pyramid
carrying charge	cylinder	installment price	referent
circle graph	dependent	interpolation	simple interest
composite	discrete	line graph	sphere
compound interest	extrapolation	low risk	surface area
cone	faces approach	net	volume
consumer loans	high risk	overdraft protection	

1. Data that can be sorted into well defined categories. _____ (Module 3, Lesson 1)
2. Purchasing by paying the taxes upfront but waiting to pay the cost of the item at a later time. _____
3. Calculate surface area by adding the area of each surface. _____
4. Investment whose interest rate does not vary much. _____
5. A 3-D object that has a polygon base and triangular sides that share a common vertex. _____
6. A graph that relates independent and dependent variables. _____
7. Credit given as a large amount of money, and you are charged interest on the *entire* amount. _____
8. In a relationship, the variable that is affected by the other. _____
9. A 3-D object that has two circular faces joined by a rectangular face that wraps around the circumference of the two circles. _____
10. Borrowing money with the promise to pay it back. _____

Name: _____

Part C: Long Answer *(80 marks)*

Show all formulas and calculations used to answer each question. Write your final answers rounded to the number of decimal places indicated in the questions. Include units where appropriate. Clearly state your final answer.

Module 1: Interest and Credit *(38 marks)*

1. Drazan takes out a short-term loan of $7300 from his financial institution. He repays the loan 21 days later. If he pays $49.35 in interest charges, calculate the rate of interest he is charged. Write your answer as a percentage rounded to the nearest whole number. *(3 marks)*

2. Due South has $100,000 to invest for a period of five years. Institution A offers him an interest rate of 7% compounded semi-annually. Institution B offers him an interest rate of $6\frac{1}{2}$% compounded monthly.

   a) Using the compound interest formula, calculate which institution offers Due South a better return on his investment. *(5 marks)*

   b) How much more interest could be earned by investing in the institution that offers a better return? *(1 mark)*

Name: _____

3. Demelza invests $12,000 for eight years. At the end of eight years, her investment is worth $24,000.

   a) Use the rule of 72 to estimate the interest rate Demelza receives. *(1 mark)*

   b) Use the compound interest formula to determine the actual value of $12,000 compounded annually for eight years at the interest rate you found in part (a). *(2 marks)*

   c) Determine the difference between the actual value of the investment after eight years and the doubled value found with the rule of 72. *(1 mark)*

d) Is the rate you found in part (a) approximately correct? Explain why. *(2 marks)*

4. Complete the Venn Diagram below, comparing compound and simple interest. *(3 marks)*

5. Doneshia needs a loan from the bank to pay for a car that costs $14 000. If Doneshia pays $450.10 per month toward the loan, what are the interest rate and term of the loan? *(2 marks)*

Name: _____

6. List the three questions you should ask yourself when deciding whether to save money by paying cash now, or whether to use a sales promotion (buy now, pay later, or installment buying) to buy an expensive item like a computer. *(3 marks)*

7. On October 5th, DuHo uses a credit card to make a purchase for $163.16. The purchase appears on his monthly statement issued October 20th. DuHo doesn't pay for the purchase by the due date indicated on the October statement. His next monthly statement is issued November 20th. Calculate the interest he is charged for the purchase on his November statement. Assume his lending institution charges him an annual interest rate of 18.25%. Assume that he is charged interest from the date of purchase. *(4 marks)*

8. The balance on Duong's credit card statement is $1435.86. The minimum payment on the statement is 5% of the statement total or $10, whichever is greater. How much will the minimum payment be on Duong's statement? *(2 marks)*

9. Daniela is buying a dining room set. The cash price of the dining room set is $2799 plus taxes. The installment terms are $100 down plus $90 per month for three years. The monthly payment includes taxes.

   a) Calculate the cash selling price of the dining room set. *(2 marks)*

   b) Calculate the installment price of the dining room set. *(3 marks)*

   c) Calculate the percent rate of the finance charge compared to the cash selling price. *(2 marks)*

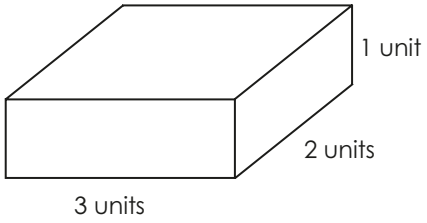
10. Complete the table below by stating one advantage and one disadvantage for each type of credit listed. *(2 marks)*

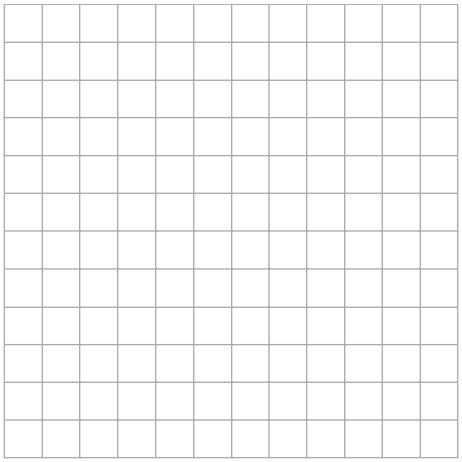
	Advantage	Disadvantage
Overdraft *(1 mark)*		
Line of Credit *(1 mark)*		

Name: _____

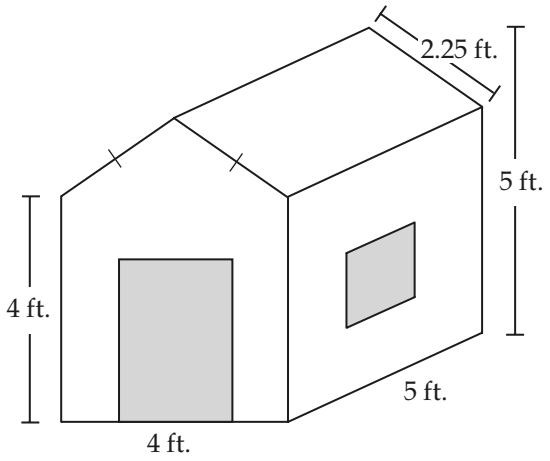
Module 2: 3-D Geometry (26 marks)

1. Find the surface area of the object using a net. *(3 marks)*





2. You have finished building a treehouse with a doorway (2 ft. wide and 3 ft. tall) and a window opening (1 ft. by 1 ft.). Now you would like to paint the outside, including the bottom. You have enough paint to cover 100 ft.^{2}. Will you need more paint? Explain. *(10 marks)*



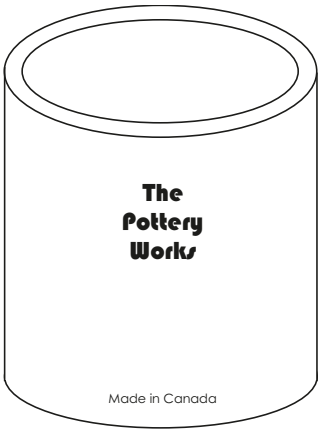
Name: _____

Continue Answer for Question 2 (if necessary)

3. Complete the table comparing volume and capacity. *(3 marks)*

How Are They Different?	
Volume	Capacity
How Are They Alike?	

4. A cylindrical clay pot has a capacity of 2617 $in.^3$. The side wall and the bottom of the pot are 1 inch thick, and the outside height of the pot is 1.5 feet.

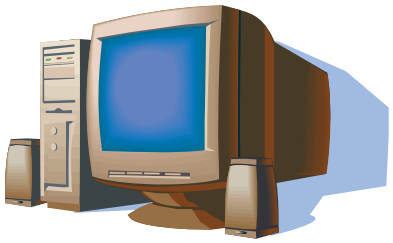


a) Calculate the outside diameter of the pot. Write your answer in inches rounded to one decimal place. *(4 marks)*

Name: _____

b) Calculate the volume of water in the pot if it is filled to three inches below the top edge. Write your answer in litres, rounded to one decimal place. Note some conversion factors: 1 litre = 61 $in.^3$, and 1 $ft.^3$ = 28.3 L. *(3 marks)*

5. You are trying to find the surface area of your computer screen and you do not have any measuring tape or ruler. You decide to use a referent to measure the screen.



a) State what you would use as a referent. *(1 mark)*

b) Explain how you would use the referent to estimate the surface area of your computer screen. *(2 marks)*

Name: _____

Module 3: Statistics ***(16 marks)***

1. Dakota is a member of the student council and is organizing the pizza lunch for all the council members. At the last meeting, the secretary surveyed everyone to find out what kind of pizza they like to eat. The following data was collected.

   Pepperoni—5

   Cheese—8

   Hawaiian—3

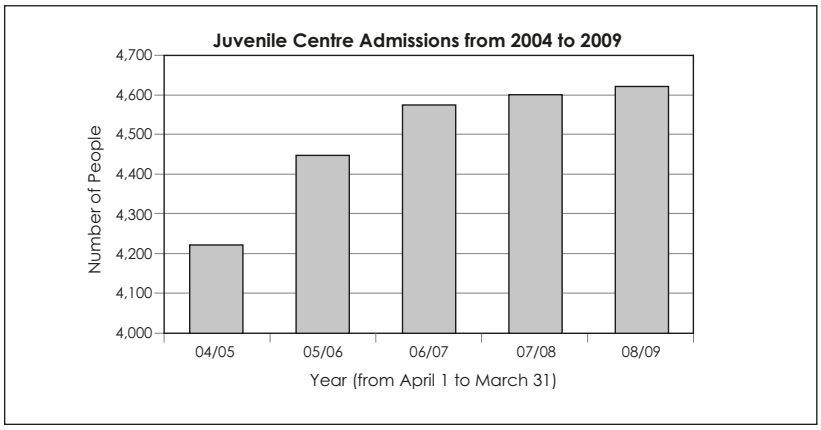
   Vegetarian—3

   Meat Lovers—7

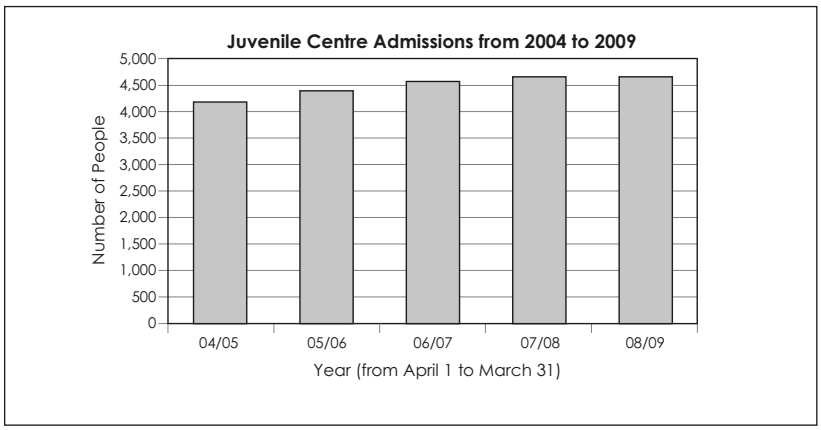
   Draw a circle graph to represent the data. Be sure to include a title and all the appropriate labels. *(13 marks)*

2. Use the Statistics Canada data depicted in the following graphs to answer the questions below.

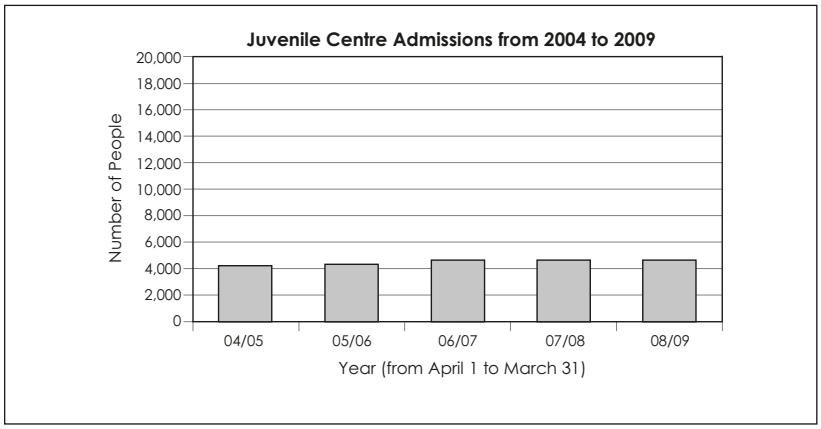
Graph A:



Graph B:



Graph C:



Name: _____

a) After a number of crimes are committed at local schools, the media is suggesting that harsher punishments are necessary to reduce the increasing amount of juvenile crime. Which graph would support this argument best? Explain why. *(2 marks)*

b) Which graph is the most "honest" graph because it does not present the data in a distorted way? *(1 mark)*

Amortization Table					
Amortization Period Monthly Payment Per $1000 Loan Proceeds					
Annual Rate	1 Year Monthly	2 Years Monthly	3 Years Monthly	4 Years Monthly	5 Years Monthly
6.00%	$86.07	$44.33	$30.43	$23.49	$19.34
6.25%	$86.18	$44.44	$30.54	$23.61	$19.46
6.50%	$86.30	$44.56	$30.66	$23.72	$19.57
6.75%	$86.41	$44.67	$30.77	$23.84	$19.69
7.00%	$86.53	$44.78	$30.88	$23.95	$19.81
7.25%	$86.64	$44.89	$31.00	$24.07	$19.93
7.50%	$86.76	$45.01	$31.11	$24.19	$20.05
7.75%	$86.87	$45.12	$31.23	$24.30	$20.16
8.00%	$86.99	$45.24	$31.34	$24.42	$20.28
8.25%	$87.10	$45.34	$31.45	$24.53	$20.40
8.50%	$87.22	$45.46	$31.57	$24.65	$20.52
8.75%	$87.34	$45.57	$31.68	$24.71	$20.64
9.00%	$87.45	$45.68	$31.80	$24.89	$20.76
9.25%	$87.57	$45.80	$31.92	$25.00	$20.88
9.50%	$87.68	$45.91	$32.03	$25.12	$21.00
9.75%	$87.80	$46.03	$32.15	$25.24	$21.12
10.00%	$87.92	$46.14	$32.27	$25.36	$21.25
10.25%	$88.03	$46.26	$32.38	$25.48	$21.37
10.50%	$88.15	$46.38	$32.50	$25.60	$21.49
10.75%	$88.27	$46.49	$32.62	$25.72	$21.62
11.00%	$88.38	$46.61	$32.74	$25.85	$21.74
11.25%	$88.50	$46.72	$32.86	$25.97	$21.87
11.50%	$88.62	$46.84	$32.98	$26.09	$21.99
11.75%	$88.73	$46.96	$33.10	$26.21	$22.12
12.00%	$88.85	$47.07	$33.21	$26.33	$22.24
12.25%	$88.97	$47.19	$33.33	$26.46	$22.37
12.50%	$89.08	$47.31	$33.45	$26.58	$22.50
12.75%	$89.20	$47.42	$33.57	$26.70	$22.63
13.00%	$89.32	$47.54	$33.69	$26.83	$22.75
13.25%	$89.43	$47.66	$33.81	$26.95	$22.88
13.50%	$89.55	$47.78	$33.94	$27.08	$23.01
13.75%	$89.67	$47.89	$34.06	$27.20	$23.14
14.00%	$89.79	$48.01	$34.18	$27.33	$23.27

GRADE 11 ESSENTIAL MATHEMATICS (30S)

Midterm Practice Exam
Answer Key

GRADE 11 ESSENTIAL MATHEMATICS

Midterm Practice Test
Answer Key

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____ / 100 = _____ %

Comments:

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The format of the examination will be as follows:

Part A: Multiple Choice	10 marks
Part B: Definitions	10 marks
Part C: Long Answer	80 marks
Total	100 marks

(see over)

Instructions (continued)

Resources Provided

- an amortization table is included at the end of this examination

Resources Required (Not Provided)

To complete this examination, you will need:

- pens/pencils (2 or 3 or each)
- blank paper
- scientific or graphing calculator
- geometry set (including a ruler, a protractor, and a compass)
- Midterm Exam Resource Sheet (The Midterm Exam Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module that is submitted for marking.)

Notes

For Part C

- show all calculations and formulas
- include units where appropriate
- clearly state your final answer
- diagrams may not be drawn to scale

Name: _____

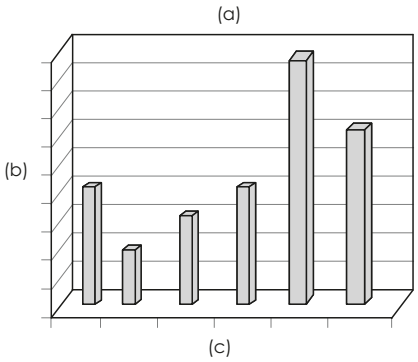
Part A: Multiple Choice *(10 x 1 = 10 marks)*

Circle the letter of the best answer for each question.

1. Draco invests $5000. Approximately how long will it take for the investment to double if the rate is 10% compounded annually?
   a) 8 years
   b) 7 years
   c) 720 years
   d) 7 months (Module 1, Lesson 2)

2. If the volume of a prism is 30 m^3, what is the volume of a pyramid with the same dimensions?
   a) 10 m^3
   b) 15 m^3
   c) 60 m^3
   d) 90 m^3 (Module 2, Lesson 3)

3. Which axis is the qualitative axis?
   a)
   b)
   c)



(Module 3, Lesson 2)

4. Which of the following is not a step in getting out of debt?
   a) Stop using your credit cards
   b) Consolidate your debts
   c) Do not tell anyone you are in debt
   d) Cash in some of your investments (Module 1, Lesson 7)

5. How much is the volume of a rectangular prism changed when you double one dimension?
   a) The volume stays the same.
   b) The volume is multiplied by 4.
   c) The volume is doubled.
   d) The volume is multiplied by 8. (Module 2, Lesson 4)

6. Which is the most cost-efficient way to pay for a big ticket item?
   a) buy now, pay later
   b) installment buying
   c) personal loan
   d) cash (Module 1, Lesson 5)

7. The surface area of a sphere is 25 cm^2. Convert this to mm^2.
   a) 2.5 mm^2
   b) 2.5 mm^3
   c) 0.25 mm^2
   d) 2500 mm^2 (Module 2, Lesson 2)

8. Which type of credit should Danuwa use to pay for his university costs when he moves to Toronto to go to university?
   a) credit card
   b) overdraft protection
   c) personal line of credit
   d) consumer loan (Module 1, Lesson 3)

9. Devdas invests $4600 for eight years. The investment earns 5% in simple interest. How much interest will Devdas earn on the investment?
   a) $1840
   b) $184 000
   c) $11 500
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Name: _____

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compound interest	extrapolation	low risk	surface area
cone	faces approach	net	volume
consumer loans	high risk	overdraft protection	

1. Data that can be sorted into well defined categories. **discrete** (Module 3, Lesson 1)
2. Purchasing by paying the taxes upfront but waiting to pay the cost of the item at a later time. **buy-now, pay-later** (Module 1, Lesson 5)
3. Calculate surface area by adding the area of each surface. **faces approach** (Module 2, Lesson 1)
4. Investment whose interest rate does not vary much. **low risk** (Module 1, Lesson 2)
5. A 3-D object that has a polygon base and triangular sides that share a common vertex. **pyramid** (Module 2, Lesson 1)
6. A graph that relates independent and dependent variables. **line graph** (Module 3, Lesson 4)
7. Credit given as a large amount of money, and you are charged interest on the *entire* amount. **consumer loans** (Module 1, Lesson 3)
8. In a relationship, the variable that is affected by the other. **dependent** (Module 3, Lesson 4)
9. A 3-D object that has two circular faces joined by a rectangular face that wraps around the circumference of the two circles. **cylinder** (Module 2, Lesson 1)
10. Borrowing money with the promise to pay it back. **credit** (Module 1, Lesson 3)

Name: _____

Part C: Long Answer *(80 marks)*

Show all formulas and calculations used to answer each question. Write your final answers rounded to the number of decimal places indicated in the questions. Include units where appropriate. Clearly state your final answer.

Module 1: Interest and Credit *(38 marks)*

1. Drazan takes out a short-term loan of $7300 from his financial institution. He repays the loan 21 days later. If he pays $49.35 in interest charges, calculate the rate of interest he is charged. Write your answer as a percentage rounded to the nearest whole number. *(3 marks)*

   Answer:

   I = \$49.35, P = \$7300, r = ?, t = 21 days ÷ 365 = 0.05753 years *(1 mark)*

   $$I = Prt$$

   $$r = \frac{I}{Pt}$$

   $$= \frac{49.35}{(7300 \times 0.05753)}$$

   $$= \frac{49.35}{420} \qquad \textit{(1 mark)}$$

   $$= 0.1175$$

   The interest rate would be $11.75 \approx 12\%$. *(1 mark)*

(Module 1, Lesson 1)

2. Due South has $100,000 to invest for a period of five years. Institution A offers him an interest rate of 7% compounded semi-annually. Institution B offers him an interest rate of $6\frac{1}{2}\%$ compounded monthly.

   a) Using the compound interest formula, calculate which institution offers Due South a better return on his investment. *(5 marks)*

   Answer:

Institution A

$A = ?$, $P = \$100{,}000$, $r = 0.07$, $n = 2$, $t = 5$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 100{,}000\left(1 + \frac{0.07}{2}\right)^{2\times 5} \quad \textit{(1 mark)}$$

$$= 100{,}000\left(1 + 0.035\right)^{10}$$

$$= 100{,}000\left(1.035\right)^{10}$$

$$= 100{,}000 \times 1.410659876$$

$$= \$141{,}059.88 \quad \textit{(1 mark)}$$

Institution B

$A = ?$, $P = \$100{,}000$, $r = 0.065$, $n = 12$, $t = 5$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 100{,}000\left(1 + \frac{0.065}{12}\right)^{12\times 5} \quad \textit{(1 mark)}$$

$$= 100{,}000\left(1 + 0.005417\right)^{60}$$

$$= 100{,}000\left(1.005417\right)^{60}$$

$$= 100{,}000 \times 1.38281732$$

$$= \$138{,}281.73 \quad \textit{(1 mark)}$$

Institution A offers Due South a better return on his investment. *(1 mark)*

   b) How much more interest could be earned by investing in the institution that offers a better return? *(1 mark)*

   Answer:

   Difference: $141,059.88 – $138,281.73 = $2,778.15 more could be earned by investing with Institution A.

(Module 1, Lesson 2)

Name: _____

3. Demelza invests $12,000 for eight years. At the end of eight years, her investment is worth $24,000.

   a) Use the rule of 72 to estimate the interest rate Demelza receives. *(1 mark)*

   Answer:

   $$rt = 72$$

   $$r = \frac{72}{t}$$

   $$= \frac{72}{8} = 9\%$$

   b) Use the compound interest formula to determine the actual value of $12,000 compounded annually for eight years at the interest rate you found in part (a). *(2 marks)*

   Answer:

   $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

   $$A = 12{,}000\left(1 + \frac{0.09}{1}\right)^{1\times 8} \qquad \textit{(1 mark)}$$

   $$= 12{,}000(1 + 0.09)^{8}$$

   $$= 12{,}000(1.09)^{8}$$

   $$= 12{,}000(1.9926)$$

   $$= \$23{,}910.75 \qquad \textit{(1 mark)}$$

   c) Determine the difference between the actual value of the investment after eight years and the doubled value found with the rule of 72. *(1 mark)*

   Answer:

   Difference: $24,000 – $23,910.75 = $89.25

(Module 1, Lesson 2)

d) Is the rate you found in part (a) approximately correct? Explain why. *(2 marks)*

Answer:

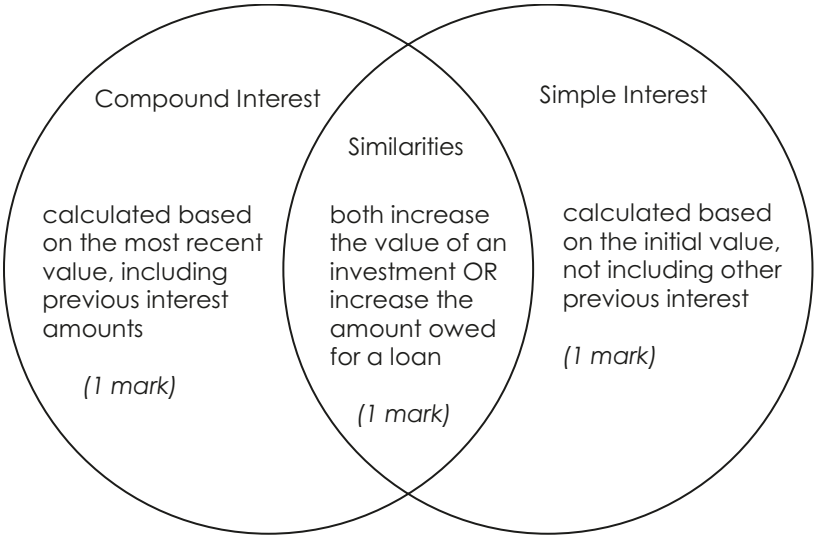
9% is approximately correct because when we applied the compound interest formula using 9%, we got approximately the same answer.

Note: $89.25 may seem like a lot of money, but remember that you are comparing values that are over $20 000. $90 is less than half of 1% of $20,000.

4. Complete the Venn Diagram below, comparing compound and simple interest. *(3 marks)*

Answer:

Answers will vary. Here is one possible answer.



(Module 1, Lesson 2)

5. Doneshia needs a loan from the bank to pay for a car that costs $14 000. If Doneshia pays $450.10 per month toward the loan, what are the interest rate and term of the loan? *(2 marks)*

Answer:

Number of $1000 dollar units: 14 000 ÷ 1000 = 14

Monthly payment per $1000: $450.10 ÷ 14 = $32.15 per month

Find this value in your amortization table.

Row: The interest rate is 9.75%

Column: The term is three years.

(Module 1, Lesson 4)

Name: _____

6. List the three questions you should ask yourself when deciding whether to save money by paying cash now, or whether to use a sales promotion (buy now, pay later, or installment buying) to buy an expensive item like a computer. *(3 marks)*

   Answer:

   1. Do you have enough money to pay cash now?
   2. Do you need the item immediately?
   3. Can you save enough money to pay cash for it within six months?

   (Module 1, Lesson 5)

7. On October 5th, DuHo uses a credit card to make a purchase for $163.16. The purchase appears on his monthly statement issued October 20th. DuHo doesn't pay for the purchase by the due date indicated on the October statement. His next monthly statement is issued November 20th. Calculate the interest he is charged for the purchase on his November statement. Assume his lending institution charges him an annual interest rate of 18.25%. Assume that he is charged interest from the date of purchase. *(4 marks)*

   Answer:

   October 5 until November 20th = 47 days

   $I = Prt$

   $$= 163.16 \times 0.1825 \times \frac{47}{365}$$

   $$= \$3.83$$

   DuHo will be charged $3.83 interest on his purchase.

   (Module 1, Lesson 6)

8. The balance on Duong's credit card statement is $1435.86. The minimum payment on the statement is 5% of the statement total or $10, whichever is greater. How much will the minimum payment be on Duong's statement? *(2 marks)*

   Answer:

   5% of $1435.86 = 71.793 ≈ $71.79 *(1 mark)*

   Duong's minimum payment will be $71.79. *(1 mark)*

   (Module 1, Lesson 6)

9. Daniela is buying a dining room set. The cash price of the dining room set is $2799 plus taxes. The installment terms are $100 down plus $90 per month for three years. The monthly payment includes taxes.

   a) Calculate the cash selling price of the dining room set. *(2 marks)*

   Answer:

   PST = 2799 × 0.07 or $195.93
   GST = 2799 × 0.05 or $139.95 *(1 mark)*

   Cash selling price = 2799 + 195.93 + 139.95 = $3134.88 *(1 mark)*

   b) Calculate the installment price of the dining room set. *(3 marks)*

   Answer:

   Number of payments = 3 years or 36 months *(1 mark)*

   Total value of monthly payments = 90 × 36 = $3240 *(1 mark)*

   Total = 3240 + 100 = $3340 *(1 mark)*

   Installment price is $3340.

   c) Calculate the percent rate of the finance charge compared to the cash selling price. *(2 marks)*

   Answer:

   Finance charge = 3340.00 – 3134.88 = $205.12 *(1 mark)*

   Percent rate = $\frac{205.12}{3134.88} \times 100 = \$6.54$ *(1 mark)*

   Percent rate of finance charge is 6.5%.

(Module 1, Lesson 5)

10. Complete the table below by stating one advantage and one disadvantage for each type of credit listed. *(2 marks)*

    Answer:

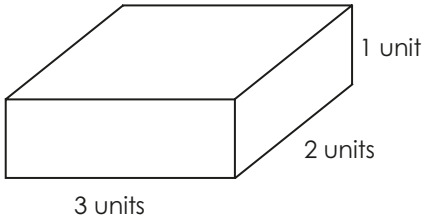
	Advantage	Disadvantage
Overdraft *(1 mark)*	It allows you to spend slightly more than you have, protecting your credit score.	You may not realize that you have gone over your limit, and some companies charge you for overdraft.
Line of Credit *(1 mark)*	You only have to pay interest on the money that you use.	You usually have to provide assets as security.

(Module 1, Lesson 3)

Name: _____

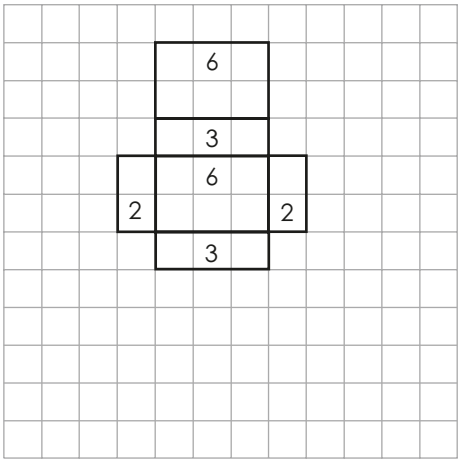
Module 2: 3-D Geometry (26 marks)

1. Find the surface area of the object using a net. *(3 marks)*



Answer:

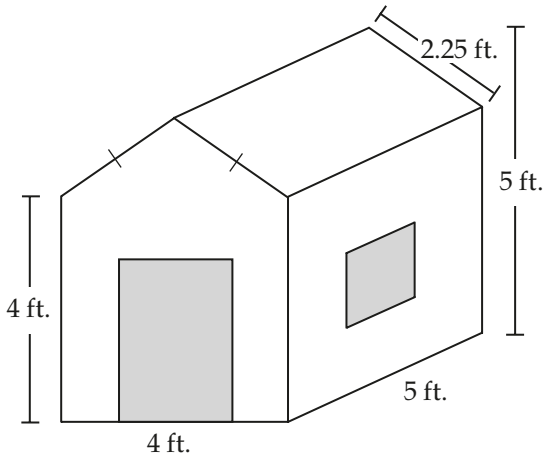
(2 marks for the net, 1 mark for the surface area)



Area: 6 + 6 + 3 + 3 + 2 + 2 = 22 units^2. (units must be squared)

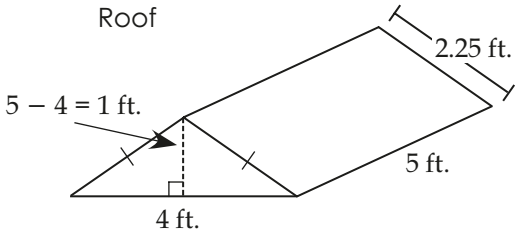
(Module 2, Lesson 1)

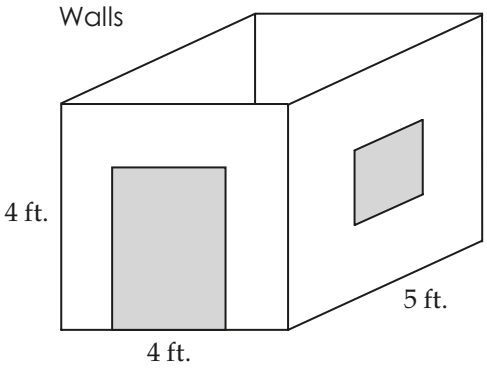
2. You have finished building a treehouse with a doorway (2 ft. wide and 3 ft. tall) and a window opening (1 ft. by 1 ft.). Now you would like to paint the outside, including the bottom. You have enough paint to cover 100 ft.^{2}. Will you need more paint? Explain. *(10 marks)*



Answer:

You may want to break apart the shapes, since this is a composite object.





Triangular prism

front/back:

$$SA = \frac{1}{2}bh$$

$$= \frac{1}{2}(4)(1) = 2 \text{ ft.}^2 \text{ each}$$

sides:

$SA = lw$

$= (5)(2.25) = 11.25$ ft.^{2} each *(2 marks)*

Name: _____

Rectangular Prism

front/back (not including door):

$SA = lh$

$= (4)(4) = 16 \text{ ft.}^2$ each

sides (not considering window):

$SA = wh$

$= (5)(4) = 20 \text{ ft.}^2$ each

bottom:

$SA = lw$

$= (4)(5) = 20 \text{ ft.}^2$ *(3 marks)*

Door: $2 \times 3 = 6 \text{ ft.}^2$

Window: $1 \times 1 = 1 \text{ ft.}^2$

Total area not needing painting $= 6 + 1 = 7 \text{ ft.}^2$ *(2 mark)*

Total $= 2(2) + 2(11.25) + 2(16) + 2(20) + 20 - 7$

$= 111.5 \text{ ft.}^2$ *(2 mark)*

No, I do not have enough paint to cover the treehouse. *(1 mark)*

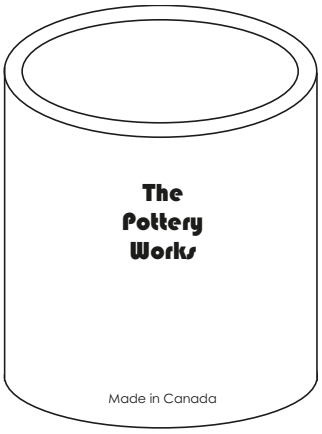
(Module 2, Lesson 2)

3. Complete the table comparing volume and capacity. *(3 marks)*

How Are They Different?	
Volume	Capacity
■ units are cm^3, m^3, $ft.^3$, $in.^3$, etc. ■ describes the amount of space an object takes up	■ units are mL, L, cups, quarts, gallons, etc. ■ describes the amount that an object can hold/contain
How Are They Alike?	
■ describes 3-D objects ■ are sometimes used interchangeably (not necessary for full marks)	

(Module 2, Lesson 5)

4. A cylindrical clay pot has a capacity of 2617 in.^{3}. The side wall and the bottom of the pot are 1 inch thick, and the outside height of the pot is 1.5 feet.



a) Calculate the outside diameter of the pot. Write your answer in inches rounded to one decimal place. *(4 marks)*

Answer:

Outside height: 1.5 ft. = 1.5 × 12 = 18 in.

Inside height: 18 in. – 1 in. = 17 in. *(1 mark)*

Capacity = $\pi r^2 h$

$2617 = \pi r^2(17)$ *(1 mark)*

$$\frac{2617}{(17\pi)} = \frac{17\pi r^2}{(17\pi)}$$

$$49 = r^2$$

$$7 = r \qquad \textit{(1 mark)}$$

Inside diameter = 2(7) = 14 in.

Outside diameter: 2(7) + 2(1) = 16 in. *(1 mark)*

Name: _____

b) Calculate the volume of water in the pot if it is filled to three inches below the top edge. Write your answer in litres, rounded to one decimal place. Note some conversion factors: 1 litre = 61 $in.^3$, and 1 $ft.^3$ = 28.3 L. *(3 marks)*

Answer:

Inside height of pot: 18 in. – 1 in. = 17 in.

Depth of water: 17 in. – 3 in. = 14 in. *(1 mark)*

$V = \pi r^2 h$

$= \pi(7)^2(14)$

$= 2155.13 \text{ in.}^3$ *(1 mark)*

Using: 1 L = 61 $in.^3$

$$61 \text{ in.}^3 = 1 \text{ L}$$

$$1 \text{ in.}^3 = \frac{1}{61} \text{ L}$$

$$2155.13 \text{ in.}^3 = \left(\frac{1}{61}\right) 2155.13 \text{ L}$$

$$= 35.33 \text{ L}$$

The volume of water = 35.3 L *(1 mark)*

OR

Using: 1 $ft.^3$ = 28.3 L

$$\text{Volume: } 2155.13 \text{ in.}^3 = \left(\frac{2155.13}{12^3}\right) \text{ft.}^3$$

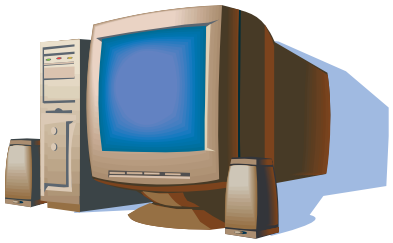
$$= 1.24718 \text{ ft.}^3$$

Volume: 1.24718 $ft.^3$ = (1.24718 × 28.3) L = 35.3 L

The volume of water = 35.3 L

(Module 2, Lesson 5)

5. You are trying to find the surface area of your computer screen and you do not have any measuring tape or ruler. You decide to use a referent to measure the screen.



a) State what you would use as a referent. *(1 mark)*

Answer:

Answers may vary. Some possibilities: computer paper, paper clip, width of hand, length of computer mouse (around 10 cm), etc.

b) Explain how you would use the referent to estimate the surface area of your computer screen. *(2 marks)*

Answer:

Answers may vary. For example, to use a piece of computer paper, you could estimate the height of the screen by comparing it to the height of the paper, and estimate the width of the screen by seeing how many widths of paper would fit across. Since we know that height and width of computer paper (11 by 8.5 inches), you can then approximate the surface area because you know the approximate dimensions.

(Module 2, Lesson 2)

Name: _____

Module 3: Statistics ***(16 marks)***

1. Dakota is a member of the student council and is organizing the pizza lunch for all the council members. At the last meeting, the secretary surveyed everyone to find out what kind of pizza they like to eat. The following data was collected.

   Pepperoni—5

   Cheese—8

   Hawaiian—3

   Vegetarian—3

   Meat Lovers—7

   Draw a circle graph to represent the data. Be sure to include a title and all the appropriate labels. *(13 marks)*

   Answer:

   Total members: 5 + 8 + 3 + 3 + 7 = 26

   Percents *(3 marks for total and percents)*

   $$\text{pepperoni} = \frac{5}{26} = 19\%$$

   $$\text{cheese} = \frac{8}{26} = 31\%$$

   $$\text{Hawaiian} = \frac{3}{26} = 12\%$$

   $$\text{vegetarian} = \frac{3}{26} = 12\%$$

   $$\text{meat lovers} = \frac{7}{27} = 27\%$$

   Note: This adds up to 101%, so students should choose one number to round down (e.g., cheese = 30%). *(½ mark for checking totals, rounding)*

Angles *(½ mark each)*

pepperoni = $0.19 \times 360^o = 68^o$

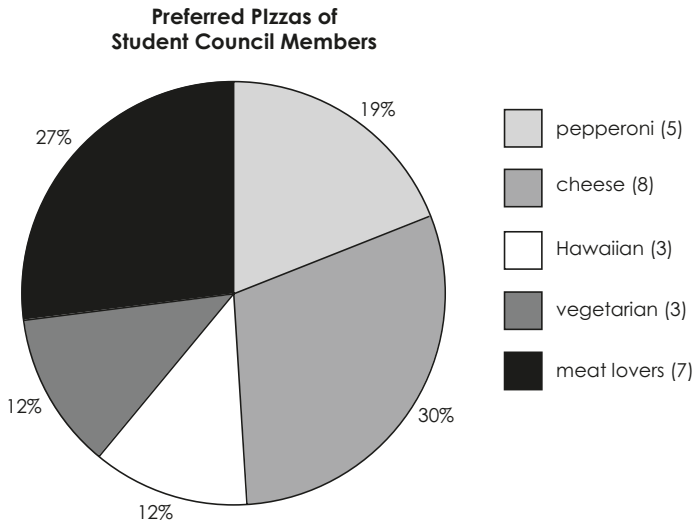
cheese = $0.30 \times 360^o = 108^o$

Hawaiian = $0.12 \times 360^o = 43^o$

vegetarian = $0.12 \times 360^o = 43^o$

meat lovers = $0.27 \times 360^o = 97^o$

Note: This adds up to 359^o, so whatever was rounded down in the previous step (cheese) should be rounded up now (e.g., cheese = 109^o). *(½ mark for checking totals, rounding)*



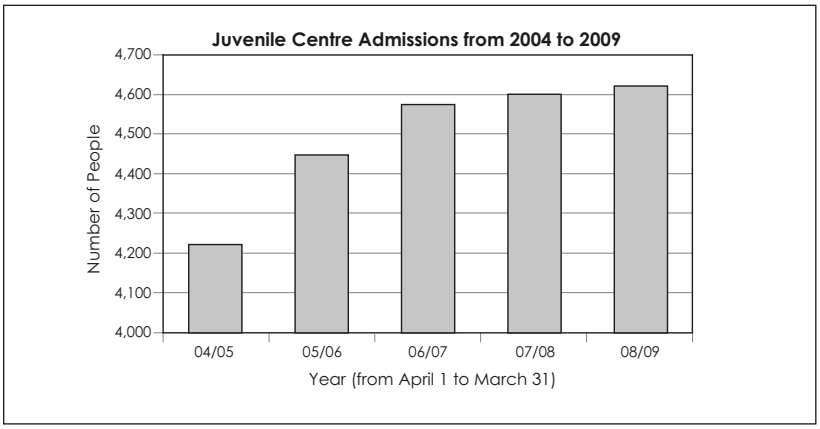
(1 mark for title, 1 mark for legend, 1 mark for data in the legend, 1 mark for % labels, ½ mark for each angle – accurate to within 1 degree)

(Module 3, Lesson 1)

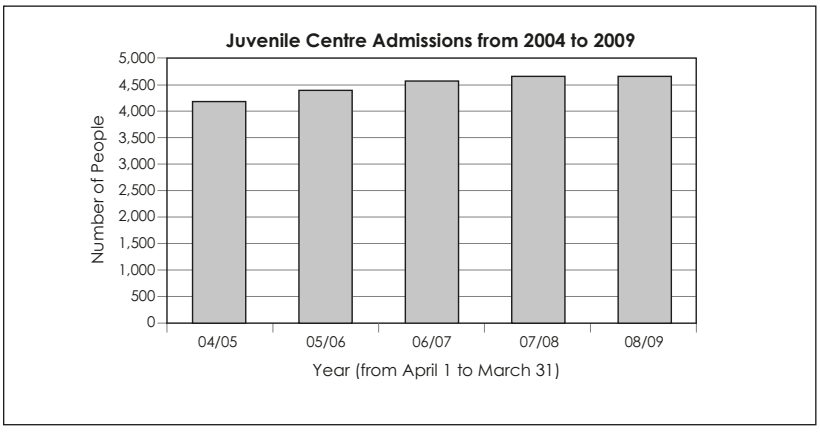
Name: _____

2. Use the Statistics Canada data depicted in the following graphs to answer the questions below.

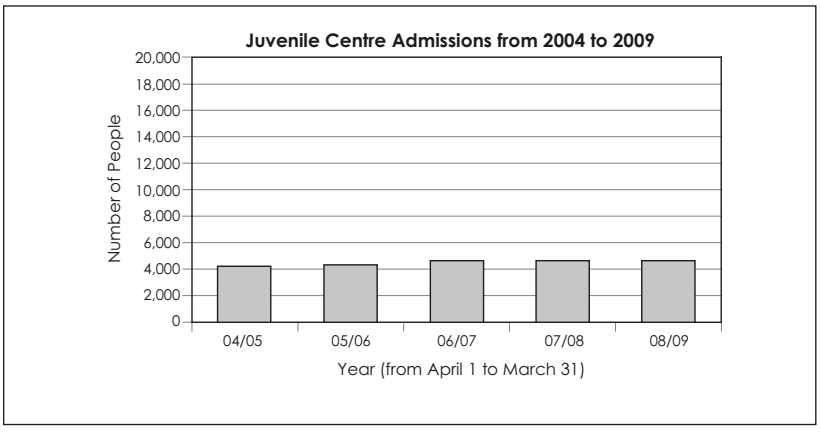
Graph A:



Graph B:



Graph C:



a) After a number of crimes are committed at local schools, the media is suggesting that harsher punishments are necessary to reduce the increasing amount of juvenile crime. Which graph would support this argument best? Explain why. *(2 marks)*

Answer:

Graph A—Because the vertical scale is so small, the change from one year to another appears quite large. This would help support the media's claim because it makes it seem as though youth crime is increasing by a lot each year. In fact, the largest growth is only (approximately) 200. This may seem like a lot, but $\frac{200}{4200}$ is less than 5%.

Graph B would not be as effective because, although it makes the number of admissions look high, it does not appear to be increasing greatly.

Graph C would not be effective because the bars are short (so it looks like there is little juvenile crime) and they do not seem to increase by very much.

b) Which graph is the most "honest" graph because it does not present the data in a distorted way? *(1 mark)*

Answer:

Graph B

(Module 3, Lesson 5)

Name: _____

Amortization Table					
Amortization Period Monthly Payment Per $1000 Loan Proceeds					
Annual Rate	1 Year Monthly	2 Years Monthly	3 Years Monthly	4 Years Monthly	5 Years Monthly
6.00%	$86.07	$44.33	$30.43	$23.49	$19.34
6.25%	$86.18	$44.44	$30.54	$23.61	$19.46
6.50%	$86.30	$44.56	$30.66	$23.72	$19.57
6.75%	$86.41	$44.67	$30.77	$23.84	$19.69
7.00%	$86.53	$44.78	$30.88	$23.95	$19.81
7.25%	$86.64	$44.89	$31.00	$24.07	$19.93
7.50%	$86.76	$45.01	$31.11	$24.19	$20.05
7.75%	$86.87	$45.12	$31.23	$24.30	$20.16
8.00%	$86.99	$45.24	$31.34	$24.42	$20.28
8.25%	$87.10	$45.34	$31.45	$24.53	$20.40
8.50%	$87.22	$45.46	$31.57	$24.65	$20.52
8.75%	$87.34	$45.57	$31.68	$24.71	$20.64
9.00%	$87.45	$45.68	$31.80	$24.89	$20.76
9.25%	$87.57	$45.80	$31.92	$25.00	$20.88
9.50%	$87.68	$45.91	$32.03	$25.12	$21.00
9.75%	$87.80	$46.03	$32.15	$25.24	$21.12
10.00%	$87.92	$46.14	$32.27	$25.36	$21.25
10.25%	$88.03	$46.26	$32.38	$25.48	$21.37
10.50%	$88.15	$46.38	$32.50	$25.60	$21.49
10.75%	$88.27	$46.49	$32.62	$25.72	$21.62
11.00%	$88.38	$46.61	$32.74	$25.85	$21.74
11.25%	$88.50	$46.72	$32.86	$25.97	$21.87
11.50%	$88.62	$46.84	$32.98	$26.09	$21.99
11.75%	$88.73	$46.96	$33.10	$26.21	$22.12
12.00%	$88.85	$47.07	$33.21	$26.33	$22.24
12.25%	$88.97	$47.19	$33.33	$26.46	$22.37
12.50%	$89.08	$47.31	$33.45	$26.58	$22.50
12.75%	$89.20	$47.42	$33.57	$26.70	$22.63
13.00%	$89.32	$47.54	$33.69	$26.83	$22.75
13.25%	$89.43	$47.66	$33.81	$26.95	$22.88
13.50%	$89.55	$47.78	$33.94	$27.08	$23.01
13.75%	$89.67	$47.89	$34.06	$27.20	$23.14
14.00%	$89.79	$48.01	$34.18	$27.33	$23.27

GRADE 11 ESSENTIAL MATHEMATICS (30S)

Final Practice Exam

GRADE 11 ESSENTIAL MATHEMATICS

Final Practice Examination

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____ /100 = _____ %

Comments:

Instructions

The final examination is based on Modules 4 to 7 of the Grade 11 Essential Mathematics course. It is worth 12.5% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the final examination.

Format

The format of the examination will be as follows:

Part A: Multiple Choice	10 marks
Part B: Definitions	10 marks
Part C: Long Answer	80 marks
Total	100 marks

(see over)

Instructions (continued)

Resources Provided

- two blank monthly budget forms are included at the end of the exam

Resources Required (Not Provided)

To complete this examination, you will need:

- pens/pencils (2 or 3 or each)
- blank paper
- scientific or graphing calculator
- geometry set (including a ruler, a protractor, and a compass)
- Final Exam Resource Sheet (The Final Exam Resource Sheet must be handed in with the exam.)

Resources Optional

- pencil crayons for section related to Module 7: Design Modelling

Notes

For Part C

- show all calculations and formulas
- include units where appropriate
- clearly state your final answer
- diagrams may not be drawn to scale

Name: _____

Part A: Multiple Choice *(10 x 1 = 10 marks)*

Circle the letter of the best answer for each question.

1. Select the account that would be appropriate for Janelle.

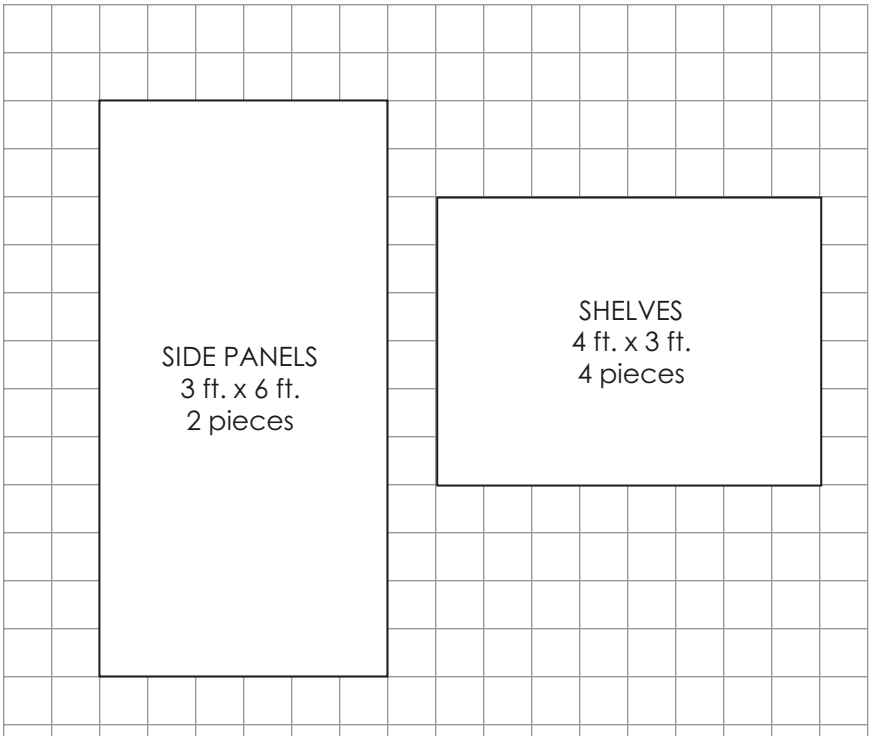
   Janelle has her first part-time job and needs to open a bank account for her money. At the moment, she is not concerned with setting any money aside for later. She just wants to have access to it for her usual spending.

   a) Savings account
   b) Chequing account
   c) Electronic savings account
   d) Combined chequing and savings account

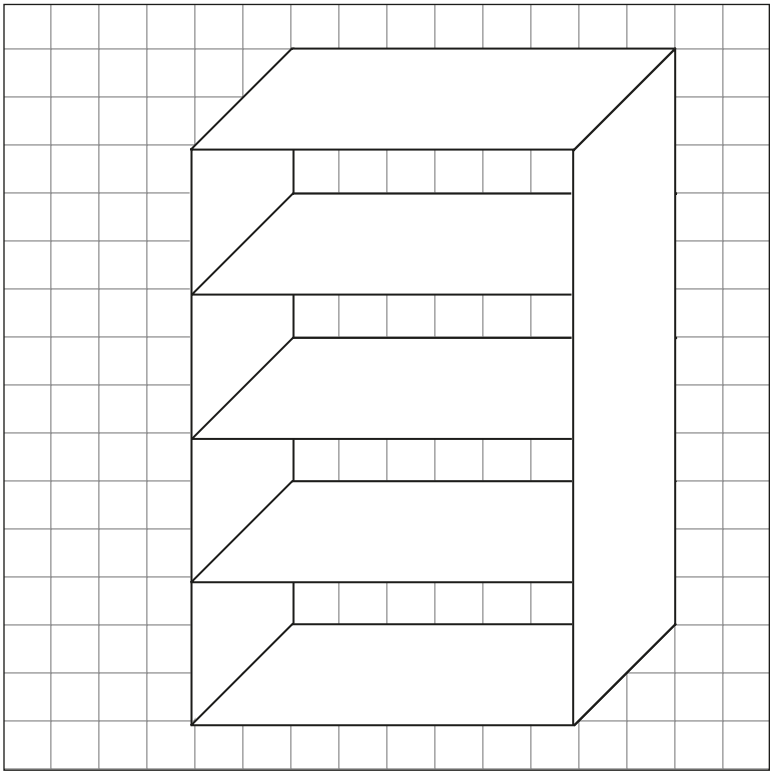
2. Calculate the slope of a line if it goes up from 3 to 8 as you move 4 units to the right.

   a) $\frac{8}{3}$
   b) $\frac{1}{2}$
   c) $\frac{4}{5}$
   d) $\frac{5}{4}$

3. Which of the following objects is made up of these component parts?

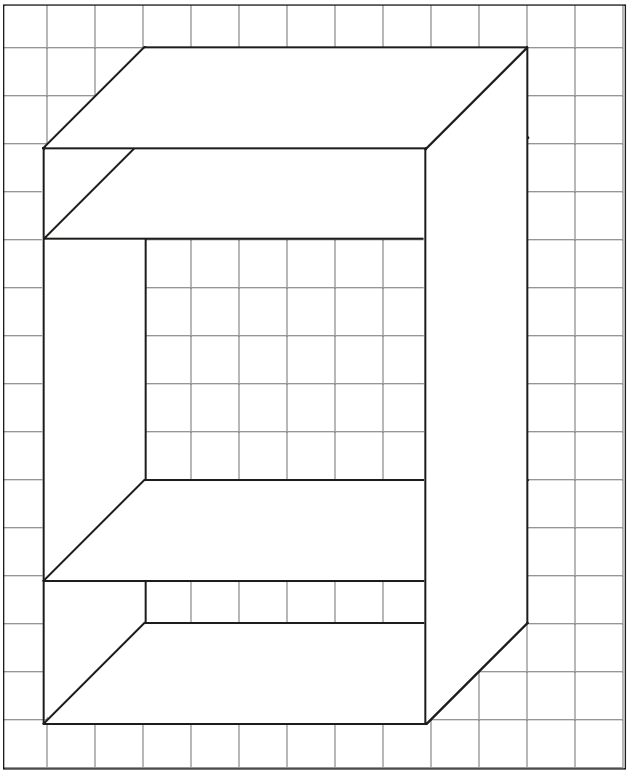


a)

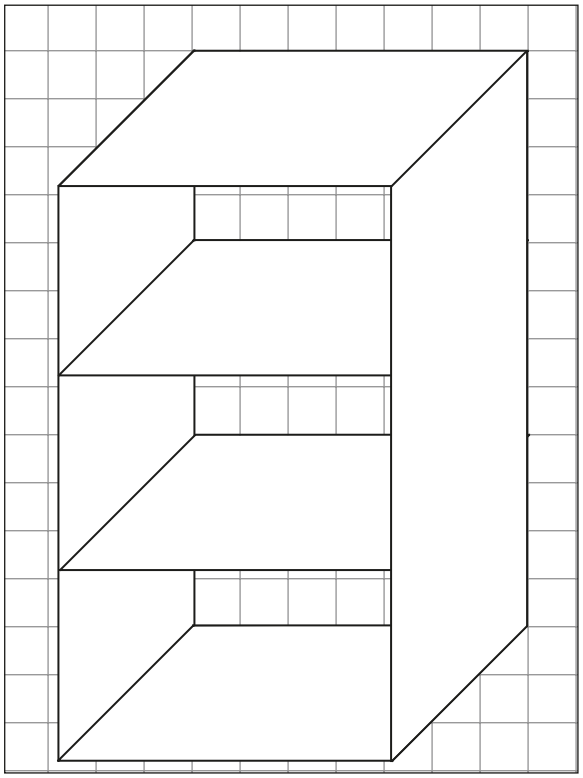


Name: _____

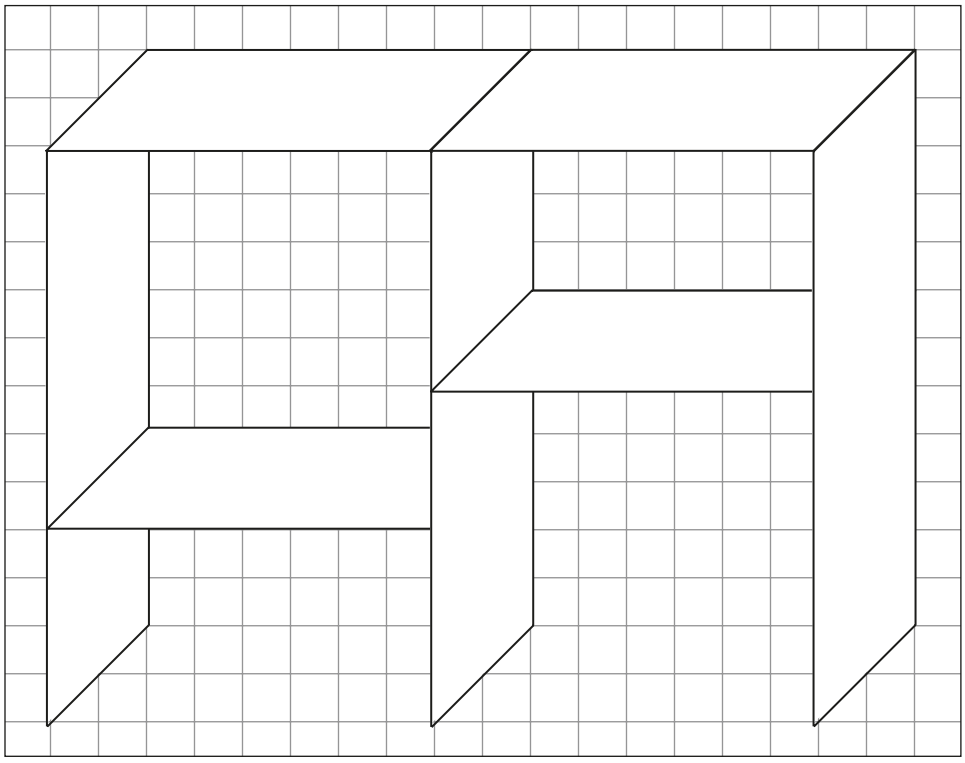
b)



c)



d)



4. Which of the following is **not** an example of a linear relation or pattern?

a)

Age (years)	5	10	15	20	25
Weight (lbs.)	60	110	150	185	185

This table is not a linear relation because the age increases by five years each time, but the weight does not always increase by the same amount each time.

b)

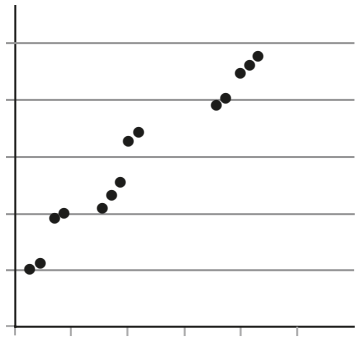
Sam's Age	2	4	6	8	10
Jay's Age	1	3	5	7	9

This table is a linear relation because for every two years that Sam ages, Jay also ages two years.

c) 2, 10, 18, 26, 34, 42…

This sequence is a linear relation because it increases by 8, but it is not a relation—a relation must have two parts that are mathematically connected in some way.

d)

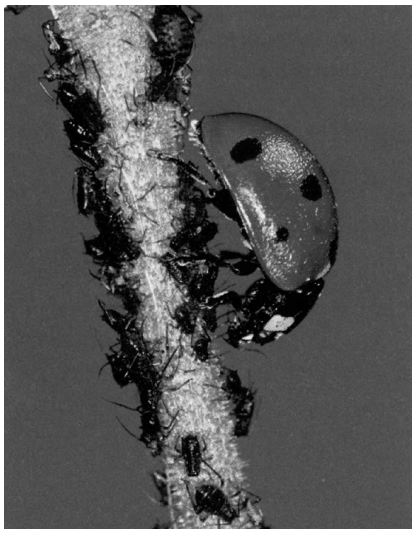


Although it doesn't look completely linear, a line of best fit could be drawn that would pass through or near all points.

Name: _____

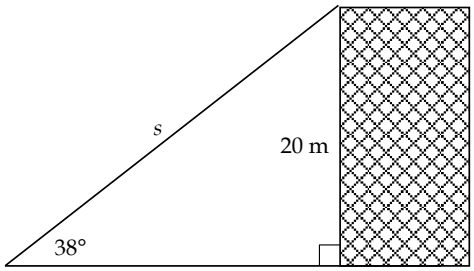
5. The scale factor of the drawing below is 1 cm : $\frac{1}{3}$ cm. What is the actual length of the ladybug?

   a) 1 cm
   b) 2 cm
   c) 9 cm
   d) 3 cm



6. Select the trigonometric ratio you would use to solve for *s*.

   a) tangent ratio
   b) cosine ratio
   c) sine ratio
   d) Pythagorean theorem

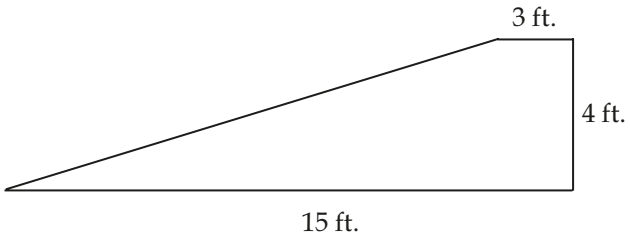


7. Identify a benefit of using your debit card to purchase goods.

   a) The interest charged for the money you spend is usually somewhat less than the rate charged by a credit card company.
   b) You are able to overdraw your bank account at any time as much as you want.
   c) If your card and PIN are stolen, you are reimbursed for the money taken by the thief.
   d) You can use it to purchase goods without using credit or adding to your debt.

8. It is important for you to keep your personal and financial information secure. Select the statement that is not a proper security precaution.
   a) Filter your email so that you can dispose of the junk mail without having to open every message.
   b) Share your password with people so that, if you forget it, they can tell you what it is.
   c) Use a combination of letters and numbers in your password.
   d) Do not share personal information on websites that are not marked as secure.

9. Calculate the slope of the ramp.
   a) $\frac{1}{3}$
   b) 3
   c) $\frac{4}{15}$
   d) $\frac{1}{5}$



10. The slope of a horizontal line is:
   a) undefined
   b) 1
   c) 0
   d) –1

Name: _____

Part B: Definitions *(10 x 1 = 10 marks)*

Match each definition with the correct term from the list below. Write the correct term on the line below each definition. Terms are used only once. Not all terms have a definition provided.

angle of depression	cosine ratio	fixed value	recurring expenses
angle of elevation	deficit	geometric sequence	savings account
arithmetic sequence	dependent variable	independent variable	scale factor
budget	direct variation	interpolation	scatterplot
cheque register	electronic savings account	linear relation	service charge
chequing account	expenses	oblique projection	sine ratio
combination account	exploded view	one-point perspective	slope
component view	extrapolation	outstanding entry	tangent ratio
constant of variation		partial variation	unexpected expenses
			variable

1. The view of an object that shows all the parts, and how they are oriented to each other. _____
2. A set of numbers such that each number is a certain amount larger/smaller than the previous number. _____
3. This account is used for day-to-day banking, and usually requires a minimum balance to pay interest. _____
4. Estimating values outside the set of data. _____
5. The ratio that compares the dependent and independent variables of a linear relation. _____
6. Money paid to another person or company to pay for an item or service. _____
7. A ratio that compares the sides opposite and adjacent to an angle in a right triangle. _____
8. An organized way to analyze and plan the way you save and spend money, and that allows you to set and achieve goals. _____
9. A symbol or letter that represents an unknown value. _____
10. A variable in a math relationship that is not affected by the other variable. _____

Part C: Long Answer *(80 marks)*

Show all formulas and calculations used to answer each question. Write your final answers rounded to the number of decimal places indicated in the questions. Include units where appropriate. Clearly state your final answer.

Module 4: Managing Money ***(22 marks)***

1. Delanie is a single mother with two children, ages 6 and 10. Delanie works 35 hours per week as an information technologist. Her net annual income is $26,945. Delanie is now renting an apartment but would like to own her own condominium. She receives a monthly child tax benefit of $72.60 for her two children. Delanie's expenses for the month of September will include the following:
   - rent of $550.00
   - adult bus pass of $55.85
   - bus tickets of $7.50 for each child
   - taxis of $14.00
   - telecommunications payment of $30.00
   - hydro payment of $75.00
   - groceries of $480.00
   - clothing of $200.00
   - eyeglass expense of $142.60
   - school supplies of $60.00
   - child-care expense of $180
   - gymnastics lessons of $30
   - personal care expense of $28.00
   - entertainment expense of $40.00
   - gifts of $30.00

   Her annual expenses will include the following:
   - life insurance premiums of $249.10
   - tenant insurance of $212
   - children's summer camp of $350
   - Christmas spending of $240
   - newspaper subscription of $170

Name: _____

a) Prepare a budget for Delanie for the month of September. Use the budget form at the end of the exam. *(10 marks)*

b) Is Delanie's budget in a deficit position? *(1 mark)*

c) Delanie would like to save $25 000 to use as a down payment on a condo. Will she be able to save this amount in the near future? *(1 mark)*

d) The cost of children's bus tickets increases to $10.00 while adult bus passes increase to $75 per month. Explain how these cost increases will affect Delanie's budget. For example, will she be in a deficit position? *(2 marks)*

2. List two advantages of online banking. *(2 marks)*

3. Datu works at a financial institution. It is his responsibility to ensure that all cheques and deposit slips are authentic (real) and correct. Today, Datu found a cheque and a deposit slip that were completed incorrectly. Identify what is wrong with each and correct the mistake. *(2 marks each × 2 = 4 marks)*

a)

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

May 31 20 12

Pay to the
order of Jane Smith $ 435.10

four hundred thirty DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

_____ 20 _____

Pay to the
order of _____ $ _____

_____ DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

Name: _____

b)

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

June 10 20 12

J. Walker
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY	20	00
COIN	452	61
CHEQUES OR COUPONS	0	90
	13	57
TOTAL	487	07
LESS Received in Cash	36	00
Please sign in presence of Teller AMOUNT OF DEPOSIT $	415	07

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

June 10 20 12

J. Walker
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY		
COIN		
CHEQUES OR COUPONS		
TOTAL		
LESS Received in Cash		
Please sign in presence of Teller AMOUNT OF DEPOSIT $		

4. Compare the following monthly account statement with its cheque register. There is one error and/or omission in the cheque register. Locate the error and/or omission and make the necessary change in the cheque register. *(2 marks)*

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Feb. 1	Balance Forward			362.54
	Deposit		400.00	762.54
Feb. 4	Cheque 183	25.16		737.38
Feb. 5	Cheque 181	32.27		
	Cheque 184	465.00		240.11
Feb. 15	Deposit		400.00	640.11
Feb. 20	Cheque 186	126.50		513.61
Feb. 22	Cheque 185	84.25		429.36
Feb. 29	Cheque 187	19.98		409.38

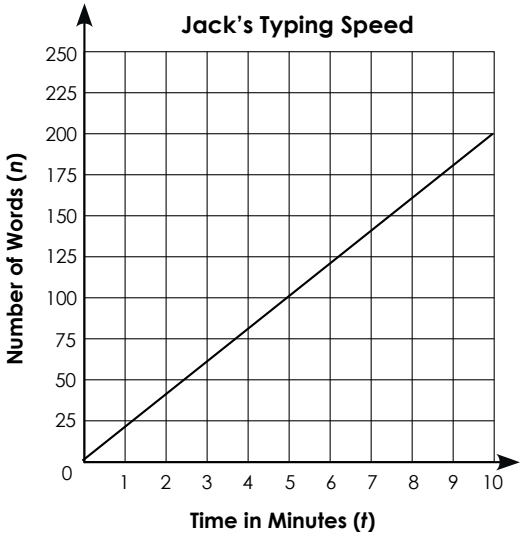
Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									362	54
Feb. 1		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	762	54
3	181	TO Photocopies For Less	32	27				CHEQUE – DEPOSIT +	32	27
		FOR						BALANCE ➜	730	27
3	182	TO M. Fenster	82	60				CHEQUE – DEPOSIT +	82	60
		FOR						BALANCE ➜	647	67
3	183	TO Wholesome Goods	25	16				CHEQUE – DEPOSIT +	25	16
		FOR						BALANCE ➜	622	51
4	184	TO O & R Rental Agency	465	00				CHEQUE – DEPOSIT +	465	00
		FOR						BALANCE ➜	157	51
15		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	557	51
19	185	TO Budget Department Store	84	25				CHEQUE – DEPOSIT +	84	25
		FOR						BALANCE ➜	473	26
19	186	TO Zack's Electronics	126	00				CHEQUE – DEPOSIT +	126	00
		FOR						BALANCE ➜	347	26
25	187	TO Terry's Photos	19	98				CHEQUE – DEPOSIT +	19	98
		FOR						BALANCE ➜	327	28
Mar. 2		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	727	28

Name: _____

Module 5: Relations and Patterns (30 marks)

1. The graph below shows Jack's typing speed.



State the independent and dependent variables. *(2 marks)*

Independent variable: _____

Dependent variable: _____

2. a) How can you tell whether a set of numbers represents a linear pattern. Use an example as part of your answer. *(2 marks)*

b) Does the table of values represent a linear relation? Explain your answer. *(3 marks)*

Number of Fish	3	6	15
Profit ($)	10	20	50

3. During the second week of November, Emma works the following hours: Monday, 2; Tuesday, 6; Wednesday, 3; Thursday, 8; Friday, 7. Emily's hourly rate is $15.

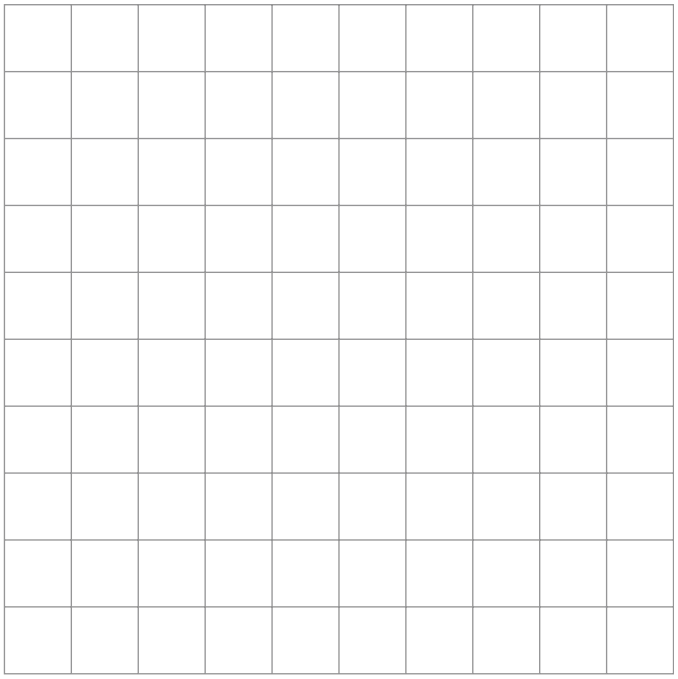
   a) Express the relation between daily gross pay and hours worked with an equation. *(2 marks)*

   b) Complete the following table of values for the relation between daily gross pay and hours worked. *(3 marks)*

	Mon.	Tues.	Wed.	Thurs.	Fri.
Hours Worked					
Gross Pay ($)					

Name: _____

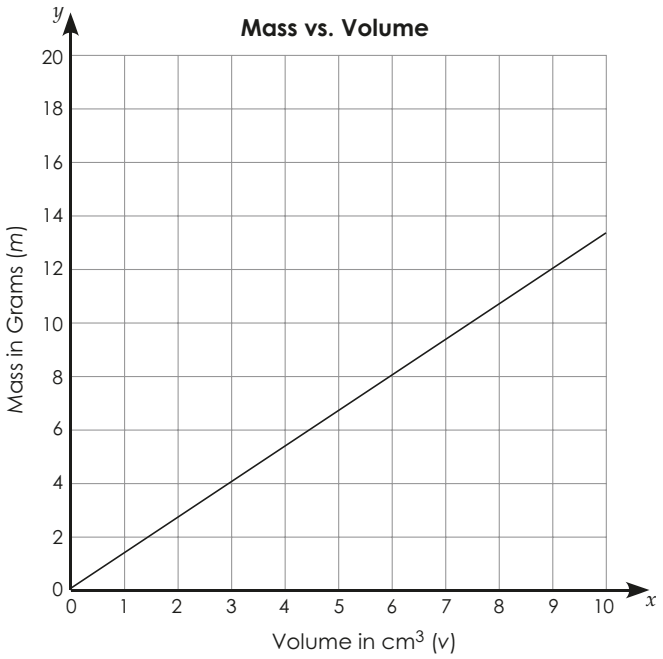
c) Draw a graph to show the relation between gross pay and the number of hours worked. Write the appropriate scales and labels onto the graph. *(5 marks)*



d) Explain how the hourly rate is related to the slope of the graph. For example, if she were paid only $10 per hour, how would this affect the graph? *(1 mark)*

e) If Emily were paid $20 per shift plus her wage of $15 per hour, how would this affect the graph? *(2 marks)*

4. The relation between the mass (m) of a substance and its volume (V) is expressed by the following graph.



a) Choose two points on the graph and use the coordinates of these points to calculate the slope. *(2 marks)*

b) Express this relation as an equation. Be sure to identify the variables. *(2 marks)*

Name: _____

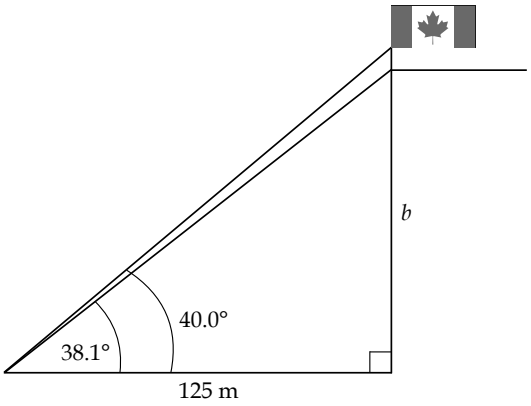
c) Find the volume of this substance with a mass of 18 g. Use the formula from part b. Write your answer rounded to one decimal place. *(2 marks)*

d) Find the mass of this substance with a volume of 9 cm^3. Show your work on the graph. *(1 mark)*

5. Sam's car has a broken speedometer, and so he is using his GPS to determine his speed. According to the GPS, his speed is 40 metres per second. The speed limit is 110 km/h. Is Sam speeding? Show how you arrive at your answer. *(3 marks)*

Module 6: Trigonometry ***(7 marks)***

1. From a point 125 m from the foot of a building, the angles of elevation of the top and bottom of the flagpole are 40.0° and 38.1° respectively. The flagpole is set on the roof of the building.

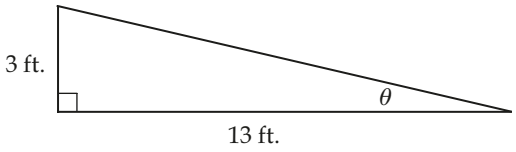


a) Calculate the height of the building *(2 marks)*

Name: _____

b) Calculate the height of the flagpole. *(3 marks)*

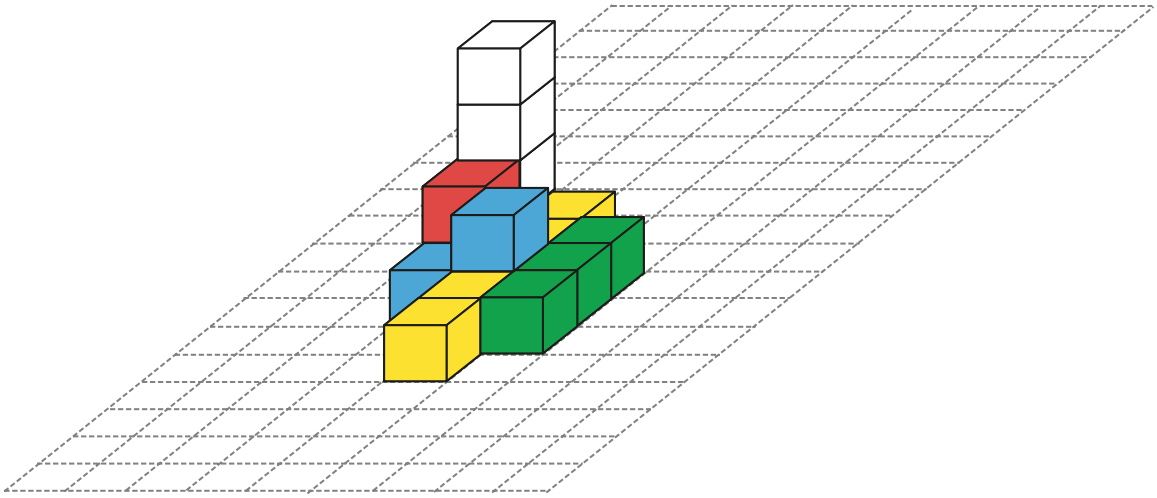
2. Calculate the measure of θ in degrees. Write your answer rounded to one decimal place. *(2 marks)*



Module 7: Design Modelling ***(21 marks)***

Blue cube = 1 unit long
Red bar = 2 units long
Green bar = 3 units long
White bar = 4 units long
Yellow bar = 5 units long

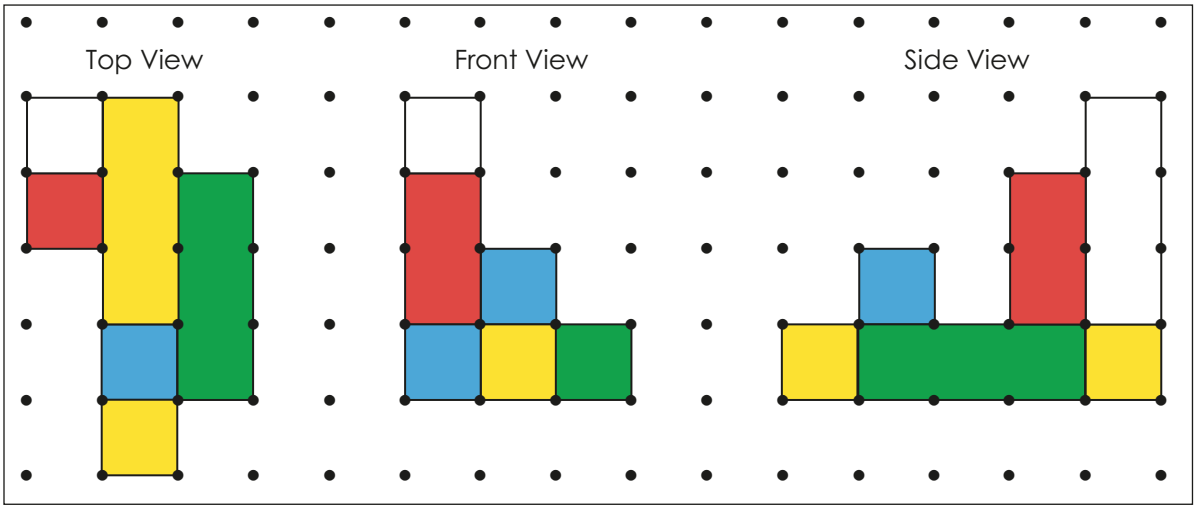
1. Use the 3-D drawing below to answer the questions that follow.



a) Select the 2-D view that matches the drawing above. *(1 mark)*

Answer: _____

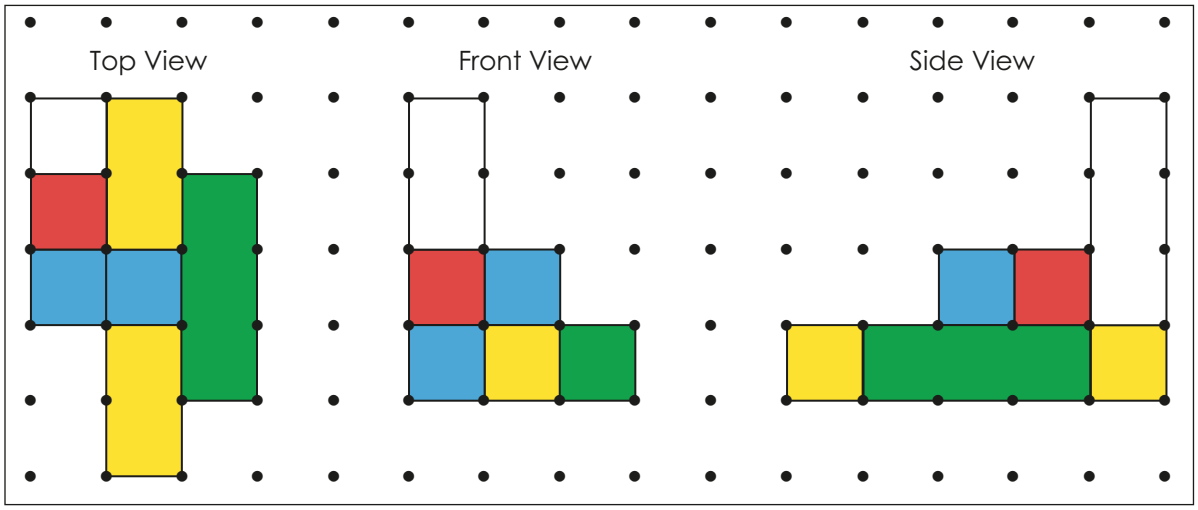
Set I:



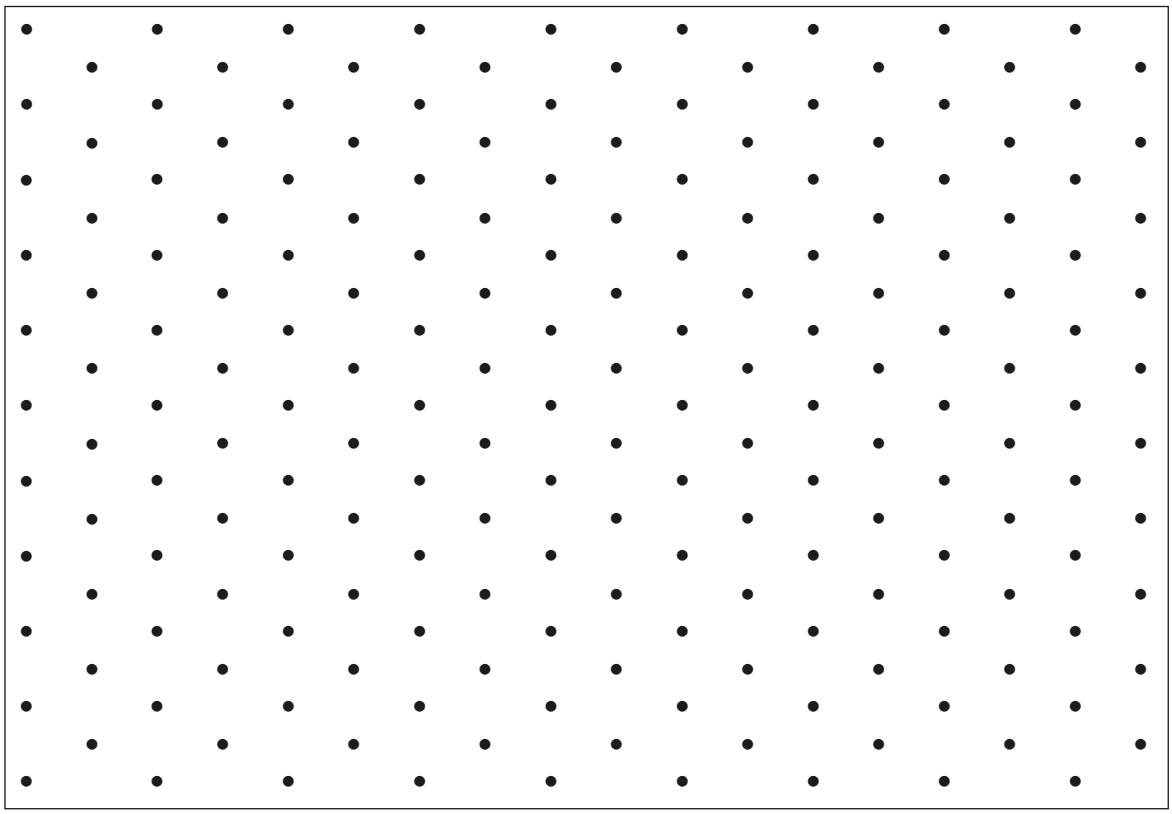
or

Name: _____

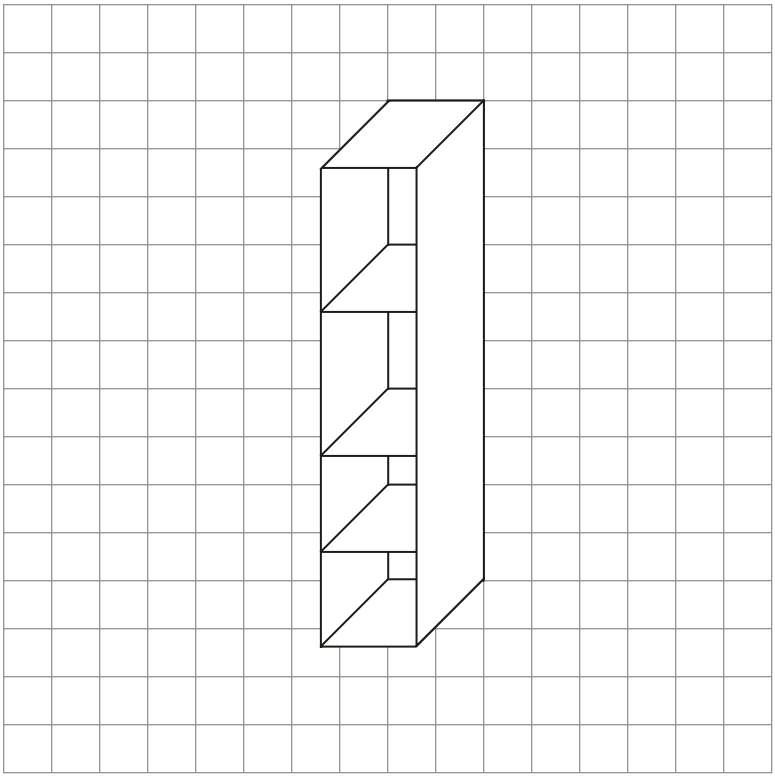
Set II:



b) Use the isometric dot paper below to draw a 3-D representation for the other set of 2-D views. Be sure to label each drawing, and label each section of each drawing with the appropriate colour if you do not have pencil crayons. *(6 marks)*



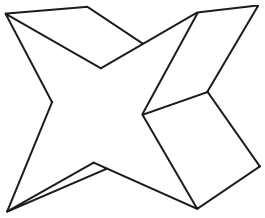
2. a) Sketch the exploded view of the following object. *(5 marks)*



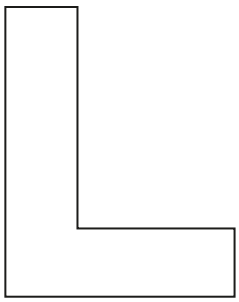
Name: _____

b) The scale of this drawing is 1 inch : $2\frac{1}{4}$ ft. What are the actual dimensions of the object? *(4 marks)*

3. a) Where is the vanishing point for the drawing shown below? *(1 mark)*



b) Complete a one-point perspective drawing of the object below, if the vanishing point is below and to the right. *(4 marks)*



Name: _____

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***		
1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Name: _____

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***		
1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

GRADE 11 ESSENTIAL MATHEMATICS (30S)

Final Practice Exam
Answer Key

GRADE 11 ESSENTIAL MATHEMATICS

Final Practice Examination
Answer Key

Name: _____

Student Number: _____

Attending ❑ Non-Attending ❑

Phone Number: _____

Address: _____

_____

_____

For Marker's Use Only

Date: _____

Final Mark: _____ / 100 = _____ %

Comments:

Instructions

The final examination is based on Modules 4 to 7 of the Grade 11 Essential Mathematics course. It is worth 12.5% of your final mark in this course.

Time

You will have a maximum of **2.5 hours** to complete the final examination.

Format

The format of the examination will be as follows:

Part A: Multiple Choice	10 marks
Part B: Definitions	10 marks
Part C: Long Answer	80 marks
Total	100 marks

(see over)

Instructions (continued)

Resources Provided

- two blank monthly budget forms are included at the end of the exam

Resources Required (Not Provided)

To complete this examination, you will need:

- pens/pencils (2 or 3 or each)
- blank paper
- scientific or graphing calculator
- geometry set (including a ruler, a protractor, and a compass)
- Final Exam Resource Sheet (The Final Exam Resource Sheet must be handed in with the exam.)

Resources Optional

- pencil crayons for section related to Module 7: Design Modelling

Notes

For Part C

- show all calculations and formulas
- include units where appropriate
- clearly state your final answer
- diagrams may not be drawn to scale

Name: _____

Part A: Multiple Choice *(10 x 1 = 10 marks)*

Circle the letter of the best answer for each question.

1. Select the account that would be appropriate for Janelle.

   Janelle has her first part-time job and needs to open a bank account for her money. At the moment, she is not concerned with setting any money aside for later. She just wants to have access to it for her usual spending.

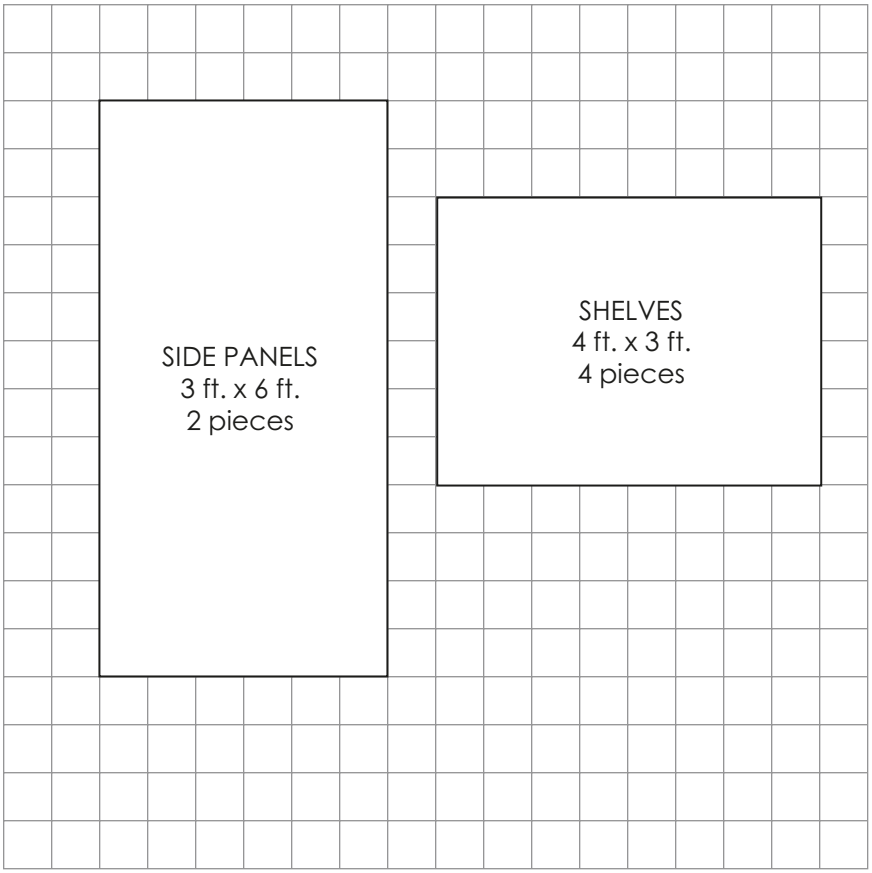
   a) Savings account
   (b) Chequing account
   c) Electronic savings account
   d) Combined chequing and savings account (Module 4, Lesson 3)

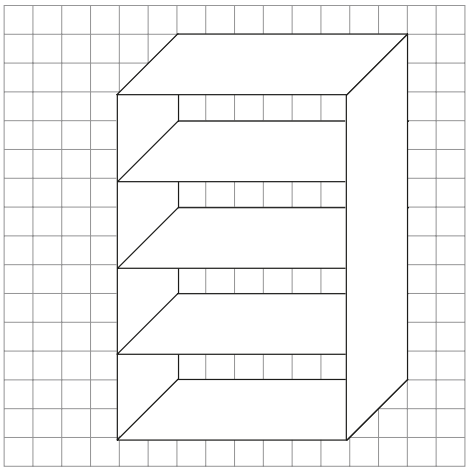
2. Calculate the slope of a line if it goes up from 3 to 8 as you move 4 units to the right.

   a) $\frac{8}{3}$
   b) $\frac{1}{2}$
   c) $\frac{4}{5}$
   (d) $\frac{5}{4}$

   (Module 5, Lesson 4)

3. Which of the following objects is made up of these component parts?



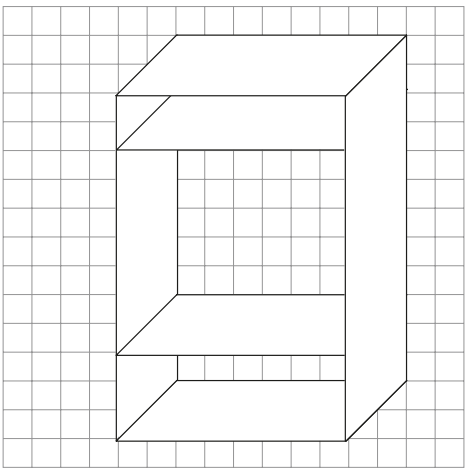
a) 

Too many shelves.

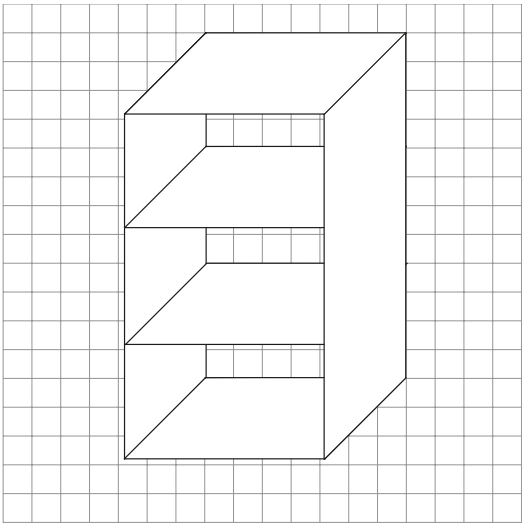
(Module 7, Lesson 4)

Name: _____

b)

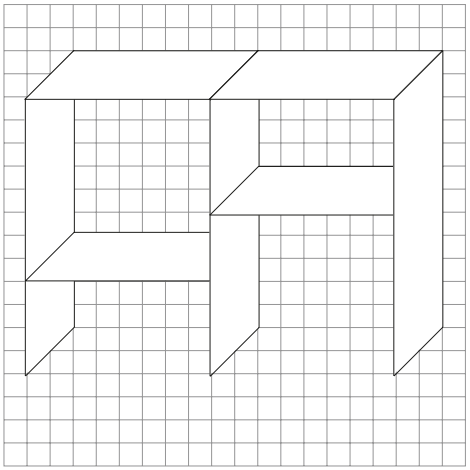


c)



Dimensions are incorrect—too narrow and too deep.

d)



Too many vertical panels.

4. Which of the following is **not** an example of a linear relation or pattern?

a)

Age (years)	5	10	15	20	25
Weight (lbs.)	60	110	150	185	185

This table is not a linear relation because the age increases by five years each time, but the weight does not always increase by the same amount each time.

b)

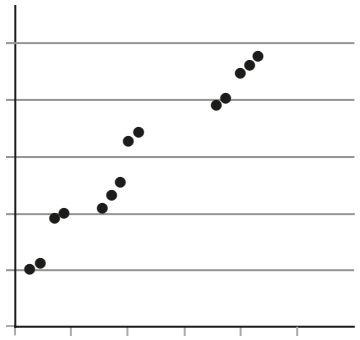
Sam's Age	2	4	6	8	10
Jay's Age	1	3	5	7	9

This table is a linear relation because for every two years that Sam ages, Jay also ages two years.

c) 2, 10, 18, 26, 34, 42…

This sequence is a linear relation because it increases by 8, but it is not a relation—a relation must have two parts that are mathematically connected in some way.

d)



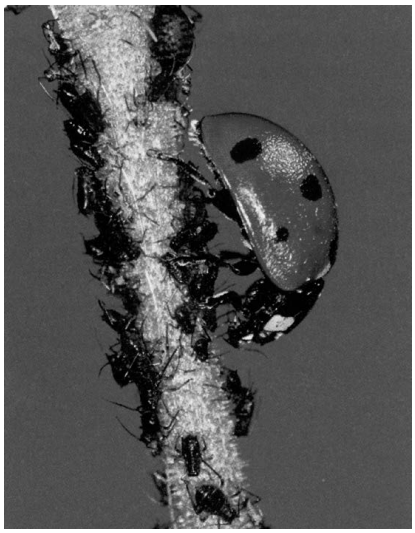
Although it doesn't look completely linear, a line of best fit could be drawn that would pass through or near all points.

(Module 5, Lesson 2)

Name: _____

5. The scale factor of the drawing below is 1 cm : $\frac{1}{3}$ cm. What is the actual length of the ladybug?

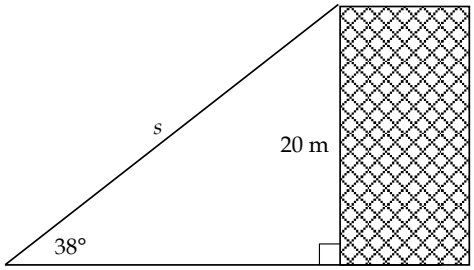
   a) 1 cm
   b) 2 cm
   c) 9 cm
   d) 3 cm



(Module 5, Lesson 7)

6. Select the trigonometric ratio you would use to solve for *s*.

   a) tangent ratio
   b) cosine ratio
   c) sine ratio
   d) Pythagorean theorem



(Module 6, Lesson 1)

7. Identify a benefit of using your debit card to purchase goods.

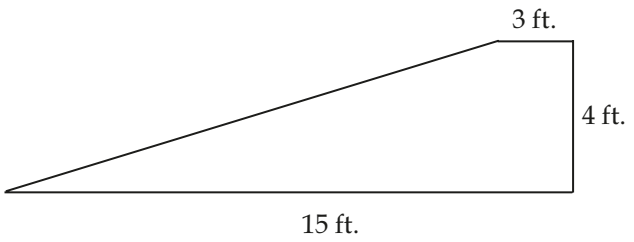
   a) The interest charged for the money you spend is usually somewhat less than the rate charged by a credit card company.
   b) You are able to overdraw your bank account at any time as much as you want.
   c) If your card and PIN are stolen, you are reimbursed for the money taken by the thief.
   d) You can use it to purchase goods without using credit or adding to your debt.

(Module 4, Lesson 3)

8. It is important for you to keep your personal and financial information secure. Select the statement that is not a proper security precaution.
   a) Filter your email so that you can dispose of the junk mail without having to open every message.
   b) Share your password with people so that, if you forget it, they can tell you what it is.
   c) Use a combination of letters and numbers in your password.
   d) Do not share personal information on websites that are not marked as secure.

(Module 4, Lesson 3)

9. Calculate the slope of the ramp.
   a) $\frac{1}{3}$
   b) 3
   c) $\frac{4}{15}$
   d) $\frac{1}{5}$



(Module 5, Lesson 5)

10. The slope of a horizontal line is:
   a) undefined
   b) 1
   c) 0
   d) –1

(Module 5, Lesson 4)

Name: _____

Part B: Definitions *(10 x 1 = 10 marks)*

Match each definition with the correct term from the list below. Write the correct term on the line below each definition. Terms are used only once. Not all terms have a definition provided.

angle of depression	cosine ratio	fixed value	recurring expenses
angle of elevation	deficit	geometric sequence	savings account
arithmetic sequence	dependent variable	independent variable	scale factor
budget	direct variation	interpolation	scatterplot
cheque register	electronic savings account	linear relation	service charge
chequing account	expenses	oblique projection	sine ratio
combination account	exploded view	one-point perspective	slope
component view	extrapolation	outstanding entry	tangent ratio
constant of variation		partial variation	unexpected expenses
			variable

1. The view of an object that shows all the parts, and how they are oriented to each other. **exploded view** (Module 7, Lesson 4)
2. A set of numbers such that each number is a certain amount larger/smaller than the previous number. **arithmetic sequence** (Module 5, Lesson 2)
3. This account is used for day-to-day banking, and usually requires a minimum balance to pay interest. **combination account** (Module 4, Lesson 3)
4. Estimating values outside the set of data. **extrapolation** (Module 5, Lesson 3)
5. The ratio that compares the dependent and independent variables of a linear relation. **slope** (Module 5, Lesson 4)
6. Money paid to another person or company to pay for an item or service. **expenses** (Module 4, Lesson 1)
7. A ratio that compares the sides opposite and adjacent to an angle in a right triangle. **tangent ratio** (Module 6, Lesson 1)
8. An organized way to analyze and plan the way you save and spend money, and that allows you to set and achieve goals. **budget** (Module 4, Lesson 1)
9. A symbol or letter that represents an unknown value. **variable** (Module 5, Lesson 3)
10. A variable in a math relationship that is not affected by the other variable. **independent variable** (Module 5, Lesson 1)

Part C: Long Answer *(80 marks)*

Show all formulas and calculations used to answer each question. Write your final answers rounded to the number of decimal places indicated in the questions. Include units where appropriate. Clearly state your final answer.

Module 4: Managing Money ***(22 marks)***

1. Delanie is a single mother with two children, ages 6 and 10. Delanie works 35 hours per week as an information technologist. Her net annual income is $26,945. Delanie is now renting an apartment but would like to own her own condominium. She receives a monthly child tax benefit of $72.60 for her two children. Delanie's expenses for the month of September will include the following:
   - rent of $550.00
   - adult bus pass of $55.85
   - bus tickets of $7.50 for each child
   - taxis of $14.00
   - telecommunications payment of $30.00
   - hydro payment of $75.00
   - groceries of $480.00
   - clothing of $200.00
   - eyeglass expense of $142.60
   - school supplies of $60.00
   - child-care expense of $180
   - gymnastics lessons of $30
   - personal care expense of $28.00
   - entertainment expense of $40.00
   - gifts of $30.00

   Her annual expenses will include the following:
   - life insurance premiums of $249.10
   - tenant insurance of $212
   - children's summer camp of $350
   - Christmas spending of $240
   - newspaper subscription of $170

Name: _____

a) Prepare a budget for Delanie for the month of September. Use the budget form at the end of the exam. *(10 marks)*

Answer:

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
1) ***NET INCOME***			1) $ 2,318.02
Primary Income	$	$ 26,945.00	
Secondary Income	$	$	
Other Income	$ 72.60/month	$ 871.20	
Total Annual Income		$ 27,816.20	
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 231.80
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ 550.00	
Car Payments (bus passes, tickets, and taxis)		$ 84.85	(55.85 + 2(7.50) + 14.00)
Telecommunications		$ 30.00	
Hydro		$ 75.00	
Other Utilities		$	
Cable		$	
Groceries		$ 480.00	
Clothing		$ 200.00	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$	
Personal Care		$ 28.00	
Entertainment		$ 40.00	
Other (glasses, school supplies, and child care)		$ 382.60	
Other (gymnastics and gifts)		$ 60.80	
TOTAL MONTHLY EXPENSES			3) $ 1,930.45

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$	$	
Life Insurance	$ 249.10	$ 20.76	
Property Taxes	$	$	
Home Insurance	$ 212.00	$ 17.67	
Vacations	$	$	
Newspapers & Periodicals	$ 170.00	$ 14.17	
Other (camp and gifts)	$ 590.00	$ 49.17	
TOTAL MONTHLY EXPENSES			4) $ 101.77

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 2,318.02
2. SAVINGS	2)	$ 231.80	
3. TOTAL MONTHLY EXPENSES	3)	$ 1930.45	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 101.77	
TOTAL AMOUNTS 2 + 3 + 4			$ 2,264.02
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 54.00

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

b) Is Delanie's budget in a deficit position? *(1 mark)*

Answer:

No, Delanie is +$54.00.

c) Delanie would like to save $25 000 to use as a down payment on a condo. Will she be able to save this amount in the near future? *(1 mark)*

Answer:

No. Although she does have some extra money, it would take a long time to save $25 000 if she is only putting $54 in the bank per month.

$\frac{25000}{54}$ = 463 months to save up $25000, or over 38 years.

Even if we include the 10% she saves, it would take

$\frac{25000}{(54 + 231.80)}$ = 88 months or over seven years.

d) The cost of children's bus tickets increases to $10.00 while adult bus passes increase to $75 per month. Explain how these cost increases will affect Delanie's budget. For example, will she be in a deficit position? *(2 marks)*

Answer:

Delanie's monthly expenses will increase, so she will have less money available for savings. In fact, she spends $1954.60 – $1930.45 = $24.15 more if the bus pass and ticket prices go up. This will not put her budget into a deficit position.

(Module 4, Lessons 1 and 2)

Name: _____

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
1) ***NET INCOME***			***AVERAGE MONTHLY INCOME***
Primary Income	$	$ 26,945.00	1) $ 2,318.02
Secondary Income	$	$	
Other Income	$ 72.60/month	$ 871.20	
Total Annual Income		$ 27,816.20	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ 231.80
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ 550.00	
Car Payments (bus passes, tickets, and taxis)		$ 109.00	(75.00 + 2(10.00) + 14.00)
Telecommunications		$ 30.00	
Hydro		$ 75.00	
Other Utilities		$	
Cable		$	
Groceries		$ 480.00	
Clothing		$ 200.00	
Car Maintenance		$	
Gasoline		$	
Health and Disability Insurance		$	
Personal Care		$ 28.00	
Entertainment		$ 40.00	***TOTAL MONTHLY EXPENSES***
Other (glasses, school supplies, and child care)		$ 382.60	
Other (gymnastics and gifts)		$ 60.80	
TOTAL MONTHLY EXPENSES			3) $ 1,954.60

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$	$	
Life Insurance	$ 249.10	$ 20.76	
Property Taxes	$	$	
Home Insurance	$ 212.00	$ 17.67	
Vacations	$	$	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ 170.00	$ 14.17	
Other (camp and gifts)	$ 590.00	$ 49.17	
TOTAL MONTHLY EXPENSES			4) $ 101.77

5) ***SUMMARY***			
1. AVERAGE MONTHLY INCOME			1) $ 2,318.02
2. SAVINGS	2)	$ 231.80	
3. TOTAL MONTHLY EXPENSES	3)	$ 1,954.60	
4. TOTAL MONTHLY CONTRIBUTIONS	4)	$ 101.77	
TOTAL AMOUNTS 2 + 3 + 4			$ 2,288.17
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)			5) $ 29.85

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

2. List two advantages of online banking. *(2 marks)*

Answer:

Answers will vary. Some advantages include:

- paying bills online
- automatic transfers are easy to set up, can automatically pay bills
- do not need to go to the bank to transfer funds from one account to another
- online statements save paper
- check balances from home or anywhere in the world
- 24-hour access to back account

(Module 4, Lesson 3)

3. Datu works at a financial institution. It is his responsibility to ensure that all cheques and deposit slips are authentic (real) and correct. Today, Datu found a cheque and a deposit slip that were completed incorrectly. Identify what is wrong with each and correct the mistake. *(2 marks each × 2 = 4 marks)*

a)

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

May 31 20 12

Pay to the order of Jane Smith $ 435.10

four hundred thirty DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

Answer:

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

May 31 20 12

Pay to the order of Jane Smith $ 435.10

four hundred thirty DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

(Module 4, Lesson 3)

Name: _____

The correct cheque looks like this:

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

May 31 20 12

Pay to the order of Jane Smith $ 435.10

four hundred thirty- five 10/100 DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

b)

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

June 10 20 12

J. Walker
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY	20	00
COIN	452	61
CHEQUES OR COUPONS	0	90
	13	57
TOTAL	487	07
LESS Received in Cash	36	00
Please sign in presence of Teller AMOUNT OF DEPOSIT $	415	07

Answer:

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

June 10 20 12

J. Walker
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY	20	00
COIN	452	61
CHEQUES OR COUPONS	0	90
	13	57
TOTAL	487	(07)
LESS Received in Cash	36	00
Please sign in presence of Teller AMOUNT OF DEPOSIT $	(415)	(07)

The correct deposit slip looks like this:

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

June 10 20 12

J. Walker
DEPOSITED BY

ANY BANK
YOUR TOWN
PROVINCE

CURRENCY	20	00
COIN	452	61
CHEQUES OR COUPONS	0	90
	13	57
TOTAL	487	08
LESS Received in Cash	36	00
Please sign in presence of Teller AMOUNT OF DEPOSIT $	451	08

Name: _____

4. Compare the following monthly account statement with its cheque register. There is one error and/or omission in the cheque register. Locate the error and/or omission and make the necessary change in the cheque register. *(2 marks)*

Bank Statement

Date	Description	Withdrawals	Deposits	Balance
Feb. 1	Balance Forward			362.54
	Deposit		400.00	762.54
Feb. 4	Cheque 183	25.16		737.38
Feb. 5	Cheque 181	32.27		
	Cheque 184	465.00		240.11
Feb. 15	Deposit		400.00	640.11
Feb. 20	Cheque 186	126.50		513.61
Feb. 22	Cheque 185	84.25		429.36
Feb. 29	Cheque 187	19.98		409.38

Cheque Register

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									362	54
Feb. 1		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	762	54
3	181	TO Photocopies For Less	32	27				CHEQUE – DEPOSIT +	32	27
		FOR						BALANCE ➜	730	27
3	182	TO M. Fenster	82	60				CHEQUE – DEPOSIT +	82	60
		FOR						BALANCE ➜	647	67
3	183	TO Wholesome Goods	25	16				CHEQUE – DEPOSIT +	25	16
		FOR						BALANCE ➜	622	51
4	184	TO O & R Rental Agency	465	00				CHEQUE – DEPOSIT +	465	00
		FOR						BALANCE ➜	157	51
15		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	557	51
19	185	TO Budget Department Store	84	25				CHEQUE – DEPOSIT +	84	25
		FOR						BALANCE ➜	473	26
19	186	TO Zack's Electronics	126	00				CHEQUE – DEPOSIT +	126	00
		FOR						BALANCE ➜	347	26
25	187	TO Terry's Photos	19	98				CHEQUE – DEPOSIT +	19	98
		FOR						BALANCE ➜	327	28
Mar. 2		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	727	28

Answer:

Cheque Register

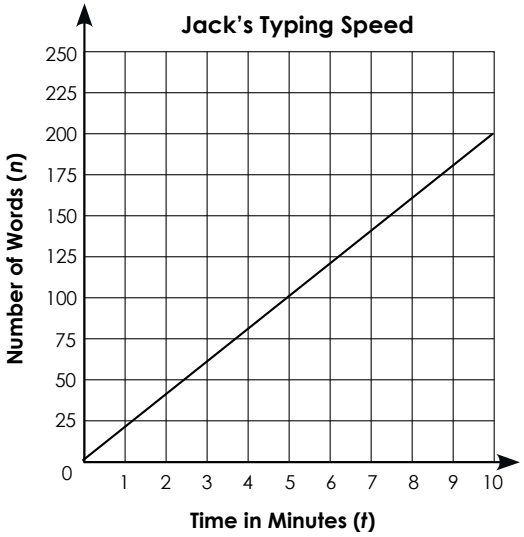
DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES AND DEPOSITS	BALANCE FORWARD	
									362	54
Feb. 1		TO Deposit			✓	400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	762	54
3	181	TO Photocopies For Less	32	27	✓			CHEQUE – DEPOSIT +	32	27
		FOR						BALANCE ➜	730	27
3	182	TO M. Fenster	82	60				CHEQUE – DEPOSIT +	82	60
		FOR						BALANCE ➜	647	67
3	183	TO Wholesome Goods	25	16	✓			CHEQUE – DEPOSIT +	25	16
		FOR						BALANCE ➜	622	51
4	184	TO O & R Rental Agency	465	00	✓			CHEQUE – DEPOSIT +	465	00
		FOR						BALANCE ➜	157	51
15		TO Deposit			✓	400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	557	51
19	185	TO Budget Department Store	84	25				CHEQUE – DEPOSIT +	84	25
		FOR						BALANCE ➜	473	26
19	186	TO Zack's Electronics	126	00				CHEQUE – DEPOSIT +	126	00
		FOR						BALANCE ➜	347	26
25	187	TO Terry's Photos	19	98	✓			CHEQUE – DEPOSIT +	19	98
		FOR						BALANCE ➜	327	28
Mar. 2		TO Deposit				400	00	CHEQUE – DEPOSIT +	400	00
		FOR						BALANCE ➜	727	28
correction: date Feb. 19 cheque (Zack's) should be $126.50				50					726	50 78

(Module 4, Lesson 5)

Name: _____

Module 5: Relations and Patterns (30 marks)

1. The graph below shows Jack's typing speed.



State the independent and dependent variables. *(2 marks)*

Answer:

Independent variable: **Time in Minutes**

Dependent variable: **Number of Words**

(Module 5, Lesson 1)

2. a) How can you tell whether a set of numbers represents a linear pattern. Use an example as part of your answer. *(2 marks)*

Answer:

A set of numbers represents a linear pattern if they increase or decrease by the same amount each time.

For example: –20, –17, –14, –11, –8… increases by 3 each time and 52, 43, 34, 25, 16… decreases by 9 (or increases by –9) each time

(Module 5, Lesson 2)

b) Does the table of values represent a linear relation? Explain your answer. *(3 marks)*

Number of Fish	3	6	15
Profit ($)	10	20	50

Answer:

Using the first two points:

$$\text{slope} = \frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{20-10}{6-3} = \frac{10}{3}$$

Using the first and third points:

$$\text{slope} = \frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{50-10}{15-3} = \frac{40}{12} = \frac{10}{3}$$

Therefore, the table of values represents a linear relation because the slope is the same between all pairs of points.

(Module 5, Lesson 2)

3. During the second week of November, Emma works the following hours: Monday, 2; Tuesday, 6; Wednesday, 3; Thursday, 8; Friday, 7. Emily's hourly rate is $15.

a) Express the relation between daily gross pay and hours worked with an equation. *(2 marks)*

Answer:

g is the gross pay ($)

h is the number of hours worked

$g = 15h$

b) Complete the following table of values for the relation between daily gross pay and hours worked. *(3 marks)*

Answer:

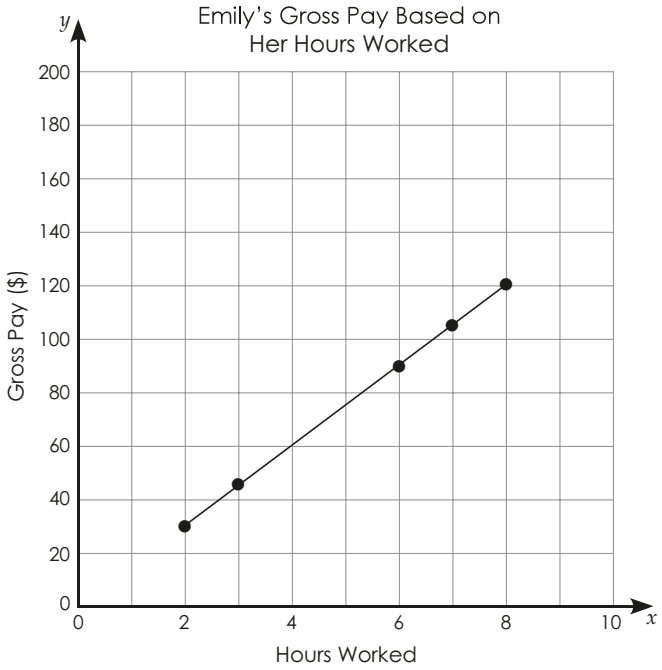
	Mon.	Tues.	Wed.	Thurs.	Fri.
Hours Worked	2	6	3	8	7
Gross Pay ($)	30	90	45	120	105

$g = 15(2) = \$30$ $g = 15(6) = \$90$ $g = 15(3) = \$45$ $g = 15(8) = \$120$ $g = 15(7) = \$105$

Name: _____

c) Draw a graph to show the relation between gross pay and the number of hours worked. Write the appropriate scales and labels onto the graph. *(5 marks)*

Answer:



d) Explain how the hourly rate is related to the slope of the graph. For example, if she were paid only $10 per hour, how would this affect the graph? *(1 mark)*

Answer:

The hourly rate determines how steep the line is. If she were paid only $10/hr., the line would be less steep. This is because the hourly wage is the slope of the graph.

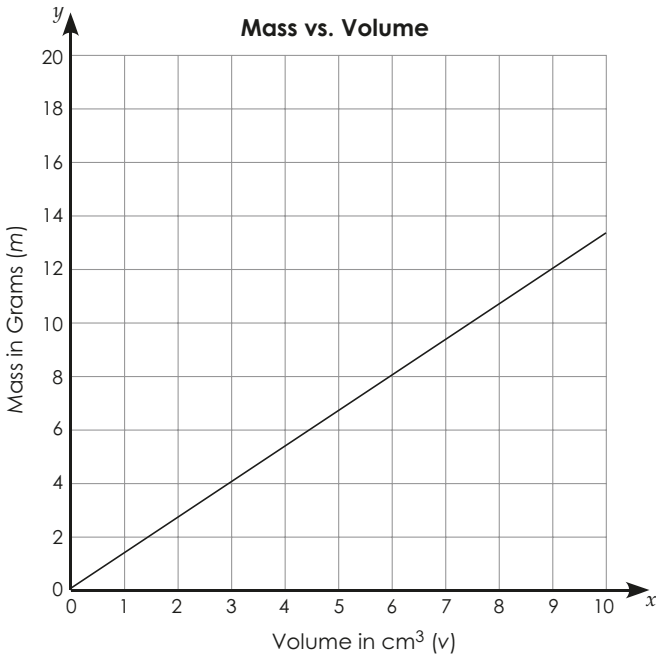
e) If Emily were paid $20 per shift plus her wage of $15 per hour, how would this affect the graph? *(2 marks)*

Answer:

If Emily were paid $20 per shift plus her hourly wage, the graph would change from a direct variation to a partial variation. The line would cross the *y*-axis (vertical axis) at (0, 20) instead of the origin (0, 0). The slope would be the same as the original graph.

(Module 5, Lessons 3 and 4)

4. The relation between the mass (m) of a substance and its volume (V) is expressed by the following graph.



a) Choose two points on the graph and use the coordinates of these points to calculate the slope. *(2 marks)*

Answer:

Point A (0, 0) Point B (6, 8)

change in mass: 8 – 0 = 8

change in volume: 6 – 0 = 6

$$\text{slope} = \frac{\text{change in mass}}{\text{change in volume}}$$

$$= \frac{8}{6}$$

$$= \frac{4}{3} \text{ or } 1.33 \text{ g/cm}^3$$

b) Express this relation as an equation. Be sure to identify the variables. *(2 marks)*

Answer:

m represents the mass of the substance

v represents the volume.

$m = 1.33v$

Name: _____

c) Find the volume of this substance with a mass of 18 g. Use the formula from part b. Write your answer rounded to one decimal place. *(2 marks)*

Answer:

$$m = 1.33v$$

$$18 = 1.33v$$

$$\frac{18}{1.33} = \frac{1.33v}{1.33}$$ Divide both sides by 1.33

$$13.5 = v$$

The volume of the substance when it has a mass of 18 g is 13.5 cm^3.

d) Find the mass of this substance with a volume of 9 cm^3. Show your work on the graph. *(1 mark)*

Answer:

Use interpolation. Find 9 cm^3 along the *x*-axis (horizontal axis). Move up along the grid line until you reach the graph. This point lines up with 12 g along the *y*-axis (vertical axis). Answer is 12 g.

(Module 5, Lesson 4)

5. Sam's car has a broken speedometer, and so he is using his GPS to determine his speed. According to the GPS, his speed is 40 metres per second. The speed limit is 110 km/h. Is Sam speeding? Show how you arrive at your answer. *(3 marks)*

Answer:

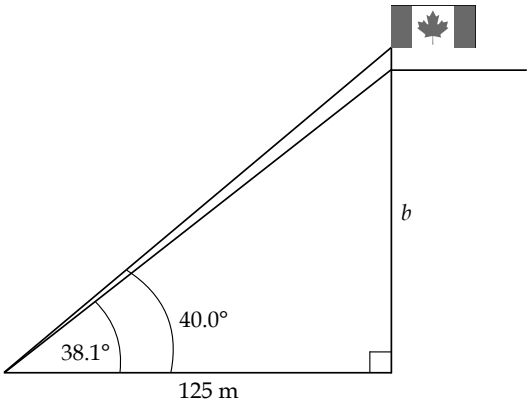
$$\frac{40\text{ m}}{1\text{ s}} = \frac{40\text{ m}}{1\text{ s}} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{60\text{ s}}{1\text{ min.}} \times \frac{60\text{ min.}}{1\text{ h}} = 144\text{ km/h}$$

Sam is speeding, because the speed limit is 110 km/h.

(Module 5, Lesson 6)

Module 6: Trigonometry *(7 marks)*

1. From a point 125 m from the foot of a building, the angles of elevation of the top and bottom of the flagpole are 40.0° and 38.1° respectively. The flagpole is set on the roof of the building.



a) Calculate the height of the building *(2 marks)*

Answer:

$$\tan 38.1° = \frac{b}{125}$$ where b is the height of the building

$$(125)\tan 38.1° = \frac{b}{125}(125)$$ multiply both sides by 125 m

$$(125)\tan 38.1° = b$$

$$98.0 = b$$

The height of the building is 98.0 m.

Be sure that your calculator is in "degrees" mode. Somewhere on the screen (if you are using a basic scientific calculator) it should say DEG. If it says GRAD or RAD, change it. If you are unsure how to change it, check your manual or look online. If your answer was 52.98…, your calculator is in RAD; if your answer was 85.23…, your calculator is in GRAD.

(Module 6, Lesson 2)

Name: _____

b) Calculate the height of the flagpole. *(3 marks)*

Answer:

Let h represent the height of the flagpole and the building.

$$\tan 40° = \frac{h}{125}$$

$$(125)\tan 40° = \frac{h}{125}(125) \qquad \text{multiply both sides by 125 m}$$

$$(125)\tan 40° = h$$

$$104.89 = h \qquad \textit{(2 marks)}$$

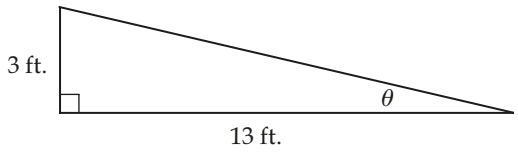
The height of the flagpole is the difference between the total height of the flagpole and building (h), and the height of the building (b).

$h - b = 104.89 - 98$

$= 6.89$ m *(1 mark)*

(Module 6, Lesson 2)

2. Calculate the measure of θ in degrees. Write your answer rounded to one decimal place. *(2 marks)*



Answer:

$$\tan\theta = \frac{3}{13}$$

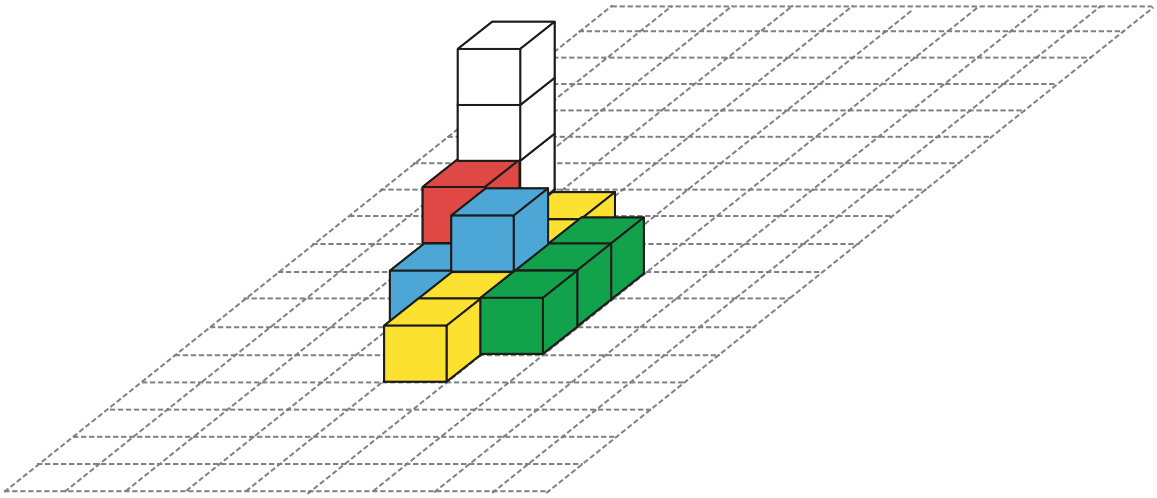
$$\theta = \tan^{-1}\left(\frac{3}{13}\right) = 13.0°$$

(Module 6, Lesson 1)

Module 7: Design Modelling ***(21 marks)***

Blue cube = 1 unit long
Red bar = 2 units long
Green bar = 3 units long
White bar = 4 units long
Yellow bar = 5 units long

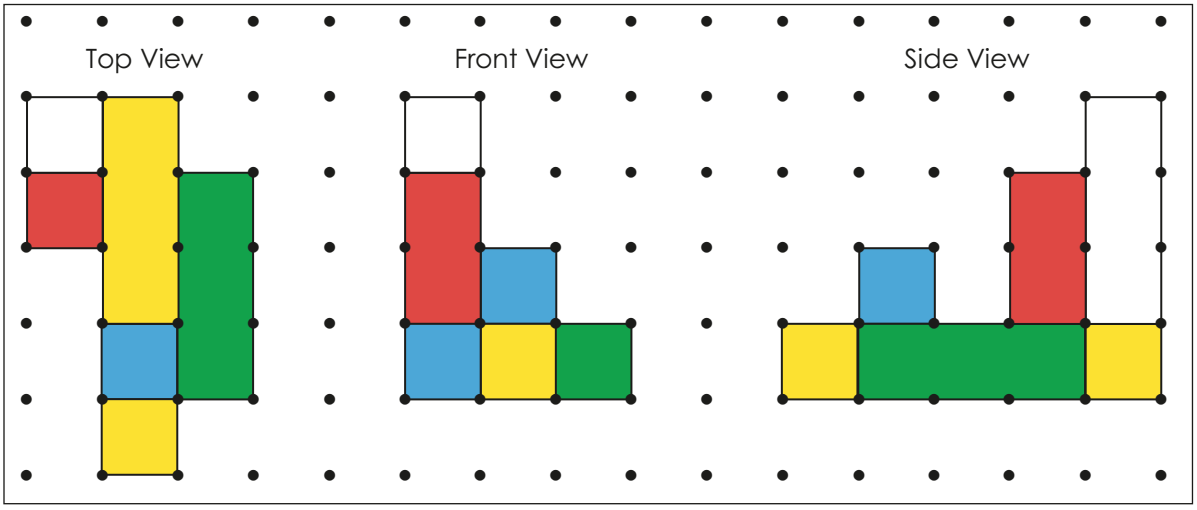
1. Use the 3-D drawing below to answer the questions that follow.



a) Select the 2-D view that matches the drawing above. *(1 mark)*

Answer: **Set II**

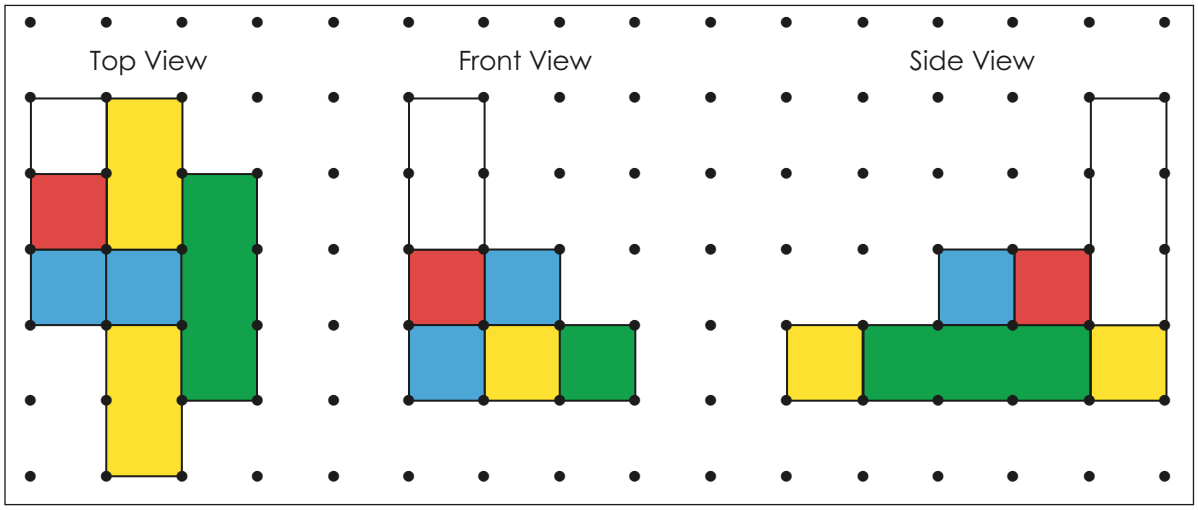
Set I:



or

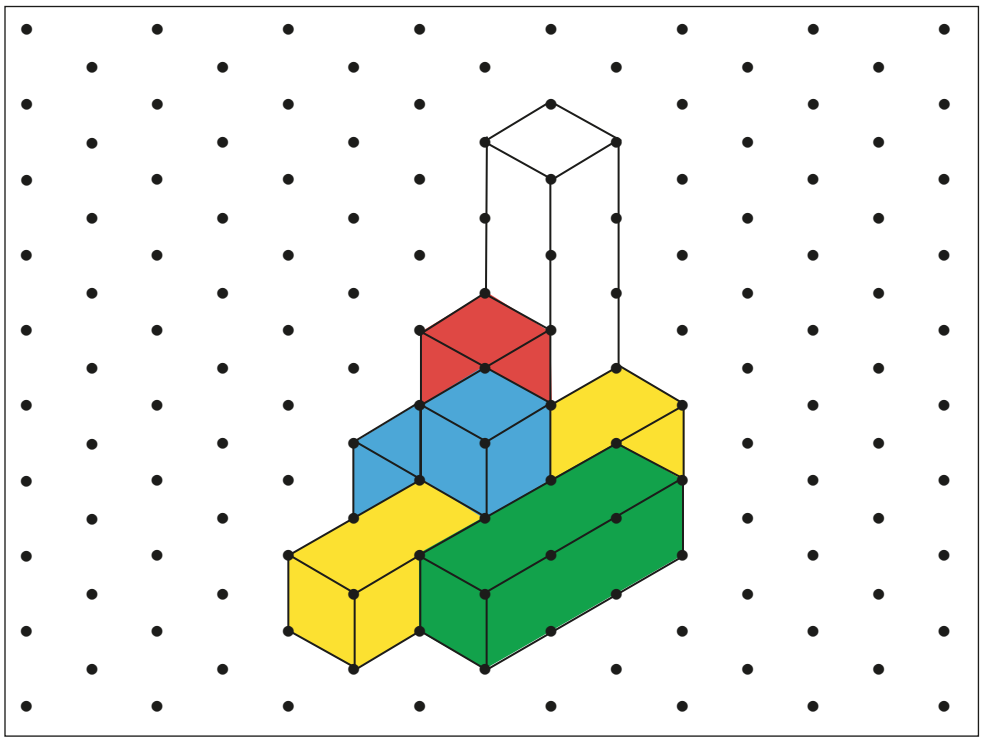
Name: _____

Set II:



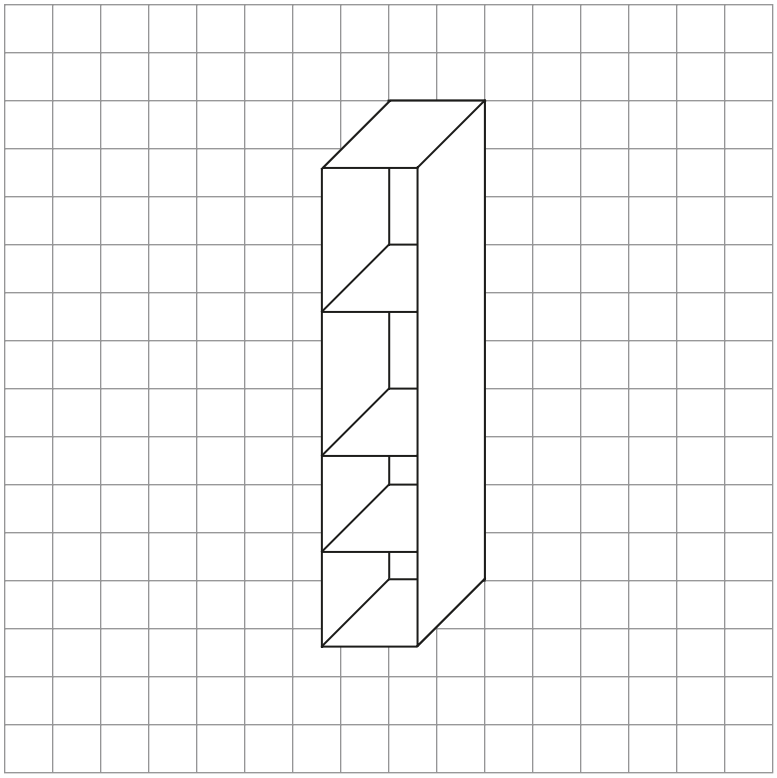
b) Use the isometric dot paper below to draw a 3-D representation for the other set of 2-D views. Be sure to label each drawing, and label each section of each drawing with the appropriate colour if you do not have pencil crayons. *(6 marks)*

Answer:

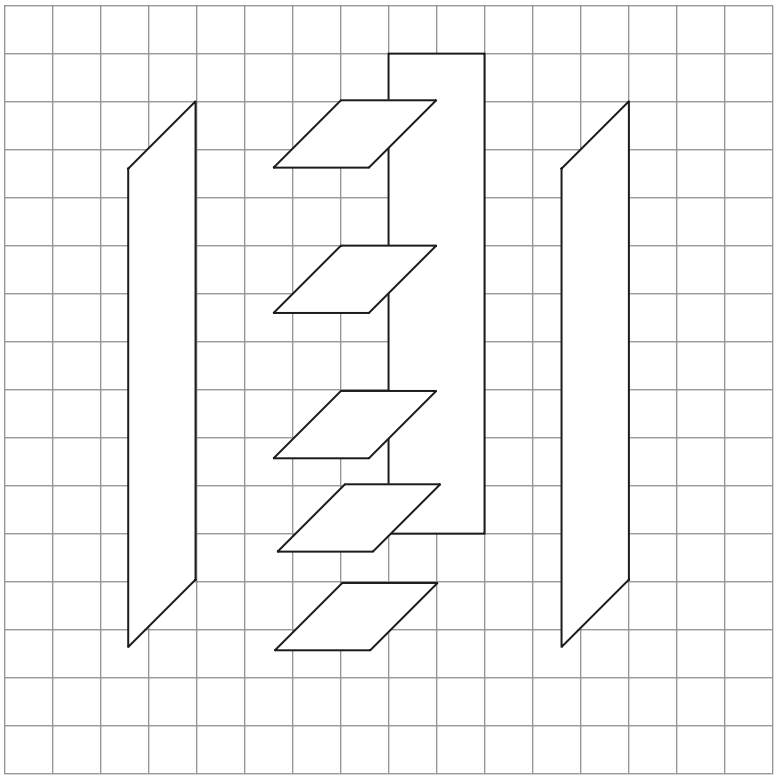


(Module 7, Lesson 2)

2. a) Sketch the exploded view of the following object. *(5 marks)*



Answer:



(Module 7, Lesson 4)

Name: _____

b) The scale of this drawing is 1 inch : $2\frac{1}{4}$ ft. What are the actual dimensions of the object? *(4 marks)*

Answer:

Length: measurement = 0.5″

0.5 × 2.25 = 1.125 ft. (actual)

Depth: measurement = 0.5″

2 × 0.5 × 2.25 = 2.25 ft. (actual)

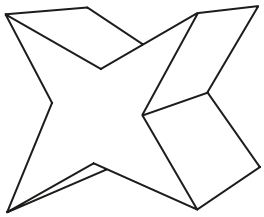
Height: measurement = 2.5″

2.5 × 2.25 = 5.625 ft. (actual)

The actual dimensions of the object are: 1.1 ft. × 1.1 ft. × 5.6 ft.

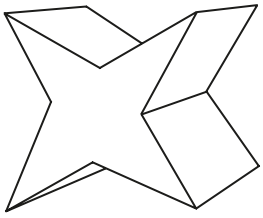
(Module 5, Lesson 7)

3. a) Where is the vanishing point for the drawing shown below? *(1 mark)*

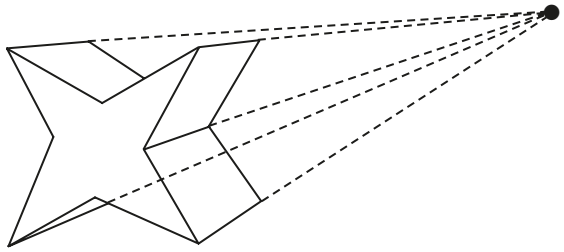


Answer:

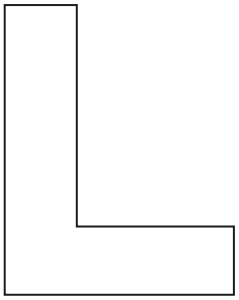
The vanishing point is to the right and slightly upward.



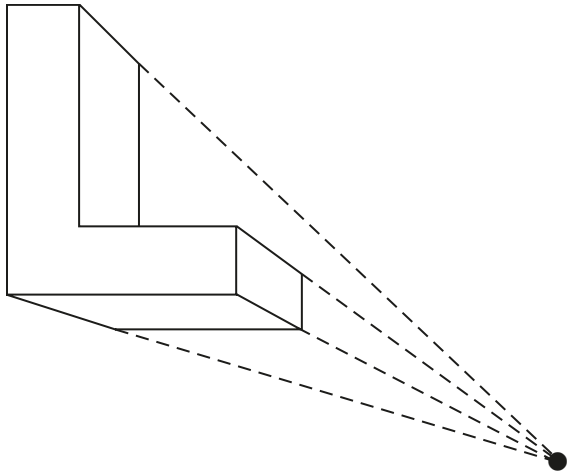
or



b) Complete a one-point perspective drawing of the object below, if the vanishing point is below and to the right. *(4 marks)*



Answer:



(Module 7, Lesson 3)

Name: _____

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***		
1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*

Name: _____

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount	
			AVERAGE MONTHLY INCOME
1) ***NET INCOME***			1) $ _____
Primary Income	$ _____	$ _____	
Secondary Income	$ _____	$ _____	
Other Income	$ _____	$ _____	
Total Annual Income		$ _____	***MONTHLY SAVINGS***
2) ***MONTHLY SAVINGS***			
(10% of Average Monthly Income)			2) $ _____
3) ***MONTHLY EXPENSES***			
Mortgage or Rent		$ _____	
Car Payments		$ _____	
Telecommunications		$ _____	
Hydro		$ _____	
Other Utilities		$ _____	
Cable		$ _____	
Groceries		$ _____	
Clothing		$ _____	
Car Maintenance		$ _____	
Gasoline		$ _____	
Health and Disability Insurance		$ _____	
Personal Care		$ _____	
Entertainment		$ _____	***TOTAL MONTHLY EXPENSES***
Other		$ _____	
Other		$ _____	
TOTAL MONTHLY EXPENSES			3) $ _____

	Annual Amount	Monthly Amount	
4) ***ANNUAL EXPENSES***			
Car Insurance	$ _____	$ _____	
Life Insurance	$ _____	$ _____	
Property Taxes	$ _____	$ _____	
Home Insurance	$ _____	$ _____	
Vacations	$ _____	$ _____	***TOTAL MONTHLY CONTRIBUTIONS***
Newspapers & Periodicals	$ _____	$ _____	
Other	$ _____	$ _____	
TOTAL MONTHLY EXPENSES			4) $ _____

5) ***SUMMARY***		
1. AVERAGE MONTHLY INCOME		1) $ _____
2. SAVINGS	2) $ _____	
3. TOTAL MONTHLY EXPENSES	3) $ _____	
4. TOTAL MONTHLY CONTRIBUTIONS	4) $ _____	
TOTAL AMOUNTS 2 + 3 + 4		$ _____
5. AMOUNT AVAILABLE FOR OTHER SAVINGS OR EXPENDITURES (DEFICIT)		5) $ _____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments to spending.*